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TIME ALLOCATION IN PROJECTS UNDER UNCERTAINTY:

A ROBUST OPTIMIZATION APPROACH

A Thesis in

Industrial Engineering and Operations Research

by

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ABSTRACT

Traditional models of project management have laid emphasis on scheduling of operations to meet deadlines. The research presented here approaches project management as a resource allocation problem where project completion is constrained by the man-hours available from those involved in the project. The variations in the relation between completion of project and time allocated to it are influenced by subjective factors. Hence there is a problem of inability to fit a standard or well-defined distribution. This thesis addresses this problem by a robust optimization approach. The budget of uncertainty is set by the decision maker, consequently controlling the extent of variation in the parameters whose bounds of uncertainty are known. The optimal allocation of time to the tasks in the project has been provided.

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Chapter I

INTRODUCTION

In the product development segment of an automotive firm, the roll-out of a new model involves the development of all the parts that go into building it. The project is broken down into sub-projects, and each is a conglomeration of parts that go through different stages of development, prototyping, testing, etc. Most of these operations are done in parallel, and are constrained not only by the deadline for completion, but also by the number of people working on them, or effectively, the number of man-hours invested in them. In the sense of a project having a specific team assigned and members allocating certain hours in working for that project, it can be seen that the more time spent on a project within the scheduled deadline, the more development or completion it attains. This encourages project management to be approached as a resource allocation problem.

Furthermore, subjective factors or human element being involved in the relationship between the completion of a project and the hours worked by the people in it, it is not always easy to assign standard properties to variations as would be observed in machine outputs or statistical data. Due to the inappropriateness of fitting a distribution to such phenomenon, it would be safer to work with the bounds of this variation rather than assuming functional properties of the randomness. This can be approached using robust optimization (RO) that uses the worst-case value of the observed parameters rather than a wrongfully measured average.

Chapter II presents an overview of contributions of operations research to automotive decision making processes, existing work that has influenced the model and the approach to the problem addressed in this paper. Chapter III explains the model in a deterministic case and the modifications made to implement a robust formulation. The subsequent chapters present the results for a sample problem, their interpretation and directions for future work in this problem.

Chapter II

LITERATURE REVIEW

a. Optimization models in the automotive industry

The automotive industry has relied heavily on operations research methodologies for its growth, development and expansion. In particular, mathematical programming has been used in making key decisions in globalizing the company's base and building an international network of operations. The approach made to the research presented here is heavily influenced by the work of Loch et al (2001) in addressing project management problems at BMW. An MILP was built for the problem of selecting projects in the company, whose objective was to minimize the shortfall or the gap in the contribution of a project to the company. The problem was constrained by the available development capacity, or the human capital in person-years required for each project. This model, although theoretically successful, was adopted only partially since the effort invested in constrained optimization required considerable resources while yielding only a marginal improvement in the solution derived. The quantitative analysis of the problem, however, did increase transparency in the project selection strategies and criteria used by the management. The research also tried best to substitute the practice of subjective, intuition-based selection of R&D projects with detailed and structured data. In yet another business model developed for BMW (Fleischmann, Ferber and Henrich, 2006), the problem of assigning long-term production of different models to the various production facilities world-wide was addressed. It was modeled on the lines of a global supply chain. The solution was broken into phases, focusing first on an MIP model that analyzed the load on a facility and the strategic planning, studying its implications within the process. The next phase developed the model further to include financial variables and study the role or the impact of investment decisions on the sites themselves. The model promised optimistic results by showing reduction in investments and costs in the supply chain by upto 7%. A GUI is proposed to implement the model in the company for use by employees who need not possess knowledge of or expertise in OR. Meyr (2004) also presented a study of German automotive supply chain planning and organization for the recent trend of mass customization of cars. The problem of reducing lead times and lowering costs on catering to end-user customization demands was focused on. Various existing strategies such as Just-in-Time delivery and Sequence-in-Line supply were listed and their impact on delivery reliability measured on new practices like online ordering and late order assignment being rapidly adopted by German carmakers.

b. Robust Optimization models

It is necessary to understand the effect of randomness on time allocation to projects depending on the environment of operations. The manufacturing of products has been addressed from different perspectives such as rate of arrival of material for processing, completion time of mechanized tasks, conformity of measurements on finished product to required standards, etc. Such aspects of a project are measurable by standard formulation and more importantly, by the availability of well-defined distributions that can be approximated to these processes. In the design phase of products, however, all uncertainty in project completion arises from subjective factors. The very measure of project completion is computed using scoring models where the input are numbers presented based on the fulfillment of criteria as observed or analyzed by the personnel involved. Completion of a design is related to the amount of time spent on it. When required to complete one phase of a project by a deadline, time-scheduling of operations is used.

However, there is a possibility that one may be contributing to multiple projects at the same time. The required level of development in one project may be specified by the number of hours of work that would need to be dedicated for it. Hence, the overall completion of a project is constrained by sum total of hours available from everyone involved in the project.

The new and fast-developing field of robust optimization was researched to present a possible solution to the problem of relating completion of project to hours worked given in intervals like "*a* hours and a possibility of $\pm\ b$ hours". Bertsimas and Thiele (2006) present the definition to robust optimization and a reasonable comparison and contrast to stochastic optimization. Their research also proposes an alternative robust modeling of various problems currently addressed by stochastic programming, eliminating their existing shortcomings or constraints. In particular, the problem of capital budgeting and allocation has been modeled by this method in Bertsimas and Sim (2003) and referenced therein. The problem considers selection of projects and then allocation of resources partially to some projects and completely to the others. Since the model considered the aspect of choosing and rejecting projects, it was modeled with binary variables. This property of the decision variables provided convenient solutions for the inner maximization or minimization problems, that is, the dual of this inner optimization problem could be represented by closed form expressions. Kachani and Langella (2005) used this technique in their approach to nominal capital rationing. New KKT conditions (complementary slackness) were derived on the original deterministic model as a result. The solution of the robust optimization model also highlighted the trade-off involved in optimal portfolio selection and robustness of the portfolio. The case of an MILP being modified or reformulated to an RO model was studied in detail in Lin, Janak and Floudas (2004). Their research problem was a scheduling model under two cases of uncertainty: with bounds and with known probability distribution. The approach of interest to RO is narrowed down to the condition of bounded uncertainty with no assumptions on the type of distribution.

c. Resource Allocation models

Analogous to the scoring and weighting of individual parts in a project, Brenner (1994) considers scoring of a set of projects to enable project selection by a company so that the limited resources available can be distributed selectively for maximizing the completion of those chosen alone. The scoring method is built on the lines of Saaty's Analytical Hierarchy Process (AHP) in stages of identifying the goals, the criteria, and rating methodology. It is implemented using the standard AHP software Expert Choice. Resource allocation, however, is done according to the classification of the final selected set of projects. Categorizing projects as one of 'winning', 'difficult' or 'eliminated', the winning are completely funded, while the difficult are funded enough to overcome their demerits. The remaining projects are simply not selected for further development. A similar approach in the healthcare industry was highlighted by Kleinmuntz (1999) that used an eight-step approach for capital allocation. This method also followed the ideas of assigning weights and rating based on the cost as well as NPV of the projects or proposals. The problem being multi-objective, traditional optimization methods were used to find combinations of projects that would maximize company success, constrained by their budget.

Chapter III

PROBLEM FORMULATION

The case of project management in an automotive product development environment can be considered as a man-hour allocation problem. Consider a set of parts to be developed in parallel for a project, and the net growth of the project is given by the aggregate development or completion of the individual parts (analogous to project management, these can be termed as sub-projects or tasks) in the project. The importance of each task to the overall project is specified by a weight associated with it. Using standard scoring models for evaluation of project completion and success, each task can also be given a score for its development. Comparing the current development of a part and the required development for an acceptable completion of the overall project, the problem becomes one of minimizing this aggregated gap weighted by the importance of each part to the product. The development of a part is constrained by the amount of time that can be spent for each of them. Using a function to represent the relationship between hours allocated and development achieved for a part, the problem now becomes one of resource allocation as the project has only a fixed number of man-hours that can be assigned to all the parts.

Let

$$\begin{aligned}
n &= \text{No. of parts} \\
w_i &= \text{Weight (importance) of part i in project} \\
D_i &= \text{Current development score of part i} \\
D_i{*} &= \text{Required development score of part i} \\
D_i{'} &= \text{Minimum acceptable development score of part i} \\
t_i &= \text{Time (man-hours) allocated to part i} \\
T &= \text{Total time}\left(\text{man-hours}\right)\text{available}
\end{aligned}$$

State of the system: D_i

Output of interest: t_i

The development to be achieved in the design of part i should meet the minimum requirements in the current stage of the project. This has been assigned as D_i'. Furthermore, in the case of expensive resources or even limited resources, the over-achievement of target development may not be required. To limit this over-use of resources, a required development score D_i* has been set. In problems where this may not be applicable, D_i* can be set to the maximum score attainable. Hence,

$$D_i' \le D_i \le D_i* \tag{1}$$

The amount of time required to achieve a certain score of development can be obtained through a study or mapping of score achieved with time input. For our problem, we will assume and approximate this relation to be linear. Setting a as the fixed amount of hours invested for any part and b_i as the scaling factor that decides how many hours it will take to achieve an incremental score of 1 for part i, we can say

$$t_i = a + b_i D_i \tag{2}$$

We assume that this scaling factor b_i is affected by uncertainty, that is, the time to complete the design for a score to be achieved will vary by the task. This uncertainty can be assumed to be symmetric and summarized as

$$b_i \in (\overline{b}_i - \hat{b}_i, \overline{b}_i + \hat{b}_i) \tag{3}$$

Furthermore, the total time available for the project is limited by the number of hours that the personnel involved are able to dedicate to the project. Hence,

$$\sum_{i=1}^{n} t_i \le T$$

or using in (2),

$$na + \sum_{i=1}^{n} b_i D_i \leq T \tag{4}$$

In light of these constraints, we seek to achieve the highest development possible. This can be modeled as trying to minimize the gap between the ideal development score and the achievable development score. Also, each part is weighted by its importance to the project. These weights (w_i) are specified by the decision maker. The objective becomes:

$$Minimize \sum_{i=1}^{n} w_i (D_i * - D_i) \tag{5}$$

where

$$0 \leq w_i \leq 1 \text{ and } \sum_{i=1}^{n} w_i = 1$$

Combining equations (1)–(5), the **nominal problem of time allocation in a project** is

(N): Minimize $\sum_{i=1}^{n} w_i (D_i * - D_i)$

subject to $na + \sum_{i=1}^{n} b_i D_i \leq T$

$b_i \in (\overline{b}_i - \hat{b}_i, \overline{b}_i + \hat{b}_i) \quad \forall i$

$D_i' \leq D_i \leq D_i * \quad \forall i$

$D_i \geq 0 \quad \forall i$

The reformulation to model (N) through robust optimization follows Bertsimas and Thiele (2006) which draws heavily upon the theory of Bertsimas and Sim (2004). These references also use two-dimensional parameter matrix **A** for the constraint of type **Ax ≤ b**, whereas the problem considers only a vector for **A**. There is also only one constraint affected by uncertainty. Taking these into consideration while following the approach suggested by Bertsimas and Sim (2004), the steps in reformulation of (N) into robust model (R) through intermediary programs (denoted by T_k, k = 1, 2, 3, 4) are presented here.

Given $b_i \in (\bar{b}_i - \hat{b}_i, \bar{b}_i + \hat{b}_i) \quad \forall i$,

Let $\quad x_i = \dfrac{b_i - \bar{b}_i}{\hat{b}_i} \quad \forall i \qquad$ be the scaled deviation of parameter b_i.

Defining

J = set of coefficients b_i subject to uncertainty

Γ = budget of uncertainty, or number of coefficients that vary from nominal value

we can set Γ to any value in $\left[0, |J|\right]$. Note that Γ need not necessarily be an integer. Specific cases include –

- $\Gamma = 0$: implies that none of the coefficients will vary from their nominal value. Hence the problem is deterministic, or the same as the nominal LP. The solution has no protection from uncertainty.
- $\Gamma = |J|$: implies all the elements of J vary to their extreme. If $|J| = n$, the total number of tasks, then this represents the worst-case of the problem where every parameter varies. The model is protected from all uncertainty, and hence provides a highly conservative solution.

The budget of uncertainty is incorporated into the formulation such that upto $\lfloor \Gamma \rfloor$ parameters can vary, and one b_i varies by $\left(\Gamma - \lfloor \Gamma \rfloor\right)\hat{b}_i$, to account for the (non-integer) Γ variations in the model. In order to maintain the feasibility of the problem, the largest such deviation should satisfy constraint (4). Including uncertainty of b_i and Γ into problem (N) yields:

(T_1): Minimize $$\sum_{i=1}^{n} w_i(D_i * - D_i)$$

subject to $$na + \sum_{i=1}^{n} \bar{b}_i D_i + \max_{t \in J}\left(\sum_{i=1}^{n} \hat{b}_i y_i + \left(\Gamma - \lfloor \Gamma \rfloor\right) \hat{b}_t y_t \right) \le T$$

$$-y_i \le D_i \le y_i \quad \forall i$$

$$D_i' \le D_i \le D_i * \quad \forall i$$

$$D_i \ge 0 \quad \forall i$$

At optimality,

$$y_i = \left| D_i^{opt} \right|$$

The inner maximization problem of T_1 has been shown (Proposition 1, Bertsimas and Sim (2004)) to be equivalent to:

(T_2): Maximize $$\sum_{i \in J} \hat{b}_i \left| D_i^{opt} \right| x_i$$

subject to $$\sum_{i \in J} x_i \le \Gamma$$

$$0 \le x_i \le 1 \quad \forall i \in J$$

In order to solve it, the dual of (T_2), the equivalent to the inner maximization problem can be written. Let z be the dual variable corresponding to the constraint $\sum_{i \in J} x_i \le \Gamma$ and p_i be the set of dual variables for the constraints $0 \le x_i \le 1 \quad \forall i \in J$. Then, the dual problem is:

(T_3): Minimize $$\Gamma z + \sum_{i \in J} p_i$$

subject to $$z + p_i \ge \hat{b}_i \left| D_i^{opt} \right| \quad \forall i \in J$$

$$p_i \ge 0 \quad \forall i \in J$$

$$z \geq 0$$

It has been shown (Theorem 1, Bertsimas and Sim (2004)) that applying strong duality, problem (T_2) is feasible and bounded, following $\Gamma \in [0, |J|]$. Hence the dual problem (T_3) is also bounded and feasible. Since their objective function values are equal, and the objective functions themselves are shown to be equivalent to the inner maximization problem of (T_1), simple substitution yields:

(T_4): Minimize $\sum_{i=1}^{n} w_i (D_i * - D_i)$

subject to
$$na + \sum_{i=1}^{n} \bar{b}_i D_i + z\Gamma + \sum_{i=1}^{n} p_i \leq T$$

$$z + p_i \geq \hat{b}_i \left| D_i^{opt} \right| \quad \forall i \in J$$

$$D_i ' \leq D_i \leq D_i * \quad \forall i$$

$$D_i \geq 0 \quad \forall i$$
$$p_i \geq 0 \quad \forall i$$
$$z \geq 0$$

Relaxing the right side of the second constraint in T_4 to linearize the problem, we replace $\left| D_i^{opt} \right|$ by the variable originally in its place, y_i. The substitution used was that, at optimality, $y_i = \left| D_i^{opt} \right|$ and it is related to the decision variable D_i by the inequality $-y_i \leq D_i \leq y_i \quad \forall i$. However, D_i is a non-negative variable, hence at optimality, $y_i = D_i^{opt}$. Therefore, in model (T_4), it is possible to replace $\left| D_i^{opt} \right|$ by D_i itself. This gives the equivalent model for **robust optimization of time allocation in a project**:

(R): Minimize $\sum_{i=1}^{n} w_i(D_i * - D_i)$

subject to $na + \sum_{i=1}^{n} \bar{b}_i D_i + z\Gamma + \sum_{i=1}^{n} p_i \leq T$

$z + p_i \geq \hat{b}_i D_i \quad \forall i$

$D_i' \leq D_i \leq D_i * \quad \forall i$

$p_i \geq 0 \quad \forall i$

$z \geq 0$

It has been shown that the choice of Γ affects the objective function, i.e. conservatism of the resulting objective value depends on the level of uncertainty in the constraints, or the possibility of its violation due to the parameter variation. This expression has also been summarized in Bertsimas and Thiele (2006), to show that for a constraint to be violated with a probability of at most ε with n sources of uncertainty, the minimum choice of Γ should be

$$\Gamma = 1 + \Phi^{-1}(1-\varepsilon)\sqrt{n} \tag{6}$$

where Φ is the CDF of the standard normal distribution. Using this expression, it was possible to narrow the guarantee of conformance to the constraints to be able to get a feasible solution. i.e., fixing ε could show the minimum protection against uncertainty in the model. For the extreme cases of Γ = 0 and Γ = n, the model gives the deterministic solution (all parameters are at their expected value) and worst case situation (all parameters are at their extremes) respectively. This relation has been plotted in Figure 1 for values n = 5 and ε upto 0.7.

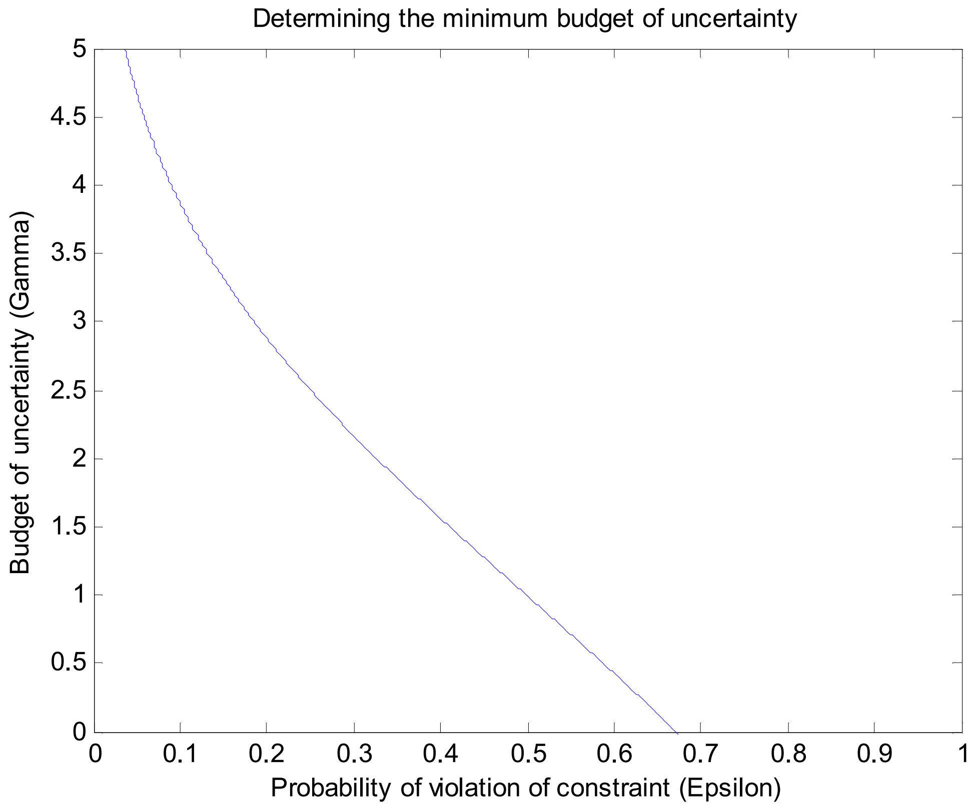

Figure 1: Level of protection of robust solution against uncertainty in the parameters

Chapter IV

IMPLEMENTATION AND RESULTS

A toy problem of n = 5 parts was taken up. The weights w_i, fixed working hours per part a, part-dependant working hours b_i, total time available T, acceptable and required levels of development were assigned fixed values. These values are presented in Table 1. The difference in performance between the nominal formulation and the robust formulation for various levels of the budget of uncertainty was studied.

Table 1: Values of parameters in toy model with n = 5

Scalars	$n = 5$	$a = 4$ (hours)

Part Index *i*	**Weight of part *i* in project w_i**	**Required development score of part *i* D_i^***	**Acceptable development score of part *i* D_i'**	**Expected value of scaled duration of work for part *i* $\bar{b}_i$ (hours)**	**Maximum deviation of scaled duration of work for part *i* $\hat{b}_i$ (hours)**
1	0.1	7	4	3	1
2	0.1	7	4	3.5	1
3	0.3	7	4	4	2.5
4	0.2	7	4	4	2
5	0.3	7	4	5	2.5

The best objective function value, i.e. the minimum gap between target completion and best possible completion is 0 for all parts achieving the required development (score = 7), and the worst case value is 3 for a feasible problem achieving only the minimum acceptable level of development (score = 4). Using $na + \sum_{i=1}^{n} b_i D_i \leq T$, the upper and lower bounds on T to have a feasible nominal problem were found to be 156.5 hours and 98 hours respectively. For various T within these bounds, the model was run. Also, for each T, different values of Γ were set. The model was run in GAMS for each pair of (T, Γ). Each run took between 6 and 10 iterations on the OSL solver. Figure 2 shows the resulting objective values for each (T, Γ) pair. Each curve represents the net difference between required development of the overall project ($\sum_{i=1}^{n} w_i D_i{}^*$) and achievable development of the project ($\sum_{i=1}^{n} w_i D_i$) under the given constraints.

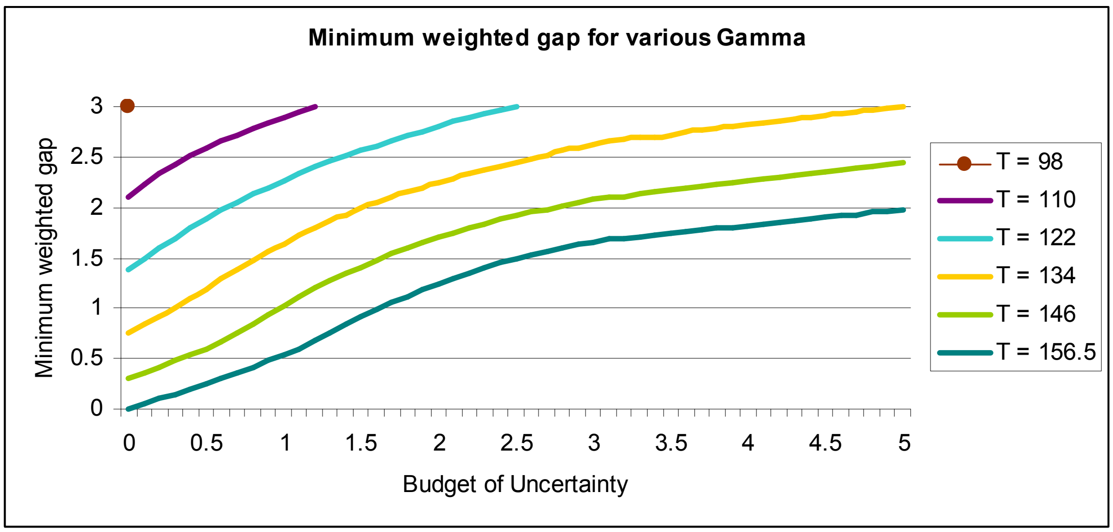

Figure 2: Objective function values for various (*T*, Γ)

Figure 3 is a useful interpretation of the same results, showing the achievable development of the overall project, taken by the weighted sum of individual parts' development score ($\sum_{i=1}^{n} w_i D_i$).

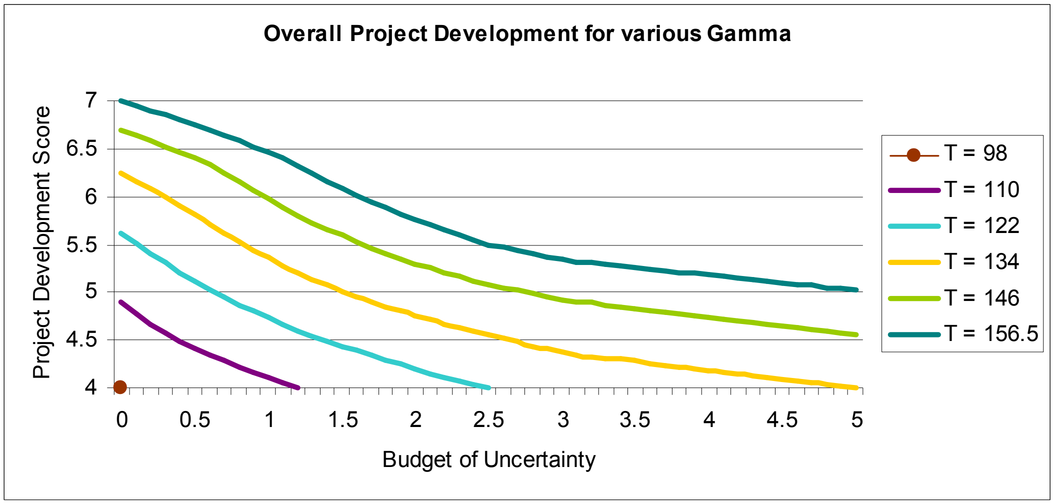

Figure 3: Overall project development for various (T, Γ)

The figures presented in this section prove that higher the number of variations in the model, lower the achievable development. This is also proof of the conservatism of the solution of the robust problem compared to nominal problem. The decline in performance is seen to be steep as the nominal model initially progresses towards uncertainty. The rate of decrease in the overall project development is lower for large Γ.

Furthermore, inadequate time (low T) coupled with large number of parameter variations (high Γ) drive the model to infeasibility. In these cases, even the minimum acceptable development cannot be achieved. Only those pairs of (T, Γ) that were feasible are represented in the graphs. The infeasibility results from constraint violation explained in Section III. It can be seen that higher the Γ, smaller the feasible region of the robust problem, thus giving a more conservative solution.

Once the optimal D_i was established, the solution of interest (time allocation) is obtained through:

$$t_i = a + b_i D_i$$

All numerical computations from the runs are presented in Appendix B.

Chapter V

FUTURE SCOPE

The robust optimization methodology helped show the impact of the number of sources of uncertainty in achieving a target performance of the objective function. Setting a low T value does not give the model adequate flexibility for time allocation. It is not possible to fulfill the minimum development requirements when there is small time available and there are a large number of parameters varying. Hence, lack of control on the parameters' variation yielded a highly conservative solution and the model was seen to be infeasible for certain pairs of (T, Γ). This preliminary model can be used to develop a multi-period approach to robust optimization of resource allocation in a project, where the cost of a resource would be taken into consideration in order to optimally use it in different phases of the project. Such an approach could stem from the theory laid out by Bertsimas and Caramanis (2007). One of the initial proposals of that research is to develop a two-stage optimization problem in order to model adaptability, and then to convert the static robust formulation into an adaptable formulation using different levels of adaptability. The scope of the problem could be expanded from symmetric uncertainty considered here to other types.

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Appendix A

COMPUTATIONAL RESULTS

Table 2: Minimum gap between required project development and achievable project development

Budget of Uncertainty Γ	Total time available (maximum amount of resource) in hours					
	T = 98	T = 110	T = 122	T = 134	T = 146	T = 156.5
0	3	2.1	1.38	0.75	0.3	0
0.1	Infeasible	2.22	1.49	0.84	0.36	0.05
0.2	Infeasible	2.33	1.59	0.92	0.42	0.1
0.3	Infeasible	2.43	1.69	1.01	0.48	0.15
0.4	Infeasible	2.52	1.8	1.1	0.53	0.2
0.5	Infeasible	2.59	1.89	1.19	0.59	0.25
0.6	Infeasible	2.66	1.97	1.29	0.67	0.3
0.7	Infeasible	2.72	2.05	1.38	0.76	0.36
0.8	Infeasible	2.78	2.13	1.47	0.85	0.42
0.9	Infeasible	2.84	2.2	1.56	0.94	0.48
1	Infeasible	2.9	2.27	1.64	1.03	0.53
1.1	Infeasible	2.95	2.34	1.72	1.11	0.59
1.2	Infeasible	3	2.4	1.8	1.2	0.68
1.3	Infeasible	Infeasible	2.46	1.86	1.27	0.76
1.4	Infeasible	Infeasible	2.51	1.93	1.34	0.84
1.5	Infeasible	Infeasible	2.56	1.99	1.41	0.92
1.6	Infeasible	Infeasible	2.61	2.04	1.48	0.99
1.7	Infeasible	Infeasible	2.66	2.1	1.54	1.06
1.8	Infeasible	Infeasible	2.71	2.15	1.6	1.12
1.9	Infeasible	Infeasible	2.75	2.2	1.65	1.18
2	Infeasible	Infeasible	2.8	2.25	1.7	1.24
2.1	Infeasible	Infeasible	2.85	2.29	1.75	1.3
2.2	Infeasible	Infeasible	2.89	2.34	1.79	1.35
2.3	Infeasible	Infeasible	2.93	2.37	1.84	1.4
2.4	Infeasible	Infeasible	2.96	2.41	1.88	1.45
2.5	Infeasible	Infeasible	3	2.45	1.92	1.5
2.6	Infeasible	Infeasible	Infeasible	2.48	1.95	1.53
2.7	Infeasible	Infeasible	Infeasible	2.52	1.98	1.56
2.8	Infeasible	Infeasible	Infeasible	2.56	2.02	1.6
2.9	Infeasible	Infeasible	Infeasible	2.59	2.05	1.63
3	Infeasible	Infeasible	Infeasible	2.63	2.08	1.66
3.1	Infeasible	Infeasible	Infeasible	2.65	2.1	1.68
3.2	Infeasible	Infeasible	Infeasible	2.67	2.11	1.69
3.3	Infeasible	Infeasible	Infeasible	2.69	2.13	1.71

3.4	Infeasible	Infeasible	Infeasible	2.7	2.15	1.72
3.5	Infeasible	Infeasible	Infeasible	2.72	2.17	1.74
3.6	Infeasible	Infeasible	Infeasible	2.74	2.19	1.76
3.7	Infeasible	Infeasible	Infeasible	2.76	2.21	1.77
3.8	Infeasible	Infeasible	Infeasible	2.78	2.22	1.79
3.9	Infeasible	Infeasible	Infeasible	2.8	2.24	1.8
4	Infeasible	Infeasible	Infeasible	2.82	2.26	1.82
4.1	Infeasible	Infeasible	Infeasible	2.83	2.28	1.84
4.2	Infeasible	Infeasible	Infeasible	2.85	2.3	1.85
4.3	Infeasible	Infeasible	Infeasible	2.87	2.32	1.87
4.4	Infeasible	Infeasible	Infeasible	2.89	2.34	1.88
4.5	Infeasible	Infeasible	Infeasible	2.91	2.35	1.9
4.6	Infeasible	Infeasible	Infeasible	2.93	2.37	1.92
4.7	Infeasible	Infeasible	Infeasible	2.94	2.39	1.93
4.8	Infeasible	Infeasible	Infeasible	2.96	2.41	1.95
4.9	Infeasible	Infeasible	Infeasible	2.98	2.43	1.96
5	Infeasible	Infeasible	Infeasible	3	2.45	1.98

Table 3: Achievable project development for a given (T, Γ)

Budget of Uncertainty Γ	**Total time available (maximum amount of resource) in hours**					
	T = 98	**T = 110**	**T = 122**	**T = 134**	**T = 146**	**T = 156.5**
0	4	4.9	5.62	6.25	6.7	7
0.1	Infeasible	4.78	5.51	6.16	6.64	6.95
0.2	Infeasible	4.67	5.41	6.08	6.58	6.9
0.3	Infeasible	4.57	5.31	5.99	6.52	6.85
0.4	Infeasible	4.48	5.2	5.9	6.47	6.8
0.5	Infeasible	4.41	5.11	5.81	6.41	6.75
0.6	Infeasible	4.34	5.03	5.71	6.33	6.7
0.7	Infeasible	4.28	4.95	5.62	6.24	6.64
0.8	Infeasible	4.22	4.87	5.53	6.15	6.58
0.9	Infeasible	4.16	4.8	5.44	6.06	6.52
1	Infeasible	4.1	4.73	5.36	5.97	6.47
1.1	Infeasible	4.05	4.66	5.28	5.89	6.41
1.2	Infeasible	4	4.6	5.2	5.8	6.32
1.3	Infeasible	Infeasible	4.54	5.14	5.73	6.24
1.4	Infeasible	Infeasible	4.49	5.07	5.66	6.16
1.5	Infeasible	Infeasible	4.44	5.01	5.59	6.08
1.6	Infeasible	Infeasible	4.39	4.96	5.52	6.01
1.7	Infeasible	Infeasible	4.34	4.9	5.46	5.94
1.8	Infeasible	Infeasible	4.29	4.85	5.4	5.88
1.9	Infeasible	Infeasible	4.25	4.8	5.35	5.82

2	Infeasible	Infeasible	4.2	4.75	5.3	5.76
2.1	Infeasible	Infeasible	4.15	4.71	5.25	5.7
2.2	Infeasible	Infeasible	4.11	4.66	5.21	5.65
2.3	Infeasible	Infeasible	4.07	4.63	5.16	5.6
2.4	Infeasible	Infeasible	4.04	4.59	5.12	5.55
2.5	Infeasible	Infeasible	4	4.55	5.08	5.5
2.6	Infeasible	Infeasible	Infeasible	4.52	5.05	5.47
2.7	Infeasible	Infeasible	Infeasible	4.48	5.02	5.44
2.8	Infeasible	Infeasible	Infeasible	4.44	4.98	5.4
2.9	Infeasible	Infeasible	Infeasible	4.41	4.95	5.37
3	Infeasible	Infeasible	Infeasible	4.37	4.92	5.34
3.1	Infeasible	Infeasible	Infeasible	4.35	4.9	5.32
3.2	Infeasible	Infeasible	Infeasible	4.33	4.89	5.31
3.3	Infeasible	Infeasible	Infeasible	4.31	4.87	5.29
3.4	Infeasible	Infeasible	Infeasible	4.3	4.85	5.28
3.5	Infeasible	Infeasible	Infeasible	4.28	4.83	5.26
3.6	Infeasible	Infeasible	Infeasible	4.26	4.81	5.24
3.7	Infeasible	Infeasible	Infeasible	4.24	4.79	5.23
3.8	Infeasible	Infeasible	Infeasible	4.22	4.78	5.21
3.9	Infeasible	Infeasible	Infeasible	4.2	4.76	5.2
4	Infeasible	Infeasible	Infeasible	4.18	4.74	5.18
4.1	Infeasible	Infeasible	Infeasible	4.17	4.72	5.16
4.2	Infeasible	Infeasible	Infeasible	4.15	4.7	5.15
4.3	Infeasible	Infeasible	Infeasible	4.13	4.68	5.13
4.4	Infeasible	Infeasible	Infeasible	4.11	4.66	5.12
4.5	Infeasible	Infeasible	Infeasible	4.09	4.65	5.1
4.6	Infeasible	Infeasible	Infeasible	4.07	4.63	5.08
4.7	Infeasible	Infeasible	Infeasible	4.06	4.61	5.07
4.8	Infeasible	Infeasible	Infeasible	4.04	4.59	5.05
4.9	Infeasible	Infeasible	Infeasible	4.02	4.57	5.04
5	Infeasible	Infeasible	Infeasible	4	4.55	5.02

Appendix B

GAMS CODE

```
sets
        i index 1 /part1 * part5/
;

positive variables
        d, z, p
;

variables
        netd
;

equations
        obj_nom
        time_nom

        obj_rob
        time_rob
        var_rob


        d_boundlow
        d_boundhigh
        timecalc
;

parameter
        n         /5/
        w(i)     /part1=0.1,part2=0.1,part3=0.3,part4=0.2,part5=0.3/
        dhigh(i) /part1=7.0,part2=7.0,part3=7.0,part4=7.0,part5=7.0/
        dlow(i)  /part1=4.0,part2=4.0,part3=4.0,part4=4.0,part5=4.0/
        a        /4/
        b_mean(i)/part1=3.0,part2=3.5,part3=4.0,part4=4.0,part5=5.0/
        b_var(i) /part1=1.0,part2=1.0,part3=2.5,part4=2.0,part5=2.5/
        budget   /1/
        maxtime  /98/
        t(i)
;



obj_nom..       netd =e= sum(i, w(i)*(dhigh(i)-d(i)));
time_nom..      n*a + sum(i, b_mean(i)*d(i)) =l= maxtime;

obj_rob..       netd =e= sum(i, w(i)*(dhigh(i)-d(i)));
```

```
time_rob..
n*a+sum(i,b_mean(i)*d(i))+z*budget+sum(i,p(i))=l=maxtime;

var_rob(i)..    z + p(i) =g= b_var(i)*d(i);

d_boundlow(i)..       d(i) =g= dlow(i);
d_boundhigh(i)..      d(i) =l= dhigh(i);


model nominal
/
        obj_nom
        time_nom
        d_boundlow
        d_boundhigh
/;

model robust
/
        obj_rob
        time_rob
        var_rob
        d_boundlow
        d_boundhigh
/;


*solve nominal using lp minimizing netd;

file output /result.txt/;

for (budget = 0 to 5 by 0.1,

solve robust using lp minimizing netd;

put output;
put /;
put budget, netd.l;


);
```