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AIRLINE REVENUE MANAGEMENT WITH CALLABLE PRODUCTS IN A MARKET OF SHRINKING MARGIN

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ABSTRACT

Since the Airline Deregulation Act was passed in 1978, revenue management (RM) system has been markedly developed, and widely utilized by airlines. They could improve their profit by maximizing revenue and minimizing cost based on the RM techniques. RM approaches were adopted even in other industries where their usefulness was well recognized. As a result, airline industry became more efficient, which caused diminished arbitrage opportunities in the market. Moreover, the advent of information era enabled customers to easily search and move to a lower-fare and higher-quality provider, using internet access. In such a tightening market situation, shrinking margin forces the management of airlines to take greater risk in order to increase their profits.

Several researchers recently introduced the concept of callable product, called a promotional ticket in this thesis, that was risky but expected to increase profit. Called a promotional ticket in this thesis, The callable product enables an airline to make money by hedging potential spoilage and yield losses while satisfying customers' time-varying demands. In this thesis, first the value of the risk premium is priced first, under assumption that the product can be called at any time until expiration, which was not addressed in previous works. Next, the expected profit increases through the promotional ticket sales is examined. In addition, the matter of risk premium pricing was facilitated by assuming that ticket price follows geometric Brownian motion. Also, customer's demand is assumed to follow non-homogeneous Poisson process.

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CHAPTER 1

INTRODUCTION AND MOTIVATION

In this chapter, a brief introduction of revenue management, which is the foundation of this research, is given first, followed by an introduction of callable products, mainly spotlighted in this work. In addition, relevant works that have been conducted so far with respect to the callable products are introduced. The purpose and contribution of this research are presented next.

1.1 Revenue Management

In 1972, Littlewood proposed his famous seat capacity control rule that discount price bookings should be accepted as long as their revenue exceeds the expected revenue of future high-fare bookings. The rule helped the start of revenue management research. In 1978, Airline Deregulation Act was passed, removing any price restriction by airlines, followed by a salient growth in revenue management techniques. Accordingly, profits of airline companies remarkably increased (Fuchs, 1987). For instance, American Airlines made an additional annual profit of more than 500 million dollars using revenue management techniques (Smith et al. 1992). Its profit in 1997 exceeded one billion dollars and it represented most of the company's revenues (Cook 1998).

The concept of revenue management is, however, not only applied to the airline industry. It can be widely utilized in other industries having similar attributes, including car rental, hotel, broadcasting, IT and internet service, railway, retailing, etc. The common characteristics in revenue management practice are relatively fixed (inventory) capacity, perishable inventory, price fluctuation, availability to sell products in advance, ability to segment market, low marginal sales cost, and high marginal capacity change cost (Kimes, 1989). The industries having limited capacity – tourism, transportation, media, internet service providers, and entertainment – consistently face revenue management problems. These industries want to maximize their total revenue, by efficiently utilizing their restricted and perishable capacity. Revenue management provides a theoretical and practical tool to resolve this sort of problems.

The revenue management problems are largely categorized into several different areas such as pricing, overbooking, forecasting, and seat inventory control. Additional issues include customer behavior analysis, performance measurement, and economics.

McGill and Van Ryzin (1999) provide a glossary of revenue management terminology in their Appendix.

1.2 Callable Products

In the financial market, it is commonplace to see financial products being traded with embedded options. Those options are issued in a rich variety of forms, depending on the underlying assets such as stock, bond, interest rate, index, or currency, to name a few. A callable bond has a unique attribute that allows the bond issuer to retain the right of redeeming the bond at some time before the bond reaches its maturity. The presence of a call option is advantageous to both the issuer and the bondholder. First, the issuer can refinance his / her liability at a cheaper price by calling the bonds if interest rates go down in the market, which results in higher current price, though the issuer pays a higher coupon rate. Next, the investor or bondholder has the benefit of having a higher coupon than with a straight (non-callable) bond. On the other hand, if interest rates fall, the bonds get called and the investor can only reinvest at the lower rate.

The callable bond has two critical characteristics similar to the characteristics of products of interest in the field of revenue management; both have maturity and both their prices fluctuate as time goes by. Due to those similarities, several recently published research papers, have tried to introduce the call property to the field of revenue management in order to exploit the advantages of the callable bond.

Gallego (2004) initially proposed the notion and concept of callable products. Under the assumption that low-fare customers arrive earlier, an airline sells both callable product and non-callable product to the low-fare customers during the first selling period. Customers who purchased the callable products would give an airline the right (option) to recall the seat at a prescribed price at any time before the departure in the future. The airline will notify the customer who bought the seat about the recall and pay the recall price. The recall price will be higher than the low fare selling price and lower than high fare selling price. Accordingly, airline will exercise the option only if it concludes that high fare demands exceed available capacity (Gallego, 2004). For the recalled seats, an airline will pay the recall price and resell them at higher fare.

The revenue management model of callable products of Gallego (2004) stems from Biyalogorsky et al. (1999) and Biyalogorsky and Gerstner (2004). Biyalogorsky et al. (1999) defined two potential losses of yield loss and spoilage loss which are trade-offs that managers face when they quote a price. Yield loss implies selling a product at low-price, losing an opportunity to sell it later at better price. Spoilage loss is the airline’s loss that occurs when it waits for high price sales, missing previous low price offer. Thus, sellers will sell some of their products in advance in order to hedge the spoilage loss and block some products against the yield loss to sell them later at higher price. Such a hedging strategy, overselling with opportunistic cancellation, allows sellers more sales than available capacity with a compensation for the customers who give up the products. Gallego (2004) extended this concept of opportunistic cancellation to the 'callable' product, which is consistent with the call provision of the callable bond in the financial industry.

Such a callable product is not usually offered to customers in the airline industry . Even so, some authors consistently claim that the new revenue management model that contains the callable products would outperform earlier models in many aspects. After Gallego (2004), Lee (2006) proposed an overbooking model of callable products based on three-fare classes. Akgunduz et al. (2007) suggested a multiple-period, multiple-fare model of the callable product and of the puttable product. Ravelojaona (2008) priced the call and put option premium using financial option pricing theory – binomial option pricing model, assuming that the ticket price movement follows a random walk.

1.3 Positioning and Outline

Akgunduz (2007) improved the two-period model of Gallego (2004) to the more generalized multi-period model, under the assumption that the call option can be exercised during the last period only. Ravelojaona (2008) also made this assumption and defined the embedded options using European-style option framework; that is, the airline which issued a callable product could not redeem it before the date of maturity. Despite many significant contributions of those works, this assumption limits the most distinct benefit of the callable products, which is an early redemption. In contrast, the models of Biyalogorsky et al. (1999) and Gallego (2004) were free from this assumption, since their models were basically of the two-period structure; sales during the first period, followed by second period recall. As a result, this thesis proposes another multi-period model that allows recall during any time period until maturity, so as to relax the assumption and bring the concept of callable products closer to the reality.

Determining demand distribution and customers' arrival behavior is one of the key issues in the field of revenue management, because of the uncertainty involved in these. Recent papers have consistently insisted that the demand which follows non-homogeneous Poisson process (NHPP) fits real demand well, and hence this thesis uses NHPP to represent the customer's demand behavior. Even though Gallego (2004) modeled the demand distribution using NHPP, its advantages were not indeed spotlighted in his work, since the model was designed as a two-period one. On the other hand, Akgunduz (2007) and Ravelojaona (2008) did not use a specific demand distribution. They used self-constructed set of customers' arrival rate for each period.

One of the biggest uncertainties comes from change in ticket price. Empirical investigations reveal that the ticket price tends to rise as time goes. In that sense, lognormal process, widely known as geometric Brownian motion (GBM), seems to fit the ticket price well. Accordingly, the ticket price movement in this thesis is assumed to follow GBM. Meanwhile, Gallego (2004) and Akgunduz(2007) simply used likely sets of ticket price for each fare class, while Ravelojaona (2008) assumed binomial movement of ticket price, called a random walk. The binomial setting of the ticket price movement facilitated valuating call- and put-option premiums.

A revenue management model is developed in Chapter 2 with an objective function and necessary constraints. The step-by-step description of the model building process will clarify the logic behind the model. Numerical results follow in Chapter 3. Also, demand and ticket price models are built, and sensitivity analyses come next.

Chapter 4 summarizes the thesis, explicitly specifies not only the contributions but also limitations of this work and makes suggestions for possible future extensions.

CHATPER 2

MODEL DEVELOPMENT

Callable ticket sales improve an airline's total expected profit from the single-flight, single-fare and multiple-period profit model that is built in this chapter. In addition, a callable ticket is called a promotional ticket and a non-callable ticket is called a general ticket in order to help understanding how the designed model runs. Both terms are employed in all the following chapters in this thesis.

2. 1. Model Description

An airline sells C tickets for one of its flights, which departs T periods later. Among the C tickets, v tickets are promotional (callable) ones and sold only during the first period, t = 1, while C-v general (non-callable) tickets are sold until the last period of flight departure, that is until t = T. It is assumed that ticket price follows a Geometric Brownian motion which shows an upward sloping trend. Promotional tickets have the following characteristics. During the first selling period, the promotional tickets are sold at a price which is less than general ones by an amount equal to the risk premium, r_p. Then, the airline can recall the outstanding tickets from the customers who purchased them at any time until the date of departure, but at prescribed price higher than initial sales price, called recall price, R. It is assumed that the airline wants to maximize its profit and hence, it would not recall outstanding promotional tickets unless ticket price goes higher than the recall price R, since otherwise the ticket sales generates negative revenue.

2.2. Model Assumptions

The assumptions made in this model are listed below.

(1) It is assumed that the promotional tickets are sold only during the first period of the ticket

sales. This is a reasonable assumption, since the model is constructed to show how the sales of promotional tickets improve the airline's overall profit. The net effect that the outstanding promotional tickets cause since the promotional tickets are distributed, can be observed.

(2) It is assumed that recall is not exercised during the first period, and that the recall price R is greater than the initial ticket price S_1. Otherwise, the airline can earn immediate profit by exercising the call option embedded in the promotional tickets, which is an arbitrage opportunity.

(3) Promotional tickets are called to meet unsatisfied high-fare demands, only if the airline possesses no general tickets available to sell during a certain period.

(4) Overbooking is allowed and it is applied only to general ticket sales. The overbooking policy enables the airline to sell more than it eventually has by the amount augmented by the no-show rate; this is done to hedge the uncertainty in demand during the entire sales period.

(5) Singe fare class is assumed for the model construction. A multiple-fare model is built with price and demand streams for each fare class, adding an additional dimension to the model.

2.3 Notations

T	The date of departure a certain flight
C	The number of available tickets (or seats) for sales - capacity - of the flight
R	Recall price of a promotional ticket (decision variable)
u	The number of outstanding promotional tickets, sold during the first period only (decision variable)
S_1	Ticket price during the first period of sales
S_t	Ticket price during each period t for all t = 1,2, ..., T

D_t	Customer demands during the period t for all t = 1,2, ..., T
v_t	The number of general tickets sold during the period t for all t = 1,2, ..., T
w_t	The number of promotional tickets recalled and resold during the period t for all t = 1,2, ..., T.
r_p	Risk premium of the promotional ticket
r_m	Market interest rate
p_{ns}	The probability of customers not showing up at the airport at the date of departure T
n_{db}	The number of denied boardings
c_{db}	Unit cost of denied boardings

2.4 Model Formulation

In this section, the model is constructed ; an objective function is formulated first followed by necessary constraints.

Objective Function

The objective of this research is to determine the optimal number of promotional tickets, u, that will be offered for sales and its recall price, R, in order to maximize the airline's expected profit. Four sources of revenue are considered. During the first period, an airline earns money from sales of general tickets equal to $S_1 \cdot v_1$, and from promotional tickets sales that amounts to $(S_1 - r_p) \cdot u$. The customers pay less for the promotional tickets because they take callable risk embedded in the tickets. Thereafter, the airline makes money from the sales of general tickets equal to $\sum_{t=2}^{T} [E(S_t) \cdot v_t] \,/\, [(1 + r_m)^{t-1}]$. When there is excess demand, the airline recalls corresponding number of outstanding promotional tickets and earn an extra revenue equal to $\sum_{t=1}^{T} [E(S_t) \cdot w_t]/[(1 + r_m)^{t-1}]$. But there is a cost associated with this which is equal to $\sum_{t=1}^{T}[R \cdot w_t]/[(1+r_m)^{t-1}]$. In addition, during the very last period, the airline incurs a cost due to customers who show up and are denied boarding, equal to $[c_{db} \cdot n_{db}]/[(1+r_m)^T]$. In summary, the expected profit, $E(P)$, is as follows:

$E(P) = (S_1 - r_p) \cdot u$; revenue that comes from promotional ticket sales during the first period

$+ (S_1 \cdot v_1)$; revenue that comes from general ticket sales during the first period

$+ \sum_{t=2}^{T} \frac{E(S_t) \cdot v_t}{(1+r_m)^{t-1}}$; revenue that comes from general ticket sales during the remaining periods

$+ \sum_{t=2}^{T} \frac{E(S_t) \cdot w_t}{(1+r_m)^{t-1}}$; revenue that comes from promotional tickets that are called and immediately resold during the remaining periods

$- \sum_{t=2}^{T} \frac{R \cdot w_t}{(1+r_m)^{t-1}}$; costs incurred from promotional tickets recalled during the remaining periods

$- \frac{c_{db} \cdot n_{db}}{(1+r_m)^T}$; cost incurred because of denied boarding on the date of departure

In this, the two decision variables are– the number of promotional tickets sold during the first period, u, and the recall price, R.

Constraints

The amount of ticket sold depends upon the expected customer demand and the remaining capacity during each time period. The expected demand will be derived in the following chapter. In this section, the number of the general tickets and promotional tickets recalled to be sold during each time period and the number of denied boarding specified at the date of departure will be derived, assuming that the expected demand is known.

1) The number of general tickets sold during sales period t, v_t, for t = 1,2, …, T.

The total number of general tickets available for the entire sales periods depends upon the seat capacity of a flight and the initial sales volume of the promotional tickets. The seat capacity, which is equivalent to the number of tickets that the airline initially holds for sale, is augmented by the no-show rate. The overbooking policy allows the airlines to accommodate more ticket requests based on the no-show rate estimated from historical data.

Total number of general (non-callable) tickets

= total capacity on sales – total number of outstanding promotional (callable) tickets

$$= C/(1-p_{ns}) - u \tag{2-1}$$

As the date of departure, T, approaches, the number of tickets held for sale steadily decreases by an amount equal to the number of general tickets sold during each period. That is, the right hand side of equation (2-1) less the number of outstanding general tickets determines the number of general tickets available for sales during time period t, given in the following.

The number of general tickets available for sale during period t

= total number of general tickets – the number of general tickets sold during the first t-1 periods.

$$= C/(1-p_{ns}) - u - \sum_{i=1}^{t-1} v_i \tag{2-2}$$

The initial sales of the general tickets are easily determined from the minimum value of the expected demand for general tickets during the first period and the total number of general tickets available for the entire sales period. The initial general ticket sales is then

$$v_1 = min\left[E(D_1), C/(1-p_{ns}) - u\right] \tag{2-3}$$

Likewise, the minimum value between the expected demand and the number of general tickets available for sale during the period t, which is specified in (2-2), determines the number of general tickets sold during period t, t = 2,3, …,T.

$$v_t = min\left[E(D_t), C/(1-p_{ns}) - u - \sum_{i=1}^{t-1} v_i\right], \forall t \in [2, T] \qquad (2\text{-}4)$$

In summary, the number of general tickets sold during each period is determined by the expected demand while not exceeding available capacity augmented by the no-show rate.

2) The number of promotional tickets recalled during sales period t, w_t, for t = 1,2, …, T.

Since it is assumed that general tickets are recalled before promotional tickets to satisfy unsatisfied demand if an airline makes a recall request, outstanding promotional tickets will be called only if the expected demand exceeds the number of general tickets available for sale during a certain period of time. Hence, the unsatisfied demand during period t is to be determined first, which is the positive difference between the expected demand and the number of general tickets for sale during the period t.

Unsatisfied demand during period t

= expected demand – number of general tickets for sale during period t

$$= E(D_t) - \left(C/(1-p_{ns}) - u - \sum_{i=1}^{t-1} v_i\right)$$

$$\text{if } E(D_t) > \left(C/(1-p_{ns}) - u - \sum_{i=1}^{t-1} v_i\right) \qquad (2\text{-}5)$$

This is summarized as

Unsatisfied demand during the period t

$$= max\left[0, E(D_t) - \left(C/(1-p_{ns}) - u - \sum_{i=1}^{t-1} v_i\right)\right] \qquad (2\text{-}6)$$

Next, in order to meet the unsatisfied demand during period t, the airline needs to have enough capacity. The capacity, which is the remaining number of promotional tickets during period t, is calculated as the difference between the total number of promotional tickets and the number of promotional tickets sold until the previous period t-1.

Remaining number of promotional tickets on sale during period t

= total number of promotional tickets outstanding – number of promotional tickets sold until period (t-1)

$$= u - \sum_{i=1}^{t-1} w_i \qquad (2\text{-}7)$$

Using (2-6) and (2-7), the number of promotional tickets to be called and immediately resold during the period can be calculated, so as to meet the unsatisfied demand. This is mathematically represented as follows.

Number of promotional tickets sold during the period t

= min (Unsatisfied demand during the period t, remaining number of promotional tickets)

$$= min\left[max\left(0, E(D_t) - C/(1-p_{ns}) + u + \sum_{i=1}^{t-1} v_i\right), u - \sum_{i=1}^{t-1} w_i\right] \qquad (2\text{-}8)$$

However, the airline would never recall the promotional tickets unless the expected ticket price during a certain period $E(S_t)$ is less than the recall price R. A binary variable y_t is introduced in equation (2-8) resulting in

$$w_t = min\left[max\left(0, E(D_t) - C/(1-p_{ns}) + u + \sum_{i=1}^{t-1} v_i\right), u - \sum_{i=1}^{t-1} w_i\right] \cdot y_t,$$

$$\forall t \in [2, T] \qquad (2\text{-}9)$$

$$y_t = \frac{max\,[0, E(S_t) - R]}{E(S_t) - R}, \forall t \in [2, T] \qquad (2\text{-}10)$$

Equation (2-10) forces y_t to have a value of either zero or one only. One additional concern is that the promotional tickets will not be recalled during the first period, which is taken care of in the assumption that the recall price R is greater than the initial ticket price S_1.

3) The number of denied boardings during the last sales period, T.

Since overbooking is allowed, airlines accommodate more booking requests than they can, and hence, denied boarding occurs. Historical data has shown that only approximately $(1 - p_{ns}) \times 100$ percent of customers will indeed show up at the time of departure. But it was assumed that the holders of promotional tickets will always show up. Therefore, the total number of tickets outstanding is determined from the sum of the total number of promotional tickets outstanding and the total number of general tickets sold during the entire periods multiplied by the probability of show-up, $(1 - p_{ns})$.

Total number of tickets outstanding

= total number of promotional tickets outstanding + total number of general tickets sold during the entire periods x probability of show-up

$$= u + \sum_{t=1}^{T} v_t \cdot (1 - p_{ns}) \tag{2-11}$$

Also, there is another flow of tickets, which happens when the outstanding promotion tickets are recalled. But, this is not taken it into account; since the net amount of the in- and out- flow of tickets is zero, recalled tickets are assumed to be resold to the customers immediately.

As a result, the denied boarding occurs only if the total number of tickets outstanding exceeds the total capacity. The net difference determines the number of customers who are denied boarding. This condition can be represented as follows.

$$n_{db} = max\left[0, \left(u + \sum_{t=1}^{T} v_t\right) \cdot (1 - p_{ns}) - C\right] \tag{2-12}$$

Alternatively, using a binary variable y_{db}, (2-12) can be rewritten as

$$n_{db} = \left[\left(u + \sum_{t=1}^{T} v_t\right) \cdot (1 - p_{ns}) - C\right] \cdot y_{db} \tag{2-13}$$

$$y_{db} = \frac{max\left[0, \left(u + \sum_{t=1}^{T} v_t\right) \cdot (1 - p_{ns}) - C\right]}{\left(u + \sum_{t=1}^{T} v_t\right) \cdot (1 - p_{ns}) - C}. \tag{2-14}$$

CHAPTER 3

NUMERICAL RESULTS AND ANALYSIS

In chapter 2, a model was built that maximizes the expected profit by determining both the optimal number of tickets to be sold and the optimal recall price of the promotional tickets. But the performance of the model depends upon the accuracy of the forecast on demand and price. Moreover, the selected ticket price model greatly affects how to price a risk premium, which is one of the dominant factors in determining the model's performance. For those reasons, it was assumed that ticket price follows a geometric Brownian motion which appropriately fits actual ticket price movement and facilitates the risk premium valuation as well.

With the values of the expected demand, ticket price and risk premium, the optimum number of promotional tickets sale and recall price can now be determined, using the model developed in Chapter 2. At the same time, is to be realized that the parameters that characterize the demand and price are exposed to many different market variations. Hence, the sensitivity of the expected profit to the change in parameters reflecting diverse market situations will be examined now. Demand and price streams, generated using random number generators instead of using the expected values, would help in studying the impact of market uncertainty.

3.1 Demand Model

Demand model plays an important role in the models of the revenue management. In this section, the demand model is defined and the expected demand of each period will be derived. Then the estimation of parameters that characterize it will be described.

Non-homogeneous Poisson process

Non-homogeneous Poisson (NHPP) process is assumed for the demand model as it has been known to be the closest model to the real demand pattern, in most recent works. In this thesis, the NHPP with the following rate is assumed for the arrival process of ticket demand.

$$\lambda_t = D^{tot} \cdot \beta_t \tag{3-1}$$

where D^{tot} is a random variable that follows the Gamma distribution with a shape parameter γ and a scale parameter δ, and β_t is assumed to follow the standardized Beta distribution over the interval $[1, T]$ with two shape parameters α, β. Although this rate function was built mostly using empirical relevance, there is also a good theoretical reason, for using Gamma distributed random variable, which is simple parameter estimation. The density function of the gamma distribution is defined as

$$f(d) = \frac{\delta^{-\gamma}}{\Gamma(\gamma)} e^{-d/\delta} d^{\gamma-1} \tag{3-2}$$

where

$$\Gamma(\gamma) = \int_0^\infty e^{-x} x^{\gamma-1} dx.$$

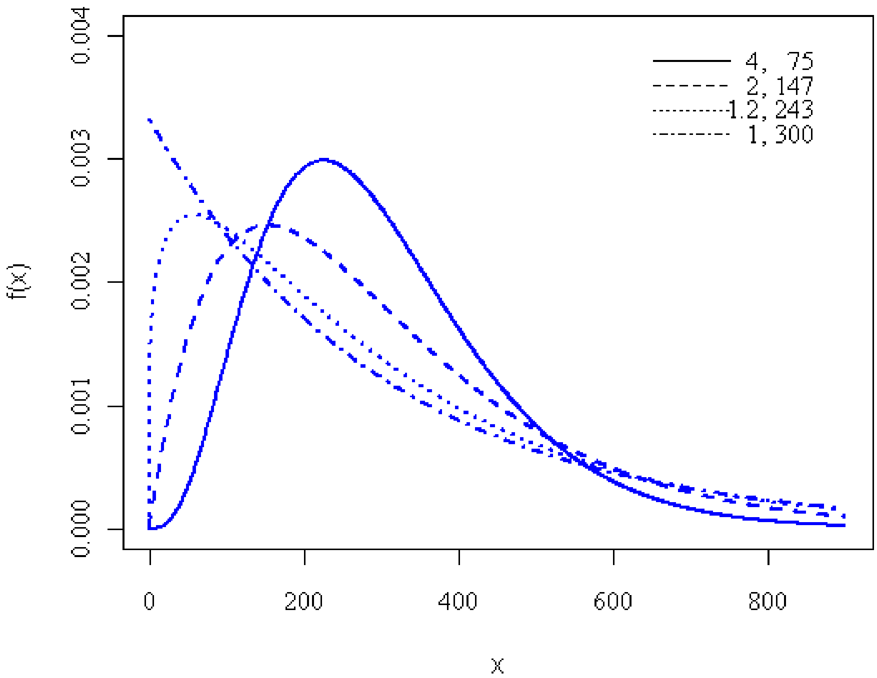

Figure 3.1: Examples of Density function of the Gamma distribution with parameters γ, δ

Several Gamma distributions are exhibited in Figure 3.1 with different sets of parameters (γ, δ). The expected value and standard deviation of the Gamma distribution are as follows.

$$E(D^{tot}) = \gamma \cdot \delta \tag{3-3}$$

$$\sigma(D^{tot}) = \sqrt{\gamma} \cdot \delta \tag{3-4}$$

Figure 3.1 displays distributions with the same expected value but different standard deviations. For instance, the solid curve with parameters (4, 75) in the figure has a mean of 300 and a standard deviation of 150. The density function of the Beta distribution with parameters α and β, is defined over the interval $[0, T]$ as

$$\beta_t = \frac{1}{T \cdot B(\alpha, \beta)} (\frac{t}{T})^{\alpha-1} (1 - \frac{t}{T})^{\beta-1} \tag{3-5}$$

in which $B(\alpha, \beta)$ is the Beta function

$$B(\alpha,\beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1}dt = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$$ [1]

Figure 3.2 depicts three sample curves of the Beta density function with respect to three different parameter sets of (α, β). All of them have the same mode of 21 over the interval of [1, 28] but different variances which determine arrival intensity. The way the parameters are estimated in terms of the known mode and variance will be discussed in detail later.

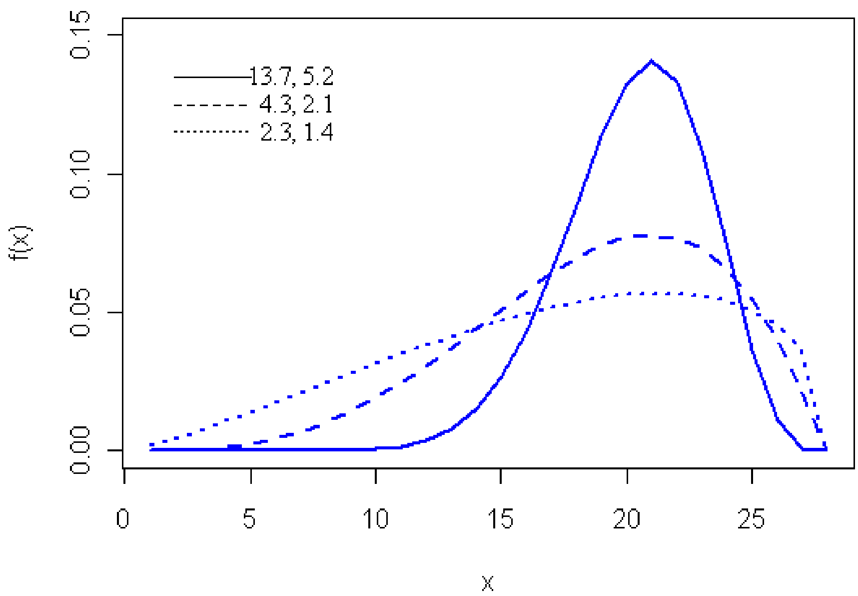

Figure 3.2: Examples of the density function of the Beta distribution with parameters α, β

Now using the formula (3-3) and (3-5), a formula can be derived which yields the expected demand of each time period. The expected value of demand (ticket purchase requests), which is Poisson distributed with the rate λ_t during period t, is specified by

$$E(D_t) = E(\lambda_t). \tag{3-6}$$

where $E(\lambda_t) = E(D^{tot}) \cdot \beta_t$

To conclude, a non-homogeneous Poisson process is characterized by its rate function λ_t, which represents arrival intensity. Defined in the formula (3-1), the rate function was introduced by Bertsimas and de Boer (2005), which follows Weatherford et al. (1993). The Beta distribution is often used to model events that are constrained to take place within an interval defined with minimum and maximum values and modeling the arrival intensity by using the beta distribution β_t allows to exhibit a wide range of unimodal arrival patterns; the higher λ_t for any time period t, the more arrivals are expected to come. In addition, the random variable D^{tot} adds an extra level of randomness to the non-homogeneous Poisson process. That is, using the Beta distribution function β_t in the formula (3-4), total demand represented by the random variable D^{tot} is distributed over the time interval $[1, T]$.

Parameter Determination

In order to generate the expected customers' demand, proper parameter estimation has to come first. The parameters γ, δ of the Gamma distribution are easily specified by defining the mean and standard deviation of D_{tot} using the formulas (3-3) and (3-4). Here, D_{tot} illustrates an expected total demand of air tickets during the whole period of ticket sales. For instance, suppose that there is an airplane that have 300 available tickets (seats) for sales. If we assume that the expected value of the total demand is 400 and its standard deviation is 200, that is

$$E(D^{tot}) = \gamma \cdot \delta = 400, \sigma(D^{tot}) = \sqrt{\gamma} \cdot \delta = 200,$$

then the parameters satisfying both equations together become γ = 4 and δ = 100, respectively. Likewise, the parameters α, β of the beta distribution function are determined by specifying a mode and a variance of the beta distribution such that

$$mode = \frac{\alpha - 1}{\alpha + \beta - 2} T \tag{3-7}$$

$$variance = \frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)} \tag{3-8}$$

Suppose that for instance that an airline company sells air tickets from January 1st to December 31st, the date of departure and expects the ticket purchase requests will be the highest during September. Then, we can simply set the mode of the Beta distribution as 270 over the interval [1,360]. If we assume, in addition, that the variance of the Beta distribution is 1%, that is

$$mode = \frac{\alpha - 1}{\alpha + \beta - 2} * 360 = 270$$

$$variance = \frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)} = .01,$$

then we can specify the value of parameters α, β which satisfies both equations together. They are α = 10 and β = 4, respectively.

3.2 Ticket Price Model

An airline company prices its tickets strategically based on the expected demand. It is an usual practice that the airline sells tickets at cheaper price in the early periods of its sales in order to draw relatively small amount of demand and accommodate high fare demands later near departure. This prevents yield loss and spoilage loss defined in Section 1.1 and enhances profit of the airline. This thesis defines a proper price model of the geometric Brownian motion that an airline's ticket price can follow. Next, the manner that its parameters are specified is illustrated and a risk premium is priced based on the model.

Geometric Brownian Motion

It is assumed that the ticket price follows geometric Brownian motion. It seems logical, because the actual ticket price tends to increase over time, but with short-term fluctuation. Given initial ticket price S_1, discrete-time ticket price model is then defined as

$$S_{t+1} = S_t + \mu dt S_t + \sigma\sqrt{dt} S_t Z_t \tag{3-9}$$

for time period t, $1 \leq t \leq T-1$, where μ is a mean drift and σ is volatility. The variable, Z_t, is a random number that follows a standard Normal distribution and the time step, dividing the whole time period into T intervals, is specified as $dt = 1/T$. This model simply becomes

$$S_{t+1} = S_t(1 + \mu dt + \sigma\sqrt{dt} Z_i), \tag{3-10}$$

which implies that the ticket price, S_t, for each period t can be simply computed with the initial sales price S_1 multiplied by $1 + \mu dt + \sigma\sqrt{dt} Z_t$ t-1 ; that is,

$$S_t = S_1 \prod_{i=1}^{t-1}(1 + \mu dt + \sigma\sqrt{dt} Z_i), \text{ for all } t,\ 2 \leq t \leq T. \tag{3-11}$$

Then, the expected value of S_t during the time period t is

$$E(S_t) = S_1(1 + \mu dt)^{t} \tag{3-12}$$[2]

for each time period t, $2 \leq t \leq T$. Therefore, the expected value of the ticket prices for each period can be calculated in a closed form, as derived in the formula (3-12).

Parameter Determination

It can be seen that the parameter estimation problem is also simplified; the mean drift μ over a predetermined time interval $[1, T]$ is to be estimated, which represents the rate of increase in ticket price between the dates of ticket sales and of departure. For instance, if it is assumed that a ticket, for a trip from New York to Paris 3 months from now, costs $200 today but its price is expected to rise by 25% for next three months, then the mean drift μ = .25.

Risk Premium Valuation

Promotional tickets, mainly spotlighted in this thesis, include call provision as defined in Chapter 1 and hence the value of the provision can be measured using financial option theory. Unlike European-style call option, however, the moment of the option exercise of the promotional tickets is not restricted to the date of maturity; that is, it can be categorized as American-style call options, which allows an exercise at any time before a prescribed expiry date. The early exercise feature of this option complicates the valuation process since the standard Black-Scholes model cannot be used. In this paper, an alternative approach is adopted; each period of the ticket sales process is considered as a maturity of European call option and have the prices of each option that expires at the end of each period is multiplied by the density function of the Beta distribution standardized on the interval $[1, T]$. This calculation provides the model with a sort of weighted-average value of the call option, which is price of the risk premium of the promotional ticket purchase.

The value of the European-style call option during the time period t is calculated using the following formula (3-13) (p.80, Higham, 2004).

$$C(S,t) = S \cdot N(d_1) - R \cdot e^{-r(T-t)} \cdot N(d_2) \qquad (3\text{-}13)$$

where $N(\cdot)$ represents the standard Normal distribution function.

$$d_1 = \frac{log(S/R) + (r_m + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = \frac{log(S/R) + (r_m - \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$$

Using the formula (3-13), the present values of European-style call options which mature each period can be calculated, simply by replacing T by t and t by 1, respectively. Then the formula becomes

$$C(S,t) = S_t \cdot N(d_1) - R \cdot e^{-rt} \cdot N(d_2) \qquad (3\text{-}14)$$

where

$$d_1 = \frac{log(S_t/R) + (r_m + \frac{1}{2}\sigma^2) \cdot (t-1)}{\sigma\sqrt{t-1}}$$

$$d_2 = \frac{log(S_t/R) + (r_m - \frac{1}{2}\sigma^2)(t-1)}{\sigma\sqrt{t-1}}$$

In order to price American-style option through a sort of "weighted-averaging" way using European-style option values, it is of mighty importance to allot proper weights for each period. As referred earlier, one of the most critical factors when a firm makes a pricing decision is the expected demand. Hence, it appears natural to employ the Beta distribution function in the formula (3-4), which has been used to determine arrival intensity of each period in the demand model, as weights in pricing the risk premiums of promotional tickets. Specifically, the formula (3-14) can be multiplied by β_t in the formula (3-4) having the same values of parameters with the demand model. It should be noted however that this multiplication should be done only if $S_t > R$, because otherwise the call exercise will result in a negative profit to the airline company.

3.3 Optimization

In sections 3.1 and 3.2, the demand and ticket price models were developed with properly estimated parameters. Using the models in the formulas (3-6) and (3-12), the expected values $E(D_t)$ and $E(S_t)$ of both models can be obtained. Accordingly, all the ingredients, that are needed to optimize the model developed in Chapter 2, are available. As defined in Chapter 2, the objective of this thesis is to determine the optimal number of promotional tickets, u, that will be offered for sale and its recall price, R, in order to maximize the airline's expected profit. To find the optimal values of u^* and R^*, cyclical coordinate search method, which is a relatively simple method, is employed. This chapter illustrates the algorithm of the search method with a flow chart in Figure 3.3, and presents a numerical example with given set of parameters so as to evaluate the performance of the model.

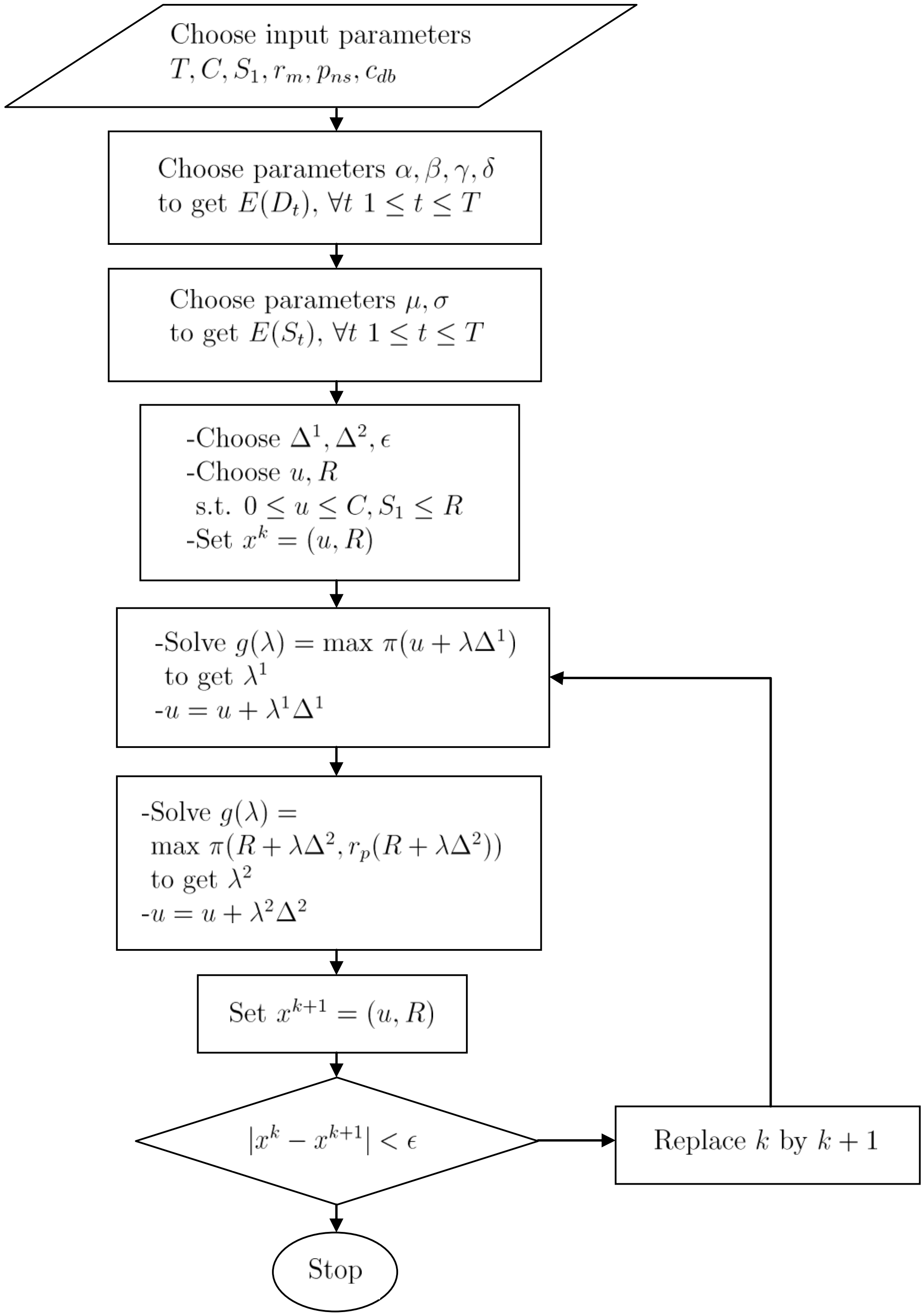

Figure 3.3: Schematic flowchart of the proposed algorithm

Algorithm

First, the input parameters: $T, C, S_1, r_m, p_{ns}, c_{db}$ need to be specified. Then, $E(D_t)\, \forall t\; 1 \leq t \leq T$ with the specified values and $\alpha, \beta, \gamma, \delta$ estimated using the formulas (3-6), (3-7), and (3-8), need to be calculated . Also, $E(S_t)$ can be calculated based on the projected parameters of μ, σ as described in Section 3.2, $\forall t\; 2 \leq t \leq T$. Now to conduct the cyclical coordinate search, increments Δ^1, Δ^2 along each direction and an arbitrary small number of ϵ and the initial value of u, R that satisfies $0 \leq u \leq C$, $S_1 \leq R$, need to be chosen.

The main problem is as follows. First, find the value of λ^1 needs to be found, that maximizes expected profit by increasing or decreasing u by the amount of $\lambda^1\Delta^1$; subsequently move u should be moved as much as $\lambda^1\Delta^1$; that is, $u^{new} = u + \lambda^1\Delta^1$. Second, the value of λ^2 needs to be found, that maximizes the expected profit by increasing or decreasing R by the amount of $\lambda^2\Delta^2$; subsequently R should be moved as much as $\lambda^2\Delta^2$; that is, $R^{new} = R + \lambda^2\Delta^2$. It should be noted that the risk premium, r_p, has to be recalculated whenever the recall price, R, has different value, since r_p is dependent on R. Hence, it is natural to have r_p altered along with a change in λ^2. Consequently, this process needs to be stopped, if the difference between the old and new sets of u and R is less than predetermined value of ϵ. Otherwise, the process of finding profit-maximizing u^* and R^* needs to be repeated, having new sets of values of u and R as the new starting point.

The entire process is schematically illustrated in Figure 3.3, and the code written is given in Appendix A. Use of this algorithm allows comparison of profits between the case of offering and not offering promotional tickets, which also reveals how good the model is.

Numerical Example

An algorithm that optimizes the expected profit is described in this section. This numerical example describes the algorithm and finds an unique solution of the given problem.

Suppose that there is an airline company called XYZ Airlines, and one of its flights which departs from Boston to Tokyo thirty days from now, has 300 seats available for sale. To enhance sales profit, the management of XYZ Airlines decides to sell promotional tickets that have several distinct features 2 days later for a duration of one day; these tickets are offered at cheaper price than the general ones. But, buyers have to assume a risk that tickets can be recalled at a certain price, suggested by the airline before purchase. The airlines wants to figure out the number of promotional tickets to offer for sale, u, and its recall price, R, that maximizes the expected profit while encouraging customers to buy the tickets, even with the perceived risk. Additional information necessary to solve this problem is as follows. The price of general rickets (non-promotional tickets) two days later will be \$600. No-show rate estimated from historical data is 30%. Denied boarding cost incurred due to overbooking as much as the no-show rate is \$200 per ticket. Prevailing monthly interest rate in the market is 4%; that is 0.2% daily. A summary of the input parameters are given in Table 3.1.

Table 3.1: Summary of input parameters for promotional ticket sales

Input parameters	Value
T	28
C	300
S_1	\$600
r_m	0.2%
p_{ns}	30%
c_{db}	\$200

The airlines also has its own forecasting division. It expected that the total demand would be as much as the number of holding tickets, C, and it is assumed to follow a Gamma distribution with a standard deviation of 150. Moreover, the division concluded that the demand would reach its peak a week before the date of departure, exhibiting the highest arrival intensity. The Beta-distributed arrival intensity is generated with a known variance of 0.01. On the other hand, ticket price is anticipated to rise by 30% during the interval between promotional ticket sales and flight departure, considering this market demand as well as diverse market factors. Subsequently, the values of indispensable parameters to solve this problem is estimated based on the forecasts, which is summarized in Table 3.2.

Table 3.2: Parameters estimated by the forecasting department of XYZ Airlines

	Forecasts	Parameters Estimation
Total Demand	$E(D^{tot})$ = 300 $\sigma(D^{tot})$ = 150	γ = 2.04 δ = 147
Arrival Intensity	Mode = 21 Variance = 0.01	α = 13.7 β = 5.2
Ticket Price Change	Markup = 30%	μ = 0.3 σ = 0.3

The problem of XYZ Airline results in the following output by using the model built in Chapter 2 and the algorithm addressed in this section.

u^* = 90 tickets,

R^* = \$ 689,

r_p = \$ 76.8,

π^p = \$ 261334.5,

π = \$ 212798.0,

improvement = 22.8%

It appears to be optimal from this output data to sell 90 promotional tickets with a stated recall price of \$689. At the same time, the price of a promotional ticket turns out to be \$523.2 as the risk premium of the promotional ticket purchase is priced as \$76.8; $S_1 - r_p$ = \$600 - \$76.8 = \$523.2, from the model proposed in Chapter 2. In other words, the airline is expected to sell 90 promotional tickets for its flight from Boston to Tokyo when it offered the ticket at $523.2 with the recall price of $689. This expected ticket sales would result in a maximized profit of $261,334.5, which gives an improved profit of 22.8%, compared to the sales of only general tickets; the airline anticipates to earn $212,798.0 when it sells all of its 300 tickets as the general price.

From this numerical example, it can be seen that that the promotional (callable) ticket sales combined with general (non-callable) ones contribute to increasing an airline's expected profit. Of course, there might be other factors that need to be included in the model. We conduct sensitivity analysis in the next section in order to measure the model's performance under diverse market conditions.

3.4 Sensitivity and Scenario Analyses

In this section, the sensitivity of several parameters reflecting market conditions is studied.

Analysis on demand forecasts

Demand for ticket sales is known to be larger as the date of departure approaches. However, the opposite trend can also be observed in the market from time to time. For the reason, the quartiles of the interval $[1, T]$ to indicate different modes are set, which represent customers' arrival intensity. This treatment is equivalent to setting each mode to mark .25, .50, and .75 spots on a given interval in order to represent skewness of the arrival intensity.

1) A case that high intensity of arrivals is expected in early periods. (mode = the 1st quartile, variance = 0.01)

Figure 3.3 illustrates the cases of early concentration of arrivals, which corresponds to the first three scenarios described in Table 3.3. In such cases, an airline would better sell its promotional tickets at a cheaper recall price. It is because the price of risk premium, affected by the arrival intensity, is being priced lower under the assumption that the ticket price goes upward as time goes by. Thereby, the probability of recall is set high around the low-fare class and it will drag the recall price down. Table 3.3 provides numerical explanation of this argument. In scenario I, which is related to mode = 7 and variance = 0.01, selling promotional tickets at \$556.3 with recall price of \$619, turns out to be optimal; the mode of 7 implies high concentration of arrival intensity in between the first and second quarters. As compared to the other scenarios, this result reveals the same result. The risk premium of \$43.7 is the lowest among all scenarios and hence the highest recall price is anticipated though it was indeed not. The recall price of \$619 is the largest among the first scenarios that have the same mode, but it is lower than in other scenarios having greater modes, due to the highest early recall probability.

Table 3.3: Change in mode and variance

Scenarios	Total demand				u^*	R^*	r_p	π^p	π	$\frac{\pi^p - \pi}{\pi}$
	mode	variance	α	β						
I	7	0.01	5.2	13.7	98	\$619	\$43.7	\$241,236	\$190,815	26.4%
II	7	0.02	2.1	4.3	93	\$606	\$69.2	\$241,390	\$193,106	25.0%
III	7	0.03	1.4	2.3	95	\$606	\$83.0	\$242,574	\$194,200	24.9%
IV	14	0.01	12.0	12.0	93	\$653	\$56.4	\$251,066	\$201,507	24.6%
V	14	0.02	3.7	3.7	92	\$626	\$84.4	\$248,515	\$201,625	23.3%
VI	14	0.03	1.6	1.6	92	\$606	\$106.8	\$246,975	\$200,762	23.0%
VII	21	0.01	13.7	5.2	90	\$689	\$76.8	\$261,335	\$212,798	22.8%
VIII	21	0.02	4.3	2.1	91	\$646	\$106.1	\$256,067	\$209,833	22.0%
IX	21	0.03	2.3	1.4	93	\$619	\$120.3	\$252,365	\$205,648	22.7%

(* with fixed γ = 4, δ = 75 and μ = 0.3, σ = 0.3)

2) A case that moderate intensity of arrivals is expected in early periods. (mode = the 1st quartile, variance = 0.02)

The scenario II in Table 3.3 which has the same mode 7 but has different variance 0.02, exhibits an increased risk premium and decreased recall price compared to scenario I. This is subject to call risk of promotional tickets, distributed more widely over low- and high-fare classes. Under such a forecast of market condition, an airline would offer promotional tickets at cheaper price, pursuing its right to exercise recall tickets at a lower price. This scenario II is depicted in Figure 3.4 with a dashed line.

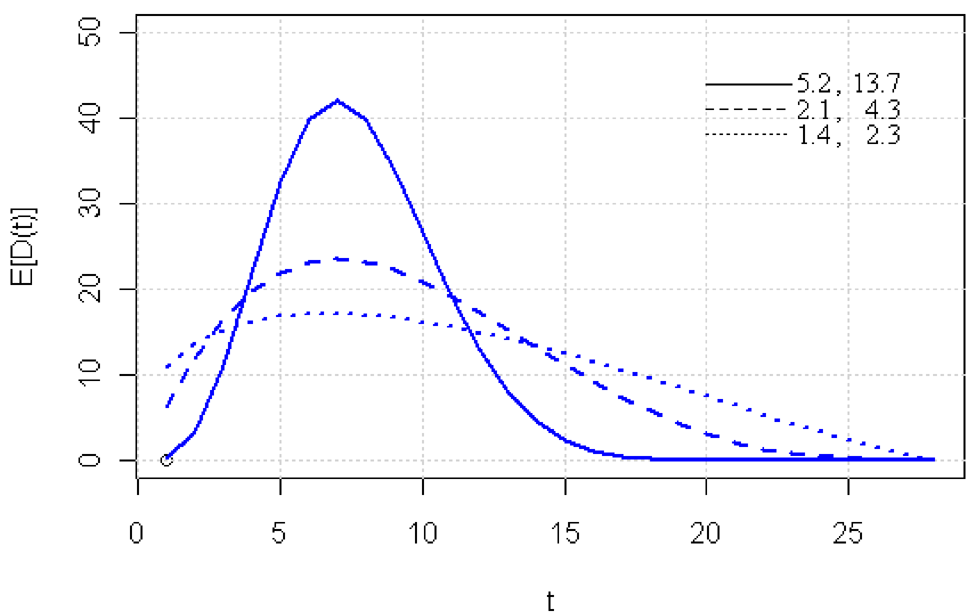

Figure 3.4: Demand distribution as mode of the Beta distribution is 7

3) A case that high intensity of arrivals is expected near the date of departure. (mode = the 3rd quartile, variance = 0.01)

The last three scenarios with modes set between 3rd and 4th quarters, provides similar comparison with those between 1st and 2nd quarters. The greater variance in arrival intensity, the higher risk premium and the lower risk price is required. Compared to other scenarios with lower modes, however, both the risk premium and recall price are more costly due to high concentration of demand around high-fare classes. Subsequently, low sales of tickets would be better and it turned out to be true as illustrated in Table 3.3. For instance, scenario VII reveals that the risk premium of $76.8 is the highest among scenarios having the same variance, and recall price, reflecting the high risk in premium as well as intensive demand around the mode, is also the highest.

4) Market indifference due to flat arrival intensity (variance = 0.03)

Scenarios III, VI, and IX with the highest variance of 0.03, exhibit the values in each category that have closer values. In Table 3.3, the gap between values of different modes is reduced as variance increases, and values associated with modes equal to 7 and to 21 converge to the values with the mode of 14. For example, when the variance is equal to 0.01, the numbers of promotional tickets to be soldl are 98, 93, 90 for each mode. But when the variance is equal to 0.03, they are 95, 92, 93 for each mode as shown in the table. The gaps between the recall prices also exhibits the same results; they are $619, $653, and $689 for the modes associated with a variance of 0.01, but are $606, $606, and $619 when the variance is equal to 0.03. If variances are further increased, the values will come closer. This is because greater variance results in relatively flat distribution in customers' arrival, as depicted in Figure 3.5. The figure illustrates demand distributions for scenarios III, VI, and IX which have the same variances but different modes. It should be noted that the solid curve with α = 1.4 and β = 2.3 in Figure 3.5 is identical with the dotted curve in Figure 3.4. The flatted curve shapes look alike in spite of different modes. Therefore, it can be concluded that the demand would be less affected by the arrival intensity when higher variance is expected in the market.

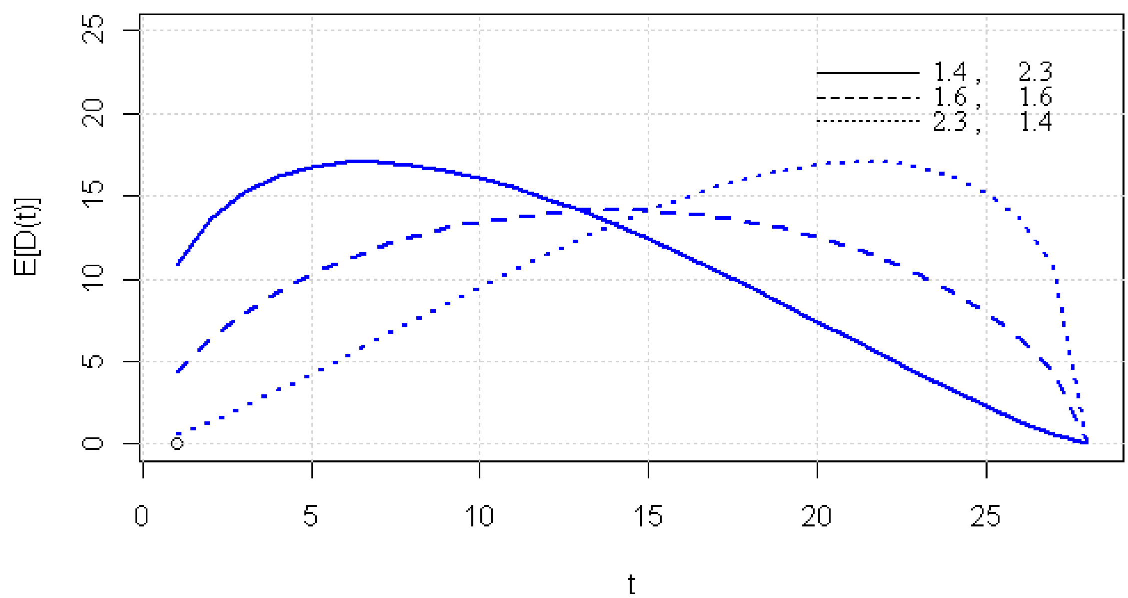

Figure 3.5: Demand distributions as variance of the Beta distribution is fixed at 0.03

5) Profit rise caused by high correlation between demand and ticket price streams.

It can be observed in Table 3.3 that the expected profit increases along with bigger modes regardless of promotional ticket sales. This seems natural since customer arrivals get concentrated around high-fare classes with respect to greater modes. For instance, expected profits as promotional tickets are not offered consistently increase from \$190,815 with mode = 7 to \$201,507 with mode = 14 and \$212,798 with mode = 21. This also holds true for promotional tickets being offered; the expected profits, π^p, are \$241,236, \$251,066, \$261,335 for the modes in the ascending order. Since ticket price is assumed to rise over time, higher mode also implies greater correlation between demand and ticket price and it can be said that the overall profit would be greater if the correlation between demand and ticket price is higher.

Analysis on ticket price forecasts

In this section, the sensitivity of the expected profits with respect to a mean drift, μ, and a volatility, σ is analyzed, which characterizes ticket price movement. It should be noted that the set of ticket price markups employed for the analysis ranges between 15% and 30%. The high price markup was assumed to reveal distinct differences in expected profits due to change in values of μ, σ.

Table 3.4: Different scenarios in terms of changes in μ, σ

Scenarios	Ticket price change			u^*	R^*	r_p	π^p	π	$\frac{\pi^p - \pi}{\pi}$
	Markup	μ	σ						
I	15%	0.15	0.15	40	$643	$48.4	$246,846	$224,011	10.2%
II			0.25	40	$646	$55.3	$246,507	$224,011	10.0%
III			0.35	40	$653	$115.1	$243,976	$224,011	8.9%
IV	20%	0.20	0.15	40	$658	$57.3	$254,593	$231,828	9.8%
V			0.25	40	$658	$58.1	$254,561	$231,828	9.8%
VI			0.35	40	$662	$76.5	$253,746	$231,828	9.5%
VII	25%	0.25	0.15	40	$673	$67.1	$262,597	$239,910	9.5%
VIII			0.25	40	$673	$67.1	$262,595	$239,910	9.5%
IX			0.35	40	$673	$72.0	$262,402	$239,910	9.4%
X	30%	0.30	0.15	40	$681	$83.9	$270,758	$248,264	9.1%
XI			0.25	40	$681	$83.9	$270,758	$248,264	9.1%
XII			0.35	40	$681	$84.7	$270,727	$248,264	9.0%

1) Effect of different expected markups

Compared with others having identical volatilities, both the risk premium and recall price can be seen to grow coincidentally as μ increases. Markup in risk premium implies cut in price of the promotional ticket. Because market price of air tickets is anticipated to increase as much as the markup amount, greater price cut and higher recall price must be offered to sell the same amount of promotional tickets. In fact, scenarios I, IV, VII, and X exhibit constant increase in price of recall and risk premium, which can be interpreted as cost which an airline has to incur. However, the increased costs are compensated with the augmented expected profit for airlines who calls outstanding promotional tickets and resells them at higher market price.

2) Effect of projected volatilities

Compared with others having identical mean drifts, we see both risk premium and recall price increase simultaneously as volatility augments. This is the same result as we change μ with fixed volatility, σ, but because of the different cause. It was because customers had to bear greater risk due to increased market uncertainty. The change in price of risk premium and recall price are observed in Table 3.4. However, the changes solely means increase in cost which results in decrease in expected profits. We can see from scenario I, II, and III that expected profits are decreasing along with increases in risk premium and recall price.

CHAPTER 4

CONCLUSION AND FUTURE WORKS

4.1 Summary

Many research topics that have been addressed in recent years in the field of revenue management; relatively new among them is an application of call feature, which is can be applied to the sale of an airline. Iin 1999, Biyalogorsky et al. introduced a conceptual framework of the callable product, though the term “callable product” was not yet introduced then. They defined two potential losses anticipated due to time-varying ticket price markup and attempted to hedge them by offering callable products. Based upon their work, Gallego established a theoretical framework of the callable product and introduced the term “callable product” in his working paper in 2004

Basically, the concept of the callable product was proposed in order to foster early tickets purchase while meeting high-fare demands that come later within the time horizon of ticket sales. Its call property benefits both the airlines and customers. Since ticket price is assumed to rise as time goes by, if an airline offers further discount on its tickets in early periods, then consumers would enjoy deeper price cut. At the same time, an airline could avoid spoilage loss caused by retaining tickets. Yet, an airline may suffer yield loss, since, if customers who concern much about future price markup purchase a majority of tickets, it would fail to satisfy high-fare demands. Therefore, if an airline has the option of calling and reselling outstanding tickets later to meet the high-fare demands, its profit would increase significantly. Besides, customers might be able to enjoy a portion of the margin generated by the high-fare sales if the compensation offered for recall of tickets is high enough. In sum, the callable product sales enables both an airline and a customer to enjoy advantages and to prevent anticipated loss simultaneously.

In 2007, Akgunduz put this callable product concept a multiple-period and multiple-fare class framework. Using this framework, Ravelojona find the optimal value of the call option tickets using binomial option pricing model, which is one of the prevailing financial option valuation techniques. But the option, which can be looked upon as an European-style one, has limited its use as it allows recall of the tickets only during the late period. In this thesis, the restriction is relaxed; outstanding promotional tickets are callable at any moment until departure. In addition, non-homogeneous Poisson process, known to be the closest to the real demand distribution in recent works, is chosen to specify demand model and geometric Brownian motion is chosen to fit ticket price movement. This is also reasonable since actual ticket price tends to rise upwards over time. These treatments are expected to improve reliability of the model.

4.2 Future works

Several further researches would improve usefulness of the model developed in this thesis. First of all, ticket price is assumed to follow geometric Brownian motion and hence risk premium of the promotional ticket was priced using applied Black-Scholes model. Unlike European-style options which are priced through standard Black-Scholes model, however, the risk premium that can be modeled as an American-style call option can be used in the model. The optimum value of the risk premium can be obtained.

The model in this thesis employed expected values of demand and ticket price to model expected profit, which subsequently facilitates solving an optimization problem. In order to reflect the model's complexity and market uncertainty, however, it would be also meaningful to find the optimal solution using simulation optimization; that is, one could generate random demand and price streams using a random number generator based on sophisticatedly estimated parameters and then use the random streams instead of expected values.

Other minor extensions include industry-specific customization of the model. Since the revenue management techniques are widely utilized in various industries, which sells so-called perishable products, customized models would increase applicability toward certain industries.

Lastly, it is recommended to use the concept of low-price guarantee, which is a popularly issued topic in marketing, into a revenue management modeling, since the concept of callable products was initially proposed in the field of marketing by Biyalgorski et al. in 1999. The low-price guarantee is usually categorized as market-based, competitor-based, or self-based pricing strategies. Among them, a self-based low-price guarantee could be combined with the revenue management model. With the guarantee, an airline can offer a price cut if price goes below initial sales price of the promotional tickets. Such an additional cost against an airline, however, would promote ticket sales by providing the additional benefit to customers. Moreover, the payout for customers can be easily priced, considering the payout as a lookback option, under the assumption that the ticket price follows geometric Brownian motion.

Figure 4.1 exhibits the pricing strategy , where $\eta_{1,t}$ denotes the actual lowest price observed in the market during the time period t. Then, the payout to the customers is

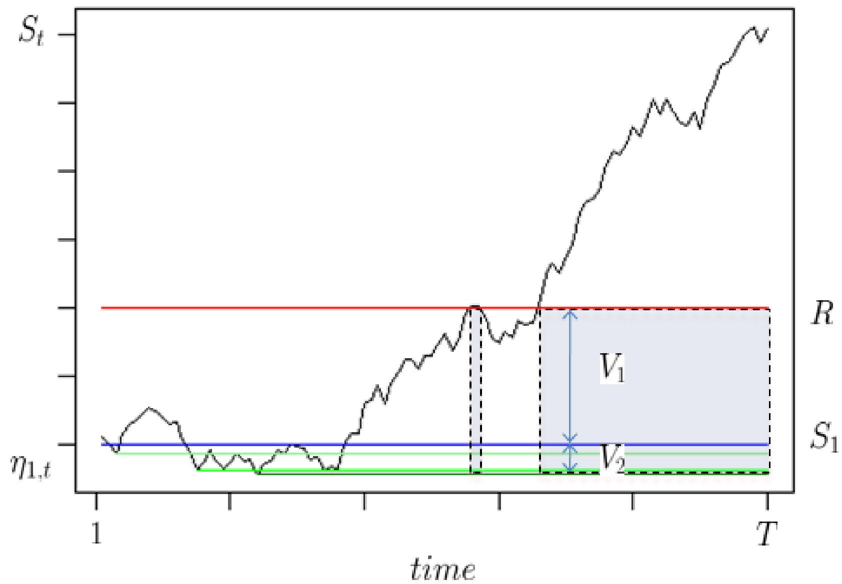

Figure 4.1: Illustration of Low-price guarantee over time-varying price

$$\text{guarantee} = max(S_1 - \eta_{1,t}, 0)$$

After all, each customer enjoys a total revenue of $V_1 + V_2$ if an airline calls outstanding promotional tickets, since recall price is set above S_1. At the same time, an airline enjoys a margin above the recall price, R, that is, $S_t - R$, by reselling the tickets recalled.

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Appendix A

R-code for the model optimization

```
#################################
##### Ticket Price Generator #####
#################################

### Expected Ticket Price ###
Exp_TkPrice <- function(s1,T,mu,sigma){
x <- rep(0,T)
dt <- 1/T
x[1] <- s1
for(t in 1:(T-1))
x[t+1] <- s1*(1+mu*dt)^t
return(x)
}

### Ticket Price ~ GBM ###
TkPrice <- function(s1,T,mu,sigma){
x <- rep(0,T)
dt <- 1/T
x[1] <- s1
for(t in 1:(T-1))
x[t+1] <- x[t]*exp((mu-.5*sigma^2)*dt+sigma*sqrt(dt)*rnorm(1,0,1))
return(x)
```

```
}

############################
##### Demand Generator #####
############################

# Beta function #
Beta <- function(t,alpha,beta){
#betafunc <- factorial(alpha-1)*factorial(beta-1)/factorial(alpha+beta-1)
beta_func <- exp(log(factorial(alpha-1))+log(factorial(beta-1))-log(factorial(alpha+beta-1)))
beta_t <- (1/(T*beta_func))*(t/T)^(alpha-1)*(1-t/T)^(beta-1)
return(beta_t)
}

### Expected Demand ###

Exp_Dem <- function(T,alpha,beta,gamma,delta){
D_tot <- gamma*delta
Demand <- D_tot*Beta(1:T,alpha,beta)
return(Demand)
}

### Demand ~ NHPP ###

Dem <- function(T,alpha,beta,gamma,delta){
```

```
D_tot <- rgamma(1,shape = gamma, scale = delta)


n <- 1000

# generating arrival times #

i <- 1

x <- 0

t <- rep(0,n)

lamb <- D_tot*max(Beta(t[i]:T,alpha,beta))

x <- x + (-1/lamb)*log(runif(1))


while(x < T){

if(.5 <= D_tot*Beta(x,alpha,beta)/lamb){

i <- i+1

t[i] <- x

}

lamb <- D_tot*max(Beta(t[i]:T,alpha,beta))

x <- x + (-1/lamb)*log(runif(1))

}

cnt <- i


# counting process #

N <- rep(0,T)

j <- 1

for(i in 1:T)

while((t[j] <= i)&&(j <= cnt)){
```

```
N[i] <- N[i]+1
j <- j+1
}
plot(N,xlab = 't',ylab = 'N(t)')

return(N)
}



#############################
### Risk premium valuation ###
#############################

Risk_prem <- function(S,R,T,mu,sigma){
for (i in 1:T)
if (S[i] > R) break

prem <- rep(0,T)
for (tau in i:T){
d1 <- (log(S[tau]/R)+(mu+.5*sigma^2)*(tau))/(sigma*sqrt(tau))
d2 <- (log(S[tau]/R)+(mu-.5*sigma^2)*(tau))/(sigma*sqrt(tau))
prem[tau] <- S[tau]*pnorm(d1) - R*exp(-r_m*(tau))*pnorm(d2)
}
wt <- Beta(1:T,alpha,beta)
avg_prem <- sum(prem[i:T]*(wt[i:T]/sum(wt[i:T])))
```

```
return (avg_prem)

}


###### Expected Revenue (with callable product) ######


Exp_Prof <- function(u,R){


# setting #

v <- rep(0,T)

w <- rep(0,T)

u_tmp <- u

C_tmp <- C*(1+p_ns) - u

r_p <- Risk_prem(S,R,T,mu,sigma)


# initial sales #

v[1] <- min(D[1],C_tmp)

w[1] <- 0

C_tmp <- C_tmp - v[1]

E_prof <- (S[1] - r_p)*u + S[1]*v[1]


for(t in 2:T){

# callable product sales #

v[t] <- min(D[t],C_tmp)

C_tmp <- C_tmp - v[t]

E_prof <- E_prof + S[t]*v[t]/(1+r_m)^(t-1)
```

```
# non-callable product sales #
if(S[t] > R){
w[t] <- min(max(0,D[t]-C_tmp),u_tmp)
u_tmp <- u_tmp - w[t]
E_prof <- E_prof +(S[t]-R)*w[t]/(1+r_m)^(t-1)
}
}

# denied boarding cost #
n_db <- 0
if (sum(v)*(1-p_ns) > C)n_db <- sum(v)*(1-p_ns)-C
E_prof <- E_prof - c_db*n_db/(1+r_m)^T

return(E_prof)
}

###### Expected Revenue (without callable product) ######

Exp_Prof_without <- function(){

# setting #
E_prof <- 0
v <- rep(0,T)
C_tmp <- C*(1+p_ns)
```

```
# ticket sales #
for(t in 1:T){
v[t] <- min(D[t],C_tmp)
C_tmp <- C_tmp - v[t]
E_prof <- E_prof + S[t]*v[t]/(1+r_m)^(t-1)
}

# denied boarding cost #
n_db <- 0
if (sum(v)*(1-p_ns) > C) n_db <- sum(v)*(1-p_ns)-C
E_prof <- E_prof - c_db*n_db/(1+r_m)^T

return(E_prof)
}

Line_search <- function(x,flag){

# setting #
u <- x[1]
R <- x[2]

# change in # of callable tickets #
if(flag == 1){
tmp = Exp_Prof(u,R); tmp1 <- Exp_Prof(u+1,R)
```

```
while(tmp < tmp1 && u < C){u <- u + 1; tmp <- tmp1; tmp1 <- Exp_Prof(u+1,R)}

tmp1 <- Exp_Prof(u-1,R)

while(tmp < tmp1 && u > 0){u <- u - 1; tmp <- tmp1; tmp1 <- Exp_Prof(u-1,R)}

}


# change in recall price #

if(flag == 2){

tmp <- Exp_Prof(u,R); tmp1 <- Exp_Prof(u,R+1);

while(tmp < tmp1 && R < S[T]){R <- R + 1; tmp <- tmp1; tmp1 <- Exp_Prof(u,R+1)}

tmp1 <- Exp_Prof(u,R-1)

while(tmp < tmp1 && R > s1){R <- R - 1; tmp <- tmp1; tmp1 <- Exp_Prof(u,R-1)}

}


x_new <- c(u,R)

return(x_new)

}


###############################################################################

################################

### Chapter 3.3 Optimization ###

################################


###############################

### Choose input parameters ###

###############################
```

```
T <- 28; C <- 300; s1 <- 600; r_m <- .002; p_ns <- .3; c_db <- 550

### Set of demand parameters ###
types <- c("solid","dashed", "dotted","dotdash")

# Gamma Distribution (gamma, delta) #
Mean_dem <- c(200,250,300,350,400)
Std_dem <- c(100,150,200,250,300)
nj <- ncol(t(Mean_dem))
ni <- ncol(t(Std_dem))
gam_para <- array(0,c(ni,2,nj))
for(j in 1:nj){
for(i in 1:ni){
gam_para[i,,j] = c(Mean_dem[j]^2/Std_dem[i]^2,Std_dem[i]^2/Mean_dem[j])
}}

# Beta Distribution (alpha, beta) #
beta_para <- array(0,c(3,2,3))
beta_para[,,1] <- matrix(c(5.2,2.1,1.4,13.7,4.3,2.3),3,2)
beta_para[,,2] <- matrix(c(12.0,3.7,1.6,12.0,3.7,1.6),3,2)
beta_para[,,3] <- matrix(c(13.7,4.3,2.3,5.2,2.1,1.4),3,2)

### Set of ticket price parameters ###
drift <- c(0.05,0.1,0.15,0.2)
volatility <- c(0.05,0.1,0.15,0.2)
```

```
###################
### Main problem ###
###################

plot(0,xlim = range(1:T), ylim = range(0:50), xlab = "t", ylab = "E[D(t)]")
grid()

# Parameter Estimation #
Mean_dem = 300
Std_dem = 150
gamma <- Mean_dem^2/Std_dem^2
delta <- Std_dem^2/Mean_dem



#gamma <- gam_para[2,1,2] # = 4
#delta <- gam_para[2,2,2] # = 75
alpha <-beta_para[1,1,3] # = 13.7
beta <- beta_para[1,2,3] # = 5.2
mu <- drift[5]
sigma <- volatility[5]

S <- Exp_TkPrice(s1,T,mu,sigma)
#S <- TkPrice(s1,T,mu,sigma)
```

```
# Cyclical Coordinate Search #

x <- rep(0,6)

x[1:2] <- c(0,s1)

D <- Exp_Dem(T,alpha,beta,gamma,delta)

#D <- Dem(T,alpha,beta,gamma,delta)

lines(D, col = 'dark blue')

while(1){

x1 <- Line_search(x[1:2],1)

x2 <- Line_search(x1,2)


if(x1[1] == x2[1] && x1[2] == x2[2]){

E_P <- Exp_Prof(x1[1],x1[2]); E_PX <- Exp_Prof_without()

R_P <- round(Risk_prem(S,x1[2],T,mu,sigma),1)

x <- c(x1,R_P,E_P,E_PX,round((E_P-E_PX)/E_PX,3))

break

}else{

E_P <- Exp_Prof(x2[1],x2[2]); E_PX <- Exp_Prof_without()

R_P <- round(Risk_prem(S,x2[2],T,mu,sigma),1)

x <- c(x2,R_P,E_P,E_PX,round((E_P-E_PX)/E_PX,3))

}}

x


################################################################################

########################################

### Chapter 3.4 Sensitivity Analysis ###
```

```
########################################

#############################
### Test on (gamma,delta) ###
#############################
plot(0,xlim = range(1:T), ylim = range(0:50), xlab = "t", ylab = "E[D(t)]")
grid()

# Fixed (alpha, beta) and (mu, sigma) #
alpha <- beta_para[1,1,3]
beta <-  beta_para[1,2,3]
mu <- .3
sigma <- .3

S <- Exp_TkPrice(s1,T,mu,sigma)
x <- array(0, c(ni,6,nj))
# j Mean(demand) : 1 = (200), 2 = (300), 3 = (400) #
# i Std(demand)  : 1 = (100), 2 = (150), 3 = (200), 4 = (250), 5 = (300), 6 = (350), 7 = (400) #
for(j in 1:nj){
for(i in 1:ni){
x[i,1:2,j] <- c(0,s1)
gamma <- gam_para[i,1,j]
delta <- gam_para[i,2,j]
D <- Exp_Dem(T,alpha,beta,gamma,delta)
```

```
while(1){
x1 <- Line_search(x[i,1:2,j],1)
x2 <- Line_search(x1,2)

if(x1[1] == x2[1] && x1[2] == x2[2]){
E_P <- Exp_Prof(x1[1],x1[2]); E_PX <- Exp_Prof_without()
R_P <- round(Risk_prem(S,x1[2],T,mu,sigma),1)
x[i,,j] <- c(x1,R_P,E_P,E_PX,round((E_P-E_PX)/E_PX,3)); break
}else{
E_P <- Exp_Prof(x2[1],x2[2]); E_PX <- Exp_Prof_without()
R_P <- round(Risk_prem(S,x2[2],T,mu,sigma),1)
x[i,,j] <- c(x2,R_P,E_P,E_PX,round((E_P-E_PX)/E_PX,3))
}}}}
x

###########################
### Test on (alpha,beta) ###
###########################
plot(0,xlim = range(1:T), ylim = range(0:50), xlab = "t", ylab = "E[D(t)]")
grid()

# Fixed (gamma, delta) and (mu, sigma) #
gamma <- gam_para[2,1,3]
delta <- gam_para[2,2,3]
mu <- .3
```

```
sigma <- .3


S <- Exp_TkPrice(s1,T,mu,sigma)
#S <- TkPrice(s1,T,mu,sigma)


x <- array(0, c(3,6,3))
# j mode : 1 = (7), 2 = (14), 3 = (21) #
# i variance : 1 = (.01), 2 = (.02), 3 = (.03) #
for(j in 1:3){
for(i in 1:3){
x[i,1:2,j] <- c(0,s1)
alpha <-beta_para[i,1,j]
beta <- beta_para[i,2,j]
D <- Exp_Dem(T,alpha,beta,gamma,delta)
#D <- Dem(T,alpha,beta,gamma,delta)
lines(D, col = 'blue',lwd = 2, lty = types[i])
lines(20:23,rep(47-i*3,4),lty = types[i])
text(c(25,25.4,27.5),rep(47-i*3,3),c(round(alpha,1),",",round(beta,1)),pos = 2, font = 6)
while(1){
x1 <- Line_search(x[i,1:2,j],1)
x2 <- Line_search(x1,2)


if(x1[1] == x2[1] && x1[2] == x2[2]){
E_P <- Exp_Prof(x1[1],x1[2]); E_PX <- Exp_Prof_without()
R_P <- round(Risk_prem(S,x1[2],T,mu,sigma),1)
```

```
x[i,,j] <- c(x1,R_P,E_P,E_PX,round((E_P-E_PX)/E_PX,3))
break
}else{
E_P <- Exp_Prof(x2[1],x2[2]); E_PX <- Exp_Prof_without()
R_P <- round(Risk_prem(S,x2[2],T,mu,sigma),1)
x[i,,j] <- c(x2,R_P,E_P,E_PX,round((E_P-E_PX)/E_PX,3))
}}}
}
x


#########################
### Test on (mu,sigma) ###
#########################
plot(0,xlim = range(1:T), ylim = range(600:800), xlab = "t", ylab = "E[D(t)]")
grid()

# Fixed (gamma, delta) and (alpha, beta) #
gamma <- gam_para[5,1,4]
delta <- gam_para[5,2,4]
alpha <- beta_para[1,1,3]
beta <-  beta_para[1,2,3]

D <- Exp_Dem(T,alpha,beta,gamma,delta)
#D <- Dem(T,alpha,beta,gamma,delta)
```

```
nj <- ncol(t(drift))
ni <- ncol(t(volatility))
x <- array(0, c(ni,6,nj))
# j Mean(demand) : 1 = (200), 2 = (300), 3 = (400) #
# i Std(demand)  : 1 = (100), 2 = (150), 3 = (200), 4 = (250), 5 = (300), 6 = (350), 7 = (400) #
for(j in 1:nj){
for(i in 1:ni){
x[i,1:2,j] <- c(0,s1)

mu <- drift[j]
sigma <- volatility[i]

S <- Exp_TkPrice(s1,T,mu,sigma)
#S <- TkPrice(s1,T,mu,sigma)

lines(S, col = 'dark blue')
while(1){
x1 <- Line_search(x[i,1:2,j],1)
x2 <- Line_search(x1,2)

if(x1[1] == x2[1] && x1[2] == x2[2]){
E_P <- Exp_Prof(x1[1],x          1[2]); E_PX <- Exp_Prof_without()
R_P <- round(Risk_prem(S,x1[2],T,mu,sigma),1)
x[i,,j] <- c(x1,R_P,E_P,E_PX,round((E_P-E_PX)/E_PX,3))
```

```
break
}else{
E_P <- Exp_Prof(x2[1],x2[2]); E_PX <- Exp_Prof_without()
R_P <- round(Risk_prem(S,x2[2],T,mu,sigma),1)
x[i,,j] <- c(x2,R_P,E_P,E_PX,round((E_P-E_PX)/E_PX,3))
}}}
}
x

################################################################################
```