MULTI-CRITERIA SUPPLY CHAIN INVENTORY MODELS WITH TRANSPORTATION COSTS

A Thesis in
Industrial Engineering

by
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ABSTRACT

This thesis deals with developing and solving tactical inventory planning models for a supply chain that will enable the individual companies to determine their ordering policies efficiently under different conditions. The supply chain is modeled as a single warehouse supplying a product to several retailers which in turn satisfies the end consumer demand. The supply chain operates under a decentralized control, i.e., each location is managed by an independent decision maker (DM).

In the first model, using the conventional single cost objective framework we propose a new coordination scheme that enables the warehouse to better manage its inventory and at the same time meet the retailers’ demands without deviating too much from their requirements. To avoid the problem of estimating marginal cost information and to incorporate the DM’s preference information, a more realistic multiple criteria model is then developed. To account for discounts in shipping, actual freight rate functions are used to model transportation costs between the stages. The conflicting criteria considered are: 1) capital invested in Inventory 2) annual number of orders 3) annual transportation costs. While the first two models deal with deterministic demand and constant lead time, the third model deals with stochastic demand and random lead time. In addition to the above three criteria, fill rate is used as a fourth criterion to measure customer satisfaction. The multiple criteria models are solved to generate several efficient solutions. The value path method, a visual tool, is used to display tradeoffs associated with the efficient solutions to the DM of each location in the supply chain.

The models are tested with real world data obtained from a Fortune 500 consumer products company. Additional problems faced while extending the theoretical models to the real world data are addressed and solved. The decision making process is simulated by using an executive from the company to be the DM for the warehouse. The preference information is obtained using standard multi-criteria techniques to generate the set of efficient solutions. The DM adjudged the multi-criteria methodology to be a more effective decision making tool since he had to evaluate tradeoffs and use his judgment to choose the most preferred solution.
# TABLE OF CONTENTS

**LIST OF TABLES**  
viii  

**LIST OF FIGURES**  
x  

1. INTRODUCTION, MOTIVATION AND PROBLEM STATEMENT ................................................................. 1  
1.1. Motivation ........................................................................................................................................ 2  
1.2. Problem Statement .......................................................................................................................... 5  

2. LITERATURE REVIEW .......................................................................................................................... 9  
2.1 Overview of Supply Chain ................................................................................................................. 9  
2.1.1 Supply Chain Structure ................................................................................................................. 9  
2.1.2 Centralized Vs. Decentralized Supply Chains ............................................................................. 10  
2.1.3 Modeling Demand, Lead Time and Lead Time Demand ............................................................ 11  
2.1.4 Ordering Policies ......................................................................................................................... 13  
2.1.5 Modeling Service Levels ............................................................................................................. 14  
2.1.6 Backorders Vs Lost Sales ............................................................................................................. 14  
2.1.7 Importance of Transportation ..................................................................................................... 16  
2.2. Review of Multi Criteria Optimization .......................................................................................... 18  
2.2.1 General Multi-Criteria Mathematical Program ......................................................................... 19  
2.2.2 Terminology Associated with MCDM ....................................................................................... 19  
2.2.3 Approaches to Solve MCMP ....................................................................................................... 20  
2.2.4 \( P, \lambda \) method ........................................................................................................................... 22  
2.3 Literature review ............................................................................................................................. 23  
2.3.1 Single Location Inventory Control Models ................................................................................. 24  
2.3.2 Supply Chain Inventory Models .................................................................................................. 27  
2.4 Contribution of the Research ....................................................................................................... 35  

3. A MODIFIED BASE PERIOD POLICY FOR A SINGLE WAREHOUSE MULTI RETAILER SYSTEM .......... 39  
3.1. Introduction ...................................................................................................................................... 39  
3.1.1 Notation ....................................................................................................................................... 40  
3.1.2 EOQ Problem ................................................................................................................................. 41  
3.1.3 Economic Reorder Interval Problem ......................................................................................... 41  
3.2. Problem Description ...................................................................................................................... 41  
3.2.1 Assumptions ................................................................................................................................ 42  
3.2.2 Base Period Policy ....................................................................................................................... 42  
3.3. Modified Base Period Policy ........................................................................................................ 42  
3.3.1 Model Formulation ...................................................................................................................... 42
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.3.2. Setting $T_B$: A Different Perspective</td>
<td>43</td>
</tr>
<tr>
<td>3.3.3. Another Interpretation of $T_B$</td>
<td>43</td>
</tr>
<tr>
<td>3.3.4. Solution to Problem ($MR_i$)</td>
<td>44</td>
</tr>
<tr>
<td>3.4. Analysis of the Modified Base Period Policy</td>
<td>46</td>
</tr>
<tr>
<td>3.4.1. Grouping Retailers</td>
<td>46</td>
</tr>
<tr>
<td>3.4.2. Effectiveness of the Policy</td>
<td>47</td>
</tr>
<tr>
<td>3.5. Warehouse’s Ordering Policy</td>
<td>53</td>
</tr>
<tr>
<td>3.5.1. Average Inventory at the Warehouse</td>
<td>53</td>
</tr>
<tr>
<td>3.5.2. Warehouse’s Optimization Problem</td>
<td>59</td>
</tr>
<tr>
<td>3.5.3. Solution to Problem ($W$)</td>
<td>59</td>
</tr>
<tr>
<td>3.6. Algorithm for the Modified Base Period Policy</td>
<td>61</td>
</tr>
<tr>
<td>3.7. Final Comments on the Modified Base Period Policy</td>
<td>62</td>
</tr>
<tr>
<td>3.8. Example Problem</td>
<td>63</td>
</tr>
<tr>
<td>3.9. Summary</td>
<td>66</td>
</tr>
<tr>
<td>4. Deterministic Multi-Criteria Model for a Supply Chain with</td>
<td>67</td>
</tr>
<tr>
<td>Transportation Costs</td>
<td></td>
</tr>
<tr>
<td>4.1. Introduction</td>
<td>67</td>
</tr>
<tr>
<td>4.2. Problem Description</td>
<td>67</td>
</tr>
<tr>
<td>4.2.1. Assumptions</td>
<td>68</td>
</tr>
<tr>
<td>4.2.2. Notation</td>
<td>68</td>
</tr>
<tr>
<td>4.3. Retailer’s Model Formulation</td>
<td>69</td>
</tr>
<tr>
<td>4.3.1 Retailer’s Multi-Criteria Problem</td>
<td>74</td>
</tr>
<tr>
<td>4.3.2 Solution Procedure to Solve Problem ($R_i$)</td>
<td>74</td>
</tr>
<tr>
<td>4.3.3 Modified Retailer’s Problem</td>
<td>77</td>
</tr>
<tr>
<td>4.3.4 Solution Procedure to Solve Problem ($MR_i$)</td>
<td>78</td>
</tr>
<tr>
<td>4.4. Warehouse’s Model Formulation</td>
<td>80</td>
</tr>
<tr>
<td>4.4.1 Warehouse’s Multi-Criteria Problem</td>
<td>82</td>
</tr>
<tr>
<td>4.4.2 Solution Procedure to Solve Problem ($W$)</td>
<td>83</td>
</tr>
<tr>
<td>4.5. Algorithm</td>
<td>86</td>
</tr>
<tr>
<td>4.6. General MCMP for Retailer and Warehouse</td>
<td>88</td>
</tr>
<tr>
<td>4.7. Example Problem</td>
<td>90</td>
</tr>
<tr>
<td>4.8. Summary</td>
<td>102</td>
</tr>
<tr>
<td>5. Stochastic Multi-Criteria Model for a Supply Chain with</td>
<td>103</td>
</tr>
<tr>
<td>Transportation Costs</td>
<td></td>
</tr>
<tr>
<td>5.1. Introduction</td>
<td>103</td>
</tr>
<tr>
<td>5.2. Problem Description</td>
<td>103</td>
</tr>
<tr>
<td>5.2.1. Assumptions</td>
<td>103</td>
</tr>
<tr>
<td>5.2.2. Notation</td>
<td>105</td>
</tr>
</tbody>
</table>
LIST OF TABLES

Table 2.1 Joint Distribution of LTD ................................................................. 12
Table 2.2 Contribution of Chapter 3 ............................................................... 35
Table 2.3 Contribution of Chapter 4 ............................................................... 36
Table 2.4 Contribution of Chapter 5 ............................................................... 37
Table 3.1 Worst Case Deviation Bounds for n≥2 Case ........................................ 49
Table 3.2 Data for Example Problem ............................................................. 63
Table 3.3 Output of Problem (R_i) ................................................................. 64
Table 3.4 Output of Problem (MR_i) ............................................................... 65
Table 4.1 Data for Example Problem ............................................................. 91
Table 4.2 Freight Rate Data ........................................................................... 91
Table 4.3 Freight Rate Data with Indifference Points ........................................ 91
Table 4.4 Summary of Curve Fitting ............................................................... 92
Table 4.5 Bounds on Order Quantity ............................................................. 93
Table 4.6 Criteria Values for the Retailers ...................................................... 95
Table 4.7 Normalized Criteria Values for the Retailers .................................... 95
Table 4.8 Most Preferred Solution for the Retailers ........................................ 98
Table 4.9 Summary of Results for the Retailers .............................................. 98
Table 4.10 Efficient Solutions for the Warehouse .......................................... 100
Table 4.11 Criteria and Normalized Criteria Values for the Warehouse .......... 101
Table 5.1 Demand Distribution for Retailer .................................................... 136
Table 5.2 Lead Time Distribution for Retailer ............................................... 137
Table 5.3 Bounds on Decision Variables ....................................................... 137
Table 5.4 Weight Combinations .................................................................. 138
Table 5.5 Sample Calculations ..................................................................... 139
Table 5.6 Criteria Values for the Retailers ..................................................... 140
Table 5.7 Normalized Criteria Values for the Retailers .................................. 140
Table 5.8 Ordering Policy of the Retailers ..................................................... 144
Table 5.9 Most Preferred Criteria Values for the Retailers ............................. 144
Table 5.10 Distribution of Lead Time at the Warehouse ................................. 145
Table 5.11 Efficient Solutions for the Warehouse ......................................... 146
Table 5.12 Criteria and Normalized Criteria Values for the Warehouse ........ 146
Table 6.1 Output of EOQ Problem for Retailers ............................................ 150
Table 6.2 Output of Modified Problem for Retailers ..................................... 151
Table 6.3 Output of Modified Problem for Retailers in Tier II ....................... 153
Table 6.4 Comparison of Results with and without Tier Approach ............... 153
LIST OF FIGURES

Figure 1.1 Logistics Cost as a Percent of GDP .................................................................................. 2
Figure 1.2 Components of the Logistics Cost .................................................................................. 3
Figure 1.3 Supply Chain System under Consideration .................................................................... 5
Figure 2.1 Supply Chain Network .................................................................................................. 9
Figure 2.2 Arborescence Structure ................................................................................................. 10
Figure 2.3 Serial Supply Chain ...................................................................................................... 10
Figure 2.4 Supported and Unsupported Efficient Points ................................................................. 23
Figure 2.5 Classification of Literature Review .............................................................................. 23
Figure 3.1 Worst Case Deviation Variation ..................................................................................... 50
Figure 3.2 Worst Case Deviation within a Cluster .......................................................................... 51
Figure 3.3 $T^*_b/T^*$ Vs. Deviation for $n=1$ Case ......................................................................... 53
Figure 3.4 Inventory Pattern of the Retailers .................................................................................. 54
Figure 3.5 Inventory Pattern of the Warehouse ............................................................................. 56
Figure 3.6 Flow Chart of the Modified Base Period Policy ............................................................ 62
Figure 4.1 Retailers Inventory Pattern ............................................................................................ 69
Figure 4.2 Freight Rate Function ................................................................................................... 72
Figure 4.3 Value Path Graphs for Retailers .................................................................................... 97
Figure 4.4 Value Path Graph for Warehouse ................................................................................. 101
Figure 5.1 Retailers Inventory Pattern ........................................................................................... 107
Figure 5.2 Value Path Graphs for Retailers .................................................................................... 142
Figure 5.3 a and b Behavior of Fill Rate/Cycle Service Level ....................................................... 143
Figure 5.4 Value Path Graph for Warehouse .................................................................................. 147
Chapter 1

INTRODUCTION, MOTIVATION AND PROBLEM STATEMENT

Over the past decade and a half there has been an increasing number of articles in the field of multi-echelon systems, popularly called supply chain management. The reason for the continuing fascination in this area among researchers is not only due to the complexity that arises from the interactions among the various stages/echelons but also due to the enormous practical applications in the real world.

A comprehensive definition of a supply chain given by Min and Zhou (2002) is as follows: the supply chain can be defined as an integrated system or network which synchronizes a series of inter-related business processes in order to:

- Acquire raw materials.
- Add value to raw materials by transforming them into finished/semi-finished goods.
- Distribute these products to distribution centers or sell to retailers or directly to the customers.
- Facilitate the flow of raw materials/finished goods, cash and information among the various partners which include suppliers, manufacturers, retailers, distributors and third-party logistic providers (3PL).

Thus the main objective of the supply chain is to maximize the profitability of not just a single firm but also all of the partners involved. This can only be done if all the partners in the supply chain think ‘Win-Win’ and are not concerned about optimizing their individual performance. The major drivers of the supply chain are (Chopra and Meindl, 2001):

- Inventory.
- Transportation.
- Facilities.
- Information.

From an optimization perspective, the main focus in the area of multi-echelon systems has been the problem of inventory management. The inventory problem consists of answering the two fundamental questions: 1. When to order? and 2. How much to
order? These two questions are conflicting in the sense that, if we decide to order frequently, then we order in lesser quantities, and similarly, if we decide to order less frequently, then we order larger quantities. Thus, within each company there are conflicting objectives. Hence, appropriate tradeoffs have to be made within each company for the inventory function alone.

Within each company, there exist inter-functional tradeoffs. For example, the tradeoff between the inventory function and the transportation function has been well studied for problems involving single stocking points (Baumol and Vinod, 1970, Das, 1974, Buffa and Reynolds, 1977, Constable and Whybark, 1978). A faster and more reliable transportation mode can be used to ship smaller batches more frequently leading to a reduction in the amount of inventory stored at the stocking point while maintaining the same service level, but the decision maker (DM) has to make sure that the increase in transportation costs is offset by reduction in inventory costs. Capturing such tradeoffs within each company for all the companies in a multi-echelon system is a challenging problem.

1.1 Motivation

To give an idea about how significant the inventory and transportation costs are, Fig. 1.1 (Cook, 2006) tracks the total logistics cost as a percent of the Gross Domestic Product (GDP) of U.S. over the past decade. The logistics cost includes cost of inventory, transportation and logistics administration.

![Fig. 1.1: Logistics Cost as a Percent of GDP](image)
The total logistics decreased from the mid nineties to 2003 mainly due to the reduction in inventory holding costs due to the Just in time (JIT) initiatives undertaken by the U.S. companies. Since 2003, the logistics costs have been increasing. This is due to the increase in inventory holding and transportation costs. Inventory holding costs increased due to higher interest rates, and the companies stocking up their warehouses to prevent disruptions in their supply after the 9/11 terrorist attacks, while transportation costs increased due to inflating gas prices and high labor turnover (Cook, 2006).

In 2005 the logistics cost as a percent of the GDP was 9.5% and was valued at $1.183 trillion. The amount spent on each component of the logistics cost is shown in Fig. 1.2 (Cook, 2006).

The inventory carrying cost was estimated to be $393 billion, considerably less than the transportation cost, which was estimated to be $744 billion. The transportation cost accounted for nearly 6% of the GDP. Out of the $744 billion, $583 billion was spent on transportation using trucks reiterating the fact that road mode is still the preferred method for transportation in the U.S.

Fig. 1.2: Components of the Logistics Cost

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There has been an increase of about $61 billion dollar in the inventory costs from the year 2004. According to Cook (2006) a bigger concern is the increase in the road transportation costs of about $74 billion dollars from 2004. The road transportation costs are expected to go up over the coming years due to increasing freight rates, aging infrastructure and improving security measures.

From the preceding paragraphs, it is clear that inventory and transportation management are crucial to reducing costs in the supply chain. The decision maker (DM) of a company has to pay careful attention while determining the ordering policy (when to order and how much to order) so as to take advantage of the freight rate discount and at the same time not stock too much while providing the necessary service to its customers. Thus, the DM has to balance several conflicting criteria before choosing the most preferred solution.

Inventory decisions made without taking into account transportation costs would fail to take advantage of the economies of scale in shipping. The first paper to address this issue was due to Baumol and Vinod (1970). Since then a lot of work has been done in determining inventory policies when transportation costs are included (Das, 1974, Buffa and Reynolds, 1977, Constable and Whybark, 1978, Lee, 1986, Tyworth and Zeng, 1998, etc.). Researchers who have incorporated transportation costs in supply chain inventory models include Ganeshan (1999), Qu et al. (1999), Chan et al. (2002), Toptal et al. (2003), etc.

With the imminent rise in transportation costs, each company in the supply chain should pay close attention to its logistics function. Cartin and Ferrin (1996) list some of the advantages to companies that manage their inbound logistics. This is discussed in more detail in Chapter 2.

The conventional way of determining ordering policies for a single location have been by optimizing a single cost objective (Hadley and Whitin, 1963). Other researchers like Brown (1961), Starr and Miller (1962), Gardner and Dannenbring (1979) and Agrell (1995) argue that marginal costs information such as ordering costs, inventory holding costs, cost of lost sales/backorders, etc. are difficult to estimate. A multi-criteria approach obviates this problem as the marginal cost information is not needed. Most multi-echelon inventory models also use a single cost objective (Sherbrooke, 1968,

Since the DM is an integral part of any system, his/her preference information must be incorporated during the decision making process. This aspect is completely ignored by the single objective methods. Single objective optimization gives the DM an optimal solution (if it exists) or a “good” solution based on some heuristic procedure, but in a multi-criteria approach the DM is provided with several efficient solutions. An efficient solution is defined as one in which any further improvement of a criterion would result in the worsening of at least one other criterion. The DM after evaluating tradeoffs among the conflicting criteria will then be able to choose the most preferred solution.

1.2 Problem Statement

The problem setting consists of a manufacturer supplying a single product to a warehouse (owned by the manufacturer) which in turn distributes the product to multiple retailers in order to meet the end customer demand as shown in Fig. 1.3. The manufacturer is assumed to have infinite capacity. The goal is to determine the ordering policies for each retailer and the warehouse efficiently under different scenarios.

![Fig. 1.3: Supply Chain System under Consideration](image)

This system operates in a decentralized framework, i.e., there are decision makers at each location trying to optimize their own objectives. The individual retailer problems
are solved, and the output of the retailer problem is used as input to solve the warehouse problem.

The transportation cost has two components, namely, the inbound transportation cost at the warehouse and the inbound transportation cost at each of the retailers. The inbound transportation cost at the warehouse is modeled using full truck load (TL) freight rates since a single supplier is supplying to the warehouse. The inbound transportation cost at each retailer is modeled using less than full truck load (LTL) freight rates, since the order quantity will be smaller. Continuous functions are used to model the transportation cost based on how well they fit the freight rate data.

Each location in the supply chain is assumed to follow a continuous review policy. A continuous review policy is one in which the decision maker places an order when on hand inventory depletes to a reorder point. Giant retailers (e.g., Walmart) use Electronic Data Interchange (EDI) to place an order to their suppliers as soon as the reorder point is reached. The inventory level is tracked using bar codes.

The external demand can be either deterministic or stochastic. We consider both cases in this thesis. The time taken by the preceding stage to replenish an order called the lead time can be instantaneous, deterministic or stochastic. If the demand and lead time are deterministic, then the new order arrives exactly when the on hand inventory reaches zero. But this is not the case when either demand or lead time or both are random variables. In such a scenario, there is a possibility that when the new order arrives the location has some inventory or is already out of stock. The latter leads to a stockout condition. In a stockout situation, excess demand can either be backordered or lost. The DM of a company has to carefully balance the amount of inventory maintained at its shelf so as to not have excess inventory and at the same time reduce stockout situations as it would lead to customer dissatisfaction

Chapter 2 starts of with a brief review of some of the important aspects that must be considered in modeling inventory and transportation components in a supply chain. Then, the important concepts in multi-criteria optimization and a review of some of the methods that are used to solve these problems are discussed. This is followed by a detailed literature review pertaining to single location and supply chain inventory models. Chapter 2 concludes with the contributions of the research.
In Chapter 3, a new control policy is proposed that enables the warehouse to better manage its inventory and at the same time meet the retailers’ demands without deviating too much from their requirements. The retailers face deterministic demand. Instantaneous replenishment is assumed at the retailers and the warehouse. Assuming the availability of marginal cost information such as inventory holding and ordering costs, a single objective cost model is developed for each location. Closed-form expressions are derived which facilitates determining the ordering policies efficiently. Using the structure of the solution, a theoretical method is developed to group retailers into clusters based on their importance to the warehouse. The effectiveness of the policy is tested by developing theoretical bounds on the deviation from the optimal solution of the retailers.

In Chapter 4, the single cost objective model of Chapter 3 is extended to the case where marginal cost information is not known. We model the supply chain problem as a multi-criteria problem with three criteria: 1) capital invested in inventory 2) annual number of orders placed and 3) annual transportation costs. Lead times are assumed to be deterministic. Transportation costs are considered between each stage and are modeled using appropriate continuous functions. Efficient solutions are generated by changing the weight assigned to each criterion. A graphical tool is used to visualize the tradeoff information which would enable the DM of each location to choose the most preferred solution.

Chapter 5 extends the deterministic multi-criteria model with transportation costs to the case where both demands and lead times are random variables. The demands faced by the retailers are independent, non-identical Poisson random variables. The case of compound Poisson demand and normal demands are also considered. The lead times are assumed to follow any discrete distribution. The demand at the warehouse is approximated by a normal distribution using renewal theory concepts. Excess demand at the retailers and the warehouse is assumed to be lost. We also discuss how the backorders case can be handled. Fill rate is the fourth criterion and is used to measure customer satisfaction. Fill rate is defined as the proportion of demand met from on-hand inventory. The solution procedure used to solve the multi-criteria model is similar to the one in Chapter 4.
In Chapter 6, we apply the methodologies developed in Chapters 3, 4 and 5 using real world data obtained from a Fortune 500 consumer products company. Some of the problems faced while extending the theoretical models to the real world data are addressed. A tool is created using Excel and Visual Basic (VB) macros to automate certain aspects of the solution methodologies. We used an executive from the Fortune 500 Company to be the DM at the warehouse to analyze the various tradeoffs and hence, choose the most preferred solution.

Chapter 7 discusses the conclusions and future work.
2.1 Overview of Supply Chain

This section provides a brief introduction to some basic terminologies and concepts of supply chains. Also, an overview of modeling the parameters that are needed in the latter chapters is given.

2.1.1 Supply Chain Structure

The most general supply chain structure is called the supply chain network where there are many companies in each stage, and each company in a particular stage is supplied by one or more companies in the preceding stage, and similarly each company can supply to one or more companies in the succeeding stage. Such a structure is very hard to analyze as the number of possible interactions within each stage is very large. A possible scenario of a supply chain network is depicted in Fig. 2.1.

![Fig. 2.1: Supply Chain Network](image)

Hence, most of the research done in this area has been restricted to arborescence structures (see Fig. 2.2). In such structures, a company in a particular stage can be supplied by at most one predecessor but it can supply to at least one company in the succeeding stage.
The simplest case of an arborescence structure is the **serial supply chain**. In a serial supply chain the various companies are arranged in series, i.e., each stage is supplied by only one predecessor stage. A 3-stage serial supply chain is shown in Fig. 2.3.

Another special case of the arborescence structure is the more complicated **single warehouse multi-retailer system** that we have described in Chapter 1. This thesis solely concentrates on such a system.

### 2.1.2 Centralized Vs. Decentralized Supply Chains

In a **centralized** supply chain there is a single decision maker who decides on the operating policy for all the companies in the supply chain whereas in a **decentralized** supply chain there are several decision makers such that each decision maker manages a single company. The latter occurs more frequently in real world (Lee and Whang, 1999).

In a centralized supply chain the ordering decisions are based on the **echelon stock** concept. Echelon stock can be defined as the inventory position at a particular location and all the inventories at the downstream echelons. In a decentralized system the ordering decisions are based on **installation stock**, i.e., stock at each installation (location).
An immediate research question is the comparison of centralized and decentralized systems. Intuitively a centralized control should dominate the decentralized control as in the centralized case decisions are made by optimizing all the objectives of the entire supply chain, while in a decentralized system each location is going to make its own independent decisions, which might be best for them, but might not be good for the supply chain.

Axsater and Rosling (1993) showed that a centralized control dominates a decentralized control for N-stage serial supply chain where each location in the system follow a (Q,r) policy. The results were extended to a general assembly system.

Axsater and Juntti (1996) determined the worst-case performance of installation stock policy when compared to echelon stock policy results for the case of a 2-stage serial supply chain under a deterministic scenario. For stochastic demand, a simulation was performed and the results indicated that a centralized control dominated a decentralized control when the warehouse lead times are long and vice-versa when the warehouse lead times are short.

DiFillipo (2003) showed through examples that the above intuitive result does not hold when transportation costs are included in the model.

### 2.1.3 Modeling Demand, Lead Time and Lead Time Demand

#### 2.1.3.1 Demand

Demand of a particular product can be deterministic or probabilistic. In the deterministic case it is possible to determine the state of the system at any given point in time. However, the same cannot be said in the probabilistic case since the demand is a random variable. The demand in such cases is assumed to have a known probability mass function (pmf) in the discrete case or a probability density function (pdf) in the continuous case. Since most companies keep track of historical data, forecasting techniques can be used to determine the distribution of future demand (Starr and Miller, 1962).
2.1.3.2 Lead Time

The lead time is the time between when an order is placed and when it is received at the customer end. It comprises mainly the ordering time and transit time. The ordering time includes preparing and processing the order. Transit time is the time spent by the goods in a particular transportation mode. We assume that the lead time equals transit time. Some researchers have emphasized including ordering time by treating it as a constant in addition to the transit time, e.g., Tyworth and Zeng (1998) and Ganeshan (1999). Including the constant ordering time poses no problem and is addressed as an extension to the basic model in Chapter 5.

Based on the lead time between two locations, inventory models can be classified as instantaneous replenishment (zero lead time), constant lead time and stochastic lead time (Ravindran, 2002). The first case simplifies the analysis a great deal. The second case is very similar to the first except that it is a more realistic approach. The third case represents real life situations where deliveries at a particular stocking point arrive late due to randomness in the lead time.

2.1.3.3 Lead Time Demand

Lead time demand (LTD) is simply the demand during the lead time. To model the LTD, we need the parameters of both demand and lead time (mean and variance). In this case the LTD distribution has to be obtained through joint distributions. Except for few distributions, the joint distributions result in forms which are not mathematically tractable. A list of the cases for which the joint distribution of LTD is known is given in Table 2.1. A proof of the first case is found in Hadley and Whitin (1963, Chapter 3, pp. 116-117). The other three cases are listed in Tyworth (1991).

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<th>Lead time</th>
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<td>Gamma</td>
<td>Negative binomial</td>
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<tr>
<td>Normal</td>
<td>Gamma</td>
<td>Approximate Gamma</td>
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</tbody>
</table>
The standard way of avoiding this issue (determining joint distributions) was to assume that the LTD is normally distributed with mean and variance given by:

\[ \mu_{LD} = \mu_D \mu_L \]  
\[ \sigma_{LD}^2 = \mu_L \sigma_D^2 + \mu_D^2 \sigma_L^2 \]

(2.1)  
(2.2)

where:

- \( \mu_D \) - Mean period demand
- \( \sigma_D \) - Standard deviation of period demand
- \( \mu_L \) - Mean lead time
- \( \sigma_L \) - Standard deviation of lead time

Eppen and Martin (1988) and Tyworth (1991) state that this approach for characterizing the LTD as a normal distribution can lead to inaccurate estimates of safety stock. Eppen and Martin (1988) and Tyworth (1992) suggest the use of a convex combination approach for modeling the LTD. Provided that the demands are independent between time periods, the LTD distribution can be modeled as a weighted sum of the conditional demand distributions over the range of the lead time values. Tyworth and Zeng (1998) state that this approach can be used for period demands that follow Normal, Poisson, Gamma and Exponential distributions and lead times having general discrete distribution or discrete approximations of continuous probability distributions. For example, if demand is normally distributed, \( N(\mu_D, \sigma_D^2) \) and the lead time takes on discrete values \( j = 1, \ldots, n \) with probability \( p_j \), then the conditional LTD \( \sim N(j\mu_D, j\sigma_D^2) \).

We use this result in Chapter 5, which deals with determining ordering policies under both stochastic demand and lead time. We also prove that the convex combination approach can be used for a special case of a compound Poisson distribution called the geometric Poisson distribution.

2.1.4 Ordering Policies

Inventory models, depending on the ordering policy followed by a location can be classified as continuous review or period review. In a continuous review policy a location places an order of \( Q \) units when a reorder point of \( r \) units is reached. For this reason it is also called a \((Q,r)\) policy. The time between orders in this case is a random variable. In a
periodic review policy a location places an order every T periods to raise the inventory level up to S units. For this reason a periodic review policy is also called a (T,S) policy. Here, the order quantity is a random variable.

A more general policy is the (S,s) policy wherein the location orders up to S units when the reorder point is less than or equal to s units. An (S,s) policy can be either continuously reviewed or periodically reviewed.

2.1.5 Modeling Service Levels

When the demand or lead time or both are random variables, LTD is also a random variable. As a result, the on-hand inventory when an order arrives is not known with certainty. When the LTD exceeds the reorder point, stock-outs occur. One obvious way of preventing stock-outs is to increase the product availability, but increasing the product availability increases the safety stock and thus the inventory holding cost. Hence, there should be a tradeoff between availability and cost.

In case of a stockout, one of the two things can happen: 1) excess demand is backordered or 2) excess demand is lost. In either case, we have to determine a stockout cost. In practice, stock-out cost is very hard to quantify as it consists of loss of goodwill with customers, downstream echelon retard, etc. An implicit way to avoid estimating stockout costs is for the DM to specify a target service level and make sure it is met by adding it as a constraint to the optimization problem.

There are two types of service level objectives:

1. Cycle service level (CSL): This is the fraction of order cycles in which all the customer demand is met.
2. Fill rate (FR): This is the proportion of demand that is filled from existing inventory.

2.1.6. Backorders Vs Lost Sales

Before we distinguish between the backorders and lost sales case, the following terms are defined:

- *On-hand Inventory*: The inventory that is available on the shelf.
- *Net Inventory*: On-hand Inventory – backorders.
• **Inventory Position:** On-hand Inventory – backorders + amount on order. The inventory level is thought in terms of inventory position for the case when more than one order is outstanding.

For the lost sales case, the number of backorders is zero. Hence, net inventory = on-hand inventory and inventory position = on-hand inventory + amount on order.

Let us assume that a location following a (Q,r) policy places an order of Q units when the on-hand inventory reaches the reorder point, r. The inventory position is Q+r units. Let demands continue to occur till a stockout condition is reached at the location. At this point the inventory position is Q units. If the location is assumed to backorder excess demand, the inventory position continues to decrease. There is a possibility that the outstanding order does not arrive after Q-r demands have occurred after the stockout. If this is the case, then another order is placed. Now, there are two outstanding orders. Thus, in the backorders case there can be more than one outstanding order.

In the lost sales case, the inventory position when a stockout situation is reached is still Q units. However, if more demands occur, the inventory position does not decrease, i.e., it remains at Q units. Hence, there can be at most one outstanding order for the lost sales case. The only way a location having lost sales can have more than one outstanding order is when r>Q. However, in the real world, the probability of having more than one order outstanding is very rare.

Hadley and Whitin (1963, Chapter 4, pp. 181-188) provide an exact analysis of the backorder case when there can be more than one outstanding order for a single location following a (Q,r) policy, facing Poisson demand and having deterministic lead times. It was based on determining the steady state distribution of the inventory position as uniform in [r+1,r+Q]. Once this was established, the steady state distributions for on-hand inventory and backorders were determined.

For the lost sales case, the inventory position will still be in [r+1,r+Q]. However, it may not be uniformly distributed. This is because when a stockout is followed by a demand, the inventory position in the lost sales will not decrease like in the backorders case. Instead of computing the steady state probabilities, they derive the expression for expected cost/cycle and multiply it by the total number of cycles. This analysis was based on the assumption that at most one order is outstanding.
Recently, Johansen and Thorstenson (2004) have solved the lost sales problem with more than one order outstanding for Poisson demands and Erlang distributed lead times.

2.1.7 Importance of Transportation

Most of the inventory models that have been developed in multi-echelon systems completely ignore transportation costs in their formulations. This is because most models implicitly assume *Freight on Board (F.O.B) destination*. Here, the supplier is responsible for shipping the products to the customer. The customer takes ownership of the products when the order is delivered at the destination. The price of the product includes the transportation cost associated with shipping the order to the destination. Alternatively, if *F.O.B origin* is assumed, the customer takes ownership of the order at the supplier. Transportation arrangements are made by the customer to ship the order to the destination.

As explained in Chapter 1, transportation costs account for nearly 6% of the GDP, and it is expected to increase in the future (Cook, 2006). Hence, it is vital for each company to be cognizant of its logistics function. Carter and Ferrin (1996) give reasons as to why a company should manage its inbound logistics. According to them a supplier can take advantage of customers by negotiating a better deal with the freight provider. Further, due to deregulation in the transportation industry, the primary way to obtain the desired transportation rates and services is through negotiation. These are some benefits to the company for managing its own inbound logistics network. However, capital will be tied up in the form of *in-transit inventory*. The expression for in-transit inventory is obtained by multiplying the fraction of year the annual demand spends in transit multiplied by the cost of the product and is given as:

\[
\frac{\mu_L}{365} \mu_D C
\]

where, \( C \) – Cost of the product.

Baumol and Vinod (1970) list the transportation factors that play an important role in determining the order quantity as:

- Freight rate.
• Speed (average transit time)
• Reliability (variance of transit time).

2.1.7.1 Impact of Freight Rate

The freight rate is a function of mode of transportation, weight shipped, distance and product. The different modes of transportation include rail, road, air and water. This thesis deals with only the road mode of transportation. Trucks are used for shipping goods between locations. Shipments can be full truck load (TL) or less than truck load (LTL). TL rates are usually stated based on distance travelled or a fixed charge/truck while LTL rates are stated per hundredweight (cwt). Note: 1 cwt = 100 lbs.

Products are classified according to several classes. There are 18 classes ranging from class 50 to class 500 (Ballou, 2003). Lower class items are charged a lower rate compared to higher class items. The rates for different classes are expressed as percentages of a base rate for a particular route. For example, a class 50 item would be charged 50% of the base rate. Rate tables are constructed that show how freight rates vary with distance and weight shipped. An excellent discussion about this topic is found in Ballou (2003).

Trucking companies offer discounts on the freight rate to encourage shippers to buy in large quantities. The details about the nature of the freight rate function and how it can be approximated by continuous functions are described in Chapter 4.

2.1.7.2 Speed

As mentioned before, transit time is the time spent by the goods in a particular transportation mode and represents a major portion of the lead time. Speed of shipping is determined by the mean transit time. If a company follows FOB origin, it will have capital tied up in the form of in-transit inventory. From Eq. 2.3, speed has an indirect relationship with the capital tied up in in-transit inventory, i.e., for low speeds more capital would be tied up and vice versa. Speed also affects the LTD variance given by Eq. 2.2. A low speed carrier would increase the variance of LTD and hence, increase safety stock.
2.1.7.3 Reliability

Reliability of a carrier is measured by the *variance of transit time*. From Eq. 2.3, it is clear that reliability does not affect the capital invested in in-transit inventory. However, from Eq. 2.2 it affects the LTD variance and hence, the safety stock.

2.2. Review of Multi-Criteria Optimization

Multi-criteria optimization deals with optimizing more than one objective function simultaneously. These functions are conflicting in nature, in the sense that increasing the value of one objective leads to a decrease in the value of the other. The decision maker has to use his/her experience to make the appropriate tradeoff and choose an efficient solution from a set of efficient solutions. Since we are dealing with a multi-functional (inventory and transportation) multi-echelon problem, each decision maker has to make tradeoffs that are not only efficient for their own company but also efficient for the entire system. A multi-criteria approach will enable the decision maker to make such decisions.

The marginal cost information needed to solve inventory models include ordering cost, inventory holding cost and cost of backorders/lost sales. Ordering costs is the cost associated with filling an order and is dependent on the labor rate and the number of labors employed. In general an order may consist of several different products. In such cases it is difficult to estimate the ordering cost for a particular product. Inventory holding costs mainly consists of *opportunity cost*, i.e., capital tied in inventory and *insurance costs* to prevent losses due to breakage or pilferage. The former is dependent on the company’s internal rate of return or market rate of return. These factors fluctuate on a regular basis and hence, affect the marginal cost information. Cost of lost sales/backorders is primarily dependent on loss of goodwill with customers which is again very hard to estimate. Modeling the inventory problem as a multi-criteria problem will circumvent this problem.

The aim of this discussion is to provide a brief overview of Multi-Criteria Decision Making (MCDM) techniques. We define the general Multi-Criteria Mathematical Program (MCMP), some terminology and a summary of the types of methods available for solving these problems.
Multi-criteria decision making (MCDM) models are broadly classified into two categories (Shin and Ravindran, 1991):

- **Multi-attribute decision analysis**: This area is applicable to problems having finite number of alternatives in a stochastic setting.
- **Multi-criteria optimization**: This is applicable to problems having infinite number of alternatives, mostly in a deterministic setting.

### 2.2.1 General Multi-Criteria Mathematical Program

The general multi-criteria mathematical problem (MCMP), also called the *vector maximization* problem is as follows:

\[
\begin{align*}
\text{Max } f_1(x) \\
\text{Max } f_2(x) \\
\vdots \\
\text{Max } f_P(x)
\end{align*}
\]

Subject to: \( x \in S \)

where:
- \( x \) – Vector of decision variables.
- \( S \) – Feasible region in decision space.
- \( S = \{ x \in \mathbb{R}^n \mid g_i(x) \leq 0, x \geq 0 \} \)
- \( Y \) – Objective space/Payoff set
  \( Y = \{ Y \mid Y = [f_1(x), f_2(x), \ldots, f_P(x)] \forall x \in S \} \)

### 2.2.2 Terminology Associated with MCDM

**Ideal Solution**

Ideal solution is defined as a vector of the individual optima (Shin and Ravindran, 1991). It is obtained by optimizing each objective function independent of the other. If the ideal solution is attainable, then the objectives are not conflicting.
Efficient Solution (Pareto optimal / Non-dominated solution)

A solution $x^e \in S$ is said to be an efficient solution if and only if for any point $x \in S$ and objective $k$ for which $f_k(x) > f_k(x^e)$, there exists at least one objective $j$ for which $f_j(x) < f_j(x^e)$, i.e., the only way of improving one objective is by doing worse on the other objective (Shin and Ravindran, 1991). The set of efficient solutions is called the efficient frontier.

Properly Efficient Solution

A point $x^e$ is said to be properly efficient if and only if (Shin and Ravindran, 1991)

- $x^e$ is efficient.
- Gain to loss rate or the tradeoff between two objectives is finite, i.e.,

$$\frac{f_k(x) - f_k(x^e)}{f_j(x^e) - f_k(x)} \leq M$$

where $M$ is a positive number.

Best Compromise Solution

The best compromise solution is one that maximizes the DM’s preferences (Shin and Ravindran, 1991). The DM’s preferences are modeled using a continuous preference function, also called the utility function which is not explicitly known. Thus, the MCMP reduces to a single objective problem that maximizes the DM’s utility function:

$$\text{Max } U \{f_1(x), f_2(x), \ldots, f_P(x)\}$$

Subject to: $x \in S$

2.2.3 Approaches to Solve MCMP

In many cases it is not possible to obtain a mathematical representation of the utility function (Steuer, 1986). The solution procedures that have been developed for MCMP are classified based on preference information from the DM (Shin and Ravindran, 1991).

- Methods requiring pre-specified preferences of the DM- In this category the DM’s preference information is known before the MCMP is actually solved. One
of the popular methods under this category is Goal Programming (GP). The DM assigns goals/target that have to be achieved for each objective. Then the DM is asked his/her preference on achieving the goals. GP tries to come up with a solution that comes as close as possible to all the stated goals in the specified preference order. Based on the preference information from the DM, GP can be classified as preemptive and non-preemptive. In preemptive GP relative priorities are assigned to each goal. The key idea here is that the highest priority goals are satisfied first, then the next higher order priority goals and so on. If the goals are set too low then GP might come up with a non-efficient solution. Solutions for linear preemptive GP are given by Lee (1972) and by Arthur and Ravindran (1978). Arthur and Ravindran (1979) also developed a solution procedure for solving linear integer preemptive GP. Non-preemptive GP is one in which weights can be specified for each goal. In this case the GP problem simplifies to a linear programming problem. More information about GP can be found in Lee (1972) and Ignizio (1976).

- **Methods that do not require any preference information from the DM** - This approach tries to generate all the points in the efficient set and then the DM has to choose his/her most preferred solution from the entire efficient set. Methods under this category are the \( P_\lambda \) problem (Geoffrion, 1968) and Compromise Programming (Zeleny, 1982). Once the efficient set is generated it can be plotted as an *efficient policy curve* if there are two criteria or *efficient policy surface* if there are three criteria.

- **Progressive articulation of preferences by the DM (Interactive method)** - The first step in this method is to generate a set of efficient solutions. Then, the tradeoff/preference information is obtained from the DM. Based on the DM’s response more efficient solutions are generated in the region of interest expressed by the DM. This process is repeated till the best compromise solution is found. The DM’s utility function is not known explicitly. An exhaustive survey of various techniques under the interactive method is given in Shin and Ravindran (1991).
We use the $P_\lambda$ method in conjunction with a graphical tool called the *value path* method to illustrate the tradeoffs to the DM. If the DM is not satisfied with the current set of efficient solution, another set of solutions is generated using the DM’s preference information. In the next section we summarize the $P_\lambda$ method. A description of the value path method is given in Chapter 4.

### 2.2.4 $P_\lambda$ Method

Consider the MCMP problem defined in Section 2.2.1. Then, the $P_\lambda$ problem assuming that the individual objectives are of the minimizing type is given as:

$$\begin{align*}
\text{Min } Z &= \sum_{k=1}^{P} \lambda_k f_k(x) \\
\text{Subject to: } & \quad x \in S \\
& \quad \sum_{k=1}^{P} \lambda_k = 1 \\
& \quad \lambda_k > 0 \forall k
\end{align*}$$

*Sufficient Condition:* If $x^*$ is an optimal solution to the $P_\lambda$ problem, then $x^*$ is an efficient solution to the MCMP problem.

*Necessary and Sufficient Condition:* If $S$ is a convex set, $f_k(x)$’s are convex (if it is of a minimizing type), then all the efficient points can be generated by the $P_\lambda$ problem.

If the set $S$ is not convex, then all the efficient points cannot be generated by the $P_\lambda$ method. Such points are called *unsupported efficient points*. This is illustrated in Fig. 2.4. The *supported efficient points* are the points that can be generated by the $P_\lambda$ problem, and they are represented by the darkened lines. The unsupported efficient points are represented by the dashed lines.
2.3 Literature Review

The articles that were reviewed can be classified as shown in Fig. 2.5.

There are some articles which fall under more than one subclass. Such articles will be classified under a particular category depending upon its contribution towards that category.
2.3.1 Single Location Inventory Control Models

2.3.1.1 With Transportation Costs

Baumol and Vinod (1970) developed the famous inventory theoretic model. They were the first to incorporate transportation factors such as cost of shipping (constant shipping cost/unit), speed (mean lead time) and reliability (variance of lead time) of the transportation mode. First, a deterministic model was developed and the optimal order quantity and cost was derived. Then demand and lead time were treated as random variables where demand is assumed to follow a Poisson distribution and lead time a normal distribution. The safety stock was approximated using the normal approximation to the Poisson distribution.

Das (1974) extended the basic inventory theoretic model on two fronts. The order quantity was determined independent of the safety stock expression. He also considered a more general expression to estimate the variance of the lead time demand. As a result the variance of the lead time demand decreases and hence the total cost.

Buffa and Reynolds (1977) generalized the inventory theoretic model to include a general freight tariff structure by including the rates for LTL, TL and Carload (CL) shipments. But the transportation cost was still a constant/unit. However, they contributed to the literature by using indifference curves to perform a sensitivity analysis by changing the parameters of the transportation factors. They concluded that the order quantity was sensitive to tariff rate, moderately sensitive to variability in lead time demand and insensitive to mean lead time.

Constable and Whybark (1978) extended the inventory theoretic model to include backorders. The problem was to determine the transportation alternative and the inventory parameters that minimize the total inventory and transportation costs. They solved the problem using a computationally intensive enumeration method to get exact solutions and a heuristic method. For the first method an iterative approach was used which decreases the value of the reorder point by one till the cost function begins to increase. This was repeated for all the transportation alternatives and the least cost alternative was chosen. The second method assumes normality of lead time demand. The inventory parameters were determined for a particular alternative. Based on this, the other
transportation alternatives were evaluated. The least cost alternative was chosen and the inventory parameters were recalculated. The heuristic results were very close to the actual solution.

Lee (1986) extended the basic EOQ model to incorporate discounts in the freight rates. He considered all units discount, incremental discount and the case of a stepwise freight cost which is proportional to the number of trucks used. He developed an efficient algorithm to solve the first case and modified it to solve the other two special cases.

Tyworth and Zeng (1998) used a sensitivity analysis approach to estimate the effects of carrier in-transit time on total logistics cost and service. A single stocking point following a continuous review policy faces stochastic demand. The lead time is assumed to have two components: transit time which is a discrete random variable and a fixed order processing time. Demand was assumed to be a gamma distributed random variable. Fill rate was used as a service measure and is arbitrarily set to a value. To incorporate transportation costs in the model, a curve was fit to the freight rates given the origin, destination and class of freight. They concluded that a power function would work well for the available data. This was implemented in a spreadsheet where the user can study the effects of the various parameters on the total cost.

Swenseth and Godfrey (2002) used the inverse and adjusted inverse freight rate function to determine the optimal order quantity. Based on these two functions the authors developed a heuristic to determine when to over-declare a shipment as a TL or to continue as a LTL shipment. This implies that in the former case the inverse function is used and in the latter the adjusted inverse function is used.

2.3.1.2 Lost Sales

Smith (1977) determines inventory policies for a location following a (S,S-1) policy, facing Poisson demand and any arbitrary distribution for lead times. A (S,S-1) policy is a special case of a (S,s) policy wherein an order of one unit is placed when the reorder point equals S-1 units. It is also called one-for-one ordering policy. He models the system as a queue with multiple servers and uses Erlang’s loss formula to compute the steady state distribution of the lead time demand. Using this, the steady state distribution for inventory and shortages were derived.
Nahmias (1979) developed approximate solutions for a location following a periodic review policy. He first extends the deterministic lead time lost sales case to incorporate fixed cost for placing an order. The second extension was to accommodate stochastic lead times. The third model accounts for partial backordering.

Archibald (1981) considered the inventory analysis of a single location for demand following a compound Poisson process and constant lead times. An (S, s) policy that is being continuously reviewed is assumed to be followed at the location. Excess demand was treated as lost sales. Using a Markovian approach, an expression for the average stationary cost was derived.

2.3.1.3 Multi-Criteria Inventory Control Theory

Brown (1961) was the first to suggest the treatment of the inventory control problem as a multi-criteria problem. The two criteria used were the annual number of orders and the quantity ordered per cycle.

Starr and Miller (1962) formulated the multi-item inventory control problem as a bi-criteria problem for a single stocking point. They used the capital invested in inventory and the annual number of orders as the two criteria. Using Lagrangian relaxation technique they proved that the product of the two criteria is a constant. Based on this result an optimal policy curve was developed which helps the decision maker in making tradeoffs.

Gardner and Dannenbring (1979) extended the work of Starr and Miller (1962) to a stochastic demand setting. In addition to the two criteria, they evaluated the customer service objective as the third criteria. The optimal policy curve was replaced by an optimal policy surface. A combination of Lagrangian relaxation and successive approximation technique was used to locate the points on the optimal policy surface.

Agrell (1995) solved the stochastic inventory problem by using three criteria: 1) expected annual cost. 2) annual number of stock-out occasions. 3) annual number of items stocked-out. An interactive decision exploration method (IDEM) was used to solve the problem. Local tradeoffs were determined by solving a generalized Lagrangian problem. The decision maker had to decide whether he/she has to improve, maintain or
sacrifice a particular objective. An Excel-based tool was then developed to solve medium range production planning problems.

2.3.2 Supply Chain Inventory Models

2.3.2.1 Deterministic Supply Chain Inventory Models

This section provides a review of some of the significant work done in single warehouse multi-retailer type of systems.

Schwarz (1973) concluded that the optimal policy for a single warehouse multi-retailer system was very complex since the order quantity varies with time even though the demand is deterministic. He concentrated on a class of policies called the basic policy and he showed that the optimal policy can be found in the set of basic policies. For the single retailer case he proved that the single cycle policy is the optimal policy. This result was not applicable for a more generalized system. A heuristic solution was proposed to solve the general problem.

Roundy (1985) concluded the ineffectiveness of nested policies. He developed a new class of policy called the power-of-two policies. The power-of-two restriction states that the time between orders of a retailer is a power-of-two multiple of a base period. An optimal solution to the relaxation of power-of-two formulation was derived. For a fixed base period, the cost of the power-of-two policy was 6% above the cost of the optimal policy whereas for a variable base period the cost was 2% above the cost of the optimal policy.

Abdul-Jalbar et al. (2003) compared the single warehouse multi-retailer problem operating under decentralized and centralized frameworks subjected to deterministic demand. When the system is operating in a decentralized framework, the warehouse faces time varying demand as the retailers order independently, and at different time periods. Hence, the order quantity at the warehouse was found over a finite planning horizon. For the centralized case the common replenishment time and different replenishment time were evaluated. The parameters were sampled from two separate uniform distributions. The number of instances where the decentralized case produced better results than the centralized cases increased with increase in the number of retailers. Also, the
decentralized case was preferred when the costs at the retailers was significantly greater than the cost at the warehouse.

Rangarajan and Ravindran (2005) introduced a base period policy for a decentralized supply chain. This policy states that every retailer orders in integer multiples of some base period, $T_B$, which is arbitrarily set by the warehouse. The warehouse problem is solved as a finite planning horizon model using Wagner-Whitin’s model.

2.3.2.2 Stochastic Supply Chain Inventory Models with Backorder Assumption

Under this category the literature pertaining to (S,S-1) and (Q,r) policies are covered. A (S,S-1) policy is a special case of a (S,s) policy wherein an order of one unit is placed when the reorder point equals S-1 units. It is also called a one-for-one ordering policy.

(S,S-1) Policies

Sherbrooke (1968) considered the case of a single warehouse supplying to several downstream retailers, where the demand faced by the retailers follow a Poisson distribution and stochastic lead times. The outstanding orders at the warehouse are characterized as a Poisson distribution. This follows from the result of Palm’s theorem. However, the same cannot be said about the outstanding orders at each retailer as they are dependent on the inventory position at the warehouse. Sherbrooke approximates the outstanding orders as a Poisson distribution which is characterized by the mean (single parameter approximation).

Graves (1985) extended Sherbrooke’s (1968) work by characterizing the outstanding orders at each retailer by using the mean and the variance (two-parameter approximation). He then fits a negative binomial distribution to these parameters to determine the ordering policy at each location.

Axsater (1990) demonstrated that the approximation of Sherbrooke (1968) underestimates the number of backorders, while the two-parameter approximation of Graves (1985) overestimates the number of backorders. Using the observation that an item ordered by a location will be used to fill the $S^{th}$ demand at that location and the
result that in a Poisson process the waiting time until the $n^{th}$ event follows an Erlang distribution, he provides an exact solution by deriving expressions for expected costs at the warehouse and the retailers. A recursive procedure is used to solve the problem efficiently.

$(Q,r)$ Policies

Deuermeyer and Schwarz (1981) developed an analytical model for predicting the service level when the retailers face identical Poisson demand. The demand at the warehouse is approximated as a stationary renewal process. They approximate the lead time demand at the warehouse as a normal distribution. The actual lead time of a retailer is the sum of the deterministic lead time plus the delay in meeting that retailer’s order as a result of a stockout situation at the warehouse. They approximate the stochastic lead time by determining only the expected delay. A simulation model was developed to check the accuracy of the analytical model.

Svoronos and Zipkin (1988) consider the same problem as Deuermeyer and Schwarz’s (1981). However, they use a different type of approximation to determine the mean and variance of the lead time. Simulation results indicate that a shifted Poisson distribution provided the best fit to the above parameters. In addition to quantifying the mean delay at the warehouse due to a stockout, they also estimate the variance of the delay.

Axsater (1993) provides an exact solution to the case of identical retailers in a single warehouse multi-retailer system. The exact cost was derived as a weighted mean of the costs for one-for-one ordering policies. This cost was evaluated by computing the probability that a system demand will trigger a retailer’s order. Since the computational effort involved in determining the probabilities is huge, he suggests the use of two approximations. Based on a simulation study, it was concluded that the approximations did give accurate results.

Forsberg (1996) provides an exact solution to the case of non-identical retailers. He uses the same methodology of Axsater (1993) to derive the expression for the exact cost. However, the probability computations were different as the previous method can
only be used for the case of two retailers. He computes the probability that the $i^{th}$ last retailer order was triggered by the $j^{th}$ last customer demand.

Ganeshan (1999) presented near optimal solutions for a two stage decentralized production-distribution system with multiple suppliers supplying a single warehouse, which in turn supplied multiple retailers. The period demand at the retailer was assumed to be a Poisson distribution. The period demand at the warehouse was then approximated to a Poisson distribution provided that the number of the retailers is at least 20. The lead time between each retailer and warehouse (fixed order processing time, fixed delay time in case of shortages and random transit time) and the lead time between the warehouse and each supplier (fixed order processing time and random transit time) are random variables with known density functions. A linear logarithmic cost function was used to estimate the transportation rates. These parameters were fed into a simulation model to determine the backorder levels at the retailers, and it was found to be very close to the actual fill rates set in the optimization model.

2.3.2.3 Stochastic Supply Chain Inventory Models with Lost Sales Assumption

Nahmias and Smith (1994) modeled the excess demand at the retailers in a two echelon system as partial sales i.e. some customers are willing to wait with a known probability, the others are treated as lost sales. Each location orders up to a base stock every review period. External demand faced by the retailers followed a negative binomial distribution. The demand at the warehouse is approximated as a Normal distribution. Instantaneous replenishment was assumed throughout the paper.

Andersson and Melchiorrs (2001) extended the model developed by Sherbrooke (1968) to the case where excess demand is considered as lost sales at the retailers. An approximate cost function was derived for each retailer based on the assumption that the lead times are i.i.d. The demand at the warehouse was approximated as a Poisson process to derive the cost function at the warehouse. Using a heuristic, the solution on an average was 0.4% above the optimal solution which was obtained using a simulation.
2.3.2.4 Supply Chain Models with Transportation Costs

Perl and Sirisoponsilp (1988) analyzed the interdependence between facility location, transportation and inventory decisions by developing an integrated model for a distribution network. A good account of how strategic, tactical and operational decisions in one function affected the various decisions in the other was outlined. The objective function was to minimize the warehousing, trucking (between supplier and warehouse), delivery (between warehouse and customer), in-transit inventory, cycle and safety stock (at both the warehouse and customer) costs. Based on the service levels (probability of a stockout), the inventory levels, the mode of transportation, assigning distribution centers (DC) to suppliers and allocating customers to DCs were determined by the model. The trucking cost comprised of a fixed cost and a variable cost which is inversely related to the amount shipped. Nothing was mentioned about how the problem was solved.

Qu et al. (1999) developed a methodology where decisions regarding both inventory and transportation could be made simultaneously. It was more of a shipment consolidation problem where a central warehouse had to dispatch a truck to various suppliers to collect different items. The total cost expression consisted of transportation cost (stopover and routing costs) and inventory costs (ordering, holding and backorder costs). The warehouse followed a modified periodic review policy where the fast moving items (base items) were ordered every base period, and the remaining items were ordered in integer multiples of the base period. The problem was solved by a decomposition procedure. Two problems, the inventory master problem and the transportation sub-problem were created. The solution was obtained by iterating between the master and the sub-problem. The sub-problem reduces to a traveling salesman problem since the vehicle was assumed to have infinite capacity. Transportation cost per unit distance was used.

Nozick and Turnquist (2001) developed an integrated framework that accounts for facility, inventory and transportation costs with service levels at the customer zones. A fixed charge model was developed in which an approximate inventory function was embedded. This was based on an earlier work done by the authors, which approximates the safety stock as a linear function of the number of facilities (distribution centers) for a given stock-out rate under the condition that demand is a Poisson random variable. To capture the fundamental tradeoff between service level and cost of opening new facilities,
they developed a multi-objective math program by introducing another objective which minimized the uncovered demand. By assigning a weight to this objective, the aforementioned tradeoff can be captured by increasing/decreasing the weights. They suggested the use of any standard algorithm that solves the fixed charge problem to solve this problem because of the same structure. However, neither actual freight rates nor functions which emulate the actual freight rates were used.

Chan et al. (2002) considered a single warehouse-multiple retailer scenario where the warehouse is supplied by a single supplier. They developed a finite horizon model for time varying demand with a piecewise linear concave transportation cost structure for the link between supplier and warehouse and a modified all unit discount structure (weight breaks) between warehouse and retailers. They evaluated Zero Inventory Ordering (ZIO) policies, i.e., the warehouse places an order only if the inventory level is zero (Wagner-Whitin rule), which implies that the warehouse is a cross-docking facility. Upper bounds were calculated on the cost of the ZIO policy in the case of time varying transportation and stationary transportation costs. After the transformation, the problem was reformulated as a shortest-path problem and solved using Dijkstra’s algorithm. However, the problem becomes NP hard (non deterministic polynomial) when the number of retailers increases. The problem was then formulated as an Integer Program (IP), which could be solved only for small scale problems. Hence, a Linear Program (LP) based heuristic was developed which gave quality solutions very quickly for large scale problems.

Geunes and Zeng (2003) presented an approach to optimize inventory and transportation costs when shortages occur at a warehouse which follows a periodic review policy, while facing stochastic demand. Complete expediting, complete backordering and a hybrid policy (partial backordering-partial expediting policy) were evaluated. A maximum current demand needs to be shipped (MCDS) every period. The warehouse used expedited shipping to ship the difference between MCDS and available stock from the supplier. The rest of the demand is backlogged. Expressions for inventory and transportation costs were developed. A heuristic approximation was used to solve the problem since a mathematically tractable expression could not be developed for the transportation cost. Results indicate that the hybrid policy outperforms the other policies.
and, in some cases where the expediting cost was greater than the backlogging cost, the solution recommended to follow a complete backordering policy. However, the authors do not consider a systems approach i.e. they solved only the warehouse problem.

Toptal et al. (2003) extended the buyer-vendor (retailer-warehouse) problem under deterministic demand by developing two models: 1) including inbound transportation costs and 2) including inbound and outbound transportation costs at the warehouse. The transportation cost structure included a fixed cost and a stepwise freight cost that is proportional to the number of trucks used. Heuristics solutions were developed for both the models, and worst case error bounds were 6% and 25% for model 1 and model 2, respectively. However, numerical analysis of a sample problem indicates that the error in the heuristic solution was much less than 25% for model 2. The model, however, would give accurate results for TL because of the transportation cost function.

Mason et al. (2004) built a mixed integer programming finite horizon model that integrated inventory and transportation decision making considering time based service levels into account for a service parts logistics industry under varying demand. The problem setting consists of a single supplier supplying different parts to multiple warehouses, which in turn supply to different customer zones. The problem was formulated for three different inventory policies: 1) \((s, S)\) policy, 2) Base stock policy, and 3) \((r, Q)\) policy. The model determined how much each warehouse ships to each customer by which transportation mode in a time period. It also determined the reorder points, order-up-to levels and the average inventory levels at each warehouse for all the parts. A full factorial experiment was conducted by varying some of the parameters to compare the dynamic model with a static one and the performances of the aforementioned policies were evaluated. As expected, the dynamic model outperformed the static model. In terms of the inventory policies, there was no significant difference between \((s, S)\) and \((r, Q)\) policy. The base stock policy performed better than the other two in the static case but not in the dynamic case.

Another stream of research in coordinating inventory-transportation decisions in supply chain focuses on vehicle routing issues. A good overview of existing literature in this area is given by Schwarz et al. (2004)
2.3.2.5 Multi-Criteria Supply Chain Inventory Models

Bookbinder and Chen (1992) considered a single warehouse single retailer problem following a continuous review policy subjected to deterministic demand. Assuming the availability of marginal cost information they developed two objectives: 1) sum of ordering and holding costs and 2) fixed transportation cost for each shipment. The analysis was then extended to probabilistic demand. Two cases were considered, one with marginal cost information and the other without. In the first case the conflicting objective functions were the sum of ordering and holding costs and the annual number of stock-out occasions. The optimal order quantity and the reorder point were obtained by varying the probability of stock-out. For case 2, in addition to the two objectives in case 1 fixed transportation cost was the third objective. An interactive method based on Lagrangian relaxation was used to solve the problem.

Thirumalai (2001) developed multi-criteria models for a serial supply chain where each company is managed by an independent decision maker. In the deterministic demand case the supply chain had two objectives, which were the annual number of orders placed and the capital invested in inventory. A collaborative optimization algorithm was suggested to bring the companies closer to supply chain efficiency. This was then extended to the stochastic demand case. In addition to the two objectives, a third objective that was considered was the probability of a stock-out.

DiFillipo (2003) extended the classic single warehouse multi-retailer problem by using a multi-criteria approach that explicitly considered freight rate functions that emulate actual freight rates for both centralized and decentralized cases. Under the decentralized case, the fixed order and non-stationary order policies at the warehouse were analyzed. Under the centralized case, common and different replenishment policies were analyzed. To compare the different policies single objective problems were solved using marginal cost information. Based on an example problem, both the policies under the centralized case were dominated by the decentralized fixed order policy case. This contradicts the well known result that centralized case outperforms the decentralized case. However, the analysis was restricted to two retailers subjected to deterministic demand.
2.4 Contributions of the Research

The contribution of this thesis is assessed chapter wise. Table 2.2 summarizes the literature survey of *deterministic base period inventory models*.

Table 2.2: Contribution of Chapter 3

<table>
<thead>
<tr>
<th>Author</th>
<th>System</th>
<th>Control</th>
<th>Base Period</th>
<th>Effectiveness</th>
<th>Grouping retailers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roundy (1985)</td>
<td>SWMR*</td>
<td>Centralized</td>
<td>Arbitrary</td>
<td>✓</td>
<td>x</td>
</tr>
<tr>
<td>Rangarajan &amp; Ravindran (2005)</td>
<td>SWMR</td>
<td>Decentralized</td>
<td>Arbitrary</td>
<td>✓</td>
<td>x</td>
</tr>
<tr>
<td>This thesis</td>
<td>SWMR</td>
<td>Decentralized</td>
<td>Specified</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

Roundy (1985) and Rangarajan and Ravindran (2005) use an arbitrary value to set the base period. Rangarajan and Ravindran (2005) addressed the issue that specifying the base period incorrectly can lead to inferior solutions. In Chapter 3, the modified base period policy is introduced wherein the base period is chosen based on sound intuitive reasoning and validation from an executive of a Fortune 500 consumer products company. This allows us to theoretically group retailers into clusters in terms of how important they are to the warehouse. The effectiveness of the policy was tested by developing theoretical bounds on the deviation from the optimal solution. The example problem and case study (with real world data) indicates that the modified base period policy performs really well for a wide range of data.

Table 2.3 summarizes the literature review associated with the *deterministic multi-criteria supply chain model with transportation costs* in Chapter 4.

---

* Single Warehouse Multi Retailer
Table 2.3: Contribution of Chapter 4

<table>
<thead>
<tr>
<th>Author</th>
<th>Location</th>
<th>System</th>
<th>Criteria</th>
<th>Transportation Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brown(1961)</td>
<td>Single</td>
<td>-</td>
<td>Multiple</td>
<td>-</td>
</tr>
<tr>
<td>Starr &amp; Miller (1962)</td>
<td>Single</td>
<td>-</td>
<td>Multiple</td>
<td>-</td>
</tr>
<tr>
<td>Thirumalai (2001)</td>
<td>Multiple</td>
<td>Serial</td>
<td>Multiple</td>
<td>-</td>
</tr>
<tr>
<td>Chan et al. (2002)</td>
<td>Multiple</td>
<td>SWMR</td>
<td>Single</td>
<td>Piecewise linear concave &amp; modified all unit discount</td>
</tr>
<tr>
<td>Toptal et al. (2003)</td>
<td>Multiple</td>
<td>Serial</td>
<td>Single</td>
<td>Stepwise cost structure</td>
</tr>
<tr>
<td>DiFillipo (2003)</td>
<td>Multiple</td>
<td>SW2R †</td>
<td>Multiple</td>
<td>Continuous functions</td>
</tr>
<tr>
<td>This thesis</td>
<td>Multiple</td>
<td>SWMR</td>
<td>Multiple</td>
<td>Continuous functions</td>
</tr>
</tbody>
</table>

Brown (1961) and Starr and Miller (1962) treated the inventory problem from a multi-criteria perspective for single locations facing deterministic demands. Thirumalai (2001) and DiFillipo (2003) have treated the deterministic supply chain inventory problem from a multi-criteria perspective. While Thirumalai’s work is restricted to serial supply chains, DiFillipo’s work is restricted to a single warehouse two retailer system. In Chapter 4, we extend the modified base period policy proposed in Chapter 3 to a multi-criteria framework. The three criteria considered are capital invested in inventory, annual number of orders placed and annual transportation costs.

Table 2.4 summarizes the relevant literature related to the stochastic multi-criteria model with transportation costs developed in Chapter 5.

† Single Warehouse Two Retailers
Table 2.4: Contribution of Chapter 5

<table>
<thead>
<tr>
<th>Author</th>
<th>Location</th>
<th>System</th>
<th>Lead times</th>
<th>BO/LS&lt;sup&gt;‡&lt;/sup&gt;</th>
<th>Criteria</th>
<th>Transportation Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gardner &amp; Dannenbring (1979)</td>
<td>Single</td>
<td>-</td>
<td>-</td>
<td>BO</td>
<td>Multiple</td>
<td>-</td>
</tr>
<tr>
<td>Deuermeyer &amp; Schwarz (1981)</td>
<td>Multiple</td>
<td>SWMR</td>
<td>Deterministic</td>
<td>BO</td>
<td>Single</td>
<td>-</td>
</tr>
<tr>
<td>Svoronos &amp; Zipkin (1988)</td>
<td>Multiple</td>
<td>SWMR</td>
<td>Deterministic</td>
<td>BO</td>
<td>Single</td>
<td>-</td>
</tr>
<tr>
<td>Bookbinder &amp; Chen (1992)</td>
<td>Multiple</td>
<td>Serial</td>
<td>Deterministic</td>
<td>BO</td>
<td>Single</td>
<td>-</td>
</tr>
<tr>
<td>Axsater (1993)</td>
<td>Multiple</td>
<td>SWMR</td>
<td>Deterministic</td>
<td>BO</td>
<td>Single</td>
<td>-</td>
</tr>
<tr>
<td>Nahmias &amp; Smith (1994)</td>
<td></td>
<td></td>
<td>Instantaneous</td>
<td>LS</td>
<td>Single</td>
<td>-</td>
</tr>
<tr>
<td>Agrell (1995)</td>
<td>Single</td>
<td>-</td>
<td>-</td>
<td>BO</td>
<td>Multiple</td>
<td>-</td>
</tr>
<tr>
<td>Forsberg (1996)</td>
<td>Multiple</td>
<td>SWMR</td>
<td>Deterministic</td>
<td>BO</td>
<td>Single</td>
<td>-</td>
</tr>
<tr>
<td>Ganeshan (1999)</td>
<td>Multiple</td>
<td>SWMR</td>
<td>Stochastic</td>
<td>BO</td>
<td>Single</td>
<td>Continuous functions</td>
</tr>
<tr>
<td>Thirumalai (2001)</td>
<td>Multiple</td>
<td>Serial</td>
<td>-</td>
<td>BO</td>
<td>Multiple</td>
<td>-</td>
</tr>
<tr>
<td>Andersson &amp; Melchior (2001)</td>
<td>Multiple</td>
<td>SWMR</td>
<td>Deterministic</td>
<td>LS</td>
<td>Single</td>
<td>-</td>
</tr>
<tr>
<td>This thesis</td>
<td>Multiple</td>
<td>SWMR</td>
<td>Stochastic</td>
<td>LS &amp; BO</td>
<td>Multiple</td>
<td>Continuous functions</td>
</tr>
</tbody>
</table>

Deuermeyer and Schwarz (1981), Svoronos and Zipkin (1988) and Ganeshan (1999) provide an approximate approach to modeling and solving a single warehouse multi-retailer system with identical stochastic demands. Ganeshan (1999) treats the lead times as random variables. Axsater (1993) and Forsberg (1996) concentrate on developing exact solutions for the identical and non-identical demands respectively. They acknowledge that extending their work to the case of stochastic lead times is extremely difficult. None of the above researchers consider the case of lost sales. Nahmias and Smith (1994) consider the lost sales case for a periodic review system, while Andersson and Melchior (2001) consider the lost sales case for a one-for-one ordering policy.

<sup>‡</sup> Backorders/Lost Sales
Bookbinder and Chen (1992) and Thirumalai (2001) have used a multi-criteria approach to the backorders case while assuming that the lead time demand is normally distributed. They treat cycle service level (CSL) as the criterion that measures customer satisfaction. In Chapter 5, we develop a multi-criteria model that accounts for both stochastic non-identical demands and random lead times. Fill rate (FR) is the fourth criterion and is used to measure customer satisfaction. We cover both the lost sales and backorders case.

To summarize, there has been a lot of work done in joint inventory-transportation decision making for single stocking points, both under the deterministic and stochastic case. Researchers have used complex step functions as well as simple continuous functions to model the freight rate costs. Multi-criteria techniques have also been used to solve the inventory problem under both deterministic and stochastic setting for single stocking points.

Multi-criteria optimization techniques have hardly been used in the field of supply chain management. Given the nature of the problem (conflicting criteria) and the human involvement in the decision making process, particularly in supply chains, there is a need for paradigm shift from single objective methods to multiple criteria methods. Since the DM analyzes the tradeoff information among the set of efficient solutions before the most preferred solution is chosen, it is a more effective decision making tool. By choosing the most preferred solution, the DM makes sure his/her preference information is incorporated in the final solution.

In chapter 3, a single cost objective model is developed and solved that enables all the companies in the supply chain to determine their inventory ordering policies efficiently. Using this model, a new inventory control policy is discussed, and its effectiveness is tested. Also, a theoretical method is developed which enables the warehouse to group retailers into clusters.
Chapter 3
A MODIFIED BASE PERIOD POLICY FOR A SINGLE WAREHOUSE MULTI-RETAILER SYSTEM

3.1 Introduction

In a single warehouse multi-retailer system operating under decentralized control, it is highly likely that each retailer is going to face different demands. Each retailer will therefore order in different time periods, totally independent of one another. The warehouse, in such a scenario faces an infinite horizon time varying demand. There has to be some sort of an inventory control policy and coordination among the various entities in the supply chain that will enable the warehouse to plan its inventory and at the same time meet the demands of the retailers without deviating too much from their actual requirements. Several such policies exist in literature, and they are listed next with respect to the single warehouse multi-retailer system:

1. **Integer policy** (Crowston et al. (1973): The order quantity of the warehouse is an integer multiple of the sum of the order quantity of the downstream retailers.

2. **Basic Policy** (Schwarz, 1973): A basic policy is one which satisfies the following properties:
   - A delivery is made to the warehouse only when the warehouse has zero inventory and at least one retailer has zero inventory.
   - A delivery is made to the retailer only when that retailer has zero inventory.
   - The deliveries made to a retailer during the ordering periods of the warehouse are equal.

3. **Myopic policy** (Schwarz, 1973): Myopic policy is a special class of basic policy where the original problem is split into N one warehouse one retailer problems.

4. **Nested policy** (Love, 1972): Nested policy is one in which each time the warehouse orders all the retailers also place an order.

5. **Stationary or fixed order policy** (Schwarz, 1973): If the order quantity and the time between orders for each retailer and the warehouse repeat itself then the policy is said to be a stationary policy.
6. **Single cycle policy** (Schwarz, 1973): A single cycle policy is another class of basic policy which is nested and stationary.

7. **Power-of-two policy** (Roundy, 1985): If the time period between orders of each retailer is restricted to the power-of-two multiple of a base period, such a policy is termed power-of-two policy.

8. **Base period policy** (Rangarajan and Ravindran, 2005): If the time period between orders of each retailer is restricted to integer multiple of a base period, such a policy is termed base period policy.

The power-of-two policy and the base period policy assume arbitrary value for the base period, for example a week or a month. Incorrectly specifying the base period can lead to inferior solutions at the retailer. To avoid this problem, we introduce a modified base period policy wherein the base period is set by the warehouse, based on a suitable performance measure. Following the notations we first discuss the economic order quantity (EOQ) model and an equivalent formulation called the economic reorder interval problem.

### 3.1.1 Notation

**Data**

- \(D_i\) – Annual demand faced by retailer \(i \in \{1, 2, \ldots, N\}\)
- \(h_i\) – Inventory holding cost at retailer ($/unit/year) \(i \in \{1, 2, \ldots, N\}\)
- \(a_i\) – Fixed order cost at retailer \(i \in \{1, 2, \ldots, N\}\)
- \(D_w\) – Annual demand faced by the warehouse.
- \(h_w\) – Inventory holding cost at the warehouse ($/unit/year).
- \(a_w\) – Fixed order cost at the warehouse.
- \(N\) – Number of retailers.

**Variables**

- \(Q_i\) – Order quantity of retailer \(i \in \{1, 2, \ldots, N\}\)
- \(T_i\) – Time between orders at retailer \(i \in \{1, 2, \ldots, N\}\)
- \(Q_w\) – Order quantity of the warehouse.
- \(T_w\) – Time between orders at the warehouse.
3.1.2 EOQ Problem

Consider the case of a single location. The notation defined previously can be used without the subscript. Then, the EOQ problem is to determine $Q$ that minimizes annual inventory cost:

$$\min_{Q \geq 0} Z = a \frac{D}{Q} + h \frac{Q}{2}$$

The optimal order quantity, $Q^* = \sqrt{\frac{2aD}{h}}$ (3.2)

The optimal reorder interval, $T^* = \frac{Q^*}{D} = \sqrt{\frac{2a}{hD}}$ (3.3)

The optimal cost, $Z^* = \sqrt{2aDh}$ (3.4)

3.1.3 Economic Reorder Interval Problem

Eq. 3.1 can also be expressed in terms of the reorder intervals, $T$ by using $T = Q/D$ as:

$$\min_{T \geq 0} Z = \frac{a}{T} + gT$$

where $g = hD/2$

Eq. 3.3 can be rewritten as $T^* = \sqrt{\frac{a}{g}}$ (3.6)

and Eq. 3.4 as $Z^* = 2\sqrt{ag}$ (3.7)

3.2 Problem Description

The supply chain system being modeled consists of a single warehouse supplying a product to several retailers. A manufacturer owns the warehouse and replenishes it on a regular basis. The system operated under a decentralized control. The goal is to efficiently determine ordering policies for each retailer and the warehouse.
3.2.1 Assumptions

- Demand faced by the retailers is assumed to be deterministic and continuous. It is further assumed that extraneous factors such as risk, seasonality, promotions, etc. have been incorporated when the forecasts were determined.
- Marginal cost information such as ordering and inventory holding costs are available.
- Instantaneous lead times between echelons.

3.2.2 Base Period Policy

This policy was introduced by Rangarajan and Ravindran (2005) for a decentralized supply chain. This policy states that every retailer orders in integer multiples, \( n \) of some base period, \( T_B \), which is arbitrarily set by the warehouse. This condition is incorporated into the economic reorder interval problem (Section 3.1.3) as follows:

\[
\begin{align*}
\min_{T \geq 0} \ Z &= \frac{a}{T} + gT \\
\text{s.t.} \quad T &= nT_B \\
\quad n &\geq 1 \text{ and integer.}
\end{align*}
\]

A line search method is used to determine the optimal integer values, \( n \). It is also shown that the policy guaranteed error bounds within 6% of the EOQ policy provided the base period is chosen carefully. The warehouse problem is solved as a finite planning horizon model using Wagner-Whitin model (Wagner and Whitin, 1958).

3.3 Modified Base Period Policy

This section introduces the modified base period policy. This policy differs in one important aspect: the base period, \( T_B \) is no longer set arbitrarily.

3.3.1 Model Formulation

Under the modified base period policy each retailer starts of by solving Eq. 3.1 or Eq. 3.5. This is called Problem \( (R_i) \). The output (optimal reorder interval) is passed on to the warehouse. The warehouse identifies a base retailer or a set of base retailers. A base
retailer(s) is one who orders *most* frequently or, equivalently, has the *least* reorder interval among all the retailers. Instead of arbitrarily specifying the base period, the warehouse sets the base period as the reorder interval of the base retailer(s).

$$T_B = \text{Min} T_i^*$$  \hspace{1cm} (3.9)

where $T_i^*$ is the economic reorder interval for retailer ‘i’.

The other retailers or the set of *non-base* retailers order in integer multiples of the base period given by Eq. 3.9, i.e., these retailers have to incorporate Eq. 3.8 to Problem (R_i), where $T_B$ is determined by the warehouse using Eq. 3.9. This problem is called the Modified Retailer’s problem denoted by Problem (MR_i) and is given as:

**Problem (MR_i)**

$$\text{Min}_{T_i^{\geq 0}} Z_i = \frac{a_i}{T_i} + g_i T_i$$

s.t.: $T_i = n_i T_B$

$n_i \geq 1$ and integer

### 3.3.2 Setting $T_B$: A Different Perspective

An intuitive reason to set the base period as in Eq. 3.9 is as follows: The warehouse receives its first order at a time period given by $\text{Min} T_i^*$. This implies that none of the other retailers are going to place an order before $\text{Min} T_i^*$. Hence, there is no need to set the base period below the value given by Eq. 3.9. Now, consider the case where $T_B$ is set such that $T_B > \text{Min} T_i^*$. This means that the orders placed by the set of base retailers will not be met on time. Hence, there is no point in setting $T_B > \text{Min} T_i^*$.

### 3.3.3 Another Interpretation of $T_B$

In addition to the interpretation of $T_B$ in Section 3.3.1, another interpretation is warranted. $T_B$ can also be thought of as a *throughput* measure. Throughput is a performance metric used in scheduling jobs in a machine shop to indicate the rate at which a part(s) exits the machine shop after finishing all the machining requirements. Increasing throughput increases the efficiency of the shop by decreasing WIP (Work In-
Process Inventory). A similar analogy can be extended to our model by viewing the supply chain as a shop floor. From the warehouse’s perspective, the base retailer(s) turns over its inventory most rapidly or has the maximum inventory velocity.

In an interview published in Harvard Business Review (Magretta (1998)), Mr. Michael Dell stresses the importance of inventory velocity as one of the performance measures that has enabled Dell computers to hold only about 3-4 days of inventory. This was more in reference to Dell’s internal operations. On interacting with executives of a Fortune 500 consumer products company, we found that they evaluate their customers using Eq. 3.9. According to them, customers having lower reorder intervals are preferred over customers having higher reorder intervals. This validates our observations in the preceding paragraphs that $T_B$ can be used to gauge each retailer’s performance relative to the base retailer(s). This leads us to another aspect, i.e., can the warehouse use this performance measure to segment its market (all the retailers) into clusters and if so how? This is answered in Section 3.4.1.

3.3.4 Solution to Problem (MRi)

Ordering in integer multiple of the base period (as set by Eq. 3.9) implies that the non-base retailer’s orders piggyback over the orders placed by the base retailers. This is apparent when we analyze the inventory pattern at the warehouse.

To determine the ordering period of the non-base retailers, the integer multiples, $n_i$’s have to be determined. The integer multiples can be determined by exploiting the point-wise convexity property of the cost function.

**Lemma 3.1** – *The objective function of Problem (MRi) is point-wise convex in $n_i$.*

**Proof:** In general, to show that an objective function is point-wise convex, we have to show that second difference with respect to the integer variable is nonnegative. Problem (MRi) can be restated as a function of the integer variable $n_i$ as:

$$\min_{n_i \geq 1, \text{integer}} Z_i = \frac{a_i}{n_i T_B} + g_i n_i T_B$$

For convenience subscript $i$ is eliminated.
\[ Z(n) = \frac{a}{nT_B} + gnT_B \]

\[ Z(n + 1) = \frac{a}{(n + 1)T_B} + g(n + 1)T_B \]

Computing the first difference:
\[ \Delta Z(n) = Z(n + 1) - Z(n) \]
\[ = -\frac{a}{T_B n(n + 1)} + gT_B \]

Similarly,
\[ \Delta Z(n + 1) = -\frac{a}{T_B (n + 1)(n + 2)} + gT_B \]

Computing the second difference:
\[ \Delta^2 Z(n + 1) = \Delta Z(n + 1) - \Delta Z(n) \]
\[ = \frac{a}{T_B} \left\{ \frac{1}{n(n + 1)} - \frac{1}{(n + 1)(n + 2)} \right\} \]
\[ = \frac{a}{T_B} \left\{ \frac{2}{n(n + 1)(n + 2)} \right\} \]
\[ > 0 \]

\textbf{Theorem 3.1} - The optimal integer value \( n_i \) for each retailer is the smallest integer that satisfies the following condition:
\[
 n_i (n_i + 1) \geq \left( \frac{T_i^*}{T_B} \right)^2 
\]  \hspace{1cm} (3.10)

\textbf{Proof}: The optimal integer multiple is determined by finding the smallest integer such that first difference is non-negative.
\[ \Delta Z(n + 1) = Z(n + 1) - Z(n) \geq 0 \]
\[ \Rightarrow -\frac{a}{T_B n(n + 1)} + gT_B \geq 0 \]
\[ \Rightarrow \frac{a}{T_B n(n + 1)} \leq gT_B \]
\[ \Rightarrow n(n+1) \geq \left( \frac{T^*}{T_B} \right)^2 \]

The right hand side (RHS) of Eq. 3.10 is always greater than 1 due to Eq. 3.9. The ratio \( \frac{T_i^*}{T_B} \) indicates how often each non-base retailer orders relative to the base retailers. It can also be thought of as how each non-base retailer performs in comparison to the base retailer in terms of inventory turns. We use \( \frac{T_B}{T^*_i} \) instead of \( \frac{T_i^*}{T_B} \) in the rest of the analysis.

3.4 Analysis of the Modified Base Period Policy

In this section we analyze the modified base period policy in more detail. The first part of the analysis deals with developing a theoretical framework to classify retailers into clusters. In the second part, bounds are developed on the deviation from the EOQ solution as a result of following this policy for the retailers.

3.4.1 Grouping Retailers

Since the concepts of revenue management are becoming prominent in supply chain management, it is important for the warehouse to differentiate among its customers by providing different quality of service in order to maximize its revenues.

We can use an analogy from Pareto’s principle: 80% of the revenue is generated from 20% of the customers. Hence, the warehouse has to provide better service to these retailers, e.g., meeting demand requirements, on time delivery, etc. To do this, the warehouse has to first segment its market into groups or clusters based on several performance measures.

As mentioned in Section 3.3.3., \( T_B \) as set by Eq. 3.9 is used by a Fortune 500 company to measure the performance of their retailers. We use this performance metric along with the result of Problem (MRi) as given by Eq. 3.10 to theoretically derive results that help the warehouse in grouping retailers into clusters.

It is of interest to find the breakpoints, i.e., at what values of \( \frac{T_B}{T^*} \) does the value of \( n \) change. This can be achieved by solving the equality form of Eq. 3.10:
The value of $n$ remains unchanged between two consecutive breakpoints. We refer to this as the \textit{breakpoint interval}. For example, the breakpoint interval for $n=2$ is $\left[ \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{2}} \right]$. At $T_B/T^* = 1/\sqrt{6}$, $n$ changes from 3 to 2 and at $T_B/T^* = 1/\sqrt{2}$, $n$ becomes 1.

More specifically, we can use these breakpoint intervals to group retailers into a cluster. For example, if $T_B/T^*$ for a set of retailers fall in the interval $\left[ \frac{1}{\sqrt{2}}, 1 \right]$, then they can be grouped together into cluster 1\(^\S\). If the ratios fall in the interval $\left[ \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{2}} \right]$, then it is grouped into cluster 2, and so on.

We believe that these results can be used in conjunction with other financial metrics, e.g., gross margins, to more effectively group retailers. Based on this, the warehouse can then decide how best to serve each group of retailers.

\section*{3.4.2 Effectiveness of the Policy}

Being a decentralized system, each retailer would like to have their ordering policy as close to the EOQ policy as possible. The base retailer’s ordering policy is going to be the EOQ solution itself. It is important to know how much the solution for the set of non-base retailers deviate from the optimal solution.

Since the objective function of Problem (MR\textsubscript{i}) is point-wise convex, we have to find the smallest integer where the objective value starts to increase, i.e.,

\[ Z\{(n-1)T_B \geq Z[nT_B] \leq Z\{(n+1)T_B \} \]

Consider $Z\{nT_B \} \leq Z\{(n+1)T_B \}$

\S The base retailers are grouped in cluster 1 as well.
\[ \Rightarrow \frac{a}{nT_B} + gnT_B \leq \frac{a}{(n+1)T_B} + g(n+1)T_B \]

\[ \Rightarrow \frac{a}{T_B n(n+1)} \leq gT_B \]

\[ \Rightarrow \frac{(T^*)^2}{T_B (n+1)} \leq T \]  (Using Eq. 3.6 and Eq. 3.8)  (3.11)

Similarly it can be shown that \( T \leq \frac{(T^*)^2}{T_B (n-1)} \)  (3.12)

Eq. 3.11 and Eq. 3.12 implies that \( \frac{(T^*)^2}{T_B (n+1)} \leq T \leq \frac{(T^*)^2}{T_B (n-1)} \)  (3.13)

Eq. 3.13 provides a lower and upper bound on the reorder interval of a retailer. The next step is to compute the objective values at these two bounds

Let \( L = \frac{(T^*)^2}{T_B (n+1)} \) and \( U = \frac{(T^*)^2}{T_B (n-1)} \)

\[ Z_L = \frac{a}{(T^*)^2 / T_B (n+1)} + g \frac{(T^*)^2}{T_B (n+1)} \]

\[ = gT_B (n+1) + \frac{a}{T_B (n+1)} \]  (Using Eq. 3.6)  (3.14)

\[ \frac{Z_L}{Z^*} = \frac{1}{2} \left\{ \frac{T_B}{T^*} (n+1) + \frac{T^*}{T_B} \frac{1}{T_B (n+1)} \right\} \]  (Multiplying and dividing by \( 2 \sqrt{ag} \))

Similarly,

\[ \frac{Z_U}{Z^*} = \frac{1}{2} \left\{ \frac{T_B}{T^*} (n-1) + \frac{T^*}{T_B} \frac{1}{T_B (n-1)} \right\} \]  (3.15)

The maximum of Eq. 3.14 and Eq. 3.15 is the worst case deviation bound from the EOQ solution when following the modified base period policy and is given as:

\[ \frac{Z}{Z^*} \leq \text{Max} \left[ \frac{1}{2} \left\{ \frac{T_B}{T^*} (n+1) + \frac{T^*}{T_B} \frac{1}{T_B (n+1)} \right\}, \frac{1}{2} \left\{ \frac{T_B}{T^*} (n-1) + \frac{T^*}{T_B} \frac{1}{T_B (n-1)} \right\} \right] \]  (3.16)

This bound could have been derived such that it is independent of \( T_B \) as given by Rangarajan and Ravindran (2005). Here, the bound has been deliberately derived with \( T_B \) so as to show the impact of \( T_B / T^* \) on the deviation bounds. This bound is valid for \( n \geq 2 \).
since for n=1, Eq. 3.16 becomes $\infty$. Hence, the deviation bound for the n=1 case is treated separately.

**Case 1: n ≥ 2**

This case corresponds to $T_B/T^* < 1/\sqrt{2}$. Using Eq. 3.10, the optimal integer multiples, n, can be found for different $T_B/T^*$ (or $T^*/T_B$) ratios. The worst case deviation bound is computed using Eq. 3.16 for $T_B/T^*$ values in the range $\left[0.05, \frac{1}{\sqrt{2}}\right]$ in increments of 0.05. This is given in Table 3.1.

<table>
<thead>
<tr>
<th>$T_B/T^*$</th>
<th>n</th>
<th>Max{$Z_L/Z^<em>, Z_U/Z^</em>$}</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>20</td>
<td>1.0013</td>
</tr>
<tr>
<td>0.1</td>
<td>10</td>
<td>1.0056</td>
</tr>
<tr>
<td>0.15</td>
<td>7</td>
<td>1.0167</td>
</tr>
<tr>
<td>0.2</td>
<td>5</td>
<td>1.025</td>
</tr>
<tr>
<td>0.25</td>
<td>4</td>
<td>1.0417</td>
</tr>
<tr>
<td>0.3</td>
<td>3</td>
<td>1.1333</td>
</tr>
<tr>
<td>0.35</td>
<td>3</td>
<td>1.0643</td>
</tr>
<tr>
<td>0.4</td>
<td>3</td>
<td>1.1125</td>
</tr>
<tr>
<td>0.45</td>
<td>2</td>
<td>1.3361</td>
</tr>
<tr>
<td>0.5</td>
<td>2</td>
<td>1.25</td>
</tr>
<tr>
<td>0.55</td>
<td>2</td>
<td>1.1841</td>
</tr>
<tr>
<td>0.6</td>
<td>2</td>
<td>1.1778</td>
</tr>
<tr>
<td>0.65</td>
<td>2</td>
<td>1.2314</td>
</tr>
<tr>
<td>0.7</td>
<td>2</td>
<td>1.2881</td>
</tr>
</tbody>
</table>

The percent deviation in some cases is more than 25%; however, this is only the upper bound on the deviation. The *actual* percent deviation can be as low as 0 as seen in the example problem. If the actual percent deviation is as high as the worst case percent deviation, a scheme is suggested later that can significantly reduce the percent deviation and hence, the cost. To provide insight into how the worst case deviation varies with $T_B/T^*$, a graph is plotted in Fig. 3.1.
Unfortunately, we cannot come to a definitive conclusion on the behavior of the worst case deviation except that it has an increasing trend. There are several spikes in the graphs, where the deviation increases and then decreases.

Fig. 3.2 shows how the worst case deviation varies in a breakpoint interval, or equivalently, within a particular cluster. From Table 3.1, the worst case deviation bound is still low for $n=4$. The graph was plotted for the values 6, 5, 4, 3 and 2 (worst case percent deviation > 2%). Additional points can be generated for each value of $n$ by dividing the range of the corresponding breakpoint interval by the number of points needed. We have generated 10 points for each value of $n$. 

**Fig. 3.1: Worst Case Deviation Variation**
Within each breakpoint interval or cluster, the worst case deviation decreases, reaches a minimum value and then starts to increase, like a convex function. Let the ratio corresponding to the minimum worst case deviation within each cluster be \( \left( \frac{T_B}{T^*} \right)_{\text{min}}^C \), where \( C \) refers to the cluster number. For example, consider the set of retailers in cluster 3, i.e., \( C=3 \). The corresponding breakpoint interval is \( \left[ \frac{1}{\sqrt{12}}, \frac{1}{\sqrt{6}} \right) \). The worst case percent deviation starts at \( 15.47\% \left( \frac{T_B}{T^*} = 0.2887 \right) \), decreases to a minimum of \( 6.59\% \left( \left( \frac{T_B}{T^*} \right)_{\text{min}}^3 = 0.3485 \right) \), and increases to \( 10.8\% \left( \frac{T_B}{T^*} = 0.3963 \right) \).

It would be advantageous for a retailer to move to the right or to the left depending on which side of the curve its \( \frac{T_B}{T^*} \) ratio falls. If the \( \frac{T_B}{T^*} \) ratio falls on the left side of the curve, then the retailer would be better off by decreasing its \( T^* \) such that its \( T_B/T^* = \left( \frac{T_B}{T^*} \right)_{\text{min}}^C \). Similarly, if the \( T_B/T^* \) ratio falls on the right side of the curve,
then the retailer would be better off by increasing its $T^*$. For example, by moving from $T_B / T^* = 0.2887$ to $0.3485$ a cost savings of approximately $10\%$ can be achieved, which is a significant reduction.

**Case 2: $n = 1$**

The integer multiple $n$ becomes $1$ for $T_B / T^*$ ratios that fall in the range $\left[ \frac{1}{\sqrt{2}}, 1 \right]$. This implies that the ordering period for the retailers whose $T_B / T^*$ ratio falls in the abovementioned range will be set as the base period. The deviation for this case is derived by computing the ratio of the cost when the ordering period is $T_B$ to the cost obtained by the EOQ solution:

$$
\frac{Z}{Z^*} = \frac{a/T_B + g T_B}{2\sqrt{ag}}
$$

$$
= \frac{1}{2 T_B} \sqrt{\frac{a}{g}} + \frac{T_B}{2} \sqrt{\frac{g}{a}}
$$

$$
= \frac{1}{2} \left\{ \frac{T^*}{T_B} + \frac{T_B}{T^*} \right\}
$$

(3.17)

Using Eq. 3.17, the deviation is computed for ratios within the range $\left[ \frac{1}{\sqrt{2}}, 1 \right]$ in increments of $0.05$ and is plotted in Fig. 3.3.
The maximum percent deviation in this case is about 6%. From Fig. 3.3, it is apparent that as the ratio $T_B/T^*$ increases to 1, the percent deviation decreases to 0. This intuitively makes sense because as $T_B/T^*$ increases we are approaching the EOQ solution of the base retailer, and hence there is no deviation from the optimal ordering policy.

3.5 Warehouse’s Ordering Policy

Being a decentralized supply chain, the warehouse will have to use the information passed on by each retailer to formulate its optimization problem and determine its ordering policy.

3.5.1 Average Inventory at the Warehouse

The warehouse receives the optimal integer multiples $n_i$ (and hence the ordering period) of the other retailers with respect to the base period. There exists a time period when all the retailers place their order at the same time. This is given by the least
common multiple of the reorder intervals, \( \text{L.C.M}(T_1, T_2, \ldots, T_N) = \text{L.C.M}(n_1, n_2, \ldots, n_N)T_B \). This also corresponds to the \textit{smallest} reorder period of the warehouse. Then, the number of orders that each retailer places during the smallest reorder period of the warehouse, denoted by \( m_i \), can be computed as:

\[
m_i = \frac{\text{L.C.M}(T_1, T_2, \ldots, T_N)}{T_i} = \frac{\text{L.C.M}(n_1, n_2, \ldots, n_N)}{n_i} \quad \forall i
\]  

(3.18)

These concepts are illustrated with the help of an example. Consider a warehouse supplying the product to three different retailers. Each retailer’s inventory follows a \textit{saw tooth} pattern and has the \textit{Zero Inventory Ordering (ZIO)} property. Retailer 1 places an order every base period, Retailer 2 every two base periods, and Retailer 3 every three base periods. Assume the base period \( T_B \) is 1 time unit. The ratios \( n_i \) are \( n_1 = 1 \), \( n_2 = 2 \), and \( n_3 = 3 \). The time when all the retailers place their order is \( \text{L.C.M}(n_1, n_2, n_3)T_B = \text{L.C.M}(1, 2, 3) = 6 \) time units. The number of orders placed by Retailer 1, Retailer 2 and Retailer 3 are \( m_1 = 6 \), \( m_2 = 3 \) and \( m_3 = 2 \), respectively, during 6 time units. The inventory pattern for the 3 retailers is shown in Fig. 3.4.

![Fig. 3.4: Inventory Pattern of the Retailers](image-url)
Let $\tau = \text{L.C.M}(n_1, n_2, ..., n_N)$ \hspace{1cm} (3.19)

Hence, the general relationship:

$$m_1 T_1 = m_2 T_2 = ... = m_N T_N = \tau T_B$$ \hspace{1cm} (3.20)

The warehouse places an order once every $\tau T_B$ periods or in integer multiples of $\tau T_B$ periods. Hence, the time between warehouse orders, denoted by $T_W$, is given by:

$$T_W = \delta(\tau T_B) \text{ where } \delta \geq 1 \text{ and integer}$$ \hspace{1cm} (3.21)

Hence, the number of orders placed by each retailer between warehouse orders is $\delta m_i \forall i$.

The annual demand faced by the warehouse is given by:

$$D_W = \sum_{i=1}^{N} D_i$$ \hspace{1cm} (3.22)

Substituting Eq. 3.21 and Eq. 3.22 in $Q_W = T_W D_W$:

$$Q_W = \delta(\tau T_B) \sum_{i=1}^{N} D_i$$

$$= \delta \sum_{i=1}^{N} m_i \frac{Q_i}{D_i} D_i$$

$$= \delta \sum_{i=1}^{N} m_i Q_i$$ \hspace{1cm} (3.23)

where $Q_i$'s are the retailer order quantities.

The next step is to derive an expression for the average inventory in the warehouse. Based on the one warehouse three retailer example given earlier (see Fig. 3.4.) an expression for the average inventory at the warehouse is derived. This expression is then generalized for the N retailer case. For the example, $\delta=1$ and $T_W=6$ time units.
Fig. 3.5: Inventory Pattern of the Warehouse
Fig. 3.5 illustrates the inventory pattern in the warehouse. Clearly, the inventory at the warehouse does not follow a triangular saw tooth pattern. Let us assume that the retailers and the warehouse have zero inventories at the beginning of the cycle. The warehouse receives $Q_W$ units at the beginning of the cycle. All three retailers place their orders for $Q_1$, $Q_2$ and $Q_3$. These orders are shipped out instantly. The inventory at the warehouse drops to $Q_W - Q_1 - Q_2 - Q_3$. After one time period ($T_B$), only Retailer 1 places an order. The inventory at the warehouse drops to $Q_W - 2Q_1 - Q_2 - Q_3$. After two time periods, Retailer 1 and Retailer 2 place orders. The inventory at the warehouse further reduces to $Q_W - 3Q_1 - 2Q_2 - Q_3$. This process continues till the inventory drops down to zero at the end of period 6. At this moment the warehouse places another order and the inventory level rises to $Q_W$ and this process repeats itself. Note that the warehouse still follows a fixed order policy.

Based on Fig. 3.5 the average inventory at the warehouse denoted by $\overline{Q_W}$ is:

$$\overline{Q_W} = \frac{(Q_W - Q_1 - Q_2 - Q_3)T_B + (Q_W - 2Q_1 - Q_2 - Q_3)T_B + (Q_W - 3Q_1 - 2Q_2 - Q_3)T_B + (Q_W - 4Q_1 - 2Q_2 - 2Q_3)T_B + (Q_W - 5Q_1 - 3Q_2 - 2Q_3)T_B + (Q_W - 6Q_1 - 3Q_2 - 3Q_3)T_B}{6T_B}$$

$$= Q_W - \left[ \frac{(Q_1 + 2Q_1 + ... + 6Q_1) + (2Q_2 + 4Q_2 + 6Q_2) + (3Q_3 + 6Q_3)}{6} \right]$$

$$= Q_W - \left[ \frac{1(Q_1 + 2Q_1 + ... + 6Q_1) + 2(Q_2 + 2Q_2 + 3Q_2) + 3(Q_3 + 2Q_3)}{6} \right]$$

$$= Q_W - \left[ \frac{7Q_1}{2} + 2Q_2 + \frac{3Q_3}{2} \right]$$

The above expression is the average inventory at the warehouse for a specific example. Observing the expression below, a pattern emerges:

$$\overline{Q_W} = Q_W - \left[ \frac{1(Q_1 + 2Q_1 + ... + 6Q_1) + 2(Q_2 + 2Q_2 + 3Q_2) + 3(Q_3 + 2Q_3)}{6} \right]$$ (3.24)

In the numerator, the numbers 1, 2 and 3 outside the three summations correspond to $n_1$, $n_2$ and $n_3$. The coefficient of the last term in the first summation corresponds to $m_1$. Similarly, the coefficient of the last term in the second and the third summations corresponds to $m_2$ and $m_3$. The denominator represents the smallest ordering cycle of the warehouse.
Let $\delta = 2$. Then, the time between orders at the warehouse is 12. The warehouse does not have to place an order at period 6 as shown in Fig. 3.5 since inventory has not reached zero. Inventory continues to deplete, and it reaches zero at period 12. At this instant the warehouse places an order, and the inventory level rises to $Q_W$. The number of orders placed by Retailer 1, Retailer 2 and Retailer 3 are $12(2*6)$, $6(2*3)$ and $4(2*2)$, respectively. Then Eq. 3.24 can be easily modified as:

$$\bar{Q}_W = Q_W - \left[\frac{1(Q_1 + 2Q_1 + \ldots + 12Q_1) + 2(Q_2 + 2Q_2 + \ldots + 6Q_2) + 3(Q_3 + 2Q_3 + \ldots + 4Q_3)}{(2)(6)}\right]$$ (3.25)

Hence, Eq. 3.25 for the three retailer case can be generalized as:

$$\bar{Q}_W = Q_W - \left[\frac{n_1(Q_1 + 2Q_1 + \ldots + \delta m_1 Q_1) + n_2(Q_2 + 2Q_2 + \ldots + \delta m_2 Q_2) + n_3(Q_3 + 2Q_3 + \ldots + \delta m_3 Q_3)}{\delta \tau}\right]$$

$$= Q_W - \left[\frac{n_1 Q_1(1 + 2 + \ldots + \delta m_1) + n_2 Q_2(1 + 2 + \ldots + \delta m_2) + n_3 Q_3(1 + 2 + \ldots + \delta m_3)}{\delta \tau}\right]$$

$$= Q_W - \left[\frac{n_1 Q_1 \delta m_1 (\delta m_1 + 1) + n_2 Q_2 \delta m_2 (\delta m_2 + 1) + n_3 Q_3 \delta m_3 (\delta m_3 + 1)}{2\delta \tau}\right]$$

$$= Q_W - \frac{1}{2} \left[\sum_{i=1}^{3} n_i Q_i (\delta m_i (\delta m_i + 1)) \right] \quad \text{[Using Eq. 3.19]}$$

$$= Q_W - \frac{1}{2} \left[\sum_{i=1}^{3} Q_i m_i (\delta m_i (\delta m_i + 1)) \right] \quad \text{[Using Eq. 3.18]}$$

$$= Q_W - \frac{1}{2} \left[\sum_{i=1}^{3} Q_i (\delta m_i + 1) \right]$$

This expression is the average inventory at the warehouse for the three retailer case. The average inventory expression at the warehouse can be generalized for the N retailer case as:

$$Q_w = \frac{1}{2} \left[\sum_{i=1}^{N} Q_i (\delta m_i + 1) \right]$$ (3.26)

Consider the case of a serial supply chain, i.e., $N=1$, and $\delta = 1$. Then, Eq. 3.26 simplifies to $Q_w = \frac{1}{2} Q_1 (m_1 + 1)$. This is the same result obtained by Thirumalai (2001) for a serial supply chain when the warehouse orders in integer multiple of the retailer.
3.5.2 Warehouse’s Optimization Problem

The optimization problem for the warehouse can now be formulated as:

Problem (W)

\[
\text{Min } Z_w = h_w \left( Q_w - \frac{1}{2} \left[ \sum_{i=1}^{N} Q_i \left( \delta m_i + 1 \right) \right] + a_w \frac{D_w}{Q_w} \right)
\]

s.t. : \( Q_w = \delta \sum_{i=1}^{N} m_i Q_i \)

\( Q_w \geq 0 \)

\( \delta \geq 1 \) and integer

3.5.3 Solution to Problem (W)

To solve Problem (W) optimally, it is enough if the integer multiple \( \delta \) is determined. Following the solution procedure of the modified retailer’s problem (Problem (MRi)), we can determine \( \delta \) as follows:

**Lemma 3.2-** The objective function of Problem (W) is point-wise convex in \( \delta \).

**Proof:** Writing the objective function in terms of \( \delta \):

\[
Z_w(\delta) = \frac{h_w}{2} \left( \delta \sum_{i=1}^{N} m_i Q_i - \sum_{i=1}^{N} Q_i \right) + a_w \frac{D_w}{\delta \sum_{i=1}^{N} m_i Q_i}
\]

\[
Z_w(\delta + 1) = \frac{h_w}{2} \left( (\delta + 1) \sum_{i=1}^{N} m_i Q_i - \sum_{i=1}^{N} Q_i \right) + a_w \frac{D_w}{(\delta + 1) \sum_{i=1}^{N} m_i Q_i}
\]

\[
\Delta Z_w(\delta) = Z_w(\delta + 1) - Z_w(\delta) = \frac{h_w}{2} \left( \sum_{i=1}^{N} m_i Q_i \right) - \frac{a_w D_w}{\left( \sum_{i=1}^{N} m_i Q_i \right) \delta (\delta + 1)}
\]

Similarly,

\[
\Delta Z_w(\delta + 1) = \frac{h_w}{2} \left( \sum_{i=1}^{N} m_i Q_i \right) - \frac{a_w D_w}{\left( \sum_{i=1}^{N} m_i Q_i \right) (\delta + 1)(\delta + 2)}
\]
Computing the second difference:

$$\Delta^2 Z_w(\delta + 1) = \Delta Z_w(\delta + 1) - \Delta Z_w(\delta)$$

$$= \frac{a_w D_w}{\sum_{i=1}^{N} m_i Q_i} \left\{ \frac{1}{(\delta)(\delta + 1)} - \frac{1}{(\delta + 1)(\delta + 2)} \right\} = \frac{2a_w D_w}{\sum_{i=1}^{N} m_i Q_i (\delta)(\delta + 1)(\delta + 2)}$$

$$> 0$$

$$\Rightarrow$$ Point-wise convex

\[\blacksquare\]

**Theorem 3.2** - The optimal integer value \(\delta\) for the warehouse is the smallest integer that satisfies the following condition:

$$\delta(\delta + 1) \geq \frac{2a_w \sum_{i=1}^{N} D_i}{h_w \left( \sum_{i=1}^{N} m_i Q_i \right)^2}$$  \hspace{1cm} (3.27)

**Proof:** The integer \(\delta\) can be found by finding the smallest integer such that the first difference is non-negative, i.e.,

$$\Delta Z_w(\delta) = Z_w(\delta + 1) - Z_w(\delta) \geq 0$$

$$\Rightarrow \frac{h_w}{2} \left( \sum_{i=1}^{N} m_i Q_i \right) - \frac{a_w D_w}{\left( \sum_{i=1}^{N} m_i Q_i \right) \delta(\delta + 1)} \geq 0$$

$$\Rightarrow \frac{h_w}{2} \left( \sum_{i=1}^{N} m_i Q_i \right) \geq \frac{a_w D_w}{\left( \sum_{i=1}^{N} m_i Q_i \right) \delta(\delta + 1)}$$

$$\Rightarrow \delta(\delta + 1) \geq \frac{2a_w \sum_{i=1}^{N} D_i}{h_w \left( \sum_{i=1}^{N} m_i Q_i \right)^2}$$

\[\blacksquare\]
3.6 Algorithm for the Modified Base Period Policy

**Step 1:** Each retailer solves its own Problem (R_i) (EOQ model). The optimal reorder interval for each retailer is found using \( T_i^* = \sqrt{\frac{2a_i}{h_iD_i}} \) (see Eq. 3.3). Each retailer passes this information to the warehouse.

**Step 2:** The warehouse identifies the base period (and hence the set of base retailers) using \( T_B = \min T_i^* \) (see Eq. 3.9). The set of non-base retailers have to solve Problem (MR_i), which includes the constraint that their ordering period is an integer multiple of the base period.

**Step 3:** The optimal integer values for the non-base retailers can be determined by finding the smallest integer that satisfies \( n_i(n_i + 1) \geq \left( \frac{T_i^*}{T_B} \right)^2 \) (see Eq. 3.10). Then, the reorder intervals can be determined using \( T_i = n_iT_B \) (see Eq. 3.8).

**Step 4:** Each retailer calculates the actual percent deviation from their EOQ solution as \( \varepsilon_i = \frac{Z_i - Z_i^*}{Z_i^*} \).

**Step 5:** Is \( \varepsilon_i \leq d_{\text{min}} \)? where, \( d_{\text{min}} \) is the minimum deviation that is acceptable to the retailer. If yes, Go to Step 6, else go to Step 8.
Note: The \( d_{\text{min}} \) value is set by the retailer.

**Step 6:** Pass on the corresponding order information to the warehouse.

**Step 7:** Using \( \delta(\delta + 1) \geq \frac{2a_w \sum_{i=1}^{N} D_i}{h_w \left( \sum_{i=1}^{N} m_i Q_i \right)^2} \) (see Eq. 3.27), the warehouse determines its optimal integer \( \delta \) and hence, its reorder interval.
Step 8: If $\varepsilon_1 > d_{\min}$, then the retailer informs the warehouse about the large deviation from its EOQ solution. The warehouse can use Fig. 3.2 to indicate in which direction the retailer should move so as to decrease the deviation.

Step 9: Repeat Step 8 until the condition $\varepsilon_1 \leq d_{\min}$ is satisfied. Go to Step 6.

This procedure is summarized as a flowchart in Fig. 3.4, where the dotted lines indicate the flow of information.

3.7 Final Comments on the Modified Base Period Policy

Though it appears initially that the warehouse is the more dominant player in the supply chain since the non-base retailers are made to order in integer multiples of a base period also set by the warehouse, it is not the case. In fact the warehouse steers each retailer according to the scheme suggested in Section 3.3.2., which results in significant cost reductions for the retailer. The retailer is also allowed the option of setting a minimum acceptable deviation from its optimal solution which the warehouse has to...
necessarily meet. If the warehouse is unable to meet this requirement using the scheme suggested, it can offer purchasing discounts to the concerned retailer(s).

This procedure is actually similar to the *Vendor Managed Inventory* (VMI). As the name suggests, VMI deals with the vendor (warehouse) managing the ordering decisions at the customers (retailers). This was first pioneered by the world’s biggest retailer, Walmart. Thus, contrary to beliefs giant retailers like Walmart are *not* inflexible in accepting changes to their ordering policies. Walton (2005) writes that Walmart is open to changes in its ordering schedule as long as there are benefits for both parties involved.

From the warehouse’s perspective, it has better visibility of all the retailers ordering process. We already noted that the orders of the non-base retailers piggyback over the base retailers. Also, all the retailers within a particular cluster place an order at the same time. As a result, the warehouse receives an aggregated order of at least one cluster of retailers every base period. Aggregating orders leads to achievement of economies of scale at the warehouse. This would result in reducing significantly the order processing and transportation costs.

Based on the preceding paragraphs, it is clear that this control policy is more of a *collaborative* rather than a *compromising* approach due to the interactions between the retailers and the warehouse. Moreover, there is no dominant player in the supply chain and hence, should result in a “*win-win*” situation for all the companies in the system.

### 3.8 Example Problem

An example problem consisting of a single warehouse and three retailers is considered to explain the methodology developed in the previous sections. The data is provided in Table 3.2.

<table>
<thead>
<tr>
<th></th>
<th>Demand (D_i)</th>
<th>Order cost (a_i)</th>
<th>Holding cost (h_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retailer 1</td>
<td>800 units</td>
<td>$42</td>
<td>$48</td>
</tr>
<tr>
<td>Retailer 2</td>
<td>525 units</td>
<td>$100</td>
<td>$21</td>
</tr>
<tr>
<td>Retailer 3</td>
<td>415 units</td>
<td>$52</td>
<td>$28</td>
</tr>
<tr>
<td>Warehouse</td>
<td></td>
<td>$37</td>
<td>$8</td>
</tr>
</tbody>
</table>
Step 1: Each retailer solves Problem(R), i.e., the EOQ model to determine the optimal order quantity, reorder interval and cost using Eq. 3.2, Eq. 3.3 and Eq. 3.4, respectively. For example, for Retailer 1:

\[ Q_1^* = \sqrt{\frac{2a_1 D_1}{h_1}} = \sqrt{\frac{2 \times 42 \times 800}{48}} = 37.4166 \text{ units} \]

\[ T_1^* = \sqrt{\frac{2a_1}{h_1 D_1}} = \sqrt{\frac{2 \times 42}{48 \times 800}} \times 52 = 2.4321 \text{ weeks} \]

\[ Z_1^* = \sqrt{2a_D h} = \sqrt{2 \times 42 \times 800 \times 48} = 1796 \text{ $} \]

This information for all the retailers is summarized in Table 3.3.

<table>
<thead>
<tr>
<th>Retailer</th>
<th>Q* (units)</th>
<th>T* (weeks)</th>
<th>Z* ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retailer 1</td>
<td>37.4166</td>
<td>2.4321</td>
<td>1796</td>
</tr>
<tr>
<td>Retailer 2</td>
<td>70.7107</td>
<td>7.0037</td>
<td>1484.92</td>
</tr>
<tr>
<td>Retailer 3</td>
<td>39.261</td>
<td>4.9195</td>
<td>1099.31</td>
</tr>
</tbody>
</table>

Each retailer passes on Q* and T* information to the warehouse.

Step 2: The warehouse identifies the base period as 2.4321 weeks because Retailer 1 has the lowest reorder interval. Retailers 2 and 3 have to solve Problem (MR), which includes the constraint that their ordering period is an integral multiple of the base period.

Step 3: Using Eq. 3.10, Retailers 2 and 3 determine their optimal integer multiples. For example, for Retailer 1: \( \left( \frac{T_1^*}{T_B} \right)^2 = 8.2926 \). The smallest integer such that the condition in Eq. 3.10 is satisfied is 3. Then the reorder interval, order quantity, and the cost can be calculated as:

\[ T_2 = n^* T_B = 3 \times 2.4321 = 7.2963 \text{ weeks} \]

\[ Q_2 = \frac{T_2 D_2}{52} = \frac{7.2963 \times 525}{52} = 73.6646 \text{ units} \]
Similarly, they can be calculated for Retailer 3. This is summarized in Table 3.4.

Table 3.4: Output of Problem (MR)

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>T (weeks)</th>
<th>Q (units)</th>
<th>Z ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retailer 2</td>
<td>3</td>
<td>7.2963</td>
<td>73.6646</td>
<td>1486.17</td>
</tr>
<tr>
<td>Retailer 3</td>
<td>2</td>
<td>4.8642</td>
<td>38.8201</td>
<td>1099.37</td>
</tr>
</tbody>
</table>

Step 4: The actual percent deviations are \( \left( Z_2 - Z^* \right) / Z^* = 0.0842\% \) and \( \left( Z_3 - Z^* \right) / Z^* = 0.0055\% \). Retailer 2 falls in cluster 3 and Retailer 3 in cluster 2. The \( T_B / T^* \) ratios for Retailers 2 and 3 are 0.3473 and 0.4944, respectively. Using Eq. 3.16 the upper bound on percent deviation for Retailer 2 is 6.71% and for Retailer 3 is 25.85%. Thus, the actual percent deviation for both the retailers is not only within the error bound but is also very close to 0. Since the actual percent deviation is very low for both retailers, there is no need to increase or decrease the \( T^* \) values.

Step 5: In this step the warehouse uses the ordering information of the retailers to solve Problem (W). The annual demand faced by the warehouse is given by Eq. 3.22.

\[
D_W = \sum_{i=1}^{N} D_i = 800 + 525 + 415 = 1740 \text{ units}
\]

Using Eq. 3.18, the number of orders that each retailer places during the smallest ordering period of the warehouse is:

\[
m_1 = \frac{\text{L.C.M}(n_1, n_2, ..., n_N)}{n_1} = \frac{\text{L.C.M}(1, 2, 3)}{1} = 6 \text{ orders}
\]

Similarly the number of orders placed by the other retailers are \( m_2 = 2 \) and \( m_3 = 3 \). To determine the integer multiple \( \delta \), the RHS of Eq. 3.27 is computed as:

\[
\frac{2a_w}{h_w} \left( \sum_{i=1}^{N} m_i Q_i \right)^2 = \frac{2 \times 37 \times 1740}{8 \times (6 \times 37.4166 + 2 \times 73.6646 + 3 \times 38.8201)^2} = 0.0675
\]
The smallest integer $\delta$ that satisfies the condition of Eq. 3.27 is 1. The order quantity at the warehouse is determined from Eq. 3.23.

$$Q_w = \delta \sum_{i=1}^{N} m_i Q_i = 488.2891 \text{ units}$$

The reorder interval at the warehouse is $T_w = \frac{Q_w}{D_w} \times 52 = \frac{488.2891}{1740} \times 52 = 14.5925 \text{ weeks}$. 

Plugging in these values in the objective function of Problem (W) given by:

$$Z_w = h_w \left( Q_w - \frac{1}{2} \left[ \sum_{i=1}^{N} Q_i (\delta m_i + 1) \right] \right) + a_w \frac{D_w}{Q_w}$$

$$= 8 \left( 488.2981 - \frac{1}{2} \left( 7 \times 37.4166 + 3 \times 73.6646 + 4 \times 38.8201 \right) \right) + 37\frac{1740}{488.2981} = $1485.47$$

**Step 6:** The total supply chain cost is $1796 + 1486.17 + 1099.37 + 1485.40 = $5867.01.

### 3.9 Summary

In this chapter, we concentrated our efforts on determining the ordering policy for each company in a single warehouse multi-retailer supply chain system operating under decentralized control in a deterministic setting. We introduced the modified base period control policy wherein each retailer orders in integer multiples of a base period. Unlike previous control policies, where the base period is set to an arbitrary value, the base period in this policy is set as the least ordering period of the retailers. Setting the base period in this manner also aided in the development of a method to group retailers in clusters. The effectiveness of the policy was tested by developing theoretical bounds on the deviation from the EOQ solution for the retailers. In cases where the deviation was high, a scheme was suggested where the retailer can obtain significant reductions in cost. Finally, an example problem was provided to explain the methodology developed.

In the next chapter, the supply chain inventory problem is formulated as a multi-criteria problem where transportation costs are explicitly considered between the stages. The weighted objective method with a graphical tool is used to solve, and present the efficient solutions to the DM.
Chapter 4

DETERMINISTIC MULTI-CRITERIA MODEL FOR A SUPPLY CHAIN WITH TRANSPORTATION COSTS

4.1 Introduction

In the previous chapter we introduced the modified base period policy which enabled us to determine the optimal ordering policies for each company in the supply chain. The model assumed the availability of marginal cost information such as ordering and holding costs. In reality, this information may not be available (as seen in Chapter 6) or difficult to estimate, e.g., costs due to loss of goodwill when excess demand is backordered or lost. Secondly, there are several criteria other than cost, which are often conflicting in nature. This cannot be directly taken care of in the single objective methodology of Chapter 3. Thirdly, for a DM to make better decisions, he/she must choose from several solutions by evaluating tradeoff information. In a single objective framework, the DM is provided with a single solution. Finally, the solutions generated using a single objective methodology does not take into account the DM’s preference information. Since the DM is an integral part of the company, it is important for the solution to reflect his/her’s preference information.

These disadvantages can be overcome by formulating the problem as a multi-criteria problem. In this chapter we extend the single objective cost model to a multi-criteria model with conflicting criteria. Transportation costs are specifically included in the model as one of the criteria.

4.2 Problem Description

The supply chain system under consideration is the same as in the previous chapter wherein a warehouse supplies a single product to several retailers. A manufacturer with infinite capacity replenishes the warehouse. The system operates under a decentralized control. The inbound transportation cost at the retailer is modeled using less than full truckload (LTL) rates while the inbound transportation cost at the warehouse is modeled using truckload (TL) rates.
4.2.1 Assumptions

- Demand faced by the retailers is assumed to be deterministic and continuous. It is further assumed that extraneous factors such as risk, seasonality, promotions, etc. have been incorporated when the forecasts were determined.
- Lead times between echelons are fixed.
- Continuous review policy is followed at each location, i.e., each location places an order when its reorder point is reached.
- Freight on board (FOB) origin is assumed for each location, i.e., the retailer takes ownership of the goods at the warehouse, while the warehouse takes ownership of the goods at the manufacturer.

4.2.2 Notation

Data

\[ D_i \] – Annual demand faced by retailer \( i \) \( \forall i=1,2,\ldots, N \)

\[ D_w \] – Annual demand faced by warehouse

\[ L_i \] – Lead time (days) between warehouse and retailer \( i \) \( \forall i=1,2,\ldots, N \)

\[ L_w \] – Lead time (days) between manufacturer and warehouse

\[ C_i \] – Cost/unit at retailer \( i \) \( \forall i=1,2,\ldots, N \)

\[ C_w \] – Cost/unit at warehouse

\[ F_{yi} \] – Freight rate function for retailer \( i \) \( \forall i=1,2,\ldots, N \)

\[ F_{yw} \] – Freight rate function for warehouse

\[ \omega \] – Weight of a single unit in lbs

\[ N \] – Number of retailers

Decision Variables

\[ Q_i \] – Order quantity of retailer \( i \) \( \forall i=1,2,\ldots, N \)

\[ T_i \] – Time between orders at retailer \( i \) \( \forall i=1,2,\ldots, N \)

\[ Q_w \] – Order quantity of warehouse

\[ T_w \] – Time between orders at warehouse
\( r_i \) – Reorder point for retailer \( i \forall i=1,2,\ldots, N \)

\( r_w \) – Reorder point for warehouse

4.3 Retailer’s Model Formulation

In a continuous review policy, the inventory at a particular location is monitored continuously, and an order is placed when the on-hand inventory reaches a reorder point (r). The reorder point is determined as follows:

\[
 r = D(L - jT) \tag{4.1}
\]

where:

- D - Annual demand
- L – Lead time.
- T – Time between orders.
- \( j \) – Largest integer less than or equal to \( L/T \)

When \( L < T \) (particularly when shipping using trucks), the reorder point is LD, which is the demand during the lead time. In this case the order is placed when the on-hand inventory reaches LD. This is shown in Fig. 4.1.

![Retailer’s Inventory Pattern](image)

**Fig. 4.1: Retailer’s Inventory Pattern**

Retailer’s Criteria

The following criteria are used in the formulation of the multi-criteria problem for all retailers:
• Capital invested in inventory (including in-transit inventory due to the FOB origin).
• Number of orders placed annually.
• Annual transportation cost.

**Criterion 1: Capital Invested in Inventory**

The inventory at the retailer follows the saw tooth pattern shown in Fig. 4.1. The capital invested in inventory consists of two components:

- *Capital invested in inventory at retailer*: It is the product of average inventory at each retailer and the cost/unit at each retailer. The average inventory from Fig. 4.1 is \( Q/2 \). Hence, the capital invested in inventory at each retailer is simply:

\[
\frac{Q}{2} C_i \forall i
\]  

(4.2)

- *Annual capital invested in In-transit inventory*: Since FOB origin is assumed, each retailer is assumed to manage their inbound logistics, i.e., the retailer takes possession of the product at the warehouse. Hence, each retailer has capital tied up in the form of in-transit inventory. Annual capital invested in in-transit inventory is the fraction of year the annual demand of each retailer is in transit multiplied by the cost/unit at each retailer:

\[
\frac{L_i}{365} D_i C_i \forall i
\]  

(4.3)

Thus, the capital invested in inventory is the sum of Eq. 4.2 and Eq. 4.3

\[
\frac{Q}{2} C_i + \frac{L_i}{365} D_i C_i \forall i
\]  

(4.4)

**Criterion 2: Number of Orders Placed Annually**

The number of orders placed annually by a retailer is the ratio of annual demand to the order quantity:

\[
\frac{D_i}{Q_i} \forall i
\]  

(4.5)
**Criterion 3: Annual Transportation Costs**

Transportation cost is an important criterion which is often overlooked in supply chain inventory models when determining the order quantity. Shipments can be full truck load (TL) or less than truck load (LTL). TL rates are usually stated based on distance traveled or a fixed charge/truck while LTL rates are stated per hundredweight (CWT).

Trucking companies offer discounts on the freight rate to encourage shippers to buy in large quantities. Since it is known that the inventory and transportation components interact, it is essential to explicitly consider transportation costs in supply chain models for determining the ordering policy followed by a company. There are two problems in trying to incorporate actual freight rates:

1. Determining exact rates between every origin and destination is time consuming and expensive.
2. The freight rate function is not differentiable.

Consider an all-units discount offered by the carrier. The cost function is:

\[ G(Q) = \begin{cases} 
  F_1Q\omega/100 & \text{if } 0 \leq Q < B_1 \\
  F_2Q\omega/100 & \text{if } B_1 \leq Q < B_2 \\
  \hspace{1cm} \ddots & \\
  F_nQ\omega/100 & \text{if } B_{n-1} \leq Q < B_n 
\end{cases} \]

This function implies that if the order quantity weighs in the range \([0, B_1)\), then the freight rate is $F_1$/cwt (1cwt = 100lbs). Similarly, if the order quantity weighs in the range \([B_1, B_2)\), the freight rate is $F_2$/cwt, and so on. Also, $F_1 > F_2 > \ldots > F_n$. This means that, discounts are given by the carrier to the shipper. $B_1, B_2, \ldots, B_n$ are called the break points.

It is clear that $G(Q)$ is not a continuous function.

An interesting feature of this function is that there would be a point in each weight category called an *indifference point* beyond which the shipper pays the same rate as that of the next weight range by *over-declaring* the shipment weight. This cost function can then be converted to Fig. 4.2 by finding the indifference points in all the weight break levels. Indifference point is the point at which the weight, when multiplied by the actual freight rate, yields the same total cost that is charged at the next break point.
Fig. 4.2: Freight Rate Function

The indifference points are denoted by $I_1$, $I_2$, ..., $I_n$ in Fig. 4.2. Any shipping weight in the interval $[I_1, B_1)$ can be over-declared to $B_1$ and charged a \textit{fixed lower} rate. It must be noted that the quantity shipped is not increased to $B_1$, i.e., the shipper still ships the required order quantity greater than or equal to $I_1$, but it is declared as $B_1$. If the weight shipped is in the interval $(B_1, I_2)$, then a \textit{variable cost} ($/\text{cwt}$) is charged. Fig. 4.2 is a stepwise linear function. Though the function is continuous, it is not differentiable due to the indifference points and hence, difficult to work with analytically.

Because of these difficulties, researchers have explored the possibility of using continuous functions to fit the actual freight rate structure. Langley (1980) was the first to use continuous functions to model transportation costs.

Ballou (1991) suggested that freight rates should be estimated as determining exact rates for combinations of origin-destination points for different weight breaks is time consuming as well as expensive. He used a simple linear regression model to relate the freight rate to the distance.

Swenseth and Godfrey (1996) used 5 different continuous functions on the actual freight rate structure for 40 different routes. For each function the mean squared error criterion was used to select the best function among the 40 routes. Once the best functions were determined, they were compared with each other to find out which function best emulates the actual freight rates. The functions that were considered were \textit{constant}, \textit{proportional}, \textit{exponential}, \textit{adjusted inverse} and \textit{inverse} functions. They
concluded that the inverse function exactly determines the freight rates when shipping weights were high emulating TL rates, whereas the proportional and the adjusted inverse functions emulated LTL rates particularly well.

Tyworth and Zeng (1998) recommend the use of a power function to model transportation costs.

Since each retailer is going to send out trucks or use a 3PL (third party logistics provider) to pick up the orders, the order quantity is most likely to be less than truck load (LTL). We use the proportional function proposed by Swenseth and Godfrey (1996) as it is easy to work with analytically. For the example problem, different continuous functions are fit to the freight rate data to determine the function that best estimates the transportation cost. The proportional function is very similar to a linear function proposed by Langley (1980). Mathematically, the freight rate function by Swenseth and Godfrey (1996) for shipping LTL for a retailer is given by:

$$F_{yi} = F_{xi} + \alpha_i(W_{xi} - W_{yi}), \forall i$$  \hspace{1cm} (4.6)

where:

- $F_{yi}$ - Freight rate for shipping a given load to the retailer ($/cwt$)
- $F_{xi}$ - TL freight rate for each retailer ($/cwt$)
- $W_{xi}$ - TL weight
- $W_{yi}$ - Weight shipped to each retailer i (lbs.)
- $\alpha_i$ - Rate at which freight rate increases per 100lbs decrease in shipping weight for retailer i.

Simplifying Eq. 4.6,

$$F_{yi} = F_{xi} + \alpha_i W_{xi} - \alpha_i Q_i, \forall i$$

Since $F_{xi}$ and $\alpha_iW_{xi}$ are constants we can replace $F_{xi} + \alpha_iW_{xi}$ by another constant $A_i$:

$$F_{yi} = A_i - \alpha_i Q_i, \forall i$$  \hspace{1cm} (4.7)

Eq. 4.7 is the proportional function proposed by Langley (1980), and it implies that the freight rate decreases at a rate $\alpha_i$ for every unit increase in $Q_i$.

The value of $\alpha_i$ can be determined from either Eq. 4.6 or Eq. 4.7. From Eq. 4.6, $\alpha_i$ is determined by minimizing the mean squared error between actual and estimated LTL freight rates for each route. From Eq. 4.7, $\alpha_i$ is determined by fitting a simple linear
regression model between the freight rate and order quantity; however, both methods require the knowledge of the actual shipping rates for a particular route.

Thus, the transportation cost for shipping an order quantity $Q_i$ for retailer $i$ using Eq. 4.6 is:

$$\{F_{xi} + \alpha_i(W_{xi} - W_{yi})\} \frac{Q_i \omega}{100}$$

The annual transportation cost for each retailer can be obtained by multiplying the above expression with the number of order cycles/year.

$$\{F_{xi} + \alpha_i(W_{xi} - W_{yi})\} \frac{Q_i \omega D_i}{100 Q_i}$$

and $W_{yi} = Q_i \omega$

Annual transportation cost = $$\{F_{xi} + \alpha_i(W_{xi} - Q_i \omega)\} \frac{\omega D_i}{100} \forall i \quad (4.8)$$

4.3.1 Retailer’s Multi-Criteria Problem

Each retailer has to minimize all the three criteria given by Eq. 4.4, Eq. 4.5 and Eq. 4.8 simultaneously. The multi-criteria mathematical program (MCMP) for each retailer denoted by problem $(R_i)$ is as follows:

**Problem (R$_i$)**

Min $Z_i^1 = \frac{Q_i}{2} C_i + \frac{L_i}{365} D_i C_i$

Min $Z_i^2 = \frac{D_i}{Q_i}$

Min $Z_i^3 = \{F_{xi} + \alpha_i(W_{xi} - Q_i \omega)\} \frac{\omega D_i}{100}$

Subject to: $Q_i \geq 0 \forall i$

4.3.2 Solution Procedure to Solve Problem (R$_i$)

The $P_\lambda$ method (Section 2.2.4) is used to solve the retailer’s multi-criteria problem. Efficient points for each retailer can be obtained by solving Problem $(R_i^\lambda)$:
Problem ($R_i^\lambda$)

\[
\text{Min } Z_i^\lambda = \lambda_i^1 \left( \frac{Q_i}{2} C_i + \frac{L_i}{365} D_i C_i \right) + \lambda_i^2 \frac{D_i}{Q_i} + \lambda_i^3 \{F_{xi} + \alpha_i (W_{xi} - Q_i \omega) \} \frac{D_i \omega}{100}
\]

Subject to:
\[
\sum_{k=1}^{3} \lambda_i^k = 1 \\
\lambda_i^k > 0 \ \forall i, k \\
Q_i \geq 0 \ \forall i
\]

$\lambda_i^k$ - Convex combination parameter or weight assigned for criterion k for retailer i.

Every optimal solution to the $R_i^\lambda$ problem is an efficient solution to the retailer’s multi-criteria problem. Also, all the criteria are convex, and the constraint set is a convex set. Thus, the $R_i^\lambda$ problem is both necessary and sufficient. Hence, all the efficient points can be generating by varying $\lambda_i^k$ between 0 and 1.

Lemma 4.1- Objective function of Problem ($R_i^\lambda$) is convex.

Proof: Taking the first and second differential of Z with respect to Q:
\[
\frac{dZ_i^\lambda}{dQ_i} = \frac{\lambda_i^1 C_i}{2} - \frac{\lambda_i^2 D_i}{Q_i^2} + \lambda_i^3 \left( -\alpha_i D_i \omega^2 \right) \\
\frac{d^2Z_i^\lambda}{dQ_i^2} = \frac{2\lambda_i^2 D_i}{Q_i^3} > 0 \text{ for positive } D_i, Q_i \text{ and } \lambda_i^2
\]

Theorem 4.1- For each retailer, in addition to $\lambda_i^k \in (0, 1)$ and $\sum_{k=1}^{3} \lambda_i^k = 1$, the following condition will also be satisfied by the efficient solution:
\[
\frac{\lambda_i^1}{\lambda_i^3} > \frac{2\alpha_i D_i \omega}{C_i}
\]
Proof: Lemma 4.1 implies that the objective function of the $R_i^\lambda$ problem for the retailer is convex and the local minimum is the global minimum. The minimum can be found by equating the first differential to zero.

$$\frac{dZ^\lambda_i}{dQ_i} = 0 \Rightarrow \frac{\lambda_i^1 C_i}{2} - \frac{\lambda_i^2 D_i}{Q_i^2} + \lambda_i^3 \left( - \frac{\alpha_i D_i \omega^2}{100} \right) = 0$$

Let $\omega^2/100 = \bar{\omega}$

$$\frac{\lambda_i^1 C_i}{2} - \frac{\lambda_i^2 D_i}{Q_i^2} + \lambda_i^3 \left( - \alpha_i D_i \bar{\omega} \right) = 0$$

$$\Rightarrow Q_i = \sqrt{\frac{2\lambda_i^2 D_i}{\lambda_i^1 C_i - 2\lambda_i^3 \alpha_i D_i \bar{\omega}}} \quad (4.9)$$

The denominator of Eq. 4.9 has to be greater than zero.

$$\lambda_i^1 C_i - 2\lambda_i^3 \alpha_i D_i \bar{\omega} > 0$$

$$\Rightarrow \frac{\lambda_i^1}{\lambda_i^3} > \frac{2\alpha_i D_i \bar{\omega}}{C_i} \quad (4.10)$$

Eq. 4.9 clearly brings out the impact of explicitly considering the transportation costs. Excluding transportation cost from the model results in an order quantity $Q_i = \sqrt{\frac{2\lambda_i^2 D_i}{\lambda_i^1 C_i}}$. Thus, when transportation costs are included in the model, the denominator decreases, as a result the order quantity increases. Further, increasing the value of $\alpha_i$ causes the order quantity to increase. (Note: Increasing $\alpha_i$ increases the transportation costs.)

By changing the value of the weights for each criterion, a set of efficient solutions can be generated. Instead of developing a methodology that steers the DM towards the best compromise solution, graphical display tools can be used to aid the DM. One such tool is the value path method developed by Schilling et al. (1983). This method is extremely useful to visualize and quantify the tradeoff information among the conflicting criteria.
For each alternative there is a value path. The basic premise here is that if two or more value paths intersect each other, then all the alternatives are efficient. If the value paths for two alternatives do not intersect, then the one lying above the other is non-efficient. If the value path for two or more alternatives intersect then the value paths that are above this intersection point are non-efficient.

The DM, after visually and quantitatively analyzing the tradeoff information, can choose the best compromise solution and hence, the weight combination for each retailer. After fixing the weight combination, each retailer can determine the time between the orders as:

$$T_i = \frac{Q_i}{D_i} \quad \forall i$$  \hspace{1cm} (4.11)

### 4.3.3 Modified Retailer’s Problem

As in the previous chapter, the warehouse identifies the base retailer by determining the least reorder interval among all the retailers:

$$T_B = \text{Min}_i T_i$$  \hspace{1cm} (4.12)

The non-base retailers order in integer multiples of the base period:

$$T_i = n_i T_B$$  \hspace{1cm} (4.13)

Substituting Eq. 4.11 into Eq. 4.13 and rewriting in terms of order quantity:

$$Q_i = n_i Q_B \frac{D_i}{D_B} = n_i Q_B R_i$$  \hspace{1cm} (4.14)

$R_i$ is referred to as the Demand Ratio and is defined as the ratio of demand of a particular retailer to the demand of the retailer who orders most frequently. The optimization problem for the base retailer, Problem $R_B^\lambda$ remains unchanged. The $R_i$ problem for the non-base retailers is modified to incorporate Eq. 4.14 and is called Problem (MR$_i$).

**Problem (MR$_i$)**

$$\text{Min } Z_i^1 = \frac{Q_i}{2} C_i + \frac{L_i}{365} D_i C_i$$

$$\text{Min } Z_i^2 = \frac{D_i}{Q_i}$$
\[
\text{Min } Z^\lambda_i = \{F_{xi} + \alpha_i (W_{xi} - Q_i \omega)\} \frac{D_i \omega}{100}
\]

Subject to:
\[Q_i = n_i Q_B R_i\]
\[Q_i \geq 0\]
\[n_i \geq 1 \text{ and integer}\]

**4.3.4 Solution Procedure to Solve Problem (MR_\lambda)**

Problem (MR_\lambda) is converted to a single objective problem by using the P_\lambda method as follows:

**Problem (MR_\lambda^*)**

\[
\text{Min } Z^\lambda_i = \lambda^1_i \left( \frac{Q_i}{2} + \frac{D_i L_i}{365} \right) C_i + \lambda^2_i \frac{D_i}{Q_i} + \lambda^3_i \{F_{xi} + \alpha_i (W_{xi} - Q_i \omega)\} \frac{D_i \omega}{100}
\]

Subject to:
\[\sum_{k=1}^{3} \lambda^k_i = 1\]
\[\lambda^k_i > 0 \ \forall i, k\]
\[Q_i = n_i Q_B R_i\]
\[Q_i \geq 0\]
\[n_i \geq 1 \text{ and integer}\]

Though the individual criteria are convex functions, the necessary condition of the general P_\lambda problem is not satisfied since the constraint set of Problem (MR_\lambda^*) is no longer a convex set. Hence, all the efficient points cannot be generated. However, an optimal solution to Problem (MR_\lambda^*) will generate an efficient solution.

We proceed in the same way as in Lemma 3.1 and Theorem 3.1 in Chapter 3 to determine the integer multiple, n_i and hence, Q_i.

**Lemma 4.2** - The objective function of Problem (MR_\lambda^*) is point-wise convex in n_i.

**Proof:** For convenience, subscript ‘i’ is eliminated.
\[ Z^\lambda(n) = \lambda^1 \left( \frac{n Q B R}{2} + \frac{D L}{365} \right) C + \lambda^2 \frac{D}{n Q B R} + \lambda^3 \left\{ F_x + \alpha \left( W_x - n Q B \omega \right) \right\} \frac{D \omega}{100} \]

\[ Z^\lambda(n + 1) = \lambda^1 \left( \frac{(n + 1) Q B R}{2} + \frac{D L}{365} \right) C + \lambda^2 \frac{D}{(n + 1) Q B R} + \lambda^3 \left\{ F_x + \alpha \left( W_x - (n + 1) Q B \omega \right) \right\} \frac{D \omega}{100} \]

Computing the first difference:

\[ \Delta Z^\lambda(n) = Z^\lambda(n + 1) - Z^\lambda(n) \]

\[ = \lambda^1 \frac{Q B R C}{2} - \frac{\lambda^2 D}{Q B R n(n + 1)} - \lambda^3 \alpha Q B R D \omega \]

where \( \omega^2/100 = \overline{\omega} \)

Similarly,

\[ \Delta Z^\lambda(n + 1) = \lambda^1 \frac{Q B R C}{2} - \frac{\lambda^2 D}{Q B R n(n + 1)(n + 2)} - \lambda^3 \alpha Q B R D \omega \]

Computing the second difference:

\[ \Delta^2 Z^\lambda(n + 1) = \Delta Z^\lambda(n + 1) - \Delta Z^\lambda(n) \]

\[ = \frac{\lambda^2 D}{Q B R} \left\{ \frac{1}{n(n + 1)} - \frac{1}{(n + 1)(n + 2)} \right\} \]

\[ = \frac{\lambda^2 D}{Q B R} \left\{ \frac{2}{n(n + 1)(n + 2)} \right\} \]

\[ > 0 \]

\[ \Rightarrow \text{Point-wise convex} \]

**Theorem 4.2-** The optimal integer value \( n_i \) for each retailer is the smallest integer that satisfies the following condition:

\[ n_i(n_i + 1) \geq \frac{\lambda^2 D_i}{(Q B R_i)^2} \left\{ \frac{\lambda^1 C_i}{2} - \lambda^3 \alpha_i D_i \frac{\overline{\omega}}{\overline{\omega}} \right\} \]

**Proof:** To determine the integer \( n \), we have to find the smallest integer such that first difference is non-negative:

\[ \Delta Z^\lambda(n + 1) = Z^\lambda(n + 1) - Z^\lambda(n) \geq 0 \]
\[
\Rightarrow \lambda^1 \frac{Q_B R C}{2} - \frac{\lambda_2 D}{Q_B R n(n + 1)} - \lambda^1 \alpha Q_B R D \omega \geq 0
\]
\[
\Rightarrow \frac{\lambda^2 D}{Q_B R n(n + 1)} \leq \lambda^1 \frac{Q_B R C}{2} - \lambda^1 \alpha Q_B R D \omega
\]
\[
\Rightarrow n(n + 1) \geq \frac{\lambda^2 D}{\left(Q_B R \right)^2 \left\{ \frac{\lambda^1 C}{2} - \lambda^1 \alpha D \omega \right\}} \tag{4.15}
\]

As in Eq. 4.9, we can bring the importance of including transportation costs in the model formulation by observing Eq. 4.15. If transportation costs were not included, then Eq. 4.15 simplifies to:
\[
\Rightarrow n(n + 1) \geq \frac{\lambda^2 D}{\left(Q_B R \right)^2 \left\{ \frac{\lambda^1 C}{2} \right\}} \tag{4.16}
\]

Clearly, the denominator of Eq. 4.15 is less than the denominator of Eq. 4.16. As a result, the integer multiple ‘n’ could be rounded off to a larger integer, which in turn will increase the order quantity.

4.4 Warehouse’s Model Formulation

Being a decentralized system, the warehouse will use the ordering information of the retailers to formulate its multi-criteria mathematical programming (MCMP) problem. For the warehouse to formulate its MCMP, it has to first determine the average inventory. This expression ** is given by:
\[
Q_w = \frac{1}{2} \left[ \sum_{i=1}^{N} Q_i (\delta m_i + 1) \right]
\]

\( \delta m_i \) is the total number of orders placed by retailer i during the reorder interval of the warehouse. The \( m_i \)'s refer to the number of orders placed by retailer i during the smallest reorder interval of the warehouse given by L.C.M(\( T_1 \), \( T_2 \),..., \( T_N \)) = L.C.M(\( n_1 \), \( n_2 \),..., \( n_N \))\( T_B \).

** For a detailed explanation of how this expression was derived please refer to Section 3.5.1.
The \( i \)'s are calculated using the formula \( \frac{L.C.M(n_1, n_2, \ldots, n_N)}{n_i} \). The warehouse optimizes the same criteria as the retailers.

**Warehouse’s Criteria**

**Criterion 1: Capital Invested in Inventory**

The capital invested in inventory at the warehouse is developed in the same way as it was developed for the retailer.

- Capital invested in inventory at warehouse: It is the product of average inventory in the warehouse and the cost/unit at the warehouse.

\[
Q_w - \frac{1}{2} \left( \sum_{i=1}^{N} Q_i (\delta m_i + 1) \right) C_w
\]  
(4.17)

- Capital invested in in-transit inventory: FOB origin is assumed again. Annual capital invested in in-transit inventory is the fraction of year the annual demand of the warehouse is in transit multiplied by the cost/unit at each retailer:

\[
\frac{L_w}{365} D_w C_w
\]  
(4.18)

The sum of Eq. 4.17 and Eq. 4.18 gives the capital invested in inventory for the warehouse.

\[
\left( Q_w - \frac{1}{2} \left( \sum_{i=1}^{N} Q_i (\delta m_i + 1) \right) \right) C_w + \frac{L_w}{365} D_w C_w
\]  
(4.19)

**Criterion 2: Number of Orders Placed Annually**

The number of orders placed annually by the warehouse is the ratio of annual demand to the order quantity:

\[
\frac{D_w}{Q_w}
\]  
(4.20)

**Criterion 3: Annual Transportation Costs**

Since the warehouse is supplied by a sole manufacturer, the order quantity is most likely to be truck load (TL). Swenseth and Godfrey (1996) advocate the use of the inverse function to model TL freight rates. In fact for all shipping weights greater than 20,000lbs
the inverse function exactly determines the actual freight rates. The freight rate function for shipping TL for a particular distance is given by Eq. 4.21.

$$F_{yw} = \frac{F_{xw} W_{xw}}{W_{yw}}$$  \hspace{1cm} (4.21)

$F_{yw}$- Freight rate for shipping a given load for the warehouse ($/cwt$)

$F_{xw}$- TL freight rate for the warehouse ($/cwt$)

$W_{xw}$ - TL weight for the warehouse

$W_{yw}$- Weight of the order quantity shipped to the warehouse (lbs)

The numerator is the total cost to transport a truck load between an origin and a destination. Hence, $F_{yw}$ can be defined as the cost of shipping an entire TL divided by the weight of the warehouse’s shipment. In some cases, TL rates are expressed in terms of freight rate/unit distance. In that case, the total cost of transporting a TL can be found by multiplying the cost/unit distance and the distance between the origin and destination.

Now the transportation cost expression for shipping an order quantity $Q_w$ to the warehouse is:

$$F_{yw} Q_w \frac{\omega}{100} = \frac{F_{xw} W_{xw}}{100}$$

The annual transportation cost for the warehouse is obtained by multiplying the above expression by the number of shipments in a year which is $D_w/Q_w$:

$$\frac{F_{xw} W_{xw}}{Q_w} \frac{D_w}{100}$$  \hspace{1cm} (4.22)

### 4.4.1 Warehouse’s Multi-Criteria Problem

The warehouse has to minimize the three criteria given by Eq. 4.19, Eq. 4.20 and Eq. 4.22 simultaneously subject to the constraint that the order quantity at the warehouse is an integer multiple of the sum of the order quantities of the retailers during the ordering
period of the warehouse††. The multi-criteria nonlinear integer program for the warehouse denoted by Problem (W) is as follows:

**Problem (W)**

\[
\begin{align*}
\text{Min } Z_W^1 &= \left\{ Q_w - \frac{1}{2} \sum_{i=1}^{N} Q_i (\delta m_i + 1) \right\} C_w + \frac{L_w}{365} D_w C_w \\
\text{Min } Z_W^2 &= \frac{D_w}{Q_w} \\
\text{Min } Z_W^3 &= \frac{F_{sw} W_{sw}}{Q_w} \frac{D_w}{100}
\end{align*}
\]

Subject to:
\[
Q_w = \delta \sum_{i=1}^{N} m_i Q_i \\
Q_w \geq 0 \\
\delta \geq 1 \text{ and integer}
\]

**4.4.2 Solution Procedure to Solve Problem (W)**

The \( P_{\lambda} \) method is again used to convert Problem (W) to a single objective math program:

**Problem (W^λ)**

\[
\begin{align*}
\text{Min } Z_W^\lambda &= \lambda_w^1 \left\{ Q_w - \frac{1}{2} \sum_{i=1}^{N} Q_i (\delta m_i + 1) \right\} C_w + \frac{L_w}{365} D_w C_w + \lambda_w^2 \frac{D_w}{Q_w} + \lambda_w^3 \frac{F_{sw} W_{sw}}{Q_w} \frac{D_w}{100}
\end{align*}
\]

Subject to:
\[
\sum_{k=1}^{3} \lambda_w^k = 1 \\
\lambda_w^k > 0 \ \forall k \\
Q_w = \delta \sum_{i=1}^{N} m_i Q_i \\
Q_w \geq 0 \\
\delta \geq 1 \text{ and integer}
\]

\( \lambda_w^k \) - Convex combination parameter or weight assigned for criterion \( k \) for the warehouse.

†† For details please see Section 3.5.1.
As in Problem (MR₁), the feasible region is no longer a convex set; hence, the necessary condition of the general Pλ problem is not satisfied, and all the efficient points cannot be generated. However, an optimal solution to Problem (Wλ) will generate an efficient solution.

The order quantity of the warehouse can be determined once the value of the integer multiple δ is determined. To determine δ, we have to prove that the objective function of Problem (Wλ) is point-wise convex.

**Lemma 4.3-** The objective function of Problem (Wλ) is point-wise convex in δ.

**Proof:** Writing the objective function in terms of δ:

\[
Z^λ_w(δ) = \frac{\lambda_w^1}{2} \left\{ \delta \sum_{i=1}^{N} m_i Q_i - \sum_{i=1}^{N} Q_i + \frac{L_w D_w}{365} \right\} C_w + \lambda_w^2 \frac{D_w}{\delta} \sum_{i=1}^{N} m_i Q_i + \lambda_w^3 \frac{F_{xw} W_{xw}}{(δ+1)} \sum_{i=1}^{N} m_i Q_i \frac{D_w}{100}
\]

\[
Z^λ_w(δ+1) = \frac{\lambda_w^1}{2} \left\{ (δ+1) \sum_{i=1}^{N} m_i Q_i - \sum_{i=1}^{N} Q_i + \frac{L_w D_w}{365} \right\} C_w + \lambda_w^2 \frac{D_w}{(δ+1)} \sum_{i=1}^{N} m_i Q_i + \lambda_w^3 \frac{F_{xw} W_{xw}}{(δ+1)(δ+2)} \sum_{i=1}^{N} m_i Q_i \frac{D_w}{100}
\]

\[
\Delta Z^λ_w(δ) = Z^λ_w(δ+1) - Z^λ_w(δ)
\]

\[
= \frac{\lambda_w^1}{2} \left( \sum_{i=1}^{N} m_i Q_i \right) C_w - \frac{\lambda_w^2}{\delta} \frac{D_w}{(δ+1)} \sum_{i=1}^{N} m_i Q_i - \frac{\lambda_w^3}{(δ+1)} \frac{F_{xw} W_{xw}}{100} \frac{D_w}{\sum_{i=1}^{N} m_i Q_i}
\]

Similarly,

\[
\Delta Z^λ_w(δ+1) = \frac{\lambda_w^1}{2} \left( \sum_{i=1}^{N} m_i Q_i \right) C_w - \frac{\lambda_w^2}{(δ+1)(δ+2)} \frac{D_w}{\sum_{i=1}^{N} m_i Q_i} - \frac{\lambda_w^3}{(δ+1)(δ+2)} \frac{F_{xw} W_{xw}}{100} \frac{D_w}{\sum_{i=1}^{N} m_i Q_i}
\]

Computing the second difference:

\[
\Delta^2 Z^λ_w(δ+1) = \Delta Z^λ_w(δ+1) - \Delta Z^λ_w(δ)
\]

\[
= \frac{\lambda_w^2}{\sum_{i=1}^{N} m_i Q_i} \frac{D_w}{\delta(δ+1)(δ+2)} + \frac{\lambda_w^3 F_{xw} W_{xw}}{\sum_{i=1}^{N} m_i Q_i} \frac{D_w}{(δ+1)(δ+2)}
\]

\[
= \frac{\lambda_w^2 D_w}{\sum_{i=1}^{N} m_i Q_i} + \frac{\lambda_w^3 F_{xw} W_{xw}}{100} \frac{D_w}{\sum_{i=1}^{N} m_i Q_i} \frac{2}{(δ+1)(δ+2)}
\]
> 0
⇒ Point-wise convex

**Theorem 4.3:** The optimal integer value $\delta$ for the warehouse is the smallest integer that satisfies the following condition:

$$\delta(\delta + 1) \geq \frac{2 \sum_{i=1}^{N} D_i}{\left( \sum_{i=1}^{N} m_i Q_i \right)^2} \left( \lambda_w^2 + \frac{\lambda_w^3 F_{sw} W_{sw}}{100} \right)$$

**Proof:** The integer $\delta$ can be found by finding the smallest integer such that the first difference is non-negative i.e.

$$\Delta Z^\lambda_w (m + 1) = Z^\lambda_w (m + 1) - Z^\lambda_w (m) \geq 0$$

$$\Rightarrow \lambda_w \frac{\left( \sum_{i=1}^{N} m_i Q_i \right) C_w}{\left( \sum_{i=1}^{N} m_i Q_i \right)} - \frac{\lambda_w^2 D_w}{\left( \sum_{i=1}^{N} m_i Q_i \right) \delta(\delta + 1)} \geq 0$$

$$\Rightarrow \lambda_w \frac{\left( \sum_{i=1}^{N} m_i Q_i \right) C_w}{\left( \sum_{i=1}^{N} m_i Q_i \right)} \geq \frac{D_w}{\left( \sum_{i=1}^{N} m_i Q_i \right) \delta(\delta + 1)} \left( \lambda_w^2 + \frac{\lambda_w^3 F_{sw} W_{sw}}{100} \right)$$

$$\Rightarrow \delta(\delta + 1) \geq \frac{2 \sum_{i=1}^{N} D_i}{\left( \sum_{i=1}^{N} m_i Q_i \right)^2} \left( \lambda_w^2 + \frac{\lambda_w^3 F_{sw} W_{sw}}{100} \right) \tag{4.23}$$

Again, observing Eq. 4.23 we can analyze the importance of including transportation costs in the model formulation. If criterion 3 (transportation cost) was not included in the model formulation, Eq. 4.23 becomes:

$$\delta(\delta + 1) \geq \frac{2 \lambda_w \sum_{i=1}^{N} D_i}{\left( \sum_{i=1}^{N} m_i Q_i \right)^2} \lambda_w^2 C_w \tag{4.24}$$
The RHS of Eq. 4.24 is less than the RHS of Eq. 4.23 due to the term $\frac{\lambda^3_w F_{xW} W_{xW}}{100}$. Hence, the integer multiple $\delta$ could be rounded off to a larger integer value as a result of which $Q_w$ will be larger.

4.5 Algorithm

Step 1: Each retailer solves their own Problem ($R_i^\lambda$) by using different set of weights for each criterion such that $\frac{\lambda^i_1}{\lambda^i_2} > \frac{2 \alpha_i D_i \bar{\omega}}{C_i}$ (see Eq. 4.10) is satisfied. The order quantity for each retailer for different combinations of weights can be found using $Q_i = \sqrt{\frac{2 \lambda^2_i D_i}{\lambda^i_1 C_i - 2 \lambda^2_i \alpha_i D_i \bar{\omega}}}$ (see Eq. 4.9). The value of each criterion is then computed. Using the value path approach, the DM chooses the most preferred solution. The time between the orders for each retailer can be found using $T_i = \frac{Q_i}{D_i}$ (see Eq. 4.11). Each retailer passes this information to the warehouse.

Step 2: The warehouse identifies the base period using $T_B = \min_i T_i$ (see Eq. 4.12). The non-base retailers order in integer multiples of the base period, $T_i = n_i T_B$ (see Eq. 4.13). These retailers have to solve Problem (MR_i), by incorporating $Q_i = n_i Q_{B_i} R_i$ (see Eq. 4.14) as a constraint to the original Problem (R_i).

Step 3: Using $n_i (n_i + 1) \geq \frac{\lambda^2_i D_i}{(Q_{B_i} R_i)^2 \left\{ \lambda^i_1 \frac{C_i}{2} - \lambda^2_i \alpha_i D_i \bar{\omega} \right\}}$ (see Eq. 4.15), the integer values for the retailers can be determined. Then, the order quantity of the retailers is found out using $Q_i = n_i Q_{B_i} R_i$ (see Eq. 4.14). The reorder point is computed using $r_i = D_i (L_i - j T_i)$ (see Eq. 4.1).

Step 4: The retailers send the corresponding order information to the warehouse.
Step 5: The warehouse uses this information to formulate its own MCMP (Problem (W)) and determines its integer multiple given by
\[
\delta(\delta + 1) \geq \frac{2\sum_{i=1}^{N} D_i}{\left(\sum_{i=1}^{N} m_i Q_i\right)^2} \left\{ \lambda_w^2 + \frac{\lambda_w^3 F_{sw} W_{sw}}{100} \right\} \] (see Eq. 4.23) and hence its order quantity for different weight combinations. Using the value path approach, the DM can choose the most preferred solution.

Note 1: A heuristic procedure to determine the number of trucks needed for each location, particularly for the warehouse is as follows. If a location orders $Q_\omega$ lbs every cycle and the maximum allowable weight is $W_x$ lbs, then the number of trucks needed is $\left\lceil \frac{Q_\omega}{W_x} \right\rceil$. If the number of TL is $\left\lceil \frac{Q_\omega}{W_x} \right\rceil$ and the weight shipped in the $\left\lceil \frac{Q_\omega}{W_x} \right\rceil^{th}$ truck is less than the weight requirement for a TL, then the firm is charged a fixed TL rate/full truck and a LTL rate for the $\left\lceil \frac{Q_\omega}{W_x} \right\rceil^{th}$ truck.

Note 2: We made an important assumption regarding the type of shipments used by the retailers and the warehouse, and hence the type of freight rate function used. For the retailers, a proportional function was used to model the transportation costs since it was assumed that shipments are LTL. For the warehouse, an inverse function was used since it was assumed to use TL shipments. If, after solving the retailer’s problem the weight of the order quantity is such that it can be declared as a TL, then the annual transportation cost might be overestimated. Similarly, if the warehouse’s shipment weight is not large enough to be declared as a TL, then the annual transportation cost is overestimated at the warehouse. This difficulty can be overcome in three ways:

1) Recalculate the annual transportation costs by plugging in the weight shipped in the appropriate freight rate function, i.e., for the retailers use the inverse function and for the warehouse use the proportional function.
2) Resolve the MCMP with the appropriate freight rate function, i.e., use the inverse function in Problem (Ri) and the proportional function in Problem (W).
3) Formulate the MCMP with an additional binary variable which will indicate when to use the proportional function and when to use the inverse function. This formulation is described in the next section.

4.6 General MCMP for Retailer and Warehouse

As noted in the previous section, the assumption that the retailers use LTL and the warehouse uses TL can be relaxed by introducing a binary variable which is activated when the weight shipped is less than the indifference point needed to declare the load as a TL, i.e.,

\[ Q\omega < I \Rightarrow \gamma = 1 \]  \hspace{1cm} (4.25)

and \[ Q\omega \geq I \Rightarrow \gamma = 0 \]  \hspace{1cm} (4.26)

\( Q\omega \) – Weight shipped

I- Weight beyond which the shipment is declared as TL (Indifference point).

\( \gamma \in (0,1) \) and integer

Either Eq. 4.25 or Eq. 4.26 can occur but not both. This can be modeled as:

\[ Q\omega - (1 - \varepsilon) \leq M(1 - \gamma) \]  \hspace{1cm} (4.27)

\[ -Q\omega + I \leq M\gamma \]  \hspace{1cm} (4.28)

M- Very large number

\( \varepsilon \) - Infinitesimally small value

To see how constraints Eq. 4.25 and Eq. 4.26 represent Eq. 4.27 and Eq. 4.28, we consider two cases:

**Case 1:** \( Q\omega \geq I \)

\( Q\omega \geq I \Rightarrow Q\omega - I \geq 0 \). The LHS of Eq. 4.28 is negative. Eq. 4.28 is satisfied when \( \gamma = 0 \) or \( \gamma = 1 \), but Eq. 4.27 is infeasible when \( \gamma = 1 \). This means that \( \gamma \) is forced to 0. Hence, Eq. 4.26 \( \equiv \) Eq. 4.28. When \( \gamma = 0 \), Eq. 4.27 is redundant.
**Case 2: Qω < 1**

Qω < 1 is an open set and is converted to a closed set by subtracting ε from I, i.e., Qω ≤ 1 − ε. In this case Eq. 4.27 is satisfied for both γ = 1 and γ = 0, but Eq. 4.28 is infeasible when γ = 0. This means that γ is forced to 1. Hence, Eq. 4.25 = Eq. 4.27. When γ = 1, Eq. 4.28 is redundant.

Assuming that each location follows the same freight rate table, we can formulate the general MCMP for each retailer and the warehouse as:

**Problem (GRi)**

\[
\begin{align*}
\text{Min } Z_i^1 &= \frac{Q_i}{2} C_i + \frac{L_i}{365} D_i C_i \\
\text{Min } Z_i^2 &= \frac{D_i}{Q_i} \\
\text{Min } Z_i^3 &= \gamma_i \left\{ F_{xi} + \alpha_i (W_{xi} - Q_i \omega) \frac{\omega D_i}{100} \right\} + (1 - \gamma_i) \left\{ F_{xi} W_{xi} D_i \frac{Q_i}{100} \right\} \\
\end{align*}
\]

Subject to:

\[
\begin{align*}
Q_i \omega - (1 - \epsilon) &\leq M(1 - \gamma_i) \\
- Q_i \omega + 1 &\leq M \gamma_i \\
Q_i &\geq 0 \forall i \\
\gamma_i &\in (0, 1) \text{ and integer } \forall i
\end{align*}
\]

The objective function of criterion 3 is changed so that if γ = 1, then LTL is used, and if γ = 0, then TL is used. Similarly, Problems (MRi) and (W) can be reformulated as:

**Problem (GMRi)**

\[
\begin{align*}
\text{Min } Z_i^1 &= \frac{Q_i}{2} C_i + \frac{L_i}{365} D_i C_i \\
\text{Min } Z_i^2 &= \frac{D_i}{Q_i} \\
\text{Min } Z_i^3 &= \gamma_i \left\{ F_{xi} + \alpha_i (W_{xi} - Q_i \omega) \frac{\omega D_i}{100} \right\} + (1 - \gamma_i) \left\{ F_{xi} W_{xi} D_i \frac{Q_i}{100} \right\} \\
\end{align*}
\]

Subject to:

\[
\begin{align*}
Q_i &= n_i Q B R_i \\
Q_i \omega - (1 - \epsilon) &\leq M(1 - \gamma_i)
\end{align*}
\]
\[ -Q_i \omega + 1 \leq My_i \]
\[ Q_i \geq 0 \ \forall i \]
\[ n_i \geq 1 \text{ and integer } \forall i \]
\[ \gamma_i \in (0,1) \text{ and integer } \forall i \]

**Problem (GW)**

\[
\text{Min } Z_1^w = \left\{ Q_w - \frac{1}{2} \sum_{i=1}^{N} Q_i (\delta m_i + 1) \right\} C_w + \frac{L_w}{365} D_w C_w
\]

\[
\text{Min } Z_2^w = \frac{D_w}{Q_w}
\]

\[
\text{Min } Z_3^w = \gamma_w \left\{ F_{xw} + \alpha_w \left( W_{xw} - Q_w \omega \right) \frac{\omega D_w}{100} \right\} + (1 - \gamma_w) \left\{ \frac{F_{xw} W_{xw}}{Q_w} \frac{D_w}{100} \right\}
\]

Subject to:
\[ Q_w = \delta \sum_{i=1}^{N} m_i Q_i \]
\[ Q_w \omega - (1 - \varepsilon) \leq M(1 - \gamma_w) \]
\[ -Q_w \omega + 1 \leq M\gamma_w \]
\[ Q_w \geq 0 \]
\[ \delta \geq 1 \text{ and integer} \]
\[ \gamma_w \in (0,1) \text{ and integer} \]

Using the \( P_2 \) problem, the general MCMP’s can be converted into a single objective model. Solving these models analytically is hard due to the presence of binary variables, but it can be solved efficiently using optimization packages such as LINGO, GAMS, Excel Solver, etc.

**4.7 Example Problem**

To illustrate the solution methodology, an example problem consisting of a one warehouse three retailer problem is provided. The weight of the product is 40lbs. The
cost, demand (same as in Chapter 3), and lead time data for each retailer and the warehouse is given in Table 4.1.

### Table 4.1: Data for Example Problem

<table>
<thead>
<tr>
<th></th>
<th>Demand ($D_i$)</th>
<th>Cost ($C_i$)</th>
<th>Lead Time ($L_i$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retailer 1</td>
<td>800 units</td>
<td>$100</td>
<td>5 days</td>
</tr>
<tr>
<td>Retailer 2</td>
<td>525 units</td>
<td>$140</td>
<td>4 days</td>
</tr>
<tr>
<td>Retailer 3</td>
<td>415 units</td>
<td>$150</td>
<td>3 days</td>
</tr>
<tr>
<td>Warehouse</td>
<td>1740 units</td>
<td>$65</td>
<td>10 days</td>
</tr>
</tbody>
</table>

The freight rate data is taken from Swenseth and Godfrey (2001) and is reproduced as Table 4.2.

### Table 4.2: Freight Rate Data

<table>
<thead>
<tr>
<th>Weight Break (lbs)</th>
<th>Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum Charge</td>
<td>$40.00</td>
</tr>
<tr>
<td>1-499</td>
<td>$17.60/cwt</td>
</tr>
<tr>
<td>500-999</td>
<td>$14.80/cwt</td>
</tr>
<tr>
<td>1000-1999</td>
<td>$13.80/cwt</td>
</tr>
<tr>
<td>2000-4999</td>
<td>$12.80/cwt</td>
</tr>
<tr>
<td>5000-9999</td>
<td>$12.40/cwt</td>
</tr>
<tr>
<td>10000-19999</td>
<td>$6.08/cwt</td>
</tr>
<tr>
<td>$\geq$ 20000 (Truckload)</td>
<td>$1110.00</td>
</tr>
</tbody>
</table>

The freight rate data with indifference points is shown in Table 4.3.

### Table 4.3: Freight Rate Data with Indifference Points

<table>
<thead>
<tr>
<th>Weight Break (lbs)</th>
<th>Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\leq$ 227</td>
<td>$40.00</td>
</tr>
<tr>
<td>228-420</td>
<td>$17.60/cwt</td>
</tr>
<tr>
<td>421-499</td>
<td>$74.00</td>
</tr>
<tr>
<td>500-932</td>
<td>$14.80/cwt</td>
</tr>
<tr>
<td>933-999</td>
<td>$138.00</td>
</tr>
<tr>
<td>1000-1855</td>
<td>$13.80/cwt</td>
</tr>
<tr>
<td>1856-1999</td>
<td>$256.00</td>
</tr>
<tr>
<td>2000-4749</td>
<td>$12.80/cwt</td>
</tr>
<tr>
<td>4750-9999</td>
<td>$608.00</td>
</tr>
<tr>
<td>10000-18256</td>
<td>$6.08/cwt</td>
</tr>
<tr>
<td>$\geq$ 18257</td>
<td>$1110.00</td>
</tr>
</tbody>
</table>
If the weight shipped, e.g., is 5000 lbs, then the shipment is over-declared to 10000 lbs and the company is charged a fixed rate of $608. The effective rate (Warsing, 2006) for this shipment is 608/5000 is $0.1216/lb or $12.16/cwt. If the weight of the shipment exceeds 18256 lbs, it is declared as a TL.

Instead of using the proportional function we decided to fit different functions to the effective rates obtained from Table 4.3. The function which has the maximum $R^2$ value is selected as the best function to estimate the freight rates. A summary of the different functions used and their $R^2$ value are given in Table 4.4.

**Table 4.4: Summary of Curve Fitting**

<table>
<thead>
<tr>
<th>Function type</th>
<th>Function</th>
<th>$R^2$ value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>-0.0487x + 17.25</td>
<td>0.2164</td>
</tr>
<tr>
<td>Quadratic</td>
<td>0.0002x^2 - 0.134x + 19.302</td>
<td>0.2929</td>
</tr>
<tr>
<td>Cubic</td>
<td>-1E-06x^3 + 0.0011x^2 - 0.2457x + 20.783</td>
<td>0.3294</td>
</tr>
<tr>
<td><strong>Power</strong></td>
<td>32.686x^{-0.3484}</td>
<td><strong>0.8798</strong></td>
</tr>
<tr>
<td>Exponential</td>
<td>16.137e^{0.005x}</td>
<td>0.74</td>
</tr>
<tr>
<td>Logarithmic</td>
<td>-4.8782Ln(x) + 28.596</td>
<td>0.5399</td>
</tr>
</tbody>
</table>

From Table 4.4, we can see that the power function provides the best estimate of the LTL rate. In fact, the linear function provides the worst fit. It is assumed that the freight rates are same for all the retailers. The results of Problem (R_i) and Problem (MR_i) were derived based on the assumption that the transportation cost was of the linear type and hence cannot be used. Re-deriving the expressions using the power function leads to mathematically intractable expressions for the order quantity. However, the general power function $a(x)^b$ is a convex function provided $b$ is negative. Hence, the objective function of Problem (R_i^λ) (Problem (MR_i^λ)) is convex (point-wise convex). Standard optimization packages like Excel Solver or Lingo can be used to solve for the ordering policy.

Table 4.5 provides the upper and lower bounds for the order quantity set by the DM of each retailer.
Table 4.5: Bounds on Order Quantity

<table>
<thead>
<tr>
<th></th>
<th>Upper bound</th>
<th>Lower bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retailer 1</td>
<td>100</td>
<td>20</td>
</tr>
<tr>
<td>Retailer 2</td>
<td>80</td>
<td>15</td>
</tr>
<tr>
<td>Retailer 3</td>
<td>70</td>
<td>10</td>
</tr>
</tbody>
</table>

A sample MCMP for Retailer 1 is given as:

Problem \((R_1)\)

\[
\text{Min } Z^1_i = \frac{Q_i}{2} 100 + \frac{5}{365} (800)(100)
\]

\[
\text{Min } Z^2_i = \frac{800}{Q_i}
\]

\[
\text{Min } Z^3_i = 32.686(Q_i\omega/100)^{-0.3484} \frac{(800)(40)}{100}
\]

Subject to: \(Q_i \geq 20\)

\(Q_i \leq 100\)

To prevent one criterion from dominating the other, normalization must be done. Normalization is done by dividing the weight assigned to a particular criterion by the ideal value of that criterion. Ideal value of each criterion is found by minimizing the individual criterion independently subjected to the upper and lower bound constraints. This is the same as assigning unit weight to a criterion and 0 to the others. The ideal values for the criteria are $2095.89, 8, and $3436.67. The next step is to convert the original MCMP to a single objective type by using the \(P_\lambda\) method as shown for Retailer 1:

Problem \((R^\lambda_1)\)

\[
\text{Min } Z^\lambda_i = \frac{\lambda^1_i}{2095.89} \left( \frac{Q_i}{2} 100 + \frac{5}{365} (800)(100) \right) + \frac{\lambda^2_i}{8} \frac{800}{Q_i}
\]

\[+ \frac{\lambda^3_i}{3436.67} 32.686(Q_i\omega/100)^{-0.3484} \frac{(800)(40)}{100}\]

Subject to: \(Q_i \geq 20\)

\(Q_i \leq 100\)
The $P_\lambda$ problem for each retailer is then solved for different combinations of weights using Excel Solver. Since there are only three criteria, the weight combinations are chosen randomly. This is shown in Table 4.6. To aid the DM in choosing the most preferred solution, the value path method is used. The first step is to divide each criterion’s value by the ideal value of that criterion for all the weight combinations. This is shown in Table 4.7. The next step is to plot graphs with the criteria along the X axis and the normalized values along the Y axis as shown in Fig. 4.3.
### Table 4.6: Criteria Values for the Retailers

<table>
<thead>
<tr>
<th>Weights</th>
<th>Retailer 1</th>
<th>Retailer 2</th>
<th>Retailer 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1,0,0)</td>
<td>2095.89</td>
<td>5068.45</td>
<td>3676.83</td>
</tr>
<tr>
<td>(0,1,0)</td>
<td>6095.89</td>
<td>2893.04</td>
<td>2052.05</td>
</tr>
<tr>
<td>(0,0,1)</td>
<td>6095.89</td>
<td>2893.04</td>
<td>2052.05</td>
</tr>
<tr>
<td>(0.33,0.33,0.33)</td>
<td>4765.2</td>
<td>3222.36</td>
<td>2388.95</td>
</tr>
<tr>
<td>(0.2,0.4,0.4)</td>
<td>6095.89</td>
<td>2893.04</td>
<td>2096.39</td>
</tr>
<tr>
<td>(0.4,0.2,0.4)</td>
<td>3876.28</td>
<td>3549.35</td>
<td>2436.43</td>
</tr>
<tr>
<td>(0.4,0.2,0.2)</td>
<td>3188.51</td>
<td>3951.90</td>
<td>2926.79</td>
</tr>
<tr>
<td>(0.6,0.2,0.2)</td>
<td>6095.89</td>
<td>2893.04</td>
<td>2096.39</td>
</tr>
<tr>
<td>(0.2,0.2,0.6)</td>
<td>5496.01</td>
<td>2987.36</td>
<td>2224.58</td>
</tr>
<tr>
<td>Ideal</td>
<td>2095.89</td>
<td>2893.04</td>
<td>2099.96</td>
</tr>
</tbody>
</table>

### Table 4.7: Normalized Criteria Values for the Retailers

<table>
<thead>
<tr>
<th>Weights</th>
<th>Retailer 1</th>
<th>Retailer 2</th>
<th>Retailer 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1,0,0)</td>
<td>1</td>
<td>5</td>
<td>1.9698</td>
</tr>
<tr>
<td>(0,1,0)</td>
<td>2.9085</td>
<td>5</td>
<td>1.7519</td>
</tr>
<tr>
<td>(0,0,1)</td>
<td>2.9085</td>
<td>5</td>
<td>1.9698</td>
</tr>
<tr>
<td>(0.33,0.33,0.33)</td>
<td>2.0236</td>
<td>1.1138</td>
<td>1.2358</td>
</tr>
<tr>
<td>(0.2,0.4,0.4)</td>
<td>2.9085</td>
<td>5</td>
<td>1.7918</td>
</tr>
<tr>
<td>(0.4,0.2,0.4)</td>
<td>5</td>
<td>1.7918</td>
<td></td>
</tr>
<tr>
<td>(0.4,0.4,0.2)</td>
<td>2.1096</td>
<td>1.5463</td>
<td></td>
</tr>
<tr>
<td>(0.6,0.2,0.2)</td>
<td>2.8287</td>
<td>1.2608</td>
<td></td>
</tr>
<tr>
<td>(0.2,0.6,0.2)</td>
<td>4.1246</td>
<td>1.0326</td>
<td></td>
</tr>
<tr>
<td>(0.2,0.2,0.6)</td>
<td>2.8287</td>
<td>1.2608</td>
<td></td>
</tr>
</tbody>
</table>
Value Path Graph for Retailer 1

Value Path Graph for Retailer 2
In Fig. 4.3 all the value paths intersect, which implies that all the alternatives belong to the efficient set. The DM by looking at Fig. 4.3 can visualize how each criterion varies with different weight combinations. Further, by looking at Table 4.7, the DM can quantify the tradeoff information among different weight combinations. For example, if the DM for Retailer 1 wants to switch from weight combination (1, 0, 0) to weight combination (0.4, 0.2, 0.4), he/she is willing to increase the capital invested in inventory by 84.95% from the ideal value in order to decrease the annual number of orders and annual transportation cost by 64.03% and 29.96%, respectively. This result intuitively makes sense as the DM’s preference for the capital invested in inventory criterion has reduced, while his/her’s preference towards number of orders, and annual transportation costs has increased. Thus, a tradeoff analysis can be used to validate the results by observing the criteria values, and verifying if their variations make sense.

This method is a simple way of incorporating the DM’s preference information. The DM is involved entirely during the decision making process. Also, the cognitive burden on the DM is less. If the DM is not satisfied with the efficient solutions, then
another set of efficient solutions can be generated by tweaking the weight combinations, and the process is repeated.

**Note:** Instead of choosing some weight combinations, one could interact with the decision maker and get the preference information on the criteria by different methods. This approach is illustrated in the case study discussed in Chapter 6.

Once the DM for each retailer chooses a weight combination and hence, the most preferred solution, the reorder interval is calculated based on the assumption that there are 52 weeks in a year. This is shown in Table 4.8.

<table>
<thead>
<tr>
<th>Table 4.8: Most Preferred Solution for the Retailers</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Weight combination</strong></td>
</tr>
<tr>
<td><strong>Retailer 1</strong></td>
</tr>
<tr>
<td><strong>Retailer 2</strong></td>
</tr>
<tr>
<td><strong>Retailer 3</strong></td>
</tr>
</tbody>
</table>

Each retailer passes the order quantity and reorder interval information in the above table to the warehouse. The warehouse identifies the base period, $T_B = 2.65541$ weeks which corresponds to the ordering period of Retailer 1. Retailers 2 and 3 have to solve the modified retailer’s problem, i.e., Problem (MR$_2$) and Problem (MR$_3$) respectively where they order in integer multiples of the base period.

Eq. 4.15 cannot be used to obtain the optimal integer values since it is applicable only when the transportation cost function is linear. However, the modified problem has only one integer variable and the objective function is point-wise convex. Hence, Excel Solver can be used to solve for the optimal integer values. Excel Solver determines the optimal integer values as $n_2 = 2$ and $n_3 = 2$. Reorder points are calculated using $r = D(L - jT)$(Eq. 4.1). A summary of the ordering policy and each criterion’s value for all the retailers is given in Table 4.9.

<table>
<thead>
<tr>
<th>Table 4.9: Summary of Results for the Retailers</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Order Quantity</strong></td>
</tr>
<tr>
<td><strong>Retailer 1</strong></td>
</tr>
<tr>
<td><strong>Retailer 2</strong></td>
</tr>
<tr>
<td><strong>Retailer 3</strong></td>
</tr>
</tbody>
</table>
The output of the retailers is used to formulate the warehouse’s problem. The annual demand faced by the warehouse is the sum of the annual demands faced by all the retailers and is 1740 units. The lead time and cost at the warehouse are 10 days and $65, respectively. Assuming a truck can carry up to 46,000lbs (460 cwt), TL rate is 1110/460 = $2.413/cwt.

The number of orders that each retailer places during the smallest ordering period of the warehouse is found by:

\[ m_1 = \frac{\text{L.C.M}(1,2,2)}{1} = 2 \]

Similarly, \( m_2 = 1 \) and \( m_3 = 1 \). The MCMP for the warehouse for the example is given as:

\[
\begin{align*}
\text{Min } Z_W^1 &= Q_w - \frac{1}{2} \left[ 40.85(2\delta + 1) + 53.62(\delta + 1) + 42.38(\delta + 1) \right] + 10.65 (1740)(65) \\
\text{Min } Z_W^2 &= \frac{1740}{Q_w} \\
\text{Min } Z_W^3 &= \frac{(2.413)(46000)(1740)}{100Q_w}
\end{align*}
\]

Subject to:
\[
\begin{align*}
Q_w &= 177.71\delta \\
Q_w &\geq 0 \\
\delta &\geq 1 \text{ and integer}
\end{align*}
\]

As in the retailer problem normalization is done by dividing the weight of a criterion by the ideal value of that criterion. The ideal value is determined by optimizing the individual criterion subjected to upper bound and lower bound constraints on either \( Q_w \) or \( \delta \), in addition to the original constraints of Problem \( (W^\lambda) \). The upper and lower bound used for \( \delta \) is \([1,5]\). The ideal values are $4426.33, 1.96 and $2173.68. This formulation is converted to a single objective problem by using the \( P_\lambda \) method and is given as:

\[
\begin{align*}
\text{Min } Z_W^\lambda &= \frac{\lambda_1^W}{4426.33} \left[ Q_w - \frac{1}{2} \left[ 40.85(2\delta + 1) + 53.62(\delta + 1) + 42.38(\delta + 1) \right] \right] + 10.65 (1740)(65) \\
+ \frac{\lambda_2^W}{1.96} \frac{1740}{Q_w} + \frac{\lambda_3^W}{2173.68} \frac{(2.413)(46000)(1740)}{100Q_w}
\end{align*}
\]
Subject to:  
\[ Q_w = 177.71\delta \]
\[ Q_w \geq 0 \]
\[ \delta \geq 1 \]
\[ \delta \leq 5 \]
\[ \delta \text{ - integer} \]

The ordering policy of the warehouse can be directly determined by computing the RHS of Eq. 4.23, \( \delta(\delta + 1) \geq \frac{2\sum_{i=1}^{N} D_i}{\sum_{i=1}^{N} m_i Q_i} \left( \frac{\lambda^2_w}{\lambda^1_w C_w} + \frac{\lambda^3_w F_w W_{sw}}{100} \right) \), for different weight combinations. Note that Eq. 4.23 cannot be used to determine the ordering policies for the weight combinations (1,0,0), (0,1,0) and (0,0,1). By changing the weight parameters, different values of \( \delta \) can be obtained. The DM can then use the value path approach explained previously to choose the most preferred solution. The \( \delta \) calculation for the weight combination (0.6,0.2,0.2) using the RHS of Eq. 4.23 is given as:

\[
\frac{2(1740)}{(177.71)^2 * (0.6/4426.33) * 65} \left( \frac{0.2/1.96 + (0.2/2173.68)(2.413)(46000)}{100} \right) = 2.5534 \Rightarrow \delta = 2
\]

The values of the decision variables for the different weight combinations are given in Table 4.10. The criteria and the normalized criteria values are given in Table 4.11. The value path graph for the warehouse is shown in Fig. 4.4.

| Table 4.10: Efficient Solutions for the Warehouse |
|-----------------|-----|---------|---------|
| Weights         | \( \delta \) | \( Q_w \) (units) | \( T_w \) (weeks) |
| (0.33,0.33,0.33) | 3   | 533.12  | 15.93   |
| (0.2,0.4,0.4)    | 4   | 710.83  | 21.24   |
| (0.4,0.2,0.4)    | 2   | 355.42  | 10.62   |
| (0.4,0.4,0.2)    | 2   | 355.42  | 10.62   |
| (0.6,0.2,0.2)    | 2   | 355.42  | 10.62   |
| (0.2,0.6,0.2)    | 4   | 710.83  | 21.24   |
| (0.2,0.2,0.6)    | 4   | 710.83  | 21.24   |
| (0.7,0.1,0.2)    | 1   | 177.71  | 5.31    |
Table 4.11: Criteria and Normalized Criteria Values for the Warehouse

<table>
<thead>
<tr>
<th>Weights</th>
<th>Criteria1</th>
<th>Criteria2</th>
<th>Criteria3</th>
<th>Normalized Criteria1</th>
<th>Normalized Criteria2</th>
<th>Normalized Criteria3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.33,0.33,0.33)</td>
<td>15977.35</td>
<td>3.26</td>
<td>3622.80</td>
<td>3.61</td>
<td>1.67</td>
<td>1.67</td>
</tr>
<tr>
<td>(0.2,0.4,0.4)</td>
<td>21752.86</td>
<td>2.45</td>
<td>2717.10</td>
<td>4.91</td>
<td>1.25</td>
<td>1.25</td>
</tr>
<tr>
<td>(0.4,0.2,0.4)</td>
<td>10201.84</td>
<td>4.90</td>
<td>5434.20</td>
<td>2.30</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td>(0.4,0.4,0.2)</td>
<td>10201.84</td>
<td>4.90</td>
<td>5434.20</td>
<td>2.30</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td>(0.6,0.2,0.2)</td>
<td>10201.84</td>
<td>4.90</td>
<td>5434.20</td>
<td>2.30</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td>(0.2,0.6,0.2)</td>
<td>21752.86</td>
<td>2.45</td>
<td>2717.10</td>
<td>4.91</td>
<td>1.25</td>
<td>1.25</td>
</tr>
<tr>
<td>(0.2,0.2,0.6)</td>
<td>21752.86</td>
<td>2.45</td>
<td>2717.10</td>
<td>4.91</td>
<td>1.25</td>
<td>1.25</td>
</tr>
<tr>
<td>(0.7,0.1,0.2)</td>
<td>4426.33</td>
<td>9.79</td>
<td>10868.39</td>
<td>1</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Ideal</td>
<td>4426.33</td>
<td>1.96</td>
<td>2173.68</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 4.4: Value Path Graph for Warehouse

If the DM of the warehouse was to choose between weight combinations (0.6,0.2,0.2) and (0.7,0.1,0.2), he/she would most likely choose (0.6,0.2,0.2). This can be concluded by observing the value paths corresponding to both the alternatives in Fig. 4.4. The distance between the capital invested in inventory criterion is less than the distance between the annual transportation cost criterion. This implies that the increase in annual
transportation cost is more than the decrease in capital invested in inventory. Assuming that the DM chooses weight combination (0.6,0.2,0.2) as the most preferred solution, the warehouse’s ordering policy is to order 355.42 units when the reorder point reaches 47.67 units. The time between orders is 10.62 weeks.

The weight shipped for the weight combination (0.7,0.1,0.2) is 7108.4 lbs which is less than 18256 lbs, the weight requirement for using the inverse function. As a result the annual transportation cost is overestimated. If a power function was used, the annual transportation cost would be $5150.08. Hence, the DM of the warehouse might have to consider using LTL shipments instead of TL shipments if they want to choose weight combination (0.7,0.1,0.2).

4.8 Summary

In this chapter we extended the single objective cost model of the previous chapter to a multi-criteria framework where marginal cost information is not known. Transportation costs were considered between the echelons. Under the assumption that the retailers’ shipments were LTL and the warehouse’s shipment was TL a MCMP was developed. The P_\lambda method in conjunction with a graphical tool called the Value Path Method was used to assist the DM in choosing the most preferred solution. The impact of transportation costs on the ordering policy of each location was studied. Relaxing the previous assumption, another MCMP was developed which not only determines the ordering policies of the companies but also indicates when to use TL and when to use LTL. An example problem was used to explain the methodology.

In the next chapter we develop and solve the multi-criteria supply chain inventory problem with transportation costs by relaxing the assumptions on demands and lead times. The external demands faced by the retailers are non-identical random variables, and the lead times follow a general discrete distribution.
Chapter 5
STOCHASTIC MULTI-CRITERIA MODEL FOR A SUPPLY CHAIN WITH TRANSPORTATION COSTS

5.1 Introduction

In this chapter we extend the basic model of Chapter 4 to the case where there is randomness in the system due to the external demand faced by the retailers and the replenishment time required to fulfill the orders placed by each company in the supply chain. We model the supply chain as a multi-criteria problem with four criteria. In addition to the three criteria of the previous chapter, capital invested in inventory, annual number of orders and annual transportation costs, a customer service criterion is included. The objective in this chapter is to develop relatively simple, yet realistic, models and solution procedures that can be implemented in practice.

5.2 Problem Description

The system under consideration is the same as in Chapters 3 and 4. We are interested in determining ordering policies for each location in a single warehouse multi-retailer system. The warehouse is supplied by a manufacturer that is assumed to have infinite capacity. The retailers face independent but non-identical stochastic demands. Further, the lead times between the stages are assumed to be discrete random variables. Transportation costs are considered between the echelons. The inbound transportation cost at the retailer and the warehouse is modeled using LTL and TL rates, respectively. The optimization problem for each location is modeled as a multi-criteria problem. A general model for determining the retailer inventory policy is presented. Depending upon the distribution of the lead time demand three variants of this model are discussed. Then, using the output of all the retailers, we formulate and solve the warehouse’s inventory problem.

5.2.1 Assumptions

- Each location follows a continuous review policy, i.e., an order is placed when the inventory level reaches the reorder point; however instead of continuous tracking,
it is enough if each transaction is recorded when it occurs, and the inventory levels are adjusted after the transaction has been finalized. This is called a transactions reporting system. For the system to place an order precisely when the inventory level reaches the reorder point the system must be monitored continuously or demand must occur one at a time. In cases where the number of units ordered when a demand occurs exceeds one or when a transaction reporting system is used, then the inventory level after the demand has been realized might have overshot the reorder point. This is called demand overshoot, which leads us to the next assumption.

• The probability of a large overshoot is very small which means that when it occurs, it is very small compared to the lead time demand.

• Excess demand at each location is treated as lost sales. While most of the literature in supply chain inventory models assume that excess demand is backordered, this may not be true particularly at the retailer for functional products since a customer who enters a retail shop is not going to wait for an item if it is not in the shelf. He/she is going to purchase that item elsewhere. Excess demand at the warehouse is treated as lost sales since the supply chain is operating under a decentralized control.

• There is no order splitting at the warehouse, i.e., if the warehouse is not able to completely meet an order, the entire order is lost. This is a realistic assumption since the freight rate is a function of the retailer’s order quantity.

• The average time that any location is out of stock is negligible compared to its reorder interval. Again, this is a realistic assumption due to the high level of service requirements imposed on each company.

• There is no outstanding order at time of reorder. This implies that the reorder point is less than the order quantity.

• The mean rate of demand does not vary with time (the demand distributions are stationary).

• Orders do not crossover, i.e., orders arrive in the same order they are placed.
• FOB origin is assumed at each location. This means that the retailers take ownership of the goods at the warehouse and the warehouse takes ownership of the goods at the manufacturer and pay for in transit inventory.

5.2.2 Notation

Data

D_i - Expected annual demand for retailer i
μ_{D_i} - Mean demand for retailer i during a time period t
σ_{D_i} - Standard deviation of demand for retailer i during a time period t

D_W - Expected annual demand at the warehouse
μ_{D_W} - Mean demand at the warehouse during a time period t
σ_{D_W} - Standard deviation of demand at the warehouse during a time period t

L_i - Lead time for retailer i with distribution \( Pr\{L_i = j\} = p_{ij} \) (random variable)
μ_{L_i} - Mean lead time for retailer i
σ_{L_i} - Standard deviation of lead time for retailer i

L_W - Lead time for the warehouse with distribution \( Pr\{L_W = j\} = p_{Wj} \) (random variable)
μ_{L_W} - Mean lead time for warehouse
σ_{L_W} - Standard deviation of lead time for warehouse

LD_i - Demand during the lead time for retailer i (random variable)
μ_{LD_i} - Mean lead time demand for retailer i
σ_{LD_i} - Standard deviation of lead time demand for retailer i

LD_W - Demand during the lead time for warehouse (random variable)
μ_{LD_W} - Mean lead time demand for warehouse
σ_{LD_W} - Standard deviation of lead time demand for warehouse

C_i - Cost/unit at retailer i \( \forall i = 1, 2, \ldots, N \)
C_W - Cost/unit at warehouse
ω - Weight of a single unit in lbs

F_{yi} - Proportional freight rate function for retailer i given by \( F_{yi} = F_{xi} + \alpha_{i}(W_{xi} - W_{yi}) \)
F_{xi} - TL freight rate for each retailer ($/cwt)
W_{xi} - TL weight
W_{yi} - Weight shipped to each retailer i (lbs)
\alpha_i - Rate at which freight rate increases / 100lbs decrease in shipping weight for retailer i.

F_{yw} - Inverse freight rate function for warehouse given by \[ F_{yw} = \frac{F_{xw} W_{sw}}{W_{yw}} \]

F_{xw} - TL freight rate for the warehouse ($/cwt)
W_{xw} - TL weight for the warehouse
W_{yw} - Weight of the order quantity shipped to the warehouse (lbs)
N - Number of retailers

**Decision Variables**

Q_i - Order quantity of retailer i
Q_w - Order quantity of warehouse
r_i - Reorder point for retailer i
r_w - Reorder point for warehouse
s_i - Safety stock for retailer i (expected on hand inventory at the time when new order arrives)
s_w - Safety stock for warehouse
ESC_i - Expected shortages/order cycle (lost sales) for retailer i
ESC_w - Expected shortages/order cycle (lost sales) for warehouse

**5.3 Retailer’s Model Formulation**

When the demand faced by a retailer that is following a continuous review policy is stochastic, the reorder interval, i.e., time between orders is a random variable but the order quantity is fixed. There is a possibility that the lead time demand (LTD), which is now a random variable, may exceed the reorder point and leading to a stockout. To minimize stockouts, a safety stock is maintained. Safety stock is defined as the *expected on-hand inventory* when an order arrives. Safety stock is usually determined based on
some service criterion. Excess safety stock leads to higher holding costs. Hence, there
should be a proper tradeoff between service provided and holding costs.

The inventory pattern for the lost sales case is shown in Fig. 5.1.

Note that the inventory level does not fall below zero since excess demand is lost. In Fig.
5.1, demands during T2-T1 are lost sales. When a new order arrives, the system can either
be out of stock or have some inventory. In Fig. 5.1, at time T2 the inventory is zero, while
at time T3 there is positive inventory. The expected number of shortages (lost sales)
during an order cycle, ESC, is given by:

$$ESC = \int_{r}^{\infty} (x - r)f_{LD}(x)dx$$  \hspace{1cm} (5.1)$$

where $f_{LD}(\bullet)$- Probability density function (pdf) of LTD for a retailer. Eq. 5.1 is
simplified as:

$$ESC = \int_{r}^{\infty} xf_{LD}(x)dx - \int_{r}^{\infty} rf_{LD}(x)dx$$

$$= \int_{r}^{\infty} xf_{LD}(x)dx - r[F_{LD}(r)]$$  \hspace{1cm} (5.2)$$

where $F_{LD}(\bullet)$- Cumulative distribution function (cdf) of LTD for a retailer.
Following Ravindran et al. (1987, Chapter 8, pp. 356-364), the expected on-hand inventory when a new order arrives is the safety stock, \( s \), and is determined as:

\[
s = \int_{0}^{\infty} (r - x)f_{LD}(x)dx
\]

Note that the inventory level at the time when new order arrives is:

\[
r - x \text{ if } x \leq r
\]

\[
0 \text{ if } x > r
\]

Hence,

\[
s = \int_{0}^{r} (r - x)f_{LD}(x)dx + \int_{r}^{\infty} f_{LD}(x)dx
\]

The second integral vanishes as any demand in excess of \( r \) is lost in the lost sales case. Hence, the expression for safety stock is:

\[
s = \int_{0}^{r} (r - x)f_{LD}(x)dx
\] (5.3)

Eq. 5.3 is simplified as:

\[
s = \int_{0}^{\infty} (r - x)f_{LD}(x)dx - \int_{r}^{\infty} f_{LD}(x)dx
\]

\[
= \int_{0}^{\infty} rf_{LD}(x)dx - \int_{0}^{\infty} xf_{LD}(x)dx - \int_{r}^{\infty} rf_{LD}(x)dx + \int_{r}^{\infty} xf_{LD}(x)dx
\]

\[
= r - \mu_{LD} - r[1 - F_{LD}(r)] + \int_{r}^{\infty} xf_{LD}(x)dx
\] (5.4)

Substituting Eq. 5.2 into Eq. 5.4:

\[
s = r - \mu_{LD} + \text{ESC}
\] (5.5)

**Retailer’s Criteria**

The following criteria are used in the retailer’s multi-objective problem:

- Capital invested in inventory (including in-transit inventory due to FOB origin)
- Number of orders placed annually
- Annual transportation cost
- Fill rate (service criterion)
**Criterion 1: Capital Invested in Inventory**

As in the previous chapter the capital invested in inventory has two components:

- **Capital invested in inventory at Retailer**: The expression for average inventory is based on the approximate lost sales model in Hadley and Whitin (1963, Chapter 4, pp. 162-172) and Ravindran et al. (1987, Chapter 8, pp. 356-364). Since the expected ending inventory level at the retailer is its safety stock, the expected inventory level when an order is received is $Q_i + s_i$. The average inventory at the retailer is:

$$i \left( \frac{Q_i}{2} + s_i \right)$$

Hence, the capital invested in inventory at the retailer is:

$$C_i \left\{ \frac{Q_i}{2} + s_i \right\}$$  \hspace{1cm} (5.6)

- **Capital invested in In-transit inventory**: Since the in-transit inventory is affected only by the mean lead time and not the standard deviation, the expression for the capital invested in In-transit inventory is the same as in Chapter 4.

$$C_i \frac{\mu_{Li} D_i}{365}$$  \hspace{1cm} (5.7)

The total capital invested in inventory is given by the sum of Eq. 5.6 and Eq. 5.7:

$$C_i \left\{ \frac{Q_i}{2} + s_i + \frac{\mu_{Li} D_i}{365} \right\}$$  \hspace{1cm} (5.8)

where $s_i = r_i - \mu_{LDi} + ESC_i$.

**Criterion 2: Number of Orders Placed Annually**

The annual number of orders is still the ratio of the expected annual demand to the order quantity as the average time spent by the system in a stockout situation is negligible due to the high level of service provided by the retailer:

$$\frac{D_i}{Q_i}$$  \hspace{1cm} (5.9)

‡‡ If the assumption is relaxed, the number of orders placed is $rac{D}{Q + \mu_D T^s}$, where $T^s (T_2 - T_1$ in Fig. 5.1.) is the average time spent by the system when there is no stock.
Criterion 3: Annual Transportation Costs

Since it is assumed that the retailer’s shipments are most likely going to be LTL, the proportional function.§§ is used to model the transportation cost. The annual transportation cost is given by:

\[ \{F_{xi} + \alpha_i (W_{xi} - Q_i \omega)\} \frac{D_i \omega}{100} \]  

(5.10)

Criterion 4: Fill Rate (FR)

As explained previously, the safety stock is set based on service commitments promised by each location. A fill rate criterion is used to model the service criterion. Fill rate is defined as the proportion of demand that is met from available stock. Mathematically it is defined as:

\[ FR_i = 1 - \frac{ESC_i}{Q_i} \]

We would like to maximize fill rate or minimize the fraction of demand lost. Hence, the fourth criterion is:

\[ 1 - FR_i = \frac{ESC_i}{Q_i} \]  

(5.11)

5.3.1 Retailer’s Multi-Criteria Problem

Each retailer’s MCMP consists of minimizing capital invested in inventory (Eq. 5.8), annual number of order (Eq. 5.9), annual transportation cost (Eq. 5.10), and fraction of demand not met (Eq. 5.11) simultaneously and is given as:

---

§§ In the example problem of Chapter 4, we concluded that a power function best estimates LTL rates, but using the proportional function enables us to develop closed form expressions. We revert to using the power function for the example problem.
Retailer’s Multi-criteria Problem (R_i)

\[ \text{Min } Z_i^1 = C_i \left\{ \frac{Q_i}{2} + r_i - \mu_{Li} + \text{ESC}_i + \frac{\mu_{Li} D_i}{365} \right\} \quad \text{\{Using Eq. 5.5\}} \]

\[ \text{Min } Z_i^2 = \frac{D_i}{Q_i} \]

\[ \text{Min } Z_i^3 = \left\{ F_{xi} + \alpha_i (W_{xi} - Q_i \omega) \right\} \frac{D_i \omega}{100} \]

\[ \text{Min } Z_i^4 = \frac{\text{ESC}_i}{Q_i} \]

s.t.: \( Q_i \geq 0 \)
\( r_i \geq 0 \)

5.3.2 Solution Procedure to Solve Problem (R_i)

Using the same P_\lambda method discussed in Chapter 4, the MCMP is converted into a single objective problem:

Problem \((R_i^\lambda)\)

\[ \text{Min } Z_i^\lambda = \lambda_1 \left\{ C_i \left\{ \frac{Q_i}{2} + r_i - \mu_{Li} + \text{ESC}_i + \frac{\mu_{Li} D_i}{365} \right\} \right\} + \lambda_2 \frac{D_i}{Q_i} + \lambda_3 \left\{ F_{xi} + \alpha_i (W_{xi} - Q_i \omega) \right\} \frac{D_i \omega}{100} + \lambda_4 \frac{\text{ESC}_i}{Q_i} \]

s.t.: \( \sum_{k=1}^{3} \lambda_i^k = 1 \)
\( \lambda_i^k > 0 \ \forall i, k \)
\( Q_i \geq 0 \ \forall i \)
\( r_i \geq 0 \)

\( \lambda_i^k \) - Convex combination parameter or weight assigned for criterion k for retailer i.

For the weighted objective method to perform well, the individual criteria must be scaled. We utilize the following normalization scheme:

\[ Z_i^{kn} = \frac{Z_i^k - Z_i^{k*}}{Z_i^{k*} - Z_i^{k*}} \]
where:

\( Z_i^{kn} \) - Normalized value of criterion k for retailer i

\( Z_i^{k*} \) - Lowest value or ideal solution of criterion k for retailer i

\( Z_i^{k*} \) - Largest value or anti-ideal solution of criterion k for retailer i

This normalization scheme guarantees \( Z_i^{kn} \) to be between 0 and 1.

Then Problem \( (R_i^\lambda) \) can be rewritten as:

\[
\text{Min } Z_i^\lambda = \lambda_i^1 \left( C_i \frac{Q_i}{2} + r_i - \mu_{iLDi} + ESC_i + \frac{\mu_i D_i}{365} - Z_i^{k*} \right) + \lambda_i^2 \left( \frac{D_i}{Q_i} - Z_i^{2*} \right) + \lambda_i^3 \left( \frac{F_{xi} + \alpha_i (W_{xi} - Q_i \omega)}{100} - Z_i^{3*} \right) + \lambda_i^4 \left( \frac{ESC_i}{Q_i} - Z_i^{4*} \right)
\]

\[
\text{Min } Z_i^\lambda = \lambda_i^{1n} \left( C_i \frac{Q_i}{2} + r_i - \mu_{iLDi} + ESC_i + \frac{\mu_i D_i}{365} - Z_i^{k*} \right) + \lambda_i^{2n} \left( \frac{D_i}{Q_i} - Z_i^{2*} \right) + \lambda_i^{3n} \left( \frac{F_{xi} + \alpha_i (W_{xi} - Q_i \omega)}{100} - Z_i^{3*} \right) + \lambda_i^{4n} \left( \frac{ESC_i}{Q_i} - Z_i^{4*} \right)
\]

where \( \lambda_i^{kn} = \frac{\lambda_i^k}{Z_i^{k*} - Z_i^{k*}} \), the normalized/scaled weight for criterion k for retailer i.

Before we solve the \( P_\lambda \) problem, we have to make sure the objective functions are convex. We know that the first three criteria are convex. Also, the annual number of orders criterion is strictly convex. From Hadley and Whitin (Chapter 4, pp. 220-221), \( \frac{ESC_i}{Q_i} \) is also convex. Since the weighted sum of convex functions is a convex function, \( Z_i^\lambda \) is convex.

Computing the partial derivative of \( Z_i^\lambda \) with respect to \( Q_i \) and setting it to zero:

\[
\frac{\partial Z_i^\lambda}{\partial Q_i} = \lambda_i^{1n} \frac{C_i}{2} - \lambda_i^{2n} \frac{D_i}{Q_i^2} + \lambda_i^{3n} \left( -\frac{\alpha_i D_i \omega^2}{100} \right) - \lambda_i^{4n} \frac{ESC_i}{Q_i^2} = 0
\]
Let $\omega^2/100 = \bar{\omega}$

$$\frac{1}{Q_i} \left\{ \lambda_i^{2n}D_i + \lambda_i^{4n}\text{ESC}_i \right\} = \lambda_i^{2n} \frac{C_i}{2} - \lambda_i^{3n} \alpha_i D_i \bar{\omega}$$

$$\Rightarrow Q_i = \frac{\sqrt{2\left(\lambda_i^{2n}D_i + \lambda_i^{4n}\text{ESC}_i \right)}}{\lambda_i^{2n}C_i - 2\lambda_i^{3n} \alpha_i D_i \bar{\omega}} \quad (5.12)$$

Computing the partial derivative of $Z_i^2$ with respect to $r_i$ and setting it to zero:

$$\frac{\partial Z_i^2}{\partial r_i} = \lambda_i^{2n} C_i \frac{\partial s_i}{\partial r_i} + \lambda_i^{4n} \frac{1}{Q_i} \frac{\partial \text{ESC}_i}{\partial r_i} = 0$$

Using Eq. 5.1 and Eq. 5.5 for $\text{ESC}_i$ and $s_i$, respectively, and using the Leibnitz rule***:

$$\lambda_i^{2n} C_i \left\{ [1 - r_i f_{LD_i}(r)] - \left[ (1 - r_i f_{LD_i}(r_i) - F_{LD_i}(r_i)) \right] \right\} + \lambda_i^{4n} \frac{1}{Q_i} \left\{ \left[ \left. s_i \right|_{r_i} - (1 - r_i f_{LD_i}(r_i) - F_{LD_i}(r_i)) \right] \right\} = 0$$

$$\lambda_i^{2n} C_i \left\{ [1 - F_{LD_i}(r_i)] \right\} = \frac{\lambda_i^{2n} C_i}{\lambda_i^{2n} C_i + \frac{\lambda_i^{4n}}{Q_i}}$$

$$\left\{ [1 - F_{LD_i}(r_i)] \right\} = \frac{\lambda_i^{2n} C_i}{\lambda_i^{2n} C_i + \frac{\lambda_i^{4n}}{Q_i}} \quad (5.13)$$

In Eq. 5.13, $F_{LD}(r)$ is called the cycle service level (CSL) and is defined as the probability of not having a stockout in a cycle. Mathematically CSL is $\int_0^r f_{LD}(x)dx$. Since $\lambda_i^{2n}$ and $\lambda_i^{4n}$ are positive and the RHS of Eq. 5.13 is between 0 and 1, there will always be a solution. The LHS of Eq. 5.13 is nothing but the probability that lead time demand exceeds the reorder point. We can make the following observations from Eq. 5.13:

$$\lambda_i^{2n} C_i \rightarrow 1 \Rightarrow r_i \rightarrow 0$$

$$\left\{ \lambda_i^{2n} C_i + \frac{\lambda_i^{4n}}{Q_i} \right\} \rightarrow 1$$

*** Leibnitz rule: $\frac{d}{dr} \int_{\chi(r)}^{\psi(r)} f(x, r) dx = \int_{\chi(r)}^{\psi(r)} \frac{df(x, r)}{dr} dx + \frac{d\psi}{dr} f(\psi(r), r) - \frac{d\chi}{dr} f(\chi(r), r)$ (Nahmias, 2001)
Eq. 5.15 implies that when transportation costs are not included, the value of $Q$ decreases, and $r$ increases as a result.

Using Hadley and Whitin’s (1963, Chapter 4, pp. 169-172) procedure, Eq. 5.12 and Eq. 5.13 can be solved iteratively for different weight combinations to determine the values of $Q$ and $r$. For a particular weight combination, this iterative procedure is guaranteed to find the absolute minimum (if there is one) due to the convexity of the objective function. According to the necessary and sufficient condition of the $P_1$ problem, the solution will be properly efficient. Alternatively, optimization tools like Excel Solver, Lingo, etc. can be used directly to solve for $Q$ and $r$.

5.4 Evaluation of Lead Time Demand (LTD) Distributions

The results derived in Section 5.3 were for any general distribution of LTD. The hardest term to evaluate is the joint distribution of LTD. There are very few distributions of demand and lead time that give mathematically tractable expressions for the distribution of LTD.

Demand for fast moving products can be modeled using a normal distribution. According to Silver and Peterson (1985, Chapter 8, p. 330), a normal assumption can be used as long as the period demand, $\mu_D \geq 10$ units. If $\mu_D < 10$ units, they recommend the use of a Poisson distribution and hence can be used to model the demand of slow moving products.

There has been a lot of debate on the assumption of normality for the LTD. Often the Central Limit Theorem is used to justify this assumption. Once the LTD is assumed to follow a normal distribution, the mean and variance (under the assumption that the demand and lead time are independent) are computed using Eq. 5.16 and Eq. 5.17 (Ravindran et al., 1987, Chapter 8, p. 368):
\[
\mu_{LD} = \mu_D \mu_L \tag{5.16}
\]
\[
\sigma_{LD}^2 = \mu_L \sigma_D^2 + \mu_D^2 \sigma_L^2 \tag{5.17}
\]

Eppen and Martin (1988) and Tyworth (1991) state that specifying the LTD incorrectly as a normal distribution can lead to inaccurate estimates of safety stock. It can also be argued, however, that even though the deviations in safety stock costs may be large, it may not be significant when compared to cycle stock costs, ordering costs, etc.

We consider three cases:

- Case 1 is for a slow moving product where the demand follows a Poisson distribution and the lead time is a general discrete distribution. This implies that when a demand occurs, only one unit is ordered by the customer.
- Case 2 is again for slow moving products where the demand follows a geometric Poisson process (special case of Compound Poisson process) and the lead time is a general discrete distribution. In this case when a demand occurs, a customer can order more than one unit.
- Case 3 is for a fast moving product and is the normal approximation for Case 1 and Case 2. Here, the demand follows a normal distribution, while the lead time is a general discrete distribution.

### 5.4.1 Case 1: Demand-Poisson Distribution, Lead Time-General Discrete Distribution

The retailer demand is assumed to follow a Poisson distribution and is denoted as \( P(\mu_{Di}) \). The mean and variance of a Poisson distribution are equal to \( \mu_{Di} \). The lead time follows a discrete distribution given by \( \Pr\{L_i = j\} = p^i_j \). Given that \( L_i = j \), the conditional distribution of LTD still follows a Poisson distribution given by \( LD_i^j \sim P(\mu_{Di}) \). Thus, the conditional probability mass function (pmf) of the LTD (omitting the subscript \( i \)) is:

\[
p_{LD}^j(x) = e^{-\mu_j} \frac{(\mu_j)^x}{x!} \tag{5.18}
\]

Eq. 5.18 is used to determine the expected shortages/reorder cycle (ESC) and hence, the safety stock. ESC is determined in two steps. First, we determine the expected shortages by conditioning on the lead time:
\[
\text{ESC}_i^j = \sum_{r_i}^{\infty} (x - r_i)p_{LDi}(x) \quad (5.19)
\]

where \(\text{ESC}_i^j\) is expected shortages/reorder cycle given that the lead time is \(j\) periods.

\(p_{LDi}(\bullet)\) - pmf of the LTD for retailer \(i\) given that lead time is \(j\) periods.

Then, the total (unconditional) ESC is determined as:

\[
\text{ESC}_i = \sum_j \text{ESC}_i^j p_i^j \quad (5.20)
\]

The safety stock is determined in a similar way:

\[
s_i^j = \sum_0^{r_i} (r_i - x)p_{LDi}(x)
= r_i - j\mu_{Di} + \text{ESC}_i^j \quad \text{(From Eq. 5.5)} \quad (5.21)
\]

where \(s_i^j\) is expected safety stock given that the lead time is \(j\) periods.

The total (unconditional) safety stock for retailer \(i\) is:

\[
s_i = \sum_j s_i^j p_i^j \quad (5.22)
\]

To evaluate Eq. 5.19, the Poisson command in Excel can be used. The syntax is Poisson \((x, \text{mean, cumulative})\), where

- \(x\)- Number of events
- Cumulative- is a logical parameter and takes either True (1) or False (0). If true, it returns the cdf, else it returns the pmf.

Since the LTD is discrete, the order quantity and the reorder point must also be discrete. They can be derived by using the point-wise convexity property discussed in Chapters 3 and 4. Since the objective of the \(P_\lambda\) problem for the retailer is point-wise convex in \(Q\) and \(r\), we have to determine the smallest integer such that the first difference is negative. The expression for \(Q\) can be modified from Eq. 5.12.

\[
Q(Q + 1) \geq 2\left\{ \frac{2^D + \lambda^4 \text{ESC}}{\lambda^3 - 2\lambda^3 \alpha D \omega} \right\} \quad (5.23)
\]
Q can be determined by finding the smallest integer that satisfies the condition given by Eq. 5.23. However, the expression for r cannot be deduced from Eq. 5.13. To determine r, we start of by computing the first difference of $Z^\lambda$ with respect to r:

$$\Delta Z^\lambda(Q, r + 1) = Z^\lambda(Q, r + 1) - Z^\lambda(Q, r) \geq 0$$

$$\lambda^n C(r + 1 - \mu_{LD} + ESC_{r+1}) + \frac{\lambda^4n}{Q}ESC_{r+1} - \lambda^n C(r - \mu_{LD} + ESC_r) - \frac{\lambda^4n}{Q}ESC_r \geq 0$$

$$\lambda^n C + \left(\lambda^n C + \frac{\lambda^4n}{Q}\right)[ESC_{r+1} - ESC_r] \geq 0 \quad (5.24)$$

r can be determined by finding the smallest integer that satisfies Eq. 5.24. Since Eq. 5.24 is more cumbersome when compared to Eq. 5.13 for determining r, a continuous approximation for the discrete case can be used. Hadley and Whitin (1963, Chapter 4, p. 190) recommend that Q can be approximated by using Eq. 5.12 when $Q \geq 10$, and r can be approximated using Eq. 5.13 when $\mu_{LD} \geq 25$.

**5.4.2 Case 2: Demand-Compound Poisson Distribution, Lead Time-General Discrete Distribution**

As mentioned before a Poisson distribution can be used to model the demand of slow moving products. When demand follows a Poisson distribution, demand occurs one at a time, and the variance to mean ratio is one. However, a customer going to a retailer may buy several quantities of the same product. Further, Nahmias and Smith (1994) assert that the variance to mean ratio is often greater than 1. A compound Poisson distribution can be used to model these two conditions.

A compound Poisson process is a stochastic process wherein demand occurs according to a Poisson distribution and the number of units demanded per demand occurrence is a positive discrete random variable. The total number of units demanded in a time period t is given by:

$$X(t) = \sum_{i=1}^{N(t)} Y_i$$

where,
N(t) – Number of demand events (i.e., orders) with mean \( \mu \) in period \( t \) and is a Poisson process.

\( Y_i \) – Number of units ordered by customer \( i \). \( Y_i \) is a discrete i.i.d random variable and is independent of \( N(t) \).

\( E(Y_i) \) – Average number of units ordered by any customer. Let \( E(Y_i) = E(Y) \) \( \forall i \).

A good discussion of this type of stochastic process is found in Ross (2000, Chapter 5, pp. 289-294), Sherbrooke (1968a) and Adelson (1966). Some important properties are listed next:

1. Given \( w \) customers during time period \( t \), let \( p^{(w)}(x) \) be the probability that a total of \( x \) units are demanded by \( w \) customers or the \( w \)-fold convolution of \( p(x) \) and is computed as:

\[
p^{(w)}(x) = \Pr \{ Y_1 + Y_2 + \ldots + Y_w = x \}
\]

Then the probability that \( x \) units are ordered (Probability mass function) is given by:

\[
\Pr[X(t) = x] = \sum_{w=1}^{x} \frac{\mu^w e^{-\mu}}{w!} p^{(w)}(x)
\]

\[
\Pr[X(t) = 0] = e^{-\mu t}
\]

2. The mean and variance of \( X(t) \) is:

\[
E[X(t)] = \mu t E[Y]
\]

\[
Var[X(t)] = \mu t E[Y^2]
\]

3. The sum of independent compound Poisson distributions is itself a compound Poisson distribution.

The difficulty in using a compound Poisson process lies in the evaluation of \( p^{(w)}(x) \). A specific case is the geometric Poisson process wherein the \( Y_i \)'s are geometric i.i.d.'s has been used in inventory theory (see Hadley and Whitin, 1963, Chapter 3, pp. 112-114 and Sherbrooke, 1968a). Such a stochastic process is also called a Stuttering Poisson process. Assume that a customer entering a location orders one unit. If \( \rho \) is the probability that the same customer orders another unit, then the probability that \( y-1 \) units are demanded by that customer is:
\[ \rho^{y-1}(1-\rho) \forall y \geq 1 \]  
(5.29)

The first and the second moment of the pmf given by Eq. 5.29 are:

\[ E[Y] = \frac{1}{1-\rho} \]  
(5.30)

\[ E[Y^2] = \frac{1+\rho}{(1-\rho)^2} \]  
(5.31)

Substituting Eq. 5.30 and Eq. 5.31 into Eq. 5.27 and Eq. 5.28, respectively:

\[ E[X(t)] = \frac{\mu t}{1-\rho} \]  
(5.32)

\[ \text{Var}[X(t)] = \frac{\mu t}{(1-\rho)^2} \]  
(5.33)

Using the result that the \( w \)-fold convolution of the geometric distribution with parameter \( \rho \) follows a negative binomial distribution (Hadley and Whitin, 1963, Chapter 3, p. 114), Eq. 5.25 can be rewritten as:

\[ \Pr[X(t) = x] = \sum_{w=1}^{x} \frac{\mu^w e^{-\mu t}}{w!} \left( \frac{x-1}{w-1} \right) \rho^{x-w}(1-\rho)^w \]  
(5.34)

Eq. 5.34 and Eq. 5.26 represent the probability mass function of the geometric Poisson process.

In Case 1, we used the result that the conditional distribution of the LTD is Poisson for a Poisson distributed demand and lead time as a discrete random variable. This is also true in the case of Normal, Gamma and Exponential demand distributions (Tyworth and Zeng, 1998); however, no such result exists for a geometric Poisson process. To prove this result we have to first determine the mean and variance of the sum of independent geometric Poisson processes. We know that the convolution of independent geometric Poisson process is a geometric Poisson process. To determine the mean and variance of the resulting convolution we have the following lemma:

**Lemma 5.1**: The sum of \( n \) independent Geometric Poisson processes has a mean and variance that is equal to the sum of the individual means and variances, respectively.

**Proof**: The moment generating function of a random variable \( X \) is defined as:

\[ This implies that whenever an order occurs, at least one unit is demanded.\]
\[ \phi(s) = E[e^{ts}] = \sum_s e^{ts} p(s) \]

The first and second moments of \( X \) can then be determined by computing the first and second differentials:

\[ E[X] = \phi'(0) \]
\[ E[X^2] = \phi''(0) \]

The variance is computed as:

\[ \text{Var}[X] = E[X^2] - \{E[X]\}^2 \]

The moment generating function of a compound Poisson process

\[ \phi(s) = \exp\{-\mu t (1 - \phi_Y(s))\} \]

(5.35)

\( \phi_Y(s) \) is the moment generating function of number of units demanded defined by random variable \( Y \). Since \( Y \) is geometrically distributed, its moment generating function is given by:

\[ \phi_Y(s) = \frac{(1-\rho)e^s}{1-\rho e^s} \]

(5.36)

Substituting Eq. 5.36 into Eq. 5.35, we get the moment generating function of the geometric compound Poisson process as:

\[ \phi(s) = \exp\left\{-\mu t \left( \frac{1-e^s}{1-\rho e^s} \right) \right\} \]

(5.37)

Let \( X_1(t) \) and \( X_2(t) \) be two independent geometric Poisson processes and \( X(t) \) be the convolution of \( X_1(t) \) and \( X_2(t) \), i.e.,

\[ X(t) = X_1(t) + X_2(t) \]

The moment generating function of \( X(t) \) is the product of the moment generating functions of \( X_1(t) \) and \( X_2(t) \):

\[ \phi(s) = \phi_1(s)\phi_2(s) \]

\[ = \exp\left\{-\mu_1 t \left( \frac{1-e^s}{1-\rho_1 e^s} \right) \right\} \exp\left\{-\mu_2 t \left( \frac{1-e^s}{1-\rho_2 e^s} \right) \right\} \]

\[ = \exp\left\{-\mu_1 t \left( \frac{1-e^s}{1-\rho_1 e^s} \right) - \mu_2 t \left( \frac{1-e^s}{1-\rho_2 e^s} \right) \right\} \]
\[ \phi'(s) = \exp\left\{ -\mu_1 t \left( \frac{1-e^s}{1-\rho_1 e^s} \right) - \mu_2 t \left( \frac{1-e^s}{1-\rho_2 e^s} \right) \right\} \frac{d}{ds} \left\{ -\mu_1 t \left( \frac{1-e^s}{1-\rho_1 e^s} \right) - \mu_2 t \left( \frac{1-e^s}{1-\rho_2 e^s} \right) \right\} \]

Consider \( \frac{d}{ds} \left\{ -\mu_1 t \left( \frac{1-e^s}{1-\rho_1 e^s} \right) \right\} \)

\[ = -\mu_1 t \left( \frac{(1-\rho_1 e^s)(-e^s) - (1-e^s)(-\rho_1 e^s)}{(1-\rho_1 e^s)^2} \right) \]

\[ = \mu_1 t \left( \frac{e^s(1-\rho_1)}{(1-\rho_1 e^s)^2} \right) \]

Similarly, \( \frac{d}{ds} \left\{ -\mu_2 t \left( \frac{1-e^s}{1-\rho_2 e^s} \right) \right\} = \mu_2 t \left( \frac{e^s(1-\rho_2)}{(1-\rho_2 e^s)^2} \right) \)

\[ \phi'(s) = \exp\left\{ -\mu_1 t \left( \frac{1-e^s}{1-\rho_1 e^s} \right) - \mu_2 t \left( \frac{1-e^s}{1-\rho_2 e^s} \right) \right\} \left\{ \mu_1 t \left( \frac{e^s(1-\rho_1)}{(1-\rho_1 e^s)^2} \right) + \mu_2 t \left( \frac{e^s(1-\rho_2)}{(1-\rho_2 e^s)^2} \right) \right\} \]

\[ \phi'(0) = E[X(t)] = \frac{\mu_1 t}{1-\rho_1} + \frac{\mu_2 t}{1-\rho_2} \]

\[ . . . E[X(t)] = E[X_1(t)] + E[X_2(t)] \quad \text{(From Eq. 5.32)} \quad (5.38) \]

\( \phi'(s) \) is rewritten as:

\[ \phi'(s) = \exp\left\{ -\mu_1 t \left( \frac{1-e^s}{1-\rho_1 e^s} \right) - \mu_2 t \left( \frac{1-e^s}{1-\rho_2 e^s} \right) \right\} \left\{ \mu_1 t \left( \frac{e^s(1-\rho_1)}{(1-\rho_1 e^s)^2} \right) + \right\} \]

\[ \exp\left\{ -\mu_1 t \left( \frac{1-e^s}{1-\rho_1 e^s} \right) - \mu_2 t \left( \frac{1-e^s}{1-\rho_2 e^s} \right) \right\} \mu_1 t \left( \frac{e^s(1-\rho_1)}{(1-\rho_1 e^s)^2} \right) \]

Differentiating the first term with respect to \( s \):

\[ \exp\left\{ -\mu_1 t \left( \frac{1-e^s}{1-\rho_1 e^s} \right) - \mu_2 t \left( \frac{1-e^s}{1-\rho_2 e^s} \right) \right\} \mu_1 t(1-\rho_1) \left( \frac{1-\rho_1 e^s}{1-\rho_1 e^s} \right)^2 + \frac{2e^s(1-\rho_1 e^s)(1-\rho_1 e^s)}{(1-\rho_1 e^s)^4} \]

\[ \mu_1 t \left( \frac{e^s(1-\rho_1)}{(1-\rho_1 e^s)^2} \right) \exp\left\{ -\mu_1 t \left( \frac{1-e^s}{1-\rho_1 e^s} \right) - \mu_2 t \left( \frac{1-e^s}{1-\rho_2 e^s} \right) \right\} \left\{ \mu_1 t \left( \frac{e^s(1-\rho_1)}{(1-\rho_1 e^s)^2} \right) + \mu_2 t \left( \frac{e^s(1-\rho_2)}{(1-\rho_2 e^s)^2} \right) \right\} \]

Putting \( s=0 \):
\[
\mu_1 t (1 - \rho_1) \left( \frac{(1 - \rho_1)^2 - 2(1 - \rho_1)(-\rho_1)}{(1 - \rho_1)^3} \right) + \frac{\mu_1 t}{1 - \rho_1} \left( \frac{\mu_1 t}{1 - \rho_1} + \frac{\mu_2 t}{1 - \rho_2} \right) \\
= \frac{\mu_1 t (1 + \rho_1)}{(1 - \rho_1)^2} + \left( \frac{\mu_1 t}{1 - \rho_1} \right)^2 + \frac{\mu_1 t \mu_2 t}{(1 - \rho_1)(1 - \rho_2)} \\
= \frac{\mu_1 t}{(1 - \rho_1)^2} (1 + \rho_1 + \mu_1 t) + \frac{\mu_1 t \mu_2 t}{(1 - \rho_1)(1 - \rho_2)}
\]

Similarly, differentiating the second term and setting \( s=0 \) leads to:
\[
\frac{\mu_2 t}{(1 - \rho_2)^2} (1 + \rho_2 + \mu_2 t) + \frac{\mu_1 t \mu_2 t}{(1 - \rho_1)(1 - \rho_2)}
\]
\[
\therefore E[X(t)^2] = \frac{\mu_1 t}{(1 - \rho_1)^2} (1 + \rho_1 + \mu_1 t) + \frac{\mu_1 t \mu_2 t}{(1 - \rho_1)(1 - \rho_2)} + \frac{\mu_2 t}{(1 - \rho_2)^2} (1 + \rho_2 + \mu_2 t) + \frac{\mu_1 t \mu_2 t}{(1 - \rho_1)(1 - \rho_2)}
\]

Eq. 5.39 gives the second moment of \( X(t) \). The variance of \( X(t) \) is computed as:
\[
\text{Var}(X[t]) = E[X(t)^2] - (E[X(t)])^2 \\
= \frac{\mu_1 t}{(1 - \rho_1)^2} (1 + \rho_1 + \mu_1 t) - \left( \frac{\mu_1 t}{1 - \rho_1} \right)^2 + \frac{\mu_2 t}{(1 - \rho_2)^2} (1 + \rho_2 + \mu_2 t) - \left( \frac{\mu_2 t}{1 - \rho_2} \right)^2
\]
\[
\text{Var}[X(t)] = \mu_1 t \frac{1 + \rho_1}{(1 - \rho_1)^2} + \mu_2 t \frac{1 + \rho_2}{(1 - \rho_2)^2}
\]
\[
\therefore \text{Var}[X(t)] = \text{Var}[X_1(t)] + \text{Var}[X_2(t)] \quad (\text{From Eq. 5.33}) \quad (5.40)
\]

Eq. 5.38 and Eq. 5.40 can be extended to any number of geometric Poisson processes. Thus, the sum of \( n \) independent geometric Poisson processes is a geometric Poisson process whose mean equals the sum of the individual means and variance equals the sum of the individual variances.

**Corollary 5.1:** If the individual geometric Poisson processes are identical then the resulting geometric compound process is the \( n \)-fold convolution, and hence Eq. 5.38 and Eq. 5.40 simplify to:
\[
E[X(t)] = n E[X_1(t)] \quad (5.41)
\]
\[
\text{Var}[X(t)] = n \text{Var}[X_1(t)] \quad (5.42)
\]
**Theorem 5.1:** If the demand process follows a geometric Poisson process with parameters \( \left( \frac{\mu}{1 - \rho}, \frac{\mu(1 + \rho)}{(1 - \rho)^2} \right) \) and the lead time follows a discrete distribution given by \( \Pr\{L = j\} = p^j \), then the conditional distribution of the lead time demand follows a geometric Poisson distribution with mean and variance \( \left( \frac{j\mu}{1 - \rho}, \frac{j\mu(1 + \rho)}{(1 - \rho)^2} \right) \).

**Proof:** If the lead time follows a discrete distribution given by \( \Pr\{L = j\} = p^j \) and demands in different periods are independent from one another, the probability mass function of lead time demand given that lead time is \( j \) time units from Feller (1957, Vol.1, Chapter 12, p. 268) is:

\[
p^{(j)}_{LD}(x) = p^{(j)}_{LD}(x)
\]

\( p^{(j)}_{LD}(\bullet) \) is the probability mass function of the \( j \)-fold convolution of demand during one unit of lead time. Since the demand during unit of lead time follows a geometric Poisson distribution, the \( j \)-fold convolution from corollary 5.1 has mean and variance given by Eq. 5.41 and Eq. 5.42. Thus, we have proved that if the demand process follows a geometric Poisson distribution and the lead time follows a discrete distribution, the conditional distribution of LTD follows a geometric Poisson distribution with mean and variance \( \left( \frac{j\mu}{1 - \rho}, \frac{j\mu(1 + \rho)}{(1 - \rho)^2} \right) \).

\[
\sum_{w=1}^{L} \binom{L}{w} p^w \left( \frac{m_j}{w!} \right) \left( \frac{x-1}{w-1} \right) \rho^{x-y} (1-\rho)^y
\]

(5.43)

Unfortunately, Excel does not have a built-in function to evaluate Eq. 5.43. However, tables like the one in Sherbrooke (1968a) can be used to evaluate Eq. 5.43 and hence, Eq. 5.19, Eq. 5.20, Eq. 5.21 and Eq. 5.22.
5.4.3 Case 3: Demand-Normal Distribution, Lead Time-General Discrete Distribution

This case can be used when demand is normally distributed as $N(\mu_{D_i}, \sigma_{D_i}^2)$ or as approximations to the previous two cases. For large values of $\mu_{D_i}$, the Poisson and compound Poisson distributions can be accurately approximated as a normal distribution. The lead time follows a discrete distribution given by $\Pr[L_i = j] = p_i^j$. Then the conditional distribution of LTD still follows a normal distribution given by $LD_i \sim N(j\mu_{D_i}, j\sigma_{D_i}^2)$ (Feller, 1957, Vol.1, Chapter 12, p. 268). The conditional pdf is:

$$f_{LD}^j(x) = \frac{1}{\sqrt{2\pi j\sigma_D}} e^{-\frac{1}{2} \left( \frac{x - j\mu_D}{\sqrt{j\sigma_D}} \right)^2} \quad (5.44)$$

The expression of the conditional ESC from Eq. 5.2 is $\int_r^\infty xf_{LD}^j(x)dx - r\{1 - F_{LD}^j(r)\}$, where $F_{LD}^j(\bullet)$ is the cdf of the lead time demand of a retailer given that lead time is $j$ periods.

Substituting Eq. 5.44 into $\int_r^\infty xf_{LD}^j(x)dx$:

$$\int_r^\infty xf_{LD}^j(x)dx = \frac{1}{\sqrt{2\pi j\sigma_D}} \int_r^\infty xe^{-\frac{1}{2} \left( \frac{x - j\mu_D}{\sqrt{j\sigma_D}} \right)^2} dx$$

Put $z = \frac{x - j\mu_D}{\sqrt{j\sigma_D}} \Rightarrow dx = dz\sqrt{j\sigma_D}$

$$\int_r^\infty xf_{LD}^j(x)dx = \frac{1}{\sqrt{2\pi j\sigma_D}} \int_r^\infty (z\sqrt{j\sigma_D} + j\mu_D)e^{-\frac{1}{2} z^2} dz$$

$$= \frac{1}{\sqrt{2\pi}} \left\{ \int_r^{\infty} z\sqrt{j\sigma_D} e^{-\frac{1}{2} z^2} dz + \int_r^{\infty} j\mu_D e^{-\frac{1}{2} z^2} dz \right\}$$

Consider the first part of the integral:
\[
\int_{r-j\mu_{D}}^{\infty} z \sqrt{j\sigma_{LD}} e^{\frac{-z^2}{2}} dz
\]

Put \( \frac{z^2}{2} = t \Rightarrow dz = \frac{dt}{z} \)

\[
\therefore \int_{r-j\mu_{D}}^{\infty} z \sqrt{j\sigma_{LD}} e^{\frac{-z^2}{2}} dz = \int e^{-t} dt = e^{-\frac{1}{2}(\frac{r-j\mu_{D}}{\sqrt{j\sigma_{D}}})^2}
\]

\[
\therefore \int_{r}^{\infty} x f_{LD}^{j}(x) dx = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{r-j\mu_{D}}{\sqrt{j\sigma_{D}}})^2} \sqrt{j\sigma_{D}} + j\mu_{D} \int_{r-j\mu_{D}}^{\infty} e^{\frac{-z^2}{2}} dz
\]

\[
= f_{LD}^{j} \left( \frac{r-j\mu_{D}}{\sqrt{j\sigma_{D}}} \right) \sqrt{j\sigma_{D}} + j\mu_{D} \left\{ 1 - F_{LD}^{j} \left( \frac{r-j\mu_{D}}{\sqrt{j\sigma_{D}}} \right) \right\}
\]

(5.45)

Plugging Eq. 5.45 into \( \int_{r}^{\infty} x f_{LD}^{j}(x) dx - r \left\{ 1 - F_{LD}^{j}(r) \right\} \), the conditional ESC is:

\[
ESC_{i}^{j} = f_{LD}^{j} \left( \frac{r_{i} - j\mu_{Di}}{\sqrt{j\sigma_{Di}}} \right) \sqrt{j\sigma_{Di}} + (j\mu_{Di} - r_{i}) \left\{ 1 - F_{LD}^{j} \left( \frac{r_{i} - j\mu_{Di}}{\sqrt{j\sigma_{Di}}} \right) \right\}
\]

(5.46)

Then the total ESC is computed by substituting Eq. 5.46 into \( \sum_{j} ESC_{i}^{j} p_{i}^{j} \) (see Eq. 5.20). The safety stock calculations are the same as in the previous cases. The first step is to determine the conditional safety stock as \( s_{i}^{j} = r_{i} - j\mu_{Di} + ESC_{i}^{j} \) (see Eq. 5.21). Then, the safety stock is determined as: \( s_{i} = \sum_{j} s_{i}^{j} p_{i}^{j} \) (see Eq. 5.22)

Eq. 5.46 can be evaluated by using the Normdist function in Excel. The syntax is: NORMDIST (x, mean, standard deviation, cumulative) where cumulative is a logical parameter and takes either True (1) or False (0). If true, it returns the cdf, else it returns the pdf. If the mean is 0 and standard deviation is 1, then the function returns the standard normal distribution.
Note: The model in the previous sections assumed that the lead time was equal to the transit time. Researchers, e.g., Tyworth and Zeng (1998) and Ganeshan (1999), have modeled the lead time as the sum of the transit time and a fixed ordering time. The fixed ordering time can be incorporated as follows. Let $P$ be the fixed order processing time. Then, the mean of the conditional distribution of LTD is shifted to the right by $P$, while the variance remains the same. For example, for Case 3, the distribution of LTD given the transit time is $j$ periods is $LD^j \sim N((j + P) \mu_D, j \sigma_D^2)$. The rest of the procedure is the same as in Section 5.4.3.

5.5 Warehouse’s Problem

As in Chapter 4, the warehouse uses the information of the individual retailers to formulate and hence, determine its ordering policy. The warehouse, like the retailers, follows a continuous review policy. Excess demand at the warehouse is treated as lost sales. This assumption was validated by a Fortune 500 company. According to the company, it does not backorder since none of its customers are willing to wait. The warehouse is assumed to fill the retailers’ orders FIFO (First in First Out). If the warehouse does not have enough to meet the demand of a particular retailer, the entire order is lost. It is assumed that the concerned retailer will purchase the order from another supplier. The lead time of the warehouse is a discrete random variable. The warehouse has the same criteria as the retailers.

Before we formulate the warehouse’s problem, it is necessary to discuss the demand process at the warehouse and hence, the LTD distribution. The demand process at the warehouse is very complicated as it depends on the inventory level of the individual retailers. Hence, the demand faced by the warehouse is non-stationary.

Deuermeyer and Schwarz (1981), under the assumption of external identical Poisson demands and complete backorders, characterize the demand process at the warehouse as an approximate renewal process since the demand process is non-stationary. After running several simulations, they concluded that the non-stationarity is not significant. Also, they claim that this approximation is accurate if the warehouse serves a large number of retailers ($N$). For $N \geq 20$, the above approximation can be safely used (Ganeshan, 1999).
Andersson and Melchiors (2001) explain that even for the one-for-one replenishment (unit order quantity) policy at the retailers, the demand at the warehouse is not Poisson under the lost sales assumption. They also approximate the demand process at the warehouse as a stationary Poisson process.

Since a retailer follows a continuous review policy and the external demands is Poisson random variables, the time between orders, $T_i$, is i.i.d. and follows an *Erlang-$Q_i$* distribution (Hadley and Whitin, 1963, Chapter 3, p. 112 and Ross, 2000, Chapter 5, pp. 262-263) whose mean and variance is given by:

\[
\mu_{T_i} = \frac{Q_i}{\mu_{D_i}} \tag{5.47}
\]

\[
\sigma_{T_i}^2 = \frac{Q_i^2}{\mu_{D_i}^2} \tag{5.48}
\]

where $\mu_{T_i}$ is mean time between orders for retailer $i$, and $\sigma_{T_i}^2$ is variance of time between orders for retailer $i$.

Let $N_i(t)$ be the number of orders placed by retailer ‘$i$’ during period $t$. Then, $N_i(t)$ is a renewal process. Let $N_W(t)$ be the total number of orders received by the warehouse during period $t$. $N_W(t)$ is not a renewal process as the order arrival process of the retailers is dependent on the inventory level at the retailers. Hence, the time between the retailer orders received by the warehouse is non-identical. In general, $N_W(t)$ is a renewal process only when each retailer’s order quantity is one. Assuming that the non-stationarity is not significant, $N_W(t)$ can be approximated as a renewal process and is given by:

\[
N_w(t) = \sum_{i=1}^{N} N_i(t) \tag{5.49}
\]

where $N$ is the number of retailers.

Determining the exact distribution of $N_W(t)$ is difficult and time consuming. Approximations have been given by Deuermeyer and Schwarz (1981) and Svoronos and Zipkin (1987) for the identical retailer case. We approximate this renewal process by applying the central limit theorem for renewal processes (Ross, 2000, Chapter 7, p. 377). Using this theorem, the renewal processes $N_i(t)$ can be approximated as a normal distribution for large values of $t$: 
Substituting Eq. 5.47 and Eq. 5.48 into Eq. 5.50:

\[ N_i(t) \sim N \left( t \frac{\mu_{Di}}{Q_i}, t \frac{\sigma_{Di}}{Q_i} \right) \]  

(5.51)

Eq. 5.51 is the distribution of the number of orders placed by each retailer. The distribution of the retailer’s orders (demands) is determined by scaling Eq. 5.51 by a factor \( Q_i \) and is given as:

\[ N(t\mu_{Di}, t\mu_{Di}) \]  

(5.52)

Then the distribution of the total number of orders received by the warehouse during time period \( t \) (using central limit theorem) from Eq. 5.49 and Eq. 5.51 is:

\[ N_W(t) \sim N \left( t \sum_{i=1}^{N} \frac{\mu_{Di}}{Q_i}, t \sum_{i=1}^{N} \frac{\mu_{Di}}{Q_i} \right) \]  

(5.53)

Similarly, using Eq. 5.52 the distribution of all the retailers’ orders (demands) during time period \( t \) is given by:

\[ N \left( t \sum_{i=1}^{N} \mu_{Di}, t \sum_{i=1}^{N} \mu_{Di} \right) \]  

(5.54)

Thus, the distribution of period demand at the warehouse is characterized as a normal distribution with mean and variance give by \( \mu_{DW} = \sum_{i=1}^{N} \mu_{Di} \) and \( \sigma_{DW}^2 = \sum_{i=1}^{N} \mu_{Di}^2 \).

The distribution of demand during the lead time of the warehouse can be characterized as in Case 3. If the lead time at the warehouse is a discrete distribution given by \( \Pr[L_W = j] = p_{Wj} \), then the conditional distribution of LTD at the warehouse given the lead time is \( j \) time units is distributed as \( LTD_W^j \sim N(j\mu_{DW}, j\sigma_{DW}^2) \). Now we can formulate the warehouse’s MCMP.

Note: The demand distribution at the warehouse is characterized as a normal distribution based on the assumption that the external demands are Poisson. We believe that the above method can be used for external demands following a geometric Poisson process.
provided that the probability of a customer ordering more than one unit is less. Since the Poisson and geometric Poisson random variables can be approximated as a normal distribution using the central limit theorem, the case of retailers facing normal demands can also be used.

Warehouse’s Criteria

Criterion 1: Capital Invested in Inventory

The capital invested in inventory consists of 2 components:

- **Capital invested in inventory at warehouse**: As in the retailer’s problem, the expected ending inventory level at the warehouse is its safety stock $w_s$. Then, the expected inventory level when an order is received is $q_w + s_w$. The average inventory at the warehouse is approximated as the average of the beginning and ending inventory levels, i.e., $\frac{q_w}{2} + s_w$. Hence, the capital invested in inventory at the warehouse is:

$$C_w \left\{ \frac{q_w}{2} + s_w \right\}$$  \hspace{1cm} (5.55)

- **Capital invested in in-transit inventory**: The capital invested in in-transit inventory is:

$$C_w \left\{ \frac{\mu_{lw}d_w}{365} \right\}$$  \hspace{1cm} (5.56)

The total capital invested in inventory at the warehouse is given by the sum of Eq. 5.55 and Eq. 5.56:

$$C_w \left\{ \frac{q_w}{2} + s_w + \frac{\mu_{lw}d_w}{365} \right\}$$  \hspace{1cm} (5.57)

Criterion 2: Number of Orders Placed Annually

The annual number of orders at the warehouse under the assumption that the average time spent by the warehouse in a stockout situation is negligible is:

$$\frac{d_w}{q_w}$$  \hspace{1cm} (5.58)
Criterion 3: Annual Transportation Costs

Since it is assumed that the warehouse’s shipments are most likely going to be TL, the inverse function is used to model transportation costs. The annual transportation cost is given by:

\[
\frac{F_{xW} W_{xW}}{Q_W} \frac{D_W}{100}
\]  

(5.59)

Criterion 4: Fill Rate

The fill rate at the warehouse is given as:

\[
FR_W = 1 - \frac{ESC_W}{Q_W}
\]

where \(ESC_W\) is expected shortages/order cycle at the warehouse.

Maximizing fill rate is equivalent to minimizing the fraction of demand not met. Hence, the fourth criterion is to minimize:

\[
\frac{ESC_W}{Q_W}
\]  

(5.60)

Since the conditional LTD has been characterized as a normal distribution, we can determine the ESC given the lead time is \(j\) units as in Case 3:

\[
ESC^j_{W} = f^j_{LDW} \left( \frac{r_{W} - j\mu_{DW}}{\sqrt{j\sigma_{DW}}} \right) \sqrt{j\sigma_{DW}} + \left( j\mu_{DW} - r_{W} \right) \left\{ 1 - F^j_{LDW} \left( \frac{r_{W} - j\mu_{DW}}{\sqrt{j\sigma_{DW}}} \right) \right\}
\]  

(5.61)

The total number of shortages at the warehouse over the range of lead time values is:

\[
ESC_W = \sum_j ESC^j_{W} p^j_{W}
\]  

(5.62)

The safety stock at the warehouse given that the lead time is \(j\) units is:

\[
s^j_{W} = r_{W} - j\mu_{D} + ESC^j_{W}
\]  

(5.63)

The safety stock at the warehouse over the range of lead time values is:

\[
s_{W} = \sum_j s^j_{W} p^j_{W}
\]  

(5.64)
where:

\( f_{LDW}^j (\bullet) \) - pdf of the lead time demand for the warehouse given that lead time is \( j \) periods.

\( F_{LDW}^j (\bullet) \) - cdf of the lead time demand for the warehouse given that lead time is \( j \) periods.

### 5.5.1 Warehouse’s Multi-Criteria Problem

The warehouse’s optimization problem is to minimize Eq. 5.57, Eq. 5.58, Eq. 5.59 and Eq. 5.60 simultaneously subjected to the constraint that its ordering quantity is a continuous multiple, \( \delta \) of the sum of the ordering quantities of all the retailers.

**Problem (W)**

\[
\text{Min } Z_W^1 = C_W \left\{ \frac{Q_W}{2} + r_W - \mu_{LDW} + \text{ESC}_W + \frac{\mu_{LW} D_W}{365} \right\}
\]

\[
\text{Min } Z_W^2 = \frac{D_W}{Q_W}
\]

\[
\text{Min } Z_W^3 = \frac{F_{xW} W_{xW} D_W}{Q_W 100}
\]

\[
\text{Min } Z_W^4 = \frac{\text{ESC}_W}{Q_W}
\]

s.t.:

\[
Q_W = \delta \sum_{i=1}^{N} Q_i
\]

\[
Q_W \geq 0
\]

\[
r_W \geq 0
\]

\[
\delta \geq 1
\]

### 5.5.2 Solution Procedure to Solve Problem (W)

The P_\lambda method is again used to solve the warehouse’s MCMP.

**Problem (W^4)**

\[
\text{Min } Z_W^\lambda = \lambda_W^1 \left\{ C_W \left\{ \frac{Q_W}{2} + r_W - \mu_{LDW} + \text{ESC}_W + \frac{\mu_{LW} D_W}{365} \right\} \right\} + \lambda_W^2 \frac{D_W}{Q_W}
\]

\[
+ \lambda_W^3 \frac{F_{xW} W_{xW} D_W}{Q_W 100} + \lambda_W^4 \frac{\text{ESC}_W}{Q_W}
\]
s.t.: \[ Q_w = \delta \sum_{i=1}^{N} Q_i \]
\[ \sum_{k=1}^{3} \lambda_k = 1 \]
\[ \lambda_k > 0 \ \forall k \]
\[ Q_w \geq 0 \]
\[ r_w \geq 0 \]
\[ \delta \geq 1 \]

where \( \lambda_k \) is the convex combination parameter or weight assigned for criterion \( k \) for warehouse.

As in Problem (R\(^k\)_\(1\)), the individual criterion must be normalized. Using the same scheme, Problem (W\(^k\)) can be reformulated as:

\[
\begin{align*}
\text{Min } Z_W^k &= \lambda_{W}^1 \left( C_W \left( \frac{Q_w}{2} + r_w - \mu_{LDW} + \text{ESC}_W + \frac{\mu_{LW} D_w}{365} \right) - Z_W^* \right) + \lambda_{W}^2 \left( \frac{D_w}{Q_w} - Z_W^* \right) \\
&+ \lambda_{W}^3 \left( \frac{F_{W} W_{xW} D_w}{Q_w} - Z_W^* \right) + \lambda_{W}^4 \left( \frac{\text{ESC}_W}{Q_w} - Z_W^* \right)
\end{align*}
\]

where:

\[ \lambda_{W}^{kn} = \frac{\lambda_{W}^{k}}{Z_W^{*k} - Z_W^*} \]

\( Z_W^* \) is the lowest or ideal value of criterion \( k \) for warehouse

\( Z_W^{*k} \) is the largest or anti-ideal value of criterion \( k \) for warehouse

Rewriting the objective function of Problem (W\(^k\)) in terms of \( \delta \):
The expression for determining reorder point is similar to that of Eq. 5.13 and can be modified for the warehouse as:

\[
\left\{ \text{1 - } F_{\text{LTD}}(r_W) \right\} &= \frac{\lambda^1_W C_W}{\lambda^1_W C_W + \frac{\lambda^4_W}{Q_W}} \quad (5.66)
\]

Eq. 5.65 and Eq. 5.66 can be solved iteratively as described for the retailer’s problem or optimization tools like Excel Solver, LINGO, etc. can be used to directly determine \(Q_W\) and \(r_W\).
5.6 Backorder Assumption

In this section we discuss the modification needed to incorporate the backorder assumption. To distinguish from the variables of the lost sales case, a superscript b is used.

5.6.1 Retailer

The optimization problem is essentially the same as given in Section 5.3.1, except for the expression of safety stock. In the backorders case safety stock is defined as the net inventory when an order arrives. The safety stock, $s_i^b$, for the backorders case is:

$$s_i^b = \int_0^\infty \left(t_i^b - x\right) f_{LDi}(x) dx = t_i^b - \mu_{LDi}$$

The order quantity expression is still $Q_i = \{\}^{\omega_{\alpha_l} \lambda - \lambda_i}_i^{n_3}$ while the reorder point formula changes to:

$$\left\{1 - F_{LDi}(t_i^b)\right\} = \frac{\lambda^{ln} Q_i^b C_i}{\lambda_i^{4n}} \quad (5.67)$$

Comparing Eq. 5.67 and Eq. 5.15 given by $\{1 - F_{LDi}(t_i)\} = \frac{\lambda_i^{ln} C_i}{\lambda_i^{ln} C_i + \lambda_i^{4n}}$, we can conclude that the reorder point in the lost sales is greater than the reorder point in the backorders cost. As a result the order quantity in the lost sales case is less than the order quantity in the backorders case.

5.6.2 Warehouse

As in the retailer problem, the only difference in the model formulation at the warehouse when excess demand is backordered is in the safety stock expression. The safety stock at the warehouse is $s_w^b = t_w^b - \mu_{LDW}$. The expression for order quantity at the warehouse is the same as in the lost sales case and is given by
\[ Q^b_W = \sqrt{\frac{2 \left( \frac{\lambda_{2n}^{n} D_W + \frac{\lambda_1^{n} D_W F_{xW} W_{xW}}{100} + \frac{\lambda_4^{n} ESC_W}{\lambda_1^{n} C_W} \right)}{\frac{\lambda_1^{n} C_W}} } \] (see Eq. 5.65). The formula to derive the reorder point changes to \[ \left\{ -F_{L_Di}(r^b_W) \right\} = \frac{\lambda_1^{n} Q^b_W C_W}{\lambda_4^{n}}. \]

An important factor that must be considered with the backorder assumption is that a stockout situation at the warehouse in the backorders case can cause a delay in fulfilling the orders of the retailers, and hence, affects the FR or CSL at the retailers. This delay is stochastic and depends on a lot of factors like the number of outstanding orders at the warehouse, the number of backorders at the warehouse, etc. Thus, the effective lead time at the retailer is the actual lead time plus the delay. This violates the assumption that the lead times of the retailers are independent. Svoronos and Zipkin (1987) claim that this violation does not pose any serious consequence, and that the retailers’ lead times can be treated as independent random variables.

Deuermeyer and Schwarz (1981) and Ganeshan (1999) approximate this delay by its expected value. This is a reasonable approximation as the delay accounts for a very small portion of the effective lead time because of the high fill rates maintained by the warehouse. Svoronos and Zipkin (1987) provide a two parameter approximation of the delay.

All the above papers deal with the case of all the retailers ordering in equal batches. Ganeshan (1999) first calculates the average number of orders that the warehouse is unable to meet as \[ O_{avg} = \frac{Q_W (1 - FR_W)}{Q_R} = \frac{ESC_W}{Q_R}, \] where \( Q_R \) is the batch size of each retailer. Then, he computes the expected delay at the warehouse as \[ \frac{ESC_W}{Q_R E[N_W(t)]}, \] where \( E[N_W(t)] \) is the mean number of orders received by the warehouse in period \( t \) and is given by Eq. 5.53.

In our case the order quantity of all the retailers need not be equal. Hence, the above method cannot be used. To account for unequal batch sizes, we provide three estimates of the expected delay. First we compute the following:
\[ Q_{\text{max}} = \max_i Q_i \quad Q_{\text{min}} = \min_i Q_i \quad Q_{\text{avg}} = \frac{1}{N} \sum_{i=1}^{N} Q_i \]

The *maximum*, *minimum* and *average* number of orders that the warehouse is unable to meet is given by \( O_{\text{max}} = \frac{\text{ESCO}_W}{Q_{\text{max}}} \), \( O_{\text{min}} = \frac{\text{ESCO}_W}{Q_{\text{min}}} \) and \( O_{\text{avg}} = \frac{\text{ESCO}_W}{Q_{\text{avg}}} \) respectively. Then, the maximum, minimum and average delay at the warehouse is:

\[ D_{\text{max}} = \frac{\text{ESCO}_W}{Q_{\text{max}} E[N_W(t)]}, \quad D_{\text{min}} = \frac{\text{ESCO}_W}{Q_{\text{min}} E[N_W(t)]} \quad \text{and} \quad D_{\text{avg}} = \frac{\text{ESCO}_W}{Q_{\text{avg}} E[N_W(t)]} \]

The warehouse provides these delay estimates to the individual retailers. The mean of the effective lead time would be shifted by \( D_{\text{max}}, D_{\text{min}} \) or \( D_{\text{avg}} \) depending on which estimate is used. The variance of the effective lead time remains the same.

The retailers will have to analyze the deterioration in their FR and CSL criteria using the above delay estimates. If a particular retailer feels that their FR or CSL falls below a certain acceptable level, then the retailer will have to inform the warehouse about this issue. The warehouse will then have to determine if it has to change its ordering policy to accommodate that retailer.

### 5.7 Example Problem

A single warehouse supplying to three retailers is used to explain the methodology developed in this chapter. Excess demand is treated as lost sales for the retailers and the warehouse. The weight of the product, the freight data, and the cost data at each location are the same as in Chapter 4. It is assumed that the demand is normally distributed and the distribution of demand for each retailer is given in Table 5.1.

<table>
<thead>
<tr>
<th>Retailer</th>
<th>Mean Daily Demand</th>
<th>Standard Deviation of Daily Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retailer 1</td>
<td>20</td>
<td>5</td>
</tr>
<tr>
<td>Retailer 2</td>
<td>15</td>
<td>10</td>
</tr>
<tr>
<td>Retailer 3</td>
<td>25</td>
<td>15</td>
</tr>
</tbody>
</table>

The lead time distribution (days) at the retailers is given in Table 5.2 and is assumed to be any discrete distribution.
Table 5.2: Lead Time Distribution for Retailer

<table>
<thead>
<tr>
<th>Retailer 1</th>
<th>Retailer 2</th>
<th>Retailer 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>L₁</td>
<td>P(L₁)</td>
<td>L₂</td>
</tr>
<tr>
<td>1</td>
<td>0.05</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0.1</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>0.2</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>0.45</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>0.1</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>0.05</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>0.05</td>
<td>7</td>
</tr>
</tbody>
</table>

Note that the distribution of lead time is skewed to the right for Retailers 2 and 3. The methods described in Sections 5.3.1, 5.3.2 and 5.3.3 can be used for these distributions as they are independent of the shape of the lead time distribution.

As in the deterministic multi-criteria model, the DM has to provide upper and lower bounds for the two decision variables, Q and r. This is given in Table 5.3.

Table 5.3: Bounds on Decision Variables

<table>
<thead>
<tr>
<th></th>
<th>Order Quantity (Q)</th>
<th>Reorder point (r)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Upper bound</td>
<td>Lower bound</td>
</tr>
<tr>
<td>Retailer 1</td>
<td>250</td>
<td>150</td>
</tr>
<tr>
<td>Retailer 2</td>
<td>215</td>
<td>120</td>
</tr>
<tr>
<td>Retailer 3</td>
<td>250</td>
<td>130</td>
</tr>
</tbody>
</table>

Normalization is an extra step that must be performed when dealing with multi-criteria problems. The scheme used in this case is as suggested in Section 5.3.2. To use this scheme, the ideal (smallest) and the anti-ideal (largest) values must be determined for each criterion. The ideal values are determined by minimizing the criterion independent of the other criteria, subjected to bounds on the decision variables. To determine the anti-ideal values, the criterion is maximized independent of the other criteria.

The next step is to solve Problem \((R^λᵢ)\) for different weight combination. Excel Solver is used to solve the problem. Since there are four criteria in this model, a standard procedure to determine the weight combinations is used. Steuer (1976) shows that if there are \(k\) criteria, then the number of efficient solutions that should be generated should lie between \(k+1\) and \(2k+1\). Since we have four criteria, the maximum number of solutions that have to be generated is 9. Starting with the weight combinations \((1,0,0,0), (0,1,0,0), \)
(0,0,1,0) and (0,0,0,1), the fifth weight combination is determined by calculating the centroid as (1/4,1/4,1/4,1/4). Using any three of the initial weight combinations and the fifth weight combination, the other four weight combinations are determined. For example, the sixth weight combination is the centroid of (1,0,0,0), (0,1,0,0), (0,0,1,0) and (1/4,1/4,1/4,1/4) which is (5/16,5/16,5/16,1/16). The rest of the weight combinations are given in Table 5.4. The $P_{\lambda}$ problem can now be solved for each retailer for the weight combinations given in Table 5.4.

Note: Instead of choosing some weight combinations, one could interact with the decision maker and get the preference information on the criteria by different methods. This approach is illustrated in the case study discussed in Chapter 6.

<table>
<thead>
<tr>
<th>Weight Combinations</th>
<th>Weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(1,0,0,0)</td>
</tr>
<tr>
<td>2</td>
<td>(0,1,0,0)</td>
</tr>
<tr>
<td>3</td>
<td>(0,0,1,0)</td>
</tr>
<tr>
<td>4</td>
<td>(0,0,0,1)</td>
</tr>
<tr>
<td>5</td>
<td>(1/4, 1/4, 1/4, 1/4)</td>
</tr>
<tr>
<td>6</td>
<td>(5/16, 5/16, 5/16, 1/16)</td>
</tr>
<tr>
<td>7</td>
<td>(5/16, 5/16, 1/16, 5/16)</td>
</tr>
<tr>
<td>8</td>
<td>(5/16, 1/16, 5/16, 5/16)</td>
</tr>
<tr>
<td>9</td>
<td>(1/16, 5/16, 5/16, 5/16)</td>
</tr>
</tbody>
</table>

We know from Section 5.3.3 that the conditional distribution of LTD for each retailer is $N\left(\mu_{Di}, \sigma_{Di}^2 \right)$. Hence, ESC and safety stock can be calculated. Sample calculations for Retailer 1 for weight combination (5/16,1/16,5/16,5/16) is tabulated in Table 5.5. The values of $Q$ and $r$ after optimization are 250 and 81.12 units.
Table 5.5: Sample Calculations

| $L_1$ | $p(L_1)$ | $j\mu_{D_1}$ | $\sqrt{j\sigma_{D_1}}$ | $(r - j\mu_{D_1})/\sqrt{j\sigma_{D_1}}$ | ESC$|j$ | $sj$ |
|-------|----------|---------------|------------------------|----------------------------------|--------|------|
| 1     | 0.05     | 20            | 5                      | 12.2331                          | 0      | 61.1654 |
| 2     | 0.1      | 40            | 7.0711                 | 5.8217                           | 0      | 41.1654 |
| 3     | 0.2      | 60            | 8.6603                 | 2.4440                           | 0.0206 | 21.1860 |
| 4     | 0.45     | 80            | 10                     | 0.1165                           | 3.4338 | 4.5992 |
| 5     | 0.1      | 100           | 11.1803                | -1.6846                          | 19.0469| 0.2123 |
| 6     | 0.05     | 120           | 12.2474                | -3.1708                          | 38.8372| 0.0025 |
| 7     | 0.05     | 140           | 13.2288                | 4.4475                           | 58.8346| 0      |

The first two columns of Table 5.5 represent the lead time distribution. The third and fourth columns are the conditional mean and standard deviation of demand during period $j$, respectively. The conditional ESC for lead time of 4 days is computed using:

$$ ESC^j_i = \frac{r_j - j\mu_{D_1}}{\sqrt{j\sigma_{D_1}}} \sqrt{j\sigma_{D_1}} + (j\mu_{D_1} - r_j) \left[ 1 - F_{LD_1} \left( \frac{r_j - j\mu_{D_1}}{\sqrt{j\sigma_{D_1}}} \right) \right] $$

(see Eq. 5.46)

The above formula is calculated in Excel as:

$\text{NORMDIST}(0.1165,0,1,0) \times 10 + (80-81.12) \times (1-\text{NORMDIST}(0.1165,0,1,1)) = 3.4338$ units.

The conditional ESC for all possible values of the lead time is computed. The total ESC for Retailer 1 for the weight combination $(5/16,1/16,5/16,5/16)$ is $ESC^1 = \sum_j ESC^j_i | p^j_i = 0 \times 0.05 + 0 \times 0.1 + \ldots + 58.8346 \times 0.05 = 8.3376$ units.

The conditional safety stock for Retailer 1 is calculated using $s^j_i = r_j - j\mu_{D_1} + ESC^j_i$ (see Eq. 5.21). These values are given in the last column of Table 5.5. The total safety stock for Retailer 1 is:

$$ s^1_1 = \sum_j s^j_i | p^j_i = 61.1654 \times 0.05 + 41.1654 \times 0.1 + \ldots + 0 \times 0.05 = 13.503 \text{ units}. $$

The output of the $P_\lambda$ problem is given in Table 5.6. The normalized criterion values are given in Table 5.7. The value path approach described in Chapter 4 is used to present the DM with the set of efficient solutions. This is shown in Fig. 5.2. If the DM is not satisfied with the current set of solutions, another set of solutions can be generated by tweaking the values of the weight parameters.
### Table 5.6: Criteria Values for the Retailers

<table>
<thead>
<tr>
<th>Weight</th>
<th>Combination</th>
<th>Retailer 1</th>
<th>Retailer 2</th>
<th>Retailer 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Criterion1</td>
<td>Criterion2</td>
<td>Criterion3</td>
</tr>
<tr>
<td>1</td>
<td>6573.56</td>
<td>561.15385</td>
<td>24092.90</td>
<td>0.357903</td>
</tr>
<tr>
<td>2</td>
<td>12573.56</td>
<td>29.2</td>
<td>19184.24</td>
<td>0.186109</td>
</tr>
<tr>
<td>3</td>
<td>12573.56</td>
<td>29.2</td>
<td>19184.24</td>
<td>0.186109</td>
</tr>
<tr>
<td>4</td>
<td>19929.55</td>
<td>29.2</td>
<td>19184.24</td>
<td>0.000349</td>
</tr>
<tr>
<td>5</td>
<td>13871.12</td>
<td>29.2</td>
<td>19184.24</td>
<td>0.03335</td>
</tr>
<tr>
<td>6</td>
<td>12927.82</td>
<td>29.2</td>
<td>19184.24</td>
<td>0.091062</td>
</tr>
<tr>
<td>7</td>
<td>13871.12</td>
<td>29.2</td>
<td>19184.24</td>
<td>0.03335</td>
</tr>
<tr>
<td>8</td>
<td>13871.12</td>
<td>29.2</td>
<td>19184.24</td>
<td>0.03335</td>
</tr>
<tr>
<td>9</td>
<td>15925.63</td>
<td>29.2</td>
<td>19184.24</td>
<td>0.009849</td>
</tr>
<tr>
<td>Ideal</td>
<td>6573.56</td>
<td>29.2</td>
<td>19184.24</td>
<td>0.000349</td>
</tr>
<tr>
<td>Anti-Ideal</td>
<td>19929.55</td>
<td>561.15385</td>
<td>24092.90</td>
<td>0.357903</td>
</tr>
</tbody>
</table>

### Table 5.7: Normalized Criteria Values for the Retailers

<table>
<thead>
<tr>
<th>Weight</th>
<th>Combination</th>
<th>Retailer 1</th>
<th>Retailer 2</th>
<th>Retailer 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Criterion1</td>
<td>Criterion2</td>
<td>Criterion3</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0.4492</td>
<td>0</td>
<td>0</td>
<td>0.5195</td>
</tr>
<tr>
<td>3</td>
<td>0.4492</td>
<td>0</td>
<td>0</td>
<td>0.5195</td>
</tr>
<tr>
<td>4</td>
<td>1.0000</td>
<td>0</td>
<td>0</td>
<td>0.0000</td>
</tr>
<tr>
<td>5</td>
<td>0.5464</td>
<td>0</td>
<td>0</td>
<td>0.0923</td>
</tr>
<tr>
<td>6</td>
<td>0.4758</td>
<td>0</td>
<td>0</td>
<td>0.2537</td>
</tr>
<tr>
<td>7</td>
<td>0.5464</td>
<td>0</td>
<td>0</td>
<td>0.0923</td>
</tr>
<tr>
<td>8</td>
<td>0.5464</td>
<td>0</td>
<td>0</td>
<td>0.0923</td>
</tr>
<tr>
<td>9</td>
<td>0.7002</td>
<td>0</td>
<td>0</td>
<td>0.0266</td>
</tr>
</tbody>
</table>
A tradeoff analysis can be used to validate the results obtained from the optimization models. For example, for Retailer 1, consider weight combinations 4 (1/4,1/4,1/4,1/4) and 5 (5/16,5/16,5/16,1/16). The capital invested in inventory decreases by 30.4%, and fill rate increases by 94.56%, while the other two criteria remain unchanged. This is because the DM’s preference for capital invested in inventory, number of orders, and transportation costs has increased, while his/her’s preference towards fill rate has reduced.

Recall that $F_{LDi}(τ_i)$ is the cycle service level for each retailer. It can be used as another measure of customer satisfaction. To capture how CSL and FR behave, we first vary the weight of the fill rate criterion, $λ_4$, and annual transportation costs, $λ_3$, while keeping the weights of capital invested in inventory, $λ_1$, and annual number of orders placed, $λ_2$, constant at 0.025. This is shown in Fig. 5.3a. The alternatives in the X axis refer to a particular combination of $(λ_3,λ_4)$, e.g., alternative 1 is (0.05,0.9), alternative 2 is (0.1,0.85), etc. In Fig. 5.3b, $λ_2$ and $λ_3$ kept constant at 0.025 while $(λ_1,λ_4)$ are varied.
Fig. 5.3a and b: Behavior of Fill Rate/Cycle Service Level
As we move across the right in Fig. 5.3a ($\lambda_3$ increases, $\lambda_4$ decreases), the CSL starts to decreases while the FR is almost constant. This is because the FR is primarily influenced by the value of Q, which increases as $\lambda_3$ increases. CSL is only influenced by the value of r and starts to decrease as $\lambda_4$ decreases. In Fig. 5.3a, the FR is 97.82% for $(\lambda_3, \lambda_4) = (0.9, 0.05)$, while CSL is 74.93%.

As we move across the right in Fig. 5.3b ($\lambda_1$ increases, $\lambda_4$ decreases) both the FR and CSL decreases; however, the decrease in CSL is significant. For example, for $(\lambda_1, \lambda_4) = (0.9,0.05)$, the FR is 75.04% while the CSL is 13.77%. Thus, in addition to looking at the 4 criteria, the DM must also observe CSL, as in some cases even when the FR is acceptable, the CSL may be unsatisfactory.

To determine the most preferred solution, the DM has to use Table 5.7 and Fig. 5.2 to analyze the tradeoff information. Observing the values in Table 5.7, the annual number of orders placed and the annual transportation costs takes only two values, the ideal and anti-ideal, for all the retailers. Capital invested in inventory is minimized when Q and r are fixed at their lower bounds. The annual number of orders and annual transportation costs are minimized when Q is fixed at the upper bound. FR is maximized (1-FR is minimized) when Q and r at their upper bounds. Since 3 out of the 4 criteria are minimized for larger values of Q, care should be taken while setting the value of $\lambda_1$ so that the capital invested in inventory criterion does not get dominated. Taking this and the CSL into account the retailers’ ordering policies are given in Table 5.8. The criteria values are given in Table 5.9.

### Table 5.8: Ordering Policy for the Retailers

<table>
<thead>
<tr>
<th></th>
<th>Weights Order Quantity Reorder point</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retailer 1</td>
<td>(0.4,0.05,0.15,0.4)</td>
</tr>
<tr>
<td>Retailer 2</td>
<td>(0.45,0.05,0.2,0.3)</td>
</tr>
<tr>
<td>Retailer 3</td>
<td>(0.3,0.05,0.1,0.55)</td>
</tr>
</tbody>
</table>

### Table 5.9: Most Preferred Criteria Values for the Retailers

<table>
<thead>
<tr>
<th></th>
<th>Capital Invested in Inventory</th>
<th>Annual Number of Orders</th>
<th>Annual Transportation Cost</th>
<th>Fill Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retailer 1</td>
<td>$12152.37</td>
<td>34.25</td>
<td>$20279.82</td>
<td>96.45%</td>
</tr>
<tr>
<td>Retailer 2</td>
<td>$16321.3</td>
<td>27.30</td>
<td>$15536.78</td>
<td>95.55%</td>
</tr>
<tr>
<td>Retailer 3</td>
<td>$18911.09</td>
<td>48.08</td>
<td>$26396.12</td>
<td>92.41%</td>
</tr>
</tbody>
</table>
Once the order policy for each retailer is established, this information is passed on to the warehouse. The distribution of the daily demand at the warehouse given by Eq. 5.54:

\[
N\left( \sum_{i=1}^{N} \mu_{Di}, \sum_{i=1}^{N} \frac{Q_i}{\mu_{Di}} \right) = N\left( 20 + 15 + 25, \frac{213.16}{20} + \frac{200.54}{15} + \frac{189.8}{25} \right) = N(60,60).
\]

The lower and upper bound for the order quantity, \( Q_W \) is 603.5‡‡‡ and 1500, respectively. The lower and upper bound for the reorder point, \( r_W \) is 250 and 500, respectively. The distribution of the lead time at the warehouse is given in Table 5.10.

<table>
<thead>
<tr>
<th>Lead time, ( L_W )</th>
<th>Probability, ( p(L_W) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.05</td>
</tr>
<tr>
<td>2</td>
<td>0.1</td>
</tr>
<tr>
<td>3</td>
<td>0.1</td>
</tr>
<tr>
<td>4</td>
<td>0.1</td>
</tr>
<tr>
<td>5</td>
<td>0.25</td>
</tr>
<tr>
<td>6</td>
<td>0.15</td>
</tr>
<tr>
<td>7</td>
<td>0.1</td>
</tr>
<tr>
<td>8</td>
<td>0.05</td>
</tr>
<tr>
<td>9</td>
<td>0.05</td>
</tr>
<tr>
<td>10</td>
<td>0.05</td>
</tr>
</tbody>
</table>

From Section 5.4, we know that the distribution of the LTD at the warehouse is \( N(\mu_{DW}, \sigma_{DW}^2) \). Hence, ESC and safety stock at the warehouse can be computed. Using the weight combinations of Table 5.4 we can determine the variables and the criteria values. These are shown in Table 5.11 and Table 5.12, respectively. Fig. 5.4 shows the value path graph of the warehouse.

‡‡‡ The lower bound is computed by adding all the retailers order quantities, i.e.,

\[213.16+200.54+189.8=603.5 \text{ units}.\]
Table 5.11: Efficient Solutions for the Warehouse

<table>
<thead>
<tr>
<th>Weight Combination</th>
<th>δ</th>
<th>Qw (units)</th>
<th>rw</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>603.5</td>
<td>250</td>
</tr>
<tr>
<td>2</td>
<td>2.4855</td>
<td>1500</td>
<td>250</td>
</tr>
<tr>
<td>3</td>
<td>2.4855</td>
<td>1500</td>
<td>250</td>
</tr>
<tr>
<td>4</td>
<td>2.4855</td>
<td>1500</td>
<td>500</td>
</tr>
<tr>
<td>5</td>
<td>2.4855</td>
<td>1500</td>
<td>385.01</td>
</tr>
<tr>
<td>6</td>
<td>2.4855</td>
<td>1500</td>
<td>272.96</td>
</tr>
<tr>
<td>7</td>
<td>2.1517</td>
<td>1298.54</td>
<td>402.06</td>
</tr>
<tr>
<td>8</td>
<td>2.1517</td>
<td>1298.54</td>
<td>402.06</td>
</tr>
<tr>
<td>9</td>
<td>2.4855</td>
<td>1500</td>
<td>500</td>
</tr>
</tbody>
</table>

Table 5.12: Criteria and Normalized Criteria Values for the Warehouse

<table>
<thead>
<tr>
<th>Weight Combination</th>
<th>Criteria1</th>
<th>Criteria2</th>
<th>Criteria3</th>
<th>Criteria4</th>
<th>Normalized Criteria1</th>
<th>Normalized Criteria2</th>
<th>Normalized Criteria3</th>
<th>Normalized Criteria4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>21667.2</td>
<td>36.29</td>
<td>40279.31</td>
<td>0.148707</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>50803.45</td>
<td>14.6</td>
<td>16205.71</td>
<td>0.05983</td>
<td>0.7281</td>
<td>0</td>
<td>0</td>
<td>0.3826</td>
</tr>
<tr>
<td>3</td>
<td>50803.45</td>
<td>14.6</td>
<td>16205.71</td>
<td>0.05983</td>
<td>0.7281</td>
<td>0</td>
<td>0</td>
<td>0.3826</td>
</tr>
<tr>
<td>4</td>
<td>61683.32</td>
<td>14.6</td>
<td>16205.71</td>
<td>0.004752</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0.0000</td>
</tr>
<tr>
<td>5</td>
<td>55494.8</td>
<td>14.6</td>
<td>16205.71</td>
<td>0.017939</td>
<td>0.8453</td>
<td>0</td>
<td>0</td>
<td>0.0916</td>
</tr>
<tr>
<td>6</td>
<td>51317.61</td>
<td>14.6</td>
<td>16205.71</td>
<td>0.049797</td>
<td>0.7410</td>
<td>0</td>
<td>0</td>
<td>0.3129</td>
</tr>
<tr>
<td>7</td>
<td>49783</td>
<td>16.87</td>
<td>18719.19</td>
<td>0.017491</td>
<td>0.7026</td>
<td>0.1044</td>
<td>0.1044</td>
<td>0.0885</td>
</tr>
<tr>
<td>8</td>
<td>49783</td>
<td>16.87</td>
<td>18719.19</td>
<td>0.017491</td>
<td>0.7026</td>
<td>0.1044</td>
<td>0.1044</td>
<td>0.0885</td>
</tr>
<tr>
<td>9</td>
<td>61683.32</td>
<td>14.6</td>
<td>16205.71</td>
<td>0.004752</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Ideal</td>
<td>21667.2</td>
<td>14.6</td>
<td>16205.71</td>
<td>0.004752</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Anti-Ideal</td>
<td>61683.32</td>
<td>36.29</td>
<td>40279.31</td>
<td>0.148707</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
As noted while solving the retailer’s problem that the weights generated by Steuer’s method (Steuer, 1976) may not be applicable to this problem as there is a possibility that the capital invested in inventory might be dominated by the other three criteria. This can be observed in Table 5.11. For weight combination 6 ($\lambda_1=0.3125$), the order quantity is set to its upper bound of 1500 units. We had also observed for the retailer’s problem that even though the FR could be at an acceptable level, the CSL can be very low. It is even more vital for the warehouse to keep track of the CSL because of the assumption that there is no lot splitting. Assuming that the DM of the warehouse keeps these two issues in mind, the most preferred weight combination is (0.35,0.1,0.1,0.45), which results in the warehouse ordering $Q_W = 918.20$ units when the reorder point $r_W$ reaches 460.45 units. The criteria values are: Capital invested in inventory = $40525.08$, Annual number of orders placed = 23.85, Annual transportation cost = $26474$ and FR = 98.68%. The CSL is computed to be 85.69%.
5.8 Summary

In this chapter we modeled and solved the inventory problem for a decentralized single warehouse multi-retailer system with transportation costs using a multi-criteria approach when demand and lead time are treated as random variables. The retailers face independent but non-identical demands. Three distributions of external demand were considered: Poisson, geometric Poisson, and normal. The lead times were assumed to follow any general discrete distribution. The demand at the warehouse was approximated using a normal distribution based on renewal theory. Fill rate was used to measure the customer service at each location. Both the lost sales and backorders case were considered. The weighted objective method was used with the value path method to help the DM choose the most preferred solution. An example problem was used to explain the methodology developed.

In the next chapter, the results obtained by applying the models developed in Chapters 3, 4 and 5 to real world data obtained from a Fortune 500 consumer products company are discussed.
Chapter 6
CASE STUDIES

6.1 Introduction
The purpose in this chapter is to validate the theoretical models developed in the previous chapters by applying them to inventory planning case studies for a product manufactured by a Fortune 500 consumer products company. It is well known that there is a gap between the theoretical models and real world problems. While some models may not represent situations faced in practice, others may be too complicated for practical implementation. This chapter bridges that gap. After explaining the details of the case, the problems are solved under the model assumptions given in Chapters 3, 4 and 5, and their results are discussed. Also, some of the practical problems faced while using these models with the real world data and their resolutions are also discussed. Implementation is done using Microsoft Excel, which is widely available.

6.2 Case Study Description
The case study deals with planning the inventory for a functional product manufactured by a Fortune 500 company with a relatively stable demand. The company wants to manage its inventory for a one year period (tactical planning) so that it can serve up to 20 retailers. Daily sales data for the past three years are available. Since the company operates at high service levels, the demand can be approximated by the sales data. The cost of the product at the warehouse, $C_W = $19.10/box. The selling price of the product (cost at the retailer), $C_i$ is roughly $21/box assuming FOB origin. The weight of a box of the product ($\omega$) is 40 lbs. The lead times for the retailer and the warehouse are in the interval [1,3] days.

6.3 Case Study 1: Chapter 3
6.3.1 Overview
The purpose of this study is to test the effectiveness of the modified base period policy developed using a single objective cost (sum of inventory holding costs and

---

§§§ The differences in the selling prices among the retailers are not significant as the product is functional.
ordering costs) framework to this data. The closed-form expressions are coded using Visual Basic (VB) macros in Excel. The demand estimate for the current year was obtained by taking the average of the last three years data. The exact marginal cost information, i.e., the ordering and opportunity costs at each retailer and warehouse location were not available. The ordering cost was assumed to be $50 and $40. The opportunity cost at each location was assumed to be $0.17/year. Thus, the inventory holding costs are computed as $3.6992/unit/year and $3.247/unit/year.

6.3.2 Results

The above information was input into Excel. The optimal results (order quantity, time between orders and annual inventory cost) for each retailer after solving the EOQ problem are as shown in Table 6.1.

Table 6.1: Output of EOQ Problem for Retailers

<table>
<thead>
<tr>
<th>Retailers</th>
<th>Demand</th>
<th>Q*(units)</th>
<th>T*(weeks)</th>
<th>Z*(S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>17102</td>
<td>679.94</td>
<td>2.0674</td>
<td>2515.23</td>
</tr>
<tr>
<td>2</td>
<td>15416</td>
<td>645.55</td>
<td>2.1775</td>
<td>2388.03</td>
</tr>
<tr>
<td>3</td>
<td>20736</td>
<td>748.70</td>
<td>1.8775</td>
<td>2769.60</td>
</tr>
<tr>
<td>4</td>
<td>47185</td>
<td>1129.40</td>
<td>1.2447</td>
<td>4177.88</td>
</tr>
<tr>
<td>5</td>
<td>5091</td>
<td>370.98</td>
<td>3.7892</td>
<td>1372.32</td>
</tr>
<tr>
<td>6</td>
<td>6648</td>
<td>423.93</td>
<td>3.3159</td>
<td>1568.19</td>
</tr>
<tr>
<td>7</td>
<td>8931</td>
<td>491.36</td>
<td>2.8609</td>
<td>1817.62</td>
</tr>
<tr>
<td>8</td>
<td>34027</td>
<td>959.09</td>
<td>1.4657</td>
<td>3547.85</td>
</tr>
<tr>
<td>9</td>
<td>64727</td>
<td>1322.78</td>
<td>1.0627</td>
<td>4893.24</td>
</tr>
<tr>
<td>10</td>
<td>38213</td>
<td>1016.37</td>
<td>1.3831</td>
<td>3759.75</td>
</tr>
<tr>
<td>11</td>
<td>6847</td>
<td>430.23</td>
<td>3.2674</td>
<td>1591.49</td>
</tr>
<tr>
<td>12</td>
<td>41243</td>
<td>1055.90</td>
<td>1.3313</td>
<td>3905.97</td>
</tr>
<tr>
<td>13</td>
<td>7210</td>
<td>441.48</td>
<td>3.1841</td>
<td>1633.13</td>
</tr>
<tr>
<td>14</td>
<td>29806</td>
<td>897.63</td>
<td>1.5660</td>
<td>3320.52</td>
</tr>
<tr>
<td>15</td>
<td>25036</td>
<td>822.68</td>
<td>1.7087</td>
<td>3043.24</td>
</tr>
<tr>
<td>16</td>
<td>8809</td>
<td>487.99</td>
<td>2.8806</td>
<td>1805.17</td>
</tr>
<tr>
<td>17</td>
<td>14274</td>
<td>621.18</td>
<td>2.2630</td>
<td>2297.88</td>
</tr>
<tr>
<td>18</td>
<td>51589</td>
<td>1180.93</td>
<td>1.1903</td>
<td>4368.50</td>
</tr>
<tr>
<td>19</td>
<td>9831</td>
<td>515.52</td>
<td>2.7268</td>
<td>1907.01</td>
</tr>
<tr>
<td>20</td>
<td>19002</td>
<td>716.71</td>
<td>1.9613</td>
<td>2651.27</td>
</tr>
</tbody>
</table>
The warehouse identifies the base retailer as Retailer 9 (the one with the lowest $T^*$), and the reorder interval corresponding to this retailer is set as the base period and is 1.0627 weeks. The next step is for the non-base retailers to solve a modified problem where their reorder interval is an integer multiple of 1.0627 weeks. The new order quantity, reorder interval and the annual inventory cost along with the cluster in which each retailer falls is given in Table 6.2.

<table>
<thead>
<tr>
<th>Retailers</th>
<th>Cluster</th>
<th>Q(units)</th>
<th>T(weeks)</th>
<th>Cost($)</th>
<th>% Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>1</td>
<td>1322.78</td>
<td>1.0627</td>
<td>4893.24</td>
<td>0.0000</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>964.29</td>
<td>1.0627</td>
<td>4230.17</td>
<td>1.2516</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>695.39</td>
<td>1.0627</td>
<td>3732.81</td>
<td>5.2132</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>780.93</td>
<td>1.0627</td>
<td>3891.04</td>
<td>3.4918</td>
</tr>
<tr>
<td>12</td>
<td>1</td>
<td>842.86</td>
<td>1.0627</td>
<td>4005.57</td>
<td>2.5499</td>
</tr>
<tr>
<td>18</td>
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<td>1054.29</td>
<td>1.0627</td>
<td>4396.64</td>
<td>0.6441</td>
</tr>
<tr>
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<td>699.00</td>
<td>2.1254</td>
<td>2516.19</td>
<td>0.0382</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>630.09</td>
<td>2.1254</td>
<td>2388.73</td>
<td>0.0294</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>847.54</td>
<td>2.1254</td>
<td>2790.91</td>
<td>0.7697</td>
</tr>
<tr>
<td>14</td>
<td>2</td>
<td>1218.25</td>
<td>2.1254</td>
<td>3476.59</td>
<td>4.7002</td>
</tr>
<tr>
<td>15</td>
<td>2</td>
<td>1023.29</td>
<td>2.1254</td>
<td>3115.99</td>
<td>2.3904</td>
</tr>
<tr>
<td>17</td>
<td>2</td>
<td>583.42</td>
<td>2.1254</td>
<td>2302.40</td>
<td>0.1968</td>
</tr>
<tr>
<td>20</td>
<td>2</td>
<td>776.66</td>
<td>2.1254</td>
<td>2659.83</td>
<td>0.3228</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>407.58</td>
<td>3.1881</td>
<td>1569.40</td>
<td>0.0773</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>547.55</td>
<td>3.1881</td>
<td>1828.29</td>
<td>0.5869</td>
</tr>
<tr>
<td>11</td>
<td>3</td>
<td>419.78</td>
<td>3.1881</td>
<td>1591.97</td>
<td>0.0302</td>
</tr>
<tr>
<td>13</td>
<td>3</td>
<td>442.04</td>
<td>3.1881</td>
<td>1633.13</td>
<td>0.0001</td>
</tr>
<tr>
<td>16</td>
<td>3</td>
<td>540.07</td>
<td>3.1881</td>
<td>1814.46</td>
<td>0.5146</td>
</tr>
<tr>
<td>19</td>
<td>3</td>
<td>602.73</td>
<td>3.1881</td>
<td>1930.35</td>
<td>1.2239</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>416.17</td>
<td>4.2508</td>
<td>1381.40</td>
<td>0.6613</td>
</tr>
</tbody>
</table>

The last column in Table 6.2 measures the percent deviation in annual inventory cost from the EOQ policy as a result of following the modified base period policy. Note that the modified base period policy performs extremely well (within 6% of EOQ cost) for a wide range of demand data. In reality, the warehouse would have to share the benefits of order consolidation with the retailers to offset the increase in their costs. This by itself is a separate research problem.

This information is used to formulate and solve the warehouse’s problem. The ordering policy of the warehouse is to order 115683.52 units every 12.75 weeks at a cost
of $163923.67. In the next section, we discuss why the reorder interval and the order quantity are on the higher side and how to reduce them.

6.3.3 Difficulties Encountered

The first problem we faced was due to the unavailability of marginal cost information. This was addressed as one of the problems of applying single objective optimization inventory models in practice. This study further corroborates that problem. To avoid this problem, we had to assume values for the ordering cost and opportunity cost at each location.

The second issue was pertaining to the high reorder interval and order quantity at the warehouse. The reason for this is due to demand data ranging from 5091 units (Retailer 5) to as high as 64727 units (Retailer 9). Most of the theoretical models in the literature fail to consider this issue. *They assume that all the retailers face identical demands, which is unrealistic in the real world.* To resolve this issue, a tier approach is proposed. In this method, the warehouse groups clusters of retailers into tiers, and solves independent optimization problems for each tier. The warehouse will have to decide on the number of clusters in each tier, and hence, the number of tiers. The range of the reorder interval is an important factor in making the above decision. For higher ranges (low number of tiers), the warehouse holds excess inventory in order to meet the demand of all the retailers. This increases the inventory holding costs. Alternatively, for lower ranges (large number of tiers), the warehouse places a lot more orders, and hence, results in an increase in ordering costs. Thus, the number of tiers should be such that there is a balance between holding and ordering costs.

Using these guidelines, we group the four clusters of retailers identified in Table 6.2 into tiers, and re-solve the optimization model for each tier. For example, let us group the retailers in clusters 1 and 2 in Tier I, and the retailers in clusters 3 and 4 in Tier II. The ordering policy for Tier I retailers is going to be the same as given in Table 6.2. For the retailers in Tier II, the base period should be identified. The base retailer in Tier II is Retailer 19 and the base period is 2.7268 weeks. The non-base retailers in Tier II have to resolve the modified problem with the constraint that their reorder interval is an integer multiple of 2.7268 weeks. The results are shown in Table 6.3.
Table 6.3: Output of the Modified Problem for Retailers in Tier II

<table>
<thead>
<tr>
<th>Retailers</th>
<th>Cluster</th>
<th>Q(units)</th>
<th>T(weeks)</th>
<th>Cost($)</th>
<th>% Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>19</td>
<td>1</td>
<td>515.52</td>
<td>2.7268</td>
<td>1907.01</td>
<td>0.00</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>266.96</td>
<td>2.7268</td>
<td>1447.28</td>
<td>5.4622</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>348.6087</td>
<td>2.7268</td>
<td>1598.291</td>
<td>1.9193</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>468.325</td>
<td>2.7268</td>
<td>1819.718</td>
<td>0.1153</td>
</tr>
<tr>
<td>11</td>
<td>1</td>
<td>359.0439</td>
<td>2.7268</td>
<td>1617.592</td>
<td>1.6401</td>
</tr>
<tr>
<td>13</td>
<td>1</td>
<td>378.08</td>
<td>2.7268</td>
<td>1652.80</td>
<td>1.2042</td>
</tr>
<tr>
<td>16</td>
<td>1</td>
<td>461.93</td>
<td>2.7268</td>
<td>1807.89</td>
<td>0.1506</td>
</tr>
</tbody>
</table>

All the retailers in Tier II are classified in one cluster. Again, the percent deviation in cost as a result of following the modified base period policy is less than 6%.

The warehouse has to solve its optimization problems for Tier I and Tier II separately. The result after solving these two optimization problems along with the result without using the tier approach is shown in Table 6.4.

Table 6.4: Comparison of Results with and without Tiered Approach

<table>
<thead>
<tr>
<th></th>
<th>Without Tier approach</th>
<th>Tier approach</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tier I</td>
<td>Tier II</td>
</tr>
<tr>
<td>Order Quantity (units)</td>
<td>115,683.52</td>
<td>17099.34</td>
</tr>
<tr>
<td>Reorder Interval (weeks)</td>
<td>12.75</td>
<td>2.1254</td>
</tr>
<tr>
<td>Annual Costs ($)</td>
<td>163,923.67</td>
<td>10,168.54</td>
</tr>
</tbody>
</table>

The total annual cost at the warehouse using the tier approach is $109,311.4 way below $163,923.67, the cost when the warehouse's problem was solved in a single step, a decrease of 93.33%. This enormous decrease is attributed to the reduction in the reorder interval and hence, reduction in inventory holding costs.

6.4 Case Study 2: Chapter 4

6.4.1 Overview

In this study we would like to demonstrate the usefulness of a multi-criteria approach to determine the ordering policies for the companies in the supply chain when transportation costs are considered between the echelons. The three criteria used in the model formulation are capital invested in inventory, annual number of orders, and annual transportation costs. FOB origin is assumed. Since the problem is formulated as a multi-
criteria problem, we do not have to make any assumptions about opportunity and ordering costs as in the previous case study. We use the same freight rate data as in Chapter 4 and Chapter 5. Based on that data, the cost of a full TL is $1110. A truck can carry up to 46,000 lbs (460 cwt). Hence, TL rate is 1110/460 = $2.413/cwt. The retailer shipments are modeled using a power function while the warehouse’s shipments are modeled using an inverse function. The lead times of the retailers are in the interval [1,3] days. The lead time of the warehouse is 3 days. The demand estimates are the same as in the previous study. To simulate the decision making process, we used an executive from the same Fortune 500 company to specify the upper and lower bounds on the order quantity and provide preference information pertaining to the three criteria using three different methods (discussed in the next section). Using this preference information, three sets of weights for the criteria were computed. These weights were used in the model to obtain efficient solutions from which the DM could analyze the tradeoffs and choose the most preferred solution. Excel Solver and VB macros were used to generate efficient solutions.

6.4.2 Results

Each retailer starts off by solving Problem (R_i) by inputting different weight combinations. Assuming that the DM has chosen the most preferred solution, the ordering policy for the retailers is given in Table 6.5.
<table>
<thead>
<tr>
<th>Retailers</th>
<th>Weight</th>
<th>Q(units)</th>
<th>T(weeks)</th>
<th>Reorder Point</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(0.5,0.2,0.3)</td>
<td>383.60</td>
<td>1.1664</td>
<td>93.71</td>
</tr>
<tr>
<td>2</td>
<td>(0.45,0.2,0.35)</td>
<td>380.82</td>
<td>1.2846</td>
<td>84.47</td>
</tr>
<tr>
<td>3</td>
<td>(0.5,0.25,0.25)</td>
<td>511.58</td>
<td>1.2829</td>
<td>113.62</td>
</tr>
<tr>
<td>4</td>
<td>(0.55,0.2,0.25)</td>
<td>557.44</td>
<td>0.6143</td>
<td>129.27</td>
</tr>
<tr>
<td>5</td>
<td>(0.45,0.17,0.38)</td>
<td>244.32</td>
<td>2.4955</td>
<td>41.84</td>
</tr>
<tr>
<td>6</td>
<td>(0.55,0.1,0.35)</td>
<td>214.29</td>
<td>1.6761</td>
<td>54.64</td>
</tr>
<tr>
<td>7</td>
<td>(0.54,0.16,0.3)</td>
<td>270.72</td>
<td>1.5762</td>
<td>73.41</td>
</tr>
<tr>
<td>8</td>
<td>(0.4,0.2,0.4)</td>
<td>407.45</td>
<td>0.6227</td>
<td>93.22</td>
</tr>
<tr>
<td>9</td>
<td>(0.4,0.2,0.4)</td>
<td>704.80</td>
<td>0.5662</td>
<td>177.33</td>
</tr>
<tr>
<td>10</td>
<td>(0.45,0.17,0.38)</td>
<td>434.07</td>
<td>0.5907</td>
<td>104.69</td>
</tr>
<tr>
<td>11</td>
<td>(0.5,0.2,0.3)</td>
<td>294.17</td>
<td>2.2341</td>
<td>56.28</td>
</tr>
<tr>
<td>12</td>
<td>(0.45,0.2,0.35)</td>
<td>490.23</td>
<td>0.6181</td>
<td>112.99</td>
</tr>
<tr>
<td>13</td>
<td>(0.54,0.16,0.3)</td>
<td>320.43</td>
<td>2.3110</td>
<td>59.26</td>
</tr>
<tr>
<td>14</td>
<td>(0.5,0.25,0.25)</td>
<td>481.35</td>
<td>0.8398</td>
<td>81.66</td>
</tr>
<tr>
<td>15</td>
<td>(0.5,0.2,0.3)</td>
<td>389.40</td>
<td>0.8088</td>
<td>137.18</td>
</tr>
<tr>
<td>16</td>
<td>(0.55,0.2,0.25)</td>
<td>258.56</td>
<td>1.5263</td>
<td>72.40</td>
</tr>
<tr>
<td>17</td>
<td>(0.5,0.17,0.33)</td>
<td>226.14</td>
<td>0.8238</td>
<td>78.21</td>
</tr>
<tr>
<td>18</td>
<td>(0.3,0.2,0.5)</td>
<td>621.34</td>
<td>0.6263</td>
<td>141.34</td>
</tr>
<tr>
<td>19</td>
<td>(0.45,0.2,0.35)</td>
<td>229.67</td>
<td>1.2148</td>
<td>80.80</td>
</tr>
<tr>
<td>20</td>
<td>(0.5,0.17,0.33)</td>
<td>439.33</td>
<td>1.2023</td>
<td>156.18</td>
</tr>
</tbody>
</table>

The warehouse identifies the base retailer as Retailer 9, and the reorder interval corresponding to this retailer is the base period and is 0.5662 weeks. Recall that in case study 1, the reorder interval for Retailer 9 was 1.0627 weeks. By specifying a weight of 0.4 to the capital invested in inventory criterion, the DM is able to reduce the average inventory and hence, the reorder interval.

The next step is for the non-base retailers to solve a modified problem where their reorder interval is an integer multiple of 0.5662 weeks. The new order quantity, reorder interval and the criteria values along with the cluster in which each retailer falls is given in Table 6.6.
Using the information of Table 6.6, the warehouse formulates and solves its multi-criteria optimization problem. For the retailers, we had assumed a weight combination as the most preferred combination. However, for the warehouse we were able to obtain the preference information of the DM (the company executive) using the following methods.

Method 1 (Rating method):

In this method the DM is asked to rate the criteria on a 1-10 scale where 1 indicates a low score and 10 indicates a high score. Then the weights can be computed as shown in Table 6.7.
Method 2 (Pairwise Comparison method):

In this method the DM is asked questions about his preference with respect to a pair of criteria. Let \( A = \{a_{ij}\} \) be the pairwise comparison matrix (Ravindran, 2003). Then we have the following:

\[
a_{ii} = 1 \forall i
\]

If \( i \succ j \) then \( a_{ij} = 1, a_{ji} = 0 \)

If \( j \succ i \) then \( a_{ji} = 1, a_{ij} = 0 \)

If \( i \Leftrightarrow j \) then \( a_{ij} = a_{ji} = 1 \)

where: \( \succ \) - More preference

\( \Leftrightarrow \) - Equal preference

Using these guidelines, the pairwise comparison matrix was obtained from the company executive and the weights were computed as shown in Table 6.8.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Score (1-10)</th>
<th>Weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital invested in inventory</td>
<td>7</td>
<td>7/15=0.47</td>
</tr>
<tr>
<td>Annual number of orders</td>
<td>3</td>
<td>3/15=0.2</td>
</tr>
<tr>
<td>Annual transportation costs</td>
<td>5</td>
<td>5/15=0.33</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>15</strong></td>
<td></td>
</tr>
</tbody>
</table>

Method 3:

It is an extension of the pairwise comparison method and is used in the Analytical Hierarchy Process (Saaty, 1980). In this method the DM has to indicate the strength of

<table>
<thead>
<tr>
<th>Capital invested in inventory</th>
<th>Annual number of orders</th>
<th>Annual transportation costs</th>
<th>Total</th>
<th>Weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>3/6=0.5</td>
</tr>
<tr>
<td><strong>Annual number of orders</strong></td>
<td><strong>0</strong></td>
<td><strong>1</strong></td>
<td><strong>1</strong></td>
<td><strong>1/6=0.17</strong></td>
</tr>
<tr>
<td><strong>Annual transportation costs</strong></td>
<td><strong>0</strong></td>
<td><strong>1</strong></td>
<td><strong>2</strong></td>
<td><strong>2/6=0.33</strong></td>
</tr>
</tbody>
</table>
the preference on a 1-9 scale. Let \( A = \{a_{ij}\} \) be the pairwise comparison strength matrix. Then we have the following:

\[ a_{ii} = 1, \forall i \]

If \( i > j \) then: \( a_{ij} = 1 \) (equal preference)

\[ a_{ij} = 3 \] (weak preference)

\[ a_{ij} = 5 \] (strong preference)

\[ a_{ij} = 7 \] (very strong preference)

\[ a_{ij} = 9 \] (extreme preference)

\[ a_{ji} = 1/a_{ij} \]

Using these guidelines, the strength of the preference among the criteria was obtained from the company executive as shown in Table 6.9.

<table>
<thead>
<tr>
<th></th>
<th>Capital invested in inventory</th>
<th>Annual number of orders</th>
<th>Annual transportation costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital invested in inventory</td>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Annual number of orders</td>
<td>1/3</td>
<td>1</td>
<td>1/2</td>
</tr>
<tr>
<td>Annual transportation costs</td>
<td>1/2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Column sum</td>
<td>1.83</td>
<td>6</td>
<td>3.5</td>
</tr>
</tbody>
</table>

Table 6.9 is normalized by dividing each cell value in a particular column by that column sum. The normalized matrix is shown in Table 6.10. The weights of the criteria are obtained by computing the row averages in Table 6.10.

<table>
<thead>
<tr>
<th></th>
<th>Capital invested in inventory</th>
<th>Annual number of orders</th>
<th>Annual transportation costs</th>
<th>Weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital invested in inventory</td>
<td>0.55</td>
<td>0.5</td>
<td>0.57</td>
<td>0.54</td>
</tr>
<tr>
<td>Annual number of orders</td>
<td>0.18</td>
<td>0.17</td>
<td>0.14</td>
<td>0.16</td>
</tr>
<tr>
<td>Annual transportation costs</td>
<td>0.27</td>
<td>0.33</td>
<td>0.29</td>
<td>0.3</td>
</tr>
</tbody>
</table>
The weight combinations that best described the company executive’s preference by the three methods were (0.47,0.2,0.33), (0.5,0.17,0.33) and (0.54,0.16,0.3). The efficient solutions for the warehouse are given in Table 6.11. Since the ordering policy is the same for the three weight combinations, we decided to generate two extreme efficient solutions, i.e., one that minimizes capital invested in inventory, and the other that minimizes annual transportation costs.

**Table 6.11: Efficient Solutions for the Warehouse**

<table>
<thead>
<tr>
<th>Weight</th>
<th>(Q_W)</th>
<th>(T_W) (weeks)</th>
<th>Criterion1($)</th>
<th>Criterion2</th>
<th>Criterion3($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.47,0.2,0.33)</td>
<td>123276.47</td>
<td>13.59</td>
<td>1175521.57</td>
<td>3.83</td>
<td>4247.39</td>
</tr>
<tr>
<td>(0.5,0.17,0.33)</td>
<td>123276.47</td>
<td>13.59</td>
<td>1175521.57</td>
<td>3.83</td>
<td>4247.39</td>
</tr>
<tr>
<td>(0.54,0.16,0.3)</td>
<td>123276.47</td>
<td>13.59</td>
<td>1175521.57</td>
<td>3.83</td>
<td>4247.39</td>
</tr>
<tr>
<td>(0.7,0.1,0.2)</td>
<td>61638.24</td>
<td>6.79</td>
<td>586876.41</td>
<td>7.65</td>
<td>8494.78</td>
</tr>
<tr>
<td>(0.35,0.2,0.45)</td>
<td>184914.71</td>
<td>20.38</td>
<td>1764166.72</td>
<td>2.55</td>
<td>2831.59</td>
</tr>
</tbody>
</table>

Observe from Table 6.11 that for weight combination (0.7,0.1,0.2), the reorder interval is 6.79 weeks. Hence, the capital invested in inventory is reduced. This is due to the importance given to the capital invested in inventory criterion. In the single objective method, the DM would not have the opportunity to look at this alternative. Before we discuss the selection of the most preferred solution, it is important to discuss some of the problems that we faced and how they were resolved.

### 6.4.3 Difficulties Encountered

As in the previous study, including all the retailers while solving the warehouse’s problem leads to an increase in the reorder interval and hence, the average inventory at the warehouse. This is due to the range of the demand data. To resolve this issue, we identify the retailers in clusters 1 and 2 from Table 6.6, and group them in Tier I. Similarly, the retailers in clusters 3 and 4 are grouped in Tier II.

The ordering policy of the retailers in Tier I is going to be the same as in Table 6.6. For the retailers in Tier II, the base period should be identified. The base retailer in Tier II is Retailer 16 and the base period is 1.5263 weeks. The non-base retailers in Tier II have to resolve the modified problem with the constraint that their reorder interval is an integer multiple of 1.5263 weeks. The results are shown in Table 6.12.
Table 6.12: Output of the Modified Problem for Retailers in Tier II

<table>
<thead>
<tr>
<th>Retailers</th>
<th>Cluster</th>
<th>Q(units)</th>
<th>T(weeks)</th>
<th>Capital invested in inventory($)</th>
<th>Annual number of orders</th>
<th>Annual Transportation costs($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>1</td>
<td>258.56</td>
<td>1.5263</td>
<td>4388.60</td>
<td>34.07</td>
<td>22879.96</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>262.14</td>
<td>1.5263</td>
<td>4449.38</td>
<td>34.07</td>
<td>23085.95</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>195.13</td>
<td>1.5263</td>
<td>3311.99</td>
<td>34.07</td>
<td>19046.13</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>298.86</td>
<td>3.0526</td>
<td>4162.10</td>
<td>17.0348</td>
<td>12572.31</td>
</tr>
<tr>
<td>11</td>
<td>2</td>
<td>401.94</td>
<td>3.0526</td>
<td>5597.70</td>
<td>17.04</td>
<td>15250.16</td>
</tr>
<tr>
<td>13</td>
<td>2</td>
<td>423.25</td>
<td>3.0526</td>
<td>5894.46</td>
<td>17.04</td>
<td>15772.22</td>
</tr>
</tbody>
</table>

The warehouse’s problem is solved separately for Tier I and Tier II retailers. For Tier I retailers, the warehouse’s efficient solutions are given in Table 6.13.

Table 6.13: Output of the Warehouse’s Problem Including Tier I Retailers

<table>
<thead>
<tr>
<th>Weight</th>
<th>Q_w</th>
<th>T_w(weeks)</th>
<th>Criterion1($)</th>
<th>Criterion2</th>
<th>Criterion3($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.47,0.2,0.33)</td>
<td>18649.85</td>
<td>2.2649</td>
<td>185075.68</td>
<td>22.96</td>
<td>25484.33</td>
</tr>
<tr>
<td>(0.5,0.17,0.33)</td>
<td>18649.85</td>
<td>2.2649</td>
<td>185075.68</td>
<td>22.96</td>
<td>25484.33</td>
</tr>
<tr>
<td>(0.54,0.16,0.3)</td>
<td>18649.85</td>
<td>2.2649</td>
<td>185075.68</td>
<td>22.96</td>
<td>25484.33</td>
</tr>
<tr>
<td>(0.7,0.1,0.2)</td>
<td>9324.93</td>
<td>1.1324</td>
<td>96022.64</td>
<td>45.92</td>
<td>50968.66</td>
</tr>
<tr>
<td>(0.35,0.2,0.45)</td>
<td>27974.78</td>
<td>3.3973</td>
<td>274128.72</td>
<td>15.31</td>
<td>16989.55</td>
</tr>
</tbody>
</table>

Note that the weight of the shipment of the warehouse for any alternative is greater than 46000 lbs, which is the maximum weight that can be shipped using a truck. The model formulation does not account for the truck capacity. Hence, the annual transportation cost at the warehouse is underestimated.

The heuristic proposed in Chapter 4, Section 4.5, Note 1, is used to calculate the actual annual transportation costs. If the warehouse orders Q_wω lbs every cycle and the maximum allowable weight is W_x lbs, then the number of trucks needed is \( \left[ \frac{Q_w \omega}{W_x} \right] \). If the number of TL is \( \left[ \frac{Q_w \omega}{W_x} \right] \) and the weight shipped in the \( \left[ \frac{Q_w \omega}{W_x} \right]^{th} \) truck is less than the weight requirement for a TL, then the firm is charged a fixed TL rate/ full truck and a LTL rate for the \( \left[ \frac{Q_w \omega}{W_x} \right]^{th} \) truck.
A sample calculation to determine the actual annual transportation costs for the weight combination (0.47,0.2,0.33) is as follows:

The number of full truck loads = \( \frac{745994.05}{46000} \) = 16. The weight shipped by the 17th truck = 745994.05-16*46000 = 9994.05 lbs. This weight is less than 18257 lbs, the weight requirement for declaring the shipment as a truck load. Hence, the power function should be used. The actual annual transportation cost is computed as (Cost of shipping full TL + Cost of shipping LTL)*Annual number of orders placed:

\[
\left\{ \frac{16*1110 + 32.686(9994.05 / 100)^{-0.3484} 9994.05}{100} \right\} * 22.96 = $422834.9
\]

Similarly the actual annual transportation costs for weight combinations (0.7,0.1,0.2) and (0.35,0.2,0.45) are $426953.5 and $420848.5.

Observe that the difference in actual annual transportation costs between different alternatives is not large when compared to the difference in capital invested in inventory even though the difference in the order quantity is large. This is because in practice there is no discount for number of trucks used in an order cycle. Since, the volume of annual demand is going to be the same, the transportation costs due to shipment using full truck loads is going to be the same. The difference in the actual annual transportation costs arises due to the amount shipped using LTL, which is minor. Hence, the company executive had no hesitation in choosing the solution corresponding to weight combination (0.7,0.1,0.2) as the most preferred solution. The ordering policy at the warehouse for Tier I retailers is to order 9324.93 units when inventory reduces to 3519.35 units. The reorder interval is 1.1324 weeks.

For Tier II the warehouse’s efficient solutions are given in Table 6.14.

**Table 6.14: Output of the Warehouse’s Problem Including Tier II Retailers**

<table>
<thead>
<tr>
<th>Weight</th>
<th>QW</th>
<th>T_W(weeks)</th>
<th>Criterion1($)</th>
<th>Criterion2</th>
<th>Criterion3($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.47,0.2,0.33)</td>
<td>5111.40</td>
<td>6.1051</td>
<td>38077.66</td>
<td>8.52</td>
<td>9454.17</td>
</tr>
<tr>
<td>(0.5,0.17,0.33)</td>
<td>5111.40</td>
<td>6.1051</td>
<td>38077.66</td>
<td>8.52</td>
<td>9454.17</td>
</tr>
<tr>
<td>(0.54,0.16,0.3)</td>
<td>2555.70</td>
<td>3.0526</td>
<td>13670.70</td>
<td>17.03</td>
<td>18908.34</td>
</tr>
<tr>
<td>(0.7,0.1,0.2)</td>
<td>2555.70</td>
<td>3.0526</td>
<td>13670.70</td>
<td>17.03</td>
<td>18908.34</td>
</tr>
<tr>
<td>(0.35,0.2,0.45)</td>
<td>5111.40</td>
<td>6.1051</td>
<td>38077.66</td>
<td>8.52</td>
<td>9454.17</td>
</tr>
</tbody>
</table>
Again, the actual annual transportation costs must be computed. The actual annual transportation costs for $Q_W = 5111.40$ units and $Q_W = 2555.70$ units are $47271.70$ and $49174.85$. In the former, all the shipments are using TL while in the latter, two full TL and one LTL is used. Since the difference in actual annual transportation is less when compared to the difference in capital invested inventory, the company executive decided to go with the alternative that has the least inventory capital which is either weight combination (0.54,0.16,0.3) or (0.7,0.1,0.2). The ordering policy for the warehouse to serve its Tier II retailers is to order 2555.70 units when inventory reduces to 357.83 units. The reorder interval is 3.0526 weeks.

To use the value path method, the ideal value of the criteria must be determined. The ideal value is obtained by optimizing the criterion independent of another. Since we use a heuristic to determine the actual annual transportation costs, the value path method cannot be used.

The executive of the Fortune 500 company who was the DM for the warehouse, preferred the multi-criteria methodology over the single objective methodology for determining ordering policies. He felt more involved in the decision making process as he had to evaluate tradeoffs and use his judgment to choose the most preferred solution. This further validated the advantages of using a multi-criteria approach. Also, the cognitive burden during the decision making process was less as he had to analyze the tradeoff information among three efficient solutions.

### 6.5 Case Study 3: Chapter 5

#### 6.5.1 Overview

In this study, the models and methodology developed in Chapter 5 are applied to solve the inventory planning problem when there is uncertainty in the system in the form of stochastic demands and stochastic lead times. In addition to the three criteria in the previous study, fill rate (FR) is the fourth criterion and is used to measure customer satisfaction. Since the locations are assumed to follow a continuous review policy, the reorder interval is a random variable. The decision variables are order quantity and reorder point. Due to high FR maintained by the company, the average time spent by the system in a stockout situation is small. In general, this is also true for the retailers. Excess
demand is treated as lost sales at all the locations. As in Case Study 2, retailers’ shipments are LTL and the warehouse’s shipment is TL. The DM for each location must specify upper and lower bounds on the order quantity and the reorder point. Since we were using monthly data, the demands were large enough to approximate either the Poisson or geometric Poisson distribution by a normal distribution. Hence, it was sufficient to check for the normality of the monthly demand data. The K-S test (Ravindran et al., 1987, pp. 622-623) in Minitab was used to check for normality. The results are given in Table 6.15.

### Table 6.15: Normality Test for Demand

<table>
<thead>
<tr>
<th>Retailers</th>
<th>K-S test</th>
<th>Mean</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>✓</td>
<td>1603</td>
<td>710.6</td>
</tr>
<tr>
<td>2</td>
<td>✓</td>
<td>1445</td>
<td>830</td>
</tr>
<tr>
<td>3</td>
<td>✓</td>
<td>1944</td>
<td>1170</td>
</tr>
<tr>
<td>4</td>
<td>✓</td>
<td>4424</td>
<td>1118</td>
</tr>
<tr>
<td>5</td>
<td>✓</td>
<td>477.3</td>
<td>166.8</td>
</tr>
<tr>
<td>6</td>
<td>✓</td>
<td>623.3</td>
<td>364.2</td>
</tr>
<tr>
<td>7</td>
<td>?</td>
<td>1083</td>
<td>1463</td>
</tr>
<tr>
<td>8</td>
<td>✓</td>
<td>3190</td>
<td>1235</td>
</tr>
<tr>
<td>9</td>
<td>✓</td>
<td>6068</td>
<td>823.3</td>
</tr>
<tr>
<td>10</td>
<td>✓</td>
<td>3583</td>
<td>899.6</td>
</tr>
<tr>
<td>11</td>
<td>✓</td>
<td>641.9</td>
<td>425.2</td>
</tr>
<tr>
<td>12</td>
<td>✓</td>
<td>3867</td>
<td>1564</td>
</tr>
<tr>
<td>13</td>
<td>✓</td>
<td>675.9</td>
<td>272.7</td>
</tr>
<tr>
<td>14</td>
<td>X</td>
<td>2794</td>
<td>1375</td>
</tr>
<tr>
<td>15</td>
<td>✓</td>
<td>2347</td>
<td>586</td>
</tr>
<tr>
<td>16</td>
<td>✓</td>
<td>825.9</td>
<td>416.8</td>
</tr>
<tr>
<td>17</td>
<td>✓</td>
<td>1338</td>
<td>360</td>
</tr>
<tr>
<td>18</td>
<td>✓</td>
<td>4836</td>
<td>833.8</td>
</tr>
<tr>
<td>19</td>
<td>X</td>
<td>921.7</td>
<td>336</td>
</tr>
<tr>
<td>20</td>
<td>✓</td>
<td>1781</td>
<td>415.5</td>
</tr>
</tbody>
</table>

Assuming a significance level of 0.05, the demand for most of the retailers can be approximately described by a normal distribution. For Retailers 14 and 19, the p values are 0.043 and 0.046, respectively. For Retailer 7, the test is inconclusive as the p value is less than 0.01. We believe that approximating the demand for these retailers as a normal distribution would not cause any serious deviation in the ordering policy.
As mentioned before, the lead time varies between one and three days. The distribution for the lead time as quantified by the company is given in Table 6.16.

Table 6.16: Distribution of Lead Time

<table>
<thead>
<tr>
<th>Lead time</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>0.3</td>
</tr>
<tr>
<td>3</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Again, Excel solver with VB macros was used to solve the optimization problems, and present the efficient solutions to the DM. The decision making process was simulated by using the company executive to identify the most preferred solution at the warehouse.

6.5.2 Results

The retailers’ most preferred solution is shown in Table 6.17.

Table 6.17: Ordering Policy of the Retailers

<table>
<thead>
<tr>
<th>Retailer</th>
<th>Weight</th>
<th>Q</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(0.35,0.1,0.2,0.35)</td>
<td>853.58</td>
<td>164.48</td>
</tr>
<tr>
<td>2</td>
<td>(0.35,0.1,0.1,0.45)</td>
<td>685.44</td>
<td>203.27</td>
</tr>
<tr>
<td>3</td>
<td>(0.35,0.05,0.2,0.4)</td>
<td>1048.02</td>
<td>278.84</td>
</tr>
<tr>
<td>4</td>
<td>(0.33,0.1,0.15,0.42)</td>
<td>969.13</td>
<td>354.10</td>
</tr>
<tr>
<td>5</td>
<td>(0.6,0.1,0.1,0.2)</td>
<td>300.00</td>
<td>63.69</td>
</tr>
<tr>
<td>6</td>
<td>(0.33,0.1,0.15,0.42)</td>
<td>345.00</td>
<td>101.47</td>
</tr>
<tr>
<td>7</td>
<td>(0.35,0.05,0.1,0.5)</td>
<td>700.00</td>
<td>239.89</td>
</tr>
<tr>
<td>8</td>
<td>(0.35,0.05,0.15,0.45)</td>
<td>877.27</td>
<td>277.26</td>
</tr>
<tr>
<td>9</td>
<td>(0.38,0.05,0.1,0.47)</td>
<td>1024.16</td>
<td>448.18</td>
</tr>
<tr>
<td>10</td>
<td>(0.35,0.05,0.1,0.5)</td>
<td>756.22</td>
<td>306.14</td>
</tr>
<tr>
<td>11</td>
<td>(0.4,0.08,0.12,0.4)</td>
<td>440.10</td>
<td>123.40</td>
</tr>
<tr>
<td>12</td>
<td>(0.3,0.05,0.15,0.5)</td>
<td>1077.40</td>
<td>423.35</td>
</tr>
<tr>
<td>13</td>
<td>(0.3,0.1,0.15,0.45)</td>
<td>351.35</td>
<td>109.13</td>
</tr>
<tr>
<td>14</td>
<td>(0.3,0.05,0.12,0.53)</td>
<td>954.00</td>
<td>362.53</td>
</tr>
<tr>
<td>15</td>
<td>(0.33,0.1,0.15,0.42)</td>
<td>719.26</td>
<td>251.79</td>
</tr>
<tr>
<td>16</td>
<td>(0.34,0.05,0.15,0.46)</td>
<td>415.51</td>
<td>157.04</td>
</tr>
<tr>
<td>17</td>
<td>(0.37,0.1,0.13,0.4)</td>
<td>477.00</td>
<td>148.41</td>
</tr>
<tr>
<td>18</td>
<td>(0.35,0.1,0.15,0.4)</td>
<td>1093.39</td>
<td>432.80</td>
</tr>
<tr>
<td>19</td>
<td>(0.41,0.1,0.1,0.39)</td>
<td>345.80</td>
<td>112.75</td>
</tr>
<tr>
<td>20</td>
<td>(0.39,0.05,0.15,0.41)</td>
<td>496.33</td>
<td>183.17</td>
</tr>
</tbody>
</table>
The criteria values are given in Table 6.18. In addition to the Fill Rate (FR), the Cycle Service Level (CSL) is also computed.

**Table 6.18: Criteria Values for Retailers**

<table>
<thead>
<tr>
<th>Retailer</th>
<th>Capital invested in Inventory</th>
<th>Annual number of orders</th>
<th>Annual transportation costs</th>
<th>Fill Rate</th>
<th>Cycle Service Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10514.24</td>
<td>22.85</td>
<td>33413.63</td>
<td>95.4352%</td>
<td>69.0440%</td>
</tr>
<tr>
<td>2</td>
<td>9750.91</td>
<td>25.65</td>
<td>32512.68</td>
<td>94.9873%</td>
<td>74.8537%</td>
</tr>
<tr>
<td>3</td>
<td>14552.91</td>
<td>22.57</td>
<td>37725.50</td>
<td>95.3436%</td>
<td>74.5777%</td>
</tr>
<tr>
<td>4</td>
<td>12361.92</td>
<td>55.54</td>
<td>88225.69</td>
<td>92.5072%</td>
<td>67.6796%</td>
</tr>
<tr>
<td>5</td>
<td>3921.02</td>
<td>19.36</td>
<td>14321.82</td>
<td>98.3637%</td>
<td>82.5708%</td>
</tr>
<tr>
<td>6</td>
<td>5013.60</td>
<td>21.98</td>
<td>17813.80</td>
<td>96.4948%</td>
<td>79.1759%</td>
</tr>
<tr>
<td>7</td>
<td>11102.35</td>
<td>18.82</td>
<td>24189.78</td>
<td>90.4562%</td>
<td>71.2450%</td>
</tr>
<tr>
<td>8</td>
<td>11247.97</td>
<td>44.24</td>
<td>65862.66</td>
<td>90.9546%</td>
<td>65.3459%</td>
</tr>
<tr>
<td>9</td>
<td>12964.46</td>
<td>72.09</td>
<td>118704.84</td>
<td>94.0042%</td>
<td>70.3183%</td>
</tr>
<tr>
<td>10</td>
<td>10117.08</td>
<td>57.65</td>
<td>77904.38</td>
<td>93.0709%</td>
<td>70.5724%</td>
</tr>
<tr>
<td>11</td>
<td>6450.63</td>
<td>17.75</td>
<td>16853.54</td>
<td>97.2766%</td>
<td>81.7393%</td>
</tr>
<tr>
<td>12</td>
<td>15614.07</td>
<td>43.67</td>
<td>74324.01</td>
<td>93.0748%</td>
<td>72.8022%</td>
</tr>
<tr>
<td>13</td>
<td>5178.84</td>
<td>23.41</td>
<td>19194.70</td>
<td>98.2906%</td>
<td>86.2082%</td>
</tr>
<tr>
<td>14</td>
<td>14314.35</td>
<td>35.63</td>
<td>56025.80</td>
<td>93.9963%</td>
<td>75.1426%</td>
</tr>
<tr>
<td>15</td>
<td>10054.65</td>
<td>39.70</td>
<td>51928.91</td>
<td>96.9958%</td>
<td>80.1861%</td>
</tr>
<tr>
<td>16</td>
<td>6680.63</td>
<td>24.18</td>
<td>22123.27</td>
<td>97.9863%</td>
<td>86.9097%</td>
</tr>
<tr>
<td>17</td>
<td>6537.29</td>
<td>34.13</td>
<td>34158.25</td>
<td>97.2903%</td>
<td>80.3036%</td>
</tr>
<tr>
<td>18</td>
<td>14830.31</td>
<td>53.81</td>
<td>92472.39</td>
<td>96.3270%</td>
<td>77.4906%</td>
</tr>
<tr>
<td>19</td>
<td>4904.79</td>
<td>32.43</td>
<td>26320.65</td>
<td>96.4371%</td>
<td>78.6908%</td>
</tr>
<tr>
<td>20</td>
<td>6944.47</td>
<td>43.66</td>
<td>44842.81</td>
<td>96.7311%</td>
<td>79.5196%</td>
</tr>
</tbody>
</table>

The daily demand at the warehouse is approximated as a normal distribution with mean 1482.30 and variance 243.59. The lead time distribution is given in Table 6.16. In Case Study 2, the company executive’s preference information was captured using three methods. Using those methods, the weights are computed as shown in Tables 6.19, 6.20, 6.21 and 6.22, respectively.
Table 6.19: Weights Using Method 1

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Score (1-10)</th>
<th>Weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital invested in inventory</td>
<td>7</td>
<td>$7/25=0.28$</td>
</tr>
<tr>
<td>Annual number of orders</td>
<td>3</td>
<td>$3/25=0.12$</td>
</tr>
<tr>
<td>Annual transportation costs</td>
<td>5</td>
<td>$5/25=0.2$</td>
</tr>
<tr>
<td>Fill rate</td>
<td>10</td>
<td>$10/25=0.4$</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>25</strong></td>
<td></td>
</tr>
</tbody>
</table>

Table 6.20: Weights Using Method 2

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Criterion 1</th>
<th>Criterion 2</th>
<th>Criterion 3</th>
<th>Criterion 4</th>
<th>Total</th>
<th>Weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>Criterion 1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td>$3/10=0.3$</td>
</tr>
<tr>
<td>Criterion 2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>$1/10=0.1$</td>
</tr>
<tr>
<td>Criterion 3</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>$2/10=0.2$</td>
</tr>
<tr>
<td>Criterion 4</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>4</td>
<td>$4/10=0.4$</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>10</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6.21: Method 3-Strength of Preferences

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Criterion 1</th>
<th>Criterion 2</th>
<th>Criterion 3</th>
<th>Criterion 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Criterion 1</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>$1/2$</td>
</tr>
<tr>
<td>Criterion 2</td>
<td>$1/3$</td>
<td>1</td>
<td>$\frac{1}{2}$</td>
<td>$1/4$</td>
</tr>
<tr>
<td>Criterion 3</td>
<td>$1/2$</td>
<td>2</td>
<td>1</td>
<td>$1/3$</td>
</tr>
<tr>
<td>Criterion 4</td>
<td>2</td>
<td>4</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Column sum</td>
<td>3.83</td>
<td>10</td>
<td>6.5</td>
<td>2.08</td>
</tr>
</tbody>
</table>

Table 6.22: Weights Using Method 3

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Criterion 1</th>
<th>Criterion 2</th>
<th>Criterion 3</th>
<th>Criterion 4</th>
<th>Weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>Criterion 1</td>
<td>0.26</td>
<td>0.33</td>
<td>0.31</td>
<td>0.24</td>
<td>0.285</td>
</tr>
<tr>
<td>Criterion 2</td>
<td>0.09</td>
<td>0.1</td>
<td>0.08</td>
<td>0.12</td>
<td>0.09</td>
</tr>
<tr>
<td>Criterion 3</td>
<td>0.13</td>
<td>0.2</td>
<td>0.15</td>
<td>0.16</td>
<td>0.16</td>
</tr>
<tr>
<td>Criterion 4</td>
<td>0.52</td>
<td>0.4</td>
<td>0.46</td>
<td>0.48</td>
<td>0.465</td>
</tr>
</tbody>
</table>

The weight combinations that best describe the company executive’s preference are $(0.28,0.12,0.2,0.4)$, $(0.3,0.1,0.2,0.4)$, and $(0.285,0.09,0.16,0.465)$. The efficient solutions for the warehouse are given in Table 6.23.
Table 6.23: Efficient Solutions for Warehouse

<table>
<thead>
<tr>
<th>Weights</th>
<th>Q</th>
<th>R</th>
<th>Criterion1</th>
<th>Criterion2</th>
<th>Criterion3</th>
<th>Criterion4</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.28,0.12,0.2,0.4)</td>
<td>23810.12</td>
<td>4479.67</td>
<td>265000.92</td>
<td>22.72</td>
<td>25222.18</td>
<td>99.9888%</td>
</tr>
<tr>
<td>(0.3,0.1,0.2,0.4)</td>
<td>22273.24</td>
<td>4479.54</td>
<td>250321.44</td>
<td>24.29</td>
<td>26962.54</td>
<td>99.9880%</td>
</tr>
<tr>
<td>(0.285,0.09,0.16,0.465)</td>
<td>20862.19</td>
<td>4493.45</td>
<td>237096.99</td>
<td>25.93</td>
<td>28786.19</td>
<td>99.9908%</td>
</tr>
</tbody>
</table>

The CSL values are 93.7683%, 93.7548% and 95.1488%, respectively. After observing the criteria values in Table 6.23, the company executive felt that the capital invested in inventory was on the higher side. By increasing the weight assigned to the capital invested in inventory and decreasing the weight assigned to FR, a new efficient solution was obtained. The new weight combination was (0.42,0.09,0.16,0.33). The criteria values corresponding to this combination were: Capital invested in inventory = $201573.31, Annual number of orders = 31.46, Annual transportation costs = $34918.55 and Fill rate = 99.9778 %. The CSL was computed to be 91.9735%. The ordering policy of the warehouse was to order 17198.39 units when the reorder point reached 4463.57 units.

As in Case Study 2, the weight of the warehouse’s shipment was greater than 46000 lbs, the weight of a single truck. Hence, the annual transportation cost was underestimated. By using the same heuristic method in Section 6.4.3, the actual annual transportation cost was calculated as $573993.71. The warehouse would use 15 trucks to transport this shipment.

6.6 Summary

In this chapter we showed that the theoretical models developed in this thesis can be used for inventory planning for a product produced by a Fortune 500 consumer products company under different scenarios. We had to make suitable modifications to the existing methodologies to account for some problems faced while adapting them to the real world data. This study further validated some of the advantages of using a multi-criteria methodology for determining inventory policies in a decentralized supply chain.

Chapter 7 discusses the conclusions of this research and future work.
Chapter 7
CONCLUSIONS AND FUTURE WORK

7.1 Overview

The highly competitive nature of today’s market makes it extremely difficult for a company to function as an independent entity. A company is better off being part of a supply chain where they can contribute to the profitability of the supply chain by concentrating on its core competencies. Thus, each company will have its own Decision maker (DM) trying to optimize its own objectives. To be successful, the company has to integrate and coordinate its activities with the activities of the rest of the supply chain. This is one of the major challenges facing supply chains today, and what is termed as “seamless” integration has still not been achieved. Achieving integration and coordination is dependent on the ability of the companies within the supply chain to share real time data. This is in turn dependent on the willingness of the companies to collaborate with each other and the IT capabilities that are in place.

Inventory management is a vital function that companies within supply chains must continually focus on as it leads to enormous cost savings not only for the individual companies but also for the entire supply chain. Each company must plan its inventory carefully so as to not stock too much and at the same time meet the demands of the downstream companies. There is a cascading effect in the sense that the inability to meet the demands of the downstream companies will translate into the end consumers not getting what they want. In the current environment where consumers are not willing to wait, they will turn to a competitor’s products. Hence, it is important to have the right inventory, in the right quantities, at the right place, at the right time.

Other functions such as transportation, facilities location, routing, etc. influence inventory ordering policies. Hence, it is important to consider one or more of these functions while determining the inventory policy at each location. As explained in Chapter 1, transportation costs contribute more to the GDP of the United States than inventory costs. Also, transportation costs are expected to rise in the future. Hence, it is of paramount importance to integrate the transportation function while determining inventory policies.
There are often several criteria of interest other than cost that conflict with each other. It is usually difficult to estimate a cost associated with these criteria in practice. One of the main advantages of using multi-criteria optimization techniques is that it can model conflicting criteria without using the cost information that is essential in solving single objective optimization models.

Inter-functional decisions are made by evaluating the tradeoffs associated with different alternatives. A multi-criteria approach offers the DM exactly this, wherein, by assigning different weights to each criterion, several efficient solutions (alternatives) can be generated from which the most preferred solution is chosen. In this way the DM incorporates his/her preference information in the final ordering policy. A single objective optimization model provides the DM with a single optimal or heuristic solution, which is independent of the DM’s preference information.

7.2 Conclusions

Keeping the importance of coordination in supply chains we first proposed a modified base period policy using a single objective framework when demands are assumed to be deterministic. The objective of the policy was to enable the warehouse to have a better visibility of all the retailer orders so that it can plan its inventory and at the same time meet the demands of the retailers without deviating too much from their requirements. Instead of specifying an arbitrary value for the base period (as is common in the literature), a performance metric that measures how quickly a retailer turns over its inventory is used. This was validated by a Fortune 500 consumer products company. Using this metric and the structure of the optimal solution, the warehouse can evaluate its retailers, and classify them into clusters based on how important they are to the warehouse. This policy was tested with real world data obtained from a Fortune 500 company for one of its functional products which is non-seasonal and non-perishable. Based on the results, it was concluded that the policy performed very well for the retailers for a wide range of demand data, i.e., the warehouse was able to meet the requirements without deviating too much from the optimal policy. However, the range of the demand data had an impact on the ordering period of the warehouse in the sense that to accommodate all the retailers, the warehouse had to order in larger quantities increasing
its inventory holding costs. One way of circumventing this problem was for the warehouse to group the clusters of retailers into Tier I, Tier II, etc. and re-solve the problem independently for each tier.

To avoid the problem of estimating marginal cost information and to incorporate the DM’s preference information, we formulated the above problem as a multi-criteria problem where transportation costs were considered between the echelons. The three criteria that were considered were the capital invested in inventory, annual number of orders placed and, annual transportation costs. Transportation costs were modeled using continuous functions that best fit the freight rate data. Under the assumption that the retailers’ shipments were LTL and the warehouse’s shipment was TL, efficient solutions were derived. The value path method was used to graphically display the tradeoff information to the DM. A general model was formulated which will indicate to each location whether to use TL or LTL depending on the weigh of the shipment. The former model (where retailers’ shipments are LTL and warehouse’s shipment is TL) was tested with the same data set used in the previous chapter. To make the decision making process more realistic, an executive from the Fortune 500 company was asked to be the DM. His preference information was obtained using standard multi-criteria techniques such as rating method, pairwise comparison method and AHP. The weights obtained from the different methods were used to generate a set of efficient solutions. Based on the feedback we received, the company executive was very comfortable with the multi-criteria model since he had the option of choosing from several different efficient solutions. He was able to analyze the tradeoffs in an easy manner. By assigning more priority to a particular criterion he was able to obtain the solution of his preference. One of the problems faced while using the theoretical model was due to the range of the retailer demand data. The other problem was due to the assumption that the truck employed by the warehouse did not have any capacity constraint. Hence, the transportation cost at the warehouse was underestimated. We provided a simple heuristic to resolve this issue.

In the last part of the thesis, we modeled and solved the multi-criteria inventory problem with transportation costs when both demand and lead times were assumed to be random variables. Fill rate (FR) was used as the fourth criterion to measure customer
service. From a modeling standpoint it is important to determine the distribution of the lead time demand (LTD). For the case of Poisson and normal demands and lead times following any discrete distribution, we used the convex combination approach to quantify the LTD as Poisson and normal respectively. We proved that the convex combination approach can also be used for the case of a geometric Poisson demand. Using renewal theory concepts and the convex combination approach, the LTD at the warehouse was characterized as a normal distribution. Efficient solutions were generated by changing the weights assigned to the criteria. We recommend the use of a cycle service level (CSL) in conjunction with the FR criterion, since there is a possibility that CSL could be very low even when the FR is at an acceptable level. Value path graphs were used to present the tradeoffs to the DM. The model was tested with same data set. Again, we used a real DM to determine the ordering policy at the warehouse by evaluating the tradeoffs associated with the four criteria.

Microsoft Excel was used to implement the models developed in this thesis. For the single objective model, Excel was used with VB macros to automate the solution methodology and generate optimal solutions. For the models developed in Chapter 4 and Chapter 5, Excel Solver was used with VB macros to solve for different efficient solutions according to the weight inputs provided by the DM.

7.3 Managerial Implications

A summary of some of the important managerial implications of the models developed in this thesis are as follows:

- The cluster analysis developed using model 1 can be used by a company with other performance metrics to identify their best customers. This is important due to the concept of revenue management gaining momentum in supply chains. Based on this cluster analysis, the company can provide better service to its most preferred customers.

- The models developed in the thesis can be used by a company to serve its customers that face non-identical demands. For example, for models 1 and 2, we use a tier based approach to reduce costs at the company, and at the same time maintain the quality of the solution at the customers. For the stochastic case in
model 3, we construct the demand process at the warehouse under the assumption that the external demands are non-identical random variables.

- Models 2 and 3 (the multi-criteria models) were specifically developed to involve the manager (DM) of a company in the solution process so that the ordering policy reflects the manager’s preference information. The preference information in the form of weights of the criteria is obtained by using standard methods discussed in Chapter 6. These weights are used in the optimization problem to generate the efficient solutions. To enable the manager choose the most preferred solution, a graphical based tool called value path method is used to present the tradeoff information. Tradeoff analysis can be performed by changing the weights assigned to the criteria. Since the manager analyzes the tradeoff information, the chosen solution reflects his/her’s preference.

- What-if analysis can be performed by changing the parameters such as the number of retailers, demand and lead time distributions, cost of the product, etc.

### 7.4 Limitations

Some of the limitations of the models developed in this thesis are:

- The models are restricted to two echelons/stages.
- The manufacturer that supplies the warehouse is assumed to have infinite capacity.
- The demands faced by the retailers are assumed to be stationary, or demand distributions that do not vary with time.

### 7.5 Future Work

This section describes some possible extensions from this thesis and in general other emerging areas in the field of supply chain management.

- This work can be extended by incorporating the facilities location function and analyze how transportation costs and inventory policies are affected. Treating the problem from a multi-criteria perspective will facilitate capturing all the necessary tradeoff information before choosing the final solution.
• An interesting problem that is still in its nascent stage is quantifying risks in supply chains. This is relevant due to global outsourcing. Outsourcing is done primarily to take advantage of cheap labor and raw materials. However, companies fail to take into account the risk factor, e.g., stability of suppliers, disruptions in the flow of goods due to uncontrollable events, long lead times, etc. Incorporating these factors in an integrated framework is essential for the supply chain to first decide if the outsourcing venture is going to be beneficial. It then has to decide how it can continue to provide the same level of service as there is an increase in transit time. Again, it would be advantageous to treat it as a multi-criteria problem due to the number of conflicting criteria and to quantify the tradeoffs associated with these criteria.

• For coordination schemes such as the one proposed in Chapter 3 to work successfully, the benefits (profits) obtained as a result of following the scheme must be shared fairly among the various partners in the supply chain. This will enable these companies to collaborate on a more frequent basis, which is a cornerstone for resolving conflicts and improving existing partnerships.
BIBLIOGRAPHY


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