CHARACTERIZATION AND MODELING OF A FLEXIBLE MATRIX
COMPOSITE MATERIAL FOR ADVANCED ROTORCRAFT DRIVELINES

A Thesis in
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by
Stanton G. Sollenberger

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The thesis of Stanton G. Sollenberger was reviewed and approved* by the following:

Charles E. Bakis  
Distinguished Professor of Engineering Science and Mechanics  
Thesis Advisor

Edward C. Smith  
Professor of Aerospace Engineering

Renata S. Engel  
Professor of Engineering Science and Mechanics

Judith A. Todd  
P. B. Breneman Department Head Chair  
Professor, Department of Engineering Science and Mechanics

*Signatures are on file in the Graduate School
ABSTRACT

Most of the current driveline designs for rotary wing aircraft (rotorcraft) consist of rigid aluminum shafts joined by flexible mechanical couplings to account for misalignment in the driveline. The couplings are heavy and incur maintenance and cost penalties because these parts wear and need frequent replacement. A possible solution is to replace the current design with a continuous, flexurally-soft, torsionally-stiff, flexible matrix composite (FMC) shaft. This design could eliminate the flexible couplings and reduce the overall weight of the driveline. Previous research studies on this topic using preliminary materials have found design solutions that meet operational criteria; however, these designs employed thick-walled, heavy shafts. In order to significantly reduce the weight of the driveline, a stiffer FMC material resulting in a lighter shaft design is required. A new FMC with a matrix material that is stiffer than the previous, preliminary material by a factor of five is characterized in this thesis. It is believed that future design solutions with this new material will increase weight savings on the final driveline design.

For rotorcraft operated in hostile environments, a topic of concern regarding FMC shafts is ballistic impact tolerance. This topic has not yet been explored for these new kinds of composite materials and is pursued here on a coupon-level basis. Results show that tubular FMC test coupons absorb more energy and suffer larger reductions in torsional strength than their conventional composite counterpart, although reductions in tensile and compressive strengths are similar in both materials. This difference is attributed to the greater pull-out of fibers in the FMC material. Coupon-level testing is a good first step in understanding ballistic tolerance in FMCs, although additional testing
using fully sized and designed shafts is suggested so that proper comparisons between driveshafts can be made.

Self-heating of FMC shafts is an important issue because many design criteria are dependent on temperature. A model has already been developed to simulate self-heating in FMC shafts, although it is limited to analyzing composite layups with one fiber angle and no ballistic damage. New closed-form and finite element models are developed in the current investigation that are capable of analyzing self-heating in FMC shafts with multiple fiber angles through the thickness. The newly developed models have been validated with comparisons against experiments and one another. Experiments using simple angle-ply tubes made with the new FMC suggest that for a shaft operating under real service strains, temperature increase due to self-heating can be less than 10°C. Material development, characterization of the ballistic tolerance of FMCs, and improved analytical tools are contributions from the current investigation to the implementation of FMC driveshafts in rotorcraft.
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Chapter 1  Introduction

Rotory wing aircraft serve many functions from news gathering to medical services and are used widely. Current driveline designs in rotorcraft utilize a system which transmits power through segmented shafts. In most instances the shafts themselves are made of aluminum and are supported by bearings and joined together by flexible couplings. Bearings prevent contact between the shaft and tailboom. Couplings accommodate the inevitable misalignment of the driveline during flight and prevent bending fatigue of the metal shaft material. There are some problems with this design, namely it is heavy, and normal operation results in constant replacement and maintenance (Hetherington et al., 1990).

A possible solution to this heavy, complex design is to replace the aluminum segments and couplings with one continuous flexible matrix composite (FMC) shaft. This would of course eliminate coupling maintenance and likely save weight. Some optimization findings by Mayrides (2005) have shown that the FMC design can reduce the number of midspan bearings as well. Flexible matrix composite materials contain strong reinforcing fibers with high stiffness that are joined together by a flexible resin with large strain capability. While the material is very compliant in the direction dominated by the flexible resin, it maintains its strength and stiffness in the fiber
direction. With proper design a flexurally soft, torsionally stiff FMC driveshaft can be found to meet design conditions that would otherwise be difficult or impossible to meet with other materials. A schematic of the current and proposed helicopter tail-rotor driveline is shown in Figure 1.

One issue of concern in maintaining the regular operation of rotorcraft is damage. When considering the use of these machines in a military setting, damage due to ballistic impact is of particular concern. For the designer, the effect of ballistic tolerance in FMC shafts in regard to stiffness, strength and stability is a valuable piece of information.

Another separate issue, since misalignment in the driveline in the proposed design is accounted for in the shaft itself, is the hysteretic self-heating of the FMC under cyclic bending strain. An analysis tool that calculates self-heating in an FMC shaft exists and has shown promise (Shan, 2006), but this tool is limited to simple angle-ply laminates.

This investigation is part of an ongoing effort to design continuous FMC driveshafts to replace current segmented metallic designs. Specifically, the purposes are
to contribute to design by means of material development, investigate the effects of a ballistic impact event on flexible matrix composites, and refine analytical tools that calculate the self-heating behavior in FMC driveshafts. The objectives are summarized in more detail following the literature review.
Chapter 2  Literature Review

The current design for a driveline in rotorcraft consists of a series of aluminum segmented shafts that are connected with flexible couplings and supported by hanger bearings. Flexible couplings are needed to account for the misalignment between the main transmission and the tail rotor transmission. Depending on the specific aircraft, this misalignment may be inherent or it could be due to aerodynamic and maneuvering loads. Due to fatigue limitations of metallic materials, the design ensures there is no bending in the aluminum segments and all of the misalignment in the driveline is accounted for in the couplings. Suggestions to improve this design have been to replace the aluminum segments with rigid matrix composite (RMC) segments – the conventional structural composite which is usually a carbon/epoxy system (Darlow and Creonte, 1995). There were some weight savings in these studies, but reduction in parts, maintenance, and reliability was not observed. A proposed design to improve and replace the segmented-coupling design is one with a continuous FMC driveshaft (Hannibal and Avila, 1984; Shan and Bakis, 2005; Mayrides, 2005). This design will reduce parts, weight (Mayrides et al., 2005) and vibrations (DeSmidt, 2005), and the high degree of anisotropy of FMCs will allow for design parameters in stiffness to be met that would otherwise be difficult or impossible to achieve with conventional RMCs.
2.1 Flexible Matrix Composites

The term flexible matrix composites (FMCs) is used to identify composite materials that are comprised of high strength, high stiffness reinforcing fibers embedded in a deformable, flexible, elastomeric polymer with a high yield strain and a glass transition temperature that is usually significantly below ambient room temperature. Examples of these matrix materials include rubbers, silicones, and polyurethanes. When combined with the stiff reinforcing fibers, the composite materials have very high degrees of anisotropy because the stiffness of the material in the fiber direction can be 500 times greater (Shan, 2002; Shan 2005) than the stiffness in the direction transverse to the fibers. This ratio for RMCs is typically around 20 (Daniel and Ishai, 2006). Also, the ultimate transverse tensile strain of FMCs has been observed to be as high as 28% (Shan, 2006) whereas for RMCs it is typically around 0.6% (Daniel and Ishai, 2006). The compressive strain at which fibers experience micro-buckling in FMCs has been observed to be as low as 900 με (Shan, 2006) whereas for RMCs it is typically around 9400 με (Gibson, 2007). The structural and principal coordinate systems for a composite ply are shown in Figure 2. For sake of comparison plots of the structural axial stiffness ($E_X$), shear stiffness ($G_{xy}$) and Poisson’s ratio ($\nu_{xy}$) of $[\pm \theta]_s$ composite laminates computed by classic lamination theory (Daniel and Ishai, 2006) are shown in Figures 3 – 5 using the in-plane major ($E_1$) and minor ($E_2$) Young’s moduli, shear modulus ($G_{12}$), and Poisson’s ratio ($\nu_{12}$) for an FMC (Shan, 2006) and an RMC (Daniel and Ishai, 2006) — shown in Table 1. When the matrix material is the primary constituent loaded in the composite, huge differences are seen in the properties of the laminates. When the fibers are loaded, similar properties can be achieved in both FMC and RMC laminates.
Table 1: Quasi-static ply elastic properties for an FMC and RMC

<table>
<thead>
<tr>
<th>Material</th>
<th>$E_1$ (GPa)</th>
<th>$E_2$ (GPa)</th>
<th>$G_{12}$ (GPa)</th>
<th>$v_{12}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon/polyurethane (T700/L100)</td>
<td>115</td>
<td>0.23</td>
<td>0.25</td>
<td>0.32</td>
</tr>
<tr>
<td>Carbon/epoxy (IM7/977-3)</td>
<td>190</td>
<td>9.9</td>
<td>7.8</td>
<td>0.35</td>
</tr>
</tbody>
</table>

Figure 2: Structural ($x$-$y$) and principal (1-2) coordinate systems

Figure 3: Axial stiffness ($E_x$) of $[\pm \theta]$, FMC and RMC laminates
Many interesting applications have been explored for these materials including composite flywheels (Gabrys and Bakis, 1997; Emerson et al., 1997), extension-twist box beams for rotor blades (Nampy, 2005) cargo restraints in rotorcraft (Tiwari et al., 2009), robotic actuators and morphing structures (Shan et al., 2006) and flexible driveshafts in
rotorcraft. In the work on the internal heating behavior of FMC driveshafts (Shan and Bakis, 2002; Shan and Bakis, 2009), a candidate material system was used for analysis and experiments. This system combined Toray T700 carbon fibers with a polyurethane matrix made of Adiprene® L100 prepolymer and Caytur® 21 curing agent.

2.2 Damage Tolerance in Fiber Reinforced Composites

A good deal of work has been done on examining the effects of impact damage in RMCs over the years. Most of the work focuses on carbon/epoxy systems in a laminated plate configuration. Many advanced numerical simulations and analytical methods that predict the effects of a ballistic impact event on a composite laminate have been developed (Lee and Sun, 1993; Silva et al., 2005; Naik and Shrirao, 2004) including damage evolution, damage size, and the ballistic limit.

Some investigators have studied the effects of impact damage on carbon/epoxy composite tubes (Christiansen et al., 1988; Christoforou and Swanson, 1988; Shivakumar et al., 1985; Valle, 1989). Most of these investigations concentrate on the reduction in strength due to low velocity impact, which increases exponentially with increased area of damage according to Valle (1989). Although a study conducted by Christiansen et al. (1988) using various impactors at hyper-velocity impact (4000 – 7500 m/s) on carbon/epoxy tubes shows that projectile parameters such as size, density, velocity, and impact inclination have notable effects on target damage.

The damage tolerance of carbon/epoxy tubes made by hand layup with drilled holes and medium-velocity (330 – 530 m/s) impact damage loaded in compression was studied by Ochoa et al. (1991) at Texas A&M University. The impact damage was introduced by a .22 caliber rifle shot at normal and oblique angles to the composite tubes.
Interestingly enough, the compressive strengths after impact for the two tubes with normal and oblique impact damage were no different. Also, the reported reduction in strength due to the ballistic impact was equal to the reduction in strength due to a drilled hole with a diameter that is 50% of the tube diameter (.22 caliber projectile diameter is 5.5% of tube diameter). This ratio (drilled hole or projectile diameter to tube diameter) is referred to as the hole-tube ratio, \( d_h/d_t \). Ochoa et al. (1991) report that for \( d_h/d_t \) values of 0.13 and 0.50, knockdowns in compressive strength for carbon/epoxy tubes were 52% and 78%, respectively. Tubes in this investigation had an outer diameter of 10.16 cm, a wall thickness of 1.40 mm, and a stacking sequence of \([±18/90_2/±18/90_2/±18]\).

A study on the failure mode of carbon/epoxy tubes made by hand layup with drilled holes in torsion was performed by Bauchau et al. at Rensselaer Polytechnic Institute (1988). In this study the authors concluded that the failure mode for tubes that had \( d_h/d_t \) values of 0.30 – 0.46 was torsional buckling. When this ratio increased to 0.60, the mode observed was material failure. This trend was observed for tubes with outer diameters of about 42 mm, wall thicknesses of about 1.1 mm, and two different stacking sequences of \([15/-15/-45/-15/15/45]\) and \([45/15/-15/-45/-15/15]\).

The effect of drilled holes as stress concentrations in filament-wound flexible matrix composite tubes was examined by Sollenberger et al. (2009). In this case, the quasi-static moduli and strength of notched and unnotched tubes were compared in tension and compression. For \( d_h/d_t \) values of 0.25 and 0.30, the maximum knockdowns in compression strength (multiple layups) for the carbon/flexible epoxy tubes were 25% and 40%, respectively. Different angle-ply layups were tested from ±23° to ±90° with the hoopwound configuration seeing the maximum knockdown. The ±23° configuration saw knockdowns of only 5 and 13%. The outer diameter of these tubes was about 22 mm and
wall thicknesses were about 1 mm. Although it is recognized that the geometries, layups, and stacking sequences are quite different in the two studies of RMCs (Ochoa et al., 1991) and FMCs (Sollenberger et al., 2009); in regard to the drilled hole-tube ratio, the knockdowns in compression strength for the rigid matrix composites seem to be higher than those for the flexible matrix composites. Fatigue testing of FMC tubes subjected to over 3 million cycles with drilled holes was also performed by Sollenberger et al. (2009). Using X-radiography, the author found no observable fatigue crack growth.

2.3 Calculating Temperature Increase due to Hysteresis

Flexible matrix composite driveshafts that are misaligned and rotating experience self-heating due to hysteretic damping and cyclic bending strain. Most of the damping occurs in the polymer matrix material, although as pointed out by many investigators (e.g. Gibson et al., 1982) fibers do have a small amount of damping that contribute to the overall damping of the composite. Hysteretic damping refers to the behavior in materials where the resultant force lags behind the applied strain. In each cycle of loading, a finite amount of energy is lost as heat due to damping. A thermomechanical model that calculates self-heating in FMC driveshafts has been developed (Shan and Bakis, 2009) and shows good correlation with experimental results — shown in Figure 6. Since the self-heating model agrees well with experimental data, the approach to model and measure the damping behavior of FMCs is chosen to be the same in the current work. A review of some of the most common material damping models is given here – including the specific damping models used in the work of Shan and Bakis (2009).
It is assumed that the composite can be modeled as a linear viscoelastic solid with material properties dependent on temperature and loading frequency. A linear elastic solid has a stress-strain relationship that is independent of time. A linear viscoelastic solid has a linear relationship between stress and strain with a viscous component and an elastic component. The viscous component of the viscoelastic solid gives the solid a strain rate dependent on time. In a cycle of loading, a viscoelastic solid loses energy due to damping whereas an elastic solid does not. During the vibration of a viscoelastic system, a finite amount of energy has to be expended in each cycle. So, if the applied strain is given as

$$\varepsilon(t) = \varepsilon_0 \sin \omega t$$  \hspace{1cm} (1)

where $\varepsilon_0$ is the applied strain amplitude and $\omega$ is the oscillation angular frequency, then the resultant stress response is given by

Figure 6: Temperature increase for a ±60° T700/L100 FMC tube for various speeds and flexural strain (Shan and Bakis, 2009)
\[ \sigma(t) = \sigma_a \sin(\omega t + \phi) \]  

(2)

where \( \sigma_a \) is the resultant stress amplitude and \( \phi \) is the phase angle difference between the applied strain and resultant stress. For an elastic material, the phase difference between the stress and strain, \( \phi \) is zero. For a viscoelastic material, \( \phi \) is non-zero. The work done per cycle is then

\[ \Delta W = \int \sigma(t)d\varepsilon(t) = \int_{0}^{2\pi/\omega} \sigma(t)\varepsilon(t)dt = \pi \sigma_a \varepsilon_a \sin \phi. \]  

(3)

and the maximum strain energy in the system can be defined as half the product of the maximum strain and the corresponding instantaneous value of stress,

\[ W = \frac{1}{2} \varepsilon_a \sin \frac{\pi}{2} \sigma_a \sin \left( \frac{\pi}{2} + \phi \right) = \frac{1}{2} \varepsilon_a \sigma_a \cos \phi. \]  

(4)

The loss factor is then defined by

\[ \eta = \frac{1}{2\pi} \frac{\Delta W}{W} = \tan \phi, \]  

(5)

which is a measure of the energy lost during each cycle. The loss factor can be determined by measuring the phase difference between the sinusoidal waveforms of the applied strain and the resulting stress. This approach to measuring the loss factor is called the phase lag method and is described by Kinra and Wren (1992a, 1992b). An FFT analysis is done on the waveforms to determine the phase difference.

Another result of the phase lag method is the complex modulus – which relates the applied strain to the resultant stress. The complex modulus is comprised of two parts, the storage modulus and the loss modulus. The storage modulus is a measurement of the ratio of the in-phase resultant stress to the applied strain. The loss modulus is a measurement of the out-of-phase ratio of the resultant stress to the applied strain. Using trigonometric identities, Equation 2 can be re-written as
\[
\sigma(t) = \sigma_a \sin \omega t \cos \phi + \sigma_a \cos \omega t \sin \phi
\]  
(6)

From Equation 6 it is clear that the stress is resolved into two quantities; one that is in phase with the applied strain and one that is out of phase with the applied strain. These quantities can be related to the storage and loss moduli by Equations 7 and 8, respectively.

\[
E' = \frac{\sigma_a}{\varepsilon_a} \cos \phi
\]  
(7)

\[
E'' = \frac{\sigma_a}{\varepsilon_a} \sin \phi
\]  
(8)

Hence, Equation 6 can be re-written as

\[
\sigma(t) = E' \varepsilon_a \sin \omega t + E'' \varepsilon_a \cos \omega t = (E' + iE'') \varepsilon_a = E^* \varepsilon_a
\]  
(9)

where \(E^*\) is called the complex modulus. By dividing Equation 8 by Equation 7, one obtains the loss factor.

\[
\frac{E''}{E'} = \tan \phi = \eta
\]  
(10)

Complex moduli can be determined from the phase lag method but also from methods like the forced vibration of a cantilever beam (viscoelastic flexural modulus), where the phase angle by which the tip deflection of the beam lags behind the displacement of a shaker at the base measures damping (Paxson, 1975).

Other measures of damping include the logarithmic decrement, \(\delta\), which is a measure of the exponential free decay of oscillation with time in a damped system. It is determined by the free vibration method which measures the decay in the vibration amplitude of an initially excited cantilever beam (Paxson, 1975, Chandra et al., 2003). Some researchers use the inverse quality factor, \(Q^{-1}\), which relates to the steady state
amplitude attained by a structure excited by a harmonically oscillating force. It is
determined by

\[ Q^{-1} = \frac{\omega_2 - \omega_n}{\omega_n}, \]  

where \( \omega_n \) is the resonant frequency and \( \omega_1 \) and \( \omega_2 \) are the frequencies on either side of \( \omega_n \) for which the response amplitude is \( \frac{1}{\sqrt{2}} \) times the resonant amplitude – i.e., the half power points. The inverse quality factor can be determined by the impulse technique where a transfer function for the specimen is found by tapping a beam or a cubic specimen with a hammer which has a force transducer attached to its head. A motion transducer is used to measure the response of the specimen. The signals from the two transducers are fed into an FFT analysis to determine the response versus frequency. The inverse quality factor, \( Q^{-1} \), is obtained by using half-power-band-width technique (Suarez et al., 1984; Crane and Gillespie, 1991; Chandra et al., 2003). The inverse quality factor can also be determined from the torsional vibration method (Adams et al., 1969; White and Abdin, 1985) where a composite rod is undergoing steady state torsional vibration. A vibrational load is applied to the bar by a two coil-magnet system undergoing an alternating current. The alternating current is compared to the response from the surface strain gages and the damping is determined by Equation 11.

In order to accurately represent the behavior of the matrix-dominated properties of the composite for ranging frequencies, a fractional derivative model (Bagley and Torvik, 1979; Bagley and Torvik, 1983; Papouilia and Kelly, 1997; Rogers, 1983) was used by Shan and Bakis (2009). The fractional derivative model is a variation from the standard linear model or the Zener model (Nashif et al., 1985; Ferry, 1970). Instead of the classic representation of the state equation between \( \sigma \) and \( \varepsilon \) which is given by
the integral derivatives are replaced by fractional derivatives,

\[ \sigma(t) + a D^\beta \sigma(t) = E \varepsilon(t) + b E D^\beta \varepsilon(t) \]

where \(0 < \beta < 1\), and the fractional derivative of order \(\beta\) is defined as

\[ D^\beta[x(t)] = \frac{1}{\Gamma(1-\beta)} \frac{d}{dt} \int_0^t (t-\tau)^{\beta-1} x(\tau) d\tau, \]

where \(\Gamma(\cdot)\) denotes the gamma function. The rheological representation of the fractional derivative standard linear model is shown in Figure 7, where the elastic elements are assumed to follow Hooke’s law and the viscous elements are assumed to be filled with a Newtonian fluid of viscosity \(\mu\). The stress-strain behavior of the springpot element is given by \(\sigma(t) = \mu D^\beta \varepsilon(t)\) where it can vary between purely elastic (\(\beta=0\)) and purely viscous (\(\beta=1\)).

\[ \begin{align*}
E & = E_\infty \\
a & = \frac{\mu}{E_1} \\
b & = \mu \left(\frac{E_\infty + E_1}{E_\infty E_1}\right)
\end{align*} \]

Figure 7: Rheological model of the fractional derivative standard linear model

The generalized stress-strain relationship for the fractional derivative model is expressed as (Nashif et al., 1985)
\[ \sigma(t) + \sum_{k=1}^{n} a_k D^\beta_k \sigma(t) = E \varepsilon + E \sum_{k=1}^{n} b_k D^\beta_k \varepsilon(t). \]  

(15)

For a harmonic response of the form \( \sigma = \sigma_0 e^{i\omega \tau} \) and \( \varepsilon = \varepsilon_0 e^{i\omega \tau} \), Equation 15 gives

\[
\sigma_0 = E \varepsilon_0 \frac{1 + \sum_{k=1}^{n} b_k (i\omega)^{\beta_k}}{1 + \sum_{k=1}^{n} a_k (i\omega)^{\beta_k}}.
\]

(16)

So the storage and loss moduli are

\[
E' = \text{Re} \left\{ \frac{1 + \sum_{k=1}^{n} b_k (i\omega)^{\beta_k}}{1 + \sum_{k=1}^{n} a_k (i\omega)^{\beta_k}} \right\} E
\]

(17)

\[
E^\ast = \text{Im} \left\{ \frac{1 + \sum_{k=1}^{n} b_k (i\omega)^{\beta_k}}{1 + \sum_{k=1}^{n} a_k (i\omega)^{\beta_k}} \right\} E
\]

(18)

In order to account for the differences in storage and loss moduli throughout varying temperature ranges, the temperature-frequency superposition principle (Ferry, 1970) is used (Shan and Bakis, 2009). Temperature shifting factors translate storage modulus and loss factor curves versus frequency at different temperatures toward a reference temperature into one master curve. The master curve contains information for the complex modulus at different temperatures and frequencies. To model damping in the composite, a macro-mechanical strain energy method developed by Adams et al. (Adams and Bacon, 1973a, 1973b; Ni and Adams, 1984) was used in the work of Shan and Bakis (2009) to calculate the dissipated strain energy. If the overall damping in the composite is a sum of all of the strain energy losses from each of the elements of the composite, then the overall dissipated strain energy is represented by
\[ \sum_i \Delta W_i = \sum_i 2\pi \eta_i W_i , \]  

(19)

where the index \(i\) represents the individual elements of the composite. Contributions from the three normal and in-plane shear stresses and strains were included in the thermomechanical model (Shan and Bakis, 2009).

Other methods to model damping in composites include the viscoelastic correspondence approach first proposed by Hashin (Hashin, 1970a; Hashin, 1970b) which says that a quasi-static linear viscoelastic analysis can be converted to a dynamic linear viscoelastic analysis by replacing static stresses and strains with the corresponding dynamic stresses and strains and by replacing elastic moduli with complex moduli. Micro-mechanical approaches to model damping in composites have been also been examined that model damping mechanisms on the fiber/matrix level. Many of these models have examined short fiber reinforced composite material (Gibson et al., 1982; Sun et al., 1987; White and Abdin, 1985) but some have also modeled continuous fiber reinforced materials (Caruso and Chamis, 1986; Saravanos and Chamis, 1992). The continuous fiber models include fiber, matrix, and volume fraction properties. They also predict damping behavior of the composite based on temperature, absorbed moisture, fiber/matrix interface friction, and fiber breakage.

### 2.4 Limitations of Previous Work

Although the flexible matrix material – Adiprene® L100 – used in Shan and Bakis (2002, 2009) exhibited good dynamic and fatigue behavior, experimental and design work at Penn State has shown that this system may be too soft for an actual shaft application. Some testing and analytical work revealed the onset of fiber micro-buckling to be about 900 \( \mu\varepsilon \) (Shan, 2006). This is a failure mode that jeopardizes the stiffness of
the composite and is the result of fibers being loaded in compression while being supported by a soft matrix. Designing with the L100 system would require a thick-walled, heavy shaft. This is not ideal when the goal of the design is to minimize weight. The micro-buckling strain and weight issues certainly have an adverse effect on design, and a stiffer matrix may improve the end result. A stiffer matrix material with a flexible epoxy formulation was used in studying the effect of stress concentrations in FMCs (Sollenberger et al., 2009), but upon further inspection the dynamic behavior of the flexible epoxy chemistry turned out to have major disadvantages. The matrix was overdamped and exhibited a large increase in stiffness with increasing frequency. For these reasons, the possibility of testing and designing with a stiffer polyurethane matrix is an attractive option. This approach would maintain the good dynamic properties of the Adiprene® L100 system while alleviating some of the issues of the very soft and compliant matrix. Because many design such as heating, strength, buckling, and whirl instability of the shaft constraints are dependent on material properties, material selection and property determination is an important task and is pursued here.

As far as the author knows, all of the previous work on ballistic testing of composites has been carried out with RMC systems like carbon/epoxy and glass/epoxy. For this reason, the ballistic tolerance of FMC materials will be investigated here. Damage tolerance in FMC tubes was studied in Sollenberger et al. (2009), but it was limited to drilled holes. A drilled hole does not necessarily represent the damage due to a ballistic impact event. The dynamics of a high velocity impact can cause material to deform and fracture in ways that are highly unpredictable. In this work (Sollenberger et al., 2009), the post-impact testing was also limited to quasi-static tension and compression and a bending fatigue test. So, in the current work the effect of ballistic
damage on the quasi-static torsional strength of a small-scale FMC tube will be studied in addition to quasi-static tension and compression along with bending fatigue. Since torsion is the primary quasi-static loading case of a driveshaft, this effect will be of key interest.

The thermomechanical model that predicts the steady state temperature of a rotating misaligned composite shaft developed by Shan and Bakis (2009) is limited to one fiber angle in the layup sequence. In the optimization work of Mayrides (2005) it was found that layups with more than one angle were sometimes preferred. So, an improvement to Shan and Bakis’s (2009) analysis tool that allows for the calculation of self-heating in a multi-angled tube seems to be relevant and is pursued. Also, a finite element model that calculates self-heating in an FMC driveshaft could easily model multi-angled layups and could prove to be valuable when considering FMC shaft design in industry. With a framework to calculate self-heating in finite elements, aspects of FMC shaft design like bearing support and complicated misalignment loadings could be introduced with some minor additional effort. For these reasons, the development of a finite element model is also pursued in the present investigation.

2.5 Problem Statement/Research Goal

The objectives of the current investigation are to characterize the material property parameters of a stiff carbon/polyurethane composite; characterize the ballistic tolerance of FMC tubes; and improve and add to analytical tools used to calculate self-heating in FMC driveshafts. A stiff polyurethane may simultaneously reduce the weight of an optimized FMC driveshaft in a final design while keeping the superior dynamic behavior observed in previous research. Although, the actual design of an FMC shaft for a rotorcraft is beyond the scope of the present investigation. Characterizing the ballistic
tolerance of FMC tubes will provide preliminary information about how an FMC driveshaft may perform during and after an impact event in service. As far as the author knows, testing the ballistic tolerance of flexible matrix composite materials is unique. Because design tools have suggested optimized layups that contain multiple fiber angles, an improvement on the established analytical tool to calculate heating in FMC driveshafts to include these conditions is considered worthwhile and relevant. These objectives will be achieved by the following task list:

1. Select an appropriate material system on which to perform quasi-static and dynamic testing for material property characterization
2. Perform ballistic testing on filament-wound tube coupons to determine the effects of a ballistic impact event on FMC tubes.
3. Alter the existing thermomechanical model that calculates self-heating of undamaged FMC driveshafts to include multi-angled layups. Also, develop a finite element model to calculate the self-heating.
4. Carry out quasi-static and dynamic testing on FMC materials to determine the material properties need to predict the quasi-static and self-heating behavior of FMC tubes.
5. Use the material property parameters and the self-heating models to compare the calculations of self-heating (from models) and the measurements of self-heating (from experiments) for undamaged FMC tubes.

The following chapters describe the approach of the current investigation. Materials and fabrication methods are covered in Chapter 3. Chapter 4 covers ballistic testing, quasi-static testing, and quasi-static material property determination. Note that
Chapter 4 is the only chapter addressing ballistic testing. Chapter 5 describes dynamic testing and material property determination. Chapter 6 reviews the self-heating results from the models and experiments. Finally, conclusions and recommendations for future work are given in Chapter 7. Computer codes used in the current research, a sensitivity analysis, and a few relevant videos are given in the appendices.
Chapter 3  Materials and Fabrication Methods

For this investigation, carbon fiber reinforced composites with a rigid matrix material and a flexible matrix material were fabricated so that comparisons in performance between RMCs and FMCs can be made. The rigid matrix material in the RMCs is an epoxy, while the flexible matrix material in the FMCs is a flexible polyurethane (described later). Tubes will be used to describe small, hollow, cylindrical test coupons and shafts will be used to describe a cylindrical, fully-designed, load-bearing structure in this thesis. Filament-wound tubes were made for spin testing, ballistic testing, as well as quasi-static and dynamic material property characterization. Flat plate coupons were also made by filament winding for additional dynamic material property characterization. For flat plate coupons, instead of winding wet carbon tows onto a cylindrical mandrel (as in tube fabrication) the tows were wound onto a flat steel plate to create unidirectional composite plies. These plies were then cut and arranged in a closed mold and cured in a hot-press.

The filament winding process is a method of fabricating fiber reinforced polymer composites. A schematic of the filament winding process can be seen in Figure 8. To start, the carbon fiber tow unwinds from the original fiber spool. The tow then runs over a set of two pulleys that are attached to the fiber tensioning machine. The normal force exerted on these pulleys from the tow is used as feedback information for the tensioner so
that it can apply the constant user-defined amount of tension on the tow. The tow then
makes a turn at the rear end of the crossfeed just before it enters the first orifice in the
resin bath. The first orifice is usually selected to be considerably larger than the second
because it does not affect the amount of resin that is impregnated into the tow – a major
factor affecting the final fiber volume fraction of the part. Once the tow enters the resin
bath, it is spread out on three polished, static rods while simultaneously being immersed
in the liquid resin. The spreading of the tow ensures proper impregnation. When the tow
leaves the resin bath, a prescribed amount of the impregnated resin is squeezed out of the
tow by the second orifice. The size of the second orifice is selected to achieve the
appropriate fiber volume fraction in the composite. Once the tow leaves the resin bath it
is fed through the payout eye. The payout eye lays the wet tow onto the mandrel in the
proper position. The linear speed at which the carriage moves relative to the rotational
speed of the mandrel dictates the fiber winding angle, which is shown as $\alpha$ in Figure 8.
Once the mandrel is completely covered with the composite material, it is wrapped with a
heat shrinking tape that consolidates the part during cure. The mandrel with the
composite and shrinking tape is then transported to an oven for curing. Once the
composite is cured, the tape is removed from the composite, and the composite is
removed from the mandrel.
Figure 8: Schematic of the filament winding machine

All composites were manufactured using HexTow® AS4D-GP 12k carbon fiber, a high strength PAN (polyacrylonitrile) based carbon fiber made by Hexcel Corporation (Stamford, CT). HexTow® is sold as a continuous strand, or tow, of fibers that is wrapped on a spool. The tow itself is comprised of 12,000 individual carbon fibers.

3.1 Matrix Materials

The optimum flexible matrix material for an FMC shaft application in rotorcraft is yet to be determined. Previous work has shown that the fatigue and self-heating properties of a polyurethane matrix (Adiprene® L100) are of good quality (Shan, 2006). Although, upon further investigation, it was found that in order to satisfy stiffness, strength, and stability design requirements while designing with the Adiprene® L100 resin, the geometry of the FMC shafts turned out to be thick-walled. So, even though the shafts would meet service loading conditions, they were heavy and were not likely to reduce the total driveline weight. A possible solution to this problem is to design shafts with a stiffer polyurethane matrix. The aim is to keep the good fatigue and self-heating properties of the polyurethane chemistry while simultaneously reducing the weight of the
final, optimally designed shaft. A stiffer matrix material will not require such a thick wall to meet all of the design requirements, although it may generate more hysteretic heating than a softer matrix.

The Chemtura Corporation (Middlebury, CT) was consulted for material development, and they suggested Adiprene® LF750D, a liquid polyether prepolymer. They also suggested using a curative with this prepolymer called Caytur® 31 DA, a delayed-action diamine curative. The polymer is made by mixing the prepolymer and the curative at a mass mixing ratio of 100 : 50.3. This mixture is then heated to 140°C for 2 hours followed by a 16 hour post-cure at 100°C. Caytur® 31 DA is described as a delayed-action curative because at room temperature it is nearly non-reactive. However, when Caytur® 31 DA is heated to 115 – 160°C, the salt complex in the curative dissolves which frees the methylene dianiline (MDA) complex. The MDA then reacts quickly with the toluene diisocyanate (TDI) terminated prepolymer to form an elastomeric polyurethane (Chemtura, 2007). The temperature of the resin bath during fabrication was kept at 50°C to reduce viscosity. Since this stiffer polyurethane may show promise in designing FMC shafts for operational rotorcraft, it will be used for all testing of flexible matrix composites in this investigation. A sample sheet of the cured polyurethane was provided by Chemtura for material property evaluation.

Another research goal for the current investigation is to make a comparative examination between the ballistic tolerance of RMCs and FMCs. For the rigid matrix composites, a low viscosity liquid epoxy resin system was selected for use because of its availability and good processing characteristics. The rigid matrix material is made by combining EPON™ 862, a liquid bisphenol A epoxide made by Hexion Specialty Chemicals Inc. (Columbus, OH), with Baytec Curative W, a liquid aromatic diamine.
curative also made by Hexion, with a mass mixing ratio of 100 : 26.4. This mixture is heated to 121°C for a 1 hour soak followed by a 2 hour cure at 177°C. The resin bath was kept at room temperature during fabrication.

3.2 Tube Fabrication

Small-scale FMC and RMC composite tubes were made by the wet filament winding fabrication process on a McLean-Anderson Filament Winder, photographs of which can be seen in Figure 9. Before winding, a target fiber volume fraction, $V_{ft}$, is chosen to calculate the size of the orifice on the resin bath. The relationship between target fiber volume fraction and orifice diameter, $D_o$, is calculated as follows

$$V_{ft} = \frac{D_f^2 \cdot N_f}{D_o^2}$$  \hspace{1cm} (20)

where $D_f$ is the diameter of the fibers, and $N_f$ is the number of fibers per tow. For a target fiber volume fraction of 0.60, calculations indicate a 0.940 mm (0.037 in) diameter orifice for 12K (12,000 filaments) tow with a fiber diameter of 6.7 μm (Hexcel, 2009). Due to limitations on orifice selection, a 0.965 mm (0.038 in) diameter orifice is used, which provides a volume fraction of 0.58. The actual fiber volume fraction of the specimen after it is cured, $V_{fa}$, is calculated using the following equation

$$V_{fa} = \frac{N_p \cdot N_f \cdot \pi D_f^2}{2b_w \cdot t}$$  \hspace{1cm} (21)

where $N_p$ is the number of plies, $b_w$ is the programmed bandwidth of each tow and $t$ is the total thickness of the wound part following cure. Each coverage of a helically wound tube is considered to be 2 layers for this calculation.
Two different sized mandrels were used for tube fabrication. A solid steel mandrel with a 19.84 mm outer diameter is used to make tubes for spin testing, ballistic testing, and quasi-static material property characterization. A smaller solid steel mandrel with an outer diameter of 9.91 mm was used to make FMC tubes for dynamic material property measurements. Both mandrels have tapped holes on either end so that end plugs can be attached. These end plugs then interface with and are gripped by the two chucks on the filament winder. From now on, tubes with winding angles between 0° and 60° will be referred to as helical tubes, while tubes with a winding angle of 0° will be referred to as unidirectional tubes, and tubes with a winding angle close to 90° will be referred to as hoopwound tubes. For helical and unidirectional tubes, pin rings are used on the end plugs (Figure 10). Pin rings eliminate the sharp curvature of the end of the cylindrical mandrel, which could fray the fibers and lead to tow breakage, and they provide a hooking point at the end of the mandrel so that the tow does not slip – a feature that is particularly important and useful for fiber winding angles less than 30°. The winding angle for hoopwound tubes is very close to 90° but is always slightly less. An illustration of the hoopwound configuration and the helical/unidirectional configuration is shown in

Figure 9: Filament winder, overview (left) and detailed view (right)
Figure 10. In preparation for each wind, the mandrel is sanded with grit cloth, and wiped down with acetone. Two layers of Ease Release™ 200, a silicone release, made by Mann Formulated Products (Easton, PA) are then applied to the mandrel to prevent bonding of the composite to the mandrel.

Different programs must be written in the computer of the filament winder for different types of tubes. For helical tubes, a program is written for the filament winder using the helical function. Inputs are bandwidth of the tow, winding angle, layers of coverage, length of mandrel and outer diameter of mandrel. Once these inputs are entered into the program, it allows the user to select the pattern. The pattern is best described as the number of cross-hatching diamond patterns that exist around the circumference of the final fully-covered part. For all helical wound tubes a pattern of two was selected for this investigation. A virtual rendering of a pattern of two, generated by the filament winder software, can be seen in Figure 11. All helical tubes were made on the 19.84 mm mandrel. The bandwidth was selected to be 2.337 mm (0.092 in) with two layers of coverage to achieve a wall thickness of roughly 1 mm.
For unidirectional tubes, since all of the fiber tows are oriented in the axial direction of the cylindrical mandrel, the carriage moves from one end of the mandrel to the other while the mandrel remains stationary. The mandrel rotates (through its index) only while the carriage dwells at one end or the other. The inputs for writing a unidirectional program are length and outer diameter of the mandrel, index, number of strokes, and the plunge distance. The index, \( i \), is measured in degrees and is expressed as follows

\[
i = 360^\circ + \frac{360^\circ}{S}
\]  

(22)

where \( S \) is the number of strokes. The number of strokes is how many total times the winder traverses the mandrel. In a unidirectional wind, the winder lays tows sequentially next to one another until the entire mandrel is covered. The plunge distance is how far the crossfeed moves in toward the mandrel when the carriage is located at either end of the mandrel. This translation of the crossfeed allows the pin rings to hook the tow at the ends of the mandrel after each pass. All unidirectional tubes were made on the 9.91 mm mandrel with an index of 366.7\(^\circ\) and 54 strokes totaling one layer.
For hoopwound tubes the inputs to the program are bandwidth of the tow, layers of coverage, length of mandrel and outer diameter of mandrel. The bandwidth of the tow for hoopwound tubes was also selected to be 2.337 mm (0.092 in). All hoopwound tubes were made on the 19.84 mm mandrel with four layers of coverage to achieve a wall thickness of roughly 1 mm. The actual fiber angle of the hoopwound tubes in this investigation was 83.2°.

For all tube types, a tension force of 4.45 N was applied to the tow. All helical and hoopwound tubes were wound on the 19.84 mm mandrel while all unidirectional tubes were wound on the 9.91 mm mandrel. Two different angles were wound with the carbon/epoxy system (Table 2). Six different fiber angles were wound with the carbon/polyurethane system (Table 3). Measured geometric properties of the tubes after cure and the actual (calculated) fiber volume fractions (assuming no voids) for the carbon/epoxy and carbon/urethane tubes are shown in Table 2 and in Table 3. The inner diameters of the ±60° and ±83.2° tubes are slightly larger than the mandrel size which is a consequence of the dependence of the thermal expansion characteristics of the tubes on the fiber winding angle.

**Table 2: Geometric properties and fiber volume fractions of the carbon/epoxy tubes**

<table>
<thead>
<tr>
<th>Fiber Angle (deg.)</th>
<th>± 45°</th>
<th>± 60°</th>
</tr>
</thead>
<tbody>
<tr>
<td>ID (cm)</td>
<td>1.98</td>
<td>1.98</td>
</tr>
<tr>
<td>OD (cm)</td>
<td>2.21</td>
<td>2.21</td>
</tr>
<tr>
<td>Cross-sectional Area (mm²)</td>
<td>75</td>
<td>74</td>
</tr>
<tr>
<td>$V_{fa}$</td>
<td>0.64</td>
<td>0.63</td>
</tr>
</tbody>
</table>
Table 3: Geometric properties and fiber volume fractions of the carbon/polyurethane tubes

<table>
<thead>
<tr>
<th>Fiber Angle (deg.)</th>
<th>± 20°</th>
<th>± 30°</th>
<th>± 45°</th>
<th>± 60°</th>
<th>± 83.2°</th>
<th>0°</th>
</tr>
</thead>
<tbody>
<tr>
<td>ID (cm)</td>
<td>1.98</td>
<td>1.98</td>
<td>1.98</td>
<td>1.99</td>
<td>1.99</td>
<td>0.991</td>
</tr>
<tr>
<td>OD (cm)</td>
<td>2.22</td>
<td>2.23</td>
<td>2.23</td>
<td>2.23</td>
<td>2.24</td>
<td>1.201</td>
</tr>
<tr>
<td>Cross-sectional Area (mm²)</td>
<td>80</td>
<td>80</td>
<td>83</td>
<td>79</td>
<td>84</td>
<td>36</td>
</tr>
<tr>
<td>$V_f$</td>
<td>0.59</td>
<td>0.60</td>
<td>0.58</td>
<td>0.59</td>
<td>0.58</td>
<td>0.65</td>
</tr>
</tbody>
</table>

After the filament winder finished covering the mandrel, the program was stopped and shrink-tape was wrapped around the specimen in order to help consolidate the composite and provide a smooth finish. The shrink-tape is called Hi-Shrink Tape: Release Coated (25 mm wide, 0.05 mm thick, 80°C activation temperature) from Dunstone Inc. (Charlotte, NC). Careful attention must be paid to the tension on the tape, the angle at which it is applied, and the width of the tape versus over-lap width when being applied. This idea is more clearly understood by examining Figure 12. Over-lap width was selected to be roughly half the width of the shrink-tape.

![Figure 12: Shrink tape application](image)

Figure 12: Shrink tape application

After the tubes were cured and the shrink tape was removed, the tubes were pulled off the mandrel. The ends were cut off using a water-cooled circular saw with a diamond tipped blade and discarded. The 19.84 mm sized tubes were 90 cm in length. From the 90 cm tubes, 7.6 cm sections were cut for compression testing, 12.7 cm sections
were cut for tension and torsion testing, and 45.7 cm sections were cut for spin testing. The 9.91 mm mandrels were 38 cm in length. From the 38 cm tubes, 25 mm sections were cut for torsional dynamic mechanical analysis.

### 3.3 Flat Plate Fabrication

Flat plate coupons were made for dynamic material property characterization for the FMC material. Pre-impregnated sheets of carbon fiber (referred to as pre-preg sheets or simply pre-pregs from now on) were made by using the wet filament winding process, much the same way tubes are made. Instead of winding the wet tow onto a cylindrical mandrel, the tow was wound onto a flat mandrel. To generate motion of the machine, a circumferential (hoopwound) winding program was chosen. Since the flat mandrel is not cylindrical in shape, the diameter input is nonsensical. So, a diameter was chosen such that the distance that the tow will travel while winding around the flat mandrel (the perimeter) is equal to the circumference of an imaginary cylindrical mandrel. Selecting the diameter and bandwidth properly ensured that fibers were spaced appropriately, with no gaps. The diameter and bandwidth used were 19.4 cm and 2.337 mm, respectively. A photograph of the set-up used to wind pre-preg sheets of carbon fiber is shown in Figure 13. In this photograph the carriage is hidden behind the flat mandrel, and the program has just started its first circuit.
Figure 13: Winding onto a flat plate mandrel to make pre-impregnated sheets of carbon fiber

Two layers of coverage were used to cover the flat mandrel. For a circumferential wind, one layer of coverage is defined as a sweep of the carriage from the headstock to tailstock. Once the flat mandrel was covered, it and the composite material wound onto it were placed between two pressure platens so that the continuous strands could be cut on the edges of the flat mandrel – resulting in two sheets of pre-preg. These two sheets of pre-preg were roughly 20 x 28 cm in size. Because the closed steel mold used to cure the flat plate coupons is roughly 10 x 56 cm in size, the two sheets of pre-preg were cut in half and stacked on top of one another so they fit the mold. The broken-fiber seam at the mid-length position of the mold was excluded from the testing program. This process is summarized in Figure 14.
A photograph of the mold used, containing only half of the pre-preg sheets, is shown in Figure 15. Two layers of Ease Release™ 200 spray were applied to all contact surfaces of the mold before the pre-pregs are placed in the mold. Once the sheets were arranged, a silicone slab (not shown in Figure 15) was placed on top of the composite material. This slab helps provide an even pressure on the part during cure and it also expands to seal the mold as it heats. After the silicone slab was in place, the male-end of the mold was fitted into the female-end, and the entire unit was positioned between the pressure platens in the hot press. The hot press was programmed to apply 241 kPa (35 psi) of pressure on the pre-preg. After pressure is applied, the platens heat and cool the mold by conductive heat transfer according to the cure schedule of the matrix material. The platens were programmed to open once the mold was cooled to room temperature. Flat plate coupons were cut to the appropriate size from the large cured composite.
The thickness for the flat plate coupons was determined to be about 1.2 mm, resulting in a fiber volume fraction of about 0.61 (assuming no voids) according to Equation 21. These values along with other geometric properties of the flat plate coupons are shown in Table 4. A rectangular coupon cut from the sample sheet of LF750DD/C31 DA provided by Chemtura is used to determine material properties for modeling. The geometric properties of this rectangular coupon are given in Table 5. A photograph of the three different composite specimen types and sizes is shown in Figure 16.

Table 4: Geometric properties of the 90° flat plate coupons used in dynamic testing

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width (mm)</td>
<td>7.07</td>
</tr>
<tr>
<td>Thickness (mm)</td>
<td>1.19</td>
</tr>
<tr>
<td>Length (mm)</td>
<td>30.0</td>
</tr>
<tr>
<td>$V_f$</td>
<td>0.61</td>
</tr>
</tbody>
</table>

Table 5: Geometric properties of neat resin testing coupon

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width (cm)</td>
<td>1.346</td>
</tr>
<tr>
<td>Thickness (cm)</td>
<td>0.305</td>
</tr>
<tr>
<td>Length (cm)</td>
<td>12.7</td>
</tr>
</tbody>
</table>
Figure 16: Composite test specimens left to right; 19.84 mm tube, 9.91 mm tube, 30 mm plate
Chapter 4  Ballistic Tolerance and Determination of Quasi-Static Properties

Several test methods and test results are described in this chapter. The ballistic impact testing of FMC and RMC materials is discussed first. The stress-strain behavior of damaged and undamaged tubes is discussed next. This behavior provides a means to quantify ballistic tolerance. Also, analysis of the quasi-static stress-strain behavior of undamaged flexible carbon/polyurethane composites yields ply elastic properties that characterize the composite material. Dynamic material properties are presented in Chapter 5.

4.1 Ballistic Impact Testing

FMC and RMC tubes of the 20 mm inner diameter type were used to characterize the ballistic tolerance of flexible and rigid matrix composites. The composite tubes were impacted with a steel ball bearing fired from a pressurized air gun. In this section, the impact testing set-up and method are described first, followed by a discussion about the behavior of the projectile and the tube before, during, and after impact as well as damage evaluation. After the specimens were impacted, they were tested in four different tests: quasi-static tension, compression and torsion as well as dynamic spin testing. These tests and their results with the damaged composite tubes are described in subsequent sections.
4.1.1 Testing Set-Up

Ballistic impact testing was performed in the ballistic lab of the NASA Glenn Research Center (GRC) in Cleveland, OH. Composite tubes of both resin types (polyurethane and epoxy) were used so that the ballistic tolerance of FMC and RMC materials could be compared. Only tubes with ±45° and ±60° fiber winding angles were used in impact testing. The quantity and length (corresponding to a planned subsequent testing method) of each type of specimen tested in ballistic impact is summarized in Table 6. Tubes 45.7 cm in length were only made with the carbon/polyurethane material because spin testing in the current rig with RMC tubes, especially under large misalignment strains, has proven difficult or impossible (Shan, 2006).

<table>
<thead>
<tr>
<th>Material</th>
<th>Fiber Angle</th>
<th>Tension †</th>
<th>Compression †</th>
<th>Torsion ‡</th>
<th>Spin §</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon/Epoxy</td>
<td>±45°</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>±60°</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>-</td>
</tr>
<tr>
<td>Carbon/Polyurethane</td>
<td>±45°</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>±60°</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

† Length of 7.62 cm  
‡ Length of 12.7 cm  
§ Length of 45.7 cm

Specimens were shot using the Large Vacuum Gun (LVG) in GRC’s Ballistic Lab (Melis et al., 2005). A photo of the LVG is shown in Figure 17. In order to execute a shot, the pressure vessel is filled with Helium gas while the chamber and barrel are evacuated of air. The pressure vessel and the chamber are separated by a thin Mylar film that acts as a seal. Whenever the projectile is ready to be shot, a trigger wire melts the Mylar sheet and releases the high pressure air. Since the projectile, a 4.763 mm steel ball bearing, is smaller than the 5.08 cm bore diameter of the barrel, a polycarbonate sabot is used to fire the projectile. The sabot is fitted with two rubber o-rings that are coated with
vacuum grease to ensure a seal in the barrel. Once the projectile and the sabot enter the evacuated chamber, the sabot is stopped and the projectile is released from it. The size of the projectile was chosen such that the ratio of the projectile’s diameter to the diameter of the test coupon is roughly 0.22.

![Large Vacuum Gun (LVG) in the NASA GRC ballistic lab](image)

Figure 17: Large Vacuum Gun (LVG) in the NASA GRC ballistic lab

A photograph of the interior of the chamber is shown in Figure 18. The specimen in Figure 18 has a length of 12.7 cm from end-to-end. The barrel and sabot stopper are shown in the left-hand portion of the figure. Two solid steel cylindrical plugs were inserted into the ends of the hollow composite specimen about 3.3 cm on each side, and the tube was gripped by rubber hose clamps during the test. The steel plugs were attached to a rigid fixture in the center of the chamber. The fixture and tube were aligned so that the projectile would impact the center of the tube. The pressure of the vessel (about 1 MPa) was chosen so that velocity of the projectile (typically around 308 m/s) will be high enough to penetrate both walls of the tube. A block of clay is used at the end of the chamber to capture the projectile after impact. High-speed cameras with a frame rate of
40,000 frames per second were used to film the impact so that observations and measurements could be made before, during, and after the impact.

![Figure 18: Chamber of the LVG](image)

### 4.1.2 Damage Evaluation

To begin with, during the impact, it was observed that the circumferential region around the impact site in FMC tubes experiences a large amount of swelling or ovalization of the cross-section. This is most easily seen in the slow-motion video of the impact (Appendices D.1 and D.2), but two still photographs were sampled at the peak and valley of the cross-section disturbance for the sake of comparison here. These photographs are shown in Figure 19 with a gage bar illustrating the peak expansion (70 µs after impact) and the peak contraction (210 µs after impact) of the tube. The projectile takes about 100 µs to pass through both walls of the tube.
Swelling or ovalization deformation of the RMC tubes under impact is not noticed in the slow-motion videos (Appendix D.3). This difference could suggest that the FMC absorbs more energy during impact, although drawing a definite conclusion from deformation information alone is difficult because the tubes have different stiffnesses. A quantifiable calculation can be made to determine just how much energy is absorbed in the FMC and RMC tubes during impact. This calculation is made by measuring the velocities of the projectiles before and after they enter the FMC and RMC tubes. The measurement is done by counting the number of pixels the projectile travels in each subsequent image captured by the camera and comparing it to the frame rate of the camera. Pixels are converted into a distance via a calibration image containing a rod with a known length. For this calculation, the interval between each image was 25 μs and the exposure time was 1 μs. Energy is calculated by Equation 23 where \( m \) is the mass of the projectile (0.444 g) and \( v \) is the velocity of the projectile. The energy absorbed in both tube types is shown in Table 7. The velocity measurements were made for ±45°, 7.62 cm long tubes of both resin types (one data point each).
\[ E = \frac{1}{2} mv^2 \]

Table 7: Pre and post impact velocities, energies

<table>
<thead>
<tr>
<th>Tube Type</th>
<th>Velocity (m/s)</th>
<th>Energy (J)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Rigid</td>
<td>Flexible</td>
</tr>
<tr>
<td>Pre</td>
<td>304</td>
<td>304</td>
</tr>
<tr>
<td>Post</td>
<td>228</td>
<td>152</td>
</tr>
<tr>
<td>Absorbed</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

The amount of energy absorbed is clearly higher in the case of the flexible tubes. As discussed, the swelling or ovalization of the tubes after impact is more easily seen in the videos. However, a sequence of multiple still photographs provides some additional information about the impact event. In Figure 20 and in Figure 21 these sequential sets are shown for an FMC and an RMC tube, respectively. The images start just prior to the projectile entering the first wall of the tube and end just after the projectile has exited the second wall of the tube. They are separated by 25 μs intervals, corresponding to the frame rate of the camera. More debris is created by impacting the rigid tubes.

Figure 20: Sequential set of stills, FMC tube during impact, 25 μs intervals
After the impact event, it is natural to remove the composite tubes and examine the outer and inner surfaces after they have been shot. On the outer surface of the tubes there are visible (although sometimes barely visible in the FMC case) openings at the entrance region of the first wall and at the exit region of the second wall (Figure 22 and Figure 23). In general, on the surface, the holes in the tubes created by the projectile are more clearly defined in RMCs. On the inner surface of the tubes, the exit region of the first wall appears to have a significant amount of damage (more so in FMCs). This kind of “bird’s nest” damage region on the inner surface of the first (convex) wall surrounds the actual hole made by the projectile. It is attributed to delamination and fiber pull-out. Less damage of this type was seen on the outer surface of the second (concave) wall. The white numbering on the surface of the tubes in Figure 22 and in Figure 23 is an irrelevant numbering scheme used to keep track of how many shots the vacuum gun has made.
In addition to observing surface damage, an invasive approach to detect damage was explored. Two FMC composite tubes (one ±45° tube and one ±60°) were used for X-radiography. After ballistic testing, a photopenetrant is applied around the damaged entrance and exit regions of the tube. The penetrant solution is a dense, low viscosity fluid that contains 60 g of zinc iodide, 10 mL of water, 10 mL of isopropyl alcohol, and 10 mL of Kodak’s “Photo-Flo 200”. The purpose of the penetrant is to migrate through the cracks and delaminations of the composite. The dense solution then shows up during
X-radiography where it penetrated the composite. X-radiography pictures were taken with an ICM X-ray emitter that was shot at the specimen with a phosphorous film that was positioned behind the specimen. The scanner, SCANX12 – Portable, the emitter and film were made by All Pro Imaging and Air Techniques (Melville, NY). The emitter was placed 38 inches from the detector in order to achieve the best resolution possible. The emitter was set at 40 kV and the exposure time was anywhere from 2.5 – 3.5 seconds. The results of the X-radiography are shown in Figure 24 and in Figure 25. The hazy lines in the photographs are micro cracks along the fiber winding angle, whereas the dark shaded areas are delaminations. Although only one FMC tube of each angle type was used in X-radiography, it would seem that the region around the impact sites in FMC tubes develops a substantial amount of matrix cracking all the way around the circumference. X-radiography was not performed on RMC tubes.

Figure 24: X-radiography image of a flexible ±45° tube after ballistic impact event
4.1.2.1 Spin Testing Damaged FMC Tubes

The details of the PSU spin rig and spin testing are described in detail in Section 6.2, but the results from spin testing a damaged FMC tube are given here for the sake of thesis structure. Only FMC tubes were tested in the spin rig because the robustness of the rig will not accommodate RMC tubes (Shan, 2006). Although the tubes see no torque in this test, the results will shed some light on how damaged shafts might perform in cyclic fatigue. The positioning of the bearings on the rig is such that they are meant to impose a constant strain along the length of the tube. For these tests, ±45° and ±60° FMC tubes with a length of 45.7 cm were used as described in Table 6. The lowest nominal strain setting on the rig, 0.25%, was used and the rotational speed of 1200 rpm was selected. A 2.54 cm clip gage extensometer is used to measure the actual strain at different positions along the length of the tube. This procedure is also described in more detail in Section 6.2.1. In the case of spin testing damaged tubes, measuring the actual strain along the length of the tube is particularly important because the damage region may be more compliant than the rest of the tube. Hence, the misalignment from tip to end may be
concentrated in the damaged region while the rest of the tube sees less strain. Measured strains on the tensile and compressive sides of the tube are shown in Figure 26. At the beginning of the spin test, the mid span position at the site of the impact damage sees about 0.26% strain while the other positions see about 0.20% strain.

![Figure 26: Strain along damaged ±45° flexible tube](image)

After the strain is measured and recorded, the motor is turned on and the temperatures at different positions along the length are recorded by optical thermocouples. For this test, the temperatures at the middle of the tube and at the quarter span position closest to the motor were recorded. The temperature and speed histories versus time are shown in Figure 27. At first, the temperature at the mid span position of the tube seems like it may reach a steady state condition. However, at around 400 seconds the tube reaches an unstable condition. The tube heats more at this location because initially there is more strain at the mid span. The heating causes the material to be more compliant, which in turn feeds more strain into that region. The result is a compounding effect that becomes unstable. The peak in the temperature at the mid span location corresponds to the point during the test where the tube essentially kinks. When
this happens, all of the misalignment from tip to end is concentrated in the kink.

Consequently, the tube cools at midspan and at quarterspan because the material in the tube is no longer experiencing cyclic strain. The tube is for all intensive purposes broken. It would no longer be able to carry a torsional load.

![Graph showing temperature and speed over time](image)

**Figure 27: Heating of damaged ±45° flexible tube**

After the misaligned spin test, the photopenetrant solution was again applied to the failed specimen so that another round of X-radiography photographs could be taken to compare the damage from the impact and the damage from the spin test after impact. This comparison is shown in Figure 28. Clearly, there is much more damage present after the misaligned spin test. All of the regions in the tube that were previously filled with matrix cracking are now filled with large delaminations. Similar undamaged tubes were tested (mentioned in Section 6.2) in the spin rig under the same conditions. These tubes survived spin tests at twice the nominal strain and twice the speed without failing. The damage in the flexible tubes due to the ballistic impact event seems to have a noteworthy effect for self-heating and cyclic fatigue. The projectile causes damage which reduces the stiffness of the tube in that localized region. Due to the reduced stiffness, more bending
strain is concentrated in that region resulting in more heating. Higher temperatures also reduce stiffness, and a compounding effect seems to take place.

**Figure 28**: X-radiography before (left) and after (right) misaligned spin test, ±45° FMC

**Figure 29**: X-radiography before (left) and after (right) misaligned spin test, ±60° FMC

### 4.2 Quasi-Static Testing

Axial tension and compression tests were carried out at two different places on two different machines. The first tension and compression tubular specimens of each type listed in Table 6 along with their undamaged counterpart were tested in the fatigue/fracture lab, Building 49, at the NASA Glenn Research Center. The test machine at GRC is a hydraulic axial-torsion machine. The remaining tension and compression tests for tubular specimens were performed at Penn State on a 267 kN screw-driven test
machine. All torsion testing was performed at Penn State on a hydraulic axial-torsion machine. Quasi-static tension testing of the neat resin material was also performed on the screw-driven test machine at Penn State.

4.2.1 Tension and Compression

At NASA GRC, load data were recorded by the load cell (22.2 kN range) in the testing machine, and strain was recorded by two pairs of cameras (two viewing the front of the specimen and two viewing the back). These cameras take successive sets of pictures during the test that are sent to a software program after the test where they are compared to one another to make full-field strain measurements on both sides of the specimen. This technique is called digital image correlation and in this case is performed by an optical deformation analysis software package called ARAMIS made by GOM International AG (Widen, Switzerland). The software can also pick two points in the field of view and measure the relative displacement between them, mimicking a strain measurement by a clip gage. For these tests longitudinal (2.54 cm) and transverse (1.27 cm) virtual clip gages were used. For the case where tubes with ballistic damage were being tested, the damage regions were aligned with the front (entrance side) and back (exit side) cameras and the virtual clip gage spanned the damage region. Since the surface of the composite is black and reflective, ultra flat white and black spray paints are used to create a speckle pattern on the surface of the composite. This pattern and correct lighting of the specimen allows for more accurate measurements by the ARAMIS software when comparing two images.

At Penn State, load data were recorded by the load cell (267 kN range) while strain data for tubular specimens was recorded by two 2.54 cm clip gage extensometers
located 180° from each other. Longitudinal strain data for the neat resin coupon was recorded by a laser extensometer whereas transverse strain data was recorded by a 1.27 cm clip gage extensometer. For the case where tubes with ballistic damage were being tested, an extensometer spanned the holes made by the projectile on either side. Voltage outputs from the extensometers and load cell were recorded by a data acquisition computer running a LabVIEW program.

An example of the gripping set-up used in tension testing of tubes is shown in Figure 30. This photograph shows a composite tube, with the black and white speckle pattern, bonded into a set of steel swivel grips that are held by the hydraulic-powered grips on the testing machine at GRC. The tension testing set-up for tubes would appear much the same at Penn State, except the testing machine grips at Penn State are mechanical wedge grips. For tension testing of tubes, the specimens were first prepared by wiping down the inside and outside of the tube with a cloth wet with acetone. This removes any dust and silicon release that is left over from fabrication. The tube specimens were then bonded into a set of grips using a bonding adhesive. The adhesive used with the carbon/epoxy system is called Loctite Weld made by Henkel Corporation (Avon, OH). The adhesive used with the carbon/polyurethane system is called Devcon Plastic Welder™ made by ITW Devco (Danvers, MA). The swivel grips are further illustrated in Figure 31. They can be taken apart and have a set of swivel bearings to eliminate spurious moments during testing. After the binding adhesive is applied, the specimen and the grips are placed in an oven at 60°C for at least 6 hours to allow the adhesive to cure. When the test is complete, the specimen is cut out of the grips, and the steel inner and outer sleeves are placed in a burnout oven at 510°C for approximately 30 minutes. Once the sleeves have cooled down to room temperature, they are taken apart,
sanded down with grit cloth, and wiped clean with a cloth wetted with acetone. They are then re-assembled for future testing. The rectangular coupon of neat resin is simply gripped by mechanical wedge grips in tension testing.

**Figure 30:** Quasi-static tension testing; gripping set-up and tubular specimen with speckle pattern

**Figure 31:** Schematic of grip used in quasi-static tension testing (Shan, 2006)
An example of the gripping set-up used in compression testing is shown in Figure 32. This photograph shows a composite tube potted into grooves in a pair of disk-shaped steel end fixtures. These end fixtures serve as platens that are compressed by the load frame of the testing machine at Penn State. The hemispherical joint shown at the bottom of the photograph was used to eliminate the possibility of moments being introduced due to misalignment of the load train. The compression testing set-up would appear much the same at GRC. For compression testing, the disk-shaped steel end fixtures are heated to about 80°C on a hot plate. As the fixtures heat, a fusible alloy contained within the grooves of the fixtures melts. The commercial name of the alloy in this case is Cerrobend. Melting of the alloy allows the composite tubes to fit into the grooves of the end fixtures. Once the tube was fitted into the groove, the fixture and the tube were removed from the hot plate and allowed to cool. When the alloy cools, it also expands which clasps the composite tube tightly and helps prevent end-brooming during compression testing. When a test is complete, the end fixtures are heated again, the alloy melts, the test specimens are removed from the fixtures, and the fixtures are ready to be used again.
The testing set-up for tension testing the neat resin material is shown in Figure 33. A laser extensometer was used to measure longitudinal strain because of the high compliance of the polymer being tested. The laser extensometer works by placing two reflective strips onto the specimen that reflect light back to the extensometer. From the incident angle of the reflected light, the laser extensometer can calculate how far apart the reflective strips are positioned from one another. This initial distance was recorded as the gage length (about 3.8 cm in this case). The sensitivity of the laser was set to the highest with $5 \text{ V} = 0.1 \text{ inches}$. The width of the specimens was selected carefully to accommodate the 1.27 cm clip gage. The clip gage was calibrated and affixed to the specimen with a set of rubber bands. The distance between the mechanical wedge grips was about 8 cm.
For the axial tension and compression tests, stress is calculated by dividing the load by the cross sectional area of the tubes. The geometric properties of each of the tubes are tabulated in Table 2 and in Table 3 and the geometric properties of the neat resin coupon are tabulated in Table 5. For tests where clip gage extensometers were used to measure deformation, strain is calculated by transforming the voltage data into displacement by the equation determined during calibration. For the laser extensometer, the strain is calculated by converting voltage data into displacement by the conversion equation mentioned above. Axial strain rates for tension and compression testing of tubes were typically about 150 $\mu$ε/s, whereas for the neat resin the strain rate was about 220 $\mu$ε/s.

### 4.2.2 Torsion

Torsion tests were carried out at Penn State on a hydraulic axial-torsion load frame. The load frame was set to angle control for the torsional actuator and force control for the axial actuator. The actuator rotates 10 degrees in 10 minutes throughout the length of the test which corresponds to a shear strain rate of about 33 $\mu$γ/s for the 98.8-mm-long gage section of the test specimen. For these pure torsion tests, the axial actuator was set at
a force value of zero. The feedback mechanism for angle control is the rotary variable differential transformer (RVDT). The feedback mechanism for force control is the load cell in the testing machine. Because the angular displacement measured by the RVDT is the displacement of the actuator, some assumptions must be made as to how much angular displacement the composite tube is actually experiencing. The assumptions made here are that the steel end grips (Figure 34) are torsionally rigid and that the tube is rigidly bonded into these grips. That is, the length of the exposed composite tube (9.88 cm) and the angular displacement of the crosshead are used to calculate shear strain.

These assumptions may introduce some inaccuracies. The adhesive material used to bond the tube into the grips certainly has some compliance. The bonding adhesives used in torsion testing are the same that were used in tension testing (mentioned in Section 4.2.1).

After testing, the tubes were burned out of the grips, and the grips were cleaned and prepared for another test in the same manner as for the tension grips. Shear stress, $\tau_{xy}$, at the outer surface of the tube is calculated by

$$\tau_{xy} = \frac{T r_0}{J} = G_{xy} \gamma_{xy} = G_{xy} \frac{\theta r_0}{l}$$

where $T$ is the torque on the tube, $r_0$ is the outermost radius of the tube, and $J$ is the polar moment of inertia for a round tube. For a circular hollow tube, $J$ is given by

$$J = \frac{\pi}{2} (r_o^4 - r_i^4)$$

where $r_i$ is the inner radius of the tube. The calculation for shear strain can also be seen in Equation 24 where $\theta$ is the angle of twist and $l$ is the gage length of the test specimen.
4.3 Quasi-Static Test Results

Stress-strain curves were generated in tension, compression, and torsion testing for damaged and undamaged tubes. Values of Young’s modulus and ultimate strength were determined from these curves and are tabulated here. In a few cases where transverse strains were measured, the Poisson’s ratios were also determined. Generally speaking, the test matrix for damaged specimens follows the outline shown in Table 6, although not all of the tests represented in Table 6 were completed successfully due to some laboratory mishaps. The test matrix for undamaged specimens is shown in Table 8. Observations about how the specimens deformed during testing and their failure modes as well as the results from testing the neat resin in quasi-static tension are covered here.

<table>
<thead>
<tr>
<th>Material</th>
<th>Winding Angle</th>
<th>Tension</th>
<th>Compression</th>
<th>Torsion</th>
<th>Spin Testing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon/Polyurethane</td>
<td>±20°, ±30°, ±45°, ±60°, ±83.2</td>
<td>3</td>
<td>3</td>
<td>1^</td>
<td>1^</td>
</tr>
<tr>
<td>Carbon/Epoxy</td>
<td>±45°, ±60°</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>-</td>
</tr>
</tbody>
</table>

1±45° and ±60° tubes only
2±30°, ±45°, and ±60° tubes

For testing performed at GRC, the information from the longitudinal and transverse virtual clip gages generated stress-strain curves and plots of negative transverse strain versus longitudinal strain. A snapshot during one of these tests is shown
in Figure 35 with the color mapping scheme (generated after testing by the software program) overlaid onto an image of the specimen. The strain in the longitudinal loading direction of the tube (vertical in this picture) is represented by $\varepsilon_x$ and the red crosses indicate the end points of the virtual clip gages. A video of a damaged ±45° FMC tube in tension is shown in Appendix D.4. For testing performed at Penn State, only longitudinal mechanical clip gages were used, so only stress-strain curves were generated. The stress-strain curves from the front and back of the tubes were averaged in case there was any bending of the tube during the test. A sample stress-strain curve for one of the specimens tested at GRC is shown in Figure 36.

![Figure 35: Snapshot, a rigid undamaged ±45° tube in tension](image)
Figure 36: Representative average tensile stress-strain data generated by a virtual 2.54 cm clip gage – rigid, undamaged, ±45° tube

A sample plot of negative transverse strain versus longitudinal strain and a sample measurement (typically in the longitudinal strain range of about 1%) of the Poisson’s ratio of one of the composite tubes are shown in Figure 37. Longitudinal and transverse strain measurements from the front and back of the tube were averaged and a straight line was fit to the plot to determine the structural Poisson’s ratio, $\nu_{xy}$. All of the Poisson’s ratio measurements are summarized later in this section.

Figure 37: Representative measurement of Poisson’s ratio – flexible, undamaged, ±45° tube in tension
A comparison of longitudinal stress-strain curves from each testing set-up, for flexible, undamaged ±45° tubes tested in tension, is shown in Figure 38. This graph shows the strain region where the modulus measurement is made. The modulus is taken as a secant modulus, calculated in the range of 0 – 600 με for all tests ($\varepsilon_x$ in tension and compression and $\gamma_{xy}$ in torsion). A linear fit to the data in this region is used to make the measurement because of data noise issues. Although this seems to be appropriate because typical $R^2$ values for the linear fit are higher than 0.95 for tests run at GRC and Penn State. Also, beyond this strain range the elastic response becomes noticeably nonlinear, especially for the axial tension and compression tests. For example, from the data shown from the DIC in Figure 38, the modulus measured in the 0 – 600 με range is 4.09 GPa while the modulus measured in the 0 – 3000 με range is 2.55 GPa – almost a 38% reduction. Operational strain ranges for $\varepsilon_x$ are expected to be less than 2000 με. All of the modulus and ultimate strength values are summarized later in this section.

Figure 38: Strain measurement comparison, DIC (GRC) and clip gage (Penn State), a flexible undamaged ±45° tube in tension
In tension and compression testing, the full-field strain measurement at GRC allows for observations to be made about deformation patterns that arise on the entire surface of the tube during loading. This information is not captured with a simple clip gage, and it provides insight into the evolution of damage in impacted and non-impacted composite tubes. Due to the size of the field of view of the cameras, the entire specimen could not be photographed during testing. The area of the full-field strain measurement and the positioning of the clip gage for a damaged tube in tension are shown in Figure 39. The configuration for tubes in compression and undamaged tubes is very similar. Sequential photographs with longitudinal strain overlay of tubes being loaded are shown in Figures 40 – 42. These photographs simply show the development of damage during loading and are not sampled at similar rates from one test to another. These images show the front side (projectile entrance) of the tubes. In both FMC and RMC tubes regions of high strain seem to accumulate in patterns that resemble the tow pattern in which the tube was fabricated. The regions of highest strain in the undamaged tubes appear to follow the last or next to last tow that was wound onto the part. Also, in undamaged tubes strain is distributed throughout the entire tube whereas in damaged tubes material deformation seems to be concentrated in the damage region. Because of this localized material deformation, the modulus measurements mentioned later in this section are really a smeared stiffness over the gage length of the extensometer. Modeling reductions in stiffness and strength may require a more in-depth analysis. When comparing the tensile behavior of the RMC tubes and FMC tubes, it appears that the damage region in the RMC is fairly localized and in the FMC it is more spread out. Similar material deformation and failure behavior is observed in tension and compression. Failure in the form of matrix cracking and fiber shearing occurs in a spiral pattern oriented along the
fiber winding angle. For damaged tubes, this spiral pattern intersects the boundary of the hole made by the projectile.

Figure 39: Schematic of full-field strain measurement and slip gage positioning for a tube in tension

Figure 40: Snapshots of longitudinal strain in rigid ±45° undamaged (top) and damaged (bottom) tubes tested in tension with stress-strain curves from digital clip gage
Figure 41: Snapshots of longitudinal strain in flexible ±45° undamaged (top) and damaged (bottom) tubes tested in tension with stress-strain curves from digital clip gage

Figure 42: Snapshots of longitudinal strain in flexible ±45° undamaged (top) and damaged (bottom) tubes in compression with stress-strain curves from digital clip gage

Figure 43 is provided in order to further illustrate the difference in the damage regions of FMC and RMC tubes. All of the tubes in Figure 43 are experiencing the same remote axial strain, 2000 με, which is a typical maximum value that an FMC tube might
experience in service. The damage in the FMC tube seems to be more spread out while the damage in the rigid tube appears to be more localized. This is consistent with the more extensive “bird’s nest” damage seen on the inner surface of the FMC tubes in comparison to the RMC tubes.

Figure 43: Full-field strain measurement under tensile loading, FMC and RMC, all ±45° front side

Failure in torsion testing occurred in one of two ways. The failure region during loading was either concentrated at the site where the impact was made (a crushing mode) or matrix cracking and fiber shearing occurred in a spiral pattern much the same way as it did in tension and compression testing. The crushing mode was only seen in damaged tubes, although not all damaged tubes saw this mode of failure. Some damaged tubes failed with the spiral pattern intersecting the hole from the projectile at its boundary. These two types of failure modes are shown in Figure 44.
Figure 44: Failure modes in torsion: crushing in a ±45° FMC (left) and spiral pattern in a ±60° (right)

All modulus and ultimate strength measurements in tension, compression and torsion of filament-wound carbon/polyurethane and carbon/epoxy composite tubes with varying fiber winding angles are shown in Tables 9 – 14 where $C_v$ is the coefficient of variation. For the most part coefficients of variation were less than 15% for undamaged tubes and considerably larger for damaged tubes. Results matched well for tension and compression testing across testing platforms.

Table 9: Quasi-static moduli in axial tension

<table>
<thead>
<tr>
<th>Fiber Angle</th>
<th>FMC</th>
<th>RMC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus (GPa)</td>
<td>Average Modulus (GPa)</td>
<td>$C_v$ (%)</td>
</tr>
<tr>
<td>±20°</td>
<td>31.1</td>
<td>32.2</td>
</tr>
<tr>
<td>±30°</td>
<td>11.6</td>
<td>10.7</td>
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<td>±45°</td>
<td>4.09</td>
<td>3.90</td>
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<td>±60°</td>
<td>1.86</td>
<td>2.46</td>
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<tr>
<td>±83.2°</td>
<td>1.45</td>
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<td>±45°</td>
<td>2.95</td>
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</tr>
<tr>
<td>±60°</td>
<td>2.03</td>
<td>-</td>
</tr>
</tbody>
</table>

1 Tested at NASA GRC
2 Data is suspect – order of magnitude differences in strain measurement on front and back of specimen
### Table 10: Quasi-static moduli in axial compression

<table>
<thead>
<tr>
<th>Fiber Angle</th>
<th>Modulus (GPa)</th>
<th>Average Modulus (GPa)</th>
<th>$C_v$ (%)</th>
<th>Modulus (GPa)</th>
<th>Average Modulus (GPa)</th>
<th>$C_v$ (%)</th>
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</thead>
<tbody>
<tr>
<td>Undamaged</td>
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<tr>
<td>$\pm 20^\circ$</td>
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<tr>
<td></td>
<td>27.3</td>
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<td>-</td>
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<tr>
<td></td>
<td>26.4</td>
<td>-</td>
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<td>$\pm 30^\circ$</td>
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<td></td>
<td>9.60</td>
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<td>$\pm 45^\circ$</td>
<td>4.28</td>
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<td>$\pm 60^\circ$</td>
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<td>$\pm 83.2^\circ$</td>
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<td></td>
<td>1.49</td>
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<td>$\pm 45^\circ$</td>
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<td>2.73</td>
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<td>3.81</td>
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<td>$\pm 60^\circ$</td>
<td>1.08</td>
<td>1.20</td>
<td>17</td>
<td>4.37</td>
<td>6.45</td>
<td>5.84</td>
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</table>

* Tested at NASA GRC

### Table 11: Quasi-static moduli in torsion

<table>
<thead>
<tr>
<th>Fiber Angle</th>
<th>Shear Modulus (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FMC</td>
</tr>
<tr>
<td>Undamaged</td>
<td>$\pm 45^\circ$</td>
</tr>
<tr>
<td></td>
<td>$\pm 60^\circ$</td>
</tr>
<tr>
<td>Damaged</td>
<td>$\pm 45^\circ$</td>
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<tr>
<td></td>
<td>$\pm 60^\circ$</td>
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### Table 12: Quasi-static ultimate strength in axial tension

<table>
<thead>
<tr>
<th>Fiber Angle</th>
<th>Strength (MPa)</th>
<th>Average Strength (MPa)</th>
<th>(C_v) (%)</th>
<th>Strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Undamaged</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>±20°</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>±30°</td>
<td>70.1</td>
<td>70.6</td>
<td>5</td>
<td>-</td>
</tr>
<tr>
<td>±45°</td>
<td>29.7</td>
<td>29.6</td>
<td>1</td>
<td>137.2(^\dagger)</td>
</tr>
<tr>
<td>±60°</td>
<td>21.5(^\dagger)</td>
<td>24.9</td>
<td>13</td>
<td>64.7(^\dagger)</td>
</tr>
<tr>
<td>±83.2°</td>
<td>10.0</td>
<td>10.1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>±45°</td>
<td>15.3(^\dagger)</td>
<td>-</td>
<td>-</td>
<td>87.1(^\dagger)</td>
</tr>
<tr>
<td>±60°</td>
<td>11.4(^\dagger)</td>
<td>-</td>
<td>-</td>
<td>36.3(^\dagger)</td>
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<tr>
<td>Damaged</td>
<td></td>
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</tr>
</tbody>
</table>

\(^\dagger\)Tested at NASA GRC

### Table 13: Quasi-static ultimate strength in axial compression

<table>
<thead>
<tr>
<th>Fiber Angle</th>
<th>Strength (MPa)</th>
<th>Average Strength (MPa)</th>
<th>(C_v) (%)</th>
<th>Strength (MPa)</th>
<th>Average Strength (MPa)</th>
<th>(C_v) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Undamaged</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>±20°</td>
<td>82.1</td>
<td>76.5</td>
<td>7</td>
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<td>-</td>
<td>-</td>
</tr>
<tr>
<td>±30°</td>
<td>50.7</td>
<td>50.1</td>
<td>4</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>±45°</td>
<td>32.8(^\dagger)</td>
<td>36.9</td>
<td>11</td>
<td>134(^\dagger)</td>
<td>145</td>
<td>139</td>
</tr>
<tr>
<td>±60°</td>
<td>32.2(^\dagger)</td>
<td>36.2</td>
<td>10</td>
<td>145(^\dagger)</td>
<td>146</td>
<td>145</td>
</tr>
<tr>
<td>±83.2°</td>
<td>30.8</td>
<td>27.5</td>
<td>23</td>
<td>-</td>
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<td>-</td>
</tr>
<tr>
<td>±45°</td>
<td>17.6(^\dagger)</td>
<td>20.6</td>
<td>20</td>
<td>85.1(^\dagger)</td>
<td>82.2</td>
<td>83.7</td>
</tr>
<tr>
<td>±60°</td>
<td>13.9(^\dagger)</td>
<td>15.4</td>
<td>10</td>
<td>42.8(^\dagger)</td>
<td>51.1</td>
<td>52.9</td>
</tr>
<tr>
<td>Damaged</td>
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<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

\(^\dagger\)Tested at NASA GRC
Table 14: Quasi-static strength in torsion for ballistic tolerance comparison

<table>
<thead>
<tr>
<th>Fiber Angle</th>
<th>Torsional Strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FMC</td>
</tr>
<tr>
<td>Undamaged</td>
<td>±45°</td>
</tr>
<tr>
<td></td>
<td>±60°</td>
</tr>
<tr>
<td>Damaged</td>
<td>±45°</td>
</tr>
<tr>
<td></td>
<td>±60°</td>
</tr>
</tbody>
</table>

The measurements of the Poisson’s ratios, $\nu_{xy}$, of the composite tubes that were obtained are shown in Table 15. Poisson’s ratio measurements were only made for undamaged tubes.

Table 15: Poisson’s ratios of the composite tubes tested at Glenn

<table>
<thead>
<tr>
<th>Fiber Angle</th>
<th>Tension</th>
<th>Compression</th>
<th>Tension</th>
<th>Compression</th>
</tr>
</thead>
<tbody>
<tr>
<td>±45°</td>
<td>0.88</td>
<td>1.00</td>
<td>0.62</td>
<td>†</td>
</tr>
<tr>
<td>±60°</td>
<td>0.60</td>
<td>0.50</td>
<td>†</td>
<td>0.31</td>
</tr>
</tbody>
</table>

†Bad transverse strain measurement

A graph of the transverse and longitudinal strains used to make the Poisson’s ratio measurement of the neat resin is shown in Figure 45. The Poisson’s ratio for the LF750DD/C31 DA neat resin is 0.38. The secant modulus ($0 – 1000 \mu e$) is 310 MPa.

Figure 45: Poisson's ratio measurement for LF750DD/C31 DA neat resin
In order to illustrate the differences in the moduli and strengths of damaged and undamaged FMC and RMC tubes, normalized bar charts are shown in Figures 46 – 49. A knockdown is the amount of reduction in a material property (modulus or strength) due to ballistic impact. For knockdowns in modulus in tension and compression, considering certain fiber angles, the FMC may perform better than the RMC or vice versa. But for the most part, there is no significant difference between the knockdowns in moduli for FMC and RMC materials in tension and compression. In torsion, however, the knockdowns in modulus for the FMC case (~50%) seem to be clearly higher than the RMC case (~20%). For knockdowns in strength in tension and compression, the findings are very similar to knockdowns in modulus. While considering certain fiber angles, the FMC may perform better than the RMC or vice versa. But for the most part, there is no significant difference between the knockdowns in strength for FMC and RMC materials in tension and compression. In torsion, however, the knockdowns in strength for the FMC case are almost 90% while the knockdowns in strength for the RMC case are about 55%.

*Data is suspect – strain measurement discrepancy
Figure 47: Moduli before and after ballistic damage, RMC

Figure 48: Ultimate strengths before and after ballistic damage, FMC
Figure 49: Ultimate strengths before and after ballistic damage, RMC

4.5 Determination of Ply Elastic Properties

The determination of the material properties for the LF750D composite is a research objective and is made here. Ply elastic properties were only determined for the flexible matrix composite material. Each ply in a laminated composite can be modeled as an orthotropic material. These orthotropic materials have four independent material properties that fully characterize the ply, namely $E_1$, $E_2$, $v_{12}$, and $G_{12}$. The $1$-$2$ coordinate system is defined in Figure 50 which shows a section of a ply where the $1$-direction is coincident with the fiber direction. The $x$-$y$ coordinate system is the structural coordinate system of the composite part with the $x$-axis the direction of applied loading. When these plies are stacked up into a laminate, the four ply elastic properties are then able to predict the structural response of the laminate through classical laminated plate theory (CLPT – Daniel and Ishai, 2006).
Material properties of the carbon/polyurethane composite were obtained through a few different ways. The major in-plane Poisson’s ratio was determined by rule of mixtures. That is,

$$\nu_{12} = V_f \nu_f + (1 - V_f) \nu_m$$  \hspace{1cm} (26)$$

where $V_f$ is the fiber volume fraction (assumed to be 0.59 – an average from the values of the 20 mm tubes in Table 3), $\nu_f$ is the Poisson’s ratio of the fiber (assumed to be 0.28 – Krucinska and Stypka, 1991), and $\nu_m$ is the Poisson’s ratio of the polyurethane matrix material, measured as 0.38. The value of $\nu_{12}$ for the FMC was determined to be 0.33. The minor Young’s modulus, $E_2$, was determined by testing undamaged hoopwound tubes in tension and compression. Although the actual winding angle for these tubes is 83.2°, the moduli measured from these tests are assumed to be $E_2$. According to classical laminated plate theory (using the values in Table 16) the difference between the axial modulus for an 83.2° tube and a 90° tube is less than 1.25%. The values of the minor Young’s modulus in tension and compression were determined to be 1.71 GPa and 1.57 GPa.
respectively. The in-plane shear modulus was determined by the [±45]_{ns} coupon test (Daniel and Ishai, 2006). In this case, the coupon was an undamaged angle-ply tube of the 20 mm inner diameter type. Longitudinal and transverse strains were measured in these kinds of tests (using the testing machine at NASA GRC), and were transformed into shear strains in the principal coordinate system. Longitudinal stress was also transformed into shear stress in the principal coordinate system. If in the linear range of stress and strain, the in-plane shear modulus can be calculated by

\[ G_{12} = \frac{\bar{\tau}_{12}}{\bar{\gamma}_{12}} = \frac{\bar{\sigma}_x}{2(\bar{\varepsilon}_x - \bar{\varepsilon}_y)} = \frac{E_x}{2(1 + \nu_{xy})}, \]  

(27)

where \( \bar{\tau}_{12} \) is the in-plane shear stress, \( \bar{\gamma}_{12} \) is the in-plane shear strain, \( \bar{\sigma}_x \) is the longitudinal stress, \( \bar{\varepsilon}_x \) is the longitudinal strain, \( \bar{\varepsilon}_y \) is the transverse strain, \( \bar{E}_x \) is the longitudinal or axial modulus, \( \nu_{xy} \) is the Poisson’s ratio obtained from the initial slopes of the curves of the ±45° tube (Daniel and Ishai, 2006). The longitudinal moduli and Poisson’s ratios are used to calculate \( G_{12} \). These values and their respective strain ranges of calculation are given in Section 4.3. The tension and compression tests at GRC yield values of 1.09 and 1.07 GPa for the in-plane shear moduli of the FMC, respectively.

The determination of the major Young’s modulus is not a trivial task. First, the rule of mixtures was used to calculate \( E_1 \) but this value appeared to be much too large when comparing CLPT theory (with the three previously mentioned ply elastic properties) and experimental data on the \( E_x \) versus fiber winding angle (\( \theta \)) curve – Figure 51. So, since the value of \( E_1 \) has only a major influence on lower fiber angles, it was adjusted manually so that the curve generated by CLPT would fit best to the tensile – CLPT (T), compressive – CLPT (C) and average of tensile and compressive moduli –
CLPT (Avg) for the ±20° tube. These three curves, generated by three values of \( E_1 \), are shown with experimental data with scatter bars in Figure 51. An expanded view of the same graph for higher fiber angles is shown in Figure 52. The ply elastic material properties used along with CLPT to generate curves are tabulated in Table 16.

![Figure 51: FMC \( E_x \) vs. \( \theta \), experiments and theory, CLPT (T, C, and Avg)](image1)

![Figure 52: FMC \( E_x \) vs. \( \theta \), experiments and theory, CLPT (T, C, and Avg), reduced range](image2)
Table 16: Ply elastic material properties for fitting routine to tension and compression testing

<table>
<thead>
<tr>
<th>Parameter</th>
<th>CLPT (T)</th>
<th>CLPT (C)</th>
<th>CLPT (Avg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_1$ (GPa)</td>
<td>69.0</td>
<td>43.0</td>
<td>51.0</td>
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<tr>
<td>$E_2$ (GPa)</td>
<td>1.71</td>
<td>1.57</td>
<td>1.64</td>
</tr>
<tr>
<td>$G_{12}$ (GPa)</td>
<td>1.09</td>
<td>1.07</td>
<td>1.09</td>
</tr>
<tr>
<td>$\nu_{12}$</td>
<td>0.33</td>
<td>0.33</td>
<td>0.33</td>
</tr>
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</table>

The fitting line, CLPT (Avg), in Figure 51 and in Figure 52 fits well to within less than 3% for all tube angles except for ±30° where it over predicts by 24% and for ±60° where it under predicts by 13%. An interesting observation to make is on how well these ply elastic properties predict the structural shear modulus, $G_{xy}$, of the undamaged 20 mm ±45° tubes in torsion testing. As it turns out, the value of $E_1$ in Table 16 is much too low to accurately predict $G_{xy}$ of the ±45° tube. So, another method was used to choose appropriate ply elastic properties that would predict shear moduli well. The values of $E_2$, $G_{12}$, and $\nu_{12}$ that were determined from the average fitting routine mentioned above were fixed while $E_1$ was allowed to vary so that a best fitting curve of $G_{xy}$ versus $\theta$ generated by CLPT could be found. The value of $E_1$ was allowed to vary because the other three ply elastic properties do not have a major influence on the $G_{xy}$ curve near ±45°. That best fit curve denoted by CLPT (Torsion) is shown in Figure 53 and the ply elastic properties are tabulated in Table 17.
Figure 53: $G_{xy}$ vs. $\theta$, experiments and theory, CLPT (Torsion)

Table 17: Ply elastic material properties for fitting routine to torsion testing

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_1$ (GPa)</td>
<td>79.0</td>
</tr>
<tr>
<td>$E_2$ (GPa)</td>
<td>1.64</td>
</tr>
<tr>
<td>$G_{12}$ (GPa)</td>
<td>1.09</td>
</tr>
<tr>
<td>$\nu_{12}$</td>
<td>0.33</td>
</tr>
</tbody>
</table>

The fitting line in Figure 53 agrees well with the limited experimental data, with deviations of 7%. However, the value of $E_1$ for CLPT (Torsion) is considerably higher than the value of $E_1$ for CLPT (Avg). For comparison, the value of $E_1$ that is chosen to fit to the $G_{xy}$ experimental data is used to generate an $E_1$ versus $\theta$ curve. This is shown in Figure 54 with an expanded view of the same graph for higher fiber winding angles shown in Figure 55.
While CLPT (Avg and Torsion) fit the experimental data well for winding angles greater than 45°, there are some major discrepancies for lower fiber winding angles. For ±30° CLPT (Avg) over predicts by 24% while CLPT (Torsion) over predicts by 41%. For ±20° CLPT (Avg) is right on the average of the tensile and compressive data points while CLPT (Torsion) over predicts by 34%. As another point of reference, the value of $E_1$ predicted by the rule-of-mixtures for $V_{fa} = 0.59$, $E_f = 245$ GPa (Young’s modulus of the
fiber – Hexcel, 2009), and $E_m = 310$ MPa (Young’s modulus of the neat resin) is 145 GPa. If a unidirectional 0° tube were to be tested in axial tension, it is believed that the modulus would be close to the value calculated by the rule-of-mixtures. For these reasons, a value of $E_1 = 79$ GPa is considered to be the major in-plane Young’s modulus of the FMC.

4.6 Summary

The results from the ballistic impact testing seem to show that more damage was created in the FMC tubes than in the RMC tubes from the steel ball bearing projectile. Most evidence from the initial inspection after impact, to the swelling observed during impact, to the amount of energy absorbed by the two different tubes, to the full-field strain measurements made by the DIC show that the damage region in the FMC was larger and more widespread. A comparison between FMC and RMC tubes was not made using X-radiography, but this technique showed extensive matrix cracking in the circumferential region around the impact site in FMC tubes. The swelling of the tube and the considerably (~50%) larger amount of energy that the FMCs absorbed during impact can be explained by the high strain to failure of elastomeric materials. The matrix material deforms much more in the FMCs during impact, absorbs more energy, resulting in what seems to be overall more material fracture. The matrix material in the RMC is rigid and has a low strain to failure. In this case the projectile fractures the material at the impact site, makes a clearly defined hole, and does not seem to propagate damage as extensively beyond the impact site. Again, only FMC tubes were used in spin testing because RMC tubes were too stiff to be misaligned in the spin rig. However, during spin testing of damaged FMC tubes, a compounding effect of localized strain and heat was
observed at the impact site where the tube eventually kinked and failed. Similar undamaged tubes were also tested (mentioned in Section 6.2) that survived tests at twice the nominal strain and twice the speed. Interestingly enough, the knockdowns in the tensile and compressive quasi-static modulus and strength for FMCs and RMCs were comparable. For some fiber angles, one material seemed to perform better than the other, but across the board there was no material that seemed to outperform the other in modulus and strength knockdowns in tension and compression.

In torsion while RMCs saw about a 20% knockdown in shear modulus, FMCs saw about a 50% knockdown. In terms of ultimate strength, RMCs saw about a 55% knockdown where FMCs saw about a 90% knockdown. An interesting result of torsion testing is that no tubes failed by torsional buckling. Either the spiral pattern of sheared or buckled fibers at a local level or the crushing of damaged material at the impact site was seen as failure modes. This contrasts to the results seen by Bauchau et al. (1988), and may be due to the differences in geometries of the test specimens. In torsion testing of the ±45° and ±60° tubes, the fibers are primarily loaded. In tension and compression testing of the same angles, the matrix tends to be the primary constituent that is loaded. For this reason, the large knockdowns in torsional modulus and strength in FMC tubes may be attributed to the larger regions of fiber pull-out seen on the inside of impacted tubes.

Quasi-static testing was carried out in tension, compression, and torsion on two different load frames. Results agreed well across testing platforms. Where applicable, coefficients of variation for undamaged tubes were less than 15% for quasi-static modulus and strength measurements (with a few minor exceptions). The variation among damaged tubes was considerably more.
It is clear that conventional laminated plate theory does not accurately predict the axial and torsional stiffnesses of angle-ply FMC tubes. First of all, the differences in the tensile and compressive axial moduli for the same fiber winding angle can be anywhere from 3 – 27% different. In general, a larger discrepancy is seen between tension and compression for lower winding angles. While appropriate values for $E_2$, $G_{12}$, and $\nu_{12}$ coupled with laminated plate theory may predict the axial moduli of higher angled tubes (above ±45°), laminated plate theory fails when predicting axial moduli of tubes with winding angles close to 0° and torsional moduli of tubes with winding angles close to 45°. The common denominator between these two scenarios is the majority of the load is applied in the fiber direction. Perhaps the fact that the fibers in these composites are woven in an undulated manner and surrounded by a soft, flexible matrix is the reason why there is such a discrepancy between CLPT and experiments. The composites in the current investigation do not have stacked, discrete composite plies comprised of stiff fibers surrounded by a stiff matrix – the situation upon which classical laminated plate theory was developed.

The values of $E_1$ shown in Tables 16 – 17 (51 and 79 GPa) are significantly less than the value of $E_1$ calculated by rule of mixtures (145 GPa). If tensile and compressive axial moduli of angle-ply tubes were measured for fiber winding angles less than 20°, it is believed that the value of $E_1$ to accurately predict the moduli of these tubes would increase (from 51 GPa for a ±20° tube). So perhaps a modified version of laminated plate theory with a major in-plane Young’s modulus that is dependent on the fiber winding angle would accurately predict the structural response of angle-ply FMC tubes for all fiber winding angles. Developing a modified version of classic lamination theory to predict the axial and torsional stiffnesses of FMC tubes is beyond the scope of the current
work, although a previous attempt effects to model axial stiffness of FMC tubes considering fiber undulation has been made with moderate success (Zindel, 2009).
Chapter 5  Determination of Viscoelastic Lamina Properties

The viscoelastic material properties in the transverse and shear directions of a composite lamina are sensitive to temperature and frequency. Because these properties are used over varying temperature and frequency ranges to calculate the steady state temperature in misaligned, spinning composite shafts, it is necessary to model these variations accurately. The approach for modeling and the experimental determination of these properties for the carbon/polyurethane composite is described in this chapter. The RMC was not included in this aspect of the investigation because its high rigidity prevented good experimental measurements from being obtained.

5.1 Material Property Model

The viscoelastic matrix dominated properties – transverse and shear moduli and loss factors – are modeled using the fractional derivative constitutive model (Bagley and Torvick, 1979). In the heating model, the fiber dominated properties in the longitudinal direction of the lamina are considered to be constant throughout the frequency range. Also, all of the Poisson’s ratios are considered to be independent of frequency. The fractional derivative model used to model frequency-dependent properties was also used in the earlier work that modeled temperature increase in FMC shafts (Shan and Bakis,
The storage modulus, loss modulus, and loss factor are determined from Equations 10, 17 and 18 and are given by

\[ E' = \frac{AC + BD}{C^2 + D^2} E \]  
(28)

\[ E'' = \frac{BC - AD}{C^2 + D^2} E \]  
(29)

and

\[ \eta = \frac{E''}{E'} = \frac{BC - AD}{AC + BD} \]  
(30)

where

\[ A = 1 + b_1 \omega^{\beta_1} \cos\left(\frac{\pi \beta_1}{2}\right) + b_2 \omega^{\beta_2} \cos\left(\frac{\pi \beta_2}{2}\right) + \ldots + b_n \omega^{\beta_n} \cos\left(\frac{\pi \beta_n}{2}\right) \]  
(31)

\[ B = b_1 \omega^{\beta_1} \sin\left(\frac{\pi \beta_1}{2}\right) + b_2 \omega^{\beta_2} \sin\left(\frac{\pi \beta_2}{2}\right) + \ldots + b_n \omega^{\beta_n} \sin\left(\frac{\pi \beta_n}{2}\right) \]  
(32)

\[ C = 1 + a_1 \omega^{\beta_1} \cos\left(\frac{\pi \beta_1}{2}\right) + a_2 \omega^{\beta_2} \cos\left(\frac{\pi \beta_2}{2}\right) + \ldots + a_n \omega^{\beta_n} \cos\left(\frac{\pi \beta_n}{2}\right) \]  
(33)

\[ D = a_1 \omega^{\beta_1} \sin\left(\frac{\pi \beta_1}{2}\right) + a_2 \omega^{\beta_2} \sin\left(\frac{\pi \beta_2}{2}\right) + \ldots + a_n \omega^{\beta_n} \sin\left(\frac{\pi \beta_n}{2}\right) \]  
(34)

The parameters \( E \), \( a_n \), \( b_n \), and \( \beta_n \) are material parameters for each lamina property, and \( \omega \) is the angular velocity of the shaft (radians per second). These are determined by matching best-fit curves (generated by Equations 28 and 30) to the experimentally determined storage modulus and loss factor master curves, respectively. Since fitting curves to the data involves multiple parameters, an optimization process must be used.

This optimization process was developed in the previous work (Shan and Bakis, 2009) and a modified version of that process developed by Dr. Shan is used here (Appendix B.1). The optimum number of fractional derivatives in the previous work was determined to be two \( (n = 2) \) and this same number was used in the present investigation. Master
curves were generated using the temperature-frequency superposition principle (Ferry, 1970). After experimental testing, plots of material properties versus frequency are adjusted for different temperatures via temperature shifting factors, $\alpha_t$. The temperature shifting factors translate the individual curves for one temperature horizontally on the logarithmic frequency axis so that they align into one master curve. Shifting factors are functions of temperature, $T$, and were determined by shifting the curves by eye in this case. The Williams-Landel-Ferry (WLF) temperature shifting function was used. It is given by

$$\log \alpha_t = \frac{C_1(T - T_0)}{C_2 + T - T_0}$$

where $C_1$ and $C_2$ are material parameters that are chosen by a curve fitting program, and $T_0$ is the reference temperature (taken in this case to be 60°C) to which all other curves are shifted. To apply temperature shifting, the angular frequency $\omega$ in Equations 31 – 34 is replaced by the reduced frequency, $\omega\alpha_t$, or $f\alpha_t$ in this case (where $f = \omega/2\pi$). By making this replacement, functions for storage modulus, loss modulus, and loss factor that are dependent on frequency and temperature are obtained.

5.2 Damping Model

Overall damping in the composite tubes was modeled the same way as in the previous work (Shan and Bakis, 2009). The approach combines the fractional derivative model for dynamic lamina properties with the strain energy structure developed by Adams et al. (Adams and Bacon, 1973a, 1973b; Ni and Adams, 1984). The loss factor is given by

$$\eta = \frac{\Delta W}{2\pi W}$$

(36)
where $\Delta W$ is the amount of energy dissipated per cycle per unit volume and $W$ is the maximum strain energy per unit volume in that cycle. So for a given volume of a ply in the shaft, the energy dissipation per unit volume is given by

$$\Delta W = 2\pi(W_1\eta_1 + W_2\eta_2 + W_3\eta_3 + W_{12}\eta_{12})$$

$$= \pi(\eta_1 E_i\varepsilon_i^2 + \eta_2 E_2\varepsilon_2^2 + \eta_3 E_3\varepsilon_3^2 + \eta_{12} G_{12}\gamma_{12}^2)$$

where $\eta_i$ are the loss factors, and $\varepsilon_i$ and $\gamma_i$ are the strains in the 1-2-3 coordinate system.

Contributions to energy dissipation from out-of-plane shear strains are disregarded due to their insignificance (Shan, 2006). For material parameters that are independent of frequency or temperature, static values were used for the modulus and loss factor. For material parameters that are dependent on temperature and frequency, the appropriate storage modulus and loss factor were used. These loss factors can be determined experimentally in dynamic mechanical testing. If stroke or angle control is used in the test, loss factors can be determined by measuring the phase difference between the waveform of the applied strain and the resulting waveform from the load or torque cell. The storage modulus can be determined by measuring the phase difference and comparing the amplitude of the resulting waveform from the load or torque cell to the amplitude of the applied strain.

5.3 Experimental Methods for Dynamic Properties

Dynamic tests were performed on two different dynamic mechanical analyzers (DMA). For determination of the temperature and frequency-dependent transverse dynamic properties ($E_2'$, $E_2''$, and $\eta_2$) a machine was used that applies an oscillating normal strain – referred to from now on as the axial DMA. For determination of the shear
properties \( (G_{12}', G_{12}'', \text{ and } \eta_{12}) \) a machine was used that applies a shearing strain to a thin-walled tube – referred to from now on as the torsional DMA.

The axial DMA is a Q800 dynamic mechanical analyzer made by TA Instruments (New Castle, DE) located in the Wood Chemistry Laboratory in the School of Forest Resources at Penn State. In the axial DMA, a tension film clamping set-up is used for the flat 90° carbon/polyurethane specimens to measure the dynamic transverse properties \( (E_2) \). The specimen was clamped by a stationary grip and by a grip attached to an oscillating driveshaft. A schematic of this clamping set-up is shown in Figure 56. One must pay proper attention when tightening the screws that adjust the grip pressure on the specimen. Too much pressure may break the specimen while too little may allow the specimen to slip during the test. For the current investigation, 2 in-lb of torque was used to tighten the screws on the tension film clamps. Specimens that are aligned well and have unvarying width and thickness contribute to good results. The driveshaft is driven by air pressure and very precise care must be taken not to damage the shaft while installing specimens or tightening grips. The grips are located inside a thermally insulated chamber so that the machine can maintain accurate control of temperature during the test. An external liquid nitrogen tank is connected to the machine to provide cool air while an electrical heater that is attached to the machine provides warm air. The liquid nitrogen tank and the axial DMA can be seen in Figure 57. The temperature range used during the tests varied from about 23°C to 100°C. As temperature was ramped, sinusoidal loading was applied at seven different frequencies in sequence: 1, 5, 10, 20, 40, 60, and 80 Hz. The strain amplitude at which the driveshaft oscillates must be chosen so that the 18 N load capacity on the load cell is not exceeded. For the current investigation, a strain amplitude of 200 με was chosen. The testing apparatus and all of the testing parameters
including strain rate, strain amplitude, temperature range, specimen modulus and temperature ramp rate conform to testing standards (ASTM, 2006a; ASTM, 2006b).

The torsional DMA is a RDS-II dynamic spectrometer made by Rheometrics Inc., which is now owned by TA Instruments (New Castle, DE), located in the Materials
Science and Engineering Department at Penn State. In the torsional DMA, the tubular specimens were first bonded into a set of steel disk-shaped grips (Figure 58). The adhesive used was Devcon 5 Minute Epoxy made by ITW Devcon (Danvers, MA). The ends of the steel disks were made to interface with the clamping fixture in the torsional DMA machine which has a stationary platform and a rotating platform. An axial-torsional force transducer attached to the stationary platform measures the dynamic torsional response of the tube. A motor attached to the rotating platform applies a dynamic shear strain to the tube. The dynamic in-plane shear properties ($G_{12}$) can be measured in this manner because torque applied to a $0^\circ$ tube is equivalent to a unidirectional ply in pure shear. The tube, grips and platforms are located inside a thermally insulated chamber so that the machine can maintain precise temperature control during the test. Cooling is provided by a liquid nitrogen tank while heating is provided by an electrical heater. The torsional DMA has a torque load capacity of 2000 g-cm. The shear strain amplitude was chosen to be 600 με.

![Figure 58: Torsional DMA grip (left) and 0° tube in grips (right)](image)

The results from the tests run in the axial DMA are shown in Figure 59 and Figure 60. The results from tests run in the torsional DMA are shown in Figure 61 and in Figure
62. Storage modulus and loss factor are plotted versus frequency for five different temperature steps: 23, 40, 60, 80, and 100°C. The storage modulus for both lamina properties increases with increasing frequency and decreases with increasing temperature. The loss factors in the transverse direction maintain constant values of around 0.2 – 0.3 for the tested temperature and frequency ranges. The loss factors in shear initially increase with increasing temperature and decrease with increasing frequency. However, somewhere between 60 – 80°C a change occurs and there is a peak in shear the loss factor. The results from the torsional DMA assume that the adhesive rigidly bonds the tube to the disk-shaped grips throughout the temperature range. The peak in the shear loss factor and the decrease in shear modulus beyond 80°C may be evidence that points toward the softening of the adhesive beyond these temperatures. If this is the case, the torsional response measured by the transducer in this temperature range could be partially influenced by the softening of the adhesive. While this is acknowledged as an issue, if a function can be fit well to the master curve, it will certainly fit well to the data that is believed to be accurate (lower temperatures). Also the temperature simulations discussed in Chapter 6 use the part of the curve that is believed to be created from accurate measurements. This part of the curve is at temperatures below 50°C and frequencies above 1200 rpm with a reduced frequency range of about 0.4 – 6.
**Figure 59: Transverse storage modulus versus frequency, at different temperatures**

**Figure 60: Transverse loss factor versus frequency, at different temperatures**
Figure 61: Shear storage modulus versus frequency, at different temperatures

Figure 62: Shear loss factor versus frequency, at different temperatures
5.4 Master Curves and Model Parameters for Dynamic Properties

Using the dynamic measurements from the previous section, each temperature data set was shifted horizontally on the logarithmic frequency axis to create continuous, smooth master curves for each dynamic lamina material property. The shifting factors are specific to one material property but are applied (by eye) simultaneously to the storage modulus and loss factors for that property. For this reason, a balance must be achieved between aligning the shifted data in the storage modulus and loss factor graphs. A reference temperature of 60°C was chosen to which all other data sets are shifted. The master curves for the transverse properties are shown in Figure 63 and in Figure 64 while the master curves for the shear properties are shown in Figure 65 and in Figure 66. Shifting factors are functions of temperature, \( T \). So, in order to determine the shifting factor for any arbitrary temperature, a shifting function must be fit to the data. The Williams-Landel-Ferry (WLF) function (Equation 36) was used for this purpose. Parameters for the WLF function were determined by linear regression. The temperature shifting function for the transverse dynamic properties is shown in Figure 67 while the shifting function for the shear dynamic properties is shown in Figure 68. Once the data is shifted into the master curves, an optimization process (Appendix B.1) is used to determine a best fit curve to the data. This process chooses the \( E , a_n , b_n \), and \( \beta_n \) material parameters for each property. These parameters for the transverse and shear dynamic lamina properties as well as the temperature shifting parameters are given in Table 18 and in Table 19 while the curves generated by these parameters for the storage modulus and loss factor are shown plotted against experimental data in Figures 69 – 72.
Figure 63: Transverse modulus versus reduced frequency, reference temperature of 60°C

Figure 64: Transverse loss factor versus reduced frequency, reference temperature 60°C
Figure 65: Shear modulus versus reduced frequency, reference temperature of 60°C

Figure 66: Shear loss factor versus reduced frequency, reference temperature of 60°C
Figure 67: Shift factor versus temperature for $E_2$ and $\eta_2$, reference temperature 60°C

\[
\log(\alpha_i) = \frac{-17.02 (T - T_0)}{230.3 + T - T_0}
\]

$R^2 = 0.9867$

Figure 68: Shift factor versus temperature for $G_{12}$ and $\eta_{12}$, reference temperature 60°C

\[
\log(\alpha_i) = \frac{-29.46 (T - T_0)}{264.1 + T - T_0}
\]

$R^2 = 0.9822$
Figure 69: Model fit and transverse modulus versus reduced frequency, reference temperature 60°C

Figure 70: Model fit and transverse loss factor versus reduced frequency, reference temperature 60°C
Figure 71: Model fit and shear modulus versus reduced frequency, reference temperature 60°C

Figure 72: Model fit and shear loss factor versus reduced frequency, reference temperature 60°C
Table 18: Fractional derivative model parameter values for transverse properties ($n = 2$)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$</td>
<td>0.2148 (GPa)</td>
</tr>
<tr>
<td>$a_1$</td>
<td>0.2120</td>
</tr>
<tr>
<td>$b_1$</td>
<td>4.9357</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.2275</td>
</tr>
<tr>
<td>$a_2$</td>
<td>$6.0 \cdot 10^{-5}$</td>
</tr>
<tr>
<td>$b_2$</td>
<td>0.0039</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>0.8106</td>
</tr>
<tr>
<td>$C_1$</td>
<td>-4.802</td>
</tr>
<tr>
<td>$C_2$</td>
<td>126.5</td>
</tr>
</tbody>
</table>

Table 19: Fractional derivative model parameter values for shear properties ($n = 2$)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
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<td>$G$</td>
<td>0.1571 (GPa)</td>
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<tr>
<td>$a_1$</td>
<td>2.8878</td>
</tr>
<tr>
<td>$b_1$</td>
<td>0.0230</td>
</tr>
<tr>
<td>$\beta_1$</td>
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</tr>
<tr>
<td>$a_2$</td>
<td>0.5170</td>
</tr>
<tr>
<td>$b_2$</td>
<td>4.6893</td>
</tr>
<tr>
<td>$\beta_2$</td>
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<tr>
<td>$C_1$</td>
<td>-24.46</td>
</tr>
<tr>
<td>$C_2$</td>
<td>264.1</td>
</tr>
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</table>

Both of the temperature shifting functions seem to fit reasonably well to the temperature shifting factors. The values of $R^2$ are both at least 0.98. The model for the dynamic shear properties fits more accurately to the data than the model for the dynamic transverse properties (specifically transverse loss factor). Herein lies the issue with adjusting the curves by eye and finding a balance between the storage modulus and loss factor curves. Temperature shifting factors must be chosen so that both the storage modulus and loss factor curves are shifted into appropriate master curves. Although the behavior beyond 80°C in the torsional DMA test is doubtful, the material model is still regarded as acceptable because the reduced frequency ranges of the spin test experiments (about 2.5) discussed in Chapter 6 lie above the range in the material model where the interference from the adhesive becomes an issue.
Chapter 6  Self-heating of Fiber Reinforced Composites

It is important to be able to predict the self-heating characteristics of misaligned, spinning FMC shafts due to internal damping for the rotorcraft application. Material properties such as stiffness, strength, and loss can be dependent on temperature. The ability to predict temperature increase in the shaft accurately will allow designers to enforce constraints on the shaft so that it will not fail during service. This calculation was explored in an earlier work (Shan and Bakis, 2002; Shan and Bakis, 2009) with a closed-form code using MATLAB. That code has been slightly modified in the current work so that it can analyze self-heating in shafts with multi-angled layups. In addition, a finite element analysis is also developed in the current investigation. The results from the analyses and experiments of spinning, misaligned FMC tubes are compared in this chapter. This comparison is of primary interest because it serves as a means of validating the newly developed self-heating analyses.

6.1 Modeling

Many aspects of the self-heating analyses in the current work are similar to the analysis developed by Shan and Bakis (2009). Each of the analyses uses the same approach to model the basic lamina elastic and damping properties of the composites. Strain energy loss caused by internal damping is calculated via Equation 38 and
converted to heat generation. Each analysis assumes the same thermal conductivity and convective coefficients of the FMC shaft. These aspects of the models are briefly explained next.

The transverse and shear properties of the composite are assumed to be temperature and frequency dependent according to the fractional derivative model as outlined in the previous chapter. Determination of the major in-plane Young’s modulus was discussed in Section 4.4 and the value used in the self-heating models is 79 GPa. The major in-plane loss factor, \( \eta_1 \), was selected to be 0.0015 (Gibson, 2007). The value of the major in-plane Poisson’s ratio is determined by rule of mixtures (discussed in Section 4.4) and is calculated to be 0.33. The composite ply is considered to be transversely isotropic, so the 2-direction is considered to be the same as the 3-direction. The out-of-plane shear modulus, \( G_{23} \), is calculated from \( E_2 \) and \( \nu_{23} \) values,

\[
G_{23} = \frac{E_2}{2 (1 + \nu_{23})}
\]  

(38)

and the out-of-plane Poisson’s ratio, \( \nu_{23} \), for the carbon/polyurethane material under investigation was determined by an interpolating function that was fitted to data of \( \nu_{23} \) versus the minor Young’s modulus for three different types of composites (Figure 73 and Table 20). The value of \( \nu_{23} \) for the carbon/polyurethane material under investigation was calculated to be 0.87. The value for the carbon/flexible epoxy material was measured in the Composites Manufacturing Technology Center (CMTC) at Penn State by Ms. Ye Zhu in separate unpublished research. All of the material properties used in the self-heating models are summarized in Table 21. The properties that would be repeated in the 3-direction due to transverse isotropy and the properties that are calculated are not shown for brevity.
Table 20: Minor in-plane Young’s modulus and out-of-plane Poisson’s ratio for different composites

<table>
<thead>
<tr>
<th>Material</th>
<th>$E_2$ (GPa)</th>
<th>$\nu_{23}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T700/L100/C21</td>
<td>0.23</td>
<td>0.93</td>
</tr>
<tr>
<td>AS4D/LF750DD/C31DA</td>
<td>1.5</td>
<td>0.87</td>
</tr>
<tr>
<td>Carbon/flexible epoxy</td>
<td>7.4</td>
<td>0.61</td>
</tr>
<tr>
<td>Carbon/epoxy</td>
<td>10.3</td>
<td>0.54</td>
</tr>
</tbody>
</table>

$^1$Interpolated Value

$$\nu_{23} = -0.0397 \times E_2 + 0.9322$$

$R^2 = 0.9892$

Figure 73: Interpolating function for $\nu_{23}$ vs. $E_2$ and four different composites
Table 21: Material properties used in the self-heating models

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
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<td>Longitudinal</td>
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<td>79 GPa</td>
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<tr>
<td>$\eta_1$</td>
<td></td>
<td>0.0015</td>
</tr>
<tr>
<td>Transverse</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E$</td>
<td></td>
<td>0.2148 GPa</td>
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<tr>
<td>$a_1$</td>
<td></td>
<td>0.2120</td>
</tr>
<tr>
<td>$b_1$</td>
<td></td>
<td>4.9357</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td></td>
<td>0.2275</td>
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<tr>
<td>$a_2$</td>
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<td>$6 \cdot 10^{-3}$</td>
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<td>$\beta_2$</td>
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<td>$a_1$</td>
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<td>$2 \cdot 10^{-11}$</td>
</tr>
<tr>
<td>$a_2$</td>
<td></td>
<td>0.5170</td>
</tr>
<tr>
<td>$b_2$</td>
<td></td>
<td>4.6893</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td></td>
<td>0.2433</td>
</tr>
<tr>
<td>$C_1$</td>
<td></td>
<td>-24.46</td>
</tr>
<tr>
<td>$C_2$</td>
<td></td>
<td>264.1</td>
</tr>
<tr>
<td>Poisson’s Ratio</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu_{12}$</td>
<td></td>
<td>0.33</td>
</tr>
<tr>
<td>$\nu_{23}$</td>
<td></td>
<td>0.87</td>
</tr>
</tbody>
</table>

The thermomechanical heating analyses assume that all strain energy loss caused by internal damping is converted to heat. In each analysis, there are a number of finite nodal points through the thickness of the shaft where heat generation rates are applied. The thermal energy generated per unit volume per unit time at each nodal point is given by

$$q'' = \frac{\omega}{2\pi} \Delta W$$  \hspace{1cm} (39)$$

where $\omega$ is the angular velocity of the spinning shaft. Strains may vary through the thickness of the shaft wall, so heat generation terms may be different at each nodal point. In each analysis the point in the cross-section of the shaft at which the strains are sampled to calculate $\Delta W$ is at the point of maximum bending strain. For example, in Figure 74, for a bending moment about the $X$-axis, the maximum bending strain would occur at the $X = 0$ and $Y = OD/2$ location, where $OD$ is the outside diameter. According to Lekhnitskii
(1963), for an orthotropic tube under pure bending, the only stresses that are non-zero at this location are $\sigma_R$, $\sigma_Z$, and $\sigma_\theta$. The interlaminar shear stress, $\tau_{R\theta}$, is non-zero at the $X = OD/2$ and $Y = 0$ location. So, the interlaminar shear stress, $\tau_{R\theta}$, is $90^\circ$ out of phase with the normal stresses which prohibits a straightforward calculation of the heat generation. An additive method could be used to include the contribution from this interlaminar shear stress, but the value is so low that it is simply neglected altogether (Shan, 2006). The coordinate systems in Figure 74 are further explained in Section 6.1.2.1.

![Global coordinate systems for a hollow shaft](image)

Figure 74: Global coordinate systems for a hollow shaft

The thermal conductivity of the composite was assumed to be constant throughout the temperature range of interest — 0.72 W/m·K. The value of thermal conductivity within cited ranges (Kalogiannakis et al., 2004) for carbon/epoxy (about 0.5 – 0.7 W/m·K) does not have a large influence on the final temperature of the shaft. The convective coefficient for a rotating shaft in a horizontal plane is determined by (Kendoush, 1996)

$$ h_r = k_a \frac{\text{Nu}}{OD} $$

(40)
where \( k_a \) is the thermal conductivity of air, taken to be 0.0251 W/m·K at 20°C. The expression for \( \text{Nu} \) (the average Nusselt number) is

\[
\text{Nu} = 0.533 \text{Re}^{1/2}
\]  

(41)

where \( \text{Re} \) is the rotation Reynolds number defined as

\[
\text{Re} = \frac{OD^2 \omega / 2 \nu}{\nu}
\]  

(42)

where \( \nu \) is the kinematic viscosity of air. At 20°C, \( \nu \) has a value of 1.57×10\(^{-5}\) m\(^2\)/s.

Finally, the emissivity, \( \varepsilon \), of the surface of the shaft was assumed to be equal to 1 for an unpainted carbon fiber composite.

### 6.1.1 Closed-Form Analysis

The closed-form analysis developed here is similar to the analysis developed in the previous work on heating in FMC shafts (Shan and Bakis, 2009). The fundamental difference between the two is the way each model calculates resulting stresses and strains in a shaft from applied bending loads. The previous analysis uses Lekhnitskii’s elastic theory (Lekhnitskii, 1963) for a thick, homogenous, cylindrically-orthotropic shaft under bending, tensile, and torsional loading to calculate laminate level stresses. This analysis cannot accommodate laminates with differing fiber angles through the wall thickness. In the present investigation, a three-dimensional elasticity solution for a thick, layered, composite shaft under bending, tensile, and torsional loading (Jolicoeur and Cardou, 1994) is used to determine layer-wise three-dimensional stress. The assembly of coaxial circular tubes is created by joining the tubes at an interface with a no slip condition. The material of each tube is considered to be cylindrically orthotropic. Therefore, the current heating analysis can calculate the temperature increase in mixed angle-ply laminates. The
layered tube elasticity solution was coded in MATLAB by Dr. Ying Shan of the CMTC at Penn State in unpublished work.

Besides the differences in the elasticity solutions, the analysis developed by Shan and Bakis (2009) and the analysis in the present investigation share many similarities. For example, both analyses use the fractional derivative model for frequency and temperature dependent lamina properties, and both analyses use a material model for thick laminates proposed by Sun and Li (1988) for calculating the three-dimensional effective elastic constants for a laminate from knowledge of transversely isotropic ply elastic properties and laminate stacking sequence. While the previous analysis (Shan and Bakis, 2009) calculates effective laminate elastic constants once for an entire $\pm \theta$ angle-ply tube, the analysis developed in the current investigation calculates effective elastic constants for each layer of the laminated tube. Lamina level stresses and strains are calculated via classical lamination theory and general coordinate transformations. Flow charts for the homogeneous and laminated heating analyses starting from basic lamina elastic properties are illustrated in Figure 75.
In order to solve the elasticity solutions though, the geometric properties of the shaft (inner and outer diameter), the fiber winding angles, and the bending loads are needed. The geometries of the small-scale tubes being tested and modeled are tabulated in Section 3.2. The fiber winding angle is used to determine the three-dimensional effective elastic constants from basic lamina elastic properties in the Sun and Li model (1988). Lastly, the bending loads are applied by a pure moment calculated by

$$M_x = \frac{2E_x I \varepsilon_x}{OD}$$  \hspace{1cm} (43)

where $E_x$ is the axial modulus, $I$ is the second moment of area, and $\varepsilon_x$ is the nominal strain at the $OD$ along the length of the shaft (assumed constant curvature). With a constant
curvature the stresses and strains, and hence temperatures, do not vary along the axial
direction of the shaft. The nominal strain is held constant (kinematic boundary condition)
in the model. Therefore, since the axial modulus of the tube is a function of temperature
and frequency, so is the applied bending moment.

In the closed-form analyses, the heat transfer problem is one-dimensional in the
radial direction. The heat transfer problem makes a few assumptions. The shaft has a
constant thermal conductivity, the inner surface is insulated, the outer surface transfers
heat to the surrounding air by convection, and heat generation rates are applied to finite
volumes through the thickness of the shaft. In the homogeneous analysis shafts with only
one fiber angle are studied, so the total number of finite volumes through the thickness
could be changed arbitrarily to improve discretization. However, in the laminated
analysis the total number of finite volumes through the wall thickness of the shaft must
be chosen in integer multiples of the number of layers.

Once the thermal segment of the model solves the heat transfer problem, the
resulting temperature distribution through the radial direction of the shaft is averaged to
one value, $T_{\text{cal}}$. On the initial iteration, this average temperature of the shaft is compared
to the starting ambient temperature, $T_{\text{amb}}$. If it is different than the starting temperature,$T_{\text{mat}}$ is assigned the value of $T_{\text{cal}}$ and the material properties in the structural segment are
changed based on the new temperature, $T_{\text{mat}}$. The structural model is then solved again
based on the new material properties. The process continues until the model converges on
a final, steady state temperature of the shaft. This process is illustrated in Figure 76.
6.1.2 Finite Element Model

The finite element code used to calculate the temperature increase in a misaligned, spinning composite shaft in the current work is ANSYS® Academic Research, Release 12.1. Overall, the approach to calculate temperature increase is similar to the closed-form approach. A static bending load is applied to a composite shaft in the structural segment of the analysis which results in lamina-level strains. Since in the actual shaft these are cyclic strains, they are used in the finite element analysis to calculate heat generation within the shaft from the strain energy framework established by Adams et al. (1973, 1984). Heat generation is then used along with conduction and convection heat transfer to solve a steady state thermal model in the thermal segment of the analysis. The average temperature of the shaft resulting from the solution of the thermal segment of the
model is then used to change the material properties in the structural segment of the model. The process iterates until the model converges on a final temperature of the shaft. The finite element self-heating analysis shares similarities with the current laminated closed-form analysis in MATLAB in that it has the ability to model self-heating in multi-angled shafts. The added advantage of the finite element code is that it contains the necessary framework for modeling full-sized shafts with any kind of boundary conditions. In the following sub-sections two different macro files that are used to run the finite element self-heating analysis are described.

6.1.2.1 The Finite Element “Building” Macro

A macro is a simple computer text file with a *.mac file extension. It contains a list of recognized commands and is written in the basic programming language used in the ANSYS® software — the ANSYS Parametric Design Language (APDL). All of the commands can also be executed by using the graphical user interface (GUI) in the finite element software. Once defined and located in the home directory, the filename can be entered and executed in the command window in ANSYS®. The first macro file described here defines the geometry, material properties, layup, loads, and meshing parameters for the finite element analysis. To begin, it is beneficial to define a few different coordinate systems in the finite element model. The global Cartesian coordinates are \(X-Y-Z\) while the global cylindrical coordinate are \(R-\Theta-Z\). The \(Z\) axis is oriented in the axial direction of the composite shaft. This coincides with the \(x\)-axis mentioned in previous chapters. The elemental coordinate system is defined by the \(x'-y'-z'\) coordinates. These three coordinate systems are summarized in Figure 77.
To start building the geometry of the shaft, eight keypoints are entered into ANSYS® to define the corners of a quarter section of the shaft. A volume is then created by these eight keypoints in the global cylindrical coordinate system resulting in a quarter volume section of the shaft (Figure 78). This quarter section volume is then mirrored three times so that four quarter volume sections combine to resemble the total volume of the shaft — also shown in Figure 78. After meshing, duplicate lines are removed so that one solid shaft exists. The inner and outer diameters are chosen to model the shafts in the spin tests, and the length is reduced by a factor of 4 to reduce model run time. The bending loads are applied in a way such that a constant nominal strain along the length of the shaft is imposed.
The element selected for the analysis is a three-dimensional layered structural and thermal solid brick element with 20 nodes. The name for this element designated by ANSYS® is SOLID186. A schematic of this kind of element is shown in Figure 79. The element allows up to 250 different layers. Each layer requires three-dimensional elastic properties. The elemental coordinate system is shown again here. This coordinate system is defined by the numbering of the nodes within the element. The layered coordinate system is denoted here by the 1-2-3 coordinates. The layered system relative to the elemental coordinate system is rotated by the fiber winding angle, $\theta$, in the $x'-y'$ plane. The three-dimensional lamina elastic properties are assigned to the layered coordinate system. The layup is chosen in the preprocessor and it is assigned to each element. Although any discrete layup can be chosen within an element, the layup chosen to model the shafts in the spin tests in the current work was $[\theta_2/-\theta_2]$, so that if more than one element were stacked through the thickness, the stacking sequence would still be balanced and symmetric. Whenever the model switches from the structural segment to the thermal or vice-versa, the node locations remain the same but the material properties, loads, and boundary conditions change. The key options (or KEYOPTs) that must be
switched on for the element to enable layer construction and to store layer data are key options 3 and 8, respectively.

The $x'$ axis for each element is oriented along the global $Z$-axis of the tube. This is achieved by defining a meshing algorithm in ANSYS® to assign the node numbers and locations within an element in a specific way. Each of the four sub-volumes must be oriented properly so that the meshing algorithm knows to align all of the elemental coordinate systems in this way. In the current work this is achieved by selecting the VEORIENT command with the keypoint option (ANSYS®, 2010). Once these meshing algorithms have been established, the tube is meshed with density parameters around the circumference and along the length that are equivalent so that each element is constructed with maximum squareness in mind. Also, all models in the current work were constructed with two elements through the wall thickness of the tube. After meshing is complete, all coincident items (lines, nodes, elements etc.) are merged.
For a frequency value of interest, the 9 three-dimensional lamina elastic material properties are either entered into ANSYS® in a table format via the graphical user interface or they are entered by a macro file. A sample macro file for the current material system at a frequency of 1200 rpm is shown in Appendix A.1, and these values are generated by a MATLAB program (Appendix B.2) from the fractional derivative model. Material properties are only able to be entered into ANSYS® as functions of temperature. So, for each frequency value of interest, a set of temperature-dependent materials properties must be entered into the analysis. The material property equivalent in the thermal segment is thermal conductivity. The value mentioned previously for thermal conductivity in the closed-form analyses is also used in the finite element analysis. The difference between the finite element and closed-form analyses is that the closed-form model solves a 1D heat transfer problem and the finite element model solves a 3D problem. The in-plane thermal conductivity would have an effect on the heat transfer surfaces at the ends of the shaft which, relative to the rest of the shaft, is a very small surface area. Although the in-plane and out-of-plane thermal conductivities of a fiber reinforced polymer composite are quite different, it is assumed that the sampling position for temperature \(Z = l/2\) is far enough away from the ends of the shaft that any effect the end surfaces of the shaft might have on the steady state temperature would be insignificant. For this reason thermal conductivity is assumed to be isotropic in the shaft.

Once the model of the shaft has been built with the appropriate geometry, layup, element orientation, mesh density, and material property set; loading is applied to both the structural and thermal segments of the model. In the structural segment, a constant bending strain is introduced into the shaft by rotating both faces of the shaft by an angle, \(\phi\), about the \(X\)-axis. This angle is given by
where $l$ is the length of the shaft in the model, and $\varepsilon$ is the nominal strain along the length of the shaft. This is a kinematic boundary condition. So regardless of the stiffness of the shaft, which is dependent on temperature, the nominal strain along the length will be the same. Rotation of the end faces is achieved by creating two pilot nodes in the center of the shaft at either end ($X = 0; Y = 0; Z = 0, l$). The node at either end is tied to all of the other nodes at that location that lie within the same $X$-$Y$ plane. After this contact is made, the rotation angle, $\phi$, is applied to the pilot node only. To summarize, all of the structural boundary conditions at both ends of the tube are as follows for pure bending:

\begin{align*}
\text{@ } Z &= 0 \\
& \quad u_x = u_y = u_z = 0 \\
& \quad rot_x = \phi \\
\text{@ } Z &= l \\
& \quad u_x = u_y = 0 \\
& \quad rot_x = -\phi
\end{align*}

\begin{equation}
\phi = \frac{\varepsilon l}{OD}
\end{equation}

where $u$ are displacements and $rot$ are angles (in radians) of rotation of the pilot nodes. In the thermal segment the inner surface and the two end surfaces of the shaft are considered to be insulated, so a zero heat flux condition is applied to these surfaces. The outer surface in the finite element model is assumed to transfer heat to its surroundings by convection. The temperature of the surroundings is assumed to be the ambient room temperature and the temperature of the shaft is calculated by the model. The convective coefficient is determined in the same way as mentioned previously in this chapter. A body load in the form of a heat generation rate is also applied to the shaft. The heat generation rates are calculated in the same way as mentioned previously in this chapter, and the details of how they are applied to the shaft are described in the following section.
6.1.2.2 The Finite Element “Simulation” Macro

The macro file described here solves the structural and thermal segments of the finite element analysis. It takes the results from one segment and uses them to calculate inputs for the next segment and continues to do so until a convergence on shaft temperature is found. It is basically exercising the flow chart shown in Figure 76. Each time the model toggles between the structural and thermal segments, it changes the SOLID186 elements to their thermal counterpart, SOLID90.

First, the temperatures of the shaft and its surroundings are set to the ambient temperature so that the structural segment knows which material properties to use and the thermal segment knows the temperature of the surroundings for the convection calculation. Then the structural segment is solved resulting in stresses and strains throughout the shaft. The strains are sampled at the midspan location at the point of maximum bending strain. At this point, there are 5 nodes through the thickness of the shaft that are shared by 8 elements which have 8 layers each (Figure 80).

![Diagram](image)

**Figure 80: Schematic, sampling elements in finite element model for heat generation calculation**

Two of these 8 elements are chosen to be the sampling elements from which the resulting strains are extracted to calculate heat generation rates. Heat generation rates must be applied as body loads to the elements because they have the units of energy per unit volume per unit time, although the heat generation rates are allowed to vary through the
volume of the element. This variation is an interpolation defined by single values of heat generation at the nodes—described in ANSYS® as a nodal body force load. Since for the most part strains only vary in the radial direction, the same heat generation rate value (calculated from Equations 37 and 39) is applied to all of the nodes that lie at the same radial distance away from the centerline of the shaft. The strains used to calculate the heat generation rates are taken from the sampling elements. For the nodes on the inner diameter of the shaft, strains from Layer 1 are used to calculate $q_1''$. For the mid-nodes of that element on the inner diameter, strains from Layer 4 and Layer 5 are used to calculate $q_2''$. The layers from which strains are sampled to calculate heat generation rates are summarized in Figure 81.

The thermal segment now has all necessary geometry, material properties, and loading cases to be solved. The thermal segment solves for the steady state temperature distribution in the shaft. The model then averages the temperatures at the 5 nodes shown in Figure 80, compares this temperature to the previous temperature of the shaft, and iterates until it finds a convergence on temperature (no variation beyond the tenths position). Since the simulation macro uses the temperature and frequency dependent
material properties to calculate heat generation rates, and the building macro uses these
same material properties to solve the structural segment of the model, they must be
updated in both files when modeling different materials.

6.2 Spin Testing

Spin testing allows for experimental validation of the heating models. Only FMC
tubes were tested. The nominal strains along the length of the tubes are measured, and
then the tubes are spun while the rig measures the change in temperature of the tube. The
strains measured are used as inputs for the heating models. The final steady state
temperature of the tubes in the models is compared to the final temperature of the tube in
the experiment. Spin testing also gives an idea about the strain, speed, and temperature at
which the tubes will fail.

6.2.1 Experimental Set-Up

The spin rig was built by Dr. Ying Shan, Penn State, during his doctorate work for
investigating dynamic behavior of FMC tubes. Basically there are four bearings, two on
each side. One set of bearings is fixed in place. These are located by the motor (Figure
82). The other set can move on a track that has different pin holes corresponding to an
intended nominal flexural strain on the tube. The flexural strain on the tube in Figure 82
is about 0.75%. The positions of the movable bearings are designed to impose pure
bending on the tube. Speed is measured by an infrared tachometer. Temperature is
measured optically, using infrared thermocouples at the various distances along the tube.
A conventional constantan-iron J-type thermocouple is used to measure the ambient air
temperature. End fittings extend from the bearings and are fit onto the ends of the tube.
There are also two position detectors on the rig each consisting of an IR emitter
underneath the tube and in IR detector on the top. When the tube is in the correct position, it blocks the IR signal to the detector. In the event that a tube should break, the transmission will be received by the detector and will trigger a relay switch to shut off the motor. Thermocouple and tachometer outputs were recorded using a data acquisition computer running a LabVIEW program, also designed by Dr. Ying Shan.

Before the motor turns on, the tubes are positioned into the rig and the actual nominal strain is measured by a clip gage (briefly described in Section 4.1.2). The gage is calibrated and then fastened to the tube at 7 different positions along the length. While the gage is fastened to one section of the tube, the movable bearings are shifted from the initial unbent configuration to the various pin holes on the track so that a measurement can be made at each of the pin hole settings. Both tensile and compressive strains are measured. An example of the results from one of these measurements is shown in a spatial plot in Figure 83 for a ±60° tube. The variation along the length is not significant for this tube or the other winding angles, so the values of strain on the tensile and
compressive sides of the tube are averaged for each pin position. These averaged values (Table 22) are then used in the temperature simulation models.

Figure 83: Nominal strain versus position in a ±60° FMC tube

Table 22: Actual and measured nominal strains for three different FMC tubes

<table>
<thead>
<tr>
<th>Fiber Winding Angle</th>
<th>Pin Hole Setting (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>±30°</td>
<td>0.19</td>
</tr>
<tr>
<td>±45°</td>
<td>0.22</td>
</tr>
<tr>
<td>±60°</td>
<td>0.22</td>
</tr>
</tbody>
</table>

6.2.2 Temperature Increase

As previously mentioned, temperatures along the tube are measured using non-contacting optical thermocouples. The approach to characterize temperature increase on the misaligned FMC tubes is to measure temperatures at different positions along the length of the tube as the tube is spun with a designated speed and nominal flexural strain defined by the position of the movable bearing. Three different fiber angles (±60, ±45, ±30), three different speeds (1200 rpm, 1800 rpm, 2400 rpm), and three different nominal misalignment strains (0.25%, 0.50%, 0.75%) were evaluated. The ±30° tube was only
evaluated at the 0.25% pin hole setting because it was believed that the high stiffness of
the tube may damage the rig with higher strain settings. For each set of conditions, the
motor was turned on and output was recorded using a data acquisition computer. After
temperatures in the tube reached steady state and were measured at all of the locations
along the tube, the motor was turned off. Sample test results are shown in Figure 84.

Figure 84: Steady state temperature at various locations and speed versus time for a ±45° FMC tube
at 0.50% flexural strain

The differences in temperature along the length shown in Figure 84 could be due
to non-uniform strain along the length of the tube, but since the strain measurements in
Figure 83 do not show non-uniform strain, this result is most likely attributed to some
kind of external heat conduction from the movable bearings or slipping in the grips. The
data presented in Figure 84 is simply one data point for one tube at one strain setting and
speed. So, it is beneficial to process the results of all the tests and arrange the steady state
temperatures at different speeds and nominal strains into spatial plots for each tube. This
kind of plot is shown in Figures 85 – 87 for ±60°, ±45°, and ±30° FMC tubes where ε₀ is
the intended nominal strain from the pin holes. The 60° tube failed at a nominal strain of
0.75%, at a speed of 1200 rpm and at the quarterspan position close to the motor. The 45°
tubeb also failed at 0.75% nominal strain but at 1800 rpm and at the midspan location. The 30° tube did not fail but saw extensive heating at the far end near the movable bearings.

![Figure 85: Temperature increase versus position for a ±60° FMC tube](image1)

![Figure 86: Temperature increase versus position for a ±45° FMC tube](image2)
All tubes saw more heating at the end near the movable bearings. The result is not a rational consequence of the quasi-static strain measurements, and it could be attributed to the tube slipping in the grip at this end, or the bearings may be heating up excessively due to friction and conducting heat to the tubes through the grips. The 30° tube saw so much heating at this location that the results are disregarded and are not used for comparison to the simulation. In any case, without considering the 30° tube, the temperature variation along the length is substantial, and one temperature must be chosen for each case so that a comparison to the predictions from the different simulations can be made. For the present work, the temperature at the midspan position and the average temperature along the length of the tubes was selected for comparison to the predictions from the simulations.

6.3 Self-Heating Analyses and Experimental Results

All simulations assume an ambient temperature of 21°C. For a first comparison, it is useful to evaluate the results for a particular set of conditions from different analyses side by side. For a ±45° FMC tube spinning at 2400 rpm, values of the calculated
temperature increase, $\Delta T$, in the tube from the homogenous and laminated closed-form and finite element analyses versus nominal strain are given in Figure 88. The results from the three analyses seem to agree well for the current tube geometry, speed, and applied strain. At all misalignment strains the finite element analysis predicts a lower $\Delta T$ than the closed-form analyses, although this effect is more pronounced at higher strains. At the lowest strain the temperature increase values calculated by the models differ by 0.32°C while at 0.66% strain they differ by about 7.59°C. These values are summarized in Table 23.

![Graph](image)

**Figure 88**: Calculated $\Delta T$ versus nominal strain, $\pm45^\circ$ FMC tube at 2400 rpm

**Table 23**: Calculated values of $\Delta T$ for different nominal strains and heating analyses, $\pm45^\circ$ FMC tube at 2400 rpm

<table>
<thead>
<tr>
<th>Self-Heating Analysis</th>
<th>Homogeneous, $\Delta T$ (°C)</th>
<th>Laminated, $\Delta T$ (°C)</th>
<th>Finite Element, $\Delta T$ (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal Strain (%)</td>
<td>0.22</td>
<td>7.98</td>
<td>7.66</td>
</tr>
<tr>
<td>0.45</td>
<td>28.77</td>
<td>28.31</td>
<td></td>
</tr>
<tr>
<td>0.66</td>
<td>47.76</td>
<td>46.91</td>
<td></td>
</tr>
</tbody>
</table>

As mentioned, the results shown in Figure 88 and Table 23 show that finite elements predict a lower value of $\Delta T$ than the closed-form analyses. An attempt at
explaining this result is given here. The differences in the two kinds of analyses is believed to originate from the calculation of heat generation rates. Dissipated heat energy is calculated via Equation 38. The loss factors and the moduli in this equation are inputs determined from the material property models. The strains are determined by solving the structural model in the finite element analysis, and from the processes shown in Figure 75 in the closed-form analyses. Since bending of the tube is defined by a kinematic boundary condition, the axial strains in both kinds of analyses should be similar. However, the hoop and out-of-plane strains are dependent on constitutive relationships. Finite elements are known to produce results that make the response of the structure appear to be stiffer than the results from continuum solutions—especially in bending. These discrepancies result from numerical problems such as shear-locking (Taylor et al., 1997) and hourglassing. If the response of the composite tube in the finite element analysis is stiffer, it would result in less strain, which means a lower value of ΔT. Also, the radial stresses vary in a parabolic way through the wall thickness of the tube. Since the quadratic through-thickness displacements of the 3D brick elements in the finite element analysis results in linear variations of stress and strain, the parabolic radial stress distribution of the radial stress may not be captured accurately. These differences in stress and strain could lead to differences in computed heat generation rates through the wall thickness of the tube.

The differences in the results from the homogeneous and laminated closed-form analyses are very minor. However, it has been observed that for very thick-walled tubes, the results start to differ slightly. Sample calculations with a few different FMC tubes (±45° winding angle) with ranging geometries spinning at 1200 rpm with a misalignment strain of 0.66% were performed to illustrate this issue (Figure 89). It is useful to define a
geometric ratio, $r_m/t$, where $r_m$ is the mean radius and $t$ is the thickness of the tube that describes how thick or thin a tube may be. This ratio is about 8.4 for the geometry of the tubes under investigation. The outer diameter from the current tube geometry was held constant. At an $r_m/t$ value of 10, the percent difference between the two final values of $\Delta T$ is only 0.3%. For an $r_m/t$ value of 2, the percent difference between the calculations from the homogenous and laminated heating models is about 11%. This difference is illustrated in Figure 89. It must originate from some slight dissimilarities of the stress distribution in the elasticity solutions for the homogeneous tube and the assembly of orthotropic hollow cylinders. The difference is substantial but not radical, and shaft designs with $r_m/t$ values of 5 or less are not likely.

A slight discrepancy between the finite element analysis and the closed-form analysis is that the average temperature is reported as the final temperature in the finite element model. In the closed-form models, the temperature at the outermost surface of the tube is reported. The temperature difference through the wall thickness of the tube is quite small in both models, however (less than 0.7°C). The temperature difference
through the thickness of a ±45° FMC tube at 1200 rpm and 0.22% misalignment strain calculated by the finite element model is shown in Figure 90.

![Figure 90: Temperature distribution through the thickness of an FMC tube](image)

The finite element analysis requires a little more computational power and time than the closed-form analyses. The finite element simulations were run on a Windows 32-bit operating system that had a processor that runs at 3.16 GHz with 3.65 GB of RAM. The closed-form models require less power and time. These were run on a Windows 32-bit operating system that had a processor that runs at 2.0 GHz with 2.05 GB of RAM. For the typical geometry and strain, closed-form simulations on MATLAB lasted less than 1 minute. For thick-walled geometries, simulations may have lasted up to 10 minutes. The CPU (central processing unit) time and the elapsed time for the finite element analysis should be distinguished. The CPU time is a measurement of how much time the processor is running during the model and the elapsed time is how long the model takes to run from start to finish. The elapsed time is always slightly longer. For example, when solving a model with a ±45° FMC tube at 0.66% misalignment strain spinning at 1200 rpm the CPU is 9 minutes 40 seconds and the elapsed time is 15 minutes 32 seconds.
Computation time is dependent only slightly on fiber angle and speed and mostly on misalignment strain. For example it takes the finite element model 17 iterations to converge on a final temperature to within 0.1°C for the conditions mentioned above and only 4 iterations for a misalignment strain of 0.22% (CPU time = 3 minutes 38 seconds). This kind of convergence is illustrated in Figure 91.

As mentioned previously, the measured temperatures for the tubes at the midspan position and the average temperature along the length of the tubes were compared against analytical results. This comparison is important because it serves as a means of validation for the closed-form and finite element analyses. These experimental data points are shown with the analytical predictions of temperature increase from the laminated closed-form code and the finite element code in Figures 92 – 94.

Figure 91: Convergence for the finite element model
Figure 92: Calculated and measured $\Delta T$ versus speed and nominal strain, $\pm 45^\circ$ FMC tube

Figure 93: Calculated and measured $\Delta T$ versus speed and nominal strain, $\pm 60^\circ$ FMC tube
Figure 94: Spatial representation of the calculated and measured $\Delta T$ for the ±30° FMC tube at 0.19% strain

The analytical predictions and the measured values of temperature increase match well for the specimens under consideration. The final value of temperature increase calculated by the finite element analysis is always slightly less than the value calculated by the laminated closed-form analysis. This difference is pronounced at higher strains.

The uneven heating distribution from spin testing the ±30° tube can be clearly seen when comparing the analytical predictions of $\Delta T$ to the experimental result in Figure 94. For the ±45° tube, the analytical predictions from finite elements align well with the temperature measured at the midspan location. Average tube temperatures align well with the predictions from the laminated closed-form analysis at higher strains. For the ±60° tube, the analyses also agree well with experimental results. In general, larger misalignment strains and higher speeds produce a larger increase in temperature.

Misalignment strain has the larger effect, though. This can be explained when looking at Equation 37 which illustrates the dependence of the heat generation rate on strain squared. Also, heat transfer by forced convection increases as the tube spins faster.
A relevant comparison to make is the difference in the heating of two FMC tubes with the stiffer LF750D matrix and the more compliant L100 matrix (reported in an earlier work) tested under the same conditions. At the nominal strain setting of 0.5% and at a speed of 2400 rpm, the ±45° FMC tube used in the present investigation shows a temperature increase of about 25°C. Under the same testing conditions, a ±45° tube with the L100 resin shows a temperature increase of about 5°C (Shan, 2006). This factor of 5 increase in temperature is a consequence of the stiffness of the LF750D tubes. Equation 37 illustrates that strain energy loss from each component in the ply is directly proportional to the stiffness of that component. For example, the transverse stiffness of the LF750D composite is about 6 times that of the L100 composite. Although the stiffer and stronger LF750D material is likely to provide optimized design solutions that are lighter than L100 solutions, they will experience more heating. However, for the low misalignment strains (maximum 2000 με) an FMC shaft would experience in a real service application, self-heating may not be a major issue.

Another factor that may be considered when modeling self-heating in FMC shafts is heat transfer from the shaft to its surroundings by radiation. The emissivity of the tubes used here has not been measured, but in (Shan, 2006) it was assumed to be equal to 1 (for an unpainted carbon fiber composite). If radiation heat transfer is included on the surface of the shaft, it can affect the results by a few degrees Celsius at higher strains. For example, the laminated closed-form analysis says that the difference between the final values of temperature increase in a ±45° FMC tube spinning at 2400 rpm and 0.66% misalignment strain with (emissivity = 1) and without radiation heat transfer is 4.8°C. The inclusion of radiation heat transfer in the laminated closed-form analysis for the testing conditions of the ±45° tube is illustrated in Figure 95.
Figure 95: Calculated $\Delta T$ with and without radiation versus speed and nominal strain, $\pm 45^\circ$ FMC tube

Finally, a comparison is made between the calculated heating results from laminated closed-form analysis and the finite element analysis for multi-angled FMC tubes. The tubes in this comparison have an inner diameter of 19.79 mm, an outer diameter of 22.30 mm, a strain of 0.66% and a speed of 2400 rpm. Layup sequences start at the inner diameter and end at the outer diameter. The finite element model currently has a limitation on the stacking sequence. Any discrete sequence can be selected within an element, but this same sequence must be applied to all elements through the thickness. Hence, the $[\pm 45/\pm 60]$ and $[\pm 60/\pm 45]$ layups are unavailable for analysis in the finite element model. First it is observed that the results (Figure 96) from the two models for single angle-ply tubes ($\pm 45^\circ$ and $\pm 60^\circ$) show that the finite element model predicts lower $\Delta T$ values than the laminated closed-form model. Both models predict a lower value of $\Delta T$ when $\pm 45^\circ$ layers are on the outer diameter. This is most likely a consequence of the layers generating more heat being located closer to the convective heat transfer surface.
6.4 Summary

Since the results from Chapter 6 show that both heating analyses developed in the current work seem to match well with the results from the previous self-heating analysis developed by Shan and Bakis (2009) and current experimental results, a few discussion points arise. First, the laminated and finite element heating codes agree with the homogeneous code, so the two newly development heating analyses that calculate the steady state temperature distribution in an FMC shaft have been validated with a previous self-heating analysis. Also, the correlation between analytical predictions and experimental results not only further validates the two new heating codes; it also suggests that the quasi-static and dynamic material properties have been accurately modeled for the new FMC material within appropriate temperature and frequency ranges.

The heating distribution along the length of the tube does not correspond with the quasi-static strain measurements. For this reason, some additional effect is believed to be the cause of the differential heating. The assumption that the excess heating on the end of
the tube located near the movable bearings was due to some kind of slipping or external heat transfer seems to be valid because the calculated values from the model align well with the temperatures measured at the midspan.

For single angle-ply laminates, the homogeneous and laminated closed-form codes are in good agreement. These codes run faster than the finite element code. For this kind of analysis either one can be selected for accuracy and convenience. The modeling of multi-angled layups has been done for both codes. No experiments exist to-date against which these calculations could be compared.
Chapter 7 Conclusions and Recommendations for Future Work

Material development, characterization of the ballistic tolerance of FMCs, and improved analytical tools are contributions from the current investigation to the implementation of FMC driveshafts in rotorcraft. Details of these contributions and recommendations for future work on this research topic are given in the following sections.

Materials and Fabrication Methods

High volume fraction filament wound tubes and flat plate coupons appeared to be of good consistency in this investigation on the basis of uniform ply thicknesses near target values. However, future work should assess void content in the FMC materials manufactured by the various employed methods since voids that vary by specimen type could lead to inconsistencies in measured properties.

The use of heat-activated shrink tape for consolidation of tubes causes slight variations in outside radius point-by-point, which may be an issue for the dynamic balance of FMC shafts in a real application. A potential solution to the uneven outer surface could be fabricating shafts in a closed mold using vacuum assisted resin transfer molding.
Another issue that can arise from the application of shrink tape is a disturbance of the 0° orientation of the fibers in thin-wall unidirectional tubes for shear DMA testing. Local disturbances can not only alter the orientation of the fibers, they can also result in an uneven final geometry of the part. Applying a more uniform pressure by the use of an automated tape wrapping device could alleviate the uneven cross-sectional geometries of the small unidirectional tubes.

**Ballistic Testing**

A few aspects of the ballistic testing program must be considered before conclusions can be drawn. The two types of tubes (with rigid and flexible matrix materials) are test coupons—not driveshafts. The tubes have the same geometry and fiber winding angles, but they were definitely not designed to carry the same axial and torsional loads. Also, the size and shape of the projectile were chosen out of convenience and do not correspond to any particular threat. So, while much was learned about the specific testing conditions carried out in this research, extrapolating the conclusions to full-scale driveshafts must be done with caution. As discussed in the literature review, Ochoa et al. (1991) observed that the effective hole diameter in ballistically impacted RMC tubes was about 10 times the diameter of the projectile. In the present investigation, the projectile diameter was around one-fourth the tube diameter, meaning that the effective hole size might be well over the diameter of the tube. Ochoa’s tests were quite different than those of the current investigation in regard to materials, geometry, layup, and projectile energy, but the effective hole diameter results of Ochoa suggest that the tube diameter selected in the current investigation was too small or the projectile diameter was too large.
From the methods of damage detection explored in the current investigation such as visual inspection, energy absorption, X-radiography, and full-field strain measurements, damage in the composite tubes due to the selected projectile was extensive. Although the number of data points is limited, the full-field strain, energy absorption, and residual strength measurements all suggested that the FMC tubes sustained more damage than the RMC tubes. In spin testing, localized heating and failure of damaged FMC tubes was observed. The FMC and RMC tubes saw knockdowns in stiffness and strength of roughly 50% in tension and compression. In torsion, the FMC knockdowns in stiffness (50%) and strength (90%) were substantially larger than the RMC knockdowns in stiffness (15%) and strength (50%). The noticeable reduction in structural properties of the FMC in torsion is attributed to the fact that the fibers take much of the torsional load with the fibers at ±45° and ±60°. Because the fibers were ripped out more in the FMC as evidenced by the “bird’s nest” failure, the reduction in properties is most easily seen when the fibers are loaded.

For future investigations, some alterations from the current testing program, namely a larger tube-to-projectile ratio, may provide a better understanding of the ballistic tolerance of flexible matrix composites. The use of different types of fibers or matrix materials at strategic locations through the thickness of the tubes is another idea that might reduce the “bird’s nest” damage mode caused by ballistic impact.

**Quasi-Static Characterization**

It is felt that adequate results for determining the minor Young’s modulus and in-plane shear modulus of undirectional FMC material were obtained by testing hoopwound and ±45° tubes in tension and compression. However, a good way to determine the
major Young’s modulus of an FMC ply that accurately predicts the axial modulus of filament wound tubes in tension and compression was not found. The classical approach of modeling the modulus of a laminated composite using lamination theory does not match experimental results of angle-ply filament-wound FMC tubes in quasi-static tension, compression, and torsion for loadings and fiber orientations that stress the fibers the most. Values of major ply modulus backed out of lamination theory can be as low as one third that predicted by the rule of mixtures for unidirectional composites. Matrix-dominated laminate stiffness behavior was able to be predicted well with lamination theory despite the questionable values for the major ply modulus.

An examination of the elastic response of composites with undulated fibers supported in a soft, flexible matrix is recommended to accurately model the behavior of filament wound FMC tubes. The assumptions made in classical lamination theory were not made in consideration of fiber undulation effects. A recent investigation by Zindel (2009) modeled with moderate success the effect of undulations in filament wound tubes with soft matrix materials. In order to improve tools used for FMC shaft design, it is suggested to implement a newly developed model for the elastic response of FMC tubes into the self-heating analysis developed in the current investigation and into the optimization analysis developed by Mayrides (2005).

**Dynamic Characterization**

The fractional derivative model developed by Shan and Bakis (2009) continues to be a good way to characterize the dependency of the material properties of flexible matrix composites on temperature and frequency. The axial dynamic mechanical analyzer used in the current investigation is a convenient tool for measuring the viscoelastic behavior of
Using a flat 90° FMC coupons, although the load limit of the machine and the difficulty of making extremely thin coupons prevented measurements of stiffer composites such as those made with epoxy. The torsional dynamic mechanical analyzer used in the current investigation is an appropriate tool for measuring the viscoelastic behavior of $G_{12}$ using a 0° filament-wound tube. However, experimental evidence suggested that the adhesive used to bond the tubes into the disk-shaped grips (Devcon 5 Minute Epoxy made by ITW Devcon) softened above 80°C and limited the temperature range of valid test results for dynamic $G_{12}$.

For stiff materials, a machine with a high load limit could be used to characterize the viscoelastic response of 90° flat plate FMC coupons or alternate gripping methods could be used on the same DMA machine—such as a bending set-up. An adhesive that has a higher glass transition temperature (perhaps cyanoacrylate) is recommended for use in the future where additional data on the dynamic shear properties of an FMC above 80°C is required. Note that, in a typical aerospace application, the highest temperature of interest is typically 71°C (160°F). In the torsional DMA, a similar approach to Mead and Cudmore’s (1992) free vibration method of measuring the two shear moduli of a transversely isotropic rectangular cross-sectioned composite bar could be used as another possible means of measuring the dynamic shear behavior of FMC materials. Also, measuring dynamic material properties as a function of strain level might lead to improved predictions for the heating models since the strain range used in the axial and torsional DMA tests in this investigation were roughly an order of magnitude less than those sustained in the tube spin tests.
Self-Heating

Misalignment strain was able to be quasi-statically measured in FMC tubes in the Penn State spin rig. At the far end of the rig (opposite the motor), tubes saw excess heating that was most likely due to slipping in the grips or heating conduction from the bearings. Temperature measurements by optical non-contacting thermocouples during the tests showed that stiffer tubes experienced more heating. Although FMC tubes with LF750D were stiffer and stronger than similar L100 tubes tested by Shan (2006), at the same speed and strain the LF750D tubes saw temperature increases that were higher than the L100 tubes by about a factor of 5. Although for speeds of up to 2400 rpm and strains of up to 2500 με, midspan temperature increases in 45 and 60 degree angle-ply FMC tubes with the LF750D matrix were less than 10°C. The 30° tubes were too stiff to be properly tested with uniform strain in the PSU spin test rig.

The two self-heating models developed in the current investigation (laminated closed-form and finite element) agree with the homogeneous self-heating model of FMC shafts developed by Shan and Bakis (2009) and with the current experimental results to within a few tenths of a degree for low strain and a few degrees for higher strains. Based on a sample calculation, the two newly developed models agree for analyzing self-heating in shafts with multiple fiber winding angles.

A few additional aspects could be included in the self-heating analyses that may contribute to improved results. One value of the major in-plane Young’s modulus, $E_1$, must be chosen to run the self-heating analyses. Quasi-static material property characterization showed that the classical approach to modeling the stiffness of FMC tubes did not work well. So, perhaps implementing a refined structural response theory into the heating code would more accurately predict stress and strain and consequently
temperature increase. The convective heat transfer coefficient determined from the model
developed by Kendoush (1996) has a large influence on the final temperature of the shaft.
An experimental measurement of the convective coefficient may also contribute to
improved results. Additional measurements of the dynamic shear properties of the
composite will also enable the improved calculation of self-heating for high temperatures.

The generality of the algorithm in the finite element model that picks strains from
the structural results to calculate heat generation could be improved. If this algorithm
were general for all through-thickness and circumferential mesh densities, it could be
useful in capturing the variation of the radial stress through the wall thickness. Additions
to the finite element framework developed in the present investigation to include
complicated boundary conditions and bearing support may be useful in the future.
References


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Appendix A  ANSYS Macros

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ESLE,5
ESURF
ALLSEL
ESEL,ALL
ESEL,S,TYPE,,4
ESEL,A,TYPE,,5
ESEL,R,REAL,,4
/PSYMB,ESYS,1
/PNUM,TYPE,1
/NUM,1

EPLOT

ESEL,ALL
ESEL,S,TYPE,,4
ESEL,A,TYPE,,5
ESEL,R,REAL,,4
CMSEL,A,_NODECM
CMDEL,_NODECM
CMSEL, A, _ELEMCM
CMDEL, _ELEMCM
CMSEL, S, _KPCM
CMDEL, _KPCM
CMSEL, S, _LINECM
CMDEL, _LINECM
CMSEL, S, _AREACM
CMDEL, _AREACM
CMSEL, S, _VOLUMCM
CMDEL, _VOLUMCM
!GRES, cwz, gsav
CMDEL, _TARGET
CMDEL, _CONTACT
//COM, CONTACT PAIR CREATION -END
epio
FLST, 2, 1, 1, ORDE, 1
FITEM, 2, 25133
!*
GO
D, P51X, , 0, , , UX, UY, UZ, , ,
FLST, 2, 1, 1, ORDE, 1
FITEM, 2, 25133
!*
GO
D, P51X, , phi, , , , ROTX, , , ,
FLST, 2, 1, 1, ORDE, 1
FITEM, 2, 25134
!*
GO
D, P51X, , -phi, , , , ROTX, , , ,
FLST, 2, 1, 1, ORDE, 1
FITEM, 2, 25134
!*
GO
D, P51X, , 0, , , UX, UY, , ,
/VIEW, 1, 1, 1, 1
/ANG, 1
/REP, FAST
/AUTO, 1
/REP, FAST
/prep7
esel, s, ename,, 186
keyopt, 1, 8, 1
!*
MPTEMP, , , , , , ,
MPTEMP, 1, 0
MPDATA, KXX, 1,, kt
!*
etchg, att
FLST, 2, 4, 5, ORDE, 4
FITEM, 2, 12
FITEM, 2, 18
FITEM, 2, 24
/GO
!*
SFA, P51X, 1, CONV, hconv, _tref
FLST, 2, 16, 5, ORDE, 11
FITEM, 2, 1
FITEM, 2, -5
FITEM, 2, 7
FITEM, 2, -8
FITEM, 2, 10
FITEM, 2, -11
FITEM, 2, 13
FITEM, 2, -16
FITEM, 2, 19
FITEM, 2, -20
FITEM, 2, 22
/GO
!*
SFA, P51X, 1, HFLUX, 0
A.2 ‘tempinr3.mac’

```plaintext
_tconv=0.1
_maxiter=5
_tol=(od-id)/20

;csys,1
nsel,,loc,x,id/2-_tol,id/2+_tol
    cm,nodes1,node
nsel,,loc,x,od/2-_tol,od/2+_tol
    cm,nodes5,node
nsel,,loc,x,(id+od)/4-_tol,(id+od)/4+_tol
    cm,nodes3,node
nsel,,loc,x,((id+od)/4+id/2)/2-_tol,((id+od)/4+id/2)/2+_tol
    cm,nodes2,node
nsel,,loc,x,((id+od)/4+od/2)/2-_tol,((id+od)/4+od/2)/2+_tol
    cm,nodes4,node

;csys,0
nsel,,loc,z,1/2-0.00001,l/2+0.00001
nsel,r,loc,x,-0.00001,0.00001
nsel,r,loc,y,0,od
    cm,tempnodes,node
esln
    cmse,,nodes1
esln,r
    _elid=elnext(0)
    cmse,,tempnodes
esln
    cmse,,nodes5
esln,r
    _elod=elnext(0)
esel,,elem,,_elid
esel,a,elem,,_elod
    cm,maxelem,elem
alls
;csys
/prep7
;_tref
;_tavg
etchnxt,txts
;keyopt,1,3,1
keyopt,1,8,1
/output,temp_iter,txt,,
*vwrite,0,_tref
;_f10.1, ',', _f10.1
/output
*do,_iter,1,_maxiter
;solu
;solve

_e=2.71828183
_E=0.2148*1e9
_a1=0.212
_b1=4.9357
_beta1=0.2275
_a2=5.99e-5
_b2=0.0039
_beta2=0.8106
```
$$\alpha_1 = \beta_1$$
$$\alpha_2 = \beta_2$$

$$c_{01} = -17.02$$
$$c_{02} = 230.3$$

$$a_{ee} = (c_{01}/(t_{avg} - 333))/c_{02}$$

$$a = b_1 \cos(\pi \beta_1/2) * f*_{at}** beta_1 + b_2 \cos(\pi \beta_2/2) * f*_{at}** beta_2$$
$$c = a_1 \cos(\pi \alpha_1/2) * f*_{at}** alpha_1 + a_2 \cos(\pi \alpha_2/2) * f*_{at}** alpha_2$$

$$A = a + 1$$
$$B = b_1 \sin(\pi \beta_1/2) * f*_{at}** beta_1 + b_2 \sin(\pi \beta_2/2) * f*_{at}** beta_2$$
$$C = c + 1$$
$$D = a_1 \sin(\pi \alpha_1/2) * f*_{at}** alpha_1 + a_2 \sin(\pi \alpha_2/2) * f*_{at}** alpha_2$$

$$Ey = E*(A*C + B*D)/(C**2 + D**2)$$
$$etay = (B*C - A*D)/(A*C + B*D)$$

$$G = 0.1571 * 1e9$$
$$a_1 = 2.8878$$
$$b_1 = 0.023$$
$$\beta_1 = 2e-11$$
$$a_2 = 0.517$$
$$b_2 = 4.6893$$
$$\beta_2 = 0.2433$$

$$\alpha_1 = \beta_1$$
$$\alpha_2 = \beta_2$$

$$c_{01} = -29.46$$
$$c_{02} = 264.1$$

$$a_{ee} = (c_{01}/(t_{avg} - 333))/c_{02}$$

$$a = b_1 \cos(\pi \beta_1/2) * f*_{at}** beta_1 + b_2 \cos(\pi \beta_2/2) * f*_{at}** beta_2$$
$$c = a_1 \cos(\pi \alpha_1/2) * f*_{at}** alpha_1 + a_2 \cos(\pi \alpha_2/2) * f*_{at}** alpha_2$$

$$A = a + 1$$
$$B = b_1 \sin(\pi \beta_1/2) * f*_{at}** beta_1 + b_2 \sin(\pi \beta_2/2) * f*_{at}** beta_2$$
$$C = c + 1$$
$$D = a_1 \sin(\pi \alpha_1/2) * f*_{at}** alpha_1 + a_2 \sin(\pi \alpha_2/2) * f*_{at}** alpha_2$$

$$G_{xy} = G*(A*C + B*D)/(C**2 + D**2)$$
$$etaxy = (B*C - A*D)/(A*C + B*D)$$

$$\eta_{xy} = 0.0011$$
$$E_x = 79e9$$
$$E_z = Ey$$
$$\eta_{ez} = etaxy$$

/ps

layer,1
cmse,maxelem
rsys,solu
layer,1
etab,epel,x
etab,epel,y
etab,epel,z
etab,epel,xy
*get, epelix1, etab, 1, elem, elid
*get, epely1, etab, 2, elem, elid
*get, epelz1, etab, 3, elem, elid
*get, epelixy1, etab, 4, elem, elid

$$q = b_1 \cos(\pi \beta_1/2) * f*_{at}** beta_1 + b_2 \cos(\pi \beta_2/2) * f*_{at}** beta_2$$
$$q = b_1 \sin(\pi \beta_1/2) * f*_{at}** beta_1 + b_2 \sin(\pi \beta_2/2) * f*_{at}** beta_2$$
$$q = a_1 \sin(\pi \alpha_1/2) * f*_{at}** alpha_1 + a_2 \sin(\pi \alpha_2/2) * f*_{at}** alpha_2$$

$$Q = (q_x + q_y + q_z + q_{xy})*f$$

layer,1
etab, epel, x
etab, epel, y
etab, epel, z
get, _epelx4, etab, 1, elem, _elid
get, _epely4, etab, 2, elem, _elid
get, _epelz4, etab, 3, elem, _elid
get, _epelxy4, etab, 4, elem, _elid

_qx = _pi * _etax * _Ex * _epelx4**2
_qy = _pi * _etay * _Ey * _epely4**2
_qz = _pi * _etaz * _Ez * _epelz4**2
_qxy = _pi * _etaxy * _Gxy * _epelxy4**2

_qq4 = (_qx + _qy + _qz + _qxy) * _f

layer, 5
etab, epel, x
etab, epel, y
etab, epel, z
get, _epelx5, etab, 1, elem, _elid
get, _epely5, etab, 2, elem, _elid
get, _epelz5, etab, 3, elem, _elid
get, _epelxy5, etab, 4, elem, _elid

_qx = _pi * _etax * _Ex * _epelx5**2
_qy = _pi * _etay * _Ey * _epely5**2
_qz = _pi * _etaz * _Ez * _epelz5**2
_qxy = _pi * _etaxy * _Gxy * _epelxy5**2

_qq5 = (_qx + _qy + _qz + _qxy) * _f

_Q2 = (_qq4 + _qq5) / 2

layer, 8
etab, epel, x
etab, epel, y
etab, epel, z
get, _epelx8, etab, 1, elem, _elid
get, _epely8, etab, 2, elem, _elid
get, _epelz8, etab, 3, elem, _elid
get, _epelxy8, etab, 4, elem, _elid

_qx = _pi * _etax * _Ex * _epelx8**2
_qy = _pi * _etay * _Ey * _epely8**2
_qz = _pi * _etaz * _Ez * _epelz8**2
_qxy = _pi * _etaxy * _Gxy * _epelxy8**2

_qq8 = (_qx + _qy + _qz + _qxy) * _f

layer, 1
etab, epel, x
etab, epel, y
etab, epel, z
get, _epelx1, etab, 1, elem, _elod
get, _epely1, etab, 2, elem, _elod
get, _epelz1, etab, 3, elem, _elod
get, _epelxy1, etab, 4, elem, _elod

_qx = _pi * _etax * _Ex * _epelx1**2
_qy = _pi * _etay * _Ey * _epely1**2
_qz = _pi * _etaz * _Ez * _epelz1**2
_qxy = _pi * _etaxy * _Gxy * _epelxy1**2

_qq1 = (_qx + _qy + _qz + _qxy) * _f

_Q3 = (_qq3 + _qq1) / 2

layer, 4
etab, epe1, x
etab, epe1, y
etab, epe1, z
get, epe1x, tab, 1, elem, elod
get, epe1y, tab, 2, elem, elod
get, epe1z, tab, 3, elem, elod
get, epe1xy, tab, 4, elem, elod

_qx = pi * etax * Ex * epe1x ** 2
_qy = pi * etay * Ey * epe1y ** 2
_qz = pi * etaz * Ez * epe1z ** 2
_qxy = pi * etaxy * Gxy * epe1xy ** 2

_qq4 = (qx + qy + qz + qxy) * f

layer, 5
etab, epe1, x
etab, epe1, y
etab, epe1, z
get, epe1x, tab, 1, elem, elod
get, epe1y, tab, 2, elem, elod
get, epe1z, tab, 3, elem, elod
get, epe1xy, tab, 4, elem, elod

_qx = pi * etax * Ex * epe1x ** 2
_qy = pi * etay * Ey * epe1y ** 2
_qz = pi * etaz * Ez * epe1z ** 2
_qxy = pi * etaxy * Gxy * epe1xy ** 2

_qq5 = (qx + qy + qz + qxy) * f

_Q4 = (qq4 + qq5) / 2

layer, 8
etab, epe1, x
etab, epe1, y
etab, epe1, z
get, epe1x, tab, 1, elem, elod
get, epe1y, tab, 2, elem, elod
get, epe1z, tab, 3, elem, elod
get, epe1xy, tab, 4, elem, elod

_qx = pi * etax * Ex * epe1x ** 2
_qy = pi * etay * Ey * epe1y ** 2
_qz = pi * etaz * Ez * epe1z ** 2
_qxy = pi * etaxy * Gxy * epe1xy ** 2

_Q5 = (qx + qy + qz + qxy) * f

/prep7
alis
etchg, stt
bf, nodes1, hgen, Q1
bf, nodes2, hgen, Q2
bf, nodes3, hgen, Q3
bf, nodes4, hgen, Q4
bf, nodes5, hgen, Q5
/solu
solve
/post1
cmse, tempnodes
n = 0
_tsum = 0
*do, i, 1, 5
_n = ndnext(_n)
_t=TEMP(_n)  
_tsum=_tsum+_t  
*enddo  
_told=_tavg  
_tavg=_tsum/5  

/output,temp_iter,txt,,append  
*vwrite,_iter,_tavg  
{ F10.1, ',', F10.3}  
/output  
_delt=abs(_tavg-_told)  
*if,_delt,LT,_tconv,then  
STOP  
*endif  

/prep7  
etchg,tts  
alls  
keyopt,1,3,1  
keyopt,1,8,1  
tunif,_tavg  
*enddo
Appendix B  MATLAB Codes

B.1 ‘main_fraction_2_term.m’

```matlab
%2-term expression optimization process to determine the fractional
derivation parameters.
function main

m=csvread('c:\temp\E2Data\E2_data.csv');
E_log_freq=m(:,1);
E_store=m(:,2);
for i=1:length(E_log_freq)
    E_freq(i)=10^E_log_freq(i);
end

m=csvread('c:\temp\E2Data\yita2_data.csv');
yita_log_freq=m(:,1);
yita=m(:,2);
for i=1:length(yita_log_freq)
    yita_freq(i)=10^yita_log_freq(i);
end

m=csvread('c:\temp\E2Data\E2_loss.csv');
disp('Input Data:');
loss_freq=m(:,1);
E_loss=m(:,2);

optoptions=optimset('TolFun', 1e-10, 'Display', 'iter', 'LevenbergMarquardt', 'on',
'MaxFunEvals',... 
    5000, 'MaxIter', 5000);
para=fminsearch(@opti_fun_2,[1,1,1,0.5,1,1,0.5], optoptions, E_freq,...
    E_store, yita_freq, yita, loss_freq, E_loss) % You might need to change these value
    for best optimization

log_freq=-2:0.1:4;
E=para(1)
a1=para(2)
b1=para(3)
beta1=para(4)
a2=para(5)
b2=para(6)
beta2=para(7)
alpha1=beta1;
alpha2=beta2;

for i=1:length(log_freq)
    freq(i)=10^log_freq(i);
    A=1+b1*cos(pi*beta1/2)*freq(i)*beta1+b2*cos(pi*beta2/2)*freq(i)*beta2;
    B=b1*sin(pi*beta1/2)*freq(i)*beta1+b2*sin(pi*beta2/2)*freq(i)*beta2;
    C=1+alpha1*cos(pi*alpha1/2)*freq(i)*alpha1+alpha2*cos(pi*alpha2/2)*freq(i)*alpha2;
    D=alpha1*sin(pi*alpha1/2)*freq(i)*alpha1+alpha2*sin(pi*alpha2/2)*freq(i)*alpha2;
    E_cal(i)=E*(A*C+B*D)/(C^2+D^2);
    loss_cal(i)=E*(B*C-A*D)/(C^2+D^2);
    yita_cal(i)=(B*C-A*D)/(A*C+B*D);
end

figure(1);
plot(log_freq, E_cal, 'g', log10(E_freq), E_store, 'rx');
```
figure(2);
plot(log_freq, yita_cal, 'g', log10(yita_freq), yita, 'rx');

m=[log_freq', E_cal', yita_cal'];
csvwrite('c:\temp\E2Data\E_2para.csv', m);
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function y=opti_fun_2(para, E_freq, E_store, yita_freq, yita, loss_freq, E_loss)
%parameter number n=2
penalty=0;
for i=1:length(para)
  if para(i)<0.0
    penalty=0.9;
  end
end
E=para(1);
a1=para(2);
b1=para(3);
beta1=para(4);
a2=para(5);
b2=para(6);
beta2=para(7);
alpha1=beta1;
alpha2=beta2;
y_total1=0;
y_total2=0;
y_total3=0;
E2_norm=0;
yita_norm=0;
E2_loss_norm=0;
for i=1:length(E_freq)
  A=1+b1*cos(pi*beta1/2)*E_freq(i)^beta1+b2*cos(pi*beta2/2)*E_freq(i)^beta2;
  B=b1*sin(pi*beta1/2)*E_freq(i)^beta1+b2*sin(pi*beta2/2)*E_freq(i)^beta2;
  C=1+a1*cos(pi*alpha1/2)*E_freq(i)^alpha1+a2*cos(pi*alpha2/2)*E_freq(i)^alpha2;
  D=a1*sin(pi*alpha1/2)*E_freq(i)^alpha1+a2*sin(pi*alpha2/2)*E_freq(i)^alpha2;
  E_cal=E*(A*C+B*D)/(C^2+D^2);
  y_total1=y_total1+(E_cal-E_store(i))^2;
  E2_norm=E2_norm+E_store(i)^2;
end
for i=1:length(yita_freq)
  A=1+b1*cos(pi*beta1/2)*yita_freq(i)^beta1+b2*cos(pi*beta2/2)*yita_freq(i)^beta2;
  B=b1*sin(pi*beta1/2)*yita_freq(i)^beta1+b2*sin(pi*beta2/2)*yita_freq(i)^beta2;
  C=1+a1*cos(pi*alpha1/2)*yita_freq(i)^alpha1+a2*cos(pi*alpha2/2)*yita_freq(i)^alpha2;
  D=a1*sin(pi*alpha1/2)*yita_freq(i)^alpha1+a2*sin(pi*alpha2/2)*yita_freq(i)^alpha2;
  yita_cal=(B*C-A*D)/(C^2+D^2);
  y_total2=y_total2+(yita_cal-yita(i))^2;
  yita_norm=yita_norm+yita(i)^2;
end
for i=1:length(loss_freq)
  A=1+b1*cos(pi*beta1/2)*loss_freq(i)^beta1+b2*cos(pi*beta2/2)*loss_freq(i)^beta2;
  B=b1*sin(pi*beta1/2)*loss_freq(i)^beta1+b2*sin(pi*beta2/2)*loss_freq(i)^beta2;
  C=1+a1*cos(pi*alpha1/2)*loss_freq(i)^alpha1+a2*cos(pi*alpha2/2)*loss_freq(i)^alpha2;
  D=a1*sin(pi*alpha1/2)*loss_freq(i)^alpha1+a2*sin(pi*alpha2/2)*loss_freq(i)^alpha2;
  loss_cal=E*(B*C-A*D)/(C^2+D^2);
  y_total3=y_total3+(loss_cal-E_loss(i))^2;
  E2_loss_norm=E2_loss_norm+E_loss(i)^2;
end
y=y_total1/E2_norm+y_total2/yita_norm+0*y_total3/E2_loss_norm+penalty;
end

function opti_fun_1
clear
close all
clc

i=0;
k=0;
j=0;
TEMP=[];
history = [];
format short e

RPM=1200;

freq=RPM/60;
Tamb=21; % Ambient Temperature in Degree C
Temp=Tamb;
i=i+1;

for j=1:1:22

    TEMP(j)=Temp;
    % all unit should be in SI unit
    % Temp=9/5*old_Tavg+32
    k=k+1;
    % E22
    E=0.2148;
a1=0.212;
b1=4.9357;
beta1=0.2275;
a2=5.99e-5;
b2=0.0039;
beta2=0.8106;
alpha1=beta1;
alpha2=beta2;

c01=-17.02; % shift factor matl const

    % shift factor matl const
    at=exp(c01*(Temp-60)/(c02+Temp-60)); % shift factor

    reduced_freq=at*freq;

    A=1+b1*cos(pi*beta1/2)*reduced_freq+2*cos(pi*beta2/2)*reduced_freq+beta2;
    B=b1*sin(pi*beta1/2)*reduced_freq+2*sin(pi*beta2/2)*reduced_freq;
    C=1+a1*cos(pi*alpha1/2)*reduced_freq+a2*cos(pi*alpha2/2)*reduced_freq;
    D=a1*sin(pi*alpha1/2)*reduced_freq+a2*sin(pi*alpha2/2)*reduced_freq;
    E2_cal=E*(A*C+B*D)/(C^2+D^2)*1e9;
    eta2_cal=(B*C-A*D)/(A*C+B*D);

    % G12
    G=0.1571;
a1=2.8878;
b1=0.023;
beta1=2e-11;
a2=0.317;
b2=4.6893;
beta2=0.2433;
alpha1=beta1;
alpha2=beta2;

c01=-29.46; % matl const

    % matl const
    at=exp(c01*(Temp-60)/(c02+Temp-60)); % shift factor

    reduced_freq=at*freq;

    A=1+b1*cos(pi*beta1/2)*reduced_freq+2*cos(pi*beta2/2)*reduced_freq+beta2;
    B=b1*sin(pi*beta1/2)*reduced_freq+2*sin(pi*beta2/2)*reduced_freq;
C=1+a1*cos(pi*alpha1/2)*reduced_freq^alpha1+a2*cos(pi*alpha2/2)*reduced_freq^alpha2;
D=a1*sin(pi*alpha1/2)*reduced_freq^alpha1+a2*sin(pi*alpha2/2)*reduced_freq^alpha2;
G12_cal=G*(A*C+B*D)/(C^2+D^2)*1e9;
eta12_cal=(B*C-A*D)/(A*C+B*D);

% all unit should be in SI unit
E11=79e9;
E22=E2_cal;%0.275e9;
E33=E22;%0.275e9;
G12=G12_cal;%0.250e9;
v12=0.33;%40;%0.38 0.3145
v21=E22*v12/E11;
v13=0.33;%40;
v23=0.87;%0.82
G23=E22/2/(1+v23);
h2=eta2_cal;
h12=eta12_cal;

prop(k,1,i)=E22;
prop(k,2,i)=G12;
prop(k,3,i)=G23;
sprop(k,1,i)=v21;
sprop(k,2,i)=h2;
sprop(k,3,i)=h12;

history=[history; Temp E22 G12 G23 v21 h2 h12];
Temp=Temp+2;

end

disp('     Temp (C)    E2 (GPa)    G12 (GPa)     G23 (GPa)       v21          h2      h12')
disp(history)
Appendix C  FEA Mesh Sensitivity Analysis

In order to verify that a circumferential mesh density of 36 (36 elements around the circumference of the tube) is satisfactory, a sensitivity analysis was carried out. The models used in the sensitivity analysis are similar to the model defined by the macro files in Appendix A.1 and A.2. These models used a slightly different material — the flexible matrix composite studied in the doctorate work of Dr. Ying Shan (2006). It is also a carbon/polyurethane composite, comprised of Toray T700 fibers in a resin made by combining an Adiprene® L100 prepolymer and a Caytur® 21 curative. The material properties of this composite are summarized in Table C.1. This composite was only used in the sensitivity analysis. Everywhere else in the thesis “the FMC” refers to the AS4D/LF750DD/C31 DA composite. In the sensitivity analysis, a ±30° L100 shaft is studied with an inner diameter of 19.84 mm and an outer diameter of 21.84 mm. The shaft has a speed of 2400 rpm and a misalignment strain of 0.7%. The results of the sensitivity analysis are shown in Figure C.1. Beyond a circumferential mesh density of 60, the final temperature does not vary in the tenths position of degrees Celsius. The final temperature value for a mesh density of 36 only differs from the value for a mesh density of 60 by 0.1°C, so it is selected to run simulations based on convenience in regard to CPU run time.
Table C.1: Material properties of T700/L100 FMC (Shan, 2006) used in FEA mesh sensitivity analysis

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$E_1$</td>
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Figure C.1: FEA mesh sensitivity analysis, final temperature and CPU time versus mesh density
Appendix D  Flash Videos

The playback features of all clips can be modified by right-clicking on the images and selecting options from the drop-down menu. The clips will play continuously and start when the reader enters the page in which they are embedded.

D.1 ‘Flexible Tube Impact.swf’

This is a side-view of a 4.763 mm steel ball bearing penetrating through both walls of a hollow ±45° FMC tube. This test was carried out in the ballistic impact lab, Building 49, at NASA Glenn Research Center. Frame rate and size are 40,000 frames per second and 320x240, respectively. The duration of the clip is 3 seconds.
D.2 ‘Flexible Tube Impact front.swf’

This is a front-view of a 4.763 mm steel ball bearing penetrating through both walls of a hollow ±45° FMC tube. This test was carried out in the ballistic impact lab, Building 49, at NASA Glenn Research Center. Frame rate and size are 40,000 frames per second and 320x240, respectively. The duration of the clip is 1 minute 12 seconds.
D.3 ‘Rigid Tube Impact.swf’

This is a front-view of a 4.763 mm steel ball bearing penetrating through both walls of a hollow ±60° RMC tube. This test was carried out in the ballistic impact lab, Building 49, at NASA Glenn Research Center. Frame rate and size are 40,000 frames per second and 320x240, respectively. The duration of the clip is 24 seconds.
D.4 ‘Flexible 45 Tension Damage strain map front.swf’

This is a front-view (entrance side) of a hollow ±45° FMC tube in tension. The color scheme overlaid onto images of the specimen during the test is a longitudinal (in the direction of loading) full-field strain measurement. The color legend shows the magnitude of longitudinal strain (Epsilon Y). Time and load data are given in the stage information section. This test was carried out in the fatigue lab, Building 49, at NASA Glenn Research Center. Frame size is 704x576. The duration of the clip is 22 seconds.
Appendix E  Nontechnical Abstract

A flexible matrix composite (FMC) is a new type of fiber-reinforced polymer composite with soft, elastomeric matrix materials such as rubbers, silicones, and polyurethanes. Conventional rigid matrix composites (RMCs) utilize stiff polymer matrix materials such as polyesters, vinyl esters, and epoxies. Each type of material has advantages and disadvantages in certain engineering applications. A promising application for FMCs is in the driveline of rotorcraft. The current driveline design for rotorcraft consists of rigid aluminum shafts joined by flexible mechanical couplings to account for misalignment in the driveline, which is either inherent or introduced by aerodynamic flight loads. The couplings are heavy and incur maintenance and cost penalties because they wear and need frequent replacement. The segmented shafts and mechanical couplings could be replaced by one continuous FMC shaft. FMC materials are both flexible (due to the polymer matrix) and rigid (due to the reinforcing fibers). So, they can be configured in a shaft such that the shaft is both bending-soft to account for misalignment and torsionally-stiff to transmit power. For the rotorcraft driveline, this design could reduce part count, maintenance, and the overall weight—an eternal goal in the aerospace field. Previous research studies on this topic have developed analytical and design tools using preliminary materials. With these tools, a more appropriate stiffness range for polymer matrix materials based on real service needs was found. One material was chosen that fits into this stiffness range, and the material characteristics of an FMC containing the new polymer matrix are determined in this investigation for future FMC shaft design.
For rotorcraft operated in hostile environments, a topic of concern regarding FMC shafts is ballistic impact tolerance. This topic has not yet been explored for these new kinds of composite materials and is pursued here on a coupon-level basis. Results show that tubular FMC test coupons absorb more energy and suffer larger reductions in torsional strength than their conventional composite counterpart, although reductions in tensile and compressive strengths are similar in both materials. This difference is attributed to the greater pull-out of fibers in the FMC material during impact. Coupon-level testing is a good first step in understanding ballistic tolerance in FMCs, although additional testing using fully sized and designed shafts is suggested so that proper comparisons between driveshafts can be made.

An FMC driveshaft rotating under a misaligned condition could experience excessive self-heating caused by internal damping primarily because of the viscoelastic nature of the polymer matrix. A model has already been developed to simulate self-heating in FMC shafts, although it is limited to analyzing composite layups with one fiber angle. New closed-form and finite element models are developed in the current investigation that are capable of analyzing self-heating in FMC shafts with multiple fiber angles through the thickness. The newly developed models have been validated with comparisons against experiments and one another. Experiments using simple angle-ply tubes made with the new FMC suggest that for a shaft operating under real service strains, temperature increase due to self-heating can be less than 10°C. Material development, characterization of the ballistic tolerance of FMCs, and improved analytical tools are contributions from the current investigation to the implementation of FMC driveshafts in rotorcraft.