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Abstract

This thesis consists of three essays. In the first essay we study monopolistic pricing, with a capacity constraint, of a good that loses its value T periods after it is put on sale for the first time. Buyers only obtain utility just before the good loses its value. Examples of such goods include airline tickets and hotel rooms. This essay presents a three-period model with a single seller facing a capacity constraint, where the good being sold loses its value after the third period. The seller offers a finite measure of units for sale in a market, where in each period a continuum of buyers, each of whom might be one of two types, enter. The seller chooses, without precommitment, price and measure of units to offer in each period. Each of the buyers chooses either to make a purchase as soon as they enter, or to wait for a lower price which might be made available in the future. This allows us to capture some features of airline ticket pricing. The equilibrium price path is obtained and found to be non-decreasing, U-shaped or horizontal for the relevant range of parameter values. Any strategy involving 'final sales' is found to be non-optimal.

In the second essay we examine the voluntary provision of a public project via binary contributions when contributions may be made over multiple rounds. In many situations, early contributors are likely to pay a higher cost than those who wait. We show that in such circumstances the provision of the project always involves delay. Since this game involves coordination on complex, dynamic strategies in the face of asymmetries in payoffs, we examine behavior in the laboratory.

In the third essay, we empirically test the theoretical predictions of the first essay. We collect airline price data from an online travel agent for 30 routes in the US, each with a different proportion of business travellers. We find that while prices never fall before departure, the price path is rising for routes with low, medium and high proportion of business travellers and that the increase in prices is sharpest for routes with the highest proportion of business travellers.

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Chapter 1

A Model of Airline Pricing: Capacity Constraints and Deadlines

1.1 Introduction

It is a well-known fact that passengers on the same flight, traveling in the same class, often end up paying different prices for their tickets. This is because the prices of such tickets vary over time, often within the span of a few hours. While buyers have the option of purchasing tickets months prior to the date of departure, casual observation suggests that the prices offered by an airline are high if a purchase is attempted too early, drop after a period of time and then prior to departure they rise again. An empirical study by Stavins (2001) indicates that five weeks prior to departure, prices start rising. Instead of monotonically reducing prices, selling every available seat and waiting for takeoff, the airline instead, chooses to save a certain number of seats for future buyers, who would be willing to pay a high price for the same seats.

This shows, that in order to solve for the optimal price path of such goods, we need a model with a finite time horizon, where one or many sellers while facing a capacity constraint, offer(s) a finite measure of units for sale. In each period, a continuum of buyers, each of whom might be one of two types, enters the market. The seller chooses, without precommitment, price and measure of units to offer in each period, while each of the buyers choose either to make a purchase as soon as they enter, or to wait for a lower price which might be made available in the future.

The operations research literature identifies airline ticket pricing as dynamic pricing (also known as yield management), where the product ceases to exist at a certain point in time and capacity can only be added at a very high marginal cost. The product being discussed here is non-durable, non-storable and cannot be resold. We consider an airline ticket to be a futures contract on a service to be provided by the airline in the future. As the airline attempts to sell tickets over time, it is in effect "signing" contracts with different customers on different terms. As the seller is unable to precommit to the terms of the contract in the future and is in effect competing against future versions of himself (herself), he (she) faces the same intertemporal and time-consistency problems as a durable-goods monopolist. Other examples of such products include hotel rooms, generated electricity or other "sell before" goods where transactions occur through a futures contract (McAfee & Velde). Given the similarities in the problems facing an agent signing multiple futures contracts (airline) and a durable goods monopolist, we can refer to the vast literature on time-consistency issues in a durable-goods monopoly.

In this chapter, we use a model which extends that of a durable-goods monopoly model by Conlisk, Gerstner and Sobel (CGS, 1984). In their infinite time horizon model, a new cohort of consumers enters the market in each period, interested in buying the product either immediately or after some time. The consumers in each group differ amongst themselves in terms of the valuation for the good. Some of these buyers choose to make a purchase in the same period, while others decide to wait for a lower price. Their decisions are based on the price in that particular period and expected prices in the future. Usually the single seller, who does not face any capacity constraint, prefers to sell the product at a price just low enough to sell immediately to consumers with a high willingness to pay (as long as revenue earned from selling to “high” type buyers exceeds revenue earned from selling to “low” type buyers). However, as sufficient number of consumers with a lower willingness to pay accumulate in the market, the seller holds a ‘sale’ by dropping the price low enough, so that buyers with lower willingness to pay can buy the product. This leads to an equilibrium where periodic ‘sales’ are held and the corresponding price path is cyclic. We extend this model, by introducing a capacity constraint for the single seller and by solving for the equilibrium for a finite time horizon. We solve for possible candidates for the subgame perfect outcome, and find that for the relevant range of parameter values, the optimal price path obtained is non-decreasing, U-shaped or horizontal. If we introduce a finite time horizon in the CGS model where the single seller does not face a capacity constraint, we can check that it is possible to derive the range of parameter values for which the optimal price path is U-shaped. This range of parameter values however is different (larger) from the range of parameter values for which we get a U-shaped price path with capacity constraint.

The intuition behind the U-shape could be as follows. Initially prices decrease in a manner similar to a Dutch auction. Incumbent buyers could choose either to make a purchase in the current period or to wait for a lower price in the future while running the risk of not getting a ticket, since in the meantime, all the remaining seats might be sold out. As the date of departure draws closer, prices are raised for tickets which remain unsold, with the expectation that high valuation buyers who enter the market in the periods just before departure, will make a purchase. The intuition behind the other possible shapes is discussed at the end of Section 4 of this chapter.

1.2 Review of Literature

As mentioned in the previous section, even though airline tickets are not durable, the intertemporal problems facing a seller of airline tickets are identical to those facing a durable goods monopolist. We thus begin the review of literature section by referring to the literature on durable goods monopoly. The problem of intertemporal price discrimination as faced by a durable-goods monopolist has been the focus of several papers over the years. In his seminal paper, Coase (1972) conjectured that a durable-goods monopolist would be unable to exert any monopoly power. This is because rational buyers would anticipate correctly that in the absence of precommitment to future prices, the monopolist would reduce prices in an attempt to cater to residual demand and would refuse to buy the product as long as prices remained above the competitive level. Papers by Stokey (1981), Bulow (1982) and Kahn (1986) have formally modeled durable-goods monopoly. Bulow constructs a two-period model and shows that inability on the part of the monopolist to credibly commit to the fact that it won't exploit residual demand in period 2 places a constraint on the price received in period 2, and that a durable-goods monopolist earns higher profits by renting the good instead of selling it. Stokey and

Kahn show that under the assumption of constant marginal cost and that the monopolist is unable to precommit to any future behavior, the Coase conjecture is true in the continuous-time limit with the discount factor approaching one.[1]

The Coase conjecture on the other hand has been shown to fail under a separate set of assumptions. Using Stokey's Rational Expectations Equilibrium (REE) concept, Bond and Samuelson (1984) show that if the length of the trading period is non-zero and if the good depreciates, the monopolist has to make replacement sales in order to maintain a fixed stock of the good and that the non-depreciation result that prices fall to the competitive level does not hold. Bagnoli, Salant and Swierzbinski (1989) show that none of the three conclusions of the durable-goods literature, namely the Coase conjecture, the result that the durable-goods monopolist can increase profits by pre-committing to a time path of prices (Stokey 1979, Sobel and Takahashi 1983), and Bulow's conclusion that a monopolist renting the product earns higher profits compared to one selling it, survive when the assumption of continuum of buyers is replaced by the assumption that the set of buyers is finite, albeit possibly very large.

There are two assumptions, which are crucial to our model. The first is that the seller faces a capacity constraint, while the second is the constant influx of new buyers. It has been found that the conjecture fails to hold under these assumptions. McAfee and

[1]Stokey initially shows that the conjecture is true in the continuous-time framework. She then proves that in the discrete-time model, as the length of the period approaches zero, the equilibrium approaches the one in the continuous-time case. However, if the length of the period does not converge to zero, the path of output chosen by the monopolist approaches the one chosen by the monopolist renter.

Kahn establishes that while the conjecture is true for the continuous-time case with constant marginal cost, the Coase intuition fails if the producer faces increasing marginal costs; even in the absence of pre-commitment, the producer produces too little. Such rising marginal costs could be attributed to the fact that the seller faces a capacity constraint.

Wiseman (2003) show that capacity costs of arbitrarily small degree can eliminate the zero profit conclusion. Capacity costs borne by the seller serve as a strong commitment device, as the choice of capacity enables the seller to slow the sales, reduce the fall in prices and thus permits the seller to set initial prices above marginal costs. They examine a model in which there is a small cost for production capacity, and the seller can augment capacity at the beginning of every period. As the time between sales periods and the gap between the lowest valuation for the good and marginal cost shrink, they show that the monopolist earns the same profit irrespective of whether he or she chose capacity ex-ante or if capacity was augmented at the beginning of each period. Such profits were found to be 29.8% of static monopoly profit.

Papers by Sobel (1984), Conlisk, Gerstner and Sobel (1984) show that the equilibrium in a model with a continual influx of new buyers involves price cycles where each seller produces a homogeneous good and sells it to consumers with different willingness to pay entering the market in each period. In Sobel's model (which has all the same features as the CGS model but has more than one seller), sellers typically vary their prices over time, charging a high price in most periods, allowing buyers with high willingness to pay to purchase the product, but occasionally they reduce the price in order to sell to the group of customers with a lower willingness to pay. In some equilibria, all sellers lower their price simultaneously and to the same level. A cyclic price path is also obtained in a paper by Narasimhan (1989), who uses a framework similar to that of CGS but assumes that the entry of new consumers is governed by a diffusion process. In his model, the number of buyers who enter the market in each period is a function of cumulative sales and is thus time variant. Unlike Conlisk et al. the market size in his model is fixed, such that after some time saturation effects set in.

Dana's paper on advance-purchase discounts (1998) has a market with individual and aggregate consumer demand uncertainty. Price-taking firms set prices before demand is known and may offer advance-purchase discounts. In this case firms discriminate between buyers who have low willingness to pay but have a better chance of buying the product and buyers who have a higher willingness to pay but have a low probability of making a purchase. Thus firms screen buyers by their demand uncertainty, offering lower prices to consumers with certain demand in order to lower the costs of holding unutilized capacity or excess inventory. This is specially true for the airline industry, with the only difference being that airline companies cannot be assumed to be competitive price takers.

A number of papers attempt to solve for conditions under which the monopolist seller will offer either 'introductory offers' or 'final sales'. Denicolo and Garella (1999) show in a two-period model similar to the one used by Bulow, that the single seller will choose to offer higher prices in the second period, while rationing in the first. However, the seller does not face any aggregate capacity constraint. They show rationing reduces the incentive to lower future prices and may allow the monopolist to increase his or her discounted profit. A similar result is obtained in a paper by Wilson (1988), where he shows that a single seller can increase profits by charging two different prices for the same good and rationing sales at the lower price. However, such a result cannot be obtained without precommitment. Dudine, Hendel and Lizzeri (2005) show that if consumers choose to purchase durables in advance (stockpile) instead of delaying such purchases (as they are expected to do with the hope that lower prices are made available later on), then in the absence of commitment, the monopolist will charge higher prices in all periods and that social welfare will be lower than in the case where the monopolist can precommit. This is relevant in our model since some of the buyers who arrive early might opt for such advance purchases.

An alternate outlook is presented in papers by Brumelle and McGill (1993) and Wollmer (1992), who solve for an optimum airline seat booking policy, where lower fare class customers book tickets before higher fare class passengers. In these papers, airlines solve for a critical number of seats in each fare class, which are reserved for potential future passengers who are willing to pay a higher price. Booking requests for a particular fare class are accepted if and only if the number of empty seats is strictly greater than its critical level and rejected otherwise. Wollmer shows that this critical value is a decreasing function of the fare price and is equal to zero for the highest fare (class). However, these papers lack the flavor of durable goods, as buyers do not have the option of staying in the market to wait for a lower price, while sellers do not compete with future incarnations of themselves.

An empirical paper that studies the issue of airline prices over time is a paper by Stavins (2001). Stavins examines how price discrimination changes with market concentration in the airline market. Price discrimination is found to increase as the markets become more competitive. The data set included fares offered 35 days prior to departure, followed by 21 days prior to departure, 14 days prior to departure and finally 2 days prior to departure. The data thus allowed for examination of how prices change as the departure date drew closer. From the OLS regression it was discovered that cheaper fares disappear, leaving only more expensive tickets for sale.

The chapter proceeds as follows. In the following section a basic three-period model is developed and strategies for both players outlined. Section 4 identifies possible candidates for subgame perfect outcomes and describes conditions under which we get the different subgame perfect outcomes. Section 5 discusses extensions and section 6 concludes.

1.3 Model

A finite horizon model, with a single seller selling a finitely durable good and facing a capacity constraint is formulated with a continuum of buyers of two types entering the market in each period. This helps us to analyze the possible shape(s) of the optimal price path for the monopoly seller.

Setting. Time is discrete. We can consider a finite horizon model of T periods, where T is a large finite number. The durable good made available by the lone seller has a lifetime of T periods, after which it is assumed to be lost forever. In order to consider a simple version, we assume that $T = 3$.

Supply side. There is a single seller of the product. The seller chooses price p_i for period i (with $i = 1, 2, 3$), nonstochastically so as to maximize sum of discounted profits earned, calculated at discount factor ρ, with $0 < \rho < 1$. We also assume that the monopolist faces constant marginal cost, assumed without loss of generality to be zero. With the assumption of zero costs, the monopolist is assumed to maximize discounted stream of revenues accruing over the three periods. The monopolist cannot rent the product; at any given date, the monopolist cannot make binding commitments about future prices. The total measure of units of the product (seats) available to the monopolist is 3. The seller chooses to offer a continuum of units of measure $q_i \in [0, 3]$ for $i = 1, 2, 3$ in period i.

Demand side. A continuum of buyers of measure 2 enter the market in each period, with each buyer having unit demand. Buyers in each cohort can be one of two types. There is thus the minimum degree of consumer heterogeneity. We assume that a continuum of buyers of measure 2α (with $0 < \alpha < 1$) enter the market in each period and have valuation for the product given by V_1, while a continuum of buyers of measure $2(1 - \alpha)$ enter the market in each period and value the good at V_2, where $V_1 > V_2 > 0$. Buyers with valuation V_1 are said to be of 'high' type, while buyers with valuation at V_2 are said to be of 'low' type. We assume that the majority of buyers entering the market in each period are of low type and hence, $\alpha \in (0, 1/2)$.

Buyers are assumed to be rational. Each buyer on entering the market decides either to purchase the product in the current period or to wait for a lower price, except for buyers in the last period, who either decide to buy or not to buy the product in the last period. In the event that the buyer is indifferent between buying in the current period and waiting (or not to buy), the buyer is assumed to make the purchase immediately. Buyers assume that their own decision as to when to buy the product has no bearing on other buyer's decision as to whether and when to buy the same product. This is a consequence of the assumption that we have a continuum of buyers in the market. We further assume that the probability that the buyer will get the product in period i is given by Φ_i which is determined endogenously. Once a consumer buys the product, he or she leaves the market forever. A consumer who has not bought the product stays in the market till period 3, regardless of when he or she first entered the market. The probability with which buyers of both types get the product in period $i+1$, Φ_{i+1} implicitly acts as the discount factor.

Finally, no resales are allowed. All consumers are price takers, and they have no bargaining power. This, once again, is a consequence of the assumption that we have a continuum of buyers in the market. This feature marks another departure from the Conlisk, Gerstner and Sobel framework, where a discrete number of buyers were assumed to enter the market in each period. Usually models with a continuum of agents yield radically different equilibrium than those with a finite number of agents (example, Coase conjecture gets violated in the Bagnoli, Salant and Swierzbinski model). Levine and Pesendorfer (1995) resolve this paradox by considering a model where a discrete number of players' actions are observed with some noise and the aggregate level of noise does not disappear too rapidly as the number of players decrease.

Timing of events. At the beginning of period 1, the seller announces the price for the first period, p_1 and the measure of units available for purchase, q_1. A continuum of buyers of measure 2 enter the market in the first period, of which buyers of measure 2α are of 'high' type and buyers of measure $2(1-\alpha)$ are of 'low' type. Each buyer decides whether to buy the product in the first period, or to wait for a lower price which might be made available in the future. If the buyer decides to purchase the good in the first period, he or she exits from the market. Based on p_1, the seller knows the measure of units that were actually sold in the first period. At the beginning of the second period, the seller announces price for period 2, p_2 and the measure of units available for sale in the second period, q_2. A new cohort of buyers (of measure 2) enter the market in the second period. These buyers along with the buyers who decided not to buy the product in period 1 and hence chose to remain in the market then constitute the total measure of buyers in the market in the second period. Each of these buyers in turn decide either to purchase the product at price p_2 or to wait for a lower price in period 3. A similar sequence of events follow in period 3, except for the fact that buyers of both types in period 3, choose either to purchase or not to purchase the good in the last period. The seller is unable to precommit to any sequence of prices and measure of units to be offered for sale over the 3 periods.

We assume that the type of each buyer is *publicly observable*, such that we have a *complete information* model. We further assume that, even though the seller knows the type of each and every 'active' buyer in the market at any point of time, he or she is unable to price discriminate and must charge (or announce) a single price in every period.[2] Since this is a model involving a finite horizon, we employ the method of *backwards induction* to solve for the *subgame perfect Nash equilibrium (SPNE)* of the game described above. Here, we should note that Gul, Sonnenschein and Wilson (1986) also use the concept of subgame perfect Nash equilibrium in their widely cited paper. Conversely, we could have assumed that the buyers' types are not observable. In that case, Perfect Bayesian equilibrium would have been the appropriate equilibrium concept, where we would have to explicitly specify how agents form beliefs for information sets off the equilibrium path. In our model, the seller chooses prices and measure of units to release in each of the 3 periods, while each buyer decides whether to buy the product or

[2] By 'active' buyers we mean buyers who have chosen not to purchase the product in previous periods and have instead chosen to remain in the market for lower prices. 'Active' buyers in a particular period also include buyers who entered the market in the same period and are about to decide either to purchase the product or to wait for a lower price in periods 1 and 2, and either to purchase or not to purchase the product in period 3.

to wait in periods 1 and 2 and whether to buy or not to buy the good in period 3. In order to describe the strategies of both types of buyers and the single seller, we need to introduce the following notation.

Notation. The following notation is introduced in order to describe the total measure of 'high' and 'low' type buyers in the market at each point of time, as well as the measure of units left with the seller at the beginning of each period.

b_i^H = Total measure of 'high' type buyers in the market including ones entering the market in period i

b_i^L = Total measure of 'low' type buyers in the market including ones entering the market in period i

s_i = Measure of units left with the seller at the beginning of period which is a function of $p_{i-1}, q_{i-1}, b_{i-1}^H, b_{i-1}^L$ and s_{i-1}, where p_{i-1}, q_{i-1} are control variables for period $i-1$ and $b_{i-1}^H, b_{i-1}^L, s_{i-1}$ are state variables for period $i-1$.

1.3.1 Backwards Induction Argument

A strategy for the monopolist specifies for each period, price and measure of units to be offered to the buyers as a function of the history of the game. A strategy for the buyer of each type on the other hand, specifies at each time and after each history (in which he or she has not previously purchased or in case he or she has just entered the market) whether to accept or to reject the monopolist's offered price.

Period 3. We begin by describing the strategies of the buyers of both types in period 3. At any time all players have perfect recall. We define h_3 as the set of all possible histories available to each buyer in period 3, which provides all possible values for $(p_1, p_2, p_3, q_1, q_2, q_3, b_1^H, b_1^L, s_1, b_2^H, b_2^L, s_2, b_3^H, b_3^L, s_3)$. For buyers who choose not to purchase the product in periods 1 and 2 and for those who entered the market in period 3, the following function specifies the action to be taken by each and every one of them:

$$f_3 : h_3 \rightarrow \{B, NB\}$$

where B denotes the decision to buy and NB represents the decision not to buy the product. Analogously, we define H_3 as the set of all possible histories available to the seller at the beginning of period 3 which consists of all possible values for $(p_1, p_2, q_1, q_2, b_1^H, b_1^L, s_1, b_2^H, b_2^L, s_2, b_3^H, b_3^L, s_3)$. The seller announces p_3 and q_3 at the beginning of period 3. A part of the strategy for the seller for the last period is described as follows:

$$F_3 : H_3 \rightarrow \Re_+^2$$

Period 2. In period 2, we define h_2 as the set of all possible histories available to the buyer of each type, which consists of all possible values for $(p_1, p_2, q_1, q_2, b_1^H, b_1^L, s_1, b_2^H, b_2^L,\ s_2)$. Buyers who chose not to purchase the product in period 1 and those who entered the market in period 2 choose either to buy the product or to wait for a lower price as specified by the following function:

$$f_2 : h_2 \rightarrow \{B, W\}$$

where B once again denotes the decision by the buyer to buy the product and W refers to the decision to wait for a lower price in period 3. At the beginning of period 2, the seller has to announce p_2 and q_2. H_2 is now defined as the set of all possible histories available to the seller at the beginning of period 2 and it provides all possible values for $(p_1, q_1, b_1^H, b_1^L, s_1, b_2^H, b_2^L, s_2)$. In period 2, the following function describes part of the strategy for the seller:

$$F_2 : H_2 \rightarrow \Re_+^2$$

Period 1. Finally, for buyers of both types in period 1, the set of all possible histories available to them is denoted by h_1 which consists of all possible values for $(p_1, q_1, b_1^H, b_1^L, s_1)$. Part of the strategy for each buyer specifies an action in period 1 described by

$$f_1 : h_1 \rightarrow \{B, W\}$$

where B and W have the same interpretation as mentioned above. H_1 is defined as the set of all possible histories available to the seller at the beginning of period 1 and consists of all possible values for (b_1^H, b_1^L, s_1), where $b_1^H = 2\alpha, b_1^L = 2(1-\alpha), s_1 = 3$. Part of a strategy for the seller in period 1, who announces p_1 and q_1 is described by the following rule:

$$F_1 : H_1 \rightarrow \Re_+^2$$

The transition equations for the state variables are described as follows.

Define $m_i = \min\{q_i, d_i\}$ where $d_i = \begin{cases} b_i^H \text{ if } V_2 < p_i \leq p_i^H \\ b_i^H + b_i^L \text{ if } p_i \leq V_2 \end{cases}$

Here, d_i denotes demand for the product in period i while m_i denotes measure of units actually sold in period i.

Then, $s_{i+1} = s_i - m_i$

$$
b_{i+1}^{H} = \begin{cases}
2\alpha + b_i^H \text{ if } p_i > p_i^H, \forall q_i \\
2\alpha + (b_i^H - m_i) \text{ if } V_2 < p_i \leq p_i^H, q_i < b_i^H \text{ such that } m_i = q_i \\
2\alpha \text{ if } V_2 < p_i \leq p_i^H, q_i \geq b_i^H \\
2\alpha + b_i^H \left(1 - \dfrac{q_i}{b_i^H + b_i^L}\right) \text{ if } p_i \leq V_2, q_i < b_i^H + b_i^L \\
2\alpha \text{ if } p_i \leq V_2, q_i \geq b_i^H + b_i^L
\end{cases}
$$

$$
b_{i+1}^{L} = \begin{cases}
2(1-\alpha) \text{ if } p_i \leq V_2, q_i \geq b_i^H + b_i^L \\
2(1-\alpha) + b_i^L \left(1 - \dfrac{q_i}{b_i^H + b_i^L}\right) \text{ if } p_i \leq V_2, q_i < b_i^H + b_i^L \\
2(1-\alpha) + b_i^L \text{ if } p_i > V_2, \forall q_i
\end{cases}
$$

where p_i^H is the price in period i which makes 'high' type buyers indifferent between buying the product in period i and waiting for a lower price in period $i+1$. Now that we've described the strategies available to the seller and to each buyer in each of the 3 periods, we are ready to discuss the optimal decision rules for the seller and for buyers for both types for each of the 3 periods.

The optimal decision rule for the seller and for the buyers in period 3 is described as follows.

In period 3, buyers in the market of the two types, choose from the following actions in period 3 {Buy in period 3, Not to buy in period 3}. The 'high' type buyer chooses according as

$$In\ period\ 3,\ chosen\ action = \begin{cases} \text{Buy in period 3 if } p_3 \leq V_1 \\ \text{Not buy otherwise} \end{cases} \tag{1.1}$$

The 'low' type buyer chooses according as,

$$In\ period\ 3,\ chosen\ action = \begin{cases} \text{Buy in period 3 if } p_3 \leq V_2 \\ \text{Not buy} \quad \text{otherwise} \end{cases} \tag{1.2}$$

The seller chooses p_3, q_3 in order to

$$\max_{p_3,q_3} p_3.\min\{q_3, d_3(p_3)\} \text{ subject to } q_3 \leq s_3 \tag{1.3}$$

where d_3 is defined as follows:

$$d_3 = \begin{cases} b_3^H \text{ if } V_2 < p_3 \leq V_1 \\ b_3^H + b_3^L \text{ if } p_3 \leq V_2 \end{cases}$$

The optimal decision rule for the seller and for the buyers in period 2 is described next.

In period 2, buyers in the market of the two types, choose from the following actions in period 2 {Buy in period 2, Wait}. The 'high' type buyer chooses according as

$$In\ period\ 2,\ chosen\ action = \begin{cases} \text{Buy in period 2 if } p_2 \leq p_2^H \\ \text{Wait otherwise} \end{cases} \tag{1.4}$$

The 'low' type buyer chooses according as,

$$In\ period\ 2,\ chosen\ action = \begin{cases} \text{Buy in period 2 if } p_2 \leq V_2 \\ \text{Wait} \quad \text{otherwise} \end{cases} \tag{1.5}$$

where p_2^H is defined by the following equation

$$(V_1 - p_2^H) = \Phi_3[V_1 - p_3^*]\ with\ \Phi_3 = \begin{cases} \frac{q_3^*}{b_3^H} \text{ if } V_2 < p_3^* \leq V_1 \\ \frac{q_3^*}{b_3^H + b_3^L} \text{ if } p_3^* \leq V_2 \end{cases} \tag{1.6}$$

Here, we use a fixed-point argument. In period 2, the 'high' type buyers know b_2^H, b_2^L. For the time being they fix the p_2^H of all other 'high' type buyers and calculate the corresponding b_3^H and b_3^L. Given b_3^H, b_3^L these buyers can calculate the price and measure of units the seller will offer in period 3, p_3^* and q_3^*. Finally, using p_3^*, q_3^* and equation (1.6) these buyers should be able to recover a p_2^H which should be equal to the one originally assumed. p_2^H is thus the price the seller can charge in order to make the 'high' type buyers indifferent between buying in period 2 and waiting for a lower price in period 3. We assume that in case the buyer is indifferent between buying and waiting, the buyer decides to purchase the product in the current period.

At the beginning of period 2, the seller announces p_2 and q_2 in order to

$$\max_{p_2, q_2} p_2 . \min\{q_2, d_2(p_2)\} + \rho W(b_3^H, b_3^L, s_3) \text{ subject to } q_2 \leq s_2 \tag{1.7}$$

where W is the continuation payoff earned by the seller in period 3 and d_2 is defined as

$$d_2 = \begin{cases} b_2^H \text{ if } V_2 < p_2 \leq p_2^H \\ b_2^H + b_2^L \text{ if } p_2 \leq V_2 \end{cases}$$

Finally, we describe the optimal decision rule for the seller and the buyers for period 1.

In period 1, buyers in the market of the two types, choose from the following actions in period 1 {Buy in period 1, Wait}. The 'high' type buyer chooses according as

$$In\ period\ 1,\ chosen\ action = \begin{cases} \text{Buy in period 1 if } p_1 \leq p_1^H \\ \text{Wait otherwise} \end{cases} \tag{1.8}$$

The 'low' type buyer chooses according as,

$$In\ period\ 1,\ chosen\ action = \begin{cases} \text{Buy in period 1 if } p_1 \leq V_2 \\ \text{Wait} \quad \text{otherwise} \end{cases} \tag{1.9}$$

where p_1^H is defined by the following equation

$$(V_1 - p_1^H) = \Phi_2(V_1 - p_2^*)\ with\ \Phi_2 = \begin{cases} \dfrac{q_2^*}{b_2^H} \text{ if } V_2 < p_2^* \leq p_2^{H*} \\ \dfrac{q_2^*}{b_2^H + b_2^L} \text{ if } p_2^* \leq V_2 \end{cases} \tag{1.10}$$

Here, the 'high' type buyers use a fixed point argument similar to the one discussed above to solve for the cutoff price, p_1^H. p_1^H is thus the price the seller can charge in order to make the 'high' type buyers indifferent between buying in period 1 and waiting for a lower price in period 2.

At the beginning of period 1, the seller announces p_1 and q_1 in order to

$$\max_{p_1, q_1} p_1 . \min\{q_1, d_1(p_1)\} + \rho W(b_2^H, b_2^L, s_2) \text{ subject to } q_1 \leq 3 \tag{1.11}$$

where W is the continuation payoff earned by the seller in period 2 and d_1 is defined as

$$d_1 = \begin{cases} b_1^H \text{ if } V_2 < p_1 \leq p_1^H \\ b_1^H + b_1^L \text{ if } p_1 \leq V_2 \end{cases}$$

A *subgame perfect Nash equilibrium* of this game will thus consist of a strategy profile, $\sigma = (S, B)$ where S specifies a strategy on the part of the seller which satisfies equations (1.3), (1.7) and (1.11) while B specifies strategies on the part of each buyer who decides either to buy or to wait for a lower price in periods 1 and 2, and either to buy or not to buy in period 3, which satisfies equations (1.1), (1.4), (1.8) for 'high' type buyers and equations (1.2), (1.5) and (1.9) for 'low' type buyers. The equilibrium is a *symmetric equilibrium* in the sense that in equilibrium all buyers of the same type, choose the same action in each period. In an equilibrium it must be the case that the strategy of each player is optimal given the history of the game. This in turn ensures that the strategies are *credible*. In order to check for subgame perfection, we consider only unilateral deviations by the agents. With non-atomic buyers, unilateral deviations made by them affect neither the actions of other buyers or those of the monopolist. Thus, only unilateral deviations by the seller needs to be considered. If the seller deviates, the players keep following the optimal rules described above from that point of time onwards. This means if a player discovers a history of the game at any stage, which is not consistent with the one expected in equilibrium, the player continues to follow his or her optimal decision rule from that time onwards.

Existence of equilibrium. In order to ensure the existence of an equilibrium for the game described above, we consider the problems faced by the agents. The seller's problem involves maximization of revenue given the history of the game, subject to a linear constraint, $q_i \leq s_i$. We should note here that prices chosen by the seller belongs to the compact set $[V_2, V_1]$ ($p_3 \in \{V_1, V_2\}$) while the measure of units offered by the seller in each period belongs to another compact set $[0, s_i]$. Since the objective function is continuous in p_i and q_i and the strategy space is compact ($[V_2, V_1] \times [0, s_i]$ is compact), we can apply the *Weierstrass Theorem* to show that a solution for the optimization problem facing the seller in each period, exists. Each buyer on the other hand chooses an action in each period which fetches the highest payoff. The payoffs available to each buyer in any period are determined by the seller's choice of price and measure of units to offer for sale in the same period, and on the buyers' own expectations of prices and measure of units to be made available in the future. Since both the problems of the seller and each buyer can be solved, this ensures the existence of an equilibrium for this game.

By using the *backwards induction* argument its possible to derive 8 possible price paths, where in each period, the seller decides either to sell only to 'high' type buyers or to sell to both 'high' and 'low' types. As the seller announces p_i and q_i at the beginning of each period i, we define a *pricing policy* $(p_1, p_2, p_3, q_1, q_2, q_3)$ which describes the prices charged and the units offered for sale in each period.

1.4 Candidates for Subgame Perfect Outcome

In this section, we examine the different possible pricing policies and the associated price paths from which the seller might choose, under different combinations of the parameters V_1, V_2, α and ρ. Since we're interested in decisions made by a patient seller, we further assume that $\rho \to 1$.

1.4.1 No 'sale' in any period

The first pricing policy we consider is one where the seller chooses to sell only to 'high' type buyers in every period. The price charged in each period is V_1, while the measure of units offered for sale in each period is 2α. The pricing policy is thus $(V_1, V_1, V_1, 2\alpha, 2\alpha, 2\alpha)$ while the associated revenue earned is given by

$$R_1 = 2V_1\alpha(1 + \rho + \rho^2) \tag{1.12}$$

For $(V_1, V_1, V_1, 2\alpha, 2\alpha, 2\alpha)$ to be a subgame perfect outcome, we first need to establish whether it is credible on the part of the seller to set $p_i = V_1$ and $q_i = 2\alpha$ for $i = 1, 2, 3$. Since the seller chooses without commitment p_i and q_i at the beginning of each period, we need to check if the seller could choose to deviate profitably at the beginning of each period for the relevant range of parameter values. For example, the seller could choose to deviate in period 3 by announcing $p_3 = V_2$ and $q_3 = 3 - 4\alpha$. The required condition which ensures that no profitable deviation exists for the seller at the beginning of period 3 is $2V_1\alpha \geq (3 - 4\alpha)V_2 \Rightarrow \alpha \geq \dfrac{3V_2}{2(V_1 + 2V_2)}$

1.4.2 'Sale' in every period

The second pricing policy $(V_2, V_2, V_2, 2, 1, 0)$ yields a horizontal price path as in case (1), but this time, the seller chooses to sell to both 'high' and 'low' type buyers in every period. Discounted sum of revenue from this pricing policy is

$$R_2 = 2V_2 + \rho V_2 \tag{1.13}$$

1.4.3 'Sale' in the first period only

The seller could choose to hold a 'sale' in the first period only, where he or she offers to charge $p_1 = V_2$ and $p_2 = p_3 = V_1$. Given the price path and the measure of units offered in the first period, q_1, the measure of units to be offered in periods 2 and 3 should be $q_2 = b_2^H$ and $q_3 = s_3$ (if $s_3 < b_3^H$) or b_3^H (if $b_3^H < s_3$).

LEMMA 1.1. *With $p_1 = V_2$ and $p_2 = p_3 = V_1$ if $\alpha \in [\frac{1}{4}, \frac{V_2}{V_1}]$, then with $\rho \to 1$ the seller offers $q_1 = \dfrac{3-6\alpha}{1-\alpha}$, $q_2 = b_2^H = \dfrac{\alpha + 2\alpha^2}{1-\alpha}$ and $q_3 = s_3 = b_3^H = 2\alpha$ and if $\alpha < \frac{1}{4} \leq \frac{V_2}{V_1}$, then he or she offers $q_1 = 2$, $q_2 = q_3 = 2\alpha$. If $\alpha > \frac{V_2}{V_1}$, then with $\rho \to 1$ the seller offers $q_1 = 0$, $q_2 = b_2^H = 4\alpha$ and $q_3 = s_3 = 2\alpha$.*

Proof. See Appendix 1. ■

From the above lemma we find that with $\alpha \leq \frac{V_2}{V_1}$, the seller chooses to offer measure q_1 units in the first period in a way which ensures that $s_3 = b_3^H$, such that the seller will have no incentive to hold a 'sale' in the last period. Discounted sum of revenue earned is thus

$$R_3 = \begin{cases} \dfrac{3-6\alpha}{1-\alpha}V_2 + \rho\dfrac{\alpha+2\alpha^2}{1-\alpha}V_1 + 2\rho^2 V_1\alpha \text{ with } \dfrac{1}{4} \le \alpha \le \dfrac{V_2}{V_1} \\ 2V_2 + 2\rho V_1\alpha(1+\rho) \text{ if } \alpha < \dfrac{1}{4} \le \dfrac{V_2}{V_1} \end{cases} \tag{1.14}$$

If $\alpha > \dfrac{V_2}{V_1}$, the sum of discounted revenue earned is given by

$$R_3^1 = 4\rho V_1\alpha + 2\rho^2 V_1\alpha \tag{1.15}$$

Thus with α high enough, the seller chooses to offer zero measure of units for 'sale' in the first period.

1.4.4 'Sale' in the first two periods

Another strategy for the seller could be to offer a measure of units at price V_2 in the first two periods, and to sell to 'high' valuation buyers in the last period. Given the price path and the measure of units offered in the first and second periods (q_1 and q_2 respectively), the seller should offer $q_3 = s_3$ (if $s_3 < b_3^H$) or b_3^H (if $b_3^H < s_3$).

LEMMA 1.2. *With $p_1 = p_2 = V_2$ and $p_3 = V_1$ if $\alpha \in [\frac{1}{4}, \frac{V_2}{V_1}]$, then with $\rho \to 1$ the seller offers $q_1 = \dfrac{3-6\alpha}{1-\alpha}$, $q_2 = 0$ and $q_3 = s_3 = b_3^H = \dfrac{3\alpha}{1-\alpha}$ and if $\alpha < \dfrac{1}{4} \le \dfrac{V_2}{V_1}$ then he or she offers $q_1 = 2$, $q_2 = \dfrac{1-4\alpha}{1-\alpha}$ and $q_3 = s_3 = b_3^H = \dfrac{3\alpha}{1-\alpha}$. On the other hand, if $\alpha > \dfrac{V_2}{V_1}$, then with $\rho \to 1$ the seller offers $q_1 = 0$, $q_2 = 0$ and $q_3 = 6\alpha$.*

Proof. See Appendix 1. ∎

Once again from the above lemma we find that with $\alpha \le \dfrac{V_2}{V_1}$, the seller chooses to offer measure q_1 and q_2 units in the first two periods respectively in a way which ensures that $s_3 = b_3^H$, such that the seller will have no incentive to hold a 'sale' in the last period. Discounted sum of revenue earned is thus

$$R_4 = \begin{cases} \frac{3-6\alpha}{1-\alpha}V_2 + \rho^2 \frac{3\alpha}{1-\alpha}V_1 \text{ if } \frac{1}{4} \leq \alpha \leq \frac{V_2}{V_1} \\ 2V_2 + \rho\frac{1-4\alpha}{1-\alpha}V_2 + \rho^2 \frac{3\alpha}{1-\alpha}V_1 \text{ if } \alpha < \frac{1}{4} \leq \frac{V_2}{V_1} \end{cases} \tag{1.16}$$

If $\frac{V_2}{V_1} < \alpha < \frac{1}{2}$, revenue earned is given by $R_4^1 = 6\rho^2 V_1 \alpha$.

Once again if α is large enough, the seller chooses to offer measure zero units for 'sale' in the first two periods.

Proposition 1. *If* $\alpha < \frac{1}{4} \leq \frac{V_2}{V_1}, \rho \to 1$ *then* $(V_2, V_1, V_1, 2, 2\alpha, 2\alpha)$ *cannot be a subgame perfect outcome.*

Proof. See Appendix 2. ■

Proposition 2. *If* $\alpha \in [\frac{1}{4}, \frac{V_2}{V_1}], \rho \to 1$ *then* $(V_2, V_2, V_1, \frac{3-6\alpha}{1-\alpha}, 0, \frac{3\alpha}{1-\alpha})$ *cannot be a subgame perfect outcome.*

Proof. See Appendix 2. ■

Proposition 3. *If* $\alpha \leq \frac{V_2}{V_1}, \rho \to 1$ *then* $(V_2, V_2, V_2, 2, 1, 0)$ *cannot be subgame perfect.*

Proof. See Appendix 2. ■

1.4.5 'Sale' in the first and last period

In case the seller chooses to hold a 'sale' in the first and last period, the corresponding price path is inverted U-shaped. The seller charges $p_1 = p_3 = V_2$ and

$p_2 = p_2^H > V_2$ in order to make the 'high' type buyers indifferent between buying the good in period 2 and waiting for the lower price of V_2 in the next period. Given the price path and the measure of units offered for sale in period 1, q_1, the seller offers $q_2 = b_2^H$ and $q_3 = s_3$.

LEMMA 1.3. *With $\rho \to 1$, $p_1 = p_3 = V_2$ and $p_2 = p_2^H$ the seller offers $q_1 = 0$, $q_2 = b_2^H = 4\alpha$ and $q_3 = s_3 = 3 - 4\alpha \;\forall\; V_1 > V_2$.*

Proof. See Appendix 1. ■

Sum of discounted revenue earned by holding a 'sale' in the first and last period is thus given by the following equation

$$R_5 = 4\rho\alpha \left[V_1 \left(1 - \frac{3-4\alpha}{6-4\alpha} \right) + \frac{3-4\alpha}{6-4\alpha} V_2 \right] + \rho^2 (3-4\alpha) V_2 \tag{1.17}$$

In order to rule out profitable deviations in period 3, the seller must have no incentive to charge $p_3 = V_1$. The required condition to ensure this is $(3-4\alpha)V_2 > 2V_1\alpha \Rightarrow \alpha < \dfrac{3V_2}{2(V_1 + 2V_2)}$. Similarly, we also have to rule out profitable deviations in period 2, given the history of the game $p_1 = V_2$ and $q_1 = 0$.

PROPOSITION 4. *With $\rho \to 1$ $\left(V_2, V_1 \left(1 - \frac{3-4\alpha}{6-4\alpha} \right) + \frac{3-4\alpha}{6-4\alpha} V_2, V_2, 0, 4\alpha, 3-4\alpha \right)$ is never subgame perfect.*

Proof. See Appendix 2. ■

1.4.6 'Sale' in the second period only

For the strategy involving a 'sale' in the second period only, the price path which is generated is U-shaped. Since the seller holds a sale in the second period only, he or she charges $p_1 = p_1^H$ (to make 'high' type buyers indifferent between waiting and purchasing in period 1), $p_2 = V_2$ and $p_3 = V_1$. Given the price path and the measure of units offered in periods 1 and 2 as q_1 and q_2 respectively, the seller offers $q_3 = s_3$ (if $s_3 < b_3^H$) or b_3^H (if $b_3^H < s_3$).

Lemma 1.4. *With $p_1 = p_1^H, p_2 = V_2$ and $p_3 = V_1$ if $\alpha \leq \frac{2V_2}{V_1+V_2}$, then with $\rho \to 1$ the seller offers $q_1 = 2\alpha$, $q_2 = \frac{6\alpha^2 - 15\alpha + 6}{2(1-\alpha)}$ and $q_3 = s_3 = b_3^H = \frac{5\alpha - 2\alpha^2}{2(1-\alpha)}$. On the other hand, if $\alpha > \frac{2V_2}{V_1+V_2}$, then with $\rho \to 1$ the seller offers $q_1 = 2\alpha$, $q_2 = 0$ and $q_3 = 4\alpha$.*

Proof. See Appendix 1. ∎

As was the case with strategies involving 'sales' in the first period only or the first two periods, the seller offers measure q_1 and q_2 units in the first and second period in a way which ensures that if $\alpha \leq \frac{2V_2}{V_1+V_2}$ and $\rho \to 1$, $s_3 = b_3^H$ such that there is no incentive for the seller to hold a 'sale' in the last period. With $\alpha > \frac{2V_2}{V_1+V_2}$ the seller chooses to offer measure zero units for 'sale' in the second period.

If $\alpha \leq \frac{2V_2}{V_1+V_2}$, then with $\rho \to 1$ sum of discounted revenue earned is

$$R_6 = \left[V_1(1 - \frac{6\alpha^2 - 15\alpha + 6}{2(1-\alpha)(4-2\alpha)}) + \frac{6\alpha^2 - 15\alpha + 6}{2(1-\alpha)(4-2\alpha)}V_2\right] 2\alpha + \rho\frac{6\alpha^2 - 15\alpha + 6}{2(1-\alpha)}V_2 + \rho^2\frac{5\alpha - 2\alpha^2}{2(1-\alpha)}V_1 \tag{1.18}$$

If $\alpha > \frac{2V_2}{V_1+V_2}$, sum of discounted revenue earned is $R_6^1 = 2V_1\alpha + \rho^2 4V_1\alpha$.

PROPOSITION 5. *If* $\alpha \in \left[\frac{1}{4}, \frac{V_2}{V_1}\right]$, *then with* $\rho \to 1$

$$\left[V_1(1 - \frac{6\alpha^2 - 15\alpha + 6}{2(1-\alpha)(4-2\alpha)}) + \frac{6\alpha^2 - 15\alpha + 6}{2(1-\alpha)(4-2\alpha)}V_2, V_2, V_1, 2\alpha, \frac{6\alpha^2 - 15\alpha + 6}{2(1-\alpha)}, \frac{5\alpha - 2\alpha^2}{2(1-\alpha)}\right]$$

cannot be subgame perfect.

Proof. See Appendix 2. ■

1.4.7 'Sale' in the last two periods

For strategies involving 'sales' in the last two periods, the seller charges $p_1 = p_1^H, p_2 = p_3 = V_2$ and offers $q_1 = 2\alpha, q_2 = 3 - 2\alpha, q_3 = 0$. These are the only $q_i s$ which are time consistent. At the beginning of period 2, given the history of the game $p_1 = p_1^H$ and $q_1 = 2\alpha$ and that $p_2 = p_3 = V_2$, the seller will choose to offer $q_2 = s_2 = 3 - 2\alpha$ since $\rho < 1$. Sum of discounted revenue earned is thus

$$R_7 = \left[V_1(1 - \frac{3-2\alpha}{4-2\alpha}) + \frac{3-2\alpha}{4-2\alpha}V_2\right] 2\alpha + \rho(3-2\alpha)V_2 \tag{1.19}$$

PROPOSITION 6. *If* $\alpha \leq \frac{V_2}{V_1} < \frac{2V_2}{V_1 + V_2}, \rho \to 1$ *then the pricing policy involving 'sale' in the last two periods cannot be subgame perfect.*

Proof. See Appendix 2. ■

1.4.8 'Sale' in the last period only

For 'sale' in the last period only, the seller sets $p_1 = p_1^H, p_2 = p_2^H$ to ensure that 'high' type buyers are indifferent between buying the good and waiting for the price V_2

in the last period. The seller offers $q_1 = q_2 = 2\alpha$ and $q_3 = s_3 = 3 - 4\alpha$. In this case, these are the only $q_i s$ which are time consistent. Given that $p_1 = p_1^H, p_2 = p_2^H$ and that $q_1 = q_2 = 2\alpha$, the seller will choose to offer $q_3 = s_3$. Sum of discounted revenue is thus

$$R_8 = \left[V_1(1 - \frac{3-4\alpha}{6-4\alpha}) + \frac{3-4\alpha}{6-4\alpha}V_2\right] 2\alpha + \rho 2\alpha \left[V_1(1 - \frac{3-4\alpha}{6-4\alpha}) + \frac{3-4\alpha}{6-4\alpha}V_2\right] + \rho^2(3-4\alpha)V_2 \tag{1.20}$$

Proposition 7. *If* $\alpha \le \frac{V_2}{V_1}, \rho \to 1$ *then* $\left[V_1(1 - \frac{3-4\alpha}{6-4\alpha}) + \frac{3-4\alpha}{6-4\alpha}V_2, V_1(1 - \frac{3-4\alpha}{6-4\alpha}) + \frac{3-4\alpha}{6-4\alpha}V_2, V_2, 2\alpha, 2\alpha, 3-4\alpha\right]$ *cannot be subgame perfect.*

Proof. See Appendix 2. ■

Proposition 8. *If* $\alpha \le \frac{V_2}{V_1}, \rho \to 1$ *then* $(V_1, V_1, V_1, 2\alpha, 2\alpha, 2\alpha)$ *cannot be subgame perfect.*

Proof. See Appendix 2. ■

So far, for a particular range of parameter values ($\alpha \le \frac{V_2}{V_1}, \rho \to 1$) we have shown which pricing policies cannot be subgame perfect. Now we turn our attention to policies which are subgame perfect for the same range of parameter values.

Proposition 9. *If* $\alpha \in \left[\frac{1}{4}, \frac{V_2}{V_1}\right], \rho \to 1$ *then* $(V_2, V_1, V_1, \frac{3-6\alpha}{1-\alpha}, \frac{\alpha+2\alpha^2}{1-\alpha}, 2\alpha)$ *is subgame perfect.*

Proof. See Appendix 3. ■

PROPOSITION 10. *If* $\alpha < \frac{1}{4}$(*for* $V_1 \leq 4V_2$) *and* $\alpha \leq \frac{V_2}{V_1}$(*for* $V_1 > 4V_2$) *and* $\frac{6\alpha^2 - 7\alpha + 4}{\alpha(4\alpha + 2)} < \frac{V_1}{V_2}$(*with* $\rho \to 1$) *then the pricing policy involving a 'sale' in the second period only is subgame perfect.*

Proof. See Appendix 3. ∎

Given the set of conditions in proposition 10 under which its possible to show that the pricing policy involving a 'sale' in the second period only is subgame perfect, we can derive the range of parameter values in which the U-shaped price path is obtainable. To do so, we should first note that the term $\frac{6\alpha^2 - 7\alpha + 4}{\alpha(4\alpha + 2)}$ is monotonically decreasing in α and that if $\widehat{\alpha}$ is defined as the value of α which makes $\frac{6\alpha^2 - 7\alpha + 4}{\alpha(4\alpha + 2)} = \frac{V_1}{V_2}$, then $\forall \alpha > \widehat{\alpha}$ $\frac{6\alpha^2 - 7\alpha + 4}{\alpha(4\alpha + 2)} < \frac{V_1}{V_2}$. Thus the above proposition can be re-stated as follows:

Given that $V_1 \leq 4V_2$, *if* $\widehat{\alpha} < \alpha < \frac{1}{4}$ *(where* $\widehat{\alpha}$ *is defined as above), then the pricing policy involving a 'sale' in the second period only is subgame perfect. Similarly given* $V_1 > 4V_2$, *if* $\widehat{\alpha} < \alpha \leq \frac{V_2}{V_1}$ *then the policy generating a U-shaped price path is subgame perfect.*

Since $\frac{6\alpha^2 - 7\alpha + 4}{\alpha(4\alpha + 2)} > 3.5$ for $\alpha < \frac{1}{4}$, the lowest value of V_1 which can support a U is greater than $3.5V_2$. Again since $\frac{6\alpha^2 - 7\alpha + 4}{\alpha(4\alpha + 2)} < \frac{V_1}{V_2}$ and $\alpha \leq \frac{V_2}{V_1}$(*i.e.* $V_1 \leq \frac{1}{\alpha}V_2$), we require that $\frac{6\alpha^2 - 7\alpha + 4}{4\alpha + 2} < 1 \Rightarrow \alpha > 0.205$. Thus the range of values of α in which the U-shaped price path might be found is $0.205 < \alpha < 0.25$. Since $\frac{6\alpha^2 - 7\alpha + 4}{\alpha(4\alpha + 2)} < 4.87$ for $\alpha > 0.205$ the corresponding range of values of V_1 which can support a U-shaped price path is $3.5V_2 < V_1 < 4.87V_2$. If the values of these parameters are outside these intervals, the pricing policy involving a 'sale' in the second period only will not be subgame perfect. This brings us to our next proposition.

PROPOSITION 11. *If* $\alpha < \frac{1}{4}$ *(for* $V_1 \leq 4V_2$*) and* $\alpha \leq \frac{V_2}{V_1}$ *(for* $V_1 > 4V_2$*) and* $\frac{6\alpha^2 - 7\alpha + 4}{\alpha(4\alpha + 2)} > \frac{V_1}{V_2}$ *(with* $\rho \to 1$*) then* $(V_2, V_2, V_1, 2, \frac{1-4\alpha}{1-\alpha}, \frac{3\alpha}{1-\alpha})$ *is subgame perfect.*

Proof. See Appendix 3. ■

If $\alpha > \frac{V_2}{V_1}, \rho \to 1$ we can show that the seller offers $q_1 = 0$ for the policy involving 'sale' in the first period only and $q_1 = q_2 = 0$ for the pricing policy involving 'sales' in the first two periods (from lemmas 1 and 2 respectively). Thus the seller chooses to offer zero measure of units in periods where he or she chooses to hold a 'sale'. From proposition 4, we already know that the strategy involving a 'sale' in the first and last period cannot be subgame perfect for the same range of parameter values. If $\alpha \in \left(\frac{V_2}{V_1}, \frac{2V_2}{V_1 + V_2}\right], \rho \to 1$ following the proofs of propositions 6 and 7 we can also show, that strategies involving 'sale' in the last period and last two periods can never be subgame perfect. Finally following the proof of proposition 3, we can show that the strategy involving a 'sale' in every period cannot be subgame perfect for the relevant range of parameter values.

For the region with $\alpha > \frac{V_2}{V_1}$ we first show that if $\alpha \in \left(\frac{V_2}{V_1}, \frac{2V_2}{V_1 + V_2}\right]$, $\rho \to 1$ strategies involving a 'sale' in the first period only and a 'sale' in the second period only can be subgame perfect. Next, we show that if $\alpha > \frac{2V_2}{V_1 + V_2}$, $\rho \to 1$ the only strategy which is subgame perfect is the one involving no 'sale' in any period. We can further check that if the seller could precommit and if $\alpha > \frac{V_2}{V_1}, \rho \to 1$ then he or she would choose the pricing policy $(V_1, V_1, V_1, 2\alpha, 2\alpha, 2\alpha)$. Since the seller cannot precommit, he or she is unable to do his or her best and is thus restricted to choosing from policies which involve a 'sale' in the first period only or a 'sale' in the second period only.

Proposition 12. *If* $\alpha \in \left(\frac{V_2}{V_1}, \frac{2V_2}{V_1+V_2}\right]$, $\rho \to 1$ *and* $\alpha \geq \frac{3V_2}{2(V_1+2V_2)}$, *then* $(V_2, V_1, V_1, 0, 4\alpha, 2\alpha)$ *is subgame perfect.*

Proof. See Appendix 3. ■

Since the seller offers $q_1 = 0$, *any price* in period 1 can be supported as a Subgame Perfect outcome. Thus, if $\alpha \in \left(\frac{V_2}{V_1}, \frac{2V_2}{V_1+V_2}\right]$, $\rho \to 1$ and $\alpha \geq \frac{3V_2}{2(V_1+2V_2)}$, then $(p_1, V_1, V_1, 0, 4\alpha, 2\alpha)$ is subgame perfect.

Proposition 13. *If* $\alpha \in \left(\frac{V_2}{V_1}, \frac{2V_2}{V_1+V_2}\right]$, $\rho \to 1$ *and* $\alpha < \frac{3V_2}{2(V_1+2V_2)}$, *then* $(V_1(1 - \frac{6\alpha^2-15\alpha+6}{2(1-\alpha)(4-2\alpha)}) + \frac{6\alpha^2-15\alpha+6}{2(1-\alpha)(4-2\alpha)}V_2, V_2, V_1, 2\alpha, \frac{6\alpha^2-15\alpha+6}{2(1-\alpha)}, \frac{5\alpha-2\alpha^2}{2(1-\alpha)})$ *is subgame perfect.*

Proof. See Appendix 3. ■

Proposition 14. *If* $\alpha > \frac{2V_2}{V_1+V_2}$, $\rho \to 1$ *then* $(V_1, V_1, V_1, 2\alpha, 2\alpha, 2\alpha)$ *will be subgame perfect.*

Proof. See Appendix 3. ■

In this case, if $\alpha > \frac{2V_2}{V_1+V_2}$ the seller chooses not to hold a 'sale' in any period. Figure 3.1 shows the pricing policies which are subgame perfect for the different combinations of parameter values. For higher values of V_1 combined with high values for α, the seller chooses not to hold a 'sale' in any period, such that only 'high' valuation buyers get to purchase the good. For $\alpha \in \left(\frac{3V_2}{2(V_1+2V_2)}, \frac{2V_2}{V_1+V_2}\right]$ the seller chooses to offer $q_1 = 0, q_2 = 4\alpha, q_3 = 2\alpha$ and to charge any price p_1, $p_2 = p_3 = V_1$ which is equivalent to not offering to hold a 'sale' in any period. For the same range of parameter values, the seller cannot choose the pricing policy $(V_1, V_1, V_1, 2\alpha, 2\alpha, 2\alpha)$ since there exists a profitable deviation for the seller by holding a 'sale' in the second period and to sell to 'high' valuation buyers in the last period. Had the seller been able to credibly precommit, he or she would have chosen the pricing policy $(V_1, V_1, V_1, 2\alpha, 2\alpha, 2\alpha)$. For lower values of α, the seller chooses to hold a 'sale' in at least one period, and being patient, chooses to hold a 'sale' in the second period. Finally, for the lowest values of V_1 and α, the seller chooses to have a 'sale' in two periods and thus charges price V_2 for the first two periods. Had we considered cases where ρ is much smaller than 1, we would have found combinations of V_1 and α which make $(V_2, V_2, V_2, 2, 1, 0)$ subgame perfect.

1.5 Extensions

The basic three-period model described above needs to be extended to a richer version in several steps, with the purpose of analyzing possible optimal price paths which might be generated in each case.

One possible extension could be to assume that the parameter α, which describes the measure of 'high' type buyers entering the market in each period, increases over time, instead of remaining constant (as it does in this model). With a larger measure of higher valuation customers entering the market in each period, the focus of the exercise will be the new range of parameter values for which pricing policies generating U-shaped price paths are subgame perfect.

The model should be extended to T ($T > 3$) periods, where a single seller offers a finite measure of units to a continuum of buyers, each of two possible types. Once again, the range of parameter values (if any) for which the optimal price path is U-shaped will be of particular interest. Here it should be noted that we already know the shape of the optimal price path for a similar model in a paper by Conlisk, Gerstner and Sobel (CGS, 1984). In their model, a single seller sells a durable good (without a capacity constraint) in a market over an infinite horizon where a cohort of N buyers enters the market in each period, with α fraction with a high valuation and $1 - \alpha$ fraction with a low valuation for the product. The equilibrium described involves periodic 'sales' by the seller who once in a while reduces the price to enable buyers with a low valuation to buy the product, but otherwise sets the price low enough so that only buyers with a high valuation can make a purchase. The optimal price path is hence *cyclic*. McAfee and Wiseman (2003) while concluding their paper, make the prediction that the presence of capacity constraint in the CGS model will enhance the seller's ability to dynamically price discriminate and lengthen the price cycle.

The U.S. airline market is far from being a monopoly (except on certain routes where only a single carrier offers flights). A model with multiple sellers thus needs to be introduced. Sobel's extension (1984) of the Conlisk, Gerstner and Sobel paper serves as a useful reference. In this paper he shows that the motivation to hold 'sales' remains in a model with n sellers selling a homogeneous product to a market in which in each period a new group of N buyers enters the market, α fraction being of high valuation and $1 - \alpha$ fraction of low valuation.

The basic model in the case where each seller faces a capacity constraint and offers units of a durable good for sale over a finite time horizon should involve a three-period model, with 2 sellers offering units of finite measure to a continuum of buyers, each of whom might be one of two possible types.[3] In this case, the price offered by the rival seller in each period should be the outside option available to each buyer. However, we also require that the price offered by the rival seller to be strictly lower than the price offered by the reference seller for each buyer in the market to have an outside option in each period (in the case with more than 2 sellers, we need the lowest price offered by any of the other sellers to be strictly lower than price offered by the reference seller in each period). In a three-period model, if we have 2 sellers with identical costs, there seems to be no reason for the 2 sellers to charge different prices in period 1. But this in turn means that buyers will not have an outside option in period 1. In period 2, however, prices will vary between the 2 sellers depending on the measure of units each sold in period 1.

In a separate chapter, which is an empirical extension of this one, we collect data on airline ticket prices for 30 routes from an online travel agent. Since our theoretical model has a single seller, we consider only those routes where a single airline offers non-stop flights. This however does not preclude the selected routes from having multiple carriers which offer one-stop or two-stop flights. Routes where a single airline offered non-stop flights which departed frequently on any given day were avoided to obviate the effects of competition between flights. We collect prices for such routes twice a day for

[3] An alternate specification involving an extension of a paper by Kreps and Scheinkman (1983) could be a three-period game, where sellers choose capacities in period 1, and select prices simultaneously in periods $2, 3$ and 4.

approximately 15 weeks, check whether the proportion of 'high' valuation buyers has any effect on the slope of the price path and empirically test the theoretical predictions of this chapter.

1.6 Conclusion

We construct a three-period model in which a single seller facing a capacity constraint offers a finite measure of units of a non-durable good to a continuum of buyers (each of whom might be one of two possible types). The seller chooses (without pre-commitment) prices and measure of units to offer for sale over the 3 periods in order to maximize discounted sum of revenue earned. We then try to determine possible shapes of the corresponding optimal price path for different values of the parameters. We find that for certain combinations of the parameters in some specific range, the optimal price path is *U-shaped*. For other combinations, we find that the optimal price path is either *non-decreasing* (which is consistent with a result in a paper by Stavins) or horizontal. It has some obvious shortcomings in the sense that it does not consider optimal price paths of models with more than three periods and more than one seller. Further extensions will be attempted in these directions.

Appendix 1

Proof of lemma 1.

Given that $p_1 = V_2$, $p_2 = p_3 = V_1$ and given the measure of units offered in period 1 $q_1 \in [0, 2]$, the seller should offer $q_2 = b_2^H$ and $q_3 = s_3$ (if $s_3 < b_3^H$) or b_3^H (if $b_3^H < s_3$). Given the price path and that the seller offers measure q_1 in period 1, $q_1 = m_1$ such that $b_2^H = 2\alpha + 2\alpha(1 - \frac{q_1}{2}) = \alpha(4 - q_1) < s_2 = 3 - q_1$ $\forall\alpha$, since $q_1 \in [0, 2]$. Since all 'high' type buyers active in the market in period 2 purchase the good in period 2, $m_2 = b_2^H$ such that $b_3^H = 2\alpha$ while $s_3 = 3 - q_1 - \alpha(4 - q_1)$. With $s_3 < b_3^H$, revenue earned is given by

$$R = q_1 V_2 + \rho\alpha(4 - q_1)V_1 + \rho^2[3 - q_1 - \alpha(4 - q_1)]V_1$$

Differentiating partially with respect to q_1,

$$\frac{\partial R}{\partial q_1} = V_2 - \rho\alpha V_1 - \rho^2(1 - \alpha)V_1$$

Since $\rho \to 1$, we take $\rho = 1$ and appeal to continuity to get $\frac{\partial R}{\partial q_1} = V_2 - V_1 < 0$. With $b_3^H < s_3$, revenue earned is given by

$$R = q_1 V_2 + \rho\alpha(4 - q_1)V_1 + 2\rho^2 V_1 \alpha$$

Differentiating partially with respect to q_1, $\frac{\partial R}{\partial q_1} = V_2 - \rho\alpha V_1$

Since $\rho \to 1$, we take $\rho = 1$ and appeal to continuity to get

$$\frac{\partial R}{\partial q_1} = V_2 - \alpha V_1 > 0 \text{ if } \alpha < \frac{V_2}{V_1}$$

With $\alpha = \frac{V_2}{V_1}$, $\frac{\partial R}{\partial q_1} = V_2(1-\rho) > 0$. Further, we can check that $s_3 < b_3^H$ iff $q_1 > \frac{3-6\alpha}{1-\alpha} = \theta$ (suppose).

Thus with $q_1 = \theta + \epsilon$, for any $\epsilon > 0$, $s_3 < b_3^H \Rightarrow \frac{\partial R}{\partial q_1} < 0$. Again with $q_1 = \theta - \epsilon$, for any $\epsilon > 0$, $b_3^H < s_3 \ni$ with $\alpha \leq \frac{V_2}{V_1}$, $\frac{\partial R}{\partial q_1} > 0$. Thus if $\alpha \leq \frac{V_2}{V_1}$, the seller should offer $q_1 = \theta = \frac{3-6\alpha}{1-\alpha}$, where $\theta \in (0,3)$ for all $\alpha \in (0,2)$.

Now, $\theta = \frac{3-6\alpha}{1-\alpha} > 2$ for all $\alpha < \frac{1}{4}$. This in turn implies that $q_1 < \theta \ni b_3^H < s_3$; if $\alpha \leq \frac{V_2}{V_1}$, $\frac{\partial R}{\partial q_1} > 0 \Rightarrow q_1 = 2$. Hence with $\alpha < \frac{1}{4} \leq \frac{V_2}{V_1}$, $q_1 = 2$, $q_2 = 2\alpha$ and $q_3 = b_3^H = 2\alpha < 1 - 2\alpha = s_3$. On the other hand, if $\alpha \in \left[\frac{1}{4}, \frac{V_2}{V_1}\right]$, then $q_1 = \theta = \frac{3-6\alpha}{1-\alpha} \leq 2$, $q_2 = b_2^H = \alpha(4-q_1) = \frac{\alpha + 2\alpha^2}{1-\alpha}$ and $q_3 = s_3 = b_3^H = 2\alpha$.

Finally, with $\alpha > \frac{V_2}{V_1}$, if $b_3^H < s_3 \Rightarrow \frac{\partial R}{\partial q_1} = V_2 - \alpha V_1 < 0$. In that case, $\frac{\partial R}{\partial q_1} < 0$ with $s_3 \gtreqless b_3^H$, such that $q_1 = 0$, $q_2 = b_2^H = 4\alpha$ and $q_3 = b_3^H = 2\alpha < 3 - 4\alpha = s_3$. Thus if $\alpha > \frac{V_2}{V_1}$, $q_1 = 0, q_2 = b_2^H = 4\alpha$ and $q_3 = s_3 = 2\alpha$. ∎

Proof of lemma 2. *Part (1)* In part 1 of this proof, we consider the case with $\alpha \leq \frac{V_2}{V_1}$.

Given that $p_1 = p_2 = V_2, p_3 = V_1$ and given the measure of units offered in period 1, $q_1 \in [0,2]$, and the measure of units offered in period 2, $0 \leq q_2 \leq s_2 = 3 - q_1$, the seller should offer $q_3 = s_3$ (if $s_3 < b_3^H$) or b_3^H (if $b_3^H < s_3$). Given the price path and that the seller offers measure q_1 in period 1, $b_2^H = \alpha(4-q_1), b_2^L = (1-\alpha)(4-q_1)$

$\ni b_2^H + b_2^L = 4 - q_1$. Given that measure of units offered in period 2 is $q_2 (= m_2)$, $b_3^H = 2\alpha + b_2^H \left(1 - \dfrac{q_2}{b_2^H + b_2^L}\right) = 2\alpha + \alpha(4 - q_1 - q_2)$ while $s_3 = 3 - q_1 - q_2$. First, we find the optimal q_2, which is chosen at the beginning of period 2. With $s_3 < b_3^H$, revenue earned from period 2 onwards is given by

$$R = q_2 V_2 + \rho(3 - q_1 - q_2)V_1$$

Differentiating partially with respect to q_2, $\dfrac{\partial R}{\partial q_2} = V_2 - \rho V_1$

Since $\rho \to 1$, we take $\rho = 1$ and appeal to continuity to get $\dfrac{\partial R}{\partial q_2} = V_2 - V_1 < 0$. With $b_3^H < s_3$, revenue earned from period 2 onwards is given by

$$R = q_2 V_2 + \rho[2\alpha + \alpha(4 - q_1 - q_2)]V_1$$

Differentiating partially with respect to q_2, $\dfrac{\partial R}{\partial q_2} = V_2 - \rho\alpha V_1$

Since $\rho \to 1$, we take $\rho = 1$ and appeal to continuity to get

$$\frac{\partial R}{\partial q_2} = V_2 - \alpha V_1 > 0 \text{ if } \alpha < \frac{V_2}{V_1}$$

With $\alpha = \dfrac{V_2}{V_1}, \dfrac{\partial R}{\partial q_2} = \rho V_2(1 - \rho) > 0$. Further, we can check that $s_3 < b_3^H$ iff $q_2 > \dfrac{3 - 6\alpha - q_1(1 - \alpha)}{1 - \alpha} = \theta$ (suppose).

Thus with $q_2 = \theta + \epsilon$, where $\epsilon > 0$, $s_3 < b_3^H \Rightarrow \dfrac{\partial R}{\partial q_2} < 0$. Again with $q_2 = \theta - \epsilon$, where $\epsilon > 0$, $b_3^H < s_3 \ni$ with $\alpha \le \dfrac{V_2}{V_1}$, $\dfrac{\partial R}{\partial q_2} > 0$. Thus if $\alpha \le \dfrac{V_2}{V_1}$, the seller should offer $q_2 = \theta = \dfrac{3 - 6\alpha - q_1(1 - \alpha)}{1 - \alpha} = \dfrac{3 - 6\alpha}{1 - \alpha} - q_1 \Rightarrow q_1 + q_2 = \dfrac{3 - 6\alpha}{1 - \alpha} = \gamma$ (suppose).

This implies that given q_1, the seller will always choose a $q_2 \ni q_1 + q_2 = \frac{3-6\alpha}{1-\alpha}$.

With $\alpha \in [\frac{1}{4}, \frac{V_2}{V_1}]$, we find that $\gamma = \frac{3-6\alpha}{1-\alpha} \leq 2 \ni$ with $\rho < 1$, the seller chooses $q_1 = \frac{3-6\alpha}{1-\alpha}$, $q_2 = 0$ and $q_3 = s_3 = b_3^H = \frac{3\alpha}{1-\alpha}$. If $\alpha < \frac{1}{4} \leq \frac{V_2}{V_1}$, $\gamma = \frac{3-6\alpha}{1-\alpha} > 2 \ni$ with $\rho < 1$ the seller chooses $q_1 = 2$, $q_2 = \frac{3-6\alpha}{1-\alpha} - 2 = \frac{1-4\alpha}{1-\alpha}$ and $q_3 = s_3 = b_3^H = \frac{3\alpha}{1-\alpha}$.

Part (2) In the second part, we consider the case with $\alpha > \frac{V_2}{V_1}$.

With $\alpha > \frac{V_2}{V_1}$, if $b_3^H < s_3 \Rightarrow \frac{\partial R}{\partial q_2} = V_2 - \alpha V_1 < 0$. In that case, $\frac{\partial R}{\partial q_2} < 0$ with $s_3 \gtrless b_3^H$, such that $q_2 = 0$. Given that $q_2 = 0$ and $s_3 < b_3^H$ revenue earned over the 3 periods is given by

$$R = q_1 V_2 + \rho^2 (3 - q_1) V_1$$

Differentiating partially with respect to q_1, $\frac{\partial R}{\partial q_1} = V_2 - \rho^2 V_1$. Since $\rho \to 1$, we take $\rho = 1$ and appeal to continuity to get $\frac{\partial R}{\partial q_1} = V_2 - V_1 < 0$. With $q_2 = 0$ and $b_3^H < s_3$, revenue earned is given by

$$R = q_1 V_2 + \rho^2 [2\alpha + \alpha(4 - q_1)] V_1$$

Differentiating partially with respect to q_1, $\frac{\partial R}{\partial q_1} = V_2 - \rho^2 \alpha V_1$. With $\rho \to 1$, we take $\rho = 1$ and appeal to continuity to get

$$\frac{\partial R}{\partial q_1} = V_2 - \alpha V_1 < 0 \text{ with } \alpha > \frac{V_2}{V_1}$$

This implies that $\frac{\partial R}{\partial q_1} < 0$ with $s_3 \gtrless b_3^H \ni q_1 = 0$. Thus if $\alpha > \frac{V_2}{V_1}$, then with $\rho \to 1$ we get $q_1 = 0, q_2 = 0, q_3 = b_3^H = 6\alpha$. ∎

Proof of lemma 3.

Since the seller offers measure q_1 units in period 1 at price $V_2, b_2^H = \alpha(4-q_1)$. Since $q_2 = b_2^H$ and $p_2 = p_2^H$, $b_3^H = 2\alpha$ while $b_3^L = 2(1-\alpha)(1-\frac{q_1}{2})+4(1-\alpha) = 6(1-\alpha)-q_1(1-\alpha)$. Thus $b_3^H+b_3^L = 6-4\alpha-q_1(1-\alpha)$ and $s_3 = 3-q_1-\alpha(4-q_1) = 3-4\alpha-q_1(1-\alpha)$. With $s_3 \leq b_3^H$ the seller will always choose not to hold a 'sale' in the last period, such that to rule out profitable deviations for the seller in the last period, we require that $q_3 = s_3 > b_3^H$ and that $s_3V_2 > b_3^H V_1$. Thus with $q_3 = s_3$, sum of discounted revenue earned is

$$R = q_1V_2+\rho\alpha(4-q_1)\left[V_1\left(1-\frac{3-4\alpha-q_1(1-\alpha)}{6-4\alpha-q_1(1-\alpha)}\right)+\frac{3-4\alpha-q_1(1-\alpha)}{6-4\alpha-q_1(1-\alpha)}V_2\right]+$$
$$\rho^2[3-4\alpha-q_1(1-\alpha)]V_2$$

$$= q_1V_2+\rho\alpha(4-q_1)[V_1-(V_1-V_2)y]+\rho^2[3-4\alpha-q_1(1-\alpha)]V_2$$

where $y = \dfrac{3-4\alpha-q_1(1-\alpha)}{6-4\alpha-q_1(1-\alpha)}$.

Differentiating partially with respect to q_1,

$$\frac{\partial R}{\partial q_1} = V_2-\rho\alpha V_1+\rho\alpha(V_1-V_2)y-\rho\alpha(4-q_1)(V_1-V_2)\frac{\partial y}{\partial q_1}-\rho^2(1-\alpha)V_2$$

where $\dfrac{\partial y}{\partial q_1} = \dfrac{3(\alpha-1)}{[6-4\alpha-q_1(1-\alpha)]^2} < 0$

Since $\rho \to 1$, we take $\rho = 1$ and appeal to continuity to get

$$\frac{\partial R}{\partial q_1} = -\alpha(V_1-V_2)+\alpha(V_1-V_2)y-\alpha(4-q_1)(V_1-V_2)\frac{\partial y}{\partial q_1}$$

Substituting $q_1 = 0$,

$$\frac{\partial R}{\partial q_1} = (V_1-V_2)\left[-\alpha+\alpha\frac{3-4\alpha}{6-4\alpha}-4\alpha\frac{3(\alpha-1)}{(6-4\alpha)^2}\right] < 0 \text{ since } \left[-\alpha+\alpha\frac{3-4\alpha}{6-4\alpha}\right.$$
$$\left.-4\alpha\frac{3(\alpha-1)}{(6-4\alpha)^2}\right] < 0 \;\forall\; \alpha \in (0,1/2).$$

Differentiating partially again with respect to q_1,

$$\frac{\partial^2 R}{\partial q_1^2} = 2\alpha(V_1 - V_2)\frac{\partial y}{\partial q_1} - \alpha(4 - q_1)(V_1 - V_2)\frac{\partial^2 y}{\partial q_1^2}$$

$$= (V_1 - V_2)[A - B]$$

where, $A = 2\alpha \left[\frac{-3(1-\alpha)}{\{6 - 4\alpha - q_1(1-\alpha)\}^2} \right] < 0$

and $B = \alpha(4 - q_1) \left[\frac{6(1-\alpha)\{6 - 4\alpha - q_1(1-\alpha)\}(-(1-\alpha))}{\{6 - 4\alpha - q_1(1-\alpha)\}^4} \right] < 0.$

Assume that $|A| < |B| \ni$

$$2\alpha \frac{3(1-\alpha)}{\{6 - 4\alpha - q_1(1-\alpha)\}^2} < \ \alpha(4 - q_1)\frac{6(1-\alpha)\{6 - 4\alpha - q_1(1-\alpha)\}(1-\alpha)}{\{6 - 4\alpha - q_1(1-\alpha)\}^4}$$

$\Rightarrow 12 < 8$ which is a contradiction. Thus $|A| > |B|$ such that

$$\frac{\partial^2 R}{\partial q_1^2} = (V_1 - V_2)[A - B] < 0 \ \forall q_1$$

We thus find that the first partial derivative is negative at $q_1 = 0$ and that the second partial derivative is negative everywhere. This implies that the optimal $q_1 = 0$. With $\rho \to 1$, the seller should thus offer $q_1 = 0, q_2 = b_2^H = 4\alpha$ and $q_3 = s_3 = 3 - 4\alpha$. ■

Proof of lemma 4.

Part (1). If $\alpha \leq \frac{2V_2}{V_1 + V_2}$

Given that the seller charges V_2 in the second period only, and that he or she offers measure q_1, q_2 in periods 1 and 2 respectively and $q_3 = s_3$ (if $s_3 < b_3^H$) or b_3^H (if $b_3^H < s_3$), the seller should charge $p_1 = p_1^H = V_1(1 - \frac{q_2}{4 - q_1}) + \frac{q_2}{4 - q_1}V_2$.

$b_2^H = 4\alpha - q_1, b_2^L = 4(1-\alpha)$ while $b_3^H = 2\alpha + (4\alpha - q_1)\left(1 - \frac{q_2}{4 - q_1}\right)$ and $s_3 = 3 - q_1 - q_2$.

First, we solve for the optimal q_2 as a function of q_1, where q_2 is chosen at the beginning of period 2. With $s_3 < b_3^H$, sum of discounted revenue earned from period 2 onwards is

$$R = q_2 V_2 + \rho(3 - q_1 - q_2)V_1$$

Differentiating partially with respect to q_2, $\dfrac{\partial R}{\partial q_2} = V_2 - \rho V_1$

Taking $\rho = 1$ and appealing to continuity provides $\dfrac{\partial R}{\partial q_2} = V_2 - V_1 < 0$

With $b_3^H < s_3$,

$$R = q_2 V_2 + \rho \left[2\alpha + (4\alpha - q_1)\left(1 - \frac{q_2}{4 - q_1}\right)\right] V_1$$

Differentiating partially with respect to q_2, $\dfrac{\partial R}{\partial q_2} = V_2 - \rho \dfrac{4\alpha - q_1}{4 - q_1} V_1$

Taking $\rho = 1$ and appealing to continuity provides

$\dfrac{\partial R}{\partial q_2} = (4 - q_1)V_2 - (4\alpha - q_1)V_1 > 0$ if $\alpha \leq \dfrac{(4 - q_1)V_2 + q_1 V_1}{4V_1}$.

Further $s_3 < b_3^H$ if $q_2 > \dfrac{12 - 24\alpha + 6\alpha q_1 - 3q_1}{4(1 - \alpha)} = \theta$ (suppose). If $q_2 = \theta + \epsilon$, for any $\epsilon > 0$, $s_3 < b_3^H \ni \dfrac{\partial R}{\partial q_2} < 0$ while if $q_2 = \theta - \epsilon$, for any $\epsilon > 0$, $b_3^H < s_3 \Rightarrow \dfrac{\partial R}{\partial q_2} > 0$. Thus if $\alpha \leq \dfrac{V_2}{V_1}$, the seller chooses $q_2 = \dfrac{12 - 24\alpha + 6\alpha q_1 - 3q_1}{4(1 - \alpha)}$ (*with* $\dfrac{\partial q_2}{\partial q_1} = \dfrac{6\alpha - 3}{4(1 - \alpha)} < 0$).

Substituting the value of q_2 into the revenue function, with $s_3 < b_3^H$

$$R = [V_1 - (V_1 - V_2)y]q_1 + \rho q_2 V_2 + \rho^2 (3 - q_1 - q_2)V_1$$

where $y = \dfrac{q_2}{4 - q_1}$ such that $\dfrac{\partial y}{\partial q_1} = \dfrac{(4 - q_1)\dfrac{\partial q_2}{\partial q_1} + q_2}{(4 - q_1)^2} = 0$.

Differentiating partially with respect to q_1 yields

$$\frac{\partial R}{\partial q_1} = V_1 - (V_1 - V_2)q_1\frac{\partial y}{\partial q_1} - (V_1 - V_2)y + \rho\frac{\partial q_2}{\partial q_1}V_2 - \rho^2 V_1 - \rho^2\frac{\partial q_2}{\partial q_1}V_1$$

Taking $\rho = 1$, we find that $\frac{\partial R}{\partial q_1} = 0\ \forall q_1$.

Since $\frac{\partial}{\partial \rho}(\frac{\partial R}{\partial q_1}) = \frac{\partial q_2}{\partial q_1}V_2 - 2\rho V_1 - 2\rho\frac{\partial q_2}{\partial q_1}V_1 < 0$ for $\rho = 1$ and $\forall\alpha$, it implies that with $\rho \to 1$, $\frac{\partial R}{\partial q_1} > 0\ \forall q_1 \ni q_1 = 2\alpha$.

Substituting value of q_2 into the revenue function with $b_3^H < s_3$ sum of discounted revenue earned is

$$R = [V_1 - (V_1 - V_2)y]q_1 + \rho q_2 V_2 + \rho^2\left[2\alpha + (4\alpha - q_1)\left(1 - \frac{q_2}{4 - q_1}\right)\right]V_1$$

where $y = \frac{q_2}{4 - q_1}$ such that $\frac{\partial y}{\partial q_1} = \frac{(4 - q_1)\frac{\partial q_2}{\partial q_1} + q_2}{(4 - q_1)^2} = 0$.

Differentiating partially with respect to q_1 yields

$$\begin{aligned}\frac{\partial R}{\partial q_1} =& V_1 - (V_1 - V_2)q_1\frac{\partial y}{\partial q_1} - (V_1 - V_2)y + \rho\frac{\partial q_2}{\partial q_1}V_2 - \rho^2\left(\frac{4 - q_1 - q_2}{4 - q_1}\right)V_1 \\ &+ \rho^2(4\alpha - q_1)\left[\frac{(4 - q_1)(-1 - \frac{\partial q_2}{\partial q_1}) + 4 - q_1 - q_2}{(4 - q_1)^2}\right]V_1\end{aligned}$$

Taking $\rho = 1$, we find that $\frac{\partial R}{\partial q_1} = 0\ \forall q_1$.

Since $\frac{\partial}{\partial \rho}(\frac{\partial R}{\partial q_1}) = \frac{\partial q_2}{\partial q_1}V_2 - 2\rho(1 - y)V_1 < 0$ for $\rho = 1$ and $\forall\alpha$, it implies that with $\rho \to 1$, $\frac{\partial R}{\partial q_1} > 0\ \forall q_1 \ni q_1 = 2\alpha$.

Substituting $q_1 = 2\alpha$ into $q_2 = \frac{12 - 24\alpha + 6\alpha q_1 - 3q_1}{4(1 - \alpha)}$ provides $q_2 = \frac{6\alpha^2 - 15\alpha + 6}{2(1 - \alpha)}$,

$q_3 = s_3 = b_3^H = \dfrac{5\alpha - 2\alpha^2}{2(1-\alpha)}$.

Thus the required condition for $q_1 = 2\alpha, q_2 = \dfrac{6\alpha^2 - 15\alpha + 6}{2(1-\alpha)}, q_3 = \dfrac{5\alpha - 2\alpha^2}{2(1-\alpha)}$ is: $\alpha \leq \dfrac{(4-q_1)V_2 + q_1 V_1}{4V_1}, \rho \to 1 \Rightarrow \alpha \leq \dfrac{2V_2}{V_1 + V_2}, \rho \to 1$

Part (2). If $\alpha > \dfrac{2V_2}{V_1 + V_2}$

From part (1) we find that if $\alpha > \dfrac{(4-q_1)V_2 + q_1 V_1}{4V_1}, \rho \to 1$ the seller offers $q_2 = 0$. Substituting the value of q_2 into the revenue function for $s_3 < b_3^H$

$$R = q_1 V_1 + \rho^2 (3 - q_1) V_1$$

Differentiating partially with respect to q_1, $\dfrac{\partial R}{\partial q_1} = V_1(1-\rho^2) > 0 \ni q_1 = 2\alpha$

Substituting the value of q_2 into the revenue function for $b_3^H < s_3$

$$R = q_1 V_1 + \rho^2 (6\alpha - q_1) V_1$$

Differentiating partially with respect to q_1, $\dfrac{\partial R}{\partial q_1} = V_1(1-\rho^2) > 0 \ni q_1 = 2\alpha$.

Substituting the value of q_1 into the required condition $\alpha > \dfrac{(4-q_1)V_2 + q_1 V_1}{4V_1}, \rho \to 1 \Rightarrow \alpha > \dfrac{2V_2}{V_1 + V_2}, \rho \to 1$ such that the seller offers $q_1 = 2\alpha, q_2 = 0$ and $q_3 = 4\alpha$ if $\alpha > \dfrac{2V_2}{V_1 + V_2}, \rho \to 1$. ■

Appendix 2

Proof of proposition 1.

For $(V_2, V_1, V_1, 2, 2\alpha, 2\alpha)$ to be subgame perfect for $\alpha < \frac{1}{4} \leq \frac{V_2}{V_1}$, there should not exist any profitable deviation for the seller in any period. This implies that the sum of discounted revenue earned from $(V_2, V_1, V_1, 2, 2\alpha, 2\alpha)$ should be greater than that earned from $(V_2, V_1, V_1, 2, \frac{1-4\alpha}{1-\alpha}, \frac{3\alpha}{1-\alpha})$. The required condition is thus

$$2V_2 + 2\rho V_1 \alpha(1+\rho) > 2V_2 + \rho\frac{1-4\alpha}{1-\alpha}V_2 + \rho^2\frac{3\alpha}{1-\alpha}V_1$$

Since $\rho \to 1$, we take $\rho = 1$ and appeal to continuity to get

$$4V_1\alpha > \frac{1-4\alpha}{1-\alpha}V_2 + \frac{3\alpha}{1-\alpha}V_1 \Rightarrow \alpha > \frac{V_2}{V_1}$$

which is a contradiction to $\alpha \leq \frac{V_2}{V_1}$. Thus if $\alpha < \frac{1}{4} \leq \frac{V_2}{V_1}$ and $\rho \to 1$, $(V_2, V_1, V_1, 2, 2\alpha, 2\alpha)$ cannot be a subgame perfect outcome. ■

Proof of proposition 2.

For $\left(V_2, V_2, V_1, \frac{3-6\alpha}{1-\alpha}, 0, \frac{3\alpha}{1-\alpha}\right)$ to be subgame perfect for $\alpha \in \left[\frac{1}{4}, \frac{V_2}{V_1}\right]$, there should not exist any profitable deviation for the seller in any period. This implies that the sum of discounted revenue earned from $\left(V_2, V_2, V_1, \frac{3-6\alpha}{1-\alpha}, 0, \frac{3\alpha}{1-\alpha}\right)$ should be greater than that earned from $\left(V_2, V_1, V_1, \frac{3-6\alpha}{1-\alpha}, \frac{\alpha+2\alpha^2}{1-\alpha}, 2\alpha\right)$. The required condition is

$$\frac{3-6\alpha}{1-\alpha}V_2 + \rho^2\frac{3\alpha}{1-\alpha}V_1 > \frac{3-6\alpha}{1-\alpha}V_2 + \rho\frac{\alpha+2\alpha^2}{1-\alpha}V_1 + \rho^2 2V_1\alpha \Rightarrow \rho > 1$$

which is a contradiction. Thus if $\alpha \in \left[\frac{1}{4}, \frac{V_2}{V_1}\right]$ and $\rho \to 1$, $\left(V_2, V_2, V_1, \frac{3-6\alpha}{1-\alpha}, 0, \frac{3\alpha}{1-\alpha}\right)$ cannot be subgame perfect outcome. ■

Proof for proposition 3.

For $(V_2, V_2, V_2, 2, 1, 0)$ to be subgame perfect for $\alpha \leq \frac{V_2}{V_1}$, it must be the case that there does not exist any profitable deviation for the seller in any period, given the history of the game. We begin by assuming that $\alpha < \frac{1}{4}$.Given that $p_1 = V_2, q_1 = 2$ the seller could choose to deviate in period 2 and charge $p_2 = V_2, p_3 = V_1$ and to offer measure $q_2, q_3 = s_3$(if $s_3 < b_3^H$) or b_3^H (if $b_3^H < s_3$) in periods 2 and 3. Then $b_3^H = \alpha(4 - q_2), s_3 = 1 - q_2$. With $s_3 < b_3^H$, $R = q_2V_2 + \rho(1 - q_2)V_1$

Differentiating partially with respect to q_2, $\frac{\partial R}{\partial q_2} = V_2 - \rho V_1 < 0$. With $b_3^H < s_3, R = q_2V_2 + \rho\alpha(4 - q_2)V_1 \Rightarrow \frac{\partial R}{\partial q_2} = V_2 - \rho\alpha V_1 > 0$ if $\alpha \leq \frac{V_2}{V_1}$. We can check that $b_3^H < s_3$ iff $q_2 < \frac{1-4\alpha}{1-\alpha} = \theta$ (suppose). Thus with $q_2 = \theta + \epsilon$, where $\epsilon > 0$, $s_3 < b_3^H \Rightarrow \frac{\partial R}{\partial q_2} < 0$. Again with $q_2 = \theta - \epsilon$, where $\epsilon > 0$, $b_3^H < s_3 \ni$ with $\alpha \leq \frac{V_2}{V_1}$, $\frac{\partial R}{\partial q_2} > 0$.Thus, the seller offers $q_2 = \theta = \frac{1-4\alpha}{1-\alpha}, q_3 = s_3 = b_3^H = \frac{3\alpha}{1-\alpha}$ (with $\alpha > \frac{V_2}{V_1}$, the seller would offer $q_2 = 0, q_3 = 4\alpha$).

For $(V_2, V_2, V_2, 2, 1, 0)$ to be subgame perfect, to rule out this deviation in period 2 the required condition is

$$V_2 > \frac{1-4\alpha}{1-\alpha}V_2 + \rho\frac{3\alpha}{1-\alpha}V_1$$

Taking $\rho = 1$ and appealing to continuity, we get $V_2 > V_1$ which is a contradiction. Thus a profitable deviation exists for the seller in period 2, such that if $\alpha \leq \frac{V_2}{V_1}, \rho \to 1$ $(V_2, V_2, V_2, 2, 1, 0)$ cannot be subgame perfect.

The above proof works for $\alpha < \frac{1}{4}$. For $\alpha \in \left[\frac{1}{4}, \frac{V_2}{V_1}\right]$, we show that there exists a profitable deviation in period 1. Here, the seller can offer $p_1 = V_2$, $p_2 = p_3 = V_1$ and $q_1 = \frac{3-6\alpha}{1-\alpha}, q_2 = \frac{\alpha+2\alpha^2}{1-\alpha}, q_3 = 2\alpha$. We assume to the contrary that there does not exist a profitable deviation such that

$$2V_2 + \rho V_2 > \frac{3-6\alpha}{1-\alpha}V_2 + \rho\frac{\alpha+2\alpha^2}{1-\alpha}V_1 + \rho^2 2V_1\alpha$$

Since $\rho \to 1$, taking $\rho = 1$ and appealing to continuity yields $V_2 > V_1$ which is a contradiction. Thus for $\alpha \in \left[\frac{1}{4}, \frac{V_2}{V_1}\right], \rho \to 1$ a profitable deviation exists in period 1 for $(V_2, V_2, V_2, 2, 1, 0)$, such that it is not subgame perfect. ■

Proof of proposition 4.

In order to prove that with $\rho \to 1$ a strategy involving a 'sale' in the first and last period can never be subgame perfect, we'll show that there exists a profitable deviation for the seller in period 2, given the history of the game $p_1 = V_2$ and $q_1 = 0$.

Part (1). If $\alpha \leq \frac{V_2}{V_1}$

Given that $p_1 = V_2$ and $q_1 = 0$, we get $b_2^H = 4\alpha$ and $b_2^L = 4(1-\alpha)$. If the seller deviates and chooses to charge $p_2 = V_2, p_3 = V_1$ and to offer measure q_2 and q_3 in periods 2 and 3, $b_3^H = 6\alpha - \alpha q_2$ and $s_3 = 3 - q_2$. With $s_3 < b_3^H$ revenue earned from the second period onwards is $R = q_2V_2 + \rho(3-q_2)V_1$, such that on partial differentiation with respect to q_2 we get $\frac{\partial R}{\partial q_2} = V_2 - \rho V_1 < 0$. With $b_3^H < s_3$, $R = q_2V_2 + \rho(6\alpha - \alpha q_2)V_1$ $\ni \frac{\partial R}{\partial q_2} = V_2 - \rho\alpha V_1 > 0$ if $\alpha \leq \frac{V_2}{V_1}$ and $\rho \to 1$. Further we can check that $s_3 < b_3^H$ iff $q_2 > \frac{3-6\alpha}{1-\alpha} = \theta$ (suppose). With $q_2 = \theta + \epsilon$, for any $\epsilon > 0$, $s_3 < b_3^H \Rightarrow \frac{\partial R}{\partial q_2} < 0$ and with $q_2 = \theta - \epsilon$, for any $\epsilon > 0$, $b_3^H < s_3 \ni \frac{\partial R}{\partial q_2} > 0$. Thus with $\alpha \leq \frac{V_2}{V_1}, \rho \to 1$ if the seller chooses to charge $p_2 = V_2$ and $p_3 = V_1$, the seller should offer $q_2 = \theta = \frac{3-6\alpha}{1-\alpha}$ and $q_3 = s_3 = b_3^H = \frac{3\alpha}{1-\alpha}$. Also with $\alpha > \frac{V_2}{V_1}, \rho \to 1$ we can check that if the seller proposes to charge $p_2 = V_2$ and $p_3 = V_1$, the seller should offer $q_2 = 0$ and $q_3 = 6\alpha$.

For $\left(V_2, V_1\left(1 - \frac{3-4\alpha}{6-4\alpha}\right) + \frac{3-4\alpha}{6-4\alpha}V_2, V_2, 0, 4\alpha, 3-4\alpha\right)$ to be subgame perfect, there should not exist any profitable deviation for the seller in any period. This implies that

$$4\alpha\left[V_1\left(1 - \frac{3-4\alpha}{6-4\alpha}\right) + \frac{3-4\alpha}{6-4\alpha}V_2\right] + \rho(3-4\alpha)V_2 > \frac{3-6\alpha}{1-\alpha}V_2 + \rho\frac{3\alpha}{1-\alpha}V_1$$

Since $\rho \to 1$, we take $\rho = 1$ and appeal to continuity to get $V_2 > V_1$ which is a contradiction. Thus, with $\alpha \leq \frac{V_2}{V_1}, \rho \to 1$ a profitable deviation exists for the seller in period 2, whereby he or she charges $p_2 = V_2, p_3 = V_1$ and offers $q_2 = \frac{3-6\alpha}{1-\alpha}, q_3 = \frac{3\alpha}{1-\alpha}$.

Part (2). If $\alpha > \frac{V_2}{V_1}$.

As was shown in part (1) above, in case the seller chooses to deviate in the second period and charge $p_2 = V_2$ and $p_3 = V_1$ and if $\alpha > \frac{V_2}{V_1}$, then the seller should offer $q_2 = 0$ and $q_3 = 6\alpha$. For $\left(V_2, V_1\left(1 - \frac{3-4\alpha}{6-4\alpha}\right) + \frac{3-4\alpha}{6-4\alpha}V_2, V_2, 0, 4\alpha, 3-4\alpha\right)$ to be subgame perfect, there should not exist any profitable deviation for the seller in any period. This implies

$$4\alpha\left[V_1(1 - \frac{3-4\alpha}{6-4\alpha}) + \frac{3-4\alpha}{6-4\alpha}V_2\right] + \rho(3-4\alpha)V_2 > \rho 6 V_1 \alpha$$

Taking $\rho = 1$ and appealing to continuity, we get $\alpha < \left(\frac{18-24\alpha}{24-24\alpha}\right)\frac{V_2}{V_1} < \frac{V_2}{V_1}$ which is a contradiction to $\alpha > \dfrac{V_2}{V_1}$. Thus with $\alpha > \dfrac{V_2}{V_1}, \rho \to 1$ a profitable deviation exists for the seller in period 2, whereby he or she charges $p_2 = V_2, p_3 = V_1$ and offers measure $q_2 = 0, q_3 = 6\alpha$. ■

Proof of proposition 5.

In order to show that with $\alpha \in \left[\dfrac{1}{4}, \dfrac{V_2}{V_1}\right]$ and $\rho \to 1$, $\left(V_1(1 - \dfrac{6\alpha^2 - 15\alpha + 6}{2(1-\alpha)(4-2\alpha)}) + \dfrac{6\alpha^2 - 15\alpha + 6}{2(1-\alpha)(4-2\alpha)}V_2, V_2, V_1, 2\alpha, \dfrac{6\alpha^2 - 15\alpha + 6}{2(1-\alpha)}, \dfrac{5\alpha - 2\alpha^2}{2(1-\alpha)}\right)$ cannot be subgame perfect, we'll show the seller can deviate profitably in period 1 by charging $p_1 = V_2, p_2 = p_3 = V_1$ and by offering $q_1 = \dfrac{3 - 6\alpha}{1-\alpha}$, $q_2 = \dfrac{\alpha + 2\alpha^2}{1-\alpha}$ and $q_3 = 2\alpha$.

We define $A = \left[V_1(1 - \dfrac{6\alpha^2 - 15\alpha + 6}{2(1-\alpha)(4-2\alpha)}) + \dfrac{6\alpha^2 - 15\alpha + 6}{2(1-\alpha)(4-2\alpha)}V_2\right] 2\alpha + \rho\dfrac{6\alpha^2 - 15\alpha + 6}{2(1-\alpha)}V_2 + \rho^2\dfrac{5\alpha - 2\alpha^2}{2(1-\alpha)}V_1 - (\dfrac{3-6\alpha}{1-\alpha}V_2 + \rho\dfrac{\alpha + 2\alpha^2}{1-\alpha}V_1 + 2\rho^2 V_1\alpha)$

For the given pricing policy to be subgame perfect for $\alpha \in \left[\dfrac{1}{4}, \dfrac{V_2}{V_1}\right], \rho \to 1$ it must be the case that $A > 0$.

With $\rho = 1$, we can check to see that $A = 0$.

Partially differentiating both sides of the above equation with respect to ρ,

$$\frac{\partial A}{\partial \rho} = \frac{6\alpha^2 - 15\alpha + 6}{2(1-\alpha)}V_2 - (\frac{\alpha + 2\alpha^2}{1-\alpha})V_1 + 2\rho(\frac{\alpha + 2\alpha^2}{2(1-\alpha)})V_1$$

$$\left.\frac{\partial A}{\partial \rho}\right|_{\rho=1} = \frac{6\alpha^2 - 15\alpha + 6}{2(1-\alpha)} > 0 \ \forall \alpha \text{ such that with } \rho \to 1,\ A < 0$$

Thus there exists a profitable deviation in period 1 for the seller, which implies that if $\alpha \in \left[\dfrac{1}{4}, \dfrac{V_2}{V_1}\right], \rho \to 1$ then $\left(V_1(1 - \dfrac{6\alpha^2 - 15\alpha + 6}{2(1-\alpha)(4-2\alpha)}) + \dfrac{6\alpha^2 - 15\alpha + 6}{2(1-\alpha)(4-2\alpha)}V_2, V_2, V_1, 2\alpha, \dfrac{6\alpha^2 - 15\alpha + 6}{2(1-\alpha)}, \dfrac{5\alpha - 2\alpha^2}{2(1-\alpha)}\right)$ cannot be subgame perfect. ■

Proof of proposition 6.

To prove that $\left[V_1\left(1-\frac{3-2\alpha}{4-2\alpha}\right)+\frac{3-2\alpha}{4-2\alpha}V_2, V_2, V_2, 2\alpha, 3-2\alpha, 0\right]$ cannot be subgame perfect if $\alpha \leq \frac{V_2}{V_1}$, we will show that there exists a profitable deviation for the seller in period 2.

Given the history of the game at the beginning of period 2 with $p_1 = p_1^H, q_1 = 2\alpha$ the seller could choose to deviate and charge $p_2 = V_2$ and $p_3 = V_1$ and to offer measure q_2 and q_3 in periods 2 and 3 respectively. Then $b_3^H = 2\alpha\left(\frac{8-4\alpha-q_2}{4-2\alpha}\right)$ and $s_3 = 3-2\alpha-q_2$.

With $s_3 < b_3^H$, revenue earned is $R = q_2V_2 + \rho[3-2\alpha-q_2]V_1$

Differentiating partially with respect to q_2, $\frac{\partial R}{\partial q_2} = V_2 - \rho V_1 < 0$ with $\rho \to 1$.

With $b_3^H < s_3$, $R = q_2V_2 + \rho 2\alpha(\frac{8-4\alpha-q_2}{4-2\alpha})V_1$

Differentiating partially with respect to q_2, $\frac{\partial R}{\partial q_2} = V_2 - \rho\frac{2\alpha V_1}{4-2\alpha}$.

Taking $\rho = 1$ and appealing to continuity yields $\frac{\partial R}{\partial q_2} = V_2 - \frac{2\alpha V_1}{4-2\alpha} > 0$ if $\alpha \leq \frac{2V_2}{V_1+V_2}$.

Since $\alpha \leq \frac{V_2}{V_1} < \frac{2V_2}{V_1+V_2}, \frac{\partial R}{\partial q_2} > 0$. Further $s_3 < b_3^H$ if $q_2 > \frac{6\alpha^2-15\alpha+6}{2(1-\alpha)} = \theta$ (suppose). Thus if $q_2 = \theta+\epsilon, \epsilon > 0, s_3 < b_3^H \Rightarrow \frac{\partial R}{\partial q_2} < 0$ while if $q_2 = \theta-\epsilon, \epsilon > 0, b_3^H < s_2 \Rightarrow \frac{\partial R}{\partial q_2} > 0$

Thus if $\alpha \leq \frac{V_2}{V_1}$ and the seller chooses to deviate in period 2 and to charge $p_2 = V_2, p_3 = V_1$ he or she should offer $q_2 = \theta = \frac{6\alpha^2-15\alpha+6}{2(1-\alpha)}$ and $q_3 = s_3 = b_3^H = \frac{5\alpha-2\alpha^2}{2(1-\alpha)}$. If $\left[V_1\left(1-\frac{3-2\alpha}{4-2\alpha}\right)+\frac{3-2\alpha}{4-2\alpha}V_2, V_2, V_2, 2\alpha, 3-2\alpha, 0\right]$ is subgame perfect, it must be the case that there does not exist any profitable deviation in period 2. Thus, we require that $(3-2\alpha)V_2 > \frac{6\alpha^2-15\alpha+6}{2(1-\alpha)}V_2 + \rho\frac{5\alpha-2\alpha^2}{2(1-\alpha)}V_1$

Since $\rho \to 1$, we take $\rho = 1$ and appeal to continuity to get $V_2 > V_1$ which is a contradiction. Thus a profitable deviation exists for the seller in period 2, such that $\left[V_1\left(1 - \frac{3-2\alpha}{4-2\alpha}\right) + \frac{3-2\alpha}{4-2\alpha}V_2, V_2, V_2, 2\alpha, 3-2\alpha, 0\right]$ cannot be subgame perfect. ■

Proof of proposition 7.

The proof is similar to the one for proposition 6. To show that $\left[V_1\left(1 - \frac{3-4\alpha}{6-4\alpha}\right) + \frac{3-4\alpha}{6-4\alpha}V_2, V_1\left(1 - \frac{3-4\alpha}{6-4\alpha}\right) + \frac{3-4\alpha}{6-4\alpha}V_2, V_2, 2\alpha, 2\alpha, 3-4\alpha\right]$ cannot be subgame perfect we show that there exists a profitable deviation in period 2. Given the history of the game at the beginning of period 2 with $p_1 = p_1^H, q_1 = 2\alpha$ the seller could choose to deviate and charge $p_2 = V_2$ and $p_3 = V_1$ and to offer measure q_2 and q_3 in periods 2 and 3 respectively.

As was shown in the proof for proposition 6, if $\alpha \leq \frac{V_2}{V_1}, \rho \to 1$ the seller offers $q_2 = \frac{6\alpha^2 - 15\alpha + 6}{2(1-\alpha)}$ and $q_3 = s_3 = b_3^H = \frac{5\alpha - 2\alpha^2}{2(1-\alpha)}$.

If $\left[V_1\left(1 - \frac{3-4\alpha}{6-4\alpha}\right) + \frac{3-4\alpha}{6-4\alpha}V_2, V_1\left(1 - \frac{3-4\alpha}{6-4\alpha}\right) + \frac{3-4\alpha}{6-4\alpha}V_2, V_2, 2\alpha, 2\alpha, 3-4\alpha\right]$ is subgame perfect, it must be the case that

$$2\alpha\left[V_1\left(1 - \frac{3-4\alpha}{6-4\alpha}\right) + \frac{3-4\alpha}{6-4\alpha}V_2\right] + \rho(3-4\alpha)V_2 > \frac{6\alpha^2 - 15\alpha + 6}{2(1-\alpha)}V_2 + \rho\frac{5\alpha - 2\alpha^2}{2(1-\alpha)}V_1$$

Since $\rho \to 1$, we take $\rho = 1$ and appeal to continuity to get $(20\alpha^2 - 8\alpha^3 - 18\alpha)V_1 > (20\alpha^2 - 8\alpha^3 - 18\alpha)V_2 \Rightarrow V_2 > V_1$ (since $(20\alpha^2 - 8\alpha^3 - 18\alpha) < 0\ \forall\alpha$) which is a contradiction.

Thus a profitable deviation exists for the seller in period 2, such that $\left[V_1\left(1-\frac{3-4\alpha}{6-4\alpha}\right)+\frac{3-4\alpha}{6-4\alpha}V_2, V_1\left(1-\frac{3-4\alpha}{6-4\alpha}\right)+\frac{3-4\alpha}{6-4\alpha}V_2, V_2, 2\alpha, 2\alpha, 3-4\alpha\right]$ cannot be subgame perfect. ■

Proof of proposition 8.

Once again the proof is similar to the one for proposition 6. To prove that if $\alpha \leq \frac{V_2}{V_1}$, $(V_1, V_1, V_1, 2\alpha, 2\alpha, 2\alpha)$ cannot be subgame perfect we show that the seller can deviate profitably in period 2 by setting $p_2 = V_2, p_3 = V_1$ and by offering $q_2 = \frac{6\alpha^2 - 15\alpha + 6}{2(1-\alpha)}$ and $q_3 = s_3 = b_3^H = \frac{5\alpha - 2\alpha^2}{2(1-\alpha)}$.

If $(V_1, V_1, V_1, 2\alpha, 2\alpha, 2\alpha)$ is subgame perfect, the following condition must hold

$$2V_1\alpha(1+\rho) > \frac{6\alpha^2 - 15\alpha + 6}{2(1-\alpha)}V_2 + \rho\frac{5\alpha - 2\alpha^2}{2(1-\alpha)}V_1$$

Since $\rho \to 1$, we take $\rho = 1$ and appeal to continuity to get $\alpha > \frac{2V_2}{V_1 + V_2} > \frac{V_2}{V_1}$ which is a contradiction to $\alpha \leq \frac{V_2}{V_1}$. Thus a profitable deviation exists for the seller in period 2, such that $(V_1, V_1, V_1, 2\alpha, 2\alpha, 2\alpha)$ cannot be subgame perfect. ■

Appendix 3

Proof of proposition 9.

In order to prove that $(V_2, V_1, V_1, \frac{3-6\alpha}{1-\alpha}, \frac{\alpha+2\alpha^2}{1-\alpha}, 2\alpha)$ is subgame perfect if $\alpha \in \left[\frac{1}{4}, \frac{V_2}{V_1}\right]$, $\rho \to 1$ we show that there does not exist any profitable deviation for the seller in any period i, where $i = 1, 2, 3$.

Period 3. As was shown in the proof for lemma 1, the seller chooses q_1 in a way which ensures that $s_3 = b_3^H = 2\alpha$. Given that $s_3 = b_3^H$ the seller has no incentive to deviate and announce $p_3 = V_2$ such that there exists no profitable deviation for the seller in period 3 when he or she announces $p_3 = V_1$ and $q_3 = 2\alpha$.

Period 2. In period 2, the seller can deviate in a number of ways. Given that $\alpha \in \left[\frac{1}{4}, \frac{V_2}{V_1}\right]$ and that the seller wants to deviate and charge $p_2 = V_2, p_3 = V_1$ we can check that given the history of the game ($p_1 = V_2, q_1 = \frac{3-6\alpha}{1-\alpha}$) $b_2^H = \alpha\left(\frac{1+2\alpha}{1-\alpha}\right), b_2^L = 1+2\alpha \ni b_2^H + b_2^L = \frac{1+2\alpha}{1-\alpha}$. Thus $b_3^H = 2\alpha + b_2^H\left(1 - \frac{q_2}{b_2^H + b_2^L}\right) = 2\alpha + \alpha\left(\frac{1+2\alpha}{1-\alpha}\right) - q_2\alpha$ while $s_3 = \frac{3\alpha}{1-\alpha} - q_2$.

With $s_3 < b_3^H, R = q_2V_2 + \rho\left(\frac{3\alpha}{1-\alpha} - q_2\right)V_1$

Differentiating partially with respect to q_2, $\frac{\partial R}{\partial q_2} = V_2 - \rho V_1 < 0$

With $b_3^H < s_3, R = q_2V_2 + \rho\left[2\alpha + \alpha\left(\frac{1+2\alpha}{1-\alpha}\right) - q_2\alpha\right]V_1$

Differentiating partially with respect to q_2, $\frac{\partial R}{\partial q_2} = V_2 - \rho\alpha V_1 > 0$ if $\alpha \leq \frac{V_2}{V_1}, \rho \to 1$. Further we can check that $s_3 > b_3^H$ if $q_2\alpha > q_2$ which is a contradiction, such that $s_3 < b_3^H \Rightarrow \frac{\partial R}{\partial q_2} < 0 \Rightarrow q_2 = 0$ and $q_3 = s_3 = \frac{3\alpha}{1-\alpha}$.

Thus in order to prove that there does not exist any profitable deviation, we assume to the contrary and show that there's a contradiction.

Required condition: $\rho\dfrac{3\alpha}{1-\alpha}V_1 > \dfrac{\alpha+2\alpha^2}{1-\alpha}V_1 + 2\rho V_1\alpha$

Since $\rho \to 1$, taking $\rho = 1$ and appealing to continuity we get $V_1 > V_1$ which is a contradiction.

Similarly we can show that the seller cannot gain by deviating and choosing $p_2 = p_3 = V_2$ and $q_2 = \dfrac{3\alpha}{1-\alpha}, q_3 = 0$. Assuming to the contrary, required condition is: $\dfrac{3\alpha}{1-\alpha}V_2 > \dfrac{\alpha+2\alpha^2}{1-\alpha}V_1 + 2\rho V_1\alpha$. Since $\rho \to 1$, taking $\rho = 1$ and appealing to continuity we get $V_2 > V_1$ which is a contradiction.

Further we can check that if the seller chooses to deviate by charging $p_2 = p_2^H, p_3 = V_2$ he or she has to offer $q_2 = b_2^H = \dfrac{\alpha+2\alpha^2}{1-\alpha} \Rightarrow s_3 = b_3^H$ which ensures that such a deviation is not credible. Setting $p_2^H < p_2 \leq V_1, p_3 = V_2$ also does not prove to be credible since none of the high type buyers buy in period 2 and in that case $b_3^H = s_3$ such that price in period 3 should always be V_1. Again $p_2 < p_2^H, p_3 = V_2$ also cannot be a credible deviation, since even though higher valuation buyers buy the product in period 2, the seller does not do as well as he or she could by charging p_2^H.

Period 1. We have already shown that for the relevant range of parameter values, none of the other pricing strategies can be subgame perfect. Thus, the seller has no profitable deviation in period 1.

Since there exists no profitable deviation in periods 1,2 or 3, the pricing strategy is $\left(V_2, V_1, V_1, \dfrac{3-6\alpha}{1-\alpha}, \dfrac{\alpha+2\alpha^2}{1-\alpha}, 2\alpha\right)$ is subgame perfect if $\alpha \in \left[\dfrac{1}{4}, \dfrac{V_2}{V_1}\right], \rho \to 1$. ∎

Proof of proposition 10.

In order to show that $\left[V_1\left(1-\frac{6\alpha^2-15\alpha+6}{2(1-\alpha)(4-2\alpha)}\right)+\frac{6\alpha^2-15\alpha+6}{2(1-\alpha)(4-2\alpha)}V_2, V_2, V_1, 2\alpha, \frac{6\alpha^2-15\alpha+6}{2(1-\alpha)}, \frac{5\alpha-2\alpha^2}{2(1-\alpha)}\right]$ is subgame perfect for the relevant range of parameter values, we show that there does not exist any profitable deviation in any period i, where $i = 1, 2, 3$.

Period 3. As shown in lemma 4, the seller offers $q_1 = 2\alpha, q_2 = \frac{6\alpha^2-15\alpha+6}{2(1-\alpha)}$ such that $q_3 = s_3 = b_3^H = \frac{5\alpha-2\alpha^2}{2(1-\alpha)}$ which ensures that the seller does not deviate in period 3 by charging $p_3 = V_2$.

Period 2. The seller could deviate in a number of ways in period 2. If the seller chooses to deviate by charging $p_2 = p_3 = V_2$ he or she should offer $q_2 = 3-2\alpha, q_3 = 0$. Assuming to the contrary,

$(3-2\alpha)V_2 > \frac{6\alpha^2-15\alpha+6}{2(1-\alpha)}V_2 + \rho\frac{5\alpha-2\alpha^2}{2(1-\alpha)}V_1$ and taking $\rho = 1$ and appealing to continuity yields $V_2 > V_1$ which is a contradiction.

If the seller announces $p_2 = p_2^H, p_3 = V_2$ and offers $q_2 = 2\alpha, q_3 = s_3 = 3-4\alpha$ we assume this is a profitable deviation and show there's a contradiction. Thus,

$$\left[V_1\left(1-\frac{3-4\alpha}{6-4\alpha}\right)+\frac{3-4\alpha}{6-4\alpha}V_2\right]2\alpha+\rho(3-4\alpha)V_2 > \frac{6\alpha^2-15\alpha+6}{2(1-\alpha)}V_2+\rho\frac{5\alpha-2\alpha^2}{2(1-\alpha)}V_1$$

Once again taking $\rho = 1$ and appealing to continuity, we get $V_2 > V_1$ which is a contradiction.

The seller could also deviate by announcing $p_2 = p_3 = V_1$ and offer $q_2 = q_3 = 2\alpha$. Assuming to the contrary

$2V_1\alpha(1+\rho) > \frac{6\alpha^2-15\alpha+6}{2(1-\alpha)}V_2 + \rho\frac{5\alpha-2\alpha^2}{2(1-\alpha)}V_1 \Rightarrow \alpha > \frac{2V_2}{V_1+V_2} > \frac{V_2}{V_1}$ which is a contradiction.

For deviations where the seller charges $p_2^H < p_2 \leq V_1, p_3 = V_2$ we can show there exists no profitable deviations for the seller and the proof is similar to the one which ruled out deviations like $p_2 = p_3 = V_2$ and $q_2 = 3-2\alpha, q_3 = 0$.

Period 1. We have shown that $\left[V_1\left(1-\frac{6\alpha^2-15\alpha+6}{2(1-\alpha)(4-2\alpha)}\right)+\frac{6\alpha^2-15\alpha+6}{2(1-\alpha)(4-2\alpha)}V_2,\right.$ $\left.V_2, V_1, 2\alpha, \frac{6\alpha^2-15\alpha+6}{2(1-\alpha)}, \frac{5\alpha-2\alpha^2}{2(1-\alpha)}\right]$ cannot be subgame perfect for $\alpha \geq \frac{1}{4}$. Further we require that $\alpha \leq \frac{2V_2}{V_1+V_2}$ for the seller to offer $q_2 = \frac{6\alpha^2-15\alpha+6}{2(1-\alpha)}, q_3 = \frac{5\alpha-2\alpha^2}{2(1-\alpha)}$ when he or she wants to hold a 'sale' in the second period only. Thus for this pricing policy to be subgame perfect, we require that $\alpha < \frac{1}{4}$ (if $\frac{1}{4} \leq \frac{V_2}{V_1}$) and $\alpha \leq \frac{V_2}{V_1}$ (if $\frac{V_2}{V_1} < \frac{1}{4}$).

We have already shown that all policies except $\left(V_2, V_2, V_1, 2, \frac{1-4\alpha}{1-\alpha}, \frac{3\alpha}{1-\alpha}\right)$ cannot be subgame perfect for the relevant range of parameter values. To show that the policy involving a 'sale' in the second period only is subgame perfect, we show that the seller cannot gain by deviating to this alternate policy in the first period.

Define $B = \left[V_1\left(1-\frac{6\alpha^2-15\alpha+6}{2(1-\alpha)(4-2\alpha)}\right)+\frac{6\alpha^2-15\alpha+6}{2(1-\alpha)(4-2\alpha)}V_2\right]2\alpha + \rho\frac{6\alpha^2-15\alpha+6}{2(1-\alpha)}V_2$ $+\rho^2\frac{5\alpha-2\alpha^2}{2(1-\alpha)}V_1 - \left(2V_2+\rho\frac{1-4\alpha}{1-\alpha}V_2+\rho^2\frac{3\alpha}{1-\alpha}V_1\right)$

Taking $\rho = 1$ we find that $B = 0$.

Partially differentiating B with respect to ρ, and taking $\rho = 1$

$$\left.\frac{\partial B}{\partial \rho}\right|_{\rho=1} = \frac{6\alpha^2-7\alpha+4}{2(1-\alpha)}V_2 - 2V_1\frac{2\alpha^2+\alpha}{2(1-\alpha)}$$

If $\frac{V_1}{V_2} > \frac{6\alpha^2 - 7\alpha + 4}{\alpha(4\alpha + 2)}$, $\frac{\partial B}{\partial \rho}|_{\rho=1} < 0 \Rightarrow$ with $\rho \to 1, B > 0$. Thus there exists no profitable deviation for the seller in period 1.

Thus if $\alpha < \frac{1}{4}$ (for $V_1 \leq 4V_2$) and $\alpha \leq \frac{V_2}{V_1}$ (for $V_1 > 4V_2$) and $\frac{6\alpha^2 - 7\alpha + 4}{\alpha(4\alpha + 2)} < \frac{V_1}{V_2}$ ($\rho \to$ 1), $\left[V_1\left(1 - \frac{6\alpha^2 - 15\alpha + 6}{2(1-\alpha)(4-2\alpha)}\right) + \frac{6\alpha^2 - 15\alpha + 6}{2(1-\alpha)(4-2\alpha)}V_2, V_2, V_1, 2\alpha, \frac{6\alpha^2 - 15\alpha + 6}{2(1-\alpha)},\right.$ $\left.\frac{5\alpha - 2\alpha^2}{2(1-\alpha)}\right]$ is subgame perfect. ■

Proof of proposition 11.

In order to show that $\left(V_2, V_2, V_1, 2, \frac{1-4\alpha}{1-\alpha}, \frac{3\alpha}{1-\alpha}\right)$ is subgame perfect for the relevant range of parameter values, we show that there does not exist any profitable deviation in any period i, where $i = 1, 2, 3$ for the same range of parameter values.

Period 3. As shown in lemma 2, the seller offers $q_1 = 2, q_2 = \frac{1-4\alpha}{1-\alpha}$ such that $q_3 = s_3 = b_3^H$ which rules out profitable deviation for the seller by charging $p_3 = V_2$.

Period 2. In period 2, the seller could deviate and announce $p_2 = p_3 = V_1$ and offer $q_2 = q_3 = 2\alpha$.

Assuming $2V_1\alpha(1+\rho) > \frac{1-4\alpha}{1-\alpha}V_2 + \rho\frac{3\alpha}{1-\alpha}V_1$ and taking $\rho = 1$ and appealing to continuity, we get $\alpha > \frac{V_2}{V_1}$ which is a contradiction.

If the seller wants to deviate and charge $p_2 = p_3 = V_2$ and offer measure $q_2 = 1, q_3 = 0$ we can once again rule out such deviations using a proof by contradiction.

Assuming to the contrary, $V_2 > \frac{1-4\alpha}{1-\alpha}V_2 + \rho\frac{3\alpha}{1-\alpha}V_1$ and taking $\rho = 1$ and appealing to continuity we get $V_2 > V_1$ which is a contradiction.

If the seller charges $p_2 = p_2^H, p_3 = V_2$ and offers $q_2 = 2\alpha, q_3 = s_3 = 1 - 2\alpha$ we can attempt a similar proof by contradiction.

Assuming to the contrary

$$\left[V_1(1-\frac{1-2\alpha}{4-2\alpha})+\frac{1-2\alpha}{4-2\alpha}V_2\right]2\alpha+\rho(1-2\alpha)V_2>\frac{1-4\alpha}{1-\alpha}V_2+\rho\frac{3\alpha}{1-\alpha}V_1$$

Taking $\rho = 1$ and appealing to continuity we get $V_2 > V_1$ which is a contradiction.

Similarly, for $p_2^H < p_2 \leq V_1, p_3 = V_2$ and $q_2 = 2\alpha, q_3 = s_3 = 1$ the deviation is not profitable, and the proof is similar to the one which ruled out $p_2 = p_3 = V_2$ and $q_2 = 1, q_3 = 0$.

Period 1. We have already shown that $\left(V_2, V_2, V_1, 2, \frac{1-4\alpha}{1-\alpha}, \frac{3\alpha}{1-\alpha}\right)$ cannot be subgame perfect for $\alpha \geq \frac{1}{4}$. Further we require that with $\alpha < \frac{1}{4} \leq \frac{V_2}{V_1}$ the seller has to offer $q_1 = 2, q_2 = \frac{1-4\alpha}{1-\alpha}$ when he or she wants to hold a 'sale' in the first and second period. Thus for this pricing policy to be subgame perfect, we require that $\alpha < \frac{1}{4}$ (if $\frac{1}{4} \leq \frac{V_2}{V_1}$) and $\alpha \leq \frac{V_2}{V_1}$ (if $\frac{V_2}{V_1} < \frac{1}{4}$).

Since with $\frac{6\alpha^2 - 7\alpha + 4}{\alpha(4\alpha+2)} > \frac{V_1}{V_2}$ the conditions of proposition 10 are no longer satisfied, such that the pricing policy involving a 'sale' in the second period only cannot be subgame perfect. All other pricing policies have already been shown not to be subgame perfect for the relevant range of parameter values. Thus if $\alpha < \frac{1}{4}$ (if $\frac{1}{4} \leq \frac{V_2}{V_1}$) and $\alpha \leq \frac{V_2}{V_1}$ (if $\frac{V_2}{V_1} < \frac{1}{4}$) and $\frac{6\alpha^2 - 7\alpha + 4}{\alpha(4\alpha+2)} > \frac{V_1}{V_2}$(with $\rho \to 1$), then $\left(V_2, V_2, V_1, 2, \frac{1-4\alpha}{1-\alpha}, \frac{3\alpha}{1-\alpha}\right)$ is subgame perfect. ■

Proof of proposition 12.

In order to show that $(V_2, V_1, V_1, 0, 4\alpha, 2\alpha)$ is subgame perfect for the relevant range of parameter values, we show that there does not exist any profitable deviation at any period i, $i = 1, 2, 3$ given the history of the game. We start with the last period.

Period 3. In order to ensure that there does exist any profitable deviation by the seller by charging price V_2 in period 3, we assume to the contrary and show that this leads to a contradiction.

Assume that $(3 - 4\alpha)V_2 > 2V_1\alpha \Rightarrow \alpha < \dfrac{3V_2}{2(V_1 + 2V_2)}$ which is a contradiction to $\alpha \geq \dfrac{3V_2}{2(V_1 + 2V_2)}$

Period 2. Given the history of the game $p_1 = V_2, q_1 = 0$ we need to show that there does not exist any profitable deviation for the seller in period 2. From the proof of proposition 4, we have already seen that given the same history, if the seller wants to deviate and offer $p_2 = V_2$ and $p_3 = V_1$ he or she must offer $q_2 = 0, q_3 = 6\alpha$ for the relevant range of parameter values. Thus for $p_2 = V_2, p_3 = V_1$ to be a profitable deviation, it must be the case that $\rho^2 6V_1\alpha > \rho 4V_1\alpha + \rho^2 2V_1\alpha \Rightarrow \rho > 1$ which is a contradiction.

Similarly, if the seller wants to deviate and offer $p_2 = p_3 = V_2$ he or she should offer $q_2 = 3, q_2 = 0$. For this to be a profitable deviation, we require that $3V_2 > \rho 4V_1\alpha + \rho^2 2V_1\alpha$. Since $\rho \to 1$, taking $\rho = 1$ and appealing to continuity yields $\alpha < \dfrac{V_2}{2V_1} < \dfrac{V_2}{V_1}$ which is a contradiction. A similar proof rules out deviations like $p_2^H < p_2 = V_1, p_3 = V_2$.

Another possible deviation for the seller in period 2 is $p_2 = p_2^H, p_3 = V_2$ and $q_2 = 4\alpha, q_3 = 3 - 4\alpha$. To rule out such deviations, we assume to the contrary that $4\alpha \left[V_1 \left(1 - \frac{3-4\alpha}{6-4\alpha} \right) + \frac{3-4\alpha}{6-4\alpha} V_2 \right] + \rho(3-4\alpha)V_2 > \rho 4 V_1 \alpha + \rho^2 2 V_1 \alpha$

Taking $\rho = 1$ and appealing to continuity, we get $\alpha < \left(\frac{18-24\alpha}{24-24\alpha} \right) \frac{V_2}{V_1} < \frac{V_2}{V_1}$ which is a contradiction. It is easy to see that deviations like $p_2 < p_2^H, p_3 = V_2$ will also not be profitable.

Period 1. The other strategies which have still not been ruled out are strategies which do not involve any 'sale' in any period, involve 'sale' in the first two periods and 'sale' in the second period only. Following the proof in proposition 10, we can rule out $(V_1, V_1, V_1, 2\alpha, 2\alpha, 2\alpha)$ to be subgame perfect since there exists a profitable deviation in period 2 for the relevant range of parameter values.

For the strategy involving 'sale' in the first two periods, we can show that it cannot be subgame perfect since there exists a profitable deviation in period 2. Given the history of the game $p_1 = V_2, q_1 = 0$, instead of charging $p_2 = V_2, p_3 = V_1$ and $q_2 = 0, q_3 = 6\alpha$ it is easy to see that the seller can do better with $p_2 = p_3 = V_1$ and $q_2 = 4\alpha, q_3 = 2\alpha$.

Thus the only strategy left is the one involving a 'sale' in the second period only. Assuming that the seller can do better with such a strategy, we have

$$\left[V_1 \left(1 - \frac{6\alpha^2 - 15\alpha + 6}{2(1-\alpha)(4-2\alpha)} \right) + \frac{6\alpha^2 - 15\alpha + 6}{2(1-\alpha)(4-2\alpha)} V_2 \right] 2\alpha + \rho \frac{6\alpha^2 - 15\alpha + 6}{2(1-\alpha)} V_2 +$$

$\rho^2 \frac{5\alpha - 2\alpha^2}{2(1-\alpha)} V_1 > \rho 4 V_1 \alpha + \rho^2 2 V_1 \alpha \Rightarrow \alpha < \frac{V_2}{V_1}$ which is a contradiction.

Thus if $\alpha \in \left(\frac{V_2}{V_1}, \frac{2V_2}{V_1+V_2}\right]$, $\rho \to 1$ and $\alpha \geq \frac{3V_2}{2(V_1+2V_2)}$, then $(V_2, V_1, V_1, 0, 4\alpha, 2\alpha)$ is subgame perfect. ■

Proof of proposition 13.

In order to show that $\left[V_1\left(1-\frac{6\alpha^2-15\alpha+6}{2(1-\alpha)(4-2\alpha)}\right)+\frac{6\alpha^2-15\alpha+6}{2(1-\alpha)(4-2\alpha)}V_2, V_2, V_1, 2\alpha,\right.$ $\left.\frac{6\alpha^2-15\alpha+6}{2(1-\alpha)}, \frac{5\alpha-2\alpha^2}{2(1-\alpha)}\right]$ is subgame perfect, we show that there does not exist any profitable deviation at any period $i, i = 1, 2, 3$ given the history of the game. We start with the last period.

Period 3. Proof similar to that of proposition 10.

Period 2. Proof similar to that of proposition 10.

Period 1. Since $\alpha < \frac{3V_2}{2(V_1+2V_2)}$, the conditions of proposition 12 are not satisfied, such that $(V_2, V_1, V_1, 0, 4\alpha, 2\alpha)$ is no longer subgame perfect. Thus, none of the other strategies can provide credible deviations in period 1.

Thus if $\alpha \in \left(\frac{V_2}{V_1}, \frac{2V_2}{V_1+V_2}\right]$, $\rho \to 1$ and $\alpha < \frac{3V_2}{2(V_1+2V_2)}$, then $\left[V_1\left(1-\frac{6\alpha^2-15\alpha+6}{2(1-\alpha)(4-2\alpha)}\right)+\right.$ $\left.\frac{6\alpha^2-15\alpha+6}{2(1-\alpha)(4-2\alpha)}V_2, V_2, V_1, 2\alpha, \frac{6\alpha^2-15\alpha+6}{2(1-\alpha)}, \frac{5\alpha-2\alpha^2}{2(1-\alpha)}\right]$ is subgame perfect. ■

Proof of proposition 14.

To prove that $(V_1, V_1, V_1, 2\alpha, 2\alpha, 2\alpha)$ is subgame perfect for $\alpha > \frac{2V_2}{V_1+V_2}$, $\rho \to 1$ we will show that there does not exist any profitable deviation for the seller in any period.

Period 3. In the last period, the seller could choose to deviate by offering $q_3 = s_3 = 3 - 4\alpha$ and price $p_3 = V_2$. Assuming that this is a profitable deviation, required condition is $(3-4\alpha)V_2 > 2V_1\alpha \Rightarrow \alpha < \frac{3V_2}{2(V_1+2V_2)} < \frac{2V_2}{V_1+V_2}$ which is a contradiction. Thus there is no profitable deviation for the seller in period 3.

Period 2. The seller could choose to deviate and offer $p_2 = V_2, p_3 = V_1$. From the proof of proposition 6, we know that if $\alpha > \dfrac{2V_2}{V_1 + V_2}$, the seller will offer $q_2 = 0, q_3 = 4\alpha$ for the proposed deviation. Assuming that this is a profitable deviation, the required condition is $\rho 4\alpha V_1 > 2V_1\alpha(1+\rho)$ which is a contradiction for $\rho < 1$.

Similarly we can show that the seller cannot gain by deviating by choosing $p_2 = p_3 = V_2$ and $q_2 = 3 - 2\alpha, q_3 = 0$ as well as by announcing $p_2^H < p_2 \leq V_1, p_3 = V_2$ and $q_2 = 2\alpha, q_3 = s_3 = 3 - 2\alpha$. Assuming to the contrary, $(3-2\alpha)V_2 > 2V_1\alpha(1+\rho)$.

Taking $\rho = 1$ and appealing to continuity gives us $\alpha < \dfrac{3V_2}{2(2V_1 + V_2)} < \dfrac{2V_2}{V_1 + V_2}$ which is a contradiction.

For $p_2 = p_2^H, p_3 = V_2$ and $q_2 = 2\alpha, q_3 = 3 - 4\alpha$ the required condition which ensures profitable deviation is as follows

$$\left[V_1\left(1 - \frac{3-4\alpha}{6-4\alpha}\right) + \frac{3-4\alpha}{6-4\alpha}V_2\right]2\alpha + \rho(3-4\alpha)V_2 > 2V_1\alpha(1+\rho)$$

Taking $\rho = 1$ and appealing to continuity yields $\alpha > \dfrac{7}{4}$ which is a contradiction. Thus the seller cannot gain by deviating in such a manner. If the seller charges $V_2 < p_2 < p_2^H, p_3 = V_2$ high type buyers choose to purchase the product in period 2. However, the seller earns lower revenue than with $p_2 = p_2^H, p_3 = V_2$.

Period 1. From previous lemmas we find that if $\alpha > \dfrac{2V_2}{V_1 + V_2} > \dfrac{V_2}{V_1}$, the seller offers $q_1 = q_2 = 0, q_3 = 6\alpha$ for the pricing policy with 'sale' in the first two periods, while for the strategy involving 'sale' in the first period only he or she offers $q_1 = 0, q_2 = 4\alpha, q_3 = 2\alpha$ and finally for 'sale' in the second period only, the seller offers $q_2 = 0, q_3 = 4\alpha$.

Since $2V_1\alpha(1+\rho+\rho^2) > \rho^2 6V_1\alpha$, $2V_1\alpha(1+\rho+\rho^2) > \rho 4V_1\alpha + \rho^2 2V_1\alpha$ and $2V_1\alpha(1+\rho+\rho^2) > 2V_1\alpha + \rho^2 4V_1\alpha$ for $\rho < 1$, none of the pricing policies mentioned above can be subgame perfect for $\alpha > \dfrac{2V_2}{V_1 + V_2}$.

To rule out 'sale' in the first and last period, we assume that such a deviation is profitable and show that there exists a contradiction. The required condition is

$$\left[V_1 \left(1 - \frac{3 - 4\alpha}{6 - 4\alpha}\right) + \frac{3 - 4\alpha}{6 - 4\alpha} V_2\right] 4\rho\alpha + \rho^2(3 - 4\alpha)V_2 > 2V_1\alpha(1 + \rho + \rho^2)$$

Taking $\rho = 1$ and appealing to continuity yields $\alpha < \left(\dfrac{18 - 24\alpha}{24 - 24\alpha}\right)\dfrac{V_2}{V_1} < \dfrac{V_2}{V_1} < \dfrac{2V_2}{V_1 + V_2}$ which is a contradiction.

A similar proof rules out 'sale' in the last period only. If the seller chooses to deviate and hold a 'sale' in the last two periods, we can show that such a deviation is not credible.

Assuming to the contrary, $(3 - 2\alpha)V_2 > 2V_1\alpha(1 + \rho)$ and taking $\rho = 1$ and appealing to continuity provides $\alpha < \dfrac{3V_2}{2(V_1 + 2V_2)} < \dfrac{2V_2}{V_1 + V_2}$ which is a contradiction. Thus there exists no profitable deviation in period 1.

Hence, if $\alpha > \dfrac{2V_2}{V_1 + V_2}, \rho \to 1$ $(V_1, V_1, V_1, 2\alpha, 2\alpha, 2\alpha)$ is subgame perfect. ■

Chapter 2

On Delays in Project Completion With Cost Reduction: An Experiment

2.1 Introduction

The focus of this chapter is on the effects of externalities on delays in completion of a public project. It is often the case that the individual cost of contribution for a public good decreases as the number of contributions already made increases. Allegations of corruption against public officials can be viewed as a public project with these features. If the corrupt official can be identified and removed, everyone receives some benefit, but this can only happen if a sufficient number of individuals are willing to implicate the official. The person bringing the first allegation not only faces the social stigma that such allegations could bring, but potentially, could also have to deal with retaliation from the person or parties against whom such allegations have been made. As more allegations are brought forward, the private cost of bringing similar allegations is reduced since these allegations become more credible. Thus, individuals have an incentive to free ride on the contributions made by others. Individuals with access to information that might bring the official to justice face a dilemma: they could contribute now with the hope that the official is brought to justice sooner rather than later, or they could choose to wait, hoping others contribute first. This process of *whistleblowing* is only one example of a public project with cost reduction; other examples include early adoption of a new technology standard and allegations of sexual harassment.

We construct a multi-period voluntary contributions public project model designed to capture the vital features of the problem described above. Agents can choose to make an irrevocable, binary contribution at any point of time before a contribution deadline. The cost of contribution decreases as the number of prior contributors increases. If a sufficient number of contributions are received, the project is completed and all agents get a benefit. The benefit of the project decreases over time. If the project is not completed before the contribution deadline, none of the agents receive any benefit, but agents who chose to contribute still incur their cost of contribution.

When there is no cost reduction, there is a Pareto-dominant, subgame perfect equilibrium where the project is completed without delay. We show that as long as cost reduction is sufficiently large, there is no pure-strategy subgame perfect equilibrium that does not involve delay. While all equilibria must result in completion of the project, the effect of cost reduction is to lead to excessive delay in project provision and, since benefits decline over time, inefficient outcomes. Both with and without cost reduction, there exists multiple pure-strategy subgame perfect Nash equilibria. We design an experiment based on the same theoretical framework, where we consider two treatments, one with and one without cost reduction. The objective of the experiment was to determine whether the actions of real, human participants are consistent with the theoretical predictions of the model. And, since their are many possible equilibria, the experiment might provide insights into which outcomes are more likely. Specifically, we designed the experiment in the hope of answering the following questions:

1. Does cost reduction result in significantly more delay?

2. Is the project completed?

3. In both treatments, do the players manage to coordinate on Pareto superior equilibria?

We find that the project is completed in the treatment with cost reduction with more delay than it is in the treatment without cost reduction. We also find that the Pareto-dominant subgame perfect outcome is played frequently in both treatments. However, the players do not appear to completely overcome the significant coordination problems prevalent in this situation. For example, the actual project completion rates are significantly below what might be expected. We hypothesize that coordination problems are exacerbated in this model due to the highly asymmetric payoffs in equilibrium.

The rest of the chapter is organized as follows. In section 2 we discuss some related literature. In section 3 we present the model and our theoretical results. The design of the experiment is described in section 4. We present the experiment results in section 5. We conclude in section 6.

2.2 Related Literature

There is a substantial theoretical and experimental literature on public projects with binary contributions. A review of the extensive experimental literature on public goods provision is provided in Kagel and Roth (1995) (chapter 2).

A series of papers by Palfrey and Rosenthal (1984, 1988, 1991, 1994) examine a model of public project completion with binary contributions. They examine the model under complete and incomplete information and examine human participants behavior in the laboratory under a number of treatments. Their models differ from ours in several key aspects. First, contributions are made simultaneously so dynamics are not considered, and, second, in most cases, each agent's cost of contribution is private information.

Seminal works by Schelling (1978) and Olson (1982) recognized that dynamics may play a vital role in problems of collective action. Bliss and Nalebuff (1984) develop a model where the public good is provided if one individual makes a contribution. With a finite population, equilibrium involves inefficient waiting, but as the population size approaches infinity, the inefficiency vanishes in the sense that the public good is provided almost immediately and by the lowest cost contributor. Our model differs from Bliss and Nalebuff in that multiple contributions may be required for completion allowing for cost reduction. We also examine the situation under the assumption of complete information. With complete information, the Bliss and Nalebuff model is a special case of our model without cost reduction, and we show that there exists an equilibrium without delay.

Gradstein (1992) examines a binary contribution model where the public benefit is strictly increasing in the number of contributions. Gradstein finds that, when two contribution periods are allowed, inefficiency in the form of delay and underprovision may persist even for infinite populations. Marx and Matthews (2000) on the other hand show that in an environment where players can make multiple contributions before a contribution horizon is reached but have incomplete information about the actions of the other players, perfect Bayesian equilibria exist which essentially complete the project. They do this by constructing an equilibria involving punishment strategies where future contributions depend upon the observed level of previous contributions.

Duffy et al. (2004) experimetally examine the Marx and Matthews model, and find that sequential play not only increases average contributions, but also increases the probability that groups reach the threshold level of the public good. While Duffy et al. focus on the potential benefits of sequential giving, our experiment highlights the potential coordination pitfalls that sequential contributions might create.

Our approach differs most substantially from the literature mentioned above on two key dimensions: First, our model has the twin features of cost reduction as other players make contributions and benefit reduction as players fail to complete the project sooner rather than later. These features are both likely to be prevalent in many public project settings and can make the efficiency issues of public project provision more salient. Second, while almost all of these models utilize an incomplete information setting, we assume complete information. Under these other models, cost differences are determined exogenously by nature. While this has the advantage of allowing one to identify a single, unique equilibrium, they potentially abstract from important coordination issues. In our model with complete information, the actual costs of each player is determined endogenously by the order of contribution. Therefore, this creates a complex coordination problem that must be solved in order for the project to be completed. This is a coordination problem that we feel is likely to be prevalent in many real-world public project applications and, since it involves potential coordination between different equilibria, is ideally suited to experimental examination.

2.3 The Model

We begin the theoretical analysis by describing a generalized version of the discrete time, finite horizon model with n players. We assume that each player $i \in \{1, ..., n\}$, must choose whether and when to contribute for a public goods project during a contribution horizon lasting T periods. In each period t, player i must make an irreversible decision to either contribute (C) or not to contribute (NC). Player i's action in period t, is denoted by $g_i(t) \in \{C, NC\}$ provided $g_i(\tau) = NC \ \forall \tau = 1, .., t-1$ and $g_i(\tau) = NC$ for all $\tau = t+1, \ldots, T$ if $g_i(t) = C$. Let $G(t)$ be the number of players who chose to contribute up to period t. The project is completed in period t if $G(t) \geq \overline{G}$, where it is assumed that $\overline{G} < n$.

The common, public benefit from the completion of the project depends on the period in which the project is completed. Each player receives the benefit $b(r)$ where r is the first period where the project is completed, or $G(r) \geq \overline{G}$. Formally, let $r = \min\left\{\left\{1 \leq t \leq T : G(t) \geq \overline{G}\right\}, T+1\right\}$ where $r = T+1$ indicates that the project was not completed. The benefit from project completion decreases over time, or $b(t) < b(t-1)$. If sufficient contributions are not made before the contribution deadline, the project remains incomplete and none of the agents receive any benefit, or $b(T+1) = 0$.

The cost of contribution for player i $c_i(m)$ depends only on the number of players who have already chosen to contribute denoted by m where $m = G(t-1)$ in period t. The agent incurs the cost of contribution, even if the project remains incomplete at the end of T periods. We assume that either $c_i(m) = c_i(m')$ for all possible m and we call this the *no cost reduction* case, or $c_i(m) < c_i(m')$ for all $m > m'$ and we call this the *cost* *reduction* case. Notice that while cost incurred by a player by making a contribution in period t depends on when the player makes the contribution (and the number of prior contributions), benefit derived from project completion depends on when the project is completed and thus cannot be directly controlled by an individual player. Payoff to player i, u_i is then a function of both player i's contribution decisions (g_i) and the total contributions made (G):

$$u_i(g_i, G) = \begin{cases} b(r) - c_i(G(t-1)) & \text{if } g_i(t) = C \\ b(r) & \text{otherwise.} \end{cases} \tag{2.1}$$

Benefits and costs are assumed to vary in such a way to ensure that it is socially optimal for the project to be completed in period $t = 1$.

We assume that this is a game of complete information; each player knows her own cost and the cost of contribution of the others at each and every subgame. Players are only informed of the total number of contributions from the previous periods. Player i's personal history at the start of period t is $h_i^{t-1} = (g_i(\tau), G(\tau))_{\tau=1}^{t-1}$, and a player's strategy $s_i : h_i^{t-1} \longrightarrow g_i(t)$. A *pure-strategy subgame perfect Nash Equilibrium (SPNE)* of this game consists of a strategy profile, $s = (s_1, ..., s_n)$ which induces a Nash equilibrium in every subgame.

For the case without cost reduction, $b(1) > b(2)$ is a sufficient and necessary condition for the existence of a SPNE outcome where $\overline{G}$ of n players contribute in period 1 and the project is completed without delay. On the other hand, for the case with cost reduction, if there exists at least $n - (\overline{G} - 1)$ players such that

$$\frac{b(1) - b(2)}{c_i(0) - c_i(\overline{G} - 1)} < 1 \tag{2.2}$$

then there does not exist a SPNE outcome in which the project is completed in the first period. Given that $\overline{G} - 1$ players contribute in period 1, condition (2.2) ensures that for all other players they would rather delay completion of the project than pay the high initial contribution costs. Therefore, in equilibrium, the project is completed with delay.

Consider the following the following example that matches cost reduction environment from the experiments. Let $n = 5$ and $T = 3$ and $\overline{G} = 3$. The project completion benefit is given by $b(r) = 1000 - (r - 1)200$ for $r \leq 3$ and $b(r) = 0$ for $r > 3$ and the common contribution costs are given by $c(0) = 400$ and $c(m) = 400/(2m)$ for $m = 1, 2$.

The Pareto-dominant SPNE outcome of the game with cost reduction involves one player contributing in period 1, two of the remaining four players contribute in period 2 and the final two players not contributing. To see why project completion in the first period is not subgame perfect for this example, consider the following feasible strategy profile that does not involves delay: players 1, 2 and 3 contribute in period 1 and players 4 and 5 choose not to contribute. The payoff for players 1, 2 and 3 is 600, and the payoff for players 4 and 5 is 1000. However, players 1, 2, and 3 all find it profitable to unilaterally deviate by contributing in period 2 rather than period 1. The payoff from such a deviation is 700, which is better than the payoff under the outcome without delay. Thus project completion in period 1 for the game with cost reduction is not subgame perfect. The total surplus generated by the SPNE outcome is 3,200 in this example, whereas the efficient allocation would prescribe contribution by exactly three players in period 1 for a surplus of 3,800.

In order to compare the effects of cost reduction on delays in project completion, we modify the previous example by making cost of contribution constant. Let $c_i(m) = 400$ for all m. Thus cost incurred by a player i in period t is independent of the number of prior contributions made. The Pareto-dominant SPNE outcomes of the game without cost reduction involves three of five players contributing in period 1 and the remaining two players not contributing. Thus, the project is completed without delay and the efficient surplus is obtained.

In both cases, there are multiple SPNE.[1] The contribution patterns that are consistent with a SPNE under both cost reduction (WCR) and no cost reduction (WOCR) are listed in Table 2.1. The total surplus of the SPNE outcomes varies considerably. Each contribution pattern is actually consistent with multiple SPNE outcomes where the identity of the contributing players varies amongst the five players. In addition to coordinating on a contribution pattern, players must coordinate on who is going to contribute and when they do so. The strategy of all players not contributing in any of the three periods is a Nash equilibrium for both cases, but it is not subgame perfect. Once any player chooses to contribute in period 1, it is a best response for two of the remaining four players to contribute over the remaining two periods and complete the project. Thus, any player should be willing to deviate from the no completion strategy.

While all players face (ex ante) symmetric costs of contributions, in equilibrium, the payoffs are asymmetric. This asymmetry takes two forms. First, in both the with and

[1] There are also likely to be mixed-strategy subgame perfect Nash equilibria.

	Contributions in $Pd1\backslash Pd2\backslash Pd3$	Total Surplus
WOCR	3\0\0	3,800
	1\2\0	3,200
	0\3\0	2,800
	0\0\3	1,800
WCR	1\2\0	3,200
	0\1\2	2,200
	1\0\2	2,200

Table 2.1. Contribution patterns and surplus for all SPNE outcomes of the example game with and without cost reduction.

without cost reduction cases, there are differential payoffs due to the lack of contribution by some players. In only the cost reduction case, differential payoffs are also generated by the timing decisions of those who decide to contribute. Both these asymmetries suggest that this situation will result in substantial coordination difficulties. Even if the player's recognize the various SPNE of the game, they must find a way to arrive at a particular selection from the set. However, obvious equity issues are likely to complicate this choice. In the extreme case, players can guarantee an equitable payoff by refusing to contribute. As mentioned earlier, while this *no provision* outcome is a Nash equilibrium, it is not subgame perfect and is highly inefficient. As in all games of coordination with Pareto-ranked equilibria, coordination failure might be of two possible types (i) none of the equilibria might be achieved and (ii) players while successful in coordinating on some equilibrium, do not coordinate on the Pareto-optimal equilibrium. Further, in games with multiple equilibria, it is difficult to predict which of these is more likely to occur. This is an empirical question that we address by examining behavior in the laboratory.

2.4 Experimental Design

For the experiment, we use the same parameterized game as in the example described above, with five players and three periods. The experiment consisted of two treatments, one with cost reduction (WCR) and one without cost reduction (WOCR), each repeated for a fixed number of rounds. In all there were three sessions. In session 1, the WOCR treatment was conducted first for 25 rounds and followed by the WCR for 25 rounds.[2] In session 2, the order of treatments was reversed. In session 3, the treatment WOCR preceded the treatment WCR, but this time the treatments involved 35 repetitions. This was done to check if increasing the number of times the game is played had any effect on the outcomes of each treatment in the last five rounds.

Each session involved 15 inexperienced subjects divided into three groups of five. Data from each five-player group for each treatment is treated as one, independent observation. Thus, we have three observations for each treatment from each session, or a total of nine observations for each treatment. Each subject was matched with the same four subjects for the entire session. We did this to enable learning over the rounds, since we were interested in studying coordination.[3]

At the conclusion of the second treatment, earnings from the both treatments plus a $5 show-up payment were paid to each subject in cash. Participants could earn

[2]Data from the last round in the WOCR treatment of the session 1 was lost. As a result, the reported results are based on 24 rounds.

[3]It was impractical to have groups of five subjects, where each and every subject met each of the other four subjects for the first time in every round. Subjects were only informed of the total number of contributions made in the previous periods and not of the identity of the contributors. Given this, it is unlikely that subjects could use the repeated nature of their interaction to support alternative outcomes.

a maximum of $10 in each treatment. Participants' earnings averaged $3.95 (standard deviation of $1.98, maximum of $8.88 and minimum of $1.15) for WOCR and $4.14 (standard deviation of $0.83, maximum of $5.96 and minimum of $2.68) for WCR.

All sessions of the experiment were computerized and were conducted in the Laboratory for Economic Management and Auctions (LEMA) at Pennsylvania State University. Participants were recruited from the student population of Pennsylvania State University. The experiment was programmed and conducted with the z-Tree software (Fischbacher 1999). Instructions from the WCR treatment are contained in the appendix.

2.5 Results

First, we present our primary results considering all the rounds for the two treatments. Then, we examine the effect of learning and experience by considering the first and last five rounds of each treatment. Finally, we analyze individual behavior and its effect on coordination and discuss extensions. Unless otherwise noted, all statistical tests utilize a Wilcoxon signed ranks test, which is a powerful non-parametric test that compares the change in an observed variable for the same group under the two treatment conditions (Siegel and Castellan 1988). The results of the tests are summarized in Table 2.2. Also, since the order of treatments was varied over the experimental sessions, one might consider potential treatment order effects. In general, these effects were found to be minimal so discussion of order is limited for the sake of brevity; discussion of order effects when potentially relevant is provided after each conclusion.

	Test (H_A)	Result p-value
1	Completion Rate $WCR \neq WOCR$	$T^+ = 31$ $p = 0.3594$
2	Contributions Per Round $WCR \neq WOCR$	$T^+ = 23$ $p = 1.0000$
3	Average Completion Period $WCR > WOCR$	$T^+ = 44$ $p = 0.0078$
4	Contribution Per Round (WOCR) $First5 \neq Last5$	$T^+ = 24.5$ $p = 0.461$
5	Contribution Per Round (WCR) $First5 \neq Last5$	$T^+ = 29.5$ $p = 0.1484$

Table 2.2. Wilcoxon signed rank test results. The left-hand column represents the measured variable and the alternative hypothesis of the statistical test. The T^+ is the test statistic and p is the p-value. In all cases, $N = 9$.

All SPNE under both treatments result in project completion and exactly three contributions. Therefore, we expect the project to be completed irrespective of the treatment and for the number of contributions to approach a degenerate distribution at three. We do not expect the numbers to be any different across the two treatments. If there are differences, we would infer that the added complexity and timing considerations of the WCR treatment resulted in greater coordination difficulties that manifest themselves with lack of project completion. At this level, there appears to be little discernible difference between the treatments.

CONCLUSION 1. *Cost reduction does not effect the rate of project completion or the number of contributions.*

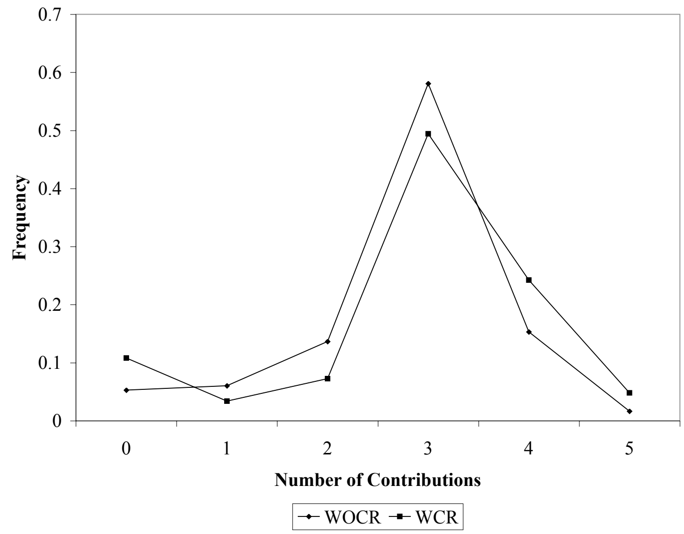


Fig. 2.1. Distribution of contributions by treatment.

Support: The average project completion rates for the treatments WOCR and WCR were 75% and 78% respectively. Under any reasonable level of significance, one cannot reject the null hypothesis that these completion rates are the same (Table 2.2). In both treatments, three of five players chose to contribute most frequently. In Figure 2.1 the distribution of contribution totals for the two treatments is displayed. For each treatment we expect a degenerate distribution at three contributions; three of five contributions is the modal choice in the data. The average number of contributions is 2.77 and 2.87 for the treatments WOCR and WCR respectively. The difference in contribution levels between the two treatments is not statistically significant (Table 2.2).

It is possible that the order in which the treatments were played might have had some effect on project completion rates; as the session progresses, participants learn that it is better to complete the project than to leave it incomplete. The strategy of no contribution by all players is not a subgame perfect equilibrium. To check whether such order effects are significant, we reversed the order of treatments in the second session and found that project completion rates were nearly identical across the two treatments (81% (WCR) versus 84% (WOCR)). There is thus little evidence of order effects on project completion rates. Increasing the number of rounds also has little effect on completion rates (74% (WCR) versus 80% (WOCR)).

The previous result indicates that, as expected, there is little difference between the two treatments in terms of coordination on project completion. However, we expect there to be substantial difference in the dynamics of project completion under the two treatments.

CONCLUSION 2. *Cost reduction results in more delay in project completion.*

Support: The project was completed in period 1 only 4% of the time on average under WCR, but completed in period 1 46% on average under WOCR. On the other hand, the project is completed in period 2 50% and 34% of the time under WCR and WOCR respectively. This results in an average project completion period under WOCR of 1.74 versus 2.41 for WCR. As reported in Table 2.2, this difference in completion time is significant. The effect of cost reduction on project completion becomes even more apparent in the final five rounds of each treatment; the project is never completed in period 1 in any session for the treatment WCR.

While this result tells us that coordination with respect to timing is largely consistent with theory, there was still a substantial amount of unexpected delay under WOCR.

There are a number of factors that might have caused such delay. First, may be coordination failure amongst players. Second, players may be playing a mixed strategy. Its possible that there are mixed strategy equilibria where given the history, players assign positive probabilities to contributing in any of the remaining periods which leave them indifferent. Finally, the order of treatments in each session suggests that experience may be determining the frequency of delays in WOCR. The project is completed more often without delay in the three groups (session 2) where WOCR was played last.

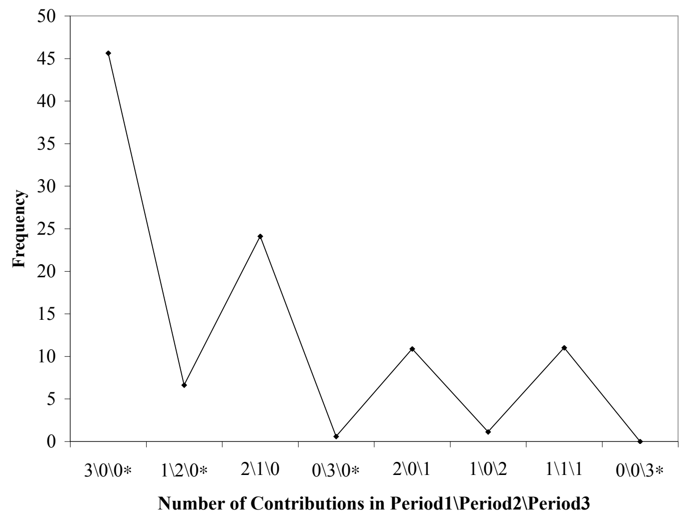
The previous two results indicate that behavior under the two treatments is at least qualitatively similar to the behavior predicted by the theory. However, project completion and delay can also be consistent with non-equilibrium play. Therefore, we examine whether play was regularly consistent with SPNE, and, if so, which SPNE outcome was most common.

Conclusion 3. *The outcome of the game is frequently consistent with SPNE. The most frequent SPNE outcome is the Pareto-dominant outcome.*

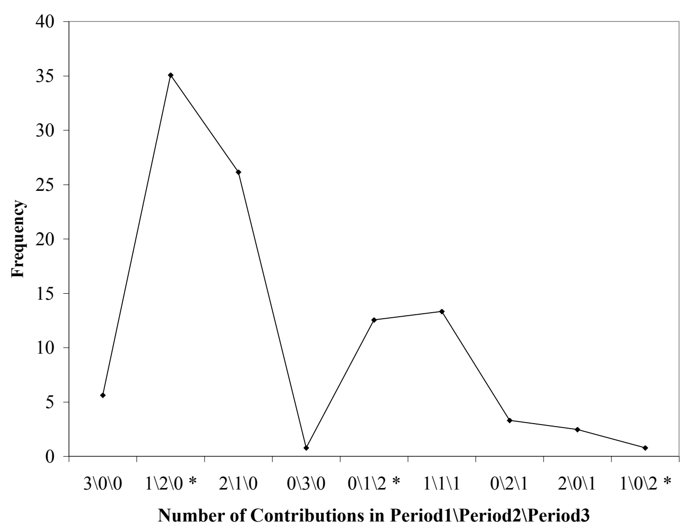
Support: Contribution choices were consistent with a SPNE 30% of the time under WOCR and 24% of the time under WCR. Further, players chose strategies consistent with the Pareto-dominant SPNE outcome 26% and 17% of the time for WOCR and WCR respectively.

Theoretically, we expect no differences in the frequencies with which players choose to play subgame perfect outcomes or the Pareto-dominant subgame perfect outcome across the two treatments. Given the complex coordination problems the players face, these numbers could be considered as fairly large. However, players are not always successful in playing the subgame perfect outcome. But there is evidence that players learn to play the Pareto-dominant subgame perfect outcome more frequently over the duration of a session, in the sense that the Pareto-dominant subgame perfect outcome for the respective treatments were played more often in sessions where they were played second. For example, given that three contributions were made, the Pareto-dominant subgame perfect outcome for the treatment WOCR was played 35% of the time when the treatment was played first as opposed to 69% when is was played second. Similarly, for the treatment WCR, the Pareto-dominant subgame perfect outcome was played 29% of the time when the treatment was played first compared to 37% of the time when the treatment was played second. The outcomes which involve exactly three contributions over the three periods under the treatment WOCR are shown in figure 2.2a. The outcomes with an asterisk correspond to outcomes consistent with SPNE, of which the one where three of the five players contribute in period 1 Pareto-dominates the others. The Pareto-dominant SPNE outcome is played most often.

Figure 2.2b reports the same information for the WCR treatment. Once again, the Pareto-dominant SPNE outcome is played most often. This implies that the subjects were able to coordinate amongst themselves at both levels (i.e., playing an outcome where three of the five participants chose to contribute and selecting the Pareto-dominant subgame perfect outcome). The outcome which was played the most frequently after the Pareto-dominant one involved two players contributing in period 1 and one of the remaining three players chose to contribute in period 2. While this outcome does not exist on the equilibrium path for any SPNE, it is sequentially rational; if players ever arrived at a subgame where two players have already contributed, it would be a Nash



(a) WOCR



(b) WCR

Fig. 2.2. Contribution patterns when the project was completed.

equilibrium of this subgame for one more player to contribute immediately. So, while this play may look inconsistent with equilibrium play, it suggests that some players may be playing in a rational manner.

While the theoretical model predicted a degenerate distribution at three for the number of players who should contribute in any equilibrium, it failed to predict which of the multiple subgame perfect outcomes would be played more often. Since the Pareto-dominant subgame perfect outcome is played most often in both the treatments, players were able to coordinate amongst themselves at both levels: (1) Three players chose to contribute most often and (2) they played the Pareto-dominant subgame perfect outcome most frequently.

2.5.1 Learning

Participants in each group were matched with the same four participants for the entire duration of each treatment. This was done to facilitate learning over the different rounds of the treatment. If there was learning, we would expect results from the last five rounds to be closer to the theoretical predictions than the first five rounds.

The project completion rates were found to decrease in the last five rounds as compared to the first five rounds. For WOCR, the project completion rate in the first five rounds was 84% and 73% in the last five rounds. For WCR, the corresponding numbers were 82% and 71% respectively. This result is similar to prior, well-known experimental findings that showed that participants in voluntary contribution games contribute less frequently over time. Such a decrease in contribution rates is supposed to be more pronounced in cases where participants are matched with the same partners for all the repetitions (Andreoni, 1988), as is true in our case. However, project completion rates went up in the second treatment for all the sessions in the last five rounds when compared to the first treatment.

The average number of contributions made in the first five rounds was 2.9 (3.1) under WOCR (WCR) and for the last five rounds the average was 2.6 (2.7) under WOCR (WCR). While the number of contributions dropped in the last five rounds under both treatments, the differences are not significant (Table 2.2). Most of the time three contributions were made in both the treatments in the first and last five rounds. There does not appear to be much evidence of learning.

The average number of contributions made across the three groups and over the three sessions in the first five rounds were found not to be significantly different from the ones made in the last five rounds for both the treatments (Wilcoxon Signed Rank Test gave values of $T^{+} = 12$ and 11 with $\alpha = 0.05, N = 6$ for treatments WOCR and WCR respectively). Most of the time three contributions were made in both the treatments in the first and last five rounds, such that there is not much evidence of learning.

With regard to project completion delay, the results are mixed. Figure 2.3 shows the average number of times the project was completed in the different periods in the first and last five rounds for the two treatments. Though the project is completed without delay for the treatment WOCR for the first five rounds, it is completed more frequently with delay in the last five rounds, which is contrary to our theoretical predictions.[4] On the other hand, the project is completed with delay for the treatment WCR both for the

[4] If we look at the same numbers for individual sessions we find that the project is completed with delay in the treatment without cost reduction in the last five rounds only in sessions 1 and 3, where it was conducted first.

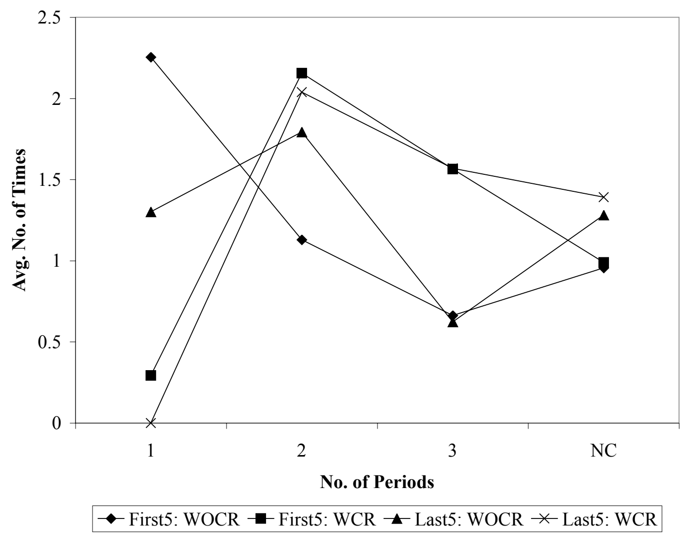


Fig. 2.3. Distribution of Project completion periods for both treatments (first and last five rounds). NC = Not Completed.

first and last five rounds. More importantly, it is never completed in the first period in any of the last five rounds in any of the sessions. There is also little evidence in favor of the players learning to play either the subgame perfect outcomes or the Pareto-dominant subgame perfect outcome in either treatment.

2.5.2 Analyzing Coordination Failures

While players were successful in coordinating on the timing of their contributions to the extent that most of the time three of the five players chose to the contribute for the public project and that given three contributions were made, the Pareto-dominant outcome was played most often in both the treatments, there were coordination failures. These failures were manifested in several ways. First, the project was not always completed in either treatment. Second, the project was completed with delay in some sessions under WOCR. Finally, non-equilibrium outcomes were played more often than equilibrium outcomes.

To analyze such coordination failures, we look at individual behavior of the players at two levels: (1) the percentage of times each player chose to contribute and (2) the percentage of times each player chose to contribute in period 1. Three of the five players in a group need to contribute for the project to be completed in both the treatments, but there are many ways such a contribution pattern could be realized. In one extreme, the same subset of players could volunteer to make contributions all the time while the others 'free-ride'. While this is easy to implement it would lead to highly asymmetric payoffs. In the other extreme, players could choose a strategy of rotation, according to which each player chooses to contribute only 60% of the time and free-rides on the contributions of others for the remaining rounds. While this outcome is equitable, it is hard to envision how the players, given the lack of direct communication, could coordination on this rotation scheme.

For the treatment WCR, choosing to contribute however, is not enough. Deciding when to contribute has important consequences. This is because the first person to contribute does not enjoy the benefits of cost reduction. By choosing to contribute first, a participant provides an incentive for the others to contribute, by reducing their costs. The Pareto-dominant outcome requires only one person needs to contribute in period 1. Once again we could either have the same player contributing in period 1 in all the rounds (the inequitable outcome) or each player contributing in period 1 only 20% of the time (the equitable outcome). A player who chooses to contribute almost all the time and frequently always chooses to contribute in the first period for the treatment with

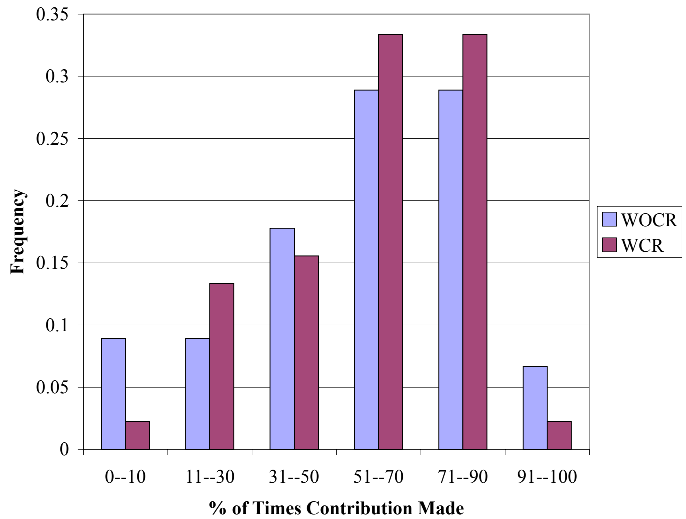


Fig. 2.4. Distribution of subject contribution frequencies.

cost reduction could be thought of as an *altruistic leader*; he sacrifices some personal earnings in the interest of completion of the project. On the other hand, a participant who chooses never to contribute in period 1 could be labeled as *selfish followers*; she is unwilling to accept a greater burden of the public project.

As is evident in Figure 2.4, most of the participants chose to contribute between 51-70% and 71-90% of the time for both the treatments. This would be consistent with individuals choosing to be neither altruists nor free-riders but instead choosing the strategy of rotation. Many subjects chose contribution rates very close to the equitable option; the number of participants who chose to contribute between 55-65% of the time for the treatments WOCR and WCR were 8 (out of 45) and 10 (out of 45) respectively. Only one participant chose never to contribute in any of the rounds under WCR. The average frequency of contributions was found to be 55 and 58 for the treatments WOCR

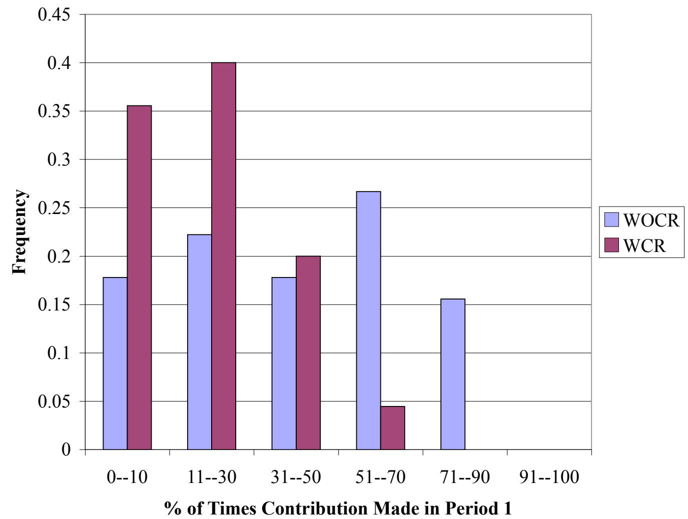


Fig. 2.5. Distribution of subject contribution frequencies in period 1.

and WCR respectively (t-statistic values were -1.3 and -0.4 for $n = 45$, $df = 44$, not significant). However, looking at only the average percentage of contributions could be misleading since a distribution where 60% of the participants contribute all the time and 40% free ride (non-rotation) would also lead to an average percentage of contribution of 60%. The variance of contribution rates provides insights into the equatability of the coordination. Under the symmetric, equitable rotation scheme, the variance of would be zero, but under the most asymmetric, inequitable scheme the variance would be 24. In the experiment, the observed variance of percentage contributions are 6.4 (WOCR) and 5 (WCR). This suggests that player strategies are more consistent with an equitable rotation scheme, but this coordination is clearly not perfect.

That most participants again chose the strategy of rotation for the role of leader is apparent from figure 2.5. Most of the subjects contributed in period 1 11 to 30% of the time for the treatment with cost reduction. However, a significant fraction chose to contribute only 0-10% of the time, preferring to "wait and watch", letting someone else to make a contribution in period 1 thereby enabling themselves to enjoy a lower cost. In WCR, 10 participants (out of 45) chose never to contribute in period 1 versus 4 (out of 45) under WOCR. For the treatment WOCR, most participants chose to contribute between 51-70% of the time in period 1 as is required for efficient completion of the project and play consistent with the Pareto-dominant SPNE. The average percentage of contributions in period 1 was found to be 40 and 18 for the treatments WOCR and WCR respectively (t-statistic for the latter with $n = 45, df = 44$ was -0.8, not significant. Corresponding variances were 6.5 and 2.4 respectively). We also calculated the correlation coefficient between the percentage of times players chose to contribute and percentage of times contributions were made by the same players in period 1 for the two treatments and found them to be 0.896 (WOCR) and 0.417 (WCR). This suggests that participants who chose to contribute for the treatment WOCR also chose to contribute early while this was not true for WCR.

A number of extensions may provide insights into behavior. First, we could allow nonbinding pre-play communication between the players and check what kind of outcomes are played more frequently. Second, we could increase the number of players in a group in the public good project model described above and check how changing the number of players affects the outcome(s) from the corresponding models. We would expect that allowing pre-play communication would obviate coordination failures while increasing the number of players would make coordination more difficult.

2.6 Conclusions

Given the asymmetric nature of the payoffs derived from project completion and the complicated nature of the game, it is remarkable that most of the time participants successfully coordinated amongst themselves at all levels to ensure that the project was completed. Given that the project was completed, the Pareto-dominant outcome was played most often. These results gain added significance in retrospect to the experimental literature that exists on coordination failures in games which involve Pareto-ranked equilibria. Most games on coordination failure refer to a "reversible safe action" and an "irreversible risky action". The intuition offered for such coordination failures is that the action which leads to the Pareto-dominant equilibrium outcome is deemed "too risky" by the players, as a result of which inefficient outcomes get played more often (Van Huyck et al. 1990, VBB).

While attempting to explain how players succeeded in coordinating their actions in this framework, as mentioned earlier, given the number of contributions made was three, while the Pareto-dominant subgame perfect outcome was played most often in both treatments, non-equilibrium outcomes together were played more often than equilibrium outcomes over all rounds in most cases. Besides, unlike the VBB model, our setup does not involve a one-shot simultaneous game. Instead, players get to choose simultaneously whether or not to contribute in each period provided they have not chosen to contribute already. This seems to suggest, that the players in a group first chose to coordinate amongst themselves in a way to ensure that exactly three of them contribute for the project in each round. This is also borne out by the fact that the fraction of times the number of contributions made was either one or five was quite low in both treatments (see figures 1 and 2). Even in the treatment WOCR, where the Pareto-dominant subgame perfect outcome required three players to contribute in period 1, players often chose to "wait and watch", letting others contribute first, even though there was no apparent benefit from waiting. This could be due to the dual effects of the "incentive to free ride" and the "incentive to coordinate to ensure exactly three contributions were made".

After comparing the results from the first five and last five rounds for each treatment across the different groups, we find that there was learning in at least some dimensions. For example, the project was never completed in period 1 in the last five rounds in any of the sessions for the treatment WCR. Participants thus realized over repeated play that completion of the project in period 1 though efficient, did not necessarily maximize individual payoffs. In fact the last round in which the project was completed in period 1 was round 15. Also, it was never the case that three of the five players in a group chose to contribute in period 1 for the treatment WCR in the last five rounds (though we had some instances where this happened in the first five rounds). However, average project completion rates and average number of contributions made actually declined from the first five to the last five rounds, while the project was completed with more delay in both treatments in the last five rounds as compared to the first five rounds.

Our main goal in this chapter was to address the following question: does a cost reduction feature induce more delay in completion of a public good project than one without it? We attempted to answer this question in the framework of coordination games with Pareto-ranked multiple equilibria, where we predicted that the Pareto-dominant subgame perfect outcome would be played most frequently, such that, the project would be completed without delay in the case where there is no cost reduction and would be completed with delay in the model with cost reduction. Experimental evidence collected supports our hypothesis that cost reduction induces more delay and that the Pareto-dominant subgame perfect outcome is played most often in both cases. While analyzing the results which back the supposition that a coordination failure was averted, we suggest that the nature of the game where a sequence of simultaneous games are played (as opposed to one shot nature as seen in VBB) could have helped players to coordinate their actions.

Appendix 1.

Instructions Used in the Experiment

The instructions used for the treatment with cost reduction are reprinted below.

WELCOME

You are about to participate in an experiment in individual and group decision making in which you will earn money based on decisions made by you and others. Your earnings are yours to keep and will be paid to you in cash at the end of the experiment. During the experiment all units of account will be in experimental dollars. Upon conclusion of the experiment, all experimental dollars will be converted into U.S. dollars at the conversion rate of 2500 experimental dollars per U.S. dollar. Your earnings, plus a lump sum amount of $5 will be paid to you in private. Please do not talk with one another for the duration of the experiment.

In this experiment you will participate in 25 periods. For the entire experiment you'll be matched with 4 other participants. The decisions you make will remain anonymous.

Earnings

In each period, consisting of three stages, you and each of the other four participants in your group will have to decide whether and when to contribute for a project. Earnings earned from the experiment will be the sum of earnings from each period, where

$$\text{Earnings from each period} = \text{Benefit from same period} - \text{Actual Cost}$$

Benefits accrue to each and every member of the group if the project is completed in three stages or less. The project is said to be completed if and only if three or more in a group of five participants choose to contribute. If the project remains incomplete, no benefits are obtained. The benefits derived from the completion of the project depend on the stage in which it is completed as follows

Benefits from Project Completion		
in Stage 1	in Stage 2	in Stage 3
1000	800	600

Individuals who choose to contribute have to bear a cost of contribution. Costs of contribution (C) for each of the five participants will initially be 400. You can choose to contribute only once in any one of the three stages. Your actual cost of contribution, however, depends on the number of participants who choose to contribute before you. Your cost of contribution remains unchanged if you're the first to contribute, gets divided by two if one participant decides to contribute before you and gets divided by four if two participants choose to contribute before you. For example, the actual cost of contribution for the participant with cost (C) = 400 will be determined as follows

Actual Cost of Contribution			
Number of participants who chose to contribute before you	0	1	2
Actual cost of contribution	400 (=C)	200 (= C/2)	100 (=C/4)

Contributions made are irreversible in the sense that even if the project is not completed and you chose to contribute, you have to incur the cost of contribution (actual) and get a negative earning (earning = 0 – cost). On the other hand, if you chose not to contribute and the project is completed in one of the three stages, you earn the corresponding benefit without incurring any costs.

Decision Stages in each Period

Stage 1: Each of the 5 participants chooses either to contribute or not to contribute in stage 1. If 3 or more than 3 individuals choose to contribute, the project is completed in stage 1 and everyone gets a benefit of 1000. Individuals who chose to contribute incur the cost of contribution, 400, while those who chose not to contribute will earn the benefit without incurring any costs. If the project remains incomplete at the end of stage 1, none of the participants get the benefit of 1000 and in that case you will move on to stage 2.

Stage 2: At the beginning of stage 2 the total number of contributions made at the end of stage 1 will be shown on your computer screen. It will however not show you who made the contribution(s). The screen will also show you your new cost of contribution in case you chose not to contribute in stage 1. If one participant chose to contribute in stage 1, your new cost of contribution will be your initial cost divided by two (C/2) and in case two participants chose to contribute your new cost will be your initial cost divided by four (C/4). If no one contributes in stage 1, your cost of contribution will remain unchanged. Once again, you should choose either to contribute or not to contribute.

The project will be completed if the total number of contributions made over stages 1 and 2 is three or more. If the project is completed in stage 2, everyone gets a benefit of 800. This means participants who chose to contribute in stage 1 (if there were any) will get a benefit of 800 only when the project is completed at the end of stage 2. If the project remains incomplete at the end of stage 2, none of the participants receive any benefit and you will move on to stage 3.

Stage 3: At the beginning of the third and last stage, you will be informed of the total number of contributions made in stages 1, 2 and your new cost of contribution in case you chose not to contribute in stages 1 and 2. If the total number of contributions made is 1, your cost of contribution will be C/2 and if the sum of contributions is 2, your cost will be C/4. If no one contributes in stages 1 and 2 your cost will remain unchanged. For the last time, you should choose whether or not to contribute.

The project is completed in stage 3 if the total number of contributions made over the 3 stages is three or more. In that case everyone gets a benefit of 600. Once again participants who chose to contribute in stages 1 or 2 will get the benefit of 600 only when the project is completed in stage 3. If the project remains incomplete, none of the participants will receive any benefit.

Computer Interface

All the information you need to participate in the experiment will be provided by the computer system. The computer automatically completes all necessary calculations according to the rules described above and displays the relevant information on your screen. However, it is important that you understand the process involved since it helps you determine how to earn money.

At the beginning of stages 2 & 3 the computer will provide you with the number of contributions which have already been made, your actual cost of contribution, your earnings in experimental dollars if you choose to contribute and the project is completed in that same stage, your earnings in experimental dollars if you choose not to contribute and the project is completed in that same stage and the time left for you to decide whether or not to contribute. If you choose to contribute, click "contribute" and if you choose not to contribute, please click "not contribute". The computer will also display your earnings at the end of each period and your total earnings at the end of the treatment (25 periods).

Negative earnings

Notice that negative earnings are possible in a given period (exercise 4 below). In that case, the earnings will be subtracted from your sum of earnings from the other periods. In the unlikely event that your total earnings from the experiment (including the $7 initial payment) fall to zero, you will not be allowed to continue with the experiment.

Exercises

To make sure you understand the instructions, please read the following exercises all of which assume that your initial cost of contribution (C) is 400.

(1) Suppose in stage 1, three other members of your group of 5 contribute. What would be your earnings?

Answer: Your earnings will be 1000 – 0 = 1000.

(2) Suppose one member contributes in stage 1, while you contribute alone in stage 2. Finally in the third stage one of the remaining three members contribute. What would be your earnings?

Answer: Your earnings would be 600 – (C/2) = 600 – 200 = 400.

(3) Suppose two other members of your group contributes in stage 1, while no one contributes in stage 2. Finally, in stage 3 you and one other member contribute. What would be your earnings?

Answer: Your earnings would be 600 – (C/4) = 600 – 100 = 500.

(4) Suppose one member contributes in stage 1, you contribute in stage 2 but no one else contributes in stages 2 or 3. What would be your earnings?

Answer: Your earnings would be 0 – (C/2) = – 200

(5) Suppose you contribute in stage 1, while two of the remaining 4 members contribute in stage 3. What would be your earnings?

Answer: Your earnings would be 600 – C = 600 – 400 = 200.

(6) Suppose you contribute in stage 1, while two of the remaining 4 members contribute in stage 2. What would be your earnings?

Answer: Your earnings would be 800 – C = 800 – 400 = 400.

If you have any questions, please raise your hand and I will come by to answer them.

Chapter 3

An Empirical Study of The Price Path of Airline Tickets

3.1 Introduction

In an earlier chapter, in order to study the behavior of prices of goods like airline tickets over time, we set up a theoretical model with the following features. A single seller facing a capacity constraint offered a finite measure of units for sale over three periods, after which the good lost its value. The seller announced without precommitment, price and measure of units for sale in each period. At the same time, a continuum of buyers entered the market in each period, each of whom was one of two types, high and low. High type buyers had a higher willingness to pay for the good than low type buyers. Both types were strategic in the sense that they could choose either to purchase the good immediately, or to wait, in case a cheaper price is made available in the future. We solved for the subgame perfect Nash outcomes of this game using backwards induction for various range of parameter values and found the price path to be non-decreasing, u-shaped or horizontal. In this chapter, we empirically test the predictions of the theoretical model and specifically verify the predicted relationship between the proportion of business travellers on a given route and the shape of the corresponding price path.

As discussed in the earlier chapter, the good being sold is non-durable, non-storable and cannot be resold. Goods like airline tickets could be considered similar to a futures contract on a service, where the airline writes contracts with different customers on different terms at different points in time. This implies that the airline faces a problem similar to the intertemporal pricing problem facing a durable goods monopolist. Our initial conjecture was that the shape of the price path of such goods would be u-shaped. This is because the seller is aware that buyers with a higher willingness to pay enter the market at all times. We hypothesized that the seller would initially set high prices and reduce prices gradually as in a Dutch auction. The seller would do this hoping that the buyers with a higher willingness to pay would purchase the good immediately, since they would be unwilling to wait for lower prices. This could be due to the fact that even though high type buyers correctly predict that prices will drop after some time they are also aware of the capacity constraint the seller faces and that while waiting for lower prices they run the risk of not getting to buy the good. Once prices drop, lower valuation buyers scramble to buy the good since they too correctly predict that prices will rise in the last period before the good will cease to have any value. Finally, in the last period, the seller increases the price again, since high valuation buyers entering the market in the last period would be willing to pay these high prices in order to purchase the good. For example, price data collected for the AirTran Airways flight 927 flying from Akron to Tampa on June 7, 2005, suggests that the price path was u-shaped. Our primary motivation in the first chapter was to construct a theoretical model which could explain such a u-shape.

The main predictions of the theoretical model were as follows. First, a sufficiently patient seller never offers any sale in the last period. This is because the seller chooses to reserve some units for sale in the last period and offer them at high prices to high valuation buyers who enter the market in that period. Second, the measure of units offered for sale in any period where the seller chose to offer the good to both types of buyers was found to be a decreasing function of the proportion of high type buyers in the market. Third, the shape of the price path was found to be horizontal, u-shaped or non-decreasing for various ranges of parameter values. For example, for routes with the highest proportion of high type buyers, sellers had no incentive to offer a sale. The price path was then found to be horizontal. Routes with lower proportions of high type buyers had price paths which were non-decreasing or u-shaped.

We collect data on prices over 15 weeks for 30 one-way, non-stop flights in the US. While the first prediction was found to be empirically valid, we found little evidence to support the hypothesis that the price path should be horizontal for routes with the highest proportion of buyers with a higher willingness to pay. We classified the routes into low, medium and high proportion of high type buyers and found that prices increased as the date of departure grew closer for all three types of routes. The rate of increase was found to be highest for routes with the highest fraction of high type buyers. We did find some evidence for a u-shaped price path for routes with low proportion of high type buyers.

3.2 Review of Literature

A number of empirical papers deal with price dispersion and discrimination in the airline market. Probably, the most cited amongst them is Borenstein and Rose (1994), who show that dispersion increases on routes with more competition. However, since the primary focus of this chapter is on the relationship between the proportion of buyers with a high willingness to pay on a particular route and the corresponding shape of the price path of airline tickets, we will refer only to those empirical papers which discuss the slope of price path of airline tickets.

Stavins (2001) addresses the issue of how airline prices move over time in a paper in which she examines how price discrimination changes with market concentration in the airline market. Price discrimination is found to increase as the markets become more competitive. The data set included fares offered 35 days prior to departure, followed by 21 days prior to departure, 14 days prior to departure and finally 2 days prior to departure. The data thus allowed for examination of how prices change as the departure date drew closer. From the OLS regression it was discovered that cheaper fares disappear, leaving only more expensive tickets for sale.

McAfee and Velde (2004) provide an extensive survey of yield management research (also called dynamic pricing, where goods are non-durable and capacity is fixed) in operations research journals and then tests the predictions of these models with airline pricing data collected from 1,260 flights. The five major propositions they test are as follows. First, prices fall as the date of departure approaches. Second, prices rise initially. Third, competition reduces the variance in prices. Fourth, prices change as the number of empty seats remaining change and finally fifth, prices of flights leaving from substitute airports or departing at substitute times are correlated.

The first proposition is a robust prediction of theories which assume that identical customer types arrive in the market over different points in time. The second prediction follows from the fact that the cost of failing to acquire a seat is negligible compared to the gains from delay when prices are expected to fall, when the time horizon is sufficiently long. McAfee and Velde found that prices increased \$50 in the week before takeoff on top of a rise of \$28.20 the previous week. This meant that the first proposition was empirically false and theories which assume that customers arriving in the market at different points in time are identical are invalid. Overall, there was scant empirical evidence in favor of the major theoretical predictions of these papers. However, McAfee and Velde do not specify as to whether the second proposition was found to be empirically valid. Also, the routes considered by them had multiple airlines serving them, such that their results are inapplicable for models with a single seller.

Etzioni et al (2003) devise an algorithm called Hamlet, which when trained on a data set comprising of over 12,000 observations over a 41 day period, was able to generate a predictive model which enabled 607 simulated passengers an average savings of 27%. They collected data for non-stop flights on two routes, Los Angeles to Boston and Seattle to Washington, DC, with departure dates spanning January, 2003. For each departure date they collected data 21 days in advance, 8 times a day. The data revealed that prices changed as often as seven times in a single day and that 63% of all such changes could be classified as dependant changes. Such price changes occur when ticket prices for flights from the same airline and having the same origin and destination changes. They classify the remaining changes to be independent, when prices change due to changes in seat availability on that particular flight. The flights were found to have discernible price tiers and the number of such tiers varied from two to four, depending on the airline and the particular flight. They classify airlines into two categories, the first one containing big players like American and United and the second containing smaller players like Air Trans and Southwest. They find that pricing policies tend to be similar for airlines belonging to the same category and that the prices fluctuate more and are more expensive for airlines in the first category. Finally, they observe that prices increase two weeks prior to departure which corroborates the empirical finding of Stavins.

The main contribution of this chapter is to provide insights into the relationship between the proportion of buyers with a higher willingness to pay and the corresponding shape of the price path. The routes we chose were hand selected to ensure that they not only had a single airline flying on them but also had a maximum of two flights operating on any given day. This ensured that there was little or no competition for each flight on each route.

The remainder of the chapter proceeds as follows. In section 3, we provide a brief overview of the predictions of our theoretical model. Section 4 describes the data, section 5 the empirical model while section 6 presents the results. Section 7 contains the summary and conclusions.

3.3 Theoretical Predictions

As mentioned earlier, the product being sold is not durable. Neither is it storable nor can it be resold. However, since the seller faces the same intertemporal and time-consistency problems as a durable goods monopolist, we followed a model constructed by Conlisk, Gerstner and Sobel (1984), where a single seller sells a durable good over an infinite time horizon in a discrete time setup. A discrete number of buyers entered the market in each period, some of whom had a higher willingness to pay for the product than the others. Buyers could choose either to purchase the product immediately or to wait. The single seller, who did not face any capacity constraint, usually sold the good to the buyers with a higher willingness to pay. Once sufficient number of buyers with a lower willingness to pay accumulated in the market, the seller chose to hold a 'sale' such that the price path generated was a cyclic one. We extended this model by considering a finite time horizon and by imposing a capacity constraint on the monopoly seller.

Model. We considered a three period model with a single seller who chose price p_i and measure of units to offer for sale $q_i, i = 1, 2, 3$ in each period. We assumed that the seller has a continuum of units of measure 3 to offer for sale in each period, such that $q_i \in [0, 3]$ and $\sum q_i \leq 3$. We assumed that the seller has a discount factor of ρ, with $0 < \rho < 1$. Marginal costs were assumed to be zero. On the demand side, a continuum of buyers of measure 2 entered the market in each period. We assumed minimum consumer heterogeneity such that buyers were one of two types. Measure 2α buyers had valuation for the good denoted by V_1 while measure $2(1 - \alpha)$ buyers had valuation V_2, with $V_1 > V_2 > 0$. Buyers with valuation V_1 were said to be of *high* type while buyers with valuation V_2 were said to be of *low* type. We also assumed that $\alpha \in (0, 1/2)$. The probability of getting to purchase the product, which was endogenously determined, acted as the discount factor for each buyer.

Timing of events. At the beginning of each period, the seller announced the price p_i and the measure of units q_i that he (she) would offer for sale in that period. Buyers of both types enter the market and based on the announced and expected future prices and units for sale chose either to purchase immediately or to wait for a lower price. These expectations turned out to be correct in equilibrium. Buyers who chose to purchase left the market immediately. Those who chose to wait, remained in the market until they chose to purchase or till the last period was reached. Based on the prices and the measure of units offered for sale in each period, the seller could deduce the measure of 'active' buyers who remain in the market.[1] The seller was assumed to be unable to precommit to any sequence of prices and measure of units that were to be offered for sale over the three periods.

We assumed that the seller knows the types of each buyer but is unable to discriminate and thus announced a single price in each period. We also assumed that the seller was patient ($\rho \to 1$) and solved for the *subgame perfect Nash outcomes* for various ranges of parameter values.

The main results of the theoretical model were as follows:

(1) Any strategy involving a 'sale' in the last period was found not to be subgame perfect. In case the seller chose to offer a 'sale' in one or both of the first two periods, the measure of units offered for 'sale' was chosen in way to ensure that the measure of units remaining with the seller at the beginning of the third period was equal to the measure of high type buyers who remained 'active' in the last period. Since $p_3 \in \{V_1, V_2\}$, revenue maximization in the last period required the seller to cater only to high valuation buyers.

(2) The total measure of units offered in any period(s) in which a 'sale' was (were) announced was a decreasing function of α. This meant that as the proportion of high valuation buyers increased, the seller chose to offer a smaller measure of units at price V_2. For example, in case the seller wanted to offer $p_1 = p_2 = V_2$ and $p_3 = V_1$, then with $\alpha \in \left[\frac{1}{4}, \frac{V_2}{V_1}\right]$ and $\rho \to 1$, the seller offered $q_1 = \frac{3-6\alpha}{1-\alpha}, q_2 = 0$ and with $\alpha < \frac{1}{4} \leq \frac{V_2}{V_1}$ then he (she) offered $q_1 = 2$ and $q_3 = \frac{1-4\alpha}{1-\alpha}$, such that $(q_1 + q_2)$ was a decreasing

[1] 'Active' buyers in any period, are those who entered the market in that same period as well as those who chose not to purchase the good in the previous period(s) and remained in the market.

function of α.Similarly, for the case where the seller chose to offer a 'sale' in the second period only, then with $\alpha \leq \frac{2V_2}{V_1+V_2}$ and $\rho \to 1$, the seller offered $q_2 = \frac{6\alpha^2 - 15\alpha + 6}{2(1-\alpha)}$ which was also decreasing in α.

(3) The price path was found to be horizontal, u-shaped or non-decreasing for various ranges of parameter values (see figure 3.1). For example, with large values for $V_1$and α the seller had no incentive to offer a 'sale' in any period and announced $p_i = V_1$ and $q_i = 2\alpha$ for $i = 1, 2, 3$. For lower values of V_1 or α, the seller chose to offer a 'sale' in one period, which was either the second or the first period. Finally, for smallest values of V_1 and α, the seller offered a 'sale' in the first two periods.

The theoretical model thus made clear-cut predictions about the shape of the price path for different parameter values. The data collected allowed us to test these predictions empirically. In the event the empirical results failed to match the theoretical predictions, we attempt to provide an intuitive explanation behind such a failure(s).

3.4 Data

While the theoretical model was highly stylized in the sense that it allowed us to capture certain features of the airline ticket pricing, it diverged from the airline ticket market in the following ways. First, we often observe last minute deals being offered by some airlines on online travel sites like Priceline. Buyers can quote their own price which might be accepted by an airline flying on that route. Entering flexible dates increases the chances of finding such an airline. Such discounts are never made available directly from the airlines themselves. In this case, airlines wait till the last few days before the flight departs and offers these seats at a discount through some online travel agents, since selling them at a lower price is preferred to flying with empty seats. Our theoretical model did not allow for such strategies. Second, airline tickets usually come with various sorts of restrictions. Travel restrictions are placed on certain tickets being offered at cheaper rates to make them unattractive to price inelastic buyers (for example, Saturday-night stay-over). Consumers thus end up self selecting the type of ticket and its price which they find most attractive. However, the theoretical model constructed, did not allow for such purchase restrictions and thus had no quality differentiation for the product being sold.

In this empirical extension, we collect price data for *economy class* tickets for one-way, non-stop flights in the US. We thus consider tickets with the least number of restrictions. Further, these routes were hand-selected such that only a single carrier offered services on each of them. This was done to ensure that the airline was a monopoly on that particular route, since the theoretical predictions are valid only for a single seller framework and we were unsure of how the predictions would change for a multiple seller setup. Even though the theoretical model made predictions about the shape of the price path and the measure of units made available for sale in each period for different range of parameter values, we could only empirically test the predictions about the shape of the price path since the number of seats made available for sale by an airline over any period of time were not observable.

The data set consists of two main components. The first component contains airline pricing data on selected routes while the second describes the proportion of "high" type buyers on each of these routes.

3.4.1 Airline Price Data

We collected pricing data for 28 one-way, non-stop flights and for 2 two-way non-stop flights from Expedia and Orbitz. The data was collected twice a day, at 8AM and 8PM, for 14-15 weeks (except for one flight for which we have 11 weeks of observations), which led to a total of 6136 observations. A total of 14 airlines operated on these routes which consisted of 44 distinct cities, of which some were major players like American and Delta, while others were smaller carriers like Midwest Airlines and Frontier Airlines. American Airlines had the maximum number of flights (four), followed by Northwest, Delta, Air Trans and Alaska which had three each. America West, United, Midwest, US Airways and Continental had two each while ATA, Aloha, Spirit and Frontier each had one.

The routes and flights selected had the following features: (1) Each route had a single airline operating on it. (2) Routes with a single airline but with more than two flights operating on a single day were excluded. Routes which had two flights which departed within a few hours of each other were also omitted. Thus, the selected routes had a maximum of two flights operating on them on any given day and in the event there was more than one flight, the flights departed at least 3 hours apart.[2] Thus, the selected flights had little or no competition.

While we managed to address the issue of competition between flights by choosing flights operating on routes which had the features described above, we were unsure of how to deal with competition between online travel agents, if any. At certain points in

[2] Of the 30 routes, only 5 had two flights operating on them on any given day.

time, we collected prices for the same flights from both Expedia and Orbitz and found that most of the time there was a fare differential of $2. Different online travel sites have been known to offer different prices for the same flights. Its unclear as to who sets the prices for such tickets sold through such online travel agents: are such prices set by the airlines themselves or is it the case that the airlines offer certain blocks of seats for sale through such online travel agents who in turn set their own prices?

The selected routes, the carriers serving them and the dates of departure, are listed in table 3.11. All flights departed in early June, 2005. The flights to Kahului and Honolulu were two-way, both having return dates on June 17, 2005. We purposely chose these dates following an observation by Etzioni at al, that prices bounce around more for flights leaving around holidays. Since there is ample evidence in favor of passengers paying higher fares at hubs, initially, we sought to avoid airports which serve as hubs for these airlines. However, we soon realized that it would be impossible to select routes connecting pairs of cities, neither of which were a hub for the airline operating on that route. This is because of the hub-and-spoke network system, whereby all airlines while traveling from one non-hub airport to another, first flies passengers from the city of origin to the nearest hub and then from that hub to the destination city. Thus, for each route, either the city of origin or the destination city serves as a hub for the airline operating on that route.

3.4.2 Data on Proportion of "High" Type Buyers

To get estimates of the proportion of "high" type buyers (α) on the different routes selected, we could have used the method used by Borenstein and Rose (1994), who first defined a tourism index for each metropolitan area (MA) as follows.

$$Tourism\ Index = \frac{\text{P x HR}}{\text{PI}}$$

where P = Proportion of hotel revenues from group/tourist customers, HR = Hotel revenues from that same MA and PI = Total personal income for the same MA. They then take the weighted average of the tourism indices of the two endpoints of a particular route to get a new variable, TOURIST, which they then used in identifying high-tourism markets. However, the sources for such data proved to be quite old and unreliable for the present context.

Instead, we chose the American Travel Survey (ATS, 1995) as the source for data on the proportion of buyers with a higher willingness to pay for the airline tickets. The ATS was developed and conducted by the Bureau of Transportation Statistics (BTS) in order to obtain information about long-distance travel characteristics of individuals living in the US. The survey contains data at the state and metropolitan area levels and describes trip characteristics for both households and individuals. Given a metropolitan area, trip characteristics for an individual person are arranged in the following sequence. First, the survey reports "person trip characteristics" given the metropolitan area as destination and the different census divisions as origin. Second, it displays the same characteristics for the same metropolitan area as origin and the various census divisions as destination. Next, taking the MA as destination, the survey presents trip characteristics for the most frequent state origins. These states are the ones with the 10 largest volumes of travel to that particular MA as origin. Fourth, taking the MA as origin, the corresponding numbers are listed for the states which are the top 10 destinations. Finally, the same order is followed for the cities which are the most frequent origins and destinations for travel to and from that particular MA.

Trip characteristics amongst others included numbers (in thousands) and percentages of individuals having certain principal means of transportation, round-trip distance, main purpose of trip and type of lodging at destination. The characteristic which was of particular interest to us was “main purpose of trip”, which was further categorized into business, pleasure and others. Ideally, we would want the percentage of travellers who traveled by plane for business purposes from one MA to another. However, these numbers are not available. Instead, we use the proportions of travellers who traveled for business purposes (includes all forms of transportation) from or to the particular MA, with the census division or state or city as origin or destination.

The survey did not have data for all the MAs which were included as endpoints in our set of routes. For example, lets consider the American Airlines flight traveling from New Orleans to Boston. While the survey reports trip characteristics for individuals traveling to and from Boston, it does not do the same for New Orleans. In fact, we failed to find data for any of the routes on which both endpoints were represented as origin and destination. Thus, while we had data for both San Francisco and Kansas City MAs individually, the proportion of business travellers traveling from San Francisco to Kansas City were unavailable. This is because on one hand San Francisco was not amongst the top 10 cities having the most travel volume going to Kansas City and on the other, Kansas City was not amongst the top 10 cities having the most travel volume coming from San Francisco.

While we could have avoided this problem by looking at routes like New York-Boston and San Francisco-Phoenix for which we would have the corresponding percentages of travellers traveling for business purposes, these routes had a number of airline carriers flying on them, which made these markets oligopolies instead of monopolies, and unsuitable for consideration. Thus we had to look at other means for calculating the proportion of business travellers traveling on the different routes. To do this, we chose to fix the destination city and looked at the proportion of business travellers traveling from the census division (CD) to which the city of origin belonged. For the flight from New Orleans to Boston, we chose to fix the destination city (Boston) and looked at the proportion of business travellers traveling from the West South Central census division to which Louisiana belongs. This meant that we could use only 19 of the original 30 routes selected for data collection. The proportions of "business" travellers traveling on the different routes are reported in table 3.12.

3.5 Empirical Model

Since the main purpose of this chapter is to test the theoretical predictions outlined in section 3, we set up a number of empirical models which when estimated, delineates the relationship between the proportion of "high" type buyers on a route and the slope of the corresponding price path. In order to check whether the proportion of "high" type buyers on a route has any effect on the slope of the price path, we begin by estimating a model which assumes that the prices on any route depend on the number of observations left for departure and the proportion of “high” type buyers on that route. We will refer to this as model 1.

$$P_{mt} = \delta_m + \beta_1(\alpha_m D_{mt}) + \gamma_1 D_{mt} + \varepsilon_{mt} \tag{1}$$

where P_{mt} is the price for route m at time t, δ_m is a route specific intercept term (dummy) which remains constant over time, α_m denotes the proportion of business travellers on route m and D_{mt} is the number of observations left for departure on route m at time t. If a route has 15 weeks of observations collected twice a day, the variable D_{mt} takes values from 210 to 1. Thus, as D_{mt} decreases, we move closer to departure. From equation (1) we get,

$$\frac{\partial P_{mt}}{\partial D_{mt}} = \beta_1 \alpha_m + \gamma_1 \tag{2}$$

which implies that if the coefficient β_1 is not significant, α_m has no effect on the slope of the price path.

Next, we construct a model where we categorize the routes into ones with ‘high’, ‘medium’ and ‘low’ proportion of business travellers and assign dummies to them as follows. Assuming that α_m represents the the proportion of business travellers on route m, we define for that route $\alpha_m = \alpha^H = 1$ if $\alpha_m > 0.45$ and 0 otherwise, $\alpha_m = \alpha^M = 1$ if $0.25 \leq \alpha_m \leq 0.45$ and 0 otherwise and finally $\alpha_m = \alpha^L = 1$ if $\alpha_m < 0.25$ and 0 otherwise. Thus, in addition to the route specific dummy variables, we construct a model with dummies which equal 1 or 0 depending on whether the route contains ‘high’, 'medium' or 'low' proportion of business travellers. We will refer to this as model 2.

$$P_{mt} = \delta_m + \beta_1(\alpha^H D_{mt}) + \beta_2(\alpha^M D_{mt}) + \gamma_1 D_{mt} + \varepsilon_{mt} \tag{3}$$

The estimates from this model will give us some idea about the slope of the price path. However, in order to get the shape of the price path we need to check how prices change over time. In the next step, we construct another model where we introduce dummies for number of weeks before departure. While the theoretical model had three periods, it is not apparent how we should define periods in the empirical counterpart. The theoretical model assumes that the measure of "high" type buyers entering the market in each period remains the same over the three periods. Typically, travellers with a higher willingness to pay for tickets enter the market in larger numbers in the weeks just prior to departure than earlier on. Thus, we introduce the dummies for number of weeks prior to departure as follows: $D^1 = 1$ for one week before departure, 0 otherwise, $D^2 = 1$ for 1 to 3 weeks before departure and 0 otherwise and $D^3 = 1$ for rest and 0 otherwise. The corresponding model (model 3) assumes the following form.

$$\begin{gathered} P_{mt} = \delta_m + \beta_1(\alpha^L D^1) + \beta_2(\alpha^L D^2) + \beta_3(\alpha^L D^3) + \gamma_1(\alpha^M D^1) + \\ \gamma_2(\alpha^M D^2) + \gamma_3(\alpha^M D^3) + \theta_1(\alpha^H D^1) + \theta_2(\alpha^H D^2) + \theta_3(\alpha^H D^3) + \varepsilon_{mt} \end{gathered} \tag{4}$$

Finally, we perform a Chow Breakpoint test to confirm whether there were structural changes in the price path before and after pre-determined cutoff points. To do this, we proceed using the following steps.

Step (1) We split the data set into two parts, such that with $D_{mt} \leq c$ (c being the pre-determined break point), the data is said to belong to group 1 and with $D_{mt} > c$ data is said to belong to group 2.

Step (2) We take $c = 105$. We then create a dummy variable which takes value 1 for $D_{mt} \geq 105$ and 0 otherwise and create another dummy variable ($time\_dummy2$) which takes value 1 for $D_{mt} < 105$ and 0 otherwise.

Step (3) Get estimates for the coefficients of the following model.

$$\begin{aligned} P_{mt} = K + \delta_m + \beta_1(\alpha_m D_{mt}) + \gamma_1(D_{mt}) + \beta_2(\alpha_m D_{mt} \times time\_dummy2) \\ +\gamma_2(D_{mt} \times time\_dummy2) + \theta_2 time\_dummy2 + \varepsilon_{mt} \end{aligned} \tag{5}$$

Since theory predicts a horizontal price path for routes with the highest α, we run the above regression only for those routes with $\alpha^H = 1$ and test for $\beta_2 = 0$, $\gamma_2 = 0$ and $\theta_2 = 0$. If the null hypothesis cannot be rejected, then there is no structural change in the model before and after the breakpoint.

3.6 Results

Table 3.1 reports the descriptive statistics for the data sets for the following two cases. (1) Includes all 30 routes for which different criteria are used for the proportion of business travellers on the different routes. For example, for the Austin-Washington DC route, we used the proportion of business travellers who traveled from Texas (state as origin) to DC and for the Seattle-Tucson route, we used the proportion of business travellers who flew from Austin. (2) Considers only 19 of the 30 routes, for which we fix the destination city and use the proportion of travellers traveling for business purposes from the census division to which to city of origin belongs, to the destination city.

	No. of Obs	Mean	St. Devn	Min	Max
All 30 Routes					
Price	6136	331.98	225.63	86	2441
Proportion of Business Travellers		0.336	0.173	0.03	0.74
19 Routes					
Price	3842	308.18	172.14	86	816
Proportion of Business Travellers		0.375	0.187	0.03	0.74

Table 3.1. Descriptive Statistics

All the equations were estimated using OLS. Route dummies were used to take into account route-specific characteristics, which remain unchanged over time. Since the use of miscellaneous criteria for the proportion of business travellers is unintuitive, we ran all the regressions for the 19 routes using the criteria as described in the second case above. Table 3.2 contains the estimates of the coefficients for equation (1).

	$Coef.$	$St.\ Error$	$t-stat$	$p-Value$
$\alpha_m \times D_{mt}$	-0.323	0.089	-3.63	0.000
D_{mt}	-0.197	0.026	-7.63	0.000

Table 3.2. Regression Results for Model 1 for 19 Routes

Since both the coefficients are negative and significant, we can conclude from equation (2) that the slope of the price path is *negative*. However, since an increase in D_{mt} signifies movement away from departure, the negative slope obtained implies that prices increase as we move closer to departure. This result corroborates earlier findings of Stavins (2001), McAfee and Velde (2004) and Etzioni et al (2003).

While the results from table 3.2 clearly show that cheaper prices disappear as the departure date comes closer, it fails to show how quickly prices increase as the proportion of "high" type buyers on a route changes. The model represented by equation (3) was constructed for this purpose. The coefficients for this model could be interpreted as follows. Each route can only have either high, medium or low proportion of travellers with a high valuation. The coefficient for D_{mt} represents the base case and denotes the slope of the price path for routes which have $\alpha_m = \alpha^L$ (second and third terms drop out). The sum of the coefficients of $\alpha^M \times D_{mt}$ and D_{mt} refers to the slope of the price path for routes with $\alpha_m = \alpha^M$, while the sum of the coefficients of $\alpha^H \times D_{mt}$ and D_{mt} represents the slope of the price path for routes with $\alpha_m = \alpha^H$.

	$Coef.$	$St.\ Error$	$t-stat$	$p-Value$
$\alpha^H \times D_{mt}$	-.215	.056	-3.85	0.000
$\alpha^M \times D_{mt}$	.060	.033	1.81	0.071
D_{mt}	-.277	.021	-13.30	0.000

Table 3.3. Regression Results for Model 2 for 19 Routes

Thus, the slopes of the price path for routes with low, medium and high proportion of travellers with a high valuation are $-0.277, -0.217$ and -0.492 respectively. Prices were thus found to increase most quickly in routes with the highest proportion of business travellers. While this provides some evidence as to the relationship between the *slope* of the price path and α_m, it does not describe the relationship between the *shape* of the price path and the corresponding α_m. To do this, we put the routes into their respective categories depending on the values of their α_m and used equation (4).

Since it is unlikely that the proportion of business travellers entering the market remains constant over time we introduced dummies for weeks before departure as described in the previous section. The estimates of the coefficients from equation (4) are reported in the following table and the corresponding plot displayed in figure 2.

	$Coef.$	$St.\ Error$	$t-stat$	$p-Value$
$\alpha^L \times D^1$	163.56	4.613	35.46	0.000
$\alpha^L \times D^2$	118.733	1.702	69.75	0.000
$\alpha^L \times D^3$	108.909	1.154	94.39	0.000
$\alpha^M \times D^1$	200.877	8.482	23.68	0.000
$\alpha^M \times D^2$	145.935	2.944	49.58	0.000
$\alpha^M \times D^3$	132.333	1.908	69.35	0.000
$\alpha^H \times D^1$	267.456	16.272	16.44	0.000
$\alpha^H \times D^2$	156.654	5.159	30.36	0.000
$\alpha^H \times D^3$	128.0702	2.319	55.22	0.000

Table 3.4. Regression Results for Model 3 for 19 Routes

The price path is found to be rising for all three categories of routes. Routes with $\alpha_m = \alpha^H$ showed the sharpest increase in prices. Thus, the theoretical prediction that the price path for routes with high α is horizontal is found to be empirically invalid. Finally, we report the results for the Chow Breakpoint test. For routes with $\alpha = \alpha^H$, theory predicts that there will be no change in the slope or the intercept before and after the break point. This implies that all three coefficients β_2, γ_2 and θ_2 need to be not significant for equation 5. However, testing for this is not the same as testing whether slope of the price path remains constant at zero. The following table reports the coefficients for the Chow Breakpoint test for different pre-determined cutoff values (c). For example, with $c = 105$, we split the data set for each route into two sets, one having number of observations left for departure larger than 105 and the other less than 105. The F-statistic is based on the null hypothesis involving the following three restrictions, $\beta_2 = 0, \gamma_2 = 0$ and $\theta_2 = 0$. The low p-values led us to conclude that the null hypothesis can be rejected for all three pre-determined cutoff points and that there is structural change in the model before and after these cutoff points.

c	β_1	γ_1	β_2	γ_2	θ_2	$F\ stat$	$p\ value$
105	0.413	−0.549	0.167	−1.043	55.908	33.99	0.000
70	0.562	−0.544	0.882	−2.774	105.632	35.66	0.000
42	0.109	−0.219	−3.148	−2.547	152.719	31.77	0.000

Table 3.5. Coefficients for Chow Breakpoint Test for Routes with $\alpha = \alpha^H$

Finally, for routes with $\alpha_m = \alpha^L$ we try to collect empirical evidence which establishes that the shape of the price path for such routes is u-shaped if we truncate the data set in the following manner. First, we discard the oldest 6 weeks of observations. The variable D_{mt} could thus take a maximum value of 126. Second, we split the price data into 2 groups, with one group containing the most recent observations with $D_{mt} < 63$ and the other group containing the rest, with $D_{mt} \geq 63$. Third, we estimate for each group the same equation as equation (1).

$$P_{mt} = \delta_m + \beta_1(\alpha_m D_{mt}) + \gamma_1 D_{mt} + \varepsilon_{mt}$$

such that the corresponding slope is

$$\frac{\partial P_{mt}}{\partial D_{mt}} = \beta_1 \alpha_m + \gamma_1$$

The estimates of the coefficients of this model for the two groups are reported in tables 3.6 and 3.7.

	$Coef.$	$St.\ Error$	$t-stat$	$p-Value$
$\alpha_m \times D_{mt}$	10.081	1.951	5.17	0.000
D_{mt}	-0.876	0.168	-5.21	0.000

Table 3.6. Regression Results for Group 1

With the average α for routes with low proportion of business travellers, $\overline{\alpha}^L = 0.11$, $\frac{\partial P_{mt}}{\partial D_{mt}} > 0$ for group 1, while for group 2, $\frac{\partial P_{mt}}{\partial D_{mt}} < 0\ \forall \alpha$. Thus, the slope of the

	$Coef.$	$St.\ Error$	$t-stat$	$p-Value$
$\alpha_m \times D_{mt}$	-5.276	0.834	-6.32	0.000
D_{mt}	-0.635	0.114	-5.55	0.000

Table 3.7. Regression Results for Group 2

price path changes from one group to another, indicating evidence of a u-shaped path. Similar evidence for a u-shaped path for routes with $\alpha_m = \alpha^L$ was also found when we considered the most recent 8 weeks and 6 weeks of observations.

In order to verify these results, we ran the following regression, where we introduced route-specific dummies and dummies for weeks as follows.

$$P_{mt} = \delta_m + \beta_1 D^1 + \beta_2 D^2 + .. + \beta_7 D^7 + \varepsilon_{mt} \tag{6}$$

where, D^1 = dummy for the last two weeks before departure, D^2 = dummy for 3 to 4 weeks before departure, D^3 = dummy for 5 to 6 weeks before departure and so on. Results for this regression equation are displayed in table 3.8.

The coefficients for dummies D^1 to D^5 shows that prices fall and then rise as the date of departure draws closer. Thus, a u-shaped pattern re-emerges once we choose to concentrate only on the last 10 weeks before take-off. We ran similar regressions for routes with medium and high proportion of business travellers. Results are reported in tables 3.9 and 3.10.

	$Coeff.$	$St.Error$	$t-stat$	$p-Value$
D^1	65.679	6.352	10.34	0.000
D^2	29.089	5.810	5.01	0.000
D^3	28.918	6.344	4.56	0.000
D^4	42.968	6.718	6.40	0.000
D^5	45.904	6.522	7.04	0.000
D^6	19.993	6.765	2.96	0.003
D^7	-6.134	6.670	-0.92	0.358

Table 3.8. Regression Results for Equation 6 for Routes with $\alpha = \alpha^L$

	$Coeff.$	$St.Error$	$t-stat$	$p-Value$
D^1	49.928	5.909	8.45	0.000
D^2	14.954	4.089	3.66	0.000
D^3	6.097	2.877	2.12	0.034
D^4	4.892	3.056	1.60	0.110
D^5	10.494	2.960	3.55	0.000
D^6	3.084	3.097	1.00	0.319

Table 3.9. Regression Results for Equation 6 for Routes with $\alpha = \alpha^M$

While these regressions allowed us to trace the price path over time for routes with low, medium and high proportion of business travellers, we must clarify that α remained constant over time for the different categories of routes. While we found evidence of a u-shape for routes with low and medium proportion of business travellers when we

	$Coeff.$	$St.Error$	$t-stat$	$p-Value$
D^1	156.131	16.401	9.52	0.000
D^2	53.894	6.536	8.25	0.000
D^3	55.281	6.494	8.51	0.000
D^4	53.109	6.731	7.89	0.000
D^5	52.097	6.541	7.97	0.000
D^6	46.364	6.674	6.95	0.000
D^7	38.068	6.766	5.63	0.000

Table 3.10. Regression Results for Equation 6 for Routes with $\alpha = \alpha^H$

looked at the last 10 weeks before departure, no such pattern emerged for routes with high α, where the price path was found to be rising.

3.7 Conclusions

In order to empirically test the theoretical predictions of an earlier chapter, we collected pricing data for several one-way non-stop flights in the US and simultaneously gathered data for the proportion of travellers traveling for business purposes on these routes. While the theoretical prediction that prices never fall before departure was corroborated, the prediction that the price path for routes with the highest proportion of "high" type buyers is horizontal was found to be empirically invalid. Instead, routes with high proportions of business travellers witnessed the steepest increase in prices. The price path for the routes with low and medium proportions of business travellers was also found to be increasing.

In our theoretical model we assumed that the proportion of buyers with a higher valuation for the good, who enters the market in each period, remains constant over the three periods. In reality, this is clearly not the case. It is our conjecture that a theoretical model which allows for variation in the proportion of high valuation buyers over the three periods, where the proportion increases from the first to the third period, will perform better in terms of providing an explanation for the empirical results. However, even if we do solve for the price paths for various parameter values for such a model, it will be difficult to access data which describe how the proportion of travellers traveling for business purposes on different routes change as the date of departure draws closer.

The theoretical model also predicted a small range of parameter values for which the price path would be u-shaped. While we did find some empirical evidence for a u-shaped price path for routes with a low proportion of high valuation buyers, we did so only after truncating the data and considering only the last 10 weeks of observations.

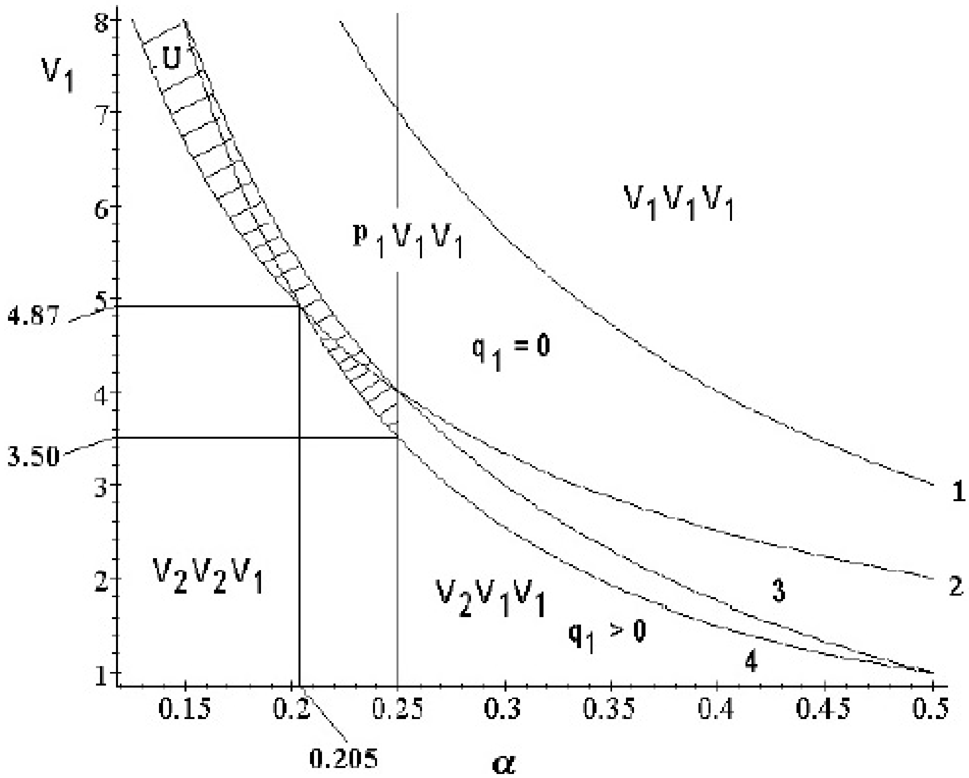


Fig. 3.1. Subgame Perfect Outcomes for different values of V_1 and α ($V_2 = 1$).

(1) $V_1 = \dfrac{2-\alpha}{\alpha}$

(2) $V_1 = \dfrac{1}{\alpha}$

(3) $V_1 = \dfrac{3-4\alpha}{2\alpha}$

(4) $V_1 = \dfrac{6\alpha^2 - 7\alpha + 4}{\alpha(4\alpha + 2)}$

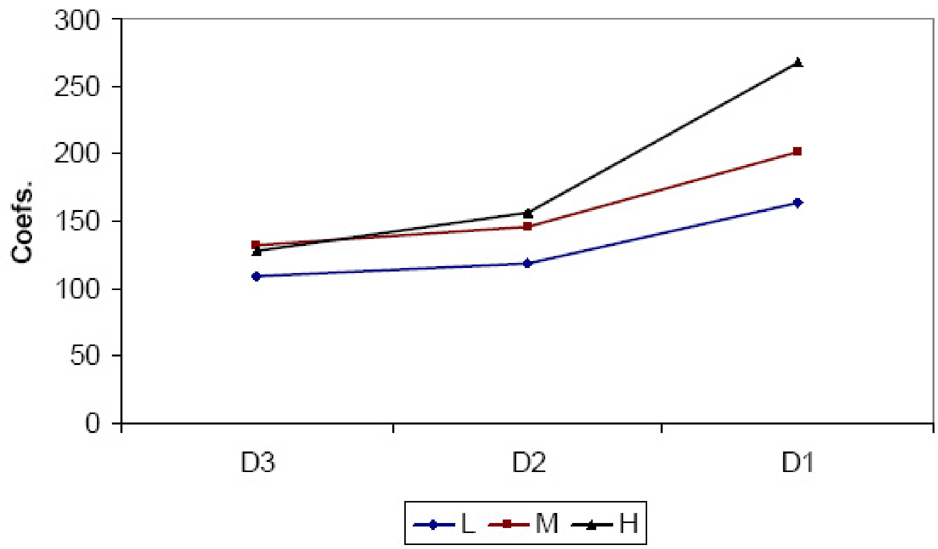


Fig. 3.2. Coefficients for Model 3

(1) D1 = 1 week before departure

(2) D2 = 1 to 3 weeks before departure

(3) D3 = Rest

City of Origin	Destination City	Airline Carrier	Date of Departure
Detroit,MI	Orange County, CA	Northwest	June 6
Spokane, WA	Las Vegas, NV	America West	June 6
Austin, TX	Washington DC (IAD)[a]	United	June 6
Orlando, FL	Rochester, NY	Air Trans	June 6
Burbank, CA	Atlanta, GA	Delta	June 6
Detroit, MI	San Diego, CA	Northwest	June 6
Portland, OR	Santa Barbara, CA	Alaska	June 7
Wrangell, AK	Petersburg, AK	Alaska	June 7
Reno, NV	Orange County, CA	Aloha	June 7
Kansas City, MO	San Antonio, TX	Midwest	June 7
Akron, OH	Tampa, FL	Air Trans	June 7
Providence, RI	Fort Myers, FL	Spirit	June 7
Denver, CO	Little Rock, AK	Frontier	June 8
San Francisco, CA	Austin, TX	United	June 8
Santa Barbara, CA	Dallas, TX	American	June 8
Akron, OH	Orlando, FL	Air Trans	June 8
Cincinnati, OH	Orange County, CA	Delta	June 8
Birmingham, AL	Washington DC (DCA) [b]	Delta	June 8
Indianapolis, IN	Miami, FL	American	June 2
New Orleans, LA	Boston, MA	American	June 2
Pittsburgh, PA	Los Angeles, CA	US Airways	June 2
Cleveland, OH	San Antonio, TX	Continental	June 2
Seattle, WA	Tucson, AZ	Alaska	June 2
Miami, FL	Phoenix, AZ	America West	June 10
Memphis, TN	Las Vegas, NV	Northwest	June 10
San Francisco, CA	Kansas City, MO	Midwest	June 10
Dallas/Fort Worth	Providence, RI	American	June 10
Portland, ME	Charlotte, NC	US Airways	June 10
Phoenix, AZ *	Kahului, HI	ATA	June 10/17
Newark, NJ *	Honolulu, HI	Continental	June 10/17

[a] There were no direct flights to Ronald Reagan Washington National Airport (DCA) from Austin.

[b] There were no direct flights to Dulles International Airport (IAD) from Birmingham.

Table 3.11. Routes, Carriers and Dates of Departure

* Two-way flights, with June 17, 2005 as return date.

City of Origin	Destination City	Criteria (Misc.)	(Origin/Destn: CD/City)
Detroit,MI	Orange County, CA	0.46	0.46
Spokane, WA	Las Vegas, NV	0.17	
Austin, TX	Washington DC (DCA)	0.7	0.62
Orlando, FL	Rochester, NY	0.03	0.03
Burbank, CA	Atlanta, GA	0.74	0.74
Detroit, MI	San Diego,CA	0.40	0.40
Portland, OR	Santa Barbara, CA	0.43	
Wrangell, AK	Petersburg, AK	0.29	
Reno, NV	Orange County, CA	0.15	0.11
Kansas City, MO	San Antonio, TX	0.52	0.41
Akron, OH	Tampa, FL	0.15	0.15
Providence, RI	Fort Myers, FL	0.17	
Denver, CO	Little Rock, AK	0.33	
San Francisco, CA	Austin, TX	0.41	
Santa Barbara, CA	Dallas, TX	0.55	0.52
Akron, OH	Orlando, FL	0.23	0.23
Cincinnati, OH	Orange County, CA	0.42	0.46
Birmingham, AL	Washington DC (DCA)	0.43	0.43
Indianapolis, IN	Miami, FL	0.06	0.45
New Orleans, LA	Boston, MA	0.55	0.55
Pittsburgh, PA	Los Angeles, CA	0.27	0.37
Cleveland, OH	San Antonio, TX	0.48	0.38
Seattle, WA	Tucson, AZ	0.25	
Miami, FL	Phoenix, AZ	0.10	0.35
Memphis, TN	Las Vegas, NV	0.15	
San Francisco, CA	Kansas City, MO	0.49	0.41
Dallas/Fort Worth	Providence, RI	0.29	
Portland, ME	Charlotte, NC	0.35	0.07
Phoenix, AZ	Kahului, HI	0.17	
Newark, NJ	Honolulu, HI	0.24	

Table 3.12. Proportion of “High” Type Buyers On Different Routes

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