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DE SITTER VACUA AND $\mathcal{N} = 2$ SUPERGRAVITY

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by
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Abstract

After reviewing the existing results we give an extensive analysis of the critical points of the potentials of the gauged $\mathcal{N} = 2$ Yang-Mills/Einstein Supergravity theories coupled to tensor- and hyper multiplets in five dimensions. Our analysis includes all the possible gaugings of all $\mathcal{N} = 2$ Maxwell-Einstein supergravity theories whose scalar manifolds are symmetric spaces. In general, the scalar potential gets contributions from R -symmetry gauging, tensor couplings and hyper-couplings. We show that the coupling of a hypermultiplet into a theory whose potential has a non-zero value at its critical point, and gauging a compact subgroup of the hyperscalar isometry group will only rescale the value of the potential at the critical point by a positive factor, and therefore will not change the nature of an existing critical point. However this is not the case for non-compact $SO(1, 1)$ gaugings. An $SO(1, 1)$ gauging of the hyper isometry will generally lead to de Sitter vacua, which is analogous to the ground states found by simultaneously gauging $SO(1, 1)$ symmetry of the real scalar manifold of the five dimensional vector multiplets with $U(1)_R$ in earlier literature. $SO(m, 1)$ gaugings with $m > 1$, which give contributions to the scalar potential only in the Magical Jordan family theories, on the other hand, do not lead to de Sitter vacua. Anti-de Sitter vacua are generically obtained when the $U(1)_R$ symmetry is gauged. We also show that it is possible to embed certain generic Jordan family theories into the Magical Jordan family while preserving the nature of the ground states. However the Magical Jordan family theories admit additional vacua which are not found in the generic Jordan family theories.

The five dimensional stable de Sitter ground states obtained by gauging $SO(1, 1)$ symmetry of the real symmetric scalar manifold (in particular a generic Jordan family manifold of the vector multiplets) simultaneously with a subgroup R_s of the R -symmetry group descend to four dimensional de Sitter ground states under certain conditions. First, the holomorphic section has to be chosen carefully by using the symplectic freedom in four dimensions; and second, a group contraction supported by a rotation of the gauge group generators that carries them into a particular basis is necessary to bring the potential into a desired form. Under this construction, stable de Sitter vacua can be obtained in dimensionally reduced theories (from 5D to 4D) if there is enough freedom to gauge a semi-direct product of $SO(1, 1)$ with $\mathbb{R}^{(1,1)}$ together with a simultaneous R_s . We review the stable de Sitter vacua

in four dimensions found in earlier literature for $\mathcal{N} = 2$ Yang-Mills Einstein supergravity with $SO(2,1) \times R_s$ gauge group in a symplectic basis that comes naturally after dimensional reduction. Although this particular gauge group does not descend directly from five dimensions, we show that, its contraction does. Hence, two different theories overlap in certain limits. Examples of stable de Sitter vacua are given for the cases: (i) $R_s = U(1)_R$, (ii) $R_s = SU(2)_R$, (iii) $\mathcal{N} = 2$ Yang-Mills/Einstein Supergravity theory coupled to a universal hypermultiplet. These are followed by a discussion regarding the extension of our results to supergravity theories with more general homogeneous scalar manifolds.

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Conventions

Indices:

- $\hat{\mu}, \hat{\nu}, \dots$ Curved space-time indices in 5 dimensions
 μ, ν, \dots Curved space-time indices in 4 dimensions
 $\tilde{I}, \tilde{J}, \dots$ Vector and tensor fields combined
 I, J, \dots Vector fields
 M, N, \dots Tensor fields
 $\tilde{a}, \tilde{b}, \dots$ Flat indices of the target manifold of real scalar fields
 $\tilde{x}, \tilde{y}, \dots$ Curved indices of the target manifold of real scalar fields
 A, B, \dots Complex coordinates
 X, Y, \dots Quaternionic coordinates

Symbols and Abbreviations:

- $P_{(d)}, P_{(d)}^{(R)}, P_{(d)}^{(T)}, P_{(d)}^{(H)}$ Potential in d dimensions
MESGT Maxwell-Einstein Supergravity Theory
YMESGT Yang-Mills-Einstein Supergravity Theory
 $G_{(d)}$ Global symmetry group of a YMESGT in d dimensions
 $K_{(d)}$ Gauge group of a YMESGT in d dimensions
 C_{IJK} Totally symmetric, G -invariant tensor

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To my parents

Introduction

1.1 A Brief History

Unifying fundamental forces of nature has been one of the biggest challenges in theoretical physics. In the first half of the 20th century most of the effort regarding this matter was spent on unifying gravity and electromagnetism. Einstein, Kaluza, Eddington, Weyl and others tried to generalize the differential geometry approach of Einstein's general theory of relativity to obtain an electro-gravitational unification theory¹. The attempts on this unification ultimately failed, mainly because the pioneers of this school could not benefit from the mainstream developments in physics. These include the discovery and understanding of the weak and strong nuclear forces, of which little was known during that time.

The first successful partial unification theory of two of the fundamental forces was formulated by Weinberg, Salam and Glashow [2, 3, 4]. In their work in the 1960's they merged electromagnetism with weak interaction and asserted that above their unification energy on the order of $10^2 GeV$ both forces combine and act as the electroweak force. This is a $SU(2) \times U(1)$ gauge theory, with the photon being the electromagnetic gauge boson and W^\pm and Z being the weak interaction gauge bosons. W^\pm and Z bosons acquire mass due to the Higgs mechanism, while the photon remains massless. Below the unification energy the theory is spontaneously broken down to $U(1)_{em}$.

Later in the early 1970's, with the development of quantum chromodynamics (QCD), the strong interaction was formulated as the theory of interactions of gluons and quarks. Together with the electroweak interaction, they formed the *Standard Model* (SM) of particle physics that has the combined gauge group $SU(3) \times SU(2) \times U(1)$. Eight massless gluons ($SU(3)$ gauge fields) mediate the strong interactions between color carrying quarks which, among other particles, combine into nucleons which together with the leptons constitute the matter we see. To date, most of the experimental data agree with the predictions of the SM. However the theory also predicts the existence of a particle, called the *Higgs*

¹For a review on *classical unified field theories* see [1].

boson which has not been detected until now. The easiest way to explain how physical matter acquires mass is established via Higgs mechanism [5]. This theory, applied to the SM, predicts a particle (Higgs boson) that has a non-zero vacuum expectation value and gives mass to the elementary particles including itself. The fact that the Higgs boson has not been experimentally observed might change once the Large Hadron Collider (LHC) at CERN starts operating later this year. With the discovery of the neutrino oscillations the SM needs to be enlarged also to accommodate the right-handed neutrinos.

Soon after the foundation of the SM, attempts have been made to embed the SM gauge group $SU(3) \times SU(2) \times U(1)$ in a larger simple group that remains unbroken at high energies. These are known as the Grand Unified Theories (GUT's). The unification is predicted to occur at an energy level on the order of $10^{15} GeV$ which is out of reach of particle accelerators. SM has 19 parameters such as the coupling constants, masses and mixing angles, that are put in the theory by hand to match the experimental results. GUT's try to reduce the number of free parameters, from which the SM parameters can be derived. Each GUT is based on a simple group that has a single coupling constant instead of the 3 in SM. In 1974 Georgi and Glashow showed that each of the three generations of chiral fermions (quarks and leptons) can be grouped into the reducible representation $\mathbf{5} \oplus \mathbf{10}$ of $SU(5)$ which contains the SM gauge group as a subgroup [6]. The other well-known GUT's are the $SO(10)$ model of Georgi [7] and Fritsch-Minkowski [8]; and the $E(6)$ model of Gürsey-Ramond-Sikivie [9]. These theories predict some of the standard model parameters, such as the weak mixing angle, with success but at the same time they brought their own problems. For instance in the Georgi-Glashow model the Higgs doublets, which give masses to chiral fermions at the scale that the electroweak symmetry breaks down, must sit in the $\mathbf{5}$ or $\bar{\mathbf{5}}$ representation of $SU(5)$. The other 3 states are color charged Higgs scalars. If these 3 scalars have low (electroweak scale) masses, then the theory predicts proton decay at a rate that contradicts the observations. In order to prevent such a decay, the 3 Higgs scalars must have much higher masses, close to the GUT scale, but then the so called "doublet-triplet splitting problem" emerges: How can the doublet stay so light while the triplet has such a high mass? The situation for the models with larger gauge groups is even less appealing because they introduce more unobserved Higgs states.

GUT's and (despite being an almost complete theory) Standard Model do not include the fourth force of nature, the gravity. The energy-scale dependence of the coupling constants for each of the four fundamental interactions suggest that a theory may be formulated at sufficiently high energies where all the interactions unify. However, the interpolation of the energy dependence curves of the coupling constants shows that the respective curves will not coincide at energy levels that the earthbound accelerator experiments can attain. Nevertheless, this theory should be capable of explaining all the particle phenomena of the Standard Model in the low energy limit. At the same time it has to be consistent with astrophysical observations of gravity.

1.2 An Accelerating Universe

Before going further into the unification theories, let us take a step back and discuss the recent experimental tests on the cosmological constant, which became the primary motivation for this thesis. The general theory of relativity, being the most successful classical theory of gravity, states that the curvature of the space-time is related to the energy-momentum content of the matter in space-time by the *Einstein's equation* [10]:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \quad (1.2.1)$$

where $G_{\mu\nu} \equiv (R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R)$ is the Einstein tensor and $T_{\mu\nu}$ is the energy-momentum tensor of matter. The Ricci curvature tensor $R_{\mu\nu}$ and the Ricci scalar R are elements of differential geometry that characterize the curvature of the space-time. The second term on the left hand side of (1.2.1) is the product of the *cosmological constant* with the space-time metric $g_{\mu\nu}$. This term was initially put there by hand to allow static homogeneous solutions in the presence of matter, although later Einstein declared that inclusion of this term was his “biggest blunder”. From the cosmological point of view, a positive cosmological constant indicates an accelerated expansion of the universe, a negative cosmological constant indicates a decelerated expansion of the universe that may end up with a “Big Crunch”. Ironically, recent astrophysical observations indicate that inclusion of a cosmological constant term in Einstein's equation might not be a blunder after all.

The Friedmann-Lemaître-Robertson-Walker (FLRW) metric describes a spatially homogeneous and isotropic space, which is a good approximation for our universe in very large scales

$$ds^2 = -dt^2 + a^2(t) R_0^2 \left[\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right], \quad (1.2.2)$$

where the scale factor $a(t) = R(t)/R_0$ characterizes the relative size of the spatial sections as a function of time (the subscript 0 refers to the present time), $d\Omega^2$ is the metric on a two-sphere and the curvature parameter k describes a positively curved, flat, and negatively curved spatial sections for values +1, 0, and -1, respectively. For this metric, one can derive the Friedmann equations from Einstein's equation for a given density ρ and pressure p

$$H^2 \equiv \left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho + \frac{\Lambda}{3} - \frac{k}{a^2 R_0^2}, \quad (1.2.3)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3p) + \frac{\Lambda}{3}, \quad (1.2.4)$$

which describe the expansion of the spatially homogeneous and isotropic universe. The expansion rate H is called the *Hubble parameter* of the universe and H_0 will refer to the *Hubble constant* of the present epoch.

Consider a standard single massless scalar field theory, with potential $V(\phi)$ that is

minimized at ϕ_0 . The variation of the action [11]

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right] \quad (1.2.5)$$

with respect to the metric $g_{\mu\nu}$ (with determinant g) gives the energy-momentum tensor

$$T_{\mu\nu} \equiv \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}} = \frac{1}{2} \partial_\mu \phi \partial_\nu \phi + \frac{1}{2} (g^{\rho\sigma} \partial_\rho \phi \partial_\sigma \phi) g_{\mu\nu} - V(\phi) g_{\mu\nu}. \quad (1.2.6)$$

The configuration with the lowest energy density, which occurs when there is no contribution from kinetic energy ($\partial_\mu \phi = 0$) implies

$$T_{\mu\nu}^{\text{vac}} = -V(\phi_0) g_{\mu\nu} \equiv -\rho_{\text{vac}} g_{\mu\nu}. \quad (1.2.7)$$

The effect of such an energy-momentum tensor is equivalent to that of a cosmological constant term of (1.2.1) if one defines the vacuum energy density as

$$\rho_{\text{vac}} \equiv \rho_\Lambda \equiv \frac{\Lambda}{8\pi G}. \quad (1.2.8)$$

On this grounds, one can say that the notion of “vacuum energy” is equivalent to that of a “cosmological constant” hence we can include the vacuum energy density ρ_{vac} into the total density ρ . Similarly, the last term of (1.2.3) can be regarded as an effective energy density in “curvature” by defining $\rho_k \equiv -(3k/8\pi G R_0^2 a^2)$. The total energy density ρ also receives contributions from radiation and matter content of the universe. One can define a density parameter for each of these components

$$\Omega_I \equiv \frac{\rho_I}{\rho_{\text{crit}}} = \left(\frac{8\pi G}{3H^2} \right) \rho_I \quad (1.2.9)$$

which are normalized with respect to the critical energy density $\rho_{\text{crit}} \equiv 3H^2/(8\pi G)$ so that $\sum_i \Omega_i = \Omega_M + \Omega_R + \Omega_\Lambda = 1 - \Omega_k$, where the density parameters on the left hand side are for matter, radiation, and vacuum, respectively. From the two Friedmann equations (1.2.3,1.2.4) one can derive the relation

$$\dot{\rho} = -3 \frac{\dot{a}}{a} (\rho + p) \quad (1.2.10)$$

which can be solved by

$$\rho_I \propto a^{-3(1+w_I)} \quad (1.2.11)$$

for each of the components with equation of state $p_I = w_I \rho_I$ that contribute to the total density, where w_I is a number. The energy density for non-relativistic matter is proportional to the number density which, as universe expands, is inversely proportional to the volume, yielding $\rho_M \propto a^{-3}$. For relativistic massive or massless particles, the energy density is the

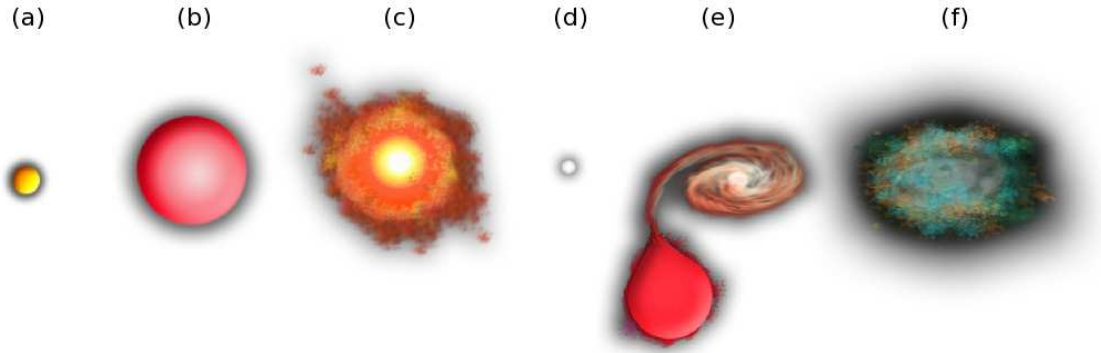


Figure 1.1. Evolution of a type Ia supernova: (a) A star that has a comparable size to the Sun, (b) expands to form a red giant, (c) sheds its outer envelope, loses mass until it can no longer continue nuclear fusion and (d) shrinks to a white dwarf. (e) It accretes mass from a binary partner (usually a red giant), until it exceeds the Chandrasekhar limit of 1.4 solar masses and (f) explodes into a type Ia supernova.

number density times ($\propto a^{-3}$) times the particle energy ($\propto a^{-1}$), yielding $\rho_R \propto a^{-4}$. The vacuum energy does not depend on the expansion of the universe, hence $\rho_\Lambda \propto a^0$; and as is defined above, the energy density in “curvature” has the proportionality relation $\rho_k \propto a^{-2}$. Altogether, the equation of state parameters for all components are summarized as [12]:

	Ω_I	w_I
matter	Ω_M	0
radiation	Ω_R	1/3
curvature	Ω_k	-1/3
vacuum	Ω_Λ	-1

Photons of the $2.73^\circ K$ cosmic microwave background (CMB), that can easily be detected, contribute $\Omega_\gamma \sim 5 \times 10^{-5}$ to the radiational energy density [13, 14]. Cosmic neutrinos contribute both to the radiational and the matter energy densities, depending on being relativistic or not; but conventional scenarios predict that they contribute about the same amount to the energy density as the photons [15]. More exotic relativistic particles give even smaller contributions, and therefore it is reasonable to believe that the universe is dominated by vacuum and matter energy. Under this assumption, the traditional deceleration parameter $q_0 \equiv \frac{1}{2} \sum_i \Omega_i (1 + 3w_i) \sim \frac{1}{2} \Omega_M - \Omega_\Lambda$ suggests that a positive cosmological constant tends to accelerate the expansion of the universe, whereas a negative cosmological constant and the matter content tend to decelerate it.

There is no method to determine the absolute values of Ω_M and Ω_Λ by direct measurement. The discovery of the consistent luminosity behavior type Ia supernovae (SNIa’e) light curves provided an indirect measurement method of these quantities. It was recognized that the vast majority of SNIa’e have similar light curve shapes [16, 17, 18, 19], spectral time series [20, 21, 22, 23], and absolute magnitudes [18, 24]. SNIa’e occur when a white

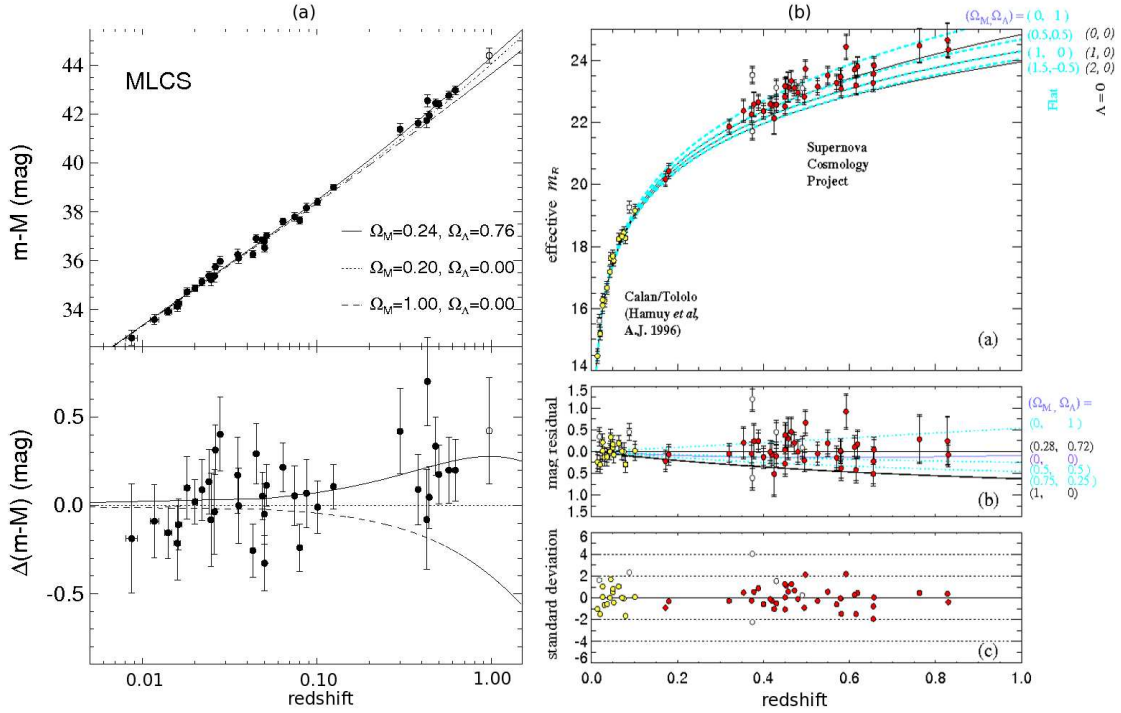


Figure 1.2. (a) Hubble diagram from high-Z Supernova Team [26]. The lines represent different cosmological models with specified parameters $(\Omega_M, \Omega_\Lambda)$. The lower plot shows the difference between observed distance modulus and the second model $(0.20, 0.00)$. (b) Hubble diagram from Supernova Cosmology Project [27]. The middle plot is normalized with respect to the $(0.28, 0.72)$ model. The bottom plot shows the deviations from the best fit curve.

dwarf gradually accretes mass from a companion star, crosses the Chandrasekhar limit and explodes, as illustrated in figure 1.1. In 1992, a review [25] of a variety of studies concluded that most of the observed SNIa'e have a characteristic luminosity and therefore they can be regarded as “standard candles”. The Chandrasekhar limit is a nearly universal quantity so it is not surprising that the resulting explosions have similar luminosities. We should note that there is still a 40% scattering in the peak brightness of SNIa'e and that can be traced to the differences in the matter compositions of the white dwarf atmospheres.

One of observable quantities that is used by astronomers to measure the motion of distant sources is called the “distance modulus”:

$$m - M = 5 \log_{10} [d_L(\text{Mpc})] + 25. \quad (1.2.12)$$

Here m is the apparent magnitude of a source and M is its absolute magnitude. SNIa were found to have the characteristic absolute magnitude of $M \sim -19.5$ [25]. The *luminosity distance*

$$d_L \equiv \sqrt{\frac{\text{intrinsic luminosity}}{4\pi \times \text{flux}}}, \quad (1.2.13)$$

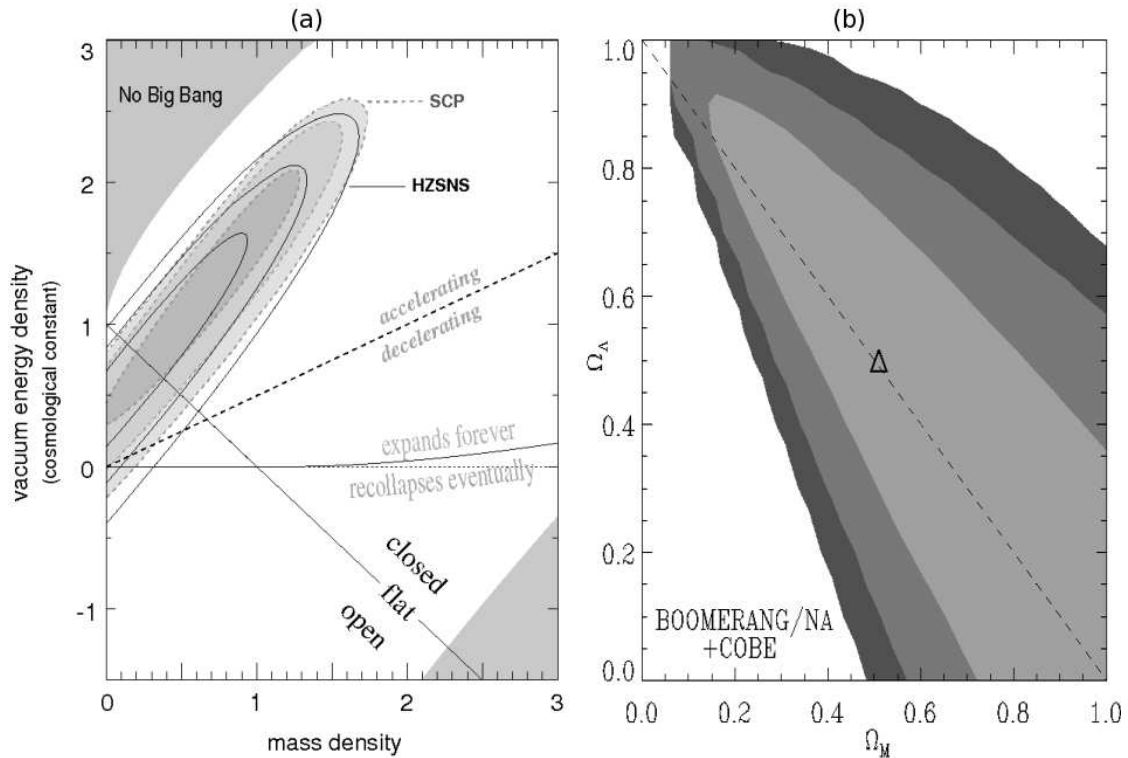


Figure 1.3. (a) The confidence regions for $(\Omega_M, \Omega_\Lambda)$ from the High-Z Supernova Team (HZSNS)[26] and Supernova Cosmology Project (SCP) [27]. The two independent experiments show that $\Omega_\Lambda > 0$ in order to reconcile observations and theory. Figure taken from [28]. (b) The confidence regions for $(\Omega_M, \Omega_\Lambda)$ from the North American flight of the BOOMERANG cosmic microwave background balloon experiment. Figure taken from [34].

expressed in terms of measurable quantities, can be evaluated numerically for any set of Ω_I 's as a function of the *redshift* $z \equiv \frac{1-a}{a}$ [12, 28] and then the experimental data can be compared to different models of space-time.

Two research groups, the High-Z Supernova Team [29, 30, 26, 31] and the Supernova Cosmology Project [32, 33, 27] observed SNIa's of $1.00 > z > 0.01$ in order to measure the density parameters. Figures 1.2a and 1.2b collect their data in distance modulus vs. redshift. These results are converted into limits on the dominant density parameters Ω_M and Ω_Λ in figure 1.3a.

The results obtained by both teams suggest a positive cosmological constant and rule out a $(\Omega_M, \Omega_\Lambda) = (1, 0)$ universe. The easiest explanation of the observed acceleration is made by adding a component of "matter" with an equation of state smaller than $w < -1/3$. The cosmological constant with static $w_\Lambda = -1$ is the most familiar solution although there are models that allow "quintessence" with a dynamic equation of state that varies through space and time. In either case, a conclusion can be made with greater than 99.9% confidence that the universe has a positive cosmological constant or some other form of dark energy.

Meanwhile, the experiments that detected the temperature anisotropies in the CMB [35] provided another method [36, 37] to constrain the cosmological density parameters and evidence for a spatially flat universe. Moreover, studies on these anisotropies showed that the universe underwent a rapid acceleration after the “Big Bang” [38, 39, 40]. However the current acceleration has a rate that is many orders smaller than the early universe inflation. Figure 1.3b shows the constraints in the $\Omega_M - \Omega_\Lambda$ plane from the data that was obtained in [34]. Comparing this with figure 1.3a, we see that their data are complementary to those obtained by SNIa experiments, in that the contours are nearly orthogonal. The confidence regions overlap in the vicinity of $(\Omega_M, \Omega_\Lambda) \sim (0.3, 0.7)$.

At the same time, an independent group of astrophysicists used extended lobes of radio galaxies as modified standard rulers to make a different distance-redshift test [41, 42, 43, 44]. They proposed a model independent approach to measure Ω_M and defined the independent “dark energy indicator” function $s(z) = y''(y')^{-3}(1+z)^{-2}$, where $y \equiv d_L/(1+z)$, that provides a measure of deviations of w from -1 . The numbers they obtained are $w_0 = -0.95 \pm 0.08$ and $(\Omega_M, \Omega_\Lambda) = (0.33 \pm 0.05, 0.64 \pm 0.10)$ if $w = -1$, which are in agreement with the combined CMB-SNIa experimental results. For a list of some other methods of constraining the cosmological density parameters, see section 3.5 of [12] and the references therein.

As of now, the latest values for the cosmological parameters that are obtained using all the data of Wilkinson Microwave Anisotropy Probe (WMAP) [45], SNIa and Baryonic Acoustic Oscillations (in the galactic power spectrum [46]) observations are [45]

$$\begin{aligned}
 \Omega_\Lambda &= 0.721 \pm 0.015 \\
 \Omega_b &= 0.0462 \pm 0.0015 \\
 \Omega_c &= 0.233 \pm 0.013 \\
 w &= -0.972 \begin{matrix} +0.061 \\ -0.060 \end{matrix} \\
 H_0 &= 70.1 \pm 1.3 \text{ km/s/Mpc}
 \end{aligned}$$

where Ω_b and Ω_c indicate the baryon and *dark matter* densities, respectively, that combine into Ω_M .

1.3 Supersymmetric Theories

The story of the unification theories continues with the birth of the *string theory*. In 1968, Veneziano constructed a scattering amplitude in [47] to interpret the strong interaction of subatomic particles. This led to the theory of bosonic open strings. It describes the time evolution of 1-dimensional objects, strings, which propagate consistently in 26 space-time dimensions. Initially, this theory had the problem of predicting a tachyon as its lightest particle. Over the next few years, physicists extended this theory to include closed strings that predicted a massless spin 2 hadron, which was not observed in the particle experiments. With the pioneering work of Ramond; Neveu and Schwarz added fermions to form the

superstring theory. This theory, which exists in 10 space-time dimensions, has remarkable properties. First, it is tachyon free. On the other hand, it was the first attempt to relate bosons to fermions with a symmetry that is called the *supersymmetry*, which asserts that every elementary particle, let it be a boson (fermion), has a superpartner that is a fermion (boson). The discovery of supersymmetric particles is the second one of the two main goals of LHC.

The idea of using string theory as a method to describe the strong interaction was abandoned but the ideas of Scherk, Schwarz and Yoneya carried it into a broader context. They proposed that the spin 2 particle of the closed string theory can be identified with the graviton and the superstring theory will describe a theory of quantum gravity, and perhaps a theory of unification [48, 49]. This idea is supported by the fact that the supersymmetric GUT scale of 10^{15} - $10^{16} GeV$ is close to the Planck mass $10^{19} GeV$ where the quantum effects of gravity are important. Green and Schwarz marked the cornerstone of the “first string theory revolution” by showing that it is possible to resolve the gravitational anomalies of perturbative string theory [50, 51]. This provided a consistent quantum mechanical framework for a possible unification of all interactions.

Five types of ten dimensional superstring theories were constructed since the mid 70’s, namely type I with gauge group $SO(32)$; type IIA and type IIB, which differ by their chirality properties; and the heterotic string theories that are hybrids of a bosonic string theory and a type I string theory, and are named after their gauge groups $E_8 \times E_8$ or $SO(32)$. A “second string theory revolution” in mid 90’s, initiated by Witten [52], revealed that the known superstring theories are related to each other by a web of dualities². T-duality transformations can act on a string if at least one of its dimensions is wrapped around a circle of radius R . If the large R limit of one theory is equivalent to small R' limit of some other theory (which can possibly be the same theory), the theories are said to be dual to each other. S-duality, on the other hand, relates the weak coupling region of one theory to the strong coupling region of the other one. For instance, the weakly coupled heterotic $SO(32)$ superstring theory is dual to the strongly coupled type I superstring theory [55]. This web of dualities also includes the 11 dimensional *supergravity*, e.g. on the strong coupling limit of the type IIA string theory, an extra dimension unfolds and the theory can be described by 11D supergravity [56].

Witten conjectured that all five superstring theories and the 11D supergravity correspond to different limits of a consistent quantum theory, called M-theory [56]. Both T- and S-dualities act on the moduli space of M-theory. These dualities can be embedded in a larger group of discrete symmetry, named as the U-duality group, which constitute an exact symmetry of the theories obtained by the M-theory compactifications. These discrete symmetries are subgroups of the continuous non-compact symmetries of the maximal supergravity theories in corresponding dimensions.

Supergravity theories are local gauge theories of supersymmetry and were first formulated in 70’s [57, 58, 59]. For a review about gauged supergravity theories of various

²For reviews and further references, see [53] and [54].

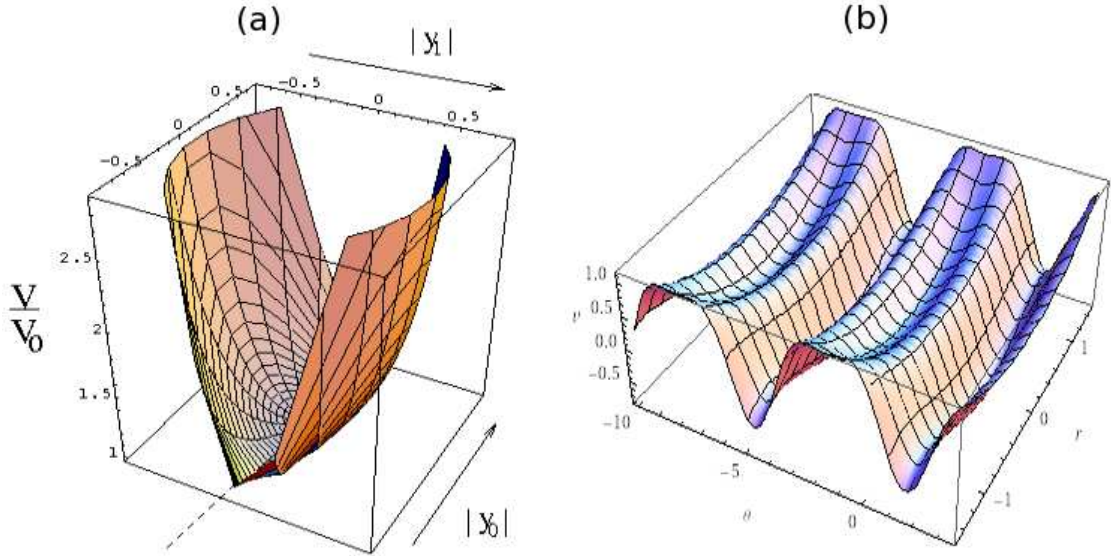


Figure 1.4. Examples of de Sitter extrema in supergravity theories: (a) Stable minima with a flat direction. The potential belongs to the $4D, \mathcal{N} = 2$ supergravity coupled to 3 vector multiplets, considered in [67]. Figure taken from [68]. (b) Saddle point, where the scalar rolls into a Minkowski minimum on one side and Anti-de Sitter minimum on the other. This is $4D, \mathcal{N} = 2$ supergravity coupled to 1 hypermultiplet, considered in [69]. The potential includes instanton corrections.

dimensions that have been studied extensively since then, see [60]. There are two ways of studying supergravity in a certain dimension. One can either construct it directly from field content and symmetries (both local and global) that the action must have; or one can obtain them from higher dimensions by dimensional reduction. Supergravity theories that are obtained purely by dimensional reduction from 10 or 11 dimensional supergravity are low energy effective limits of some superstring theory/M-theory. In such cases, their scalar manifold is the moduli space of the compactification.

Scalar fields play a fundamental role in the description of cosmological models. In fact, the assumption that the energy-momentum tensor is dominated by scalar potential energy density has been the starting point of many inflationary models³ [62, 63]. The case $\dot{\phi} = 0$ corresponds to de Sitter space with a positive cosmological constant. The current accelerated expansion of the universe can be explained either by a positive vacuum energy $V(\phi_0)$, or a scalar field in a slow-roll regime $\dot{\phi}^2/2 \ll V(\phi)$ on a near de Sitter background (quintessence) [64, 65, 66]. There has been a tremendous effort to interpret the fundamental interactions in terms of supergravity. It is important to find supergravity theories whose scalar potentials admit de Sitter or near de Sitter ground states.

There are two possible ways of explaining the positive vacuum energy in terms of scalar potentials. The observed cosmological constant may correspond to the minimum of a scalar potential, in which case the universe will continue to accelerate forever. However the de

³For a general review and further references on inflationary cosmology, see [61].

Sitter regime might be transient, i.e. it might correspond to a local maximum or a saddle point of the scalar potential. Models with slow-roll inflation ($|V''| \ll |V|$) and fast-roll inflation ($|V''| \sim |V|$) have been considered in [70]. In such cases the scalar potential either vanishes as the field rolls to $\phi \rightarrow \infty$ and the universe reaches a Minkowski stage; or the scalar field rolls to the minimum of the potential with $V(\phi) < 0$ (or $V(\phi) \rightarrow -\infty$, such that the potential does not have a minimum at all) and the universe may eventually collapse. Figure 1.4 shows two examples obtained from $4D, \mathcal{N} = 2$ supergravity theories.

The evidence of a small positive cosmological constant attracted interest in finding stable de Sitter ground state solutions in supersymmetric theories. In the context of supersymmetric theories, Anti-de Sitter ground states emerge naturally in contrast to de Sitter ground states. This is due to the fact that the de Sitter superalgebras usually have non-compact R -symmetry subalgebras, which leads to existence of ghosts if the supersymmetry is to be fully preserved. Nevertheless exact supersymmetry is not observed in nature and supersymmetry must be a broken symmetry. There are two main approaches to study de Sitter ground state solutions of supersymmetric theories. One can start from a fundamental theory (a superstring or M-theory), study compactifications on various internal manifolds and, with the combined effects of the warped geometries of the internal manifold and tree-level corrections to the $4D$ Kähler potential, obtain a potential in four dimensions that admits de Sitter critical points [71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81]. On the other hand one can search for such potentials in the extended gauged supergravity theories [82, 83, 84, 70, 85, 86, 67, 68, 69, 87, 88]. A novel result of these studies is that the mass squared of the scalar fields is quantized in units of the cosmological constant, i.e. the value of the scalar potential at its extremum,

$$m_\phi^2 = n \Lambda. \quad (1.3.1)$$

Any quantum corrections to the scalar masses will be related to the cosmological constant $\Lambda = 3H_0^2 \sim 10^{-120} M_{\text{Planck}}^4$ and will be very small and hence the extended supergravities are “protected” against quantum corrections [70].

In this thesis, we will take the second approach and start with studying five dimensional gauged supergravity theories [89, 90, 91, 92] that received renewed attention more recently due their role within the AdS/CFT correspondences in string theory [93, 94, 95, 96], Randall-Sundrum (RS) braneworld scenario [97, 98, 99] and M-/superstring theory compactifications on Calabi-Yau manifolds with fluxes [100, 101, 102, 103]. It is believed that the $5D, \mathcal{N} = 8$ gauged supergravity [90, 91, 92] is a consistent nonlinear truncation of the lowest lying Kaluza-Klein modes of type IIB supergravity on $AdS_5 \times S^5$ [104, 105, 106, 107, 108]. Moreover, certain brane world scenarios based on M-theory compactifications have $5D, \mathcal{N} = 2$ gauged supergravity as their effective field theories [55, 109, 110, 111, 112].

We adopt the convention introduced in [113] to classify the gaugings of supergravity theories. The ungauged $\mathcal{N} = 2$ supergravity coupled to vector- and/or hypermultiplets is referred as (ungauged) Maxwell-Einstein supergravity theory (MESGT). Theories obtained

by gauging a $U(1)_R$ subgroup of $SU(2)_R$ by coupling a linear combination of vector fields to the fermions [89], which are the only fields that transform nontrivially under $SU(2)_R$, are called gauged Maxwell-Einstein supergravity theories (gauged MESGT). On the other hand, if only a subgroup K of the symmetry group of the action is being gauged, the theory is referred as a Yang-Mills/Einstein supergravity theory (YMESGT). Note that the theories which include tensor fields fall into this category. A theory with a gauge group $K \times U(1)_R$ is called gauged Yang-Mills/Einstein supergravity theory (gauged YMESGT).

Pure $5D, \mathcal{N} = 2$ supergravity was constructed in [114, 115], coupling to vector multiplets was done in [116, 89] and tensor fields were added to the theory in [113]. Coupling of hypers to these theories was done in [117]. Vacua of $U(1)_R$ gauged $5D, \mathcal{N} = 2$ MESGT's and YMESGT's without hypers and tensors were studied in [89]. Vacua of the generic Jordan family models, which will be defined in the next chapter, with Abelian gaugings and tensors have been investigated in [118], the full R -symmetry group gauging was done in [119] and a study for vacua of some other gauged theories were carried out in [120]. We will give a general analysis for the vacua of $5D, \mathcal{N} = 2$ supergravity theories coupled to vector, tensor multiplets and a universal hypermultiplet. Then, following the dimensional reduction process of [116, 121], we will look for de Sitter ground states in four dimensions. The analysis in $5D$ is somewhat easier than in $4D$, mainly because in $4D$, the U -duality is an on-shell symmetry, whereas in $5D$, it is a symmetry of the Lagrangian. Moreover $5D$ theories have real geometry while the geometry in $4D$ is complex. Therefore our study in $5D$ will cover all possible ground states whereas in $4D$, motivated by experimental observations, we will concentrate only on de Sitter solutions.

The organization of this thesis is as follows. In chapter 2, we start with reviewing the field content of the $5D, \mathcal{N} = 2$ supergravity, its possible gaugings and the potential terms arising from these gaugings. Sections 2.2, 2.3 and 2.4 deal with the ground states of the generic Jordan family, magical Jordan family and generic non-Jordan family theories, respectively, that are subject to such gaugings that give non-trivial potential terms. The critical points, if they exist, of these theories are given and their stability is discussed. The addition of hypers in the theory generally makes the equations for the stability calculations very complicated. Hence, in certain cases we will just give particular numerical examples that show that it is possible to obtain stable vacua when hypers are coupled to the theory. Chapter 3 takes the story down to 4 dimensions. The symplectic freedom related to the de Roo-Wagemans rotations will be used to find de Sitter ground states. In fact, there is a relation between some of the stable five dimensional de Sitter ground states we will demonstrate in chapter 2 and those found in [67, 68]. This relation is revealed by introducing contractions on the gauge groups. Most of the calculations of this chapter uses the symmetric generic Jordan family as the scalar manifold, although in the last section we discuss about extending our results to the more general homogeneous scalar manifolds. Chapter 4 collects the summary of all of our the results and proposes future directions. In the first appendix, one can find the bosonic part of the four and five dimensional Lagrangians, the elements of the very special geometry and the derivation of the potential terms from more fundamental

quantities. In appendix B, we list the Killing vectors and their corresponding prepotentials of the hyperscalar manifold isometries that will be used to carry out the hyper-gaugings throughout the thesis. Appendix C gives the quadratic coordinate transformations between the parametrization we use in the thesis and Calabi-Vesentini coordinates that was used in [67, 68]. Certain scalar potential terms are given in the last appendix in their full form due to their lengthiness. They will be referred within the text in chapter 3.

Ground States of $\mathcal{N} = 2$ Supergravity Theories with Symmetric Scalar Manifolds in 5 Dimensions

Starting with an outline of the theory and our conventions, we will study the possible gaugings and outcoming scalar potentials of $\mathcal{N} = 2$ supergravity theories coupled to tensor- and/or hypermultiplets in five dimensions. A different section is dedicated to each of the three families of symmetric spaces; namely the generic Jordan family, magical Jordan family and generic non-Jordan family; and at the end of each section a brief summary of the ground states obtained is given. The gaugings and the resulting potentials are studied in full generality in most cases. However, for those cases where a general analytic solution cannot be obtained, we give numerical examples. Most of the content of this chapter and section 4.1 that summarizes this chapter are taken from the author's paper [122].

2.1 The Basics

The field content of the ungauged (before tensor- or hypermultiplet coupling) $\mathcal{N} = 2$ MESGT is

$$\{e_{\hat{\mu}}^{\hat{m}}, \Psi_{\hat{\mu}}^i, A_{\hat{\mu}}^I, \lambda^{i\tilde{a}}, \varphi^{\tilde{x}}\} \quad (2.1.1)$$

where

$$\begin{aligned} i &= 1, 2, \\ I &= 0, 1, \dots, \tilde{n}, \\ \tilde{a} &= 1, 2, \dots, \tilde{n}, \\ \tilde{x} &= 1, 2, \dots, \tilde{n}. \end{aligned}$$

The 'graviphoton' is combined with the \tilde{n} vector fields of the \tilde{n} vector multiplets into a single $(\tilde{n}+1)$ -plet of vector fields $A_{\tilde{\mu}}^I$ labelled by the index I . The indices $\tilde{a}, \tilde{b}, \dots$ and $\tilde{x}, \tilde{y}, \dots$ are the flat and the curved indices, respectively, of the \tilde{n} -dimensional target manifold \mathcal{M}_{VS} of the real scalar fields, which we will define below.

The bosonic part of the Lagrangian is given in the appendix A. The global symmetries of these theories are of the form $SU(2)_R \times G_{(5)}$, where $SU(2)_R$ is the R -symmetry group of the $\mathcal{N} = 2$ Poincare superalgebra and $G_{(5)}$ is the subgroup of the group of isometries of the scalar manifold that extends to the symmetries of the full action. Gauging a subgroup $K_{(5)}$ of $G_{(5)}$ requires dualization of some of the vector fields to self-dual tensor fields if they are transforming in a non-trivial representation of $K_{(5)}$. More formally, the field content, when $2n_T$ of the vector fields are dualized to tensor fields, becomes

$$\{e_{\tilde{\mu}}^{\hat{m}}, \Psi_{\tilde{\mu}}^i, A_{\tilde{\mu}}^I, B_{\tilde{\mu}\tilde{\nu}}^M, \lambda^{i\tilde{a}}, \varphi^{\tilde{x}}\} \quad (2.1.2)$$

where now

$$\begin{aligned} i &= 1, 2, \\ I &= 0, 1, \dots, n_V, \\ M &= 1, 2, \dots, 2n_T, \\ \tilde{I} &= 0, 1, \dots, \tilde{n}, \\ \tilde{a} &= 1, 2, \dots, \tilde{n}, \\ \tilde{x} &= 1, 2, \dots, \tilde{n}, \end{aligned}$$

with $\tilde{n} = n_V + 2n_T$. Tensor multiplets come in pairs with four spin-1/2 fermions (i.e. two $SU(2)_R$ doublets) and two scalars. Tensor coupling generally introduces a scalar potential of the form [113]:

$$P_{(5)}^{(T)} = \frac{3\sqrt{6}}{16} h^I \Lambda_I^{MN} h_M h_N. \quad (2.1.3)$$

Here Λ_I^{MN} are the transformation matrices of the tensor fields and $h_{\tilde{I}}, h^{\tilde{I}}$ are elements of the "very special" geometry of the scalar manifold \mathcal{M}_{VS} that has the metric $\overset{\circ}{a}_{\tilde{I}\tilde{J}}$ which is used to raise and lower the indices $\tilde{I}, \tilde{J}, \dots$.

When the full R -symmetry group $SU(2)_R$ is being gauged the potential gets the contribution

$$P_{(5)}^{(R)} = -4C^{ij\tilde{K}} \delta_{ij} h_{\tilde{K}}, \quad (2.1.4)$$

where i, j are adjoint indices of $SU(2)$. If instead, the $U(1)_R$ subgroup is being gauged, the contribution to the potential becomes

$$P_{(5)}^{(R)} = -4C^{IJ\tilde{K}} V_I V_J h_{\tilde{K}}. \quad (2.1.5)$$

The expressions that lead to the derivation of the above potential terms can be found in appendix A.

We will look at the cases, where the scalar manifold \mathcal{M}_{VS} is a symmetric space. Such spaces are further divided in two categories, depending whether they are associated with a Jordan algebra or not. The spaces that are associated with Jordan algebras are of the form $\mathcal{M}_{VS} = \frac{Str_0(J)}{Aut(J)}$, where $Str_0(J)$ and $Aut(J)$ are the reduced structure group and the automorphism group, respectively, of a real, unital Jordan algebra J , of degree three [116, 123]. More specifically,

- Generic Jordan Family:

$$J = \mathbb{R} \oplus \Sigma_{\tilde{n}} : \mathcal{M}_{VS} = \frac{SO(\tilde{n}-1, 1) \times SO(1, 1)}{SO(\tilde{n}-1)}, \quad \tilde{n} \geq 1.$$

- Magical Jordan Family:

$$\begin{aligned} J_3^{\mathbb{R}} : \quad \mathcal{M}_{VS} &= \frac{SL(3, \mathbb{R})}{SO(3)}, & \tilde{n} &= 5, \\ J_3^{\mathbb{C}} : \quad \mathcal{M}_{VS} &= \frac{SL(3, \mathbb{C})}{SU(3)}, & \tilde{n} &= 8, \\ J_3^{\mathbb{H}} : \quad \mathcal{M}_{VS} &= \frac{SU^*(6)}{Usp(6)}, & \tilde{n} &= 14, \\ J_3^{\mathbb{O}} : \quad \mathcal{M}_{VS} &= \frac{E_{6(-26)}}{F_4}, & \tilde{n} &= 26. \end{aligned} \tag{2.1.6}$$

- Generic non-Jordan Family:

$$\mathcal{M}_{VS} = \frac{SO(1, \tilde{n})}{SO(\tilde{n})}, \quad \tilde{n} \geq 1.$$

In addition to the supergravity multiplet, n vector multiplets and $2n_T$ tensor multiplets one can couple hypermultiplets into the theory. A universal hypermultiplet

$$\{\zeta^a, q^X\} \tag{2.1.7}$$

contains a spin-1/2 fermion doublet $A = 1, 2$ and four real scalars $X = 1, \dots, 4$. The total manifold of the scalars $\phi = (\varphi, q)$ then becomes

$$\mathcal{M}_{scalar} = \mathcal{M}_{VS} \otimes \mathcal{M}_Q$$

with $dim_{\mathbb{R}} \mathcal{M}_{VS} = n_V + 2n_T$ and $dim_{\mathbb{Q}} \mathcal{M}_Q = 1$. The quaternionic hyperscalar manifold \mathcal{M}_Q of the scalars of a single hypermultiplet has the isometry group $SU(2, 1)$. Gauging a subgroup of this group introduces an extra term in the scalar potential [117]

$$P_{(5)}^{(H)} = 2\mathcal{N}_{iA} \mathcal{N}^{iA} \tag{2.1.8}$$

where $\mathcal{N}^{iA} = \frac{\sqrt{6}}{4} h^I K_I^X f_X^{iA}$ with f_X^{iA} being the quaternionic vielbeins, $f_X^{iA} f_{YiA} = g_{XY}$, and

g_{XY} is the metric of the quaternionic-Kähler hypermultiplet scalar manifold [124]

$$ds^2 = \frac{dV^2}{2V^2} + \frac{1}{2V^2}(d\sigma + 2\theta d\tau - 2\tau d\theta)^2 + \frac{2}{V}(d\tau^2 + d\theta^2), \quad (2.1.9)$$

and K_I^X being the Killing vectors given in appendix B together with their corresponding prepotentials. The determinant of the metric is $1/V^6$ and it is positive definite and well behaved everywhere except $V = 0$. But since in the Calabi-Yau derivation V corresponds to the volume of the Calabi-Yau manifold [111], we restrict ourselves to the positive branch $V > 0$.

When the R -symmetry is gauged in a theory that contains hypers, the potential $P_{(5)}^{(R)}$ gets some modification due to the fact that the fermions in the hypermultiplet are doublets under the R -symmetry group $SU(2)_R$. It becomes

$$P_{(5)}^{(R)} = -4C^{IJ\tilde{K}}\vec{P}_I \cdot \vec{P}_J h_{\tilde{K}} \quad (2.1.10)$$

where \vec{P}_I are the prepotentials corresponding to the Killing vectors K_I^X .

The total scalar potential, which includes terms from tensor coupling, R -symmetry gauging and hyper coupling, is given by

$$\begin{aligned} P_{(5)} \equiv e^{-1}\mathcal{L}_{pot} &= -g^2 P_{(5)}^{(T)} - g_R^2 P_{(5)}^{(R)} - g_H^2 P_{(5)}^{(H)} \\ &\equiv -g^2 P_{(5)} \\ &= -g^2(P_{(5)}^{(T)} + \lambda P_{(5)}^{(R)} + \kappa P_{(5)}^{(H)}), \end{aligned} \quad (2.1.11)$$

where $\lambda = g_R^2/g^2$, $\kappa = g_H^2/g^2$; g_R , g_H and g are coupling constants, which need not to be all independent.

Supersymmetry of the solutions:

Demanding supersymmetric variations of the fermions vanish at the critical points of the theory, the conditions that need to be satisfied are found as [118, 124]

$$\langle W^{\tilde{a}} \rangle = \langle P^{\tilde{a}} \rangle = \langle \mathcal{N}_{iA} \rangle = 0 \quad (2.1.12)$$

where $W^{\tilde{a}}$ and $P^{\tilde{a}}$ are defined in (A.4). Any ground state that does not satisfy all of these conditions are not supersymmetric. One can see that any supersymmetric solution must be of the form

$$P_{(5)}|_{\phi^c} = -4\lambda \vec{P} \cdot \vec{P}(\phi^c) \quad (2.1.13)$$

which is negative semi-definite. Hence we know from beginning that any de Sitter type ground state of the theories we will consider will have broken supersymmetry. The parametrization of the Killing vectors of the hyperscalar manifold, which is outlined in appendix B, yields $K_I^X|_{q^c} \neq 0$, for non-compact generators. Here, the point $q^c = \{V = 1, \sigma = \theta = \tau = 0\}$

is the base point of the hyperscalar manifold, i.e. the compact Killing vectors of the hyperisometry generate the isotropy group of this point. This point will be used as the hypercoordinate candidate of the critical points. As a consequence $\langle \mathcal{N}_{iA} \rangle \neq 0$; and hence theories including non-compact hyper-gauging will not have supersymmetric critical points either.

2.2 Generic Jordan Family

The theory being considered is $\mathcal{N} = 2$ supergravity coupled to \tilde{n} Abelian vector multiplets and with real scalar manifold $\mathcal{M}_{VS} = SO(\tilde{n} - 1, 1) \times SO(1, 1)/SO(\tilde{n} - 1)$, $\tilde{n} \geq 1$. The cubic polynomial can be written in the form [118]

$$N(h) = \frac{3\sqrt{3}}{2} h^0 [(h^1)^2 - (h^2)^2 - \dots - (h^{\tilde{n}})^2]. \quad (2.2.1)$$

The non-zero $C_{\tilde{I}\tilde{J}\tilde{K}}$'s are

$$C_{011} = \frac{\sqrt{3}}{2}, \quad C_{022} = C_{033} = \dots = C_{0\tilde{n}\tilde{n}} = -\frac{\sqrt{3}}{2}$$

and their permutations. The constraint $N = 1$ can be solved by

$$h^0 = \frac{1}{\sqrt{3} \|\varphi\|^2}, \quad h^a = \sqrt{\frac{2}{3}} \varphi^a$$

with $a, b = 1, 2, \dots, \tilde{n}$ and $\|\varphi\|^2 = \varphi^a \eta_{ab} \varphi^b$, where $\eta_{ab} = (+ - - \dots -)$. The scalar field metric $g_{\tilde{x}\tilde{y}}$ and vector field metric $\tilde{a}_{\tilde{I}\tilde{J}}$ that appear in the kinetic terms in the Lagrangian are positive definite in the region $\|\varphi\|^2 > 0$. In order to have theories that have a physical meaning, our investigation is restricted to this region. As a consequence one must have $\varphi^1 \neq 0$.

The isometry group of the real scalar manifold \mathcal{M}_{VS} is $G_{(5)} = SO(\tilde{n} - 1, 1) \times SO(1, 1)$. This is the symmetry group of the full action modulo the isometry group of the hyperscalar manifold. The gauging of an $SO(1, 1)$ or an $SO(2)$ subgroup of $SO(\tilde{n} - 1, 1)$ will lead to dualization of vectors to tensor fields and this gives a scalar potential term. In the generic Jordan family there are no vector fields that are nontrivially charged when the gauge group is non-Abelian, and hence gauging a non-Abelian subgroup of $G_{(5)}$ will not give a scalar potential term. It is also possible to gauge the R -symmetry group $SU(2)_R$ or its subgroup $U(1)_R$; or one can introduce a hypermultiplet in the theory and gauge its symmetries to get additional scalar terms in the potential. We will look at each case in turn.

2.2.1 Maxwell-Einstein Supergravity

2.2.1.1 No R -symmetry Gauging

Without hypermultiplets:

There is no scalar potential and the vacuum is Minkowskian.

With a universal hypermultiplet:

One can gauge $U(1) \subset SU(2) \times U(1)$ or a non-compact subgroup $SO(1, 1)$. To gauge the $U(1)$ symmetry one has to take the Killing vector \vec{K} as a linear combination of $\vec{T}_1, \vec{T}_2, \vec{T}_3$ and \vec{T}_8 of (B.3) whereas one has to take a linear combination of $\vec{T}_4, \vec{T}_5, \vec{T}_6$ and \vec{T}_7 if he is to gauge the $SO(1, 1)$. We take a linear combination of the vector fields $(V_I A_\mu^I)$ from the vector multiplet as our gauge field. For $U(1)$ gauging, the corresponding Killing vector and hence the term $P_{(5)}^{(H)} = 2\mathcal{N}_{iA}\mathcal{N}^{iA}$ vanishes at its critical point ($V = 1, \sigma = \theta = \tau = 0$). An important consequence of this is, for the generic family, $U(1)$ gauging of the hyper isometry will not change the sign of the critical points of the theory. Simultaneous gauging of $U(1) \subset SU(2)$ with $U(1)_R$ will only rescale $P_{(5)}^{(R)}$ by a positive factor. Such a scaling can be absorbed by redefining V_I 's. But the stability of the vacuum will still need to be checked. We will see an example to this in subsection 2.2.3.3.

The situation is slightly different when a non-compact gauging of hyper isometry is done. A linear combination of all vector fields at hand ($A_\mu[SO(1, 1)] = V_I A_\mu^I$) is taken as the gauge field. More precisely,

$$\mathcal{N}^{iA} = \frac{\sqrt{6}}{4}(V_I h^I)(W^k T_k^X) f_X^{iA} \quad k = 4, \dots, 7; \quad I = 0, \dots, \tilde{n}. \quad (2.2.2)$$

At the base point $q^c = \{V = 1, \sigma = \theta = \tau = 0\}$ of hyperscalar manifold, one finds

$$\begin{aligned} \partial_{\varphi^1} P_{(5)}|_{q^c} &= \frac{1}{4}\kappa \left(2(W^4)^2 + 2(W^5)^2 + (W^6)^2 + (W^7)^2 \right) \left(\sqrt{2}V_1 - \frac{2V_0\varphi^1}{\|\varphi\|^4} \right) \\ &\quad \left(\sqrt{2}(V_1\varphi^1 + \dots + V_{\tilde{n}}\varphi^{\tilde{n}}) + \frac{V_0}{\|\varphi\|^2} \right), \\ \partial_{\varphi^a} P_{(5)}|_{q^c} &= \frac{1}{4}\kappa \left(2(W^4)^2 + 2(W^5)^2 + (W^6)^2 + (W^7)^2 \right) \left(\sqrt{2}V_a + \frac{2V_0\varphi^a}{\|\varphi\|^4} \right) \\ &\quad \left(\sqrt{2}(V_1\varphi^1 + \dots + V_{\tilde{n}}\varphi^{\tilde{n}}) + \frac{V_0}{\|\varphi\|^2} \right), \quad (a = 2, \dots, \tilde{n}), \\ \partial_V P_{(5)}|_{q^c} &= \partial_\sigma P_{(5)}|_{q^c} = 0, \\ \partial_\theta P_{(5)}|_{q^c} &= \frac{1}{4}\kappa W^4 W^6 \left(\sqrt{2}(V_1\varphi^1 + \dots + V_{\tilde{n}}\varphi^{\tilde{n}}) + \frac{V_0}{\|\varphi\|^2} \right)^2, \\ \partial_\tau P_{(5)}|_{q^c} &= \frac{1}{4}\kappa W^4 W^7 \left(\sqrt{2}(V_1\varphi^1 + \dots + V_{\tilde{n}}\varphi^{\tilde{n}}) + \frac{V_0}{\|\varphi\|^2} \right)^2. \end{aligned}$$

These expressions simultaneously vanish by letting

$$\frac{V_1}{\varphi^1} = -\frac{V_2}{\varphi^2} = \dots = -\frac{V_{\tilde{n}}}{\varphi^{\tilde{n}}} = \frac{\sqrt{2}V_0}{\|\varphi\|^4} \quad (2.2.3)$$

and by setting either $W^6 = W^7 = 0$ or $W^4 = 0$. In the former case, the potential at the

critical point becomes

$$P_{(5)}|_{\phi^c} = \frac{6\kappa ((W^4)^2 + (W^5)^2) (V_0)^2}{4\|\varphi\|^4}, \quad (2.2.4)$$

which is positive definite if not all W^4, W^5, V_0 are zero. The condition $\|\varphi\|^2 > 0$ together with the equation (2.2.3) determine the constraint on V_I 's as

$$(V_1)^2 - (V_2)^2 - \dots - (V_{\tilde{n}})^2 > 0. \quad (2.2.5)$$

Stability of this critical point is checked by calculating the Hessian of the potential at the critical point. Using the $SO(\tilde{n} - 1, 1)$ symmetry one can rotate the fields such that $\varphi^2 = \dots = \varphi^{\tilde{n}} = 0$. In particular for $\tilde{n} = 3$, the eigenvalues of the Hessian are found to be

$$\left(0, \tilde{A}, \tilde{A}, 3\tilde{A}, \tilde{A}\tilde{B}, \tilde{A}\tilde{B}, \frac{3\tilde{A}}{2(\varphi^1)^2} \right) \quad (2.2.6)$$

where

$$\begin{aligned} \tilde{A} &= \frac{27\kappa^2 (V_0)^4 ((W^4)^2 + (W^5)^2)^2}{4(\varphi^1)^{10}}, \\ \tilde{B} &= \frac{3}{4}(\varphi^1)^2 ((W^4)^2 + (3W^5)^2)^2. \end{aligned} \quad (2.2.7)$$

These eigenvalues are all non-negative, hence the critical point of the potential corresponds to a stable de Sitter vacuum.

The same result can be obtained by letting $W^4 = 0$ instead of $W^6 = W^7 = 0$.

2.2.1.2 $SU(2)_R$ Symmetry Gauging

In order to have $SU(2) \sim SO(3)$ to be a subgroup of the isometry $SO(\tilde{n} - 1, 1) \times SO(1, 1)$, one obviously needs $\tilde{n} \geq 4$.

Without hypermultiplets:

The calculation has been done in [119] with $A_{\tilde{\mu}}^2, A_{\tilde{\mu}}^3, A_{\tilde{\mu}}^4$ taken as gauge fields and the potential was found to be

$$P_{(5)} = \lambda P_{(5)}^{(R)} = 6\lambda\|\varphi\|^2. \quad (2.2.8)$$

This potential does not have any critical points¹ in the physically relevant region $\|\varphi\|^2 > 0$.

With a universal hypermultiplet:

The gauging of $SU(2)_R$ must be done simultaneously with the gauging of $SU(2) \subset SU(2, 1)$ of the hyperscalar manifold. Hence one has $\lambda = \kappa$ in this case. Without loss of generality, one can choose $A_{\tilde{\mu}}^2, A_{\tilde{\mu}}^3$ and $A_{\tilde{\mu}}^4$ as our gauge fields and identify the Killing vectors as

$$K_2^X = T_1^X \quad K_3^X = T_2^X \quad K_4^X = T_3^X \quad (2.2.9)$$

¹One can take $\lambda = 0$ but this will make the potential vanish everywhere.

and the prepotentials are taken accordingly. The scalar potential is now

$$P_{(5)} = \lambda(P_{(5)}^{(R)} + P_{(5)}^{(H)}) \quad (2.2.10)$$

with $P_{(5)}^{(R)}$ defined as in (2.1.10). The derivative of the total potential with respect to φ^1 is given by

$$\begin{aligned} \frac{\partial P_{(5)}}{\partial \varphi^1} &= \lambda \left\{ \left(V^4 + 4(\theta^2 + \tau^2 + 11)V^3 \right. \right. \\ &\quad + 2(3\theta^4 + (6\tau^2 + 46)\theta^2 + 3\tau^4 + \sigma^2 + 46\tau^2 + 51)V^2 \\ &\quad + 4(\theta^2 + \tau^2 + 11)\left(\theta^4 + 2(\tau^2 + 1)\theta^2 + \sigma^2 + (\tau^2 + 1)^2\right)V \\ &\quad \left. \left. + \left(\theta^4 + 2(\tau^2 + 1)\theta^2 + \sigma^2 + (\tau^2 + 1)^2\right)^2 \right) \varphi^1 \right\} / (32V^2) \\ &\xrightarrow{q^c} 6\lambda\varphi^1 \end{aligned}$$

and it cannot be brought to zero in the physically relevant region, unless $\lambda = 0$, but that turns off the potential and leads to Minkowski vacuum, hence the potential has no critical points for this case. However one can gauge an additional $U(1)$ and/or $SO(1, 1)$ symmetry of the hyperscalar manifold to have extra contributions to the scalar potential.

$SU(2)_R \times U(1)_H$ gauging:

A similar situation occurs as in the last case. The potential has no critical points.

$SU(2)_R \times SO(1, 1)_H$ gauging:

We choose the linear combination $V_b A_{\tilde{\mu}}^b$, $b = 0, 1, 5, 6, \dots, \tilde{n}$ as the $SO(1, 1)$ gauge field and the non-compact T_4^X as the Killing vector for this gauging. The potential is given by

$$P_{(5)} = \lambda(P_{(5)}^{(R)} + 2\mathcal{N}_{iA}\mathcal{N}^{iA}) \quad (2.2.11)$$

where now $\mathcal{N}^{iA} = \frac{\sqrt{6}}{4}(h^a K_a^X + (V_b h^b)T_4^X)f_X^{iA}$ with $a = 2, 3, 4$; K_a^X were defined in (2.2.9); the coupling constant for the $SO(1, 1)$ gauging is absorbed in V_b 's.

At the base point of the hyperscalar manifold $q^c = \{V = 1, \sigma = \theta = \tau = 0\}$ the derivatives of the potential are evaluated as,

$$\begin{aligned} \partial_{\varphi^1} P_{(5)}|_{q^c} &= \lambda \left(3\varphi^1 + \frac{1}{\sqrt{2}} \left(V_1 - \frac{\sqrt{2}V_0\varphi^1}{\|\varphi\|^4} \right) \tilde{C} \right), \\ \partial_{\varphi^a} P_{(5)}|_{q^c} &= \lambda \varphi^a \left(-3 + \frac{V_0 \tilde{C}}{\|\varphi\|^4} \right), \\ \partial_{\varphi^d} P_{(5)}|_{q^c} &= \lambda \left(-3\varphi^d + \frac{1}{\sqrt{2}} \left(V_d + \frac{\sqrt{2}V_0\varphi^d}{\|\varphi\|^4} \right) \tilde{C} \right), \quad d = 5, 6, \dots, \tilde{n}, \\ \partial_V P_{(5)}|_{q^c} &= 0, \\ \partial_\sigma P_{(5)}|_{q^c} &= \frac{\sqrt{2}}{4} \lambda \varphi^4 \tilde{C}, \\ \partial_\theta P_{(5)}|_{q^c} &= \frac{\sqrt{2}}{2} \lambda \varphi^3 \tilde{C}, \\ \partial_\tau P_{(5)}|_{q^c} &= \frac{\sqrt{2}}{2} \lambda \varphi^2 \tilde{C}, \end{aligned} \quad (2.2.12)$$

where

$$\tilde{C} = \sqrt{2}(V_e \varphi^e) + \frac{V_0}{\|\varphi\|^2}, \quad e = 1, 5, 6, \dots, \tilde{n}.$$

In order to set the last three equations of (2.2.12) to zero one might set $\tilde{C} = 0$, but applying this to the first equation makes it impossible to vanish, unless $\lambda = 0$, but that makes the overall potential zero. Hence we set $\varphi_c^2 = \varphi_c^3 = \varphi_c^4 = 0$. Then all left to solve are the first and the third equations. Motivated by (2.2.3) we set

$$V_1 \varphi^d = -\varphi^1 V_d \quad \forall d = 5, 6, \dots, \tilde{n}. \quad (2.2.13)$$

This reduces the first and third equations of (2.2.12) to

$$\lambda \varphi^e \left(3 + \frac{(-2V_0 \varphi^1 + \sqrt{2}V_1 \|\varphi\|^4)(V_0 \varphi^1 + \sqrt{2}V_1 \|\varphi\|^4)}{2(\varphi^1)^2 \|\varphi\|^6} \right) = 0. \quad (2.2.14)$$

Solving this for φ^1 yields

$$\varphi^1 = \frac{\sqrt{2}\|\varphi\|^4 V_0 V_1 \pm \sqrt{6\|\varphi\|^8 V_1^2 (3(V_0)^2 - 8\|\varphi\|^6)}}{12\|\varphi\|^6 - 4(V_0)^2}. \quad (2.2.15)$$

The constraint on V_I 's is

$$(V_1)^2 - (V_5)^2 - \dots - (V_{\tilde{n}})^2 > 0, \quad (2.2.16)$$

and since φ 's are real, by (2.2.15)

$$(V_0)^2 > \frac{8}{3}\|\varphi\|^6. \quad (2.2.17)$$

The potential evaluated at the critical point is given by

$$P_{(5)}|_{\phi^c} = \frac{\lambda}{4} \left(6\|\varphi\|^2 + \frac{(V_0 \varphi^1 + \sqrt{2}V_1 \|\varphi\|^4)^2}{(\varphi^1)^2 \|\varphi\|^4} \right) \quad (2.2.18)$$

which is positive definite. Now, given a set of V_I 's subject to the constraint (2.2.16), the critical point is determined by $\tilde{n} - 4$ equations (2.2.13) together with equation (2.2.15).² Note that, in some cases, there may be more than one solution because of multi-valuedness of (2.2.15). We calculated the Hessian of the potential and showed that it is possible to obtain positive eigenvalues and hence one can have stable de Sitter vacua. Because of the lengthiness of the expressions we give a particular example here.

Example: Suppose

$$V_0 = 2, \quad V_1 = 1, \quad V_5 = \dots = V_{\tilde{n}} = 0.$$

²One has to make sure that the equation (2.2.17) holds.

There are two critical points, given by

$$\begin{aligned}\phi_1^c & : \varphi^1 = -\frac{(\sqrt{33}-1)^{1/3}}{2^{5/6}}, \quad \varphi^5 = \dots = \varphi^{\tilde{n}} = 0, \\ \phi_2^c & : \varphi^1 = \frac{(\sqrt{33}+1)^{1/3}}{2^{5/6}}, \quad \varphi^5 = \dots = \varphi^{\tilde{n}} = 0.\end{aligned}$$

The values of the potential at these critical points read

$$\begin{aligned}P_{(5)}|_{\phi_1^c} & = \frac{3}{4}\lambda \left(\frac{3}{2}(69 - 11\sqrt{33})\right)^{1/3}, \\ P_{(5)}|_{\phi_2^c} & = \frac{3}{4}\lambda \left(\frac{3}{2}(69 + 11\sqrt{33})\right)^{1/3}\end{aligned}$$

and the numerical values for the eigenvalues of the Hessian (for $\tilde{n} = 6$) are

$$(-0.799\lambda, -0.799\lambda, -0.743\lambda, -0.686\lambda, -0.686\lambda, 0.667\lambda, 1.142\lambda, 2.991\lambda, 2.991\lambda, 29.058\lambda)$$

at ϕ_1^c and

$$(0.843\lambda, 1.102\lambda, 1.102\lambda, 1.876\lambda, 2.186\lambda, 2.186\lambda, 6.526\lambda, 7.143\lambda, 7.143\lambda, 20.441\lambda)$$

at ϕ_2^c . Hence the second critical point is stable whereas the first one is not.

2.2.1.3 $U(1)_R$ Symmetry Gauging

Without hypermultiplets:

See [89] for a complete analysis for the cases without tensors for all symmetric Jordan theories. Here we will review a specific result, which will be relevant when we will add a hypermultiplet into the theory. As the $U(1)_R$ -gauge field, a linear combination $V_I A^I$ of all the vectors in the theory will be taken. Using (2.1.5), the potential is given by

$$P_{(5)} = \lambda P_{(5)}^{(R)} = -2\lambda \left(\|V\|^2 \|\varphi\|^2 + \frac{2\sqrt{2}V_0 V_i \varphi^i}{\|\varphi\|^2} \right) \quad (2.2.19)$$

where $i = 1, \dots, \tilde{n}$ and $\|V\|^2 = (V_1)^2 - (V_2)^2 - \dots - (V_{\tilde{n}})^2$. The derivatives of this potential are calculated as

$$\begin{aligned}\partial_{\varphi^1} P_{(5)} & = -2\lambda \left(\varphi^1 \tilde{A} + \frac{\sqrt{2}V_0 V_1}{\|\varphi\|^2} \right), \\ \partial_{\varphi^a} P_{(5)} & = -2\lambda \left(-\varphi^a \tilde{A} + \frac{\sqrt{2}V_0 V_a}{\|\varphi\|^2} \right), \quad a = 2, \dots, \tilde{n}.\end{aligned} \quad (2.2.20)$$

where

$$\tilde{A} = \|V\|^2 - \frac{2\sqrt{2}V_0 V_i \varphi^i}{\|\varphi\|^4}. \quad (2.2.21)$$

A trivial way of making the derivatives (2.2.20) vanish is to set $V_i = 0$. This leads to a Minkowski ground state with broken supersymmetry ($P_1 \neq 0$) as long as $V_0 \neq 0$, i.e. the $U(1)_R$ gauging is nontrivial.

The easiest way to solve the equations nontrivially, after the derivatives are set to zero, is to solve the last equation for V_0 ; plug the resulting expression into the other equations; solve the equation before the last equation for $V_{\tilde{n}}$; plug the resulting expression into the remaining equations; solve the last of the remaining equations for $V_{\tilde{n}-1}$; plug the resulting expression into the remaining equations and so forth... At the end one finds

$$\begin{aligned}\sqrt{2}V_0\varphi^1 &= V_1\|\varphi\|^4, \\ \varphi^1V_a &= -\varphi^aV_1.\end{aligned}\tag{2.2.22}$$

V_i 's satisfy the following constraint

$$(V_1)^2 - (V_2)^2 - \dots - (V_{\tilde{n}})^2 > 0.\tag{2.2.23}$$

By plugging in (2.2.22) into the potential (2.2.19), one evaluates its value at the critical point as

$$P_{(5)}|_{\varphi^c} = -6\lambda(V_1)^2\frac{\|\varphi\|^4}{(\varphi^1)^2},\tag{2.2.24}$$

which is negative and therefore corresponds to an AdS critical point. Calculating the Hessian of the potential, one finds that it always has the negative eigenvalue

$$-4\lambda(V_1)^2(\varphi^1)^2\frac{2((\varphi^1)^2 + \dots + (\varphi^{\tilde{n}})^2) + \sqrt{\|\varphi\|^4 + 16(\varphi^1)^2((\varphi^2)^2 + \dots + (\varphi^{\tilde{n}})^2)}}{(\varphi^1)^4}$$

for any \tilde{n} , hence the critical point is not a minimum. Moreover we found that, up to $\tilde{n} = 4$, the eigenvalues of the Hessian are all negative. This means that the critical point is a maximum rather than a minimum.³ The unboundedness of the potential from below may lead someone to think that the critical point is unstable. But an analysis of small fluctuations of $\varphi^{\tilde{x}}$ around the critical point shows that, at least perturbatively, instabilities need not occur [125]. It was shown in [126] and demonstrated in [89] that a potential of the form (2.1.5) is sufficient to ensure the positivity of the energy and thereby the stability, about the AdS background at a critical point. To have supersymmetry at the critical point one needs to have $\langle P^i \rangle = 0$. We calculated $P_i = -\sqrt{\frac{3}{2}}P_{,i} = -\sqrt{\frac{3}{2}}(V_I h^I)_{,i}$ as

$$\begin{aligned}P_1 &= \frac{\sqrt{2}V_0\varphi^1}{\|\varphi\|^4} - V_1, \\ P_a &= -\frac{\sqrt{2}V_0\varphi^a}{\|\varphi\|^4} - V_a\end{aligned}$$

and equations (2.2.22) assure that these quantities vanish and hence the critical point is supersymmetric.

³See [89] for the general proof that this is the case for arbitrary \tilde{n} .

With a universal hypermultiplet:

The total potential is of the form $P_{(5)} = P_{(5)}^{(R)} + P_{(5)}^{(H)}$. The most general way of doing simultaneous $U(1)_R$ gauging together with $U(1)$ gauging of the hypermultiplet isometry is done by selecting a linear combination of compact Killing vectors from (B.3). One can easily see that at the base point $q^c = \{V = 1, \sigma = \theta = \tau = 0\}$ of the hyperscalar manifold all these compact generators vanish. Therefore one has $\mathcal{N}^{iA} = 0$ and as a consequence [120]

$$P_{(5)}^{(H)}|_{q^c} = \frac{\partial P_{(5)}^{(H)}}{\partial \varphi^I}|_{q^c} = \frac{\partial P_{(5)}^{(H)}}{\partial q}|_{q^c} = 0. \quad (2.2.25)$$

On the other hand, $P_{(5)}^{(R)}$ of (2.1.10) is of the form $P_{(5)}^{(R)} \sim f(\varphi)g(q)$, where $g(q) = \vec{P}_I \cdot \vec{P}_J(q)\delta^{IJ}$ for the generic family. $g(q)$ has an extremum point at the base point of the hyperscalar manifold (i.e. $\frac{dg}{dq}|_{q^c} = 0$). This leads to

$$\frac{\partial P_{(5)}^{(R)}}{\partial q}|_{q^c} = \frac{\partial P_{(5)}^{(R)}}{\partial q}|_{q^c} = \frac{\partial^2 P_{(5)}^{(R)}}{\partial \varphi \partial q}|_{q^c} = 0$$

and hence the Hessian is in block diagonal form. We already showed that the pure $U(1)_R$ gauging lead to at least one negative eigenvalue of the Hessian. The fact that $g(q) \geq 0$ makes it impossible to convert the non-minimum critical points that correspond to the upper block of the Hessian ($\frac{\partial^2 P_{(5)}^{(R)}}{(\partial \varphi)^2}$) to minimum points of the potential or change its sign at the critical point. Therefore a $U(1)_H$ gauging will not change the nature of an existing critical point.

However, one has to check what the non-compact generators would do for which (2.2.25) does not hold.

$U(1)_R \times SO(1,1)_H$ gauging:

For the $SO(1,1)$ gauging, a linear combination $W_I A^I$ of all the vectors of the theory will be taken as the gauge field. The $U(1)_R$ gauge field must be orthogonal to the $SO(1,1)$ gauge field. This leads to the condition

$$V_I W_I = 0. \quad (2.2.26)$$

The potential is again given by

$$P_{(5)} = \lambda(P_{(5)}^{(R)} + 2\mathcal{N}_{iA}\mathcal{N}^{iA}) \quad (2.2.27)$$

where this time $\mathcal{N}^{iA} = \frac{\sqrt{6}}{4}(V_I h^I Y^a T_a^X + W_I h^I T_4^X) f_X^{iA}$ and $Y^a T_a^X$, with $a = 1, 2, 3$, defines the linear combination of compact Killing vectors to be used; the $SO(1,1)$ coupling constant is absorbed in W_I 's and the $P_{(5)}^{(R)}$ term is

$$P_{(5)}^{(R)} = -4C^{IJK} V_I V_J h_K (Y^a \vec{P}_a) \cdot (Y^b \vec{P}_b). \quad (2.2.28)$$

The first derivatives of the potential vanish by using (2.2.22) and by setting

$$W_1 = \frac{-\sqrt{2}W_0 - 2W_b\varphi^b\|\varphi\|^2}{2\varphi^1\|\varphi\|^2}, \quad b = 2, \dots, \tilde{n}. \quad (2.2.29)$$

Plugging in everything into the potential, one finds

$$P_{(5)}|_{\phi^c} = -\frac{3}{2}\lambda \left(\frac{V_1}{\varphi^1}\right)^2 \|\varphi\|^4 (Y^a Y^a) \quad (2.2.30)$$

which is manifestly negative and therefore this corresponds to an AdS ground state. Note that all the $\varphi^{\tilde{x}}$'s in this equation are fixed by (2.2.22). The stability is checked by calculating the eigenvalues of the Hessian of the potential. The calculation is tedious and although we were not able to prove generally, all the gauge field combinations subject to (2.2.22), (2.2.26) and (2.2.29) that we tried lead to negative eigenvalues for the Hessian and hence the corresponding critical points were not minima. However the potential is in the form suggested by [126] plus a term that is quadratic in h^I hence it is possible to obtain stable AdS vacua with proper choices of V_I and W_I , provided that the eigenvalues of the Hessian that belong to the hyperscalar sector are positive.

2.2.2 YMESGT with Compact ($SO(2)$) Gauging, Coupled to Tensor Fields

The calculation was done in [118] for $\tilde{n} = 3$. Let us trivially generalize their results to arbitrary $\tilde{n} \geq 3$. The $SO(2)$ subgroup of the isometry group of the scalar manifold acts nontrivially on the vector fields A_μ^2 and A_μ^3 . Hence these vector fields must be dualized to antisymmetric tensor fields. The index \tilde{I} is decomposed as

$$\tilde{I} = (I, M)$$

with $I, J, K = 0, 1, 4, \dots, \tilde{n}$ and $M, N, P = 2, 3$. The fact that the only nonzero C_{IMN} are C_{0MN} for the theory at hand requires A_μ^0 to be the $SO(2)$ gauge field because of $\Lambda_{IN}^M \sim \Omega^{MP} C_{IPN}$ (c.f equation (2.1.3)). All the other A_μ^I with $I \neq 0$ are spectator vector fields with respect to the $SO(2)$ gauging. The potential term (2.1.3) that comes from the tensor coupling is found to be (taking $\Omega^{23} = -\Omega^{32} = -1$)

$$P_{(5)}^{(T)} = \frac{1}{8} \frac{[(\varphi^2)^2 + (\varphi^3)^2]}{\|\varphi\|^6}. \quad (2.2.31)$$

For the function $W_{\tilde{x}}$ that enters the supersymmetry transformation laws of the fermions, one obtains

$$\begin{aligned} W_1 = W_4 = \dots = W_{\tilde{n}} &= 0, \\ W_2 &= \frac{\varphi^3}{4\|\varphi\|^4}, \\ W_3 &= -\frac{\varphi^2}{4\|\varphi\|^4}, \end{aligned} \quad (2.2.32)$$

so one must have $\varphi_c^2 \varphi_c^3 = 0$ to preserve supersymmetry.

2.2.2.1 No R -symmetry Gauging

Without hypermultiplets:

Taking the derivative of the total potential $P_{(5)} = P_{(5)}^{(T)}$ with respect to $\varphi^{\tilde{x}}$, one finds

$$\begin{aligned}\partial_{\varphi^1} P_{(5)} &= -\frac{3}{4} \frac{[(\varphi^2)^2 + (\varphi^3)^2]}{\|\varphi\|^8} \varphi^1, \\ \partial_{\varphi^a} P_{(5)} &= A \varphi^a, \quad a = 2, 3; \\ \partial_{\varphi^b} P_{(5)} &= \frac{3}{4} \frac{[(\varphi^2)^2 + (\varphi^3)^2]}{\|\varphi\|^8} \varphi^b, \quad b = 4, \dots, \tilde{n}\end{aligned}$$

where

$$A = \frac{1}{4} \frac{\|\varphi\|^2 + 3 [(\varphi^2)^2 + (\varphi^3)^2]}{\|\varphi\|^8} > 0.$$

$\partial_{\varphi^a} P_{(5)} = 0$ then implies $\varphi_c^2 = \varphi_c^3 = 0$ (which then also implies $\partial_{\varphi^{\tilde{x}}} P_{(5)}|_{\varphi^c} = 0, \forall \tilde{x}$). But then $P_{(5)}|_{\varphi^c} = 0$ and we have a $\tilde{n} - 2$ parameter family of supersymmetric Minkowski ground states, given by $\langle \varphi^2 \rangle = \langle \varphi^3 \rangle = 0$ and arbitrary $\langle \varphi^d \rangle$, $d = 1, 4, 5, \dots, \tilde{n}$.

With a universal hypermultiplet:

The compact generators of (B.3) vanish at the base point of the hyperscalar manifold. Hence a $U(1)$ gauging of the hyper isometries will not introduce a non-Minkowski ground state.

$SO(2) \times SO(1, 1)_H$ gauging:

The $SO(1, 1)$ gauge field is chosen as a linear combination of all vector fields that are not dualized to tensor fields. The total potential therefore is

$$P_{(5)} = P_{(5)}^{(T)} + \kappa P_{(5)}^{(H)}$$

where $P_{(5)}^{(T)}$ was given in (2.2.31) and $P_{(5)}^{(H)} = 2\mathcal{N}_{iA} \mathcal{N}^{iA}$ where

$$\mathcal{N}^{iA} = V_e h^e T_4^X f_X^{iA}, \quad e = 0, 1, 4, 5, \dots, \tilde{n} \quad (2.2.33)$$

with T_4^X given in (B.3). At the base point of the hyperscalar manifold the derivatives of the total potential with respect to q^X vanish. One can calculate the φ^a -derivatives as

$$\partial_{\varphi^a} P_{(5)}|_{q^c} = \frac{\varphi^a \tilde{B}}{4\|\varphi\|^8} \quad (2.2.34)$$

where

$$\begin{aligned}\tilde{B} &= \|\varphi\|^2 + 3 [(\varphi^2)^2 + (\varphi^3)^2] \\ &\quad + 4\kappa V_0 \|\varphi\|^2 \{V_0 + \sqrt{2} (V_1 \varphi^1 + V_4 \varphi^4 + \dots + V_{\tilde{n}} \varphi^{\tilde{n}}) \|\varphi\|^2\}.\end{aligned}$$

There are two possible ways to make (2.2.34) vanish.

Case 1: $\tilde{B} = 0$

One can solve the equation $\tilde{B} = 0$ for V_1 and plug that into $\partial_{\varphi^b} P_{(5)}|_{q^c} = 0$ to get

$$-\frac{2V_0\varphi^b + \sqrt{2}V_b\|\varphi\|^2 \{ \|\varphi\|^2 + 3 [(\varphi^2)^2 + (\varphi^3)^2] \}}{8V_0\|\varphi\|^6} = 0.$$

Solving this for V_b and plugging the resulting expression together with the $\tilde{B} = 0$ equation into $\partial_{\varphi^1} P_{(5)}|_{q^c}$, one finds

$$\partial_{\varphi^1} P_{(5)}|_{q^c} \rightarrow \frac{9 [(\varphi^2)^2 + (\varphi^3)^2] + 2(3 + 10\kappa(V_0)^2) [(\varphi^2)^2 + (\varphi^3)^2] \|\varphi\|^2 + (1 + 12\kappa(V_0)^2) \|\varphi\|^4}{32\kappa(V_0)^2 \varphi^1 \|\varphi\|^8}$$

which cannot be brought to zero. Hence there is no solution for this case.

Case 2: $\tilde{B} \neq 0$

One has to have $\varphi^a = 0$. Applying this to the remaining first derivative equations, all there are left to solve are the following expressions

$$\begin{aligned} \frac{\kappa}{\|\varphi\|^2} \left\{ \frac{V_1}{\sqrt{2}} - \frac{V_0\varphi^1}{\|\varphi\|^4} \right\} \tilde{C} &= 0, \\ \frac{\kappa}{\|\varphi\|^2} \left\{ \frac{V_b}{\sqrt{2}} + \frac{V_0\varphi^b}{\|\varphi\|^4} \right\} \tilde{C} &= 0 \end{aligned} \quad (2.2.35)$$

with

$$\tilde{C} = V_0 + \sqrt{2} (V_1\varphi^1 + V_4\varphi^4 + V_5\varphi^5 + \dots + V_{\tilde{n}}\varphi^{\tilde{n}}) \|\varphi\|^2. \quad (2.2.36)$$

It is possible to set both expressions to zero by letting $\tilde{C} = 0$, but this will make the potential vanish at the critical point. Setting $\kappa = 0$ will turn off the hyper-gauging. Instead letting

$$\frac{\sqrt{2}V_0}{\|\varphi\|^4} = \frac{V_1}{\varphi^1} = -\frac{V_4}{\varphi^4} = -\frac{V_5}{\varphi^5} = \dots = -\frac{V_{\tilde{n}}}{\varphi^{\tilde{n}}} \quad (2.2.37)$$

will make them vanish and the value of the potential at the critical point becomes

$$P_{(5)}|_{\phi^c} = \frac{9\kappa(V_0)^2}{4\|\varphi\|^4}, \quad (2.2.38)$$

which is positive definite and hence the critical point is a de Sitter ground state. The equations (2.2.37) set the restrictions on choosing V_I as

$$\begin{aligned} (V_1)^2 - (V_4)^2 - (V_5)^2 - \dots - (V_{\tilde{n}})^2 &> 0, \\ V_0 &\neq 0. \end{aligned}$$

Given a set of V_I 's subject to these constraints, the coordinates of the critical point is totally determined by (2.2.37). The non-negativeness of the eigenvalues of the Hessian of the potential assures the stability of the vacuum. For the special case $V_4 = \dots = V_{\tilde{n}} = 0$ the

Hessian is calculated as

$$\partial\partial P_{(5)}|_{\phi^c} = \text{diag} \left(\frac{9\kappa(V_0)^2}{(\varphi^1)^6}, \frac{1+12\kappa(V_0)^2}{4(\varphi^1)^6}, \frac{1+12\kappa(V_0)^2}{4(\varphi^1)^6}, \overbrace{\frac{3\kappa(V_0)^2}{(\varphi^1)^6}, \dots, \frac{3\kappa(V_0)^2}{(\varphi^1)^6}}^{(\tilde{n}-3) \text{ times}}, \right. \\ \left. 0, \frac{9\kappa(V_0)^2}{(2\varphi^1)^4}, \frac{9\kappa(V_0)^2}{(4\varphi^1)^4}, \frac{9\kappa(V_0)^2}{(4\varphi^1)^4} \right)$$

and therefore the ground state is stable. For the more general case the Hessian is not diagonal, but we were able to show that the eigenvalues of the Hessian are non-negative up to at least $\tilde{n} = 6$.

2.2.2.2 $SU(2)_R$ Symmetry Gauging

Without hypermultiplets:

The gauge group is $SO(2) \times SU(2)_R$. For such a gauging one needs at least $\tilde{n} \geq 6$. Choosing $A_{\tilde{\mu}}^4, A_{\tilde{\mu}}^5$ and $A_{\tilde{\mu}}^6$ as the $SU(2)_R$ gauge fields one finds [119]

$$P_{(5)} = P_{(5)}^{(T)} + \lambda P_{(5)}^{(R)}$$

with

$$P_{(5)}^{(R)} = 6\|\varphi\|^2$$

and $P_{(5)}^{(T)}$ given in (2.2.31). It is easy to verify that the total potential does not have any non-Minkowskian ground states. In particular, in order to set the first derivatives to zero, one must have $\varphi_c^2 = \varphi_c^3 = \lambda = 0$ which means the $SU(2)_R$ gauging is turned off and this case was already covered in the previous subsection.

With a universal hypermultiplet:

Inclusion of a hypermultiplet in the theory will change the potential to

$$P_{(5)} = P_{(5)}^{(T)} + \lambda(P_{(5)}^{(R)} + P_{(5)}^{(H)})$$

with now

$$\begin{aligned} P_{(5)}^{(R)} &= -4C^{IJK} \vec{P}_I \cdot \vec{P}_J h_K, \\ P_{(5)}^{(H)} &= \frac{3}{4} h^I h^J K_I^X K_J^Y g_{XY} \end{aligned} \quad (2.2.39)$$

where K_I^X are defined as

$$K_4^X = T_1^X \quad K_5^X = T_2^X \quad K_6^X = T_3^X \quad (2.2.40)$$

and \vec{P}_I are defined accordingly. Remember that $K_I^X = 0$ for compact generators at the base point of the hyperscalar manifold and therefore one has $P_{(5)}^{(H)}|_{q^c} = 0$. It is easy to see that this case is very similar to the case before adding the hypermultiplet and the only possibility is to have Minkowski vacuum. An additional $U(1)_H$ gauging will not change the

situation but let us see what would the $SO(1,1)_H$ gauging do.

$SO(2) \times SU(2)_R \times SO(1,1)_H$ gauging:

The $SO(1,1)$ gauge field is chosen as the linear combination $V_a A_{\tilde{\mu}}^a$, $a = 0, 1, 7, 8, \dots, \tilde{n}$. The total potential is

$$P_{(5)} = P_{(5)}^{(T)} + \lambda(P_{(5)}^{(R)} + P_{(5)}^{(H)})$$

where $P_{(5)}^{(T)}$ and $P_{(5)}^{(R)}$ are as given as in the last case and $P_{(5)}^{(H)} = 2\mathcal{N}_{iA}\mathcal{N}^{iA}$ is modified with

$$\mathcal{N}^{iA} = \frac{\sqrt{6}}{4}(h^I K_I^X + h^a V_a T_4^X) f_X^{iA}. \quad (2.2.41)$$

The calculations for finding the critical points is overly complicated and the expressions are lengthy. Here, we will show a particular example where a stable de Sitter vacuum is found. The first derivatives of the total potential vanish at⁴ $V = 1, \sigma = \theta = \tau = \varphi^2 = \dots = \varphi^8 = 0$ except the φ^1 -derivative

$$\partial_{\varphi^1} P_{(5)}|_{\varphi^e} = 9\lambda \left\{ (1 + (V_1)^2)\varphi^1 - \frac{(V_0)^2}{(\varphi^1)^5} - \frac{V_0 V_1}{\sqrt{2}(\varphi^1)^2} \right\}.$$

Setting this to zero determines the φ^1 -coordinates of the critical points as a function of V_0 and V_1

$$\varphi^1 = \frac{1}{\sqrt{2}} \left(\frac{V_0 V_1 \pm \sqrt{(V_0)^2(8 + 9(V_1)^2)}}{1 + (V_1)^2} \right)^{1/3}. \quad (2.2.42)$$

The values of the total potential at these critical points are

$$P_{(5)}|_{\varphi^e} = \frac{27\lambda V_0 \left(3V_0(\varphi^1)^3 \pm \sqrt{9(V_0)^2(\varphi^1)^6 - 8(\varphi^1)^{12}} \right)}{8(\varphi^1)^7}$$

where φ^1 was given in (2.2.42). The values of the potential are positive definite and therefore the critical points correspond to de Sitter vacua.

Example: In particular, we look at the $\tilde{n} = 8$ theory by taking $V_0 = 1$ and $V_1 = 4$. There is a critical point located at $\varphi^1 = \left(\frac{2 + \sqrt{38}}{17\sqrt{2}} \right)^{1/3}$. The value of the potential at this point is $\frac{27}{4}(937 + 152\sqrt{38})^{1/3}\lambda$. With these choices one can calculate the eigenvalues of the Hessian of the potential numerically as

$$\{1.095\lambda, 19.574\lambda, 19.574\lambda, 127.337\lambda, 218.959\lambda, 218.959\lambda, 254.777\lambda, 284.796\lambda, 284.796\lambda, 693.122\lambda, 2.168 + 218.959\lambda, 2.168 + 218.959\lambda\}.$$

This shows that the critical point is stable.

⁴There are other critical points where φ^e ($e = 2, \dots, \tilde{n}$) are not all zero, but we found that they are unstable.

2.2.2.3 $U(1)_R$ Symmetry Gauging

Without hypermultiplets:

The gauge group is $SO(2) \times U(1)_R$. For such a gauging one needs at least $\tilde{n} \geq 3$. A linear combination $A_{\hat{\mu}}[U(1)_R] = V_I A_{\hat{\mu}}^I$ of vector fields will be used as the $U(1)_R$ gauge field. The total scalar potential in this case is

$$P_{(5)} = P_{(5)}^{(T)} + \lambda P_{(5)}^{(R)} \quad (2.2.43)$$

with

$$P_{(5)}^{(R)} = -2|V|^2 \|\varphi\|^2 - 4\sqrt{2}V_0 \frac{V_i \varphi^i}{\|\varphi\|^2} \quad (2.2.44)$$

where $i = 1, 4, 5, \dots, \tilde{n}$, $|V|^2 = (V_1)^2 - (V_4)^2 - \dots - (V_{\tilde{n}})^2$ and $P_{(5)}^{(T)}$ given in (2.2.31). The first derivatives of the potential

$$\begin{aligned} \partial_{\varphi^1} P_{(5)} &= -(D\varphi^1 + 4\lambda C V_1), \\ \partial_{\varphi^a} P_{(5)} &= \varphi^a \left(D + \frac{1}{4\|\varphi\|^6} \right), \quad a = 2, 3; \\ \partial_{\varphi^b} P_{(5)} &= (D\varphi^b - 4\lambda C V_b), \quad b = 4, \dots, \tilde{n} \end{aligned} \quad (2.2.45)$$

must simultaneously vanish at the critical point(s). Here we defined

$$\begin{aligned} C &= \frac{\sqrt{2}V_0}{\|\varphi\|^2}, \\ D &= \frac{6P_{(5)}^{(T)}}{\|\varphi\|^2} + 4\lambda \left(|V|^2 - \frac{2\sqrt{2}V_0 V_i \varphi^i}{\|\varphi\|^4} \right). \end{aligned}$$

There are two possibilities to set the second equation to zero.

Case 1: $\varphi^a = 0$

This means that $P_{(5)}^{(T)}|_{\varphi^c} = \partial_{\varphi^{\bar{x}}} P_{(5)}^{(T)}|_{\varphi^c} = 0$ and consequently $\partial_{\varphi^{\bar{x}}} P_{(5)}^{(R)}|_{\varphi^c} = 0$. Thus we are dealing with simultaneous critical points of the individual potentials $P_{(5)}^{(T)}$ and $P_{(5)}^{(R)}$. These have already been discussed above. In particular, the coordinates of the critical points are entirely determined by (2.2.22, with $a = 4, \dots, \tilde{n}$) and the potential corresponds to a supersymmetric Anti-de Sitter vacuum with the value given in (2.2.24). Also, it is possible to have a Minkowski ground state with broken supersymmetry by letting all V_I vanish, except V_0 .

Case 2: $\varphi^a \neq 0$

In this case one must have

$$D = -\frac{1}{4\|\varphi\|^6}. \quad (2.2.46)$$

The first and the last equations tell that

$$-\frac{V_1}{\varphi^1} = \frac{V_b}{\varphi^b} = \frac{D}{4\lambda C} \quad (2.2.47)$$

which means $|V|^2 > 0$ and hence

$$D = -\frac{4\lambda V_1}{\varphi^1} C = -\frac{4\sqrt{2}\lambda V_0 V_1}{\varphi^1 \|\varphi\|^2}. \quad (2.2.48)$$

This leads to

$$\varphi^1 = 16\sqrt{2}\lambda V_0 V_1 \|\varphi\|^4. \quad (2.2.49)$$

Plugging (2.2.48) and (2.2.49) into (2.2.46) one arrives at

$$\frac{1}{2\|\varphi\|^6} = 384\lambda^2 (V_0)^2 (V_1)^2 + 4\lambda |V|^2 (1 - 64\lambda (V_0)^2). \quad (2.2.50)$$

Using $|V|^2 > 0$ together with

$$(32\lambda (V_0)^2 - 1)(V_1)^2 > (64\lambda (V_0)^2 - 1)(V_b V_b)$$

which can be derived from (2.2.50), one obtains the condition

$$32\lambda (V_0)^2 > 1. \quad (2.2.51)$$

If V_0 is chosen big enough to satisfy this, new non-trivial critical points exist. Eq. (2.2.50) fixes $\|\varphi_c\|^2$ so that eq.'s (2.2.47) and (2.2.49) fix φ_c^1 and φ_c^b . This in turn fixes $[(\varphi_c^2)^2 + (\varphi_c^3)^2]$ but not φ_c^2 and φ_c^3 individually. Therefore we have a one parameter family of critical points. The value of the potential at the critical points is

$$P_{(5)}|_{\varphi^c} = -32\lambda^2 (V_0)^2 (V_1)^2 \|\varphi\|^2 - \lambda \|\varphi\|^2 |V|^2 (3 + 64\lambda (V_0)^2) \quad (2.2.52)$$

which corresponds to a non-supersymmetric Anti-de Sitter solution. This result agrees with [118] in the $\tilde{n} = 3$ limit⁵. As pointed out in that work, these critical points are saddle points of the total potential. The potential is in the form suggested in [126] plus the semi-positive definite paraboloidlike $P_{(5)}^{(T)}$ term. This tells us that the ground state is stable.

With a universal hypermultiplet:

Inclusion of a hypermultiplet in the theory changes the total potential to

$$P_{(5)} = P_{(5)}^{(T)} + \lambda (P_{(5)}^{(R)} + 2\mathcal{N}_{iA} \mathcal{N}^{iA}) \quad (2.2.53)$$

with now

$$\begin{aligned} P_{(5)}^{(R)} &= -4C^{IJK} \vec{P}_I \cdot \vec{P}_J h_K \\ \mathcal{N}^{iA} &= \frac{\sqrt{6}}{4} (V_I h^I Y^d T_d^X) f_X^{iA} \end{aligned} \quad (2.2.54)$$

where $Y^d T_d^X$ with $d = 1, 2, 3, 8$ defines the linear combination of compact Killing vectors to be used. Remember that $K_I^X = 0$ for compact Killing vectors at the base point of the

⁵ $P_{(5), \tilde{n}=3}|_{\varphi^c} = -\frac{3}{8} \frac{1}{\|\varphi\|^4}$

hyperscalar manifold and therefore one has $P_{(5)}^{(H)}|_{q^c} = 0$. In the last subsection we showed that a $U(1)_H$ gauging will not change the nature of the existing critical points in the theory and hence the critical points are saddle points in this case too.

$SO(2) \times U(1)_R \times SO(1,1)_H$ gauging:

This is very similar to the previous case. The only difference is

$$\mathcal{N}^{iA} = \frac{\sqrt{6}}{4} (V_I h^I Y^d T_d^X + W_I h^I T_4^X) f_X^{iA}. \quad (2.2.55)$$

The linear combination $W_I h^I$ of the vector fields is used as the $SO(1,1)$ gauge field. The $SO(1,1)$ coupling constant is absorbed in W_I 's and the fact that the $U(1)_R$ gauge vector field must be orthogonal to the $SO(1,1)_H$ gauge field tells the orthogonality condition

$$V_I W_I = 0. \quad (2.2.56)$$

The only nontrivial way to set the first derivatives of the potential to zero we found was done by using (2.2.29 with $b = 4, \dots, \tilde{n}$), (2.2.46) and (2.2.47) but this means that $\partial_{\varphi^{\tilde{x}}} (P_{(5)}^{(T)} + \lambda P_{(5)}^{(R)})$ and $\partial_{\varphi^{\tilde{x}}} P_{(5)}^{(H)}$ must vanish separately. Thus we are dealing with simultaneous critical points of the individual potentials $P_{(5)}^{(T)} + \lambda P_{(5)}^{(R)}$ and $P_{(5)}^{(H)}$ which have already been discussed above. In particular, the value of the potential at the one parameter family of critical points becomes

$$P_{(5)}|_{\phi^c} = -32\tilde{\lambda}^2 (V_0)^2 (V_1)^2 \|\varphi\|^2 - \tilde{\lambda} \|\varphi\|^2 |V|^2 (3 + 64\lambda (V_0)^2) \quad (2.2.57)$$

where $\tilde{\lambda} = \frac{\lambda}{4} [(Y^2)^2 + (Y^3)^2 + (Y^4)^2]$ and it corresponds to an Anti-de Sitter ground state, which is of the same form (up to a positive rescaling of λ) as before the hypermultiplet was added to the theory. We expect that it may be possible to obtain stable vacuum with proper choices of the gauge parameters V_I and W_I , provided that the eigenvalues of the Hessian that belong to the hyperscalar sector are positive.

2.2.3 YMESGT with non-compact ($SO(1,1)$) Gauging, Coupled to Tensor Fields

The calculation was done in [118] for $\tilde{n} = 3$. Let us trivially generalize their results to arbitrary $\tilde{n} \geq 2$. The $SO(1,1)$ subgroup of the isometry group of the scalar manifold acts nontrivially on the vector fields $A_{\tilde{\mu}}^1$ and $A_{\tilde{\mu}}^2$. Hence these vector fields must be dualized to antisymmetric tensor fields. The index \tilde{I} is decomposed as

$$\tilde{I} = (I, M)$$

with $I, J, K = 0, 3, 4, \dots, \tilde{n}$ and $M, N, P = 1, 2$. The fact that the only nonzero C_{IMN} are C_{0MN} for the theory at hand requires $A_{\tilde{\mu}}^0$ to be the $SO(1,1)$ gauge field because of $\Lambda_{IN}^M \sim \Omega^{MP} C_{IPN}$ (c.f equation (2.1.3)). All the other $A_{\tilde{\mu}}^I$ with $I \neq 0$ are spectator vector

fields with respect to the $SO(1, 1)$ gauging. The potential term (2.1.3) that comes from the tensor coupling is found to be (taking $\Omega^{12} = -\Omega^{21} = -1$)

$$P_{(5)}^{(T)} = \frac{1}{8} \frac{[(\varphi^1)^2 - (\varphi^2)^2]}{\|\varphi\|^6}. \quad (2.2.58)$$

For the function $W_{\tilde{x}}$ that enters the supersymmetry transformation laws of the fermions, one obtains

$$\begin{aligned} W_3 = W_4 = \dots = W_{\tilde{n}} &= 0, \\ W_1 &= -\frac{\varphi^2}{4\|\varphi\|^4}, \\ W_2 &= \frac{\varphi^1}{4\|\varphi\|^4}. \end{aligned} \quad (2.2.59)$$

Since W_2 can never vanish, there can be no $\mathcal{N} = 2$ supersymmetric critical point.

2.2.3.1 No R -symmetry Gauging

Without hypermultiplets:

Taking the derivative of the total potential $P_{(5)} = P_{(5)}^{(T)}$ with respect to $\varphi^{\tilde{x}}$, one finds

$$\begin{aligned} \partial_{\varphi^1} P_{(5)} &= B\varphi^1, \\ \partial_{\varphi^2} P_{(5)} &= -B\varphi^2, \\ \partial_{\varphi^b} P_{(5)} &= -B\varphi^b + \frac{\varphi^b}{4\|\varphi\|^6}, \quad b = 3, \dots, \tilde{n} \end{aligned}$$

where

$$B = -\frac{3}{4} \frac{(\varphi^1)^2 - (\varphi^2)^2}{\|\varphi\|^8} + \frac{1}{4\|\varphi\|^6} < 0. \quad (2.2.60)$$

Since $\partial_{\varphi^1} P_{(5)}$ cannot be brought to zero there are no critical points.

With a universal hypermultiplet:

The compact generators of (B.3) vanish at the base point of the hyperscalar manifold. Hence a $U(1)$ gauging of the hyper isometries will not introduce critical points.

$SO(1, 1) \times SO(1, 1)_H$ gauging:

The $SO(1, 1)_H$ gauge field is chosen as a linear combination of all vector fields that are not dualized to tensor fields. The total potential therefore is

$$P_{(5)} = P_{(5)}^{(T)} + \kappa P_{(5)}^{(H)}$$

where $P_{(5)}^{(T)}$ was given in (2.2.31) and $P_{(5)}^{(H)} = 2\mathcal{N}_{iA}\mathcal{N}^{iA}$ where

$$\mathcal{N}^{iA} = V_e h^e T_4^X f_X^{iA}, \quad e = 0, 3, 4, 5, \dots, \tilde{n}$$

with T_4^X given in (B.3). At the base point of the hyperscalar manifold the q -derivatives of the total potential vanish. One can calculate the φ^1 -derivative as

$$\partial_{\varphi^1} P_{(5)}|_{q^c} = -\frac{\varphi^1 (|\varphi|^2 - 3[(\varphi^1)^2 - (\varphi^2)^2] + 4\kappa V_0 |\varphi|^2 (V_0 + \sqrt{2}V_i \varphi^i |\varphi|^2))}{4|\varphi|^8}$$

where $i = 3, \dots, \tilde{n}$. Setting this expression to zero and solving for $V_{\tilde{n}}$ and plugging the resulting expression into the $\partial_{\varphi^j} P_{(5)}|_{q^c} = 0$, $j = 2, \dots, \tilde{n} - 1$ equations gives

$$\begin{aligned} \partial_{\varphi^2} P_{(5)}|_{\phi^c} &= 0, \\ \partial_{\varphi^k} P_{(5)}|_{\phi^c} &= \frac{2V_0 \varphi^k + \sqrt{2}V_k (|\varphi|^2 (|\varphi|^2 - 3[(\varphi^1)^2 - (\varphi^2)^2]))}{8V_0 |\varphi|^6} = 0; \quad k = 3, \dots, \tilde{n} - 1. \end{aligned}$$

Solving the equations in the second line for V_k and plugging in everything into the $\varphi^{\tilde{n}}$ -derivative of the potential gives

$$\partial_{\varphi^{\tilde{n}}} P_{(5)}|_{\phi^c} = \frac{(3[(\varphi^1)^2 - (\varphi^2)^2] - |\varphi|^2)^2 + 4|\varphi|^2 (5[(\varphi^1)^2 - (\varphi^2)^2] - 3|\varphi|^2) \kappa (V_0)^2}{32\kappa (V_0)^2 \varphi^{\tilde{n}} |\varphi|^8}$$

and this cannot be brought to zero. Therefore there are no critical points for this type of gauging either.

2.2.3.2 $SU(2)_R$ Symmetry Gauging

Without hypermultiplets:

The gauge group is $SO(1,1) \times SU(2)_R$. For such a gauging one needs at least $\tilde{n} \geq 5$. Choosing $A_{\tilde{\mu}}^3, A_{\tilde{\mu}}^4, A_{\tilde{\mu}}^5$ as the $SU(2)_R$ gauge fields one finds

$$P_{(5)} = P_{(5)}^{(T)} + \lambda P_{(5)}^{(R)}$$

with

$$P_{(5)}^{(R)} = 6|\varphi|^2 \tag{2.2.61}$$

and $P_{(5)}^{(T)}$ given in (2.2.58). Taking the derivative of the total potential with respect to $\varphi^{\tilde{x}}$ one finds

$$\begin{aligned} \partial_{\varphi^1} P_{(5)} &= (B + 12\lambda)\varphi^1 \\ \partial_{\varphi^2} P_{(5)} &= -(B + 12\lambda)\varphi^2 \\ \partial_{\varphi^b} P_{(5)} &= -(B + 12\lambda)\varphi^b + \frac{\varphi^b}{4|\varphi|^6}, \quad b = 3, \dots, \tilde{n} \end{aligned} \tag{2.2.62}$$

with B defined in (2.2.60). Setting the first equation to zero means

$$B = -12\lambda \tag{2.2.63}$$

since $\varphi^1 \neq 0$. The last equation then implies $\varphi_c^b = 0$. From (2.2.63) we find

$$\frac{1}{\|\varphi_c\|^6} = 24\lambda. \quad (2.2.64)$$

The value of $\|\varphi_c\|^2 = (\varphi_c^1)^2 - (\varphi_c^2)^2$ is fixed by λ but not φ_c^1 and φ_c^2 individually. The value of the potential at these critical points is

$$P_{(5)}|_{\varphi^c} = \frac{3}{8\|\varphi_c\|^4} \quad (2.2.65)$$

and therefore it corresponds to a one parameter family of de Sitter ground states. The stability of the critical points is checked by calculating the eigenvalues of the Hessian of the potential, which are easily found as

$$\left\{ 0, \frac{3 [(\varphi_c^1)^2 + (\varphi_c^2)^2]}{\|\varphi_c\|^8}, \underbrace{\frac{1}{4\|\varphi_c\|^6}, \dots, \frac{1}{4\|\varphi_c\|^6}}_{(\tilde{n}-2) \text{ times}} \right\}.$$

The eigenvalues are all non-negative, thus the one parameter family of de Sitter critical points is found to be stable.

With a universal hypermultiplet:

Since a $U(1)_H$ hyper-gauging will not change the nature of the critical points we will just do the $SO(1,1)_H$ hyper-gauging.

$SO(1,1) \times SU(2)_R \times SO(1,1)_H$ gauging:

The $SO(1,1)$ gauge field is chosen as the linear combination $V_a A_{\tilde{\mu}}^a$, $a = 0, 6, 7, 8, \dots, \tilde{n}$. The total potential is

$$P_{(5)} = P_{(5)}^{(T)} + \lambda(P_{(5)}^{(R)} + P_{(5)}^{(H)})$$

where $P_{(5)}^{(T)}$ is as given as in the last case; $P_{(5)}^{(R)}$ and $P_{(5)}^{(H)} = 2\mathcal{N}_{iA}\mathcal{N}^{iA}$ is modified with

$$\begin{aligned} P_{(5)}^{(R)} &= -4C^{IJK}\vec{P}_I \cdot \vec{P}_J h_K, \\ \mathcal{N}^{iA} &= \frac{\sqrt{6}}{4}(h^I K_I^X + h^a V_a T_4^X) f_X^{iA}. \end{aligned}$$

At the base point of the hyperscalar manifold, the q -derivatives of the total potential are found as

$$\begin{aligned} \partial_V P_{(5)}|_{q^c} &= 0 \\ \partial_\sigma P_{(5)}|_{q^c} &= \frac{27}{4}\lambda\varphi^5 \left\{ 2V_b\varphi^b + \frac{\sqrt{2}V_0}{\|\varphi\|^2} \right\} \\ \partial_\theta P_{(5)}|_{q^c} &= \frac{27}{2}\lambda\varphi^4 \left\{ 2V_b\varphi^b + \frac{\sqrt{2}V_0}{\|\varphi\|^2} \right\} \\ \partial_\tau P_{(5)}|_{q^c} &= \frac{27}{2}\lambda\varphi^3 \left\{ 2V_b\varphi^b + \frac{\sqrt{2}V_0}{\|\varphi\|^2} \right\} \end{aligned}$$

with $b = 6, \dots, \tilde{n}$. There are two ways of setting these expressions to zero. The first one is to set $\varphi_c^3 = \varphi_c^4 = \varphi_c^5 = 0$ and the second one is to set $2V_b\varphi^b + \frac{\sqrt{2}V_0}{\|\varphi\|^2} = 0$. One can show that the first case leads to the second one (and vice versa), and we choose to proceed with the second case. With this choice the $\varphi^{\tilde{x}}$ -derivatives of the potential (at the base point of the hyperscalar manifold) are evaluated as

$$\begin{aligned}\partial_{\varphi^1} P_{(5)}|_{\phi^c} &= \varphi^1 \left(9\lambda - \frac{3[(\varphi^1)^2 - (\varphi^2)^2] - \|\varphi\|^2}{4\|\varphi\|^8} \right), \\ \partial_{\varphi^2} P_{(5)}|_{\phi^c} &= -\varphi^2 \left(9\lambda - \frac{3[(\varphi^1)^2 - (\varphi^2)^2] - \|\varphi\|^2}{4\|\varphi\|^8} \right), \\ \partial_{\varphi^d} P_{(5)}|_{\phi^c} &= \frac{3}{4}\varphi^d \left(-12\lambda + \frac{(\varphi^1)^2 - (\varphi^2)^2}{\|\varphi\|^8} \right), \quad d = 3, \dots, \tilde{n}.\end{aligned}$$

The only way to set these equations to zero is to have $\varphi_c^d = 0$ together with $\lambda = \frac{1}{18\|\varphi\|^6}$. Note that, setting $\varphi_c^d = 0$ implies $V_0 = 0$. So, in order to have the potential term coming from the $SO(1, 1)_H$ gauging not vanish we must have at least $\tilde{n} \geq 6$. Plugging in everything into the total potential, one finds that

$$P_{(5)}|_{\varphi^c} = \frac{3}{8\|\varphi_c\|^4}.$$

The value of λ determines $\|\varphi_c\|^2 = (\varphi_c^1)^2 - (\varphi_c^2)^2$ but not the φ_c^1 and φ_c^2 individually. Therefore we found a one parameter family of de Sitter ground states. The eigenvalues of the Hessian of the potential, evaluated at the family of critical points, are found to be

$$\left\{ \overbrace{0, \frac{1}{4\|\varphi\|^6}, \dots, \frac{1}{4\|\varphi\|^6}}^{(\tilde{n}-3) \text{ times}}, \frac{1 + 2V_b V_b}{4\|\varphi\|^6}, \frac{1}{8\|\varphi\|^4}, \frac{1}{2\|\varphi\|^4}, \frac{1}{2\|\varphi\|^4}, \frac{3[(\varphi^1)^2 + (\varphi^2)^2]}{\|\varphi\|^8} \right\}.$$

The eigenvalues are all non-negative and therefore the critical points are stable.

2.2.3.3 $U(1)_R$ Symmetry Gauging

Without hypermultiplets:

The calculation in [118] for $\tilde{n} = 3$ was later generalized to arbitrary $\tilde{n} \geq 3$ in [120]. Let us briefly quote their results. A linear combination $A_{\tilde{\mu}}[U(1)_R] = V_I A_{\tilde{\mu}}^I$ of the vector fields is taken as the $U(1)_R$ gauge field. The scalar potential is now

$$P_{(5)} = P_{(5)}^{(T)} + \lambda P_{(5)}^{(R)}$$

where

$$P_{(5)}^{(R)} = -4\sqrt{2}V_0V_i\varphi^i\|\varphi\|^{-2} + 2|V|^2\|\varphi\|^2 \quad (2.2.66)$$

with $i = 3, \dots, \tilde{n}$ and $|V|^2 = V_i V_i$. Demanding $\partial_{\varphi^{\tilde{x}}} P_{(5)} = 0$, one obtains the following conditions

$$\begin{aligned} \frac{\varphi_c^i}{\|\varphi_c\|^4} &= 16\sqrt{2}\lambda V_0 V_i \\ \frac{1}{\|\varphi_c\|^6} &= -\frac{1}{2}(16\sqrt{2}\lambda V_0 |V|)^2 + 8\lambda |V|^2 \end{aligned} \quad (2.2.67)$$

with the constraints

$$\begin{aligned} |V|^2 &> 0 \\ 32\lambda(V_0)^2 &< 1. \end{aligned} \quad (2.2.68)$$

Given a set of V_I subject to (2.2.68), we see that $\|\varphi\|^2$ and φ^i (and thus $(\varphi^1)^2 - (\varphi^2)^2$) are completely determined by (2.2.67) but φ^1 and φ^2 are otherwise undetermined. The value of the potential at these one parameter family of critical points becomes

$$P_{(5)}|_{\varphi^c} = 3\lambda\|\varphi\|^2|V|^2(1 - 32\lambda(V_0)^2) \quad (2.2.69)$$

and this corresponds to de Sitter vacua. The stability is checked by calculating the eigenvalues of the Hessian of the potential at the critical point. We can use the $SO(1, 1)$ invariance together with the $SO(\tilde{n} - 2)$ of the φ^i to take for any critical point $\varphi_c = (\varphi^1, 0, \varphi^3, 0, \dots, 0)$. With these choices the Hessian becomes block diagonal at the critical point. φ^2 is a zero mode and the sector $\varphi^4, \dots, \varphi^{\tilde{n}}$ consists of a unit matrix times $\frac{1}{4}\|\varphi\|^{-6}$. The only non-diagonal part of the Hessian is

$$\partial_{\tilde{x}}\partial_{\tilde{y}}P_{(5)}|_{\tilde{x}, \tilde{y}=1,3} = \gamma \begin{pmatrix} (\varphi^1)^2[6(\varphi^1)^2 + 5(\varphi^3)^2] & -\varphi^1[8(\varphi^1)^2\varphi^3 + 3(\varphi^3)^3] \\ -\varphi^1[8(\varphi^1)^2\varphi^3 + 3(\varphi^3)^3] & \frac{1}{4}[2(\varphi^1)^4 + 37(\varphi^1)^2(\varphi^3)^2 + 5(\varphi^3)^4] \end{pmatrix}$$

with $\gamma = \|\varphi\|^{-8}[2(\varphi^1)^2 - (\varphi^3)^2]^{-1}$. The determinant and the trace of this part of the Hessian are

$$\begin{aligned} \det \partial\partial P_{(5)} &= \frac{12(\varphi^1)^6 - 12(\varphi^1)^4(\varphi^3)^2 + 11(\varphi^1)^2(\varphi^3)^4}{4\|\varphi\|^{14}[2(\varphi^1)^2 - (\varphi^3)^2]^2} \\ \text{tr} \partial\partial P_{(5)} &= \frac{26(\varphi^1)^4 + 57(\varphi^1)^2(\varphi^3)^2 + 5(\varphi^3)^4}{4\|\varphi\|^8[2(\varphi^1)^2 - (\varphi^3)^2]} \end{aligned}$$

which are both positive because of $(\varphi^1)^2 > (\varphi^3)^2$ and therefore the family of critical points is found to be stable. We note that, although the above quantities are both positive, they are slightly different than the ones found in [120], where the authors fixed the coupling constants with $\lambda = 1$. Figure 2.1 shows the plot of the potential (2.2.66) for the special case $\tilde{n} = 3$, $V_0 = 0$ and $\lambda = 1$.

At this point we want to emphasize that the stable dS vacua found by gauging $SO(1, 1) \times SU(2)_R$ or $SO(1, 1) \times U(1)_R$ will play an important role in the four dimensional stable dS vacua calculations in chapter 3.

With a universal hypermultiplet:

Inclusion of a hypermultiplet in the theory changes the total potential to

$$P_{(5)} = P_{(5)}^{(T)} + \lambda(P_{(5)}^{(R)} + 2\mathcal{N}_{iA}\mathcal{N}^{iA}) \quad (2.2.70)$$

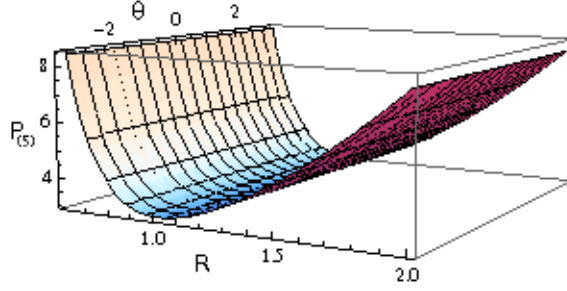


Figure 2.1. The extrema of the potential $P_{(5)}(R, \theta)$ due to $SO(1, 1) \times U(1)_R$ gauging, evaluated at $\varphi^3 = 0$; $V_0 = 0$ and $\lambda = 1$; with parametrization $\varphi^1 = R \cosh \theta$, $\varphi^2 = R \sinh \theta$. The zero eigenvalue of the Hessian corresponds to the flat direction of the potential at its minima.

with now

$$\begin{aligned} P_{(5)}^{(R)} &= -4C^{IJK} \vec{P}_I \cdot \vec{P}_J h_K \\ \mathcal{N}^{iA} &= \frac{\sqrt{6}}{4} (V_I h^I Y^d T_d^X) f_X^{iA} \end{aligned} \quad (2.2.71)$$

where $Y^d T_d^X$ with $d = 1, 2, 3, 8$ defines the linear combination of compact Killing vectors to be used. This potential also has a one parameter family of de Sitter critical points. The $U(1)$ gauging in the hyper sector will scale $P_{(5)}^{(R)}$ by a positive factor at the base point of the hyperscalar manifold, which can be embedded in V_I 's. Because of $\frac{\partial^2 P_{(5)}}{\partial \varphi \partial q} |_{q^c} = 0$, as we discussed before, the only thing remains to be checked is the stability of the hyper sector. Due to the lengthiness of the expressions, we give a particular example. Taking $Y^1 = Y^2 = 0, Y^4 = -\sqrt{3}Y^8$ we arrive at a further restriction on V_0 in order to maintain stability:

$$64\lambda(V_0)^2 > 1$$

This restriction (together with (2.2.68)) is necessary and sufficient to obtain stable dS vacua.

Example: Using the $SO(1, 1)$ and the $SO(\tilde{n} - 2)$ invariances as in the previous case before we added hypers, we calculated the coordinates of the critical point for the specific case $V_0 = \frac{3}{2}, V_3 = 1, \lambda = \frac{1}{96}, Y^8 = 1$ as

$$\varphi^1 = \sqrt{7}\sqrt[6]{48}, \quad \varphi^3 = \sqrt{2}\sqrt[3]{36}, \quad \varphi^2 = \varphi^4 = \dots = \varphi^{\tilde{n}} = 0.$$

The value of the potential at this critical point is $\frac{1}{8\sqrt[3]{36}}$ and the eigenvalues of the $\frac{\partial^2 P_{(5)}}{\partial q^2}$ part of the Hessian at this critical point are found to be

$$\frac{4\sqrt{3} - 3}{64\sqrt[3]{36}}, \quad \frac{4\sqrt{3} - 3}{64\sqrt[3]{36}}, \quad \frac{2505 - 128\sqrt{3}}{512\sqrt[3]{36}}, \quad \frac{2505 - 128\sqrt{3}}{512\sqrt[3]{36}}$$

which are all positive. Note that it is also possible to obtain unstable critical points with different choices of Y^d and V_I . To conclude this subsection we investigate the situation with an additional non-compact hyper-gauging.

	No R sym. gauging	$SU(2)_R$ gauging	$U(1)_R$ gauging
MESGT	Minkowski (supersymmetric) arbitrary $\tilde{n} \geq 0$	- $\tilde{n} \geq 4$	AdS (supersymmetric, stable) +Minkowski (nonsuper- symmetric) $\tilde{n} \geq 1$
YMESGT with tensors and gauge group $SO(2)$	Minkowski (supersymmetric) $\tilde{n} \geq 3$	- $\tilde{n} \geq 6$	AdS (supersymmetric, stable + nonsuper- symmetric, stable) +Minkowski (nonsuper- symmetric) $\tilde{n} \geq 3$
YMESGT with tensors and gauge group $SO(1,1)$ (broken susy)	- $\tilde{n} \geq 2$	dS (stable) $\tilde{n} \geq 5$	dS (stable) $\tilde{n} \geq 3$

Table 2.1. Ground states of $d = 5, \mathcal{N} = 2$ supergravity *without hypermultiplets*. The columns represent different R -symmetry gaugings whereas the rows represent different tensor couplings. \tilde{n} denotes the minimum number of vector multiplets that must be coupled to the theory in order to make the respective gauging possible. “-” means there are no ground states.

$SO(1,1) \times U(1)_R \times SO(1,1)_H$ gauging:

This is very similar to the previous case. The only difference is

$$\mathcal{N}^{iA} = \frac{\sqrt{6}}{4}(V_I h^I Y^d T_d^X + W_I h^I T_4^X) f_X^{iA}. \quad (2.2.72)$$

The linear combination $A_{\hat{\mu}}[SO(1,1)] = W_I A_{\hat{\mu}}^I$ of the vector fields is used as the $SO(1,1)$ gauge field. The $SO(1,1)$ coupling constant is absorbed in W_I 's and the fact that the $U(1)_R$ gauge vector field must be orthogonal to the $SO(1,1)_H$ gauge field tells the orthogonality condition

$$V_I W_I = 0. \quad (2.2.73)$$

The first derivatives of the potential can be set to zero by using (2.2.46), (2.2.47) and $W_0 = -\sqrt{2}W_b \varphi^b / \|\varphi\|^2$, $b = 3, \dots, \tilde{n}$. We found that the potential has a one parameter family of de Sitter ground states. The stability again depends on the values taken for the constants. The calculation is quite messy and here we will look at a particular example with stable ground state.

	No R sym. gauging	$SU(2)_R$ gauging	$U(1)_R$ gauging
MESGT	dS (stable)	dS (stable + unstable)	AdS (stable ^b + unstable)
YMESGT with tensors and gauge group $SO(2)$	dS (stable ^a)	dS (stable + unstable)	AdS (stable ^b + unstable)
YMESGT with tensors and gauge group $SO(1, 1)$	-	dS (stable)	dS (stable + unstable)

Table 2.2. Ground states of $d = 5, \mathcal{N} = 2$ supergravity *with one hypermultiplet and with non-compact $SO(1, 1)$ gauging of the hyper sector*. The columns represent different R -symmetry gaugings whereas the rows represent different tensor couplings. Note that non-compact hyper-gauging implies broken supersymmetry. The Minkowskian ground states are not listed. “-” means there are no ground states.

^aup to $\tilde{n} = 6$ at least

^bWe haven’t found explicit results but the form of the expressions suggests that it is possible to obtain stable vacua.

Example: For the constants in the theory, we take the following numbers: $V_0 = \frac{1}{2\sqrt{2}}$; $V_3 = \frac{1}{4}$; $V_e = W_e = 0$, ($e = 4, \dots, \tilde{n}$); $\lambda = 2 + \sqrt{3}$; $W_0 = \frac{1}{96\sqrt{2(2+\sqrt{3})}}$; $W_3 = -\frac{(2+\sqrt{3})^{3/2}}{96(7+4\sqrt{3})}$; $Y^1 = Y^2 = 0$; $Y^5 = -\frac{Y^4}{\sqrt{3}} = \frac{1}{3}\sqrt{\frac{5}{2}(2 - \sqrt{3})}$. The point $\varphi^1 = 2, \varphi^2 = 1, \varphi^3 = 1, \varphi^e = 0$ is a critical point. The value of the potential at this critical point is $\frac{7}{128}$ and the eigenvalues of the Hessian at this critical point are $\frac{481}{128}, \frac{481}{128}, \frac{1}{576} (490 \pm \sqrt{216430})$, $0, \dots, 0$, which are all non-negative and hence this corresponds to a stable de Sitter vacuum.

2.2.4 Summary

Our results are summarized in *Table 2.1* for the theories that do not include hypers and *Table 2.2* for theories that include a universal hypermultiplet and have non-compact $SO(1, 1)_H$ gauging of hyper isometries.

2.3 Magical Jordan Family

It is possible to apply the results obtained for the generic Jordan family to the magical Jordan family, provided that there are enough vector fields in the magical family member to do the gauging that is being “imported” from the generic family. For instance, the $SO(2) \times SU(2)_R$ gauging requires at least $\tilde{n} = 6$ vector multiplets and thus this generic family model cannot be embedded in the smallest member of the magical Jordan family $\mathcal{M} = \frac{SL(3, \mathbb{R})}{SO(3)}$. It can only be embedded into the bigger members of the family. In this

section we will first see two examples of such embeddings. These models also contain other critical points, that are special to the magical family case, i.e. that were not obtained in the generic case.

What is more interesting for the magical Jordan family is, a non-Abelian gauging of the isometry group will introduce a potential term due to the tensor coupling. This is because of the fact that, unlike in the generic Jordan family theories, in the magical Jordan family theories there are vector fields that are nontrivially charged under the non-Abelian gauge group. By “nontrivial”, we mean that there are other vectors, than the ones in the adjoint representation of the gauge group $K_{(5)}$, that are not singlets. These vector fields should be dualized to tensor fields and this dualization introduces a scalar potential $P_{(5)}^{(T)}$. An example to such a gauging will be investigated in the last part of this section.

2.3.1 $\mathcal{M} = SL(3, \mathbb{R})/SO(3)$

\mathcal{M} is described by the hypersurface $N(h) = C_{\tilde{I}\tilde{J}\tilde{K}} h^{\tilde{I}} h^{\tilde{J}} h^{\tilde{K}} = 1$ of the cubic polynomial

$$N(h) = \frac{3}{2}\sqrt{3}h^3\eta_{ij}h^i h^j + \frac{3\sqrt{3}}{2\sqrt{2}}\gamma_{imn}h^i h^m h^n, \quad (2.3.1)$$

where

$$\begin{aligned} i, j &= 0, 1, 2 & m, n &= 4, 5 \\ \eta_{ij} &= \text{diag}(+, -, -) & \gamma_0 &= -\mathbb{1}_2 \\ \gamma_1 &= \sigma_1 & \gamma_2 &= \sigma_3. \end{aligned}$$

In this parametrization the non-vanishing $C_{\tilde{I}\tilde{J}\tilde{K}}$'s are

$$\begin{aligned} C_{003} = -C_{113} = -C_{223} &= \frac{\sqrt{3}}{2} \\ C_{044} = C_{055} &= -\frac{\sqrt{3}}{2\sqrt{2}} \\ C_{244} = -C_{255} &= \frac{\sqrt{3}}{2\sqrt{2}} \\ C_{145} &= \frac{\sqrt{3}}{2\sqrt{2}} \end{aligned}$$

and their permutations. $N(h)$ indeed is the determinant of the Jordan algebra $J_3^{\mathbb{R}}$ element

$$\tilde{\mathbf{h}} = \frac{\sqrt{3}}{\sqrt{2}} \begin{pmatrix} h^0 + h^2 & h^1 & h^4 \\ h^1 & h^0 - h^2 & h^5 \\ h^4 & h^5 & \sqrt{2}h^3 \end{pmatrix}.$$

To solve $N(h) = 1$ we take the parametrization:

$$h^i = \sqrt{\frac{2}{3}}x^i, \quad h^m = \sqrt{\frac{2}{3}}b^m, \quad h^3 = \frac{1 - b^T \bar{x} b}{\sqrt{3}||x||^2}$$

where $||x||^2 = \eta_{ij}x^i x^j$ and $b^T \bar{x} b = b^m x^i \gamma_{imn} b^n$.

2.3.1.1 $SO(2) \times U(1)_R$ Gauging

The fields h^0 and h^3 are chargeless under the action of the compact $SO(2)$ generator

$$\tilde{\sigma}_2 = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

while h^1 and h^2 are forming a doublet with charge 2; and h^4 and h^5 are forming another doublet with charge 1. Therefore the vector fields $A_{\bar{\mu}}^1, A_{\bar{\mu}}^2, A_{\bar{\mu}}^4, A_{\bar{\mu}}^5$ need to be dualized to tensor fields. To gauge the $SO(2)$ subgroup of the symmetry group we will use a linear combination of the fields $A_{\bar{\mu}}^0$ and $A_{\bar{\mu}}^3$. The $U(1)_R$ symmetry will be gauged by another linear combination of the same fields. We will split the vector and tensor indices as $I, J = 0, 3$; $M, N = 1, 2, 4, 5$, respectively.

The symplectic matrix Ω_{MN} that appears in the potential is given by

$$\Omega_{MN} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

and the non-vanishing Λ_I^{MN} 's are

$$\Lambda_0^{44} = \Lambda_0^{55} = -\frac{1}{2}, \quad \Lambda_3^{11} = \Lambda_3^{22} = \frac{1}{\sqrt{2}}.$$

Now the potential terms $P_{(5)}^{(T)} = \frac{3\sqrt{6}}{16}h^I \Lambda_I^{MN} h_M h_N$ and $P_{(5)}^{(R)} = -4C^{IJK} V_I V_J h_K$ become:

$$P_{(5)}^{(T)} = \frac{3\sqrt{6}}{16}[-h^0[(h_4)^2 + (h_5)^2] + \sqrt{2}h^3[(h_1)^2 + (h_2)^2]]|_{N(h)=1}, \quad (2.3.2)$$

$$P_{(5)}^{(R)} = -2V_0 \left\{ V_0 ||x||^2 + \sqrt{2}V_3 \left(2x^0 \frac{1 - b^T \bar{x} b}{||x||^2} - [(b^4)^2 + (b^5)^2] \right) \right\}. \quad (2.3.3)$$

Here we defined $h_{\bar{I}} \equiv \frac{1}{3} \frac{\partial}{\partial h^{\bar{I}}} N|_{N=1}$. One should note that determinants of the vector/tensor field metric $\hat{a}_{\bar{I}\bar{J}}$ and the hypersurface metric $g_{\bar{x}\bar{y}}$ are given by 1 and $\frac{243}{16||x||^4}$, respectively. This tells us that both metrics are positive definite on the scalar manifold when $||x||^2 \neq 0$.

The total potential is:

$$P_{(5)} = P_{(5)}^{(T)} + \lambda P_{(5)}^{(R)} \quad (2.3.4)$$

where $P_{(5)}^{(T)}$ and $P_{(5)}^{(R)}$ were given in (2.3.2),(2.3.3). It is quite difficult to calculate the most general solution for the critical points of this potential. Instead, we look at specific sectors.

Sector 1: $b^4 = b^5 = 0$ at the critical point

This sector looks quite similar to the generic case with $\frac{SO(2,1) \times SO(1,1)}{SO(2)}$ as the scalar manifold with $SO(2) \times U(1)_R$ gauging, yet there is an important difference. In the generic case, to have the metrics $\overset{\circ}{a}_{\bar{i}\bar{j}}$ and $g_{\bar{x}\bar{y}}$ positive definite, we were forced to look at the sector $h^0 \neq 0$. Now this restriction does not apply anymore and this will help us find more ground states.

To find the critical points of the scalar manifold, we take the derivatives of the total potential with respect to all the scalars of the manifold (i.e. x^0, x^1, x^2, b^4, b^5) and set them equal to zero. These derivatives are given by

$$\begin{aligned} P_{(5),0}|_{b^4=b^5=0} &= -(A + \lambda B)x^0 + \frac{x^0}{4||x||^6} - 4\sqrt{2}\lambda \frac{V_0 V_3}{||x||^2}, \\ P_{(5),1}|_{b^4=b^5=0} &= (A + \lambda B)x^1, \\ P_{(5),2}|_{b^4=b^5=0} &= (A + \lambda B)x^2, \\ P_{(5),4}|_{b^4=b^5=0} &= P_{(5),5}|_{b^4=b^5=0} = 0 \end{aligned} \quad (2.3.5)$$

where

$$\begin{aligned} A &= \frac{3(x^1)^2 + (x^2)^2}{4||x||^8} + \frac{1}{4||x||^6}, \\ B &= 4V_0(V_0 - \frac{2\sqrt{2}V_3x^0}{||x||^4}). \end{aligned}$$

The case with $x^0 \neq 0$ is almost equivalent to the generic case and it was studied in subsection 2.2.2.3. There are two possibilities for setting the first derivatives of the potential to zero.

- **Case 1: $x_c^1 = x_c^2 = 0$**

Letting $V_0 = 0$, one can get a Minkowski ground state with broken supersymmetry. Or one can set the above equations to zero by letting $\sqrt{2}V_3 = V_0(x^0)^3$. The value of the potential then becomes $P_{(5)}|_{\varphi_c} = -6\lambda(V_0x^0)^2$ and this corresponds to a stable, supersymmetric Anti-de Sitter critical point. The same analysis we did in subsection 2.2.2.3 shows that this critical point is a saddle point.

- **Case 2: $(x_c^1)^2 + (x_c^2)^2 \neq 0$**

Setting the first derivatives of the potential to zero, one arrives at (c.f. equations

(2.2.49, 2.2.50))

$$\begin{aligned}\frac{x^0}{\|x\|^4} &= 16\sqrt{2}\lambda V_0 V_3, \\ \frac{1}{\|x\|^6} &= \frac{1}{2}(16\sqrt{2}\lambda V_0 V_3)^2 + 8\lambda(V_0)^2.\end{aligned}\quad (2.3.6)$$

In order to have these equations consistent, one needs

$$32\lambda(V_3)^2 > 1, \quad V_0 V_3 \neq 0. \quad (2.3.7)$$

There is a one parameter family of Anti-de Sitter ground states which do not preserve the full $\mathcal{N} = 2$ supersymmetry, and the value of the potential at these critical points is given by

$$P_{(5)}|_{\varphi_c} = -\frac{3}{8} \frac{1}{\|x\|^4} < 0. \quad (2.3.8)$$

Now we come to the case with $x^0 = 0$. Equations (2.3.5) reduce to

$$\begin{aligned}V_0 V_3 &= 0, \\ \frac{1}{2}x^a \left(8\lambda(V_0)^2 + \frac{1}{\|x\|^6} \right) &= 0, \quad a = 1, 2.\end{aligned}\quad (2.3.9)$$

It's not possible to solve the second equation by letting V_0 vanish, so one sets $V_3 = 0$. The second equation then is solved by $8\lambda(V_0)^2\|x\|^6 = -1$. The potential evaluated at the critical point is given by $P_{(5)}|_{\varphi_c} = -\frac{3}{8} \frac{1}{[(x^1)^2 + (x^2)^2]}$ and this is a one parameter family of stable and AdS vacua with broken supersymmetry ($x_c^a \neq 0 \Rightarrow P^a \neq 0$).

Sector 2: $x^1 = x^2 = 0$ at the critical point

The derivative of the potential with respect to the scalar x^2 is

$$P_{(5),2} = -\frac{[(b^4)^2 - (b^5)^2][4 + 8D + 5D^2 - 128\sqrt{2}\lambda V_0 V_3(x^0)^3 - 8(x^0)^6]}{32(x^0)^4} \quad (2.3.10)$$

where $D = [(b^4)^2 + (b^5)^2]x^0$.

- **Case 1:** $(b^4)^2 - (b^5)^2 = 0$

Setting $b^4 = b^5$, the b^4 and b^5 derivatives of the potential become

$$P_{(5),4} = P_{(5),5} = \frac{b^5 \{ (b^5)^2 + 3(b^5)^4 x^0 - (x^0)^2 (16\sqrt{2}\lambda V_0 V_3 + (x^0)^3) \}}{4(x^0)^2}. \quad (2.3.11)$$

One way to set this equal to zero is to make $b^5 = 0$. But this means $b^4 = 0$ and this

case was already covered in Sector 1. Instead, we set

$$(b^5)^2 + 3(b^5)^4 x^0 = (x^0)^2 (16\sqrt{2}\lambda V_0 V_3 + (x^0)^3). \quad (2.3.12)$$

Plugging this in the x^1 derivative of the potential yields

$$P_{(5),1} = \frac{(b^5)^2 \{-1 + (b^5)^2 x^0 (-2 + (b^5)^2 x^0)\}}{4(x^0)^2}. \quad (2.3.13)$$

This vanishes if we set $x^0 = \frac{1 \pm \sqrt{2}}{(b^5)^2}$. Plugging this into the x^0 derivative of the potential, one finds

$$P_{(5),0} = -\frac{10 \pm 7\sqrt{2} + 16(1 \pm \sqrt{2})\lambda(V_0)^2}{4(b^5)^2}. \quad (2.3.14)$$

This can only vanish if one selects the lower sign. One finds the value of the ratio λ of the coupling constants as a function of V_0 as

$$\lambda = \frac{10 - 7\sqrt{2}}{16(\sqrt{2} - 1)(V_0)^2}. \quad (2.3.15)$$

The value of the potential at the critical point is given by

$$P_{(5)}|_{\varphi_c} = \frac{3(10 - 7\sqrt{2} - (b^5)^{12})}{8(b^5)^4} \quad (2.3.16)$$

and b^5 is determined as a solution to the equation

$$2V_3(b^5)^6 = (\sqrt{2} - 1 - (4 + 3\sqrt{2})(b^5)^{12}) V_0. \quad (2.3.17)$$

So, the sign of the value of the potential at the critical point can be tuned by carefully choosing V_0 and V_3 . Using Mathematica, we found that the Hessian has at least one positive and one negative eigenvalue for any choice of V_0 and V_3 and hence the critical point is a saddle point. The same result can be obtained by setting $b^4 = -b^5$.

- **Case 2:** $4 + 8D + 5D^2 = 128\sqrt{2}\lambda V_0 V_3 (x^0)^3 + 8(x^0)^6$

In this case the x^1 -derivative of the potential vanishes and the b^4 and b^5 -derivatives reduce to

$$\frac{P_{(5),4}}{b^4} = \frac{P_{(5),5}}{b^5} = \frac{-4 + [(b^4)^2 + (b^5)^2]x^0 \{-4 + [(b^4)^2 + (b^5)^2]x^0\}}{32(x^0)^3}. \quad (2.3.18)$$

Setting $x^0 = \frac{2 \pm 2\sqrt{2}}{(b^4)^2 + (b^5)^2}$ makes this expression vanish. Plugging this into the x^0 -

derivative equation yields

$$P_{(5),0} = -\frac{10 \pm 7\sqrt{2} + 16(1 \pm \sqrt{2})\lambda(V_0)^2}{2[(b^4)^2 + (b^5)^2]}. \quad (2.3.19)$$

Again, as in the last case, this can only vanish by selecting the lower sign. This sets the value of λ as given in (2.3.15). The value of the potential at the critical point is found as

$$P_{(5)}|_{\varphi_c} = \frac{3(640 - 448\sqrt{2} - [(b^4)^2 + (b^5)^2]^6)}{128[(b^4)^2 + (b^5)^2]^2}. \quad (2.3.20)$$

$(b^4)^2 + (b^5)^2$ can be tuned by carefully choosing V_0 and V_3 as in the last case, but b^4 and b^5 are otherwise not determined. Using Mathematica, we found that the Hessian of the potential at this one parameter family of critical points has both positive and negative eigenvalues for any choice of V_0 and V_3 , as in the last case; so the ground states are ‘‘saddle curves’’, i.e. they are neither minima or maxima.

2.3.1.2 $SO(1,1) \times U(1)_R$ Gauging

The fields h^1 and h^3 are chargeless under the action of the non-compact $\frac{SL(2,\mathbb{R})}{SO(2)}$ generator

$$\tilde{\sigma}_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

while h^0 and h^2 are forming a doublet with charge 2; and h^4 and h^5 are forming another doublet with charge 1. Therefore the vector fields $A_{\tilde{\mu}}^0, A_{\tilde{\mu}}^2, A_{\tilde{\mu}}^4, A_{\tilde{\mu}}^5$ need to be dualized to tensor fields. To gauge the $SO(1,1)$ subgroup of the symmetry group we will use a linear combination of the fields $A_{\tilde{\mu}}^1$ and $A_{\tilde{\mu}}^3$. The $U(1)_R$ symmetry will be gauged by another linear combination of the same fields. We will split the vector and tensor indices as $I, J = 1, 3; M, N = 0, 2, 4, 5$, respectively.

The symplectic matrix Ω_{MN} that appears in the potential is given by

$$\Omega_{MN} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

and the non-vanishing Λ_I^{MN} 's are

$$\Lambda_3^{00} = \frac{1}{\sqrt{2}}, \quad \Lambda_3^{22} = -\frac{1}{\sqrt{2}}, \quad \Lambda_1^{45} = \frac{1}{2}, \quad \Lambda_1^{54} = \frac{1}{2}.$$

Now the potential terms $P_{(5)}^{(T)} = \frac{3\sqrt{6}}{16} h^I \Lambda_I^{MN} h_M h_N$ and $P_{(5)}^{(R)} = -4C^{IJK} V_I V_J h_K$ become

$$P_{(5)}^{(T)} = \frac{3\sqrt{6}}{16} [h^1 h_4 h_5 + \frac{h^3}{\sqrt{2}} ((h_0)^2 - (h_2)^2)]|_{N(h)=1}, \quad (2.3.21)$$

$$P_{(5)}^{(R)} = 2V_1 [V_1 ||x||^2 + 2V_3 (-\sqrt{2}x^1 \frac{1 - b^T \bar{x} b}{||x||^2} + \sqrt{2}b^4 b^5)]. \quad (2.3.22)$$

Here we defined $h_{\bar{I}} \equiv \frac{1}{3} \frac{\partial}{\partial h^{\bar{I}}} N|_{N=1}$. One should note that determinants of the vector/tensor field metric $\overset{\circ}{a}_{\bar{I}\bar{J}}$ and the hypersurface metric $g_{\bar{x}\bar{y}}$ are given by 1 and $\frac{243}{16||x||^4}$, respectively. This tells us that both metrics are positive definite on the scalar manifold when $||x||^2 \neq 0$.

The total potential is

$$P_{(5)} = P_{(5)}^{(T)} + \lambda P_{(5)}^{(R)} \quad (2.3.23)$$

where $P_{(5)}^{(T)}$ and $P_{(5)}^{(R)}$ were given in (2.3.21),(2.3.22). It is quite difficult to calculate the most general solution for the critical points of this potential. Instead, we look at specific sectors.

Sector 1: $b^4 = b^5 = 0$ at the critical point

This sector looks quite similar to the generic case with $\frac{SO(2,1) \times SO(1,1)}{SO(2)}$ as the scalar manifold with $SO(1,1) \times U(1)_R$ gauging, yet there is an important difference. In the generic case, to have the metrics $\overset{\circ}{a}_{\bar{I}\bar{J}}$ and $g_{\bar{x}\bar{y}}$ positive definite, we were forced to look at the sector $h^0 \neq 0$. Now this restriction does not apply anymore and this will help us find more ground states.

To find the critical points of the scalar manifold, we take the derivatives of the total potential with respect to all the scalars of the manifold (i.e. x^0, x^1, x^2, b^4, b^5) and set them equal to zero. These derivatives are given by

$$\begin{aligned} P_{(5),0}|_{b^4=b^5=0} &= (A + \lambda B)x^0, \\ P_{(5),1}|_{b^4=b^5=0} &= -(A + \lambda B)x^1 + \frac{x^1}{4||x||^6} - 4\sqrt{2}\lambda \frac{V_1 V_3}{||x||^2}, \\ P_{(5),2}|_{b^4=b^5=0} &= -(A + \lambda B)x^2, \\ P_{(5),4}|_{b^4=b^5=0} &= P_{(5),5}|_{b^4=b^5=0} = 0 \end{aligned} \quad (2.3.24)$$

where

$$\begin{aligned} A &= \frac{||x||^2 - 3[(x^0)^2 - (x^2)^2]}{4||x||^8}, \\ B &= 4V_1(V_1 + \frac{2\sqrt{2}V_3 x^1}{||x||^4}). \end{aligned}$$

The case with $x^0 \neq 0$ is almost equivalent to the generic case and it was studied in subsection 2.2.3.3. Setting the first derivatives of the potential to zero, one arrives at (c.f. equation

(2.2.67))

$$\begin{aligned}\frac{1}{\|x\|^6} &= -256(\lambda V_1 V_3)^2 + 8\lambda(V_1)^2, \\ \frac{x^1}{\|x\|^4} &= 16\sqrt{2}\lambda V_1 V_3.\end{aligned}\tag{2.3.25}$$

There is a one parameter family of ground states where the potential at the critical point is given by

$$P_{(5)}|_{\varphi_c} = 3\lambda\|x\|^2(V_1)^2(1 - 32\lambda(V_3)^2),\tag{2.3.26}$$

with $V_1 \neq 0$. Choosing $1 > 32\lambda(V_3)^2$ leads to $\|x\|^2 > 0$, whereas $1 < 32\lambda(V_3)^2$ leads to $\|x\|^2 < 0$, therefore this family of ground states correspond to de Sitter vacua.

It was found in the generic case that this family of de Sitter vacuum is stable. Let us check if the stability applies to the magical model at hand. The stability is manifested by the positivity of the eigenvalues of the Hessian of the potential. The Hessian, evaluated at the family of critical points, is of the block diagonal form

$$(\partial\partial P_{(5)})|_{\varphi_c} = \left[\begin{array}{c|c} E_{3\times 3} & 0 \\ \hline 0 & F_{2\times 2} \end{array} \right].\tag{2.3.27}$$

E has one zero eigenvalue. The product of the remaining two eigenvalues is $\frac{3((x^0)^2 + (x^2)^2)}{4\|x\|^{14}}$. This tells us one must have $\|x\|^2 > 0$ in order to have positive eigenvalues. Note the fact that in the generic case this restriction came from the positivity rule of the metrics $\overset{o}{a}_{\bar{i}\bar{j}}$ and $g_{\bar{x}\bar{y}}$ whereas now it is a requirement to have the eigenvalues of the Hessian positive definite. The sum of the eigenvalues of E is

$$Tr(E) = \frac{13(x^0)^4 - 3(x^1)^4 - 14(x^1)^2(x^2)^2 - 11(x^2)^2 + (x^0)^2(22(x^1)^2 - 2(x^2)^2)}{4\|x\|^{10}}.$$

This is positive definite in the region $\|x\|^2 > 0$.

The product of the eigenvalues of F is

$$Det(F) = \frac{1}{16}[-(x^1)^2\|x\|^4 + \frac{1}{\|x\|^6}(\frac{(x^1)^2}{\|x\|^2} + 2)^2 - \frac{2(x^1)^2}{\|x\|^2}(\frac{(x^1)^2}{\|x\|^2} + 2)]$$

and their sum is

$$Tr(F) = \frac{1}{2}x^0[\frac{2}{\|x\|^4} + (x^1)^2(\frac{1}{\|x\|^6} - 2)].\tag{2.3.28}$$

These two quantities are not positive definite everywhere on the domain of the family of de Sitter vacua. In the region where x^1 and x^2 are close to zero their limits are

$$\lim_{x^1, x^2 \rightarrow 0} Det(F) = \frac{1}{4(x^0)^6}, \quad \lim_{x^1, x^2 \rightarrow 0} Tr(F) = \frac{1}{(x^0)^3}.$$

These are positive in the region $x^0 > 0$. In the region where $\|x\|^2$ is close to zero the limits

become

$$\lim_{\|x\|^2 \rightarrow 0} \text{Det}(F) \rightarrow \frac{1}{16\|x\|^6} \left(\frac{(x^1)^2}{\|x\|^2} + 2 \right)^2, \quad \lim_{\|x\|^2 \rightarrow 0} \text{Tr}(F) \rightarrow \frac{x^0(x^1)^2}{2\|x\|^6}.$$

Again, these are positive in the region $x^0 > 0$. Thus these two regions on the scalar manifold contain stable de Sitter vacua. There are also the relations (2.3.25) that tell us x^1 and $[(x^0)^2 - (x^2)^2]$ can be tuned by a careful choice of V_i 's. But x^0 and x^2 are otherwise not fixed. Although x^0 is not fixed, it is not possible to make a transitions between $x^0 > 0$ and $x^0 < 0$ and at the same time to keep $\|x\|^2 > 0$ fixed. Hence, in reality only the second case, where $\|x\|^2$ is small and positive, and $x^0 > 0$, is a stable de Sitter ground state.

We now look at the case where $x^0 = 0$ at the critical point. To make the third expression in (2.3.24) zero there are two possibilities:

- **Case 1:** $A + \lambda B = 0$

The second expression gives us

$$\frac{x^1}{\|x\|^4} = 16\sqrt{2}\lambda V_1 V_3. \quad (2.3.29)$$

Plugging this back into $A + \lambda B = 0$ we find

$$\frac{1}{\|x\|^6} = -256(\lambda V_1 V_3)^2 + 8\lambda(V_1)^2. \quad (2.3.30)$$

The left hand side of this equation is negative definite. Hence $V_1 \neq 0$ and also $32\lambda(V_3)^2 > 1$ and by (2.3.29), $x^1 \neq 0$. The potential evaluated at the critical point is given by

$$P_{(5)}|_{\varphi_c} = 3\lambda(V_1)^2(1 - 32\lambda(V_3)^2)\|x\|^2 \quad (2.3.31)$$

which is positive, hence we have another de Sitter ground state, but this is unstable as explained before, for it leads to the same Hessian (2.3.27).

- **Case 2:** $x_c^2 = 0$

In this case $\|x\|^2 = -(x^1)^2$. From the second expression in (2.3.24) we find

$$V_1 = 0 \quad \text{or} \quad (x^1)^3 = -\frac{\sqrt{2}V_3}{V_1} \quad (2.3.32)$$

at the critical point. Setting $V_1 = 0$ leads to a one parameter family of Minkowski ground states with broken supersymmetry (unless $V_3 = 0$, which turns off the $U(1)_R$ potential). For the other case the potential at this point becomes

$$P_{(5)}|_{\varphi_c} = \frac{6\sqrt{2}\lambda V_1 V_3}{x^1} \quad (2.3.33)$$

which is negative definite because of (2.3.32) and hence this corresponds to a supersymmetric Anti-de Sitter critical point. The Hessian of the scalar potential evaluated at the critical point is given by

$$\text{diag}\left(-\frac{1+32\lambda(V_3)^2}{4(x^1)^6}, -\frac{24\lambda(V_3)^2}{(x^1)^6}, \frac{1+32\lambda(V_3)^2}{4(x^1)^6}, \frac{8\lambda(V_3)^2}{(x^1)^3} + \frac{(x^1)^3}{4}, \frac{8\lambda(V_3)^2}{(x^1)^3} + \frac{(x^1)^3}{4}\right),$$

which has both positive and negative eigenvalues and therefore the critical point is a saddle point of the potential.

Sector 2: $x^1 = x^2 = 0$ at the critical point

The derivative of the potential with respect to the scalar x^2 is

$$P_{(5),2}|_{\varphi^c} = -\frac{[(b^4)^2 - (b^5)^2][2 + 2x^0[(b^4)^2 + (b^5)^2] + (x^0)^2(b^4)^2(b^5)^2]}{8(x^0)^2}. \quad (2.3.34)$$

There are 4 possible ways of setting this equal to zero, i.e.

$$b^4 = \pm b^5, \quad b^4 = \pm \frac{\sqrt{-2 - 2(b^5)^2 x^0}}{\sqrt{2x^0 + (b^5)^2 (x^0)^2}}. \quad (2.3.35)$$

Inserting either of the last two values for b^4 into the x^0 -derivative of the potential, one finds

$$P_{(5),0}|_{\varphi^c} = 4\lambda(V_1)^2 x^0. \quad (2.3.36)$$

But since $x^0 \neq 0$, because otherwise the potential diverges at this point, one must have $V_1 = 0$, i.e. the $U(1)_R$ potential must be turned off. Applying this to the b^5 -derivative of the potential, one finds

$$P_{(5),5}|_{\varphi^c} = -\frac{(b^5)^3(1 + (b^5)^2 x^0)^2}{2(x^0)^2(2 + (b^5)^2 x^0)^2}. \quad (2.3.37)$$

Setting this equal to zero requires either $b^5 = 0$ or $1 + (b^5)^2 x^0 = 0$ but both options make the potential vanish at the critical point, so these critical points are Minkowskian.

For the $b^4 = b^5$ case we first note that $P_{(5),2}|_{\varphi^c} = 0$. Furthermore

$$P_{(5),4}|_{\varphi^c} = P_{(5),5}|_{\varphi^c} = \frac{b^5[2 + 5(b^5)^2 x^0 + 3(b^5)^4 (x^0)^2 + 16\sqrt{2}\lambda V_1 V_3 (x^0)^3]}{4(x^0)^3}. \quad (2.3.38)$$

One way to set this equal to zero is to make $b^5 = 0$. But this means $b^4 = 0$ and this case was already covered in Sector 1. In particular, the potential at the critical point in this case is (note that the restrictions $V_1 \neq 0$, $1 > 32\lambda(V_3)^2$ apply),

$$P_{(5)}|_{\varphi^c} = 3\lambda(V_1)^2 (x^0)^2 \quad (2.3.39)$$

which is non-negative.

Another way to set (2.3.38) equal to zero is to let

$$2 + 5(b^5)^2 x^0 + 3(b^5)^4 (x^0)^2 = -16\sqrt{2}\lambda V_1 V_3 (x^0)^3. \quad (2.3.40)$$

Plugging this in $P_{(5),1} = 0$ and solving the resulting expression together with $P_{(5),0} = 0$ we find

$$x^0 = -\frac{a}{(b^5)^2} \quad (2.3.41)$$

and

$$(x^0)^6 = \frac{2 - 6a + 5a^2 - a^3}{a} \quad (2.3.42)$$

where $a = 16\lambda(V_1)^2$. Plugging (2.3.40), (2.3.41) and (2.3.42) into the potential we get

$$P_{(5)}|_{\varphi_c} = \frac{3 - 6a + 3a^3}{8(x^0)^4}. \quad (2.3.43)$$

Since the left hand side of (2.3.42) is positive definite, a is constrained as

$$0 < a < 2 - \sqrt{2} \quad \text{or} \quad 1 < a < 2 + \sqrt{2}.$$

In both regions the value of the potential at the critical point (2.3.43) is positive, thus the critical point corresponds to a de Sitter vacuum. Unfortunately, using Mathematica we found that none of these choices for a yields non-negative eigenvalues for the Hessian of the potential (some eigenvalues are always negative); therefore this de Sitter vacuum is unstable.

For the $b^4 = -b^5$ case the value of the potential at the critical point is the same and the critical point is unstable as well.

2.3.2 $\mathcal{M} = SL(3, \mathbb{C})/SU(3)$

\mathcal{M} is described by the hypersurface $N(h) = C_{\tilde{I}\tilde{J}\tilde{K}} h^{\tilde{I}} h^{\tilde{J}} h^{\tilde{K}} = 1$ of the cubic polynomial

$$N(h) = \frac{3}{2}\sqrt{3}h^4 \eta_{IJ} h^I h^J + \frac{3\sqrt{3}}{2\sqrt{2}} \gamma_{IMN} h^I h^M h^N, \quad (2.3.44)$$

where

$$\begin{aligned} I, J &= 0, 1, 2, 3 & M, N &= 5, 6, 7, 8 \\ \eta_{IJ} &= \text{diag}(+, -, -, -) \\ \gamma_0 &= -\mathbb{1}_4 & \gamma_1 &= \mathbb{1}_2 \otimes \sigma_1 \\ \gamma_2 &= \sigma_2 \otimes \sigma_2 & \gamma_3 &= \mathbb{1}_2 \otimes \sigma_3. \end{aligned} \quad (2.3.45)$$

The non-vanishing $C_{\bar{I}\bar{J}\bar{K}}$'s are

$$\begin{aligned}
C_{004} = -C_{114} = -C_{224} = -C_{334} &= \frac{\sqrt{3}}{2} \\
C_{055} = C_{066} = C_{077} = C_{088} &= -\frac{\sqrt{3}}{2\sqrt{2}} \\
C_{355} = C_{377} = -C_{366} = -C_{388} &= \frac{\sqrt{3}}{2\sqrt{2}} \\
C_{156} = C_{178} = -C_{258} = C_{267} &= \frac{\sqrt{3}}{2\sqrt{2}}
\end{aligned} \tag{2.3.46}$$

and their permutations. $N(h)$ is the determinant of the Jordan algebra $J_3^{\mathbb{C}}$ element

$$\tilde{\mathbf{h}} = \frac{\sqrt{3}}{\sqrt{2}} \begin{pmatrix} h^0 - h^3 & h^1 + ih^2 & h^6 - ih^8 \\ h^1 - ih^2 & h^0 + h^3 & h^5 - ih^7 \\ h^6 + ih^8 & h^5 + ih^7 & \sqrt{2}h^4 \end{pmatrix}.$$

To solve $N(h) = 1$ we take the parametrization:

$$h^I = \sqrt{\frac{2}{3}}x^I, \quad h^M = \sqrt{\frac{2}{3}}b^M, \quad h^4 = \frac{1 - b^T \bar{x}b}{\sqrt{3}||x||^2}.$$

where $||x||^2 = \eta_{IJ}x^I x^J$ and $b^T \bar{x}b = b^M x^I \gamma_{IMN} b^N$.

2.3.2.1 $SU(2) \times U(1)$ Gauging

This is the smallest member of the magical Jordan family that admits $SU(2) \sim SO(3)$ gauging. Here we will gauge a $SU(2) \times U(1)$ subgroup of the isometry group $SL(3, \mathbb{C})$. The vector fields $A_{\bar{\mu}}^1, A_{\bar{\mu}}^2, A_{\bar{\mu}}^3$ will be used to gauge $SU(2)$ and the vector $A_{\bar{\mu}}^0$ will be the $U(1)$ gauge field. The vector fields $A_{\bar{\mu}}^M$ are charged under $SU(2) \times U(1)$ and must be dualized to tensor fields. The vector field $A_{\bar{\mu}}^4$ is a spectator vector field. The dualization of the vector fields to tensor fields introduces the scalar potential [119]

$$P_{(5)}^{(T)} = \frac{1}{8} b^M \bar{x}_{MP} \Omega^{PR} \bar{x}_{RS} \Omega^{ST} \bar{x}_{TN} b^N \tag{2.3.47}$$

where $\bar{x}_{MN} = \gamma_{IMN} x^I$ and the symplectic invariant matrix is

$$\Omega_{PR} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}.$$

The gauge fields $A_{\hat{\mu}}^1, A_{\hat{\mu}}^2, A_{\hat{\mu}}^3$ can be used to simultaneously gauge $SU(2)_R$ and this gauging leads to the potential term

$$P_{(5)}^{(R)} = 6\|x\|^2. \quad (2.3.48)$$

The total potential $P_{(5)} = P_{(5)}^{(T)} + P_{(5)}^{(R)}$ does not admit any ground states. However $P_{(5)}^{(T)}$ itself does admit a Minkowski ground state at $b^M = 0$. Using the $SU(2)$ symmetry, one can take rotate the fields such that $x^2 = x^3 = 0$ at the critical point. With this choice the eigenvalues of the Hessian of the potential are found to be⁶

$$\{0, 0, 0, 0, 2(x^0 - x^1)^3, 2(x^0 - x^1)^3, 2(x^0 + x^1)^3, 2(x^0 + x^1)^3\}$$

and it is easy to see that the ground state is a minimum in the region $\|x\|^2 > 0, x^0 > 0$ only.

In the above model, one can simultaneously gauge $U(1)_R$ instead of the full $SU(2)_R$ by taking a linear combination of $A_{\hat{\mu}}^0$ and $A_{\hat{\mu}}^4$ as $U(1)_R$ gauge field. $P_{(5)}^{(T)}$ is still given by (2.3.47), but $P_{(5)}^{(R)}$ now is

$$P_{(5)}^{(R)} = -2V_0 \left\{ V_0 \|x\|^2 + \sqrt{2}V_4 \left(2x^0 \frac{1 - b^T \bar{x} b}{\|x\|^2} - b^T b \right) \right\} \quad (2.3.49)$$

where $b^T b = b^M b^N \delta_{MN}$. There are three ways of making the b^M -derivatives of the total potential $P_{(5)} = P_{(5)}^{(T)} + \lambda P_{(5)}^{(R)}$ vanish.

- **Case 1:** $b^M = 0$

The x^I -derivatives of the potential become

$$\begin{aligned} P_{(5),0} &= -2\lambda V_0 \left(2V_0 x^0 - \frac{2\sqrt{2}V_4 [2(x^0)^2 - \|x\|^2]}{\|x\|^4} \right), \\ P_{(5),a} &= 4\lambda V_0 x^a \left(V_0 - \frac{2\sqrt{2}V_4 x^0}{\|x\|^4} \right), \quad a = 1, 2, 3. \end{aligned} \quad (2.3.50)$$

These vanish if one sets $x^a = 0$ and $\sqrt{2}V_4 = V_0(x^0)^3$. The value of the potential at this critical point is

$$P_{(5)}|_{\varphi_c} = -6\lambda(V_0)^2(x^0)^2. \quad (2.3.51)$$

which is negative definite, hence the critical point is an Anti-de Sitter ground state. The Hessian of the potential evaluated at the critical point is given by

$$\begin{aligned} \partial\partial P_{(5)}|_{\varphi_c} &= \text{diag} \left(-12\lambda(V_0)^2, -4\lambda(V_0)^2, -4\lambda(V_0)^2, -4\lambda(V_0)^2, \right. \\ &\quad \left. 2(1 - 2\lambda(V_0)^2)(x^0)^3, 2(1 - 2\lambda(V_0)^2)(x^0)^3, \right. \\ &\quad \left. 2(1 - 2\lambda(V_0)^2)(x^0)^3, 2(1 - 2\lambda(V_0)^2)(x^0)^3 \right). \end{aligned} \quad (2.3.52)$$

⁶One can arrive at the same result by doing a non-compact $SO(2,1) \times U(1)$ (without R -symmetry) gauging.

Depending on the choice of V_0 and V_4 this can be either an Anti-de Sitter maximum or saddle point.

Another way of making these derivatives vanish is to set $V_0 = 0$. This case was already covered before. It lead to a Minkowski minimum with broken supersymmetry (unless also $V_4 = 0$).

- **Case 2: $b^5 = b^6$ and $b^7 = b^8$**

Using the $SU(2)$ invariance, the scalar fields can be rotated such that $x^2 = x^3 = 0$. The first derivatives of the potential vanish with

$$\begin{aligned} 2\sqrt{2}\lambda V_0 V_4 &= (x^0 - x^1)^2(x^0 + x^1), \\ 2\lambda(V_0)^2 &= \frac{x^0 - 3x^1}{x^0 + x^1}, \\ x^1 &= -[(b^6)^2 + (b^8)^2](x^0 - x^1)^2. \end{aligned} \tag{2.3.53}$$

Given a set of V_I 's, the values for x^0 , x^1 and $(b^6)^2 + (b^8)^2$ are uniquely determined by these equations. The value of the potential at the critical point is

$$P_{(5)}|_{\varphi_c} = -3(x^0 - x^1)^2 \tag{2.3.54}$$

and this correspond to an Anti-de Sitter vacuum. Considering the fact that we already used two thirds of the gauge freedom by choosing $x^2 = x^3 = 0$, we conclude that this actually is a three-parameter family of ground states. Using Mathematica, we found that this is a maximum of the total potential.

- **Case 3: $b^5 = -b^6$ and $b^7 = -b^8$**

This is very similar to the last case. We again use the $SU(2)$ invariance to set $x^2 = x^3 = 0$. The first derivatives of the potential vanish with

$$\begin{aligned} 2\sqrt{2}\lambda V_0 V_4 &= (x^0 - x^1)(x^0 + x^1)^2, \\ 2\lambda(V_0)^2 &= \frac{x^0 + 3x^1}{x^0 - x^1}, \\ x^1 &= [(b^6)^2 + (b^8)^2](x^0 + x^1)^2 \end{aligned} \tag{2.3.55}$$

and again, this is a three-parameter family of Anti-de Sitter ground states, with the value of the potential at the critical points is

$$P_{(5)}|_{\varphi_c} = -3(x^0 + x^1)^2. \tag{2.3.56}$$

With hypermultiplets:

One can add a universal hypermultiplet to the theory and gauge simultaneously the subgroup $SU(2) \times U(1)$ of the hyperscalar isometry group $SU(2, 1)$ together with the $SU(2)_R$. The total potential $P_{(5)} = P_{(5)}^{(T)} + P_{(5)}^{(R)} + P_{(5)}^{(H)}$ is then given by

$$\begin{aligned} P_{(5)}^{(T)} &= \frac{1}{8} b^M \bar{x}_{MP} \Omega^{PR} \bar{x}_{RS} \Omega^{ST} \bar{x}_{TN} b^N, \\ P_{(5)}^{(R)} &= -4 C^{IJK} \vec{P}_I \vec{P}_J h_K, \\ P_{(5)}^{(H)} &= 2 \mathcal{N}_{iA} \mathcal{N}^{iA} \end{aligned} \quad (2.3.57)$$

with

$$\mathcal{N}^{iA} = \frac{\sqrt{6}}{4} h^I K_I^X f_X^{iA}$$

where we defined (B.3)

$$K_0^X = T_8^X, \quad K_1^X = T_1^X, \quad K_2^X = T_2^X, \quad K_3^X = T_3^X. \quad (2.3.58)$$

This theory does not admit any ground states. One can gauge an additional $SO(1, 1)_H$ symmetry. This type of gauging admits stable and unstable de Sitter vacua. But this type of calculation has been done various times in the previous subsection and therefore we skip it here.

2.3.2.2 $SO(2, 1) \times U(1)$ Gauging

We will gauge a $SO(2, 1) \times U(1)$ subgroup of the isometry group $SL(3, \mathbb{C})$. The vector fields $A_{\hat{\mu}}^0, A_{\hat{\mu}}^1, A_{\hat{\mu}}^2$ will be used to gauge $SO(2, 1)$ and the vector $A_{\hat{\mu}}^3$ will be the $U(1)$ gauge field. The vector fields $A_{\hat{\mu}}^M$ are charged under $SO(2, 1) \times U(1)$ and must be dualized to tensor fields. The vector field $A_{\hat{\mu}}^4$ is a spectator vector field. The dualization of the vector fields to tensor fields introduces the scalar potential [119]

$$P_{(5)}^{(T)} = \frac{1}{8} b^M \bar{x}_{MP} \Omega^{PR} \bar{x}_{RS} \Omega^{ST} \bar{x}_{TN} b^N \quad (2.3.59)$$

where $\bar{x}_{MN} = \gamma_{IMN} x^I$ and the symplectic invariant matrix is

$$\Omega_{PR} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}.$$

$P_{(5)}^{(T)}$ itself admits a Minkowski ground state as in the last case. In this model, one can gauge the $U(1)_R \subset SU(2)_R$ symmetry by taking a linear combination of $A_{\hat{\mu}}^0$ and $A_{\hat{\mu}}^4$ as

$U(1)_R$ gauge field. A scalar potential

$$P_{(5)}^{(R)} = 2V_3 \left\{ V_3 \|x\|^2 + \sqrt{2}V_4 \left(-2x^3 \frac{1 - b^T \bar{x} b}{\|x\|^2} + b^T \gamma_3 b \right) \right\} \quad (2.3.60)$$

is introduced. The only critical points of the total potential $P_{(5)} = P_{(5)}^{(T)} + \lambda P_{(5)}^{(R)}$ are found at $b^M = 0$ by setting $V_3 = 0$, i.e. by turning off the $U(1)_R$ potential. These are four-parameter family of Minkowski ground states and they are minima only in the region $\|x\|^2 > 0$, $x^0 > 0$. Supersymmetry is broken unless also $V_4 = 0$.

2.3.3 Summary

The gaugings of certain theories in the generic Jordan family can be reproduced in the magical Jordan family, provided there are enough vector fields to do the respective gaugings. The stability of the ground states of these theories still needs to be checked and in some cases the stability puts constraints on the gauge parameters. In this section we reproduced the Minkowski and Anti-de Sitter ground states for $SO(2) \times U(1)_R$ gauging and the de Sitter ground states for $SO(1,1) \times U(1)_R$ gauging that were already found in the generic Jordan family case. In addition to the existing ground states, we encountered other ground states that are special to the magical Jordan family case, such as de Sitter and Anti-de Sitter saddle points and curves. Although we did not do a complete analysis, we can conclude that the magical Jordan family theories are richer than the generic Jordan family theories in the numbers and properties of ground states.

The compact non-Abelian $SU(2) \times U(1)$ gauging leads to Minkowski vacua; and also Anti-de Sitter vacua when accompanied by a simultaneous $U(1)_R$ symmetry gauging. However the simultaneous $SU(2)_R$ gauging does not admit any critical points, even after including hypers in the model. The model with non-compact non-Abelian $SO(2,1) \times U(1)$ gauging has Minkowski vacuum, but doing a simultaneous $U(1)_R$ gauging results in a theory with no ground states.

The other members of the magical Jordan family ($\mathcal{M} = SU^*(6)/USp(6)$ and $\mathcal{M} = E_{6(-26)}/F_4$) have a very similar structure to the above theories and contain them as subsectors. Although they admit gaugings of bigger subgroups, such as $SO(m+1)$ or $SO(m,1)$ with $m \geq 3$, the form of the scalar potentials corresponding to the $SO(3)$ and $SO(2,1)$ gaugings of the above model suggests that it is not likely for the bigger members of the magical Jordan family, subject to $SO(m+1)$ or $SO(m,1)$ gaugings, to have ground states of different nature than the ones found in this section.

2.4 Generic non-Jordan Family

The scalar manifold $\mathcal{M} = SO(1, \tilde{n})/SO(\tilde{n})$ can be described by the hyper surface $N(h) = C_{\tilde{I}\tilde{J}\tilde{K}} h^{\tilde{I}} h^{\tilde{J}} h^{\tilde{K}} = 1$ of the cubic polynomial

$$N(h) = \frac{3\sqrt{3}}{2\sqrt{2}} \left(\sqrt{2} h^0 (h^1)^2 - h^1 [(h^2)^2 + \dots + (h^{\tilde{n}})^2] \right). \quad (2.4.1)$$

The non-vanishing $C_{\tilde{I}\tilde{J}\tilde{K}}$'s are

$$C_{011} = \frac{\sqrt{3}}{2}, \quad C_{122} = C_{133} = \dots = C_{1\tilde{n}\tilde{n}} = -\frac{\sqrt{3}}{2\sqrt{2}}$$

and their permutations. To solve $N(h) = 1$ we take the parametrization

$$\begin{aligned} h^0 &= \sqrt{\frac{2}{3}} \left(\frac{1}{\sqrt{2}(\varphi^1)^2} + \frac{1}{\sqrt{2}} \varphi^1 [(\varphi^2)^2 + \dots + (\varphi^{\tilde{n}})^2] \right), \\ h^1 &= \sqrt{\frac{2}{3}} \varphi^1, \\ h^a &= \sqrt{\frac{2}{3}} \varphi^1 \varphi^a, \quad a = 2, \dots, \tilde{n}. \end{aligned}$$

In contrast to the Jordan families, $C^{\tilde{I}\tilde{J}\tilde{K}}$'s are no longer constant or equal to $C_{\tilde{I}\tilde{J}\tilde{K}}$'s. The scalar field dependent $C^{\tilde{I}\tilde{J}\tilde{K}}$ are defined as

$$C^{\tilde{I}\tilde{J}\tilde{K}} = \overset{o}{a}{}^{\tilde{I}\tilde{I}'} \overset{o}{a}{}^{\tilde{J}\tilde{J}'} \overset{o}{a}{}^{\tilde{K}\tilde{K}'} C_{\tilde{I}'\tilde{J}'\tilde{K}'}$$

where the inverse of the vector field metric $\overset{o}{a}{}_{\tilde{I}\tilde{J}}$ is given by $\overset{o}{a}{}^{\tilde{I}\tilde{J}} = h^{\tilde{I}} h^{\tilde{J}} + h_{\tilde{x}}^{\tilde{I}} h_{\tilde{y}}^{\tilde{J}} g^{\tilde{x}\tilde{y}}$. For the symmetric non-Jordan family, the scalar field metric $g_{\tilde{x}\tilde{y}}$ is diagonal

$$g_{\tilde{x}\tilde{y}} = \text{diag} \left[\frac{3}{(\varphi^1)^2}, (\varphi^1)^3, \dots, (\varphi^1)^3 \right]$$

which is positive definite for $\varphi^1 > 0$.

2.4.1 Maxwell-Einstein Supergravity

2.4.1.1 No R -symmetry Gauging

We add one hypermultiplet to the theory and gauge a non-compact $SO(1, 1)_H$ symmetry of the hyperscalar manifold. As the $SO(1, 1)$ gauge field, we take a linear combination $W_I A_{\mu}^I$ of all the vectors in the theory. The potential is given by

$$P_{(5)} = P_{(5)}^{(H)} = 2\mathcal{N}_{iA} \mathcal{N}^{iA} \quad (2.4.2)$$

where $\mathcal{N}^{iA} = \frac{\sqrt{6}}{4}(W_I h^I) T_4^X f_X^{iA}$. The only way to make the first derivatives of the potential vanish at the base point of the hyperscalar manifold without making the potential itself vanish is to set

$$\begin{aligned} W_1 &= \frac{W_0 (2 + (\varphi^1)^3 [(\varphi^2)^2 + \dots + (\varphi^{\tilde{n}})^2])}{\sqrt{2}(\varphi^1)^3}, \\ W_a &= -\sqrt{2}\varphi^a W_0, \quad a = 2, \dots, \tilde{n}. \end{aligned} \quad (2.4.3)$$

The coordinates of the critical point is entirely determined by W_I 's. The value of the potential at the critical point becomes

$$P_{(5)}|_{\varphi^c} = \frac{9(W_0)^2}{4(\varphi^1)^4} \quad (2.4.4)$$

and the Hessian of the potential at the critical point is given by

$$\partial\partial P_{(5)}|_{\varphi^c} = \text{diag} \left[\frac{9(W_0)^2}{(\varphi^1)^6}, \underbrace{\frac{3(W_0)^2}{\varphi^1}, \dots, \frac{3(W_0)^2}{\varphi^1}}_{\tilde{n}-1 \text{ times}}, 0, \frac{9(W_0)^2}{2(\varphi^1)^4}, \frac{9(W_0)^2}{4(\varphi^1)^4}, \frac{9(W_0)^2}{4(\varphi^1)^4} \right]$$

which is semi-positive definite in the physically relevant region $\varphi^1 > 0$, therefore the critical point is a stable de Sitter vacuum. We already had many examples of having $SO(1, 1)_H$ gauging mixed with other gaugings in the generic Jordan family section. Similar analysis for the non-Jordan family shows that it is possible to obtain de Sitter ground states with other gauge groups that include $SO(1, 1)_H$. Therefore we will omit the results for $K_{(5)} \times SO(1, 1)_H$ gaugings of generic non-Jordan family theories.

2.4.1.2 $SU(2)_R$ Symmetry Gauging

This calculation was done in [119]. Let us briefly quote their results. The vectors $A_{\tilde{\mu}}^2, A_{\tilde{\mu}}^3, A_{\tilde{\mu}}^4$ are chosen as the $SU(2)$ gauge fields. This group rotates h^2, h^3, h^4 together but the other scalars are not charged under the action of this $SU(2)$, therefore no tensor fields need to be introduced. The scalar potential (2.1.4) becomes

$$P_{(5)} = P_{(5)}^{(R)} = -\frac{1}{2}(\varphi^1)^2 [(\varphi^2)^2 + (\varphi^3)^2 + (\varphi^4)^2] + \frac{3}{2\varphi^1}. \quad (2.4.5)$$

It's easy to verify that this potential does not have any critical points.

2.4.1.3 $U(1)_R$ Symmetry Gauging

This calculation was done in [120] for $\tilde{n} = 3$. Let us trivially generalize their results to arbitrary \tilde{n} . A linear combination $V_I A_{\tilde{\mu}}^I$ of all the vectors in the theory is taken as the

$U(1)_R$ gauge field. The scalar potential (2.1.5) is given by

$$P_{(5)} = P_{(5)}^{(R)} = \frac{1}{\varphi^1} \left\{ -2\sqrt{2}V_0V_1 + 2|V|^2 - (\varphi^1)^3 [V_0\|\tilde{\varphi}\|^2 + \sqrt{2}(V_1 + V_2\varphi^2 + \dots + V_{\tilde{n}}\varphi^{\tilde{n}})]^2 \right\} \quad (2.4.6)$$

where we defined $|V|^2 = (V_2)^2 + \dots + (V_{\tilde{n}})^2$ and $\|\tilde{\varphi}\|^2 = (\varphi^2)^2 + \dots + (\varphi^{\tilde{n}})^2$. The only way to make the first derivatives of the potential vanish without making the potential itself vanish is to set

$$\begin{aligned} V_1 &= \frac{V_0(2 + (\varphi^1)^3\|\tilde{\varphi}\|^2)}{\sqrt{2}(\varphi^1)^3}, \\ V_a &= -\sqrt{2}\varphi^a V_0, \quad a = 2, \dots, \tilde{n}. \end{aligned} \quad (2.4.7)$$

The coordinates of the critical point is entirely determined by V_I 's. The value of the potential at the critical point becomes

$$P_{(5)}|_{\varphi^c} = -\frac{24(\varphi^1)^2(V_1)^2}{(2 + (\varphi^1)^3\|\tilde{\varphi}\|^2)^2} \quad (2.4.8)$$

and the Hessian of the potential at the critical point is given by

$$\partial\partial P_{(5)}|_{\varphi^c} = \text{diag} [-3\tilde{A}, \underbrace{-\tilde{A}(\varphi^1)^5, \dots, -\tilde{A}(\varphi^1)^5}_{\tilde{n}-1 \text{ times}}]$$

where $\tilde{A} = \frac{(V_1)^2}{(2 + (\varphi^1)^3\|\tilde{\varphi}\|^2)^2} > 0$. Thus the critical point is an Anti-de Sitter maximum.

2.4.2 Yang-Mills/Einstein Supergravity with Tensor Coupling

The Lagrangian of the theory is not invariant under full the isometry group $SO(1, \tilde{n})$, but it is invariant under the subgroup $G_{(5)} = [SO(\tilde{n} - 1, 1) \times SO(1, 1)] \times T_{\tilde{n}-1}$, where $T_{\tilde{n}-1}$ is the group of translations in an $\tilde{n} - 1$ dimensional Euclidean space. Having a closer look at N , we see that the subgroup $SO(1, 1)$ cannot be gauged because all the vector fields are charged under it and there are no vector field to be used as the gauge field. Only the gauging of the subgroup $SO(2) \subset SO(\tilde{n} - 1, 1)$ will result in a potential term due to dualization of the vector fields to tensor fields.

2.4.2.1 No R -symmetry Gauging

The group $SO(2)$ rotates h^2 and h^3 into each other and therefore acts nontrivially on the vector fields $A_{\tilde{\mu}}^2$ and $A_{\tilde{\mu}}^3$. These fields must be dualized to tensor fields. The field $A_{\tilde{\mu}}^1$ is chosen as the $SO(2)$ gauge field. The index \tilde{I} is decomposed as

$$\tilde{I} = (I, M)$$

where $I, J, K = 0, 1, 4, 5, \dots, \tilde{n}$ and $M, N, P = 2, 3$. The scalar potential (2.1.3) is found as

$$P_{(5)} = P_{(5)}^{(T)} = \frac{(\varphi^1)^5}{8} [(\varphi^2)^2 + (\varphi^3)^2]. \quad (2.4.9)$$

This potential has an $\tilde{n} - 2$ parameter family of Minkowski minima at $\varphi^2 = \varphi^3 = 0$.

2.4.2.2 $SU(2)_R$ Symmetry Gauging

The vector fields $A_{\tilde{\mu}}^4, A_{\tilde{\mu}}^5, A_{\tilde{\mu}}^6$ are chosen as $SU(2)_R$ gauge fields, whereas $A_{\tilde{\mu}}^1$ will be used to gauge $SO(2)$. The vectors $A_{\tilde{\mu}}^2, A_{\tilde{\mu}}^3$ transform nontrivially under $SO(2)$, therefore they are dualized to tensor fields. The total potential $P_{(5)} = P_{(5)}^{(T)} + \lambda P_{(5)}^{(R)}$ is given by

$$\begin{aligned} P_{(5)}^{(T)} &= \frac{(\varphi^1)^5}{8} [(\varphi^2)^2 + (\varphi^3)^2], \\ P_{(5)}^{(R)} &= -\frac{1}{2}(\varphi^1)^2 [(\varphi^4)^2 + (\varphi^5)^2 + (\varphi^6)^2] + \frac{3}{2\varphi^1}. \end{aligned} \quad (2.4.10)$$

It is easy to verify that the total potential does not have any critical points.

2.4.2.3 $U(1)_R$ Symmetry Gauging

As in the last model, $A_{\tilde{\mu}}^1$ is the $SO(2)$ gauge field and because the vectors $A_{\tilde{\mu}}^2, A_{\tilde{\mu}}^3$ transform nontrivially under $SO(2)$, they are dualized to tensor fields. A linear combination $V_I A_{\tilde{\mu}}^I$, $I = 0, 1, 4, 5, \dots, \tilde{n}$ of vector fields is used as the $U(1)_R$ gauge field. The total potential $P_{(5)} = P_{(5)}^{(T)} + \lambda P_{(5)}^{(R)}$ is given by

$$\begin{aligned} P_{(5)}^{(T)} &= \frac{(\varphi^1)^5}{8} [(\varphi^2)^2 + (\varphi^3)^2], \\ P_{(5)}^{(R)} &= \frac{1}{\varphi^1} \{ -2\sqrt{2}V_0V_1 + 2|V|^2 \\ &\quad - (\varphi^1)^3 [V_0\|\tilde{\varphi}\|^2 + \sqrt{2}(V_1 + V_4\varphi^4 + \dots + V_{\tilde{n}}\varphi^{\tilde{n}})]^2 \}, \end{aligned} \quad (2.4.11)$$

where $|V|^2 = (V_4)^2 + \dots + (V_{\tilde{n}})^2$ and $\|\tilde{\varphi}\|^2 = (\varphi^2)^2 + \dots + (\varphi^{\tilde{n}})^2$. The first derivatives of the potential are given by

$$\begin{aligned} \partial_{\varphi^1} P_{(5)} &= \frac{5}{8}(\varphi^1)^4 [(\varphi^2)^2 + (\varphi^3)^2] + \frac{4\sqrt{2}\lambda V_0 V_1 - 2|V|^2}{(\varphi^1)^2} - 2\lambda\varphi^1 A^2, \\ \partial_{\varphi^a} P_{(5)} &= \frac{1}{4}(\varphi^1)^2 \varphi^a ((\varphi^1)^3 - 16\lambda V_0 A), \quad a = 2, 3, \\ \partial_{\varphi^b} P_{(5)} &= -2\sqrt{2}\lambda(\varphi^1)^2 (\sqrt{2}\varphi^b V_0 + V_b) A, \quad b = 4, \dots, \tilde{n}, \end{aligned} \quad (2.4.12)$$

where

$$A = \|\tilde{\varphi}\|^2 V_0 + \sqrt{2}(V_1 + \varphi^4 V_4 + \dots + \varphi^{\tilde{n}} V_{\tilde{n}}). \quad (2.4.13)$$

There are three ways of making these expressions vanish.

Case 1: $\varphi^a = A = 0$

In this case we have

$$P_{(5)}|_{\varphi^c} = -\varphi^1 \partial_{\varphi^1} P_{(5)}|_{\varphi^c} \quad (2.4.14)$$

which means the potential vanishes at the critical point.

Case 2: $\varphi^a = 0, A \neq 0$

In this case one must have $V_b = -\sqrt{2}\varphi^b V_0$ to make the third expression in (2.4.12) vanish. Plugging this into the first expression and setting it equal to zero, one finds the conditions

$$V_1 = \frac{\|\tilde{\varphi}\|^2 V_0}{\sqrt{2}}, \quad \text{or} \quad V_1 = \frac{2 + (\varphi^1)^3 \|\tilde{\varphi}\|^2 V_0}{\sqrt{2}(\varphi^1)^3}. \quad (2.4.15)$$

The first of these leads to a Minkowski minimum. The second choice gives the value of the potential at the critical point as

$$P_{(5)}|_{\varphi^c} = -\frac{12\lambda(V_0)^2}{(\varphi^1)^4} \quad (2.4.16)$$

and the Hessian of the potential at the critical point is

$$\begin{aligned} \partial\partial P_{(5)}|_{\varphi^c} = \text{diag} \quad & \left[-\frac{24\lambda(V_0)^2}{(\varphi^1)^6}, \frac{(\varphi^1)^5}{4} - \frac{8\lambda(V_0)^2}{\varphi^1}, \frac{(\varphi^1)^5}{4} - \frac{8\lambda(V_0)^2}{\varphi^1}, \right. \\ & \left. \underbrace{-\frac{8\lambda(V_0)^2}{\varphi^1}, \dots, -\frac{8\lambda(V_0)^2}{\varphi^1}}_{(\tilde{n}-3) \text{ times}} \right] \end{aligned} \quad (2.4.17)$$

which means the critical point can be a maximum or a saddle point depending on the choice of V_I 's.

Case 3: $\varphi^a \neq 0$

In this case one must have $V_b = -\sqrt{2}\varphi^b V_0$ together with $(\varphi^1)^3 = 16\lambda V_0 A$. Plugging these in the first equation in (2.4.12), solving this for V_1 one finds the value of the potential at the critical point as

$$P_{(5)}|_{\varphi^c} = -\frac{(\varphi^1)^5 (-3(\varphi^1)^3 + 32\lambda(5 + \lambda)[(\varphi^2)^2 + (\varphi^3)^2](V_0)^2)}{256\lambda^2(V_0)^2} \quad (2.4.18)$$

which might correspond to de Sitter or Anti-de Sitter, depending on the choice of V_I 's. It was shown in [120] that the de Sitter solution is a saddle point when $\tilde{n} = 3$. The calculation for the stability of the solutions are tedious but using Mathematica, we confirmed that the de Sitter solutions are saddle points for any \tilde{n} and we showed that the Anti-de Sitter solutions are either maxima or saddle points, again depending on the choice of V_I 's.

2.4.3 Summary

$SU(2)_R$ gauging does not lead to any critical points, even with the addition of tensors; whereas the model with pure $U(1)_R$ gauging has Minkowski and AdS critical points. The only way of adding tensors to the theory is done by gauging the $SO(2)$ subgroup of the isometry group. Pure $SO(2)$ gauging leads to Minkowski minima. $U(1)_R \times SO(2)$ gauging has Minkowski, dS and AdS critical points. The dS solution is always unstable but the AdS solution can be made stable by properly choosing V_I 's (c.f. [126]). Coupling hypers to the theory and gauging $SO(1,1)_H$ leads to stable de Sitter vacua as in the generic Jordan case.

4 Dimensional Story

Having discussed all the possible gaugings and the nature of the resulting ground states from the scalar potentials of $\mathcal{N} = 2$ supergravity theories with symmetric scalar manifolds in 5 dimensions, we now move on to ground states of the 4 dimensional $\mathcal{N} = 2$ supergravity theories obtained by dimensional reduction. The complexity of the scalar geometry in 4 dimensions makes it hard, but not impossible, to make a general analysis of ground states as was done in 5 dimensions. Therefore our primary concentration in this chapter will be on de Sitter ground states.

We start by giving an extract of the structure of the 5 dimensional theories, which we have discussed extensively in the previous chapter. The notation is the same, except the internal vector/tensor indices \tilde{I} are shifted by one unit to make room for the traditional 0-index for the graviphoton in 4 dimensions. The details of the dimensional reduction process can be found in [116, 121]. Here we quote the necessary tools for the calculation of the scalar potentials. The bosonic sectors of the Lagrangians before and after the dimensional reduction are given in appendix A.

Before we begin, let us see what kind of ground states we would get just by considering ordinary dimensional reduction. The dimensionally reduced potential derived from (2.1.11), in the absence of hypers¹ reads (A.25, A.27)

$$P_{(4)} = e^{-\sigma} P_{(5)}^{(T)} + \lambda e^{-\sigma} P_{(5)}^{(R)} + \frac{3}{4} e^{-3\sigma} a_{\tilde{I}\tilde{J}}^o (A^I M_{I\tilde{K}}^{\tilde{I}} h^{\tilde{K}})(A^J M_{J\tilde{L}}^{\tilde{J}} h^{\tilde{L}}), \quad (3.0.1)$$

where $M_{I\tilde{K}}^{\tilde{I}}$ are the $K_{(5)}$ -transformation matrices defined in (A.28). The scalars of the above potential are $\varphi^{\tilde{x}}$, A^I and σ . Taking the σ -derivative of the potential, setting it equal to zero and plugging the result back into the potential gives us the value of the potential at

¹Adding hypers results in an additional $P_{(4)}^{(H)}$ in the dimensionally reduced potential (3.0.1), which is given in (A.26). The two terms of $P_{(4)}^{(H)}$ have the same powers of σ and A^I as the first and third terms above and can be absorbed in them by proper field redefinitions and hence it will not change our result.

the critical point ϕ^c as

$$P_{(4)}|_{\phi^c} = -\frac{3}{2}e^{-3\sigma} \overset{\circ}{a}_{\tilde{I}\tilde{J}} (A^I M_{I\tilde{K}}^{\tilde{I}} h^{\tilde{K}})(A^J M_{J\tilde{L}}^{\tilde{J}} h^{\tilde{L}}). \quad (3.0.2)$$

The derivative of the potential with respect to any A^I must vanish at the critical point. Hence we arrive at

$$A^I \frac{\partial P_{(4)}}{\partial A^I} |_{\phi^c} = \frac{3}{2}e^{-3\sigma} A^I A^J \overset{\circ}{a}_{\tilde{I}\tilde{J}} M_{I\tilde{K}}^{\tilde{I}} M_{J\tilde{L}}^{\tilde{J}} h^{\tilde{K}} h^{\tilde{L}} = 0. \quad (3.0.3)$$

So if a critical point exists the potential vanishes there (c.f. equation (3.0.2)), and there is no possibility for an (Anti-)de Sitter ground state. Since cosmological observations imply that the universe has a very small positive cosmological constant, we must find a way around this problem.

It was shown in [121] that the dimensionally reduced 5D Yang-Mills-Einstein supergravity theories coupled to tensor multiplets result in 4D theories that have gauge groups of the form $K_{(4)} = K_{(5)} \ltimes \mathcal{H}^{n_T+1}$, where \mathcal{H}^{n_T+1} is a Heisenberg group of dimension $n_T + 1$ and \ltimes denotes semi-direct product. On the other hand, stable de Sitter vacua were found for 4D, $\mathcal{N} = 2$ theories in [67], where the authors showed that the three necessary ingredients to obtain stable de Sitter vacua are non-Abelian, non-compact gauge groups, $SO(2, 1)$ in particular; Fayet-Iliopoulos (FI) terms that are possible only for $SU(2)$ or $U(1)$ factors, which can be identified by the $SU(2)_R$ or $U(1)_R$ gaugings; and the de Roo-Wagemans (dRW) rotation. The last ingredient uses additional symmetries in 4 dimensions, where the isometry group is larger than in 5 dimensions. In order to make use of these symmetries we first need to review the structure of the complex geometry of 4 dimensional $\mathcal{N} = 2$ supergravity theories. Once this is achieved it will be easier to see the 5 dimensional origins of de Sitter ground states that we will show how to obtain in 4 dimensions.

3.1 The Geometry in 4 Dimensions

In 5 dimensions generic Jordan family has a scalar manifold \mathcal{M}_{VS}^5 given by the symmetric coset space[116, 89]

$$\mathcal{M}_{VS}^5 = \frac{SO(\tilde{n} - 1, 1) \times SO(1, 1)}{SO(\tilde{n} - 1)}, \quad \tilde{n} \geq 1 \quad (3.1.1)$$

where \tilde{n} denotes the number of vector fields $A_{\mu}^{\tilde{I}}$ and scalars $\varphi^{\tilde{x}}$ of the \tilde{n} vector multiplets. Together with the graviphoton the theory has $\tilde{n} + 1$ vector fields. \mathcal{M}_{VS}^5 is totally determined by the totally symmetric tensor $C_{\tilde{I}\tilde{J}\tilde{K}}$ which defines the hypersurface

$$N(h) = C_{\tilde{I}\tilde{J}\tilde{K}} h^{\tilde{I}} h^{\tilde{J}} h^{\tilde{K}} = 1, \quad \tilde{I}, \tilde{J}, \tilde{K} = 1, \dots, \tilde{n} + 1 \quad (3.1.2)$$

of the $\tilde{n}+1$ dimensional space $M = \{h^{\tilde{I}} \in \mathbb{R}^{\tilde{n}+1} | N(h) = C_{\tilde{I}\tilde{J}\tilde{K}} h^{\tilde{I}} h^{\tilde{J}} h^{\tilde{K}} > 0\}$. For the generic Jordan family, the cubic polynomial can be written in the form [118]

$$N(h) = \frac{3\sqrt{3}}{2} h^1 [(h^2)^2 - (h^3)^2 - \dots - (h^{\tilde{n}+1})^2]. \quad (3.1.3)$$

The non-zero $C_{\tilde{I}\tilde{J}\tilde{K}}$'s are

$$C_{122} = \frac{\sqrt{3}}{2}, \quad C_{133} = C_{144} = \dots = C_{1,\tilde{n}+1,\tilde{n}+1} = -\frac{\sqrt{3}}{2}$$

and their permutations. The constraint $N = 1$ is solved by

$$h^1 = \frac{1}{\sqrt{3} \|\varphi\|^2}, \quad h^a = \sqrt{\frac{2}{3}} \varphi^a \quad (3.1.4)$$

with $a, b = 2, 3, \dots, \tilde{n}+1$ and $\|\varphi\|^2 = \varphi^a \eta_{ab} \varphi^b$, where $\eta_{ab} = (+ - - \dots -)$.

The scalar manifold of this theory, when reduced to 4 dimensions, is the special Kähler manifold [127, 116]

$$\mathcal{M}_{VS}^4 = \mathcal{ST}[2, n-1] = \frac{SU(1,1)}{U(1)} \times \frac{SO(2, n-1)}{SO(2) \times SO(n-1)}. \quad (3.1.5)$$

In 4 dimensions, there are $n = \tilde{n}+1$ vector multiplets and n complex scalars. The $(n+1)$ field strengths $\mathcal{F}^{A\mu\nu}$ and their magnetic duals $\mathcal{G}_{A\mu\nu}$ transform in the $(2, n+1)$ representation of the U -duality group $\mathcal{U} = SU(1,1) \times SO(2, n-1)$. The models with stable de Sitter vacua that we will discuss in this chapter originate from the 5 dimensional YMESGT's with gauge groups $SO(1,1) \times U(1)_R$ or $SO(1,1) \times SU(2)_R$. The $SO(1,1)$ factor, as we will show, will become a subgroup of $SO(2, n-1)$ in 4 dimensions. This is similar to the models with stable de Sitter vacua found in [67] where the full $SO(2,1)$ is gauged. Note that the $SU(1,1)_G$ symmetry of the pure 5D, $\mathcal{N} = 2$ supergravity reduced to 4 dimensions is not the $SU(1,1) = SO(2,1)$ factor in the 4 dimensional U -duality group \mathcal{U} . It is rather a diagonal subgroup of $SU(1,1)$ times an $SO(2,1)$ subgroup of $SO(2, n-1)$ under which the following decompositions occur [128]:

$$\begin{aligned} SO(2,1) \times SO(2, n-1) &\supset SO(2,1) \times SO(2,1) \times SO(n-2) &\supset SO(2,1)_G \times SO(n-2) \\ (2, n+1) &= (2,3,1) \oplus (2,1, n-2) &= (4,1) \oplus (2,1) \oplus (2, n-2) \end{aligned}$$

Note that the four dimensional graviphoton transforms in the spin-3/2 representation of $SO(2,1)_G$ along with some linear combination of the other vectors in the theory and due to the mixing, one can say that it does not descend directly from the five dimensional graviphoton. Instead, it is a linear combination of the vector that comes from the dimensional reduction of the fünfbein and the vector that is obtained by the dimensional reduction of the five dimensional graviphoton. We will address this issue in subsection 3.3.2.

The scalars can be used to define the complex coordinates [116, 121]

$$z^{\tilde{I}} = \frac{1}{\sqrt{3}}A^{\tilde{I}} + \frac{ie^\sigma}{\sqrt{2}}h^{\tilde{I}}. \quad (3.1.6)$$

These n complex coordinates can be interpreted as the inhomogeneous coordinates of the $(n+1)$ -dimensional complex vector ($\tilde{I} = 1, \dots, n$)

$$X^A = \begin{pmatrix} X^0 \\ X^{\tilde{I}} \end{pmatrix} = \begin{pmatrix} 1 \\ z^{\tilde{I}} \end{pmatrix}. \quad (3.1.7)$$

One can introduce the prepotential²

$$F(X^A) = -\frac{1}{3\sqrt{3}}C_{\tilde{I}\tilde{J}\tilde{K}}\frac{X^{\tilde{I}}X^{\tilde{J}}X^{\tilde{K}}}{X^0} \quad (3.1.8)$$

to write the holomorphic (symplectic) section

$$\begin{aligned} \Omega_0 &= \begin{pmatrix} X^A \\ F_B \end{pmatrix} = \begin{pmatrix} X^A \\ \partial_B F \end{pmatrix} \\ &= \begin{pmatrix} X^0 \\ X^I \\ X^M \\ F_0 \\ F_I \\ F_M \end{pmatrix} = \begin{pmatrix} 1 \\ z^I \\ z^M \\ \frac{1}{3\sqrt{3}}[C_{IJK}z^I z^J z^K + 3C_{IMN}z^I z^M z^N] \\ -\frac{1}{\sqrt{3}}[C_{IJK}z^J z^K + C_{IMN}z^M z^N] \\ -\frac{2}{\sqrt{3}}C_{MNI}z^N z^I \end{pmatrix} \end{aligned} \quad (3.1.9)$$

with $\tilde{I} = (I, M)$. The reason why the above manifold is called a *special Kähler manifold* is that one can write a Kähler potential in terms of the holomorphic section Ω

$$\mathcal{K} = -\log(i\langle\Omega|\bar{\Omega}\rangle) = -\log[i(\bar{X}^A F_A - \bar{F}_A X^A)]. \quad (3.1.10)$$

The Kähler potential is used to form the Kähler metric on the scalar manifold $\mathcal{M}_{V_S}^4$ of the four-dimensional theory as

$$g_{\tilde{I}\tilde{J}} \equiv \partial_{\tilde{I}}\partial_{\tilde{J}}\mathcal{K}. \quad (3.1.11)$$

It is also possible to introduce the covariantly holomorphic section[129, 130, 131, 132]

$$V = \begin{pmatrix} L^A \\ M_B \end{pmatrix} \equiv e^{\mathcal{K}/2}\Omega = e^{\mathcal{K}/2} \begin{pmatrix} X^A \\ F_B \end{pmatrix} \quad (3.1.12)$$

²Note that the prepotential given here differs by a factor $\sqrt{6}$ from that of [121].

which obeys

$$\nabla_{\bar{I}} V = \left(\partial_{\bar{I}} - \frac{1}{2} \partial_{\bar{I}} \mathcal{K} \right) V = 0. \quad (3.1.13)$$

By defining

$$U_{\bar{I}} = \nabla_{\bar{I}} V = \left(\partial_{\bar{I}} + \frac{1}{2} \partial_{\bar{I}} \mathcal{K} \right) V \equiv \begin{pmatrix} f_{\bar{I}}^A \\ h_{B|\bar{I}} \end{pmatrix} \quad (3.1.14)$$

the *period matrix* is introduced via relations

$$\bar{M}_A = \bar{\mathcal{N}}_{AB} \bar{L}^B \quad ; \quad h_{A|\bar{I}} = \bar{\mathcal{N}}_{AB} f_{\bar{I}}^B \quad (3.1.15)$$

which can be solved by introducing two $(n+1) \times (n+1)$ vectors

$$f_{\bar{C}}^A = \begin{pmatrix} f_{\bar{I}}^A \\ \bar{L}^A \end{pmatrix} \quad ; \quad h_{A|\bar{C}} = \begin{pmatrix} h_{A|\bar{I}} \\ \bar{M}_A \end{pmatrix} \quad (3.1.16)$$

and setting

$$\bar{\mathcal{N}}_{AB} = h_{A|\bar{C}} \cdot (f^{-1})^{\bar{C}}_B. \quad (3.1.17)$$

Whenever the prepotential F exists, the period matrix has the form [133, 134, 135]

$$\mathcal{N}_{AB} = \bar{F}_{AB} + 2i \frac{\text{Im}(F_{AC}) \text{Im}(F_{BD}) L^C L^D}{\text{Im}(F_{CD}) L^C L^D} \quad (3.1.18)$$

where $F_{AB} = \partial_A \partial_B F$.

A symplectic rotation C of the holomorphic section obeys $C^T \omega C = \omega$ for

$$\omega = \begin{pmatrix} 0 & \mathbb{1}_{n+1} \\ -\mathbb{1}_{n+1} & 0 \end{pmatrix}.$$

3.2 Gauge Group Representation and dRW Angles

The special Kähler manifold (3.1.5) of vector multiplets has the isometry group $G_{(4)} = SU(1,1) \times SO(2, n-1)$. If we are to gauge a subgroup $K_{(4)} \subset G_{(4)}$, then the symplectic representation R of $G_{(4)}$, under which the electric field strengths and their magnetic duals transform must be decomposed as

$$\begin{aligned} G_{(4)} &\supset K_{(4)}, \\ R &= \text{adj.} + \text{adj.} + \text{singlets} + \text{singlets}. \end{aligned} \quad (3.2.1)$$

The electric and magnetic field strengths are in the doublet representation of $SU(1,1)$ and in $n+1$ vector representation of $SO(2, n-1)$. The non-compact non-Abelian gauge group $K_{(4)} = SO(2,1)$ which is a necessary ingredient to obtain stable de Sitter vacua in 4D, $\mathcal{N} = 2$ supergravity is embedded in $SO(2, n-1)$. The $SO(2,1)$ generators t_A

form an adjoint representation. The symplectic embedding of this representation into the fundamental representation of $Sp(2(n+1), \mathbb{R})$ is given by

$$T_A = \begin{pmatrix} t_A & 0 \\ 0 & -t_A^T \end{pmatrix} \in Sp(2(n+1), \mathbb{R}), \quad A = 0, 2, 3, \quad (3.2.2)$$

and the corresponding algebra $[T_A, T_B] = f_{AB}^C T_C$ is

$$[T_0, T_2] = T_3, \quad [T_2, T_3] = -T_0, \quad [T_3, T_0] = -T_2. \quad (3.2.3)$$

Here, f_{AB}^C are the structure constants of the algebra.

In addition to the $SO(2, 1)$, one can gauge a $U(1)_R$ (or $SU(2)_R$) R -symmetry group for theories with $n > 2$ (or $n > 4$) vector multiplets using the remaining vectors (or a linear combination of them) as gauge fields. The de Roo - Wagemans (dRW) angles, as first introduced for $\mathcal{N} = 4$ supergravity [136, 137] and later used in $\mathcal{N} = 2$ supergravity as an ingredient to obtain de Sitter vacua [67, 68], parametrize the relative embedding of the R -symmetry group within $Sp(2(n+1), \mathbb{R})$. They mix the electric and magnetic components of the symplectic section prior to the gauging by a “non-perturbative” rotation. The dRW rotation matrix has to be chosen in such a way that it commutes with $SO(2, 1)$ symmetry gauging. For example, we will use the following dRW matrix for the models where we gauge a $SO(2, 1) \times U(1)_R$ symmetry [67, 68]:

$$\mathcal{R} = \begin{pmatrix} \mathbb{1}_n & 0 & 0 & 0 \\ 0 & \cos \theta & 0 & \sin \theta \\ 0 & 0 & \mathbb{1}_n & 0 \\ 0 & \sin \theta & 0 & \cos \theta \end{pmatrix}. \quad (3.2.4)$$

The holomorphic section and the covariantly holomorphic section are rotated via

$$\begin{aligned} \Omega &\rightarrow \Omega_R = \mathcal{R} \Omega, \\ V &\rightarrow V_R = \mathcal{R} V. \end{aligned} \quad (3.2.5)$$

3.3 Symplectic Rotation

The symplectic section (3.1.9) is written in the most natural way when one comes from 5 down to 4 dimensions. But it has shortcomings. The translations $z^M \rightarrow z^M + b^M$ act on the symplectic section in such a way that the electric components mix with magnetic ones so that the transformation matrix is not block-diagonal, which is not suitable if symmetries are to be gauged in the standard way. In this section we will give two inequivalent examples of symplectic rotations that will bring Ω_0 in bases where this problem does not occur.

3.3.1 Günaydın-McReynolds-Zagermann (GMZ) Rotation

We start with observing how Ω_0 varies under the infinitesimal translation $z^M \rightarrow z^M + b^M$ [121]:

$$\Omega_0 = \begin{pmatrix} X^0 \\ X^I \\ X^M \\ F_0 \\ F_I \\ F_M \end{pmatrix} \rightarrow \begin{pmatrix} X^0 \\ X^I \\ X^M \\ F_0 \\ F_I \\ F_M \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ b^M X^0 \\ -b^M F_M \\ -\frac{2}{\sqrt{3}} b^M C_{IMN} X^N \\ -\frac{2}{\sqrt{3}} b^N C_{IMN} X^I \end{pmatrix} \quad (3.3.1)$$

In the original basis a combined infinitesimal translation and infinitesimal K transformation with parameter α^I is generated by

$$\mathcal{O} = \mathbb{1}_{2n+2} + \begin{pmatrix} B & 0 \\ C & -B^T \end{pmatrix}, \quad (3.3.2)$$

with

$$B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \alpha^I f_{IJ}^K & 0 \\ b^M & 0 & \alpha^I \Lambda_{IN}^M \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & B_{IM} \\ 0 & B_{MI} & 0 \end{pmatrix}, \quad (3.3.3)$$

where

$$B_{IM} := \frac{-2}{\sqrt{3}} C_{IMN} b^N. \quad (3.3.4)$$

By having a closer look at (3.3.1) we see that (X^0, F_I, X^M) transform among themselves, as do (F_0, X^I, F_M) . In order to make the translations block diagonal we exchange F_0 with X^0 and F_M with X^M . The symplectic rotation

$$\begin{pmatrix} X^A \\ F_B \end{pmatrix} \rightarrow \begin{pmatrix} \check{X}^A \\ \check{F}_B \end{pmatrix} \equiv \mathcal{S} \begin{pmatrix} X^A \\ F_B \end{pmatrix}, \quad \begin{pmatrix} F_{\mu\nu}^A \\ G_{\mu\nu B} \end{pmatrix} \rightarrow \begin{pmatrix} \check{F}_{\mu\nu}^A \\ \check{G}_{\mu\nu B} \end{pmatrix} \equiv \mathcal{S} \begin{pmatrix} F_{\mu\nu}^A \\ G_{\mu\nu B} \end{pmatrix},$$

$$\begin{pmatrix} L^A \\ M_B \end{pmatrix} \rightarrow \begin{pmatrix} \check{L}^A \\ \check{M}_B \end{pmatrix} \equiv \mathcal{S} \begin{pmatrix} L^A \\ M_B \end{pmatrix} \quad (3.3.5)$$

that achieves this is [121]

$$\mathcal{S} = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \delta^J_I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & D^{MN} \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \delta_I^J & 0 \\ 0 & 0 & D_{MN} & 0 & 0 & 0 \end{pmatrix}. \quad (3.3.6)$$

where $D_{MN} = -\sqrt{2}\Omega_{MN}$ and $D_{MN}D^{NP} = \delta_M^P$.

The Potential Terms:

The holomorphic Killing vectors

$$K_A^{\tilde{I}} = ig^{\tilde{I}\tilde{J}}\partial_{\tilde{J}}P_A, \quad (3.3.7)$$

that are determined in terms of the Killing prepotentials [138, 133, 139, 140, 135]

$$P_A = e^{\mathcal{K}}(\check{F}_B f_{AC}^B \check{X}^C + \check{\bar{F}}_B f_{AC}^B \check{X}^C) \quad (3.3.8)$$

can be used to show that the potential in the canonical form

$$V = e^K(\check{X}^A \bar{K}_A^{\tilde{I}})g_{\tilde{I}\tilde{J}}(\check{X}^B K_B^{\tilde{J}}) \quad (3.3.9)$$

is indeed equal to $P_{(4)}^{(T)}$ of (A.25) [121]. Here, f_{BC}^A 's are the structure constants of the gauge group.

Now we turn to the calculation of the potential $P_{(4)}^{(R)}$ rising from the R -symmetry gauging. For gauge groups with $U(1)$ or $SO(3) = SU(2)$ factors there is a superrenormalizable term, known as a Fayet-Iliopoulos (FI) term [141, 142] that can be added to the Lagrangian. The variation of this term under a supersymmetry transformation is a total derivative and it yields a supersymmetric term in the action. FI terms are used in effective field theories for standard model building or cosmology quite often. It has been recently emphasized that these terms in $\mathcal{N} = 1$ or $\mathcal{N} = 2, D = 4$ supersymmetric models are related to R -symmetry gauging [143, 144]. Here we will verify this statement by reformulating an already known $P_{(4)}^{(R)}$ potential, coming from 5 dimensions, in terms of complex geometry elements and comparing the expressions. The potential term we will consider is given by [138]

$$V' = (U^{(AB)} - 3\check{\bar{L}}^{(A}\check{L}^{B)})\mathcal{P}_A^x\mathcal{P}_B^x \quad (3.3.10)$$

where U^{AB} is defined as

$$U^{AB} \equiv f_{\tilde{I}}^A f_{\tilde{J}}^B g^{\tilde{I}\tilde{J}} = -\frac{1}{2}(\text{Im}\mathcal{N})^{-1|AB} - \check{\bar{L}}^A\check{L}^B. \quad (3.3.11)$$

The negative definite term in (3.3.10) is the gravitino mass contribution, while the U^{AB} term is the gaugino shift contribution. \mathcal{P}_A^x are called the *triholomorphic moment maps* for the gauge group action on quaternionic scalars with x being an $SU(2)$ index. When a hypermultiplet is coupled to the theory, the potential (3.3.10) carries contact interactions between the real- and hyperscalars. In this case the triholomorphic moment maps \mathcal{P}_A^x that describe the action of the R -symmetry gauge group on the quaternionic scalars are associated to the Killing prepotentials of the isometries of the hyperscalar manifold [138, 145]. This is analogous to the 5 dimensional theory (c.f. appendices A and B). An FI

term can be assigned to the moment maps if (and only if [144]) hypers are absent from the theory. For such models the triholomorphic moment maps satisfy the equivariance condition [138, 67, 145]

$$-\epsilon^{xyz}\mathcal{P}_A^y\mathcal{P}_B^z = f_{AB}^C\mathcal{P}_C^x. \quad (3.3.12)$$

In the $SU(2)_R$ case one can set $f_{yz}^x = e\epsilon_{xyz}$, where e is some number, and this condition is satisfied via

$$\mathcal{P}_A^x = \begin{cases} -\delta_y^x & \text{for } A = 3 + y \\ 0 & \text{otherwise,} \end{cases} \quad (3.3.13)$$

whereas in the $U(1)_R$ case, for each generator one can set an FI term

$$\mathcal{P}_A^x = \begin{cases} e\delta_3^x & A: \text{index for the } U(1)_R \text{ gauge vector} \\ 0 & \text{otherwise.} \end{cases} \quad (3.3.14)$$

Example: Let us now calculate the V' potential for a specific model with $n = 4$ vector multiplets where the $U(1)_R$ gauge field is a linear combination of A_μ^1 and A_μ^4 . This is indeed the model we discussed in subsection 2.2.3.3 before the dimensional reduction. Using (2.2.66) and (A.27) one can write the $U(1)_R$ potential in 4 dimensions as

$$P_{(4)}^{(R)} = e^{-\sigma}P_{(5)}^{(R)} = e^{-\sigma} \left(-4\sqrt{2}V_1V_4\varphi^4\|\varphi\|^{-2} + 2(V_4)^2\|\varphi\|^2 \right). \quad (3.3.15)$$

On the other hand, the moment map for this type of gauging can be written as

$$\mathcal{P}_A^x = \delta^{x3} (e_1\delta_{A1} + e_4\delta_{A4}) \quad (3.3.16)$$

where e_1 and e_4 parametrize the linear combination of the gauge fields. Then the potential (3.3.10) becomes

$$V'_{n=4} = e_1^2(U^{(11)} - 3\check{L}^{(1)\check{L}1}) + 2e_1e_4(U^{(14)} - 3\check{L}^{(1)\check{L}4}) + e_4^2(U^{(44)} - 3\check{L}^{(4)\check{L}4}) \quad (3.3.17)$$

and after some calculation one can find

$$\begin{aligned} U^{(11)} - 3\check{L}^{(1)\check{L}1} &= 0, \\ U^{(14)} - 3\check{L}^{(1)\check{L}4} &= -\frac{\text{Im}z^4}{(\text{Im}z^2)^2 - (\text{Im}z^3)^2 - (\text{Im}z^4)^2}, \\ U^{(44)} - 3\check{L}^{(4)\check{L}4} &= \frac{1}{2\text{Im}z^1}. \end{aligned} \quad (3.3.18)$$

By using (3.1.6) and (3.1.4) on (3.3.18), we conclude that $V'_{n=4} = P_{(4)}^{(R)}$ if we identify $e_1 = \pm(\frac{8}{3})^{\frac{1}{4}}V_1$ together with $e_4 = \pm(\frac{8}{3})^{\frac{1}{4}}V_4$.

One can arrive at a similar conclusion by gauging the full $SU(2)_R$ instead. In this case $P_{(4)}^{(R)} = 6e^{-\sigma}\|\varphi\|^2$ and $V' = 3/(2\text{Im}z_1)$ which are again directly proportional to each other.

3.3.2 A New Basis

The GMZ rotation we discussed in the last subsection resolves the block-diagonality problem of translational symmetries but there are a few more steps to take in order to find a symplectic section that will allow us to find de Sitter vacua. First, it is convenient to work in a symplectic section that satisfies the constraint

$$X^A \eta_{AB} X^B = F_A \eta^{AB} F_B = 0 \quad (3.3.19)$$

for $\eta_{AB} = \text{diag}(+ + \dots -)^3$ so that the $SO(2, n-1)$ invariance is evident. Note that we restrict our analysis to the generic Jordan family (3.1.5). Other types of scalar manifolds will be discussed in section 3.7.

Under infinitesimal translations $z^M \rightarrow z^M + b^M$, Ω_0 transforms as in (3.3.1). We noted that (X^0, F_I, X^M) transform among themselves, as do (F_0, X^I, F_M) . This time we are exchanging some of X^I with F_I keeping in mind that we are constrained by (3.3.19). Exchanging all of X^I with F_I will not leave this equation invariant. Therefore we decompose the index I as $I = (1, i)$ and swap X^1 with F_1 and keep the other X^i and F_i intact. By looking at (3.3.1) we see that one must have

$$b^M C_{iMN} X^N = 0 \quad (3.3.20)$$

in order to keep the translations block diagonal. This is indeed satisfied for all types of gaugings of the generic Jordan family isometries.

As we discussed earlier, the bare graviphoton in 4 dimensions is a linear combination of the vectors A_μ^0 and A_μ^1 which are obtained by reduction from 5 dimensions. By taking a linear combination of X^0 and F_1 (F_0 and X^1) for \tilde{X}^0 (\tilde{F}_0) we isolate the bare graviphoton as \tilde{A}_μ^0 . The new symplectic section $\tilde{\Omega}$ is given by the rotation of Ω_0 by

$$\tilde{S} = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & \tilde{\Lambda}_{1N}^M & 0 & 0 & 0 & 0 \\ 0 & 0 & \delta_i^j & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{\Lambda}_{1M}^N \\ 0 & 0 & 0 & 0 & 0 & 0 & \delta_j^i & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \end{pmatrix}. \quad (3.3.21)$$

The rescaling $\tilde{\Lambda}_{IN}^M \equiv \sqrt{2} \Lambda_{IN}^M = \frac{2}{\sqrt{3}} \Omega^{MP} C_{INP}$ is done for future convenience. It is easy to verify that the matrix S is symplectic. More explicitly, we have

³In general, the order of the + and - entries depend on the type of gauging but their numbers are fixed.

$$\begin{aligned}
\tilde{\Omega} &= \begin{pmatrix} \tilde{X}^0 \\ \tilde{X}^M \\ \tilde{X}^j \\ \tilde{X}^1 \\ \tilde{F}_0 \\ \tilde{F}_M \\ \tilde{F}_j \\ \tilde{F}_1 \end{pmatrix} = \tilde{\mathcal{S}}\Omega_0 = \tilde{\mathcal{S}} \begin{pmatrix} X^0 \\ X^1 \\ X^i \\ X^N \\ F_0 \\ F_1 \\ F_i \\ F_N \end{pmatrix} \\
&= \begin{pmatrix} \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{6}}(C_{1JK}z^Jz^K + C_{1MN}z^Mz^N) \\ \tilde{\Lambda}_{1N}^M z^N \\ z^i \\ -\frac{1}{\sqrt{2}}z^1 + \frac{1}{\sqrt{6}}(C_{1JK}z^Jz^K + C_{1MN}z^Mz^N) \\ -\frac{1}{\sqrt{2}}z^1 + \frac{1}{3\sqrt{6}}(C_{IJK}z^Iz^Jz^K + 3C_{IMN}z^Iz^Mz^N) \\ -\frac{2}{\sqrt{3}}\tilde{\Lambda}_{1M}^P C_{PNI}z^Nz^I \\ -\frac{1}{\sqrt{3}}C_{iJK}z^Jz^K \\ \frac{1}{\sqrt{2}}z^1 + \frac{1}{3\sqrt{6}}(C_{IJK}z^Iz^Jz^K + 3C_{IMN}z^Iz^Mz^N) \end{pmatrix}. \tag{3.3.22}
\end{aligned}$$

Here 0 is now the graviphoton index. The combined infinitesimal $z^M \rightarrow z^M + b^M$ translation and infinitesimal $K_{(5)}$ transformation with parameter α^I is generated by the symplectic matrix

$$\tilde{\mathcal{O}} \equiv \tilde{\mathcal{S}}\mathcal{O}\tilde{\mathcal{S}}^{-1} = \mathbb{1}_{2n+2} + \begin{pmatrix} \tilde{B} & \tilde{C} \\ \tilde{C}^T & -\tilde{B}^T \end{pmatrix}, \tag{3.3.23}$$

with

$$\begin{aligned}
\tilde{B} &= \begin{pmatrix} 0 & \frac{1}{\sqrt{2}}\tilde{\Lambda}_{1M}^P B_{P1} & 0 & 0 \\ \frac{1}{\sqrt{2}}\tilde{\Lambda}_{1N}^M b^N & \alpha^I \tilde{\Lambda}_{IN}^M & 0 & \frac{1}{\sqrt{2}}\tilde{\Lambda}_{1N}^M b^N \\ 0 & 0 & \alpha^I f_{Ij}^k & 0 \\ 0 & -\frac{1}{\sqrt{2}}\tilde{\Lambda}_{1M}^P B_{P1} & 0 & 0 \end{pmatrix}, \\
\tilde{C} &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & -\alpha^I f_{Ij}^1 & 0 \\ 0 & 0 & 0 & 0 \\ -\alpha^I f_{I1}^j & 0 & 0 & \alpha^I f_{I1}^j \\ 0 & 0 & \alpha^I f_{Ij}^1 & 0 \end{pmatrix} \tag{3.3.24}
\end{aligned}$$

where $B_{IM} := -\frac{2}{\sqrt{3}}C_{IMN}b^N$. In order to represent the combined translations and $K_{(5)}$ transformations by block diagonal matrices one must have an algebra with $f_{Ij}^1 = f_{I1}^j = 0$. Here the index 1 corresponds to the five dimensional graviphoton, which can only be a gauge field if the gauge group is Abelian because it is a singlet under the action of five dimensional isometry group $SO(\tilde{n} - 1, 1)$. Therefore this condition is automatically satisfied and hence

$\tilde{C} = 0$. Next, by setting $\tilde{B}^C{}_B = \alpha^A f^C_{AB}$ one can find

$$f^j_{ik}, \quad f^M_{IN} = \Lambda^M_{IN}, \quad f^0_{MN} = -f^1_{MN} = -\frac{1}{\sqrt{3}}\Lambda_{1M}{}^P C_{1PN}, \quad f^M_{N0} = f^M_{N1} = -\Lambda^M_{1N} \quad (3.3.25)$$

as non-vanishing components, as well as $\alpha^M = -b^M$.

3.4 de Sitter Vacua

We will now demonstrate how to obtain stable de Sitter vacua by starting with the holomorphic section (3.3.22). The model to be considered is $4D$, $\mathcal{N} = 2$ supergravity coupled to $n = 4$ vector multiplets with gauge group $K_{(4)} = SO(2, 1) \times U(1)_R$. This model can be trivially extended to arbitrary n as we will discuss at the end of this section. Note that this type of gauging was first used in [67, 68] to obtain de Sitter vacua where the authors preferred to use Calabi-Vesentini coordinates to parametrize the complex scalars. The mapping between our notation and theirs can be found in appendix C.

3.4.1 Potential $P_{(4)}^{(T)}$ from Global Isometry Gauging

The global isometry group $G_{(4)}$ for the model with 4 vector multiplets is $SU(1, 1) \times SO(2, 3)$. A potential is introduced by gauging the subgroup $SO(2, 1) \subset SO(2, 3)$.

$$P_{(4)}^{(T)} = e^K (\tilde{X}^A \tilde{K}_A^{\tilde{I}}) g_{\tilde{I}\tilde{J}} (\tilde{X}^B \tilde{K}_B^{\tilde{J}}). \quad (3.4.1)$$

The structure constants f^A_{BC} of the $SO(2, 1)$ algebra (3.2.3) read

$$f^3_{02} = f^2_{03} = -f^3_{20} = -f^2_{30} = 1, \quad f^0_{32} = -f^0_{23} = 1. \quad (3.4.2)$$

The gauge fields are the ‘‘timelike’’ $\tilde{A}^0_\mu, \tilde{A}^2_\mu$ and the ‘‘spacelike’’ \tilde{A}^3_μ with respect to $SO(2, 3)$ with signature $(+ + - - -)$; and the Killing vectors determined by (3.3.8) and (3.3.7) are given by

$$\begin{aligned} \vec{K}_0 &= \begin{pmatrix} 0 \\ -w_3 \\ -w_2 \\ 0 \end{pmatrix}, \quad \vec{K}_2 = \begin{pmatrix} 0 \\ -\frac{1}{\sqrt{2}}w_2w_3 \\ \frac{1}{2\sqrt{2}}(2 - w_2^2 - w_3^2 + w_4^3) \\ -\frac{1}{\sqrt{2}}w_3w_4 \end{pmatrix}, \\ \vec{K}_3 &= \begin{pmatrix} 0 \\ \frac{1}{2\sqrt{2}}(2 + w_2^2 + w_3^2 + w_4^3) \\ \frac{1}{\sqrt{2}}w_2w_3 \\ \frac{1}{\sqrt{2}}w_2w_4 \end{pmatrix} \end{aligned} \quad (3.4.3)$$

where we defined $w_{\tilde{I}} \equiv z^{\tilde{I}}$. The full potential term is given in (D.1). It simplifies significantly when evaluated at $\text{Re}(w_i) = 0$

$$P_{(4)}^{(T)}|_{\text{Re}(w_i)=0} = \frac{(\text{Im}w_2^2 - \text{Im}w_3^2)(2 + \|\text{Im}w\|^2)^2}{16 \text{Im}w_1 \|\text{Im}w\|^4} \quad (3.4.4)$$

with $\|\text{Im}w\|^2 \equiv (\text{Im}w_2^2 - \text{Im}w_3^2 - \text{Im}w_4^2)$. Note also that this potential term satisfies

$$\frac{\partial P_{(4)}^{(T)}}{\partial \text{Re}(w_i)}|_{\text{Re}(w_i)=0} = 0. \quad (3.4.5)$$

3.4.2 $U(1)_R$ Potential

We are considering a theory with $n = 4$ vector multiplets, and the vector field that gauges the $U(1)_R$ -symmetry is \tilde{A}_μ^1 . Hence we choose the moment map to be

$$\mathcal{P}_A^x = \delta^{x3} \delta_{A1}. \quad (3.4.6)$$

Then the $U(1)_R$ potential term is given by

$$P_{(4)}^{(R)} = U^{11} - 3\tilde{L}^1 \tilde{L}^1 \quad (3.4.7)$$

with the following definitions

$$\begin{aligned} \tilde{L}^A &\equiv S L^A \\ U^{AB} &\equiv f_{\tilde{I}}^A f_{\tilde{J}}^B g^{\tilde{I}\tilde{J}} = -\frac{1}{2} (\text{Im}\mathcal{N})^{-1|AB} - \tilde{\tilde{L}}^A \tilde{L}^B, \end{aligned} \quad (3.4.8)$$

$A, B = (0, 2, 3, 4, 1)$.

3.4.2.1 No dRW-rotation

For simplicity let's assume no de Roo-Wagemans rotation. One can show that [138]

$$U^{AB} - 3\tilde{\tilde{L}}^A \tilde{L}^B = -\frac{\eta^{AB}}{2\text{Im}w_1} \quad (3.4.9)$$

with $\eta^{AB} := \text{diag}(+ + - -)$. Then the potential (3.4.7) is

$$P_{(4)}^{(R)} = \frac{1}{2\text{Im}w_1} \sim \frac{1}{e^\sigma h^1} \sim e^{-\sigma} \|\varphi\|^2 \quad (3.4.10)$$

where an overall positive multiplier is neglected. We note that this potential is proportional to the last term of (3.3.15) and because of the diagonality of (3.4.9), one cannot get a term proportional to the first term by using a linear combination of vectors as the gauge field. One way to interpret this is: Due to the symplectic rotation (3.3.21), the five dimensional

gauge field $A_{\tilde{\mu}}^1$ is decomposed in two parts. One part contributes to the four dimensional gauge vector \tilde{A}_{μ}^1 and the other to the four dimensional bare graviphoton \tilde{A}_{μ}^0 . It is this second part of $A_{\tilde{\mu}}^1$ that leads to the first term of (3.3.15), which does not contribute to the four dimensional gauge field in this particular choice of the holomorphic section.

3.4.2.2 dRW-rotation

The de Roo-Wagemans matrix (3.2.4) rotates the symplectic section (3.3.22, with $n = 4$) to

$$\left(\begin{array}{c} \frac{1}{2\sqrt{2}} (2 - \|w\|^2) \\ w_2 \\ w_3 \\ w_4 \\ \frac{1}{2\sqrt{2}} (2 + \|w\|^2) (\cos \theta + w_1 \sin \theta) \\ -\frac{1}{2\sqrt{2}} w_1 (2 - \|w\|^2) \\ -w_1 w_2 \\ w_1 w_3 \\ w_1 w_4 \\ -\frac{1}{2\sqrt{2}} (2 + \|w\|^2) (\sin \theta - w_1 \cos \theta) \end{array} \right), \quad (3.4.11)$$

where $\|w\|^2 \equiv [w_2^2 - w_3^2 - w_4^2]$. Using Mathematica we evaluated the potential as

$$P_{(4)}^{(R)} = \frac{|\cos \theta + w_1 \sin \theta|^2}{2\text{Im}w_1}. \quad (3.4.12)$$

This potential agrees with [67] by applying the coordinate transformations outlined in appendix C.

3.4.3 Critical Points

The total potential of the current model with $n = 4$ vector multiplets and $K_{(4)} = SO(2, 1) \times U(1)_R$ gauge group evaluated at $\text{Re}(w_i) = 0$ is given by

$$\begin{aligned} P_{(4)}|_{\text{Re}(w_i)=0} &= (P_{(4)}^{(T)} + \lambda P_{(4)}^{(R)})|_{\text{Re}(w_i)=0} \\ &= \frac{1}{2\text{Im}w_1} \left(\frac{(\text{Im}w_2^2 - \text{Im}w_3^2)(2 + \|\text{Im}w\|^2)^2}{8\|\text{Im}w\|^4} + \lambda |\cos \theta + w_1 \sin \theta|^2 \right). \end{aligned} \quad (3.4.13)$$

The critical points of this potential have coordinates which obey

$$w_1 = -\cot \theta + \frac{i \csc \theta}{\sqrt{\lambda}}, \quad (\text{Im}w_2)^2 - (\text{Im}w_3)^2 = 2, \quad \text{Re}w_i = 0, \quad \text{Im}w_4 = 0 \quad (3.4.14)$$

and the potential evaluated at these points is

$$P_{(4)}|_{\phi^c} = \sqrt{\lambda} \sin \theta = \frac{1}{\text{Im}w_1^c} \quad (3.4.15)$$

which is positive definite in the physically relevant region⁴ ($0 < \theta < \pi$). Writing (3.4.14) in terms of real scalar fields we obtain the conditions

$$A_c^i = \varphi_c^4 = 0, \quad A_c^1 = -\sqrt{3} \cot \theta, \quad e^{3\sigma_c} = \frac{6\sqrt{6} \csc \theta}{\sqrt{\lambda}}, \quad [(\varphi_c^2)^2 - (\varphi_c^3)^2] = 6 e^{-2\sigma_c}. \quad (3.4.16)$$

We see that for a given θ , the values of all the scalars, including the dilaton σ , at the critical point are fixed. The only exception is that the term $[(\varphi_c^2)^2 - (\varphi_c^3)^2]$ is fixed whereas φ_c^2 and φ_c^3 are not, individually. Observe that this was also the case in five dimensions when the gauge group was $K_{(5)} = SO(1, 1) \times U(1)_R$ (c.f. subsection 2.2.3.3).

The stability of this family of critical points can be studied by calculating the eigenvalues of the Hessian of the potential evaluated at the extremum. When this is normalized by the inverse of the metric (3.1.11)

$$g^{\bar{I}\bar{J}}|_{\phi^c} = \begin{pmatrix} 4\text{Im}w_1^2 & 0 & 0 & 0 \\ 0 & 4(\text{Im}w_2^2 - 1) & 4\text{Im}w_2\text{Im}w_3 & 0 \\ 0 & 4\text{Im}w_2\text{Im}w_3 & 4(\text{Im}w_2^2 - 1) & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix}, \quad (3.4.17)$$

it gives the mass matrix of the scalar fields

$$\frac{\partial_{\bar{I}}\partial^{\bar{J}}P_{(4)}|_{\phi^c}}{P_{(4)}} = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & \frac{\text{Im}w_2^2}{2} & \frac{1}{2}\text{Im}w_2\text{Im}w_3 & 0 \\ 0 & -\frac{1}{2}\text{Im}w_2\text{Im}w_3 & -\frac{\text{Im}w_3^2}{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (3.4.18)$$

with “complex” eigenvalues $(2, 1, 1, 0)$ ⁵. Thus the family critical points corresponds to stable de Sitter vacua.

One can extend this result to a theory coupled to an arbitrary number $n > 2$ of vector multiplets by trivially extending the holomorphic section and the value of the potential at the extremum will not change. The mass matrix will contain $n - 3$ diagonal entries with the value 1 and the values of the extra scalars at the extremum will be zero.

⁴The imaginary part of w_1 is proportional to $1/|\varphi|^2$ which has to be positive definite in order to have positive kinetic terms in the Lagrangian. See section 2.2 for a more thorough discussion.

⁵The reason why we called these “complex” eigenvalues is based on the fact that the derivatives $\partial_{\bar{I}}$ are with respect to the complex scalars $z^{\bar{I}}$. The same mass matrix can be obtained by taking the derivatives with respect to $\bar{z}^{\bar{I}}$.

3.5 The Five Dimensional Connection

Dimensionally reducing $5D, \mathcal{N} = 2$ YMESGT with isometry gauging group $K_{(5)}$ yields a $4D, \mathcal{N} = 2$ YMESGT with an isometry gauging group $K_{(4)} = K_{(5)} \ltimes \mathcal{H}^{n_T+1}$ [121] where n_T is the number of tensor multiplets coupled to the theory and \mathcal{H}^{n_T+1} is the Heisenberg group generated by translations and the central charge. This Heisenberg group factor exist only if tensors are coupled to the theory.

The model discussed in the last section with gauge group $K_{(4)} = SO(2, 1) \times U(1)_R$ has stable de Sitter vacua. Unfortunately, it cannot be obtained from five dimensions directly. One can immediately think of gauging a subgroup $K_{(5)} = SO(1, 2)$ of the global isometry group for one of the three families (2.1.6) in five dimensions. For the generic Jordan Family, the resulting theory after dimensional reduction still has the gauge group $K_{(4)} = SO(1, 2)$. This type of gauging does not yield a scalar potential in five dimensions because tensors are absent from the theory, but it does in four dimensions due to the last term of (3.0.1). For the magical Jordan family there will be tensors transforming under $SO(1, 2)$ hence the gauge group in four dimensions is $SO(1, 2) \ltimes \mathcal{H}^{n_T+1}$; and for the generic non-Jordan family $SO(1, 2)$ is not gaugable because one cannot find vector fields that transform under the adjoint representation of $SO(1, 2)$ to use as the gauge fields. The first two of these families allow for four dimensional theories with an $SO(1, 2)$ factor in the gauge group but this is not the same $SO(2, 1)$ gauge group factor we discussed in the last section. The former one is a subgroup of $SO(1, 2) \times SO(1, n-3) \subset SO(2, n-1)$ and has one timelike and two spacelike dimensions whereas the latter is a subgroup of $SO(2, 1) \times SO(n-2) \subset SO(2, n-1)$ and has two timelike and one spacelike dimensions. Therefore the model with $SO(2, 1)$ gauge group factor we discussed in the last section does not originate from five dimensions.⁶

Nevertheless, this is not the end of the story. In five dimensions, de Sitter vacua were found for the $SO(1, 1) \times U(1)_R$ gauging and in four dimensions they were found for the $SO(2, 1) \times U(1)_R$ gauging. In this section, we will show that under an appropriate group contraction of $SO(2, 1)$ one can find a theory, which can be obtained from the five dimensional $SO(1, 1) \times U(1)_R$ theory under another appropriate group contraction, and that has a potential that allows stable de Sitter ground states.

3.5.1 Contracting the Algebra

A geometrical interpretation for the contraction can be given by introducing the n -dimensional inhomogeneous coordinates u_a ($a = 0, 2, 3, \dots, n$) that parametrize a hyperboloid embedded in n -dimensional space by $u_a \eta^{ab} u_b = R^2$ where $\eta_{ab} = \text{diag}(+ + \dots -)$ and R is the radius of curvature. The scalars v_k ($k = 2, 3, \dots, n$) parametrize an $(n-1)$ -dimensional hypersurface. This hypersurface is mapped onto the hyperboloid embedded in n -dimensional

⁶However this does not rule out the possibility that the $SO(1, 2)$ gauging may result in non-Minkowski ground states in four dimensions. See subsection 3.6.5 for this type of gauging.

space by the stereographical projection

$$\begin{aligned} u_0 &= \frac{R^2 - \|v\|^2}{R^2 + \|v\|^2} R, \\ u_k &= \frac{2R^2 v_k}{R^2 + \|v\|^2} \end{aligned} \quad (3.5.1)$$

where $\|v\|^2 = [v_2^2 - v_3^2 - \dots - v_n^2]$. The inverse mapping is

$$v_k = \frac{R u_k}{R + u_0}. \quad (3.5.2)$$

For $n = 4$, the $SO(2, 1)$ symmetry on the 4-dimensional hyperboloid is generated by the Killing vectors, which in terms of homogeneous hypersurface coordinates are formulated by

$$\vec{K}_0 = \begin{pmatrix} 0 \\ -w_3 \\ -w_2 \\ 0 \end{pmatrix}, \vec{K}_2 = \begin{pmatrix} 0 \\ -\frac{w_2 w_3}{R} \\ \frac{R^2 - w_2^2 - w_3^2 + w_4^2}{2R} \\ -\frac{w_3 w_4}{R} \end{pmatrix}, \vec{K}_3 = \begin{pmatrix} 0 \\ \frac{R^2 + w_2^2 + w_3^2 + w_4^2}{2R} \\ \frac{w_2 w_3}{R} \\ \frac{w_2 w_4}{R} \end{pmatrix}. \quad (3.5.3)$$

Note that if the real v_i are extended to the complex w_i and $R = \sqrt{2}$, these are the same Killing vectors we evaluated in (3.4.3). By taking the large R limit, the hyperboloid is locally flattened and the group is contracted [146, 147] to $SO(1, 1) \times \mathbb{R}^{(1,1)}$. Let us observe this by defining the new generators as

$$\vec{K}'_0 \equiv \vec{K}_0, \quad \vec{K}'_2 \equiv \frac{2\vec{K}_2}{R}, \quad \vec{K}'_3 \equiv \frac{2\vec{K}_3}{R} \quad (3.5.4)$$

and evaluating the Lie brackets

$$[\vec{K}'_0, \vec{K}'_2] = \vec{K}'_3, \quad [\vec{K}'_0, \vec{K}'_3] = \vec{K}'_2, \quad [\vec{K}'_2, \vec{K}'_3] = -\frac{4}{R^2} \vec{K}'_0. \quad (3.5.5)$$

By taking the limit $R \rightarrow \infty$, the last of these Lie brackets vanishes and we see that the new Killing vectors generate the Lie algebra of the Poincare group in two dimensions which is the semi-direct product of ‘‘Lorentz boosts’’ $SO(1, 1)$ with ‘‘translations’’ $\mathbb{R}^{(1,1)}$.

Meanwhile, for the five dimensional gauge group $K_{(5)} = SO(1, 1)$ the structure constants (3.3.25) determine the following algebra in four dimensions:

$$\begin{aligned} \left[\frac{T_0 - T_1}{\sqrt{2}}, T_2\right] &= 0, & \left[\frac{T_0 + T_1}{\sqrt{2}}, T_3\right] &= T_2, \\ \left[\frac{T_0 - T_1}{\sqrt{2}}, T_3\right] &= 0, & \left[\frac{T_0 + T_1}{\sqrt{2}}, T_2\right] &= T_3, \\ [T_2, T_3] &= \frac{T_0 - T_1}{\sqrt{2}}. \end{aligned} \quad (3.5.6)$$

They define the Lie algebra of a central extension of the Lie algebra $SO(1, 1) \oplus \mathbb{R}^{(1,1)}$, with central charge corresponding to the generator $\frac{1}{\sqrt{2}}(T_0 - T_1)$. Here “ \oplus ” denotes “semi-direct sum”. $\frac{1}{\sqrt{2}}(T_0 + T_1)$ rotates T_2 and T_3 into each other and corresponds to the bare graviphoton in 5 dimensions which acted as the $SO(1, 1)$ gauge field. Note that this result parallels completely the situation in the subsection 3.3.1 (c.f. [121]).

By defining the new generators

$$\begin{pmatrix} W_0 \\ W_1 \\ W_2 \\ W_3 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ -\beta & \beta & 0 & 0 \\ 0 & 0 & \beta & 0 \\ 0 & 0 & 0 & \beta \end{pmatrix} \begin{pmatrix} (T_0 - T_1)/\sqrt{2} \\ (T_0 + T_1)/\sqrt{2} \\ T_2 \\ T_3 \end{pmatrix} \quad (3.5.7)$$

one can rewrite the algebra as

$$\begin{aligned} [W_2, W_3] &= \frac{1}{2}(\beta^2 W_0 - \beta W_1) \\ [W_0, W_2] &= W_3 \\ [W_0, W_3] &= W_2 \\ [W_1, W_2] &= \beta W_3 \\ [W_1, W_3] &= \beta W_2. \end{aligned} \quad (3.5.8)$$

In the limit $\beta \rightarrow 0$ the transformation matrix above becomes noninvertible, but this is expected since information is generically lost during group contractions, and the algebra reduces to $SO(1, 1) \oplus \mathbb{R}^{(1,1)}$ without central charge. This is the same algebra as (3.5.5) in the large R region. Thus the two different limits of the two different theories overlap. Now, we will calculate the extrema of the scalar potential they will generate.

3.5.2 Potential by $(SO(1, 1) \times \mathbb{R}^{(1,1)}) \times U(1)_R$ Gauging

Using the Killing vectors (3.5.4) in the large R limit, the potential (3.4.1) is calculated as in (D.2). When evaluated at $\text{Re}(w_k) = 0$, ($k = 2, 3, \dots, n$) it takes the form

$$P_{(4)}^{(T)}|_{\text{Re}(w_k)=0} = \frac{(\text{Im}w_2^2 - \text{Im}w_3^2)(4 + \|\text{Im}w\|^2)^2}{64 \text{Im}w_1 \|\text{Im}w\|^4} \quad (3.5.9)$$

where $\|\text{Im}w\|^2 \equiv (\text{Im}w_2^2 - \text{Im}w_3^2 - \dots - \text{Im}w_n^2)$. This potential term satisfies

$$\frac{\partial P_{(4)}^{(T)}}{\partial \text{Re}(w_k)} \Big|_{\text{Re}(w_k)=0} = 0. \quad (3.5.10)$$

The dRW-rotation is done prior to the gauging. One must choose the $U(1)_R$ gauge field \tilde{A}_μ^b among \tilde{A}_μ^i ($i = 4, \dots, n$)⁷ and dRW-rotate \tilde{X}^b and \tilde{F}_b into each other. The dRW-matrix is given by

$$\mathcal{R} = \begin{pmatrix} \mathbb{1}_{n-1} & 0 & 0 & 0 & 0 \\ 0 & \cos \theta & 0 & \sin \theta & 0 \\ 0 & 0 & \mathbb{1}_n & 0 & 0 \\ 0 & \sin \theta & 0 & \cos \theta & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (3.5.11)$$

where we chose $b = n$. Note that with this type of gauging one must have $n > 3$ vector multiplets (c.f. $n > 2$ for the $SO(2,1) \times U(1)_R$ gauging after the dRW-rotation (3.2.4)). The calculation of the $U(1)_R$ potential is similar to the last case but it has the same expression

$$P_{(4)}^{(R)} = \frac{|\cos \theta + w_1 \sin \theta|^2}{2 \text{Im}w_1}. \quad (3.5.12)$$

The critical points of the total potential $P_{(4)} = P_{(4)}^{(T)} + \lambda P_{(4)}^{(R)}$ are given by

$$w_1^c = -\cot \theta + \frac{i \csc \theta}{\sqrt{2\lambda}}, \quad (\text{Im}w_2^c)^2 - (\text{Im}w_3^c)^2 = 4, \quad \text{Re}w_k^c = 0, \quad \text{Im}w_i^c = 0 \quad (3.5.13)$$

and the value of the potential evaluated at these points is

$$P_{(4)}|_{\phi^c} = \sqrt{\frac{\lambda}{2}} \sin \theta = \frac{1}{2 \text{Im}w_1^c}. \quad (3.5.14)$$

Writing these in terms of real scalars, we again see that for a given θ , the values of all the scalars, including the dilaton σ , at the critical point are fixed. The only exception is that the term $[(\varphi_c^2)^2 - (\varphi_c^3)^2]$ is fixed whereas φ_c^2 and φ_c^3 are not, individually.

The mass matrix for this potential evaluated at the family of critical points is

$$\frac{\partial_{\tilde{I}} \partial^{\tilde{J}} P_{(4)}}{P_{(4)}} \Big|_{\phi^c} = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & \frac{\text{Im}w_2^2}{4} & \frac{1}{4} \text{Im}w_2 \text{Im}w_3 & 0 \\ 0 & -\frac{1}{4} \text{Im}w_2 \text{Im}w_3 & -\frac{\text{Im}w_3^2}{4} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (3.5.15)$$

⁷Choosing \tilde{A}_μ^1 as the $U(1)_R$ gauge field as we did in subsection 3.4.2.2 would result rotating \tilde{X}^1 and \tilde{F}_1 into each other. But in this case, the presence of tensors makes it impossible to keep the translations block diagonal.

which has eigenvalues $(2, 1, 1, 0)$ and hence the extrema correspond to stable de Sitter vacua. The zero eigenvalue is due to the remaining $SO(1, 1)$ symmetry and means that there is a flat direction along the extrema.

The effect of group contraction to the potential

Without the contraction outlined in the last subsection, i.e. using the structure constants of the algebra (3.5.6), the potential $P_{(4)}^{(T)}$ evaluated at $\text{Re}(w_i) = 0$ is given by

$$\frac{\text{Im}w_2^2 - \text{Im}w_3^2}{2\text{Im}w_1 \|\text{Im}w\|^4}. \quad (3.5.16)$$

Subtracting this expression from (3.5.9) will give the contribution of the group contraction to the scalar potential as

$$\frac{(\text{Im}w_2^2 - \text{Im}w_3^2) P_+ P_-}{64 \text{Im}w_1 \|\text{Im}w\|^4}, \quad (3.5.17)$$

where $P_{\pm} = \|\text{Im}w\|^2 + 4(1 \pm \sqrt{2})$. This term is positive definite in the neighborhood of the extrema, where $\|\text{Im}w\|^2 \sim 4$. A quick calculation shows that the $P_{(4)}^{(T)}$ potential (3.5.16), together with the $P_{(4)}^{(R)}$ potential (3.5.12) does not have any critical points.

3.6 More Examples

3.6.1 $(SO(1, 1) \times \mathbb{R}^{(1,1)}) \times SU(2)_R$ gauging

In order to do such a gauging along with a dRW-rotation one must have $n > 5$ vector multiplets. $P^{(T)}$ is as given in (3.5.9).

For the $SU(2)_R$ gauging, the moment map is as defined in (3.3.13) and the gauge fields are chosen to be \tilde{A}_{μ}^b ($b = n - 2, n - 1, n$). \tilde{X}^b and \tilde{F}_b are rotated into each other via the dRW-matrix

$$\mathcal{R} = \begin{pmatrix} \mathbb{1}_{n-3} & 0 & 0 & 0 & 0 \\ 0 & \cos \theta \mathbb{1}_3 & 0 & \sin \theta \mathbb{1}_3 & 0 \\ 0 & 0 & \mathbb{1}_{n-2} & 0 & 0 \\ 0 & -\sin \theta \mathbb{1}_3 & 0 & \cos \theta \mathbb{1}_3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \quad (3.6.1)$$

The resulting $SU(2)_R$ potential is given by

$$\begin{aligned} P_{(4)}^{(R)} &= \left(U^{(AB)} - 3\tilde{\tilde{L}}^{(A}\tilde{L}^{B)} \right) \mathcal{P}_A^x \mathcal{P}_B^x \\ &= \sum_{y=n-2}^n \left(U^{(yy)} - 3\tilde{\tilde{L}}^{(y}\tilde{L}^{y)} \right) \\ &= \frac{3 |\cos \theta + w_1 \sin \theta|^2}{2\text{Im}w_1} \end{aligned} \quad (3.6.2)$$

which differs from (3.5.12) only by a factor 3. Each $SU(2)_R$ generator gives the same contribution as the $U(1)_R$ generator in the Abelian case. The total potential is

$$\begin{aligned} P_{(4)}|_{\text{Re}(w_k)=0} &= (P_{(4)}^{(T)} + \lambda P_{(4)}^{(R)})_{\text{Re}(w_k)=0} \\ &= \frac{(\text{Im}w_2^2 - \text{Im}w_3^2)(4 + \|\text{Im}w\|^2)^2}{64 \text{Im}w_1 \|\text{Im}w\|^4} + \frac{3\lambda |\cos\theta + w_1 \sin\theta|^2}{2\text{Im}w_1}. \end{aligned} \quad (3.6.3)$$

The critical points of the total potential $P_{(4)} = P_{(4)}^{(T)} + \lambda P_{(4)}^{(R)}$ are given by

$$w_1^c = -\cot\theta + \frac{i \csc\theta}{\sqrt{6\lambda}}, \quad (\text{Im}w_2^c)^2 - (\text{Im}w_3^c)^2 = 4, \quad \text{Re}w_k^c = 0, \quad \text{Im}w_i^c = 0 \quad (3.6.4)$$

and the value of the potential evaluated at these points is

$$P_{(4)}|_{\phi^c} = \sqrt{\frac{3\lambda}{2}} \sin\theta = \frac{1}{2 \text{Im}w_1^c}. \quad (3.6.5)$$

Writing these in terms of real scalars, we again see that for a given θ , the values of all the scalars, including the dilaton σ , at the critical point are fixed. The only exception is that the term $[(\varphi_c^2)^2 - (\varphi_c^3)^2]$ is fixed whereas φ_c^2 and φ_c^3 are not, individually.

The mass matrix for this potential evaluated at the family of critical points is

$$\frac{\partial_{\bar{I}} \partial^{\bar{J}} P_{(4)}}{P_{(4)}}|_{\phi^c} = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & \frac{\text{Im}w_2^2}{4} & \frac{1}{4} \text{Im}w_2 \text{Im}w_3 & 0 \\ 0 & -\frac{1}{4} \text{Im}w_2 \text{Im}w_3 & -\frac{\text{Im}w_3^2}{4} & 0 \\ 0 & 0 & 0 & \mathbb{1}_{n-3} \end{pmatrix} \quad (3.6.6)$$

which has eigenvalues $(2, \underbrace{1, \dots, 1}_{(n-2) \text{ times}}, 0)$ and hence the extrema correspond to stable de Sitter vacua with a flat direction due to the remaining $SO(1, 1)$ symmetry.

3.6.2 $(SO(1, 1) \times \mathbb{R}^{(1,1)}) \times U(1)_R$ gauging with hyps

The authors of [67] studied a model with 5 vector multiplets and 4 hypermultiplets and the scalars of the hypermultiplets spanned the hyperbolic space $\frac{SO(4,2)}{SO(4) \times SO(2)}$ and the gauge group was $SO(2, 1) \times SU(2)$. Here we shall consider the coupling of a single hypermultiplet to supergravity and arbitrary number n of vector multiplets. We use the same symmetric space $\mathcal{M}_Q = \frac{SU(2,1)}{SU(2) \times U(1)}$ formalism for the scalar manifold of a single hypermultiplet that we also studied on five dimensions in chapter 2. The scalars that span this space are $q^X = (V, \theta, \tau, \sigma)$. Gauging $(SO(1, 1) \times \mathbb{R}^{(1,1)}) \times U(1)_R$ gives three contributions to the scalar potential.

$P_{(4)}^{(T)}$ is not affected by the hyper coupling so we take it as given in (3.5.9). Mean-

while, gauging hyper isometries introduces the potential term (A.26) which is written in the canonical form as[138, 148]

$$P_{(4)}^{(H)} = 4 e^{\mathcal{K}} (K_A^X \tilde{X}^A) g_{XY} (K_B^Y \tilde{X}^B) \quad (3.6.7)$$

with $K_A^X = V_A Y^a T_a^X$ ($a = 1, 2, 3$), where V_A determine the linear combination of vectors to use as the $U(1)_R$ gauge field. Y^a , on the other hand, determine the linear combination of the hyper-isometries T_a^X that are gauged. T_a^X , the Killing vectors that generate the symmetries of the isometry group $SU(2, 1)$ are given in appendix B. At the base point $q^c = (V = 1, \theta = \tau = \sigma = 0)$ of the hyperscalar manifold, where the hyperspace metric g_{XY} (2.1.9) becomes diagonal, this potential satisfies

$$P_{(4)}^{(H)}|_{q^c} = \frac{\partial P_{(4)}^{(H)}}{\partial w_{\tilde{I}}} |_{q^c} = \frac{\partial P_{(4)}^{(H)}}{\partial q} |_{q^c} = 0 \quad (3.6.8)$$

because of the vanishing Killing vectors at that point. The third contribution is the $U(1)_R$ potential

$$P_{(4)}^{(R)} = (U^{(AB)} - 3\tilde{L}^{(A}\tilde{L}^{B)})\vec{\mathcal{P}}_A\vec{\mathcal{P}}_B \quad (3.6.9)$$

where the momentum map is written in terms of the Killing prepotentials as $\vec{\mathcal{P}}_A = V_A Y^a \vec{P}_a$. We choose \tilde{A}_μ^n as the $U(1)_R$ gauge field and set $V_n = 1$. The dRW-rotation matrix that mixes the electric and the magnetic components of the holomorphic section is given in (3.5.11).

The total potential

$$\begin{aligned} P_{(4)}|_{\text{Re}(w_i)=0, q^c} &= \left[P_{(4)}^{(T)} + \lambda (P_{(4)}^{(R)} + P_{(4)}^{(H)}) \right]_{\text{Re}(w_k)=0, q^c} \\ &= \frac{(\text{Im}w_2^2 - \text{Im}w_3^2) (4 + \|\text{Im}w\|^2)^2}{64 \text{Im}w_1 \|\text{Im}w\|^4} + \frac{\lambda |\cos\theta + w_1 \sin\theta|^2 (Y^a Y^a)}{8 \text{Im}w_1} \end{aligned}$$

has extrema at

$$\begin{aligned} \phi^c = \left\{ w_1^c = -\cot\theta + \frac{i\sqrt{2} \csc\theta}{\sqrt{\lambda(Y^a Y^a)}}, \quad (\text{Im}w_2^c)^2 - (\text{Im}w_3^c)^2 = 4, \right. \\ \left. \text{Re}w_k^c = 0, \quad \text{Im}w_i^c = 0, \quad V^c = 1, \quad \theta^c = \tau^c = \sigma^c = 0 \right\}, \end{aligned}$$

where it takes the value

$$P_{(4)}|_{\phi^c} = \frac{\sqrt{\lambda(Y^a Y^a)}}{2\sqrt{2}} \sin\theta = \frac{1}{2 \text{Im}w_1^c}.$$

The values of all the scalars at the critical point are fixed, except w_2 and w_3 , which satisfy

Observe that, when $\theta = \pi/2$, apart from the $X^0 \& F_1$ and $X^1 \& F_0$ mixing in the transformation (3.3.21), this corresponds to the GMZ holomorphic section we introduced in subsection 3.3.1. Although this seems like just a specific case, it will play an important role in finding stable de Sitter vacua when we study general homogeneous scalar manifolds below.

3.6.4 de Sitter Vacua from GMZ Holomorphic Section

The procedure of obtaining de Sitter ground states using the GMZ holomorphic section, which is obtained by acting on Ω_0 by the transformation matrix (3.3.6), involves a similar group contraction. Consider five dimensional $SO(1, 1)$ gauged YMESGT coupled to $\tilde{n} = 3$ vector multiplets that has A_μ^1 as the gauge field. The vectors A_μ^2 and A_μ^3 are charged under the gauge group and need to be dualized to tensors. After the dimensional reduction this becomes a four dimensional theory coupled to $n = 4$ vector multiplets with a gauge group $K_{(4)} = SO(1, 1) \times \mathbb{R}^{(1,1)}$ with central charge[121]. The structure constants are

$$f_{23}^0 = -\sqrt{2}, \quad f_{13}^2 = f_{12}^3 = \frac{1}{\sqrt{2}} \quad (3.6.13)$$

which are antisymmetric in the lower indices. With these structure constants one can calculate the Killing vectors (3.3.7) that generate $K_{(4)}$ as

$$\vec{K}_0 = 0, \quad \vec{K}_1 = \begin{pmatrix} 0 \\ w_3/\sqrt{2} \\ w_2/\sqrt{2} \\ 0 \end{pmatrix}, \quad \vec{K}_2 = \begin{pmatrix} 0 \\ -1 \\ 0 \\ 0 \end{pmatrix}, \quad \vec{K}_3 = \begin{pmatrix} 0 \\ 0 \\ -1 \\ 0 \end{pmatrix}. \quad (3.6.14)$$

The contraction will be done by going to a basis with the following rotation of the Killing vectors:

$$\vec{K}'_0 = \vec{K}_0 - \vec{K}_1, \quad \vec{K}'_1 = \vec{K}_0 + \vec{K}_1, \quad \vec{K}'_2 = \vec{K}_2, \quad \vec{K}'_3 = \vec{K}_3. \quad (3.6.15)$$

It is straightforward to show that the new Killing vectors generate $SO(1, 1) \times \mathbb{R}^{(1,1)}$ without central charge. After some calculation we found that the potential $P^{(T)}$, defined in (3.4.1), is indeed equal to the $P^{(T)}$ given in (3.6.11).

In addition to $K_{(4)}$, one can also gauge the $U(1)_R$ symmetry. Choosing the gauge field as A_μ^4 , this will result in a potential term $P_{(4)}^{(R)} = 1/(2 \text{Im}w_1)$ that is the $P_{(4)}^{(R)}$ given in (3.6.11) when $\theta = \pi/2$. The calculation in the last subsection shows that the total potential $P_{(4)} = P_{(4)}^{(T)} + \lambda P_{(4)}^{(R)}$ has de Sitter minima with a flat direction.

3.6.5 $SO(1, 2)$ Gauging from 5 Dimensions

One can start with a gauged YMESGT in five dimensions with a isometry gauging group $K_{(5)} = SO(1, 2)$. For the generic Jordan Family, the only charged vector fields are the gauge fields $A_{\hat{\mu}}^2, A_{\hat{\mu}}^3$ and $A_{\hat{\mu}}^4$ which transform under the adjoint representation of this $SO(1, 2)$. There are no tensor fields and no scalar potential is introduced.

After the dimensional reduction, the gauge group is still $SO(1, 2)$ but this is a different subgroup of the global isometry group in four dimensions than what we gauged in section 3.4. The former one is a subgroup of $SO(1, 1) \times SO(1, n - 2) \subset SO(2, n - 1)$ and has one timelike and two spacelike dimensions whereas the latter is a subgroup of $SO(2, 1) \times SO(n - 2) \subset SO(2, n - 1)$ and has two timelike and one spacelike dimensions. In contrast to the case before the dimensional reduction, gauging $SO(1, 2)$ results in a scalar potential in four dimensions due to the second term in (A.25). Taking the structure constants as $f_{34}^2 = -f_{42}^3 = -f_{23}^4 = 1$ this potential is evaluated to be

$$P_{(4)}^{(T)} = \frac{Q_{23} + Q_{24} - Q_{34}}{2|\text{Im}w_1||\text{Im}w|^4} \quad (3.6.16)$$

with $Q_{kl} = (w_k \bar{w}_l - w_l \bar{w}_k)^2$. Unfortunately, this potential does not admit any ground states other than Minkowskian. One can gauge $U(1)_R$ (in four dimensions) in addition to the $SO(1, 2)$ symmetry which adds the term (3.5.12) to the potential. But it is easy to verify that the total potential does not have any critical points in this case.

At this point, perhaps it is worth mentioning that the four dimensional theories that have different holomorphic sections as their starting points, which are related by just a symplectic transformation, describe different physics. For the generic Jordan family, an $K_{(5)} = SO(1, 2) \times U(1)_R$ gauged YMESGT has Minkowski and Anti-de Sitter ground states in five dimensions. The Minkowski ground states survive in four dimensions if one works with the GMZ holomorphic section, due to a term in the $U(1)_R$ potential that doesn't exist in the potential that is derived from our holomorphic section. We stressed this issue below equation (3.4.10).

3.7 Beyond Generic Jordan Family

For the holomorphic section we obtained by the rotation (3.3.21), satisfying the equation (3.3.20) was crucial to keep the translations block diagonal. This equation is trivially satisfied for the generic Jordan family ($C_{iMN} = 0$) but for other types of scalar manifolds, such as the magical Jordan family, it does not hold in general. This problem can be evaded by dRW-rotating all of \tilde{X}^i and \tilde{F}_i by $\pi/2$ radians. The entire symplectic transformation

matrix, including the dRW-rotation with $\theta = \pi/2$,

$$\check{\mathcal{S}} = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & D^{MN} \\ 0 & 0 & \delta^j_i & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & D_{MN} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \delta_j^i & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 \end{pmatrix} \quad (3.7.1)$$

acts on Ω_0 in the following way

$$\check{\Omega} = \begin{pmatrix} \check{X}^0 \\ \check{X}^M \\ \check{X}^j \\ \check{X}^1 \\ \check{F}_0 \\ \check{F}_M \\ \check{F}_j \\ \check{F}_1 \end{pmatrix} = \check{\mathcal{S}}\Omega_0 = \check{\mathcal{S}} \begin{pmatrix} X^0 \\ X^1 \\ X^i \\ X^N \\ F_0 \\ F_1 \\ F_i \\ F_N \end{pmatrix}.$$

Here $D_{MN} = -\sqrt{2}\Omega_{MN}$ and $D_{MN}D^{NP} = \delta_M^P$ and again, we decomposed the index I as $I = (1, i)$.

Furthermore, in order to gauge $\check{K} \equiv K_{(4)} \times U(1)_R = (SO(1,1) \ltimes \mathbb{R}^{(1,1)}) \times U(1)_R$ which was the four dimensional gauge group for the theories with de Sitter solutions that originate from five dimensions, the isometry group needs to contain a subgroup $SO(2, r-1)$ with $r \geq 3$.

So far, we studied *symmetric* space scalar manifolds only. Now we relax this restriction and look for *homogeneous* (but not necessarily symmetric) space scalar manifolds that have $SO(2, r-1)$, $r \geq 3$ as a subsector. We have to re-analyze how the holomorphic section transforms under the translations $z^M \rightarrow z^M + b^M$ because C_{IJM} does not necessarily vanish

in homogeneous spaces:

$$\check{\Omega} \rightarrow \check{\Omega} + \begin{pmatrix} \frac{1}{\sqrt{2}}b^M D_{MN}\check{X}^N \\ \sqrt{\frac{2}{3}}D^{MN}b^P \{C_{1NP}(\check{X}^1 - \check{X}^0) + \sqrt{2}C_{iNP}\check{X}^i\} \\ 0 \\ \frac{1}{\sqrt{2}}b^M D_{MN}\check{X}^N \\ -\sqrt{\frac{2}{3}}b^M \left\{ C_{1MN}D^{NP}\check{F}_P + C_{11M}\frac{\check{X}^0 - \check{X}^1}{\sqrt{2}} - C_{1jM}\check{X}^j \right\} \\ \frac{1}{\sqrt{2}}D_{MN}b^N (\check{F}_0 + \check{F}_1) \\ -\sqrt{\frac{2}{3}}b^M \left\{ C_{iMN}D^{NP}\check{F}_P + C_{i1M}\frac{\check{X}^0 - \check{X}^1}{\sqrt{2}} - C_{ijM}\check{X}^j \right\} \\ -\sqrt{\frac{2}{3}}b^M \left\{ C_{1MN}D^{NP}\check{F}_P + C_{11M}\frac{\check{X}^0 - \check{X}^1}{\sqrt{2}} - C_{1jM}\check{X}^j \right\} \end{pmatrix}$$

Observe that in order to keep the translations block diagonal, i.e. to have \check{X}^A and \check{F}_A transform among themselves,

$$C_{IJM} \stackrel{!}{=} 0 \quad (3.7.2)$$

must hold.

de Wit and Van Proeyen classified homogeneous very special manifolds and gave their corresponding cubic polynomials in [149, 150]. These spaces are of the form G/H where G is the isometry group and H is its isotropy subgroup. G is not necessarily semi-simple, thus not all the homogeneous spaces have a clear name. In their classification, the homogeneous spaces are denoted as $L(q, P)$. Here, q characterizes the real Clifford algebras ($\mathcal{C}(q+1, 0)$) that are in one-to-one correspondence with homogeneous special manifolds. These have signatures $(q+1, 1)$ for real (in five dimensions), $(q+2, 2)$ for Kähler (in four dimensions) manifolds, which are related to each other with what is called **r**-map. The non-negative integer P denotes the multiplicity of the representation of the Clifford algebra. For $q \neq 4m$ (m is a non-negative integer), P is unique. When $q = 4m$, there are two inequivalent representations. In this case the homogeneous space is denoted by $L(4m, P, \dot{P})$. Note that $L(4m, P, 0) = L(4m, 0, P) \equiv L(4m, P)$. Table 3.7 lists the special cases where $L(q, P)$ are symmetric manifolds. The cubic polynomial that has an invariance group that acts transitively on the special real manifolds can be specified in the general form

$$N(h) = C_{\check{I}\check{J}\check{K}} h^{\check{I}} h^{\check{J}} h^{\check{K}} = 3 \left\{ \hat{h}^1 (\hat{h}^2)^2 - \hat{h}^1 (\hat{h}^\beta)^2 - \hat{h}^2 (\hat{h}^m)^2 + \gamma_{\beta mn} \hat{h}^\beta \hat{h}^m \hat{h}^n \right\}, \quad (3.7.3)$$

where the index $\check{I} = 1, \dots, n$ is decomposed into $I = 1, 2, \beta, m$, with $\beta = 3, \dots, (q+3)$ and $m = (q+4), \dots, (q+3 + (P + \dot{P})\mathcal{D}_{q+1} = n)$. The dimension \mathcal{D}_{q+1} of the irreducible representation of the Clifford algebra with positive signature in $q+1$ dimensions is given by

$$\begin{aligned} \mathcal{D}_{q+1} = 1 \text{ for } q = -1, 0, & \quad \mathcal{D}_{q+1} = 2 \text{ for } q = 1, & \quad \mathcal{D}_{q+1} = 4 \text{ for } q = 2, \\ \mathcal{D}_{q+1} = 8 \text{ for } q = 3, 4, & \quad \mathcal{D}_{q+1} = 16 \text{ for } q = 5, 6, 7, 8, & \quad \mathcal{D}_{q+8} = 16\mathcal{D}_q. \end{aligned}$$

$L(q, P)$	n	real	Kähler
$L(-1, 0)$	2	$SO(1, 1)$	$\left[\frac{SU(1,1)}{U(1)}\right]^2$
$L(-1, P)$	$2 + P$	$\frac{SO(P+1,1)}{SO(P+1)}$	
$L(0, P)$	$3 + P$	$SO(1, 1) \times \frac{SO(P+1,1)}{SO(P+1)}$	$\frac{SU(1,1)}{U(1)} \times \frac{SO(P+2,2)}{SO(P+2) \times SO(2)}$
$L(1, 1)$	6	$\frac{Sl(3, R)}{SO(3)}$	$\frac{Sp(6)}{U(3)}$
$L(2, 1)$	9	$\frac{Sl(3, C)}{SU(3)}$	$\frac{SU(3,3)}{SU(3) \times SU(3) \times U(1)}$
$L(4, 1)$	15	$\frac{SU^*(6)}{Sp(3)}$	$\frac{SO^*(12)}{SU(6) \times U(1)}$
$L(8, 1)$	27	$\frac{E_6}{F_4}$	$\frac{E_7}{E_6 \times U(1)}$

Table 3.1. *Symmetric very special manifolds.* $L(-1, P)$, which correspond to the generic non-Jordan Family, are symmetric in five dimensions, but not their images under the \mathbf{r} -map. $L(0, P)$ is the generic Jordan family and the last 4 entries are the magical Jordan family manifolds. The number n is the complex dimension of the Kähler space, which also is the number of vector multiplets in 4 dimensions. Table is adapted from [149].

The constraint $r \geq 3$ translates into $q \geq 0$. Hence we immediately see that \check{K} is not gaugable for generic non-Jordan family $L(-1, P)$. Let us investigate the cases ($q = 0$) and ($q > 0$) separately.

Case 1: ($q = 0$)

If either of P or \dot{P} vanishes the homogeneous space corresponds to the symmetric generic Jordan family, which we have studied already. For non-vanishing P and \dot{P} one can write the cubic polynomial as

$$N(h) = 3 \{h^1 [(h^2)^2 - (h^3)^2 - (h^x)^2] - (h^2 - h^3)(h^{\dot{x}})^2\} \quad (3.7.4)$$

after the reparametrization

$$\hat{h}^1 = h^2 + h^3, \quad \hat{h}^2 = \frac{h^1 + h^2 - h^3}{2}, \quad \hat{h}^3 = \frac{-h^1 + h^2 - h^3}{2}, \quad \hat{h}^x = h^x, \quad \hat{h}^{\dot{x}} = h^{\dot{x}}$$

where the index m is decomposed into P indices x and \dot{P} indices \dot{x} . The fields h^2 and h^3 are charged under the gauge group $K_{(4)}$ and the corresponding vector fields A_μ^2 and A_μ^3 need to be dualized to tensor fields. Hence the index $\tilde{I} = (I, M)$ is split as follows: $I = 1, x, \dot{x}$; $M = 2, 3$. But then $C_{\dot{x}yM} \neq 0$ and hence the translations will not remain block diagonal, i.e. $K_{(4)}$ is not gaugable in the standard way.

Case 2: ($q > 0$)

All of these spaces $L(q > 0, P, \dot{P})$, which also include the symmetric magical Jordan family for $(q = 1, 2, 4, 8; P = 1)$, contain $K_{(4)} = SO(1, 1) \times \mathbb{R}^{(1,1)}$ subsectors. Consider the cubic form in the most general form as given in (3.7.3). Choosing A_μ^1 as the gauge field one can find a $K_{(4)}$ -generator such that \hat{h}^2 and \hat{h}^3 rotate into each other keeping $(\hat{h}^2)^2 - (\hat{h}^3)^2$ fixed. Because they are charged under the gauge group the corresponding vector fields need to be dualized to tensor fields. The rest of \hat{h}^β are $K_{(4)}$ -singlets and a linear combination of the corresponding vector fields can be used as the $U(1)_R$ (or even $SU(2)_R$ if $q \geq 3$) gauge field(s). h^m form $(P + \dot{P})\mathcal{D}_{q+1}/2$ doublets under $K_{(4)}$ and their corresponding vector fields are dualized to tensor fields. All the conditions are satisfied and we conclude that the homogeneous spaces of the type $L(q > 0, P)$ admit stable de Sitter vacua when \check{K} is gauged.

Summary and Proposals

4.1 5 Dimensions

In chapter 2, after reviewing the ground state solutions of the $5D, \mathcal{N} = 2$ supergravity theories with symmetric scalar manifolds that had been discovered earlier, we studied the vacua of the gauged $5D, \mathcal{N} = 2$ supergravity theories that had not been discussed in the literature. Consistent with earlier results, in the absence of hypers, we showed that all the generic Jordan family, the magical Jordan family and the generic non-Jordan family theories admit stable Anti-de Sitter vacua, whereas only the theories of the first two families admit stable de Sitter vacua and all the above families have unstable de Sitter and Anti-de Sitter ground states.

For the generic Jordan family, the only gauge groups $K_{(5)}$ that lead to the introduction of tensor fields are the Abelian groups $SO(2)$ and $SO(1, 1)$. The former leads to supersymmetric Minkowski ground states only, unless accompanied by a simultaneous $U(1)_R$ gauging. With $U(1)_R$ gauging one can obtain nonsupersymmetric Minkowski ground states, and moreover, there are supersymmetric and nonsupersymmetric Anti-de Sitter critical points resulting from the combined scalar potential of the $SO(2)$ and $U(1)_R$ gaugings. The $SO(1, 1)$ gauging, on the other hand, breaks the supersymmetry and leads to stable de Sitter vacua by a simultaneous R -symmetry ($SU(2)_R$ or $U(1)_R$) gauging. Pure $SO(1, 1)$ gauging does not lead to any critical points. It is interesting to observe that whereas the $SO(1, 1) \times SU(2)_R$ gauging has stable de Sitter vacua, its compact counterpart, namely $SO(2) \times SU(2)_R$ gauging admits Minkowski vacua only. We also note that some of the stable de Sitter models we studied; such as the one with $SO(1, 1) \times SU(2)_R$ gauging and no hypermultiplets, or the one with $SO(2) \times SO(1, 1)_H$ gauging, which has one hypermultiplet; have critical points, for which the Hessians of the potential evaluated at these points have zero eigenvalues. This is related to the fact that the potential has a family of critical points rather than an isolated critical point and therefore it has flat direction(s) at these points.

We showed that it is possible to embed certain generic Jordan family models into the

magical Jordan family theories, provided that there are sufficiently many vector fields in the magical theory to do the respective gauging. However, in some cases the stability puts additional constraints on the gauge parameters. In these models, we encountered other critical points than the ones obtained in the generic case, such as de Sitter and Anti-de Sitter saddle points and curves. These are special to the magical Jordan family. Although we found numerous critical points of these models, we could not do a complete analysis due to the complexity of the magical Jordan family theories. In addition to this, coupling a hypermultiplet and gauging a subgroup of its scalar manifold might lead to nontrivial critical points that are beyond those found in this work. These are left as open questions for future investigation.

Other than the embeddings of the generic Jordan family cases, one can gauge non-Abelian subgroups of the isometry groups of the magical theories and dualize non-trivially charged vector fields to tensor fields which yields additional contributions to the scalar potential. For the J_3^C case, the compact $SO(3) \times U(1)$ gauging leads to a Minkowski vacuum. A simultaneous $SU(2)_R$ gauging leads to a theory with no critical points whereas a simultaneous $U(1)_R$ gauging has an Anti-de Sitter solution. On the other hand, the non-compact non-Abelian $SO(2, 1) \times U(1)$ gauging only leads to a Minkowski ground state and adding a simultaneous R -symmetry gauging results in a theory with no ground states.

For the generic non-Jordan family, the model with the full R -symmetry gauged does not have any critical points, even after adding tensor coupling. The pure $U(1)_R$ gauging leads to Minkowski and Anti-de Sitter ground states. Tensor coupling to these models can only be achieved by gauging the compact $SO(2)$. A simultaneous $SO(2) \times U(1)_R$ gauging results in Minkowski, Anti-de Sitter and de Sitter ground states. The de Sitter solutions are found to be unstable whereas the Anti-de Sitter solutions can be made stable by proper choices of the parameters V_I that define the linear combination of the vector fields that is used as the $U(1)_R$ gauge field.

We also added a universal hypermultiplet to the models we considered and investigated the potentials coming from the gauging of the hyper isometries. For the generic Jordan family, we showed that a simultaneous compact $U(1)_H$ gauging does not change the sign of the potential at the existing critical points of the models that the hypermultiplet is added to, but a non-compact $SO(1, 1)_H$ gauging generally leads to de Sitter vacua. It is interesting to see that the $SO(1, 1)$ gaugings of both real and hyperscalar isometries help finding de Sitter ground states. This result is not limited to the generic Jordan family and applies to the other families.

4.2 4 Dimensions

Stable de Sitter vacua of $4D, \mathcal{N} = 2$ YMESGT's were found in [67, 68]. The main goal of chapter 3 was to relate these four dimensional theories to the theories in five dimensions with various gaugings. The authors of these papers asserted that three ingredients are necessary to obtain de Sitter vacua:

- non-compact gauge groups,
- Fayet-Iliopoulos (FI) terms,
- de Roo-Wagemans (dRW) rotation.

The non-compact gauge group they used in the three models they studied is $SO(2, 1)$. We showed that this is *not* the only gauge group that admits a potential that one needs to obtain de Sitter vacua. One can indeed contract this group to $SO(1, 1) \ltimes \mathbb{R}^{(1,1)}$ and de Sitter vacua is preserved under this contraction. We need to emphasize that whereas the $SO(2, 1)$ gauged theories do not directly descend from five dimensions, their contracted counterparts do. FI terms are available for gauge groups that have $U(1)$ or $SU(2)$ factors. The variation of such terms in the Lagrangian is a total derivative and they yield supersymmetric terms in the action. [143, 144] point out that adding FI terms to the Lagrangian is indeed equivalent to gauging R -symmetry. In the three models they studied; Fre, Trigiante and van Proeyen considered $\mathcal{N} = 2$ supergravity with a complex scalar manifold of the form $\mathcal{M}_{VS}^4 = \mathcal{ST}[2, n - 1] = \frac{SU(1,1)}{U(1)} \times \frac{SO(2, n-1)}{SO(2) \times SO(n-1)}$, parametrized by Calabi-Vesentini coordinates. These correspond to symmetric generic Jordan family which describes the geometry of a real manifold of the form $\mathcal{M}_{VS}^5 = \frac{SO(\tilde{n}-1, 1) \times SO(1, 1)}{SO(\tilde{n}-1)}$ in five dimensions. Here n and \tilde{n} denote the number of vector multiplets coupled to supergravity in four and five dimensions, respectively and they are related by $n = \tilde{n} + 1$. For such theories one has a certain amount of freedom to choose a holomorphic (symplectic) section upon dimensional reduction. This freedom is parametrized by dRW-angles θ . Different choices of θ yield different gauged models with different physics. We use the notation of [127, 121] to parametrize the complex manifold instead of Calabi-Vesentini coordinates for two main reasons. First, in the former parametrization the five dimensional connection is as clear as it could be, as the complex scalar fields are obtained directly from dimensional reduction and second, generalizing the results to homogeneous manifolds is significantly easier. The mapping between two parametrizations can be found in appendix C.

As we stressed earlier, stable de Sitter vacua exist in five dimensional $SO(1, 1) \times R_s$ gauged YMESGT's where the R_s denotes a subgroup of the full R -symmetry group $SU(2)_R$ [119, 120, 122]. These theories descend to four dimensional theories that have the gauge group $(SO(1, 1) \ltimes \mathbb{R}^{(1,1)}) \times R_s$ with a central charge [121]. The procedure in establishing de Sitter ground states in four dimensions from these theories include finding an appropriate holomorphic section by means of a dRW-rotation, and contracting the gauge group. The contraction rotates some of the group generators into each other, eliminates the central charge and gives a positive definite contribution to the potential. Without this contribution the potential does not have any ground states and that makes the group contraction essential. We showed that these theories can also be obtained from four dimensional $SO(2, 1) \times R_s$ gauged YMESGT's, which were considered in [67, 68] to have stable de Sitter vacua, by means of a different contraction.

In analogy to five dimensions, the theories with generic Jordan family scalar manifolds have stable de Sitter vacua for $(SO(1, 1) \ltimes \mathbb{R}^{(1,1)}) \times R_s$ gaugings. R_s can be either $U(1)_R$

or $SU(2)_R$. In either case, the de Sitter minima in four dimensions we found has a flat direction. Recall that this was also the case in five dimensions before the dimensional reduction. In addition to vector/tensor multiplets, one can couple a universal hypermultiplet and simultaneously gauge $U(1)$ or $SU(2)$ symmetry of its quaternionic hyperscalar manifold. We showed that, again in analogy to five dimensions, this type of extra gauging preserves the nature of the de Sitter ground states. The theories with non-compact hyper isometry gauging, which lead to stable de Sitter ground states in five dimensions, still need to be checked in four dimensions to complete the analogy. This topic is not covered in this thesis and we leave it for future investigation.

Same results can be achieved by starting either with the Günaydin-McReynolds-Zagernmann (GMZ) symplectic section [121] or with the symplectic section we introduced in (3.3.22) which has a closer connection to the Calabi-Vesentini basis used in [67, 68]. While in either case a gauge group contraction that rotates some of the generators into each other and eliminates the central charge is essential, it should be noted that one does not need an extra dRW-rotation for the GMZ symplectic section, because it is already “dRW-rotated” by $\theta = \pi/2$ radians with respect to our symplectic section.

4.3 Extension to Homogeneous Spaces

In four dimensions, general homogeneous (but not necessarily symmetric) scalar manifolds $L(q, P)$ admit de Sitter vacua provided that they contain a $(SO(1, 1) \times \mathbb{R}^{(1,1)}) \times R_s$ subsector. These spaces are limited to $L(q \geq 0, P)$. For the symmetric generic Jordan family spaces $L(0, P)$, one has the freedom to choose the dRW-angle from $0 < \theta < \pi$. This choice affects the values of the scalar fields and the value of the potential at the de Sitter minima. For the spaces of type $L(q > 0, P)$, on the other hand, the value of the dRW-angle has to be fixed to $\theta = \pi/2$ because otherwise the translational variations of the holomorphic section do not become block diagonal and one cannot gauge the theory in the standard way. Observe that the GMZ symplectic section carries this rotation to begin with.

The spaces of the type $L(q \geq 0, P)$ have de Sitter minima but one can analyze them for other ground states. However, the analysis of extrema of the homogeneous spaces in their full generality is involved and requires a separate study.

4.4 Embedding in String Theory?

Having found the recipe that starts with five dimensional $SO(1, 1) \times U(1)_R$ gauged $\mathcal{N} = 2$ YMESGT and ends with stable de Sitter vacua in four dimensions, one can ask the question: Is it possible to embed the theory into a fundamental superstring theory or M-theory? There are several directions one can take to answer this question. Compactifications of Type IIA and type IIB superstring theories on Calabi-Yau threefolds yield ungauged supergravity theories in four dimensions. Using the same method, it was shown in [100] that $5D, \mathcal{N} = 2$ MESGT coupled to hypers can be obtained by compactifying 11 dimensional supergravity.

In particular, the Hodge number $h_{(1,1)}$ of the threefold corresponds to the number of vector fields (including graviphoton) in the resulting $5D, \mathcal{N} = 2$ MESGT, whereas $h_{(2,1)} + 1$ corresponds to the number of hypermultiplets. Type IIA or Type IIB supergravity in ten dimensions compactified on T^6 results in $N = 8$ supergravity in four dimensions. Similarly, $5D, \mathcal{N} = 8$ supergravity can be obtained by compactifying eleven dimensional supergravity on T^6 . By orbifolding (modding out by discrete groups) the four dimensional theory, Sen and Vafa considered examples of models with broken supersymmetries [151]. In one of the several models the scalar manifold belongs to the generic Jordan family. As was pointed out in [152], another model they considered is the J_3^{H} of the magical Jordan family. These results can be extended to the $11D$ -to- $5D$ compactifications. However these are ungauged theories. Whether one can obtain gauged versions of these theories by turning on fluxes is an open problem to be investigated.

Meanwhile, after solving the stabilization problem of compactification of internal dimensions [153, 71], it was possible to find de Sitter vacua from string theory. This moduli stabilization fixes the runaway behavior of the axion-dilaton fields, which was also a problem we encountered upon dimensional reduction in the beginning of chapter 3. In the original KKLT scenario, the moduli stabilization brings the minimum of the scalar potential to a finite negative value. Then the addition of an anti- $D3$ brane lifts this minimum to a state with positive vacuum energy. In our construction, on the other hand, a similar effect was established through dRW-rotation, and gauging the non-compact $SO(1,1)$ subgroup of the global isometry group of the scalar manifold simultaneously with a subgroup of the R -symmetry group. Finding a relation between our and a KKLT-like scenario is an interesting problem and we leave this for a future study.

The “Very Special Geometry”, The Lagrangians in 5 and 4 Dimensions and The Potentials

The bosonic sector of the $5D, \mathcal{N} = 2$ gauged Yang-Mills-Einstein supergravity¹ coupled to tensor- and hypermultiplets is described by the Lagrangian (with metric signature $(-+++)$) [113, 154, 117]

$$\begin{aligned} \hat{e}^{-1} \mathcal{L}^{(5)} = & -\frac{1}{2} \hat{R} - \frac{1}{4} \hat{a}_{\tilde{I}\tilde{J}} \mathcal{H}_{\hat{\mu}\hat{\nu}}^{\tilde{I}} \mathcal{H}^{\tilde{J}\hat{\mu}\hat{\nu}} - \frac{1}{2} g_{XY} \mathcal{D}_{\hat{\mu}} q^X \mathcal{D}^{\hat{\mu}} q^Y \\ & - \frac{1}{2} g_{\tilde{x}\tilde{y}} \mathcal{D}_{\hat{\mu}} \varphi^{\tilde{x}} \mathcal{D}^{\hat{\mu}} \varphi^{\tilde{y}} + \frac{\hat{e}^{-1}}{6\sqrt{6}} C_{IJK} \epsilon^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}\hat{\tau}} F_{\hat{\mu}\hat{\nu}}^I F_{\hat{\rho}\hat{\sigma}}^J A_{\hat{\tau}}^K \\ & + \frac{\hat{e}^{-1}}{4g} \epsilon^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}\hat{\tau}} \Omega_{MN} B_{\hat{\mu}\hat{\nu}}^M \mathcal{D}_{\hat{\rho}} B_{\hat{\sigma}\hat{\tau}}^N - P_{(5)}(\varphi, q). \end{aligned} \quad (\text{A.1})$$

Here, non-Abelian field strengths $\mathcal{F}_{\hat{\mu}\hat{\nu}}^I \equiv F_{\hat{\mu}\hat{\nu}}^I + g f_{JK}^I A_{\hat{\mu}}^J A_{\hat{\nu}}^K$ ($I = 0, 1, \dots, n_V$) of the gauge group $K_{(5)}$ and the self-dual tensor fields $B_{\hat{\mu}\hat{\nu}}^M$ ($M = 1, 2, \dots, 2n_T$) are grouped together to define the tensorial quantity $\mathcal{H}_{\hat{\mu}\hat{\nu}}^{\tilde{I}} \equiv (\mathcal{F}_{\hat{\mu}\hat{\nu}}^I, B_{\hat{\mu}\hat{\nu}}^M)$ with $\tilde{I} = 0, 1, \dots, n_V + 2n_T$. The potential term $P_{(5)}(\varphi, q)$ is given by

$$P_{(5)}(\varphi, q) = g^2 (P_{(5)}^{(T)}(\varphi) + \lambda P_{(5)}^{(R)}(\varphi, q) + \kappa P_{(5)}^{(H)}(q)) \quad (\text{A.2})$$

where

$$\begin{aligned} P_{(5)}^{(T)} &= 2W_{\tilde{x}} W^{\tilde{x}}, \\ P_{(5)}^{(R)} &= -4\vec{P} \cdot \vec{P} + 2\vec{P}^{\tilde{x}} \cdot \vec{P}_{\tilde{x}}, \\ P_{(5)}^{(H)} &= 2\mathcal{N}_X \mathcal{N}^X \end{aligned} \quad (\text{A.3})$$

¹For the full Lagrangian, see [118, 117]

and $\lambda = g_R^2/g^2$, $\kappa = g_H^2/g^2$. The quantities given in the above expression are defined as

$$\begin{aligned} W_{\tilde{x}} &\equiv -\frac{\sqrt{6}}{8}\Omega^{MN}h_{M\tilde{x}}h_N = \frac{\sqrt{6}}{4}h^I K_I^{\tilde{x}}, \\ \vec{P} &\equiv h^I \vec{P}_I, \\ \vec{P}_{\tilde{x}} &\equiv h_{\tilde{x}}^I \vec{P}_I, \\ \mathcal{N}^X &\equiv \frac{\sqrt{6}}{4}h^I K_I^X, \end{aligned} \tag{A.4}$$

where $K_I^{\tilde{x}}$ and K_I^X are Killing vectors acting on the scalar and the hyperscalar parts of the total scalar manifold $\mathcal{M}_{scalar}^5 = \mathcal{M}_{VS}^5 \otimes \mathcal{M}_Q$; \vec{P}_I are the Killing prepotentials which will be defined below; Ω^{MN} is the inverse of Ω_{MN} , which is the constant invariant anti-symmetric tensor of the gauge group $K_{(5)}$; and h^I and $h_{\tilde{x}}^I$ are elements of the very special manifold \mathcal{M}_{VS}^5 described by the hypersurface

$$N(h) = C_{\tilde{I}\tilde{J}\tilde{K}}h^{\tilde{I}}h^{\tilde{J}}h^{\tilde{K}} = 1, \quad \tilde{I}, \tilde{J}, \tilde{K} = 0, \dots, \tilde{n} \tag{A.5}$$

of the $\tilde{n} + 1$ dimensional space $M = \{h^{\tilde{I}} \in \mathbb{R}^{\tilde{n}+1} | N(h) = C_{\tilde{I}\tilde{J}\tilde{K}}h^{\tilde{I}}h^{\tilde{J}}h^{\tilde{K}} > 0\}$ with metric

$$a_{IJ} = -\frac{1}{3}\partial_I\partial_J\ln N(h). \tag{A.6}$$

The terms $P_{(5)}^{(T)}$ and $P_{(5)}^{(H)}$ are semi-positive definite in the physically relevant region, whereas $P_{(5)}^{(R)}$ can have both signs. \mathcal{M}_{VS}^5 is determined completely by the totally symmetric tensor $C_{\tilde{I}\tilde{J}\tilde{K}}$. The scalar field metric on this hypersurface is the induced metric from the embedding space, which is given by

$$g_{\tilde{x}\tilde{y}} = \frac{3}{2}a_{\tilde{I}\tilde{J}}h_{\tilde{x}}^{\tilde{I}}h_{\tilde{y}}^{\tilde{J}}|_{N=1} = -3C_{\tilde{I}\tilde{J}\tilde{K}}h_{\tilde{x}}^{\tilde{I}}h_{\tilde{y}}^{\tilde{J}}h_{\tilde{y}}^{\tilde{K}}|_{N=1} \tag{A.7}$$

where “ \tilde{x} ” denotes a derivative with respect to $\varphi^{\tilde{x}}$. The definitions

$$\begin{aligned} \overset{\circ}{a}_{\tilde{I}\tilde{J}} &\equiv a_{\tilde{I}\tilde{J}}|_{N=1} = -2C_{\tilde{I}\tilde{J}\tilde{K}}h^{\tilde{K}} + 3h_{\tilde{I}}h_{\tilde{J}}, \\ h_{\tilde{I}} &\equiv C_{\tilde{I}\tilde{J}\tilde{K}}h^{\tilde{J}}h^{\tilde{K}} = \overset{\circ}{a}_{\tilde{I}\tilde{J}}h^{\tilde{J}}, \\ h_{\tilde{x}}^{\tilde{I}} &\equiv -\sqrt{\frac{3}{2}}h_{\tilde{x}}^{\tilde{I}}, \\ h_{\tilde{I}\tilde{x}} &\equiv \overset{\circ}{a}_{\tilde{I}\tilde{J}}h_{\tilde{x}}^{\tilde{J}} = \sqrt{\frac{3}{2}}h_{\tilde{I},\tilde{x}} \end{aligned} \tag{A.8}$$

help us write the algebraic constraints of the very special geometry

$$\begin{aligned} h^{\tilde{I}}h_{\tilde{I}} &= 1, \\ h_{\tilde{x}}^{\tilde{I}}h_{\tilde{I}} &= h_{\tilde{I}\tilde{x}}h^{\tilde{I}} = 0, \\ h_{\tilde{x}}^{\tilde{I}}h_{\tilde{y}}^{\tilde{J}}\overset{\circ}{a}_{\tilde{I}\tilde{J}} &= g_{\tilde{x}\tilde{y}}. \end{aligned} \tag{A.9}$$

There are also differential constraints to be satisfied:

$$\begin{aligned} h_{\tilde{I}\tilde{x};\tilde{y}} &= \sqrt{\frac{2}{3}} \left(g_{\tilde{x}\tilde{y}} h_{\tilde{I}} + T_{\tilde{x}\tilde{y}\tilde{z}} h_{\tilde{I}}^{\tilde{z}} \right), \\ h_{\tilde{x};\tilde{y}}^{\tilde{I}} &= -\sqrt{\frac{2}{3}} \left(g_{\tilde{x}\tilde{y}} h^{\tilde{I}} + T_{\tilde{x}\tilde{y}\tilde{z}} h^{\tilde{I}\tilde{z}} \right) \end{aligned} \quad (\text{A.10})$$

where “;” is the covariant derivative using the Christoffel connection calculated from the metric $g_{\tilde{x}\tilde{y}}$ and

$$T_{\tilde{x}\tilde{y}\tilde{z}} \equiv C_{\tilde{I}\tilde{J}\tilde{K}} h_{\tilde{x}}^{\tilde{I}} h_{\tilde{y}}^{\tilde{J}} h_{\tilde{z}}^{\tilde{K}}. \quad (\text{A.11})$$

Using (A.7),(A.8) and (A.9) one can derive

$$\overset{\circ}{a}_{\tilde{I}\tilde{J}} = h_{\tilde{I}} h_{\tilde{J}} + h_{\tilde{I}}^{\tilde{x}} h_{\tilde{J}\tilde{x}}, \quad (\text{A.12})$$

$$h_{\tilde{I}}^{\tilde{x}} h_{\tilde{J}\tilde{x}} = -2C_{\tilde{I}\tilde{J}\tilde{K}} h^{\tilde{K}} + 2h_{\tilde{I}} h_{\tilde{J}}. \quad (\text{A.13})$$

The indices $\tilde{I}, \tilde{J}, \tilde{K}$ are raised and lowered by $\overset{\circ}{a}_{\tilde{I}\tilde{J}}$ and its inverse $\overset{\circ}{a}^{\tilde{I}\tilde{J}}$. $P_{(5)}^{(T)}$ can now be written in a more compact form

$$\begin{aligned} P_{(5)}^{(T)} &= \frac{3}{8} \Omega^{MN} \Omega^{PR} C_{MRI} h_N h_P h^I \\ &= \frac{3\sqrt{6}}{16} \Lambda_I^{MN} h_M h_N h^I. \end{aligned} \quad (\text{A.14})$$

with Λ_{IN}^M being the transformation matrices of the tensor fields under the gauge group $K_{(5)}$

$$\Lambda_I^{MN} = \Lambda_{IP}^M \Omega^{PN} = \frac{2}{\sqrt{6}} \Omega^{MR} C_{IRP} \Omega^{PN}. \quad (\text{A.15})$$

Gauging the R -symmetry introduces the potential term $P_{(5)}^{(R)} = -4\vec{P} \cdot \vec{P} + 2\vec{P}^{\tilde{x}} \cdot \vec{P}_{\tilde{x}}$, where $\vec{P} = h^I \vec{P}_I$ and $\vec{P}_{\tilde{x}} = h_{\tilde{x}}^I \vec{P}_I$ are vectors that transform under the R -symmetry group that is being gauged. For the $SU(2)_R$ gauging one can take

$$\vec{P}_I = \vec{e}_I$$

where \vec{e}_I satisfy $\vec{e}_i \times \vec{e}_j = d_{ij}^k \vec{e}_k$ and $\vec{e}_i \cdot \vec{e}_j = \delta_{ij}$ when i, j, k are the $SU(2)_R$ adjoint indices (d_{ij}^k are the $SU(2)$ structure constants); and $\vec{e}_I = 0$ otherwise. With this convention and the use of (A.8) and (A.9) the potential term simplifies to

$$P_{(5)}^{(R)} = -4C^{ij\tilde{K}} \delta_{ij} h_{\tilde{K}}. \quad (\text{A.16})$$

If the $U(1)_R$ subgroup of $SU(2)_R$ is being gauged one can take

$$\vec{P}_I = V_I \vec{e},$$

where \vec{e} is an arbitrary vector in the $SU(2)$ space and V_I are some constants that define the linear combination of the vector fields A_{μ}^I that is used as the $U(1)_R$ gauge field

$$A_{\mu}[U(1)_R] = V_I A_{\mu}^I.$$

The potential term then can be written as

$$P_{(5)}^{(R)} = -4C^{IJK}V_IV_Jh_{\tilde{K}}. \quad (\text{A.17})$$

If tensors are coupled to the theory the V_I have to be constrained by

$$V_I f_{JK}^I = 0$$

with f_{JK}^I being the structure constants of $K_{(5)}$. When the target manifold \mathcal{M}_{VS} is associated with a Jordan algebra, the following equality holds componentwise

$$C^{\tilde{I}\tilde{J}\tilde{K}} = C_{\tilde{I}\tilde{J}\tilde{K}} = \text{const.}$$

After the dimensional reduction from 5 to 4, the Lagrangian (A.1) becomes [121]

$$\begin{aligned} e^{-1}\mathcal{L}^{(4)} &= -\frac{1}{2}R - \frac{3}{4}\overset{\circ}{a}_{\tilde{I}\tilde{J}}(\mathcal{D}_{\mu}\tilde{h}^{\tilde{I}})(\mathcal{D}^{\mu}\tilde{h}^{\tilde{J}}) - \frac{1}{2}e^{-2\sigma}\overset{\circ}{a}_{IJ}(\mathcal{D}_{\mu}A^I)(\mathcal{D}^{\mu}A^J) \\ &\quad - \frac{1}{2}e^{-2\sigma}g_{XY}(\mathcal{D}_{\mu}q^X)(\mathcal{D}^{\mu}q^Y) - e^{-2\sigma}\overset{\circ}{a}_{IM}(\mathcal{D}_{\mu}A^I)B^{\mu M} - \frac{1}{2}e^{-2\sigma}\overset{\circ}{a}_{MN}B_{\mu}^MB^{\mu N} \\ &\quad + \frac{e^{-1}}{g}\epsilon^{\mu\nu\rho\sigma}\Omega_{MN}B_{\mu\nu}^M(\partial_{\rho}B_{\sigma}^N + gA_{\rho}^I\Lambda_{IP}^NB_{\sigma}^P) \\ &\quad + \frac{e^{-1}}{g}\epsilon^{\mu\nu\rho\sigma}\Omega_{MN}W_{\mu\nu}B_{\rho}^MB_{\sigma}^N + \frac{e^{-1}}{2\sqrt{6}}C_{MNI}\epsilon^{\mu\nu\rho\sigma}B_{\mu\nu}^MB_{\rho\sigma}^NA^I \\ &\quad - \frac{1}{4}e^{\sigma}\overset{\circ}{a}_{MN}B_{\mu\nu}^MB^{N\mu\nu} - \frac{1}{2}e^{\sigma}\overset{\circ}{a}_{IM}(\mathcal{F}_{\mu\nu}^I + 2W_{\mu\nu}A^I)B^{M\mu\nu} \\ &\quad - \frac{1}{4}e^{\sigma}\overset{\circ}{a}_{IJ}(\mathcal{F}_{\mu\nu}^I + 2W_{\mu\nu}A^I)(\mathcal{F}^{J\mu\nu} + 2W^{\mu\nu}A^J) - \frac{1}{2}e^{3\sigma}W_{\mu\nu}W^{\mu\nu} \\ &\quad + \frac{e^{-1}}{2\sqrt{6}}C_{IJK}\epsilon^{\mu\nu\rho\sigma}\left\{\mathcal{F}_{\mu\nu}^I\mathcal{F}_{\rho\sigma}^JA^K + 2\mathcal{F}_{\mu\nu}^IW_{\rho\sigma}A^JA^K + \frac{4}{3}W_{\mu\nu}W_{\rho\sigma}A^IA^JA^K\right\} \\ &\quad - g^2P_{(4)}, \end{aligned} \quad (\text{A.18})$$

where

$$\tilde{h}^{\tilde{I}} \equiv e^{\sigma}h^{\tilde{I}}, \quad (\text{A.19})$$

$$\mathcal{D}_{\mu}A^I \equiv \partial_{\mu}A^I + gA_{\mu}^Jf_{JK}^IA^K, \quad (\text{A.20})$$

$$\mathcal{F}_{\mu\nu}^I \equiv 2\partial_{[\mu}A_{\nu]}^I + gf_{JK}^IA_{\mu}^JA_{\nu}^K, \quad (\text{A.21})$$

$$\mathcal{D}_{\mu}\tilde{h}^{\tilde{I}} \equiv \partial_{\mu}\tilde{h}^{\tilde{I}} + gA_{\mu}^IM_{\tilde{I}\tilde{K}}^{\tilde{I}}\tilde{h}^{\tilde{K}}, \quad (\text{A.22})$$

$$\mathcal{D}_{\mu}q^X \equiv \partial_{\mu}q^X + g_HA_{\mu}^IK_I^X, \quad (\text{A.23})$$

and the total scalar potential, $P_{(4)}$, is given by

$$P_{(4)} = P_{(4)}^{(T)} + \frac{g_H^2}{g^2} P_{(4)}^{(H)}, \quad (\text{A.24})$$

where

$$P_{(4)}^{(T)} \equiv e^{-\sigma} P_{(5)}^{(T)} + \frac{3}{4} e^{-3\sigma} \tilde{a}_{\tilde{I}\tilde{J}} (A^I M_{(I)\tilde{K}}^{\tilde{I}} h^{\tilde{K}}) (A^J M_{(J)\tilde{L}}^{\tilde{J}} h^{\tilde{L}}) \quad (\text{A.25})$$

and

$$P_{(4)}^{(H)} \equiv e^{-\sigma} P_{(5)}^{(H)} + \frac{1}{2} e^{-3\sigma} (A^I K_I^X) g_{XY} (A^J K_J^Y), \quad (\text{A.26})$$

which would get an additional term of the form

$$P_{(4)}^{(R)} \equiv e^{-\sigma} P_{(5)}^{(R)} (h^{\tilde{I}}) \quad (\text{A.27})$$

if the R -symmetry is being gauged. The transformation matrices $M_{(I)\tilde{K}}^{\tilde{J}}$ that correspond to the gauge group $K_{(5)}$ are decomposed as follows

$$M_{(I)\tilde{K}}^{\tilde{J}} = \begin{pmatrix} f_{IK}^J & 0 \\ 0 & \Lambda_{IM}^N \end{pmatrix}. \quad (\text{A.28})$$

f_{IK}^J are always antisymmetric in the lower two indices.

Appendix B

Killing vectors of the hyper-isometry

The eight Killing vectors k_a^X that generate isometry group $SU(2,1)$ of the hyperscalar manifold are given by [124]

$$\begin{aligned}
\vec{k}_1 &= \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, & \vec{k}_2 &= \begin{pmatrix} 0 \\ 2\theta \\ 0 \\ 1 \end{pmatrix}, & \vec{k}_3 &= \begin{pmatrix} 0 \\ -2\tau \\ 1 \\ 0 \end{pmatrix}, & \vec{k}_4 &= \begin{pmatrix} 0 \\ 0 \\ -\tau \\ \theta \end{pmatrix}, \\
\vec{k}_5 &= \begin{pmatrix} V \\ \sigma \\ \theta/2 \\ \tau/2 \end{pmatrix}, & \vec{k}_6 &= \begin{pmatrix} 2V\sigma \\ \sigma^2 - (V + \theta^2 + \tau^2)^2 \\ \sigma\theta - \tau(V + \theta^2 + \tau^2) \\ \sigma\tau + \theta(V + \theta^2 + \tau^2) \end{pmatrix}, \\
\vec{k}_7 &= \begin{pmatrix} -2V\theta \\ -\sigma\theta + V\tau + \tau(\theta^2 + \tau^2) \\ \frac{1}{2}(V - \theta^2 + 3\tau^2) \\ -2\theta\tau - \sigma/2 \end{pmatrix}, & \vec{k}_8 &= \begin{pmatrix} -2V\tau \\ -\sigma\tau - V\theta - \theta(\theta^2 + \tau^2) \\ -2\theta\tau + \sigma/2 \\ \frac{1}{2}(V + 3\theta^2 - \tau^2) \end{pmatrix}.
\end{aligned} \tag{B.1}$$

The corresponding prepotentials are

$$\begin{aligned}
\vec{p}_1 &= \begin{pmatrix} 0 \\ 0 \\ -\frac{1}{4V} \end{pmatrix}, & \vec{p}_2 &= \begin{pmatrix} -\frac{1}{\sqrt{V}} \\ 0 \\ -\frac{\theta}{V} \end{pmatrix}, & \vec{p}_3 &= \begin{pmatrix} 0 \\ \frac{1}{\sqrt{V}} \\ \frac{\tau}{V} \end{pmatrix}, & \vec{p}_4 &= \begin{pmatrix} -\frac{\theta}{\sqrt{V}} \\ -\frac{\tau}{\sqrt{V}} \\ \frac{1}{2} - \frac{\theta^2 + \tau^2}{2V} \end{pmatrix}, \\
\vec{p}_5 &= \begin{pmatrix} -\frac{\tau}{2\sqrt{V}} \\ \frac{\theta}{2\sqrt{V}} \\ -\frac{\sigma}{4V} \end{pmatrix}, & \vec{p}_6 &= \begin{pmatrix} -\frac{1}{\sqrt{V}}[\sigma\tau + \theta(-V + \theta^2 + \tau^2)] \\ \frac{1}{\sqrt{V}}[\sigma\theta - \tau(-V + \theta^2 + \tau^2)] \\ -\frac{V}{4} - \frac{1}{4V}[\sigma^2 + (\theta^2 + \tau^2)^2] + \frac{3}{2}(\theta^2 + \tau^2) \end{pmatrix}, \\
\vec{p}_7 &= \begin{pmatrix} \frac{4\theta\tau + \sigma}{2\sqrt{V}} \\ \frac{3\tau^2 - \theta^2}{2\sqrt{V}} - \frac{\sqrt{V}}{2} \\ -\frac{3}{2}\tau + \frac{1}{2V}[\sigma\theta + \tau(\theta^2 + \tau^2)] \end{pmatrix}, & \vec{p}_8 &= \begin{pmatrix} -\frac{3\theta^2 - \tau^2}{2\sqrt{V}} + \frac{\sqrt{V}}{2} \\ \frac{\sigma - 4\theta\tau}{2\sqrt{V}} \\ \frac{3}{2}\theta + \frac{1}{2V}[\sigma\tau - \theta(\theta^2 + \tau^2)] \end{pmatrix}.
\end{aligned} \tag{B.2}$$

It is easier to see that the Killing vectors close to the $SU(2, 1)$ algebra if they are recasted in the following combinations

$$\begin{aligned}
SU(2) & \quad \begin{cases} T_1 = \frac{1}{4}(k_2 - 2k_8), \\ T_2 = \frac{1}{4}(k_3 - 2k_7), \\ T_3 = \frac{1}{4}(k_1 + k_6 - 3k_4), \end{cases} & \frac{SU(2, 1)}{U(2)} & \quad \begin{cases} T_4 = k_5, \\ T_5 = -\frac{1}{2}(k_1 - k_6), \\ T_6 = -\frac{1}{4}(k_3 + 2k_7), \\ T_7 = -\frac{1}{4}(k_2 + 2k_8). \end{cases} & (B.3) \\
U(1) & \quad \begin{cases} T_8 = \frac{\sqrt{3}}{4}(k_4 + k_1 + k_6), \end{cases}
\end{aligned}$$

This basis is chosen for convenience such that the generators T_1, T_2, T_3 and T_8 are the isotropy group of the point $(V, \sigma, \theta, \tau) = (1, 0, 0, 0)$. The metric hyperscalar manifold becomes diagonal at this point. In all the theories that have hyper coupling, we will take this basis point q^C for a possible candidate of the hyper-coordinates of a critical point. The Killing vectors K_I^X are then given by $V_I^\alpha k_\alpha^X$ and the corresponding prepotentials \vec{P}_I are $V_I^\alpha \vec{p}_\alpha$, where V_I^α are constants that determine which isometries are being gauged and what linear combination of vector fields being used. In particular,

$$K_I^X = \begin{cases} T_1^X, T_2^X, T_3^X & \text{for } SU(2) \text{ gauging,} \\ V_I W^k T_k^X, \quad k = 1, 2, 3, 8 & \text{for } U(1) \text{ gauging,} \\ V_I W^k T_k^X, \quad k = 4, 5, 6, 7 & \text{for } SO(1, 1) \text{ gauging,} \end{cases}$$

where V_I and W^k are constants depending on the model.

Transformations Between Two Parametrizations

For $\mathcal{N} = 2$ supergravity coupled to n vector multiplets and no tensors, the symplectic section (3.1.9) takes the following form[121]

$$\Omega_0 = \begin{pmatrix} 1 \\ z_1 \\ z_2 \\ z_a \\ \frac{1}{2}z_1||z||^2 \\ -\frac{1}{2}||z||^2 \\ -z_1z_2 \\ z_1z_a \end{pmatrix}. \tag{C.1}$$

Here $||z||^2 = [(z_2)^2 - (z_3)^2 - \dots - (z_n)^2]$ and $a = 3, \dots, n$. FTP[67] use Calabi-Vesentini coordinates for which $(X^\Lambda, F_\Lambda = \eta_{\Lambda\Sigma}SX^\Sigma ; X^\Lambda X^\Sigma \eta_{\Lambda\Sigma} = 0, \eta_{\Lambda\Sigma} = \text{diag}(+, +, -, \dots, -))$ holds. More explicitly [67],

$$\Omega_{CV} = \begin{pmatrix} X^\Lambda \\ F_\Sigma \end{pmatrix} = \begin{pmatrix} \frac{1}{2}(1 + ||y||^2) \\ \frac{1}{2}i(1 - ||y||^2) \\ y_1 \\ y_{a-1} \\ \frac{1}{2}S(1 + ||y||^2) \\ \frac{1}{2}iS(1 - ||y||^2) \\ -Sy_1 \\ -Sy_{a-1} \end{pmatrix} \tag{C.2}$$

where $\|y\|^2 = y_1^2 + \dots + y_{n-1}^2$. The transformations between the two notations are given by

$$\begin{aligned}
 \frac{1}{2}(1 + \|y\|^2) &= \frac{1}{2\sqrt{2}}(2 - \|z\|^2) \\
 \frac{1}{2}i(1 - \|y\|^2) &= z_2 \\
 y_{a-2} &= z_a \\
 y_{n-1} &= \frac{1}{2\sqrt{2}}(2 + \|z\|^2) \\
 S &= -z_1.
 \end{aligned} \tag{C.3}$$

The matrix for the symplectic rotation $C \Omega_{CV} = \Omega_0$ is given by

$$C = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & \mathbb{1}_{n-1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathbb{1}_{n-1} & 0 \end{pmatrix} \tag{C.4}$$

It is easy to see that the symplectic section (C.2) together with the coordinate transformations (C.3) is a particular case of (3.3.22) and also that $C = S^{-1}$.

Various Potential Terms

The $P^{(T)}$ potential terms given here are calculated for $\mathcal{N} = 2$, 4D YMESGT coupled to $n = 4$ vector/tensor multiplets.

Gauging $K_{(4)} = SO(2, 1)$ symmetry results in the following potential

$$\begin{aligned}
& -i \left[(\bar{w}_2^2 - \bar{w}_3^2)w_2^4 + 2\bar{w}_2(-\bar{w}_2^2 + \bar{w}_3^2 + \bar{w}_4^2 + 2)w_3^3 + (\bar{w}_2^4 - 2(w_3^2 - \bar{w}_3w_3 + w_4^2 + \bar{w}_3^2 + \bar{w}_4^2 + 4)\bar{w}_2^2 + 2w_3^2\bar{w}_3^2 + 2w_4^2\bar{w}_3^2 \right. \\
& \quad + (\bar{w}_3^2 + \bar{w}_4^2)^2 + 4(\bar{w}_4^2 + 1) - 2w_3\bar{w}_3(\bar{w}_3^2 + \bar{w}_4^2 + 2))w_2^2 + 2\bar{w}_2((\bar{w}_2^2 - \bar{w}_3^2 - \bar{w}_4^2 - 2)w_3^2 + 8\bar{w}_3w_3 \\
& \quad + (w_4^2 + 2)(\bar{w}_2^2 - \bar{w}_3^2 - \bar{w}_4^2 - 2))w_2 - w_3^2\bar{w}_2^4 - 4w_3^2 - (\bar{w}_3(w_3^2 - \bar{w}_3w_3 + w_4^2 + 2) - w_3\bar{w}_4^2)(\bar{w}_3w_3^2 - (\bar{w}_3^2 + \bar{w}_4^2 + 4)w_3 \\
& \quad \left. + (w_4^2 + 2)\bar{w}_3) + \bar{w}_2^2(w_4^2 - 2\bar{w}_3w_3^2 + 2(w_4^2 + \bar{w}_3^2 + \bar{w}_4^2)w_3^2 - 2(w_4^2 + 2)\bar{w}_3w_3 + (w_4^2 + 2)^2) \right] \\
& \quad / \left[2(w_1 - \bar{w}_1)((w_2 - \bar{w}_2)^2 - (w_3 - \bar{w}_3)^2 - (w_4 - \bar{w}_4)^2)^2 \right]. \tag{D.1}
\end{aligned}$$

Gauging $K_{(4)} = SO(1, 1) \times \mathbb{R}^{(1,1)}$ (no central charge) symmetry, on the other hand, results in the following potential

$$\begin{aligned}
& -i \left[-2(-(w_2 - \bar{w}_2)^2 - (w_3 - \bar{w}_3)^2 + (w_4 - \bar{w}_4)^2)(-\bar{w}_2^2 + \bar{w}_3^2 + \bar{w}_4^2 + 2)w_2^2 + 8(w_2 - \bar{w}_2)(w_3 - \bar{w}_3)\bar{w}_3w_2 \right. \\
& \quad + 4\bar{w}_2(-(w_2 - \bar{w}_2)^2 - (w_3 - \bar{w}_3)^2 + (w_4 - \bar{w}_4)^2)w_2 - 8w_3(w_2 - \bar{w}_2)(w_3 - \bar{w}_3)(-\bar{w}_2^2 + \bar{w}_3^2 + \bar{w}_4^2 + 2)w_2 \\
& \quad + 2(-w_2^2 + w_3^2 + w_4^2 + 2)(w_2 - \bar{w}_2)(w_3 - \bar{w}_3)\bar{w}_3(-\bar{w}_2^2 + \bar{w}_3^2 + \bar{w}_4^2 + 2)w_2 \\
& \quad + (-w_2^2 + w_3^2 + w_4^2 + 2)\bar{w}_2(-(w_2 - \bar{w}_2)^2 - (w_3 - \bar{w}_3)^2 + (w_4 - \bar{w}_4)^2)(-\bar{w}_2^2 + \bar{w}_3^2 + \bar{w}_4^2 + 2)w_2 \\
& \quad + 8w_3(w_2 - \bar{w}_2)\bar{w}_2(w_3 - \bar{w}_3) - 8(-w_2^2 + w_3^2 + w_4^2 + 2)(w_2 - \bar{w}_2)\bar{w}_2(w_3 - \bar{w}_3)\bar{w}_3 \\
& \quad - 2(-w_2^2 + w_3^2 + w_4^2 + 2)\bar{w}_3^2(-(w_2 - \bar{w}_2)^2 - (w_3 - \bar{w}_3)^2 - (w_4 - \bar{w}_4)^2) \\
& \quad + 4w_3\bar{w}_3(-(w_2 - \bar{w}_2)^2 - (w_3 - \bar{w}_3)^2 - (w_4 - \bar{w}_4)^2) \\
& \quad - 2(-w_2^2 + w_3^2 + w_4^2 + 2)\bar{w}_2^2(-(w_2 - \bar{w}_2)^2 - (w_3 - \bar{w}_3)^2 + (w_4 - \bar{w}_4)^2) \\
& \quad + 2w_3(-w_2^2 + w_3^2 + w_4^2 + 2)(w_2 - \bar{w}_2)\bar{w}_2(w_3 - \bar{w}_3)(-\bar{w}_2^2 + \bar{w}_3^2 + \bar{w}_4^2 + 2) \\
& \quad - 2w_3^2(-(w_2 - \bar{w}_2)^2 - (w_3 - \bar{w}_3)^2 - (w_4 - \bar{w}_4)^2)(-\bar{w}_2^2 + \bar{w}_3^2 + \bar{w}_4^2 + 2) \\
& \quad \left. + w_3(-w_2^2 + w_3^2 + w_4^2 + 2)\bar{w}_3(-(w_2 - \bar{w}_2)^2 - (w_3 - \bar{w}_3)^2 - (w_4 - \bar{w}_4)^2)(-\bar{w}_2^2 + \bar{w}_3^2 + \bar{w}_4^2 + 2) \right] \\
& \quad / \left[2(w_1 - \bar{w}_1)((w_2 - \bar{w}_2)^2 - (w_3 - \bar{w}_3)^2 - (w_4 - \bar{w}_4)^2)^3 \right]. \tag{D.2}
\end{aligned}$$

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