HIGHER ORDER MODULATION RECOGNITION USING APPROXIMATE
ENTROPY

A Thesis in

Electrical Engineering

by

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ABSTRACT

Modulation recognition finds its application in today’s cognitive systems ranging from civilian to military installations. Existing modulation classification algorithms include classic likelihood approaches and feature based approaches. In this study, Approximate Entropy (ApEn), which is a non-linear method to analyze a time series, is proposed as a robust feature of a modulation scheme. ApEn is used as a feature to identify parameters such as number of symbol levels, pulse lengths and modulation indices of a continuous phase modulation (CPM) signal. The method is then extended to classify CPM signals with differing pulse shapes which include raised cosine and Gaussian pulses with varying roll-off factors and bandwidth-time products respectively. The extracted features result in high classification accuracies for a variety of signals and performs robustly even in the presence of synchronization errors and carrier phase offsets. The ApEn scheme is further applied to other modulation scheme as well such as frequency shift keying (FSK), phase shift keying (PSK) and quadrature amplitude modulations (QAM) and performance results for intra-class and inter-class classification are presented.
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1. The utility of ApEn, as an important feature of a signal modulation type is demonstrated in classifying a variety of CPM signals.

2. By varying the number of symbol levels, pulse lengths and modulation indices, an infinite number of CPM signals can be generated. The scheme of ApEn developed in this study results in high classification accuracies for a large number of signals. Results are presented for classifying among 45 different CPM signals and acceptable performance is obtained in the broad range of signal to noise ratio. This is an improvement over existing modulation approaches where the maximum number of signals considered for classification remains less than 10.

3. In previous modulation recognition techniques, the symbol pulse shape of the incoming signal is assumed to be typically rectangular. However, in this study, the ability of ApEn to parse minor perturbations of the signal in terms of varying pulse shapes is demonstrated. CPM signals with differing pulse shape parameters are identified. In the case of signals with raised cosine pulses, signals with varying roll off factors are classified whereas in case of Gaussian pulses, signals with varying bandwidth time products are classified.

4. The signal model used incorporate channel residual effects such as the carrier phase offsets, phase jitters and complete absence of synchronization. This is a major improvement over some of the existing modulation techniques which use symbol sampled signals for classification.

5. Further the scheme of ApEn is extended to classify higher order modulation from other constellation schemes such as FSK, PSK and QAM. Results for inter-class modulation identification are presented.
Chapter 1

Introduction to Modulation Classification

Modulation classification or recognition is the process of identifying the modulation type of a received communication signal in a general operating environment. The concept of modulation recognition in a non-cooperative environment, also known as automatic modulation recognition (AMR) has been studied for two decades and new results are being published which highlights the importance this particular topic has received in the recent years. AMR techniques are particularly employed in cognitive systems which are installed in civilian or military applications. In civilian applications, where bandwidth remains a valuable commodity for the cellular carriers, cognitive radios are involved in “spectrum sensing” in which the traffic free frequency bands are identified and based on the result of AMR of the current transmission scheme, alternate transmission techniques are used which enhances the capacity of the system. This concept is primarily used in adaptive modulation techniques [1] implemented through software defined radios (SDR). Thus, one can see an important application of AMR in civilian applications. On the other hand, as more sophisticated warfare technologies evolve; military applications increasingly demand the use of AMR. Signal interception and parameter identification are important tasks in electronic warfare which the AMR techniques can perform. Signals identified as friendly should be allowed to pass, whereas hostile signals should be subjected to jamming. With systems being developed which require certain amount of cognition, AMR techniques constitute an important block.

AMR techniques involve the use of a classifier, which identifies the modulation type of the incoming signal from the given set of candidate modulations or decides that the signal does not belong to the given set. Before outlining the traditional approaches to modulation classification, an understanding of the general requirement of a good classifier is necessary. Apart from being able to identify the incoming signal from a large set of constellation types, it has to be
able to determine the modulation order of the signal of the detected modulation type. With low computational complexity and a broad operating signal to noise ratio (SNR) range, it also should be able to provide a real time decision in all operating environments.

A classifier generally consists of two steps before identifying the modulation type. First is the preprocessing step, which determines various signal parameters such as carrier frequency, baud rate, carrier phase, signal to noise ratio and timing offsets. The next step after preprocessing is the classification algorithm which proceeds with the identification of the modulation type.

1.1 Approaches to Modulation Classification

Existing modulation classification algorithms include classic likelihood approaches and feature based approaches. The likelihood based (LB) framework computes the likelihood functions of the received signal and compares it against a threshold. The feature based (FB) framework extracts characteristic features from the signal to classify the modulation type. In general the blocks of a classifier can be shown as,

![Figure 1-1: Block diagram of a classifier](image-url)

Figure 1-1: Block diagram of a classifier
Before proceeding to review the techniques developed under these two frameworks, it is important to present a general signal model which takes into consideration all the parameters of the signal and the assumptions that this study makes.

1.2 Signal Model

In the case of modulation recognition, a general received baseband signal \([2]\) can be given as below,

\[
\begin{align*}
    s(t) &= a_t e^{j2\pi \Delta f t} e^{j\theta} \sum_{k=1}^{K} e^{j\theta_k} s_k^{(l)} g(t - (k - 1)T - \varepsilon T) \\
    \Delta f &= \text{carrier frequency offset}, \quad \theta = \text{carrier phase which is time invariant}, \\
    \{\theta_k\}_{k=1}^{K} &= \text{phase jitter in each of the } K \text{ received symbols}, \\
    \{s_k^{(l)}\}_{k=1}^{K} &= \text{the } K \text{ complex symbols transmitted from the alphabet set of the } l^{th} \text{ modulation format}, \\
    T &= \text{symbol period}, \quad \varepsilon = \text{the timing offset}, \\
    g(t) &= p(t) * h(t) \text{ where } p(t) = \text{pulse shape and } h(t) = \text{channel response}, \\
    a_t &= \frac{E_s}{\sqrt{\sigma_s^{(l)} E_p}} \quad \text{where } E_s = \text{signal baseband energy, } E_p = \text{energy in the pulse given by} \\
    E_p &= \int_{-\infty}^{\infty} p(t) dt, \quad \text{and } \sigma_s^{(l)} = \text{variance of the constellation given by} \\
    \sigma_s^{2(l)} &= |M_l^{-1} \sum_{m=1}^{M_l} s_m^{l}|^2 \quad \text{where } M_l = \text{number of symbols in the } l^{th} \text{ modulation format.}
\end{align*}
\]

We assume that we are operating in an additive white Gaussian noise channel (AWGN) and in this case the received signal can be written as \(r(t) = s(t) + n(t)\) where \(n(t)\) is the lowpass noise process with variance \(N_0\). It is assumed that the carrier frequency and symbol period are estimated. The pulse shape of the transmitting scheme is also known. Further, the signal model and simulations incorporate carrier phase offsets and complete absence of synchronization for the incoming received baseband sequence. Parameter \(\theta\) is uniformly distributed over \([-\pi, \pi]\). Thus, the problem statement is defined as identifying the higher order modulations of a given constellation type. No prior study has been conducted in order to identify the parameters of a...
CPM signal such as the number of symbol levels, pulse lengths and modulation indices. The parameters of the CPM signal will be explained in chapter 3.

1.3 Likelihood Based Approach

This approach is a multiple hypotheses testing problem, with each hypothesis being the modulation type of the received signal. Likelihood functions (LF) with respect to each hypothesis are calculated and as in the case of likelihood functions, the hypothesis is chosen such that probability to observe the signal with that particular modulation type is increased. As stated in [3], the principle behind the multiple hypotheses is that the probability density function (PDF) of the observed waveform, conditioned on the signal contains all the information required for classification. Depending upon the model chosen by the LB approach, three likelihood tests are possible namely

1. ALRT: Average Likelihood Ratio Test
2. GLRT: General Likelihood Ratio Test
3. HLRT: Hybrid Likelihood Ratio Test

1.3.1 ALRT

In this type of test, the unknown quantities in the vector are modeled as random variables with a certain PDFs. In such a situation, when the hypothesis $H_l$ corresponds to the $l^{th}$ modulation type, the likelihood function $\Lambda^l[r(t)]$ of $r(t)$ under $H_l$ is given by,

$$\Lambda^l[r(t)] = \int A[r(t)|U_l, H_l] * p(U_l|H_l) \, dU_l$$  \hspace{1cm} (1.2)

where $A[r(t)|U_l, H_l]$ is the conditional LF under $H_l$ and conditioned on the vector $U_l$. Note that $U_l$ is the vector which is populated by all the unknown parameters introduced in the signal model. The probability density function $p(U_l|H_l)$ is the priori PDF of the unknown vector $U_l$ under $H_l$. 5
Since the calculation of the likelihood function requires multidimensional integration, closed form solutions are very difficult to reach which leads to high computational complexity. Reference [3] uses ALRT to identify between Phase Shift Keying (PSK) and Quadrature Amplitude Modulation (QAM) signals. Perfect knowledge of all the other unknown parameters is assumed except the carrier phase. The probability of false classification between the modulations approaches zero at SNR values greater than 11 dB.

1.3.2 GLRT

In this type of approach, the unknown parameters are modeled as deterministic quantities. Assuming a given hypothesis $H_l$, these parameters are given as maximum likelihood estimates. In this situation the likelihood function under the hypothesis $H_l$ becomes,

$$L^l[r(t)] = \max_{U_l} L[r(t)|U_l, H_l]$$

GLRT avoids the multidimensional integration but is substituted by multidimensional maximization which reduces the complexity to some extent but introduce the nesting problem which is described in [4]. General likelihood ratio tests (GLRT) are employed in [4] for classifying higher order QAM modulations. Carrier phase and the data symbols are assumed to be the unknown quantities. Since all other signal parameters are assumed to be known, the output is obtained at the matched filter. After calculating the likelihood function from the output of the matched filter, the ratio test is performed between the functions corresponding to two modulation types. The thresholds for the test depend on the number of symbols and the SNR of the signal.

1.3.3 HLRT

This test is a combination of GLRT and ALRT. In this type of approach, some of the quantities are treated as deterministic unknowns whereas some of the quantities are modeled as
random variables with certain PDFs. The unknown vector \( U_t \) is written as \( U_t = [U_{t1}, U_{t2}] \). Elements of \( U_{t1} \) are modeled as random variables and elements of \( U_{t2} \) are modeled as deterministic unknowns. The likelihood function can be written as

\[
A^T[r(t)] = \max_{U_{t2}} \left\{ \int A[r(t)|U_{t1}, U_{t2}, H_1] p(U_{t1}|H_1) dU_{t1} \right\}
\]  

(1.4)

Generally, data symbols are modeled as random variables in the above test. Thus, HLRT removes the computational complexity of the ALRT considerably and removes the nesting problem of GLRT. Reference [5] uses hybrid likelihood ratio test (HLRT) to identify between PSK and QAM signals and also provides a comprehensive comparison of the likelihood tests in terms of the complexity and robustness to signal parameter model mismatch. To reduce the complexity further, approximations of ALRT in the form of quasi-log likelihood ratio test is conducted to classify between PSK and QAM modulations in [6] and [7] respectively.

### 1.4 Feature Based Approach

The signal features selected in an ad-hoc manner form the basis for modulation classification. Generally, the feature based approaches subject the feature values to a thresholding function or to a trained classifier. In cases where a trained classifier is used, support vector machines (SVM) or artificial neural networks (ANN) [8] are widely used. Since different modulation types differ in the parameter of signal it modulates; the parameters being the amplitude, phase or the frequency, the most intuitive way to extract features is to examine the instantaneous frequency, amplitude and the phase of the signal. Reference [9] calculates the variance of the magnitude of the discrete Fourier transform (DFT) of the centered normalized instantaneous amplitude to distinguish between FSK and ASK/PSK classes. FSK signal is characterized by a smaller variance and hence is easily classified. Further, investigating the
absolute and direct phases, classification between ASK and PSK signals is done. Subjecting the variance of the absolute centered instantaneous phase to a threshold, MPSK signals are separated from the ASK and BPSK signals. Also, looking at the variance of the direct phase allows to separate between ASK and BPSK signals. The above procedure of calculating the variances of the instantaneous features logically builds into a hierarchical classifier as described in the study. Complete knowledge of the channel is assumed including noise variance, baud rate and pulse shapes.

Apart from the direct examination of these properties of the signal, a group of studies focus on characterizing the wavelet transform of the signal. Magnitude of the Haar wavelet transform [10] of the received signal is used to distinguish between PSK and FSK signals. Since wavelets localize the phase, amplitude and frequency transitions in time, the magnitude of the Haar Wavelet transform (HWT) forms a classifying feature. Since PSK signals are characterized by constant amplitude, the magnitude of HWT remains constant when the wavelet covers a symbol change but peaks are observed when a phase transition occurs. Similarly, the HWT magnitude of FSK signals is a multistep function corresponding to each frequency of the signal with peaks occurring on a symbol or a phase change. Thus, to classify between PSK and FSK signals, the variance of the HWT magnitude is compared against a threshold, which is calculated depending on the noise variance and the number of symbols. However, synchronization and symbol time estimation is a major requirement for observing the peaks of the HWT magnitude since the HWT noise transients can mask themselves as peaks. The results present in the above studies assume complete knowledge of the symbol timing and synchronization. Reference [11] is an extension of the above study in which it classifies between QPSK, 4FSK and 4QAM signals.

Higher order signal statistics in the form of cumulants and cyclic cumulants (CCs) were examined in [12]-[17]. Normalized fourth order cumulant estimates of signals were used as a
classifying feature in [12]. Since fourth order cumulants characterize the shape of the noisy baseband signal, their values can be exploited for modulation classification. The theoretical values of the cumulants of the noise free signal are calculated which naturally divide the candidate modulations into real valued and complex valued constellations. Thus, a hierarchical classifier is built with the cumulant values. QPSK, 4PAM, QAM (4, 4) and 8PSK signals are classified perfectly at SNR values 12dB or higher. In this case, perfect knowledge of the channel is assumed. Preprocessing step includes symbol timing estimation, synchronization, determination of baud rate, and noise variance estimation. Analysis of classification accuracy is performed with respect to phase errors, frequency offsets and timing offsets. Extension of the above study is performed in [13] which attempts to classify signals in frequency selective channels.

Reference [14] uses the CCs of order two and four to classify modulation signals in co-channel interference. Cyclic cumulants are the coefficients of the Fourier series of cumulant functions. Since communication signals can be modeled as cyclostationary processes, the cumulant expressions can be expanded in the form of Fourier series. In this study, magnitude of warped cyclic cumulants i.e. CCs or order $n$ raised to order $\frac{2}{n}$, are used as a classifying feature between signals. Since warping projects the magnitudes of CCs in a linear space, a Euclidean minimum distance classification algorithm is proposed which classifies between BPSK, QPSK, 8PSK and 8 QAM. Classification in [15] was done between QPSK, 16QAM, and 64QAM signals. However, knowledge of cyclic frequencies and frequency offsets is necessary prior to estimation.

Extending the concept of CCs, [16] proposed the use of higher order CCs. Since the estimation of higher order CC incorporates a large variance, these cumulants were not used in practice. The study showed that use of warped sixth order CCs do not add a significant variance
to the estimated values and also the improved the classification accuracy. In this study, 16QAM, 64 QAM and V.29 was classified pairwise. Results for classification of four signals QPSK, 8QAM, 16QAM, V.29 in the presence of interference was also presented.

Similarly eighth order CCs in [17] were efficient to classify pairwise between 4PAM and 8PAM, 16QAM and 64 QAM, 64 QAM and 256 QAM, QPSK and 8PSK. A comparatively large number of symbols (orders of magnitude $10^3$) are required to perform intra class modulation classification. The number of symbols depends on the order of the modulation signals to be classified and the SNR level. Variance of the zero crossing interval sequence [18], and statistical moments of phase [19] are among the other features used for classification.

### 1.5 Pattern Recognition Approach to Modulation Classification

As mentioned earlier some of the techniques incorporate pattern recognition approaches to modulation classification, where the characteristic features of a signal are fed to a trained classifier. In this study, a pattern recognition approach is employed. The feature used is the approximate entropy of the received signal and the classifier used is a probabilistic neural network (PNN) which is trained using these extracted features. A general block diagram of a pattern recognition system can be given as below.
1.6 Organization

- Chapter 2 introduces the concept of approximate entropy and explains its significance.
- Chapter 3 explains the continuous phase modulation scheme and the problem statement of classifying different types of continuous phase modulation signals is stated.
- Chapter 4 develops a classification scheme to identify various pulse shapes such as raised cosine and Gaussian of the CPM signals.
- Chapter 5 extends the ApEn classification scheme to include signals from other modulation schemes such as frequency shift keying, phase shift keying and quadrature shift keying. Intra and inter class modulation classification is considered.
- Chapter 6 concludes the study.
Chapter 2

Introduction to Approximate Entropy

ApEn is one of the many non-linear methods used to analyze data from dynamical and chaotic systems. Traditional methods such as Fourier analysis, correlation analysis, moment statistics have not been successful in analyzing the dynamic nature of these chaotic systems [20]. In such cases, one has to resort to non-linear techniques [21].

ApEn is a regularity statistic. It gives an index which corresponds to the regularity of a time series. In the original paper by Pincus [22], ApEn is described as a family of parameters and statistics to quantify any regularity of the data without \textit{a priori} knowledge of the process generating them. The capability of ApEn to parse the complexity index of a stochastic or a deterministic processes have resulted in the application of ApEn algorithm in a lot of engineering disciplines which include biomedical engineering, image processing and power engineering among other growing disciplines.

- ApEn is used to discriminate heart and respiratory patterns during certain body conditions such as sitting and sleeping which deviate from the normal patterns prevalent when the body is standing [23].
- Using the technique of ApEn, the influence of various aging related issues are studied using gait patterns [24].
- In analyzing EEG patterns of the body during epileptic seizures and the abdominal electrical activity of the uterus, ApEn has been a useful tool [25].
- ApEn is also used to provide a diagnosis of the machine health. In this case, the vibration signals from the machine are irregular if the machine shows the effects of aging or fault conditions [26].
- Location and identification of the type of faults in an AC power line has been carried out using the approach of nonlinear time series methods [27].

- In the analysis of the dynamics of sap flow in tropical trees during water deficit, ApEn has lent its discerning characteristics [28].

Thus the diversity of the above applications illustrates the popularity of the statistic.

### 2.1 Approximate Entropy Algorithm

Committing this explanation to a more quantitative approach, the approximate entropy of a time series can be represented as,

\[
ApEn(X) = ApEn(m, r, N) \tag{2.1}
\]

where \( X \) is the time series consisting of \( N \) samples, \( m \) is the embedding dimension, and \( r \) is the criterion of similarity. Since ApEn is governed by the parameters \( N, m \) and \( r \), it is important to compare ApEn values for different systems with the same parameter values. The algorithm for extracting the ApEn index for a time series consisting of \( N \) samples represented by \( X(n) = [x(1), x(2), x(3), \ldots, x(N)] \) can be described as follows.

1.) Vectors or subsequences of length \( m \) are constructed.

2.) These vectors can be represented as \( P_m(1) = [x(1), x(2), \ldots, x(m)] \), \( P_m(2) = [x(2), x(3), \ldots, x(m+1)] \), \( P_m(3) = [x(3), x(4), \ldots, x(m+2)] \), and so on.

3.) I denote the pattern of \( m \) consecutive and discrete samples, beginning at sample \( i \), by the vector \( P_m(i) \). The number of such patterns is \( N - m + 1 \).

4.) According to criterion of similarity \( r \), two patterns of length \( m \), \( P_m(i) \) and \( P_m(j) \) are similar if the difference between the corresponding set of measurements in the pattern is less than \( r \), i.e. if \( |x(i+k) - x(j+k)| \leq r \) for \( 0 < k < m \).
5.) A statistic is now defined as $C_r^m(i) = \frac{N_r^m(i)}{N-r+1}$ where $N_r^m(i)$ is the number of similar patterns to $P_r(i)$ in the set of vectors.

6.) The natural logarithm of each $C_r^m(i)$ is calculated and averaged over $i$ to form $\theta^m(r)$ which is given by $\theta^m(r) = \frac{1}{N-r+1} \sum_{i=1}^{N-r+1} \ln(C_r^m(i))$. The statistic $\theta^m(r)$ is the frequency or the fraction of patterns of length $m$ that are similar to each other.

7.) The dimension is increased to $m + 1$ and steps 1 to 6 are repeated to form the statistic $\theta^{m+1}(r)$.

Theoretically, ApEn is defined as $ApEn(m,r) = \lim_{N \to \infty} [\theta^m(r) - \theta^{m+1}(r)]$

ApEn can be seen as the logarithmic likelihood that patterns which are similar at length $m$ remain similar at length $m + 1$. Essentially by calculating this logarithmic likelihood, it produces a regularity statistic. Thus, a highly regular time series is characterized by a low ApEn value, whereas a high ApEn value is produced by an irregular and chaotic time series. Since in practice, $N$ can only be finite, the statistic of the time series, $ApEn(m,r)$ should actually be given as $ApEn(m,r,N)$.

2.2 Parameters of ApEn

As seen, the ApEn statistic is governed by two main parameters namely the embedding dimension $m$ and the criterion of similarity, $r$.

1. Embedding Dimension: The parameter $m$ dictates the construction of vectors called the state space vectors. The embedding dimension is independent of the dynamics of the time series. As stated above, the vectors can be constructed so that the dynamics of the signal can be examined directly on these vectors.
2. **Criterion of Similarity**: The parameter \( r \) is called the criterion of similarity. Since the algorithm requires some sort of regularity measurement, the parameter \( r \) is the only parameter which allows us to capture the similarity between the state vectors.

It becomes imperative that in order to characterize the true complexity of a time series, a notion about the values of \( m \) and \( r \) has to be developed. Although there has been no theoretical guidance on the selection of the values of these parameters for specific applications, previous applications in studies [29], [30], [31] have provided good statistical validity for values of \( m = 2,3 \) and \( r = 0.15 - 0.25\sigma \), where \( \sigma \) is the standard deviation of the time series. Large values of \( r \) will ignore the underlying dynamics of the signal and will not reflect its true complexity. On the other hand, small values of \( r \) will increase the contribution of noise in the ApEn calculation.

Similarly, in the case of selection of \( m \), a small value would increase the amount of self matches whereas a large value would reduce the amount of self matches, (for a given value of \( r \)) thereby not reflecting its true complexity. Also, increasing the value of \( m \) would increase the computation time. Summarizing the algorithm of ApEn, the entire process can be represented in a flowchart in the following figure.
Figure 2-1: Flowchart of ApEn

1. Obtain the N samples of time
2. Forming the state space vectors $P_m(1), P_m(2), P_m(3), ..., P_m(N)$
3. Calculating the number of similar vectors starting at measurement $i$, according to $r$ as $N_{im}(r)$
4. Calculating $C_{im}(r) = N_{im}(r) / N + m - 1$ and $C_m(r)$ as the mean of $C_{im}(r)$
5. Repeat above process to calculate $C_{m+1}(r)$
6. ApEn = $-\ln \left( \frac{C_m(r)}{C_{m+1}(r)} \right)$
2.3 ApEn of a simple sinusoid

The concept of the regularity of a time series and its corresponding ApEn statistic can be clarified by the following example. In this example, a simple sinusoid is generated and its approximate entropy is characterized with respect to various parameters of the sinusoid. Note that, since this example is only used for clarification and explanation purposes, the recommended values of $m$ and $r$ as used in the previous literature of ApEn are used to calculate the statistic. The sinusoid is of the form

$$Y(t) = A \sin(2\pi ft + \varnothing_0)$$  (2.2)

where $A$ is the amplitude, $f$ is the frequency and $\varnothing_0$ is the initial phase of the sinusoid.

2.3.1 Correlation between ApEn and Frequency

To generate the test signal, the frequency of the sinusoid was increased linearly from 1 Hz to 15 Hz with a constant phase and constant amplitude. The sampling frequency was maintained at 1500 Hz. The number of points considered in the above time series was 1500. Values of $m = 2$ and $r = 0.2\sigma$ are maintained.
Interpretation:

The increasing frequency of the sinusoid can be seen as increasing the complexity of the signal and hence the increasing irregularity. As explained earlier, higher irregularities translate to a higher ApEn index.

2.3.2 Correlation between ApEn and the Amplitude

In this experiment, the frequency of the sinusoid was maintained at 15 Hz with the same sampling frequency, same number of data points, and same ApEn parameters as above. The amplitude of the waveform was varied linearly from 1V to 25 V.
Figure 2-3: Effect of amplitude on ApEn

Interpretation:

The approximate entropy of the sinusoid remains constant with respect to the amplitude of the signal. This is due to that fact that the parameter $r$ is normalized with respect to the amplitude of the signal, by the virtue that it is dependent on the standard deviation of the signal. This normalization with respect to the amplitude imparts scale invariance to the ApEn statistic.

2.3.3 Correlation between ApEn and Initial Phase of the Sinusoid

In this experiment, the frequency of the sinusoid was maintained at 15 Hz with the same sampling frequency, same number of data points, and same ApEn parameters as above. The amplitude of the wave was fixed at 1V. The phase was uniformly varied between 0 to $2\pi$. 
Interpretation:

The maximum percent change in the ApEn value from the mean value was observed to be 5.39 %, which indicates that ApEn value does not depend significantly on the initial phase angle of the sinusoid.

2.3.4 Correlation between ApEn and Noise Power of the Signal

The sinusoid in the presence of noise is given as

\[ Y(t) = A \sin(2\pi f t + \phi_0) + n(t) \]  

(2.3)

where \( n(t) \) is the additive white Gaussian noise (AWGN) process. In this experiment, the frequency of the sinusoid was maintained at 15 Hz with the same sampling frequency, same
number of data points, and same ApEn parameters as before. The phase of the signal was kept constant. The signal to noise ratio (SNR) of the signal was increased linearly from 1dB to 25dB.

![Figure 2-5: Effect of noise power on ApEn](image)

**Interpretation:**

As the SNR increases, the noise power decreases and hence the amount of complexity in the signal decreases. Correspondingly, the ApEn value decreases. In communication signals, the noise power will significantly affect the ApEn value.

The above experiments were conducted to provide a brief and an intuitive feel about the algorithm illustrating how the regularity statistic of the time series depends on the parameters of the signals. Projecting the regularity statistic as an important feature of the signal, a systematic investigation is conducted in applying ApEn to classify a particular modulation signal.
Chapter 3

Approximate Entropy on Continuous Phase Modulated signals

In the literature that was reviewed, no study was found that identified the parameters of a CPM signal. As will be explained subsequently, a CPM signal can be characterized by a variety of parameters namely the number of symbol levels, pulse lengths, modulation indices, and pulse shapes. Thus the problem statement is defined as follows:

**Problem Statement:** To develop a classification scheme to identify various parameters of a continuous phase modulated signal.

Before any scheme is developed, it is instructive to understand the parameters of a CPM signal.

3.1 Introduction to Continuous Phase Modulation

The present scenario is witnessing an exponential growth in communications and networking technology which has resulted in an increased capacity of communication networks, advanced interference cancellation systems, channel monitoring and improvement techniques, frequency re-use methods to incorporate more users thereby generating more revenues for the service providers. In this type of situation, where bandwidth is increasingly growing into a simulacrum of real estate, spectral efficient modulations are considered imperative. One such class of spectral efficient modulation is the Continuous Phase Modulation. In addition to the spectral efficiency, lower side lobes and lower error probability make it a lucrative scheme to be employed in many practical communication systems. Continuous Phase Modulation is implemented in GSM telephony, 802.11 FHSS, Bluetooth, proprietary modems and satellite communication systems. CPM signals come from a class of nonlinear constant envelope
modulations in which the amplitude of the carrier remains constant. The characteristic of the modulation is the continuity of the phase of the carrier. Since the phase of the carrier is constrained to be continuous, the process introduces a memory in the modulation scheme.

3.2 Representation of CPM signals

The class of constant amplitude modulations can be represented as

\[ s(t) = \sqrt{\frac{2S}{T}} \cos(2\pi f_c t + \varnothing(t, I_k)) \]

where \( S \) is the energy per symbol, \( T \) is the symbol period, \( f_c \) is the carrier frequency and \( \varnothing(t, I_k) \) is the time varying phase corresponding to \( I_k \) input data sequence. In the case of CPM modulations, the time varying phase in the interval \( nT \leq t \leq (n + 1)T \) is given as [32],

\[ \varnothing(t, I_k) = 2\pi f_d T \sum_{k=-\infty}^{n-L} I_k + 2\pi f_d \sum_{k=n-L+1}^{n} I_k q(t - kT) \]  

(3.1)

where \( L \) is the pulse length, \( f_d \) is the peak frequency deviation and \( q(t) \) is the integral of the pulse shape which exists from \( 0 \leq t \leq LT \). The product \( 2f_d T \) is the modulation index \( h \) and \( I_k \) is the input data sequence which depends upon the number of symbol levels \( M \). inspecting the above equation, it is apparent that an infinite number of CPM signals using different number of symbol levels \( (M) \), modulation indices \( (h) \), pulse lengths \( (L) \), and pulse shapes can be created. Since the utility of ApEn as an important feature of the signal has been demonstrated in the previous chapter, we aim to develop a system in which we identify various parameters of the signal such as number of symbol levels, pulse lengths, modulation index, and pulse shapes. Without loss of generality, it is assumed that the symbols are drawn out of unit variance constellations, where the average power of the symbols of the constellation is unity.
3.3 Application of ApEn on CPM Signals

The scheme for ApEn on CPM signals is developed by first introducing the classification scheme of a base system which classifies a smaller number of signals by assuming the knowledge of some of the parameters. Later the condition on these parameters is relaxed and the base system is extended to a complete classification system to classify a large number of signals.

3.3.1 Developing a base system in noiseless environment

An incremental approach is taken to identify the parameters of a CPM signal. Thus, a base system is first defined as a system in which the pulse shape, number of symbol levels \( M \), and the pulse length \( L \) of a signal is known. The signals in the base system differ only in the modulation indices \( h \). Initially, classification between CPM signals with only different modulation indices \( h \) is done. Let us consider the case where, \( M = 2, L = 1 \), and the pulse shape is assumed to be rectangular. The modulation indices considered are \( h = \frac{3}{4}, \frac{2}{3}, \frac{1}{2}, \frac{2}{5}, \frac{1}{4} \).

The baseband representation of the signal allows us to obtain the signal in quadrature and in-phase forms.

\[
Y_{bb}(t) = Y_I(t) + Y_Q(t)
\]  

(3.2)

where \( Y_{bb} \) is the complex lowpass equivalent signal, \( Y_I \) is the in-phase and \( Y_Q \) is the quadrature signal. The knowledge of the carrier frequency is assumed to be known \textit{a priori}, which enables us to obtain the quadrature and the in-phase signals. The classification scheme for the base system is developed for the baseband representation of the CPM signal. In view of this scheme, the block diagram can be shown as below.
As depicted in the block diagram, ApEn indices are extracted from both the inphase and the quadrature data streams. These two values are formed as a two dimensional feature vector. Such feature vectors from each type of the signal are fed to a probabilistic neural network classifier which classifies between CPM signals with different modulation indices. Throughout this study, the classifier is trained with 500 feature vectors. The choice of the ApEn parameters is critical in order to ensure maximum classification accuracy. As stated before, most of the studies have recommended the values of $m = 2$ and $r = 0.2\sigma$ for calculation of ApEn values. However, in this application, it is imperative to develop the notion of these values with a view to maximize the classification accuracy of the base system.
3.3.1.1 Determination of the number of samples per symbol

1. In the study of respiratory and heart signals [30], [31], experimental data is obtained at a higher sampling frequency, with the sampling frequency being much greater than the required Nyquist rate. However, none of these studies have addressed the sampling frequency effects on ApEn values. Reference [33] has analyzed the effect of sampling frequency on the complexity of phrenic nerve discharges.

2. We need to develop the notion of the sampling frequency of CPM signals with respect to the ApEn scheme in order to obtain maximum classification accuracy. Since the values of $m = 2$ and $r = 0.2\sigma$ have provided good statistical validity, these values are initially used in order to determine the effect of sampling frequency or the number of samples per symbol on the classification accuracy.

3. Reference [22] maintains that the data points required to characterize the complexity of the signal is between 200-5000. In this experiment, the total number of data points is maintained at 8000. Table 3-1 shows the effect of the number of samples per symbol on the classification accuracy. As seen, the classification accuracy ($A_c$) increases as the samples per symbol ($N_s$) increases. In general, the classification accuracy between two classes $U$ and $V$ is given as $A_c = \frac{U_p + V_p}{U_n + V_n}$, where $U_p$ is the number of samples correctly identified out of $U_n$ input samples and $V_p$ is the number of samples correctly identified out of $V_n$ input samples. Similarly, the calculation of classification accuracy can be extended to include more than two classes.

4. To illustrate the point, the cluster diagrams corresponding to two different values of $N_s$ are shown, which indicates a high degree of overlap between clusters at lower samples per symbol ($N_s = 8$), where each cluster corresponds to a collection of complex ApEn values (in-phase and quadrature) for a particular modulation index. For a given number of data points, the ApEn values of the clusters decrease as observed from the two figures, Fig. 3-2
and Fig. 3-3. The average ApEn value of the CPM signal of modulation index $h = 1/2$, decreases as $N_s$ increases.

Table 3-1: Effect of the samples per symbol on classification accuracy

<table>
<thead>
<tr>
<th>$N_s$</th>
<th>$A_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>58</td>
</tr>
<tr>
<td>12</td>
<td>62.12</td>
</tr>
<tr>
<td>16</td>
<td>61.2</td>
</tr>
<tr>
<td>20</td>
<td>81.34</td>
</tr>
<tr>
<td>24</td>
<td>91.21</td>
</tr>
<tr>
<td>28</td>
<td>100</td>
</tr>
<tr>
<td>32</td>
<td>100</td>
</tr>
<tr>
<td>36</td>
<td>100</td>
</tr>
<tr>
<td>40</td>
<td>100</td>
</tr>
</tbody>
</table>
Figure 3-2: Cluster Organization at $N_s = 8$

Figure 3-3: Cluster organization at $N_s = 40$
5. The above phenomenon can be attributed to the fact that, as the number of samples per symbol increases, the number of self matches for a vector of length $m$ increases for a given value of $r$ (note that $r$ remains same in both cases of $N_s$). Correspondingly, the decrease in the number of matches of patterns is less when the pattern length increases to $m + 1$. Thus, we obtain a lower ApEn value. This phenomenon is validated by Fig. 3-4 which shows decreasing values of the centroids of ApEn clusters with increasing sampling frequency.

![Figure 3-4: Effect of $N_s$ on ApEn values](image_url)

6. From the figures, we can see that maintaining a high sampling frequency also captures the complexity of the signal in a true sense. Fig. 3-3 shows the signals arranged in the decreasing order of ApEn values as their modulation indices. This corroborates with the fact that signals with a larger modulation index experience a larger phase shift per symbol and thus are more irregular than the signals with smaller modulation indices. The phase shift per symbol is given by $\Delta \phi = (M - 1)h\pi$. 
3.3.1.2 Relation between $r$ and the sampling frequency

1. In general, the criterion of similarity $r$ is given as $r = c\sigma$, $c$ being a constant. In the above analysis, $c = 0.2$ and $N_s = 40$ is maintained for perfect classification. However, the analysis remains incomplete without investigating the effect of $c$ in conjunction with $N_s$ on the classification rates. Fig. 3-6 plots the minimum value of $c$ required for perfect classification at various $N_s$. Thus, at a particular $N_s$, a minimum value of $c$ has to be maintained for perfect classification which results in the separation of clusters as shown in Fig. 3-5. Of course, an upper bound on the value of $c$ exists. As mentioned earlier, large values of $c$ would result in losing detailed information about the signal which leads to overlapping clusters, due to similar ApEn values of clusters. In this experiment, the upper bound was observed to be 2.67 for all values of $N_s$.

2. This phenomenon explains the overlapping of clusters in Fig. 3-2 where the signals were maintained at $N_s = 8$ and the value of $c$ used was less than the minimum (0.65) required to obtain perfect classification. Note that in the above result, the number of symbols was adjusted in order to keep the same number of data points.

3. The results from the above analysis differ significantly from the available literature [34] on the usage of values of $c$ in biomedical applications. Values of $c = 0.15 - 0.2$, irrespective of the sampling frequency provided sufficient statistical validity in these applications [35], whereas a relationship between $c$ and $N_s$ is observed in order to obtain maximum classification accuracy of CPM signals.
3.3.1.3 Determination of the number of symbols

1. Keeping in with the spirit of choosing ApEn parameters with a view to maximize classification performance of a base system, it is imperative to analyze the effect of the number of data symbols \(N_{sym}\) on the classification performance for given values of \(c\) and \(N_s\). In Fig. 3-2, where overlapping clusters gave rise to low accuracies, the effect of \(N_{sym}\) is shown in table 3-2, for the same values of \(c\) and \(N_s\).

2. Theoretically, infinite number of data points is required for ApEn calculation. Thus, in this case, one would hope that increasing the number of data points in the form of \(N_{sym}\) would achieve theoretical values of ApEn, which might lead to the separation of clusters.
3. Analysis shows that at higher $N_{\text{sym}}$ the classification rates still remain low. The centroids ($\mu$) of the clusters remain constant with the maximum difference between the corresponding clusters being less than 1%. Value of $N_{\text{sym}}$ does not affect the centroid of the clusters. The effect of increasing $N_{\text{sym}}$ is only the decrease in the variance ($\rho$) of the clusters. Thus, along with a combination of $c$ and $N_s$, a sufficient number of symbols is required to produce clusters with small variance, in order to maintain high classification accuracy. Table 3-3 shows similar effect of $N_{\text{sym}}$ on the centroid and variances for clusters, for $N_s = 40$.

<table>
<thead>
<tr>
<th>$N_{\text{sym}}$</th>
<th>5000</th>
<th>8000</th>
<th>10000</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_c$</td>
<td>66.34</td>
<td>68.23</td>
<td>68.88</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$h=3/4$</th>
<th>$\rho$</th>
<th>1.0998e-4</th>
<th>7.8956e-4</th>
<th>7.0645e-5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>0.4336</td>
<td>0.4342</td>
<td>0.4340</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$h=1/2$</th>
<th>$\rho$</th>
<th>5.8914e-4</th>
<th>2.1014e-4</th>
<th>1.9664e-5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>0.3915</td>
<td>0.3983</td>
<td>0.3890</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$h=1/4$</th>
<th>$\rho$</th>
<th>3.9655e-5</th>
<th>3.5323e-5</th>
<th>3.1859e-5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>0.3932</td>
<td>0.3945</td>
<td>0.3937</td>
<td></td>
</tr>
</tbody>
</table>

Table 3-2: Effect of $N_{\text{sym}}$ at $N_s = 8$
Table 3.3: Effect of $N_{sym}$ at $N_s = 40$

<table>
<thead>
<tr>
<th>$N_{sym}$</th>
<th>5000 $A_c = 100$</th>
<th>8000 $A_c = 100$</th>
<th>10000 $A_c = 100$</th>
</tr>
</thead>
<tbody>
<tr>
<td>h=3/4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>2.0578e-4</td>
<td>5.8341e-5</td>
<td>4.1379e-5</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.1813</td>
<td>0.1818</td>
<td>0.1823</td>
</tr>
<tr>
<td>h=1/2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>6.8164e-4</td>
<td>5.1344e-4</td>
<td>2.7364e-5</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.1551</td>
<td>0.1562</td>
<td>0.1548</td>
</tr>
<tr>
<td>h=1/4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>5.6755e-5</td>
<td>3.6712e-5</td>
<td>3.1997e-5</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.1227</td>
<td>0.1236</td>
<td>0.1219</td>
</tr>
</tbody>
</table>

3.3.1.4 Determination of $m$

1. Previous experiments have used the recommended values of $m = 2$ to analyze the performance with respect to $N_s, N_{sym}$, and $c$. In this experiment, analysis is performed to validate the value of $m$ used. Fig. 3-6 shows the effect of $m$ on $A_c$ for different $N_s$. Note that the value of $c$ is same for all $N_s$ which ensures perfect classification at a particular $N_s$. 

33
Figure 3-6: Classification accuracy ($A_c$) versus $m$. The curves parameterized by $N_s$.

2. At a large value of $m$, it is difficult for corresponding samples within any two patterns of length $m$ to be within $r$ due to the variation of the waveform, since the large pattern length significantly captures high variation. Thus, higher values of $m$ lead to similar ApEn values for all the waveforms since the number of pattern matches remain similar (number of matches attain low values, since $m$ is large), which ignores the underlying complexity of the waveforms and translates to a lower $A_c$.

3. Values of $m$ for which $A_c$ drops from the maximum accuracy (100%) increases with increase in $N_s$. Thus, at high $N_s$, because of a large number of samples, characteristic pattern matches are formed for a given value of $r$. These pattern matches give rise to characteristic ApEn values thereby producing well defined clusters, even when $m$ is increased. However, as $m$ increases further, the pattern length becomes too large to produce any characteristic pattern matches and similar ApEn values are produced for different waveforms. At $m = 2$, not only
is the performance superior but also the computational time is lower. In future settings, value of $m$ used is 2. Note that, in the above analysis, the number of data points is same.

### 3.3.2 Extension of the base system in the presence of noise

1. The ApEn parameters ($m$ and $r$), number of samples per symbol ($N_s$) and the number of data symbols ($N_{sym}$) are chosen for a base system to satiate the goal of maximum classification accuracy in the absence of noise. It is imperative to analyze the performance of the system in the presence of noise using the parameters that were set in the previous section. From Fig. 3-5 it is apparent that the received signal can be sampled at different sampling rates, with changes in $c$ to obtain perfect classification. Consider a base system, for which $N_s = 8, N_{sym} = 1000, m = 2$, and $c > 0.66$.

2. Fig. 3-7 shows the centroids of the ApEn clusters versus the signal to noise ratio (SNR) in dB, where $SNR = 10 \log_{10} \frac{S}{N_o}$. As the noise level increases, the signal is overwhelmed by the noise and the ApEn index fails to capture any regularity from the signal which was its characteristic feature. Any semblance to the regularity of the signal is eroded by the noise addition. Since all the signals resemble a noisy process, the ApEn index reflects the same value for all the signals of different modulation indices.
3. With the chosen parameters, CPM signals with different values of $M$, $L$ and $h$ are classified. In the classification problem that we have considered, signals with values of $M = [2,4,8]$, $L = [1,2,3]$, and $h = \left\{ \frac{1}{2}, \frac{1}{4}, \frac{1}{3}, \frac{2}{5}, \frac{3}{4} \right\}$ are present. Thus, 45 different signal classes are considered here, whereas in the base system only 5 different signals were considered in the classification. Fig. 3.8 shows overall $A_c$ versus SNR.
4. Considerable accuracy is obtained at higher SNRs while classifying such a large number of signals. In the case, where pulse length is assumed to be known (in this case, $L = 1$ and the number of signal classes is 15), the classification accuracy increases by almost 20%. The maximum accuracy in Fig. 3-8 is constrained by the number of data symbols and does not achieve error free performance even at higher SNRs. Table 3-4 shows the results for the classification of 15 signal classes with respect to the number of data symbols ($N_{sym}$).
Table 3-4. Effect of $N_{sym}$ on $A_c$ at SNR=15 dB

<table>
<thead>
<tr>
<th>$N_{sym}$</th>
<th>$A_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>97.477</td>
</tr>
<tr>
<td>1200</td>
<td>98.355</td>
</tr>
<tr>
<td>1400</td>
<td>99.87</td>
</tr>
<tr>
<td>1600</td>
<td>100</td>
</tr>
</tbody>
</table>

3.3.3 Use of Instantaneous Frequency

1. When the signal pool consists of unknown values of $L$, $A_c$ is lower as compared to the case when $L$ is known, as observed in Fig. 3-8. To mitigate the problem of identifying additional pulse lengths, we focus on the instantaneous frequency [36] of the waveform, which can be viewed as another feature of the signal.

2. The rationale in applying ApEn to the instantaneous frequency (IF) of the CPM signal is the ability of ApEn to detect “clinical” changes in the waveform according to the parameters of the underlying process. Since the parameters of the signal govern the IF, the extracted ApEn index is a function of these parameters.

3. Mathematically the equation of the instantaneous frequency is given as,

$$f_i = \frac{1}{2\pi} \frac{d}{dt} \arg \{z_a(t)\}$$

(3.3)  

where, $z_a(t)$ is the analytic signal of the signal received. In the context of the CPM signal, let us obtain the expression of the instantaneous frequency and indicate the parameters which affect it. The time varying phase of the CPM signal given in equation 3.2. Following equation 3.2, the instantaneous frequency of the CPM signal can be given as,

$$f_i = \frac{1}{2\pi} \frac{d}{dt} \{\varnothing(t, l_k)\}$$

(3.4)
\[
fi = f_d \sum_{k=n-L+1}^{n} l_k \frac{d}{dt} \{q(t-kT)\} 
\]  
\[ (3.5) \]

Now for a partial response CPM signal \((L > 1)\), any pulse is defined as,

\[
g(t) = p(t) \quad 0 \leq t \leq LT 
\]
\[ (3.6) \]

\[
q(t) = \int_{-\infty}^{t} g(t) \, dt 
\]
\[ (3.7) \]

In the formulation of the CPM signals, the maximum value of \(q(t)\) was normalized to be \(\frac{1}{2}\). This normalization ensures the maximum phase change over symbol to be \((M - 1)h\pi\). In our case, since the pulse shape is assumed to be rectangular and can have values between 0 and \(LT\), instantaneous frequency can be reduced to

\[
f_i = f_d \sum_{k=n-L+1}^{n} \frac{l_k}{2LT} 
\]
\[ (3.8) \]

\[
f_i = \frac{f_d}{2LT} \sum_{k=n-L+1}^{n} l_k 
\]
\[ (3.9) \]

However, we have \(h = 2f_dT\). Hence the above equation reduces to the following simplified form.

\[
f_i = \frac{h}{L(2T)^2} \sum_{k=n-L+1}^{n} l_k 
\]
\[ (3.10) \]

Without loss of generality, assuming \(T=1\), we have,

\[
f_i = \alpha \frac{h}{L} \sum_{k=n-L+1}^{n} l_k 
\]
\[ (3.11) \]
where $\alpha$ is a constant. Inspecting the equation we note that all the parameters of the CPM signal namely the modulation index ($h$), pulse length ($L$) and the number of symbols ($l_k$ is one of the symbols from the $M$ signal levels) affect the instantaneous frequency of the signal. Thus, the ApEn index on the IF of the signal can be utilized as a valuable feature.

4. Using the additional dimension of ApEn on the IF of a signal, the feature vector is now modified to span three dimensions and classification performance results are compared with the results obtained using the two dimensional feature vector in Fig. 3-10. Fig. 3-9 shows the scheme of ApEn on baseband and IF of the signal.

**ApEn Vector**

![Figure 3-9: Scheme of ApEn in three dimensions](image)

Figure 3-9: Scheme of ApEn in three dimensions
Figure 3-10: $A_c$ versus SNR for ApEn on baseband sequence and IF. The number in the bracket represents the number of signal classes.

5. In practice, it is highly unlikely that such large number of signals needs to be classified. Nevertheless, the above results illustrate an overall performance when classifying different CPM signals. Of course, as the number of unknown parameters decrease, the performance of signal classification increases. It can be insightful to see the performance when only one of the parameters is known. Fig. 3-11 shows these results.

6. Let $\lambda_1 = (M_1, L_1, h_1)$ and $\lambda_2 = (M_2, L_2, h_2)$ be two different sets consisting of CPM signals with different parameters. For $\lambda_1$, we have $(2, 3, 1/4), (4, 1, 3/4), (8, 2, 3/4), (2, 1, 2/5)$, and $(4, 3, 2/3)$ as the parameter sets. For $\lambda_2$, we have $(2, 2, 2/3), (4, 1, 3/4)$, and $(8, 3, 1/4)$. Performance for the two signal sets is shown in Fig. 3-12.
Figure 3-11: $A_c$ when one of the parameters is known. Number in bracket is no. of signal classes.

Figure 3-12: $A_c$ for signal sets $\lambda_1$ and $\lambda_2$. 
3.3.4 A comment on the relative ApEn values

1. It is obvious that $M$ affects the ApEn values of signals in a more significant way, than $L$ and $h$. In the noiseless case, the centroids of ApEn clusters for a particular modulation index for different values of $M$ and $L$ is shown in table 3-5.

<table>
<thead>
<tr>
<th>$h = \frac{2}{3}$</th>
<th>$M = 2$</th>
<th>$M = 4$</th>
<th>$M = 8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L = 1$</td>
<td>0.0582</td>
<td>0.1261</td>
<td>0.1876</td>
</tr>
<tr>
<td>$L = 2$</td>
<td>0.0422</td>
<td>0.0857</td>
<td>0.1601</td>
</tr>
<tr>
<td>$L = 3$</td>
<td>0.0331</td>
<td>0.0703</td>
<td>0.1365</td>
</tr>
</tbody>
</table>

2. For a given value of $M$ and $h$, the ApEn value decreases with the increase in the length of the pulse. In the CPM scheme, partial response signals (pulse length greater than one symbol) have memory incorporated in them by the virtue of the continuity of the pulse, thus the phase change is smoother in a symbol period, as compared to full response (pulse length equal to one symbol) CPM signal and hence the ApEn values are lower.

3. The signal resulting from a particular parameter set ($M, L, h$) will have a different regularity index from the signal resulting from a different parameter set. It is this difference in regularities, corresponding to each of the ApEn parameters that form the basis for the entire classification scheme.
3.3.5 Case for a high number of samples per symbol

1. As stated before, Fig. 3-5 provides us with a variable operating range of $N_s$ provided that the value of $c$ is adjusted to obtain perfect classification. In this section, the symbols are sampled at a high value ($N_s = 40$) and the analysis is done. The value of $c$ used in the previous section is used here. A scheme is now developed which classifies CPM signals with different $M, L, h$ parameters.

2. To proceed with the analysis, we consider a base system which was defined in the previous sections. Since $N_s$ is high, the effect of the noise is going to be much more pronounced on the classification accuracy of the base system. One of the simplest ways to alleviate the effect of noise is the use of a low pass filter with a cutoff frequency depending upon the bandwidth of the signal. In this case, the cut off frequency is set according to the value of $M$ (in this case, $M = 2$). The value of $M$ affects the bandwidth of the signal more [32] than the values of $L$ and $h$. In this case, a fifth order Butterworth filter is designed with cut off frequency set according to the bandwidth of the CPM signal with the parameters $M, L$ known beforehand.

3. Fig. 3-13 compares the classification accuracy of the signals in the base system for unfiltered and filtered signal samples. As seen, the filtering process which removes the excess noise due to oversampling improves $A_c$, when signals in base system are passed through a filter corresponding to $M = 2$. Note that $L = 1$. Through filtering, the signal preserves its characteristics and the ApEn captures the existing, non-eroded regularity of the signal to provide maximum classification accuracy. The noise in the frequency band of the signal increases the ApEn values of the signals from the values in the noise less case. It should be noted that the analysis of only one of the base system is shown. It can be shown that even for base systems with a fixed value of $M$ but $L$ other than 1, high classification performance was observed. Thus, the same filter which was designed considering the value of $M = 2, L = 1, h$ under consideration, was also able to maintain high accuracy for base
systems with $M = 2$ and $L = 2$ or 3. This type of scheme is possible because $M$ affects the bandwidth of a CPM signal more than $L$ and $h$. It should be noted that the analysis of only one of the base systems is shown here. Same analysis can be performed for other base systems as well, for e.g. a base system with $M = 4$ and $L = 2$ or a base system with $M = 8$ and $L = 3$.

4. To maintain classification in the presence of noise, use of filters which utilized the knowledge of $M$ is considered. But, during modulation identification, since we do not know the value of $M$ of the received signal a priori, use of filters is not possible. Use of a filter corresponding to largest value of $M$ under consideration, $M_{\text{max}}$, is also not suitable for classifying signals with a lower value of $M \leq M_{\text{max}}$. If a filter corresponding to $M = 8$ is used to classify signal corresponding to $M = 2$, then corresponding $A_c$ versus the SNR can be shown in Fig. 3-13.

![Effect of filtering on $A_c$ of a base system in presence of noise](image)

Figure 3-13: Effect of filtering on $A_c$ of a base system in presence of noise
5. As seen in Fig 3-13, the filter employed is not able to remove the excess noise. By virtue of the above problem, a scheme needs to be devised which would ensure classification for signals with all the values of $M$. Thus, if the signals under consideration correspond to $M$ values of $M_1, M_2, M_3$, up to $M_{\text{max}}$, then filters corresponding to those values are used. Use of three filters circumvents the above problems mentioned when using a single filter. Since the value of $M$ is unknown for the incoming signal, the signal when filtered through the corresponding filter, the clusters of different modulation indices separate out. Thus, out of the six dimensions as denoted in the Fig 3-14, in at least two dimensions, high classification performance is obtained. Since the accuracy is high in at least two of the dimensions, overall high accuracy is obtained in the six dimensions, even if the clusters are not separated in the remaining four dimensions. It should be noted here that the use of filters prior to ApEn computation effectively increases the SNR by removing the out-of-band noise. However, we use the pre-filtered SNR in the presented graphs for comparison purposes. Such a scheme of ApEn on baseband sequences is shown as a flowchart in Fig 3-15 which summarizes the classification process.
Figure 3-14: Graphical Representation of the filter system
Figure 3-15: Flowchart for the classification system
6. Comparative results are shown in Fig. 3-16 for the two systems at different number of samples per symbol. Note that, in the both the schemes the ApEn values are extracted from the IF waveform of the signal along with the baseband sequences. As seen from the results, the system with the use of filters (at $N_s = 40$) provide higher classification rates among a large number of signals at lower SNRs than the system without the use of filters (at $N_s = 8$). However, the feature vector in the former system is in nine dimensions whereas the feature vector in the latter is only in three dimensions. Thus, a tradeoff can be seen here in terms of the computational complexity and the classification rates at lower SNRs. At higher SNRs, the system at $N_s = 8$, provides excellent results among a high number of signal classes. In the remaining chapters, for the ease of referencing, the classification system involving the use of filters as described above at $N_s = 40$ is denoted by $\beta$ and the system without the filters at $N_s = 8$ is denoted by $\lambda$. 

Figure 3-16: Comparison of two systems. ApEn on baseband and IF.

7. Complete analysis for the system at $N_g = 40$ is shown in Fig. 3-17. Results are shown for the system when classifying between 45 signal classes and 15 signal classes by using ApEn feature vectors on both the IF and the baseband sequences. In this system involving the use of filters, as mentioned earlier, corresponding to each filter output, we have three elements of the feature vector; the first two being the ApEn of the in phase and the quadrature streams and the third being the ApEn of the instantaneous frequency. The feature vector now consists of nine elements, three elements corresponding to each filter output. Results for ApEn only on IF sequences are also shown. In this case, the feature vector is in three dimensions (IF output from each of the filter output). Even without the baseband sequences of the signal, the accuracy improved significantly. Thus, the IF is a more robust feature of the CPM signal with respect to the approximate entropy as it captures the minor variations as imposed by the parameters.
Figure 3-17: $A_c$ versus SNR for system $\beta$. The number in the bracket represents the number of signal classes.

8. As is evident from the above results, application of ApEn on the IF waveforms and the baseband sequences increases the classification performance considerably when classifying between multiple signal classes. However, it can be seen from Fig. 3-17 that the performance, when only IF is considered, decreases at higher SNR. This can be explained by the following phenomenon. Let us consider a base system as defined in the previous section. For the same modulating data sequence, the IF waveforms for different $h$ can be shown in Fig. 3-18.
The shape of the waveforms is identical to each other differing only in the scale factor. Since a larger modulation index produces a larger phase change over a symbol period, this translates into a larger amplitude IF waveform. Thus, barring for the scale factor, the IF waveforms of different modulation indices are identical. By virtue of the parameter $r$ which imparts scale invariance to the data series, the ApEn values of different $h$, for a given value of $M$ and $L$, remain same. Thus, at higher SNRs, the clusters of modulation indices overlap thereby giving low classification rates. Fig. 3-19 plots the centroid of the ApEn clusters with respect to the SNR. As predicted, the centroids coincide when the SNR increases.
Figure 3-19: Centroids of ApEn clusters of IF waveforms

9. It can also be seen that ApEn values of the IF waveforms of larger modulation indices are lower than ApEn values of smaller modulation indices. Since the IF waveforms are identical, the tolerance criterion $r$ is larger for a larger modulation index IF waveform than for a smaller modulation index. As the noise power is same in all the waveforms, the pattern matches in smaller modulation index waveform is lower for the value of $r$. Correspondingly, ApEn values are higher. In the case of higher modulation indices, since $r$ is large and the excursion of samples are almost the same as in the case of smaller modulation index IF waveforms (noise power being same), pattern matches are higher for that value of $r$. Correspondingly, ApEn values are lower. Note that, this is a special scenario in which the waveform nature is identical for all $h$. This obviously does not hold true when the waveform nature is not identical, as in the case of the baseband sequences for different modulation indices in the base system. In that case, the ApEn values reflect the complexity of the
waveform according to the modulation indices (see Fig. 3-3). It should be noted that this effect of IF on higher SNR on $A_c$ is also observed on system $\lambda$. However, results of only system $\beta$ are shown.
Chapter 4

Identification of pulse shapes

4.1 Pulse shape as unknown parameters

One of the unknown parameters in the signal model is pulse shape. Previous studies in modulation recognition have assumed the knowledge of pulse shape before proceeding with the classification process [2]-[19]. In the previous chapters, the pulse shape of the received signal was assumed to be rectangular. In this chapter, the scheme of ApEn is extended to classify CPM signals with varying pulse shapes. Raised cosine (RC) pulses with roll-off factors ($\alpha$) of 0.3, 0.5, and 0.7 and Gaussian pulses with bandwidth-time (BT) products of 0.3, 0.5, and 0.7 are considered. Thus, a CPM signal with a particular value of $M$, $L$, and pulse shape parameter is generated. It can be seen that by varying the values of the above parameters, different types of CPM signals can be generated. Results are shown for the identification of the above parameters. The plots in Fig. 4-1 classify between 135 signals with different pulse shape parameters, $M$, $L$, and $h$, for each of the pulse shape type (RC and Gaussian). Results are shown for both the systems $\beta$ and $\lambda$. It can be seen that system $\beta$ completely outperforms system $\lambda$ when dealing with pulse shape parameters. Acceptable performance is obtained given the large number of signals and the small difference in the pulse shape parameter values. The advantage of system $\beta$ over system $\lambda$ rises from the fact that because of the filters, corresponding to each $M$, the ApEn is able to discern the regularity index of the signal which differentiates itself from other signals. Results for both the systems are shown for classification between $M$, $L$, and type of pulse shapes. Signals with different pulse shape parameters but with the same pulse shape type are classified into one type of pulse shape family. A CPM signal with $M = 4$, $L = 3$, $h = 2/5$, $\alpha = 0.5$ and a CPM signal $M = 4$, $L = 3$, $h = 2/5$, $\alpha = 0.7$ is classified as a CPM signal with
$M = 4, L = 3, h = 2/5$ and pulse shape type as raised cosine. However, a CPM signal with $M = 8, L = 1, h = 2/3, \alpha = 0.5$ and a CPM signal $M = 4, L = 3, h = 2/5, \alpha = 0.5$ is classified as two different CPM signals with the same pulse shape type as raised cosine. Similar is the case for Gaussian pulse shapes. Rectangular pulse shapes are also included to give an overall classification performance. Acceptable performance is obtained given the large number of signals in the unknown signal pool. Since in practical situations, the number of signals to be identified remains less 135, it can be seen that the accuracy remains fairly high in the case where only three signals differing in pulse shape types are considered. The ability of ApEn to distinguish between signals across a varied number of parameters such as $M, L, h$, and pulse shape type emphasizes its ability to detect minor variations in the underlying process.

![Figure 4-1](image-url)  

Figure 4-1: $A_c$ between $M, L, h$ and pulse shape parameter. The number in the bracket represents the number of signal classes.
Figure 4-2: $A_c$ versus SNR between $M, L, h$ and pulse shape families. The number in the bracket represents the number of signal classes.

The legend in Fig. 4-2 is explained as follows. A, D consists of signals with different $M, L, h$, Gaussian pulses with BT products of 0.3, 0.5, 0.7, RC pulses with $\alpha$ of 0.3, 0.5, 0.7, and rectangular pulses. Thus the total number of signal classes is 315. The curves identify $M, L, h$, and the pulse shape type. B, E consists of signals with different $M, L, h$, Gaussian pulse with BT of 0.3, RC pulse with $\alpha$ of 0.3, and rectangular pulse. Even in this case, The curves attempt to identify $M, L, h$, and the pulse shape type, however only a single pulse shape parameter is considered. C consists of signals with $M, L, h$ fixed, Gaussian pulse with BT of 0.3, RC pulse with $\alpha$ of 0.3, and rectangular pulse. Accordingly, only 3 signals are present.
4.2 A comment on SNR estimation

As explained in the introduction, the ApEn value of a signal is affected significantly by noise. Thus, the ApEn values of the signal change with the change in the noise power. In the analysis so far, classification performance has been shown for a large SNR range. The classification scheme developed involves the formation of a feature vector which is fed to a previously trained classifier. Since the ApEn values in the feature vector change in accordance with the noise power, the feature vector formed at a particular SNR is fed to a classifier which is trained using the feature vectors at that particular SNR. Thus, a pool of classifiers trained at a specific SNR value is created. Graphically, the process can be shown in Fig. 4-3.

Figure 4-3: Classifiers trained at a specific SNR value

In practice, since the SNR of the received signal is unknown, it is apparent that the signal level be estimated before choosing any classifier from the classifier pool. This invokes the need to investigate various SNR estimation techniques for an unknown modulation type in a modulation recognition problem. Among the SNR estimation techniques for unknown modulation signals that have been reviewed, the signal subspace approach [37] for SNR estimation has been used, details
of which are given in Appendix B. The following approach is proposed for presenting the results of classification performance when SNR estimation techniques are used prior to choosing a classifier.

1. The SNR range (-5dB to 25dB) is quantized into levels that differ by 0.5 dB. Thus the estimated SNR which is less than +/- 0.25 dB from the quantized level is mapped to that quantized SNR level. The value 0.5 dB is chosen with due consideration the number of classifiers in the classifier pool and the SNR estimation error in [37]. Classifiers are trained with feature vectors at these quantized SNR values.

2. The SNR of the received signal is estimated and quantized to a particular value.

3. Feature vector is formed and fed to the classifier which is trained at the value in step 2.

   Performance results are then presented.

The process can be shown in Fig. 4-4.

![Classification scheme after SNR estimation technique](image)
Results for classifying 45 and 15 signal classes after SNR estimation are shown in Fig.4-5. Note that the details of the SNR estimation algorithm are given in Appendix B. In this scheme, $N_s = 8$.

Figure 4-5: $A_c$ after SNR estimation. ApEn on baseband.
Chapter 5

Extension to other modulation schemes

The analysis in the above chapters outlines the parameter identification process for CPM signals. From the results and observations, it is apparent that ApEn is able to identify a large number of CPM signals with different parameters with considerable classification accuracy. This chapter extends the ApEn classification process developed until now to other modulation schemes. Signals from PSK, QAM and FSK modulations are considered and classification performance results for intra class classification are presented. To proceed further, it is imperative to set the ApEn parameters, number of symbols and the number of samples per symbol for each type of modulation scheme. This is to ensure that the selection of parameters is not arbitrary and is consistent with the method followed for CPM signals in the previous chapters.

5.1 ApEn on PSK signals

As emphasized earlier, the parameters for ApEn are chosen with a view to maximize the classification accuracy. 2PSK, 4PSK and 8PSK signals are considered. The ApEn and signal parameters are set with a view to maximize the classification rate in a noiseless environment.

5.1.1 Determination of \( N_s \) and \( r \)

1. A relation between \( c \) where \( c = \frac{r}{\sigma} \), and \( N_s \) is established to obtain maximum classification rate. Table 5-1 shows the relation between \( c \) and \( N_s \). The total number of data points considered here is 1000. The number of data symbols is adjusted to keep the number of data points same in each case.
Table 5-1: Relation between $c$ and $N_s$ for maximum classification accuracy

<table>
<thead>
<tr>
<th>$N_s$</th>
<th>$c_{max}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>0.51</td>
</tr>
<tr>
<td>12</td>
<td>0.52</td>
</tr>
<tr>
<td>16</td>
<td>0.49</td>
</tr>
<tr>
<td>24</td>
<td>0.5</td>
</tr>
<tr>
<td>28</td>
<td>0.5</td>
</tr>
<tr>
<td>32</td>
<td>0.53</td>
</tr>
<tr>
<td>36</td>
<td>0.51</td>
</tr>
<tr>
<td>40</td>
<td>0.5</td>
</tr>
</tbody>
</table>

2. From the table it is observed that the maximum value of $c$ is around 0.5. This is the maximum value of $c$ beyond which the classification accuracy drops from the maximum rate i.e. near 100%. Values of $c \leq c_{max}$ also ensure maximum classification accuracy. Of course, there exists a lower bound, below which the classification rate is not maximum. At these low values of $c$, much detailed information about the underlying signal is lost and the ApEn clusters form overlapping regions and translate to a lower classification rate. In this experiment, the value of $c$ used was 0.1, which gave perfect classification rate (to within simulation accuracy) at all $N_s$. Note that the value of $m$ is 2.

5.1.2 Determination of $N_{sym}$

1. Tables 5-2 and 5-3 illustrate the effect of the number of data symbols on the classification rate. Two values of $c$ are used. One value guarantees a perfect classification and the other does not. It can be seen that increasing the number of symbols does not have an effect on the classification rate, for the latter value of $c$, in Table 5-3. As was observed in the case of CPM signals, a proper value of $c$ is required for perfect classification. Increasing $N_{sym}$ decreases the variance ($\rho$) of the clusters. However, as observed, the centroids ($\mu$) of the ApEn clusters do not change much, with the difference between the corresponding centroids being less than 3%. Note that the value of $m$ is 2.
### Table 5-2: Effect of $N_{sym}$ : $c = 0.5$

<table>
<thead>
<tr>
<th>$N_{sym}$</th>
<th>500</th>
<th>1500</th>
<th>2000</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$A_c = 100$</td>
<td>$A_c = 100$</td>
<td>$A_c = 100$</td>
</tr>
<tr>
<td>$M=2$</td>
<td>$\rho$</td>
<td>$5.8336e-5$</td>
<td>$5.4856e-4$</td>
</tr>
<tr>
<td></td>
<td>$\mu$</td>
<td>0.1724</td>
<td>0.1721</td>
</tr>
<tr>
<td>$M=4$</td>
<td>$\rho$</td>
<td>$7.1614e-5$</td>
<td>$6.27014e-5$</td>
</tr>
<tr>
<td></td>
<td>$\mu$</td>
<td>0.3451</td>
<td>0.3485</td>
</tr>
<tr>
<td>$M=8$</td>
<td>$\rho$</td>
<td>$3.3055e-5$</td>
<td>$1.5523e-5$</td>
</tr>
<tr>
<td></td>
<td>$\mu$</td>
<td>0.4490</td>
<td>0.4522</td>
</tr>
</tbody>
</table>

### Table 5-3: Effect of $N_{sym}$ : $c = 0.9$

<table>
<thead>
<tr>
<th>$N_{sym}$</th>
<th>500</th>
<th>1500</th>
<th>2000</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$A_c = 54.23$</td>
<td>$A_c = 65.23$</td>
<td>$A_c = 61.23$</td>
</tr>
<tr>
<td>$M=2$</td>
<td>$\rho$</td>
<td>$6.4291e-5$</td>
<td>$4.4312e-5$</td>
</tr>
<tr>
<td></td>
<td>$\mu$</td>
<td>0.1861</td>
<td>0.1871</td>
</tr>
<tr>
<td>$M=4$</td>
<td>$\rho$</td>
<td>$2.3303e-5$</td>
<td>$9.7106e-6$</td>
</tr>
<tr>
<td></td>
<td>$\mu$</td>
<td>0.3229</td>
<td>0.3267</td>
</tr>
<tr>
<td>$M=8$</td>
<td>$\rho$</td>
<td>$7.8405e-3$</td>
<td>$6.4523e-4$</td>
</tr>
<tr>
<td></td>
<td>$\mu$</td>
<td>0.3427</td>
<td>0.3446</td>
</tr>
</tbody>
</table>
5.1.3 Classification results

1. With $c = 0.5$, $N_s = 8$, $N_{sym} = 1000$ and $m = 2$, results for intra class PSK modulation in an AWGN channel are presented in Figure 5-1. The ApEn feature vector consists of values from the baseband sequences and the IF waveforms of the signal. Figure 5-2 presents the results with the curves parameterized by $N_{sym}$. The signal model includes the effects of timing jitter, lack of synchronization and random signal phase.

![Figure 5-1: $A_c$ between 2PSK, 4PSK and 8PSK. $N_{sym} = 1000$](image-url)
5.2 ApEn on QAM signals

4QAM, 16QAM and 64QAM signals are considered for classification. As in the case of PSK signals, ApEn parameters are determined for QAM scheme in a noiseless environment.

5.2.1 Determination of $N_s$ and $r$

1. Figure 5-3 reports the classification accuracies for various values of $c$ at multiple $N_s$. It is observed that the range of $c$ for which maximum performance rate (100%) is obtained is between 0.2-0.3. Number of data points remains same in each case.
5.2.2 Determination of $N_{sym}$

1. Table 5-4 shows the effect of $N_{sym}$ on the classification rates for two values of $c$. As observed, increasing $N_{sym}$ does not improve the classification rate between clusters at $c = 0.9$. Effect of $N_{sym}$ is seen on the variance of clusters.

<table>
<thead>
<tr>
<th>$N_{sym}$</th>
<th>$A_c$ (c = 0.3)</th>
<th>$A_c$ (c = 0.9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>98.23</td>
<td>86.23</td>
</tr>
<tr>
<td>1000</td>
<td>99.97</td>
<td>88.45</td>
</tr>
<tr>
<td>1500</td>
<td>100</td>
<td>87.12</td>
</tr>
<tr>
<td>2000</td>
<td>100</td>
<td>87.65</td>
</tr>
</tbody>
</table>
5.2.3 Classification results

1. With $c = 0.3$, $N_s = 8$, $N_{sym} = 1000$ and $m = 2$, results for intra class QAM modulation in an AWGN channel are presented in Figure 5-4. Figure 5-5 presents the results with the curves parameterized by $N_{sym}$.

![Figure 5.4: $A_c$ between 4QAM, 16QAM and 64QAM. $N_{sym} = 1000$](image)

5.3 ApEn on FSK signals

2FSK, 4FSK, 8FSK signals at a specific frequency deviation (in this case 20 Hz) are considered for classification.

5.3.1 Determination of $N_s$, $r$ and $N_{sym}$

It is observed that range of values of $c$ for which the classification accuracy remains maximum (100%) at various $N_s$ is 0.05 to 2.67. Thus for FSK signals, the operating range for $c$ is higher than CPM, PSK, and QAM signals. Similar effect, as in the case of PSK and QAM signals, is observed with increasing values of $N_{sym}$. Fig 5-6 shows the cluster organization for the three FSK signals in the noiseless case.
Figure 5-6: Cluster organization for FSK signals at $c = 1.2$, $N_s = 8$, and $N_{sym} = 1000$

5.3.2 Classification results

1. With $c = 0.3$, $N_s = 8$, $N_{sym} = 1000$ and $m = 2$, results for intra class FSK modulation in an AWGN channel are presented in Figure 5-7. Figure 5-8 presents the results with the curves parameterized by $N_{sym}$. 
Figure 5-7: $A_c$ between 2FSK, 4FSK and 8FSK at a specific frequency deviation. ApEn on IF and baseband.

Figure 5-8: $A_c$ between 2FSK, 4FSK and 8FSK. ApEn on IF and baseband.
5.4 Inter Class Modulation Classification

ApEn renders high classification performance in intra class modulation recognition as seen in the case of CPM, PSK, QAM and FSK signals. It is interesting to see performance for intra class modulation signals in an AWGN environment.

5.4.1 Determination of ApEn parameters

As seen in the previous section, analysis for choosing the optimum value of \( c \) for intra class modulation for PSK, QAM and FSK signals was done. To be able to perform classification for inter class modulation, value of \( c \) for which perfect intra class classification performance was observed is selected. Table 5-5 gives us the value of \( c \) at \( N_s = 8 \).

<table>
<thead>
<tr>
<th></th>
<th>QAM</th>
<th>FSK</th>
<th>PSK</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.25 &lt; ( c ) &lt; 0.35</td>
<td>0.05 &lt; ( c ) &lt; 2.6</td>
<td>( c ) &lt; 0.5</td>
</tr>
<tr>
<td></td>
<td>( c = 0.3 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5.4.2 Classification Performance

With \( c = 0.3 \), \( m = 2 \), \( N_s = 8 \), \( N_{sym} = 1000 \) classification results for three multi signal sets \( \Delta_1 \), \( \Delta_2 \) and \( \Delta_3 \) are presented.

\[
\Delta_1 = \{4PSK, 16QAM, 8FSK\}
\]

\[
\Delta_2 = \{2FSK, 16QAM, 64QAM, 4PSK, 8PSK\}
\]

\[
\Delta_3 = \{2PSK, 4PSK, 8PSK, 4QAM, 16QAM, 64QAM, 2FSK, 4FSK\}
\]
Figure 5-9: $A_c$ for $\Delta_1$ and $\Delta_2$. ApEn on baseband and IF.

Figure 5-10: $A_c$ for $\Delta_3$. ApEn on baseband and IF.
Chapter 6

Conclusions

1. Classification accuracies as high as 90 % with a large number of signal classes (45) is observed at 15dB SNR in both the systems, β and γ. Even at low SNR in the range of 0-5dB, performance greater than 80 % is observed in the classification system. Correspondingly, when the pulse length of the signal is assumed to be known, high classification performance greater than 93 % is observed in the SNR range of 0 to 5dB.

2. The ability of ApEn to observe minor perturbations of the signal can be seen when pulse shapes are considered. Accuracies greater than 90 % at SNR greater than 15 dB are obtained while classifying signals with different pulse shapes (in this case, the total number of signals is 135 with rectangular, raised cosine with α = 0.3, and Gaussian pulse shape with BT = 0.3 considered in system β). Results are also shown when classifying between different pulse shape types, with a total of 315 signal classes in the unknown signal pool. Particularly, classification system β performs better than λ, in the identification of pulse shape parameters.

3. Analyses for two different systems, which are differently sampled, are presented. The classification system β lends its performance advantage at lower SNRs when classifying a large number of signals. At higher SNRs, the performance is comparable to the system λ. Thus, there is a tradeoff between the classification performance between a large number of signals and the computational complexity. In practical scenarios dealing with smaller number of unknown signals, excellent results (Ac > 96%) are obtained for SNR greater than 4 dB with system λ.

4. The utility of the instantaneous frequency of the signal being a robust domain in addition to the time domain for ApEn analysis has been shown. Since a simple central finite difference estimator [36], was used, which exhibits high variance for noisy signals, better results would be obtained at lower SNRs if the instantaneous frequency is estimated by methods which
show less variance. The choice of the parameters of ApEn is critical in order to obtain high classification accuracy and accordingly a prudent choice for superior classification performance is presented.
APPENDIX

A. Comparison of modulation classification techniques

$P_c$: Probability of correct classification.

<table>
<thead>
<tr>
<th>Feature Based Methods</th>
<th>Features Used and Rationale</th>
<th>Unknown Parameters</th>
<th>Modulations Identified</th>
<th>Parameters and Performance Measures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelet Transform of the Received Signal</td>
<td>- Symbol changes give rise to transients. - Variances of magnitude of Haar wavelet transform for PSK and FSK signals at a particular scale compared to a threshold.</td>
<td>- None. - Insensitive to synchronization at high SNR. - Accurate symbol period estimate required</td>
<td>2FSK, 4FSK, 8FSK, 2PSK, 4PSK, 8PSK</td>
<td>- $N_{sym} = 100$, $N_s = 126$ - For MPSK, $P_c = 99$ at 6dB SNR - For MFSK, $P_c = 99$ at 15dB SNR - Results for inter class classification shown only at 13 dB SNR - In all above cases, perfect symbol timing and synchronization assumed.</td>
</tr>
<tr>
<td>Wavelet transform</td>
<td>- HWT of normalized QAM signals same as PSK signals. A two branch classifier provided</td>
<td>- None</td>
<td>4QAM, QPSK, QFSK</td>
<td>- $N_{sym} = 50$, $N_s = 330$ - $P_c$ between the three signals = 99 at 15dB SNR - Between 0-10dB SNR, 20&lt;$P_c$&lt;70 - Extension of above conditions</td>
</tr>
<tr>
<td>Normalized Cumulants of received</td>
<td>- Fourth order cumulants of signal</td>
<td>- None - But at 12dB SNR, effects of $\theta$,</td>
<td>BPSK, 4PAM, 16QAM, 8PSK (four class problem)</td>
<td>- $N_{sym} = 100, 250$ and 500 considered at 10dB SNR</td>
</tr>
<tr>
<td>Fourth order Cyclic Cumulants of the received signal</td>
<td>16QAM, 64QAM, 8PSK, 8QAM</td>
<td>- SNR=3, 6, 9 dB considered for ( N_{sym}=1000 ) to 7000.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>--------------------------------------------------</td>
<td>-----------------------------</td>
<td>---------------------------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Previously only order 4 CCs considered</td>
<td>- None</td>
<td>- ( P_c=99 ) for 9dB for ( N_{sym}=7000 ) 20&lt;( P_c&lt;70 ) for 0 &lt; SNR &lt; 9dB</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Utility of order 6 CCs shown. CCs are warped so that features are projected in Euclidean</td>
<td>- 16QAM, 64QAM and V.29 (Considered pairwise)</td>
<td>- ( N_{sym}=8000 ),</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Cumulant function expressed as a Fourier series</td>
<td>- Co-channel interference assumed in AWGN environment</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Coefficients are cyclic cumulants.</td>
<td>BPSK, QPSK, 8PSK, 8QAM</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Exploits the selectivity property of cyclic cumulants</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Maximum of fourth order CCs for all delays used as features.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sixth order Cyclic Cumulants</th>
<th>None</th>
<th>- SNR=3, 6, 9 dB considered for ( N_{sym}=1000 ) to 7000.</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Previously only order 4 CCs considered</td>
<td>- None</td>
<td>- ( P_c=99 ) for 9dB for ( N_{sym}=7000 ) 20&lt;( P_c&lt;70 ) for 0 &lt; SNR &lt; 9dB</td>
</tr>
<tr>
<td>- Utility of order 6 CCs shown. CCs are warped so that features are projected in Euclidean</td>
<td>- 16QAM, 64QAM, QPSK</td>
<td>- ( N_{sym}=8000 ),</td>
</tr>
</tbody>
</table>

- Higher order cumulants of Gaussian process are zero
- Characterize the shape of the noisy signal

<table>
<thead>
<tr>
<th>T, ( \epsilon ), non-Gaussian noise, shown on ( P_c ).</th>
</tr>
</thead>
</table>

- \( P_c \) of four class problem= 99 at 12 dB SNR for \( N_{sym}=200 \)
- 60<\( P_c<80 \) for 0< SNR < 10dB
- For eight class problem, results shown only for 12dB SNR, \( N_{sym}=1000 \) \( P_c=99 \).
space.

(Three class problem) \( P_c = 99 \) at 9dB SNR

- In all the above estimation, cyclic frequencies and frequency offsets already known.

<table>
<thead>
<tr>
<th>Eight order Cyclic Cumulants</th>
<th>- Extension of above concept. Generally diminishing returns with increasing order. Tradeoff between mathematical, computational complexity and performance gain</th>
<th>- None</th>
<th>- Higher order modulations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>- QPSK and 16QAM</td>
<td></td>
<td>- Signals are over sampled. Over sampling factor not provided.</td>
</tr>
<tr>
<td></td>
<td>- 4ASK and 8ASK</td>
<td></td>
<td>- ( P_c = 99 ) at 5dB SNR for ( N_{sym} = 3000 )</td>
</tr>
<tr>
<td></td>
<td>- 16QAM and 64QAM</td>
<td></td>
<td>- ( P_c = 85 ) at 5dB SNR for ( N_{sym} = 30000 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- ( P_c = 99 ) at 10dB SNR for ( N_{sym} = 20000 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- ( P_c = 85 ) at 5dB SNR for ( N_{sym} = 20000 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- ( P_c = 99 ) at 10dB SNR for ( N_{sym} = 20000 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Likelihood Based Methods</th>
<th>Features Used and Rationale</th>
<th>Unknown Parameters</th>
<th>Modulations Identified</th>
<th>Parameters and Performance Measures</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALRT</td>
<td>- Maximum Likelihood function of received demodulated matched filter outputs formed Coherent and Non coherent cases considered</td>
<td>- None</td>
<td>- BPSK, QPSK, 8PSK, 16QAM, 32 QAM in both cases</td>
<td>- ( P_c = 99 ) at 10 dB Eb/No for ( N_{sym} = 256 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- Carrier Phase ( \theta )</td>
<td></td>
<td>- ( P_c = 99 ) at 13dB Eb/No for ( N_{sym} = 256 )</td>
</tr>
</tbody>
</table>
| Quasi-ALRT | - Approximations of infinite series of likelihood functions are calculated | - Carrier Phase $\theta$ | - BPSK and QPSK | - $P_c = 99$ at 4 dB SNR for $N_{sym} = 50$
- $P_c = 99$ at -1 dB SNR for $N_{sym} = 100$
- Perfect Synchronization and carrier frequency assumed |
| Quasi-ALRT | - Approximations of infinite series of likelihood functions are calculated | - Carrier Phase $\theta$ | - 16PSK vs 16QAM
- 16QAM vs V.29 | - $P_c = 99$ at 15dB SNR for $N_{sym} = 100$
- $P_c = 99$ at 26dB SNR for $N_{sym} = 100$
- Perfect Synchronization assumed |
| HLRT | - Formation of log likelihood functions
- Treat one set of unknown parameters as deterministic unknowns and find their maximum likelihood estimates
- Treat the rest of the parameters as random variables with known PDFs. | - Carrier Phase $\theta$ | - 16PSK vs 16QAM
- 16QAM vs V.29
- 16PSK vs V.29 | - $P_c = 99$ at 8 dB SNR for $N_{sym} = 100$
- $P_c = 99$ at 10dB SNR for $N_{sym} = 100$
- $P_c = 99$ at 7dB SNR for $N_{sym} = 100$
- Improvement in classifying non constant envelop modulations than Quasi-ALRT as seen from above results
- Perfect |
Quasi-HLRT - Instead of maximum likelihood (ML) estimates of unknown parameters, non-ML estimates used
- Rayleigh Fading Channel
- Channel Amplitude $\alpha$ and Carrier Phase $\theta$
- 16QAM, 32QAM, 64QAM
- $P_e=90$ at 10dB SNR for $N_{sym}=500$
- $P_e=93$ at 20dB SNR for $N_{sym}=500$

B. SNR Estimation by Signal Subspace Approach

The method described below has been given in [37]. In this approach, the autocorrelation matrix of the signal can help us project signal subspace efficiently. To derive the autocorrelation matrix, the signal model considered is as given below

$$r(t) = s(t) + z(t)$$  \hspace{1cm} (1)

Assume that the modulation signal $s(t)$ is transmitted through a complex Additive White Gaussian Noise channel $z(t)$. The signal is received as $r(t)$. Let $T_s$ be the sampling period. If the baseband signal $r(t)$ is sampled at $T_s$, then the signal can be represented as

$$r(kT_s) = s(kT_s) + z(kT_s)$$  \hspace{1cm} (2)

$$r(k) = s(k) + z(k)$$  \hspace{1cm} (3)

Let $R$, $S$ and $Z$ be the vectors comprised of $N$ samples from the received, transmitted and noise signal respectively, such that,

$$R = [r(1), r(2), r(3), \ldots, r(N)]$$
\[ S = [s(1), s(2), s(3), \ldots, s(N)] \]

\[ Z = [z(1), z(2), z(3), \ldots z(N)] \]

Assuming that the signal is uncorrelated with the noise, the autocorrelation of the signal can be given as

\[ E[RR^H] = E[SS^H] + E[ZZ^H] \] \hspace{1cm} (4)

where, \( R^H \) is the hermitian transpose of the vector \( R \). Using the Eigenvalue decomposition of the autocorrelation matrix we have,

\[ R_r = APA^H = AQA^H + \sigma_n^2 I \] \hspace{1cm} (5)

where \( P \) and \( Q \) are \( N \times N \) diagonal matrices containing the eigenvalues of \( R_s \) and \( R_r \), in the form of \( P=\text{diag}\{b_1, b_2, b_3, \ldots, b_N\} \) and \( Q=\text{diag}\{\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_p, 0, 0, 0, 0\} \), respectively. The eigenvalues are arranged in descending order as \( \lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \ldots \geq \lambda_p \). \( A \) is an orthogonal \( N \times N \) matrix comprising of the current eigenvectors. \( \sigma_n^2 I \) is the autocorrelation matrix of the noise. Thus the above equation can be re-written as

\[ R_r = APA^H = \text{Adiag}\{\lambda_1 + \sigma_n^2, \lambda_2 + \sigma_n^2, \ldots, \lambda_p \}
+ \sigma_n^2, \ldots, \sigma_n^2, \sigma_n^2\}A^H \] \hspace{1cm} (6)

where \( \lambda_i \) is the power along \( i^{th} \) eigenvector. The equation shows us that the signal subspace is defined by only the first \( p \) eigenvectors and the rest \( N-p \) eigenvectors correspond to the noise subspace. Thus, if \( P_S \) represents the total power in the signal subspace and \( P_N \) is the total power in the noise subspace the SNR can be accurately estimated as

\[ SNR_{\text{estimated}} = 10\log \left( \frac{P_S}{P_N} \right) \]

However, the autocorrelation matrix of the signal needs to be estimated in order to estimate the SNR. If we use \( K \) observation vectors, then the matrix can be estimated as
\[ R_{est(r)} = \frac{1}{K} \sum_{k=1}^{K} r(k)r^H(k) \]  

(7)

But the power in the signal subspace can only be estimated by finding the correct value of \( p \), the signal space dimension. The value of \( p \) can be found by information theoretic criterion which were described in (add references). Criterion such as Minimum Description Length (MDL), Akaike Information Criteria (AIC) and Combined Information Criteria (CIC) have been used to calculate \( p \). We have used the MDL criterion to determine \( p \).

The MDL can be given as,

\[ MDL(k) = -\log \left( \frac{\prod_{i=k+1}^{N} b_i^{1/(N-k)}}{1/N - \sum_{i=k+1}^{N} b_i} \right)^{(N-k)K} + \frac{1}{2} k(2N-k)\log K \]  

(8)

The signal subspace dimension is the value of \( p \) which evaluates to the minimum of the argument of equation no. 8,

\[ p = \arg\{\min(MDL(k))\} \]  

(9)

Thus, the SNR can be estimated using the following steps:

1. The autocorrelation matrix can be computed from the equation no.
2. Using the Eigenvalue Decomposition of the autocorrelation matrix, the eigenvalues are arranged in descending order.
3. The signal subspace dimension \( p \) is computed according to equation no. 9.
4. Then form the above steps, the noise power is calculated as

\[ \sigma_n^2 = \frac{1}{N-p} \sum_{i=p+1}^{N} b_i \]  

(10)
5. The signal power can then be estimated as

\[ P_s = \sum_{i=1}^{p} (b_i - \sigma_n^2) \]  \hspace{1cm} (11)

6. Using the above equations, the SNR can be estimated as

\[ SNR_{estimated} = 10 \log \left( \frac{P_s}{N\sigma_n^2} \right) \]

Since this is a computationally expensive operation, a tradeoff is seen between the size of the autocorrelation matrix \(N\), number of observation vectors \(K\) and the mean squared error. In my simulation, \(K=2500\) and \(N=50\).
REFERENCES


