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FINANCIAL OPTION METHOD FOR REVENUE MANAGEMENT IN THE AIRLINE INDUSTRY

A Thesis in

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by

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ABSTRACT

The airline industry is an area where fierce competition exists. Maximizing revenue is an important target for the airlines. This is known as revenue management or yield management. A lot of research has been done in this area to help the airlines achieve their goals.

Recently, some of the researchers switched their focus from the traditional and most mainstream methods, such as stochastic and dynamic models, to financial option methods. From the works they have done, it is shown that the financial option concept, which requires easier analysis, could provide more revenue in an easier calculation method.

The purpose of this thesis is to advance the application of the financial option theory in revenue management for the airline industries. The whole booking process is divided into several exclusive stages, based on the different demand scenarios. According to financial theory, both call and put options are used to offer different types of tickets to both customer and travel agent. The American Option (an American Option can be exercised any time before the expiring date) is adopted in the booking process and this gives more flexibility to the airlines to manage the whole booking process. The model finds the optimal booking limitation for call and put option tickets, and the prices for the striking prices of the two options.

Because the model proposed has significant practical meaning, a general description of how to use the model is presented. Finally, a numerical example is given to demonstrate the utilization of the model being proposed.

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Chapter 1

Introduction

1.1 Introduction on Revenue Management and Financial Option Theory

1.1.1 Revenue Management in the Airline Industry

The research on Revenue Management (RM) could be dated back to more than 40 years. However, at the beginning when such research was started, there was no a precise term for it. According to Weatherford and Bodily (1992), the concept of RM, or more precisely Yield Management at the beginning period, was not the same or as integrated as it is nowadays. Yield management, together with overbooking and pricing, which are included in the concept of RM nowadays, were considered separately. So Weatherford and Bodily proposed a new term - PARM, abbreviation for Perishable Asset Revenue Management, to replace the traditional separated definition. From then on, the research on Revenue Management became more integrated and accurate.

Revenue management (RM) has already been defined in many ways from different perspectives. In general, revenue management could be explained as the application of quantitative and qualitative methods when making specific decisions, in order to maximize the revenue for the provided services. In fact, there are many interpretations behind its board definition. Weatherford and Bodily (1992) summarized the following characteristics for revenue management:

1. One date on which the product or service becomes available and after which it is either not available or it ages. (Product is perishable.)
2. A fixed number of resource units.
3. The possibility of segmenting price-sensitive customers.

Because of the inherent nature of RM in practical application, researchers have started to use the concept in practice at the same time as the definition was brought up. According to Chiang et al. (2007), the application and research on RM could be dated back to 1966, when a computer reservation system (SABRE) was adopted by American Airline to replace the manual reservation process being used. Then, after 1978, when the Airline Deregulation Act was adopted and airline prices were largely relaxed, the established carriers were free to change prices, schedules and service without approval from the U.S Civil Aviation Board (CAB). From then on, the RM concept was studied and used more actively. Also, following the airlines, other industries began to resort to revenue management, expecting to improve their revenue. The airline industry is the earliest user of RM, and it is still its largest user.

Nowadays, RM is being used in a wide range of areas, such as the airlines, hotels and restaurants, media and broadcasting, resorts and attractions, car rentals, energy industry, and so on. Also, the increment in revenue of companies showed the great success of RM concept and techniques. Cross (1997) gave many examples that used RM methods. One of them is related to the Marriott Hotel gaining additional annual revenue after introducing RM practices. In another example, during the 1994-1995 season, the Washington Opera increased its revenue by about 9%, compared to the non-RM result of a 5% increase.

As the original field where RM was brought up, the airline industry is still depending on it to maximize the revenue nowadays. Smith (1992) claimed that RM helped American Airlines to generate an increasing revenue of $500 million from 1989 to 1992 annually, and Delta Airlines a $300 million increase in 2004. With such attractive benefits in real-life applications, the research in RM has accelerated.

1.1.2 Financial Option Theory

In finance, an option is defined as a certain right, not the obligation, to buy or sell an asset at a certain price on or before a given date. It is an effective tool confronting uncertainty and risk. There are two kinds of options which are mutually exclusive. One is the Call Option, and the other is the Put Option. The particular option selected depends on the buyer's judgment of the fluctuation of market value of the asset.

Take call options for example. A person, who is interested in a particular asset which could be both physical and non-physical, could only pay a small amount of money, called a Premium, to gain the right to own the wanted assets on or before a Maturity Date. According to the specific contract when she/he bought the right (option), this person could decide whether or not to execute the right to pay at a price, which is also pre-decided on the contract and is called Striking Price. This decision depends on the numerical relation among the market value of the asset, the strike price and the premium. In this process, there is often a time lag between the time of buying options and the date when the options could be executed. During this lag, the market value of the asset is always changing. Before buying the option, in order to gain more money, the person could “bet” on the price fluctuation. If on the maturity date, the market price is higher than the aggregation of the premium and the striking price, she/he could execute the option and pay the striking price. The difference between the market price and the aggregation of the two parts is what the person would gain. On the other hand, if the market price is lower than the aggregation, she/he needs not to execute, and what the person loses is just the premium. In this way, the person only needs to take a risk of small amount of money to win the asset whose real value is higher than the money paid for the options. So the risk is buffered and the person would benefit

Regarding airline tickets, these could be treated as options. The market values of the tickets are fluctuating as time goes by, for the demand and supply are changing. The airline could take some risk in order to maximize its revenue; for example, it could pay some money to gain the rights to buy or sell tickets. At the beginning period of selling a specific flight, the airline pays some money to buy the rights from customers. As the demand for the tickets increases, considering the market value of the tickets, the airline could make decisions such as whether to call back the tickets sold to customers previously, with the objective of maximizing its revenue. Here, customers could be individual customers or travel agents.

The above just provides general and brief insights to the adoption of option theory in the airline industry. In the later part of this thesis, a detailed and in-depth explanation will be given. But before getting into the details, it is necessary to define some terms used in option theory, in order to help understand the thesis. Some related terms are listed and explained.

Call option: A call option gives its holder the right to purchase an asset for a specific price, called the exercise or strike price, on or before some specified expiration date.

Put Option: A put option gives its holder the right to sell an asset for a specified exercise or strike price on or before some expiration date.

Exercise or Strike Price: price set for calling (buying) an asset or putting (selling) an asset

Premium: purchase price of an option

American option: an American option allows its holder to exercise the right to purchase or sell the underlying asset on or before the expiration date.

European option: A European option allows for exercise of the option only on the expiration date.

Callable: It is a word to describe a bond or a financial security, which means being able to be redeemed prior to maturity. The issuer of a callable product has to state the conditions under which the security may be called at the time of issue.

1.2 Literature Review

RM in the airline industry focuses on setting up models in areas such as pricing, capacity allocation, demand forecasting, overbooking techniques and so on. These are different perspectives addressing RM, but it is not the case that they are unrelated. In fact, sometimes, some of these research areas have to be considered together to achieve a specific goal. This classification is just aimed to help understand the problems in revenue management better and clearer. According to the literature, pricing techniques and capacity control are two areas where substantial work has been done.

1.2.1 Pricing in RM

In the airline industry, prices are adjusted dynamically as a function of inventory level and time left in the selling season (Fleischmann et al. 2004). So pricing concerns how to set the price along time considering the current inventory level. The uncertainty of demand is a great challenge in pricing the tickets following the real market value, and a dynamic programming method is adopted that provides the airlines a convenient way to adjust the price on time. The most commonly used method regarding pricing is stochastic dynamic programming. This method provides flexibility to airlines to react actively to the market and to inventory.

The most recent work in dynamic pricing is from Marcus. et al. (2008). These arthurs presented a general revenue management model for discounted tickets for single-leg flight, in which a low-cost firm captures exogenous demand by under pricing a full-service firm. They considered two formulations: one is a deterministic price and continuous-time demand version; the other is a stochastic price and discrete-time demand version. For the first formulation, structural results are derived; a simulation method is used to give the results for the latter one. For both versions, demand is assumed to be stochastic. They found an optimized discount solution for the low-cost providers to improve their revenue.

Other research discussed pricing issues in more general cases. For example, Feng and Gallego (2000), assuming a general stochastic demand which with Markovian, time-dependent, price-sensitive and predictable intensities, proposed an efficient algorithm to compute the optimal value functions and the optimal pricing policy, in order to maximize the expected revenue.

However, because of the inherently complex nature of dynamic programming, it is usually difficult to solve and implement it in real-life problems and hence the practical application is limited. A general literature review regarding pricing could be found in Fleischmann, et al. (2004) and Elmaghraby and Keskinocak (2003).

1.2.2 Capacity Allocation

Another important research area is the capacity allocation problem. It concerns the setting of limitations for different fares for both the single leg and network flights. Based on the techniques used, models regarding capacity allocation could be divided into two general categories, namely static model and dynamic model (Birbil et al. 2008). The static model attempts to give a specified limitation for different fares, based on the historical data or forecasted demand. The dynamic model takes the actual demand into consideration, and the limitations are adjusted dynamically.

For static models, Littlewood (1972) was the pioneer who proposed the EMSR (Expected Marginal Seat Revenue) model, to give an approximated booking limits for the two fare tickets on a single leg situation. He proposed that the booking limitation should satisfy the following:

$$f_2 >= f_1 * Pr\ (d_1 > p_1),$$

where f_i is the fare for class i, i = 1,2…, d_1 is the demand for first class, and p_1 is the booking limit for the first class. Then, Belobaba (1987, 1989) furthered the results above so that it can be used for multiple classes.

However, according to Brumelle and McGill (1993), for multiple fares, the EMSR model may underestimate the optimal protection levels. So they improved the model and gave the following results:

$$Pr\ (d_1 > p_1) = f_2/f_1,$$

$$Pr\ (d_1 > p_1 \cap d_1 + d_2 > p_2) = f_3/f_1.$$

Most recently, adopting simulation techniques, Birbil et al. (2009) proposed a mixed integer-linear programming model to get approximate booking levels for different fares.

For the dynamic model, van Ryzin and McGill (2000) proposed an adaptive algorithm for determining airline seat protection levels. Using historical observations of the relative frequencies of certain seat-filling events, they found optimized limits in accordance with some optimality conditions. Simulation should be used to solve the model. Another work regarding the dynamic model in capacity allocation is by Gosavi et al. (2002), who considered a single-leg, multi-fare problem as a semi-Markov decision problem under the optimizing criterion over an infinite time horizon. They solved the model using a stochastic optimization technique called Reinforcement Learning, which also uses simulation as the solving technique. However, for both static and dynamic models, they used the same assumption, that the prices for different fares are constant and pre-specified. This may be a simplification of the real-life situation.

1.2.3 Financial Option Theory

Recently, some experts switched their attention from the prevalent DP method and began to consider some new directions. One is financial option theory. They use the concept from option theory, treating the tickets as the options in finance and try to maximize the airline revenue.

Gallego et al. (2008) are the first who proposed to treat the airline ticket as “callable products”, which is a relevant concept in financial option theory. As mentioned in their research, a “callable product” is a unit of capacity sold at the low fare to self-selected buyers who willingly grant the capacity provider the option to “call” the capacity at a pre-specified recall price. More specifically, the airline could sell a discounted ticket to its customers, with the condition that the ticket could be “recalled” sometime in the future, and the discounted fare is a kind of compensation to the customers when the tickets are called back. They claim that under mild and reasonable conditions, “offering callable products can generate a riskless increase in revenue, that is, they never reduce and can increase it with positive probability.”

Their research addressed a problem where the booking process could be divided into two consecutive periods, namely low-fare and high-fare period, with low-fare customers’ requests handled first, because they are sensitive to price. They claim that "a major function of airline revenue management system is to calculate and apply booking limits on early-booking low-fare customers, to maximize revenue by reserving sufficient space for later booking high-fare customers." These in the low-fare customer group are divided into two sub-groups. One is the group where customers prefer the callable products and the other where customers prefer the standard low-fare tickets. They mainly discuss how to set two important variables, one being the limit between the low-fare and high-fare tickets, and the other being the call-back price paid to the customers. They derive the globally optimization formula for these two variables.

However, there are some limitations in their creative research. First, the market prices for the tickets are treated as constant as time goes by, although different values are used in numerical examples after the model is proposed. This may not be true in reality, for the demand and supply of the tickets cause fluctuation in the prices. In reality, the prices always change during the whole booking process. Second, they just adopt the "callable" concept in financial theory, instead of using option theory in the pricing method. Third, they do not take into consideration different demand scenarios into consideration. Finally, no-show, which is a common occurrence, is not considered in their research.

The work by Ching et al. (2007) combines the airline revenue management and option theory together, using the perspective of the travel agent. They consider a problem where the travel agents purchase a call option from the customers, and the related premium is in the form of a discount on the regular ticket fare. In their research, they are concerned more about the time value of the revenue the travel agent may gain. Because of the known distribution of the price and the fluctuation of the interest rate, the final market value of the tickets could be estimated. Their model solves for a price for the tickets which could both improve the revenue and decrease the risk of losing revenue.

However, although option concept is adopted, time value is what they are more concerned with. How to implement the call option and put option in the airline booking process is not discussed. Also, other realistic scenarios, such as different demand situations, overbooking and denied boarding, strategies on manipulating the tickets are not included.

Also, Akgunduz et al. (2007) use financial option theory. They consider a multiple-fare-classes single-leg system. Besides using the “callable product” idea, they propose that, depending on the relations between the forecasted demand and the total capacity, a call option or put option could be used as the method in the booking process. The European option concept was used in their model. They claim that if the total forecast exceeds the total capacity, a call option could be used in the booking process, whereas on the contrary, a put option would be the better choice. No-show, overbooking and denied boarding are considered in their research. They use simulation to give some solutions to different scenarios, assuming independent fares among classes. The objective function is based on a specific fare, instead of including all the fares into one model. Another limitation of their work is that the market prices of the tickets are still treated as constant.

Recently, Ravelojaona (2008) worked on a similar topic, and advanced the model from Akgunduz et al. (2007). Her main assumptions are the following:

(1) there is only a single-fare class.

(2) the ticket price is assumed to follow a random walk and has an associated distribution.

(3) a binomial option pricing model was used to derive the premiums of both the call option and put option approach.

The European option concept is adopted by both Akgunduz et al. (2007) and Ravelojaona (2008), which means that the options (tickets) are available only at some specific date. This concept is not always flexible in practice, for it requires that, only at some specific dates, the options (the right to buy or sell the tickets) could be executed. Based on the European option pricing model, the decision of whether to call back the option tickets sold before or not depends only on the price at the maturity date, which is the last booking period. This does not cover the case where the demand has already exceeded the capacity before the maturity date. Also, the number of the tickets to be called back depends only on the market price for the last booking period. This practice may not result in the optimal solution all the times. Finally, she proposed to use the call option and put option separately, which depends on the forecasted demand for the flight. But in fact, these two option methods do not have to be mutually exclusive, and it is reasonable to combine the two options together.

Although some limitations exist in the research mentioned above, these are the pioneers who used the financial option theory in revenue management in the airline industry. McGill and van Ryzin (1999), Pak and Piersma (2002) and Chiang at el. (2007) provide a thorough literature review for research in revenue management.

1.3 Contribution of the Thesis

Based on the earlier discussion, there are two important questions that need to be answered.

⑴ The American Option concept could provide more flexibility in the financial market, compared to the European Option. So how should the American Option be adopted, instead of the European option?

⑵ The binomial pricing method for the American option is used to find the premium for the tickets.

⑶ Different demand scenarios are considered in the model.

Based on the literature review, there are no papers addressing these questions as one problem. But the answers for these questions will be explored in this thesis. Also, this thesis will provide some guidelines to help the airlines improve their revenue in a more practical sense.

The following is the overview of the main remaining chapters. Chapter 2 is the main part of the thesis; it contains the problem description and model formulation. Chapter 3 gives a numerical example of the application of the model proposed. Chapter 4 contains the summary and conclusions and provides some ideas for future research.

Chapter 2

Model Formulation

2.1 Problem Description

This thesis discusses a single-class single-leg problem in the airline tickets booking process. “Single-fare single-leg” means that there is only one class on the plane, and this plane only offers services between one origin and one destination. Although there is only one class on the plane, the airline would like to offer a certain number of discounted tickets, in order to attract some customers, who have no urgent need for the ticket but may be attracted by the low fare for the discounted tickets. So at the beginning of the period, the airline should have to figure out how many tickets should be sold at the discounted price.

In reality, the selling period is continuous. However, for the convenience of setting up the model, the selling period is divided into several periods and treated as a discrete-time problem. Customers arrive independently at the beginning of each period. Each customer orders one ticket at a time.

The airline provides three types of tickets: call option tickets, put option tickets and standard tickets. Out of these, the put option tickets are provided only to the travel agents, while the other two types are offered to customers.

The model can be analyzed in four separate stages. Before the booking process begins, the airline has to figure out how many tickets to allocate to call option, put option, and standard tickets.

Stage 1

This is the first stage of the booking period, where only call option tickets sold at discounted price are offered to the customers. These tickets can be called back anytime in the future from the customers, but a pre-determined compensation will be given to the customers, which is the striking price for the call option.

At the same time, the airline promises some tickets to travel agents that they can sell at a determined price in the future, which uses the put option concept. However, these tickets are kept by the airline and not sold to the travel agents. Also, at the end of the booking process, it is possible that not all the promised tickets could be sold to the travel agents. The airline pays a premium to the travel agents who agree to be a part of put options.

Stage 2

When stage 1 is over, standard tickets are sold to customers at market prices, which fluctuate according to a binomial distribution. The airline cannot call back such tickets. When all the allocated standard tickets are sold out, this stage ends.

Stage 3

If after all the allocated call option tickets and standard tickets are sold, there is still demand from the customers, stage 3 will begin. The airline sells the put option tickets promised to the travel agents, and to the customers. At this stage, the price of the tickets sold to the customers is still the market price. To improve the revenue, the airline would prefer to do this only when the current market price is higher than the striking prices of the put options, which are specified beforehand. Only when the market price is much higher than the striking prices for put options, the airline would earn more money by selling the tickets itself; otherwise, the airline gives the tickets to the travel agents.

Stage 3 will continue until all the allocated put option tickets are sold out. If there are still some put option tickets remaining before the departure of the flight, they will be dumped to the travel agents.

Stage 4

If all the put option tickets are sold out, and the demand for tickets continues, then the airline begins to recall the tickets sold to the customers under the call option to meet the new demand. The tickets called back will be sold to the customers also at market price. To enhance the revenue, the airline will only recall when the current market price is very high, so that they could gain extra revenue after paying the promised compensation to the customers who bought call option tickets.

In fact, for a specific flight, not all the stages could be used. Depending on the actual demand, the booking process may stop at any stage. For example, if the total demand is smaller than the call option tickets and standard tickets, the process will stop at stage 2, and stages 3 and 4 will not be used.

In addition, in reality, it is possible that some customers will not show up on time for the flight. Because of this, the airline would set the booking limit higher than the actual capacity; this is called overbooking. However, if all the customers who bought the tickets show up, there will not be sufficient seats on the plane for all the customers.

Then the airline will have to deny some customers boarding rights and offer compensation to these customers. Although such strategies may result in unsatisfied customers, no-shows do happen, and hence overbooking is included in the model. In order to maximize the revenue, the airline would like to take some risk using overbooking. In fact, some customers who are denied seats on a flight may find the compensation attractive. Overbooking is frequently used in practice and it turns out to be an effective way to enhance revenues.

The objective of the problem is to maximize the revenue for the airline.

This thesis will focus on two problems:

(1) How to determine the optimum number of call and put option tickets?

(2) How to determine the striking prices for both call and put option tickets?

2.2 Model Formulation

2.2.1 Variables

A: booking limitation for call option tickets

B: booking limitation for put option tickets

C: total capacity of a single plane. Because no-show is considered in the model, the total booking limit for both discounted tickets and the standard tickets is (1+q)*C.

c_c: premium for call option

c_p: premium for put option

D: total demand of the tickets

d_j: demand for tickets in period j, j=0, 1 …T

n_j: number of tickets sold at period j. $n_j = \min\{d_j, l_j\}$, j=0, 1 …T

n_{db}: number of customers being denied boarding at the time of departure

n^c_j: number of tickets sold as call option tickets in period j, j=0, 1 …T

n^p_j: number of tickets sold as put option tickets in period j, j=0, 1 …T

n^r_j: number of tickets recalled in period j, j=0, 1 …T

n^s_j: number of standard tickets sold in period j, j=0, 1 …T

P_j: price of a standard ticket in time period j, j=0, 1 …T

P_{db}: compensation paid to customers who are denied boarding

q: probability of no-show of customers

S_c: striking prices for call option tickets

S_p: striking prices for put option tickets

T: total number of periods

2.2.2 Assumptions

The following assumptions are made to build the model:

(1) The selling period is discrete.

(2) Demand for each period is known at the beginning of the period.

(3) Although in reality customers have the right to choose between discounted and standard tickets, in order to make the model numerically easy to solve, it is assumed that all customers prefer the discounted tickets. After such tickets are sold out, the customers have no choice but to buy the standard tickets.

⑷ Tickets sold to customers as call option could be called back at any period.

⑸ The market price of the tickets varies according to a binomial distribution.

⑹ Striking prices are constant for both call and put option tickets.

2.2.3 Put option and call option

As explained in Chapter 1, call option buyers (customers) have the right to buy the tickets at a pre-specified price, while the buyers of put options (travel agents) have the right to sell the tickets. Put option tickets would be only sold to the travel agents, while the call option tickets are sold only to customers. The two options are explained in detail now.

The call option tickets are actually sold to customers, but the airline has the right to call back such tickets at any time. In order to gain such rights, the airline has to pay a premium, c_c, to the customers. This premium results in the discounted prices of such tickets. Sometime in the future, if the airline decides to call back such tickets from customers, the airline has to give compensation to them. This compensation, $S_{c,}$ is the striking price for the call option. Call options are used to not only attract customers, but also to gain extra revenue by calling back tickets to be sold at a higher price.

Put options are used to buffer the risk of the airline when there is a high possibility that the actual total demand is less than the total capacity. If it is true, some of the seats will be empty, and it is a “loss” to the airline. In order to buffer such risk, the airline promises to sell such tickets beforehand to travel agents. So at the end of the selling period, if there are unsold tickets, the airline has the right to “force” the travel agent to buy them. It should be noted that at the beginning, the tickets are not sold to the agent physically; instead, the options are meant to be “promises” to the travel agents, that in the future, the travel agent could get the tickets at the specified price, S_p. At the end of the selling period, the airline has the right to decide whether to execute such options, depending on the current selling conditions. If the actual demand is less than the total capacity, the airline does not exercise the put options, as this will result in a loss to the airline.

2.2.4 General model objective function

In this model, the selling period is divided into 4 stages. Stage 1 is the period when the call option tickets are sold; Stage 2 sells the standard tickets; Stage 3 is for selling the put option tickets; and Stage 4 is the recall period of the call option tickets. Next, the stages are discussed one by one.

Before the selling period begins

Here, using put options, the airline just makes promises to the travel agents. The number of tickets hedged with put option is B. The premium per ticket is c_p. So the current cash flow is negative, which could be expressed as

$$R_0 = - B * c_p. \qquad [2.1]$$

- **Stage 1**

The first period is the beginning of the selling period. In this stage, A tickets are hedged with call options. In period j, the number of call option tickets sold is n_j^c. The premium for the airline buying such tickets is c_c per ticket. For each period, the current market price of a ticket is P_j. So the revenue for this period is

$$R_1 = \sum_{j=1}^{T} n_j^c * \left(P_j - c_c\right). \qquad [2.2]$$

The variable n_j^c is >=0, and should either be the demand for period j or the balance of the call option tickets remaining. So it has the following expression:

$$n_j^c = \min\{d_j, \max\left[0, A - \sum_k^{j-1} n_k\right]\}. \qquad [2.3]$$

Then the revenue is

$$R_1 = \sum_{j=1}^{T} \min\{d_j, \max\left[0, A - \sum_k^{j-1} n_k\right]\} * \left(P_j - c_c\right). \qquad [2.4]$$

It should be noted here that n_k is the number of tickets sold by the airline, regardless of the type of ticket. Because the four stages are mutually exclusive, and because the call option tickets are sold first, this number is subtracted from the total of allocated call option tickets, A, to obtain the remaining call option tickets unsold.

- **Stage 2**

This is the stage of the standard tickets. The number of tickets sold at standard price P_j, for period j is n_j^s. So the revenue for this stage is

$$R_2 = \sum_{j=1}^{T} n_j^s * P_j. \qquad [2.5]$$

The variable n_j^s is also >=0 and it should either be the demand for the current period or the rest of the standard tickets remaining like n_j^c. So

$$n_j^s = \min\left\{d_j, \max\left[0, C - B - \sum_k^{j-1} n_k\right]\right\}. \qquad [2.6]$$

Here, (C-B) is the total of call option tickets and standard tickets. The number of tickets sold is subtracted from (C-B) to get the remaining standard tickets, if any. Then the revenue for this stage is

$$R_2 = \sum_{j=1}^{T} \min\{d_j, \max[0, C - B - \sum_{k}^{j-1} n_k]\} * P_{\cdot j} \qquad [2.7]$$

- **Stage 3**

In this stage, the airline sells the tickets hedged with put option at market price. The number of tickets sold like this for period j is n_j^p, and the selling price of put tickets is P_j. The revenue could be expressed as

$$R_3 = \sum_{j=1}^{T} n_j^p * P_{j.} \qquad [2.8]$$

The variable n_j^p is >=0. Similar to n_j^c, it should either be the demand for the current period or the rest of the standard tickets remaining. So

$$n_j^p = \min\{d_j, \max[0, C - \sum_{k}^{j-1} n_k]\}. \qquad [2.9]$$

Here, C is the total capacity of the plane. The total number of tickets sold is subtracted from C to obtain the remaining put option tickets, if any. Then, the revenue in this stage is

$$R_3 = \sum_{j=1}^{T} \min\{d_j, \max[0, C - \sum_{k}^{j-1} n_k]\} * P_{j.} \qquad [2.10]$$

- **Stage 4**

In this stage, because the excessive demand for the tickets is still coming, the airline begins to recall the tickets sold as call options. For period j, the number of tickets recalled is n_j^r, and they will be sold at market price P_j. However, for each ticket called back, the airline has to pay the customer compensation, which is S_c. So the revenue for this stage is

$$R_4 = \sum_{j=1}^{T} n_j^r * (P_j - S_c). \qquad [2.11]$$

Similarly, expression for n_j^r is the following:

$$n_j^r = \min\{d_j, \max\left[0, C + A - \sum_k^{j-1} n_k\right]\}. \qquad [2.12]$$

(C+A) is the summation of the capacity and the number is the call option tickets. Since call option tickets can be recalled, the actual number of tickets that could be sold is (C+A). Subtracting the number of tickets sold already could result in the number of call option tickets not recalled yet. Then, the revenue could be expressed as

$$R_4 = \sum_{j=1}^{T} \min\{d_j, \max\left[0, C + A - \sum_k^{j-1} n_k\right]\} * \left(P_j - S_c\right). \qquad [2.13]$$

After all or some of the stages come to an end, the whole selling season may stop. At the end, the airline also has to make decisions regarding the number of tickets that could be sold to travel agents. Because some of the hedged put option tickets might have been sold already, the rest of the tickets, if any, would be sold to the travel agents at the striking prices S_p. Then the revenue is

$$R_5 = \max[0, B - \sum_{j=1}^{T} n_j^p] * S_p). \qquad [2.14]$$

As explained above, R_5 can be written as

$$R_5 = \max[0, B - \sum_{j=1}^{T} \min\{d_j, \max\left[0, C - \sum_k^{j-1} n_k\right]\}] * S_p. \qquad [2.15]$$

However, it should be ensured that the different stages are mutually exclusive, or there will be duplicate calculations for different stages. Hence some binary variables are introduced to make sure that along the whole selling process, only one of the 4 stages could occur:

$$a_1 = \begin{cases} 1 & A-\sum_{k=0}^{j-1} n_k > 0 \\ 0 & \text{Otherwise} \end{cases} \quad [2.16]$$

$$a_2 = \begin{cases} 1 & C-B-\sum_{k=0}^{j-1} n_k > 0 \\ 0 & \text{Otherwise} \end{cases} \quad [2.17]$$

$$a_3 = \begin{cases} 1 & C-\sum_{k=0}^{j-1} n_k > 0 \\ 0 & \text{Otherwise} \end{cases} \quad [2.18]$$

$$a_4 = \begin{cases} 1 & C+A-\sum_{k=0}^{j-1} n_k > 0 \\ 0 & \text{Otherwise} \end{cases} \quad [2.19]$$

$$a = a_1 + a_2 + a_3 + a_4 \quad [2.20]$$

$$I_1 = \begin{cases} 1 & a = 4 \\ 0 & \text{Otherwise} \end{cases} \quad [2.21]$$

$$I_2 = \begin{cases} 1 & a = 3 \\ 0 & \text{Otherwise} \end{cases} \quad [2.22]$$

$$I_3 = \begin{cases} 1 & a = 2 \\ 0 & \text{Otherwise} \end{cases} \quad [2.23]$$

$$I_4 = \begin{cases} 1 & a = 1 \\ 0 & \text{Otherwise} \end{cases} \quad [2.24]$$

Then, the expressions of the number of tickets sold and the corresponding revenue for the periods are modified as follows:

Stage 1

$$n_j^c = \min\{d_j, \max[0, A - \sum_k^{j-1} n_k]\} * I_1 \quad [2.25]$$

$$R_1 = \sum_{j=1}^T \min\{d_j, \max[0, A - \sum_k^{j-1} n_k]\} * I_1 * (P_j - c_c) \quad [2.26]$$

Stage 2

$$n_j^s = \min\{d_j, \max[0, C - B - \sum_k^{j-1} n_k]\} * I_2 \quad [2.27]$$

$$R_2 = \sum_{j=1}^T \min\{d_j, \max[0, C - B - \sum_k^{j-1} n_k]\} * I_2 * P_j \quad [2.28]$$

Stage 3

$$n_j^p = \min\{d_j, \max[0, C - \sum_k^{j-1} n_k]\} * I_3 \quad [2.29]$$

$$R_3 = \sum_{j=1}^T \min\{d_j, \max[0, C - \sum_k^{j-1} n_k]\} * I_3 * P_j \quad [2.30]$$

Stage 4

$$n_j^r = \min\{d_j, \max[0, C + A - \sum_k^{j-1} n_k]\} * I_4 \quad [2.31]$$

$$R_4 = \sum_{j=1}^T \min\{d_j, \max[0, C + A - \sum_k^{j-1} n_k]\} * I_4 * (P_j - c_c) \quad [2.32]$$

Now, the overbooking and denied boarding are taken into consideration. The number of customers being denied boarding is n_{db}, and for each such customer, the airline has to pay compensation equal to P_{db}. Hence:

$$R_{db} = - n_{db} * P_{db} \quad [2.33]$$

Then, the total revenue could be written as

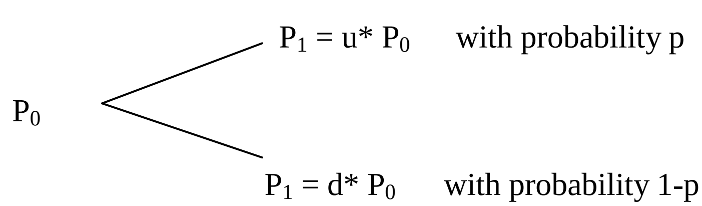
$$R = R_0 + R_1 + R_2 + R_3 + R_4 + R_5 + R_D \qquad [2.34]$$

$$= - B * c_p + \sum_{j=1}^{T} n_j^c * \left(P_j - c_c\right) + \sum_{j=1}^{T} n_j^s * P_j + \sum_{j=1}^{T} n_j^r * \left(P_j - S_c\right) + \sum_{j=1}^{T} n_j^r * \left(P_j - S_c\right) + \max[0, B - \sum_{j=1}^{T} n_j^p] * S_p - n_{db} * P_{db}$$

$$= - B * c_p + \sum_{j=1}^{T} \min\{d_j, \max[0, A - \sum_{k}^{j-1} n_k]\} * \left(P_j - c_c\right) + \sum_{j=1}^{T} \min\{d_j, \max[0, C - B - \sum_{k}^{j-1} n_k]\} * P_j + \sum_{j=1}^{T} \min\{d_j, \max[0, C - \sum_{k}^{j-1} n_k]\} * P_j + \sum_{j=1}^{T} \min\{d_j, \max[0, C + A - \sum_{k}^{j-1} n_k]\} * \left(P_j - S_c\right) + \max[0, B - \sum_{j=1}^{T} \min\{d_j, \max[0, C - \sum_{k}^{j-1} n_k]\}] * S_p - n_{db} * P_{db}.$$

2.2.5 Binomial Option Pricing Formula (BOPF)

As suggested by Cox et al. (1979), it is assumed that the ticket price follows a multiplicative binomial process during the selling periods. At the beginning of the selling time, the current market price is P_0. At the next period, there are two possible values of the tickets, increased value u^*P_0 $(u>1)$ with possibility p, and decreased value $d^* P_0$ $(d<1)$ with probability 1-p. The following graph depicts this process:



So P_1 is calculated as follows:

$$P_1 = p*u*P_0 + (1-p)*d*P_0 \qquad [2.35]$$

$$= \sum_{k=0}^{1} \binom{1}{k} p^k (1-p)^{1-k} * u^k * d^{1-k} * P_0.$$

Similarly, P_2 can be calculated based on P_0 as

$P_2 = u*u*P_0$ with probability p^2

$P_1 = u*P_0$

P_0 $\qquad$ $P_2 = d*u*P_0$ with probability $2*(1-p)*p$

$P_1 = d*P_0$

$P_2 = d*d*P_0$ with probability $(1-p)^2$

with

$$P_2 = [p^2*u^2 + 2*(1-p)*p*d*u + (1-p)^2*d^2]*P_0 \qquad [2.36]$$

$$= \sum_{k=0}^{2} \binom{2}{k} p^k (1-p)^{2-k} * u^k * d^{2-k} * P_0.$$

From the example shown above, P_j, for j = 1, 2 …T, is obtained as

$$P_j = \sum_{k=0}^{j} \binom{j}{k} p^k (1-p)^{j-k} * u^k * d^{j-k} * P_0. \qquad [2.37]$$

Once the price evolution is specified, the premiums are set for the tickets. According to Cox et al. (1979), the premiums for a call option or a put option are as follows:

$$\text{Put option: } c_p = \max\{0, S-P\}, \qquad [2.38]$$

$$\text{Call option: } c_c = \max\{0, P-S\}. \qquad [2.39]$$

Here, P is the current market price of the asset, and S is the striking price, specified when the options are bought.

In this problem, with the price evolution discussed above, the premiums for call and put option are as follows:

Put option: $c_{pj} = \max\{0, S-P_j\}$ [2.40]

$$= \max\{0, S - \sum_{k=0}^{j} \binom{j}{k} p^k (1-p)^{j-k} * u^k * d^{j-k} * P_0\},$$

Put option: $c_{cj} = \max\{0, P_j - S\}$ [2.41]

$$= \max\{0, \sum_{k=0}^{j} \binom{j}{k} p^k (1-p)^{j-k} * u^k * d^{j-k} * P_0 - S\}$$

for j=1…T.

2.2.6 Option Pricing

2.2.6.1 American option VS. European option

In our model, the American option concept is adopted for modeling the call and put options. Compared with the European option, which executes the option at the expiration date, the American option allows the execution at any time prior to the expiration date. In the airline ticket scenario, the word “execution” means selling the tickets to the customers or agents and the airline would never recall back these tickets again. In financial theory, the decisions, such as when or whether or not to execute the options are actually price driven. To explain this idea in a more explicit way, a call option using the European option is given as an example. For a specific asset, at the end of the maturity date, once its market price is higher than the pre-decided striking price, the owner of the asset has the right to sell the asset in order to gain more money;

on the other hand, if the market price is lower, the owner has the right not to sell the asset, thus avoiding a loss. So for the European option, whether it executes the price or not only depends on the price and the maturity date. However, while using the American option, such decisions are made at any time before the end of the maturity date.

Also, from the example given above, it is possible to find that the decision regarding whether or not to execute the options is price-driven for both the American and European options. However in this demand-driven model, once the demand for the standard tickets exceeds the capacity, the recall process should begin. Once this happens in the American option method, the recalled call option tickets are sold directly to the market according to the current market price. In the European option, only this is taken care of, which seems not practical and convenient. So in this sense, the American option method could provide a greater flexibility.

2.2.6.2 American option pricing model

As pointed above in the binomial pricing model, the premium value, c_j, for both call and put options for different periods can be obtained. The problem is how the premium value for all the tickets which are possible to be recalled back at any time could be obtained? Instead of adopting the complicated financial model to calculate the premiums for American option, it is suggested that a proper linear combination of c_j to get the exact values of c^c and c^p for call and put options, is found.

Here, for easy calculation, it is assumed that the weights for both call and put options are the same, and the weight sequence for different periods should be W = $\{w_1, w_2 \dots w_j, w_T\}$. So the final premiums for put option should be

$$c_p = \sum_{j=1}^{T} c_{pj} * w_j \qquad [2.42]$$

$$= \sum_{j=1}^{T} \max\{0, S - \sum_{k=0}^{j} \binom{j}{k} p^k (1-p)^{j-k} * u^k * d^{j-k} * P_0\} * w_j,$$

$$c_c = \sum_{j=1}^{T} c_{cj} * w_j \qquad [2.43]$$

$$= \sum_{j=1}^{T} \max\{0, \sum_{k=0}^{j} \binom{j}{k} p^k (1-p)^{j-k} * u^k * d^{j-k} * P_0 - S\} * w_j.$$

2.3 Model Objective Function

As the result of the earlier discussion, the final objective function is as follows:

Objective: max (E (R))

$$R = -\{\sum_{j=1}^{T} \max\left\{0, S - \sum_{k=0}^{j} \binom{j}{k} p^k (1-p)^{j-k} * u^k * d^{j-k} * P_0\right\} * w_j\} * B +$$

$$\sum_{j=1}^{T} \min\{d_j, \max[0, A - \sum_{k}^{j-1} n_k]\} * \left((\sum_{k=0}^{j} \binom{j}{k} p^k (1-p)^{j-k} * u^k * d^{j-k} * \right.$$

$$\left. P_0) - \{\sum_{j=1}^{T} \max\{0, \sum_{k=0}^{j} \binom{j}{k} p^k (1-p)^{j-k} * u^k * d^{j-k} * P_0 - S\} * w_j\} \right) +$$

$$\sum_{j=1}^{T} \min\{d_j, \max[0, C - B - \sum_{k}^{j-1} n_k]\} * (\sum_{k=0}^{j} \binom{j}{k} p^k (1-p)^{j-k} * u^k * d^{j-k} *$$

$$P_0) + \sum_{j=1}^{T} \min\{d_j, \max[0, C - \sum_{k}^{j-1} n_k]\} * (\sum_{k=0}^{j} \binom{j}{k} p^k (1-p)^{j-k} * u^k *$$

$$d^{j-k} * P_0) + \sum_{j=1}^{T} \min\{d_j, \max\left[0, C + A - \sum_{k}^{j-1} n_k\right]\} * \left((\sum_{k=0}^{j} \binom{j}{k} p^k (1 - p)^{j-k} * u^k * d^{j-k} * P_0) - S_c \right) + \max[0, B - \sum_{j=1}^{T} \min\{d_j, \max\left[0, C - \sum_{k}^{j-1} n_k\right]\}] * S_p - n_{db} * P_{db}$$

[2.44]

Chapter 3

Numerical Example

3.1 Model Application

Based on the model proposed in Chapter 2, a numerical example will be given in this chapter to show how we could use the model in practice. The method used is numerical search, which may only result in a local optimum. It is suggested that using a wide range of different initial values for the variables will help the user find a solution as close to the global optimal solution as possible. After the numerical result is presented, sensitivity on the initial values of the variables will be given.

3.1.1 Input Parameters

- **Demand Parameters**

The demand is an important parameter in our model and it is the factor which drives the whole process and all the stages. In this model, the demand during period j, d_j, j=1…T, is distributed as follows:

Table **3-1**: Demand Parameters					
Demand (unit)	20	30	40	50	60
Probability	0.10	0.25	0.30	0.25	0.10

- **Price Parameters**

The market price for the tickets is assumed to follow the binomial distribution. As indicated before, the ticket price in period j is P_j and it could be calculated as follows:

$$P_j \begin{cases} P_{j+1} = u^* P_j & \text{with probability } p \\ P_{j+1} = d^* P_j & \text{with probability } 1-p \end{cases}$$

Here, the parameters are set as indicated in the following table:

Table **3-2**: Price Parameters

u	1.10
d	0.90
p	0.60

- **Other Parameters**

In this model, our objective is to find the optimal values for premiums and booking limitations for both call and put option tickets. Also, because of the numerical search used, the increment of the variables should be given.

Table **3-3**: General Parameters

Total number of periods	T=6
Probability of no-show of customers	q=0.1
Total capacity of a flight	C=200
Compensation paid to customers who denied boarding	P_{db} = \$250
Initial ticket price	P_0 = \$200

3.1.2 Initial Values and Stepping Size for Variables

Because a numerical search method is used in the example, the global optimal values for decision variables may not be found. Initial values and step sizes for the variables should be given beforehand. The initial values and step sizes are given in Table 3-4.

Table **3-4**: Initial Values and Step Size for Variables

Booking limitation for call option tickets	A = 50
Booking limitation for put option tickets	B = 50
Striking prices for call option tickets	SC = $210
Striking prices for put option tickets	SP = $210
Step size of booking limitation for call option tickets	ΔA = 5
Step size of booking limitation for put option tickets	ΔB = 5
Step size of striking price for call option tickets	ΔSC = $5
Step size of striking price for put option tickets	ΔSP =$5

3.1.3 Optimal Results

The numerical search algorithm yielded the following results:

Table **3-5**: Results

	Expected Revenue	Optimal Values
Without Option*	E[R] = $41340.00	A = 0

		B = 0
		SC = 0
		SP = 0
With Option	E[R] = $42996.18	A=10
		B=30
		SC= $200
		SP=$200

* During the process of selling tickets without options, the tickets are sold at the current market prices till all the tickets are sold out. Tickets will not be called back or dumped on the travel agents.

As we could see from the table above, using financial option theory, the airline could improve its revenue by 4.01%. So it is recommended that the airline may consider adopting financial option theory into practice in order to improve its profit.

3.2 Sensitivity Analysis

In the example given above, the numerical search method is used to calculate the expected revenue for the airline. Some initial values are used because it is possible that the initial value may have an impact on the results. So in this section, a sensitivity analysis, regarding some important parameters such as the price parameter u, d and p, is conducted.

u, d and p are parameters for the distribution of the market prices for the tickets. In real life scenario, the prices fluctuate because of the relation between demand and supply. So airlines could not forecast the parameters accurately, and they face the risk of adopting wrong parameter values, which may influence the final results. So, a sensitivity analysis regarding the price parameters is conducted in order to help the users understand the influence of these price parameters. Also, it should be noted that other parameters are kept at the same values as the values used in the numerical example.

3.2.1 Price parameter, u

In the model, u is one of the binomial parameters for market price distribution, and it represents the increment percentage of the price. The values used in the sensitivity analysis are summarized in Table 3-6, and the results are graphically represented in Figure 3-1.

Table **3-6**: Sensitivity analysis on u

u	d	p	ER
1.05	0.9	0.6	43015.25
1.06	0.9	0.6	43023.91
1.07	0.9	0.6	43026.07
1.08*	0.9*	0.6*	43027.16*
1.09	0.9	0.6	43012.05
1.1	0.9	0.6	42996.18
1.11	0.9	0.6	42979.54
1.12	0.9	0.6	42951.38
1.13	0.9	0.6	42920.86
1.14	0.9	0.6	42889.28
1.15	0.9	0.6	42852.74

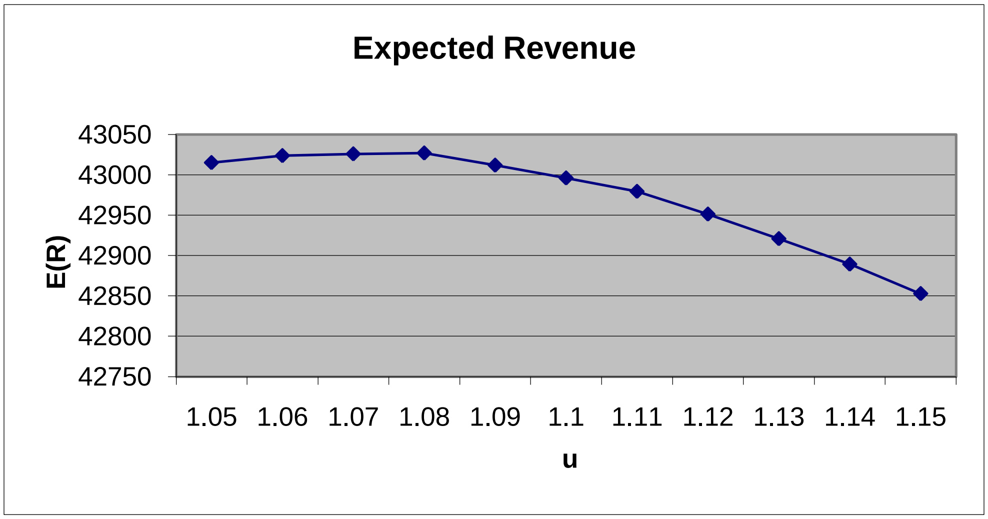


Figure 3-1 Sensitivity Analysis on u

As we see from Figure 3-1, as the value of u increases, the expected revenue increases slightly first and then declines significantly. Based on the parameters given, the optimal value for u is 1.08, whereas the expected optimal revenue is $ 43027.16*. Intuitively, it may be expected that the higher the increment of the price, the higher the expected revenue, for high values of u will help to increase the market price. However, from the sensitivity analysis, it can be seen that is not always the case. The increment, u, not only influences the price, but it will also impact the premiums for put and call options. So if u is too large, the premiums for call and put options will also be increased, which will decrease the revenue. So the users of this model should be careful in selecting the value of u.

3.2.2 Price parameter, d

Here, d is the other binomial parameter for market price distribution. It represents the decrement percentage of the price. The values used in the sensitivity analysis are summarized in Table 3-7, and the results are graphically represented in Figure 3-2.

Table **3-7**: Sensitivity analysis on d

d	u	p	ER
0.85	1.1	0.6	42926.04
0.86	1.1	0.6	42945.64
0.87	1.1	0.6	42962.61
0.88	1.1	0.6	42973.21
0.89	1.1	0.6	42984.4
0.9	1.1	0.6	42996.18
0.91*	1.1*	0.6*	43007.25*
0.92	1.1	0.6	43005.48
0.93	1.1	0.6	43004.02
0.94	1.1	0.6	43000.45
0.95	1.1	0.6	42983.88

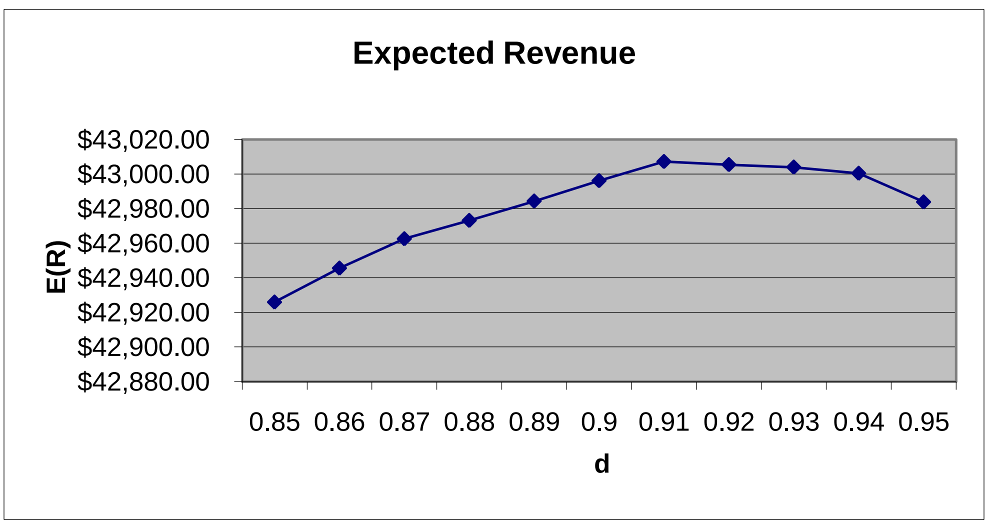


Figure 3-2 Sensitivity Analysis on d

From Figure 3-2, it can be seen that the revenue first increases as d increases, and then decreases. Based on the given parameters, the optimal value for d is 0.91, and the corresponding expected revenue is $ 43007.25*. So in the case of u, it may be expected that as d increases, the expected revenue increases, because a higher value of d means that the decrease of price is smaller. So larger d may result in higher market price, and the revenue should be larger. However, d will also influence the premiums for both call and put options. So it is also necessary to find a reasonable value for d, not only to make it realistic, but also to make sure that the expected revenue is larger.

3.2.3 Price parameter, p

The price parameter, p, is the binomial parameter for market price distribution. It represents the probability of price increasing. The values used in the sensitivity analysis are summarized in Table 3-8, and the results are graphically represented in Figure 3-3.

Table **3-8**: Sensitivity analysis on p

p	**u**	**d**	**ER**
0.55	1.1	0.9	42980.76
0.56	1.1	0.9	42984.57
0.57	1.1	0.9	42988.02
0.58	1.1	0.9	42991.11
0.59	1.1	0.9	42993.83
0.6	1.1	0.9	42996.18
0.61	1.1	0.9	42998.16
0.62	1.1	0.9	42999.77
0.63	1.1	0.9	43001
0.64	1.1	0.9	43001.87
0.65	1.1	0.9	43002.39

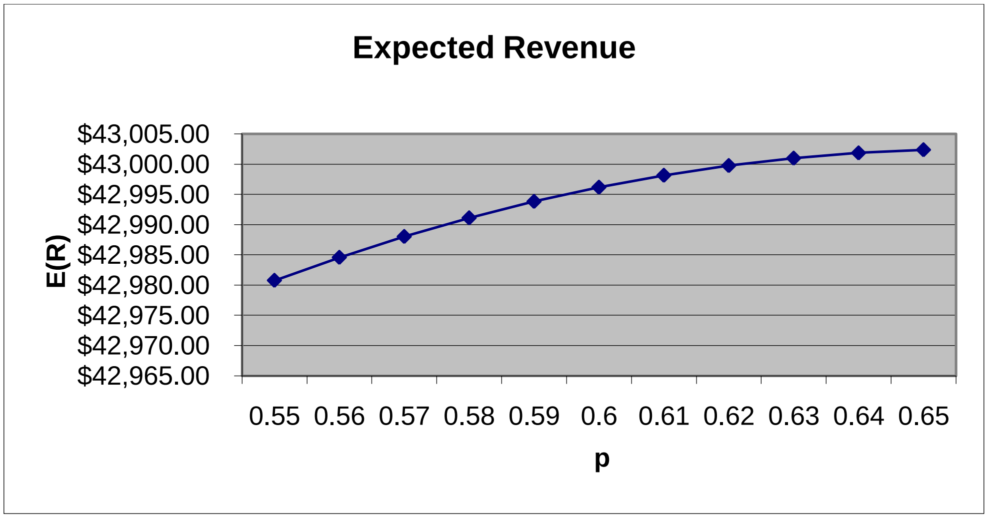


Figure 3-3 Sensitivity Analysis on p

Figure 3-3 shows a different trend. As the value of p increases, the expected revenue also increases, though the rate of increase changes. The probability of the price increasing is p. Larger value of p means that it is more probable for the price to increase. So the expected revenue may be larger with a higher p. However, it may not be realistic to have a large value of p, because the price could never be increasing all the time. It is necessary to choose a value of p that accurately represents the market price fluctuations.

Chapter 4

Conclusion and Future work

4.1 Conclusions

Adopting the financial option theory, this thesis provides a model to maximize the revenue for airlines. Airline tickets are treated like options in finance, and call and put option concepts are applied to customers and travel agents. At the beginning, knowing the demand forecast, the airline will set booking limits for both call and put option tickets. As the selling process moves on, the airline will make decisions, such as to call back the call option tickets sold to customers and how many put option tickets could be given to the travel agents. When hedging the tickets with call and put options, the airline has to pay the customers and the travel agents premiums in order to gain the rights to the tickets. Both the limitations and premiums for the call and put option tickets are important variables in this model.

Because the airline could either call back the call option tickets from the customers or give the excess tickets to the travel agents as put option tickets, it would gain more flexibility to manipulate the tickets in order to buffer the risk from demand uncertainty and to maximize the expected revenue. Also, by using the American option, instead of waiting until the departure of the flight, the airline could call back the call option tickets or sell the put option tickets to travel agents at any time. This will result in much more convenience and flexibility to the airline.

In addition, the fluctuation of the market price of the tickets is also taken into consideration. The binomial distribution is used to model the changes of the prices.

The model developed in this thesis is much easier to analyze, compared to stochastic dynamic programming models.

4.2 Future work

A wide range of work still could be done regarding adopting financial theory in revenue management for the airline industry. First, in our model, it is assumed that the airline just offers one kind of ticket to customers at a specific time period, either the call option ticket or the standard ticket. However, it is very reasonable for the airline to modify the choices to customers; for example, it could offer the two kinds of tickets to its customers at any period. Modeling this situation could be an interesting future research. Second, in our example, a numerical search method is used, which may result in a local optimum, depending on the initial value and search steps of the variables. Developing a global optimum solution is suggested for future work. Overbooking, included in this model, could result in customer dissatisfaction. A possible additional objective to be considered in the future is minimizing the dissatisfaction of customers.

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Appendix A

VBA Code*

* Part of the calculation was done on the spreadsheet directly.

```
Option Explicit '/* variables used in the model */

Private A As Integer '/* number of call option tickets*/

Private B As Integer '/* number of put option tickets*/

Private C As Integer '/* flight capacity*/

Private T As Integer '/* total selling periods */

Private n(10) As Integer '/* number of tickets sold */

Private nc(10) As Integer '/* number of call option tickets sold */

Private ns(10) As Integer '/* number of standard tickets sold */

Private np(10) As Integer '/* number of put option tickets sold */

Private nr(10) As Integer '/* number of tickets recalled */

Private dmd(10) As Integer  '/* demands for different periods */

Private ER As Single '/* expected revenue*/

Private p(10) As Single  '/* prices of tickets for each periods */
```

```
Private P0 As Single '/* prices of tickets for each periods */

Private cc As Single '/* premium for call option */

Private cp As Single '/* premium for put option */

Private SC As Single '/* striking price of call option */

Private SP As Single '/* striking price of put option */

Private j As Integer '/* index for period */

Private q As Single '/* probability of no-show */

Private w(10) As Single '/* weights for different periods */

Private DBP As Single '/* denied boarding cost for each customer */

Private DB As Integer '/* denied boarding customers */

Private CT As Integer '/* total booking limitation */

Dim u As Single

Dim d As Single

Dim prob As Single


Private Sub generalparameters() '/* values for general parameters */

    T = Worksheets("Optimal").Range("T").Value
```

```
P0 = Worksheets("Optimal").Range("PO").Value

C = Worksheets("Optimal").Range("Cap").Value

q = Worksheets("Optimal").Range("q").Value

DB = Worksheets("Optimal").Range("DB").Value

j = 0

CT = C * (1 + q)

n(T) = 0 '/* initial value is 0 */

nc(T) = 0

ns(T) = 0

np(T) = 0

nr(T) = 0

w(1) = 0.1

w(2) = 0.1

w(3) = 0.15

w(4) = 0.15

w(5) = 0.25

w(6) = 0.25
```

```
End Sub

Private sub demandparameters()
    Worksheets("Demand").Range("Demand1") = demand(1)
    Worksheets("Demand").Range("Demand2")= demand(2)
    Worksheets("Demand").Range("Demand3")= demand(3)
    Worksheets("Demand").Range("Demand4")= demand(4)
    Worksheets("Demand").Range("Demand5")= demand(5)
    Worksheets("Demand").Range("Demand6")= demand(6)
    dmd(1) = Worksheets("Demand").Range("Demand1")
    dmd(2) = Worksheets("Demand").Range("Demand2")
    dmd(3) = Worksheets("Demand").Range("Demand3")
    dmd(4) = Worksheets("Demand").Range("Demand4")
    dmd(5) = Worksheets("Demand").Range("Demand5")
    dmd(6) = Worksheets("Demand").Range("Demand6")
End Sub
```

```
Private Sub priceparameters()

        u = Worksheets("Optimal").Range("u").Value

        d = Worksheets("Optimal").Range("d").Value

        prob = Worksheets("Optimal").Range("p").Value

End Sub



Private Sub initial_variables() '/* initial value */

        A = Worksheets("Optimal").Range("A_ini").Value

        B = Worksheets("Optimal").Range("B_ini").Value

        SC = Worksheets("Optimal").Range("SC_ini").Value

        SP = Worksheets("Optimal").Range("SP_ini").Value

End Sub



Function binoCoeff(n, k)

        Dim i As Integer

        Dim bn As Double

        bn = 1
```

```
    For i = 0 To k - 1
    bn = bn * (n - i) / (k - i)
    Next i
    binoCoeff = bn
End Function

Function Max(x, y)
    If x < y Then
    Max = y
    Else
    Max = x
    End If
End Function

Function Min(x, y)
    If x > y Then
    Min = y
```

```
        Else
        Min = x
        End If
End Function

Private Sub ticketprices()
Dim k As Integer
Dim i As Integer
Dim inc_price As Single
Dim t_price As Single
Dim pp(10) as single
t_price = 0
For i = 1 To T
For k = 0 To i
inc_price = binoCoeff(i, k) * prob ^ k * u ^ k * (1 - prob) ^ (i - k) * d ^ (i - k) * P0
t_price = t_price + inc_price
Next k
```

```
pp(i) = t_price

Next i

p(1) = pp(1)

p(2) = pp(2)-pp(1)

p(3) = pp(3)-pp(2)

p(4) = pp(4)-pp(3)

p(5) = pp(5)-pp(4)

p(6) = pp(6)-pp(5)

End Sub



Private Sub call_option_price()

Dim k As Integer

Dim cc_inc As Single

Dim ccarray(10) As Single

cc = 0

ccarray(0) = 0

For k = 1 To T
```

```
cc_inc = binoCoeff(T, k) * (prob ^ k) * ((1 - prob) ^ (T - k)) * (P0 * (u ^ k) * (d ^ (T - k)) - SC)
If cc_inc < 0 Then
cc_inc = 0
End If
ccarray(k) = ccarray(k - 1) + cc_inc
Next k
For k = 1 To T
cc = ccarray(k) * w(k) + cc
Next k
End Sub


Private Sub put_option_price()
Dim k As Integer
Dim cp_inc As Single
Dim cparray(10) As Single
```

```
cp = 0
cparray(0) = 0
For k = 1 To T
cp_inc = binoCoeff(T, k) * (prob ^ k) * ((1 - prob) ^ (T - k)) * (SP - (P0 * (u ^ k) * (d
^ (T - k))))
If cp_inc < 0 Then
cp_inc = 0
End If
cparray(k) = cparray(k - 1) + cp_inc
Next k
For k = 1 To T
cp = cparray(k) * w(k) + cp
Next k
End Sub

Private Sub expectedrevenue()
Dim jj As Integer
```

```
Dim ST As Integer

Dim STC(10) As Integer

Dim STS(10) As Integer

Dim STP(10) As Integer

Dim revenue(10) As Single

ST = 0

revenue(0) = -B * cp

For jj = 1 To T

STC(jj) = Min(dmd(jj), Max(0, (A - ST)))

STS(jj) = Min(dmd(jj), Max(0, ((CT - B) - ST)))

STP(jj) = Min(dmd(jj), Max(0, CT - ST))

If STC(jj) > 0 Then

STS(jj) = 0

STP(jj) = 0

ElseIf STS(jj) > 0 Then

STC(jj) = 0

STP(jj) = 0
```

```
ElseIf STP(jj) > 0 Then

STC(jj) = 0

STS(jj) = 0

Else

STC(jj) = 0

STS(jj) = 0

STP(jj) = 0

End If

ST = ST + STC(jj) + STS(jj) + STP(jj)

revenue(jj) = STC(jj) * ((p(jj) - cc)) + STS(jj) * p(jj) + STP(jj) * p(jj)

ER = revenue(0) + revenue(jj) + ER

Next jj

DB = Max(0, (CT * (1 - q) - C))

ER = ER - DB * DBP

End Sub



Private Sub report()
```

```
With Worksheets("Optimal")
.Range("ER").Value = ER
.Range("A_opt").Value = A
.Range("B_opt").Value = B
.Range("SC_opt").Value = SC
.Range("SP_opt").Value = SP
.Range(“Price1”) = p(1)
. Range(“Price2”) = p(2)
. Range(“Price3”) = p(3)
. Range(“Price4”) = p(4)
. Range(“Price5”) = p(5)
. Range(“Price6”) = p(6)
. Range(“callpremium”) = cc
. Range(“putpremium”) = cp
End With
End Sub
```

```
Function demand(j)

    Dim z As Single

    z = Rnd()

    If z < Worksheets("Demand").Range("Prob1").Value Then

        dmd(j) = Worksheets("Demand").Range("D_1").Value

    End If

    If z > Worksheets("Demand").Range("Prob1").Value And z <
   Worksheets("Demand").Range("Prob2").Value Then

          dmd(j) = Worksheets("Demand").Range("D_2").Value

    End If

    If z > Worksheets("Demand").Range("Prob2").Value And z <
   Worksheets("Demand").Range("Prob3").Value Then

          dmd(j) = Worksheets("Demand").Range("D_3").Value

    End If

    If z > Worksheets("Demand").Range("Prob3").Value And z <
   Worksheets("Demand").Range("Prob4").Value Then

          dmd(j) = Worksheets("Demand").Range("D_4").Value
```

```
    End If
   If z > Worksheets("Demand").Range("Prob4").Value And z <
  Worksheets("Demand").Range("Prob5").Value Then
        dmd(j) = Worksheets("Demand").Range("D_5").Value
    End If
End Function
```