VISCOELASTIC FLYWHEEL ROTORS: MODELLING AND MEASUREMENT

A Thesis in
Engineering Science and Mechanics

by

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ABSTRACT

An investigation of the creep behavior of polymer composites was performed in developing a closed-form structural model of advanced polymer matrix composite (PMC) flywheels. The model is of the linear-viscoelastic type, and is applicable to the design and analysis of multiple concentric-ring PMC flywheel rotors in a state of plane-stress. This model incorporates techniques for approximating the quasi-static response to general time-varying loads including rotation, temperature change, and interference-fits between adjacent rings. The model accounts for the effect of temperature on the material response using the time-temperature superposition principle. The quasi-elastic technique was used to discretize the linear viscoelastic constitutive law, allowing the derivation of approximate solutions for the stress and strain field variables. Experimental work performed in support of this model includes thermo-viscoelastic characterization of a unidirectional glass/epoxy composite. Experimental measurement of pressure loss and strain redistribution in interference-fitted filament-wound glass and carbon fiber PMC ring pairs was performed using moiré interferometry and electrical resistance strain gages. Good agreement between the data and the plane-stress model at locations away from the ring interfaces was obtained. With the purpose of making creep measurements through the radial thickness of high-speed rotating flywheels, a new optical displacement measurement method was developed. Notable improvements over a known related method include greater displacement sensitivity, the ability to measure rigid body vibrations and separate the associated vibration-induced displacement from the strain-induced displacement, and the ability to compensate for sensor drift during flywheel operation. Displacement measurements made on an aluminum rotor operating at a maximum speed of 16 krpm (255 m/s at the point of measurement) were made with ±1-micrometer accuracy. At this speed, hoop strains were found to be within 40 to 125×10⁻⁶ of theoretical predictions. Relative to the theoretical hoop strains, the measured hoop strains differed by 5-6% at 16 krpm.
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NOMENCLATURE

$A$ a parameter in the equation for radial displacement
$A_0$ axisymmetric radial displacement of rotor
$A_1$ amplitude of in-plane rigid body radial displacement of rotor
$B$ a parameter in the equation for radial displacement
$C$ a parameter in the equation for radial displacement
$D$ a parameter in the equation for radial displacement
$E$ isotropic elastic modulus
$E_1$ fiber direction elastic modulus
$E_2$ transverse direction elastic modulus
$H_i$ parameters in the equations for radial stress
$K_t$ transverse sensitivity of an electrical resistance strain gage
$L$ creep spectrum
$R$ universal gas constant
$S_i$ time-dependent coefficients in the Prony series creep compliance
$S_c$ counter frequency (Hz)
$S_0$ elastic component of the compliance (Findlay power law and Prony series)
$\tilde{S}(t)$ compliance matrix
$S_1$ time coefficient in the Findlay power law creep compliance
$S_{11}$ fiber-direction compliance
$S_{12}$ cross term compliance
$S_{21}$ cross term compliance
$S_{22}$ transverse-direction compliance
$S_{66}$ shear compliance
$T$ temperature
$T_g$ glass transition temperature
$T_0$ reference temperature (general)
$T_{\text{ref}}$ reference temperature of a master creep compliance curve
$V_f$ volumetric fraction of fibers in a PMC
$V_m$ volumetric fraction of matrix in a PMC
$V_v$ volumetric fraction of voids in a PMC
$W$ stored energy in the Reiner-Weisenberg creep rupture theory
$W_c$ critical stored energy in the Reiner-Weisenberg creep rupture theory
$\alpha_T$ temperature-shift factor
$\alpha_{\text{te}}$ aging shift factor
$d$ shortest distance between edge of a reflective patch & center of illuminated spot
$n$ time exponent in the Findlay power law
$p_{\text{mf}}$ final pressure predicted by rotor model for the interference-fit experiments
$p_{\text{mi}}$ initial pressure predicted by rotor model for the interference-fit experiments
$r$ radial coordinate on rotor
$r'$ a particular radial location on the rotor
$r_i$ inner radius
$r_{\text{inst}}$ radial location on rotor at an instantaneous speed
$r_{\text{max}}$ maximum radial location at which a hoop strain sensitivity can be achieved
$r_o$ outer radius
\( r_{\text{ref}} \) radial location on rotor at reference (negligible deformation) speed
\( t \) time
\( t_c \) instantaneous time since quench (for isothermal aging conditions)
\( t_{\text{cref}} \) reference material age (for isothermal aging conditions)
\( t_f \) time at rupture in the Reiner-Weisenberg creep rupture theory
\( t_i \) times at which responses are calculated in the viscoelastic solution
\( t_j \) times at which step loads are applied in the viscoelastic solution
\( u \) radial displacement
\( u_{\text{min}} \) minimum detectable radial displacement
\( u_{\text{rack}} \) radial displacement of sensor rack due to temperature change
\( u_{\text{tot}} \) total radial displacement including thermal effects
\( \Gamma \) phase of a rigid body vibration of the rotor
\( \Delta H \) activation energy
\( \Delta M \) instantaneous water content in a PMC relative to a reference water content
\( \Delta \varepsilon_{h,s} \) (viscoelastic) change in hoop strain at the ID of the inner ring or OD of the outer ring after the high temperature soak (NIF tests only)
\( \Delta p_{\text{expt}} \) experimentally measured pressure loss in the interference-fit experiments
\( \Delta p_{\text{model}} \) model-predicted pressure loss in the interference-fit experiments
\( \Delta \Theta \) temperature change of rotor
\( \Theta \) instantaneous temperature relative to a reference temperature
\( \Lambda \) temperature-dependent scaling factor for critical stored energy
\( \Phi \) azimuthal coordinate on the rotor
\( \Psi_i \) parameters in the equations for radial stress
\( \alpha \) linear coefficient of thermal expansion for an isotropic material
\( \alpha_1 \) linear coefficient of thermal expansion in the fiber-direction
\( \alpha_2 \) linear coefficient of thermal expansion in the transverse-direction
\( \alpha_0 \) linear coefficient of thermal expansion in the fiber-direction
\( \alpha_r \) linear coefficient of thermal expansion in the transverse-direction
\( \beta \) coefficient of hygrothermal expansion
\( \gamma_{12} \) shear stress
\( \delta_a \) displacement boundary condition on the inside of a ring
\( \delta_b \) displacement boundary condition on the outside of a ring
\( \hat{e} \) uncorrected reading from a strain gage
\( \varepsilon_1 \) fiber-direction strain
\( \varepsilon_2 \) transverse-direction strain
\( \varepsilon_{12} \) shear strain
\( \varepsilon_f \) strain at failure
\( \varepsilon_h \) hoop strain
\( \varepsilon_{h,a} \) change in hoop strain at the ID of the inner ring or the OD of the outer ring due to assembly in IIF and NIF tests
\( \varepsilon_{h,d} \) change in hoop strain at the ID of the inner ring or the OD of the outer ring due to disassembly in IIF and NIF tests
\( \varepsilon_r \) radial strain
\( \varepsilon_{r,a} \) (elastic) radial strain measured upon assembly (NIF tests only)
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Explanation</th>
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<tbody>
<tr>
<td>$\varepsilon_{r,s}$</td>
<td>(viscoelastic) radial strain measured after the temperature soak (NIF tests only)</td>
</tr>
<tr>
<td>$\eta$</td>
<td>the so-called anisotropy ratio $\sqrt{S_{22}/S_{11}}$</td>
</tr>
<tr>
<td>$\theta_{\text{min}}$</td>
<td>rotor rotation during one counter increment (rad)</td>
</tr>
<tr>
<td>$\theta_a$</td>
<td>apparent measured compensation patch angle (rad)</td>
</tr>
<tr>
<td>$\theta_c$</td>
<td>correct compensation patch angle (rad)</td>
</tr>
<tr>
<td>$\mu$</td>
<td>temperature-dependent aging shift rate (for isothermal aging conditions)</td>
</tr>
<tr>
<td>$\nu_0$</td>
<td>reference Poisson's ratio in the transverse sensitivity correction (0.285)</td>
</tr>
<tr>
<td>$\nu_{12}$</td>
<td>major Poisson's ratio ($-\varepsilon_2/\varepsilon_1$)</td>
</tr>
<tr>
<td>$\nu_{21}$</td>
<td>minor Poisson's ratio ($-\varepsilon_1/\varepsilon_2$)</td>
</tr>
<tr>
<td>$\xi$</td>
<td>effective time</td>
</tr>
<tr>
<td>$\rho$</td>
<td>mass density</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>fiber-direction stress</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>transverse-direction stress</td>
</tr>
<tr>
<td>$\sigma_{12}$</td>
<td>shear stress</td>
</tr>
<tr>
<td>$\sigma_f$</td>
<td>failure stress in the Reiner-Weisenberg creep rupture theory</td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>radial stress</td>
</tr>
<tr>
<td>$\sigma_h$</td>
<td>hoop stress</td>
</tr>
<tr>
<td>$\sigma_{\text{ult}}$</td>
<td>ultimate strength</td>
</tr>
<tr>
<td>$\tau_i$</td>
<td>material constants associated with Prony series</td>
</tr>
<tr>
<td>$\phi$</td>
<td>duty cycle (rad)</td>
</tr>
<tr>
<td>$\phi_{\text{inst}}$</td>
<td>duty cycle at instantaneous speed</td>
</tr>
<tr>
<td>$\phi_{\text{ref}}$</td>
<td>duty cycle at reference (negligible deformation) speed</td>
</tr>
<tr>
<td>$\psi$</td>
<td>acute angle btwn. trajectory of illuminated spot &amp; displacement patch boundary</td>
</tr>
<tr>
<td>$\omega$</td>
<td>angular speed of rotor (rad/s)</td>
</tr>
</tbody>
</table>
ACKNOWLEDGMENTS

First thanks go to a true role model, Charles E. Bakis, who always steered an even keel and never failed to give me his utmost sincere support during the pursuit of this research. Dr. Bakis’ professional dedication is certainly inspiring to anyone who works with him, especially his advisees.

Thanks also go to many friends who managed to remain loyal despite the fact that I neglected them to various degrees (for years!) during my commitment to this research. Most notable among these friends is the one that finally joined me as my wife, Abby.
In industrialized nations, most human activities depend on the availability of energy from non-human sources. Traveling, working, communicating, recreating, eating, and residing in a temperature-controlled dwelling almost always involve the use of energy. There is an ever-growing demand to increase the efficiency and performance of the energy systems of the technologies associated with many of these activities (here, systems encompass the transmission and/or storage and/or conversion of energy). As an example, environmental reasons for reducing the use of fossil fuels for energy have motivated the development of electric and hybrid-electric vehicle technologies for more energy-efficient transportation. Also the advanced computer systems in developed nations for large-scale computing and hospital life-support equipment, for example, require high-quality un-interrupted power.

The flywheel battery is a kinetic energy storage and conversion device, which combines a rotating mass (the rotor, the primary storage component) with a motor/generator (the conversion component). A schematic illustration of a flywheel battery is shown in Fig. 1.1.

![Flywheel Battery Components](image)

Figure 1.1. Schematic illustration of flywheel battery components.

Mass is a primary consideration in the design of energy systems for many technologies (e.g., automobiles and earth-orbiting satellites). High strength polymer
matrix composite (PMC) materials are used in the rotor in order to attain high mass-specific energy (or energy per unit mass) of the device. Compared to steel flywheels, carbon-fiber PMC rotors achieve an order of magnitude higher mass-specific energy. Put simply, this is because the inertial stresses in a lightweight carbon-fiber PMC rotor are lower than the inertial stresses in a steel rotor of the same size and speed. Thus, the carbon-fiber PMC rotor, which possesses material strength equal to or greater than steel, can spin faster before the stresses in the rotor reach the material strength limit. Because rotational kinetic energy is proportional to mass and proportional to the square of the speed, composite rotors are able to store a greater mass-specific energy. Comparison of the strength-to-density ratios ($\sigma_{ul}/\rho$) of different materials provides a “rule of thumb” measure of the relative mass-specific energy storage capacities for different materials.

The evaluation of the competitiveness of PMC flywheels compared to other available energy storage and conversion technologies (e.g., nickel-metal hydride batteries) based on mass-specific energy alone is difficult. This is because some flywheel applications require the use of containment—usually a relatively thick steel enclosure—to ensure the protection of personnel or nearby equipment in the event of a rotor burst. When other operational factors are considered, however, the performance of PMC flywheels often exceeds that of electrochemical batteries. With an appropriate design of the motor/generator components, flywheels achieve higher and more efficient mass-specific power (power per unit mass) and/or depth-of-discharge performance compared to advanced electrochemical batteries. The use of low-friction (magnetic) bearings in a flywheel battery reduces the drag losses, effectively extending the range over which charge and discharge rates are efficient (electrochemical batteries are efficient over a relatively small range of charge/discharge rate). Also, the kinetic energy-conversion used by flywheel batteries allows them to operate efficiently over a larger temperature range in comparison to electrochemical batteries (which rely on highly temperature-dependent chemical conversion process). Finally, the cycle life of flywheel batteries is superior to that of electrochemical batteries.

Advances in flywheel engineering must continually be made for flywheels to remain competitive. This thesis is concerned with advancing the ability to design and characterize polymer matrix composite (PMC) material rotors for a flywheel battery.
1.2 Motivation

In order for PMC flywheels to achieve the order of magnitude increase in mass-specific energy in comparison to steel flywheels, it is necessary to take advantage of the fiber-direction strength of composites without suffering from the low transverse strength of composites. This has been the classic challenge of designing composite flywheel rotors. Consider two limiting cases that illustrate a primary trade-off in rotor structural design. In the first case of a spinning ring that is radially very thin, hoop-direction stresses are dominant and, therefore, a hoop-direction fiber architecture would perform well from a stress standpoint. The energy storage capacity of such a ring would be low (due to a small moment of inertia), as would be the volumetric specific energy storage capability of the rotor. Another practical consideration with the radially-thin rotor is the problem of connecting this ring to the drive shaft. Alternatively, consider the case of the solid disk. While the volumetric specific energy of this disk would be higher, significant radial stresses would develop—requiring significant radial-direction reinforcement and an associated reduction of the hoop-direction strength.

The majority of publications on specific rotor designs and general design strategies have been concerned with different ways to deal with these radial stresses and still achieve high mass-specific energy storage in the rotor. It is important to note that these designs have been based on elastic equations. The rotor designs that have been proposed include:

- combinations of concentric rings, each made of different materials with or without radial interference [Danfelt et al. 1977, Nimmer et. al. 1980, Flanagan et al. 1982, Coppa 1984, Olszewski and O’Kain 1988, Gabrys and Bakis, 1996, Ha et al. 1999], such that compressive or very small tensile radial stresses develop during operation. Because the rotor comprises multiple components—each of which is likely manufactured separately and (especially for interference fits) must be precision machined— the number of manufacturing steps involved in such rotors is considerable.
- loose fibers and/or very compliant matrix material [Ferrero et al. 1983, 1985, Ochan and Protsenko, 1987, Gabrys and Bakis, 1997, Portnov and Bakis, 2000] such that zero (loose fibers) or very small (compliant matrix) radial stresses develop. These designs theoretically fail “safely” which may allow the use of lightweight safety containment, thus increasing the mass-specific metrics of the complete flywheel system. Additionally, these rotor designs have less manufacturing complexity compared to the multiple-component rotor designs.

- more advanced techniques for handling low transverse strength (variable density and/or thickness as a function of radial location in the rotor) such that radial stress is zero everywhere [Genta 1984, Christensen and Wu 1977, Padroen et al., 1981]. Presently these designs are not considered practical because the manufacturing of such rotors is rather complex.

The designs that have emerged as the most viable include one or more unidirectional hoop-wound constant-thickness rings [e.g., the recently published designs of production flywheels by Dettmer 1997, Bowler 1997]. The tendency for a unidirectional PMC to exhibit viscoelastic behavior, however, can translate to significant redistribution of the radial stresses and strains in these rings—which have no radial-direction reinforcement. The general flywheel design publications mentioned above do not address viscoelastic effects. A recent report prepared by the Abaqus Technology Corporation [Abaqus 1996] for the U.S. Dept. of Energy, with information sources “…primarily based on telephone and personal interviews with representatives of the teams developing advanced flywheel energy storage systems in the U.S.” states that creep “…has also been identified by some flywheel researchers as a potentially significant problem for high speed fiber-composite rotors… Most researchers believe that creep effects can be managed satisfactorily however.” In light of the fact that the viscoelastic behavior of PMC materials is well known to exist but, at the same time, not considered in published flywheel designs, it is likely that flywheel designers absorb the effects of viscoelastic material behavior in a safety factor, as conjectured by Thoolen [1993].
Some researchers have developed viscoelastic models with some relevance to flywheel rotors. These models will be discussed in detail in Chapter 2, where it will be shown that none of these models are sufficient for predicting the time dependent redistribution of stress and strain in rotors subjected to realistic load histories.

A complete viscoelastic flywheel rotor model will allow predictions of the long-term stresses and strains in rotors due to creep effects and, when combined with an appropriate failure criterion (that may also include fatigue considerations and creep/fatigue interaction), would be an integral part of the flywheel design process that considers component lifetime. Such a model could indicate unfavorable combinations of ring geometries, ring materials, and operating conditions. The incorporation of durability considerations in the flywheel design process will allow the reduction of performance-restricting safety factors that are currently in place to account for viscoelastic effects—thus increasing the performance of flywheels for energy storage.

For verification of a rotor creep model, there is a need to make creep measurements on PMC rings (the fundamental shape of flywheel rotors) directly instead of conventional tensile specimens. The differences in manufacturing procedures between flat specimens and rings result in different fiber architectures. Also, the biaxial stress state in a flywheel rotor ring is very difficult to achieve with standard tensile coupons. To make meaningful creep measurements on spinning rotors (for the purpose of rotor creep model verification), long-term full-field strain measurements on spinning rotors are required. The measurement of strain on any high-speed rotating machine presents some special problems to the experimentalist. The high peripheral velocities (upwards of 1 km/s), high rotational speeds (20-100 krpm), and associated centripetal accelerations (upwards of 1 million g’s) require that the measurement be made without attaching bulky instruments, coatings, or wires to the rotor. In Chapter 2, existing techniques for measuring strain on rotating objects will be reviewed and it will be shown that none of these techniques are immediately suitable for making long-term full-field creep measurements of flywheel rotors.
1.3 Objectives

Recognizing the absence of a published viscoelastic rotor model capable of being used in the design of advanced flywheel rotors subjected to realistic loads, the objectives of this research are:

1. develop a viscoelastic rotor model and verify it with experiments on filament-wound FRP rings.
2. develop and evaluate a non-contact strain measurement technique for measuring creep strains on high-speed flywheel rotors.

1.4 Approach and Organization

To meet the first objective, several tasks were performed. First, the time and temperature-dependent constitutive behavior of the unidirectional PMCs used in advanced flywheel rotor designs was reviewed. Mathematical techniques developed for handling this constitutive behavior was incorporated into a viscoelastic flywheel rotor model that accommodates time-varying spinning and temperature loads as well as interference fits. Basic material tests were performed to provide input to the rotor model so that predictions could be made. Isothermal and non-isothermal creep experiments involving interference-fitted ring pairs were performed, and the results were compared to model predictions.

To meet the second objective, a previously-developed system for non-contact optoelectronic strain measurement (OESM) of rotating objects was improved in several fundamental ways. The OESM system developed in this investigation enables long-term, reliable, non-contact rotor strain measurements. Experiments were performed to evaluate the relevant aspects of the system.

The chapters describing these tasks are organized as follows:

1. Chapter 2: Literature Review. Findings in the current literature are reviewed in these areas:
- The general understanding and fundamental mathematical treatment of the viscoelastic behavior of polymers and PMCs
- Experimental methods for measuring the viscoelastic behavior of PMCs
- Previously developed flywheel rotor models that incorporate viscoelastic constitutive behavior
- Creep failure criteria applicable to PMCs
- Techniques for measuring rotor strains

2. Chapter 3: Viscoelastic Flywheel Rotor Model. The derivation of the rotor deformation model is presented.

3. Chapter 4: Optoelectronic Strain Measurement Method. The theoretical development of the OESM system used for the spin tests is described.

4. Chapter 5: Experimental Procedures. Procedures are presented for the following:
   - basic material creep tests
   - isothermal interference-fit relaxation experiments
   - non-isothermal interference-fit ring relaxation experiments
   - OESM validation experiments
   - setup for the creep test of a spinning PMC rotor.

5. Chapter 6: Results / Discussion. Results are presented and discussed for the experiments described in Chapter 5.

6. Chapter 7: Model Simulations. The viscoelastic rotor model is used to simulate the stress and strain redistributions in flywheel rotors subjected to several load histories.

7. Chapter 8: Conclusions and Recommendations for Future Work.

8. Chapter 9: Appendices. Appendices are presented for the following:
   - Code for the viscoelastic model
   - Error calculation for the quasi-elastic approximation
   - Drawings of the grips for the creep-tests
   - Material properties table
   - Design and assembly of the rotor hub
   - Electrical connections of the spin pit instrumentation
   - OESM LabView code.
   - Creep tube bending analysis
   - OESM vibration tolerance
   - Surface preparation for the OESM pattern
Chapter 2
LITERATURE REVIEW

In this chapter, findings in the current literature pertinent to this thesis are reviewed. In Section 2.1, the fundamental one-dimensional isothermal linear viscoelastic constitutive law for polymeric materials is presented. In Section 2.2, the literature is reviewed regarding the constitutive law for PMC laminae subjected to plane stresses (of interest for the analysis of hoop-wound ring-shaped flywheel rotors). The basic mathematical treatment in the literature regarding PMC laminae is discussed, including commonly-assumed simplifications. The incorporation of additional factors to the linear viscoelastic constitutive law (e.g., temperature, age, moisture, and stress level and sign) of PMC laminae is described in Section 2.3. Experimental validations for the mathematical treatment of most of these factors are presented throughout Section 2.3. In Section 2.4, experiments are mentioned that indicate similarity in the time and temperature characterization of neat resin and the matrix-dominated composite properties. Experimental work is presented in Section 2.5 that supports the validity of using the above-mentioned (uniaxial) constitutive characterization of PMC laminae to predict the response of such laminae under biaxial stresses. In Section 2.6, previously developed viscoelastic models that incorporate time-dependent constitutive behavior and are relevant to PMC flywheel rotors are reviewed. Creep failure criteria applicable to PMCs (to be used in conjunction with the flywheel rotor model) are reviewed in Section 2.7. Rotor strain measurement techniques are reviewed in Section 2.8.

2.1 Molecular Understanding and Macroanalytical Treatment of Polymer Viscoelasticity

Presently, myriad texts and journal reviews provide detailed discussions of polymer viscoelastic behavior from the molecular viewpoint—most notably Flory 1959, Ferry 1961 and Smith 1962. Such works generally conceptualize the polymeric solid as an entanglement of long polymer molecule chains, with some amount of free volume between neighboring chains. The viscoelastic behavior of a polymeric solid subjected to
an applied stress is qualitatively understood in terms of spatial rearrangements of these polymer chains. Thermodynamic principles dictate that these spatial rearrangements will tend towards a minimum energy equilibrium configuration over time. The mobility of polymer molecules is strongly temperature dependent: at high temperatures the rate of molecular rearrangements is high; vice versa at low temperatures.

Of more use to the engineer are mathematical characterizations of the macroscopic-level viscoelastic mechanical properties that are necessary to incorporate viscoelastic behavior into structural models. The phenomenological approach to characterizing viscoelastic constitutive behavior is based on the conceptualization of polymeric solids as combinations of mechanical springs and dampers (e.g., the classic Maxwell and Kelvin mechanical models). In the present-day literature, engineering models for polymeric (viscoelastic) materials loaded in the linear regime employ a constitutive law essentially unchanged from these classic mechanical analogs. For a viscoelastic solid subjected to time-variable loading, constitutive behavior relating stress $\sigma$, and strain $\varepsilon$, may be represented with the linear hereditary integral (or "Boltzmann superposition integral") in one-dimension by,

$$\varepsilon(t) = \int_0^t S(t-t') \frac{\partial \sigma(t')}{\partial t'} dt'.$$

Here, $S$ is the creep compliance, $t$ is time, and $t'$ is a placeholder in the integral. This convolution integral captures the "fading memory" constitutive behavior of linear viscoelastic materials. Ferry [1961] was among the first to propose the linear hereditary integral as a valid constitutive law for polymers subjected to small stresses.

The use of the classical spring-and-damper elements as mechanical analogs for viscoelastic solids results in an exponential summation for the creep compliance,

$$S(t) = S_0 + \sum_{i}^{m} S_i (1 - e^{-t/\tau_i}).$$
In Eq. 2.2 (which is also commonly referred to as a "Prony series"), $S_0$ is an instantaneous or elastic compliance, and $\tau$, and $S_i$ are material constants. When the number of elements used to model the material response becomes large (i.e. large $m$ and closely-spaced $\tau_i$), the compliance may be described by the creep spectrum. The relationship between the creep spectrum, $L(t)$, and creep compliance is given by,

$$S(t) = S_0 + \int_0^\infty L(\tau)(1 - e^{-t/\tau})d\tau.$$  \hfill (2.3)

Another mathematical form very often used to represent polymer composite creep compliance is the (Findlay) power-law expression,

$$S(t) = S_0 + S_1 t^n.$$  \hfill (2.4)

In Eq. 2.4, $S_0$ is the initial elastic compliance, $S_1$ is the time coefficient, and $n$ is the time exponent.

The above discussion of the constitutive law is in terms of the one-dimensional response of a polymeric solid under isothermal conditions. In the next section, the existing literature is reviewed that extends the one-dimensional isothermal treatment of linear viscoelasticity to PMCs subjected to biaxial stresses and variable temperatures that are of interest in this research.

### 2.2 The Two Dimensional Constitutive Law Relevant to a Unidirectional PMC Lamina

#### 2.2.1 Unidirectional PMC Lamina

Because it is desirable to validate the viscoelastic model with strain measurements of composite rings during creep tests, the viscoelastic rotor model (presented in the next chapter) is derived for rotors with unidirectional filament-wound fiber structure under plane-stress conditions. Neglecting torsion and assuming axisymmetric loads on the rotor
eliminates shear stress from consideration. Thus, the corresponding form of the two-
dimensional constitutive law for the PMC laminae of interest is given by,

\[
\begin{bmatrix}
e_1 \\
e_2
\end{bmatrix} =
\begin{bmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{bmatrix}
\begin{bmatrix}
\sigma_1 \\
\sigma_2
\end{bmatrix},
\]

(2.5)

where the 1-direction and 2-direction are parallel and perpendicular, respectively, to the
fiber. Note that the shear compliance, \(S_{66}\) (that relates shear stress, \(\tau_{12}\) to shear strain,
\(\gamma_{12}\)), has not been included in the compliance matrix. Henceforth, an abbreviated notation
will be used (a single overbar for the stress and strain vectors, a double overbar for the
compliance matrix) unless a specific component is under consideration. The components
of the above compliance matrix, \(S\), are defined in terms of engineering constants for an
orthotropic lamina according to,

\[
S_{11}=1/E_1, \quad S_{22}=1/E_2, \quad S_{12}=-\nu_{12}/E_1,
\]

(2.6a,b,c)

where \(E_1\) and \(E_2\) are the major and minor Young’s moduli and \(\nu_{12}\) is the major Poisson’s
ratio.

Much experimental work for characterizing the viscoelastic behavior of polymer
composite laminae exists in the published literature. The lack of testing standards for
measuring composite material viscoelastic properties is conspicuous and often precludes
the comparison of results from different researchers. For example, there are no standards
specifying specimen sizes or types, stress and temperature ranges, mechanical
conditioning, and test durations—all of which could affect the test results. Several
independent researchers, however, have employed consistent creep testing procedures
and report results that merit mention in this review.

Most of the published experimental work on composite viscoelastic behavior has
concerned the characterization of the time and temperature dependence of the
components in the so-called principal compliance matrix for a unidirectional lamina
under plane stress. Here and henceforth the term “principal” describes the compliance
matrix with nonzero \(S_{11}, S_{12}, S_{22}\), and \(S_{66}\) terms and zero shear-extension coupling terms,
$S_{16}$ and $S_{26}$ (Tsai and Hahn, 1980). Principal compliances are measured with strain gages on flat unidirectional laminae, as illustrated in Fig. 2.1.

![Diagram of specimen configurations for principal compliance measurements](image)

**Figure 2.1.** Specimen configurations for principal compliance measurements (adopted from Sullivan, Blais and Houston, 1993).

### 2.2.2 Simplifications to the Compliance Tensor

In general, any component of the compliance tensor for a PMC lamina (Eqn. 2.5) may be time and temperature dependent. Simplifications to the compliance tensor are assumed in this research, based on the findings of previously published experimental studies during the last 3-4 decades for characterizing the thermo-viscoelastic (T-VE) properties of composite laminae. For unidirectional glass and carbon fiber-reinforced polymer composites, the fiber-direction compliance, $S_{11}$, has been found to exhibit very little time or temperature dependence (Yeow et al. [1979], Morris et al. [1980], Sullivan [1990]). Experimental measurements of the cross compliance, $S_{12}$, of glass and carbon fiber composites indicate that the time and temperature dependences are orders of magnitude smaller than the transverse compliance, $S_{22}$ (Yeow et al. [1979], Sullivan [1990]). Also, several experiments indicate symmetry of the viscoelastic compliance matrix for polymer composites (Halpin and Pagano [1967], Sullivan [1990]). Based on the above findings, it is common for researchers to simplify models of PMC laminae by considering T-VE effects in only the transverse and shear compliances and treating the
fiber-direction and cross-compliance as time and temperature independent (Tuttle & Brinson [1986], Bradshaw & Brinson [1999]). Hence, for unidirectional filament-wound PMC rotors, the transverse compliance $S_{22}$ is the only component of the compliance tensor that is considered time and temperature dependent in the present investigation.

The parameters for the functions chosen to represent the material compliances are generally determined by a least-squares fit to experimental data from creep tests on unidirectionally reinforced laminae. A sample of published composite creep experiments and the corresponding creep compliances are summarized in Table 2.1. Note that different test temperatures were often used for measuring the creep compliances reported in Table 2.1, as indicated in the "Reported Compliance(s)" column. Because characterizations of the temperature-shift and physical aging behaviors (explained in Sections 2.3.2-3) were omitted for most of the reported compliances in Table 2.1, only limited comparisons of the compliance function parameters between the different researchers can be made. Table 2.1 mainly serves to illustrate that many researchers use a power-law expression for creep compliance, which was also used in this research.
Table 2.1. A sample of creep compliance expressions for various polymer matrix materials.

<table>
<thead>
<tr>
<th>Researcher(s)</th>
<th>Creep Compliance Form</th>
<th>Exp. data?</th>
<th>Material System(s)</th>
<th>Layup(s) (deg)</th>
<th>Test type</th>
<th>Reported Compliance(s) (1/GPa, and $t$ in minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lou &amp; Schapery [1971]</td>
<td>Power law</td>
<td>Yes</td>
<td>Glass/epoxy</td>
<td>0/30/45/60/90</td>
<td>Tension</td>
<td>$n=0.27$ for all layups (at 73°C)</td>
</tr>
<tr>
<td>Togui et al. [1983]</td>
<td>Power law</td>
<td>Yes</td>
<td>Neat polycarbonate</td>
<td>-</td>
<td>Tension</td>
<td></td>
</tr>
<tr>
<td>Beckwith [1984]</td>
<td>Power law</td>
<td>Yes</td>
<td>S901Glass/epoxy</td>
<td>0, 20, 45, 90, 0/90, ±30, ±45, ±60, ±80</td>
<td>Tension &amp; Flexure</td>
<td>$S_{22}=0.048 + 0.000118 \rho^{0.19}$ $S_{66} n/a$ (at 24°C)</td>
</tr>
<tr>
<td>Tuttle &amp; Brinson [1986]</td>
<td>Power law</td>
<td>Yes</td>
<td>T300/5208</td>
<td>0, 10, 90</td>
<td>Tension</td>
<td>$S_{22}=0.106 + 0.00136 \rho^{0.289}$ $S_{66}=0.0156 + 0.00332 \rho^{0.247}$ (at 149°C)</td>
</tr>
<tr>
<td>Lin &amp; Hwang [1989]</td>
<td>Power law</td>
<td>No, (FEA)</td>
<td>Carbon/epoxy</td>
<td>[45/-45], [0/45/90/-45], [0/90],</td>
<td>Tension (simulated)</td>
<td>$S_{22}=0.104 + 0.00056 \rho^{0.33}$ $S_{22}=0.161 + 0.00099 \rho^{0.31}$ (at 24°C)</td>
</tr>
<tr>
<td>Yen and Morris [1989]</td>
<td>Power law</td>
<td>Yes</td>
<td>Random orientation glass/polyester</td>
<td>2-D random</td>
<td>Tension</td>
<td>parameters varied with temperature $T$, $n=0.23$ for all $T [23&lt;T&lt;100]°C$</td>
</tr>
<tr>
<td>Chien &amp; Tzeng [1995]</td>
<td>Power law</td>
<td>Yes⁷</td>
<td>AS4/PEEK</td>
<td>90, [±45]₀, [0/±45/90], [±45/90],</td>
<td>Tension</td>
<td>$S_{22}=0.109 +0.1863 \rho^{0.1954}$ $S_{66}=0.201 +2.27 \rho^{0.2771}$ (at 93°C)</td>
</tr>
<tr>
<td>Liou &amp; Tseng [1997]</td>
<td>Power law</td>
<td>Yes</td>
<td>Carbon/nylon6</td>
<td>0, 15, 30, 45, 60, 90</td>
<td>Tension</td>
<td>$S_{22}=0.148 + 0.0077 \rho^{0.237}$ $S_{66}=0.334 + 0.9481 \rho^{0.212}$ (at 25°C)</td>
</tr>
<tr>
<td>Bradley et al. [1998]</td>
<td>Power law</td>
<td>Yes</td>
<td>Neat resin and E-glass-reinforced vinyl ester</td>
<td>'knitted' fabric</td>
<td>Flexure</td>
<td>$n$ varied from 0.13 to 0.20, other parameters n/a (at 23°C)</td>
</tr>
<tr>
<td>Bradshaw &amp; Brinson [1999]</td>
<td>Prony series</td>
<td>Yes</td>
<td>IM7/K3B</td>
<td>[+45/-45/90]₀, [+45/-45], [+45/-45/90/0],</td>
<td>Tension</td>
<td>30-term Prony series (not given) for $S_{22}$ and $S_{66}$</td>
</tr>
</tbody>
</table>

⁷data actually from Kim and Hartness [1987]
2.3 Factors That Affect the Transverse Compliance

Molecular-level theories for explaining the effects of temperature, magnitude and sign of the applied stress, material aging, degree of cure, and moisture on the time dependent constitutive behavior of polymers have been under development for the past 50 years. Mathematical treatments for rationally incorporating most of these phenomena into constitutive laws for PMCs have been developed as well. Some of these mathematical treatments have reached a mature state of development for characterizing certain classes of materials under certain loading conditions, and extensive experimentation has validated such treatments. Following are discussions of these factors that affect the time-dependent behavior of the transverse compliance.

2.3.1 Stress Magnitude

Linear VE behavior is generally observed at low stress levels and temperature well below the glass-transition temperature, $T_g$. Conversely, nonlinear VE behavior is generally observed at higher stress levels and temperatures near and above $T_g$. For models of structural applications that load composite components at high stress and temperature levels, a linear constitutive law (e.g., Eq. 2.1) may not be sufficiently accurate. Nonlinear viscoelastic constitutive laws exist for such cases (e.g., Schapery [1969]), which require experimental characterization that includes the effects of stress magnitude.

With regard to the stress and temperature levels that define a linear-response-regime, experimental findings in the literature vary among material system. Currently, no “rules-of-thumb” exist for predicting an expected linear viscoelastic response regime based on simple lamina properties such as transverse strength and material $T_g$. Table 2.2 provides a sample of published tests on different PMC materials and stress/temperature regimes where linear behavior was observed.
Table 2.2. A sample of tests where linear viscoelastic behavior was observed.

<table>
<thead>
<tr>
<th>Researcher(s)</th>
<th>Material System(s)</th>
<th>Test Type</th>
<th>Temperature (°C)</th>
<th>Stress Level (MPa / %(\sigma^{int}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Halpin &amp; Pagano [1967]</td>
<td>unidirectional nylon-reinforced rubber and steel wire-reinforced rubber</td>
<td>Transverse tension</td>
<td>NR*</td>
<td>NR² / NR</td>
</tr>
<tr>
<td>Lou and Schapery [1971]</td>
<td>Glass/epoxy “Scotchply”</td>
<td>30-degree off-axis tension</td>
<td>73 ((T_g)-70)</td>
<td>9.35 / 14%</td>
</tr>
<tr>
<td>Yeow et al. [1979]</td>
<td>T300/934 prepreg</td>
<td>Transverse tension</td>
<td>200 ((T_g)+20)</td>
<td>NR / 10%</td>
</tr>
<tr>
<td>Tuttle &amp; Brinson [1986]</td>
<td>T300/5208 0.65 (V_f) laminae</td>
<td>Transverse tension</td>
<td>149 ((T_g)-30)</td>
<td>15.6 / NR</td>
</tr>
<tr>
<td>Yen and Morris [1989]</td>
<td>Random orientation glass/polyester SMC-R50</td>
<td>Tension</td>
<td>100 ((T_g)-65)</td>
<td>NR / 36%</td>
</tr>
<tr>
<td>Sullivan [1990]</td>
<td>glass/Dow Derakane 470-36 0.3 (V_f) laminae</td>
<td>Transverse tension</td>
<td>115 ((T_g)-10)</td>
<td>8 / NR</td>
</tr>
<tr>
<td>Sullivan et al. [1993]</td>
<td>glass/vinyl/ester 0.48 (V_f) laminae</td>
<td>(\sigma_2) tension</td>
<td>90 ((T_g)-63)</td>
<td>8 / NR</td>
</tr>
<tr>
<td>Veazie and Gates [1997]</td>
<td>IM7/K3B</td>
<td>Transverse tension &amp; compression and shear</td>
<td>230 ((T_g)-10)</td>
<td>3.56 / NR ((\sigma_1)) 8.34 / NR ((\sigma_{12}))</td>
</tr>
</tbody>
</table>

*NR indicates that the value was not reported †at room-temperature

For unidirectional glass and carbon/epoxy materials to achieve a fatigue life of \(10^5\) to \(10^6\) cycles, the operational transverse stress levels should be limited to roughly 50% of the room-temperature transverse strength [Kim 1987]. Also, the well-known tendency for PMC transverse strength to decrease with increasing temperature should limit rotor temperatures during operation to below the glass transition temperature (\(T_g\)) [Yeow et al. 1979, Brinson et al. 1981]. Results will be presented later from an experiment for determining the linearity of the VE behavior of \(S_{22}(t,T)\) for an Eglass/epoxy material at different applied stress levels.
2.3.2 Temperature

In polymers, temperature changes cause free thermal expansion and significant changes in the time-dependent and time-independent components of the creep compliance. Under an applied stress, motion of the polymer chains is the mechanism that causes creep of polymers. This polymer chain mobility increases with temperature. Correlations between the effect of time and the effect of temperature on polymer creep led to the development of the time-temperature superposition principle (TTSP) for polymers and polymer composites. To illustrate the way that time and temperature are handled within the context of the TTSP, consider the simulation of two ten-minute duration creep tests (performed on identical specimens and at identical stress levels) at two temperatures $T_1$ and $T_2$ ($T_2 > T_1$). The simulated compliances versus time at the two temperatures are illustrated in Fig. 2.2. It can be seen that elevated temperature results in increased compliance. The fundamental mathematical expression of the TTSP for a thermo-rheologically simple material relates this temperature-dependent behavior to time according to,

$$S(t,T_2) = S(aT_2 t, T_1). \quad (2.7)$$

In this expression the time-dependent compliance at $T_2$ is effectively equal to the compliance at $T_1$ at longer times (if it were to be measured) as indicated by the temperature-dependent time shift factor $aT_2$. A master curve can then be constructed from these two data sets such that the compliance data from the tests at the two different temperatures overlap, as shown in Fig. 2.3. In the example shown in Figs 2.2 and 2.3, the shift factor $aT_2$ necessary to achieve overlap of the two data sets is 100 (or two decades on the log-time plot). The temperature shift factor appears as a horizontal shift on the master curve, where the amount of shift is measured relative to a reference temperature $T_{\text{ref}}$. Reference temperature data is not shifted (i.e. $aT_1 = aT_{\text{ref}} = 1$). Master curves are generally constructed after compliance data have been measured at more than just two temperatures. In a log-time scale plot of compliance, the log of the shift factor $aT$ is a
measure of the amount of horizontal shift resulting from the difference between the current temperature and the temperature at which the master curve is referenced.

![Figure 2.2. Simulated compliance data at two temperatures.](image)

![Figure 2.3. “Shifted” data from Fig. 2.2.](image)

Accelerated material characterization (e.g., the construction of master or “reference” curves) is made possible by the use of the TTSP—allowing effective measurements of the creep compliance over a time scale many orders of magnitude greater than the actual test duration. This is illustrated in Figs. 2.2 and 2.3, where the compliance value at $T=T_1$ and $t=10^3$ minutes is effectively measured at $T=T_2$ and $t=10^1$ minutes. More generally, the TTSP is used in a T-VE model to calculate creep compliance at an arbitrary time and temperature, $S(t,T)$, based on a master curve.
Several functional forms for the temperature shift factor $a_T$ have been considered. The WLF (Williams, Landel, Ferry) equation is considered appropriate at temperatures above the $T_g$, and is given by

$$\log a_T = \frac{C_1 (T - T_r)}{C_2 + (T - T_r)},$$

(2.8)

where $C_1$, $C_2$, and $T_r$ are experimentally-determined constants [Ferry 1961]. An Arrhenius relation for the shift factor is sometimes considered below the $T_g$,

$$\log a_T = \frac{\Delta H}{2.303R} \left( \frac{1}{T} - \frac{1}{T_0} \right),$$

(2.9)

where $\Delta H$ is an activation energy, $R$ is the universal gas constant and $T_0$ is a reference temperature. Some researchers have determined that neither the Arrhenius function nor the WLF equation accurately represents the temperature shift behavior of composite laminae in the respective applicable temperature regimes (Yeow et al. [1979], Sullivan [1990]). Bradshaw and Brinson [1999] use an exponential shift factor equation of the form

$$a_T = 10^k(T-T_{ref}),$$

(2.10)

where $k$ is an experimentally-determined constant and $T_{ref}$ is an arbitrary reference temperature. Equation 2.10 gives the form of the temperature shift factor equation used in this research. This equation is a straight line on a plot of $\log_{10}(a_T)$ versus temperature, and will be experimentally shown to fit the temperature shift behavior well for the materials and temperature range of interest in this investigation.

The two-dimensional thermo-viscoelastic constitutive law, including temperature shifts, is given by
\[ \varepsilon(\xi) = \int_0^\xi \bar{\alpha}(\xi - \xi') \frac{\partial \bar{\sigma}}{\partial \xi'} d\xi' + \int_0^\xi \bar{\alpha} \frac{\partial \Theta}{\partial \xi'} d\xi' + 2 \int_0^\xi \bar{\alpha} \frac{\partial \Theta}{\partial \xi'} d\xi', \]  

(2.11)

where \( \bar{\alpha} \) are the material coefficients of thermal expansion (treated as time-independent in this research), and \( \alpha_1 \) and \( \alpha_2 \) correspond to the longitudinal and transverse directions, respectively. The effective time \( \xi \) (sometimes called “reduced time”), and temperature change \( \Theta \), are defined by

\[ \Theta = T - T_0, \]  

(2.12)

\[ \xi = \int_0^t \alpha_1 \, dt, \quad \xi = \int_0^t \alpha_2 \, dt. \]  

(2.13a,b)

In this research, the reference temperature, \( T_0 \), in Eq. 2.12 is considered to be the stress-free temperature, and is approximated as the cure temperature of the composite ring. The lower time limit of integration of zero in Eq. 2.11 assumes that the material is stress-free prior to time zero. Correspondingly, in the viscoelastic rotor model presented in Chapter 3, time zero is taken as the moment after which the temperature of the freshly-fabricated composite ring is first reduced from the cure temperature.

### 2.3.3 Moisture

Moisture absorption in composites results in a swelling of the polymer microstructure similar to the swelling induced by a temperature increase. This swelling increases chain mobility and changes the material VE response similarly to a temperature increase and, hence, is called the hygrothermal response. The mathematical incorporation of hygrothermal effects into the VE constitutive law is analogous to the treatment of temperature effects. In the absence of stress, the linear hygro-thermo-viscoelastic constitutive law is given by,

\[ \varepsilon(\xi) = \int_0^\xi \bar{\alpha} \frac{\partial \Theta}{\partial \xi'} d\xi' + \int_0^\xi \bar{\alpha} \frac{\partial M}{\partial \xi'} d\xi', \]  

(2.14)
where $\bar{\beta}$ are the coefficients of hygroscopic expansion ($\beta_1$ and $\beta_2$), and $\Delta M$ is the change in moisture relative to a stress-free reference value. When hygroscopic effects are considered in the constitutive law, the shift factor used to calculate the effective time, $\xi$, in Eq. 2.14 must include the combined horizontal shifts in the VE properties due to temperature and moisture and is called the temperature/moisture shift factor, $a_{TM}$. Linear constitutive characterization of unidirectional lamina including hygrothermal effects [Flaggs and Crossman 1981] has been shown to accurately predict the curvature of nonsymmetric GY70/339 laminates after a one-year exposure to elevated humidity at several temperatures.

Because advanced composite flywheel rotors are operated in vacuum, thermoviscoelastic material characterization should be performed on appropriately dried specimens. Moisture absorption and desorption during the operational lifecycle would then be minimal. For this reason, moisture content was not considered in the constitutive law used for developing the rotor model in this investigation.

2.3.4 Age

Aging of polymers is caused by the tendency for polymer chains to gradually evolve towards an equilibrium configuration after being cooled (quenched) to temperatures below $T_g$. This equilibrium configuration is associated with a lower volume per unit mass—resulting in decreased chain mobility and, hence, a lower compliance. The effect of aging is incorporated into the linear VE constitutive law by an effective time contribution, i.e. a horizontal shift of the log-time scale of the VE properties. This is similar to the effect of a temperature change, with the exception that aging always advances at temperatures below the $T_g$—causing continually decreasing compliance—whereas compliance can increase or decrease depending on the sign of $\Theta$.

Mathematical characterization of aging is based on the so-called momentary response. Because the aging shift factor is itself time-dependent, it is only valid for calculating compliance relative to a reference compliance curve in a momentary time.
interval over which the aging time remains approximately constant. A rule-of-thumb
definition for a momentary time interval is an interval less than $\frac{1}{10}$ of the aging time.

An expression relating the momentary creep compliance at a given time, temperature, and age, $S(t)_{T,e}$, to a reference creep compliance curve, $S_{\text{ref}}$, is given by

$$S(t)_{T,e} = S_{\text{ref}}(a_T a_e t)_{T_{\text{ref}} e_{\text{ref}}}$$ \hspace{1cm} (2.15)

In Eq. 2.15, $t_e$ is the instantaneous material age (since quench), $T_{\text{ref}}$ and $e_{\text{ref}}$ are the
temperature and age at which the reference curve was created, and the temperature shift
factor, $a_T$, and aging shift factor, $a_e$, give the log-time scale horizontal shift relative to the
reference curve.

The aging shift factor for materials aged under isothermal conditions is given by

$$a_e = \left( \frac{t_{\text{ref}}}{t_e} \right)^{\mu(T)}$$ \hspace{1cm} (2.16)

where $t_{\text{ref}}$ is the aging time of the reference compliance curve, and $\mu$ is the temperature-
dependent shift rate [Bradshaw and Brinson, 1997]. The results of a set of experiments
for determining the shift rate $\mu(T)$ for $S_{22}$ of the composite used in the present
investigation are given later in Section 6.1, Basic Material Tests.

As a practical matter, before putting a composite rotor into service, it is beneficial
to stabilize the material mechanical properties by aging at a temperature slightly below $T_g$
for a period of time after manufacture. Aging stiffens the material, thereby enhancing the
resistance of the material to creep deformation. Sullivan [1990] reports that effective
age-equilibrium for a PMC can be achieved after about 1000 hours at a temperature of
about $T_g - 15°C$. This time and temperature profile for aging to “effective equilibrium”
was first discussed by Struik [1978], who tested the aging behavior of dozens of
polymers. While no discussion of aging considerations could be found in the published
literature on rotor designs, it is a reasonable assumption that material aging stabilization
would be conducted for practical designs. Therefore aging effects are not included in the
flywheel rotor model presented in Chapter 3. In the event that inclusion of aging effects
is desired, it should be noted that Eq. 2.16 is not sufficient for determining the aging contribution to effective time for non-isothermal aging histories. Non-isothermal aging effects can be incorporated into a model with effective-time theory (experimentally shown by Bradshaw & Brinson [1999]) similar to the way in which temperature effects are incorporated with the TTSP.

2.3.5 Stress Sign

The vast majority of PMC viscoelastic characterization has been done using tensile specimens. Gates et al. [1997] and Veazie and Gates [1997] showed that the TTSP can be used to construct compressive master curves—allowing accelerated testing and construction of master curves from momentary creep data under compressive loads. Experiments performed by these researchers for characterizing the creep response of PMC in compression indicate that the time-dependent response is different than the tensile response. An example plot of $S_{22}(t)$ of IM7-K3B specimens ($T_g$=240°C) measured under either tensile or compressive applied stresses is shown in Fig. 2.4.

In many structures (including flywheels), the sign of stress may change over time under normal operational conditions. Modeling such structures with the linear viscoelastic constitutive law (the Boltzmann hereditary integral), however, requires that a single
compliance matrix be used to represent the material response to the load history. In the context of linear viscoelasticity, there are modeling inaccuracies in using either the tensile or compressive compliance to represent the material response in a structure subjected to loads of variable sign. Such inaccuracies were neglected in the present investigation.

### 2.3.6 Degree of Cure

For thermosetting polymers, the advancement of the polymerization process (curing) decreases the volume per unit mass and, hence, affects chain mobility in a fashion similar to aging. When the degree of cure (DOC) is considered, it is incorporated into the constitutive law with a time-shift factor. The DOC is a primary consideration for calculating the stress evolution in a structure as the DOC advances (e.g., for modeling residual stresses due to manufacturing).

When the DOC is combined with physical aging and temperature changes, there is a coupling of the effects that is difficult to model. Consequently, most material characterization is performed on systems that have a very advanced DOC. Because the DOC monotonically increases rapidly towards complete cure during the composite-manufacturing phase, the associated effects diminish with time and, therefore, are not necessarily considered in models of stresses and strains due to operational loads. As with aging, it is generally advantageous to advance the DOC as far as possible before putting the material into service to maximize material stability.

### 2.4 Polymer vs. Polymer Composite Characterization

There is evidence in the literature supporting the validity of performing creep tests on neat resin to characterize certain matrix-dominated time- and temperature-dependencies of the mechanical properties of the resin with fiber reinforcement (e.g., the dependence of $S_{22}$ on physical age and temperature). The advantage of testing the neat polymer is that it is easier and less expensive to fabricate compared to the corresponding PMC. Sullivan [1990] noted that the "shape" of a glass/epoxy $S_{22}$ master curve was
similar to the shape of the corresponding curve of the neat resin. For neat bisphenol-A epoxy and vinyl ester, Sullivan [1995] also noted that the retardation spectra (the stress-relaxation analogue to the creep spectrum given in Eqn. 2.2) are "essentially the same in shape" with the corresponding composites. Sullivan also found that, below the \( T_g \), the temperature shifting behavior of composites and their resins are very similar. Kim and Daniel [1995] found that the creep spectra of composites and the corresponding neat resins had roughly identical peaks. Bradley et al. [1998] measured approximately identical power exponents fitted to creep curves of neat vinyl ester and glass-reinforced vinyl ester. This equivalence of creep exponents was also noted by Beckwith [1984] for epoxy and a corresponding glass/epoxy composite. Bradshaw and Brinson [1997] noted "similar aging behavior" of IM7/K3B composite and the neat polymer during viscoelastic characterization of the transverse and shear moduli of the composite. As a final note, Sullivan [1995] mentioned that a micromechanical model can be successfully used to calculate scaling factors for estimating composite properties based on resin characterization.

No experiments were performed in the present research to test similarities between the time/temperature dependency of the composite and the corresponding neat resin. Experiments were performed, however, to characterize the time and temperature behavior of the transverse compliance of a unidirectional glass-fiber composite. In the rotor model predictions presented in Chapters 6 and 7, the transverse compliance of a unidirectional carbon-fiber composite (with the same matrix as the characterized glass-fiber composite) is assumed to possess the same time and temperature dependence as the glass-fiber composite.

## 2.5 Biaxial Stress

While the above discussion of PMC viscoelastic characterization considers only uniaxial applied stress, it should be noted that experiments have validated the direct use of such characterizations in predicting the time-dependent response of PMCs under biaxial stresses. Using the TTSP, Yeow et al. [1979] generated master curves for \( S_{22} \) and \( S_{66} \) of unidirectional carbon/epoxy specimens in the linear viscoelastic regime. Using the
$S_{22}$ and $S_{66}$ master curves and applying the standard orthotropic transformation equation commonly used in elastic characterizations [Jones, 1975], Yeow et al. were able to predict transformed master curves of $[30]_{16}$ and $[60]_{16}$ laminates. The conclusion that “a lamination theory computational procedure including viscoelastic properties is feasible,” implies that properties characterized with individually applied uniaxial stresses can be used to predict deformations in a biaxial stress field. In a similar set of experiments, Tuttle and Brinson [1986] determined the viscoelastic behavior of unidirectional T300/5208 composite specimens in uniaxial stress fields and then used classical thin lamination theory (CTLT) to predict the time-dependent linear and nonlinear viscoelastic response of specimens cut from $[0/30/-60/0]_S$ and $[-80/-50/40/-80]_S$ laminates and subjected to uniaxial stress. “Reasonably accurate,” results were obtained using this approach. More recently, Bradshaw and Brinson [1999] characterized the principal linear viscoelastic creep compliance including aging effects for uniaxially stressed IM7/K3B laminae. Applying CTLT to the principal linear viscoelastic compliance, they were able to predict the non-isothermal aging response of $[+60/-60]_S$ and $[+45/-45/90/0]_S$ laminates. Experimental results indicated that aging must be considered and, furthermore, that it is necessary to consider separate aging shift rates for $S_{22}$ and $S_{66}$.

The above experimental findings indicate that the uniaxial creep tests performed in this investigation (for linear viscoelastic characterization of $S_{22}$ of unidirectional E-glass/epoxy) are appropriate for the characterization of a composite material used in a flywheel in which biaxial stresses are present.

### 2.6 Viscoelastic Models Related to PMC Rotors

Williams’ [1964] review paper is among the earliest known articles concerning PMC materials in a structural viscoelastic model related to flywheel rotors. Williams presented the solution for the (radial) contact stress at the interface of an internally-pressurized isotropic viscoelastic cylinder and an external, elastic, isotropic casing under plane-strain, isothermal conditions. Schapery [1967] presented the solution for hoop stress at the inside radius of an internally pressurized anisotropic viscoelastic cylinder in plane-strain and isothermal conditions. In both of these publications, single step loads at
time zero were applied—allowing an expression for hoop stress (at the respective radial locations mentioned above) to be derived using the elastic-viscoelastic correspondence principle (EVECP). The EVECP is discussed in more detail in Chapter 3. Full field solutions for the hoop stresses and strains were not derived in either of these publications.

Feng [1985] derived an isothermal viscoelastic field solution for a disk of isotropic rubber and presented plots of stress and strain field redistribution over time due to a step spinning load applied at time zero. A recurrent algebraic formulation of the linear constitutive law (the convolution integral) was used, enabled by employing an exponential form of the material time dependence. The major benefit of this technique was reduced computation time. While this model was derived for isotropic rubber materials, Feng noted that it could be modified for PMC composites as well.

Several examples of viscoelastic flywheel models are found in the Russian literature. Gurvich [1987] gave an example of a flywheel model with an anisotropic elastic annular rotor and viscoelastic filamentary spokes (for a hub) that were wrapped over the rim. Stresses in the rim were evaluated using a plane stress elasticity solution, whereas stresses in the spokes were evaluated using a strength-of-materials approach (they were treated as a statically determinate system of rods). Displacement compatibility at the spoke/rim interface was enforced. The quasi-elastic solution procedure (developed by Schapery [1965]) was then used to solve the equations of viscoelasticity for stresses and strains in the spokes and the rim. Antsilevich [1992] presented an analysis of a similar rotor-to-shaft connection, consisting of a composite overwrap shell with variable-angle fiber architecture. The rotor modeled by Antsilevich had a rounded outer diameter and the shell containing the rotor was in the shape of an ellipsoid of revolution. The composite consisted of a linearly viscoelastic fiber and a nonlinearly viscoelastic matrix. The hereditary integral constitutive equations were converted to approximate summation forms and a time-stepping method was used to solve for the full-field displacements in the hub subjected to rotational loads. Trufanov and Smetannikov [1990] analyzed another overwrap shell structure using an axisymmetric finite-element method and Schapery's quasi-elastic solution technique to solve for time-dependent displacements and stresses at several geometric locations in
both the rotor and the shell. A linear viscoelastic constitutive law was assumed for the filamentary composite rotor and shell.

The research by Gurvich, Antsilevich, and Trufanov and Smettanikov cited above was aimed at solving the ongoing challenge of the shaft-rotor connections in early-1990s Russian flywheels. A recent paper by Portnov [1999] summarized experimental spin test results of such flywheels. Portnov cited dynamic problems with the spoke and overwrap rotor-to-shaft connection types that effectively render such designs impractical. He concluded that the most effective design is a unidirectional filament-wound rim connected to the shaft with simpler planar construction. Spoke and overwrap flywheel designs are nonexistent in the recently-published literature.

Chien and Tzeng [1995] presented a thermo-viscoelastic generalized plane-strain model for multiple-layer anisotropic cylinders and used the EVECP to solve the equations of viscoelasticity. A technique for greatly reducing the computation time for rotors with a large number of layers was demonstrated. In Chien and Tzeng [1995] and Tzeng [2001(a)], rotation was not considered, however the other necessary components for a complete thermo-viscoelastic flywheel model were included. In the most recent publication by Tzeng [2001(b)], the rotor model was modified to include inertial loading, but thermal loading and thermal effects on the creep compliance were removed.

2.7 Critical Strain Energy Creep Failure Criterion

When stressed, viscoelastic materials undergo time-dependent deformation, part of the energy used to deform the material is stored and part of the energy is dissipated during this deformation. The critical strain energy failure criterion (CSEFC), originally attributed to Reiner and Weisenberg [1939], was derived on the assumption that, in viscoelastic materials, the stored energy component is available for the formation of cracks that can lead to rupture. In the context of the CSEFC, Hiel et al. [1984] derived an expression for the stored energy of a given viscoelastic material that is approximated with the same parameters used to characterize the creep compliance of the material. Based on a power law expression for the creep compliance of the form,
\[ S(t,T) = S_0 + S_1 t^n, \]  

(2.17)

Hiel et al. derived an expression for the stored energy of the form,

\[ W = \sigma_f^2 \left[ \left( \frac{1}{2} \right) S_0 + (1-2^n-1) S_1 t^n \right], \]  

(2.18)

where \( W \) is the stored energy. Hiel performed tensile creep experiments on \([90]_{8s}\) carbon/epoxy lamina and found that the stored energy reaches a constant value at rupture. This critical stored energy, \( W_c \), is of the form,

\[ W_c = \sigma_f^2 \left[ \left( \frac{1}{2} \right) S_0 + (1-2^n-1) S_1 t_f^n \right], \]  

(2.19)

where \( W_c \) is the critical stored energy, \( \sigma_f \) is the failure stress, and \( t_f \) is the time at rupture.

Yen and Morris [1989] used this same CSEFC technique to investigate the creep rupture of SMC-R50, a polyester matrix sheet molding compound which contains 50% by weight chopped glass fibers randomly oriented in the plane of the sheet. A power-law expression was fit to the compliance data of the SMC-R50 specimens, which were found to behave linearly at stresses of 11, 20, 30 and 36% of the room-temperature ultimate tensile strength and temperatures as high as 100°C (polyester \( T_g \approx 150°C \)). Yen and Morris assumed that creep rupture curves obey the same temperature-time shift behavior as the compliance curves, resulting in a general expression for the critical stored energy,

\[ \frac{W_c(T_0)}{W_c(T)} = \sigma_f^2 \left[ \frac{1}{2} S_0 + (1-2^n-1) S_1 (a_T t_f)^n \right], \]  

(2.20)

where \( \Lambda = \frac{W_c(T_0)}{W_c(T)} \) is the temperature dependent scaling factor for the critical stored energy and \( a_T \) is the temperature dependent time-shift factor. Note that experiments have shown that \( W_c(T) \) is a decreasing function of temperature—meaning that materials are able to store more energy to rupture at lower temperatures.

The creep rupture prediction process described above is defined in the context of linear viscoelasticity and uniaxial stress fields. The \([90]_{16}\) carbon/epoxy specimens tested by Hiel and the SMC-R50 specimens tested by Yen and Morris were characterized with a
linear viscoelastic compliance. The creep rupture master curves, which were based on the associated compliance master curves, matched experimental creep rupture results well over times as long as $10^4$ minutes.

Raghavan and Meshii [1995] also used a CSEFC to predict the creep rupture of AS4/3501-6 carbon/epoxy [90]$_{16}$ laminae. Like the method described above, they evaluated the stored component of the strain energy. Unlike the method described above, they aimed to develop a fracture criterion for composites applicable both below and above the $T_g$ and for a wide range of strain rates. Activation theory and DMTA testing were used to determine the critical stored energy for the lamina. Raghavan and Meshii criticized the approach of Hiel et al. from the standpoint that different mechanisms are responsible for fracture above the $T_g$ versus below the $T_g$. In other words, a rupture master curve based on a power law cannot accurately predict rupture if the material has become rubbery.

Brinson [1995] also discussed the CSEFC and the approach of Hiel et al. in a review of failure predictions for PMCs. Brinson mentioned that a Prony series could be used instead of a power law for the creep compliance expression, resulting in an expression for the stored energy of the form

$$W = \sigma^2 \left[ \frac{S_0}{2} + \sum_{i=1}^{n} \frac{S_i}{2} \left( 1 - e^{-t/\tau_i} \right) \right],$$

where $S_0$, $S_i$, and $\tau_i$ are the parameters of the Prony series fit to the creep compliance for the material. The Prony series exponents can be chosen such that the compliance master curve spans the glassy and rubbery regions of the compliance curve, effectively allowing rupture predictions for a given material under glassy and rubbery conditions. Very limited experimental data have been collected to investigate this approach.

It is important to note that the above-mentioned research involved isothermal creep tests under constant, unidirectional applied stress. Under such conditions, the critical value of stored energy—including its dependence on temperature—is easily calculated. Only one paper has been found in which a strain-energy failure characterization has been incorporated into a rotor model. In the viscoelastic rotor model
presented by Antsilevich [1992], the “area under the stress-strain curve,” of the material was used to define a rupture energy. Failure was predicted to occur when the cumulative work of deformation equaled the pre-determined rupture energy of the material. While it is straightforward to keep track of this value of (total) strain energy ($\frac{1}{2} \sigma(t) \varepsilon(t)$ at all times), it is the stored component of the strain energy that is the quantity of interest, as mentioned above. Indeed, experimental results of ruptured creep specimens presented in Chapter 6 show that there is a large difference between the values of critical stored energy calculated with Eq. 2.19 and with $\frac{1}{2} \sigma(t) \varepsilon(t)$.

No publications on the characterization of the creep rupture behavior of PMCs under biaxially applied stress have been found. A biaxial creep rupture criterion for composites (the determination of which is beyond the scope of the work presented here) is recognized as a very important complement to the viscoelastic rotor model that is presented in this investigation and indeed necessary to predictions of flywheel rotor life.

2.8 Rotor Strain Measurement Techniques

2.8.1 Electronic Speckle Pattern Interferometry (ESPI)

Pulsed laser ESPI has been employed by Preater [1980] to measure radial and hoop strains on the axial surface of rotating disks. The system was initially developed for the gas-turbine industry and gives real-time full-field information. Two cylindrical mirrors were used to illuminate the surface of the disk, thereby creating an interference pattern related to radial displacement. To consistently illuminate the same segment of the disk and freeze the motion for photographic storage, a 1-J, 20-ns pulsed laser was used as the coherent light source. The displacement information was converted to strain using strain-displacement relations. The ESPI technique was shown to be capable of measuring strain on a component rotating at 23,842 rpm (corresponding to a maximum tangential speed of 364 m/s at the outer radius of the component), with sensitivity better than 10 $\mu$ε and accuracy within 10 $\mu$ε of the theoretical strain [Preater 1993].

While the pulsed laser ESPI technique is capable of excellent strain sensitivity for high-speed rotors under general states of deformation, it has several disadvantages. The
The foremost disadvantage is the expense of a Q-switched laser. The fact that the laser beams must be diverged over a distance of at least 4 meters to reduce optical errors makes the apparatus rather large. In-plane vibrations, especially at high speeds, can cause significant or complete image degradation and loss of fringe contrast, making displacement measurements impossible. Also, a large area of the rotor surface must be illuminated by the interferometer to obtain deformation information over the entire radius of the flywheel with one pair of mirrors. This requirement could be difficult to satisfy for flywheel rotors operated in tightly confined spaces, although the use of multiple miniaturized interferometers could potentially mitigate the problem.

### 2.8.2 Strain Gages

Several investigators have used electrical resistance strain gages for strain measurements on flywheels (Ferrero et al. [1983, 1985], Morgenthaler and Bonk [1967]). The use of strain gages on a rotating object could require a slip ring to transfer the measured resistance changes to stationary instrumentation. The sensitivity of strain gages in this application is good, and top reported speeds using strain gage measurements reported by Ferrero et al. are 5000 rpm. The top speed at which strain was measured by Morgenthaler and Bonk was 17,260 rpm (the rotor failed shortly thereafter at 18,640 rpm, before another set of measurements was made). Another investigation by Hoeltzel et al. [1993] employing a single rotating strain gage used a small onboard battery powered circuit containing a Wheatstone bridge and a voltage to frequency conversion circuit to eliminate the need for a slip ring. Strain data were used to modulate the frequency of a pulsed optical signal transmitted to a fixed optical receiver via an axially oriented LED at the spin axis. The signal was then converted back to a voltage proportional to the strain. The application reported for this technique was a belt-driven universal joint with no maximum realized speed given, although it was mentioned that this technique should be useful at speeds up to 120 krpm.

Disadvantages of using strain gages to measure flywheel ring strains include independent deformation of the gage relative to the underlying rotor surface and lead wire deformation during high centripetal accelerations. In cases where slip rings are used
to transmit excitation and signal voltages, slip ring life and speed capability are significant limitations for practical operating speeds of flywheel rotors.

2.8.3 X-ray Diffraction (XRD)

X-ray diffraction (XRD) is an important non-contact strain measurement method for crystalline materials. With this technique, the change of spacing of atomic planes within a collection of randomly oriented crystals near the illuminated surface of the specimen is sampled and mathematically related to a change of local strain. In the context of flywheels, XRD has been used to measure the strains of polycrystalline foil bonded to the surface of noncrystalline rotating specimens [Saito et al. 1993]. This same technique could be used to measure strains in fiber-reinforced polymer (noncrystalline) flywheels.

Saito et al. used XRD to measure the strains of three 170×10×2-mm rotating epoxy bars spinning about the major principal axis and compared the measurements to theoretical strain predictions. The ends of the bars were loaded with weights (to increase stress), producing a tensile state of longitudinal strain that is approximately uniform along the length. The rotating bars were instrumented with 30-μm-thick aluminum foil for diffraction of X-rays at the radial location of 45 mm. Maximum rotational speed for the experiments was 1400 rpm. Over the maximum strain range of 2000 με, measurements generally differed from theoretical strain predictions by less than 100 με. Provided the metallic foil used in this method does not deform independently of or reinforce the flywheel, this technique could prove to be useful for flywheels.
2.8.4 Optoelectronic Strain Measurement (OESM)

Simpson and Welch [1989] devised a rudimentary non-contact optoelectronic strain measurement (OESM) technique for application to strain measurement on axisymmetric rotating disks. In this technique, a stationary light source projects a spot on a painted or etched pattern on the axial surface of a spinning rotor. A stationary receiver generates a logic signal according to the brightness of the illuminated spot (e.g., “on” when brightness exceeds some value, and “off” when brightness is below some value). The pulse width as the spot passes over the pattern in the “on” state is mathematically related to the angular width of the bright portion of the pattern. As the disk and pattern deform in unison, the change in angle measured by the optical sensor is related to radial displacement based on a geometric description of the pattern and viewed spot. The Simpson-Welch OESM system was shown to be able to measure radial displacements as small as 9.5 μm in a disk rotating at 5 krpm. Drift on a constant-speed disk over a 5-day period was 19 μm. The purported advantages of this technique over other optical methods include low cost, easy miniaturization, and relatively loose alignment requirements between the optics and rotor. Simpson and Welch discussed performance-improving modifications for almost every aspect of the OESM technique, making it theoretically capable of measuring strains at many locations on high-speed flywheels with acceptable sensitivity. Mentioned areas for improvement of the technique included a faster timing device for improved displacement sensitivity and a smaller spot size, possibly achieved with a laser, for reduced sensitivity of the system to electronic drift in long term use. The Simpson-Welch approach forms the basis of the improved OESM method that was developed and demonstrated in this research and is discussed in Chapter 4.
Chapter 3

VISCOELASTIC FLYWHEEL ROTOR MODEL

The model presented in this chapter considers axisymmetric, polar orthotropic, multiple-concentric-ring rotors subjected to time-variable thermal and spinning loads as well as interference fits. Plane-stress conditions are assumed, and the thermal loads are assumed to be uniform in each ring. The mechanics-of-materials approach (constitutive equations, stress-strain relations, force-equilibrium equations, and boundary conditions) and the quasi-elastic solution technique are used to develop an approximate viscoelastic solution based on the associated elastic solution. Temperature effects on the material creep properties (i.e. constitutive law) are accounted for with time-shifting according to the time-temperature superposition principle. Model predictions are shown to match (in the elastic limit) published elastic solutions for single ring and multiple-ring assemblies subjected to interference-fit, rotational, and thermal loading and to match a published viscoelastic solution of a single internally pressurized ring. As well, model predictions are compared to published plane-strain predictions for a flywheel subjected to rotational loads. The computer code for the model was written in MATLAB and is given in Appendix A.

3.1 Model Derivation

The algebraic elastic constitutive law allows structural models to be developed using the mechanics of materials equations in a straightforward fashion. There are difficulties in developing a structural model using the integral viscoelastic constitutive law (see Eq. 2.11) directly. To avoid these difficulties, approximate techniques have been developed by others to effectively convert the integral viscoelastic constitutive law to an algebraic relation. These techniques allow existing elastic structural solutions to be used in deriving approximate viscoelastic structural solutions. Schapery [1965] is the most notable early researcher who applied these approximate techniques to composite materials. One restriction in using these approximate techniques is that the loading should be nearly constant (i.e. a step load). This restriction presents little practical
limitation because variable loads can be approximated as time-discretized step loads, and the response to each of these steps can be superposed. Such a superposition method is used in the present research. The other restriction is that a plot of the solved quantity of interest (e.g., radial stress at a given radial location) should have small curvature when plotted against \( \log_{10} t \) [Schapery 1965], or else the approximation becomes inaccurate. An example presented in Appendix B addresses this second restriction and indicates that using the quasi-elastic solution technique results in good approximate solutions of the stresses and strains for the rotors, materials, and loadings of interest in this investigation.

### 3.1.1 Elastic Equations

The elastic equations describing the stresses and strains are now derived. The appropriate elastic constitutive law for a polar orthotropic material in plane stress is

\[
\begin{bmatrix}
\varepsilon_h \\
\varepsilon_r 
\end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{12} & S_{22} \end{bmatrix} \begin{bmatrix} \sigma_h \\ \sigma_r \end{bmatrix} + \begin{bmatrix} \alpha_h \\ \alpha_r \end{bmatrix} \Theta, \tag{3.1}
\]

where the subscripts \( h \) and \( r \) indicate the hoop and radial directions, respectively, which are associated with the tensor ‘1’ and ‘2’ subscripts, respectively. The strain-displacement relations are given by

\[
\varepsilon_h = r^{-1} u_r, \tag{3.2}
\]

\[
\varepsilon_r = u_r, \tag{3.3}
\]

where \( u \) is radial displacement, \( r \) is the radial coordinate, and \( (\cdot) \) implies differentiation with respect to \( r \). The plane-stress, axisymmetric equilibrium equation is given by,

\[
\sigma_{rr} + r^{-1}(\sigma_r - \sigma_h) + \rho r \omega^2 = 0, \tag{3.4}
\]
where $\rho$ is the mass density and $\omega$ is the rotational speed, both of which are assumed constant. Substituting Eqs. 3.2 and 3.3 into Eq. 3.1, the stresses are expressed in terms of displacement and temperature change:

$$
\sigma_h = \frac{\Theta S_{12} \alpha_h - \Theta S_{22} \alpha_h - S_{12} u_{1r} + S_{22} r^{-1} u}{S_{11} S_{22} - S_{12}^2},
$$

(3.5)

$$
\sigma_r = \frac{\Theta S_{12} \alpha_h + S_{11} u_{1r} - S_{12} r^{-1} u - \Theta S_{11} \alpha_r}{S_{11} S_{22} - S_{12}^2}.
$$

(3.6)

Substituting Eqs. 3.5 and 3.6 into Eq. 3.4 provides the second order differential equation for displacement in an isolated ring:

$$
u_{rr} + r^{-1} u_{r} + \frac{S_{22}}{r^2 S_{11}} u + \frac{\Theta (\alpha_h (-S_{11} - S_{12}) + \alpha_b (S_{12} + S_{22}))}{r S_{11}} + \frac{r}{S_{11}} (S_{11} S_{22} - S_{12}^2) \rho \omega^2 = 0.
$$

(3.7)

If displacements are specified on the inner radius $r_i$ and outer radius $r_o$ of the ring,

$$
u_{i} = \delta_a, \quad \nu_{o} = \delta_b,
$$

(3.8a,b)

the solution for displacement is:

$$
u = r^{-\eta} (A + B + C) D^{-1},
$$

(3.9a)

$$
\eta = \sqrt{S_{22}/S_{11}},
$$

(3.9b)
3.1.2 Boundary Conditions

The displacement formulation of the problem given above requires specification of displacements on the inner and outer radii of the ring to arrive at a full field solution. For flywheels consisting of multiple concentric rings, continuity of radial stresses and displacements between neighboring rings is enforced while either radial stress or displacement is specified at the inner and outer radii of the assembly. The radial stress field equation, expressed in terms of boundary displacements, is obtained by substituting \( u \) from Eq. 3.9 into Eq. 3.6:

\[
\sigma_r = \frac{\Psi_1 + (S_{11}\alpha_r - S_{12}\alpha_h) \Theta + \Psi_2 + \Psi_3}{S_{11}S_{22} - S_{12}^2},
\]

(3.10a)
\[ \psi_1 = \left\{ \begin{array}{c} H_1 \left[ \left( \frac{r_1}{r} \right)^\eta - \left( \frac{r_1}{r_0} \right)^\eta \right] + H_2 \left[ \left( \frac{r_0}{r} \right)^\eta - \left( \frac{r_0}{r_0} \right)^\eta \right] + \\
H_3 \left[ \left( \frac{r_1}{r_0} \right)^\eta - \left( \frac{r_1}{r_0} \right)^\eta \right] \end{array} \right\} - \frac{S_{12} r_1^\eta}{H_4 r_0^{\eta}}. \]  

(3.10b)

\[ \psi_2 = \left\{ \begin{array}{c} H_1 \left[ 1 + \left( \frac{r}{r} \right)^{2\eta} \right] \left( \frac{r_1}{r} \right)^\eta - H_3 \left( \frac{r_1}{r_0} \right)^\eta \right] - \\
H_3 \left[ 1 + \left( \frac{r_0}{r} \right)^{2\eta} \right] \left( \frac{r_0}{r} \right)^\eta \right] \end{array} \right\} - \frac{S_{12} r_1^\eta}{r_0^{\eta} H_4}. \]  

(3.10c)

\[ \psi_3 = \left[ \left( \frac{r_1}{r_0} \right)^{2\eta} - 1 \right] 2S_{11} r_1^2 \left( S_{11} - S_{22} \right) \left( S_{11} S_{22} - S_{12} \right) \rho_1^2 \]  

(3.10d)

\[ H_1 = \left\{ \begin{array}{c} S_{11} \left[ 9\delta_9 - 9r_1 \Theta + r_1^3 S_{22} \rho_1^2 \right] - \\
S_{22} \left[ 10\delta_6 - r_0 (\alpha_z + 9\alpha_h) \Theta \right] + \\
S_{22} \left[ \delta_9 - r_0 \alpha_h \Theta \right] + r_0 S_{12} \left[ \alpha_z \Theta - 9\alpha_h \Theta + r_0^2 S_{12} \rho_1^2 \right] \end{array} \right\} + \]  

(3.10e)

\[ H_2 = \left\{ \begin{array}{c} S_{11} \left[ 9\delta_9 - 9r_1 \Theta + r_1^3 S_{22} \rho_1^2 \right] - \\
S_{22} \left[ 10\delta_6 - r_0 (\alpha_z + 9\alpha_h) \Theta \right] + \\
S_{22} \left[ \delta_9 - r_0 \alpha_h \Theta \right] + r_0 S_{12} \left[ \alpha_z \Theta - 9\alpha_h \Theta + r_0^2 S_{12} \rho_1^2 \right] \end{array} \right\} + \]  

(3.10f)

\[ H_3 = \left\{ \begin{array}{c} S_{22} \left[ \alpha_z - \alpha_h \right] \Theta - S_{22} \alpha_h \Theta + r_1^2 S_{12} \rho_1^2 \right] + \\
S_{11} \left[ 9S_{12} \left[ \alpha_h \Theta - \alpha_z \Theta - r_1^2 S_{12} \rho_1^2 \right] \right] + \\
S_{11} \left[ 9\alpha_h \Theta - \alpha_z \Theta + r_1^2 S_{12} \rho_1^2 \right] \end{array} \right\} + \]  

(3.10g)

\[ H_4 = \left( \frac{r_1}{r_0} \right)^\eta - 1 \right) \left( \frac{r_1}{r_0} \right)^\eta + 1 \left( 9S_{11} - S_{22} \right) \left( S_{11} - S_{22} \right). \]  

(3.10h)
Considering an assembly of \( z \) concentric rings, the \( 2z \) boundary conditions selected for this investigation are,

\[
\begin{align*}
\sigma_t & \bigr|_{r_{i,k}} = 0 \\
\sigma_t & \bigr|_{r_{o,k}} = \sigma_t \bigr|_{r_{i,k+1}} \\
u & \bigr|_{r_{o,k}} = u \bigr|_{r_{i,k+1}} \\
\sigma_t & \bigr|_{r_{i,k,z}} = 0
\end{align*}
\]  

(3.11)

defined for \([k=1, 2, 3, \ldots (z-1)]\), with subscripts 1 and 2 referring to the innermost and outermost rings, respectively. In Eq. 3.11, \( r_{i,k} \) and \( r_{o,k} \) respectively refer to the inner and outer radii of the \( k^{th} \) ring. The resulting set of \( 2z \) equations is solved for boundary displacements \( \delta_{z,k} \) and \( \delta_{b,k} \) for \([k=1,2,3\ldots z]\), after which the displacement field (Eq. 3.9) is completely specified and can be substituted into Eqs. 3.2, 3.3, 3.5, and 3.6 to determine the strain and stress fields in each ring.

### 3.1.3 Viscoelastic Solution Procedure

The quasi-elastic solution procedure is used to calculate the viscoelastic response due to general time-varying rotational, thermal, and press-fit loads. Because the quasi-elastic method is only valid for constant loads, general time-varying loads are approximated by a number of discrete load steps applied at times \( t_j \). The responses (stress, strain, or displacement) to each of these constant load steps are then superposed to calculate the total response at some future times of interest, \( t_i \).

To illustrate the relationship between the times of discrete load steps, \( t_j \), and the times of interest \( t_i \), examples of speed and temperature histories for a flywheel are shown in Figure 3.1. Index \( j \) increments whenever temperature or speed changes, whereas index \( i \) increments only when a response is computed (circled points in Fig. 3.1). Gradual changes in speed or temperature are approximated as a series of constant-load steps in accordance with the quasi-elastic method. A flowchart is also given (Figure 3.2) to
illustrate the quasi-elastic solution procedure used in solving for the viscoelastic displacements, strains and stresses at the times of interest.

Figure 3.1. Illustration of times $t_i$ and $t_j$ in an example load history.

Considering first the “INPUT” section of the flowchart, it is assumed that the inner and outer radii for each ring $k$ in a $z$-ring assembly [$z \geq 1$] have been specified. The material properties $S$, $\rho$, $\alpha$, and $a_T$ also need to be specified for all $z$ rings. The time scale and the thermal and rotational loads must be discretized at time steps, $t_j$. In practice, this discretization can be refined in order to achieve reasonable convergence of the stresses and strains that are finally evaluated at the times of interest. Some (at least one) “times of interest” $t_i$ must be specified before the solution process can begin.

Considering next the “Solution” section of the flowchart, the spinning and thermal load steps are calculated at each time step, $t_j$, in a differential fashion by,

\[
\Delta \omega_j = \omega_j^2 - \omega_{j-1}^2, \quad (3.12\ a)
\]

\[
\Delta \Theta_j = T_j - T_{j-1}. \quad (3.12\ b)
\]
For each ring $k$, effective time at each load step $t_j$ is calculated by Eq. 3.13 (a time-discretized form of Eq. 2.13b),
\[ \xi_j = \sum_{m=1}^{j} (a_T)_m (t_m - t_{m-1}) . \]  

(3.13)

In this equation, \((a_T)_m\) is calculated according to \(T_m\) at each step \(t_m [0< t_m \leq t_j]\), and is assumed to be uniform for all rings. Assuming for convenience that times of interest \(t_i\) coincide with certain times of load change, the effective times of interest \(\xi_i\) are given by,

\[ \xi_i = \xi_j \bigg|_{t_j = t_i} . \]  

(3.14)

The solution process now consists of stepping through times of interest (“LOOP 1” in Figure 3.2) and, nested within this loop, calculating \(\bar{S}(\xi_j-\xi_i)\) (using Eq. 3.13 and a mathematical form for compliance, e.g., Eq. 2.4) at each load step \(j\) on the interval \([0< t_j < t_i]\) (“LOOP 2”). At each step \(j\), \(\bar{S}(\xi_j-\xi_i)\) is treated as a “quasi-elastic” compliance and, using load steps \(\Delta \omega_j\) and \(\Delta \Theta_j\), the displacement, strain, and stress increments \((\Delta u_j, \Delta \xi_j, \Delta \sigma_j, \text{respectively})\) are calculated using the elastic equations (Eqs. 3.1 to 3.9).

After all of the increments have been calculated on the interval \([0< t_j < t_i]\), they are summed to yield the field quantities at the times of interest: \(u_i, \xi_i, \text{and } \sigma_i\).

Note in Fig. 3.1 that the times of interest \(t_i\) are limited to “interesting” times (e.g., at peak speeds or the beginning or end of dwell times). If instead the stresses and strains were evaluated at every \(t_j\), computer simulation times would be unnecessarily long. Also note that the first load experienced by a PMC ring is a negative temperature load (“cool-down”) from the elevated manufacturing temperature. In a unidirectional filament wound ring, this cool-down load produces a state of tensile radial stress through the thickness of the ring and generally comprises the major portion of the unavoidable residual stresses resulting from any elevated temperature manufacturing process. Such residual stresses can be quite significant – especially for radially thick rings and/or high manufacturing temperatures. Other contributors to residual stresses (e.g., resin cure shrinkage, tow tension, aging) can be important as well, but are beyond the scope of the current investigation.
3.1.4 Interference Fits

Loads due to interference fits are input to the model by the boundary conditions, and require special treatment. At some time $t_f > 0$, two or more rings that were independently manufactured are assembled with radial interferences $\phi_k [k=1, 2, 3, \ldots, z - 1], \quad \phi_k = r_{a,k} - r_{i,k+1} \quad (3.15)$

In place of Eqs. 3.11 in the model, at times $[0 < t_f < t_g]$ Eqs. 3.16 are used $[k=1, 2, 3, \ldots, z],

\begin{align*}
\sigma_{t,n,k} &= 0 \\
\sigma_{t,n,k} &= 0
\end{align*} \quad (3.16)

and at times $[t_f < t_f < t_i]$ Eqs. 3.17 are used $[k=2, 3, 4, \ldots, z],

\begin{align*}
\sigma_{t,n,1} &= 0 \\
\sigma_{t,n,k-1} &= \sigma_{t,n,k} \\
u_{t,n,k-1} - u_{t,n,k} &= -\phi_k \\
\sigma_{t,n,z} &= 0
\end{align*} \quad (3.17)

3.2 Model Verification

Verification of the complete viscoelastic model as described above is not possible because no thermo-viscoelastic solutions exist in the literature. Verification of the present model in the elastic limit, however, has been performed by comparing predictions with several published solutions for simulated plane-stress cases. To verify the ability to correctly model loads caused by rotation and interference-fits, model predictions were compared to two cases by Arnold [2001], both of which used unidirectional carbon-fiber
PMC rings (see Table 3.1 for “Rotation and Interference” material properties). These two cases are:

- a single ring subjected to rotational loads. Arnold plots radial and hoop stress (normalized by multiplying by $S_{11}/(\rho \omega^2 (r_o)^2)$) versus normalized radial position

$$\frac{2r}{r_o + r_i},$$

for four values of

$$\frac{2(r_o - r_i)}{r_o + r_i}$$

where $r$ denotes the radius, $r_i$ and $r_o$ are the inner and outer radii of the rotor, respectively. The present model predictions (circle symbols) are plotted on top of Arnold’s predictions in Figs. 3.3-4.

- assemblies of three-rings, subjected to rotational loads and interference-fit loads. All three rings are the same carbon FRP material. One set (Fig. 3.5) has no interference and, hence, behaves like a single ring. The other set (Fig. 3.6) has radial interfaces of 0.1% of the nominal radial positions at the two respective interfaces.

For thermal loads, the model was compared in the elastic limit to Genta’s solution [1985]. The case investigated was a single ring with inner and outer radii of 50 and 100 mm, respectively. The “Thermal” material properties listed in Table 3.1 were used for this simulation. Normalized radial and hoop stresses are plotted in Figs. 3.7 and 3.8.

<table>
<thead>
<tr>
<th>Verification Case</th>
<th>$S_{11}$ (GPa$^{-1}$)</th>
<th>$S_{12}$ (GPa$^{-1}$)</th>
<th>$S_{22}$ (GPa$^{-1}$)</th>
<th>$\alpha_1$ ($\mu e/\circ C$)</th>
<th>$\alpha_2$ ($\mu e/\circ C$)</th>
<th>$\rho$ (kg/m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotation and interference</td>
<td>0.006283</td>
<td>-0.001772</td>
<td>0.1152</td>
<td>0.484</td>
<td>45.7</td>
<td>1583</td>
</tr>
<tr>
<td>Thermal</td>
<td>0.01923</td>
<td>-.005769</td>
<td>0.04951</td>
<td>6.6</td>
<td>25.6</td>
<td>2138</td>
</tr>
</tbody>
</table>
Figure 3.3 Normalized radial stress vs. position for simulated carbon/polymer composite rings under rotational loads—comparison of present solution and Arnold’s solution.

Figure 3.4 Normalized hoop stress vs. position for simulated carbon/polymer composite rings under rotational loads—comparison of present solution and Arnold’s solution.
Figure 3.5. Normalized radial stress versus position for three 3-ring assemblies of different mean radii without interference fits—comparison of present solution and Arnold’s solution.

Figure 3.6. Normalized radial stress versus position for three 3-ring assemblies of different mean radii with interference fits—comparison of present solution and Arnold’s solution.
Figure 3.7. Normalized radial stress versus position for thermal loading—comparison of present solution and Genta’s solution.

Figure 3.8 Normalized hoop stress versus position for thermal loading—comparison of present solution and Genta’s solution.
The present viscoelastic model solution was compared to Schapery’s published solution [1967] for a single anisotropic ring subjected to a single step-application of internal pressure ($P_i$) under isothermal conditions. Although plane strain conditions were assumed in Schapery’s model, the solution for hoop stress at the inner boundary of the ring is identical to the present plane stress solution. The example gives radial and hoop-direction relaxation moduli as,

$$E_r = [100(t/\tau_0)^{-0.5}+1]E_e,$$  \hspace{1cm} (3.18)

$$E_\theta = [100(t/\tau_0)^{-0.1}+1]E_e,$$  \hspace{1cm} (3.19)

respectively, where $\tau_0$ and $E_e$ are constants. For the present model, the relaxation moduli were converted to creep compliances according to $S_{11} = 1/E_r$, $S_{22} = 1/E_r$, and $S_{12} = -0.3/E_r$.

Figure 3.9 shows a plot of the normalized hoop stress at the inner radius of the cylinder versus log of normalized time.

---

**Figure 3.9.** Plot of normalized hoop stress at the inner radius versus log-time for an internally-pressurized anisotropic cylinder (from Schapery [1967]).
Figures 3.3-9 verify the accuracy of the elastic model to be within graphical accuracy for all of the loading types of interest in this thesis, and Fig. 3.9 further verifies proper implementation of the quasi-elastic solution technique, albeit for isothermal conditions and a single load step.
Chapter 4
OPTOELECTRONIC STRAIN MEASUREMENT METHOD

4.1 Optoelectronic Strain Measurement (OESM) Concept

4.1.1 OESM Components

Simpson and Welch [1987] devised a rudimentary non-contact optoelectronic strain measurement (OESM) technique for application to strain measurement on axisymmetric rotating disks. In this technique, a stationary light source projects a spot on a painted or etched pattern on the axial surface of a spinning rotor. A stationary photodetector “looks” at this projected spot and generates a logic signal according to the brightness of the spot (e.g., “on” when brightness exceeds some value, and “off” when brightness is below some value). The pulse width as the spot passes over the pattern in the “on” state is mathematically related to the angular width of the reflective portion of the pattern.

Figures 4.1a-c illustrate the pattern on the rotor face, the orientation of the sensor, and the sensor output pulse (in terms of the rotor angle swept when the logic signal is “on”, called the duty cycle, $\phi$) for fixed sensors at three different locations above the rotor. The dashed lines that indicate azimuth angle on the rotor face in Fig. 4.1a are fixed on the rotor. The pattern used in Fig. 4.1 is a spiral boundary (SB) pattern (white indicates the reflective portion), and was the type used in the Simpson and Welch investigation [1987]. The SB pattern results in a linear relationship between duty cycle and radial position. Figure 4.1a shows a sensor located near the outer radius of the rotor. The corresponding duty cycle is a small angle, centered on angle $\pi$ (radians) on the rotor azimuth. For the sensor located at mid-radius in Fig. 4.1b, the duty cycle has angular width of $\pi$ radians. With the sensor located near the outer radius in Fig. 4.1c, the duty cycle is almost a full $2\pi$ radians. This pattern is a so-called “one-lobe” design, because a fixed sensor pointed at the surface would receive only one reflected pulse during a rotor revolution.
Figure 4.1. Illustration of reflective pattern on the rotor face, optical sensor orientation, and sensor output (duty cycle) for a fixed sensor located (a) near the outer radius, (b) at mid-radius, (c) near the inner radius.
4.1.2 Measuring Displacement with OESM

To extend the duty cycle/radial position concept illustrated in Figs. 4.1a-c to the actual measurement of rotor deformation, consider the illustration in Fig. 4.2. In this example, a sensor is fixed at the mid-radial position, \( r_1 \), with corresponding duty cycle, \( f_1 \), before deformation. The dashed circle indicates the path that the sensor traces on the rotor surface, and the shaded area indicates the angle swept. The sensor is at the same fixed radial position (dashed circle) after the deformation, but the pattern—which has enlarged—causes a new duty cycle, \( f_2 \), to be swept. Inspection of the plot of duty cycle versus radial position in Fig. 4.2 shows that this \( f_2 \) corresponds to a different radial location, \( r_2 \), on the rotor. The displacement during this deformation is then calculated as \( u_{r_1} = r_1 - r_2 \). A method of using discrete OESM measurements at many radii to calculate quasi full-field strains is given in Section 4.2.4.

Figure 4.2. Illustration of change in duty cycle during an axisymmetric rotor deformation.
4.2 Application of OESM to Rotor Strain Measurement

4.2.1 Sensitivity to Strain

Due to the small radial displacements expected in realistic flywheel applications, the sensitivity requirement for the OESM device is derived using linear strain-displacement relationships. The target hoop-strain sensitivity is \( \pm 10 \, \mu \varepsilon \). At a fixed radial location \( r' \) on an axisymmetric ring rotating at constant speed, the radial and hoop strains are given in terms of the radial displacement field \( u \) by Eqs. 4.1 and 4.2, respectively

\[
\varepsilon_r = u_{r,|_{r=r'}} \quad (4.1)
\]

\[
\varepsilon_h = \frac{u}{r_{|_{r=r'}}} \quad (4.2)
\]

where the displacement and strains vary with coordinate \( r \), in general. These strain-displacement relations allow the determination of the minimum displacement, \( u_{\text{min}} \), necessary to achieve strain measurements of desired accuracy for a given application. The requirement of a given \( u_{\text{min}} \) places requirements on the shape of the reflective pattern and the accuracy (or speed) of the pulse-duration measurement device. The relationships among pattern shape, pulse-measurement speed, and duty cycle are illustrated with the following example.

Pulse width (elapsed time) detected by the optical sensor is measured by a digital counter with a temporal resolution of \( 1/S_c \), where \( S_c \) is the counter frequency (Hz). During one time increment, a rotor with angular speed \( \omega \) (rad/sec) rotates through the angle \( \theta_{\text{min}} = \omega/S_c \) (rad). At a radial location on the rotor where the slope of the duty cycle is \( \phi_r \) (radians swept per unit radial distance), radial displacements can be resolved with a minimum value, \( u_{\text{min}} \), of \( \theta_{\text{min}} / \phi_r \) (in units of radial displacement). A useful form of this relationship is given by Eq. 4.3,
\[
\phi_r = \frac{\omega}{S_c u_{\text{min}}}.
\] (4.3)

Equation 4.3 shows that the duty cycle slope (\(\phi_r\)) and the digital counter speed are inversely related, given a required displacement sensitivity and angular speed. It is therefore beneficial to maximize the duty cycle slope and the digital counter speed to maximize sensitivity to radial displacement (by minimizing \(u_{\text{min}}\)).

### 4.2.2 Maximizing Sensitivity to Displacement

One way to maximize the duty cycle slope is to use a reflective pattern with boundaries that are tangential to several points on a fixed radius of the rotor. Figure 4.3a illustrates such a reflective (white) tangential boundary (TB) pattern on a rotor. The TB pattern in Fig. 4a is shown as a “four-lobe” design for the simple reason that there is room around the circumference for multiple reflective patches. A four-lobe SB pattern is also shown in Fig. 4.3 for comparison purposes. It is briefly mentioned now (and discussed in more detail in Section 4.4) that multiple lobes enable displacement measurements at multiple azimuthal locations—which is central to the technique developed in this investigation for separating flexible and rigid body displacements (vibration).

![Figure 4.3. Illustration of a four-lobe TB (a) and four-lobe SB (b) pattern.](image-url)
Figure 4.4 shows the duty cycle vs. radial position for one lobe on the TB and SB patterns, which are mathematically described by Eqs 4.4 and 4.5, respectively.

\[
\phi = 2[\cos^{-1}(r_i/r_o) - \cos^{-1}(r_i/r)],
\]

(4.4)

\[
\phi = \frac{\pi}{2} \left( \frac{r_o - r}{r_o - r_i} \right),
\]

(4.5)

In Eqs. 4.4-5, \( r_i \) and \( r_o \) are the inner and outer radii of the annular region of the rotor containing the reflective pattern.

![Figure 4.4. Duty cycle versus position for one lobe of the SB and TB patterns shown in Fig. 4.3.](image)

The slope of the TB duty cycle vs. radial position,

\[
\phi_r = \frac{-2r_i}{r^2 \sqrt{1 - \frac{r_i^2}{r^2}}},
\]

(4.6)
is significantly higher than the slope of the SB pattern near the inner radius. Compared to the SB pattern, the TB pattern allows radial displacement measurement with greater sensitivity (i.e. smaller $u_{\text{min}}$) near the inner radius and less sensitivity near the outer radius. This is because the slope, $\phi_{r}$, of the TB duty cycle is higher than that of the SB duty cycle near the inner radius, and vice-versa near the outer radius (Eq. 4.3 illustrates the inverse relation between $\phi_{r}$ and $u_{\text{min}}$).

To determine the maximum radial location, $r_{\text{max}}$, at which a desired hoop strain sensitivity can be achieved on a TB pattern, Eq. 4.3 is rewritten in terms of a desired minimum hoop strain, $\varepsilon_{h \text{min}}$, as follows

$$\phi_{r} = \frac{\omega}{S_{c} r \varepsilon_{h \text{min}}}.$$  \hspace{1cm} (4.7)

Setting Eq. 4.6 equal to Eq. 4.7 and setting $r$ equal to $r_{\text{max}}$, gives

$$r_{\text{max}} = r_{i} \sqrt[4]{\frac{4 \varepsilon_{h \text{min}}^{2} S_{c}^{2} + \omega^{2}}{\omega}},$$  \hspace{1cm} (4.8)

which is the theoretical maximum radial location at which an optical sensor can be located to resolve $\varepsilon_{h \text{min}}$.

### 4.2.3 Optical Sensor

In this investigation, custom reflection-detecting optical sensors were built for measuring rotor displacements. For illumination, the optical sensor uses a 24-mW infrared (IR) light emitting diode (Photonic Detectors, part # PDI-E805, 880 nm peak wavelength), mounted to a 40-mm-long, 9-mm-dia. stainless steel tube containing a fixed circular aperture and a double-convex lens (JP Manufacturing part # 90-1201). The circular IR spot projected onto the rotor surface is about 0.4 mm in diameter. An optical logic detector (Fairchild Optoelectronics Group, part # QSE157, 935 nm, peak sensitivity) is mounted in a similar stainless steel tube with a lens to focus the reflected
IR image onto the detector. The detector has a specified rise/fall time of 70 ns, and is used to provide a logic signal to the data acquisition system. An illumination source and detector comprise a sensor, which is mounted with a stand-off distance from the rotor of approximately 4.5 mm. A picture of a sensor is shown in Fig. 4.5 (view is in the negative radial direction).

![Diagram of OESM sensor components]

Figure 4.5. Labeled side-view picture of the OESM sensor.

If the size of the spot projected by the emitter is made very small, it is possible to make measurements very near the inner radius of the TB pattern and, hence, take advantage of the increased sensitivity to displacement resulting from higher duty cycle slope. A problem with a very small sensor spot size is that surface and pattern roughness with dimensions of the order of the spot size cause excessive noise in the measured duty cycle. The use of a spot with a diameter about ten times larger than surface and pattern roughness dimensions was experimentally found to yield satisfactory results.
4.2.4 OESM Scheme

The overall aim of the OESM scheme is to measure radial displacements of the rotor. To this end, Eq. 4.5 is recast to provide the original radius on the pattern (and the rotor) corresponding to a measured duty cycle,

\[
r = \frac{r_i \cos \left[ \cos^{-1}\left( \frac{r_i}{r_o} \right) - \frac{\phi}{2} \right]}{\cos \left[ \cos^{-1}\left( \frac{r_i}{r_o} \right) - \frac{\phi}{2} \right]}. \tag{4.9}
\]

Using a very low rotor speed (and, hence, very low deformation), a fixed reference sensor position, \( r_{\text{ref}} \), is determined by using the reference duty cycle, \( \phi_{\text{ref}} \), in Eq. 4.9. At higher rotor speeds, the rotor deforms and the duty cycle changes to the instantaneous value, \( \phi_{\text{inst}} \). The instantaneous duty cycle is then used in Eq. 4.9 to calculate the instantaneous radial position on the rotor, \( r_{\text{inst}} \), that has displaced radially outward to the fixed sensor location. Instantaneous radial displacement (at the location \( r_{\text{inst}} \) on the rotor) is then calculated according to Eq. 4.10,

\[
u = r_{\text{ref}} - r_{\text{inst}}. \tag{4.10}
\]

While one sensor that is fixed in space can measure deformation at one point in the radial coordinate, an array of such sensors can provide measurements at multiple radial locations. A pattern with several concentric regions, each containing a set of TB reflective patches, allows sensitive displacement measurements at several radial locations (near the inner radius of each region), allowing the construction of a quasi full-field deformation plot. Figure 4.6 shows an illustration of one lobe of the four-lobe pattern used in the present investigation, consisting of ten concentric regions. The so-called compensation patch adjacent to each TB patch in Fig. 4.6 will be explained in Section 4.3.2.
The OESM scheme presented here takes advantage of separate duty cycle measurements for each lobe—a significant improvement over the Simpson-Welch method of summing multiple duty cycles during a complete rotor revolution. Considering the kinematics of a disk experiencing an in-plane rigid body vibration with a perfectly circular orbit, it is necessary to measure at least two duty cycles at orthogonally-unique azimuths to determine the phase and amplitude of the vibration. If the disk experiences axisymmetric flexible-body displacements simultaneous to such a vibration, then an additional displacement measurement at a third unique azimuth is necessary in order to separate the in-plane rigid body vibration from flexible body displacement at a given radial location. The OESM scheme presented next also allows the use of constant-angle patches to compensate for out-of-plane displacement and sensor drift.
4.3 Errors and Compensation

There are several sources for error in duty cycle measurement in an OESM system. In this investigation, several techniques were incorporated to compensate for these errors. The different sources of error and the corresponding compensation techniques are presented and discussed below. Experiments for validating these compensation techniques are described in the Chapter 5 and results of the validation are given in Chapter 6.

4.3.1 Pattern Edge-Roughness and Misalignment

Pattern edge-roughness and misalignment between the center of rotation and the center of the pattern cause differences between the real and theoretical relationships between duty cycle and radius. To avoid errors due to these differences, a calibration is performed to create an empirical “look-up” table relating $f$ and $r$. Calibration data are determined by stepping the sensor across the rotor in the radial direction and recording $f$ and $r$ at each radial location. These data describe the reference (i.e. undeformed) state of the rotor.

4.3.2 Sensor Effects

Besides the TB patches used for displacement measurement, constant-angle patches placed immediately adjacent to each TB patch in each region are used to compensate for errors in the measurement of $\phi$ due to potentially significant changes in light intensity, e.g., from heating or aging of the emitter (Fig. 4.6). Changes in the spot intensity cause errors in the time intervals during which the photodetector is in either an “on” or “off” state (i.e. error in $\phi$). A constant-angle patch that does not change in angular width due to axisymmetric displacement can be used for compensation of such errors in $\phi$ because these incorrect intervals would cause changes in the measured angle of the constant-angle (compensation) patches. These measurable changes could then be
used to compensate for the related error in $\phi$ on the adjacent TB patch, provided that the illumination spot is nearly circular, as was verified to be the case in this investigation.

Figure 4.7 is an illustration of the effect of spot intensity on the “on” intervals for a compensation patch traversing a sensor. In Case 1 (high incident intensity), less spot area is required to activate the photodetector than in Case 2 (correct intensity). The apparent compensation patch angle, $q_a$, that is swept is therefore larger than the correct compensation patch angle, $q_c$. Similarly, more spot area is required to activate the photodetector with Case 3 (low incident intensity), so a smaller-than-correct apparent angle is swept. While not labeled in Fig. 4.7, the apparent angle for Case 3 is smaller than $q_c$.

![Figure 4.7. Illustration of effect of spot intensity on apparent compensation patch angle.](image)

Figure 4.8 illustrates the angle swept and the associated apparent radial location on a TB patch when the intensity is correct and when it is too high. Assuming that the spot has a radially symmetric intensity profile, the distance between the edge of the TB patch and the center of the spot, labeled $d$ in Fig. 4.8, is easily calculated from $r_{ref}$ along with $q_a$ and $q_c$ from the compensation patch,

$$d = r_{ref} \left( \frac{q_a - q_c}{2} \right).$$

(4.11)
Figure 4.8. Illustration of effect of spot intensity on duty cycle measurement on displacement patch.

When the spot toggles the sensor at distance \( d \) from the TB patch edge, the apparent radial location, \( r_{\text{app}} \), and the correct radial location, \( r \), are related by,

\[
r = r_{\text{app}} + \frac{d}{\cos(\psi)} ,
\]

where \( \psi \) is the angle between the \( r_{\text{app}} \) line and the line normal to the TB patch edge. The angle \( \psi \) is computed directly from the apparent duty cycle of the TB patch,

\[
\psi = \cos^{-1}\left( \frac{r_i}{r_o} \right) - \frac{\phi}{2} .
\]

The absolute values for displacement during operation (~300 \( \mu \)m) and correction for displacement due to sensor intensity fluctuations (~10 \( \mu \)m) are small relative to the reference location of the sensor, \( r_{\text{ref}} \) (which is in the range of 61,000-151,000 \( \mu \)m for the rotors in this investigation). Because of this, the simple use of \( r_{\text{ref}} \) in Eq. 4.11 introduces negligible error (<1%) in the calculation of \( d \). This simple method was used in the present research. A more precise approach to correcting \( r_{\text{app}} \) involves iterating a two-step process; for step one, use \( r_{\text{app}} \) in place of \( r_{\text{ref}} \) in Eq. 4.11 (to calculate an initial value for
for step two, use this $d$ with Eq. 4.12 to calculate a new value for $r_{ref}$ (for used in Eq. 4.11). With this iterative approach, however, it would still be necessary to guess an initial value for $r_{app}$, say, $r_{ref}$.

4.4 Vibration

In-plane and out-of-plane rigid body vibrations generally arise in the various components of a flywheel system (rotor, shaft, bearing/support structure) at different speeds during operation. Generally, the flywheel center of rotation (CR) varies as a function of speed. The resulting vibrations cause synchronous displacements of the rotor that can be easily accounted for and separated from axisymmetric displacements due to rotor material strain. Other types of vibrations may also occur, such as flexible body vibrations or asynchronous whirl. These types of vibration are theoretically measurable by slightly more elaborate optical sensor arrays and data processing techniques than those investigated presently. All subsequent discussion of vibrations is limited to synchronous rigid body displacements that are either in or out of the plane of the rotor.

4.4.1 In-Plane Rigid Body Displacements

A qualitative illustration of the effect of an in-plane vibration on separately-measured duty cycles is shown in Fig. 4.9 for a rotor with three equally-spaced TB patches. The path that the sensor follows in the absence of vibration is concentric with the rotor, as shown by the solid line. The corresponding sensor output is three equal-width duty cycles. Conversely, during vibration the duty cycles are generally not equal. These unequal duty cycles are converted to radial displacement values with the lookup table created during low-speed calibration.
The in-plane vibration amplitude and phase are determined by fitting the three displacements to the general function,

\[
u = A_0 + A_1 \sin(F + G).
\] (4.14)

In Eq. 4.14, \(F\) is the azimuthal coordinate on the rotor and \(A_0\) corresponds to the axisymmetric component of radial displacement experienced at all TB patches due to material strain. For equally spaced patches, \(A_0\) is easily calculated as the mean of the displacements measured at each patch. In-plane vibration is then characterized by the amplitude \(A_1\) and the phase \(G\). These in-plane rigid body vibrations were measured during spin tests as described above.

### 4.4.2 Out-of-Plane Rigid Body Displacements

Vibration and thermal expansion of the flywheel—including the rotor support structure—may cause out-of-plane displacements, which effect a de-focusing of the projected spot on the rotor surface and a corresponding error in duty cycle. The compensation for the resulting error in angle measurement can be performed as described for intensity variations in the “Sensor Effects” section. To correct out-of-plane displacements that vary around the rotor circumference (e.g., during a synchronous rigid body wobble) it is necessary to locate compensation patches in close circumferential proximity to the associated TB patches.
Chapter 5

EXPERIMENTAL PROCEDURES

5.1 Basic Material Property Tests

Creep tests were performed to provide basic material properties for input to the viscoelastic rotor model. In this section, a description of the specimens, grips, strain gage instrumentation, and test procedures used during the basic material property tests of E-glass/Dow 383 epoxy composite is given.

5.1.1 Specimens

E-glass/Dow 383 tubes were wet-filament-wound with the processing parameters listed in Table 5.1. Each layer of deposited material was ~0.196-mm thick, therefore the final specimen (1-mm wall thickness) comprises ~4.9 material layers (~2.5 circuits). It was found that manufacturing the part to ten-layers (5 circuits) of thickness greatly facilitated removal of the part from the mandrel. Winding time for such ten-layer-thick tubes is approximately 40 minutes with the filament winder set at maximum speed (60 rpm). After curing in a mechanical convection oven, the 760-mm long tube was cut with a diamond saw into four shorter equal-length tubes. These tubes were clearance-fit onto a steel jig that was live-centered and carefully aligned on a lathe such that radial runout measured everywhere on the jig was less than 0.0005 in. A tool-post grinder with a high-speed diamond wheel was then used to uniformly machine the OD and axial ends of these shorter tubes. A recirculating water bath was used during grinding to reduce dust and cool the diamond saw and composite.

After machining, the tubes were dried in a mechanical oven at 90 °C for 24 hours. The fiber and void contents were determined from burnoff tests of three 10×10×1-mm pieces from one of the machined specimens, following ASTM D 2734 procedure [Annual Book of ASTM Standards, 1999].
Table 5.1. Filament winding parameters for E-glass/Dow 383 tubular specimens

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiber</td>
<td>Owens Corning Type 30 E-glass, 158B-AB-450, inside-pull</td>
</tr>
<tr>
<td>Matrix</td>
<td>Dow 383 resin &amp; Jeffamine T403 curative (47 parts T403 : 100 parts 383 by weight)</td>
</tr>
<tr>
<td>Gel Time</td>
<td>20 min at 85°C</td>
</tr>
<tr>
<td>Resin Bath Temp</td>
<td>23°C</td>
</tr>
<tr>
<td>Mandrel OD</td>
<td>19.84 mm</td>
</tr>
<tr>
<td>Mandrel Temp</td>
<td>85°C</td>
</tr>
<tr>
<td>Mandrel Speed</td>
<td>60 rpm</td>
</tr>
<tr>
<td>Mandrel Material</td>
<td>Stainless Steel</td>
</tr>
<tr>
<td>Part Length</td>
<td>760 mm</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>3.05 mm</td>
</tr>
<tr>
<td>Fiber Angle</td>
<td>87.2 deg.</td>
</tr>
<tr>
<td>Post-Cure</td>
<td>16 hours at 85°C</td>
</tr>
<tr>
<td>As-Wound Wall Thickness</td>
<td>2 mm</td>
</tr>
<tr>
<td>Machined Wall Thickness</td>
<td>1 mm</td>
</tr>
</tbody>
</table>

5.1.2 Grips

Two sets of grips were designed and machined, similar to the grips described in the ASTM D5450 test method. Drawings of the grip parts are given in Appendix C. The load train integral to the grips employed a spherical washer set to allow (limited) swiveling such that the specimen could self-align with the applied load to reduce bending.

Prior to potting the specimen in the grips, three 1-mm² pieces of brass shim (equally spaced in a circumferential sense) were bonded to the grip section of the specimen OD using cyanoacrylate. These small pieces of shim helped to minimize relative misalignment of the specimen and grip axes. Specimens were potted in the grips using Dexter Hysol¹ 3101 methacrylate adhesive which has a reported [Hysol 2000] shear strength of 14 MPa at 85°C. The potted length of the specimen at each gripped end was 2.54 cm, giving 1600 mm² of bond area. Assuming a composite transverse tensile strength of 77.0 MPa (based on unpublished pilot tests of E-glass/Dow 383 beam specimens in 4-pt. bending), this area gave a factor of safety of 3

¹ Loctite Corp, Rocky Hill, CT, 06067.
against pullout. No problems with specimen pullout were experienced at any time during the tests.

The grips were put in a ventilated ceramic element oven at 510ºC (950 ºF) for a 30-minute period before potting a new specimen. This exposure was found sufficient to completely burn off the methacrylate adhesive and composite matrix within the potted length. Following burn-off, the grips were lightly sanded and wiped clean with a paper towel to remove residue.

5.1.3 Instrumentation

Most composite specimens were instrumented with three Measurements Group CEA-06-375UW-120 strain gages (9.53 mm grid length), bonded with M-Bond AE-10 adhesive according to the manufacturer-recommended bonding procedure. Strain gages were bonded in the middle of the gage section, equally spaced around the tube OD and aligned in the transverse material direction (axial to the specimen). In this configuration, an average of the three gage readings, in the presence of potential bending, served to provide the axial strain. For some of the specimens, one of these three gages was substituted with a CEA-06-250UT-120 biaxial gage, so that fiber-direction strains could also be measured. Transverse sensitivity corrections were made with the equations from section 12 of the ASTM D5450 test procedure,

\[
\varepsilon_1 = \frac{\hat{\varepsilon}_1(1 - \nu_0 K_{11}) - K_{11} \hat{\varepsilon}_3(1 - \nu_0 K_{13})}{1 - K_{11} K_{13}}, \quad (5.1)
\]

\[
\varepsilon_3 = \frac{\hat{\varepsilon}_3(1 - \nu_0 K_{33}) - K_{33} \hat{\varepsilon}_1(1 - \nu_0 K_{11})}{1 - K_{11} K_{33}}. \quad (5.2)
\]

In these expressions from the ASTM procedure, subscripts 1 and 3 actually correspond to the material 2 and 1 directions respectively. The corrected transverse and fiber-direction strains are therefore given by \( \varepsilon_1 \) and \( \varepsilon_3 \), \(^\wedge\) indicates an uncorrected quantity, The Poisson’s ratio, \( \nu_0 \), is equal to 0.285, and \( K_{11} \) and \( K_{13} \) are the transverse sensitivity factors (equal to 0.8 ± 0.2 percent and 1.2 ± 0.2 percent, respectively). For a typical set of uncorrected strains \( \hat{\varepsilon}_1 = 847.0 \, \mu \varepsilon \) and \( \hat{\varepsilon}_3 = -75.0 \, \mu \varepsilon \),
the corrected strains are 845.7 με and –84.9 με (or 0.1% and 13.2% correction, respectively).

Vishay model 2120 strain-conditioning modules were used, with voltage output sent to a data acquisition computer capable of 20-Hz, 8-bit, 8-channel sampling. Thermal compensation during the creep tests was achieved by using a dummy specimen (without grips) connected with the test specimen to the strain conditioner in a half-bridge setup. The dummy specimen had identical instrumentation to the test specimens, and was positioned directly alongside the test specimen in the creep frame oven. The creep tests were conducted using an ATS model 2330 lever-arm creep frame with a temperature-controlled oven, as shown in Fig. 5.1. Pictures of an instrumented tubular specimen in the grips and in the oven are shown in Figs. 5.2, and 5.3, respectively.

Figure 5.1. Lever-arm creep frame used in the creep tests.
5.1.4 Quasi-Static Tests

To determine the room temperature compliance and ultimate transverse strength, $\sigma_{ult}^2$, of the E-glass/Dow 383 specimens, quasi-static tensile tests on three specimens were performed using a 5-kip ATS screw-driven load frame. The strength value measured from these quasi-static tests (45.6 MPa) served to determine the stress limit on subsequent creep tests.
5.1.5 Creep Tests

5.1.5.1 Procedural Overview

For constructing a master compliance curve for the E-glass/Dow 383 lamina, creep tests were performed on the tubular specimens. The compliance data measured from multiple tests (each test conducted at a different temperature) were plotted on a base-ten log-time scale (log\(_{10}(t)\)) and, within the context of the TTSP, these data were shifted to construct a master curve. The test plan in Table 5.2 indicates the identification scheme for temperature and stress (\(\sigma_2\)) loading conditions that were used for the creep tests. The maximum test temperature of 50°C was established in order to stay sufficiently below the approximate material \(T_g\) of 70°C (from unpublished tests of neat cured resin using dynamic mechanical analysis). In order to generate measurable creep strains, 30% of room temperature strength was chosen as the lowest stress level. It should be noted that, while some creep tests were conducted with transverse stress levels of 60% of \(\sigma_{ult}^2\), transverse stress levels above 50% are generally avoided in flywheel designs for durability reasons (fatigue performance).

<table>
<thead>
<tr>
<th>Stress level (%(\sigma_{ult}^2)) / MPa</th>
<th>Temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>30</td>
</tr>
<tr>
<td>30 / 13.7</td>
<td>30°_30%</td>
</tr>
<tr>
<td>45 / 20.5</td>
<td>30°_45%</td>
</tr>
<tr>
<td>60 / 27.4</td>
<td>30°_60%</td>
</tr>
</tbody>
</table>

Creep testing procedures discussed in the literature were varied with regards to the use of virgin specimens. Lou and Schapery [1971] found that mechanical conditioning was necessary to achieve repeatable results during isothermal creep tests at 73°C in the *nonlinear* viscoelastic range. Such mechanical conditioning consisted of ten cycles of creep (1 hr) and recovery (23 hr) at 73°C. Tuttle and Brinson [1986] did not perform specimen conditioning prior to creep testing (multiple tests per specimen) in the *nonlinear* viscoelastic range, and noted the presence of permanent strain after recovery. They attributed the permanent strain to damage accumulated during creep testing—which complicated accurate data reduction (correlation of data...
using the TTSP). In the experimental program carried out by Yeow et al. [1979] for creep testing in the linear viscoelastic range, a thermal and mechanical conditioning regimen is described:

“…It will be shown that neither type of conditioning substantially altered the mechanical properties.

“Specimens were subjected to a constant load and were thermally conditioned by slowly raising the temperature from room temperature to 210°C, then slowly cooling back to room temperature. When this process was repeated five times it was found that the stress-strain curve was practically the same for the fifth cycle as for the first…

“Mechanical conditioning was accomplished by first subjecting specimens to creep loads of approximately 50 percent of the room temperature ultimate strength. After 16 min the load was removed, the specimen was allowed to recover to approximately zero strain, and the process was repeated five times. The same specimens were then subjected to fifty low frequency cyclic loads. The stress-strain curve measured after the cyclic loading was essentially the same as the stress-strain curve obtained after the first thermal cycle. Thus, conditioning of the specimens was unnecessary, and it was established that the same specimen could be used repeatedly.”

Considering the three reports above, and the fact that characterization of the flywheel rotor material in the linear viscoelastic range was desired, mechanical conditioning was not employed for the creep testing reported in this research.

Several pilot creep tests that were performed in the present research did not yield meaningful results, but served to guide subsequent creep testing. These tests indicated:

- Material damage occurred during two repetitions of the 50%-60% condition (both specimens ruptured after 18 minutes after load application). The 60% stress level (at all temperatures) was avoided in subsequent tests.
- Damage (indicated by permanent accumulated strain) was not observed during multiple creep test repetitions of 30-minute duration on a single specimen for all remaining test conditions.
- To best eliminate the time-shifting effects due to material aging during measurements of the temperature time-shifting behavior, a well-aged specimen should be used, i.e. a material that has been aged “to effective equilibrium” (discussed in Section 2.3.4)
Taking these above three items into consideration, the procedure followed for the viscoelastic characterization of the E-glass/Dow 383 material system was to use a single tube specimen for the creep testing.

5.1.5.2 Master Curve for Transverse Compliance

Prior to the start of the creep tests, this specimen was aged at a temperature of 55°C for ~1100 hours, and then allowed to come to thermal equilibrium at 30°C. At this time, the strain gage outputs were set to zero. The first set of creep tests was performed at the applied stress level of 30% (of $\sigma_{ult}$) and temperatures of 30, 40, and 50°C. Figure 5.4 shows an illustration of the time and temperature cycles used for this set of creep tests.

![Time and Temperature Cycles](image)

Figure 5.4. Illustration of the time periods and temperatures during the creep and recovery cycles used to acquire master curve data for transverse compliance.

The momentary creep tests were limited to 15-minute duration to minimize the time necessary for recovery before starting a subsequent creep test at another temperature level. After the completion of each short-term test, the specimen was
subjected to a two-hour period at 50°C in order to accelerate recovery of the creep strains incurred during the previous test. The creep strains were of the order of 20-100 μm/m (depending on the temperature level). Following this high-temperature recovery, the specimen was allowed one additional hour at the next test temperature to achieve thermal equilibrium. It was found that this dwell period (three hours total) resulted in strains that recovered within ±5 μm/m of the initial strain (prior to the creep test). Following the procedure adopted by Sullivan et al. [1993] regarding residual strain recovery, this strain at the end of each recovery period was simply subtracted prior to beginning the next 15-minute creep test. At the end of the three-hour recovery period, the rate of recovery (<1 μm/m per 15 minutes) of this small residual strain does not significantly contribute to the compliance measured during the subsequent short-term (15-minute) test.

At the completion of the three 30% tests, the specimen was allowed to recover overnight at 50°C. At an applied stress of 45% (of $σ_{ult}$), the specimen was then subjected to the same creep and recovery schedule shown in Fig. 5.3 with one difference—the duration of the 3rd creep cycle (at the test temperature of 50°C) was extended to 139 minutes in order to extend the time period on the master curve by approximately one decade.

At the end of the creep and recovery cycle tests at 30% and 45%, the specimen was allowed to recover at 50°C for 12 hours and then was allowed several hours to cool to equilibrium at 30°C. At the end of this (final) recovery period, the indicated strains were within ±5 με of zero strain. This indicated that no significant material damage or aging occurred during the testing. Master curves constructed from the tests described above are given in section 6.1.2.1.

5.1.5.3 Temperature Shift Factor

With a reference temperature, $T_{ref}$, of 30°C in the master-curve tests described in 5.1.5.2, the 40°C and 50°C tests allow the measurement of the temperature-time shift factor $α_T$ (see Section 2.3.2 for significance of $α_T$), at only two temperatures—$T_{ref}+10°C$ and $T_{ref}+20°C$. The tests described in this section were performed simply to provide more data points for characterizing the temperature shift factor in the temperature range of interest (room-temperature to 50°C). All tests were
performed at the stress level of 45% of $\sigma_{ult}^2$. Starting at room temperature (21.1°C), the strain gage outputs were zeroed and a creep test was run with 5-minute duration. Immediately after unloading, the oven temperature was increased by several degrees and the specimen was allowed to simultaneously recover and come to thermal equilibrium for a 45-minute period. This cycle was repeated until the final test temperature of 49.9°C was reached. Results of the temperature-shift measurement tests are given in Section 6.1.2.2.

5.1.5.4 Aging Shift Factor and Shift Rate

The isothermal aging behavior in the transverse direction of the E-glass/Dow 383 material system was investigated on a different specimen to compare the aging behavior of this material system to the behavior of other PMC composite materials in the literature. Specifically, the procedures described here are for the measurement of the aging shift factor, $\alpha_{te}$, and the temperature-dependent shift rate, $\mu$, of the transverse compliance for E-glass/Dow 383 (see the equations in Section 2.3.4 for the significance of these aging parameters).

The procedure first requires that the specimen age be “erased” prior to testing (referred to as the “rejuvenation”). This was achieved by subjecting the unloaded specimen to a temperature of 85°C (e.g., $T_g+15^\circ C$) for a 2-hour period—which is similar to the temperatures and times used in other published investigations [Bradshaw and Brinson 1997, Sullivan et al. 1995, Sullivan 1990]. Following this age-erasure period (which took place in the ATS creep frame oven), the temperature control for the oven heater was set at 30°C, the oven door was opened (as shown in Fig. 5.3), and a flowing stream of room-temperature air was gently directed at the open oven chamber in order to accelerate cooling to 30°C for the subsequent aging tests. This cooling procedure is referred to as the “quench”. A very short temperature transient during the quench is desirable in order to isolate the isothermal aging behavior. The transient achieved by the procedure described above took between twenty and thirty minutes. At the end of the transient period (as indicated by the strain gage instrumentation) the strains were zeroed. Considering the start of the quench to be “zero” time, several momentary creep tests were then performed over the time interval of approximately $10^2$ to $10^3$ minutes (momentary implies test
duration less than \(1/20\)th of the aging time), while the specimen was maintained at 30°C. After the last momentary test at 30°C, the specimen was rejuvenated and quenched to 40°C, then the 40°C isothermal aging tests were performed over the time interval of approximately \(10^2\) to \(10^4\) minutes. After the last momentary test at 40°C, the specimen was rejuvenated and quenched to 50°C, then the 50°C isothermal aging tests were performed over the time interval of approximately \(10^2\) to \(10^4\) minutes. Using this procedure, aging shift factor \(a_t\) and temperature dependent aging shift rate \(\mu\) were measured at 30°, 40°, and 50°C. Applied stress in all tests was 45% of \(\sigma_{ult}^2\) (16 MPa). Results are given in Section 6.1.2.3.

5.1.5.5 Cross-Term Compliance, \(S_{12}\)

Cross-term compliance, \(S_{12}\), was measured on some of the creep specimens, using a strain gage orientated along the fiber direction (like that shown in Fig. 2.1b). Results of the measurement of the cross-term compliance are given in Section 6.1.2.4.

5.1.5.6 Stress Magnitude

An experiment was performed to investigate the effect of stress magnitude on the compliance of the aged E-glass/Dow 383 specimen. Four creep tests at applied stress levels of 17, 34, 42, and 51% of \(\sigma_{ult}^2\) were chosen for these experiments, which were each conducted at 30°C for a 15-minute duration. Recovery between successive tests was performed for 2 hours at 50°C, after which time the specimen was allowed 1 hour to achieve thermal equilibrium at 30°C. The results of this experiment are presented in Section 6.1.2.5.

5.1.5.7 Strain Response Due to Combined Variable Thermal and Stress Loads

An experiment was performed on a tubular specimen to verify the accuracy of the creep characterization of the E-glass/Dow 383 PMC. This experiment also served to validate that the time-temperature superposition principal and the quasi-elastic solution technique were properly implemented in the computer code for predicting strain under the combination of variable thermal and stress loads. The experiment
was conducted in the creep frame (Fig. 5.3) using a specimen that had been aged to effective equilibrium. The specimen was instrumented as described in Section 5.1.3. Uniaxial stress was applied and removed at random times during the test in a step-wise fashion and was kept at or below 20 MPa, and temperature was varied with the controller and was kept below 50°C during the test. The temperature of the specimen was measured with a thermocouple attached to the specimen, and was recorded with computer data acquisition. A plot of the applied stress and temperature and the predicted and measured strain is given in Section 6.1.2.7.

5.2 Isothermal Interference-Fit (IIF) Experiments

5.2.1 Overview

The objective of the IIF experiments is to use an interference fit to impose a biaxial stress state on a pair of concentric rings and to measure the resulting time-dependent deformation and pressure loss when one or more of the rings is made of a PMC material. The results of this experiment serve to validate the viscoelastic rotor model.

The approach taken in the experiments was to use a composite ring configuration that is commonly encountered in flywheel designs. Many flywheel designs employ multiple concentric rings--usually with a relatively inexpensive glass/epoxy (henceforth “gl/ep”) composite inner ring, and a stiffer, stronger, carbon/epoxy (henceforth “c/ep”) composite outer ring. Accordingly, this investigation uses gl/ep and c/ep rings under external and internal pressure loads, respectively.

Some of the ring pairs used an aluminum alloy (henceforth “al”) ring in place of one or the other composite component. The aluminum alloy is assumed to have time-independent properties (at the stresses and temperatures of interest in this investigation). Using aluminum as one of the two interference-fitted rings isolates the time-dependent response of the composite, and provides a different pressure vs. time loading profile on the composite ring. The radial thickness of each aluminum ring was chosen to provide the same effective elastic stiffness as the corresponding composite component. Hence, the radial interferences are the same in all experiments with a particular target pressure.
In the interest of gathering as much information as possible during the experiment, point-wise strains were recorded with electrical resistance strain gages and full-field displacements were recorded with moiré interferometry. The ring pairs were first press-fitted and the strains of the free inside diameter (ID) and outside diameter (OD) surfaces were recorded immediately completion of the assembly. These strains were used to calculate initial interfacial pressure using a one-dimensional closed-form, plane-stress, orthotropic elasticity solution for assemblies of multiple concentric rings [Lekhnitskii, 1963]. Full-field strains were also recorded at this time using moiré interferometry. After three weeks the rings were un-pressed, and the instantaneous strains upon disassembly were measured—yielding the corresponding relieved pressure. Full-field strains were again recorded, showing the creep strains that accrued during the experiment.

5.2.2 Materials and Ring Manufacture

The materials and manufacturing parameters for the composite rings used in this set of experiments are summarized in Table 5.3. Note that the gl/ep rings are precisely the same material system as the tubular creep specimens from Section 5.1. The composite rings were wet-filament wound using a continuous in-situ curing process [Gabrys and Bakis (1994)]. Four aluminum alloy rings were also used. Material properties for these materials are listed in Appendix D.

An 80°C mandrel temperature and 5 cm/hr radial deposition rate was chosen to manufacture all rings. The c/ep rings were wound on a mandrel that gave a 2 deg. taper on the ID. The gl/ep parts were wound on an untapered mandrel. Following gellation on the mandrel, all parts were maintained at 80°C in a forced-air oven for an additional 30 hours. Machining of the composite rings was done on a lathe with a tool post grinder. A re-circulating water bath was used to cool the diamond grinding wheel. Machining was done to all composite axial surfaces and ODs. The OD of the glass/epoxy rings was tapered 2 deg., matching the carbon/epoxy ring IDs. The rings were re-dried after machining by heating to 100°C for 48 hours. During the experiments, the rings were stored in ambient laboratory environments (10-40% relative humidity, 23-27°C).
Table 5.3. Filament winding parameters for the PMC specimens used in the interference-fit experiments.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glass Fiber</td>
<td>Owens Corning Type 30 E-glass, 158B-AB-450, inside-pull</td>
</tr>
<tr>
<td>Carbon Fiber</td>
<td>Toray T700S 24K</td>
</tr>
<tr>
<td>Matrix</td>
<td>Dow 383 resin &amp; Jeffamine T403 curative (47 parts T403 : 100 parts 383 by weight)</td>
</tr>
<tr>
<td>Gel Time</td>
<td>24 min at 80°C</td>
</tr>
<tr>
<td>Resin Bath Temp</td>
<td>23°C</td>
</tr>
<tr>
<td>Mandrel OD for the gl/ep rings</td>
<td>60 mm</td>
</tr>
<tr>
<td>Mandrel OD for the c/ep rings</td>
<td>85.49 mm</td>
</tr>
<tr>
<td>Mandrel Temp</td>
<td>80°C</td>
</tr>
<tr>
<td>Mandrel Speed</td>
<td>34 rpm</td>
</tr>
<tr>
<td>Mandrel Material</td>
<td>6061 Aluminum alloy</td>
</tr>
<tr>
<td>Mandrel Length</td>
<td>30 mm</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>3.05 mm</td>
</tr>
<tr>
<td>Fiber Angle</td>
<td>89.1 to 89.35 deg.</td>
</tr>
<tr>
<td>Post-Cure</td>
<td>30 hours at 80°C</td>
</tr>
</tbody>
</table>

*The T700/Dow 383 rings were tapered on the ID, so this number represents the mean OD.

5.2.3 Specimens and Instrumentation

The specimens consisted of six ring pairs, three designed with a "low" interface pressure of 27 MPa, and three with a "high" 54 MPa interface pressure. The high pressure, in particular, is high in comparison to typical press-flitting operations for flywheels, but was chosen to provide enough strain at the composite interface to reduce error in the collected data. The ring pair configurations are identified and summarized in Table 5.4. Henceforth, the abbreviations LP and HP stand for low pressure and high pressure, respectively. The abbreviations H, G, and C denote the hybrid (gl/ep and c/ep), glass (gl/ep and al), and carbon (c/ep and al) ring pairs, respectively. The precise dimensions of each of the rings are given in the next section (Load History) as these dimensions are directly related to the interface pressures in the rings upon loading with the interference fit.
Table 5.4. Ring pair configurations in the IIF tests.

<table>
<thead>
<tr>
<th>Ring Pair</th>
<th>Inner Ring</th>
<th>Outer Ring</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low Pressure (LP)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LP-H</td>
<td>gl/ep</td>
<td>c/ep</td>
</tr>
<tr>
<td>LP-G</td>
<td>gl/ep</td>
<td>al</td>
</tr>
<tr>
<td>LP-C</td>
<td>al</td>
<td>c/ep</td>
</tr>
<tr>
<td>High Pressure (HP)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HP-H</td>
<td>gl/ep</td>
<td>c/ep</td>
</tr>
<tr>
<td>HP-G</td>
<td>gl/ep</td>
<td>al</td>
</tr>
<tr>
<td>HP-C</td>
<td>al</td>
<td>c/ep</td>
</tr>
</tbody>
</table>

All rings were instrumented with Measurements Group CEA-06-125UN-120 electrical resistance strain gages in the hoop direction. Redundant gages were used to minimize the risk of data loss due to a malfunctioning strain gage and to detect possible nonalignment of the rings during the pressing process. Figure 5.5 schematically illustrates the gage locations, including the radially-oriented gages on each composite ring. Strains were monitored during the press-fit operation and thereafter for 24 to 30 days using two Measurements Group SB-10 switch and balance units and P-3500 strain gage conditioners.

Moiré interferometry was employed to provide full-field hoop and radial strains in the composite components. The moiré technique uses the interference between two incident, collimated, coherent laser beams and a diffraction grating on the surface of the specimen to obtain a fringe pattern representing the displacement field on the specimen [Post et al., 1994]. The equations governing the formation of fringes are identical to those used in moiré methods that employ amplitude gratings.
rather than diffraction gratings. Thus, the analogy of a specimen grating and a “virtual” reference grating may be applied in the case of moiré interferometry. In this analogy, the specimen grating deforms with the specimen. The reference is projected onto the specimen grating and does not deform with the specimen. When the specimen is not deformed, the fringe pattern created by the superposition of the virtual and specimen gratings is a field of uniformly spaced parallel lines. When the specimen deforms, this fringe pattern changes. A simple analysis of an image of this fringe pattern gives full-field displacements and strains [Post et al. 1994]. The gratings used in this experiment had a frequency of 2400 lines/mm, providing a displacement resolution of 417 nm/fringe. The orientation and location of the gratings is shown in Fig. 5.5, and the picture in Fig. 5.6 shows a close-up of some of the moiré and strain gage instrumentation on the HP-H ring pair.

Figure 5.6. Close-up of instrumentation on the HP-H ring pair, IIF tests.

5.2.4 Load History

The interference between the OD of the inner ring and the ID of the outer ring dictates the pressure achieved upon assembly. The mean radial dimensions, interferences, and target pressures for all ring pairs are given in Table 5.5. The pressure at the interface of two elastic rings was calculated based on the measured hoop strain at either the ID of the inner ring or at the OD of the outer ring. If a ring has time-dependent deformation, then the elastic approximation at very short times (e.g., \( t = 0.01 \) min.) must be used to calculate interference pressure changes. The target pressures and strains listed in Table 5.5 were calculated using Lekhnitskii’s [1963]
elastic solution (prior to the development of the present viscoelastic model). The same target strains are, however, also calculated with the present viscoelastic rotor model, using a time value of \( t=0.01 \) minutes.

The final machined axial thickness for all composite rings was chosen as 25.4 mm. All aluminum rings were made 31- mm-thick to accommodate any possible additional axial distance necessary to achieve the desired interface pressure during the pressing process. It should be noted that—because the aluminum alloy rings were axially oversized (22% more axial thickness compared to the composite rings)—the “extra” axial thickness had to be considered in the target strain and pressure calculations. This extra axial thickness was considered to effectively stiffen the aluminum alloy rings by 22%. During model calculations for target strains, therefore, the Young’s modulus of the aluminum alloy rings was increased by 22%.

Table 5.5. Ring dimensions, target pressures and target strains in the IIF tests.

<table>
<thead>
<tr>
<th>Ring Pair</th>
<th>Material</th>
<th>Inner Radius (mm)</th>
<th>Outer Radius (mm)</th>
<th>Radial Interference (mm)</th>
<th>Target Pressure (MPa)</th>
<th>Target Strain (( \mu )e)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LP-H</td>
<td>gl/ep</td>
<td>60.00</td>
<td>83.49</td>
<td>83.74</td>
<td>0.25</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>c/ep</td>
<td>83.49</td>
<td>95.49</td>
<td>95.49</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LP-G</td>
<td>gl/ep</td>
<td>60.00</td>
<td>83.49</td>
<td>83.74</td>
<td>0.25</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>al</td>
<td>83.49</td>
<td>11.195</td>
<td>11.195</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>c/ep</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LP-C</td>
<td>al</td>
<td>66.04</td>
<td>83.49</td>
<td>83.74</td>
<td>0.25</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>c/ep</td>
<td>83.49</td>
<td>95.49</td>
<td>95.49</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HP-H</td>
<td>gl/ep</td>
<td>60.00</td>
<td>83.49</td>
<td>84.00</td>
<td>0.51</td>
<td>54</td>
</tr>
<tr>
<td></td>
<td>c/ep</td>
<td>83.49</td>
<td>95.49</td>
<td>95.49</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HP-G</td>
<td>gl/ep</td>
<td>60.00</td>
<td>83.49</td>
<td>84.00</td>
<td>0.51</td>
<td>54</td>
</tr>
<tr>
<td></td>
<td>al</td>
<td>83.49</td>
<td>11.195</td>
<td>11.195</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>c/ep</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HP-C</td>
<td>al</td>
<td>66.04</td>
<td>83.49</td>
<td>84.00</td>
<td>0.51</td>
<td>54</td>
</tr>
<tr>
<td></td>
<td>c/ep</td>
<td>83.49</td>
<td>95.49</td>
<td>95.49</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The press-fitting was done on a displacement-controlled 300 kN universal load frame. Normally, this is done with catalyzed epoxy at the interface to provide lubrication during the pressing, and for added interfacial shear strength during later operation of a flywheel. The need to un-press the ring pairs after a long period of time necessitated the use of uncatalyzed epoxy as a lubricant. A no-slip condition of the rings following pressing and during the long-term experiments was verified to within 30 \( \mu \)m using a depth gage on the assembled rings. The required un-press loads were in
all cases about twice the press loads. Figure 5.7 is a schematic illustration of the hybrid ring pair being pressed.

![Figure 5.7. Cross-section illustration of ring pair during the press-fit operation.](image)

The LP-H ring pair was assembled first and the instantaneous elastic strains of the free ID and OD surfaces were measured. These compressive strains on the ID of the LP-H glass ring served as target strains during the subsequent assembly of the LP-G ring (which was designed for adjustable interference capability). Likewise the tensile strains on the OD of the LP-H carbon ring served as a target strain during the subsequent assembly of the LP-C ring pair. The HP-H, HP-G and HP-C rings were all assembled in the same fashion as their low-pressure counterparts. Strain gage data and radial and hoop-direction moiré displacement field images were taken immediately after each of the ring pairs mentioned above were assembled.

After three weeks each of the ring pairs were disassembled. For each of the six ring pairs, the strain gage and moiré data were again taken immediately after disassembly. Subsequent recovery of strains in the rings was not monitored.

**5.3 Non-Isothermal Interference-Fit (NIF) Experiments**

**5.3.1 Overview**

To appreciate the reasons for performing the NIF experiments, it is helpful to examine the results from the set of isothermal interference fit (IIF) experiments. The significant difference between the expected and observed deformation state in the IIF experiments was unforeseen. A finite element model (created and analyzed post-experiment) of the assembled composite rings used in the IIF experiment showed that the elastic strains on the surface of the rings (where the strain gage and moiré
instrumentation was located) deviated significantly from the strains predicted by the closed-form plane-stress elastic model that was used to design the experiment. This deviation was especially severe near the tapered interface of the assembled rings, where the free axial surface allows out-of-plane deformations.

Three reasons existed for performing the NIF experiments. First, it was desired to place instrumentation radially far enough away from the ring interface so as to be located in a region where the elastic FEA predictions and the moiré results from the IIF experiments indicated reasonable agreement of strain predictions with the closed-form model. In IIF experiments, all of the radially-oriented strain gages were located very close to the ring interface.

The second reason for performing the NIF experiments was that a better understanding of the composite constitutive behavior had been gained after IIF experiments were performed. Specifically, the effect of aging on the material compliance was not considered in the IIF experiments. Aging causes the value and rate of change of material compliance to decrease considerably over time. Hence, it is desirable in a creep test to be able to account for aging effects when performing a mechanical creep test. In the NIF experiments, aging effects were eliminated by thermal conditioning prior to applying the interference-fit loads.

The third reason for performing the NIF tests is related to the second reason in that knowledge had been gained in understanding the non-isothermal constitutive behavior of the E-glass/Dow 383 composite, and in being able to incorporate that behavior in the viscoelastic model. By exposing the specimen to elevated temperatures, the NIF tests could be performed in a much shorter time than the IIF tests (~2 hours versus ~3 weeks) while still achieving a measurable amount of creep strain.

The LP-H and LP-G rings from the IIF tests were used in this non-isothermal re-investigation of the Interference-Fit experiment. These ring pairs were chosen because each of these pairs contained an E-glass/Dow 383 PMC ring, and the non-isothermal viscoelastic properties of this material system had been characterized extensively since the IIF tests (Section 5.1). The composite rings used in the NIF experiments were exactly those rings from the IIF tests, but the instrumentation and load history were different and are described below.
5.3.2 Specimen Instrumentation

The LP-H and LP-G ring pairs were instrumented with strain gages only. Figures 5.8-9 show photographs of the specimens with labeling to indicate the orientation and location of each strain gage in this set of experiments. Table 5.6 gives the precise radial location of the gages in the two experiments. All of the radial gages had a 1.6-mm (0.062 inch) gage length (Measurements Group CEA-13-062UW-120), and all hoop gages had a 3.2-mm gage length (CEA-06-125UN-120). It should be noted that the 5r and 6r gages in the LP-G specimen were intentionally placed at r=79 mm. This is a radial location close to the interface of the mating rings—where the moiré results from the IIF tests indicated that experimental strains deviated significantly from plane-stress model predictions. These two gages were placed at this location simply to confirm the accuracy of the moiré results.

Figure 5.8. Photograph of the LP-G specimen with labeling to indicate gage locations and orientations, NIF tests.
Figure 5.9. Photograph of the LP-H specimen with labeling to indicate gage locations and orientations, NIF tests.

The “signal conditioner” column in Table 5.6 indicates which gages were connected to each of the two types of Vishay signal conditioners that were used in these tests. The eight outputs from the 2120 strain conditioner in each test were connected to a data acquisition computer that recorded the voltages (strains) at frequencies that ranged from 1 Hz (during pressing) to 0.05 Hz (during dwell times at elevated temperature). Strain data indicated on the P3500/SB10 unit were recorded by hand at various times during the load history, which is described next.

5.3.3 Load History

5.3.3.1 Specimen conditioning

The prior creep strains in the composite rings in the LP-H and LP-G specimens were relieved by raising the temperature of the rings to the cure temperature of 85°C, (approximately 15°C higher than the glass transition temperature of the Dow 383 matrix) for a period of twelve-hours. Upon cooling to
Table 5.6. Radial locations of the gages used in the NIF tests.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Ring</th>
<th>Gage ref. number</th>
<th>Radial location (mm)</th>
<th>Signal Conditioner</th>
</tr>
</thead>
<tbody>
<tr>
<td>LP-G gl/ep</td>
<td>$1r$</td>
<td>65.9</td>
<td></td>
<td>2120</td>
</tr>
<tr>
<td></td>
<td>$2r$</td>
<td>65.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$3r$</td>
<td>69.4</td>
<td></td>
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<tr>
<td></td>
<td>$4r$</td>
<td>69.7</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>$5r$</td>
<td>78.5</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>$6r$</td>
<td>78.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$1\theta$</td>
<td>60.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$2\theta$</td>
<td>60.0</td>
<td></td>
<td>P3500/ SB10</td>
</tr>
<tr>
<td></td>
<td>$3\theta$</td>
<td>60.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$4\theta$</td>
<td>111.95</td>
<td></td>
<td>2120</td>
</tr>
<tr>
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<td>$5\theta$</td>
<td>111.95</td>
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<td></td>
<td>$6\theta$</td>
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</tr>
<tr>
<td>LP-H gl/ep</td>
<td>$1r$</td>
<td>65.9</td>
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<td>2120</td>
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<td></td>
<td>$2r$</td>
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<td>$2\theta$</td>
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<td>P3500/ SB10</td>
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<td></td>
<td>$3\theta$</td>
<td>95.49</td>
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<td></td>
<td>$5r$</td>
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<tr>
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<td>$6r$</td>
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<td></td>
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<tr>
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<td>$7r$</td>
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<td></td>
<td>$8r$</td>
<td>89.9</td>
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<td>$4\theta$</td>
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<td></td>
<td>$6\theta$</td>
<td>95.49</td>
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</tbody>
</table>

Room temperature after this “strain-erasure” period, the strain gages were installed on the rings using Measurements Group AE-10 adhesive system. Following the manufacturer’s instructions precisely, all rings were heated to 65°C (maximum test temperature plus 15°C) for two-hours in order to ensure creep-free performance of the gage adhesive at the test temperature of 50°C.

Following the gage installation and adhesive post-cure, the composite rings were heated to 55°C for 1 week. Aging the specimens prior to performing the non-isothermal interference (NIF) experiments represents a very significant difference in experimental procedure compared to the isothermal interference (IIF) tests. This exposure advanced the state of aging to a degree such that aging effects would be
negligible (see discussion of “momentary test” in Section 2.3.4) during the time period when the rings would be assembled and heated to a maximum temperature of 50°C for a two-hour period.

After aging of the rings was complete, the strain gages were connected to the signal conditioning and recording equipment as described above and the strains were zeroed. It is noted here that the availability of only 20 strain gage conditioning circuits necessitated that the LP-G and LP-H ring pairs (a total of 26 gages) be tested separately.

Each pair of disassembled rings was then subjected to thermal cycles in a mechanical convection oven from room-temperature (21°C) to 50°C for 90 minutes and then back to 21°C to investigate the thermal stability of the gages and to make sure that the indicated strains returned to a value (±10 με) reasonably close to zero. For both ring pairs, it was found that after one such thermal cycle, the strains returned to ±100 με of zero. After re-zeroing the gages and subjecting the rings to a second thermal cycle, the strains returned to within 10 με of zero. This indicated that strains accumulated prior to the start of the test were relieved to a degree so as not to affect the strains measured during subsequent interference-fit tests.

5.3.3.2 Interference-Fit Loads

As in the IIF experiments described in Section 5.2.2, the pressing of the rings was done on a displacement-controlled 300-kN universal load frame. A thin coat of uncatalyzed epoxy was used as a lubricant at the ring interface. In both the LP-G and LP-H tests, the gl/ep ring was pressed down into the aluminum or c/ep ring, respectively (opposite of that which is illustrated in Fig. 5.4). A set of 1.27-cm-thick polycarbonate shims were used to provide a means to transmit axial press-force to the gl/ep rings without damaging the strain gages. A picture of the instrumented LP-G ring with these shims just prior to assembly is shown in Fig. 5.10.

For both ring pairs, the strains at the inner radius of the gl/ep ring were monitored during the pressing and dictated when the pressing was considered complete. As in the IIF experiments, the ability to advance the radial interference until target strains were met was enabled by the use of an axially-oversized aluminum
ring. For the LP-H ring pair, press-fit force was applied until the bottom face of the gl/ep ring contacted the base on which the c/ep ring rested in the test frame (i.e. until the bottom faces of the gl/ep and c/ep rings were co-planar). The assembly operation typically took about three minutes. The viscoelastic rotor model from Chapter 3 was used to predict the target strains for room-temperature assembly of the LP-G and LP-H ring pairs. These target and actual strains upon assembly are presented in the Results chapter.

Following each of the interference fits, the assembled ring pair was placed in the mechanical convection oven (which was pre-heated at 50°C), for three hours (henceforth called the “temperature soak”). Computer-recorded strain data indicated that the thermal transient experienced by each ring assembly was between 45 minutes and 1 hour. Thus, each ring assembly was exposed to a minimum of two hours at the maximum target temperature of 50°C.

The ring assemblies were then carefully removed from the oven and cooled. The cooling transient was hastened by using a household-sized fan that blew room-temperature (21°C) air across the ring assembly. The strain data indicated that a uniform temperature in the rings was realized after approximately 40-50 minutes. The cooled rings were then taken back to the load frame and “un-pressed”, which again took approximately three minutes.
The disassembled rings were then placed back in the oven, for a 12-hour period at 50°C for strain recovery. Note that strain recovery was not necessary for the aluminum ring in the LP-G test.

5.4 Preparation of the Optical Pattern for the OESM System

5.4.1 Reflective Pattern Design

Four-lobe patterns (lobe type shown in Fig. 4.6) with a 1-cm radial spacing of ten concentric annular regions were applied to the S2-glass/polyurethane and aluminum rotors. The annuli spanned a 6- to 16-cm radial range.

The target hoop strain resolution of ±10 με at all ten annular regions requires a displacement resolution, \( u_{\text{min}} \) (calculated using Eq. 4.2), which varies as a function of radius. Sensor reference positions, \( r_{\text{ref}} \), were established at radial locations less than \( r_{\text{max}} \) (typically 1-2 mm from the inner radius of each region—see Eq. 4.8) assuming \( S_c=20 \) MHz and \( \omega=2100 \) rad/s (20,000 rpm). With the sensors placed at these locations, hoop strains of 14,000 to 16,000 με can be measured before the illuminated spot encounters the inside radii of the TB patches (rendering further measurements impossible).

5.4.2 Pattern Generation and Application

Currently, the photodetectors with the quickest response time operate in the infrared region of the electromagnetic spectrum. Thus, it was decided to produce a pattern that reflects highly contrasting amounts of incident infrared radiation. For the range of incident radiation intensities necessary to operate the available photodetectors, matte silver is sufficiently reflective and matte black is sufficiently non-reflective.

The pattern was electronically generated using rudimentary graphics commands (e.g., Circle[], Polygon[]) in the Mathematica® programming language, then rastered at 57.7 dot/mm (1440 dots/in) in Adobe Photoshop®. Each lobe was printed on glossy transparency film using an Epson Stylus Color 800® ink-jet printer that has 57.7 by 28.3 dot/mm (1440 by 720 dot/in) resolution. The printed lobes were
assembled on a rigid, clear, acrylic disk that was used as a template to expose the pattern shape onto the rotor using standard photographic contact print techniques. Detailed specification of the materials and procedures used to apply the pattern to the rotor surface are outlined in Appendix J.

The final pattern was about 45 μm in thickness—slightly more than half of the total thickness of a typical encapsulated electrical resistance strain gage. Because the majority of the thickness is polyurethane and exposed emulsion, the reinforcement effect of this layer is negligible for flywheel applications. Assuming that the coating has a density of 1 g/cc and a shear modulus of 37 MPa and that self-reinforcement is absent, the shear deformation of the top surface of this coating at a 15-cm radius, due to inertial loading at 20 krpm, is conservatively estimated to be less than 1 μm relative to the interface with the rotor. Hence, errors in radial displacement measurements due to coating thickness are expected to be less than 1 μm under the conditions of this investigation.

5.5 Validation of the OESM System

5.5.1 Overview

Experiments were performed on two disk-shaped flywheel rotors with reflective patterns to evaluate different aspects of the OESM method. An S2-glass/polyurethane composite rotor was used in low speed spin tests to evaluate the methods of compensation for vibrations, pattern misalignment, and illumination intensity. An aluminum rotor was used in higher speed spin tests to evaluate the ability of the OESM system to measure axisymmetric radial displacements, which were validated by comparisons with theoretical values for an aluminum rotor subjected to measured changes in speed and temperature. The higher-speed rotor spin tests were carried out in an armored steel chamber that was evacuated to reduce drag and frictional heating.
5.5.2 Experimental Setup

5.5.2.1 Low-Speed Test Setup

The composite rotor used in the low-speed tests was a 2.5-cm-thick disk with a 3-cm inside radius and a 16-cm outside radius, consisting of hoop-wound S2-glass fibers, ~62% by volume, in a polyurethane matrix [Gabrys and Bakis, 1997]. This rotor was attached to a motor, which was mounted with rubber vibration absorbers to a massive steel base. A frame bolted to the steel base provided a rigid foundation on which to mount one optical sensor. Precision axial and radial positioning stages controlled the sensor location relative to the rotor surface during calibration. Two axially spaced inductive position sensors with 1-µm-resolution were aimed radially at the motor for vibration measurements and a third inductive position sensor was used to determine the radial position of the optical sensor during calibration (Fig. 5.11).

![Figure 5.11. Low-speed spin test setup.](image)

5.5.2.2 High-Speed Test Setup

The 2024-T351 aluminum alloy rotor used for the high-speed tests was an 8.5-mm-thick disk with a 6-cm inside radius, and a 16-cm outside radius. The relevant properties of this alloy are given in Appendix D. The rotor was attached to an 11.4-cm-OD aluminum hub by a flexible polyurethane interlayer that served to isolate the aluminum rotor from radial stresses that would otherwise result from a rigid hub connection. A steel sleeve lightly press-fitted into the aluminum hub provided the
connection to the 6.35-mm-OD drive shaft (see Appendix E for a detailed drawing of the hub). The drive shaft was connected to a hydraulic motor through a magnetic clutch developed in-house. The magnetic clutch transmitted torque into the evacuated spin pit without a mechanical connection and protected the hydraulic motor in the event of a rotor failure. Figure 5.12 shows a cross-section schematic of the motor, clutch, and rotor components in the high-speed test setup.

Figure 5.12. High-speed spin test setup.

Figure 5.13 shows a picture of the aluminum rotor setup, suspended from the spin pit lid (lid not shown). To minimize air friction and consequent heating of the rotor, and to ensure safety, tests were performed in an evacuated armored spin pit. In an attempt to avoid sensor displacements due to thermal effects during an experiment, the racks on which the sensors were mounted were made of a unidirectional carbon/epoxy composite having a low coefficient of thermal expansion (<1 με/°C) in the radial direction of the rotor. These two (redundant) racks of sensors, henceforth
referred to as “rack A” and “rack B,” were positioned at 90-degree intervals on the rotor, and were connected to the underside of the steel spin pit lid. The A1 and B1 sensors were located at the radial position of 60.9 mm, A2 and B2 were at 70.7 mm, A9 and B9 were at 142.0 mm, and A10 and B10 were at 152.2 mm.

![Photograph of the high-speed spin test setup and OESM sensors.](image)

Integral to the connection of each rack with the lid was a precision linear slide that allowed the sensor rack to be moved in the radial direction for calibration purposes. The radial positions of the racks relative to the lid were monitored with inductive sensors. Three IR thermocouples measured the temperature of the surface of the rotor at radii of 85, 115, and 145 mm. Two copper feed-throughs (one 12-conductor and one 27-conductor) were used to transmit the power to and signals from the instruments in the spin pit. A bank of electrical relays installed in the pit reduced the number of conductors required to operate the optical sensors from 26 to 12. Additionally, one 9-conductor J-type thermocouple feedthrough was used to conduct temperature signals from inside the pit. Appendix F provides a detailed description of the electrical connections in the high-speed setup.
5.5.3 Experimental Procedures for OESM

5.5.3.1 Balancing

The rotors in both the low and high-speed setups were balanced prior to testing the different aspects of the OESM system. The position signals from the inductive sensors provided the vibration data (amplitude and phase) used in these balancing procedures. For the low-speed test setup (Fig. 5.11), a simple graphical one-plane balancing method [Ehrich 1999] was used in order to limit vibration amplitudes to less than 10 \( \mu \text{m} \) over the speed range of interest. The single plane balancing procedure requires the amplitude of vibration to be recorded at a “target speed” during an initial run and then three subsequent “trials.” The target speed should be near a structural resonance of the flywheel. During each of the trials, a small “trial mass” is added to a different azimuthal location on the rotor. The weight of the trial mass and the azimuths at which the mass is placed during these trials should be chosen such that a significantly different amplitude of vibration is observed for each of the trials. For a 2-kg rotor, a typical trial mass for the initial balancing run is on the order of 1 to 10 grams, and the most reasonable set of azimuthal locations should be spaced 120-degrees apart. The different amplitudes observed during the trials and the initial run are then used to construct a graph (see [Ehrich 1999]) that reveals a solution mass and azimuth to counteract the imbalance that exists at the target speed. Typically, vibration amplitudes are reduced two to ten-fold after applying the solution mass.

On the low-speed test setup, two-part quick-dry epoxy applied to the outer diameter of the rotor was used as the balance mass. This method sufficed to balance the low-speed test setup over the speed range of interest (<1000 rpm). To balance the high-speed test setup, the procedure followed was to first balance the rotor using the same one-plane method mentioned above (after the bottom clutch and rotor shaft shown in Fig. 5.12 had been balanced beforehand) at low speeds (<3 krpm). The “Rotor Balance Mass” location is shown in Fig. 5.12. The masses used were short pieces of 2.3-mm diameter steel wire that weighed 0.5-2 grams. At higher speeds, a two-plane influence-coefficient balancing method was used [Ehrich 1999]. The two-plane procedure requires the amplitude and phase to be recorded during an initial run and two trials wherein trial mass is added to two different balancing planes. The
solution mass is then calculated with a mathematical formula. Small pieces of 0.13-mm thick brass shim weighing less than 20 mg served as rotor balance masses at the higher speeds (>3 krpm). The two sets of threaded holes in the bottom clutch served as the second balance mass plane during this two-plane balancing procedure. Placing 70% of the mass in the top row (nearest the magnets) and 30% in the bottom row effectively added mass on the plane through the center of mass of the clutch. Hence, these two rows were treated as one plane in the clutch-shaft-rotor assembly.

5.5.3.2 Calibration

The first step optimum axial location of the sensor is determined by the alignment of the tubes that hold the electronic components of the sensor. In practice, rough axial positioning of the sensor was performed by adjusting the pieces of the sensor support structure (which was designed with adjustable geometry—see “Position adjustment rods” at the top of the photograph in Fig. 5.13). More precise axial positioning was then performed with the fine adjustment screw (indicated in Fig. 4.5) on the support arm. Correct axial position was thus determined using a spacer between the rotor surface and the sensor as an indicator (for the sensors used in this investigation the stand-off distance was 4.5 mm). It was then necessary to align the photodetector with the image (created by the receiver lens) of the projected IR spot. This was performed by moving the support plate along and around the axis of the support bolt. Precise location of the photodetector was thus enabled by switching on the LED, and monitoring the photodetector output while performing the positioning adjustments. The correct position of the photodetector was found in an iterative fashion by adjusting the position of the sensor then slightly decreasing the voltage to the LED until a minimum value was reached that still activated the photodetector.

Creation of the empirical look-up table of duty cycle versus radial position (discussed in 4.3.1) was performed one-sensor-at-a-time by moving the sensor in the radial direction using a screw to push the sensor rack. With the rotor spinning at a sufficiently low speed to avoid measurable flexible body displacement (~500 rpm), duty cycles of the TB patches were recorded at 10-15 μm radial intervals over an appropriate range of radial position such that maximum expected rotor displacements would be measurable at all times. For the aluminum rotor used in the high-speed test,
each sensor was calibrated over a 300-μm range. The determination of an optimum radial location of the sensor ($r_{\text{max}}$ from Eq. 4.8 is recommended) was discussed in Section 4.2.2. A set of calibration data from the low-speed rotor is plotted in Fig. 6.21 (Section 6.4.1).

5.5.3.3 Data Acquisition

Data acquisition is performed in a momentary fashion throughout a spin test. During these momentary data acquisition events, the rotor is held at a constant speed. One sensor at a time, the logic signal output from a given sensor is input to the gate of a 20 MHz counter that measures the pulse width of the TB patch and the compensation patch in each of four lobes in a given radial region (i.e. 8 pulses per revolution). Simultaneously, another 20 MHz counter is used as a precision tachometer to convert these sensor logic signal pulse widths to the angles $\phi_{\text{inst}}$ and $\theta_r$. These angle values are acquired for twenty (not necessarily consecutive) rotor revolutions. For each set of twenty angle measurements, the highest and lowest-five are discarded and the mean of the remaining ten is calculated to reduce the effects of noise. The acquisition and processing of twenty sets of eight angles as described above requires approximately three seconds. The Labview VI for processing the sensor output signals as described above includes on-the-fly calculations of the corrected radial position (Eq. 4.12), as well as the vibration phase and amplitude (from Eq. 4.14). A logical flowchart and the LabView “Wiring Diagram” for this VI are given in Appendix G.

5.5.4 Low-Speed Experiments

5.5.4.1 Sensor Effects

Measurement drift was assessed by making measurements on the low speed rotor over a 20-hr period while the rotor was spun at a constant speed of 550 rpm. This speed is slow enough that the rotor would not exhibit appreciable ($<<1$ μm) time-dependent deformation, and would thereby serve to provide a baseline against which drift could be measured.
To supplement the simple drift experiment, the intensity compensation algorithm described in Section 4.3.2 was evaluated on the low-speed rotor by recording two sets of duty cycle data versus radial position using different LED intensities, choosing one set as the “correct” set, and then applying the LED intensity compensation algorithm (Eqs. 4.11-13) to the other set. The results of the intensity compensation test are given in Section 6.4.2.

5.5.4.2 Rigid Body Displacements

The ability of the optical sensor to measure in-plane rigid body displacements was evaluated on the low-speed rotor by recording radial displacements at the nominal position of 151.6 mm at speeds spanning a rigid body resonance. The resonance amplitude, which was previously reduced to below 10 μm by balancing the rotor, was temporarily increased for this test by attaching a 0.15-g piece of vinyl electrical tape to the OD of the rotor at the 35-deg. location. The first mode (cylindrical) of the rotor/motor assembly was at approximately 850 rpm. Displacement data from two inductive position sensors (see Fig. 5.12) that recorded the radial position of the motor housing were used for comparison with the data from the optical sensor. The sensitivity of the OESM method to out-of-plane displacements and the ability to compensate for out-of-plane displacements were investigated by fixing the sensor in the radial direction, translating the sensor in the axial direction, and recording the angles swept on the displacement and compensation patches at each step. Applying the LED intensity compensation algorithm to these data allowed calculations of corrected radial locations at each axial position. Results of these rigid-body displacement experiments are given in Section 6.4.3.

5.5.5 High-Speed Experiment

A high-speed spin test was performed on the aluminum alloy rotor to verify the ability of the OESM system to measure flexible body axisymmetric displacements due to inertial forces. A maximum rotational speed of 16 krpm was used to obtain reasonably large elastic deformations without surpassing the 325 MPa yield stress of the aluminum.
After balancing and calibrating the rotor as described above, the sensor reference positions were established (see Section 5.5.3.2). Speed was then incremented and held constant while rotor temperature and displacements (rigid body vibration and flexible body) were measured. These displacement and temperature data were used to calculate the separate contributions of rigid and flexible body displacements and temperature to the total radial displacements measured by the sensors at the different radial and azimuthal locations on the rotor. Results of this experiment are presented in Section 6.4.4.

5.6 Creep Test of Spinning Rotor

5.6.1 Experimental Setup

5.6.1.1 Rotor and Instrumentation

The rotor built for the spinning creep test had an inner radius of 6 cm and an outer radius of 16 cm, and was 12 mm in thickness. The rotor material was the same E-glass/Dow 383 as that listed in Table 5.1. The rotor was manufactured at 85°C and post-cured for 16 hours. Machining of the faces and OD of the rotor was performed with the same grinding procedure described in Section 5.1. A 3.2-mm radially thick polyurethane (Adiprene L100/Caytur-21) interlayer was cast between the rotor and an aluminum hub as described in A5.2. A drawing of the hub is also given in Appendix E. The reflective TB pattern was applied to the rotor with the procedure described in Appendix J in preparation for making optical displacement measurements using the OESM system.

The high-speed test setup and instrumentation described in Section 5.5.2.2 was used in the composite creep test setup. Additionally, a 100-watt light bulb was placed under the rotor at the bottom of the spin pit. This bulb was used to provide light (heating) during the test to accelerate the creep behavior. The bottom of the rotor was painted black to increase the absorption of thermal energy from the lamp.
5.6.1.2 Experimental Procedure

The rotor was balanced using the same technique described for the high-speed test setup in 5.5.3.1. At the end of the final high-speed balancing run (near 13 krpm), an attempt was made to spin up to the desired test speed of 15.1 krpm in order to ensure that there were no vibration modes in that speed range that would require further balancing. At a speed of 14.2 krpm, the rotor cracked due to radial stress, broke the metallic drive shaft, and significantly damaged the high speed test setup. The OESM instruments had not yet been calibrated for measuring displacements and were offline at the time of the fracture. The fact that the rotor fractured (i.e. ending the creep test before any measurements could be made) is mentioned here because this result indicates a poorly designed experiment. The remaining discussion regarding this experiment is in the Results/Discussion chapter, Section 6.5.
Chapter 6
RESULTS / DISCUSSION

6.1 Characterization of E-glass/Dow 383 Material

6.1.1 Physical and Mechanical Properties

The fiber and void content of the E-glass/Dow 383 hoop-wound tubes determined from the burnoff tests were 69% and 2.0%, respectively. A summary of the room-temperature quasi-static test results is given in Table 6.1. The last row of Table 6.1 is the coefficient of variation, CV%, which is a measure of the dispersion within a data set and is calculated as

\[
CV\% = \frac{\text{standard deviation}}{\text{mean}} \times 100.
\]  

Table 6.1 Summary of the quasi-static tensile test results

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<tr>
<th>Specimen</th>
<th>Compliance ( S_{22} ) (1/GPa)</th>
<th>Ultimate Strength ( \sigma_{ult} ) (MPa)</th>
<th>Failure Strain ( \varepsilon_{ult} ) (( \mu \varepsilon ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0551</td>
<td>46.66</td>
<td>2807</td>
</tr>
<tr>
<td>2</td>
<td>0.0547</td>
<td>46.70</td>
<td>2803</td>
</tr>
<tr>
<td>3</td>
<td>0.0555</td>
<td>43.51</td>
<td>2643</td>
</tr>
<tr>
<td>Mean</td>
<td>0.0551</td>
<td>45.62</td>
<td>2751</td>
</tr>
<tr>
<td>CV%</td>
<td>5.9</td>
<td>3.3</td>
<td>2.8</td>
</tr>
</tbody>
</table>

The compliances reported in Table 6.1 represent the chord compliance in the 500 to 1500-\( \mu \varepsilon \) strain range, according to the ASTM procedure D 5450. Though not indicated in Table 6.1, the static tests revealed a significant disparity between the gage readings on a single specimen. This disparity was also evident in the creep tests (the results of which are presented in the next section), and could have come from several sources in the specimen including bending, variable wall thickness, and material heterogeneity. After a
careful analysis, it was concluded the disparity was likely caused by a combination of the three above-mentioned effects. Appendix H contains an assessment of the sources for discrepancy among strain gage readings on a tubular specimen (applicable to the quasi-static and creep tests).

6.1.2 Creep Test Results

6.1.2.1 Master Curve for Transverse Compliance

The $S_{22}(t)$ master curves constructed from creep data measured at 30% and 45% of $\sigma_2^{ult}$ are shown in Fig 6.1. Each of the curves is comprised of three momentary creep tests run at 30, 40, and 50°C. As shown in Fig. 6.1, the power-law creep function that was used to represent $S_{22}(t)$ in the flywheel model was fit to the 45% master curve data. For the 45% master curve, the 40°C and 50°C data have been horizontally shifted relative to the 30°C data by $a_{(40)} = 10^{1.54}$ and $a_{(50)} = 10^{3.84}$, respectively. The 50°C creep test ran for 139 minutes, resulting in an effective test duration of $139 \times 10^{3.84}$ minutes (or 3.2 years) at 30°C.

![Figure 6.1. Master curve at 30°C for $S_{22}$ of E-glass/Dow 383 measured at the 30% stress level.](image_url)
The time exponent of the power law fit to the $S_{22}$ data here is approximately half those reported by other researchers (see Table 2.1). This difference is believed to be the result of the thermal conditioning (1000 hours at $T_g-15^\circ C=55^\circ C$) that was used to age the specimens prior to creep testing. Unpublished preliminary creep tests for $S_{22}(t)$ at the 45% stress level using E-glass/Dow 383 specimens that were aged only 24 hours at 40°C, resulted in a master curve with a best fit power law of $S_{22}(t) = 0.0495 + 0.00307 t^{0.156}$ (1/GPa). The difference in exponent ($n=0.105$ versus $n=0.156$) immediately indicates that the reported mathematical fits to the compliances in Table. 2.1 are of limited use in a structural model because thermal conditioning procedures were not specified (with the exception of Bradshaw and Brinson [1999]). To appreciate the implications of a 0.1 versus a 0.2 time exponent, consider the two theoretical compliance curves plotted in Fig. 6.2.

![Figure 6.2. Example illustration of the sensitivity of compliance to the time exponent.](image)

In Fig 6.2, the 100,000-min (69-day) creep compliance for $n=0.2$ is 30% higher than the $n=0.1$ compliance. Sullivan [1990]—while not providing functional fits to any master curves—reported that the momentary creep compliances ($S_{22}$ and $S_{66}$) of a vinyl-ester composite (completely post-cured prior to testing) decrease by 20% after 860 hours of aging at $T_g-15^\circ C$. Bradley et al. [1998] report Findlay power law exponents in fitting
the creep compliance data of glass knit-reinforced vinyl ester with different degrees of

cure. Room-temperature (RT) creep tests performed on the specimens resulted in $n=0.20$.
After a 4-hour postcure at 93°C ($T_g-25^\circ C$), $n$ decreased to 0.13. While Bradley et al.
attributed this decrease solely to the advancement of cure (determined with differential
scanning calorimetry to be at 99% after the 4-hour 93°C postcure and less than 90% for
the RT test), this author believes that concurrent aging effects also contributed to the
decrease in time exponent.

6.1.2.2 Temperature Shift Factor

The horizontally-shifted compliance data measured during the temperature-shift
experiment (Section 5.1.5.3) are shown in Fig. 6.3. A plot of the log of $\alpha_T$ (temperature-
time shift factor) versus departure from reference temperature is shown in Fig. 6.4. The
data in Fig. 6.3 has been horizontally shifted to the reference temperature of 21.1°C
and, hence, differs from the data plotted in Fig. 6.1—which was shifted to the reference
temperature of 30°C.

Figure 6.3. $S_{22}(t)$ data from temperature-shift experiment, shifted to 21.1°C.
The temperature shift data in Fig. 6.4 was fit with an exponential function, $\alpha_T = 10^{0.18(T-30^\circ C)}$, and this function was used in the viscoelastic rotor model. Note that the temperature range investigated for this measurement of $\alpha_T$ is limited to a maximum of 50°C, which corresponds to approximately $T_g - 20^\circ C$. In this investigated temperature range, it can be seen that the shift data are well fit by the exponential expression. For comparison, Bradshaw and Brinson [1999] fit an exponential function of the form $\alpha_T = 10^{0.1071(T-225^\circ C)}$ to temperature shift data of the transverse compliance of an IM7/K3B composite.

The four square symbols shown in Fig. 6.4 indicate the temperature shifts used to align the 40°C and 50°C data (relative to the 30°C data) from both master curves in Fig. 6.1. As can be seen, the shift factors used in constructing the master curves match the data from the shift factor experiment quite well, which demonstrates repeatability.
6.1.2.3 Isothermal Aging Behavior of $S_{22}$

The momentary compliance curves measured during the isothermal aging tests are given in Fig. 6.5 (procedures are described in Section 5.1.5.4). For each of the three test temperatures, the maximum aging time was chosen as the reference aging time, $t_{ref}$ (see Eq. 2.15).

Figure 6.5. Momentary creep curves during isothermal aging tests of $S_{22}$ of the E-glass/Dow 383 composite.

Figure 6.6 shows the aging shift factors, $a_{te}$, that were used at each of the three temperatures in order to shift the “younger” momentary curves to align with the “oldest” reference curve of each temperature. The aging shift rate, $\mu$, from Eq. 2.16 is calculated as the (negative) slope of the best-fit line to the data plotted in Fig. 6.6.

Considered in terms of temperature relative to $T_g$, the temperature-dependent aging shift behavior reported here on the E-glass/Dow 383 material system compares well with the behavior reported by other researchers for similar material systems. For example, at $T_g-25^\circ C$, Bradshaw and Brinson [1999] measured an aging shift rate of 0.82 for $S_{22}$ of an IM7/K3B lamina. While aging-time shifts are neglected in the present rotor model, the characterization is presented here to provide a basis for future viscoelastic rotor models that may include aging effects.
6.1.2.4 Cross-term Compliance, $S_{12}$

The cross-term compliance, $S_{12}$, was measured during the first round of creep tests (described in Section 5.1.5.1) and the results of these tests are shown in Fig. 6.7. The results of the three tests at 30°C support the modeling simplification that $S_{12}$ is time independent. This finding is consistent with the findings of other researchers, as mentioned in Section 2.2.2. The apparent time-dependence observed during the 50°-60% test is inconsistent with the other tests, but may be due to damage in the specimen. This specimen ruptured after 18 minutes.
6.1.2.5 Stress Magnitude

Recall that the creep compliance of an ideal linear viscoelastic material does not exhibit any stress dependency. The results of the experiment for investigating the effect of stress magnitude on the measured compliance $S_{22}$ are shown in Fig. 6.8. It was found that $S_{22}$ exhibited only a 3% change in magnitude over an applied stress range of 7.79 MPa to 23.60 MPa. This result supports the validity of approximating the material constitutive law as a linear relationship when the maximum stress is less than or equal to 51% of room-temperature $\sigma^\text{ult}_2$. 
Figure 6.8. Effect of stress magnitude on $S_{22}(t)$ of E-glass/Dow 383 composite for test temperature of 30°C.

6.1.2.6 Critical Stored Energy, $W_c$, of Ruptured Specimens

Several of the creep specimens tested during the first round of testing ruptured, enabling a limited evaluation of a creep rupture criterion. The data analyzed in this section are considered preliminary and are not suitable for design purposes. The transverse strains, $\varepsilon_2$, versus time for the specimens of interest, along with several quasi-statically tested specimens, are plotted in Fig. 6.9.

Failure occurred in the gage section in all quasi-static and creep tests. To calculate the critical stored energy, $W_c$, (see Eq. 2.19) at the time of rupture in each test, Findlay power laws were fit to each of the curves. Using Eq. 2.19, $W_c$ was then calculated directly from these power law fit parameters. The power law fit parameters and the associated $W_c$ for each of the creep tests shown in Fig. 6.9 are summarized in Table 6.2. Also calculated for each of the creep tests and shown as the last column in
Figure 6.9. Strain versus time for E-glass/Dow 383 composite specimens that ruptured during testing.

Table 6.2 is the strain energy at failure,

$$W_{se} = \frac{1}{2} \sigma_f \varepsilon_f,$$

which is provided for comparison with the critical stored energy, $W_c$. The strain energy at failure allows the static test results (from 6.1.1) to be compared with the creep test results.

Table 6.2. Power law parameters and critical stored energy for ruptured specimens.

<table>
<thead>
<tr>
<th>Test</th>
<th>$S_0$ (GPa$^{-1}$)</th>
<th>$S_1$ (GPa$^{-1}$)</th>
<th>$n$</th>
<th>$\sigma_f$ (MPa)</th>
<th>$t_f$ (min)</th>
<th>$W_c$ (kPa)</th>
<th>$W_{se}$ (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40°_60%</td>
<td>0.0563</td>
<td>0.00764</td>
<td>0.197</td>
<td>28.02</td>
<td>2805</td>
<td>34.3</td>
<td>35.7</td>
</tr>
<tr>
<td>50°_60%</td>
<td>0.0649</td>
<td>0.01212</td>
<td>0.271</td>
<td>28.02</td>
<td>15.0</td>
<td>33.3</td>
<td>34.6</td>
</tr>
<tr>
<td>50°_60%, #2</td>
<td>0.0552</td>
<td>0.00938</td>
<td>0.244</td>
<td>28.02</td>
<td>18.0</td>
<td>27.7</td>
<td>28.5</td>
</tr>
<tr>
<td>54°_50%</td>
<td>0.0474</td>
<td>0.00537</td>
<td>0.142</td>
<td>23.35</td>
<td>259</td>
<td>15.8</td>
<td>15.9</td>
</tr>
<tr>
<td>Quasi-static #1, 23°C</td>
<td>0.0551</td>
<td>0</td>
<td>0</td>
<td>46.66</td>
<td>2</td>
<td>60.0</td>
<td>65.5</td>
</tr>
<tr>
<td>Quasi-static #2, 23°C</td>
<td>0.0547</td>
<td>0</td>
<td>0</td>
<td>46.70</td>
<td>2</td>
<td>59.6</td>
<td>65.5</td>
</tr>
<tr>
<td>Quasi-static #3, 23°C</td>
<td>0.0555</td>
<td>0</td>
<td>0</td>
<td>43.51</td>
<td>2</td>
<td>52.5</td>
<td>59.9</td>
</tr>
</tbody>
</table>
The power law parameters listed in Table 6.2 were fit directly to the data measured during these rupture tests only (i.e. these do not correspond to master curve parameters). These results show that the E-glass/Dow 383 material system exhibits a critical strain energy in the 30-35 kPa range under creep conditions (for the temperature and stress levels investigated). Under room-temperature quasi-static conditions, this material system exhibits a critical strain energy of 60-65 kPa.

Using creep rupture experiments on random-orientation chopped glass fiber composite specimens, Yen and Morris [1989] experimentally determined that $W_c$ has an inverse dependence on temperature. Lacking a thorough characterization of this temperature-dependence for the present material system, a comparison of only the 40°_60% test with the two 50°_60% tests can be made with regard to temperature. Indeed, the values of $W_c$ and $W_{se}$ (34.3 and 35.7 kPa, respectively) calculated for the 40°_60% test are higher than the corresponding average values of $W_c$ and $W_{se}$ for the two 50°C tests (30.5 and 31.6 kPa, respectively).

It is interesting to note that the 54°_50% test exhibited a significantly lower value for $W_c$ than the other three creep tests. This specimen was the one that was used in the creep tests for determining the master curves and the temperature and aging shift behavior and, hence, had been subject to several dozen creep/recovery cycles. While the fact is that the strains during the testing of this specimen were recoverable (as reported in Section 5.1.5.2) as indicated by the strain gage instrumentation, the possibility of unnoticed damage accumulation exists.

Another major difference between the specimens is that the 54°_50% specimen had been aged for over 1000 hours at $T_g-15^\circ$C. The decreased compliance caused by aging is also accompanied by an increased brittleness (i.e. reduced failure strain)—a universal finding for un-reinforced polymeric materials qualitatively reported by Struik [1978]. Interestingly, Struik also reported that aging “PVC samples” at 50°C for 250 hours increased the yield stress by 10%. The competing effects of aging-induced increased brittleness and yield stress on the strain energy was not addressed by Struik, nor by other researchers (e.g., Sullivan, Bradshaw and Brinson) studying the effects of aging on PMCs.
6.1.2.7 Strain Response Due to Combined Variable Thermal and Stress Loads

The results from the test of the strain response due to combined variable thermal and stress loads on the tube specimen (see Section 5.1.5.7) are given in Fig. 6.10. The applied stress and temperature history is also shown in Fig. 6.10. Given this applied stress and temperature history, axial strain at times $t_i$ were calculated using a time-discretized form of Eq. 2.1,

$$\varepsilon_i = \sum_j S(\xi_i - \xi_j)\Delta\sigma_j,$$

(6.3)

where $\xi_i$ and $\xi_j$ are defined in Eqs. 3.13-14.

Fig. 6.10. Plot of the load history and predicted and measured strain for the test of the strain response under combined variable stress and temperature loads.

These axial strains were calculated using the creep compliance formula given in Fig. 6.1 and the quasi-elastic solution procedure described in Section 3.1.3—except that the applied axial stresses (as opposed to the spin and thermal loads used in the rotor model)
admitted the use of Eq. 6.3 directly for predicting strains. The intervals between successive $t_i$ in the load history used for the calculations were as small as 0.01 minutes at times when there was a significant rate of change in either temperature or stress (e.g. $[0<t<216]$ and $[t>934]$ minutes) and as large as 718 minutes in the time interval $[216<t<934]$ minutes. The good correlation between the predicted and measured strains shown in Fig. 6.10 yields confidence that the creep characterization procedures are accurate, and the solution procedure has been properly implemented in the flywheel rotor code.

### 6.2 Isothermal Interference-Fit Experiment

#### 6.2.1 Loss of Interference Pressure

Experimentally measured percent pressure loss, $\Delta p_{\text{expt}}$, and percent pressure loss predicted by the model, $\Delta p_{\text{model}}$, are listed in Table 6.3 for the six ring pairs that were tested as described in Section 5.2.

<table>
<thead>
<tr>
<th>Ring</th>
<th>$\Delta p_{\text{expt}}$ (%)</th>
<th>$\Delta p_{\text{model}}$ (%)</th>
<th>Assembly Duration (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LP-H</td>
<td>-2.9</td>
<td>-4.0</td>
<td>42800</td>
</tr>
<tr>
<td>LP-G</td>
<td>-5.1</td>
<td>-2.3</td>
<td>41500</td>
</tr>
<tr>
<td>LP-C</td>
<td>-4.2</td>
<td>-1.7</td>
<td>37200</td>
</tr>
<tr>
<td>HP-H</td>
<td>-2.7</td>
<td>-3.8</td>
<td>35700</td>
</tr>
<tr>
<td>HP-G</td>
<td>-4.0</td>
<td>-2.2</td>
<td>35100</td>
</tr>
<tr>
<td>HP-C</td>
<td>-4.4</td>
<td>-1.7</td>
<td>35300</td>
</tr>
</tbody>
</table>

The values in the $\Delta p_{\text{expt}}$ column were calculated from the instantaneous changes in hoop strains measured during assembly and disassembly, $\varepsilon_{h,a}$ and $\varepsilon_{h,d}$, respectively, by
In Eq. 6.4 the overbar represents the arithmetic mean of the strains indicated by the three individual gages at either the ID or the OD of the ring (see Fig. 5.5). The pressure loss predicted by the model was calculated by

\[ \Delta p_{\text{model}} = \left( \frac{p_{m,f} - p_{m,i}}{p_{m,i}} \right) 100, \]  

(6.5)

where \( p_{m,i} \) is the initial model pressure at each ring interface, and \( p_{m,f} \) is the final interface pressure calculated by the model. To calculate \( p_{m,i} \) and \( p_{m,f} \) for a given ring pair, the geometry for the assembly (from Table 5.5) was input to the viscoelastic rotor model (from Chapter 3) with a load history given in Table 6.4.

Table 6.4. Load history input to the rotor model for the IIF tests.

<table>
<thead>
<tr>
<th>Time, ( t ) (min)</th>
<th>Temperature, ( T ) (°C)</th>
<th>Speed, ( \omega ) (rad/s)</th>
<th>Boundary conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>80</td>
<td>0</td>
<td>Eq. 3.11</td>
</tr>
<tr>
<td>0.01</td>
<td>23</td>
<td>0</td>
<td>Eq. 3.11</td>
</tr>
<tr>
<td>43200</td>
<td>23</td>
<td>0</td>
<td>Eq. 3.11</td>
</tr>
<tr>
<td>43201</td>
<td>23</td>
<td>0</td>
<td>Eq. 3.17</td>
</tr>
<tr>
<td>43201+Assembly</td>
<td>23</td>
<td>0</td>
<td>Eq. 3.17</td>
</tr>
<tr>
<td>Duration</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Also, for the model predictions, the room-temperature transverse compliances of the gl/ep and c/ep components were, respectively, given by

\[ S_{22}(t) = 0.0484 + 0.0034 t^{0.26}, \]  

(6.6)

\[ S_{22}(t) = 0.09 + 0.0062 t^{0.26}. \]  

(6.7)
The compliances in Eqs. 6.6-6.6 are based on compliances measured with DMA on un-aged monolithic Dow 383 epoxy specimens at 30°C. The compliance values of the neat epoxy specimens were scaled in magnitude such that they matched the elastic compliance values of gl/ep and c/ep laminae at \( t=0.01 \) minutes. Because the epoxy specimens used in the DMA measurements were not aged, the \( \Delta P_{\text{model}} \) in Table 6.3 represent the pressure loss of ring pairs with composites that do not stiffen with (aging) time. The magnitudes of the \( \Delta P_{\text{model}} \) predictions, therefore, represent an upper bound to predicted pressure loss for the ring pairs.

In the case of the hybrid composite ring-pairs, the pressure loss predictions were approximately a factor of 1.4 higher than experimental pressure losses. In the ring pairs with only one composite ring, the predictions were low by a factor of 2 to 3. Little difference existed between the predicted pressure changes in analogous high and low pressure cases. The differences between the model predictions and experimental results are unexplainable considering only these results from the hoop-oriented strain gages. Insight into an explanation for the disparity is, however, gained from inspection of the full-field strains, presented next.

### 6.2.2 Strain Distribution Results

The radial and hoop strain distributions through the radial thickness of the ring pairs were recorded at the time of assembly, and immediately prior to disassembly. Additionally, the residual strain distributions immediately after disassembly were recorded. Figures 6.11-14 show plots of the measured residual radial strain distributions for all composite rings, with the corresponding residual strain distributions predicted by the model. One radially-oriented strain gage was placed on all composite rings, next to the moiré grid and as close as possible to the interface of the mating rings (see Fig. 5.6). According to standard procedure for moiré interferometry, strains from the moiré grids were matched to those of the gages at the locations of the gages shown in Figs. 6.11-14.
Figure 6.11. Experimental and predicted residual creep strains in LP gl/ep rings, IIF tests.

Figure 6.12. Experimental and predicted residual creep strains in HP gl/ep rings, IIF tests.
Inspection of Figs. 6.11 and 6.12 show that the measured residual strain distributions are fairly well predicted by the model away from the OD of the inner composite ring. Similarly, the residual strain distributions shown in Figs. 6.13 and 6.14 are fairly well predicted by the model away from the ID of the outer composite ring. The
locations where the measured strains deviate from the predictions are all near the interface with the opposing ring, and the strains deviate in a positive direction at these locations.

To investigate the strains in a press-fitted ring pair, a two-dimensional axisymmetric elastic finite-element analysis (FEA) of the LP-H ring-pair (ring geometry given in Table 5.5) was performed using ANSYS Version 6.1. An illustration of the mesh is shown in Fig. 6.15. Elements of the type PLANE42 with the axisymmetric option were used to model the gl/ep and c/ep rings, and CONTAC12 elements were used to connect the interface between the rings with an interference fit. Note that the $x$ and $y$ directions defined for this FEA correspond to the $r$ and $z$ polar coordinate directions, respectively, used everywhere else in this research. The only boundary condition imposed on the mesh was a zero value for $y$-direction displacement of the innermost, lowest node in the gl/ep ring, indicated by the pin and roller symbol in Fig. 6.15. Material properties used for these rings are given in Table 6.5, and the real constant set for the CONTAC12 elements is given in Table 6.6.

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Fig. 6.15. Illustration of the mesh used for the FEA press-fit investigation.

---

1 ANSYS Inc., Canonsburg, PA.
Table 6.5. Material properties used in the FEA analysis of the press-fit.

<table>
<thead>
<tr>
<th>Property</th>
<th>gl/ep</th>
<th>c/ep</th>
<th>Contact Elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_x$ (GPa)</td>
<td>20.1</td>
<td>10.8</td>
<td>-</td>
</tr>
<tr>
<td>$E_y$ (GPa)</td>
<td>20.1</td>
<td>10.8</td>
<td>-</td>
</tr>
<tr>
<td>$E_z$ (GPa)</td>
<td>52.1</td>
<td>152.9</td>
<td>-</td>
</tr>
<tr>
<td>$v_{xy}$</td>
<td>0.35</td>
<td>0.35</td>
<td>-</td>
</tr>
<tr>
<td>$v_{yz}$</td>
<td>0.13</td>
<td>0.024</td>
<td>-</td>
</tr>
<tr>
<td>$v_{xz}$</td>
<td>0.13</td>
<td>0.024</td>
<td>-</td>
</tr>
<tr>
<td>$G_{xy}$ (GPa)</td>
<td>7.4</td>
<td>4</td>
<td>-</td>
</tr>
<tr>
<td>$G_{yz}$ (GPa)</td>
<td>10</td>
<td>15</td>
<td>-</td>
</tr>
<tr>
<td>$G_{xz}$ (GPa)</td>
<td>10</td>
<td>15</td>
<td>-</td>
</tr>
<tr>
<td>$\rho$ (kg m⁻³)</td>
<td>2138</td>
<td>1580</td>
<td>-</td>
</tr>
<tr>
<td>$\mu^*$</td>
<td>-</td>
<td>-</td>
<td>0.2</td>
</tr>
</tbody>
</table>

*The symbol $\mu$ is defined here as the coefficient of friction for the CONTAC12 elements.

Table 6.6. Real constant set used with the CONTAC12 elements.

<table>
<thead>
<tr>
<th>Orientation angle angle</th>
<th>Normal Stiffness $K_N$ (N/mm)</th>
<th>Interference INTF (mm)</th>
<th>Gap Status START</th>
<th>Sticking Stiffness $K_S$ (N/mm)</th>
<th>Reduction Factor REDFACT</th>
</tr>
</thead>
<tbody>
<tr>
<td>THETA (deg)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-88</td>
<td>$2.1 \times 10^9$</td>
<td>0.25</td>
<td>1</td>
<td>$1 \times 10^8$</td>
<td>0</td>
</tr>
</tbody>
</table>

Precise descriptions of the real constants for the CONTAC12 element are found in the ANSYS Elements Reference Manual. The values for $K_N$ and $K_S$ were determined by trial and error such that the two nodes defining each of the contact elements exhibited relative $x$-direction displacement equal to the interference (0.25 mm), and identical $y$-direction displacements after the solution was done (i.e. mimicking an interference-fitted ring pair without a “lip” on the axial face).

The results of the FEA show that the elastic radial strain fields on the top and bottom axial surfaces of the gl/ep ring (indicated in the inset in Fig. 6.16 by “top plane” and “bottom plane”, respectively) deviate significantly in the positive direction relative to
the strain fields elastically predicted by FEA at the "mid-plane". The FEA mid-plane radial strain field is closer to matching the strain field predicted by the closed-form plane stress elastic solution, but also exhibits an exaggerated positive concavity relative to the closed form strain. In the c/ep ring of the LP-H ring pair, the FEA shows that the elastic radial strain fields on the top and bottom axial surfaces of the c/ep ring (indicated in the inset in Fig. 6.17 by "top plane" and "bottom plane", respectively) deviate significantly in the positive direction relative to the strains predicted by FEA at the "mid-plane".

The viscoelastic model predictions show that the plotted shape of the time dependent strain redistribution “follows” the shape of the elastic strains (even in a biaxial strain field) in the sense that a monotonic elastic strain field will remain monotonic after creep, and an elastic strain field with local extremes will retain the extremes in approximately the same location after creep. This phenomenon is briefly mentioned here and will be illustrated in Chapter 7. It is therefore reasonable to expect that the experimental residual viscoelastic radial strains measured with a moiré grid on the top-plane of the rings (Figs. 6.11-14) would assume a shape similar to that of the corresponding elastic strains (Figs. 6.16-17) over the same range of radius. Because the closed-form elastic solution is the basis of the viscoelastic model, the FEA elastic radial strain predictions offer the most likely explanation for why the shape of the predicted radial creep strains deviates from the shape of the measured radial strains near the interface.

6.2.3 Isothermal Interference-Fit Experiment Conclusions

One useful and important result was gleaned from these experiments, which is that creep effects caused only minor degradation of the compressive pre-stress that was imparted to an interference-fitted pair of hoop-wound composite rings. The understanding of the effects of material aging as well as the understanding of the non-planar strain state at the interface of the rings—unfortunately gained after completion of the experiments—begged for a repetition of the experiment with a
Figure 6.16. Comparison of mid-plane and top-plane elastic FEA predictions with closed-form elastic model predictions in LP-H gl/ep ring.

Figure 6.17. Comparison of mid-plane and top-plane elastic FEA predictions with closed-form elastic model predictions in LP-H c/ep ring.

controlled age condition of the composites and more careful placement of instrumentation (specifically, placement of the radial strain gages away from the ring interfaces). This second set of interference-fit experiments is described next.
6.3 Non-Isothermal Interference-Fit Experiment

6.3.1 Thermal Conditioning

Figure 6.18 shows the strains versus time measured during the 50°C thermal conditioning of the disassembled LP-G ring pair prior to imposing interference-fit loads. At $t=0$ min., the rings were in thermal equilibrium at the ambient laboratory temperature of 23°C. At $t=1$ min., the rings were placed in a mechanical convection oven that was preheated at 50°C. The transient time for heat-up is about 60 minutes. The rings were removed from the oven at $t=92$ min., and were allowed to cool to room temperature. The cool-down transient time is about 100 minutes. While not shown, the transient times for the LP-H ring pair were similar to those for the LP-G ring pair. The strains return to ±10 με of zero after the temperature returned to 23°C at $t=200$ minutes. This indicated that strains due to prior loads had been effectively erased (see Section 5.3.3.1) and, thus, strains observed during subsequent testing would be due only to subsequent loads.

![Figure 6.18. Strains on the disassembled LP-G ring pair during thermal conditioning cycle, NIF tests.](image-url)
The NIF experiments were designed such that strains could be measured immediately after assembling the rings (at ambient laboratory temperature) and again after cooling to room temperature following the temperature soaks. Thus, the experimental setup did not include the use of dummy specimens (standard for compensating apparent strains in composites due to non-isothermal conditions). While performing the experiments, however, it was found that the ambient laboratory temperature varied over time and at various locations in the laboratory by about one degree Celsius. Specifically, in both the LP-G and LP-H tests, the final temperature reached upon cooling following the high-temperature soak was one degree Celsius cooler than the temperature at which the gages were zeroed immediately prior to the assembly. This temperature difference was included in the model strain predictions, and required experimentally measured apparent strain values to be adjusted by –23 and –11 με for the radial and hoop-gages, respectively, which were respectively self-temperature compensated for aluminum and steel.

**6.3.2 Loss of Interference Pressure**

The predicted and measured elastic hoop strains upon assembly of the LP-G and LP-H ring pairs are listed in Table 6.7 in the Assembly Strains column. Each “Experimental $\bar{\varepsilon}_{h,a}$” represents the mean value of three hoop gages (equally-spaced in the circumferential sense either at the ID of the inner ring, or the OD of the outer ring). The small coefficients of variation associated with these mean values indicate that the ring pairs were assembled with a high degree of parallelism and coaxiality of the ring components. The change in these hoop strains after cooling to room temperature (following the 50°C “soak”), $\Delta \bar{\varepsilon}_{h,s}$, are also listed in Table 6.7. The coefficients of variation associated with these “after temperature soak” values represent standard deviations in the $\Delta \bar{\varepsilon}_{h,s}$ data of less than 10 με.

Experimentally measured pressure loss, $\Delta p_{\text{expt}}$ (Eq. 6.4), and pressure loss predicted by the model, $\Delta p_{\text{model}}$ (Eq. 6.5), are listed in Table 6.7 for the LP-H and LP-G
ring pairs that were tested as described in Section 5.3. As mentioned in Section 5.3.1, the composite rings had been aged and, hence, the material properties assumed in making the model predictions are those listed for “Aged E-glass/Dow 383” and “Aged T700/Dow 383” in Appendix D. The values in the experimental pressure change column were calculated from the elastic instantaneous ring strains measured immediately after assembly and disassembly of the ring-pairs.

Table 6.7. Predicted and experimental hoop strains and change in pressure in the NIF rings.

<table>
<thead>
<tr>
<th>Assembly strains</th>
<th>Change in strain after temperature soak</th>
<th>Pressure change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ring (location)</td>
<td>Model $\varepsilon_{h,a}$ (µε)</td>
<td>Experimental $\varepsilon_{h,a}$ / CV% (µε) / %</td>
</tr>
<tr>
<td>LP-G gl/ep ($r_l$) al ($r_o$)</td>
<td>-2015</td>
<td>-2017 / 2.2</td>
</tr>
<tr>
<td>772</td>
<td>763 / 1.8</td>
<td>-6</td>
</tr>
<tr>
<td>LP-H gl/ep ($r_l$) c/ep ($r_o$)</td>
<td>-1305</td>
<td>-1271 / 1.6</td>
</tr>
<tr>
<td>700</td>
<td>735 / 1.2</td>
<td>-15</td>
</tr>
</tbody>
</table>

The load histories used to make the model predictions for the LP-G and LP-H tests consist only of temperature loads (including cooldown from manufacturing) and imposition of the interference-fit boundary conditions at the time of the assembly. These histories are listed in Tables 6.8 and 6.9. Notably absent from the load histories of the rings are the loads from the IIF experiments, which are omitted for modeling simplicity. This is a fair simplification because, as mentioned in Section 6.3.1, the creep strains due to these loads were effectively erased by the thermal conditioning.
Table 6.8. Load history used in the simulation of the LP-G test, NIF tests.

<table>
<thead>
<tr>
<th>Notes</th>
<th>( t ) (min.)</th>
<th>( \Delta t ) (min.)</th>
<th>( T ) (°C)</th>
<th>( \Delta T ) (°C)</th>
<th>B.C. Eq.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manufacturing cooldown</td>
<td>0</td>
<td>80</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Four years since IIF tests</td>
<td>0.01</td>
<td>23</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Thermal conditioning</td>
<td>2,000,002</td>
<td>53</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Cooldown ramp</td>
<td>2,002,890</td>
<td>10</td>
<td>53( \rightarrow ) 23</td>
<td>-2</td>
<td>3.16</td>
</tr>
<tr>
<td>Dwell time</td>
<td>2,011,112</td>
<td>23</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Perform press-fit</td>
<td>2,011,113</td>
<td>23</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Heating ramp</td>
<td>2,011,121( \rightarrow )</td>
<td>1</td>
<td>23.5( \rightarrow ) 50</td>
<td>27/60</td>
<td>3.17</td>
</tr>
<tr>
<td>High-temperature soak</td>
<td>2,011,325</td>
<td>50</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Cooldown ramp</td>
<td>2,011,326( \rightarrow )</td>
<td>1</td>
<td>49.5( \rightarrow ) 22</td>
<td>-28/55</td>
<td></td>
</tr>
</tbody>
</table>

Table 6.9. Load history used in the simulation of the LP-H test, NIF tests.

<table>
<thead>
<tr>
<th>Notes</th>
<th>( t ) (min.)</th>
<th>( \Delta t ) (min.)</th>
<th>( T ) (°C)</th>
<th>( \Delta T ) (°C)</th>
<th>B.C. Eq.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manufacturing cooldown</td>
<td>0</td>
<td>80</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Four years since IIF tests</td>
<td>0.01</td>
<td>23</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Thermal conditioning</td>
<td>2,000,002</td>
<td>53</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Cooldown ramp</td>
<td>2,002,890( \rightarrow )</td>
<td>10</td>
<td>53( \rightarrow ) 23</td>
<td>-2</td>
<td>3.16</td>
</tr>
<tr>
<td>Dwell time</td>
<td>2,012,555</td>
<td>23</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Perform press-fit</td>
<td>2,012,560</td>
<td>23</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Heating ramp</td>
<td>2,012,561( \rightarrow )</td>
<td>1</td>
<td>23.7( \rightarrow ) 50</td>
<td>27/40</td>
<td>3.17</td>
</tr>
<tr>
<td>High-temperature soak</td>
<td>2,012,727</td>
<td>50</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Cooldown ramp</td>
<td>2,012,728( \rightarrow )</td>
<td>1</td>
<td>49.5( \rightarrow ) 22</td>
<td>-28/55</td>
<td></td>
</tr>
</tbody>
</table>

6.3.3. Strain Distribution Results

Figure 6.19 shows the predicted radial (\( \varepsilon_{r,a} \)) and hoop strains (\( \varepsilon_{h,a} \)) immediately after assembly (at ambient laboratory temperature) as well as after cooldown (\( \varepsilon_{r,s} \) and \( \varepsilon_{h,s} \)) following the two-hour, 50°C soak. Considering the good correlation between the experimental and theoretical elastic hoop strains shown at the ring boundaries in Fig.
6.19, it appears that the plane-stress equations accurately predicted the midplane hoop strains (hoop gages were placed at the midplane). Hoop strains at the inner radius of the gl/ep ring immediately after assembly were used as target strains during the pressing, and the corresponding hoop strains at the outer radius of the aluminum ring were within 2% of the predictions.

The initial experimental elastic radial strains in the gl/ep rings at the \( r=66 \) mm and \( r=70 \) mm locations in Fig. 6.19 show good correlation with the plane stress predictions away from the interface, but deviate significantly from the predictions at the 78-mm location. This deviation is consistent in character (positive-direction) with that observed in the isothermal experiments (Fig. 6.11).

![Strain graph](image)

Figure 6.19. Strains in the LP-G ring pair immediately after assembly and after cooldown following two-hour exposure to 50°C, NIF tests.

To better illustrate the correlation between the predicted and measured viscoelastic behavior during this experiment, the difference between the strains immediately after assembly and the strains after cooling from the soak are plotted in Fig. 6.20. Figure 6.20 shows that, following the soak, the experimental differences in hoop strains at the ID of the glass and at the OD of the aluminum were +16 \( \mu \varepsilon \) and −6 \( \mu \varepsilon \).
relative to the corresponding predicted values at those respective locations. These small strain differences are close to the accuracy of the instrumentation (±10 με). In the gl/ep ring, the measured average change in radial strain at \( r=66 \) mm was 30 με higher than predicted. At \( r=70 \) mm, the measured average change in radial strain was 3 με higher than predicted. The experimental radial strains at \( r=78 \) mm were 75 με too high. The disagreement between the plane stress viscoelastic prediction and the measured strains at this location is not surprising, however, considering the strong evidence for the existence of significant three-dimensional stresses at this location.

![Graph showing strain differences](image)

Figure 6.20. Difference in the room-temperature strains before and after the 50°C soak for the assembled LP-G ring pair, NIF tests.

The experimental radial and hoop strains in the assembled LP-H ring pair (Fig. 6.21) were lower in magnitude compared to the LP-G assembly (Fig. 5.17) due to a smaller amount of interference achieved during assembly. This LP-H assembly did not have adjustable interference. As with the LP-G assembly, all elastic hoop strains in the LP-H assembly and the radial strains in the gl/ep ring of the LP-H assembly showed good correlation with the plane-stress model predictions. The significant difference between
experimental and predicted radial strains in the c/ep ring in the LP-H remains unexplained.

![Strain graph](image)

Figure 6.21. Strains in the LP-H ring pair immediately after assembly and after cooldown following two-hour exposure to 50°C, NIF tests.

The viscoelastic redistribution of strain in the LP-G experiment is more clearly illustrated in a plot of the difference in strain before and after the soak, which is shown in Fig. 6.22. The distributions of the difference in radial strain before and after the soak in both experiments show that the experimentally-observed changes in radial strain were everywhere smaller in magnitude than the predictions. In the gl/ep ring, the measured average change in radial strain at \( r=66 \text{ mm} \) was 28 \( \mu \varepsilon \) higher than predicted, and at \( r=70 \text{ mm} \), was 35 \( \mu \varepsilon \) higher than predicted. In the c/ep ring, the average change in radial strain at \( r=90 \text{ mm} \) was 28 \( \mu \varepsilon \) higher than predicted, and at \( r=87.7 \text{ mm} \) was 95 \( \mu \varepsilon \) higher than predicted.
One explanation for the small differences between experimental and predicted strains may be that the radial stresses were compressive in all cases, whereas the constitutive law used to represent the composite rings came from tensile creep tests. As discussed in Section 2.3.5, it is possible for composite laminae to exhibit different time-dependent responses under compressive and tensile loads.

6.3.4. Non-Isothermal Interference-Fit Experiment Conclusions

In conclusion, the NIF tests showed that the loss of pressure due to interference-fit loads on aged specimens is relatively small, no more than 2.5%, approximately half the pressure loss observed in the IIF experiments. While the un-aged specimens in the IIF tests were assembled for a longer period of time (~40,000 min) compared to the NIF tests (~120 minutes), the fact that the NIF tests were soaked at 50°C translates to an equivalent loading time of 13.6 years at room temperature. This result indicates the effectiveness of aging PMC composites in order to reduce the time-dependent behavior.
The deviation from plane-stress predictions near the ring interface was again observed in these experiments. These deviations show that the press-fitted ring pair is not an ideal specimen for studying the biaxial creep behavior of composites because the strains are non-planar at the free surfaces near the interface of the rings. This result also means that out-of-plane stress effects at the interface of neighboring components may be significant and should be considered in the final analysis of such a flywheel design.

The viscoelastic strains accrued in all of the composite rings during the LP-G and LP-H tests were recoverable to within ±5 με of zero after subjecting the disassembled rings to 50°C for a 12-hour period. This result suggests that, in the stress and temperature range investigated, the materials are described well by a viscoelastic constitutive law and do not require the consideration of plastic deformations.

### 6.4 OESM System Validation Results

#### 6.4.1 Pattern Generation

Calibration was performed with a single optical sensor over a 0.6-mm range in the 150-160 mm annular region. With the rotor spinning at a sufficiently low speed to avoid measurable flexible body displacement (580 rpm), duty cycles of the TB patches were recorded at 10-15 μm radial intervals (Fig. 6.23). The TB patch data in Fig. 6.23 exhibit small local deviations from the theoretical values. These deviations are attributed to roughness of the rotor surface or pattern edge. The 0.3 to 0.5 degree difference in duty cycle between the four TB patches is most likely due to small misalignments of the lobes on the rotor.

#### 6.4.2 Sensor Effects

At the beginning and end of the 20-hr drift experiment, duty cycles at all locations differed by less than 0.005 deg. and compensation patch angles differed by less than
Figure 6.23. Calibration data on the low-speed rotor in the 150-160 mm annular region.

0.001 deg. In terms of radial position, these small variations correspond to less than 1 µm of drift. Over the 20-hour duration of the drift experiment, the sensor transmitted >5 million logic pulses to the data acquisition computer.

The intensity compensation algorithm was tested as described in Section 5.5.4.1. Figure 6.24 shows that the apparent radial positions of the “high intensity” data exceed the “correct” positions by about +6 µm and are correctable to within ±1.5 µm of the “correct” values over the entire range interrogated. Such corrected radial position accuracy ensures a hoop strain accuracy of ±10 µε at this radial location on the rotor. This amount of correction represents a change in light intensity slightly higher than values experimentally observed due to drift during longer duration tests. It seems reasonable, therefore, to expect that this compensation technique is appropriate for use in long term spin tests.

Experience has shown that while the radial position correction is quite insensitive to the value chosen for the actual compensation patch angle, $\theta_c$, it is sensitive to $|\theta_a - \theta_c|$. When $|\theta_a - \theta_c|$ is more than about 0.004 deg., errors in the correction exceed the targeted
<table>
<thead>
<tr>
<th>Radial Position, $r$ (mm)</th>
<th>Duty Cycle, $\phi$ (deg.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Correct Intensity</td>
</tr>
<tr>
<td></td>
<td>High Intensity</td>
</tr>
<tr>
<td></td>
<td>Corrected</td>
</tr>
</tbody>
</table>

Figure 6.24. Results of the sensor intensity-compensation experiment.

minimum displacement resolution, $u_{\text{min}}$. It is suspected that this error is caused by local pattern edge roughness, which invalidates some of the geometrical assumptions made in the derivation of Eqs. 4.11-12. Such a limitation on $|\theta_s - \theta_c|$ is easily handled in practice since the changes in LED intensity occur gradually. Incident intensity can be continually monitored and adjusted in order to yield an acceptably low value of $|\theta_s - \theta_c|$ for any one correction.

6.4.3 Rigid Body Displacements

The results of the in-plane rigid body displacement experiment described in Section 5.5.4.2 are given below. Displacement amplitude and phase versus speed are shown in Figs. 6.25 and 6.26, respectively.

It can be seen that agreement between the optical and inductive sensors is quite good, i.e. the data exhibit peak amplitudes and phase change at the same speed. Note that data acquisition hardware limitations prevented simultaneous optical and inductive position measurements of displacement and phase. Specifically, the optical measurements are performed with the optical sensor and tachometer sensor outputs connected to the gates of the two high-speed counter timer circuits. This optical setup
encodes rotor displacement information in the width of the sensor output pulse, and is operated with a LabView Virtual Instrument (VI) that controls the counter circuits. The counters are digital in the sense that they are either on or off (counting or not counting—depending on whether the TTL gate voltage is high or low). The inductive position sensors each provide displacement information directly in the voltage of an analog output signal, which is recorded with a different VI. It was found that simultaneously connecting the tachometer output to a (digital) counter gate and an analog
input produced an invalid signal in both of the above-mentioned VIs. Therefore, the “optical” and “inductive” rigid-body displacement measurements in Figs. 6.25-26 were made during separate runs, and small disagreements in the plots (especially at the peak) are expected.

The results of the experiment for testing the sensitivity of the OESM system to out-of-plane displacements, as well as the ability to compensate for such displacements, are now given. Figure 6.27 shows the apparent and corrected radial displacements indicated by the sensor versus the axial position of the sensor relative to the optimal position.

![Figure 6.27. Apparent and corrected radial displacement versus axial position of the sensor.](image)

Over this ±50-μm displacement of the sensor in the axial direction, the apparent radial displacement of the sensor varied by ±2 μm but was correctable to within ±1 μm of the correct value of zero. The corrected radial displacements for all four lobes over this same axial position range had sufficient accuracy for a ±10 με strain accuracy at the 151.6-mm measurement position. For changes of axial position larger than about ±50 μm, the LED intensity compensation equations failed to provide proper corrections due to the image of the spot moving out of the active area of the sensor photodetector.
The next-generation optical sensor, on which some work has already been performed [Bakis et al., 2002], incorporates an illumination source with an optical axis normal to the rotor surface, instead of the ~10-degree angle of incidence as shown in Fig. 4.5. As well, the next-generation sensor has a dramatically increased operational stand-off distance (enabled by the use of a laser). Both of these changes are designed to reduce sensitivity to out-of-plane displacements.

6.4.4 Flexible Body Displacements

The results of the high-speed spin test of the aluminum rotor are now given. The high-speed setup is described in Section 5.5.2.2, and the experimental procedures are given in Section 5.5.5. The vacuum level in the spin pit was maintained in the 0.05 to 0.1-torr range during this experiment. The temperature increase of the rotor, measured by three IR thermocouples, was found to be uniform within ±1°C at all times during the test. The speed and temperature data versus the elapsed time of the experiment are given in Fig. 6.28.

![Figure 6.28. Rotor temperature and speed versus time in the high-speed test of the aluminum rotor.](image-url)
Since the low thermal gradient in the isotropic rotor implies very low thermal stresses, the total (apparent) displacement measurements were corrected for thermal expansion according to the following formula:

\[ u = u_{\text{tot}} - r_{\text{ref}} \alpha (\Delta T) - u_{\text{rack}}, \]  

(6.8)

where \( u \) is the corrected radial displacement, \( u_{\text{tot}} \) is the total radial displacement including thermal effects, \( r_{\text{ref}} \) is the reference radial location where the measurement was made, \( \alpha \) is the linear coefficient of thermal expansion of aluminum (Table D.1), and \( \Delta T \) is the change in temperature relative to the initial temperature. The \( u_{\text{rack}} \) term corrects for radial displacement of the optical sensor rack due to temperature changes in the rack support structure. For an isotropic annulus with inner radius \( r_i \) and outer radius \( r_o \), the theoretical plane stress radial displacement under only spinning loads can be calculated by [Timoshenko, 1934],

\[ u_{\text{theory}} = \frac{\rho \omega^2}{8E} \left[ (1 - \nu)(3 + \nu)(r_i^2 + r_o^2)r + \frac{(1 + \nu)(3 + \nu)r_i^2r_o^2}{r} - (1 - \nu^2)r^3 \right]. \]  

(6.9)

Figure 6.29 is a plot of the hoop strains \( \varepsilon_r = u/r \) measured during the test. The A2 and B2 sensor readings were averaged to obtain displacements at a radius \( r = 70.7 \) mm and the A10 and B10 sensor readings were averaged for the displacements at a radius of \( r = 152.2 \) mm. The A9 and B9 sensor strain readings were omitted from this plot for clarity, as these readings were virtually indistinguishable (experimentally and theoretically) from the A10 and B10 readings. A small amount of damage to the reflective pattern near the inner radius of the rotor precluded the A1 and B1 sensors from use in this test. Calculated experimental displacements (shown as strains in Fig. 6.29) were corrected for rotor expansion and rack displacement due to thermal effects (Eq. 6.8). The average rigid-body vibration (henceforth simply referred to as “vibration”) amplitude and phase indicated by the sensors at each of the hold speeds is indicated in brackets in
Fig. 6.29. The flexible body displacement and hoop strain data from this spin test, including the data from the A9 and B9 sensors, are summarized in Table 6.10.

![Graph showing hoop strain vs speed](image)

Figure 6.29. Experimental and theoretical hoop strain versus speed at two radial locations on the aluminum rotor during the high-speed spin test.

Each data point shown in Fig. 6.29 is an average of three or more successive measurements made while the speed was held constant (each measurement itself an average computed as described in the procedure). The scatter within any set of measurements was less than ±1 µm. The measured displacements exceeded theoretical values by ~0-10 µm at all radii and all speeds, indicating that errors are likely independent of rotor speed and radial location. At the maximum speed of 16 krpm, the experimental strains at the 70.7- and 152.2-mm positions were 6.4% and 5.0% greater than the respective theoretical values.
Table 6.10. Flexible body displacement and hoop strain measurements during high-speed spin test.

<table>
<thead>
<tr>
<th>Speed (krpm)</th>
<th>Theoretical* ( u ) and ( \varepsilon_h ) at 70.7 mm</th>
<th>Experimental** ( u ) and ( \varepsilon_h ) at 70.7 mm</th>
<th>Theoretical* ( u ) and ( \varepsilon_h ) at 141.2 mm</th>
<th>Experimental* ( u ) and ( \varepsilon_h ) at 141.2 mm</th>
<th>Theoretical* ( u ) and ( \varepsilon_h ) at 152.2 mm</th>
<th>Experimental** ( u ) and ( \varepsilon_h ) at 152.2 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(( \mu )m)</td>
<td>(( \mu )m)</td>
<td>(( \mu )m)</td>
<td>(( \mu )m)</td>
<td>(( \mu )m)</td>
<td>(( \mu )m)</td>
</tr>
<tr>
<td>4</td>
<td>8.6</td>
<td>121</td>
<td>9.1</td>
<td>129</td>
<td>8.2</td>
<td>58</td>
</tr>
<tr>
<td>7</td>
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<td>430</td>
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<td>178</td>
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<tr>
<td>9</td>
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<td>16</td>
<td>137.9</td>
<td>1950</td>
<td>146.7</td>
<td>2075</td>
<td>132.1</td>
<td>935</td>
</tr>
</tbody>
</table>

*Calculated using Eq. 6.9
**Corrected for measured temperature change and rack displacement (Eq. 6.8)
The vibration amplitude and phase indicated by the A2, B2, A10, and B10 sensors agreed within ±7 µm and ±20 degrees, respectively, at all speeds. Plotted in Fig. 6.30 is an example set of vibration data, recorded at the maximum speed of 16 krpm. These data (symbols) are the individual lobe displacements measured by each of the six sensors at azimuthal locations of 0, π/2, π, and 3π/2 radians. The sinusoidal curves in Fig. 6.30 represent best-fits of Eq. 4.14 to each set of four data points for the six sensors. The amplitude and phase and $R^2$ (goodness of fit) value for each of the fits are summarized in the inset table in Figure 6.30.

![Figure 6.30. In-plane vibration data at 16 krpm during the high-speed test of the aluminum rotor.](image)

This data set was chosen as it represents the worst-case set of data (i.e. having the highest amount of disagreement between the vibration phase and amplitude indicated by each of the sensors). Vibration phase and amplitude at the constant speed of 16 krpm (at which all six sensors readings were taken) should be consistent among all six sensors. Inspection of the curves fit to the data shows that the vibration amplitudes indicated by
the A9 and B9 sensors (dashed lines) are significantly lower than the amplitudes indicated by the other four sensors (solid lines), which are all very similar to each other. As well, the phase indicated by the A9 sensor is significantly less than that indicated by the other sensors. This disagreement is likely caused by pattern edge roughness at the radial location of the A9 and B9 sensors (141.2 mm). Considering the above discussion and the low $R^2$ values calculated for the sine curves fitted to the A9 and B9 sensor data, it is reasonable to disregard the A9 and B9 sensor readings when calculating the average phase and amplitude at this speed. Interestingly, despite the poor correlation of the vibration data indicated by the A9 and B9 sensors, the flexible-body displacement (strain) agreed with the theoretical prediction at 141.2 mm (see Table 6.10).

In this high-speed test, the experimental flexible body displacements exceeded the theoretical displacements by approximately 1 to 9 μm at all speeds (see Table 6.10). As well, during previous (unpublished) tests on this rotor at speeds as high as 10 krpm (with the current pattern and two other applications of the TB pattern), the experimental displacements consistently exceeded theoretical displacements by approximately the same amount. Therefore, this error appears to be systematic. The electronics are not responsible, because this error is speed independent. Drift is not responsible, because the $\phi$ versus r calibration curves are consistently repeatable to within 1 μm at the calibration speed—even “before and after” a high-speed test run has been performed. A geometric analysis (the results of which are given in Appendix I) based on perfectly smooth edges of the TB reflective pattern predicts that errors in the flexible body displacements due to vibration are negligible. Because perfectly-smooth TB patch edges are impossible to generate, the combination of pattern edge roughness with unavoidable rigid body vibrations that occur at all speeds other than the calibration speed is most likely responsible for the observed error in displacement. Additional (future) work is required to explore, confirm, and remedy this error.

6.4.5 Conclusions for the OESM System Validation Experiments

In conclusion, the validation experiments show that the OESM system is a viable and attractive method for making long-term strain measurements on flywheel rotors. The
results of the OESM experiments on the low-speed rotor validate the novel concepts and techniques for compensating for drift and for measuring rigid-body displacements (vibration). The results of the experiments on the high-speed rotor validate that the OESM system can effectively measure flexible body displacements (strain) and can effectively separate vibration from this strain.

6.5 Rotor Creep Spin Test

6.5.1 Overview

As mentioned in Section 5.6, the gl/ep rotor, which was intended to be spun and monitored for creep with the optoelectronic strain measurement system for validation of the viscoelastic rotor model, sustained a fracture (transverse to the hoop-wound fibers) very shortly before the creep spin test. A picture of the cracked rotor, with enlarged detail of the crack location, is shown in Fig. 6.31. A labeled arrow points to the crack in the inset in Fig. 6.31. There are actually six cracks that are all located at approximately \( r=99\pm1.5 \text{ mm} \), each of which exhibits slight overlap with the neighboring crack (0.5 to 2 mm in the radial sense; 0 to 1 cm in the circumferential sense). A sketch of the six cracks (with exaggerated overlaps) is shown in Fig. 6.32. Also shown in Fig. 6.31 is the black mark on the hub used as the target for the optical tachometer.
Figure 6.31. Cracked E-glass/Dow 383 rotor.

Figure 6.32. Illustration of cracks on the E-glass/Dow 383 rotor.
6.5.2 Vibration Data During the Rotor Creep Spin Test

6.5.2.1 Vibration Data Immediately Before and After the Event

The inductive position sensors targeted at the bearing housing (sensor positions are shown in Fig. 5.12) provided vibration data. Snapshots of the data (at 14,230 rpm) recorded immediately prior to the supposed formation of the crack, as well as the data recorded (at 14,286 rpm) immediately after the crack are shown in Figs. 6.33 and 6.34, respectively. In this experiment, analog vibration data were recorded at 60,000 Hz. The presence of the large-amplitude sub-synchronous component in the vibration signals is believed to be evidence of the formation of the crack. The crack event was not accompanied by any detectable irregular noise during the time that these snapshots were taken. The vertical axes in Figs. 6.33 and 6.34 indicate the standoff distance of the two inductive positions sensor from their respective targets, which are the top and the bottom, respectively, of the outside of the bearing housing (see Fig. 5.12). The dashed line labeled “Tachometer” in Figs. 6.33 and 6.34 is not in units of millimeters, but is shown simply to provide a reference for the “once-per-revolution” harmonic of the vibration. (see appendix F.2.1 for a description of the operation of the tachometer).

Fig. 6.33. Vibration signal “snapshot” before the rotor cracked.
Upon observing the large amplitude component in both vibration signals at 14,286 rpm, it was known that there was a malfunction with the rotor. The drive motor was immediately de-throttled so that the rotor could be slowed down and the problem could be assessed and fixed. In a vacuum of 0.1 torr, the rotor decelerates at only about 25 rpm/sec. and, because there was a resonance at about 13 krpm, the vent on the spin pit was opened to let air into the chamber to increase the deceleration to 100-150 rpm/sec. While the rotor decelerated, the vibration amplitude continually increased. At around 13 krpm, the vibration was large enough so as to be audible. The noise and vibration continued to worsen through 12 krpm and, shortly thereafter, the tachometer stopped functioning. A few seconds after that, several sharp noises were heard, which were probably when the shaft snapped and the rotor came into contact with the bore of the spin pit. This was followed by a scraping sound, which was likely the rotor coming to rest on the bottom of the pit. The entire event lasted 30-60 seconds.

6.5.2.2 Vibration Data During the Last Two Runs

The data plotted in Figs. 6.33 and 6.34 show only the data immediately before and after the cracking “event” at 14,286 rpm. Sixty-five vibration data “snapshots” were,
however, recorded over the 8500 to 12,800 rpm speed range during this run (hereafter called “Run 3”) and two balancing runs (hereafter called “Run 1” and “Run 2”) that were performed in a two-hour time period prior to Run 3. The vibration amplitude and phase was extracted from these snapshots, each of which was \(1/12^{th}\) of a second in duration. These snapshots were recorded as a matter of balancing the rotor, and were acquired during the acceleration portions of these final three runs (see Section 5.4.4.1 for a description of rotor balancing). The amplitude and phase data from the Run 2 and Run 3 snapshots are given now “for the record”. These data could contain information that would signal the impending initiation of a crack and may provide useful input to future rotor-health monitoring algorithms, though no health monitoring programs (e.g., Shiue, 2001) were used during these spin tests.

The vibration amplitude versus speed that was measured by the two sensors is shown in the top set of axes in Fig. 6.35 during Runs 2 and 3. It is interesting to note that the amplitude measured by both sensors sharply decreased immediately after the crack occurred (indicated with an arrow in Fig. 6.35). The phase of the vibration is shown for Runs 2 and 3 in the bottom set of axes in Fig. 6.35, and is measured in degrees relative to the fall of the tachometer signal from “High” to “Low” (see Fig. 6.33). The divergence in phase behavior indicated by Sensor 1 between Runs 2 and 3 at approximately 11 krpm is also interesting (labeled in Fig. 6.35). The difference in the state of the rotor between these two runs was the addition of a small amount of balance mass. Experience has shown that when a solution mass is attached to the rotor, it typically causes only a small shift in phase at speeds below the target speed (the target speed in this case was 12.8 krpm). Inspection of the plot of the phase indicated by Sensors 1 and 2 at speeds less than 11 krpm illustrate this “typical behavior.” It is believed, therefore, that the divergence in phase indicated by Sensor 1 at 11 krpm may indicate crack initiation. Because health monitoring was not used during these runs, this potential “warning sign” was not noticed when it occurred.
6.5.3 Stress and Strain Analysis

The viscoelastic rotor model was used to estimate the state of the stress and strain in the gl/ep rotor during the above spin tests to determine the distribution of strain energy in the rotor at the time of rupture. Well after the time of the manufacture of the rotor (at cure temperature of 80°C), the rotor was heated to 100°C for 12 hours during the casting of the polyurethane interlayer (100 wt.% Adiprene-L100 / 21 wt% Caytur-21) that connected the hub to the rotor. The load history was assumed to start upon cooling below
80°C after the curing of this polyurethane interlayer, and is given in Table 6.11. Boundary condition equation 3.11 was used at all steps in the simulation. Several balancing runs (all at less than 7 krpm) that were performed in the time period denoted by “One month dwell at 23°C” are omitted from this simulation because the stress and temperature loads associated with these runs are negligible. This is a fair assumption because stresses and strains are proportional to the square of the speed.

### Table 6.11. Load history used in the simulation of the rotor creep spin test.

<table>
<thead>
<tr>
<th>Notes</th>
<th>t (min.)</th>
<th>Δt (min.)</th>
<th>T (°C)</th>
<th>ω (krpm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cooldown after interlayer cure</td>
<td>0 → 0.01</td>
<td>0.01</td>
<td>80 → 55 (constant (\dot{T}))</td>
<td>0</td>
</tr>
<tr>
<td>One month age at 55°C</td>
<td>43,200</td>
<td>-</td>
<td>55</td>
<td>0</td>
</tr>
<tr>
<td>Cooldown after aging</td>
<td>43,201 → 43,260</td>
<td>1</td>
<td>55 → 23 (constant (\dot{T}))</td>
<td>0</td>
</tr>
<tr>
<td>One month dwell at 23°C</td>
<td>86,400</td>
<td>-</td>
<td>23</td>
<td>0</td>
</tr>
<tr>
<td>Run 1 spin up</td>
<td>86,400.5 → 86,415</td>
<td>0.5</td>
<td>23 → 26 ((\dot{T} \propto t)^* (\dot{\omega} \propto t^{1/2})^*</td>
<td>0 → 10.2</td>
</tr>
<tr>
<td>Run 1 spin down &amp; cool</td>
<td>86,415.25 → 86,418</td>
<td>0.25</td>
<td>26 → 25 ((\dot{T} \propto t))</td>
<td>10.2 → 0</td>
</tr>
<tr>
<td>Run 1 continue cool</td>
<td>86,419 → 86,430</td>
<td>1</td>
<td>25 → 23 ((\dot{T} \propto t))</td>
<td>0</td>
</tr>
<tr>
<td>Dwell after Run 1</td>
<td>86,460</td>
<td>-</td>
<td>23</td>
<td>0</td>
</tr>
<tr>
<td>Run 2 spin up</td>
<td>86,460.5 → 86,480</td>
<td>0.5</td>
<td>23 → 26 ((\dot{T} \propto t))</td>
<td>0 → 12.8</td>
</tr>
<tr>
<td>Run 2 spin down &amp; cool</td>
<td>86,480.25 → 86,483</td>
<td>0.25</td>
<td>26 → 25 ((\dot{T} \propto t))</td>
<td>12.8 → 0</td>
</tr>
<tr>
<td>Run 2 continue cool</td>
<td>86,483 → 86,495</td>
<td>1</td>
<td>25 → 23 ((\dot{T} \propto t))</td>
<td>0</td>
</tr>
<tr>
<td>Dwell after Run 2</td>
<td>86,520</td>
<td>-</td>
<td>23</td>
<td>0</td>
</tr>
<tr>
<td>Run 3 spin up</td>
<td>86,520.5 → 86,545</td>
<td>0.5</td>
<td>23 → 26 ((\dot{T} \propto t))</td>
<td>0 → 14.286</td>
</tr>
</tbody>
</table>

*Rates of change of T and ω are expressed relative to the beginning of each time sub-interval*

During the “spin-up” portions of the three runs, the speed was assumed to vary with the square-root of time (i.e. a constant-power speed input). After reaching each peak speed, a three-minute deceleration with \(\dot{\omega}\) proportional to the inverse square root of time (i.e. deceleration due to drag force) was imposed for each of the runs. The rotor temperature, T during these cycles was assumed to rise from 23°C to 26°C with \(\dot{T}\)
proportional to \( t \) until the maximum speed was reached, after which time the temperature decreases to 23°C with \( T \) proportional to \( t \). This temperature versus time profile closely approximates that which actually occurs during a single run (see Fig. 6.28). An illustration of the speed and temperature load profile starting at the beginning of the three balancing runs in this simulation is shown in Fig. 6.36. Labeled circles in Fig. 6.36 correspond to times on interest at which the radial stress and strain field variables were solved.

![Graph showing speed and temperature profile](image)

Fig. 6.36. Speed and temperature load profile used in the simulation of the ruptured E-glass/Dow 383 rotor.

A 30-minute dwell time (with \( \omega=0 \) and \( T=23^\circ C \)) was imposed after the first two cycles, simulating the activity of opening the pit and adding balance masses. During changes in speed and/or temperature in the simulation, a step size of 0.25 minutes was used and found to yield satisfactory convergence of the solved field variables. Hoop stresses will not be considered, as the predicted values of hoop stress are, at most, 5% of the fiber-direction strength of the composite. The radial stress and strain distributions are plotted in Figs. 6.37 and 6.38, respectively, at several times in the load history described above. Note that the stress-free thermal strain (\( \alpha_r \Theta \)) has been subtracted from the radial strain plots. Figure 6.37 shows that the peak radial stress of 21.9 MPa occurs at \( r=97.5 \text{ mm} \) at time \( t_6 \). The peak radial strain in Fig. 6.38 occurs at \( r=96.8 \text{ mm} \), with a value of 784 \( \mu \varepsilon \).
Figure 6.37. Radial stress at several times for the E-glass/Dow 383 rotor rupture simulation.

Figure 6.38. Radial strain at several times for the E-glass/Dow 383 rotor rupture simulation.
Because the stresses and strains at time $t_1$, $t_3$, and $t_5$ are indistinguishable from each other in Figs. 6.37 and 6.38, a closeup plot is provided in Fig. 6.39. In Fig. 6.39 it can be seen that at times $t_3$ and $t_5$, the radial strains—which have been allowed to recover after the first and second spin cycles—are predicted to be only 1-2 microstrain higher than the so-called “residual” strain field at time $t_1$. The predicted radial stresses at times $t_3$ and $t_5$ are practically indistinguishable from the residual radial stress at $t_1$, even in this closeup.

![Fig 6.39. Radial stresses and strains at the end-of-dwell times for the E-glass/Dow 383 rotor rupture simulation.](image)

Another useful piece of information is the amount of accumulated viscoelastic redistribution of stress and strain predicted by the model to have occurred during all of the balancing runs. This was calculated by adding an extra load point to the history shown in Fig. 6.36. This load point consists of a 0.01 minute duration ramp down to zero speed and 23°C following $t_6$—effectively subtracting the loads in an elastic (instantaneous) fashion. The difference between the resulting stress and strain fields at the end of this ramp and those at time $t_1$ are considered to represent the viscoelastic component of radial stress and strain, $\sigma_r^{VE}$ and $\varepsilon_r^{VE}$ at the time of rupture, which are shown in Fig 6.40. The approximate maximum values of these components, respectively, are -0.10 MPa at 93 mm, and +18.4 $\mu$e at 100 mm. This very small amount of time-
dependent deformation predicted by the model at the time of the crack does not support creep rupture as the most likely mechanism for the failure. Rather, this result points to a transverse strength (~22 MPa) of the PMC in rotor form that is significantly lower than the PMC strength measured in coupon form (e.g., the tube strength results presented earlier showed transverse material strength of 45.6 MPa).

Figure 6.40. Radial stress and strain in the E-glass/Dow 383 rotor, illustrating the predicted viscoelastic stress and strain components at the time of rupture.

6.5.4 Failure Analysis

6.5.4.1 Strain Energy

As discussed in Section 2.7, a failure criterion based on strain-energy is considered appropriate for predicting the delayed failure (i.e. creep rupture) of PMCs. A substantial number of researchers have performed isothermal, uniaxial, constant-stress creep tests on laminae, and have effectively validated models that predict that failure will occur when the stored strain energy component in such a creep test reaches a critical value. However, no methodologies have been proposed in the literature for incorporating the critical stored strain energy for a given material into a structural model, where
conditions are generally very different from the controlled creep test. In a structural model (e.g., the flywheel rotor model), applied loads—including temperature—vary with time, and the stresses are biaxial.

In the absence of a methodology for calculating the stored component of the strain energy in the E-glass/Dow 383 rotor at the time of failure, the energy corresponding to the area under the radial stress-strain curve (or the uniaxial strain energy at failure, Eq. 6.2) is now considered. This quantity reaches a maximum value of approximately 8585 Pa at \( r = 97 \text{ mm} \), which is only slightly more than half of the value of strain energy at rupture achieved by the 54° 50% tubular specimen (which had the lowest strain energy at failure for all of the tests listed in Table 6.2). Based on the assumption that strain energy at failure is a decreasing function of temperature (see Section 2.7), the rotor should have achieved an even higher value of strain energy than that achieved by the tubular specimen. This is because the estimated temperature at failure in the E-glass/Dow 383 rotor was 27°C—considerably lower than the temperatures for the ruptured tube tests reported in Table 6.2. Admittedly, the energy calculated using Eq. 6.2 is not the stored energy that is of interest for making suitable rupture predictions based on the critical strain energy work outlined in Section 2.7. There is, however, a large difference between the rupture test results from Table 6.2 and the rotor tests if area under the stress-strain curve is considered.

6.5.4.2 Maximum Stress or Strain

Considering only the radial stress at rupture, the value of 21.9 MPa is 48% of the transverse strength of the composite measured during room-temperature quasi-static strength tests on the tubular specimens (Section 6.1.1). Considering only the radial strain at rupture (excluding free thermal strain), the radial strain in the rotor at the radial location of fracture, or 784 µε, is 58% of the transverse failure strain of the 54° 50% tubular specimen (1362 µε). From either a maximum stress or maximum strain standpoint, this rotor failed at a significantly lower value than those measured on the tubular specimens of the same material.
6.5.5 Rotor Creep Spin Test Conclusions

In conclusion, the rotor failed before creep measurements could be made and, according to the analysis, before significant creep deformation occurred. Based on the quasi-static strengths and the values of strain energy at rupture for the tubular specimens from Section 6.1, the stresses and strains calculated with the viscoelastic rotor model predicted that this failure should not have occurred. The fact that the fiber architecture of the tubular specimens is different than that of the rotor may explain the difference, though more experiments would be necessary to explore and test this idea. Figure 6.41 is a schematic illustration of the orientation of the E-glass fiber tow in the tubular specimens and in the rotor, with respect to the tensile stresses. The volume effect (a larger volume of material increases the likelihood of encountering a material defect) may also be responsible for the premature failure of the rotor. The failure of the rotor shows that the development of tension in the transverse material direction is not a very safe approach to performing a spin creep test.

Figure 6.41. Illustration of the different orientations of the E-glass fiber tow in the tubes and the rotor.
Chapter 7
MODEL SIMULATIONS

7.1 Overview

Three example model simulations are now presented which illustrate the effects of viscoelastic behavior on the stress and strain distributions in hoop-direction filament wound flywheel rotors. In these simulations, two materials described in Table 7.1 were used. Both materials have virtually identical properties at \( T=23^\circ\text{C} \) and \( t=0.01 \) minutes (e.g., the materials are virtually identical in the elastic sense), and are representative of a unidirectional carbon/epoxy material with \(~70\%\) fiber volume fraction. The parameters of the power law for \( S_{22} \) of Material 1 are based on the creep tests of unidirectional E-glass/Dow 383 epoxy specimens. In these creep tests, a power law (Eq. 2.4) was fit to the experimentally determined \( S_{22} \) creep data, giving a time exponent, \( n \), of 0.105 and a value of 20 for the ratio \( S_0/S_1 \). The same time exponent and ratio were used to estimate \( S_{22} \) of a carbon fiber composite (Material 1) with the same resin. The validity of this estimation, which assumes commonality of the time dependence of neat resin and the associated composites, has been discussed in Section 2.4. The time exponent in the power law representing \( S_{22} \) of Material 2 was arbitrarily set slightly higher than that of Material 1, and serves to illustrate the effects of the material time-dependency on the stress and strain distributions in the rotor simulations. The equation used for the temperature shift factor, \( \alpha_1 \), of Materials 1 and 2 is exactly that which was fit to experimentally measured data from the unidirectional E-glass/Dow 383 specimens. For simplicity, the composite rotors were assumed to be spun in isolation (i.e., the hub of the flywheel has no influence on the stresses in the rotor).

Table 7.1. Material properties used in the model simulations.

<table>
<thead>
<tr>
<th>Material</th>
<th>( S_{22}(t) ) (GPa(^{-1}))</th>
<th>( a_T )</th>
<th>( S_{11} ) (GPa(^{-1}))</th>
<th>( S_{12} ) (GPa(^{-1}))</th>
<th>( \rho ) (g/cm(^3))</th>
<th>( \alpha_1 ) (ppm/(^\circ\text{C}))</th>
<th>( \alpha_2 ) (ppm/(^\circ\text{C}))</th>
<th>( T_0 ) (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.09+0.0045(^{0.105}) (10^{0.18(T-30)})</td>
<td>0.00653</td>
<td>-0.00196</td>
<td>1.580</td>
<td>-2.3</td>
<td>-2.3</td>
<td>80</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.09+0.0045(^{0.130}) (10^{0.18(T-30)})</td>
<td>0.00653</td>
<td>-0.00196</td>
<td>1.580</td>
<td>32.3</td>
<td>32.3</td>
<td>80</td>
<td></td>
</tr>
</tbody>
</table>
7.2 Example 1

In Example 1, a PMC ring of Material 1 and a PMC ring of Material 2 are (independently) subjected to an identical load history. The rings each have an inner radius of 120 mm and outer radius of 140 mm. These dimensions were chosen such that the radial and hoop stress limits of \([-75 < \sigma_r < 21.4]\) and \([-2,450 < \sigma_h < 2,450]\) MPa, respectively, are not exceeded during operation. The simple load history to which the rings are subjected begins with a temperature decrease from the stress-free temperature of 80°C to 23°C at \(t=0.01\) min, which is then held for 30 days (\(t=43,200\) min). At \(t=43,200.01\) min, the rotor is subjected to a spin load of 9,300 rad/s and a temperature of 55°C that are held constant for ten years. The radial and hoop stresses and strains are plotted at times of interest of 43,200.01 min (initial spin-up) and 10 years in Fig. 7.1.

![Figure 7.1](image)

Figure 7.1. Stress (a) and strain (b) distributions at different times in Example 1 rotors.

Figure 7.1(a) shows that the peak radial stress takes on an initial value of 21.4 MPa in both materials and decreases by 2.3% in Material 1 and 4.2% in Material 2 after ten years. At the inner radius, the hoop stress has an initial value of 2,452 MPa that decreases by 0.4% in Material 1 and 0.8% in Material 2 after ten years. The time-
dependent change in radial strain (Figure 7.1b) is much more pronounced compared to the changes in the stresses. Note that the radial and hoop strain distributions shown in Figure 7.1b include the effects of temperature change.

### 7.3 Example 2

The rotor considered in Example 2 is designed under the presumption that the rotor material possesses zero tensile strength transverse to the fibers (radial direction). To counteract the radial tensile stresses that develop due to rotational loads, the rotor is comprised of a pair of interference-fitted rings. The interference fit imparts a state of radial compression through the radial thickness of the rotor before the rotor is subjected to rotational loads. The radial compressive strengths of Materials 1 and 2 are both assumed to be -75 MPa, which governs the maximum amount of radial interference that can be applied between the two rings. The tensile and compressive hoop strengths of both materials are assumed to be ±2,450 MPa, respectively.

The first load on the rings is a temperature decrease from the stress-free cure temperature, $T_0$, to 23°C at time 0.01 min, which is held for 43,200 min (30 days). At 43,200.01 min, the rings are assembled with radial interference (such that the radial stress in the assembly is –75 MPa) until $t=86,400$ min (60 days). At 86,400.01 min, the rings are subjected to a step increase in temperature to 55°C and spin loaded such that the radial strength limit is reached in both rings, and the hoop strength limit is reached in the outer ring only. The ring dimensions and spin load that satisfy the above design specification are an inner ring with an inner radius of 100.00 mm and outer radius of 114.29 mm, an outer ring with an inner radius of 113.29 mm and outer radius of 126.07 mm, and a spin load of 9,360 rpm.

Figure 7.2 shows the radial stress versus radial position in each of the two rings (made of Material 2) at various times in the load history described above. Figure 7.2a shows that the press-fit relaxation over $[43,200.01<t<86,400]$ min. is very small (<1% for both materials). With the temperature and spin load applied at 86,400.01 min, the radial stress is still compressive everywhere, but approaches zero near the inner and outer radii.
of the assembly (see close-up, Fig. 7.2b). If the speed and temperature are held constant, viscoelastic effects cause the radial stress to increase, becoming positive near the inner and outer radii of the assembly, as shown in Figure 7.2b for \( t = 5,259,600 \) minutes (10 years).

![Radial Stress Diagram](image)

Figure 7.2. Radial stress at different times in Example 2 using Material 2 in both rings.

In order to keep the radial stress at or below zero in the assembly during operation, it is necessary to continuously decrease the rotational speed over time. For the Material 2 rotor, it is necessary to reduce the speed to 9,170 rad/s after 10 years, corresponding to energy storage reduction of 4%. For the rotor assembly made of two rings of Material 1, it is necessary to reduce the speed to 9,270 rad/s after 10 years, corresponding to an energy storage reduction of 2%.

### 7.4 Example 3

In Example 3, the rotor from Example 2 will be used, and only Material 2 will be analyzed. The load history from Example 2 will also be used except that, instead of a step increase of speed and temperature at \( t = 86,400.01 \), the rotor is subjected to cyclic loads of speed and temperature. Figure 7.3 shows a plot of the first two 100-minute cycles of simulated speed and temperature.
Figure 7.3. Cyclic speed and temperature loads in Example 3.

Over the time interval $[86,400 < t < 86,400.01]$ minutes, the speed is increased from zero to 3,120 rad/s, and the temperature is increased from 23°C to 40°C. After this, the rotor is subjected to 100-minute cycles of speed and temperature with symmetric “charge” and “discharge” portions. During the “charge” portion of the cycle the temperature varies linearly with $t$ from 40 to 55°C, and the speed varies proportionally to $\sqrt{t}$ (i.e. constant power input) to the maximum speed of 9,360 rad/s. Immediately following the “charge” half-cycle is a symmetric “discharge” half-cycle. The 100-minute cycle length and 3:1 speed ratio is representative of a low-earth-orbit satellite with an 89% depth-of-discharge.

Figure 7.4 shows the radial strain profile in the rings at several points in time in the previous example. This plot is used as the basis for selecting radial locations of interest in the current example. It can be seen that the interface of the two rings is the radial location where the highest (absolute) amount of time-dependent change in radial strain occurs after the step spin and temperature load is applied at $t = 86,400.01$ min. Consequently, the outer radius of the inner ring and the inner radius of the outer ring are selected as radial locations of interest for the current example. The times corresponding to the “peaks” and “valleys” in Fig. 7.3 are chosen as the times of interest, $t_i$. Figure 7.5 shows a plot of the radial strains at the locations of interest versus cycle number (log
scale). Time step size \((t_j - t_{j-1})\) was set at 1.0 min in this example. All four traces in the Figure 7.5 follow a similar trend versus cycle number, each decreasing approximately 480 \(\mu\varepsilon\) per cycle decade. While strain predictions from only three hundred cycles of 100-minute duration (i.e. 21 days total) are presented in Fig. 7.5, extrapolation of the curves can be made for several more decades of cycle time as a first approximation of long term response. Using such an approximation, the highest (absolute) radial strains in the outer ring at the peak loading are expected to reach approximately \(-12,500 \, \mu\varepsilon\) after 10 years, which is 12% less in magnitude than the highest radial strain calculated in Example 2 after a similar period of time at a fixed rotational speed.

Figure 7.4. Radial strain profiles at different times in Example 2.  
Figure 7.5. Peak and valley radial strains at outer radius of inner ring and inner radius of outer ring for cyclic loading history of Example 3.
Chapter 8
SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS
FOR FUTURE WORK

One objective of this research was to develop a closed-form viscoelastic rotor model. The model that was developed is the first published model that includes techniques for approximating the (quasi-static) response to time-varying spinning and thermal loads, and that includes the effects of material time-temperature shifting for multiple-ring polar-orthotropic rotors. Future work towards developing the model should include consideration of time-shifting due to material physical aging effects. Also, a rational methodology for incorporating a viscoelastic failure criterion into a structural flywheel model is needed for using the viscoelastic rotor model for rotor-life prediction. A widely accepted failure criterion of this type does not exist for PMCs, so the development of a viscoelastic biaxial failure criterion is another recommendation for future work. The effect of material age on the viscoelastic failure behavior (specifically the critical strain energy) of PMCs subjected to uniaxial and biaxial stress is another area where research is recommended.

Creep tests that were originally performed with the intent of simply providing viscoelastic material property input to the rotor model served that function, and additionally revealed a remarkable and significant lack of standardized test methods for determining viscoelastic material properties of PMCs. Meaningful characterization of the creep behavior of the composite specimens in this research was achievable only after experimentally determining the usefulness of various procedures followed in the published literature on creep testing of PMCs. To achieve reliable and repeatable creep test results in the sub-$T_g$ temperature range of interest in this research, it was found to be of critical importance to age the material in order to separate the associated time-shifting effects of aging from the time-shifting effects of temperature. Future work in the area of creep testing should include characterization of viscoelastic behavior of PMCs under compressive transverse stress. To date, only one group of researchers [Gates et al., 1997; Veazie and Gates, 1997] is known to have published compressive creep data, and this was for only one PMC material system. If the disparity between tensile and compressive
creep behavior is significant for many PMC systems, then it is inappropriate to use a single creep compliance to model structures that possess spatial regions where compressive or tensile stresses develop at different times. While the characterization of the creep rupture of PMCs was not an intended area of investigation in this research, the fact that rupture was encountered (in the tube tests as well as in the spin test of the composite rotor for creep measurements) motivated an attempt to explain the failures in terms of several available theories. The stress and strain results from four composite tubes that failed in creep-rupture were analyzed in terms of a one-dimensional strain energy failure criterion derived by Hiel [1984] and based on the Reiner-Weisenburg critical (stored) strain energy. Because only four data points of creep rupture were available from the creep tests, a complete characterization of the critical strain energy for the E-glass/Dow 383 material system was not possible. The results did, however, allow the calculation of an estimated range of critical strain energy for this material system, though the strain energy in the gl/ep rotor at failure were well below this estimated range.

The isothermal interference-fit experiments were originally chosen as an inexpensive and relatively simple means (compared to a spin test) for developing a biaxial stress state in PMC rings. These experiments showed that only modest (less than 5%) pressure losses are sustained by the interference-fitted glass and carbon fiber PMCs over a one-month period. At the end of this period, the rate of change of strains indicated by the gages had decreased to near zero, showing that redistribution due to viscoelastic effects was nearly complete (i.e. not much more than the 5% drop in pressure was expected to occur even at longer times) at room temperature. The full-field strain redistributions measured with moiré interferometry during these experiments revealed that the state of strain on the face of the specimens near the interface of the mating rings differed significantly from both the elastic and viscoelastic plane-stress predictions. A three-dimensional elastic finite element model of an interference-fitted ring pair (created after this result was observed) added confidence in the experimental observations. Away from the ring interface, the strains obtained by moiré and by the rotor model predictions were in reasonable agreement. The experimentally measured interfacial pressure loss, however, was in most cases about twice the predicted value. This pressure loss was
consistent for two disparate interface pressures (27 and 54 MPa), spanning a very wide practical range for hoop-wound flywheels.

After performing the above set of isothermal interference fit experiments, knowledge gained on the state of strain near the interface of the ring pair and on the material constitutive behavior served to guide the design of a second set of interference-fit experiments. With strain gages placed away from the locations where strains deviate from plane-stress predictions, interference fit loads were again imposed on two ring pairs, and elevated temperature was used to accelerate the subsequent time-dependant behavior. The model predictions of the redistributions of radial strain were 3 to 30 με higher in magnitude than the measured strains for the low-pressure glass/aluminum ring pair, and were 28 to 95 με higher than the measured strains for the low-pressure glass/carbon ring pair. As in the isothermal interference fit experiments, the hoop-oriented gages on the inner and outer radii of the rings indicated that the experimentally measured change in pressure over the duration of the experiment was approximately twice the predicted value. One explanation for the disagreement may be that the material characterization used in the model, which was performed in tension, may not accurately characterize the response of the interference-fit rings, which were in states of radial compression.

As a final note on the interference-fit experiments, the non-planar state of stress in the specimens requires a non-planar stress model. Some general trends of the stress and strain behavior of composite flywheels can be obtained with the plane stress model, but a recommendation for future work is to compare the plane stress and plane strain predictions with corresponding models that can handle three-dimensional stress states (e.g., using two-dimensional axisymmetric FEA) of multi-ring structures in order to evaluate how significantly the results of these models differ.

The second objective of this work was to develop and experimentally validate a non-contact optical strain measurement method for high-speed flywheel rotors. This objective was met, as demonstrated during the high-speed spin test on the aluminum rotor. The validation results demonstrated that the deviations between the experimental and theoretical hoop strains were at most 6.4% (i.e. predicted strain was 1950 με and experimental was 2075 με) at a rotational speed of 16 krpm. The tangential rotor speeds at which these measurements were made were in the range of 118 to 253 m/s). The
reflective pattern incorporated novel geometric features designed to maximize
displacement sensitivity and to compensate for drift of the sensor over time, making this
measurement technique attractive for long term spin tests of flywheels. Another novel
and very useful feature of the OESM system is the technique for measuring the amplitude
and phase of rigid-body in-plane displacements (vibration) simultaneously and separately
from flexible body displacements (strain). These aspects of the OESM system were also
successfully validated through experiments. Recommendations for future work on the
OESM system include the development of an easier technique for applying the reflective
pattern to the rotor, improving the smoothness of the edges of the reflective features, and
improving the mechanical durability of the pattern coating materials. Improvements have
already been made in developing a sensor with a larger stand-off distance that is also
aimed normal to the rotor face, in order to reduce sensitivity to axial rigid body
displacements [Bakis et al. 2002]. Improvements can also be made in designing and
evaluating a sensor support structure that is immune from thermal deformations. Finally,
improvements can be made in designing compensation patches centered on the
displacement patches—because each compensation patch (by design) provides lobe-
specific compensation for intensity and, thus, benefits from being as close as possible to
the center of the lobe.

Regarding the attempt to perform a creep test on a spinning rotor, the monolithic
glass/epoxy rotor was found to be a poor choice for a specimen—precisely because it
develops tension in the transverse material direction (i.e., the radial direction on the
rotor). The rotor failed at a significantly lower speed than that predicted by the maximum
stress and maximum strain failure criteria that were used in the specimen design. Despite
the lack of a proper methodology for incorporating the critical strain energy failure
criterion of the E-glass/Dow 383 composite into the rotor model (to make a proper creep
rupture prediction), the exceedingly low values of viscoelastic strain predicted by the
model at the instant of failure practically discounts creep rupture as an explanation
therein. Hoop-wound rotors tested in the radial direction are hypothesized to possess a
much lower strength than hoop-wound tubes tested in the axial direction. The strength
difference could be due to the different nesting of fibers in the two cases.
A recommended alternative design for a creep spin test rotor is one that develops a compressive state of stress in the radial direction prior to or during spinning. Employing an interference-fit is also a possible approach, although care should be taken to avoid large “un-usable” regions where three-dimensional stress effects cause deformations to deviate from either plane-stress or plane-stress solutions (if it is desired to validate such models). A multi-ring, multi-material rotor and a mass-loaded rotor can both develop radial compression but can also develop some radial tension under spin loads. Not only does this tension provide a driving force for rupture, but recently recognized concerns on the potentially significant difference between compressive and tensile creep compliance may prove the present viscoelastic rotor model (based on a single creep compliance) inaccurate for predicting strain distributions in a ring with some portion in radial tension and some portion in radial compression.

Concerning the long-term creep of rotors, there were several practical outcomes of the research presented in this thesis. Aging the E-glass/Dow 383 composite was found to be very effective at reducing the rate of creep of the material under load and at elevated temperatures. Aging the composite rings prior to the NIF tests resulted in pressure losses of only a few percent over simulated times of 13.6 years at room temperature. This result speaks well for the feasibility of using unidirectional filament epoxy-matrix materials in flywheel rotors under stress and temperature conditions where linear viscoelastic behavior is observed. Using the viscoelastic rotor model, simulations of realistic rotor geometries and loads show that very modest losses in energy storage capacity result from de-rating rotor speeds such that stress or strain levels remain below some pre-defined level. Finally, the premature failure of the rotor to be used for measuring creep under spin loads revealed that the development of radial tension in hoop-wound rotors should be avoided in conservative rotor designs.
REFERENCES


Tsai, S. W., and Hahn, H. T., 1980, Introduction to Composite Materials, Technomic, Lancaster, PA.


Appendix A

CODE FOR THE VISCOELASTIC ROTOR MODEL

The viscoelastic rotor code, which was written in MATLAB version 5.1, is given below. The code execution closely follows that which is shown in the flowchart of the viscoelastic solution procedure (Fig. 3.2).

A.1 ‘example.m’

The batch file ‘example.m’ contains two of the four components of the INPUT block of the flowchart in Fig. 3.2—specifically the radial dimensions and the solution times of interest, $t_i$. The other two components of the INPUT block are not addressed until the solution commences and, hence they are called by ‘stepica.m’, which primarily handles the tasks associated with the SOLUTION block in Fig. 3.2. After the ‘stepica.m’ subprogram (given in Section A.2) runs, OUTPUT information is returned back to ‘example.m’. Plotting is then performed by ‘example.m’ if there are two to four solution times, $t_i$. The output field variables are saved in ‘output.dat’ in ASCII format in the same directory as that in which this code resides.

```matlab
%--------------- START INPUT ------------------
%----------- define the materials & -----------
%-------------- ring radii here ---------------
% 'MATS' is a matrix of text with each row a name of a “.dat” material
% file. Row 1 is the innermost ring in a multiple-ring assembly, or row 1 of
% a single ring assembly). Rings are listed in order from inside to outside.

MATS = ['carbon' % ! Pad material
    'carbon' % names to a common
    '' % length for all rings !
    '' % The characters comprising
    '' % the names are stored in a
    '' % matrix, so no blanks
    '' % are allowed.
    ''
    ];

RSIR=0; RSOR=0; % radial stress on the
    % inner and outer radii

rad(1)=100; rad(2)=rad(1)/.8; % Inner & outer radii for ring 1
rad(3)=rad(2)-.5; rad(4)=rad(3)/.85; % Inner & outer radii for ring 2
```
solve_at=[50000.01 100000.01 200000 200000.01]; % Define the
% solution
% times of interest.

start=1; % Ring at which to start calculating.
stop=1; % Ring at which to stop calculating.
fine=19; % # of radial pos. points at which vars are calc'd.
tog=[1;1;1;1;1]; % Toggle the fields of interest: 1=solve, 0=don't solve.

%------------ END OF USER INPUT -----------------------
%------------ DO NOT EDIT BELOW -----------------------

[rpos,rsig,reps,ur,hsig,heps]=stepica(start,stop,fine,tog,rad,MATS,solve_at,
RSIR,RSOR);

% PLOTTING ROUTINES BELOW

if length(solve_at) == 2,
  if tog(1) == 1,
    figure
    plot(rpos,rsig(:,1),rpos,rsig(:,2))
    title('Radial Stress (MPa)')
    Legend('Time 1','Time 2')
  end
  zoom
  if tog(2) == 1,
    figure
    plot(rpos,reps(:,1)*1e6,rpos,reps(:,2)*1e6)
    title('Radial Strain (microstrain)')
    Legend('Time 1','Time 2')
  end
  zoom
  if tog(3) == 1,
    figure
    plot(rpos,ur(:,1),rpos,ur(:,2))
    title('Radial Displacement (mm)')
    Legend('Time 1','Time 2')
  end
  zoom
  if tog(4) == 1,
    figure
    plot(rpos,hsig(:,1),rpos,hsig(:,2))
    title('Hoop Stress (MPa)')
    Legend('Time 1','Time 2')
  end
  zoom
  if tog(5) == 1,
    figure
    plot(rpos,heps(:,1)*1e6,rpos,heps(:,2)*1e6)
    title('Hoop Strain (microstrain)')
    Legend('Time 1','Time 2')
  end
  zoom
end

if length(solve_at) == 3,
if tog(1) == 1,
figure
plot(rpos,rsig(:,1),rpos,rsig(:,2),rpos,rsig(:,3))
title('Radial Stress (MPa)')
Legend('Time 1','Time 2','Time 3')
end

if tog(2) == 1,
figure
plot(rpos,reps(:,1)*1e6,rpos,reps(:,2)*1e6,rpos,reps(:,3)*1e6)
title('Radial Strain (microstrain)')
Legend('Time 1','Time 2','Time 3')
end

if tog(3) == 1,
figure
plot(rpos,ur(:,1),rpos,ur(:,2),rpos,ur(:,3))
title('Radial Displacement (mm)')
Legend('Time 1','Time 2','Time 3')
end

if tog(4) == 1,
figure
plot(rpos,hsig(:,1),rpos,hsig(:,2),rpos,hsig(:,3))
title('Hoop Stress (MPa)')
end

if length(solve_at) == 4,
    if tog(1) == 1,
    figure
    plot(rpos,rsig(:,1),rpos,rsig(:,2),rpos,rsig(:,3),rpos,rsig(:,4))
    title('Radial Stress (MPa)')
    Legend('Time 1','Time 2','Time 3','Time 4')
    end
    if tog(2) == 1,
    figure
    plot(rpos,reps(:,1)*1e6,rpos,reps(:,2)*1e6,rpos,reps(:,3)*1e6,rpos, ...
    reps(:,4)*1e6)
    title('Radial Strain (microstrain)')
    Legend('Time 1','Time 2','Time 3','Time 4')
    end
    if tog(3) == 1,
    figure
    plot(rpos,ur(:,1),rpos,ur(:,2),rpos,ur(:,3),rpos,ur(:,4))
    title('Radial Displacement (mm)')
    Legend('Time 1','Time 2','Time 3','Time 4')
    end
    if tog(4) == 1,
    figure
    plot(rpos,hsig(:,1),rpos,hsig(:,2),rpos,hsig(:,3),rpos,hsig(:,4))
    title('Hoop Stress (MPa)')
end
function [rpos,rsigout,repsout,urout,hsigout,hepsout]=stepica(start,stop,fine,tog,rad,MATS,solve_at,RSIR,RSOR)
% The STEPICA function assumes that a file has been created
% called "loads.dat", with time increments in the first column
% (units consistent with the units  
% used for the time-dependent compliance), rotational speed 
% in the second column (units of rad/sec), Temperature  
% (units of degrees Celsius) in the third column, and in the 
% the fourth column either a zero (0) or a one (1) indicating 
% the state of the assembly--with "zero" indicating disassembled 
% and a "one" indicating that the rings are assembled. This column 
% vector is called "assy"
% The file loads.dat must include a "time-zero" entry, with the 
% zero speed and the cure temperature on the first line. Generally 
% a multiple-ring assembly will not be assembled at time zero, so assy 
% at time zero would also have a value of zero. 
% for example loads.dat for a multiple ring assembly (with time column 
% 1 given in minutes) may look like:
% % ------loads.dat------- 
% 0 0 80 0 <--time zero
% .01 0 50 0 <--quick ramp down to age temperature
% 50000 0 50 0 <--age rings for a month at 50 C
% 50000.01 0 23 0 <--ramp down to room temperature
% 51440 0 23 0 <--let sit for a day at RT
% 51440.01 0 23 1 <--assemble rings
% 52880 0 23 1 <--let sit for another day
% 52880.01 5000 23 1 <--start spinning
% 102880 5000 23 1 <--let spin for a month
%
% (et-cetera)
% -----------------------
%
% The STEPICA arguments pertain to the way in which calculations
% of the field variables of interest are to be carried out.
% The ’start’ and ’stop’ arguments tell the program for which
% (contiguous) range of rings that field variables are to be
% calculated. The arg. ’fine’ indicates the discretization
% of the field vars., e.g., a value of 10 for fine means that
% field vars. will be calculated at ten points in each ring. A
% value of 10 is generally a sufficient discretization.
%
% The rows of the ’tog’ vector contains either a ones or a zero
% depending on whether or not it is desired to calculate
% the field variables. The order of these toggles in ’tog’:
%        tog(1) ==> radial stress
%        tog(2) ==> radial strain
%        tog(3) ==> radial displacement
%        tog(4) ==> hoop displacement
%        tog(5) ==> hoop strain
%
% E.G., : stepica(1,3,10,[1 1 0 0 0]) calculates radial stress and
% radial strain at ten points in each ring, for rings 1 thru 3.
% The displacement field, hoop stress, and hoop strain are not
% calculated because the respective toggles are set to zero.
%
% See the comments in ‘example.m’ for a description of ‘rad’, ‘MATS’,
% ‘solve_at’, ‘RSIR’, and ‘RSOR’.

format long

%------- SOLUTION CODE BELOW, DO NOT EDIT -------
load loads.dat;
t=loads(:,1); % first column of loads.dat is time,
Omega=loads(:,2); % second column is rotational speed,
Temp=loads(:,3); % third column is temperature,
assy=loads(:,4); % fourth column indicates state of the assembly.
nurming=size(MATS,1); holder=rad(:,1:nurming*2);
clear rad; rad=holder; clear holder
rsigout=zeros(nurming*(fine+1),length(solve_at)); % initialize
repsout=zeros(nurming*(fine+1),length(solve_at)); % the output
urout=zeros(nurming*(fine+1),length(solve_at)); % matrices.
hsigout=zeros(nurming*(fine+1),length(solve_at));
hepsout=zeros(nurming*(fine+1),length(solve_at));

%_________________________________________________
for i = 1:length(Omega)-1, % calculate
    dOmega(i)=sqrt((Omega(i+1))^2-... % the dOmega
                   (Omega(i))^2); % and deltaT
    (Omega(i))^2); % the dOmega
%(Omega(i))^2); % and deltaT

%------- SOLUTION CODE BELOW, DO NOT EDIT -------
dtemp(i)=Temp(i+1)-Temp(i); % vector.
end

for m=1:size(MATS,1), % calculate the
    [eff_time,mat]=feval(MATS(m,:),t,Temp);
    eff_t(:,m)=eff_time;
end

for i=1:length(solve_at), % this finds index values in the
    ind(i)=find(t==solve_at(i));
    end % real time vector corresponding to
    end % the desired solution times

if tog(1)|tog(2)|tog(3)|tog(4)|tog(5) == 1, %
    n=1; %
    for m=1:size(MATS,1), % create a
        for mm = 0:fine, % radial position
            rpos(n)=rad(2*m-1)+(mm/fine)*...
            (rad(2*m)-rad(2*m-1)); % (for all
        n=n+1; % rings)
    end %
end %
end %
radhold=rad;

for i = 1:length(solve_at),
    j=1;
    while solve_at(i) > t(j), % this sets up the limits of summation
        % based on the real-times
            for m=1:size(MATS,1), % Effective material
                [tef,ring(:,m)]=...
                feval(MATS(m,:),eff_t(ind(i),m)-...
                eff_t(j,m),1); % during the
            end % time step t.i - t.j.
            Omega=dOmega(j); % incremental w
            deltaT=dtemp(j); % and deltaT.
            assd=assy(j+1); % State of the assembly
            if j>1,
                assc=assy(j);
            else
                assc=3; % dummy value
            end
            disphold(:,j)=bcbuild(rad,ring,... %
            numring,RSIR,RSOR,Omega,deltaT,assd,assc);
            end %

if tog(1) == 1, %
    rp=1;
    for m=1:size(MATS,1), %
        a=rad(2*m-1); b=rad(2*m);
        da=disphold(2*m-1,j); db=disphold(2*m,j); % radial
        for mm = 0:fine, % stresses
            rsig(rp,j)=da*dacof(rpos(rp),rad,ring,m)+... %
            db*dbcof(rpos(rp),rad,ring,m)+...
            remaind(rpos(rp),rad,ring,m,Omega,deltaT); %
rp=rp+1; %
end %
end %
end %

if tog(2) == 1, %
  rp=1; %
  for m=1:size(MATS,1), %
    a=rad(2*m-1); b=rad(2*m); %
    da=disphold(2*m-1,j); %
    db=disphold(2*m,j); % radial
    for mm = 0:fine, % strains
      reps(rp,j)=strainr(rpos(rp),a,b,da,...
      db,ring,m,Omega,deltaT); %
      rp=rp+1; %
    end %
  end %
end %

if tog(3) == 1, %
  rp=1; %
  for m=1:size(MATS,1), %
    a=rad(2*m-1); b=rad(2*m); %
    da=disphold(2*m-1,j); %
    db=disphold(2*m,j); % radial
    for mm = 0:fine, % displacements
      ur(rp,j)=raddisp(rpos(rp),a,b,da,...
      db,ring,m,Omega,deltaT); %
      rp=rp+1; %
    end %
  end %
end %

if tog(4) == 1, %
  rp=1; %
  for m=1:size(MATS,1), %
    a=rad(2*m-1); b=rad(2*m); %
    da=disphold(2*m-1,j); %
    db=disphold(2*m,j); % hoop
    for mm = 0:fine, % stresses
      hsig(rp,j)=hoops(rpos(rp),a,b,da,...
      db,ring,m,Omega,deltaT); %
      rp=rp+1; %
    end %
  end %
end %

if tog(5) == 1, %
  rp=1; %
  for m=1:size(MATS,1), %
    a=rad(2*m-1); b=rad(2*m); %
    da=disphold(2*m-1,j); %
    db=disphold(2*m,j); % hoop
    for mm = 0:fine, % strains
      heps(rp,j)=strainh(rpos(rp),a,b,da,...
      db,ring,m,Omega,deltaT); %
      rp=rp+1; %
    end %
  end %
end %
j=j+1;
end

[r,c]=size(disphold);
if c==1,
    dispout(:,i)=disphold;
else
    dispout(:,i)=sum([disphold]')';
end
clear disphold;
% ring boundary
% displacements at
% the load steps j
% are summed to give
% displacements at
% time of interest i.

if tog(1) ==1,
    [r,c]=size(rsig);
    if c==1,
        rsigout(:,i)=rsig;
    else
        rsigout(:,i)=sum([rsig]')';
    end
clear rsig time of interest i.
end
%
%
if tog(2)==1,
    [r,c]=size(reps);
    if c==1,
        repsout(:,i)=reps;
    else
        repsout(:,i)=sum([reps]')';
    end
clear reps
end
% ditto
% radial strains
%
if tog(3)==1,
    [r,c]=size(ur);
    if c==1,
        urout(:,i)=ur;
    else
        urout(:,i)=sum([ur]')';
    end
clear ur
end
% ditto
% field displacements
%
if tog(4)==1,
    [r,c]=size(hsig);
    if c==1,
        hsigout(:,i)=hsig;
    else
        hsigout(:,i)=sum([hsig]')';
    end
clear hsig
end
% ditto
% hoop stresses
%
if tog(5)==1,
    [r,c]=size(heps);
    if c==1,
        hepsout(:,i)=heps;
    else
        hepsout(:,i)=sum([heps]')';
    end
clear heps
end
% ditto
% hoop strains

end
rpos=rpos';
function [disps] = bcbuild(rad,ring,numring,RSIR,RSOR,Om,dT,assd,assc)

% 'bcbuild.m' is called exclusively from 'stepica.m', and its function
% is to build and solve the boundary condition equations for the ring
% boundary displacements, using either Eq 3.11 or 3.17 (as appropriate).
% These boundary displacements are then returned to 'stepica.m'

format long

LL=numring*2;
BC=zeros(LL); % initialize the BC matrix
RR=zeros(LL,1); % initialize the remainder matrix

%__________________________build BC and RR__________________________

if numring == 1, % if only analyzing one ring,
    a=rad(1); b=rad(2); % only have stress free inner and outer
    % radii
    BC(1,1)=dacof(a,rad,ring,1); % stress
    BC(1,2)=dbcof(a,rad,ring,1); % free
    RR(1)=RSIR-remaind(a,rad,ring,1,Om,dT); % radius
    BC(LL,LL-1)=dacof(b,rad,ring,numring); % stress
    BC(LL,LL)=dbcof(b,rad,ring,numring); % inner
    RR(LL)=RSOR-remaind(b,rad,ring,numring,Om,dT); % radius
end

if numring > 1 & assd == 0, % if analyzing multiple-ring
    % assemblies that have not been
    % assembled yet
    for i = 1:2:LL,
        a=rad(i); b=rad(i+1); % stress
        BC(i,i)=dacof(a,rad,ring,(i+1)/2); % free
        BC(i,i+1)=dbcof(a,rad,ring,(i+1)/2); % inner
        RR(i)=-remaind(a,rad,ring,(i+1)/2,Om,dT); % radius
        BC(i+1,i)=dacof(b,rad,ring,(i+1)/2); % stress
        BC(i+1,i+1)=dbcof(b,rad,ring,(i+1)/2); % outer
        RR(i+1)=-remaind(b,rad,ring,(i+1)/2,Om,dT); % radius
    end
end

if numring > 1 & assd == 1, % if analyzing multiple-ring
    % assemblies that are
    % assembled
    if assc == 0,
a=rad(1); b=rad(2); % stress
BC(1,1)=dacof(a,rad,ring,1); % free
BC(1,2)=dbcof(a,rad,ring,1); % inner
RR(1)=RSIR-remaind(a,rad,ring,1,Om,dT); % radius

a=rad(LL-1); b=rad(LL); % stress
BC(LL,LL-1)=dacof(b,rad,ring,numring); % free
BC(LL,LL)=dbcof(b,rad,ring,numring); % outer
RR(LL)=RSOR-remaind(b,rad,ring,numring,Om,dT); % radius

for i = 1:2:LL-3,
    BC(i+1,i+1)=-1; % displacement
    BC(i+1,i+2)=1; % compatibility
    RR(i+1)=rad(i+1)-rad(i+2);

    a=rad(i); b=rad(i+1); % stress
    BC(i+2,i)=dacof(b,rad,ring,(i+1)/2); % compat.
    BC(i+2,i+1)=dbcof(b,rad,ring,(i+1)/2); % conditions
    RR(i+2)=remaind(b,rad,ring,(i+1)/2,Om,dT); % ...

    a=rad(i+2); b=rad(i+3); % at
    BC(i+2,i+2)=dacof(b,rad,ring,(i+1)/2+1); % ring
    BC(i+2,i+3)=dbcof(b,rad,ring,(i+1)/2+1); % interfaces
    RR(i+2)=RR(i+2)+remaind(b,rad,ring,(i+1)/2+1,Om,dT);
end

else

    a=rad(1); b=rad(2); % stress
    BC(1,1)=dacof(a,rad,ring,1); % free
    BC(1,2)=dbcof(a,rad,ring,1); % inner
    RR(1)=RSIR-remaind(a,rad,ring,1,Om,dT); % radius

    a=rad(LL-1); b=rad(LL); % stress
    BC(LL,LL-1)=dacof(b,rad,ring,numring); % free
    BC(LL,LL)=dbcof(b,rad,ring,numring); % outer
    RR(LL)=RSOR-remaind(b,rad,ring,numring,Om,dT); % radius

for i = 1:2:LL-3,
    BC(i+1,i+1)=-1; % displacement
    BC(i+1,i+2)=1; % compatibility
    RR(i+1)=0;

    a=rad(i); b=rad(i+1); % stress
    BC(i+2,i)=dacof(b,rad,ring,(i+1)/2); % compat.
    BC(i+2,i+1)=dbcof(b,rad,ring,(i+1)/2); % conditions
    RR(i+2)=remaind(b,rad,ring,(i+1)/2,Om,dT); % ...

    a=rad(i+2); b=rad(i+3); % at
    BC(i+2,i+2)=dacof(b,rad,ring,(i+1)/2+1); % ring
    BC(i+2,i+3)=dbcof(b,rad,ring,(i+1)/2+1); % interfaces
    RR(i+2)=RR(i+2)+remaind(b,rad,ring,(i+1)/2+1,Om,dT);
end
end
A.4 A Sample Material Property File: ‘eglass.m’

Each of the material property subprograms perform two functions, depending on whether it is called (from ‘stepica.m’) with a single value or a vector of values in the time argument. In the first case, this subprogram will return a set of material properties evaluated at the time given in the time argument. In the latter case, the vector of input times (in real time, $t_j$, from the ‘loads.dat’) and the accompanying temperature at each of the times is converted to effective times, $\xi_j$, using a material-specific temperature-shift equation, $a_T$ (e.g., Eq. 2.10), and this effective time vector is then returned (to ‘stepica.m’).

```matlab
function [eff_t,mat] = eglass(t,Temp);
format long

% this material property file actually performs one of two
% functions depending on the format of the input time vector:
%
% 1. it returns an instantaneous material property vector, (mat(i)),
% given an input time value. this time value should
% be an effective time as calculated based on the thermal history.
%
% 2. It calculates an effective-time vector
% given a real-time history and the associated thermal history vector

if length(t) == 1 % for calculating item '1' above
    mat(1)=4.84e-005 + 2.3e-6*t^(.105); % S22 (1/GPa x 1e-3), form of Eq. 2.4
    mat(2)=-5.76923e-006; % S12 (1/GPa x 1e-3)
    mat(3)=1.9231e-005; % S11 (1/GPa x 1e-3)
    mat(4)=2.138e-9; % DENSITY (G/CC X 1e-9)
    mat(5)=25e-6; % TRANSVERSE CTE
    mat(6)=6.6e-6; % FIBER DIRECTION CTE
    mat(7)=80; % STRESS-FREE TEMPERATURE (CURE TEMP)

    mat=mat';
    eff_t=0; % pass a dummy effective time to stepica to avoid warning
else
    mat=0; % Pass a value of 'mat' to stepica to avoid warning

    eff_t(1)=0;
    aT=Zeros(length(t),1);
    for i = 2:length(t),
```

```
aT(i)=10^(0.18*(Temp(i)-30)); % Calculation of the
end
eff_t(i)=eff_t(i-1)+aT(i)*(t(i)-t(i-1)); % effective-time vector
% per item '2' above.
end
eff_t=eff_t';
end

A.5 ‘dacof.m’

function [value] = dacof(r,rad,mat,k)

% This function is the coefficient of the 'delta_a' term
% in the radial stress eqn 3.10 .

a=rad(2*k-1); b=rad(2*k); % inner and outer radii of the ring

format long

s11=mat(1,k); % transverse compliance
s12=mat(2,k); % cross term compliance
s14=mat(3,k); % fiber direction compliance

exp1=sqrt(s11/s14); %
cof1=(-s12+sqrt(s11)*sqrt(s14)); % local definitions
cof2=(s12+sqrt(s11)*sqrt(s14)); %
cof5=-s12^2+s11*s14; %

value=((a^exp1*r^((-1)-exp1)*...((b^(2*exp1)*cof2 + cof1*r^(2*exp1))))/...(((a^(2*exp1) - b^(2*exp1))*cof5));

A.6 ‘dbcof.m’

function [value] = dbcof(r,rad,mat,k)

% This function is the coefficient of the 'delta_b' term
% in the radial stress eqn 3.10 .

a=rad(2*k-1); b=rad(2*k); % inner and outer radii of the ring

format long

s11=mat(1,k); % transverse compliance
s12=mat(2,k); % cross term compliance
s14=mat(3,k); % fiber direction compliance

exp1=sqrt(s11/s14); %
cof1=(-s12+sqrt(s11)*sqrt(s14)); % local definitions
cof2=(s12+sqrt(s11)*sqrt(s14)); %
cof5=-s12^2+s11*s14; %

value=((b^exp1*r^((-1)-exp1)*...((a^(2*exp1)*cof2 + cof1*r^(2*exp1))))/...(((a^(2*exp1) - b^(2*exp1))*cof5)));
A.7 'remaind.m'

```matlab
function [value] = remaind(r,rad,mat,k,Omega,deltaT)
% This function is the remainder term (after delta_a x dacof and
% delta_b x dbcof have been subtracted from the radial stress eqn 3.10.
% a=rad(2*k-1); b=rad(2*k); % inner and outer radii of the ring

format long
s11=mat(1,k); % transverse compliance
s12=mat(2,k); % cross term compliance
s14=mat(3,k); % fiber direction compliance
Rho1=mat(4,k); % material density
alphar1=mat(5,k); % transverse-direction CTE
alphac1=mat(6,k); % fiber-direction

exp1=sqrt(s11/s14); %
cof1=(-s12+sqrt(s11)*sqrt(s14)); %
cof2=(s12+sqrt(s11)*sqrt(s14)); %
cof3=s11-9*s14; %
cof4=s11+s12; %
cof5=-s12^2+11*s14; % local definitions
cof6=s12+s14; %
cof7=s11-s14; %
cof8=-s12^2+3*s14; %
cof9=sqrt(s11)-3*sqrt(s14); %
cof10=sqrt(s11)/sqrt(s14); %
cof11=sqrt(s11)+3*sqrt(s14); %
cof12=sqrt(s11)+3*sqrt(s14); %

value=(((r^(( - 1) - exp1)*((( - a^(1 + exp1))*cof3*...(( - alphac1)*cof3*deltaT*((( - b^exp1)*cof2*r + b*cof4*r^exp1)) +...alphar1*cof3*deltaT*((( - b^exp1)*cof2*r + b*cof4*r^exp1)) +...cof2*cof7*((b^exp1)*cof8*r^3 - b^3*cof1*r^exp1))*Rho1*Omega^2)) +...a^(2*exp1)*cof2*(((alphac1*cof3*deltaT*(((b^1 + exp1)*cof4 + cof1*cof7*(b^2 + exp1)*cof2 + cof1*r^2)*Omega2)) +...alpha1*cof3*deltaT*((( - b^exp1)*cof2*r + b*cof4*r^exp1)) +...cof2*cof7*((b^exp1)*cof8*r^3 - b^3*cof1*r^exp1))*Rho1*Omega^2)) +...a^2*cof2*(((alphac1*cof3*deltaT*(((b^1 + exp1)*cof4 + cof1*r^2)*Omega2)) +...alpha1*cof3*deltaT*(((b^1 + exp1)*cof4 + cof1*r^2)*Omega2)) +...cof2*cof7*(((b^exp1)*cof8*r^3 - b^3*cof1*r^exp1))*Rho1*Omega^2)) +...a^2*cof2*(((alphac1*cof3*deltaT*(((b^1 + exp1)*cof4 + cof1*r^2)*Omega2)) +...alpha1*cof3*deltaT*(((b^1 + exp1)*cof4 + cof1*r^2)*Omega2)) +...cof2*cof7*(((b^exp1)*cof8*r^3 - b^3*cof1*r^exp1))*Rho1*Omega^2))); %
```

A.8 'strainr.m'

```matlab
function [rstrain] = strainr(r,a,b,da,db,mat,k,Omega,deltaT)
% This program calculates and returns radial strain at radial location 'r'

format long
S11=mat(1,k); % transverse compliance
S12=mat(2,k); % cross term compliance
S14=mat(3,k); % fiber direction compliance
Rho1=mat(4,k); % material density
alphar1=mat(5,k); % transverse-direction CTE
alphac1=mat(6,k); % fiber-direction

exp1=sqrt(S11)/sqrt(S14);
```
\[ p_1 = R_1 \Omega^2 \left( (r^{-1} - r) \right) \left( a^e b^e \left( \sqrt{S_{11}} \right) \right) \]

\[ p_2 = \left( r^{-1} - r \right) \left( a^e b^e \left( \sqrt{S_{11}} \right) \right) \]

\[ r_{strain} = p_1 + p_2; \]

A.9 'raddisp.m'

function [value] = raddisp(r,a,b,da,db,mat,k,Omega,deltaT)

% This program calculates and returns radial displacement at
% radial location 'r'

format long

S11=mat(1,k); % transverse compliance
S12=mat(2,k); % cross term compliance
S14=mat(3,k); % fiber direction compliance
Rho1=mat(4,k); % material density
alphar1=mat(5,k); % transverse-direction CTE
alphac1=mat(6,k); % fiber-direction

exp1=sqrt(S11)/sqrt(S14);

p1=Rho1*Omega^2*(((r^(-exp1))*((2*a^3*b^exp1)*((S11-S14))*(((-S12^2)+S11*S14))+2*a^3*b^exp1*(((S11^2)*S14-S12^2*S14+S11*((S12^2+S14^2))))+sqrt(S11)*(((-2)*a^(3+exp1)*((S11-S14))*(((-S12^2)+S11*S14))-2*a^(3+exp1)*(((S12^2+S14^2)))))*r^2))/((2*((a^(2*exp1)-b^(2*exp1)))*((sqrt(S11)-3*sqrt(S14)))*((sqrt(S11)-sqrt(S14)))*((sqrt(S11)+sqrt(S14)))*((sqrt(S11)+3*sqrt(S14))))*sqrt(S14)));

p2=(((r^(-1)-r)*((a^e b^e \sqrt{S_{11}})*((a^e b^e (S11-S14))-a^e b^e ...\]

\[ \text{pl}=801 \Omega^2 (((r^{-1}) - r^2) \left( a^e b^e \left( \sqrt{S_{11}} \right) \right) \]

\[ \text{pl}=801 \Omega^2 (((r^{-1}) - r^2) \left( a^e b^e \left( \sqrt{S_{11}} \right) \right) \]

\[ \text{p2}=(((r^{-1}) - r^2) \left( a^e b^e \left( \sqrt{S_{11}} \right) \right) \]

\[ \text{p2}=(((r^{-1}) - r^2) \left( a^e b^e \left( \sqrt{S_{11}} \right) \right) \]

\[ \text{value}=p1+p2; \]

A.10 'hoops.m'

function [value] = hoops(r,a,b,da,db,mat,k,Omega,deltaT)

% This program calculates and returns hoop stress at
% radial location ‘r’

format long

S11=mat(1,k); % transverse compliance
S12=mat(2,k); % cross term compliance
S14=mat(3,k); % fiber direction compliance
Rho1=mat(4,k); % material density
alphar1=mat(5,k); % transverse-direction CTE
alphacl1=mat(6,k); % fiber-direction

pl=Rhol*Omega^2*(((r^((-1)-sqrt(S11)/sqrt(S14))*((2*a^(3+sqrt(S11)/sqrt(S14))*...)
sqrt(S11)*((r^((2*sqrt(S11))/sqrt(S14))*((S12-sqrt(S11)*sqrt(S14)))+...
b^((2*sqrt(S11))/sqrt(S14))*((S12-sqrt(S11)*sqrt(S14)))))*((S11-S14))*...)
(((-S12^2)*S11*S14)-b^2*(sqrt(S11)/sqrt(S14))/sqrt(S11))*r^2*(sqrt(S11)/sqrt(S14))*...)
(((S11-S14)^2)-b^2*(sqrt(S11)/sqrt(S14))/sqrt(S11))*r^2*(sqrt(S11)/sqrt(S14))*...)

p2=((r^((-1)-sqrt(S11)/sqrt(S14))*((a^2)^2*(sqrt(S11)/sqrt(S14)))*...)
((S12+sqrt(S11)*sqrt(S14)))*r^2*(sqrt(S11)/sqrt(S14))*...)
((S11-S14)^2)-b^2*(sqrt(S11)/sqrt(S14))/sqrt(S11))*r^2*(sqrt(S11)/sqrt(S14))*...)

value=pl+p2;

A.11 'strainh.m'

function [rstrain] = strainr(r,a,b,da,db,mat,k,Omega,deltaT)
% This program calculates and returns hoop strain at
% radial location ‘r’

format long

S11=mat(1,k); % transverse compliance
S12=mat(2,k); % cross term compliance
S14=mat(3,k); % fiber direction compliance
Rho1=mat(4,k); % material density
alphar1=mat(5,k); % transverse-direction CTE
alphac1=mat(6,k); % fiber-direction

expl=sqrt(S11)/sqrt(S14);

p1=8bol*Omega^2*(((r^((-1)-expl))*((a^expl*b)^expl*sqrt(S11)*((2*a^expl*b)^3*(S11-S14))*((-S12^2)*...
S14-S14)+S14-S14+S14+((S12^2+S14^2))))+sqrt(S11)*...
((-2)*a^((3+expl))*(S11+3*S14)+((-S12^2)+S11+3*S14)-2*b^-((3+expl))*((S11+3*S14)+S14-S14+((S12^2+S14^2))))+r^((2+expl)*6)*((a^((2+expl)-b)^((2+expl)))*(S11-S14)*... 
 sqrt(S14)*((S12+2*S14)*S14)*r^-((3+expl)))/((2^((a^((2+expl)-b)^((2+expl)))(sqrt(S11)+3*sqrt(S14)))*... 
((sqrt(S11)-sqrt(S14))*)(sqrt(S11)+sqrt(S14)))*((sqrt(S11)+3*sqrt(S14)))*sqrt(S14))); 

p2=(((r^((-1)-expl))*(((-a^expl)*b^expl*sqrt(S11)*((a^expl*db*((S11-S14))-a^expl*b*deltaT*((alphac1*((S11+S12))-alphar1*((S12+S14))))+b^expl*(((S11-S14)))+(a^expl*da*((S11-...S14)))+a^(-1+expl)*da*((alphac1*((S11+S12))-alphar1*((S12+S14)))))/(S11-S14)+b^expl*(((alphac1*((S11+S12))-alphar1*((S12+S14)))))*r^((2+expl)*... 
((a^((2+expl)-b)^((2+expl))))*deltaT*sqrt(S14)*((alphac1*((S11+S12))-alphar1*((S12+S14))))))))*r^-((1+expl))));

rstrain=p1+p2
Appendix B

ERROR CALCULATION FOR THE QUASI-ELASTIC APPROXIMATION

Schapery [1965] developed several techniques for deriving approximate solutions for structural models (such as flywheel rotors) that incorporate anisotropic linear viscoelastic PMCs. In one of these techniques, the elastic-viscoelastic correspondence principle (EVECP) was used to arrive at Laplace-domain forms of the stress and strain field equations, and the direct-inversion method was used to transform these Laplace-domain equations to the time-domain (where the stresses and strains were calculated at some “time of interest”). Schapery presented an error calculation for the direct-inversion method whereby the solved function of interest, \( f \), (e.g., stress or strain at some point in the model) is plotted versus \( \log_{10}(t) \), due to a step input load:

“When \( f \) is plotted against \( \log(t) \) and a tangent is drawn to \( f \) at any point, the net algebraic area \( A_{\Delta} \) enclosed by the tangent line, the function \( f \), and about three-fourths to one decade on each side of the tangency point should be small relative to the area \( A_T \) under \( f \) in the same interval. If \( f \) has constant curvature over a 1.8 decade interval, one can show that the relative error in \( f \) is essentially equal to the area ratio \( A_{\Delta}/A_T \) for this interval.” -- [Schapery, 1967]

Referring to this “small curvature condition” in the same paper, Schapery states, “…most viscoelastic materials … are characterized by operational moduli whose variation is spread smoothly over several decades… This spread usually provides the small curvature condition.” The term “operational moduli” can equivalently be replaced by “power-law forms of creep compliances” in the last sentence (i.e. the characterization of the materials of interest in this thesis).

Because the quasi-elastic calculations (used for the rotor model in this thesis) for deriving approximate viscoelastic solutions is derived from (and, more relevantly, algebraically proportional) to the direct-inversion method described above, the above error calculation is performed below as an approximation of the error associated with the quasi-elastic technique. The example used for this calculation consists of a pair of E-
glass/Dow 383 rings subjected to a step-load of interference-fit, rotation, and temperature change. The inner ring has inner and outer radii of 100 mm and 133.33 mm, respectively, and the outer ring has inner and outer radii of 132.33 mm and 176.44 mm, respectively. The load history used for the simulation is listed in Table B.1.

Table B.1. Load history used in the example of the quasi-elastic error approximation.

<table>
<thead>
<tr>
<th>Notes</th>
<th>$t$ (min.)</th>
<th>$T$ (°C)</th>
<th>$\omega$ (rad/s)</th>
<th>B.C. Eq.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>80</td>
<td>0</td>
<td>3.11</td>
</tr>
<tr>
<td>Menufacturing cooldown load</td>
<td>0.001</td>
<td>50</td>
<td>0</td>
<td>3.11</td>
</tr>
<tr>
<td>Impose spin and interference-fit loads</td>
<td>0.003</td>
<td>50</td>
<td>3400</td>
<td>3.17</td>
</tr>
<tr>
<td>Loads held constant over this period, and solved (for the plots, below) every half–decade</td>
<td>0.010→100000</td>
<td>50</td>
<td>3400</td>
<td>3.17</td>
</tr>
</tbody>
</table>

The radial stress distribution in the assembly at $t=0.01$ minutes is shown in Fig. B.1. The locations where radial stresses are plotted versus $\log_{10}t$ (for the error calculation) are the mid-radii of each ring—indicated by circles in Fig. B.1 and plotted in Fig. B.2. To better illustrate the curvature of the plots in Fig. B.2, the data are re-plotted in Fig. B.3.

![Figure B.1. Radial stress at $t=0.01$ min in the example of the quasi-elastic approximation.](image-url)

Locations where radial stress will be plotted versus $log_{10}t$
Figure B.2. Radial stress versus time at two locations in the example of the quasi-elastic approximation.

Figure B.3. Close-up plot of the radial stress data from Fig. A2.2.

The 1.8-decade length of time used for the error estimation is chosen to be centered at $\log_{10} t = 4$. The areas used as $A_A$ for Schapery’s error calculation are shown as gray shaded areas in Fig. B.4.
Figure B.4. Areas used as $A_A$ in Schapery’s quasi-elastic error calculation.

The two (gray) $A_A$ in Fig. B.4 are approximately 0.01 MPa-decades in area. The two $A_T$ in the same interval are approximately 24 and 38 MPa-decades in area. The relative errors in radial stress then, according to Schapery’s estimation $A_A/A_T$ given above, are 0.04% and 0.03%.

Using this same technique, the errors for the other field variables (radial strain, displacement, hoop stress and strain) were also calculated. The largest error in these field variables was also less than 0.1%. In conclusion, the errors associated with using the quasi-elastic solution technique for the flywheel problems in this thesis are reasonably small according to Schapery’s “small curvature” estimation procedure.
Appendix C

DRAWINGS OF GRIPS FOR CREEP TESTS

All units in inches

SHELL

Drilled & tapped through for 10-32, 2 places
1/64 drill through, 4 places
45 deg

Bottom view

Hole for 1/16 Dowel pin, 2 places

Section A-A

Figure C.1. “Shell” component of the creep tube grips.
Figure C.2. “Insert” component of the grips used for the tubular specimens.
Figure C.3. Assembly drawing of “Shell” and “Insert”.
### Appendix D

**MATERIAL PROPERTIES**

Table D.1 Mechanical and physical properties of the materials used in this document.

<table>
<thead>
<tr>
<th>Material</th>
<th>$S_{11}$ (1/GPa)</th>
<th>$S_{12}$ (1/GPa)</th>
<th>$S_{22}$ (1/GPa)</th>
<th>$E_\text{tensile}$ (GPa)</th>
<th>$\nu_{12}$</th>
<th>$\rho$ (kg/m$^3$)</th>
<th>$\alpha_T$</th>
<th>$V_f$ (%)</th>
<th>$V_m$ (%)</th>
<th>$V_v$ (%)</th>
<th>$T_{ref}$ (°C)</th>
<th>$\alpha_1$ ($10^{-6}$/°C)</th>
<th>$\alpha_2$ ($10^{-6}$/°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum 6061-T6 alloy</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>68.3</td>
<td>0.33</td>
<td>2700</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>23.6</td>
<td>23.6</td>
<td></td>
</tr>
<tr>
<td>Aluminum 2024-T351</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>71.7</td>
<td>0.33</td>
<td>2780</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>23.2</td>
<td>23.2</td>
<td></td>
</tr>
<tr>
<td>316-Stainless Steel</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>200</td>
<td>0.30</td>
<td>7900</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>16.6</td>
<td>16.6</td>
<td></td>
</tr>
<tr>
<td>Aged E-glass/Dow 383</td>
<td>0.0192$^1$</td>
<td>-0.00577</td>
<td>0.0484+0.0023$^{10_5}$</td>
<td>-</td>
<td>0.30</td>
<td>2138</td>
<td>$10^{0.18(T-T_{ref})}$</td>
<td>69.0</td>
<td>2.0</td>
<td>29</td>
<td>30</td>
<td>6.6</td>
<td>25</td>
</tr>
<tr>
<td>Aged T700/Dow 383</td>
<td>0.00654$^1$</td>
<td>-0.00196</td>
<td>0.090+0.0045$^{10_5}$†</td>
<td>-</td>
<td>0.30</td>
<td>1520</td>
<td>$10^{0.18(T-T_{ref})}$</td>
<td>66.3</td>
<td>4.4</td>
<td>29.3</td>
<td>30</td>
<td>-2.3</td>
<td>32.3</td>
</tr>
<tr>
<td>Adiprene L100/Caytur 21</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>7.7</td>
<td>0.42</td>
<td>1100</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>50$^2$</td>
<td>50$^2$</td>
<td></td>
</tr>
</tbody>
</table>

$^1$estimated from rule-of-mixtures

$^2$estimated

$^†$estimated such that the time-dependent behavior is proportional to that of E-glass/Dow 383
Table D.2. Strength properties of the materials used in this document.

<table>
<thead>
<tr>
<th>Material</th>
<th>Tensile Strength, fiber direction (MPa)</th>
<th>Tensile Strength, transverse direction (MPa)</th>
<th>Compressive Strength, fiber direction (MPa)</th>
<th>Compressive Strength, transverse direction (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum 6061-T6 alloy</td>
<td>275&lt;sup&gt;a&lt;/sup&gt;</td>
<td>275&lt;sup&gt;a&lt;/sup&gt;</td>
<td>275&lt;sup&gt;a&lt;/sup&gt;</td>
<td>275&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>Aluminum 2024-T351</td>
<td>325&lt;sup&gt;a&lt;/sup&gt;</td>
<td>325&lt;sup&gt;a&lt;/sup&gt;</td>
<td>325&lt;sup&gt;a&lt;/sup&gt;</td>
<td>325&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>316-Stainless Steel</td>
<td>931&lt;sup&gt;a&lt;/sup&gt;</td>
<td>931&lt;sup&gt;a&lt;/sup&gt;</td>
<td>931&lt;sup&gt;a&lt;/sup&gt;</td>
<td>931&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>Aged E-glass/ Dow 383</td>
<td>1400&lt;sup&gt;b&lt;/sup&gt;</td>
<td>45.6</td>
<td>-1400&lt;sup&gt;c&lt;/sup&gt;</td>
<td>-144.5</td>
</tr>
<tr>
<td>Aged T700/ Dow 383</td>
<td>2610&lt;sup&gt;b&lt;/sup&gt;</td>
<td>54.8&lt;sup&gt;d&lt;/sup&gt;</td>
<td>-2610&lt;sup&gt;c&lt;/sup&gt;</td>
<td>-140</td>
</tr>
<tr>
<td>Adiprene L100/ Caytur 21</td>
<td>20-30&lt;sup&gt;e&lt;/sup&gt;</td>
<td>20-30&lt;sup&gt;e&lt;/sup&gt;</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

<sup>a</sup>yield
<sup>b</sup>hoop strength calculated by multiplying hoop modulus by fiber elongation given in composite form given by fiber manufacturers.
<sup>c</sup>estimated
<sup>d</sup>from a four-point bend test
<sup>e</sup>estimated room temperature strength from the manufacturer.
Appendix E

DESIGN AND ASSEMBLY OF THE ROTOR HUB

The design of the hub assembly used for the 2024-T351-aluminum alloy and E-glass/Dow 383 rotors discussed in Chapter 5 is presented in this section. The dimensions for the hub components are given and the features that permit alignment and balancing are discussed. The stress analyses of the urethane interlayers between the hubs and the rotors are presented, and the steps for assembling the rotors and hubs are also described.

E.1 Hub Design

E.1.1 Hub Dimensions

An exploded-view illustration of the hub assembly, consisting of a 6061-T6-aluminum alloy hub and a 316-stainless steel sleeve, is shown in Fig. E.1. The outer radius of the hub (and, therefore, the radial thickness of the polyurethane interlayer connecting the hub to the above-mentioned rotors—both of which had a 60-mm inner radius) was determined using an elastic analysis of the rotor/hub assembly (given in Section E.1.2). The radial thicknesses for the Adiprene-L100/Caytur-21 (Crompton Corporation – Uniroyal Chemical, Middlebury, CT) polyurethane interlayer for the aluminum alloy and E-glass/Dow 383 rotors were 3.6 and 3.2 mm thick, respectively.

The 0.02-mm radial interference at the interface of the sleeve and hub ensured compressive radial stress at this interface at operational speeds. The 1-degree taper at this interface was shallow enough that the frictional force developed at the interface between the steel sleeve and the aluminum hub was larger than the (opposing) axial component of the radial pressure at the interface (otherwise the sleeve would “pop out”). Two-part, five-minute epoxy, which was applied to the interface between the sleeve OD and the hub ID with the primary purpose of lubrication during the press-fit of the two components, provided an extra measure of adhesion between the components. If a more robust sleeve-to-hub connection is desirable, a washer that spans the underside of the hub
could be bolted to the three axially oriented 8-32 holes in the sleeve. The thru-hole in the bottom of the steel sleeve and the three radially-oriented 8-32 set screws serve to attach the hub to the shaft, which has a 0.250-inch outer diameter.

The hub incorporates features for aligning the hub/rotor assembly with the spin axis of the shaft. By machining the bore in the steel sleeve larger than the shaft by roughly 0.001 inch (25 μm), the three radially-oriented 8-32 set screws can be independently tightened to eliminate axial runout as measured with a dial indicator on surface “A” in Fig. E.1. In this way, the inevitable mass imbalance of the hub/rotor assembly is moved closer to a plane perpendicular to the shaft spin axis, and a simple single-plane balancing procedure [Ehrich 1999] can be used prior to operation.

A manufacturing technique was developed to assist in the geometric alignment of the hub/sleeve assembly with the spin axis of the shaft. This technique requires the hub to be initially machined with slightly oversized axial and radial dimensions. Once the hub/sleeve assembly was mounted on the drive shaft on which it will spin during operation, the inevitable axial (or “wobble”) runout was measured with a dial indicator on the surface “A” on Fig. E.1, and recorded on the hub with a marker. Likewise, the radial runout was measured with a dial indicator on either surface “B” or “C”. It is noteworthy
that the relative angular positions (about the spin axis) of the shaft and hub/sleeve assembly generally affect the magnitudes of the runouts. It was often possible at this point to determine an optimal relative angular position such that one or both types of runout were minimized or eliminated. This position was recorded with a marker on the shaft and hub/sleeve. The hub/sleeve assembly was then chucked on a lathe with a 4-jaw chuck and the position of the hub/sleeve was until the runouts matched the runouts recorded while the assembly was mounted to the shaft. The hub/sleeve assembly was thus essentially chucked with an alignment matching that when it was mounted to the shaft. Material subsequently removed from the hub (to achieve final dimensions), therefore, results in a hub with an outer radius that is concentric with the spin axis of the shaft and axial faces that are perpendicular to the spin axis, provided the hub/sleeve assembly is re-attached to the shaft as marked earlier.

E.1.2 Stress Analyses of the Rotor /Hub Assemblies

The rotor model was used to perform elastic analyses of the stresses in the Adiprene-L100 polyurethane interlayers of the aluminum rotor/hub and E-glass/Dow 383 rotor/hub assemblies after they were cooled from 100°C—the temperature at which the urethane interlayers were cast and cured—to 23°C (i.e. $\Delta T = -77^\circ$C). It is assumed that the stress-free reference temperature for the urethane interlayer in these assemblies is 100°C, though some of the radial and hoop stresses that develop in the interlayer during cool-down will relax before the assembly is spun. The model was also used to calculate the stresses due to a spin load of 16 krpm superposed on the temperature load.

Radial and hoop stresses for the aluminum rotor/hub assembly are shown in Fig. E.2. The stresses in the interlayer are practically independent of temperature in this assembly, as the radial and hoop stresses due to the cooling load are practically zero throughout. With the superimposed spin load of 16 krpm, only a very small amount of tensile hoop and radial stress is developed at the interface with the rotor (at $r = 60$ mm).
The stresses in the E-glass/Dow 383 rotor/hub assembly are plotted in Fig. E.3 for the same set of loading conditions. Small amounts of tensile radial and hoop stresses develop in this assembly due to the $\Theta=-77^\circ$C temperature change, and these increase only slightly at $r=60$ mm due to the additional spin load of 16 krpm.

**E.2 Assembly of the Rotor and Hub**

For both the aluminum alloy and the E-glass/Dow 383 rotors, a polyurethane interlayer (Adiprene L100) was cast directly in the gap between the rotor and the hub components. A heated aluminum plate with a 1500-watt ring heater placed on a turntable provided a means to align the components immediately prior to casting the interlayer, and served as a temporary seal for the uncured polyurethane during casting. Figure E.4 shows pictures of the setup used to make the hub and E-glass/Dow 383 rotor. A centrally-located 10-32 threaded rod was used to clamp the hub and rotor to the heated
plate. The turntable and heated plate are more clearly illustrated in Fig. E.5. Axial runout of the heated plate was eliminated by adjusting three 3/8-16 bolts with nuts positioned between the turntable and plate. A bubble indicator was used for leveling the turntable and ensuring an interlayer of uniform axial thickness.

Figure E.4. Setup used to align and cast the polyurethane interlayer between the hub and rotor.
To enhance the bond strength of the polyurethane, a thin coating of Lord Corp. Chemlok 213 was applied to the hub OD and the rotor ID. After conditioning the Chemlok according to the manufacturer's instructions, the components were lightly clamped to the turntable and tapped until the OD of the hub and the ID of the rotor were concentric to within 0.0005 inch (12 \( \mu \)m). The Adiprene L100 prepolymer and Caytur 21 curative (20% by wt.) were pre-heated and mixed at 40°C and the rotor/hub assembly was heated to 50°C to maintain the low viscosity of the urethane as it flowed into the radial gap. A stream of the urethane was injected into the radial gap with a small plastic syringe. Several small air bubbles that formed during injection were popped with a steel wire immediately after the interlayer was cast (but still liquid). For curing, the temperature of the heated plate was increased to the recommended cure temperature of 100°C. A blanket was placed over the entire setup for insulation. After the interlayer cured for 24 hours, the temperature was slowly decreased in 10°C increments over an additional 24 hour period to room temperature to enhance the relief of residual stresses in the interlayer.
Appendix F

ELECTRICAL CONNECTIONS OF THE SPIN PIT INSTRUMENTATION

This appendix shows the locations of the different types of instrumentation that have been installed on the PSU spin pit. These instrumentation are divided into “Basic Spin Pit Instrumentation” (tachometers, inductive position sensors for vibration measurement, thermocouples), and OESM instrumentation. Schematics of the electrical connections (power supplies and signals) to the various instruments and the data acquisition are also given.

F.1 Basic Spin Pit Instrumentation

Figure F.1 shows a schematic illustration of the layout of the spin pit hardware and basic instrumentation. Figure F.2 shows a picture of the hardware and instrumentation (temporarily pulled out of the bore of the lid) on the top-side of the vacuum barrier.
F.2 Electrical Connections for the Basic Spin Pit Instrumentation

F.2.1 Tachometer

The non-contact optical tachometers used for speed measurement were constructed by adhering an LED and a logic detector to the head of a 1/4-20 hex head bolt (convenient for mounting purposes). The radiation emitted by the LED illuminates a rotating reflective target (e.g., a metal shaft or hub) and then is reflected back into the detector. A black mark (e.g., from a permanent marker) on the rotating target provided modulation of the reflected light. Power to the LED was varied (e.g., by a variable power supply, or with a potentiometer) such that the incident light was detectable by the logic detector. The logic detector output (either zero or 10 volts) was then be connected to a standard digital tachometer gage and/or a data acquisition channel. The specifications on the LED emitter and logic detector are listed in Table F.1. In practice, for the two types
of LEDs listed in Table F.1, the necessary voltage is in the 1.1 to 1.3 V range. Figure F.3 shows a picture of an assembled tachometer.

![LED emitter and logic detector](image)

Figure F.3. An assembled optical tachometer.

**F.2.2 Inductive position sensor**

The inductive position sensors (Fargo Controls, Inc. \(^1\)) used on the PSU spin pit setup consist of a threaded barrel (with internal circuitry) that is mounted such that it points perpendicularly at the target. A flat piece of mild steel has been found to provide the largest change in output current with change in sensor position (i.e., the most sensitive position measurements). To provide a voltage output suitable for a data acquisition channel (0-5 volts), a 330 \( \Omega \) load is placed between the sensor (current) output and electrical ground. The voltage across the resistor (in the 0-5 VDC range) is then used for data acquisition. Fig. F.4 shows a picture of the inductive position sensor and the tag provided with the sensor, which schematically illustrates the electrical connections. Because the current output is significantly non-linear with position, a calibration is always required. A typical calibration curve is shown in Fig. F.5. The sensor output is also temperature-dependent.

![Inductive position sensor and tag](image)

Figure F.4. Picture of Fargo Controls, Inc. position sensor, and electrical connection tag.
F.2.3 Thermocouples

Thermocouples (J-type contact and non-contact infrared) were placed at locations shown in Fig. F.1. These were connected to a thermocouple data acquisition board (EXP-16/DAS 801, Computer Boards, Inc.\(^2\)), which was used to monitor the temperatures during the spin tests.

F.3 OESM Instrumentation

A detailed drawing of the optical sensor is given in Fig. 5.10. Figure F.6 below shows a schematic of the electrical connections for the sensor.

An arrangement of switches was used to connect the powers and signals from the OESM sensors (in the pit) to the data acquisition board (outside of the pit). The switches were needed because the available data acquisition board had only two counter channels—meaning that a maximum of two streams of pulse-widths can be measured

---

\(^1\) Fargo Controls, Inc., Eatontown, NJ 07724
\(^2\) Measurement Computing, Middleboro, MA 02346
simultaneously. The tachometer occupies one of the counters, leaving only one counter available for pulse-width measurement of the reflective patches on the rotor (for displacement measurements). Because a total of eight sensors were used during rotor strain-measurement experiments, the switches provided the means to connect all of the sensor outputs to the one available counter.

Switches also were used to provide power to only one sensor at a given time (while that particular sensor was being used to make pulse-width measurements) instead of powering all sensor electronics during a spin test. Sequential energizing of the sensors is advantageous because the LEDs and logic detectors generate heat. During spin-test experiments that are run in a vacuum, the heat generated from these components is conducted to the other structural components in the pit. This is undesirable, because the thermal expansion of these structural components causes motion of the sensors—which is manifested as false rotor displacement. Temporarily switching on only one sensor at a time minimizes such heating. Turn-on transients of the sensors used in vacuum in the present research were found to be 15 to 30 seconds.

Because the number of conductors in the feed-throughs was limited, an arrangement of relays (see Table F.1 for relay specifications) inside the pit was used to reduce the number of required conductors. Besides an electrical ground, each sensor requires three conductors—one to power to the LED, a second to power the photodetector, and a third to carry the logic signal out to the data acquisition board. With eight sensors in the pit, this is a total of 24 conductors. By toggling the relays between either of two states, 12 conductors in the feedthrough can intermittently connect 24 conductor paths. Fig. F.7 shows a schematic of the relay connections in both of the two states. Descriptions of the pin abbreviations are given in Table F.2.
The voltage to the IR LEDs was controlled with an analog voltage output channel from a PCI-6024E board\(^3\). Because the LEDs draw more than 5 mA of current (which is the current limit of each of the two analog-out channels), it was necessary to create an LED driver circuit. This circuit is shown in Fig. F.8. The typical control voltage was approximately 6 VDC.

![Infrared LED driver circuit](image)

**Figure F.8.** Infrared LED driver circuit.

As a side note, a counter board with more channels could be used to circumvent the need for switching. The maximum number of counters on currently available boards is eight—which would allow one tachometer and seven strain sensor measurements without switching. However, the continuous processing of pulse-widths for just two counter streams places a significant burden on the processor of a present-day computer, limiting the (present) viability of making more simultaneous measurements.

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\(^3\) National Instruments, Austin, TX.
Table F.1. Specifications for the spin pit electronics.

<table>
<thead>
<tr>
<th>Part</th>
<th>Manufacturer / part no.</th>
<th>Power requirement</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tachometer emitter</td>
<td>Panasonic / (Digikey p.n. LN54PA-ND)</td>
<td>1.1-1.2 VDC</td>
<td>17-deg. beam angle, 4.6 mW, 950-nm peak emission</td>
</tr>
<tr>
<td>Tachometer and OESM photodetector</td>
<td>QT Optoelectronics / (Digikey p.n. QSE157QT-ND)</td>
<td>4-16 VDC</td>
<td>±25-deg. reception, 70-ns rise/fall time, 935 peak wavelength sensitivity</td>
</tr>
<tr>
<td>Potentiometer</td>
<td>Bourns Inc. / (Digikey p.n. 3006P-101-ND)</td>
<td>0-100 Ω</td>
<td></td>
</tr>
<tr>
<td>Position Sensor</td>
<td>Fargo Controls I-AOTM/XIP</td>
<td>18-30 VDC</td>
<td></td>
</tr>
<tr>
<td>Opto emitter</td>
<td>Photonic Detectors / (Digikey p.n. PDI-E805)</td>
<td>1.2 - 1.5 VDC</td>
<td>5-deg. beam angle, 24 mW, 880 nm peak emission</td>
</tr>
<tr>
<td>transistor (npn)</td>
<td>Radio Shack TIP3055</td>
<td>-</td>
<td>Needs a heat sink and about 30 seconds to warm up</td>
</tr>
<tr>
<td>Opto relay</td>
<td>Aromat p.n. TQ2E-L-12V</td>
<td>±12 V</td>
<td>Latching type, DPDT</td>
</tr>
</tbody>
</table>

Table F.2. Description of the pin abbreviations in Fig. F.7.

<table>
<thead>
<tr>
<th>Pin</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CP</td>
<td>Coil power (either +/- 12 V)</td>
</tr>
<tr>
<td>CG</td>
<td>Coil ground</td>
</tr>
<tr>
<td>LPA1 / A2 / A9 / A10</td>
<td>LED power to A1 / A2 / A9 / A10 sensor</td>
</tr>
<tr>
<td>LPB1 / B2 / B9 / B10</td>
<td>LED power to B1 / B2 / B9 / B10 sensor</td>
</tr>
<tr>
<td>LP1 / 2 / 3 / 4</td>
<td>LED power from power supply, switch at 1 / 2 / 3 / 4 position</td>
</tr>
<tr>
<td>PPA1 / A2 / A9 / A10</td>
<td>Photodetector power to A1 / A2 / A9 / A10 sensor</td>
</tr>
<tr>
<td>PP1 / 2 / 3 / 4</td>
<td>Photodetector power from power supply, switch at 1 / 2 / 3 / 4 position</td>
</tr>
<tr>
<td>SA1 / A2 / A9 / A10</td>
<td>Photodetector from A1 / A2 / A9 / A10 sensor</td>
</tr>
<tr>
<td>S1 / 2 / 3 / 4</td>
<td>Photodetector to counter, switch at 1 / 2 / 3 / 4 position</td>
</tr>
</tbody>
</table>
Figure F.7. Relay electrical connections used in the high-speed spin test setup.
Appendix G
OESM LABVIEW CODE

The computer program that was part of the OESM system was written in LabView version 5. A flowchart of the functions of this VI is given in Fig. G.1. The wiring diagram for the LabView VI is shown in Fig. G.2(a). The three dashed boundaries superposed over the wiring diagram in Fig. G.2(a) indicate portions of the diagram that are detailed in Figs. G.2(b-e). Figures G.3-4 are sub-VIs that are called from the wiring diagram in Figure G.2, and the front panel is shown in Fig. G.5.

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Fig. G.1 Flowchart for “optical5.vi” LabView program.
Fig. G.2(a). LabView wiring diagram for “optical5.vi”
Figure G.2(b) shows the portion of the complete “optical5.vi” wiring diagram that contains the code for controlling the counter/timer circuits of the 6024E board in order to measure the pulse widths from the optical sensor. This portion of the code (indicated in Fig. G.2(b) by the box labeled “Pulse Width Measurement”) came from the LabView 5.0 counter example “Measure Buffered Pulse.vi”. Using the digital counters precluded the ability to trigger the measurement of the pulse stream starting at the same pulse, e.g., the pulse from the displacement patch at the zero-degree azimuth. Hence, the displacement-patch and compensation-patch pulses are always initially acquired from the stream of pulses (output from the sensor) in a sequential order with a random start point.

The pulse widths, in units of seconds, are converted to angles (four calibration-patch angles and four displacement-patch angles) by multiplying them with the instantaneous RPMx6 (code shown near the box labeled “Tachometer”). The sub-VI shown by the box labeled “SORT angles” (see Fig. G.3) contains this factor of six but, more importantly, serves to provide an order to the buffered pulse stream. This “SORT angles” sub-VI relies on the compensation patches of different size (3, 3, 4, and 5 deg.) that were incorporated in the pattern for this purpose.

The eight individual angles are extracted from the two angle arrays (each of size 4x1) in the upper right of Fig G.2(b). At this point, the angles are ready for buffering (see Fig. G.1 “Start buffering data?”) if the operator decides to initiate an acquisition event. If the acquisition is initiated (by pressing the “Grab and Display 20 values” button on the front panel), these individual data angles are used to populate eight 20x1 arrays shown in the while-loop labeled “Grab and Display 20 values” in the upper left corner of Fig. G.2(c). This while loop also samples the analog channel corresponding to the inductive sensor that gives the position of either the “A” or “B” rack (changes in this position during a test correspond to urack used in Eq. 6.5). Though not shown, the “false” state of this while loop in Fig. G.2(c) does not allow the buffered angle data to pass through. If, however, the “Grab and Display 20 values” loop is allowed to finish, the data do pass through and are available to the rest of the VI. If the operator desires, this collected data can be used as a data point (see “Use this data as a sample?” in Fig. G.1). In this case, the front-panel button labeled “write to raw data file” is pressed, which engages the larger “while” loop shown at the bottom of Fig. G.2(c) to calculate the mean value and standard deviation of
the median 10 values in each of the eight angles, as well as the mean value and standard
deviation in the rack position sensor reading. These “raw data” are then written
(appended) to a tab-delimited ASCII file, which is named on the front-panel by the
operator. A time stamp (computer system time) is also included in this file.

After the raw data has been calculated as described above, it is available to the
portion of the code shown in Fig. G.2(d). This part of the VI processes the raw angle
data into displacement data by “looking-up” the instantaneously measured angles in the
calibration files for the sensors. Calibration must always be performed beforehand, and
part of a sample calibration file is given in Table G.1. The while-loop in Fig. G.2(d) is
executed by the “update pos and vibe” button on the front panel. Compensations for
sensor intensity (Eqs. 4.11-4.13) are performed within this while-loop. Given the
instantaneous compensated displacements, the flexible and rigid-body displacements are
calculated and then displayed on the front panel in terms of flexible-body displacement
and vibration amplitude and phase. For this calculation, the “phase and amp.vi” (shown
as a box on the right side of the while-loop in Fig. G.2(d), and shown explicitly in Fig.
G.4) takes advantage of the built-in LabView fast-Fourier-transform function to
effectively fit the displacement data to the sine function given in Eq. 4.14. To calculate
instantaneous flexible body displacement, it is necessary to specify a reference position
for each sensor (see Eq. 4.10). These reference positions are specified by the operator on
the front panel as “r0 for all sensors.” All processed displacements, as well as the
vibration amplitude and phase, are written to a second tab-delimited ASCII file with a
name specified by the operator.
Fig. G.2(b). LabView wiring diagram for “optical5.vi”.
Fig. G.2(c). LabView wiring diagram for “optical5.vi”.
Fig. G.2(d). LabView wiring diagram for “optical5.vi”.
Figure G.3. LabView wiring diagram for “SORT angles.vi”
Table G.1. A sample calibration file (for input to “optical5.vi”.

<table>
<thead>
<tr>
<th>$\phi_0$ (deg.)</th>
<th>$\phi_{90}$ (deg.)</th>
<th>$\phi_{180}$ (deg.)</th>
<th>$\phi_{270}$ (deg.)</th>
<th>$\theta_0$ (deg.)</th>
<th>$\theta_{90}$ (deg.)</th>
<th>$\theta_{180}$ (deg.)</th>
<th>$\theta_{270}$ (deg.)</th>
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<td>2.97</td>
<td>3.46</td>
<td>3.99</td>
<td>888.81</td>
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<td>2.98</td>
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<td>39.27</td>
<td>3.03</td>
<td>2.98</td>
<td>3.48</td>
<td>4.01</td>
<td>843.36</td>
</tr>
<tr>
<td>39.88</td>
<td>39.44</td>
<td>38.96</td>
<td>39.35</td>
<td>3.03</td>
<td>2.98</td>
<td>3.48</td>
<td>4.01</td>
<td>833.59</td>
</tr>
<tr>
<td>39.94</td>
<td>39.59</td>
<td>39.09</td>
<td>39.44</td>
<td>3.04</td>
<td>2.99</td>
<td>3.48</td>
<td>4.02</td>
<td>825.78</td>
</tr>
<tr>
<td>39.99</td>
<td>39.65</td>
<td>39.18</td>
<td>39.48</td>
<td>3.04</td>
<td>2.99</td>
<td>3.49</td>
<td>4.02</td>
<td>820.73</td>
</tr>
<tr>
<td>40.05</td>
<td>39.73</td>
<td>39.27</td>
<td>39.54</td>
<td>3.04</td>
<td>2.99</td>
<td>3.49</td>
<td>4.02</td>
<td>814.05</td>
</tr>
<tr>
<td>40.10</td>
<td>39.76</td>
<td>39.32</td>
<td>39.58</td>
<td>3.04</td>
<td>2.99</td>
<td>3.49</td>
<td>4.02</td>
<td>808.60</td>
</tr>
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<td>40.20</td>
<td>39.83</td>
<td>39.42</td>
<td>39.65</td>
<td>3.04</td>
<td>2.99</td>
<td>3.49</td>
<td>4.03</td>
<td>799.48</td>
</tr>
<tr>
<td>40.29</td>
<td>39.93</td>
<td>39.60</td>
<td>39.73</td>
<td>3.05</td>
<td>3.00</td>
<td>3.49</td>
<td>4.03</td>
<td>785.12</td>
</tr>
</tbody>
</table>

In Table G.1, $\phi$ and $\theta$ refer to the displacement and compensation patch angles, respectively, and the numerical subscript refers to the azimuth. The radial position at which these angles were measured during calibration is given in the $r$ column.

![Diagram](image)

Fig. G.4. “Phase and Amp.vi” sub-VI from Fig. G.2(d).
Figure G.5. Front panel for “optical5.vi”.
Control of the analog voltage output by the PCI-6024E board to the sensor illumination LEDs is also provided in “optical5.vi”, in the area is shown on the front panel as “voltage out to LEDs”. As discussed in 6.4.2, these voltage values are adjusted by the operator “on the fly” such that the difference between the “Current cal patch angle” and “Correct' cal patch angle” is as small as possible (<0.003 degrees). Depending on which sensor (1 thru 8) is selected on the “SELECT SENSOR” control, the computer adjusts the analog-output voltage to the value listed in the box next to “a1”, “a2”, etc. These values are not all identical because each illumination LED has a slightly different intensity versus voltage characteristic and needs to be adjusted independently from the other LEDs. When a sensor is selected in “optical5.vi”, it is necessary to also select that sensor on the physical electrical switch on the spin pit (hardware wiring is discussed in Appendix F).
Appendix H

SOURCES FOR DISPARITY IN STRAIN GAGE READINGS ON THE TUBULAR SPECIMENS DURING CREEP TESTS

Three strain gages, axially oriented and equally spaced (in the circumferential sense), were used on each tubular specimen. On each specimen, discrepancy occurred between the three individual strain gage readings. The difference between any single gage reading and the mean gage reading on a given specimen was as much as 11%. A summary of the error in the individual strain gage readings, $\varepsilon_{i}^{err}$, (given as percent difference from the mean) on the specimens is given in Table H.1, and calculated according to

$$
\varepsilon_{i}^{err} = \left( \frac{3\varepsilon_{i}}{\sum_{i=1}^{3}\varepsilon_{i}} - 1 \right) \times 100.
$$

(H.1)

It should be noted that the values listed in Table H.1 are for readings taken ~5 minutes into each test. For each specimen, the differences diverged slightly with time. For example, at the end of the test of specimen $50^\circ_30\%$ (1400-minute duration), $\varepsilon_{1}^{err}$, $\varepsilon_{2}^{err}$, and $\varepsilon_{3}^{err}$ were +3.6, -0.8, and -2.8 percent, respectively.

Table H.1. Difference between individual and mean strain gage readings during the creep tests.

<table>
<thead>
<tr>
<th>Test</th>
<th>$\varepsilon_{1}^{err}$ (%)</th>
<th>$\varepsilon_{2}^{err}$ (%)</th>
<th>$\varepsilon_{3}^{err}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$30^\circ_30%$</td>
<td>-6.7</td>
<td>-3.9</td>
<td>+10.6</td>
</tr>
<tr>
<td>$40^\circ_30%$</td>
<td>+0.2</td>
<td>-3.5</td>
<td>+3.3</td>
</tr>
<tr>
<td>$50^\circ_30%$</td>
<td>+3.4</td>
<td>-0.8</td>
<td>-2.6</td>
</tr>
<tr>
<td>$30^\circ_45%^*$</td>
<td>+5.9</td>
<td>+7.3</td>
<td>-13.2</td>
</tr>
<tr>
<td>$40^\circ_45%$</td>
<td>+3.5</td>
<td>+1.0</td>
<td>-4.5</td>
</tr>
<tr>
<td>$50^\circ_45%$</td>
<td>-1.4</td>
<td>-1.4</td>
<td>+2.8</td>
</tr>
<tr>
<td>$30^\circ_60%$</td>
<td>-1.2</td>
<td>-7.7</td>
<td>8.9</td>
</tr>
<tr>
<td>$40^\circ_60%$</td>
<td>-0.5</td>
<td>+6.1</td>
<td>-5.6</td>
</tr>
<tr>
<td>$50^\circ_60%$</td>
<td>+7.2</td>
<td>-1.5</td>
<td>-5.7</td>
</tr>
</tbody>
</table>

*It was determined that this specimen had a bad $\varepsilon_{3}$. 

-
There are four ways in which variability among the strain gage readings could be introduced into these experiments. These ways are: bad gages, specimen bending, variable specimen wall thickness, and variations in the composite structure local to the gage. Experience has shown that the error attributable to “bad” gages (usually due to imperfect installation) is of the order of 1-2%. Each of the remaining three potential causes of strain gage reading variability are discussed below, and estimates of the potential contribution of each cause to total strain reading variability are given.

**H.1 Bending**

To calculate the effect of misalignment of the grips relative to the tube, the extensional and bending contributions to strain must be separated. The final strain state is assumed as a superposition of bending and extensional strains. Because the grips incorporate spherical washers for alignment purposes, the worst-case scenario of alignment occurs when the specimen is laterally offset in the grips. With a load $P$ applied along the grip centerline, the offset $\delta$ gives rise to a bending moment $M$ and extensional load $X$ as shown in Fig. H.1.

![Effect of grip and specimen misalignment.](image)

Figure H.1. Effect of grip and specimen misalignment.

The bending moment in terms of applied load is given as

$$M = P\delta$$  \hspace{1cm} (H.2)
and the moment of inertia, \( I \), of a thin-walled tubular beam with radius \( r \) and wall thickness \( t \) is given as

\[
I = \pi r^3 t. \tag{H.3}
\]

The thin beam bending equations are used to approximate the radius of curvature, \( R \), experienced by a beam with Young’s modulus, \( E \), under an applied bending moment,

\[
R = \frac{EI}{M}. \tag{H.4}
\]

Extensional strain due to bending, \( \varepsilon_{\text{bend}} \), along the outer curved surface of the tubular beam is then given as

\[
\varepsilon_{\text{bend}} = \frac{r}{R}. \tag{H.5}
\]

Combining Eqs. H.1-5 gives

\[
\varepsilon_{\text{bend}} = \frac{P\delta}{E\pi r^2 t}. \tag{H.6}
\]

Extensional strain, \( \varepsilon_{\text{ext}} \), is given as

\[
\varepsilon_{\text{ext}} = \frac{P}{2E\pi rt}. \tag{H.7}
\]

The fractional strain due to bending, \( \varepsilon_{bf} \), is defined as

\[
\varepsilon_{bf} = \frac{\varepsilon_{\text{bend}}}{\varepsilon_{\text{bend}} + \varepsilon_{\text{ext}}}. \tag{H.8}
\]

Combining Eqs. H.6, 7, and 8 results in

\[
\varepsilon_{bf} = \frac{\delta/r}{\delta/r + 1/2}. \tag{H.9}
\]

The geometry of the specimen and grips is such that the maximum value of \( \delta \) is 0.2 mm and \( r \) is 9.92 mm. Using Eq. H.8, the maximum \( \varepsilon_{bf} \) is 0.038, or 3.8\% of the total strain during a tensile test. While 3.8\% (worst-case) is a large percentage of many of the experimentally measured \( \varepsilon_{\text{err}} \) listed in Table 1, it is believed that bending is not the only contributor. Halfway through the testing program, the grip/tube assembly procedure was modified in order to minimize \( \delta \) to no more than 0.025 mm. This was accomplished by bonding three small (approximately 1 by 2 mm) pieces of 125 \( \mu \text{m} \)-thick brass shim to the OD of the potted portion of the specimen so that it mated with the grips with a slip-fit.
This procedure did not significantly affect the frequency of encountering high values of $e_{ij}$ in subsequent tests. The conclusion on the effects of bending alone cannot fully explain the discrepancy among individual strain gage readings.

**H.2 Variable Wall Thickness of the Specimen**

During the specimen machining process, it is possible to introduce variable wall thickness (VWT) into the tubular specimens—both in the axial and circumferential sense—due to the inability to perfectly align the tube with the spin axis of the lathe during the machining process. Locating all strain gages at a common axial position eliminates errors due to the component of VWT along the longitudinal axis of the tubular specimen, because this type of VWT simply results in a tapered but axisymmetric specimen. For a specimen with VWT in the circumferential sense, strain gage readings at different circumferential locations will exhibit variability. While no model of such a condition is presented here, a reasonable approximation of the effect of VWT is that a strain gage reading will be inversely proportional to the wall thickness local to the gage. Measurements of specimen wall thickness local to the gages (to an accuracy of 0.001 inches) after completing each test showed that the variation in wall-thickness within a particular specimen never exceeded 2.5%. The contribution of VWT to strain gage reading variability is therefore expected to be less than 2.5%.

**H.3 Composite Heterogeneity**

The filament wound tube specimens contain resin-rich regions between adjacent fiber tows. The local strains in these resin rich regions are different from the average strain in the specimen. In the design-of-experiment phase, it was decided to use strain gages with grid section lengths that spanned three bandwidths. It was believed that this would provide sufficient averaging of inevitable local strain variations on the specimen surface. The Measurements-Group CEA-06-375UW-120 gages have a grid length of 0.375 inches. Depending on circumferential location, this length will span either three or
four resin rich layers. An illustration of the gage and composite structure geometry is shown in Fig. H.2.

![Diagram](image)

Figure H.2. Illustration of composite structure and strain gage for the tube specimens.

An experiment was performed on an E-glass/Dow 383 tube specimen under axial loading to quantify the differences between the strains in the resin-rich regions and the average strain of the specimen. The specimen was fitted with end caps and subjected to axial compression by way of a central threaded rod that provided clamping force to the caps, as illustrated in the cross section view of the assembled test fixture in Fig. H.3.

![Diagram](image)

Figure H.3. Cross section view of the tube specimen fixture for testing composite heterogeneity.

The specimen was instrumented with a moiré interferometry grid and analyzed in a fashion identical to that described with the IIF ring experiments (see Section 5.2.3). The
conclusion of this analysis was that the worst-case variability in strain gage readings resulting from composite heterogeneity was approximately 4%.

**H.4 Conclusion**

Explanations for disparity among strain gage readings during a test have been presented. The above-mentioned mechanisms likely superpose during a test, resulting in strain gage readings that are sometimes in good agreement (e.g., specimen 40°_30% or 50°_45%), and sometimes not in such good agreement (e.g., specimen 30°_30%).
Appendix I

OESM VIBRATION TOLERANCE

In this section, the theoretical tolerance of the OESM system to in-plane rigid body vibrations for a four-lobe TB pattern is derived and discussed. Here, a “tolerable” vibration is one where the rigid and flexible-body displacements are determinable using the simple averaging technique of Eq. 4.14 and errors associated with this simple equation are negligible (e.g., error in the resulting indicated displacements are smaller than the desired displacement resolution $u_{\text{min}}$). Using the Mathematica programming language, a parametric study was carried out to calculate the actual angles swept on a set of four equally-spaced TB patches during a vibration, and to calculate the error resulting from using such angles with Eq. 4.14 for separating flexible and rigid body displacements.

I.1 Geometry of the TB Patches and Sensor Path During a Vibration

The geometry of the TB patches and the sensor path during a vibration will now be presented. The letter and number designations discussed in the text correspond to those used in the Mathematica file where the actual calculations are presented.

Consider the angles swept by the sensor spot across the four equally-spaced TB patches during an in-plane rigid body displacement defined by a center of rotation, $C_{\text{ROT}}$, offset from the center of the pattern, $C_{\text{PAT}}$. For simplicity, the vibration shown in Figs. I.1 and I.2 coincides with the $y$-axis. In general, however, a vibration is defined by the vector pointing from $C_{\text{ROT}}$ to $C_{\text{PAT}}$. The “paths” across the lobes can be collapsed onto a single TB patch, for computational purposes. Path 1 is a circular arc on Lobe 1, and has a center $O_1$, and Path 2 is a circular arc on Lobe 2 and has a center $O_2$, etc. To calculate the actual angles swept during a vibration, it is necessary to calculate the intersections of these paths with the lines that define the TB patch. Fig. I.2 is a close-up view of the four paths on the TB patch, with labeled points of intersection.
Figure I.1. Paths swept across the four TB patches during an in-plane vibration.

Figure I.2. Definition of geometric points used in the analysis of an in-plane vibration.
Consider now the general case of vibration, defined by some amplitude, \( \rho \) and some phase \( \theta \), measured relative to the x-axis of the rotor. Greek “rho” is defined in this appendix as the vibration amplitude, and is not to be confused with the symbol for material density used in the body of this document. Figure I.3 shows the path C-D and angle (“Angle 1”) swept across lobe 1, which is centered at \((\rho, \theta_1)\), or \(O'_1\). For clarity, the other three paths and angles are omitted from Fig. I.3. The angles Angle 1 – Angle 4 are the quantities of interest in this analysis as they correspond to the angles measured by the OESM sensor during a general vibration. The parametric calculation of these four angles in terms of a general vibration \((\rho, \theta)\) and an evaluation of the error in the displacement indicated by simply using these angles with Eq. 4.14 are given in the Mathematica notebook in Section I.2.

Fig. I.3. Geometric definitions describing “angle1” for a general vibration of amplitude “\( \rho \)” and phase “\( \theta_1 \)”.
I.2 Computer Code used in the Parametric Study

The code used to perform the parametric study of vibration-induced error was written in the Mathematica programming language. An example calculation of the error in displacement is given below, for the 70 to 80-mm radius annular region of the pattern and a nominal radial location of the sensor at \( r = 70.7 \text{ mm} \). This corresponds to the location of the A2 sensor in the high-speed spin test setup (see Section 6.4.4). A vibration amplitude of 250 \( \mu \text{m} \) and a phase and 0 degrees (relative to the x-axis as defined in Section I.1) is used in these example calculations. Commentary is given throughout the program. The results of the study using this program are given in Section I.3.

\[
\begin{align*}
\text{In[1]:=} & \quad (\text{This file calculates the error that occurs in the flexible body displacements when the center of rotation is offset from the center of the pattern. These calculations assume that the pattern is of the four-foil tangential boundary type, and is perfectly symmetric and equally-spaced (i.e. 90 degrees) in the circumferential sense. The equations are performed in the cartesian 'x-y' plane, not polar coordinates.)} \\
& r_i = 70000; \quad (\text{for this example, the 70 to 80-mm region is considered}) \\
& r_o = 80000; \\
& \phi = 2 \text{ ArcCos}[r_i / r_o] \\
\text{In[2]:=} & \quad (\text{x and y coords defining the TB vertices 'A' and 'B' that define 'line1' and 'line2', see Fig. A9.2} \\
& \quad \text{that define 'line1' and 'line2', see Fig. A9.2}) \\
& x_a = -r_i \text{Sin} [\phi/2] \\
& x_b = r_i \text{Sin} [\phi/2] \\
& y_a = r_i \text{Cos} [\phi/2] \\
& y_b = r_i \text{Cos} [\phi/2] \\
\text{In[3]:=} & \quad (\text{slopes of 'line1' and 'line2'}) \\
& \text{slope1} = (r_o - y_a) / (-x_a) \\
& \text{slope2} = (y_b - r_o) / x_b \\
\text{In[10]:=} & \quad (\text{equations for 'line1' and 'line2'}) \\
& \text{line1}[x_] = \text{slope1} x + r_o \\
& \text{line2}[x_] = \text{slope2} x + r_o
\end{align*}
\]
\(\text{In[12]} := \) offset radius in micrometers, relative to pattern origin.

Note that the greek "\(\rho\)" is a local definition, not to be confused with material density, which this symbol represents in the main part of the thesis.\(\rho = 250; \) \(\text{(* 250-micrometer vibration amplitude *)}\)

(\(\text{(* offset angle used to calculate the 'path 1', considering the x-axis to coincide with the zero degree line, see Fig. A9.1 and A9.2, and the general offset case in Fig. A9.3 *)}\)

\(\text{\(\vartheta_1 = 45 \pi /160;\)}\)

\(\text{In[14]} := \) based on the assumption that the lobes are spaced 90 degrees apart, the paths on the four lobes can be effectively collapsed onto a single lobe in order to reduce computations. See Fig. A9.1.

\(\vartheta_2 = \vartheta_1 + \pi/2; \)
\(\vartheta_3 = \vartheta_2 + \pi/2; \)
\(\vartheta_4 = \vartheta_3 + \pi/2; \)

(\(\text{(* radial location of the sensor *)}\)

\(r = 70700; \) (this just happens to be the location of the "AZ" sensor discussed in the high-speed test of the aluminum rotor in Section 6.4.4.\)

(\(\text{(* the following 'Solve' command creates an expression for 'path 1' (an offset circle) in the form of 'y = f(x)' and calls it 'eq[11]' *)}\)

\(\text{eq[11]} := \text{Solve}\left[ \begin{array}{c}
0 = \rho \cos(\vartheta_1)^2 + (\rho - \rho \sin(\vartheta_1))^2 = r^2; \ y
\end{array} \right] \)

\(\text{Out[18]} := \left\{ \begin{array}{c}
y = 125 \sqrt{2} - \sqrt{4998458750 + 250 \sqrt{2} x - x^2} \\
y = 125 \sqrt{2} + \sqrt{4998458750 + 250 \sqrt{2} x - x^2}
\end{array} \right\} \)

\(\text{In[19]} := \) the output of the above 'Solve' command yields two solutions for 'y' but we are interested only in the positive part of the circle, so we use the expression immediately below to 'extract' the equation of interest from the eq[11] output and assign it to 'path[1]' as a function of 'x'.\(\)

\(\text{path[1][x_\_]} := \text{eq[11]}[[2, 1, 2]] \)

\(\text{Out[19]} := 125 \sqrt{2} + \sqrt{4998458750 + 250 \sqrt{2} x - x^2} \)

\(\text{In[20]} := \) solve for the \(x\)-coordinate values of the intersections of 'line1' and 'line2' with 'path1'.\(\)

These intersection points are labeled in Fig. A9.2 by 'C' and 'D', respectively.\(\)

\(\text{x11 := Solve[\{line1[x] - path[1][x] == 0, x \_][2, 1, 2]]; \)
\(\text{x21 := Solve[\{line2[x] - path[1][x] == 0, x \_][1, 1, 2]]; \)

\(\text{Out[21]} := \frac{175}{64} \left( \begin{array}{c}
35 \sqrt{2} + 3200 \sqrt{15} - 5 \sqrt{30} - \\
\sqrt{10062176 + 1566000 \sqrt{2} - 700 \sqrt{15} + 224000 \sqrt{30}}
\end{array} \right) \)
In[23] = (* the above calculations are carried out for the three
other paths, and corresponding points 'E' through 'J' below,
output is suppressed *)

\[ eq12 = \text{Solve}\left\{ x - p \cos[\theta 2] y + (y - p \sin[\theta 2] y)^2 = r^2, y \right\} \]
path2[\_{\text{x}}] = eq2[2, 1, 2] \]
\[ x12 = \text{Solve[\{list[1\_{\text{x}}] - path2[\_{\text{x}}] = 0, x\_{\text{I}}[2, 1, 2] \} \}} \] (* x-coordinate for point 'E' *)
\[ x23 = \text{Solve[\{list[2\_{\text{x}}] - path2[\_{\text{x}}] = 0, x\_{\text{I}}[1, 1, 2] \} \}} \] (* x-coordinate for point 'F' *)

\[ eq13 = \text{Solve}\left\{ x - p \cos[\theta 3] y + (y - p \sin[\theta 3] y)^2 = r^2, y \right\} \]
path3[\_{\text{x}}] = eq3[2, 1, 2] \]
\[ x13 = \text{Solve[\{list[1\_{\text{x}}] - path3[\_{\text{x}}] = 0, x\_{\text{I}}[2, 1, 2] \} \}} \] (* x-coordinate for point 'G' *)
\[ x23 = \text{Solve[\{list[2\_{\text{x}}] - path3[\_{\text{x}}] = 0, x\_{\text{I}}[1, 1, 2] \} \}} \] (* x-coordinate for point 'H' *)

\[ eq14 = \text{Solve}\left\{ x - p \cos[\theta 4] y + (y - p \sin[\theta 4] y)^2 = r^2, y \right\} \]
path4[\_{\text{x}}] = eq4[2, 1, 2] \]
\[ x14 = \text{Solve[\{list[1\_{\text{x}}] - path4[\_{\text{x}}] = 0, x\_{\text{I}}[2, 1, 2] \} \}} \] (* x-coordinate for point 'I' *)
\[ x24 = \text{Solve[\{list[2\_{\text{x}}] - path4[\_{\text{x}}] = 0, x\_{\text{I}}[1, 1, 2] \} \}} \] (* x-coordinate for point 'J' *)

In[34] = (* Classic 'head to tail' construction of the vectors
that define the lines from 'O' to the eight intersection points.
See Fig 4.9.3 for illustration of 'vec1' and 'vec2' *)
vec1 = N [\{x11, \text{line1}[11] \}] - (p \cos[\theta 1], p \sin[\theta 1]) \] (* vector from O-prime_1 to 'C' *)
vec2 = N [\{x21, \text{line2}[21] \}] - (p \cos[\theta 2], p \sin[\theta 1]) \] (* vector from O-prime_1 to 'D' *)
vec3 = N [\{x12, \text{line1}[12] \}] - (p \cos[\theta 2], p \sin[\theta 2]) \] (* vector from O-prime_2 to 'E' *)
vec4 = N [\{x22, \text{line2}[22] \}] - (p \cos[\theta 2], p \sin[\theta 2]) \] (* vector from O-prime_2 to 'F' *)
vec5 = N [\{x13, \text{line1}[13] \}] - (p \cos[\theta 3], p \sin[\theta 3]) \] (* vector from O-prime_3 to 'G' *)
vec6 = N [\{x23, \text{line2}[23] \}] - (p \cos[\theta 3], p \sin[\theta 3]) \] (* vector from O-prime_3 to 'H' *)
vec7 = N [\{x14, \text{line1}[14] \}] - (p \cos[\theta 4], p \sin[\theta 4]) \] (* vector from O-prime_4 to 'I' *)
vec8 = N [\{x24, \text{line2}[24] \}] - (p \cos[\theta 4], p \sin[\theta 4]) \] (* vector from O-prime_4 to 'J' *)

In[42] = (* Use the dot product to calculate the included angle between the appropriate
vectors. Note that the vector normals are calculated explicitly, as the
current version of Mathematica does not have a function for calculating a vector normal *)
ang1 = N[\{\text{ArcCos[\{vec1 \cdot vec2\]/(sqrt[vec1[1][1] + vec1[1][2]] - sqrt[vec2[1][1] + vec2[1][2]]) \}]]} \] 180/Pi;
ang2 = N[\{\text{ArcCos[\{vec3 \cdot vec4\]/(sqrt[vec3[1][1] + vec3[1][2]] - sqrt[vec4[1][1] + vec4[1][2]]) \}]]} \] 180/Pi;
ang3 = N[\{\text{ArcCos[\{vec5 \cdot vec6\]/(sqrt[vec5[1][1] + vec5[1][2]] - sqrt[vec6[1][1] + vec6[1][2]]) \}]]} \] 180/Pi;
ang4 = N[\{\text{ArcCos[\{vec7 \cdot vec8\]/(sqrt[vec7[1][1] + vec7[1][2]] - sqrt[vec8[1][1] + vec8[1][2]]) \}]]} \] 180/Pi;
In the example code given above, the output of \(-2.43 \mu m\) is the difference between the apparent radial location of the sensor and the actual location of the sensor (which was the input parameter \('r'\)) for a vibration of 250 \(\mu m\) and a phase of 45 deg.

Theoretically, therefore, such a vibration (and sensor setup) would result in an error in the indicated displacement, \(r_{\text{inst}}\), of positive 2.43 \(\mu m\).

**I.3 Results of the Parametric Study**

Relevant to the current research, the reference radial location of the sensor, \(r_{\text{ref}}\), will be set at \((r_{\text{max}}-300 \mu m)\) in all radial regions of the TB pattern used in this research.
(see Eq. 4.8 for a definition of $r_{\text{max}}$). This corresponds to a reasonable radial location during a spin test of a rotor with the TB pattern. It should be noted that symmetry in the pattern reduces the dependence of the results on $\theta$ to a range of 45 degrees. The error was found to be very weakly dependent on $\theta$, with the worst-case error occurring for $\theta$ equal to 45 degrees. A vibration phase angle of 45 degrees will be used in the calculations.

Vibration amplitude is the only remaining independent parameter. Figure I.4 shows a plot of the error in versus vibration amplitude at three radial regions on the rotor. The plus (+) symbol shows the vibration amplitude (130 $\mu$m) necessary to cause a displacement error of $u_{\text{min}}$ (0.6 $\mu$m—corresponding to a hoop strain error of 10 $\mu$e) at the 60.8-mm location. The vibration amplitudes necessary to cause displacement errors of $u_{\text{min}}$ (1.0 and 1.5 $\mu$m) at the other two locations ($r=101.5$ and $r=152.4$ mm, respectively) are higher than 250 $\mu$m.

![Figure I.4. Error in indicated displacement versus vibration amplitude at three locations on the rotor with ten-region reflective TB pattern.](image)

In conclusion, this analysis shows that reasonable amounts of vibration theoretically cause very little error in the flexible body displacements calculated if Eq. 4.14 is applied to the individual displacements at the four lobes of the TB pattern, and the pattern has perfectly smooth boundaries. Relevant to the high-speed spin test results
presented in Section 6.4.4, the vibration level of 65 μm theoretically causes errors of less than 0.5 μm at all regions on the rotor.
Appendix J
SURFACE PREPARATION AND APPLICATION OF THE OESM PATTERN

Rotor surface preparation is required prior to developing the contact print pattern on the rotor. It is necessary to apply reflective paint as well as a clear overcoat that protects the paint and is also compatible with the emulsion. After reading the Rockland Liquid Light emulsion instructions and performing experiments on test pieces of E-glass/epoxy composite, a successful recipe was found. The materials and approximate accumulated thickness after the application of the final coat of each material (measured with a micrometer with 1-μm precision) are given in order and thickness in Table J.1.

<table>
<thead>
<tr>
<th>Material</th>
<th># Coats</th>
<th>Type</th>
<th>Approximate Accumulated Thickness (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light Gray Primer 2081 RUST-OLEUM¹</td>
<td>2</td>
<td>Spray can</td>
<td>2</td>
</tr>
<tr>
<td>Silver Paint 19101 Plasti-Kote²</td>
<td>2</td>
<td>Spray can</td>
<td>4</td>
</tr>
<tr>
<td>Clear Lacquer 1906 RUST-OLEUM¹</td>
<td>2</td>
<td>Spray can</td>
<td>6</td>
</tr>
<tr>
<td>Fast Drying Polyurethane, Clear Gloss MINWAX³</td>
<td>6</td>
<td>Hand application</td>
<td>20</td>
</tr>
<tr>
<td>Liquid Light Photo Emulsion Rockland⁴</td>
<td>10-20</td>
<td>Airbrush</td>
<td>45</td>
</tr>
</tbody>
</table>

With the rotor temporarily mounted to a turntable rotating at about 200 rpm, the primer, silver paint, and lacquer were sprayed onto the clean rotor surface directly from the spray cans. Each of these coats was applied and allowed to dry according to

¹ RUST-OLEUM CORPORATION, Vernon Hills, IL, 60061.
³ Minwax Company, Upper Saddle River, NJ.
⁴ Rockland Colloid Corp., Piermont, NY.
manufacturers’ recommendations. The best method for applying the glossy polyurethane is to apply an excessive amount by hand and then remove much of that with a paper towel, leaving a thin and uniform layer after each coat. Before each re-coat of glossy polyurethane, the rotor surface was lightly sanded with 400-grit sandpaper according to the manufacturer’s recommendations. The rotor was slowly rotated by hand during and in-between the application of these glossy polyurethane coats. The shallow-angle reflection of a light bulb on the rotor surface can be used to aid inspection of the coat as it is applied in order to achieve a uniform coat. It is important that the manufacturer’s recommended setting times be precisely followed.

After the final coat of glossy polyurethane was applied, the rotor was removed from the temporary motor and put in an oven for 24 hours at 40°C in order to remove volatiles in the coatings. The rotor was then re-mounted to the turntable and centered in the radial direction to within ±0.01 mm (0.001 inch) of the axis of rotation of the motor. The following portion of the procedure must be done in a darkroom.

In the darkroom, the light-sensitive emulsion was sprayed onto the rotor surface with a small airbrush (such as those typically used for model airplanes). To achieve a workable viscosity, it was necessary to dilute the emulsion with water to 1/3rd strength. With the rotor slowly rotating in a horizontal plane in the darkroom, the diluted emulsion was airbrushed onto the rotor surface by directing the spray at the rotor surface at an approximately 30-degree angle of incidence (90-degrees being perpendicular). During each pass, the stream was slowly moved from the outside radius inward—resulting in a spiral deposition pattern. Each pass took approximately 5-7 seconds to apply and 1-2 minutes to dry, and deposited approximately 1.5 cm³ of the diluted emulsion. A complete rotor surface (691 cm²) required approximately 100-110 cm³ of the diluted emulsion. The shallow-angle reflection of the red darkroom safelight on the rotor surface was used to inspect each coat as it was applied in order to achieve a uniform coat.

While still in the darkroom, the rotor was allowed to dry for 15 minutes after the final coat of emulsion was applied. The acrylic disk template described in Section 5.4.2 was placed on the rotor surface and the outer radial surface was centered to within ±0.01-mm (0.0005-inch) of the rotor outer radius using precision dial indicator. Once the rotor and template disk were aligned, a standard 100-watt bulb—mounted in a common
paraboloid-shaped utility fixture—was used to expose the negative of the template pattern. An aperture was used on the utility fixture to limit the exit path of the light to a 1-cm diameter hole in order to maximize the sharpness of the exposed image. The exposure time was approximately 40 seconds. Following this exposure, the pattern was immediately developed by immersion in a bath of Kodak Dektol Developer for approximately 45 seconds, briefly rinsed, fixed with Kodak Fixer for 7 minutes, and then washed for 30 minutes in a cool water bath as recommended in the emulsion instructions. The rotor and pattern were dried for at least 12 hours so that the emulsion could strengthen sufficiently.
VITA

Ryan P. Emerson was born in Valdosta, GA, on May 11, 1973. He spent his first 7 years (as the son of an active-duty U.S. Air Force pilot) living in as many places throughout the U.S. When Ryan was 8 years of age, the Emerson family moved to Sparta, NJ, where they stayed until Ryan was 15, at which time they moved to Bucks County, PA. Ryan attended grades 10-12 at Central Bucks H.S. West in Doylestown, PA.

After graduating high school in 1991, Ryan began undergraduate studies at Penn State University, University Park. Ryan entered the Engineering Science undergraduate program in his third year at Penn State, and worked for two semesters at Martin Marietta Astro Space in Cranbury, NJ, through the Engineering Cooperative Education program at Penn State. Ryan graduated in December 1995 with a B.S. degree in Engineering Science.