The Pennsylvania State University
The Graduate School
Department of Aerospace Engineering

REDUCING HELICOPTER MAIN ROTOR POWER REQUIREMENTS USING
MULTIPLE TRAILING EDGE FLAPS AND EXTENDABLE CHORD SECTIONS

A Thesis in
Aerospace Engineering

by

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Reducing helicopter rotor power requirements is of much interest in terms of payload, ceiling altitude, maximum speed and maximum range. Rotorcraft power reduction can be tackled using a large variety of design solutions, ranging from passive blade twist design to active on-blade devices. The present study examines power reductions achievable using two different on-blade active methods. A rotorcraft model was developed in order to realistically assess the performance improvements. A propulsive trim analysis is performed on a UH-60 type helicopter model featuring advanced rotor and fuselage geometry, rigid blades free to flap and a rigid prescribed wake.

The first concept, referred to as Static Extended Trailing Edge (SETE), aims at expanding the flight envelope by quasi-statically increasing the chord through the extension of a flat plate through a slit trailing edge over a section of the blade. This device used on the flight envelope boundaries for stall alleviation appears to be a better high-lift device than trailing-edge flaps or even Gurney flaps in that it results in higher lift-to-drag coefficients at high $C_L$. Simulation results indicate that in stall-dominant conditions (high gross-weigh, altitude, sufficient available power) increases of up to 3,000 ft in the maximum altitude, 2,400 lbs in the maximum gross-weight, 26 knots in the maximum speed, and reductions up to 33.4% in the rotor power can be obtained. The blade section with the SETE mechanism was fabricated and appeared to operate well.

The second concept aims at reducing the rotor power requirements for a range of advance ratios – from 0 to 0.4 – using multiple spanwise-segmented Trailing Edge Flaps (TEFs). 4 TEFs located from 50% to 90% span actuated up to 2/rev with a maximum amplitude of 5 degrees are optimally deflected to reduce the total average torque. Power reductions ranging from 1.61% to 5.65% depending on airspeed and thrust levels were observed with the UH-60 non-linear twist distribution. Using a -8 degrees linear twist blade, power reductions ranging from 2.74% to
7.98% were observed. Such reductions are obtained by redistributing the lift inboard and reducing the trim controls, globally off-loading the tip of the rotor blade and reducing its drag.
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<th>Description</th>
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<tbody>
<tr>
<td>b</td>
<td>Semi-chord length</td>
</tr>
<tr>
<td>c</td>
<td>Blade chord</td>
</tr>
<tr>
<td>$C_D$</td>
<td>Section drag coefficient</td>
</tr>
<tr>
<td>$C_L$</td>
<td>Section lift coefficient</td>
</tr>
<tr>
<td>$C_M$</td>
<td>Section pitching moment coefficient</td>
</tr>
<tr>
<td>$C_T$</td>
<td>Thrust coefficient</td>
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<tr>
<td>d</td>
<td>Distance of quarter chord behind the pitch axis</td>
</tr>
<tr>
<td>$dD$</td>
<td>Sectional drag</td>
</tr>
<tr>
<td>$D_F$</td>
<td>Fuselage drag</td>
</tr>
<tr>
<td>$D_{HT}$</td>
<td>Horizontal tail drag</td>
</tr>
<tr>
<td>$dL$</td>
<td>Sectional lift</td>
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<tr>
<td>$dM$</td>
<td>Sectional pitching moment</td>
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<tr>
<td>e</td>
<td>Dimensional flap hinge location</td>
</tr>
<tr>
<td>$F$</td>
<td>Prandtl’s tip-loss function</td>
</tr>
<tr>
<td>$GF$</td>
<td>Gurney Flap</td>
</tr>
<tr>
<td>$H$</td>
<td>Rotor drag force (positive toward the tail)</td>
</tr>
<tr>
<td>HHC</td>
<td>Higher Harmonic Control</td>
</tr>
<tr>
<td>$IBC$</td>
<td>Individual Blade Control</td>
</tr>
<tr>
<td>$I_x$</td>
<td>Inertial flap-pitch coupling</td>
</tr>
<tr>
<td>$I_\beta$</td>
<td>Blade flapping moment of inertia</td>
</tr>
<tr>
<td>$J$</td>
<td>Jacobian matrix</td>
</tr>
<tr>
<td>$k_x, k_y$</td>
<td>Longitudinal and lateral inflow gradients</td>
</tr>
<tr>
<td>$L_F$</td>
<td>Fuselage lift</td>
</tr>
<tr>
<td>$L_{HT}$</td>
<td>Horizontal tail lift</td>
</tr>
<tr>
<td>$M$</td>
<td>Mach number</td>
</tr>
<tr>
<td>MiTE</td>
<td>Miniature Trailing-Edge Effector</td>
</tr>
<tr>
<td>$M_x$</td>
<td>Rotor roll moment (positive roll left)</td>
</tr>
<tr>
<td>$M_y$</td>
<td>Rotor pitch moment (positive pitch up)</td>
</tr>
<tr>
<td>$M_\beta$</td>
<td>Total blade flapping moment due to aerodynamic forces</td>
</tr>
<tr>
<td>$N_b$</td>
<td>Number of blades</td>
</tr>
<tr>
<td>$N_F$</td>
<td>Blade root flapwise moment</td>
</tr>
</tbody>
</table>
N_L  Blade root lagwise moment
N_P  Blade root pitchwise moment
P    Rotor power
P_0  Profile power
P_i  Induced power
P_P  Propulsive power
Q    Rotor torque
R    Rotor radius
r    Dimensional radial location
R_Co Blade root cut out
SETE Static Extended Trailing Edge
S_r  Blade root radial shear force
S_x  Blade root in-plane shear force
S_z  Blade root vertical shear force
S_β First moment of flap mode
T    Rotor thrust
TEF  Trailing Edge Flap
U    Resultant velocity at blade element
U_p  Out-of plane velocity normal to rotor disk plane
U_T  In-plane velocity parallel to rotor disk plane
V    Freestream velocity
v_i  Induced velocity
V_0  Tangential velocity component
W, GW Helicopter gross weight
x_a  Non-dimensional distance from the blade mid-chord to the pitch axis
x_c  Non-dimensional distance from the blade mid-chord to the TEF hinge
x_CG, y_CG, z_CG Location of the center of gravity with respect to the hub
x_HT, y_HT, z_HT Location of the horizontal tail with respect to the hub
x_i  Chordwise offset of blade center of gravity behind pitch axis
x_TR, y_TR, z_TR Location of the tail rotor with respect to the hub
Y    Rotor side force (positive out the right door)
α, AoA Blade section angle of attack
α_WL Fuselage angle of attack (positive nose down)
\( \beta \) Blade flapping angle, positive up
\( \beta_0 \) Blade coning angle
\( \beta_{1c} \) Longitudinal flapping angle
\( \beta_{1s} \) Lateral flapping angle
\( \Gamma \) Vortex circulation strength
\( \delta \) TEF deflection, positive down
\( \delta_{0}^{(n)} \) TEFn static deflection angle
\( \delta_{1c}^{(n)} \) TEFn first harmonic cosine deflection component
\( \delta_{1s}^{(n)} \) TEFn first harmonic sine deflection component
\( \delta_{2c}^{(n)} \) TEFn second harmonic cosine deflection component
\( \delta_{2s}^{(n)} \) TEFn second harmonic sine deflection component
\( \delta_{SETE} \) SETE inclination angle, positive down
\( \varepsilon \) Dimensional SETE chord extension
\( \theta \) Blade pitch control
\( \theta_0 \) Blade collective pitch
\( \theta_{1c} \) Lateral cyclic pitch
\( \theta_{1s} \) Longitudinal cyclic pitch
\( \theta_{TR} \) Tail rotor pitch control
\( \theta_w \) Blade twist rate
\( \kappa \) Induced power correction factor
\( \lambda \) Rotor inflow ratio
\( \lambda_i \) Induced inflow ratio
\( \mu \) Rotor advance ratio
\( \nu_{\beta} \) Blade flapping natural frequency (non-dimensional in per revolution)
\( \rho \) Air density
\( \sigma \) Blade solidity
\( \phi \) Inflow angle of attack
\( \phi_F \) Fuselage roll angle (positive roll right)
\( \chi \) Wake slew angle
\( \psi \) Rotor azimuth angle
\( \psi_b \) Blade azimuth angle
\( \psi_w \) Wake age
\( \Omega \) Rotor angular velocity
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This work would have never been completed without the support of many great colleagues and friends. First of all, many thanks to my advisor Dr. Gandhi who supported me and guided me during those two past years. Working with him has been an exceptional experience and I am extremely thankful for his great mentorship.

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Chapter 1

Introduction

Helicopters have proven over the past years their undeniable usefulness over fixed-wing aircraft for specific flight conditions. Indeed, rotorcraft’s unique abilities to hover, to take-off and to land vertically, to fly in any direction or to climb and to descend, have made it an indispensable part of the modern aviation. However, compared to fixed-wing aircraft, such versatility comes at a cost: greater mechanical complexity, greater complex aerodynamics taking place when considering a rotary-wing system, and higher power requirements are inherent to rotorcraft designs.

Reducing helicopter main rotor power requirements is then of much interest in terms of maximum gross-weight capabilities and payload, ceiling altitude, maximum attainable speed, maximum range and endurance. Many efforts have been made by the rotorcraft research community in order to improve the performance of helicopters, and this present work follows the same idea by examining the helicopter rotor power reductions achievable using two different techniques. The first one focuses on performance improvements when flying close the flight envelope boundaries by the means of a chord-extension device mounted on a section of the rotor blades, allowing alleviation of retreating blade stall by increasing the lift capabilities of the rotor. The second one aims at redistributing the lift across the helicopter rotor in the most optimal way using multiple spanwise-segmented trailing-edge flaps mounted on the rotor blades.

The following sections provide more details about the physics and the motivations behind this work. A literature survey also presents previous research conducted in this area of rotor
power reduction. Finally the objectives of this present study are described and an overview of this thesis is presented.

1.1 Background and Motivation

1.1.1 Rotor power requirements

On a helicopter, the main rotor provides all lift, propulsion and control – except yaw – during flight. Such a coupling between lift forces and control makes the design of a helicopter a mechanical challenge in terms of flight capabilities and efficiency. In terms of power requirements, great improvements have been made over the past years and on-going research still provides a large scope of different design solutions to study. Indeed, in order to reduce the required torque on the rotor shaft, passive methods, such as advanced blade design, or active methods such as higher harmonic blade control or high-lift devices can be used.

The total power required on a helicopter rotor can be separated in different parts: the induced power, the propulsive power and the profile power (Refs. 1, 2). The first one represents the energy left in the wake of the rotor due to the acceleration of the air necessary to generate the required lift. The second one represents the power necessary to provide propulsion and to overcome the drag and the parasitic forces acting on the fuselage of the helicopter. Finally the third one is the contribution of the rotor profile drag, thus the power necessary to move the blades and to overcome the associated viscous forces. A mathematical formulation can be derived looking at Figure 1-1 and Figure 1-2 which represent a rotor-blade and a blade-element.
Figure 1-1: Blade element analysis, top view of a blade showing local air velocities.

Figure 1-2: Blade element subject to aerodynamic forces.

Figure 1-1 and Figure 1-2 show the aerodynamic environment and the different airflow velocities on a rotor blade, as well as the induced aerodynamic forces and moments created on the blade elements. Considering a blade element of length \( dr \), the forces generated on the airfoil are basically a function of the angle of attack \( \alpha \), which is the difference between the local pitch angle \( \theta \) and the inflow angle \( \phi \), and the Mach number \( M \). The resultant force acting on the airfoil can be decomposed in the incremental lift \( dL \) and drag \( dD \). Since the lift and drag forces are defined in
the plane of incoming airflow, a change of basis is necessary to obtain the forces acting in the hub-plane. The incremental lift and drag forces are given by equation 1-1 and 1-2 and the resulting forces in the hub-plane are given by equation 1-3 and 1-4.

\[
dL = \frac{1}{2} \rho U^2 c_{L}(\alpha, M) dr
\]
\[
dD = \frac{1}{2} \rho U^2 c_{D}(\alpha, M) dr
\]
\[
dFz = dL \cos \varphi - dD \sin \varphi
\]
\[
dFx = dL \sin \varphi + dD \cos \varphi
\]

As a consequence, the required torque and power on the rotor are a function of \(dF_x\), the in-plane force, and the azimuthally-averaged torque and power can be written as in equation 1-5 and 1-6.

\[
Q = \frac{N_k}{2\pi} \int_{\Psi_y} (dL \sin(\varphi) + dD \cos(\varphi)) r d\Psi
\]
\[
P = Q \Omega
\]

Using a small-angle approximation, since generally the out-of-plane velocity \(U_p\) is much smaller than \(U_T\), we can rewrite this expression as in equation 1-7.

\[
P = \Omega \frac{N_k}{2\pi} \int_{\Psi_y} (\varphi dL + dD) r d\Psi
\]

In equation 1-7, one can now see the contributions of the induced velocity and the lift to the total power, more specifically to the induced and propulsive power, and the power contribution of the rotor drag. As a consequence, reducing the total required power can be seen as an optimization problem where, for a given flight condition, the best inflow and lift-to-drag distributions across the rotor are sought. Such a problem is clearly not straightforward since many parameters have to be taken into account and the aerodynamic environment around a helicopter...
rotor is complex. For instance, the vortices trailed by a blade are interacting with the other rotor-blades as well as with other vortices, globally modifying the inflow distribution across the rotor disk, and thus the local lift and circulation distribution. Furthermore, the lift and the drag depend on the airfoil characteristics and the blade geometry, which in turn also have an impact on the way the vortices are trailed.

The total rotor power can then be written as in equation 1-8, with the following definitions.

\[
P_{\text{total}} = P_i + P_p + P_0
\]

(1-8)

\[
P_i + P_p = \Omega \frac{N_b}{2\pi} \int \int (\rho \cdot dL) \, r \, d\Psi
\]

(1-9)

\[
P_0 = \Omega \frac{N_b}{2\pi} \int \int (dD) \, r \, d\Psi
\]

(1-10)

The definition of the induced power alone can however be subject to confusion and is actually not straightforward when developing a comprehensive helicopter model (Ref. 3). Indeed, using an energy approach, one can define the induced power as in equation 1-11 which involves the definition of the airloads and the induced velocity components across the rotor.

\[
P_i = \int v_i \, dF_z
\]

(1-11)

Such a definition can lead to some inconsistencies and a modified version of Ormiston’s definition of the induced power, taking into account the rotor profile power and the propulsive power as written in Equation 1-12, is used in the present work. Indeed, defining the propulsive power as the power required to move the helicopter forward at a velocity \( V \), thus \( F_{\text{prop}} \cdot V \), the definition of induced power becomes unambiguous and is defined by Equation 1-13. The definition of the propulsive force is shown in Equation 1-14.
\[ P_t = P_{total} - P_p - P_0 \]  
(1-12)

\[ P_t = \left( \Omega \frac{N_b}{2\pi} \int \int (\varphi \cdot dL) \cdot r \partial \Psi \right) - \left( F_{prop} \cdot V \right) \]  
(1-13)

\[ F_{prop} = F_{Zr} \sin \alpha_{WL} - F_{Xr} \cos \alpha_{WL} \]  
(1-14)

Once these conventions are set, the power as a function of the airspeed for a typical UH-60A helicopter can be examined, as presented in Figure 1-3. As one can observe, the contribution of each component changes throughout the entire range of airspeeds considered: in hover, the induced power is predominant due to the strong influence of the trailed vortices that remain close to the rotor. Increasing the airspeed, those vortices are swept away from the rotor and the induced power decreases. However, the forces required for propulsion to overcome the parasitic forces due to the fuselage drag are increasing with the velocity, requiring more power. The rotor profile drag, which is low in hover condition, remains almost constant, slowly increasing with the airspeed due to viscosity effects. For high velocities however, compressibility effects and retreating blade stall are approached, suddenly increasing the rotor profile power.
Some observations can be made here. For hover conditions, in order to reduce power requirements, the main objective is to reduce the induced power. This objective can also be interesting in forward flight, as mentioned by Wachspress in Ref. 4. Previous work has been done in this direction, considering the problem as an optimization problem and looking for the best circulation distribution across a rotor for different flight conditions (Refs. 5-6). Those results can then be used in association with passive or active controls to tailor the lift distribution accordingly. For instance, a remarkable result (Ref. 4) is the design of a non-linear blade twist distribution resulting in low induced power requirements in hover, as designed for the UH-60 helicopter blade. In forward flight, power reductions can be mainly obtained by reducing the propulsive and the rotor profile power. Reductions in propulsive power can be achieved using advanced fuselage designs while reductions in profile power can be achieved through lift
redistribution using high-lift devices or stall-alleviation methods. The present work does not address the reduction of propulsive power but concentrates on reducing the total main rotor power by improving the lift-generation efficiency of the rotor in forward flight using two different design, thus reducing profile and induced power.

1.1.2 Quasi-Static Extended Trailing Edge

The first method explored in this work concentrates on reducing power requirements when operating close to the flight envelope boundaries and expanding the flight envelope. In such a case the aerodynamic limitations such as retreating blade stall and compressibility effect on the advancing tip, as depicted in Figure 1-4, become important and limit the performance of the helicopter. Retreating blade stall is observed for high thrust requirements, when high angles of attack are necessary on the retreating side of the rotor, thus for high altitudes or high gross-weights for example. When a rotor undergoes retreating blade stall, flow separation induces high drag penalties and a sharp decrease in lift. Figure 1-5 shows the lift and drag coefficients of SC-1094R8 airfoil, used on the UH-60 blade, at a Mach number M of 0.3 obtained through wind-tunnel tests (Ref. 7). Stall can clearly be observed for and angle of attack greater than 16 degrees in that case.
Figure 1-4: Aerodynamic environment on a rotor in forward flight.

Figure 1-5: (a) Lift coefficient and (b) Drag coefficient as a function of the angle of attack of a SC-1094R8 airfoil at a Mach number of 0.3.
Due to high drag penalties associated with retreating blade stall, its alleviation is of much interest in terms of power reduction. An approach presented by Liu et al in Ref. 8 to generate additional lift near stall is to extend the chord at the trailing edge of the airfoil using a thin splitting plate as illustrated in Figure 1-6. The authors have referred to this concept as a Static Extended Trailing Edge (SETE). The plate can be deployed either at zero angle or at some non-zero inclination angle as shown in the next section in Figure 2-8. Figure 1-7 (from Ref. 8) shows wind-tunnel measurements for the lift coefficient versus angle of attack of a baseline NACA 0012 airfoil, as well as for the cases of the NACA 0012 with a 10% chord SETE deployed at 0 deg and 6 deg. A 6 deg inclination showed a marked improvement in the lift coefficient, but interestingly, even a 0 deg inclination (essentially a NACA 0012 with a trailing-edge flat-plate tab, a non-optimal airfoil shape) showed no degradation in lift-coefficient. Furthermore, the drag penalty of the 0 deg SETE was very low, making the SETE a promising trailing-edge lift augmentation device.

In the present study the performance of a Static Extended Trailing Edge (SETE) device, deployed at 0 deg deflection angle, is investigated for stall alleviation and flight envelope expansion. The chord is quasi-statically extended over a section of the blade span when operating close to the stall boundary. Under normal flight condition, i.e. moderate speed, moderate altitude and medium gross weight, a fixed-geometry large-chord blade would add unnecessary extra profile drag and thus is not desired. The present SETE is then a stall alleviation device only quasi-statically deployed close to the flight envelope boundary.

A SETE is expected to have low drag penalties compared to other devices such as trailing edge flaps and Gurney flaps, since the device is embedded in the wake. This would result in low-to-moderate drag increment for a unit increment of lift. Furthermore, the actuation energy
requirements are expected to be modest as the SETE does not have to overcome aerodynamic pressure differences, as required by a TEF, for example.

Figure 1-6: Static Extended Trailing Edge prototype device built at the Pennsylvania State University.

Figure 1-7: Lift coefficient versus angle of attack wind tunnel data for baseline NACA 0012 airfoil and airfoil with SETE (M < 0.1), from Liu et al. (Ref. 8).
1.1.3 Trailing-Edge Flaps

The second method studied in the present research focuses on lift redistribution across the rotor disk using 4 spanwise-segmented Trailing Edge Flaps (TEFs) extending over 40% of the blade span from 50% to 90% radius actuated up to 2 per revolution. Redistributing the lift in the most optimal way using high lift devices is expected to reduce the rotor power requirements over the entire range of airspeeds, reducing the rotor profile drag and the induced power by tailoring the airloads distribution in both the radial and azimuthal directions.

High-lift devices such as Trailing-Edge Flaps (TEFs) have shown promising results in terms of rotorcraft performance enhancement (Refs. 9, 10, 17, 18, 25, 28). A TEF as depicted in Figure 1-8 is embedded in the blade and is actuated using smart-material actuators in order to deflect the flap and to modify the effective camber of an airfoil. High lift-to-drag ratios can be obtained using such a device as depicted in Figure 1-9 and give the opportunity to redistribute the airloads across the rotor efficiently. Furthermore, one of the most attractive features of TEFs is the low actuation forces and power requirements compared to Higher Harmonic Control or Individual Blade Control technologies presented later.

This approach, however, has its limitations and challenges including high-frequency actuation requirement, large actuation force (to overcome the hinge moment), limited stroke, complexity in implementation, and high drag or limited effectiveness in certain conditions. For these reasons, TEFs are still a subject of on-going research. A limited number of studies have been conducted on rotor power reduction using trailing edge flaps. It should be noted however that many studies have been conducted on rotor noise and vibration reduction using TEFs (Refs. 10-17), as well as primary control using TEFs (Ref. 18-25), with very interesting results making TEFs a promising technology for a wide range of rotorcraft applications.
Figure 1-8: Trailing Edge Flap System Developed by Boeing (Ref. 4)

Figure 1-9: Comparison of lift-to-drag ratios obtained through CFD at Mach 0.3 for different TEF deflections, between a clean SC-1094R8 airfoil and 20% c TEF mounted on the same airfoil (Ref. 29)
1.2 Literature Review

Achieving helicopter rotor power reduction can be tackled using a large variety of design solutions. Many different studies have been conducted over the past 50 years, exploring active methods such as Individual Blade Control (IBC), Higher-Harmonic Control (HHC), Trailing Edge Flaps (TEFs) or Miniature Trailing Edge Effectors (MiTEs). This survey intends to cover the contributions related to the present study and to report the current state of the art in terms of power reduction using active controls.

The idea of tailoring the lift distribution across the rotor to improve the performances of a helicopter has long been recognized and early studies include the Goldstein rotor theory dealing with optimal loading of lightly loaded rotors in axial flow (Refs. 2, 28). More recently, many studies have been focusing on reducing the rotor power requirements in hover and forward flight using different methods.

1.2.1 Induced power reduction

The problem of reducing the rotor induced power via circulation or lift distribution optimization has been addressed in few studies. In Ref. 31, Moffitt and Bissell aimed at determining the optimum airloads for rotors in hover and forward flight to minimize the power requirements. The conclusions drawn out of their results include the use of a non-linear twist similar to the twist distribution found on a UH-60 Black Hawk for optimized hover performances. In forward flight, power reductions of about 10% at 175 knots, 18,000 lbs gross weight and an altitude of 4,000 ft were observed comparing the optimal airloads distribution solution with a baseline UH-60 helicopter. A redistribution of the airloads forward and aft of the rotor can be observed at an airspeed of 175 knots for the optimized case described in their study.
Similarly, Hall et al in Ref. 6 developed a minimum induced power optimization problem solving for the optimum lift distribution using a finite-element method approach for axial and forward flight conditions. Despite some issues arising from the wake model making the finite elements stiff, a typical 4-bladed rotor is shown to have about 10%-15% more induced power requirements than an optimized solution at an advance ratio of 0.25. Considering a rotor in axial flight, or a propeller, results closely match Goldstein’s theory. In forward flight conditions, the results for minimum induced power show a longitudinal and lateral symmetry of lift distribution – with and without constraints on the maximum lift coefficient. This result suggests that improved helicopter performances can be obtained using a 2 per revolution control input.

In a similar fashion, Rand & Khromov in Ref. 5 determined the lower limit of induced power attainable for a helicopter rotor operating in hover and forward flight solving a circulation distribution optimization problem. Using a formulation only based on radial and azimuthal distributions of the bound vortices in order to solve a problem not depending on the rotor design parameters such as chord, twist or airfoil characteristics, and using an advanced free-wake model via CAMRAD II (Comprehensive Analytical Model of Rotorcraft Aerodynamics and Dynamics) and a prescribed wake model via RAPID (Rotorcraft Analysis for Preliminary Design) for the initial studies, they showed that compared to a typical helicopter rotor the lower limit of the induced power is about 10% lower for advance ratios below 0.25. The results suggest that in hover a good blade design can reduce the rotor induced power requirements thanks to the uniformity of the optimum circulation distribution across the rotor. In forward flight however, it is observed that such a result can only be attained using active methods due to the more complex circulation distributions involved. One can also observe in those results an increase of the circulation forward and aft of the rotor, substantiating Hall’s work.

In Ref. 4 Wachspress et al. examined then the influence of different parameters such as planform characteristics, circulation distribution and other modeling assumptions on the rotor
induced power using advanced rotorcraft analysis. Although the model used does not account for stall or compressibility effects for high advance ratios, conclusions on the optimum twist distributions are drawn for hover with a non-linear twist distribution and for advance ratios between 0.1 and 0.5 using a linear twist. Using higher harmonic cyclic pitch controls (HHC), the induced power was reduced by 2% to 4% depending on the blade twist, the number of blades (2 or 4) and the HHC configuration. A proper phasing of the 2/rev or 3/rev pitch inputs led to these observations.

Finally, still dealing with induced power, Ormiston in Ref. 3 investigated the characteristics of rotor induced power over a large range of advance ratios. The study considered a simple rotor with four rigid blades modeled through RCAS (Rotorcraft Comprehensive Analysis System). A prescribed wake model was used to compute the inflow distribution and blade lift stall was included in the airfoil lift model – although no description of the drag model was presented. Also using HHC to optimize the loading distribution across the rotor in a radial and azimuthal manner, interesting reductions in induced power were observed. At a high advance ratio of 0.5 and with -15 deg linear twist, a 20% induced power reduction was reported with a 2/rev input of 3 deg amplitude.

1.2.2 Power reduction through active control

As presented in the last two references, the idea of using active control technology in order to approach the ideal lift distribution leading to minimum power requirements for a given advance ratio seems natural due to the non-uniformities of the optimum airloads distribution. Higher Harmonic Control (HHC, depicted in Figure 1-10) was the first active control technology considered for rotor power reduction even though its main application in previous research has been for vibration reduction. Many studies have been conducted but the results are still somehow
inconclusive. Indeed, while negligible power reductions are observed in some studies using HHC for vibration suppression (Refs. 32, 33 & 34), other studies only predict increases in power requirements (Refs. 35-36). On the other hand, Shaw et al in Ref. 37 and Nguyen & Chopra in Ref. 38 observed interesting performance improvements. Shaw et al performed a wind-tunnel test on a 3-bladed articulated CH-47D Chinook rotor scaled model. If this study is mainly looking at vibration reduction, Shaw also examined the power reductions achievable using a 2/rev input of 2 degrees amplitude leading to power reductions of 6% at 135 knots and 4% at 160 knots. Using a comprehensive analysis of the same CH-47D rotor model, Nguyen and Chopra observed a power reduction up to 3.8% at high speeds with a 2/rev input of 2 degrees amplitude. However, a significant increase in 2/rev in-plane blade root shear loads was also reported and the study did not account for actuation power requirements.

![Figure 1-10: Schematic of Higher Harmonic Control (HHC)](image)

Individual Blade Control method (IBC, depicted in Figure 1-11), which can be seen as a generalization of the HHC concept (Ref. 39) since each blade is excited in the rotating frame with individual actuators, was first developed by Guinn (Ref. 40) and Ham (Refs. 41, 42) for vibration
reduction. This concept has also received some attention from the rotorcraft community in order to reduce power requirements. The effect of 2/rev root pitch IBC was studied by Jacklin et al in Refs. 43-44 on a 4-bladed hingeless full-scale BO-105 rotor. Power reductions of 4% with 1 degree amplitude and 7% with 2 degrees were observed for high advance ratios above 0.3. When actuation power requirements were included, the reductions were reevaluated at about 2.5%. Cheng et al observed similar results in Ref. 45 while examining IBC on a 4-bladed articulated UH-60 type rotor model. Using a 2/rev input of amplitude 1 degree, up to 1.5% power reduction was observed for a medium gross weight configuration and 3.8% power reduction for high gross weight. Those results however were obtained using a simplified structural model and a linear inflow model, limiting their accuracy. Indeed, using a free-wake model with the same helicopter model in Ref. 46, the previously observed power reductions were significantly reduced and became almost negligible. Kessler on the other hand (Ref. 47) examined the effect of 2/rev root pitch IBC input on power requirements during a flight test on a CH-53G helicopter. At an airspeed of 130 knots and using a IBC amplitude of 0.67 degrees, a power reduction of 2% was observed, which is reevaluated by the authors after correction for trim to be up to 6%.

Figure 1-11: Schematic of Individual Blade Control (IBC)
Another way to control the load distribution across the rotor is to use Active Trailing Edge Flaps (TEFs) along the blades as shown in Figure 1-12. In Ref. 17, Liu et al examined the power reductions achievable on a 4-bladed hingeless BO-105 type rotor using an analytical model and 1 or 2 Trailing Edge Flaps actuated at 2-5/rev. For an advance ratio of 0.35, power reductions of 1.73% with 1 TEF and 1.76% using 2 TEFs were observed, although these results were accompanied with an increase in vibrations by 100%. When aiming at reducing both vibrations and power requirements, a dual flap configuration led to a 0.67% power reduction. A power reduction of 1.46% was observed for a high thrust level and even more, about 1.82% reduction, for very high thrust levels. For an advance ratio of 0.4 the authors reported up to 4.04% power reductions when again vibrations are also reduced. These results however are based on a very simple TEF model where the drag varies linearly with the magnitude of the flap deflection, without accounting for the airfoil angle of attack or the Mach number which leads to a loss of accuracy in power reduction predictions. In a similar fashion, Bae et al in Ref. 49 conducted a study on rotor power reduction using multiple spanwise-segmented MiTEs (Miniature Trailing-Edge Effectors) on a 4-bladed articulated UH-60 type rotor. Using 4 MiTEs located from 50% to 90% of the blade and actuated at 1/rev and 2/rev, the optimal deployments were sought using a gradient-based optimization procedure for low, moderate and high gross weight configurations and moderate to high advance ratios – from 0.3 to 0.4. The results show that when stall is experienced by the baseline rotor, MiTEs have the ability to alleviate the phenomenon and large power reductions up to 40% were observed. When stall is not significant, MiTEs still give the opportunity to redistribute the airloads across the rotor and power reductions up to 8.74% were reported. However, the study used a rigid-blade model limiting the validity to high torsion frequency rotors. Moreover, vibration levels were not considered. A large part of the present work is very similar to the work performed by Bae et al.
In Ref. 9, Yeo compared a variety of active control methods for helicopter performance improvement. The active methods considered include leading edge slat, variable droop leading edge, oscillatory jet, Gurney flap, IBC, active twist and TEF. An AH-64 Apache rotor system with blades incorporating VR-12 airfoil characteristics was considered and the analysis was performed using CAMRAD II. The conclusions drawn from this study were that using IBC, active twist and TEF concepts can improve the rotor lift-to-drag ratio with a 2/rev harmonic control, and leading edge slat, variable droop leading edge, oscillatory jet and Gurney flap concepts can increase the loading capability when used over the retreating side of the rotor.

1.2.3 Stall Alleviation and Chord Extension Concept

Yeo in Ref. 9 also observed that the best performance improvements are obtained when the blade experiences stall. Indeed, the high-lift devices examined in this study increase the stall angle of attack, thus the maximum lift coefficient attainable. As a consequence, large power reductions can be observed when stall is alleviated since when stall occurs the slope of the power
curve quickly steepens. This observation is supported by Bae et al in Ref. 49, where power reductions of 28.1% and 40% were observed for flight conditions exhibiting retreating blade stall and combining respectively high advance ratio of 0.4 and moderate gross weight, and medium advance ratio of 0.3 and high gross weight. In Ref. 32, Kinzel et al also investigated MiTEs as a way to alleviate stall. Deploying the MiTEs when stall is approached, thus mostly on the third and fourth quadrants of the rotor, significant improvements in the helicopter flight envelope were observed. Indeed, increase in maximum flight speed up to 20%, increase in the maximum lift-to-drag ratio up to 16% and increase in the achievable thrust up to 10% were observed using this stall alleviation method.

Alleviating retreating blade stall is then an important way to improve the performances of a rotor. Such an objective can be attained using high-lift devices. Recently, a concept presented in Ref. 8 by Liu et al and referred to as Static Extended Trailing Edge (SETE) has shown promising characteristics in terms of airfoil lift generation. This concept, however, was not considered for stall alleviation and rotorcraft applications. Liu et al attached a static extended trailing edge to a NACA 0012 airfoil section and performed a set of wind-tunnel tests at low mach numbers as well as CFD simulations. Deploying the SETE up to 10% of the chord with an inclination angle varying from 0 degree to 14 degrees, the aerodynamic performances of the SETE were measured and compared to results obtained with a MiTE (also referred to as Gurney flap). The conclusions drawn from the results are indicating that using a SETE can increase the lift with low drag penalties. While exploring BVI noise reduction, Noboru et al reported supporting results in Ref. 51 by through a set of wind-tunnel tests at low Mach number on a same NACA 0012 airfoil section mounted with a tab – similar as a SETE. Extensions up to 15% of the chord and inclination angles from 0 degree to 6 degrees were investigated. Similar trends with and without inclination angle are observed for the lift and the drag coefficients, showing promising lift generation and rather low drag penalties. Consequently, the present study introduces a new
concept of quasi-static extended trailing edge device as a lift-augmentation and stall alleviation method for helicopter flight envelope expansion and explores the possibilities offered by such a device.

1.3 Problem Statement and Thesis Overview

Reducing rotor power requirements is of great interest in order to improve the global performances of a helicopter, in terms of range, endurance, speed, payload and ceiling altitude. As detailed previously, many studies have explored different ways of achieving this objective and many different design solutions have shown promising results. This thesis intends on continuing such a work by analyzing power reductions achievable using two different techniques, one dealing with stall-alleviation issues near the flight envelope boundaries with a newly introduced concept of trailing-edge extension and one aiming at redistributing the airloads across the rotor in the most efficient way throughout different flight conditions using multi-segmented trailing edge flaps.

The first step in this process is to develop a sufficiently accurate helicopter model in order to perform these two different studies. Chapter 2 deals with the formulation of the helicopter model. The physics behind the blade aerodynamics, the rotor model, the wake model, the helicopter model as well as a description of the optimization procedure used is presented. A validation concludes this chapter, showing the good performances of the model developed here.

A first set of results is then presented in Chapter 3 which examines the expansion of the flight envelope that can be achieved when reducing the power requirements while alleviating stall with a quasi-static extended trailing-edge device. The performance enhancement in terms of increased altitudes and maximum speeds are presented, as well as the possible power reductions. A prototype design is also introduced.
Following, a second set of results are presented in Chapter 4. This chapter examines the power reduction achievable using 4 spanwise-segmented TEFs actuated up to 2/rev for an advance ratio going from 0 to 0.4 and three different gross weights. This study examines the optimum flap deflections required and the associated power reductions, as well as the impact of using a non-linear twist and a linear twist.

Finally, the conclusions drawn from the three preceding chapters are summarized in Chapter 5. Recommendations for future work are listed at the end of this chapter.
Chapter 2

Formulation

In order to realistically assess the performance improvements on a helicopter using active methods such as Trailing Edge Flaps or Static Extended Trailing Edge, accurate helicopter model and propulsive trim analysis are necessary. The model used in this present research is essentially based on Bluman (Ref. 18), Johnson (Ref. 2) and Leishman (Ref. 1) and consists of a 4-bladed fully articulated rotor model configured to represent a UH-60A helicopter.

At any flight condition, the helicopter is fully trimmed using a forward-difference Newton-Raphson iterative method in order to determine the controls leading to equilibrium. The forces and moments generated by the rotor, the fuselage, the horizontal tail and the tail rotor are taken into account to calculate the net resultants acting on the helicopter. Blade response is solved using a time integration scheme until steady-state solution is reached. The blades are undergoing rigid-body flapping about the flap hinge and other degrees of freedom such as lag and feathering motions are not considered. Elastic deformations are also neglected since the study does not focus on loads and vibrations. The UH-60 blade properties such as chord span-wise variations, nonlinear twist, tip-sweep and use of different airfoil sections (SC-1095 and SC-1094R8) along the blade are taken into account. The aerodynamic model is based on blade element theory and the sectional lift coefficients are calculated using C81 airfoil tables based on the sectional angle of attack and Mach number. Finally a rigid prescribed wake is used for the inflow calculation.

This chapter presents the derivations, the implementation and the validation of this analytical helicopter model.
2.1 Inertial Blade Model

In the present articulated rotor model, only rigid-flapping blade motion is considered. The flapping response and the hub loads are shortly derived in the following sections. Small angle assumption is used and no hinge spring is considered.

2.1.1 Flapping Response

In order to determine the flapping response of the rigid blade, the equilibrium of inertial, centrifugal and aerodynamic moments about the flap hinge have to be considered as shown in Figure 2-1. Examining a small blade section, the following forces are to be considered:

- The inertial force \( \ddot{z}dm = \ddot{\beta}(r - e)dm \), with moment arm \( (r - e) \).
- The aerodynamic force \( F_z \), with moment arm \( (r - e) \).
- The centrifugal force \( \Omega^2 rdm \), with moment arm \( z = (r - e)\beta \).

![Figure 2-1: Rotor blade flapping forces and moments.](image-url)
However, such a formulation does not account for the blade pitch motion, which is constrained by the control input. Since there is an offset $x_i$ between the feathering axis and the center of gravity of each blade section, as shown in Figure 2-2 and Figure 2-3, the pitch motion contributes to the equilibrium equation and the actual forces to consider are:

- The inertial force $\left( z - x_i, \dot{\theta} \right) dm = \left( \dot{\beta}(r - e) - x_i, \dot{\theta} \right) dm$, with moment arm $(r - e)$.
- The aerodynamic force $F_z$, with moment arm $(r - e)$.
- The centrifugal force $\Omega^2 r dm$, with moment arm $z - x_i, \theta = (r - e)\beta - x_i, \theta$.

![Figure 2-2: Articulated rotor blade with flap and pitch degrees of freedom.](image)

![Figure 2-3: Blade section showing feathering axis and center of gravity locations.](image)
Integrating the moments generated by those forces over the blade span, the equilibrium can be written as in Equation 2.15.

\[
\begin{aligned}
\left( \int_{e}^{R} (r-e)^2 \, dm \right) \dddot{\beta} - \left( \int_{e}^{R} (r-e)x_i \, dm \right) \ddot{\theta} + \\
\left( \int_{e}^{R} r(r-e) \, dm \right) \beta \Omega^2 - \left( \int_{e}^{R} rx_i \, dm \right) \Omega^2 = \left( \int_{e}^{R} (r-e)F_{z} \, dm \right)
\end{aligned}
\] (2.15)

Using the notations as written in Equations 2.16 to 2.20, Equation 2.15 can be simplified and written as in Equation 2.21.

\[
\begin{aligned}
\left( \int_{e}^{R} (r-e)^2 \, dm \right) = I_{\beta} \\
\left( \int_{e}^{R} (r-e)x_i \, dm \right) = I_{x} \\
\left( \int_{e}^{R} r(r-e) \, dm \right) = S_{\beta} \\
\left( \int_{e}^{R} rx_i \, dm \right) = S_{i} \\
\left( \int_{e}^{R} (r-e)F_{z} \, dm \right) = M_{\beta}
\end{aligned}
\] (2.16) (2.17) (2.18) (2.19) (2.20)

\[
I_{\beta} \dddot{\beta} + (I_{\beta} + eS_{\beta}) \beta \Omega^2 - I_{x} \ddot{\theta} - (I_{x} + eS_{i}) \Omega^2 = M_{\beta}
\] (2.21)

In order to use a non-dimensional form of Equation 2.21, the notations \( \dot{X} = \Omega \dddot{X} \) and \( \dddot{X} = \Omega^2 \dddot{X} \) are generally used, with \( X \) the variable considered. Using this non-dimensional notation, dividing Equation 2.21 by \( I_{\beta} \Omega^2 \) and neglecting the term \( eS_{i} \) which is much smaller than \( I_{x} \), the following linear second-order ODE Equation 2.22 is obtained.
Where the non-dimensional natural flapping frequency $\nu_{\beta}$ is given by $\nu_{\beta}^2 = 1 + \frac{eS}{I_{\beta}}$ and

$I_x = \frac{I_e}{I_{\beta}}$. This Equation 2.22 is solved using a numerical time integration Runge-Kutta scheme until steady state flapping response is found.

It should be noted here that there is no ODE describing the pitching motion of the blade since in the current model the blade is considered rigid and no torsional root spring is used. Consequently the pitch angle and its derivatives are simply prescribed by the control inputs and Equations 2.23 and 2.24 are used to determine their values.

$$\theta = \theta_0 + \theta_{lc} \cos \psi + \theta_{ls} \sin \psi$$  \hspace{1cm} (2.23)

$$\theta = -\theta_{lc} \cos \psi - \theta_{ls} \sin \psi$$  \hspace{1cm} (2.24)

### 2.1.2 Hub Loads

The calculation of the loads acting on the hub is done once the blade response has been determined. These loads transmitted through the pitch bearing and the flap hinge are calculated by doing a force summation, integrating the resultant forces along the blade span. It should be pointed out that the flapping moments are not transmitted through the flapping hinge.

The definition of the root shears and moments on the rotating blade, as well as the forces and moments on the hub in the non-rotating frame is presented in Figure 2-4. As a consequence, we calculate the rotating loads as presented in Equations 2.25 to 2.29.

$$S_x = \int_e^r (F_x) dr$$  \hspace{1cm} (2.25)
Performing a Fourier Coordinate Transform from the rotating frame to the non-rotating frame attached to the hub, the total forces and moments acting on the hub are obtained, integrating over the azimuth to get the average results over one revolution. Equations 2.30-2.35 define the hub loads.
\[ H = \frac{N_B}{2\pi} \int_{0}^{2\pi} (S_r \cos \psi + S_z \sin \psi) d\psi \]  
(2.30)

\[ Y = \frac{N_B}{2\pi} \int_{0}^{2\pi} (S_r \sin \psi - S_z \cos \psi) d\psi \]  
(2.31)

\[ T = \frac{N_B}{2\pi} \int_{0}^{2\pi} (S_z) d\psi \]  
(2.32)

\[ M_x = \frac{N_B}{2\pi} \int_{0}^{2\pi} (eS_z \sin \psi + N_p \cos \psi) d\psi \]  
(2.33)

\[ M_y = \frac{N_B}{2\pi} \int_{0}^{2\pi} (-eS_z \cos \psi + N_p \sin \psi) d\psi \]  
(2.34)

\[ M_z = \frac{N_B}{2\pi} \int_{0}^{2\pi} (eS_x + N_q) d\psi \]  
(2.35)

### 2.2 Aerodynamic Model

The Blade Element Theory (BET) is used in the present study to model the aerodynamics of the main rotor. This theory assumes that each section of the blade acts as a quasi-2D airfoil. As seen in the previous section, the sectional forces and moments generated can then be integrated over the blade span to calculate the hub loads and the helicopter performances. In this study, the blade is divided in 40 blade elements and an azimuth step of 5 degrees was selected for time integration schemes.

A blade element subject to a free stream of velocity \( U \) is depicted in Figure 2-5. The local free stream velocity \( U \) can be separated in two components, the out-of-plane velocity \( U_p \) and the in-plane component \( U_T \). Both tangential and perpendicular velocities are a function of the angular rotation, the blade motion and the flight speed, as described by Equations 2.36 and 2.37.

\[ U_p = \dot{\lambda} \Omega R + r \dot{\Omega} \cos \beta + \mu \dot{\Omega} R \sin \beta \cos \psi \]  
(2.36)

\[ U_T = \Omega r + \mu \dot{\Omega} R \sin \psi \]  
(2.37)
The inflow angle $\varphi$ is then defined by Equation 2.38 and the effective angle of attack by Equation 2.39, being the difference between the local pitch angle and the inflow angle. This definition of the angle of attack is used to define the rates of change $\dot{\alpha}$ and $\ddot{\alpha}$ used in the quasi-steady theory. The rates of change of the plunging motion are defined by Equations 2.40 to 2.42.

$$\varphi = \tan^{-1} \left( \frac{U_p}{U_T} \right)$$  \hspace{1cm} (2.38)

$$\alpha = \theta_{loc} - \varphi$$  \hspace{1cm} (2.39)

$$\dot{h} = (r - e) \beta$$  \hspace{1cm} (2.40)

$$\ddot{h} = (r - e) \ddot{\beta}$$  \hspace{1cm} (2.41)

$$\dddot{h} = (r - e) \dddot{\beta}$$  \hspace{1cm} (2.42)

Figure 2-5: Aerodynamic environment and velocity perturbations around a blade element.

The local pitch angle $\theta_{loc}$ is different from the control pitch $\theta_{con}$ since it takes into account the blade twist $\theta_{tw}$ as described by Equation 2.43.
\[ \theta_{loc} = \theta_{con} + \theta_{tw} \]  

(2.43)

The UH-60 blade twist design is unique. Indeed, it features a non-linear down-up twist schedule at the tip of the blade and is presented in Figure 2-6. This schedule is used in the present model.

![Figure 2-6: UH-60 non-linear blade twist schedule.](image)

### 2.2.1 Quasi-steady Airloads

Considering an airfoil of chord-length c, a free stream of velocity U coming at an angle of attack \( \alpha \) on the blade section as shown in Figure 2-5, the sectional aerodynamic lift, drag and moment are given by Equations 2.44 to 2.46, with \( \rho \) the air density.

\[
dL = \frac{1}{2} \rho U^2 c C_L dr
\]

(2.44)

\[
dD = \frac{1}{2} \rho U^2 c C_D dr
\]

(2.45)
\[ dM = \frac{1}{2} \rho U^2 c^2 C_M \, dr \]  

(2.46)

It should be noted that the lift and drag forces are defined respectively perpendicular and parallel to the incoming flow velocity \( U \). A change of basis by the inflow angle \( \phi \) is necessary to obtain the resultant components \( dF_x \) and \( dF_z \) in the hub plane, as given by Equations 2.47 and 2.48.

\[
dF_x = dL \sin \phi + dD \cos \phi \quad (2.47)
\]

\[
dF_z = dL \cos \phi - dD \sin \phi \quad (2.48)
\]

In those equations, \( C_L \), \( C_D \) and \( C_M \) are respectively the lift, the drag and the pitching moment coefficients, representing the aerodynamic characteristics of the airfoil considered. Such coefficients are mainly dependant on the angle of attack and the Mach number. However, the physics described by those coefficients is not simple: it includes flow separation, stall, compressibility effects, etc. As a consequence, an accurate analytical representation of those coefficients is usually not available. This study uses C81 aerodynamic tables generated by the US Army Aeroflightdynamics Directorate (Ref. 7) and obtained through wind-tunnel tests to determine the aerodynamic characteristics of the steady airfoils. Two different airfoils are used on a UH-60A helicopter rotor blade: a SC-1094R8 airfoil and a SC-1095 airfoil.

Look-up tables give an accurate representation of a steady airfoil and capture flow separations, viscous and compressibility effects. However, a rotor-blade has pitching and plunging motions as it rotates azimuthally, as shown in Figure 2-5. Such an unsteady motion produces a distribution of perturbations that modifies the airloads. A quasi-steady formulation can be developed as detailed in Reference 1 and additional lift and pitching moment generated by circulation and apparent mass effects are predicted. Such a theory for flapped airfoil was first developed by Theodorsen (Ref. 52) who analyzed lift attenuation and delay due to flow
unsteadiness. A part of his work, representing the circulatory and non-circulatory airloads is used in this study, omitting however the lift deficiency function \((C(k)=1)\) here.

For a flapped airfoil, it is then convenient to separate the different contributors when examining the lift and pitching moment coefficients as presented in Equations 2.49 to 2.51.

\[
C_L = C_{L,\text{BASE}} + C_{L,\text{CIRC}} + C_{L,\text{NON-CIRC}} + \Delta C_{L,\text{TEF}} \tag{2.49}
\]
\[
C_D = C_{D,\text{BASE}} + \Delta C_{D,\text{TEF}} \tag{2.50}
\]
\[
C_M = C_{M,\text{BASE}} + C_{M,\text{CIRC}} + C_{M,\text{NON-CIRC}} + C_{L,\text{BASE}} \frac{d}{c} + \Delta C_{M,\text{TEF}} \tag{2.51}
\]

In the three expressions below, \(C_{L,\text{BASE}}, C_{D,\text{BASE}}\) and \(C_{M,\text{BASE}}\) are the aerodynamics characteristics of the steady airfoil extracted from the previously mentioned C81 tables. The circulatory and non-circulatory terms arise from the airfoil motion and the complete derivations can be found in Theodorsen’s theory.

![Figure 2-7: Definition of parameters for Theodorsen’s theory.](image)

It is convenient to define the following parameters as depicted in Figure 2-7:

- \(b\) the semi-chord, thus \(b = c/2\).
- $x_a$ the non-dimensional location of the pitch axis with respect to the semi-chord location – should have a negative value.

- $d$ the dimensional distance between the pitch axis location and the $\frac{1}{4}$-chord.

- $x_c$ the non-dimensional location of the flap hinge with respect to the semi-chord location.

It should be noted that since the pitching moment coefficients are not used in the rigid-blade model, the expressions for the pitching moment components are not presented here.

The circulatory term arising in the definition of the lift coefficient is given in Equation 2.52 and is a function of the pitching rate. One could notice that in the actual Theodorsen’s theory, the circulatory term is also a function of the angle of attack $\alpha$ and the flapping rate: those components are taken into account in the base lift coefficient $C_{L, \text{BASE}}$ since the local angle of attack $\alpha$ used account for the flapping rate – see the definition of $U_p$.

$$C_{L, \text{CIRC}} = 2\pi \left( \frac{1}{2} - x_a \right) \frac{b\dot{\alpha}}{U}$$  \hspace{1cm} (2.52)

The non-circulatory term is given by Equation 2.53. This term arises from the physical displacement of the air around the airfoil, air that has a certain inertia imparting reaction forces on the airfoil.

$$C_{L, \text{NON-CIRC}} = \frac{\pi b}{U^2} \left( \dot{h} + U\dot{\alpha} - bx_a \dot{\alpha} \right)$$  \hspace{1cm} (2.53)

The increments in lift and drag coefficients provided by the flap are obtained through CFD as detailed in Section 2.2.1.2.
2.2.1.1 Static Extended Trailing Edge Model

Based on the limited wind-tunnel test data, presented in Section 1.1.2, which indicated little to no change in the airfoil aerodynamic coefficient associated with the extension of a flat-plate tab through the trailing-edge of a NACA 0012 airfoil, and in the absence of any CFD data, the aerodynamics of the SETE are modeled based on some relatively simple assumptions. The SETE is composed of a thin rigid horizontal plate being deployed or retracted depending on the flight condition. This research assumed that the addition of this thin extended trailing edge did not significantly modify the aerodynamics coefficients compared to a clean SC-1094R8 with the same chord (the envelope expansion simulations are based on the UH-60 helicopter which uses the SC-1094R8 airfoil along the spanwise sections where the SETE would be introduced). Using the wind tunnel aerodynamic coefficient tables for a clean SC-1094R8 and the previous definitions of the aerodynamic coefficients, only the chord term was modified when calculating the lift and drag, as shown in Equations 2.54 and 2.55. The quasi-steady contributions will be neglected in this study when using the SETEs, so that only the airfoil base aerodynamic coefficients are used.

\[
L = \frac{1}{2} \rho V^2 (c + \varepsilon) C_L(a, M) \tag{2.54}
\]

\[
D = \frac{1}{2} \rho V^2 (c + \varepsilon) C_D(a, M) \tag{2.55}
\]

The chord extension denoted as \( \varepsilon \) is added to the baseline airfoil chord length \( c \) and the aerodynamic coefficients of the baseline airfoil, which depend on the angle of attack and Mach number, are used with no alteration.

The additional pitching moment generated by the device was not taken into account in the model since the blade was assumed torsionally rigid. However, increasing the chord moves the
center of pressure while the pitch axis location remains the same. Thus a non negligible nose-down pitching moment increment may be expected.

The SETE model used is simple but reasonable, based on the data from Ref. 8. Perhaps two points of concern are that the data in Ref. 8 are obtained for very low air speed and for symmetric NACA 0012 airfoils, and while SETE deployed at 0 deg inclination angle shows no reduction in lift coefficient, that may not hold true for a non-symmetric airfoil, such as the SC-1094R8. For the non-symmetric airfoil, extending the chord to increase the airfoil surface results in a reduction in the effective camber, which could decrease the lift coefficient. In that event, the predicted benefits with the SETE, using this model, would likely be optimistic. However, it is noted that the lift coefficient of the SETE can be significantly improved (Figure 1-7) if the extended trailing-edge is tilted at a small non-zero deployment angle as shown in Figure 2-8. It is then assumed that the results obtained using the current model are, if optimistic for a horizontal SETE, easily achievable with a SETE deployed at a small non-zero angle. Generation of aerodynamic data for non-symmetric airfoils with SETEs deployed at 0 deg and at non-zero inclination is a subject of future research.

![Figure 2-8: Static Extended Trailing Edge with (a) 0 deg deployment, and (b) non-zero deployment angle, \( \delta_{\text{SETE}} \).](image)
Figure 2-9: CFD Database Mach, AoA, and Flap Deflection Limits (Ref. 28)
2.2.1.2 Trailing-Edge Flap Contributions

The aerodynamic characteristics of a 20% TEF mounted on a SC-1094R8 airfoil were computed in Refs. 28 and 29 by Duling. The results were obtained for a range of angle of attacks, Mach numbers and deflection angles as depicted in Figure 2-9. Duling’s analysis provides lift, drag and pitching moment coefficients using a 2D Navier-Stokes CFD code (OVERTURNS) modified for TEF problems.

The increment in lift and drag are directly provided through a table and after interpolation the ‘deltas’ are added to the lift and drag coefficients as described in Equations 2.49 and 2.50. A simple tricubic interpolation scheme is used here since the table giving the aerodynamic coefficients depends on 3 variables – angle of attack, Mach number and TEF deflection.

2.2.2 Inflow Model

This study focuses on performance improvements using on-blade mounted devices such as TEFs and SETEs. Those high-lift devices will alter the lift distribution along the blade, and thus the circulation. As a consequence, the trailed vortices created at the edge of these devices will significantly modify the inflow distribution. If a simple linear inflow model can be used as a first approximation, a wake model taking account for those trailed vortices is required. Consequently, a rigid prescribed wake is developed for this present work. Such a prescribed wake is considered sufficient in this study for moderate to high advance ratios, since self-induced distortions in wake geometry are important only at low speeds and in descent conditions. In addition, such a wake is far less computationally demanding than a free-wake model, in view of the large amount of data generated in the present research and the use of the model in optimization procedures.
2.2.2.1 Linear Inflow

A simple inflow model, derived from momentum theory, first developed by Glauert and then modified by Drees, is to assume a linear variation of the inflow in both longitudinal and lateral directions. This model is presented in Equation 2.56 with $\lambda_0$ the mean average induced velocity at the center of the rotor, derived from uniform momentum theory (2.57), $C_T$ the rotor thrust coefficient and $\alpha_{HUB}$ the angle of the hub to the free stream velocity.

\[
\lambda_i = \lambda_0 \left( 1 + k_x \frac{r}{R} \cos \psi + k_y \frac{r}{R} \sin \psi \right)
\]

\[
\lambda_0 = \mu \tan \alpha_{HUB} + \frac{C_T}{2\sqrt{\mu^2 + \lambda_0^2}}
\]

The coefficients $k_x$ and $k_y$ derived by Drees based on vortex theory are expressed in Equation 2.58, with $\chi$ the rotor wake skew angle given in Equation 2.59.

\[
k_x = \frac{4}{3} \left( \frac{1 - \cos \chi - 1.8\mu^2}{\sin \chi} \right)
\]

\[
k_y = -2\mu
\]

\[
\chi = \tan^{-1} \left( \frac{\mu}{\lambda_0} \right)
\]

Such a model is attractive due to its simplicity. The results however are expected to be poor since the wake and thus the vortices trailed by the blades are not accurately modeled. An attempt to improve this inflow model that is worth mentioning is the application of the Prandtl’s tip-loss function (Ref. 2) that tries to account for the large increase in induced velocity at the tip of the blades due to the strong tip vortices that are typically trailed, considerably reducing the lift generated at the tip. This function is given in Equation 2.60 and modifies the lift and moment expressions – but not the drag – as written in Equations 2.61.
2.2.2.2 Rigid Prescribed Wake

2.2.2.2.1 Wake Geometry

A rigid prescribed wake gives a more accurate inflow model, taking account for the generation of vortices along the blade, even if it also has important limitations. A rigid prescribed wake assumes no mutual interactions between the vortices trailed by the blades as they rotate. Such an assumption makes the geometry of a rotor wake fixed, or prescribed, only depending on the advance ratio and the average inflow (Ref. 1), compared to a Free-Vortex Method which calculates the actual geometry of the wake due to self-distortion. This assumption holds for advance ratios typically greater than 0.1. The geometry of the wake relative to the Tip Path Plane (TPP) is defined by Equation 2.62 which is the mathematical expression of a helix as shown in Figure 2-10.

\[
F = \frac{2}{\pi} \cos^{-1}\left[ \exp\left( N_b \left( \frac{r - 1}{R - \frac{2}{2}\lambda_0} \right) \right) \right] 
\]

\[
dL = \left( \frac{1}{2} \rho U^2 c C_L dr \right) F 
\]

\[
dM = \left( \frac{1}{2} \rho U^2 c^2 C_M dr \right) F 
\]

In Equation 2.62, \( \psi_b \) is the azimuth of the blade, \( \psi_w \) the age of the vortex filament considered with respect to the blade and \( r_p \) the radial position of the vortex as depicted in Figure 2-11.
Figure 2-10: Prescribed geometry of the vortex filaments trailed by a blade.

Figure 2-11: Definition of the wake parameters.
2.2.2.2 Biot-Savart Law and Vortex Model

The Biot-Savart law is used in order to calculate the induced velocity due to a vortex filament at a given point on the blade. The wake is discretized in a finite number of vortex filaments delimited by collocation points. For simplicity those filaments are usually represented by straight line vortex segments on which one can apply the Biot-Savart law, as presented in Figure 2-12. Those filaments representing the vortices path are depicted in Figure 2-10 by the blue lines in the azimuthal direction. The radial lines only represent the time steps or the bound vortices that give the strength of the trialed vortices as explained later.

![Figure 2-12: Straight line vortex filament and parameters for application of Biot-Savart law.](image)

Equations 2.63 to 2.65 are used to calculate the contribution of a single filament to the induced velocity at a single point on the rotor. It is then necessary to sum the contributions of every filament at a single point on the rotor, all over the rotor, which requires a formidable amount of computation. In Equation 2.63, \( \Gamma_v \) is the strength of the vortex element, \( h \) the perpendicular distance to the line AB, and the collocation points A and B are given by the wake geometry described earlier.
\[ V = \frac{\Gamma_v}{4\pi h} \left( \cos \theta_1 - \cos \theta_2 \right) \frac{r_{AB} \times r_{AP}}{r_{AB} \times r_{AP}} \]  

\[ h = r_{AB} \sin \theta_1 = r_{BP} \sin \theta_2 \]  

\[ \cos \theta_1 = \frac{r_{AB} \cdot r_{AP}}{|r_{AB}||r_{AP}|} \]  

\[ \cos \theta_2 = \frac{r_{AB} \cdot r_{BP}}{|r_{AB}||r_{BP}|} \]  

Only the vertical component of Equation 2.63 is actually calculated to get the induced inflow across the rotor. Looking at the Equation 2.63, a singularity appears for \( h \) equals to 0, thus when \( P \) is located on the segment AB. This doesn’t have any physical meaning and a more realistic model has to be considered in order to prevent unreasonably high induced velocities in the inflow calculation when a vortex filament is near a control point. Moreover, those singularities can lead to convergence issues.

A 2D tangential velocity profile is generally used to model trailed vortices, neglecting radial and axial components. On Figure 2-13, two distinct regions can be observed when looking at the vortex representation. The first, the core region, can almost be modeled as the rotation of a solid body and is characterized by a given core radius. The second, the outer part, behaves as a potential flow.
Figure 2-13: Velocity field of trailed vortex.

Different models can be found in the literature – Rankine, Lamb-Oseen and Scully models for example. Vatistas gave the Equation 2.66 for the tangential velocity.

\[ V_\theta(r) = \left( \frac{\Gamma_c}{2\pi} \right) \frac{r}{(r_{c}^{2n} + r^{2n})^{\frac{1}{n}}} \]  \hspace{1cm} (2.66)

Equation 2.66 is only meaningful for n equals to 1 which is the Scully model and n equals to 2 which is close to the Lamb-Oseen model. The Scully model was used in the code for its simplicity; however a Lamb-Oseen may be used for more accuracy.

The Biot-Savart law can then be modified to take account for this vortex core model, giving Equation 2.67.

\[ \vec{V} = \left( \frac{\Gamma_c}{4\pi} \right) \frac{h}{(h_{c}^{2n} + r_{c}^{2n})^{\frac{1}{n}}} \left( \cos \theta_1 - \cos \theta_2 \right) \frac{\vec{r}_{AB} \times \vec{r}_{AP}}{|\vec{r}_{AB} \times \vec{r}_{AP}|} \]  \hspace{1cm} (2.67)
The vortex core radius is an important parameter for the calculation of the inflow and the loads. Based on experimental observations and measurements, the vortex core radius depends on many parameters and grows with wake age. Here again different models can be found to represent the behavior of the core radius. Bhagwat and Leishman (Ref. 53) give an expression for the core growth as written in Equation 2.68 and depicted in Figure 2-14.

\[
r_c(\psi_w) = \sqrt{r_0^2 + \frac{4\alpha\delta v\psi_w}{\Omega}}
\]

(2.68)

The initial vortex core radius \( r_0 \) can be taken equals to 5% of the chord, \( \alpha \) is a constant equal to 1.25643, \( \delta \) is the turbulent viscosity coefficient which can be considered as constant and approximately equals to 1000 for full-scale rotors, \( v \) is the kinematic viscosity coefficient, \( \psi_w \) is the age of the considered vortex element and \( \Omega \) is the rotational velocity of the rotor.

![Figure 2-14: Growth of a viscous vortex](image)
2.2.2.2.3 Vortex Strength

The strength of a vortex, denoted $\Gamma_v$, is obtained using the Kutta-Joukovski theorem that relates the lift to the circulation as shown in Equation 2.69.

$$L = \rho V \Gamma_v \quad \text{(2.69)}$$

The prescribed wake developed in this work takes only into account the trailed vortices. The impact of the shed vortices should be captured by the unsteady aerodynamic model. The strength of these trailed vortices is defined by the strength of the bound vortices. Indeed, the trailed vortices are generated by the variations of lift along the blade, thus the radial derivatives of the circulation, as opposed to the strength of a shed vortex that is given by the time or azimuthal derivative. Since the blade is segmented in the radial direction, the strength of a trailed vortex is defined as the difference of circulation between the 2 discrete bound vortices that generate it as shown in Figure 2-15 and defined in Equation 2.70. The strength of a bound vortex is given by Equation 2.69 and the lift is obtained using a simple lifting-line theory.

$$\left(\Gamma_n\right)_{\text{trailed}} = \left(\Gamma_n\right)_{\text{bound}} - \left(\Gamma_{n-1}\right)_{\text{bound}} \quad \text{(2.70)}$$

Figure 2-15: Bound circulation and vortices strength.
2.2.2.4 Inflow Calculation Procedure

The procedure to calculate the inflow distribution for a given set of rotor control inputs can then be explained. Using the specified rotor controls, the lift distribution across the rotor can be calculated using the rotor model developed previously and a linear inflow model as an initial guess for the inflow distribution. The lift distribution gives the circulation distribution that is used to calculate the strength of the trailed vortices. The geometry of the wake is already given analytically as described previously and no computation is required for this part, as opposed to a free-wake model. Using the Biot-Savart law modified to take into account the vortex model, the inflow distribution across the rotor can be computed. However, the new inflow distribution changes the actual lift generated across the rotor since the inflow angles are now modified. The process is then iterative: in order to find a converged solution, this procedure has to be repeated and the inflow distribution gradually modified until a stop criterion is reached – see Equation 2.71.

\[
\sqrt{\sum (\lambda_i^2)_{n+1}} - \sqrt{\sum (\lambda_i^2)_n} \over \sqrt{\sum (\lambda_i^2)_n} < 0.05\% \quad (2.71)
\]

2.2.2.5 Wake Age and Meshing

Another significant parameter in the wake modeling process is the meshing. For the present work, the choice has been made to use a fully meshed wake in order to capture the effects of strong vortices released by the edges of the on-blade devices. The number of vortex filaments can thus be very large as seen in Figure 2-10 and the computation cost huge. The meshing parameters have to be carefully selected in order to keep the computation time as low as possible and the accuracy of the calculation as good as possible. In order to avoid any interpolation issue that could lead to convergence problems, the same discretization as for the rotor forces and
moments integration scheme was selected (40 blade elements and 5° azimuth steps). The number of wake revolutions to take into account was selected based on a convergence study looking at the mean inflow across the rotor as seen in Figure 2-16. Convergence is obtained after different numbers of wake revolutions depending on the advance ratio, which is to be expected since the vortices are trailed away much faster for high advance ratios and have then less influence on the rotor induced velocities. After 3 revolutions a converged solution (greater than 99.5%) is generally fully obtained and this parameter will always be set to 3 in this study. The convergence is obtained when Equation 2.71 is satisfied.

![Figure 2-16: Mean inflow convergence for different wake revolutions using a rigid prescribed wake](image)
It should be noted however that using a fully meshed wake is clearly prohibitive in terms of computation time. A way to capture the effects of the vortices trailed by the high-lift devices mounted on the blade while using the same approach is to consider a full mesh on a fraction of the wake (near-wake) and then only the tip-vortex for the rest of the wake (far-wake), as depicted in Figure 2-17. Considerable computation time can then be saved without losing too much accuracy on the inflow calculation. Indeed, in the two following cases studied and presented in Figure 2-18 and Figure 2-19, for two advance ratios of 0.2 and 0.3, a roll-up angle of 360 degrees reduces the computation time by at least a factor 3 while the accuracy of the model in terms of power predictions decreases by less than 0.4%. Such a model with a roll-up azimuth angle set to 1 revolution and presented in Figure 2-17 is then used in the present work due to the large amount of data generated.

Figure 2-17: Prescribed wake mesh model with full-mesh on 1 revolution and rolled-up tip-vortex on the last 2 revolutions
Figure 2-18: Wake performance analysis at $\mu=0.3$ on a baseline rotor

Figure 2-19: Wake performance analysis at $\mu=0.2$ on a baseline rotor
2.2.2.6 Validation

The inflow distribution across the rotor disk given by the rigid prescribed wake model described in the previous section is compared with results obtained with CAMRAD II in Ref. 54 for 4 different advance ratios (0.1 to 0.4). The characteristics of the rotor used in this analysis are presented in Table 2-1 and a longitudinal trim is performed in order to obtain figures Figure 2-20 to Figure 2-23. It should be noted that CAMRAD II uses an advanced free-wake model that can be taken as a reference for the present analysis. These sets of figures present both CAMRAD II and the rigid prescribed wake model inflow distributions with the same scale for direct qualitative comparison.

It can be seen that the inflow distributions obtained using the rigid prescribed wake model for advance ratios greater than 0.1 are comparable to CAMRAD II results, and good agreement is generally observed. The main discrepancies arise in the reverse flow region, where the vortices are more likely to interact with each other, physics that is not captured by the prescribed wake model. These discrepancies however have a minor impact on the power predictions as discussed later and the model presented and used in the present research is considered accurate enough. These results give a good confidence in the model used and will be confirmed in the Section 2.4.
Figure 2-20: Inflow distribution across the rotor disk at $\mu=0.1$ given by (a) CAMRAD II and (b) the rigid prescribed wake.

Figure 2-21: Inflow distribution across the rotor disk at $\mu=0.2$ given by (a) CAMRAD II and (b) the rigid prescribed wake.
Figure 2-22: Inflow distribution across the rotor disk at $\mu=0.3$ given by (a) CAMRAD II and (b) the rigid prescribed wake.

Figure 2-23: Inflow distribution across the rotor disk at $\mu=0.4$ given by (a) CAMRAD II and (b) the rigid prescribed wake.
Table 2-1: Rotor parameters for inflow model validation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of blades</td>
<td>$N_b$</td>
<td>4</td>
</tr>
<tr>
<td>Rotor radius</td>
<td>$R$</td>
<td>20 ft</td>
</tr>
<tr>
<td>Root cutout</td>
<td>$R_{co}$</td>
<td>2 ft</td>
</tr>
<tr>
<td>Airfoil</td>
<td></td>
<td>NACA 0012</td>
</tr>
<tr>
<td>Blade solidity</td>
<td>$\sigma$</td>
<td>0.1</td>
</tr>
<tr>
<td>Linear twist</td>
<td>$\theta_{tw}$</td>
<td>-6°</td>
</tr>
<tr>
<td>Tip speed</td>
<td>$V_{tip}$</td>
<td>600 ft/s</td>
</tr>
<tr>
<td>Angular velocity</td>
<td>$\Omega$</td>
<td>30 rd/s</td>
</tr>
<tr>
<td>Air density</td>
<td>$\rho$</td>
<td>0.0023769 slug/ft$^3$</td>
</tr>
<tr>
<td>Weight</td>
<td>$W$</td>
<td>7527 lb</td>
</tr>
<tr>
<td>Flat plate drag coefficient</td>
<td>$S.C_D$</td>
<td>15 ft$^2$</td>
</tr>
</tbody>
</table>

2.3 Helicopter Model

The rotor structural and aerodynamic model has been described in the previous sections. In addition to the loads generated by the rotor, the fuselage, tail rotor and horizontal tail also contribute to the global equilibrium equations of the helicopter. The models used to compute these additional forces and moments generated on the helicopter are directly adapted from Bluman (Ref. 18) and Steiner (Ref. 26). The trim procedure is however different due to the addition of the wake model.

2.3.1 Rotor Forces and Moments

As previously detailed, the forces and moments acting on the rotor are computed using Equations 2.30 to 2.35. However because the rotor shaft on the UH-60 is tilted by an angle $\alpha_{sx}$ relative to the hub, a base transformation is required to have the reactions in the correct reference frame using Equation 2.72.
2.3.2 Tail Rotor Forces and Moments

The tail rotor model used in the present study is very simple. A uniform inflow is assumed and a Newton-Raphson procedure is used to determine its value. The tail rotor power requirements are not calculated and consequently are not added to the total helicopter power requirements. The tail rotor thrust is calculated using Equation 2.73.

\[ T_{TR} = \sigma_{TR} A_{TR} \rho \pi \Omega_{TR}^2 R_{TR}^4 \left( \theta \left( \frac{2}{3} + \mu_{TR}^2 \right) - \lambda_{TR} \right) \]  
\[ (2.73) \]

In addition, the tail rotor is tilted by an angle \( \varphi_{TR} \), and we have the transformation 2.74 in order to move the reaction forces and moments to the hub reference frame.

\[
\begin{bmatrix}
F_{XTR} \\
F_{YTR} \\
F_{ZTR}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & -\sin \varphi_{TR} & \cos \varphi_{TR} & 0 \\
0 & \cos \varphi_{TR} & \sin \varphi_{TR} & T_{TR}
\end{bmatrix}
\]  
\[ (2.74) \]

The properties of the tail rotor are summarized in Table 2-2.

**Table 2-2: Tail rotor properties**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angular velocity</td>
<td>( \Omega_{TR} )</td>
<td>150 rad/s</td>
</tr>
<tr>
<td>Rotor radius</td>
<td>( R_{TR} )</td>
<td>5.5 ft</td>
</tr>
<tr>
<td>Cant angle</td>
<td>( \varphi_{TR} )</td>
<td>20 deg</td>
</tr>
<tr>
<td>Blade solidity</td>
<td>( \sigma_{TR} )</td>
<td>0.1875</td>
</tr>
</tbody>
</table>
2.3.3 Horizontal Tail Model

The horizontal tail model uses a NACA 0012 airfoil C81 table to calculate the forces and moments generated, even though the actual airfoil is a NACA 0014. Lift and drag are computed using an angle of attack modified by the main rotor wake velocities. Such an angle is the result of a series of trials and errors by Bluman (Ref. 18) to match flight test. The lift and drag are then calculated using Equations 2.75 and 2.76, with \( A_{HT} = 45 \text{ ft}^2 \).

\[
L_{HT} = \frac{1}{2} \rho V_{\infty}^2 A_{HT} C_{L,HT} \tag{2.75}
\]

\[
D_{HT} = \frac{1}{2} \rho V_{\infty}^2 A_{HT} C_{D,HT} \tag{2.76}
\]

For the horizontal tail one has to consider the angle \( \eta_{HT} = \chi_{HT} + \alpha_{WL} \), with \( \chi_{HT} \) the wake slew angle and \( \alpha_{WL} \) the helicopter pitch attitude, to move the forces to the hub reference frame using then Equation 2.77.

\[
\begin{bmatrix}
F_{X,HT} \\
F_{Y,HT} \\
F_{Z,HT}
\end{bmatrix} =
\begin{bmatrix}
\cos \eta_{HT} & 0 & \sin \eta_{HT} \\
0 & 1 & 0 \\
-\sin \eta_{HT} & 0 & \cos \eta_{HT}
\end{bmatrix}
\begin{bmatrix}
D_{HT} \\
L_{HT}
\end{bmatrix} \tag{2.77}
\]

2.3.4 Fuselage Model

The lift induced by the fuselage pitch attitude was approximated in Ref. 55 using a least-square method on wind tunnel data. Equation 2.78 provides then the lift as a function of the helicopter pitch attitude in radian and positive nose down and Equation 2.79 provides the fuselage drag.

\[
L_F = \frac{1}{2} \rho V_{\infty}^2 \left( -1.0239 \alpha_{WL}^5 + 12.841 \alpha_{WL}^4 + 39.558 \alpha_{WL}^3 - 30.214 \alpha_{WL}^2 - 106.09 \alpha_{WL} \right) \tag{2.78}
\]

\[
D_F = \frac{1}{2} \rho V_{\infty}^2 \left( 35.14 + 1447.376 \alpha_{WL}^2 \right) \tag{2.79}
\]
Finally, the fuselage lift, drag and weight acting on the center of gravity are transformed using Equation 2.80.

\[
\begin{bmatrix}
F_{XF} \\
F_{YF} \\
F_{ZF}
\end{bmatrix} =
\begin{bmatrix}
\cos \alpha_{WL} & 0 & \sin \alpha_{WL} \\
0 & 1 & 0 \\
-\sin \alpha_{WL} & \cos \alpha_{WL}
\end{bmatrix}
\begin{bmatrix}
D_F \\
W \sin \varphi_F \\
L_F - W \cos \varphi_F
\end{bmatrix}
\]  

(2.80)

Figure 2-24: Side view of helicopter forces, moments and dimensions

2.3.5 Vehicle Force and Moment Sums

The forces and moments acting on the helicopter as shown in Figure 2-24 and Figure 2-25 are summed at the hub in order to calculate the total forces and moments. As a consequence, a series of vector transports are required to carry the forces and moments from a reference
frame to the hub frame denoted as \((XYZ)_h\) in the following figures. To perform these transformations, the helicopter dimensions are required and are represented in Table 2-3. It should be noted that the dimensions in the \(x\)-direction are positive from the hub to the tail, in the \(y\)-direction positive from the hub to the right door, in the \(z\)-direction from the hub to above. The helicopter pitch attitude \(\alpha_{\text{WL}}\) is positive nose-down and the roll attitude \(\varphi_F\) positive roll-right.

Figure 2-25: Front view of helicopter longitudinal forces, moments and dimensions
Table 2-3: Helicopter geometrical parameters (‘Distance wrt the hub)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shaft Forward Tilt</td>
<td>(a_{sx})</td>
<td>3°</td>
</tr>
<tr>
<td>Tail Rotor Cant Angle</td>
<td>(\varphi_{tr})</td>
<td>20°</td>
</tr>
<tr>
<td>Longitudinal CG Offset</td>
<td>(X_{cg})</td>
<td>1.525 ft</td>
</tr>
<tr>
<td>Lateral CG Offset</td>
<td>(Y_{cg})</td>
<td>0 ft</td>
</tr>
<tr>
<td>Vertical CG Offset</td>
<td>(Z_{cg})</td>
<td>-5.825 ft</td>
</tr>
<tr>
<td>Lon. Stabilator offset</td>
<td>(X_{ht})</td>
<td>29.925 ft</td>
</tr>
<tr>
<td>Lat. Stabilator offset</td>
<td>(Y_{ht})</td>
<td>0 ft</td>
</tr>
<tr>
<td>Vert. Stabilator offset</td>
<td>(Z_{ht})</td>
<td>-5.915 ft</td>
</tr>
<tr>
<td>Lon. Tail Rotor offset</td>
<td>(X_{tr})</td>
<td>32.565 ft</td>
</tr>
<tr>
<td>Lat. Tail Rotor offset</td>
<td>(Y_{tr})</td>
<td>0 ft</td>
</tr>
<tr>
<td>Vert. Tail Rotor offset</td>
<td>(Z_{tr})</td>
<td>0.805 ft</td>
</tr>
</tbody>
</table>

Using the previous definitions, the following equations summing the forces and moments at the hub can be written as shown in Equations 2.81 to 2.86. Finally, the total forces and moments acting on the helicopter are represented by vector \(\{F\}\) in Equation 2.87.

\[
\sum F_X = F_{XR} + F_{XTR} + F_{XHT} + F_{XF} \tag{2.81}
\]

\[
\sum F_Y = F_{YR} + F_{YTR} + F_{YHT} + F_{YF} \tag{2.82}
\]

\[
\sum F_Z = F_{ZR} + F_{ZTR} + F_{ZHT} + F_{ZF} \tag{2.83}
\]

\[
\sum M_X = M_{XR} + M_{XHT} + F_{YTR}Z_{TR} - F_{YR}Z_{CG} \tag{2.84}
\]

\[
\sum M_Y = M_{YR} + F_{ZT}X_{CG} - F_{X}Z_{CG} + F_{ZTR}X_{TR} + F_{ZHT}X_{HT} - F_{XHT}Z_{HT} \tag{2.85}
\]

\[
\sum M_Z = M_{ZB} + M_{ZHT} + F_{YR}X_{CG} + F_{YTR}X_{TR} \tag{2.86}
\]

\[
\{F\} = \left[ F_X, F_Y, F_Z, M_X, M_Y, M_Z \right]^T \tag{2.87}
\]

2.3.6 Trim Convergence Procedure

Once the forces and moments acting on the entire helicopter are determined as described in the previous section, an iterative relaxed forward-difference Newton-Raphson procedure is used to calculate the set of controls \(\{\theta\} = [\theta_u, \theta_\psi, \theta_{\alpha}, \alpha, \varphi, \varphi_{tr}]^T\) giving a trimmed helicopter.
The objective is to determine the input controls giving zero net vehicle loads and moments. This procedure is presented in Equation 2.88 where J is the Jacobian of the system, thus a 6x6 matrix giving the gradients as shown in Equation 2.89. A typical value for the relaxing parameter \( \eta \) is 0.2 or less.

\[
\{ \theta \}_{n+1} = \{ \theta \}_n - \eta [J]^{-1} \{ F \} 
\]

(2.88)

\[
[J]_{6x6} = 
\begin{bmatrix}
\frac{\partial F_X}{\partial \theta_0} & \frac{\partial F_X}{\partial \theta_{ic}} & \ldots & \frac{\partial F_X}{\partial \theta_{tr}} \\
\frac{\partial F_Y}{\partial \theta_0} & \frac{\partial F_Y}{\partial \theta_{ic}} & \ldots & \frac{\partial F_Y}{\partial \theta_{tr}} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial M_Z}{\partial \theta_0} & \frac{\partial M_Z}{\partial \theta_{ic}} & \ldots & \frac{\partial M_Z}{\partial \theta_{tr}}
\end{bmatrix}
\]

(2.89)

The convergence is obtained when Equation 2.90 is satisfied with a \textit{Tolerance} of 15, thus when the maximum force is less than 15 lbs and the maximum moment less than 15 ft.lbs which is a negligible quantity compared to the helicopter weight or torque – less than 0.1%.

\[
\max[\| F \|] \leq \text{\textit{Tolerance}}
\]

(2.90)

The trim procedure can then be summarized by a flow chart as depicted in Figure 2-26. In such a procedure however, it can be noted that the prescribed wake is recalculated for every helicopter controls update. This method is often quite time-consuming because of the large number of iterations sometimes required to trim the helicopter and trim convergence issues were observed. Consequently, a procedure where the inflow is computed out of the trim-loop was developed as presented in Figure 2-27. This second method appears to be more robust and faster since the wake is only calculated using trimmed input controls solutions. In this second flow chart, the ‘global convergence’ refers to the convergence of both the inflow and the helicopter controls, which means that any improvement in the controls does not change significantly the inflow distributions and reciprocally.
Figure 2-26: Trim procedure flow chart
Figure 2-27: Variation of the trim procedure flow chart
2.4 Validation

The model described in the previous sections is compared with CAMRAD II simulation results and flight test data obtained for a UH60-A helicopter. Consequently, the model is tailored to represent such a helicopter using the parameters presented in Table 2-4.

Table 2-4: UH-60A Helicopter properties

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight</td>
<td>( W )</td>
<td>18,300 lbs</td>
</tr>
<tr>
<td>Rotor radius</td>
<td>( R )</td>
<td>26.83 ft</td>
</tr>
<tr>
<td>Root cutout</td>
<td>( R_{co} )</td>
<td>3.83 ft</td>
</tr>
<tr>
<td>Blade chord(^+)</td>
<td>( c )</td>
<td>1.73 ft</td>
</tr>
<tr>
<td>Blade twist(^+)</td>
<td>( \theta_{tw} )</td>
<td>-16(^\circ), non-linear (Figure 2-6)</td>
</tr>
<tr>
<td>Rotor angular velocity</td>
<td>( \Omega )</td>
<td>258 RPM</td>
</tr>
<tr>
<td>Solidity</td>
<td>( \sigma )</td>
<td>0.0822</td>
</tr>
<tr>
<td>Induced power correction</td>
<td>( \kappa )</td>
<td>1.15</td>
</tr>
<tr>
<td>factor</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Airfoil(^+)</td>
<td></td>
<td>SC-1095 SC-1094R8</td>
</tr>
<tr>
<td>First flap non-dimensional</td>
<td></td>
<td></td>
</tr>
<tr>
<td>rotating frequency</td>
<td>( \nu_{\beta} )</td>
<td>1.04</td>
</tr>
<tr>
<td>Flapping moment of inertia</td>
<td>( I_{\beta} )</td>
<td>1861 slug.ft(^2)</td>
</tr>
<tr>
<td>Inertial flap-torsion coupling</td>
<td>( I_x )</td>
<td>1.5147 slug.ft(^2)</td>
</tr>
</tbody>
</table>

\(^+\)Varies span-wise

Comparing the blade input controls in Figure 2-28, Figure 2-29 and Figure 2-30, it can be seen that the trends are generally well captured compared to flight tests and CAMRAD II simulation results with both linear inflow and prescribed wake (PW) models. Discrepancies arise due to model simplifications. Indeed, the torsional root spring has been removed from the model and no elastic blade model was considered which would improve the blade controls predictions.
Figure 2-28: Collective pitch comparison over a range of airspeeds

Figure 2-29: Longitudinal cyclic pitch comparison over a range of airspeeds
Figure 2-30: Lateral cyclic pitch comparison over a range of airspeeds

The blade control inputs results obtained with the current model are however satisfactory for the present study compared to CAMRAD II results: the collective pitch follows the trend with a constant offset which is due to the lack of torsional root spring and the lateral cyclic pitch trend and values are well captured. One can note the large discrepancies (over 1 degree) between the linear inflow and prescribed wake models in Figure 2-28 for airspeeds lower than 40 knots.

The comparisons of the aerodynamic response of the blade are presented in Figure 2-31 to Figure 2-33. The coning angle is well captured compared to CAMRAD II results. The trend of the longitudinal cyclic flap is considered satisfactory for the purpose of the present study. The lateral cyclic flap however is not well captured, which can be explained again by the choice of a rigid-blade model and the use of a prescribed wake.
Figure 2-31: Coning angle comparison over a range of airspeeds

Figure 2-32: Longitudinal cyclic flap angle comparison over a range of airspeeds
However, for the purpose of this work, which look at the main power requirements, these results are considered good enough. Indeed, as presented in Figure 2-34, the power requirements are very well captured over the considered range of airspeeds compared both to flight tests and CAMRAD II simulation results. The current model is then considered valid for airspeeds greater than 40 knots using the prescribed wake model and a good confidence in power predictions is expected.

The helicopter pitch attitude closely matches CAMRAD II since it is mainly dominated by the horizontal tail slew schedule which was modeled using CAMRAD II results. The helicopter roll attitude trend and values are also well captured.
Figure 2-34: Helicopter power requirements comparison over a range of airspeeds

Figure 2-35: Helicopter pitch angle comparison over a range of airspeeds
Finally, very good correlation is observed between the current model and CAMRAD II or flight tests results. This was made possible by modeling the unique features of the UH-60A helicopter even if simplifications were made – no torsional root spring, no elastic blade model, no free-wake model for example. The power requirements are very well captured, which is mandatory for the present work. The current model using a prescribed wake is then validated for airspeeds over 40 knots.

2.5 Optimization Procedure for TEF Deflection Schedules

One of the objectives of this present work is to find the optimal deflection schedule of the 4 TEFs mounted on the rotor blades so that minimum power requirement is achieved. In order to
2.5.1 Variables, Objective Function and Constraints

One of the objectives of the present work aims at reducing the rotor power requirements using multiple-segmented TEFs located from 50% to 90% of the rotor blade. In order to perform such a task, 4 TEFs actuated up to 2/rev are used. This way the results obtained by other researchers when studying the effects of 2/rev inputs on rotor power reduction (Refs. 5, 4) are expected to be captured. Consequently, each flap deflection schedule is defined by 5 variables:

- The mean deflection $\delta_0$
- The first harmonic that can be defined by $\delta_{1c}$ and $\delta_{1s}$
- The second harmonic that can be defined by $\delta_{2c}$ and $\delta_{2s}$

The total deflection schedule is then given by Equation 2.91 with $N$ the TEF considered – from 1 to 4.

\[ \delta_N = \delta_0^{(N)} + \delta_{1c}^{(N)} \cos \psi + \delta_{1s}^{(N)} \sin \psi + \delta_{2c}^{(N)} \cos 2\psi + \delta_{2s}^{(N)} \sin 2\psi \quad (2.91) \]

With 4 TEFs, the number of variables in the optimization procedure is 20. For a given set of 20 deflection variables, the helicopter is trimmed and the power is calculated. The objective function is then a single scalar $P$ and is a function of 20 variables that can be grouped in a single vector that will be written as \{δ\}. The maximum deflection that can be achieved using a trailing edge flap is however limited and the current maximum stroke using piezoelectric material is about 5 degrees (Ref. 18). As a consequence, each flap is subject to a constraint function limiting the maximum value as written is Equation 2.92.

\[ \max |\delta_{\nu}| \leq 5^\circ \quad (2.92) \]
The optimization problem can then be summarized and written as in Equation 2.93.

\[
\begin{aligned}
\text{minimize } & P(\{\delta\}_{201}) \\
\text{with } & \{\delta\}_{201} = [\delta^{(i)}_0, \delta^{(i)}_{iC}, \delta^{(i)}_{iS}, \delta^{(i)}_{2C}, \delta^{(i)}_{2S}, \ldots, \delta^{(i)}_4, \delta^{(i)}_{iC}, \delta^{(i)}_{iS}, \delta^{(i)}_{2C}, \delta^{(i)}_{2S}]^T \\
\text{such that } & \forall N \in [1:4], \max|\delta_N| \leq 5^* 
\end{aligned}
\] (2.93)

### 2.5.2 Optimization Procedure

Using a simple gradient-based method, the optimal deflection schedule can be computed iteratively. Indeed, from a given deflection schedule \(\{\delta\}\), the gradient with respect to each variable gives the direction in which a local minimum can be found. The gradient is determined numerically by perturbing slightly the control vector \(\{\delta\}\) as written in Equation 2.94. The vector \(\{\tilde{\delta}\}\) is the vector \(\{\delta\}\) plus a perturbation \(\varepsilon\) on the \(i^{th}\) variable – from 1 to 20.

\[
\frac{\partial P}{\partial \delta_i} \approx \frac{P(\{\tilde{\delta}\}) - P(\{\delta\})}{\varepsilon}
\] (2.94)

The procedure used here is then to calculate the gradient for each variable and then to update the TEF deflection control vector \(\{\delta\}\) gradually using a relaxation parameter \(\alpha\) usually set to 3%. This method is described in Equation 2.95. The constraints are applied when calculating the total power for a given deflection perturbation so that the global deflections never exceed the limit.

\[
\{\delta\}_{n+1} = \{\delta\}_n - \alpha \left\{ \frac{\partial P}{\partial \delta_i} \right\}
\] (2.95)

Such a method is very simple and robust – although not efficient in terms of number of function evaluations. It requires the computation of every gradient for each step, which is
computationally demanding because of the 20 design variables but because of its robustness, this method was selected over other optimization algorithm that were giving inconsistent results.

In some cases where the helicopter flight conditions were extreme (high advance ratio and high gross weight) the trim procedure was not able to converge for certain deflection schedules \( \{\delta\} \) making the results diverging. It was noticed that using the built-in MATLAB function FMINCON could bypass this issue since it is less sensitive to such issues. This optimization function is also a gradient-based method, but uses much advanced optimization algorithms. However, using this function in a simple case did not give a correct converged solution or was getting close but not satisfactory enough. Further investigations using this method are worth studying due to its potential efficiency in terms of number of function evaluations.

2.5.3 Initial Guess

In order to start the optimization problem, an initial guess is used. In the present work, it was noticed that the TEF essentially work in a positive deflection mode – positive downward. An initial guess with a positive static deflection of 2 degrees was then assumed for the 4 TEFs.

Due to the large amount of time required to compute the optimal deflection schedule and the large number of cases studied, only one initial guess was used and obviously there is no guaranty that the solutions found are global minima but rather local minima. This issue is discussed however in the following section.

It can also be noticed that for a real-time implementation of this concept of TEF deflection optimum for power reduction, the starting point is a 0 degree deflection of every flap, not a 2 degrees static deflection. However Figure 2-37, Figure 2-38 and Figure 2-39 show the converged solutions obtained using these 2 different initial guesses for a gross weight of 18,300
lbs and 3 different advance ratios (0.3, 0.35 and 0.4). Very little difference is obtained in term of power reduction or optimal deflection solution for these 3 different cases.

Figure 2-37: Optimal deflections for $\mu = 0.3$ and $W = 18,300$ lbs using an initial guess of (a) 2 deg and (b) 0 deg.

Figure 2-38: Optimal deflections for $\mu = 0.35$ and $W = 18,300$ lbs using an initial guess of (a) 2 deg and (b) 0 deg.
2.5.4 On Finding the Global Minimum

As noted previously, there is no assurance that the global minimum is found using this method and it is more likely that a local minimum will be obtained. However, one specific case was studied in order to investigate this issue. For an advance ratio of 0.3 and a gross weight of 18,300 lbs, 1 uniform and 4 random initial guesses were used to generate the solutions presented in Figure 2-40 to Figure 2-44. The solutions obtained are extremely similar as well as the power reductions obtained despite the significant differences in the initial guesses, and tend to imply that this solution could be a global minimum. Obviously this does not mean that such a result is valid for every case studied in the next sections, however it seems that consistent solutions were obtained every time (see Sections 4.1 and 4.2) and that if better solutions can be found they would not be very different from the ones presented later.
Figure 2-40: (a) 1st initial guess and (b) solution obtained for $\mu=0.3$ and $W=18,300$ lbs.

Figure 2-41: (a) 2nd initial guess and (b) solution obtained for $\mu=0.3$ and $W=18,300$ lbs.

Figure 2-42: (a) 3rd initial guess and (b) solution obtained for $\mu=0.3$ and $W=18,300$ lbs.
Figure 2-43: (a) 4th initial guess and (b) solution obtained for $\mu=0.3$ and $W=18,300$ lbs.

Figure 2-44: (a) 5th initial guess and (b) solution obtained for $\mu=0.3$ and $W=18,300$ lbs.
Chapter 3

Power Reduction and Flight Envelope Expansion using Extendable Chord Sections

The first sets of results obtained in the present work analyze the power reductions achievable with the use of SETEs. Since this device is mostly considered here as a lift-augmentation or stall-alleviation device, the power reductions obtained will be presented in terms of flight envelope expansions, thus in terms of maximum speed and maximum altitude attainable. First of all, a close look at the aerodynamic characteristics of the SETE based on the previously described model compared to other high-lift devices will be presented. Then, flight envelope expansion results will be analyzed.

3.1 Comparison of the Aerodynamics of Trailing Edge Devices

In this section the aerodynamic performance of the SETE is compared with that of a TEF and a Gurney flap. TEF and Gurney flap aerodynamic data was obtained using CFD calculations on an SC-1094R8 airfoil. A 20% chord TEF was considered and lift and drag coefficient data (from Refs. 28, 29) is presented for TEF deflections, $\delta$, of 4, 8 and 12 deg. Gurney flap lift and drag coefficients are calculated in-house using the University of Maryland TURNS code and presented for deflections of h = 0.5%, 1% and 2% chord. In the Introduction and Analysis sections it was discussed that the SETE did not affect the lift and drag coefficients so much as
produce increments in lift and drag due to effective chord increase. However, for comparison purposes, equivalent aerodynamic coefficients for the SETE are calculated based on the reference chord length of the baseline airfoil as derived in Equations 3.96 to 3.99. As stated previously, it should be recalled that the additional pitching moment generated by the device was not taken into account in the model since the blade was assumed torsionally rigid.

\[
(C_L)_{SETE} = \left(1 + \frac{\varepsilon}{c}\right)\left(C_L\right)_{BASE} \quad (3.96)
\]

\[
(\Delta C_L)_{SETE} = \frac{\varepsilon}{c}\left(C_L\right)_{BASE} \quad (3.97)
\]

\[
(C_D)_{SETE} = \left(1 + \frac{\varepsilon}{c}\right)\left(C_D\right)_{BASE} \quad (3.98)
\]

\[
(\Delta C_D)_{SETE} = \frac{\varepsilon}{c}\left(C_D\right)_{BASE} \quad (3.99)
\]

Static trailing-edge extensions of $\varepsilon = 10\%$, $20\%$ and $30\%$ chord are considered in the present study. Since the primary interest is in the use of trailing-edge devices for stall alleviation, and retreating blade stall typically occurs at low Mach numbers, aerodynamic data for the TEF, Gurney flap and SETE are presented only at Mach 0.3, for comparison.

While $10\%$ and $20\%$ extensions appear to be quite reasonable, a $30\%$ extension may be considered large. This could make practical realization more challenging. Further, aerodynamic penalties arising from nonlinear phenomena not captured in the present analysis may start to become more significant. However, as noted previously in Section 2.2.1.1, the desired increment in lift could be achieved even with a smaller extension, if the extendable section is deployed at a small, non-zero, inclination angle (Figure 1-7).
Figure 3-1 and Figure 3-2, respectively, show the lift coefficient and the increment in lift coefficient, versus angle of attack for the three trailing-edge mechanisms, relative to the baseline airfoil at Mach 0.3.

At low angles of attack, the increment in lift coefficient of a 0.5% – 1% Gurney flap is comparable to that of a 4 deg TEF deflection, and that of a 2% Gurney flap is comparable to a 8 deg TEF deflection. However, at higher angles of attack, the effectiveness of a TEF in incrementing lift rapidly decreases due to flow separation and earlier onset of stall, and the Gurney flap performs better. The SETE ranks lowest in effectively incrementing the lift coefficient at low angles of attack, but its effectiveness increases almost linearly and it
outperforms both the TEF and the Gurney at very high angles of attack. From Figures 4a and 4b, a 10% SETE is comparable to a 0.5% GF, a 20% SETE to a 1% GF and a 30% SETE to a 2% GF in terms of lift coefficient improvements near stall.

Figure 3-2: Lift coefficient increments as a function of angle of attacks for trailing edge devices on a SC-1094R8 airfoil at Mach 0.3.

Figure 3-3 and Figure 3-4 show the corresponding drag coefficients and the increments in drag coefficient, respectively. While the increase in drag coefficient for the 0.5% Gurney flap may be modest and that for the 1% Gurney flap may be marginally acceptable, that for the 2% Gurney flap is extremely large. This is true both in absolute terms, as well as in comparison to other approaches at low-to-moderate angles of attack. At higher angles of attack, however, the
increment in drag coefficient associated with TEF deflection increases very rapidly and exceeds that of the Gurney flap (due to the earlier occurrence of stall). The effective increments in drag coefficient associated with the SETE are generally lower, and do not show rapid increase at high angles of attack.

Figure 3-3: Drag coefficients as a function of angle of attacks for trailing edge devices on a SC-1094R8 airfoil at Mach 0.3.
Figure 3-4: Drag coefficient increments as a function of angle of attacks for trailing edge devices on a SC-1094R8 airfoil at Mach 0.3.

Figure 3-1 to Figure 3-4 show the lift increments and corresponding drag penalties associated with the three trailing-edge devices, but the ratio allows the best comparison. Figure 3-5 and Figure 3-6 show the lift-to-drag ratios and lift-to-drag increment ratios, respectively, versus lift coefficient. For a given $C_L$, the more efficient device in terms of lift generation is the one producing less drag, thus high lift-to-drag ratio. Up to $C_L$ values of about 1.3, TEFs provide the best lift-to-drag ratio, whereas Gurney flaps show a significant reduction, relative to the baseline airfoil (Figure 3-5). However, the maximum $C_L$ that can be achieved with TEFs is lower than that with Gurney flaps or SETEs. Overall, the lift-to-drag ratios achieved with SETEs at high $C_L$ are better than those possible with Gurney flaps. With the high $C_L$ values possible with
Gurney flaps and SETEs, flight conditions that were not accessible with baseline airfoils are expected to be reachable using these devices. This would be at a lower cost using SETEs due to the better $C_L/C_D$ values. Supporting data is presented in Figure 3-6, where the lift-to-drag increment ratios are plotted versus the lift coefficient of the baseline SC-1094R8 airfoil. Thus, for any baseline airfoil $C_L$ (corresponding to a given angle of attack), the increment in lift per unit increment in drag for the various devices can be compared. Note that from Equations 3.97 and 3.99, the ratio is independent of the magnitude of the extension, $\varepsilon$. Above a $C_L$ of 1.2 on the baseline airfoil, deploying the SETE appears to be the most efficient means of increasing lift, of the three trailing edge devices compared.

![Figure 3-5: Lift-to-drag coefficient ratio versus lift coefficient for trailing edge devices on a SC-1094R8 airfoil at Mach 0.3.](image-url)
Flight Envelope Expansion

Flight envelope expansion is explored using the previously described UH-60 helicopter and SETE aerodynamic models. For this study, a SETE of 20% span length, located from 70–90% radius is considered.
3.2.1 Increase in Maximum Speed and Altitude

Envelope expansion is first considered for a moderate vehicle gross weight of 18,000 lbs. For this weight, the maximum attainable speeds at given altitudes for the baseline rotor and using SETE devices are shown in Figure 3-7. The boundaries of the flight envelope are depicted in this plot and correspond to an engine power limitation. It is assumed that the available power to the main rotor for this helicopter model is 3,000 HP at mean sea level, which is representative of a UH-60A double-turboshaft capability. Furthermore, increasing the altitude reduces the air density which also decreases the engine power output. A good approximation for a turboshaft engine power output is given by Equation 3.100, with the altitude H in feet.

\[
P_{av} \approx P_{MSL} \sigma \\
\text{with } \sigma = \frac{\rho}{\rho_0} = \left(1 - 6.8756 \times 10^{-6} H\right)^{4.255876} \tag{3.100}
\]

At altitudes below 10,000 ft where the maximum airspeed is greater than 150 knots, no improvements are observed with chord extension. With high drag penalties due to advancing blade compressibility being the dominant cause for increase in rotor power, the SETEs are not able to improve performance. For altitudes above 12,000 ft, the balance between compressibility and retreating blade stall is changing. Indeed, as altitude increases, air density decreases and higher angles of attack are required to provide lift. Increasing angles of attack gets the retreating blade closer to stall, where SETEs can be helpful. Compared to the baseline flight envelope, moderate improvements are observed when using the SETEs. A rotor is limited by the amount of power necessary to produce the required lift or by the impossibility to produce the required thrust because of the maximum aerodynamic lift characteristics of the rotor. In this case, for a medium gross weight helicopter, the aerodynamic limits are not reached, but the power necessary to meet the requirements is attained and the SETEs are not fully exploited, since they are used away from
stall. As an example, the flight condition represented by the point A in Figure 3-7 is detailed. For an altitude of 15,000 ft, the baseline maximum speed is 131 knots. A 30% SETE increases the speed by 8 knots, which represents a modest 6% improvement. Similarly, at an airspeed of 131 knots, the baseline maximum altitude is 15,000 ft, while a 30% SETE increases the maximum altitude by 1,500 ft, thus 10% improvement. With this UH-60 configuration and medium gross weight, the SETE device gives moderate altitude improvements.

Figure 3-7: UH-60A 18,000 lbs gross weight, altitude versus maximum speed envelope extension using SETEs.
Figure 3-8: UH-60A 24,000 lbs gross weight, altitude versus maximum speed envelope extension using SETEs.

Figure 3-8 plots the maximum altitudes and airspeed for a gross weight of 24,000 lbs. For larger gross weight, higher angles of attack are necessary to meet thrust requirements. Consequently, the rotor gets close to retreating blade stall sooner and benefits of the SETE are seen to be greater. In Figure 3-8, at altitudes below 4,000 ft where maximum airspeeds are higher than 140 knots, compressibility effects are again significant factors and the rotor reaches its power limit. However for altitudes above 4,000 ft the SETEs significantly expand the flight envelope boundaries. In those regions retreating blade stall is approached and the power available is still large enough not to be as constraining as in the previous example. The flight envelope expansion observed using the SETEs is then greater. Point B in Figure 3-8 is taken as an example. For an altitude of 8,000 ft, the baseline maximum airspeed is 112 knots. With the 30% SETE the
maximum speed is increased by 15 knots, which represents 13% increment. Similarly, for an airspeed of 112 knots, the baseline maximum altitude is 8,000 ft while the 30% SETE increases it by 2,000 ft, which gives a 25% increment. Conclusively, SETEs show significant flight envelope expansion capability when retreating blade stall is approached, i.e. for high gross weight and high altitudes.

However, the flight envelopes presented in Figure 3-8 are still power limited and do not completely exhibit stall on the retreating side of the considered rotors. Assuming a greater installed power, the retreating blade can have higher angles of attack and gets closer to stall. Figure 3-9 shows the flight envelope expansions for an installed power of 4,000 HP at mean sea level, representative of a UH-60L engine capabilities. Looking at point C, which is at the same altitude of 8,000 ft as in the previous example, it can be seen that a 30% SETE increases the maximum speed by 26 knots, a 22% improvement over the baseline. At an airspeed of 118 knots, an increase in altitude by 3,000 ft is observed (point D), which represents 37% improvement. Figure 3-10 shows the percent increase in ceiling altitude compared to the baseline rotor when using the SETEs. A larger relative percent increase is observed for high speeds even though the increments in altitude are almost constant. Indeed, for an airspeed of 140 knots, the increment in altitude is still 3,000 ft, but represents now a 50% increase.

The flight conditions attainable using the SETEs in this example cannot be reached using a baseline rotor with a larger available power installed since stall is occurring and no additional lift can be extracted from the blade. The chord extensions provide here the necessary lift increments to go beyond this aerodynamic limit and to expand the flight envelope.
Figure 3-9: Altitude versus maximum speed envelope extension using SETEs for a UH-60A 24,000 lbs gross weight, with increased installed power of 4,000 HP at sea level.

Figure 3-10: Percent increase in ceiling altitude as function of maximum speed using SETEs for a UH-60A 24,000 lbs gross weight, with increased installed power of 4,000 HP at sea level.
Figure 3-11: Baseline rotor disk angle of attack distribution for 24,000 lbs gross weight, at 118 knots, 8,000 ft altitude condition.

Figure 3-12: Lift distribution difference between 30% SETE rotor and baseline rotor at 24,000 lbs gross-weight, 118 knots, maximum altitude (11,000 ft for 30% SETE rotor, 8,000 ft for baseline rotor).
The angle of attack distribution of a baseline rotor operated at point C in Figure 3-9, thus at an altitude of 8,000 ft and an airspeed of 118 knots, for a high gross weight and high installed power helicopter configuration is presented in Figure 3-11. The rotor is stalled almost entirely over its retreating side. The trailed vortices generated by lift differences along the blade create peak regions of high angles of attack since the baseline blade geometry is not uniform due to varying airfoils, non linear twist and the presence of the trim tab. The high lift increment delivered by the SETE close to stall condition is then fully exploited when the chord is extended leading to significant altitude improvements presented in Figure 3-9. At an airspeed of 118 knots, Figure 3-12 shows the difference in lift around the rotor disk at maximum ceiling altitude between the rotor with SETEs (point D on Figure 3-9) and the baseline rotor (point C on Figure 3-9). Clearly, the SETE provides a large lift increase over an entire annulus of the rotor, with larger increments on the retreating side, enabling the helicopter to be operated at higher altitude before being stall or power limited.

3.2.2 Increase in Gross-Weight

As seen previously, a large gross weight implies high thrust requirements leading the rotor closer to stalled condition. Figure 3-13 shows the impact of increasing gross weight on the rotor power requirements for an airspeed of 112 knots and an altitude of 8,000 ft (point B on Figure 3-8 and Figure 3-13). For gross weights lower than 20,000 lbs, no improvements in power are observed since retreating blade stall does not occur. For higher gross weight, power improvements are progressively more observable until the baseline stalls, which is characterized by the sharp increase in power requirements. Above 24,500 lbs, the baseline cannot be operated at this flight condition. However, using SETEs leads to greater possible gross weights until the modified rotors stall as well. Operating with SETEs gives the opportunity to increase the
helicopter payload at a given power. Point B on Figure 3-13, which represents a 24,000 lbs gross weight helicopter operated at 8,000 ft altitude and 112 knots, requiring 2360 HP, is used as an example. For a power of 2360 HP, the maximum gross weight using a 30% SETE is increased by 1,600 lbs, a 6.7% improvement. Stall is then clearly alleviated using SETEs for high gross weights. With a greater available power of 4,000 HP at sea-level, the maximum gross-weight for the baseline rotor at this flight condition is 24,300 lbs (point E on Figure 3-13). In this case, using a 30% SETE increases the maximum payload even more by 2,400 lbs, a 10% improvement. This plot also provides an insight on the power required to operate the rotor at a given gross weight. Rotor power reductions can be realized when using SETEs. For a 24,000 lbs gross-weight for example, using a 30% SETE reduces the power requirements by 450 HP, while for a 24,300 lbs gross weight, the power is reduced by 1050 HP.

Figure 3-13: Rotor power required as a function of gross weight, for 8,000 ft altitude, 112 knots airspeed.
3.2.3 Power Reduction

Another way of examining the effects of the SETE is to consider the power reduction achievable when getting away from retreating blade stall. For this purpose, the cases of a rotor operated at 24,000 lbs gross weight and 8,000 ft altitude with 3,000 HP and 4,000 HP available power at sea-level are examined in Figure 3-14. This example was discussed previously (points B and C on Figure 3-8 and Figure 3-9). An airspeed sweep was performed for the baseline and for different SETE deployments in order to plot the rotor power requirements. The 26 knots increment in maximum attainable speed can again be observed for the higher available power case. In addition, it is noted that the benefits obtained with a 20% SETE are fairly close to those obtained with a 30% SETE for this flight condition. For 2360 HP available to the main rotor, the maximum speed of the baseline is 112 knots, and at this speed the 30% chord extension reduces the power requirements by 450 HP, which represents a 16.7% power reduction. For 3140 HP available to the main rotor, the maximum speed of the baseline is 118 knots, and at this speed the 30% chord extension reduces the power by 1050 HP, giving a 33.4% power reduction. Since stall is characterized by a large increase in drag, such a number is expected when stall is alleviated. As a consequence, these airspeeds above 110 knots that a UH-60 baseline helicopter would reasonably never be operated at under this flight condition are accessible with SETE devices as seen in Figure 3-14. Approaching stall, significant power reduction is achievable using SETEs and flight conditions not reachable with a baseline rotor due to retreating blade stall are attainable thanks to the lift increment and stall alleviation provided by those devices leading to a large alleviation of stall and associated penalties. This conclusion is supported by Figure 3-15 to Figure 3-17. At the flight condition of 112 knots, 8,000 ft and 24,000 lbs, the difference in rotor disk lift, drag and angles of attack distribution between the baseline and a 30% SETE rotor are plotted. The lift increment over an annulus described by the SETE in Figure 3-15 presents significant
improvements, especially along the retreating side of the rotor disk. Although a drag increment is observed over the annulus where the SETE is deployed (Figure 3-16), this is more than offset by drag reductions elsewhere to result in a net reduction in rotor power. Figure 3-17 shows reductions in angles of attack over most of the rotor disk with SETE deployment, with the largest reductions on the retreating side. The power reduction observed are occurring on the flight envelope boundaries through stall alleviation, which implies that the power reductions are mainly coming from profile power reductions.

Figure 3-14: Rotor power required versus airspeed for 24,000 lbs gross weight aircraft at 8,000 ft altitude (horizontal lines correspond to 3,000 HP and 4,000 HP available sea level power).
Figure 3-15: Lift difference between 30% SETE rotor and baseline rotor at 24,000 lbs gross-weight, 112 knots, 8,000 ft altitude.

Figure 3-16: Drag difference between 30% SETE rotor and baseline rotor at 24,000 lbs gross-weight, 112 knots, 8,000 ft altitude.
Figure 3-17: Angle of attack difference between 30% SETE rotor and baseline rotor at 24,000 lbs gross-weight, 112 knots, 8,000 ft altitude.

3.2.4 Dependence on SETE Location and Span Length

All the preceding results were presented for a 20% span SETE located between 70–90% radius (mid-point location, x_{SETE} = 0.8). In this section, the influence of change in SETE size and location is examined. The effectiveness of the SETE is most pronounced in stalled flight conditions, so a condition approaching retreating blade stall on the baseline rotor (8,000 ft altitude, 112 knots, 24,000 lbs gross weight) was selected for consideration, and change in power requirements with SETE size and location were examined.
10%, 20% and 30% chord extensions showed results and led to conclusions that were qualitatively similar. Consequently, only the results for 20% extension are presented in Figure 3-18. In this figure, the upper region represents non-feasible designs based on the geometrical constraint described by Equation 3.101. The SETE was not allowed to extend into the blade tip because the sweep and the highly non-linear twist in this region would not provide an appropriate location for such a device.

\[ x_{\text{SETE}} + \frac{l_{\text{SETE}}}{2} \leq 0.95 \quad (3.101) \]
In Equation 3.101, $x_{\text{SETE}}$ is the location of the mid-point of the SETE and $l_{\text{SETE}}$ is the spanwise length, both non-dimensionalized by rotor radius. From Figure 3-18 it is observed that for a given SETE size, $l_{\text{SETE}}$, an outboard location results in greater power reductions. Similarly, for a given SETE location, $x_{\text{SETE}}$, a larger size is preferable. To the extent possible, then, the larger and the farther outboard the SETE, the better. Of course, reduced power of a larger SETE has to be balanced against the practicality and weight of a moderately sized SETE, and other such considerations. From Figure 3-18, it is observed that the 20% span SETE located at 80% radius (used throughout this study), is a near-optimal selection, although slightly larger reductions in power could have been obtained increasing the span length to 25%. Small SETEs located inboard (bottom left of Figure 3-18) would actually result in power increase (note that the baseline power is 2360 HP) since the profile drag increase is greater than their effectiveness in stall alleviation.

### 3.3 Prototype SETE Device

A prototype SETE device was designed and built by Eric Hayden at the Pennsylvania State University as detailed in Ref. 56. Practical realization of the concept appears feasible. The device in its retracted and fully deployed configurations is shown in Figure 3-19. After studying different practical solutions, a morphing X-truss mechanism was designed to actuate the extendable plate through the trailing-edge. Figure 3-20 shows the device without the top skin and in different phases of the deployment. The blade section with the SETE mechanism was fabricated and appeared to operate well.
Figure 3-19: Assembled prototype with SETE in (a) fully retracted, and (b) fully extended configurations.
Figure 3-20: Prototype with top skin removed showing morphing truss in (a) fully retracted, (b) partially extended, and (c) fully extended configurations.
Chapter 4

Rotor Power Reduction using Multiple Spanwise-Segmented Trailing Edge Flaps

This chapter presents a second set of results focusing on rotor power reduction using multiple spanwise-segmented Trailing Edge Flaps (TEFs) actuated up to 2/rev in the most optimal manner using a gradient-based optimization scheme detailed previously in Section 2.5. The first part deals with power reductions on a UH-60 helicopter model including the non-linear twist of the blade. The mechanisms of power reductions are explained for a large variety of flight conditions. The same procedure is then repeated with a -8 deg linear blade twist in order to determine the importance of the initial rotor design. The effect of the number of TEFs is also investigated. Nominally 4 TEFs optimally scheduled up to 2/rev and located from 50% to 90% rotor radius are used.

The following TEF numbering scheme is used in the present work as depicted in Figure 4-1:

- TEF1 is located from 50% to 60% of the rotor radius.
- TEF2 is located from 60% to 70% of the rotor radius.
- TEF3 is located from 70% to 80% of the rotor radius.
- TEF4 is located from 80% to 90% of the rotor radius.
4.1 Results for a Non-Linear Twist Distribution

Using the UH-60 helicopter model described previously with the original non-linear twist, power reduction mechanisms are analyzed in this section for three different gross weights: a low gross weight of 16,000 lbs, a medium gross weight of 18,300 lbs and a high gross weight of 22,000 lbs, in order to study the impact of thrust requirements on power consumption. For each gross weight case, a series of power optimizations was performed over a range of advance ratios ranging from 0 to 0.4. However, due to the large amount of data generated, only the results concerning the medium gross weight case are detailed, the results obtained for the lower and higher gross weights are summarized and more results can be observed in the Appendix.

4.1.1 Medium Gross Weight, 18,300 lbs

4.1.1.1 $W = 18,300$ lbs, $\mu = 0$

First of all, an important note has to be made here: as explained in Section 2.2.2.2, for an advance ratio lower than 0.1 the rigid prescribed wake produces poor results in terms of inflow...
and thus power predictions because of its inability to account for self-induced distortions. As a consequence, the results observed in this section are presented for sake of completeness, but no serious deductions should come out of this hover case.

The optimal TEFs schedules are presented in Figure 4-2a. It can be observed that in hover, the optimal schedules are not constant deflections as one could expect: when considering the entire UH-60 helicopter model, the required thrust is not quite vertical and cyclic controls are actually needed. The use of TEFs provides a better airloads distribution across the rotor, mainly using static and 2/rev deflections. This 2/rev deflection scheme makes the TEFs deploy on the advancing ($\Psi = 90$ deg.) and retreating side ($\Psi = 270$ deg.) of the rotor. The total power convergence history is shown in Figure 4-2b and a total power reduction of 4.14% is observed once the optimization process has converged. Again, little confidence is put in this percent reduction for this specific case, but the physics behind it is worth studying.

In order to understand the mechanism behind this power reduction and for all the other cases, it is convenient to look at some disk plots. Figure 4-3 shows the lift distributions of the rotor with TEFs optimally actuated in figure a, the baseline rotor in figure b and the lift difference between these two cases in figure c. Clearly, examining Figure 4-3c, the TEFs redistribute the airloads over the entire rotor, with increases in lift on the advancing and retreating side as expected when looking at the TEFs deflection schedules in Figure 4-4a. The drag for the optimally flapped rotor, for the baseline rotor and their difference can be seen in Figure 4-4. An important observation can be made in Figure 4-4c: even if the drag is increased on the inboard regions where the flaps are deflected, the extreme outer portion of the blade (the last 10%) undergoes a significant decrease in drag. The lift is also reduced on the outer portion of the blade, which can be explained looking at Table A-3 in the Appendix. Indeed the trim controls are reduced when using the optimally deflected TEFs which decreases the lift on the outer part while the TEFs increase the lift inboard. The decrease in drag on the outer portion of the blade
compensates the increment seen inboard due to the greater moment arm and globally reduces the power requirements.

Figure 4-5 breaks down the sources of drag difference observed between the optimally flapped rotor and the baseline. Indeed, as discussed in Section 1.1.1 the drag can be decomposed in an induced plus propulsive drag (the $dL \sin \phi$ term) and a profile drag (the $dD \cos \phi$ term). Since the propulsive power does not change when comparing the baseline and the optimally flapped rotors (very small differences in the helicopter pitch attitudes $\alpha_{WL}$ can be seen in the Appendix for all the different cases), looking at the difference between them shows directly the differences of induced drag and profile drag – the propulsive one being removed by subtraction. As a consequence, Figure 4-5a gives the induced drag difference between the optimally flapped rotor and the baseline, while Figure 4-5b provides the profile drag difference in a similar fashion.

Compared to Figure 4-5a, the scale of Figure 4-5b is significantly lower by an order of magnitude, meaning that the profile power has little impact on power reduction in this case. This is to be expected since the main source of power is the induced one in hover (Refs. 1, 5). Figure 4-5a shows then that the power reductions achieved arise from the reduction in induced drag on the outer portion of the rotor disk. However these figures only give a qualitative answer on the how power reductions are achieved and it is hard to tell how much induced power is saved since Figure 4-5a shows decreases on the outer region of the blade but also increases inboard. More details on the exact power reductions observed are presented in Section 4.1.3.
Figure 4-2: Non-linear twist, GW = 18,300 lbs and μ = 0, (a) optimal TEFs deflection schedules and (b) total power convergence history.
Figure 4-3: Non-linear twist, GW = 18,300 lbs and μ = 0, (a) Optimal TEFs lift, (b) Baseline lift and (c) Lift difference between a and b.

Figure 4-4: Non-linear twist, GW = 18,300 lbs and μ = 0, (a) Optimal TEFs drag, (b) Baseline drag and (c) Drag difference between a and b.

Figure 4-5: Non-linear twist, GW = 18,300 lbs and μ = 0, difference between optimal TEFs and baseline rotor of (a) induced drag, (b) profile drag.
4.1.1.2 $W = 18,300$ lbs, $\mu = 0.1$

The reliability of the wake model increases with the advance ratio. For $\mu$ equals 0.1, the results may still be subject to debate, but small differences in terms of power reduction percents and mechanisms are reasonably expected compared to a more advanced wake model. For an advance ratio greater than 0.1, the inflow predictions are much more reliable and the following results are considered sufficiently accurate.

Figure 4-6a shows the optimal TEFs deflection schedules obtained for $\mu$ equals 0.1 and Figure 4-6b shows the convergence history of the optimization procedure, leading to a 2.12% total power reduction. The deflection schedules of the 4 TEFs are in phase and TEF4 contains a significant amount of 2/rev control. Large deflections are observed on the front and the rear of the rotor disk. As shown in Figure 4-7c, the optimal TEF deflections increase the amount of lift produced on the front and the rear of the rotor, especially from 70% to 90% span, while the outer ring is off-loaded. Figure 4-8c also shows an increase in drag especially on the rear of the disk, with however a large reduction of the drag on the outer portion of the rotor disk. This drag reduction compensates the increase observed inboard and leads to a global power reduction of 2.12%.

As in hover, Figure 4-9 shows a difference by an order of magnitude in the scales of Figure 4-9a and Figure 4-9b, respectively the induced and the profile power differences, which leads to the conclusion that the main source of power reduction comes here from induced power reductions obtained through lift redistribution.
Figure 4-6: Non-linear twist, GW = 18,300 lbs and $\mu = 0.1$, (a) optimal TEFs deflection schedules and (b) total power convergence history.
Figure 4-7: Non-linear twist, GW = 18,300 lbs and μ = 0.1, (a) Optimal TEFs lift, (b) Baseline lift and (c) Lift difference between a and b.

Figure 4-8: Non-linear twist, GW = 18,300 lbs and μ = 0.1, (a) Optimal TEFs drag, (b) Baseline drag and (c) Drag difference between a and b.

Figure 4-9: Non-linear twist, GW = 18,300 lbs and μ = 0.1, difference between optimal TEFs and baseline rotor of (a) induced drag, (b) profile drag.
4.1.1.3  $W = 18,300$ lbs, $\mu = 0.2$

For an advance ratio of 0.2, a power reduction of 3.21% is observed as pointed out in Figure 4-10b which also shows the convergence history. The optimal TEF deflections are presented in Figure 4-10a. Significant 2/rev components are observed, globally showing large deflections on the front and the rear of the rotor disk, with a slightly lower magnitude at the front.

Looking at the lift distributions and the difference between the optimally flapped rotor and the baseline in Figure 4-11, lift redistribution all over the rotor can be observed, with major increases on the inboard sections at the front and the rear. Such an increase in lift on these regions is accompanied by a corresponding increase in total drag as depicted in Figure 4-12c. However, the same mechanism as described previously can be observed: on the outer portion of the blade, the drag is significantly reduced and globally compensates for the increase inboard, reducing the power requirements by 3.21%. Indeed, deflecting the TEFs reduces the collective and cyclic pitch controls of the rotor as detailed in the Appendix and lower angles of attack are then observed on the outer portion of the blade, producing then less drag.

The scale of Figure 4-13b is now comparable to the one of Figure 4-13a, showing the increasing importance of the profile drag compared to the induced drag as the advance ratio is increasing. However, due to the large induced drag reductions on the outer ring of the rotor disk, and the uniformity of the profile drag disk plot it can be concluded that power improvements mainly arise from induced power reductions.
Figure 4-10: Non-linear twist, 18,300 lbs and $\mu = 0.2$, (a) optimal TEFs deflection schedules and (b) total power convergence history.
Figure 4-11: Non-linear twist, GW = 18,300 lbs and μ = 0.2, (a) Optimal TEFs lift, (b) Baseline lift and (c) Lift difference between a and b.

Figure 4-12: Non-linear twist, GW = 18,300 lbs and μ = 0.2, (a) Optimal TEFs drag, (b) Baseline drag and (c) Drag difference between a and b.

Figure 4-13: Non-linear twist, GW = 18,300 lbs and μ = 0.2, difference between optimal TEFs and baseline rotor of (a) induced drag, (b) profile drag.
4.1.1.4 $W = 18,300$ lbs, $\mu = 0.3$

With the advance ratio increasing, the characteristics of the optimal TEFs deployments change as depicted in Figure 4-14 where a power reduction of 2.83% is observed. Indeed, if 2/rev inputs can still be observed, their amplitudes are decreasing compared to lower advance ratios cases. This qualitative observation is confirmed later in Figure 4-34 to Figure 4-37 showing a global decrease of the amplitude of the 2/rev components as the advance ratio increases. Large deflections are observed on the retreating side of the rotor disk around $\Psi = 270$ deg. One can also observed negative deflections on the advancing side for the 2 inboard flaps, TEF1 and TEF2. The convergence history plots are from now on omitted since convergence is always reached as seen in the last 3 cases.

Such a deflection schedule is expected since for increasing airspeeds, low angles of attack are required on the advancing side and lift generation capabilities are not an issue there. Consequently TEFs should not be much deployed on the advancing side. However on the front, the rear and the retreating side of the rotor, TEFs are expected to deflect in order to improve the lift generation capabilities of these 3 regions where the air pressure is lower.

Comparing the lift distributions in Figure 4-15, it is observed that the TEFs redistribute the lift all over the rotor on the inboard regions. This redistribution is generally off-loading the blade tip. Furthermore, Figure 4-15a presents less negative lift on the advancing blade tip than Figure 4-15b which represents the baseline rotor. Figure 4-16 gives the drag distributions and clearly shows a large reduction of the total drag on the outer portion of the blade and a moderate increase of the drag on the retreating side of the rotor disk where the flaps are deflected, globally contributing to lower power requirements. Examining Figure 4-17, it is observed that power reductions coming from induced drag are obtained through the exact same mechanism, while increases in profile drag are concentrated on very small regions, the majority of the disk
sustaining a decrease in profile drag globally reducing the total power requirements. Thus in this case power reductions are a combination of induced and profile power reductions – statement supported in Section 4.1.3.

Figure 4-14: Non-linear twist, GW = 18,300 lbs and μ = 0.3, optimal TEFs deflection schedules.

Figure 4-15: Non-linear twist, GW = 18,300 lbs and μ = 0.3, (a) Optimal TEFs lift, (b) Baseline lift and (c) Lift difference between a and b.
Figure 4-16: Non-linear twist, GW = 18,300 lbs and μ = 0.3, (a) Optimal TEFs drag, (b) Baseline drag and (c) Drag difference between a and b.

Figure 4-17: Non-linear twist, GW = 18,300 lbs and μ = 0.3, difference between optimal TEFs and baseline rotor of (a) induced drag, (b) profile drag.

4.1.1.5  W = 18,300 lbs, μ = 0.35

Similar observations can be made in this case where μ equals 0.35. Figure 4-18 shows a total power reduction of 3.13% and deflection schemes supporting the comments made in the previous case: the 1/rev components become predominant and the deflections occur on the front, the retreating side and the rear of the rotor disk. As depicted in Figure 4-19c, large lift
redistributions arise all over the rotor. The blade tips are off-loaded due to decrease in pitch and cyclic rotor controls as observed in supporting data in the Appendix. Figure 4-20 shows large drag reductions on the outer ring of the rotor while a moderate increase is observed where the flaps are deflected. The growing importance of profile drag reductions can be observed in Figure 4-21b, where small regions of increased drag are noticed but with generally much greater regions where the profile drag is significantly reduced – on the rear of the rotor. The induced drag presented in Figure 4-21a has not changed much and if power reductions are still obtained thanks to the decreasing drag at the tip, it is compensated by increased drag where the flaps are deployed. It will be seen later that the actual induced power reductions are rather constant with the increasing advance ratio while significant increasing differences are observed on the profile power reductions.

Figure 4-18: Non-linear twist, GW = 18,300 lbs and μ = 0.35, optimal TEFs deflection schedules.
Figure 4-19: Non-linear twist, GW = 18,300 lbs and μ = 0.35, (a) Optimal TEFs lift, (b) Baseline lift and (c) Lift difference between a and b.

Figure 4-20: Non-linear twist, GW = 18,300 lbs and μ = 0.35, (a) Optimal TEFs drag, (b) Baseline drag and (c) Drag difference between a and b.

Figure 4-21: Non-linear twist, GW = 18,300 lbs and μ = 0.35, difference between optimal TEFs and baseline rotor of (a) induced drag, (b) profile drag.
4.1.1.6 \( W = 18,300 \) lbs, \( \mu = 0.4 \)

This last study case at \( \mu = 0.4 \) shows a total power reduction of 3.80% associated with the optimal TEF deployment schedules in Figure 4-22. Flaps 1, 2 and 3 show large downward deflections over the entire retreating side of the rotor disk while TEF4 deploys down at the rear of the disk. No negative upward deflections are observed anywhere around the disk.

The lift redistribution is presented in Figure 4-23c, showing larger increase in lift on the rear, the advancing side and the front of the rotor disk. The blade tip once again is off-loaded. It should be noted however that reducing the rotor controls to reduce the power requirements leads to an increase in negative lift produced on the advancing tip due to the high twist on this region as seen in Figure 4-23a and Figure 4-23b. Such a trade-off is most likely why the TEFs tend to increase the lift on the inboard advancing side too, in order to compensate for this additional negative lift generated by the blade tip.

Figure 4-24 emphasizes the drag distributions and shows drag reductions on the outer ring of the rotor disk compensating for the increases observed where the flaps are deflected and globally reducing the power requirements because of the greater moment arm. In Figure 4-25a and Figure 4-25b it can be noticed once again the increased importance of profile drag reductions occurring almost all over the rotor while induced drag reductions are constant as detailed later since no qualitative results can be extracted directly from these two figures. For high advance ratios, the predominant source of power reductions comes from profile power reduction since stall is approached. Indeed, as shown in Figure 4-26, the use of TEFs reduces the angles of attack mainly on the 4th quadrant where the baseline rotor shows angles of attack up to 9 degrees. On these regions, the Mach number is about 0.5. Figure 4-27 shows that stall in indeed approached for such angles of attack on a SC-1094R8 airfoil and similar observations can be made on SC-1095R8 airfoils. The TEFs are then used here as stall-alleviation devices.
Figure 4-22: Non-linear twist, GW = 18,300 lbs and μ = 0.4, optimal TEFs deflection schedules.

Figure 4-23: Non-linear twist, GW = 18,300 lbs and μ = 0.4, (a) Optimal TEFs lift, (b) Baseline lift and (c) Lift difference between a and b.
Figure 4-24: Non-linear twist, GW = 18,300 lbs and μ = 0.4, (a) Optimal TEFs drag, (b) Baseline drag and (c) Drag difference between a and b.

Figure 4-25: Non-linear twist, GW = 18,300 lbs and μ = 0.4, difference between optimal TEFs and baseline rotor of (a) induced drag, (b) profile drag.

Figure 4-26: Non-linear twist, GW = 18,300 lbs and μ = 0.4, (a) Optimal TEFs AoA, (b) Baseline AoA and (c) AoA difference between a and b.
4.1.2 Lower and Higher Gross Weights

The previous analysis has been repeated for a lower gross weight of 16,000 lbs and for a higher gross weight of 22,000 lbs. The power reductions and lift redistributions mechanisms are however the exact same as the ones observed in the case of a medium gross weight of 18,300 lbs. Consequently, no discussion on the disk plots is made since the same conclusions can be drawn. One can refer to the Appendix if necessary, where lift and drag distributions are included for higher and lower gross weights.

Figure 4-28 to Figure 4-33 show the optimal TEFs deflection schedules obtained for the 3 gross weights and for advance ratios ranging from 0 to 0.4. Many observations can be made when examining these figures. First, a trend appears when examining the different gross weights: the deflection amplitudes are generally higher for larger gross weights due to higher thrust requirements. Second, considering one advance ratio, the respective deflection profiles remain usually very similar for different gross weights – with some rare exceptions. This leads to the observation that the TEFs controls generally keep the same phase as the gross weight increases.
The phase of the TEFs deflection components are thus mainly a function of the advance ratio while their different amplitudes are mainly a function of the gross weight. This observation could be useful to reduce the number of design variables for further optimization analysis. Finally, as the advance ratio increases, it is noted that the amplitudes of the 2/rev components decrease. Indeed, examining Figure 4-34 to Figure 4-37 which show the amplitude of the different harmonics for the 4 TEFs over the entire range of advance ratios and for the 3 gross weights, the amplitude of the 2/rev components are globally greater for advance ratios of 0.1 and 0.2 than at 0.3, 0.35 and 0.4. If for low advance ratios the TEFs tend to increase the lift on the front and the rear of the disk leading to a 2/rev input, for higher advance ratios the TEFs tend to increase the lift on the retreating side, deflecting the flaps at $\Psi = 270$ deg. This usually prevents the use of 2/rev deflections since very little deployment on the advancing side, at $\Psi = 90$ deg. is required due to high adverse pressure. The 1/rev component is then favored as observed for advance ratios greater than 0.3.

Such an observation is supported by Bae et al in Reference 49 where the same study was performed with MiTEs instead of TEFs – with however a slightly different rotor model stalling for earlier flight conditions and using a linear twist blade design. His study looked at power reductions on a UH-60 helicopter model using 4 MiTEs actuated at 1 and 2/rev, for advance ratios greater than 0.3. Strong 1/rev components were also reported. The arising of 2/rev components for lower advance ratios, typically 0.1 and 0.2, is supported by studies conducted by Hall et al (Ref. 6) and Wachspress et al (Ref. 4) where lift redistributions on the front and rear of the rotor disk were also reported for induced power reductions.
Figure 4-28: Non-linear twist, deflections at $\mu = 0$ for (a) 16,000 lbs, (b) 18,300 lbs and (c) 22,000 lbs

Figure 4-29: Non-linear twist, deflections at $\mu = 0.1$ for (a) 16,000 lbs, (b) 18,300 lbs and (c) 22,000 lbs

Figure 4-30: Non-linear twist, deflections at $\mu = 0.2$ for (a) 16,000 lbs, (b) 18,300 lbs and (c) 22,000 lbs

Figure 4-31: Non-linear twist, deflections at $\mu = 0.3$ for (a) 16,000 lbs, (b) 18,300 lbs and (c) 22,000 lbs
Figure 4-32: Non-linear twist, deflections at $\mu=0.35$ for (a) 16,000 lbs, (b) 18,300 lbs and (c) 22,000 lbs

Figure 4-33: Non-linear twist, deflections at $\mu=0.4$ for (a) 16,000 lbs, (b) 18,300 lbs and (c) 22,000 lbs

Figure 4-34: TEF1 deflections harmonic components as function of $\mu$ for a non-linear twist.
4.1.3 Power Reductions Comparisons

In order to have a more quantitative description of the power reductions, Table 4-1, Table 4-2 and Table 4-3 respectively present the total, profile and induced power reductions for all the different flight conditions examined previously. Two sets of data are presented. The ‘a’ tables
show the power reduction percents while the ‘b’ tables show the actual power difference in horsepower.

Once again, the numbers presented here for hover conditions are not reliable and should not be seriously considered. However for moderate to high advance ratios, many observations can be made. Table 4-1a shows an average power reduction for advance ratios ranging from 0.2 to 0.35 of 2.6% for low gross weight, 3.06% for medium gross weight and 4.31% for high gross weight. For a high advance ratio of 0.4, total power reductions of up to 3.37%, 3.8% and 5.65% respectively are observed. Thus the higher the thrust, the higher the percent reduction. Table 4-1b shows that for an increasing gross weight and an increasing advance ratio, the rotor power saved increases in the same manner – not considering hover – a minimum power difference of 15.68 HP being observed at \( \mu \) equals 0.1 and a gross weight of 16,000 lbs, and a maximum power difference of 227.05 HP being observed at 0.4 and 22,000 lbs.

Table 4-2 and Table 4-3 break apart the source of power reductions and describe the importance of the profile power and the induced power in terms of power reductions. Table 4-2 gives then the profile power reductions compared to the baseline rotor, while Table 4-3 gives the induced power reductions – both in percents and power differences. Examining Table 4-2, it is observed that profile power reductions significantly increase with the airspeed and gross weight. This is expected since higher thrust requirements lead to an increase in profile power since stall is approached as described previously in Section 4.1.1.6. Alleviating stall provides large profile power reductions. On the other hand, the induced power reductions remain rather constant over the variations in gross weight or airspeed: looking at Table 4-3b, induced power differences ranging from 15 to 25 HP can be observed with no actual trend. The profile power differences presented in Table 4-2b show low values – less than 10 HP – for low gross weight and low airspeeds, but become comparable to induced power reductions until an advance ratio of 0.2 or
0.3. For higher airspeed and thrust levels, the profile power reductions become predominant, which was not obvious previously when only looking at the disk plots in Section 4.1.1.

Table 4-1: Total power (a) percent reduction and (b) difference in HP, between the optimally flapped and baseline rotors with a non-linear twist.

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(b)

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Table 4-2: Profile power $P_0$ (a) percent reduction and (b) difference in HP between the optimally flapped and baseline rotors with a non-linear twist.

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Table 4-3: Induced power $P_i$ (a) percent reduction and (b) difference in HP between the optimally flapped and baseline rotors with a non-linear twist.

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4.1.4 On the Number of TEFs

In order to investigate the importance of the different trailing edge flaps in the power reductions process, power reductions were acquired when only 3 and 2 of the 4 TEFs were actuated. In these simulations, the inboard flaps (respectively TEF1, and then TEF1 and TEF2) were removed from the model. The remaining flaps were actuated using the results of the optimization schemes using 4 TEFs, which suggests that when using 2 TEFs for example, slightly better power reductions could actually be observed using an optimization procedure.

The results for all the different cases can be observed in Figure 4-38. It can be seen that the drop in percent power reduction when removing TEF2 is always greater than when TEF1 is removed. Indeed, when only TEF1 is removed, the differences in power reductions generally drop by less than 0.5% - except for extreme cases and hover. But considering the 3 flaps cases as reference, removing TEF2 usually leads to a drop of 0.8% - which gives more than 1.3% difference considering the 4 flaps cases as a reference. It is then concluded that TEF1 does not have a major impact in terms of rotor power reductions and can be removed if necessary – which would decrease the complexity of the system and its cost. TEF2 however provides more significant contributions to the total power reductions.

Even though the rotor models are different, it can be noted that the use of 2 TEFs for power reduction studies was conducted by Liu et al in Ref. 17. Using a maximum deflection of 3 degrees, they reported a rotor power reduction of up to 1.76% using 2 flaps. Such a number is quite comparable to the reductions obtained at this advance ratio with 2 TEFs, ranging from 1.5% to 2.9% depending on the gross weight and airspeed. However, considering a linear twist blade design, much greater reductions are observed later, which can be explained by the higher maximum deflection amplitude used in this study.
4.2 Results for a Linear Twist Distribution

The UH-60A blade has a very unusual non-linear twist distribution (see Section 2.1 showing the down-up twist distribution) whereas linear twist distribution is quite common in many helicopter blades. As a consequence, it is also interesting to study power reductions with spanwise-segmented TEFs on a linearly twisted blade and to compare those results with the ones obtained previously with a non-linear blade twist. The same optimization procedure has then been performed for 3 gross weights (16,000 lbs, 18,300 lbs and 22,000 lbs) over the same range of advance ratios going from 0 to 0.4 with the same helicopter model but this time considering a -8 degrees linear twist. However, due to the large similarities in terms of power reduction mechanisms as observed previously, only 2 advance ratios are discussed: 0.2 and 0.35. The complementary results (other advance ratios, lower and higher gross weights) are summarized and additional data can be found in the Appendix. It should be pointed out that 2 cases did not
converge and are then not presented: at an advance ratio of 0.4, the 18,300 lbs and the 22,000 lbs cases presented convergence issues.

4.2.1 Medium Gross Weight, 18,300 lbs

4.2.1.1 W = 18,300 lbs, μ = 0.2

Similarly to the non-linear twist case, large deflections on the front and rear of the rotor are observed in Figure 4-39, giving a strong 2/rev control. The power reduction (5.09%) is however much larger than the one obtained previously (3.21%). The power reduction mechanism is similar to the one described in Section 4.1.1.3: the lift redistribution on the inboard sections at the front and the rear of the rotor disk off-loads the blade tip (Figure 4-40) and significantly reduces the drag on the outer ring of the rotor.

In order to understand the power reduction difference observable when comparing the linear twist and the non-linear twist case, Figure 4-41 and Figure 4-12 can be compared. The scales are almost the same which facilitates the following observation: the baseline rotor with a linear twist in Figure 4-41b sustains much more drag on the 4th quadrant than the non-linear twist case in Figure 4-12b. Indeed, the non-linear twist used features a down-up twist design at the tip which generates less drag on the outer portion of the 4th quadrant where angles of attack are increasing to overcome the low dynamic pressure. As seen in Figure 4-43c, with the linear twist configuration the tip of the blade sustains much higher angles of attack and stalled conditions are approached leading to a large increase in drag. This phenomenon is avoided with the non-linear twist which mainly modifies the outer 10% of the rotor blade, finally giving this power reduction difference of 1.88% between the linear twist and the non-linear twist configurations.
Figure 4-39: Linear twist, GW = 18,300 lbs and $\mu = 0.2$, optimal TEFs deflection schedules.

Figure 4-40: Linear twist, GW = 18,300 lbs and $\mu = 0.2$, (a) Optimal TEFs lift, (b) Baseline lift and (c) Lift difference between a and b.
Figure 4-41: Linear twist, GW = 18,300 lbs and $\mu = 0.2$, (a) Optimal TEFs drag, (b) Baseline drag and (c) Drag difference between a and b.

Figure 4-42: Linear twist, GW = 18,300 lbs and $\mu = 0.2$, difference between optimal TEFs and baseline rotor of (a) induced drag, (b) profile drag.

Figure 4-43: Angles of attack distribution for the (a) linearly twisted and (b) non-linearly twisted rotor, and (c) the difference, for $W = 18,300$ lbs and $\mu = 0.2$. 
4.2.1.2 \( W = 18,300 \) lbs, \( \mu = 0.35 \)

At a higher advance ratio of 0.35, the TEFs tend to deflect over the entire retreating side of the rotor, avoiding large deflections on the advancing side as observed in Figure 4-44. However, the larger lift increments are observed on the front, the rear and the advancing side of the rotor as shown in Figure 4-45. A power reduction of 6.57% is obtained, which again is greater than the 3.13% obtained when a non-linear blade twist design was used. The power reduction mechanism is the same as previously described: off-loading the tip (Figure 4-45) reduces the drag on the outer ring leading to a global power reduction. Figure 4-48 shows the angles of attack distributions for the optimally flapped rotor, the baseline rotor and their difference. Similarly to the case with non-linear twist and an advance ratio of 0.4, it can be seen that angles of attack of 8 degrees can be observed on the retreating side of the rotor which are approaching stall conditions. Using the TEFs clearly reduces the angles of attack and stall is alleviated. Power reductions are then made by reducing the profile power as detailed in the next section.

As in the previous section, a similar explanation on why this greater relative power reduction is observed can be made examining the baseline drag distributions in Figure 4-46b and Figure 4-20b. The scales are the same and it is clearly observed that a much higher drag is generated on the outer part of the 4\(^{th}\) quadrant on the rotor with a linear twist compared to the one with the non-linear twist. Consequently, a greater relative power reduction is coming out from the alleviation of this larger drag penalty. The angles of attack distributions are compared in Figure 4-49 and it can be seen that using the TEFs significantly reduces the angles of attack on the outboard of the blade. The UH-60 non-linear twist seems then a better design choice compared to a simple -8 degrees linear twist, even for high advance ratios. It should also be noticed, looking at the baseline lift distributions in Figure 4-45 and Figure 4-19, that the non-linear twist also produces a region of negative lift on the advancing tip. However compared to penalty induced by
the high drag on the 4th quadrant, such a negative lift seems less significant in terms of power penalties.

Figure 4-44: Linear twist, GW = 18,300 lbs and μ = 0.35, optimal TEFs deflection schedules.

Figure 4-45: Linear twist, GW = 18,300 lbs and μ = 0.35, (a) Optimal TEFs lift, (b) Baseline lift and (c) Lift difference between a and b.
Figure 4-46: Linear twist, GW = 18,300 lbs and $\mu = 0.35$, (a) Optimal TEFs drag, (b) Baseline drag and (c) Drag difference between $a$ and $b$.

Figure 4-47: Linear twist, GW = 18,300 lbs and $\mu = 0.35$, difference between optimal TEFs and baseline rotor of (a) induced drag, (b) profile drag.

Figure 4-48: Linear twist, GW = 18,300 lbs and $\mu = 0.35$, (a) Optimal TEFs AoA, (b) Baseline AoA and (c) AoA difference between $a$ and $b$. 
Figure 4-49: Angles of attack distribution for the (a) linearly twisted and (b) non-linearly twisted rotor, and (c) the difference, for \( W = 18,300 \) lbs and \( \mu = 0.35 \).

### 4.2.2 Lower and Higher Gross Weight

The optimal deflections schedules obtained with a linear blade twist are presented in Figure 4-50 to Figure 4-54. The corresponding power reductions are shown in the Table 4-4. A general similarity with the deflections obtained with the previous non-linear twist can be observed and similar comments can be made: at a given advance ratio, the TEFs controls generally keep the same phase as the gross weight increases while the amplitudes vary. In addition, the 2/rev control components, significantly present for low to medium advance ratios, are progressively reduced for medium to high advance ratios (\( \mu \) of 0.3 and 0.35) where the 1/rev controls are generally more dominant as depicted in Figure 4-55 to Figure 4-58. Thus for low to medium advance ratio, the lift redistributions occurs on the front and the rear of the rotor while for medium to high advance ratios lift is redistributed over the entire retreating side of the rotor – deflections on the advancing side being avoided.

The similarity in the deflections between the linear twist and the non-linear twist cases is expected since the real non-linearity of the twist distribution is located on the outer 10% of the rotor disk where the flaps are not located, as depicted in Figure 2-6.
Figure 4-50: Linear twist, deflections at $\mu = 0$ for (a) 16,000 lbs, (b) 18,300 lbs and (c) 22,000 lbs.

Figure 4-51: Linear twist, deflections at $\mu = 0.1$ for (a) 16,000 lbs, (b) 18,300 lbs and (c) 22,000 lbs.

Figure 4-52: Linear twist, deflections at $\mu = 0.2$ for (a) 16,000 lbs, (b) 18,300 lbs and (c) 22,000 lbs.
Figure 4-53: Linear twist, deflections at $\mu = 0.3$ for (a) 16,000 lbs, (b) 18,300 lbs and (c) 22,000 lbs.

Figure 4-54: Linear twist, deflections at $\mu = 0.35$ for (a) 16,000 lbs, (b) 18,300 lbs and (c) 22,000 lbs.

Figure 4-55: TEF1 deflections harmonic components as function of $\mu$ for a -8 deg linear twist.
4.2.3 Comparison with Non-Linear Twist Results

The total power reductions in percents and power differences in horsepower for a rotor configured with a linear twist are presented in Table 4-4. Compared to Table 4-1, large differences over all the different flight conditions are observed. These differences can be seen in
Figure 4-60, which presents the percent power reductions for both the non-linear blade twist and the linear blade twist cases. For an advance ratio of 0.35, total power reductions of up to 5.70% for low gross weight, 6.57% for medium gross weight and 7.98% for high gross weight are observed.

These large differences in terms of relative power reductions come from the fact that the baselines with a linear twist have greater power requirements due to the high drag penalties coming from the 4th quadrant of the rotor – due to high angles of attack approaching stall sooner than with a down-up non-linear twist blade design. Thus large power reductions can be obtained relative to the baseline rotors depending on the specific twist distribution of the baseline blade. In this study it was observed (Figure 4-59) that the non-linear blade twist design generally has lower power requirements and then the power reductions achievable were moderate. However, using a linear-twist blade design, the baseline rotor being less power efficient, larger relative power reductions were obtained.

Figure 4-59: Baseline total power comparison between the non-linear twist and the linear twist cases.
Table 4-4: Total power (a) percent reduction and (b) difference in HP, between the optimally flapped and baseline rotors with a linear twist.

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(b)  

Figure 4-60: Comparison of percent power reductions between the rotor with a non-linear twist and with a linear twist.

Table 4-5 and Table 4-6 show respectively the profile and the induced power reductions in percent and in power difference for every flight condition. These tables are compared with the previously described Table 4-2 and Table 4-3, presenting the same set of results for the non-linear blade twist rotor, in Figure 4-61 and Figure 4-62. These 2 figures show the power reductions in profile and induced power for the non-linear twist and the linear twist configurations. Clearly, the larger differences arise from the profile power for high advance ratios, while the reductions in
induced drag remain quasi constant – except for hover. The larger reductions are then coming from stall alleviation as described in previous cases. Indeed, the high drag penalty observed previously on the 4th quadrant is due to retreating blade stall occurring at the tip of the blade. Using the TEFs, the rotor controls are reduced as detailed in the Appendix and stall is alleviated. This phenomenon is much more visible with a linear twist since the down-up twist schedule of the UH-60 non-linear twist minimizes this effect.

**Table 4-5: Profile power \( P_0 \) (a) percent reduction and (b) difference in HP, between the optimally flapped and baseline rotors with a linear twist.**

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**Figure 4-61: Comparison of profile power differences between the non-linear twist and the linear twist cases**
Table 4-6: Induced power $P_i$ (a) percent reduction and (b) difference in HP, between the optimally flapped and baseline rotors with a linear twist.

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Figure 4-62: Comparison of induced power differences between the non-linear twist and the linear twist cases
Chapter 5

Summary and Conclusions

Reducing rotor power requirements is of great interest in order to improve helicopter performances in terms of range, endurance, payload, speed and altitude. Active methods such as HHC, IBC, TEFs or MiTEs have been shown to have the ability to decrease rotorcraft power requirements. In hover and for moderate advance ratios, the induced power is a significant contributor while for moderate to high advance ratios profile and parasite power become more significant. This present work focused on the analysis of two different active methods that aimed at reducing the rotor power requirements of a UH-60 type helicopter. To that end, a helicopter model was developed and validated in order to realistically capture the improvements that can be obtained. Based on this model, a first design solution referred to as Static Extended Trailing Edge (SETE) was studied for flight envelope expansion. A SETE was used in this study as a lift augmentation or stall alleviation device, mainly working on the flight envelope boundaries – thus high altitudes, high speeds or high gross weight. Following this work, a different design solution aiming at redistributing the airloads across the rotor over a large range of advance ratios and for different gross weight levels was studied. Using CFD results, this work used multiple spanwise-segmented TEFs actuated up to 2/rev and located from 50% to 90% of the rotor radius. Optimally deflecting these TEFs, power reductions were sought and the aerodynamic mechanisms behind the observed improvements were explained.
5.1 Model Summary

A model configured to represent a UH-60 type helicopter was developed in order to predict blade, hub and vehicle behavior in propulsive trim. This model includes a rigid-blade free to flap about hinge and a time-integration scheme is used to compute the flap response and the loads generated on the hub. The blade element lift and drag were computed using C81 tables combined with Theordorsen’s theory to account for quasi-steady contributions and coupled with a rigid prescribed wake model to calculate the inflow distribution associated with spanwise and azimuthal variation in airloads. A simple analytical SETE model was developed based on wind-tunnel observations in order to provide a model for the increments in lift and drag when such a device is deployed. CFD data were used to model the TEFs aerodynamic characteristics and to accurately represents the lift and drag increments varying with angles of attack, Mach numbers and the deflection angles. The tail rotor, the horizontal tail and the fuselage were properly modeled based on previous work in order to realistically represent a UH-60 helicopter design and all the forces and moment are summed at the hub. The trim procedure uses a forward difference Jacobian scheme to perform a propulsive trim analysis. Such a model showed good correlation with flight test and CAMRAD II results, capturing accurately the power requirement and other trim variables as a function of airspeed.

5.2 Results Summary and Conclusions

A first study was conducted to assess the capability of a quasi-statically extended trailing-edge device for retreating blade stall alleviation and flight envelope expansion. The SETE is a flat plate extended through a slit trailing-edge of the airfoil. The aerodynamic characteristics of other high-lift devices such as a trailing-edge flaps (TEFs) and Gurney flaps on a SC-1094R8 airfoil
were examined using CFD and the aerodynamic characteristics of the SETE were compared to those of the TEF and Gurney flap. A propulsive trim analysis, based on the UH-60 helicopter, was used to determine the maximum speeds, altitudes and gross weights attainable with 10%, 20% and 30% chord extensions, showing the flight envelope boundaries expansion. Possible rotor power reductions with the use of the SETE device near the envelope boundaries were also examined.

The following conclusions are drawn from the study:

1. Gurney flaps and SETEs are comparable in term of lift increment near stall at low Mach numbers. However, the lift-to-drag ratios are higher for the SETEs at high $C_L$, which makes them attractive trailing-edge devices for stall alleviation. TEFs provide the largest lift-to-drag ratios for moderate $C_L$ requirements, but cannot reach the high $C_{L\text{MAX}}$ values that Gurney flaps and SETEs can, and are generally less competitive at high lift coefficients.

2. SETEs show the potential to significantly increase the maximum altitude of the UH-60A helicopter. For a medium gross weight of 18,000 lbs, moderate improvements of 1,500 ft in ceiling altitude are observed. Here the aircraft is power limited rather than lift limited, so SETEs have only a modest effect. For a larger gross weight of 24,000 lbs, retreating blade stall appears sooner and provision of additional lift by the SETEs increases the maximum altitude by 2,000 ft. Assuming a greater available engine power so that the rotor is truly and completely lift limited, even larger altitude increases of up to 3,000 ft are observed, with the stall alleviation capability of the SETEs fully exploited.

3. Increase in maximum speed with the deployment of the SETEs is relatively modest since the SETEs can alleviate stall but do not address the compressibility problems on the advancing blade tips. In stall-dominant conditions (high gross-weight, high-altitude, higher available power), increases in maximum speed of up to 26 knots were observed.
4. At high altitudes/gross-weight where the baseline rotor is close to stall, the SETEs can offer large increases in gross-weight capabilities by alleviating stall. The actual increases depend on the available power and are greater if the available power is larger. For 3000 HP available at sea level, the maximum gross-weight capability increased by 1,600 lbs (or 6.7%) for operation at high altitude (8,000 ft). For 4000 HP available at sea level, the increase in maximum gross-weight capability at high altitude was an even more impressive 2,400 lbs (or 10%). The increases in gross-weight capabilities observed translate to large increases in aircraft payload (or range, if the extra available weight is used to carry more fuel).

5. At high-altitudes or gross-weights (close to stall), significant power reductions are achievable through the use of a SETE. The actual power reductions depend on the available power. For 3,000 HP available at sea level, a 450 HP (16.7%) reduction was observed with a 30% chord extension, when operating at 8,000 ft, 24,000 lbs gross-weight. For 4000 HP available at sea level, an even larger 1050 HP (33.4%) reduction was observed with 30% chord extension. These power reductions are mainly coming from profile power reductions.

6. The position and size of the SETE device has a significant impact on the possible benefits it can produce. In general, larger span and a more outboard location is most effective.

Although the results obtained in this study look very promising, the aerodynamic characteristics of the SETE were based on a couple of low-speed wind-tunnel tests of a symmetric NACA 0012 airfoil. More detailed work (CFD and wind-tunnel tests) is required to characterize the aerodynamic properties of more advanced airfoils with flat-plates at the trailing-edge, over a wider range of Mach numbers. Such data will then allow a more accurate assessment of the potential of SETEs in terms of retreating blade stall alleviation resulting in flight envelope expansion or power reduction near the envelope boundaries.
A second study was conducted to assess the potential of multiple spanwise-segmented trailing edge flaps actuated up to 2/rev for total power reductions over a range of airspeeds and thrust levels. Four TEFs located from 50% to 90% of the rotor radius were used. CFD data were included to model the aerodynamic characteristics of 20% TEFs mounted on a SC-1094R8 airfoil. Optimal flap deflection schedules were computed using a gradient-based method while propulsive trim was satisfied. The study examined the power reductions achievable for advance ratios ranging from 0 to 0.4 and for gross weights of 16,000 lbs, 18,300 lbs and 22,000 lbs. This analysis took into account the UH-60 non-linear twist schedule which features a down-up twist distribution at the tip of the blade. In order to study a more common case, the same analysis was performed on the same model with a -8 degrees linear blade twist. The sources of power reductions were investigated looking at the induced and profile power reductions. The power reductions differences between the non-linear twist distribution and the linear one were examined. Finally, optimal TEFs deflection schemes were analyzed and the power reduction mechanisms investigated.

The following conclusions can be drawn from this second study:

1. Depending on the advance ratio and the thrust level, the optimal TEFs deflection schedules vary. Indeed, the amplitude of the 2/rev components are greater for advance ratios lower that 0.2 while 1/rev deflections generally become predominant for high airspeeds. For moderate to high advance ratios, the TEFs are deflected down over the entire retreating side.

2. The optimal TEFs deflections redistribute the airloads all across the rotor. The lift redistributions mainly happen in the inboard sections on front and the rear of the rotor for the majority of the flight conditions studied. This observation is substantiated by other works where similar redistributions where reported. However, for higher advance ratio significant lift increase was observed on the advancing side as well, which is likely generated to compensate for the negative lift observed at the advancing blade tip.
3. The power reductions observed all arise from the same mechanism: the outer most 10% of the blade is off-loaded due to the lift redistribution inboard, where the TEFs are located. This lift redistribution reduces the rotor controls, thus collective, which decrease the angles of attack at the tip of the blade. The decrease in drag observed at the tip is more than compensating for the additional drag generated on the inboard sections and globally reduces the power requirements.

4. For advance ratios up to 0.2, the main source of power reduction comes from the induced power. For higher advance ratios, power reductions are obtained by reducing the rotor profile power. For high advance ratios, large reductions are then realized while approaching and alleviating stall.

5. Using the UH-60 non-linear twist schedule, power reductions ranging from 1.61% to 5.65% were observed depending on the advance ratio and the gross weight. The higher the gross weight and the airspeed, the larger the power reductions. Using a -8 deg linear twist, even larger power reductions were reported, ranging from 2.74% to 7.98% total power reductions.

6. The greater power reductions observed with a linear blade twist design arise from the large drag penalties occurring on the 4th quadrant at the tip of the blade compared to the non-linear twist. The non-linear twist features a down-up twist schedule that reduces the angles of attack on the retreating blade tip. When retreating blade stall occurs, this design then produces less drag than a simple linear twist. This design is however producing more negative lift on the advancing side.

7. The number of TEFs has a significant impact of the power reductions achievable. A comparison of the rotor power reductions obtained when using only 3 and 2 TEFs showed that if the 3 outboard TEFs give most of the power reductions obtained with all 4 TEFs, using only the 2 outboard TEFs considerably reduces the performances observed with 4 TEFs. Using only 3 TEFs
would then be preferable in order to reduce the number of design variables and the complexity of
the system.

5.3 Recommendations

The present work has developed the area of rotor power reduction using two different
methods: flight envelope expansion using SETEs and total power reduction using multiple
spanwise-segmented TEFs. However the following further analysis is recommended.

The helicopter model itself is to be improved in order to increase the accuracy of the
predicted results and to confirm the results observed in this thesis. A free-wake model would
improve the performances of the helicopter model for low advance ratios, taking into account the
self-induced distortions. The results in hover would be more pertinent than the ones presented in
this study. For high advance ratios, a dynamic stall model would improve the fidelity of the
power reduction results. Furthermore, the present model only takes into account the rigid flapping
motion of the blade. Blade flexibility through a coupled elastic flap, lag and torsion rotor can be
added and an elastic blade model should be included in order to compare the results to those
obtained in the present work.

Furthermore, including an elastic blade model is critical in order to account for the effects
of additional pitching moments in the two studies. Indeed, the SETEs are expected to have
significant additional pitching moments since the chord is extended at the trailing edge and the
TEFs pitching moment increments are available and should not be neglected. An elastic blade
model gives the opportunity to study vibration levels, which is necessary for a more detailed
analysis. Looking only at power reductions without considering vibrations, as performed in the
present thesis, is a preliminary study and it should be verified that similar power reduction results
are obtained with a more advance rotor model. It should also be verified that the vibration levels are not increased while using these devices.

Concerning the SETEs, the analytical model used is simple and is based on reasonable assumptions. In order to improve the accuracy of the predictions made, a CFD analysis is to be performed and included in the helicopter model. Different CFD tools should be considered for validation purpose. Reliable codes include MSES, which is a subsonic incompressible Euler code that provides fast and reliable results for steady-flow predictions, and OVERFLOW, which is a Reynolds-averaged Navier-Stokes solver. A comparison with wind-tunnel experiments would also be of great interest in order to confirm or infirm previous observations. Indeed, the wind-tunnel tests conducted by Liu et al (Ref. 8) and Noboru et al (Ref. 51) were performed for very low Mach numbers and symmetrical airfoils. Further analysis of the real potential of SETEs for rotorcraft application is then necessary.

In addition, a practical implementation of power reduction using multiple spanwise-segmented trailing edge flaps should be considered. Indeed, this thesis presents the optimal TEFs deflections obtained using an off-line optimizer coupled with the helicopter model. Using a T-matrix approach, an on-line optimization algorithm can be developed. The optimal deflections could then be computed in real time at a given flight condition. The results obtained through such a process should then be compared to those obtained in the present work.
Bibliography


Appendix A

Optimal Trailing Edge Flaps Controls

Table A-1: Optimal trailing edge flaps controls for the non-linear blade twist configuration.

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Table A-2: Optimal trailing edge flaps controls for the -8 deg. linear blade twist configuration.

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Appendix B

Helicopter Trim Controls

Table B-1: Helicopter controls for the baseline rotor and the optimally flapped rotor for a non-linear blade twist schedule.

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Table B-2: Helicopter controls for the baseline rotor and the optimally flapped rotor for a -8 deg. linear blade twist schedule.

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Appendix C

Additional Lift and Drag Distributions
Disk Plots

C.1. Results for the Rotor with Non-Linear Twist

C.1.1 Gross Weight of 16,000 lbs

Figure C-1: Non-linear twist, GW=16,000 lbs and μ=0, Lift (a) Optimal, (b) Baseline, (c) Difference.

Figure C-2: Non-linear twist, GW=16,000 lbs and μ=0, Drag (a) Optimal, (b) Baseline, (c) Difference.
Figure C-3: Non-linear twist, GW=16,000 lbs, μ=0.1, Lift (a) Optimal, (b) Baseline, (c) Difference.

Figure C-4: Non-linear twist, GW=16,000 lbs, μ=0.1, Drag (a) Optimal, (b) Baseline, (c) Difference.

Figure C-5: Non-linear twist, GW=16,000 lbs, μ=0.2, Lift (a) Optimal, (b) Baseline, (c) Difference.
Figure C-6: Non-linear twist, GW=16,000 lbs, $\mu=0.2$, Drag (a) Optimal, (b) Baseline, (c) Difference.

Figure C-7: Non-linear twist, GW=16,000 lbs, $\mu=0.3$, Lift (a) Optimal, (b) Baseline, (c) Difference.

Figure C-8: Non-linear twist, GW=16,000 lbs, $\mu=0.3$, Drag (a) Optimal, (b) Baseline, (c) Difference.
Figure C-9: Non-linear twist, GW=16,000 lbs, μ=0.35, Lift (a) Optimal, (b) Baseline, (c) Difference.

Figure C-10: Non-linear twist, GW=16,000 lbs, μ=0.35, Drag (a) Optimal, (b) Baseline, (c) Difference.

Figure C-11: Non-linear twist, GW=16,000 lbs, μ=0.4, Lift (a) Optimal, (b) Baseline, (c) Difference.
C.1.2 Gross Weight of 22,000 lbs

Figure C-13: Non-linear twist, GW=22,000 lbs, μ=0, Lift (a) Optimal, (b) Baseline, (c) Difference.

Figure C-14: Non-linear twist, GW=22,000 lbs, μ=0, Drag (a) Optimal, (b) Baseline, (c) Difference.
Figure C-15: Non-linear twist, GW=22,000 lbs, μ=0.1, Lift (a) Optimal, (b) Baseline, (c) Difference.

Figure C-16: Non-linear twist, GW=22,000 lbs, μ=0.1, Drag (a) Optimal, (b) Baseline, (c) Difference.

Figure C-17: Non-linear twist, GW=22,000 lbs, μ=0.2, Lift (a) Optimal, (b) Baseline, (c) Difference.
Figure C-18: Non-linear twist, GW=22,000 lbs, $\mu=0.2$, Drag (a) Optimal, (b) Baseline, (c) Difference.

Figure C-19: Non-linear twist, GW=22,000 lbs, $\mu=0.3$, Lift (a) Optimal, (b) Baseline, (c) Difference.

Figure C-20: Non-linear twist, GW=22,000 lbs, $\mu=0.3$, Drag (a) Optimal, (b) Baseline, (c) Difference.
Figure C-21: Non-linear twist, GW=22,000 lbs, \( \mu=0.35 \), Lift (a) Optimal, (b) Baseline, (c) Difference.

Figure C-22: Non-linear twist, GW=22,000 lbs, \( \mu=0.35 \), Drag (a) Optimal, (b) Baseline, (c) Difference.

Figure C-23: Non-linear twist, GW=22,000 lbs, \( \mu=0.4 \), Lift (a) Optimal, (b) Baseline, (c) Difference.
C.2. Results for the Rotor with -8 deg Linear Twist

C.2.1 Gross Weight of 16,000 lbs

Figure C-25: Linear -8deg twist, GW=16,000 lbs, μ=0, Lift (a) Optimal, (b) Baseline, (c) Difference.

Figure C-26: Linear -8deg twist, GW=16,000 lbs, μ=0, Drag (a) Optimal, (b) Baseline, (c) Difference.
Figure C-27: Linear -8deg twist, GW=16,000 lbs, μ=0.1, Lift (a) Optimal, (b) Baseline, (c) Difference.

Figure C-28: Linear -8deg twist, GW=16,000 lbs, μ=0.1, Drag (a) Optimal, (b) Baseline, (c) Difference.

Figure C-29: Linear -8deg twist, GW=16,000 lbs, μ=0.2, Lift (a) Optimal, (b) Baseline, (c) Difference.
Figure C-30: Linear -8deg twist, GW=16,000 lbs, μ=0.2, Drag (a) Optimal, (b) Baseline, (c) Difference.

Figure C-31: Linear -8deg twist, GW=16,000 lbs, μ=0.3, Lift (a) Optimal, (b) Baseline, (c) Difference.

Figure C-32: Linear -8deg twist, GW=16,000 lbs, μ=0.3, Drag (a) Optimal, (b) Baseline, (c) Difference.
Figure C-33: Linear -8deg twist, GW=16,000 lbs, μ=0.35, Lift (a) Optimal, (b) Baseline, (c) Difference.

Figure C-34: Linear -8deg twist, GW=16,000 lbs, μ=0.35, Drag (a) Optimal, (b) Baseline, (c) Difference.

Figure C-35: Linear -8deg twist, GW=16,000 lbs, μ=0.4, Lift (a) Optimal, (b) Baseline, (c) Difference.
Figure C-36: Linear -8deg twist, GW=16,000 lbs, μ=0.4, Drag (a) Optimal, (b) Baseline, (c) Difference.

C.2.2 Gross Weight of 18,300 lbs

Figure C-37: Linear -8deg twist, GW=18,300 lbs, μ=0, Lift (a) Optimal, (b) Baseline, (c) Difference.

Figure C-38: Linear -8deg twist, GW=18,300 lbs, μ=0, Drag (a) Optimal, (b) Baseline, (c) Difference.
Figure C-39: Linear -8deg twist, GW=18,300 lbs, μ=0.1, Lift (a) Optimal, (b) Baseline, (c) Difference.

Figure C-40: Linear -8deg twist, GW=18,300 lbs, μ=0.1, Drag (a) Optimal, (b) Baseline, (c) Difference.

Figure C-41: Linear -8deg twist, GW=18,300 lbs, μ=0.3, Lift (a) Optimal, (b) Baseline, (c) Difference.
C.2.3 Gross Weight of 22,000 lbs
Figure C-45: Linear -8deg twist, GW=22,000 lbs, μ=0.1, Lift (a) Optimal, (b) Baseline, (c) Difference.

Figure C-46: Linear -8deg twist, GW=22,000 lbs, μ=0.1, Drag (a) Optimal, (b) Baseline, (c) Difference.

Figure C-47: Linear -8deg twist, GW=22,000 lbs, μ=0.2, Lift (a) Optimal, (b) Baseline, (c) Difference.
Figure C-48: Linear -8deg twist, GW=22,000 lbs, μ=0.2, Drag (a) Optimal, (b) Baseline, (c) Difference.

Figure C-49: Linear -8deg twist, GW=22,000 lbs, μ=0.3, Lift (a) Optimal, (b) Baseline, (c) Difference.

Figure C-50: Linear -8deg twist, GW=22,000 lbs, μ=0.3, Drag (a) Optimal, (b) Baseline, (c) Difference.
Figure C-51: Linear -8deg twist, GW=22,000 lbs, $\mu=0.35$, Lift (a) Optimal, (b) Baseline, (c) Difference.

Figure C-52: Linear -8deg twist, GW=22,000 lbs, $\mu=0.35$, Drag (a) Optimal, (b) Baseline, (c) Difference.