A DESIGN APPROACH FOR A CONFIGURABLE HIGH-POWER MAGNETOSTRICTIVE DRIVE MADE FROM IRON-GALLIUM ALLOY (GALFENOL)

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by
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Abstract

Magnetostrictive gallium-iron alloys (collectively known as Galfenol) have been identified as a candidate active materials for use in sonar transducers. Among the interesting characteristics of Galfenol are its competitive strain capability, attractive magnetic permeability, and mechanical robustness. The latter property is especially noteworthy as it allows access to a design space inaccessible to other high-strain active materials, both magnetostrictive and piezoelectric.

This dissertation develops a design approach for leveraging Galfenol’s unique properties to create a self-contained magnetostrictive drive constructed from finely laminated structures. The approach relies on using a generalized design framework that is flexible enough to be configured for different applications. Flexibility is achieved through the adjustment of the laminated structure dimensions, the modular assembly of these structures, and the ability to bias the system with either permanent magnets or direct current. A key outcome of this study is a one-dimensional design model that simplifies the process of tailoring the drive to potential applications.

Validation of this configurable drive concept is accomplished through designing, fabricating, and modeling a prototype drive that seeks to optimize the material’s use. The one-dimensional design model is used to produce the prototype design and the results are corroborated with three-dimensional, nonlinear finite element analysis. Fabrication of the device is accomplished by assembling thin layers of Galfenol steel into two laminated structures, winding drive coils on these structures, and placing permanent magnets in between. Finally, a full one-dimensional, multi-domain model is created to simulate the behavior of the device.

To investigate the performance of Galfenol in sonar applications, the drive is assembled into a tonpilz-style transducer and tested in a water-filled anechoic tank. Simulation is accomplished by updating the one-dimensional model of the drive to represent the testing scenario and by employing finite element modeling. The
model results are then compared to measurement with good agreement.

Finally, measured values for the quality factor, electromechanical coupling coefficient, and efficiency of the Galfenol tonpilz are compared to published data for various nickel and Terfenol-D designs. The result is that the Galfenol transducer achieves comparable results in all three areas. It is concluded that Galfenol is competitive with other magnetostrictive materials and merits consideration for future magnetostrictive designs.
# Table of Contents

**List of Figures** xiii

**List of Tables** xxiv

**List of Symbols** xxv

**Acknowledgments** xxxii

**Chapter 1**

<table>
<thead>
<tr>
<th>Introduction and objectives</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1 Magnetostriction</td>
<td>1</td>
</tr>
<tr>
<td>1.1.1 Sonar</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Historical background</td>
<td>2</td>
</tr>
<tr>
<td>1.2.1 Origins</td>
<td>2</td>
</tr>
<tr>
<td>1.2.2 Work at Harvard University</td>
<td>3</td>
</tr>
<tr>
<td>1.2.2.1 Early work by G. W. Pierce</td>
<td>3</td>
</tr>
<tr>
<td>1.2.2.2 The Harvard Underwater Sound Laboratory</td>
<td>3</td>
</tr>
<tr>
<td>1.2.3 Competition from piezoelectric crystals</td>
<td>4</td>
</tr>
<tr>
<td>1.2.4 Discovery of giant magnetostrictive materials</td>
<td>5</td>
</tr>
<tr>
<td>1.2.5 Galfenol</td>
<td>5</td>
</tr>
<tr>
<td>1.3 Literature review</td>
<td>7</td>
</tr>
<tr>
<td>1.3.1 Galfenol material studies</td>
<td>7</td>
</tr>
<tr>
<td>1.3.1.1 Discovery and early observations</td>
<td>7</td>
</tr>
<tr>
<td>1.3.1.2 Quenching</td>
<td>8</td>
</tr>
<tr>
<td>1.3.1.3 Mechanical properties</td>
<td>8</td>
</tr>
<tr>
<td>1.3.1.4 Ternary Fe-Ga-X alloys</td>
<td>8</td>
</tr>
<tr>
<td>1.3.1.5 Shear modulus softening</td>
<td>9</td>
</tr>
</tbody>
</table>
1.3.1.6 Magnetostrictive behavior with temperature variation ........................................... 9
1.3.1.7 Thermal properties ........................................................................................................ 9
1.3.1.8 Crystalline structure ....................................................................................................... 9
1.3.1.9 Crystal growth ................................................................................................................ 10
1.3.1.10 Magnetomechanical coupling ...................................................................................... 10
1.3.1.11 Annealing ...................................................................................................................... 10
1.3.1.12 Galfenol steel .............................................................................................................. 11
1.3.1.13 Texturing ...................................................................................................................... 11
1.3.1.14 Rolling .......................................................................................................................... 11
1.3.1.15 Punching ...................................................................................................................... 12
1.3.2 Galfenol sample characterization ..................................................................................... 12
1.3.2.1 Quasi-static characterization ....................................................................................... 12
1.3.2.2 Dynamic characterization ............................................................................................. 12
1.3.3 Galfenol device modeling ................................................................................................ 13
1.3.4 Galfenol devices ................................................................................................................ 13
1.3.4.1 Underwater transducers ............................................................................................... 13
1.3.4.2 Sensor and actuators .................................................................................................... 13
1.3.5 Modular approaches to magnetostrictive transducers ....................................................... 14
1.3.6 High power sonar transducers ......................................................................................... 14
1.3.7 Transducer comparison .................................................................................................... 15
1.4 Problem statement ............................................................................................................... 16
1.5 Proposed solution .................................................................................................................. 16

Chapter 2

Theory .................................................. 18

2.1 Magnetism ........................................... 18
2.1.1 Fundamentals of magnetism ............................................................................................. 20
2.1.1.1 Magnetic field produced by a long, straight wire .......................................................... 20
2.1.1.2 Magnetic field produced by a circular loop of wire ....................................................... 21
2.1.2 Magnetic materials ......................................................................................................... 21
2.1.2.1 Diamagnetic materials ............................................................................................... 22
2.1.2.2 Paramagnetic materials ............................................................................................. 22
2.1.2.3 Ferromagnetic materials ............................................................................................ 23
2.1.2.4 The $B-H$ curve ........................................................................................................... 23
2.1.3 Permanent magnets .......................................................................................................... 26
2.1.4 Electromagnets ................................................................................................................ 27
2.1.5 Eddy currents .................................................................................................................. 28
2.1.5.1 Skin effect ................................................................................................................... 28
2.1.5.2 Lamination .................................................................................................................. 29
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1.5.3</td>
<td>Critical frequency</td>
<td>30</td>
</tr>
<tr>
<td>2.1.5.4</td>
<td>Power loss</td>
<td>30</td>
</tr>
<tr>
<td>2.1.6</td>
<td>Demagnetizing fields</td>
<td>30</td>
</tr>
<tr>
<td>2.2</td>
<td>Mechanics</td>
<td>31</td>
</tr>
<tr>
<td>2.2.1</td>
<td>Stress</td>
<td>31</td>
</tr>
<tr>
<td>2.2.2</td>
<td>Strain</td>
<td>32</td>
</tr>
<tr>
<td>2.2.3</td>
<td>Elasticity</td>
<td>33</td>
</tr>
<tr>
<td>2.2.4</td>
<td>Poisson's ratio</td>
<td>34</td>
</tr>
<tr>
<td>2.3</td>
<td>Acoustics</td>
<td>34</td>
</tr>
<tr>
<td>2.3.1</td>
<td>Modes of bars</td>
<td>34</td>
</tr>
<tr>
<td>2.3.2</td>
<td>Quality factor</td>
<td>35</td>
</tr>
<tr>
<td>2.3.3</td>
<td>Radiation impedance</td>
<td>36</td>
</tr>
<tr>
<td>2.3.4</td>
<td>Radiated power</td>
<td>37</td>
</tr>
<tr>
<td>2.3.5</td>
<td>Cavitation</td>
<td>38</td>
</tr>
<tr>
<td>2.4</td>
<td>Transduction</td>
<td>39</td>
</tr>
<tr>
<td>2.4.1</td>
<td>Electroacoustic transducers</td>
<td>40</td>
</tr>
<tr>
<td>2.4.1.1</td>
<td>Tonpilz transducers</td>
<td>40</td>
</tr>
<tr>
<td>2.4.1.2</td>
<td>Flextensional transducers</td>
<td>41</td>
</tr>
<tr>
<td>2.4.1.3</td>
<td>Split cylinder transducers</td>
<td>41</td>
</tr>
<tr>
<td>2.4.2</td>
<td>Relating electrical and magnetic variables</td>
<td>41</td>
</tr>
<tr>
<td>2.4.3</td>
<td>Relating mechanical and acoustic variables</td>
<td>42</td>
</tr>
<tr>
<td>2.4.4</td>
<td>Coupling coefficient</td>
<td>42</td>
</tr>
<tr>
<td>2.4.4.1</td>
<td>Material coupling coefficient</td>
<td>43</td>
</tr>
<tr>
<td>2.4.4.2</td>
<td>Coil coupling coefficient</td>
<td>44</td>
</tr>
<tr>
<td>2.4.4.3</td>
<td>Effective coupling coefficient</td>
<td>44</td>
</tr>
<tr>
<td>2.5</td>
<td>Magnetostriction</td>
<td>45</td>
</tr>
<tr>
<td>2.5.1</td>
<td>Piezomagnetism and piezoelectricity</td>
<td>45</td>
</tr>
<tr>
<td>2.5.2</td>
<td>Magnetoelasticity</td>
<td>45</td>
</tr>
<tr>
<td>2.5.3</td>
<td>Magnetostrictive constitutive equations</td>
<td>47</td>
</tr>
<tr>
<td>2.5.4</td>
<td>Prestress</td>
<td>50</td>
</tr>
<tr>
<td>2.5.5</td>
<td>The $S-H$ curve</td>
<td>51</td>
</tr>
</tbody>
</table>

Chapter 3

Modeling

3.1 Introduction                                      53
3.2 Modeling mathematics                             54
  3.2.1 Numerical methods                             54
  3.2.2 The finite element method                     55
  3.2.3 Simulation (model analyses)                   56
3.3 Magnetostrictive materials modeling              58
3.3.1 Constitutive equations modeling ........................................... 58
3.3.2 Thermodynamic energy-based modeling ............................... 59
3.4 One-dimensional device modeling ........................................... 59
3.4.1 Static and dynamic systems ............................................... 59
3.4.2 Analogies ........................................................................ 62
3.4.3 Graphical representation of differential equations .................. 63
3.4.4 Lumped element modeling ............................................... 63
3.4.5 Segmented modeling ....................................................... 65
3.4.6 Distributed element modeling ........................................... 66
3.5 SPICE ................................................................................ 66
3.5.1 Multi-port elements ......................................................... 68
3.5.2 Analog behavioral modeling ............................................. 69
3.5.3 Coupling domains with ABM ........................................... 70
3.5.4 Magnetic circuit modeling ................................................ 72
3.5.5 Permanent magnet modeling ............................................ 77
3.5.6 Eddy current loss modeling ............................................. 77
3.5.7 Active material modeling ................................................ 79
3.5.7.1 Piezoelectric piece ..................................................... 80
3.5.7.2 Piezomagnetic piece .................................................. 82
3.5.8 One-dimensional modeling example .................................... 83
3.5.8.1 Model complexity ..................................................... 87
3.6 Finite element analysis in device modeling .............................. 90
3.6.1 COMSOL Multiphysics .................................................... 90
3.7 Efficiency through progressive model complexity ........................ 91

Chapter 4
Configurable magnetostrictive drive ........................................... 92
4.1 Introduction ........................................................................ 92
4.2 Magnetostrictive design ...................................................... 94
4.2.1 Nickel design ................................................................... 95
4.2.2 Terfenol-D design ......................................................... 97
4.2.3 Galfenol design ............................................................. 98
4.2.4 Decoupled DC and AC magnetic circuits ............................ 99
4.3 Configurable drive .............................................................. 100
4.3.1 Piezoelectric parallel ...................................................... 103
4.4 Applications ....................................................................... 103

Chapter 5
Drive design, fabrication, modeling, and measurement .................. 106
5.1 Design .............................................................................. 106
6.6.1.3 Electroacoustic efficiency ............................................. 170
6.6.1.4 Beam patterns ......................................................... 171
6.6.2 Transducer failure and rebuild ....................................... 172
6.6.3 Increasing drive measurements ..................................... 175
   6.6.3.1 Transmitting current response ................................. 175
   6.6.3.2 Source level ....................................................... 176
   6.6.3.3 Total harmonic distortion .................................... 177
   6.6.3.4 Source level versus volt-amps ................................ 178
   6.6.3.5 Strain ............................................................. 179
   6.6.3.6 Temperature rise ................................................ 180
6.6.4 Cavitation .................................................................... 181
6.7 Evaluation of GCD tonpilz ................................................ 182

Chapter 7
Conclusions and recommendations ............................................ 184
7.1 Summary ........................................................................... 184
7.2 Conclusions ................................................................. 185
   7.2.1 Assessment of Galfenol material ................................. 186
   7.2.2 Assessment of one-dimensional modeling techniques .... 187
   7.2.3 Assessment of prototype drive ..................................... 188
7.3 Contributions .................................................................... 188
7.4 Future work ..................................................................... 189

Appendix A
GCD tonpilz ATF testing results ................................................ 192
A.1 Test 1 – Sweeps, 15 kHz to 100 kHz at 48 mA (50 mA attempted) ............................................. 192
A.2 Test 2 – Sweeps, 5 kHz to 20 kHz at 48 mA (50 mA attempted) ............................................. 198
A.3 Test 3 – Beam pattern, 10 kHz horizontal at 48 mA (50 mA attempted) .................................... 204
A.4 Test 4 – Beam pattern, 20 kHz horizontal at 48 mA (50 mA attempted) .................................... 204
A.5 Test 5 – Beam pattern, 30 kHz horizontal at 48 mA (50 mA attempted) .................................... 204
A.6 Test 6 – Sweeps, 5 kHz to 25 kHz at 176 mA (200 mA attempted) ............................................. 208
A.7 Test 7 – Beam pattern, 10 kHz horizontal at 176 mA (200 mA attempted) .................................... 214
A.8 Test 8 – Beam pattern, 20 kHz horizontal at 176 mA (200 mA attempted) .................................... 214
A.9 Test 9 – Beam pattern, 30 kHz horizontal at 176 mA (200 mA attempted) .................................... 214
A.10 Test 10 – Beam pattern, 10 kHz vertical at 48 mA (50 mA attempted) ...................................... 218
A.11 Test 11 – Beam pattern, 20 kHz vertical at 48 mA (50 mA attempted) ...................................... 218
A.12 Test 12 – Beam pattern, 30 kHz vertical at 48 mA (50 mA attempted) ...................................... 218
| Appendix A | Test 13 – Beam pattern, 10 kHz vertical at 176 mA (200 mA attempted) | 222 |
| Appendix A | Test 14 – Beam pattern, 20 kHz vertical at 176 mA (200 mA attempted) | 222 |
| Appendix A | Test 15 – Beam pattern, 30 kHz vertical at 176 mA (200 mA attempted) | 222 |
| Appendix A | Test 16 – Sweeps, 5 kHz to 25 kHz at approximately 200 mA | 226 |
| Appendix A | Test 17 – Sweeps, 5 kHz to 25 kHz at 90 mA (100 mA attempted) | 229 |
| Appendix A | Test 18 – Sweeps, 5 kHz to 25 kHz at 189 mA (200 mA attempted) | 235 |
| Appendix A | Test 19 – Sweeps, 5 kHz to 25 kHz at 264 mA (300 mA attempted) | 241 |
| Appendix A | Test 20 – Sweeps, 5 kHz to 25 kHz at 370 mA (400 mA attempted) | 247 |
| Appendix A | Test 21 – Sweeps, 5 kHz to 25 kHz at 323 mA (500 mA attempted) | 253 |

**Appendix B**

Material characterization curves  260

**Appendix C**

Useful derivations  271

C.1 Two-port piezomagnetic magnetomechanical bar analytical model 271
C.2 Piezomagnetic plate 274
C.3 Piezomagnetic bar 280
C.4 Estimation of mechanical losses in a piezomagnetic bar 285
C.5 Eddy current losses in laminated stacks 285
   C.5.1 Representing eddy current losses as an electrical impedance 290
   C.5.2 Representing eddy current losses as a magnetic reluctance 293

**Appendix D**

SPICE netlists and MATLAB scripts  296

D.1 SPICE Netlists 296
   D.1.1 Ideal transformer 296
   D.1.2 Ideal gyrator 296
   D.1.3 Time-differentiating gyrator 297
   D.1.4 Mechanical transmission line 297
   D.1.5 Piezoelectric piece 297
   D.1.6 Piezomagnetic piece 298
   D.1.7 Reluctance with eddy current losses 298
   D.1.8 Piezomagnetic piece with eddy current losses 299
   D.1.9 HUSL nickel transducer model 300
   D.1.10 GCD design model 303
   D.1.11 GCD drive model 306
## List of Figures

1.1 Tetragonal magnetostriction as a function of atomic percent gallium for slow cooled (blue circles) and quenched (red squares) samples, showing a peak magnetostriction of 400 ppm. Only the first peak is useful for transduction. Figure from [1]. ........................................ 6

2.1 Magnetic hysteresis loop. Schematic recreated from [2] and [3]. . . . . . . . . . 25

2.2 Normalized real and imaginary parts of the radiation impedance for a baffled, circular piston. .......................................................... 37

2.3 Schematic representation of magnetostriction, recreated from [4]. Original caption: “Cartoon of changing strain $S$ and magnetic induction $B$ in a magnetostrictive element subjected to a constant compressive stress, $T$. The applied magnetic field $H$ increases from (a) $-H_s$ through (c) $H = 0$ to (e) $H_s$. (Symbols changed for consistancy)” .......................................................... 46

2.4 Cartoon illustrating effect of compressive prestress. Strains shown are greatly exaggerated. Based on schematics by [5] and [6]. . . . . . 51

2.5 Typical strain versus field plot, showing application of unbiased and biased fields with resultant strains. ................................. 52

3.1 Numerically calculating $\pi$ using the finite element method. Notice that the more elements (finer mesh), the more accurate the $\pi$ calculation. ............................................................... 57

3.2 A forced, damped simple harmonic oscillator. ................................. 61

3.3 An LRC circuit. ........................................................................ 61

3.4 A generic two-port element. ...................................................... 68

3.5 A generic model of a magnetostrictive bar transducer. ..................... 69

3.6 An implementation of an ideal transformer SPICE subcircuit .......... 71

3.7 An implementation of an ideal gyrator SPICE subcircuit ............... 72
3.8 Schematic of lumped magnetic reluctances: (a) flux traveling along a constant cross-section, (b) flux traveling across a semi-circular path, (c) flux traveling around a quarter annulus (corner), and (d) flux traveling radially in a annulus. .......................... 74

3.9 An implementation of a modified gyrator SPICE subcircuit for connecting electrical and magnetic reluctance domains using the ddt command. ........................................... 75

3.10 The author’s symbol for representing the two-port coupling piece for connecting the electrical and magnetic reluctance domains in SPICE ...................................................... 75

3.11 Magnetic reluctance with eddy current losses piece implemented in SPICE. The continued fraction approximation has been truncated at the fifteenth term. The netlist for this circuit is included in Appendix D ........................................... 79

3.12 The author’s SPICE symbol for magnetic reluctances with eddy current losses .................................................... 79

3.13 Transmission line symbol in LTspice ................................................. 80

3.14 The author’s symbol for a mechanical transmission line .................. 80

3.15 The piezoelectric piece implemented in SPICE. The netlist for this circuit is included in Appendix D ........................................... 81

3.16 The piezomagnetic piece implemented in SPICE. The netlist for this circuit is included in Appendix D ........................................... 83

3.17 The piezomagnetic piece with eddy current losses (15-term approximation) implemented in SPICE. The netlist for this circuit is included in Appendix D ........................................... 84

3.18 The author’s SPICE symbol for representing the three-port lossy piezomagnetic piece. In this symbol, mechanical ports are at either end of the bar, and the magnetic port is in the middle. .................. 84

3.19 One-dimensional model of the HUSL nickel transducer in SPICE. Parameter statements have been omitted but are contained in Appendix D ........................................... 85

3.20 Electrical input impedance measurement of the HUSL nickel transducer compared to the simulation of the one-dimensional model ........................................... 87

3.21 An elementary circuit model of a magnetostrictive transducer ............... 88

3.22 Simulated electrical input impedance of the the nickel transducer for the full and simple circuit models .................. 89
4.1 The HUSL nickel transducer has 60 laminates, each 0.15 mm thick. Overall, the transducer is 46.6 mm long and 16.5 mm wide with a stack height of 11.7 mm. Each coil has 30 winds and the bias is achieved with what is surmised to be an Alnico magnet.

4.2 A HUSL drawing for a series of magnetic circuits with no biasing mechanism (other than possible biasing from remanent magnetization). The dashed line represents the nodal plane for the first mode of vibration and the white/black dots represent coil winds in/out of the page. Figure recreated from the NDRC report on magnetostriction transducers.

4.3 A HUSL drawing for a magnetic circuit that has a central segment of twice the width of the outside segments. This arrangement gives all segments equal flux density. The dashed line represents the nodal plane for the first mode of vibration and the white/black dots represent coil winds in/out of the page. Figure recreated from the NDRC report on magnetostriction transducers.

4.4 A HUSL drawing for a series of magnetic circuits that are biased by wrapping DC coils around high permeability cores (cross-hatched areas) that are inserted into the laminated structure. The dashed line represents the nodal plane for the first mode of vibration and the white/black dots represent coil winds in/out of the page. Figure recreated from the NDRC report on magnetostriction transducers.

4.5 A simple schematic showing one possible configuration for parallel magnetization in a magnetostrictive transducer.

4.6 A schematic of the dogbone shaped lamination geometry used in the configurable drive. Because of the flexibility of the drive, each realization of the design will have different dimensions, but should retain the basic geometrical features.

4.7 A generic manifestation of the Galfenol configurable drive biased with permanent magnets. Lamination detail is not shown. In this particular drawing, round holes are shown as a concept for lamination alignment during stacking.

4.8 A typical piezoelectric tonpilz transducer shown in cross-section, displaying the piezoelectric stack drive. Note the wiring of the electrodes and the ring polarities. Figure reproduced from Sherman and Butler (With kind permission from Springer Science+Business Media: Transducers and Arrays for Underwater Sound, Chapter 3, 2007, p. 93, C. H. Sherman and J. L. Butler, Figure 3.17).

4.9 A mockup of a tonpilz type transducer using configurable drives for the motor section.
4.10 A cutaway mockup of a flexextensional type transducer incorporating configurable drives.

5.1 $S-B$ and $B-H$ curves for the Galfenol steel reference data (red) showing desired bias point (black circle) and drive swing (black line).

5.2 A reduced design model concept that defines the 2-D geometry as 22 lumped reluctances, all of which can be defined with eight dimensions. Flux paths are illustrated with dashed lines and show leakage across the circuit and around the permanent magnets.

5.3 A one-dimensional model that realizes the reduced design model framework using two independent circuit networks to simulate DC and AC conditions. Parameter statements are not shown, but included in the Appendix D.

5.4 Dimensions from the 1-D lumped reluctance model defines a 3-D geometry that can be input into an FEA physics model.

5.5 Schematic cross section of the GCD’s drive segments (in gray), showing simplified coil modeling using four sheets of current.

5.6 A plot of the magnetic induction, $B_z$, in the center of each drive portion. Because of the loop, the two drive portion experience the field in an opposite spatial directions.

5.7 Anhysteretic fit to measured B-H data for Galfenol steel.

5.8 HUSL Galfenol reproduction transducer being measured with thermal camera. The transducer is suspended on a thin wire.

5.9 Measurement versus simulation for the heatup of the HUSL Galfenol reproduction transducer. The transducer is driven at CW for 90 s and allowed to cool for an additi.

5.10 Simulation of the GCD heat up for 100% (CW) and 50% duty cycles. After 90 s of CW operation and 180 s operation at 50% duty cycle, the transducer temperature does not exceed 45° C.

5.11 Galfenol steel boules to be used in the GCD prototype. On the left is shown the original shipment of six boules. A single boule is shown in detail on the right. A US quarter is shown for scale.

5.12 Part drawing for configurable drive lamination showing top and side views. All dimensions are in millimeters.

5.13 Individual laminations shown on the left with a US nickel and a $\frac{1}{32}" \times \frac{1}{2}" \times \frac{1}{2}"$ NdFeB biasing magnet. On the right a single lamination is shown in detail. These are lamina made to the drawing shown in Figure 5.12.

xvi
5.14 Reproduction of HUSL transducer. On the left are shown three lamina made from rolled Galfenol material. The final transducer is shown on the right in front of the HUSL original.

5.15 Compression and alignment jig created for assembling laminated stacks. The picture on the left shows lamina placed on the base plate with a few alignment pins. On the right the assembled jig is seen.

5.16 Two Galfenol steel laminations – the one on the right has been coated in insulating varnish. Oxide on the uncoated laminations was not removed before applying the varnish. A U.S. nickel is shown for scale.

5.17 Stack assembly jig. Two pieces with sharp right angles are bonded to a plate so that laminations can be aligned using their notch feature. A layer of Teflon® tape is used to prevent the stack from being bonded to the plate.

5.18 Reference B-H data shown with the $\mu_r = 100$ linear approximation used in the one-dimensional design model. Also plotted are the curves for boules D1-9-42-1 and D1-9-39-3, illustrating that the reference data serves as a good average of the two.

5.19 Quasi-static B-H measurements on all nine boules. Least-squares estimates for the second region magnetic permeability are shown for several boules. Because of the similarity in first-region permeability, only one fit is shown to represent all boules. Measurements courtesy of ETREMA Products.

5.20 Measured magnetic permeabilities of laminated stacks as a function of bias field. Maximum permeability occurs at the optimum bias.

5.21 A study to determine what thickness NdFeB would be required to provide 1T bias to the magnetic circuit for a 1/2" × 1/2” cross-section.

5.22 Drive Circuit. Parameter statements are not shown, but included in Appendix D.

5.23 Electrical input impedance for just the Galfenol drive as 1) measured on the device and 2) simulated with the one-dimensional model.

6.1 GCD tonpilz endmasses. On the left is shown the rectangular magnesium headmass; markings on the mating face were used to align the headmass during tonpilz assembly. The rectangular tungsten tailmass is shown on the right.
6.2 The first three modes of the GCD headmass computed with COMSOL Multiphysics ........................................ 148
6.3 One dimensional model for the tonpilz transducer in water. Parameter statements are not shown, but are included in Appendix D ......................................................... 150
6.4 Real and imaginary parts of the normalized radiation impedance for a rectangular piston. Six different aspect ratios, a/b, are shown. ......................................................... 151
6.5 Plot of cavitation threshold, recreated from [9]. Original caption: “Frequency dependence of the cavitation threshold. CW data on fresh water at atmospheric pressure.” The author has added points that show predicted intensities of the GCD tonpilz at different drive amplitudes, derived from the linear one-dimensional model. ......................................................... 153
6.6 The assembled GCD tonpilz transducer. A US nickel is shown for scale ......................................................... 155
6.7 Demagnetization curves for grade N42 neodymium permanent magnets. Curves reproduced from [10]. ......................................................... 157
6.8 Measured electrical input impedance for the GCD drive before and after building it into the tonpilz transducer. ......................................................... 160
6.9 Measured versus simulated electrical input impedance for the GCD tonpilz. ......................................................... 161
6.10 Mounting plate. On the left the GCD tonpilz is shown for scale. Details of the lip, O-ring channel, and a threaded bolt-hole are shown on the right ......................................................... 162
6.11 Transducer headmass held in place with bubble rubber. ......................................................... 163
6.12 Transducer radiating face sealed with a thin layer of urethane. ......................................................... 163
6.13 Acoustic window created by filling the mold with urethane. ......................................................... 164
6.14 Finished acoustic window with sanded face. ......................................................... 164
6.15 Transducer support brackets. ......................................................... 165
6.16 Additional test instrumentation. The picture on the left shows thermocouple placement. On the right is the leak detector. ......................................................... 165
6.17 Shell assembly. The shell attached to the mounting plate is shown on the left. On the right is the bulkhead. Two thermocouple modules are resting on the bulkhead: one is in a grey plastic case and the other has been removed from its case and is used as just a bare circuit board ......................................................... 166
6.18 Transducer housing assembly in 0° roll-plane orientation, ready for testing. ......................................................... 167
6.19 Measured versus simulated results for the GCD tonpilz in water. Shown here are the transmitting current response, impedance magnitude, and impedance phase. ......................................................... 168
6.20 Measured versus simulated results for the GCD tonpilz in water with $s_3$ set to 18 TPa$^{-1}$ in the one-dimensional model. Shown here are the transmitting current response, impedance magnitude, and impedance phase.

6.21 Measured versus simulated TCR curves for the GCD tonpilz in water comparing the one-dimensional and finite element modeling techniques. The FEA model is shown on the right for a transient analysis.

6.22 In-water and in-air electrical input impedance circles for the GCD tonpilz. Values for the diameter in water, $D_w$, and the diameter in air, $D_a$, are shown.

6.23 30 kHz beam pattern simulation in COMSOL Multiphysics. The quantity plotted is sound pressure level (SPL). In this picture, the radiating face is centered on the upper edge of the rectangular cutout.

6.24 Horizontal and vertical beam patterns for the GCD tonpilz. Simulated patterns are shown in red, measured data are in blue. All patterns have been scaled at 0 dB at 0$^\circ$, except for the vertical pattern at 30 kHz which is set to -10 dB at 0$^\circ$. Data measured at 48 mA drive current.

6.25 In-water electrical input impedance measurements of the GCD tonpilz before and after suspected damage.

6.26 In-air electrical input impedance for the first and second assembly of the GCD tonpilz.

6.27 TCR curves for the GCD tonpilz at varying drive levels.

6.28 Source level at resonance as a function drive level.

6.29 Total harmonic distortion as a function drive level.

6.30 TCR and volt-amps as a function of frequency for the 323 mA drive level.

6.31 Transducer strain as a function of frequency for the various drive levels.

6.32 Cavitation. A profile of the housing is shown on the left with cavitation bubbles visible on the face of the acoustic window. Pictured on the right is the setup for observing cavitation.

A.1 GCD tonpilz ATF testing - transmitting power response, 15 kHz to 100 kHz at 48 mA (50 mA attempted).

A.2 GCD tonpilz ATF testing - transmitting voltage response, 15 kHz to 100 kHz at 48 mA (50 mA attempted).

A.3 GCD tonpilz ATF testing - transmitting current response, 15 kHz to 100 kHz at 48 mA (50 mA attempted).
<table>
<thead>
<tr>
<th>Section</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.4</td>
<td>GCD tonpilz ATF testing - impedance magnitude, 15 kHz to 100 kHz at 48 mA (50 mA attempted)</td>
</tr>
<tr>
<td>A.5</td>
<td>GCD tonpilz ATF testing - impedance phase, 15 kHz to 100 kHz at 48 mA (50 mA attempted)</td>
</tr>
<tr>
<td>A.6</td>
<td>GCD tonpilz ATF testing - transmitting power response, 5 kHz to 20 kHz at 48 mA (50 mA attempted)</td>
</tr>
<tr>
<td>A.7</td>
<td>GCD tonpilz ATF testing - transmitting voltage response, 5 kHz to 20 kHz at 48 mA (50 mA attempted)</td>
</tr>
<tr>
<td>A.8</td>
<td>GCD tonpilz ATF testing - transmitting current response, 5 kHz to 20 kHz at 48 mA (50 mA attempted)</td>
</tr>
<tr>
<td>A.9</td>
<td>GCD tonpilz ATF testing - impedance magnitude, 5 kHz to 20 kHz at 48 mA (50 mA attempted)</td>
</tr>
<tr>
<td>A.10</td>
<td>GCD tonpilz ATF testing - impedance phase, 5 kHz to 20 kHz at 48 mA (50 mA attempted)</td>
</tr>
<tr>
<td>A.11</td>
<td>GCD tonpilz ATF testing - beam pattern, 10 kHz horizontal at 48 mA (50 mA attempted)</td>
</tr>
<tr>
<td>A.12</td>
<td>GCD tonpilz ATF testing - beam pattern, 20 kHz horizontal at 48 mA (50 mA attempted)</td>
</tr>
<tr>
<td>A.13</td>
<td>GCD tonpilz ATF testing - beam pattern, 30 kHz horizontal at 48 mA (50 mA attempted)</td>
</tr>
<tr>
<td>A.14</td>
<td>GCD tonpilz ATF testing - transmitting power response, 5 kHz to 25 kHz at 176 mA (200 mA attempted)</td>
</tr>
<tr>
<td>A.15</td>
<td>GCD tonpilz ATF testing - transmitting voltage response, 5 kHz to 25 kHz at 176 mA (200 mA attempted)</td>
</tr>
<tr>
<td>A.16</td>
<td>GCD tonpilz ATF testing - transmitting current response, 5 kHz to 25 kHz at 176 mA (200 mA attempted)</td>
</tr>
<tr>
<td>A.17</td>
<td>GCD tonpilz ATF testing - impedance magnitude, 5 kHz to 25 kHz at 176 mA (200 mA attempted)</td>
</tr>
<tr>
<td>A.18</td>
<td>GCD tonpilz ATF testing - impedance phase, 5 kHz to 25 kHz at 176 mA (200 mA attempted)</td>
</tr>
<tr>
<td>A.19</td>
<td>GCD tonpilz ATF testing - beam pattern, 10 kHz horizontal at 176 mA (200 mA attempted)</td>
</tr>
<tr>
<td>A.20</td>
<td>GCD tonpilz ATF testing - beam pattern, 20 kHz horizontal at 176 mA (200 mA attempted)</td>
</tr>
<tr>
<td>A.21</td>
<td>GCD tonpilz ATF testing - beam pattern, 30 kHz horizontal at 176 mA (200 mA attempted)</td>
</tr>
<tr>
<td>A.22</td>
<td>GCD tonpilz ATF testing - beam pattern, 10 kHz vertical at 48 mA (50 mA attempted)</td>
</tr>
<tr>
<td>A.23</td>
<td>GCD tonpilz ATF testing - beam pattern, 20 kHz vertical at 48 mA (50 mA attempted)</td>
</tr>
<tr>
<td>A.24</td>
<td>GCD tonpilz ATF testing - beam pattern, 30 kHz vertical at 48 mA</td>
</tr>
<tr>
<td>------</td>
<td>---------------------------------------------------------------</td>
</tr>
<tr>
<td></td>
<td>(50 mA attempted)</td>
</tr>
<tr>
<td></td>
<td>221</td>
</tr>
<tr>
<td>A.25</td>
<td>GCD tonpilz ATF testing - beam pattern, 10 kHz vertical at 176 mA</td>
</tr>
<tr>
<td></td>
<td>(200 mA attempted)</td>
</tr>
<tr>
<td></td>
<td>223</td>
</tr>
<tr>
<td>A.26</td>
<td>GCD tonpilz ATF testing - beam pattern, 20 kHz vertical at 176 mA</td>
</tr>
<tr>
<td></td>
<td>(200 mA attempted)</td>
</tr>
<tr>
<td></td>
<td>224</td>
</tr>
<tr>
<td>A.27</td>
<td>GCD tonpilz ATF testing - beam pattern, 30 kHz vertical at 176 mA</td>
</tr>
<tr>
<td></td>
<td>(200 mA attempted)</td>
</tr>
<tr>
<td></td>
<td>225</td>
</tr>
<tr>
<td>A.28</td>
<td>GCD tonpilz ATF testing - impedance magnitude, 5 kHz to 25 kHz at approximately 200 mA</td>
</tr>
<tr>
<td></td>
<td>227</td>
</tr>
<tr>
<td>A.29</td>
<td>GCD tonpilz ATF testing – impedance phase, 5 kHz to 25 kHz at approximately 200 mA</td>
</tr>
<tr>
<td></td>
<td>228</td>
</tr>
<tr>
<td>A.30</td>
<td>GCD tonpilz ATF testing - transmitting power response, 5 kHz to 25 kHz at 90 mA (100 mA attempted)</td>
</tr>
<tr>
<td></td>
<td>230</td>
</tr>
<tr>
<td>A.31</td>
<td>GCD tonpilz ATF testing - transmitting voltage response, 5 kHz to 25 kHz at 90 mA (100 mA attempted)</td>
</tr>
<tr>
<td></td>
<td>231</td>
</tr>
<tr>
<td>A.32</td>
<td>GCD tonpilz ATF testing - transmitting current response, 5 kHz to 25 kHz at 90 mA (100 mA attempted)</td>
</tr>
<tr>
<td></td>
<td>232</td>
</tr>
<tr>
<td>A.33</td>
<td>GCD tonpilz ATF testing - impedance magnitude, 5 kHz to 25 kHz at 90 mA (100 mA attempted)</td>
</tr>
<tr>
<td></td>
<td>233</td>
</tr>
<tr>
<td>A.34</td>
<td>GCD tonpilz ATF testing - impedance phase, 5 kHz to 25 kHz at 90 mA (100 mA attempted)</td>
</tr>
<tr>
<td></td>
<td>234</td>
</tr>
<tr>
<td>A.35</td>
<td>GCD tonpilz ATF testing - transmitting power response, 5 kHz to 25 kHz at 189 mA (200 mA attempted)</td>
</tr>
<tr>
<td></td>
<td>236</td>
</tr>
<tr>
<td>A.36</td>
<td>GCD tonpilz ATF testing - transmitting voltage response, 5 kHz to 25 kHz at 189 mA (200 mA attempted)</td>
</tr>
<tr>
<td></td>
<td>237</td>
</tr>
<tr>
<td>A.37</td>
<td>GCD tonpilz ATF testing - transmitting current response, 5 kHz to 25 kHz at 189 mA (200 mA attempted)</td>
</tr>
<tr>
<td></td>
<td>238</td>
</tr>
<tr>
<td>A.38</td>
<td>GCD tonpilz ATF testing - impedance magnitude, 5 kHz to 25 kHz at 189 mA (200 mA attempted)</td>
</tr>
<tr>
<td></td>
<td>239</td>
</tr>
<tr>
<td>A.39</td>
<td>GCD tonpilz ATF testing - impedance phase, 5 kHz to 25 kHz at 189 mA (200 mA attempted)</td>
</tr>
<tr>
<td></td>
<td>240</td>
</tr>
<tr>
<td>A.40</td>
<td>GCD tonpilz ATF testing - transmitting power response, 5 kHz to 25 kHz at 264 mA (300 mA attempted)</td>
</tr>
<tr>
<td></td>
<td>242</td>
</tr>
<tr>
<td>A.41</td>
<td>GCD tonpilz ATF testing - transmitting voltage response, 5 kHz to 25 kHz at 264 mA (300 mA attempted)</td>
</tr>
<tr>
<td></td>
<td>243</td>
</tr>
<tr>
<td>A.42</td>
<td>GCD tonpilz ATF testing - transmitting current response, 5 kHz to 25 kHz at 264 mA (300 mA attempted)</td>
</tr>
<tr>
<td></td>
<td>244</td>
</tr>
<tr>
<td>A.43</td>
<td>GCD tonpilz ATF testing - impedance magnitude, 5 kHz to 25 kHz at 264 mA (300 mA attempted)</td>
</tr>
<tr>
<td></td>
<td>245</td>
</tr>
<tr>
<td>A.44</td>
<td>GCD tonpilz ATF testing - impedance phase, 5 kHz to 25 kHz at 264 mA (300 mA attempted)</td>
</tr>
<tr>
<td>A.45</td>
<td>GCD tonpilz ATF testing - transmitting power response, 5 kHz to 25 kHz at 370 mA (400 mA attempted)</td>
</tr>
<tr>
<td>A.46</td>
<td>GCD tonpilz ATF testing - transmitting voltage response, 5 kHz to 25 kHz at 370 mA (400 mA attempted)</td>
</tr>
<tr>
<td>A.47</td>
<td>GCD tonpilz ATF testing - transmitting current response, 5 kHz to 25 kHz at 370 mA (400 mA attempted)</td>
</tr>
<tr>
<td>A.48</td>
<td>GCD tonpilz ATF testing - impedance magnitude, 5 kHz to 25 kHz at 370 mA (400 mA attempted)</td>
</tr>
<tr>
<td>A.49</td>
<td>GCD tonpilz ATF testing - impedance phase, 5 kHz to 25 kHz at 370 mA (400 mA attempted)</td>
</tr>
<tr>
<td>A.50</td>
<td>GCD tonpilz ATF testing - transmitting power response, 5 kHz to 25 kHz at 323 mA (500 mA attempted)</td>
</tr>
<tr>
<td>A.51</td>
<td>GCD tonpilz ATF testing - transmitting voltage response, 5 kHz to 25 kHz at 323 mA (500 mA attempted)</td>
</tr>
<tr>
<td>A.52</td>
<td>GCD tonpilz ATF testing - transmitting current response, 5 kHz to 25 kHz at 323 mA (500 mA attempted)</td>
</tr>
<tr>
<td>A.53</td>
<td>GCD tonpilz ATF testing - impedance magnitude, 5 kHz to 25 kHz at 323 mA (500 mA attempted)</td>
</tr>
<tr>
<td>A.54</td>
<td>GCD tonpilz ATF testing - impedance phase, 5 kHz to 25 kHz at 323 mA (500 mA attempted)</td>
</tr>
</tbody>
</table>

<p>| B.1 | ETREMA Products, Inc. measurement - Boule D1-9-40-76. S-B and B-H curves are shown both before and after stress annealing as a function of preload. | 261 |
| B.2 | ETREMA Products, Inc. measurement - Boule D1-9-42-153. S-B and B-H curves are shown both before and after stress annealing as a function of preload. | 262 |
| B.3 | ETREMA Products, Inc. measurement - Boule D1-9-42-1. S-B and B-H curves are shown both before and after stress annealing as a function of preload. | 263 |
| B.4 | ETREMA Products, Inc. measurement - Boule D1-9-40-1. S-B and B-H curves are shown both before and after stress annealing as a function of preload. | 264 |
| B.5 | ETREMA Products, Inc. measurement - Boule D1-9-40-153. S-B and B-H curves are shown both before and after stress annealing as a function of preload. | 265 |</p>
<table>
<thead>
<tr>
<th>Section</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>B.6</td>
<td>ETREMA Products, Inc. measurement - Boule D1-9-42-76. S-B and B-H curves are shown both before and after stress annealing as a function of preload.</td>
<td>266</td>
</tr>
<tr>
<td>B.7</td>
<td>ETREMA Products, Inc. measurement - Boule D1-9-39-3. S-B and B-H curves are shown both before and after stress annealing as a function of preload.</td>
<td>267</td>
</tr>
<tr>
<td>B.8</td>
<td>ETREMA Products, Inc. measurement - Boule D1-9-39-78. S-B and B-H curves are shown both before and after stress annealing as a function of preload.</td>
<td>268</td>
</tr>
<tr>
<td>B.9</td>
<td>ETREMA Products, Inc. measurement - Boule D1-9-39-153. S-B and B-H curves are shown both before and after stress annealing as a function of preload.</td>
<td>269</td>
</tr>
<tr>
<td>C.1</td>
<td>Example outputs of the two-port model for several different electromagnetic coupling coefficients.</td>
<td>274</td>
</tr>
<tr>
<td>C.2</td>
<td>Cauer circuit.</td>
<td>291</td>
</tr>
</tbody>
</table>
## List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Mechanics notation (follows [11])</td>
<td>32</td>
</tr>
<tr>
<td>3.1</td>
<td>Components, symbols, variables, and units for selected impedance analogies</td>
<td>64</td>
</tr>
<tr>
<td>4.1</td>
<td>Room temperature properties of magnetostrictive materials</td>
<td>93</td>
</tr>
<tr>
<td>5.1</td>
<td>Comparison of permanent magnet materials. Alnico values were retrieved from [12] and NdFeB properties from [10]. Recoil permeabilities are relative. Maximum energy product values are also shown for comparison.</td>
<td>109</td>
</tr>
<tr>
<td>5.2</td>
<td>Galfenol magnetic circuit performance calculated with the design model</td>
<td>114</td>
</tr>
<tr>
<td>5.3</td>
<td>Values for the B-H model fit shown in Figure 5.7</td>
<td>120</td>
</tr>
<tr>
<td>5.4</td>
<td>Summary of wire EDM machining on stress annealed highly-textured polycrystalline Galfenol steel boules</td>
<td>127</td>
</tr>
<tr>
<td>5.5</td>
<td>Details of lamination assignment and stack consolidation</td>
<td>133</td>
</tr>
<tr>
<td>6.1</td>
<td>Real and imaginary parts of normalized radiation impedance values calculated for a rectangular piston with an aspect ratio of 1.5.</td>
<td>152</td>
</tr>
<tr>
<td>6.2</td>
<td>Plane-wave intensities calculated with the one-dimensional model</td>
<td>154</td>
</tr>
<tr>
<td>6.3</td>
<td>Comparison of published single-element magnetostrictive sonar transducers that are biased with permanent magnets</td>
<td>183</td>
</tr>
</tbody>
</table>
## List of Symbols

- **$A$** Area, p. 31
- **$A_m$** Cross-sectional area of the permanent magnet, p. 77
- **$a$** Piston radius, p. 36
- **$B$** Magnetic induction also called the magnetic flux density, p. 19
- **$B_{pk}$** Peak magnetic induction, p. 30
- **$B_r$** Remanent induction, p. 24
- **$B_s$** Saturation induction, p. 24
- **$C$** Generalized compliance variable, p. 63
- **$C_e$** Electrical capacitance, p. 61
- **$C_m$** Mechanical compliance, p. 60
- **$c$** Speed of sound, p. 35
- **$c$** Elastic stiffness, p. 16
- **$\chi$** Magnetic susceptibility, p. 23
- **$\chi$** $B$-$H$ model parameter, p. 118
- **$D$** Electric displacement field, p. 19
- **$D$** Diameter, p. 28
- **$D_a$** Impedance circle diameter in-air, p. 170
$D_w$ Impedance circle diameter in-water, p. 170

d Magnetostrictive coefficient, p. 48

d Laminate thickness, p. 77

\(\delta\) Eddy current penetration depth, p. 28

\(E\) Electric field strength, p. 19

\(E\) Elastic modulus, p. 33

\(\mathcal{E}\) Energy, p. 44

\(e\) Piezomagnetic $e$ parameter, p. 49

\(e\) Generalized effort variable, p. 63

\(\eta\) Efficiency, p. 170

\(\epsilon\) Back electromotive force (emf), p. 139

\(\varepsilon_0\) Permittivity of free space, p. 19

\(\varepsilon^S\) Permittivity of piezoelectric material at constant $S$, p. 81

\(F\) Force, p. 31

\(f\) Frequency, p. 15

\(f\) Generalized flow variable, p. 63

\(f_1\) Lower half-power frequency, p. 35

\(f_2\) Upper half-power frequency, p. 35

\(f_a\) Antiresonance frequency, p. 44

\(f_c\) Eddy current critical frequency, p. 30

\(f_r\) Resonance frequency, p. 35

\(f_n\) Resonance frequency at $n$-th mode, p. 35

FOM Figure of merit, p. 13

\(\Phi\) Magnetomotive force (mmf), p. 42

xxvi
$\mathcal{F}_c$  Coercive magnetomotive force, p. 77
$g$  Piezomagnetic $g$ parameter, p. 48
$g$  Piezoelectric $g$ parameter, p. 81
$\gamma$  Piezomagnetic inverse permeability parameter, p. 48
$H$  Magnetic field strength, p. 19
$H_0$  Applied magnetic field, p. 31
$H_C$  $B$-$H$ model corner location parameter, p. 118
$H_c$  Coercive field, p. 24
$H_d$  Demagnetizing field, p. 31
$H_{\text{int}}$  Internal magnetic field of a sample, p. 31
$H_s$  Saturation field, p. 46
$h$  Piezomagnetic $h$ parameter, p. 49
$I$  Generalized inertance variable, p. 63
$I$  Acoustic intensity, p. 152
$i$  Electrical current, p. 20
$i_{\text{in}}$  Input electrical current, p. 154
$J$  Current density, p. 19
$j$  Imaginary number, $\sqrt{-1}$, p. 36
$k$  Wavenumber, p. 36
$k$  Coupling coefficient, p. 42
$k_{33}$  Magnetomechanical coupling coefficient, p. 42
$k_c$  Electromagnetic (coil) coupling coefficient, p. 42
$k_{\text{eff}}$  Effective electromechanical coupling coefficient, p. 44
$k_{\text{em}}$  Electromechanical coupling coefficient, p. 42
$k_m$ Mechanical stiffness, p. 33

$k_m^H$ Mechanical stiffness of magnetostrictive material at constant $H$, p. 273

$ka$ Helmholtz number, p. 36

$L$ Electrical inductance, p. 61

$\ell$ Length, p. 20

$\lambda$ Wavelength, p. 35

$\lambda$ Magnetostriction, p. 45

$\lambda_s$ Saturation magnetostriction, p. 45

$M$ Magnetization, p. 31

$m$ Mass, p. 15

$m$ Slope, p. 118

$\mu$ Magnetic permeability, p. 21

$\mu_0$ Permeability of free space, p. 21

$\mu_r$ Relative permeability, p. 21

$\mu_{rec}$ Recoil permeability, p. 109

$\mu^T$ Incremental permeability at constant $T$, p. 48

$\mu^S$ Incremental permeability at constant $S$, p. 48

$N_d$ Demagnetizing factor, p. 31

$N_L$ Number of laminations, p. 77

$n$ Turn count, p. 28

$n$ Mode number, p. 35

$\omega$ Angular frequency, p. 37

$\omega_0$ Angular resonance frequency, p. 143

$p$ Pressure, p. 42
$p_{pk}$ Peak pressure, p. 152

$p_{ref}$ Pressure reference, p. 154

$p$ Permeance, p. 75

$\Pi$ Power, p. 37

$\phi$ Elevation angle, p. 154

$\phi_r$ Transduction coefficient, p. 279

$\varphi$ Magnetic flux, p. 42

$\varphi_r$ Remanent flux, p. 77

$\dot{\varphi}$ Magnetic flux rate, p. 72

$\varnothing$ Diameter symbol, p. 123

$Q$ Quality factor, p. 35

$Q_m$ Mechanical quality factor, p. 15

$\dot{Q}_{ec}$ Heat generation arising from eddy current losses, p. 30

$R$ Vector from element to point of interest, p. 20

$R$ Generalized resistance variable, p. 63

$R_0$ Electrical resistance at resonance, p. 170

$R_1$ Real part of the normalized radiation impedance, p. 36

$R_e$ Electrical resistance, the real part of the electrical impedance, p. 28

$R_m$ Mechanical resistance, p. 60

$R_{rad}^m$ Radiation resistance, p. 37

$r$ Radial distance, p. 20

$\Re$ Magnetic reluctance, p. 73

$\Re_m$ Magnetic reluctance of the permanent magnet, p. 77

$\rho$ Mass density, p. 36

xxix
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varrho$</td>
<td>Electric charge density</td>
<td>19</td>
</tr>
<tr>
<td>$S$</td>
<td>Strain</td>
<td>16</td>
</tr>
<tr>
<td>$s$</td>
<td>Elastic compliance</td>
<td>47</td>
</tr>
<tr>
<td>$s_B^E$</td>
<td>Elastic compliance at constant $B$</td>
<td>47</td>
</tr>
<tr>
<td>$s_D^E$</td>
<td>Elastic compliance in piezoelectric material at constant $D$</td>
<td>81</td>
</tr>
<tr>
<td>$s_H^E$</td>
<td>Elastic compliance at constant $H$</td>
<td>47</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Electrical conductivity</td>
<td>29</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Poisson’s ratio</td>
<td>34</td>
</tr>
<tr>
<td>$T$</td>
<td>Stress</td>
<td>31</td>
</tr>
<tr>
<td>$T_C$</td>
<td>Curie temperature</td>
<td>23</td>
</tr>
<tr>
<td>$T_{em}$</td>
<td>Transduction coefficient, electrical-to-mechanical</td>
<td>39</td>
</tr>
<tr>
<td>$T_{me}$</td>
<td>Transduction coefficient, mechanical-to-electrical</td>
<td>39</td>
</tr>
<tr>
<td>$t$</td>
<td>Time</td>
<td>19</td>
</tr>
<tr>
<td>$t$</td>
<td>Thickness</td>
<td>30</td>
</tr>
<tr>
<td>$\tau_d$</td>
<td>Time delay</td>
<td>79</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Azimuth angle</td>
<td>154</td>
</tr>
<tr>
<td>$U$</td>
<td>Volume velocity</td>
<td>42</td>
</tr>
<tr>
<td>$V$</td>
<td>Volume</td>
<td>16</td>
</tr>
<tr>
<td>$V$</td>
<td>Voltage</td>
<td>39</td>
</tr>
<tr>
<td>$v$</td>
<td>Velocity</td>
<td>37</td>
</tr>
<tr>
<td>$v_t^B$</td>
<td>Sound speed at constant $B$</td>
<td>82</td>
</tr>
<tr>
<td>$v_t^D$</td>
<td>Sound speed in piezoelectric material at constant $D$</td>
<td>81</td>
</tr>
<tr>
<td>$w$</td>
<td>Width</td>
<td>78</td>
</tr>
<tr>
<td>$X_1$</td>
<td>Imaginary part of the normalized radiation impedance</td>
<td>36</td>
</tr>
</tbody>
</table>

xxx
\( x \) Displacement, p. 33

\( \xi \) B-H model corner sharpness parameter, p. 118

\( Z_0 \) Characteristic impedance, p. 66

\( Z_e \) Electrical impedance, p. 39

\( Z_{ee} \) Electrical input impedance, p. 273

\( Z_{\text{rad}}^a \) Acoustical radiation impedance, p. 36

\( Z_{\text{rad}}^m \) Mechanical radiation impedance, p. 36

\( z_m \) Mechanical impedance, p. 39

\( z_L \) Mechanical load impedance, p. 273

\( Z_{\text{mot}} \) Motional impedance, p. 273

\( Z_{\text{m}} \) Complex magnetic reluctance, p. 78
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Chapter 1

Introduction and objectives

1.1 Magnetostriction

Magnetostriction is the change in a material’s dimensions resulting from a change in its magnetic state, or *vice versa*. The deformation is extremely small, with strains on the order of tens of parts-per-million in elemental magnetostrictives and over a thousand of parts-per-million for giant magnetostrictives. In most applications, the change in shape is too small to be observed with the naked eye or detected through touch. For this reason, it is probable that magnetostriction was first detected by its audible effects. In fact, the reader is probably already acquainted with this phenomenon — magnetostriction is the cause of the familiar “hum” produced by electrical transformers.

1.1.1 Sonar

Locating underwater objects is a difficult task. Visible light is quickly attenuated, rendering it practically useless for any sort of long-range detection. Likewise, magnetic detection of metal objects is also extremely limited in range. Fortunately, water is an excellent fluid for the propagation of sound.

To take advantage of this fact, underwater echo-location is used. Electroacoustic transducers are first operated as sound projectors to emit acoustic energy into the water and then switch to receive mode to detect the echoes. If a reflective object is nearby, the sound will be returned. This system is known as *active sonar*. 
When the object ensonifies the water of its own accord, transducers can locate it just by “listening” – this is called *passive sonar*.

Sonar was originally an acronym standing for **S**ound **N**avigation **A**nd **R**anging, following the use of RADAR (also no longer an acronym). It is “bio-inspired,” being found in nature: bats and dolphins use these echo-ranging techniques with great accuracy and discrimination. Even blind humans have acquired the ability to use clicking noises to navigate around furniture and other obstacles [13].

However, even with sonar equipment, underwater detection is a daunting task. For perspective, Gannon [14] cites a 1958 GE publication that compares sonar to trying to find a black object suspended in the middle of a completely dark room that has small mirrors covering the ceiling and floor using only a flashlight with a low battery.

Clearly, improved transduction techniques are necessary for better detection. Detection can be enhanced through higher radiated power and wider bandwidth. This dissertation will address a newly introduced magnetostrictive material called Galfenol and investigate its application to sonar systems.

### 1.2 Historical background

Understanding the significance of Galfenol requires a historical review of magnetostriction and its relation to piezoelectricity. A summary of this history is presented here, starting with the origins of magnetostriction and ending with the discovery of Galfenol.

#### 1.2.1 Origins

A number of early experimenters investigated the magnetostrictive effect with little in the way of substantial evidence [15] [16]. As a result, it is James Prescott Joule’s authoritative studies in the 1840s that are generally credited with the conclusive discovery of the direct magnetostrictive effect, i.e. a deformation resulting from a change in the material’s magnetic state. It was not until the 1860s, however, that Villari proved the existence of the reverse effect, namely that altering the material’s magnetic state can be accomplished by deforming the sample. Today,
the direct effect is known as *Joule magnetostriction* and the reverse phenomenon is called the *Villari effect*. In his book on acoustics, Raichel [17] heralds Joule’s work on magnetostriction as the genesis of ultrasonic acoustics.

In 1880, Jacques and Pierre Curie discovered the piezoelectric effect [16]. This development would have an important bearing on the course of magnetostriction.

Full details of the early history of magnetostriction are well-recorded in McKeehan [15] and Hunt [16].

### 1.2.2 Work at Harvard University

Magnetostrictive materials became aligned with underwater transduction applications largely through work conducted at Harvard University. This work may be conveniently divided into two phases: first the peace-time research of Professor Pierce, and second the war efforts of the Harvard Underwater Sound Laboratory.

#### 1.2.2.1 Early work by G. W. Pierce

Application of magnetostriction to underwater acoustics was largely pioneered by G. W. Pierce at Harvard University in the late 1920s. Pierce was motivated to find a practical use after reading McKeehan’s [15] observation that “no important application of the fact of magnetostriction... has yet been made” [18]. In between the World Wars, Professor Piece developed several types of underwater magnetostrictive transducers and was so successful that at the beginning of World War II, nearly all anti-submarine U.S. warships were equipped with nickel magnetostrictive projectors for echolocation [18]. His notes formed the basis for the magnetostrictive transducers developed during World War II at the Harvard Underwater Sound Laboratory (HUSL) [7 14].

#### 1.2.2.2 The Harvard Underwater Sound Laboratory

During World War II, magnetostrictive transducers were developed and researched at HUSL, from 1941 to 1945 under the direction of F. V. Hunt [19 14]. This laboratory was organized by the National Defense Research Committee, a government initiative to incorporate civilian scientists and engineers into the war effort [7 14].

---

1Hunt pronounced the HUSL acronym as “hustle” [14].
At the start of World War II, America’s torpedo technology had grown woefully stagnant and obsolete, and HUSL, working in parallel with Bell Laboratories, is credited with a major success in developing America’s first acoustic homing torpedo, the air-dropped “Fido” Mark-24 Mine. Even though the final version of the Mark-24 design used Bell Laboratory’s piezoelectric transducers, a later version using HUSL’s nickel magnetostrictive transducers was eventually introduced as the Mark-34.

Once the war had ended, Harvard decided against supporting a military laboratory on its campus. Thus HUSL closed and half of the personnel transferred to the Navy Underwater Sound Laboratory in New London, Connecticut and the other half to the newly-established Ordnance Research Laboratory at the Pennsylvania State University under the direction of Dr. Eric Walker. A much smaller group from HUSL established the Defense Research Laboratory at the University of Texas at Austin. Since this time, all three institutions have been renamed: they are now known as the Naval Undersea Warfare Center, the Applied Research Laboratory, and the Applied Research Laboratories, respectively.

1.2.3 Competition from piezoelectric crystals

Quartz and Rochelle salt were the leading materials for the first generation of piezoelectrics. Nickel was regarded as being competitive with these materials because what it lacked in performance, it made up in ruggedness and durability.

In the 1940s and 1950s, the development of poled piezoelectric ceramics, most notably barium titanate, BaTiO$_3$, and lead zirconate titanate (PZT), Pb$(Zr_xTi_{1-x})O_3$, signaled a turning point in the course of active materials. With these ceramics, piezoelectric materials demonstrated clear advantages over magnetostrictives in both strain capability and electromechanical coupling coefficient. In addition, design complications inherent to magnetic devices further favored the piezoelectric approach. The combination of these factors led to the piezoelectric technology being widely preferred over magnetostriction, a situation that persists to the present day.

\footnote{Because of secrecy concerns, the Mark-24 was designated as a mine to confuse enemy intelligence.}
1.2.4 Discovery of giant magnetostrictive materials

In the 1960s, certain compounds were observed to exhibit very large magnetostrictive strains at cryogenic temperatures. In 1970, Clark, et al., discovered a new material called Terfenol-D, Tb\textsubscript{0.3}Dy\textsubscript{0.7}Fe\textsubscript{1.92}, that produced “giant” magnetostriction at room temperature.\(^3\) It is capable of regularly producing strains in excess of 1200 ppm. However, Terfenol is a material of extremes — its huge strain capability is tempered by acute mechanical brittleness and very low magnetic permeability. Subsequent work on Terfenol suggests that it may be used very effectively in specialized applications, but that it will probably not replace the widespread use of piezoelectric transducers. There are many cases in which a magnetostrictive material with a more balanced set of properties would be desirable.

1.2.5 Galfenol

In 1999, Clark et al.\(^1\)\(^23\) discovered large magnetostriction in iron-gallium alloys, Fe\textsubscript{100-x}Ga\textsubscript{x}, called Galfenol.\(^7\) The magnetostriction of Galfenol as a function of gallium content is shown in Figure 1.1. Although Galfenol’s strain is only a fraction of Terfenol’s (250-400 ppm compared to 1200 ppm), it possesses other favorable characteristics related to mechanical strength and magnetic performance. Thus, Galfenol occupies a middle ground between the traditional magnetostrictives with high strength and low strains and Terfenol-D which has low strength and high strain. In addition, the raw material cost for Galfenol is about $0.08/g compared to approximately $0.50/g for Terfenol-D.\(^{24}\)

Galfenol has been grown as both single crystal and polycrystal. While single crystal has higher strains, the polycrystalline form is mechanically stronger and is faster to manufacture. Different degrees of texturing of the polycrystals are available based on the crystal growth rate.\(^{25}\)

Galfenol’s mechanical strength allows it to be machined and welded. Piezoelectric materials and Terfenol-D have very low tensile strengths which necessitates the use of a stress bolt to apply a stress bias to the material to prevent the material from cracking.

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\(^3\)The name Terfenol-D stands for Terbium, Iron (element symbol Fe), the Naval Ordnance Laboratory (the facility where Terfenol was invented), and Dysprosium.

\(^4\)Like Terfenol-D, the name Galfenol stands for Gallium, Iron (Fe), and the Naval Ordnance Laboratory.
Figure 1.1. Tetragonal magnetostriction as a function of atomic percent gallium for slow cooled (blue circles) and quenched (red squares) samples, showing a peak magnetostriction of 400 ppm. Only the first peak is useful for transduction. Figure from [1].

from experiencing tension during operation; the mechanical strength of Galfenol allows it to operate in tension, so no stress bolt is required. The strength of Galfenol also allows it be used as a structural member. From a transducer design standpoint, the durability and machinability of Galfenol provides access to a design space that has traditionally been inaccessible to other active materials.

Another strong attribute of Galfenol is its high magnetic permeability that makes it easier to get magnetic flux into the material – saturation magnetostriction may be achieved with moderate fields (about 30 kA/m for binary Galfenol, approximately 15 kA/m for Galfenol steel). This leads to low material reluctances that allows the drive and bias fields to be generated in a different part of the mag-
netic circuit and not directly at the Galfenol element. One downside to the high magnetic permeability is that it leads to smaller eddy current penetration depths that require finer lamination of the material for effective mitigation.

1.3 Literature review

In this section, the literature dealing with Galfenol is surveyed and is divided into four main parts: first, investigations into the material properties of Galfenol; second, methods used to characterize Galfenol samples; third, transducers that are built around Galfenol as the active material; and fourth, efforts to model Galfenol-based devices.

1.3.1 Galfenol material studies

The majority of work on Galfenol has gone into understanding the material itself. Top priorities for this research are 1) to obtain material properties of Galfenol and 2) to maximize strain capability.

1.3.1.1 Discovery and early observations

Large magnetostriction in iron-gallium alloys, Fe_{100-x}Ga_x (Galfenol), was discovered by Clark et al. in 1999 [1]. The initial publications in 2000 reported magnetostriction with strain values in excess of 200 ppm for polycrystalline and 300 ppm for single crystal and with low temperature dependence [26, 27, 23]. These initial studies explored the magnetostriction as a function of gallium content, compressive stress, and temperature. The material exhibits two peaks in magnetostriction (shown in Figure [1,1]): one at 18 at. % gallium and the other at 27%. Also noted in these studies was Galfenol’s comparatively low raw material cost. Early work done by Guruswamy et al. [28], lists Galfenol’s attractive features: high strength, good ductility, negligible hysteresis losses, and attractive magnetostrictive properties. As Galfenol is the first magnetostrictive material to possess large strain capability and high mechanical strength, the authors of [28] concluded that the new material might “be especially suited for high-power transduction and actuator applications, particularly those that are employed in abusive or explosive environments.”
1.3.1.2 Quenching

Under large compressional stresses, single crystal Galfenol was observed by Clark et al. in 2001 to exhibit tetragonal magnetostrictive strains \((\frac{3}{2}\lambda_{100})\) greater than 400 ppm \([29]\). These values were achieved with quenched samples that were shown to have higher magnetostriction than furnace cooled samples for gallium contents of approximately 25 at.\% \([30, 31]\). This experiment was repeated for temperature and stress variations by Kellogg et al. in 2002 with similar results \([32]\). While quenching enhances the magnetostriction, the rapid cooling decreases the magnetic permeability and magnetostrictive constant. Lograsso et al. used X-ray diffraction to study this effect and suggest that a tetragonal lattice distortion is responsible for the magnetostriction increase with quenching \([33]\).

1.3.1.3 Mechanical properties

Guruswamy et al. made the first Galfenol tensile strength measurement of 576 MPa along the rolling direction ([001]) of their sample in 2000. Refined measurements in the [100] direction were conducted by Kellogg et al. in 2004, resulting in discontinuous yielding beginning at 515 MPa and a Young’s modulus of 65 GPa \([34]\). These measurements were used to calculate the stiffness values \(c_{11}, c_{12},\) and \(c_{44}\) for varying gallium content. During this same year, Summers et al. observed a polycrystalline Galfenol tensile strength of 440 MPa with 0.25% elongation before failure and Vickers hardness values between 200 and 250 for gallium content between 15-20 at.\% \([25]\). In 2009, Wun-Fogle et al. measured the soft and hard moduli of Galfenol samples; for the case of constant induction, measured values for binary Galfenol ranged from 73 to 79 GPa and 66 GPa was measured for Galfenol steel \([35]\).

1.3.1.4 Ternary Fe-Ga-X alloys

Small additions of nickel and molybdenum to Galfenol by Clark et al. \([31]\) in 2001 were expected to decrease the material anisotropy; instead, the magnetostriction suffered severely. This experiment was repeated by Restorff et al. in 2002 with nickel, molybdenum, tin, and aluminum with similar results except that with tin, the magnetostriction was relatively unchanged \([36]\). Later, in 2007, small amounts
of carbon, vanadium, chromium, manganese, cobalt, and rhodium were added to Galfenol with the intent of increasing the saturation magnetostriction, but in each case the magnetostriction was decreased [37]. The addition of carbon did, however, increase the magnetostriction for the unquenched case, and it was suggested that carbon addition could be substituted in place of the quenching process. Gallium addition to steel is used to create Galfenol Steel, which is discussed in a later section.

1.3.1.5 Shear modulus softening

In 2003, the effects of added gallium was studied by Clark et al. since this results in an unusual double peak feature is seen in the plot of magnetostriction versus gallium content (Figure 1.1) [38, 39]. The second peak feature is attributed to a softening of the shear modulus and not to magnetoelastic effects.

1.3.1.6 Magnetostrictive behavior with temperature variation

A 2005 paper by Clark et al. shows temperature variation of the anisotropy and magnetostriction of Galfenol with three different gallium contents [39]. Samples were tested over a range of temperatures from cryogenic to room temperature. For 8.6% gallium content the magnetostriction increased with temperature (a trend also seen in iron), for 16.6% gallium content magnetostriction decreased and for the 28% gallium content magnetostriction also decreased. At cryogenic temperatures, the 28% gallium exhibited strains of nearly 800 ppm.

1.3.1.7 Thermal properties

Little investigation has been conducted into the thermal characteristics of Galfenol. Clark et al. reported Galfenol’s Curie temperature to be $T_C \approx 700^\circ$C [31]. Other than this, the specific heat of Galfenol at very low temperatures has been investigated by Hill et al. [40].

1.3.1.8 Crystalline structure

The structural complexity of Galfenol has been a topic of much interest since it does not have conventional magnetoelastic effects [30, 31]. In 2001, Clark et al.
suggest that the unusual behavior may be due to clusters of gallium atoms \cite{30}.

1.3.1.9 Crystal growth

In 2004, Summers \textit{et al.} compared Bridgman growth to Free Stand Zone Melt (FSZM) processed Galfenol and found that Bridgman single crystals had the highest magnetostriction, followed by a research grade FSZM, Bridgman multi-crystal, and production grade FSZM (in that order) \cite{25}.

1.3.1.10 Magnetomechanical coupling

Restorff \textit{et al.}, in 2005, reported enhanced magnetomechanical coupling in Galfenol elements operated under tension, with an average coupling coefficient of $k_{33} = 0.66$ \cite{41}. Work detailed in a 2006 paper by Wun-Fogle \textit{et al.} studies coupling as a function of stress and magnetic field and reports a coupling coefficient in excess of 0.6 for Galfenol polycrystals \cite{42}. In a 2009 study of the elastic moduli of Galfenol, Wun-Fogle \textit{et al.} determined coupling values ranging from 0.61 to 0.84 for binary Galfenol and 0.48 for Galfenol steel \cite{35}.

1.3.1.11 Annealing

Stress annealing Galfenol enhances the material’s high-power capability by increasing the material anisotropy such that it enhances the strain capabilities \cite{43} \cite{44}. For unannealed samples, saturation magnetostriction is only accessible under high loading; with annealing, a built-in stress allows saturation magnetostriction to be attained with low loads. A 2005 paper by Wun-Fogle \textit{et al.} reports full magnetostrictive performance of stress annealed samples for compressive loads of 100 MPa to tensile loads of 20 MPa \cite{44}. In 2006, Clark \textit{et al.} presented results for the magnetostriction of stress annealed Galfenol with compressive loads greater than 100 MPa to tensile loads of 40 MPa \cite{43}. At the same time, Wun-Fogle, Restorff, and Clark \cite{6} investigated the underlying mechanism and found that stress annealing holds the magnetic moments perpendicular to the stress axis. In a different study, these authors also found that stress annealing has little impact on the magnetic permeability, piezomagnetic $d$ constant, and coupling coefficient, but affects the stress and field levels where the maximum values occur \cite{42}.
1.3.1.12 Galfenol steel

In 2008, Clark, Wun-Fogle, and Lograsso patented Galfenol Steel [45]. Substituting low carbon steel for the pure electrolytic iron [46] increases the material’s strength and ductility, lowers its magnetostriction and magnetic permeability, and does not affect its ability to be prestressed [47]. The added ductility is useful for applications that require significant metal working (machining, rolling, forging, etc.).

1.3.1.13 Texturing

It is advantageous to develop a preferred crystallographic orientation in Galfenol; this preference is referred to as texture. Because Galfenol’s magnetic easy axis and maximum magnetostriction are along the \(<100>\) direction, this preferred orientation for texturing. Therefore, for this orientation, higher texture increases the strain of the material.

1.3.1.14 Rolling

One disadvantage of the high permeability of Galfenol is that it requires finer lamination for eddy current mitigation. Punching laminations from thin sheets of rolled Galfenol should be economical and efficient – this process could reduce the cost to manufacture these transducers. Much work has focused on developing the material processes necessary for rolling. In 2000, Guruswamy et al. hot rolled Galfenol for tensile testing but did not report the magnetostriction of the rolled material or comment on its texture [28]. A 2003 paper by Kellogg et al. details rolling polycrystalline Galfenol and attaining magnetostriction of approximately 170 ppm. This report is accompanied by an FEA model of the rolled material [48]. Srisukhumnowornchai and Guruswamy [49] examined rolled Ga-Fe texturing in detail in 2004, using a combination of hot and warm rolling processes and achieved a near-complete \([001]\) orientation but did not give magnetostriction values for the rolled material. Na and Flatau (2005) [50] report on rolling Galfenol at 1000 °C and achieving a maximum magnetostriction of 200 ppm in the rolling direction after introducing boron to prevent cracking. In 2007, Chang et al. reported on efforts to roll Galfenol specimens: severe edge cracks developed during rolling at temperatures less than 900 °C, the most successful rolling occurred at 1000 °C [51].
This study did not comment on the effect of rolling on magnetostriction.

1.3.1.15 Punching

As it is expected to enable economical manufacture of magnetostrictive transducers, punching thin-rolled Galfenol is of interest. In 2009, Brooks et al. tried punching disks from rolled Galfenol but reported that heat-treatments in the rolling process embrittled the samples and led to fracture and crumbling [46]. Other than this, the author is unaware of any investigation of punched Galfenol.

1.3.2 Galfenol sample characterization

The only standard method for characterizing magnetostrictive samples is to use thin sheets of material to form a laminated torus. Because they are difficult to roll, this is problematic to do with materials like Terfenol-D and Galfenol. Other methods have been tried, but unfortunately no standard measurement setup has been established. Efforts to measure the magnetostrictive parameters of bar samples are reviewed here.

1.3.2.1 Quasi-static characterization

Quasi-static, major-loop characterization of Galfenol samples is common practice. Summers et al. describe a setup in which a Galfenol sample is placed in a load frame, stressed, and excited by a solenoid at very low frequency (0.1 [Hz]) [25]. A magnetic return path completes the magnetic circuit. In this setup, three sensors are used: a Hall-effect generator to measure the magnetic field, a sense coil around the sample to detect changes in the magnetic induction, and a linear variable displacement transducer (LVDT) to sense the displacement caused by magnetostriction. A similar setup is reported by Wun-Fogle and Restorff, except that a coil is employed instead of a Hall-effect sensor and strain gauges are used in place of the LVDT [6, 44].

1.3.2.2 Dynamic characterization

In 2008, Scott reported on a dynamic, minor-loop characterization scheme [52]. This setup was intended to be used for rapid testing of Galfenol in an industrial
setting. Two pairs of Helmholtz coils were employed in this setup to create volumes of constant-magnitude static and dynamic magnetic fields. Sensors employed were a Hall-effect sensor to determine the magnetic field, a sense coil wound on (or slipped over) the sample to measure the magnetic induction, and a laser doppler vibrometer (LDV) to measure the displacement of the free sample.

### 1.3.3 Galfenol device modeling

Practical transducer design will benefit from device-level modeling of Galfenol. Finite element analysis approaches have been investigated by Graham and Evans. Graham achieved bidirectional coupling in *COMSOL Multiphysics* using a script to connect magnetostatic and static structural mechanic application modes [53]. Evans developed a thermodynamic framework for modeling the nonlinear material behavior of Galfenol and demonstrated a linear implementation in *COMSOL* [54].

### 1.3.4 Galfenol devices

As a relatively new material, only a handful of Galfenol devices have been built. This section reviews the incorporation of Galfenol into a variety of different transduction and sensor applications.

#### 1.3.4.1 Underwater transducers

Very little work has been conducted to study the applicability of Galfenol for underwater sonar transducers. As such, the author is aware of only one previous study. In 2006, Meyer, Slaughter, and Scott [55] built and tested a tonpilz transducer constructed around two rods of highly textured polycrystalline Galfenol and successfully tested the device both in-air and in-water.

#### 1.3.4.2 Sensor and actuators

Investigations into other applications of Galfenol are plentiful; several successful efforts are listed here. In 2008, Ueno *et al.* described a micro-sized (5.8 mm long) magnetostrictive longitudinal vibrator that uses a U-shaped Galfenol element [56] and Ueno and Higuchi presented a micro bending actuator using a nickel-Galfenol
bimorph [57]. A 2009 thesis by Mahadevan details torque sensing on a rotating shaft with a polycrystalline Galfenol patch using a magnetic circuit and sense coil detection scheme [58]. Also in 2009, Ueno et al. reported on a miniature spherical motor that uses four Galfenol rods to actuate a spherical joint for micro-actuation [59].

### 1.3.5 Modular approaches to magnetostrictive transducers

The Summary Report of the National Defense Research Committee [7] published in 1946 describes several designs that are not strictly modular, but anticipate groupings of magnetic circuits in a way that is similar to the configurable drive design proposed in this dissertation. A 1989 patent by Porzio et al. describes a modular magnetostrictive driver section comprised of two rods made from interleaved Terfenol-D and samarium cobalt disks and joined at the ends with high-permeability return path pieces [60, 61]. In this way, the length and resonance of the drive section could be adjusted by removing or adding disks. Galfenol’s higher permeability does not require as much permanent magnet material to achieve proper bias and permits locating the magnets away from the mechanical action. Another end-to-end modular design is proposed by Engdahl and Kvarnsjö [62] having separate AC and DC flux paths (separate AC and DC circuits are discussed in section 4.2.4).

### 1.3.6 High power sonar transducers

Every transducer has acoustical, thermal, mechanical, and electromagnetic limitations. Of these, it is usually the mechanical or electromagnetic considerations that limit the operation of the device [63]. Woollett, in the 1960s, presented equations for the electrical and mechanical power limits based on one-dimensional transducer models to arrive at expressions based on fundamental transducer quantities [63, 64].

In 1993, Moffett et al. demonstrated a source level of 203 dB ref. 1 μPa at 1 m at 1.48 kHz with a high power Terfenol-D flexextensional transducer [65], but their efforts were limited by mechanical attachments. Also at this time Claeyssen et al. [66] presented several kilowatt transducer designs based on Terfenol-D (predicted source levels of 205, 209, 208, 223 dB ref. 1 μPa at 1 m for frequencies ranging
between 305 - 1200 Hz). The paper, perhaps optimistically, concludes that “in any case, magnetostrictive transducers are better than piezoelectric transducers.”

A 1997 article by Joshi, Lindberg, and Clark \[67\] investigates a high-power (181 dB, 430 Hz) Tb\(_{0.6}\)Dy\(_{0.4}\) (not Terfenol-D) sonar transducer based on cryogenically-cooled magnetostriction excited with superconducting coils.

Larson, Rogers, and Munk published a 1998 paper describing a “state-switched” transducer that employed variable mass or variable compliance transducers to simultaneously achieve high power, low frequency, and respectable bandwidth by operating a high-\(Q_m\) transducer a tunable resonance \[68\]. For this study, open-circuit and short-circuit conditions were applied to PZT rings to vary the transducer compliance. While the actual implementation of this concept was not especially compact or practical, state-switching was successfully demonstrated. A similar scheme could be enacted with magnetostrictive material.

In 2007, ETREMA Products, Inc. developed a commercial, off-the-shelf 1 kW Terfenol-D transducer \[69\].

### 1.3.7 Transducer comparison

The fundamental differences of magnetostrictive and piezoelectric transducers have led many to seek a single value to express the worthiness of a transducer. The result is figure of merit (FOM) expressions to compare devices. In the literature, several figures of merit have been suggested:

In the 1960s, Woollett advocated the use of the coupling coefficient, \(k\), as a figure of merit for preliminary comparison between transducers of different types \[70\].

A direct experimental comparison of magnetostriction to piezoelectricity (Terfenol-D versus PZT) was accomplished by Moffett and Clay \[65\] in 1993 who suggested transducers be compared by a “power handling” figure of merit (FOM) that divides the maximum radiated power by the mass, \(m\), frequency, \(f\), and mechanical quality factor, \(Q_m\) of the transducer:

\[
\Pi_{\text{rad, max}} \frac{1}{mfQ_m}
\]

Caveats regarding coupling and efficiency were given with this figure of merit. Also in 1993, Claeyssen \textit{et al.} \[66\] employed a similar figure of merit that uses the
transducer volume, $V$, instead of the transducer mass:

$$\Pi_{acs} = \frac{\Pi_{acs}}{V f Q_m}$$

(1.2)

to compare Terfenol-D transducers to piezoelectrics.

Joshi et al. [67] suggest that the field-limited energy density – calculated with the elastic stiffness in the 3-direction, $c_{33}$, and the strain, $S$ – is a useful metric for comparing active materials:

$$\frac{1}{2} c_{33} S_{\text{max}}^2$$

(1.3)

Unfortunately, the lack of consensus suggests that when comparing magnetostrictives to piezoelectric technologies, attaining a high figure of merit in favor of magnetostriction may simply lie in choosing a favorable expression.

### 1.4 Problem statement

Many attractive features of Galfenol have been identified, but it remains unclear in which scenarios this material may be used advantageously over competing materials in underwater transduction applications. Although considerable effort has focused on improving Galfenol as a material, very little work has been done to leverage its unique features to achieve good magnetostrictive design. It has already been mentioned that one of Galfenol’s strengths is its well-rounded property set, but this means that, in the absence of a single standout property, Galfenol will need to demonstrate its many attractive features in concert to compare favorably against other technologies. Moreover, it must be demonstrated that design complications do not overshadow the material advantages of designing with Galfenol.

### 1.5 Proposed solution

This dissertation aims to demonstrate a design approach that takes advantage of all Galfenol’s unique properties in a way that is easy to design, simple to build, and general enough to apply to a wide range of scenarios. Surveying legacy HUSL designs will serve as an initial starting point for this project since nickel and Galfenol
are both machinable and have magnetic permeabilities on the same order of magnitude. Simplified design tools for adjusting the generalized framework and multidomain models will be important outcomes of this work. The practicality of this design approach will be tested by designing and building a prototype Galfenol drive and using it as the motor section for a simple transducer. This transducer will then be measured in-water to characterize the performance of the drive and assess the design and modeling efforts.
Chapter 2

Theory

2.1 Magnetism

Nearly everyone is familiar with the effects of magnetism through the invisible attraction and repulsion of certain metals. Just as familiar is the compass that has a needle that will point north despite the compass’ orientation. Magnetism is also incorporated into many aspects of our daily lives: magnetic latches, computer hard drives, and power transformers, just to name a few.

Humans have known of the basic physical phenomenon of magnetism since antiquity. The ancient Greeks were aware of the magnetic attraction of the lodestone to iron at least 2500 years ago [2]. There is, however, evidence to suggest that the ancient Chinese were probably the first to discover magnetism around 4000 B.C. [71].

Magnetism played a central role at the birth of western science. During the Renaissance, magnetism became one of the first topics to be studied in a scientific manner when, in 1600, William Gilbert of England published his book, De Magnete Magneticisque Corporibus, et de Magno Magnete Tellure (Concerning Magnetism, Magnetic Bodies, and the Great Magnet Earth). This volume recorded seventeen years of Gilbert’s experimental work on magnetism [72].

As an interesting sidenote, Gilbert is considered by some to be the first true scientist, his work only slightly preceding that of Galileo Galilei. In fact, De Magnete not only inspired Galileo to study magnetism on his own, but Galileo himself credits Gilbert with establishing the scientific method [72].
Several centuries later, Danish scientist Hans Christian Ørsted made what is arguably the greatest experimental discovery in science when, in 1820, he observed that a compass needle is deflected by the nearby flow of electrical charges (current) in a conductor, proving a connection between electricity and magnetism. Up until this time, a piece of iron could only be magnetized by rubbing it with a lodestone or with another piece of iron (or other ferromagnetic material) that had already been magnetized. In the flurry of activity that followed, Ampère was quick to take Ørsted’s result and postulate that permanent magnetism is in fact caused by internal currents within the material.

The pinnacle achievement in electromagnetism occurred in 1861 and 1862 when Scottish physicist James Clerk Maxwell published a series of papers that combined the laws of Gauss, Faraday, and Ampère (with his own modification) to arrive at a unified theory of electromagnetism, which indicated that the nature of light is an electromagnetic wave. A complete statement of this theory is expressed by the four formulas collectively known as Maxwell’s equations.

\[ \nabla \cdot D = \varrho_{\text{enc}} \tag{2.1a} \]
\[ \nabla \cdot B = 0 \tag{2.1b} \]
\[ \nabla \times E = -\frac{\partial B}{\partial t} \tag{2.1c} \]
\[ \nabla \times H = J + \varepsilon_0 \frac{\partial E}{\partial t} \tag{2.1d} \]

Here \( E \) is the electric field strength, \( D \) is the electric displacement field, \( H \) is the magnetic field strength, \( B \) is the magnetic induction, \( J \) is the current density, \( \varrho_{\text{enc}} \) is the enclosed electric charge density, \( \varepsilon_0 \) is the permittivity of free space, and \( t \) is time. Individually, these equations are known as Gauss’ law for electric fields, Gauss’ law for magnetic fields, Faraday’s law, and the Ampère-Maxwell law. These four equations describe the solution for all problems in electricity and magnetism with the exception of those containing quantum mechanical effects.

For a complete account of the rich history of magnetism, the reader is referred to the first chapter in Mattis’ book. In English, Ørsted is commonly spelled as Oersted.
2.1.1 Fundamentals of magnetism

Ampère’s assessment of Ørsted’s discovery was correct: *all magnetism is caused by charges in motion*. Conversely, we can observe that all charges in motion produce magnetic fields. The magnetic field is described by two variables: the *magnetic field strength*, $H$, and the *magnetic flux density* (also called the *magnetic induction*), $B$. The SI units for these quantities are amperes per meter (A/m) and the tesla (T), respectively. When a magnetic field is applied to a medium, the result is magnetic induction.

As soon as Ørsted published his findings, scientists in England and France began to work on a mathematical description of the phenomenon. One of the first was an empirical relationship between magnetic field strength and current developed in France. Today it is known as the Biot-Savart law in honor of its discoverers. In the case of steady currents it is equivalent to Ampère’s law (equation 2.1c).

$$\delta H = \frac{i}{4\pi} \frac{d\vec{\ell} \times \vec{R}}{|\vec{R}|^3}$$

(2.2)

Here $i$ is the electric current, $d\ell$ is an elemental length of the conductor, and $R$ is the vector from the element to the point of interest.

2.1.1.1 Magnetic field produced by a long, straight wire

The simplest case to consider is that of the magnetic field induced by current carried in a long, straight wire. Integration of equation 2.2 for this case shows that at a point a radial distance, $r$, from the wire the magnitude of the magnetic field strength will be

$$H = \frac{i}{2\pi r}$$

(2.3)

Notice that the cross product in equation 2.2 denotes a tangentially circulating magnetic field around the wire.
2.1.1.2 Magnetic field produced by a circular loop of wire

Understanding the field created by a straight wire gives insight into the case in which the wire is wound into a circular loop. Clearly, a greater magnetic field is produced within the loop than outside it. Mathematically, this can be seen as the superposition of the tangential fields created by the all the elementary conductors in the wire loop. From the Biot-Savart law (equation 2.2), the field strength at the center of a loop of radius $r$ is

$$H = \frac{i}{2r} \quad (2.4)$$

For physical interpretation, if loop of wire with a $r = 0.5$ m radius carries $i = 1$ A of current, the field produced at the center is $1$ A/m [74].

2.1.2 Magnetic materials

All materials respond to the application of magnetic fields. When a magnetic field of strength $H$ is present in a medium it induces a certain flux density. The amount of flux density produced is dependent on the strength of the magnetic field and a property of the medium called the magnetic permeability, $\mu$:

$$B = \mu H \quad (2.5)$$

In a vacuum, $\mu$ is a constant and is denoted $\mu_0$. This quantity is referred to as the permeability of free space and has a value of $4\pi \times 10^{-7}$ H/m. Often the permeability of other media are given in terms of a relative permeability, $\mu_r$ such that $\mu_r \mu_0 = \mu$. The permeability of air is nearly the same as the permeability of free space: air has $\mu_r = 1.00000037$ [2]. In many materials, the permeability is not a constant at all, but a nonlinear function of $H$; this behavior will be discussed further in later sections.

Based on their response, magnetic materials are classified into five groups: diamagnetic, paramagnetic, ferromagnetic, antiferromagnetic, and ferrimagnetic. This dissertation considers materials that fall into the first three types. For information regarding antiferromagnetism and ferrimagnetism, the reader is referred to Cullity and Graham [2].
2.1.2.1 Diamagnetic materials

Since magnetic fields are caused by charges in motion, all materials should exhibit some minute magnetic response simply due to the motion of electrons at the atomic level. This type of magnetism has been observed and is termed diamagnetism because the net effect is that the material responds to an applied magnetic field by generating an opposing field. For this reason, diamagnetism may be thought of as “negative magnetism” \[2\]. The most strongly diamagnetic material is bismuth, which has a relative permability of 0.9998, so the effect is small.

Within the atom, electrons make two contributions to the magnetic field: one from orbital motion of the electron around the nucleus and the other from the spin. In diamagnetic materials, the atom or molecule has no net magnetic moment because of cancellation. When an external magnetic field is applied, it slows down the orbital motion of the electrons so that the effective current is less \[2\]. This change causes an imbalance in magnetic moment cancellation, resulting in a net moment in the opposite direction of the applied field.

The diamagnetic effect is common to all materials and is not a temperature-dependent effect (the permeability is not a function of temperature). Familiar examples of diamagnetic materials include copper and water.

2.1.2.2 Paramagnetic materials

In paramagnetism, the magnetic moments of the orbital motion and spin do not cancel out, leaving each atom or molecule with a net magnetic moment. Thermal agitation, however, causes these magnetic moments to be oriented randomly so that the material has a net moment of zero. Application of an external magnetic field will tend to cause these moments to align themselves with the field. When the contributions of these moments are summed up, the net effect is the generation of a molecular magnetic field that points in the same direction as the applied field and reinforces it. The molecular alignment is opposed by the disordering effects of thermal agitation, leading to a decrease in the susceptibility of a bulk sample as its temperature increases.

Paramagnetism, like diamagnetism, is considered to be a weak magnetic response. While weak, the paramagnetic effect is still much greater than diamag-
netism. Aluminum and magnesium are familiar examples of paramagnetic materials.

Diamagnetic and paramagnetic materials have permeabilities values that are very close to the permeability of free space so that the response of these materials is often given as a magnetic susceptibility, \( \chi \), where

\[
\mu = \mu_0 (1 + \chi)
\]  

(2.6)

2.1.2.3 Ferromagnetic materials

Ferromagnetism is a very strong magnetic effect – more than a million times stronger than paramagnetism [2]. Above a certain temperature, however, ferromagnetic materials become paramagnetic. This temperature is different for different materials and is called the Curie temperature, denoted \( T_C \).

Like paramagnetism, ferromagnetic materials have an aligned magnetic field. The molecular field produced by ferromagnetic materials, however, is so strong that the material is self-saturating [2]. In bulk form, the material does not exhibit saturation because it is naturally subdivided into domains, which are regions of aligned atoms. These domains form spontaneously as the material cools past its Curie temperature and the net magnetic moments form in opposing directions so that the material sample has no net magnetic moment.

When an external field is applied to a ferromagnetic sample, domains that are most in line with the applied field direction appear to “grow” at the expense of domains that are oriented against the field. This domain growth continues until the entire piece of material is one domain. At this point the material is magnetically saturated. Once a magnetic field is applied, it is not easy to return the material to its original state; this is discussed more fully in the next section. Common examples of ferromagnetic materials are iron, cobalt, and nickel.

2.1.2.4 The \( B-H \) curve

One of the key representations of ferromagnetic material behavior is the \( B-H \) curve, sometimes called the hysteresis loop. A generic curve is shown in Figure 2.1. If a ferromagnetic material is heated above its Curie temperature and allowed to cool
in the absence of magnetic fields, it is *demagnetized*. The demagnetized state is indicated by point $O$ in Figure 2.1. Consider a quadrant-by-quadrant examination of the material’s *major-loop* behavior:

1. When a magnetic $H$-field is applied to a demagnetized sample, the material experiences an induction ($B$-field) such that it follows the initial magnetization curve. The material initially responds with a slope corresponding to the permeability of free space, $\mu_0$, but gradually increases. At some field value, the material reaches magnetic saturation (represented by point $B_s$). Once saturated, further increases of the $H$-field result in diminished gains in the $B$-field. If the magnetic field is decreased, the $B$ vs. $H$ curve does not follow the same path – this phenomenon is known as *hysteresis*. The result is a remanent flux density, $B_r$, at zero field.

2. To return the $B$-field to zero, a magnetic field must be applied in the reverse direction. The field magnitude required to affect this change is called the *coercive field*, represented by point $H_c$. This quadrant is commonly called the *demagnetization curve* and is often presented by itself to characterize permanent magnets.

3. Increasing the magnitude of the reverse magnetic field will eventually result in another saturation point, $-B_s$. Removing the magnetic field from the saturated state results in the remanent magnetization $-B_r$.

4. Application of a field of magnitude $H_c$ is required to traverse the third quadrant. Notice that the material has returned to zero-induction.

5. When the curve returns to the first quadrant, a new path, different from the initial magnetization curve is taken. Along this path, a line traced out between $O$ and the points along the major loop will experience a maximum slope that represents the material’s maximum permeability, $\mu_m$.

To return the material to its initial state (point $O$) without heating the material above its Curie temperature, a large, saturating sinusoidal field may be applied

---

3 Jiles [74] offers this helpful insight: “The term ‘hysteresis,’ [means] to lag behind.” To illustrate this, a Lissajous figure (two sinusoids plotted against each other with one lagging in phase behind the other) may be considered as a first-order approximation to the hysteresis loop.
such that the material response follows the major loop and its magnitude slowly decreased to zero.

Notice that Figure 2.1 shows three minor loops. The first, loop $a \rightarrow b \rightarrow c \rightarrow d \rightarrow e \rightarrow a$, is generated by applying a moderate sinusoidal signal around point $O$.

If the demagnetized material is magnetized to point $g$ and then the field is decreased, the material will follow path $g \rightarrow f$. Applying the magnetic field again
will cause the material to follow path \( f \to g \). Increasing the field beyond \( g \) will cause the material to return to the initial magnetization curve and follow it to saturation.

Following the major loop to point \( h \) and decreasing the magnetic field results in path \( h \to k \). The slope of this minor loop is the incremental permeability, \( \mu_\Delta \). For permanent magnet materials, this quantity is called the recoil permeability and implies operation in the second quadrant (the demagnetization curve).

### 2.1.3 Permanent magnets

Permanent magnets are ferromagnetic materials with large remanent magnetizations and are useful for supplying magnetic bias fields. Along with the remanent flux and coercive force, an important figure of merit for permanent magnets is the energy product that indicates the amount of work a magnet can do by integrating the \( B \) and \( H \) fields over the magnet’s volume:

\[
\int B \cdot H \, dV \quad (2.7)
\]

A particular concern for permanent magnets is the degradation in magnetization caused by (1) fields high enough to demagnetize and (2) high temperatures. Although some authors suggest that permanent magnets should not be heated above 75% of their Curie temperature [74], a better metric for actual use is the maximum operating temperature [75], which can be substantially less than the Curie temperature.

Numerous materials have been used as permanent magnets and regular improvements have made many of the older types obsolete. These are all well-reviewed in Jiles [74]. In this dissertation, three types of permanent magnets were considered:

**Alnico** magnets are a family of alloys consisting largely of aluminum, iron, nickel, and cobalt. The first Alnico magnets were developed during the 1930s and were state of the art in World War II. Some of the HUSL magnetostrictive designs rely on these magnets to provide the transducer’s bias field. Even though newer materials have significantly higher energy products, Alnico has
a comparatively high recoil permeability of $\mu_r \approx 5$. The Curie temperature for Alnico is $T_C = 860^\circ C$.

**Samarium cobalt** (SmCo) magnets were developed in the 1960s. In spite of the moderately superior magnetic properties of the newer neodymium magnets, the high power transducer designer should be aware of samarium cobalt’s high Curie temperature ($T_C = 820^\circ C$), low recoil permeability ($\mu_r \approx 1$), and corrosion resistance. Unfortunately, samarium cobalt magnets are mechanically brittle, so they may be ill-suited for transducers designed to withstand shock.

**Neodymium** (NdFeB) magnets are the newest permanent magnetic material, developed in the 1980s. These magnets were originally intended to replace samarium cobalt in response to a cobalt shortage. Although neodymium magnets have the highest energy products, they also have a recoil permeability $\mu_r \approx 1$ and a low Curie temperature of $312^\circ C$. Also, corrosion issues mean that NdFeB magnets are usually coated with zinc. Like samarium cobalt, neodymium magnets are brittle.

Design and/or selection of permanent magnets for a given application is a complex problem and must consider the magnetic circuit. For more information on practical design with permanent magnets, the reader is referred to Moskowitz [76], Jiles [74], and Campbell [77].

### 2.1.4 Electromagnets

It has already been established that electrical currents in straight wires create circulating magnetic fields. Also, it has been shown that when the wire is bent into a loop that the contributions from all the infinitesimal current-carrying elements add up to result in a larger field in the center. These make up the basics of electromagnets.

If wire is wound into a helix, it forms an array of loops called a coil. When a tightly-wound coil is much longer than its diameter, it is referred to as a solenoid. In this case,
\[ H = \frac{ni}{\ell} \]  

(2.8)

where \( \ell \) is the coil’s axial length in meters, \( n \) is the number of turns, and \( i \) is the current in amps. The magnitude of the magnetic induction follows equation 2.5 – if the coil is wrapped on a rod of high-permeability steel, for example, a greater flux density will result than if the coil is empty (air-cored). In fact, equation 2.8 is only exactly true if the coil is infinitely long. For a finite length coil the expression is modified by geometry terms:

\[
H(x) = \frac{ni}{\ell} \times \left( \frac{\ell + 2x}{2\sqrt{D^2 + (\ell + 2x)^2}} + \frac{\ell - 2x}{2\sqrt{D^2 + (\ell - 2x)^2}} \right)
\]  

(2.9)

Here \( x \) is the axial distance from the coil’s center and \( D \) is the coil diameter. Depending on whether direct or alternating current is impressed on the electromagnet, both static and oscillating magnetic fields may be produced.

### 2.1.5 Eddy currents

Faraday’s Law (equation 2.1c) states that when an electrical conductor experiences a changing magnetic field, electric currents are set up in the conductor such that they create a magnetic field. The changing magnetic field may be caused by either relative motion between the conductor and field or by virtue of a time-varying magnetic field. The negative sign shown in equation 2.1c denotes the fact that the induced currents set up a field that opposes the change in magnetic flux. This negative sign is known as Lenz’s Law.

Clearly this scenario of changing magnetic flux in an electrical conductor is very relevant to magnetostrictive transducers and this phenomenon will be examined in detail. Of particular concern is the fact that these induced currents generate \( i^2 R_e \) resistive heating losses.

#### 2.1.5.1 Skin effect

Eddy currents do not exist uniformly throughout the cross-section of an electrical conductor: the induced eddy currents circulate in a direction such that they
reinforce the applied current on the outer part of the loop, but oppose it on the inward half. Thus, the electrical current density and magnetic flux tend to be concentrated near the conductor’s surface. This is known as the **skin effect**. The magnitude of the eddy currents is described by an evanescent wave, exponentially decaying beneath the surface. For a time harmonic magnetic field, a penetration depth, $\delta$, may be defined as the distance at which the wave has decayed to $e^{-1}$ ($\approx 37\%$) of its surface value:

$$\delta = \sqrt{\frac{1}{\pi f \mu \sigma}} \quad (2.10)$$

Here $f$ is the frequency of the magnetic field, $\sigma$ is the electrical conductivity of the conductor, and $\mu$ is its magnetic permeability. A derivation of this expression is given in Appendix C. Notice that the penetration depth decreases as frequency increases (i.e. the eddy currents are more concentrated at the surface for higher frequencies).

### 2.1.5.2 Lamination

An effective scheme to mitigate eddy current losses is to divide the cross-section of the conductor (as seen by the magnetic flux) into several smaller conductors, each electrically insulated from the others. A key requirement for these sub-conductors is that the smallest dimension be less than the penetration depth for the maximum frequency of interest. Litz wire is a prime example of this strategy – at high frequencies losses are reduced by using a bundle of many small diameter strands as opposed to a single large diameter wire.

Another way this is practically implemented is by slicing the conductor into very thin sheets and reassembling it with electrical insulation between the sheets. This process is known as **lamination** and the sheets are called **lamina**. When reassembled, only a portion of the cross-sectional area is actually made up of the conductive metal; the rest is taken up by insulation and epoxy. This gives rise to a common metric in laminated structures, the **fill (or stacking) factor**, which is the ratio of the conductive metal area in the cross-section to the total cross-sectional area. In practice, stacking factors between 0.85 and 0.99 are common. Lower values are generally indicative of thinner laminations that require more epoxy layers.
Instead of slicing the lamina from a solid piece, it is usually more economical to first roll the material into thin sheets and then punch laminations from the sheet and stack the laminations to achieve the final structure. This, however, is not always possible due to limitations of certain materials.

The most prevalent use of laminated structures is in electrical transformer cores and electric motor assemblies. The bulk of the literature on laminated structures is therefore to be found in the electrical engineering field and not in reference to magnetostrictive transducers. Just as Lindenberg et al. [78] wryly described electrostatic loudspeakers as being “merely a noisy capacitor,” sometimes it is useful to think of the laminated Galfenol structures presented in this dissertation as merely being noisy transformer cores.

2.1.5.3 Critical frequency

Lamination is only effective for frequencies for which the lamination thickness is less than the penetration depth. This limitation imposes an upper limit on frequency that may be readily calculated. For rectangular laminations, the accepted definition of the onset of full eddy current losses occurs when \( d = \sqrt{2\delta} [7, 8, 79] \). From this relationship, it is straightforward to define, for a lamination thickness \( t \), the critical frequency at which eddy current losses begin:

\[
f_c = \frac{2}{\pi\mu\sigma t^2}
\]  

(2.11)

2.1.5.4 Power loss

The power loss (heat generation) due to eddy currents, \( \dot{Q}_{ec} \), is given by Foucault’s equation [80]. For a peak magnetic induction \( B_{pk} \),

\[
\dot{Q}_{ec} = \frac{\pi^2\sigma f^2 t^2 B_{pk}^2}{6}
\]

(2.12)

2.1.6 Demagnetizing fields

When a magnetic material is introduced within a magnetic field, \( H_0 \), poles are formed in the material that setup a secondary demagnetizing field, \( H_d \), around the
sample in opposition to the original field. Thus, the internal field experienced by the material is

\[ H_{\text{int}} = H_0 - H_d \quad (2.13) \]

No simple correction exists for the field surrounding the material. This is a concern when trying to experimentally determine the field inside a sample with an external sensor. \( H_d \) is sometimes expressed as \( N_d M \), where \( M \) is the magnetization of the material and \( N_d \) is a dimensionless, geometry-based demagnetizing factor. For only a couple of special cases do exact analytic solutions for \( N_d \) exist; most values are approximate calculations or experimental values \[74\]. Tabulated \( N_d \) values may be found in Bozorth \[3\] and Jiles \[74\]. A special case is ellipsoidal samples in which the demagnetizing fields are uniform.

2.2 Mechanics

Having just considered the basics of electromagnetism, attention is now given to the mechanics of materials. The focus of this section is the relationship between the application of forces to a body and the response of the material through deformation, i.e. elasticity. While mechanical interactions between bodies are generally expressed in terms of force and velocity, inside the bodies it is much more convenient to use the specific variables stress and strain.

2.2.1 Stress

Stress is defined as force, \( F \), per unit area, \( A \). Following the established piezomagnetic nomenclature \[81, 82\], stress is represented by the symbol \( T \).

\[ T = \frac{F}{A} \quad (2.14) \]

By inspection, stress has units of pascals, \( (\text{Pa}) \). The concept of stress carries with it an important sign convention: tensile stresses are denoted with a positive sign while compressive stresses are indicated with a negative sign.

Stress in three dimensions may be resolved into six components of two types:
normal stress and shear stress. If an infinitesimal volume \( dV = dx \, dy \, dz \) is conceptually removed from the area of interest, these stresses may be applied to the faces of the volume. As the name implies, normal stresses act in a direction perpendicular to the face that they originate on. Shear stresses act in the plane of the face on which they are applied. This dissertation uses a common shorthand notation for the normal and shear stresses shown in Table 2.1.

<table>
<thead>
<tr>
<th>Stress</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal stress in 1-direction</td>
<td>( T_1 )</td>
</tr>
<tr>
<td>Normal stress in 2-direction</td>
<td>( T_2 )</td>
</tr>
<tr>
<td>Normal stress in 3-direction</td>
<td>( T_3 )</td>
</tr>
<tr>
<td>Shear stress “hinging” around 1-axis</td>
<td>( T_4 )</td>
</tr>
<tr>
<td>Shear stress “hinging” around 2-axis</td>
<td>( T_5 )</td>
</tr>
<tr>
<td>Shear stress “hinging” around 3-axis</td>
<td>( T_6 )</td>
</tr>
</tbody>
</table>

This same notation is applied to strain variables. In piezoelectricity, the 3-direction is conventionally chosen to be the same as the poling direction. Similarly, in magnetostriction the 3-direction usually denotes the direction of the bias field.

### 2.2.2 Strain

Deformation is the response of an elastic body subjected to stress. In order to quantify the deformation in an intensive manner, it is usually expressed as the fractional change in dimension. This quantity is known as the \textit{strain}, and in piezomagnetism is symbolized as \( S \) \[81, 82\].

\[
S = \frac{\Delta \ell}{\ell} \tag{2.15}
\]

where \( \ell \) is the length of the original dimension and \( \Delta \ell \) is the measured deformation. As a ratio of lengths, strain is dimensionless. Since strain values are typically small, they are often expressed as a percentage or, as is common in active materials, in parts-per-million, (ppm).
### 2.2.3 Elasticity

Classical elasticity defines the relationship between the stress and strain in a material with an *elastic modulus*, $E$.

$$E = \frac{T}{S}$$  \hspace{1cm} (2.16)

Like stress, $E$ also has the units of pascals. This expression is equivalent to Hooke’s Law, which relates the displacement, $x$, of a spring to an applied force, $F$, through a mechanical stiffness, $k_m$, so that $F = -k_m x$.

Because of their crystalline nature, active materials are strongly anisotropic. For an anisotropic material, the elasticity is expressed in terms of a $6 \times 6$ stiffness matrix, $c$.

$$T = cS$$  \hspace{1cm} (2.17)

where this equation represents the matrices

$$
\begin{bmatrix}
T_1 \\
T_2 \\
T_3 \\
T_4 \\
T_5 \\
T_6
\end{bmatrix} =
\begin{bmatrix}
c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\
c_{12} & c_{11} & c_{13} & 0 & 0 & 0 \\
c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & c_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & c_{44} & 0 \\
0 & 0 & 0 & 0 & 0 & c_{66}
\end{bmatrix}
\begin{bmatrix}
S_1 \\
S_2 \\
S_3 \\
S_4 \\
S_5 \\
S_6
\end{bmatrix}
$$  \hspace{1cm} (2.18)

Note that this form of the $c$ matrix is common for many magnetostrictive materials. Being that $c_{66} = 2(c_{11} - c_{12})$, equation 2.18 shows five independent terms. Equivalently, the elasticity matrix is sometimes written in terms of compliance, $s$, where

$$S = sT$$  \hspace{1cm} (2.19)

In order to convert $c$ to $s$, or *vice versa*, the entire matrix must be known so that a matrix inversion may be performed, i.e. $c = s^{-1}$. It is incorrect to invert single matrix elements, e.g. $c_{11} \neq (s_{11})^{-1}$. 
2.2.4 Poisson’s ratio

When a solid body is stressed along one dimension, it will deform not only in the same direction as the stress, but also in the lateral dimensions. The dimensionless ratio of the lateral strain to the axial strain is called Poisson’s ratio, $\sigma$:

$$\sigma = -\frac{S_{\text{lateral}}}{S_{\text{axial}}}$$

(2.20)

Galfenol exhibits an unusual negative Poisson’s ratio in some directions, but a positive Poisson’s ratio in others. The effects of these ratios cancel out, so that there is practically zero volume change during magnetostriction [24].

2.3 Acoustics

Acoustics is the study of sound waves - disturbances that travel in time and space within elastic media. While wave phenomena are not exclusive to acoustics (i.e. electromagnetic waves), all acoustic wave propagation occurs within a continuum of matter.

A hallmark of acoustics is the highly interdisciplinary nature of the field [83]. This is generally viewed as one of its strengths, but it means that the fundamental discoveries relevant to acoustics often occurred in other fields. Although acoustics has ancient origins, the field was not fully established until the 19th century when John William Strutt, third Baron Rayleigh, published his Theory of Sound [84]. Beyond this, the history of acoustics relevant to this dissertation has already been addressed in Chapter 1. A short summary the history of acoustics may be found in Raichel’s book [17] and the definitive history of electroacoustics (up until the 1950s) is detailed by Hunt [16].

2.3.1 Modes of bars

Magnetostrictive transducers often rely on the extensional motion of bar-like geometries. For this reason, it is important to consider the theory of compressional waves in bars for three fundamental cases.

Free-free and fixed-fixed bars have a first extensional mode at the frequency where its length, $\ell$, equals a half-wavelength so that $\ell = \lambda/2$. The second mode
occurs when a full wavelength equals the bar length. Thus the $n$-th mode occurs at a frequency $f_n$ such that:

$$f_n = \frac{c}{\lambda} = \frac{nc}{2\ell}$$  \hspace{1cm} (2.21)

where $c$ is the speed of sound in the bar material and $n$ is the mode number. In the case where one end of the bar is fixed and the other is free, it becomes a quarter wavelength resonator. This arrangement can only support odd multiples of quarter-wavelength and is described by:

$$f_n = \frac{(2n-1)c}{4\ell}$$  \hspace{1cm} (2.22)

There are many scenarios in which these relationships are useful in both design and evaluation. Equation 2.21 has application to the characterization of magnetostrictive samples and equation 2.22 forms a basic approximation of a tonpilz transducer motor section.

### 2.3.2 Quality factor

An important metric for characterizing the resonances of oscillating systems is a dimensionless quantity known as the quality factor, $Q$. In the time domain, the quality factor is a measure of how long the system will ring: high $Q$ signifies ringing for many cycles whereas a low $Q$ means the vibrations will be quickly damped. $Q$ can also be interpreted in the frequency domain as the sharpness of the resonance peak where a sharp peak is associated with high $Q$. Although many definitions for $Q$ exist [85], one especially useful expression is:

$$Q = \frac{f_r}{f_2 - f_1}$$  \hspace{1cm} (2.23)

where $f_r$ is the resonance frequency, $f_1$ is the frequency below the resonance at half power referenced to the power at resonance (-3dB), and similarly $f_2$ is the half-power point above resonance.
2.3.3 Radiation impedance

An important acoustical effect to consider in transducer design is the radiation impedance on the radiating face. When a piston moves against a fluid, it experiences a complex radiation impedance. Mathematically, the acoustical radiation impedance of a rigid, baffled, circular piston is calculated as

$$Z_{rad}^a = \frac{\rho_0 c_0}{A} [R_1 (2ka) + jX_1 (2ka)]$$  \hspace{1cm} (2.24)

where

$$R_1 (2ka) = 1 - \frac{2J_1 (2ka)}{(2ka)}$$  \hspace{1cm} (2.25)

$$X_1 (2ka) = \frac{2H_1 (2ka)}{(2ka)}$$  \hspace{1cm} (2.26)

$k$ is the wavenumber, $a$ is the piston radius, $A$ is area of the piston face, $j = \sqrt{-1}$ is an imaginary number, and $\rho_0$ and $c_0$ are the density and sound speed of the fluid, respectively. $J_1$ is the first order Bessel function and $H_1$ is the first order Struve function. The quantity $ka$ (sometimes called the Helmholtz number) is a dimensionless number that effectively compares the size of the piston to the frequency at which it is radiating. $R_1$ and $X_1$ are the real and imaginary parts of the normalized radiation impedance and are plotted in Figure 2.2. Note that for large values of $ka$, $X_1$ goes to zero and $R_1$ approaches unity. Many approximation to these quantities have been formulated; these are further discussed in Chapter 3. For a more thorough examination of the radiation impedance, the reader is referred to [86].

It is important to note that the radiation impedance is often expressed in mechanical terms as the mechanical radiation impedance, $Z_{rad}^m$, where

$$Z_{rad}^m = Z_{rad}^a A^2$$  \hspace{1cm} (2.27)

This is simply a transformation of the impedance quantity from the acoustical to mechanical domain. The coupling relations employed in this transformation are presented in section 2.4.3.
2.3.4 Radiated power

Radiated power is an important performance metric for transducers used as projectors. High power, low frequency, and high bandwidth (low-$Q_m$) are key to long range sonar detection [68, 67] but are difficult to achieve simultaneously. The power radiated by a piston source moving at velocity, $v$, is given by [87]:

$$\Pi_{\text{rad}} = \frac{1}{2} v^2 R_{\text{rad}}^m$$

(2.28)

Here, $R_{\text{rad}}^m$ is the real part of the mechanical radiation impedance. Assuming harmonicity, this may be expressed as

$$|\Pi_{\text{rad}}| = \frac{1}{2} \omega^2 x^2 R_{\text{rad}}^m = \frac{1}{2} \omega^2 (S\ell)^2 R_{\text{rad}}^m$$

(2.29)

where $\omega$ is the angular frequency, $x$ is the displacement, and $\ell$ is the length of the magnetostrictive element. This last form shows why the strain capability is
of such importance to the transducer designer: the radiated power depends on the strain squared. The difficulty is that sonar transducers must be compact (having dimensions that are much smaller than the wavelength produced) and the combination of small size and low frequency yields small $ka$ and thus low values of $R_{\text{rad}}^m$.

### 2.3.5 Cavitation

The amount of power that can be radiated in water is limited by cavitation, a phenomenon in which tiny bubbles of distilled gas are pulled out of the liquid by negative pressures\(^4\). One way to conceptualize cavitation is to think of the water as being torn apart when the negative pressure exceeds the “tensile strength” of fluid. Since cavitation originating from harmonic excitation only occurs during the negative half of the cycle, it can be identified by the presence of a sub-harmonic at $\frac{1}{2} f_d$, where $f_d$ is the driving frequency.

Cavitation causes many undesirable effects for sonar. Most significantly, cavitation can damage and/or destroy a transducer due to the air bubbles unloading the headmass while it is operating at high power\(^5\). The radiating face can also be eroded from the shockwaves produced by nearby imploding bubbles. In addition to physical damage to the transducer, cavitation severely degrades the transducer’s performance since the waveform undergoes significant nonlinear distortion. Moreover, the acoustic transmit and receive signals must pass through the cavitation bubble screen which can muddle echo-location results. Finally, cavitation represents an additional loss mechanism for the transducer since some of the input energy must go into the formation of bubbles\[^8\]. There are a number of ways to raise the cavitation threshold: increasing the drive frequency, decreasing pulse lengths (since cavitation is a nucleation process), operating the transducer in a fluid with a higher viscosity (such as castor oil\[^9\]), and increasing the hydrostatic pressure (accomplished by operating the transducer in a pressurized vessel or by submerging the transducer to greater depths).

---

\(^4\) Cavitation is seen whenever sufficiently large negative pressures exist, not just for acoustic waves. For example, cavitation also is a significant issue for propellers.

\(^5\) The term “unload” here is meant to signify a substantial decrease in the radiation impedance seen by the headmass.
The topic of cavitation is discussed in detail by Urick [9] and Stansfield [88].

2.4 Transduction

Transduction is the process of converting energy from one form to another [10]. Energy may exist in many different “domains” – electrical, magnetic, optical, mechanical, thermal, and chemical. A device that takes in one form of energy and converts it to another form is known as a transducer. In practice this conversion is never perfect and a portion of the energy is always trapped in other domains or lost.

In some cases, a transducer might employ more than one transduction process to connect the domains. For example, a certain transducer might use a two-step approach, first converting the input energy into an intermediate form that is then converted to the output form. This is true for electromechanical transducers using magnetostrictive elements since energy being transferred between the electrical and mechanical domains must pass through an intermediate magnetic domain.

A simple model of transduction may be developed that simply considers the input and output types of energy. Following Hunt [10], an electromechanically coupled transducer may be described as

\[
V = Z_e i + T_{me} v
\]

\[
F = T_{em} i + z_m v
\]

The electrical variables (voltage, \(V\), and current, \(i\)) are related to the mechanical variables (force, \(F\), and velocity, \(v\)) by the transduction coefficients \(T_{me}\) and \(T_{em}\). Observe that if these transduction coefficients are set to zero, two familiar impedance relations result: \(V = Z_e i\) (Ohm’s law) and \(F = z_m v\). In this case, no transduction occurs and the two domains operate independently of each other. If \(T_{me}\) and \(T_{em}\) are returned to non-zero values, the electrical voltage is not only determined by the electrical current but also in part by the mechanical velocity. Similarly, there is a mechanical force contribution dependent on the electrical current. Transduction may therefore be defined as this interaction between domains.
This dissertation is broadly concerned with the conversion of electrical signals into acoustical signals and vice versa—a topic referred to as electroacoustics. This topic has been the focus of much work, resulting in a wide array of devices based on several different physical mechanisms.

2.4.1 Electroacoustic transducers

The past two centuries have seen the invention of countless forms of the electroacoustic transducer. Most devices for underwater sound may be categorized into five general types, each based on different physical principles:

- Electrostatic (variable capacitance)
- Variable reluctance
- Electrodynamic (moving-coil)
- Piezoelectric/electrostrictive
- Magnetostrictive

More thorough discussions of these electroacoustic transducer types may be found in [8] and [16]. A few transducer designs that may be suitable for underwater Galfenol applications are briefly introduced in the following sections.

2.4.1.1 Tonpilz transducers

Active material transducers (i.e. piezoelectric and magnetostrictive transducers) for underwater applications often employ a “tonpilz” design configuration. In German, tonpilz means “singing mushroom,” and it is an apt description of the transducer’s shape and function [89][88]. The tonpilz is a longitudinal vibrator, so a reasonable description of the system can be realized with plane-wave (one-dimensional) analysis.

Tonpilz transducers sandwich an active material motor section between two unequal end-masses. Together these masses act to lower the resonance frequency of the motor section so that the overall effect is to create a transducer with dimensions much smaller than the wavelength it produces. The first of these masses is the
headmass and it serves as an area transformer to couple the mechanical motion of the motor section into the radiating fluid. The second mass, called the tailmass, is heavy and inertial such that it approximates a fixed condition at the back of the motor section.

In this unbalanced arrangement, the headmass displacement is maximized. This is advantageous because it increases the volume velocity (introduced shortly in section 2.4.3) produced by the transducer. Enlarging the radiating face can provide additional volume velocity, however, making the headmass too large can cause flexural modes to place a null in the response curve, limiting the effective bandwidth.

The reader interested in examining the tonpilz design in more detail is referred to the writings of Wilson, Stansfield, Woollett, and Sherman and Butler [89, 88, 90, 91, 8].

2.4.1.2 Flextensional transducers

Flextensional transducers use a longitudinal vibrator to drive a flexible shell that serves as the acoustic radiator. The shell acts as a mechanical lever to amplify the displacement generated by the stack, which is a beneficial arrangement since active materials typically have a high mechanical impedance (large force, low displacement). This transducer type is discussed at length in [8].

2.4.1.3 Split cylinder transducers

Another transducer type for which Galfenol may present interesting possibilities is the split cylinder transducer (sometimes called a slotted cylinder). As the name implies this transducer that has a hollow cylinder geometry, except that a there is a break in the cylinder wall that extends along the length of the cylinder. A motor mechanism drives this cylinder in a “clamshell” mode, that opens and closes the gap formed by the slot. This transducer is also detailed in [8].

2.4.2 Relating electrical and magnetic variables

Because the $H$ and $B$ are geometry-related variables, magnetism is more specifically expressed in terms of magnetomotive force (mmf), $\mathcal{F}$, and flux, $\varphi$. Practical
electromagnetic coupling involves the use of a coil, so the coil’s number of turns, \( n \), cross-sectional area, \( A \), and axial length, \( \ell \), are needed to relate the magnetic variables to the electric variables.

\[
V = -n \frac{d\phi}{dt} = -nA \frac{dB}{dt} \quad \text{(2.31a)}
\]

\[
\vec{F} = ni = H\ell \quad \text{(2.31b)}
\]

### 2.4.3 Relating mechanical and acoustic variables

The acoustical effort variable is the pressure, \( p \), and the flow variable is volume velocity, \( U \). Units for these quantities are pascals (Pa) and volume per second (m\(^3\)/s), respectively. Coupling acoustical and mechanical domains is comparatively straightforward – it is simply an area transformation:

\[
pA = F \quad \text{(2.32a)}
\]

\[
U = Av \quad \text{(2.32b)}
\]

### 2.4.4 Coupling coefficient

During transduction it is likely that some of the input energy will become stored in such a way that it does not contribute to the output. This concept is quantified with a dimensionless measure known as the coupling coefficient, \( k \). Physically realizable values of \( k \) range from zero (no coupling) to one (perfect coupling). Confusingly, both \( k \) and \( k^2 \) are referred to as the coupling coefficient. Of the two, \( k^2 \) is the more intuitive and useful concept as it represents the ratio of energy transduced to the total energy stored in the system [89]. Additionally, the coupling coefficient is a measure of effective bandwidth (see section [2.4.4.3]). For magnetostrictive projectors, this may be expressed as the energy stored in the magnetostrictive material as mechanical deformation over the total energy stored in the device.

A useful way to conceptualize the coupling coefficient is to consider a very
simple mechanical system: a spring, $\beta$, with one free end and one fixed end. Storing energy in $\beta$ only requires displacing the free end. When this occurs, all the input energy is stored in $\beta$ and the coupling is perfect, $k^2 = \frac{E_\beta}{E_\beta} = 1$. Unfortunately, this model is not representative of an actual energy storage device. A more realistic (but still simple) scenario is achieved by attaching a second spring, $\alpha$, to the free end of $\beta$. In this configuration, it is impossible to get all the input energy into $\beta$ by displacing the free end of $\alpha$ because some will always be stored in $\alpha$. The coupling of this system, therefore, is given by $k^2 = \frac{E_\beta}{E_\alpha + E_\beta} < 1$. If $\alpha$ is very stiff compared to $\beta$, $k^2$ will approach one. Otherwise, if $\alpha$ is very compliant compared to $\beta$, $k^2$ will go to zero. It should be stressed that the coupling coefficient is not a measure of efficiency.

Woolett [70] provides an authoritative examination of the coupling coefficient for all five basic transducer types. He derives the coupling coefficient equations for each type based on simplified lumped-element analyses.

Practical magnetostriction requires the consideration of two couplings: the coil coupling (electromagnetic coupling, $k_c$) and the material coupling (magneto-mechanical coupling, $k_{33}$), both of which will be discussed in upcoming sections. This is an inherent disadvantage for magnetostriction because the electromechanical coupling is $k_{em} = k_c \times k_{33}$. Multiplying two positive numbers less than one results in an even smaller value. Even minor decreases in $k_{em}$ are significant because the coupling coefficient is a squared quantity: mathematical expressions generally use $k^2$, not $k$. Piezoelectrics do not suffer this setback as the transduction process goes straight from electrical to mechanical with no intermediate stage. And while real piezoelectric transducers do have $k_{em} < k_{33}$ because of certain design features, the reduction is typically not as great as multiplying the $k_{33}$ by another coupling value.

2.4.4.1 Material coupling coefficient

For a magnetostrictive element operated in its (3,3) mode, the material coupling coefficient may be expressed as, [22]:

---

This paragraph closely follows Robert Gerson’s explanation of the coupling coefficient [92].
where $d$ is the magnetostrictive parameter, $s$ is the material compliance (inverse of elastic modulus), and $\mu$ is the permeability, all in the (3,3) direction. These variables will be explained in more detail in section 2.5.3.

### 2.4.4.2 Coil coupling coefficient

Not all electric energy supplied to an electromagnetic coil is converted into magnetic field in the region of interest within the magnetostrictive material. Some of the magnetic field is manifested in other portions of the magnetic circuit. Therefore a coil coupling may be expressed as the ratio of magnetic energy in the motor section(s), $\mathcal{E}_{\text{motor}}$, to the total magnetic energy in the circuit:

\[
k_c^2 = \frac{\mathcal{E}_{\text{motor}}}{\mathcal{E}_{\text{motor}} + \mathcal{E}_{\text{return path}}}\quad (2.34)
\]

where $\mathcal{E}_{\text{return path}}$ is the total magnetic energy in the magnetic circuit not contained in the motor section(s). The magnetic energy in a volume, $V$ is calculated as

\[
\mathcal{E}_{\text{magnetic}} = \frac{1}{2} \mu H^2 V = \frac{1}{2} BHV\quad (2.35)
\]

### 2.4.4.3 Effective coupling coefficient

The effective (actual) coupling of a transducer may be measured from the resonance and antiresonance frequencies of the device ($f_r$ and $f_a$). The effective electromechanical coupling coefficient is given by:

\[
k_{\text{eff}}^2 = 1 - \left(\frac{f_r}{f_a}\right)^2\quad (2.36)
\]

The development of this expression can be found in Stansfield [88]. In a magnetostrictive sample, the resonance and antiresonance frequencies originate from the two elastic moduli of the material (discussed in section 2.5.2). Thus the coupling may be viewed as
2.5 Magnetostriction

Magnetostriction, $\lambda$, is a deformation in a material resulting from a change in its magnetic state, or vice versa. All materials exhibit magnetostriction, but for many the effect is too small to be useful. Materials may exhibit either positive magnetostriction (expanding with an applied magnetic field) or negative magnetostriction (contracting along an applied magnetic field). Only with an appropriate bias will the material expand and contract with alternating fields.

2.5.1 Piezomagnetism and piezoelectricity

Magnetostrictive transducers are often said to be effectively piezomagnetic when the material is operated under the condition of magnetic biasing. This is meant to signify that when the direction of an applied magnetic field is reversed, the magnitude of the material deformation will change signs. However, it should be noted that biased magnetostriction is not true piezomagnetism. Much confusion exists over this distinction and Cullity [93] points out that incorrect usage of the term “piezomagnetic” in transducer literature is widespread.

The correct criterion for identifying a piezomagnetic material is that an initially demagnetized crystal can be magnetized solely through an applied stress [93]. Only a handful of true piezomagnetic materials are known to exist and the effect is so exceedingly small that it is not useful for transduction. Since piezomagnetism is strongly analogous to piezoelectricity, the key benefit to viewing biased magnetostriction as effective piezomagnetism is that it allows the familiar mathematics of piezoelectricity to be applied to analysis of magnetostriction.

2.5.2 Magnetoelasticity

Due to the magnetomechanical coupling, the elastic behavior of magnetostrictives is affected by its magnetic state. Following Bozoth’s explanation [3], when a magnetostrictive material is subject to an applied stress, the material will lengthen due
to the combination of two phenomenon:

- classical elasticity typical of all solids and
- rotation of magnetic domains

In fact, this second mechanism is magnetostriction itself – a deformation caused by magnetic domain rotation. The elastic behavior of magnetostrictive materials is characterized by specifying two modulii: $E^H$ and $E^B$. In each case, the superscript refers to the variable held constant. These are sometimes referred to as the open-circuit and short-circuit conditions, respectively. In the literature, the change of elastic modulus is designated as the $\Delta E$ effect.

When the material experiences constant induction, the magnetic moments are “locked” in place so that the material behaves likes an ordinary solid. Thus, the material is stiffer at constant induction than at constant field.

Bidirectional coupling means that the magnetic state of the material is likewise affected by the mechanical conditions. This is seen in the magnetic permeability – magnetostrictive materials are more permeable at constant stress ($\mu^T$) than at constant strain ($\mu^S$). Considering the cartoon of Figure 2.3, it is clear that at
constant stress – the condition that allows bar magnets to rotate – that the magnets will align to an applied field so that a higher magnetic permeability is realized [5].

2.5.3 Magnetostrictive constitutive equations

Linear transduction models of magnetostriction use a pair of piezomagnetic constitutive equations (sometimes these are referred to as the governing equations or equation of state). For one dimension,

\[
dS = \frac{\partial S}{\partial T} \bigg|_H dT + \frac{\partial S}{\partial H} \bigg|_T dH \tag{2.38a}
\]

\[
 dB = \frac{\partial B}{\partial T} \bigg|_H dT + \frac{\partial B}{\partial H} \bigg|_T dH \tag{2.38b}
\]

It is convenient to define these partial derivates as magnetostrictive parameters. In some texts, these are referred to as “magnetostrictive constants,” but this is a misnomer as they are functions of mechanical preload, magnetic bias, magnetic drive, frequency, and temperature. Only under very small signal conditions is it appropriate to consider these parameters as constants.

The elastic compliance parameter, \( s \), is the inverse of the elastic modulus \( (s = E^{-1}) \), clearly seen by its mathematical definition: the slope of strain vs stress. While \( H \) is held constant:

\[
\frac{\partial S}{\partial T} \bigg|_H = s^H \tag{2.39}
\]

The elastic parameter may also be determined for the case of constant \( B \).

The two are related by:

\[
s^B = s^H \left(1 - k_{33}^2\right) \tag{2.40}
\]

The incremental permeability, \( \mu \), is the change in magnetic flux density per change in magnetic field intensity for an AC signal superimposed around a DC bias [3]. At constant stress the permeability is:
\[
\left. \frac{\partial B}{\partial H} \right|_T = \mu^T
\] (2.41)

This is related to the permeability measured at constant \( S \) by:

\[
\mu^S = \mu^T \left(1 - k_{33}^2\right)
\] (2.42)

Although the incremental permeability is generally denoted \( \mu_\Delta \), in the magnetostrictive nomenclature [81, 82] it is simply written as \( \mu \) and is differentiated from other permeabilities by context.

The magnetostrictive parameter, \( d \), couples the governing equations. Thus, \( d \) relates magnetic changes to mechanical changes and \textit{vice versa}.

\[
\frac{\partial B}{\partial T} = \frac{\partial S}{\partial H} = d
\] (2.43)

Assuming small signals, equation 2.38 can be rewritten and generalized for three dimensions by using matrix quantities (represented here by boldface type).

\[
S = s^H T + d^{tr} H
\] (2.44a)
\[
B = d T + \mu^T H
\] (2.44b)

If \( d = 0 \), the equations are no longer coupled and revert to familiar forms \( T = cS \) and \( B = \mu H \). Also, this pair of equations may be equivalently expressed in four ways. Depending on the scenario, certain paired arrangements of the constitutive equations are easier to solve. The other three pairs of piezomagnetic constitutive equations [22] are:

\[
S = s^B T + g^{tr} B
\] (2.45a)
\[
H = -g T + \gamma^T B
\] (2.45b)
These equations are known as the constitutive equations because they deal with material properties. The additional variables used here are:

- \( c^H = (s^H)^{-1} \)
- \( c^B = (s^B)^{-1} \)
- \( \gamma^T = (\mu^T)^{-1} \)
- \( \gamma^S = (\mu^S)^{-1} \)
- \( e = d \ c^H \)
- \( h = e^{ir} \gamma^S \)
- \( g = \gamma^T \ d \)

Notice that these are all matrix quantities. For a typical magnetostrictive material, equation (2.44) represents the matrices
For magnetostrictive elements of high aspect ratio (i.e. \( \ell \gg w \) and \( \ell \gg h \)), a bar approximation can be applied. In the bar approximation the stress in the direction of \( \ell \) is the only non-zero stress. Most magnetostrictive bar elements are operated in a (3,3) mode, in which case \( T_3 \neq 0 \) and \( T_1 = T_2 = T_4 = T_5 = T_6 = 0 \) \[11\]. Also in this mode of operation, \( H_3 \neq 0 \) and \( H_1 = H_2 = 0 \). This reduces equation 2.48 to the form

\[
\begin{bmatrix}
S_1 \\
S_2 \\
S_3 \\
S_4 \\
S_5 \\
S_6 \\
B_1 \\
B_2 \\
B_3
\end{bmatrix} =
\begin{bmatrix}
\mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{s}_{11}^H & \mathbf{s}_{12}^H & \mathbf{s}_{13}^H \\
\mathbf{s}_{12}^H & \mathbf{s}_{11}^H & \mathbf{s}_{13}^H \\
\mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{s}_{13}^H \\
\mathbf{d}_{15} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{d}_{31} & \mathbf{d}_{31} & \mathbf{d}_{33}
\end{bmatrix}
\begin{bmatrix}
\mathbf{T}_1 \\
\mathbf{T}_2 \\
\mathbf{T}_3 \\
\mathbf{T}_4 \\
\mathbf{T}_5 \\
\mathbf{T}_6 \\
\mathbf{H}_1 \\
\mathbf{H}_2 \\
\mathbf{H}_3
\end{bmatrix}
\] (2.48)

that is the basis for the one-dimensional piezomagnetic bar modeling discussed in Chapter 3 and derived in Appendix C.

### 2.5.4 Prestress

Most piezoelectric and Terfenol-D transducers use a compressive prestress mechanism to prevent the material from experiencing tension. Galfenol, however, is often compressively prestressed to increase the strain capability and to allow the material to operate under moderate tensile stress \[6\]. A schematic representation of this alteration is shown in Figure 2.4.

Under low stress conditions, the material does not achieve the magnetostriction...
saturation value. Stress annealing allows the material to have a built-in compressive stress so that saturation strains are available even under low loads [43].

2.5.5 The $S$-$H$ curve

For magnetostrictive materials, not only is the material characterized by its $B$ vs $H$ plot, but also the $S$ versus $H$ curve, shown in Figure 2.5. Notice that at the origin the curve exhibits quadratic behavior that eventually levels off at the saturation magnetostriction, $\lambda_s$.

This plot illustrates the need for a bias field. In the unbiased case (shown in green), low strains and frequency doubling result from the nonlinear operation.
Figure 2.5. Typical strain versus field plot, showing application of unbiased and biased fields with resultant strains.

When the alternating $H$-field is applied around a magnetic bias, the magnetostriction operates in an approximately linear region yielding large strains. Very similar to the $S-H$ curve is the plot of $S$ versus $B$; the main difference is that since the $B$ field in magnetic materials saturates, the quadratic behavior in the $S-B$ curve has saturated endpoints instead of regions of saturated strain.
Modeling

3.1 Introduction

Modeling is the process of using a mathematical framework to predict the behavior of a physical system. Often this mathematical description is simplified or reduced. Reddy \[94\] defines a model as “a set of equations that [express] the essential features of a physical system in terms of variables that describe the system.” The accuracy of the predication depends on three factors: the sophistication of the model, the validity of the inputs, and the resolution of parameterized variables. In this dissertation, modeling is used to simulate the performance of electroacoustic devices.

There are many merits to modeling transduction devices. First, modeling allows designs to be explored without resorting to a costly and inefficient cut-and-try approach. In many cases, models solve quickly enough to try a large number of different designs in a small fraction of the time it would take to build a single prototype. Modeling is also valuable in situations in which materials are expensive or difficult to obtain. By modeling, the designer can evaluate different design scenarios and purchase the materials to build a device that corresponds to the best performing case. Most importantly, modeling can confirm a physical understanding of the system. After the device is built, the model is still useful to identify opportunities for improvement and to evaluate modifications to the design.

This chapter begins with a brief overview of the numerical mathematics that underlie computerized modeling. Next, three types of modeling are discussed:
modeling the magnetomechanical coupling in magnetostrictive materials, modeling
devices with rudimentary one-dimensional analysis, and modeling devices with
sophisticated finite element codes. The chapter ends with a discussion of combining
these techniques so as to efficiently model systems.

For any model it is critical to remember that the results are based on a simplified
mathematical description – therefore every model has limitations. Understanding
these limitations is part of the art of modeling. George Box observed that, “[a]ll
models are wrong, but some are useful” [95].

3.2 Modeling mathematics

The advent of the computer era has made solving large sets of differential equations
with numerical methods an attractive means to evaluating physics-based problems.
With a suitable formulation, these iterative techniques can find approximate so-
lutions to an equation without requiring step-by-step algebraic manipulation from
the user. This type of repetitive process is well-suited for a computerized solution.

3.2.1 Numerical methods

Differential equations may be numerically solved with iterative methods. By ap-
proximating derivatives with finite differences, these differential equations are con-
verted to algebraic formulas.

To illustrate, consider a basic case in which a single equation is a function of
one variable. Numerical solution involves seeking an output target value, $t$, with a
series of input guesses, $g$. Successive guesses become more accurate since they are
based on evaluation of the previous guesses. The process iterates until the guess
is sufficiently refined and a tolerance, $l$, is met

$$|t - f(g)| \leq l \times t \quad (3.1)$$

where $f$ is the function to be solved. For simultaneous solution of multiple equa-
tions, matrix methods (linear algebra) are used.

Readers unfamiliar with basic numerical methods may find more complete
treatment in a textbook that deals with numerics, such as [96].
3.2.2 The finite element method

An extremely powerful method for applying numerical techniques to complex geometries is finite element analysis (FEA). The real value of this method is that it can iteratively solve partial differential equations for real-world geometries that have no analytical solution. At times, the advantages of this technique can be so compelling that the user risks overlooking the drawbacks: increased model complexity and larger computation times. Thus, it is important to be cautious of situations where FEA is overkill.

Modern FEA is sufficiently complex that the full details of its operation are beyond the scope of this dissertation. However, it is important to grasp basic principles underlying the method. Following [94], the method may be divided into three steps:

• First, the problem domain is discretized into a set of simple elements that approximate the geometry.

• Second, the mathematical equations to be solved are evaluated on each element in order to evaluate desired quantities.

• Third, all the individual element results are assembled to represent the solution on the entire domain.

Collectively, the discrete elements form a mesh and the quality of the mesh approximation depends on the size of the elements. Smaller elements create a finer mesh with more accurate results but this comes at the cost of increased computation time. It is always desired to use the coarsest mesh that produces the required accuracy.

Wherever possible, the FEA model should be defeatured to include only the essential elements of the design. In some cases, symmetry and 2D modeling may also be exploited to simplify the model. The goal is to remove unnecessary or redundant elements so that computation time is decreased.

At the heart of FEA is the numerical solution of partial differential equations; this gives the method its ability to model many complex physical situations. Examples of the application of FEA to physical phenomena include structural mechanics, electromagnetism, heat transfer, fluid transport, acoustics, and chemical reactions.
A classic example of the finite element method numerically calculates the value of $\pi$ by approximating a circle with regular polygons (each line segment forms a one-dimensional element). The coarsest polygon mesh is an equilateral triangle inscribed on the circle (three elements, or $n = 3$). By adding more elements (refining the mesh), the numerical calculation becomes more accurate because the polygon becomes successively more circle-like: adding successive elements forms a square ($n = 4$), then a pentagon ($n = 5$), a hexagon ($n = 6$), heptagon ($n = 7$), octagon ($n = 8$), up to the $n$-gon that achieves the desired accuracy. This example is shown graphically in Figure 3.1. A more thorough treatment on the theory of finite element analysis may be found in an introductory textbook on the subject, such as [94].

There are many FEA software packages available that differ in cost and complexity. Although software has traditionally been specialized for modeling a particular type of problem, generalized codes such as ANSYS and COMSOL Multiphysics have recently received considerable attention for being able to simulate many different types of physics and solve problems with coupled physics. Specific FEA codes for active material electroacoustic transduction exist; both PZFlex and ATILA advertise built-in capabilities for analyzing magnetostrictives.

### 3.2.3 Simulation (model analyses)

Once a model is built, several different types of simulation may be run to study the system.

**Operating point** (or DC) analysis solves a static or quasi-static problem (i.e. variables do not change with time or change very slowly with time). This is a basic analysis and usually solves quickly; often the operating point calculation is a precursor for another analyses. Linearizing the operating point for nonlinear DC problems is typically accomplished with iterative methods such as the Newton-Raphson method.

**Small signal** (or linear AC) analysis starts with an evaluation of the model’s operating point and then assumes small, steady-state variations around the operating point. Although the analysis is monofrequency, the system’s fre-
Figure 3.1. Numerically calculating $\pi$ using the finite element method. Notice that the more elements (finer mesh), the more accurate the $\pi$ calculation.

The frequency response may be constructed using a parameterized frequency vector. The variations are qualified as “small” because, for nonlinear elements, a linear approximation around the operating point is used – this analysis approximates any nonlinear functions by taking their derivative as evaluated at the operating point. If the system being modeled has signal levels that are large enough to experience nonlinear behavior, this analysis type will be of limited utility.

Transient analysis is a powerful method for evaluating models by solving for the
system’s behavior as a function of time. Beginning with a parameterized time vector, operating point and small signal analyses are completed at each time step. Because of this, transient solutions are computationally demanding and time intensive. An advantage of the transient analysis is the ability to include nonlinearities. Sensitivity to initial conditions may cause the solver to not converge. One way to sidestep this issue is to ramp sources from a zero initial condition using a Heaviside (or similar) function.

3.3 Magnetostrictive materials modeling

Before it is possible to consider modeling Galfenol-based devices, the material itself must be modeled. This is critical because the magnetic domain interactions within the material form the basis for magnetostrictive transduction. This task is complicated due to the fact that many of the material’s properties are strongly nonlinear functions of stress, temperature, and frequency.

3.3.1 Constitutive equations modeling

The constitutive equations have already been discussed in section 2.5.3. Implementing magnetomechanical coupling with these equations is straightforward, but limited. In particular, the magnetic bias has to be introduced through the magnetostrictive parameters, which are functions of the magnetic bias.

Often it is convenient to model the material at an optimum bias where the piezomagnetic coefficient is at a maximum since the material is usually intended to operate around this point. Single-valued representation of these parameters restricts the model to a linear description. Strictly speaking, this model is only valid for very small signals, since magnetostrictive materials quickly become nonlinear. This statement should be qualified, however, by recognizing that much useful information may be gained by using linear modeling as an approximation for large-signal scenarios.

A true nonlinear magnetostriction model built on the constitutive equation approach requires the parameters to be defined as functions of amplitude. This is certainly feasible using a lookup table based on material measurements, but it
departs from a first-principles approach. As such, energy-based materials modeling has garnered much support.

### 3.3.2 Thermodynamic energy-based modeling

An alternate scheme for modeling magnetostrictive materials is found in energy-based methods. Armstrong [97] developed such an approach for Terfenol-D that determines the magnetization and magnetostriction of the material as a function of applied stress and magnetic field based on the minimization of magnetic free energy. An extension of Armstrong’s method for Galfenol has been developed by Evans [54].

### 3.4 One-dimensional device modeling

With two material descriptions in hand, it is now appropriate to consider the modeling of devices based on the magnetostrictive effect. Even devices built primarily out of magnetostrictive material (such as the HUSL laminated transducers), require careful device-level modeling. A basic, but useful, modeling technique, one-dimensional modeling, is now presented.

One-dimensional modeling (or plane-wave modeling) radically simplifies the description of a system by considering wave-propagation in only one direction (i.e. no cross modes). Essentially, this model discards the full device geometry and describes the system in terms of propagation distances. The actual utility of this method depends on the system being modeled, but for devices operated longitudinally (such as tonpilz transducers), it forms a good description. As is demonstrated with some of the magnetics modeling in this dissertation, one-dimensional models also work well for describing single variable systems that are not necessarily constrained to a single space dimension.

### 3.4.1 Static and dynamic systems

The analysis of systems is often decomposed into a time-independent (zero-frequency) problem and a time-dependent problem. Generally speaking, these two parts are the bias and signal. In mechanical terminology, the terms statics
and *dynamics* are used. Electrical terminology is *DC* and *AC*. When discussing interdisciplinary devices (such as electroacoustic transducers), all of these terms tend to be used interchangeably.

A simple example of a system with static and dynamic solutions is a mass placed on top of a vertically-oriented spring. If the mass is initially held in place so that the spring is uncompressed but then suddenly released, both types of behavior will be exhibited. In the static solution, the mass compresses the spring by a certain length as described by Hooke’s Law, \( F = -k_m x \). Dynamically, however, the system oscillates: the mass compresses the spring but overshoots, causing the system to ring. Assuming some small resistance, the oscillations will decay and eventually the system will come to rest at the static solution.

In transduction, the dynamic solution is of primary concern, but in some cases, especially in magnetism, the static solution is critical as it describes the conditions under which the dynamic problem takes place. Dynamic systems are characterized by having a time dependence. To illustrate, two basic dynamic systems are shown here: one mechanical and one electrical.

A cornerstone concept for understanding mechanical vibrations (and by extension, acoustic oscillation) is the idea of a forced, damped, simple harmonic oscillator. This system consists of a sinusoidal force of magnitude \( F \) applied to a point mass, \( m \), situated on a linear spring of stiffness, \( k_m \), and a dashpot with mechanical resistance (damping), \( R_m \). This is shown schematically in Figure 3.2. A reference point is established by fixing one end of the spring and dashpot. All elements in this system are considered to be ideal, not having the characteristics of the other elements.

Noting that the mechanical stiffness, \( k_m \), may be equivalently expressed as a mechanical compliance, \( C_m \) (where \( C_m = k_m^{-1} \)), the equation of motion for this system is:

\[
F = m \frac{dv}{dt} + R_m v + \frac{1}{C_m} \int v \, dt
\]

(3.2)

It is very natural to label the simple harmonic oscillator as a dynamic system because clearly the system is in motion. However, this term may also accurately describe electrical systems with time-varying behavior. An example of a dynamic
system in the electrical domain is the basic LRC circuit, so named the circuit is solely comprised of an inductance, $L$, a resistance, $R_e$, and a capacitance, $C_e$ energized by a sinusoidal voltage of magnitude $V$. Figure 3.3 shows this circuit. The ground symbol is used to establish an electrical reference point. Again, all elements are considered to be ideal. Kirchoff’s loop law for the LRC circuit is:

$$V = L \frac{di}{dt} + R_e i + \frac{1}{C_e} \int i \, dt$$  \hspace{1cm} (3.3)

Figure 3.2. A forced, damped simple harmonic oscillator.

Figure 3.3. An LRC circuit.
3.4.2 Analogies

Mathematical analogy is a powerful technique in the analysis of multi-domain devices. It is especially useful in constructing and analyzing one-dimensional models. Analogy provides a generic set of physical and mathematical concepts that can be applied to fundamentally different problems. Olson [98] states that “[a]n analogy is a recognized relationship of consistent and mutual similarity between the equations and structures appearing in two or more fields of knowledge, and an identification and association of the quantities and structural elements that play mutually similar roles in these equations or structures for the purpose of facilitating the transfer of knowledge of mathematical or other procedures of analysis and behavior of the structure between these fields.” To summarize, analogies provide insights and intuition in unfamiliar situations. And while the old adage that “all analogies break down” holds true even in this context, it is nevertheless an indispensable means for interpreting physical systems.

The most fundamental concept in analogy is that energy exists in each physical domain and that the transfer of this energy can be characterized by two variables: an effort variable, $e$, and a flow variable, $f$. Alternatively, these variables are also called the across and through variables. It is convenient to require all domains to handle energy in a consistent manner; therefore, the standard requirement is that the product of the effort and flow variables have units of power (watts in this document) [99].

Assigning physical meaning to the effort and flow variables such that the ratio of effort to flow results in an impedance quantity is called an impedance analogy. In some cases, models are simplified by designating the effort and flow variables such that their ratio is an admittance quantity; this is known as the mobility analogy. For clarity, all circuits in this document are presented in the impedance analogy.

Analogy may be demonstrated by examining the differential equations describing the two simple systems presented in section 3.4.1. Comparison of equations 3.2 and 3.3 reveals that they are isomorphic. Thus, an analogy may be drawn between the two systems – the behavior of the LRC circuit can be studied to understand the simple harmonic oscillator, and vice versa. Equations 3.2 and 3.3 may be rewritten in the generalized form:
Both these systems include all three passive (unpowered) elements: the iner-
tance, \( I \), describes the derivate term; the resistance, \( R \), defines the proportional
term; and the compliance, \( C \), details the integral term. In true analogies, iner-
tances and compliances represent energy storage while resistances indicate energy
dissipation. The SI impedance-analogy variables for the electrical, mechanical, and
acoustic domains are presented in Table 3.1 along with their units and the circuit
symbols used in this document.

It is clear that systems combining these three elements are described by differ-
etial equations. Conversely, it is possible to use these elements to represent first-
and second-order differential equations.

### 3.4.3 Graphical representation of differential equations

With the development of electric circuit theory, circuit diagrams have become
a standard means to express differential equations graphically. This technique
allows a skillful practitioner to quickly understand complex systems by inspection.\(^1\)
Circuit models of electroacoustic systems are well-established \([16, 101]\) and are a
suitable means for implementing the one-dimensional techniques under discussion.

In circuit theory, the iner-
tance, resistance, and compliance terms are pictori-
ally represented as elements that are “wired” together to denote the configuration.
Although separate symbol sets have been suggested for mechanical and acoustical
elements \([101]\), usually inertances, compliances, and resistances are simply repre-
sented with electrical symbols.

### 3.4.4 Lumped element modeling

An indispensable technique for forming simple models is the concept of *lumped
elements*. This assumption simplifies a system by grouping distributed quantities
into a small number point quantities, usually just one.

\[ e = I \frac{df}{dt} + Rf + \frac{1}{C} \int f \, dt \]  

(3.4)

\(^1\) Alternatively, bond graphs may be used to graphically portray differential equations. Readers
seeking additional information on bond graph modeling of transducers are referred to \([100]\).
Table 3.1. Components, symbols, variables, and units for selected impedance analogies

<table>
<thead>
<tr>
<th>Effort Source</th>
<th>Flow Source</th>
<th>Inertance</th>
<th>Resistance</th>
<th>Compliance</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Electrical Symbol" /></td>
<td><img src="image" alt="Current Symbol" /></td>
<td><img src="image" alt="Inductance Symbol" /></td>
<td><img src="image" alt="Resistance Symbol" /></td>
<td><img src="image" alt="Capacitance Symbol" /></td>
</tr>
<tr>
<td><strong>Voltage</strong> $V$ [V]</td>
<td><strong>Current</strong> $i$ [A]</td>
<td><strong>Inductance</strong> $L$ [H]</td>
<td><strong>Resistance</strong> $R_e$ [Ω]</td>
<td><strong>Capacitance</strong> $C_e$ [F]</td>
</tr>
<tr>
<td><strong>Force</strong> $F$ [N]</td>
<td><strong>Velocity</strong> $v$ [m/s]</td>
<td><strong>Mass</strong> $m_m$ [kg]</td>
<td><strong>Resistance</strong> $R_m$ [kg/s]</td>
<td><strong>Compliance</strong> $k^{-1} = C_m$ [m/N]</td>
</tr>
<tr>
<td><strong>Pressure</strong> $P$ [Pa]</td>
<td><strong>Volume velocity</strong> $U$ [m$^3$/s]</td>
<td><strong>Mass</strong> $m_a$ [kg/m$^4$]</td>
<td><strong>Resistance</strong> $R_a$ [kg/(s·m$^4$)]</td>
<td><strong>Compliance</strong> $C_a$ [m$^5$/N]</td>
</tr>
</tbody>
</table>
A prime example of the lumped element approach is the classic circuit model of a Helmholtz resonator. Although the resonator is filled entirely with air, in the neck region the air tends to behave in a mass-like manner and in the cavity the air functions primarily as a compliance. The complexities of the fluid dynamics are avoided since the resonator is well-modeled with one lumped mass, one lumped compliance, and one lumped resistance (notice that this system is also analogous to the simple harmonic oscillator and LRC circuit).

The validity of the lumped element approach is determined by comparing the wavelength of interest, $\lambda_i$, to the largest dimension of the system, $\ell_L$. When $\lambda_i \gg \ell_L$, the approximation generally works well. However, as $\lambda_i$ approaches $\ell_L$, the model begins to break down as the waveform is no longer well-represented by a single value. To accurately resolve higher frequencies, it is necessary to segment structures into multiple elements.

### 3.4.5 Segmented modeling

When the wavelength of interest is no longer much greater than the largest dimension in the system, it becomes necessary to “un-lump” elements in order to preserve the model accuracy. In other words, multiple elements are used to resolve one system component.

The distributed approach can be illustrated by modeling the mechanical vibrations of a bar. In a longitudinal mode, bars are essentially stiff springs with non-negligible mass. If we follow a lumped element approach, the bar is simply modeled with one mass and one compliance. This proves to be a very poor model, unless the length is much less than a wavelength, $L \ll \lambda$. To improve the model, it is necessary to distribute elements along its length by considering the bar to be made up of a large collection of tiny, identical segments connected in series, each modeled with lumped elements. The more sections used to model the bar, the more accurate the model – finite element analysis is essentially a manifestation of this approach in multiple dimensions. Adding more sections improves the high-frequency response without affecting low-frequency performance.

This approach is also useful in modeling systems in which the properties of the component vary along its length. In this case, the lumped elements that model
each segment are not identical. Gabrielson [102] uses this approach to approximate a horn in which acoustic propagation occurs along a changing cross-section.

### 3.4.6 Distributed element modeling

Segmented modeling provides a conceptual basis for understanding the development of *waveguides*. A waveguide model represents a continuum with the analytical solution found in the limit of an infinite number of infinitesimally thick sections. The distributed nature of the waveguides mean that they have infinite degrees-of-freedom (infinite resonances) and can resolve all longitudinal modes. A waveguide of length $\ell$ with a propagation wavenumber $k$ is described by the equations:

\[
\begin{bmatrix}
e_1 \\
e_2
\end{bmatrix} = \begin{bmatrix}
\frac{Z_0}{j \tan(\ell k)} & \frac{Z_0}{j \sin(\ell k)} \\
\frac{Z_0}{j \sin(\ell k)} & \frac{Z_0}{j \tan(\ell k)}
\end{bmatrix} \times \begin{bmatrix}
f_1 \\
f_2
\end{bmatrix}
\]

where $Z_0$ is the characteristic impedance of the waveguide and $e_1$, $e_2$, $f_1$, and $f_2$ are the efforts and flows at the two ends of the waveguide. This relationship is developed as part of the piezomagnetic plate derivation in Appendix C (section piezoplate).

In electrical engineering, waveguides are practically realized in transmission lines, and sometimes the two terms are used interchangeably. Just as a mechanical waveguide models the vibration of a bar, an acoustic waveguide models wave propagation in a tube.

The waveguide concept is particularly useful in this dissertation for modeling the active materials in transducers. By adding coupling terms to equation 3.5, domains can be connected in a way that provides generalized models of piezoelectric and magnetostrictive materials (discussed further in section 3.5.7). Waveguides are also employed to model the resonant behavior of non-active mechanical pieces such as the headmass and tailmass.

### 3.5 SPICE

A particularly attractive computer program for solving lumped-element and distributed element systems is SPICE [103]. The core code was partially developed by
L. W. Nagel as his doctoral project at the University of California, Berkeley in the early 1970s. **SPICE** was created to numerically solve electrical circuits, as indicated by the **SPICE** acronym that stands for *Simulation Program with Integrated Circuit Emphasis*.

The core **SPICE** code specifies the circuit as a text file that lists components with their values and the electrical nodes they connect to; this table is known as the *netlist*. This is a valuable feature since identical circuits will always have identical netlists, even if they are graphically different. Netlists of **SPICE** circuits shown in this document are contained in Appendix D.

Nagel and his advisor, D. O. Pederson, decided to openly distribute the core Berkeley **SPICE** code, which has greatly promoted its proliferation. At present, the core capabilities have been integrated into a plethora of commercial circuit-solving packages. Typically these programs modify the core code, add features, and bookend the solver between a pre-processor (that converts user-drawn circuits into netlists) and a post-processor (to plot the calculated results).

**SPICE** is often viewed as tool for generating equivalent circuits with approximate solutions. In many cases this is true because of the simplifying assumptions used to construct a particular model. But it is important to realize that **SPICE** is essentially a numerical solver of differential equations where the solutions are only limited by the numerical error, which in most cases is negligible.

Although **SPICE** was originally intended to solve electrical circuits, consideration of circuit analogies and the graphical representation of differential equations with circuit schematics justifies using this program to solve dynamic systems in other domains. The application of **SPICE** to the analysis electroacoustic systems has been advocated by Leach [104, 105].

For this dissertation, the author chose to use **SPICE** because of its established performance and low cost. Specifically, the author used a fully-featured **SPICE** variant called **LTspice** that is freely available from Linear Technologies Corporation [106]. It should be stressed, however, that there are many other software packages besides **SPICE** that are equally suited to solve systems of differential equations. A key benefit of using **SPICE** is the ability to assemble a custom set of...

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2It is interesting to point out that this name does not preclude the possibility of using it to solve non-electrical or multi-domain problems as the author has done.
subcircuits into a user-defined library to form an environment for drag-and-drop electroacoustic modeling.

### 3.5.1 Multi-port elements

In order to quickly and intuitively build circuit models, a set of generalized subcircuits is introduced that can easily be rearranged and reconfigured to represent a wide range of electroacoustic device configurations. One way to achieving this is by introducing the concept of the **multi-port element**, a “black box” that relates different sets of effort and flow variables via some internal mechanism. Both two- and three-port elements are common in transducer modeling. The schematic of the two-port (four-terminal) element is shown in Figure 3.4.

Even though an entire electroacoustic transducer can be represented with a device-level two-port element that has electrical terminals on one side and acoustic terminals on the other, this is an oversimplification for the transducer designer. Usually, it is most useful to analyze the transducer as a set of component-level multi-port elements. A basic model of a generic magnetostrictive transducer is shown in Figure 3.5. This model uses a two-port electromagnetic piece to connect the electrical domain to the magnetic reluctance domain and a three-port magnetomechanical piece that couples the magnetic reluctance domain to the mechanical outputs at both ends of a magnetostrictive bar. The rest of the model may be built upon this basic framework: the electric driving circuit, magnetic reluctances, and mechanical pieces may be modeled to the desired detail.

In approaching one-dimensional modeling, the author has sought to implement commonly-used components as generalized, robust multi-port elements that can
be wired together in any configuration. Several of these components are shown in the following sections. By storing these custom components as subcircuits in a SPICE library, new models may be readily created using multi-port elements in a drag, drop, and wire approach.

### 3.5.2 Analog behavioral modeling

A useful feature of SPICE and one that is regularly exploited by the author is controlled sources, also called analog behavioral modeling (ABM). These are effort and flow sources that are controlled by either the effort or the flow in a different part of the circuit. These sources can be used to describe coupling in multi-domain models, implement complicated mathematical functions, and enable nonlinear modeling. SPICE features the four standard controlled sources plus arbitrary sources:

- voltage dependent voltage sources (E-sources)
- current dependent current sources (F-sources)
- voltage dependent current sources (G-sources)
- current dependent voltage sources (H-sources)
- arbitrary voltage/current sources (B-sources)

The input variable is related to the output variable through a user-specified gain value or expression. As will be discussed shortly in section 3.5.3, these sources can be used to model coupling between domains – an essential feature for transducer design. In addition, these sources are also valuable for modeling nonlinearities and performing secondary calculations on the model outputs.

Arbitrary sources (B-sources) are not explicitly controlled and allow for the user to enter arbitrary functions, which may include efforts and flows from the circuit and model parameters, making it easier to create circuits that express algebraic and differential equations. The generality of B-sources makes them an especially powerful tool for directly writing out expressions, avoiding the need to interpret equations into a circuit diagram.

Despite being convenient, the convergence properties of B sources can sometimes be problematic. In the event that a more robust subcircuit is needed, the mathematics can be represented with E, F, G, and H sources along with passive elements.

### 3.5.3 Coupling domains with ABM

Circuit models of electroacoustic devices need, by definition, to model coupling between at least two physical domains. While it is possible to mathematically represent the entire model in a single domain, models are more easily understood when separate domains are represented as separate circuit meshes that are coupled together. Most types of coupling can be represented with two circuit components: the ideal transformer and the ideal gyrator. Transformers are devices that relate efforts to efforts and flows to flows:

$$
\begin{bmatrix}
e_2 \\
f_2
\end{bmatrix} = T \times \begin{bmatrix}
e_1 \\
f_1
\end{bmatrix} = \begin{bmatrix}
1/\alpha & 0 \\
0 & \alpha
\end{bmatrix} \times \begin{bmatrix}
e_1 \\
f_1
\end{bmatrix}
$$

(3.6)

It is important to qualify these components as being ideal, since real-world transformer and gyrator element are subject to a variety of loss mechanisms.
Gyrators, on the other hand, relate efforts to flows and flows to efforts:

\[
\begin{bmatrix}
  e_2 \\
  f_2
\end{bmatrix}
= \mathbb{G} \times
\begin{bmatrix}
  e_1 \\
  f_1
\end{bmatrix}
= \begin{bmatrix}
  0 & -\beta \\
  -\frac{1}{\beta} & 0
\end{bmatrix}
\times
\begin{bmatrix}
  e_1 \\
  f_1
\end{bmatrix}
\]

(3.7)

Most SPICE packages do not include ideal transformer and ideal gyrator elements, so this is practically accomplished with controlled sources in SPICE. SPICE’s ABM is flexible enough to allow these components to be modeled with a variety of different controlled sources configurations. One possible way of modeling an ideal transformer is shown in Figure 3.6.

Here equation 3.6 is implemented with a voltage dependent voltage source and a current dependent current source. An important feature of this subcircuit is the very small series resistances (\(G_{\text{min}}\)) and the very large parallel resistances (\(1/G_{\text{min}}\)) connected to the controlled sources – these resistances aid in convergence and prevent errors from sources being wired in series. The four terminals of this two-port element (\(A^+, A^-, B^+, \text{ and } B^-\)) provide generalized connectivity so that Figure 3.6 can be called as a subcircuit stored in a SPICE library. Mechanoacoustic coupling (equation 2.32) can be implemented with the ideal transformer, where \(\alpha = A\).

Modeling an ideal gyrator is done in a similar fashion. A circuit implementation of equation 3.7 is shown in Figure 3.7. To model this piece, two current dependent voltage sources are used. Similar to the transformer subcircuit, resistances are used to assist with convergence. The gyrator element is often used to couple electrical

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Figure 3.6. An implementation of an ideal transformer SPICE subcircuit

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4\(G_{\text{min}}\) is a value that LTspice uses internally to represent very small conductances and that are automatically applied to aid in convergence. The default value is \(1e-12\), but it may be adjusted by the user.
and mechanical domains in loudspeaker models where it is used to implement the equations:

\[ F = B\ell i \]  \hspace{1cm} (3.8a)

\[ v = -\frac{V}{B\ell} \]  \hspace{1cm} (3.8b)

### 3.5.4 Magnetic circuit modeling

Systems incorporating magnetic features will often channel the flux using high-permeability flux paths. As a consequence of the apparent absence of magnetic monopoles in the universe, magnetic flux lines always form closed loops (as dictated by equation 2.1b). It benefits the design, therefore, to contain these flux lines in a magnetic circuit. The purpose of a magnetic circuit is to provide a closed loop for the flux that routes the flux to where it is useful while eliminating or minimizing low-permeability regions (such as air gaps).

Since magnetic circuits play a prominent role in this dissertation, it is appropriate to examine the modeling of these systems in detail. The traditional method of analyzing magnetic circuits is to draw an analogy to electrical circuit theory. Following the guidelines set forth in section 3.4.2, an impedance analogy is constructed using the magnetomotive force, \( \mathcal{F} \), as the effort variable and the flux rate, \( \dot{\phi} \), as the flow variable. With this choice of modeling variables, the electrical and magnetic domains are coupled via the gyrator component such that \( \beta = n \), where \( n \) is the number of turns on the coupling coil. A magnetic domain with these modeling variables is referred to as the permeance analogy. The difficulty with using
this analogy is that magnetic nonlinearities are often functions of $\varphi$, not $\dot{\varphi}$.

Choosing $\mathfrak{F}$ and $\varphi$ as the modeling variables results in a framework referred to as the \textit{reluctance analogy}. It is not a true circuit analogy, but it is a valid mathematical description. With these variables, an Ohm’s law type of equation can be written as

$$\mathfrak{F} = \varphi \mathcal{R}$$  \hspace{1cm} (3.9)

where $\mathcal{R}$ is the \textit{magnetic reluctance}. This relationship is known as Hopkinson’s law \cite{109}, after John Hopkinson who pioneered the application of circuit techniques to the analysis of magnetic systems \cite{110}. For flux traveling along a flux conductor of constant cross-section, the reluctance is calculated in an analogous manner to electrical resistance:

$$\mathcal{R} = \frac{\ell}{\mu A}$$  \hspace{1cm} (3.10)

as shown schematically in Figure 3.8a. Here $\ell$ is the distance traveled by the flux in the material, $A$ is the cross-sectional area, and $\mu$ is the material’s magnetic permeability.

A one-dimensional model of a magnetic circuit may be constructed using a lumped-reluctance approach. Constructing this model, however, requires thoughtful consideration of all relevant flux paths, many of which are not obvious to those without magnetics experience. Unlike electricity, magnetic fields are prone to stray from the circuit and so cannot be assumed to be confined solely to circuit elements. The reason for this is that the conductivity of a moderate electrical insulator and a good electrical conductor differ by about 20 orders of magnitude whereas in the best case for magnetic circuits, the flux conductor and flux insulator have a ratio of permeabilities that is no greater than 5 orders of magnitude and often much less \cite{80}. Therefore, it is important to model leakage flux paths. Other useful reluctance formulas used in this document are the reluctance of a semicircular cylinder (Figure 3.8b) \cite{111},

$$\mathcal{R} = \frac{1}{0.26\mu h}$$  \hspace{1cm} (3.11)
Figure 3.8. Schematic of lumped magnetic reluctances: (a) flux traveling along a constant cross-section, (b) flux traveling across a semi-circular path, (c) flux traveling around a quarter annulus (corner), and (d) flux traveling radially in a annulus.

the reluctance of flux travelling azimuthally through a quarter-annulus (Figure 3.8c),

\[ R = \frac{\pi}{2 \mu \ln \left( \frac{r_2}{r_1} \right)} \]  \hspace{1cm} (3.12)

and the reluctance of flux traveling radially in an annulus (Figure 3.8d),

\[ R = \frac{\pi}{2 \mu h \ln \left( \frac{r_2}{r_1} \right)} \]  \hspace{1cm} (3.13)

The variables used in equations 3.10, 3.11, 3.12, and 3.13 are defined graphically in Figure 3.8. Thorough and complete examples of reluctance modeling can be found in [111, 76].

The price of working in the magnetic reluctance domain is that coupling an electrical circuit model to a magnetic reluctance model requires a time-differentiating gyrator — a non-standard circuit element. Implementing this component in SPICE is essentially the same as a standard gyrator except that it incorporates SPICE’s
numerical time derivative command, \( \text{ddt} \). This is so that equation 2.31 can be
implemented with \( \varphi \) being the magnetic flow variable instead of \( \dot{\varphi} \). One possible
subcircuit for a time-differentiating gyrator to couple electrical and magnetic reluctance domains is shown in Figure 3.9.

![Figure 3.9. An implementation of a modified gyrator SPICE subcircuit for connecting
electrical and magnetic reluctance domains using the \text{ddt} command.]

To be able to drag and drop the time-differentiating gyrator as a subcircuit
in a larger circuit model, a symbol must be assigned. Although the choice of
symbols is arbitrary, the author has chosen to use the one shown in Figure 3.10.
In this symbol, the electrical port is represented by the coil leads, and the magnetic
terminals are on either end of the coil.

![Figure 3.10. The author’s symbol for representing the two-port coupling piece for
connecting the electrical and magnetic reluctance domains in SPICE.]

Returning to the permeance analogy, one benefit of this framework is that it
couples the electrical and magnetic domains with a standard gyrator component.
Permeance, \( p \), may therefore calculated by simply taking the inverse of reluctance.
In the permeance analogy, however, magnetic elements are modeled with capacitors since

\[
\vec{\mathbf{\tilde{f}}} = \frac{1}{j \omega p} \dot{\varphi} \tag{3.14}
\]
Although both the reluctance and permeance analogies have been documented in the literature, there is still some debate as to which method is to be preferred. In particular, Hamill [112] criticizes reluctance modeling as being unphysical on two counts: first, the resistor symbol, normally denoting an energy-dissipating element, represents a magnetic phenomenon which is physically energy-storing; second, the product of the effort and flow variables ($\mathbf{F} \times \mathbf{q}$) has the units of energy (joules) not power (watts) as was stipulated as a requirement for a true analogy in section 3.4.2. Each of these objections may be addressed.

It is possible to accurately model magnetics with the reluctance analogy because the resistors experience a time-integrated flow ($\mathbf{q} = \int \dot{\mathbf{q}} dt$). Discomfort with the resistor symbol is purely symbolic; the mathematical equations are still correct. Another way to understand this is that in the reluctance analogy the time integration occurs once at the flow source so that it does not need to occur at every individual component as is the case with the permeance analogy. This also addresses the second concern – since the circuit uses a time-integrated flow, it makes sense that the effort-flow product has the units of time-integrated power: energy. As was stressed before, in the reluctance domain the mathematical underpinnings are valid and correct, even if symbol meanings are skewed. To summarize, it is possible for this domain to accurately interact with other domains based on standard analogies.

Therefore, if the reluctance and permeance analogies are mathematically equivalent, what advantage is there to using reluctance rather than permeance? The benefits are threefold: 1) the reluctance analogy is an easier framework for the inclusion of magnetic nonlinearities, 2) using flux as the flow variable instead of flux rate tends to be more intuitive, and 3) it is more common to find the reluctance analogy used in practice and in the literature. These advantages come at the cost of using the time-differentiating gyrator piece instead of a standard gyrator, but a method for easily accomplishing this in SPICE has been shown (Figure 3.9). Although the one-dimensional models presented in this dissertation do not include magnetic nonlinearities, they are based on the reluctance analogy in anticipation that some designs will need to include nonlinear effects.
3.5.5 Permanent magnet modeling

Traditionally, permanent magnet design has been accomplished by using a load line to determine the magnets operating point as dictated by the reluctance of the magnetic circuit less the magnet reluctance (i.e. the load reluctance). This technique is explained in detail in Moskowitz [76]. It may be helpful to examine the analogous mechanical case, which is presented by Meeker [113], to help conceptualize the magnetic scenario.

Alternatively, permanent magnets may be designed using circuit modeling techniques. This is especially straightforward for ideal permanent magnets that have a straight-line demagnetization curve (the second quadrant of the B-H major loop) and a relative permeability of one. Samarium cobalt and neodymium magnets are accurately described as ideal magnets at room temperature. In this state, these magnets can be described in two equivalent ways in the magnetic reluctance scheme [114, 115]: either as a source of magnetic flux or a source of mmf. Consider an ideal magnet having a thickness $t$ and a cross-sectional area $A_m$ magnetized in the thickness direction. The first method is accomplished by specifying a remanent flux source, where $\varphi_r = B_r A_m$, in parallel with the magnet’s reluctance, $R_m = t/(\mu A_m)$. In the second approach, a coercive mmf source with $F_c = H_c t$ is placed in series with the magnet’s reluctance.

Even though the load line approach is not used here for designing with permanent magnets, it is useful for understanding the onset of non-ideal behavior that occurs at elevated temperatures. This is discussed fully in section 6.3.1.

3.5.6 Eddy current loss modeling

Magnetically-induced eddy currents are an $i^2 R_e$ loss mechanism for magnetostrictive transducers. As was discussed previously in section 2.1.5.2, magnetic structures are often laminated to mitigate these losses. This technique, however, only acts to decrease losses and does not completely eliminate them. It is therefore necessary to account for eddy current losses. The author has chosen to do this with a nonlinear magnetic reluctance model. In Appendix C an analytical expression is

\[ 5 \text{Alnico is an example of a non-ideal permanent magnet as it exhibits a corner feature in the second quadrant and is operated along minor loops.} \]
derived for a complex magnetic reluctance, \( Z_{\text{M}} \):

\[
Z_{\text{M}} = \frac{3}{\varphi} = \frac{\ell \zeta}{\mu wdN_L} \coth (\zeta) \tag{3.15}
\]

Here \( \zeta = \frac{3}{2} \sqrt{j\omega \mu \sigma} \) and \( \ell \) is the length of the flux path, \( w \) is the width of a lamination, \( d \) is the laminate thickness (smallest dimension), \( N_L \) is the number of laminations, and \( \omega \) is the frequency of interest. \( \mu \) and \( \sigma \) are the magnetic permeability and electrical conductivity of the flux conductor, respectively.

Following the technique of Warren and LoPresti [116], a continued fraction expansion [117] allows equation 3.15 to be re-expressed as

\[
Z_{\text{M}} = R_0 + \frac{1}{\frac{3}{j\omega L_0} + \frac{1}{5R_0 + \frac{1}{\frac{7}{j\omega L_0} + \frac{1}{9R_0 + \ldots}}}} \tag{3.16}
\]

where

\[
R_0 = \frac{\ell}{\mu wdN_L} \tag{3.17}
\]

\[
L_0 = \frac{\mu \sigma d^2}{4R_0} \tag{3.18}
\]

This form bears a resemblance to the impedance of a Cauer L-R (inductor-resistor) ladder network. It is therefore possible to approximate this equation with the circuit of Figure 3.11. Notice that this is a 15-term approximation (there are fifteen passive elements representing the continued fraction). As more L-R loops are included, the approximation becomes more accurate. Since this circuit is intended to be stored as a netlist in a SPICE library and since passive networks generally solve quickly, it makes sense to use a very large number of terms so that the approximation is in good agreement with the analytical solution for a large frequency range. The eddy current loss piece is a one-port element and the author has chosen to represent it with the symbol shown in Figure 3.12.

So far, eddy current losses have only been considered for the scenario of Figure
The continued fraction approximation has been truncated at the fifteenth term. The netlist for this circuit is included in Appendix D.

Figure 3.11. Magnetic reluctance with eddy current losses piece implemented in SPICE. The author’s SPICE symbol for magnetic reluctances with eddy current losses.

3.8a – a straight flux conductor with a constant cross-section. The circuit as developed may be used in a more general fashion. Instead of specifying the flux conductor dimensions, the user may instead directly input a value for $R_0$ based on a more refined calculation in order to model the reluctance with eddy current losses.

It is worth noting that the eddy current phenomenon is analogous to the thermoviscous boundary layers seen in acoustical systems. The techniques in this section may therefore be applied to modeling these losses as well as eddy currents [116]. For more details on the implementation shown here, the reader may find a full derivation of both this piece and an equivalent electrical impedance in Appendix C.

3.5.7 Active material modeling

The waveguide equations (equation 3.5) are conveniently and natively implemented in SPICE via the lossless transmission line piece. Inputs for this element are a propagation time-delay, $\tau_d$, and the waveguide’s characteristic impedance, $Z_0$, but these can be readily replaced with functions that instead describe the system in
terms of the sound speed, \( v \), waveguide length, \( \ell \), density, \( \rho \), and cross-sectional area, \( A \). Since LTspice uses a non-standard symbol for its transmission line piece, it is shown for reference in Figure 3.13.

Figure 3.14 shows the author’s symbol for a mechanical transmission line piece that is set up to accept \( \ell \), and \( A \) as inputs and calls a database of material properties using SPICE’s .include command to get \( v \) and \( \rho \). The details of the implementation of this block piece and the material database are included in Appendix D.

3.5.7.1 Piezoelectric piece

The piezoelectric transmission line model was developed by Redwood [118] in the 1960s. This is presented here as a familiar example to acquaint the reader with the author’s method of modeling active materials with SPICE subcircuits. Following the derivation of Berlincourt et al. [22] leads to equation 3.19.

\[^6\] In this context, \( c \) is avoided as a variable for the sound speed since the stiffness matrix is also represented by \( c \).
\[
\begin{bmatrix}
F_1 \\
F_2 \\
V
\end{bmatrix} =
\begin{bmatrix}
Z_0 & Z_0 & g_{33} \\
\frac{j \pi}{t} & j \sin \left( \frac{\pi}{t} \right) & \frac{g_{33}}{j \omega s_{33}} \\
\frac{1}{j \omega s_{33}} & \frac{\pi}{t} & \frac{g_{33}}{j \omega s_{33}} \\
\end{bmatrix}
\times
\begin{bmatrix}
v_1 \\
v_2 \\
i
\end{bmatrix}
\tag{3.19}
\]

Essentially, the system is modeled by relating the efforts (forces \( F_1 \) and \( F_2 \) and voltage \( V \)) to the flows (velocities \( v_1 \) and \( v_2 \) and current \( i \)) through an impedance matrix. The four matrix elements in the upper-left corner implement the waveguide guide equations (equation 3.5); in the lower-right is the blocked electrical impedance. The remaining elements describe the electromechanical coupling.

One possible SPICE implementation is shown as a three-port element in Figure 3.15. Here, the transmission line piece represents the waveguide equations, a capacitor introduces the blocked electrical impedance, and the three B-sources model the coupling. The six terminals (Mech1+, Mech1-, Mech2+, Mech2-, Elec+, and Elec-) make it suitable for generalized use as a library subcircuit. Notice that the lower terminals on the transmission line have not been wired together.

**Figure 3.15.** The piezoelectric piece implemented in SPICE. The netlist for this circuit is included in Appendix D.
as is often seen in the literature [118, 119, 22, 91]. This allows for multiple instances of the piezoelectric piece to be interconnected in any configuration. This particular implementation employing B-sources with laplace is shown for clarity. However, the reader should be aware that it is difficult to get this scheme to converge in transient analyses. These problems may be overcome by using inertance and compliance components to achieve differentiation and integration operations. As shown, the circuits for this and the following active material models function correctly for small signal analysis.

### 3.5.7.2 Piezomagnetic piece

Woollett [91] derives the description of the piezomagnetic piece in a similar fashion to Berlincourt’s piezoelectric piece derivation. This yields an electromagnetic description that does not explicitly consider the magnetic domain. To investigate this domain, equation 2.31 is applied to convert this one-step mathematical simplification into the two-steps that mirror the physical processes. When this is done, the magnetomechanical piece is described by:

\[
\begin{bmatrix}
F_1 \\
F_2
\end{bmatrix} = \begin{bmatrix}
\frac{Z_0}{j\tan\left(\frac{\omega}{\sqrt{\ell}}\right)} & \frac{Z_0}{j\sin\left(\frac{\omega}{\sqrt{\ell}}\right)} & \frac{g_{33}}{s_{33}} \\
\frac{Z_0}{j\sin\left(\frac{\omega}{\sqrt{\ell}}\right)} & \frac{Z_0}{j\tan\left(\frac{\omega}{\sqrt{\ell}}\right)} & \frac{g_{33}}{s_{33}} \\
-\frac{g_{33}}{j\omega s_{33}} & -\frac{g_{33}}{j\omega s_{33}} & \frac{f}{\mu s_{33} A}
\end{bmatrix} \times \begin{bmatrix}
v_1 \\
v_2 \\
\varphi
\end{bmatrix}
\]  

(3.20)

This equation is derived in Appendix C. Representing equation 3.20 with a circuit may be accomplished with controlled sources – a possible configuration is shown in Figure 3.16. Notice that it is very similar to the circuit shown in Figure 3.15, except that the Laplace statements are different and that instead of a blocked electrical capacitance, there is a blocked magnetic reluctance (if translated to the electrical domain, this quantity appears as an electrical inductance).

Notice that the blocked magnetic reluctance does not include eddy current losses. Since many magnetostrictive elements are laminated, it is practical to include these losses in the piece. Adding in the ladder network of Figure 3.11, the piezomagnetic piece with eddy current losses may be represented with the subcircuit shown in Figure 3.17.
Figure 3.16. The piezomagnetic piece implemented in SPICE. The netlist for this circuit is included in Appendix D.

The circuit shown in Figure 3.17 is incorporated as a subcircuit in a SPICE library as a three-port element. Any arbitrary three-port symbol may be used to represent this; the author has chosen to use the symbol shown in Figure 3.18.

### 3.5.8 One-dimensional modeling example

The use of one-dimensional modeling is illustrated here for the HUSL nickel transducer shown in Figure 4.1. This transducer is a good choice for illustrating these techniques because it includes physics from electricity, magnetism, mechanics, and acoustics.

Many properties in this model were measured from the device. All dimensions were determined with a caliper. A stereoscope was used to count the number of laminations and coil turns. The DC resistance of the coils was measured with a multimeter. Values for nickel’s $Q_m$, $S^{H}_{33}$, $d_{33}$, and $\mu_{33}$ were taken from Wilson’s book [89]. Other material properties are from widely available sources.

All values in the model are stored as parameters using `.param` statements. SPICE recognizes standard SI unit prefixes as value modifiers so that $-3.1n$ is
Figure 3.17. The piezomagnetic piece with eddy current losses (15-term approximation) implemented in SPICE. The netlist for this circuit is included in Appendix D.

Figure 3.18. The author’s SPICE symbol for representing the three-port lossy piezomagnetic piece. In this symbol, mechanical ports are at either end of the bar, and the magnetic port is in the middle.

interpreted as $-3.1 \times 10^{-9}$, etc.\footnote{If a .param statement is set equal to an expression, that expression must be flagged with curly braces. Notice that this rule also holds for circuit component values.}

Simulating the device as a projector, the excitation comes from the electrical domain (the innermost loop). Here a 1 A current source is used in order to provide easy access to the electrical input impedance (simply the voltage at node $Z$). Also modeled in the electrical domain is the electrical resistance, $R_{coil}$, of both drive

\footnote{In the absence of a $\mu$ character on the computer keyboard, SPICE allows $u$ to represent $1 \times 10^{-6}$. This notation is used for some netlist parameter statements in Appendix D.}
Figure 3.19. One-dimensional model of the HUSL nickel transducer in SPICE. Parameter statements have been omitted but are contained in Appendix D.
coils. These coils are wound in both electrical and magnetic series.

Wrapped around the electrical domain is the magnetic reluctance domain. Coupling is achieved with the time-differentiating gyrator discussed in section 3.5.3. In this domain there are three types of magnetic impedances: lossy reluctances included in the piezomagnetic pieces, lossy reluctances representing the laminated circuit and the unlaminated magnet, and leakage reluctances. The goal of the magnetic circuit design is to deliver magnetic energy to the piezomagnetic piece that couples the reluctance domain into the mechanical domain.

The mechanical domain is intuitive because it pictures the actual device geometry: two drive sections that are situated between a headmass and a tailmass. Two material block pieces are used to model the tailmass and the magnet is modeled as a point mass. Mechanical losses, \( R_m \), are calculated for this domain from the mechanical quality factor of nickel with some simplifying assumptions that are presented in section 5.4.

Acoustically, only two elements are used: the radiation impedance on the radiating face and the end face of the tailmass. Rather than add two more domains to the model, these impedances have been translated into the mechanical domain with equation 2.32. Also, the radiation impedance has been simplified to be the characteristic impedance of propagating fluid, \( Z_0 = \rho c \), which, when multiplied by the radiator area, is equivalent to the mechanical radiation resistance, \( R_{\text{rad}} \), at high \( ka \) values. For this model in which the transducer is considered to be in air, the radiation impedance is very small and can be overlooked without much error. It cannot be neglected, however, when modeling the transducer in-water because the characteristic impedance of water is approximately 3,500 times the characteristic impedance of air.

To compare the model and measurement, the electrical input impedance is simulated as a function of frequency. This is accomplished with an AC analysis consisting of a 2000-point linear frequency sweep between 1kHz to 100kHz, indicated by the directive \texttt{.ac lin 2e3 1e3 1e5}.

Both measured and simulated electrical input impedances for the device are shown in Figure 3.20. The impedance was measured on an HP 8194 impedance analyzer. Notice that the model is not perfect, but it is in good agreement with the model and includes the major features. The inductance in the magnetic circuit is
Figure 3.20. Electrical input impedance measurement of the HUSL nickel transducer compared to the simulation of the one-dimensional model.

indicated by the slope of the impedance. First and second longitudinal resonances are reasonably well-predicted (within 10% of the actual value) and the effective coupling is similar (indicated by the separation of the first resonance and anti-resonance frequencies). The $90^\circ$ phase angle is indicative of the inductive nature of the device. As frequency increases, however, the complex eddy current losses increase, pulling the phase angle down toward $45^\circ$ — a characteristic of fully-formed eddy currents [120].

3.5.8.1 Model complexity

The one-dimensional model shown in Figure 3.19 is quite complex and it is worth noting that much simpler models may be used to achieve very similar behavior. To illustrate, the most basic model of a magnetostrictive transducer consists of two inductors, a capacitor, and a resistor as shown in Figure 3.21. In this model, the inductive rise in electrical input impedance is modeled with inductor $\text{LSX1}$. This
Figure 3.21. An elementary circuit model of a magnetostrictive transducer.

An inductor connects via a gyrator to an LRC circuit that represents the motional impedance (modeled as a simple harmonic oscillator), but to simplify, the gyrator component is removed by translating the LRC circuit across the gyrator into the electrical domain. This translation shifts the LRC circuit from the impedance analogy into the mobility analogy, replacing the components with their “duals” – LSX1, RSX1, and CSX1 – and rearranging them into a parallel configuration. The resonance frequency, $\omega_0$, is determined by $\omega_0 = \sqrt{1/(LXM \times CXM)}$, and the spacing between the resonance and antiresonance frequencies is controlled by the coupling coefficient, $k^2 = (CXM)/(CXM + LXE)$. The sharpness of the resonance peak and antiresonance dip is determined by the damping, modeled with the resistor RSX1.

With appropriate values for these four circuit components, the simplified model can produce very similar results to the full one-dimensional model of Figure 3.19. A comparison of the simple model to the full model is presented in Figure 3.22.8 Even though the model has been significantly reduced in complexity, many important features are still resolved. One practical use of this model is to create simple electrical circuits that simulate the electrical load of real transducers for testing purposes. Notice, however, that in the simplified model only the first resonance is captured and that the phase is an ideal +90° away from resonance.

The real drawback to the simplified model lies in determining appropriate values for the four components. Experimentally adjusting the values to match an existing impedance curve is relatively straightforward, but calculating these component values from basic measurements is a much more daunting task. This is one of the advantages to the full model: many such calculations are included so that

---

8The values used in this model are $LXE=75e-6$, $LXM=5e-6$, $RXM=32e-3$, $CXM=252e-8$. 
Figure 3.22. Simulated electrical input impedance of the nickel transducer for the full and simple circuit models.

The inputs do not need to be effective values for the whole system. Similarly, the outputs of the full model are much more accessible; extracting certain quantities from the simplified model would also require involved calculations.

This leads to considering the two models in light of their intended use. While the simplified model can be used to quickly form a rough model of an existing transducer, it would be a very poor choice for design work. The full model, however, is intended to be used for this purpose and a major reason for its complexity is simplifying the design process by working with design variables.

With so many components, the most important circuit elements in the full model are easily obscured. There are two main drivers of the impedance in the full model: first, the magnetic reluctances model the inductive rise and phase droop and secondly the lossy piezomagnetic bar pieces resolve the resonant features.
3.6 Finite element analysis in device modeling

Finite element analysis is able to give very detailed description of transduction devices in one, two, or three dimensions. Fundamentally, the model includes less simplification and assumption, providing a more authoritative simulation of device behavior. The downsides are that 1) the model setup is more complex and 2) the models require many more material inputs (full material matrices for three-dimensional models); these values are often difficult to obtain.

Because of the increased complexity, in most cases it is extremely impractical to develop a custom FEA modeling environment. This makes commercial software packages with predefined application modes extremely attractive. Therefore, this examination of FEA methods will not be as exhaustive as the preceding discussion on one-dimensional models since the author did not develop his own framework. Due to the lack of three-dimensional material properties and because of the use of a well-documented, commercially-available finite element code (COMSOL Multiphysics), an FEA model of the HUSL nickel transducer is not presented.

3.6.1 COMSOL Multiphysics

The software package the author used for the finite element analyses in this dissertation was COMSOL Multiphysics, versions 3.5a and 4.0. COMSOL was selected for its generality, flexibility, and competitive cost. However, the reader should be aware that other feasible options exist (see section 3.2.2).

COMSOL assists the user by employing predefined setups for different physical scenarios, called application modes. In these application modes, the software developers have already implemented the mathematical framework so that the end-user only needs to specify the model geometry, define subdomain properties and physics, apply boundary conditions, and select a suitable solver routine. The base package of COMSOL includes a variety of fundamental application modes, and specialized application modes are distributed in separate, physics-specific add-on modules. To achieve the detailed electroacoustic modeling in this dissertation, the author made extensive use of the AC/DC and acoustics modules.
3.7 Efficiency through progressive model complexity

Two basic types of models have been discussed thus far: one-dimensional and FEA. Instead of using one method at the expense of the other, the author uses both in a two-step approach.

First, one-dimensional modeling is used to generate a basic model of the system. The advantage here is the very short simulation times, allowing the modeler to rapidly implement many small successive changes until the model performs satisfactorily. This is especially useful for design work in which a set of performance criteria need to be met. A designer can try out many different configurations in the span of a few minutes until the design satisfies the requirements. It is important to remember that there are very real limitations of the one-dimensional model.

Second, once the one-dimensional model has been iterated to within $\approx 20\%$ of the expected performance/design criteria, the model is ready to be improved with more sophisticated FEA techniques. Because additional details of the model are taken into account, this leads to a better prediction of the design’s performance. At this point in the modeling process, the increased computational cost is acceptable since the basic model mechanisms have already been verified with the one-dimensional case.

This two-step modeling approach allows for efficient model creation and device design. Using basic models, a basic description of the system is quickly generated with little time lost waiting on the numeric solver. Eventually, a more detailed FEA solution is desired, but with this approach the amount of time spent on intensive simulation is minimized.

A final benefit of this two-step approach is that it allows the designer to corroborate results. When both the one-dimensional and FEA models are in reasonable agreement, this imparts a degree of confidence in the design and it is often a milestone in progressing toward actual device fabrication.
Chapter 4

Configurable magnetostrictive drive

4.1 Introduction

Two barriers must be overcome for magnetostriction to be competitive with other active materials used in underwater electroacoustics. The first barrier magnetostriction faces deals with the material itself. Although in the past, some magnetostrictives have exhibited a single stellar property — e.g. high strain for Terfenol-D and high coupling for Metglas [89] — other unfavorable material characteristics have offset the advantages. A summary of the room temperature properties of magnetostrictive materials is given in Table 4.1. Finding better materials has been the primary concern of magnetostrictive research and Galfenol is an attractive discovery because of its well-rounded set of properties. And although it does not have a single standout property, neither does Galfenol suffer from any significant material setbacks. Compared to other high-strain materials, Galfenol has a modest strain capability and a reasonable material coupling coefficient. Perhaps the most attractive characteristic of the material is its magnetic permeability: with $\mu_r \approx 100$ the permeability is high enough for effective magnetic circuits but not so high as to require excessive lamination. Galfenol’s unique mechanical strength provides access to an unexplored design space in which transducers can be high-strain, mechanically robust, and free from prestress mechanisms. Therefore, it seems that Galfenol may be a viable solution in high-stress and abusive scenarios where other high-strain materials cannot function.

The second — and often overlooked — barrier for magnetostrictives is the state
Table 4.1. Room temperature properties of magnetostrictive materials.

<table>
<thead>
<tr>
<th>Material</th>
<th>$\lambda_s$ [ppm]</th>
<th>$\mu_{33}^T$</th>
<th>$d_{33}$ [nm A]</th>
<th>$s_{33}^H$ [TPa$^{-1}$]</th>
<th>$k_{33}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Galfenol</td>
<td>250$^a$, 400$^b$</td>
<td>65-70$^c$</td>
<td>46$^d$</td>
<td>15-45$^e$</td>
<td>0.61–0.84$^e$</td>
</tr>
<tr>
<td>Galfenol steel$^f$</td>
<td>250</td>
<td>100</td>
<td>21$^c$</td>
<td>18$^e$</td>
<td>0.48$^e$</td>
</tr>
<tr>
<td>Nickel$^g$</td>
<td>-33</td>
<td>20</td>
<td>-3.1</td>
<td>5</td>
<td>0.15–0.31</td>
</tr>
<tr>
<td>Metglas$^h$</td>
<td>50–55</td>
<td>5000–7000</td>
<td>300–400</td>
<td>30</td>
<td>0.84–0.90</td>
</tr>
<tr>
<td>Terfenol-D</td>
<td>1084–1658$^i$</td>
<td>9.3$^d$</td>
<td>15$^d$</td>
<td>38$^d$</td>
<td>0.72$^d$</td>
</tr>
</tbody>
</table>

$^a$Highly-textured polycrystalline. Clark, $^{23}$
$^b$Quenched single crystal. Clark, $^{20}$
$^c$Summers, $^{121}$
$^d$Sherman and Butler, $^8$
$^e$Wun-Fogle, et al, $^{35}$
$^f$Values used for models in this dissertation
$^g$Berlincourt, et al, $^{22}$
$^h$Wilson, $^{89}$
$^i$Engdahl, $^{122}$

of magnetostrictive transducer design art. There is relatively little in the way of a standard magnetostrictive transducer design that Galfenol can take advantage of because the other magnetostrictive materials that have been extensively investigated (nickel and Terfenol-D) have had very different properties and characteristics than Galfenol. Also, many of the processes and techniques for assembling the older nickel designs have been lost. In contrast, numerous piezoelectric transducer designs have been established through decades of widespread use, yielding refined assembly techniques. Most importantly, the design of piezoelectric transducers has been simplified to a degree that allows them to be easily engineered without the need for advanced analysis. It is highly desirable to create this same situation for transducers based on magnetostriction.

The aim of this effort is to introduce a design concept that makes the best use of Galfenol in real devices for a wide range of scenarios pertaining to underwater electroacoustic transduction. Doing this will help to identify applications in which Galfenol may be used advantageously.
4.2 Magnetostrictive design

A good magnetostrictive transducer cannot be achieved by simply copying an established piezoelectric design. This is because of the fundamental differences in the nature of electric and magnetic fields and because of the differences in the ability of available materials to conduct these fields. Therefore, optimized magnetostrictive design differs considerably from designs based on piezoelectricity. In practice, these differences are encountered as three features that must be considered in magnetostrictive transducers but not for piezoelectrics:

1. **Coil coupling:** Magnetostriction is a two-step transduction process (electrical ⇔ magnetic ⇔ mechanical) as opposed to the one-step transduction of piezoelectricity (electrical ⇔ mechanical). This is indicated by the two different coupling coefficients discussed in section 2.4.4. While the magnetomechanical coupling is largely material dependent, the electromagnetic coil coupling is determined by the design of the magnetic circuit. Often this coupling is discussed indirectly in terms of a requirement for providing a magnetic return path, but the concept has broader implications. An attentive regard for the coil coupling (section 2.4.4.2) will suggest other opportunities to improve magnetic circuit design.

2. **Magnetic biasing:** A bias magnetic field may be accomplished in three ways: by remanent magnetic flux in the magnetostrictive material [7], by placing permanent magnet material in the magnetic circuit, or by impressing a DC electrical signal on a coil of wire wrapped around the magnetic circuit. For most magnetostrictive materials, the first method is completely impractical since the remanent flux is typically very small. Permanent magnet biasing is practical, but it puts a large reluctance in the magnetic circuit, lowering the coil coupling and raising the drive requirements. In contrast, direct current biasing does not add reluctance, but Joule heating in the bias coil raises several concerns. An additional cost of using DC biasing is the need for additional electronic equipment.

3. **Eddy current mitigation:** Joule heating also occurs within the magnetic circuit as a result of induced eddy currents. This can be a significant loss
mechanism if measures to impede the formation of these current loops are neglected. Common mitigation schemes are laminating the flux conductor (as in electrical transformer cores) and raising the electrical resistivity of the material (such as the inclusion of silicon in electrical steel).

There are other secondary (but significant) difficulties with magnetostrictive design. First, electric fields are much better channeled by electrical conductors and electrical insulators than magnetic fields are channeled by flux conductors and flux insulators. This causes undesirable flux leakage. Second, there is no standard experimental setup for measuring magnetostrictive parameters for materials that cannot be formed into a laminated toroid [52, 7]. Third, the operation of magnetostrictive transducers will be affected by the presence of any nearby ferromagnetic materials (such as steel).

Cumulatively, these challenges have generally made magnetostrictive design prohibitively complicated and generally unattractive in comparison to the competing piezoelectric technology. The following sections will review how these issues have been typically handled for nickel and Terfenol-D and then consider how to most effectively design with Galfenol.

4.2.1 Nickel design

Previous sections have discussed the groundbreaking work accomplished by HUSL in developing nickel-based magnetostrictive sonar transducers during World War II. The HUSL transducer shown in Figure 4.1 is an elegant design that incorporates all three of the features discussed in the previous section. First, parallel drive segments form a magnetic circuit, allowing the magnetic flux lines to form a closed loop in the nickel and permanent magnet; using two drive sections also increases the fraction of the magnetic circuit where useful magnetostriction occurs, thus raising the coil coupling. Second, a permanent magnet serves the dual purpose of biasing the magnetic circuit and adding mass to the tail segment of the transducer. Third, the single nickel structure that forms the head, tail, and drive segments is laminated, raising the onset frequency for full eddy current losses to above the operating band. With proper manufacture and assembly techniques, this method of transducer construction is both rapid and economical.
Figure 4.1. The HUSL nickel transducer has 60 laminates, each 0.15 mm thick. Overall, the transducer is 46.6 mm long and 16.5 mm wide with a stack height of 11.7 mm. Each coil has 30 winds and the bias is achieved with what is surmised to be an Alnico magnet.

Other creative configurations by HUSL for nickel laminations are considered in the NDRC report [7]. A few of these configurations are shown in Figures 4.2, 4.3, and 4.4. These ideas do not appear to have actually been built as successful transducers, but are important because conceptually they are precursors to the Galfenol design presented in this chapter. Moreover, they illustrate the flexibility available to a magnetics designer using a strong, machinable material. Figure 4.2 illustrates how adjacent magnetic circuits might be arranged into a single piece. Consideration of this scenario indicates that the inside segments will have twice the flux as the two outside segments — this is summarily addressed in Figure 4.3 in which equal magnetic flux density is achieved by making the outside segments half the width of the inside segment. An interesting DC biasing arrangement is shown in Figure 4.4, whereby coils wrapped on high permeability cores are inserted between faces of the laminated structure.
4.2.2 Terfenol-D design

Terfenol-D transducers cannot exploit the HUSL design (feasibly) because of the material’s brittleness; the HUSL design can only be implemented with materials that can be rolled or cut into thin sheets and machined into the final geometry. The low permeability of Terfenol-D ($\mu_r \approx 10$) causes flux leakage to be a greater concern. To cope with these issues, Terfenol-D designs often comprise a coarsely laminated rod surrounded by drive and/or biasing coils, encased with
a high-permeability material that serves as the magnetic return path [123, 124]. Alternatively, some designs use an axially-magnetized cylindrical permanent magnet around the coil to provide the magnetic bias [123, 125]. However, the low magnetic permeability of Terfenol has also made it common for drive rods to be segmented and interspersed with permanent magnets for better uniformity of the bias field [123, 60, 126].

4.2.3 Galfenol design

The discovery of magnetostriction in Galfenol represents an opportunity to re-explore the HUSL design and investigate new possibilities based on these types of laminated magnetostrictive structures. It seems likely that Galfenol will be best used in transducers resembling the HUSL designs. At present, efforts to roll Galfenol have encountered difficulty with material cracking and degradation of the magnetostrictive properties and/or crystalline texture. However, it is anticipated that continued work in this area will yield processes that will overcome these
obstacles. In the meantime, Galfenol laminations can be created with wire electric discharge machining (wire EDM) from highly-textured polycrystalline boules. This is how the drive mechanism detailed in this dissertation was constructed; in discussions with the material manufacturer, it is anticipated that the currently available boule-grown material will be representative of rolled Galfenol once the rolling processes have been sufficiently refined [121].

4.2.4 Decoupled DC and AC magnetic circuits

Forcing AC flux to traverse the low-permeability (high reluctance) region presented by permanent magnet material lowers the coil coupling. To avoid this, some designs attempt to use separate magnetic circuits for the DC and AC flux paths. Engdahl et al. presents two examples of this “parallel magnetism” [122, 62]. Also, decoupled static and dynamic magnetic circuits have been incorporated successfully in several balanced-armature transducer designs [127, 128, 129].

A simple example of how this might be accomplished for a magnetostrictive transducer is shown in Figure 4.5. The DC and AC flux are coincident in the magnetostrictive material; elsewhere they follow separate circuits. The AC flux is reluctant to cross the low-permeability permanent magnet material so it is primary confined to the AC return path. On the other hand, the DC flux is generated by the permanent magnet material and completes its circuit through the magnetostrictive material. This implies that the AC return path must be engineered to have a higher reluctance than the segment of active material, otherwise the DC flux will bypass the magnetostrictive, leaving it unbiased.

Notice that the DC circuit could easily be rotated out-of-plane and a second DC return path could be placed immediately opposite it to give improved DC flux uniformity. Better coil coupling could be achieved by adding a second magnetostrictive element on the opposite side of the AC return path with accompanying DC return path(s) circulating flux in the opposite spatial direction. Adding these features, however, makes the magnetic system more elaborate and more difficult to manufacture.

Ultimately, decoupled flux paths were not deemed appropriate for the Galfenol drive in this study due to the added complexity. At present, a streamlined so-
A simple schematic showing one possible configuration for parallel magnetization in a magnetostrictive transducer.

Figure 4.5. A simple schematic showing one possible configuration for parallel magnetization in a magnetostrictive transducer.

Solution for parallel magnetization in a self-contained magnetostrictive drive is not immediately obvious. Therefore, the drive design has coincident DC and AC flux paths. If, however, future work can cleverly decouple the DC and AC circuits, it will raise the electromagnetic (and by extension, the electromechanical) coupling of the device.

4.3 Configurable drive

As has already been stressed, for magnetostriction to gain traction in transduction applications, refined designs are needed to complement the merits of the materials. This is especially true for a relatively new material like Galfenol that has an unique set of properties.

This section details a concept for a self-contained Galfenol drive incorporating all three of the features listed in section 4.2 so that additional structures are not needed for the magnetic system. These features are considered individually below. The key element in this design is a dogbone-shaped Galfenol geometry, shown in Figure 4.6. Combining two such laminated structures forms the basic structure of
Figure 4.6. A schematic of the dogbone shaped lamination geometry used in the configurable drive. Because of the flexibility of the drive, each realization of the design will have different dimensions, but should retain the basic geometrical features.

the drive. The entire concept for the drive is shown in Figure 4.7 and the author has chosen to designate this design as the *Galfenol configurable drive* (GCD).

1. **Coil coupling** is addressed by creating a magnetic circuit with two dogbone-shaped Galfenol structures to form two parallel drive segments. Biasing with permanent magnets placed between the dogbones decreases the coil coupling, but may be necessary to avoid the complications of a DC biasing scheme.

2. **Magnetic biasing** is accomplished with either direct current or permanent magnets, the design is equally suited to both. A pair of permanent magnets placed between the two Galfenol structures may be used or a DC coil may be wound around part of the magnetic circuit.

3. **Eddy current mitigation** is effected by building the Galfenol structures from laminated stacks in which adjacent laminations are electrically insulated from one another.

Dogbone-shaped structures have many benefits. First, the long and slender geometry allows laminations to be created from narrow material forms such as Bridgman grown boules or rolled strips. Second, having the ability to separate the two halves of the magnetic circuit greatly eases the winding of drive coils since threading or routing of the magnet wire is not required. However, cutting the magnetic circuit in half creates small air gaps that act to decrease the coil coupling.

The proposed drive is configurable because it may be tailored to specific applications: the designer can adjust the dimensions of the drive and/or use multiple
drives in a modular fashion. Because the dimensions of the drive may be adjusted, the proposed design is not intended to be an “off-the-shelf” solution. Connecting drives modularly may be done in either a side-by-side or end-on-end fashion. They can also be arranged by stacking, but this is equivalent to just building up the stacks to a greater height with more laminates. An important consideration with connecting drives side-by-side is that the flux levels in the outside drive segments will be less than for the inside segments unless their areas are varied accordingly (refer to Figure 4.3). One might question why drives would ever be used in an end-on-end configuration since a single long drive is ideal for coil coupling. The answer is that an elongated drive is more susceptible to leakage flux so, if the gap spacing between drive segments cannot be increased, end-on-end drives may be an appropriate solution.

Most importantly, the configurable drive concept is intended to reduce the
complexity of the magnetic circuit design problem so that it is more accessible to designers, recasting the physical scenario into a simplified engineering problem that avoids the complications of the full analysis.

4.3.1 Piezoelectric parallel

One way to understand the Galfenol configurable drive concept is to consider an existing design with a similar form and function. Within the scope of sonar transducers, a very common extensional drive is the piezoelectric stack [89, 88, 90, 91, 8]. This drive is used in a variety of transducers, most prominently in tonpilz designs. The design consists of a stack of piezoelectric rings or plates that are assembled such that the direction of the rings’ polarity alternates. Electrodes interspersed between the rings allow an AC electric field to be applied so that the entire assembly will expand and contract in unison. This arrangement is shown in Figure 4.8 for a typical tonpilz design.

The advantage of the piezoelectric stack design is that the designer can tweak the drive to meet the design requirements of a particular transducer. The number of rings, the ring dimensions, and the piezoelectric material that the rings are made from are all selected by the designer, but the basic drive design is retained: the mechanism does not need to be redesigned from scratch each time it is used.

The author’s vision for a Galfenol drive parallels the piezoelectric stack design. Instead of using rings of piezoelectric material, dogbone-shaped laminated stacks of Galfenol are assembled to form magnetic circuits. Just as for the piezoelectric stack, the Galfenol configurable drive is not a one-size-fits-all approach, but rather a general design concept that may be specifically applied to specific applications. The designer must select the drive dimensions, the number of circuits, and the bias mechanism(s), but does not need reinvent the system.

4.4 Applications

It is envisioned that, the Galfenol configurable drive could serve as the motor section for a variety of transducers. Obvious applications are in tonpilz, flextensional, and split cylinder designs. Mockups of how the drive might be incorporated into
Figure 4.8. A typical piezoelectric tonpilz transducer shown in cross-section, displaying the piezoelectric stack drive. Note the wiring of the electrodes and the ring polarities. Figure reproduced from Sherman and Butler § (With kind permission from Springer Science+Business Media: Transducers and Arrays for Underwater Sound, Chapter 3, 2007, p. 93, C. H. Sherman and J. L. Butler, Figure 3.17).

the tonpilz and flextensional transducers are shown in Figures 4.9 and 4.10 below. These images show an early concept of the drive, but the final design is also applicable.
Figure 4.9. A mockup of a tonpilz type transducer using configurable drives for the motor section.

Figure 4.10. A cutaway mockup of a flextensional type transducer incorporating configurable drives.
This chapter details the practical realization of the Galfenol configurable drive (GCD) concept presented in Chapter 4 through the construction of a prototype drive. Discussed are the design methods, fabrication techniques, drive models, and qualifying measurements.

5.1 Design

Designing satisfactory magnetostrictive motors requires consideration of the electrical, magnetic, mechanical, and thermal performance of the device [122]. This section describes the analytical approaches and modeling efforts used to create a design that performs satisfactorily in each of these four areas. The design process consists of identifying design criteria, constructing a one-dimensional design model, verifying the results with a three-dimensional FEA model, and investigating the thermal considerations of the design.

5.1.1 Design criteria

A Galfenol device must take full advantage of the material’s strain capabilities in order to be competitive against other active materials that have similar or higher strain capabilities, e.g. single crystal piezoelectrics, piezoceramics, and Terfenol-
D. Therefore, the author chose access to full magnetostriction as the number one design criterion for the prototype drive. Having established this goal, identification of the magnetic design requirements is straightforward through the use of the $S-B$ curve. $S-B$ and $B-H$ measurements by ETREMA Products, Inc. on a rod of Bridgman-grown Galfenol steel (18.4 at. % Ga + 1002.5 steel) material at 45.5 MPa prestress [121] served as the reference data for the drive design.

By examining the reference $S-B$ curve shown in Figure 5.1, it is seen that nearly the entire linear strain region can be is accessible with a $1 \, \text{T}_{\text{pp}}$ drive operating around a $1 \, \text{T}_{\text{DC}}$ bias. Outside of this range, the material’s strain response is strongly nonlinear: magnetic saturation begins around 1.5 T and below 0.5 T the material exhibits quadratic behavior. Deviation of the reference data from the swing line shows that the material is not perfectly linear in this region, but only approximate. With no major nonlinearity found in the $S-B$ relationship, the desired operating range was then examined on $B-H$ curve to confirm the absence of significant magnetic nonlinearity. The second design goal for the prototype drive is to engineer a coil that uses 1 A of drive current to induce the $1 \, \text{T}_{\text{pp}}$ of magnetic excitation.

### 5.1.2 Biasing scheme selection

Achieving the desired $1 \, \text{T}_{\text{DC}}$ bias requires using either permanent magnets or direct current coils. The relative merits of both biasing schemes have already been discussed in section 4.2. Although the GCD concept is able to easily incorporate either system, permanent magnets were used primarily to avoid thermal issues that might arise from the Joule heating of a biasing coil. Thermal considerations for the GCD drive are discussed in detail in section 5.1.7. Another benefit of choosing this scheme is that without a biasing coil, there is more room on the magnetic circuit for winding drive coils, which will aid in attaining the $1 \, \text{T}_{\text{pp}}$ drive design goal.

A secondary design choice for the biasing scheme was the distribution of the permanent magnet material in the magnetic circuit. Although using two magnets increased the number of low-permeability gaps in the magnetic circuit, this arrangement provided a significant improvement in the uniformity of the bias field.
Figure 5.1. \(S-B\) and \(B-H\) curves for the Galfenol steel reference data (red) showing desired bias point (black circle) and drive swing (black line).

along the length of the drive segment. Using a single magnet in a slot at one end of the circuit (as in the HUSL design) imposes a bias field that decreases with distance from the magnet. This type of non-uniform bias is avoided because such an arrangement prevents the entire drive segment from operating around the optimum bias, meaning that some portions of the Galfenol are being under-utilized. Also, a variable bias device will be difficult to model properly.

5.1.3 Permanent magnet material selection

An important part of the drive design is the selection of the permanent magnet material. Because the magnets in the GCD concept are situated in the AC flux path it is advantageous to minimize their magnetic reluctance. Since the required thickness of the magnet is inversely proportional to its coercive field and \(\mathfrak{R} = \ell / (\mu A)\), an ideal material will have a large coercive field and high magnetic permeability.
However, a tradeoff is presented by the available materials: high coercive fields are usually found in materials with low magnetic permeabilities.

Specifically, the author wished to compare Alnico and neodymium (NdFeB) magnets. Neodymium has a greater coercive field but Alnico has a higher permeability. To determine which material fulfills the magnetic requirements with the lowest reluctance, the mmf source reluctance model of a permanent magnet (section 3.5.5) is considered. It seems reasonable to expect the best permanent magnet to produce a large magnetomotive force, $\mathbf{F}$, while maintaining a low magnet reluctance, $\mathcal{R}_m$, so that for a thickness $t$ and cross-section $A$ it maximizes the figure of merit (FOM):

$$\text{FOM} = \frac{\mathbf{F}}{\mathcal{R}_m} = \frac{H_c t}{(\frac{t}{\mu_{\text{rec}}A})} = \mu_{\text{rec}} H_c A$$  \hspace{1cm} (5.1)

where $\mu_{\text{rec}}$ is the recoil permeability of the permanent magnet. For a given cross-section, the figure of merit reduces to $\mu_{\text{rec}} H_c$. Application of this selection criterion to Alnico and NdFeB yields the values shown in Table 5.1. With this choice of FOM, there appears to be a clear advantage to using neodymium over Alnico.

<table>
<thead>
<tr>
<th>Material</th>
<th>$\mu_{\text{rec}}$</th>
<th>$H_c$ [kA/m]</th>
<th>$\mu_{\text{rec}} H_c$ [kA/m]</th>
<th>$(BH)_{\text{max}}$ [kJ/m$^3$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alnico 2</td>
<td>6.8</td>
<td>45</td>
<td>306</td>
<td>12.7</td>
</tr>
<tr>
<td>Alnico 3</td>
<td>5.1</td>
<td>38</td>
<td>194</td>
<td>11.1</td>
</tr>
<tr>
<td>Alnico 4</td>
<td>4.1</td>
<td>57</td>
<td>234</td>
<td>10.7</td>
</tr>
<tr>
<td>Alnico 5</td>
<td>3.7</td>
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<td>189</td>
<td>43.8</td>
</tr>
<tr>
<td>Alnico 6</td>
<td>5.6</td>
<td>60</td>
<td>336</td>
<td>31.0</td>
</tr>
<tr>
<td>N42 NdFeB</td>
<td>1.05</td>
<td>875</td>
<td>919</td>
<td>334.2</td>
</tr>
</tbody>
</table>

Table 5.1. Comparison of permanent magnet materials. Alnico values were retrieved from [12] and NdFeB properties from [10]. Recoil permeabilities are relative. Maximum energy product values are also shown for comparison.

Of course, this comparison has been made solely on the basis of magnetic performance. In practical magnet selection, there are many other factors that must also be considered such as Curie temperature, brittleness, and corrosion resistance. Using the FOM as an initial guide, the author reviewed the other key material characteristics. Since no critical limitations were found for NdFeB, it was selected for the Galfenol drive design.
5.1.4 One-dimensional design model

An important outcome of this project is the development of simplified tools that enable rapid tailoring of the GCD concept. The purpose of these design tools is to allow the magnetic circuit and geometry to be quickly adjusted to fit a specific application without the need for intensive FEA modeling. The one-dimensional modeling techniques discussed in Chapter 3 are well-suited to meet the requirements for simplicity and speed. Figure 5.2 illustrates a scheme for dividing the generalized magnetic circuit into a collection of lumped elements that are geometrically defined by eight dimensions. This setup can be used to analyze both the DC and AC performance. Notice that a critical – but easily overlooked – component in the magnetic system is air used as the flux insulator in the center of the circuit (shown in blue in Figure 5.2). Therefore, completely defining the model requires specifying 1) the eight dimensions for the circuit geometry, 2) the magnetic permeabilities of the materials in the system, and 3) either the remanent induction or coercive field of the permanent magnets (see section 3.5.5). Most reluctances are calculated with equation 3.10, but the reluctance of the permanent magnet short-circuit paths are determined with equation 3.11 and the corner reluctances use equation 3.12. It is important to note that this model is invariant to the stack height (the out-of-the-page dimension in Figure 5.2). This means a design based on this model is valid no matter how many lamina are used, assuming the permanent magnets are the same height as the stack.

Numerical solution of this model framework is accomplished by constructing a resistor network in SPICE based upon the magnetic reluctance analogy. Two such networks are shown in Figure 5.3 which allow for the simultaneous solution of the DC and AC systems with a single operating point analysis. Although it is possible to simulate the DC and AC flux on a single network, the advantage of this method is that the output is immediate and consists of single values for $B_{DC}$ and $B_{AC}$, making the model easier to use and understand.

Because permanent magnet biasing is employed, the DC network models a single magnetic reluctance domain. For the AC network, however, an electrical domain is also present with the magnetic domain so that the AC flux is derived from an applied current. The benefit of this arrangement is it includes the drive coils in the magnetic circuit design. Note that in avoiding an .ac analysis, the AC
Figure 5.2. A reduced design model concept that defines the 2-D geometry as 22 lumped reluctances, all of which can be defined with eight dimensions. Flux paths are illustrated with dashed lines and show leakage across the circuit and around the permanent magnets.

current is specified with a DC source.

SPICE B-sources are used to perform output calculations. The static and dynamic $B$ and $H$ fields in the drive segments are determined with sources $B\_HDC$, $B\_BDC$, $B\_HAC$, and $B\_BAC$. These geometry-scaling calculations are required since the magnetic reluctance analogy has mmf, $\mathcal{F}$, and flux, $\varphi$, for the effort and flow
Figure 5.3. A one-dimensional model that realizes the reduced design model framework using two independent circuit networks to simulate DC and AC conditions. Parameter statements are not shown, but included in the Appendix D.
variables. A fifth B-source, BX3, is written in netlist form for compact notation and estimates the coil coupling by calculating the magnetic energy in each lumped element.

Now the design process is reviewed. The first step is to select a permanent magnet to bias the circuit. This step serves as the starting point in order to incorporate off-the-shelf magnets into the design so custom-sized magnets are avoided. In general, it is desirable to maximize the area normal to the magnetization direction – this decreases the magnet’s reluctance and increases the flux. However, a tradeoff occurs in the third dimension: increasing the magnet thickness will raise the mmf but also increases the magnet’s reluctance. Therefore, trial-and-error may be required to determine the minimum magnet thickness needed to achieve the bias design goal. Once the magnet is selected, two of the eight model dimensions (tmag and wmag) are established along with the height dimension of the stack. For the prototype drive, the thinnest readily-available NdFeB magnet (1/32” thick) was chosen with a 1/2” × 1/2” cross-section. Even though all neodymium magnets are brittle, the thin geometry made these prone to cracking during handling, especially when magnetic attraction caused them to forcefully snap against other magnetic materials.

The next consideration in the design is to create maximum spacing in between the two drive segments in order to limit flux leakage. Being manufactured from Bridgman-grown material, the maximum lamina width for the prototype drive was constrained to be 15 mm[12]. Because a device built from Galfenol is expected to be mechanically rugged, a drive segment width (wgf) of 5 mm was selected to avoid artificially weakening the structure. Experimentation with the model showed that a 2mm fillet radius (rfillet) provided satisfactory performance, thereby constraining t2 to be 3mm.

In this approach, the most important lamination dimension, lcir, is determined last. From a coil coupling perspective, the larger lcir, the better, but flux leakage limits this length. As lcir increases, so does its magnetic reluctance. At some point the circuit becomes so elongated that a portion of the flux prefers to traverse a low permeability path through the air between the drive segments rather than travel the full length of the circuit. With the design model accounting for the effects of leakage, iterative evaluation found 35 mm to be a reasonable value
for $l_{cir}$. An assumed value of 0.1 mm for the thickness of the low-permeability regions (comprised of air, epoxy, and corrosion-resistant magnet plating) on either side of the permanent magnets ($l_{ag}$) completes the specification of the DC problem.

Coil design requires specifying the diameter of the magnet wire, $d_{wire}$. For the 29 American wire gauge (AWG) magnet wire used in the prototype, the diameter of the wire with insulating enamel was $\varnothing 0.32$ mm ($\varnothing 0.29$ mm for the bare wire). The process of wire gauge selection is explored later in section 5.1.6. This dimension includes the insulating enamel so that the number of turns that fit in a single layer can be estimated by calculating $l_{cir}/d_{wire}$ and rounding down. However, when 1 A of current was evaluated for the prototype drive with a single-layer coil, the AC induction falls short of the design goal. To overcome this, multiple coil layers must be used. The model was employed to determine the minimum layer count ($layers$) required to achieve an AC amplitude of $> 0.5$ T (corresponding to the $1T_{pp}$ design goal). For the prototype drive design, three layers were found to be sufficient.

Only the benefit of hindsight allows the design method to be presented in such a straightforward and linear manner. In actuality some experimentation was needed to find feasible values and, in the end, iterative adjustment of dimensions was used to tweak the magnetic performance of the circuit to meet the design criteria. In SPICE, it is sometimes useful to employ the .step command for parameterizing variables. This feature allows optimum values to be quickly explored for a given variable.

Using the values just discussed, the design model yields the magnetic performance results shown in Table 5.2. Predicted results for both the DC and AC induction slightly exceed the design goals.

Table 5.2. Galfenol magnetic circuit performance calculated with the design model.

<table>
<thead>
<tr>
<th></th>
<th>(B_{HDC}[kA/m])</th>
<th>(B_{BDC}[T])</th>
<th>(B_{HAC}[kA/m])</th>
<th>(B_{BAC}[T])</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>8.266</td>
<td>1.039</td>
<td>4.164</td>
<td>0.523</td>
</tr>
</tbody>
</table>

\(^{1}\text{The NdFeB magnets used in this dissertation had a 15-21 \(\mu m\) nickel-copper-nickel plating. Because this was only about 5\% of the total magnet thickness, the plating was not modeled separately.}\)
The fact that this is a very reduced model should not be overlooked. Mechanical and acoustical effects are intentionally neglected in order to focus on quickly establishing a basic magnetic circuit design. Once satisfactory performance is achieved, the results should be implemented in a more sophisticated model to verify the performance.

5.1.5 FEA magnetic circuit design verification

Combining stack height with the dimensions and parameters from the one-dimensional model completely defines geometry of the drive, making it possible for three-dimensional modeling with FEA software. The geometry modeled in COMSOL Multiphysics is shown in Figure 5.4. Because of the complexity of the software package, all the details of the model’s implementation cannot be examined here. However, several important aspects of the model are considered:

First, the drive coil is represented with four discrete isosceles-trapezoidal subdomains that describe sheets of current. An alternating current density, \( J = \frac{i_{\text{drive}}}{\left( \frac{\pi}{4} d_{\text{wire}}^2 \right)} \), is imposed in a coordinate direction for each subdomain such that...
Figure 5.5. Schematic cross section of the GCD’s drive segments (in gray), showing simplified coil modeling using four sheets of current.

it forms a piecewise approximation to the circulating current paths. This setup is shown in Figure 5.5.

Second, the electrical conductivities of the Galfenol and copper subdomains are reduced to keep simulated eddy current losses in check. Version 3.5 of COMSOL does not include the ability to impose an electrical insulation condition on internal boundaries which makes modeling coils and laminated structures difficult. Thin air domains can be included between lamina, but the small dimensions lead to excessive mesh densities which significantly increase the demand for computational resources. Therefore, since these thin, electrically-insulating layers are not resolved by the model, decreasing the material conductivity is used to represent an effective value.

COMSOL allows for implementation of a nonlinear $B$-$H$ curve. To take advantage of this capability, a least-squares fit of experimental data was used to create a model curve. The complete details of this fit are discussed in the next section.

Transient simulation of the drive was performed to corroborate with the outputs of the one-dimensional model. To aid in convergence, the model starts with all magnetic sources set to zero. Once the simulation begins, the remanent induction of the permanent magnets is smoothly ramped up using a Heaviside function. The model is allowed to stabilize at this bias point. Next, the sinusoidal drive current in the coils is increased with a similar function and allowed to continue for
Figure 5.6. A plot of the magnetic induction, $B_z$, in the center of each drive portion. Because of the loop, the two drive portion experience the field in an opposite spatial directions.

several cycles. A 1 A, 16 kHz sine wave excitation is used since this frequency had been predicted as the resonance of the device based on an eigenfrequency analysis. Simulation results are shown in Figure 5.6 showing the value of the longitudinal component of the $B$ field measured in the center of both drive sections. This shows a bias of just under 1 T<sub>DC</sub> and a drive amplitude slightly above 1 T<sub>pp</sub>. Both of these values are in reasonable agreement with the one-dimensional model results.

5.1.5.1 Nonlinear $B$-$H$ modeling

Nonlinear magnetic modeling in the COMSOL model was implemented with a least-squares fit to the reference $B$-$H$ curve for Galfenol steel (presented earlier in Figure 5.1 shown here over an extended range in Figure 5.7). Because of the very narrow hysteresis loop, an anhysteretic fit to measured data was judged to be sufficient.

In each direction, Galfenol’s $B$-$H$ curve exhibits three distinct slopes (three
regions) separated by two-corners making it impossible to model with Langevin’s function \cite{3}. Instead, a ten-parameter model was devised:

\[
B = B_0 + \mu_0 \frac{m_1 (H - H_0) + m_2 (H - H_0) \chi_1 + \left( m_3 (H - H_0) + \frac{B_3 - H_0}{\mu_0 |H - H_0|} \right) \chi_2}{1 + \chi_1 + \chi_2}
\]  
(5.2)

where

\[
\chi_1 = \left[ \frac{H - H_0}{H_{C1}} \right]^{\xi_1}
\]  
(5.3)

and

\[
\chi_2 = \left[ \frac{H - H_0}{H_{C2}} \right]^{\xi_2}
\]  
(5.4)

The ten parameters are used in this model are: \(B_0\), the \(B\) offset; \(H_0\), the \(H\) offset; \(m_1\), the slope in the first region; \(m_2\), the slope in the second region; \(m_3\), the slope in the third region; \(B_3\), the \(B\) value of the second corner; \(H_{C1}\), the value of \(H\) at which the first corner occurs; \(H_{C2}\), the value of \(H\) at which the second corner occurs; \(\xi_1\), a variable controlling the sharpness of the first corner; and \(\xi_2\), which controls the sharpness of the second corner. The data and the ten parameter fit are shown in Figure 5.7. Overall, the model represents the measured data well, but for this case the fit degrades slightly at the saturating corners.

Initial guesses for each parameter (except \(\xi_1\) and \(\xi_2\)) may be extracted from measured data. Then, a MATLAB routine adjusts the parameters to find the best fit to the measured data (this script is included in Appendix D). The model values returned by the routine are given in Table 5.3. Although COMSOL could use these values to calculate the \(B-H\) curve on-the-fly, it is more convenient to tabulate the model and interpolate.

### 5.1.6 Coil design

A particularly complicated aspect of the design is engineering the drive coils to achieve the AC magnetic induction target \((1 \text{Tpp})\). While the one-dimensional model is able to produce a rough design, it assumes that the magnet wire gauge
Figure 5.7. Anhysteretic fit to measured B-H data for Galfenol steel.

has already been selected. Choosing a suitable gauge is not as straightforward as it seems: lower gauge wire has less resistance per unit length but higher gauge wire can achieve a higher mmf by fitting more turns in the available space. Since $\mathcal{F} = n i$, the magnetomotive force generated by the coil can be increased by either winding more turns or impressing more current. However, Joule heating is based on $i^2 R_e$ losses so to minimize these losses it is clearly preferable to increase $\mathcal{F}$ by adding more turns (more $R_e$) than to increase the drive level (more $i$). Proper resolution of this tradeoff involves finding the highest gauge wire for which the electrical power supplied during normal operation does not cause deleterious effects.

Approaching this problem from a purely electrical standpoint is difficult since there are very few guidelines based on the electrical scenario. Onderdonk’s [131] equation may be used to calculate the short time it takes to melt a copper wire carrying a very high current. However, melting the copper is a very unlikely (and catastrophic) failure mode – other limiting effects will occur at much lower currents. Watson’s [132] treatment of the issue is more applicable, but only offers
Table 5.3. Values for the B-H model fit shown in Figure 5.4.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_0$</td>
<td>-1.647E-02</td>
</tr>
<tr>
<td>$H_0$</td>
<td>-6.682E+02</td>
</tr>
<tr>
<td>$m_1$</td>
<td>6.323E+01</td>
</tr>
<tr>
<td>$m_2$</td>
<td>7.138E+01</td>
</tr>
<tr>
<td>$m_3$</td>
<td>1.117E+00</td>
</tr>
<tr>
<td>$B_3$</td>
<td>1.578E+00</td>
</tr>
<tr>
<td>$H_{C1}$</td>
<td>4.304E+03</td>
</tr>
<tr>
<td>$H_{C2}$</td>
<td>5.508E+03</td>
</tr>
<tr>
<td>$\xi_1$</td>
<td>1.409E+01</td>
</tr>
<tr>
<td>$\xi_2$</td>
<td>2.009E+01</td>
</tr>
</tbody>
</table>

It is more appropriate to view this problem from a thermal perspective. At some point the electrical power will cause excessive heating to the degree that the drive is permanently damaged. Understanding the relationship between the electrical power and the coil temperature necessitates a broader view of the thermal scenario that encompasses the entire transducer and its surrounding environment. This will be discussed in the next section.

Eventually, 29 AWG magnet wire was selected since it was readily available and it could achieve enough windings to reach the magnetic drive goal. Also, this wire has a copper diameter that is small enough to raise the critical frequency to $f_c = 104$ kHz (calculated with equation 2.11) — well above the anticipated band of interest. The next step in the design process was to confirm that thermal effects would not be an issue. Although Montgomery [133] provides some analytical methods for calculating the thermal performance of a coil, for the prototype drive it was easiest to employ FEA modeling.

5.1.7 Coil design investigation with thermal modeling

Before building to the drive design, it first had to be established that the predicted magnetic performance would not be limited by thermal effects. Joule heating occurs in the coils due to the drive current and within the lamina from the induced
eddy currents. To investigate the thermal behavior, an FEA model was used. The goal of this model was not to provide an exact description of the drive’s thermal performance, but rather to estimate whether the thermal effects would interfere with the magnetic system. Thermal limitations for the Galfenol drive are:

- exceeding the maximum operating temperature of the neodymium magnets (45-80°C [130])
- burning the insulating enamel off the magnet wire (approx. 100°C)
- removing the stress annealing (120-150°C [121])
- melting the electrical insulation between lamina (160°C [121])
- exceeding the Curie temperature of the neodymium magnets (312°C [74])
- exceeding the Curie temperature of the Galfenol (700°C [31])

Building the thermal simulation on top of existing electromagnetic models is impractical since the physical processes occur at radically different time scales. Therefore, heat sources were specified in a heat conduction analysis and natural convection was simulated by coupling into a Navier-Stokes fluid transport analysis. In the model, as the transducer heats up it warms the nearby air, lowering its density and making it buoyant compared to the surrounding, cooler air.

In the design phase it is impossible to compare simulation results to measurements on the finished device. To work around this, a Galfenol reproduction of the HUSL transducer (discussed in more detail in section 5.2.3.1) was modeled and measured. The measurement setup suspended the transducer on a thin wire in front of a Mikron TS7300/TS7302 thermal camera. This is shown in Figure 5.8.

Test conditions for the transducer were continuous wave (CW) operation for 90 s at 12 kHz, 1 A drive followed by 90 s of cooling. The measurement point was at the center of the drive coil. Since Galfenol’s thermal properties are undetermined, the model used the heat capacity and thermal conductivity of iron. Heat generation in the coils was specified by Joule heating arising from the drive current. In the lamina, heat generation was calculated using equation 2.12.

Measurement and simulation results are shown in Figure 5.9 for three different frequencies. Investigating these three frequencies yielded insight into the model’s
treatment of resistive heating from induced eddy currents in the lamina. When eddy currents are negligible (the 10 Hz case), the model is more accurate. Interestingly, the model also does not accurately simulate cooling for the 100 Hz and 1000 Hz cases, despite using the Navier-Stokes analysis for natural convection. Even though differences between the model and measurement can be clearly seen because of the close vertical axis, it is important to note that all error is less than 1.5°C. Ultimately, the goal of conducting the thermal modeling is to establish a rough prediction of the thermal performance, so this degree of error is tolerable.

With these measurements supporting the model results, the same analysis was applied to the GCD geometry to predict the heating for different drive conditions. This is shown in Figure 5.10. Because of the limitations discussed previously, the goal of the model was to show the transducer could operate for 90 s and stay below 80°C. Even for duty cycles that are well above what is typically used in practice, the coil temperature was predicted not to exceed 45°C.

Based on the results of this study, it is concluded that the temperature rise in the prototype drive design is not enough to limit magnetic performance during normal operation.
Figure 5.9. Measurement versus simulation for the heatup of the HUSL Galfenol reproduction transducer. The transducer is driven at CW for 90 s and allowed to cool for an additi

5.2 Fabrication

Having arrived at a suitable design through modeling efforts, the next step was to fabricate prototype drives. Galfenol material for the drives was supplied by ETREMA Products, Inc.

5.2.1 Galfenol steel material

For the prototype drive, nine cylindrical boules 25 mm in diameter and 75 mm long (\(\varnothing 25 \text{ mm} \times 75 \text{ mm}\)) of highly-textured Galfenol steel were grown with the advanced Bridgman method. Six of the boules are pictured in Figure 5.11. This material was used because it is believed that future work on rolling processes will yield similar performance from rolled Galfenol [12]. ETREMA Products, Inc. supplied quasi-static characterization curves for all nine boules before and after stress annealing;
Figure 5.10. Simulation of the GCD heat up for 100% (CW) and 50% duty cycles. After 90 s of CW operation and 180 s operation at 50% duty cycle, the transducer temperature does not exceed 45°C.

these data are plotted in Appendix B.

5.2.2 Lamina manufacture and preparation

Traditionally, stacked lamina are manufactured by rolling the material into thin sheets and creating the final geometry through a punching process [7, 80]. The unavailability of high-strain rolled Galfenol has already been discussed in section 1.3.1.14. In large quantities, punching is advantageous since this should allow laminates to be made quickly and inexpensively. For the small-scale work presented here, these methods were avoided, primarily because of the costs associated with tooling. There was also concern over the possibility of cracking and fatigue hardening the material for both the rolling and punching processes, especially in light of the embrittlement that has been observed in heat-treated rolled Galfenol by Brooks et al. [46] and Kellogg [24]. Instead, wire electrical discharge machining
(EDM) was used to cut the boules into the final lamination geometry, shown in Figure 5.12. The thickness was chosen to be 10 mil (0.254 mm) — the minimum machinable thickness [121]. For a relative permeability of $\mu_r = 100$ and an electrical conductivity of $\sigma = 1.2 \times 10^6 \text{S/m}$, this thickness corresponds to a critical frequency of 65 kHz (equation 2.11). This is a satisfactory value as it means that the drive performance will not be degraded by the full onset of eddy current losses for two full octaves above the initial 16 kHz resonance prediction. With such a high degree of lamination, other factors are likely to limit the upper extent of the drive’s bandwidth before eddy currents are fully formed.

The boules were machined by ETREMA Products, Inc. with a 6 mil wire. Using the smallest wire possible was critical to maximizing lamina yield by decreasing kerf loss. It was initially projected that each boule would yield 50 laminations, but, in practice, a maximum of 40 laminations per boule was attained. Moreover, internal cracking in one boule resulted in zero yield and machining-induced cracking along grain boundaries limited the yield in other boules [121]. Similar cracking along grain boundaries in polycrystalline Galfenol has been observed by Kellogg [24]. The results of the machining operation are summarized in Table 5.4 and the machined lamina are shown in Figure 5.13.
Figure 5.12. Part drawing for configurable drive lamination showing top and side views. All dimensions are in millimeters.
Table 5.4. Summary of wire EDM machining on stress annealed highly-textured polycrystalline Galfenol steel boules.

<table>
<thead>
<tr>
<th>Boule</th>
<th>Good parts</th>
<th>Damaged parts</th>
<th>Strain [ppm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1-9-40-1</td>
<td>23</td>
<td>17</td>
<td>221</td>
</tr>
<tr>
<td>D1-9-40-76</td>
<td>21</td>
<td>18</td>
<td>203</td>
</tr>
<tr>
<td>D1-9-40-153</td>
<td>0</td>
<td>0</td>
<td>234</td>
</tr>
<tr>
<td>D1-9-42-1</td>
<td>38</td>
<td>0</td>
<td>252</td>
</tr>
<tr>
<td>D1-9-42-76</td>
<td>36</td>
<td>1</td>
<td>200</td>
</tr>
<tr>
<td>D1-9-42-153</td>
<td>40</td>
<td>0</td>
<td>194</td>
</tr>
<tr>
<td>D1-9-39-3</td>
<td>36</td>
<td>0</td>
<td>229</td>
</tr>
<tr>
<td>D1-9-39-78</td>
<td>20</td>
<td>0</td>
<td>196</td>
</tr>
<tr>
<td>D1-9-39-153</td>
<td>19</td>
<td>0</td>
<td>215</td>
</tr>
</tbody>
</table>

*Reduced yield because of crack near boule end
*No yield because of crack down middle interior of boule that was probably created during stress annealing

Figure 5.13. Individual laminations shown on the left with a US nickel and a \( \frac{1}{32}'' \times \frac{1}{2}'' \times \frac{1}{2}'' \) NdFeB biasing magnet. On the right a single lamination is shown in detail. These are lamina made to the drawing shown in Figure 5.12.

During machining, the workpiece was cooled by immersion in a water bath. This resulted in a layer of surface rust on the laminations. No attempt was made to remove this rust since it aids in electrically insulating adjacent laminations.

Upon receipt of the completed laminations, all pieces were cleaned in a vapor degreaser. The laminations were placed in a heated vapor (105°C) of trichloroethylene for approximately 20 minutes. ETREMA Products, Inc. advised not heating laminations over 120°C since degradation of the stress annealing would occur.
around 150° C [121].

Special care was required to handle the lamina - the average lamina thickness was measured to be 0.262±0.001 mm – roughly three times the thickness of piece of office paper. Lamina could be easily bent or creased by careless handling. During cleaning and assembly, a small number of laminations cracked easily at the ends. Remarkably, these cracks did not pass through the alignment notch feature.

5.2.3 Stack assembly

Assembling the laminates into stacks proved to be a difficult endeavor. The final stack assembly should achieve a good fill factor, provide satisfactory electrical insulation between all lamina, and provide a solid mechanical bond. The HUSL report [7] describes three methods of consolidating the laminations into stacks:

1. An epoxy spray can be used to coat the laminations on both sides. Doing this necessitates laying out the laminations to spray one side and then flipping them over to spray the other side. After a minute or two, the epoxy tacks up so the pieces may be handled and stacked.

2. Laminations can be coated by dipping them into the epoxy. This method requires withdrawing the laminations from the epoxy at a constant linear rate to avoid a bead forming at the trailing edge. The coated lamina are then assembled in the same manner as the first method.

3. Capillary action can be employed to draw epoxy in between stacked lamina. To do this, a clean stack of laminations is brushed with epoxy around the edges. The NDRC report states that “[t]his procedure results in stacks that are uniform in frequency behavior and of high potential efficiency but moderate strength.”

In all three cases, the epoxy is cured while the stack is under compressive stress.

The “Cycleweld” epoxy described in the NDRC report [7] is no longer available, requiring the use of a different epoxy. A suitable replacement epoxy must meet several requirements. First, the epoxy should allow sufficient working time and demonstrate good working abilities — assembling very sticky laminations is challenging because of their fragility. Second, the epoxy must form an appropriate
mechanical bond. This is complicated by the HUSL NDRC report warning that it was “repeatedly demonstrated that too strong a bond between laminations is a source of trouble.” Finally, it should be remembered that the epoxy will be used in a mechanical parallel configuration, and not in series as it is with piezoelectric rings. For the latter case, a high elastic modulus is desirable whereas a low modulus is best suited for the laminated stack so as to not clamp the active material.

5.2.3.1 Assembly study

Before building the configurable drive, an assembly test was conducted to experiment with building a laminated Galfenol structure. This was important because even though the NDRC report [7] details many assembly techniques, a great deal of practical experience and general know-how for constructing these transducers has been lost. The HUSL transducer was used as the design template, but lamina were made from rolled Galfenol steel material that was cut into its final form with wire EDM. Galfenol steel lamina and the finished Galfenol reproduction transducer are pictured in Figure 5.14.

In this project, the author investigated duplicating the sprayed-epoxy approach using an aerosol adhesive to consolidate stacks. This scheme uses the adhesive as both mechanical bond and electrical insulation. Many industrial aerosol adhesives have a coarse spray pattern that results in very uneven coverage on small pieces; an acceptably fine spray pattern was achieved with an aerosol paper adhesive. For the 0.5 mm-thick laminates used on the Galfenol HUSL reproduction transducer, this appeared to be an adequate solution.

A special assembly jig was created to align the lamina and compress the stack while the adhesive dried. This jig is shown in Figure 5.15. The construction of this jig was based on descriptions in the NDRC report and consisted of a base-plate, mid-plate, and top-plate. Adhesive-sprayed lamina were stacked on the base-plate and aligned with dowel pins that were inserted into the plate. Next, the mid-plate with oversized through-holes was placed over the dowel pins and sat directly on top of the stack. A large ball bearing was placed between this mid-plate and the top-plate to allow the mid-plate to pivot flush against the top of the stack. Finally, the top-plate aligned the free ends of the dowel pins and was pressed down by a pair of compression springs.
Figure 5.14. Reproduction of HUSL transducer. On the left are shown three lamina made from rolled Galfenol material. The final transducer is shown on the right in front of the HUSL original.

Figure 5.15. Compression and alignment jig created for assembling laminated stacks. The picture on the left shows lamina placed on the base plate with a few alignment pins. On the right the assembled jig is shown – the base-plate, top-plate, springs, and dowel pins can be seen.

5.2.3.2 GCD stack assembly

The spray-on adhesive approach was not appropriate for the configurable drive. Because of the finer laminations, a higher fraction of adhesive made the constructed stack very flexible, especially in bending and shear. Instead, the GCD lamina were consolidated with a new process that separated the electrical insulation from the mechanical bonding. To do this, Sprayon S00609 green insulating varnish was
applied to both sides of the laminations. This particular type of varnish was chosen for its waterproof and oil-resistant characteristics and had a dielectric breakdown of 1900 V/mil. Varnish was limited to one coat in order to preserve good fill factor. The average coat thickness was measured to be approximately 20 µm. An uncoated lamination and a varnish-coated lamination are shown side-by-side in Figure 5.16.

A new assembly jig, shown in Figure 5.17, was created to orient the GCD lamina by taking advantage of the alignment notch feature. This new jig was much simpler than the HUSL-style jig (Figure 5.15) and was consequently less expensive to produce. Once the lamina were stacked, EPON 828-3140 epoxy was brushed onto edges in a manner similar to the third HUSL method. The low viscosity of this epoxy was the chief criterion in its selection (over other on-hand epoxies) since this attribute aids in delivering the epoxy in between laminations through capillary action.

Although the jig aligned the lamina centers, the outer edges did not always match up well because of machining variances. This poor alignment lead to appreciable air gaps being adjacent to the permanent magnets. As a first step to reduce these differences, laminates where individually oriented in the jig such that the variation on these faces was minimized.

Because of the varying magnetostriction between boules, lamina stacks were organized by originating boule to avoid abrupt differences in the magnetostriction

Figure 5.16. Two Galfenol steel laminations – the one on the right has been coated in insulating varnish. Oxide on the uncoated laminations was not removed before applying the varnish. A U.S. nickel is shown for scale.
of adjacent laminations as shown in Table 5.5. This approach led stacks 5 and 4 to have the highest magnetostriction material and the largest fraction of material from a single boule.

The epoxy was prepared with a 100:33 mix ratio and placed in a vacuum to evacuate air bubbles. A small paint brush was used to apply the epoxy sparingly to the sides of the stack, carefully avoiding the area around the notch feature (so as not to cement the assembly to the jig). Once the epoxy had been brushed on, a compressive stress was placed on the stack with a large binder clip while the epoxy cured. Cure conditions were a 22-hour temperature soak at 60\(^\circ\)C to 65\(^\circ\)C.

In spite of attempts to align the lamina, some irregularity remained in the faces that would be pressed flush against the permanent magnets. Achieving the best surface finish was essential to minimize low-permeability regions of air and epoxy on either side of the magnets. After the epoxy had cured, these faces were belt-sanded. This was effective to quickly reduce the ragged edges caused by lamination variation, but the faces were still somewhat uneven because the sanding removed more material around the edges and corners. To improve the flatness of the stack, it was sanded faces-down on a sheet of sand paper on a flat surface. This two-step sanding process yielded an acceptable surface flatness quality that greatly reduced

\footnote{As an interesting side-note, a fully-cured stack constructed in this manner can be disassembled by placing the stack in a beaker of ACL-23 EpoxSol and using an ultrasonic cleaner. After a very long soak, the laminates can be easily separated with a razor blade.}
Table 5.5. Details of lamination assignment and stack consolidation.

<table>
<thead>
<tr>
<th>Stack</th>
<th>Boule</th>
<th>Laminations</th>
<th>Strain [ppm]</th>
<th>h [mm]</th>
<th>Fill factor (^a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>D1-9-42-76</td>
<td>15</td>
<td>200</td>
<td>11.93</td>
<td>87.9%</td>
</tr>
<tr>
<td></td>
<td>D1-9-42-153</td>
<td>25</td>
<td>194</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>D1-9-40-76</td>
<td>8</td>
<td>203</td>
<td>11.96</td>
<td>87.6%</td>
</tr>
<tr>
<td></td>
<td>D1-9-42-76</td>
<td>21</td>
<td>200</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>D1-9-39-78</td>
<td>11</td>
<td>196</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>D1-9-40-1</td>
<td>16</td>
<td>221</td>
<td>12.71</td>
<td>82.5%</td>
</tr>
<tr>
<td></td>
<td>D1-9-39-153</td>
<td>19</td>
<td>215</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>D1-9-40-76</td>
<td>5</td>
<td>203</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>D1-9-39-3</td>
<td>33</td>
<td>229</td>
<td>12.05</td>
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<td></td>
<td>D1-9-40-1</td>
<td>7</td>
<td>221</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>D1-9-39-3</td>
<td>2</td>
<td>229</td>
<td>12.24</td>
<td>85.6%</td>
</tr>
<tr>
<td></td>
<td>D1-9-42-1</td>
<td>38</td>
<td>252</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^a\)Calculated from a measurement of 33 laminates having a mean lamination thickness of 0.262±0.001 mm

the low-permeability gaps on either side of the permanent magnets.\(^3\)

Once the assembly stage was complete, stacks 1 and 2 were paired to form one drive and stacks 4 and 5 became another. After initial impedance measurements showed that neither drive had serious defects, stacks 4 and 5 were selected to be the focus of the remaining study because of their higher magnetostriction. Stack 3 was not used at all because its low fill factor gave it a significantly different height from the other stacks.

### 5.2.4 Drive coil assembly

The drive coil design produced by the one-dimensional model specified three-layer windings of 29 AWG magnet wire for the full length of both drive sections. Although calculations predicted that three layers would yield 327 turns, in practice

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\(^3\)On several occasions it was suggested to the author that ferrofluid could be placed in between the stacks and the magnets to further reduce air gaps. Unfortunately, this scheme is not feasible as ferrofluid typically experiences magnetic saturation at a few tens of milliteslas – at the intended GCD magnetic operating conditions, the ferrofluid would be completely saturated and have essentially the same permeability as air.
quantities closer to 300 were achieved with careful hand winding. This is consistent with Roters’[111] assumption that only about 95% of calculated turns are realized in practice. To correct for this discrepancy, a fourth layer was added to the coil, bringing the total number of turns per coil to 410 (giving the GCD a total turn count of \( n = 820 \)). Coil layers were wound tightly and directly on top of each other – tape and glue in between layers were omitted to minimize heating.

It is important to mechanically decouple the coil from the magnetostrictive material. In the author’s first attempt, the coil was wound directly on Teflon® tape wrapped around the stack.[4] This proved to be insufficient – drives with coils wound this way exhibited a second resonance slightly above the main resonance that greatly diminished the effective coupling coefficient. This spurious resonance was effectively removed by wrapping the coil on a layer of \( \frac{1}{32} \)” corprene.

5.3 Magnetic state study

After constructing the laminated stacks and assembling the prototype drive, it became apparent that the state of the magnetic circuit would need to be verified experimentally. Of concern was the fact that the electrical impedance sweeps on the drives demonstrated a lower-than-expected inductance based on the one-dimensional model. This seemed to suggest that either the actual permeability of the stacks was lower than the value used in the design stage or the magnetic bias was different than the predicted value. The design models specified \( \mu_r = 100 \) for Galfenol, which was consistent with reference data supplied by ETREMA Products. Figure 5.18 shows the reference data plotted against the linearized permeability. Notice that this single value is a good first-order approximation for the material below saturation.

To investigate the stack permeabilities, the major-loop quasi-static \( B-H \) boule measurements made by ETREMA Products, were plotted on the same axes. These

---

[4]After the coils were wound, small drops of five minute epoxy were placed at each end of the coils to prevent them from unwinding or slipping.

[5]It has been repeatedly demonstrated that thin laminations can easily cut through the insulating enamel on magnet wire (especially when tightly wound). The result is that turns are shorted as the applied current instead travels through the electrically conductive core. As a precaution against this, coils are generally wound on a layer of Teflon® tape. Wrapping with 18 AWG wire around sharp corners required 10 mil thick tape to protect the enamel.
Figure 5.18. Reference $B$-$H$ data shown with the $\mu_r = 100$ linear approximation used in the one-dimensional design model. Also plotted are the curves for boules D1-9-42-1 and D1-9-39-3, illustrating that the reference data serves as a good average of the two.

are shown in Figure 5.19 with the permeability estimated with a least-squares fit. Significant variation is seen in the second-region of the hysteresis loop: $\mu_r$ ranges from 50 to 200. ETREMA indicated that the large difference in the D1-9-39-X boules was due to a different stress annealing process and that the permeability can be better controlled in future material by adjusting the internal stress state.

Fortunately, because of the way lamina were sorted for the stack assemblies, most stacks were constructed with material that was predominately from a single boule. Referring to Table 5.4, 82% of stack 4 lamina originated from boule D1-9-39-3 and 95% of stack 5 material came from boule D1-9-42-1. In Figure 5.18 it can be seen that the reference data forms a reasonable average of the two boule curves.

Also, the effective permeability of each constructed stack was investigated by placing them in the magnetic fields generated by DC and AC Helmholtz coils and
Figure 5.19. Quasi-static $B$-$H$ measurements on all nine boules. Least-squares estimates for the second region magnetic permeability are shown for several boules. Because of the similarity in first-region permeability, only one fit is shown to represent all boules. Measurements courtesy of ETREMA Products.
Figure 5.20. Measured magnetic permeabilities of laminated stacks as a function of bias field. Maximum permeability occurs at the optimum bias.

measuring the $B$ and $H$ fields with a sense coil and Hall probe sensor, following the method of Scott [52]. These permeabilities are effective values because they include the effect of finite thickness bond lines that form low-permeability regions between the lamina. Stacks 1, 2, 4, and 5 were measured in this manner, and the results are shown in Figure 5.20. Notice that stacks 1 and 2 have relative permeabilities close to 70. Stacks 4 and 5, however, have an average optimum permeability of 106 – close to the $\mu_r = 100$ used in the design models.

Thus, having two different perspectives on the average magnetic permeability of stacks 4 and 5 that both suggest that the design data should be representative of the material used, it seems reasonable to rule out decreased permeability as a cause of the lower-than-expected inductance in stacks 4 and 5. The next step was to investigate the magnetic bias of the circuit.

To assess the static induction in a drive, the author devised a measurement technique that consisted of measuring the back emf in a drive coil resulting from
the two stacks being suddenly separated. As the two halves are pulled apart, the measured stack sees a change in induction from its bias value to zero. By Faraday’s Law, this time-varying flux generates a back emf in the coil. With an oscilloscope, this voltage spike may be captured. Integrating over the spike feature and dividing the result by the coil cross-section (this is discussed further in the next section) and the number of turns yields the magnetic induction. In order for this method to work correctly, the permanent magnets must be fixed to the stack that is not being measured (the author used a small amount of cyanoacrylate). Experimental error was reduced by performing this measurement multiple times. The error was then estimated using the standard deviation of the mean. Early measurements indicated that the bias level was indeed lower than expected. Approximately 0.7 T was generated with a \( \frac{1}{2}'' \times \frac{1}{2}'' \times \frac{1}{16}'' \) NdFeB magnet. To determine how much magnetic material would be required to achieve the design goal of 1 T, measurements were made with different numbers of \( \frac{1}{32}'' \) and \( \frac{1}{16}'' \) magnets placed in between the stacks. The results are shown in Figure 5.21. An interesting feature of this plot is the difference in slope seen for stacking \( \frac{1}{32}'' \) magnets and \( \frac{1}{16}'' \) magnets. The lower slope is due to stacks of \( \frac{1}{32}'' \) magnets having more air gaps.

Since the author only had \( \frac{1}{32}'' \) and \( \frac{1}{16}'' \) magnets available, it was decided to investigate whether a combination of these two geometries could be used to achieve an approximately 1 T bias. Judging from Figure 5.21, three \( \frac{1}{32}'' \) magnets would achieve slightly less than the desired bias. To compensate, it was decided to try combining one \( \frac{1}{32}'' \) and one \( \frac{1}{16}'' \) magnet (eliminating one air gap). Measurement of this configuration yielded a bias of \( 1.071 \pm 0.002 \) T that was deemed acceptable for the prototype drive.

It is important to question why the two design models (one-dimensional and FEA) both predicted that \( \frac{1}{32}'' \) NdFeB magnets would be sufficient for the 1 T bias, but in practice \( \frac{3}{32}'' \) of material had to be used. One possible cause of this discrepancy was that the \( B_r \) value used from the manufacturer’s data sheet (1.32 T) was higher than what was seen in practice. The \( \frac{3}{32}'' \) result is consistent

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6 This technique could also be accomplished by bringing the two halves of the drive together, but the strong attraction of the NdFeB magnets caused the pieces to snap together, which sometimes caused damage to the magnets and/or the stacks.

7 Interestingly, a designer with considerable experience in using Terfenol-D for sonar transducers related to the author that he always needed to double the predicted amount of magnet material to achieve proper bias in the actual devices.
5.3.0.1 Applying Faraday’s law to the laminated stacks

Measuring the magnetic induction in laminated stacks can be done using the drive coils and Faraday’s Law of Induction. Equation 2.1c may be rearranged to express the magnetic induction, $B$, in the stack as a function of the back emf, $\epsilon$:

$$B = -\frac{1}{nA} \int \epsilon \, dt$$

(5.5)
where \( n \) is the number of turns on the coil, and \( A \) is cross sectional area inside the coil. If a harmonic magnetic excitation of angular frequency \( \omega \) is applied, then the expression is simplified to:

\[
B = -\frac{\epsilon}{j\omega nA}
\]  \hspace{1cm} (5.6)

For the prototype drives, accurate determination of the induction required the application of two area correction factors:

1. The first correction factor was due to the coils being wound on the \( \frac{1}{32} \)-thick corprene. Comparing permeability measurements made in the Helmholtz coil setup that measured the back emf with a sense coil and drive coil, the correction factor was experimentally determined to be 1.23 - nearly the same as the ratio of areas with and without the corprene.

2. The second correction was due to not all the cross-sectional area of the stack being made up of Galfenol: thus a correction of one over the fill factor was required. For stack number 4, the correction was \( \frac{1}{0.87} = 1.15 \). This correction is only needed to determine the induction in the Galfenol. For measuring the effective permeability of the assembled stack, this correction would not be applied.

For permeability measurements also incorporating an experimental determination of the \( H \)-field, a demagnetization correction factor would also need to be applied.

5.3.0.2 Correcting for the demagnetization effect

For measurements made in the Helmholtz coil setup (Figure 5.20), the demagnetization effect must be taken into account. A correction for the demagnetization effect was estimated by the sampling the \( H \)-field radially outward from the sample. Recording the position of the center of the Hall probe sensor tip and the measured \( H \), the \( H \) field at the sample surface may be extrapolated. Comparing this extrapolated value to the standard measurement placement (the center of the Hall probe sensor positioned 3.18 mm away from the top of the sample) allows a correction
factor to be calculated. The author’s measurements determined this value to be 0.82.

### 5.4 One-dimensional drive model

Modeling the drive in one-dimension is accomplished by extending the design model so that the magnetic reluctance domain couples into a mechanical domain. This portion of the model simulates the motional behavior including resonant features. The full one-dimensional drive model is shown in Figure 5.22. Essentially, this model is the AC network from the design model updated in a number of ways:

1. **DC and AC magnetic reluctance networks are combined.** Updating the AC network from the design model with flow sources that represent the permanent magnets allows the calculation of a DC solution. This is done by using the `.savebias` command in SPICE that writes the operating point solution to a text file at the beginning of the `.ac` analysis. Analog behavior modeling is used to calculate the $B$ and $H$ fields in the drive segments using $B$-sources. Since the magnetostrictive parameters $\mu$, $d$, and $s$ are functions of the magnetic bias the values used imply a certain bias that is not the same as the calculated bias. In other words, the calculated magnetic bias is not used to update the value of the magnetostrictive parameters. The values used were $\mu_{33}^T = 100$, $s_{33}^H = 15.1 \text{TPa}^{-1}$ [35] and $d_{33} = 21 \text{nm/A}$ [121] and $Q_m = 28$ [121].

2. **Magnetic reluctances are modified to include eddy current losses.** This is done by using the lossy magnetic reluctance components described in section 3.5.6 and 3.5.7 to represent the high electrical conductivity flux conductors. When considering eddy current losses in the prototype drive, the neodymium magnets should not be overlooked as they have a high electrical conductivity but are not laminated.

3. **The mechanical domain is added to the model.** This is done using the lossy piezomagnetic piece described in Section 3.5.7 to connect to the magnetic reluctance domain. These two pieces only resolve the narrow drive
Figure 5.22. Drive Circuit. Parameter statements are not shown, but included in Appendix D.
segments; the ends of the dogbone shape are represented with non-active material waveguide components. Point mass representation of the permanent magnets is accomplished with inertance components. The radiation loading that occurs at the ends of the oscillating drive is included with simplified $\rho c A$ mechanical resistances.

4. **Mechanical losses are estimated using a measured $Q_m$ value.** This is done with a very rough model that considers a free-free bar of the active material and represents each half of the bar as a simple harmonic oscillator. With elementary relations the following relationship may be derived:

$$R_m = \frac{2A}{\omega_0 s_{33} Q_m \ell} \quad (5.7)$$

where $Q_m$ is the mechanical quality factor, $A$ is the drive segment cross-section, $\omega_0$ is the resonance frequency, $s_{33}$ is the elastic compliance, and $\ell$ is the total length of the drive segment (a complete derivation can be found in Appendix [C]). This resistance is placed at both ends of the lossy piezomagnetic piece.

5. **Epoxy stiffening effects can be included.** The author investigated mechanical stiffening of the laminated stack due to the contribution of epoxy layers. Although this effect turned out to be negligible for the prototype drive, the circuit structure that represents it has been retained in the model in case it may be important in other GCD designs. This feature is modeled in essentially the same way a stress-bolt would be for a piezoelectric stack: a waveguide in parallel with the active element. The feature may be activated by removing the shorting wire at either end of the structure.

### 5.5 Drive measurements

Having fabricated the prototype drive and created a model, it is logical to compare the measurements of the device to the simulated results. This is an opportunity to check that the model represents the actual device and that the actual device is functioning as expected. Both magnetic and electrical measurements were made.
Measurements on the magnetic state of the circuit were discussed previously in section 5.3 so only the electrical input impedance is considered here.

### 5.5.1 Electrical input impedance

Verification of the one-dimensional model is accomplished by comparing simulation results to measurements of the actual drive. Figure 5.23 shows the simulated and measured electrical input impedance are in generally good agreement. The overall inductive rise in impedance magnitude is well-modeled with the magnetic reluctance components (notice that no inductors are present in the electrical domain). Excellent agreement in trends indicates that the magnetic reluctance modeling is accurately describing the electromagnetic behavior of the device. Also, the slow decrease in phase — due to the presence of eddy currents — is resolved by the lossy reluctances.

The effective coupling coefficient also shows reasonable agreement: 39% simulated, 46% measured. This value for measured effective coupling is somewhat unexpected since a magnetomechanical coupling of $k_{33} = 0.48$ was reported by Wun-Fogle et al. [35] for Galfenol steel. If this value were representative of the material used in the GCD, it would suggest a coupling coefficient of 95% which seems highly unlikely. As Galfenol steel is a relatively young material that is still being experimented with, it seems more probable that the sample measured by Wun-Fogle was not representative of the GCD material. While a direct comparison cannot be made because only the drive is considered here, effective coupling coefficients for entire transducers reported in the literature range from 20% [65] to 53% [123].

There are several discrepancies between the measured and simulated electrical input impedances that must be noted. First, the predicted resonance frequency has an error of 6%. This is probably due to differences in the actual elastic modulus to the value used in the model. However, this degree of variation is expected in one-dimensional models because of their many simplifying assumptions. Better results may be beyond the limitations of the one-dimensional model, so more sophisticated modeling may be the appropriate next step. Second, there is clearly a difference in the amount of damping, with the actual device exhibiting more damping than
Figure 5.23. Electrical input impedance for just the Galfenol drive as 1) measured on the device and 2) simulated with the one-dimensional model.

the model. This result is not unexpected because it can be difficult to accurately capture all the various loss mechanisms in the model.
Chapter 6

Drive characterization with a tonpilz transducer

Investigation of Galfenol’s suitability for sonar projectors culminated with in-water testing of the configurable drive. For this to happen, the drive had to first be assembled into a transducer and placed in a watertight housing. Because it is simple to build and analyze, the tonpilz design was selected — creating a tonpilz only required bonding headmass and tailmass components to the ends of the drive. This chapter discusses designing the endmasses, fabricating and modeling the transducer, assembling the housing, and the characterizing the device in a water-filled test tank.

6.1 Tonpilz design

The first step was to design the GCD into a tonpilz transducer. An introduction to the tonpilz has already been provided in section 2.4.1.1. The GCD tonpilz was designed with two goals in mind. First, the transducer should be simple to allow for straightforward analysis of the drive — this means that the transducer itself is not optimized for performance. Second, the first longitudinal resonance of the device should be well above 5 kHz so that operation in the ARL tank facility could be qualified as anechoic.

The main design task was to produce a suitable headmass and tailmass to place on the drive. Due to the drive’s irregular shape, rectangular endmasses
were created for uniform load distribution. One-dimensional modeling was a key component in this design process as it made it possible to quickly assess different headmass and tailmass configurations and predict the behavior of the transducer both in-air and in-water.

6.1.1 Headmass design

Of the two endmasses, the headmass is the more difficult to design. As the acoustic radiator, the primary purpose of the headmass is to match the impedance of the drive to the propagating fluid, which leads to competing requirements for low mass, large area, and high flexural resonance. Consideration of these requirements realizes light and stiff materials as the best for headmass construction. For this reason — and because the material was available — magnesium was selected for the GCD headmass. It should be recognized, however, that a more careful material search is likely to find better-suited materials (a method to do this is described in [135]) but, for a simple transducer, magnesium is sufficient.

The GCD headmass was machined from a block of leftover magnesium that had a measured density of 1784 kg/m$^3$. Working within the constraints of the available material, a headmass was designed with a 25.40 mm $\times$ 38.10 mm radiating face that was extruded 3.18 mm before being tapered down over another 3.18 mm to a 19.50 mm $\times$ 31.75 mm face that mated to the end of the drive. The finished part had a measured mass of 9.8 g and is shown in on the left in Figure 6.1. With the drive segment cross-section of the GCD being 2 $\times$ 5 mm $\times$ 12 mm, the headmass provides an effective area transformation of 8.

Since headmass flexure can limit the operating band of a tonpilz transducer [136, 89, 88, 8], a calculation of the GCD headmass modes was conducted to confirm these frequencies were above the upper-limit of the transducer’s operating band (30 kHz, based on the one-dimensional simulation). Creating an FEA model in COMSOL Multiphysics was the most convenient method for determining the eigenfrequencies and mode shapes of the headmass. The simulation results for the first three modes are shown in Figure 6.2; all three are at frequencies well above 30 kHz.
Figure 6.1. GCD tonpilz endmasses. On the left is shown the rectangular magnesium headmass; markings on the mating face were used to align the headmass during tonpilz assembly. The rectangular tungsten tailmass is shown on the right.

Figure 6.2. The first three modes of the GCD headmass computed with COMSOL Multiphysics.

6.1.2 Tailmass design

Tailmass design is comparatively straightforward. A good rule of thumb for starting the design process is to simply make the tailmass have about 5 times the mass of the head \((5 \times 9.8 \text{ g} = 49 \text{ g})\) \[123\]. Experimentation with the one-dimensional model confirmed diminishing gains in headmass velocity for tailmass designs more massive than this value.

Compactness was achieved by using a high-density material. A 3.2 mm-thick disk of tungsten with a measured density of 16440 kg/m\(^3\) was cut into two 32.7 mm \(\times\) 14.0 mm plates. The two pieces were then stacked and bonded together with EPON 828 epoxy (in a 100:33 mix ratio) after the appropriate surfaces had been prepared with sandblasting. The completed tailmass assembly had a measured mass of 48.17 g and is shown on the right in Figure 6.1.
6.2 One dimensional tonpilz model

Having designed the tonpilz headmass and tailmass, creating a one-dimensional model of the GCD tonpilz was a simple modification of the drive model presented in section 5.4. This updated model is shown in Figure 6.3. The headmass and tailmass components were modeled with transmission line pieces to resolve their resonant behavior and they were wired in series to the ends of the drive to impose a continuity of velocity condition.

Complex radiation loads were applied to the headmass and tailmass components. Modeling these impedances is critical because water-loading on the radiating face significantly alters the transducer’s performance. In SPICE, implementing the real part of the radiation impedance is readily effected with resistor components (R_RRADH and R_RRADT). Adding the imaginary part, however, is more complicated. B-sources (B_XRADH and B_XRADT) are used to impose Ohm’s law by creating a voltage drop proportional to the current passing through them. To make this impedance imaginary, the laplace statement s/abs(s) is used.\footnote{This technique may be modified for creating frequency-dependent resistances in SPICE. As was discussed in section 3.5.7.1 using laplace commands in transient analyses can be problematic.} A complicating factor for determining actual values for the radiation impedances was the rectangular shape of the endmasses. This is examined in the next section. Another B-Source, B_TCR1, calculated the transducer’s transmitting current response (TCR). A full discussion of the TCR metric and the details of its calculation are included in section 6.2.0.3.

6.2.0.1 Radiation impedance of a rectangular piston

The concept of mechanical radiation impedance, $Z_{rad}^m$, is discussed in section 2.3.3 for the classic case of a harmonically-oscillating circular piston in an infinite rigid baffle. Consideration of a rectangular piston is not as common. This is not because it is an unusual scenario to find in practice, rather the mathematical description is somewhat complicated and unwieldy.

While several approaches are presented in the literature, the author implemented the method of Stepanishen [137], with a MATLAB routine (included in Appendix D). Figure 6.4 shows normalized results from this routine for rectangular
Figure 6.3. One dimensional model for the tonpilz transducer in water. Parameter statements are not shown, but are included in Appendix [D].
Figure 6.4. Real and imaginary parts of the normalized radiation impedance for a rectangular piston. Six different aspect ratios, $a/b$, are shown.

Radiating faces of varying aspect ratios having side lengths $a$ and $b$. There are two important aspect ratios to consider. The first, $a/b = 1$, represents a square and its radiation impedance agrees closely with the case of a circular piston [138]. The second case, when $a/b = 1.5$, corresponds to the GCD headmass. This case is summarized for frequencies around the expected GCD tonpilz resonance in Table 6.1. Because these functions do not change abruptly, the values calculated for 11 kHz were used in the one-dimensional tonpilz model because these most closely corresponded to the simulated resonance frequency.

Although the radiating load is approximated here with single values for $R_1$ and $X_1$, a framework for calculating the radiation impedance as a function of frequency for .ac analysis in SPICE is outlined in [139].
Table 6.1. Real and imaginary parts of normalized radiation impedance values calculated for a rectangular piston with an aspect ratio of 1.5.

<table>
<thead>
<tr>
<th>Frequency [kHz]</th>
<th>$R_1$</th>
<th>$X_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.2498</td>
<td>0.5270</td>
</tr>
<tr>
<td>11</td>
<td>0.2957</td>
<td>0.5605</td>
</tr>
<tr>
<td>12</td>
<td>0.3436</td>
<td>0.5893</td>
</tr>
</tbody>
</table>

6.2.0.2 Cavitation evaluation

With a realistic simulation of water loading, the SPICE model was used to compare the transducer’s predicted performance to the cavitation threshold. Figure 6.5 shows the behavior of the cavitation threshold as a function of frequency for continuous wave (CW) operation. The cavitation threshold is very imprecise [88], denoted by the large uncertainty band (shown in gray).

Using the simulated peak output force, $F_{pk}$ (measured as the potential of the wire connecting elements $VRAD$ and $R_{RRADH}$), the acoustic plane-wave intensity, $I$, is calculated as [87]:

$$I = \frac{p_{pk}^2}{2\rho c} = \frac{F_{pk}^2}{2\rho c (\ell w)^2}$$  \hspace{1cm} (6.1)

Here $p_{pk}$ is the peak pressure, $\rho$ is the density of the propagating fluid, $c$ is its sound speed, and $\ell$ and $w$ are the dimensions of the radiating face. The results of this calculation are presented in Table 6.2.

Plotting a few of these points at the predicted resonance frequency (11 kHz) on Figure 6.5 gives a sense of the likelihood of cavitation. From these predictions, it appears that at a 200 mA drive the GCD tonpilz will just exceed the upper-limit for cavitation. The actual threshold, however, should be higher than what appears on the plot since the actual test conditions call for placing the transducer at an 8 ft depth and using short pulses. However, even with these changes it seems improbable that the transducer will achieve the full 1 A drive without cavitation.

The reader must understand the large degree of ambiguity in these results: the uncertainty of the plot is compounded by several differences in assumed versus actual operating conditions.
Figure 6.5. Plot of cavitation threshold, recreated from [9]. Original caption: “Frequency dependence of the cavitation threshold. CW data on fresh water at atmospheric pressure.” The author has added points that show predicted intensities of the GCD tonpilz at different drive amplitudes, derived from the linear one-dimensional model.

6.2.0.3 Transmitting current response

An important metric for assessing the performance of magnetostrictive projectors is the transmitting current response (TCR), defined as the sound pressure at a distance of 1 m divided by the amplitude of the signal current [140]. At a point \((r, \theta, \phi)\) from the center of the radiating face, the pressure, \(p\), generated by a baffled rectangular piston of length \(\ell\) and width \(w\) moving harmonically with a velocity \(v_0\) at frequency \(\omega\) against a fluid with density \(\rho\) is [141]:

\[
p(r, \theta, \phi) = \frac{j\omega \rho}{2\pi} w \ell v_0 \text{sinc} \left( \frac{1}{2} k_x \ell \right) \text{sinc} \left( \frac{1}{2} k_y w \right) \frac{e^{-jkr}}{r} \quad \text{(6.2)}
\]
Table 6.2. Plane-wave intensities calculated with the one-dimensional model.

<table>
<thead>
<tr>
<th>Simulated drive [mA]</th>
<th>Predicted Intensity [W/m³]</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.49</td>
</tr>
<tr>
<td>200</td>
<td>1.94</td>
</tr>
<tr>
<td>300</td>
<td>4.37</td>
</tr>
<tr>
<td>400</td>
<td>7.76</td>
</tr>
<tr>
<td>500</td>
<td>12.1</td>
</tr>
<tr>
<td>600</td>
<td>17.5</td>
</tr>
<tr>
<td>700</td>
<td>23.8</td>
</tr>
<tr>
<td>800</td>
<td>31.1</td>
</tr>
<tr>
<td>900</td>
<td>39.3</td>
</tr>
<tr>
<td>1000</td>
<td>48.5</td>
</tr>
</tbody>
</table>

where $k_x = k \sin \theta \cos \phi$ and $k_y = k \sin \theta \sin \phi$. For the on-axis case ($\theta = 0^\circ$, $\phi = 0^\circ$) with input current $i_{in}$, the ratio of pressure to current is therefore

$$\frac{p}{i_{in}} = \frac{\omega \rho w \ell v_0}{2\pi i_{in}}$$

This statement is incorporated into SPICE using a B-source (B.TCR1) that generates a voltage representing this ratio. The Laplace statement `abs(s)` must be used to include the frequency dependence. Notice that a zero-output force source, VRAD, is used as a velocity monitor for the TCR calculation [110].

If the B-source expression is divided by the standard pressure reference for water, $p_{ref} = 1 \mu Pa$, it can then be plotted with the LTspice post-processor on a decibel scale to show the response level in dB so that

$$TCR = 20 \log_{10} \left( \frac{\omega \rho w \ell v_0}{2\pi i_{in}p_{ref}} \right)$$

For any SPICE model, it is convenient to simulate a 1 A input current so that 1) the electrical input impedance is equal to the voltage drop across the current source and 2) the current variable may be omitted from the TCR calculation.
6.3 Transducer assembly

Construction of the tonpilz element began with gluing the permanent magnets into place with cyanoacrylate. Doing this also had the effect of cementing the two drive halves together. Cyanoacrylate is not as permanent as a regular epoxy, but it sets much more quickly. Securing the magnets in this way avoided any misalignment of the magnetic circuit during the long epoxy curing process.

The bonding surfaces on the headmass and tailmass were sandblasted for better adhesion. Similarly, the ends of the drive were sanded on a flat surface to promote proper mating. Sanding also removed the electrical insulating varnish from the end surfaces that would otherwise weaken the bond.

EPON 828 epoxy was used in a 100:33 mix ratio to bond the headmass and tailmass to the drive. The epoxy cured with the transducer situated vertically (headmass down) and with a 547 g weight placed on top of the tailmass to provide constant compression. During the epoxy cure, a special concern was not to exceed the maximum operating temperature of the magnets; a complete discussion of this topic is presented in the next section. The transducer was built twice using two different epoxy cures: the original assembly used a 22-hour temperature soak at 60°C and was later rebuilt (see section 6.6.2) with a 4 day cure at 40°C. Pictures of the completed tonpilz assembly are shown in Figure 6.6.
6.3.1 Maximum operating temperature for permanent magnets

Elevated temperatures can degrade the magnetic performance of permanent magnets. Heating concerns for permanent magnets in transducers arise because 1) transducers are often subjected to long heat soaks during the assembly stage in order to cure epoxies and 2) transducers heat-up during high-power operation. To avoid degradation, the permanent magnets must only experience reversible demagnetization during operation. Therefore, in a given magnetic circuit in which the permanent magnet material, magnet geometry, and maximum magnetic excitation are known, a temperature may be specified that denotes the onset of irreversible demagnetization. This is called the maximum operating temperature. Continued heating above the maximum operating temperature will cause further demagnetization until, at its Curie temperature, the magnet becomes fully demagnetized.

In practice, the maximum operating temperature is more of a guideline than an absolute value since it is often roughly inferred from inspecting the demagnetization curves of the permanent magnet material. Another reason that this value is approximate is that the maximum operating temperature is influenced by the magnet’s shape, flux return path(s), and operating conditions. Manufacturers will often specify maximum operating temperatures based on open-circuit conditions, i.e. no flux return path other than the surrounding air. The traditional method for assessing the impact of these factors on the magnet’s operating state is through the use of a load line \[76\], a construct to determine the magnetic operating point.

Normal demagnetization curves for N42-grade NdFeB are shown in Figure 6.7 for six different temperatures. Notice that at 20°C the neodymium behaves ideally — the demagnetization curve is a straight line. When an AC magnetic field is impressed on the magnet, the operating point swings along this line.

As temperature increases, however, a corner feature occurs in the demagnetization curve that reduces the useful region (the portion of the curve still parallel to the 20°C curve). The higher the temperature, the more this useful region is reduced. To avoid irreversible demagnetization, the operating point must not “fall” over the corner. Because of hysteresis, if the operating point goes below the corner

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\(^2\)This section draws on the explanation of the maximum operating temperature for neodymium magnets given by \[75\].
it cannot return to the straight-line region without the magnet being recharged. Instead, it will swing along a narrow minor loop that has a slope parallel to the ideal portion of the curve.

Determining the magnet’s load line requires calculating the $B/H$ ratio (also called the permeance coefficient). For a high-permeability (low reluctance) return path, this ratio will have a high value signifying that the magnetic system is approaching a short-circuit condition where the flux produced by the magnet is nearly equal to its remanent flux. In the case of a low-permeability (high reluctance) return path, the $B/H$ ratio will be small and the demagnetization field will be so large that the operating point will approach the coercive field (zero-flux open circuit condition). Calculation of the $B/H$ ratio (with $B$ expressed in units of gauss and $H$ in oersteds) for the magnets in the GCD with the one-dimensional design model (section 5.1.4) is straightforward. This yields a result of 1059 G/
1067 Oe ≈ 1 G/Oe. The load line is then drawn from the origin to the 1.0 permeance coefficient tick mark (these tick marks are on the top and left edges of Figure 6.7 and are a standard feature of manufacturer’s demagnetization curves). This load line is shown in orange and the operating point is at its intersections with the appropriate demagnetization curve. By visual inspection of Figure 6.7 irreversible demagnetization will occur if the unenergized transducer is heated to 100°C since the operating point is already slipping down the corner. At 1 A drive current, the GCD has an \( H \)-field amplitude of 4.2 kA/m (section 5.1.4); examination of Figure 6.7 shows that operating the GCD at full drive at 60°C will also cause irreversible demagnetization. Based on this plot, it seems that specifying 40°C as the maximum operating temperature for the GCD operating at full drive leaves a reasonable margin for error. For the static case, it appears that the drive can achieve temperatures up to 90°C without degrading the magnets.

It is worth noting that if the magnets are removed from the low-reluctance magnetic circuit provided by the stacks, their \( B/H \) ratios (and, by extension, their maximum operating temperatures) will drop dramatically. Without any sort of return path (i.e. the flux circulating through the surrounding air), the temperature of these magnets should definitely not be elevated to 60°C as shown by the blue and red load lines in Figure 6.7.

### 6.4 Tonpilz in-air measurements

With the tonpilz assembled, in-air measurements were conducted to characterize the transducer and verifying proper assembly and expected operation. Overheating during the epoxy cure was still a concern and impedance measurements were an easy way to check for damage. This was also an opportunity to compare to the results of the one-dimensional model.

#### 6.4.1 Electrical input impedance

Figure 6.8 shows two measured in-air electrical input impedance curves: one for the drive alone and the other for the drive assembled into the tonpilz transducer.

\[ 1 \text{T} = 1 \times 10^4 \text{ G} \text{ and } 79.578 \text{ A/m} = 1 \text{ Oe}. \]
Notice that the mass-loading lowers the resonance frequency as expected: the 14.4 kHz resonance of the drive is lowered to 11.0 kHz for the tonpilz. Also expected is the moderate decrease in the effective electromechanical coupling coefficient: this value changed from 40% for the drive to 33% for the entire transducer.\footnote{The effective coupling of the drive in this measurement varies from the 46% value observed in Figure 5.23. Some adjustments were made to the drive in preparation for the tonpilz assembly, including swapping out the permanent magnets and cementing the magnets to the stacks. Magnet-to-magnet variance caused the measured coupling coefficient changes on the order of 5%. Also, in assembling the drives, it was observed that small changes in the alignment of the permanent magnets could alter the coupling value.} The mechanical quality factor was qualitatively assessed using the derivative of the phase angle (in degrees) with respect to frequency at resonance:\footnote{This expression is intended for use with the mechanical impedance, not the electrical input impedance. As such the values produced are inaccurate, but allow for a comparison of $Q_m$ values to be made.}

\[ Q_m \propto - \frac{d(\angle Z)}{df} \bigg|_{f_0} \]  \hspace{1cm} (6.5)

Using this method, it is determined that adding the headmass and tailmass doubled the drive’s $Q_m$. The increase agrees with expectations.

The electrical input impedance may also be used to assess the operation of the drive. The smooth and familiar shape of the resonant features point to the absence of significant mechanical defects. Away from resonance, the two impedance curves coincide, indicating no damage to the coils and magnetic circuit during assembly.

Having measured the completed transducer, the results were compared to the one-dimensional tonpilz model discussed in section 6.2. Doing this required the head radiation load to be modified for air. As shown in Figure 6.9, overall good agreement is achieved. Many of the comparisons made in section 5.5.1 are applicable to the interpretation of this plot, so these comments are not repeated here. Overall, these measurements are a positive indicator of acceptable device performance: changes are as expected and no unusual characteristics are observed.

### 6.5 Transducer housing

To operate the GCD tonpilz in the water, it had to be assembled into an watertight housing. Because of the low resonance of the device, a large diameter housing was
needed to approximate an infinite baffle. After surveying the available housings, a ø12.75" shell was selected. Stansfield [88] notes that a circular baffle having a diameter greater than $2\lambda$ approximates the case of an infinitely large baffle. Assuming the speed of sound in fresh water to be 1481 m/s, a housing with this diameter will act as an infinite baffle down to 9.2 kHz. This is a satisfactory value — the primary resonance of the GCD tonpilz is above this frequency so the infinite baffle approximation applies for the entire operating band of the transducer. Having an effectively infinite baffle is important so that the measurement setup corresponds to the assumptions for the radiation impedance calculations made in section 6.2.0.1 which will allow the in-water measurements to be directly compared to the one-dimensional simulations.

To implement the baffle, the author designed a special mounting plate to accommodate the GCD’s irregular headmass. Aluminum was selected for the plate construction since it is paramagnetic and would not interfere with the drive. The
plate thickness was made to be 0.75” so that the fundamental plate mode would occur well below the transducer’s resonance. This frequency was estimated by calculating the first resonance of a $\varnothing 12.75” \times 0.75”$ plate analytically [142] and with a COMSOL model. These methods were found to be in good agreement: the fundamental plate resonance was determined to be 456 Hz with the analytical calculation and 457 Hz from the FEA simulation. However, this geometry is just an approximation: the actual mounting plate had a rectangular hole placed in the center in addition to a lip feature with an O-ring groove around the entire inside edge. This rectangular hole accommodated the radiating face of the transducer. The corners of the rectangular hole were drilled out to $\varnothing^3/16”$ to aid in manufacturing. Also, 0.020” clearance was added to each side of the hole to allow for the edges of the headmass to be sealed with strips of bubble rubber. The final machined plate is shown in Figure 6.10.

Mounting the transducer to the plate began by supporting the plate and transducer on blocks and then adjusting both with shims until the radiating face of the
transducer was flush with the top surface of the plate. Bubble rubber was placed around the edges of the headmass as shown in Figure 6.11.

The next step was the creation of an acoustic window using CPD 9107 urethane. Essentially, the acoustic window creates a seal around the transducer face while acting as a layer of “solid water” because it has a similar density and similar sound speed to water. Because of this impedance-match, the acoustic window does not need to be resolved in the modeling beyond including a water load on the radiating face. Since the transducer was primarily supported by its attachment to the acoustic window, a good bond between the window and headmass was critical. To enhance this bond, the radiating face was sandblasted and pickled (chemically etched by dipping the part in an acidic solution) and the mounting plate’s front surface was prepared with Cytec PR-1167 primer. Air bubbles in the acoustic window were prevented by applying a small amount of urethane on top of the bubble rubber as shown in Figure 6.12. This figure also shows the dam created with a plastic ring and duct seal to mold the acoustic window on just the center portion of the plate. Once the thin layer had dried, the rest of the mold was filled with urethane (Figure 6.13). The finished window is shown in Figure 6.14 and was about $\frac{1}{4}$” thick.

Mechanical support for the transducer was primarily provided by the bond between the headmass and the acoustic window, but extra support was added to prevent dislocation in the event of mechanical shock. Four brackets were fashioned out of aluminum scrap to hold the transducer in place. As before, aluminum
Figure 6.11. Transducer head mass held in place with bubble rubber.

Figure 6.12. Transducer radiating face sealed with a thin layer of urethane.

was chosen because of its paramagnetic ordering. The brackets contacted the transducer at the tail (low-displacement) section using corprene wedges. With double-sided tape, the finished supports were secured in place by sticking them to the inside surface of the mounting plate. These are shown in Figure 6.15.

Additional instrumentation was placed in the housing: two thermocouples were mounted on the transducer and a leak detector was put in the bottom of the shell. The location of the thermocouples is shown on the left in Figure 6.16. One thermocouple was placed in the center of the coil and the other was mounted directly on the Galfenol at the end of the drive segment. This arrangement was chosen so that the temperature of the coils and of the Galfenol could be monitored while the transducer was under test. Also, a leak detector was placed in the bottom of the shell so that an alarm would be triggered if water entered the housing. The leak detector is also shown in Figure 6.16 on the right.
Having sealed and secured the transducer and rigged up the extra instrumentation, the mounting plate was bolted onto the ®12.75” shell section. Two battery-powered Fluke 80TK thermocouple modules were placed inside the housing; these generated a voltage proportional to the temperature at the thermocouples. These modules are shown in Figure 6.17. Before closing the housing, the loose cables and thermocouple modules were duct taped to the bulkhead to prevent shifting during testing. Figure 6.18 shows the shell assembly ready for testing. Once closed, the seals were checked by drawing a vacuum on the housing and observing the pressure over a period of time (10 to 15 minutes) to make sure the vacuum did not slowly decrease from air leaks.
6.6 Tonpilz in-water measurements

In-water testing of the GCD tonpilz was done in the Applied Research Laboratory’s anechoic tank facility (ATF), a 18’ deep $\times$ 26’ long $\times$ 17.5’ wide test tank. Testing occurred on September 30, 2010 and October 27, 2010. Drive conditions are specified in milliamps at resonance since the amplifier was operated at constant voltage. Full results from the tests can be found in Appendix A.
6.6.1 Small-signal measurements

Small signal measurements of the transducer are presented in this section. The TCR, electrical input impedance, and beam patterns were measured at a drive current of 48 mA. Model results are compared to the TCR and impedance measurements.

6.6.1.1 Transmitting current response

The measured TCR for the GCD tonpilz is shown in Figure 6.19 alongside the simulated response of the one-dimensional model. Although there is significant deviation from the smooth simulation curve, the general trend, peak level, and resonance show considerable agreement. This demonstrates the utility of using these simplified models to predict the response of complicated transducers. A measured $Q_m$ of 6.4 (8.2 simulated) is fairly in-line with other published magnetostrictive transducers. Both model and measurement show an effective coupling coefficient of 44%, which is an above-average value for magnetostrictive transducers.

The largest single source of error in the model appears to originate from the $s_{33}$ (elastic compliance) parameter that strongly affects the resonance frequency. In Figure 6.19, the resonance peaks in the TCR are 10.4 kHz (measured) and 11.4 kHz (modeled) — approximately 10% error. This discrepancy in predicted versus measured resonance has been noted throughout the development of the one-
dimensional model. If the $s_{33}$ parameter is adjusted from a value of $15.1 \text{TPa}^{-1}$ up to $18 \text{TPa}^{-1}$, the model fits the data better as shown in Figure 6.20. Showing this adjustment is done to support the possibility that the difference is caused by a poor choice of inputs and not errors in the model.

There is significant detail in the measured TCR that is not captured in the one-dimensional model. Some of these features may be attributed to flexural and cross modes in the transducer itself. Others are likely resonances of the transducer housing and tank facility. In order to better investigate these features, a three-dimensional FEA simulation of the device was undertaken in COMSOL Multiphysics. Magnetostrictive coupling between the mechanical and magnetic domains was achieved with a technique for COMSOL 3.5 developed by [143]. A significant limitation to this model was the need for full material property matrices — these can be difficult to obtain. In this case, estimated matrices for binary Galfenol were used in the absence of three-dimensional properties for Galfenol steel. The
Figure 6.19. Measured versus simulated results for the GCD tonpilz in water. Shown here are the transmitting current response, impedance magnitude, and impedance phase.

results of the FEA simulation are shown on the left in Figure 6.21. Interestingly, while the one-dimensional model over-predicts the main resonance frequency, the FEA model under-predicts and significantly over-estimates the peak value. It does, however, capture the peak below the main resonance, which it shows to be caused by a bending mode. Also, the FEA simulation does not include the erroneous band-limiting resonance feature predicted around 37 kHz by the one-dimensional model.

The COMSOL model was run for both frequency response and transient analyses. A single time-step result from the transient analysis is shown in Figure 6.21 on
Figure 6.20. Measured versus simulated results for the GCD tonpilz in water with $s_{33}$ set to 18 TPa$^{-1}$ in the one-dimensional model. Shown here are the transmitting current response, impedance magnitude, and impedance phase.

the right. This image illustrates the different physical processes at work in the GCD: the red arrows represent the magnetic flux circulating around the drive, the color on the transducer denotes the displacement in the axial direction (red is displacement toward the propagating fluid, blue is displacement in the opposite direction), and the acoustic pressure field is illustrated in the fluid domain (red is positive pressure, blue is negative).
6.6.1.2 Electrical input impedance

Also shown in Figure 6.19 is the magnitude and phase of the electrical input impedance measured in water. As with previous comparisons, the overall agreement is reasonable although it degrades at higher frequencies. From this plot, the effective electromechanical coupling coefficient is measured to be 44%. Figure 6.20 shows the impedance for the case in which $s_{33}$ has been adjusted to 18 TPa$^{-1}$ (see previous section).

6.6.1.3 Electroacoustic efficiency

Efficiency is estimated at resonance from impedance circles obtained by plotting the electrical input reactance against the electrical input resistance. In-air and in-water impedance data for the GCD tonpilz are presented in this manner in Figure 6.22. Notice that the in-water loop has a diameter, $D_w$, that is significantly less than the in-air loop diameter, $D_a$. This is due to the larger radiation impedance in-water and it allows the electroacoustic efficiency, $\eta$, to be estimated as [7, 140, 89]:

$$\eta = \frac{D_w (D_a - D_w)}{R_0 D_a} \quad (6.6)$$

where $R_0$ is the in-water resistance at resonance. Based on the measured diameters in Figure 6.22 and $R_0 = 1230 \Omega$, the GCD tonpilz has an efficiency of $\eta \approx 50\%$. 

**Figure 6.21.** Measured versus simulated TCR curves for the GCD tonpilz in water comparing the one-dimensional and finite element modeling techniques. The FEA model is shown on the right for a transient analysis.
Values for the diameter in water, $D_w$, and the diameter in air, $D_a$, are shown.

6.6.1.4 Beam patterns

Horizontal and vertical beam patterns of the GCD tonpilz were measured at 10, 20, and 30 kHz. The horizontal plane is perpendicular to the long dimension of the radiating face. These measurements are compared to simulated beam patterns in Figure 6.24 (all patterns have been scaled at 0 dB at 0°, except for the vertical pattern at 30 kHz which is set to -10 dB at 0° for better comparison). The simulated results were calculated in COMSOL Multiphysics using the acoustics module to accomplish a simplified two-dimensional simulation of the housing and piston geometries, shown in Figure 6.23. Reasonable agreement is observed for all cases except the 30 kHz vertical pattern. In this measurement plane (parallel

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6To create this model, the housing geometry was subtracted from a 1 m-radius circle. Having this radius set the arc length along the circumference equal to the angle in radians. The domain was assigned the properties of water, an acceleration condition was applied to a line segment representing the width of the piston face, a $\rho c$ characteristic impedance was specified along the circumference, and rigid boundary conditions were imposed on the housing edges. The beam pattern was calculated by measuring the sound pressure level along the circumference.
Figure 6.23. 30 kHz beam pattern simulation in COMSOL Multiphysics. The quantity plotted is sound pressure level (SPL). In this picture, the radiating face is centered on the upper edge of the rectangular cutout.

to the long dimension of the radiating face), the transducer appears prone to creating rocking motion in the radiating face due to differences in the displacements produced by the two drive segments. While seems like a plausible explanation for the poor quality of this pattern, the FEA model (which does not account for these types of imperfections) did not predict this type of mode at this frequency.

### 6.6.2 Transducer failure and rebuild

The original test plan called for calibrating the transducer at a 50 mA drive level and then characterizing the transducer’s performance at nominal drive currents of 200, 500, 700, 800, 900, 1000, and 1100 mA. During testing on September 30th, shortly after increasing the drive current to 500 mA, the transmit signal was observed to drop 4 dB in the receive spectrum. Damage to the transducer was immediately suspected. To verify, the transducer was taken back to 200 mA and the impedance sweep measurement was repeated. The results, presented in Figure 6.25, show a deterioration of the motional impedance of the drive, suggesting
Figure 6.24. Horizontal and vertical beam patterns for the GCD tonpilz. Simulated patterns are shown in red, measured data are in blue. All patterns have been scaled at 0 dB at 0°, except for the vertical pattern at 30 kHz which is set to -10 dB at 0°. Data measured at 48 mA drive current.
mechanical damage to the device. The agreement away from resonance seems to indicate that no degradation of the electromagnetic system (coils, magnets, etc.) occurred.

![Graph showing in-water electrical input impedance measurements of the GCD tonpilz before and after suspected damage.]

Figure 6.25. In-water electrical input impedance measurements of the GCD tonpilz before and after suspected damage.

Progressive disassembly of the transducer suggested that bonding between the drive and an endmass had been damaged. After removing the headmass and tailmass, the electrical input impedance of the drive matched measurements made before the failure. In typical piezoelectric tonpilz transducers, a secondary func-

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7The headmass and tailmass were removed one at a time. First, the transducer was clamped
tion of the stress bolt is keeping the headmass and tailmass attached; the GCD transducer relies solely on the epoxy joint.

Upon close inspection, it appeared that the most likely cause of failure was the headmass bond. The transducer was then reassembled with extra care to create strong bonds between the drive and endmasses. To enhance the strength of the bond during reassembly several changes were made to the bonding process: 1) more epoxy was used, 2) the entire headmass was pickled, not just the radiating face, 3) a 3 mil nylon mesh was placed in between the headmass and drive to hold the epoxy in place while curing, and 4) the epoxy was cured for four days at 40 °C, as opposed to an overnight cure at 60 °C.

A comparison of measured electrical input impedances for the original transducer assembly and the reassembled transducer are shown in Figure 6.26. It appears that the longer cure made the epoxy harder as evidenced by the slight upward shift in resonance frequency and the decrease in mechanical losses. Also, the original coupling is preserved.

6.6.3 Increasing drive measurements

With the tonpilz transducer reassembled, in-water testing was resumed with a focus on characterizing the performance at increasing drive levels. The transducer was successfully operated at 48 mA, 90 mA, 176 mA, 189 mA, 264 mA, 323 mA, and 370 mA. Measurements from these tests are presented in this section.

6.6.3.1 Transmitting current response

TCR measurements are shown in Figure 6.27. The most significant observation is that a large difference is seen in the region around resonance between the measurements made during the first set of testing and the second set; this indicates that

into a vise with aluminum blocks situated immediately behind the endmass. This provided a high thermal conductivity path into the vise (the heatsink) and away from the Galfenol and neodymium magnets. Next, with the transducer sandwiched between the aluminum blocks, a set of locking pliers were used to apply a slight torque on the endmass while simultaneously heating the end face with a heat gun. Within seconds, the heat weakened the epoxy bond, allowing a clean break to be achieved between the endmass and the drive.

These were the currents that resulted from attempting to set the drive levels to nominal values of 50, 100, 200, 200, 300, 500, and 400 mA, respectively.
some change took place during the disassembly/reassembly process. The change is unfavorable since it places a dip in the response curve right at the resonance peak (this change is also observed in the impedance data around antiresonance for the rebuilt transducer, see Appendix A). Interestingly, the apparent resonance frequency decreases while in Figure 6.26 there is a slight increase. This suggests that there might be an issue with the transducer’s mounting. Outside of the region around resonance, there is very little change in the TCR curves, indicating that no significant nonlinearities are encountered at these drive amplitudes. A prominent peak around 7 kHz is seen in all measurements — previously this was found to be a flexural mode of the drive (section 6.6.1.1).

6.6.3.2 Source level

Another way to examine the transducer’s output is to look at its source level, the decibel quantity of the on-axis pressure at 1 m referenced to 1 μPa. Figure 6.28
Figure 6.27. TCR curves for the GCD tonpilz at varying drive levels.

shows the constant-voltage maximum source level produced by the transducer as a function of drive amplitude. As expected for a linear system, each doubling of drive amplitude results in a 6 dB increase.

6.6.3.3 Total harmonic distortion

Because of the material nonlinearities in both the $S-B$ and $B-H$ curves, the distortion produced by the transducer is an important consideration. Although not a comprehensive metric, a common way of characterizing distortion is to calculate the total harmonic distortion (THD). If a sinusoidal input is fed into a real device, the output spectrum will have a peaks at the input frequency and its harmonics. Taking the Fourier transform of a voltage waveform, the THD is calculated as

$$\text{THD} = \sqrt{\frac{V_2^2 + V_3^2 + V_4^2 + V_5^2 + \cdots + V_\infty^2}{V_1}}$$ (6.7)

where $V_1$ is the voltage at the input (fundamental) frequency and and $V_n$ is the
voltage at the $n$th harmonic. Because summing to infinity is not practical and only the first handful of harmonics make significant contributions to the THD value, it is common to only include a certain number of harmonics in the calculation. Figure 6.29 shows the THD values calculated for the GCD tonpilz using the first eight harmonics.

### 6.6.3.4 Source level versus volt-amps

One important way of assessing a transducer’s performance is to compare the source level to the product of the drive voltage and drive current magnitudes. This metric is useful for understanding the electronic amplifier requirements. Units of $V \cdot A$ are used instead of watts because the phase information has been discarded. A plot of constant-voltage maximum source level versus volt-amps for the GCD tonpilz is shown for the 323 mA drive level in Figure 6.30. The general trend is for the electrical drive requirements to decrease as frequency is increased, except
in the region of the fundamental antiresonance.

### 6.6.3.5 Strain

The strain produced by the GCD tonpilz can be estimated from the measured TCR by using the definition of strain and substituting in equation \(6.3\) :

\[
S = \frac{x}{\ell_0} = \frac{2\pi i_{\text{in}} p_{\text{ref}} \times 10^{\left(\frac{\text{TCR}}{20}\right)}}{(j\omega \rho w \ell) \times \ell_0}
\]  

(6.8)

Again, \(x\) is the displacement, \(\ell_0\) is the length of the drive segment, \(i_{\text{in}}\) is the drive current, \(p_{\text{ref}} = 1 \mu\text{Pa}\) is the reference pressure, \(\omega\) is the angular frequency, \(\rho\) is the mass density of the propagating fluid, and \(w\) and \(\ell\) are the width and length of the rectangular piston face. The results of this calculation are presented in Figure \(6.31\). Several comments must be made about this plot. First, the selection of \(\ell_0\) is somewhat arbitrary — in the actual transducer, the active strain is not confined
Figure 6.30. TCR and volt-amps as a function of frequency for the 323 mA drive level.

to the narrow portions of the stack but, for the purposes of Figure 6.31 it is assumed that all the strain does occur in this region so that $\ell_0 = 39$ mm. Second, equation 6.8 assumes the transducer operates with purely longitudinal motion, an assumption that is clearly invalid for the 7 kHz resonance (a bending mode). Finally, this plot shows the dynamic strain, whereas the strains indicated in the $S$-$B$ curves elsewhere in this document are quasi-static strains. Thus, in Figure 6.31 there are two contributions in each of the strain curves: 1) the magnetostrictive strain of the material and 2) the strain that any dynamically excited elastic sample will exhibit.

6.6.3.6 Temperature rise

During testing, no significant rise in temperature was observed from the thermocouple measurements. This absence of significant heating is due to both the low drive levels and low duty cycles. The maximum observed temperature rise was approximately $2^\circ$C.
6.6.4 Cavitation

As expected, testing of the GCD tonpilz was limited by the onset of cavitation. In the anechoic tank facility, cavitation occurred randomly at drive levels around 300 mA. The presence of cavitation was verified by observing bubble plumes at the face of the transducer (shown on the left in Figure 6.32) and seeing subharmonic generation in the receive signal. Although it cannot be stated with absolute certainty, it seems very likely that the GCD tonpilz failure discussed in section 6.6.2 occurred because cavitation bubbles unloaded the radiating face.

Ultimately, cavitation signifies that the GCD transducer encountered a limitation of the available test facility. Testing was discontinued since further increases in the drive level would 1) risk damaging the transducer and 2) produce poor results. Although it was desired to characterize the GCD tonpilz at higher drive levels, the fact that cavitation occurred is not necessarily a negative outcome: the transducer’s ability to cause cavitation at a fraction of its full-strain drive level
points to the high-power capability of the device.

It is the author’s hope that there will be future opportunities for this design to be characterized up to the full 1 A drive. The most practical way to do this will be to test at a different facility in which the cavitation threshold can be significantly increased by raising the hydrostatic pressure.

6.7 Evaluation of GCD tonpilz

The mechanical quality factor, effective electromechanical coupling coefficient, and efficiency of the GCD tonpilz are compared to other magnetostrictive transducers in Table 6.3. Values that were not directly provided by the indicated references were estimated using impedance data and equations 2.36 and 6.6. The reported $Q_m$ values are calculated from response curves using equation 2.23. Table 6.3 includes Galfenol, nickel, and Terfenol-D transducer of both tonpilz and flexextensional designs. Output values (such as TCR and source level) are not compared here due to the wide range of resonance frequencies.

Having assembled these metrics, it is appropriate to comment on the performance of the GCD tonpilz. First, its mechanical quality is on-par with the nickel design and some Terfenol-D devices. Second, the electromechanical coupling coefficient of the GCD tonpilz is better than the other devices considered, except those for which a high coupling coefficient was the primary design objective. Third,
Table 6.3. Comparison of published single-element magnetostrictive sonar transducers that are biased with permanent magnets.

<table>
<thead>
<tr>
<th>Transducer</th>
<th>$f_0$ [kHz]</th>
<th>$Q_m$</th>
<th>$k_{eff}$</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSU ARL Galfenol Configurable Drive Tonpilz</td>
<td>10</td>
<td>6.4</td>
<td>44%</td>
<td>50%</td>
</tr>
<tr>
<td>HUSL Nickel SPEP-PM1-1</td>
<td>60</td>
<td>7.5</td>
<td>35%</td>
<td>43%</td>
</tr>
<tr>
<td>NUWC Mod IV Terfenol-D Flextensional</td>
<td>1.5</td>
<td>3.8</td>
<td>20%</td>
<td>15%</td>
</tr>
<tr>
<td>NUWC Terfenol-D Tonpilz</td>
<td>2.5</td>
<td>2.5</td>
<td>36%</td>
<td>60%</td>
</tr>
<tr>
<td>CEDRAT Terfenol-D Class IV Flextensional</td>
<td>0.3</td>
<td>5.0</td>
<td>30%</td>
<td>46%</td>
</tr>
<tr>
<td>PSU ARL Terfenol-D High-Coupling Tonpilz</td>
<td>6.4</td>
<td>6.1</td>
<td>54%</td>
<td>–</td>
</tr>
</tbody>
</table>

*Data from [7], p. 214-219. Coupling coefficient estimated from Figure 82 on p. 218.

The GCD transducer has an electroacoustic efficiency that ranks well against the other transducers. Overall, Galfenol is shown to compare favorably with nickel and Terfenol-D in these areas. Even though Terfenol has a much higher strain capability, Galfenol is still competitive and should be considered for transducer work in which the raw power output is not the sole performance criterion.
Conclusions and recommendations

This dissertation has examined the Galfenol configurable drive from initial concept to in-water testing of a prototype device. The present chapter reviews these efforts, makes some conclusions regarding the outcomes of the study, discusses the contributions made by this project, recommends areas for improvement, and suggests topics for future work.

7.1 Summary

In investigating the use of Galfenol for sonar transducers, this study has:

- Proposed a configurable drive concept that incorporates the essential features for effective magnetostrictive design: good coil coupling, a mechanism for magnetic biasing, and eddy current mitigation.
- Presented a design that incorporates either DC or permanent magnet biasing, allows for easy winding of the drive coils, and anticipates high-strain rolled Galfenol to make the manufacture of magnetostrictive drives rapid and inexpensive.
- Developed simplified modeling tools for tailoring the geometry and magnetic circuit of the drive to specific applications.
- Designed a prototype drive with the goal of achieving the full magnetostriction of Galfenol; the design was completed with a simplified one-dimensional
model and verified with finite element analysis.

- Fabricated the prototype drive from Galfenol steel lamina and developed techniques for insulating, aligning, and assembling the laminated stacks.

- Simulated the performance of the drive with one-dimensional modeling that includes the magnetic domain and eddy current losses and compared this model to finite element analysis.

- Built the completed drive into a tonpilz transducer and mounted it in a watertight housing for in-water testing.

- Characterized the transducer’s in-water performance in an anechoic tank facility and analyzed the results.

- Modeled the behavior of the transducer in-air and in-water with good agreement with experimental results.

### 7.2 Conclusions

A configurable Galfenol drive concept is proposed and investigated in this dissertation. The goal of this project is to lower the design barriers associated with magnetostrictive transducers by introducing a flexible drive that is configured with a set of simplified design tools. The attractive material properties of Galfenol are successfully exploited: 1) the material’s full strain capability is capable of being fully accessed 2) the mechanical strength eliminates the need for a stress-bolt (although difficulty with endmass bonding has been found to be a related issue), and 3) the high permeability has been used to create a low-leakage circuit that allows the $1 \text{T}_{\text{DC}}$ bias to be achieved with permanent magnets located away from the drive segment. In addition, this design achieves the necessary features for magnetostrictive design without resorting to auxiliary structures. Conclusions from this study are divided into three groups that deal with the material, the modeling, and the prototype drive.
7.2.1 Assessment of Galfenol material

Galfenol has a property set that suggests that the material is well-suited for sonar transducer design. The motivation for this project was to investigate this possibility and compare Galfenol to other magnetostrictive materials. It is tempting to judge Galfenol as being inferior to Terfenol-D simply on the basis of strain capabilities. However, this takes a very narrow view of the materials and fails to consider the whole set of properties. Based on the three performance metrics presented in Table 6.3, Galfenol compares favorably with Terfenol. Instead of ranking one above the other, it seems more appropriate to consider these two materials as being of equal merit for sonar applications but suited to different tasks. This leads to the conclusion that Galfenol is one of the best currently-available magnetostrictive materials and deserves to be considered for future magnetostrictive designs.

On the other hand, Galfenol suffers many of the same setbacks as other magnetostrictive materials: it has a low $Q_m$, significant magnetic nonlinearities, and its otherwise attractive material coupling coefficient tends to be diminished in real-world designs by the coil coupling. These material difficulties are compounded by the design hurdles inherent to the magnetic design. External biasing, magnetic circuit design, the absence of excellent flux conductors/flux insulators, the need to avoid nearby ferromagnetic materials, and the lack of industry familiarity with magnetostriction are definite challenges. While the configurable drive concept is meant to reduce these barriers, they cannot be completely eliminated.

However, Galfenol’s unique properties may make it especially attractive for certain applications. Galfenol’s mechanical strength could be leveraged for transducers that are designed to survive shock and explosive environments. While there are many systems that might encounter these conditions, a specific application for which Galfenol may be well-suited is air-dropped platforms that house transducers that must withstand impact with the water’s surface. Additionally, the absence of fatigue in magnetostrictive materials may make Galfenol transducers appropriate for clandestine platforms that are infrequently or never serviced.
7.2.2 Assessment of one-dimensional modeling techniques

Based on the modeling efforts in this dissertation it is concluded that one-dimensional modeling is effective for rapid design and simulation of magnetostrictive devices. For a relatively new material like Galfenol, this is beneficial as only the material properties in the (3,3) direction are needed, i.e. this technique does not require the full property matrices.

Progressive comparison of the one-dimensional model to the actual device throughout the assembly process provided much insight into the validity of using these modeling techniques, but raises questions over the predictive abilities of the model. Ultimately it seems that the model is limited more by the sample-to-sample variation in material properties than the assumptions inherent in the one-dimensional scheme. However, there were some changes that had to be made to the model that were not apparent from the beginning of the modeling process. These are documented here in hopes that future modeling efforts will be improved. Specifically, there were three items that needed to be reworked in order to achieve a good match between the model and measurement:

1. The remanent induction, $B_r$, of the permanent magnets appeared to be less than the value specified by the manufacturer. A good fit between model and measurement was observed for the case that the remanence value used in the model was 75% of the manufacturer’s specified remanence flux density. The reason for this discrepancy is unknown and it is suggested that future efforts experimentally verify the remanent induction of permanent magnets before assembling them into a device.

2. Correctly modeling the length of the flux paths in the leakage reluctance value, $R_{\text{leak}}$, was also critical to good model performance. This flux path length is the short dimension across the air domain in the center of the magnetic circuit. It was found that this reluctance is most accurate when the path length is taken to be the distance between the upper coil layers rather than from stack-to-stack.

3. Use of Roter’s [111] 95% criteria for expected-versus-actual coil winds will provide a better estimate of the coil during the design stage. Using $n =$
\[ \ell/d_{\text{wire}} \] to calculate windings assumed that the turns were perfectly placed, which, of course, was not true in practice. Although this factor is not exact, it is useful. This can be readily included in the \text{SPICE} parameter statements that calculate the number of turns.

### 7.2.3 Assessment of prototype drive

The GCD prototype demonstrates that the configurable drive concept is realizable and that finely laminated Galfenol structures are feasible for transducer motor sections. In Table 6.3, the finished prototype was shown to be comparable to other magnetostrictive transducers.

### 7.3 Contributions

Several contributions to the field of electroacoustics are contained in this dissertation. These contributions are:

- An original concept for a configurable drive that can be tailored to specific applications by altering 1) the geometry, 2) the configuration of modules, and 3) the biasing scheme.

- A reduced-model method for designing magnetic circuits to make magnetostrictive design more accessible.

- \text{SPICE} modeling of magnetostrictive devices in which the magnetic domain and eddy current losses are included.

- An anhysteretic mathematical expression for describing a three-region \( B-H \) curve.

- A practical investigation into the construction of finely laminated Galfenol structures in non-rod geometries and the documentation of updated assembly techniques.

- Experimental data for a Galfenol sonar transducer tested in-water.
7.4 Future work

Several areas have been identified as potential topics for future work:

- The foremost concern for future work is testing the GCD tonpilz in a high-pressure facility in order to obtain a better understanding of the high-power operation and nonlinear behavior. Doing this will require modifying or redesigning the mounting plate to hold the transducer rigidly in place, probably best accomplished by supporting the transducer at the back of the headmass. A compliant material will need to be used to mechanically decouple the headmass motion from the plate [88].

- Better material property measurements are essential to improved model accuracy, yet no standard measurement method exists for determining the magnetostrictive parameters of non-toroidal geometries. Several approaches have been tried (see review in [52]), but further work is required. This is a significant obstacle for the adoption of magnetostrictive materials and should be considered for additional investigation.

- Based on the material measurements that have been made, a more consistent material is needed. At present, Galfenol’s properties can vary widely (see, for example, Figure 5.19). Since many of the properties can be controlled through material processing, it seems that standard formulations are required for effective predictive modeling of Galfenol devices.

- One troubling aspect of Galfenol is the grain boundary embrittlement that has been observed to cause cracking in rolling and stamping processes. Similar cracking was observed during the manufacture of the GCD lamina. Although this effect has already been studied by Brooks et al. [46] for the cases of rolling and stamping, it would be useful to more fully understand the cracking seen in thin wire-cut sheets. Also, rolling and stamping processes for Galfenol should be further studied since the ability to form stamped laminations from rolled Galfenol will make drive manufacture much more economical.
• In the evaluation of the drive’s magnetic state, the bias value was found to be lower than expected. It was suggested without proof that this might be caused by a discrepancy in reported versus actual remanence induction. While this seems quite plausible, a full investigation into this topic is appropriate. If some other effect is responsible, a designer following the GCD approach should be aware of it.

• The coil design included in the one-dimensional design model is oversimplified since it does not consider the power delivery limitations of actual amplifiers. A more comprehensive coil design that takes this into account will be important for developing transducers that will be integrated into real-world systems. Incorporation of this feature into the one-dimensional design model should be pursued.

• Very little data is available on the thermal properties of Galfenol. This information will be very valuable for design work and may suggest new possibilities for Galfenol applications. Also, not having these values discourages designing with Galfenol. A high emphasis should be placed on this topic.
GCD tonpilz ATF testing results

This appendix contains the complete test results from the Applied Research Laboratory’s anechoic tank facility (ATF).

A.1 Test 1 – Sweeps, 15 kHz to 100 kHz at 48 mA (50 mA attempted)

- Figure A.1 - Transmit power response.
- Figure A.2 - Transmit voltage response.
- Figure A.3 - Transmit current response.
- Figure A.4 - Impedance magnitude.
- Figure A.5 - Impedance phase.
Figure A.1. GCD tonpilz ATF testing - transmitting power response, 15 kHz to 100 kHz at 48 mA (50 mA attempted)
Figure A.2. GCD tonpilz ATF testing - transmitting voltage response, 15 kHz to 100 kHz at 48 mA (50 mA attempted)
Figure A.3. GCD tonpilz ATF testing - transmitting current response, 15kHz to 100kHz at 48mA (50mA attempted)
Figure A.4. GCD tonpilz ATF testing - impedance magnitude, 15 kHz to 100 kHz at 48 mA (50 mA attempted)
Figure A.5. GCD tonpilz ATF testing - impedance phase, 15 kHz to 100 kHz at 48 mA (50 mA attempted)
A.2 Test 2 – Sweeps, 5 kHz to 20 kHz at 48 mA
(50 mA attempted)

- Figure A.6 - Transmit power response.
- Figure A.7 - Transmit voltage response.
- Figure A.8 - Transmit current response.
- Figure A.9 - Impedance magnitude.
- Figure A.10 - Impedance phase.
Figure A.6. GCD tonpilz ATF testing - transmitting power response, 5 kHz to 20 kHz at 48 mA (50 mA attempted)
Figure A.7. GCD tonpilz ATF testing - transmitting voltage response, 5 kHz to 20 kHz at 48 mA (50 mA attempted)
Figure A.8. GCD tonpilz ATF testing - transmitting current response, 5kHz to 20kHz at 48mA (50mA attempted)
Figure A.9. GCD tonpilz ATF testing - impedance magnitude, 5 kHz to 20 kHz at 48 mA (50 mA attempted)
Figure A.10. GCD tonpilz ATF testing - impedance phase, 5 kHz to 20 kHz at 48 mA (50 mA attempted)
A.3  Test 3 – Beam pattern, 10 kHz horizontal at 48 mA (50 mA attempted)
   • Figure A.11

A.4  Test 4 – Beam pattern, 20 kHz horizontal at 48 mA (50 mA attempted)
   • Figure A.12

A.5  Test 5 – Beam pattern, 30 kHz horizontal at 48 mA (50 mA attempted)
   • Figure A.13
Figure A.11. GCD tonpilz ATF testing - beam pattern, 10 kHz horizontal at 48 mA (50 mA attempted)
Figure A.12. GCD tonpilz ATF testing - beam pattern, 20 kHz horizontal at 48 mA (50 mA attempted)
Figure A.13. GCD tonpilz ATF testing - beam pattern, 30 kHz horizontal at 48 mA (50 mA attempted)
A.6 Test 6 – Sweeps, 5 kHz to 25 kHz at 176 mA (200 mA attempted)

- Figure A.14 - Transmit power response.
- Figure A.15 - Transmit voltage response.
- Figure A.16 - Transmit current response.
- Figure A.17 - Impedance magnitude.
- Figure A.18 - Impedance phase.
Figure A.14. GCD tonpilz ATF testing - transmitting power response, 5 kHz to 25 kHz at 176 mA (200 mA attempted)
Figure A.15. GCD tonpilz ATF testing - transmitting voltage response, 5 kHz to 25 kHz at 176 mA (200 mA attempted)
Figure A.16. GCD tonpilz ATF testing - transmitting current response, 5 kHz to 25 kHz at 176 mA (200 mA attempted)
Figure A.17. GCD tonpilz ATF testing - impedance magnitude, 5 kHz to 25 kHz at 176 mA (200 mA attempted)
Figure A.18. GCD tonpilz ATF testing - impedance phase, 5 kHz to 25 kHz at 176 mA (200 mA attempted)
A.7 Test 7 – Beam pattern, 10 kHz horizontal at 176 mA (200 mA attempted)

• Figure A.19

A.8 Test 8 – Beam pattern, 20 kHz horizontal at 176 mA (200 mA attempted)

• Figure A.20

A.9 Test 9 – Beam pattern, 30 kHz horizontal at 176 mA (200 mA attempted)

• Figure A.21
Figure A.19. GCD tonpilz ATF testing - beam pattern, 10 kHz horizontal at 176 mA (200 mA attempted)
Figure A.20. GCD tonpilz ATF testing - beam pattern, 20 kHz horizontal at 176 mA (200 mA attempted)
Figure A.21. GCD tonpilz ATF testing - beam pattern, 30 kHz horizontal at 176 mA (200 mA attempted)
A.10 Test 10 – Beam pattern, 10 kHz vertical at 48 mA (50 mA attempted)

• Figure A.22

A.11 Test 11 – Beam pattern, 20 kHz vertical at 48 mA (50 mA attempted)

• Figure A.23

A.12 Test 12 – Beam pattern, 30 kHz vertical at 48 mA (50 mA attempted)

• Figure A.24
Figure A.22. GCD tonpilz ATF testing - beam pattern, 10 kHz vertical at 48 mA (50 mA attempted)
Figure A.23. GCD tonpilz ATF testing - beam pattern, 20 kHz vertical at 48 mA (50 mA attempted)
Figure A.24. GCD tonpilz ATF testing - beam pattern, 30 kHz vertical at 48 mA (50 mA attempted)
A.13  Test 13 – Beam pattern, 10 kHz vertical at 176 mA (200 mA attempted)

• Figure A.25

A.14  Test 14 – Beam pattern, 20 kHz vertical at 176 mA (200 mA attempted)

• Figure A.26

A.15  Test 15 – Beam pattern, 30 kHz vertical at 176 mA (200 mA attempted)

• Figure A.27
Figure A.25. GCD tonpilz ATF testing - beam pattern, 10kHz vertical at 176 mA (200 mA attempted)
Figure A.26. GCD tonpilz ATF testing - beam pattern, 20kHz vertical at 176 mA (200 mA attempted)
Figure A.27. GCD tonpilz ATF testing - beam pattern, 30 kHz vertical at 176 mA (200 mA attempted)
A.16 Test 16 – Sweeps, 5 kHz to 25 kHz at approximately 200 mA

Measured after suspected damage to transducer caused by increasing drive level to approximately 500 mA. Compare to Test 6.

- Figure A.28 - Impedance magnitude.

- Figure A.29 - Impedance phase.
Figure A.28. GCD tonpilz ATF testing - impedance magnitude, 5 kHz to 25 kHz at approximately 200 mA
Figure A.29. GCD tonpilz ATF testing—impedance phase, 5 kHz to 25 kHz at approximately 200 mA.
A.17 Test 22 – Sweeps, 5 kHz to 25 kHz at 90 mA
(100 mA attempted)

• Figure A.30 - Transmit power response.
• Figure A.31 - Transmit voltage response.
• Figure A.32 - Transmit current response.
• Figure A.33 - Impedance magnitude.
• Figure A.34 - Impedance phase.
Figure A.30. GCD tonpilz ATF testing - transmitting power response, 5 kHz to 25 kHz at 90 mA (100 mA attempted)
Figure A.31. GCD tonpilz ATF testing - transmitting voltage response, 5 kHz to 25 kHz at 90 mA (100 mA attempted)
**Figure A.32.** GCD tonpilz ATF testing - transmitting current response, 5 kHz to 25 kHz at 90 mA (100 mA attempted)
Figure A.33. GCD tonpilz ATF testing - impedance magnitude, 5 kHz to 25 kHz at 90 mA (100 mA attempted)
Figure A.34. GCD tonpilz ATF testing - impedance phase, 5 kHz to 25 kHz at 90 mA (100 mA attempted)
A.18  Test 23 – Sweeps, 5 kHz to 25 kHz at 189 mA 
(200 mA attempted)

• Figure A.35 - Transmit power response.

• Figure A.36 - Transmit voltage response.

• Figure A.37 - Transmit current response.

• Figure A.38 - Impedance magnitude.

• Figure A.39 - Impedance phase.
Figure A.35. GCD tonpilz ATF testing - transmitting power response, 5 kHz to 25 kHz at 189 mA (200 mA attempted)
Figure A.36. GCD tonpilz ATF testing - transmitting voltage response, 5 kHz to 25 kHz at 189 mA (200 mA attempted)
Figure A.37. GCD tonpilz ATF testing - transmitting current response, 5 kHz to 25 kHz at 189 mA (200 mA attempted)
Figure A.38. GCD tonplz ATF testing - impedance magnitude, 5 kHz to 25 kHz at 189 mA (200 mA attempted)
Figure A.39. GCD tonpilz ATF testing - impedance phase, 5 kHz to 25 kHz at 189 mA (200 mA attempted)
A.19 Test 24 – Sweeps, 5 kHz to 25 kHz at 264 mA (300 mA attempted)

- Figure A.40 - Transmit power response.
- Figure A.41 - Transmit voltage response.
- Figure A.42 - Transmit current response.
- Figure A.43 - Impedance magnitude.
- Figure A.44 - Impedance phase.
Figure A.40. GCD tonpilz ATF testing - transmitting power response, 5 kHz to 25 kHz at 264 mA (300 mA attempted)
Figure A.41. GCD tonpilz ATF testing - transmitting voltage response, 5 kHz to 25 kHz at 264 mA (300 mA attempted)
Figure A.42. GCD tonpilz ATF testing - transmitting current response, 5 kHz to 25 kHz at 264 mA (300 mA attempted)
Figure A.43. GCD tonpilz ATF testing - impedance magnitude, 5 kHz to 25 kHz at 264 mA (300 mA attempted)
Figure A.44. GCD tonpilz ATF testing - impedance phase, 5 kHz to 25 kHz at 264 mA (300 mA attempted)
A.20 Test 25 – Sweeps, 5 kHz to 25 kHz at 370 mA (400 mA attempted)

- Figure A.45 - Transmit power response.
- Figure A.46 - Transmit voltage response.
- Figure A.47 - Transmit current response.
- Figure A.48 - Impedance magnitude.
- Figure A.49 - Impedance phase.
Figure A.45. GCD tonpilz ATF testing - transmitting power response, 5 kHz to 25 kHz at 370 mA (400 mA attempted)
Figure A.46. GCD tonpilz ATF testing - transmitting voltage response, 5 kHz to 25 kHz at 370 mA (400 mA attempted)
Figure A.47. GCD tonpilz ATF testing - transmitting current response, 5 kHz to 25 kHz at 370 mA (400 mA attempted)
Figure A.48. GCD tonpilz ATF testing - impedance magnitude, 5 kHz to 25 kHz at 370 mA (400 mA attempted)
Figure A.49. GCD tonpilz ATF testing - impedance phase, 5 kHz to 25 kHz at 370 mA (400 mA attempted)
A.21 Test 22 – Sweeps, 5 kHz to 25 kHz at 323 mA (500 mA attempted)

- Figure A.50 - Transmit power response.
- Figure A.51 - Transmit voltage response.
- Figure A.52 - Transmit current response.
- Figure A.53 - Impedance magnitude.
- Figure A.54 - Impedance phase.
Figure A.50. GCD tonpilz ATF testing - transmitting power response, 5 kHz to 25 kHz at 323 mA (500 mA attempted)
Figure A.51. GCD tonpilz ATF testing - transmitting voltage response, 5 kHz to 25 kHz at 323 mA (500 mA attempted)
Figure A.52. GCD tonpilz ATF testing - transmitting current response, 5 kHz to 25 kHz at 323 mA (500 mA attempted)
Figure A.53. GCD tonplz ATF testing - impedance magnitude, 5 kHz to 25 kHz at 323 mA (500 mA attempted)
Figure A.54. GCD tonpilz ATF testing - impedance phase, 5 kHz to 25 kHz at 323 mA (500 mA attempted)
Appendix B

Material characterization curves

This appendix contains the material characterization curves for the nine Galfenol steel boules used in the construction of the Galfenol configurable drive. $S$-$B$ and $B$-$H$ data are shown both pre- and post-stress annealing for four different preloads.

- Figure B.1 - Boule D1-9-40-76
- Figure B.2 - Boule D1-9-42-153
- Figure B.3 - Boule D1-9-42-1
- Figure B.4 - Boule D1-9-40-1
- Figure B.5 - Boule D1-9-40-153
- Figure B.6 - Boule D1-9-42-76
- Figure B.7 - Boule D1-9-39-3
- Figure B.8 - Boule D1-9-39-78
- Figure B.9 - Boule D1-9-39-153
Figure B.1. ETREMA Products, Inc. measurement - Boule D1-9-40-76. $S$-$B$ and $B$-$H$ curves are shown both before and after stress annealing as a function of preload.
Figure B.2. ETREMA Products, Inc. measurement - Boule D1-9-42-153. \( S \)-\( B \) and \( B \)-\( H \) curves are shown both before and after stress annealing as a function of preload.
Figure B.3. ETREMA Products, Inc. measurement - Boule D1-9-42-1. S-B and B-H curves are shown both before and after stress annealing as a function of preload.
Figure B.4. ETREMA Products, Inc. measurement - Boule D1-9-40-1. S-B and B-H curves are shown both before and after stress annealing as a function of preload.
Figure B.5. ETREMA Products, Inc. measurement - Boule D1-9-40-153. S, B and B-H curves are shown both before and after stress annealing as a function of preload.
Figure B.6. ETREMA Products, Inc. measurement - Boule D1-9-42-76. $S-B$ and $B-H$ curves are shown both before and after stress annealing as a function of preload.
Figure B.7. ETREMA Products, Inc. measurement - Boule D1-9-39-3. B-H curves are shown both before and after stress annealing as a function of preload.
Figure B.8. ETREMA Products, Inc. measurement - Boule D1-9-39-78. $S$, $B$, and $B-H$ curves are shown both before and after stress annealing as a function of preload.
Figure B.9. ETREMA Products, Inc. measurement - Boule D1-9-39-153. S-B and B-H curves are shown both before and after stress annealing as a function of preload.
Appendix C

Useful derivations

C.1 Two-port piezomagnetic magnetomechanical bar analytical model

Following Hunt [16] and Hall [5], this analytical approach creates a simple mathematical model of the transducer. An electromechanical transducer can be most generally described by the following equations:

\[ E = Z_e i + T_{em} v \] \hspace{1cm} (C.1)

\[ F = T_{me} i + z_m v \] \hspace{1cm} (C.2)

Recall that piezomagnetism can be described by 2.44a and 2.44b. The aim here is to manipulate 2.44a and 2.44b to match the form of C.1 and C.2 so that traditional, analytical tools may be used.

First, solve 2.44a for T (the stress)

\[ T = E^H S - dE^H H \] \hspace{1cm} (C.3)

Substitute C.3 into 2.44b

\[ B = d (E^H S - dE^H H) + \mu^T H \] \hspace{1cm} (C.4)

Remembering that \( s^H = \frac{1}{E^H} \) and \( k^2 = \frac{d^2}{s^H \mu^H} \), simplify C.4
and use $2.42$ to arrive at:

$$B = dE^H S + \mu^S H$$  \hfill (C.6)

Next, recognize that $S = \frac{v}{\ell} = \frac{v}{j\omega \ell}$ and $H = \frac{ni}{\ell}$ and substitute these into $C.3$ and $C.6$:

$$T = \frac{E^H}{j\omega \ell} v - \frac{dE^H n}{\ell} i$$  \hfill (C.7)

$$B = \frac{dE^H}{j\omega \ell} v + \frac{\mu^S n}{\ell} i$$  \hfill (C.8)

Combine Ohm's law with Faraday's law of induction $(2.1c)$ to get:

$$V = R_e i + j\omega n AB$$  \hfill (C.9)

Substitute this expression into $C.8$:

$$V = R_e i + j\omega n A \left( \frac{dE^H}{j\omega \ell} v + \frac{\mu^S n}{\ell} i \right)$$  \hfill (C.10)

Next, recall that

$$F = AT$$  \hfill (C.11)

and substitute $C.7$ to get

$$F = A \left( \frac{E^H}{j\omega \ell} v - \frac{dE^H n}{\ell} i \right)$$  \hfill (C.12)

At this point the equation can be rearranged to demonstrate that they have successfully been cast into the form of $C.1$ and $C.2$:

$$V = \left( R_e + \frac{j\omega \mu^S n^2 A}{\ell} \right) i + dnk_m^H v$$  \hfill (C.13)
\[ F = -dnk_m^H + \frac{k_m^H}{j\omega} \]  

(C.14)

where \( k_m^H = \frac{\Delta E^H}{\ell} \) is the lumped mechanical stiffness of the material at constant field. By comparison:

\[ T_{\text{em}} = dnk_m^H \]  

(C.15)

\[ T_{\text{me}} = -dnk_m^H \]  

(C.16)

\[ Z_e = R_e + j\omega\mu n^2 A \frac{\ell}{\ell} \]  

(C.17)

\[ z_m = \frac{k_m^H}{j\omega} \]  

(C.18)

Note that \( T_{\text{em}} = -T_{\text{me}} \). If we define the motional impedance, \( Z_{\text{mot}} \), as

\[ Z_{\text{mot}} = \frac{-T_{\text{em}}T_{\text{me}}}{z_m + z_L} \]  

(C.19)

\((z_L \text{ is the mechanical load impedance})\) we find that

\[ Z_{\text{mot}} = \frac{d^2n^2A^2 (E^H)^2}{\ell^2 (z_m + z_L)} \]  

(C.20)

which can be expressed as

\[ Z_{\text{mot}} = \frac{k^2\mu T n^2 (k_m^H)^2}{E^H (z_m + z_L)} \]  

(C.21)

Finally, the electrical input impedance, \( Z_{ee} \), is defined as:

\[ Z_{ee} = Z_e + Z_{\text{mot}} \]  

(C.22)

or

\[ Z_{ee} = R + \frac{j\omega\mu n^2 A}{\ell} + \frac{k^2\mu T n^2 (k_m^H)^2}{E^H (z_m + z_L)} \]  

(C.23)
Implementing this model in MATLAB and running it for variety of electromechanical coupling coefficients results in the impedance curves shown in Figure C.1. Notice that there is no mechanical damping included in this model.

### C.2 Piezomagnetic plate

Consider a piezomagnetic plate (i.e. the thickness, \( \ell \), is much less than any other dimension) and constant cross-sectional area \( A \). Being a (3,3) mode transducer,
the B-field does not vary in the 3-direction. Therefore, \( \frac{\partial B_3}{\partial x_3} = 0 \).

Using the plate approximation, \( S_1 = S_2 = S_4 = S_5 = S_6 = 0 \) and \( S_3 \neq 0 \) [?]. Knowing the most about stress, a straightforward pair of constitutive equations can be selected.

\[
T_3 = c_{33}^B S_3 - h_{33} B_3 \quad \text{(C.24)}
\]
\[
H_3 = -h_{33} S_3 + \gamma_{33}^S B_3 \quad \text{(C.25)}
\]

Assume an equation of motion in the form,

\[
\xi_3 = \left[ C_1 \sin \left( \frac{\omega x_3}{v_l^B} \right) + C_2 \cos \left( \frac{\omega x_3}{v_l^B} \right) \right] e^{j\omega t} \quad \text{(C.26)}
\]

where \( \xi \) is the displacement, \( x_3 \) is the position along the thickness direction, and \( v_l^B \) is the sound speed. \( C_1 \) and \( C_2 \) are constants to be solved for. Velocity may be expressed as

\[
\dot{\xi}_3 = j\omega \left[ C_1 \sin \left( \frac{\omega x_3}{v_l^B} \right) + C_2 \cos \left( \frac{\omega x_3}{v_l^B} \right) \right] e^{j\omega t} \quad \text{(C.27)}
\]

Evaluate this at the boundary conditions. First set \( \dot{\xi} = v_1 \) at \( x_3 = 0 \).

\[
\dot{\xi}_3 \bigg|_{x_3=0} = v_1 = j\omega \left[ C_2 \right] e^{j\omega t} \quad \text{(C.28)}
\]

Solving for \( C_2 \) yields

\[
C_2 = \frac{1}{j\omega} v_1 e^{-j\omega t} \quad \text{(C.29)}
\]

Apply the boundary condition \( \dot{\xi}_3 = v_2 \) at \( x_3 = \ell \):

\[
\dot{\xi}_3 \bigg|_{x_3=\ell} = -v_2 = j\omega \left[ C_1 \sin \left( \frac{\omega \ell}{v_l^B} \right) + C_2 \cos \left( \frac{\omega \ell}{v_l^B} \right) \right] e^{j\omega t} \quad \text{(C.30)}
\]

Substitute in \( \text{C.29} \) and solve for \( C_1 \).

\[
C_1 = -\frac{1}{j\omega} \left( \frac{v_2}{\sin \left( \frac{\omega \ell}{v_l^B} \right)} + \frac{v_1}{\tan \left( \frac{\omega \ell}{v_l^B} \right)} \right) e^{-j\omega t} \quad \text{(C.31)}
\]

Now, define the effort variable (force) at the first mechanical port as the product
of stress and area:

\[ F_1 = -T_3 A \big|_{x_3=0} \]  

(C.32)

Substitute in (C.24).

\[ F_1 = -A \left( c_{33}^B S_3 - h_{33} B_3 \right) \big|_{x_3=0} \]  

(C.33)

Rearrange Faraday’s law of induction to arrive at

\[ B_3 = -\frac{1}{j\omega n A} V_3 \]  

(C.34)

Now, substitute (C.34) into (C.33).

\[ F_1 = -A \left( c_{33}^B S_3 + h_{33} \frac{V_3}{j\omega n A} \right) \big|_{x_3=0} \]  

(C.35)

Evaluate the strain in the 3-direction at this boundary.

\[ S_3 = \frac{\partial \xi}{\partial x_3} = \frac{\omega}{v_3^B} \left[ C_1 \cos \left( \frac{\omega x_3}{v_3^B} \right) - C_2 \sin \left( \frac{\omega x_3}{v_3^B} \right) \right] e^{j\omega t} \]  

(C.36)

\[ S_3 \big|_{x_3=0} = \frac{\omega}{v_3^B} C_1 e^{j\omega t} \]  

(C.37)

Substitute in (C.31) and simplify.

\[ S_3 \big|_{x_3=0} = \frac{j}{v_3^B} \left[ \frac{v_2}{\sin \left( \frac{\omega \ell}{v_3^B} \right)} + \frac{v_1}{\tan \left( \frac{\omega \ell}{v_3^B} \right)} \right] \]  

(C.38)

Take this result and substitute it into (C.38)

\[ F_1 = -A \left[ \frac{j}{v_3^B} \left( \frac{v_2}{\sin \left( \frac{\omega \ell}{v_3^B} \right)} + \frac{v_1}{\tan \left( \frac{\omega \ell}{v_3^B} \right)} \right) + \frac{h_{33}}{j\omega n A} V_3 \right] \]  

(C.39)

Define the effort variable (force) at the second mechanical port.

\[ F_2 = -T_3 A \big|_{x_3=\ell} \]  

(C.40)

Substitute in (C.24) and apply Faraday’s law of induction.
\[ F_2 = -A \left( c_{33}^B S_3 + \frac{h_{33}}{j\omega n A} V_3 \right) \Bigg|_{x_3=\ell} \]  

(C.41)

Evaluate \( S_3 \) at this boundary.

\[ S_3 = \frac{\partial \xi}{\partial x_3} = \frac{\omega}{v_t^B} \left[ C_1 \cos \left( \frac{\omega x_3}{v_t^B} \right) - C_2 \sin \left( \frac{\omega x_3}{v_t^B} \right) \right] e^{j\omega t} \]  

(C.42)

\[ S_3|_{x_3=\ell} = \frac{\omega}{v_t^B} \left[ C_1 \cos \left( \frac{\omega \ell}{v_t^B} \right) - C_2 \sin \left( \frac{\omega \ell}{v_t^B} \right) \right] e^{j\omega t} \]  

(C.43)

Substitute in [C.31] and [C.29] and cancel time dependent terms.

\[ S_3|_{x_3=\ell} = \frac{\omega}{v_t^B} \left[ \left( -\frac{1}{j\omega} \left( \frac{v_2}{\sin \left( \frac{\omega \ell}{v_t^B} \right)} + \frac{v_1}{\tan \left( \frac{\omega \ell}{v_t^B} \right)} \right) \right) \cos \left( \frac{\omega \ell}{v_t^B} \right) - \left( \frac{1}{j\omega} v_1 \right) \sin \left( \frac{\omega \ell}{v_t^B} \right) \right] \]  

(C.44)

Simplify the expression.

\[ S_3|_{x_3=\ell} = \frac{j}{v_t^B} \left[ \left( \frac{1}{\tan \left( \frac{\omega \ell}{v_t^B} \right)} \right) v_2 + \left( \frac{\cos^2 \left( \frac{\omega \ell}{v_t^B} \right) + \sin^2 \left( \frac{\omega \ell}{v_t^B} \right) }{\sin \left( \frac{\omega \ell}{v_t^B} \right)} \right) v_1 \right] \]  

(C.45)

\[ S_3|_{x_3=\ell} = \frac{j}{v_t^B} \left[ \left( \frac{1}{\tan \left( \frac{\omega \ell}{v_t^B} \right)} \right) v_2 + \left( \frac{1}{\sin \left( \frac{\omega \ell}{v_t^B} \right)} \right) v_1 \right] \]  

(C.46)

Substitute this expression for strain into [C.59]

\[ F_2 = -A \left( \frac{j c_{33}^B}{v_t^B} \left[ \frac{v_2}{\tan \left( \frac{\omega \ell}{v_t^B} \right)} + \frac{v_1}{\sin \left( \frac{\omega \ell}{v_t^B} \right)} \right] + \frac{h_{33}}{j\omega n A} V_3 \right) \]  

(C.47)

Next it is easiest to define the flow variable (current) in the electrical port rather than the effort variable (voltage). This preference is due to the form of Ampere’s law\(^1\)

\(^1\)In the piezoelectric analysis, Kirchhoff’s Voltage Law, \( V_3 = \int E_3 \cdot dl \), is used which makes it convenient to find the effort variable across the electrical terminals. The fact that electrical flow is used in the present derivation puts to the fact that the electrical domain will either be
\[ i_3 = -\frac{1}{n} \int H_3 dx_3 \]  
(C.48)

Note that the direction for positive flow of current is arbitrarily chosen. By selecting this direction, however, the final matrix (in equation C.67) will be symmetric.

\[ i_3 = -\frac{1}{n} \int_0^\ell H_3 dx_3 + \frac{1}{n} H_{rp} \ell_{rp} \]  
(C.49)

\( H_{rp} \) and \( \ell_{rp} \) are the H-field and length of the return path, respectively. Assume that the second term is negligible. Now the equation is again analogous in form to that of the piezoelectric case. \( H_3 \) can be found from C.25.

\[ i_3 = -\frac{1}{n} \int_0^\ell h_{33} S_3 + \gamma_{33}^S B_3 dx_3 \]  
(C.50)

Define \( S_3 \) from C.42, C.31 and C.29

\[ S_3 = \frac{\partial \xi}{\partial x_3} = \frac{j}{v_t^B} \left[ \left( \frac{v_2}{\sin \left( \frac{\omega}{v_t^B} \right)} + \frac{v_1}{\tan \left( \frac{\omega}{v_t^B} \right)} \right) \cos \left( \frac{\omega x_3}{v_t^B} \right) - \sin \left( \frac{\omega x_3}{v_t^B} \right) v_1 \right] (C.51) \]

C.31 and Faraday’s law may be substituted into C.50

\[ i_3 = -\frac{1}{n} \int_0^\ell \frac{h_{33}}{jv_t^B} \left[ \left( \frac{v_2}{\sin \left( \frac{\omega}{v_t^B} \right)} + \frac{v_1}{\tan \left( \frac{\omega}{v_t^B} \right)} \right) \cos \left( \frac{\omega x_3}{v_t^B} \right) + v_1 \sin \left( \frac{\omega x_3}{v_t^B} \right) \right] - \frac{\gamma_{33}^S}{j\omega n A} V_3 dx_3 \]  
(C.52)

Break-up the integral into three pieces. For the last term the integral is trivial.

\[ i_3 = -\frac{h_{33}}{jnv_t^B} \left[ \left( \frac{v_2}{\sin \left( \frac{\omega}{v_t^B} \right)} + \frac{v_1}{\tan \left( \frac{\omega}{v_t^B} \right)} \right) \int_0^\ell \cos \left( \frac{\omega x_3}{v_t^B} \right) dx_3 + v_1 \int_0^\ell \sin \left( \frac{\omega x_3}{v_t^B} \right) dx_3 \right] + \frac{\gamma_{33}^S}{j\omega n^2 A} V_3 \]  
(C.53)

Evaluate the integrals represented as a mobility-type circuit connected to the mechanical domain by a transformer or the electrical domain can be left in an impedance-type circuit but connected to the mechanical domain with a gyrator.
\[
i_3 = -\frac{h_{33}}{j n v_t^B} \left[ \left( \frac{v_2}{\sin \left( \frac{\omega \ell}{v_t^B} \right)} + \frac{v_1}{\tan \left( \frac{\omega \ell}{v_t^B} \right)} \right) \sin \left( \frac{\omega \ell}{v_t^B} \right) v_t^B \omega + v_1 \left( 1 - \cos \left( \frac{\omega \ell}{v_t^B} \right) \right) v_t^B \omega \right] + \frac{\gamma_{33}^S \ell}{j \omega n^2 A} V_3 \tag{C.54}
\]

and simplify.

\[
i_3 = -\frac{h_{33}}{j \omega n} \left[ v_2 + \cos \left( \frac{\omega \ell}{v_t^B} \right) v_1 + v_1 - \cos \left( \frac{\omega \ell}{v_t^B} \right) v_1 \right] + \left( \frac{\gamma_{33}^S \ell}{j \omega n^2 A} V_3 \right) \tag{C.55}
\]

Canceling the cosine terms yields the final form of the electrical equation.

\[
i_3 = -\frac{h_{33}}{j \omega n} v_1 - \frac{h_{33}}{j \omega n} v_2 + \frac{\gamma_{33}^S \ell}{j \omega n^2 A} V_3 \tag{C.56}
\]

Note that the mechanical characteristic impedance of the active material in the 3-direction is

\[
Z_0 = \rho v_t^PA = \rho \sqrt{\frac{c_{33}^B}{\rho}} A = \sqrt{\rho c_{33}^B A} = c_{33}^B \sqrt{\frac{\rho}{c_{33}^B} A} = \frac{c_{33}^B}{v_t^P} A \tag{C.57}
\]

All three ports of the transduction element are now defined:

\[
F_1 = \frac{Z_0}{j \tan \left( \frac{\omega \ell}{v_t^P} \right)} v_1 + \frac{Z_0}{j \sin \left( \frac{\omega \ell}{v_t^P} \right)} v_2 - \frac{h_{33}}{j \omega n} V_3 \tag{C.58}
\]

\[
F_2 = \frac{Z_0}{j \sin \left( \frac{\omega \ell}{v_t^P} \right)} v_1 + \frac{Z_0}{j \tan \left( \frac{\omega \ell}{v_t^P} \right)} v_2 - \frac{h_{33}}{j \omega n} V_3 \tag{C.59}
\]

\[
i_3 = -\frac{h_{33}}{j \omega n} v_1 - \frac{h_{33}}{j \omega n} v_2 + \frac{\gamma_{33}^S \ell}{j \omega n^2 A} V_3 \tag{C.60}
\]

The transduction coefficient, \( \phi \) can be determined from the first two equations since

\[
\phi = \frac{F}{V} \bigg|_{\text{blocked}} \tag{C.61}
\]

The blocked condition implies that \( v_1 = v_2 = 0 \), so that
\[
F_1|_{\text{blocked}} = F_2|_{\text{blocked}} = -\frac{h_{33}}{j\omega n} V_3 \tag{C.62}
\]

and thus
\[
\phi = -\frac{h_{33}}{j\omega n} \tag{C.63}
\]

For the blocked condition also note that
\[
i_3|_{\text{blocked}} = \frac{\gamma_{33} S}{j\omega n^2 A} V_3 \tag{C.64}
\]

If re-arranged into an electrical impedance this gives
\[
Z_{eb} = j\omega \left( \frac{n^2 A}{\gamma_{33} S \ell} \right) \tag{C.65}
\]

This is clearly an inductive impedance. It is convenient, for the purposes of constructing an analogous circuit, to define an electrical blocked inductance\footnote{Since in this derivation the electrical flow variable, \( I_3 \), is naturally grouped with the mechanical effort variables, \( F_1 \) and \( F_2 \) and the electrical effort variable, \( V_3 \) finds itself on the same side of the equation as the mechanical flow variables, \( v_1 \) and \( v_2 \), some derivations find the mobility analogy useful for the electrical portion of the analogous circuit. In this form, an blocked electrical \emph{capacitance} must be used where \( C_{eb} = \frac{n^2 A}{\gamma_{33} S \ell} \).}:
\[
L_{eb} = \frac{n^2 A}{\gamma_{33} S \ell} \tag{C.66}
\]

Organizing the three transduction equations into matrix form
\[
\begin{bmatrix}
F_1 \\
F_2 \\
i_3
\end{bmatrix} =
\begin{bmatrix}
Z_0 & Z_0 \\
\frac{Z_0}{j\tan(\frac{\omega}{\ell})} & \frac{Z_0}{j\sin(\frac{\omega}{\ell})} \\
\frac{Z_0}{j\sin(\frac{\omega}{\ell})} & \frac{Z_0}{j\tan(\frac{\omega}{\ell})}
\end{bmatrix}
\begin{bmatrix}
\phi \\
\phi
\end{bmatrix}
\times
\begin{bmatrix}
v_1 \\
v_2 \frac{1}{j\omega L_{eb}} \\
V_3
\end{bmatrix} \tag{C.67}
\]

### C.3 Piezomagnetic bar

The derivation for the piezomagnetic bar piece is very similar to that of the piezomagnetic plate just considered - the many of the mathematical steps are the same.
Therefore, the derivation here only includes key steps.

Consider a piezomagnetic bar of constant cross-sectional area, \( A \), and of length, \( \ell \), much greater than any other dimension. As with the plate, \( \frac{\partial B_3}{\partial x_3} = 0 \) since we are considering (3,3) operation.

Apply the bar approximation, \( T_3 \neq 0 \) and \( T_1 = T_2 = T_4 = T_5 = T_6 = 0 \), because the most is known about stress. The best pair of constitutive equations in this case is 2.45a and 2.45b, yielding

\[
S_3 = s_{33}^B T_3 + g_{33} B_3 \quad \text{(C.68)}
\]

\[
H_3 = -g_{33} T_3 + \gamma_{33}^T B_3 \quad \text{(C.69)}
\]

The force at the first mechanical port is

\[
F_1 = -T_3 A \bigg|_{x_3=0} = - \left[ \frac{1}{s_{33}^B} S_3 \bigg|_{x_3=0} - \frac{g_{33}}{s_{33}^B} B_3 \right] A \quad \text{(C.70)}
\]

Referencing equations C.38 and Faraday’s law, arrive at

\[
F_1 = - \left[ \frac{j}{s_{33}^B \sqrt{\omega}} \left( \frac{v_2}{\sin \left( \frac{\omega t}{v_B} \right)} + \frac{v_1}{\tan \left( \frac{\omega t}{v_B} \right)} \right) - \frac{g_{33}}{s_{33}^B} \left( -\frac{1}{j\omega n} V_3 \right) \right] A \quad \text{(C.71)}
\]

Note that

\[
Z_0 = \rho v_b^B A = \rho \sqrt{\frac{1}{\rho s_{33}^B}} A = \sqrt{\frac{\rho}{s_{33}^B}} A = \frac{1}{s_{33}^B} \sqrt{\rho s_{33}^B} A = \frac{A}{s_{33}^B v_b^B} \quad \text{(C.72)}
\]

Simplifying C.71 gives

\[
F_1 = \frac{Z_0}{j\tan \left( \frac{\omega t}{v_B} \right)} v_1 + \frac{Z_0}{j\sin \left( \frac{\omega t}{v_B} \right)} v_2 - \frac{g_{33}}{j\omega n s_{33}^B} V_3 \quad \text{(C.73)}
\]

Likewise, the force at the second mechanical port is

\[
F_2 = -T_3 A \bigg|_{x_3=\ell} = - \left[ \frac{1}{s_{33}^B} S_3 \bigg|_{x_3=\ell} - \frac{g_{33}}{s_{33}^B} B_3 \right] A \quad \text{(C.74)}
\]

Which by substitution of C.46 and Faraday’s law becomes
\[ F_2 = - \left[ \frac{j}{s_{33} v_t} \left( \frac{1}{\tan \left( \frac{\omega B}{v_t} \right)} v_2 + \frac{1}{\sin \left( \frac{\omega B}{v_t} \right)} v_1 \right) - \frac{g_{33}}{s_{33}} \left( - \frac{1}{j \omega n A} V_3 \right) \right] \quad (C.75) \]

Simplification yields

\[ F_2 = \frac{Z_0}{j \sin \left( \frac{\omega B}{v_t} \right)} v_1 + \frac{Z_0}{j \tan \left( \frac{\omega B}{v_t} \right)} v_2 - \frac{g_{33}}{j \omega n s_{33} B} V_3 \quad (C.76) \]

Now the third equation is

\[ i_3 = - \frac{1}{n} \int_0^\ell H_3 dx_3 = - \frac{1}{n} \int_0^\ell \left( - g_{33} T_3 + \gamma_{33}^T B_3 \right) dx_3 \quad (C.77) \]

Again, the direction of the current is chosen so that the final matrix will be symmetric.

\[ i_3 = \frac{1}{n} \int_0^\ell g_{33} \left( \frac{1}{s_{33}} S_3 - \frac{g_{33}}{s_{33}} B_3 \right) - \gamma_{33}^T B_3 dx_3 \quad (C.78) \]

\[ i_3 = \frac{1}{n} \int_0^\ell \frac{g_{33}}{s_{33}} S_3 - \left( \frac{g_{33}^2}{s_{33}} + \gamma_{33}^T \right) B_3 dx_3 \quad (C.79) \]

Include \[ C.51 \] and Faraday’s law:

\[ i_3 = \int_0^\ell \frac{j g_{33}}{n s_{33} v_t} \left[ \left( \frac{v_2}{\sin \left( \frac{\omega B}{v_t} \right)} + \frac{v_1}{\tan \left( \frac{\omega B}{v_t} \right)} \right) \cos \left( \frac{\omega x_3}{v_t} \right) - \sin \left( \frac{\omega x_3}{v_t} \right) v_1 \right] \quad (C.80) \]

Break into three integrals and evaluate as in the plate derivation.

\[ i_3 = - \frac{g_{33}}{j \omega n s_{33} B} v_1 - \frac{g_{33}^2}{j \omega n s_{33} B} v_2 + \left( \frac{g_{33}^2}{s_{33}} + \gamma_{33}^T \right) \left( \frac{\ell}{j \omega n^2 A} V_3 \right) \quad (C.81) \]

At this point, recognize that

\[ s_{33}^B = s_{33}^H (1 - k_{33}^2) \quad (C.82) \]

and
\[ k_{33}^2 = \frac{g_{33}^2}{s_{33} H_{133}} \]  

(C.83)

\[ i_3 = -\frac{g_{33}}{j\omega n s_{33}^B} v_1 - \frac{g_{33}}{j\omega n s_{33}^B} v_2 + \left( \frac{g_{33}^2}{s_{33} H_{133}^2 (1 - k_{33}^2)} + 1 \right) \left( \frac{\gamma_{33}^T \ell}{j\omega n^2 A} V_3 \right) \]  

(C.84)

\[ i_3 = -\frac{g_{33}}{j\omega n s_{33}^B} v_1 - \frac{g_{33}}{j\omega n s_{33}^B} v_2 + \left( \frac{k_{33}^2}{1 - k_{33}^2} + 1 - k_{33}^2 \right) \left( \frac{\gamma_{33}^T \ell}{j\omega n^2 A} V_3 \right) \]  

(C.85)

\[ i_3 = -\frac{g_{33}}{j\omega n s_{33}^B} v_1 - \frac{g_{33}}{j\omega n s_{33}^B} v_2 + \frac{\gamma_{33}^T \ell}{j\omega n^2 A (1 - k_{33}^2)} V_3 \]  

(C.86)

Now having the three equations, we can examine either blocked force to determine that

\[ \phi = -\frac{g_{33}}{j\omega n s_{33}^B} \]  

(C.87)

The electrical blocked impedance has an inductance of

\[ L_{eb} = \frac{n^2 A (1 - k_{33}^2)}{\gamma_{33}^T \ell} = \frac{n^2 A}{\ell} \mu_{33} \]  

(C.88)

Having new definitions for \( \phi \) and \( L_{eb} \), we can write a similar matrix equation as was used in the piezomagnetic plate.

\[
\begin{bmatrix}
F_1 \\
F_2 \\
i_3
\end{bmatrix} =
\begin{bmatrix}
\frac{Z_0}{j\tan(\frac{\omega \ell}{v_t})} & \frac{Z_0}{j\sin(\frac{\omega \ell}{v_t})} & \phi \\
\frac{Z_0}{j\sin(\frac{\omega \ell}{v_t})} & \frac{Z_0}{j\tan(\frac{\omega \ell}{v_t})} & \phi \\
\phi & \phi & \frac{1}{j\omega L_{eb}}
\end{bmatrix}
\begin{bmatrix}
v_1 \\
v_2 \\
V_3
\end{bmatrix}
\]

(C.89)

With this set of equations the bar can be modeled by writing out the equations as a circuit schematic. However, the magnetic domain is not explicitly shown. This is not suitable for magnetic modeling. To create a magnetic domain in the piezomagnetic bar piece the third port can be converted from electrical variables.
to magnetic variables. Then a two-port element can be created to convert the magnetic variables back to electrical ones.

To explicitly show the magnetic domain, the electrical effort and flow variables, \( V \) and \( i \), must be converted to similar magnetic effort and flow quantities, \( F \) and \( \phi \). For the reluctance analogy the relationships between the magnetic and electric variables are described in section 2.4.2. Using these equations to replace the electrical variables yields

\[
\begin{bmatrix}
F_1 \\
F_2 \\
\frac{1}{n} F
\end{bmatrix} = \begin{bmatrix}
\frac{Z_0}{j \tan \left( \frac{\omega \ell}{v_B t} \right)} & \frac{Z_0}{j \sin \left( \frac{\omega \ell}{v_B t} \right)} & \phi \\
\frac{Z_0}{j \sin \left( \frac{\omega \ell}{v_B t} \right)} & \frac{Z_0}{j \tan \left( \frac{\omega \ell}{v_B t} \right)} & \phi \\
\phi & \phi & 0
\end{bmatrix}
\times
\begin{bmatrix}
v_1 \\
v_2 \\
-\frac{d \phi}{dt}
\end{bmatrix}
\] (C.90)

Notice that now the left hand matrix contains only effort variables and the right hand matrix contains flow variables. The center matrix is an impedance matrix.

\[
\begin{bmatrix}
F_1 \\
F_2 \\
\mathbf{\mathfrak{F}}
\end{bmatrix} = \begin{bmatrix}
\frac{Z_0}{j \tan \left( \frac{\omega \ell}{v_B t} \right)} & \frac{Z_0}{j \sin \left( \frac{\omega \ell}{v_B t} \right)} & \frac{-n^2}{j \omega L_e b} \\
\frac{Z_0}{j \sin \left( \frac{\omega \ell}{v_B t} \right)} & \frac{Z_0}{j \tan \left( \frac{\omega \ell}{v_B t} \right)} & \frac{-n^2}{j \omega L_e b} \\
n \phi & n \phi & \frac{d}{dt} \left( \frac{n^2}{j \omega L_e b} \right)
\end{bmatrix}
\times
\begin{bmatrix}
v_1 \\
v_2 \\
\phi
\end{bmatrix}
\] (C.91)

Substitute [C.87] and [C.88] back in.

\[
\begin{bmatrix}
F_1 \\
F_2 \\
\mathbf{\mathfrak{F}}
\end{bmatrix} = \begin{bmatrix}
\frac{Z_0}{j \tan \left( \frac{\omega \ell}{v_B t} \right)} & \frac{Z_0}{j \sin \left( \frac{\omega \ell}{v_B t} \right)} & \frac{g_{33}}{s_{33}} \\
\frac{Z_0}{j \sin \left( \frac{\omega \ell}{v_B t} \right)} & \frac{Z_0}{j \tan \left( \frac{\omega \ell}{v_B t} \right)} & \frac{g_{33}}{s_{33}} \\
-\frac{g_{33}}{j \omega s_{33}} & -\frac{g_{33}}{j \omega s_{33}} & \frac{\ell}{\mu_{33} A}
\end{bmatrix}
\times
\begin{bmatrix}
v_1 \\
v_2 \\
\phi
\end{bmatrix}
\] (C.92)

These equations are implemented in SPICE with the piezomagnetic bar circuit shown in Figure 3.16.
C.4 Estimation of mechanical losses in a piezomagnetic bar

In order to relate a measured $Q_m$ into a mechanical resistance value, $R_m$, that can be used in one-dimensional modeling efforts, the following technique is employed. Several rather large assumptions are made, so the result should be treated as a very crude approximation. First a free-free bar sample of the material is considered where the ends are situated at $x = -\ell/2$ and $x = \ell/2$. In this configuration, a nodal plane occurs at $x = 0$ when the bar is excited at its primary resonance, $\omega_0$. Thus, each half may be considered separately as a free-fixed bar of length $\ell/2$. As a first approximation, the free-fixed bar may be modeled as a simple harmonic oscillator. In this scenario,

$$Q_m = \frac{\omega_0 m}{R_m}$$  \hfill (C.93)

where $m$ is the lumped mass of the half-bar and $\omega_0^2 = k_m/m$, so that

$$R_m = \frac{k_m}{\omega_0 Q_m}$$  \hfill (C.94)

Here $k_m$ is the lumped mechanical stiffness. The effective stiffness of an elastic solid is $k_m = \frac{EA}{L}$ where $E$ is the elastic modulus, $L$ is the length of the solid, and $A$ is the cross-sectional area. Therefore,

$$R_m = \frac{(2EA)}{\omega_0 Q_m}$$  \hfill (C.95)

In a piezomagnetic bar operated in its (3,3) mode, the elastic modulus is $E = s_{33}$, which yields:

$$R_m = \frac{2A}{\omega_0 s_{33} Q_m \ell}$$  \hfill (C.96)

C.5 Eddy current losses in laminated stacks

Consider a high permeability bar made up of thin sheets, called laminates, of width $w$, thickness $d$, and length $\ell$. A magnetic field is incident on the stack in
the negative z-direction.

Following Vanderkooy [120], begin with Maxwell’s Equations.

\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (C.97) \]
\[ \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \quad (C.98) \]

Making use of \( B = \mu H \) and \( J = \sigma E \), express \( C.97 \) and \( C.98 \) as

\[ \nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t} \quad (C.99) \]
\[ \nabla \times \mathbf{H} = \sigma \mathbf{E} + \frac{\partial \mathbf{D}}{\partial t} \quad (C.100) \]

Rearrange \( C.100 \).

\[ \mathbf{E} = \frac{1}{\sigma} \left( \nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} \right) \quad (C.101) \]

Now, substitute \( C.101 \) into \( C.99 \)

\[ \nabla \times \left( \nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} \right) = -\mu \sigma \frac{\partial \mathbf{H}}{\partial t} \quad (C.102) \]

Next, make use of the identity

\[ \nabla \times (\nabla \times \mathbf{Z}) = \nabla (\nabla \cdot \mathbf{Z}) - \nabla^2 \mathbf{Z} \quad (C.103) \]

to get

\[ \nabla (\nabla \cdot \mathbf{H}) - \nabla^2 \mathbf{H} - \nabla \times \frac{\partial \mathbf{D}}{\partial t} = -\mu \sigma \frac{\partial \mathbf{H}}{\partial t} \quad (C.104) \]

Again, use \( B = \mu H \).

\[ \frac{1}{\mu} \nabla (\nabla \cdot \mathbf{B}) - \nabla^2 \mathbf{H} - \nabla \times \frac{\partial \mathbf{D}}{\partial t} = -\mu \sigma \frac{\partial \mathbf{H}}{\partial t} \quad (C.105) \]

Due to \( 2.1b \)

\[ \nabla^2 \mathbf{H} + \nabla \times \frac{\partial \mathbf{D}}{\partial t} = \mu \sigma \frac{\partial \mathbf{H}}{\partial t} \quad (C.106) \]
Vanderkooy notes that since the $\frac{\partial D}{\partial t}$ term is less than $J$ by a factor of $\omega / \sigma$, it can usually be ignored. This leaves

$$\nabla^2 H = \mu \sigma \frac{\partial H}{\partial t}$$

(C.107)

which is in the form of a diffusion equation. Solving this equation requires a different general solution than most differential equations encountered in acoustics. We apply this magnetic situation to thin laminations such that the thickness, $d$, is much less than either the width, $w$, or the length, $\ell$. Assume a time-harmonic solution:

$$\frac{\partial^2 H}{\partial z^2} = j \omega \mu \sigma H$$

(C.108)

Define a variable, $k$, such that

$$k^2 = j \omega \mu \sigma$$

(C.109)

This can be expressed as

$$k = \pm \sqrt{j \omega \mu \sigma}$$

(C.110)

Using the equality $\sqrt{j} = \frac{1}{\sqrt{2}} (1 + j)$ an alternate expression can be obtained for $k$.

$$k = \frac{1 + j}{\sqrt{\frac{2}{\mu \sigma \omega}}}$$

(C.111)

Notice that here the wave has equal real and imaginary parts which, by definition, is an evanescent wave. This the eddy-current penetration depth, $\delta$ can be described as

$$\delta = \sqrt{\frac{2}{\mu \sigma \omega}}$$

(C.112)

so that

$$k = \frac{1 + j}{\delta}$$

(C.113)

$k$ represents an imaginary (evanescent) wavenumber.
Now, assume a solution of the form

\[ \mathbf{H} = H_1 \sinh (kz) + H_2 \cosh (kz) \]  \hspace{1cm} (C.114)

where \( H_1 \) and \( H_2 \) are constants. Following Avila-Rosales et al [144], apply the following boundary conditions which assume that the field distribution is constant over all laminations.

\[ \mathbf{H} \bigg|_{z = \frac{d}{2}} = H_1 \sinh \left( \frac{kd}{2} \right) + H_2 \cosh \left( \frac{kd}{2} \right) = H_0 \]  \hspace{1cm} (C.115)

\[ \mathbf{H} \bigg|_{z = -\frac{d}{2}} = H_1 \sinh \left( -\frac{kd}{2} \right) + H_2 \cosh \left( -\frac{kd}{2} \right) = H_0 \]  \hspace{1cm} (C.116)

Making use of the relationships \( \sinh(-\theta) = -\sinh(\theta) \) and \( \cosh(-\theta) = \cosh(\theta) \), we can re-express (C.116) as

\[ \mathbf{H} \bigg|_{z = -\frac{d}{2}} = -H_1 \sinh \left( \frac{kd}{2} \right) + H_2 \cosh \left( \frac{kd}{2} \right) = H_0 \]  \hspace{1cm} (C.117)

Setting the hyperbolic expressions of (C.115) and (C.117) equal to each other results in

\[ 2H_1 \sinh \left( \frac{kd}{2} \right) = 0 \]  \hspace{1cm} (C.118)

This implies that

\[ H_1 = 0 \]  \hspace{1cm} (C.119)

Knowing this, either (C.115) or (C.117) yields

\[ H_2 = \frac{H_0}{\cosh \left( \frac{kd}{2} \right)} \]  \hspace{1cm} (C.120)

Having solved for these constants, the solution becomes

\[ \mathbf{H} = H_0 \frac{\cosh (kz)}{\cosh \left( \frac{1}{2}kd \right)} \]  \hspace{1cm} (C.121)
Which, through \( B = \mu H \), becomes

\[
B = \mu H_0 \frac{\cosh (kz)}{\cosh (\frac{1}{2}kd)} \tag{C.122}
\]

The flux in a single lamination can be determined from the integral

\[
\varphi = \int_{-\frac{d}{2}}^{+\frac{d}{2}} wB dx \tag{C.123}
\]

\[
\varphi = \frac{w\mu H_0}{k \cosh (\frac{1}{2}kd)} \left[ \sinh (kz) \right]_{-\frac{d}{2}}^{+\frac{d}{2}} \tag{C.124}
\]

\[
\varphi = \frac{w\mu H_0}{k \cosh (\frac{1}{2}kd)} \left[ \sinh \left( \frac{kd}{2} \right) + \sinh \left( \frac{kd}{2} \right) \right] \tag{C.125}
\]

\[
\varphi = \frac{2w\mu H_0}{k} \tanh \left( \frac{kd}{2} \right) \tag{C.126}
\]

\[
\varphi = w\mu dH_0 \frac{1}{\zeta} \tanh (\zeta) \tag{C.129}
\]

It is useful to set the hyperbolic tangent argument equal to some constant, \( \zeta \).

\[
\zeta = \frac{(1 + j)d}{2\delta} = \frac{kd}{2} = \frac{d}{2} \sqrt{j\omega \mu \sigma} \tag{C.128}
\]

so that

\[
\varphi = \frac{w\mu d}{\zeta} H_0 \tanh (\zeta) \tag{C.129}
\]

Following Lammeraner and Stafl [145], a stack of laminations having a combined thickness of \( T \) consists of \( N_L = \frac{T}{d} \) laminations. For this stack, the total area seen by the flux is \( A = (wd) N_L \).

\[
\varphi_t = H_0 \frac{\mu wd N_L}{\zeta} \tanh (\zeta) \tag{C.130}
\]
### C.5.1 Representing eddy current losses as an electrical impedance

Often in electrical circuits it is desirable to represent the eddy current phenomenon as an electrical impedance. To do this, Ohm’s Law, Faraday’s Law of Induction, and $H_0 = \frac{n_i}{\ell}$ can be used to show that

\[
Z_e = \frac{V}{i} = \left( \frac{j\omega n^2}{H_0 \ell} \right) \varphi_t = \left( \frac{j\omega n^2}{H_0 \ell} \right) \left( N_L H_0 \frac{w \mu d}{\zeta} \tanh (\zeta) \right) \tag{C.131}
\]

Simplifying,

\[
Z_e = \frac{j\omega n^2 \mu w d N_L}{\ell \zeta} \tanh (\zeta) \tag{C.132}
\]

Using \( C.128 \), the expression may be rewritten to match \( [144] \).

\[
Z_e = N_L \frac{2w n^2}{\ell} \sqrt{\frac{j\omega \mu}{\sigma}} \tanh \left( \sqrt{\frac{j\omega \mu \sigma d}{2}} \right) \tag{C.133}
\]

This form is equivalent to the results given in \( [145, 146] \). The hyperbolic tangent function can be represented \( [147] \) by a continued fraction of the form

\[
\tanh (x) = \frac{x}{1 + \frac{x^2}{3 + \frac{x^2}{5 + \frac{x^2}{7 + \ldots}}}} \tag{C.134}
\]

which is convenient since a truncated expression can be represented by the Cauer ladder network \( [116] \) in Figure C.2. This circuit has an impedance of the form

\[
Z = \frac{1}{\frac{1}{j\omega L_1} + \frac{1}{R_1 + \frac{1}{\frac{1}{j\omega L_2} + \frac{1}{R_2 + \ldots}}}} \tag{C.135}
\]

To obtain a continued fraction expression for the electrical impedance in the form of \( C.135 \) start by representing \( C.133 \) as:
Multiplying the leading terms by the first numerator yields

\[
Z_e = N_L \frac{2wn^2}{\ell} \sqrt{\frac{j\omega \mu}{\sigma}} \frac{\sqrt{j\omega \mu \sigma \frac{d^2}{4}}}{1 + \frac{j\omega \mu \sigma \frac{d^2}{4}}{3 + \frac{j\omega \mu \sigma \frac{d^2}{4}}{5 + \frac{j\omega \mu \sigma \frac{d^2}{4}}{7 + \ldots}}}
\]  

(C.136)

Since we want to force each successive numerator to be one,

\[
Z_e = \frac{j\omega \mu N_L wn^2d}{\ell} \left( \frac{\ell}{j\omega \mu N_L wn^2d} \right)
\]

(C.137)

\[
1 \left( \frac{\ell}{j\omega \mu N_L wn^2d} \right) + \left( \frac{\ell}{j\omega \mu N_L wn^2d} \right) \frac{j\omega \mu \sigma \frac{d^2}{4}}{3 + \frac{j\omega \mu \sigma \frac{d^2}{4}}{5 + \frac{j\omega \mu \sigma \frac{d^2}{4}}{7 + \ldots}}}
\]

(C.138)
\( Z = \frac{1}{\ell j \omega \mu \nu \ell N w^2 d + \frac{\ell \sigma d}{4 \mathcal{N}_L w^2} \left( 4 \mathcal{N}_L w^2 \frac{\ell \sigma d}{4 \mathcal{N}_L w^2} \right)} \) \( + \frac{j \omega \mu \sigma \frac{d^2}{4}}{3} + \frac{j \omega \mu \sigma \frac{d^2}{4}}{5} + \frac{j \omega \mu \sigma \frac{d^2}{4}}{7} + \ldots \) \hfill (C.139)

\[ Z = \frac{1}{\ell j \omega \mu \nu \ell N w^2 d + \frac{\ell \sigma d}{4 \mathcal{N}_L w^2} \left( 4 \mathcal{N}_L w^2 \frac{\ell \sigma d}{4 \mathcal{N}_L w^2} \right)} \] \[ + \frac{1}{3 \left( 4 \mathcal{N}_L w^2 \frac{\ell \sigma d}{4 \mathcal{N}_L w^2} \right) + \frac{1}{5 \left( \frac{\ell j \omega \mu \nu \ell N w^2 d}{\ell \sigma d} \right)} + \frac{j \omega \mu \sigma \frac{d^2}{4}}{7} + \ldots \] \hfill (C.140)

\[ Z = \frac{1}{\ell j \omega \mu \nu \ell N w^2 d + \frac{\ell \sigma d}{4 \mathcal{N}_L w^2} \left( 4 \mathcal{N}_L w^2 \frac{\ell \sigma d}{4 \mathcal{N}_L w^2} \right)} \] \[ + \frac{1}{3 \left( 4 \mathcal{N}_L w^2 \frac{\ell \sigma d}{4 \mathcal{N}_L w^2} \right) + \frac{1}{5 \left( \frac{\ell j \omega \mu \nu \ell N w^2 d}{\ell \sigma d} \right)} + \frac{1}{7 \left( 4 \mathcal{N}_L w^2 \frac{\ell \sigma d}{4 \mathcal{N}_L w^2} \right)} + \ldots \] \hfill (C.141)

\[ Z = \frac{1}{\ell j \omega \mu \nu \ell N w^2 d + \frac{\ell \sigma d}{4 \mathcal{N}_L w^2} \left( 4 \mathcal{N}_L w^2 \frac{\ell \sigma d}{4 \mathcal{N}_L w^2} \right)} \] \[ + \frac{1}{3 \left( 4 \mathcal{N}_L w^2 \frac{\ell \sigma d}{4 \mathcal{N}_L w^2} \right) + \frac{1}{5 \left( \frac{\ell j \omega \mu \nu \ell N w^2 d}{\ell \sigma d} \right)} + \frac{1}{7 \left( 4 \mathcal{N}_L w^2 \frac{\ell \sigma d}{4 \mathcal{N}_L w^2} \right)} + \ldots \] \hfill (C.142)

From this form, it is clear that

\[ L_M = \frac{\mu w n^2 d}{\ell (4M - 3) \mathcal{N}_L} \] \hfill (C.143)

\[ R_M = \frac{4 w n^2 (4M - 1) \mathcal{N}_L}{\ell \sigma d} \] \hfill (C.144)
where $M = 1, 2, 3, \ldots$

Other methods of approximating the eddy current losses in circuit models are discussed in [144, 148, 149].

C.5.2 Representing eddy current losses as a magnetic reluctance

For models that explicitly include the magnetic domain, it seems most natural to include the eddy current losses as a magnetic component since since eddy currents are an artifact of the magnetic circuit and magnetic flux. We can change variables in equation C.132 to derive a complex reluctance (a magnetic impedance, so to speak) which will be denoted $Z_{2\Omega}$.

\[
Z_{2\Omega} = \frac{3}{\varphi} = \frac{\ell \zeta}{\mu wdN_L} \coth(\zeta) \tag{C.145}
\]

As in the previous section, this expression can be represented with an circuit by expanding the hyperbolic cotangent [117] so that it can be approximated with a Cauer ladder network.

\[
\coth(x) = \frac{1}{x} - \frac{x}{\frac{3}{2}} + \frac{(\frac{x}{2})^2}{\frac{5}{2}} - \frac{(\frac{x}{2})^2}{\frac{7}{2}} + \frac{(\frac{x}{2})^2}{\frac{9}{2}} + \ldots \tag{C.146}
\]

Expanding equation C.145 following the procedure in the preceding section results in

\[
Z_{2\Omega} = R_o + \frac{1}{\frac{3}{j\omega L_0}} + \frac{1}{5R_o} + \frac{1}{\frac{7}{j\omega L_0}} + \frac{1}{9R_o} + \ldots \tag{C.147}
\]

where
$$\mathcal{R}_0 = \frac{\ell}{\mu wdN_L} \quad (C.148)$$

$$L_0 = \frac{\sigma ld}{4wN_L} \quad (C.149)$$

Note the familiar form of $\mathcal{R}_0$, albeit scaled by $1/N_L$. From this observation, it appears that $C.147$ may be generalized for use with any reluctance calculation by simply dividing the reluctance value by $N_L$ and using it in $C.147$ as $\mathcal{R}_0$. Then let

$$L_0 = \frac{\mu \sigma d}{4} \mathcal{R}_0 \quad (C.150)$$
Appendix D

SPICE netlists and MATLAB scripts

D.1 SPICE Netlists

Using these netlists, the reader can recreate the SPICE models presented in this document. Continued lines are indicated here with an indentation. Subcircuits were called into models using a custom component library in LTspice.

D.1.1 Ideal transformer

```plaintext
R2 N001 A- {1/Gmin}
R3 N002 B- {1/Gmin}
R1 N001 A+ {Gmin}
R4 B+ N002 {Gmin}
E1 N001 A- N002 B- {1/alpha}
F1 B- N002 value={I(E1)/alpha}
.params alpha=10
.backanno
.end
```

D.1.2 Ideal gyrator

```plaintext
R2 N001 A- {1/Gmin}
R3 N002 B- {1/Gmin}
```
D.1.3 Time-differentiating gyrator

\[ B_E \text{ Elec}^- \text{ Elec}^+ V = -n \frac{d}{dt}(I(B_{\text{mmf}})) \]
\[ B_{\text{mmf}} \text{ Mag}^- \text{ Mag}^+ V = n I(B_E) \]
[param n=100]
.backanno
.end

D.1.4 Mechanical transmission line

\[ T1 \text{ UL LL UR LR Td} = \{\text{length}/c\} \]
\[ Z0 = \{\text{rho}*c*area\} \]
.include materials.txt
.backanno
.end

D.1.5 Piezoelectric piece

\[ T1 \text{ Mech1}+ \text{ N001 Mech2}+ \text{ N002 Td} = \{\text{length}/v_D\} \]
\[ Z0 = \{\text{rho}*v_D*area\} \]
\[ C1 \text{ Elec}+ \text{ N003 \{area*e33S/length\}} \]
\[ B1 \text{ N001 Mech1}^- V = I(B3)*\{g33/s33D\} \text{ laplace} = \{1/s\} \]
\[ B2 \text{ N002 Mech2}^- V = I(B3)*\{g33/s33D\} \text{ laplace} = \{1/s\} \]
\[ B3 \text{ N003 Elec}^- V = (I(B1)+I(B2))*\{g33/s33D\} \text{ laplace} = \{1/s\} \]
[param g33=k33*sqrt(s33E/e33T)\]
.param s33D={s33E*(1-k33*k33)}
.param e33S={e33T*(1-k33*k33)}
.param v.D={1/sqrt(s33D*rho)}
.backanno
.end

D.1.6 Piezomagnetic piece

T1 MECH1+ N001 MECH2+ N002 Td={length/v_B} Z0={rho*v_B*area}
B2 N002 MECH2- V=I(B3)*{g33/s33B}
B1 N001 MECH1- V=I(B3)*{g33/s33B}
R1 N003 MAG+ {length/(area*mu33S)}
B3 N003 MAG- V=(I(B1)+I(B2))*{g33/s33B} laplace={1/s}
.param g33=d33/mu33T
.param k33={sqrt(d33*d33/(s33H*mu33T))}
.param mu33S={mu33T*(1-k33*k33)}
.param s33B={s33H*(1-k33*k33)}
.param v.b={1/sqrt(s33B*rho)}
.backanno
.end

D.1.7 Reluctance with eddy current losses

L1 N001 Mag- {L0/3}
L2 N002 Mag- {L0/7}
L3 N003 Mag- {L0/11}
L4 N004 Mag- {L0/15}
L5 N005 Mag- {L0/19}
R2 N002 N001 {R0*5}
R3 N003 N002 {R0*9}
R4 N004 N003 {R0*13}
R5 N005 N004 {R0*17}
D.1.8 Piezomagnetic piece with eddy current losses

T1 MECH1+ N001 MECH2+ N002 Td={length/v_B} Z0={rho*v_B*area}
B2 N002 MECH2- V=I(B3)*{g33/s33B}
B1 N001 MECH1- V=I(B3)*{g33/s33B}
B3 N011 MAG- V=(I(B1)+I(B2))*{g33/s33B} laplace=1/s
L1 N003 N011 {L0/3}
L2 N004 N011 {L0/7}
L3 N005 N011 {L0/11}
L4 N006 N011 {L0/15}
L5 N007 N011 {L0/19}
R1 N004 N003 {R0*5}
R2 N005 N004 {R0*9}
R3 N006 N005 \{R0*13\}
R4 N007 N006 \{R0*17\}
L6 N008 N011 \{L0/23\}
R5 N008 N007 \{R0*21\}
R8 N003 MAG+ \{R0\}
L9 N010 N011 \{L0/31\}
R9 N010 N009 \{R0*29\}
L10 N009 N011 \{L0/27\}
R10 N009 N008 \{R0*25\}
.param g33=d33/mu33T
.param k33={\sqrt(d33*d33/(s33H*mu33T))}
.param mu33S={mu33T*(1-k33*k33)}
.param s33B={s33H*(1-k33*k33)}
.param v_b={1/\sqrt(s33B*rho)}
.param R0=length/mu33T/w/d/NL
.param L0=R0*d**2*mu33T*sigma/4
.backanno
.end

D.1.9 HUSL nickel transducer model

I1 0 Z 0 AC 1
XU_Headmass N005 N012 N009 0 matl_block
  area=tcir*16.8m length=2.7m material=nickel
XU2 0 N002 0 N001 matl_block
  area=tcir*6.37m length=26.8m material=nickel
RACS1 N009 0 \{Rrad\}
RACS2 N010 N016 \{Rrad\}
RMECH1 N005 N004 \{Rm\}
RMECH3 N003 N002 \{Rm\}
RMECH2 N012 N019 \{Rm\}
RMECH4 N018 N017 \{Rm\}
XU_ECR1 N015 N008 EddyCurrents R0=\{2*Rcorner\}
    NL=\text{lams}, \mu=\{\text{muni*mu0}\}, \sigma=\text{sigmani} \ L=\text{lcir}, \ w=\text{wcir}, \ d=\text{lt}
RE1 N011 N013 \{Rcoil\}
RMAG1 N008 N015 \{Rleak\}
XU_ECR2 P001 N006 EddyCurrents R0=\{2*Rcorner+2*Rag\}
    NL=\text{lams}, \mu=\{\text{muni*mu0}\}, \sigma=\text{sigmani} \ L=\text{lcir}, \ w=\text{wcir}, \ d=\text{lt}
RMAG2 N006 0 \{Rleak\}
XU10 N003 0 N004 0 N007 N008 MechMag_EC length=\text{lcir}, \ w=\text{wcir}, \ d=\text{lt}
    \mu33T=\text{muni*mu0}, \ s33H=\text{s33ni}, \ d33=\text{d33ni} \ area=\text{acir}, \ \rho=\rho_\text{ni}, \ n=\text{nx}
    NL=\text{lams}, \ \sigma=\text{sigmani}
XU6 N006 N007 Z N011 MagElec \ n=\text{nx}
XU5 0 N014 0 N013 MagElec \ n=\text{nx}
XU_ECR3 0 P001 EddyCurrents R0=\{Rmag\}
    NL=1, \mu=\{\text{mumag*mu0}\}, \sigma=\text{sigmamag} \ L=\text{lcir}, \ w=\text{wcir}, \ d=\text{tmag}
XU3 N017 0 N016 0 mat1_block
    \text{area}=\text{tcir*6.37m} \ \text{length}=26.8m \ \text{material}=\text{nickel}
LMagnet N010 N001 7.6m
XU1 N018 0 N019 0 N014 N015 MechMag_EC length=\text{lcir}, \ w=\text{wcir}, \ d=\text{lt}
    \mu33T=\text{muni*mu0}, \ s33H=\text{s33ni}, \ d33=\text{d33ni} \ area=\text{acir}, \ \rho=\rho_\text{ni}, \ n=\text{nx}
    NL=\text{lams}, \ \sigma=\text{sigmani}
.ac \ lin \ 2e3 \ 1e3 \ 1e5
; Geometry
.param \text{lmag}=25.4m; \text{Magnet length}
.param \text{wmag}=3.5m; \text{Magnet width}
.param \text{tmag}=12m; \text{Magnet thickness}
.param \text{lcir}=17.2m; \text{Length of nickel leg section}
.param \text{wcir}=2.85m; \text{Width of nickel leg section}
.param \text{tcir}=12m; \text{Thickness of nickel leg section}
.param \text{rfillet}=2.5m; \text{Radius of fillets}
.param \text{lag}=0.1m; \text{Length of air gaps}
.param \text{acir}=\{\text{wcir*tcir}\}; \text{Area of nickel leg section}
.param \text{lt}=0.15m; \text{Lamination thickness}
.param \text{lams}=60; \text{Number of laminations}
.param nx=30; Number of turns per coil

; Reluctances
.param Rmag={wmag/(mumag*mu0*lmag*tmag)}; Reluctance of magnet
.param Rleak={5m/(mu0*(17.4/2)*tmag)}; Reluctance across circuit
.param Rcorner={pi/(2*muni*mu0*tcir*ln((rfillet+wcir)/rfillet))};
    Reluctance of corner
.param Rag={lag/(mu0*wmag*tmag)}; Reluctance of air gap
.param Rrad={rho_air*c_air*12m*16.8m};
    Mechanical radiation resistance
.param Rcoil=1; Electrical resistance of both coils in series
.param Rm=2*acir/(omega0*Qm*s33ni*lcir/2);
    Mechanical resistance calculation

; Material Properties
.param mu0=4e-7*pi; Permeability of free space
.param muni=22; Relative permeability of nickel
.param mumag=5; Relative permeability of Alnico
.param rho_air=1.21; Density of air
.param c_air=345; Speed of sound in air
.param sigmani=1.4e7; Conductivity of nickel
.param sigmamag=2.1e6; Conductivity of Alnico V
.param rho_ni=8800; Density of nickel
.param Qm=150; Qm for nickel
.param s33ni=5p; s33 for nickel
.param omega0=47k*2*pi; Resonance frequency
.param d33ni=-3.1n; Piezomagnetic coefficient
.lib ARL_LTSpice_SP.lib
.backanno
.end
D.1.10 GCD design model

B_HAC  H_AC  0  V=abs(V(N1)-V(N2))/\{lcir\}
B_BAC  B_AC  0  V=abs(I(Rgf_AC1))/(wgf*h)
Rmag_AC1 N022  0  \{Rmag\}
Rleck_AC1 N2  N014  \{Rleck\}
Rleck_AC2 N020 N019  \{Rleck\}
Rag_AC4 N029  0  \{Rag\}
Rag_AC1 N022 N030  \{Rag\}
Rmag_AC2 N004 N003  \{Rmag\}
Rag_AC3 N007 N003  \{Rag\}
Rag_AC2 N004 N008  \{Rag\}
R_GF2_AC3 N007 N011  \{Rgf2\}
Rgf2.AC2 N008 N012  \{Rgf2\}
Rgf2.AC1 N026 N030  \{Rgf2\}
Rgf2.AC4 N025 N029  \{Rgf2\}
I1  0  N016  1  AC  0
Rmag_DC1 N021  0  \{Rmag\}
I_MAG.1  0  N021  \{Br*wmag*h\}
RSC_DC1  0  N021  \{Rsc\}
RSC_DC2 N021  0  \{Rsc\}
Rleck_DC1 M2  N013  \{Rleck\}
Rleck_DC2 M1  N018  \{Rleck\}
Rag_DC4 N027  0  \{Rag\}
Rag_DC1 N021 N028  \{Rag\}
Rmag_DC2 N002 N001  \{Rmag\}
I_MAG.2  N002  N001  \{Br*wmag*h\}
RSC_DC3 N001 N002  \{Rsc\}
RSC_DC4 N002 N001  \{Rsc\}
Rag_DC3 N005 N001  \{Rag\}
Rag_DC2 N002 N006  \{Rag\}
Rgf2_DC3 N005 N009  \{Rgf2\}
Rgf2_DC2 N006 N010  \{Rgf2\}
Rgf2_DC1 N024 N028  \{Rgf2\}
.param mu0=4e-7*pi; [H/m] Permeability of free space
.param mugf=100; [dimensionless] Relative permeability of Galfenol
.param mumag=1.05; [dimensionless] Relative permeability of NdFeB
.param Hc=12.4k; [A/m] Coercive field for NdFeB Grade 42
.param Br=1.32; [T] Remanent induction for NdFeB Grade 42

; Geometry
.param tmag={1/32*25.4m}; [m] Thickness of the biasing magnets (flux traversing dimension)
.param wmag={1/2*25.4m}; [m] Width of biasing magnets
.param h={1*25.4m}; [m] Height of stack/magnet
.param dwire=0.32m; [m] Diameter of the drive coil 29 AWG magnet
wire
.param lcir=35m; [m] Length of drive segment
.param rfillet=2m; [m] Radius of fillets at ends of drive segment
.param wgf=5m; [m] Width of drive segment
.param lag=0.1m; [m] Length of air gap (assumed)
.param t2=3m; [m] Length of Galfenol piece from fillet to magnet face

; Reluctances
.param Rmag={tmag/(mumag*mu0*wmag*h)}; [1/H] Reluctance seen by flux traversing the permanent magnets
.param Rgf={lcir/(mugf*mu0*wgf*h)}; [1/H] Reluctance of the Galfenol drive segment
.param Rgf2={t2/(mugf*mu0*wmag*h)}; [1/H] Reluctance of the Galfenol region between the fillet and magnet face
.param Rcorner={pi/(2*mugf*mu0*h*ln((rfillet+wgf)/rfillet))}; [1/H] Reluctance of flux traveling around corners in Galfenol piece
.param Rleak={(2*rfillet+2*t2+tmag)/(mu0*(lcir+2*rfillet)/2*h)}; [1/H] Leakage reluctance for half cutout
.param Rag={lag/(mu0*wmag*h)}; [1/H] Reluctance of air gaps
.param Rsc={1/(0.26*h*mu0)}; [1/H] Reluctance of short circuit leakage paths (see Roters)

; Counts
.param layers=3; [layers] Number of wire layers in drive coil
.param nx={floor((lcir)/dwire)*layers}; [turns] Number of turns on each coil

.op BX3 keff 0 V=sqrt((
+2*abs(I(Rgf_AC1))**2*Rgf
+/((
+2*abs(I(Rmag_AC1))**2*Rmag
++4*abs(I(Rcnr_AC1))**2*Rcorner
++2*abs(I(Rgf_AC1))**2*Rgf
++4*abs(I(Rgf2_AC1))**2*Rgf2

)}
D.1.11 GCD drive model

\[
++4*\text{abs}(I(Rag_{AC1}))**2*Rag \\
++2*\text{abs}(I(R\text{leak}_{AC1}))**2*R\text{leak} \\
++4*\text{abs}(I(Rsc_{AC1}))**2*Rsc \\
+)
\]

.lib ARL_LTSpice_SP.lib
.lib backanno
.end

B1 Hz 0 V=(V(0)-V(N1))/\{lcir\}
RLeak2 0 N011 \{R\text{leak}\}
RLeak1 N020 N009 \{R\text{leak}\}
I1 0 Z 0 AC 1
RSC_5 N018 N013 \{Rsc\}
RSC_6 N018 N013 \{Rsc\}
RSC_7 N017 N012 \{Rsc\}
RSC_8 N017 N012 \{Rsc\}
H1 Bz 0 value=\{I(V1)/(wgf*h)\}
V1 N021 N1 0 AC 0
XU1 N013 N011 EddyCurrents R0=\{(R\text{corner}+Rgf2)+Rag\} d=l, l=1, w=1,
NL=40 mu=mugf*mu0, sigma=sigmagf
XU2 N018 0 EddyCurrents R0=\{(R\text{corner}+Rgf2)+Rag\} d=l, l=1, w=1,
NL=40 mu=mugf*mu0, sigma=sigmagf
XU3 N009 N012 EddyCurrents R0=\{(R\text{corner}+Rgf2)+Rag\} d=l, l=1, w=1,
NL=40 mu=mugf*mu0, sigma=sigmagf
XU4 N020 N017 EddyCurrents R0=\{(R\text{corner}+Rgf2)+Rag\} d=l, l=1, w=1,
NL=40 mu=mugf*mu0, sigma=sigmagf
XU5 N010 N009 0 N014 MagElec n=nx
XU9 N006 0 N007 0 N010 N011 MechMag_EC NL=lams, d=l, n=nx,
area=\{wgf*h\} mu33T=\{mugf*mu0\}, d33=d33gf, s33H=s33gf
length=\{lcir+2*rfilet\},
\[ w = w_{gf}, \quad \rho = \rho_{hgf}, \quad \sigma = \sigma_{hgf} \]

\[ XU7 \quad N024 \quad 0 \quad N025 \quad 0 \quad N021 \quad 0 \quad \text{MechMag}_EC \quad NL = \text{lams}, \quad d = l_t, \quad n = n_x, \]
\[ \text{area} = \{w_{gf} \cdot h\} \quad \mu_{33T} = \{\mu_{g} \cdot \mu_0\}, \quad d_{33} = d_{33g}, \quad s_{33H} = s_{33g}, \]
\[ \text{length} = \{l_{cir} + 2 \cdot r_{fillet}\}, \]
\[ w = w_{gf}, \quad \rho = \rho_{hgf}, \quad \sigma = \sigma_{hgf} \]

\[ XU10 \quad N005 \quad 0 \quad N003 \quad 0 \quad \text{matl_block} \quad \text{area} = \text{w} \cdot \text{h} \quad \text{length} = \text{wmag} \]
\[ \text{material} = \text{galfenolsteel} \]

\[ \text{RACS1} \quad N027 \quad N015 \quad \{\text{Rrad}\} \]
\[ \text{RE} \quad N019 \quad N014 \quad \{\text{Rcoil}\} \]
\[ \text{RM1} \quad N004 \quad N007 \quad \{\text{Rm}\} \]
\[ \text{RM2} \quad N006 \quad N003 \quad \{\text{Rm}\} \]
\[ \text{RM3} \quad N024 \quad N023 \quad \{\text{Rm}\} \]
\[ \text{RM4} \quad N026 \quad N025 \quad \{\text{Rm}\} \]
\[ T1 \quad N001 \quad N003 \quad N002 \quad N004 \quad \text{Td} = \{(l_{cir} + r_{fillet}) / \text{cep}\} \quad Z_0 = \{\rho_{e} \cdot \text{cep} \cdot \text{A}_e\} \]
\[ T2 \quad N029 \quad N026 \quad N028 \quad N023 \quad \text{Td} = \{(l_{cir} + r_{fillet}) / \text{cep}\} \quad Z_0 = \{\rho_{e} \cdot \text{cep} \cdot \text{A}_e\} \]
\[ L1 \quad N008 \quad N015 \quad \{\text{mmag}\} \]
\[ L2 \quad N016 \quad N005 \quad \{\text{mmag}\} \]
\[ R1 \quad N001 \quad N003 \quad \{\text{Rep}\} \]
\[ R2 \quad N002 \quad N004 \quad \{\text{Rep}\} \]
\[ R3 \quad N023 \quad N028 \quad \{\text{Rep}\} \]
\[ R4 \quad N026 \quad N029 \quad \{\text{Rep}\} \]

\[ XU6 \quad N020 \quad N1 \quad N019 \quad \text{Z MagElec} \quad n = n_x \]
\[ \text{I}_1 \quad \text{Mag1} \quad N017 \quad N012 \quad \{\text{Br} \cdot \text{wmag} \cdot \text{h}\} \]
\[ \text{I}_2 \quad \text{Mag2} \quad N013 \quad N018 \quad \{\text{Br} \cdot \text{wmag} \cdot \text{h}\} \]

\[ XU11 \quad N008 \quad 0 \quad N004 \quad 0 \quad \text{matl_block} \quad \text{area} = \text{w} \cdot \text{h} \quad \text{length} = \text{wmag} \]
\[ \text{material} = \text{galfenolsteel} \]

\[ XU12 \quad N012 \quad N017 \quad \text{EddyCurrents} \quad R0 = \{\text{Rmag}\} \quad d = \text{h}, \quad l = 1, \quad w = 1, \quad \text{NL} = 1 \]
\[ \mu = \mu_{m} \cdot \mu_0, \quad \sigma = \sigma_{m} \]

\[ XU13 \quad N013 \quad N018 \quad \text{EddyCurrents} \quad R0 = \{\text{Rmag}\} \quad d = \text{h}, \quad l = 1, \quad w = 1, \quad \text{NL} = 1 \]
\[ \mu = \mu_{m} \cdot \mu_0, \quad \sigma = \sigma_{m} \]

\[ XU8 \quad N027 \quad 0 \quad N026 \quad 0 \quad \text{matl_block} \quad \text{area} = \text{w} \cdot \text{h} \quad \text{length} = \text{wmag} \]
\[ \text{material} = \text{galfenolsteel} \]

\[ XU14 \quad N022 \quad 0 \quad N023 \quad 0 \quad \text{matl_block} \quad \text{area} = \text{w} \cdot \text{h} \quad \text{length} = \text{wmag} \]
material=galfenolsteel
RACS2 N022 N016 {Rrad}

; Physical Properties
.param mu0=400n*pi; [H/m] Permeability of free space
.param mugf=100; [dimensionless] Relative permeability of Galfenol
.param mumag=1.05; [dimensionless] Relative permeability of NdFeB
.param sigmagf=1.2e6; [S/m] Electrical conductivity of Galfenol
.param sigmamag=6.25e5; [S/m] Electrical conductivity of NdFeB
.param Eep=4G; [Pa] Elastic modulus of Epon 828 epoxy in 100/33 mix ratio
.param d33gf=21n; [m/A] Magnetostrictive parameter for Galfenol
.param Qm=20; [dimensionless] Mechanical quality factor of Galfenol
.param Qep=475; [dimensionless] Mechanical quality factor of Epon 828 epoxy
.param rhogf=7900; [kg/m^3] Density of Galfenol
.param rhoair=1.21; [kg/m^3] Density of air
.param rhoep=1160; [kg/m^3] Density of Epon 828 epoxy
.param cair=345; [m/s] Speed of sound in air
.param cep=sqrt(Eep/rhoep); Speed of sound in Epon 828 epoxy
.param Hc=12.4k; [A/m] Coercive field for NdFeB Grade 42
.param Br=1; [T] Remanent induction for NdFeB Grade 42
.param omega0=14k*2*pi; [rad/s] Resonance frequency of the drive

; Geometry
.param lcir=35m; [m] Length of drive segment
.param rfillet=2m; [m] Radius of fillets at ends of drive segment
.param wgf=5m; [m] Width of drive segment
.param lag=0.1m; [m] Length of air gap (assumed)
.param t2=3m; [m] Length of Galfenol piece from fillet to magnet face
.param lt=0.262m; [m] Lamination thickness
.param dwire=0.32m; [m] Diameter of the drive coil 29 AWG magnet
wire
.param h=12m; [m] Measured stack height
.param h0=lt*lams; [m] Height contribution from Galfenol
.param hbl=h-h0; [m] Total height contribution from bondlines
.param Aep=hbl*wgf; [m^2] Cumulative area of epoxy
.param wend={wgf+2*rfillet+2*t2}; [m] Width of end segments
.param tmag={1/32*25.4m*nummag}; [m] Thickness of the biasing magnets (flux traversing dimension)
.param wmag={1/2*25.4m}; [m] Width of biasing magnets
.param mmag={0.92m*nummag}; [kg] Mass of biasing magnet
.param tsl=25.4m/32; [m] Thickness of corprene sleeve under coil
; Reluctances
.param Rmag={tmag/(mumag*mu0*wmag*h)}; [1/H] Reluctance seen by flux traversing the permanent magnets
.param Rgf={(1cir)/(mugf*mu0*wgf*h)}; [1/H] Reluctance of the Galfenol drive segment
.param Rgf2={t2/(mugf*mu0*wmag*h)}; [1/H] Reluctance of the Galfenol region between the fillet and magnet face
.param Rcorner={pi/(2*mugf*mu0*h*ln((rfillet+wgf)/rfillet))}; [1/H] Reluctance of flux traveling around corners in Galfenol piece
.param Rleak={2*rfillet+2*t2-3.5m+tmag-8*dwire-2*tsl}/
(mu0*(1cir+2*rfillet)/2*h); [1/H] Leakage reluctance for half cutout, -1mm b/c of face sanding
.param Rag={lag/(mu0*wmag*h)}; [1/H] Reluctance of air gaps
.param Rsc={1/(0.26*h*mu0)}; [1/H] Reluctance of short circuit leakage paths (see Roters)
; Resistances
.param Rm=2*wgf*h0/(omega0*Qm*s33gf*1cir/2); [kg/s] Mechanical resistance calculation
.param Rep=2*wgf*hbl*Eep/(omega0*Qep*1cir/2); [kg/s] Mechanical resistance calculation
.param Rrad=rhoair*cair*h*(2*wend+tmag); [kg/s] Radiation resistance in air
.param Rcoil={2*nx*(2*(wgf+2*dwire)+2*(h+2*dwire))*0.2684024};

[Ohms] DC Resistance of both coils

; Counts

.param nx=410; [turns] Number of turns on each coil
.param lams=40; [laminations] Number of laminations in each stack
.param nummag=3; [magnets] Number of permanent magnets at each end
.ac lin 801 2k 40k
.savebias bias.txt
;step param mugf list 13 14
.lib ARL_LTSpice_SP.lib
.backanno
.end

D.1.12 GCD tonpilz model

B1 Hz 0 V=(V(0)-V(N1))/lcir
RLeak2 0 N011 {Rleak}
RLeak1 N026 N009 {Rleak}
I1 0 Z 0 AC 1
RSC_5 N024 N013 {Rsc}
RSC_6 N024 N013 {Rsc}
RSC_7 N023 N012 {Rsc}
RSC_8 N023 N012 {Rsc}
H1 Bz 0 value=I(V1)/(wgf*h)
V1 N027 N1 0 AC 0
XU1 N013 N011 EddyCurrents R0={(Rcorner+Rgf2)+Rag} d=lt, l=1, w=1,
NL=40 mu=mugf*mu0, sigma=sigmagf
XU2 N024 0 EddyCurrents R0={(Rcorner+Rgf2)+Rag} d=lt, l=1, w=1,
NL=40 mu=mugf*mu0, sigma=sigmagf
XU3 N009 N012 EddyCurrents R0={(Rcorner+Rgf2)+Rag} d=lt, l=1, w=1,
NL=40 mu=mugf*mu0, sigma=sigmagf
XU4 N026 N023 EddyCurrents R0={(Rcorner+Rgf2)+Rag} d=lt, l=1, w=1,
NL=40 mu=mugf*mu0, sigma=sigmagf
XU5 N010 N009 0 N020 MagElec n=n\text{x}
XU9 N006 0 N007 0 N010 N011 MechMag_EC NL=lams, d=lt, n=n\text{x},
\hspace{1em} area={wgf}*h \hspace{1em} \mu33T=\{mugf*\mu0\}, \hspace{1em} d33=d33gf, \hspace{1em} s33H=s33gf
\hspace{1em} length={}l_{cir}+2*r_{fillet}, \hspace{1em} w=wgf, \hspace{1em} rho=rhogf, \hspace{1em} sigma=sigmagf
XU7 N030 0 N031 0 N027 0 MechMag_EC NL=lams, d=lt, n=n\text{x},
\hspace{1em} area={wgf}*h \hspace{1em} \mu33T=\{mugf*\mu0\}, \hspace{1em} d33=d33gf, \hspace{1em} s33H=s33gf
\hspace{1em} length={}l_{cir}+2*r_{fillet}, \hspace{1em} w=wgf, \hspace{1em} rho=rhogf, \hspace{1em} sigma=sigmagf
XU10 N005 0 N003 0 matl_block\hspace{1em} area=w\text{end}*h \hspace{1em} length=w\text{mag}
\hspace{1em} \hspace{1em} material=galfenolsteel
RE N025 N020 \{\text{Rcoil}\}
RM1 N004 N007 \{\text{Rm}\}
RM2 N006 N003 \{\text{Rm}\}
RM3 N030 N029 \{\text{Rm}\}
RM4 N032 N031 \{\text{Rm}\}
T1 N001 N003 N002 N004 Td={l_{cir}+r_{fillet}}/cep \hspace{1em} Z0=\{\rho_{eop}\text{*}\text{cep}\text{*}\text{Aep}\}
T2 N034 N032 N033 N029 Td={l_{cir}+r_{fillet}}/cep \hspace{1em} Z0=\{\rho_{eop}\text{*}\text{cep}\text{*}\text{Aep}\}
L2 N018 N005 \{\text{mmag}\text{*numtm}\}
R1 N001 N003 \{\text{Rep}\}
R2 N002 N004 \{\text{Rep}\}
R3 N029 N033 \{\text{Rep}\}
R4 N032 N034 \{\text{Rep}\}
XU6 N026 N1 N025 Z MagElec n=n\text{x}
I_Mag1 N023 N012 \{B\text{r}*w\text{mag}*h\}
I_Mag2 N013 N024 \{B\text{r}*w\text{mag}*h\}
XU11 N008 0 N004 0 matl_block\hspace{1em} area=w\text{end}*h \hspace{1em} length=w\text{mag}
\hspace{1em} \hspace{1em} material=galfenolsteel
XU12 N012 N023 EddyCurrents R0=\{\text{Rmag}\} \hspace{1em} d=h, \hspace{1em} l=1, \hspace{1em} w=1, \hspace{1em} NL=1
\hspace{1em} \hspace{1em} \mu=mumag*\mu0, \hspace{1em} \sigma=sigmamag
XU13 N013 N024 EddyCurrents R0=\{\text{Rmag}\} \hspace{1em} d=h, \hspace{1em} l=1, \hspace{1em} w=1, \hspace{1em} NL=1
\hspace{1em} \hspace{1em} \mu=mumag*\mu0, \hspace{1em} \sigma=sigmamag
R_{RRADH} N021 N015 \{0.2957*R_{radh}\}
VRAD N015 N014 0
XUM2 N018 N028 N016 0 matl_block\hspace{1em} area=htm*w\text{tm} \hspace{1em} length={ltm*numtm\}}
Material={tungsten}
B_TCR1 TCR 0 V=I(VRAD)*rhowtr*whm*hhm/2/pi/1e-6 laplace=abs(s)
XU8 N019 0 N032 0 matl_block area=wend*h length=wmag
material=galfenolsteel
XU14 N028 0 N029 0 matl_block area=wend*h length=wmag
material=galfenolsteel
XUM3 N019 N017 0 N014 matl_block area=whm*hhm length=lhm
material={magnesium}
L1 N008 N017 {mmag*numtm}
B_XRADH NO21 0 V={I(B_XRADH)*0.5605*Rradh} laplace=s/abs(s)
R_RRADT N022 N016 {0.4085*Rradt}
B_XRADT N022 0 V={I(B_XRADT)*0.5892*Rradt} laplace=s/abs(s)
; Physical Properties
.param mu0=400n*pi; [H/m] Permeability of free space
.param mugf=106; [dimensionless] Relative permeability of Galfenol
.param mumag=1.05; [dimensionless] Relative permeability of NdFeB
.param sigmagf=1.2e6; [S/m] Electrical conductivity of Galfenol
.param sigmamag=6.25e5; [S/m] Electrical conductivity of NdFeB
.param Eep=4G; [Pa] Elastic modulus of Epon 828 epoxy in 100/33 mix ratio
.param d33gf=21n; [m/A] Magnetostrictive paramater for Galfenol
.param Qm=28; [dimensionless] Mechanical quality factor of Galfenol
.param Qep=475; [dimensionless] Mechanical quality factor of Epon 828 epoxy
.param rhowtr=1000; [kg/m^-3] Density of water
.param rhogf=7900; [kg/m^-3] Density of Galfenol
.param rhoair=1.21; [kg/m^-3] Density of air
.param rhoep=1160; [kg/m^-3] Density of Epon 828 epoxy
.param cair=345; [m/s] Speed of sound in air
.param cwtr=1481; [m/s] Speed of sound in water
.param cep=sqrt(Eep/rhoep); Speed of sound in Epon 828 epoxy
.param Hc=12.4k; [A/m] Coercive field for NdFeB Grade 42
.param Br=1; [T] Remanent induction for NdFeB Grade 42
.param omega0=14k*2*pi; [rad/s]Resonance frequency of the drive
; Geometry
.param lcir=35m; [m] Length of drive segment
.param rfillet=2m; [m] Radius of fillets at ends of drive segment
.param wgf=5m; [m] Width of drive segment
.param lag=0.1m; [m] Length of air gap (assumed)
.param t2=3m; [m] Length of Galfenol piece from fillet to magnet face
.param lt=0.262m; [m] Lamination thickness
.param dwire=0.32m; [m] Diameter of the drive coil 29 AWG magnet wire
.param h=12m; [m] Measured stack height
.param h0=lt*lams; [m] Height contribution from Galfenol
.param hbl=h-h0; [m] Total height contribution from bondlines
.param Aep=hbl*wgf; [m^2] Cumulative area of epoxy
.param wend={wgf+2*rfillet+2*t2}; [m] Width of end segments
.param tmag={1/32*25.4m*nummag}; [m] Thickness of the biasing magnets (flux traversing dimension)
.param wmag={1/2*25.4m}; [m] Width of biasing magnets
.param mmag={0.92m*nummag}; [kg] Mass of biasing magnet
.param tsl=25.4m/32; [m] Thickness of corprene sleeve under coil
.param hhm=3/2*25.4m; [m] Height of headmass
.param whm=25.4m; [m] Width of headmass
.param lhm=25.4m/4; [m] Thickness of headmass
.param htm=32.8m; [m] Height of tailmass
.param wtm=13.9m; [m] Width of tailmass
.param ltm=6.37m; [m] Length of tailmass
; Reluctances
.param Rmag={tmag/(mumag*mu0*wmag*h)}; [1/H] Reluctance seen by flux traversing the permanent magnets
.param Rgf={(lcir)/(mugf*mu0*wgf*h)}; [1/H] Reluctance of the
Galfenol drive segment

.param Rgf2={t2/(mugf*mu0*wmag*h)}; [1/H] Reluctance of the Galfenol region between the fillet and magnet face

.param Rcorner={pi/(2*mugf*mu0*h*ln((rfillet+wgf)/rfillet))}; [1/H] Reluctance of flux traveling around corners in Galfenol piece

.param Rleak={(2*rfillet+2*t2-3.5m+tmag-8*dwire-2*tsl)/
(mu0*(1cir+2*rfillet)/2*h)}; [1/H] Leakage reluctance for half cutout, -1mm b/c of face sanding

.param Rag={lag/(mu0*wmag*h)}; [1/H] Reluctance of air gaps

.param Rsc={1/(0.26*h*mu0)}; [1/H] Reluctance of short circuit leakage paths (see Roters)

; Resistances

.param Rm=2*wgf*h0/(omega0*Qm*s33gf*lcir/2); [kg/s] Mechanical resistance calculation

.param Rep=2*wgf*hbl*Eep/(omega0*Qep*lcir/2); [kg/s] Mechanical resistance calculation

.param Rradh={rhowtr*cwtr*(whm*hhm)}; Radiation resistance of headmass into water

.param Rradt={rhoair*cair*(wtm*htm)}; Radiation resistance of tailmass into air

.param Rcoil={2*nx*(2*(wgf+2*dwire)+2*(h+2*dwire))*0.2684024}; [Ohms] DC Resistance of both coils

; Counts

.param nx=410; [turns] Number of turns on each coil

.param lams=40; [laminations] Number of laminations in each stack

.param nummag=3; [magnets] Number of permanent magnets at each end

.param numtm=1; [tailmasses] Number of tungsten tailmass pieces

.ac lin 5e3 2 40k

.savebias bias.txt

.lib ARL_LTSpice_SP.lib

.backanno

.end
D.1.13 Materials.txt contents for mechanical transmission line piece

.param E=youngsmodulus*1e9
.param rho=density
.param sigma=poisson
.param c=soundspeed

.param tungsten=1
.param nickel=2
.param carbon_steel=3
.param stainless_steel=4
.param alumina=5
.param becu=6
.param beryllia=7
.param brass=8
.param ferrite=9
.param pzt4=10
.param titanium=11
.param galfenol=12
.param albemet=13
.param terfenold=14
.param aluminum=15
.param lead=16
.param macor=17
.param glass=18
.param magnesium=19
.param pmn33pt=20
.param grp_alongfiber=21
.param grp_crossfiber=22
.param a2epoxy=23
.param pvdf=24
.param lucite=25
.param nylon66=26
.param nylon6=27
.param syntacticfoam=28
.param hardrubber=29
.param mdf=30
.param corprene500psi=31
.param kraftpaper=32
.param sadm=33
.param neoprene=34
.param phenolicresincotton=35
.param onionskin=36
.param corprene100psi=37
.param polyurethane=38
.param siliconerubber=39
.param T4=40
.param NST=41
.param Magnet=42
.param galfenolsteel=43
.param tungsten2=44

.param youngsmodulus=table(material,1,362,2,210,3,207,4,193,5,300,6,125,7,345,8,104,9,140,10,65,11,104,12,57,13,200,14,26,15,71,16,16.5,17,66.9,18,62,19,44.8,20,8.4,21,16.4,22,11.9,23,5.8,24,3,25,4,26,3.3,27,2.8,28,4,29,2.3,30,3,31,1.58,32,1.14,33,0.55,34,0.5,35,0.55,36,0.56,37,0.49,38,0.3,39,0.12,40,90,41,210,42,150,43,66,44,362)

.param density=table(material,1,19350,2,8800,3,7860,4,7900,5,3690,6,8200,7,2850,8,8500,9,4800,10,7550,11,4500,12,7900,13,2100,14,9250,15,2700,16,11300,17,2520,18,2300,19,1770,20,8040,21,2020,22,2020,23,1770,24,1600,25,1200,26,1140,27,1130,28,690,29,1100,
.param poisson=table(material,1,0.17,2,0.31,3,0.28,4,0.28,5,0.21,  
6,0.33,7,0.33,8,0.37,9,0.29,10,0.34,11,0.36,12,0.44,13,0.17,  
14,0.43,15,0.33,16,0.44,17,0.29,18,0.24,19,0.33,20,0.47,21,0.44,  
22,0.37,23,0.34,24,0.34,25,0.4,26,0.4,27,0.39,28,0.35,29,0.4,  
30,0.2,31,0.45,32,0.35,33,0.4,34,0.48,35,0.43,36,0.35,37,0.45,  
38,0.48,39,0.48,40,0.33,41,0.31,42,0.33,43,0.44,44,0.17)

.param soundspeed=table(material,1,4320,2,4890,3,5130,4,4940,  
5,9020,6,3900,7,11020,8,3500,9,5400,10,2930,11,4810,12,2690,  
13,9760,14,1680,15,5150,16,1200,17,5150,18,5200,19,5030,20,1020,  
21,2850,22,2430,23,1810,24,1370,25,1800,26,1700,27,1570,28,2410,  
29,1450,30,1940,31,1200,32,974,33,524,34,524,35,613,36,750,  
37,700,38,526,39,319,40,3497.43,41,4890,42,4580,43,2890,44,4692)

D.2 MATLAB Scripts

A couple of useful MATLAB scripts are included here. The first fits the tenparameter anhysteretic $B$-$H$ curve model (presented in section 5.1.5.1) to measured data; the second implements Stepanishen’s method for calculating the radiation impedance of a baffled rectangular piston (discussed in section 6.2.0.1). Continued lines are indicated by the ellipsis.

D.2.1 Anhysteretic $B$-$H$ model

close all
clear all
clc

% Written by Scott Porter, 24 Nov 2009
%% Parameters
mu0=4e-7*pi;
fval=99;

%% Read and sort measured data
raw=textread('GalfenolSteelBHMeasured.txt'); % H (Oe) and B (Gauss)
data(:,1)=raw(:,1)*79.578; % Converts to [A/m]
data(:,2)=raw(:,2)*1e-4; % Converts to [T]
sorted=sortrows(data);
H=sorted(:,1);
B=sorted(:,2);

%% Initial guesses
a=64;
b=72;
c=1.2;
d=14;
e=20;
f=4300;
g=5600;
h=1.58;
i=-0.02;
j=-645;

%% Evaluate initial guesses
gr=i+mu0.*(a.*(H-j)+b.*(H-j).*abs((H-j)/f).^d+(h/mu0*sign(H-j)+c.*(H-j)).*abs((H-j)/g).^e)./(1+abs((H-j)/f).^d+abs((H-j)/g).^e);

%% Fit
while fval>14.8 % 14.8 is an arbitrary end condition
fit=@(x)sum(abs(B-(x(9)+mu0.*(x(1).*(H-x(10))+x(2).*(H-x(10)).*...
(((H-x(10))/x(6)).^x(4)+(x(8)/mu0*sign(H-x(10))+x(3).*((H-x(10)))*...((H-x(10))/x(7)).^x(5))./(1+((H-x(10))/x(6)).^x(4)+...((H-x(10))/x(7)).^x(5))));

[x,fval,exitflag]=fminsearch(fit,[a b c d e f g h i j],...
optimset('MaxIter',20000,'MaxFunEvals',20000,'TolX',1e-12,...
'TolFun',1e-12));

disp(['Sum of error: ' num2str(fval)]);
a=x(1); b=x(2); c=x(3); d=x(4); e=x(5); f=x(6); g=x(7); h=x(8);
i=x(9); j=x(10);
outp=[fval x];
dlmwrite('xvalues.txt',outp,'append','newline','pc','delimiter',' ');
clear outp
end
toc

%% Evaluate
model(:,1)=[linspace(min(H),-25.1e3,20)...
linspace(-25e3,25e3,300)...
linspace(25.1e3,max(H),20)];

model(:,2)=real(x(9)+mu0.*(x(1).*((model(:,1)-x(10)))+x(2).*...
(model(:,1)-x(10)).*((model(:,1)-x(10))/x(6)).^x(4)+... 
(x(8)/mu0*sign(model(:,1)-x(10))+x(3).*((model(:,1)-x(10)))*... 
((model(:,1)-x(10))/x(7)).^x(5))./(1+((model(:,1)-x(10))/x(6)).^x(4)+... 
((model(:,1)-x(10))/x(7)).^x(5))));

D.2.2 Radiation impedance of a rectangular piston

close all
clear all
clc
% Written by Scott Porter, 22 Sep 2010


%%% Parameters
a=3/2*25.4e-3; %[m] Long dimension
b=25.4e-3; %[m] Short dimension
c=1481; %[m/s] Speed of sound
f=linspace(0,100e3,2000); %[Hz]
n=[1 1.5 2 3 4 5];% a/b; Aspect Ratio

%%% Radiation impedance calculations
for jj=1:length(n) % Calculate multiple aspect ratios
    for ii=1:length(f) % Calculate multiple frequencies
        k(ii)=2*pi*f(ii)/c; % Wavenumber calculation
        kb(ii)=k(ii)*b; % kb parameter
        % Evaluate C1 & C2
        x=kb(ii);
        y=acosh(sqrt(n(jj)^2+1));
        F2 = @(beta)sin(x.*cosh(beta))./(cosh(beta).^2);
        C21 = -1*quadl(F2,0,y); % Solve first instance of C2
        clear x y F2
        x=n(jj)*kb(ii);
        y=acosh(sqrt(n(jj)^2+1)/n(jj));
        F2 = @(beta)sin(x.*cosh(beta))./(cosh(beta).^2);
        C22 = -1*quadl(F2,0,y); % Solve second instance of C2
        clear x y F2
        x=kb(ii);
        y=acosh(sqrt(n(jj)^2+1));
F1 = @(beta)cos(x.*cosh(beta))./(cosh(beta).^2);
C11 = quadl(F1,0,y); % Solve first instance of C1
clear x y F1
x=n(jj)*kb(ii);
y=acosh(sqrt(n(jj)^2+1)/n(jj));
F1 = @(beta)cos(x.*cosh(beta))./(cosh(beta).^2);
C12 = quadl(F1,0,y); % Solve second instance of C1
clear x y F1

%% Calculate
Rn(:,ii,jj)=1-((2/(n(jj)*pi))/(kb(ii)^2))*...
(1-cos(kb(ii))-cos(n(jj)*kb(ii))+cos(sqrt(n(jj)^2+1)*kb(ii)))+...
((2/pi)/kb(ii))*C21+((2/(n(jj)*pi))/kb(ii))*C22; %Eq. 12a
Xn(:,ii,jj)=((2/(n(jj)*pi))/kb(ii))*(1+n(jj))-...
((2/(n(jj)*pi))/(kb(ii)^2))*(sin(kb(ii))+sin(n(jj)*kb(ii)))-...
sin(sqrt(n(jj)^2+1)*kb(ii)))-((2/pi)/kb(ii))*C11-...
((2/(n(jj)*pi))/kb(ii))*C12; %Eq. 12b
end
end
Bibliography


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Vita
Scott P. Porter

Born into a Navy family, Scott Porter grew up in military housing along both the east and west coasts. After moving to Michigan, he graduated from Kettering University with B.S. degrees in Applied Physics and Mechanical Engineering in 2006. Immediately afterward, he began studies in The Graduate Program in Acoustics at Penn State. While at Penn State, Scott had opportunities to teach five weeks of an introductory course on acoustics for undergraduates (ACS 402), to serve as chair and vice-chair of the Penn State Audio Engineering Society Student Section, and to help transition the Acoustical Society of America’s Central Pennsylvania regional chapter into the Penn State Student Chapter of the Acoustical Society of America. In addition, he was involved with the Acoustical Society of America’s Student Council, representing the Engineering Acoustics technical committee. In 2009, Scott completed a M.Eng. degree based on a project in which he built his own set of loudspeakers for 5.1 multichannel audio reproduction. Scott and his wife Betsie have two children, Brooklyn and Shiloh.