GEOMETRICAL MAGNETIC FRUSTRATION AND DEMAGNETIZATION OF ARTIFICIAL SPIN ICE

A Thesis in

Physics

by

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ABSTRACT

Ice is a common material that has unusual properties. The hydrogen ions in ice keep in disordered states even at the extremely low temperatures. Thus ice has the so-called zero point entropy. The disordered states in ice are a consequence of geometrical frustration, a fascinating phenomenon that attracts not only considerable interest in basic physics but also provides a novel platform for important applications, such as data storage and neural networks.

Geometrical frustration also occurs in magnetic materials, in which the geometry of an ordered lattice prohibits simultaneous minimization of all magnetic interactions. Spin ice is a class of geometrically frustrated materials in which the magnetic ions mimic the frustration of hydrogen ion positions in frozen water. However, such chemically synthesized materials put severe limitations on probing the individual magnetic ions and tuning the magnetic interactions.

We used electron beam lithographic patterning to create square arrays of single-domain permalloy ($Ni_{0.8}Fe_{0.2}$) nanomagnets in which the dipolar interactions displayed two-dimensional frustration analogous to spin ice. Magnetic force microscopic (MFM) images of individual magnetic moments directly displayed the local accommodation of frustration. We saw both ice-like short-range correlations and an absence of long-range correlations, behavior which is strikingly similar to the low-temperature state of spin ice.

The second part of this thesis is about our investigations on demagnetization on the nanometer scale. We studied demagnetization protocols for artificial spin ice by rotating it in a changing magnetic field. To demagnetize the sample, we find that the most
effective demagnetization is achieved by not only stepping the field strength down while
the sample is rotating, but by combining each field step with an alternation in the field
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moment. These results suggest that non-monotonic variations in field magnitude around
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1.1 Frustration in magnetism

Frustration arises when a system cannot satisfy all the competing interactions [1] [2]. In magnetic materials, certain geometrical arrangements of spins prohibit the minimization of all spin-spin interactions and this kind of frustration is called geometrical magnetic frustration.

The simplest example of a geometrically frustrated magnetic system contains three Ising spins sitting on the vertices of an equilateral triangle, as shown in figure 1-1. When the interactions between each spin pair are antiferromagnetic, the system finds no way to minimize the three interactions while the system is in the lowest energy state. In fact, its ground state is six-fold degenerate and all of the three spins are frustrated in each ground state, because none of them can satisfy its two neighbors simultaneously.

This simple triangle unit can then be repeated to construct many kinds of two-dimensional lattices, as shown in figure 1-2. Triangles can also construct the three-dimensional frustrated garnet lattice as shown in figure 1-3a. Another often-studied 3D frustrated lattice is the pyrochlore lattice (see figure 1-3b) in which the corner-sharing tetrahedra are the lattice units.

The 2D and 3D geometrically frustrated magnetic materials have the characteristic of a huge number of degenerate ground states, comparable to the number of
magnetic ions in the material. Because of the large number of accessible ground states, these materials can hop from one ground state to another, with little energy cost, even at cryogenic temperatures. This fluctuation results in a much lower phase transition temperature than the one predicted by the Curie-Weiss law.

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1.2 Water ice and the ice rule

Water ice was the first frustrated system studied in physics. In an ice crystal, each oxygen atom is tetrahedrally surrounded by four other oxygen atoms with a distance of 2.76 Å. In between this center oxygen atom and each of the four surrounding oxygen atoms, there stays a hydrogen atom as shown in Fig. 1-4. These four hydrogen atoms are not at equal distance to the hydrogen atom they surround. Two are near to and two are further away from each oxygen atom, and this hydrogen atom arrangement is called the ice rule. The shorter H-O bond is 0.95 Å and the longer one is 1.81 Å. So for the 4 hydrogen atoms around an oxygen atom, there are 6 energetically equivalent arrangements complying with the ice rule. Water ice retains this crystal structure, which has the disordered hydrogen atom arrangement, even at cryogenic temperatures. This disorder leads to residue entropy [3] [4] at zero Kelvin. Pauling [5] was the first to explain the residue entropy of water ice in 1935.

Figure 1-4: Oxygen and hydrogen ion arrangement in water ice. Each oxygen ion (large circle) is surrounded by four hydrogen ions (solid black dots), in which two are closer to the oxygen ion and two others are farther away. Reproduced from Snyder (reference 9) and Bramwell (reference 11).
For a water ice crystal with N oxygen atoms, there are 2N hydrogen atoms. Each hydrogen atom has two possible positions to choose from, namely either close to or further away from an oxygen atom. There would be $2^{2N}$ possible proton configurations in such a system if each proton were free to choose between these two positions. But because of the ice rule, the four hydrogen ions surrounding each oxygen ions have only 6, instead of $2^4 = 16$, allowed site configurations. Therefore, the total allowed number of proton configurations is $2^{2N}(6/16)^N$ or $(3/2)^N$. This leads to an entropy of

$$S = K_B \times (3/2)^N$$ \hspace{1cm} 1.1

Where $K_B$ is Boltzmann’s constant.

Or

$$S = R \times \ln(3/2) = 0.806 \text{cal} \cdot k \cdot mol$$ \hspace{1cm} 1.2

Where $R$ is the gas constant that equals $1.987 \text{cal} \cdot K^{-1} \cdot \text{Mol}^{-1}$.

The prediction in equation 1.2 is in good agreement with the experimental value of $0.82 \pm 0.05 \text{cal} \cdot K^{-1} \cdot \text{Mol}^{-1}$ at 10 Kelvin.

Pauling based his calculation on the assumption that the hydrogen sites are independent. In real water ice, the hydrogen sites are interdependent since they have to comply with the ice rule. J. F. Nagle [6] gave a rigorous calculation of the residue entropy as

$$S = R \times \ln(1.50685 \pm 0.00015)$$
$$= 0.8145 \pm 0.0002 \text{cal} \cdot K^{-1} \cdot \text{Mol}^{-1}$$ \hspace{1cm} 1.3
This result is remarkably close to the experimental value.

The disordered hydrogen atoms in water ice need to overcome an energy barrier of the order of 1 eV or $10^4$ K in order to establish long range ordering [7]. This large energy barrier effectively prevents the system from entering into a zero entropy state because the process is extremely slow. From this point of view, the residue entropy of water ice is not a violation of the third law of thermodynamics.

1.3 Spin ice magnetic material

In 1997, M. J. Harris, et al. [8], discovered that Ho$_2$Ti$_2$O$_7$ displayed geometrical frustration. Researchers found similar phenomena in Dy$_2$Ti$_2$O$_7$ [7] [9] and Ho$_2$Sn$_2$O$_7$ [10] later. In all these materials, the rare earth ions sit on the corners of corner-sharing tetrahedrons, as shown in figure 1-3b. These rare earth ions have large magnetic moments, $\sim 10$ $\mu_B$, and they point either toward or away from the center of each tetrahedron because of large crystal field effects (on the scale of 300 Kelvin). In other words, these magnetic ions are essentially Ising spins. The dipolar interactions among these ions lead to macroscopic degenerate ground states. In the ground states, on each tetrahedron, there are always two spins pointing toward to the center of the tetrahedron and the other two away from the center (see figure 1-5). This spin moment configuration is similar to the hydrogen proton arrangement in water ice, as shown in figure 1.4. Because of this similarity, these frustrated materials are called “spin ice” and “the two spins in and two spins out” configuration the “spin ice rule”.
S. T. Bramwell, et al. [11] [12], studied the spin correlations in Ho$_2$Ti$_2$O$_7$ at zero magnetic field using neutron scattering technique. Figure 1-6a shows the scattering pattern at $T \sim 50$ mK. It displays a “four-leaf clover” of intense scattering around [0, 0, 0]. There is also strong scattering around [0, 0, 3] and a broad region of slightly weaker scattering around [3/2, 3/2, 3/2]. The width of the intense regions indicates short-range correlations on the order of one lattice spacing.

M. J. Harris, S. T. Bramwell and their collaborators advocated nearest-neighbor ferromagnetic exchange-only model [8] [13] [14] to explain the frustration in spin ice materials. In this model, they think that the exchange interaction between nearest neighbor rare earth ions is ferromagnetic, which favor the two-spin-in / two-spin-out structure in each tetrahedron. And this model totally neglects the contribution of dipolar interactions between spins. Calculations on the neutron scattering pattern (see figure 1-6b), using the nearest-neighbor ferromagnetic exchange-only model, reproduces some features of the experimental pattern. However, this result has notable differences in the extension of the 0, 0, 0 intense region around [hhh] and the relative intensities of the
regions around 0, 0, 3 and 3/2, 3/2, 3/2. Contrary to the nearest-neighbor ferromagnetic exchange-only model, experimental data on magnetization [15] suggests that the exchange interaction between rare earth ions is antiferromagnetic, instead of ferromagnetic. Furthermore, the dipolar interactions, among rare earth ions in the spin ice materials, are comparable to, if not bigger than, their exchange interaction counterpart. The reason is that the rare earth ions have magnetic moment of 10 µB or so. Based on these considerations, Gingras, et al. suggested a dipolar spin ice model [11] [16] to explain the geometrical frustration in spin ice materials. This model takes into account both the dipolar interactions and the antiferromagnetic nature of the exchange interactions in artificial spin ice. This model more successfully replicates the experimental neutron scattering pattern of Ho$_2$Ti$_2$O$_7$, as we can see in figure 1-6c and agrees very well with the experimental data of specific heat and entropy of Dy$_2$Ti$_2$O$_7$ [7].
1.4 Two dimensional ice type model

The real ice rule can also be constructed in the two dimensional "square ice" [17-23], as shown in figure 1-7. On this lattice, each spin is only allowed to point along the
sides of the lattice and, on each vertex, there are precisely two spins pointing toward to the center of the vertex and two others pointing away.

Like the three dimensional spin ice material, on each vertex of such a square lattice, there are 6 possible spin configurations which agree with the ice rule. E. H. Lieb [19] [20] calculated the exact entropy of the square ice and its value is

$$S = R \times \ln\left(\frac{4}{3}\right)^{3/2}$$

$$= 1.987 \text{cal} \cdot K^{-1} \cdot \text{Mol}^{-1} \times \ln 1.5396$$

$$= 0.8574 \text{cal} \cdot K^{-1} \cdot \text{Mol}^{-1}$$

For the six configurations in figure 1-7b, we assign their energy as $\varepsilon_1$, $\varepsilon_2$, ..., $\varepsilon_6$ respectively. The Boltzmann weights of the six vertex arrangements are:

$$a = e^{-\varepsilon_1/K_B T} = e^{-\varepsilon_2/K_B T} = e^{-\varepsilon_3/K_B T}$$

$$b = e^{-\varepsilon_4/K_B T} = e^{-\varepsilon_5/K_B T} = e^{-\varepsilon_6/K_B T}$$
\( \varepsilon_1, \varepsilon_2, \ldots, \varepsilon_6 \) can be different in real system and the ice type model can be classified into three important special cases according to the vertex energies. The ice type model can be solved exactly when the nearest neighbor interaction is the only interaction present in the system [17] [18].

In the ice model, \( \varepsilon_1 = \varepsilon_2 = \ldots = \varepsilon_6 = 0 \), all the six ice rule configurations are equivalent and they have the same possibility to present in the disordered ground state. But in material like KH\(_2\)PO\(_4\) (KDP), which is a hydrogen-bonded crystal of coordination number four,

\[
\varepsilon_1 = \varepsilon_2 = 0, \varepsilon_3 = \varepsilon_4 = \varepsilon_5 = \varepsilon_6 > 0
\]

Therefore its ground states prefer to have all the vertex configurations as either type 1 or type 2. The two possible ground states, which are long range ordered ferromagnetic, are shown in figure 1-8.

Figure 1-8: The two ferromagnetic ground states of the KDP model. (a) contains only type 1 vertex and (b) only type 2 vertex.

There is also a so-called anti-ferromagnetic F-model, when a, b, and c choose to be

\[
\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon_4 > 0, \varepsilon_5 = \varepsilon_6 = 0.
\]
In this case, the ground state has the alternative placement of vertex 5 and 6. There are only two possible ground states as shown in figure 1-9.

Figure 1-9: Two possible ground states of the F model shown in (a) and (b). They are both long range ordered anti-ferromagnetic states. Vertex configuration 5 and 6 sit on the lattice alternatively.

More generally, the spin ice type materials undergo several phases depending on the relative value of a, b and c. Its phase diagram is shown in figure 1-10.
The four phases are:

I. **Ferromagnetic, \( a > b + c \).** The lowest energy state is one in which all vertices are either type 1 or type 2, as shown in figure 1-6. The system freezes in the ground state once it chooses one.

II. **Ferromagnetic, \( b > a + c \).** The lowest energy state is one in which either all vertices are type 3 or type 4.

III. **Disordered, \( a, b, c < \frac{1}{2} (a + b + c) \).** All correlations decay to zero with increasing distance.

Figure 1-10: The phase diagram of the zero field ice-type model, in terms of the Boltzmann weights \( a, b, c \). The dashed circular quadrant corresponds to the \( a^2 + b^2 = c^2 \) case, which is a disordered state with infinite correlation length. Reproduced from Baxter (reference 14).
IV. Antiferromagnetic, \( c > a + b \). The ground state is one of the vertex configurations shown in figure 1-7.

1.5 Summary

Geometrical magnetic frustration occurs in some magnetic materials in which the geometrical arrangement of magnetic spins prevents the simultaneous satisfaction of all spin-spin interactions. Spin ice is a class of geometrically frustrated magnetic materials and its spin configurations in ground states are very similar to the proton arrangements in water ice. In spin ice, the magnetic ions sit on the corners of corner-sharing tetrahedrons and the dipolar interactions outpower the antiferromagnetic exchange interactions among neighboring spins. The ice-like geometrical frustration can be constructed in the two dimensional square lattice. Theoretical calculations suggest that the two dimensional square lattice can enter ferromagnetic, antiferromagnetic, or disordered states, depending on the energy distributions of the possible spin configurations on one vertex of the square lattice. The main goal of our research is to create a two dimensional square lattice which can enter a disordered state and mimics the geometrical frustration in spin ice.

1.6 References


Chapter 2

Fabrication of Artificial Spin Ice and Experimental Techniques

2.1 Short Review of Patterned Magnetic Nanostructures

Nanometer sized ferromagnets attracted significant amount of research interest in recent years because they are promising candidates for the data storage industry [1] [2], magnetoresistive random access memory (MRAM) [3], and data processing technology [4] [5], and because they are model systems [6] [7] to study micromagnetism in confined geometries. A single domain nanomagnet typically has size of around or below 100 nm [6] [8] [9] [10] [11] with elongated shape because such geometry favors nearly uniform spin structure. In a discrete pattern of such single domain nanomagnets, each nanomagnet behaves like a giant spin. By properly designed shape anisotropy, the magnetic moment of the nanomagnet will align with the long axis of the nanomagnet [6] [8] [12] [13] [14]. A combination of small size and large shape anisotropy can make a nanomagnet act like a giant Ising spin.

Studies on patterned arrays of nanomagnets also suggest that the dipolar interaction between the nanomagnets is significant when they are in close proximity [4] [6] [15-27]. The dipolar interaction is anisotropic. This is best explained by considering the potential energy $U$ of two dipoles $\vec{M}_1$ and $\vec{M}_2$ with a separation of $\vec{r}$ [28].

$$U = \frac{1}{4\pi\mu_0 r^3} [\vec{M}_1 \cdot \vec{M}_2 - \frac{3}{r^2} (\vec{M}_1 \cdot \vec{r})(\vec{M}_2 \cdot \vec{r})]$$
Because of this anisotropic nature of the dipolar interaction, experimentally researchers observed both antiferromagnetic [5] [12] [13] [21] and ferromagnetic orderings [4] [11] [16] [21] [24] in chains of nanomagnets depending on the lateral arrangement inside the chain. Figure 2-1 schematically displays these two ordering structures in one dimensional chains.

![Antiferromagnetic and Ferromagnetic Orderings](image)

**Figure 2-1:** Two ordering structures in a one dimensional system of nanomagnets. a). Antiferromagnetic ordering occurs when the magnets are side by side. b). Ferromagnetic ordering happens when the magnets are aligned along their long axis. These two ordering structures appear only when the distance between neighboring magnets is smaller than or comparable to the size of the nanomagnets.

For arrays of single-domain nanomagnets placed on the lattice sites of a regular square lattice, a checkerboard pattern [15] [27] is observed after the sample is AC demagnetized. Such a checkerboard pattern demonstrates the antiferromagnetic ordering in this two dimensional magnetic system. In both the one and two dimensional systems
discussed above, the long range ordering appears typically when the spacing between neighboring nanomagnets is smaller than or comparable to the lateral dimension of the nanomagnets themselves.

2.2 Limitations of Conventional Geometrically Frustrated Magnets

As we have discussed in chapter 1, geometrically frustrated magnets exist in many chemical compounds, in which the magnetic ions sit on triangle or tetrahedron based lattices. While these frustrated materials exhibit rich physics, there are also significant limitations inherent in such systems. Such limitations include: 1. difficulties in discovering and synthesizing the frustrated magnetic compounds. 2. difficulties in controlling the lattice constant or introducing defects into the lattices. 3. absence of an effective probe for investigating individual magnetic moments of magnetic ions. But, by arranging single-domain nanomagnets in frustrated lattices, we can fabricate artificial geometrically frustrated magnets, in which each nanomagnet behaves just like a giant electronic spin. These frustrated lattices can be made by electron beam lithography, with which we can tune the lattice constant, size and shape of each nanomagnet and can design defects in the lattice. Examples of proposed two-dimensional artificial frustrated lattices are shown in figure 2-2. In these lattices, the dipolar interactions among neighboring nanomagnets cannot be minimized simultaneously, and this leads to geometrical magnetic frustration.

Figure 2-3 explains in detail the geometrical magnetic frustration in the square array shown in figure 2.2. On a vertex of the square array, according to equation 2.1, two
neighboring nanomagnets minimize their interaction energy by having the north (or south) pole of one nanomagnet closer to the south (or north) pole of another, as shown in figure 2-3a. Otherwise, as in figure 2-3b, the interaction energy is increased. For the four nanomagnets in figure 2-3c, there are 6 pairs of dipolar interactions, but only 4 of them can be minimized simultaneously, and as this situation happens, two of the four magnetic moments point toward the center of the vertex while the another two point away from the center. This situation is similar to the four magnetic ions on a tetrahedron in spin ice materials. Because of this similarity between the square array and the spin ice material, we refer to the two-dimensional square array as artificial spin ice.
Figure 2-2: Schematics of frustrated two-dimensional arrays of single-domain nanomagnets.  a). frustrated square array. b). frustrated honey-comb array.
Figure 2-3: Schematic drawings explaining the geometrical magnetic frustration in a square lattice. a). two neighboring nanomagnets minimize their dipolar interaction energy by pointing one’s magnetization to the other’s south pole. b). two neighboring nanomagnets maximize their dipolar interaction energy by pointing one’s magnetization away from the other’s south pole. c). The best compromises for the six pairs of dipolar interactions is that 4 of them are minimized, leaving two others maximized.
2.3 Design of Artificial Spin Ice

In chapter 1, we discuss that a square array of spins, as shown in figure 1-10, may exhibit frustration when the energy scale of different vertex configuration is in a certain region. To realize such a scenario, we need to make an artificial structure that mimics the spin configurations in the ice type model. In order to do this, we need to choose a ferromagnetic material to make such an artificial structure in the first place. The novel ferromagnetic semiconductor GaMnAs [29] and MnAs [30] were our initial choices. Patterning such materials into nanostructures is similar to well known processes for semiconductors. However such materials have disadvantages such as cryogenic or slightly above room temperature Curie point, relatively small saturated magnetization and, most importantly, complicated magnetic properties. Our final choice is the well known ferromagnetic material called permalloy [31] [32] [33], which contains roughly 80% Ni and 20% Fe. Permalloy has a Curie temperature of about 600 Celsius, permeability of about 10000 or above, nearly zero crystalline anisotropy, and low coercivity of around 1 Oe. Such properties enable us to realize strong interactions between permalloy nanomagnets, to make each permalloy nanomagnet behave as an Ising spin by simply designing proper shape anisotropy, and to conduct experimental study in room temperature.

2.3.1 OOMMF Simulation on Single Permalloy Nanomagnet

In order to have a guidance of selecting proper size and shape of permalloy nanomagnet, we used OOMMF [34] to run micromagnetic simulations prior to making
the sample. The first two objects we designed were ellipses, with their long and short axes as (50 nm, 60 nm) and (100 nm, 120 nm) respectively. Their thickness was 25 nm. The third object we designed is shown in figure 2-4. This object has a shape like a stadium field with lateral size of 80 nm by 220 nm. Its two ends are half circles. The thickness is also 25 nm.

---

Figure 2-4: Object of the third OOMMF simulation. It has a stadium field shape with lateral size of 80 nm by 220 nm. Its thickness is 25 nm.

---

In these simulations, the objects are divided into 5 nm*5 nm*5 nm cells. We use an exchange stiffness of $1.3 \times 10^{11}$ J/m, saturation magnetization of $8.6 \times 10^{5}$ A/m, zero crystalline anisotropy and a damping coefficient of 0.5 for the permalloy. These simulation parameters can be found in reference 34. Figure 2.5, 2.6 and 2.7 display the OOMMF simulation results for these three objects respectively.
Figure 2-5: OOMMF simulation result for a permalloy ellipse with lateral size of 50 nm by 60 nm and a thickness of 25 nm. The top panel is the hysteresis loop obtained in simulation. Field is along the ellipse’s long axis during simulation. The middle and bottom panel are spin configurations that correspond to stage A to F indicated in the hysteresis loop.
Figure 2-6: OOMMF simulation result for a permalloy ellipse with lateral size of 100 nm by 120 nm and a thickness of 25 nm. The top panel is the hysteresis loop obtained in simulation. Field is along the ellipse’s long axis during simulation. The middle and bottom panels are spin configurations that correspond to stage A to H indicated in the hysteresis loop.
Because the permalloy islands used in simulations are symmetric, an initial randomness of spin configuration is used to eliminate the symmetry of the system.

Figure 2-7: OOMMF simulation result for a permalloy nanomagnet with a shape of stadium field, lateral size of 80 nm by 220 nm, and a thickness of 25 nm. The top panel is the hysteresis loop obtained in simulation. Field is along the ellipse’s long axis during simulation. The bottom panel shows spin configurations that correspond to stage A to D indicated in the hysteresis loop.
Otherwise, the simulation would reach a marginally stable state at the end of its first simulation stage and the final result of the simulation could be unreliable.

Figure 2-5 shows that the ellipse with lateral size of 50 nm by 60 nm relaxes into a vortex state (as shown in the spin configuration of stage A of figure 2.3) in zero field. Under an external field along the ellipse’s long axis, the vortex core moves toward one edge (stage B and C of figure 2-5) of the ellipse until the field reaches 500 Oe, where this permalloy ellipse fully saturates and enters the single domain state (stage D of figure 2-5). After this initial saturation, this permalloy ellipse stays in a single domain state during the rest of the hysteresis loop. This single domain state switches it on orientation sharply when the magnetic field decreases from -1400 Oe to -1500 Oe (stage E and F of figure 2-5). This rotation of magnetization suggests that the permalloy ellipse behaves like an Ising spin in external field.

For the ellipse with lateral size of 100 nm by 120 nm, its relaxed state in zero field is also a vortex state (stage A in figure 2-6). Under external field along the long axis of the ellipse, the vortex moves toward the edge until it annihilates at H = 1000 Oe (see stage B to E in figure 2-6 for demonstration of the movement of the vortex core). After the field decreases to a small negative value of -100 Oe, the magnetization of this ellipse starts to rotate by forming a curling state (stage F in figure 2-6) first followed by vortex states (stage G and H in figure 2-6). At -1000 Oe, the magnetization finally reaches its negative saturation. These spin configurations under different magnetic field clearly show that this ellipse does not mimic the behavior of an Ising spin.

The stadium shaped permalloy island relaxes into a state with two vortices of opposite chiralities in zero field. A 200 Oe magnetic field along its long axis fully repels
the vortices and saturates the permalloy island. This permalloy island stays in a single
domain state in the rest of the hysteresis loop and a sharp reversion of magnetization
occurs at $H = -1000 \text{ Oe}$ (stage C and D of figure 2-7).

In conclusion, the initial vortex state of the stadium shaped permalloy magnet and
the smaller permalloy ellipse can be removed by a moderate magnetic field. Their
remnant state is a single-domain state and their magnetization switching is like that of an
Ising spin. Such characteristics make these two magnets suitable for constructing an
artificial spin ice, in which each magnet behaves like an Ising spin. We decided to
fabricate the stadium shaped magnet in our frustrated arrays because the stadium shaped
magnet has larger shape anisotropy, meaning more like an Ising spin, and larger lateral
dimension, which can be fabricated using electron beam lithography with higher
precision than the ellipse.

### 2.3.2 Dipolar Interaction and Spin Configurations on a Vertex of Square Lattice

As we learn from equation 2.1, the dipolar interaction energy depends on the
magnetic moment, separation, and relative orientation of two interacting dipoles. At room
temperature, the intensity of magnetization of our permalloy sample, as measured by
SQUID magnetometer, is

$$I = 650 \text{ emu/cm}^3$$

$$= 650 \times 4\pi \times 10^{-4} \text{ Wb/m}^2$$

$$= 0.82 \text{ Wb/m}^2$$
For a permalloy island with the shape and size as shown in figure 2.2, its volume is

\[ V = \left( \frac{\pi \times (80 \text{nm})^2}{4} + 80 \text{nm} \times 140 \text{nm} \right) \times 25 \text{nm} \]

\[ = 4.06 \times 10^{-22} \text{ m}^3 \]

So, the magnetic moment of an island is

\[ M = I \times V \]

\[ = 3.33 \times 10^{-22} \text{ Wb} \times \text{m} \]

\[ = 2.86 \times 10^7 \text{ Bohr – magneton} \]

Suppose we arrange these magnetic moments on a triangular lattice and set their orientations as shown in figure 2-8. Site 1 and 2 are nearest neighbors. Sites (1, 3) and (2, 4) are two different kinds of second-nearest neighbors, which we name as the linear type and the transverse type respectively. Using equation 2.1, we get the interaction energy of these three pairs.

\[ E_{12} = E_{\text{nearest neighbor}} \]

\[ = \frac{1}{4 \pi \mu_0} \left( 0 - 3 \frac{(M \times \sqrt{2}a \times \cos \frac{\pi}{4}) \times (M \times \sqrt{2}a \times \cos \frac{\pi}{4})}{(\sqrt{2}a)^3} \right) \]

\[ = - \frac{1}{4 \pi \mu_0} \frac{3 \sqrt{2} M^2}{8a^3} \]

\[ E_{13} = E_{\text{second-nearest neighbor – L}} \]

\[ = \frac{1}{4 \pi \mu_0} \frac{M^2}{(2a)^3} - 3 \frac{(M \times 2a) \times (M \times 2a)}{(2a)^3} \]

\[ = - \frac{1}{4 \pi \mu_0} \frac{M^2}{4a^3} \]
If we set \( 2a = 320 \text{ nm} \), then we get

\[
E_{24} = E_{\text{second-nearest-neighbor--T}} \\
= \frac{1}{4\pi\mu_0} \left( -\frac{M^2}{(2a)^3} - 0 \right) \\
= -\frac{1}{4\pi\mu_0} \frac{M^2}{8a^3}
\]

If we set \( 2a = 320 \text{ nm} \), then we get

\[
E_{12} = E_{\text{Nearest-neighbor}} \\
= -\frac{1}{4\pi \times 4\pi \times 10^{-7} \text{ H/m}} \times \frac{3\sqrt{2} \times (3.33 \times 10^{-22} \text{ Wb} \cdot \text{m})^2}{8 \times (1.6 \times 10^{-7} \text{ m})^3} \\
= -9.092 \times 10^{-19} \text{ Joule} = -6.58 \times 10^4 \text{ Kelvin}
\]

\[
E_{13} = E_{\text{second-nearest-neighbor--L}} \\
= -3.1 \times 10^4 \text{ Kelvin}
\]

\[
E_{24} = E_{\text{second-nearest-neighbor--T}} \\
= -1.55 \times 10^4 \text{ Kelvin}
\]

---

Figure 2-8: Schematic of four point dipoles placed on a frustrated square lattice. The dipolar interaction energy between two magnetic dipoles is shown in equation 2.5 to 2.10.
For a vertex of a square lattice, there are 16 possible spin configurations as shown in figure 2-9. According to equation 2.5 to 2.10, we can classify them into four groups and configurations within a group have the same dipolar energy. Still, we suppose that the lattice constant is \(2a\). The energies of these four groups are

\[
E_{\text{type-I}} = -\frac{1}{4\pi\mu_0} \times \frac{3\sqrt{2} \times M^2}{2a^3}
\]

\[
E_{\text{type-II}} = -\frac{1}{4\pi\mu_0} \times \frac{M^2}{2a^3}
\]

\[
E_{\text{type-III}} = 0
\]

\[
E_{\text{type-IV}} = \frac{1}{4\pi\mu_0} \times \frac{(3\sqrt{2} + 1) \times M^2}{2a^3}
\]

\[\text{2.11}\]
From equation 2.11, we conclude that the dipolar interaction energy of the 4 spins on a vertex strongly depends on the lattice constant. By varying the lattice constant, we can therefore tune the dipolar interaction. Table 2-1 summarizes the dipolar interaction energies for the four types of vertices under 8 different lattice constants.

Figure 2-9: The 16 possible spin configurations on a vertex of the square lattice. These configurations are grouped into 4 types according to their energy level. Type I and II are lower energy states and have the two-spin-in / two-spin-out structure. Note that, if the dipolar interaction is zero and thus the moment configuration is random, the possibility of type I, II, III, and IV is 12.5%, 25%, 50%, and 12.5% respectively.
The calculations above are based on the assumptions that each permalloy island is a perfect point dipole. This simple model tells the correct trend of energy scales. A better model treats each island as a three-dimensional entity, since, in our lattices, the island size is comparable to the distance between islands.

Figure 2-10 is a schematic of permalloy islands on a square lattice with lattice constant of 320 nm. We assume that the blue island, in figure 2-10, has magnetization direction as shown and the magnetization is uniform inside the island. We then calculate the field generated by this island at 6 different points: $A_c$, $A_t$, $B_c$, $B_t$, $C_c$ and $C_t$. Details of this calculation is included in appendix A. Table 2-2 lists the summary of the results when lattice constant is 320 nm, 680 nm, and 880 nm respectively.

### Table 2-1: Dipolar interaction and vertex energy versus lattice constant (using point dipole model)

<table>
<thead>
<tr>
<th>Lattice Constant</th>
<th>$E_{\text{type-I}}$ ($10^4 K$)</th>
<th>$E_{\text{type-II}}$ ($10^4 K$)</th>
<th>$E_{\text{type-III}}$ ($10^4 K$)</th>
<th>$E_{\text{type-IV}}$ ($10^4 K$)</th>
<th>$E_{12}$ ($10^4 K$)</th>
<th>$E_{13}$ ($10^4 K$)</th>
<th>$E_{24}$ ($10^4 K$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>320 nm</td>
<td>-26.32</td>
<td>-6.2</td>
<td>0</td>
<td>32.52</td>
<td>-6.58</td>
<td>-3.1</td>
<td>-1.55</td>
</tr>
<tr>
<td>360 nm</td>
<td>-18.48</td>
<td>-4.35</td>
<td>0</td>
<td>22.84</td>
<td>-4.62</td>
<td>-2.18</td>
<td>-1.09</td>
</tr>
<tr>
<td>400 nm</td>
<td>-13.48</td>
<td>-3.17</td>
<td>0</td>
<td>16.65</td>
<td>-3.37</td>
<td>-1.59</td>
<td>-0.79</td>
</tr>
<tr>
<td>440 nm</td>
<td>-10.12</td>
<td>-2.38</td>
<td>0</td>
<td>12.51</td>
<td>-2.53</td>
<td>-1.19</td>
<td>-0.60</td>
</tr>
<tr>
<td>480 nm</td>
<td>-7.80</td>
<td>-1.84</td>
<td>0</td>
<td>9.64</td>
<td>-1.95</td>
<td>-0.92</td>
<td>-0.46</td>
</tr>
<tr>
<td>560 nm</td>
<td>-4.91</td>
<td>-1.16</td>
<td>0</td>
<td>6.07</td>
<td>-1.23</td>
<td>-0.58</td>
<td>-0.29</td>
</tr>
<tr>
<td>680 nm</td>
<td>-2.74</td>
<td>-0.65</td>
<td>0</td>
<td>3.39</td>
<td>-0.69</td>
<td>-0.32</td>
<td>-0.16</td>
</tr>
<tr>
<td>880 nm</td>
<td>-1.26</td>
<td>-0.30</td>
<td>0</td>
<td>1.56</td>
<td>-0.32</td>
<td>-0.15</td>
<td>-0.075</td>
</tr>
</tbody>
</table>

Table 2-2: Stray field generated by the blue island in figure 2-10. The permalloy island is 80 nm by 220 nm laterally. Its thickness is 25 nm.

<table>
<thead>
<tr>
<th>Lattice Constant</th>
<th>$A_c$</th>
<th>$A_t$</th>
<th>$B_c$</th>
<th>$B_t$</th>
<th>$C_c$</th>
<th>$C_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>320 nm</td>
<td>36.7 Oe</td>
<td>207.5 Oe</td>
<td>7.1 Oe</td>
<td>7.17 Oe</td>
<td>5.5 Oe</td>
<td>14.6 Oe</td>
</tr>
<tr>
<td>680 nm</td>
<td>3.79 Oe</td>
<td>7.0 Oe</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>880 nm</td>
<td>2.68 Oe</td>
<td>2.77 Oe</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 2-10: Schematic of permalloy islands in a square lattice with lattice constant of 320 nm. The stray field of the blue island at points $A_c$, $A_t$, $B_c$, $B_t$, $C_c$ and $C_t$ is summarized in table 2-2.
Table 2.2 shows a strong nearest neighbor dipolar field, when the lattice constant is only 320 nm. This dipolar field, 207.5 Oe, on the tip of nearest neighbor is significant considering that the permalloy islands have a coercive field of 900 Oe or so, suggested by the OOMMF simulation. Table 2.2 also tells that the second nearest neighbor dipolar field is much smaller than the dipolar field exerted by nearest neighbors. In addition, when lattice constant increases to 880 nm, the nearest neighbor dipolar field decreases to merely 3 Oe or so, indicating that these islands are nearly independent from each other at this lattice constant.

This three-dimensional model yields \( E_{12} = -9.37 \times 10^4 \, \text{K} \), \( E_{13} = -4.95 \times 10^4 \, \text{K} \), and \( E_{24} = -1.25 \times 10^4 \, \text{K} \). The values in table 2.1, obtained by point dipole model, agree with this result qualitatively, with noticeable difference of up to 50 percent.

2.4 Sample Fabrication

The sample fabrication process is a combination of electron beam lithography, thin film deposition, and lift-off. Figure 2-11 illustrates the process of our fabrication protocol. First, polymethylglutarimide (PMGI) [35] [36] [37] and polymethyl methacrylate (PMMA) resist are applied on the Si substrate prior to electron beam exposure. PMGI resist does not mix with PMMA. Because of this characteristic of PMGI, PMMA / PMGI double layer often delivers cleaner lift-off than the common choice of PMMA / MMA double layer. A solution of methyl isobutyl ketone (MIBK) and isopropanol (IPA), with relative volume ration of 1:3, is used to develop PMMA after electron beam exposure. Then we use MCC developer 101 (tetra ethyl ammonium
hydroxide) to resolve the PMGI layer from PMMA openings, and this operation creates
undercut beneath the PMMA pillars. Such an undercut is an advantageous character of
this double layer resist for better lift-off output. Then we grow 25 nm permalloy
(Ni$_{0.80}$Fe$_{0.20}$) and 3 nm aluminum capping layer on the sample using molecular beam
epitaxy with a deposition rate of 0.1 Å/s at ambient temperature. Our permalloy film has
a grain size of about 5 nm. In the final step, we use acetone to remove the PMMA and
PMGI layer. This lift-off process leaves only permalloy islands (with Al capping layer)
on the Si substrate. The detailed recipe of lithography processes is in appendix B.

Figure 2-12 to 2-15 display SEM images of square arrays with lattice constant of
320 nm, 360 nm, 480 nm, and 880 nm. In these images, the permalloy islands have edge
roughness of about 5-10 nm. Their overall lateral size is about 80 nm by 220 nm,
although some size distribution is noticeable due to the finite resolution of electron beam
lithography. We also would like to point out that SEM images in figure 2-12 to 2-15 were
taken 18 months after the permalloy arrays were fabricated. We also made other
permalloy patterns later (using the same fabrication techniques) and imaged these
patterns within three months of their fabrication date. Figure 2-16 shows the SEM image
of one such rather “fresh” permalloy pattern. The SEM image displays much smaller
edge roughness than the ones in figure 2-12 to 2-15. We believe that longer exposure in
open air, thus more permalloy oxidation, contributes to more edge roughness of our
permalloy islands.
Figure 2-11: Illustrations of sample fabrication process. a) Electron beam exposure on the double layered resist of PMGI and PMMA. b) Develop PMMA after electron beam exposure. c) Develop PMGI layer and to create a slight undercut beneath the PMMA layer. d) Grow 25 nm thick permalloy film and 3 nm capping layer of Al. e) “Lift-off” the PMMA and PMGI layers by acetone. The permalloy pattern (with Al capping layer) stands on the Si substrate after this process.
Figure 2-12: SEM image of a square array with lattice constant of 320 nm.
Figure 2-13: SEM image of a square array with lattice constant of 360 nm.
Figure 2-14: SEM image of a square array with lattice constant of 480 nm.
Figure 2-15: SEM image of a square array with lattice constant of 880 nm.
2.5 Summary

We used electron beam lithography and lift-off techniques to fabricate square lattices of permalloy nanomagnets. The permalloy nanomagnets are 80 nm by 220 nm laterally and have a thickness of about 25 nm. Micromagnetic simulations show that such nanomagnets enter the single domain state under a 200 Oe magnetic field along its long
axis. When the magnetic field sweeps through the coercive field of the permalloy nanomagnet, its magnetization reverses abruptly, behaving like an artificial Ising spin.

The dipolar interactions among the permalloy nanomagnets are geometrically frustrated. The low energy spin configurations (type I and II) for a vertex of the square lattice comply with the spin ice rule, which has the two-spin-in / two-spin-out structure. The energy difference between the low energy configurations and the high energy ones (type III and IV) strongly depends on the lattice constant, which implies that the vertex’s possibility of entering the low energy state (spin ice rule) can be tuned by changing the lattice constant. Calculations also show that the stray field from a permalloy island on its neighbors changes in strength by nearly 2 orders when the lattice constant varies from 320 nm to 880 nm.

2.6 References


35. PMGI is a positive resist, see www.microchem.com for detailed information.


3.1 Demagnetization of Artificial Spin Ice in a Rotational Magnetic Field

For frustrated magnets, the ground state usually has zero magnetization \([1][2][3]\), when the sample is cooled in zero magnetic field. Therefore, for artificial spin ice, we also expect zero magnetization in the ground state. In order to achieve this zero magnetization state, we could elect to demagnetize the artificial spin ice by thermal annealing. However as stated in chapter 2, the interaction energy between two neighboring permalloy islands is on the order of \(10^4\) Kelvin. This indicates that an effective thermal annealing of artificial spin ice requires cooling the sample from at least \(10^4\) Kelvin, a temperature that obviously would melt the sample, and is not realistic for normal laboratory conditions. Considering the dipolar interaction energy scale, we choose to demagnetize the artificial spin ice by rotating it in a changing magnetic field (see the schematic of demagnetization experiment in figure 3-1), an approach similar to the one used in elsewhere [4].

Magneto optic Kerr effect (MOKE) [5] measurements (figure 3-2) on the artificial spin ice show that all the eight lattices have nearly the same coercive field, about 770 Oe. Therefore, when we demagnetize the sample, we always start from a field, much higher than the coercive field, of 1300 Oe or higher. Our sample rotates at 1000 RPM while the magnetic field changes polarity with each step down of its magnitude. In reference 4, the
authors demagnetized their sample without changing the polarity of the external field. We give a detailed discussion on demagnetization in chapter 4.

Figure 3-1: Schematic of demagnetization set up for the artificial spin ice. The magnetic field is applied in the plane of the sample.
After demagnetization, the lattices of artificial spin ice are measured at room temperature, in absence of external magnetic field, using magnetic force microscopy (MFM)[6][7][8]. MFM utilizes a ferromagnetic tip attached at the end of a micron scale cantilever to detect the derivative of interaction force between the tip and the stray field of the magnetic sample. The cantilever vibrates at (or near) its resonant frequency, if no interaction exists between the tip and the sample. A non-zero derivative of the interaction force between the tip and the sample shifts the frequency away from the tip’s resonance. By monitoring the magnitude and sign of this shift in frequency, we obtain qualitative information about the orientation of magnetic domains in the sample.

Figure 3-2: Hysteresis loops of artificial spin ice patters, which have lattice constant from 320 nm to 880 nm, measured by MOKE technique.

### 3.2 Investigation on Artificial Spin Ice Using Magnetic Force Microscopy

After demagnetization, the lattices of artificial spin ice are measured at room temperature, in absence of external magnetic field, using magnetic force microscopy (MFM)[6][7][8]. MFM utilizes a ferromagnetic tip attached at the end of a micron scale cantilever to detect the derivative of interaction force between the tip and the stray field of the magnetic sample. The cantilever vibrates at (or near) its resonant frequency, if no interaction exists between the tip and the sample. A non-zero derivative of the interaction force between the tip and the sample shifts the frequency away from the tip’s resonance. By monitoring the magnitude and sign of this shift in frequency, we obtain qualitative information about the orientation of magnetic domains in the sample.
Our magnetic force microscope is the Multimode IV model made by Veeco. The magnetic tip has Co-Cr coating with coercivity of about 400 Oe [9]. This MFM works in lift-mode, in which the instrument obtains the topography of the sample surface first, just like a normal atomic force microscope, then lifts the tip at a certain distance (50 nm in our case) above the sample in order to get information about phase shift of the cantilever’s resonance. Our MFM has a spatial resolution of about 50 nm. Figure 3-3 to 3-10 display the AFM-MFM images of our artificial spin ice with lattice constant of 320 nm to 880 nm. Before taking these AFM/MFM images, our sample was demagnetized by rotating in a varying magnetic field as mentioned above. The topographic images display individual permalloy islands with high resolution. The corresponding MFM images show that each island has a simple black-white contrast, which reveals the single-domain nature of the island. These black-white regions correspond to the north-south poles of permalloy islands respectively. Figure 3-11 displays a high quality AFM/MFM image set of a square lattice with lattice constant of 400 nm. In the MFM image, three vertices, which are types I, II, and III as defined in chapter 2, are highlighted.
Figure 3-3: a) Three-dimensional view of the AFM image of an artificial spin ice with lattice constant of 320 nm. The scanning range is 8 micron by 8 micron. b).The corresponding MFM image covering the same area as of the AFM image.
Figure 3-4: a). 3-D view of the AFM image of an artificial spin ice with lattice constant of 360 nm. The scanning range is 10 micron by 10 micron. b). The corresponding MFM image covering the same area as of the AFM image.
Figure 3-5: a). 2-D view of the AFM image of an artificial spin ice with lattice constant of 400 nm. The scanning range is 10 micron by 10 micron. b). The corresponding MFM image covering the same area as of the AFM image.
Figure 3-6: a). 2-D view of the AFM image of an artificial spin ice with lattice constant of 440 nm. The scanning range is 10 micron by 10 micron. b). The corresponding MFM image covering the same area as of the AFM image.
Figure 3-7: a). 2-D view of the AFM image of an artificial spin ice with lattice constant of 480 nm. The scanning range is 10 micron by 10 micron. b). The corresponding MFM image covering the same area as of the AFM image.
Figure 3-8: a). 3-D view of the AFM image of an artificial spin ice with lattice constant of 560 nm. The scanning range is 10 micron by 10 micron. b). The corresponding MFM image covering the same area as of the AFM image.
Figure 3-9: a). 3-D view of the AFM image of an artificial spin ice with lattice constant of 680 nm. The scanning range is 10 micron by 10 micron. b). The corresponding MFM image covering the same area as of the AFM image.
Figure 3-10: a). 3-D view of the AFM image of an artificial spin ice with lattice constant of 880 nm. The scanning range is 10 micron by 10 micron. b). The corresponding MFM image covering the same area as of the AFM image.
Figure 3-11: a). AFM of a square array of permalloy nanomagnets with lattice constant of 400 nm. b). Corresponding MFM image covering the same area of the array. All permalloy islands are in the single domain state. The magnetization direction points from white to black. The three highlighted vertices are Type I, II, and III respectively.
3.3 OOMMF Simulation of a Vertex’s Relaxation in Zero Field

While MFM measurements show that permalloy islands enter single-domain state, OOMMF simulations of a vertex provide additional perspective on the spin configurations in those nano-islands. We performed OOMMF simulations on vertices with lattice constant of 320 nm and each island is 80 nm by 220 nm laterally, with thickness of 25 nm. (For OOMMF simulations on vertices with other lattice constants, please refer to Appendix C). The cell size in simulation is 5 nm * 5 nm * 5 nm. In these simulations, the initial state of the four types of vertices has spin orientation perfectly aligned with the long axis of each island. Such initial states then relax in zero external magnetic field. Figures 3-12 to 3-15 display the final states after the type I, II, III, and IV vertices undergo such relaxation. These figures show that the magnetization of each island is a very good approximation of the perfect single domain state. We only find spin tilting at the tips of the islands because the dipolar interaction field is stronger at these areas.

OOMMF simulation also provides the energy of these four vertex types after their relaxation. Table 3.1 summaries the results.

<table>
<thead>
<tr>
<th>Vertex Type</th>
<th>Total Energy ($10^{-17}$ J)</th>
<th>Exchange Energy ($10^{-18}$ J)</th>
<th>Dipolar Interaction Energy ($10^{-17}$ J)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type I</td>
<td>5.122</td>
<td>1.072</td>
<td>5.015</td>
</tr>
<tr>
<td>Type II</td>
<td>5.386</td>
<td>1.912</td>
<td>5.195</td>
</tr>
<tr>
<td>Type III</td>
<td>5.675</td>
<td>1.885</td>
<td>5.487</td>
</tr>
<tr>
<td>Type IV</td>
<td>6.873</td>
<td>2.533</td>
<td>6.620</td>
</tr>
</tbody>
</table>

Table 3-1 shows that exchange interaction, a quantum effect that tries to align neighboring electron spins in ferromagnetic materials, contributes less than 4 percent to
the total interaction energy, for all four types of vertices. This predominance of dipolar interaction in artificial spin ice is very similar to spin ice materials. In both cases, the dipolar interaction leads to geometrical magnetic frustration. Table 3-1 also shows that type I and II are lowest energy states. If we set the energy of type I as zero, then the energies of type II, III, and IV are $0.264 \times 10^{-17}$ J, $0.553 \times 10^{-17}$ J, and $1.751 \times 10^{-17}$ J respectively. Because type III and IV cost significantly more energy than type I and II, we anticipate observing more type I and type II than type III and type IV.
Figure 3-12: Spin configuration of a type I vertex after relaxation in zero magnetic field. The lattice constant is 320 nm and each island is 80 nm * 220 nm * 25 nm.
Figure 3-13: Spin configuration of a type II vertex after relaxation in zero magnetic field. The lattice constant is 320 nm and each island is 80 nm * 220 nm * 25 nm.
Figure 3-14: Spin configuration of a type III vertex after relaxation in zero magnetic field. The lattice constant is 320 nm and each island is 80 nm * 220 nm * 25 nm.
In order to probe the nature of frustration in the square lattices, we studied how the properties varied with the spacing between the islands, counting between 1,000 and
3,000 islands in measurements of 2 to 4 different arrays for each lattice spacing. This allowed us direct control over the frustrated interactions, something that is not easily attainable in geometrically frustrated magnetic materials.

Since we can determine the vertex types in the MFM images, an immediate question is whether our arrays obeyed the ice rules, that is, did a preponderance of the vertices fall into a two-in/two-out configuration (type I or II). By simple counting arguments (see figure 2-9) we can predict the expected distribution of different vertex types if the moments were non-interacting and randomly oriented. One would expect only 37.5% of the vertices to have a two-in/two-out configuration if the orientations were random; an excess of such vertices would indicate that interactions are determining the moment configuration. We compute the excess percentage for each type of vertex, defined as the difference between the percentage observed and that expected for a random distribution. We plot this excess versus lattice spacing in figure 3-16 for each of the four vertex types, as well as for types I and II combined. The excess percentage of vertices with a two-in/two-out configuration (types I and II) was well over 30% for the smallest lattice; in other words, over 70% of all vertices had a spin-ice-like configuration. This excess percentage decreased monotonically with increasing lattice spacing (decreasing interactions), approaching zero for our largest lattice spacing, as would be expected for non-interacting (randomly oriented) moments. In fact, the excess for all vertex types approached zero as the lattice spacing increased, lending credence both to our understanding of the system and to the effectiveness of the rotating-field method in enabling facile local re-orientation of the moments.
To further understand the nature of frustration in this system, we also studied the pairwise correlations between the Ising-like moments of the islands. Defining a correlation function is somewhat complicated by the anisotropic nature of our lattice and that of the dipole interaction. We thus define a set of correlation functions between distinct types of neighboring pairs. The closest pairing is labelled “NN” for the nearest neighbor; “L” denotes the next nearest neighbor pairing which is in the longitudinal direction of the island; and “T” denotes the next nearest neighbor in the transverse direction from the island (see Figure 3-18 inset). We define a correlation, C, such that C = +1 if two moments are aligned to minimize the dipole interaction energy, and C = -1 if two moments are aligned to maximize the dipole energy. In this way, if the moments for a particular type of neighboring pair were uncorrelated on the lattice, the average value of C would be zero. Figure 3-17 shows schematics of different pair configurations in which C equals +1 or -1.

Figure 3-16: The excess percentage of different vertex types, plotted as a function of the lattice spacing of the underlying square array lattice. Note that the excess percentages approach zero for the largest lattice spacing.
Figure 3-17: Schematics of correlation for different magnetic moment pairs. (a), (b), and (c) shows magnetic moment pairs that have correlation $C = 1$. In (d), (e), and (f), the correlation $C = -1$. 
We find that the island pairs which were further separated than the L and T neighbors had weak or zero correlations ($|C| < 0.1$) for all lattices. We do see correlations for the NN and T neighbors as shown in Fig. 3b, but somewhat surprisingly, the correlations for the L neighbors were relatively small. We can understand this as a direct consequence of the frustration in the system. Interaction between the NN neighbors is the strongest, and therefore it is predominant. A pair of islands of type L has a direct

Figure 3-18: The correlations between different pairs of the islands as a function of the lattice spacing of the underlying square lattice. The inset shows our definitions of the near neighbor pairs from the grey central island (see text for details). For both the correlations and the vertex statistics, the typical variation between images for nominally identically prepared samples was <10% for the closely spaced lattices in which we had more than 1000 islands in a single image, but up to 50% for the more widely spaced lattices in which we had only a few hundred islands per image.
interaction (which is somewhat weaker than that of the NN pair), but also an indirect interaction, since the two islands in an L pairing share two NN neighbors. If all of the NN pair energies are minimized, then the L pair energy is maximized, and we believe that this frustration leads to the surprisingly weak correlation between the L neighbors. By the same logic, the relatively strong correlations between the T neighbor pairs also arise from indirect interactions via NN intermediaries. In the case of the T neighbor pairs, if the NN neighbor pair interaction energy is minimized, the indirect interaction energy will also be minimized, and thus the combined effect is to increase correlations as we observe. For all of the neighbor types, we find that the correlations approached zero for the largest lattice parameters, as expected since the interactions should strongly decrease as the islands are separated.

The existence of only short range order and ice-like correlations on the lattice is precisely analogous to the behavior of the spin ice materials, in which there is also no experimental evidence for long range order, only ice-like short range correlations. While there are theoretical long-range ordered low-energy states for spins on either our lattice or the pyrochlore spin ice lattice \cite{10}, the complex energy landscape associated with the frustration leads to a disordered state when thermal or magnetic-field-induced excitations are removed. This is in sharp contrast with unfrustrated lines of ferromagnetic islands, in which longer range correlations are observed. It is interesting that the relative populations of different types of vertices reaches the randomly oriented limit rapidly as the lattice constant increases, and that even within the regime of closely-spaced and therefore strongly interacting islands, the system can access a very wide range of nearly degenerate states. This wide range of accessible states has the potential for importance to
applications [11], since, if information were encoded within a low energy configuration of the moments, the energetic driving force for local magnetization reversals could be suppressed by this near-degeneracy, even for highly dense arrays.

3.5 References


9. See www.veeco.com for detailed information.


11. V. H. Crespi, private communication.
Chapter 4
Demagnetization of Artificial Spin Ice

4.1 Demagnetization Protocols

Chapter 3 described briefly the demagnetization of artificial spin ice samples by rotating in a changing magnetic field. Demagnetization using external magnetic field is widely used in both bulk ferromagnetic materials and nanostructured ferromagnets. Previous studies showed that arrays or chains of nanomagnets [1] [2] [3] [4] displayed various magnetization patterns after they were demagnetized and these patterns were closely related to the nature of the magnetic interactions between neighboring nanomagnets.

We used several demagnetization protocols for our artificial spin ice systems. We find that rotation in a magnetic field with uniformly diminishing amplitude leaves significant magnetization, but that a magnetic field that alternates in direction while its amplitude is decreased can effectively demagnetize the arrays.

During demagnetization, our sample was placed on a 1000 rpm rotating stage (as shown in figure 3-1) inside a changing magnetic field, an approach similar to the one used in reference 4. The demagnetization procedure always began at a field of 1,300 Oe or higher. We tested four different demagnetization protocols, as shown in figure 4-1.
In protocol 1, the magnetic field strength was stepped down in magnitude and switched polarity with each step. In protocol 2, the magnetic field was stepped down in magnitude without changing polarity, but was held at zero field temporarily between each step. In protocols 3 and 4, the magnetic field was decreased to zero either linearly (protocol 3) or by steps (protocol 4) without changing polarity. We could adjust independently the field ramping rate ($R_m$), the field step size ($S_H$), and the time that the array stayed at each field ($T_s$).

After each demagnetization procedure, we took magnetic force microscope (MFM) images of the square lattices in zero magnetic field. Each MFM image covered an area of 10 by 10 microns (including about 280 to 1300 permalloy islands in each MFM image, depending on lattice constant). Because the moment of each island is parallel to its long axis, we assigned a value of ±1 to each island moment (positive being defined as upward or to the right, depending on the island orientation). After mapping the moments of all islands in the MFM image, we could count the numbers of upward and downward
moments ($N_y$ and $N_y^\prime$) as well as rightward and leftward moments ($N_x$ and $N_x^\prime$). For each MFM image, we calculated the remanent magnetization as:

$$m_y = \frac{(N_y - N_y^\prime)}{(N_y + N_y^\prime)}$$

$$m_x = \frac{(N_x - N_x^\prime)}{(N_x + N_x^\prime)}$$

Total remanent magnetization: $$m_{tot} = \sqrt{m_x^2 + m_y^2} / \sqrt{2}$$

We then used $m_{tot}$ to characterize the efficiency of the demagnetization protocols.

4.2 Experimental Results of Protocol 1 and 2

4.2.1 Protocol 1

In protocol 1, we set the magnetic field sequence as $H_1, -H_2, H_3, -H_4, ..., 0$ where the negative field values had switched polarity in the laboratory frame. We defined the field step as $S_H = |H_i| - |H_{i+1}|$ in this case. We used $R_m = 24,000$ Oe/second and $T_s = 1$ s for most of these tests (a small number of tests with slower ramp rate and longer hold times suggest that these factors do not strongly affect the demagnetization). In this protocol we used $S_H \sim 32.6$ Oe (for $H = 1308$ Oe to -816 Oe ), followed by steps to 800 Oe, -767 Oe, 734 Oe, and then $S_H \sim 16.3$ Oe down to $H = 0$. We used protocol 1 to demagnetize our samples multiple times for each lattice spacing, measuring the orientation of over 2000 islands for each array lattice spacing except the largest (880 nm) where we imaged only 1100 due to the smaller number in each MFM image.
Repeated runs on an array with lattice parameter of 320 nm revealed that 49.2% of 3568 islands kept their initial orientation after repeating the protocol (the corresponding number was 49.3% in an array with lattice parameter of 560 nm). This result is very close to the ideal value of 50%, and it strongly suggests that there is no sample history dependence to the results of the demagnetization. As shown in figure 4-2, \( m_{\text{tot}} \) ranges between 0.056 and 0.152 for the 8 lattices, indicating rather good demagnetization, although there is an apparent slight increase in \( m_{\text{tot}} \) as the lattice constant increases.

---

**Figure 4-2:** Comparison between experimental value of \( m_{\text{tot}} \) and its expectation value using binominal functions.
This apparent increase may simply be a statistical effect associated with the 10 x 10 micron size of all of the MFM images. This fixed size results in about 1300 permalloy islands (about 650 islands in one subset) in one MFM image for the 320 nm array, comparing to only 280 (about 140 islands in one subset) in the 880 nm array. See table 4-1 for details. We can calculate the expectation value of $M_{tot}$ using binominal functions.

This statistical calculation is base on such assumptions:

1. The number of islands along X (or Y) direction in a MFM image is $n$.
2. The array’s initial state is a fully saturated one in which all horizontal islands have their moments rightward and all vertical islands have their moments downward.
3. Each island has a possibility of $p$ to flip its orientation after the demagnetization process.
4. The flip of one island’s moment is independent of its neighbors.

Then after a demagnetization by protocol 1, the possibility that $\nu$ out of $n$ islands flip their moment is described by a binominal function in equation 4.1, in which $q = 1 - p$.

Table 4-1: Experimental and calculated value of $m_{tot}$ for different lattices. The detail about statistical calculation is below this table.

<table>
<thead>
<tr>
<th>Lattice Constant (nm)</th>
<th>Average # of islands along X / Y direction</th>
<th>Experiment value of $m_{tot}$</th>
<th>Expectation value of $m_{tot}$ ($p = 0.5$)</th>
<th>Expectation value of $m_{tot}$ ($p = 0.508$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>320</td>
<td>646</td>
<td>0.05645</td>
<td>0.03487</td>
<td>0.03769</td>
</tr>
<tr>
<td>360</td>
<td>670</td>
<td>0.05889</td>
<td>0.03424</td>
<td>0.03711</td>
</tr>
<tr>
<td>400</td>
<td>546</td>
<td>0.04738</td>
<td>0.03793</td>
<td>0.04053</td>
</tr>
<tr>
<td>440</td>
<td>464</td>
<td>0.06812</td>
<td>0.04114</td>
<td>0.04355</td>
</tr>
<tr>
<td>480</td>
<td>360</td>
<td>0.07195</td>
<td>0.04671</td>
<td>0.04884</td>
</tr>
<tr>
<td>560</td>
<td>272</td>
<td>0.08222</td>
<td>0.05375</td>
<td>0.0556</td>
</tr>
<tr>
<td>680</td>
<td>200</td>
<td>0.15159</td>
<td>0.06268</td>
<td>0.06427</td>
</tr>
<tr>
<td>880</td>
<td>140</td>
<td>0.10978</td>
<td>0.07492</td>
<td>0.07625</td>
</tr>
</tbody>
</table>
Thus, the possibility that \( x_+ \) islands, out of \( n \) horizontally placed islands, flip their moment after demagnetization is:

\[
P(x_+) = B_{n,p}(x) = C_n^{x_+} p^{x_+} q^{n-x_+}.
\]  \hspace{1cm} 4.1

The magnetization along \( x \) direction is:

\[
m_x = \frac{(x_+ - x_-)}{n} = \frac{(x_+ - n + x_+)}{n} = \frac{x_+}{n} - 1 \frac{n}{2}
\]  \hspace{1cm} 4.3

\( x_+ \) can also be represented by \( m_x \) as in equation 4.4.

\[
x_+ = \frac{n}{2} (m_x + 1)
\]  \hspace{1cm} 4.4

Since \( x_+ \) and \( m_x \) have the one-to-one mapping, the possibility of magnetization along \( x \) be \( m_x \) is:

\[
P(m_x) = P(x_+) = C_n^{x_+} p^{x_+} q^{n-x_+}.
\]  \hspace{1cm} 4.5

In equation 4.5,

\[
m_x = 1, \frac{n-2}{n}, \frac{n-4}{n}, ..., \frac{2-n}{n}, -1
\]  \hspace{1cm} 4.6

And we assumed \( n \) be an even number for simplicity.

Similarly, the possibility of magnetization along \( y \) be \( m_y \) is:

\[
P(m_y) = P(y_+) = C_n^{y_+} p^{y_+} q^{n-y_+}.
\]  \hspace{1cm} 4.7
In equation 4.7, \( y^+ \) is the number of vertically placed islands that flip their moments to the upward direction. And,

\[
m_y = 1, \frac{n-2}{n}, \frac{n-4}{n}, ..., \frac{2-n}{n}, -1
\]  \hspace{1cm} 4.8

Using equation 4.5 and 4.7, we can calculate the possibility of \( m_x^2 \) and \( m_y^2 \).

When \( m_x = 0 \) and \( m_y = 0 \),

\[
P(m_x^2 = 0) = P(m_x = 0) = C_n^{n/2} p^{n/2} q^{n/2}
\]  \hspace{1cm} 4.9

\[
P(m_y^2 = 0) = P(m_y = 0) = C_n^{n/2} p^{n/2} q^{n/2}
\]  \hspace{1cm} 4.10

When \( 0 < m_x, m_y \leq 1 \),

\[
P(m_x^2) = P(m_x) + P(m_{-x}) = C_n^{2} \left( \frac{n}{2} \right)^{(1+m_x)} \left( \frac{n}{2} \right)^{(1-m_x)} p^{2} q^{2}
\]  \hspace{1cm} 4.11

\[
+ C_n^{2} \left( \frac{n}{2} \right)^{(1-m_x)} \left( \frac{n}{2} \right)^{(1+m_x)} p^{2} q^{2}
\]

\[
P(m_y^2) = P(m_y) + P(m_{-y}) = C_n^{2} \left( \frac{n}{2} \right)^{(1+m_y)} \left( \frac{n}{2} \right)^{(1-m_y)} p^{2} q^{2}
\]  \hspace{1cm} 4.12

\[
+ C_n^{2} \left( \frac{n}{2} \right)^{(1-m_y)} \left( \frac{n}{2} \right)^{(1+m_y)} p^{2} q^{2}
\]

The total magnetization \( m_{\text{tot}} \) is:
Because $m_x^2$ and $m_y^2$ are independent, the possibility of $m_{tot}$ is:

$$P(m_{tot}) = P(m_x^2) \times P(m_y^2)$$  \hspace{1cm} 4.14

The expectation of $m_{tot}$ is thus:

$$\bar{E}_{m_{tot}} = \sum m_{tot} \cdot P(m_{tot})$$  \hspace{1cm} 4.15

The summation in equation 4.15 is over all possible values of $m_{tot}$.

Table 4-1 listed the expectation values of $m_{tot}$ when $p$ equals 0.5 and 0.508 respectively. In figure 4.1, these calculated expectation values and the experimental data of $m_{tot}$ have very similar trends, that is, arrays with larger lattice constant have bigger $m_{tot}$. The good agreement between our calculation and the experiment suggests that the number of islands in one MFM image gives rise to significant statistical effect. For a limited number of islands, $m_{tot}$ is a non-zero value even when the demagnetization is perfect, because of the above statistical effect. It is also noticeable in figure 4.2 that the experimental data and the calculated expectations have a difference of about 0.02 to 0.03, except for a difference of 0.09 for the lattice parameter of 680 nm. This difference may be caused by the assumption of independent flipping of magnetic moments in the calculation above. This assumption obviously neglects the fact that near neighbors in our sample have short-range correlations. Therefore, the flipping of magnetic moments in artificial spin ice is not completely independent.

Figure 4-3a displays an array with a lattice constant of 360 nm after demagnetization by protocol 1. Figure 4-3b shows a two-dimensional discrete Fourier
transform of the same image with each magnetic moment represented by ±1. The small remanent magnetization for protocol 1 is seen in the weak intensity in the center of the Fourier transformed image.
Figure 4-3: a). MFM image of an array, with lattice constant of 360 nm, after demagnetization by protocol 1. The image covers an area of 10 micron by 10 micron. b). Fourier transform of the MFM image in figure 4.3a, after each magnetic moment is represented by 1 or -1.
4.2.2 Protocol 2

In contrast with protocol 1, protocol 2 generally resulted in a significant magnetization. For $S_H$ of 32 Oe, $m_{tot}$ ranged from 0.17 to 0.64 for lattice constant of 440 nm and 560 nm. Figure 4-4a displays a MFM image of an array with lattice constant of 360 nm, after it was demagnetized by protocol 2. The significant remnant magnetization appears as a strong peak in the middle of the Fourier transform image (figure 4-4b).

A fine step size of 3.2 Oe with this protocol also resulted in $m_{tot}$ of 0.13 for the 400 nm array and 0.27 for the 560 nm array. But, in one experiment, we also found small remnant magnetization ($m_{tot} = 0.03$) for an array with lattice constant of 360 nm.
Figure 4-4: a). MFM image of an array, with lattice constant of 360 nm, after demagnetization by protocol 2. The image covers an area of 10 micron by 10 micron. b). Fourier transform of the MFM image in figure 4.4a, after each magnetic moment is represented by 1 or -1.
4.3 Experimental Results of Protocols 3 and 4

In contrast to the effective and reproducible demagnetization from protocol 1, protocols 3 consistently did not demagnetize the arrays for any lattice constant. In the linear ramp of protocol 2, we tested \( R_m = 8, 0.32, \) and 0.08 Oe/second, which produced \( m_{tot} \sim 0.2 - 0.5 \).

Figure 4.5a shows the MFM image of an array with lattice constant of 360 nm after demagnetization by protocol 3; note the large central peak in the Fourier transform (figure 4-5b). Protocol 4, with the same step sizes as protocol 1, resulted in a similarly large magnetization range \( m_{tot} \sim 0.2 - 0.5 \).

While this thesis was being prepared, some new experimental data taken by Xianglin Ke and Jie Li [5] shows significantly better demagnetization by protocol 4 when the field step size is only 3.2 Oe. These experiments show that \( m_{tot} \) is about 0.05 and 0.17 for arrays with lattice constant of 400 nm and 880 nm respectively. Their sample has slightly smaller island size, about 80 nm by 200 nm laterally with thickness of 25 nm.

For complete list of all the original MFM data for the demagnetization experiments, please refer to appendix D.
Figure 4-5: a). MFM image of an array, with lattice constant of 360 nm, after demagnetization by protocol 3. The image covers an area of 10 micron by 10 micron. b). Fourier transform of the MFM image in figure 4.4a, after each magnetic moment is represented by 1 or -1.
4.4 Discussion

The results of our investigation indicate that nonmonotonic excursions in the magnetic field strength within a decreasing field envelope substantially improve demagnetization of the arrays for protocols of the step size studied. This suggests that the process of magnetizing the arrays, taking the field below the coercive field, and then remagnetizing with a field near the coercive field is the important factor in optimizing demagnetization. The efficacy of sweeping through zero field further suggests that there is a broad range of time and length scales which are of importance to the demagnetization process. This would be expected in a glassy system, as our frustrated arrays may be, but a more detailed local investigation of the collective dynamics or studies of well-isolated islands could shed considerable additional light on whether this is relevant here. The present results do, however, demonstrate the need to follow a careful protocol in the demagnetization of arrays of single-domain nanomagnets – a necessary step in order to study the important many-body effects resulting from magnetostatic interactions within such arrays.

4.5 References


5. Xianglin Ke, Jie Li, and William McConville. Unpublished experimental data.
Appendix A

Dipolar Interactions in the Square Lattices

Part I: The case of a square lattice with lattice constant of 320nm.

A square lattice with lattice constant of 320nm is shown in the schematic above. Each dot is 80 nm by 220 nm with half circular ends. Dot thickness is 25 nm.

Now we assume the blue particle has magnetization direction as shown and it is uniform inside the particle. We want to calculate the field generated by this particle at 6 different points: $A_c$, $A_t$, $B_c$, $B_t$, $C_c$ and $C_t$. The lattice constant in consideration is 320 nm.
The intensity of magnetization of Permalloy, at room temperature, is

$$I = 650 \text{emu/cm}^3 = 650 \times 4\pi \times 10^{-4} \text{Wb/m}^2 = 0.82 \text{Wb/m}^2$$ \hspace{1cm} \text{(A16)}

Volume of single nano-dot is

$$V = (\frac{\pi \times (80 \text{nm})^2}{4} + 80 \text{nm} \times 140 \text{nm}) \times 25 \text{nm} = 4.06 \times 10^{-22} \text{m}^3$$ \hspace{1cm} \text{(A17)}

So, the magnetic moment of single nano-dot is

$$M = I \times V = 3.33 \times 10^{-22} \text{Wb} \cdot \text{m}$$ \hspace{1cm} \text{(A18)}

Or

$$\frac{3.33 \times 10^{-22} \text{Wb} \cdot \text{m}}{1.165 \times 10^{-29} \text{Wb} \cdot \text{m}} = 2.86 \times 10^7 \text{Bohr magneton}$$ \hspace{1cm} \text{(A19)}

The coordinates of the six field points are listed below

<table>
<thead>
<tr>
<th></th>
<th>((x, y))</th>
<th>((x', y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A_c)</td>
<td>160 nm, 90 nm</td>
<td>160 nm, -230 nm</td>
</tr>
<tr>
<td>(A_t)</td>
<td>50 nm, 90 nm</td>
<td>50 nm, -230 nm</td>
</tr>
<tr>
<td>(B_c)</td>
<td>320 nm, -70 nm</td>
<td>320 nm, -70 nm</td>
</tr>
<tr>
<td>(B_t)</td>
<td>320 nm, 40 nm</td>
<td>320 nm, -180 nm</td>
</tr>
<tr>
<td>(C_c)</td>
<td>0, -390 nm</td>
<td>0, 250 nm</td>
</tr>
<tr>
<td>(C_t)</td>
<td>0, -280 nm</td>
<td>0, 140 nm</td>
</tr>
</tbody>
</table>

Particle thickness is

$$T = 25 \text{ nm}$$ \hspace{1cm} \text{(A20)}
Radius of the half circles on each particle’s end is

\[ R = 40 \text{ nm} \] \hspace{1cm} \text{(A21)}

The surface charge density on the half circle I is

\[ \sigma = \vec{I} \cdot \vec{n} = I \cdot \sin \theta, \text{ (we use the x-y coordinate here)} \] \hspace{1cm} \text{(A22)}

Where, \( \vec{n} \) is the surface vector of half circle I.

Surface charge density on the half circle II is

\[ \sigma' = \vec{I} \cdot \vec{n}' = -I \cdot \sin \theta', \text{ (we use the x'-y' coordinate here)} \] \hspace{1cm} \text{(A23)}

Where, \( \vec{n}' \) is the surface vector of half circle II.

1. Field on point \( A_c \).

\[ dH_I = \frac{dm_I}{4\pi\mu_0 r^2} \cdot \vec{r} = \frac{\sigma ds}{4\pi\mu_0 r^2} \hat{r} \] \hspace{1cm} \text{(A24)}

\[ = \frac{\sigma TRd\theta}{4\pi\mu_0 r^2} \hat{r} = \frac{ITR \sin \theta d\theta}{4\pi\mu_0 r^2} \hat{r} \]

\[ dH_{II} = \frac{dm_{II}}{4\pi\mu_0 r'^2} \cdot \vec{r}' = \frac{\sigma' ds'}{4\pi\mu_0 r'^2} \hat{r}' \] \hspace{1cm} \text{(A25)}

\[ = \frac{\sigma' TRd\theta'}{4\pi\mu_0 r'^2} \hat{r}' = -\frac{ITR \sin \theta' d\theta'}{4\pi\mu_0 r'^2} \hat{r}' \]

In which,

\[ r^2 = (x - 160)^2 + (y - 90)^2 \]
\[ = x^2 - 320x + 160^2 + y^2 - 180y + 90^2 \]
\[ = R^2 - 320x - 180y + 160^2 + 90^2 \] \hspace{1cm} \text{(A26)}
\[ r'^2 = (x' - 160)^2 + (y' + 230)^2 = R^2 - 320x' + 460y' + 160^2 + 230^2 \]  \hspace{1cm} \text{A27}

Then,

\[
dH_{ix} = \frac{I TRS \sin \theta d\theta}{4\pi \mu_0 r^2} \cdot \frac{160 - x}{r} \]  \hspace{1cm} \text{A28}

\[
dH_{iy} = \frac{I TRS \sin \theta d\theta}{4\pi \mu_0 r^2} \cdot \frac{90 - y}{r} \]  \hspace{1cm} \text{A29}

\[
dH_{ix'} = \frac{I TRS \sin \theta' d\theta'}{4\pi \mu_0 r'^2} \cdot \frac{160 - x'}{r'} \]  \hspace{1cm} \text{A30}

\[
dH_{iy'} = \frac{I TRS \sin \theta' d\theta'}{4\pi \mu_0 r'^2} \cdot \frac{-230 - y'}{r'} \]  \hspace{1cm} \text{A31}
\[ H_x = \int dH_{ix} + \int dH_{ilx} \]
\[ = \frac{1}{4\pi\mu_0} \left( \int_0^\pi 0.82 \times 25 \times 40 \times \sin\theta \times (160 - 40\cos\theta) \right) \frac{(40^2 - 320 \times 40\cos\theta - 180 \times 40\sin\theta + 160^2 + 90^2)^{3/2}}{d\theta} \]
\[ + \int_0^\pi \frac{-0.82 \times 25 \times 40 \times \sin\theta' \times (160 - 40\cos\theta') \times (40^2 - 320 \times 40\cos\theta' + 460 \times 40\sin\theta + 160^2 + 230^2)^{3/2}}{d\theta'} \]
\[ = \frac{1}{4\pi\mu_0} \left( \int_0^\pi 32.8(4 - \cos\theta)\sin\theta \times (353 - 128\cos\theta - 72\sin\theta)^{3/2} d\theta \right) \]
\[ + \int_0^\pi \frac{-32.8(4 - \cos\theta')\sin\theta' \times (801 - 128\cos\theta' + 184\sin\theta')^{3/2}}{d\theta'} \]
\[ = \frac{1}{4\pi\mu_0} \left( 0.0547 - 0.009 \right) A/m \]
\[ = \frac{1}{4\pi \times 10^{-4}} \times 0.0457 \text{Oe} = 36.4 \text{Oe} \]

\[ H_y = \int dH_{iy} - \int dH_{ily} \]
(Because \( y \) and \( y' \) have opposite direction, we use minus sign here.)
\[ = \frac{1}{4\pi\mu_0} \left( \int_0^\pi 32.8 \cdot \sin\theta \cdot (2.25 - \sin\theta) \times (353 - 128\cos\theta - 72\sin\theta)^{3/2} d\theta \right) \]
\[ - \int_0^\pi \frac{-32.8 \cdot \sin\theta' \cdot (-5.75 - \sin\theta') \times (801 - 128\cos\theta' + 184\sin\theta')^{3/2}}{d\theta'} \]
\[ = \frac{1}{4\pi \times 10^{-4}} \left( 0.0211155 - 0.0149598 \right) \]
\[ = 4.9 \text{Oe} \]

So,
\[ \sqrt{H_x^2 + H_y^2} = 36.7 \text{Oe} \]

2. Field on point \( A_t \).
\[ r^2 = (x - 50)^2 + (y - 90)^2 \]
\[ = R^2 - 100x - 180y + 50^2 + 90^2 \]
\[ r'^2 = (x' - 50)^2 + (y' + 230)^2 \]
\[ = R^2 - 100x' + 460y' + 50^2 + 230^2 \]

Then,

\[
H_x = \int dH_{hc} + \int dH_{hc} \\
= \frac{1}{4\pi\mu_0} \left( \int_0^\pi \frac{ITR\sin\theta d\theta}{r^2} \cdot \frac{50-x}{r} + \int_0^\pi \frac{-ITR\sin\theta'}{r'^2} \cdot \frac{50-x'}{r'} \right) \]
\[
= \frac{1}{4\pi\mu_0} \left( \int_0^\pi \frac{8.2 \cdot \sin\theta \cdot (5 - 4\cos\theta)}{(122 - 40\cos\theta - 72\sin\theta)^{\frac{3}{2}}} d\theta \right) + \int_0^\pi \frac{-8.2 \cdot \sin\theta' \cdot (5 - 4\cos\theta')}{(570 - 40\cos\theta' + 184\sin\theta')^{\frac{3}{2}}} d\theta' \]
\[
= \frac{1}{4\pi \times 10^{-4}} (0.165742 - 0.00422113) Oe \\
= 128.6 Oe \]

\[
H_y = \int dH_{hy} - \int dH_{hy} \\
= \frac{1}{4\pi\mu_0} \left( \int_0^\pi \frac{8.2 \cdot \sin\theta \cdot (9 - 4\sin\theta)}{(122 - 40\cos\theta - 72\sin\theta)^{\frac{3}{2}}} d\theta \right) - \int_0^\pi \frac{-8.2 \cdot \sin\theta' \cdot (-23 - 4\sin\theta')}{(570 - 40\cos\theta' + 184\sin\theta')^{\frac{3}{2}}} d\theta' \]
\[
= \frac{1}{4\pi \times 10^{-4}} (0.227333 - 0.022579) Oe \\
= 162.9 Oe \]

\[
\sqrt{128.6^2 + 162.9^2} = 207.5 Oe \]

3. Field on point B_c.
\[ r^2 = (x - 320)^2 + (y + 70)^2 \]
\[ = R^2 - 640x + 140y + 320^2 + 70^2 \] \hspace{1cm} A40

\[ r'^2 = (x' - 320)^2 + (y' + 70)^2 \]
\[ = R^2 - 640x' + 140y' + 320^2 + 70^2 \] \hspace{1cm} A41

So,

\[
H_x = \int dH_{tx} + \int dH_{lx} \\
= \frac{1}{4\pi\mu_0} \left( \int_0^\pi 8.2 \cdot \sin\theta \cdot (32 - 4\cos\theta) \frac{dx}{(1089 - 256\cos\theta + 64\sin\theta)^{3/2}} \, d\theta \right) \\
+ \int_0^\pi -8.2 \cdot \sin\theta' \cdot (32 - 4\cos\theta') \frac{dx'}{(1089 - 256\cos\theta' + 64\sin\theta')^{3/2}} \, d\theta' \\
= 0 \]

\[
H_y = \int dH_{ty} - \int dH_{ly} \\
= \frac{1}{4\pi\mu_0} \left( \int_0^\pi 8.2 \cdot \sin\theta \cdot (-7 - 4\sin\theta) \frac{dx}{(1089 - 256\cos\theta + 64\sin\theta)^{3/2}} \, d\theta \right) \\
- \int_0^\pi -8.2 \cdot \sin\theta' \cdot (-7 - 4\sin\theta') \frac{dx'}{(1089 - 256\cos\theta' + 64\sin\theta')^{3/2}} \, d\theta' \\
= \frac{1}{4\pi \times 10^{-4}} (-0.004455 \times 2) Oe \\
= -7.1 Oe \hspace{1cm} A42 \hspace{1cm} A43

4. Field on point B_t.

\[ r^2 = (x - 320)^2 + (y - 40)^2 \]
\[ = R^2 - 640x - 80y + 320^2 + 40^2 \] \hspace{1cm} A44
\[ r^{t2} = (x' - 320)^2 + (y' + 180)^2 \]
\[ = R^2 - 640x' + 360y' + 320^2 + 180^2 \]

\[ H_x = \frac{1}{4\pi \mu_0} \left( \int_0^\pi \frac{8.2 \cdot \sin \theta \cdot (32 - 4\cos \theta)}{(1056 - 256\cos \theta - 32\sin \theta)^{3/2}} d\theta + \int_0^\pi \frac{-8.2 \cdot \sin \theta' \cdot (32 - 4\cos \theta')}{(1364 - 256\cos \theta' + 144\sin \theta')^{3/2}} d\theta' \right) \]
\[ = \frac{1}{4\pi \times 10^{-4}} (0.0162297 - 0.00933592) \text{Oe} \]
\[ = 5.5 \text{Oe} \]

\[ H_y = \frac{1}{4\pi \mu_0} \left( \int_0^\pi \frac{8.2 \cdot \sin \theta \cdot (4 - 4\sin \theta)}{(1056 - 256\cos \theta - 32\sin \theta)^{3/2}} d\theta - \int_0^\pi \frac{-8.2 \cdot \sin \theta' \cdot (-18 - 4\sin \theta')}{(1364 - 256\cos \theta' + 144\sin \theta')^{3/2}} d\theta' \right) \]
\[ = \frac{1}{4\pi \mu_0} (0.00045 - 0.006226) \text{Oe} \]
\[ = -4.6 \text{Oe} \]

\[ \sqrt{5.5^2 + 4.6^2} = 7.17 \text{Oe} \]

5. Field on point C_c.

\[ r^2 = (x - 0)^2 + (y + 390)^2 \]
\[ = R^2 + 780y + 390^2 \]
\[ r'^2 = (x' - 0)^2 + (y' - 250)^2 \]
\[ = R'^2 - 500y' + 250^2 \]

Then,

\[ H_x = \frac{1}{4\pi\mu_0} \left( \int_0^\pi \frac{8.2 \cdot \sin\theta \cdot (-4\cos\theta)}{(1537 + 312\sin\theta)^{3/2}} \, d\theta \right) \]
\[ + \int_0^\pi \frac{8.2 \cdot \sin\theta' \cdot (-4\cos\theta')}{(641 - 200\sin\theta')^{3/2}} \, d\theta' \]
\[ = 0 \]

\[ H_y = \frac{1}{4\pi\mu_0} \left( \int_0^\pi \frac{8.2 \cdot \sin\theta \cdot (-39 - 4\sin\theta)}{(1537 + 312\sin\theta)^{3/2}} \, d\theta \right) \]
\[ - \int_0^\pi \frac{8.2 \cdot \sin\theta' \cdot (25 - 4\sin\theta')}{(641 - 200\sin\theta')^{3/2}} \, d\theta' \]
\[ = \frac{1}{4\pi\mu_0} (-0.009203 + 0.01606) \text{Oe} \]
\[ = 5.5 \text{Oe} \]

6. Field on point Ct.

\[ r^2 = (x - 0)^2 + (y + 280)^2 \]
\[ = R^2 + 560y + 280^2 \]

\[ r'^2 = (x' - 0)^2 + (y' - 140)^2 \]
\[ = R'^2 - 280y' + 140^2 \]
\[ H_y = \frac{1}{4\pi\mu_0} \left( \int_{0}^{\pi} 8.2 \cdot \sin\theta \cdot (-28 - 4\sin\theta) \frac{d\theta}{(800 + 244\sin\theta)^{3/2}} \right) \\
-\int_{0}^{\pi} 8.2 \cdot \sin\theta' \cdot (14 - 4\sin\theta') \frac{d\theta'}{(212 - 112\sin\theta')^{3/2}} \]  \\
= \frac{1}{4\pi\mu_0} (-0.0168 + 0.0352) Oe \\
= 14.6 Oe
Calculation of magnetostatic interaction energy

A. Nearest neighbors

Still, the particles in consideration have size of 80 nm by 220 nm. Thickness is 25 nm. Lattice constant is 320 nm.

First, calculate the energy between two nearest neighbors. Their magnetization is shown in the figure above. We need to calculate the magnetostatic interaction energies among half circle 1, 2, 3, and 4 one by one but exclude the self magnetostatic energy. Then add them up to get the total. The general equation used to calculate the magnetostatic energy is:

\[ U = \frac{m_1 \cdot m_2}{4\pi\mu_0 r} \]
Where, \( m_1 \) and \( m_2 \) are two magnetic poles with distance of \( r \).

1. Magnetiostatic energy between half circle 1 and 2.

\[
dm_1 = \sigma_1 \cdot dS_1 = I \cdot \sin \theta_1 \cdot dz_1 \cdot R \cdot d\theta_1
\]

\[
dm_2 = \sigma_2 \cdot dS_2 = I \cdot \cos \theta_2 \cdot dz_2 \cdot R \cdot d\theta_2
\]

\[
r = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}
\]

\[
x_1 = R \cdot \cos \theta_1, \quad y_1 = R \cdot \sin \theta_1
\]

\[
x_2 - a = R \cdot \cos \theta_2 \quad \text{or} \quad x_2 = a + R \cdot \cos \theta_2
\]

\[
y_2 - b = R \cdot \sin \theta_2 \quad \text{or} \quad y_2 = b + R \cdot \sin \theta_2
\]

\[
dU_{12} = \int I^2 R^2 \sin \theta_1 \cos \theta_2 d\theta_1 d\theta_2 dz_1 dz_2
\]

\[
= \frac{I^2 R^2}{4\pi \mu_0} \int_0^\pi d\theta_1 \int_{\pi/2}^{3\pi/2} d\theta_2 \int_0^{25\text{mm}} dz_1 \int_0^{25\text{mm}} dz_2
\]

\[
\frac{\sin \theta_1 \cos \theta_2}{\sqrt{(R \cos \theta_1 - R \cos \theta_2 - a)^2 + (R \sin \theta_1 - R \sin \theta_2 - b)^2 + (z_1 - z_2)^2}}
\]
\[ I = 0.82 \text{Wb/m}^2, \quad R = 40 \text{nm} \quad \text{A65} \]

\[ a = 90 \text{nm}, \quad b = 90 \text{nm} \quad \text{A66} \]

So,

\[
U_{12} = \frac{(0.82 \text{Wb/m}^2)^2 \times (40 \text{nm})^2}{4\pi \times 4\pi \times 10^{-7} \text{H/m} \times \frac{1}{40 \text{nm}}} \times \frac{1}{40 \text{nm}} \\
\times \int_0^\pi d\theta_1 \int_{\pi/2}^{3\pi/2} d\theta_2 \int_0^{25 \text{nm}} dz_1 \int_0^{25 \text{nm}} dz_2 \\
\frac{\text{Sin}\theta_1 \cdot \text{Cos}\theta_2}{\text{Sin}\theta_1 \cdot \text{Cos}\theta_2} \\
\sqrt{(\text{Cos}\theta_1 - \text{Cos}\theta_2 - 2.25)^2 + (\text{Sin}\theta_1 - \text{Sin}\theta_2 - 2.25)^2} + \left(\frac{\frac{z_1}{40} - \frac{z_2}{40}}{40}\right)^2 \\
= 0.1703 \times 10^{-2} \text{Joule} \cdot \frac{1}{m^2} \times (-1236.84 \times 10^{-18} \text{m}^2) \\
= -2.106 \times 10^{-18} J \quad \text{A67} \]

2. Magnetostatic energy between half circle 1 and 4.

\[ dm_1 = \sigma_1 \cdot dS_1 = I \cdot \text{Sin}\theta_1 \cdot T \cdot R \cdot d\theta_1 \quad \text{A68} \]

\[ dm_4 = \sigma_4 \cdot dS_4 = I \cdot \text{Cos}\theta_4 \cdot T \cdot R \cdot d\theta_4 \quad \text{A69} \]
\[ U_{14} = 0.1703 \times 10^{-2} \frac{\text{Joule}}{m^2} \int_0^\pi d\theta_1 \int_{-\pi/2}^{\pi/2} d\theta_2 \int_0^{25 \text{nm}} dz_1 \int_0^{25 \text{nm}} dz_4 \]
\[
\frac{\text{Sin}\theta_1 \cdot \text{Cos}\theta_2}{\sqrt{(\text{Cos}\theta_1 - \text{Cos}\theta_2 - 5.75)^2 + (\text{Sin}\theta_1 - \text{Sin}\theta_2 - 2.25)^2 + \left(\frac{z_1}{40} - \frac{z_2}{40}\right)^2}} \]
\[= 0.1703 \times 10^{-2} \frac{\text{Joule}}{m^2} \times (374.14 \times 10^{-18} \text{ m}^2)\]
\[= 0.6372 \times 10^{-18} \text{ J} \]

3. Magnetostatic interaction energy between half circle 3 and 2.

\[ U_{32} = 0.1703 \times 10^{-2} \frac{\text{Joule}}{m^2} \int_0^{2\pi} d\theta_1 \int_{-\pi/2}^{\pi/2} d\theta_2 \int_0^{25 \text{nm}} dz_1 \int_0^{25 \text{nm}} dz_4 \]
\[
\frac{\text{Sin}\theta_1 \cdot \text{Cos}\theta_2}{\sqrt{(\text{Cos}\theta_1 - \text{Cos}\theta_2 - 2.25)^2 + (\text{Sin}\theta_1 - \text{Sin}\theta_2 - 5.75)^2 + \left(\frac{z_1}{40} - \frac{z_2}{40}\right)^2}} \]
\[= 0.1703 \times 10^{-2} \frac{\text{Joule}}{m^2} \times (374.414 \times 10^{-18} \text{ m}^2)\]
\[= 0.6376 \times 10^{-18} \text{ J} \]

4. Magnetostatic energy between half circle 3 and 4.

\[ U_{34} = 0.1703 \times 10^{-2} \frac{\text{Joule}}{m^2} \int_0^{2\pi} d\theta_1 \int_{-\pi/2}^{\pi/2} d\theta_2 \int_0^{25 \text{nm}} dz_1 \int_0^{25 \text{nm}} dz_4 \]
\[
\frac{\text{Sin}\theta_1 \cdot \text{Cos}\theta_2}{\sqrt{(\text{Cos}\theta_1 - \text{Cos}\theta_2 - 5.75)^2 + (\text{Sin}\theta_1 - \text{Sin}\theta_2 - 5.75)^2 + \left(\frac{z_1}{40} - \frac{z_2}{40}\right)^2}} \]
\[= 0.1703 \times 10^{-2} \frac{\text{Joule}}{m^2} \times (-271.708 \times 10^{-18} \text{ m}^2)\]
\[= -0.4627 \times 10^{-18} \text{ J} \]

So, the total magnetostatic interaction energy is
\[ U = U_{12} + U_{14} + U_{32} + U_{34} \]
\[ = (-2.106 + 0.6376 \times 2 - 0.4627) \times 10^{-18} \, J \]
\[ = -1.294 \times 10^{-18} \, J \]
\[ = -9.37 \times 10^4 \, K \]
B. Second nearest neighbor (L type)
\[ U = \frac{I^2 R}{4 \pi \mu_0} \left( \int_{-\pi/2}^{\pi/2} d\theta_1 \int_{-\pi/2}^{\pi/2} d\theta_2 \int_0^{25 \text{nm}} dz_1 \int_0^{25 \text{nm}} dz_2 \right) \]
\[
\cos\theta_1 \cdot \cos\theta_2
\]
\[
\sqrt{(\cos\theta_1 - \cos\theta_2 - 4.5)^2 + (\sin\theta_1 - \sin\theta_2)^2 + \left( \frac{z_1}{40} - \frac{z_2}{40} \right)^2}
\]
\[
+ \left( \int_{-\pi/2}^{\pi/2} d\theta_1 \int_{-\pi/2}^{\pi/2} d\theta_3 \int_0^{25 \text{nm}} dz_1 \int_0^{25 \text{nm}} dz_3 \right)
\]
\[
\cos\theta_1 \cdot \cos\theta_3
\]
\[
\sqrt{(\cos\theta_1 - \cos\theta_3 - 4.5)^2 + (\sin\theta_1 - \sin\theta_3)^2 + \left( \frac{z_1}{40} - \frac{z_3}{40} \right)^2}
\]
\[
+ \left( \int_{-\pi/2}^{\pi/2} d\theta_4 \int_{-\pi/2}^{\pi/2} d\theta_2 \int_0^{25 \text{nm}} dz_4 \int_0^{25 \text{nm}} dz_2 \right)
\]
\[
\cos\theta_4 \cdot \cos\theta_2
\]
\[
\sqrt{(\cos\theta_4 - \cos\theta_2 - 8)^2 + (\sin\theta_4 - \sin\theta_2)^2 + \left( \frac{z_4}{40} - \frac{z_2}{40} \right)^2}
\]
\[
+ \left( \int_{-\pi/2}^{\pi/2} d\theta_4 \int_{-\pi/2}^{\pi/2} d\theta_3 \int_0^{25 \text{nm}} dz_4 \int_0^{25 \text{nm}} dz_3 \right)
\]
\[
\cos\theta_4 \cdot \cos\theta_3
\]
\[
\sqrt{(\cos\theta_4 - \cos\theta_3 - 11.5)^2 + (\sin\theta_4 - \sin\theta_3)^2 + \left( \frac{z_4}{40} - \frac{z_3}{40} \right)^2}
\]
\[
= 0.1703 \times 10^{-2} \text{ Joule/m}^2 \times
\]
\[
(-833.352 + 311.212 \times 2 - 190.955) \times 10^{-18} \text{ m}^2
\]
\[
= -0.684 \times 10^{-18} \text{ J}
\]
\[
= -4.95 \times 10^4 \text{ K}
\]
C. Second nearest neighbor (T type)
\[ U = \frac{I^2 \cdot R}{4\pi\mu_0} \left( \int_{\theta_1}^{\pi} d\theta_1 \int_{\theta_2}^{\pi} d\theta_2 \int_{0}^{25\text{nm}} dz_1 \int_{0}^{25\text{nm}} dz_2 \right. \]

\[ \left. \sqrt{\left(\cos \theta_1 - \cos \theta_2 - 8\right)^2 + \left(\sin \theta_1 - \sin \theta_2\right)^2 + \left(\frac{z_1 - z_2}{40}\right)^2} \right) \]

\[ + \int_{\pi}^{2\pi} d\theta_1 \int_{\theta_1}^{\pi} d\theta_2 \int_{0}^{25\text{nm}} dz_1 \int_{0}^{25\text{nm}} dz_2 \]

\[ \left. \sqrt{\left(\cos \theta_1 - \cos \theta_4 - 8\right)^2 + \left(\sin \theta_1 - \sin \theta_4 + 3.5\right)^2 + \left(\frac{z_1 - z_4}{40}\right)^2} \right) \]

\[ + \int_{\pi}^{2\pi} d\theta_2 \int_{\theta_2}^{2\pi} d\theta_4 \int_{0}^{25\text{nm}} dz_2 \int_{0}^{25\text{nm}} dz_4 \]

\[ \left. \sqrt{\left(\cos \theta_2 - \cos \theta_4 - 8\right)^2 + \left(\sin \theta_2 - \sin \theta_4\right)^2 + \left(\frac{z_2 - z_4}{40}\right)^2} \right) \]

\[ + \int_{\pi}^{2\pi} d\theta_2 \int_{\theta_3}^{\pi} d\theta_3 \int_{0}^{25\text{nm}} dz_3 \int_{0}^{25\text{nm}} dz_3 \]

\[ \left. \sqrt{\left(\cos \theta_2 - \cos \theta_3 - 8\right)^2 + \left(\sin \theta_2 - \sin \theta_3 - 3.5\right)^2 + \left(\frac{z_2 - z_3}{40}\right)^2} \right) \]

\[ = 0.1703 \times 10^{-2} \text{ Joule/m}^2 \times (-315.406 + 265.03) \]

\[ -315.406 + 265.03) \times 10^{-18} \text{ m}^2 \]

\[ = -0.172 \times 10^{-18} J \]

\[ = -1.25 \times 10^4 K \]
Part II: The case of lattice constant of 880nm.

Calculations in this section are very similar to part I. The coordinates of \(A_c\), \(A_t\), \(B_c\), \(B_t\), \(C_c\) and \(C_t\) are listed in table below.

<table>
<thead>
<tr>
<th></th>
<th>((x, y))</th>
<th>((x', y'))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A_c)</td>
<td>440 nm, 370 nm</td>
<td>440 nm, -510 nm</td>
</tr>
<tr>
<td>(A_t)</td>
<td>330 nm, 370 nm</td>
<td>330 nm, -510 nm</td>
</tr>
</tbody>
</table>

1. Field on point \(A_c\).

\[
H_x = \int dH_{ix} + \int dH_{ixc}
\]

\[
= \frac{1}{4\pi \mu_0} \left(\int_0^\frac{\pi}{2} \frac{0.82 \times 25 \times 40 \times \sin \theta \times (440 - 40 \cos \theta)}{(40^2 - 880 \times 40 \cos \theta - 740 \times 40 \sin \theta + 370^2 + 440^2)^{3/2}} d\theta \right.
\]

\[
+ \int_0^\frac{\pi}{2} \frac{-0.82 \times 25 \times 40 \times \sin \theta' \times (160 - 40 \cos \theta')}{(40^2 - 880 \times 40 \cos \theta' + 1020 \times 40 \sin \theta + 510^2 + 440^2)^{3/2}} d\theta'\right)
\]

\[
= \frac{1}{4\pi \mu_0} \left(\int_0^\frac{\pi}{2} \frac{32.8(1 - \cos \theta) \sin \theta}{(3321 - 352 \cos \theta - 296 \sin \theta)^{3/2}} d\theta \right.
\]

\[
+ \int_0^\frac{\pi}{2} \frac{-32.8(1 - \cos \theta') \sin \theta'}{(4553 - 352 \cos \theta' + 408 \sin \theta')^{3/2}} d\theta'\right)
\]

\[
= \frac{1}{4\pi \mu_0} \left(0.004219 - 0.002123\right) A / m
\]

\[
= \frac{1}{4\pi \times 10^{-4}} \times 0.002096 Oe = 1.67 Oe
\]
\[ H_y = \int dH_{ly} + \int dH_{ly} \]
\[ = \frac{1}{4\pi\mu_0} \left( \int_0^\pi \frac{0.82 \times 25 \times 40 \times \sin\theta \times (370 - 40\sin\theta)}{(40^2 - 880\times 40\cos\theta - 740\times 40\sin\theta + 370^2 + 440^2)^{3/2}} d\theta \right. \]
\[ \left. - \int_0^\pi \frac{-0.82 \times 25 \times 40 \times \sin\theta' \times (-510 - 40\sin\theta')}{(40^2 - 880\times 40\cos\theta' + 1020\times 40\sin\theta' + 510^2 + 440^2)^{3/2}} d\theta' \right) \]
\[ = \frac{1}{4\pi\mu_0} \left( \int_0^\pi \frac{32.8(9.25 - \sin\theta)\sin\theta}{(3321 - 352\cos\theta - 296\sin\theta)^{3/2}} d\theta \right. \]
\[ \left. - \int_0^\pi \frac{-32.8(-12.75 - \sin\theta')\sin\theta'}{(4553 - 352\cos\theta' + 408\sin\theta')^{3/2}} d\theta' \right) \]
\[ = \frac{1}{4\pi\mu_0} (0.00326 - 0.00062) A/m \]
\[ = \frac{1}{4\pi \times 10^{-4}} \times 0.00264 Oe = 2.10 Oe \]

\[ \sqrt{1.67^2 + 2.1^2} = 2.68 Oe \]

**2. Field on point \( A_t \)**

\[ H_x = \int dH_{lx} + \int dH_{lx} \]
\[ = \frac{1}{4\pi\mu_0} \left( \int_0^{\pi} \frac{32.8(8.25 - \cos\theta)\sin\theta}{(2474 - 264\cos\theta - 296\sin\theta)^{3/2}} d\theta \right. \]
\[ \left. - \int_0^{\pi} \frac{32.8(8.25 + \cos\theta')\sin\theta'}{(3706 - 264\cos\theta' + 408\sin\theta')^{3/2}} d\theta' \right) \]
\[ = \frac{1}{4\pi\mu_0} (0.005114 - 0.002135) A/m \]
\[ = \frac{1}{4\pi \times 10^{-4}} \times 0.002979 Oe = 2.37 Oe \]
\[ H_y = \int dH_{y'} + \int dH_{Hy} \]
\[ = \frac{1}{4\pi\mu_0} \left( \int_0^\pi \frac{32.8(9.25 - \sin \theta) \sin \theta}{(2474 - 264 \cos \theta - 296 \sin \theta)^{3/2}} d\theta \right. \]
\[ - \int_0^\pi \frac{32.8(12.75 + \sin \theta') \sin \theta'}{(3706 - 264 \cos \theta' + 408 \sin \theta')^{3/2}} d\theta' \right) \quad \text{A80} \]
\[ = \frac{1}{4\pi\mu_0} (0.005278 - 0.003487) \text{A/m} \]
\[ = \frac{1}{4\pi \times 10^{-4}} \times 0.002791 \text{Oe} = 1.43 \text{Oe} \]
\[ \sqrt{2.37^2 + 1.43^2} = 2.77 \text{Oe} \quad \text{A81} \]
Part III: The case of lattice constant of 680nm.

The coordinates of $A_c$ and $A_t$ are listed in the table below.

<table>
<thead>
<tr>
<th></th>
<th>$(x, y)$</th>
<th>$(x', y')$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_c$</td>
<td>340 nm, 270 nm</td>
<td>340 nm, -410 nm</td>
</tr>
<tr>
<td>$A_t$</td>
<td>230 nm, 270 nm</td>
<td>230 nm, -410 nm</td>
</tr>
</tbody>
</table>

1. Field on point $A_c$.

\[
H_x = \int dH_{x_c} + \int dH_{y_c}
\]

\[
= \frac{1}{4\pi\mu_0} \left( \int_0^\pi \frac{32.8(8.5 - \cos\theta)\sin\theta}{(1901 - 272\cos\theta - 216\sin\theta)^{3/2}} d\theta + \int_0^\pi \frac{-32.8(8.5 - \cos\theta')\sin\theta'}{(2853 - 272\cos\theta' + 328\sin\theta')^{3/2}} d\theta' \right)
\]

\[
= \frac{1}{4\pi\mu_0} (0.00779 - 0.00322) A/m
\]

\[
= \frac{1}{4\pi \times 10^{-3}} \times 0.00457 Oe = 3.64 Oe
\]
\[ H_y = \int dH_{y} + \int dH_{\perp y} \]
\[ = \frac{1}{4\pi\mu_0} \left( \frac{32.8(6.75 - \sin\theta)\sin\theta}{(1901 - 272\cos\theta - 216\sin\theta)^{3/2}} d\theta \right) \]
\[ - \int_0^\pi \frac{32.8(10.25 + \sin\theta')\sin\theta'}{(2853 - 272\cos\theta' + 328\sin\theta')^{3/2}} d\theta' \]  
\[ = \frac{1}{4\pi\mu_0} (0.00551 - 0.0042) A/m \]
\[ = \frac{1}{4\pi \times 10^{-3}} 	imes 0.00131 Oe = 1.04 Oe \]

\[ \sqrt{3.64^2 + 1.04^2} = 3.79 Oe \]  

A83

2. Field on point \( A_l \).

\[ H_x = \int dH_{x} + \int dH_{\perp x} \]
\[ = \frac{1}{4\pi\mu_0} \left( \frac{32.8(5.75 - \cos\theta)\cos\theta}{(1274 - 184\cos\theta - 216\sin\theta)^{3/2}} d\theta \right) \]
\[ + \int_0^\pi \frac{-32.8(5.75 - \cos\theta')\cos\theta'}{(2226 - 184\cos\theta' + 328\sin\theta')^{3/2}} d\theta' \]  
\[ = \frac{1}{4\pi\mu_0} (0.0103 - 0.00304) A/m \]
\[ = \frac{1}{4\pi \times 10^{-3}} \times 0.00726 Oe = 5.80 Oe \]

A85
\[ H_y = \int dH_{y} + \int dH_{\theta y} \]
\[ = \frac{1}{4\pi\mu_0} \left( \int_0^\pi \frac{32.8(6.75 - \sin\theta)\sin\theta}{(1274 - 184\cos\theta - 216\sin\theta)^{3/2}} d\theta \right. \]
\[ \left. - \int_0^\pi \frac{32.8(10.25 + \sin\theta)\sin\theta'}{(2226 - 184\cos\theta' + 328\sin\theta')^{3/2}} d\theta' \right) \]
\[ = \frac{1}{4\pi\mu_0} (0.0108 - 0.00588) \text{A/m} \]
\[ = \frac{1}{4\pi \times 10^{-3}} \times 0.00492 \text{Oe} = 3.92 \text{Oe} \]

\[ \sqrt{5.8^2 + 3.92^2} = 7.0 \text{Oe} \]
Appendix B

Recipe for Fabrication of Permalloy Pattern Using PMMA/PMGI

1. Apply PMGI (SF1) photo resist onto 3-inch silicon wafer. Set the spinner ramping rate at 500 rpm/second and do dynamic dispense at 500 rpm for 5 seconds. Then the resist spins at 1200 rpm for 45 seconds. The ramping rate of spinner from 500 rpm to 1200 rpm is 10,000 rpm/second. Finally the spinner stops at a rate of 5000 rpm/second.

2. Bake the wafer on hot plate for 5 minutes at a temperature of 190 Celsius.

3. Apply PMMA resist on top of PMGI. The PMMA has molecular weight of 950 K and is dissolved in anisole with a volume concentration of 2%. Ramp the spinner from zero to 500 rpm at a rate of 1000 rpm/second and stay at 500 rpm for 10 seconds while dipping PMMA resist onto the wafer. Then ramp the spinner to 3000 rpm at a rate of 1000 rpm/second and stay at 3000 rpm for 90 seconds before the spinner ramps to zero at a rate of 5000 rpm/second.

4. Bake the wafer on a hot plate for 5 minutes at 180 Celsius.

5. Expose the PMMA/PMGI resist using 100 KV electron beam with a dosage of 800 μC/cm².

6. Develop PMMA at 20 Celsius for 90 seconds using MIBK : IPA (1 : 3).

7. Rinse the wafer in IPA for 1 minute.

8. Bake the wafer on a hot plate for 2 minutes at 120 Celsius.

10. Rinse the wafer in DI water for 1 minute.

11. Bake the wafer on a hot plate for 2 minutes at 120 Celsius.

12. Deposit 25nm permalloy and 3nm aluminum on the wafer.

13. Lift-off PMMA and PMGI resist using acetone with ultra sonic agitation for about 10 minutes. This will leave clean permalloy patterns on the silicon wafer.

### Appendix C

**Energies of Four Types of Vertices Before and After Relaxation in Zero Applied Field.**

<table>
<thead>
<tr>
<th>Object</th>
<th>Lattice Parameter (nm)</th>
<th>State</th>
<th>Total Energy (J)</th>
<th>Exchange Energy (J)</th>
<th>Demagnetization Energy (J)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single Dot (Initial state has spins along positive Y direction. Dot’s long axis is also along Y)</td>
<td>Initial</td>
<td>1.516E-17</td>
<td>1.627E-25</td>
<td>1.516E-17</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Final</td>
<td>1.467E-17</td>
<td>2.740E-19</td>
<td>1.439E-17</td>
<td></td>
</tr>
<tr>
<td>Single Dot (Initial state has random spin configuration)</td>
<td>Initial</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Final</td>
<td>1.467E-17</td>
<td>2.963E-19</td>
<td>1.437E-17</td>
<td></td>
</tr>
<tr>
<td>One Ying-Yang vertex (Initial state is 320)</td>
<td>Initial</td>
<td>5.738E-17</td>
<td>6.550E-25</td>
<td>5.738E-17</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Final</td>
<td>5.386E-17</td>
<td>1.912E-18</td>
<td>5.195E-17</td>
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</tr>
<tr>
<td></td>
<td>360</td>
<td>Initial</td>
<td>5.830E-17</td>
<td>6.611E-25</td>
<td>5.830E-17</td>
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<tr>
<td></td>
<td>Final</td>
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<td>1.734E-18</td>
<td>5.393E-17</td>
<td></td>
</tr>
<tr>
<td></td>
<td>400</td>
<td>Initial</td>
<td>5.948E-17</td>
<td>6.743E-25</td>
<td>5.948E-17</td>
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<tr>
<td></td>
<td>Final</td>
<td>Initial</td>
<td>Final</td>
<td></td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>--------</td>
<td>----------</td>
<td>--------</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>440</strong></td>
<td>5.706E-17</td>
<td>1.756E-18</td>
<td>5.531E-17</td>
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<td></td>
</tr>
<tr>
<td><strong>480</strong></td>
<td>5.978E-17</td>
<td>6.749E-25</td>
<td>5.978E-17</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>560</strong></td>
<td>5.753E-17</td>
<td>1.652E-18</td>
<td>5.588E-17</td>
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</tr>
<tr>
<td><strong>680</strong></td>
<td>5.997E-17</td>
<td>6.751E-25</td>
<td>5.997E-17</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>880</strong></td>
<td>5.781E-17</td>
<td>1.575E-18</td>
<td>5.623E-17</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**One Bow-Tie vertex (Initial state is 3)**

<table>
<thead>
<tr>
<th></th>
<th>Final</th>
<th>Initial</th>
<th>Final</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>320</strong></td>
<td>5.306E-17</td>
<td>6.569E-25</td>
<td>5.306E-17</td>
</tr>
<tr>
<td><strong>360</strong></td>
<td>5.122E-17</td>
<td>1.072E-18</td>
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<tr>
<td><strong>400</strong></td>
<td>5.556E-17</td>
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<td>5.556E-17</td>
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<tr>
<td><strong>440</strong></td>
<td>5.367E-17</td>
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<td>5.253E-17</td>
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<tr>
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<td>5.757E-17</td>
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<td>5.757E-17</td>
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<tr>
<td><strong>560</strong></td>
<td>5.557E-17</td>
<td>1.179E-18</td>
<td>5.439E-17</td>
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<tr>
<td><strong>680</strong></td>
<td>5.839E-17</td>
<td>6.745E-25</td>
<td>5.836E-17</td>
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<tr>
<td><strong>880</strong></td>
<td>5.638E-17</td>
<td>1.188E-18</td>
<td>5.519E-17</td>
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</tbody>
</table>

**One Pac-man vertex (Initial state is 880)**

<table>
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<th></th>
<th>Final</th>
<th>Initial</th>
<th>Final</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>320</strong></td>
<td>5.984E-17</td>
<td>6.385E-25</td>
<td>5.984E-17</td>
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<td><strong>880</strong></td>
<td>6.051E-17</td>
<td>6.755E-25</td>
<td>6.051E-17</td>
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<tr>
<td>Time (ms)</td>
<td>Initial</td>
<td>Final</td>
<td>OR State</td>
</tr>
<tr>
<td>----------</td>
<td>---------</td>
<td>-------</td>
<td>-----------</td>
</tr>
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<td>320</td>
<td>7.142E-17</td>
<td>6.873E-17</td>
<td>2.533E-18</td>
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<td>6.550E-17</td>
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<td>6.409E-17</td>
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<td>6.090E-17</td>
<td>6.090E-17</td>
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</tbody>
</table>
Appendix D

Summary of the Experimental Data of the Demagnetization Protocols

Protocol 1: The magnetic field was stepped down in strength and switched polarity with each step.

We start from a field as high as 2016 Oe, which is well above Permalloy islands’ coercive field. Then this field gradually damps to zero while its polarity changes between positive and negative. The schematic of the field ramping process is shown in figure D-1.

Figure D-1: Schematic of the field ramping process of protocol 1.
1. Sequence 1-1.

Field steps are: 32 Oe from 1280 Oe to -800 Oe, 16 Oe from -800 Oe to 784 Oe, 32 Oe from 784 Oe to 720 Oe, 16 Oe from 720 Oe to 16 Oe, and 8 Oe from 16 Oe to 0.

<table>
<thead>
<tr>
<th>Image</th>
<th>Number of islands</th>
<th>Lattice Spacing (nm)</th>
<th>$M_{\text{tot}}$</th>
<th>Angle</th>
<th>Step time (s)</th>
<th>Rotation rate (RPM)</th>
<th>Ramping Rate (Oe/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>813</td>
<td>1115</td>
<td>320</td>
<td>0.05303</td>
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</tbody>
</table>
2. Sequence 1-2.

Field step: 32 Oe from 1280 Oe to -800 Oe, 16 Oe from -800 Oe to 784 Oe, 32 Oe from 784 Oe to 720 Oe, 16 Oe from 720 Oe to 16 Oe, and 8 Oe from 16 Oe to 0. Only one power supply is used for this series of experiments.
<table>
<thead>
<tr>
<th>Image</th>
<th>Number of islands</th>
<th>Lattice Spacing (nm)</th>
<th>$M_{tot}$</th>
<th>Angle</th>
<th>Step time (s)</th>
<th>Rotation Rate(RPM)</th>
<th>Ramping Rate(Oe/s)</th>
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<td>0.06152</td>
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<td>954</td>
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<td>963</td>
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<td>0.03606</td>
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</table>

3. Sequence 1-3

The field step is 160 Oe from 1208 Oe to 0.

<table>
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<tr>
<th>Image</th>
<th>Number of islands</th>
<th>Lattice Spacing (nm)</th>
<th>$M_{tot}$</th>
<th>Angle</th>
<th>Step time (s)</th>
<th>Rotation Rate(RPM)</th>
<th>Ramping Rate(Oe/s)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>749</td>
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<td>1000</td>
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<td>1000</td>
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</tbody>
</table>

4. Sequence 1-4

The field step is 3.2 Oe from 1280 Oe to 0.

<table>
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<tr>
<th>Image</th>
<th>Number of islands</th>
<th>Lattice Spacing (nm)</th>
<th>$M_{tot}$</th>
<th>Angle</th>
<th>Step time (s)</th>
<th>Rotation Rate(RPM)</th>
<th>Ramping Rate(Oe/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nm020</td>
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<td>0.12869</td>
<td>297.1</td>
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<td>24000</td>
</tr>
<tr>
<td>Nm019</td>
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<td>400</td>
<td>0.09192</td>
<td>317.3</td>
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<td>1000</td>
<td>24000</td>
</tr>
</tbody>
</table>
5. Sequence 1-5

The field step is 3.2 Oe from 2016 Oe to 0.

<table>
<thead>
<tr>
<th>Image</th>
<th>Number of islands</th>
<th>Lattice Spacing (nm)</th>
<th>$M_{tot}$</th>
<th>Angle</th>
<th>Step time (s)</th>
<th>Rotationa l Rate(RPM)</th>
<th>Ramping Rate(Oe/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nm021</td>
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<td>400</td>
<td>0.03677</td>
<td>280.5</td>
<td>10</td>
<td>1000</td>
<td>24000</td>
</tr>
<tr>
<td>Nm022</td>
<td>681</td>
<td>480</td>
<td>0.04172</td>
<td>267.1</td>
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<td>1000</td>
<td>24000</td>
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</tbody>
</table>

6. Sequence 1-6

The magnetic field step decreases to 1008 Oe from 2016 Oe then it decreases with field steps of 3.2 Oe from 1008 Oe to 0.

<table>
<thead>
<tr>
<th>Image</th>
<th>Number of islands</th>
<th>Lattice Spacing (nm)</th>
<th>$M_{tot}$</th>
<th>Angle</th>
<th>Step time (s)</th>
<th>Rotationa l Rate(RPM)</th>
<th>Ramping Rate(Oe/s)</th>
</tr>
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<tbody>
<tr>
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<td>1000</td>
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<tr>
<td>Nm024</td>
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<td>480</td>
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<td>232.4</td>
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<td>1000</td>
<td>24000</td>
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</tbody>
</table>

7. Sequence 1-7

The magnetic field decreases to 512 Oe from 2016 Oe then it decreases with field steps of 3.2 Oe from 512 Oe to 0.

<table>
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<tr>
<th>Image</th>
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<th>Lattice Spacing (nm)</th>
<th>$M_{tot}$</th>
<th>Angle</th>
<th>Step time (s)</th>
<th>Rotationa l Rate(RPM)</th>
<th>Ramping Rate(Oe/s)</th>
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8. Sequence 1-8

The magnetic field decreases to 752 Oe from 2016 Oe then it decreases with field steps of 3.2 Oe from 752 Oe to 0.

<table>
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<th>Number of islands</th>
<th>Lattice Spacing (nm)</th>
<th>$M_{tot}$</th>
<th>Angle</th>
<th>Step time (s)</th>
<th>Rotation Rate (RP M)</th>
<th>Ramping Rate (Oe/s)</th>
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<tr>
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<tr>
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<td>0.12304</td>
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9. Sequence 1-9

The magnetic field decreases to 640 Oe from 2016 Oe then it decreases with field steps of 3.2 Oe from 640 Oe to 0.

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<th>Number of islands</th>
<th>Lattice Spacing (nm)</th>
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<th>Step time (s)</th>
<th>Rotation Rate (RP M)</th>
<th>Ramping Rate (Oe/s)</th>
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<tr>
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<tr>
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</table>
10. Sequence 1-10

The magnetic field decreases to 704 Oe from 2016 Oe then it decreases with field steps of 3.2 Oe from 704 Oe to 0.

<table>
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<th>Image</th>
<th>Number of islands</th>
<th>Lattice Spacing (nm)</th>
<th>$M_{tot}$</th>
<th>Angle</th>
<th>Step time (s)</th>
<th>Rotationa l Rate(RPM)</th>
<th>Ramping Rate(Oe/ s)</th>
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11. Sequence 1-11

The magnetic field decreases to 848 Oe from 2016 Oe then it decreases with field steps of 3.2 Oe from 848 Oe to 752 Oe. The field stays at 0 Oe for 20 seconds afterwards before it drops to -748.4 Oe, after which the field ramps with step size of 3.2 Oe before it finally settles at 0 Oe.

<table>
<thead>
<tr>
<th>Image</th>
<th>Number of islands</th>
<th>Lattice Spacing (nm)</th>
<th>$M_{tot}$</th>
<th>Angle</th>
<th>Step time (s)</th>
<th>Rotationa l Rate(RPM)</th>
<th>Ramping Rate(Oe/ s)</th>
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<tr>
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<td>1000</td>
<td>24000</td>
</tr>
</tbody>
</table>
12. Sequence 1-12

Field steps: directly down to 848 Oe from 2016 Oe, 3.2 Oe from 848 Oe to 640 Oe, stay at 0 for 20 sec then down to 0 by 3.2 Oe from -636.8 Oe.

<table>
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<th>Image</th>
<th>Number of islands</th>
<th>Lattice Spacing (nm)</th>
<th>$M_{\text{tot}}$</th>
<th>Angle</th>
<th>Step time (s)</th>
<th>Rotational Rate (RP M)</th>
<th>Ramping Rate (Oe/s)</th>
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<tbody>
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<td>1000</td>
<td>24000</td>
</tr>
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<td>1000</td>
<td>24000</td>
</tr>
</tbody>
</table>
Protocol 2: The magnetic field strength was stepped down without polarity alternation and was held at zero temporarily between each step.

Figure D-2 displays the schematic of the field ramping of protocol 2.

![Magnetic Field Strength vs Time](image)

Figure D-2: Schematic of the field ramping process of protocol 2.

1. Sequence 2-1

Field step: 32Oe from 1280Oe to 0

<table>
<thead>
<tr>
<th>Image</th>
<th>Number of islands</th>
<th>Lattice Spacing (nm)</th>
<th>$M_{tot}$</th>
<th>Angle</th>
<th>Step time (s)</th>
<th>Rotationa l Rate(RPM)</th>
<th>Ramping Rate(Oe/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>990</td>
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<td>6</td>
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</tr>
</tbody>
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---
2. Sequence 2-2

Field step: 32Oe from 1280Oe to 800Oe, 16 Oe from 800Oe to 784 Oe, 32Oe from 784Oe to 720Oe, 16 Oe from 720 to 16Oe, 8 Oe from 16 Oe to 0.

<table>
<thead>
<tr>
<th>Image</th>
<th>Number of islands</th>
<th>Lattice Spacing (nm)</th>
<th>$M_{\text{tot}}$</th>
<th>Angle</th>
<th>Step time (s)</th>
<th>Rotation Rate (RP M)</th>
<th>Ramping Rate (Oe/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nm003</td>
<td>1131</td>
<td>360</td>
<td>0.0297</td>
<td>177.6</td>
<td>1</td>
<td>1000</td>
<td>24000</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nm001</td>
<td>836</td>
<td>440</td>
<td>0.14779</td>
<td>169.5</td>
<td>1</td>
<td>1000</td>
<td>24000</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nm004</td>
<td>441</td>
<td>560</td>
<td>0.36062</td>
<td>216.5</td>
<td>1</td>
<td>1000</td>
<td>24000</td>
</tr>
</tbody>
</table>

3. Sequence 2-3

Field step: 3.2Oe from 2016Oe to 0Oe

<table>
<thead>
<tr>
<th>Image</th>
<th>Number of islands</th>
<th>Lattice Spacing (nm)</th>
<th>$M_{\text{tot}}$</th>
<th>Angle</th>
<th>Step time (s)</th>
<th>Rotation Rate (RP M)</th>
<th>Ramping Rate (Oe/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nm102</td>
<td>966</td>
<td>400</td>
<td>0.13223</td>
<td>136.7</td>
<td>10</td>
<td>1000</td>
<td>24000</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nm104</td>
<td>449</td>
<td>560</td>
<td>0.27011</td>
<td>188.1</td>
<td>10</td>
<td>1000</td>
<td>24000</td>
</tr>
</tbody>
</table>
Protocol 3: The magnetic field was decreased to zero linearly.

Figure 3 displays the field ramping process of protocol 3.

<table>
<thead>
<tr>
<th>Image</th>
<th>Number of islands</th>
<th>Lattice Spacing (nm)</th>
<th>M_{tot}</th>
<th>Angle</th>
<th>Step time (s)</th>
<th>Rotationa l Rate (RP M)</th>
<th>Ramping Rate (Oe/ s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>964</td>
<td>914</td>
<td>440</td>
<td>0.52679</td>
<td>275.4</td>
<td>na</td>
<td>1000</td>
<td>8</td>
</tr>
<tr>
<td>974</td>
<td>1057</td>
<td>360</td>
<td>0.32456</td>
<td>112.5</td>
<td>na</td>
<td>1000</td>
<td>0.08</td>
</tr>
<tr>
<td>973</td>
<td>758</td>
<td>440</td>
<td>0.40093</td>
<td>112.4</td>
<td>na</td>
<td>1000</td>
<td>0.08</td>
</tr>
<tr>
<td>Nm033</td>
<td>967</td>
<td>400</td>
<td>0.25809</td>
<td>100.1</td>
<td>na</td>
<td>1000</td>
<td>0.32</td>
</tr>
<tr>
<td>Nm034</td>
<td>715</td>
<td>480</td>
<td>0.21496</td>
<td>104.5</td>
<td>na</td>
<td>1000</td>
<td>0.32</td>
</tr>
<tr>
<td>Nm035</td>
<td>674</td>
<td>480</td>
<td>0.29698</td>
<td>101.5</td>
<td>na</td>
<td>1000</td>
<td>0.32</td>
</tr>
</tbody>
</table>

Figure D-3: Schematic of the magnetic field ramping process of protocol 3.
Protocol 4: The magnetic field was decreased to zero by steps.

Figure D-4 displays the schematic of the field ramping process of protocol 4.

1. Sequence 4-1

Field step: 32Oe from 1280Oe to 800Oe, 16 Oe from 800 Oe to 784Oe, 32Oe from 784Oe to 720 Oe, 16 Oe from 720 Oe to 16Oe, 8Oe from 16Oe to 0.

<table>
<thead>
<tr>
<th>Image</th>
<th>Number of islands</th>
<th>Lattice Spacing (nm)</th>
<th>$M_{\text{tot}}$</th>
<th>Angle</th>
<th>Step time (s)</th>
<th>Rotationa l Rate(RPM)</th>
<th>Ramping Rate(Oe/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>845</td>
<td>1097</td>
<td>400</td>
<td>0.29062</td>
<td>62.2</td>
<td>10</td>
<td>1000</td>
<td>24000</td>
</tr>
</tbody>
</table>
2. Sequence 4-1

Field step: 3.2Oe from 2016Oe to 0Oe

<table>
<thead>
<tr>
<th>Image</th>
<th>Number of islands</th>
<th>Lattice Spacing (nm)</th>
<th>( M_{\text{tot}} )</th>
<th>Angle</th>
<th>Step time (s)</th>
<th>Rotation Rate (RP M)</th>
<th>Ramping Rate (Oe/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nm036</td>
<td>1053</td>
<td>400</td>
<td>0.20365</td>
<td>304.4</td>
<td>10</td>
<td>1000</td>
<td>24000</td>
</tr>
</tbody>
</table>

We also carried out demagnetization experiments using a field ramping process as outlined in figure D-5. In these experiments, the magnetic field ramps from about 1300 Oe to values around the coercive field and stays at these values for 15 minutes. Then the field decrease to zero directly.
<table>
<thead>
<tr>
<th>Image</th>
<th>Number of islands</th>
<th>Lattice Spacing (nm)</th>
<th>$M_{tot}$</th>
<th>Angle</th>
<th>Step time (s)</th>
<th>Rotation Rate (RP M)</th>
<th>Ramping Rate (Oe/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>975</td>
<td>922</td>
<td>400</td>
<td>0.34719</td>
<td>262.4</td>
<td>na</td>
<td>1000</td>
<td>24000</td>
</tr>
<tr>
<td>976</td>
<td>748</td>
<td>400</td>
<td>0.40659</td>
<td>137.1</td>
<td>na</td>
<td>1000</td>
<td>24000</td>
</tr>
<tr>
<td>978</td>
<td>922</td>
<td>400</td>
<td>0.577</td>
<td>95.2</td>
<td>na</td>
<td>1000</td>
<td>24000</td>
</tr>
<tr>
<td>979</td>
<td>967</td>
<td>400</td>
<td>0.53669</td>
<td>224.1</td>
<td>na</td>
<td>1000</td>
<td>24000</td>
</tr>
<tr>
<td>980</td>
<td>860</td>
<td>400</td>
<td>0.7955</td>
<td>324.6</td>
<td>na</td>
<td>1000</td>
<td>24000</td>
</tr>
<tr>
<td>981</td>
<td>922</td>
<td>400</td>
<td>0.83439</td>
<td>318.5</td>
<td>na</td>
<td>1000</td>
<td>24000</td>
</tr>
</tbody>
</table>

Figure D-5: Schematic of the field ramping process of protocol 5.
VITA

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Representative Publications

1. "Demagnetization protocols for frustrated interacting nanomagnet arrays."
   Accepted by JOURNAL of APPLIED PHYSICS, to be published in June 2007

   NATURE, January 2006, 439(7074), 303-306