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DISRUPTION AND OPERATIONAL RISK QUANTIFICATION AND MITIGATION MODELS FOR OUTSOURCING OPERATIONS

A Dissertation in

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Abstract

More companies choose outsourcing to gain cost advantages, focus on their core competencies and maintain competitive edge. Although outsourcing provides many benefits, it also makes the buyer more dependent to the outside firms and increases his exposure to supply chain risks. This dissertation provides quantitative techniques to measure those risks and mathematical models to incorporate risks in supply chain decision making. Outsourcing risks are grouped under two main categories: *operational risks* which represent risks due to day to day global supply chain operations and *disruption risks* which are related to rare, but catastrophic events that may disrupt supply chains and cause heavier damage than the operational risks. In this dissertation, we first present a general risk quantification scheme and a classification based on four major risk components: severity of impact, frequency of occurrence, detection time and recovery time, and implement this scheme to quantify disruption risks. Severity of impact is modeled using the *Generalized Extreme Value Distribution* which is appropriate for modeling minima and maxima of rare events. Frequency of occurrence is modeled as a Poisson process. A Markov chain is used to model information propagation in supply chains and detection time is modeled using the *mean first passage time* concept. Risk recovery time is assumed to be exponentially distributed and a conceptual model to compute the parameter of the exponential distribution is also developed.

Another important issue in risk management is mitigation plans. Once a supplier faces a disruption, the buyer should have an alternative strategy to follow. In this dissertation, we propose two multiobjective mathematical models to optimally generate supplier assignment and mitigation plans under two different purchasing strategies. The first strategy, called *single sourcing*, assumes that the buyer assigns an order for a product to one and only one supplier; that is, order splitting among suppliers is not allowed. The second strategy, called *multiple sourcing*, is a generalization of the single sourcing model where the buyer can split an order among a predetermined number of suppliers. Both models consider four objectives: minimizing total cost, lead time and risk value, and maximizing quality of purchased items. The multiobjective models are solved using four variants of goal programming and their solutions are discussed.

Operational risks are more common in supply chains and can be modeled using traditional probability distributions. In this dissertation, we extend the multiobjective mathematical models developed earlier to stochastic programming models. Uncertainty in customer demand and production capacity are assumed to cause operational risk. Initially, demand and capacity parameters are modeled as normal random variables and chance constraints are formed to include those stochastic data in the mathematical models. Deterministic equivalents of those chance constraints are derived to numerically solve the models. When no correlation among random variables is assumed, the deterministic equivalent models are linear mixed integer programs which can be solved efficiently using commercial optimization software. If correlation is included, deterministic equivalent models become nonlinear mixed integer programs which are computationally more challenging. Alternative linearization procedures are proposed to transform the deterministic equivalents of the nonlinear models to linear mixed integer programs. This results in an increase in the problem size. Deterministic equivalent models are also solved using goal programming techniques. It is observed that the optimal solutions to the deterministic models are infeasible to the stochastic models. This indicates that previous supply chain decisions are no longer valid when uncertainty is considered in decision making. We also present a robust model where the normality assumption on demand and capacity random variables is removed. This robust model is valuable when the decision makers have information only on the mean and the variance of demand and capacity. The robust model is more conservative and provides poorer results compared to the stochastic models under normality.

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R. Ufuk Bilsel
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Dedication

To my parents Hikmet and Nafiz Bilsel.

Chapter 1

Introduction

1.1 Supply Chain Setup and Basics

A supply chain is a set of business units involved directly or indirectly in fulfilling a customer request (Chopra & Meindl 2006). Chopra & Meindl (2006) argue that a typical supply chain involves customers, retailers, wholesalers/distrubutors, manufacturers and suppliers. A generic supply chain can be schematized as in Figure 1.1.

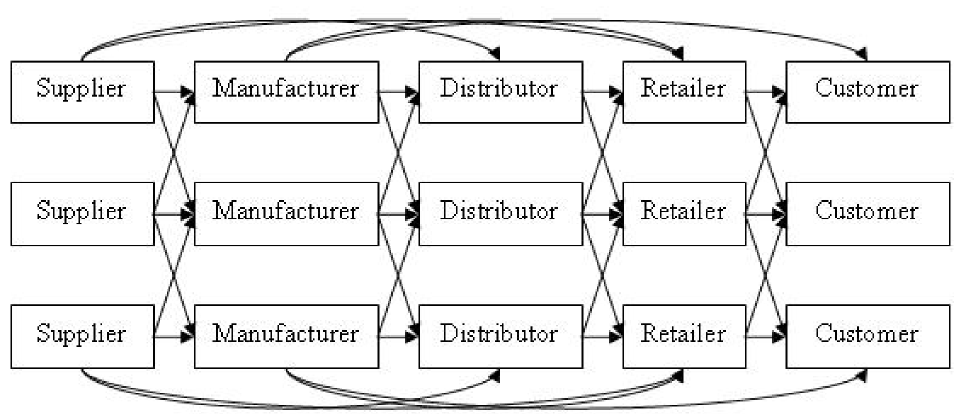

Figure 1.1. Layout of a typical supply chain

Note that not all supply chains have to adhere to the settings of Figure 1.1. For instance, some supply chains may not include all the business entities in Figure 1.1 or may include different ones. Similarly, not all links in Figure 1.1 have to exist in every supply chain. Clearly, effective management of such a large structure is a challenging undertaking, but vital for success. Chopra & Meindl (2006) define *supply chain management* as the management of flows between and among stages in a supply chain to maximize total profitability. Simchi-Levi et al. (2008) provide a broader definition; according to them supply chain management consists of a set of approaches utilized to efficiently integrate suppliers, manufacturers, warehouses and stores, so that merchandize is produced and distributed at the right quantities, to the right locations, and at the right time, in order to minimize system-wide costs while satisfying service level requirements.

The former statement defines the objective of supply chain as maximizing the total profitability. The latter describes the path to achieve this goal as successfully integrating all entities in the supply chain and optimizing the flows among them. We believe both definitions are incomplete since they ignore a key contributor to supply chain operations: *the risk*. Even though one might argue that the definition brought by Simchi-Levi et al. (2008) inherently involves risk by including efficiency of coordination and optimization of flows, it leaves out the risk of rare events. We will discuss more about the impact of risks on supply chains in Section 1.2.

Supply chain management consists of a variety of activities. These activities include locating facilities and selecting suppliers to create a supply chain network, forecasting and managing customer demand, organizing distribution channels, synchronizing transportation and scheduling manufacturing activities. There exist a large body of literature on all supply chain activities and their synchronization. Interested reader can refer to recent books by Chopra & Meindl (2006) and Simchi-

Levi et al. (2008) for a broad discussion of supply chain activities.

1.2 Risk Management in Supply Chains

Risk management can be defined as a process of balancing potential risks against potential rewards to achieve favorable outcomes in terms of overall business performance (Ritchie & Brindley 2007). Although generally accepted supply chain management definitions leave out the risk component, supply chain risks have been acknowledged and investigated by many researchers (see for instance Chopra & Sodhi (2004), Chopra & Meindl (2006), Tang (2006), Yang (2006) and Ravindran et al. (2009)). Ritchie & Brindley (2007) argue that there are five drivers of supply chain risks: external environment, industry, supply chain configuration, partners and supply chain nodes. Furthermore, Handfield (2008) asserts that supply chain success depends on a formal process of assessment of risks, identification of potential impacts and development of a set of contingency plans to mitigate risks. Simchi-Levi et al. (2008) identifies five major sources of uncertainty that potentially bring risk to supply chain management:

- Matching supply and demand,
- Fluctuation of inventory levels and backorders across the supply chain,
- Amplified forecasting errors,
- Uncertain lead times, manufacturing yields and parts availability,
- Uncertainty imposed by lean manufacturing, outsourcing and offshore operations.

There exists other drivers of supply chain risks. For instance misalignment of manufacturing and marketing strategies is argued to be another source of supply chain risk (Handfield 2008).

The last bullet in the above classification includes risks brought by outside suppliers, which is of primary interest in this dissertation. Supply risks can be grouped under two broad categories: risks due to ordinary operations in the supply chain and risks due to man made or natural disasters. Yang (2006) calls the operational risks as *Miss-the-Target (MtT)* type risks and the man made and natural risks as *Value-at-Risk (VaR)* type risks. These names come from the methodologies he proposes in quantifying each type of risk. Chopra et al. (2007) use the terms *recurrent risks* for the former and *disruption risks* for the latter risk groups. Yang (2006) distinguishes between four different aspects of supply chain risks: severity, likelihood, internality and manageability and develops a broad supply chain risk classification matrix. According to his classification, disruption risks fall under the category *Large Impact and Small Possibility*. We will discuss more about Yang's classifications in the following pages. Chopra & Sodhi (2004) categorize supply chain risks and their causes as in Table 1.1.

Effect of disruptions on supply chains can be crucial. In February 1997, a fire in one of Toyota's brake supplier's facility resulted in a two week cease of operations at Toyota plants in Japan and caused losses around $195 million. Again in 1997, Boeing lost an estimate of $2.6 Billions due to a delivery failure of two outsourced critical parts (Radju et al. 2002). A brief fire in March 2000 at a Philips semiconductor plant in Albuquerque, NM resulted in a six-week shut down. The plant was the sole supplier of the Swedish telecommunication company Ericsson which came up millions of chips short of what it needed for a key new product and lost more than $400 millions in revenue (Latour 2001). As a result, Ericsson lost the North American cellular phone market. After hurricane Katrina struck the gulf coast, the US realized shortages in fuel, coffee and several food products from that region. More strikingly, average gas prices in the US immediately increased about 50 cents per gallon, which corresponded to a sharp jump of more than 20% (*Fuel Economy Report* 2008). Griffy-Brown (2003) cites a list of disruptive events that happened between late 1990s and 2002 and report their impact: in 1998, hurricane

Table 1.1. Supply chain risks and risk drivers (adapted from Chopra & Sodhi (2004))

Category of Risk	Drivers of Risk
Disruption	Natural disaster Labor strike Supplier bankruptcy Terrorism Leave of key personnel
Delays	High capacity utilization at suppliers Inflexibility at suppliers Poor yield quality at suppliers Transportation, materials handling and customs delays
Systems	Information systems breakdown Systems integration issues E-commerce
Forecasting	Forecasting errors Bullwhip effect and information distorsion
Intellectual Property	Vertical integration of supply chain Global outsourcing
Procurement	Exchange rates Single sourcing Capacity restrictions Supply contracts and supplier relationships
Receivables	Number of customers Financial strength of customers
Inventory	Obsolescence Holding costs Product value Demand and supply uncertainty
Capacity	Cost of capacity Capacity flexibility

Mitch hit Guatemala and Honduras and destroyed banana fields in the region and cut up to 10% of world's banana production. Chiquita leveraged alternative sources of banana supply and managed to maintain deliveries whereas Dole failed to find replacements and lost 70% of its regional supply. Consequently, Dole lost over $100 million. The Taiwan earthquake in 1999 caused power shortages that harmed computer parts to be supplied to US manufacturers. Dell managed the demand by canalizing customers to then-currently-available products. Apple, on the other hand, was unable to alter neither the demand nor the product configurations and suffered backlogs. A substantial number of cattle were destroyed after the Mad Cow disease outbreak in England in 2001. Leather good manufacturers were among those affected from the disease. Following the 9/11 attacks, the U.S. government imposed stricter border security regulations that caused shipment delays. As a consequence, Ford had to shut down five plants for several days.

The above figures demonstrate the significance of supply chain disruption risks. Despite the importance of the subject, only 5 to 25% of Fortune 500 companies have implemented measures to handle disruption risks (Mitroff & Alpaslan 2003). Furthermore, several references including Tang & Tomlin (2008), and Zsidisin et al. (2004) report that there has not been significant investment in risk reduction and mitigation programs. They argue that firms' lack of knowledge of supply chain risk management and unavailability of analytical tools for conducting risk estimation are the major reasons behind firms' disinterest with disruption risks. We will discuss risk analysis and quantification more in-depth in Chapter 3 with a focus on disruption risks. At this point it would be sufficient to recognize different categories of risks and provide a general discussion. For this purpose, we refer to Yang (2006) where the author used four dimensions to classify supply chain risks. His classification is presented in Figure 1.2.

	Large Impact & Small Possibility	Large Impact & Large Possibility	Small Impact & Small Possibility	Small Impact & Large Possibility
External & Manageable	A1 Bankruptcy of a major supplier	B1 A car accident temporarily causes a traffic jam on a road through which incoming trucks have to pass for a manufacturing facility using JIT	C1 A long-waiting customer service phone line operated by a third-party technical service provider in India	D1 Higher than expected defective rate from a T-shirt manufacturer in China which supplies Wal-Mart
External & Unmanageable	A2 An serious earthquake destroys one of the manufacturing facilities	B2 An Car bomb attached to an American petroleum production facility in Iraq	C2 Product returns from customers	D2 A car accident temporarily causes a traffic jam on a road through which incoming trucks have to pass for a manufacturing facility not using JIT
Internal & Manageable	A3 Labor strikes	B3 Too many defective products from a newly built production line with new employees	C3 Breakdown of a machine	D3 Misplacement of an inspection instrument in a hospital
Internal & Unmanageable	A4 A serious fire in a major warehouse	B4 Miscommunication between Engineers in an overseas manufacturing facility and foreign workers caused by language barrier	C4 Defective films from Kodak's production lines	D4 Defective CPUs from Intel's production lines

Figure 1.2. Risk classification of Yang (2006)

We argue that risks falling under category C (small impact, small possibility) are the least harmful ones. Firms may not even need to consider these risks since they occur with very small probability and their impacts are insignificant. Risks under the D category are operational risks. They occur in day to day operations, but have small impacts. When economically feasible, firms should work on eliminating these risk since their outcome may accumulate and lead to significant supply chain disarrays. Risks under the B category, on the other hand, should be given priority since they occur frequently and halt supply chain operations. Firms facing these type of risks may not survive in a competitive business environment. Finally, risks under the A category are the so-called *disruption risks*. They happen very infrequently, but can cause significant damage to supply chains (the previously mentioned Nokia-Ericsson case is a classic example). Parallel to our discussion, supply chain risk management literature is generally focused on the analysis of operational risks. Despite its significance, disruption risks are far less studied compared to operational risks. An extensive review of risk management in supplier selection is presented in Chapter 2.

1.3 Sourcing in Supply Chain

Procurement is one of the major cost drivers in supply chains. Degraeve et al. (2000) report that Cockerill Sambre, a Belgian steel manufacturer, spends about £0.6 billion annually (more than 70% of total costs) for procurement. For the U.S. automotive industry, the cost of components and parts from outside suppliers may exceed 50% of sales (Wadhwa & Ravindran 2007). Furthermore, Favre & Easton (2006) claim that more and more businesses realize that outsourcing is an opportunity to increase efficiency. Chopra & Meindl (2006) and Favre & Easton (2006) list some benefits of oursourcing as follows,

- Achieving better economies of scale,
- Driving contract compliances,
- Improving forecasting and planning via better integration with suppliers,
- Sharing risks through supply contracts,
- Focusing on core competencies,
- Reducing service costs via outsourcing services.

According to Favre & Easton (2006), firms can cut costs up to 5% - 18% via outsourcing. Quinn & Hilmer (1994) report that Apple Computer outsources about 70% of its components to focus on operating system and software development. Simchi-Levi et al. (2008) note that between 1998 and 2000 outsourcing in the electronics industry has increased from 15% to 40%. Apart from the benefits cited above and despite the heavy trend, outsourcing brings several risks to companies:

- Loss of competitive knowledge (Simchi-Levi et al. 2008),
- Conflicting objectives of suppliers and buyers (Simchi-Levi et al. 2008),
- Increased exposure to supply risks.

Efficient supplier selection practices would potentially help companies to enjoy the benefits of outsourcing while remaining hedged against its risks. Tang (2006) depicts supplier selection as a three step process. Fist, the buyer has to define its supplier selection criteria and a set of candidate suppliers. Then, these criteria should be used to select appropriate supplier(s) from the candidate set. Finally, the buyer places orders with the selected supplier(s). Note that the first step in Tang's description points out that supplier selection should indeed be a multiple criteria problem as opposed to the significant portion of the supplier selection literature focusing only on outsourcing costs. Moreover, according to Ritchie & Brindley (2007), purchasing components primarily based on cost considerations may bring short term benefits; but in the long run, this choice may increase the buyers' exposure to supply risks. However, as we will see in Chapter 2, the significance of other decision making criteria in supplier selection is often omitted and most of the literature is concerned with single criterion supplier selection models.

Another issue in outsourcing is the categorization of items to be purchased. Several authors propose classifications for items to procure and claim that different techniques might be appropriate for each class. To our knowledge, Faris et al. (1967) are the first to propose a classification of purchasing situations. Their categorization involves three groups: *new task situation* which corresponds to the procurement of entirely new products from new suppliers, *modified rebuy* which is the procurement of new products from known suppliers and *straight rebuy* which is the procurement of known products from known suppliers. Later, in early 80s, Kraljic (1983) came up with the purchasing portfolio approach where procurement situations are grouped with respect to two aspects: supply risk and profit impact. We provide Kraljic's purchasing portfolio matrix in Table 1.2.

Table 1.2. Purchasing portfolio (adapted from Kraljic (1983))

	Low supply risk	High supply risk
Low profit impact	Routine items	Bottleneck items
High profit impact	Leverage items	Strategic items

Routine items in Table 1.2 refer to purchasing situations where many suppliers can provide the required items. These items have low profit impacts; therefore, it would not be logical to constantly spend effort to search for new and better suppliers. *Leverage items* can be procured from many suppliers with low risk. The abundance of suppliers brings possibilities of cost saving for buyers through supplier competition; therefore, buyers should frequently look for alternative suppliers. *Bottleneck items* are delivered by a single or by a limited number of suppliers. Even though these items have low profit margins, buyers might still need to spend effort on finding alternative supply options since these items are subject to elevated risks. de Boer et al. (2001) argue that buyers should keep inventory on hand for these items in case of supply disruptions. Similar to the bottleneck items, procurement of *strategic items* carry high supply risk. Since these items have a high profit impact, buyers should continuously monitor suppliers and invest in contingency plans. de Boer et al. (2001) combine the classifications proposed in Faris et al. (1967) and Kraljic (1983) to create hybrid purchasing situations such as *straight rebuy of routine items* or *modified rebuy of strategic items*. We will refer to these classifications to position the models we develop in this dissertation.

1.4 Global Sourcing and Global Operations

Global sourcing is an extension of local procurement operations to a worldwide scale. Although global sourcing offers significant cost reductions and an expanded market access, it also increases the variety and magnitude of risks faced by a local supply chain. Many large companies today have operations overseas and their supply chain covers multiple continents. For these companies, global sourcing is more of a consequence than a choice. Small companies, on the other hand, often choose global sourcing to increase the variety of products and resources and perceive global operations as a growth opportunity. In either case, global sourcing needs to be managed carefully since it significantly increases the exposure of supply chains to all kinds of risks. For example, Handfield et al. (2008) cites several reasons of global sourcing amplifying supply chain disruptions. Wadhwa (2008) and Ravindran et al. (2009) present models tailored for global supply chain management and outsourcing. Models proposed in this dissertation are closely related to global operations and can be extended to assist global supply chain decision making. We discuss necessary modifications in relevant chapters.

1.5 Resources on Risk Management

There exists a large number of resources for supply chain researchers and practitioners to manage supply chain risks related to procurement activities. Apart from the large body of academic literature surveyed in Chapter 2, professional organizations, governments and consulting firms offer valuable resources. For instance the PMI (Producer Price Index) index provided by the Institute for Supply Management is a leading index for tracking fluctuations in the U.S. total purchasing expenditures (Handfield 2008). PMI is created by polling purchasing managers at American companies in a monthly basis. PMI data is available on the Institute for Supply Management's website (http://www.ism.ws). JP Morgan also reports global PMI results on a monthly basis. These reports are freely accessible on the Institute for Supply Management's website as well. More resources include:

- U.S. State Department,
- World Bank,
- Standard & Poor's country profiles,
- Moody's country profiles,

Several other resources are available through databases under the Penn State Libraries website. These include:

- Gartner reports on outsourcing,
- China Data Online,
- Digital National Security Archive,
- Global Development Finance Online,
- Homeland Security Digital Library,
- IBIS World,
- International Financial Statistics Online,
- Plunkett Research Online,

- Standard & Poor's NetAdvantage,
- Wharton Research Data Service,
- World Bank e-Library.

Governments as well have recognized the risks associated with sourcing, especially in global operations, and started to implement new measures to minimize adverse effects. A new initiative by the U.S. Customs and Border Protection, Customs-Trade Partnership Against Terrorism (C-TPAT) addresses inbound cargo security at U.S. ports. C-TPAT is an accreditation program which allows verified companies to have faster customs clearance. More details on C-TPAT can be found online at http://www.cbp.gov/xp/cgov/trade/.

1.6 Additional Related Concepts

Several other concepts are related to risk assessment and management in supply chains. These concepts are listed below:

- *Robustness*: Robustness is the supply chain's ability to resist a risk event and return to do its intended mission under the same configuration as it had before the event (Asbjornslett 2008).
- *Resilience*: Resilience is the ability of the supply chain to adapt to a new environment after a risk event happens (Asbjornslett 2008). A resilient supply chain is expected to restructure itself with regards to the new conditions of the environment.
- *Vulnerability*: Vulnerability is a concept to be used for assessing a supply

chain's lack of robustness or resilience with respect to risk events (Asbjornslett 2008).

- *Flexibility*: Flexibility is the ability of a supply chain to moderate the impact of risk events (Tang & Tomlin 2008).

1.7 Research Statement

This dissertation explores supplier selection from a risk management perspective. As mentioned in Section 1.3 and further emphasized in Chapter 2, supplier selection is a multicriteria optimization problem; therefore, mathematical models we build in this dissertation are multicriteria optimization models. Furthermore, as discussed in Section 1.2, operational and disruption risks are inherent in supplier selection and models we develop should address these risks. The literature review of Chapter 2 reveals that the majority of the work in risk quantification and management is concerned with operational risks. Disruption risks, on the other hand, have only recently been recognized as significant; but, there is a lack of analytical models addressing these risks. Hence, a part of this dissertation is devoted to modeling and quantifying disruption risks. Chapter 3 takes over where several previous studies have left off and addresses the disruption risk quantification issue. In short, we propose a risk quantification scheme by breaking the risk down in components and analytically modeling each component. The end product of risk quantification is a final numerical value of risk, obtained through aggregation of the quantified components, which is later used in our multiobjective optimization models.

The models we develop in this research are powerful enough to act as decision making tools for purchasing practitioners. There are two major approaches in addressing supply chain risks. The first approach consists of holding excess resources available and implementing systems to increase visibility in operations. This is a reactive approach in that it focuses on observing the risk event and responding swiftly with additional capacity. The second approach attempts to prevent disruptions from happening by proactively reshaping the supply chain structure. As discussed in Handfield et al. (2008), the reactive approach focuses on minimizing the impact of risk events whereas the proactive strategy reduces the probability of occurrence. Models developed in this dissertation are of proactive nature and restructure the supply chain by optimally positioning suppliers at primary and backup levels and by optimally allocating orders to primary suppliers. That is, the models not only solve the supplier selection / order allocation problem but also provide mitigation strategies against supply chain risks. Hence, this dissertation develops multicriteria optimization models to coordinate procurement activities while managing and mitigating operational and disruption risks in supply chains. In the models presented in Chapter 4, we assign a primary supplier (or a set of primary suppliers) and a set of backup suppliers to the buyer. Primary supplier(s) will be the primary source for procurement and satisfy the buyer's demand. If on the other hand a primary supplier might fail, then that supplier would be replaced by a backup supplier, who is also assigned in the multiobjective optimization model. Therefore, with the solution of the model at hand, the buyer will not only have an order allocation chart, but also a risk mitigation plan.

Models formulated in Chapter 4 provide an introduction to the realm of supplier selection and risk mitigation. In Chapter 5, we extend these models to much more realistic cases where the input data is assumed to be uncertain. We present stochastic counterparts for all of the models developed in Chapter 4 where uncertainty is introduced using chance constraints. Deterministic equivalents to these chance-constrained stochastic optimization models are presented with numerical applications. Chapter 6 summarizes our research findings and concludes the dissertation with a future research agenda.

Chapter 2

Literature Review

Several different techniques have been applied to address the supplier selection problem. We group these techniques under three categories:

- Mathematical programming techniques,
- Game theoretic methods,
- Artificial intelligence techniques.

2.1 Mathematical Programming Techniques

Mathematical programming is one of the most widely applied techniques to address the supplier selection problem. There exist two different approaches adopted by researches. Some supplier selection models assume a single objective function which mostly involves minimizing costs or maximizing profits. We group these publications under *Single objective models* in Section 2.1.1. Decision makers (DMs), on the other hand, may consider additional selection criteria with along with costs, which is hard to practically handle with single objective models. Instead, multiple objective mathematical models have been proposed and became increasingly popular in the supplier selection literature. We review these multiple objective models in Section 2.1.2.

2.1.1 Single Objective Models

A single objective mathematical program has the following general formulation.

$$\begin{aligned} &\min\ z = f(x) \\ &\text{s.t.} \\ &x \in X \end{aligned} \tag{2.1}$$

Properties of the objective function f, the feasible region X and decision variables x define the nature of the single objective mathematical program in (2.1). For instance, if the objective and feasible region consist of linear functions and if the decision variables admit continuous values, then the mathematical program (2.1) is called a *linear program* (LP). In case where the objective function and/or any of the constraints are nonlinear with continuous decision variables the resulting program is called a *nonlinear program* (NLP). If the decision variables are restricted to integer values, the mathematical program in (2.1) becomes an *integer program* (IP). Mixed integer linear programs (MILP) are the most commonly utilized single objective models in supplier selection. Arguably, Gaballa (1974) is the first to use mathematical programming for modeling a supplier selection problem. The MILP model he proposes assumes multiple items, multiple suppliers, quantity discounts and minimizes purchasing costs. The model was applied to Qantas Airways. Bender et al. (1985) report a MILP application to select suppliers for IBM's Poughkeepsie manufacturing plant. They don't provide the actual mathematical model, but argue that the implementation leads to 5% - 10% cost savings. Tang (1988) presents a supplier selection model that considers price and cost of quality. The paper focuses on expressing the cost of quality via Bayesian inference on past data. Turner (1988) defines an LP formulation for multiple item purchasing at British Coal. Instead of more common MILP formulations, he proposes solving a number of LPs in a repeated interactive manner to generate several solutions that can be presented to purchasing managers. Benton (1991) formulates a supplier selection problem with quantity discount and storage space restrictions at buyers. The model considers multiple suppliers and items and is solved via a Lagrangian relaxation heuristic. Hong & Hayya (1992) develop a nonlinear mixed integer programming (NLMIP) formulation for the supplier selection problem with the objective of minimizing costs. They test the model with convex, concave and staircase cost functions. The model treats multiple vendors and yields the optimal order quantities as result. Chaudry et al. (1993) present all-units and incremental price breaks models for supplier selection. Their model minimizes total procurement costs and recognizes delivery time and quality as supplier performance constraints. Sadrian & Yoon (1994) formulate an MILP model to select suppliers in business volume discount environments. As opposed to the more frequently considered quantity discount, in volume discount practices suppliers offer price reductions over the aggregated purchase volume. The model in Sadrian & Yoon (1994) considers multiple items and multiple suppliers. Resenthal et al. (1995) present a supplier selection model where a single buyer is offered products in bundles if the total purchase amount surpasses some threshold quantity. They formulate the models as MILPs with multiple suppliers and items. Degraeve & Roodhooft (2000) develop an MILP formulation for supplier selection to minimize total cost of ownership. Their model assumes multiple items, multiple suppliers and multiple periods. The paper is among the rare references which incorporates inventory management in supplier selection (see Table 2.3 for a comprehensive list). Ghodsypour & O'Brien (2001) formulate an NLMIP for the supplier selection problem. Their model accommodates multiple suppliers and incorporates inventory management concepts. The single objective model minimizes total purchasing costs. They also propose a two objective extension to the model which minimizes cost and deviation from quality standards. The two objective model is transformed to a single objective model by weighting the objective functions. Talluri & Narasimhan (2003) present a max-min approach for supplier selection. Their approach consist of solving fractional LPs to find best and worst performing vendors with respect to a performance criterion. The vendors are then arranged into different groups that will be used in procurement decision making. Talluri & Narasimhan (2004) present a model similar to Talluri & Narasimhan (2003) where the final grouping of suppliers is evaluated via a technique called *cross efficiency method*.

Recently, Burke et al. (2007) present a supplier selection model to determine the optimal number of suppliers and the order size to allocate to each supplier in order to minimize total procurement costs. Their model assumes a single product in a two echelon supply chain with demand uncertainty. They first define optimality conditions for their model using Karush-Kuhn-Tucker conditions and then propose several extensions that they treat numerically. The Burke et al. (2007) paper mentions that some suppliers in the model solution should be used in emergency situations, but this point is not further elaborated.

2.1.2 Multiple Criteria Models

Multicriteria decision making (MCDM) models are commonly divided into two categories as *Multicriteria Selection (MCS)* models and *Multiobjective Mathematical Programming (MOMP)* models (see for instance Masud & Ravindran (2008)). MCS deals with problems where alternatives can be discretely identified and are of finite numbers. In MOMP models on the other hand, alternatives are continuous and infinitely many. MCDM problems have several important distinctions from single objective problems that we shall later use in our discussion:

1. There is practically no optimal solution in MCDM problems. In stead, they admit best solutions, also known as Pareto - optimal, efficient or non dominated solutions. On the other hand, MCDM problems have an ideal solution which can be obtained by optimizing each objective independent of others. The ideal solution is not feasible to the MCDM problem.

2. Efficient solutions in an MCDM problem depend on the preferences of DMs on each criterion. Therefore, the same problem may have different solutions when evaluated by different DMs.

3. Some MCDM methods are general enough to accept goals or aspiration levels from DMs. These methods seek to reach the provided goals for each objective.

We will proceed by grouping supplier selection models as MCS and MOMP problems.

2.1.2.1 Multicriteria Selection Models

The Multicriteria Selection Problem (MCSP) deals with the selection of the best alternative from a set of known candidate alternatives. Because of the discrete form of the alternative set, Masud & Ravindran (2008) classify methods treating the MCSP as discrete alternative MCDM methods. The MCSP problem can be defined as in (2.2)

$$\begin{aligned} & \max z = [c_1(x), c_2(x), ...c_k(x)] \\ & \text{s.t.} \\ & x \in X \end{aligned} \tag{2.2}$$

where $c_i(x)$ is the score of alternative x with respect to the i-th criterion and the set X of alternatives is discrete and finite. MCS methods are among the most widely utilized techniques in supplier selection. One major strength of MCS methods is that they provide an intuitive framework for considering a large number of criteria, both quantitative and qualitative, for all decision problems. The seminal paper by Dickinson (1966) proposes a set of 23 criteria that can be used in supplier selection. Dickinson's work is the first publication that justifies the application of multicriteria methods in supplier selection. Later, Timmerman (1986) uses a multicriteria linear averaging model for selecting suppliers. He further provides an application framework for mid-size companies. Several well known MCS techniques have been applied to tackle the supplier selection problem. We begin with a brief presentation of these techniques and then proceed with reviewing the relevant literature.

Analytic Hierarch Process (AHP) (Saaty 1990) is arguably the most commonly applied MCS method. In AHP, the decision problem at hand is first structured as a hierarchy: the goal at the top, followed by a main set of criteria, other levels of sub-criteria and the alternatives at the bottom. Then, the DM(s) provides judgements of alternatives with respect to the next higher level of subcriteria. Usually, these judgements are given as pairwise comparisons and they are assigned a strength of preference value ranging from 1 to 9 (1 meaning equally important, 9 being extremely more important). Same judgements are asked for comparing sub-criteria with the sub-criteria at the next higher level, all the way up the the highest level of criteria. The DM's preferences are then synthesized using AHP formulas (see Masud & Ravindran (2008)) to obtain criteria weights and final ranking of alternatives.

Other MCS methods including ELECTRE, PROMETHEE and TOPSIS have been used in supplier selection. ELECTRE (Roy 1991) is a pairwise outranking method that eliminates alternatives using a dominance criterion derived from outranking relationships. There exists several variations of ELECTRE, namely, ELECTRE I, II, III, IV, TRI and IS, which all together are referred to as the ELECTRE family. PROMETHEE, developed by Brans & Vincke (1985), is based on the extension of the concept of criterion to pseudo-criterion. In PROMETHEE, each criterion is assigned an increasing evaluation function to model the performance of an alternative with respect to that particular criterion and then multiplied by criteria weights. Two different ranking schemes are used in PROMETHEE: PROMETHEE I rankings are based on leaving and entering flows which can be interpreted as the strength and weakness of an alternative. PROMETHEE II rankings are established according to net flows of alternatives which are calculated by aggregation of leaving and entering flows. Finally, TOPSIS proposed by Hwang & Yoon (1981) operates using ideal and anti ideal solutions. It uses an index that considers closeness to the ideal and distance from the anti ideal solution at the same time. An alternative that maximizes the index is declared as the best choice. Below, we present a review of supplier selection literature that makes use of the MCS methods.

Barbarosoglu & Yazgac (1997) use AHP to select suppliers for a Turkish electric motor manufacturer. They used a hierarchy of 16 criteria and 61 subcriteria. Ghodsypour & O'Brien (1998) combine AHP and linear programming to select suppliers. Their approach first calculates supplier rates using AHP and then constructs an LP formulation that maximizes the products of AHP rates and order quantities assigned to suppliers. Fittipaldi et al. (2001) present an application of ELECTRE I and II to select electric power suppliers. Lee et al. (2001) propose an AHP based supplier selection framework. They use AHP to determine weights of supplier evaluation criteria and later they conduct a Pareto analysis to determine the most important criteria. Bhutta & Huq (2002) compare and contrast AHP and total cost of ownership approaches within the context of supplier selection. Chan (2003) proposes a two step methodology for supplier selection. In the first step, he implements an interactive method to determine supplier evaluation criteria. The second step is an AHP application for supplier selection. Dulmin & Mininno (2003) apply PROMETHEE so select suppliers at an Italian public transportation company.

Recently, Wang et al. (2004) combine AHP and goal programming to integrate product type and life cycle concerns into supplier selection. They borrow supplier selection criteria from the well known SCOR (Supply Chain Operations Reference) model. Liu & Wu (2005) apply data envelopment analysis (DEA) to supplier selection. They first use AHP to determine criteria weights and then apply DEA to select suppliers. Kokangul & Susuz (2009) present a similar framework where the multiobjective model includes price discounts as well. Almeida (2007) uses ELECTRE in supplier selection. He proposes a method that first generates a utility function related to each evaluation criterion and then applies ELECTRE using these utility functions. Araz & Ozkarahan (2007) discuss a multicriteria supplier sorting method for strategic procurement activities. Their approach uses the PROMETHEE method.

Alternatively, it has been recognized that contribution of some criteria in supplier selection problems such as reputation of the supplier, desire to outsource with a specific supplier, etc. may be difficult to be quantitatively expressed. Fuzzy sets, introduced by Zadeh (1965), provide a convenient way to quantify these vague preferences. Fuzzy numbers, a special case of fuzzy sets, have become a popular tool used to incorporate imprecise and vague linguistic judgements in decision making problems. Recently, Chen et al. (2006) utilized a fuzzy version of TOPSIS to integrate linguistic assessments of DMs into supplier selection problems. Haq & Kannan (2006) use a fuzzy AHP and a genetic algorithm (GA) to tackle the supplier selection problem. They define a set of 26 criteria and evaluate candidate suppliers with respect to these criteria using fuzzy AHP. Then, they transfer these suppliers to a large multi-echelon supply chain model and determine the optimal order amounts, transportation schedule and optimal inventory policy to minimize total cost. Chou et al. (2007) develop a fuzzy multicriteria supplier rating system. The criteria and subcriteria weights are first assigned as fuzzy numbers, aggregated and then defuzzyfied to be used in the actual supplier ranking. Lin & Chang (2008) develop a hybrid supplier selection and order allocation model by combining fuzzy TOPSIS and MILP models. They use two MILP formulations to determine the price and amount of outsourced items and use the closeness to the ideal value criterion in fuzzy TOPSIS to adjust pricing decisions. Boran et al. (2009) extends fuzzy TOPSIS to group decision making and presents a supplier selection application with multiple decision makers. Montazer et al. (2009) presents a fuzzy ELECTRE moldel for supplier selection with an application to an Iranian oil company.

2.1.2.2 Multiobjective Mathematical Programming Models

Seldom purchasing managers select suppliers based on one single criterion. In most practical situations several criteria are used together to determine the appropriate set of suppliers. Multiobjective mathematical programming (MOMP) offers tools to mathematically formulate and solve such problems. A general form for MOMP is as follows (Masud & Ravindran 2008),

$$\begin{aligned} &\min\ [f_1(x), f_2(x), ..., f_k(x)] \\ &\text{s.t.} \\ &x \in X \end{aligned} \tag{2.3}$$

We can easily demonstrate the concepts of efficient solution, dominated solution and ideal solution discussed earlier in a bi-criteria MOMP. Consider the problem in (2.4) (from Masud & Ravindran (2008)),

$$\begin{aligned} &\max\ z_1 = 5x_1 + x_2 \\ &\max\ z_2 = x_1 + 4x_2 \\ &\text{s.t.} \\ &x_1 \leq 5 \\ &x_2 \leq 3 \\ &x_1 + x_2 \leq 6 \\ &x_1, x_2 \geq 0 \end{aligned} \tag{2.4}$$

The MOMP in (2.4) is a two-variable bi-criteria linear problem which can be easily sketched in both the decision space (space of decision variables) and in the objective space (space of objective functions) as in Figure 2.1.

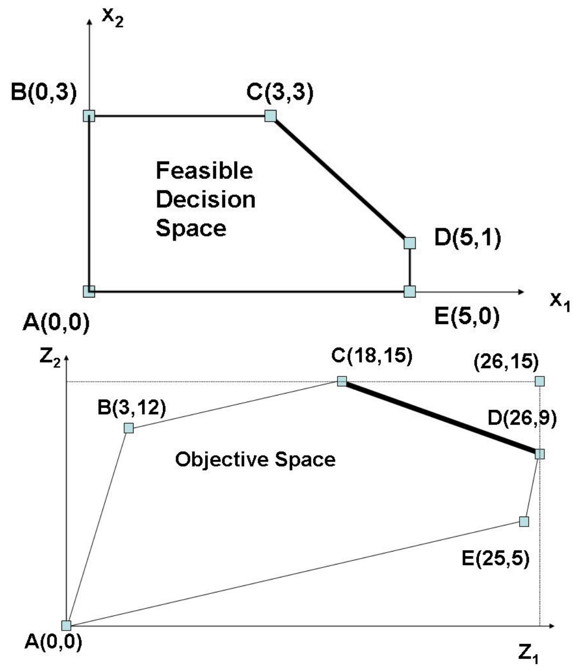

Figure 2.1. Problem (2.4) in Decision and Objective Spaces

Points D and C of the decision space in Figure 2.1 respectively correspond to optimal solutions when objective functions z_1 and z_2 are optimized independently. The optimal objective value for z_1 is 26 and for z_2 is 15. Note that the point (26, 15) of the objective space is the ideal solution for problem (2.4) and is not attainable. The points on the line segment joining points C and D of the objective space are the best possible solutions for problem (2.4), they are called the *efficient solutions, Pareto - optimal solutions, non-dominated solutions* or simply *the efficient set*. All other solutions in the achievable objective values set are worse than these efficient solutions; therefore, there is no need to consider these inferior points. Hence, solving the MOMP problem boils down to determining the efficient solutions. Once some or all efficient solutions are determined, the DM(s) chooses one or several of these efficient solutions as their best solution for a given problem.

Similar to the single objective case, objective functions f_i, feasible region X, and decision variables x in the generic MOMP model determine the type of the problem. To our knowledge, Buffa & Jackson (1983) are the first to use MOMP in supplier selection. They use a goal programming (GP) model and set target values for cost, quality and delivery criteria. Akinc (1993) proposes a MILP formulation for supplier selection. He considers cost, lead time and quality of purchased items as criteria and uses utility function to combine all model criteria. The actual model he proposes maximizes the utility function. Weber & Current (1993) present a multiobjective model with three criteria: minimizing costs, percentage of late deliveries and defective items. They solve the model using the weighted objectives method and applied it to a Fortune 500 company. Weber et al. (1998) use DEA together with a MOMP formulation to provide a negotiation scheme for purchasing activities. Karpak et al. (1999) formulate a goal programming (GP) model for selecting suppliers in an equipment company. They consider procurement cost, product quality and delivery reliability as objective functions and solve the resulting GP model using interactive techniques. Talluri (2002*a*) presents several multiobjective models for estimating efficient marginal costs of supplier outputs. He argues that the estimated values might be used to increase the effectiveness of procurement decision making by using the optimal output prices during negotiations with suppliers. He presents GP and DEA formulations for his models. Dahel (2003) gives a multiobjective MILP formulation for a more general procurement case where he considers multiple items, multiple suppliers and multiple buyers. The model also incorporates volume discounts and is solved via transforming the model to a single objective problem by weighting the objective functions.

Recently, Yang (2006) includes risk into multiobjective supplier selection problem. Risk is assumed to be associated to manmade or natural rare events (disruption risks) and to daily operations (operational risks). Disruption risks are modeled using the *Extreme Value Theory* and operational risks are quantified via *Taguchi's loss functions*. He proposes GP formulations to solve the supplier selection problem. Saen (2007) applies imprecise DEA (IDEA) for supplier selection. IDEA is argued to be useful when input data are not known with certainty. In IDEA, imprecise data are assumed to be bounded within some interval and a nonlinear formulation is required to solve the model. Saen (2007) proposes a linearization of the IDEA nonlinear model for an application in supplier selection. Wadhwa & Ravindran (2007) formulate a multiobjective model for vendor selection. They incorporate quantity discount breaks and consider price, quality and lead time as objective functions. The model presented is solved using different techniques including weighed objectives method, GP and compromise programming. Xia & Wu (2007) develop a four-objective MIP formulation for supplier selection under volume price discounts. Their model also includes an AHP based preference calculation method used for determining weights of all candidate suppliers. They report that the model is solved on MATLAB but do not precisely indicate using which solution technique. Demirtas & Ustun (2008) propose an integrated approach for supplier selection. They first calculate criteria weights using the *Analytical Network Process* and then carry these weights to a multiobjective MILP. They solve the optimization model via the ϵ-constrained method where they optimize one of the three proposed objective functions and constrain the others to be smaller than some preset ϵ value.

Fuzzy optimization techniques have also been proposed for supplier selection. Erol & Ferrell (2003) present a fuzzy quality function deployment (fuzzy QFD) framework to select supplier using both quantitative and qualitative preference inputs. Amid et al. (2006) develop a fuzzy multicriteria linear model for supplier selection. They start by formulating a basic multiobjective LP which they solve to obtain upper and lower bounds for each objective. Using these bounds, they formulate a fuzzy LP, derive the crisp equivalent of the fuzzy LP and solve it to determine the order amounts. Kumar et al. (2004) propose a fuzzy GP formulation for supplier selection. They note that in real life situations supplier performance parameters may not be known with certainty and argue that fuzzy numbers would be good approximations in those cases. Later, Kumar et al. (2006) analyze the same problem with a more general fuzzy multiobjective formulation.

Table 2.1 provides a classification of the most commonly used multicriteria methods in supplier selection.

Table 2.1. Classification of multi criteria supplier selection methods (* indicates fuzzy models)

	MCSP Methods	
AHP	*ELECTRE*	*PORMETHEE*
Barbarosoglu & Yazgac (1997)	Fittipaldi et al. (2001)	Dulmin & Mininno (2003)
Ghodsypour & O'Brien (1998)	Almeida (2007)	Araz & Ozkarahan (2007)
Lee et al. (2001)		
Bhutta & Huq (2002)		*TOPSIS*
Chan (2003)		Chen et al. (2006)*
Wang et al. (2004)		Lin & Chang (2008)*
Liu & Wu (2005)		
Haq & Kannan (2006)*		
Xia & Wu (2007)		
	MOMP Methods	
Weighted objective	*DEA*	*Goal Programming*
Akinc (1993)	Weber et al. (1998)	Buffa & Jackson (1983)
Weber & Current (1993)	Talluri (2002*b*)	Karpak et al. (1999)
Dahel (2003)	Liu & Wu (2005)	Talluri (2002*b*)
	Saen (2007)	Kumar et al. (2004)*
		Wang et al. (2004)
		Yang (2006)
		Ravindran et al. (2009)
		Wadhwa & Ravindran (2007)

2.2 Game Theoretic Methods

Several researchers considered aspects of competition and collaboration in supplier selection. Game theory provides a rich theoretical framework to incorporate competition and collaboration into mathematical models. Talluri (2002*a*) presents a procurement negotiation model. The negotiation scheme is used to generate a number of rates which are later used in an MILP formulation to select the optimal bids. Agrell et al. (2004) analyze a two-period game concerned with information sharing in the context of an investment problem in the telecommunications industry, which involves supplier selection as well. A tournament method to select the best set of suppliers is proposed in Deng & Elmaghraby (2005). According to the tournament approach, a firm starts with a number of suppliers, tests them and allows them to improve their specifications without offering any financial support. The suppliers compete against each other to receive a supply contract. The proposed model is solved in a game setting where the firm has incomplete information on costs and performance of suppliers. Recently, Swinney & Netessine (2009) formulated a two phase game to compare short and long term contracts under supplier bankruptcy risk. Under the assumptions of single sourcing, single buyer and single component to procure, they showed that long term contracts enable the buyer to coordinate its supply chain under supplier bankruptcy risk.

2.3 Artificial Intelligence Techniques

Additional to the previously mentioned methods, state-of-art computational techniques have been applied to solve the supplier selection problem. Choy et al. (2003) propose an artificial neural network model to select and benchmark suppliers. The model was applied to reduce supplier selection time in a firm. Simple computational frameworks to select suppliers for a manufacturing firm are presented in Emerson & Piramuthu (2004) and Piramuthu (2005). These models assume two suppliers and involve learning without assuming competition or collaboration between suppliers. An extension of the model to three levels - suppliers, assemblers and a factory - is also proposed. Ding et al. (2005) develop a GA based approach to tackle the supplier selection problem. In their model, the GA searches over different supplier portfolios and related operation parameters. Simulation models are then created to estimate key performance indicators with respect to which suppliers are selected. Hong et al. (2005) propose a three step supplier selection methodology. In the first step, they determine supplier selection criteria. The second step clusters suppliers using a GA and in the third step, they solve a MILP model to determine order allocations. A reinforcement learning framework for supplier selection is developed in Valluri & Croson (2005). Their model selects suppliers based on two different exploration policies: first one oriented to maximize the quality and a second one to maximize profit. Altinparmak et al. (2006) use a GA to solve a three objective NLMIP supply network problem. They used a tree representation, uniform crossover and mutation and a selection operator based on high selective pressure. Multiple objectives are combined into one single objective using weights.

Recently, Kim et al. (2007) combine reinforcement learning and fictitious game playing to model the supplier selection problem as a repetitive non-collaborative game. Their model studies the case of a two-echelon manufacturing supply chain where the suppliers bid according to their own utility functions to receive an order from the buyer. Another multiobjective supplier selection model is presented in Liao & Rittscher (2007). They develop a GA to solve the model. Wang & Che

(2007) use a blend of fuzzy decision making and GAs to solve a multicriteria supplier selection problem for procuring in rapidly changing environments.

2.4 Surveys

Choi & Hartley (1996) conducted a survey study to assess the importance of a set of supplier selection criteria used in the US car manufacturing industry. They show that car manufacturers adopt very similar supplier selection practices. de Boer et al. (2001) present an extensive literature survey on supplier selection methods prior to year 2001. They also provide several useful classifications of the surveyed papers with respect to methodologies used and aspects of the supplier selection problems addressed. More recently, Aissaoui et al. (2007) present a review of the literature on supplier selection and order allocation models. Huang & Keskar (2007) present a set of supplier performance metrics that can be used as criteria in supplier selection problems. They categorize these metrics under several groups and provide insights about the cases where it would be appropriate to use them.

2.5 Supplier Selection Models with Uncertainty

As discussed in Section 1.2, risk is an inherent part of supply chain operations and presence of outside suppliers is a major driver of supply chain risks. Several researchers (see for instance Yang (2006) and Chopra et al. (2007)) separate supply chain risks in two broad categories: operational risks and disruption risks. In this section, we review supplier selection literature addressing these risks.

2.5.1 Models with Operational Risks

Glitches in daily supply chain operations are quite common. Suppliers may delay shipments, deliver inadequate amounts or received items may not all match quality restrictions. Buyers usually have some knowledge of their suppliers' operational performance, mostly in terms of historical data, but do not know their actual performance with certainty. The uncertainty about suppliers' yields causes the so-called operational risks. There exists a vast body of literature on operational risks in supply chain. Tang (2006) observes four drivers of operational risks, *uncertain demand*, *uncertain supply*, *uncertain lead times* and *uncertain costs*. We shall follow his classification and cite the literature addressing each risk driver below.

2.5.1.1 Demand Uncertainty

Kasilingam & Lee (1996) formulate a supplier selection MILP with demand risk. They assume that the demand is normally distributed and introduce a probabilistic demand constraint in their model. Peleg et al. (2002) consider three different outsorucing decisions: single sourcing for long term supplier relationship, multiple suppliers for short term and a combination of these two. For a fixed number of suppliers, they analytically drive the optimal order amounts for each scenario under demand uncertainty. Then they assume the number of suppliers as a variable and derive formulas for the optimal number of suppliers to ask for price quotes. Nagurney et al. (2005) introduce a bi-criteria supply chain model that considers both supply and demand risks. The supply chain consists of manufacturers, distributors and buyers. They express each party's problem in variational inequalities and derive optimality conditions. Then, they derive equilibrium conditions for the whole supply chain and propose an algorithm to solve the equilibrium problem.

Lam & Tang (2006) formulate a large scale two-objective linear supplier selection problem. They assume uncertain demand but feed the model with the expected value of random demand. Their model considers inventory holding decisions at distribution centers and is solved by the weighted objectives method. Ernst et al. (2007) analyze the effects of lead time demand and demand uncertainty in supplier selection. They show that the supplier with the shortest expected lead time may not always be the best choice if the variation in demand is also considered. Liao & Rittscher (2007) develop a multiobjective supplier selection model where they treat demand uncertainty. They model the demand as a normal random variable and derive closed for expressions. Awasthi et al. (2009) develop a supplier selection and order allocation model under uncertain demand where suppliers have restrictions on minimum and maximum order sizes they can accept. They also propose a heuristic algorithm to solve the model.

2.5.1.2 Supply Uncertainty

Anupindi & Akella (1993) analyze an inventory management problem with two unreliable suppliers. They consider three procurement cases which model different situations of supply uncertainty. They assume a continuous demand distribution and provide optimal order quantities for each procurement case. One major finding of their paper is that it is never optimal to purchase all items from the more reliable expensive supplier. Parlar & Perry (1996) formulate a multiple supplier inventory model where the suppliers are subject to breakdowns and buyers face a probability of not receiving the order. They argue that the model covers supply unavailability from breakdowns, labor strikes or other uncertain events and introduce the supply risk via exponential distribution. Later research however (see for instance Yang (2006) and Chopra et al. (2007)) argue that supply risks should be broken down in two as operational and disruption risks and separate models are required for each type of risk. Tagaras & Lee (1996) examine quality risks in supplier selection. They provide several probabilistic quality cost functions that might be used in estimating cost of quality when selecting suppliers. Swaminathan & Shanthikumar (1999) analyze a supplier selection and inventory problem with two unreliable suppliers. In their probabilistic scheme, suppliers either deliver the entire order or do not ship anything at all. Unlike Anupindi & Akella (1993), they assume a discrete unknown customer demand and show that in some settings it is optimal to order everything from the more reliable supplier; but in most cases it is optimal to order a large quantity from the reliable supplier and the rest from the unreliable one. Chen et al. (2001) consider supply uncertainty in outsourcing. They allow suppliers to rework non-qualifying items to increase yields. They derive conditions to determine cases when it's optimal to use a single supplier and multiple suppliers. Feng et al. (2001) use the process capability index (PCI) and Taguchi's loss function to introduce quality uncertainty in supplier selection. PCI is a measure of yield quality of a process with respect to previously defined quality restrictions. They formulate a stochastic IP to select suppliers based on their yield quality. Chen et al. (2003) address the quality uncertainty in supplier selection via fuzzy PCI. They propose a fuzzy inference rule set to compare PCI's of different suppliers and assess the quality of their production yield.

Recently, Burke et al. (2007) consider a supplier selection model with demand uncertainty and unreliable suppliers. They model the demand as a continuous random variable and assign a reliability index varying in the interval [0, 1] to each supplier. They further introduce a diversification benefit function that encourages buyers to procure from different suppliers. Chopra et al. (2007) consider both operational risks and disruption risks in inventory problems. They consider two suppliers (reliable and unreliable) and derive expressions of optimal order quantities under different conditions. The major contribution of the paper is that the authors prove that joint consideration of disruption risks and recurrent risks leads to sub-optimal procurement practices and that these two types of risk should be disaggregated. Dada et al. (2007) analyze a supplier selection problem with both reliable and unreliable suppliers. Unreliable suppliers are prone to deliver less than the amount determined in the supply contract. They examine a case where item costs are considered to choose qualifying suppliers and reliability is considered to determine the winners. Yang et al. (2007) formulate the supplier selection problem of a buyer facing random supplier yields and customer demand. Randomness in yield of a supplier is a function of the size of an order placed at that particular supplier. The model assumes minimizing total costs as the single objective and is solved via a Newton search procedure.

2.5.1.3 Lead Time Uncertainty

Sedarage et al. (1999) formulate a multiple-supplier single item inventory model with lead time and demand uncertainty. Their model minimizes total expected cost. They test different distributions for demand and run simulation to determine the optimal number of suppliers and order allocations in different cases. Kelle & Miller (2001) analyze a two-supplier inventory problem with random lead times and lead time demand. Their model minimizes the stockout risk due to random lead times, which in turns is modeled as exponentially distributed. They provide closed form expressions for the exponential case and use approximations for more general lead time distributions. Ryu & Lee (2003) treat a two-supplier inventory problem with exponentially distributed lead times. Their model allows lead time reduction through investments and minimizes expected costs. Smith et al. (2006)

model interactions between suppliers in a multiple supplier inventory problem. Their model assumes that suppliers may face disruptions and thus the lead time is uncertain. They provide several models including an order splitting example which selects suppliers and allocates order amounts.

2.5.1.4 Cost Uncertainty

Alonso-Ayuso et al. (2003) analyze a large supply chain problem where they consider plant selection, product allocation to plants and supplier selection. They assume uncertainty in costs, product prices and demand. They model the problem as a two-phase stochastic program. The first stage of the model solves the plant sizing, production allocation and supplier selection problems. The second stage treats more tactical decisions such as order allocation to suppliers and production volumes at different plants. Kumar et al. (2006) consider uncertainty in costs, delivery, quality and supplier capacity. They express each term as a fuzzy number and formulate a fuzzy supplier selection problem. They recast the fuzzy model as a crisp model and solve for determining the optimal set of suppliers and order amounts. Li et al. (2009) present a short term periodical purchasing and a long term commitment model to select suppliers under price and demand uncertainty. They then develop a stochastic dynamic program to combine both strategies to derive optimal supplier selection decisions.

2.5.2 Models with Disruption Risks

Disruptions refer to rare but catastrophic events that halt supply chain operations. A detailed discussion on disruptive events and their consequences can be found in Section 1.2. Disruption risks have become popular in the last decade, especially after the Nokia-Ericsson supplier fire case and the 9/11 attacks. Early literature on disruption analysis consists of conceptual papers where the necessity of analyzing disruptions and severity of outcomes are emphasized. Zsidisin et al. (2004) explore different supplier risk assessment methods. They survey several companies about their risk assessment methods and report that only some use formal risk assessment techniques. They also interviewed companies about their plans to mitigate disruptions and found out that companies have adopted the following practices to avoid disruptions: building mitigation plans, developing demand forecasts and modeling supply processes. Hendricks & Singhal (2005) run an empirical study using data from 885 publicly traded firms and reported performance changes. They use inventory level and economic growth as performance indicators and statistically show that independent of its causes, disruptions have negative effects on the performance of the studied firms. Kleindorfer & Saad (2005) introduce a framework to be used in managing disruption risks in supply chains. The conceptual methodology consists of two steps: specifying sources of risk and vulnerability and risk assessment and mitigation. The proposed framework is based upon 10 principles including risk diversification, preventive actions, information sharing and total quality management practices. More recently, Ritchie & Brindley (2007) provide an excellent discussion of supply chain risks. The paper covers both operational and disruption risks, proposes an overall supply chain risk management framework and discusses an actual implementation.

Recently, the emphasis of supply chain disruption research has shifted from conceptual risk frameworks towards analytical models. Hopp & Yin (2006) use a NLMIP formulation to account for supply disruptions due to catastrophic failure and solve the model for minimizing total cost defined as the sum as the inventory cost and protection cost. They analyze the effects of several different protection policies to mitigate disruption risks. In short, they consider three policies: holding inventory, using high capacity (which is only valid for service supply chains) and a third policy where both previous policies are considered at the same time. Tomlin (2006) analyzes the supplier selection problem under supply disruption through stochastic optimization. He assumes that there are two suppliers, a reliable but more expensive and a less reliable but cheaper and argues that a supplier's percentage uptime and the nature of the disruption are key determinants of supplier selection. He also introduces the expected duration of disruption in the analysis. Yang (2006) proposes a disruption risk quantification methodology based on Extreme Value Distributions. His generic risk model assumes frequency and severity of impact as two dimensions of risk for quantification purposes. He also proposes methods for quantifying operational supply chain risks. Snyder & Shen (2007) develop a simulation model to come up with the optimal order frequency, inventory policy and supply chain structure under demand uncertainty and supply disruptions. Wu et al. (2007) develop a Petri-net based model to analyze propagation of disruptions in large supply chains. Their work motivates the disruption delay model presented in Chapter 3 of this dissertation. Kull & Closs (2008) uses a simple supply chain model and simulation analysis to demonstrate effects of disruptions at second tier suppliers. An interesting result in their paper is that under certain settings in decentralized supply chains, increasing inventory at first tiers also increases the supply chain's exposure to risk if second tier supplier are prone to disruptions. Xu & Nozick (2009) formulated and solved a two stage stochastic MIP to select suppliers under two purchasing options: a fixed initial cost an an option to extend orders up to a maximum number at a piece-cost. Disruptions at suppliers are introduced as scenarios with different known probabilities of occurrence. The model minimizes the total cost of supplier selection and option exercising under considered scenarios. Yang et al. (2009) develop a single supplier - single buyer supply chain model under information asymmetry. The single supplier can be of high or low reliable type and can either subcontract or pay penalty to the buyer in case of disruptions at his facility. Yu et al. (2009) presents a two stage supply chain model to compare single versus dual sourcing strategies under disruption risks. Disruptions are assumed to occur with a given probability.

Table 2.2 presents a classification of the supplier selection literature considering different aspects of risk.

Table 2.2. Classification of supplier selection papers with respect to risk

Operational risks			
Demand	*Supply*	*Lead time*	*Costs*
Kasilingam & Lee (1996)	Anupindi & Akella (1993)	Sedarage et al. (1999)	Alonso-Ayuso et al. (2003)
Sedarage et al. (1999)	Tagaras & Lee (1996)	Kelle & Miller (2001)	Kumar et al. (2006)
Kelle & Miller (2001)	Swaminathan & Shanthikumar (1999)	Ryu & Lee (2003)	
Peleg et al. (2002)	Chen et al. (2001)	Smith et al. (2006)	
Nagurney et al. (2005)	Feng et al. (2001)	Yang (2006)	
Lam & Tang (2006)	Chen et al. (2003)		
Ernst et al. (2007)	Yang (2006)		
Liao & Rittscher (2007)	Burke et al. (2007)		
	Chopra et al. (2007)		
	Dada et al. (2007)		
	Yang et al. (2007)		

Disruption Risks	
Conceptual	*Analytic*
Zsidisin et al. (2004)	Hopp & Yin (2006)
Hendricks & Singhal (2005)	Tomlin (2006)
Kleindorfer & Saad (2005)	Yang (2006)
Ritchie & Brindley (2007)	Snyder & Shen (2007)

2.6 Observations and Research Opportunities

In summary, the first work in supplier selection was on defining multiple selection criteria (Dickinson 1966). Interestingly however, most of the earlier work in supplier selection used single objective models. To our knowledge, the first multiobjective supplier selection model was introduced in Buffa & Jackson (1983); still the single objective models dominated the supplier selection literature until early 90's. The Weber & Current (1993) paper started a new era in the supplier selection research. After 1993, we can observe two major research directions in terms of mathematical modeling: single objective models, which started to focus on other issues in supplier selection such as inventory (see Table 2.3 for a comparison of supplier selection literature integrating inventory management) and supply chain coordination and competition among suppliers; and multiple objective models that arguably treat more realistic supplier selection problems with conflicting objectives.

Table 2.3. Supplier selection papers integrating inventory management

Reference	Uncertainty	Objective
Buffa & Jackson (1983)	Deterministic	Min cost
Anupindi & Akella (1993)	Demand - supply	Min cost
Sedarage et al. (1999)	Lead time - demand	Min cost
Swaminathan & Shanthikumar (1999)	Demand - supply	Min cost
Degraeve & Roodhooft (2000)	Deterministic	Min cost
Ghodsypour & O'Brien (2001)	Deterministic	Min cost
Kelle & Miller (2001)	Lead time - demand	Min stockout risk
Haq & Kannan (2006)	Deterministic	Min cost
Lam & Tang (2006)	Demand	Min cost - min rejects
Smith et al. (2006)	Lead time - supply	Min cost
Burke et al. (2007)	Demand - supply	Max profit
Chopra et al. (2007)	Supply - disruption	Min cost
Ernst et al. (2007)	Demand - lead time	Min cost

Again in the early 90s, risk starts to be an essential part in supplier selection models, at least in single objective models. To date, most authors treated different perspectives of operational risks in single objective models (see Section 2.5.1). The introduction of risk in multi objective models has been limited compared to single objective literature. Furthermore, events in the last decade have opened up a brand new research dimension in supply chain risks. The well known Nokia-Ericsson case, Taiwan earthquake, 9/11 attacks and similar unexpected events disrupted supply chains of many companies. Such disruption risks only recently have started to appear in the literature and have been mostly considered in conceptual models (see Section 2.5.2). Based on our literature survey we can identify the following research opportunities concerned with supplier selection:

1. Most of the supplier selection papers falling under the Multicriteria Selection (MCS) category apply AHP. There are two drawbacks of this method: first, it poses significant cognitive burden on the DMs as the problem grows larger and second AHP has been found to be cumbersome in real applications Ravindran et al. (2009). Other methods should be tried out to compromise these drawbacks of AHP.

2. The fuzzy MCS literature is still ripe and open for improvements. For instance the fuzzy PORMETHEE method has never been applied to the supplier selection problem.

3. Most Multiobjective Mathematical Programming (MOMP) models ignore inventory issues. Concepts from inventory theory and single objective supplier selection literature can be borrowed and integrated in MOMP problems.

4. Most MOMP models ignore operational risks. Again, relevant concepts can be borrowed from other domains and integrated in the multi objective supplier selection models.

5. Disruption risks have only recently been studied analytically. There is a good

opportunity both in modeling and in integrating disruption risks in supplier selection models, either in single or multiobjective models.

6. Following the quantification of disruption risks, there is need for models to propose mitigation strategies for firms under the risk of disruption. Again, there is only one reference we are aware of that treats this issue in facility location literature (Snyder & Daskin 2005), and none specifically in supplier selection.

Table 2.4 gives a classification of the key publications in supplier selection. The last row in Table 2.4 underline sthe contributions of this dissertation.

Table 2.4. Classification of recent key publications in supplier selection

Paper	Multiobjective	Operational risks	Disruption risks	Mitigation
Lam & Tang (2006)	✓	✓		
Burke et al. (2007)		✓		
Chopra et al. (2007)		✓	✓	
Dada et al. (2007)		✓		
Snyder & Shen (2007)		✓		
Wadhwa & Ravindran (2007)	✓			
Xia & Wu (2007)	✓			
Demirtas & Ustun (2008)	✓			
Ravindran et al. (2009)	✓	✓	✓	
Proposed research	✓	✓	✓	✓

Chapter 3

Management and Quantification of Disruption Risks in Supply Chains

3.1 Introduction

A supply chain is a set of business units involved, directly or indirectly, in fulfilling a customer request. A generic manufacturing supply chain consists of raw material suppliers, manufacturing facilities, warehouses, retailers and consumers. In large supply chains, a manufacturer may have a large number of suppliers (also called tier one suppliers), who may have their own suppliers (tier two suppliers). Furthermore, the tier two suppliers may procure from their suppliers (tier three) and so on. Therefore, suppliers connect procurement at business units of the supply chain and their performance is critical to the overall success of the supply chain. Performance of suppliers can be measured in a number of different ways. For instance on-time delivery, quality of the delivered products and cost of the procured material are among the frequently used performance criteria. Recently, risks related to uncertainty in demand and production and risks related to unforeseen supply disruptions due to extreme events have been introduced to the supplier selection problems. However, supply chain literature is still short of research on risk quantification methods and strategies to mitigate supply disruptions.

The purpose of this chapter is to fill this gap by proposing analytical methods of risk quantification as a basis for better risk management and mitigation. The methods make use of and combine several techniques of probability and mathematical statistics, including Extreme Value Distributions and Markov chains, together with concepts from Reliability Engineering. The chapter begins with a short discussion of disruptive events. A general risk function is introduced in Section 3.2. The risk function defines risk as a function of impact and occurrence. Impact of a risk event is modeled using the Generalized Extreme Value Distributions. Occurrence of risk events is modeled as a Poisson random variable. Closed form expressions of expected value and variance parameters of the combined risk function are then derived. Section 3.3 provides several theoretical results for combining multiple risk functions under mild assumptions with a priori knowledge on the number of occurrences. Closed form expressions obtained in Section 3.3 involve integration of complicated functions which could be computed using numerical techniques. Section 3.4 discusses the detectability aspect of disruptive events. Detectability is modeled using the Mean Fist Passage Time concept of Markov chains. A discussion and examples of several ways to incorporate detectability to risk quantification are also provided. Section 3.5 discusses a conceptual model to include disruption recovery time in risk analysis. Section 3.6 combines all of our findings in this chapter and presents a comprehensive example. Section 3.7 summarizes our findings and concludes the chapter.

3.2 Disruptive Events and Risk Quantification

Disruption risk is among the most significant threats to supply chains. Although disruptive events (also called as *rare events* or *extreme events* due to their infrequent nature) seldom occur, they have caused major losses to various companies in different business segments (see Section 1.2). A number of researchers have recognized the significance of disruption risks and concentrated their work on the analysis of disruptions. There is a growing body of literature on qualitative techniques and frameworks to be used in disruption risk analysis and assessment. Past and current work on this topic is thoroughly discussed in Section 2.5.2 of Chapter 2. There is also a trend towards the quantification of disruption risks, although research towards this aim, at least in supply chain management, is still at its infancy. Other domains of science, e.g. civil engineering and statistics, are much more advanced compared to operations research in modeling disruptive events. A recent book by Castillo et al. (2005) overviews the body of statistical techniques available to model extreme events from a civil engineering perspective. We believe that valuable insights can be drawn from the available statistical techniques to devise a disruption risk quantification methodology for use in supply chain management.

Several references, including Yang (2006) and Tang & Tomlin (2008), state that all risks, including disruptions, should be measured in two dimensions: likelihood of occurrence and impact. Although this assertion is valid, it leaves out several other dimensions that might affect the nature of risk events. Indeed, there exist more comprehensive conceptual models in the literature. For instance, Ayyub et al. (2007) define risk as the product of three terms, consequence, vulnerability and threat; where consequence is the maximum possible loss, vulnerability is total weakness of the system over all hazard scenarios and threat is the rate of hazard occurrence per period. Moreover, Castillo et al. (2005) propose models for the *duration* of a risk event. Finally, one can argue that in large supply chains it may be challenging to detect the occurrence of a risk event; hence, *detection* may be considered as a component of risk as well. Handfield et al. (2008) gives the following example to emphasize the significance of disruption detection: An appliance manufacturer had developed a new oven that began selling quickly. Distributors were not able to meet demand because of a disruption at an oversees tier-two supplier of the manufacturer. It had taken the manufacturer several weeks to discover the source of the disruption and resulted in a loss of market share to competitors.

Within the risk classification scheme presented in Chapter 1 (see Figure 1.2), disruptive events fall under category A; that is, events with small possibility but large impact. The nature of disruptive events makes it harder to develop mitigation strategies because of several reasons:

- Disruptive events are related to the distributions of extreme values in samples; and hence, traditional statistical tools would be insufficient in modeling them.
- It is hard to collect data on disruptive events since they happen infrequently.

In this and following sections, we provide statistical means of modeling disruptive events. The proper implementation of the presented models rely on the availability of past data on disruptive events. We begin by presenting the general risk quantification model of Yang (2006),

$$\text{Risk}=\text{Impact}\times\text{Occurrence}$$

This research proposes a broader view and more general quantification schemes.

We model risk as

$$\text{Risk}=f(\text{Impact, Occurrence, Detectability}) \tag{3.1}$$

In the next subsections, we discuss methods to quantify components of risk in disruptive events and aggregation methods to have a final expression of risk.

3.2.1 Modeling the Impact

Impact of disruptive events is the financial loss due to extreme events such as earthquakes, fires, hurricanes and labor strikes. In general, the impact of such extreme events is modeled using heavy-tailed distributions including Weibull, Gumbel and Frechet distributions. Heavy tails of these distributions are appropriate for maximum or minimum extreme values; wave heights or low temperatures for instance (Castillo et al. 2005). A general family of distributions called the *Generalized Extreme Value Distributions (GEVD)* arises as the limit distributions of Weibull, Gumbel and Frechet. In fact, Castillo et al. (2005) show that GEVDs are the only feasible limit distributions for both maximal and minimal extremes. There are two GEVD distributions, one for maximal extremes, denoted as $GEVD_M$, and another for minimal extremes, $GEVD_m$. Cumulative distribution functions (cdf) for the $GEVD_M$ are given in Equations (3.2) and (3.3).

$$\text{for } \kappa \neq 0, \quad F_\kappa^M(x;\lambda,\delta) = exp\left(-\left[1-\kappa\left(\frac{x-\lambda}{\delta}\right)\right]^{\frac{1}{\kappa}}\right) \tag{3.2}$$

$$\text{for } \kappa = 0, \quad F_0^M(x;\lambda,\delta) = exp\left[-exp\left(\frac{\lambda-x}{\delta}\right)\right] \tag{3.3}$$

Cdfs for the $GEVD_m$ are given in Equations (3.4) and (3.5).

$$\text{for } \kappa \neq 0, \quad F_\kappa^m(x;\lambda,\delta) = 1 - exp\left(-\left[1+\kappa\left(\frac{x-\lambda}{\delta}\right)\right]^{\frac{1}{\kappa}}\right) \tag{3.4}$$

$$\text{for } \kappa = 0, \quad F_0^m(x;\lambda,\delta) = 1 - exp\left[-exp\left(\frac{x-\lambda}{\delta}\right)\right] \tag{3.5}$$

Table 3.1 provides definitions of the GEVD parameters.

Table 3.1. GEVD Parameters

Parameter	**Interpretation**
K	Shape parameter
	$K > 0$, corresponds to a Frechet distribution,
	$K = 0$, corresponds to a Gumbel distribution,
	$K < 0$, corresponds to a Weibull distribution.
δ	Scale parameter
λ	Location parameter

Probability density functions (pdf) of the GEVD distributions can be calculated by differentiating Equations (3.2) to (3.5) with respect to x to obtain,

$$\text{for } \kappa \neq 0, \quad f_\kappa^M(x;\lambda,\delta) = \frac{1}{\delta}exp\left(-\left[1-\kappa\left(\frac{x-\lambda}{\delta}\right)\right]^{\frac{1}{\kappa}}\right)\left[1-\kappa\left(\frac{x-\lambda}{\delta}\right)\right]^{\frac{1}{\kappa}-1} \tag{3.6}$$

$$\text{for } \kappa = 0, \quad f_0^M(x;\lambda,\delta) = \frac{1}{\delta}exp\left[-exp\left(\frac{\lambda-x}{\delta}\right)\right]exp\left(\frac{\lambda-x}{\delta}\right) \tag{3.7}$$

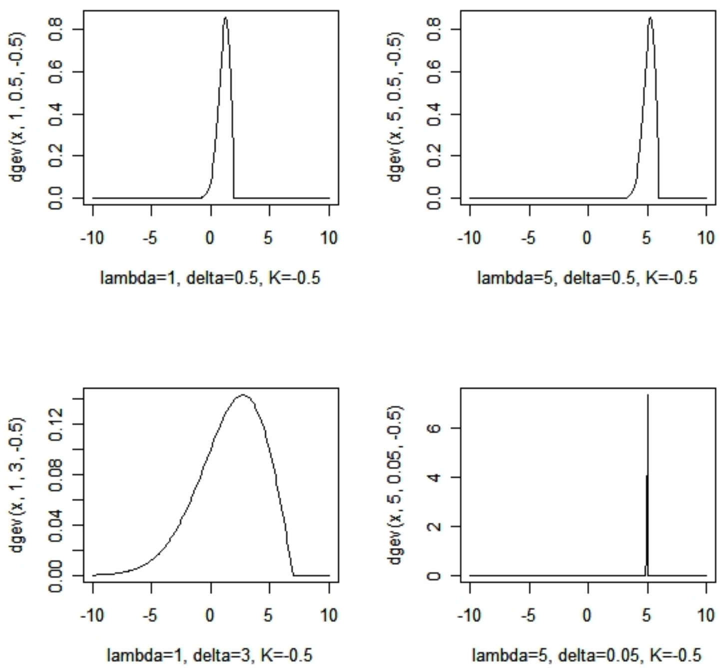

Figure 3.1. $GEVD_M$ pdfs for $\kappa = -0.5$

$$\text{for } \kappa \neq 0, \quad f_\kappa^m(x;\lambda,\delta) = \frac{1}{\delta} exp\left(-\left[1+\kappa\left(\frac{x-\lambda}{\delta}\right)\right]^{\frac{1}{\kappa}}\right)\left[1+\kappa\left(\frac{x-\lambda}{\delta}\right)\right]^{\frac{1}{\kappa}-1} \tag{3.8}$$

$$\text{for } \kappa = 0, \quad f_0^m(x;\lambda,\delta) = -\frac{1}{\delta} exp\left[-exp\left(\frac{x-\lambda}{\delta}\right)\right] exp(\frac{x-\lambda}{\delta}) \tag{3.9}$$

As noted previously, the subscripts M and m denote maximal and minimal GEVDs in Equations (3.6) through (3.9). Plots of the pdf of $GEVD_M$ for different combinations of parameters are given in Figures 3.1 through 3.4

The mean of the GEVD can be computed using Equation (3.10) if the parameter

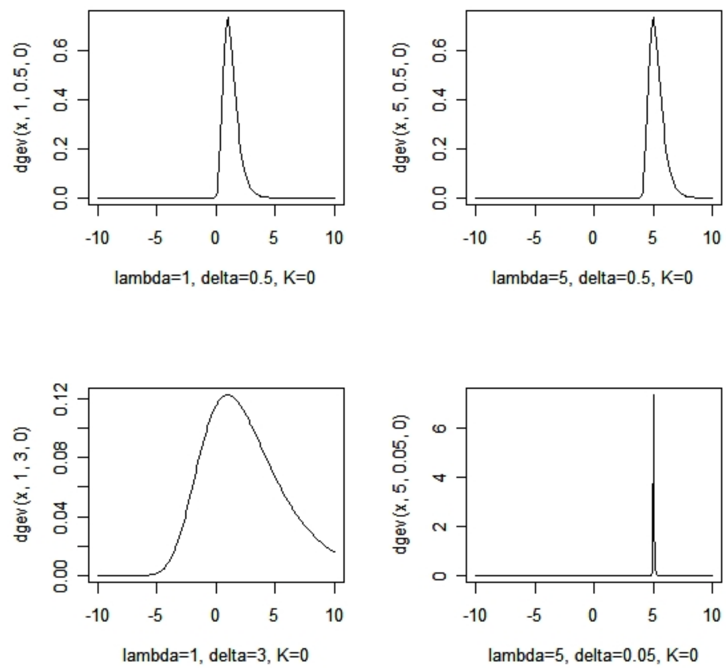

Figure 3.2. $GEVD_M$ pdfs for $\kappa = 0$

$\kappa \neq 0$,

$$E(X) = \lambda - \frac{\delta}{\kappa} + \frac{\delta}{\kappa}\Gamma(1-\kappa) \tag{3.10}$$

If, on the other hand $\kappa = 0$, then GEVD boils down to a Gumbel distribution and the mean can be approximated as $\lambda + \xi\delta$, where $\xi = 0.57721...$ is the Euler-Mascheroni constant.

3.2.2 Modeling the Occurrence

Occurrence of risk is the number of times a particular type of disruptive event (e.g. hurricane, flood, storm, etc.) happens over a period of time. The period of time can be taken as a year, two years or several years depending on the event of interest (see for instance Castillo et al. (2005) and Yang (2006)). Occurrence can

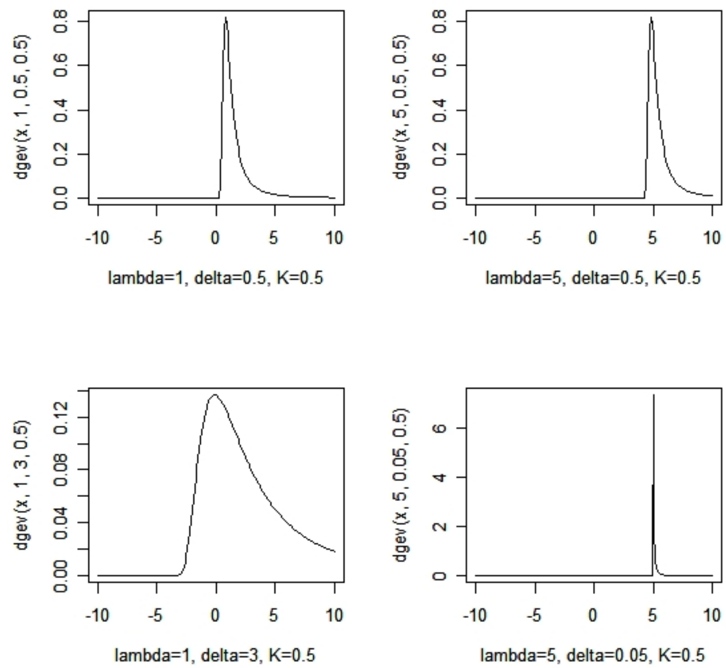

Figure 3.3. $GEVD_M$ pdfs for $\kappa = 0.5$

be modeled using a Poisson distribution with the assumption that disruptive events are not correlated. For convenience, we present the probability mass function of the Poisson distribution in (3.11).

$$P(x) = \frac{e^{-\lambda}\lambda^x}{x!} \tag{3.11}$$

The λ parameter is the expected number of occurrences over a given period. Mean and variance of the Poisson distribution are both equal to λ.

Note that several references, including Olsen et al. (1998) and Bogachev et al. (2008), cite that the independence assumption may not hold for all disruptive events and some events show long term correlation. However, we do not consider long term correlations in our models.

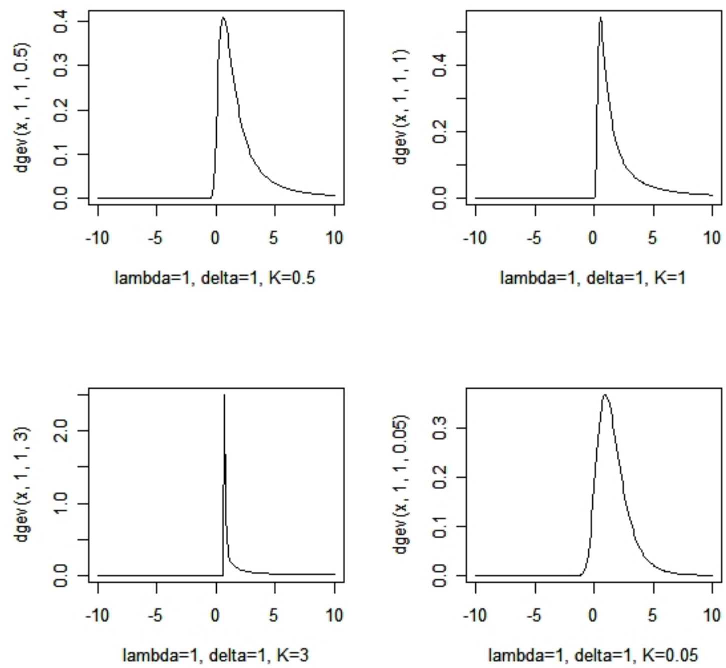

Figure 3.4. Effects of varying κ on the $GEVD_M$ pdf

3.2.3 The Disruption Risk Function

The disruption risk function is obtained by combining the impact and occurrence functions described in Sections 3.2.1 and 3.2.2 respectively. Aggregation can be carried out in two ways: analytically or using simulation. Here, we detail the analytical aggregation of impact and occurrence. We also briefly discuss the simulation method previously introduced in Tang (2006).

The analytical method relies on the following two assumptions:

- The number of events occurring over a given period of time follows a Poisson distribution.
- Impact and occurrence random variables are independent.

Under these assumptions, let N be the number of occurrences of a same rare event over a given period of time and X_i be the impact of event i. We can represent N in years and X_i in monetary units. Within this scheme, a random number of the same rare event occurs in a year, each causing a random financial loss. The financial loss follows an appropriate GEVD. Then, the risk function is the sum of random variables X_i where the upper bound of the sum N is also a random variable. We derive the expressions of the expected value and variance of risk in the following proposition.

Proposition 3.1. *Consider a collection of independent random variables X_i, $i = 1, ..., N$, following identical GEVDs where N is a random variable independent of all X_i. Then, the expected value and variance of the compound random variable $C = \sum_{i=1}^{N} X_i$ are*

$$E(C) = E(N)E(X) \tag{3.12}$$

and

$$var(C) = E(N)var(X) + E(X)^2 var(N) \tag{3.13}$$

Proof. Let us start by proving (3.12). Following Ross (2003), we can use conditional expectations to derive the result below,

$$\begin{aligned} E\left(\sum_{i=1}^{N} X_i\right) &= E\left[E\left(\sum_{i=1}^{N} X_i | N\right)\right] \\ &= E\left[E\left(\sum_{i=1}^{n} X_i | N = n\right)\right] \\ &= E\left[E\left(\sum_{i=1}^{n} X_i\right)\right] \\ &= E[nE(X)] \end{aligned}$$

Hence $E\left[\sum_{i=1}^{N} X_i | N\right] = NE(X_i)$ and $E(C) = E\left[\sum_{i=1}^{N} X_i\right] = E(N)E(X)$

We need the following result from Casella & Berger (2001) to prove the expression of the variance,

$$var(X) = E(var(X|Y)) + var(E(X|Y)) \tag{3.14}$$

Here, we have

$$\begin{aligned} var(C|N=n) &= var\left(\sum_{i=1}^{N} X_i|N=n\right) \\ &= var\left(\sum_{i=1}^{n} X_i|N=n\right) \\ &= var\left(\sum_{i=1}^{n} X_i\right) \\ &\sum_{i=1}^{n} var(X_i) = nvar(X) \end{aligned}$$

Then,

$$E(var(C|N)) = E(Nvar(X)) = E(N)var(X)$$

On the other hand,

$$E(C|N) = E\left(\sum_{i=1}^{N} X_i|N=n\right) = NE(X)$$

Thus,

$$var(E(C|N)) = var(NE(X)) = E(X)^2 var(N)$$

Putting together all expressions in Equation (3.14), we obtain

$$var(C) = E(N)var(X) + E(X)^2 var(N)$$

□

If we further assume a Poisson distribution $P(\lambda)$ for occurrence as in Section 3.2.2, then $var(C) = \lambda var(X) + \lambda E(X)^2$. In this case, expectation of N is the sample mean and the expectation of X_i can be obtained using the formula (3.10).

Note that the proposition we have just proved makes three key assumptions about the extreme events. First, it assumes that the damage from the events is non-correlated. This assumption is reasonable as long as we can rule out the possibility that the damage of an extreme event being related to the damages of previous events. Second, we assume that the number of extreme events is independent of the individual impact of each event. This assumption is also reasonable since the number of extreme events affects the total impact, not individual impacts. The last assumption is the identical distribution of extreme events. This assumption should hold as long as the supply chain is hit by similar extreme events during the period of analysis. If, on the other hand, extreme events disrupting the supply chain are dissimilar, the third assumption may not be verified. We treat this situation in Section 3.3.

The simulation model described in Yang (2006) generates the entire distribution of risk as opposed to just the expectation. Variance is not a direct output of the simulation, but can be computed if necessary. The simulation technique uses percentiles of the distribution to set risk tolerance levels. Yang's method operates as follows:

1. Generate Poisson random numbers to simulate the occurrence function over each period. A large number of periods should be considered for a proper application.

2. Generate as many impact values as the simulated number of occurrences.

3. Sum the impact values in each period. Use the inverse of the GEVD cdf to

generate percentile values.

The simulation model requires extensive calculations to generate the distribution of the total risk, which might be prohibitive in large problems. On the other hand, the analytical methods proposed in this dissertation can be used to quickly compute the mean and variance of compound disruption risks, given risks are of the same type, and to determine the cdf of the convolution of several different type of disruption risk functions as shown in the following.

3.3 Quantile Estimation

This section presents closed form expressions of cdfs of the convolution of a finite number of independent random variables distributed with respect to several types of GEVDs. Results on the cdfs are especially interesting since the cdf completely describes the probability distribution of a random variable. The number of random variables involved in the convolution represents the number of extreme events occurring over a fixed period of time. We assume a priori knowledge of the number of extreme events over the fixed period of time under consideration. The number of extreme events can be varied to carry out what-if type analyzes. The cdf of the convolution then provides the probability of having a total disruption damage less than some dollar value of interest. The inverse of the cdf, although difficult to compute in this case, is called the *Quantile Function* and provides the maximum value due to disruptions at a probability level of interest.

3.3.1 Theoretical Results

Results in this section are proven for maximum-type GEVD distributions, but can be extended to minimum-type GEVDs as well. The following theorem is introduced as a basis for computing the cdf of the sum of a finite number of independent GEVD random variables.

Theorem 3.1. *The cumulative distribution function of the convolution of a finite number of random variables distributed with respect to independent GEVD distributions with the parameter* $\kappa = 0$ *is*

$$F_{X_1+..+X_n}(a) = \mathrm{P}(X_1 + .. + X_n \leq a) = \int_{-\infty}^{\infty} F_{Y_{n-1}}(a - X_n) f_{X_n}(X_n) dX_n \quad (3.15)$$

where Y_{n-1} *is the random variable defined by the convolution of the first* $n-1$ *random variables, that is,* $Y_{n-1} = X_1 + .. + X_{n-1}$ *and* $f_{X_i}(X_i)$ *is the pdf of the* i*-th random variable.*

Proof. The proof is by induction. Let us first assume $n = 2$ to analyze the convolution of two random variables X_1 and X_2 following independent GEVDs with $\kappa = 0$. Expressions of the pdf and cdf of GEVD random variables with $\kappa = 0$ are given in Equations (3.7) and (3.3). Pdf and cdf functions are defined over $\mathbb{R}$ when $\kappa = 0$. The cdf of the convolution of X_1 and X_2 can be obtained as below.

$$\begin{aligned}
F_{X_1+X_2}(a) = \mathrm{P}(X_1 + X_2 \leq a) &= \iint_{X_1+X_2 \leq a} f_{X_1}(X_1) f_{X_2}(X_2) dX_1 dX_2 \\
&= \int_{-\infty}^{\infty} \int_{-\infty}^{a-X_2} f_{X_1}(X_1) f_{X_2}(X_2) dX_1 dX_2 \\
&= \int_{-\infty}^{\infty} \left[\int_{-\infty}^{a-X_2} f_{X_1}(X_1) dX_1 \right] f_{X_2}(X_2) dX_2 \\
&= \int_{-\infty}^{\infty} F_{X_1}(a - X_2) f_{X_2}(X_2) dX_2
\end{aligned}$$

The expression of the function to integrate is

$$F_{X_1}(a - X_2) f_{X_2}(X_2) = \frac{1}{\delta_2} e^{-\left[exp\left(\frac{\lambda_1 + X_2 - a}{\delta_1}\right) + exp\left(\frac{\lambda_2 - x_2}{\delta_2}\right)\right]} e^{\left(\frac{\lambda_2 - x_2}{\delta_2}\right)}$$

where $exp(.)$ is the exponential function.

Now, let us consider the convolution of three independent GEVD random variables, X_1, X_2 and X_3 with $\kappa = 0$. We introduce the notation Y_{n-1} to denote the sum of the first $n-1$ terms in the convolution. Hence, $F_{X_1+X_2+X_3}(a) = \mathrm{P}(X_1 + X_2 + X_3 \leq a) \equiv F_{Y_2+X_3}(a) = \mathrm{P}(Y_2 + X_3 \leq a)$. Then, $F_{X_1+X_2+X_3}(a)$ can be calculated as below,

$$\begin{aligned} F_{X_1+X_2+X_3}(a) &= \iint_{Y_2+X_3 \leq a} f_{Y_2}(Y_2) f_{X_3}(X_3) dY_2 dX_3 \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{a-X_3} f_{Y_2}(Y_2) f_{X_3}(X_3) dY_2 dX_3 \\ &= \int_{-\infty}^{\infty} \left[\int_{-\infty}^{a-X_3} f_{Y_2}(Y_2) dY_2 \right] f_{X_3}(X_3) dX_3 \\ &= \int_{-\infty}^{\infty} F_{Y_2}(a - X_3) f_{X_3}(X_3) dX_3 \end{aligned}$$

In the last equation above $F_{Y_2} = F_{X_1+X_2}$, which was calculated previously. Also, note that we actually do not need to compute the pfd of the convolution, f_{Y_2}, to have the cdf of $F_{X_1+X_2+X_3}$; although f_{Y_2} can be calculated by differentiating $F_{X_1+X_2}(a)$ with respect to a.

Now, let us assume that Equation (3.15) holds for the convolution of n independent GEVD random variables and extend the result to $n+1$ to complete the proof. Let $F_{X_1+\ldots+X_{n+1}}(a) = F_{Y_n+X_{n+1}}(a) = \mathrm{P}(Y_n + X_{n+1} \leq a)$ be the cdf of the convolution. Then,

$$\begin{aligned} F_{Y_n+X_{n+1}}(a) &= \iint_{Y_n+X_{n+1}\leq a} f_{Y_n}(Y_n) f_{X_{n+1}}(X_{n+1}) dY_n dX_{n+1} \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{a-X_{n+1}} f_{Y_n}(Y_n) f_{X_{n+1}}(X_{n+1}) dY_n dX_{n+1} \\ &= \int_{-\infty}^{\infty}\left[\int_{-\infty}^{a-X_{n+1}} f_{Y_n}(Y_n) dY_n\right] f_{X_{n+1}}(X_{n+1}) dX_{n+1} \\ &= \int_{-\infty}^{\infty} F_{Y_n}(a-X_{n+1}) f_{X_{n+1}}(X_{n+1}) dX_{n+1} \end{aligned}$$

The expression of the cdf function $F_{Y_n} = F_{X_1+\ldots+X_{n+1}}$ is given in Equation (3.15) and hence can be evaluated at $(a - X_{n+1})$. □

A result similar to what has been asserted in Theorem 3.1 can be obtained using the Central Limit Theorem (CLT). As stated in Casella & Berger (2001), CLT assures that a random variable $\frac{\sqrt{n}(\overline{X}_n-\mu)}{\sigma}$ converges to a standard normal distribution where $\overline{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$ and X_i are independent and identically distributed random variables with finite mean and variance. The result in CLT is stronger because it provides a normal approximation with very few assumptions. On the other hand, Casella & Berger (2001) note that the quality of the approximation depends on the original distribution and is not guaranteed to be satisfactory. Moreover, a relatively large sample size is required for CLT to hold, which may not be always available in disruptive events. Finally, CLT requires the underlying random variables to follow an identical distribution, whereas this assumption is not necessary in Theorem 3.1.

Obtaining closed form expressions when $\kappa \neq 0$ is more challenging since the pdf functions to be integrated are not defined for all real numbers. Instead, the GEVD pdf is defined for $x \leq \lambda + \frac{\delta}{\kappa}$ when $\kappa > 0$ and for $x \geq \lambda + \frac{\delta}{\kappa}$ when $\kappa < 0$. Let us first consider the convolution of two random variables following independent GEVDs with strictly positive κ parameters. The following proposition provides the cdf of the convolution.

Proposition 3.2. *Consider two GEVD random variables, $X_1 \sim GEVD(\lambda_1, \delta_1, \kappa_1)$ and $X_2 \sim GEVD(\lambda_2, \delta_2, \kappa_2)$ with κ_1, $\kappa_2 > 0$. Assume that X_1 and X_2 are independent. Then, the cdf of the convolution of X_1 and X_2 is given in Equation (3.16).*

$$F_{X_1+X_2}(a) = \int_{-\infty}^{\lambda_2+\frac{\delta_2}{\kappa_2}} F_{X_1}(a - X_2) f_{X_2}(X_2) dX_2 \tag{3.16}$$

Proof. We use an approach similar to what we have used to prove Theorem 3.1 for $n = 2$. Pdf and cdf of a GEVD random variable with $\kappa \neq 0$ are given in Equations (3.6) and (3.2). Note that the pdf is defined for $x \leq \lambda + \frac{\delta}{\kappa}$ when $\kappa > 0$. The cdf of the convolution of X_1 and X_2 can be calculated as below,

$$F_{X_1+X_2}(a) = \iint_{X_1+X_2 \leq a} f_{X_1}(X_1) f_{X_2}(X_2) dX_1 dX_2$$

Note that we have two upper bounds on X_1. First, $X_1 \leq \lambda_1 + \frac{\delta_1}{\kappa_1}$ by the definition of f_{X_1}; and second, $X_1 \leq a - X_2$ by the integration. If $\lambda_1 + \frac{\delta_1}{\kappa_1} \geq a - X_2$, then the upper bound on X_1 would be $a - X_2$. Otherwise, the upper bound should be $\lambda_1 + \frac{\delta_1}{\kappa_1}$. The issue with this reasoning is that the exact value of X_2 is not known since it is a random variable. Nevertheless, it can still be argued that the upper bound on X_1 can be taken as $a - X_2$ when $\lambda_1 + \frac{\delta_1}{\kappa_1} \leq a - X_2$. The integrand, f_{X_1}, will then be integrated to its upper bound and the remainder will integrate to zero. Thus,

$$\begin{aligned} F_{X_1+X_2}(a) &= \int_{-\infty}^{\lambda_2+\frac{\delta_2}{\kappa_2}} \left[\int_{-\infty}^{a-X_2} f_{X_1}(X_1) dX_1 \right] f_{X_2}(X_2) dX_2 \\ &= \int_{-\infty}^{\lambda_2+\frac{\delta_2}{\kappa_2}} F_{X_1}(a - X_2) f_{X_2}(X_2) dX_2 \end{aligned}$$

□

The following Corollary provides an interesting result for a special case of Proposition 3.2.

Corollary 3.1. *Cumulative distribution function of the convolution of two GEVD random variables* $X_1 \sim GEVD(\lambda_1, \delta_1, \kappa_1)$ *and* $X_2 \sim GEVD(\lambda_2, \delta_2, \kappa_2)$*, assuming* X_1 *and* X_2 *are independent, with strictly positive* κ *parameters verifying* $\lambda_1 + \frac{\delta_1}{\kappa_1} \leq a - \lambda_2 - \frac{\delta_2}{\kappa_2}$ *is*

$$F_{X_1+X_2}(a) = F_{X_1}\left(\lambda_1 + \frac{\delta_1}{\kappa_1}\right) F_{X_2}\left(\lambda_2 + \frac{\delta_2}{\kappa_2}\right)$$

Proof. The result follows from Equation (3.16) and the bounds on X_1 and X_2. Note that $X_2 \leq \lambda_2 + \frac{\delta_2}{\kappa_2}$ by the definition of f_{X_2}. Then, $X_1 + X_2 \leq a$ translates in to $X_1 \leq a - \lambda_2 - \frac{\delta_2}{\kappa_2}$. If the upper bound on X_1 is smaller than the known upper bound on X_2, that is $\lambda_1 + \frac{\delta_1}{\kappa_1} \leq a - \lambda_2 - \frac{\delta_2}{\kappa_2}$, then we can use $\lambda_1 + \frac{\delta_1}{\kappa_1}$ as the upper bound on the integral of f_{X_1}. Then, Equation (3.16) can be further simplified as below,

$$\begin{aligned} F_{X_1+X_2}(a) &= \int_{-\infty}^{\lambda_2+\frac{\delta_2}{\kappa_2}} F_{X_1}\left(\lambda_1 + \frac{\delta_1}{\kappa_1}\right) f_{X_2}(X_2) dX_2 \\ &= F_{X_1}\left(\lambda_1 + \frac{\delta_1}{\kappa_1}\right) \int_{-\infty}^{\lambda_2+\frac{\delta_2}{\kappa_2}} f_{X_2}(X_2) dX_2 \\ &= F_{X_1}\left(\lambda_1 + \frac{\delta_1}{\kappa_1}\right) F_{X_2}\left(\lambda_2 + \frac{\delta_2}{\kappa_2}\right) \end{aligned}$$

Note that in this case the cdf of the convolution does not depend on a. □

Let us now analyze the convolution of two independent random variables $X_1 \sim GEVD(\lambda_1, \delta_1, \kappa_1)$ and $X_2 \sim GEVD(\lambda_2, \delta_2, \kappa_2)$ with strictly negative κ. The pdf of a GEVD with strictly negative κ is defined for $x \geq \lambda + \frac{\delta}{\kappa}$. Cdf of the convolution is derived in the following proposition.

Proposition 3.3. *Consider two independent GEVD random variables,* $X_1 \sim GEVD(\lambda_1, \delta_1, \kappa_1)$ *and* $X_2 \sim GEVD(\lambda_2, \delta_2, \kappa_2)$ *with* κ_1, $\kappa_2 < 0$. *Then, the cdf of the convolution of* X_1 *and* X_2 *is given in Equation* (3.17)

$$F_{X_1+X_2}(a) = \int_{\lambda_2+\frac{\delta_2}{\kappa_2}}^{\infty} F_{X_1}(a - X_2) f_{X_2}(X_2) dX_2 \tag{3.17}$$

Proof. Again we use a similar approach to what we have used to prove Theorem 3.1 for $n = 2$. Pdf and cdf of a GEVD random variable with $\kappa \neq 0$ are given in Equations (3.6) and (3.2). Note that the pdf is defined for $x \geq \lambda + \frac{\delta}{\kappa}$ when $\kappa < 0$. The cdf of the convolution of X_1 and X_2 can be calculated as below,

$$\begin{aligned} F_{X_1+X_2}(a) &= \iint_{X_1+X_2 \leq a} f_{X_1}(X_1) f_{X_2}(X_2) dX_1 dX_2 \\ &= \int_{\lambda_2+\frac{\delta_2}{\kappa_2}}^{\infty} \underbrace{\left[\int_{\lambda_1+\frac{\delta_1}{\kappa_1}}^{a-X_2} f_{X_1}(X_1) dX_1 \right]}_{I} f_{X_2}(X_2) dX_2 \end{aligned}$$

$$\begin{aligned} I &= \underbrace{\int_{-\infty}^{a-X_2} f_{X_1}(X_1) dX_1}_{\text{Exists when } a - X_2 \geq \lambda_1 + \frac{\delta_1}{\kappa_1}} - \underbrace{\int_{-\infty}^{\lambda_1+\frac{\delta_1}{\kappa_1}} f_{X_1}(X_1) dX_1}_{f_{X_1} \text{ not defined over this range}} \\ &= F_{X_1}(a - X_2) \end{aligned}$$

substituting I, we get

$$F_{X_1+X_2}(a) = \int_{\lambda_2+\frac{\delta_2}{\kappa_2}}^{\infty} F_{X_1}(a - X_2) f_{X_2}(X_2) dX_2$$

□

All possible cases of the convolution of two independent GEVD random variables with κ parameters verifying the same conditions have been treated in the previous propositions. Now we turn our attention to the convolution of GEVD random variables where κ parameters obey different conditions. The following three propositions treat all possible cases.

Proposition 3.4. *Consider two independent GEVD random variables,* $X_1 \sim GEVD(\lambda_1, \delta_1)$ *and* $X_2 \sim GEVD(\lambda_2, \delta_2, \kappa_2)$ *with* $\kappa_2 > 0$*. Then, the cdf of the convolution of* X_1 *and* X_2 *is given in Equation* (3.18).

$$F_{X_1+X_2}(a) = \int_{-\infty}^{\lambda_2+\frac{\delta_2}{\kappa_2}} F_{X_1}(a - X_2) f_{X_2}(X_2) dX_2 \tag{3.18}$$

Proof. Pdf and cdf of a GEVD random variable with $\kappa \neq 0$ are given in Equations (3.6) and (3.2) and those for $\kappa = 0$ are given in (3.7) and (3.3). Note that the pdf is defined for $x \leq \lambda + \frac{\delta}{\kappa}$ when $\kappa > 0$ and over $\mathbb{R}$ when $\kappa = 0$. The cdf of the convolution of X_1 and X_2 can be calculated as below,

$$\begin{aligned} F_{X_1+X_2}(a) &= \iint_{X_1+X_2 \leq a} f_{X_1}(X_1) f_{X_2}(X_2) dX_1 dX_2 \\ &= \int_{-\infty}^{\lambda_2+\frac{\delta_2}{\kappa_2}} \left(\int_{-\infty}^{a-X_2} f_{X_1}(X_1) dX_1 \right) f_{X_2}(X_2) dX_2 \\ &= \int_{-\infty}^{\lambda_2+\frac{\delta_2}{\kappa_2}} F_{X_1}(a - X_2) f_{X_2}(X_2) dX_2 \end{aligned}$$

□

Proposition 3.5. *Consider two independent GEVD random variables,* $X_1 \sim GEVD(\lambda_1, \delta_1)$ *and* $X_2 \sim GEVD(\lambda_2, \delta_2, \kappa_2)$ *with* $\kappa_2 < 0$*. Then, the cdf of the convolution of* X_1 *and* X_2 *is given in Equation* (3.19).

$$F_{X_1+X_2}(a) = \int_{\lambda_2+\frac{\delta_2}{\kappa_2}}^{\infty} F_{X_1}(a - X_2) f_{X_2}(X_2) dX_2 \tag{3.19}$$

Proof. Pdf and cdf of a GEVD random variable with $\kappa \neq 0$ are given in Equations (3.6) and (3.2) and those for $\kappa = 0$ are given in (3.7) and (3.3). Note that the pdf is defined for $x \geq \lambda + \frac{\delta}{\kappa}$ when $\kappa < 0$ and over $\mathbb{R}$ when $\kappa = 0$. The cdf of the convolution of X_1 and X_2 can be calculated as below,

$$\begin{aligned} F_{X_1+X_2}(a) &= \iint_{X_1+X_2 \leq a} f_{X_1}(X_1) f_{X_2}(X_2) dX_1 dX_2 \\ &= \int_{\lambda_2+\frac{\delta_2}{\kappa_2}}^{\infty} \left(\int_{-\infty}^{a-X_2} f_{X_1}(X_1) dX_1 \right) f_{X_2}(X_2) dX_2 \\ &= \int_{\lambda_2+\frac{\delta_2}{\kappa_2}}^{\infty} F_{X_1}(a - X_2) f_{X_2}(X_2) dX_2 \end{aligned}$$

□

Proposition 3.6. *Let two GEVD random variables, $X_1 \sim GEVD(\lambda_1, \delta_1, \kappa_1)$ and $X_2 \sim GEVD(\lambda_2, \delta_2, \kappa_2)$ with $\kappa_1 < 0$ and $\kappa_2 > 0$. Assume that X_1 and X_2 are independent of each other. Then, the cdf of the convolution of X_1 and X_2 is given in Equation* (3.20)*.*

$$F_{X_1+X_2}(a) = \int_{-\infty}^{\lambda_2+\frac{\delta_2}{\kappa_2}} F_{X_1}(a - X_2) f_{X_2}(X_2) dX_2 \tag{3.20}$$

Proof. Pdf and cdf of a GEVD random variable with $\kappa \neq 0$ are given in Equations (3.6) and (3.2). Note that the pdf is defined for $x \geq \lambda + \frac{\delta}{\kappa}$ when $\kappa < 0$ and for $x \leq \lambda + \frac{\delta}{\kappa}$ when $\kappa > 0$. Then, the convolution formula can be obtained as follows,

$$\begin{aligned} F_{X_1+X_2}(a) &= \iint_{X_1+X_2 \leq a} f_{X_1}(X_1) f_{X_2}(X_2) dX_1 dX_2 \\ &= \int_{-\infty}^{\lambda_2+\frac{\delta_2}{\kappa_2}} \left(\underbrace{\int_{\lambda_1+\frac{\delta_1}{\kappa_1}}^{a-X_2} f_{X_1}(X_1) dX_1}_{I} \right) f_{X_2}(X_2) dX_2 \end{aligned}$$

Note that the integral I exists if $\lambda_1 + \frac{\delta_1}{\kappa_1} < a - X_2$. Then,

$$I = \int_{-\infty}^{a-X_2} f_{X_1}(X_1)dX_1 - \underbrace{\int_{-\infty}^{\lambda_1+\frac{\delta_1}{\kappa_1}} f_{X_1}(X_1)dX_1}_{\text{does not exist for } x \leq \lambda_1 + \frac{\delta_1}{\kappa_1}}$$

$$= F_{X_1}(a - X_2)$$

Replacing the expression of I we get,

$$F_{X_1+X_2}(a) = \int_{-\infty}^{\lambda_2+\frac{\delta_2}{\kappa_2}} F_{X_1}(a - X_2) f_{X_2}(X_2)dX_2$$

□

3.3.2 Numerical Examples

Several numerical examples are provided in this section to illustrate the implementation of the theoretical results derived in Section 3.3. Let us consider a supplier who faces several disruptive events every year and assume that two type of events, floods and blizzards, cause the most damage to the supplier. Assume a buyer considers including this supplier into his supplier portfolio and wants to evaluate the supplier's potential risk to his operations.

To implement our results developed earlier, we consider three different cases where the aforementioned disruptive events follow different distributions. First, let us assume that the risk from floods over a period of one year, X_1, follows $GEVD(\lambda_1 = 500, \delta_1 = 350)$ and risk from blizzards over a period of one year, X_2, follows $GEVD(\lambda_2 = 750, \delta_1 = 450)$. Using the results of Theorem 3.1, the distribution of the total damage from floods and blizzards over a year can be

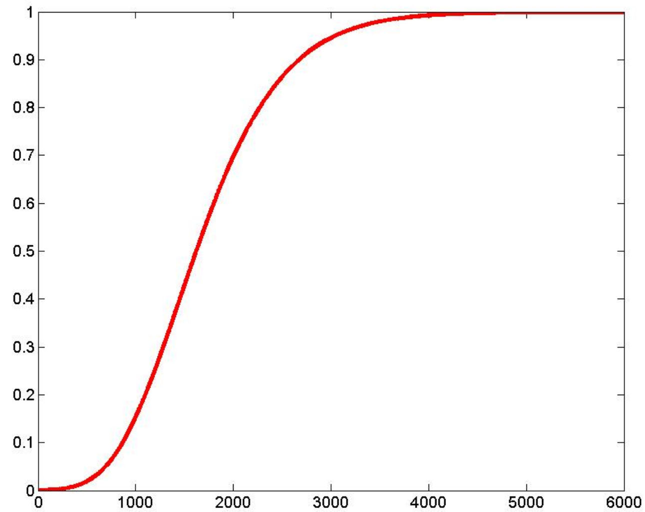

Figure 3.5. Plot of $F_{X_1+X_2}(a)$ with $\kappa_1 = \kappa_2 = 0$

calculated by evaluating the following integral,

$$F_{X_1+X_2}(a) = \frac{1}{750}\int_{-\infty}^{\infty} e^{-\left[exp\left(\frac{500+x-a}{350}\right)+exp\left(\frac{750-x}{450}\right)\right]} e^{\left(\frac{750-x}{450}\right)} dx$$

The integral is hard to calculate analytically, but it can be evaluated numerically using available computation software such as MATLAB or MATHEMATICA. An algorithm is written to evaluate the integral using quadrature functions available in MATLAB where the value of a is varied to evaluate the entire cdf. As a result, the plot of the cdf of the convolution is obtained and is shown in Figure 3.5.

Cdf of the convolution is the probability that the convolution of two random variables is smaller than some value a; that is, $F_{X_1+X_2}(a) = \mathrm{P}(X_1 + X_2 \leq a)$. Therefore, the plot in Figure 3.5 defines the entire distribution of the convolution and can be used to compute probabilities of upper bounds on the maximum combined damage from disruptive events. For instance, it can be read that there is 0.9453 probability that total the damage will be less than $3,000. We will provide

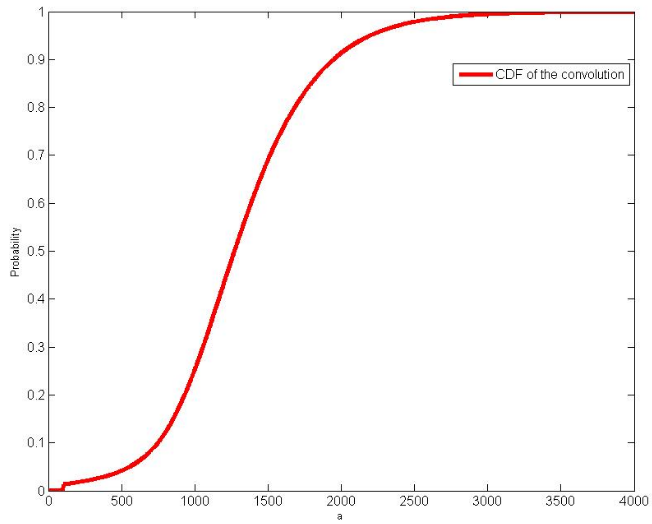

Figure 3.6. Plot of $F_{X_1+X_2}(a)$ with $\kappa_1 = 0$ and $\kappa_2 > 0$

two more examples to conclude this section.

Let us now consider that yearly damage from floods, X_1, follows $GEVD(\lambda_1 = 500, \delta_1 = 350)$ and let yearly damage from blizzards, X_2, follow a different distribution, $GEVD(\lambda_2 = 650, \delta_2 = 200, \kappa = 1.5)$. According to Equation (3.18), the total yearly damage can be evaluated by computing the following integral,

$$F_{X_1+X_2}(a) = \frac{1}{650} \int_{-\infty}^{783.33} e^{-\left[exp\left(\frac{500+x-a}{350}\right)\right]} e^{-\left(1-1.5\frac{x-650}{200}\right)^{\frac{2}{3}}} \left(1 - 1.5\frac{x-650}{200}\right)^{-\frac{1}{3}} dx$$

Using the same MATLAB algorithm as before, the cdf plot of the total damage in Figure 3.6 is obtained. As it can be observed from Figure 3.6, the combined damage from two disruptive events does not exceed \$3,000 with 0.9947 probability.

Finally, let us assume that yearly damage from floods follows $GEVD(\lambda_1 = 500, \delta_1 = 350, \kappa_1 = -1)$ and yearly damage from blizzards follows $GEVD(\lambda_2 = 650, \delta_2 = 200, \kappa_2 = 1.5)$. According to Equation (3.20), the total yearly damage

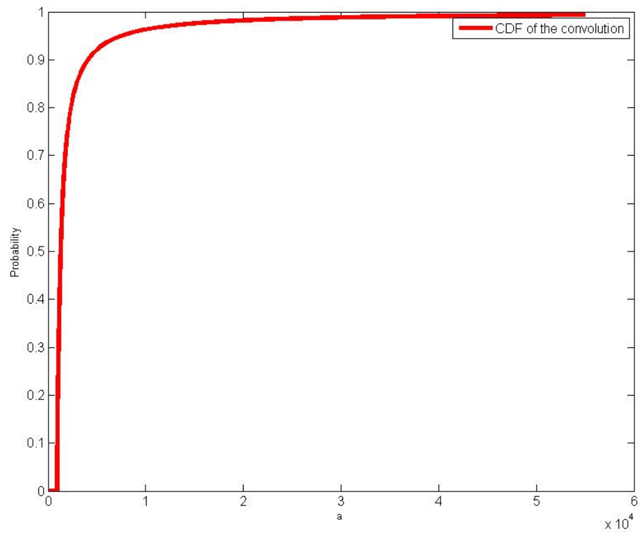

Figure 3.7. Plot of $F_{X_1+X_2}(a)$ with $\kappa_1 < 0$ and $\kappa_2 > 0$

can be evaluated by computing the following integral,

$$F_{X_1+X_2}(a) = \frac{1}{650}\int_{-\infty}^{783.3} e^{-\left(1+\left(\frac{a-x-500}{350}\right)\right)^{-1}} e^{-\left(1-\frac{3(x-650)}{400}\right)^{\frac{2}{3}}} \left(1-\frac{3(x-650)}{400}\right)^{-\frac{1}{3}} dx$$

The above integral is evaluated on MATLAB and the cdf plot of the convolution is given in Figure 3.7.

3.4 Detectability of Disruptive Events

Supply chains of today feature large and complex structures that pose additional difficulties to supply chain management. One challenge is the transfer of information among nodes in a supply chain. Although information sharing has proven to enhance effective supply chain practice (see for instance Zhou & Benton (2007) for recent survey results of U.S. companies), it also has undesirable effects and brings additional risks, including loss of bargaining power and sharing of business secrets as listed in Yuan & Qiong (2008). Information sharing becomes even more intricate when nodes in a supply chain does not belong to a same organization due to trust issues and unwillingness to share corporate information (Fawcett et al. 2007) and possible ERP software mismatches. This is often the situation in supply chains where some activities, e.g., services, manufacturing operations, transportation and logistics, are outsourced to different companies. Our interest in this section is modeling the propagation of disruption information from a failed supplier to the buyer of the end product. The failed supplier can be located at any tier with respect to the buyer. Obviously, information from lower tier suppliers will take longer to reach the buyer. An example of delay of information and its results were discussed in Section 3.2.

Modeling the propagation of disruption information in a supply chain depends on the way nodes communicate with one another. Under the best case scenario, each supplier, independent of its tier, communicates directly with the buyer. This is a very optimistic scenario since it requires every supplier to know the end destination of its product and to establish a direct connection with the buyer. A second-to-best scenario assumes every disrupted supplier reporting to its immediate buyer. This situation may be more realistic; however, it does not ensure that the information will reach the primary buyer promptly since the immediate buyer of the disrupted supplier may not share the information with his customers or with the main buyer. Another scenario would be assuming that the information flows randomly in the supply chain network. This may sound like a pessimistic scenario at first, but the method can be customized to better model the reality.

Information flow is related to the *mean first passage time (MFPT)* concept used in Markov chain analysis. MFPT represents the mean time it takes for a signal released at a node to reach another node in the Markov chain. In the following subsections, we will present some Markov chain properties required to hold in order to compute the MFPT values, provide formulas to compute the MFPT matrix of a Markov chain and illustrate the concept with an example.

3.4.1 Some Basic Properties of Markov Chains

Consider a stochastic process $\{X_n,\ n = 0, 1, 2, ...\}$ that takes a finite or countable number of possible values. $X_n = i$ indicates that the process is at state i at time (or stage) n. A stochastic process defined as above is called a Markov chain if the probability of being in a certain state at a stage depends only on the state at the immediately previous stage. That is, $\Pr[X_{n+1} = i_{n+1}|X_n = i_n, X_{n-1} = i_{n-1}, ..., X_0 = i_0] = \Pr[X_{n+1} = i_{n+1}|X_n = i_n]$ where Pr denotes probability.

Two states accessible from each other in a Markov chain are said to communicate. A class of states is a group of states that communicate with each other. If a Markov chain has only one class, then it is said to be *irreducible*. The period in a Markov chain is the minimum number of transitions required to return to a state upon leaving it. A Markov chain of period one is called *aperiodic*.

Let $\mathbf{P} = (p_{ij})$ be the transition probability matrix of a Markov chain. p_{ij} is the probability of moving to a state j in the next stage while the process is at state i at the current stage; that is, $p_{ij} = \Pr[X_{t+1} = j|X_t = i]$. A Markov chain is said to have steady state probabilities if the transition probability matrix converges to a constant matrix. Every Markov chain with a finite state set has a unique stationary distribution. If in addition the Markov chain is aperiodic, then it admits steady state probabilities. The method described below can be used to compute steady state probabilities given the transition probability matrix of a Markov chain (Chandra 2005):

1. Let $\mathbf{P_{m\times m}} = (p_{ij})$ be a transition probability matrix and $\mathbf{\Pi_{1\times m}}$ be the unknown steady state probability vector. We need to calculate the elements of $\mathbf{\Pi}$ such that $\mathbf{\Pi P} = \mathbf{\Pi} \Leftrightarrow \mathbf{\Pi(I - P)} = \mathbf{0}$, where $\mathbf{I}$ is the identity matrix of appropriate size. Let $\mathbf{Q} = \mathbf{I} - \mathbf{P}$ and compute $\mathbf{Q}$.

2. Delete one row or column of $\mathbf{Q}$. Deletion is allowed since the system has one redundant equation. Assume that we delete the last column of $\mathbf{Q}$ to have $\mathbf{R_{m\times(m-1)}}$. The system to solve becomes $\mathbf{\Pi R} = \mathbf{0}$

3. Note that $\mathbf{\Pi} = (\pi_j)_{j=1,\dots,m}$ is a vector of probabilities in the same space, and hence, the sum of the elements in $\mathbf{\Pi}$ should equal one. This condition transforms the system we need to solve to the following:

$$\begin{cases} \mathbf{\Pi R} = \mathbf{0} \\ \sum_{j=1}^{m} \pi_j = 1 \end{cases}$$

   We can add a column of ones ($\mathbf{1_{m\times 1}}$) to $\mathbf{R}$ to include the sum of the elements of $\mathbf{\Pi}$ and change the right hand side from $\mathbf{0}$ to $\widehat{\mathbf{e_m}}$ to make sure that π_j sum up to one. The system can be recast as $\mathbf{\Pi S} = \widehat{\mathbf{e_m}}$ where $\mathbf{S}$ is the matrix $\mathbf{R}$ augmented with a column of ones added as the last column and $\widehat{\mathbf{e_m}}$ is a vector of zeros except a one at position m.

4. Invert the $\mathbf{S}$ matrix. The last row of $\mathbf{S^{-1}}$ has the π_j values due to the way we define $\widehat{\mathbf{e_m}}$.

Note that aperiodicity is often a strong assumption in Markov chains. One can still carry out the above calculations even if a Markov chain has a period greater than one. Components of the vector $\mathbf{\Pi}$ can then be interpreted as long run proportions of time that the underlying stochastic process would be in a given state (Chandra 2005).

3.4.2 Computing the MFPT Matrix

Once $\mathbf{\Pi}$ is calculated, we can compute the MFPT matrix $\mathbf{M}$ as described in Kemeny & Snell (1976). Several auxiliary matrices have to be computed to reach $\mathbf{M}$. Let $\mathbf{Z} = \left(\mathbf{I} - \mathbf{P} + \mathbf{e}\mathbf{\Pi}^T\right)^{-1}$ be the *fundamental matrix* where $\mathbf{I}$ is the identity matrix, $\mathbf{P}$ the transition probability matrix, $\mathbf{e}$ a vector of all ones and $\mathbf{\Pi}$ the vector of steady state probabilities[1]. Let also $\mathbf{Z_{dg}}$ be the matrix that has the same elements as $\mathbf{Z}$ on the diagonal and zeros elsewhere and $\mathbf{D}$ be the matrix that has $\frac{1}{\pi_j}$ on the diagonal and zeros elsewhere. Then, $\mathbf{M}$ can be computed using Equation (3.21) as below,

$$\mathbf{M} = \left(\mathbf{I} - \mathbf{Z} + \mathbf{E}\mathbf{Z_{dg}}\right)\mathbf{D} \tag{3.21}$$

where $\mathbf{E}$ is a matrix of all ones. We can also compute the variance of the first passage time once we have computed the MFPT matrix $\mathbf{M}$. Equation (3.22) below provides the formula for computing the variance of the first passage time matrix $\mathbf{V}$.

$$\mathbf{V} = \mathbf{M}\left(2\mathbf{Z_{dg}}\mathbf{D} - \mathbf{I}\right) + 2\left(\mathbf{ZM} - \mathbf{E}(\mathbf{ZM_{dg}})\right) \tag{3.22}$$

where $\mathbf{ZM_{dg}}$ is a matrix that agrees with the product matrix $\mathbf{ZM}$ on the diagonal and has zeros elsewhere.

[1]The term *steady state probability* is used in a rather loose sense since only aperiodic recurrent Markov chains admit this property.

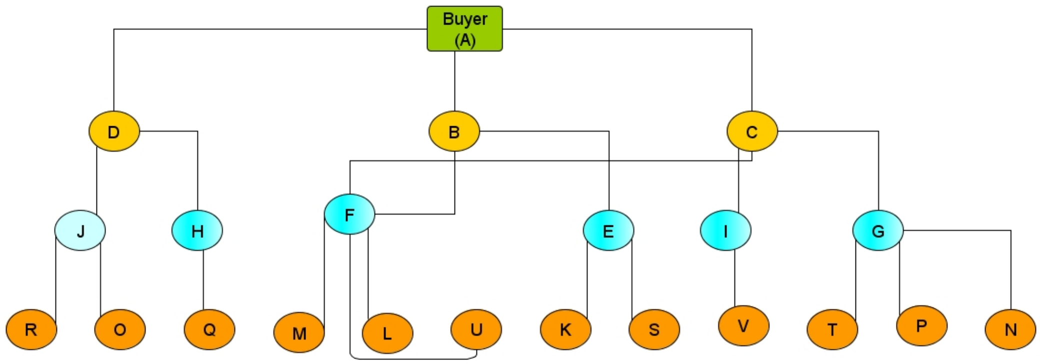

Figure 3.8. Example supply network

3.4.3 Numerical Examples

Let us assume the supply chain network on Figure 3.8 where we have one buyer (node A), three tier 1 suppliers (B,C,D) color coded in yellow, six tier 2 suppliers (E,F,G,H,I,J) in blue and twelve tier 3 suppliers (K,L,M,N,O,P,Q,R,S,T,U,V) in orange. We assume that information can flow in both directions in the supply network; in other words, edges between the nodes are undirected.

Under the best case scenario discussed above, a disruption news at any tier would reach the buyer (node A) in one transition. This is possible in the network presented in Figure 3.8 only if the disrupted supplier creates a direct link to the buyer; unless he is in the first tier. Under the second-best scenario, the number of transitions that a disruption news would reach the buyer is proportional to the disrupted supplier's tier. That is, it would take one transition for a tier-one supplier, two transitions for a tier-two supplier and three a transitions for tier-three supplier to pass a signal through.

To examine the random walk information propagation scenario, we need to create a matrix of transition probabilities that can mimic the propagation of information following a random walk. One way to do this is using the inverse of node degrees as transition probabilities (White & Smyth 2003). Degree of node v in an undirected graph is the number of edges that have v as the end node (Brandes & Erlebach 2005). In-degree of a node v is the number of edges that end at v and out-degree is the number of edges emanating from v. A useful concept in graph theory is the *adjacency matrix* **A** defined as

$$a_{ij} = \begin{cases} 1 & \text{if nodes } i \text{ and } j \text{ are connected with an edge,} \\ 0 & \text{otherwise.} \end{cases}$$

Adjacency matrix is a concise way to present a graph or a network using matrices and can be used to compute degrees of nodes in the network. For instance, the supply network of Figure 3.8 can be presented as in Table 3.2. Adjacency matrix of a graph can also be used to compute the random walk transition probability matrix. First calculate the out degree, $deg_{out}(i)$, of each node, which is the sum of the elements in each row of the matrix. Then, under the random walk model of White & Smyth (2003), the probability of moving to any adjacent node j from i is $p_{ij} = \frac{1}{deg_{out}(i)}$. For example, the transition probability from node C to node A is $p_{CA} = \frac{1}{4} = 0.25$. Similarly, $p_{FC} = \frac{1}{5} = 0.20$ and $p_{LF} = \frac{1}{1} = 1$. By definition, transition probability between two non adjacent nodes, e.g. from node L to node A is zero. The complete random walk transition probability matrix of the supply network in Figure 3.8 is given in Table 3.3.

Table 3.2. Adjacency matrix of the supply network in Figure 3.8

	A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**	**L**	**M**	**N**	**O**	**P**	**Q**	**R**	**S**	**T**	**U**	**V**
A	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
B	1	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
C	1	0	0	0	0	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
D	1	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0
E	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0
F	0	1	1	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	1	0
G	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	1	0	0
H	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
I	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
J	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0
K	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
L	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
M	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
N	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
O	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
P	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Q	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
R	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
S	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
T	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
V	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 3.3. Transition probability matrix of the supply network in Figure 3.8

	A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**	**L**	**M**	**N**	**O**	**P**	**Q**	**R**	**S**	**T**	**U**	**V**
A	0.00	0.33	0.33	0.33	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
B	0.33	0.00	0.00	0.00	0.33	0.33	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
C	0.25	0.00	0.00	0.00	0.00	0.25	0.25	0.00	0.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
D	0.33	0.00	0.00	0.00	0.00	0.00	0.00	0.33	0.00	0.33	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E	0.00	0.33	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.33	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.33	0.00	0.00	0.00
F	0.00	0.20	0.20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.20	0.20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.20	0.00
G	0.00	0.00	0.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.00	0.25	0.00	0.00	0.00	0.25	0.00	0.00
H	0.00	0.00	0.00	0.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.50	0.00	0.00	0.00	0.00	0.00
I	0.00	0.00	0.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.50
J	0.00	0.00	0.00	0.33	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.33	0.00	0.00	0.33	0.00	0.00	0.00	0.00
K	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
L	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
M	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
N	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
O	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
P	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Q	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
R	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
S	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
T	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
U	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
V	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 3.4. Steady state probabilities of nodes in the supply network of Figure 3.8

A	**B**	**C**	**D**	**E**	**F**
0.068	0.068	0.091	0.068	0.068	0.114
G	**H**	**I**	**J**	**K**	**L**
0.091	0.045	0.045	0.068	0.023	0.023
M	**N**	**O**	**P**	**Q**	**R**
0.023	0.023	0.023	0.023	0.023	0.023
	S	**T**	**U**	**V**	
	0.023	0.023	0.023	0.023	

Following the 4-step methodology detailed in Section 3.4.1 and based on the transition probability matrix of Table 3.3, the steady state probability matrix of the supply network can be computed as in Table 3.4. Using Equation (3.21) and the definitions in Section 3.4.2, the MFPT matrix of the supply network of Figure 3.8 is computed and given in Table 3.5.

Table 3.5. Mean first passage time (MFPT) matrix of the supply network in Figure 3.8

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
A	15	20	17	33	59	25	54	74	57	72	102	68	68	96	115	96	117	115	102	97	68	101
B	14	15	19	47	39	17	56	88	60	86	82	60	60	99	129	99	131	129	82	99	60	103
C	17	25	11	50	64	20	37	91	41	89	107	63	63	80	132	80	134	132	107	80	63	84
D	11	31	28	15	70	36	65	41	68	39	113	79	79	108	82	108	84	82	113	108	79	112
E	19	5	24	52	15	22	61	93	65	91	43	65	65	104	134	104	136	134	43	104	65	108
F	19	17	14	52	56	9	51	93	54	91	99	43	43	93	134	93	136	134	99	94	43	97
G	24	32	7	57	71	27	11	98	48	96	114	70	70	43	139	43	141	139	114	43	70	91
H	14	34	31	3	73	39	68	22	71	42	116	82	82	111	85	111	43	85	116	111	82	115
I	20	28	3	53	67	23	40	94	22	92	110	66	66	83	135	83	137	135	110	83	66	43
J	16	36	33	5	75	41	70	46	74	15	118	84	84	113	43	113	89	43	118	113	84	117
K	20	6	25	53	1	23	62	94	66	92	44	66	66	105	135	105	137	135	44	105	66	109
L	20	18	15	53	57	1	52	94	55	92	100	44	44	94	135	94	137	135	100	95	44	98
M	20	18	15	53	57	1	52	94	55	92	100	44	44	94	135	94	137	135	100	95	44	98
N	25	33	8	58	72	28	1	99	49	97	115	71	71	44	140	44	142	140	115	44	71	92
O	17	37	34	6	76	42	71	47	75	1	119	85	85	114	44	114	90	44	119	114	85	118
P	25	33	8	58	72	28	1	99	49	97	115	71	71	44	140	44	142	140	115	44	71	92
Q	15	35	32	4	74	40	69	1	72	43	117	83	83	112	86	112	44	86	117	112	83	116
R	17	37	34	6	76	42	71	47	75	1	119	85	85	114	44	114	90	44	119	114	85	118
S	20	6	25	53	1	23	62	94	66	92	44	66	66	105	135	105	137	135	44	105	66	109
T	25	33	8	58	72	28	1	99	49	97	115	71	71	44	140	44	142	140	115	44	71	92
U	20	18	15	53	57	1	52	94	55	92	100	44	44	94	135	94	137	135	100	95	44	98
V	21	29	4	54	68	24	41	95	1	93	111	67	67	84	136	84	138	136	111	84	67	44

Values in the MFPT matrix in Table 3.5 can be interpreted as the mean number of transitions that it takes for a message sent from a node in the first column to reach another node listed in the first row. For instance, $m_{VA} = 21$ indicates that it would take 21 transitions before the buyer (A) learns about a disruption at his tier-3 supplier V. Note that this does not indicate that the disruption would reach the buyer in 21 transitions. It may take longer or never reach the buyer for several reasons including possible inventories at tier-2 or tier-1 suppliers or corrective activities taken by these suppliers.

Even though the random walk model provides valuable insight, the assumptions it requires do not necessarily fit with real life supply chains. The strongest assumption may be the the random propagation of information from the source of the disruption. The most critical weakness of the random walk model is that there is equal probability at a node to transfer the signal upstream (towards suppliers) and downstream (towards the buyer) in the supply network[2]. We arbitrarily assign a probability of 0.8 for downstream information transfer and distributed the remaining 0.2 among the number of upstream links. A drastically different MFPT matrix is computed and given in Table 3.6.

[2]It is important to note at this point that the term moving downstream in our supply network is moving from suppliers to the buyer, hence it actually corresponds to moving from bottom to top in Figure 3.8. The opposite is valid for moving upstream.

Table 3.6. Mean first passage time (MFPT) matrix of the supply network in Figure 3.8 with modified transition probabilities

	A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**	**L**	**M**	**N**	**O**	**P**	**Q**	**R**	**S**	**T**	**U**	**V**
A	3	6	6	6	69	39	98	61	92	67	585	687	687	1683	564	1683	211	564	585	1683	687	364
B	2	6	7	8	63	36	99	62	94	68	579	684	684	1684	565	1684	212	565	579	1684	684	365
C	2	7	6	8	70	37	92	62	87	68	586	685	685	1677	565	1677	212	565	586	1677	685	358
D	2	8	7	6	71	40	99	55	94	61	587	688	688	1685	558	1685	208	558	587	1685	688	366
E	3	2	9	9	52	37	100	63	95	70	516	685	685	1686	567	1686	213	567	516	1686	685	367
F	3	5	5	9	68	32	97	60	92	70	584	648	648	1682	566	1682	201	566	584	1682	648	363
G	3	9	2	9	72	39	79	64	83	70	588	687	687	1585	567	1585	214	567	588	1585	687	338
H	3	9	9	2	72	42	101	45	96	62	588	690	690	1686	559	1686	167	559	588	1686	690	367
I	3	9	2	9	72	39	93	64	71	70	588	687	687	1679	567	1679	214	567	588	1679	687	287
J	3	9	9	2	72	42	101	57	96	50	588	690	690	1686	497	1686	209	497	588	1686	690	367
K	4	3	10	10	1	38	101	64	96	71	517	686	686	1687	568	1687	214	568	517	1687	686	368
L	4	6	6	10	69	1	98	61	93	71	585	649	649	1683	567	1683	202	567	585	1683	649	364
M	4	6	6	10	69	1	98	61	93	71	585	649	649	1683	567	1683	202	567	585	1683	649	364
N	4	10	3	10	73	40	1	65	84	71	589	688	688	1586	568	1586	215	568	589	1586	688	339
O	4	10	10	3	73	43	102	58	97	1	589	691	691	1687	498	1687	210	498	589	1687	691	368
P	4	10	3	10	73	40	1	65	84	71	589	688	688	1586	568	1586	215	568	589	1586	688	339
Q	4	10	10	3	73	43	102	1	97	63	589	691	691	1687	560	1687	168	560	589	1687	691	368
R	4	10	10	3	73	43	102	58	97	1	589	691	691	1687	498	1687	210	498	589	1687	691	368
S	4	3	10	10	1	38	101	64	96	71	517	686	686	1687	568	1687	214	568	517	1687	686	368
T	4	10	3	10	73	40	1	65	84	71	589	688	688	1586	568	1586	215	568	589	1586	688	339
U	4	6	6	10	69	1	98	61	93	71	585	649	649	1683	567	1683	202	567	585	1683	649	364
V	4	10	3	10	73	40	94	65	1	71	589	688	688	1680	568	1680	215	568	589	1680	688	288

Observing the first column of Table 3.6, the mean number of transitions it takes for a disruption news to arrive at the buyer has decreased significantly compared to the data in Table 3.5. This is due to the way the probabilities were modified to push for downward information propagation. As a secondary consequence, information sharing between suppliers at the same tier is much less significant and takes many more transactions compared to the random walk MFPTs of Table 3.5. Finally, the variance of the first passage times can be computed using Equation (3.22). Standard deviation of the first passage times can be obtained by computing the square root of the values in the variance matrix. Standard deviations of the first passage times under the modified transition probabilities are given in Table 3.7.

Table 3.7. Standard deviations of the first passage times of the supply network in Figure 3.8 with modified transition probabilities

	A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**	**L**	**M**	**N**	**O**	**P**	**Q**	**R**	**S**	**T**	**U**	**V**
A	3	9	8	9	97	54	138	85	130	94	826	970	970	2379	796	2379	298	796	826	2379	970	513
B	2	9	9	10	93	52	138	86	131	95	822	968	968	2380	797	2380	298	797	822	2380	968	514
C	2	10	9	10	98	53	133	86	126	95	827	969	969	2375	797	2375	298	797	827	2375	969	509
D	2	10	10	9	98	55	139	81	131	90	827	971	971	2380	792	2380	295	792	827	2380	971	514
E	4	2	11	11	84	53	140	87	132	96	775	969	969	2381	798	2381	299	798	775	2381	969	515
F	4	8	8	11	97	49	137	85	130	96	825	942	942	2378	798	2378	290	798	825	2378	942	513
G	4	11	2	11	99	54	124	87	123	96	828	970	970	2309	798	2309	299	798	828	2309	970	495
H	4	11	11	2	100	57	140	73	132	91	828	972	972	2381	793	2381	265	793	828	2381	972	515
I	4	11	2	11	99	54	134	87	113	96	828	970	970	2376	798	2376	299	798	828	2376	970	456
J	4	11	11	2	100	57	140	82	132	81	828	972	972	2381	747	2381	296	747	828	2381	972	515
K	5	3	11	12	1	54	140	88	133	97	776	969	969	2381	799	2381	300	799	776	2381	969	516
L	5	9	9	12	98	1	138	85	130	97	826	943	943	2379	798	2379	291	798	826	2379	943	514
M	5	9	9	12	98	1	138	85	130	97	826	943	943	2379	798	2379	291	798	826	2379	943	514
N	5	12	3	12	100	55	1	88	124	97	828	970	970	2309	799	2309	300	799	828	2309	970	495
O	5	12	12	3	100	57	141	83	133	1	829	973	973	2382	748	2382	297	748	829	2382	973	516
P	5	12	3	12	100	55	1	88	124	97	828	970	970	2309	799	2309	300	799	828	2309	970	495
Q	5	12	12	3	100	57	141	1	133	91	829	973	973	2382	793	2382	265	793	829	2382	973	516
R	5	12	12	3	100	57	141	83	133	1	829	973	973	2382	748	2382	297	748	829	2382	973	516
S	5	3	11	12	1	54	140	88	133	97	776	969	969	2381	799	2381	300	799	776	2381	969	516
T	5	12	3	12	100	55	1	88	124	97	828	970	970	2309	799	2309	300	799	828	2309	970	495
U	5	9	9	12	98	1	138	85	130	97	826	943	943	2379	798	2379	291	798	826	2379	943	514
V	5	12	3	12	100	55	135	88	1	97	828	970	970	2376	799	2376	300	799	828	2376	970	457

It is important to note that all values in Tables 3.5, 3.6, 3.7 and 3.7 are in the number of transitions; therefore, the actual time it takes a message to reach another node depends on the duration of a transition in the network. This issue may be significant when including detection in disruption risk quantification and will be discussed in the next section.

3.4.4 Using MFPT in Disruption Risk Quantification

There are several ways to include detectability of disruption risks in supply chain management models. One way is to directly use the values in the MFPT matrix, the m_{ij} values, and create an objective function to minimize the number of transitions between suppliers and the buyer. Otherwise, as discussed at the end of Section 3.4.3, the MFPT values may not be suitable to use directly in risk quantification since values in the MFPT matrix are in the number transitions and need to be transformed to actual time units (e.g. hours, days or weeks) for proper use in disruption quantification. This transformation is instance specific since the speed with which the information spreads through the nodes depends on the information technology systems implemented at each node and the availability and strength of connection among the nodes. For instance, if a buyer has implemented an ERP system to communicate with all tiers of his supply chain, he would have much better connectivity to any supplier and the transition times would be much shorter than buyers that do not have a similar visibility. For general modeling purposes, however, it is reasonable to assume that suppliers who are closer to the buyer have better ERP systems than suppliers far upstream in the supply network, that is, supplier B in tier 1 may process information much faster than supplier U in tier 3; and hence, B provides a faster transition time. Therefore, we assume that transition time is a function of the supplier's tier in the supply network and use the notation $t_{tran}(i)$ to denote transition time at supplier i. If we consider a signal to be transmitted between nodes i and j, then the average transition time would be $t_{tran}(i)m_{ij}$ where m_{ij} is the mean number of transitions to reach node j from node i given in Table 3.6 (MFPT matrix). Since we differentiate transition speed with respect to tiers, we need to consider MFPT between tiers rather than between the source and the buyer.

Consider the following example. Assume that we want to model the spread of a disruption news from supplier T of Figure 3.8. Let the transition time at tier 3 suppliers be three days, at tier 2 suppliers two days and at tier 1 suppliers one day. Once T suffers a disruption, the news will reach the tier 2 supplier G in $t_{tran}(T) \times m_{TG} = 3 \times 1 = 3$ days, from G to the tier 1 supplier C in $t_{tran}(G) \times m_{GC} = 2 \times 2 = 4$ days and from C to the buyer A in $t_{tran}(C) \times m_{CA} = 2 \times 1 = 2$ days. In total, the buyer will receive the news in 3+4+2=9 days. We call this result as the *disruption delay between nodes i and j* and denote it as Δ_{ij}.

Detectability of disruption risks can be included in the general risk model we developed in Section 3.2.3 in several different ways. We can assume that the magnitude of disruption will increase with the time it takes for the news to reach the buyer, but only up to a certain level. That is, the damage from delay of disruption news should have a maximum level, which will indeed be the point where the buyer bankrupts and cannot be further hurt by the disruption. In mathematical terms, we can use an increasing function of the disruption delay with a flat horizontal tail. One can use the natural logarithm function, $f_1(\Delta) = \ln(\Delta)$ even though it tends towards infinity as $\Delta \to \infty$. We believe that the function $f_1(\Delta)$ is appropriate for a resilient supply chain which can reconfigure itself under heavy disruptions without breaking down. Another option is using a function with an horizontal

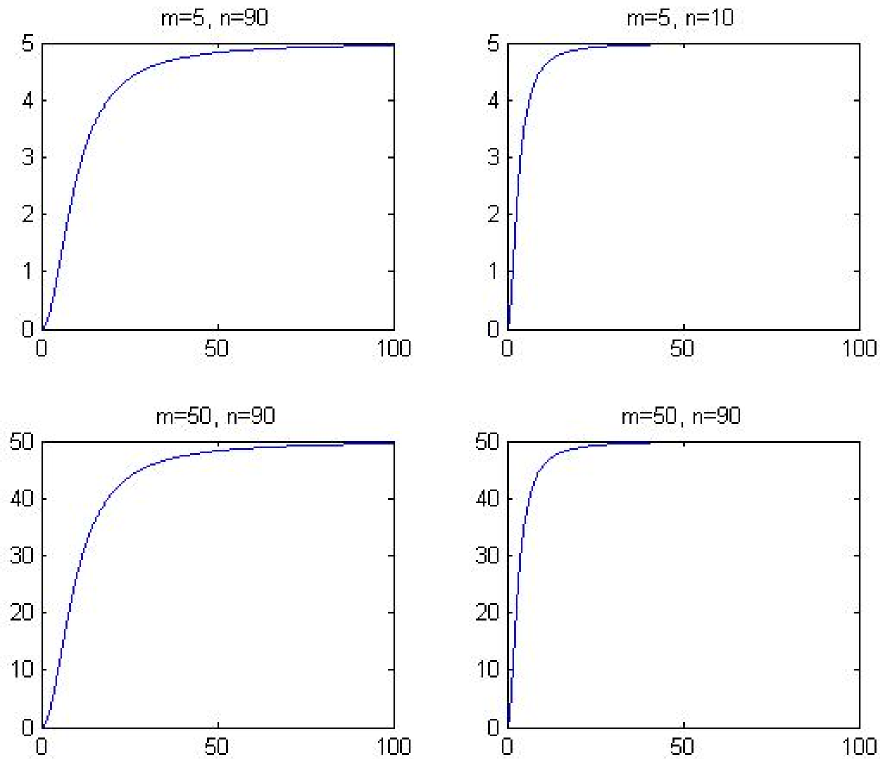

Figure 3.9. Plots of $f_2(\Delta)$ for different n and m values

asymptote such as $f_2(\Delta) = \frac{m\Delta^2}{\Delta^2+n}$. This function has an horizontal asymptote at m which is reached at n. Several plots of $f_2(\Delta)$ with different m and n parameter values are given in Figure 3.9.

We believe that the function $f_2(\Delta)$ is more suitable for robust supply chains which can resist disruptions up to a certain degree, but then break down. As in Figure 3.9, the parameter n actually acts as a measure of supply chain robustness.

Detectability of disruption risks can be included in the general risk function proposed in Equation (3.1) as a multiplier. It can be associated with a coefficient, say $\alpha \in [0, 1]$ to control the weight of detectability in the overall risk value.

3.5 A Conceptual Model of Risk Recovery

It is a vital phase in a company's life to get back to its previous state or to adjust itself to a new environment following a disruption. Additional to the aspects of risk discussed throughout this chapter, risk recovery plays a key role in a company's survival. It is important to note that recovery can start only after the occurrence of the risk event has been detected. We call the total time it takes from the occurrence of disruption to its neutralization as the *risk time*, which is the sum of detection time and recovery time.

This section proposes a conceptual model of risk recovery based on concepts borrowed from queuing theory and reliability engineering. In queuing theory, it is customary to model a system using a Poisson process and assume exponential interarrival times. Once an arriving customer is served, the system transfers to another state and the whole process can be modeled as a Markov chain. These concepts are used also in reliability engineering, especially for modeling the reliability of component systems. For instance, Mohammed et al. (2002) modeled multi-component serial and parallel systems and used exponentially distributed random variables to include system recovery time from a failure. We adopt a similar model in this section with an emphasis on the parameter of the exponential distribution. Here, the exponential distribution is used to model the recovery time from disruptive events; that is, the time between the disruptive event is detected and is completely neutralized.

Exponential distribution is used to model the time between occurrences of two events and has a single parameter, μ. μ can be interpreted as the rate of occurrence of events. In our models, μ is the recovery rate of the failed node and $\frac{1}{\mu}$ is the recovery time at the failed node.

The exponential cdf reaches to 1 faster as the μ parameter gets larger, that is, a higher μ leads to a faster recovery time. Therefore, there is a direct relationship between μ and recovery time. Note that recovery time, hence μ, is a function of at least the following,

- Inventory level (I),
- Availability of mitigation plans (δ),
- Impact of the risk event (S).

The higher the inventory levels along the supply chain, the better chances a company has in meeting the customer demand through available inventory and alleviate the effects of disruption. Therefore, μ increases with the inventory level. It is assumed that mitigation plans will greatly alleviate effects of disruption risks. A mitigation plan can be a backup supplier plan such as the outcome of the optimization models introduced in Chapter 4. The buyer can immediately contact a backup supplier upon disruption if a backup supplier is available to replace the failed supplier. The backup supplier may still need some time to react to the buyer's request, but we argue that this will be much quicker compared to waiting for the failed supplier to recover. We therefore define δ as a continuous variable in]0,1] to capture different degrees of mitigation. A δ parameter close to 1 should be used for buyers that have good mitigation plans. Cdf of a probability distribution or a fuzzy number can be used to model δ as a function of mitigation plans. Impact of the risk event S, on the other hand, is assumed to increase the recovery time, that is, the higher the impact, the longer it takes to recover from the risk event. Within this scheme, the recovery rate μ of the disruption recovery time model can be modeled as in Equation (3.23).

$$\mu(I, \delta, S) \propto (\delta)\frac{I}{S} \tag{3.23}$$

Let us give a numerical example to conclude this section. Consider the supply chain given in Figure 3.8 and consider a disruption happening at supplier T. It has been shown in Section 3.4.4 that it takes nine days for the buyer to detect the disruption. Let us assume that the buyer has a total inventory worth of \$1,000/day and the impact of risk does not exceed \$5,000 with 0.99 probability. Total inventory information can be collected through ERP systems, whereas risk percentiles can be estimated as in Section 3.2.3 for a single risk event or as in Section 3.3 for multiple risk events. Assuming $\delta = 1$, the μ parameter can be calculated as 0.2 using Equation (3.23). With $\mu = 0.2$, it can be easily estimated that the mean time it will take five days ($\frac{1}{\mu}$) the buyer to recover from disruption. The total risk time is 9+5=14 days.

Disruption recovery can also be included in the supply chain optimization models. One can first create risk scenarios based on previously computed risk values (using the techniques described in Sections 3.2.3 and 3.3). Then, assuming certain inventory levels, the μ parameter for each scenario can be calculated and the sum of μ parameters of each supplier can be maximized.

3.6 An Application to Supply Chain Disruption Modeling

This section combines all models proposed in this chapter and provides an integrated disruption risk quantification example that can be used for risk management and mitigation. We start with a real storm damage data set retrieved from Pielke et al. (2008) that shows normalized storm damage data in the U.S. from 1900 to 2005. We consider the storms that hit Florida and fit a GEVD distribution to the data set using the *Probability Weighted Moments (PWM)* method detailed in Hosking et al. (1985). The method works on an ordered data set $x_1 < x_2 < ... < x_n$. The PWM method requires moment estimates that can be computed using plotting positions. A plotting position is a distribution free estimate of the empirical cdf of a data set. For a data set of size n, plotting position p_i of a data point i is $p_i = \frac{i-a}{n}$ with $-0.5 < a < 0.5$. Then, the estimate of the r-th moment is $\hat{\beta}_r(p_i) = n^{-1}\sum_{i=1}^{n} p_i^r x_i$. An intermediary parameter $c = \frac{2b_1-b_0}{3b_2-b_0} - \frac{log2}{log3}$ has to be computed to estimate GEVD parameters. Then, the following formulas can be used to estimate κ, δ and λ parameters of a GEVD,

$$\hat{\kappa} = 7.859c + 2.9554c^2$$

$$\hat{\delta} = \frac{(2b_1 - b_0)\hat{\kappa}}{\Gamma(1+\hat{\kappa})(1-2^{-\hat{\kappa}})}$$

$$\hat{\lambda} = b_0 + \frac{\hat{\delta}\,[\Gamma(1+\hat{\kappa}) - 1]}{\hat{\kappa}}$$

where $\Gamma(.)$ is the Gamma function. We arbitrarily set $a = 0.25$ to compute plotting points and b_0, b_1 and b_2 values in Table 3.8. Using parameter estimation formulas above, a GEVD with $\kappa = -0.68$, $\delta = 2,205,657,132.59$ and $\lambda = 3,527,790,956.87$ can be fitted to the data set in Table 3.8.

When data are available, the above method can be used to estimate parameters of a GEVD distribution. We are not going to use the fitted distribution in our example since the data are collected for the total damage faced at the entire state of Florida and do not represent a single supplier. Rather, we will use artificial data

Table 3.8. Plotting position and b_r computations for the normalized Florida storm damage data set

n	*Damage*	*Plotting Position (PP)*	*PP× Damage*	PP^2	$PP^2\times$ *Damage*
1	924,871	0.009	8,780	0.000	83
2	2,649,286	0.022	58,687	0.000	1,300
3	3,245,588	0.035	112,979	0.001	3,933
4	3,782,009	0.047	179,526	0.002	8,522
5	8,102,891	0.060	487,199	0.004	29,294
6	13,574,119	0.073	987,990	0.005	71,911
7	14,316,188	0.085	1,223,219	0.007	104,516
8	23,238,836	0.098	2,279,759	0.010	223,647
9	24,022,769	0.111	2,660,750	0.012	294,703
10	30,828,136	0.123	3,804,738	0.015	469,572
11	34,977,305	0.136	4,759,570	0.019	647,663
12	40,210,382	0.149	5,980,658	0.022	889,528
13	43,535,305	0.161	7,026,268	0.026	1,133,986
14	49,977,884	0.174	8,698,682	0.030	1,514,011
15	69,346,467	0.187	12,947,600	0.035	2,417,432
16	96,054,210	0.199	19,150,048	0.040	3,817,889
17	120,235,627	0.212	25,492,997	0.045	5,405,161
18	125,914,494	0.225	28,290,915	0.050	6,356,503
19	126,232,889	0.237	29,960,337	0.056	7,110,840
20	130,663,800	0.250	32,665,950	0.063	8,166,487
21	132,753,509	0.263	34,868,801	0.069	9,158,578
22	153,918,449	0.275	42,376,282	0.076	11,666,888
23	160,414,410	0.288	46,195,289	0.083	13,303,074
24	235,746,491	0.301	70,873,154	0.090	21,306,803
25	251,656,977	0.313	78,841,901	0.098	24,700,469
26	255,541,019	0.326	83,293,433	0.106	27,149,442
27	306,339,577	0.339	103,728,907	0.115	35,123,396
28	328,708,795	0.351	115,464,165	0.123	40,558,615
29	330,417,454	0.364	120,246,858	0.132	43,760,724
30	361,980,468	0.377	136,315,429	0.142	51,333,975
31	371,014,964	0.389	144,414,052	0.152	56,211,799
32	495,114,445	0.402	198,985,869	0.162	79,972,169
33	520,502,973	0.415	215,778,131	0.172	89,452,327
34	565,294,842	0.427	241,502,543	0.183	103,173,555
35	624,127,559	0.440	274,537,123	0.193	120,761,582
36	703,954,462	0.453	318,561,671	0.205	144,159,237
37	783,043,644	0.465	364,263,974	0.216	169,451,912
38	972,672,385	0.478	464,789,652	0.228	222,098,853
39	991,035,044	0.491	486,108,961	0.241	238,439,522
40	1,017,035,501	0.503	511,736,217	0.253	257,487,527
41	1,088,138,521	0.516	561,286,642	0.266	289,524,439
42	1,178,217,658	0.528	622,665,661	0.279	329,066,979
43	1,180,430,774	0.541	638,777,413	0.293	345,667,524
44	1,361,044,627	0.554	753,743,069	0.307	417,421,003
45	1,375,861,447	0.566	779,364,554	0.321	441,475,491
46	1,961,924,843	0.579	1,136,177,995	0.335	657,976,497
47	2,193,891,626	0.592	1,298,283,968	0.350	768,288,298
48	2,230,000,000	0.604	1,347,879,747	0.365	814,699,467
49	2,834,851,643	0.617	1,749,354,653	0.381	1,079,506,827
50	3,079,860,592	0.630	1,939,532,462	0.397	1,221,414,430
51	3,105,119,662	0.642	1,994,744,593	0.413	1,281,434,027
52	3,124,397,145	0.655	2,046,677,877	0.429	1,340,703,546
53	3,571,129,627	0.668	2,384,520,099	0.446	1,592,195,382
54	3,599,433,103	0.680	2,448,981,384	0.463	1,666,237,334
55	3,619,996,492	0.693	2,508,795,037	0.480	1,738,690,232
56	3,725,252,507	0.706	2,628,896,548	0.498	1,855,202,311
57	3,769,907,746	0.718	2,708,129,931	0.516	1,945,397,134
58	4,205,750,228	0.731	3,074,456,654	0.534	2,247,466,731
59	4,653,166,631	0.744	3,460,424,551	0.553	2,573,416,992
60	5,554,281,127	0.756	4,200,864,524	0.572	3,177,236,143
61	6,168,924,124	0.769	4,743,824,564	0.591	3,647,941,041
62	6,339,955,773	0.782	4,955,598,342	0.611	3,873,521,489
63	6,577,589,728	0.794	5,224,604,499	0.631	4,149,923,194
64	7,496,264,391	0.807	6,049,200,696	0.651	4,881,475,245
65	9,648,997,103	0.820	7,908,513,449	0.672	6,481,977,795
66	10,147,966,555	0.832	8,445,934,189	0.693	7,029,369,278
67	13,468,454,482	0.845	11,379,991,604	0.714	9,615,372,652
68	13,770,172,464	0.858	11,809,230,183	0.735	10,127,536,011
69	14,756,390,795	0.870	12,841,795,787	0.757	11,175,613,422
70	15,514,011,620	0.883	13,697,497,601	0.780	12,093,676,680
71	16,297,047,080	0.896	14,595,140,265	0.802	13,070,964,225
72	18,438,963,151	0.908	16,746,779,823	0.825	15,209,891,802
73	20,600,000,000	0.921	18,970,253,165	0.848	17,469,441,996
74	23,044,661,003	0.934	21,513,212,013	0.872	20,083,536,531
75	31,842,991,619	0.946	30,129,919,285	0.895	28,509,005,906
76	31,898,632,986	0.959	30,586,347,452	0.919	29,328,048,348
77	35,587,059,722	0.972	34,573,504,224	0.944	33,588,815,812
78	54,337,237,494	0.984	53,477,471,078	0.969	52,631,308,561
79	139,497,413,165	0.997	139,055,965,655	0.994	138,615,915,130
b_0	**6,878,082,219**	b_1	**6,192,759,497**	b_2	**5,685,623,966**

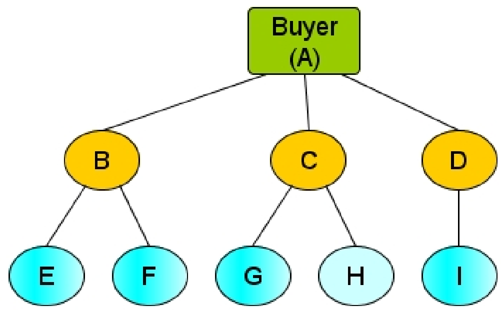

Figure 3.10. A three-tier supply chain

we created for numerical examples in Section 3.3.2.

Now, consider a small supply chain of three tiers, a buyer, three tier 1 suppliers and five tier 2 suppliers as shown in Figure 3.10. Assume that the buyer wants to have a single supplier and further assume that all tier 1 and their tier 2 suppliers have enough capacity to meet the buyer's demand. Further, assume that the buyer does not want to suffer more than \$3,000 in losses due to disruptions at his suppliers. Considering the examples in Section 3.3.2, (all supplier face two disruptive events over a given period) supplier B will not cause losses greater than \$3,000 with 0.9453 probability. Similarly, losses from supplier C's disruption will not exceed \$3,000 with 0.9947 probability and losses from supplier D will be below \$3,000 with 0.8542 probability. This result can be used as a criterion in a supplier selection model to pick suppliers that would provide the best possible risk hedging.

To include detectability, let us assume that with 0.8 probability, disruption information will be shared with nodes downstream (towards the buyer) and with 0.2 probability information will be shared with nodes upstream in the supply chain of Figure 3.10. Then, the transition probability matrix of the supply chain in Figure 3.10 is given in Table 3.9 and the relevant MFPT matrix is given in Table 3.10.

Let us assume that the transition time (e.g. in days) between a tier 1 supplier

Table 3.9. Transition probability matrix of the supply chain network in Figure 3.10

	A	B	C	D	E	F	G	H	I
A	0	0.33	0.33	0.33	0	0	0	0	0
B	0.8	0	0	0	0.1	0.1	0	0	0
C	0.8	0	0	0	0	0	0.1	0.1	0
D	0.8	0	0	0	0	0	0	0	0.2
E	0	1	0	0	0	0	0	0	0
F	0	1	0	0	0	0	0	0	0
G	0	0	1	0	0	0	0	0	0
H	0	0	1	0	0	0	0	0	0
I	0	0	0	1	0	0	0	0	0

Table 3.10. MFPT matrix of the supply chain network in Figure 3.10

	A	B	C	D	E	F	G	H	I
A	2.5	6	6	6	65	65	65	65	35
B	1.5	6	7.5	7.5	59	59	66.5	66.5	36.5
C	1.5	7.5	6	7.5	66.5	66.5	59	59	36.5
D	1.5	7.5	7.5	6	66.5	66.5	66.5	66.5	29
E	2.5	1	8.5	8.5	60	60	67.5	67.5	37.5
F	2.5	1	8.5	8.5	60	60	67.5	67.5	37.5
G	2.5	8.5	1	8.5	67.5	67.5	60	60	37.5
H	2.5	8.5	1	8.5	67.5	67.5	60	60	37.5
I	2.5	8.5	8.5	1	67.5	67.5	67.5	67.5	30

and the buyer follows a Uniform(1,2) distribution and transition time between a tier 2 and a tier 1 supplier follows a Uniform(2,4) distribution. Sampling from these uniform distributions, let $t_{tran}(B) = 2$, $t_{tran}(C) = 2$, $t_{tran}(D) = 1$, $t_{tran}(E) = 4$, $t_{tran}(F) = 2$, $t_{tran}(G) = 2$, $t_{tran}(H) = 4$ and $t_{tran}(I) = 3$. We finally have the Δ_{iA} disruption delay values between suppliers and the buyer as given in Table 3.11.

Values in the Δ_{iA} column of Table 3.11 can be used in supply chain decision making. For example, a buyer may consider minimizing the transition times of his supply chain as a supplier selection criterion to address disruption risks. Taking our analysis one step further, we can use one of the functions $f_1(\Delta)$ or $f_2(\Delta)$ introduced in Section 3.4.4 to characterize losses to the company due to disruption delays. The

Table 3.11. Disruption delay times of the supply chain network in Figure 3.10

Source (i)	Path	Δ_{iA}	$f_1(\Delta_{iA})$
B	B $\rightarrow$ A	3	1.098
C	C $\rightarrow$ A	3	1.098
D	D $\rightarrow$ A	3	1.098
E	E $\rightarrow$ B $\rightarrow$ A	4+3=7	1.946
F	F $\rightarrow$ B $\rightarrow$ A	2+3=5	1.609
G	G $\rightarrow$ C $\rightarrow$ A	2+3=5	1.609
H	H $\rightarrow$ C $\rightarrow$ A	4+3=7	1.956
I	I $\rightarrow$ D $\rightarrow$ A	3+3=6	1.609

last column in Table 3.11 shows the magnitude of losses that the buyer would suffer from supplier disruptions under the $f_1(\Delta)$ function. As previously noted, buyers can use the $f_1(\Delta)$ values (or the $f_2(\Delta)$ values given that they come up with the required parameters) to select among different suppliers.

Another important aspect in dealing with disruptions is the risk recovery time discussed in Section 3.5. We use an exponential model to compute recovery time where the parameter μ of the exponential distribution can be computed as in Equation (3.23). Let us assume that tier 1 suppliers have good backup plans and set $\delta = 1$. Assume that supplier B has \$3,000 worth of inventory, supplier C has \$5,000 worth of inventory and supplier D has \$1,000 worth of inventory. Then, $\mu_B = 3,000/3,000 = 1$, $\mu_C = 5,000/3,000 = 1.67$ and $\mu_D = 1,000/3,000 = 0.33$. Therefore, it would take supplier B $\frac{1}{\mu_B} = 1$ day, supplier C $\frac{1}{1.67} = 0.6$ days and supplier C $\frac{1}{0.33} = 3$ days to recover from disruptions. Recovery time can be used as a stand alone criterion to evaluate suppliers. Also, the total risk time, which is the summation of the detection and recovery times, can be calculated and used as another criterion. In our example, risk time for supplier B is 3+1=4 days. For supplier C risk time is 3+0.6=3.6 days and supplier D's risk time is 3+3=6 days.

An interesting situation would arise when some suppliers work in a just-in-time framework and practically carry no inventory. In such a case, Equation (3.23) would yield $\mu = 0$ for which $\frac{1}{\mu}$ is not finite. It may be assumed that for such a supplier, recovery time would equal the maximum lead time between that supplier and his own suppliers given that the disrupted supplier would be able to manufacture, and hence recover, once all required items reach his facility.

3.7 Conclusions

This chapter presents theoretical results on risk quantification and several conceptual models for disruption detection and disruption recovery in supply chains. Theoretical results proved in the first part of the chapter provide the mean and variance of compound risk functions and the entire distribution of convolution of several different risk functions. Results presented in Sections 3.2.3 and 3.3, to our knowledge, are the first analytical models for computing damage from disruption risks. The theoretical part is followed by two sections presenting conceptual work on disruption detection (Section 3.4) and disruptive event recovery (Section 3.5) in supply chains. Although there are several papers on information, disease or disruption propagation in supply chains, to our knowledge, the work in Section 3.4 is the first of its kind. The concept of failure recovery, on the other hand, is well known in reliability engineering; however, to our knowledge, this research is the first work to connect the recovery concept with supply chain disruptions. Section 3.6 provides a comprehensive example where all risk concepts developed in this chapter are applied to a supply chain network and interpretation and future use of the derived results are discussed.

Results presented throughout the chapter can be used in supply chain performance improvement studies, i.e., in risk assessment, vulnerability analysis and optimization models. Chapter 4 presents multicriteria optimization models where results of this chapter are used to optimally select suppliers, allocate orders and generate risk mitigation strategies.

Chapter 4

Multiple Objective Supplier Selection Models

4.1 Introduction

This chapter presents multiobjective mathematical models for assigning one (or a group) of primary supplers and several backup suppliers to a buyer and for determining supplier order quantities in supply chain problems. Primary suppliers are those suppliers that ship orders to buyers, whereas backup suppliers are used only when a primary supplier faces disruption. Models developed in this chapter are motivated by discussions in the first three chapters on supply chain disruption, effects and destructive outcomes of those disruptive events and necessity for contingency plans and mitigation strategies. Mitigation is included in the models by sequentially assigning backup suppliers to primary suppliers. We therefore name our problem as the *Sequential Supplier Allocation (SSA) Problem* and the related mathematical models as SSA models.

As discussed in Chapters 1 and 2, supplier selection practitioners often consider more than a single measure to compare and evaluate potential suppliers. Therefore, models developed in this chapter are multicriteria mathematical programming models where several objectives are jointly considered for optimal supplier and order allocation. To our knowledge, Buffa & Jackson (1983) is the first paper to apply multiobjective optimization models to a supplier selection problem and the literature on the topic has flourished since then. Previous work on multiobjective supplier selection models is surveyed in Section 2.1.2 of Chapter 2.

Several objectives have already been studied in the supplier selection literature; either as part of multiobjective problems or individually in single objective applications (see Sections 2.1.1 and 2.1.2 of Chapter 2 for examples). Among these, minimizing the total operations costs, maximizing the quality of procured products, minimizing the lead time of procurement are selected as objectives to be included in the model formulations since these are the most common criteria used in practice and literature. Minimizing loss due to disruption risks is included as the fourth objective following our developments in Chapter 3. Note that the order in which these objectives are to be optimized in the multiobjective program solution process depends on the decision makers' (DM) preferences. Additionally, alternative criteria might be appended to the multiobjective formulation depending on the domain of operation of the supply chain. For instance, maximizing the service rate or fill rate can be considered as alternative objectives.

Another important aspect of the models proposed in this chapter is the distinction between single versus multiple sourcing. As outlined in Section 4.2, single sourcing refers to the practice of gathering all units of a particular item or an entire service plan from a single supplier. In the case of multiple sourcing, on the other hand, buyers are allowed to use more than one supplier to allocate an order. Burke et al. (2007) provides a discussion of pros and cons of these two strategies through a single objective mathematical model. It is important to note that there exists another mode of replenishment called *sole sourcing*. Sole sourcing is similar to single sourcing; however, buyers are using a single source not because of their business strategy, but since there exists only one supplier that can provide the required service or item. For instance, several chemical composites required to fabricate diapers and tissue paper are manufactured and distributed by only one firm in the world. Hygiene product manufacturers do not have any other option than purchasing those chemicals from the sole supplier.

Based on the classification above, the SSA problem is analyzed under two distinct operating scenarios: *single sourcing* and *multiple sourcing*. In the case of single sourcing, a buyer constrains his business relationships with one supplier for each product. Different suppliers may provide different products, but a single supplier is responsible of fulfilling the entire demand for a product. Even though single sourcing can be considered as unsafe from a risk management perspective, firms may prefer single sourcing due to patent issues, expertise of suppliers or competitive secrecy. Moreover, dealing with multiple suppliers may increase cost and bring additional complexity to supply chain operations. Single sourcing is frequently preferred in high tech industries. Multiple sourcing mode is a generalization of single sourcing where a buyer can split orders among several suppliers.

Models proposed in this chapter address disruption risks and provide risk mitigation plans together with traditional supplier-order assignments. As previously discussed, disruption risk management is currently a growing field of research in supply chain management. Literature in this topic is surveyed in Section 2.5.2 of Chapter 2. Currently available models in supplier disruption management do not provide analytical schemes for quantifying effects of disruptions or estimating probability of occurrence of disruptive events. Methods addressing these points are developed in Chapter 3 of this dissertation and are used in the multiobjective SSA models of this chapter.

Chapter 4 is organized as follows. Section 4.2 introduces two basic SSA optimization models and presents multiobjective formulations for each model. Section 4.3 discusses several methods to solve multiobjective optimization problems with an emphasis on *goal programming*. Section 4.4 presents numerical applications to the basic SSA models presented in Section 4.2. Extensions to the basic SSA models with additional numerical examples are discussed in Section 4.5. Section 4.6 summarizes our findings and concludes this chapter.

4.2 Sequential Supplier Assignment Models with Backup Suppliers

Performance of suppliers can be measured in a number of different ways. For instance on-time delivery, quality of the delivered products and cost of the supplied material are among the frequently used performance criteria. Recently, risks related to uncertainty in demand and production and risks related to unforeseen disruptions due to extreme events have been introduced to the supplier selection problems. In this chapter, we propose two multiobjective optimization models that fit different sourcing strategies considering disruption risks. A common trait of the proposed models is the sequential assignment of backup suppliers which will replace primary suppliers in case of disruption. Mathematical models we present here can be used to build emergency plans and a more robust supply chain structure at firms that actually face or may face disruptions.

4.2.1 The Sequential Supplier Assignment (SSA) Problem

SSA models we propose yield a sequence of suppliers from primary to backups and their order quantities. We first decide on the objectives to consider in the SSA formulations. Minimizing total operations costs, maximizing quality of procured products, minimizing lead time of procurement and minimizing losses due to disruption risks are selected to be optimized in the basic SSA models of this chapter. The order in which these objectives will be treated in the multiobjective program solution process depends on the DM's preferences.

The SSA problem is analyzed under two distinct operating scenarios: *single sourcing* and *multiple sourcing*. In the case of single sourcing, the buyer constrains his business relationship with one supplier for each product. Different suppliers may provide different products, but a single supplier is responsible of fulfilling the entire demand for a product. Even though single sourcing can be considered as unsafe from a risk management perspective, firms may prefer this option due to patent issues, expertise of suppliers or competitive secrecy. Moreover, dealing with multiple suppliers may increase overhead costs and bring additional complexity to supply chain operations. Single sourcing is frequently preferred in high tech industries. Multiple sourcing mode is a generalization of single sourcing where the buyer can split orders among several suppliers.

4.2.2 Single Sourcing Model (SSM)

Consider a supply chain where a buyer purchases multiple products from different suppliers. Let $J = 1,...,m$ represent the set of suppliers and $K = 1,...,n$ the set of products. Indices j and k represent suppliers and products respectively. Under the SSM assumption, once a supplier is chosen for a particular product, he will provide the entire demand for that product. Hence, the SSM assumes one set of decision variables, x_{jkr}, defined as follows,

$$x_{jkr} = \begin{cases} 1 & \text{if supplier } j \text{ is assigned as a level } r \text{ supplier of product } k, \\ 0 & \text{otherwise.} \end{cases}$$

Table 4.1. SSM Parameters

Parameter	**Description**
D_k	Demand for product k
Cap_{jk}	Capacity at which the supplier j can provide product k
c_{jkr}	Unit cost supplier j charges for product k when j is a level r supplier
F_{jr}	Fixed cost of operating with supplier j at level r
Q_{jkr}	Quality of product k procured from supplier j when j is assigned at level r
L_{jkr}	Lead time of product k from supplier j when j is a level r supplier
ρ_j	Mean loss due to disruption risks

The concept of level r supplier is related to the concept of *level r facility* introduced in Snyder & Daskin (2005). They use level r facilities to handle sequential allocation of facilities to customers in an uncapacitated facility location problem. Similar to their work, in our models a supplier assigned to level one is the main supplier for the given product. Other suppliers are then sequentially assigned to levels from two to m', $m' \leq m$. In the case of a disruption at the current supplier, the backup supplier assigned at one level below replaces the failed supplier. The model is designed to assign suppliers at consecutive levels for providing a plan to mitigate disruptions in the supply chain. The SSM formulation requires the parameters defined in Table 4.1.

We assume that suppliers would perform differently depending on their assignment levels; therefore, quality, lead time and cost parameters are indexed over levels. That is, in the SSA models, a supplier when assigned as a backup would ask for a higher price, provide longer lead times and lower quality. In other words, a supplier when assigned at level r, would impose higher cost, lower quality and longer lead times than what would be if he were assigned at level $r-1$. This assumption is made since a backup supplier would be used only when a primary supplier fails and would be on a short-notice to satisfy an order. The case for fixed costs, on the other hand, is the opposite. Now that the buyer will not order from a backup supplier until he is needed, it is assumed that the fixed costs of backup suppliers would be smaller than those of primary suppliers. The buyer would still need to cover fixed costs for backups as he may still have to sign a contract with backup suppliers to be able to use them in case of disruptions. Fixed cost of backup suppliers would include overhead costs as well. It is assumed that the fixed cost would decrease with the backup levels; that is, a supplier assigned at level r would be associated with a smaller fixed cost than a supplier at level $r-1$. Lastly, note that the risk value depends on the business environment. Therefore, independent of its assignment level, a supplier has the same risk value. The multiobjective mathematical formulation of the SSM is as follows.

$$\min z_1^{SSM} = \sum_{j\in J}\sum_{k\in K}\sum_{r=1}^{m'} c_{jkr} D_k x_{jkr} + \sum_{j\in J}\sum_{k\in K}\sum_{r=1}^{m'} F_{jr} x_{jkr} \tag{4.1}$$

$$\max z_2^{SSM} = \sum_{j\in J}\sum_{k\in K}\sum_{r=1}^{m'} Q_{jkr} x_{jkr} \tag{4.2}$$

$$\min z_3^{SSM} = \sum_{j\in J}\sum_{k\in K}\sum_{r=1}^{m'} L_{jkr} x_{jkr} \tag{4.3}$$

$$\min z_4^{SSM} = \sum_{j\in J}\sum_{k\in K}\sum_{r=1}^{m'} \rho_j x_{jkr} \tag{4.4}$$

s.t.

$$\sum_{j \in J} x_{jkr} = 1 \quad \forall k \in K,\ r = 1, ..., m' \tag{4.5}$$

$$\sum_{r=1}^{m'} x_{jkr} = 1 \quad \forall j \in J,\ \forall k \in K \tag{4.6}$$

$$x_{jkr}(Cap_{jk} - D_k) \geq 0 \quad \forall j \in J,\ \forall k \in K,\ r = 1, ..., m' \tag{4.7}$$

$$x_{jkr} \in \{0,\ 1\} \quad \forall j \in J,\ \forall k \in K,\ r = 1, ..., m' \tag{4.8}$$

The first objective function in (4.1) minimizes total variable and fixed costs. The first summation captures variable cost and the second summation represents fixed cost. Let us analyze the first objective function in parts. It can be separated in two as $\sum_{j \in J} \sum_{k \in K} c_{jk1} D_k x_{jk1} + \sum_{j \in J} \sum_{k \in K} \sum_{r=2}^{m'} c_{jkr} D_k x_{jkr}$ where the first part represents the variable cost associated with primary suppliers. The second part can be considered as the opportunity cost of assigning a supplier as backup at some level $r > 1$. Hence, the objective function in Equation (4.1) minimizes the sum of variable cost of primary suppliers, opportunity cost of backup suppliers and fixed cost of the overall supplier assignment. The second objective function in (4.2) maximizes the quality resulting from supplier assignments. The third objective function in (4.3) minimizes the lead time resulting from the supplier assignments and the last objective function in (4.4) minimizes the total risk value of suppliers. Note that all objective functions can be broken down similarly as the first objective.

Constraints in Equation (4.5) ensure that the buyer is assigned with a level r supplier for each product. These constraints guarantee that there will be one supplier at each level for all products. Constraints (4.6) prohibit assigning the same supplier to more than one level for the same product. However, the model allows a supplier to be a backup supplier for more than one product. Constraints (4.7) relate the supplier allocation to capacity restrictions. Given supplier j and product k, if the difference between supply capacity Cap_{jk} and demand D_k is greater than 0; that is if the supplier has enough capacity to satisfy the demand, the SSM formulation allows the respective assignment variable x_{jkr} to be either 0 or 1. It is important to note that constraints in (4.7) can also be considered as logical relations: *if* $Cap_{jk} - D_k \geq 0$ *then* create the relevant x_{jkr} variables; *else* do not introduce those x_{jkr} to the model. We shall use this logical relation when coding and solving the model to eliminate unnecessary x_{jkr} variables. Constraints in (4.7) are given for completeness of the SSM formulation. Lastly, constraints (4.8) restrict the model variables to binary values.

4.2.3 Multiple Sourcing Model (MSM)

A possible extension to the SSM presented in Section 4.2.2 is the *multiple sourcing model (MSM)* where the buyer does not need to rely on the supply capability of just one supplier; but rather, can split an order among several suppliers. Those suppliers replenishing the buyer are called the *primary suppliers* and the set containing the primary suppliers is called the *primary set*. Several modifications to the SSM formulation have to be made for building a multiobjective mathematical formulation that conforms the MSM requirements. Let us introduce a new set of continuous decision variables, h_{jkr}, to model the amount of product k shipped from supplier j where j is a level r supplier. This modification allows incorporating order splitting; and hence, formulating a more general model. Moreover, assume that the buyer requires having no more than p suppliers in the primary set. This assumption is not apart from reality in that most buyers have restrictions on the number of suppliers that they want to operate with. The remaining suppliers are to be assigned as backups. Other model parameters and variables are the same as in the SSM and were detailed in Section 4.2.2. The MSM formulation is as follows.

$$\min\ z_1^{MSM} = \left[\sum_{j\in J}\sum_{k\in K} c_{jk1}h_{jk1} + \sum_{j\in J}\sum_{k\in K}\sum_{r=2}^{m'-p+1} c_{jkr}x_{jkr}\right] + \sum_{j\in J}\sum_{k\in K}\sum_{r=1}^{m'-p+1} F_{jr}x_{jkr} \tag{4.9}$$

$$\max\ z_2^{MSM} = \sum_{j\in J}\sum_{k\in K} Q_{jk1}h_{jk1} + \sum_{j\in J}\sum_{k\in K}\sum_{r=2}^{m'-p+1} Q_{jkr}x_{jkr} \tag{4.10}$$

$$\min\ z_3^{MSM} = \sum_{j\in J}\sum_{k\in K} L_{jk1}h_{jk1} + \sum_{j\in J}\sum_{k\in K}\sum_{r=2}^{m'-p+1} L_{jkr}x_{jkr} \tag{4.11}$$

$$\min z_4^{MSM} = \sum_{j\in J}\sum_{k\in K} \rho_j h_{jk1} + \sum_{j\in J}\sum_{k\in K}\sum_{r=2}^{m'-p+1} \rho_j x_{jkr} \tag{4.12}$$

s.t.

$$\sum_{j\in J} x_{jk1} \le p \quad \forall k \in K \tag{4.13}$$

$$\sum_{j\in J} x_{jkr} = 1 \quad \forall k \in K,\ r = 2, ..., m' - p + 1 \tag{4.14}$$

$$\sum_{r=1}^{m'-p+1} x_{jkr} = 1 \quad \forall j \in J,\ \forall k \in K \tag{4.15}$$

$$\sum_{j\in J} h_{jk1} = D_k \quad \forall k \in K \tag{4.16}$$

$$h_{jk1} \le Cap_{jk}x_{jk1} \quad \forall j \in J,\ \forall k \in K \tag{4.17}$$

$$x_{jkr} \in \{0,\ 1\} \quad \forall j \in J,\ \forall k \in K,\ r = 1, ..., m' - p + 1 \tag{4.18}$$

$$h_{jkr} \ge 0 \quad \forall j \in J,\ \forall k \in K,\ r = 1, ..., m' - p + 1 \tag{4.19}$$

The first objective function minimizes total cost. The terms in brackets in Equation (4.9) correspond to variable cost and the summation left out of the brackets represents fixed cost. The variable cost expression has already been broken down in two. The level index r is fixed at 1 in the first summation that corresponds to the variable cost of primary suppliers (different from the SSM, the buyer is allowed to have more than one primary supplier in the MSM) and uses the h_{jk1} variables that model the amount of product k ordered from primary supplier j. The second term covers r from 2 to $m' - p + 1$ and can be considered as the *opportunity cost* related to backup suppliers. It involves the x_{jkr} assignment variables since there will be no shipment from backup suppliers; i.e. $h_{jkr} = 0$ for $r > 1$. The fixed cost term covers all assignment levels to account for both primary and backup suppliers. The second objective function in (4.10) maximizes the total quality of purchased products and backup supplier quality. The third objective function in (4.11) minimizes the total lead time and the last objective function in (4.12) minimizes the risk value associated with the supplier selection.

Constraints in (4.13) limit the number of suppliers in the primary set to p where $p \leq m' \leq m$. We assume that p is determined exogenously by the buyer based on his outsourcing strategy. Constraints (4.14) ensure that after including up to p suppliers in the primary set, the remaining suppliers are assigned as backups. Note that once the p primary suppliers are determined, there remains $m' - p$ suppliers; and therefore, these suppliers can be assigned up to level $m' - p + 1$. Constraints (4.15) operate the same way as constraints (4.6). Constraints in (4.16) ensure that shipments from the primary suppliers cover the entire demand for each product. Constraints in (4.17) restrict the amount shipped from the primary suppliers to their capacity. Also note that an h_{jk1} variable would be active only when the corresponding x_{jk1} binary variable is set to 1; that is, only when supplier j is in the primary set for product k. Lastly, constraints in (4.18) and (4.19) force binary and non negativity requirements on the MSM decision variables.

4.3 Solution Techniques

Multiobjective mathematical programs (MOMP) can be solved using several different techniques. The most straightforward method is the *weighted objectives method*, where an MOMP is transformed to a single objective problem by assigning a weight to each objective. Weights indicate the DM's strength-of-preference for the objective functions. Generally, weights are normalized to sum up to one. Objective functions are first scaled, multiplied with their respective weights, summed up and the problem is solved as a single objective optimization model. One can obtain the efficient frontier to visualize the tradeoffs between the objectives by changing the weights. Other methods, such as *compromise programming* and *method of global criterion* can also be used (see Masud & Ravindran (2008) for detailed explanation of these methods). However, the most popular MOMP method is *goal programming* (GP). In GP, each objective is assigned a target value which represents the desired achievement for that objective. Target values are more natural to managers since they have a level of aspiration in mind regarding the performance of their supply chain. In practice, target values can be provided by DMs or can be calculated as the ideal solution for an objective plus or minus some spread. The aim of GP is to minimize deviations from targets which is calculated using deviational variables assigned to each objective function. There are four GP techniques distinguished by the way in which objective functions are prioritized and deviations from targets are treated. These techniques are called preemptive GP, non-preemptive GP, MinMax GP and fuzzy GP. GP formulations of the SSM and

MSM are discussed in the following sections.

4.3.1 Preemptive GP Formulation

Preemptive GP ranks objective functions with respect to ordered preferences of the DMs and minimizes deviations from target values associated with each objective in the provided preemptive order. Several different techniques can be used to derive preemptive priorities. One convenient way is to use discrete alternative MCDM methods such as rating, Borda count, pairwise comparison or the AHP method (see Ravindran et al. (2009) for an application). These methods also provide a numerical strength-of-preference value that can be used in non-preemptive GP models. The preemptive GP model formulation, assuming that the preference ordering of the objectives is z_1, z_2, z_3 and z_4, of the Single Sourcing Model (SSM) is as follows,

$$\begin{aligned}
&\min P_1 d_1^+ + P_2 d_2^- + P_3 d_3^+ + P_4 d_4^+ \\
&\quad \text{s.t.} \\
&\quad z_1^{SSM} + d_1^- - d_1^+ = T_1^{SSM} \\
&\quad z_2^{SSM} + d_2^- - d_2^+ = T_2^{SSM} \\
&\quad z_3^{SSM} + d_3^- - d_3^+ = T_3^{SSM} \\
&\quad z_4^{SSM} + d_4^- - d_4^+ = T_4^{SSM} \\
&\quad d_i^-, d_i^+ \geq 0 \ \ \forall i
\end{aligned} \tag{4.20}$$

where P_1, P_2, P_3 and P_4 represent the respective priorities.

All other SSM constraints (4.5) through (4.8) are also included.

The newly introduced T_i^{SSM} parameters are target values for the SSM objectives. Additional variables d_i^-, d_i^+ in (4.20) respectively represent negative and positive deviations from the SSM target values. Preemptive objective function in (4.20) minimizes the sum of deviations from the target values specified in the additional constraints appended above. The P_i parameters represent preemptive priority of each objective function. For instance, $P_1 d_1^+$ can be interpreted as minimizing the positive deviation from the cost target having the highest priority. Note that the second objective (Quality) is to maximize; hence, we minimize the negative deviation variable d_2^-.

Preemptive GP formulation of the Multiple Sourcing Model (MSM) is very similar to (4.20) and is presented below.

$$
\begin{aligned}
&\min P_1 d_1^+ + P_2 d_2^- + P_3 d_3^+ + P_4 d_4^+ \\
&\quad \text{s.t.} \\
&\quad z_1^{MSM} + d_1^- - d_1^+ = T_1^{MSM} \\
&\quad z_2^{MSM} + d_2^- - d_2^+ = T_2^{MSM} \\
&\quad z_3^{MSM} + d_3^- - d_3^+ = T_3^{MSM} \\
&\quad z_4^{MSM} + d_4^- - d_4^+ = T_4^{MSM} \\
&\quad d_i^-, d_i^+ \geq 0 \;\; \forall i \\
&\quad \text{All other MSM constraints (4.13) through (4.19) are also included}
\end{aligned}
\tag{4.21}
$$

4.3.2 Non-Preemptive GP

The second GP solution method is the non-preemptive GP where preferences on objective functions are imposed using numerical weights rather than ordinal priorities. Methods used in Ravindran et al. (2009) can be applied to derive DMs' weights for each objective function. Also, in contrast with preemptive GP, objective functions in non-preemptive GP models need scaling for accurate calculations since they will be optimized all at once. Scaling can be done using one of the methods detailed in Masud & Ravindran (2008). Non-preemptive GP formulation for the SSM model is as follows,

$$
\begin{aligned}
\min\ & w_1 d_1^+ + w_2 d_2^- + w_3 d_3^+ + w_4 d_4^+ \\
& \text{s.t.} \\
& z_1^{SSM} + d_1^- - d_1^+ = T_1^{SSM} \\
& z_2^{SSM} + d_2^- - d_2^+ = T_2^{SSM} \\
& z_3^{SSM} + d_3^- - d_3^+ = T_3^{SSM} \\
& z_4^{SSM} + d_4^- - d_4^+ = T_4^{SSM} \\
& d_i^-, d_i^+ \geq 0 \ \ \forall i \\
& \text{All other SSM constraints (4.5) through (4.8) are also included}
\end{aligned}
\tag{4.22}
$$

where w_i represents the weight assigned to objective function i. Non-preemptive GP formulation for the MSM is very similar. The objective function is the same as in (4.22), but the problem needs to be restricted over constraints (4.13) through (4.19) and MSM objectives and targets should be used.

4.3.3 MinMax GP

Also known as Tchebysheff GP, the MinMax GP minimizes the maximum deviation from target values. The MinMax GP formulation for the SSM is as in (4.23).

$$
\begin{aligned}
&\min\max\ \left(d_1^+, d_2^-, d_3^+, d_4^+\right) \\
&\qquad \text{s.t.} \\
&\qquad z_1^{SSM} + d_1^- - d_1^+ = T_1^{SSM} \\
&\qquad z_2^{SSM} + d_2^- - d_2^+ = T_2^{SSM} \\
&\qquad z_3^{SSM} + d_3^- - d_3^+ = T_3^{SSM} \\
&\qquad z_4^{SSM} + d_4^- - d_4^+ = T_4^{SSM} \\
&\qquad d_i^-, d_i^+ \geq 0 \ \ \forall i \\
&\qquad \text{All other SSM constraints (4.5) through (4.8) are also included}
\end{aligned} \tag{4.23}
$$

The MinMax GP program above has a nonlinear objective function which can be linearized by setting $M = max\left(d_1^+, d_2^-, d_3^+, d_4^+\right)$ and restricting the deviations to be less than or equal to M. As in non-preemptive GP, objective functions need to be scaled for accurate calculations. The modified MinMax GP formulation of the SSM problem is given below,

$$
\begin{aligned}
&\min M \\
&\quad \text{s.t.} \\
&\quad M \geq d_1^+ \\
&\quad M \geq d_2^- \\
&\quad M \geq d_3^+ \\
&\quad M \geq d_4^+ \\
&\quad z_1^{SSM} + d_1^- - d_1^+ = T_1^{SSM} \\
&\quad z_2^{SSM} + d_2^- - d_2^+ = T_2^{SSM} \\
&\quad z_3^{SSM} + d_3^- - d_3^+ = T_3^{SSM} \\
&\quad z_4^{SSM} + d_4^- - d_4^+ = T_4^{SSM} \\
&\quad d_i^-, d_i^+ \geq 0 \ \ \forall i \\
&\quad \text{All other SSM constraints (4.5) through (4.8) are also included}
\end{aligned}
\tag{4.24}
$$

MinMax GP formulation for the MSM is very similar. The objective function is the same as in (4.24), but the problem needs to be restricted over constraints (4.13) through (4.19) and MSM objectives and targets should be used.

4.3.4 Fuzzy GP

Fuzzy GP minimizes the maximum fractional deviation from ideal solutions. Ideal solutions are obtained by optimizing each objective independently. Fuzzy GP uses the following scheme to calculate the fractional deviation from ideal values,

$$\begin{cases} \dfrac{I_i - f_i}{I_i - N_i} & \text{if } f_i \text{ is to be maximized} \\ \dfrac{f_i - I_i}{N_i - I_i} & \text{if } f_i \text{ is to be minimized} \end{cases} \tag{4.25}$$

In (4.25), I_i is the ideal solution and N_i is the anti-ideal solution for objective i. Note that the objective functions are scaled through (4.25). The Fuzzy GP formulation for the SSM is given in (4.26).

$$\min \max \left(\frac{f_{cost} - I_{cost}}{N_{cost} - I_{cost}}, \frac{I_{quality} - f_{quality}}{I_{quality} - N_{quality}}, \frac{f_{lt} - I_{lt}}{N_{lt} - I_{lt}}, \frac{f_{risk} - I_{risk}}{N_{risk} - I_{risk}} \right)$$
$$\text{s.t.} \tag{4.26}$$

All other SSM constraints (4.5) through (4.8) are also included

The nonlinear program in (4.26) can be linearized as in the MinMax GP by introducing a variable M and constraining the fractional deviation values to be less then or equal to M. Fuzzy GP formulation of MSM is very similar, with the difference of constraining the model over (4.13) through (4.19) and using the MSM objectives, ideal and anti-ideal values.

4.4 Numerical Applications

In this section, we present numerical examples for the SSM and MSM formulations. We begin with an SSM example.

4.4.1 An SSM Example

We consider a supplier selection problem with a single buyer, five suppliers and three products. Fixed cost, variable cost, quality, lead time, capacity and demand data presented in Tables 4.2 through 4.8 are used to implement the SSM application.

Table 4.2. Demand data for the SSM and MSM (in units)

Product	1	2	3
Demand	210	250	250

Table 4.3. Capacity data for the SSM (in units)

	Product		
Supplier	1	2	3
1	220	250	300
2	250	350	250
3	250	270	260
4	300	400	0
5	200	300	300

Table 4.4. Quality data for the SSM and MSM (in % of good items)

		Level		
	Supplier	*1*	*2*	*3*
Product 1	*1*	0.95	0.9025	0.857375
	2	0.95	0.9025	0.857375
	3	0.9	0.855	0.81225
	4	0.9	0.855	0.81225
	5	0.9	0.855	0.81225
Product 2	*1*	0.95	0.9025	0.857375
	2	0.97	0.9215	0.875425
	3	0.9	0.855	0.81225
	4	0.93	0.8835	0.839325
	5	0.92	0.874	0.8303
Product 3	*1*	0.93	0.8835	0.839325
	2	0.99	0.9405	0.893475
	3	0.9	0.855	0.81225
	4	0.9	0.855	0.81225
	5	0.97	0.9215	0.875425

Risk values are calculated using the method presented in Section 3.2.3. Risk values in Table 4.8 represent the mean loss due to disruptions over one year at

Table 4.5. Lead time data for the SSM and MSM (in days)

		Level			
	Product	*1*	*2*	*3*	*4*
Supplier 1	*1*	10	10.5	11.025	11.57625
	2	9	9.45	9.9225	10.41863
	3	1	1.05	1.1025	1.157625
Supplier 2	*1*	5	5.25	5.5125	5.788125
	2	2	2.1	2.205	2.31525
	3	8	8.4	8.82	9.261
Supplier 3	*1*	8	8.4	8.82	9.261
	2	3	3.15	3.3075	3.472875
	3	9	6	6.3	6.615
Supplier 4	*1*	3	3.15	3.3075	3.472875
	2	4	4.2	4.41	4.6305
	3	6	6.3	6.615	6.94575
Supplier 5	*1*	8	8.4	8.82	9.261
	2	2	2.1	2.205	2.31525
	3	4	4.2	4.41	4.6305

Table 4.6. Variable cost data for the SSM and MSM (in $)

		Level			
	Product	*1*	*2*	*3*	*4*
Supplier 1	*1*	15	15.75	16.5375	17.36438
	2	10	10.5	11.025	11.57625
	3	12	12.6	13.23	13.8915
Supplier 2	*1*	15	15.75	16.5375	17.36438
	2	8	8.4	8.82	9.261
	3	9	9.45	9.9225	10.41863
Supplier 3	*1*	10	10.5	11.025	11.57625
	2	9	9.45	9.9225	10.41863
	3	5	5.25	5.5125	5.788125
Supplier 4	*1*	15	15.75	16.5375	17.36438
	2	16	16.8	17.64	18.522
	3	9	9.45	9.9225	10.41863
Supplier 5	*1*	6	6.3	6.615	6.94575
	2	8	8.4	8.82	9.261
	3	18	18.9	19.845	20.83725

each supplier. We assume only one disruptive event affects suppliers every period; however, more general models, as discussed in Chapter 3, can be used as well. Objective weight values and priorities shown in Table 4.9 are taken from an actual case study presented in Ravindran et al. (2009) and are used in the non-preemptive and preemptive GP examples. We set $m' = 4$ to allow only four levels of assignment. We present the SSM objective achievements obtained by solving the different GP models in Tables 4.10 through 4.13. Ideal values are calculated by optimizing each objective independently and targets are set at 5% from the ideal values. For an objective function to be maximized, the target is set at 5% less than the respective ideal value. For minimization objectives, targets are set at 5% greater than respective ideal values.

The *Status* column in Tables 4.10 through 4.13 reports the target achievements. A target is achieved if the objective value at optimality, reported in the *Achievement* column, is between the ideal value and the target value. The first value in each row of the *Achievement* column is the objective value achieved in the optimal solution. The values in parentheses are the portions of the objective value corresponding to the primary suppliers. Note that there are two values in parentheses for the cost objective: the first value is the variable cost of primary suppliers and

Table 4.7. Fixed cost data for the SSM and MSM (in $)

	Supplier				
Level	1	2	3	4	5
1	100	200	150	150	120
2	75	150	113	113	90
3	56	113	84	84	68
4	42	84	63	63	51

Table 4.8. Risk data for the SSM and MSM (in $)

Supplier	1	2	3	4	5
ρ	400707.6	496028.7	360772.8	937732.7	968961.9

the second value is the fixed cost of primary suppliers.

Table 4.9. Weights and priorities used in the SSM and MSM solutions

Criteria	**Weight**	**Preemptive Priority**
Cost	0.343	P_1
Quality	0.338	P_2
Lead time	0.246	P_3
Risk	0.073	P_4

Table 4.10. Non-preemptive GP results for the SSM

Objective	**Ideal**	**Target**	**Status**	**Achievement**
Cost	34445.23	36167.49	Achieved	36053.35; (11650, 420)
Quality	10.15	9.64	Achieved	10.08; (2.80)
Lead time	58.62	61.55	Achieved	60.26; (11)
Risk	6616955.00	6947802.75	Not Achieved	7185208.86; (2844427.29)

Table 4.11. Preemptive GP results for the SSM

Objective	**Ideal**	**Target**	**Status**	**Achievement**
Cost	34445.23	36167.49	Achieved	36058.58; (11650, 370)
Quality	10.15	9.64	Achieved	10.08; (2.70)
Lead time	58.62	61.55	Achieved	59.56; (18)
Risk	6616955.00	6947802.75	Not Achieved	7185208.86; (2307402.18)

Table 4.12. MinMax GP results for the SSM

Objective	**Ideal**	**Target**	**Status**	**Achievement**
Cost	34445.23	36167.49	Achieved	34990.22; (7900.01, 450)
Quality	10.15	9.64	Achieved	10.09; (2.84)
Lead time	58.62	61.55	Not Achieved	63.35; (18.99)
Risk	6616955.00	6947802.75	Achieved	6648187.80; (1834472.97)

Table 4.13. Fuzzy GP results for the SSM

Objective	**Ideal**	**Anti-Ideal**	**Achievement**	**Dev. from Ideal (%)**
Cost	34445.23	37093.07	34797.45; (7900, 450)	1.02
Quality	10.15	10.06576	10.11; (2.84)	0.39
Lead time	58.62	71.61312	63.85; (19)	8.92
Risk	6616955.00	7225144	6648183.75; (1834468.92)	0.47

We can see that the most promising results are obtained using non-preemptive, preemptive and MinMax GP solution techniques (see Tables 4.10 through 4.12) where three out of four targets have been achieved. Fuzzy GP, on the other hand, yields cost, quality and risk results close to the ideal values, but the solution presents significant deviations from ideal values for the lead time objective. For illustration, we present the preemptive GP optimal solution of the SSM in Table 4.14. The solution can be interpreted as follows: The first row indicates that supplier 1 is going to be assigned as a level 1 supplier for product 1. The second and third rows further indicate that supplier 1 will not be assigned at any level for product 2 and will be a level 3 supplier of product 3. Analyzing further, we can observe that suppliers 4 and 5 are the other primary suppliers of the buyer and will provide products 2 and 3 respectively. Suppliers 2 and 3 act as backup suppliers only. The buyer is assigned a single backup supplier for each product at each level. These backup suppliers will not ship any products unless a primary supplier fails. Finally, note that each product has four suppliers, one primary and three backups, as expected from the SSM formulation.

4.4.2 An MSM Example

We consider the same problem solved in the SSM application in Section 4.4.1 with the same weights and preemptive priorities given in Table 4.9. We use capacity values given in Table 4.15 below.

We set the p value to three; that is, the primary set will allow up to three suppliers. We further fixed $m' = 5$ to allow a total of five supplier assignments (three in the primary set and two as backups). We obtain the solutions in Tables 4.16 through 4.19. The first value in each row of the *Achievement* column is the objective value at optimality. Values in parentheses are the portion of the objective value corresponding to primary suppliers. Note that for the cost objective there are

Table 4.14. Preemptive GP solution of the SSM

	Levels				
Supplier	*1*	*2*	*3*	*4*	**Product**
1	1	0	0	0	1
	0	0	0	0	2
	0	0	1	0	3
2	0	0	1	0	1
	0	0	0	1	2
	0	1	0	0	3
3	0	0	0	1	1
	0	1	0	0	2
	0	0	0	1	3
4	0	1	0	0	1
	1	0	0	0	2
	0	0	0	0	3
5	0	0	0	0	1
	0	0	1	0	2
	1	0	0	0	3

two values in parentheses: the first value is the variable cost of primary suppliers and the second value is the fixed cost of primary suppliers.

Preemptive GP yields the best solution for the MSM example (see Table 4.17),

Table 4.15. Capacity data for the MSM (in units)

	Product		
Supplier	1	2	3
1	50	45	100
2	90	100	20
3	70	50	150
4	50	200	50
5	60	100	60

Table 4.16. Non-preemptive GP results for the MSM model

Objective	**Ideal**	**Target**	**Status**	**Achievement**
Cost	8322.93	8739.07	Not Achieved	8785.64; (6850, 1370)
Quality	670.71	637.17	Achieved	660.77; (655.50)
Lead time	2908.20	3053.60	Not Achieved	3206.20; (3160)
Risk	350243600.00	367755780.00	Not Achieved	379680796.46; (375507696.13)

Table 4.17. Preemptive GP results for the MSM model

Objective	**Ideal**	**Target**	**Status**	**Achievement**
Cost	8322.93	8739.07	Achieved	8693.71; (6757.25, 1340)
Quality	670.71	637.17	Achieved	662.87; (657.66)
Lead time	2908.20	3053.60	Achieved	3053.60; (3005.41)
Risk	350243600.00	367755780.00	Not Achieved	398453557.24; (394311686.15)

Table 4.18. MinMax GP results for the MSM model

Objective	**Ideal**	**Target**	**Status**	**Achievement**
Cost	8322.93	8739.07	Achieved	8697.37; (6778.87, 1290)
Quality	670.71	637.17	Achieved	664.82; (659.66)
Lead time	2908.20	3053.60	Not Achieved	3432.03; (3391.44)
Risk	350243600.00	367755780.00	Not Achieved	367756158.42; (363077262.21)

Table 4.19. Fuzzy GP results for the MSM model

Objective	**Ideal**	**Anti - Ideal**	**Achievement**	**Deviation from Ideal (%)**
Cost	8322.93	8739.07	8593.44; (6674.94, 1290)	3.25
Quality	670.71	637.17	667.25; (662.08)	0.05
Lead time	2908.20	3053.60	3258.81; (3218.24)	12.06
Risk	350243600.00	367755780.00	388824748.94; (384145852.72)	11.02

Table 4.20. Preemptive GP solution of the MSM model

	Levels			
Supplier	*1*	*2*	*3*	**Product**
1	0	1	0	1
	0	1	0	2
	1	0	0	3
2	1	0	0	1
	1	0	0	2
	0	1	0	3
3	1	0	0	1
	1	0	0	2
	1	0	0	3
4	1	0	0	1
	0	0	1	2
	0	0	1	3
5	0	0	1	1
	1	0	0	2
	1	0	0	3

	Levels			
Supplier	*1*	*2*	*3*	**Product**
1	0	0	0	1
	0	0	0	2
	100	0	0	3
2	90	0	0	1
	100	0	0	2
	0	0	0	3
3	70	0	0	1
	50	0	0	2
	119	0	0	3
4	50	0	0	1
	0	0	0	2
	0	0	0	3
5	0	0	0	1
	100	0	0	2
	31	0	0	3

where the first three targets are achieved. Only the second target can be achieved in the non preemptive GP solution and the first two targets are achieved in the Min-Max GP solution. Fuzzy GP solution presents small deviations from the cost and quality ideal values. Deviations from lead time and risk ideal values, on the other hand, are larger. Optimal solution to the MSM preemptive GP is given in Table 4.20. The optimal solution can be interpreted as follows. The table on the left shows the supplier assignments. The first row of has one entry equal to 1 at level 2; meaning that supplier 1 is a level 2 supplier (a backup supplier) of product 1. The second and third rows of supplier 1 indicate that supplier 1 is a level 2 supplier of product 2 and a primary supplier of product 3. We can further observe that the buyer has nine primary suppliers (number of primary suppliers $= p^*$number of products $\equiv\ 3 \times 3$), three for each product. Moreover, the buyer has three level 2 and three level 3 backup suppliers, each assigned for different products. The table on right presents the optimal order allocations. Note that only the primary suppliers have been assigned with positive orders and entries greater than zero in the *Level 1* column match the supplier assignment decisions; verifying that an order is allocated to a supplier only if he has been assigned as a primary supplier. As an illustration, for product 1, there are three primary suppliers (S2, S3 and S4) and their respective order quantities are 90, 70 and 50 units.

4.4.3 Visualization of the Solutions

It is often challenging for managers to interpret numerical solutions of multiobjective optimization problems. An effective visualization tool is required to compare and contrast results obtained using different solution techniques. In this section, we use the *value path approach (VPA)* proposed by Schilling et al. (1983) to display the SSM and MSM solutions computed by different GP techniques. VPA begins with determining the best objective value obtained. Then, each objective value is divided by the best objective for scaling. The best objective gets a scaled value of 1 while all others are scaled to a value greater than one. The larger the scaled objective value, the worse a GP method performs for that objective. Value path calculations for the SSM and MSM solutions are given in Tables 4.21 and 4.22 where the first value in each column is the actual objective value and the second value is the scaled objective value.

Table 4.21. Value path calculations for the SSM solutions

	Cost	**Quality**	**Lead time**	**Risk**
Non Preemptive	36053	10	60	7185209
	1.036	1.003	1.000	1.081
Preemptive	36059	10	60	7185209
	1.036	1.003	1.012	1.081
MinMax	34990	10	63	6648188
	1.006	1.002	1.064	1.000
Fuzzy	34797	10	64	6648184
	1.000	1.000	1.072	1.000

Table 4.22. Value path calculations for the MSM solutions

	Cost	**Quality**	**Lead time**	**Risk**
Non Preemptive	8786	661	3206	379680796
	1.02	1.01	1.05	1.03
Preemptive	8694	663	3054	398453557
	1.01	1.01	1.00	1.08
MinMax	8697	665	3432	367756158
	1.01	1.00	1.12	1.00
Fuzzy	8593	667	3259	388824749
	1.00	1.00	1.07	1.06

Figures 4.1 and 4.2 show the actual value paths for the SSM and MSM solutions. These graphs can be used to determine dominated and non-dominated solutions.

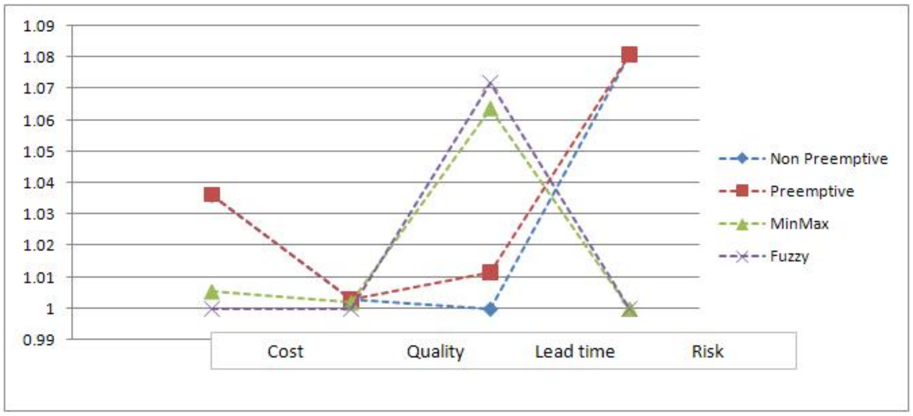

Figure 4.1. Value path comparison of SSM solutions

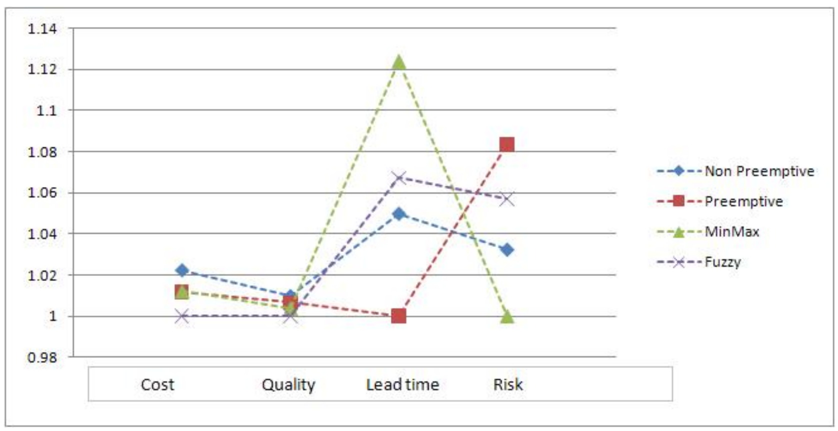

Figure 4.2. Value path comparison of MSM solutions

If the value path of one solution is above that of another, then the solution is dominated. If value paths of two solutions cross each other, then these solutions do not dominate one another. None of the GP solutions computed in the numerical applications dominate each other (see Figures 4.1 and 4.2). On the other hand, certain GP methods perform better on a particular objective than others. For instance, as seen on Figure 4.1, all methods perform very well for the quality objective; however, Fuzzy and MinMax GP yield poor results for the lead time objective. VPA enables managers to easily perform tradeoff analyses to compare different GP solutions and select the best compromise solution.

4.5 Extensions to the SSA Models

SSA models presented in Section 4.2 can be extended in several ways. This section discusses and illustrates several possible extensions.

4.5.1 Conditional Contracts

Suppliers may be reluctant to enter in a business relationship with a buyer under the SSA settings given in Section 4.2. For instance, a supplier may not accept being a backup for a certain product unless he is guaranteed a primary supplier spot for another product. We name such restrictions as *conditional contracts* with the buyer. Conditional contracts can be handled by adding extra constraints to SSM and MSM formulations. Let us illustrate conditional contracts with a simple example before providing a general formulation. Consider a supplier, say supplier 1, accepts being a backup supplier for a product, say product 3, only if he is assigned as a primary supplier for product 2. Assuming that the buyer has four levels of assignment (one primary and three backups), the supplier's restriction can be modeled as follows,

$$x_{132} + x_{133} + x_{134} = x_{121}$$

Note that x_{jkr} are binary variables in the SSM and MSM formulations. In the above constraint, $x_{121} = 1$ if supplier 1 is a primary supplier of product 2. The term on the left hand side is the sum of variables that assign supplier 1 as a level 2, level 3 and level 4 supplier for product 3. Therefore, if supplier 1 is assigned as a primary supplier of product 2, than he has to be assigned as level 2, level 3 or level 4 supplier of item 3; and vice versa.

For a general formulation, let us assume supplier j agrees to be at a backup position for product k' only if he gets a primary position for product k^*. Further assume that the buyer admits m' levels of supplier assignments. Then, the general constraint for the SSM can be formulated as follows,

$$\sum_{r=2}^{m'} x_{jk'r} = x_{jk^*1} \tag{4.27}$$

Assuming p primary suppliers, the MSM counterpart of constraint (4.27) would be,

$$\sum_{r=p+1}^{m'} x_{jk'r} = \sum_{r=1}^{p} x_{jk^*r} \tag{4.28}$$

4.5.2 Inclusion of Risk Detection and Risk Recovery

Another possible extension to the SSA models is to include detectability of disruption risks and risk recovery into the model formulations. We have given an integrated risk quantification example in Chapter 3, Section 3.6. Risk detection and risk recovery can be used as separate criteria in a multiobjective optimization model or all risk values can be normalized and weighted into a single risk objective, which in turns can be used in a different multiobjective optimization model. We treat detectability and recovery separately and provide a numerical example in the following section.

4.5.3 Numerical Example

Consider the same supply chain example solved in Section 4.4. Additionally, let the supply network of the five candidate suppliers be as in Figure 4.3. In this example, the buyer can select his tier 1 suppliers, but cannot affect tier 2 and tier

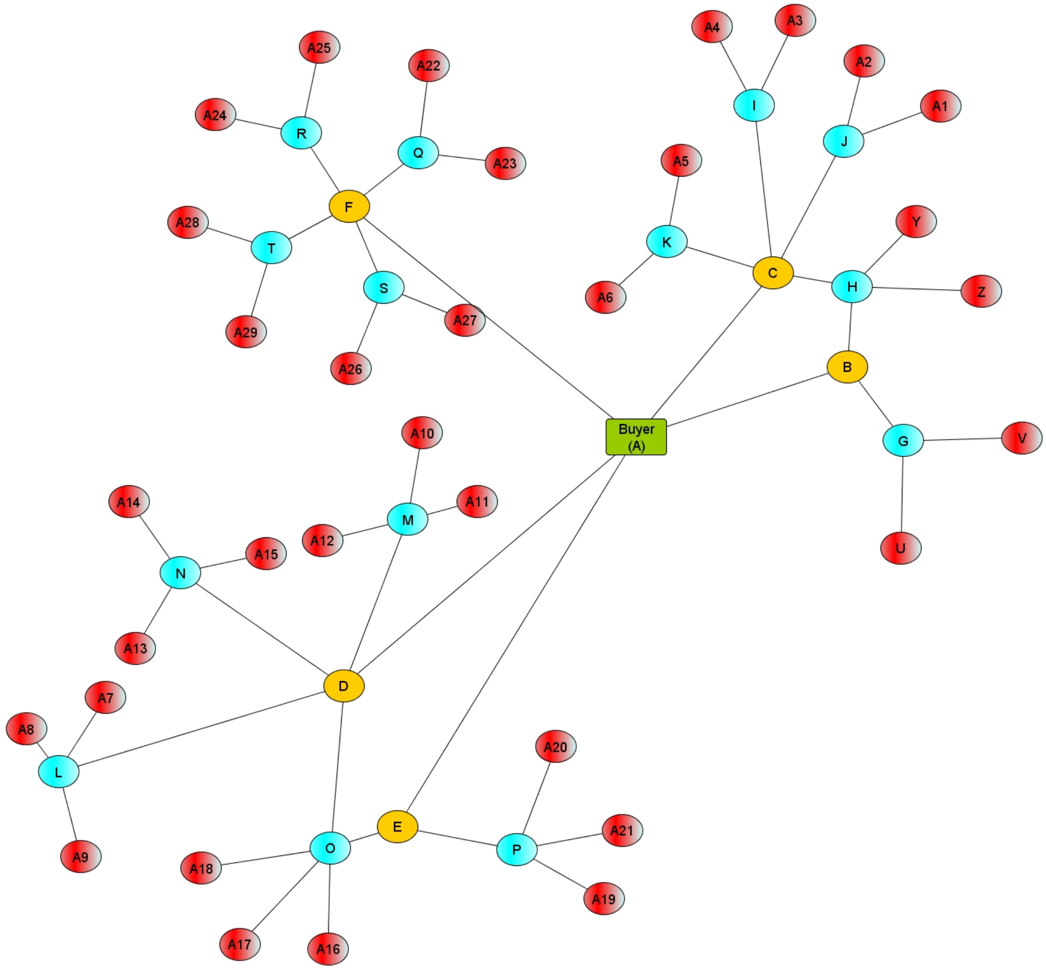

Figure 4.3. Example supply chain network to implement SSA extensions

3 supplier selections. Nevertheless, the buyer knows his tier 2 and tier 3 suppliers which have previously been selected by his tier 1 and tier 2 suppliers respectively.

The partial MFPT matrix of the supply network of Figure 4.3 is given in Table 4.23 where the elements m_{ij} represent the mean number of transitions required to transmit a signal from node i to node j.

Table 4.23. Partial MFPT matrix of the supply network in Figure 4.3

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	Y	Z
A	2.6	11.1	10.8	10.8	11.1	11.5	118	74	213	213	213	213	213	213	74	118	220	220	220	220	981	981	638	638
B	1.6	10.8	11.7	12.4	12.7	13.1	107	67	214	214	214	215	215	215	76	119	222	222	222	222	970	970	632	632
C	1.6	12.4	10.2	12.4	12.7	13.1	119	71	203	203	203	215	215	215	76	119	222	222	222	222	983	983	636	636
D	1.6	12.7	12.4	10.2	12.4	13.1	119	76	215	215	215	203	203	203	71	119	222	222	222	222	983	983	640	640
E	1.6	12.7	12.4	11.7	10.8	13.1	119	76	215	215	215	214	214	214	67	107	222	222	222	222	983	983	640	640
F	1.6	12.7	12.4	12.4	12.7	10.5	119	76	215	215	215	215	215	215	76	119	209	209	209	209	983	983	640	640
G	3.1	1.5	13.2	13.9	14.2	14.6	86	69	216	216	216	217	217	217	77	121	223	223	223	223	864	864	633	633
H	3.1	7.7	7.3	13.9	14.2	14.6	114	57	210	210	210	217	217	217	77	121	223	223	223	223	978	978	564	564
I	3.1	13.9	1.5	13.9	14.2	14.6	120	73	163	204	204	217	217	217	77	121	223	223	223	223	984	984	637	637
J	3.1	13.9	1.5	13.9	14.2	14.6	120	73	204	163	204	217	217	217	77	121	223	223	223	223	984	984	637	637
K	3.1	13.9	1.5	13.9	14.2	14.6	120	73	204	204	163	217	217	217	77	121	223	223	223	223	984	984	637	637
L	3.1	14.2	13.9	1.5	13.9	14.6	121	77	217	217	217	163	204	204	73	120	223	223	223	223	985	985	642	642
M	3.1	14.2	13.9	1.5	13.9	14.6	121	77	217	217	217	204	163	204	73	120	223	223	223	223	985	985	642	642
N	3.1	14.2	13.9	1.5	13.9	14.6	121	77	217	217	217	204	204	163	73	120	223	223	223	223	985	985	642	642
O	3.1	14.2	13.9	7.3	7.7	14.6	121	77	217	217	217	210	210	210	57	114	223	223	223	223	985	985	642	642
P	3.1	14.2	13.9	13.2	1.5	14.6	121	77	217	217	217	216	216	216	69	86	223	223	223	223	985	985	642	642
Q	3.1	14.2	13.9	13.9	14.2	1.5	121	77	217	217	217	217	217	217	77	121	168	210	210	210	985	985	642	642
R	3.1	14.2	13.9	13.9	14.2	1.5	121	77	217	217	217	217	217	217	77	121	210	168	210	210	985	985	642	642
S	3.1	14.2	13.9	13.9	14.2	1.5	121	77	217	217	217	217	217	217	77	121	210	210	168	210	985	985	642	642
T	3.1	14.2	13.9	13.9	14.2	1.5	121	77	217	217	217	217	217	217	77	121	210	210	210	168	985	985	642	642
U	4.1	2.5	14.2	14.9	15.2	15.6	1	70	217	217	217	218	218	218	78	122	224	224	224	224	865	865	634	634
V	4.1	2.5	14.2	14.9	15.2	15.6	1	70	217	217	217	218	218	218	78	122	224	224	224	224	865	865	634	634
Y	4.1	8.7	8.3	14.9	15.2	15.6	115	1	211	211	211	218	218	218	78	122	224	224	224	224	979	979	565	565
Z	4.1	8.7	8.3	14.9	15.2	15.6	115	1	211	211	211	218	218	218	78	122	224	224	224	224	979	979	565	565

A supplier's transition time is assumed to be proportional to its tier level; that is, transition time is 1 time unit at a tier 1 supplier , 2 time units at a tier 2 supplier and 3 time units at a tier 3 supplier. The disruption delay value at a tier 1 supplier is calculated as the maximum transition time along all paths connecting tier 2 and tier 3 suppliers to the buyer through that particular tier 1 supplier. For instance, tier 1 supplier B connects tier 2 suppliers G and H and tier 3 suppliers U, V, Y and Z to the buyer. Note that in Chapter 3 we define $t_{tran}(i)$ as the transition time at node i and Δ_{ij} as the time time it takes to send a signal from node i to node j (disruption delay). Using our notation, disruption delay along the path $U \rightarrow G \rightarrow B \rightarrow A$ is $\Delta_{UA} = t_{tran}(U) \times m_{UG} + t_{tran}(G) \times m_{GB} + t_{tran}(B) \times m_{BA} = 3 \times 1 + 2 \times 1.5 + 1 \times 1.6 = 7.6$ days. Similarly, disruption delay along the path $V \rightarrow G \rightarrow B \rightarrow A$ is $\Delta_{VA} = 3 \times 1 + 2 \times 1.5 + 1 \times 1.6 = 7.6$ days, disruption delay along the path $Y \rightarrow H \rightarrow B \rightarrow A$ is $\Delta_{YA} = 3 \times 1 + 2 \times 7.7 + 1 \times 1.6 = 20$ days and disruption delay along the path $Z \rightarrow H \rightarrow B \rightarrow A$ is $\Delta_{ZA} = 3 \times 1 + 2 \times 7.7 + 1 \times 1.6 = 20$ days. Under the assumption that the buyer wants to hedge himself against the worst possible disruption delay, the maximum of all disruption delays along a path crossing a tier 1 supplier is assigned as that tier 1 supplier's disruption delay value; thus, $\tilde{\Delta}_B = 20$. Other disruption delay values can be computed as $\tilde{\Delta}_C = 19.2$, $\tilde{\Delta}_D = 7.6$, $\tilde{\Delta}_E = 20$ and $\tilde{\Delta}_F = 7.6$.

Another important risk concept introduced in Chapter 3 is the risk recovery time at a supplier. The risk recovery model introduced in Chapter 3 assumes that the time between disruptive event occurrences follows an exponential distribution. The μ parameter of the exponential distribution for a supplier depends on the inventory level at the supplier (I), on the cumulative impact of risk events that may hit the supplier over a fixed period of time (S) and on a parameter δ that models the availability of risk mitigation plans at the supplier. Let tier 1 candidate suppliers carry the following amount of inventory given in dollars and the values of δ parameters: $I_B = \$100,000$, $I_C = \$110,000$, $I_D = \$80,000$, $I_E = \$100,000$, $I_F = \$95,000$; $\delta_B = 0.3$, $\delta_C = 0.4$, $\delta_D = 0.8$, $\delta_E = 0.1$ and $\delta_F = 0.1$. It can be observed that candidate supplier D has a strong mitigation plan and carries less inventory compared to other candidate suppliers. Using formula (3.23) of Chapter 3 and risk values given in Table 4.8, the average recovery time for supplier i, $RT_i = \frac{1}{\mu_i}$, are computed as $RT_B = 13.36$, $RT_C = 7.52$, $RT_D = 5.63$, $RT_E = 93.77$ and $RT_F = 101.99$. Supplier D has the shortest recovery time mainly because it has a large δ value representing a good mitigation plan and a low risk value. Suppliers E and F have significantly larger recovery times since they both have high risk values and inadequate mitigation plans to address disruptions (i.e. small δ values).

We assume that the buyer can split orders among several suppliers and needs to have three suppliers as primary; that is, $p = 3$. Furthermore, let us assume that the buyer does not want to have more than five suppliers for a product (three primary and two backup suppliers) in his portfolio, that is $m' = 5$. We use the same data as in the examples solved in Section 4.4 and demonstrate an integrated application of MSM using non-preemptive goal programming. In addition to the previously defined data, we consider disruption delay times and risk recovery times computed above. Disruption delay time and risk recovery time are formulated in two separate objectives as below,

$$\min \ z_5^{MSM} = \sum_j \sum_k \tilde{\Delta}_j x_{jk1} \tag{4.29}$$

$$\min \ z_6^{MSM} = \sum_j \sum_k RT_j x_{jk1} \tag{4.30}$$

Table 4.24. MSM results for the supply chain network in Figure 4.3

Objective	Ideal	Target	Status	Achievement
Cost	8420.72	8841.76	Achieved	8800.69; (6850, 1370)
Quality	670.71	637.17	Achieved	660.77; (655.50)
Lead time	2908.20	3053.60	Not Achieved	3205.99; (3160)
Risk	350243600.00	367755780.00	Not Achieved	379680796.47; (375507696.13)
Dis. delay	104.8	110.04	Not Achieved	128.8
Rec. time	159.94	167.937	Not Achieved	334.82

where $\tilde{\Delta}_j$ is the disruption delay time and RT_j is the recovery time at supplier j. Note that the risk time value, which is the sum of transition and recovery times, can be used as an alternative objective as well. Also, note that objective functions in (4.29) and (4.30) consider primary suppliers only. More general objective functions considering all assignment levels can be implemented as well.

Let us finally assume that supplier 4 agrees to be a backup supplier of product 2 only if he is assigned as a primary supplier of product 1. This conditional contract can be modeled as follows,

$$x_{422} + x_{423} = x_{411} \tag{4.31}$$

The MSM formulation we solve here is the same as in Equations (4.9) to (4.19) with two additional objectives introduced in Equations (4.29) and (4.30) and the additional constraint (4.31). The model is solved using non-preemptive GP with normalized random weights drawn from a Uniform(0,1) distribution, $w_{z_1^{MSM}} = 0.233$, $w_{z_2^{MSM}} = 0.127$, $w_{z_3^{MSM}} = 0.133$, $w_{z_4^{MSM}} = 0.306$, $w_{z_5^{MSM}} = 0.117$, $w_{z_6^{MSM}} = 0.084$.

With these two new objectives and one new constraint, the MSM formulation now has six objective functions and eight sets of constraints. We solve a non-preemptive MSM example using the data presented in Section 4.4 and additional disruption delay time, risk recovery time and objective function weight data presented above. The MSM result is given in Table 4.24 and the solution is given in Table 4.25.

The non-preemptive MSM result presented in Table 4.24 achieves only the first two objectives whereas the non-preemptive MSM result presented in Table 4.16 achieved all objectives. The difference between these two results is partially due to the increase in the number of objective functions and partially to the new random weight values used in the latter example. It can also be observed in Table 4.25 that the conditional contract imposed by supplier 4 is satisfied.

4.6 Conclusions and Future Work

This chapter presents two multiobjective supplier selection models, called the *Sequential Supplier Allocation* (SSA) models, to generate mitigation plans against disruption risks. The basic SSA models incorporate minimizing cost, lead time and risk and maximizing quality as objective functions and determine supplier

Table 4.25. Non-preemptive GP solution for the extended MSM model

	Levels			
Supplier	*1*	*2*	*3*	**Product**
1	0	1	0	1
	0	1	0	2
	1	0	0	3
2	1	0	0	1
	1	0	0	2
	0	1	0	3
3	1	0	0	1
	1	0	0	2
	1	0	0	3
4	1	0	0	1
	0	0	1	2
	1	0	0	3
5	0	0	1	1
	1	0	0	2
	0	0	1	3

	Levels			
Supplier	*1*	*2*	*3*	**Product**
1	0	0	0	1
	0	0	0	2
	100	0	0	3
2	90	0	0	1
	100	0	0	2
	0	0	0	3
3	70	0	0	1
	50	0	0	2
	150	0	0	3
4	50	0	0	1
	0	0	0	2
	0	0	0	3
5	0	0	0	1
	100	0	0	2
	0	0	0	3

and order allocations for procurement of multiple products. Models assign both primary and backup suppliers under two different procurement scenarios. The first model, the *Single Sourcing Model (SSM)*, implements a single supplier case; whereas the second model, the *Multiple Sourcing Model (MSM)*, is suitable for order splitting among several suppliers. Both models are solved using four different GP solution techniques: preemptive GP, non-preemptive GP, MinMax GP and fuzzy GP. Solutions are displayed using the value path approach and performance of the solutions is discussed. We observe that for the data set we test, preemptive GP and non-preemptive GP yield the best solutions in terms of the number of objectives achieved.

Two extensions to the basic SSA models are also presented. The first extension incorporates conditional contracts to model situations where certain suppliers are willing to accept backup spots for some products only with the condition of having primary supplier spots for other products. The second extension incorporates disruption risk detectability and risk recovery concepts developed in Chapter 3. An MSM example with both extensions is solved at the end of the chapter and the model solution is discussed.

Chapter 5

Stochastic Extensions to the SSA Models

5.1 Introduction

Models developed in Chapter 4 of this dissertation provide tools for optimally mitigating disruption risks in outsourcing. As per our discussion in Chapter 1, supply chains, especially at the global level, are subject to two major types of risk: disruption and operational risks. This chapter provides extensions to incorporate operational risks into the mathematical models of Chapter 4. Study of operational risks in Chapter 5 is limited to risks due to capacity and demand uncertainty (readers can refer to Chapter 2 for literature on other drivers of operational risks). Capacity risks cover risks due to uncertain manufacturing or service output levels at suppliers. Raw material defects, machine performances, delivery delays from lower tier suppliers and transportation disruptions are among the factors which make capacity at suppliers random. Demand uncertainty, on the other hand, represents unknown customer demand for the products or services offered by the buyer. Consumer preferences, competition and economic uncertainty are among the factors that contribute to demand uncertainty. Forecasting methods can be used to have estimates of demand and capacity and those estimates can be used in decision making. Most forecasting methods provide both the actual forecast and a measure of forecasting error. Using these two parameters, forecasting outputs can be used as inputs to optimization models. However, Chopra & Meindl (2006) makes the following comment about the use of demand forecasts in actual decision making practices,

> *"...thus the forecast error (of demand) must be a key input into most supply chain decisions. Unfortunately most firms do not maintain any estimate of forecast error."*

Models developed in this chapter assume that the buyer has some information about the uncertain capacity and demand; either the entire distribution governing these uncertain inputs or at least the mean and the standard deviation of capacity and demand random variables. Under this assumption, one can use *Stochastic Programming* (SP) approaches to extend the SSA models to treat cases with operational risks.

There exists two major approaches used in modeling SPs. The first approach, *chance constrained programming* (also called as *probabilistically constrained programming*) introduced by Charnes & Cooper (1959), relaxes the deterministic constraints in traditional mathematical programming and replaces them with probabilistic constraints, where some or all data elements are random and the constraints are required to hold with at least some level of reliability $\alpha < 1$. Note that a chance constraint with $\alpha = 1$ is equivalent to a deterministic constraint. Chance constraints are used in this chapter to incorporate capacity and demand uncertainty into the deterministic SSA models of Chapter 4. A detailed introduction to chance constraints is given in Section 5.2. The second SP approach that can be used is called *recourse models*. Recourse programs are two-stage (or multi-stage) mathematical models where decisions made in the first stage are corrected in the second stage (or later stages) based on the realizations of the problem's random components. Correction in these models, or the recourse, is done at the expense of some cost [1]. Recourse models are not used in this chapter; however, the same SSA models can also be extended to recourse problems as well. Interested readers are referred to Kall & Wallace (1994) and Birge & Louveaux (1997) for an introduction to recourse problems and references therein for more advanced topics related to that SP approach.

This chapter is organized as follows. The following section introduces chance constrained models and covers the literature on chance constrained programming methods and applications. Note that a review of the literature on models with operational uncertainty is given in Chapter 2. Section 5.3 extends the SSM formulation to a chance constrained optimization model under distribution assumptions. Section 5.4 proposes a linearization procedure to transform the nonlinear deterministic equivalents to the stochastic SSM model and provides a numerical example. Sections 5.5 and 5.6 provide similar extensions to the MSM formulation. Section 5.7 relaxes the assumptions on the capacity and demand distributions and presents more general extensions with a numerical example. Section 5.8 concludes the chapter.

[1] Cost is used as a general terms that decreases the quality of a model's solution

5.2 A Introduction to Chance Constraints and Literature Review

The generic form of probabilistic constraints can be stated as in Equations (5.1) and (5.2) (Kall & Wallace 1994).

$$\{x|P\left[(\xi \mid T_i(\xi)x \geq b_i(\xi))\right] \geq \alpha_i\} \tag{5.1}$$

and

$$\{x|P\left[(\xi \mid T(\xi)x \geq b(\xi))\right] \geq \alpha\} \tag{5.2}$$

Equation (5.1) presents *separate chance constraints* where each chance constraint needs to hold independent of others. In (5.1), ξ is the random variable affecting the **T** matrix (also called the *technology matrix*) and the right-hand-side vector **b**. Separate chance constraints are introduced in Charnes & Cooper (1959). Equation (5.2) represents *joint chance constraints* where all chance constraints need to hold together. Joint chance constraints are introduced in Miller & Wagner (1965). There exists deterministic equivalent transformations for both type of chance constraints under some assumptions discussed in Kall & Wallace (1994), Prekopa (1995) and Birge & Louveaux (1997). One particular problem structure of interest, chance constrained programming with binary and continuous decision variables, is first studied in Hillier (1967). Several papers to date have been published on the latter topic, including Armstrong & Balintfy (1975), Colome et al. (2003) and Agpak & Gokcen (2007).

Chance constraints can be separated in at least two categories with respect to the components of the model affected by uncertainty. The fist category deals with problems where uncertainty is only in the right-hand-side of a stochastic constraint. The seminal paper by Charnes & Cooper (1959) considers such chance constraints. When uncertainty affects the right-hand-side parameters only, and if constraints of the problem are to be treated separately, deterministic equivalent transformations do not introduce additional nonlinearities to the original problem. However, when chance constraints need to hold together, deterministic equivalents of the joint chance constraints become nonlinear. The second category is concerned with problems where uncertainty is in the left-hand-side of the stochastic constraint. This case is first studied in Charnes & Cooper (1963). When the technology matrix $\mathbf{T}$ is random, deterministic equivalent transformations introduce additional nonlinearity to the original problem. That is, a stochastic LP or MIP would yield a deterministic NLP or NLMIP after the transformation. Note that in equations (5.1) and (5.2) above, both the technology matrix and the right-hand-side vectors are random since they are functions of the random variable ξ. A third possible category covers problems where objective function coefficients are random. A convenient way to handle the third case is to use the expected value of the random coefficients in the objective function and carry out an expected value optimization as suggested in Charnes & Cooper (1963). Recently, robust optimization techniques have been developed to optimize expected values while minimizing the variance (Greenberg & Morrison 2008).

Recent developments in chance constrained programming are mainly in generalization of chance constraints and relaxation of restrictive distribution assumptions. There has also been some previous work on bounds that we will review first. Research on bounds was motivated by the distribution restrictions that initial chance constrained models had. The aim of bounds is to provide upper and lower limits to chance constraints without any major assumptions on the distributions of random variables appearing in the constraints. The work in this area is motivated by Hoeffding's work on bounds on sums of random variables (Hoeffding 1963). The major reference on bounds is Pinter (1989), where the author provides upper bounds based on Chebyshev's and Benstein's inequalities for constraints involving both a single random variable and sums of random variables. A discussion on lower bounds can be found in Birge & Louveaux (1997). Although bounds present a relatively easy way to remove restrictive distribution assumptions, they only provide approximations and the quality of the approximation is debatable. Recently, several researchers developed exact formulations and computational techniques to accommodate any distribution in chance constrained programming. Calafiore & El Ghaoui (2006) first generalize chance constraints to a large family of distributions called *radial distributions*. They then provide deterministic equivalents for cases where the mean and the variance of the distribution is known and later when only the range of the distribution is known. Klopfenstein (2009) develops a linear deterministic equivalent under normality assumption for a simple chance constraint with binary variables. He then provides a branch and cut algorithm to solve chance constrained programs with less restrictive assumptions. The work in this chapter is a generalization of the linearization in Klopfenstein (2009) to mixed integer programming problems. The new deterministic model is linear in the decision variables (all decision variables will be binary). Linearization requires some knowledge of the distribution function but does not necessarily require the Normality assumption. Extensions that work under less restrictive conditions are also formulated using the results in Calafiore & El Ghaoui (2006).

The following sections present several chance constrained extensions to the SSM and MSM formulations developed in Chapter 4. We assume that demand for products and capacity at suppliers are random variables. Chance constraints in

SSM are of category two, that is, the problem uncertainty affects the technology matrix. Hence, deterministic equivalents for SSM, in general, are nonlinear MIPs. Further note that demand uncertainty in SSM affects the first objective function (minimization of cost); therefore, the deterministic equivalent of the SSM objective function will be an expected value optimization. In the case of MSM, demand uncertainty affects the right-hand-side of constraint (5.29); hence this is a category one chance constraint and the deterministic equivalent will be linear for the separate case. Capacity uncertainty, on the other hand, affects the technology matrix and will yield a nonlinear deterministic equivalent. Linearization of all nonlinear deterministic equivalents is discussed later in the chapter.

5.3 Extensions to the SSM Formulation under the Normality Assumption

5.3.1 Demand Uncertainty

Demand uncertainty refers to uncertainty in the market where the buyer is operating. The buyer has no control over demand fluctuations unless effective policies such as promotions, sales and marketing campaigns are implemented. A good forecasting system can help the buyer to predict demand more accurately; however, would not completely remove uncertainty.

Both separate and joint chance constraints can be used to model demand uncertainty. Practical implications of each type of constraint would be the following:

- If demand for every product should be met at a different level of reliability α, then *separate chance constraints* may be more suitable.

- If demand for all products follows the same distribution and has to be met at the same α level, then *joint chance constraints* may be more suitable.

- Moreover, if products can be grouped in such a way that demand for products in a group can assumed to be the same, then *joint chance constraints* can be applied to individual groups. This may be useful in practice since large firms tend to cluster their products (e.g. ABC classification) and serve each cluster's demand separately.

For illustration, we start by demonstrating chance constraints and deterministic equivalents of the SSM formulation. Let us present the SSM formulation below for completeness.

$$\text{min} z_1^{SSM} = \sum_{j\in J}\sum_{k\in K}\sum_{r=1}^{m'} c_{jkr} D_k x_{jkr} + \sum_{j\in J}\sum_{k\in K}\sum_{r=1}^{m'} F_{jr} x_{jkr} \tag{5.3}$$

$$\text{max} z_2^{SSM} = \sum_{j\in J}\sum_{k\in K}\sum_{r=1}^{m'} Q_{jkr} x_{jkr} \tag{5.4}$$

$$\text{min} z_3^{SSM} = \sum_{j\in J}\sum_{k\in K}\sum_{r=1}^{m'} L_{jkr} x_{jkr} \tag{5.5}$$

$$\text{min} z_4^{SSM} = \sum_{j\in J}\sum_{k\in K}\sum_{r=1}^{m'} \rho_j x_{jkr} \tag{5.6}$$

s.t.

$$\sum_{j\in J} x_{jkr} = 1 \quad \forall k \in K,\ r = 1, ..., m' \tag{5.7}$$

$$\sum_{r=1}^{m'} x_{jkr} = 1 \quad \forall j \in J,\ \forall k \in K \tag{5.8}$$

$$x_{jkr}(Cap_{jk} - D_k) \geq 0 \quad \forall j \in J,\ \forall k \in K,\ r = 1, ..., m' \tag{5.9}$$

$$x_{jkr} \in \{0,\ 1\} \quad \forall j \in J,\ \forall k \in K,\ r = 1, ..., m' \tag{5.10}$$

In the above formulation, constraints (5.9) guarantee that if a supplier j is chosen for product k at level $r = 1$ (primary), then he should have enough capacity to supply the entire demand for that product. We can expand constraints (5.9) for the first product, that is for $k = 1$, as below.

$$\begin{aligned}
&x_{111}(Cap_{11} - D_1) \geq 0 \\
&x_{112}(Cap_{11} - D_1) \geq 0 \\
&... \\
&x_{11m'}(Cap_{11} - D_1) \geq 0 \\
&x_{211}(Cap_{21} - D_1) \geq 0 \\
&x_{212}(Cap_{21} - D_1) \geq 0 \\
&... \\
&x_{21m'}(Cap_{21} - D_1) \geq 0 \\
&...
\end{aligned}$$

Decision variables x have the same coefficient $Cap_{11} - D_1$ in the first m' equations above. We can observe the same pattern in the remaining equations: For instance, decision variables x_{211} to $x_{21m'}$ have a same coefficient of $Cap_{21} - D_1$. We can use this pattern to re-index the problem data to fit the general form of chance constraints [2]. Let $\theta \equiv (j, k)$; therefore, we can now denote x_{jkr} as $x_{\theta r}$ when

[2]Indeed, the change of index is not essential. It is enough to note that the decision variable coefficient depends on indices (j, k) and is independent of index r. When the stochastic extension is carried out, the mean value of the coefficient will be the same through each (j, k) combination. That is, the mean parameter of the distribution defining the stochastic part can be indexed over combinations of (j, k). The θ index introduced next replaces combinations of (j, k). This transformation will help to explicitly have a mean vector rather than a mean matrix for the multivariate distribution required to model joint chance constraints

necessary. Care must be taken to map all (j, k) index combinations to θ. The size of the index set $\theta \in \Theta$ is $|J| \times |K|$ so that all (j, k) combinations can be properly represented using θ.

Following our discussion above, constraint (5.9) can be extended to accommodate demand and capacity uncertainty. For now, let us assume that capacity is deterministic and demand for product k is a normal random variable with mean μ_{D_k} and covariance matrix Σ_{D_k}. Then, the decision variable coefficient in constraint (5.9) is a normally distributed random variable $\mathrm{Y}_{jk}x_{jkr}$ with mean $\mu_{Y_{jk}} = (Cap_{jk} - \mu_{D_k})x_{jkr} = \mu x$ and covariance $\Sigma_{Y_{jk}}x = x^T\Sigma_{D_k}x$. Moreover, let us impose the condition that constraint (5.9) should hold with at least $100\alpha\%$ probability where $\alpha \in (0, 1)$. Under these conditions we can write the chance constrained counterpart of (5.9) as follows,

$$P[(Y_{jk}x_{jkr} \geq 0)] \geq \alpha \quad \forall j \in J,\ \forall k \in K,\ r = 1, ..., m' \tag{5.11}$$

We can now use the θ notation to simplify (5.11) to

$$P[(Y_\theta x_{\theta r} \geq 0)] \geq \alpha \quad \forall \theta \in \Theta,\ r = 1, ..., m' \tag{5.12}$$

We can write a deterministic equivalent for constraint (5.12) after dropping the indices,

$$\begin{aligned} P[\mathbf{Y}x \geq 0)] &= P\left[\frac{\mathbf{Y}x - \mu x}{\sqrt{x^T\Sigma x}} \geq \frac{-\mu x}{\sqrt{x^T\Sigma x}}\right] \\ &= 1 - \Phi\left(\frac{-\mu x}{\sqrt{x^T\Sigma x}}\right) \end{aligned} \tag{5.13}$$

According to constraint (5.12), the deterministic equivalent in (5.13) should hold with probability greater than or equal to $100\alpha\%$. Then, we can rearrange the terms in (5.13) as follows,

$$
\begin{aligned}
1 - \Phi\left(\frac{-\mu x}{\sqrt{x^T \Sigma x}}\right) \geq \alpha \Leftrightarrow \Phi\left(\frac{-\mu x}{\sqrt{x^T \Sigma x}}\right) &\leq (1-\alpha) \\
\Leftrightarrow \frac{-\mu x}{\sqrt{x^T \Sigma x}} &\leq \Phi^{-1}(1-\alpha) \\
\Leftrightarrow -\mu x &\leq \Phi^{-1}(1-\alpha)\sqrt{x^T \Sigma x} \\
\Leftrightarrow &\boxed{\mu x + \Phi^{-1}(1-\alpha)\sqrt{x^T \Sigma x} \geq 0}
\end{aligned}
$$

Note that the deterministic equivalent obtained above is nonlinear since it involves the product of decision variables under the square root sign. However, it has been proved that (see for instance Kall & Wallace (1994)) if the decision variables were continuous, then the deterministic equivalent would define a convex region under the assumptions we make, provided $\alpha > 0.5$, which usually is the case. This implies that any linear relaxation of the SSM formulation with the above nonlinear constraint would be a continuous convex nonlinear program; nevertheless this does not necessarily imply any triviality in optimization. Furthermore, note that constraints (5.11) and (5.12); and therefore the deterministic equivalent, assumes that the chance constraints would hold jointly. Translating to supply chain operations, the deterministic equivalent asks for a minimum $100\alpha\%$ service level for all items from all suppliers. This restriction can be relaxed by setting different service level requirements for every item by grouping supply constraints with respect to index k and by setting an α_k service level for each group. We can as well carry out another relaxation with respect to suppliers in a similar manner.

5.3.2 Demand and Capacity Uncertainties

This section extends the demand uncertainty model introduced in Section 5.3.1 to accommodate uncertainty both in demand and capacity within the SSM framework. Capacity uncertainty introduces variance in output performance of suppliers and reflects operational uncertainty introduced through outsourcing. Like the demand uncertainty, capacity uncertainty is external to the buyer, but can be managed through accurate supplier selection and mitigation practices.

Let us assume that output capacity at supplier j for product k, Cap_{jk} follows a normal distribution with mean $\mu_{Cap_{jk}}$ and standard deviation $\sigma_{Cap_{jk}}$. We also assume, as in Section 5.3.1, that demand is normally distributed with mean μ_{D_k} and standard deviation σ_{D_k}. Hence, the random variable $G_{jk} = (Cap_{jk} - D_k)$ obeys the normal distribution

$$N\left(\mu_G, \sigma_G^2\right) = N\left(\mu_{Cap_{jk}} - \mu_{D_k}, \sigma^2_{Cap_{jk}} + \sigma^2_{D_k}\right)$$

We can then apply the index transformation and calculation steps as in Section 5.3.1 to write the deterministic equivalent to constraint (5.9) as

$$P\left[Gx \geq 0\right] \geq \alpha \Leftrightarrow \boxed{\mu_G x + \Phi^{-1}(1-\alpha)\sqrt{x^T \Sigma_G x} \geq 0} \qquad (5.14)$$

where Σ_G is the covariance matrix of the random variable G. Same remarks on convexity and extensions made at the end of Section 5.3.1 are valid for the deterministic equivalent in (5.14).

5.4 Linearization of the SSM with Separate Chance Constraints

5.4.1 SSM with Demand and Supply Uncertainty

This section provides a linearization scheme for separate chance constraints involving the SSM binary variables under normal demand and capacity assumptions. We first present a general linearization method which can be used to linearize any deterministic equivalent of type (5.14) and then provide a simpler linearization to accommodate the stochastic SSM formulation.

Consider a pure binary integer stochastic constraint and its deterministic equivalent below where stochastic elements a_{ij} follow a normal distribution $N(\mu_{ij}, \sigma_{ij}^2)$,

$$\begin{aligned} &\mathbf{P}\left(\sum_j a_{ij}x_{ij} \geq 0\right) \geq \alpha_i \\ &\Rightarrow \sum_j \mu_{ij}x_{ij} + \Phi^{-1}(1-\alpha_i)\sqrt{\sum_j \sum_k cov_{ijk}x_{ij}x_{ik}} \geq 0 \quad \forall i \end{aligned} \tag{5.15}$$

In (5.15), Φ is the standard normal cumulative distribution function and cov_{ijk} is the covariance between a_{ij} and a_{jk} The deterministic equivalent in (5.15), assuming uncorrelated random variables, can be linearized as below,

$$\begin{aligned} &\sum_j \mu_{ij}x_{ij} + \Phi^{-1}(1-\alpha_i)\sqrt{\sum_j \sum_k cov_{ijk}x_{ij}x_{ik}} \geq 0 \\ &\Leftrightarrow \sum_j \mu_{ij}x_{ij} + \Phi^{-1}(1-\alpha_i)\sqrt{\sigma_{ij}^2 x_{ij}^2} \geq 0 \\ &\Leftrightarrow \sum_j \mu_{ij}x_{ij} + \Phi^{-1}(1-\alpha_i)\sigma_{ij}x_{ij} \geq 0 \quad \forall i \end{aligned} \tag{5.16}$$

x_{ij}^2 terms in the last equivalence in (5.16) can be taken out of the square root as

x_{ij} (without absolute value) since these are binary variables, therefore are positive. The standard deviation term σ_{ij} is also positive.

Assuming no correlation between random variables can be acceptable in several cases in the SSA models. First, capacity for different products at a same supplier can be assumed independent if for instance production or service lines are dedicated. Demand and capacity for a product can be assumed independent if production or service providing capacity for that product does not change with the demand (at least in short term) or if the amount of output does not influence the demand. In other cases, assuming no correlation may lead to incorrect solutions. The nonlinear deterministic equivalent in (5.15), when correlation among random variables is verified, can be linearized as follows,

$$\begin{aligned} & \sum_j \mu_{ij} x_{ij} + \Phi^{-1}(1-\alpha_i)\sqrt{\sum_j \sum_k cov_{ijk} x_{ij} x_{ik}} \geq 0 \\ \Leftrightarrow & \sum_j \mu_{ij} x_{ij} \geq -\Phi^{-1}(1-\alpha_i)\sqrt{\sum_j \sum_k cov_{ijk} x_{ij} x_{ik}} \\ \Leftrightarrow & \left(\sum_j \mu_{ij} x_{ij}\right)^2 \geq \left(\Phi^{-1}(1-\alpha_i)\right)^2 \sum_j \sum_k cov_{ijk} x_{ij} x_{ik} \end{aligned} \tag{5.17}$$

Note that the inverse of the standard normal CDF, $\Phi^{-1}(t)$, is less than zero for $t < 0.5$, which will be verified in our models since we assume a reliability level α close to one. If $\Phi^{-1} < 0$, then the right-hand-side of the second inequality in (5.17) is greater than or equal to zero, and therefore, we can write the last inequality by squaring both sides.

The terms on the left and right hand sides in the last inequality in (5.17) are nonlinear in the decision variables as they are stated. Although the model can be solved as a nonlinear binary program, a linearization method is provided in the following to solve the model as a mixed integer binary program. It is important to note that the deterministic equivalent in (5.17) is convex as long as $\alpha_i > 0.5$ [3]. Note that nonlinear binary programs are in general less tractable compared to mixed integer binary programs and algorithms for the former can yield suboptimal solutions. The linearization procedure described below resolves this issue at the expense of model size, nevertheless commercially available optimization software can efficiently optimize large size mixed integer binary programs.

Let us consider the term on the left-hand-side first. When the square of the summation is expended, $|J|$ terms involving x_{ij}^2 and $\binom{|J|}{2}$ terms involving $x_{ij}x_{ik}$ $j \neq k$, where $|J|$ is the cardinality of the index set j, will appear. The x_{ij}^2 terms are equivalent to x_{ij} $\forall i, j$ since x_{ij} are binary variables. The product terms can be linearized as discussed in Glover & Woolsey (1974) and Ravindran et al. (1987) by replacing the product with a new positive continuous variable x_{ijk} and by adding the following set of constraints for each product term,

$$
\begin{aligned}
x_{ij} + x_{ik} - x_{ijk} &\leq 1 \\
x_{ijk} &\leq x_{ij} \\
x_{ijk} &\leq x_{ik}
\end{aligned}
$$

The above method can also be used to linearize the cross products at the right-hand-side of Equation (5.17). Note that this linearization method introduces additional constraints and continuous variables to the original model formulation. As discussed above, the expansion of the square of the summation will add $\binom{|J|}{2}$ $x_{ij}x_{ik}$ terms. Each product term will require one new continuous variable and three constraints as shown above. Therefore, the linearization will bring in $\binom{|J|}{2}$ new continuous variables and $3\binom{|J|}{2}$ new constraints. Note that the product terms

[3]More accurately, the continuous linear relaxation of (5.17) obtained during the solution procedure is convex

from the left-hand-side and right-hand-side expansions are the same. The left-hand-side expansion will result in x_{ij}^2 terms which can be linearized without any additional variables or constraints.

Results derived above can be extended to the SSM with stochastic capacity and demand data. Let Cap_{jk} follow the normal distribution $N\left(\mu_{jk}^{Cap}, \left(\sigma_{jk}^{Cap}\right)^2\right)$ and let D_k be normally distributed as $N\left(\mu_k^D, \left(\sigma_k^D\right)^2\right)$. Note that under these settings, uncertainty affects the cost objective function in (5.3) and constraint (5.9). The cost objective can be written as an expected value function as suggested in Charnes & Cooper (1963),

$$\min \mathbf{E}\left(z_1^{SSM}\right) = \sum_{k \in K} \mu_k^D \sum_{j \in J} \sum_{r=1}^{m'} c_{jkr} x_{jkr} + \sum_{j \in J} \sum_{k \in K} \sum_{r=1}^{m'} F_{jr} x_{jkr} \tag{5.18}$$

A linear deterministic equivalent for constraints (5.9) can be derived when stochastic constraints are to hold separately using relations in Equations (5.16) or (5.17), depending on the correlation assumptions. Note that decision variables are not summed in constraints (5.9), making the linearization simpler than discussed above. The probabilistic counterpart of (5.9) is $\mathbf{P}\left[\left(Cap_{jk} - D_k\right) x_{jkr}\right] \geq \alpha_{jkr}$ where α_{jkr} is the level of reliability. Let $g_{jk} = \left(Cap_{jk} - D_k\right) \ \forall (j,k)$. Then, $g_{jk} \sim N(\mu_{jk}, \sigma_{jk}^2)$ where $\mu_{jk} = \mu_{jk}^{Cap} - \mu_k^D$ and $\sigma_{jk}^2 = (\sigma_{jk}^{Cap})^2 + (\sigma_k^D)^2$. Using g_{jk} and assuming uncorrelated random variables, we can write a linear deterministic equivalent for the stochastic counterpart of constraint (5.9) as below,

$$\begin{aligned} &\mu_{jk} x_{jkr} + \Phi^{-1}\left(1 - \alpha_{jkr}\right) \sqrt{\sigma_{jk}^2 x_{jkr}^2} \geq 0 \\ &\Rightarrow \mu_{jk} x_{jkr} + \left(\Phi^{-1}\left(1 - \alpha_{jkr}\right)\right) \sigma_{jk} x_{jkr} \geq 0 \quad \forall j \in J,\ \forall k \in K,\ r = 1, ..m' \end{aligned} \tag{5.19}$$

The deterministic equivalent obtained in (5.19) is linear in the decision variables.

5.4.2 Stochastic SSM Numerical example

This section presents a numerical implementation of the chance constrained SSM formulation. Linearized deterministic equivalents discussed in Section 5.4.1 are used to formulate a non-preemptive GP version of the SSM as below,

$$\min w_1 d_1^+ + w_2 d_2^- + w_3 d_3^+ + w_4 d_4^+$$

subject to

$$\mathbf{E}\left(z_1^{SSM}\right) + d_1^- - d_1^+ = T_1^{SSM}$$

$$z_2^{SSM} + d_2^- - d_2^+ = T_2^{SSM}$$

$$z_3^{SSM} + d_3^- - d_3^+ = T_3^{SSM}$$

$$z_4^{SSM} + d_4^- - d_4^+ = T_4^{SSM}$$

Constraints in (5.7), (5.8), (5.10) and (5.19) are to be included as well.

As in Chapter 4, T_i^{SSM} represent target values and d_i^- and d_i^+ are decision variables to model negative and positive deviations from targets. The parameters w_i are weights associated with the deviation variables. Cost, quality, lead time, risk and weight parameters introduced in Tables 4.4 to 4.9 in Chapter 4 are used in the stochastic SSM example. Stochastic capacity and demand data given in Tables 5.1 and 5.2 are used instead of deterministic data of Chapter 4.

Let us present the linearization of a chance constraint to illustrate our methodology. Consider the deterministic capacity-demand constraint in Equation (5.9) for supplier 1, product 1 at level 1. The constraint is written as below,

$$x_{111}\left(Cap_{11} - D_1\right) \geq 0$$

The probabilistic counterpart of the deterministic constraint can be expressed as,

$$P(G_{111}x_{111}) \geq \alpha_{111} \tag{5.20}$$

where the random variable G introduced in Section 5.3.2 denotes the difference between the capacity and demand random variables. Using the transformation in Equation (5.19) with the stochastic capacity and demand data in Tables 5.1 and 5.2 and assuming $\alpha_{111} = 0.95$, the probabilistic constraint in Equation (5.20) can be deterministically expressed ad follows:

$$\begin{aligned} &(220 - 210)x_{111} + \left[\Phi^{-1}(1 - 0.95)\right]\left(\sqrt{25 + 36}\right)x_{111} \geq 0 \\ &\Rightarrow -2.847x_{111} \geq 0 \end{aligned} \tag{5.21}$$

The deterministic equivalent constraint in Equation (5.21) can hold if and only if the binary x_{111} variable equals 0; that is, only if supplier 1 is not a level 1 supplier of product 1. Note that capacity and demand data do not depend on assignment levels. Furthermore, since we assume the same confidence level α at all levels for supplier 1 - product 1 assignments, Equation (5.21) indicates that supplier 1 will not be at any level of product 1. This result can also be verified from the problem's optimal solution given in Table 5.3.

For the stochastic SSM example, all α_{jkr} values are set at 0.95, meaning that suppliers assigned at all levels should have enough capacity to meet the demand with at least 95% probability. We set $m' = 4$ to allow four levels of supplier assignment; one primary and three backups. Target values are set at 90% of ideal values. Ideal values are calculated by optimizing each objective function independently of others. Each objective function is divided by its ideal value for scaling in the non preemptive GP solution process. The optimal solution is displayed in Table 5.3.

Table 5.1. SSM Stochastic Capacity Data

	Product		
Supplier	1	2	3
1	N(220,25)	N(250,25)	N(300,25)
2	N(250,36)	N(350,36)	N(300,36)
3	N(250,9)	N(270,9)	N(260,9)
4	N(300,49)	N(400,49)	0
5	N(230,4)	N(300,4)	N(300,4)

Table 5.2. SSM Stochastic Demand Data

Product	1	2	3
Demand	N(210,36)	N(250,49)	N(250,64)

Table 5.3. Stochastic SSM Solution

	Levels				
Supplier	*1*	*2*	*3*	*4*	**Product**
1	0	0	0	0	1
	0	0	0	0	2
	0	1	0	0	3
2	0	0	0	1	1
	0	1	0	0	2
	1	0	0	0	3
3	0	1	0	0	1
	0	0	1	0	2
	0	0	1	0	3
4	0	0	1	0	1
	0	0	0	1	2
	0	0	0	0	3
5	1	0	0	0	1
	1	0	0	0	2
	0	0	0	1	3

The optimal solution can be interpreted as follows. The *Level 1* column of Table 5.3 lists the primary suppliers. Supplier 2 is the primary supplier for product 3 and supplier 5 is primary for products 1 and 2. As enforced by constraint (5.8), supplier 2 does not appear at any other level of product 3 and supplier 5 is not a backup supplier of products 1 and 2. Third, fourth and fifth columns of Table 5.3 display backup supplier assignments. It can be observed that there is one and only one backup supplier at each level for each product (as enforced by constraints (5.7) and (5.8)). In summary, product 1 is provided by supplier 5 and suppliers 3, 4 and 2 are backups in that order. Let us denote the assignment for product 1 as $(\mathbf{5}; 3-4-2)$. Assignments for product 2 are $(\mathbf{5}; 2-3-4)$ and for product 3 we have $(\mathbf{2}; 1-3-5)$.

The optimal solution in Table 5.3 achieves all objective targets. It can be observed that suppliers 2 and 5 are used as primary suppliers, whereas suppliers 1, 3 and 4 are used only as backups. The optimal assignment arises from the combination of various factors including input data, target values and DM weights. Also, note that supplier 4 does not appear at any level of product 3 since that supplier does not manufacture product 3 (see Table 5.1). A detailed solution report with ideal values, targets, objective achievements and optimal values is given in Table 5.4.

Since we assume no correlation, the problem's size is the same as the deterministic problem solved in Chapter 4. We transformed all probabilistic capacity - demand constraints to deterministic equivalents to have 60 deterministic equivalent constraints. The stochastic model was initially solved using the deterministic demand and capacity data presented in Chapter 4 as the mean of the stochastic data and with randomly generated integer variance values. The initial data yielding an infeasible solution, two capacity data elements, Cap_{23} and Cap_{51} were

Table 5.4. Stochastic SSM Results

Objective	**Ideal**	**Target**	**Status**	**Achievement**
Cost	33911.92	37303.11	Achieved	35011.86; (5510, 440)
Quality	10.19	9.17	Achieved	10.17; (2.81)
Lead time	56.62	62.29	Achieved	57.51; (18)
Risk	7753463.00	8528809.30	Achieved	7753463.21; (2433952.50)

increased to 300 and 230 respectively. (These two data elements were set at 250 and 200 in the deterministic data set used in Chapter 4). The deterministic model, when solved with this new data set, achieves all objective targets (all but the risk target were achieved in the original optimal solution). Finally, we put the deterministic non preemptive SSM optimal solution into the stochastic MSM described here to test the deterministic model's performance under stochastic conditions. The deterministic optimal solution turns out to be infeasible for the stochastic model. We then solve the deterministic SSM with the increased mean capacity data used in the stochastic SSM model and plug the new deterministic optimal solution into the stochastic model. The deterministic optimal supplier assignments remain infeasible under stochastic conditions.

5.5 Extensions to the MSM Formulation under the Normality Assumption

Let us present below the MSM formulation for convenience.

$$\min\ z_1^{MSM} = \left[\sum_{j\in J}\sum_{k\in K} c_{jk1}h_{jk1} + \sum_{j\in J}\sum_{k\in K}\sum_{r=2}^{m'-p+1} c_{jkr}x_{jkr}\right] + \sum_{j\in J}\sum_{k\in K}\sum_{r=1}^{m'-p+1} F_{jr}x_{jkr} \tag{5.22}$$

$$\max\ z_2^{MSM} = \sum_{j\in J}\sum_{k\in K} Q_{jk1}h_{jk1} + \sum_{j\in J}\sum_{k\in K}\sum_{r=2}^{m'-p+1} Q_{jkr}x_{jkr} \tag{5.23}$$

$$\min\ z_3^{MSM} = \sum_{j\in J}\sum_{k\in K} L_{jk1}h_{jk1} + \sum_{j\in J}\sum_{k\in K}\sum_{r=2}^{m'-p+1} L_{jkr}x_{jkr} \tag{5.24}$$

$$\min z_4^{MSM} = \sum_{j\in J}\sum_{k\in K} \rho_j h_{jk1} + \sum_{j\in J}\sum_{k\in K}\sum_{r=2}^{m'-p+1} \rho_j x_{jkr} \tag{5.25}$$

s.t.

$$\sum_{j\in J} x_{jk1} \leq p \quad \forall k \in K \tag{5.26}$$

$$\sum_{j\in J} x_{jkr} = 1 \quad \forall k \in K,\ r = 2, ..., m' - p + 1 \tag{5.27}$$

$$\sum_{r=1}^{m'-p+1} x_{jkr} = 1 \quad \forall j \in J,\ \forall k \in K \tag{5.28}$$

$$\sum_{j\in J} h_{jk1} = D_k \quad \forall k \in K \tag{5.29}$$

$$h_{jk1} \leq Cap_{jk}x_{jk1} \quad \forall j \in J,\ \forall k \in K \tag{5.30}$$

$$x_{jkr} \in \{0,\ 1\} \quad \forall j \in J,\ \forall k \in K,\ r = 1, ..., m' - p + 1 \tag{5.31}$$

$$h_{jkr} \geq 0 \quad \forall j \in J,\ \forall k \in K,\ r = 1, ..., m' - p + 1 \tag{5.32}$$

Following our discussion in Section 5.2, constraints (5.29) and (5.30) can be extended to probabilistic constraints.

5.5.1 Demand Uncertainty

Assume that there are m suppliers ($j \in \{1, ..., m\}$) and n products ($k \in \{1, ..., n\}$). Demand constraints in Equation (5.29) can be expanded as follows,

$$\begin{aligned} &h_{111} + h_{211} + h_{311} + ... + h_{m11} = D_1 \\ &h_{121} + h_{221} + h_{321} + ... + h_{m21} = D_2 \\ &... \\ &h_{1n1} + h_{2n1} + h_{3n1} + ... + h_{mn1} = D_n \end{aligned} \tag{5.33}$$

Separate chance constraints

Assume that demand for product k, D_k, follows a probability distribution with cdf F_{D_k} and that we are concerned with satisfying the demand for each product independently. Using (5.33) we can model this situation with the separate chance constrains as below,

$$P\left(\sum_{j \in J} h_{jk1} \geq D_k\right) \geq \alpha_k \quad \forall k \in K \tag{5.34}$$

Deterministic equivalent to the chance constraints in (5.34) is simply

$$F_{D_k}\left(\sum_{j \in J} h_{jk1}\right) \geq \alpha_k \Leftrightarrow \sum_{j \in J} h_{jk1} \geq F_{D_k}^{-1}(\alpha_k) \quad \forall k \in K \tag{5.35}$$

Note that $F_{D_k}^{-1}$ is a constant for a given α_k. The deterministic equivalent in (5.35) preserves the linearity of (5.34).

Joint chance constraints

In the case of joint chance constraints, we enforce the demand for all products (or at least for a group of products) to be satisfied together with some probability $100\alpha\%$. We can model this situation using the joint chance constrains as below,

$$P\left(\sum_{j\in J} h_{jk1} \geq D_k\right) \geq \alpha \quad \forall k \in K \tag{5.36}$$

To write the matrix notation of (5.36), let us note that the technology matrix $\mathbf{T}$ in Equation (5.2) corresponds to a matrix of ones in our case, i.e., $\mathbf{T} \equiv 1_{n,m}$. Again, following the notation in (5.2), decision variable x refers to h in our model and the right hand side vector $\mathbf{b}(\xi)$ is the random demand vector which can be properly expressed as $\mathbf{D}(\xi)$. Finally, the random variable ξ represents the demand process. Joint chance constraints for demand can be written as follows,

$$P(1_{n,m}h \geq \mathbf{D}) \geq \alpha \tag{5.37}$$

Under the assumption of logconcave distributions (see Prekopa (1995) for instance), the deterministic equivalent to (5.37) is

$$\ln\left[F_D(h)\right] \geq \ln(\alpha) \tag{5.38}$$

Obviously, the deterministic equivalent in (5.38) is no longer linear since decision variables appear under the logarithm function.

5.5.2 Capacity Uncertainty

Lastly, capacity in the MSM formulation can be assumed uncertain. Note that in MSM, capacity and demand constraints are separated; therefore, any deterministic equivalent transformation we carry out here will not affect our previous results in Section 5.5.1. Let Ψ_{jk} be the random variable denoting the uncertain capacity at which supplier j can provide product k. Setup changeovers, batch quality and worker performance are among the reasons for uncertain capacity. We assume Ψ_{jk} follows the normal distribution $N(\mu_{\Psi_{jk}}, \sigma^2_{\Psi_{jk}})$. Let us assume that the buyer needs an overall safety level α regarding his supply. This situation can be modeled using joint chance constraints as follows:

$$P[\Psi x \geq h] \geq \alpha \tag{5.39}$$

Let μ_Ψ be the mean vector for the multidimensional random variable Ψ and let $\Sigma_\Psi = E[(\Psi - \mu_\Psi)(\Psi - \mu_\Psi)^T]$ be its covariance matrix. Then, after dropping the subscript on the mean and covariance terms, we have,

$$\begin{aligned} P[(\Psi x \geq h)] \geq \alpha = P\left[\frac{(\Psi-\mu)^T x}{\sqrt{x^T \Sigma x}} \geq \frac{h - \mu^T x}{\sqrt{x^T \Sigma x}}\right] \geq \alpha \\ \Leftrightarrow 1 - \Phi\left(\frac{h - \mu^T x}{\sqrt{x^T \Sigma x}}\right) \geq \alpha \\ \Leftrightarrow \Phi\left(\frac{h - \mu^T x}{\sqrt{x^T \Sigma x}}\right) \leq (1-\alpha) \\ \Leftrightarrow \frac{h - \mu^T x}{\sqrt{x^T \Sigma x}} \leq \Phi^{-1}(1-\alpha) \\ \Leftrightarrow h - \mu^T x \leq \Phi^{-1}(1-\alpha)\sqrt{x^T \Sigma x} \\ \Leftrightarrow \boxed{\mu^T x - h + \Phi^{-1}(1-\alpha)\sqrt{x^T \Sigma x} \geq 0} \end{aligned} \tag{5.40}$$

As previously, the deterministic equivalent derived in (5.40) is nonlinear. If the buyer needs separate safety levels for each product from all suppliers or for all products from each supplier, the probabilistic constraints in (5.39) should be written using separate constraints for each product.

5.6 Linearization of the MSM with Separate Chance Constraints

5.6.1 MSM with Demand and Supply Uncertainty

This section addresses the linearization of chance constraints in the MSM formulation. As discussed in Section 5.5, two constraints in the MSM, Equations (5.29) and (5.30), can be extended to chance constraints and their general counterparts are discussed in Equations (5.35) and (5.36) for demand uncertainty and in Equation (5.40) for capacity uncertainty. Note that the deterministic equivalent of the separate demand chance constraints in Equation (5.35) is already linear in the model's decision variables. The deterministic equivalent to the capacity chance constraints, on the other hand, is not linear (see Equation (5.40)) and the resulting model can be solved as a nonlinear mixed integer program (NLMIP). However, NLMIPs are in general computationally expensive to solve and available algorithms do not often guarantee optimality. We present next an alternative method to linearize the MSM NLMIP using a method similar to the linearization of Section 5.4.1. The resulting program is a linear mixed integer program which is more tractable compared to NLMIPs. The tradeoff of the linearization is an increase in the model size.

Let us consider the nonlinear deterministic equivalent in Equation (5.40) which can be written in summation form, assuming separate chance constraints and $\alpha_i > 0.5$, as below,

$$\sum_j \mu_{ij}x_{ij} - \sum_j h_{ij} \geq -\Phi^{-1}(1-\alpha_i)\sqrt{\sum_j \sum_k cov_{ijk}x_{ij}x_{ik}}$$
$$\Leftrightarrow \underbrace{\left(\sum_j \mu_{ij}x_{ij}\right)^2}_{1} - \underbrace{2\sum_j \mu_{ij}x_{ij}h_{ij}}_{2} + \underbrace{\left(\sum_j h_{ij}\right)^2}_{3} \geq \tag{5.41}$$
$$\left(\Phi^{-1}(1-\alpha_i)\right)^2 \sum_j \sum_k cov_{ijk}x_{ij}x_{ik}$$

where $x_{ij} \in \{0,1\}$ and $h_{ij} \geq 0\ \forall i,j$. Linearization of (1) in Equation (5.41) has already been discussed in Section 5.4.1. Term (2) involves the product of binary and continuous decision variables and term (3) has the square of a summation of continuous variables. Both terms (2) and (3) in (5.41) can be linearized by replacing the continuous variables with a binary representation and taking advantage of the product of binary variables. The linearization process is described below.

It is a known result in integer programming that any bounded positive integer variable can be represented as a combination of binary decision variables (Salkin & Mathur 1989). Let $h_i \leq u_i$ be a bounded positive integer variable [4]. Then, h_i can be replaced by $\sum_{k=0}^{l_i} 2^k t_{ki}$ where t_{ki} are binary variables and l_i is the smallest integer such that $\sum_{k=0}^{l_i} 2^k = 2^{l_i+1} - 1 \geq u_i$. Thus, instead of using $h_i \leq u_i$, one can use the binary representation of h_i and add the constraint $\sum_{k=0}^{l_i} 2^k t_{ki} \leq u_i$ to remove the continuous variables.

Let us go back to the MSM formulation and develop this idea through an example. Note that we were treating h_{jk1} as continuous variables. For the remainder of this section, we will assume that h_{jk1} are integer variables, which is more natural since h_{jk1} represents the amount of products shipped from a supplier to the buyer, and replace them with a binary representation. Consider supplier 5 and product

[4] We drop the index j for ease of presentation.

1. Assume that the capacity at which supplier 5 can provide product 1, Cap_{51}, follows a $N(200, 4)$ distribution. When capacity is assumed to follow a normal distribution, one can use mean plus three times the standard deviation (note that the probability that the normal random variable is smaller than this quantity is 0.9973) of the capacity random variable as a bound on the relevant h_{jk1}. We therefore can use $u_{51} = 200 + 3 \times 4 = 212$ as an upper bound on Cap_{51}. The next step is to find an integer l_{51} that verifies $2^{l_{51}+1} - 1 \geq u_{k1} = 212$. $l_{51} = 7$ verifies the required criterion [5]; thus, we can replace h_{511} with the following binary expression,

$$h_{511} = \sum_{k=0}^{l_{51}=7} 2^k t_k = t_0 + 2t_1 + 4t_2 + 8t_3 + 16t_4 + 32t_5 + 64t_6 + 128t_7 \tag{5.42}$$

where $t_k \in \{0, 1\}\ \forall k$.

Consider now the product $x_{511}h_{511}$. Using the discretized expression of h_{511} in (5.42), $x_{511}h_{511}$ can be expressed as below,

$$\begin{aligned} x_{511}h_{511} &= x_{511}\left(t_0 + 2t_1 + 4t_2 + 8t_3 + 16t_4 + 32t_5 + 64t_6 + 128t_7\right) \\ &= x_{511}t_0 + 2x_{511}t_1 + 4x_{511}t_2 + 8x_{511}t_3 + 16x_{511}t_4 + 32x_{511}t_5 \\ &+ 64x_{511}t_6 + 128x_{511}t_7 \end{aligned}$$

Now, let us define another set of decision variables, $s_k \geq 0$ for $k \in \{0, ..., 7\}$, such that $s_k \leq x_{511}$, $s_k \leq t_k\ \ \forall k$ and $x_{511} + t_k - s_k \leq 1\ \forall k$. Then,

$$x_{511}h_{511} = s_0 + 2s_1 + 4s_2 + 8s_3 + 16s_4 + 32s_5 + 64s_6 + 128s_7 \tag{5.43}$$

is linear in all new decision variables. The linearization of the product required an

[5]Note that $2^7 = 256$ and the upper bound on $h_{511} = 212 = \mu_{511} + 3\sigma_{511}$. 2^7 is actually larger than $\mu_{511} + 18\sigma_{511}$, which is enough to cover almost 100% of the normal range in question.

additional eight binary decision variables and 24 new constraints.

Theorem 5.1 proves a general result to linearize term (2) in Equation (5.41) using the binary representation of continuous variables as discussed above.

Theorem 5.1. *Consider two variables, a binary variable x and a bounded positive integer variable h discretized as $h = \sum_{k=0}^{l} 2^k t_k$ with $t_k \in \{0,1\}$ and l verifying $2^{l+1} - 1 \geq u$ where u is the smallest integer equal to a power of two with $u \geq h$. Then, the product of x and h can be linearly expressed as*

$$xh = \sum_{k=0}^{l} 2^k s_k \tag{5.44}$$

where $s_k \geq 0$ and $s_k \leq x$, $s_k \leq t_k$ and $x + t_k - s_k \leq 1 \; \forall k \in \{0, ..., l\}$.

Proof. We have previously shown that a positive continuous variable h can be discretized as $h = \sum_{k=0}^{l} 2^k t_k$ with $t_k \in \{0,1\} \; \forall k \in \{0, ...l\}$ where l verifies the Theorem's conditions. Then, the product of x and h can be expressed as below,

$$xh = x\left(\sum_{k=0}^{l} 2^k t_k\right) = \sum_{k=0}^{l} 2^k t_k x \tag{5.45}$$

Let us define $s_k \geq 0$ such that $s_k \leq x$, $s_k \leq t_k$ and $x + t_k - s_k \leq 1 \; \forall k \in \{0, ..., l\}$. Then, using the result in Glover & Woolsey (1974), (5.45) can be linearized as,

$$xh = \sum_{k=0}^{l} 2^k s_k \tag{5.46}$$

□

Term (3) in Equation (5.41), that involves the square of a sum of continuous decision variables, can be also be linearized. Expanding term (3) yields terms in h_i^2 and $h_i h_k$ $i \neq k$. The latter product terms can be treated using Theorem 5.1.

The h_i^2 terms can be linearized using results in Theorem 5.2. Let us first go back to the MSM formulation to motivate Theorem 5.2 and then cite and prove it.

Consider again the decision variable h_{511} as discretized in Equation (5.42).

$$
\begin{aligned}
&h_{511}^2 = (t_0 + 2t_1 + 4t_2 + 8t_3 + 16t_4 + 32t_5 + 64t_6 + 128t_7)^2 \\
&= t_0^2 + 2t_1(2t_0 + 2t_1) + 4t_2(2t_0 + 4t_1 + 4t_2) + 8t_3(2t_0 + 4t_1 + 8t_2 + 8t_3) \\
&+ 16t_4(2t_0 + 4t_1 + 8t_2 + 16t_3 + 16t_4) + 32t_5(2t_0 + 4t_1 + 8t_2 + 16t_3 + 32t_4 + 32t_5) \\
&+ 64t_6(2t_0 + 4t_1 + 8t_2 + 16t_3 + 32t_4 + 64t_5 + 64t_6) + 128t_7(2t_0 + 4t_1 + 8t_2 + 16t_3 \\
&+ 32t_4 + 64t_5 + 128t_6 + 128t_7) \\
&= t_0^2 + 4t_0t_1 + 4t_1^2 + 8t_0t_2 + 16t_1t_2 + 16t_2^2 + 16t_0t_3 + 32t_1t_3 + 64t_2t_3 \\
&+ 64t_3^2 + 32t_0t_4 + 64t_1t_4 + 128t_2t_4 + 256t_3t_4 + 256t_4^2 + 64t_0t_5 + 128t_1t_5 \\
&+ 256t_2t_5 + 512t_3t_5 + 1024t_4t_5 + 1024t_5^2 + 128t_0t_6 + 256t_1t_6 + 512t_2t_6 \\
&+ 1024t_3t_6 + 2048t_4t_6 + 4096t_5t_6 + 4096t_6^2 + 256t_0t_7 + 512t_1t_7 + 1024t_2t_7 \\
&+ 2048t_3t_7 + 4096t_4t_7 + 8192t_5t_7 + 16384t_6t_7 + 16384t_7^2
\end{aligned}
$$

Squared decision variables above can be easily linearized since t_k are binary. To linearize the product of t_it_j where $i \neq j$, let us define a new set of decision variables $y_{ij} \geq 0$ such that $y_{ij} \leq t_i$, $y_{ij} \leq t_j$ and $t_i + t_j - y_{ij} \leq 1$ where $i \in \{0, ..., 6\}$ and $j \in \{1, ..., 7\}$. Then, the above nonlinear equation is equivalent to,

$$
\begin{aligned}
h_{511}^2 = {} & t_0 + 4t_1 + 16t_2 + 64t_3 + 256t_4 + 1024t_5 + 4096t_6 + 16384t_7 \\
& + 4y_{01} + 8y_{02} + 16y_{03} + 32y_{04} + 64y_{05} + 128y_{06} + 256y_{07} \\
& + 16y_{12} + 32y_{13} + 64y_{14} + 128y_{15} + 256y_{16} + 512y_{17} \\
& + 64y_{23} + 128y_{24} + 256y_{25} + 512y_{26} + 1024y_{27} \\
& + 256y_{34} + 512y_{35} + 1024y_{36} + 2048y_{37} \\
& + 1024y_{45} + 2048y_{46} + 4096y_{47} \\
& + 4096y_{56} + 8192y_{57} \\
& + 16384y_{67}
\end{aligned}
\tag{5.47}
$$

The Equation in (5.47) is linear in all decision variables. The linearization operation required an additional 36 binary decision variables.

Theorem 5.2. *Let $h \in \mathbb{R}^+$ be discretized as $h = \sum_{k=0}^{l} 2^k t_k$, where $t_k \in \{0,1\}\ \forall k$, and let l be such that $2^{l+1} - 1 \geq u$ where u is the smallest integer equal to a power of two with $u \geq h$. Then, the square of the continuous variable h can be computed using Equation (5.48),*

$$
h^2 = \sum_{i=0}^{l} 2^{2i} t_i + \sum_{a=1}^{l} \left[\sum_{v=a}^{l} 2^{2v-a+1} y_{(v-a),v} \right] \tag{5.48}
$$

where $y_{(v-a),v} \in \{0,1\}$, $y_{(v-a),v} \leq t_{v-a}$, $y_{(v-a),v} \leq t_v$ and $t_{v-a} + t_v - y_{(v-a),v} \leq 1\ \forall v, a$.

Proof. The proof is by induction. Let us first consider $l = 1$. With $t_0,\ t_1 \in \{0,1\}$, $h = \sum_{k=0}^{1} 2^k t_k = t_0 + 2t_1$ and $h^2 = (t_0 + 2t_1)^2 = t_0^2 + 4t_1^2 + 4t_0 t_1$. Let $y_{01} \geq 0$ such that $y_{01} \leq t_0$, $y_{01} \leq t_1$ and $t_0 + t_1 - y_{01} \leq 1$. Then, h^2 can be linearized as

$h^2 = t_0 + 4t_1 + 4y_{01}$. Now, let us expand (5.48) for $l = 1$,

$$\begin{aligned} h^2 &= \sum_{i=0}^{1} 2^{2i} t_i + \sum_{a=1}^{1} \left[\sum_{v=a}^{1} 2^{2v-a+1} y_{(v-a),v} \right] \\ &= t_0 + 4t_1 + \sum_{v=1}^{1} 2^{2v} y_{(v-1),v} = t_0 + 4t_1 + 4y_{01} \end{aligned}$$

Therefore, we conclude that Equation (5.48) holds for $l = 1$.

Let us now assume that (5.48) holds for $l' = l$ and prove that it holds for $l' = l+1$ [6]. Let $h = \sum_{k=0}^{l+1} 2^k t_k = \sum_{k=0}^{l} 2^k t_k + 2^{l+1} t_{l+1}$ where $t_j \in \{0,1\}\ \forall j \in \{1, ..., l+1\}$. Squaring this term, we get,

$$\begin{aligned} h^2 &= \left(\sum_{k=0}^{l} 2^k t_k + 2^{l+1} t_{l+1} \right)^2 \\ &= \left(\sum_{k=0}^{l} 2^k t_k \right)^2 + 2^{2l+2} t_{l+1}^2 + 2\left(2^{l+1} t_{l+1}\right) \left(\sum_{k=0}^{l} 2^k t_k \right) \\ &= \sum_{k=0}^{l} 2^{2k} t_k + \sum_{a=1}^{l} \left[\sum_{v=a}^{l} 2^{2v-a+1} y_{(v-a),v} \right] + 2^{2l+2} t_{l+1} + 2\left(2^{l+1} t_{l+1}\right) \left(\sum_{k=0}^{l} 2^k t_k \right) \\ &= \sum_{k=0}^{l+1} 2^{2k} t_k + \sum_{a=1}^{l} \left[\sum_{v=a}^{l} 2^{2v-a+1} y_{(v-a),v} \right] + \underbrace{\left(2^{l+2} t_{l+1}\right) \left(\sum_{k=0}^{l} 2^k t_k \right)}_{A} \end{aligned} \tag{5.49}$$

(5.48) is used to deduce the third equality from the second. Square on t_{l+1} is dropped since $t_{l+1} \in \{0,1\}$. To move from the third to the fourth equality, we included the term $2^{2l+2} t_{l+1}$ to the first summation and combined the $2\left(2^{l+1} t_{l+1}\right)$ term to get $2^{l+2} t_{l+1}$. Let us now analyze the term A.

$$\begin{aligned} A &= 2^{l+2} t_{l+1} \left(t_0 + 2t_1 + 4t_2 + ... + 2^l t_l \right) \\ &= 2^{l+2} t_0 t_{l+1} + 2^{l+3} t_1 t_{l+1} + ... + 2^{2l+2} t_l t_{l+1} \end{aligned} \tag{5.50}$$

[6] For ease of presentation, we keep l and $l+1$ as summation upper bounds in what follows.

Let $y_{j,l+1} \geq 0$, $y_{j,l+1} \leq t_j$, $y_{j,l+1} \leq t_{l+1}$ and $t_j + t_{l+1} - y_{j,l+1} \leq 1 \; \forall j \in \{0, ..., l\}$. Then, we can rewrite (5.50) as in (5.51)

$$A = 2^{l+2} y_{0,l+1} + 2^{l+3} y_{1,l+1} + ... + 2^{2l+2} y_{l,l+1} = \sum_{i=0}^{l} 2^{l+2+i} y_{i,l+1} \tag{5.51}$$

We will now show that the summation in (5.51) is equal to the sum of additional terms when the second summation in (5.48) is expanded with $l' = l + 1$. To see that, let us expand the second summation in (5.48) and illustrate the pattern,

$$\begin{aligned}
&\sum_{a=1}^{l+1} \left[\sum_{v=a}^{l+1} 2^{2v-a+1} y_{(v-a),v} \right] \\
&= \sum_{v=1}^{l+1} 2^{2v} y_{(v-1),v} + \sum_{v=2}^{l+1} 2^{2v-1} y_{(v-2),v} + \sum_{v=3}^{l+1} 2^{2v-2} y_{(v-3),v} + ... + \sum_{v=l}^{l+1} 2^{2v-l+1} y_{(v-l),v} \\
&\quad + \sum_{v=l+1}^{l+1} 2^{2v-l} y_{(v-l-1),v} \\
&= \underbrace{4y_{01} + 16y_{12} + ... + 2^{2l} y_{(l-1),l} + \boxed{2^{2l+2} y_{l,(l+1)}}}_{a=1} \\
&\quad + \underbrace{8y_{02} + ... + 2^{2l-1} y_{(l-2),l} + \boxed{2^{2l+1} y_{(l-1),(l+1)}}}_{a=2} \\
&\quad + \underbrace{16y_{03} + ... + 2^{2l-2} y_{(l-3),l} + \boxed{2^{2l} y_{(l-2),(l+1)}}}_{a=3} + \\
&\quad ... \\
&\quad + \underbrace{2^{l+1} y_{0l} + \boxed{2^{l+3} y_{1,(l+1)}}}_{a=l} + \underbrace{\boxed{2^{l+2} y_{0,(l+1)}}}_{a=l+1}
\end{aligned} \tag{5.52}$$

We can observe that the sum of the boxed terms in (5.52) is the same as the summation in (5.51), which also equals the term A in equations (5.49) and (5.50).

To wrap up the proof, let us return to (5.49). We have shown that the term

A in deed represents the sum of additional terms resulting from expanding the second summation of (5.48) for $l' = l + 1$ in stead of $l' = l$. Adding A back to the second summation in (5.49), we get,

$$h^2 = \sum_{i=0}^{l+1} 2^{2i} t_i + \sum_{a=1}^{l+1} \left[\sum_{v=a}^{l+1} 2^{2v-a+1} y_{(v-a),v} \right]$$

which concludes the proof. □

Finally, the right-hand-side of Equation (5.41) can be linearized as discussed in Section 5.4.1

Linearization of h^2 in Equation (5.48) brings an additional $l + 1$ binary t variables for each h variable transformed. Moreover, this transformation adds another $l+(l-1)+(l-2)+...+1 = \frac{l(l+1)}{2}$ binary y variables for each h variable. In total, one continuous h variable can be discretized at the expense of $l + 1 + \frac{l(l+1)}{2} = \frac{(l+1)(l+2)}{2}$ binary variables. In a model with $j \times k$ h_{jk1} variables, the total number new binary variables will be $\sum_{j \in J} \sum_{k \in K} \frac{(l_{jk}+1)(l_{jk}+2)}{2}$. The transformation in Equation (5.44) introduces an additional $l + 1$ binary s variables for each h. In a model with $j \times k$ h_{jk1} variables, total number new variables will be $\sum_{j \in J} \sum_{k \in K} (l_{jk} + 1)$.

Transformations developed in this chapter will also increase the number of constraints. There will be an additional $j \times k$ constraints to define h_{jk1} as a linear combination of binary decision variables. Transformation in Equation (5.48) will require a total of $3 \sum_{j \in J} \sum_{k \in K} \frac{(l_{jk}+1)(l_{jk}+2)}{2}$ new constraints to restrict all new y variables to be smaller than or equal to t variables. The transformation in Equation (5.44) will require a total of $3 \sum_{j \in J} \sum_{k \in K} (l_{jk} + 1)$ new constraints to restrict all new s variables to be smaller than or equal to x and t variables and additional $l + 1$ constraints to restrict the sum of model variables minus the new variable to be less than or equal to one.

Linearization of the capacity constraint deterministic equivalents of the MSM is a special case of the framework discussed above. The nonlinear deterministic equivalent in Equation (5.40) can be expressed in summation form, assuming separate chance constraints, as below,

$$\mu_{jk}x_{jk1} - h_{jk1} + \Phi^{-1}(1-\alpha_{jk})\sqrt{\sum_k \sum_l cov_{jkl}x_{jk1}x_{jl1}} \geq 0 \quad \forall j \in J,\ k \in K$$

Assuming $\alpha_{jk} \geq 0.5\ \forall j \in J,\ k \in K$, we can attempt to linearize the deterministic equivalent obtained above. Let us first assume that capacity for products k and l are not correlated and replace the covariance term with the variance of capacity of product k, σ_{jk}^2. Then, the deterministic equivalent becomes

$$\begin{aligned} &\mu_{jk}x_{jk1} - h_{jk1} + \Phi^{-1}(1-\alpha_{jk})\sqrt{\sigma_{jk}^2 x_{jk1}^2} \geq 0 \\ &\mu_{jk}x_{jk1} - h_{jk1} + \Phi^{-1}(1-\alpha_{jk})\sigma_{jk}x_{jk1} \geq 0 \quad \forall j \in J,\ k \in K \end{aligned} \tag{5.53}$$

where we are allowed to take the $\sigma_{jk}^2 x_{jk1}^2$ out of the square root and maintain its sign since $\sigma_{jk} \geq 0$ and $x_{jk1} \in \{0, 1\}$ and hence greater than or equal to zero. If the assumption on uncorrelated capacity random variables is relaxed, the deterministic equivalent above can be linearized as follows,

$$\mu_{jk}x_{jk1} - h_{jk1} \geq -\Phi^{-1}(1-\alpha_{jk})\sqrt{\sum_k \sum_l cov_{jkl}x_{jk1}x_{jl1}}$$
$$\Rightarrow (\mu_{jk}x_{jk1} - h_{jk1})^2 \geq \left(\Phi^{-1}(1-\alpha_{jk})\right)^2 \sum_k \sum_l cov_{jkl}x_{jk1}x_{jl1}$$
$$\Rightarrow \mu_{jk}^2 x_{jk1}^2 - 2x_{jk1}h_{jk1} + h_{jk1}^2 \geq \left(\Phi^{-1}(1-\alpha_{jk})\right)^2 \sum_k \sum_l cov_{jkl}x_{jk1}x_{jl1} \tag{5.54}$$
$$\Rightarrow \mu_{jk}^2 x_{jk1} - 2x_{jk1}h_{jk1} + h_{jk1}^2 - \left(\Phi^{-1}(1-\alpha_{jk})\right)^2 \sum_k \sum_l cov_{jkl}x_{jk1}x_{jl1} \geq 0$$
$$\forall j \in J,\ k \in K$$

The nonlinear deterministic equivalent in (5.54) can be linearized using the results in Theorems 5.1 and 5.2. Corollary 5.1 below provides the linear deterministic equivalent to (5.54).

Corollary 5.1. *The nonlinear deterministic equivalent in Equation* (5.54) *can be linearly expressed as in* (5.55) *below.*

$$\begin{aligned} \mu_{jk}^2 x_{jk1} - 2\mu_{jk}\sum_{i=0}^{l_{jk}} 2^i s_{jki} + \sum_{i=0}^{l_{jk}} 2^{2i} t_{jki} + \sum_{a=1}^{l_{jk}} \left[\sum_{v=a}^{l_{jk}} 2^{2v-a+1} y_{j,k,(v-a),v}\right] \\ - \left(\Phi^{-1}(1-\alpha_{jk})\right)^2 \sum_k \sum_l cov_{jkl}x_{jk}x_{jl} \geq 0 \quad \forall j,k \end{aligned} \tag{5.55}$$

where parameter l and variables t and y are defined in Theorem 5.2 and variables s are defined in Theorem 5.1.

Proof. Follows directly by replacing the results of Theorems 5.2 and 5.1 in Equation (5.54). □

5.6.2 Stochastic MSM Numerical Example

Let us conclude this section with a numerical example. We consider the same cost, quality, lead time and risk data used in the deterministic MSM example discussed in Section 4.4.2 of Chapter 4 and introduce the stochastic capacity and demand data sets in Tables 5.5 and 5.6. Mean demand and capacity data are the same as the deterministic data in Chapter 4 except for Cap_{41}, Cap_{51} and Cap_{53} which have been increased respectively by 30, 10 and 10 units to have a feasible problem. The reliability level is set at $\alpha_{jk} = 0.95$, meaning that at least 95% of demand should be met, and we set $p = 3$ to allow 3 primary suppliers for each product in the solution. Since no correlation is assumed among capacity and demand random variables the simple linearization in Equation 5.53 can be used to generate linear deterministic equivalents for capacity constraints. Let us illustrate the linearization of the capacity constraint for supplier 1 and product 3.

$$\begin{aligned} &Cap_{131}x_{131} - h_{131} + \Phi^{-1}(1 - 0.95)\sigma_{131}x_{131} \geq 0 \\ &\Rightarrow 100x_{131} - h_{131} - (1.645)5x_{131} \geq 0 \\ &\Rightarrow 91.78x_{131} \geq h_{131} \end{aligned} \tag{5.56}$$

The last inequality in (5.56) appears in the MSM formulation as the deterministic equivalent of the probabilistic capacity constraint for supplier 1 - product 3 combination. Note that in the optimal solution (see Table 5.8) supplier 1 is chosen as a primary supplier of product 3 ($x_{131} = 1$) and ships 89 units of product 3 ($h_{131} = 89$). Deterministic equivalent for the demand constraint, on the other hand, is already linear. The deterministic demand constraint for product 1 can be

Table 5.5. Stochastic MSM capacity data

	Product		
Supplier	1	2	3
1	N(50, 6.25)	N(45, 5.06)	N(100, 25)
2	N(90,20.25)	N(100, 25)	N(20, 1)
3	N(70,12.25)	N(50, 6.25)	N(150, 56.25)
4	N(80, 6.25)	N(200, 100)	N(50, 6.25)
5	N(70,12.25)	N(100, 25)	N(70, 12.25)

Table 5.6. Stochastic MSM demand data

Item		
1	2	3
N(210, 36)	N(250, 49)	N(250, 64)

generated using Equation (5.35) as below,

$$\sum_{j=1}^{5} h_{1j1} \geq F_{D_1}^{-1}(0.95) \Rightarrow \sum_{j=1}^{5} h_{1j1} \geq 219.87$$

We solve a non-preemptive GP example using the weights in Table 4.9 in Chapter 4. Table 5.7 gives the ideal values, goal targets, achievement results and objective values at optimality. A goal target is achieved if the optimal solution of that objective is between the ideal value and the target. All targets for this problem have been achieved. Table 5.8 shows the optimal solution.

As in the MSM example of Chapter 4, it can be observed that the buyer has three primary suppliers for each product (since $p = 3$) and one level two and one level three backup supplier for each product. Orders are assigned only to the

Table 5.7. Stochastic MSM results using non-preemptive GP

Objective	**Ideal**	**Target**	**Status**	**Achievement**
Cost	8970.60	9867.66	Achieved	9575.77; (7628.18, 1340)
Quality	698.96	629.07	Achieved	696.31; (691.13)
Lead time	3170.77	3487.85	Achieved	3487.85; (3440.34)
Risk	433110600	476421660	Achieved	476421660; (472856748.74)

primary suppliers.

We solved the deterministic non preemptive MSM GP model of Chapter 4 with the new mean capacity data. As in the original data set, only the quality objective is achieved. The main reason behind this poor performance is that target values in Chapter 4 are set at 5% of the ideal solutions, whereas targets in Chapter 5 are at 10%. We finally put the optimal non preemptive MSM GP solution of Chapter 4 into the stochastic non preemptive MSM GP of Chapter 5 to test the deterministic solution's performance in a stochastic environment. The problem turns out to be infeasible since the deterministic solution does not satisfy the demand requirements of the stochastic model. We then solve the deterministic MSM with the increased mean capacity values used in this chapter and plug the new deterministic optimal solution into the stochastic model. The deterministic optimal solution remains infeasible under stochastic conditions.

Table 5.8. Non-preemptive GP solution of the MSM model

	Levels			
Supplier	*1*	*2*	*3*	**Product**
1	0	0	1	1
	0	0	1	2
	1	0	0	3
2	1	0	0	1
	1	0	0	2
	0	1	0	3
3	1	0	0	1
	0	1	0	2
	1	0	0	3
4	1	0	0	1
	1	0	0	2
	0	0	1	3
5	0	1	0	1
	1	0	0	2
	1	0	0	3

	Levels			
Supplier	*1*	*2*	*3*	**Product**
1	0	0	0	1
	0	0	0	2
	89	0	0	3
2	80	0	0	1
	78	0	0	2
	0	0	0	3
3	64	0	0	1
	0	0	0	2
	125	0	0	3
4	76	0	0	1
	184	0	0	2
	0	0	0	3
5	0	0	0	1
	0	0	0	2
	49	0	0	3

5.7 Generalizations of the Chance Constrained Models

Deterministic equivalents presented in Sections 5.3 and 5.5 assume normally distributed random variables. Although normal distribution arises in applications, it is generally a strong assumption to impose normal distribution for every case. This section presents results that extend our findings in Sections 5.3 and 5.5 without assuming normal distribution. In other words, we discuss situations where there are no restrictive assumptions on the distributions governing random variables in chance constrained programming and presents results on the reliability of chance constraints in the most general case. We restrict our attention to the results presented in Calafiore & El Ghaoui (2006) on distributionally robust chance constraints (DRCC). A DRCC is obtained by enforcing a chance constraint (see Equation (5.1)) to hold for any distribution belonging to a family of distributions, that is,

$$\{x \in \mathbb{R}^n, \xi \in \Xi | \inf_{\xi \sim \Xi} P\left[(\xi \mid T_i(\xi)x \geq b_i(\xi))\right] \geq \alpha_i\} \tag{5.57}$$

where Ξ is a family of distributions which have a same mean μ_Ξ and covariance matrix Σ_Ξ. Calafiore & El Ghaoui (2006) proved the following Theorem after rearranging the terms in (5.57),

Theorem 5.3. *For any $\alpha \in (0,1)$, the DRCC*

$$\inf_{d \sim \mathcal{D}} P\left(\mathbf{d^T}\tilde{\mathbf{x}} \leq 0\right) \geq 1 - \alpha \tag{5.58}$$

is equivalent to the convex second order cone constraint

$$\mathcal{K}_\alpha \sigma(x) + \hat{\phi}(x) \leq 0 \quad \mathcal{K}_\alpha = \sqrt{\frac{1-\alpha}{\alpha}} \tag{5.59}$$

In (5.58), $d_i = [T_i(\xi),\ b_i(\xi)]$ and $\tilde{x}_i = [x_i,\ 1]$. Both vectors $\mathbf{d}$ and $\tilde{\mathbf{x}}$ are in $\mathbb{R}^{n+1}$. In (5.59), $\sigma(x)$ and $\hat{\phi}(x)$ are elements of the covariance matrix and mean vector of $\mathbf{d}^{\mathbf{T}}\tilde{\mathbf{x}}$ respectively.

The DRCC and its equivalent given in Equations (5.57) - (5.59) are more conservative compared to chance constraints in (5.1) since in chance constraints the inequality needs to hold only for the considered distribution; whereas in DRCC, the inequality needs to be verified for an entire family of distributions that can have the same mean and covariance matrix. In that sense, the DRCC represents restrictions that need to be verified in the worst case (or at least a case worse than treated in chance constraints).[7]

Theorem 5.3 is applied to the SSA models to have a chance constrained model that requires much milder assumptions on distributions compared to previous sections. In the following, we present DRCC counterparts and deterministic equivalents of the stochastic demand and capacity constraints of the MSM.

Uncertainty affects two sets of constraints in the MSM: demand constraints in (5.29) and capacity constraints in (5.30). Using the result in Theorem 5.3, demand and capacity constraints can be deterministically expressed as in Equations (5.60) and (5.61) respectively.

[7] Robust constraints and robust optimization models are often criticized for focusing on the worst possible outcomes and being overly conservative. A recent article, Bertsimas & Sim (2004), proposes a new robust formulation where the decision maker can control the degree of conservativeness of the model.

$$\begin{aligned} &\mathbf{P}\left(D_k - \sum_k h_{jk1} \le 0\right) \ge 1 - \epsilon_k \ \ \forall k \\ &\Leftrightarrow \sqrt{\frac{1-\epsilon_k}{\epsilon_k}}\sigma_k^D + \mu_k^D - \sum_j h_{jk1} \le 0 \ \ \forall k \end{aligned} \tag{5.60}$$

$$\begin{aligned} &\mathbf{P}\left(h_{jk1} - Cap_{jk}x_{jk1} \le 0\right) \ge 1 - \epsilon_{jk} \ \forall j,k \\ &\Leftrightarrow \sqrt{\frac{1-\epsilon_k}{\epsilon_k}}\sqrt{\sum_k \sum_l cov_{jkl}x_{jk1}x_{jl1}} + h_{jk1} - \mu_{jk}^{Cap}x_{jk1} \le 0 \ \ \forall j,k \end{aligned} \tag{5.61}$$

Note that the deterministic equivalent in Equation (5.60) is linear in the MSM decision variables. The capacity deterministic equivalent in (5.61) can be linearized by squaring the inequality and using the results in Theorems 5.2 and 5.1. Furthermore, if correlation between capacities devoted to different products at a supplier is assumed zero, Equationn (5.61) can be linearly expressed as in Equation (5.62)

$$\sqrt{\frac{1-\epsilon_k}{\epsilon_k}}\sigma_{jk}^{Cap}x_{jk1} + h_{jk1} - \mu_{jk}^{Cap}x_{jk1} \le 0 \ \ \forall j,k \tag{5.62}$$

We now present a numerical example where we do not have any restrictions on the demand and capacity distributions, except assuming that the mean and covariance of the random variables are known. We set $\epsilon_k = 0.05 \ \forall k$ for enforcing the probabilistic constraints in (5.60) and (5.61) to hold with 95% probability. Finally, we set $p = 3$ to allow three primary suppliers in the MSM solution. We first use the random demand and capacity data given in Tables 5.5 and 5.6 without imposing the normal distribution restriction. That is, capacity for product 1 at supplier 1 follows an unknown distribution with mean 50 and standard deviation equal to $\sqrt{6.25} = 2.5$. All remaining data elements were retrieved from Tables 5.5 and 5.6 are incorporated the same way into the example. We found that the

Table 5.9. Mean capacity data used in the MSM example with DRCC

	Item		
Supplier	1	2	3
1	**100.00**	45.00	100.00
2	90.00	100.00	**95.00**
3	70.00	50.00	150.00
4	**100.00**	200.00	50.00
5	70.00	100.00	70.00

Table 5.10. MSM with DRCC results under non preemptive GP

Objective	**Ideal**	**Target**	**Status**	**Achievement**
Cost	9825.94	10808.5	Achieved	10322.33; (8384.99, 1370)
Quality	798.13	718.3	Achieved	798.13; (793.02)
Lead time	4083.39	4491.7	Achieved	4348.6; (4307.81)
Risk	494427500.00	543870250.0	Achieved	538028599.25; (534030689.46)

problem does not have a feasible solution with the discussed data set. Taking a closer look at the new constraints, the $\sqrt{\frac{1-\epsilon_k}{\epsilon_k}}$ multiplier equals 4.359 when $\epsilon = 0.05$ and the constraints become much more tighter compared the normally distributed random variables case, where the multiplier $\left(\Phi^{-1}(1-\alpha)\right)^2$ equals 2.705 with $\alpha = 0.95$. Therefore, we increased the mean capacity at some suppliers as in Table 5.9 to have a feasible solution to the problem. Modified data elements are shown in bold.

Ideal values, targets, objective achievements and objective values at optimality for the problem solved with mean capacity data in Table 5.9 are given in Table 5.10. The optimal solution to the problem is displayed in Table 5.11.

The ideal and target values in Table 5.10 are worse compared to the ideal values of the instances solved under the Normality assumption (see Table 5.7). This is mainly due to relaxing the Normality assumption which requires higher capacity values to obtain feasibility and thus allows higher order quantities. This result should be expected since the model is solved with much less information and the same level of reliability, 95%, is required. Note that higher ideal values results in

Table 5.11. MSM with DRCC solution under non preemptive GP

	Levels			
Supplier	*1*	*2*	*3*	**Product**
1	1	0	0	1
	0	1	0	2
	1	0	0	3
2	1	0	0	1
	1	0	0	2
	1	0	0	3
3	0	0	1	1
	0	0	1	2
	1	0	0	3
4	1	0	0	1
	1	0	0	2
	0	0	1	3
5	0	1	0	1
	1	0	0	2
	0	1	0	3

	Levels			
Supplier	*1*	*2*	*3*	**Product**
1	89	0	0	1
	0	0	0	2
	78	0	0	3
2	70	0	0	1
	78	0	0	2
	91	0	0	3
3	0	0	0	1
	0	0	0	2
	117	0	0	3
4	89	0	0	1
	156	0	0	2
	0	0	0	3
5	0	0	0	1
	78	0	0	2
	0	0	0	3

more relaxed targets (since targets are set at a percentage of ideal values); and therefore the solution achieves all objective targets in Table 5.10.

5.8 Conclusions

This chapter presents chance constrained extensions to the *Single Sourcing Model* (SSM) and *Multiple Sourcing Model* (MSM) formulations introduced in Chapter 4. Uncertainty affecting demand and capacity parameters reflects operational risks, and therefore, models in Chapter 5 provide a simultaneous treatment of disruption risks (previously introduced in Chapters 3 and 4) and operational risks. Deterministic equivalents for capacity and demand chance constraints are derived under the Normality assumption. These deterministic equivalents are then linearized by the introduction of additional binary variables and numerical examples are provided. Stochastic optimal solutions are tested in the deterministic models and are observed to provide equal or better quality results in terms of target achievements than deterministic optimal solutions. The deterministic optimal solutions, when tested in the stochastic models, yield infeasible solutions. This indicates that the deterministic optimal solutions are inferior to the stochastic optimal solutions and supplier - order assignments under deterministic scenarios are not feasible when uncertainty is considered.

The linearization techniques described in this chapter are not limited to cases with the Normality assumption. Calafiore & El Ghaoui (2006) showed that deterministic equivalents of type (5.15) can be obtained when the underlying distribution belongs to the family of radial distributions. Resulting deterministic equivalents can be linearized in the same way since radial distributions are symmetrically distributed around the origin (similar to the standard normal distribution used in Sections 5.3 and 5.5). Finally, we present a more general framework where the assumption of the underlying distribution is relaxed to a case where only the knowledge of mean and covariance is required. A numerical example is provided to show that the solution in the general case is worse than the solution obtained under the Normality assumption. On the other hand, the general case is less restrictive and has more practical value.

Chapter 6

Summary of the Research and Future Directions

6.1 Summary of the Research

This dissertation provides multiobjective mathematical models to enhance supplier selection and analytical models of risk quantification to include disruption and operational risks into supplier selection. Key contributions of this dissertation are:

- Analytical models to quantify risk impact, occurrence, detection and recovery with an emphasis on disruption risks (Chapter 3),
- Multiobjective models to enhance supplier selection with disruption risk mitigation (Chapter 4),
- Chance constrained optimization models to include operational risks and theoretical results to linearize chance constraints in nonlinear mixed integer programming problems (Chapter 5).

The first two chapters of this dissertation act as a general introduction to supply chains, supply chain management, outsourcing, risks and analytical models used in improving supply chain management and oursourcing practices. One of the key ideas discussed in the first two chapters is the distinction between disruption and operational risks. Operational risks are related to day-to-day supply chain operations, whereas disruption risks are due to rare but extreme events that can bring the supply chain into disarray. A general risk classification scheme developed in Yang (2006) is discussed in Chapter 1 to position disruption and operational risks in a 4-by-4 matrix. Chapter 2 surveys the literature on analytical methods with a focus on optimization models addressing the supplier selection problem. Chapter 2 also surveys the literature on supply chain risk management and covers both operational and disruption risks. One of the key findings of the survey is the lack of analytical techniques to model disruption risks, which motivates Chapter 3 of this dissertation.

Chapter 3 presents analytical models to quantify disruption risks. Risks are broken down into several components, including severity of impact, frequency of occurrence, detectability and risk recovery time. We develop a quantification scheme for each of these risk components with a focus on disruption risks. Severity of impact of disruption risks is modeled using the Extreme Value Distributions, which have been used in civil engineering to tests the resilience of structures exposed to extreme natural events. Frequency of occurrence is modeled using Poisson random variables. Chapter 3 presents analytical results to estimate the expected loss and its variance resulting from disruptive events. We then provide analytical results to generate the entire distribution of loss from disruptive events over a fixed period of time. The next development in Chapter 3 is a Markov chain based approach to model the propagation of disruption information in a supply chain. The motivation behind this model is the time it can take to discover the occurrence of a disruption, especially in lower tiers, in large supply chains. The time it takes to discover the occurrence of a disruption at a supply chain node is called the disruption detection time and the mean time of detection is modeled using the mean first passage time concept in Markov chains. We provide several models to mimic information sharing in supply chains and compare the detection times. The final development in Chapter 3 is a risk recovery time model. Recovery time is the time it takes the supply chain to return to an operational state after suffering from a disruption. Similar to the available literature, recovery time at a node is modeled as an exponential random variable. The parameter of the random variable is defined as a function of inventory available at that node, severity of the disruptive events and availability of risk mitigation plans at the node. Chapter 3 is concluded with a comprehensive example where all risk models are integrated to treat a supply chain disruption problem.

The available literature also lacks quantitative models to mitigate disruption risks. Chapter 4 aims at filling this gap with two multiobjective mathematical models. The first mathematical model considers a single sourcing strategy where the buyer satisfies the entire demand for a product through a single buyer. The second model allows splitting orders among multiple suppliers. Both models provide risk mitigation plans by assigning primary supplier(s) and a number of backup suppliers for each product. Primary suppliers are those suppliers used to meet the demand for a product and backup suppliers are reserves that will be used in case a primary supplier fails due to a disruption. Both mathematical models consider four objective functions: minimizing total costs, lead time and risk value (computed following the results in Chapter 3) and maximizing the quality of procured products. Four goal programming techniques (preemptive, non-preemptive, minmax and fuzzy goal programming) are used to implement numerical solutions. Objective priorities and weights are taken from a previous study where we interacted with real decision makers working for a global IT company. Solutions to the numerical examples are compared using the Value Path Approach. Two extensions to the basic mathematical models are proposed at the end of the chapter. The first extension considers conditional contracts where a supplier agrees to be a backup supplier for a product only if he is guaranteed a primary position for another product. Additional constraints to include conditional contracts are developed. The second extension incorporates risk detection and risk recovery concepts developed in Chapter 3. Risk detection and recovery times are included as two additional objective functions to be minimized. A comprehensive example to incorporate these two extensions is solved to conclude Chapter 4.

The last chapter of this dissertation includes operational risks to supplier selection and extends the deterministic multiobjective mathematical models of Chapter 4 to stochastic programming models. Operational risks related to customer demand and supplier capacity are introduced using chance constraints. Deterministic equivalents of these stochastic programs are given for both separate and joint chance constraints. In most cases, deterministic equivalent transformations add nonlinearity to the models. Although the models can be solved as nonlinear mixed integer programs (NLMIP), this approach may cause numerical problems since algorithms to solve NLMIPs may not be tractable and may yield suboptimal solutions especially when the problem size is large. Therefore, an alternative linearization procedure is presented in Chapter 5 to transform the deterministic NLMIPs with separate chance constraints to linear MIPs at the expense of the model size. The new linear models can be solved using MIP solvers which are much more common and reliable than NLMIP solvers. Deterministic equivalents are computed assuming capacity and demand are normally distributed random variables. A more general formulation called *Distributionally Robust Chance Constrained (DRCC) Programming* (Calafiore & El Ghaoui 2006), where no distribution assumption is made, is used in this dissertation as an extension to the deterministic equivalent models of Chapter 5. Examples with and without the Normality assumption are solved and discussed. The model without the Normality assumption yields poorer solutions since it operates with less information on the distribution of random variables.

6.2 Future Directions

Research presented in this dissertation can be extended in several ways. Firstly, research directions discussed in Section 2.6 of Chapter 2 can be followed. Among those, the following two directions have not been followed in this dissertation and are open for research:

1. Emphasizing fuzzy multicriteria decision making (MCDM) methods in supplier selection,
2. Integrating inventory decisions to multiobjective supplier selection models,

Moreover, the following ideas can be elaborated:

1. The risk quantification results in Section 3.3 of Chapter 3, except for Theorem 3.1, are proven for the convolution of two different types of disruptive events. Although we believe that the probability of more than two disruptive events happening over a reasonable period of time is quite small, further effort can be spent to extend the theoretical results to treat more than two events.

2. All risk quantification results in Chapter 3 assume no correlation between risk events. Models with correlation can be developed in case the correlation between risk events is significant. A discussion on this subject is provided in Chapter 3.

3. Multiobjective mathematical models of Chapter 4 assume a single buyer. Although models with multiple buyers are not customary, such extensions can be developed.

4. Data used in the mathematical models of Chapter 4 are independent of the order amounts. Models where suppliers offer price breaks regarding order amounts are available in the literature (see for instance Wadhwa & Ravindran (2007)). Lead time and quality data also can be made a function of the order amounts. This will be a straightforward extension to the single sourcing model since the buyer purchases the entire demand from a single supplier, and therefore, the order amounts are not decision variables. In the multiple sourcing model, on the other hand, order amounts are decision variables and such an extension would be more challenging to implement.

5. Models in Chapter 5 are mainly developed for separate chance constraints. Although separate chance constraints are useful, there may be supplier selection cases where joint chance constraints would be more appropriate (see for instance the discussion in Section 5.3.1). Effort can be spent to derive deterministic equivalents for joint chance constraints and to linearize those joint chance constraints.

6. Chance constrained programming is only one way to include uncertainty to the SSA models. Developing recourse models and comparing their solutions

with the solutions in Chapter 5 would be an interesting direction of research. Furthermore, the same SSA models can be formulated as robust optimization models and further insight might be obtained.

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Vita

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