DEVELOPMENT OF A MULTI-MECHANISTIC, DUAL-POROSITY, DUAL-PERMEABILITY NUMERICAL FLOW MODEL FOR COALBED METHANE RESERVOIRS ACCOUNTING FOR COAL SHRINKAGE AND SWELLING EFFECTS

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ABSTRACT

Most existing coalbed methane (CBM) simulators often treat coal seams as dual-porosity, single-permeability systems, which ignore the effects of water presence in the coal matrix. In this study, a compositional dual-porosity, dual-permeability CBM simulator has been developed. The CBM reservoir is treated as a dual-porosity, dual-permeability system consisting of coal matrix and fracture network. The development of the proposed numerical model incorporates the effects of water presence in the coal matrix and those of coal shrinkage and swelling. The transport of gas follows a multi-mechanistic flow mechanism, which is triggered by pressure and concentration gradients. The proposed simulator was successfully validated against a dual-porosity, single-permeability CBM model (PSU-COALCOMP) and a single-porosity, single-permeability model (GEM). The proposed simulator was also tested against the existing commercial and research CBM simulators for CO$_2$-enhanced CBM recovery process. The simulator was also used in parametric sensitivity studies to investigate the validity of the flow mechanism, the effects of water in the coal matrix and coal shrinkage and swelling conditions. History matching was performed on pure CO$_2$ injection and flue gas injection tests using the actual field data.

The governing flow equations of the proposed simulation model were further used in deriving a material balance equation and a dimensionless form of the transport equation for CBM reservoirs exhibiting homogeneous and isotropic properties. The developed material balance equation was successfully tested and implemented in predicting the reservoir size, production performance and average reservoir pressure using the simulated production data. Lastly, the developed dimensionless production rates and dimensionless time were used in the production type curves application for reservoir characterization. A series of production type curves were generated using the proposed simulator. Several sets of type-curve matching exercises were successfully performed to predict reservoir properties on different systems exhibiting a variety of reservoir and fluid properties.
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<td>$B_g$</td>
<td>Formation volume factor of gas phase</td>
<td>RB/SCF</td>
</tr>
<tr>
<td>$B_w$</td>
<td>Formation volume factor of water phase</td>
<td>RB/STB</td>
</tr>
<tr>
<td>$c_1$</td>
<td>Curve-fitting coefficient</td>
<td>SCF/ton</td>
</tr>
<tr>
<td>$c_2$</td>
<td>Curve-fitting coefficient</td>
<td>psia</td>
</tr>
<tr>
<td>$c_m$</td>
<td>Matrix shrinkage compressibility</td>
<td>$\text{psi}^{-1}$</td>
</tr>
<tr>
<td>$c_p$</td>
<td>Pore volume compressibility</td>
<td>$\text{psi}^{-1}$</td>
</tr>
<tr>
<td>$c_t$</td>
<td>Total compressibility</td>
<td>$\text{psi}^{-1}$</td>
</tr>
<tr>
<td>$c_f$</td>
<td>Formation compressibility</td>
<td>$\text{psi}^{-1}$</td>
</tr>
<tr>
<td>$c_F$</td>
<td>Compressibility of the fracture</td>
<td>$\text{psi}^{-1}$</td>
</tr>
<tr>
<td>$c_M$</td>
<td>Compressibility of the matrix</td>
<td>$\text{psi}^{-1}$</td>
</tr>
<tr>
<td>$C_f$</td>
<td>Cleat-volume compressibility</td>
<td>$\text{psi}^{-1}$</td>
</tr>
<tr>
<td>$C_g$</td>
<td>Concentration of gas phase</td>
<td>lb-mole/ft$^3$</td>
</tr>
<tr>
<td>$C_m$</td>
<td>Seidle and Huitt’s matrix swelling coefficient</td>
<td>US-ton/scf</td>
</tr>
<tr>
<td>$c_\phi$</td>
<td>Formation compressibility</td>
<td>$\text{psi}^{-1}$</td>
</tr>
<tr>
<td>$D$</td>
<td>Diffusion coefficient</td>
<td>$\text{ft}^2$/day</td>
</tr>
<tr>
<td>$E$</td>
<td>Young’s modulus</td>
<td>psi</td>
</tr>
<tr>
<td>$f$</td>
<td>Fraction from 0 – 1</td>
<td>fraction</td>
</tr>
</tbody>
</table>
\( f_c \) Critical sorption capacity factor fraction

\( f_i \) Fugacity of component \( i \) in gas phase psia

\( f_i^0 \) Fugacity of component \( i \) in gas phase at standard state psia

\( \tilde{f} \) Sorption capacity factor fraction

\( g \) Local gravitational acceleration \( \text{ft}^2/\text{sec} \)

\( g_c \) Conversion factor for gravitational acceleration \( \text{lb}_m\text{ft}/\text{lb}_f\text{sec}^2 \)

\( G \) Depth from datum to the center of the block ft

\( G_A \) Amount of current adsorbed gas SCF

\( G_{A,i} \) Amount of original adsorbed gas SCF

\( G_c \) Gas content SCF/ton

\( G_F \) Original free gas in the fracture SCF

\( G_M \) Original free gas in the matrix SCF

\( h \) Thickness ft

\( IMBC_g \) Incremental material balance check for gas phase -

\( IMBC_w \) Incremental material balance check for water phase -

\( k \) Absolute permeability perms

\( k_i \) Initial absolute permeability perms

\( k_{rg} \) Relative permeability to gas phase fraction

\( k_{rw} \) Relative permeability to water phase fraction

\( k_{x}, k_y, k_z \) Absolute permeability in \( x-, y- \) and \( z- \) directions perms

\( K \) Bulk modulus psi

\( L_{x}, L_{y}, L_{z} \) Fracture spacing in \( x-, y- \) and \( z- \) directions ft

\( m \) Moisture content wt. percent
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_c )</td>
<td>Critical moisture content</td>
<td>wt. percent</td>
</tr>
<tr>
<td>( M )</td>
<td>Constrained axial modulus</td>
<td>psi</td>
</tr>
<tr>
<td>( M_g )</td>
<td>Molecular weight of gas</td>
<td>lb/lb-mole</td>
</tr>
<tr>
<td>( n_i^0 )</td>
<td>Pure component adsorption capacity at standard state condition</td>
<td>SCF/ton</td>
</tr>
<tr>
<td>( OGIP )</td>
<td>Original gas in place</td>
<td>SCF</td>
</tr>
<tr>
<td>( OWIP )</td>
<td>Original water in place</td>
<td>STB</td>
</tr>
<tr>
<td>( n_d )</td>
<td>Adsorption capacity of dried coals</td>
<td>mmole/g-coal</td>
</tr>
<tr>
<td>( n_w )</td>
<td>Adsorption capacity of wet coals</td>
<td>mmole/g-coal</td>
</tr>
<tr>
<td>( p )</td>
<td>Pressure</td>
<td>psia</td>
</tr>
<tr>
<td>( p_{cgw} )</td>
<td>Capillary pressure</td>
<td>psia</td>
</tr>
<tr>
<td>( p_g )</td>
<td>Pressure of gas phase</td>
<td>psia</td>
</tr>
<tr>
<td>( p_i )</td>
<td>Initial pressure</td>
<td>psia</td>
</tr>
<tr>
<td>( p_{sat} )</td>
<td>Saturated pressure</td>
<td>psia</td>
</tr>
<tr>
<td>( p_w )</td>
<td>Pressure of water phase</td>
<td>psia</td>
</tr>
<tr>
<td>( p_{DF} )</td>
<td>Dimensionless fracture pressure</td>
<td>-</td>
</tr>
<tr>
<td>( p_{DM} )</td>
<td>Dimensionless matrix pressure</td>
<td>-</td>
</tr>
<tr>
<td>( p_R )</td>
<td>Average reservoir pressure</td>
<td>psia</td>
</tr>
<tr>
<td>( p_{sf} )</td>
<td>Sandface pressure</td>
<td>psia</td>
</tr>
<tr>
<td>( p_L )</td>
<td>Langmuir pressure</td>
<td>psi</td>
</tr>
<tr>
<td>( q_g )</td>
<td>Gas flow rate</td>
<td>SCF/day</td>
</tr>
<tr>
<td>( q_{srp} )</td>
<td>Sorption flow rate</td>
<td>SCF/day</td>
</tr>
<tr>
<td>( q_w )</td>
<td>Water flow rate</td>
<td>STB/day</td>
</tr>
<tr>
<td>( q_{D,g} )</td>
<td>Dimensionless gas flow rate</td>
<td>-</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Unit</td>
</tr>
<tr>
<td>--------</td>
<td>--------------------------------------------------</td>
<td>---------------</td>
</tr>
<tr>
<td>$q_{D,w}$</td>
<td>Dimensionless water flow rate</td>
<td>-</td>
</tr>
<tr>
<td>$S$</td>
<td>Salinity</td>
<td>% wt.</td>
</tr>
<tr>
<td>$S_g$</td>
<td>Gas saturation</td>
<td>fraction</td>
</tr>
<tr>
<td>$S_{girr}$</td>
<td>Irreducible Gas saturation</td>
<td>fraction</td>
</tr>
<tr>
<td>$S_w$</td>
<td>Water saturation</td>
<td>fraction</td>
</tr>
<tr>
<td>$S_{wirr}$</td>
<td>Irreducible water saturation</td>
<td>fraction</td>
</tr>
<tr>
<td>$r_c$</td>
<td>Drainage radius</td>
<td>ft</td>
</tr>
<tr>
<td>$r_w$</td>
<td>Wellbore radius</td>
<td>ft</td>
</tr>
<tr>
<td>$r_D$</td>
<td>Dimensionless radius</td>
<td>-</td>
</tr>
<tr>
<td>$R$</td>
<td>Gas constant (10.731)</td>
<td>ft$^3$·psi/</td>
</tr>
<tr>
<td>$R_g$</td>
<td>Residual of gas flow equation</td>
<td>SCF/day</td>
</tr>
<tr>
<td>$R_{sw}$</td>
<td>Solution gas-water ratio</td>
<td>SCF/STB</td>
</tr>
<tr>
<td>$R_{sw,i}$</td>
<td>Solution gas-water ratio at initial conditions</td>
<td>SCF/STB</td>
</tr>
<tr>
<td>$R_w$</td>
<td>Residual of water flow equation</td>
<td>SCF/day</td>
</tr>
<tr>
<td>$t$</td>
<td>Time</td>
<td>day</td>
</tr>
<tr>
<td>$t_D$</td>
<td>Dimensionless time</td>
<td>-</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature</td>
<td>R</td>
</tr>
<tr>
<td>$v_g^F$</td>
<td>Fickian velocity of gas phase</td>
<td>ft/day</td>
</tr>
<tr>
<td>$v_g^D$</td>
<td>Darcian velocity of gas phase</td>
<td>ft/day</td>
</tr>
<tr>
<td>$v_g^T$</td>
<td>Total velocity of gas phase</td>
<td>ft/day</td>
</tr>
<tr>
<td>$V_a$</td>
<td>Volume adsorbed</td>
<td>SCF</td>
</tr>
<tr>
<td>$V_b$</td>
<td>Bulk volume</td>
<td>ft$^3$</td>
</tr>
</tbody>
</table>
\( V_{b2} \)  
Bulk volume of the fracture system  
\( \text{ft}^3 \)

\( V_e \)  
Adsorption capacity  
\( \text{SCF} \)

\( V_m \)  
Langmuir isotherm surface constant  
\( \text{SCF/ton} \)

\( V_L \)  
Langmuir volume  
\( \text{SCF/ft}^3 \)

\( W_e \)  
Water influx  
\( \text{STB} \)

\( W_p \)  
Cumulative water production  
\( \text{STB} \)

\( W_f \)  
Original free water in the fracture  
\( \text{STB} \)

\( W_M \)  
Original free water in the matrix  
\( \text{STB} \)

\( X \)  
Mole fraction of the gas phase  
fraction

\( Y \)  
Mole fraction of the adsorbed phase  
fraction

\( Z \)  
Compressibility factor  
-

**Greek**

\( \alpha_s \)  
Matrix coefficient for Shi and Durucan’s model  
\( \text{ft}^3/\text{SCF} \)

\( \beta \)  
Matrix shrinkage-swelling coefficient  
\( \text{ft}^3/\text{SCF} \)

\( \Delta \)  
Finite-difference grid size  
\( \text{ft} \)

\( \Delta t \)  
Finite-difference time step  
\( \text{day} \)

\( \varepsilon_{exp} \)  
Experimental strain  
-

\( \varepsilon_i \)  
Parameter of Langmuir curve match to volumetric strain change  
-

\( \phi \)  
Porosity  
fraction

\( \phi_i \)  
Initial porosity  
fraction

\( \Phi_g \)  
Flow potential of gas phase  
\( \text{psia} \)
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Phi_w )</td>
<td>Flow potential of water phase</td>
<td>psia</td>
</tr>
<tr>
<td>( \varphi )</td>
<td>Fugacity coefficient</td>
<td>-</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Grain compressibility</td>
<td>psi(^{-1})</td>
</tr>
<tr>
<td>( \mu_g )</td>
<td>Gas viscosity</td>
<td>cp</td>
</tr>
<tr>
<td>( \mu_w )</td>
<td>Water viscosity</td>
<td>cp</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Coal density</td>
<td>ton/ft(^3)</td>
</tr>
<tr>
<td>( \rho_g )</td>
<td>Gas density</td>
<td>lb/ft(^3)</td>
</tr>
<tr>
<td>( \rho_w )</td>
<td>Water density</td>
<td>lb/ft(^3)</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>Shape factor</td>
<td>1/ft(^2)</td>
</tr>
<tr>
<td>( \tau )</td>
<td>Sorption time constant</td>
<td>day</td>
</tr>
<tr>
<td>( \Gamma_g )</td>
<td>Interporosity term of gas phase</td>
<td>SCF/day</td>
</tr>
<tr>
<td>( \Gamma_w )</td>
<td>Interporosity term of water phase</td>
<td>STB/day</td>
</tr>
<tr>
<td>( \nu )</td>
<td>Poisson's ratio</td>
<td>-</td>
</tr>
<tr>
<td>( \omega )</td>
<td>Fracture storage factor</td>
<td>-</td>
</tr>
<tr>
<td>( \Psi^0 )</td>
<td>Pure component spreading pressure group at standard state condition</td>
<td>SCF/ton</td>
</tr>
<tr>
<td>( \Psi^0_i )</td>
<td>Spreading pressure group of pure component ( i ) in adsorbed phase at standard state condition</td>
<td>SCF/ton</td>
</tr>
</tbody>
</table>

**Subscripts**

- \( F \) Index for fracture system
- \( g \) Index for gas phase
Index for $i$-direction of the finite-difference grid

Index for $j$-direction of the finite-difference grid

Index for $k$-direction of the finite-difference grid

Index for gas component $l$

Index for matrix system

Number of finite-different blocks

Number of gas components

Standard conditions (60 °F and 14.7 psia)

Index for upstream weighting transmissibility

Index for water phase

$x$-direction spatial coordinate

$y$-direction spatial coordinate

$z$-direction spatial coordinate
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Chapter 1

Introduction

As conventional gas supplies have declined over the last few years, many exploration and production activities have focused on coalbed methane (CBM) production. At the present time, many of the CBM developments are driven by high demand for energy from all over the world, especially China and India. At the current rate of consumption, the life time for the world’s coal reserves is estimated to last over 150 years; therefore, CBM is considered as an important component of the energy budget of the world. In the U.S. and Australia, CBM exploration and production over the course of years have been successful. A total of 10% of gas production in the US is from CBM and this number is expected to increase in the years to come. In Australia, CBM reserves are approximately 60% of the total gas reserves including CBM and conventional gas resources. CBM developments in China have been growing significantly due to its high CBM resources estimated at 1,200 tcf (Chakhmakhchev, 2007).

CBM reservoirs are dual porosity systems, which consist of coal matrix and fracture network. At the initial conditions methane gas is stored in the coal surfaces in the adsorbed state and in the coal matrix as free phase and the cleat structure is saturated with water. As methane gas is released, the adsorption capacity of the coal decreases due to the reduction of system pressure. Water in the pore spaces is initially drained and the methane gas is delivered from the matrix via the fracture network to the wellbore. There are numerous mathematical and numerical models that capture the behavior of the CBM reservoirs. Typical CBM models have been developed using dual-porosity/single-permeability domain characteristics. Single permeability in this case represents the fracture permeability, which is assumed to be the only system that water and gas can flow simultaneously before they enter the wellbore. This assumption typically yields
the gas production to start too soon, resulting in over-prediction of gas production and under-prediction of water production. In addition, the coal permeability changes as a function of pressure. The changes in permeability are influenced by internal stresses, and release or adsorption of gas in the coal matrix in conjunction with production/injection processes. Such mechanisms are referred to as coal shrinkage and swelling.

The proposed work involves developing a comprehensive CBM model that captures the fluid flow dynamics in a coal seam, which exhibits dual-porosity, dual-permeability characteristics and accounts for the effects of coal shrinkage and swelling and potential flow hindrance due to the presence of water in the coal matrix. The gas flow in the coal matrix and fracture network follows a dual-mechanism (multi-mechanistic) model as proposed by Ertekin et al. (1986) considering the two drive mechanisms including flow within the pressure field (Darcian) and flow within the concentration field (Fickian). The inclusion of Fickian flow component increases the capability of the model in simulating gas flow in ultra-tight reservoirs where the gas transport by diffusion becomes increasingly more predominant. In addition, the proposed model considers the presence of water in the coal matrix, which is generally neglected in typical dual-porosity, single-permeability CBM models. This study categorizes water in the coal matrix under two groups including bulk water and bound water and presence of water in each of these categories has varying effects on gas and water production rates.

The proposed model was successfully tested against the dual-porosity, single-permeability CBM model (PSU-COALCOMP) and a commercial single-porosity, single-permeability model (GEM). Also, a comparison study was conducted using the proposed model and the existing commercial and research CBM simulators for enhanced CBM recovery process.

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1 PSU-COALCOMP is a compositional numerical simulator for coal seams developed by Manik and Ertekin (1999), The Pennsylvania State University.
2 GEM is a generalized equation-of-state model compositional reservoir simulator developed by CMG (Computer Modelling Group Ltd.).
with pure CO$_2$ injection. A number of simulation studies were conducted to investigate the effects of multi-mechanistic flow behavior, water presence in the coal matrix and coal shrinkage and swelling on overall production characteristics of coal seams. History matches were performed on pure CO$_2$ and flue gas injection tests to show the capabilities of the proposed model.

In addition, the proposed simulation model can be further implemented in other applications of CBM reservoir engineering such as material balance equation and production type curve analysis. In this study, flow equations describing gas and water flows in fracture and matrix systems were utilized in deriving the material balance equation, which can be used for predicting drainage area, production performance and average reservoir pressure. Also, a series of production type curves were generated using the proposed simulation model. This series of production type curves can be a useful pragmatic tool for determining reservoir properties including the fracture permeability, fracture porosity, matrix porosity and water saturations in the fracture and matrix systems.
Coalbed methane (CBM) reservoirs are naturally fractured systems consisting of two porosity systems, namely coal matrix and fracture network. Adsorbed gas, which is the major portion of gas in CBM reservoirs, is stored on the internal surface areas of coal grains in the coal matrix. The fracture network spans throughout coal seams and connects the surface area of the coal matrix. The pore spaces are initially occupied by water. Water and gas transport in coal seams is a sequential process. As reservoir pressure decreases, free water in the pore spaces is drained out and the adsorption capacity of coal decreases causing the gas to desorb from the surface of the coal matrix and flow through the interconnected coal matrix into fracture networks. Gas, then, flows through the fracture network into the wellbore. Transport of gas in both coal matrix and fracture network domains is driven by the influence of two parallel flow fields: a pressure field and a concentration field. Gas transport under the pressure field follows Darcy's law and under concentration field follows Fick's law of diffusion simultaneously (Ertekin et al., 1986). Figure 2-1 illustrates the transport of gas in coal seams.

Figure 2-1: Transport of gas in coal seams (after Remner et al., 1986)
Numerous studies have been conducted to understand the flow mechanism of gas in CBM reservoirs. King et al. (1986) developed a conventional mathematical and numerical model for coal seam degasification. In his development, the gas is flowing under a multi-mechanistic model that follows two driving mechanisms including flow through the pressure field (Darcian flow) and flow through the concentration field (Fickian flow). The model was developed based on the non-equilibrium sorption using pseudo steady-state formulation. It was also derived for a continuum exhibiting dual-porosity, single-permeability characteristics with a single well in rectangular, cylindrical or elliptical coordinate geometry. The model also assumes that no water is present in the coal matrix. Remner et al. (1986) proposed a two-dimensional (Cartesian coordinate system), two-phase (gas-water), multi-mechanistic flow model in coal seams. This model also treats the coal seam as a dual-porosity, single-permeability system and utilizes a quasi-steady kinetic-type state model developed by King et al. (1986) and Langmuir isotherms to calculate the desorption rate. Extending Remner et al.’s model, Sung et al. (1986) developed a numerical model describing the simultaneous flow of methane and water in hydraulically fractured coal seams. The hydraulic fracture was represented by superimposing fracture nodes on the matrix nodes. Flow within the fractures is assumed to be one-dimensional while flow in the matrix is two-dimensional. The fracture and matrix are connected by a “fluid loss” term. Sawyer et al. (1990) proposed a three-dimensional, multi-well, two-phase, gas-water flow model of CBM reservoirs. Sawyer et al.’s model also accounts for changes in cleat permeability due to matrix shrinkage/swelling. Although these models were able to describe flow in the coal, they were not applicable for enhanced CBM recovery studies as they considered a single-component gas.

Manik et al. (2002) developed a comprehensive computer model of compositional water-gas transport in CBM reservoirs. The model was constructed as a three-dimensional, two-phase (single component water and multi-component gas), dual-porosity, compositional CBM simulator, which was used to model carbon dioxide and nitrogen injections for enhanced CBM
recovery. The thermodynamically consistent ideal absorbed solution (IAS) theory and the Peng-Robinson equation of state were used in developing the multi-component sorption model.

Reeves and Pekot (2001) proposed a triple-porosity dual-permeability compositional model where desorbed gas travels from the coal through the coal matrix into the fractures and the wellbore, respectively. Cicek (2003) also developed a dual-porosity, dual-permeability compositional and non-isothermal simulation model of CO₂ sequestration in fractured CBM reservoirs. However, in these models the flow in the system accounts for only Darcian flow and neglects diffusional flow (Fickian flow).

In addition, Clarkson and Bustin (1999) developed a pressure-dependent diffusion/adsorption model of gas transport in coal matrix. The model accounts for a nonlinear adsorption in microporosity and a multimodal pore volume distribution. The model was found to be sufficient for dull or banded coals, which have complicated pore structure. Fathi and Akkutlu (2009) proposed a gas transport and sorption model in coal seams accounting for the effects of local spatial heterogeneities in porosity using statistical approach. Wei et al. (2007) developed a CBM model based on bidisperse diffusion mechanism and Maxwell-Stefan counter-diffusion.

2.1 Dual-porosity, dual-permeability Characteristics

CBM reservoirs are naturally fractured formations, which consist of two porosity/two permeability systems, including coal matrix and fractures. Adsorbed gas is stored on the exposed surface areas of the coal grains in the coal matrix. Fracture network connects the surface of the coal matrix and distributed throughout the reservoirs. Early CBM models (King et al., 1986; Remner et al., 1986; Sung et al., 1986; Sawyer et al., 1990; Manik et al., 2002) treat CBM reservoirs as dual-porosity, single-permeability systems where gas is transported directly to the cleat network and subsequently to the wellbore as it desorbs. This approach has been proved to
suffer from over-prediction of gas production and under-prediction of water production due to the lack of consideration of the presence of water in the coal matrix as well as the gas and water transport through the coal matrix.

As the coal matrix and fracture systems exhibit different flow characteristics, flow equations for both systems are developed individually so that the system can be treated utilizing a dual-porosity and dual-permeability representation. The corresponding governing equations describe the flow of gas and water through the coal matrix and fracture systems. The coal matrix and fracture flow domains are coupled to each other using transfer terms, which consider fluid transfer between the two systems. Porosity and permeability characteristics of the coal matrix system are different from those of the fracture system. This dual-porosity, dual-permeability representation has been implemented in the more recent fracture models of fractured systems (Al-Shaalan et al., 2003; Ayala et al., 2005; Fung and Al-Shaalan, 2005; Gong et al., 2006) and CBM systems (Reeves and Pekot, 2001; Cicek, 2003).

2.2 Multi-mechanistic Model of Gas Flow

A mathematical formulation describing the gas flow through tight porous media was first proposed by Ertekin et al. (1986). The flow of gas is considered to be driven by: 1) a pressure field and 2) a concentration field. Flow in the pressure field is formulated using Darcy’s law, which states that the fluid velocity is directly proportional to the pressure gradient:

\[ v^D_g = -\frac{k}{\mu_g} \frac{\partial p}{\partial L} \]  \hspace{1cm} (2.1)

The expression above contains a number of assumptions including 1) fluid flowing under steady state conditions and in laminar regime, 2) one-dimensional single-phase flow, 3) flow through
homogeneous media, 4) no gas slippage (Klingkenberg effect), 5) no change in fluid density and viscosity and 6) no chemical reaction between fluid and porous media.

Flow in the concentration field is modeled via Fick’s law of diffusion, which states that the fluid velocity is directly proportional to the concentration gradient as shown below:

$$v^F_g = -D \frac{M_g}{\rho_g} \frac{\partial C_g}{\partial L}$$  \hspace{1cm} (2.2)

In addition, the concentration of gas, $C_g$, can be written as (Ertekin et al., 1986):

$$C_g = \frac{p_g S_g}{ZRT}$$  \hspace{1cm} (2.3)

Thus, the gas flow in the concentration field can be re-written as:

$$v^F_g = -D \frac{M_g}{\rho_g} \frac{1}{RT} \frac{\partial}{\partial L} \left( \frac{p_g S_g}{Z} \right)$$  \hspace{1cm} (2.4)

Assuming that the gas density and concentration gradients are parallel and both Darcian and Fickian flow mechanisms are acting independently, the multi-mechanistic superficial gas velocity of the fluid can be expressed by superimposition of the flow fields as follows:

$$v^T_g = -\frac{k}{\mu_g} \frac{\partial p}{\partial L} - D \frac{M_g}{\rho_g} \frac{1}{RT} \frac{\partial}{\partial L} \left( \frac{p_g S_g}{Z} \right)$$  \hspace{1cm} (2.5)

Eq.(2.5) describes the flow in porous media driven by the influence of a pressure field and a concentration field. In applying Eq.(2.5) to CBM reservoirs, which are in general naturally tight, the mechanism of gas diffusion is assumed to be independent of the flowing pressure. This flow mechanism is referred as “Knudsen flow” where the interactions between molecule and surface are dominant.

Multi-mechanistic flow approach has been widely used by a number of researchers. For instance, Chawathe et al. (1996a) developed a multi-mechanistic gas-water flow model in fractured reservoirs. Ayala et al. (2005) developed a compositional multi-mechanistic gas-
condensate flow model in naturally-fractured reservoirs. However, the incorporation of diffusional flow has been neglected by a number of existing CBM models (Manik et al., 2002; Cicek, 2003). The diffusional flow (Fickian flow) can become significant in fluid transport in tight porous media where permeability is less than 0.1 md. Bezilla et al. (1989) showed that the conventional simulators that neglected the diffusional flow, under-predicted recovery and shortened the predicted production life of the reservoir under consideration.

2.3 Water Presence in the Coal Matrix

The mechanisms of water entrapment in the coal matrix can be classified under two broad categories including bound water and bulk water (Unsworth et al., 1988; Norinaga et al., 1998). Bound water is referred to as immobile water molecules in a vapor phase tightly adsorbed on the hydrophilic parts of the coal matrix while bulk water is referred as free mobile water in the coal matrix. Both types of water yield different effects on gas sorption capacity. As moisture is added to a dried coal, the bound water is held to the coal structure by hydrogen bonding between the oxygen functionality in the coal structure and the hydrogen molecules of the water (Mahajan and Walker, 1971). As the moisture content increases, the gas sorption capacity in the coal matrix decreases. After the bound water is formed on all of the accessible hydrophilic parts of the coal surface, excess water is developed in the coal matrix in the form of bulk water. The amount of bound water on the coal surface depends on the amount of oxygen atoms in the functional group on the coal surface and the availability of hydrogen atoms in water (Allardice and Evans, 1971). The maximum amount of bound water formed on all of the accessible hydrophilic parts of the coal surface, which relates to the amount of oxygen atoms in the functional group on the coal surface, can be referred as the critical moisture content, $m_c$. As moisture content becomes larger
than $m_c$, the sorption capacity of gas in the coal matrix remains the same, which indicates that bulk water does not affect the gas sorption capacity.

Joubert et al. (1973, 1974) performed experimental work on sorption of methane in moist coals. Coal samples were dried and evacuated to remove residual gas and moisture. The moisture content was obtained based on the weight measurement, and the void-space measurement was conducted to determine the sorption of methane for the moist coal samples. The results showed that methane sorption at a given pressure decreases as moisture content increases up to a critical value. No further reduction of methane sorption was found despite increasing moisture content over this critical value.

Experimental studies on the adsorption of pure gas and binary mixtures on a wet coal have been conducted by a number of researchers (Hall et al., 1994; Fitzgerald et al., 2005). Clarkson and Bustin (2000) studied the effect of moisture content on the gas adsorption/desorption isotherms. The results suggest that the selectivity for CO$_2$ is decreased by the presence of coal moisture. Krooss et al. (2002) conducted experiments on moisture-equilibrated coals and concluded that the sorption capacity of CO$_2$ decreases due to the moisture in coal. Jahediesfanjani and Civian (2006) conducted studies on the effect of water on enhanced coal gas recovery. They reported that the existence of water lowers the CO$_2$ and N$_2$ adsorption rates. Some of the accessible sorption sites on the coal surface are occupied by the bound water. This reduces sorption sites for gas molecules; however, these sorption sites cannot be reduced further as critical moisture content is reached. Ozdemir (2004) conducted similar experiments on the effect of moisture on CO$_2$ adsorption capacity of coal. It was found that water in the coal matrix significantly decreased the CO$_2$ adsorption capacity of coals. The relationship between the CO$_2$ adsorption capacity of coals and moisture content is represented by a linear equation below:

$$n_w = n_d - cm$$

(2.6)
In Eq.(2.6), \( n_w \) and \( n_d \) are adsorption capacities of wet and dried coals, respectively and \( m \) represents the moisture content \( (m \leq m_c) \). The sorption capacity factor is introduced in this work as a parameter accounting for the reduction of gas sorption capacity due to the presence of water in the coal matrix. The gas sorption capacity of dried coal is equal to unity. The sorption capacity factor decreases due to the effect of bound water as moisture content is introduced into the coal matrix; however, the sorption capacity factor remains the same at the critical value, \( f_c \), as the moisture increases above the critical moisture content, \( m_c \). In the proposed model, for CBM reservoir applications, bound water is considered immobile at all times and moisture content is always above the critical moisture content. Figure 2-2, which is created based on the findings of the previous work (Joubert et al., 1973, 1974; Ozdemir, 2004), describes the behavior of the gas sorption capacity affected by the presence of water in the coal matrix at a given pressure. Bulk water, on the other hand, does not have an effect on the sorption capacity but it forms the blocking mechanism in the coal matrix that reduces the mean radius of the gas flow path.

![Figure 2-2: Sorption capacity of coal as a function of moisture content](image)
Day et al. (2008) conducted experiments on the effects of moisture on the CO$_2$ and CH$_4$ sorption capacity of bituminous coals. It was found that the reduction of sorption capacity depended on the coal rank and the sorbate gas (CO$_2$ and CH$_4$). Moisture in the coals had more effect on the sorption capacity of low rank coals than that of high rank coals as low rank coals contain larger proportion of polar sites (hydroxyl groups) on the coal surface where water molecules are attached to. The decrease in gas sorption capacity while increasing moisture under critical moisture content was caused by the volumetric displacement of sorbate gas molecules by the water molecules. In addition, adsorbed water has larger effects on the reduction of CH$_4$ sorption capacity than that of CO$_2$ sorption capacity as CH$_4$ molecules can only attach on the surface of hydrophobic sites, which is not occupied by water molecules, while CO$_2$ molecules can form a layer over the adsorbed water.

In addition to the effects of water presence in the coal matrix, the adsorbate molecular size and pore structure of coal seams were found to have strong impact on the adsorption and diffusion of gases. Cui et al. (2003) conducted experiment study on the selective adsorption and diffusion of gases including CO$_2$, CH$_4$ and N$_2$ in coal seams. The results showed the selective of gas diffusion was strongly dependent on the relative adsorbate molecular size and adsorbent pore structure. CO$_2$ was found to permeate both macropores and ultra micropores due to its smaller diameter and found adsorbed in smaller pores due to its larger adsorption affinity.

### 2.4 Coal Shrinkage and Swelling

For nearly two decades, the effects of coal shrinkage and swelling on gas sorption have been studied. Harpalani and Schraufnagel (1990) conducted experiments on the effect of methane desorption on gas flow through coal. The results showed that, as pressure decreases below the desorption pressure, the adsorbed methane starts desorbing and the coal permeability
starts increasing sharply. Simultaneously, as pore pressure decreases, there is an increase in effective stress that leads to a reduction in permeability. These two responses from desorption and effective stress changes have opposing effects on permeability changes. A number of permeability models have been proposed to capture the influence of shrinkage and swelling on the permeability of coal. Sawyer et al. (1990) developed a coal permeability model based on the assumption that the permeability of coalbeds is sensitive to changes in internal stresses and the release of gas from the coal matrix. By superimposing the two effects, Sawyer et al. (1990) proposed the following permeability model:

\[
\frac{k}{k_i} = \left( \frac{\phi}{\phi_i} \right)^3 = \left[ 1 + c_p \left( p - p_i \right) - \frac{c_m \left( 1 - \phi_i \right)}{\phi_i} \frac{\Delta p_i}{\Delta C_i} (C - C_i) \right]^3 \tag{2.7}
\]

where \( C(p) = V_L \left( \frac{p}{p + p_L} \right) \tag{2.8} \)

Seidle and Huit (1995) proposed their model using the assumption that the ratio of the current permeability to the initial permeability is directly proportional to the cube of the ratio of the current porosity to the initial porosity. The porosity ratio was derived under the assumptions that the matrix shrinkage and swelling is proportional to the amount of gas adsorbed on the coal matrix and that the adsorbed gas is related to pressure via Langmuir’s equation. However, the elastic properties of coal are neglected in the development of this model. Eqs.(2.9) and (2.10) show the Seidle-Huitt permeability model:

\[
\frac{k}{k_i} = \left( \frac{\phi}{\phi_i} \right)^3 = \left[ 1 + \left( 1 + \frac{2}{\phi_i} \right) C_m \left( 1 \times 10^{-6} \right) V_m \left( \frac{p_i}{p_L + p_i} - \frac{p}{p_L + p} \right) \right]^3 \tag{2.9}
\]

where \( C_m = \frac{\varepsilon_{exp} + c_m p}{V_m \frac{p}{p_L + p}} \tag{2.10} \)
Palmer and Mansoori (1996) proposed another model derived using a cubic relationship between permeability and porosity and the equation of elasticity for strain in porous rock. This model was developed based on a theoretical formulation for stress-dependent permeability of coalbeds that includes (1) the effect of cleat compression causing a decrease in permeability and (2) the effect of the desorption of gas in the coal matrix that causes matrix shrinkage resulting in an increase in permeability. Eqs. (2.11) and (2.12) summarizes the Palmer-Mansoori model:

\[
\frac{k}{k_i} = \left(\frac{\phi}{\phi_i}\right)^3 = \left[1 + \tilde{c}_m \frac{(p - p_i)}{\phi_i} + \varepsilon_i \frac{K}{M} \left(\frac{p}{p_L + p} - \frac{p_i}{p_L + p_i}\right)\right]^3
\] (2.11)

where \(\tilde{c}_m = \frac{1}{M} \left[\frac{K}{M} + f - 1\right]\gamma\) (2.12)

Pekot and Reeves (2003) reported that Sawyer et al.’s model was equivalent to the Palmer-Mansoori model; however, Sawyer et al.’s model is considered to be more accurate than Palmer-Mansoori model for undersaturated CBM reservoirs.

Shi and Durucan’s model (2005) directly considers a relationship between the matrix shrinkage and the gas desorption during reservoir drawdown. The model uses an exponential relationship between the ratio of permeability and the change in effective stress and stress/strain relationship for a linear thermoelastic porous medium to derive the change in effective stress. Then, the assumption that volumetric-shrinkage strain is correlated directly to the amount of gas desorbed is implemented. The Shi-Durucan model can be written as follows:

\[
\frac{k}{k_i} = \exp \left\{-3C_f \left[-\frac{\nu}{1-\nu} (p - p_i) + \frac{E\alpha_3 V_L}{3(1-\nu)} \left(\frac{p}{p_L + p} - \frac{p_i}{p_L + p_i}\right)\right]\right\}
\] (2.13)

Permeability change due to the adsorption/release of the gas in the coal matrix is considered in developing all of the existing coal shrinkage/swelling models; however, none of the existing models accounts for the effects of water in the coal matrix on the gas sorption. In this work, such effects are incorporated in predicting the coal shrinkage/swelling behavior.
Chapter 3

Problem Statement

The transport of gas and water in coalbed methane reservoirs involves three different physical processes in two flow domains: fracture and matrix. At the initial stage, methane is adsorbed on the surface of coal adsorbent while the pores of fracture and matrix systems are filled with water. As the reservoir pressure decreases, water in the pores is drained out to the wellbore. As a result, adsorbed gas is released and diffuses in the matrix system and flows within the fracture network. Then, gas flows through the fracture network by convection towards the wellbore. However, a typical CBM reservoir model treats the CBM system as a dual-porosity, single-permeability scheme with the implication that the matrix acts as storage and serves as localized sources/sinks to the fractures while the fluids can only flow through the fractures. Such a concept ignores the existence of water in the coal matrix, which has also been known to decrease the gas sorption capacity. This often results in over-prediction of gas production and under-prediction of water production. In addition, two conflicting effects on coal permeability due to the changes in internal stress and the release of gas from the coal occur during pressure drawdown process. This behavior is referred as the coal shrinkage and swelling which also has effects on gas production.

All the key characteristics of CBM systems mentioned above are not captured by the existing numerical models for coal seams (King, 1986; Remner et al., 1986; Sung et al., 1986; Sawyer et al., 1990; Manik et al., 2002; Reeves and Pekot, 2001; Cicek, 2003). Therefore, the primary objective of this study is to develop a numerical model that simulates the two-phase gas-water flow in a dual-porosity, dual-permeability coalbed methane systems honoring the effects of the existence of water in the coal matrix and the coal shrinkage and swelling due to the coupled
effects of changes in the internal stress and the desorption/adsorption of gas in the coal. In addition, the transport of gas in the proposed model is governed by the multi-mechanistic flow dynamics that consider the influence of pressure and concentration gradients. The flow under pressure gradient is assumed to obey Darcy’s law while the flow under concentration gradients is assumed to be a Knudsen flow and obey the Fick’s law of diffusion. The application of multi-mechanistic flow is expected to increase the capability of the model in simulating gas flow in very tight reservoirs where the effect of gas diffusivity is significant. Moreover, the proposed model is developed compositionally and is applicable for enhanced recovery and CO₂ sequestration studies. Table 3-1 summarizes the key aforementioned characteristics accounted in the existing and proposed numerical CBM models

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Although a simulation model is often considered the most accurate tool used for understanding of the reservoir characteristics and predicting production performance, building and maintaining a reliable simulation model is time-consuming. In addition, unavailability and uncertainty of certain properties affect the accuracy of the simulation model. In this study, further applications of CBM reservoir engineering are proposed including a material balance equation and production type curves. Both applications simplify the reservoir systems by assuming that the reservoir system exhibits homogeneous and isotropic reservoir and fluid properties and can be utilized together with production data for production forecast and reservoir characterization. The proposed material balance equation is modified and used for predicting drainage area, production performance and average reservoir pressure. Moreover, a series of production type curves generated using the proposed simulation model is used for predicting reservoir properties.
Chapter 4

Formulation of the Compositional Fluid Flow in Dual-porosity, Dual-permeability Domains

The mathematical formulation is derived based on the multi-mechanistic two-phase (gas and water) flow model in a dual-porosity, dual-permeability system. The governing equations are developed for the simultaneous flow of gas and water in two domains: fracture network and coal matrix. It is assumed that water can be present in the matrix. In this development, gas phase is treated as a multi-component system while water phase is assumed to be a single component system. In addition, gas transport is considered to be multi-mechanistic in both fracture and matrix domains while water transport is considered to take place by Darcian flow only.

4.1 Governing Equations

Using the multi-mechanistic superficial gas velocity in Eq.(2.5), the gas flow equation for component \( l \) in the fracture domain can be written as follows:

\[
\frac{\partial}{\partial x} \left[ \frac{D_{l,F} \phi_F S_{g,F} A_x}{5.615} \frac{\partial}{\partial x} \left( X_{l,F} \frac{S_{g,F}}{B_{g,F}} \right) \right] + \frac{\partial}{\partial y} \left[ \frac{D_{l,F} \phi_F S_{g,F} A_y}{5.615} \frac{\partial}{\partial y} \left( X_{l,F} \frac{S_{g,F}}{B_{g,F}} \right) \right] + \frac{\partial}{\partial z} \left[ \frac{D_{l,F} \phi_F S_{g,F} A_z}{5.615} \frac{\partial}{\partial z} \left( X_{l,F} \frac{S_{g,F}}{B_{g,F}} \right) \right]
\]

\[
= X_{l,F} \left( \frac{F_g}{F_r} \frac{A_x k_{x,F} k_{rg,F}}{\mu_g B_{g,F}} \frac{\partial \Phi_{g,F}}{\partial x} + R_{sw,F} \frac{A_y k_{y,F} k_{rw,F}}{\mu_w B_{w,F}} \frac{\partial \Phi_{w,F}}{\partial y} \right) + R_{sw,F} \frac{A_y k_{y,F} k_{rw,F}}{\mu_w B_{w,F}} \frac{\partial \Phi_{w,F}}{\partial y} \Delta x
\]

\[
+ X_{l,F} \left( \frac{F_g}{F_r} \frac{A_y k_{y,F} k_{rg,F}}{\mu_g B_{g,F}} \frac{\partial \Phi_{g,F}}{\partial y} + R_{sw,F} \frac{A_z k_{z,F} k_{rw,F}}{\mu_w B_{w,F}} \frac{\partial \Phi_{w,F}}{\partial z} \right) \Delta y
\]

\[
+ X_{l,F} \left( \frac{F_g}{F_r} \frac{A_z k_{z,F} k_{rg,F}}{\mu_g B_{g,F}} \frac{\partial \Phi_{g,F}}{\partial z} + R_{sw,F} \frac{A_z k_{z,F} k_{rw,F}}{\mu_w B_{w,F}} \frac{\partial \Phi_{w,F}}{\partial z} \right) \Delta z
\]

\[
+ \Gamma_{g,F} + q_{g,F} + R_{sw,F} q_w = \frac{1}{5.615} \frac{\partial}{\partial t} \left[ X_{l,F} \frac{V_g \phi_F S_{g,F}}{B_{g,F}} + R_{sw,F} \frac{V_w \phi_F S_{w,F}}{B_{w,F}} \right]
\]

(4.1)
In Eq.(4.1) the term $\phi_F S_{g,F} A$ represents the effective area for gas to diffuse as it is transported through porous media as there are two phases in the system. Due to small fracture spacing of coal matrix, the gas flow equation for the same component $l$ in the matrix domain can be written as follows:

$$\Gamma_{g,l,M} - q_{srp,l} = \frac{1}{5.615} \frac{\partial}{\partial t} \left[ X_{l,M} \frac{V_b \phi_M S_{g,M}}{B_{g,M}} + R_{sw,l,M} \frac{V_b \phi_M S_{w,M}}{B_{w,M}} \right] \tag{4.2}$$

The depth is assumed to be positive downward from the datum plane; therefore, the gas flow potentials, $\Phi_{g,F}$ and $\Phi_{g,M}$ in Eqs(4.1) and (4.2) can be expressed as:

$$\Phi_{g,F} = p_{g,F} - \frac{1}{144} g \frac{\rho_g}{\rho_c} G$$ \tag{4.3}

$$\Phi_{g,M} = p_{g,M} - \frac{1}{144} g \frac{\rho_g}{\rho_c} G$$ \tag{4.4}

As stated earlier that water flow is controlled by Darcy’s law only and water flow equation in the fracture domain can be written as follows:

$$\frac{\partial}{\partial x} \left[ A_x k_{w,F} k_{rw,F} \frac{\partial \Phi_{w,F}}{\partial x} \right] + \frac{\partial}{\partial y} \left[ A_y k_{w,F} k_{rw,F} \frac{\partial \Phi_{w,F}}{\partial y} \right] + \frac{\partial}{\partial z} \left[ A_z k_{w,F} k_{rw,F} \frac{\partial \Phi_{w,F}}{\partial z} \right] \Delta x + \frac{\partial}{\partial y} \left[ A_y k_{w,F} k_{rw,F} \frac{\partial \Phi_{w,F}}{\partial y} \right] \Delta y + \frac{\partial}{\partial z} \left[ A_z k_{w,F} k_{rw,F} \frac{\partial \Phi_{w,F}}{\partial z} \right] \Delta z$$

$$+ \Gamma_{w,F} + q_w = \frac{1}{5.615} \frac{\partial}{\partial t} \left[ V_b \phi_F S_{w,F} \frac{\partial \Phi_{w,F}}{\partial t} \right] \tag{4.5}$$

Also, water flow equation in the matrix domain can be written as:

$$\Gamma_{w,M} = \frac{1}{5.615} \frac{\partial}{\partial t} \left[ V_b \phi_M S_{w,M} \frac{\partial \Phi_{w,M}}{\partial t} \right] \tag{4.6}$$
The water flow potentials, $\Phi_{w,F}$ and $\Phi_{w,M}$, in Eqs.(4.5) and (4.6) can be written as:

$$\Phi_{w,F} = p_{w,F} - \frac{1}{144} g \frac{\rho_w}{g_c} G$$  (4.7)

$$\Phi_{w,M} = p_{w,M} - \frac{1}{144} g \frac{\rho_g}{g_c} G$$  (4.8)

The necessary equations/correlations for rock and fluid properties are presented in Appendix A.

### 4.2 Auxiliary Equations

There are a total of $2NC+8$ unknown variables. These variables are $p_{g,F}$, $p_{g,M}$, $p_{w,F}$, $p_{w,M}$, $S_{g,F}$, $S_{g,M}$, $S_{w,F}$, $S_{w,M}$, $X_{l,F}$ and $X_{l,M}$, where $l = 1, 2, \ldots NC$. A set of the same number of equations is required to solve for those variables. A total of $2NC+2$ equations can be obtained from gas and water equations as stated in Eqs.(4.1), (4.2), (4.5) and (4.6). The remaining six equations can be obtained from capillary pressure relationship, saturation and mole fraction constraints for the fracture and matrix domains:

**Capillary pressure relationships:**

$$p_{cwg,F} \left( S_{g,F} \right) = p_{g,F} - p_{w,F}$$  (4.9)

$$p_{cgw,M} \left( S_{g,M} \right) = p_{g,M} - p_{w,M}$$  (4.10)

**Saturation constraints:**

$$S_{g,F} + S_{w,F} = 1$$  (4.11)

$$S_{g,M} + S_{w,M} = 1$$  (4.12)
Mole fraction constraints:

\[
\sum_{i=1}^{NC} X_{i,F} = 1 \\
\sum_{i=1}^{NC} X_{i,M} = 1
\] (4.13) (4.14)

4.3 Finite-Difference Approximations

The partial differential equations (PDE’s) describing the simultaneous flow of gas and water in the fracture and matrix domains shown in Eqs.(4.1), (4.2), (4.5) and (4.6) contain derivatives with respect to the space and time. These derivatives can be approximated with finite differences. The finite-difference approximation of the gas flow equation in the fracture domain can be written in residual form as:
\[
R_{g,F}^{n+1}_{i,j,k} = \frac{D_{l,F} \phi_F S_{g,F} A_x}{5.615 \Delta x} \left[ \left( X_{l,F} \frac{S_{g,F}}{B_{g,F}} \right)_{i,j,k}^{n+1} - \left( X_{l,F} \frac{S_{g,F}}{B_{g,F}} \right)_{i+1,j,k}^{n+1} \right] \\
- \frac{D_{l,F} \phi_F S_{g,F} A_x}{5.615 \Delta x} \left[ \left( X_{l,F} \frac{S_{g,F}}{B_{g,F}} \right)_{i-1,j,k}^{n+1} - \left( X_{l,F} \frac{S_{g,F}}{B_{g,F}} \right)_{i+1,j,k}^{n+1} \right] \\
+ \frac{A_x}{\Delta x} \left[ \frac{1}{2} k_x F_F \left( \frac{R_{x,F}^{n+1}}{B_{g,F}} \right)_{i+1,j,k}^{n+1} - \left( \frac{R_{x,F}^{n+1}}{B_{g,F}} \right)_{i,j,k}^{n+1} \right] \\
- \frac{A_x}{\Delta x} \left[ \frac{1}{2} k_x F_F \left( \frac{R_{x,F}^{n+1}}{B_{g,F}} \right)_{i-1,j,k}^{n+1} - \left( \frac{R_{x,F}^{n+1}}{B_{g,F}} \right)_{i,j,k}^{n+1} \right] \\
+ \frac{A_x}{\Delta x} \left[ \frac{1}{2} k_x F_F \left( \frac{R_{x,F}^{n+1}}{B_{g,F}} \right)_{i+1,j,k}^{n+1} - \left( \frac{R_{x,F}^{n+1}}{B_{g,F}} \right)_{i,j,k}^{n+1} \right] \\
- \frac{A_x}{\Delta x} \left[ \frac{1}{2} k_x F_F \left( \frac{R_{x,F}^{n+1}}{B_{g,F}} \right)_{i-1,j,k}^{n+1} - \left( \frac{R_{x,F}^{n+1}}{B_{g,F}} \right)_{i,j,k}^{n+1} \right] \\
+ \frac{D_{l,F} \phi_F S_{g,F} A_y}{5.615 \Delta y} \left[ \left( X_{l,F} \frac{S_{g,F}}{B_{g,F}} \right)_{i,j+1,k}^{n+1} - \left( X_{l,F} \frac{S_{g,F}}{B_{g,F}} \right)_{i,j,k}^{n+1} \right] \\
- \frac{D_{l,F} \phi_F S_{g,F} A_y}{5.615 \Delta y} \left[ \left( X_{l,F} \frac{S_{g,F}}{B_{g,F}} \right)_{i,j-1,k}^{n+1} - \left( X_{l,F} \frac{S_{g,F}}{B_{g,F}} \right)_{i,j,k}^{n+1} \right] \\
+ \frac{A_y}{\Delta y} \left[ \frac{1}{2} k_y F_F \left( \frac{R_{y,F}^{n+1}}{B_{g,F}} \right)_{i,j+1,k}^{n+1} - \left( \frac{R_{y,F}^{n+1}}{B_{g,F}} \right)_{i,j,k}^{n+1} \right] \\
- \frac{A_y}{\Delta y} \left[ \frac{1}{2} k_y F_F \left( \frac{R_{y,F}^{n+1}}{B_{g,F}} \right)_{i,j-1,k}^{n+1} - \left( \frac{R_{y,F}^{n+1}}{B_{g,F}} \right)_{i,j,k}^{n+1} \right] \\
+ \frac{A_y}{\Delta y} \left[ \frac{1}{2} k_y F_F \left( \frac{R_{y,F}^{n+1}}{B_{g,F}} \right)_{i,j+1,k}^{n+1} - \left( \frac{R_{y,F}^{n+1}}{B_{g,F}} \right)_{i,j,k}^{n+1} \right] \\
- \frac{A_y}{\Delta y} \left[ \frac{1}{2} k_y F_F \left( \frac{R_{y,F}^{n+1}}{B_{g,F}} \right)_{i,j-1,k}^{n+1} - \left( \frac{R_{y,F}^{n+1}}{B_{g,F}} \right)_{i,j,k}^{n+1} \right]
\]
In the right-hand side of Eq.(4.15), the first, second and third terms of the derivative expression describes the Fickian flow, the Darcian flow and the solution flow of gas component $l$, respectively.

The finite-difference approximation of the gas flow equation in the matrix domain can be written in residual form as:
Similarly, the finite-difference approximation of the water flow equation in the fracture domain can be written in residual form as:

\[
R_{w,F}^{n+1}_{i,j,k} = \frac{A_x}{\Delta x} \left( k_{x,F}^{n+1} \frac{1}{\mu_w B_{w,F(i+1/2,j,k)}} \right) \left( \Phi_{w,F(i+1,1/2,j,k)}^{n+1} - \Phi_{w,F(i+1/2,j,k)}^{n+1} \right) - \frac{A_y}{\Delta y} \left( k_{y,F}^{n+1} \frac{1}{\mu_w B_{w,F(i,j+1/2,k)}} \right) \left( \Phi_{w,F(i,j+1,1/2)k}^{n+1} - \Phi_{w,F(i,j+1/2,k)}^{n+1} \right)
\]

\[
+ \frac{A_z}{\Delta z} \left( k_{z,F}^{n+1} \frac{1}{\mu_w B_{w,F(i,j,k+1/2)}} \right) \left( \Phi_{w,F(i,j,k+1,1/2)}^{n+1} - \Phi_{w,F(i,j,k+1/2)}^{n+1} \right) - \frac{A_z}{\Delta z} \left( k_{z,F}^{n+1} \frac{1}{\mu_w B_{w,F(i,j,k-1/2)}} \right) \left( \Phi_{w,F(i,j,k-1,1/2)}^{n+1} - \Phi_{w,F(i,j,k-1/2)}^{n+1} \right)
\]

\[
+ \Gamma_{w,F}^{n+1}_{i,j,k} + q_{w}^{n+1}_{i,j,k} - \frac{1}{5.615\Delta t} \left( V_b\Phi_b S_{w,F} \right)^{n+1}_{i,j,k} + \frac{1}{5.615\Delta t} \left( V_b\Phi_b S_{w,F} \right)^{n}_{i,j,k}
\]

\[(4.17)\]
The finite-difference approximation of the water flow equation in the matrix domain can be written in residual form as:

\[
R_{w,M}^{n+1}_{i,j,k} = \Gamma_{w,M}^{n+1}_{i,j,k} - \frac{1}{5.615\Delta t} \left( \frac{V_b \phi_M S_{w,M}}{B_{w,M}} \right)^{n+1}_{i,j,k} + \frac{1}{5.615\Delta t} \left( \frac{V_b \phi_M S_{w,M}}{B_{w,M}} \right)^{n}_{i,j,k}
\]  (4.18)

### 4.4 Calculation of the Transmissibility Terms

The transmissibility terms represent the fluid movement at the interface between two neighboring grid blocks. The transmissibility expressions generally contain three different groups of terms including grid block properties, fluid properties and rock properties. Consider the following gas transmissibility terms from Eq.(4.15) at the interface between grid block \(i,j,k\) and \(i+1,j,k\) in the fracture system as shown below:

\[
\left[ \frac{A_x}{\Delta x} \bigg|_{i+\frac{1}{2},j,k} \bigg] \cdot \left[ \frac{1}{\mu g} \frac{B_{g,F}}{\bigg|_{i+\frac{1}{2},j,k}} \right] \cdot \left[ k_{rg,F}^{n+1} \right]
\]

The first group, \( \left( \frac{A_x}{\Delta x} \bigg|_{i+\frac{1}{2},j,k} \right) \cdot \left( k_{x,F}^{n+1} \bigg|_{i+\frac{1}{2},j,k} \right) \), is related to the grid block dimensions and rock property (permeability). The grid block dimensions are constant. The permeability, however, is a function of pressure as it changes due to the effects of coal shrinkage and swelling. The harmonic average is used to calculate the first group and it can be written as follows:

\[
\left[ \frac{A_x}{\Delta x} \bigg|_{i+\frac{1}{2},j,k} \bigg] \cdot \left[ \frac{2A_x}{A_x} \bigg|_{i,j,k} A_x \bigg|_{i+1,j,k} k_{x,F}^{n+1} \bigg|_{i,j,k} k_{x,F}^{n+1} \bigg|_{i+1,j,k} \right] = \frac{2A_x}{A_x} \bigg|_{i,j,k} A_x \bigg|_{i+1,j,k} \left( \frac{k_{x,F}^{n+1} \bigg|_{i,j,k} k_{x,F}^{n+1} \bigg|_{i+1,j,k}}{\Delta x_{i+1,j,k}} \right)
\]  (4.19)
The second group, \( \left[ \frac{1}{\mu_B B_{g,F}} \right]^{n+1}_{i+\frac{1}{2},j,k} \), is a rather weak non-linear function of fluid properties (viscosity and formation volume factor). Each property in this group can be calculated using the arithmetic averaging technique as follows:

\[
\frac{1}{\mu_B B_{g,F}} \left|_{i+\frac{1}{2},j,k} \right|^{n+1} = \frac{1}{2} \left( \mu_B \left|_{i,j,k} \right|^{n+1} + \mu_B \left|_{i+1,j,k} \right|^{n+1} \right) \cdot \frac{1}{2} \left( B_s \left|_{i,j,k} \right|^{n+1} + B_s \left|_{i+1,j,k} \right|^{n+1} \right)
\]

(4.20)

The third group, \( k_{rg,F} \left|_{i,j,k} \right|^{n+1} \), is a strongly non-linear expression. The single-point upstream weighting technique is used to determine the gas mole fraction and relative permeability. The other averaging techniques for relative permeability could cause early disappearance of saturation in the production wells.

- If \( \Phi \left|_{i,j,k} \right|^{n+1} \geq \Phi \left|_{i+1,j,k} \right|^{n+1} \), then \( k_{rg,F} \left|_{i,j,k} \right|^{n+1} = k_{rg,F} \left|_{i+1,j,k} \right|^{n+1} \)

(4.21)

- If \( \Phi \left|_{i,j,k} \right|^{n+1} < \Phi \left|_{i+1,j,k} \right|^{n+1} \), then \( k_{rg,F} \left|_{i,j,k} \right|^{n+1} = k_{rg,F} \left|_{i+1,j,k} \right|^{n+1} \)

Note that similar to the transmissibility terms of Darcian flow above, the transmissibility group of the diffusion flow such as \( \left[ D_{i,F} A_x / \Delta x \right]^{n+1}_{i+\frac{1}{2},j,k} \) in Eq.(4.15) is calculated using Eq.(4.22) as the diffusion coefficient is considered to be constant:

\[
\left[ D_{i,F} A_x \right]^{n+1}_{i+\frac{1}{2},j,k} = D_{i,F} \frac{2 A_x \left|_{i,j,k} \right| A_x \left|_{i+1,j,k} \right|}{A_x \left|_{i,j,k} \right| \Delta x \left|_{i+1,j,k} \right| + A_x \left|_{i+1,j,k} \right| \Delta x \left|_{i,j,k} \right|}
\]

(4.22)

In addition, harmonic average is used for the calculations of the non-linear term \( \left[ \phi_F S_{g,F} \right]^{n+1}_{i+\frac{1}{2},j,k} \):

\[
\left[ \phi_F S_{g,F} \right]^{n+1}_{i+\frac{1}{2},j,k} = \frac{2 \left[ \phi_F S_{g,F} \right]_{i,j,k} \left[ \phi_F S_{g,F} \right]_{i+1,j,k}}{\left( \phi_F S_{g,F} \right)_{i,j,k} + \left( \phi_F S_{g,F} \right)_{i+1,j,k}}
\]

(4.23)
Calculations of the transmissibility terms in other flow equations including Eq.(4.17) are analogous to Eqs.(4.19), (4.20) and (4.21) where the harmonic averaging technique is used to calculate the group of grid block dimensions and permeability and the arithmetic averaging technique is used for the weakly non-linear fluid properties including viscosity and formation volume factor.

In addition, single-point upstream weighting technique is used in calculating the mole fraction at the interface between two blocks as suggested by Manik et al. (2002).

\[ \text{If, } \Phi_{g,F}^{n+1}_{i,j,k} \geq \Phi_{g,F}^{n+1}_{i+1,j,k} \text{ then } X_{i,j,k}^{n+1} = X_{i,j,k}^{n+1} \]  

\[ \text{If } \Phi_{g,F}^{n+1}_{i,j,k} < \Phi_{g,F}^{n+1}_{i+1,j,k} \text{, then } X_{i+1,j,k}^{n+1} = X_{i+1,j,k}^{n+1} \]  

4.5 Source/Sink Terms

In the dual-porosity, dual-permeability CBM reservoir system, the source/sink terms can be categorized under two groups including the implicit and explicit source/sink terms.

4.5.1 Implicit Source/Sink Terms

The term, \( \Gamma_{g,F}^{n+1}_{i,j,k} \) in Eq.(4.15) represents the gas transfer from the fracture to the matrix. Chawathe et al. (1996) proposed the following implicit source relationship that describes the gas transfer between the fracture and the matrix domains in fractured reservoirs:
The average gas pressure, \( \bar{p}_{g}^{n+1}_{i,j,k} \), is an arithmetic average of gas pressures of the fracture and matrix systems. Since the diffusion in the tight matrix is dominant, the matrix diffusion coefficient, \( D_{i,M} \), is used instead of the average diffusion coefficient. The average permeability, \( \bar{k}_{i,j,k}^{n+1} \), can be calculated using the geometric mean permeability obtained from the \( x \)- and \( y \)-directional permeabilities. In this study, the shape factor, \( \sigma \), developed by Chang (1993) for unsteady flow in fractured rock is used.

\[
\sigma = \pi^2 \left( \frac{1}{L_x^2} + \frac{1}{L_y^2} + \frac{1}{L_z^2} \right)
\]  

In Eq.(4.26), \( L_x, L_y \) and \( L_z \) are the lengths of the coal matrix in \( x \)-, \( y \)- and \( z \)-directions, respectively.

The gas mole fraction and relative gas permeability are determined using the one-point upstream weighting technique as:

- If \( p_{g,M}^{n+1}_{i,j,k} \geq p_{g,F}^{n+1}_{i,j,k} \), then \( X_{l,M}^{n+1}_{i,j,k} = X_{l,F}^{n+1}_{i,j,k} \) and \( k_{rg,F}^{n+1}_{i,j,k} = k_{rg,M}^{n+1}_{i,j,k} \)
- If \( p_{g,M}^{n+1}_{i,j,k} < p_{g,F}^{n+1}_{i,j,k} \), then \( X_{l,F}^{n+1}_{i,j,k} = X_{l,M}^{n+1}_{i,j,k} \) and \( k_{rg,F}^{n+1}_{i,j,k} = k_{rg,M}^{n+1}_{i,j,k} \)

Assuming that all the gas transferred from matrix goes to fracture, one can obtain the following equation:

\[
\Gamma_{g,F}^{n+1}_{i,j,k} = -\Gamma_{g,F}^{n+1}_{i,j,k}
\]  

(4.25)
The water transfer term, $\Gamma_{w,F}^{n+1}_{i,j,k}$, in Eq.(4.17) can be written as:

$$
\Gamma_{w,F}^{n+1}_{i,j,k} = \sigma V_b \left( \frac{k_{rw,m}^{n+1}_{i,j,k} k_{mj}^{n+1}_{i,j,k}}{\mu_w \left( p_w^{n+1}_{i,j,k} - p_{w,m}^{n+1}_{i,j,k} \right) B_w \left( p_{w,m}^{n+1}_{i,j,k} \right)} \right) p_{w,m}^{n+1}_{i,j,k} - p_{w,F}^{n+1}_{i,j,k}
$$  (4.29)

The average pressure in the water phase, $p_{w,m}^{n+1}_{i,j,k}$, is an arithmetic average of water pressures of the fracture and matrix systems. Similar to the relative gas permeability, the relative permeability to water is also determined using the one-point upstream weighting technique as follows:

- If $p_{w,m}^{n+1}_{i,j,k} \geq p_{w,F}^{n+1}_{i,j,k}$ then $k_{rw,m}^{n+1}_{i,j,k} = k_{rw,m}^{n+1}_{i,j,k}$  (4.30)

- If $p_{w,m}^{n+1}_{i,j,k} < p_{w,F}^{n+1}_{i,j,k}$, then $k_{rw,m}^{n+1}_{i,j,k} = k_{rw,F}^{n+1}_{i,j,k}$

Assuming that all the water transferred from matrix goes to fracture, we obtain the following equation:

$$
\Gamma_{w,M}^{n+1}_{i,j,k} = \Gamma_{w,F}^{n+1}_{i,j,k}
$$  (4.31)

### 4.5.2 Explicit Source/Sink Terms

(a) **Gas flow rate term**

The gas flow rate term, $q_{x}^{n+1}_{i,j,k}$, in Eq.(4.15) can be expressed by the Peaceman’s wellbore equation as follows:
The average fracture permeability, $\bar{k}_F$, is calculated using the geometric mean of the permeabilities in the plane that is perpendicular to the well.

**b) Water flow rate term**

The water flow rate term, $q_w$, in Eq.(4.17) can be written as:

$$q_{w, i,j,k}^{n+1} = -X_{i,F}^{n+1} \frac{2\pi T_{sc}}{T_p} \left( k_{rg,F}^{n+1} k_F^{n+1} h_{i,j,k} \left( p_{g,F}^{n+1} - p_{wf}^{n+1} \right) \right)$$

$$- \frac{2\pi T_{sc}}{T_p} \cdot D_{l,F} h_{i,j,k} X_{i,F} \left( p_{g,F}^{n+1} S_{g,F}^{n+1} - p_{wf}^{n+1} S_{g,F}^{n+1} \right)$$

$$- \frac{Z(p_{g,F}^{n+1})}{Z(p_{wf}^{n+1})} \ln \left( \frac{r_e^{i,j,k}}{r_w^{i,j,k}} \right) + S_{i,j,k}$$

**c) Gas sorption rate term**

King et al. (1986) proposed a non-equilibrium sorption rate model for calculating the gas sorption term, $q_{sg,F}^{n+1}$. The non-equilibrium sorption model is pressure and time dependent and is derived on the basis of the rate of gas adsorbed in the coal:

$$\frac{dV_{a,f}}{dt} = \frac{1}{\tau} (V_{a,f} - V_{c,f})$$
\( V_{a,l} \) and \( V_{c,l} \) are the amount of gas adsorbed and adsorption capacity of the gas component \( l \), respectively. \( \tau \) is the pseudo-steady state sorption time constant, which represents the time delay of sorbed gas transport through the micropores in the coal matrix. Rearranging Eq.(4.34), we can obtain:

\[
\int_{V_{a,l}^{n+1}}^{V_{a,l}^n} \frac{dV_{a,l}}{V_{a,l} - V_{c,l}} = \int_{\tau}^{\tau + t} \frac{dt}{\tau} \tag{4.35}
\]

In order for the sorption rate formulation to be applicable for the instantaneous sorption process, the adsorption capacity, \( V_{c,l} \), in Eq.(4.35) is proposed to be evaluated at the time level \( n+1 \) (Manik et al., 2002). By integrating both sides of Eq.(4.35), one obtains the following non-equilibrium sorption rate formulation:

\[
V_{a,l}^{n+1}_{i,j,k} = V_{a,l}^n_{i,j,k} \exp\left(-\frac{\Delta t}{\tau}\right) + V_{c,l}^n_{i,j,k} \left[1 - \exp\left(-\frac{\Delta t}{\tau}\right)\right] \tag{4.36}
\]

The gas sorption rate for gas component \( l \) can be calculated as:

\[
q_{\text{ssp},l}^{n+1}_{i,j,k} = \frac{V_{a,l}^{n+1}_{i,j,k} - V_{a,l}^n_{i,j,k}}{\Delta t} \tag{4.37}
\]

### 4.6 Calculations of Multi-component Sorptions

In this work, the partial adsorption capacity of gas components is calculated based on the thermodynamic equilibrium between gas components in the free and adsorbed phase (Myers and Prausnitz, 1965). Using the ideal adsorbed solution model, the equilibrium relations between gas components in the free phase (\( X_l \)) and adsorbed phase (\( Y_l \)) can be written as:

\[
f_i = f_i^0 Y_l \tag{4.38}
\]

or

\[
pX_i \varphi_i = p_i^0 \varphi_i^0 Y_l \tag{4.39}
\]
where \( f_i^0 \) is fugacity of pure component \( l \) in gas phase at standard conditions, \( p_i^0 \) is pressure at a standard state condition of pure component \( l \) in the adsorbed phase and \( \varphi_i^0 \) is pure component fugacity coefficient at standard conditions. The relationship between the pure component adsorption capacity, \( n_i^0 \), and the spreading pressure group at the standard state, \( \psi_i^0 \), can be written as:

\[
\psi_i^0 = \int_0^{f_i^0} n_i^0 \, df_i
\]

(4.40)

where \( f_i \) is fugacity of component \( l \) in gas phase. At standard conditions, spreading pressure values for all components are equal; therefore, one can obtain:

\[
\psi_1^0 = \psi_2^0 = \ldots = \psi_{NC}^0
\]

(4.41)

In calculating the multi-component sorption, there are a total of \( 2 \times NC \) unknowns including \( Y_1, Y_2, \ldots, Y_{NC} \) and \( f_1^0, f_2^0, \ldots, f_{NC}^0 \). Eqs.(4.38), (4.39) and (4.41) provides \( 2 \times NC - 1 \); therefore, the mole fraction constraint of the adsorbed phase is needed.

\[
\sum_{i=1}^{NC} Y_i = 1
\]

(4.42)

To solve for the unknowns \( Y_1, Y_2, \ldots, Y_{NC} \) and \( f_1^0, f_2^0, \ldots, f_{NC}^0 \), Manik et al. (2002) proposed an iterative method using the following objection function:

\[
\Re(\psi) = \sum_{i=1}^{NC} Y_i - 1 = \sum_{i=1}^{NC} \left( \frac{f_i}{f_i^0} \right) - 1 = 0
\]

(4.43)

Eq.(4.43) is solved by the Newton-Raphson procedure as follows:

\[
\psi^0|_{k+1} = \psi^0|_k - \frac{\Re(\psi^0)}{d\Re(\psi^0)/d\psi^0} |_k
\]

(4.44)
Substituting Eq.(4.40) into the derivative \( \frac{d\mathcal{R}(\psi^0)}{d\psi^0} \), one can obtain:

\[
\frac{d\mathcal{R}(\psi^0)}{d\psi^0} = \frac{d}{d\psi^0} \left[ \sum_{i=1}^{NC} \left( \frac{f_i}{f_i^0} \right) - 1 \right] = \sum_{i=1}^{NC} \left[ - f_i \frac{df_i^0}{d\psi^0} \right] \left( \frac{f_i^0}{f_i^0} \right)^2
\]

(4.45)

where \( \frac{df_i^0}{d\psi^0} = \frac{f_i^0}{n_i^0} \)  

(4.46)

Substituting Eqs.(4.45) and (4.46) into Eq.(4.44) gives:

\[
\psi^0 \bigg|_{k+1} = \psi^0 \bigg|_k - \frac{\sum_{i=1}^{NC} \left[ f_i \frac{f_i}{f_i^0} \right] - 1}{- \sum_{i=1}^{NC} \left[ f_i n_i^0 \frac{f_i}{f_i^0} \right]} \]

(4.47)

The iterative procedure developed by Manik et al. (2002) to solve Eq.(4.47) starts by calculating the fugacity of gas in the free phase \( (f_i) \) using Eqs.(A-2) and (A-3) and adsorption capacity \( (n_i) \) using any sorption isotherm for a specified range of pressure. The calculated \( f_i \) and \( n_i \) are fitted to the following equation:

\[
n_i = \frac{c_1 f_i}{c_2 + f_i}
\]

(4.48)

where \( c_1 \) and \( c_2 \) are curve-fitting coefficients. Substituting Eq.(4.48) into Eq.(4.40) yields an expression of spreading pressure group as:

\[
\psi^0_i = \int_0^{f_i^0} \frac{c_1}{c_2 + f_i} df_i = c_1 \ln \left( \frac{c_2 + f_i^0}{c_2} \right)
\]

(4.49)

Re-writing Eq.(4.49), one can obtain an expression of fugacity as:

\[
f_i^0 = c_2 \exp \left( \frac{\psi^0_i}{c_1} \right) - c_2
\]

(4.50)
Also, \( n_i^0 \) can be calculated as:

\[
n_i^0 = \frac{c_if_i^0}{c_2 + f_i^0}
\]  \hspace{1cm} (4.51)

The convergence criterion for Eq.(4.47) is set as:

\[
\left| \psi^{k+1} - \psi^k \right| \leq 10^{-15}
\]  \hspace{1cm} (4.52)

### 4.7 Generalized Newton-Raphson Procedure

The developed numerical model contains a system of \( NB \times (2NC+2) \) non-linear equations. The generalized Newton-Raphson procedure is used to linearize the non-linear equations and form a system of linear equations. The system of linear equation is then solved using an iterative protocol. By implementing the six auxiliary equations mentioned previously into the residual equations, including Eq.(4.15), (4.16), (4.17) and (4.18), we can reduce the number of primary unknowns for each block to \( 2NC+2 \) (\( p_{g,F}, p_{g,M}, S_{w,F}, S_{w,M}, X_{1,F}, X_{2,F}, \ldots, X_{NC-1,F} \) and \( X_{1,M}, X_{2,M}, \ldots, X_{NC-1,M} \)). By linearizing the residual equations for gas flow in the fracture system using the generalized Newton-Raphson procedure, we can obtain the following residual equation for the block \( m \):
\[
[R_{g,f,t}]_{m}^{(k)} \mid_{n+1} = \\
- \left( \left[ \frac{\partial R_{g,f,t}}{\partial p_{f,t}} \right]_{m}^{(k)} \bigg)_{n+1} \left[ \Delta p_{g,f} \right]_{m}^{(k+1)} + \left[ \frac{\partial R_{g,f,t}}{\partial p_{g,M}} \right]_{m}^{(k)} \bigg)_{n+1} \left[ \Delta p_{g,M} \right]_{m}^{(k+1)} \right) \\
+ \left[ \frac{\partial R_{g,f,t}}{\partial S_{w,F}} \right]_{m}^{(k)} \bigg)_{n+1} \left[ \Delta S_{w,F} \right]_{m}^{(k+1)} + \left[ \frac{\partial R_{g,f,t}}{\partial S_{w,M}} \right]_{m}^{(k)} \bigg)_{n+1} \left[ \Delta S_{w,M} \right]_{m}^{(k+1)} \right) \\
+ \left[ \frac{\partial R_{g,f,t}}{\partial X_{1,F}} \right]_{m}^{(k)} \bigg)_{n+1} \left[ \Delta X_{1,F} \right]_{m}^{(k+1)} \right) + \left[ \frac{\partial R_{g,f,t}}{\partial X_{2,F}} \right]_{m}^{(k)} \bigg)_{n+1} \left[ \Delta X_{2,F} \right]_{m}^{(k+1)} + \left[ \frac{\partial R_{g,f,t}}{\partial X_{NC_{1,F}}} \right]_{m}^{(k)} \bigg)_{n+1} \left[ \Delta X_{NC_{1,F}} \right]_{m}^{(k+1)} \right) \\
+ \left[ \frac{\partial R_{g,f,t}}{\partial X_{1,M}} \right]_{m}^{(k)} \bigg)_{n+1} \left[ \Delta X_{1,M} \right]_{m}^{(k+1)} \right) + \left[ \frac{\partial R_{g,f,t}}{\partial X_{2,M}} \right]_{m}^{(k)} \bigg)_{n+1} \left[ \Delta X_{2,M} \right]_{m}^{(k+1)} + \left[ \frac{\partial R_{g,f,t}}{\partial X_{NC_{1,M}}} \right]_{m}^{(k)} \bigg)_{n+1} \left[ \Delta X_{NC_{1,M}} \right]_{m}^{(k+1)} \right) \\
+ \left( - \left[ \frac{\partial R_{g,f,t}}{\partial p_{f,t}} \right]_{m}^{(k+1)} \bigg)_{n+1} \left[ \Delta p_{g,f} \right]_{m}^{(k+1)} + \left[ \frac{\partial R_{g,f,t}}{\partial p_{g,M}} \right]_{m}^{(k+1)} \bigg)_{n+1} \left[ \Delta p_{g,M} \right]_{m}^{(k+1)} \right) \\
+ \left[ \frac{\partial R_{g,f,t}}{\partial S_{w,F}} \right]_{m}^{(k+1)} \bigg)_{n+1} \left[ \Delta S_{w,F} \right]_{m}^{(k+1)} + \left[ \frac{\partial R_{g,f,t}}{\partial S_{w,M}} \right]_{m}^{(k+1)} \bigg)_{n+1} \left[ \Delta S_{w,M} \right]_{m}^{(k+1)} \right) \\
+ \left[ \frac{\partial R_{g,f,t}}{\partial X_{1,F}} \right]_{m}^{(k+1)} \bigg)_{n+1} \left[ \Delta X_{1,F} \right]_{m}^{(k+1)} \right) + \left[ \frac{\partial R_{g,f,t}}{\partial X_{2,F}} \right]_{m}^{(k+1)} \bigg)_{n+1} \left[ \Delta X_{2,F} \right]_{m}^{(k+1)} + \left[ \frac{\partial R_{g,f,t}}{\partial X_{NC_{1,F}}} \right]_{m}^{(k+1)} \bigg)_{n+1} \left[ \Delta X_{NC_{1,F}} \right]_{m}^{(k+1)} \right) \\
+ \left[ \frac{\partial R_{g,f,t}}{\partial X_{1,M}} \right]_{m}^{(k+1)} \bigg)_{n+1} \left[ \Delta X_{1,M} \right]_{m}^{(k+1)} \right) + \left[ \frac{\partial R_{g,f,t}}{\partial X_{2,M}} \right]_{m}^{(k+1)} \bigg)_{n+1} \left[ \Delta X_{2,M} \right]_{m}^{(k+1)} + \left[ \frac{\partial R_{g,f,t}}{\partial X_{NC_{1,M}}} \right]_{m}^{(k+1)} \bigg)_{n+1} \left[ \Delta X_{NC_{1,M}} \right]_{m}^{(k+1)} \right) + \ldots \right)
\[
- \left\{ \frac{\partial R_{g,l,F}}{\partial p_{g,F}} \right\}_{m}^{(k)} \left[ \Delta p_{g,F} \right]_{NB}^{(k)} + \frac{\partial R_{g,l,F}}{\partial p_{g,M}} \left[ \Delta p_{g,M} \right]_{NB}^{(k)} + \frac{\partial R_{g,l,F}}{\partial S_{w,F}} \left[ \Delta S_{w,F} \right]_{NB}^{(k)} + \frac{\partial R_{g,l,F}}{\partial S_{w,M}} \left[ \Delta S_{w,M} \right]_{NB}^{(k)} + \frac{\partial R_{g,l,F}}{\partial X_{1,F}} \left[ \Delta X_{1,F} \right]_{NB}^{(k)} + \frac{\partial R_{g,l,F}}{\partial X_{2,F}} \left[ \Delta X_{2,F} \right]_{NB}^{(k)} + \frac{\partial R_{g,l,F}}{\partial X_{1,M}} \left[ \Delta X_{1,M} \right]_{NB}^{(k)} + \frac{\partial R_{g,l,F}}{\partial X_{2,M}} \left[ \Delta X_{2,M} \right]_{NB}^{(k)} \right\}^{(k+1)}_{n+1} \\
\text{where } l = 1, 2, \ldots, NC \text{ and } m = 1, 2, \ldots, NB. \tag{4.53}
\]
Similarly, one can obtain the linear gas residual equation for the matrix system for block $m$ as follows:

$$
[R_{g,l,M}]_{m}^{(k)} =
- \left\{ \left[ \frac{\partial R_{g,l,M}}{\partial p_{g,F}} \right]_{m}^{(k)} \left[ \Delta p_{g,F} \right]_{m}^{(k+1)} + \left[ \frac{\partial R_{g,l,M}}{\partial p_{g,M}} \right]_{m}^{(k)} \left[ \Delta p_{g,M} \right]_{m}^{(k+1)} \right\} 
+ \left\{ \left[ \frac{\partial R_{g,l,M}}{\partial S_{w,F}} \right]_{m}^{(k)} \left[ \Delta S_{w,F} \right]_{m}^{(k+1)} + \left[ \frac{\partial R_{g,l,M}}{\partial S_{w,M}} \right]_{m}^{(k)} \left[ \Delta S_{w,M} \right]_{m}^{(k+1)} \right\} 
+ \left\{ \left[ \frac{\partial R_{g,l,M}}{\partial X_{1,F}} \right]_{m}^{(k)} \left[ \Delta X_{1,F} \right]_{m}^{(k+1)} + \left[ \frac{\partial R_{g,l,M}}{\partial X_{2,F}} \right]_{m}^{(k)} \left[ \Delta X_{2,F} \right]_{m}^{(k+1)} \right\} + \ldots
+ \left\{ \left[ \frac{\partial R_{g,l,M}}{\partial X_{NC-1,F}} \right]_{m}^{(k)} \left[ \Delta X_{NC-1,F} \right]_{m}^{(k+1)} \right\} 
- \left\{ \left[ \frac{\partial R_{g,l,M}}{\partial p_{g,F}} \right]_{2}^{(k)} \left[ \Delta p_{g,F} \right]_{2}^{(k+1)} + \left[ \frac{\partial R_{g,l,M}}{\partial p_{g,M}} \right]_{2}^{(k)} \left[ \Delta p_{g,M} \right]_{2}^{(k+1)} \right\} 
+ \left\{ \left[ \frac{\partial R_{g,l,M}}{\partial S_{w,F}} \right]_{2}^{(k)} \left[ \Delta S_{w,F} \right]_{2}^{(k+1)} + \left[ \frac{\partial R_{g,l,M}}{\partial S_{w,M}} \right]_{2}^{(k)} \left[ \Delta S_{w,M} \right]_{2}^{(k+1)} \right\} 
+ \left\{ \left[ \frac{\partial R_{g,l,M}}{\partial X_{1,F}} \right]_{2}^{(k)} \left[ \Delta X_{1,F} \right]_{2}^{(k+1)} + \left[ \frac{\partial R_{g,l,M}}{\partial X_{2,F}} \right]_{2}^{(k)} \left[ \Delta X_{2,F} \right]_{2}^{(k+1)} \right\} + \ldots
+ \left\{ \left[ \frac{\partial R_{g,l,M}}{\partial X_{NC-1,F}} \right]_{2}^{(k)} \left[ \Delta X_{NC-1,F} \right]_{2}^{(k+1)} \right\} + \ldots
$$
\[
\left. - \left[ \frac{\partial R_{g,i,M}}{\partial p_{g,F}} \right]_{n+1}^{(k)} \Delta p_{g,F}^{(k+1)} \right|_{N_{B}}^{n+1} + \left[ \frac{\partial R_{g,i,M}}{\partial p_{g,M}} \right]_{n+1}^{(k)} \Delta p_{g,M}^{(k+1)} \right|_{N_{B}}^{n+1} \\
+ \left[ \frac{\partial R_{g,i,M}}{\partial s_{w,F}} \right]_{n+1}^{(k)} \Delta s_{w,F}^{(k+1)} \right|_{N_{B}}^{n+1} + \left[ \frac{\partial R_{g,i,M}}{\partial s_{w,M}} \right]_{n+1}^{(k)} \Delta s_{w,M}^{(k+1)} \right|_{N_{B}}^{n+1} \\
+ \left[ \frac{\partial R_{g,i,M}}{\partial x_{1,F}} \right]_{n+1}^{(k)} \Delta x_{1,F}^{(k+1)} \right|_{N_{B}}^{n+1} + \left[ \frac{\partial R_{g,i,M}}{\partial x_{2,F}} \right]_{n+1}^{(k)} \Delta x_{2,F}^{(k+1)} \right|_{N_{B}}^{n+1} + \ldots + \left[ \frac{\partial R_{g,i,M}}{\partial x_{NC-1,F}} \right]_{n+1}^{(k)} \Delta x_{NC-1,F}^{(k+1)} \right|_{N_{B}}^{n+1} \\
+ \left[ \frac{\partial R_{g,i,M}}{\partial x_{1,M}} \right]_{n+1}^{(k)} \Delta x_{1,M}^{(k+1)} \right|_{N_{B}}^{n+1} + \left[ \frac{\partial R_{g,i,M}}{\partial x_{2,M}} \right]_{n+1}^{(k)} \Delta x_{2,M}^{(k+1)} \right|_{N_{B}}^{n+1} + \ldots + \left[ \frac{\partial R_{g,i,M}}{\partial x_{NC-1,M}} \right]_{n+1}^{(k)} \Delta x_{NC-1,M}^{(k+1)} \right|_{N_{B}}^{n+1} \right] \\
(4.54)
\]

where \( l = 1, 2, \ldots, NC \) and \( m = 1, 2, \ldots, NB \).
The linearization of the residual equations for water flow in the fracture system yields the following linear equation for block \( m \).

\[
[R_{w,F}]_{m}^{(k)} = \left\{ \left. \left( \frac{\partial R_{w,F}}{\partial p_{g,F}} \right)_{m}^{(k)} \right|_{n+1} \Delta p_{g,F} \right)_{n+1}^{(k+1)} + \left. \left( \frac{\partial R_{w,F}}{\partial p_{g,M}} \right)_{m}^{(k)} \right|_{n+1} \Delta p_{g,M} \right)_{n+1}^{(k+1)} \right\} \\
+ \left\{ \left. \left( \frac{\partial R_{w,F}}{\partial S_{w,F}} \right)_{m}^{(k)} \right|_{n+1} \Delta S_{w,F} \right)_{n+1}^{(k+1)} + \left. \left( \frac{\partial R_{w,F}}{\partial S_{w,M}} \right)_{m}^{(k)} \right|_{n+1} \Delta S_{w,M} \right)_{n+1}^{(k+1)} \right\} \\
- \left\{ \left. \left( \frac{\partial R_{w,F}}{\partial p_{g,F}} \right)_{NK}^{(k)} \right|_{n+1} \Delta p_{g,F} \right)_{NK}^{(k+1)} + \left. \left( \frac{\partial R_{w,F}}{\partial p_{g,M}} \right)_{NK}^{(k)} \right|_{n+1} \Delta p_{g,M} \right)_{NK}^{(k+1)} \right\} \\
+ \left\{ \left. \left( \frac{\partial R_{w,F}}{\partial S_{w,F}} \right)_{NK}^{(k)} \right|_{n+1} \Delta S_{w,F} \right)_{NK}^{(k+1)} + \left. \left( \frac{\partial R_{w,F}}{\partial S_{w,M}} \right)_{NK}^{(k)} \right|_{n+1} \Delta S_{w,M} \right)_{NK}^{(k+1)} \right\} + \ldots +
\]

\[
(4.55)
\]

where \( m = 1, 2, \ldots, NB \).
Similarly, one can obtain the linearized water residual equation for the matrix system for block $m$ as follows:

\[
\left[ R_{w,M} \right]_{m}^{(k)} =
\begin{cases}
\left[ \frac{\partial R_{w,M}}{\partial p_{g,F}} \right]_{m}^{(k)} \left[ \Delta p_{g,M} \right]_{m}^{(k+1)} + \left[ \frac{\partial R_{w,M}}{\partial S_{w,M}} \right]_{m}^{(k)} \left[ \Delta S_{w,M} \right]_{m}^{(k+1)}
\end{cases}
\]

\[
\begin{align*}
\left[ \frac{\partial R_{w,M}}{\partial S_{w,F}} \right]_{m}^{(k)} & \left[ \Delta S_{w,F} \right]_{m}^{(k+1)} + \left[ \frac{\partial R_{w,M}}{\partial S_{w,M}} \right]_{m}^{(k)} \left[ \Delta S_{w,M} \right]_{m}^{(k+1)} \\
\left[ \frac{\partial R_{w,M}}{\partial S_{w,F}} \right]_{2}^{(k+1)} & \left[ \Delta S_{w,F} \right]_{2}^{(k+1)} + \left[ \frac{\partial R_{w,M}}{\partial S_{w,M}} \right]_{2}^{(k)} \left[ \Delta S_{w,M} \right]_{2}^{(k+1)} \\
\left[ \frac{\partial R_{w,M}}{\partial S_{w,F}} \right]_{NB}^{(k+1)} & \left[ \Delta S_{w,F} \right]_{NB}^{(k+1)} + \frac{\partial R_{w,M}}{\partial S_{w,M}} \left[ \Delta S_{w,M} \right]_{NB}^{(k+1)}
\end{align*}
\]

(4.56)

where $m = 1, 2, \ldots, NB$. 
Chapter 5
Development of a Coal Shrinkage-Swelling Model accounting for Water in the Coal Matrix

As previously discussed in Section 2.3, the water content in the coal matrix can be grouped into two types: bound water and bulk water. Bound water is considered to be immobile as it is firmly bonded to the hydrophilic surface of the coal. Bulk water is free water which can be mobile in the coal matrix. Experimental studies (Joubert et al., 1973; Clarkson and Bustin, 2000; Krooss et al., 2002; Ozdemir, 2004; Jahediesfanjani and Civan, 2006) showed that bound water reduced the gas sorption capacity by taking up the sorption sites from gas molecules; however, there existed a critical value of water content in the coal matrix where all the accessible hydrophilic sites are occupied by bound water and the gas sorption capacity cannot be reduced further. The critical value is referred as moisture content, $m_c$. The new parameter is defined as sorption capacity factor and used to account for the decrease in gas sorption capacity due to existence of water in the coal matrix. The gas sorption capacity of a dried coal is equal to unity. The sorption capacity factor decreases due to the effect of bound water as water is introduced into the coal matrix; however, the sorption capacity factor remains the same at the critical value, $f_c$, after the critical moisture content is reached. The relationship between the moisture content and the sorption capacity factor is shown in Figure 2-2. To account for such effects of bound water in the coal matrix on gas sorption capacity, the Langmuir isotherm can be modified as follows:

$$V_e = \tilde{f}V_m \left( \frac{p}{p_L + p} \right)$$

(5.1)

where $\tilde{f}$ is the sorption capacity factor.
The existing coal shrinkage-swelling models (Sawyer et al., 1990; Seidle and Huit, 1995; Palmer and Mansoori, 1996; Shi and Durucan, 2005) do not incorporate the effects of water in the coal matrix and may result in overestimation of gas productivity/injectivity due to the absence of moisture as inhibiting the sorption phenomena. The proposed coal shrinkage-swelling model incorporates the effect of moisture content in the coal matrix.

Considering a saturated coal sample with an initial volume of \( V_i \), the change in cleat volume, \( \Delta V_i \), caused by rock expansion/compression, \( \Delta V_p \), and the release/adsorption of gas in the coal matrix, \( \Delta V_g \), can be written as follows:

\[
\Delta V_i = \Delta V_p + \Delta V_g \tag{5.2}
\]

The volume change due to rock expansion/compression can be expressed as:

\[
\Delta V_p = \phi_i V_i \left( e^{\frac{c_p}{p_i}} - 1 \right) \tag{5.3}
\]

where \( c_p \) is the cleat volume compressibility, calculated using the following equation (Palmer and Mansoori, 1996; Pekot and Reeves, 2003):

\[
c_p = \frac{1}{2E\phi_i} \tag{5.4}
\]

In addition, the changes in cleat volume due to the release/adsorption of gas in the coal matrix can be expressed using the Langmuir sorption isotherm as follows:

\[
\Delta V_g = f \beta \rho V_m \left( \frac{p_i}{p_l + p_i} - \frac{p}{p_l + p} \right) \tag{5.5}
\]

where \( f \) is the sorption capacity factor, and \( \beta \) is the matrix shrinkage-swelling coefficient which quantifies the cleat volume change due to the release/adsorption of gas in the coal matrix. The coal density is given by \( \rho \), while \( V_m \) and \( p_l \) are the Langmuir volume and the Langmuir pressure constants, respectively. Substituting Eqs.(5.3) and (5.5) into Eq.(5.2), one can obtain:
\[ \Delta V_i = \phi V_i - \phi V_i = \phi V_i \left( e^{c_s(p-p_i)} - 1 \right) + \tilde{f} \beta \rho V_m \left( \frac{p_i}{p_L + p_i} - \frac{p}{p_L + p} \right) \]  \hspace{1cm} (5.6)

Re-arranging Eq.(5.6) yields:

\[ \frac{\phi}{\phi_i} = e^{c_s(p-p_i)} + \frac{\tilde{f} \beta \rho V_m}{\phi_i} \left( \frac{p_i}{p_L + p_i} - \frac{p}{p_L + p} \right) \]  \hspace{1cm} (5.7)

It is assumed that permeability is related to porosity as described by Reiss (1980):

\[ \frac{k}{k_i} = \left( \frac{\phi}{\phi_i} \right)^3 = \left[ e^{c_s(p-p_i)} + \frac{\tilde{f} \beta \rho V_m}{\phi_i} \left( \frac{p_i}{p_L + p_i} - \frac{p}{p_L + p} \right) \right]^3 \]  \hspace{1cm} (5.8)

For an undersaturated coal, the volume change due to the release/adsorption of gas in the coal matrix during drawdown is given by:

\[ \Delta V_g = 0 \]  \hspace{1cm} \text{for } p > p_{sat} \]  \hspace{1cm} (5.9)

\[ \Delta V_g = \tilde{f} \beta \rho V_i V_m \left( \frac{p_{sat}}{p_L + p_{sat}} - \frac{p}{p_L + p} \right) \]  \hspace{1cm} \text{for } p \leq p_{sat} \]  \hspace{1cm} (5.10)

During the gas injection, the volume change, \( \Delta V_g \), is given by:

\[ \Delta V_g = \tilde{f} \beta \rho V_i V_m \left( \frac{p_{sat}}{p_L + p_{sat}} - \frac{p}{p_L + p} \right) \]  \hspace{1cm} (5.11)

For multi-component systems, the volume change due to the release/adsorption of multi-component gas can be expressed using partial pressure as:

\[ \Delta V_g = \sum_{l=1}^{NC} \tilde{f}_l \beta_l \rho V_{m,l} \left( \frac{Y_{l,i} p_i}{p_{L,l} + p_i} - \frac{Y_l p}{p_{L,l} + p} \right) \]  \hspace{1cm} (5.12)

As a result, the coal shrinkage and swelling model for multi-component systems can be written as:

\[ \frac{k}{k_i} = \left( \frac{\phi}{\phi_i} \right)^3 = \left[ e^{c_s(p-p_i)} + \sum_{l=1}^{NC} \tilde{f}_l \beta_l \rho V_{m,l} \left( \frac{Y_{l,i} p_i}{p_{L,l} + p_i} - \frac{Y_l p}{p_{L,l} + p} \right) \right]^3 \]  \hspace{1cm} (5.13)
Chapter 6

Computer Model

The proposed dual-porosity, dual-permeability CBM reservoir model is an extension of the compositional dual-porosity, single-permeability model developed by Manik et al. (2002). The proposed model is capable of simulating the flow of multi-component gas and water in fracture and matrix systems in CBM reservoirs in three-dimensional, body-centered, Cartesian coordinate system. The flow of gas follows the multi-mechanistic flow mechanism while the water flow obeys the Darcy’s law. In addition, the effects of coal shrinkage and swelling and the presence of water in the coal matrix are implemented in the model. The highly nonlinear formulations are linearized by Newton-Raphson procedure and solved in a fully implicit form using a direct matrix solver with a standard Gaussian elimination without pivoting. The thermodynamically consistent ideal absorbed solution (IAS) theory and the Peng-Robinson equation of state are used to develop the multi-component sorption model. The sorption model accounts for the non-ideality of the free gas phase as partial fugacity of the gas component is calculated using curve-fitting method of an analytical expression of fugacity as a function of spreading pressure group. The model is developed with no-flow outer boundary condition. The inner boundary condition options include water flow rate, gas flow rate and sandface pressure specifications, which are permitted to change to accommodate well schedules. The model offers vertical and horizontal options for well orientation. Also, the grid system can have more than one layer and wells can penetrate through more than one grid block. Incremental material balance and residual checks are performed at the end of each time step.
The computer model was developed using C++ programming language. The code consists of a main program, seven subroutines and 24 functions. Figure 6-1 shows the flow diagram of the computer program.

![Flow diagram of computer model](image)

**Figure 6-1**: Flow diagram of computer model

**Main Program**

The main program coordinates the sequence of the simulation from start to end. Also, the main program performs improvement and oscillation checks to determine the time step size for next time step calculations. The initial time step size is set as 0.1 day. The time step size is set to
reduce by 150% if 1) the maximum saturation improvement ($\Delta S_{\text{max}}$) is larger than 0.5 or 2) the maximum mole fraction improvement ($\Delta X_{\text{max}}$) is larger than 0.5 or 3) the maximum residual for iteration $k+1$ is larger than that for iteration $k$. If the time step size is reduced to less than $1 \times 10^{-10}$ days, the program will halt. Moreover, if the above criteria are not met, the time step size is set to increase by 50% until it reaches the user-specified time step size.

Subroutine read

This subroutine reads and prints input data from an input file. The input data include reservoir properties, rock/fluid properties, well data and well schedule. The rock/fluid properties, including (1) water relative permeability, (2) gas relative permeability, (3) capillary pressure, (4) gas viscosity, (5) gas formation volume factor, (6) gas density, water viscosity, (7) water formation volume factor, (8) water density and (9) solubility of gas in water, can be given in a tabular form or using empirical correlations which are presented in Appendix A.

Subroutine reservoirInit

This subroutine initializes the reservoir properties after reading the input data. It calculates constants in rock/fluid properties, constant transmissibility terms and original gas and water in place. The original gas in place (OGIP) for gas component $l$ and water in place (OWIP) can be calculated using Eqs. (6.1) and (6.2).

$$OGIP = 0.028317 \sum_{i=1}^{NC} \sum_{j=1}^{NB} V_{i,j} \rho G_{c,j} + \sum_{i=1}^{NC} \sum_{j=1}^{NB} \left[ \frac{X_{l,p} V_{b,j} \phi_{F,j} (1 - S_{w,F,j})}{5.615 B_{g,F,j}} + \frac{X_{l,M} V_{b,j} \phi_{M,i} (1 - S_{w,M,i})}{5.615 B_{g,M,j}} \right]$$

(6.1)

$$OWIP = \sum_{i=1}^{NR} \left[ V_{b,j} \phi_{F,i} S_{w,F,i} + V_{b,j} \phi_{M,i} S_{w,M,i} \right]$$

(6.2)

where $G_{c,l}$ is gas content for gas component $l$ in scf/ton and $\rho$ is coal density in g/cc.
Subroutine **newtonRaphson**

This subroutine calculates the residual terms in Eqs.(4.15) through Eqs.(4.18) and the partial derivative terms in Eqs.(4.53) through (4.56) and also constructs a jacobian matrix and right-hand-side vector required for solution by the generalized Newton-Raphson procedure.

Subroutine **mtxslv**

This subroutine solves the system of equations constructed in subroutine newtonRaphson using a band matrix solver with a standard Gaussian elimination without pivoting (Aziz, 1979).

Subroutine **convCriteria**

This subroutine checks for convergence and number of iterations. If convergence or maximum number of iterations is achieved, the subroutine terminates the iteration loop and performs incremental material balance checks for both gas and water. The convergence criteria and the number of maximum number of iterations can be specified in the input file. However, if they are not specified, the convergence criteria for water residuals and gas residuals in this work are $1 \times 10^{-5}$ and $1 \times 10^{-1}$, respectively and the default maximum number of iterations is 10. The incremental material balance check for gas ($IMBC_g$) can be calculated as follows:

$$IMBC_g = \frac{GIP^{n+1} - GIP^n}{G_p}$$  \hspace{1cm} (6.3)

where

$$GIP = \sum_{i=1}^{NC} \sum_{m=1}^{mz} \sum_{j=1}^{ny} \sum_{k=1}^{nx} \left( \frac{X_{i,F} V_b \phi_F S_{g,F}}{5.615B_{g,F}} + \frac{R_{w,F} V_b \phi_F S_{w,F}}{5.615B_{w,F}} + \frac{X_{i,M} V_b \phi_M S_{g,M}}{5.615B_{g,M}} + \frac{R_{w,M} V_b \phi_M S_{w,M}}{5.615B_{w,M}} \right)_{i,j,k}$$ \hspace{1cm} (6.4)

and
The incremental material balance check for water (IMBC\(_w\)) can be calculated as follows:

\[
IMBC_w = \left| \frac{WIP^{n+1} - WIP^n}{W_p} \right|
\]

(6.6)

where

\[
WIP = \sum_{k=1}^{nz} \sum_{j=1}^{ny} \sum_{i=1}^{nx} \left( V_b \phi_F S_{w,F} + \frac{V_b \phi_M S_{w,M}}{5.615 B_{w,F}} \right)_{i,j,k}
\]

(6.7)

and

\[
W_p = \sum_{k=1}^{nz} \sum_{j=1}^{ny} \sum_{i=1}^{nx} (q_{w,i,j,k}^{n+1}) \Delta t
\]

(6.8)

**Subroutine nextTimeStep**

This subroutine is used to handle production schedule for each well. It checks well criteria for the next time step. If the well criteria are satisfied, it will change well propertiesSpecification as specified in the input data file. The well criteria include simulation time, water flow rate, gas flow rate and sandface pressure.

**Subroutine incReport**

This subroutine prints output data which include (1) pressure distribution, (2) saturation distribution, (3) mole fraction distribution, (4) residual check distribution, (5) volume of adsorbed gas distribution, (6) sorption capacity distribution, (7) well production of water and gas for each component, (8) sandface pressure for each well and (9) water and gas material balance checks.
**Functions lookup**

This function performs linear interpolation for the rock/fluid properties that are specified in tabular forms.

**Function Z_factor**

This function calculates compressibility factor using Peng-Robinson EOS presented in Appendix A.

**Function Zsc_factor**

This function calculates compressibility factor at $T_{sc}$ and $p_{sc}$ using Peng-Robinson EOS presented in Appendix A.

**Function gasFVF**

This subroutine calculates gas formation volume factor using Peng-Robinson EOS presented in Appendix A.

**Function gasMW**

This subroutine calculates gas molecular weight.

**Function gasDen**

This subroutine calculates gas density using Peng-Robinson EOS presented in Appendix A.
Function `gasVis`

This subroutine calculates gas viscosity using Lee, Gonzales and Eakin correlation presented in Appendix A.

Function `waterFVF`

This subroutine calculates water formation volume factor using McCain correlation presented in Appendix A.

Function `waterDen`

This subroutine calculates water density using McCain correlation presented in Appendix A.

Function `waterVis`

This subroutine calculates water viscosity using McCain correlation presented in Appendix A.

Function `krw`

This subroutine calculates relative permeability to water using Corey’s equation presented in Appendix A.

Function `krg`

This subroutine calculates relative permeability to gas using Corey’s equation presented in Appendix A.
Function **pcgw**

This subroutine calculates capillary pressure using Corey’s equation presented in Appendix A.

Function **porosity**

This subroutine calculates porosity and permeability using rock compressibility.

Function **porosity_1**

This subroutine calculates porosity and permeability using Sawyer’s model.

Function **porosity_2**

This subroutine calculates porosity and permeability using Palmer and Mansoori’s model.

Function **porosity_3**

This subroutine calculates porosity and permeability using Seidle’s model.

Function **porosity_4**

This subroutine calculates porosity and permeability using the proposed coal shrinkage and swelling model.

Function **perm_SD**

This subroutine calculates permeability using Shi and Durucan’s model.
Function `gasFCPure`

This subroutine calculates pure component fugacity coefficients using Peng-Robinson EOS.

Function `gasFCPartial`

This subroutine calculates partial component fugacity coefficients using Peng-Robinson EOS presented in Appendix A.

Function `x_IAS`

This subroutine calculates spreading pressure group, equilibrium free gas mole fraction, partial sorption capacity and total sorption capacity if equilibrium adsorbed gas mole fraction is given. The calculation of multi-component sorption is presented in Section 4.6.

Function `y_IAS`

This subroutine calculates spreading pressure group, equilibrium adsorbed gas mole fraction, partial sorption capacity and total sorption capacity if equilibrium free gas mole fraction is given. The calculation of multi-component sorption is presented in Section 4.6.

Function `qSorption`

This subroutine calculates sorption rate at time level $n+1$ using Eqs.(4.36) and (4.37).
7.1 Validation of Numerical Model

In this section, the proposed dual-porosity, dual-permeability CBM model is tested against the dual-porosity, single-permeability coalbed methane model (PSU-COALCOMP) and the single-porosity, single-permeability model (GEM). The proposed dual-porosity, dual-permeability model is also used in a comparison study for enhanced coalbed methane recovery where the results are compared against those obtained using the existing CBM models including GEM, ECLIPSE, COMET2, SIMED II, GCOMP and PSU-COALCOMP.

7.1.1 Validation against the Dual-porosity, Single-permeability Model

The proposed dual-porosity, dual-permeability computer code developed for CBM reservoirs is tested against a dual-porosity, single-permeability model. The compositional dual-porosity, single-permeability CBM simulator (PSU-COALCOMP) developed by Manik et al. (2002) is used for validation purposes.

The concept of the flow in dual-porosity, single-permeability domains implies that the matrix is a system providing storage and behaves as localized sources/sinks to the fractures. Therefore, the fluid flow only occurs in the fracture network. In order to capture such mechanism by the proposed dual-porosity, dual-permeability model, fracture porosity ($\phi_f$) and matrix permeability ($k_m$) are set close to zero. The reservoir system described in Table 7-1 and Figures 7-1 through 7-3 is used for this test. In the dual-porosity, single-permeability model, fractures are
assumed to be the only flow field in which gas travels through after desorption. This model speculates that it is possible for the desorbed gas to flow through both matrix and fracture in the dual-porosity, single-permeability model. Thus, the predicted initial gas production from the dual-porosity, single-permeability model is expected to be higher than that from the dual-porosity, dual-permeability model. In this exercise, initial water saturation is set as 100% for the dual-porosity, single-permeability model; however, to accelerate the gas production in dual-porosity, dual-permeability model, some initial gas is placed in the matrix ($S_{wi,M} = 95\%$). Figures 7-4 and 7-5 show the predicted gas flow rate and water flow rate profiles from the two models, respectively. It can be observed that both gas flow rate and water flow rate profiles from the two models are in close agreement.

Table 7-1: Input data for validation test against dual-porosity, single-permeability model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Dual-$\phi$ single-$k$ model</th>
<th>Dual-$\phi$ dual-$k$ model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir size</td>
<td>1,100 ft $\times$ 1,100 ft</td>
<td>1,100 ft $\times$ 1,100 ft</td>
</tr>
<tr>
<td>Reservoir thickness</td>
<td>10 ft</td>
<td>10 ft</td>
</tr>
<tr>
<td>Fracture porosity</td>
<td>8%</td>
<td>0.0000001 %</td>
</tr>
<tr>
<td>Matrix porosity</td>
<td>8%</td>
<td>8%</td>
</tr>
<tr>
<td>Fracture permeability</td>
<td>80 md</td>
<td>80 md</td>
</tr>
<tr>
<td>Matrix permeability</td>
<td>0.000001 md</td>
<td></td>
</tr>
<tr>
<td>Coal density</td>
<td>1.435 g/cm$^3$</td>
<td>1.435 g/cm$^3$</td>
</tr>
<tr>
<td>Sorption time constant</td>
<td>0 day</td>
<td>0 day</td>
</tr>
<tr>
<td>Initial pressure</td>
<td>1,100 psia</td>
<td>1,100 psia</td>
</tr>
<tr>
<td>Fracture spacing</td>
<td>-</td>
<td>0.01 ft</td>
</tr>
<tr>
<td>Initial water saturation, fracture</td>
<td>100 %</td>
<td>100 %</td>
</tr>
<tr>
<td>Initial water saturation, matrix</td>
<td>-</td>
<td>95%</td>
</tr>
<tr>
<td>Irreducible water saturation</td>
<td>20%</td>
<td>20%</td>
</tr>
<tr>
<td>Reservoir temperature</td>
<td>80 ºF</td>
<td>80 ºF</td>
</tr>
<tr>
<td>Number of grid blocks in $x, y$ and $z$ directions</td>
<td>11, 11, 1</td>
<td>11, 11, 1</td>
</tr>
<tr>
<td>Well location</td>
<td>6, 6, 1</td>
<td>6, 6, 1</td>
</tr>
<tr>
<td>Well radius</td>
<td>0.25 ft</td>
<td>0.25 ft</td>
</tr>
<tr>
<td>Sandface pressure</td>
<td>14.7 psia</td>
<td>14.7 psia</td>
</tr>
<tr>
<td>Skin factor</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Figure 7-1: Methane sorption isotherm for validation test against the dual-porosity, single-permeability model \((V_m = 906 \text{ SCF/ton, } p_L = 625 \text{ psia})\)

Figure 7-2: Water and gas relative permeabilities for validation test against the dual-porosity, single-permeability model
Figure 7-3: Gas-water capillary pressure for validation test against the dual-porosity, single-permeability model

Figure 7-4: Gas flow rate profiles from validation test against the dual-porosity, single-permeability model
7.1.2 Validation against the Single-porosity, Single-permeability Model

The proposed dual-porosity, dual-permeability model can also represent flow in a single-porosity, single-permeability domain. The commercially developed single-porosity, single-permeability simulator developed by the Computer Modelling Group (CMG) is used for validation purpose.

For a typical sandstone reservoir, the produced gas and water come from the “free” phase in the pores. In order to model such behavior using the dual-porosity, dual-permeability CBM model, the sorption time constant is set to a large value to ensure that no appreciable gas desorption takes place. In addition, the fracture porosity ($\phi_F$) and matrix permeability ($k_M$) are set close to zero to simulate the flow taking place in a single-porosity, single-permeability domain. The reservoir system described in Table 7-2 and Figures 7-1 through 7-3 is used for this test.
Figure 7-6 shows the test results. It can be observed that the predicted gas profiles from the two models are in good agreement. The test shows the proposed model’s capability to collapse into a single-porosity, single-permeability formulation as it mimics a typical sandstone reservoir.

Table 7-2: Input data for validation test against single-porosity, single-permeability model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Single-ϕ single-k model</th>
<th>Dual-ϕ dual-k model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir size</td>
<td>1,100 ft × 1,100 ft</td>
<td>1,100 ft × 1,100 ft</td>
</tr>
<tr>
<td>Reservoir thickness</td>
<td>10 ft</td>
<td>10 ft</td>
</tr>
<tr>
<td>Fracture porosity</td>
<td>8%</td>
<td>0.0000001%</td>
</tr>
<tr>
<td>Matrix porosity</td>
<td>8%</td>
<td></td>
</tr>
<tr>
<td>Fracture permeability</td>
<td>80 md</td>
<td>80 md</td>
</tr>
<tr>
<td>Matrix permeability</td>
<td></td>
<td>0.00001 md</td>
</tr>
<tr>
<td>Coal density</td>
<td>-</td>
<td>1.435 g/cm³</td>
</tr>
<tr>
<td>Sorption time constant</td>
<td>-</td>
<td>1,000,000 day</td>
</tr>
<tr>
<td>Initial pressure</td>
<td>1,100 psia</td>
<td>1,100 psia</td>
</tr>
<tr>
<td>Fracture spacing</td>
<td>-</td>
<td>0.01 ft</td>
</tr>
<tr>
<td>Initial water saturation, fracture</td>
<td>15 %</td>
<td>15 %</td>
</tr>
<tr>
<td>Initial water saturation, matrix</td>
<td>-</td>
<td>15 %</td>
</tr>
<tr>
<td>Reservoir temperature</td>
<td>80 ºF</td>
<td>80 ºF</td>
</tr>
<tr>
<td>Number of grid blocks in x, y and z directions</td>
<td>11, 11, 1</td>
<td>11, 11, 1</td>
</tr>
<tr>
<td>Well location</td>
<td>6, 6, 1</td>
<td>6, 6, 1</td>
</tr>
<tr>
<td>Well radius</td>
<td>0.25 ft</td>
<td>0.25 ft</td>
</tr>
<tr>
<td>Sandface pressure</td>
<td>14.7 psia</td>
<td>14.7 psia</td>
</tr>
<tr>
<td>Skin factor</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
In this part of the study, the proposed dual-porosity, dual-permeability CBM model is validated against the existing commercial and research CBM models including (1) GEM, Computer Modelling Group (CMG) Ltd., (2) ECLIPSE, Schlumberger GeoQuest, (3) COMET2, Advance Resources International (ARI), (4) SIMED II, Commonwealth Scientific and Industrial Research Organization (CSIRO) and the Netherlands Institute of Applied Geoscience TNO, (5) GCOMP, BP and (6) PSU-COALCOMP, the Pennsylvania State University. The comparison study deals with enhanced coalbed methane (ECBM) recovery process with pure CO$_2$ injection in a five-spot pattern. The results obtained from the existing commercial and research models are presented by Law et al. (2002). Figure 7-7 shows the schematic diagram of the five-spot pattern. Tables 7-3 and 7-4 show the properties used in this comparison study.
Figure 7-8 shows the comparisons of methane production profiles for the primary and ECBM recovery with CO₂ injection processes. Figure 7-9 shows the comparisons of the total gas and CO₂ production profiles. It can be observed that enhanced methane production begins after only a few days of CO₂ injection. This short delay of the enhanced methane production is caused by the dewatering process. The enhancement of methane production reaches a maximum rate after approximately 8 days of production and it continues until CO₂ breakthrough which occurs at the producer after approximately 60 days of production. At that point, the methane production rate decreases sharply while the CO₂ production rates increases. Figure 7-10 shows the comparisons of production gas compositions of CH₄ and CO₂. After approximately 100 days the CO₂-ECBM recovery process is completed as the methane composition becomes zero. Figure 7-11 shows the comparisons of the bottom-hole pressure at the injector. The initial injection bottom-hole pressure decreases as water is being produced in the dewatering process. After the pressure’s initial decline, the injection bottom-hole pressure rises slowly after CO₂ breakthrough as a small amount of the injected CO₂ can only be adsorbed in the coal matrix leaving the remaining of the injected CO₂ to flow to the producer.

The overall results from the existing and proposed CBM models are in good agreement. Law et al. (2002) reported that the discrepancies between the results from different models could come from a number of reasons including differences in (1) initialization process, (2) reservoir representation, (3) methods of handling wells in 5-spot pattern, (4) convergence parameters, (5) numerical control parameters. In addition, the differences between the proposed model and other commercial simulators could result from differences in mechanisms used in describing the flow mechanism of gas in CBM reservoirs, treatments of the presence of water in the coal matrix and sorption isotherm theories. Most existing simulators treat CBM systems as a dual-porosity, single-permeability system and ignore the effects of water presence in the coal matrix, which was found to over predict gas production rates. In addition, most of the existing simulator use the
extended Langmuir isotherm, which was found to provide less accurate predictions of multi-component gas adsorption than the ideal adsorbed solution (IAS) theory (Jessen et al., 2007). The lack of the aforementioned characteristics and poor adsorption predictions suggests that more accurate predictions are obtained by the proposed simulator that incorporates dual-porosity, dual-permeability characteristics, effects of water presences in the coal matrix and the IAS theory.
Table 7-3: Coalbed properties (Law et al., 2002)

<table>
<thead>
<tr>
<th>Coalbed Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coal seam thickness = 29.527 ft</td>
</tr>
<tr>
<td>Top of coal seam = 4112.8 ft</td>
</tr>
<tr>
<td>Initial absolute permeability of natural fracture = 3.65 md</td>
</tr>
<tr>
<td>Porosity of natural fracture = 0.1%</td>
</tr>
<tr>
<td>Effective coalbed compressibility = $1 \times 10^{-6}$ psi$^{-1}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Initial Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature = 113°F</td>
</tr>
<tr>
<td>Average pressure = 1109.5 psia</td>
</tr>
<tr>
<td>Gas saturation = 40.8%</td>
</tr>
<tr>
<td>Water saturation = 59.2%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Water Properties at 45°C (113°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density = 61.8 lb/ft$^3$</td>
</tr>
<tr>
<td>Viscosity = 0.607 cp</td>
</tr>
<tr>
<td>Compressibility = $4 \times 10^{-6}$ psi$^{-1}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Pure Gas Adsorption Isotherms at 45°C (113°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average in-situ coal density = 89.5 lb/ft$^3$</td>
</tr>
<tr>
<td>Average in-situ moisture content (by wt.) = 0.0672</td>
</tr>
<tr>
<td>Average in-situ ash content (by wt.) = 0.156</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sorption parameter</th>
<th>CH$_4$</th>
<th>CO$_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Langmuir pressure (psia)</td>
<td>680</td>
<td>276</td>
</tr>
<tr>
<td>Langmuir volume (scf/ton)</td>
<td>486.0</td>
<td>993.8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Grid System</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area = ¼ of a 2.5 acres pattern</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Operating Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Well radius = 0.11975 ft</td>
</tr>
<tr>
<td>Well skin factor = 0</td>
</tr>
<tr>
<td>182.5-day continuous CO$_2$ injection/production period (0 – 182.5 days)</td>
</tr>
<tr>
<td>CO$_2$ injection rate (full well) = 1,000,000 scf/d</td>
</tr>
<tr>
<td>Bottom-hole pressure at producer = 39.885 psia</td>
</tr>
</tbody>
</table>
Table 7-4: Relative permeability data (Law et al., 2002)

<table>
<thead>
<tr>
<th>Water saturation</th>
<th>Relative permeability to water</th>
<th>Relative permeability to gas</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>0.975</td>
<td>0.814</td>
<td>0.0035</td>
</tr>
<tr>
<td>0.950</td>
<td>0.731</td>
<td>0.007</td>
</tr>
<tr>
<td>0.90</td>
<td>0.601</td>
<td>0.018</td>
</tr>
<tr>
<td>0.85</td>
<td>0.490</td>
<td>0.033</td>
</tr>
<tr>
<td>0.80</td>
<td>0.392</td>
<td>0.051</td>
</tr>
<tr>
<td>0.75</td>
<td>0.312</td>
<td>0.070</td>
</tr>
<tr>
<td>0.70</td>
<td>0.251</td>
<td>0.090</td>
</tr>
<tr>
<td>0.65</td>
<td>0.200</td>
<td>0.118</td>
</tr>
<tr>
<td>0.60</td>
<td>0.154</td>
<td>0.147</td>
</tr>
<tr>
<td>0.55</td>
<td>0.116</td>
<td>0.180</td>
</tr>
<tr>
<td>0.50</td>
<td>0.088</td>
<td>0.216</td>
</tr>
<tr>
<td>0.45</td>
<td>0.067</td>
<td>0.253</td>
</tr>
<tr>
<td>0.40</td>
<td>0.049</td>
<td>0.295</td>
</tr>
<tr>
<td>0.35</td>
<td>0.035</td>
<td>0.342</td>
</tr>
<tr>
<td>0.30</td>
<td>0.024</td>
<td>0.401</td>
</tr>
<tr>
<td>0.25</td>
<td>0.015</td>
<td>0.466</td>
</tr>
<tr>
<td>0.20</td>
<td>0.007</td>
<td>0.537</td>
</tr>
<tr>
<td>0.15</td>
<td>0.002</td>
<td>0.627</td>
</tr>
<tr>
<td>0.10</td>
<td>0.0013</td>
<td>0.720</td>
</tr>
<tr>
<td>0.05</td>
<td>0.0006</td>
<td>0.835</td>
</tr>
<tr>
<td>0.00</td>
<td>0.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Figure 7-7: Schematic diagram of the five-spot pattern used for model validation against the existing CBM numerical models.
Figure 7-8: Methane production rate for primary and ECBM recovery with CO$_2$ injection processes

Figure 7-9: Total gas and CO$_2$ production rates for ECBM recovery with CO$_2$ injection process
Figure 7-10: Production gas compositions of CH$_4$ and CO$_2$

Figure 7-11: Bottom-hole pressure at the injector
7.2 Parametric Studies on the Primary CBM Production

Parametric studies are conducted to determine the sensitivity of the model. This part of the study illustrates how the model responds to the effects of a multi-mechanistic flow mechanism, presence of water phase in the coal matrix and coal shrinkage and swelling phenomena on the primary CBM production.

7.2.1 Multi-mechanistic Flow

In the proposed dual-porosity, dual-permeability CBM model, the transport of gas in both fracture and matrix systems follows a multi-mechanistic flow mechanism, which considers the simultaneous influences of pressure and concentration gradients. The flow under the pressure field obeys the Darcy’s law while under the concentration field it follows the Fick’s law of diffusion. The implementation of the two flow mechanisms have been indicated to provide significant increase in cumulative gas production (Chawathe et al., 1996b). The multi-mechanistic flow of gas in the CBM reservoirs is investigated by evaluating permeable, moderately tight and ultra tight systems with diffusion coefficients of \(0, 10^{-8}, 10^{-6}, 10^{-5}, 10^{-4}\) and \(10^{-3}\) sq.ft/day. The range of diffusion coefficients of gas in coal was reported to be between \(10^{-3}\) and \(10^{-10}\) sq.ft/day (Sevenster, 1958; Karn et al., 1970; Thimons and Kissel, 1973; Olague and Smith, 1989; Pone et al, 2009). The diffusion coefficients in this analysis represent the diffusional flow from the coal matrix to fracture network while sorption time constant represents the diffusion process in the micropores. The case where the diffusion coefficient is 0 ft\(^2\)/day represents the single-mechanistic (Darcian) flow. The parameters that are used in identifying these three systems are fracture permeability and matrix permeability values. Table 7-5 and Figures 7-1 through 7-3 summarize the reservoir properties used in this study. Figures 7-12 through 7-17 depict the gas flow rate
profiles and cumulative gas production of the three systems with different diffusion coefficients. The overall results are in agreement with observations from the study in multi-mechanistic flow in fractured reservoirs conducted by Chawathe et al. (1996b). The diffusion coefficient does not have a significant effect on the gas flow rates in the permeable system as the Darcian flow is vastly dominant. The permeable system gives the same cumulative gas production for all the diffusion coefficients. However, the Fickian flow becomes more dominant for tighter systems, particularly at early times. The initial gas rates and cumulative gas production are higher for the systems with larger diffusion coefficients for the tight and ultra tight systems. The additional gas production for these two systems is caused by the effect of diffusional flow.

Table 7-5: Input data for parametric study on the effect of multi-mechanistic flow

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Permeable system</th>
<th>Tight system</th>
<th>Ultra tight system</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir size</td>
<td>1,100 ft × 1,100 ft</td>
<td>1,100 ft × 1,100 ft</td>
<td>1,100 ft × 1,100 ft</td>
</tr>
<tr>
<td>Reservoir thickness</td>
<td>20 ft</td>
<td>20 ft</td>
<td>20 ft</td>
</tr>
<tr>
<td>Fracture porosity</td>
<td>0.9%</td>
<td>0.9%</td>
<td>0.9%</td>
</tr>
<tr>
<td>Matrix porosity</td>
<td>8.0%</td>
<td>8.0%</td>
<td>8.0%</td>
</tr>
<tr>
<td>Fracture permeability</td>
<td>1,000 md</td>
<td>10 md</td>
<td>0.1 md</td>
</tr>
<tr>
<td>Matrix permeability</td>
<td>1 md</td>
<td>0.01 md</td>
<td>0.0001 md</td>
</tr>
<tr>
<td>Coal density</td>
<td>1.435 g/cm³</td>
<td>1.435 g/cm³</td>
<td>1.435 g/cm³</td>
</tr>
<tr>
<td>Sorption time constant</td>
<td>10 day</td>
<td>10 day</td>
<td>10 day</td>
</tr>
<tr>
<td>Diffusion coefficient</td>
<td>0 – 10⁻³ ft²/day</td>
<td>0 – 10⁻³ ft²/day</td>
<td>0 – 10⁻³ ft²/day</td>
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<tr>
<td>Initial pressure</td>
<td>1,100 psia</td>
<td>1,100 psia</td>
<td>1,100 psia</td>
</tr>
<tr>
<td>Fracture spacing</td>
<td>1 ft</td>
<td>1 ft</td>
<td>1 ft</td>
</tr>
<tr>
<td>Initial water saturation, fracture</td>
<td>95%</td>
<td>95%</td>
<td>95%</td>
</tr>
<tr>
<td>Initial water saturation, matrix</td>
<td>70%</td>
<td>70%</td>
<td>70%</td>
</tr>
<tr>
<td>Reservoir temperature</td>
<td>80 °F</td>
<td>80 °F</td>
<td>80 °F</td>
</tr>
<tr>
<td>Number of grid blocks in x, y, and z directions</td>
<td>11, 11, 1</td>
<td>11, 11, 1</td>
<td>11, 11, 1</td>
</tr>
<tr>
<td>Well location</td>
<td>6, 6, 1</td>
<td>6, 6, 1</td>
<td>6, 6, 1</td>
</tr>
<tr>
<td>Well radius</td>
<td>0.25 ft</td>
<td>0.25 ft</td>
<td>0.25 ft</td>
</tr>
<tr>
<td>Sandface pressure</td>
<td>14.9 psia</td>
<td>14.9 psia</td>
<td>14.9 psia</td>
</tr>
<tr>
<td>Skin factor</td>
<td>0</td>
<td>0</td>
<td>0</td>
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</tbody>
</table>
Figure 7-12: Gas flow rate profiles of the permeable system ($k_F = 1,000$ md and $k_M = 1$ md) for parametric study on the effect of multi-mechanistic flow

Figure 7-13: Cumulative gas production of the permeable system ($k_F = 1,000$ md and $k_M = 1$ md) for parametric study on the effect of multi-mechanistic flow
Figure 7-14: Gas flow rate profiles of the tight system ($k_F = 10$ md and $k_M = 0.01$ md) for parametric study on the effect of multi-mechanistic flow.

Figure 7-15: Cumulative gas production of the tight system ($k_F = 10$ md and $k_M = 0.01$ md) for parametric study on the effect of multi-mechanistic flow.
Figure 7-16: Gas flow rate profiles of the ultra tight system ($k_F = 0.1$ md and $k_M = 0.0001$ md) for parametric study on the effect of multi-mechanistic flow.

Figure 7-17: Cumulative gas production of the ultra tight system ($k_F = 0.1$ md and $k_M = 0.0001$ md) for parametric study on the effect of multi-mechanistic flow.
7.2.2 Water Presence in the Coal Matrix

It has been reported that the lack of water presence in the coal matrix in the typical dual-porosity, single-permeability model generally results in over-predicting gas production and under-estimating water production (Reeves and Pekot, 2001). Water is assumed to be present only in the fracture system. In the proposed dual-porosity, dual permeability model, the presence of water in the coal matrix is implemented by assigning water saturation within the matrix system. As mentioned earlier in Section 2.3, there are mainly two groups of water existing in the coal matrix which are bound water and bulk water. Bound water is considered as immobile water molecules adsorbed on the surface of the coal matrix while bulk water is the free water which has mobility in the coal matrix. Bound water decreases gas sorption capacity of coals yielding low gas production. Bulk water, on the other hand, has no effect on gas sorption capacity; however, it blocks the flow path of gas in the coal matrix, which reduces the gas flow rates. Although the existence of both types of water in the coal matrix results in low gas production rates, it is caused by different mechanisms. Hence, the analysis on the effects of water presence in the coal matrix is performed in two parts.

Effect of bulk water

In the dual-porosity, single-permeability, water in the coal matrix is neglected. There is no blocking mechanism due to the presence of bulk water. In this analysis, the proposed dual-porosity, dual-permeability model is compared against the dual-porosity, single-permeability model (PSU-COALCOMP) to examine the effect of bulk water in the coal matrix.

In this exercise, the water saturation in the matrix system is set as 100% to represent the existence of water in the coal matrix. Bound water is considered as part of the irreducible water which is immobile for the entire production period. The fracture porosity ($\phi_f$) and matrix
permeability \((k_M)\) are set close to zero. The matrix porosity \((\phi_M)\) is varied from 0.5% to 10%. Table 7-6 and Figures 7-1 through 7-3 describe the reservoir system used in the analysis. Figures 7-18 and 7-20 show the predicted gas flow profiles from the two models for the systems with matrix porosity of 0.5% and 10%, respectively. Similarly, Figures 7-19 and 7-21 show the predicted water flow profiles from the two models for the systems with matrix porosity of 0.5% and 10%, respectively. It can be observed that the dual-porosity, dual-permeability model gives lower initial gas production rate and higher initial water production rate than the dual-porosity, single-permeability model. In addition, the discrepancy becomes more significant for the system with larger matrix porosity \((\phi_M = 10\%)\). As the pore space in the matrix becomes larger, more water in the coal matrix can be stored and it takes longer for water to be produced. Not only does this blocking mechanism blocks the gas flow path, it also delays the gas desorption from the coal resulting in less gas production at early times after the well is put on production. At late times, it can be observed that the gas production rates predicted from the dual-porosity, dual-permeability model start to catch up as lower amount of water is left in the coal matrix and the effect of blocking mechanism becomes rather insignificant. The overall results reported here agree with the field observations (Reeves and Pekot, 2001). Incorporating the presence of water in the coal matrix in the proposed dual-porosity, dual-permeability model incorporates a necessary correction to the over-predicted gas and the under-predicted water production rates.
Table 7-6: Input data for parametric study on the effect of bulk water in the coal matrix

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Dual-ϕ single-k model</th>
<th>Dual-ϕ dual-k model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir size</td>
<td>3,000 ft × 3,000 ft</td>
<td>3,000 ft × 3,000 ft</td>
</tr>
<tr>
<td>Reservoir thickness</td>
<td>20 ft</td>
<td>20 ft</td>
</tr>
<tr>
<td>Fracture porosity</td>
<td>0.5 - 10%</td>
<td>0.0000001%</td>
</tr>
<tr>
<td>Matrix porosity</td>
<td></td>
<td>0.5 - 10%</td>
</tr>
<tr>
<td>Fracture permeability</td>
<td>350 md</td>
<td>350 md</td>
</tr>
<tr>
<td>Matrix permeability</td>
<td>0.00001 md</td>
<td></td>
</tr>
<tr>
<td>Coal density</td>
<td>1.435 g/cm³</td>
<td>1.435 g/cm³</td>
</tr>
<tr>
<td>Sorption time constant</td>
<td>0 day</td>
<td>0 day</td>
</tr>
<tr>
<td>Initial pressure</td>
<td>1,100 psia</td>
<td>1,100 psia</td>
</tr>
<tr>
<td>Fracture spacing</td>
<td>0.5 ft</td>
<td>0.5 ft</td>
</tr>
<tr>
<td>Initial water saturation, fracture</td>
<td>100 %</td>
<td>100 %</td>
</tr>
<tr>
<td>Initial water saturation, matrix</td>
<td>-</td>
<td>100 %</td>
</tr>
<tr>
<td>Reservoir temperature</td>
<td>80 ºF</td>
<td>80 ºF</td>
</tr>
<tr>
<td>Number of grid blocks in x, y and z directions</td>
<td>15, 15, 1</td>
<td>15, 15, 1</td>
</tr>
<tr>
<td>Well location</td>
<td>8, 8, 1</td>
<td>8, 8, 1</td>
</tr>
<tr>
<td>Well radius</td>
<td>0.25 ft</td>
<td>0.25 ft</td>
</tr>
<tr>
<td>Sandface pressure</td>
<td>14.9 psia</td>
<td>14.9 psia</td>
</tr>
<tr>
<td>Skin factor</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Figure 7-18: Gas flow rate profiles for parametric study on the effect of bulk water in the coal matrix for the system with the matrix porosity of 0.5%.

Figure 7-19: Water flow rate profiles for parametric study on the effect of bulk water in the coal matrix for the system with the matrix porosity of 0.5%.
Figure 7-20: Gas flow rate profiles for parametric study on the effect of bulk water in the coal matrix for the system with the matrix porosity of 10%.

Figure 7-21: Water flow rate profiles for parametric study on the effect of bulk water in the coal matrix for the system with the matrix porosity of 10%.
Effect of bound water

To examine the effect of bound water in the coal matrix on gas sorption capacity and productivity, the modified Langmuir equation below is utilized:

\[
V_c = \frac{W f V_m}{p} \left( \frac{p}{p_c + p} \right)
\]  

(7.1)

where \( f \) is the sorption capacity factor. It has been discussed that gas sorption capacity in coalbeds is controlled by the amount of bound water in the coal matrix. Figure 7-22 shows the Langmuir sorption isotherms with the effect of water in the coal matrix. The Langmuir volume and pressure constants used are 904 scf/ton and 625 psia. Figure 7-22 shows that the sorption capacity becomes smaller when larger amounts of bound water are found in the coal matrix, this is represented by the smaller sorption capacity factor (inversely representing amount of bound water in the coal matrix).

Figure 7-22: Effect of water in the coal matrix on sorption isotherms
The effect of bound water in the coal matrix on gas production is investigated by evaluating the systems with different amounts of bound water in the coal matrix. The sorption capacity factor is varied from 0.2 to 1.0 to represent the effect of different amounts of bound water in the coal matrix on the gas sorption capacity. The sorption capacity factor of 1.0 represents a system with no bound water in the coal matrix. Also, the systems with lower sorption capacity factors contain larger amounts of bound water. The initial water saturation in the matrix is set to be equal to the irreducible water saturation to remove the effect of bulk water in the coal matrix. Bound water is considered as part of irreducible water. The matrix porosity is set as low as 0.1% to reduce the influence of the free gas in the coal matrix. Table 7-7 and Figures 7-2, 7-3 and 7-22 describe the properties of the five systems used in this study. Figures 7-23 and 7-24 show the gas flow rate profiles and cumulative gas production of the five systems with different sorption capacity factors. It can be observed that the gas flow profiles differ from the previous cases as in this exercise the initial gas production rates are dominated by the flow of free gas in the matrix. The gas production rates with higher amount of bound water in the coal matrix (smaller sorption capacity factor) are lower than those with lower amount of bound water in the coal matrix (larger sorption capacity factor). This is in good agreement with our previous observation that higher the amount of bound water in the coal matrix is smaller the amount of gas that can be released from the coal matrix resulting in smaller gas production rates. Figure 7-24 shows a significant difference of cumulative gas production from systems with different sorption capacity factors. Ignoring the bound water in the coal matrix could highly over-predict the gas production rates.
Table 7-7: Input data for parametric study on the effect of bound water in the coal matrix

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir size</td>
<td>3,000 ft × 3,000 ft</td>
</tr>
<tr>
<td>Reservoir thickness</td>
<td>20 ft</td>
</tr>
<tr>
<td>Fracture porosity</td>
<td>0.01%</td>
</tr>
<tr>
<td>Matrix porosity</td>
<td>0.1%</td>
</tr>
<tr>
<td>Fracture permeability</td>
<td>10 md</td>
</tr>
<tr>
<td>Matrix permeability</td>
<td>0.01 md</td>
</tr>
<tr>
<td>Coal density</td>
<td>1.435 g/cm³</td>
</tr>
<tr>
<td>Sorption time constant</td>
<td>0 day</td>
</tr>
<tr>
<td>Initial pressure</td>
<td>1,100 psia</td>
</tr>
<tr>
<td>Fracture spacing</td>
<td>0.5 ft</td>
</tr>
<tr>
<td>Initial water saturation, fracture</td>
<td>100%</td>
</tr>
<tr>
<td>Initial water saturation, matrix</td>
<td>20%</td>
</tr>
<tr>
<td>Reservoir temperature</td>
<td>80 ºF</td>
</tr>
<tr>
<td>Number of grid blocks in x, y and z directions</td>
<td>15, 15, 1</td>
</tr>
<tr>
<td>Well location</td>
<td>8, 8, 1</td>
</tr>
<tr>
<td>Well radius</td>
<td>0.25 ft</td>
</tr>
<tr>
<td>Sandface pressure</td>
<td>14.9 psia</td>
</tr>
<tr>
<td>Skin factor</td>
<td>0</td>
</tr>
</tbody>
</table>
Figure 7-23: Gas flow rate profiles for parametric study on the effect of bound water in the coal matrix

Figure 7-24: Cumulative gas production for parametric study on the effect of bound water in the coal matrix
7.2.3 Coal Shrinkage and Swelling

Model implementation on field data

A history match of field permeability data from the San Juan basin coalbeds was performed using the proposed coal shrinkage-swelling model. The permeability curve used was provided by McGovern (2004) and was also used for matching purposes by Shi and Durucan’s model. The coal parameters used in this match are shown in Table 7-8. The values of initial pressure, $p_i$, initial porosity, $\phi$, and Young modulus of the coalbed, $E$, obtained from the match are within the typical ranges in the San Juan basin (1,300 – 1,650 psia, 0.1 – 0.5% and 124000 – 445000 psi, respectively (Palmer and Mansoori, 1996; Shi and Durucan, 2005)). Also, the value of $\rho V_m \beta$ is obtained from the large-scale reservoir parameters that are appropriate to coalbeds in the San Juan basin reported by Palmer and Mansoori (1996).

Table 7-8: Matched coal parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
<td>0.30 %</td>
</tr>
<tr>
<td>$p_i$</td>
<td>1,300 psia</td>
</tr>
<tr>
<td>$E$</td>
<td>421,000 psi</td>
</tr>
<tr>
<td>$\rho V_m \beta$</td>
<td>0.01266</td>
</tr>
<tr>
<td>$p_L$</td>
<td>625 psia</td>
</tr>
</tbody>
</table>

The permeability curves shown in Figure 7-25 illustrate predictions of the proposed model with a variation of sorption capacity factor. It is observed that the predicted coalbed permeability is highly dependent on the sorption capacity factor. At pressures below initial pressure (1,300 psia), the predicted coalbed permeability becomes smaller as the sorption capacity factor decreases. This observation highlights the effect of the presence of water in the coal matrix on the coalbed permeability. Bound water adsorbed on the surface of coal matrix prevents the gas in the coal matrix from being released into the coal matrix. The amount of bound
water controls the amount of gas desorption, and consequently the change in permeability due to the coal shrinkage. Larger amount of bound water represented by smaller sorption capacity factors hinders the release of gas from the coal matrix causing smaller changes in the cleat volume. At pressures above initial pressure, the gas adsorption process takes place. The gas adsorption causes coalbed permeability to decrease as pressure increases. The larger decrease in permeability due to gas adsorption occurs in the coal with larger sorption capacity factor (smaller amount of bulk water). However, the effect due to the change in internal stress is more prominent than that due to the gas adsorption to the coal as the pressure increases. This causes the total permeability to increase. A good match between the predicted permeability using the proposed model and the actual field permeability is obtained with a sorption capacity factor of 0.50.

Figure 7-25: Predictions of coalbed permeability relative to actual field data for sorption capacity factors ranging from 0.20 to 1.00
A comparative analysis of coal shrinkage-swelling models

Coalbed reservoir parameters from the San Juan Basin are used in this section of the paper for model comparison purposes. Table 7-9 summarizes the coalbed parameters used in each model.

Table 7-9: San Juan Basin coalbed parameters used in each model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
<th>Sawyer</th>
<th>Seidle-Huit</th>
<th>Palmer-Mansoori</th>
<th>Shi-Durucan</th>
<th>This work</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_i$</td>
<td>%</td>
<td>0.1 – 0.5</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>$p_L$</td>
<td>psi</td>
<td>625</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>$V_L$</td>
<td>scf/cuft</td>
<td>33.68</td>
<td>×</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p_i$</td>
<td>psia</td>
<td>1,100</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>$c_m$</td>
<td>psi$^{-1}$</td>
<td>0.000001784</td>
<td>×</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\varepsilon_{exp}$</td>
<td>dimensionless</td>
<td>150</td>
<td>×</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$V_m$</td>
<td>scf/ton</td>
<td>521</td>
<td>×</td>
<td></td>
<td></td>
<td></td>
<td>×</td>
</tr>
<tr>
<td>$K$</td>
<td>psi</td>
<td>676,400</td>
<td>×</td>
<td>×</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M$</td>
<td>psi</td>
<td>890,000</td>
<td>×</td>
<td></td>
<td>×</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>psi$^{-1}$</td>
<td>0</td>
<td>×</td>
<td>×</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f$</td>
<td>dimensionless</td>
<td>0.5</td>
<td>×</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\varepsilon_l$</td>
<td>dimensionless</td>
<td>0.0128</td>
<td>×</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$v$</td>
<td>dimensionless</td>
<td>0.35</td>
<td>×</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E$</td>
<td>psi</td>
<td>445,000</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha_S$</td>
<td>cuft/scf</td>
<td>0.00038</td>
<td>×</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C_f$</td>
<td>psi$^{-1}$</td>
<td>0.00096</td>
<td>×</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f$</td>
<td>dimensionless</td>
<td>1.00</td>
<td>×</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho V_m \beta$</td>
<td>dimensionless</td>
<td>0.00325</td>
<td>×</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The effects of bound water in the coal matrix in the proposed model is neglected (sorption capacity factor = 1.0) in order to establish a more meaningful comparative analysis. Figures 7-26 and 7-27 shows the porosity changes predicted by the existing and proposed coal shrinkage-swelling models for systems with initial porosity values of 0.1% and 0.5%, respectively. Similarly, Figures 7-28 and 7-29 show the permeability changes predicted by the existing and proposed coal shrinkage-swelling models for the system with initial porosity values of 0.1% and 0.5%, respectively. It is observed that the change in permeability becomes
increasingly significant with lower initial porosity for nearly every model. On the other hand, Shi and Durucan’s permeability model shows independence from initial porosity. Sawyer et al.’s and Palmer and Mansoori’s models show comparable results for all values of initial porosity while the proposed model provides similar predictions to Sawyer et al.’s and Palmer and Mansoori’s models for initial porosity value of 0.5%. Another observation is that in Seidle and Huitt’s approach permeability increases significantly as system pressure is approaching to zero. Also, changes in cleat porosity and permeability as predicted by Sawyer et al.’s, Seidle and Huitt’s, Palmer and Mansoori’s and the proposed models are found to be increasingly insignificant at higher initial porosity values. In addition, a qualitative comparison between the volume change due to the rock expansion/compression and the release/adsorption of gas in the coal matrix is displayed in Figure 7-27. From the predictions of Sawyer et al.’s, Palmer and Mansoori’s, Shi and Durucan’s and the proposed models for the coalbed sample studied, the cleat volume change due to rock expansion is dominant at first as pressure decreases below initial pressure (1,100 psia). However, as pressure is continuously reduced the cleat volume change due to the release of gas from the coal matrix becomes increasingly more dominant. For the region above initial pressure, Sawyer et al.’s, Palmer and Mansoori’s and the proposed models predict that the cleat volume change due to internal stress is dominant while Shi and Durucan’s model yields the decrease of cleat volume due to adsorption of the gas in the coal matrix at first and the cleat volume becomes larger due to the change in internal stress as pressure continuously increases. In addition, Seidle and Huitt’s model predicts a decrease in cleat volume at pressure above initial pressure as it only takes into account the adsorption of the gas in the coal matrix but ignores the effect of internal stress.
Figure 7-26: Comparison of predicted porosity change with pressure for the system with initial porosity of 0.1%

Figure 7-27: Comparison of predicted porosity change with pressure for the system with initial porosity of 0.5%
Figure 7-28: Comparison of predicted permeability change with pressure for the system with initial porosity of 0.1%

Figure 7-29: Comparison of predicted permeability change with pressure for the system with initial porosity of 0.5%
To investigate the effects of the coal shrinkage and swelling on the productivity of CBM reservoirs, the proposed dual-porosity, dual-permeability model is used. The existing and proposed coal shrinkage-swelling permeability models are implemented to the simulator. The properties described in Tables 7-9, 7-10 and Figures 7-1 through 7-3 are used in this exercise. A number of simulation runs using different coal shrinkage-swelling models were performed on two simulated CBM reservoir systems with initial fracture porosity of 0.1% and initial matrix porosity of 0.5%. Figures 7-30 and 7-31 show the gas flow rate profiles and well block pressure profiles of the CBM reservoir systems with different coal shrinkage-swelling models. The predicted production results showed a reasonable agreement with predicted changes in porosity and permeability, particularly with the porosity and permeability in the fracture (Figures 7-26 and 7-28). As compared to the results of the case with no shrinkage and swelling effects, the predicted gas production rates using Shi-Durucan and Seidle-Huitt models are found to be higher. This can also be observed from the rapid decline of the reservoir pressure as shown in Figure 7-31 and the increase of permeability as the pressure decreases as shown in Figure 7-28. The proposed model predicted slightly lower gas production rate at the beginning but the production rate becomes larger as reservoir pressure declines. Palmer and Mansoori’s and Sawyer et al.’s models predicted lower gas production rates at the beginning as their models predict a slight decrease in permeability; however, their predicted production rates appear to sustain for longer periods due to the slow decline of reservoir pressure and the increase in permeability at low pressure.
Table 7-10: Input data for comparative study on coal shrinkage-swelling effects

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir size</td>
<td>3,000 ft × 3,000 ft</td>
</tr>
<tr>
<td>Reservoir thickness</td>
<td>20 ft</td>
</tr>
<tr>
<td>Fracture porosity</td>
<td>0.1%</td>
</tr>
<tr>
<td>Matrix porosity</td>
<td>0.5%</td>
</tr>
<tr>
<td>Fracture permeability</td>
<td>5 md</td>
</tr>
<tr>
<td>Matrix permeability</td>
<td>0.5 md</td>
</tr>
<tr>
<td>Coal density</td>
<td>1.435 g/cm³</td>
</tr>
<tr>
<td>Sorption time constant</td>
<td>0 day</td>
</tr>
<tr>
<td>Initial pressure</td>
<td>1,100 psia</td>
</tr>
<tr>
<td>Fracture spacing</td>
<td>0.5 ft</td>
</tr>
<tr>
<td>Initial water saturation, fracture</td>
<td>100%</td>
</tr>
<tr>
<td>Initial water saturation, matrix</td>
<td>100%</td>
</tr>
<tr>
<td>Reservoir temperature</td>
<td>80 ºF</td>
</tr>
<tr>
<td>Number of grid blocks in x, y and z directions</td>
<td>15, 15, 1</td>
</tr>
<tr>
<td>Well location</td>
<td>8, 8, 1</td>
</tr>
<tr>
<td>Well radius</td>
<td>0.25 ft</td>
</tr>
<tr>
<td>Sandface pressure</td>
<td>100 psia</td>
</tr>
<tr>
<td>Skin factor</td>
<td>0</td>
</tr>
</tbody>
</table>
Figure 7-30: Gas flow rate profiles of the systems with different coal shrinkage-swelling models

Figure 7-31: Well block pressure of the systems with different coal shrinkage-swelling models
Effects of coal shrinkage and swelling and water in the coal matrix on the productivity

Thus far, the effects of coal shrinkage-swelling mechanism and bound water in the coal matrix have been studied separately. The changes in reservoir porosity and permeability (or coal shrinkage and swelling) are caused by two conflicting actions, which are the rock expansion/compression and the release/adsorption of gas in the coal matrix. However, the dynamics of gas sorption in the matrix is also influenced by the existence of bound water in the coal matrix. It is necessary to account for the effects of bound water in the coal matrix when coal shrinkage-swelling formulation is implemented. In this part of the study, the proposed coal shrinkage and swelling, which accounts for bound water in the coal matrix is investigated. The system previously used in the comparative study of coal shrinkage-swelling model is again used here. The sorption capacity factor is varied from 0.2 to 1.0 to observe the effects of bound water in the coal matrix. The reservoir properties used in this study are presented in Tables 7-9, 7-10 and Figures 7-2, 7-3 and 7-22. Gas flow rate profiles and well block pressure for this study are presented in Figures 7-32 and 7-33, respectively. The circles represent the results of the system with no shrinkage/swelling effects and no presence of water in the coal matrix. A good degree of discrepancy can be observed between the results of the system with shrinkage/swelling effects and presence of water in the coal matrix and that without shrinkage/swelling effects and absence of water in the coal matrix. The overall results agree with previous analysis on the effects of bound water in the coal matrix. The gas flow rates of the coalbed methane system with higher amount of bound water in the coal matrix (smaller sorption capacity factor) are lower than that with lower amount of bound water in the coal matrix (larger sorption capacity factor). In addition, even though the presence of bound water in the coal matrix reduces gas sorption in the coal matrix and results in decreasing gas flow rates, the higher gas flow rates could be experienced for the systems with high sorption factor capacity (sorption capacity close to unity) compared to the case with no shrinkage and swelling effects. This is because the effects of porosity and
permeability increase due to the release of gas in the coal matrix on the gas production is more
dominant than those of bound water in the coal matrix.

Figure 7-32: Gas flow rate profiles of the systems with the effects of coal shrinkage-swelling
mechanism and water in the coal matrix
7.3 Parametric Studies on the Enhanced CBM Recovery Process

This part of the study illustrates how the model responds to the effects of presence of water in the coal matrix and coal shrinkage and swelling on the CO\textsubscript{2} enhanced coalbed methane (CO\textsubscript{2}-ECBM) recovery process or CO\textsubscript{2} sequestration.

7.3.1 Water Presence in the Coal Matrix

As mentioned earlier in Section 2.3, water in the coal matrix can be classified in two types including bulk water and bound water. Bulk water blocks the flow path of gas while bound water reduces the gas sorption capacity. As illustrated in Sections 7.2.2, the presence of both types of water in the coal matrix reduces the gas flow rates. In this part of the study, the proposed
compositional dual-porosity, dual-permeability CBM model is used to determine the effects of both types of water in the coal matrix on enhanced methane production and CO₂ breakthrough.

Effect of bulk water

In this analysis, the proposed compositional dual-porosity, dual-permeability CBM model is compared against the compositional dual-porosity, single-permeability CBM model (PSU-COALCOMP) where the water in the coal matrix is neglected. The five-spot pattern of CO₂ injection in Section 7.1.3 is used in this exercise; however, water saturation in the matrix is increased to 80% to enhance the effect of bulk water in the coal matrix. Tables 7-3, 7-4 and 7-11 describe the reservoir system used in this analysis. Figure 7-34 shows the comparisons of the primary and enhanced methane production rates while Figure 7-35 shows the comparison of the methane cumulative production. Figure 7-36 shows the comparison of the CO₂ production rates. For the primary process without CO₂ injection, the early gas production predicted by the dual-porosity, single-permeability model is larger than that predicted by the proposed model, which agrees with the observations in Section 7.2.2. In the primary CBM production process, the existence of bulk water in the coal matrix blocks the flow path of gas from the matrix to the fractures causing lower gas production rates at early time compared to the case without bulk water in the coal matrix. On the contrary, for the CO₂ injection process, the gas production rates predicted by the dual-porosity, single-permeability model is lower than that predicted by the proposed model during the period before CO₂ breakthrough. Also, an earlier breakthrough time of CO₂ at the producer is predicted by the proposed model. This is caused by the blocking mechanism of the bulk water, which prevents some amount of the injected CO₂ from flowing into the matrix to get adsorbed on the coal surface. The injected CO₂ that could not enter the matrix would then transport through the fractures towards the producer causing an increase in gas saturation in the fractures. This yields larger gas relative permeability in the fractures around the
producer resulting in higher gas production compared to the system without bulk water in the coal matrix. The injected CO\textsubscript{2} bypassing through the fractures also provides an earlier CO\textsubscript{2} breakthrough. Figure 7-37 shows the comparison of the production gas compositions, which also illustrates an earlier CO\textsubscript{2} breakthrough observed in the proposed model.

Table 7-11: Additional input data for sensitivity analysis on the effect of bulk water

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Dual-(\phi) single-(k) model</th>
<th>Dual-(\phi) dual-(k) model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fracture porosity</td>
<td>0.1%</td>
<td>0.01%</td>
</tr>
<tr>
<td>Matrix porosity</td>
<td></td>
<td>0.1%</td>
</tr>
<tr>
<td>Fracture permeability</td>
<td>3.65 md</td>
<td>3.65 md</td>
</tr>
<tr>
<td>Matrix permeability</td>
<td></td>
<td>0.00001 md</td>
</tr>
<tr>
<td>Fracture spacing</td>
<td>-</td>
<td>0.06 ft</td>
</tr>
<tr>
<td>Initial water saturation, fracture</td>
<td>80%</td>
<td>80%</td>
</tr>
<tr>
<td>Initial water saturation, matrix</td>
<td>-</td>
<td>80%</td>
</tr>
<tr>
<td>Irreducible water saturation</td>
<td>20%</td>
<td>20%</td>
</tr>
</tbody>
</table>

Figure 7-34: CH\textsubscript{4} production rates for parametric study on the effect of bulk water in the coal matrix
Figure 7-35: CH₄ cumulative production rates for parametric study on the effect of bulk water in the coal matrix

Figure 7-36: CO₂ production rates for parametric study on the effect of bulk water in the coal matrix
Effect of bound water

The effect of bound water in the coal matrix on the CO₂-ECBM recovery process is investigated here. As mentioned earlier, the existence of bound water in the coal matrix results in the reduction of gas sorption capacity. In addition, bound water has a larger proportional effect on CH₄ sorption than CO₂ sorption as CH₄ can only adsorb on the hydrophobic sites of the coal surface while CO₂ can also form a layer around adsorbed water. Two simulation runs were conducted. The first run was conducted on a system without the existence of the bound water in the coal matrix. The sorption capacity factors for CH₄ and CO₂ are set as unity. The second run considers the effect of the existence of the bound water in the coal matrix. The sorption capacity factors of 0.50 and 0.70 are used for CH₄ and CO₂, respectively. These sorption capacity factors are taken from the reduction of CO₂ and CH₄ sorption capacities for the Hunter Valley sample reported by Day et al. (2008). Tables 7-3, 7-4 and 7-12 describe the reservoir system used in this
analysis. Figures 7-38 and 7-39 show the comparisons of the CH$_4$ production rates and CH$_4$ cumulative production of the CO$_2$-ECBM recovery process. The results agree with those presented in 7.2.2 as the CH$_4$ production rates for the system considering bound water in the coal matrix are lower than those for the system without it. Figure 7-40 shows the comparisons of the CO$_2$ production rates and Figure 7-41 shows the comparison of production gas compositions. An earlier CO$_2$ breakthrough can be observed from the system with bound water in the coal matrix as less CO$_2$ can be adsorbed on the coal surface. As there are less accessible sites for CO$_2$ to get adsorbed, the faster the coal surface becomes saturated with CO$_2$. This results in an earlier breakthrough of CO$_2$. In addition, after the CO$_2$ breakthrough occurs, mole fraction of CH$_4$ decreases more sharply for the system with bound water in the coal matrix. Ignoring the effect of bound water in the coal matrix could result in over-prediction of the CH$_4$ production and late CO$_2$ breakthrough.

Table 7-12: Additional input data for sensitivity analysis on the effect of bound water

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Without bound water</th>
<th>With bound water</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fracture porosity</td>
<td>0.01%</td>
<td>0.01%</td>
</tr>
<tr>
<td>Matrix porosity</td>
<td>0.1%</td>
<td>0.1%</td>
</tr>
<tr>
<td>Fracture permeability</td>
<td>3.65 md</td>
<td>3.65 md</td>
</tr>
<tr>
<td>Matrix permeability</td>
<td>0.00001 md</td>
<td>0.00001 md</td>
</tr>
<tr>
<td>Fracture spacing</td>
<td>0.06 ft</td>
<td>0.06 ft</td>
</tr>
<tr>
<td>Initial water saturation, fracture</td>
<td>20%</td>
<td>20%</td>
</tr>
<tr>
<td>Initial water saturation, matrix</td>
<td>20%</td>
<td>20%</td>
</tr>
<tr>
<td>Sorption capacity factor – CH$_4$</td>
<td>1.00</td>
<td>0.50</td>
</tr>
<tr>
<td>Sorption capacity factor – CO$_2$</td>
<td>1.00</td>
<td>0.70</td>
</tr>
</tbody>
</table>
Figure 7-38: CH$_4$ production rates for parametric study on the effect of bound water in the coal matrix

Figure 7-39: CH$_4$ cumulative production for parametric study on the effect of bound water in the coal matrix
Figure 7-40: CO$_2$ production rates for parametric study on the effect of bound water in the coal matrix

Figure 7-41: Production gas compositions for parametric study on the effect of bound water in the coal matrix
7.3.2 Coal Shrinkage and Swelling

The effects of coal shrinkage and swelling on the enhanced CBM recovery process are investigated in this section. In the primary production process of CBM systems, it was found in Section 7.2.3 that as pressure decreased, changes in porosity and permeability were caused by the rock expansion and the release of methane from the coal surface. However, for enhanced CBM recovery process with pure CO$_2$ injection, the porosity and permeability changes caused by the adsorption of injected CO$_2$ in the coal matrix is necessary to be considered. In this analysis, the five-spot pattern of pure CO$_2$ injection is studied. Tables 7-3, 7-4 and 7-13 describe the reservoir system used in this analysis. Figures 7-42 and 7-43 show the comparisons of the CH$_4$ production rates and CH$_4$ cumulative production of the CO$_2$-ECBM recovery process. Figure 7-44 shows the comparisons of the CO$_2$ production rates and Figure 7-45 shows the comparison of production gas compositions. At the early period of production before CO$_2$ breakthrough, it can observed that the methane production rates for the system with coal shrinkage and swelling effects are slightly lower than those for the system without coal shrinkage and swelling effects. This is caused by a decrease in fracture permeability as rock expansion is more dominant than the release of methane in the coal matrix. This can be observed from Figure 7-46, which illustrates the changes of fracture permeability of the producer block versus time. This decrease in permeability also sustains the methane production rates for the system with coal shrinkage and swelling effects longer and causes a delay of CO$_2$ breakthrough (see Figures 7-44 and 7-45). From Figure 7-46, after CO$_2$ breakthrough occurs (approximately 60 days after the production starts), a sharp reduction of fracture permeability occurs. It can also be observed from Figure 7-47 that pressure at the producer block increases after CO$_2$ breakthrough. This increase in pressure at the producer block slightly increases and causes rock to compress yielding larger porosity and permeability after the CO$_2$ breakthrough. However, as the injected CO$_2$ molecules get adsorbed on the coal
surface, the coal grains swell causing smaller porosity and lower permeability. Thus, the sharp
dercrease in fracture permeability is caused by the overbearing effect of the adsorption of CO$_2$
in the coal matrix. At later times, the fracture permeability remains constant as the coal surface is
saturated with CO$_2$ molecules leaving the injected CO$_2$ to be produced.

Table 7-13: Additional input data for sensitivity analysis on the effect of coal shrinkage and
swelling

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fracture porosity</td>
<td>0.01%</td>
</tr>
<tr>
<td>Matrix porosity</td>
<td>0.1%</td>
</tr>
<tr>
<td>Fracture permeability</td>
<td>3.65 md</td>
</tr>
<tr>
<td>Matrix permeability</td>
<td>0.000001 md</td>
</tr>
<tr>
<td>Fracture spacing</td>
<td>0.06 ft</td>
</tr>
<tr>
<td>Initial water saturation, fracture</td>
<td>20%</td>
</tr>
<tr>
<td>Initial water saturation, matrix</td>
<td>20%</td>
</tr>
<tr>
<td>Matrix shrinkage-swelling coefficient</td>
<td>0.000007 ft$^3$/SCF</td>
</tr>
<tr>
<td>Young’s modulus</td>
<td>286,000 psi</td>
</tr>
</tbody>
</table>

Figure 7-42: CH$_4$ production rates for parametric study on the effects of coal shrinkage and
swelling
Figure 7-43: CH₄ cumulative production for parametric study on the effects of coal shrinkage and swelling

Figure 7-44: CO₂ production rates for parametric study on the effects of coal shrinkage and swelling
Figure 7-45: Production gas compositions for parametric study on the effects of coal shrinkage and swelling.

Figure 7-46: Fracture permeability at the producer block for parametric study on the effects of coal shrinkage and swelling.
7.4 Comparative Study on the Existing and Proposed Simulation Models

In addition to the parametric studies, a comparative study between the existing and proposed simulation models is conducted here to investigate their prediction capabilities. The existing simulator used in this study is the generalized equation-of-state model compositional reservoir simulator or GEM developed by the Computer Modelling Group. The GEM for coalbed methane reservoirs is developed using the dual-porosity, single-permeability formulation and it contains the Palmer and Mansoori’s coal shrinkage and swelling representation. The GEM assumes that there is only gas phase in the coal matrix and the flow of gas in the coal matrix is only driven by diffusion mechanism. Also, the GEM uses the extended Langmuir equation in determining the multi-component gas adsorption/desorption. On the contrary, the proposed simulation model treats the coalbed methane reservoirs as the dual-porosity, dual-permeability
system and takes into account the water presence in the coal matrix and the coal shrinkage and swelling. In addition, the flow of gas is considered to be driven by the multi-mechanistic mechanism. The proposed model uses the ideal adsorbed solution (IAS) theory for multi-component sorption calculations.

In this comparative study, each simulator was used at its fullest capabilities in predicting gas production for the primary methane production process and the enhanced CBM recovery process with CO$_2$ injection. To illustrate the discrepancies between the fullest capacities of the two simulators, a complex reservoir system exhibiting multiple CBM characteristics was used. Tables 7-3, 7-4 and 7-14 describe the reservoir system. Figure 7-48 shows the comparison between the gas production profiles predicted from the GEM and the proposed simulators for the primary methane production process. It can be observed that the gas production rates predicted from the proposed simulator are lower than those predicted from the GEM simulator. Figure 7-49 shows the comparison of the methane production rates while Figure 7-50 shows the comparison of the CO$_2$ production rates. Figure 7-51 shows the comparison of production gas compositions. From the results of the CO$_2$-enhanced CBM recovery process, the proposed simulator provides smaller methane production rates and predicts earlier CO$_2$ breakthrough compared to those predicted from the GEM simulator. These differences observed in the primary and enhanced CBM recovery processes are caused by the different coalbed methane reservoir descriptions used in developing the two simulators models, which include (1) the representation of CBM reservoirs (dual-porosity, single-permeability or dual-porosity, dual-permeability), (2) the gas flow mechanism (single-mechanistic or multi-mechanistic), (3) the incorporation of water presence in the coal matrix, (4) multi-component gas sorption theory (extended Langmuir equation or IAS theory) and (5) the coal shrinkage and swelling model. In particular, it can be observed that among others, the effects of water presence in the coal matrix are most dominant and result in a significant decrease in gas production rates and in early CO$_2$ breakthrough as shown and
discussed in Section 7.3. This exercise demonstrates the considerable differences that can occur between the GEM and the proposed simulators, which are caused by the different assumptions and limitations of the simulators.

Table 7-14: Additional input data for comparative study

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sorption capacity factor – CH\textsubscript{4}</td>
<td>0.5</td>
</tr>
<tr>
<td>Sorption capacity factor – CO\textsubscript{2}</td>
<td>0.7</td>
</tr>
<tr>
<td>Matrix shrinkage-swelling coefficient</td>
<td>1.5×10\textsuperscript{-6} ft\textsuperscript{3}/SCF</td>
</tr>
<tr>
<td>Rock compressibility</td>
<td>6×10\textsuperscript{-6} 1/psi</td>
</tr>
</tbody>
</table>

Figure 7-48: Gas production rates of the primary methane production process for comparative study between the existing and proposed simulation models
Figure 7-49: Methane production rates of the enhanced CBM recovery process for comparative study between the existing and proposed simulation models

Figure 7-50: CO₂ production rates of the enhanced CBM recovery process for comparative study between the existing and proposed simulation models
Figure 7-51: Production mole fractions of the enhanced CBM recovery process for comparative study between the existing and proposed simulation models

7.5 Validation of the Model via History Matches

In this part of the study, the application of the proposed CBM reservoir model is implemented on field history matching. History matching studies are conducted using the Fenn Big Valley field data (Ertekin, 2003). The first history match case is an injection test of pure CO\textsubscript{2}. The pure CO\textsubscript{2} injection test involved 4 stages including (1) CO\textsubscript{2} injection, (2) shut-in for pressure falloff, (3) gas production, and (4) shut-in for pressure buildup. The second case is an injection test of flue gas involving 3 stages: (1) flue gas (12.4% CO\textsubscript{2} and 87.6% N\textsubscript{2}), (2) shut-in for pressure falloff and (3) gas production. The pure CO\textsubscript{2} injection test and flue gas injection test schedules are shown in Tables 7-15 and 7-16, respectively. The coalbed properties of the Fenn Big Valley field are shown in Tables 7-4 and 7-17. Table 7-18 and Figure 7-52 summarize the field and history match parameters used for both cases. For the pure CO\textsubscript{2} injection case, CO\textsubscript{2}
injection rate is specified in the injection periods while water production rate is specified in the production period. For the flue gas injection case, flue gas injection rate is specified in the injection periods while sandface pressure is specified in the production period. The results of history matches for the pure CO$_2$ injection case are presented in Figures 7-53 through 7-56. Also, the results of history matches for the flue gas injection case are presented in Figure 7-57 through 7-60. The overall history matches for both cases are satisfactory considering the actual field fracture permeability value was used and the matched porosity values are reasonably close to the field data. Relative permeability curves were adjusted to match the actual gas production rates in both cases.

The history match results for the pure CO$_2$ injection test show reasonable matches using the field fracture permeability and gas relative permeability curve. Figure 7-53 shows the history match between the predicted and the actual bottom-hole pressure during injection/shut-in process at early times (from 0 to 0.30 day of the test) for the pure CO$_2$ injection case while Figure 7-54 shows the bottom-hole pressure match during the drawdown process (from 12 to 15 days of the test). Discrepancy between the predicted and the actual bottom-hole pressure can be observed in the injection/shut-in process; however, a better pressure match is found during drawdown process and also the buildup period after the production. The match of gas production rates is presented in Figure 7-55. The results show an under-prediction of gas rate at the beginning of the production (first 4.8 hrs of production period); however, a better match is obtained at later times. Figure 7-56 presents a reasonable match of mole fractions of gas production after pure CO$_2$ injection process. Eq.(7.2) shows the formula used in determine the accuracy of the history matches. $\chi_{\text{hist}}^i$ and $\chi_{\text{sim}}^i$ are the actual and predicted values at the $i$-th period, respectively, and $N$ is the total number of periods during production. Errors of gas production and mole fraction history matches were found to be 18% and 8%, respectively.
\[
\text{Error} = \frac{1}{N} \sum_{i=1}^{N} \frac{|X_{\text{hist}}^i - X_{\text{sim}}^i|}{X_{\text{hist}}^i} \times 100
\]  

(7.2)

Satisfactory history matches were obtained for the flue gas injection test using the actual fracture permeability. The initial gas mole fractions are modified to match the actual mole fraction of the gas production for the flue gas injection case. Figure 7-57 shows the bottom-hole pressure match during injection/shut-in process. A good match was obtained during the period of injection/shut-in test by incorporating the coal shrinkage and swelling model. Figures 7-58 and 7-59 present the history matches for gas and water production rates, respectively. The predicted initial gas rate is found to be smaller than the actual rate; however, a better match was obtained at later times. The overall predicted water production rate is higher than the actual rate; however, it is considered to be reasonable due to its small magnitude. Figure 7-60 shows a good agreement in predicted and actual mole fraction of gas production after flue gas injection process. Errors of gas production and mole fraction history matches using Eq.(7.2) were found to be 11% and 5%, respectively. It is noted that the proposed simulator was able to provide excellent matches of production mole fractions for the flue gas injection test, which was found to be difficult for most of the existing simulators.
Table 7-15: Pure CO$_2$ injection test schedule for history match study

<table>
<thead>
<tr>
<th>Time</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000000 - 0.016667 days</td>
<td>Pure CO$_2$ Injection</td>
</tr>
<tr>
<td>0.016667 - 0.077775 days</td>
<td>Shut-in</td>
</tr>
<tr>
<td>0.077775 - 0.161108 days</td>
<td>Pure CO$_2$ Injection</td>
</tr>
<tr>
<td>0.161108 - 12.000000 days</td>
<td>Shut-in (Pressure Falloff)</td>
</tr>
<tr>
<td>12.000000 - 14.741917 days</td>
<td>Production</td>
</tr>
<tr>
<td>14.741917 - 20.553333 days</td>
<td>Shut-in (Pressure Build-up)</td>
</tr>
</tbody>
</table>

Table 7-16: Flue gas injection test schedule for history match study

<table>
<thead>
<tr>
<th>Time</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0000 - 5.1410 days</td>
<td>Shut-in</td>
</tr>
<tr>
<td>5.1410 - 5.6201 days</td>
<td>Flue Gas Injection</td>
</tr>
<tr>
<td>5.6201 - 5.8910 days</td>
<td>Shut-in</td>
</tr>
<tr>
<td>5.8910 - 8.6618 days</td>
<td>Flue Gas Injection</td>
</tr>
<tr>
<td>8.6618 - 8.9326 days</td>
<td>Shut-in</td>
</tr>
<tr>
<td>8.9326 - 10.0785 days</td>
<td>Flue Gas Injection</td>
</tr>
<tr>
<td>10.0785 - 79.4951 days</td>
<td>Shut-in (Pressure Falloff)</td>
</tr>
<tr>
<td>79.4951 - 101.4951 days</td>
<td>Production</td>
</tr>
</tbody>
</table>
Table 7-17: Coalbed properties of the Fenn Big Valley field

<table>
<thead>
<tr>
<th>Coalbed Properties</th>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Coal seam thickness</td>
<td>13.025 ft</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Top of coal seam</td>
<td>4112.8 ft</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial absolute permeability of natural fracture</td>
<td>3.65 md</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Porosity of natural fracture</td>
<td>0.1%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Effective coalbed compressibility</td>
<td>$1 \times 10^{-6}$ psi$^{-1}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Initial Conditions</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature</td>
<td>113°F</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average pressure</td>
<td>1109.68 psia</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gas saturation</td>
<td>40.8%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Water saturation</td>
<td>59.2%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gas composition</td>
<td>92% CH$_4$, 1.5% CO$_2$, 5% N$_2$, 1.5% C$_2$H$_6$, C$_3$H$_8$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Water Properties at 45°C (113°F)</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>61.8 lb/ft$^3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Viscosity</td>
<td>0.607 cp</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Compressibility</td>
<td>$4 \times 10^{-6}$ psi$^{-1}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Pure Gas Adsorption Isotherms at 45°C (113°F)</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Average in-situ coal density</td>
<td>91.14 lb/ft$^3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average in-situ moisture content (by wt.)</td>
<td>0.025</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average in-situ ash content (by wt.)</td>
<td>0.25</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sorption parameter</th>
<th>CH$_4$</th>
<th>CO$_2$</th>
<th>N$_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Langmuir pressure (psia)</td>
<td>1,034</td>
<td>439.5</td>
<td>1,540</td>
</tr>
<tr>
<td>Langmuir volume (scf/ton)</td>
<td>531.8</td>
<td>1210.4</td>
<td>224.3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Grid System</th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Area</td>
<td>160 acres</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Parameter</td>
<td>Field data</td>
<td>Pure CO₂ injection</td>
<td>Flue gas injection</td>
</tr>
<tr>
<td>-----------------------------------------------</td>
<td>------------</td>
<td>-------------------</td>
<td>-------------------</td>
</tr>
<tr>
<td>Fracture permeability (md)</td>
<td>3.65</td>
<td>3.65</td>
<td>3.65</td>
</tr>
<tr>
<td>Matrix permeability (md)</td>
<td>-</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Fracture porosity (%)</td>
<td>0.1</td>
<td>0.5</td>
<td>0.15</td>
</tr>
<tr>
<td>Matrix porosity (%)</td>
<td>-</td>
<td>1.3</td>
<td>3.0</td>
</tr>
<tr>
<td>Desorption time constant (day)</td>
<td>-</td>
<td>6.9</td>
<td>0.0</td>
</tr>
<tr>
<td>Initial water saturation, fracture (%)</td>
<td>59.2</td>
<td>52.0</td>
<td>55.0</td>
</tr>
<tr>
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<td>$k_{rg}$-I</td>
<td>$k_{rg}$-II</td>
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<td>Relative permeability to water model</td>
<td>$k_{rw}$-I</td>
<td>$k_{rw}$-II</td>
<td>$k_{rw}$-III</td>
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<tr>
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<td>385.6</td>
<td>385.6</td>
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<td>-</td>
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<td>15.0</td>
<td>18.0</td>
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<td>-</td>
<td>19.0</td>
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<td>18.0</td>
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<td>19.0</td>
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<tr>
<td>Skin factor</td>
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<tr>
<td>Matrix shrinkage-swelling coefficient (ft³/SCF)</td>
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<td>-</td>
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<tr>
<td>Young’s modulus (psi)</td>
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Figure 7-52: Relative permeability values used in history matching studies

Figure 7-53: Well bottom-hole pressure at early times for pure CO₂ injection history match
Figure 7-54: Well bottom-hole pressure at late times for pure CO$_2$ injection history match

Figure 7-55: Gas production rates for pure CO$_2$ injection history match
Figure 7-56: Mole fractions of gas production for pure CO₂ injection history match

Figure 7-57: Well bottom-hole pressure at early times for flue gas injection history match
Figure 7-58: Gas production rates for flue gas injection history match

Figure 7-59: Water production rates for flue gas injection history match
Figure 7-60: Mole fractions of gas production for flue gas injection history match
Chapter 8

Material Balance Equation for Coalbed Methane Reservoirs

The material balance equation has been widely used by reservoir engineers for reserve estimation and production performance predictions. The general material balance equation, which is known as the conventional material balance equation (CMBE), was initially developed by Schilthuis (1936) for conventional reservoirs. The development of the CMBE is based on a volumetric balance. As reservoir pressure is reduced, the changes in the oil and free gas volumes in the reservoirs are equal to the changes in the water and rock volumes. Havlena and Odeh (1963) developed a method of applying the material balance equation as a straight line. This method has been found to be effective for black-oil and dry gas reservoirs. Walsh et al. (1994) incorporated a straight-line method and proposed the generalized material balance equation (GMBE), which also takes into account volatized-oil. The GMBE can be applied to all reservoir types, especially volatized oil, gas-condensate and wet gas reservoirs.

King (1993) developed the material balance equation for coal-seam and Devonian shale gas reservoirs to estimate original gas in place and predict reservoir performance by incorporating adsorbed gas into the material balance equation of dry gas reservoirs. Adsorbed gas (gas stored by adsorption) is expressed using the Langmuir’s equation. Eqs.(8.1) to (8.4) show the material balance equation developed by King (1993) for CBM reservoirs:

\[
G_p = \frac{V_{b2\phi_1}Z_{sc}T_{sc}}{p_wT} \left[\left(1-S_{wi}\right)p_i + \frac{RTC_{MEi}}{\phi_i}\right] - \left[\left(1-c_\phi(p_i - p)\right)\left(1-S_{wi}\right)p + \frac{RTC_{ME}}{\phi_i}\right] \tag{8.1}
\]

\[
\frac{S_{wi}}{S_w} = \frac{(1+c_w(p_i - p)) + \frac{5.615\left(W_e - B_wW_{fp}\right)}{\phi_i V_{b2}}}{(1-c_\phi(p_i - p))} \tag{8.2}
\]
where $V_{b2}$ is the bulk volume of the fracture system. $c_\phi$ and $c_w$ are compressibility of the formation and compressibility of water, respectively. $G_p$ and $W_p$ are cumulative gas production and cumulative water production, respectively. $S_w$ represents the average water saturation remaining in the cleats.

Ahmed et al. (2006) proposed the generalized material balance equation that can be used to estimate drainage area and predict reservoir pressure and production performance for CBM reservoirs. The generalized material balance equation accounts for free gas, adsorbed gas, water expansion and formation compaction. Similar to King’s material balance equation, Langmuir’s equation is used to characterize gas sorption. The generalized material balance equation developed by Ahmed et al. (2006) is shown below:

$$G_p + \frac{B_a W_p}{B_a (1 - c_\phi (p_i - p))} = \frac{A_h}{1359.7 \rho_B \left( G_e - \frac{V_m p}{p + p_L} \right)} + \frac{7758 \phi \left[ (p_i - p)(c_\phi + c_w S_w) - (1 - S_w) \right]}{B_a (1 - c_\phi (p_i - p))} + \frac{7758 A_h \phi (1 - S_w)}{B_{gi}}$$

where $G_e$ is initial adsorbed gas content in scf/ton.

The two existing material balance equations mentioned above have been tested against numerical simulation model; however, they ignore the presence of water in the coal matrix and coal shrinkage and swelling effects, which could provide inaccuracy of reserve estimation and production performance predictions.
8.1 Formulation

The development of the proposed material balance equation for CBM systems is demonstrated in this part of the study. The proposed material balance equation accounts for water in the coal matrix and coal shrinkage-swelling effects, which are ignored by the existing material balance equations. The development of the proposed material balance equation involves the derivation of the gas and water flow equations in the fracture and matrix domains of CBM systems (Eqs.(4.1), (4.2), (4.5) and (4.6)). The derivation of the proposed material balance equation follows a mathematical technique used in demonstrating relationship between numerical-simulation and material-balance approaches. This mathematical technique was proposed by Ertekin et al. (2001). The derivation assumes that (1) the reservoir is holding an average pressure with no pressure gradients, (2) there is no capillary effect, (3) the potential gradients are negligible and (4) the reservoir exhibits homogeneous and isotropic properties. These assumptions can be stated as:

\[ p_{g,F} = p_{g,M} = p_{w,F} = p_{w,M} = p \]  
\[ \frac{\partial p}{\partial x} = \frac{\partial p}{\partial y} = \frac{\partial p}{\partial z} = 0 \]  
\[ \nabla \Phi = \nabla p - \frac{\rho}{144 g_c} \nabla G = 0 \]

Substituting Eqs. (8.6) to (8.8) into Eqs.(4.1), (4.2), (4.5) and (4.6) results in:

\[ \Gamma_{g,F} + q_g + R_{sw} q_w = \frac{1}{5.615} \frac{\partial}{\partial t} \left[ \frac{V_g \phi_p s_{g,F}}{B_g} \frac{1}{R_{sw}} + \frac{V_w \phi_p s_{w,F}}{B_w} \right] \]  
\[ \Gamma_{g,M} - q_{swp} = \frac{1}{5.615} \frac{\partial}{\partial t} \left[ \frac{V_g \phi_M s_{g,M}}{B_g} + R_{sw,M} \frac{V_w \phi_M s_{w,M}}{B_w} \right] \]
\[ \Gamma_{w,v} + q_w = \frac{1}{5.615} \frac{\partial}{\partial t} \left[ \frac{V_b \phi_{r,s} S_{w,F}}{B_w} \right] \]  
(8.11)

\[ \Gamma_{w,M} = \frac{1}{5.615} \frac{\partial}{\partial t} \left[ \frac{V_b \phi_{M,s} S_{w,M}}{B_w} \right] \]  
(8.12)

Multiplying Eq.(8.9) by \( dt \) and integrating it. This results in:

\[ \int_0^t \Gamma_{g,v} dt + \int_0^t q_g dt + \int_0^t R_{sw} q_w dt \]

\[ = \frac{1}{5.615} \left[ \frac{V_b \phi_{r,s} S_{g,F}}{B_g} - \frac{V_b \phi_{F,i} S_{g,F,i}}{B_{g,i}} + R_{sw} \frac{V_b \phi_{r,s} S_{w,F}}{B_w} - R_{sw,i} \frac{V_b \phi_{F,i} S_{w,F,i}}{B_{w,i}} \right] \]  
(8.13)

Let us consider the left-hand side of Eq.(8.13). The integration of gas flow rate (free gas and solution gas in water phase) over a period of time is equivalent to gas cumulative production, \( G_p \), thus we obtain:

\[ \int_0^t q_g dt + \int_0^t R_{sw} q_w dt = -G_p \]  
(8.14)

Also, the integration of fracture-matrix transfer term of gas can be expressed as:

\[ \int_0^t \Gamma_{g,v} dt = -\int_0^t \Gamma_{g,M} dt = G_T \]  
(8.15)

The original free gas in the fracture system, \( G_f \), and original water in place in the fracture system, \( W_f \), can be written as:

\[ G_F = \frac{1}{5.615} \left[ \frac{V_b \phi_{F,i} S_{g,F,i}}{B_{g,i}} \right] \]  
(8.16)

\[ W_F = \frac{1}{5.615} \left[ \frac{V_b \phi_{F,i} S_{w,F,i}}{B_{w,i}} \right] \]  
(8.17)
Substituting Eqs. (8.14) to (8.17) into Eq. (8.13), we obtain:

\[
G_F - G_p = \frac{1}{5.615} \left[ \frac{V_b \phi_p S_{g,F}}{B_g} - 5.615G_F + R_{sw} \frac{V_b \phi_p S_{w,F}}{B_w} - 5.615R_{sw,i} W_F \right]
\]  
(8.18)

Multiplying Eq. (8.10) by \( dt \) and integrating it. This results in:

\[
\int_0^I \Gamma_{g,M} dt - \int_0^I q \cdot dt = \frac{1}{5.615} \left[ \frac{V_b \phi_{M,i} S_{g,M,i}}{B_{g,i}} - \frac{V_b \phi_{M,i} S_{g,M,i}}{B_{g,i}} + R_{sw} \frac{V_b \phi_M S_{w,M}}{B_w} - R_{sw,i} \frac{V_b \phi_{M,i} S_{w,M,i}}{B_{w,i}} \right]
\]  
(8.19)

The integration of sorption flow rate in the left-hand side of Eq. (8.19) can be expressed as follows:

\[
\int_0^I q \cdot dt = V_a \bigg|^{v_i} - V_a \bigg|^{v_0} = G_A - G_{A,i}
\]  
(8.20)

where \( G_A \) and \( G_{A,i} \) are amount of current adsorbed gas and amount of original adsorbed gas, respectively. The original free gas in the matrix system, \( G_M \), and original water in place in the matrix system, \( W_M \), can be written as:

\[
G_M = \frac{1}{5.615} \left[ \frac{V_b \phi_M S_{g,M,i}}{B_{g,i}} \right]
\]  
(8.21)

\[
W_M = \frac{1}{5.615} \left[ \frac{V_b \phi_M S_{w,M,i}}{B_{w,i}} \right]
\]  
(8.22)

Substituting Eqs. (8.20) to (8.22) into Eq. (8.19), we obtain:

\[
- G_F - G_A + G_{A,i} = \frac{1}{5.615} \left[ \frac{V_b \phi_M S_{g,M}}{B_g} - 5.615G_M + R_{sw} \frac{V_b \phi_M S_{w,M}}{B_w} - 5.615R_{sw,i} W_M \right]
\]  
(8.23)
Multiplying Eq.(8.11) by $dt$ and integrating it. This results in:

$$\int_0^t \Gamma_{w,F} dt + \int_0^t q_w dt = \frac{1}{5.615} \left[ \frac{V_b \phi_F S_{w,F}}{B_w} - \frac{V_b \phi_{F,i} S_{w,F,i}}{B_{w,i}} \right]$$

(8.24)

The left-hand side of Eq.(8.24) can be expressed in terms of water production, $W_p$, as:

$$\int_0^t q_w dt = -W_p$$

(8.25)

Also, the integration of fracture-matrix transfer term of water can be expressed as:

$$\int_0^t \Gamma_{w,M} dt = \int_0^t \Gamma_{w,M} dt = W_T$$

(8.26)

Substituting Eq.(8.25) and (8.26) into Eq.(8.24), we obtain:

$$W_T - W_p = \frac{1}{5.615} \left[ \frac{V_b \phi_F S_{w,F}}{B_w} - 5.615 W_F \right]$$

(8.27)

Similarly, multiplying Eq.(8.12) by $dt$ and integrating it, we obtain:

$$\int_0^t \Gamma_{w,M} dt = \frac{1}{5.615} \left[ \frac{V_b \phi_M S_{w,M}}{B_w} - \frac{V_b \phi_{M,i} S_{w,M,i}}{B_{w,i}} \right]$$

(8.28)

Substituting Eq.(8.26) into Eq.(8.28), we obtain:

$$-W_T = \frac{1}{5.615} \left[ \frac{V_b \phi_M S_{w,M}}{B_w} - 5.615 W_M \right]$$

(8.29)

Rearranging Eq.(8.27), we obtain:

$$\frac{V_b \phi_F S_{w,F}}{B_w} = 5.615 \left[ W_T - W_p + W_F \right]$$

(8.30)

Also, rearranging Eq.(8.29), we obtain:

$$\frac{V_b \phi_M S_{w,M}}{B_w} = 5.615 \left[ -W_T + W_M \right]$$

(8.31)
Substituting Eq.(8.30) into Eq.(8.18) and rearranging the equation, we obtain:

\[ V_b \phi_F S_{g,F} = 5.615 B_g \left[ G_T - G_p + G_F - R_{sw} \left( W_T - W_p + W_F + R_{sw} W_F \right) \right] \quad (8.32) \]

Substituting Eq.(8.31) into Eq.(8.23) and rearranging the equation, we obtain:

\[ V_b \phi_M S_{g,M} = 5.615 B_g \left[ -G_T - (G_A - G_{A,j}) + G_M - R_{sw} \left( -W_p + W_F + W_M + R_{sw} W_M \right) \right] \quad (8.33) \]

Rearranging Eq.(8.30), we obtain:

\[ V_b \phi_F S_{w,F} = 5.615 B_w \left[ W_T - W_p + W_F \right] \quad (8.34) \]

Rearranging Eq.(8.31), we obtain:

\[ V_b \phi_M S_{w,M} = 5.615 B_w \left[ -W_T + W_M \right] \quad (8.35) \]

Summing Eqs.(8.32) through (8.35) results in

\[ V_b \phi_F \left( S_{g,F} + S_{w,F} \right) + V_b \phi_M \left( S_{g,M} + S_{w,M} \right) = 5.615 B_g \left[ -G_p + G_F - (G_A - G_{A,j}) - R_{sw} \left( -W_p + W_F + W_M + R_{sw} W_M \right) \right] + 5.615 B_w \left[ -W_p + W_F + W_M \right] \quad (8.36) \]

Substituting the following saturation constraints: \( S_{g,F} + S_{w,F} = 1 \) and \( S_{g,M} + S_{w,M} = 1 \) into Eq.(8.36) yields:

\[ V_b \left( \phi_F + \phi_M \right) = 5.615 B_g \left[ -G_p + G_F - (G_A - G_{A,j}) - R_{sw} \left( -W_p + W_F + W_M \right) + R_{sw} \left( W_F + W_M \right) \right] + 5.615 B_w \left[ -W_p + W_F + W_M \right] \quad (8.37) \]

At initial conditions, the pore volume can be written as:

\[ V_b \left( \phi_{F,i} + \phi_{M,i} \right) = 5.615 B_{g,i} \left( G_F + G_M \right) + 5.615 B_{w,i} \left( W_F + W_M \right) \quad (8.38) \]
Subtracting Eq.(8.38) from Eq.(8.37) results in

\[
V_b \left[ (\phi_F - \phi_{F,i}) + (\phi_M - \phi_{M,i}) \right] \\
= -5.615 B_g G_p - 5.615 B_g \left( G_A - G_{A,i} \right) + 5.615 B_g R_{sw} W_p - 5.615 B_g W_p \\
+ 5.615 (G_F + G_M) (B_g - B_{g,i}) - 5.615 B_g \left( W_F + W_M \right) (R_{sw} - R_{sw,i}) \\
+ 5.615 (W_F + W_M) (B_w - B_{w,i})
\]  

\quad \text{(8.39)}

The changes in porosity in the fracture and matrix systems can be written using the coal shrinkage and swelling model that accounts for rock compression/expansion and the release or adsorption of gas in the coal matrix, which is mentioned in Chapter 5, as:

\[
(\phi_F - \phi_{F,i}) + (\phi_M - \phi_{M,i}) = \phi_{F,i} \left( e^{\varepsilon_{p,r}(p-p_i)} - 1 \right) + \phi_{M,i} \left( e^{\varepsilon_{p,u}(p-p_i)} - 1 \right) \\
+ \beta \rho b V_L \left( \frac{p_i}{p_L + p_i} - \frac{p}{p_L + p} \right)
\]  

\quad \text{(8.40)}

Substituting Eq.(8.40) into Eq.(8.39) yields

\[
V_b \left[ \phi_{F,i} \left( e^{\varepsilon_{p,r}(p-p_i)} - 1 \right) + \phi_{M,i} \left( e^{\varepsilon_{p,u}(p-p_i)} - 1 \right) \\
+ \beta \rho b V_L \left( \frac{p_i}{p_L + p_i} - \frac{p}{p_L + p} \right) \right] \\
= -5.615 B_g G_p - 5.615 B_g \left( G_A - G_{A,i} \right) + 5.615 B_g R_{sw} W_p - 5.615 B_g W_p \\
+ 5.615 (G_F + G_M) (B_g - B_{g,i}) - 5.615 B_g \left( W_F + W_M \right) (R_{sw} - R_{sw,i}) \\
+ 5.615 (W_F + W_M) (B_w - B_{w,i})
\]  

\quad \text{(8.41)}

The bulk volume, \( V_b \), in Eq.(8.41) can be expressed using the drainage area in acre, \( A \), and thickness in ft, \( h \), as:

\[
V_b = 43,560 Ah
\]  

\quad \text{(8.42)}
Substituting Eq.(8.42) into (8.41) results in:

\[
7758Ah \left[ \phi_{F,i} \left( e^{x_{p,i}(p-p_i)} - 1 \right) + \phi_{M,i} \left( e^{x_{p,i}(p-p_i)} - 1 \right) + \beta \rho_b V_L \left( \frac{p_i}{p_L + p_i} - \frac{p}{p_L + p} \right) \right] \\
= -B_g G_p - B_g \left( G_A - G_{A,i} \right) + B_g R_w W_p - B_w W_p \\
+ \left( V_F + G_M \right) \left( B_g - B_{g,i} \right) - B_g \left( W_p + W_M \right) \left( R_w - R_{sw,i} \right) \\
+ \left( W_F + W_M \right) \left( B_w - B_{w,i} \right)
\]  

(8.43)

The amount of current adsorbed gas, \( G_{A,i} \), can be expressed using Langmuir sorption equation as:

\[
G_{A,i} = 1359.7 Ah \bar{\rho}_b V_L \left( \frac{p}{p_L + p} \right)
\]  

(8.44)

where \( \rho_b \) is the bulk density of coal in g/cc, \( V_L \) is Langmuir volume in SCF/ton and \( p_L \) is Langmuir pressure in psi. Also, the amount of original adsorbed gas, \( G_{A,o} \), can be expressed as:

\[
G_{A,o} = 1359.7 Ah \bar{\rho}_b G_c
\]  

(8.45)

where \( G_c \) is initial gas content in SCF/ton. For a saturated reservoir, \( G_c \) can be written using Langmuir sorption equation as:

\[
G_c = \tilde{f} V_L \left( \frac{p_i}{p_L + p_i} \right)
\]  

(8.46)

In addition, water compression/expansion can be expressed as:

\[
B_w = B_w [1 + c_w (p_i - p)]
\]  

(8.47)

Dividing Eq.(8.43) by \( B_g \) and substituting Eqs.(8.16), (8.17), (8.21), (8.22), (8.42), (8.44), (8.45), (8.46) and (8.47) into it yields the proposed material balance equation for coalbed methane systems as:
\[
G_p + \frac{B_w W_p}{B_g} - R_{sw} W_p
\]

\[
= -7758 \frac{A_h}{B_g} \left[ \phi_{F,i} \left( e^{e_{r,i}(p-p)} - 1 \right) + \phi_{M,i} \left( e^{e_{r,u}(p-p)} - 1 \right) + f\beta \rho_v V_L \left( \frac{p_i}{p_L + p_i} - \frac{p}{p_L + p} \right) \right]
\]

\[
-1359.7 A_h f\beta \rho_v V_L \left[ \frac{p}{p_L + p} - \left( \frac{p_i}{p_L + p_i} \right) \right]
\]

\[
+ 7758 \frac{A_h}{B_g} \left[ \phi_{F,i} (1 - S_{w,F,i}) + \phi_{M,i} (1 - S_{w,M,i}) \right] \left( \frac{1}{B_{g,i}} - \frac{1}{B_g} \right)
\]

\[
- 7758 \frac{A_h}{B_{w,i}} \left( \phi_{F,i} S_{w,F,i} + \phi_{M,i} S_{w,M,i} \right) \left( R_{sw} - R_{sw,i} \right)
\]

\[
+ 7758 \frac{A_h}{B_g} c_w \left( \phi_{F,i} S_{w,F,i} + \phi_{M,i} S_{w,M,i} \right) \left( p_i - p \right)
\]

Eq. (8.48) can be expressed as a straight-line equation:

\[
y = mx + c
\]  

where

\[
y = G_p + \frac{B_w W_p}{B_g} - R_{sw} W_p
\]  

\[
x =
\]

\[
-7758 \frac{1}{B_g} \left[ \phi_{F,i} \left( e^{e_{r,i}(p-p)} - 1 \right) + \phi_{M,i} \left( e^{e_{r,u}(p-p)} - 1 \right) + f\beta \rho_v V_L \left( \frac{p_i}{p_L + p_i} - \frac{p}{p_L + p} \right) \right]
\]

\[
-1359.7 f\beta \rho_v V_L \left[ \frac{p}{p_L + p} - \left( \frac{p_i}{p_L + p_i} \right) \right]
\]

\[
+ 7758 \left( 1 - S_{w,F,i} \right) + \phi_{M,i} \left( 1 - S_{w,M,i} \right) \left( \frac{1}{B_{g,i}} - \frac{1}{B_g} \right)
\]

\[
+ 7758 \frac{1}{B_g} c_w \left( \phi_{F,i} S_{w,F,i} + \phi_{M,i} S_{w,M,i} \right) \left( p_i - p \right)
\]

Plotting the variables \( y \) vs. \( x \) using field production data would yield a straight line with

\[
m = Ah
\]
and

\[ c = -7758 \frac{Ah}{B_{w,j}} \left( \phi_{F,j} S_{w,F,j} + \phi_{M,j} S_{w,M,j} \right) \left( R_{w} - R_{sw,j} \right) \]  \hspace{1cm} (8.53)

From the slope, \( m \), we can obtain the size of the reservoir in acre-ft and the drainage area can be calculated as:

\[ A = \frac{m}{h} \]  \hspace{1cm} (8.54)

The original gas in place (OGIP) and water in place (OWIP) can be calculated using the size of reservoir, \( Ah \), obtained from the slope.

### 8.2 Comparative Analysis of the Proposed and the Existing Material Balance Equations

The proposed material balance equation is compared to material balance equations developed by King (1993) and Ahmed et al. (2006). All three material balance equations share similar assumptions as all of them account for free gas, adsorbed gas and water and rock expansion/compression; however, the effects of water presence in the coal matrix and the coal shrinkage and swelling effects are not incorporated in the two existing material balance equations. In this part of the study, four comparative cases are conducted to determine the effects of water presence in the coal matrix and that of the coal shrinkage and swelling. Each material balance equation is used to predict the size of coalbed methane reservoir using numerically-simulated production data generated from the proposed dual-porosity, dual-permeability CBM reservoir model.
No effects of water presence in the coal matrix and coal shrinkage and swelling

The first set of comparison is performed under no effects of the water presence in the coal matrix and the coal shrinkage and swelling to validate the proposed material balance equation against material balance equations of King (1993) and Ahmed et al. (2006). Table 8-1 presents the reservoir properties used in this comparison. Figure 8-1 demonstrates the comparison of the proposed and existing material balance equations used in predicting the size of reservoir. A linear regression model is used to fit each curve and determine the slope and intercept. Table 8-2 summarizes the actual and predicted sizes of reservoir. It can be observed that the predictions of the reservoir size obtained using the three aforementioned material balance equations are in good agreement. In addition, all the proposed and existing material balance equations provided satisfactory results with error margin less than 2%.

Table 8-1: Reservoir properties used in the comparative study of the proposed and existing material balance equations without considering the effects of water in the coal matrix and coal shrinkage and swelling

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir area</td>
<td>573.9 acres</td>
</tr>
<tr>
<td>Reservoir thickness</td>
<td>20 ft</td>
</tr>
<tr>
<td>Fracture porosity</td>
<td>0.1%</td>
</tr>
<tr>
<td>Matrix porosity</td>
<td>3%</td>
</tr>
<tr>
<td>Coal density</td>
<td>1.435 g/cm³</td>
</tr>
<tr>
<td>Initial pressure</td>
<td>1,100 psia</td>
</tr>
<tr>
<td>Initial water saturation, fracture</td>
<td>0%</td>
</tr>
<tr>
<td>Initial water saturation, matrix</td>
<td>0%</td>
</tr>
<tr>
<td>Reservoir temperature</td>
<td>80 °F</td>
</tr>
<tr>
<td>Langmuir’s volume constant</td>
<td>526.3 SCF/ton</td>
</tr>
<tr>
<td>Langmuir’s pressure constant</td>
<td>535.3 psi</td>
</tr>
<tr>
<td>Formation compressibility</td>
<td>$6 \times 10^{-6}$ psi$^{-1}$</td>
</tr>
<tr>
<td>Water compressibility</td>
<td>$3 \times 10^{-6}$ psi$^{-1}$</td>
</tr>
<tr>
<td>Matrix shrinkage-swelling coefficient ($\beta$)</td>
<td>0.0 ft/SCF</td>
</tr>
</tbody>
</table>
Table 8-2: Predictions of reservoir size using the proposed and existing material balance equations

<table>
<thead>
<tr>
<th>Method</th>
<th>Size of reservoir (acre-ft)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual size</td>
<td>11,478</td>
<td>-</td>
</tr>
<tr>
<td>Proposed model</td>
<td>11,281</td>
<td>1.72</td>
</tr>
<tr>
<td>Ahmed et al. (2006)</td>
<td>11,277</td>
<td>1.75</td>
</tr>
<tr>
<td>King (1993)</td>
<td>11,277</td>
<td>1.75</td>
</tr>
</tbody>
</table>

Figure 8-1: Comparison of the proposed and existing material balance equations used in predicting the size of reservoir

Effects of water presence in the coal matrix

The second set of comparison between the material balance equations for CBM systems is conducted to study the effects of water presence in the coal matrix. Table 8-3 shows the reservoir properties used in this comparison. Figure 8-2 illustrates the comparison of the proposed and existing material balance equations used in predicting the size of reservoir. Each curve was fitted with a straight line and slopes and intercepts were obtained using linear regression. Table 8-
summarizes the actual and predicted sizes of the coalbed methane reservoir under consideration. It is observed that all three material balance equations under-predicted the size of the reservoir, particularly Ahmed et al. and King’s material balance equations. However, the prediction from the material balance equation proposed in this study is considered to be more accurate due to its smaller error of 1.79%. The Ahmed et al.’s and King’s material balance equations, which do not incorporate the presence of water in the coal matrix, both produced similar results with an error of 15.02%. As the presence of water in the coal matrix is ignored, the volume changes of free gas in the coal matrix, which is represented by the third term of the right-hand side of the proposed material balance equation (Eq.(8.48)), is over-predicted. This caused the slopes of the straight lines in Figure 8-2 predicted by the existing material balance equations to drop resulting in an under-prediction of the reservoir size.

Table 8-3: Reservoir properties used in the comparative study of the proposed and existing material balance equations on the effects of water in the coal matrix

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir area</td>
<td>573.9 acres</td>
</tr>
<tr>
<td>Reservoir thickness</td>
<td>20 ft</td>
</tr>
<tr>
<td>Fracture porosity</td>
<td>0.1%</td>
</tr>
<tr>
<td>Matrix porosity</td>
<td>3%</td>
</tr>
<tr>
<td>Coal density</td>
<td>1.435 g/cm³</td>
</tr>
<tr>
<td>Initial pressure</td>
<td>1,100 psia</td>
</tr>
<tr>
<td>Initial water saturation, fracture</td>
<td>0%</td>
</tr>
<tr>
<td>Initial water saturation, matrix</td>
<td>80%</td>
</tr>
<tr>
<td>Reservoir temperature</td>
<td>80 °F</td>
</tr>
<tr>
<td>Langmuir’s volume constant</td>
<td>526.3 SCF/ton</td>
</tr>
<tr>
<td>Langmuir’s pressure constant</td>
<td>535.3 psi</td>
</tr>
<tr>
<td>Formation compressibility</td>
<td>6×10⁻⁶ psi⁻¹</td>
</tr>
<tr>
<td>Water compressibility</td>
<td>3×10⁻⁶ psi⁻¹</td>
</tr>
<tr>
<td>Matrix shrinkage-swelling coefficient (β)</td>
<td>0.0 ft³/SCF</td>
</tr>
</tbody>
</table>
Table 8-4: Predictions of reservoir size using the proposed and existing material balance equations

<table>
<thead>
<tr>
<th>Method</th>
<th>Size of reservoir (acre-ft)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual size</td>
<td>11,478</td>
<td>-</td>
</tr>
<tr>
<td>Proposed model</td>
<td>11,290</td>
<td>1.79</td>
</tr>
<tr>
<td>Ahmed et al. (2006)</td>
<td>9,754</td>
<td>15.02</td>
</tr>
<tr>
<td>King (1993)</td>
<td>9,754</td>
<td>15.02</td>
</tr>
</tbody>
</table>

Figure 8-2: Comparison of the proposed and existing material balance equations used in predicting the size of reservoir (effects of water in the coal matrix)

Effects of coal shrinkage and swelling

The third set of comparison between the proposed and existing material balance equations emphasizes on the effects of coal shrinkage and swelling. Table 8-5 shows the reservoir properties used in this comparison. The coal shrinkage and swelling model described in Chapter 5 was implemented. Water saturation in the matrix was set to zero to eliminate the effects of water in the coal matrix. Figure 8-3 shows the comparison of the proposed and existing material
balance equations used in predicting the size of the reservoir. Linear regression was used to obtain the slope of each curve, which is later used in determining the size of the coalbed methane reservoir. Table 8-6 summarizes the actual and predicted sizes of the reservoir under consideration. The best prediction was obtained from the proposed material balance equation with an error of 2.00%. The Ahmed et al.’s and King’s material balance equations yielded errors of 4.27% and 3.33%, respectively. Although the Ahmed et al.’s and King’s material balance equations take into account rock expansion during pressure drawdown, excluding the effect of the release of gas in the coal matrix, which causes an increase in coal porosity, resulted in an under-prediction of reservoir size. As the changes in porosity due to the release of gas in the coal matrix is ignored, the right-hand side of the material balance equations (Eq.(4.48)), which represents the volume changes in the coal matrix, is over-predicted. This caused a decrease in the slope of the straight line as generated by the proposed material balance equation (see Figure 8-3) resulting in an under-prediction of the reservoir size.

Table 8-5: Reservoir properties used in the comparative study of the proposed and existing material balance equations on the effects of coal shrinkage and swelling

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir area</td>
<td>573.9 acres</td>
</tr>
<tr>
<td>Reservoir thickness</td>
<td>20 ft</td>
</tr>
<tr>
<td>Fracture porosity</td>
<td>0.1%</td>
</tr>
<tr>
<td>Matrix porosity</td>
<td>3%</td>
</tr>
<tr>
<td>Coal density</td>
<td>1.435 g/cm³</td>
</tr>
<tr>
<td>Initial pressure</td>
<td>1,100 psia</td>
</tr>
<tr>
<td>Initial water saturation, fracture</td>
<td>0%</td>
</tr>
<tr>
<td>Initial water saturation, matrix</td>
<td>0%</td>
</tr>
<tr>
<td>Reservoir temperature</td>
<td>80 ºF</td>
</tr>
<tr>
<td>Langmuir’s volume constant</td>
<td>526.3 SCF/ton</td>
</tr>
<tr>
<td>Langmuir’s pressure constant</td>
<td>535.3 psi</td>
</tr>
<tr>
<td>Water compressibility</td>
<td>3×10⁻⁶ psi⁻¹</td>
</tr>
<tr>
<td>Matrix shrinkage-swelling coefficient (β)</td>
<td>0.00012 ft²/SCF</td>
</tr>
<tr>
<td>Young’s modulus (E)</td>
<td>600,000 psi</td>
</tr>
</tbody>
</table>

Note: Rock compressibility was calculated using Eq.(5.4).
Table 8-6: Predictions of reservoir size using the proposed and existing material balance equations

<table>
<thead>
<tr>
<th>Method</th>
<th>Size of reservoir (acre-ft)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual size</td>
<td>11,478</td>
<td>-</td>
</tr>
<tr>
<td>Proposed model</td>
<td>11,248</td>
<td>2.00</td>
</tr>
<tr>
<td>Ahmed et al. (2006)</td>
<td>10,988</td>
<td>4.27</td>
</tr>
<tr>
<td>King (1993)</td>
<td>11,096</td>
<td>3.33</td>
</tr>
</tbody>
</table>

Simultaneous effects of water presence in the coal matrix and coal shrinkage/swelling phenomena

From the previous two sets of comparison, it was found that ignoring each of the two phenomena including the effects of water presence in the coal matrix and coal shrinkage/swelling resulted in under-predictions of the reservoir size. The fourth set of comparison was conducted to analyze the effects of both phenomena simultaneously. Table 8-7 shows the reservoir properties used in this comparison. Water saturation in the matrix was set to 80% and the coal shrinkage and
swelling model was implemented. Figure 8-4 shows the comparison of the proposed and existing material balance equations used in predicting the size of the reservoir. Table 8-8 summarizes the actual and predicted sizes of reservoir. The best prediction was obtained from the proposed material balance equation with an error margin of 2.23%. The Ahmed et al.’s and King’s material balance equations provided approximate errors of 17%. In addition, the results agree with those from the previous two sets of comparison as both Ahmed et al.’s and King’s material balance equations under-predicted the reservoir size and provided larger errors in this set of comparison than the previous two where the effects of each of the two phenomena were individually studied. It can be deduced that ignoring both water presence in the coal matrix and coal shrinkage and swelling phenomena could provide even poorer predictions of the reservoir size.

Table 8-7: Reservoir properties used in the comparative study of the proposed and existing material balance equations on the effects of water in the coal matrix and coal shrinkage/swelling

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir area</td>
<td>573.9 acres</td>
</tr>
<tr>
<td>Reservoir thickness</td>
<td>20 ft</td>
</tr>
<tr>
<td>Fracture porosity</td>
<td>0.1%</td>
</tr>
<tr>
<td>Matrix porosity</td>
<td>3%</td>
</tr>
<tr>
<td>Coal density</td>
<td>1.435 g/cm³</td>
</tr>
<tr>
<td>Initial pressure</td>
<td>1,100 psia</td>
</tr>
<tr>
<td>Initial water saturation, fracture</td>
<td>0%</td>
</tr>
<tr>
<td>Initial water saturation, matrix</td>
<td>80%</td>
</tr>
<tr>
<td>Reservoir temperature</td>
<td>80 °F</td>
</tr>
<tr>
<td>Langmuir’s volume constant</td>
<td>526.3 SCF/ton</td>
</tr>
<tr>
<td>Langmuir’s pressure constant</td>
<td>535.3 psi</td>
</tr>
<tr>
<td>Water compressibility</td>
<td>$3 \times 10^{-6}$ psi$^{-1}$</td>
</tr>
<tr>
<td>Matrix shrinkage-swelling coefficient ($\beta$)</td>
<td>0.00012 ft$^3$/SCF</td>
</tr>
<tr>
<td>Young’s modulus ($E$)</td>
<td>600,000 psi</td>
</tr>
</tbody>
</table>

Note: Rock compressibility was calculated using Eq.(5.4).
Table 8-8: Predictions of reservoir size using the proposed and existing material balance equations

<table>
<thead>
<tr>
<th>Method</th>
<th>Size of reservoir (acre-ft)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual size</td>
<td>11,478</td>
<td>-</td>
</tr>
<tr>
<td>Proposed model</td>
<td>11,222</td>
<td>2.23</td>
</tr>
<tr>
<td>Ahmed et al. (2006)</td>
<td>9,476.2</td>
<td>17.44</td>
</tr>
<tr>
<td>King (1993)</td>
<td>9,497.8</td>
<td>17.25</td>
</tr>
</tbody>
</table>

Figure 8-4: Comparison of the proposed and existing material balance equations used in predicting the size of reservoir (effects of water presence in the coal matrix and coal shrinkage/swelling)

8.3 Applications

This section demonstrates a number of reservoir engineering techniques using the proposed material balance equation that can be implemented to perform reserves and production performance predictions. The first application of the proposed material balance equation is reserve estimation using a straight-line method, which is presented in the previous section. This
method is straightforward and undemanding as no iterative method is required. Further applications of the proposed material balance equation include the predictions of production performance and average reservoir pressure. The prediction techniques involve using the Newton-Raphson iterative method, which initiated by Ahmed et al. (2006).

8.3.1 Predictions of Production Performance

Methodology

Production forecasting is considered one of the most critical tasks in field development studies. In this section, an iterative method is developed using the proposed material balance equation, which accounts for the presence of water in the coal matrix, and coal shrinkage and swelling effects, to predict production performance. For simplicity, solution gas in the water phase is considered negligible. The development of production performance predictions begins by differentiating the proposed material balance equation (Eq.(8.47)) with respect to pressure, $p$. 
\[
\frac{\partial}{\partial p}\left( G_p + \frac{B_w W_p}{B_s} \right) \\
= 7758 Ah \left[ \phi_{F,j} \left( 1 - e^{e_{p,u}(p-p_i)} \right) + \phi_{M,j} \left( 1 - e^{e_{p,M}(p-p_i)} \right) - \tilde{f} \beta \rho_b V_l \left( \frac{p_i}{p_i + p} - \frac{p}{p + p_L} \right) \right] \frac{\partial}{\partial p} \left( \frac{1}{B_s} \right) \\
- 7758 Ah \left[ \frac{\phi_{F,j} C_p e_{p,F}(p-p_i)}{B_s} + \frac{\phi_{M,j} C_p e_{p,M}(p-p_i)}{B_s} - \tilde{f} \beta \rho_b V_l \frac{\partial}{\partial p} \left( \frac{p}{p_i + p} \right) \right] \\
- 1359.7 Ah \rho_b V_l \frac{\partial}{\partial p} \left( \frac{p}{p_L + p} \right) \\
- 7758 Ah \left( \phi_{F,j} \left( 1 - S_{w,F,i} \right) + \phi_{M,j} \left( 1 - S_{w,M,i} \right) \right) \frac{\partial}{\partial p} \left( \frac{1}{B_s} \right) \\
+ 7758 Ah \left[ \frac{C_w}{B_s} \left( \phi_{F,j} S_{w,F,i} + \phi_{M,j} S_{w,M,i} \right) \left( p - p \right) \right] \frac{\partial}{\partial p} \left( \frac{1}{B_s} \right) \\
- 7758 Ah \left[ \frac{C_w}{B_s} \left( \phi_{F,j} S_{w,F,i} + \phi_{M,j} S_{w,M,i} \right) \right]
\]

(8.55)

Finite-difference approximation of Eq.(8.55) can be written as:
\[
\left( G_p^{n+1} + \frac{B_w^{n+1}W_p^{n+1}}{B_g^{n+1}} \right) = \left( G_p^n + \frac{B_w^nW_p^n}{B_g^n} \right)
+ 7758Ah \left[ \phi_{F,i} \left( 1 - e^{c_{r,F}(p^{n+1} - p_i)} \right) + \phi_{M,i} \left( 1 - e^{c_{r,M}(p^{n+1} - p_i)} \right) - \tilde{f} \beta \rho_b V_L \left( \frac{p_i}{p_L + p_i} - \frac{p^{n+1}}{p_L + p^{n+1}} \right) \right] \left( \frac{1}{B_g^{n+1}} - \frac{1}{B_g^n} \right)
- 7758Ah \left[ \frac{\phi_{F,i}c_{r,F}}{B_g^{n+1}} \left( e^{c_{r,F}(p^{n+1} - p_i)} \right) + \frac{\phi_{M,i}c_{r,M}}{B_g^{n+1}} \left( e^{c_{r,M}(p^{n+1} - p_i)} \right) - \tilde{f} \beta \rho_b V_L \left( \frac{p_L}{p_L + p^{n+1}} \right)^2 \right] \left( p^{n+1} - p^n \right)
- 1359.7Ah \tilde{f} \rho_b V_L p_L \left( \frac{p^{n+1} - p^n}{p_L + p^{n+1}} \right)^2
- 7758Ah \left( \phi_{F,i} \left( 1 - S_{w,F,i} \right) + \phi_{M,i} \left( 1 - S_{w,M,i} \right) \right) \left( \frac{1}{B_g^{n+1}} - \frac{1}{B_g^n} \right)
+ 7758Ah \left[ c_w \left( \phi_{F,i}S_{w,F,i} + \phi_{M,i}S_{w,M,i} \right) (p_i - p^{n+1}) \right] \left( \frac{1}{B_g^{n+1}} - \frac{1}{B_g^n} \right)
- 7758Ah \left[ \frac{c_w}{B_g^{n+1}} \left( \phi_{F,i}S_{w,F,i} + \phi_{M,i}S_{w,M,i} \right) \right] \left( p^{n+1} - p^n \right)
\]

where superscript \( n+1 \) and \( n \) represent future and current time levels, respectively. Eq.(8.56) can be used to solve for future cumulative gas production, \( G_p^{n+1} \), and future cumulative water production, \( W_p^{n+1} \), using the following iterative method.

(1) Select future reservoir pressure, \( p^{n+1} \).

(2) Estimate \( W_p^{n+1} \) by guessing average water saturation, \( S_w^{(k)} \).

\[
W_p^{(k)} = 7758 \frac{Ah}{B_w^{n+1}} \left( \phi_{F,i}S_{w,F,i} + \phi_{M,i}S_{w,M,i} \right) (1 + c_w \left( p_i - p^{n+1} \right))
- 7758 \frac{Ah}{B_w^{n+1}} \left[ \phi_{F,i}e^{c_{r,F}(p^{n+1} - p_i)} + \phi_{M,i}e^{c_{r,M}(p^{n+1} - p_i)} + \tilde{f} \beta \rho_b V_L \left( \frac{p_i}{p_L + p_i} - \frac{p^{n+1}}{p_L + p^{n+1}} \right) \right] S_w^{(k)}
\]

\[(8.57)\]
where the following assumption is made:

\[
S_w^{(k)}_{n+1} = S_w^{n+1} = S_w^{n+1}_{F} = S_w^{n+1}_{M} \tag{8.58}
\]

This assumption is considered reasonable even for the systems where initial water saturation in the fracture and that in the matrix are different since they equilibrate rapidly due to gas diffusion.

(3) Calculate \( G_p^{(k) n+1} \) using the relationship between \( G_p^{(k) n+1} \) and \( W_p^{(k) n+1} \)

\[
G_p^{(k) n+1} = G_p^n + \left( \frac{GWR_{n+1}^{(k)} + GWR^n}{2} \right) \left( W_p^{(k) n+1} + W_p^n \right) \tag{8.59}
\]

where gas-water ratio can be calculated using Eq.(8.60).

\[
GWR = \frac{k_{rg}}{k_{rw}} \left( \frac{\mu_w B_w}{\mu_g B_g} \right) \tag{8.60}
\]

(4) Calculate \( G_p^{(k) n+1} \) using the proposed material balance equation (Eq.(8.56)).
\[
G_p^{(k)} \bigg|_{II} = - \left( \frac{B_w^{n+1} W_p^{(k)}}{B_g^{n+1}} \right) + \left( G_p^n + \frac{B_w^n W_p^n}{B_g^n} \right) \\
+ 7758Ah \left[ \phi_{F,i} \left( 1 - e^{-c_{F,i} (p^{n+1} - p_i)} \right) + \phi_{M,i} \left( 1 - e^{-c_{M,i} (p^{n+1} - p_i)} \right) \right] - \tilde{f} \beta p_b V_L \left( \frac{p_i}{p_L + p_i} - \frac{p^{n+1}}{p_L + p^{n+1}} \right) \left( \frac{1}{B_g^{n+1}} - \frac{1}{B_g^n} \right) \\
- 7758Ah \left[ \frac{\phi_{F,i} C_{p,F}}{B_g^{n+1}} \left( e^{-c_{F,i} (p^{n+1} - p_i)} \right) + \phi_{M,i} C_{p,M} \left( e^{-c_{M,i} (p^{n+1} - p_i)} \right) \right] - \tilde{f} \beta p_b V_L \frac{p_L}{p_L + p^{n+1}} \left( p^{n+1} - p^n \right) \\
- 1359.7 Ah \tilde{f} \rho_b V_L p_L \left( \frac{p^{n+1} - p^n}{p_L + p^{n+1}} \right)^2 \\
- 7758Ah \left[ \phi_{F,i} \left( 1 - S_{w,F,i} \right) + \phi_{M,i} \left( 1 - S_{w,M,i} \right) \right] \left( \frac{1}{B_g^{n+1}} - \frac{1}{B_g^n} \right) \\
+ 7758Ah \left[ \frac{C_w}{B_g^{n+1}} \phi_{F,i} S_{w,F,i} + \phi_{M,i} S_{w,M,i} \right] \left( p_i - p^{n+1} \right) \left( \frac{1}{B_g^{n+1}} - \frac{1}{B_g^n} \right) \\
- 7758Ah \left[ \frac{C_w}{B_g^{n+1}} \phi_{F,i} S_{w,F,i} + \phi_{M,i} S_{w,M,i} \right] \left( p^{n+1} - p^n \right) \\
\] (8.61)

(5) From steps (3) and (4), the future cumulative gas production values \[ G_p^{n+1} \bigg|_{II} \] and \[ G_p^{n+1} \bigg|_{II} \] are calculated using two independent equations including Eq.(8.59) and (8.61). The error function, \( F \), is formed as the difference between the values of \[ G_p^{n+1} \bigg|_{II} \] and \[ G_p^{n+1} \bigg|_{II} \].

\[
F \left( \overline{S}_w^{n+1} \right) = G_p^{(k)} \bigg|_{II} - G_p^{(k)} \bigg|_{II} = 0 \] (8.62)

Eq.(8.62) can be solved for \( \overline{S}_w^{n+1} \) using the Newton-Raphson protocol as follows:

- If the value of \( F \) is smaller than the tolerance, \( \varepsilon \), the values of \( \overline{S}_w^{n+1} \) and \( G_p^{(k)} \) are satisfactory and we can continue with future reservoir pressure in step (1).
• If not, repeat steps (2), (3), (4) and (5) with the new $S_w^{(k)}$ obtained using the following Newton-Raphson expression:

$$\frac{S_w^{(k+1)}}{S_w^{(k+1)}} = \frac{S_w^{(k+1)}}{\partial F(S_w^{(k+1)})} - \frac{F(S_w^{(k+1)})}{\partial S_w^{(k+1)}}$$

(8.63)

where

$$F(S_w^{(k+1)}) = \left(\frac{GWR^{n+1} + GWR^n}{2}\right)\left(W_p^{n+1} + W_p^n\right)$$

$$+ \left(\frac{B_w^{n+1}W_p^{(k+1)}}{B_g^{n+1}}\right) - \left(G^n_p + \frac{B_w^nW_p^n}{B_g^n}\right)$$

$$- 7758Ah \left[\phi_{F,i}\left(1 - e^{-e_{r,s}(p^{n+1} - p_i)}\right) + \phi_{M,i}\left(1 - e^{-e_{r,m}(p^{n+1} - p_i)}\right) - \tilde{f}_{Bp_vL}\left(\frac{p_i}{p_L} + \frac{p_{n+1}}{p_L + p^{n+1}}\right)\right] \left(\frac{1}{B_g^{n+1}} - \frac{1}{B_g^n}\right)$$

$$\left(\frac{1}{B_g^{n+1}} - \frac{1}{B_g^n}\right)$$

$$+ 1359.7Ah \tilde{f}_{Bp_vL} p_{n+1} - p^n \left(\frac{B_g^n}{B_g^{n+1}}\right)^2$$

$$+ 7758Ah \left(\phi_{F,i}(1 - S_{w,F,i}) + \phi_{M,i}(1 - S_{w,M,i})\right) \left(\frac{1}{B_g^{n+1}} - \frac{1}{B_g^n}\right)$$

$$- 7758Ah \left[c_w(\phi_{F,i}S_{w,F,i} + \phi_{M,i}S_{w,M,i})\right] \left(\frac{1}{B_g^{n+1}} - \frac{1}{B_g^n}\right)$$

$$+ 7758Ah \left[c_w^{(k)}(\phi_{F,i}S_{w,F,i} + \phi_{M,i}S_{w,M,i})\right] \left(p^{n+1} - p^n\right) = 0$$

(8.64)
\[ \frac{\partial F\left(S_{w}^{(k)}\right)}{\partial S_{w}^{(k)}} = \frac{F\left(S_{w}^{(k)}\right) - F\left(S_{w}^{(k-1)}\right)}{S_{w}^{(k)} - S_{w}^{(k-1)}} \]  

(8.65)

Validation of the Methodology

The proposed iterative methodology for gas and water production predictions was validated against the dual-porosity, dual-permeability coalbed methane numerical simulator presented in Chapter 4. Table 8-9 and Figures 7-2 to 7-3 summarize the reservoir properties used in the validation test. Figures 8-4 and 8-5 show good matches of cumulative gas production and cumulative water production, respectively. Figure 8-6 shows the comparison of water saturation in the fracture and that in the matrix from the numerical simulation and average water saturation from the proposed material balance equation. This confirms that the assumption in Eq.(8.58) is practical. This is evident as the values of water saturation in the fracture and matrix become close to each other after a short period of time and stay comparable throughout the entire production period.
Table 8-9: Reservoir properties used for the validation of production performance

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir size</td>
<td>573.9 acres</td>
</tr>
<tr>
<td>Reservoir thickness</td>
<td>20 ft</td>
</tr>
<tr>
<td>Fracture porosity</td>
<td>0.1%</td>
</tr>
<tr>
<td>Matrix porosity</td>
<td>3%</td>
</tr>
<tr>
<td>Fracture permeability</td>
<td>8 md</td>
</tr>
<tr>
<td>Matrix permeability</td>
<td>0.0001 md</td>
</tr>
<tr>
<td>Coal density</td>
<td>1.435 g/cm$^3$</td>
</tr>
<tr>
<td>Initial pressure</td>
<td>1,100 psia</td>
</tr>
<tr>
<td>Fracture spacing</td>
<td>1 inch</td>
</tr>
<tr>
<td>Initial water saturation, fracture</td>
<td>50%</td>
</tr>
<tr>
<td>Initial water saturation, matrix</td>
<td>95%</td>
</tr>
<tr>
<td>Reservoir temperature</td>
<td>80 °F</td>
</tr>
<tr>
<td>Langmuir’s volume constant</td>
<td>526.3 SCF/ton</td>
</tr>
<tr>
<td>Langmuir’s pressure constant</td>
<td>535.3 psi</td>
</tr>
<tr>
<td>Water compressibility</td>
<td>$3 \times 10^{-6}$ psi$^{-1}$</td>
</tr>
<tr>
<td>Matrix shrinkage-swelling coefficient ($\beta$)</td>
<td>0.00012 ft$^3$/SCF</td>
</tr>
<tr>
<td>Young’s modulus ($E$)</td>
<td>445000 psi</td>
</tr>
</tbody>
</table>

Figure 8-5: Validation of cumulative gas production prediction of the proposed material balance equation
Figure 8-6: Validation of cumulative water production prediction of the proposed material balance equation

Figure 8-7: Comparison of fracture and matrix saturation from the numerical simulation and the average water saturation from the proposed material balance equation
8.3.2 Prediction of Reservoir Pressure

Methodology

Reservoir pressure is an important parameter that can be used in reservoir management and optimization. A development for average reservoir pressure prediction using the proposed material balance equation and historical gas and water production data is presented next. The proposed material balance equation is evaluated as function, \( F \), below:

\[
F(p) = G_p + \frac{B_w W_p}{B_g} + 7758 \frac{Ah}{B_g} \left[ \phi_{F,j} \left( e^{\phi_{r,L} (p - p_i)} - 1 \right) + \phi_{M,j} \left( e^{\phi_{r,M} (p - p_i)} - 1 \right) + \tilde{f} \beta \rho_b V_L \left( \frac{p_i}{p_L + p_i} - \frac{p}{p_L + p} \right) \right] \\
+ 1359.7 Ah \tilde{f} \rho_b V_L \left[ \left( \frac{p}{p_L + p} \right) - \left( \frac{p_i}{p_L + p_i} \right) \right] \\
- 7758 Ah \phi_{F,j} \left( \frac{1}{B_{g,i} B_{g,j}} \right) + \phi_{M,j} \left( 1 - S_{w,M,j} \right) \left( \frac{1}{B_{g,i} B_{g,j}} - \frac{1}{B_{g,j}} \right) \\
- 7758 \frac{Ah}{B_g} c_w \phi_{r,M} S_{w,F,j} + \phi_{r,M} S_{w,M,j} \left( p_i - p \right)
\]

(8.66)

Eq.(8.66) can be solved for average reservoir pressure using the Newton-Raphson iterative method. The Newton-Raphson expression can be written as:

\[
p^{k+1} = p^k - \frac{F(p)}{\partial F(p)}
\]

(8.67)
where

\[ F(p) = W_{p} \left[ - \frac{B_{w} c_{w}}{B_{g}} + B_{w} \cdot 5.615 \frac{Z_{sc} T_{sc}}{p_{sc} T} \left( \frac{1}{Z} - \frac{p}{Z^2} \frac{\partial Z}{\partial p} \right) \right] \]

\[ - 7758 Ah \left[ \phi_{F,i} \left( 1 - e^{\varepsilon_{p,F} (p-p_i)} \right) + \phi_{M,i} \left( 1 - e^{\varepsilon_{p,M} (p-p_i)} \right) \right] \cdot 5.615 \frac{Z_{sc} T_{sc}}{p_{w} T} \left( \frac{1}{Z} - \frac{p}{Z^2} \frac{\partial Z}{\partial p} \right) \]

\[ + 7758 Ah \left[ \tilde{f} \beta p_{b} V_{L} \left( \frac{p_i}{p_L + p_i} - \frac{p}{p_L + p} \right) \right] \cdot 5.615 \frac{Z_{sc} T_{sc}}{p_{sc} T} \left( \frac{1}{Z} - \frac{p}{Z^2} \frac{\partial Z}{\partial p} \right) \]

\[ + 7758 Ah \left[ \frac{\phi_{F,i} c_{p,F} (e^{\varepsilon_{p,F} (p-p_i)}) + \phi_{M,i} c_{p,M} (e^{\varepsilon_{p,M} (p-p_i)}) - \tilde{f} \beta p_{b} V_{L} \frac{p_L}{(p_L + p)^2} \right] \]

\[ + 1359.7 Ah \beta p_{b} V_{L} \frac{p_L}{(p_L + p)^2} \]

\[ + 7758 Ah \left( \phi_{F,i} \left( 1 - S_{w,F,i} \right) + \phi_{M,i} \left( 1 - S_{w,M,i} \right) \right) \cdot 5.615 \frac{Z_{sc} T_{sc}}{p_{sc} T} \left( \frac{1}{Z} - \frac{p}{Z^2} \frac{\partial Z}{\partial p} \right) \]

\[ - 7758 Ah \left[ c_{w} \left( \phi_{F,i} S_{w,F,i} + \phi_{M,i} S_{w,M,i} \right) (p_i - p) \right] \cdot 5.615 \frac{Z_{sc} T_{sc}}{p_{w} T} \left( \frac{1}{Z} - \frac{p}{Z^2} \frac{\partial Z}{\partial p} \right) \]

\[ + 7758 Ah \left[ \frac{c_{w}}{B_{g}} \left( \phi_{F,i} S_{w,F,i} + \phi_{M,i} S_{w,M,i} \right) \right] \]

\[ \text{(8.68)} \]

and

\[ \frac{\partial Z}{\partial p} = \frac{Z(p^k) - Z(p^{k-1})}{p^k - p^{k-1}} \]

\[ \text{(8.69)} \]

**Validation of the Methodology**

In this part of the study, gas and water production data used in this validation were generated using the dual-porosity, dual-permeability coalbed methane numerical simulator and the reservoir properties shown in Table 8-9 and Figures 7-2 to 7-3. The average reservoir pressure from the numerical model is calculated using the pore volume weighted averaging technique as follows:
where $p_R$ is the average reservoir pressure. Figure 8-8 shows a satisfactory match of the average reservoir pressure predicted from the proposed material balance equation and that from the numerical simulation. This technique is considered useful and economical for reservoir engineers as reservoir pressure can simply be estimated by using historical production data without conducting pressure surveys.

![Graph showing validation of cumulative water production prediction of the proposed material balance equation.](image)

Figure 8-8: Validation of cumulative water production prediction of the proposed material balance equation
Chapter 9

Production Type-Curves for Coalbed Methane Reservoirs

Type-curve analysis is one of the fundamental tools in well test analysis to determine reservoir characteristics such as reservoir permeability and reservoir boundaries. There are two applications of type curves: pressure type curves and production type curves. Pressure type curves, which consist of a series of dimensionless pressure drop values plotted against dimensionless time, can be used for pressure transient analysis when a well testing is conducted under a specified constant flow rate and the corresponding pressure response is measured. In contrast, production type curves consist of a series of dimensionless production rate values versus dimensionless time and can be used for decline curve analysis purposes where the well is flowing at constant sandface pressure. Both applications should yield similar accuracy levels as flow rate and pressure are related and controlled by individual reservoir characteristics.

Da Prat et al. (1981) proposed decline curve analysis using type curves for naturally fractured systems. The type curves were generated from partial differential equations for a single phase flow in a one-dimensional, radial, two-porosity system originally developed by Warren and Root (1963) and extended by Mavor and Cinco-Ley (1979).

\[
\frac{\partial^2 p_{JD}}{\partial r_D^2} + \frac{1}{r_D} \frac{\partial p_{JD}}{\partial r_D} = (1 - \omega) \frac{\partial p_{mD}}{\partial t_D} + \omega \frac{\partial p_{fD}}{\partial t_D} \tag{9.1}
\]

where

\[
\omega = \frac{(\phi Vc)_f}{(\phi Vc)_f + (\phi Vc)_w} \tag{9.2}
\]
\( \omega \) is the dimensionless fracture storage parameter. \( V_f \) and \( V_m \) are the ratios of the total volume of fracture system to the bulk volume and that of total volume of matrix system to the bulk volume, respectively. Eq.(9.1) was solved analytically and used to generate decline type curves. In this application, the log-log production type curves were used to determine fracture permeability, \( k_F \), and total storage, \( (\phi V_c)_f + (\phi V_c)_m \).

A type-curve solution was proposed for CBM reservoirs by Mohaghegh and Ertekin (1991). The gas and water transport equations in radial, single-porosity coalbed methane systems proposed by King (1986) were used and combined to derive a single expression, Eq.(9.3), that describe the flow of both phases under constant pressure specification.

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial p^2}{\partial r} \right) = \frac{\phi c_t}{\lambda_t} \frac{\partial p^2}{\partial t} \quad (9.3)
\]

where \( \lambda_t \) is the total system mobility and the amount of adsorbed gas, \( V_a \), is taken into account in the total system compressibility term as follows:

\[
c_t = \frac{\gamma}{\phi} B_g \frac{\partial V_a}{\partial p} = S_g \frac{\partial B_g}{\partial p} - S_w \frac{\partial B_w}{\partial p} \quad (9.4)
\]

The dimensionless equation can be written as:

\[
\frac{\partial^2 p_D}{\partial r_D^2} + \frac{1}{r_D} \frac{\partial p_D}{\partial r_D} = \frac{\partial p_D}{\partial t_D} \quad (9.5)
\]

The type curves were generated using a two-phase, one-dimensional, radial numerical simulator (King, 1986) while sandface pressure was specified. The production type curves appear to be sensitive to ratio of sandface pressure to initial pressure, initial water saturation and reservoir size. This approach was tested to be a reasonable tool for determining permeability and porosity of the system.

Gerami et al. (2008) proposed type curves for dry CBM reservoirs. The model presented below was developed for the single-phase, radial flow using pseudo-pressure approach.
\[
\frac{1}{r} \left( \frac{\partial}{\partial r} \left( r \frac{\partial \psi}{\partial r} \right) \right) = \frac{\phi \mu c^*_g \partial \psi}{k \partial t}
\] (9.6)

The gas desorption term is accounted for in the total compressibility, \( c_t^* \), expressed below:

\[
c_t^* = c_g + \frac{p_{sc} T V_L p_L Z}{T_c \phi(p + p_L)^2}
\] (9.7)

where \( c_g \) is the real gas compressibility. The gas desorption term in Eq.(9.7) was introduced by Bumb and McKee (1988). The solution to the diffusivity equation was obtained by linearizing the non-linear product of viscosity and compressibility term and using the modified pseudo-time.

In addition, Clarkson et al. (2009) proposed a two-phase CBM production type-curve solution for hydraulically-fractured and horizontal wells.

In this study, a dimensionless diffusivity equation that describes the two-phase flow in dual-porosity CBM systems is developed using the gas and water flow equations in the fracture and matrix domains of CBM systems (Eqs.(4.1), (4.2), (4.5) and (4.6)). The proposed dimensionless diffusivity equation combines two applications developed by Da Prat et al. (1981) and Mohaghegh and Ertekin (1991), which account for dual-porosity characteristics and gas sorption in coal seams, respectively. The gas sorption phenomenon is taken into account through the matrix compressibility. The dual-porosity, dual-permeability numerical simulator proposed in Chapter 4 will be used in generating production type curves and testing the methodology.

### 9.1 Formulation

The formulation of the dimensionless equation that describes the flow of gas and water in the dual-porosity, dual-permeability coalbed methane systems is presented here. The following assumptions were made in deriving the dimensionless equation: (1) reservoir exhibits homogeneous and isotropic properties, (2) there is no capillary effect, (3) potential gradients are
negligible, (4) diffusion in the fracture system can be ignored as convection is dominant in the
fracture system, (5) gas and water flow rate terms are treated as inner boundary condition, and (6)
there is no gas dissolved in water phase.

Gas flow equations in fracture and matrix systems can be re-written as:

\[
\nabla \cdot \left[ \frac{k_F k_{rg,F}}{\mu_g B_{g,F}} \nabla p_F \right] + \frac{\Gamma_{g,F}}{V_b} = \frac{1}{5.615 \partial t} \left[ \phi_F S_{g,F} \right]
\]

(9.8)

\[
\frac{\Gamma_{g,M}}{V_b} - \frac{q_{srp}}{V_b} = \frac{1}{5.615 \partial t} \left[ \phi_M S_{g,M} \right]
\]

(9.9)

where \( q_{srp} = \frac{\partial V_a}{\partial t} \)

Water flow equations in fracture and matrix systems can be re-written as:

\[
\nabla \cdot \left[ \frac{k_F k_{rw,F}}{\mu_w B_{w,F}} \nabla p_F \right] + \frac{\Gamma_{w,F}}{V_b} = \frac{1}{5.615 \partial t} \left[ \phi_F S_{w,F} \right]
\]

(9.11)

\[
\frac{\Gamma_{w,M}}{V_b} = \frac{1}{5.615 \partial t} \left[ \phi_M S_{w,M} \right]
\]

(9.12)

The sorption functional group, \( \psi \), is introduced as:

\[
\psi = 1 + \frac{5.615 V_a B_{g,M}}{V_b \phi_M S_{g,M}}
\]

(9.13)

Substituting Eq.(9.13) into Eq.(9.9) yields:

\[
\frac{\Gamma_{g,M}}{V_b} = \frac{1}{5.615 \partial t} \left[ \psi \phi_M S_{g,M} \right]
\]

(9.14)

Multiplying Eq.(9.8) by \( B_{g,F} \) yields:

\[
B_{g,F} \nabla \cdot \left[ \frac{k_F k_{rg,F}}{\mu_g B_{g,F}} \nabla p_F \right] + B_{g,F} \frac{\Gamma_{g,F}}{V_b} = B_{g,F} \frac{1}{5.615 \partial t} \left[ \phi_F S_{g,F} \right]
\]

(9.15)
Multiplying Eq.(9.14) by $B_{g,M}$ yields:

$$B_{g,M} \frac{\Gamma_{g,M}}{V_b} = \frac{B_{g,M}}{5.615} \frac{\partial}{\partial t} \left[ \frac{\psi \phi_{M} S_{g,M}}{B_{g,M}} \right]$$  \hspace{1cm} (9.16)

Multiplying Eq.(9.11) by $B_{w,F}$ yields:

$$B_{w,F} \nabla \cdot \left[ \frac{k_f k_{w,F}}{\mu_w B_{w,F}} \nabla \cdot p_F \right] + B_{w,F} \frac{\Gamma_{w,F}}{V_b} = \frac{B_{w,F}}{5.615} \frac{\partial}{\partial t} \left[ \frac{\phi_{F} S_{w,F}}{B_{w,F}} \right]$$  \hspace{1cm} (9.17)

Multiplying Eq.(9.12) by $B_{w,M}$ yields:

$$B_{w,M} \frac{\Gamma_{w,M}}{V_b} = \frac{B_{w,M}}{5.615} \frac{\partial}{\partial t} \left[ \frac{\phi_{M} S_{w,M}}{B_{w,M}} \right]$$  \hspace{1cm} (9.18)

Adding Eqs.(9.15), (9.16), (9.17) and (9.18) results in the following equation:

$$B_{g,F} \nabla \cdot \left[ \frac{k_f k_{g,F}}{\mu_g B_{g,F}} \nabla \cdot p_F \right] + B_{w,F} \nabla \cdot \left[ \frac{k_f k_{w,F}}{\mu_w B_{w,F}} \nabla \cdot p_F \right] + B_{g,F} \frac{\Gamma_{g,F}}{V_b} + B_{g,M} \frac{\Gamma_{g,M}}{V_b} + B_{w,F} \frac{\Gamma_{w,F}}{V_b} + B_{w,M} \frac{\Gamma_{w,M}}{V_b}$$

$$= \frac{B_{g,F}}{5.615} \frac{\partial}{\partial t} \left[ \frac{\phi_{F} S_{g,F}}{B_{g,F}} \right] + \frac{B_{g,M}}{5.615} \frac{\partial}{\partial t} \left[ \frac{\psi \phi_{M} S_{g,M}}{B_{g,M}} \right] + \frac{B_{w,F}}{5.615} \frac{\partial}{\partial t} \left[ \frac{\phi_{F} S_{w,F}}{B_{w,F}} \right] + \frac{B_{w,M}}{5.615} \frac{\partial}{\partial t} \left[ \frac{\phi_{M} S_{w,M}}{B_{w,M}} \right]$$  \hspace{1cm} (9.19)

Let us consider the right-hand side (RHS) of Eq.(9.19). Applying chain rule in the right-hand side of Eq.(9.19) yields:

$$\text{RHS} = \frac{B_{g,F}}{5.615} \frac{\partial}{\partial t} \left[ \frac{\phi_{F} S_{g,F}}{B_{g,F}} \right] \frac{\partial p_F}{\partial t} + \frac{B_{g,M}}{5.615} \frac{\partial}{\partial t} \left[ \frac{\psi \phi_{M} S_{g,M}}{B_{g,M}} \right] \frac{\partial p_M}{\partial t}$$

$$+ \frac{B_{w,F}}{5.615} \frac{\partial}{\partial t} \left[ \frac{\phi_{F} S_{w,F}}{B_{w,F}} \right] \frac{\partial p_F}{\partial t} + \frac{B_{w,M}}{5.615} \frac{\partial}{\partial t} \left[ \frac{\phi_{M} S_{w,M}}{B_{w,M}} \right] \frac{\partial p_M}{\partial t}$$  \hspace{1cm} (9.20)
Eq.(9.20) can be expanded as:

\[
RHS = \frac{1}{5.615} \left[ S_{g,F} \frac{\partial \phi_F}{\partial p_F} + \frac{\partial S_{g,F}}{\partial p_F} \frac{B_F}{B_{g,F}} \right] \frac{\partial p_F}{\partial t} 
+ \frac{1}{5.615} \left[ \psi S_{g,M} \frac{\partial \phi_M}{\partial p_M} + \psi \frac{\partial S_{g,M}}{\partial p_M} - \frac{\partial B_{g,M}}{B_{g,F}} \frac{\partial p_F}{\partial p_F} \right] \frac{\partial p_M}{\partial t} 
+ \frac{1}{5.615} \left[ S_{w,F} \frac{\partial \phi_F}{\partial p_F} + \frac{\partial S_{w,F}}{\partial p_F} - \frac{\partial B_{w,F}}{B_{w,F}} \frac{\partial p_F}{\partial p_F} \right] \frac{\partial p_F}{\partial t} 
+ \frac{1}{5.615} \left[ S_{w,M} \frac{\partial \phi_M}{\partial p_M} + \frac{\partial S_{w,M}}{\partial p_M} - \frac{\partial B_{w,M}}{B_{w,M}} \frac{\partial p_M}{\partial p_M} \right] \frac{\partial p_M}{\partial t}
\]

(9.21)

Differentiating Eq.(9.13) with respect to \( p_M \), one obtains:

\[
\frac{\partial \psi}{\partial p_M} = \frac{5.615}{V_b} \left[ \phi_m \frac{\partial S_{g,M}}{\partial p_M} + \frac{\partial \phi_M}{\partial p_M} \frac{B_{g,M}}{B_{g,F}} \frac{\partial p_F}{\partial p_F} - \frac{V_a B_{g,M} S_{g,M}}{\phi_M S_{g,M} \partial p_M} + \frac{V_a}{\phi_M S_{g,M} \partial p_M} \frac{\partial B_{g,M}}{\partial p_M} \right]
\]

(9.22)

Substituting Eq.(9.22) into Eq.(9.21) yields:

\[
RHS = \frac{1}{5.615} \left[ \left( S_{g,F} + S_{w,F} \right) \frac{\partial \phi_F}{\partial p_F} + \phi_F \left( \frac{\partial S_{g,F}}{\partial p_F} + \frac{\partial S_{w,F}}{\partial p_F} \right) - \frac{\partial B_{g,F}}{B_{g,F}} \frac{\partial p_F}{\partial p_F} + S_{w,F} \frac{\partial B_{w,F}}{B_{w,F}} \frac{\partial p_F}{\partial p_F} \right] \frac{\partial p_F}{\partial t} 
+ \frac{5.615}{V_b} \frac{\partial V_a}{\partial p_M} \left( S_{g,M} + S_{w,M} \right) \frac{\partial \phi_M}{\partial p_M} \frac{\partial p_M}{\partial t} 
+ \frac{1}{5.615} \left[ \phi_m \left( \frac{\partial S_{g,M}}{\partial p_M} + \frac{\partial S_{w,M}}{\partial p_M} \right) - \phi_M \left( \frac{\partial B_{g,M}}{B_{g,F}} + \frac{\partial B_{w,M}}{B_{w,M}} \frac{\partial p_M}{\partial p_M} \right) \right] \frac{\partial p_M}{\partial t}
\]

(9.23)

Recall the following saturation constraints:

\[
S_{g,F} + S_{w,F} = 1 \quad (9.24)
\]

\[
S_{g,M} + S_{w,M} = 1 \quad (9.25)
\]

Differentiating Eqs.(9.24) and (9.25) with respect to \( p_F \) and \( p_M \), respectively, one obtains:
\[
\frac{\partial S_{w,F}}{\partial p_F} + \frac{\partial S_{w,F}}{\partial p_M} = 0
\]  
(9.26)

\[
\frac{\partial S_{w,M}}{\partial p_F} + \frac{\partial S_{w,M}}{\partial p_M} = 0
\]  
(9.27)

Substituting Eqs. (9.24), (9.25), (9.26) and (9.27) into Eq. (9.23) results in:

\[
RHS = \frac{1}{5.615} \left[ \frac{\hat{\phi}_F}{\partial p_F} - \phi_F \left( \frac{S_{w,F}}{B_{w,F}} \frac{\hat{B}_{w,F}}{\partial p_F} + \frac{S_{w,F}}{B_{w,F}} \frac{\hat{B}_{w,F}}{\partial p_F} \right) \right] \frac{\partial p_F}{\partial t} + \frac{1}{5.615} \left[ \frac{5.615 B_{w,M} \partial V_a}{\partial p_M} + \frac{\hat{\phi}_M}{\partial p_M} - \phi_M \left( \frac{S_{w,M}}{B_{w,M}} \frac{\hat{B}_{w,M}}{\partial p_M} + \frac{S_{w,M}}{B_{w,M}} \frac{\hat{B}_{w,M}}{\partial p_M} \right) \right] \frac{\partial p_M}{\partial t}
\]  
(9.28)

For simplicity, let us write Eq. (9.28) as follows:

\[
RHS = \phi_F c_F \frac{\partial p_F}{\partial t} + \phi_M c_M \frac{\partial p_M}{\partial t}
\]  
(9.29)

where \(c_F\) and \(c_M\) can be expressed as:

\[
c_F = \frac{1}{5.615} \phi_F \left[ \frac{\hat{\phi}_F}{\partial p_F} - \phi_F \left( \frac{S_{w,F}}{B_{w,F}} \frac{\hat{B}_{w,F}}{\partial p_F} + \frac{S_{w,F}}{B_{w,F}} \frac{\hat{B}_{w,F}}{\partial p_F} \right) \right]
\]  
(9.30)

\[
c_M = \frac{1}{5.615} \phi_M \left[ \frac{5.615 B_{w,M} \partial V_a}{\partial p_M} + \frac{\hat{\phi}_M}{\partial p_M} - \phi_M \left( \frac{S_{w,M}}{B_{w,M}} \frac{\hat{B}_{w,M}}{\partial p_M} + \frac{S_{w,M}}{B_{w,M}} \frac{\hat{B}_{w,M}}{\partial p_M} \right) \right]
\]  
(9.31)

Substituting Eq. (9.29) into Eq. (9.19), one obtains:

\[
B_{g,F} \nabla \cdot \left[ \frac{k_F k_{g,F}}{\mu_g B_{g,F}} \nabla \cdot p_F \right] + B_{w,F} \nabla \cdot \left[ \frac{k_F k_{w,F}}{\mu_g B_{w,F}} \nabla \cdot p_F \right] + B_{g,F} \frac{\Gamma_{g,F}}{V_b} + B_{g,M} \frac{\Gamma_{g,M}}{V_b} + B_{w,F} \frac{\Gamma_{w,F}}{V_b} + B_{w,M} \frac{\Gamma_{w,M}}{V_b} = \phi_F c_F \frac{\partial p_F}{\partial t} + \phi_M c_M \frac{\partial p_M}{\partial t}
\]  
(9.32)

Since

\[
\Gamma_{g,F} = -\Gamma_{g,M}
\]  
(9.33)

and
\[ \Gamma_{w,F} = -\Gamma_{w,M} \] (9.34)

Substituting Eqs.(9.33) and (9.34) into Eq.(9.32), one obtains:

\[ B_{g,F} \nabla \left[ \frac{k_F k_{rg,F}}{\mu_g B_{g,F}} \nabla \cdot p_F \right] + B_{w,F} \nabla \left[ \frac{k_F k_{rw,F}}{\mu_w B_{w,F}} \nabla \cdot p_F \right] + (B_{g,F} - B_{g,M}) \frac{\Gamma_{g,F}}{V_b} + (B_{w,F} - B_{w,M}) \frac{\Gamma_{w,F}}{V_b} = \phi_F c_F \frac{\partial p_F}{\partial t} + \phi_M c_M \frac{\partial p_M}{\partial t} \] (9.35)

As the terms \((B_{g,F} - B_{g,M}) \frac{\Gamma_{g,F}}{V_b}\) and \((B_{w,F} - B_{w,M}) \frac{\Gamma_{w,F}}{V_b}\) are small and can be ignored, Eq.(9.35) becomes.

\[ B_{g,F} \nabla \left[ \frac{k_F k_{rg,F}}{\mu_g B_{g,F}} \nabla \cdot p_F \right] + B_{w,F} \nabla \left[ \frac{k_F k_{rw,F}}{\mu_w B_{w,F}} \nabla \cdot p_F \right] = \phi_F c_F \frac{\partial p_F}{\partial t} + \phi_M c_M \frac{\partial p_M}{\partial t} \] (9.36)

Expanding the left-hand side of Eq.(9.36), one obtains:

\[ \left( \frac{k_F k_{rg,F}}{\mu_g} + \frac{k_F k_{rw,F}}{\mu_w} \right) \nabla^2 \cdot p_F + B_{g,F} \nabla \cdot \left[ \frac{k_F k_{rg,F}}{\mu_g B_{g,F}} \nabla \cdot p_F \right] + B_{w,F} \nabla \cdot \left[ \frac{k_F k_{rw,F}}{\mu_w B_{w,F}} \nabla \cdot p_F \right] = \phi_F c_F \frac{\partial p_F}{\partial t} + \phi_M c_M \frac{\partial p_M}{\partial t} \] (9.37)

Let

\[ \lambda_g = \frac{k_F k_{rg,F}}{\mu_g} \] (9.38)

and

\[ \lambda_w = \frac{k_F k_{rw,F}}{\mu_w} \] (9.39)

Substituting Eqs.(9.38) and (9.39) into Eq.(9.37) yields:

\[ (\lambda_g + \lambda_w) \nabla^2 \cdot p_F + B_{g,F} \nabla \cdot \left[ \frac{\lambda_g}{B_{g,F}} \nabla \cdot p_F \right] + B_{w,F} \nabla \cdot \left[ \frac{\lambda_w}{B_{w,F}} \nabla \cdot p_F \right] = \phi_F c_F \frac{\partial p_F}{\partial t} + \phi_M c_M \frac{\partial p_M}{\partial t} \] (9.40)
Rewriting Eq.(9.40) yields:

\[
\left( \lambda_{g} + \lambda_{w} \right) \nabla^{2} \cdot p_{F} + \lambda_{g} \frac{B_{g,F}}{\lambda_{g}} \nabla \cdot \left( \frac{\lambda_{g}}{B_{g,F}} \nabla \cdot p_{F} \right) + \lambda_{w} \frac{B_{w,F}}{\lambda_{w}} \nabla \cdot \left( \frac{\lambda_{w}}{B_{w,F}} \nabla \cdot p_{F} \right) = \phi_{F} c_{F} \frac{\partial p_{F}}{\partial t} + \phi_{M} c_{M} \frac{\partial p_{M}}{\partial t}
\]

(9.41)

Since

\[
\nabla \cdot \ln u = \frac{1}{u} \nabla \cdot u
\]

(9.42)

Eq.(9.41) can be written as:

\[
\left( \lambda_{g} + \lambda_{w} \right) \nabla^{2} \cdot p_{F} + \lambda_{g} \nabla \cdot \ln \left( \frac{\lambda_{g}}{B_{g,F}} \right) + \lambda_{w} \nabla \cdot \ln \left( \frac{\lambda_{w}}{B_{w,F}} \right) = \phi_{F} c_{F} \frac{\partial p_{F}}{\partial t} + \phi_{M} c_{M} \frac{\partial p_{M}}{\partial t}
\]

(9.43)

Numerical exercises conducted by Mohaghegh and Ertekin (1991) suggested the following relationships:

\[
\frac{\lambda_{g}}{B_{g,F}} = a p_{F}
\]

(9.44)

and

\[
\frac{\lambda_{w}}{B_{w,F}} = b p_{F}
\]

(9.45)

where \(a\) and \(b\) are constant. Thus, Eq.(9.43) becomes:

\[
\left( \lambda_{g} + \lambda_{w} \right) \nabla^{2} \cdot p_{F} + \left( \lambda_{g} + \lambda_{w} \right) \frac{1}{p_{F}} (\nabla \cdot p_{F})^{2} = \phi_{F} c_{F} \frac{\partial p_{F}}{\partial t} + \phi_{M} c_{M} \frac{\partial p_{M}}{\partial t}
\]

(9.46)

Re-arranging Eq.(9.46) yields:

\[
\nabla^{2} \cdot p_{F} + \left( \frac{1}{p_{F}} (\nabla \cdot p_{F})^{2} \right) = \phi_{F} c_{F} \frac{\partial p_{F}}{\partial t} + \phi_{M} c_{M} \frac{\partial p_{M}}{\partial t}
\]

(9.47)

Eq.(9.47) can be written as:
\[ \nabla^2 \cdot p_F^2 = \frac{\phi_F c_F}{\lambda_g + \lambda_w} \frac{\partial p_F^2}{\partial t} + \frac{\phi_M c_M}{\lambda_g + \lambda_w} \frac{\partial p_M^2}{\partial t} \]  

(9.48)

Eq. (9.48) in two-dimensional Cartesian coordinates can be written as:

\[ \frac{\partial^2 p_F^2}{\partial x^2} + \frac{\partial^2 p_F^2}{\partial y^2} = \frac{\phi_F c_F}{\lambda_g + \lambda_w} \frac{\partial p_F^2}{\partial t} + \frac{\phi_M c_M}{\lambda_g + \lambda_w} \frac{\partial p_M^2}{\partial t} \]  

(9.49)

Transforming Eq. (9.49) into dimensionless form yields:

\[ \frac{\partial^2 p_{D,F}}{\partial x_D^2} + \frac{\partial^2 p_{D,F}}{\partial y_D^2} = \omega \frac{\partial p_{D,F}}{\partial t_D} + (1 - \omega) \frac{\partial p_{D,M}}{\partial t_D} \]  

(9.50)

where

\[ \omega = \frac{\phi_F c_F}{\phi_F c_F + \phi_M c_M} \]  

(9.51)

\[ x_D = \frac{x}{r_w} \]  

(9.52)

\[ y_D = \frac{y}{r_w} \]  

(9.53)

\[ t_D = \frac{2.637 \times 10^{-4} (\lambda_g + \lambda_w)}{(\phi_F c_F + \phi_M c_M) r_w^2} \]  

(9.54)

\[ p_{D,F} = \frac{p_i^2 - p_F^2}{(p_i^2 - p_{sF}^2)} q_{D,F} \]  

(9.55)

\[ p_{D,F} = \frac{p_i^2 - p_M^2}{(p_i^2 - p_{sF}^2)} q_{D,F} \]  

(9.56)

\[ q_{D,g} = \frac{1.424 q_{g,sc} ZT}{\lambda_s h (p_i^2 - p_{sF}^2)} \]  

(9.57)

and
The dimensionless time in Eq.(9.54) and gas and water flow rates in Eqs.(9.57) and (9.58) are expressed in the field units shown in Table 9-1.

Table 9-1: Description of units used in the proposed type curves

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>x, y</td>
<td>ft</td>
</tr>
<tr>
<td>k</td>
<td>md</td>
</tr>
<tr>
<td>t</td>
<td>day</td>
</tr>
<tr>
<td>m</td>
<td>cp</td>
</tr>
<tr>
<td>p</td>
<td>psia</td>
</tr>
<tr>
<td>q_g</td>
<td>SCF/D</td>
</tr>
<tr>
<td>q_w</td>
<td>STB/D</td>
</tr>
</tbody>
</table>

With the key assumptions presented earlier, the relationship between the dimensionless diffusivity equation and dimensionless groups and the proposed numerical dual-porosity, dual-permeability coalbed methane reservoir model presented in Chapter 4 can be established as demonstrated from Eqs.(9.8) to (9.57). Thus, in this study the proposed numerical model will be used to generate production type curves and validate the performance of the type curves.

9.2 Sensitivity Analysis of the Proposed Production Type Curves

Sensitivity of the proposed gas and water production type curves is investigated in this section. The numerical simulation model presented in Chapter 4 was used to generate gas and water flow rates versus time information. Such production data were used to calculate the dimensionless time, dimensionless gas and water rate groups. All parameters in the dimensionless groups including gas and water formation volume factors, compressibility factor (Z), pressure, gas saturation and water saturation were evaluated at the initial conditions. In addition, the derivative terms, which appears in Eqs.(9.30) and (9.31), were evaluated as follows:
\[
\frac{\partial B_{g,F}}{\partial p_F} = \frac{B_{g,F}(p_{F,i}) - B_{g,F}(p_{sf})}{p_{F,i} - p_{sf}} \tag{9.59}
\]
\[
\frac{\partial B_{w,F}}{\partial p_F} = \frac{B_{w,F}(p_{F,i}) - B_{w,F}(p_{sf})}{p_{F,i} - p_{sf}} \tag{9.60}
\]
\[
\frac{\partial B_{g,M}}{\partial p_M} = \frac{B_{g,M}(p_{M,i}) - B_{g,M}(p_{sf})}{p_{M,i} - p_{sf}} \tag{9.61}
\]
\[
\frac{\partial B_{w,M}}{\partial p_M} = \frac{B_{w,M}(p_{M,i}) - B_{w,M}(p_{sf})}{p_{M,i} - p_{sf}} \tag{9.62}
\]
\[
\frac{\partial V_a}{\partial p_M} = \frac{V_a(p_{M,i}) - V_a(p_{sf})}{p_{M,i} - p_{sf}} \tag{9.63}
\]

For simplicity, the value of \( V_a \) is calculated using the Langmuir sorption isotherm, Eq.(5.1). Thus, the dimensionless production type curves were constructed on a log-log scale. Several reservoir and well properties were varied for sensitivity analysis. It was found that production type curves were strongly sensitive to fracture and matrix porosities, initial pressure, sandface pressure, drainage area, wellbore radius, fracture and matrix water saturations and relative permeability. In this study, functional groups of these parameters with strong sensitivity to the proposed production type curves were developed and used to reduce the number of variables in the system.

The fracture storage (\( \omega \)) was used to capture the unique results corresponding to different fracture and matrix porosities. The ratio of sandface pressure to initial pressure used by Monhaghegh and Ertekin (1991) was used to characterize the unique results of different initial pressure and sandface pressure. Also, the ratio of the distance between the wellbore and the reservoir boundary to wellbore radius was used to characterize the unique results of different drainage area and wellbore radius.

The production type curves were found to be slightly sensitive to certain parameters during the intermediate period between early-time and late-time production periods. These
parameters with weak sensitivity to the proposed production type curves include matrix permeability, diffusion coefficient and sorption time constant, all of which are related to the multi-mechanistic flow in the matrix system and the delay of gas sorption. Figures 9-1, 9-2 and 9-3 show the sensitivity results of the type curves to matrix permeability, diffusion coefficient and sorption time constant, respectively. It can be observed that the production type curves are slightly sensitive to these three parameters during the intermediate production period. However, each parameter shows no effect on the production type curves during the early-time and late-time production periods. This phenomenon is caused by the dominant influence of the flow from the fracture system or fracture permeability on the production during early time and the flow equilibrium between the fracture and matrix systems at late time. In addition, the proposed production type curves were found to be sensitive to relative permeability; however, this problem can be solved by a simple procedure proposed by Monhaghegh and Ertekin (1991), which is presented in Section 9.3.

From the results of the sensitivity analysis, the gas and water production type curves for coalbed methane systems are generated for different values of fracture storage, water saturations in the fracture and matrix systems, ratio of initial pressure to sandface pressure and ratio of the distance between the wellbore and the reservoir boundary to wellbore radius. Although matrix permeability, diffusion coefficient and sorption time constant are excluded from the generation of the type curves, the application of the production type curves is still valid particularly at early time and late time as not only is the sensitivity of the production type curves to these three parameters small at the intermediate production period but also the three parameters show no sensitivity to the production type curves during the early-time and late-time periods. Thus, it is important to note that the application of the proposed type curves should be emphasized at the early and late periods of the production to avoid the effects of the matrix permeability, diffusion coefficient and sorption time constant.
Figure 9-1: Gas production type curves with different matrix permeabilities

Figure 9-2: Gas production type curves with different diffusion coefficients
9.3 Application

In this part of the study, the use of the proposed production type curves for two-phase flow in CBM systems for reservoir characterization is validated. The reservoir and fluid properties used in generating the proposed production type curves are presented in Table 9-2 and Figure 9-4. The dimensionless time and flow rate groups were calculated using Eqs.(9.54), (9.57) and (9.58), respectively, and plotted on a log-log scale. Thus, the proposed production type curves were tested against several systems with a wide range of reservoir and fluid properties. The dual-porosity, dual-permeability numerical simulation model in Chapter 4 was used to generate the gas and water flow rates versus time data while sandface pressure was specified. The production rate versus time data were plotted on a log-log scale and used to match with the log-log production type curves. A set of type curves, which provided the best match, was selected and used for determining some reservoir properties. The calculated reservoir properties were then compared against the actual ones for validation. It is noted that all case studies conducted provided satisfactory results; however, a few cases are presented in this section.
Table 9-2: Reservoir and fluid properties used in generating the proposed production type curves

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir thickness</td>
<td>20 ft</td>
</tr>
<tr>
<td>Matrix porosity</td>
<td>6%</td>
</tr>
<tr>
<td>Fracture permeability</td>
<td>20 md</td>
</tr>
<tr>
<td>Matrix permeability</td>
<td>0.0001 md</td>
</tr>
<tr>
<td>Diffusion coefficient</td>
<td>( 1 \times 10^{-8} ) sq.ft/day</td>
</tr>
<tr>
<td>Coal density</td>
<td>1.435 g/cc</td>
</tr>
<tr>
<td>Langmuir volume constant</td>
<td>1,134 SCF/ton</td>
</tr>
<tr>
<td>Langmuir pressure constant</td>
<td>625 psi</td>
</tr>
<tr>
<td>Sorption time constant</td>
<td>50 days</td>
</tr>
<tr>
<td>Initial pressure</td>
<td>1,100 psia</td>
</tr>
<tr>
<td>Fracture spacing</td>
<td>1 inch</td>
</tr>
<tr>
<td>Relative permeability</td>
<td>Set A</td>
</tr>
<tr>
<td>Reservoir temperature</td>
<td>80 ºF</td>
</tr>
<tr>
<td>Well radius</td>
<td>0.25 ft</td>
</tr>
</tbody>
</table>

Note: Fracture porosity is varied for different values of fracture storage (\( \omega \)).

Figure 9-4: Relative permeability curves: Set A
Cases 1 and 2 are presented to illustrate the validation of the proposed production type curves for determining reservoir properties including fracture permeability, fracture porosity, matrix porosity and initial water saturations. Case 3 demonstrates the procedure for the systems with different relative permeability curves. All three cases exhibit a variety of reservoir and fluid properties, which are presented in Table 9-3 and Figure 9-5.

Table 9-3: Reservoir and fluid properties used in validating the proposed production type curves

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir size</td>
<td>2,500 ft × 2,500 ft</td>
<td>3,100 ft × 3,100 ft</td>
<td>1,500 ft × 1,500 ft</td>
</tr>
<tr>
<td>Reservoir thickness</td>
<td>10 ft</td>
<td>15 ft</td>
<td>24 ft</td>
</tr>
<tr>
<td>Fracture porosity</td>
<td>0.108%</td>
<td>0.094%</td>
<td>0.22%</td>
</tr>
<tr>
<td>Matrix porosity</td>
<td>2.1%</td>
<td>8.2%</td>
<td>9.5%</td>
</tr>
<tr>
<td>Fracture permeability</td>
<td>4.9 md</td>
<td>62.1 md</td>
<td>11.3 md</td>
</tr>
<tr>
<td>Matrix permeability</td>
<td>0.000014 md</td>
<td>0.002 md</td>
<td>0.00032 md</td>
</tr>
<tr>
<td>Diffusion coefficient</td>
<td>$5 \times 10^{-9}$ sq.ft/day</td>
<td>$1 \times 10^{-9}$ sq.ft/day</td>
<td>$1 \times 10^{-8}$ sq.ft/day</td>
</tr>
<tr>
<td>Coal density</td>
<td>1.435 g/cc</td>
<td>1.435 g/cc</td>
<td>1.435 g/cc</td>
</tr>
<tr>
<td>Langmuir volume constant</td>
<td>281 SCF/ton</td>
<td>1,397 SCF/ton</td>
<td>675 SCF/ton</td>
</tr>
<tr>
<td>Langmuir pressure constant</td>
<td>987 psi</td>
<td>853 psi</td>
<td>1,086 psi</td>
</tr>
<tr>
<td>Sorption time constant</td>
<td>225 days</td>
<td>15 days</td>
<td>87 days</td>
</tr>
<tr>
<td>Initial pressure</td>
<td>750 psia</td>
<td>900 psia</td>
<td>500 psia</td>
</tr>
<tr>
<td>Initial water saturation, fracture</td>
<td>40%</td>
<td>60%</td>
<td>60%</td>
</tr>
<tr>
<td>Initial water saturation, matrix</td>
<td>60%</td>
<td>20%</td>
<td>60%</td>
</tr>
<tr>
<td>Fracture spacing</td>
<td>1 inch</td>
<td>1 inch</td>
<td>1 inch</td>
</tr>
<tr>
<td>Relative permeability</td>
<td>Set A</td>
<td>Set A</td>
<td>Set B</td>
</tr>
<tr>
<td>Reservoir temperature</td>
<td>80 °F</td>
<td>80 °F</td>
<td>80 °F</td>
</tr>
<tr>
<td>Sandface pressure</td>
<td>525 psia</td>
<td>450 psia</td>
<td>50 psia</td>
</tr>
<tr>
<td>Well radius</td>
<td>0.5 ft</td>
<td>0.31 ft</td>
<td>0.2 ft</td>
</tr>
</tbody>
</table>
Case 1

In this case study, a simulation run was conducted for a rather tight CBM system with $x_{eD} = y_{eD} = 5,000$ and $p_{wf}/p_i = 0.7$. The gas and water flow rates versus time data were plotted on a log-log scale and a type-curve matching was performed on several sets of production type curves with different values of $\omega$ and initial water saturations presented in Appendix B. The best match was obtained from the production type curves with the values of $\omega$, initial water saturation in the fracture and that in the matrix of 0.01, 40% and 60%, respectively. Figures 9-6 and 9-7 show the matches of gas and water production type curves, respectively.

For the match of gas production rate, the coordinates of the match point, which is denoted in Figure 9-6 with a cross sign, are:

$$
\begin{bmatrix}
q_{D,g} = 1 \\
t_D = 0.9
\end{bmatrix}_{Type\ curve} \quad \text{and} \quad \begin{bmatrix}
q_g = 800,000 \\
t = 0.00001
\end{bmatrix}_{Actual\ data}
$$
The fracture permeability can be calculated using the definition of the dimensionless gas flow rate.

\[ k_F = \frac{1.424 q_{g,sc} Z T \mu_g}{q_{D,g} h \left( p_i^2 - p_{wf}^2 \right) k_{rg}} \]  

(9.64)

In addition, the fracture porosity and matrix porosity can be calculated using the definition of the dimensionless time.

\[ \phi_F = \frac{2.637 \times 10^{-4} (\lambda_g + \lambda_w) \phi}{t_D c_F r_w^2} \]  

(9.65)

\[ \phi_M = \frac{2.637 \times 10^{-4} (\lambda_g + \lambda_w) (1 - \omega) t}{t_D c_M r_w^2} \]  

(9.66)

Using the match point data and Eqs.(9.64) to (9.66), the permeability of the fracture is calculated as 4.81 md and the fracture porosity and matrix porosity are calculated as 0.117% and 2.27%, respectively.

A similar exercise was performed using the water production data to validate the results obtained from the gas production match. For the water production match, the coordinates of the match point of the match between the water production data and the water production type curves, which is denoted in Figure 9-7 with a cross sign, are:

\[
\begin{bmatrix}
q_{D,w} = 0.08 \\
t_D = 0.8
\end{bmatrix}_{Type\ curve} \quad \text{and} \quad \begin{bmatrix}
q_w = 0.1 \\
t = 0.00007
\end{bmatrix}_{Actual\ data}
\]

The fracture permeability can be calculated using the definition of the dimensionless water flow rate.

\[ k_F = \frac{282.53 q_{w,sc} B_{w,F} p_F \mu_w}{q_{D,w} h \left( p_i^2 - p_{wf}^2 \right) k_{rw}} \]  

(9.67)

Using the match point data and Eqs.(9.67), (9.65) and (9.66), the permeability of the fracture is calculated as 5.26 md and the fracture porosity and matrix porosity are calculated as 0.101% and
1.96%, respectively. Table 9-4 summarizes the values of reservoir parameters obtained from the gas and water production type curves. The results were considered reasonable as the fracture permeability, fracture porosity and matrix porosity from the type-curve matches and those from the actual data were found to be in good agreement. Also, it can be observed in this case that the average values of the fracture permeability, fracture porosity and matrix porosity obtained using both gas and water type-curve approaches yield even better results as the errors are smaller than 3%.

Figure 9-6: Gas production type-curve match for Case 1
Figure 9-7: Water production type-curve match for Case 1

Table 9-4: Reservoir parameters obtained from the type-curve match for Case 1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Simulation</th>
<th>Gas production type curve</th>
<th>Water production type curve</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Value</td>
<td>Error</td>
<td>Value</td>
</tr>
<tr>
<td>Fracture permeability</td>
<td>4.90 md</td>
<td>4.81 md</td>
<td>1.84%</td>
<td>5.26 md</td>
</tr>
<tr>
<td>Fracture porosity</td>
<td>0.108%</td>
<td>0.117%</td>
<td>8.33%</td>
<td>0.101%</td>
</tr>
<tr>
<td>Matrix porosity</td>
<td>2.10%</td>
<td>2.27%</td>
<td>8.10%</td>
<td>1.96%</td>
</tr>
</tbody>
</table>

Case 2

A simulation run was conducted for a typical CBM system with $x_{eD} = y_{eD} = 10,000$ and $p_{wf}/p_i = 0.5$. Similar to Case 1, a type-curve matching exercise was performed. The best match was obtained from the production type curves with the values of $\omega$, initial water saturation in the fracture and that in the matrix of 0.001, 60% and 20%, respectively. Figures 9-8 and 9-9 show the matches of gas and water production type curves, respectively. In this case, it should be noted that the use of the water production type curves was crucial for obtaining the accurate initial water saturation in the fracture and that in the matrix as the gas production type curves for initial water saturation in the matrix of 20% and that of 40% are nearly indistinguishable. Using the match from
gas production type curves alone could provide inaccuracy of the initial water saturation in the fracture and that in the matrix.

For the match of gas production rate, the coordinates of the match point, which is denoted in Figure 9-8 with a cross sign, are:

\[
\begin{bmatrix}
q_{D,g} = 0.01 \\
t_D = 30
\end{bmatrix}_{\text{Type curve}} \quad \text{and} \quad \begin{bmatrix}
q_g = 200,000 \\
t = 0.0008
\end{bmatrix}_{\text{Actual data}}
\]

Using the match point data above and Eqs.(9.64) to (9.66), the fracture permeability can be calculated as 66.4 md and the fracture porosity and matrix porosity are calculated as 0.095% and 8.27%, respectively. In addition, the coordinates of the match point of the match between the water production data and the water production type curves, which is denoted in Figure 9-9 with a cross sign, are:

\[
\begin{bmatrix}
q_{D,w} = 0.02 \\
t_D = 3 \times 10^8
\end{bmatrix}_{\text{Type curve}} \quad \text{and} \quad \begin{bmatrix}
q_w = 7 \\
t = 8,000
\end{bmatrix}_{\text{Actual data}}
\]

Using the match point data and Eqs.(9.67), (9.65) and (9.66), the permeability of the fracture is calculated as 63.8 md and the fracture porosity and matrix porosity are calculated as 0.091% and 7.94%, respectively. Table 9-5 summarizes the values of reservoir parameters obtained from the gas and water production type curves for Case 2. The results were satisfactory as the errors range from less than 1% to 7%.
Figure 9-8: Gas production type-curve match for Case 2

Figure 9-9: Water production type-curve match for Case 2

Table 9-5: Reservoir parameters obtained from the type-curve match for Case 2

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Simulation</th>
<th>Gas production type curve</th>
<th>Water production type curve</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Value</td>
<td>Error</td>
<td>Value</td>
<td>Error</td>
</tr>
<tr>
<td>Fracture permeability</td>
<td>62.1 md</td>
<td>66.4 md</td>
<td>63.8 md</td>
<td>65.1</td>
</tr>
<tr>
<td>Fracture porosity</td>
<td>0.094%</td>
<td>0.095%</td>
<td>0.091%</td>
<td>0.093%</td>
</tr>
<tr>
<td>Matrix porosity</td>
<td>8.2%</td>
<td>8.27%</td>
<td>7.94%</td>
<td>8.11%</td>
</tr>
</tbody>
</table>
Case 3

In this case, a simulation run was conducted on a coalbed methane system with different relative permeability characteristics. In Section 9.2, the production type curves were found to be sensitive to different initial water saturations. This implies that the production type curves are expected to be sensitive to relative permeability due to their intrinsic dependence on saturation. Mohaghegh and Ertekin (1991) proposed a simple procedure to solve this problem.

1. Find the gas relative permeability at the initial water saturation.

2. Find the water saturation that corresponds to the same value of gas relative permeability obtained from step (1) using the relative permeability curves used in generating the production type curves (Figure 9-4).

3. Use the production type curves with the same value of water saturation obtained from step (2) for the analysis.

In this exercise, the relative permeability curves in Figure 9-5 were used. The initial water saturations in the fracture and matrix systems are both 60%. The gas relative permeability at water saturation of 60% was found as 0.50. The corresponding water saturation from the relative permeability curves used in generating the production type curves is 40%; therefore, the gas production type curve with initial water saturations in the fracture and matrix of 40% were used to perform the type-curve matching. The best match was obtained from the gas production type curve with the value of \( \omega \) of 0.001, \( x_{eD} = y_{eD} = 7,500 \) and \( \frac{p_{wf}}{p_i} = 0.1 \). Figure 9-10 shows the match of gas production type curve. The coordinates of the match point, which is denoted in Figure 9-10 with a cross sign, are:

\[
\begin{align*}
q_{D,g} &= 1 \\
q_D &= 6 \times 10^9 \\
q_g &= 4,000,000 \\
t_D &= 100,000
\end{align*}
\]

The fracture permeability can be calculated as 11.78 md and the fracture porosity and matrix porosity are calculated as 0.208% and 9.01%, respectively. Table 9-6 show the results from the
gas type curve matching for Case 3, which are considered satisfactory compared with the actual data.

![Gas production type-curve match for Case 3](image)

**Figure 9-10**: Gas production type-curve match for Case 3

**Table 9-6**: Reservoir parameters obtained from the type-curve match for Case 3

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Simulation</th>
<th>Gas production type curve</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Value</td>
</tr>
<tr>
<td>Fracture permeability</td>
<td>11.30 md</td>
<td>11.78 md</td>
</tr>
<tr>
<td>Fracture porosity</td>
<td>0.219%</td>
<td>0.208%</td>
</tr>
<tr>
<td>Matrix porosity</td>
<td>9.50%</td>
<td>9.01%</td>
</tr>
</tbody>
</table>
Chapter 10
Conclusions

This thesis presents the development and implementation of compositional, dual-porosity, dual-permeability, two-phase flow formalisms applicable to coalbed methane reservoirs. The transport of gas follows the multi-mechanistic flow mechanism which considers the influence of pressure and concentration gradients. CBM reservoirs are treated as dual-porosity, dual-permeability systems formed by fracture network and coal matrix. The model also incorporates the effects of coal shrinkage and swelling and the presence of water in the coal matrix. In situations when developing a simulation model can be time-consuming and cost intensive, other reservoir engineering analysis tools may be considered. For that reason material balance equation and production type curves specifically developed for CBM reservoirs are proposed in this study.

The governing equations of the two-phase flow in dual-porosity, dual-permeability CBM reservoirs form the basis in the development of a new CBM reservoir material balance equation accounting for the effects of water in the coal matrix and coal shrinkage/swelling phenomena. The applications of the MBE are extended for production performance and average reservoir pressure predictions. Furthermore, the governing equations are collapsed into the diffusivity equation for CBM systems. The diffusivity equation was transformed into a dimensionless form and additional dimensionless groups were developed to construct a type-curve solution. Thus, the proposed numerical model was used to generate gas and water production type curves, which can be used for reservoir characterization.

The conclusions drawn from the study are as follows:

1. The proposed dual-porosity, dual-permeability CBM model can be collapsed to simpler model representations including the dual-porosity, single-permeability model and the
single-porosity, single-permeability model by ignoring the contributions of fracture porosity and matrix permeability. In particular, the proposed model can be collapsed to the single-porosity, single-permeability model, which is often used to represent typical sandstone reservoirs, by ignoring adsorbed gas.

2. The proposed CBM simulator provided satisfactory results when compared to the existing commercial and research CBM simulators for an enhanced CBM recovery process in CO$_2$ injection. Discrepancies could be due to differences in reservoir representation, gas flow mechanism, treatment of the presence of water in the coal matrix and gas sorption theory and among others. In particular, the proposed simulator is considered more robust as it incorporates ultimate characteristics of coal seams ignored by most existing simulators.

3. Darcian flow is dominant in the permeable system as expected. The multi-mechanistic flow mechanism is more pronounced in the tighter systems and Fickian flow component becomes increasingly more dominant as the permeability of the system decreases, particularly at early times. From this study, the effect of the Fickian flow becomes observable for the system with the fracture and matrix permeability values of 10 md and 0.01 md, respectively.

4. Bound water in the coal matrix reduces the gas sorption capacity as it occupies hydrophilic part of the gas sorption sites while bulk water in the coal matrix blocks the gas flow path in the pore spaces. The presence of both types of water was found to decrease the gas production rate and increases the water production rate. In addition, during enhanced CBM recovery process with pure CO$_2$ injection, the presence of both types of water in the coal matrix, particularly the bound water, was found to force an early CO$_2$ breakthrough.
5. The proposed coal shrinkage and swelling model, which incorporates the effects of internal stress, release/adsorption of gas in the coal and water in the coal matrix, was successfully used to match the field data from the San Juan basin. This model was also compared with the existing models and a larger discrepancy in permeability changes in permeability between the coal shrinkage and swelling models was found for the system with lower initial porosity.

6. For enhanced CBM recovery process with pure CO$_2$ injection, the reduction of fracture permeability caused by coal swelling was found to delay CO$_2$ breakthrough. In particular, the adsorption of CO$_2$ in the coal matrix after CO$_2$ breakthrough occurred resulted in coal swelling and caused a sharp reduction of fracture permeability.

7. The proposed simulator and the generalized equation-of-state model compositional reservoir simulator (GEM) were used at their fullest capabilities to predict production performance of a complex CBM reservoir system. Significant discrepancies between the predictions of the two simulators were observed and found to be caused by the different assumptions and limitations of each simulator, particularly the treatment of water presence in the coal matrix.

8. Good history matches were obtained for the pure CO$_2$ injection and the flue gas injection tests conducted in the Fenn Big Valley field (Alberta, Canada) using the proposed dual-porosity, dual-permeability simulator. In particular, the proposed simulator was able to provide excellent matches of mole fractions produced for the flue gas injection test, which was found to be difficult if not impossible for most of the existing simulators.

9. Comparative studies were conducted between the proposed and existing material balance equations and more accurate predictions were obtained from the proposed MBE due to the incorporation of the effects of water presence in the coal matrix and coal shrinkage and swelling. Exclusion of these two phenomena in the material balance equation was
found to reduce the estimated reservoir capacity, resulting in under-predictions of reservoir size.

10. Iterative methods for predicting production performance and average reservoir pressure using the proposed MBE were developed and successfully tested against the simulation model. In particular, these methods are considered more economical and less time-demanding when compared to the simulation approach.

11. The proposed production type curves were found to be strongly sensitive to fracture and matrix porosities, initial pressure, sandface pressure, drainage area, wellbore radius, fracture and matrix water saturations. Additional parameters including the matrix permeability, diffusion coefficient and sorption time constant were found to have small effects on the production type curves as the flow contribution from the matrix system is minor during the early and late time periods. Thus, it is reasonable to ignore these effects and the application of the proposed type-curve match should be emphasized at the early and late periods of the production.

12. The type-curve solution for reservoir characterization was tested against a wide range of reservoir and fluid properties using the simulation data. The production type curves were used to estimate reservoir properties including fracture permeability, fracture porosity, matrix porosity and water saturations in the fracture and matrix systems, which were found to be in good agreement with target properties.
References


Schilthuis, R. J., Active Oil and Reservoir Energy, Trans. AIME (1936), 148, pp. 33-52.


Appendix A

Correlations for Rock and Fluid Properties

The rock and fluid properties in this study can be obtained using the empirical correlations presented in this section if data for table-lookup is not provided. Peng-Robinson equation of state (EOS) is utilized to calculate the compressibility factor and fugacity coefficients.

**Compressibility Factor (Z-Factor)**

The Peng-Robinson cubic expression:

\[
Z^3 - (1 - B)Z^2 + \left( A - 2B - 3B^2 \right)Z - \left( AB - B^2 - B^3 \right) = 0
\]  

(A.1a)

where \( A = \sum_i \sum_j X_i X_j A_{ij} \)  

(A.1b)

\( A_{ij} = (1 - \delta_{ij}) \sqrt{A_i A_j} \)  

(A.1c)

\[ A_i = \Omega_{ai} \frac{P_{ri}}{T_{ri}^2} \]  

(A.1d)

\[ B = \sum_{i=1}^{n} C_i B_i \]  

(A.1e)

\[ B_i = 0.077796074 \frac{P_{ri}}{T_{ri}} \]  

(A.1f)

\[ P_{ri} = \frac{P}{P_{ci}} \]  

(A.1g)

\[ T_{ri} = \frac{T}{T_{ci}} \]  

(A.1h)
\[
\Omega_{ai} = 0.45723552\left[1 + m_i(1 - \sqrt{T_{ni}})\right]^2
\]  
(A.1i)

- If \( \omega_i \leq 0.49 \) then 
  \[ m_i = 0.374640 + 1.54226\omega_i - 0.26992\omega_i^2 \]  
  (A.1j)

- If \( \omega_i > 0.49 \) then 
  \[ m_i = 0.379642 + 1.48503\omega_i - 0.164423\omega_i^2 + 0.016666\omega_i^3 \]  
  (A.1k)

\( \delta_{ij} \) = Binary mixture coefficient between the i-th and j-th components

Analytical Solution of cubic polynomials:

For the generic polynomial: 
\[ x^3 + ax^2 + bx + c = 0 \]  
(A.11)

Calculate 
\[ Q = \frac{a^2 - 3b}{9} \]  
(A.1m)

\[ R = \frac{2a^3 - 9ab + 27c}{54} \]  
(A.1n)

\[ M = R^2 - Q^3 \]  
(A.1o)

a) If \( M \leq 0 \), the polynomial has three real roots.

\[ x_1 = -(2\sqrt[3]{Q}\cos\frac{\theta}{3}) - \frac{a}{3} \]  
(A.1p)

\[ x_2 = -(2\sqrt[3]{Q}\cos\frac{\theta + 2\pi}{3}) - \frac{a}{3} \]

\[ x_3 = -(2\sqrt[3]{Q}\cos\frac{\theta - 2\pi}{3}) - \frac{a}{3} \]

where \( \theta = \arccos\left(\frac{R}{\sqrt[3]{Q^3}}\right) \)  
(A.1q)

b) If \( M > 0 \), the polynomial has only one real root equal to

\[ x = S + T - \frac{a}{3} \]  
(A.1r)

where 
\[ S = \sqrt[3]{-R + \sqrt{M}} \]  
(A.1s)

\[ T = \sqrt[3]{-R - \sqrt{M}} \]  
(A.1t)
Fugacity Coefficients and Fugacity

Fugacity Coefficients and Fugacity can be calculated using Peng-Robinson EOS as follows:

\[
\ln \varphi_i = -\ln(Z - B) + \frac{A}{2.828427125B} \left[ \left( \sum_{j=1}^{n} A_{ij} X_j \right) - \frac{B_i}{B} \right] \ln \left[ \frac{Z - 0.414213562B}{Z + 2.414213562B} \right] + \frac{B_i}{B} (Z - 1) 
\]

(A.2)

\[
f_i = \varphi_i X_i p
\]

(A.3)

Gas Formation Volume Factor

\[
B_g = \frac{p_{sc} Z (T + 460)}{5.615 p Z_{sc} (T_{sc} + 460)}
\]

(A.4)

Gas Density

\[
\rho_g = \frac{p M_g}{Z R (T + 460)}
\]

(A.5)

Gas Viscosity (Lee et al., 1966)

\[
\mu_g = 1 \times 10^{-4} k_v \exp \left( x_v \left( \frac{\rho_g}{62.4} \right)^{y_v} \right)
\]

(A.6a)

where

\[
k_v = \frac{(9.4 + 0.02 M_g) T^{1.5}}{209 + 19 M_g + T}
\]

(A.6b)

\[
y_v = 2.4 - 0.2 x_v
\]

(A.6c)

\[
x_v = 3.5 + \frac{986}{T} + 0.01 M_g
\]

(A.6d)
Water Formation Volume Factor (McCain, 1991)

\[ B_w = \left( 1 + \Delta V_{wt} \right) \left( 1 + \Delta V_{wp} \right) \] (A.7a)

where \[ \Delta V_{wt} = -1.00010 \times 10^{-2} + 1.33391 \times 10^{-4} T + 5.50645 \times 10^{-7} T^2 \] (A.7b)

\[ \Delta V_{wp} = -1.95301 \times 10^{-9} pT - 1.72834 \times 10^{-13} p^2 T - 3.58922 \times 10^{-7} p \] 
\[ - 2.25341 \times 10^{-10} p^2 \] (A.7c)

Water Density (McCain, 1991)

\[ \rho_w = 62.368 + 0.438603S + 1.60074 \times 10^{-3} S^2 \] (A.8)

Water Viscosity (McCain, 1991)

\[ \mu_w = AT^8 \left( 0.9994 + 4.0295 \times 10^{-5} p + 3.1062 \times 10^{-9} p^2 \right) \] (A.9a)

where \[ A = 109.574 - 8.40564S + 0.313314S^2 + 8.72213 \times 10^{-3} S^3 \] (A.9b)

\[ B = -1.12166 + 2.63951 \times 10^{-2} S - 6.79461 \times 10^{-4} S^2 - 5.47119 \times 10^{-5} S^3 \] 
\[ + 1.55586 \times 10^{-6} S^4 \] (A.9c)

Relative Permeability to Gas

Corey’s correlation:

\[ k_{rg} = k_{rgc} \left( 1 - (S_{wn})^2 \right) \left( 1 - S_{wn} \right)^2 \] (A.10a)

where \[ k_{rgc} = k_{rg} \text{ at } S_g = 1 - S_{wirr} \] (A.10b)

\[ S_{wn} = \frac{S_w - S_{wirr}}{1 - S_{girr} - S_{wirr}} \] (A.10c)
Relative Permeability to Water

Corey’s correlation:

\[ k_{rw} = k_{rwc} \left( S_{wn} \right)^4 \]  \hspace{1cm} (A.11a)

where \[ k_{rwc} = k_{rw} \text{ at } S_w = 1 - S_{girr} \] \hspace{1cm} (A.11b)

\[ S_{wn} = \frac{S_w - S_{wirr}}{1 - S_{girr} - S_{wirr}} \] \hspace{1cm} (A.11c)

Capillary Pressure

Corey’s correlation:

\[ P_{cgw} = \frac{1}{\sqrt{1.448(S_w - S_{wirr})}} \] \hspace{1cm} (A.12)
Appendix B

Production Type-Curves for Coalbed Methane Systems

Figure B-1: Gas production type-curve for the system with $\omega = 0.001$, $p_{sf}/p_i = 0.1$, $x_{eD} = y_{eD} = 7,500$ and $S_{wF,i} = 80\%$ and different $S_{wM,i}$

Figure B-2: Water production type-curve for the system with $\omega = 0.001$, $p_{sf}/p_i = 0.1$, $x_{eD} = y_{eD} = 7,500$ and $S_{wF,i} = 80\%$ and different $S_{wM,i}$
Figure B-3: Gas production type-curve for the system with $\omega = 0.001$, $p_{sf}/p_i = 0.1$, $x_{eD} = y_{eD} = 7,500$ and $S_{wF,i} = 60\%$ and different $S_{wM,i}$.

Figure B-4: Water production type-curve for the system with $\omega = 0.001$, $p_{sf}/p_i = 0.1$, $x_{eD} = y_{eD} = 7,500$ and $S_{wF,i} = 60\%$ and different $S_{wM,i}$.
Figure B-5: Gas production type-curve for the system with $\omega = 0.001$, $p_s/p_i = 0.1$, $x_{eD} = y_{eD} = 7,500$ and $S_{wF,i} = 40\%$ and different $S_{wM,i}$

Figure B-6: Water production type-curve for the system with $\omega = 0.001$, $p_s/p_i = 0.1$, $x_{eD} = y_{eD} = 7,500$ and $S_{wF,i} = 40\%$ and different $S_{wM,i}$
Figure B-7: Gas production type-curve for the system with $\omega = 0.001$, $p_{sf}/p_i = 0.1$, $x_{eD} = y_{eD} = 7,500$ and $S_{wF,i} = 20\%$ and different $S_{wM,i}$.

Figure B-8: Water production type-curve for the system with $\omega = 0.001$, $p_{sf}/p_i = 0.1$, $x_{eD} = y_{eD} = 7,500$ and $S_{wF,i} = 20\%$ and different $S_{wM,i}$.
Figure B-9: Gas production type-curve for the system with $\omega = 0.001$, $p_{sf}/p_i = 0.5$, $x_eD = y_eD = 10,000$ and $S_{w_{F,i}} = 80\%$ and different $S_{w_{M,i}}$

Figure B-10: Water production type-curve for the system with $\omega = 0.001$, $p_{sf}/p_i = 0.5$, $x_eD = y_eD = 10,000$ and $S_{w_{F,i}} = 80\%$ and different $S_{w_{M,i}}$
Figure B-11: Gas production type-curve for the system with $\omega = 0.001$, $p_{sf}/p_i = 0.5$, $x eD = y eD = 10,000$ and $S_{wF,i} = 60\%$ and different $S_{wM,i}$.

Figure B-12: Water production type-curve for the system with $\omega = 0.001$, $p_{sf}/p_i = 0.5$, $x eD = y eD = 10,000$ and $S_{wF,i} = 60\%$ and different $S_{wM,i}$.
Figure B-13: Gas production type-curve for the system with $\omega = 0.001$, $p_{sf}/p_i = 0.5$, $x_{eD} = y_{eD} = 10,000$ and $S_{wF,i} = 40\%$ and different $S_{wM,i}$.

Figure B-14: Water production type-curve for the system with $\omega = 0.001$, $p_{sf}/p_i = 0.5$, $x_{eD} = y_{eD} = 10,000$ and $S_{wF,i} = 40\%$ and different $S_{wM,i}$. 
Figure B-15: Gas production type-curve for the system with $\omega = 0.001$, $p_{sf}/p_i = 0.5$, $x_{eD} = y_{eD} = 10,000$ and $S_{wF,i} = 20\%$ and different $S_{wM,i}$

Figure B-16: Water production type-curve for the system with $\omega = 0.001$, $p_{sf}/p_i = 0.5$, $x_{eD} = y_{eD} = 10,000$ and $S_{wF,i} = 20\%$ and different $S_{wM,i}$
Figure B-17: Gas production type-curve for the system with $\omega = 0.01$, $p_{sf}/p_i = 0.7$, $x_e D = y_e D = 5,000$ and $S_{wF,i} = 80\%$ and different $S_{wM,i}$

Figure B-18: Water production type-curve for the system with $\omega = 0.01$, $p_{sf}/p_i = 0.7$, $x_e D = y_e D = 5,000$ and $S_{wF,i} = 80\%$ and different $S_{wM,i}$
Figure B-19: Gas production type-curve for the system with $\omega = 0.01$, $p_{sf}/p_i = 0.7$, $x_{cD} = y_{cD} = 5,000$ and $S_{wF,i} = 60\%$ and different $S_{wM,i}$.

Figure B-20: Water production type-curve for the system with $\omega = 0.01$, $p_{sf}/p_i = 0.7$, $x_{cD} = y_{cD} = 5,000$ and $S_{wF,i} = 60\%$ and different $S_{wM,i}$. 
Figure B-21: Gas production type-curve for the system with $\omega = 0.01$, $p_{sf}/p_i = 0.7$, $x_{eD} = y_{eD} = 5,000$ and $S_{wF,i} = 40\%$ and different $S_{wM,i}$

Figure B-22: Water production type-curve for the system with $\omega = 0.01$, $p_{sf}/p_i = 0.7$, $x_{eD} = y_{eD} = 5,000$ and $S_{wF,i} = 40\%$ and different $S_{wM,i}$
Figure B-23: Gas production type-curve for the system with $\omega = 0.01$, $p_{sf}/p_i = 0.7$, $x_{eD} = y_{eD} = 5,000$ and $S_{wF,i} = 20\%$ and different $S_{wM,i}$

Figure B-24: Water production type-curve for the system with $\omega = 0.01$, $p_{sf}/p_i = 0.7$, $x_{eD} = y_{eD} = 5,000$ and $S_{wF,i} = 20\%$ and different $S_{wM,i}$
Appendix C

Graphical User Interface

In this work, a Graphical User Interface (GUI) was designed and developed using Visual Basic (VB) to offer a user-friendly way to interact with the simulator. The development of the GUI was conducted in two parts including (1) the pre-processing unit and (2) the post-processing unit. The pre-processing unit allows the users to provide input data and generate an input data file. The post-processing unit provides graphical representations of simulation results. Figure C-1 shows the main panel of the GUI containing eight parts; each part is represented by different tab.

Figure C-1: GUI main panel
Pre-processing GUI

The pre-processing unit is used to gather input data from the users and generate the input data file. The input data can be categorized into seven groups including (1) grid properties, (2) rock & fluid properties, (3) component properties, (4) initial conditions, (5) sorption properties, (6) wells & recurrent and (7) numerical parameters. Each group is represented by a different tab as shown in Figure C-1.

Grid properties

The grid properties section allows the users to input number of grid blocks, size of grid blocks, thickness information and depth of the top of the formation (top depth). The size of grid blocks, thickness and top depth can be uniform or non-uniform. It is noted that the data for the case of non-uniform properties need to be entered in a matrix form. In addition, surface maps of thickness and top depth can be obtained. Figure C-2 shows an example of grid properties section with data entries.
Figure C-2: Grid properties section

Rock and fluid properties

The rock and fluid properties section contains five sub-sections including (1) porosity, fracture spacing and coal density, (2) permeability, (3) relative permeability and capillary pressure, (4) viscosity, formation volume factor and fluid density, and (5) coal shrinkage and swelling. Fracture porosity, matrix porosity, coal density and permeabilities in $x$-, $y$- and $z$-directions can be uniform throughout the reservoir, uniform through each layer or non-uniform. In addition, surface maps of porosities and permeabilities are available. Relative permeability, capillary pressure, fluid viscosity, formation volume factor can be calculated using the correlations presented in Appendix A or provided by the users. The users have several options for coal shrinkage and swelling effects. Such options include (1) ignoring coal shrinkage and swelling effects, (2) using the existing coal shrinkage and swelling models (Sawyer et al., 1990;
Seidle and Huitt, 1995; Palmer and Mansoori, 1996; Shi and Durucan, 2005) and (3) using the proposed coal shrinkage and swelling model. Figure C-3 shows an example of rock and fluid properties section with data entries.

Figure C-3: Rock/fluid properties section

**Component properties**

The component properties section allows the users to input number of components, physical properties of each component and sorption equilibrium method. The number of components in gas phase can be up to 10. The options for sorption equilibrium method include ideal and real gas approaches. Figure C-4 shows an example of component properties section with data entries.
Initial conditions

The initial conditions section allows the users to input initial pressure, initial water saturation and initial mole fraction in the fracture and matrix domains. Each initial parameter can be uniformly distributed through the entire reservoir, uniformly distributed in each layer or non-uniform. Figure C-5 shows an example of initial conditions section with data entries.
Figure C-5: Initial conditions section

**Sorption properties**

The sorption properties section allows the users to input Langmuir sorption properties of each gas component, sorption capacity factor accounting for the effect of bound water in the coal matrix and sorption time constant. Figure C-6 shows an example of sorption properties section with data entries.
The wells and recurrent section allows the users to input well information including number of wells in the reservoir, well locations, well specification and well schedule. In addition, locations of inactive blocks can be identified in this section along with well location information. Well specification sub-section allows the users to provide well name, well mode (active or inactive), well type (producer, water injector, pure gas injector or mixed gas injector), well orientation (vertical or horizontal) and well specification (sandface pressure, water flow rate or gas flow rate). Well schedule for each well can be generated using well schedule sub-section, which contains well criteria and well command. Well criteria element offers several condition options where the users can select to specify the criterion used in changing well specification. Condition options include simulation time, sandface pressure, water flow rate and gas flow rate. Well command element is similar to well specification where the users can identify well
specification for the period after the condition specified in the well criteria element is met. Figure C-7 shows an example of wells & recurrent section with data entries.

Figure C-7: Wells & recurrent section

Numerical

The numerical parameter section allows the users to input criteria for numerical derivative calculation, convergence criteria, maximum time step size and total simulation time. The users have options to use the default criteria for numerical derivative calculation and convergence criteria presented in Chapter 6 or specify those parameters. Figure C-8 shows an example of numerical section with data entries.
The pre-processing unit also provides default properties available for the users. This option can be obtained using the “Default System” button at the bottom of the panel. The properties of the default system are presented in Table C-1, Figures 7-2 and 7-3. After all the input data are entered into the GUI panel, the input data file can be generated using the “Generate Input” button. This action will generate the input data called “data.dat” for the simulator.
Table C-1: Properties of the default system

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir size</td>
<td>1,100 ft × 1,100 ft</td>
</tr>
<tr>
<td>Reservoir thickness</td>
<td>10 ft</td>
</tr>
<tr>
<td>Top depth</td>
<td>1,000 ft</td>
</tr>
<tr>
<td>Fracture porosity</td>
<td>0.5%</td>
</tr>
<tr>
<td>Matrix porosity</td>
<td>3%</td>
</tr>
<tr>
<td>Fracture permeability</td>
<td>8 md</td>
</tr>
<tr>
<td>Matrix permeability</td>
<td>0.001 md</td>
</tr>
<tr>
<td>Coal density</td>
<td>1.435 g/cm³</td>
</tr>
<tr>
<td>Sorption time constant</td>
<td>1 day</td>
</tr>
<tr>
<td>Initial pressure</td>
<td>1,100 psia</td>
</tr>
<tr>
<td>Fracture spacing</td>
<td>0.5 ft</td>
</tr>
<tr>
<td>Initial water saturation, fracture</td>
<td>100 %</td>
</tr>
<tr>
<td>Initial water saturation, matrix</td>
<td>100 %</td>
</tr>
<tr>
<td>Langmuir volume constant</td>
<td>0.0010826 lb-mole/lb</td>
</tr>
<tr>
<td>Langmuir pressure constant</td>
<td>625 psi</td>
</tr>
<tr>
<td>Reservoir temperature</td>
<td>110 °F</td>
</tr>
<tr>
<td>Number of grid blocks in x, y and z directions</td>
<td>11, 11, 1</td>
</tr>
<tr>
<td>Well location</td>
<td>6, 6, 1</td>
</tr>
<tr>
<td>Well radius</td>
<td>0.25 ft</td>
</tr>
<tr>
<td>Well orientation</td>
<td>Vertical</td>
</tr>
<tr>
<td>Sandface pressure</td>
<td>14.7 psia</td>
</tr>
<tr>
<td>Skin factor</td>
<td>0</td>
</tr>
<tr>
<td>Maximum time step size</td>
<td>30 days</td>
</tr>
<tr>
<td>Total simulation time</td>
<td>6,000 days</td>
</tr>
</tbody>
</table>

After the input data file is generated, the users can make a simulation run using the “Run Simulation” button. This action will create a command prompt window shown in Figure C-9, which shows the progress of the simulation run. The default system is used for this example.
The post-processing unit of the GUI developed in this work offers graphical representations of the simulation results via MATLAB® platform. After the simulation run is completed, the users can generate the graphical representations of the following information for each phase: production rate, cumulative production, injection rate, cumulative injection, sorption rate, cumulative sorption, volume in place, maximum residual and incremental material balance. Also, the users can obtain the graphical representations the following information for each gas component: production rate, cumulative production, production mole fraction, injection rate, cumulative injection, sorption rate, cumulative sorption and volume in place. The aforementioned information can be obtained for the field and for each well. The field results can be obtained in the Phase Results and Compositional Results sections while the results for each well can be obtained in the Well Results section. Figure C-10 shows an example for the simulation results.
section. Figure C-11 shows the plot of gas production rate versus time obtained using the post-processing unit.

Figure C-10: Simulation results section
Figure C-11: Gas production rate versus time plot
Vita

Prob Thararoop was born in Bangkok, Thailand on June 16, 1981. He attended Triam Udom Suksa School and graduated in April 1998. Mr. Thararoop enrolled at the Chulalongkorn University in June 1998 and graduated with a Bachelor of Engineering degree in petroleum engineering in April 2002. He started working in June 2002 at Unocal Thailand, Ltd. He worked for Unocal Thailand, Ltd. for 3 years as a petroleum engineer. Responsibilities included reserves estimation, project economic analysis, well depletion plan for infill drilling projects, capacity/production forecast, well surveillance, production optimization and well problem analysis. Then, he attended the Pennsylvania State University and received a Master of Science in petroleum and natural gas engineering in May 2007. He continued his education at the Pennsylvania State University where he pursued a doctoral degree in petroleum and natural gas engineering.

During his studies at the Pennsylvania State University, he worked as a teaching/research assistant and he was responsible for assisting undergraduate and graduate courses including energy and environment, environment protection, petroleum engineering design, reservoir modeling, numerical reservoir simulation and well testing. Mr. Thararoop received a Graduate Student Merit Award in April 2007 and he received another Graduate Student Merit Award and Graduate Assistant of the Year Award in April 2009.

Mr. Thararoop also worked as a petroleum engineering intern for Chevron Energy Technology Company during summers of 2008 and 2009. He has authored and co-authored multiple publications in the areas of artificial neural network applications development for field development and development of the coal shrinkage and swelling model.