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INTRODUCTION TO DATA ENVELOPMENT ANALYSIS AND A CASE STUDY IN HEALTH CARE PROVIDERS

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ABSTRACT

Data Envelopment Analysis is a methodology which is used to determine relative efficiencies between decision making units (DMU). It can be used in various industries such as hospitals, banks and schools to name a few. The objective of this thesis is to provide a comparative analysis of different DEA models using an actual application to the health care industry. A detailed literature review of DEA models is provided which covers research on weight restriction methods, models incorporating multiple criteria with DEA, super efficiency in DEA and ranking methods in DEA.

Four important models in DEA are discussed in detail. They include the CVDEA model, MCDM DEA model, AHP DEA model and the Super Efficiency model. DEA is a very useful tool but it has its disadvantages including the existence of extreme outliers in a set of efficient DMUs and weight restriction capabilities of certain models. The models chosen in this research focuses on methods to eliminate the drawbacks of earlier DEA models. Multi criteria decision making (MCDM) and the Analytical Hierarchy Processes (AHP) are two of the methods which when combined with DEA provide added functionality. The concept of Super Efficiency in DEA is also used in one of the chosen models.

A case study has been conducted to determine the efficiency of health care providers. The four DEA models are evaluated for 44 health care providers with real data and meaningful results are obtained. The results for the efficiency classification provided by

the models are similar but the four models have provided some additional information unique to their approaches. The AHP DEA model and the Super Efficiency model are judged to be better models on the basis of the practical insights they provided to the health care provider.

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CHAPTER 1

INTRODUCTION

1.1 Introduction

Data Envelopment Analysis is a methodology which is used to determine relative efficiencies between decision making units (DMUs). It was first developed by Charnes, Cooper and Rhodes (1978). A DMU can be any entity like a hospital, school or bank. Among a group of DMUs, DEA helps to distinguish between the efficient and the inefficient DMUs.

DEA uses the mathematical method of linear programming. It uses a non-parametric method or a method which does not need a production function to determine efficiency. DEA utilizes the inputs and outputs of the DMUs to determine its output: input ratio. The efficiency of the DMUs are determined by their place on the efficient frontier which is a graphical representation of all the DMUs with their respective inputs and outputs. The slope of the line joining the DMU to the point of origin will determine its output: input ratio. The highest slope formed by a DMU is therefore called the efficient frontier. Hence all the DMUs which fall on this line are deemed efficient and the ones below the line, inefficient. The further a DMU is located from the efficient frontier, the more inefficient it is. In fact the term ‘Envelopment’ in DEA comes due to the property of the efficient frontier to ‘envelop’ all the efficient and inefficient points.

This can be explained by determining the weights for the output: input ratios. An inefficient DMU will have a lesser weight on the ratio than an efficient one. Hence DEA uses the weighted sum of the outputs to the weighted sum of the inputs to determine the performance between DMUs.

The linear program used will have the weights as the decision variables and they are determined in a way such that it gives each DMU the highest efficiency score. The number of linear programs which will need to be run will depend on the number of DMUs because each DMU is compared to the rest of the DMU in one formulation to see how efficient it is compared to the others. The weights derived from this process will be the DMUs optimal weights. The linear program can be input or output oriented. An input oriented model will have an objective function which will generate a value of 1.0 if a DMU is efficient. The closer this value is to 1.0 the more efficient the DMU. An output oriented linear program will have its opposite logic and hence the lower the value of the objective functions, the more efficient the DMU will be.

This is related to input reduction and output augmentation. The desired outcome for a DMU will be a way to reduce its inputs to get more output. For example a hospital may want to reduce its inputs like nurses, doctors and still get more patient hours. This is the methodology used by DEA.

DEA can be compared to statistical regression analysis as it has similar objectives. Regression gives the “average” performance of a DMU. Like the efficient frontier regression analysis uses the regression line. All units above it are deemed efficient and below it are deemed inefficient. The magnitude of inefficiency is determined from the distance to this line. DEA is similar but it compares all the DMUs to the most efficient DMU in its group. Hence an advantage comes out of this method. The most efficient DMU can serve as a “benchmark” for improvements. Regression analysis does not exclude the efficient from the inefficient when providing suggestions for improvement. DEA measures performances relative to all the other DMUs.

The efficiency derived in DEA is in a sense “technical efficiency” compared to “economic efficiency”. This is because its objective does not use its inputs and outputs in a production function. Hence the objective is not cost reduction by a combination of inputs and outputs and unit cost saved by a set of inputs is not the focus. DEA actually identifies target for achievement for a DMU compared to the others in the reference set. In other words it calculates a method to eliminate “waste”. Hence the term “technical efficiency” has been given to it.

DEA has many advantages why it has been a popular method for evaluating efficiencies and they have been tabulated below

Table 1-1:

Advantages of Data Envelopment Analysis
1. It has the capability of handling multiple inputs and outputs 2. Inputs and outputs can have different units of measure. 3. It is a non parametric method which does not need a functional form for computing efficiency 4. It can calculate the sources and the extent of inefficiency in inputs and outputs 5. It can use benchmarking techniques to use the efficient units as a benchmark to evaluate inefficient units 6. It can be used in the measurement of productivity in addition to efficiency analysis. 7. It can be used as an "what-if" analysis tool to include certain inputs and exclude outputs for a DMU

But DEA has its share of disadvantages. Most of them have been addressed in past literature and many researchers have formulated solutions to these problems. The main disadvantages are tabulated in Table 1-2.

Table 1-2:

Limitations of Data Envelopment Analysis
1. Extensive linear program formulations can make the analysis of all the DMUs lengthy and tedious 2. Only the relative efficiency is calculated and the absolute or maximal efficiency is not addressed 3. The possibility of extreme outliers DMUs to be termed efficient exists and hence weights have to be derived carefully. Hence discrimination between DMUs is poor. 4. It is difficult to perform statistical hypothesis testing as it is a non-parametric method

DEA has the ability to incorporate multiple inputs and outputs but it may weaken the formulation. In Meng et al [2008], a model which can group inputs or outputs of same priority to use in DEA is explained. Hierarchical structures of these inputs and outputs were incorporated in their DEA model. Grouping of inputs and outputs which have same characteristics for computational benefits was also shown in Kao [2008] with a linear two level DEA model. Also sometimes, some of the inputs of a DMU cannot be controlled or changed to increase performance and productivity. Banker and Morey [1986a] and Fried and Lovell [1996] developed a one-stage and a three-stage model respectively to address this issue. Muniz [2001] made some changes to the three-stage model. All three models are effective tools to incorporate DEA with uncontrollable units. In fact Muniz mentioned that to get better results it is better to check the consistency of the results using both models.

Initially one of the disadvantages of DEA was the inability to influence weights with a decision maker's preference. This was overcome by many methods in recent literature. One such example is the Value Efficiency Analysis method by Halme et al [1999]. Their model derives a 'Most Preferred Solution' (MPS) by using an interactive multi objective linear program. The MPS incorporates the decision maker's judgment and then the inefficient units are compared to this solution.

1.2 Classical DEA Models:

In this section the classical DEA models which led the foundation for future research is explained in detail.

The first model developed by Charnes, Cooper and Rhodes [1978] is the CCR model. It incorporates a weighted output to input ratio for each DMU to determine its relative efficiency. The basic model is setup as follows:

Let x_i and y_r represent the inputs and outputs where i= 1,2,…I and r= 1,2,…R represent specific inputs and outputs.

Let u and v be the output and inputs weights respectively.

Efficiency is calculated using the ratio given below:

$$Efficiency = \frac{Virtual\_Output}{Virtual\_Input} = \frac{\sum_{r=1}^{R} u_r y_r}{\sum_{i=1}^{I} v_i x_i}$$

The decision variables are the weights for this equation and they are derived by solving the linear program for each DMU which needs to be analyzed. Assume there are j=1, 2,…, n DMUs.

For $DMU_{o,}$ which is the test DMU, the linear program is as follows:

$$\max E_o = \frac{\sum_{r=1}^{R} u_{ro} y_{ro}}{\sum_{i=1}^{I} v_{io} x_{io}}$$

Subject to:

$$0 \leq \frac{\sum_{r=1}^{R} u_{ro} y_{ro}}{\sum_{i=1}^{I} v_{io} x_{io}} \leq 1; j = 1, 2, ..., n \qquad (1.1)$$

$$v_{io,} u_{ro} \geq 0; i = 1, 2, ..., I; r = 1, 2, ..., R$$

Where v_{ro} and u_{io} are the decision variables and E_o is the efficiency score for DMU_o

y_{ro} is the r th output of DMU_o

u_{ro} is the r th output weight for DMU_o

x_{io} is the i th input for DMU_o

v_{io} is the weight of i th input for DMU_o

y_{rj} and x_{ij} are the r th output and the i th input, respectively for DMU_j, j=1,2,…,n

The objective is a fractional linear objective and hence nonlinear. To convert it to a linear program, the model is reformulated as follows:

Objective Function:

$$\max z = \sum_{r=1}^{R} u_{ro} y_{ro}$$

Subject To:

$$\sum_{i=1}^{I} v_{io} x_{io} = 1 \qquad (1.2)$$

$$\sum_{r=1}^{R} u_{ro} y_{rj} - \sum_{i=1}^{I} v_{io} x_{ij} \leq 0; j = 1, 2, ..., n$$

$$u_{ro,} v_{io} \geq 0; i = 1, 2, ...I; r = 1, 2, ..., R$$

The Banker, Charnes and Cooper [1984] model is an extension of the CCR model. It is essentially the dual of the original DEA model.

Objective Function:

$$\min \theta_o$$

Subject to:

$$\sum_{j=1}^{n} \lambda_{ij} x_{ij} \leq \theta_o x_{io}; i = 1, 2, ..., I$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}; r = 1, 2, ..., R \qquad (1.3)$$

$\lambda_j \geq 0; j = 1, 2, ... n$

θ_o unrestricted

Where

θ_o = the efficiency score of the DMU_o

λ_j = weight of the j th DMU (j=1,2,…,n) with respect to DMU_o

y_{rj} = output measure of the r th output (r=1,2,…,R) of the j th DMU (j=1,2,…,n)

x_{io} =input measure of the i th input (i=1,2,…,I) of DMU_o

x_{ij} = input measure of the i th input (i=1,2,…,I) for the j th DMU (j=1,2,…,n)

The BCC has fewer decision variables compared to the CCR model. In the BCC models the decision variables are only λ_j, j=1,…,n. That is, it provides a unique set of weights for each DMU compared to the CCR model, where each DMU has weights for its inputs and outputs. In practical cases, the numbers of DMUs are usually more than the number of inputs and outputs and hence the BCC is more efficient.

The concept of Returns to Scale (RTS) comes into light when comparing these two basic models. The CCR model is a Constant Returns to Scale (CRS) whereas the BCC model incorporates a Variable Returns to Scale (VRS) method. This simply means that the production frontier or efficient frontier created by the DMU make a straight line with CRS employed. Under VRS conditions seen in the BCC model have piecewise linear and concave functions. That is, it initially increases, remains constant and then decreases.

Another interesting difference between the two models is based on the concept of "translation invariance" put forth by Cooper [2000]. This is a phenomenon which occurs in data which makes it shift laterally so that negative data will become positive to make sure all data is non negative. This is a common constraint in DEA model formulations. According to Cooper, the CCR model is non-translation invariant but the BCC model to some extent is shown to be translation invariant.

Though the CCR and the BCC models were the first models to be developed in DEA, a lot of research has been done thereafter which made changes to the basic models. Although all models cannot be mentioned here , there are two more models worth mentioning as they have been used widely in research.

The first one is the Additive DEA model put forth by Charnes, Cooper, Golany and Seiford [1985]. The main feature of this model is that, unlike the classical DEA model, it does not distinguish between input oriented and output oriented models it combines both these orientations into one objective function. The advantage of this model is that it provides measures of slack and surplus associated with each input and output. Additive DEA models can be of different forms and one such form is given below:

$$\begin{aligned}
&\max z = es^{+} + es^{-} \\
&X\lambda + s^{-} = x_o \\
&Y\lambda - s^{+} = y_o \qquad (1.4) \\
&e\lambda = 1 \\
&\lambda \geq 0, s^{-} \geq 0, s^{+} \geq 0
\end{aligned}$$

Where

s^+ = the surplus in the output of a DMU_o

s^- = the slack in the input of a DMU_o

X = the input vector for DMU_o

Y = the output vector for DMU_o

λ = the weights associated with DMU_o

e = the column vector of ones

Hence the values of s^+ and s^- can be useful information in performance enhancement of the DMUs. The Additive model is also translation invariant. This model has been combined with the Analytical Hierarchy Process to incorporate value judgments. This is discussed more in detail in Chapter 3.

1.3 Thesis Objective and Organization:

The objective of this thesis is to provide a comparative analysis of different DEA models using an actual case study in the health care area. The literature review on DEA models covers research work which helps in solving common DEA problems. It also throws light to the areas where DEA has been unified with other methodologies like Multi Criteria Decision Making (MCDM) and Analytical Hierarchy Process (AHP). The development of the concept of Super Efficiency in DEA is also elaborated. A case study using data from a health care provider has been used to illustrate the ability of the models chosen in this thesis to perform with real world data.

The rest of thesis is organized as follows: Chapter 2 deals with an extensive literature review. Chapter 3 introduces the DEA models which have been chosen for detailed investigation. A small data set is used to illustrate the models and show their functionality. Chapter 4 will cover the case study using real world data from a health care provider to see how the models introduced in Chapter 3 produce results. Finally, Chapter 5 is a conclusion on the findings from Chapter 3 and 5 describing the advantage and disadvantages of the various DEA models.

CHAPTER 2

LITERATURE REVIEW

The concept of DEA was first introduced by Charnes, Cooper and Rhodes [1978] as explained in detail in the previous chapter. Their model proposed a way to measure performance of entities (e.g. hospitals, schools) called as Decision Making Units (DMU) which usually has multiple inputs and outputs and performance was based on relative efficiency. There have been many models proposed which are slight variations from the Charnes, Cooper and Rhodes (CCR) model. However classical DEA models have some disadvantages and recent literature tries to solve the problems which include (i) highly flexible weight restrictions (ii) poor discrimination among DMUs. The result of these problems can lead to rendering inefficient units as efficient and defeat the purpose of calculating the efficiency of units. The following literature review gives an insight into previous literature aimed at solving these problems and enhancing the classical DEA models. The role of Multi Criteria Decision Making (MCDM) in DEA is elaborated and an extension of it, the Analytical Hierarchy Process applications with DEA has also been researched. The concept of Super efficiency and its connection with DEA in previous literature has been researched. Finally a brief review of the ranking methods that use DEA and super efficiency techniques has been included.

2.1 Weight restriction methods in DEA:

One of the problems faced by traditional DEA methods is the dispersion of weights. This concept was first introduced by Dyson and Thanassoulis[1988]. Lower bounds on weight restrictions were constructed which can be specifically applied to a single input case. Here the author explains that the output weights are seen as resources needed per unit of the respective output.

It has been further explained in Roll, Cook and Golany[1991]. They also used a single input multiple output model. Pedraja-Chaparro et al [1997] showed that this model had problems as the DEA Linear Programming model would lead to infeasible solutions. Their model involves the process of first running the model without weight constraints and then finding the feasible range and analyzing the results.

In Jahanshahloo et al [2003] the possibility of infeasibility because of adding multiple weight restrictions is eliminated. They achieve this by using goal programming and Big M techniques. If the alteration of the weights is small, infeasibility can be avoided. Another method, which made sure the weights are consistent with DMU objectives, is the Cone Ratio method by Charnes, Cooper, Huang and Sun [1990]. It is a modification of the traditional CCR model using the mathematical concepts of polyhedral cones for the virtual multiplier or weights. The polyhedral cones are utilized to mathematically alter the input and output values. It provides a constraint cone which the user can change to get a desired pattern of input usage and output production to get the desired efficiency. This is also useful to incorporate expert opinion. It provides better efficiency outputs compared to the traditional CCR model. The disadvantage is that it needs to be converted to the cone ratio form and back to its original form for computational purposes. The advantages of this model are that it can be used on software in which weights cannot be incorporated.

The concept of using assurance regions to increase weight restrictions was introduced by Thompson, Singleton, Thrall and Smith [1986]. The decision maker decides on the values α and β which will restrict the values of the input and output weights u and v .The concept involves increasing the assurance region or the region where the DMU will be efficient till the decision maker is satisfied with the efficiency levels generated. The restrictions was put in a linear homogenous equation as follows

$$\alpha_r u_1 \leq u_r \leq \beta_r u_1$$

$$\alpha_r v_i \leq v_i \leq \beta_i v_i$$

This gives rise to AR1 or Assurance Region 1 .The restrictions are dependent on the values of input and output weight and like the Cone ratio method it will lead to at least one efficient DMU. A modification was made which lead to the AR 11 where the ratio of the output to the input was restricted. The difference between the Cone ratio method and AR method was that latter may lead to an infeasible solution and hence having no DMU as efficient was possible.

Another method which uses assurance region concepts is given in Thanassoulis and Allen [1998]. Their method is another weight restriction method which uses 'unobserved DMUs' which are used to incorporate value judgments in DEA.

All these methods involve knowing a priori information about the weights. This might be a disadvantage as errors might be introduced if the weights had some human error or were inconsistent. Hence the following methods incorporate weight restrictions without it.

Li and Reeves [1998] formulated a multi criteria model using multi objective linear programming. It uses three objective functions based on deviational variables or a measure of inefficiency. They are (i) minimizing the deviational variable (ii) minimizing the maximum of the deviational variables (iii) minimizing the sum of the deviational variables. The use of the minmax and minsum criteria provides greater restrictions and renders fewer efficient DMUs.

Cross evaluation (Silkman, 1986) is another method of increasing discrimination among DMUs. It involves a method of peer evaluation among different efficient DMUs than the traditional DEA evaluation of a single DMU. This can be done by two different types of formulations. They are the 'aggressive' and 'benevolent' formulations. The aggressive formulation uses a multi objective model. The first objective is the efficiency calculated by a classical DEA model like the CCR model. The second objective is used to minimize the cross efficiency of all other DMUs other than the DMU used in the classical DEA model. The aim is to get a weighting scheme which is optimal in the classical model and also involves all other DMUs. The benevolent formulation is the same except it minimizes the cross efficiency of the DMUs.

A cross efficiency matrix is formed with the diagonal elements being the optimal value using classical DEA methods and rest of the elements are the cross efficiency for the respective elements. For example, for a DMU k, E_{ks} is defined as the efficiency of DMU s calculated using the weighted scheme of DMU k and E_{kk} is its own efficiency score. Using this value for e_k which is the mean cross efficiency and M_k, which is the greatest difference between the standard cross efficiency and mean cross efficiency, can be calculated.

$$e_k = \frac{1}{n}\sum_{\forall s} E_{sk}$$

$$M_k = (\frac{E_{kk} - e_k}{e_k})$$

These can be used as measures to distinguish between efficient DMUs. The advantage is that this method does not need prior information of weights but it is a complex method compared to others.

The model given by Bal,Orcku and Celebioglu [2007], also called the CVDEA model, uses a different approach to solve the problem of discrimination among efficient DMUs. They include coefficient of variation in the objective function of the basic DEA model for

both the inputs and the output weights. It compares the relative dispersion between two set of data. It includes the CV variable in the objective function of the CCR model.
It has the same purpose as the Li and Reeves model but it is a greatly simplified version using a single objective instead of multi objective and it does not need a priori information from a decision maker.

2.2 Multi criteria methods in DEA:

Multi criteria decision making methods and DEA have been used together in many situations for performance measurement. The first method integrating both concepts was put forth by Golany [1988] where he uses an interactive multi linear programming model. This model helps DEA to choose the effective DMU rather than just the efficient DMU, the difference being the former will be able to achieve its objective more closely. Given a set of input and output vectors from previous experience for a DMU it aims at arriving at an efficient output level which a DMU can achieve for a given input level. The further a DMU is away from its DEA production function the more inefficient it is deemed. A series of sequential linear programs are solved to get a set of efficient points. Combining the use of MCDM and DEA is also seen in Doyle and Green [1993] where they extended the work of Stewart [1992] by using a multi objective model with the DEA output to input ratio in it. The objective is to maximize D_{kk} which is for DMU k, the ratio of output weights to input weights and minimize D_{kj} which is its cross evaluation with respect to another DMU j.

A visual approach was given by Belton and Vickers [1993] where they used DEA and MCDM to represent the efficient DMUs on the efficient frontier. While plotting the aggregate input versus the output measure they found that the efficient DMU all lie on the northwest efficient frontier. But this visual approach was found to be useful only when applied to a small set of units. The use of a reference point model using multi objective linear programming (MOLP) along with the CCR model in DEA was shown in Joro, Korhonen and Wallenius[1998]. Here the reference point model, which is the

MOLP discussed in the paper, is shown to be structurally similar to the CCR output oriented model. Hence they conclude that formulating a DEA model using MOLP gives more flexibility by finding a way to make inefficient DMUs efficient.

A Multi Objective Linear Fractional Program (MOLFP) was developed by Kornbluth[1991] which uses the cone ratio method of restricting weights. Here they prove that using MOLFP to solve DEA models gives more information than the standard DEA model. They argue that the standard DEA model gives the efficiency of the DMU under consideration but does not give direct efficiency evaluations of other DMUs for a set of optimal weights. The MOLFP model gives this evaluation directly.

Bouyssou [1999] shows the common violations which might occur when integrating MCDM and DEA techniques. Using three DEA based models he shows that using MCDM along with DEA may not give the right results as some normative properties may be violated. Stewart [1996] contrasts the concept of relative efficiency in DEA with that of Pareto optimality in MCDM and discusses some issues in applying interactive MCDM techniques for solving the weight restriction problem in DEA. The paper focuses on how MCDM can solve the problems in DEA model particularly when it comes to setting bounds on weights. It is shown that MCDM helps in setting more realistic judgment when it comes to assigning weights.

2.3 AHP and DEA:

The Analytical Heirarchy Process (AHP) has also been combined with DEA in past research. The AHP is a MCDM tool used for selecting and ranking of alternatives introduced by Saaty [1980]. It compares the various alternatives available for making a decision with respect to conflicting multiple criteria and ranks the alternatives so that the best alternative can be identified easily.

The role of AHP in DEA problems have been studied in different ways. Primarily it has been used to derive weights for use in the DEA models. Qualitative factors to be used in DEA model were quantified in Shang and Sueyoshi [1995]. Their goal was to find the best Flexible Manufacturing System and they used AHP to analyze only tangible factors such as long term goals and strategies and analyze monetary goals using a simulation model. The efficiency of the system is then calculated by combining the AHP and simulation results by using a cross efficiency method to determine the most efficient system. The AHP helped the model by providing upper and lower bounds for the weights. The drawback of this model is that it does not provide any preference structure for the decision maker in a linear relationship.

In Seiford and Zhu [1998], the AHP and DEA methods are integrated through the assurance region concept explained earlier. Their aim was to find ways to improve the industrial productivity in China by analyzing past data. The weights are incorporated in the DEA formulation in order to apply preference information of the decision maker or expert. They used a scale of 1 to 9 to give priorities to the input and output variations and formed a pairwise comparison matrix in order to get the final weights.

Even though AHP and DEA are integrated for efficiency analysis there are some inherent problems. A variation in the use of AHP and DEA to address the problems is explained in Sinuany-Stern, Mehrez and Hadad [2000]. They prove that their model, the AHP/DEA model does uses AHP in a more quantifiable manner than using preference information and also the ranking is more accurate version when compared to the traditional DEA models. They conduct a pairwise comparison between two DMUs at a time providing the cross evaluation for all the DMUS. Then AHP uses this matrix to rank them by comparing the values in the matrix. But the problem of rank reversal exists in this method. When one of the alternatives is removed the ranking of all the alternatives could change.

The Voting AHP method by Liu and Hai [2005] is different from the previous model as it removes the effort of making pairwise comparisons for providing weights to criteria. In the paper, a six step procedure is provided which will enable ranking using AHP as well as DEA methods.

Wang, Liu and Elhag [2008] propose a more efficient way to combine AHP and DEA methods. They use AHP to determine weights for the decision criteria and decision criteria for the problem. Then they use assessment levels provided by expert opinion for each criterion and solve the model they proposed for each criterion to get the local weights. These local weights are then aggregated to get the ranking. The advantages of this mentioned when compared to the voting AHP method is that it has lesser computation and lesser pairwise comparisons.

Recent research includes the paper by Ramanathan [2006] in which he develops a model called the DEAHP method. They eliminate the rank reversal problem that happens when an irrelevant alternative is removed. It uses AHP to make judgment matrices which give information on alternatives. This matrix is then used by DEA to get the local weights. The final weights are derived after aggregation for the different criteria. Hence the combined logic used here led to the DEAHP method.

There has been criticism of the DEAHP method in Wang and Chin [2008]. The authors believe that DEAHP method provides erroneous results when inconsistent matrices arise in the pairwise comparisons. They propose two DEA models which overcome this problem and prove mathematically the flaws of DEAHP. They were able to derive weights for inconsistent matrices and used the Simple Additive Weighting method (SAW) for determining the local weights. They extend their model to a group AHP scenario too where many pairwise comparison matrices are involved.

2.4 Super efficiency:

Super efficiency is a measure used in DEA as a sensitivity analysis tool or a ranking tool for DMUs which are deemed efficient.

The concept of super efficiency in DEA started when the idea of discriminating between efficient DMUs was brought to light by Andersen and Petersen [1993]. Here they put forth a ranking method for efficient DMUs. They compare the DMU to be evaluated against the combinations of all the other DMUs. In essence it is a measure of the radial distance of that DMU from the efficient frontier. It is calculating how the inputs can be increased proportionally but still preserving the efficiency of the DMU under consideration. It is very similar to the BCC model explained earlier. In the BCC model, efficiency is given by an index value equal to one but in Andersen and Petersen's model (AP model) the value is greater than one. In the AP model they determine the technical efficiency of all the efficient units and then determine the super efficiency of one unit compared to the others. They do this by excluding the unit under study from the reference set. It has its own disadvantages. One of them is that in the process of ranking the efficient units, it may provide an efficiency score to a unit which may not be that useful in other measures and methods. In other words it may be ranked too high than what is necessary. The author suggests that this is due to the lack of prior knowledge of the weights used in the model. Hence this model essentially does not distinguish between economically efficient units and technically efficient units. The model discussed earlier which solves this problem is the cone ratio method and the assurance region methods for determining efficiency.

Seiford and Zhu [1998a] proposed a relationship between infeasibility and efficiency classification. Essentially the paper concludes that the CCR efficiency of a DMU is stable to changes. Seiford and Zhu [1998b] showed that this extended to other DEA models like BCC model and the additive model. In Seiford and Zhu [1999] he uses a worst case scenario for a DMU when its efficiency is deteriorating when all other DMUs efficiency increase. The paper conducts a sensitivity analysis of the scenario and provides the necessary and sufficient conditions for preserving a DMUs efficiency when inputs and outputs of all other DMUs are changed simultaneously.

Super efficiency has been used in models which use a different base logic other than the CCR model. The CCR model uses the Farrell approach of proportional improvements between DMUs by using relative efficiency concepts. Bogetoft and Hougaard[2002]use the proportional improvement method introduced by Bogetoft[1999] .This method lets the production outputs determine which direction it wants to improve. In other words it improves depending on actual possibilities when compared to the Farrel method [1957] where the changes are structured proportionally. Using this theory they developed a super efficiency measure which they call the potential slack super efficiency measure. It differs from the original super efficiency measures as they can be invariant to linear and non-linear transformations made.

Many papers have focused on deriving super efficiency measures but the problem of infeasibility occurs when the problems uses a different returns to scale method other than the constant returns to scale (CRS) method. Xue and Harker[2002] show that ranking of

DMUs was still possible regardless of the infeasibility problem encountered. They obtained a ranking from the subset of the efficient DMUs called the strongly efficient DMU (E) and super efficient DMU (SE). A further subset was derived from this to get the strongly super efficient DMUs (SSE).

2.5 Ranking in DEA:

In Adler et al [2002], the authors have listed and explained six different types of ranking methods which include the super efficiency method. He explains in detail six specific methods in previous literature for ranking in DEA and gives examples of which industries use these methods.

The six methods include cross efficiency evaluation, super efficiency concepts, benchmarking techniques, multivariate statistics tools, ranking of inefficient units and ranking using DEA and MCDM. Cross efficiency and super efficiency have been described earlier.

Ranking using two stage benchmarking was explained in Torgesen et al [1996]. The model uses the additive model to evaluate the set of efficient DMUs. It then uses the concept aggregation of weights to determine a benchmark value. This value essentially is the fraction of the total aggregated possible increase in an output for the DMU. However many units may get the same rank using this method. Sinuany-Stern et al [1994] also use a similar two stage technique of finding the efficient units and comparing the inefficient units to the efficient.

The multivariate statistical methods include the method put forth by Friedman et al [1997] called Canonical Correlation analysis. It is an extension of regression analysis but it uses multiple inputs and multiple outputs unlike regression. Linear discriminate analysis method is another multivariate statistical technique by Sinuany-Stern et.al [1994] where they use a one dimensional linear equation to rank units according to the value D_j determined by the equation. Its disadvantage is that it can be used only on non-negative weights. Discriminant analysis using ratios was developed by Sinuany-Stern et al (1998) is the same as the previous method but it does not use a linear equation but a ratio of the linear combination of inputs to the outputs. The infeasibility problem which arises in the previous two methods is resolved in this. However the author mentions that the optimal solution reached using this model using non-linear search optimization techniques may not be globally optimal.

CHAPTER 3

MODELING AND ANALYSIS

To explain the versatility of DEA, four models have been chosen for this chapter to be explained in detail. The first model uses the measure of Coefficient of Variance (CV) to improve discrimination among DMUs. In the second model, DEA and MCDM are combined together to get the MCDEA model. The use of AHP along with DEA is chosen as the third model. This model helps incorporate decision maker's preferences over inputs and outputs. Finally in the fourth model, the concept of super efficiency is discussed using information provided by the DMUs and the DMUs are ranked according to their super efficiency.

All the four DEA models use the sample data given in Table 3-1 for illustration.

Table 3-1: Sample Data

DMU	Output 1	Output 2	Output 2	Input 1	Input 2	Input 3
A	86	75	71	0.06	260	11.3
B	82	72	67	0.05	320	10.5
C	81	79	80	0.08	340	12
D	81	73	69	0.06	460	13.1

3.1 CVDEA model [Hasan, B., Hasan, O. & Salih, C., 2008] :

One of the major problems in DEA analysis is its discriminating power among different DMUs. This is caused when the relative efficiencies of the different DMUs are calculated as the ratio of the weighted sum of outputs to the weighted sum of inputs. Here there is a probability for unrealistic weights to be given to the inputs and the outputs. Sometimes impractical weights like giving a higher weight to a less important input/ouptut and vice versa can cause problems along with the case of zero weights. Many methods have been developed to overcome this problem as described earlier in the literature review. One such method is by using the concept of minimizing the coefficient of variation (CV) put forth by Hasan et al (2008).

The basic model used in this method is the CCR model put forth by Charnes et al (1978). The relative efficiencies of DMUs are calculated by using their weighted output to weighted input ratios for each DMU. This is given as a fractional programming problem as shown below:

Let

w_0 = efficiency of DMU_o

u_r = weight of the outputs; r=1,2…s

v_i = weight of the inputs; i=1,2,…m

y_{ro} = is the r th output for DMU_o

x_{io} = is the i th input for DMU_o

y_{rj} = is the r th output for DMU_j; j= 1,2,..,n

x_{ij} = is the i th input for DMU_j; j= 1,2,..,n

$$w_0 = \max \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}}$$

$$s.t \quad \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1,\ j = 1,2,....n, \qquad (3.1)$$

$$u_r \ge 0, r = 1,2,....s,$$

$$v_i \ge 0, i = 1,2,....m$$

The fractional programming model is converted to a linear program in the CCR model by adding a constraint such that the weighted sum of inputs for the DMU under consideration is equal to 1. Hence the model will change as follows:

$$wo = \max \sum_{r=1}^{s} u_r y_{ro}$$

$$s.t \quad \sum_{i=1}^{m} v_i x_{io} = 1$$

$$\sum_{i=1}^{s} u_r y_{rj-} \sum_{i=1}^{m} v_i x_{ij} \le 0,\ j = 1,2....,n \qquad (3.2)$$

$$u_r \ge 0, r = 1,2,....s$$

$$v_i \ge 0, i = 1,2,....m$$

The optimal value of this model will give the relative efficiency of DMU_o, which is the DMU under consideration. It is efficient only if wo =1.

The coefficient of variation (CV) will help minimize the changes in the input and output weights. The definition of CV is the ratio of the sample standard deviation to the sample mean. It is given by the basic formula:

$$CV = \frac{\text{standard deviation}}{\text{mean}}$$

The standard deviation in the term is the sample standard deviation and in this case it will be the measure the variability of the weights to their average.

For the output weights u_r , CV is given by :

$$CV_o = \sqrt{\frac{\sum_{r=1}^{s}(u_r - \bar{u})^2 / s-1}{\bar{u}}}$$

Similarly for inputs weights v_i it is given by the following formula:

$$CV_i = \sqrt{\frac{\sum_{i=1}^{m}(v_i - \bar{v})^2 / m-1}{\bar{v}}}$$

In this model the objective is to minimize the coefficient of variation of the input and output weights. Hence in the objective function our aim is to incorporate this as a minimization function with the existing objective function. Hence the CCR model changes to the following non-linear optimization model, which the authors call the

CVDEA model:

$$wo = \max \sum_{r=1}^{s} u_r y_{ro} - \sqrt{\frac{\sum_{r=1}^{s}(u_r - \bar{u})^2 \Big/ s-1}{\bar{u}}} - \sqrt{\frac{\sum_{i=1}^{m}(v_i - \bar{v})^2 \Big/ m-1}{\bar{v}}}$$

$$\begin{aligned} s.t \quad & \sum_{i=1}^{m} v_i x_{io} = 1 \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, j = 1, 2...., n \\ & u_r \geq 0, r = 1, 2, ...s \\ & v_i \geq 0, i = 1, 2, ...m \end{aligned} \tag{3.3}$$

The minimization of the CV here is achieved by the condition min= (- max) for the coefficient of variation terms of the objective function.

The model is applied to our sample data set and the results are obtained are shown in Table 3-2. The variables here are u and v which are the output and input weights.

Table 3-2: Results of the CCR DEA model

DMU	Efficiency	u_1	u_2	u_3	v_1	v_2	v_3
A	0.999	0.0116	0	0	0.9963	0.0003	0.0743
B	0.999	0.0021	0	0.0123	0.9940	0.0002	0.0816
C	0.999	0	0	0.0125	2.2443	0.000328	0.0590
D	0.858	0	0	0.0124	16.6666	0	0

Table 3-3: Results of the CVDEA model

DMU	Efficiency	u_1	u_2	u_3	v_1	v_2	v_3
A	1	0.0043	0.0043	0.0043	0.0790	0.0003	0.0810
B	0.999	0.0021	0	0.0123	0.9940	0.0002	0.08164
C	0.955	0.0039	0.0039	0.0039	0.0766	0.0002	0.0748
D	0.796	0.003	0.003	0.003	0.0665	0.0002	0.0671

Table 3-2 gives the result when the sample data was run in the classical CCR model. Table 3-3 gives the results of the CVDEA with the same data. It can be noticed that the weights and efficiencies obtained in Table 3-3 is more dispersed without any extreme outliers and the efficiency of the four DMUs are more restricted. DMUs A, B and C in Table 3-2 have the efficiency of 0.999 but using the CVDEA method it can be seen that the efficiency is different for all the four DMUs and hence a better ranking can be derived from it.

By comparing both the results we can see that the CVDEA model gives better discrimination among the DMUs and more dispersed input and output weights.

3.2 MCDM DEA model [Xiao-Bai, L., Gary, R. R., 1999]:

The next model which will be described is a Multi Objective model which builds upon the CCR model in DEA as its basis. In the CCR model the DMU is considered to be efficient if the objective function gives a value 1. This is its relative efficiency compared to other DMUs. In this MCDM model, a new variable is defined to express relative efficiency, called the deviational variable d_0.

The DMU will be termed efficient if and only if the deviational variable for the DMU under consideration, $d_0 = 0$. Hence d_0 can be used as a measure of inefficiency; the lesser the value of d_o, the more efficient the DMU is. The deviational variable for an inefficient DMU expresses how far the DMU is from being efficient and hence efficiency of a DMU can be expressed as: $h_0 = 1 - d_0$

Thus, the CCR model is modified by adding the deviation variable as follows:

$$
\begin{aligned}
&\min d_o \\
&s.t \quad \sum_{i=1}^{m} v_i x_{io} = 1 \\
&\qquad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + d_j = 0, \; j = 1,2....,n \qquad (3.4) \\
&\qquad u_r \geq 0, r = 1,2,....s \\
&\qquad v_i \geq 0, i = 1,2,....m \\
&\qquad d_j \geq 0, \; for \; all \; j
\end{aligned}
$$

where

d_o = deviational variable of DMU_o

v_i = weight of the i th input

x_{io} = i th input of DMU_o

u_r = weight of the r th output

y_{rj} = r th output of DMU_j; j= 1,2,...,n

x_{ij} = i th input of DMU_j; j= 1,2,…,n

d_j = deviational variable for DMU_j; j= 1,2,...,n

Since the deviational variable cannot be the only measure of efficiency, two more objectives are added to this model as a function of the deviational variable. They include (i) minimizing the maximum of the deviational variable, otherwise called the *minmax* objective and (ii) minimizing the sum of the deviational variables derived using model 3.4 also called the *minsum* objective.

The minmax objective is given by the variable M which represents the maximum quantity among all deviation variables d_j (j= 1…n).

The minsum criterion is given by the summation of all the deviational variables and hence can be represented as:

$$\text{Min } \sum_{j=1}^{n} d_j$$

Hence the resulting model is a Multi objective linear programming model which includes three criteria as follows:

$$\begin{aligned}
&\min d_o \\
&\min M \\
&\min \sum_{j=1}^{n} d_j \\
&s.t \quad \sum_{i=1}^{m} v_i x_{io} = 1 \\
&\qquad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + d_j = 0, j = 1,2....,n \qquad (3.5) \\
&\qquad M - d_j \geq 0, j = 1,2...,n \\
&\qquad u_r \geq 0, r = 1,2,....,s \\
&\qquad v_i \geq 0, i = 1,2,....,m \\
&\qquad d_j \geq 0, \text{ for all } j
\end{aligned}$$

Where

d_o = deviational variable of DMU_o

M = Maximum value of the deviational variables for DMU_j; j= 1,2,...,n

d_j = deviational variable for DMU_j; j= 1,2,…,n

v_i = weight of the i th input

x_{io} = i th input of DMU_o

u_r = weight of the r th output

y_{rj} = r th output of DMU_j; j= 1,2,…,n

x_{ij} = i th input of DMU_j; j= 1,2,…,n

In multiple objective programming models, it is generally not possible to find one optimal solution. This is due to the conflicting nature of the objectives. Hence it usually finds several non dominated solutions to help the decision maker to choose the most preferred solution.

A non dominated solution in multi criteria programming has the property that an improvement in any one objective can only be achieved by sacrificing on at least one of the other objectives.

To solve the multi objective model, the problem is converted to a single objective by using weights for each of the three objectives. The weights of the three objectives will be referred to as λ_1, λ_2 and λ_3

The weights are used in two scenarios. Scenario 1 is when all three objectives have equal weight. Scenario 2 uses unequal weights for the three objectives and hence gives them unequal priorities.

Thus, for computational purposes, the objective function will be formulated as follows.

$$\lambda_1 d_o + \lambda_2 (\sum d_j) + \lambda_3 M \qquad (3.6)$$

Where d_o is the deviational variable of the test DMU. $\sum d_j$ is the summation of all the deviational variables and M is the maximum value of the deviational variables.
Hence all the three objectives in the original problem given by model (3.5) are incorporated.

Tables 3.4 and 3.5 give the results of the analysis for scenarios 1 and 2 respectively.

Table 3-4: Results of the MCDEA model with equal weights

$\lambda_1 = 0.33, \lambda_2 = 0.33, \lambda_3 = 0.33$													
DMU	Deviational Variables				Sum of deviations	Maximum deviation	Input weights			Output weights		
	d_1	d_2	d_3	d_4	$\sum d_j$	M	v_1	v_2	v_3	u_1	u_2	u_3
A	0	0	0	0.221	0.221	0.22	2.53	0	0.06	0	0	0.01
B	0	0	0	0.235	0.235	0.23	2.67	0	0.07	0	0	0.01
C	0.025	0	0	0.177	0.20	0.17	1.87	0	0.07	0	0	0.01
D	0.024	0	0	0.171	0.19	0.17	1.80	0	0.06	0	0	0.01

Table 3-5: Results of the MCDEA model with unequal weights

$\lambda_1 = 0.6, \lambda_2 = 0.3, \lambda_3 = 0.1$												
DMU	Deviational Variables				Sum of deviations	Maximum deviation	Input weights			Output weights		
	d_1	d_2	d_3	d_4	$\sum d_j$	M	v_1	v_2	v_3	u_1	u_2	u_3
A	0	0	0	0.221	0.221	0.221	2.52	0	0.06	0	0	0.01
B	0.030	0	0	0.21	0.242	0.212	2.23	0	0.08	0	0	0.01
C	0.025	0	0	0.177	0.203	0.177	1.87	0	0.07	0	0	0.01
D	0.024	0	0	0.171	0.195	0.171	1.80	0	0.06	0	0	0.01

In Table 3-4, we see that DMUs A and B are efficient but there is a slight change in the results when the weights of the objective change in Table 3-5. DMU B is deemed inefficient using unequal priorities for the three objectives.

Model (3.5) is also called the MCDEA model but it is to be noted that it is not unique to these three criteria and the MCDEA model's results may change according to the efficiency criteria incorporated in it.

3.3 AHP with DEA model [Seiford, M.L., & Zhu, J., 1998]

The next model will focus on the use of the Analytical Hierarchy Process (AHP). AHP is a multi criteria decision making tool which is widely used to incorporate human judgment in quantitative evaluations. In DEA it has been used in different ways to improve analysis as explained in Chapter 2. The advantage of using AHP is that it allows for the decision maker's judgments to be incorporated as part of the analysis.

One such model, which combines AHP and DEA, was proposed by Seiford and Zhu who used it in their paper "Identifying Excesses and Deficits in Chinese Industrial Productivity" [1997]. The DEA model they use is the 'Additive DEA model' proposed by Charnes, Cooper, Golany, Seiford and Stutz [1985].

The weighted CRS additive DEA model combines the concept of both the input oriented and output oriented DEA models. Usually the input based models calculate efficiency by considering the possible input level reductions while keeping output levels constant and the output based models keep the input levels constant while considering the possible output increase which can be obtained from a model. This model by Seiford and Zhu has an advantage over other models as it incorporates both.

The model is described as follows:

$$\begin{aligned}
&\max \sum_{i=1}^{m} w_i^- s_i + \sum_{r=1}^{s} w_r^+ e_r \\
&s.t \quad \sum_{j=1}^{n} \lambda_j x_{ij} + s_i = x_{io},\ i = 1,2.....m; \\
&\qquad \sum_{j=1}^{n} \lambda_j y_{rj} - e_r = y_{ro},\ r = 1,2.....s; \\
&\qquad \lambda_j, s_i, e_r \geq 0
\end{aligned} \tag{3.7}$$

Where

x_{ij} = the i th input for DMU_j; j= 1,2,…,n

y_{rj} = the r th output for DMU_j; j= 1,2,…,n

w_i^- = user specified weights for x_{ij}; i=1,2,…,m, j=1,2,…,n

w_r^+ = user specified weight for y_{rj} ; r=1,2,…s, j=1,2,…,n

s_i = excess utilization of x_{ij}; i=1,2,…m, j=1,2…,n

e_r = deficit in $y_{rj\ r}$; r= 1,2,…s, j=1,2,...n

x_{ij} = i th input for DMU_j; j= 1,2,…,n

y_{rj} = r th output for DMU_j; j= 1,2,…,n

x_{io} = i th input for DMU_o

y_{ro} = r th output for DMU_o

λ_j = weight associated with DMU_j; j= 1,2,…,n

AHP is used to determine the user specified weights. The additive model used here is a Constant Returns to Scale or CRS additive model. This is used because this original additive model will assume equal weights for the input and output slacks. To avoid discrepancy different weights are assigned to inputs and outputs otherwise non-commensurable measures will be added.

If the above model identifies an efficient DMU then the objective function value of the model will be zero. In other words, $s_i^*=0$ and $e_r^*=0$. If a DMU is termed inefficient then the non zero optimal values of s_i^* will give the excess utilization of i^{th} input and e_r^* will give the deficit in the r^{th} output. In other words, it gives a value to show how the input can be reduced and the output can be increased. It is useful for setting targets for the inefficient DMUs.

The results heavily depend upon the decision maker's weights. The efficiency scores from this model do not differ from the classical CCR model but this model provides added functionality.

Steps to determine weights for model (3.7) are given below:

- Using a rating scale of 1-9 given in Table 3-6, experts state their preferences for input and output adjustments with 1 being least priority and 9 being the highest priority
- Pairwise comparison is performed between the inputs and outputs
- The judgment matrix M is derived after the comparisons.
- $M = \left(m_{ij}\right)_{p\times p}$ where p= m+s and p is total number of inputs.
- This can be extended to a group judgment matrix using geometric mean to combine the matrices derived for each decision maker using the equation

$M^G = \left(m^G_{ij}\right)_{p\times p} = \left[\left(\prod_{i=1}^{5} m^l_{ij}\right)^{1/5}\right]_{p\times p}$. Here l=1, 2….L, where L is the number of decision makers.

- Using the judgement matrix the weights can be determined by using the following equation given by Saaty [1980].

  $M^G w = \lambda_{\max} w$

Table 3-6: Rating Scale

Score	Definitions
1	Lowest priority
2	Below average priority
3	Average priority
4	Above average priority
5	Highest priority

$$M = \begin{array}{c} \\ I_1 \\ I_2 \\ I_3 \\ O_1 \\ O_2 \\ O_3 \end{array} \begin{array}{c} \begin{array}{cccccc} I_1 & I_2 & I_3 & O_1 & O_2 & O_3 \end{array} \\ \begin{bmatrix} 1 & 2 & 4 & 3 & 2 & 2 \\ 0.5 & 1 & 5 & 1 & 2 & 2 \\ 0.25 & 0.20 & 1 & 3 & 2 & 3 \\ 0.33 & 1 & 0.33 & 1 & 5 & 4 \\ 0.5 & 0.5 & 0.5 & 0.20 & 1 & 5 \\ 0.5 & 0.5 & 0.33 & 0.25 & 0.20 & 1 \end{bmatrix} \end{array}$$

In our example we consider the case of only one decision maker and we performed the pair wise comparison on the judgment matrix given below. Each element m_{ij} is the decision maker's preference over a particular input reduction or output augmentation.

The matrix is then normalized by dividing each element m_{ij} of the matrix by its column sum. This gives rise to the normalized matrix M_{Norm}.

$$M_{Norm} = \begin{array}{c} \\ I_1 \\ I_2 \\ I_3 \\ O_1 \\ O_2 \\ O_3 \end{array} \begin{array}{c} \begin{array}{cccccc} I_1 & I_2 & I_3 & O_1 & O_2 & O_3 \end{array} \\ \begin{bmatrix} 0.320 & 0.380 & 0.358 & 0.355 & 0.163 & 0.117 \\ 0.162 & 0.192 & 0.440 & 0.118 & 0.163 & 0.117 \\ 0.081 & 0.038 & 0.089 & 0.355 & 0.163 & 0.176 \\ 0.107 & 0.192 & 0.029 & 0.118 & 0.400 & 0.235 \\ 0.162 & 0.096 & 0.044 & 0.059 & 0.088 & 0.294 \\ 0.162 & 0.096 & 0.029 & 0.029 & 0.016 & 0.058 \end{bmatrix} \end{array}$$

The weights are derived by averaging across each row as given below.

W_1=0.282

W_2=0.198

W_3=0.150

W_4=0.180

W_5=0.078

W_6=0.065

For example,

$$W_1 = \frac{0.32 + 0.38 + 0.358 + 0.355 + 0.163 + 0.117}{6} = 0.282$$

Using model (3.7) and the sample data in Table 3-1, the results are obtained in Table 3-7.

Table 3-7: Results of the weighted CRS additive model

DMU	Input excesses			Output deficits		
	s_1	s_2	s_3	e_1	e_2	e_3
A	0	0	0	0	0	0
B	0	0	0	0	0	0
C	0.0016	206.933	2.1013	2.7066	0	0.1066
D	0	0	0	0	0	0

From Table 3-7 we can see that DMU A,B and D are efficient and hence they do not have excessive inputs and deficit outputs. But as suggested by this model we can interpret the results for DMU C and D in order to improve their productivity. DMU C can lower their input by 0.0016, 206.933 and 2.1013 units. The first and third output of DMU C can be increased by 2.7066 and 0.1066 units respectively.

3.4: Super Efficiency with infeasibility model [Chen, 2004]

The next model to be discussed incorporates the concept of super efficiency in DEA. Super efficiency is used to discriminate between DMUs. Hence it can be used to rank the efficient DMUs. Super efficiency was first introduced by Andersen and Petersen [1993] They calculate super efficiency by removing the DMU under consideration from the reference set when calculating the relative efficiency of that DMU.

The CRS or Constant Returns to Scale model is used, It considers simultaneously changes in inputs and outputs when calculating the relative efficiency for a DMU. But when one changes only the inputs while keeping the outputs constant or vice versa, the infeasibility problem occurs. This type of model is called the VRS or Variable Returns to Scale model.

Seiford and Zhu [1998b] proved that super efficiency can be used as an efficiency stability measure. After arriving at the efficiency classification of a set of DMUs, its stability to input and output changes can be tested. If an input oriented model is used then it means that the model will be stable to any input changes. If an output oriented model is used, it is stable to output changes. Infeasibility can be represented as the highest super efficiency.

When a VRS model is used the inputs are not kept constant when the output changes. Similarly the outputs are also not kept constant when the inputs change. Therefore in this case super efficiency can be interpreted as input saving or output surplus. This is proven in the work Chen [2004] where it is shown that super efficiency can exist in either input oriented VRS model or the output oriented model but not in both. Input super efficiency is input savings and output super efficiency is output surplus. Hence the classification of DMUs can be done by using both these models.

A ranking of DMUs using the VRS model is now possible because both the input and output oriented models are both considered and super efficiency can be fully characterized. This is done by integrating both scenarios into one formula for the purpose of ranking.

The following models will give a detailed description on how ranking can be achieved even when infeasibility exists in a model. It is followed by formula to enable the ranking of DMUs.

The VRS model which exhibits input orientation as given by Banker et al [1994] is as follows:

$$
\begin{aligned}
&\min \theta_0 \\
&s.t \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta_0 x_{io}, i = 1,2...,m \\
&\sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro}, r = 1,2...,s \\
&\sum_{j=1}^{n} \lambda_j = 1 \\
&\lambda_j \ge 0, j = 1,...,n
\end{aligned}
\tag{3.8}
$$

Where

θ_0 = input oriented efficiency of DMUo

x_{ij} = i th input of DMU_j; j= 1,2,..,.n

y_{rj} = r th output of DMU_j; j= 1,2,..,.n

x_{io} = i th output of DMU_o

y_{ro} = r th output of DMU_o

λ_j = weight associated with DMU_j; j= 1,2,..,.n

The output of this model gives us the relative efficiency of each DMU.

The super efficiency for each DMU can be found and the model related to the VRS model as given by Seiford and Zhu [1999] is used as follows:

$$\begin{aligned}
&\min \theta_0^{VRS-\text{super}} \\
&s.t \sum_{j=1, j\neq 0}^{n} \lambda_j x_{ij} \leq \theta_0^{VRS-\text{super}} x_{io},\ i = 1,2..,\ m \\
&\sum_{j=1, j\neq 0}^{n} \lambda_j y_{rj} \geq y_{ro},\ r = 1,2..,\ s \\
&\sum_{j=1, j\neq 0}^{n} \lambda_j = 1 \\
&\theta_0^{VRS-\text{super}} \geq 0 \\
&\lambda_j \geq 0, j \neq 0
\end{aligned} \tag{3.9}$$

Here

$\theta_0^{VRS-\text{super}} =$ input oriented super efficiency of DMU_o

x_{ij} = i th input of DMU_j; j= 1,2,…,n

y_{rj} = r th output of DMU_j; j= 1,2,…,n

x_{io} = i th output of DMU_o

y_{ro} = r th output of DMU_o

λ_j = weight associated with DMU_j; j= 1,2,…,n

Here, DMU_o under evaluation will be excluded from the reference set of DMUs.

The output oriented model for VRS super efficiency can also be formulated as shown below:

$$\max \phi_o^{VRS-\text{super}}$$
$$s.t \sum_{j=1, j\neq 0}^{n} \lambda_j x_{ij} \leq x_{io},\ i = 1, 2.., m$$
$$\sum_{j=1, j\neq 0}^{n} \lambda_j y_{rj} \geq \phi_o^{VRS-\text{super}} y_{ro},\ r = 1, 2.., s \tag{3.10}$$
$$\sum_{j=1, j\neq 0}^{n} \lambda_j = 1$$
$$\phi_o^{VRS-\text{super}} \geq 0$$
$$\lambda_j \geq 0, j \neq 0$$

Where

$\phi_o^{VRS-\text{super}}$ = output oriented super efficiency for DMU_o

x_{ij} = i th input of DMU_j; j= 1,2,…,n

y_{rj} = r th output of DMU_j; j= 1,2,…,n

x_{io} = i th output of DMU_o

y_{ro} = r th output of DMU_o

λ_j = weight associated with DMU_j; j= 1,2,…,n

The following model, which derives $\tilde{\theta}_0^{VRS-\sup er}$, is a useful variation of model (3.8) but the optimal value to the VRS output oriented model is incorporated into it as shown below.

$$\begin{aligned}
&\min \tilde{\theta}_0^{VRS-\sup er} \\
&s.t \sum_{j=1}^{n} \lambda_j x_{ij} \le \tilde{\theta}_0^{VRS-\sup er} x_{io}, i = 1,2...,m \\
&\sum_{j=1}^{n} \lambda_j \hat{y}_{rj} \ge \hat{y}_{ro}, r = 1,2...,s \qquad (3.11) \\
&\sum_{j=1}^{n} \lambda_j = 1 \\
&\lambda_j \ge 0, j = 1,...,n
\end{aligned}$$

Where

$\tilde{\theta}_0^{VRS-\sup er}$ = input oriented super efficiency after output augmentation

x_{ij} = i th input of DMU j=1,2,...n

$\hat{y}_{rj} = \Phi^* y_{rj}$

Φ^* = optimal value to model (3.12)

x_{io} = i th input for DMU_o

$\hat{y}_{ro}$ = r th output for DMU_o = y_{ro}

λ_j = weight associated with DMU_o

Model (3.12) is similar to model (3.8) except that it is for a output oriented model.

$$
\begin{aligned}
&\phi_o^* = \max \phi_o \\
&s.t \sum_{j=1, j\neq 0}^{n} \lambda_j x_{ij} \leq x_{io},\ i = 1,2..,\ m \\
&\quad \sum_{j=1, j\neq 0}^{n} \lambda_j y_{rj} \geq \phi_o y_{ro},\ r = 1,2..,s \\
&\quad \sum_{j=1, j\neq 0}^{n} \lambda_j = 1 \\
&\quad \phi_o^{VRS-\text{super}} \geq 0 \\
&\quad \lambda_j \geq 0, j \neq 0
\end{aligned}
\qquad (3.12)
$$

This change is done to incorporate a case when model (3.9) can be infeasible for a particular DMU but it can be found to be feasible by incorporating the output super efficiency. In other words its super efficiency can be realized by applying model (3.11)

Similar to model (3.11) the output oriented super efficiency incorporating the efficient input oriented DMUs can be realized by model (3.13)

$$
\begin{aligned}
&\max \tilde{\phi}_o^{VRS-\text{super}} \\
&s.t \sum_{j=1, j\neq 0}^{n} \lambda_j \hat{x}_{ij} \leq \hat{x}_{io},\ i = 1,2..,\ m \\
&\quad \sum_{j=1, j\neq 0}^{n} \lambda_j y_{rj} \geq \tilde{\phi}_o^{VRS-\text{super}} y_{ro},\ r = 1,2..,\ s \\
&\quad \sum_{j=1, j\neq 0}^{n} \lambda_j = 1 \\
&\quad \phi_o^{VRS-\text{super}} \geq 0 \\
&\quad \lambda_j \geq 0, j \neq 0
\end{aligned}
\qquad (3.13)
$$

Where

$\tilde{\phi}_o^{VRS-\text{super}}$ = super efficiency of DMU_o after input augmentation

$\widehat{x}_{ij} = \widehat{x}_{ij} = \theta_j^* x_{ij}$

y_{rj} = r th output of DMU_j; j= 1,2,…,n

$\widehat{x}_{io}$ = i th input for $DMU_o = x_{io}$

y_{ro} =r th output for DMU_o

λ_j = weight associated of DMUj; j= 1,2,…,n

Where $\widehat{x}_{ij} = \theta_j^* x_{ij}$ and θ_j^* is the optimal value to model (3.8) when DMU j is considered. Ranking of the DMUs using super efficiency can be done by realizing which models render them inefficient. The two variables γ_o and τ_o can help in deciding the super efficiency score using the following conditions:

The possible values for γ_o are

$$\gamma_o = \begin{cases} \theta_o^{VRS-\text{super}} & \text{if model } (2.8) \text{ is feasible} \\ \tilde{\theta}_o^{VRS-\text{super}} & \text{if model} (2.8) \text{ is feasible and model } (2.10) \text{ is feasible} \\ 1 & \text{if model } (2.10) \text{ is infeasible} \end{cases} \tag{3.14}$$

$$\tau_o = \begin{cases} \Phi_o^{VRS-\text{super}} & \text{if model } (2.9) \text{ is feasible} \\ \tilde{\Phi}_o^{VRS-\text{super}} & \text{if model } (2.9) \text{ is feasible and model} (2.12) \text{ is feasible} \\ 1 & \text{if model } (2.12) \text{ is infeasible} \end{cases} \tag{3.15}$$

Now a ranking of the DMUs can be got by evaluating S_0 and $\widehat{S}_0$ which integrates the values of γ_o and τ_o in order to get one super efficiency score.

$$S_0 = w_\gamma \gamma_0 + w_\tau \frac{1}{\tau_0} \tag{3.16}$$

$$\widehat{S}_0 = w_\gamma \frac{1}{\gamma_0} + w_\tau \tau_0 \tag{3.17}$$

Also w_γ and w_τ are selected such that $w_\gamma + w_\tau = 1$ and $S_0 \geq 1$ and $\widehat{S}_0 \leq 1$.

Larger values of S_0 will give high super efficiency scores and lower values of $\widehat{S}_0$ will give higher super efficiency performance scores.

Table 3-8: Results of input oriented super efficiency model

DMU	θ_0	θ_0^{VRS}
A	1	1.26 (infeasible)
B	1	1.2
C	1	0.941 (infeasible)
D	1	0.910

Table 3-9: Results of the output oriented super efficiency model

DMU	Φ_0	Φ_0^{VRS}
A	1	0.766 (infeasible)
B	1	0.866 (infeasible)
C	1	0.8875
D	1.02	0.822

Table 3-10: Conditions for super efficiency based ranking

DMU	θ_0	θ_0^{VRS}	$\tilde{\theta}_0^{VRS}$	Φ_0	Φ_0^{VRS}	$\tilde{\Phi}_0^{VRS}$	γ_0	τ_0	S_0	$\widehat{S}_0$
A	1	infeasible	infeasible	1	infeasible	infeasible	1	1	1	1
B	1	1.2	1.2	1	infeasible	infeasible	1.2	1	1.8	0.8
C	1	infeasible	infeasible	1	0.88	NA	1	0.88	1.9	0.22

We can see that DMU A is found to be infeasible for both input and output oriented models. This is a rare case and it occurs due to the nature of the data used. This means that it has no super efficiency in terms of input saving or output surplus. DMU D is not efficient in terms of its output oriented model and hence is not included in the super efficiency ranking calculations.

Table 3-11: Ranking of DMUs using super efficiency

DMU	Rank
A	3
B	2
C	1

Hence we can see that the ranking in Table 3-12 does not get affected because infeasibility exists due to VRS conditions. It includes super efficiency in terms of input oriented and output oriented models.

Therefore, according to table three DMU C is the most super efficient among the three DMUs which satisfied the conditions in equations (3.14) and (3.15).

CHAPTER 4

CASE STUDY ON HEALTH CARE PROVIDER

4.1 Introduction to efficiency for Health Care Providers:

This chapter elaborates the need for efficiency in the health care industry and the use of DEA to improve it. The chapter is organized as follows: first, we explain the need and the difficulties of measuring hospital efficiency. We then discuss the use of DEA in computing hospital efficiency in the past by bringing out the important factors affecting hospital efficiency and its advantages over other techniques. Finally we use a case study with real data from a health care provider to evaluate the models we introduced in chapter 3.

4.1.1 History of the Health Care Industry:

Hospitals earlier in the 19th and 20th century were considered social institutions and expenditure was not a problem. The Medicare program developed by the government did not urge providers to improve their efficiency. Then the Prospective Pay System (PPS) based on diagnostic related groups (DRG) was introduced and the reimbursement was based on case by case rather than cost per day. These groups were made to classify hospital cases into sub groups according to patient history, sex, age etc. Hospitals started facing competition from multi owned private hospitals. Businesses which provide health care started using contracts from Preferred Provider Organization (PPO) in order to get discounts.

4.2 Health Care efficiency:

4.2.1 Problems associated with hospital efficiency:

The concern over the efficiency of health care providers materialized when the federal spending decreased and competition became tougher. As the cost of health care kept rising, the cost reimbursement structure became narrower. Hence more health care providers started to focus on ways to cut costs and make their clinics more efficient and effective.

There has been some difficulty in assessing hospital efficiency. One of them is the fact there are numerous outputs for a single provider. Also the efficiency of the clinic cannot be easily realized as a function of inputs and outputs alone. Hospital Efficiency is mainly composed of two types of efficiency, C*linical* and *Managerial Efficiency* (Donabedian and Wyszewianski, 1982). *Clinical Efficiency* is the one viewed by physicians. For example, it is related to how the services are performed and how long it took to complete a medical procedure in order to increase the health of a patient. *Managerial Efficiency* is related to ancillary services which are intermediate products of a hospital such as the number of lab tests, X rays etc. (Johnson and Johnson, 1986) Hence hospital efficiency is the technical efficiency which is a combination of clinical and managerial efficiency (Fetter, 1991a). Hospital efficiency has its limitations because of the number of outputs each health provider produces. Combining clinical and managerial efficiency may also be difficult to achieve.

4.2.2 Methods used to determine hospital efficiency:

DEA is one of the methods to measure hospital efficiency along with regression analysis and ratio analysis although DEA has its advantages over the other two methods. Regression analysis usually deals with multiple inputs and one output. Even though multiple regression can be used but multiple residuals are formed and hence some a priori information needs to be used again for the inputs and outputs. *Ratio analysis* might create ambiguity by deeming one provider efficient with respect to one dimension without considering all the dimensions. For example, a hospital can be efficient if the patient days are less but no importance is given to the quality of care as a factor. Hence the results may be biased to one factor. On the other hand DEA might give a very deterministic output which does not provide for an error term. Hence random variation might not favor in DEA. Also the inherent problem of multiple optima in DEA is another disadvantage. There has been speculation on which methods are better (Kooreman , 1994).

4.2.3 Factors affecting health care efficiency evaluations

The factors which affect the efficiency of hospitals using various DEA models have been researched and one such factor is its dependency on the number of inputs and outputs. Too many or too few inputs and outputs affect the output of the DEA models. Reducing the inputs and outputs will eliminate detail which might be necessary. Too many inputs and outputs might increase the complexity for no reason.

Health care providers can be of different types. The most common is the community hospital which provides generalized services. There are also public hospitals run by government, private for profit, and private nonprofit hospitals (William and Torrens, 1993). Hospitals can also be classified into teaching and non teaching hospitals. Teaching hospitals are those connected to medical schools. The output of these different units is dependent on several factors and each of them functions differently. There might be regulatory and other local factors like the state where it is situated. (Ozcan, 1992). Community and not-for-profit hospitals might have fewer funds and even if the quality of care is good, they might not get enough resources to be deemed efficient. Hence when comparing health care providers the category of hospitals should be taken into consideration.

Effectiveness is the next factor concerning health care efficiency. Even though a health care provider might be efficient he might not be effective. Research on this topic (Chillingerian and Sherman, 1990) has shown that there is always a tradeoff between *Effectiveness* and *Efficiency*. Being effective is related to using more resources and hence reduces a provider's efficiency.

Quality of care is another important factor in health care evaluations. It is highly dependent on the cost of care. Ozcan, Luke and Haksever(1992) conducted a study and showed that government hospitals were more efficient than private hospitals and this was achieved at the expense of quality. Generally in health care, the repercussions of bad quality of health care are large and hospitals are forced to spend more to maintain the quality of service.

4.3 Case study:

Real data from a health care provider was used in the models discussed in chapter 3 to analyze the performance of the models and a ranking is provided using the Super Efficiency model. The data consists of three inputs and two outputs. The inputs include the (i) input providers, (ii) support staff and (iii)Total building expenses. Outputs include the (i) Relative Value Unit (RVU) and (ii) the Total Net Revenue. These inputs and outputs are few among the various factors which affect a health care units performance and they were chosen by the health care provider.

Two of the inputs selected affect the day to day work of a unit. This involves the number of doctors and nurses. This is included in the input measures called input providers and support staff respectively. Total building expenses cover all the expenses incurred in the unit. Relative value unit is a measure used in health care by insurance providers to evaluate the health care providers. This value determines the reimbursement rate for these providers taking into consideration factors for example geographic location. Hence it is an important output measure. Finally the profit a health care unit must be taken into consideration. Hence the Total Net Revenue is chosen as one of the output measures.

Forty four locations of the health care provider are the 44 DMUs which will be used in the evaluation. The four models used were solved using excel solver software.

4.3.1 CVDEA Model:

The first model is the CVDEA model explained in model (3.3) which helps in better discrimination among the decision making units. Using the data for the case study, 46 constraints were solved in the LP problem for the CVDEA model. This includes 2 non negativity constraints for the input and output weights u and v and the 44 health care providers chosen for the study.

The results of the model are shown in Table 4-1.

Table 4-1: Results of the CVDEA model for the Case Study:

DMU	Efficiency	u_1	u_2	v_1	v_2	v_3
1	0.6731	0.0000	0.0000	0.0425	0.0428	0.0000
2	0.6674	0.0000	0.0000	0.0564	0.0572	0.0000
3	1.0000	0.0000	0.0000	0.0762	0.0755	0.0000
4	0.5895	0.0000	0.0000	0.0313	0.0338	0.0000
5	0.5114	0.0000	0.0000	0.0890	0.0849	0.0000
6	0.7989	0.0000	0.0000	0.0402	0.0410	0.0000
7	0.7293	0.0000	0.0000	0.0565	0.0580	0.0000
8	0.6685	0.0000	0.0000	0.0207	0.0189	0.0000
9	1.0000	0.0000	0.0000	0.1013	0.0988	0.0000
10	0.6531	0.0000	0.0000	0.0895	0.0865	0.0000
11	1.0000	0.0000	0.0000	0.0021	0.0021	0.0000
12	0.6684	0.0001	0.0000	0.1153	0.1171	0.0000

13	0.9542	0.0000	0.0000	0.1378	0.1353	0.0000
14	0.6708	0.0000	0.0000	0.0311	0.0320	0.0000
15	0.6228	0.0000	0.0000	0.0790	0.0800	0.0000
16	0.7169	0.0000	0.0000	0.0436	0.0453	0.0000
17	0.5578	0.0000	0.0000	0.0574	0.0374	0.0000
18	0.6898	0.0000	0.0000	0.0700	0.0748	0.0000
19	0.6347	0.0000	0.0000	0.0660	0.0338	0.0000
20	0.7266	0.0000	0.0000	0.0746	0.0744	0.0000
21	0.7120	0.0000	0.0000	0.0841	0.0816	0.0000
22	0.5600	0.0000	0.0000	0.1773	0.1646	0.0000
23	0.7391	0.0000	0.0000	0.0469	0.0424	0.0000
24	0.6695	0.0000	0.0000	0.1305	0.1439	0.0000
25	0.8124	0.0000	0.0000	0.0070	0.0070	0.0000
26	0.7654	0.0000	0.0000	0.1251	0.1506	0.0000
27	0.7793	0.0000	0.0000	0.0501	0.0500	0.0000
28	0.7537	0.0000	0.0000	0.1337	0.1312	0.0000
29	0.8789	0.0000	0.0000	0.0307	0.0306	0.0000
30	0.7178	0.0000	0.0000	0.0502	0.0427	0.0000
31	0.7029	0.0000	0.0000	0.0402	0.0413	0.0000
32	0.6187	0.0000	0.0000	0.2516	0.0114	0.0000
33	0.5777	0.0000	0.0000	0.1224	0.1254	0.0000
34	0.9152	0.0000	0.0000	0.1608	0.1697	0.0000

35	0.7189	0.0000	0.0000	0.0496	0.0496	0.0000
36	0.7389	0.0000	0.0000	0.0272	0.0270	0.0000
37	0.7193	0.0000	0.0000	0.0903	0.0928	0.0000
38	0.8335	0.0000	0.0000	0.0478	0.0500	0.0000
39	1.0000	0.0000	0.0000	0.1929	0.1961	0.0000
40	0.6247	0.0000	0.0000	0.1934	0.1931	0.0000
41	0.9319	0.0000	0.0000	0.1526	0.1529	0.0000
42	0.6902	0.0001	0.0000	0.0748	0.0749	0.0000
43	0.7286	0.0000	0.0000	0.0617	0.0577	0.0000
44	0.7588	0.0000	0.0000	0.0571	0.0573	0.0000

The results show that only four DMUs are deemed efficient using this model. They are DMUs 3, 9, 11 and 39. Also the output weights are extremely low. Most of the weight is placed on input 1 and input 2 which are the total building expenses and the Relative Value Units (RVU's). Almost no importance is given to the other inputs and outputs. The fact that out of 44 DMUs only four are deemed efficient shows the model's discrimination power.

4.3.2 MCDM DEA Model:

The MCDM model given in model (3.5) was tested with the same data. In this model, as explained earlier, the deviational variable expressed as d_0 for an inefficient DMU expresses how far the DMU is from being efficient. Hence efficiency of a DMU can be expressed as: $h_0 = 1 - d_0$

Scenario 2, where equal priorities are given to the three objectives, was used using equation (3.6). This scenario was chosen so that there will be no discrimination among the three objectives and the case study can be evaluated in an unbiased manner. The results are shown in Table 4-2

Table 4-2: Results of the MCDM DEA model for the Case Study

DMU	d_o	$\sum d_j$	M	u_1	u_2	v_1	v_2	v_3
1	0.0000	8.4729	0.5534	0.0000	0.0000	0.0000	0.0591	0.0000
2	0.3240	0.9484	0.5134	0.0000	0.0000	0.0961	0.0336	0.0000
3	0.0000	15.1947	0.9923	0.0000	0.0000	0.0000	0.1060	0.0000
4	0.4037	6.4386	0.4146	0.0000	0.0000	0.0000	0.0443	0.0000
5	0.5077	16.6985	0.5077	0.0000	0.0000	0.0000	0.1165	0.0000
6	0.1751	8.1372	0.4443	0.0000	0.0000	0.0938	0.0225	0.0000
7	0.2955	11.1902	0.6110	0.0000	0.0000	0.1290	0.0310	0.0000
8	0.1632	6.3704	0.3478	0.0000	0.0000	0.0734	0.0176	0.0000
9	0.3349	19.7860	1.2923	0.0000	0.0000	0.0000	0.1380	0.0000
10	0.3564	14.1000	0.9209	0.0000	0.0000	0.0000	0.0983	0.0000
11	0.0683	8.7870	0.4582	0.0000	0.0000	0.0858	0.0300	0.0000
12	0.8976	22.9960	1.2556	0.0000	0.0000	0.2651	0.0636	0.0000
13	0.0648	26.5670	1.7351	0.0000	0.0000	0.0000	0.1853	0.0000
14	0.3178	6.2803	0.3429	0.0000	0.0000	0.1643	0.0574	0.0000
15	0.3580	16.8280	0.8775	0.0000	0.0000	0.1643	0.0574	0.0000
16	0.3030	8.4740	0.4627	0.0000	0.0000	0.0977	0.0234	0.0000
17	0.4708	7.2084	0.4708	0.0000	0.0000	0.0000	0.0503	0.0000
18	0.2584	13.7154	0.7747	0.0000	0.0000	0.0831	0.0653	0.0000
19	0.3457	7.0080	0.4577	0.0000	0.0000	0.0000	0.0926	0.0000
20	0.2759	13.2750	0.8670	0.0000	0.0000	0.0000	0.0926	0.0000

21	0.3348	17.6574	1.1532	0.0000	0.0000	0.0000	0.1231	0.0000
22	0.4875	28.4530	1.8583	0.0000	0.0000	0.0000	0.1984	0.0000
23	0.2585	8.2783	0.4317	0.0000	0.0000	0.0808	0.0282	0.0000
24	0.3489	25.9758	1.4183	0.0000	0.0000	0.2995	0.0719	0.0000
25	0.1674	8.0143	0.4179	0.0000	0.0000	0.0782	0.0273	0.0000
26	0.3145	25.3670	1.4707	0.0000	0.0000	0.3334	0.0554	0.0000
27	0.1924	10.8420	0.7081	0.0000	0.0000	0.0000	0.0756	0.0000
28	0.2586	24.4690	1.4453	0.0000	0.0000	0.3052	0.0732	0.0000
29	0.1445	5.6700	0.3703	0.0000	0.0000	0.0000	0.0395	0.0000
30	0.3017	8.0470	0.5256	0.0000	0.0000	0.0000	0.0561	0.0000
31	0.2928	8.0370	1.0097	0.0000	0.0000	0.0000	0.0561	0.0000
32	0.2557	13.8630	0.9037	0.0000	0.0000	0.0000	0.0965	0.0000
33	0.4953	18.7410	1.0097	0.0000	0.0000	0.2211	0.0425	0.0000
34	0.1543	25.1840	1.6448	0.0000	0.0000	0.0000	0.1756	0.0000
35	0.2738	9.9070	0.6471	0.0000	0.0000	0.0000	0.0691	0.0000
36	0.2880	4.6300	0.3024	0.0000	0.0000	0.0000	0.0323	0.0000
37	0.3266	14.3440	0.7480	0.0000	0.0000	0.1400	0.0489	0.0000
38	0.1874	9.3720	0.5117	0.0000	0.0000	0.1081	0.0259	0.0000
39	0.0000	37.2730	2.0425	0.0000	0.0000	0.0000	0.2181	0.0000
40	0.3850	32.3820	1.8291	0.0000	0.0000	0.1962	0.1542	0.0000
41	0.0890	28.1620	1.5377	0.0000	0.0000	0.3247	0.0779	0.0000
42	0.3729	15.2460	0.7950	0.0000	0.0000	0.1488	0.0520	0.0000

Here only DMU 1 and DMU 39 are considered efficient as the value of d_o for these two DMUs is zero.

4.3.3 AHP DEA Model

The main advantage of the AHP DEA model introduced earlier is the ability of performance measurement of the DMUs. Pairwise comparisons were calculated using the rating scale provided in given in Table 4-3 for the inputs and outputs. Using this scale the matrices M and M_{Norm} were derived.

Table 4-3: Rating Scale for the AHP model in the case study

Score	Definitions
1	Lowest priority
2	Below average priority
3	Average priority
4	Above average priority
5	Highest priority

$$
M = \begin{array}{c} \\ I_1 \\ I_2 \\ I_3 \\ O_1 \\ O_2 \end{array}
\begin{array}{c}
\begin{array}{ccccc} I_1 & I_2 & I_3 & O_1 & O_2 \end{array} \\
\begin{bmatrix}
1 & 2 & 1 & 2 & 1 \\
0.5 & 1 & 3 & 2 & 1 \\
1 & 0.33 & 1 & 3 & 3 \\
0.5 & 0.5 & 0.33 & 1 & 2 \\
1 & 1 & 0.33 & 0.5 & 1
\end{bmatrix}
\end{array}
$$

The matrix is then normalized by dividing each element m_{ij} of the matrix by its column sum to get the normalized matrix M_{Norm}.

$$M_{Norm} = \begin{array}{c} \\ I_1 \\ I_2 \\ I_3 \\ O_1 \\ O_2 \end{array} \begin{array}{c} \begin{array}{ccccc} I_1 & I_2 & I_3 & O_1 & O_2 \end{array} \\ \begin{bmatrix} 0.25 & 0.41 & 0.17 & 0.23 & 0.12 \\ 0.12 & 0.20 & 0.52 & 0.23 & 0.12 \\ 0.25 & 0.06 & 0.17 & 0.35 & 0.37 \\ 0.12 & 0.10 & 0.05 & 0.11 & 0.25 \\ 0.25 & 0.20 & 0.05 & 0.05 & 0.12 \end{bmatrix} \end{array}$$

The average across each row of M_{Norm} is found to derive the weights.

W_1=0.238

W_2= 0.244

W_3=0.245

W_4= 0.13

W_5=0.139

Hence using the weights, we incorporate preference information in the AHP DEA model given by a decision maker with knowledge about the inputs and outputs.

In this case study, the rating for comparing each input and output were chosen by me. In a real life scenario it will be useful to use the knowledge of people working in the health care industry.

Table 4-4 gives the results of the forty four DMUs and their input excesses (s_i) and output deficits (e_r) as defined below:

s_1= Excess in the number of input providers

s_2= Excess in the number of input staff

s_3= Excess in the total building expenses

e_1= Deficit in the total RVU

e_2= Deficit in the total net revenue

Table 4-4: Results of the AHP DEA model for the Case Study

DMU	Input Excess			Output Deficits	
	s_1	s_2	s_3	e_1	e_2
1	0.15	0.00	62257.87	3429.51	542409.94
2	0.00	0.00	0.00	6902.04	704423.69
3	0.00	0.00	0.00	0.00	0.00
4	0.00	0.51	4502.92	11683.00	956544.60
5	0.00	1.19	17688.67	4381.67	332472.33
6	0.62	0.00	0.00	2521.27	390676.71
7	0.76	0.00	0.00	2953.48	338994.32
8	1.44	0.00	0.00	7801.00	758569.40
9	0.00	0.00	0.00	0.00	0.00
10	0.00	0.00	54119.42	5008.42	330901.33
11	0.00	0.00	0.00	0.00	0.00

12	0.28	0.00	23368.76	840.34	708956.07
13	0.00	0.00	0.00	0.00	0.00
14	1.10	0.00	0.00	7929.34	757902.35
15	0.04	0.00	13186.35	2750.37	403701.79
16	1.46	0.00	60110.05	69.88	300002.37
17	0.00	2.08	133240.98	6233.50	652334.90
18	0.04	0.00	37749.08	2440.48	317263.64
19	0.00	0.00	75173.93	6759.38	845531.16
20	0.02	0.00	59440.05	2370.56	327652.40
21	0.00	1.30	0.00	3001.59	149204.83
22	0.00	1.07	32735.33	1617.33	100765.67
23	0.10	0.00	10256.60	3512.85	601560.80
24	0.14	0.00	15031.79	1618.40	216597.36
25	0.24	0.00	0.00	5947.75	335540.55
26	0.47	0.00	0.00	813.19	124164.80
27	0.00	0.00	14619.40	1838.04	272280.51
28	0.13	0.00	0.00	975.90	169019.21
29	0.00	2.00	23913.86	0.00	7800.15
30	0.00	0.00	57540.75	7265.35	474951.45
31	0.00	0.00	0.00	8820.88	668179.84
32	0.33	1.32	0.00	1478.19	139277.38
33	0.27	0.00	85623.72	2307.15	314320.73

34	0.00	0.00	65368.79	0.00	0.00
35	0.00	0.00	10940.37	3100.50	525645.63
36	0.00	0.00	224038.74	10211.05	616346.77
37	0.11	0.00	106553.86	659.69	281892.87
38	0.94	0.00	0.00	1569.31	185356.49
39	0.00	0.00	0.00	0.00	0.00
40	0.01	0.00	31468.84	1440.73	189413.75
41	0.00	0.00	0.00	0.00	0.00
42	0.09	0.00	0.00	3281.17	479515.67
43	0.57	0.00	0.00	2634.08	236864.58
44	0.04	0.00	0.00	5235.74	392981.73

In order to determine which DMUs are efficient, the values of input excess and output deficits should be zero.

Hence the results show that DMUs 3, 9, 11, 13, 39 and 41 are efficient without any input excesses and output deficits according to this model.

4.3.4 Super Efficiency Model:

The final model explained incorporates evaluating super efficiency of the efficient DMUs in the presence of infeasibility. The forty four DMUs were analysed using the four models to determine the efficiency and subsequently their super efficiency using both input and output oriented models. Finally a ranking is derived from the results.

The results of the input oriented model to determine efficient and super efficient DMUs are given in Table 4-5. These correspond to model (3.8) and model (3.9)

Table 4-5: Results of input oriented super efficiency for the Case study

DMU	θ_0	θ_0^{VRS}
1	0.7551	-
2	0.830	-
3	1	Infeasible
4	0.7199	-
5	0.6704	-
6	0.9184	-
7	0.8063	-
8	1	1.0492
9	1	1.2071
10	0.7113	-
11	1	1.3302

12	0.7990	-
13	1	1.2119
14	0.8200	-
15	0.6574	-
16	0.6629	
17	0.8032	-
18	0.8097	-
19	0.8046	-
20	0.8040	-
21	0.8408	-
22	1	1.3765
23	0.8482	-
24	0.8866	-
25	0.8918	-
26	1	1.08998
27	0.8784	-
28	0.9598	-
29	1	Infeasible
30	0.8019	-
31	0.7522	-
32	0.9278	-
33	0.6618	-
34	0.9421	-

35	0.7990	-
36	1	Infeasible
37	0.7967	-
38	0.9583	-
39	1	1.2331
40	1	1.2443
41	1	1.0846
42	0.7409	-
43	0.9453	-
44	0.7762	-

As shown in the Table 4-5, only the DMUs which are efficient were evaluated using model (3.9) and their super efficiency was determined.

Table 4-6 gives the result for the output oriented model using models (3.12) and (3.10).

Table 4-6: Results of output oriented super efficiency

DMU	Φ_0	Φ_0^{VRS}
1	1.2847	-
2	1.2827	-
3	1	0.9285
4	1.2890	-
5	1.6079	-
6	1.0789	-
7	1.1990	-
8	1	1.2247
9	1	0.7386
10	1.3844	-
11	1	-
12	1.2893	-
13	1	Infeasible
14	1.1823	-
15	1.4411	-
16	1.0805	-
17	1.2297	-
18	1.2544	-
19	1.3076	-

20	1.2576	-
21	1.1998	-
22	1	Infeasible
23	1.154	-
24	1.4402	-
25	1.0713	-
26	1	Infeasible
27	1.1174	-
28	1.2211	-
29	1	0.6992
30	1.2135	-
31	1.2998	-
32	1.0864	-
33	1.5600	-
34	1.058	-
35	1.2144	-
36	1	0.9675
37	1.2045	-
38	1.0369	-
39	1	Infeasible
40	1	Infeasible
41	1	Infeasible
42	1.2848	-

43	1.2148	-
44	1.2614	-

In order to obtain a combined ranking the steps in equation (3.14) and (3.15) need to be applied.

Out of 44 DMUs, 12 DMUs were considered efficient by both input and oriented models.

Super efficiency is then determined for these 12 DMUs

Then these DMUs are ranked by applying equations (3.16) and (3.17) which gives a combined ranking of input and output oriented models.

The final results are tabulated in Table 4-7.

Table 4-7: Ranking of super efficient units

DMU	θ_0	θ_0^{VRS}	$\tilde{\theta}_0^{VRS}$	Φ_0	Φ_0^{VRS}	$\tilde{\Phi}_0^{VRS}$	γ_0	τ_0	S_0	$\widehat{S}_0$	Rank
3	1	Infeasible	Infeasible	1	0.9285	NA	1	0.9285	1.0387	0.9640	10
8	1	1.0492	NA	1	1.1147	NA	1.0492	1.1147	0.9733	1.0336	12
9	1	1.2071	NA	1	0.7386	NA	1.2071	0.7386	1.2810	0.7832	2
11	1	1.3302	NA	1	0.8138	NA	1.3302	0.8138	1.2800	0.7824	1
13	1	1.2119	NA	1	Infeasible	Infeasible	1.2119	1	1.1059	0.9125	7
22	1	1.3765	NA	1	Infeasible	Infeasible	1.3765	1	1.1880	0.8633	4
26	1	1.0899	NA	1	Infeasible	Infeasible	1.0899	1	1.0445	0.9591	8
29	1	Infeasible	Infeasible	1	0.699203	NA	1	0.699203	1.2153	0.8495	3
36	1	Infeasible	Infeasible	1	0.96755	NA	1	0.96755	1.0170	0.9835	11
39	1	1.2331	NA	1	Infeasible	Infeasible	1.2331	1	1.1165	0.9055	6
40	1	1.2644	NA	1	Infeasible	Infeasible	1.2644	1	1.1320	0.8955	5
41	1	1.0846	NA	1	Infeasible	Infeasible	1.0846	1	1.0420	0.9612	9

After analyzing the four models we can conclude that no one model gives you the same result. Each of the four models provides different functionality and different results.

4.4 Comparisons of DEA models:

In this section, the results of each model are analyzed and the advantages and limitations for them are explained. A comparative study is made between the models to better understand the use of each model.

4.4.1 Summary of Results:

4.4.1.1 Results of CVDEA Model:

This primary use of this model is its ability to discriminate better than older DEA models like the CCR models. From the results, only 4 out of the 44 DMUs are deemed efficient. The efficient DMUs are 3, 9, 11 and 39.

When determining the relative efficiency the weights are chosen by the linear program such that a DMU is given the most beneficial efficiency score. Hence an efficient DMU will be operating with a lesser weighted sum of inputs per weighted sum of outputs. Hence the results of the weights for the 4 relatively efficient DMUs are determined when comparing each one of them with the rest of the DMUs and there is no correlation between their weights.

The only advantage of this method is the fact it renders fewer DMUs efficient than older DEA models. The limitations would be the number of constraints involved which increases with the number of DMUs when compared to the MCDM model whose number of constraints is determined by the number of inputs and outputs. Also the fact that too many DMUs have their output weight as zero throws light on the fact that equal importance has been given to the achievement of the two outputs which are RVU and total revenue.

4.4.1.2 Results of MCDM Model:

The three objectives in the MCDM model were given equally priorities in the case study to avoid a biased result. A DMU is deemed efficient only if d_o=0. Hence only DMUs 1 and 39 are efficient.

The advantage of this model is it provides better discrimination and also it incorporates three objectives. This additional restrictiveness is provided by the *minmax* and *minsum* objectives. As put forth by Li and Reeves [1998], if a DMU is *minmax* and *minsum efficient*, then it should be DEA efficient with d_o=0 as *minmax* and *minsum* conditions require d_o=0. But also, a DMU can be DEA efficient and still not be *minmax* and *minsum* efficient. That is, d_o=0 does not signify that M or $\sum d_j$ is minimized. Hence *minmax* and *minsum* objectives yield more restrictive results.

The disadvantage of this model is that one optimal solution cannot be generated. But the decision maker can always choose from the non dominated solutions generated. Secondly, the output weights still show insignificant weights when compared to the input weights. Also, from Table 3-5, it can be noted that even with unequal weights there was no significant difference results between scenario 1 and scenario 2.

4.4.1.3 Results of the AHP DEA Model:

The objective of the AHP DEA model is to suggest methods for performance improvement for a DMU. A DMU is deemed efficient if it does not have any input excesses, s_i where i=1, 2, 3 or output deficits e_r where r=1, 2. The results of the case study for this model showed DMUs 3, 9, 11, 13, 39 and 41 as efficient.

It can be noted that the model suggested an output augmentation for 42 out of the 44 DMUs while simultaneously reducing their inputs in order to make them more efficient. This reflects the weights chosen for the pairwise comparisons made earlier.

The advantage of this model is that it provides more than a classification of efficient and inefficient DMUs. The concept of performance measurement in the model will help decision makers greatly. Also it incorporates value judgment in the model and hence a decision maker has the power to give an educated preference to one input or output over the others. The disadvantage however is that this preference information should be provided by experienced people. Otherwise, the model might not provide meaningful results.

4.4.1.4 Results of the Super Efficiency Model:

The ranking provided by this model incorporates both input and output oriented models. The main objective of this model is to show that ranking of super efficient DMUs can be done even in the presence of infeasibility. Hence a multi stage procedure was put forth by Chen [2004].

The results of the input and output oriented ranking for efficient DMUs yielded same results with the exact 12 DMUs deemed efficient in both models. Then the variable returns to scale super efficiency was determined for both sets of efficient DMUs. This reduced the number of efficient DMUs from 12 to 9. These nine DMUs are then ranked according to their super efficiency. γ_o and τ_o represent the scores for characterizing the input and output super efficiencies. S_o and $\hat{S}_o$ provide scores to integrate these two values to provide a super efficiency ranking. The results give DMU 11 the highest rank among the 12 super efficient DMUs.

It should be noted that if a DMU is deemed infeasible by $\tilde{\theta}_o^{VRS-\sup er}$ and $\tilde{\Phi}_o^{VRS-\sup er}$, then its super efficiency score is 1. This is how the infeasible DMUs are incorporated in the model.

4.4.2 Comparison of DEA models:

The four DEA models discussed in the case study have different objectives and generate different results. Hence a comparison of the models based on its functionality is given in Table 4-8.

Table 4-8: Comparison of DEA models

Capability	CVDEA model	MCDM model	AHP model	Super efficiency model
Improved discrimination between DMU's	✓	✓	✓	✓
Input and output weights	✓	✓		
Preference information			✓	
Multi objective		✓		
Ranking				✓
Performance Measurement			✓	

As shown in Table 4-8, all four models can be used to improve discrimination among the 44 DMUs. But the best discrimination capability can be attributed to the MCDM as it rendered only two DMUs efficient. The AHP DEA model gives additional capabilities like performance measurement and the ability to add preference information which is not present in the rest of the models. The ability of the MCDM model to analyze multiple objectives is also unique, when comparing with the other models. Finally, incorporating super efficiency and ranking is done only in the Super Efficiency model, which makes it different from the rest of the models.

Similar results were also generated by the models. For example the CVDEA model and the AHP DEA model both determine that DMU 3, 9, 11 and 39 as efficient DMUs. The Super Efficiency model determined 12 efficient DMUs and it did include the DMUs 3, 9, 11 and 39. Hence there is consistency of results among the models.

4.4.3 Recommendations to Health Care Providers and Managerial Insights:

After conducting an analysis on the results of the four models in the case study, the author recommends the use of the AHP DEA model as it has the added ability to suggest performance improvement in terms of input excesses and outputs deficits. This can help a health care provider run its units more efficiently. For example, DMU 14 is an inefficient DMU according to the case study. But it has been suggested by the model that it can become efficient if it makes the following changes:

s_1: increase the number of input providers by about 2 units

s_2: make no change to the total building expenses

s_3: make no change to the number of support staff

e_1: increase the RVU’s by 7929.34 units

e_2: increase the total net revenue by 757902.35 units

Also, if a narrower ranking of units is needed, the Super Efficiency model is very useful. Many health care units might be deemed efficient as they use similar inputs and outputs. Also the weights attached to them also might be similar since most health care providers function alike. To provide the best among a group of efficient units the Super Efficiency method can be used.

The CVDEA model, although useful for improved discrimination, is outperformed by the other models as it provides a generic efficiency classification. The AHP model provides the same classification with added functionality and hence the CVDEA model might deliver redundant results.

The MCDM model can be used by health care providers only if multiple objectives are to be incorporated. Otherwise, preference information, such as the reduction of expenses and increase of revenue for a health care unit, can always be added in the form of pairwise comparison in the AHP DEA model.

To conclude the analysis of the case study, the AHP DEA model and the super efficiency model come off better to determine the efficiency and ranking of health care providers.

Chapter 5

CONCLUSION AND FUTURE RESEARCH

5.1 Summary

The objective of this thesis was to provide a comparative study between the chosen DEA models. Chapter 1 gave an introduction to the concept of DEA and the classical DEA models developed initially, such as the CCR, BCC and the Additive models. An extensive literature review was conducted in chapter 2 on some of the previous work in DEA. It covered topics related to the four models chosen in the thesis which are (i) CVDEA model, (ii) MCDM DEA model, (iii) AHP DEA model, (iv) Super Efficiency model. It also covered papers which addressed weight restriction concepts and ranking methods in DEA. The rise in the concept of super efficiency and the main problem associated with it, namely infeasibility, was addressed. It was proven that even with infeasibility, ranking of units could be performed and the model chosen for this successfully ranks the units. Also, models proposing methods to overcome the weight restriction problem in DEA were also detailed in the literature review.

Chapter 3 explained in detail the four models used in this thesis. An illustrative data set is used to evaluate them and understand their functionality. Chapter 4 introduced a case study on the efficiency of health care providers along with a brief introduction to health care industry concepts. The rise in the use of mathematical tools to rank and measure performance of health care providers and the reasons behind it were speculated. The importance of using tools like DEA in the health care industry was elaborated. The four models were used along with the data to provide meaningful results on the provider's efficiency and performance measurement.

5.2 Conclusions

A comparison between the results of the case study for the four models showed consistent and meaningful results. But from a management perspective in the health care industry, two models were found to be more useful among the four models. They are the AHP DEA model and the Super Efficiency model. The AHP DEA model is versatile as it allows for a decision maker to incorporate value judgments in the model. It also provides a health care provider with performance measurement techniques. It can help a provider to determine which inputs are being over utilized and which outputs are underperforming. The super efficiency model is useful as it provides a detailed ranking of the health care units based on super efficiency. Its advantage is that it finds the best provider among a group of already efficient providers.

The CVDEA model is a basic DEA model which provides better discrimination than older DEA model and does not provide more functionality like the AHP DEA model. The MCDM model is useful in terms of reducing the number of DMUs under consideration or increasing discriminating characteristics when evaluating DMUs. If multiple objectives are present, they can be incorporated as preferences in the AHP DEA model.

Finally, the fact that DEA is a non parametric model helps greatly in the case study as we involve inputs and outputs with very different units of measurement. Its ability to handle large number of inputs and outputs also is a plus to use DEA in the health care industry.

5.3 Future Research

Data envelopment analysis has evolved into a vast topic with a multitude of applications. Even though researchers question the robustness of DEA models and compare it with regression analysis and stochastic methods, it still proves to be a good choice.

The health care industry has been the most active users of DEA but it still can exploit the method to a larger extent. It is noted by many researchers that very little research has been done in the areas of quality of health care providers. Morey et al (1992) is one of the few papers which incorporate the concept of quality in health care evaluation.

The MCDM DEA model used in this thesis can be extended by solving the three objectives simultaneously to provide multiple non dominated solutions. The decision maker can then choose from these results. The AHP DEA model can use preference information from experienced health care providers and can be evaluated again to compare the current efficiency classification.

The increased use of stochastic methods and hypothesis testing combined with DEA is another option for evaluation. It has been researched in the past and has more scope for improvement. The main disadvantage to this approach is the fact that DEA is a non parametric method and implementation of hypothesis testing is difficult.

Also, DEA will be more useful if it is able to incorporate economical efficiency instead of just producing technical efficiency. This is due to the fact it does not have a production function. If a ‘cost saving’ analysis can be incorporated, it will improve the usability of the model. Finally even with these problems, DEA is an extremely useful tool for analyzing entities to determine the relative efficiency among the entities of an industry.

APPENDIX

References:

Adler, N., Friedman, L. & Siuany-Stern, Z., (2002), *Review of Ranking Methods in the Data Envelopment Analysis Context, European* Journal of Operations Research, Vol. 140, pp. 249-265

Andersen, P. & Petersen, N.C., (1993), *A Procedure for Ranking Efficient Units in Data Development Analysis*, Management Science, Vol. 39, No. 10, pp. 1261-1264

Angulo-Meza & Estelliantos, (2002), M.P., *Review of methods for increasing discrimination in data envelopment analysis*, Annals of Operations Research, Vol. 116, pp. 225-242

Banker, R.D., Charnes, A., Cooper, W.W., (1984), *Some models for estimating technical and scale inefficiencies in data envelopment analysis*, Vol. 30, No. 9, pp. 1078-1092

Banker, R.D., Morey, R., (1986a), *Efficiency Analysis for exogenously fixed inputs and outputs*, Operations Research, Vol. 34, No.4, pp. 513-521

Belton, V. & Vickers, S.P., (1993) *Demystifying DEA- A visual interactive approach based on Multiple Criteria Analysis*, The Journal of the Operational Research Society, Vol. 44, No. 9, pp. 883-896

Bogetoft, P. & Hougaard, J.L., (1999), *Efficiency evaluations based on potential (non-proportional) improvements*, Journal of Productivity analysis, Vol. 12, No. 3

Bogetoft, P. & Hougaard, J.L., (2002), *Super efficiency evaluations based on potential slack*, European Journal of Operations Research, Vol. 152, pp. 14-21

Bouyssou, D., (1999), *Using DEA as a tool for MCDM: Some Remarks*, The Journal of the Operations Research Society, Vol. 50, No. 9, pp. 974-978

Charnes, A., Cooper, W. W., Golany, B., Seiford, L., (1985), Stutz, J., *Foundations of data envelopment analysis for Pareto-Koopman's efficient empirical production functions*, Journal of Econometrics, Vol. 30, pp. 1-17

Charnes, A., Cooper, W.W., Huang, Z.M., & Sun, D.B., (1990) , *Polyhedral cone-ratio models with an illustrative application to large commercial banks,* Journal of Econometrics, Vol. 46, pp. 73-91

Charnes, A., Cooper, W. W., Lewin, A.Y., Seiford, L., (1994), *Data Envelopment Analysis: Theory, Methodology and Applications*, Kluwer Academic Publishers, Boston, MA

Charnes, A., Cooper, W. W., & Rhodes, E., (1978), *Measuring the efficiency of decision making units*, European Journal of Operational Research, Vol. 2, pp. 429–444

Chen, Y., (2004), *Ranking efficient units in DEA*, The International Journal of Management Science, Vol. 32, pp. 213-219

Chillingerian, J. & Sherman, D., (1990), *Managing Physician efficiency and effectiveness in providing hospital services,* Health Services Management Research, Vol. 3, pp. 3-15

Donabedian, A. Wheeler & Wyszewianski, J. R., (1982), *Quality, cost and health: An integrative model*, Medical Care Vol. 20, pp. 975-992

Doyle, J. & Green, R., (1993), *Data envelopment analysis and Multi Criteria Decision Making*, International Journal of Management Science, Vol. 21, No. 6, pp. 713-715

Dyson, R.G. & Thanassoulis, E., (1998), *Reducing weight flexibility in data envelopment analysis*, The Journal of Operations Research, Vol. 39, No. 6, pp. 563-576

Eric, M.G., (2004), *Data Envelopment Analysis: Benchmarking and Comment*, The Pennsylvania State University

Farell, M.J., (1957), *The measurement of productive efficiency*, Journal of Royal Society, Vol. 120, No. 3

Fetter, R.B., (1991a), *Diagnosis Related Groups Understanding hospital Performance, Interfaces* Vol. 21, No. 1, pp. 6-26

Francisco, P., Javier, S. & Peter, S., (1997), *On the role of weight restrictions in data envelopment analysis*, Journal of Productivity, Vol. 8, pp. 215-230

Frank, R.G., (1988), *On making the illustration illustrative: a comment on Banker, Conrad and Strauss,* Management Science, Vol. 34, pp. 1026-1029

Franklin Liu, F.H. & Hai, H.L., (2005), *The Voting Analytical Hierarchy Process method for selecting supplier*, International Journal of Production Economics, Vol. 97, pp. 308-317

Fried, H.O., Lovell, C.A.K., (1996), Searching the Zeds, Working paper presented at II Georgia productivity workshop

Friedman, L. & Siuany-Stern, Z., (1997), *Scaling units via the canonical correlation analysis in the DEA context,* European Journal of Operations Research, Vol. 100, No. 3, pp. 629-637

Golany, B., (1988), *An Interactive MOLP Procedure for the Extension of DEA to Effective Analysis*, The Journal of the Operational Research Society, Vol. 39, No. 4, pp. 725-734

Halme, M., Joro, T., Korhonen, P., Salo, S., Wallenius, J., (1999), *A Value Efficiency Approach to Incorporating Preference Information in Data Envelopment Analysis*, Management Science, Vol.45, No. 1, pp. 103-115

Hasan, B., Hasan, O. & Salih, C., (2008), *A new method based on dispersion of weights in data envelopment analysis,* Computers and Industrial Engineering, Vol. 54, pp. 502-512

Jahanshaloo, G.R., Memarirani, A., Hosseinzadeh, F. & Shoja, N., (2005), *A feasible interval for weights in data envelopment analysis*, Applied Mathematics and Compuation, Vol. 159, pp. 155-168

Johnson, E.A & Johnson, R.L., (1986), *Hospitals under fire: Strategies for success,* Rockville, Maryland: Aspen

Joro, T., Korhonen, P. & Wallenius, J., (1998), *Structural comparison of data envelopment analysis and multiple objective linear programming*, Management Science, Vol. 44, No. 7, pp. 962-970

Kao, C., *A Linear Formulation of the Two-Level DEA Model*, (2008), The International Journal of Management Science, Vol.36, pp. 958-962

Kooreman, P., (1994), *Data envelopment analysis and parametric front estimation: complimentary tools,* Journal of health Economics, Vol. 13, pp. 345-346

Kornbluth, J.S.H., (1991), *Analyzing Policy Effectiveness using Cone Restricted Data Envelopment Analysis*, The Journal of the Operational Research Society, Vol. 42, No. 12, pp. 1097-1104

Liam O' N., (1997), *Nonparametric diagnostics in Data Envelopment Analysis with an application to hospital efficiency*, The Pennsylvania State University

Meng, W., Zhang.D., Qi.L., Liu.W., (2008), *Two-level DEA approaches in research evaluation, The internal Journal of Management Science*, Vol. 36, pp. 950-957

Morey, R.C., Fine, D., Loree, S.W., Retzlaff-Roberts, D., & Tsubakitani, S., (1992), *The Trade-Off between Hospital Quality and Cost: An Exploratory Empirical Analysis*, Medical Care, Vol. 30, No.8, pp. 677-698

Muniz, M.A., (2001), *Separating managerial inefficiency and external conditions in data envelopment analysis*, European Journal of Operations Research, Vol. 143, pp. 623-643

Ozcan, Y.A., (1993), *Sensitivity analysis of hospital efficiency under alternative output/input and peer groups: A Review,* The International Journal of Knowledge and Public Policy, Vol. 1, pp. 1-31

Ozcan, Y.A., Luke, R., Haksever, C., (1992), *Ownership and organizational performance, A comparison of technical efficiency across hospitals*, Medical Care, Vol. 30, pp. 781-794

Pedraja-Chaparro, F., Salinas-Jimenez, J., Smith, P., (1997), *On the Role of Weight Restrictions in Data Envelopment Analysis*, Journal of Productivity Analysis, Vol. 8, pp. 215-230

Ramanathan, R., (2006), *DEA for weight derivation and aggregation in the analytical hierarchy process*, Computers and Operations Research, Vol. 33, pp. 1289-1302

Roll, Y., Cook, W., Golany, B., (1991), *Controlling Factor Weights in DEA*, IIE Transactions, Vol. 23, pp. 2-9

Saaty, J.L., (1980), *The Analytical Heirarchy Process*, European Journal of Operations Research, Vol. 48, No.1, pp. 9-26

Seiford, L.M., Zhu, J., (1998a), *Stability Regions for maintaining efficiency in Data Envelopment Analysis,* European Journal of Operational Research, Vol. 108, pp. 127-139

Seiford, L.M., Zhu, J., (1998b), *Sensitivity Analysis of DEA models for simultaneous changes in all the data*, Journal of the Operational Research Society, Vol. 49, pp. 1060-1071

Seifert, M.L., Zhu, J., (1998), *Identifying Excesses and Deficits in Chinese Industrial Productivity (1953-1980): A Weighted Data Envelopment Analysis Approach*, International Journal of Management Science, Vol. 26, No. 2, pp. 279-296

Seiford, L.M., Zhu, J., (1999), *Infeasibility of Super Efficiency Data Envelopment Analysis models*, INFOR, Vol.37, pp. 174-187

Shang, J., Sueyoshi, T., (1995), *A unified framework for the selection of a Flexible Manufacturing System*, European Journal of Operations Research, Vol. 85, pp. 297-315

Silkman, R.H., (1986), *Measuring Efficiency: An Assessment of Data Envelopment Analysis*, Jossey-Bass Inc. Publications

Sinuany-Stern, Z., Friedman, L., (1998), *Data Envelopment Analysis and the discriminant analysis of ratios for ranking units*, European Journal for Operational Research, Vol. 111, pp. 470-478

Sinuany-Stern, Z., Mehrez, A. & Hadad, Y., (2000), *An AHP/DEA methodology for ranking decision making units*, International Transactions in Operations Research, Vol.7, pp. 109-204

Sinuany-Stern, Z., Weiner, I., (1994), *The one dimensional cutting stock problem using two objectives*, Journal of the Operations Research Soceity, Vol. 45, No. 2, pp. 231-236

Stewart, T.J., (1992), *A critical survey on the status of multiple criteria decision making theory and practice*, Omega, Elsevier, Vol. 20, pp. 569-586

Stewart, T.J., (1996), *Relationship between Data Envelopment Analysis and Multi criteria decision Analysis*, The Journal of the Operational Research Society, Vol. 47, No.5, pp. 654-665

Thanassoulis, E., Allen, R., (1998), *Simulating weight restrictions in data envelopment analysis by means of unobserved decision making units*, Management Science, Vol. 44, No. 4, pp. 586-594

Thanassoulis, E., Dyson, R.G., (1992), *Estimating preferred target input-output levels using DEA*, European Journal of Operations Research, Vol.56, pp. 80-97

Thomson, R.G., Singelton, F.D. Jr., Thrall R.M., Smith, B.A., (1986), *Comparative site evaluations for locating a high-energy physics lab in Texas*, Interfaces, Vol. 16, pp. 35-49

Torgesen, A.M., Forsend, F.R., Kittelsen, S.A.C., (1996), *Slack-adjusted efficiency measures and ranking of efficient units*, The Journal of Productivity Analysis, Vol. 7, pp. 379-398

Wang, Y.M., Chin, K.S., (2008), *A new Data Envelopment Analysis method for priority determination and group decision making in the Analytical Hierarchy process*, European Journal of Operations Research, Vol. 195, No. 1

Wang, Y.M., Liu, J., Elhag, T.M.S., (2008), *An integrated AHP- DEA methodology for bridge risk assessment*, Computers and Industrial Engineering, Vol. 54, pp. 513-525

William, S., Torrens, P., (1993), *Introduction to Heath Services*, 4th edition

Wong, H.B., Beasley, J.E., (1990), *Restricting weight flexibility in DEA*, Journal of Operations Research, Vol. 41, No. 9, pp. 829-835

Xiao-Bai, L., Gary, R. R., (1999), *A multiple criteria approach to data envelopment analysis,* European Journal of Operational Research, Vol. 115, pp. 507- 517

Xue, M., Harker, P.T., (2002), *Ranking DMUs with Infeasible Super Efficiency DEA models*, Management Science, Vol. 48, No. 5, pp. 705-710

Zhu, J., (2001), *Super Efficiency and DEA sensitivity analysis*, European Journal of Operations Research, Vol. 129, pp. 443-455