AN ANALYSIS OF BIDDING STRATEGIES IN REVERSE AND
COMBINATORIAL AUCTIONS

A Dissertation in
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by
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Abstract

This dissertation deals with the problem bidders have to solve to formulate their bids in two types of auctions: reverse and combinatorial auctions.

In reverse procurement auctions the object for sale is a contract, bidders are suppliers, and the bid taker is a buyer. The suppliers bidding for the contract are usually the current supplier (the incumbent) and a group of potential new suppliers (the entrants). Since the buyer has an ongoing relationship with the incumbent, he needs to adjust the bids of the entrants to include non-price attributes. The buyer can run a scoring auction, in which suppliers compete on the adjusted bids or scores, or, he can run a buyer-determined auction, in which suppliers compete on the price, and the buyer adjusts the bids with the non-price attributes after the auction to determine the winner. In the second chapter I study the bidding strategies in a buyer-determined auction in which an incumbent and a group of suppliers compete for a contract.

When different types of suppliers compete (for example domestic and foreign suppliers), practitioners have observed that domestic suppliers stop bidding after observing a low bid from the foreign competitors, even though the domestic suppliers dominate their foreign competitors in non-price dimensions. To mitigate this anticompetitive behavior, the feedback given to suppliers in the auction changed from the bid price to the bid rank. In the third chapter, I study the bidding strategies when two suppliers of different type compete in a reverse auction with rank feedback.

Finally, in the fourth chapter, I study the bidding strategies in sealed-bid combinatorial auctions. Combinatorial auctions are auctions of multiple heterogeneous objects that allow bids on subsets of the objects, giving bidders the flexibility to express if the objects in a set are more valuable together than separate. This added flexibility makes it possible for the bidders to express a variety of preferences, but also complicates the problem they need to solve to find their bidding strategies. I present the problem a bidder has to solve in a combinatorial auction of two objects, in which bidders submit mutually exclusive bids once and winners pay their bids.
# Table of Contents

List of Figures vi

List of Tables vii

Acknowledgments viii

Chapter 1
  Introduction 1
    1.1 Reverse auctions ........................................ 1
    1.2 Combinatorial auctions ................................. 4

Chapter 2
  An analysis of bidding strategies in buyer-determined procurement auctions 6
    2.1 Literature review .......................................... 7
    2.2 The model .................................................. 10
      2.2.1 One incumbent versus one entrant .................. 11
      2.2.2 One incumbent versus multiple entrants ........... 21
    2.3 Conclusions ................................................ 29

Chapter 3
  An analysis of bidding strategies in procurement auctions with rank feedback 31
    3.1 Bidding strategies for rank feedback with non-overlapping costs and known opponent identity .......................... 32
    3.2 Bidding strategies for rank feedback with overlapping costs and known opponent identity .......................... 34
    3.3 Bidding strategies for rank feedback with overlapping costs and unknown opponent identity .......................... 36
    3.4 Simulation results ........................................ 40
### Chapter 4
An analysis of bidding strategies in sealed-bid combinatorial auctions

<table>
<thead>
<tr>
<th>Section</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1</td>
<td>The auction</td>
<td>45</td>
</tr>
<tr>
<td>4.2</td>
<td>Bidding strategies</td>
<td>45</td>
</tr>
<tr>
<td>4.2.1</td>
<td>Bidding for the individual objects</td>
<td>46</td>
</tr>
<tr>
<td>4.2.2</td>
<td>Bidding for the package</td>
<td>48</td>
</tr>
<tr>
<td>4.2.3</td>
<td>Bidding for the package and something else</td>
<td>50</td>
</tr>
<tr>
<td>4.3</td>
<td>The Uniform case</td>
<td>52</td>
</tr>
<tr>
<td>4.3.1</td>
<td>Results for the bidding strategies</td>
<td>53</td>
</tr>
<tr>
<td>4.3.2</td>
<td>Bidding for overlapping sets</td>
<td>55</td>
</tr>
<tr>
<td>4.4</td>
<td>Competing against oneself in the lab</td>
<td>57</td>
</tr>
<tr>
<td>4.5</td>
<td>Conclusions</td>
<td>66</td>
</tr>
</tbody>
</table>

### Appendix A
Appendix for Chapter 1

<table>
<thead>
<tr>
<th>Section</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.1</td>
<td>Simulation results</td>
<td>68</td>
</tr>
</tbody>
</table>

### Appendix B
Appendix for Chapter 3

<table>
<thead>
<tr>
<th>Section</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>B.1</td>
<td>Event trees</td>
<td>72</td>
</tr>
<tr>
<td>B.2</td>
<td>CRRA expected profit calculations</td>
<td>75</td>
</tr>
<tr>
<td>B.3</td>
<td>Individual logistic regression results</td>
<td>75</td>
</tr>
<tr>
<td>B.4</td>
<td>Instructions for experiments</td>
<td>77</td>
</tr>
</tbody>
</table>

### Bibliography

81
List of Figures

2.1 difference=average cost in buyer-determined auction (BD) - average cost in scoring auction (scoring) .......................... 28
2.2 difference=average cost in buyer-determined auction (BD) - average cost in scoring auction (scoring) .......................... 29

3.1 Average cost for the buyer from 100,000 simulations of the bidding strategies ......................................................... 41

4.1 Event tree when the bidder bids $b_\alpha$ ........................................... 47
4.2 Event tree when the bidder bids $b_{\alpha\beta}$ ........................................ 48
4.3 Event tree when the bidder bids $b_\alpha$ and $b_{\alpha\beta}$ .................. 50
4.4 Probability that the best competitive bid be superadditive ......... 55
4.5 Optimal bids when $n_\alpha = n_\beta = 3, n_{\alpha\beta} = 2, k = 1.2$ ........... 56
4.6 (-- Optimal expected profit of bidding only for the package. (-- --)
    Optimal expected profit of bidding for the package and one object. ... 57
4.7 Cumulative distributions of profits .............................................. 62
4.8 Cumulative distributions of profits .............................................. 63
4.9 Percentage of rounds each bidding strategy was chosen ........... 64
4.10 Cumulative distribution of rounds .............................................. 65
4.11 Percentage of subjects that bid for the object in each round ....... 65

B.1 Event tree when the bidder bids for all ...................................... 72
B.2 Event tree when the bidder bids for all - I ................................. 73
B.3 Event tree when the bidder bids for all - II ............................... 73
B.4 Event tree when the bidder bids for all - III .............................. 74
B.5 Event tree when the bidder bids for all - IV .............................. 74
List of Tables

2.1 Scenario 1 .................................................. 19
2.2 Scenario 2 .................................................. 20
2.3 Results from simulations ................................. 20
2.4 Notation ..................................................... 24

4.1 Optimal bidding strategies assuming Uniform ....................... 54
4.2 Expected (risk neutral) profit for bidding strategies in the experiments 60
4.3 Average profit in T-71 and T-91 ............................... 61

A.1 Results for the average cost (and its variance) – UC1 ............... 68
A.2 p-values for difference with scoring – UC1 ............................. 69
A.3 Results for the average cost (and its variance) – BC1 ............... 69
A.4 p-values for difference with scoring – BC1 ............................. 69
A.5 Results for the average cost (and its variance) – UC2 ............... 70
A.6 p-values for difference with scoring – UC2 ............................. 70
A.7 Results for the average cost (and its variance) – BC2 ............... 70
A.8 p-values for difference with scoring – BC2 ............................. 71

B.1 Expected CRRA utility function for bidding strategies in the experiments ........................................... 75
B.2 Expected CRRA utility function for bidding strategies in the experiments ........................................... 75
B.3 Expected CRRA utility function for bidding strategies in the experiments ........................................... 75
B.4 Results for T-71 ................................................. 76
B.5 Results for T-91 ................................................. 76
Acknowledgments

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In memory of Michael H. Rothkopf
1939–2008
Chapter 1

Introduction

The introduction is divided in two sections, one related to reverse auctions (which corresponds to the setting studied in Chapters 2 and 3), and one related to combinatorial auctions (which corresponds to the setting studied in Chapter 4).

1.1 Reverse auctions

In the last two decades, electronic (reverse) auctions started to revolutionize procurement practices. Instead of having separate time-consuming negotiations with a group of suppliers, a buyer can make them compete in an auction to award a contract.

In buyer-determined auctions, which are the most common type of procurement auction, suppliers compete on price and after the bidding is over, the buyer considers non-price attributes to award the contract (Jap 2002). These auctions are used broadly, spanning procurement for a wide range of products and services; examples include contracts to provide complex machined metal components, critical printed circuit board assemblies, marketing services and legal services (Furey 2009).

Because bidding starts at a high price that goes down these mechanisms are known as dynamic reverse auctions, as opposed to traditional dynamic forward auctions, in which bidding starts at a low price that goes up.

Like traditional forward auctions, reverse auctions have the capacity to reduce transaction costs and improve competition through dynamic bidding, making them an attractive tool for procurement managers. Furey (2009) reports that in the last
ten years, reverse auctions run by Ariba—a company specializing in procurement services—drove incremental savings of five to seven percent on average, versus quotations obtained without dynamic bidding.

An accepted rule in these auctions is that the buyer may retain the right not to award the contract to the supplier who submitted the lowest bid, based on final qualification results (Freemarkets 2001), (Jap 2002). Therefore, allocation is not their main objective, opposite to traditional forward auctions, which are primarily designed to allocate objects (some examples of objects are art paintings, cars, wines, horses, fish and flowers). Since buyer-determined auctions do not guarantee that the lowest bidder wins the contract, this lack of commitment can lessen competition, because a supplier does not need to win the auction to win the contract.

Usually, non-price factors such as experience, quality, and switching costs, need to be considered by the buyer before awarding the contract. If the buyer can quantify the value of the non-price attributes for each supplier, he can run a scoring auction, in which suppliers compete on the scores or adjusted bids (for example, the bid plus the non-price attributes). In this case, the supplier with the lowest adjusted bid will correspond to the best supplier, and the result of the auction will dictate who wins the contract. If those non-price attributes are not available and are expensive to determine, the buyer can run a buyer-determined auction, in which suppliers compete on the price, and after the auction is over, the buyer selects some of the bidders and adjusts their bids -by adding a factor that accounts for the non-price attributes- to determine the winner. In this case, the buyer can choose to adjust some or all bids.

Regarding the type of feedback given in the auction, the buyer can choose to provide bidders with the rank of the bids instead of the price bids. This type of auction format is becoming popular, because as auctions have expanded to include both domestic and foreign suppliers, practitioners have noted that domestic suppliers stop bidding early on the auction, when they see a low bid submitted by the foreign competitors (Carter, Kaufmann, Beall, Carter, Hendrick & Petersen 2004), even though they can dominate the foreign suppliers in non-price dimensions. The objective of rank feedback is to overcome this anticompetitive effect.

Whether a buyer does better by (1) running a scoring or a buyer-determined
auction, and (2) using rank feedback instead of full feedback, are relatively un-studied questions in the literature. To understand if the scoring or the buyer-determined auction generates the lowest cost for the buyer, and the effect of rank feedback, we first need to understand the suppliers’ bidding strategies in those settings.

To model the strategies we can assume fully rational bidders or (myopic) best-responding bidders. Under the full rationality assumption, bidders decide how to bid given the optimal strategies used by their competitors, which they have to calculate. The resulting bidding strategies in this case are (Bayes-Nash) equilibrium bidding strategies. Under the best-response assumption, bidders decide how to bid given the bids they observe from their competitors, assuming the competitors do not respond strategically.

When suppliers are fully rational, the analysis of bidding strategies in buyer-determined auctions and in auctions with rank feedback is difficult; in these settings bidders do not have a dominant strategy, and that complicates the search for equilibrium strategies. An alternative approach, is to use decision theory and assume bidders best-respond to the bids they see at any given time. Turns out, that even with this simplifying assumption, the problem to characterize the bidding strategies is still challenging, but it is workable enough to gain useful practical insight.

Using decision theory to model bidding strategies in auctions has a long history. Friedman (1956) proposed a model for bidding strategies in sealed-bid auctions, where the number of bidders could become large and unknown, and Capen, Clapp & Campbell (1971) identified the “winner’s curse”, one of the key ideas of auction theory. More recently, Parkes & Ungar (2000) and Parkes & Kalagnanam (2005) use myopic best-response bidding strategies in the design of combinatorial auctions, and multiattribute procurement auctions correspondingly.

As Rothkopf (2007) discussed, one of the advantages that decision theory offers to model and think about auctions is that “... it brings the ability to model many issues and contextual matters that are too hard for game theory.” ((Rothkopf 2007), p. 170). Moreover, from the practitioner’s perspective, Elmaghraby (2007) asked practitioners involved in procurement auctions, whether suppliers incorporate the strategic behavior of their competitors to formulate their bids, and
“... the resounding answer was no.” (Elmaghraby 2007, p. 413).

Here, I use decision theory to model the bidding strategies in: (1) a buyer-determined auction, in which an incumbent competes with a group of entrants, and (2) a reverse auction with rank feedback, in which a domestic supplier competes with a foreign supplier.

### 1.2 Combinatorial auctions

Combinatorial auctions are auctions of multiple heterogeneous objects where bidders can bid for combinations of the objects. For example, if two objects (a and b) are being auctioned, bidders in the auction can bid for a, b, and for the package that contains a and b. When a bidder is willing to pay more for a and b together, than the sum of what he would pay for a alone and b alone, the objects have synergy for the bidder, since they are more valuable together than separate. In general, if a set of objects has synergy for a bidder, his value for the set is greater than the sum of the values of the individual objects contained in the set. If the set does not have synergy for a bidder, his value for it is less than the sum of the individual values of the objects contained in the set. These properties are also referred in the literature as the objects being complements or substitutes correspondingly (Krishna 2002).

When objects in an auction have synergy for a bidder, and bids for combinations of the objects are not allowed, the bidder can bid for the objects separately to try to win the package. However, if he wins some objects but not the whole package, and his bids included the synergistic gains, he will likely incur a loss because those gains were not realized. This problem is known as the exposure problem (Cramton, Shoam & Steinberg 2006).

The use of combinatorial bidding compared to non-combinatorial bidding, not only reduces the risk of exposure for the bidders, it also improves the efficiency of the auction, and the seller’s revenue when the objects have synergy for the bidders (Milgrom 2004). But these gains come at a price. The problem the auctioneer has to solve to determine the winner(s) belongs to the NP-hard family of optimization problems (Rothkopf, Pekec & Hastard 1998), and the bidders face a set of potential bids that increases exponentially with the number of ob-
jects. However, despite their complexity, both public and private sectors are using combinatorial auctions in their procurement operations in sealed-bid formats (Epstein, Henríquez, Catalán, Weintraub & Martínez 2002), (Sheffi 2004), (Cantillon & Pesendorfer 2006) and in multi-round formats (Hohner, Rich, Ng, Reid & Davenport 2003), (Sandholm, Levine, Concordia, Martyn, Hughes, Jacobs & Begg 2006), (Goeree & Holt 2008).

The research related to combinatorial auctions has been concerned with the design of the auction (Cramton et al. 2006), and the complexity of the winner determination problem (de Vries & Vohra 2003), leaving aside the bid determination problem faced by the bidders: what to bid for and how much.

If there are $m$ objects being auctioned, a bidder has $2^m - 1$ potential sets he can bid for (assuming all sets have value for him). Once the bidder identifies the sets he wants to bid on, he needs to determine the bidding strategy for each of them, which is a non-trivial problem given the interaction between sets. Suppose a bidder in an auction of two objects, bids for the package -a and b- and a. If his bid for the package, is less than his bid for a, plus the competitors’ highest bid for b, he will lose the package because of his bid for a. Therefore, when a bidder bids for overlapping sets, he can compete against himself. The consequences that competing against oneself might have in the bidding strategies in combinatorial auctions have been unstudied in the literature, so it is not clear if bidders will do better (or worse) by avoiding to bid for overlapping sets.

Using decision analysis, I explore the bidding strategies in a combinatorial auction of two objects, where bidders submit their bids once and the winners pay their bids (first-price, sealed-bid combinatorial auction). A bidder can bid for all sets, but only one of his bids can win, hence bids are mutually exclusive. As discussed by Park & Rothkopf (2003), dynamic auctions have a sharing information advantage for the bidders that single-round auctions do not have, but they provide opportunities for collusion and signaling (Brusco & Lopomo 2002), which are limited in single-round formats.
Chapter 2

An analysis of bidding strategies in buyer-determined procurement auctions

In this chapter I study bidding strategies in a dynamic procurement auction, in which suppliers (an incumbent and a group of entrants), compete on price during the auction and once the auction is over, the buyer adjusts the price bids to allocate the contract. Note that in this case the winner of the auction is not necessarily the winner of the contract.

As a benchmark, I consider a scoring auction, in which suppliers compete on the scores or adjusted bids and therefore the winner of the auction is the winner of the contract.

Whether a buyer does better by running a scoring or a buyer-determined auction, is a relatively unstudied question in the literature. To understand if the scoring or the buyer-determined auction generates the lowest cost for the buyer, we first need to understand the suppliers’ bidding strategies in buyer-determined auctions.

When the buyer runs a scoring auction, since the winner of the auction is also the winner of the contract, standard theory prescribes that bidders should decrease their bids until they are winning or reach their cost. However, when the buyer runs a buyer-determined auction, since it is possible for a bidder to lose the auction and win the contract, the standard theory does not prescribe how suppliers should bid.
I find that in a buyer-determined auction suppliers bid down to cost plus a markup. The markup is different for the type of supplier (incumbent or entrant), and it depends on the number of bids the buyer adjusts after the auction. According to my results, whether the buyer does better by running a scoring auction instead of a buyer-determined auction, depends on the difference between the cost of the incumbent with respect to the cost of the entrants, the number of entrants and the non-price attributes.

2.1 Literature review

Procurement auctions have gained popularity opening a variety of research areas. Elmaghraby (2007) and Rothkopf & Whinston (2007) provide detailed comparisons and discussions between practice and the issues addressed in the literature.

In particular, Elmaghraby (2007) reports a strong tendency for buyers to avoid committing to the result of the auction and instead selecting the winner using the auction’s results as an input, in which case winning the auction does not mean winning the contract. To understand why a buyer would prefer not to commit to the result of the auction, Engelbrecht-Wiggans, Haruvy & Katok (2007) propose a model (which is also tested with laboratory experiments) to compare a price-based mechanism in which the auctioneer commits to awarding the contract to the winner of the auction, to a buyer-determined mechanism in which the auctioneer is free to select the bid that maximizes his surplus (which is not necessarily the lowest bid from the auction). Results show that the buyer-determined mechanism is better for situations with many bidders, but in situations with few bidders, the price-based mechanism performs better. The format Engelbrecht-Wiggans et al. (2007) study, corresponds to a sealed-bid auction with symmetric bidders. In contrast, I consider an open-descending reverse auction, with asymmetric bidders.

Rezende (2009) follows a different approach, and studies the use of a bias function to determine the allocation in a reverse auction, allowing the buyer to bias the result of the auction towards a specific supplier. In his model, Rezende (2009) considers two symmetric sellers who compete for a contract under a bias function established by the buyer, and he studies the outcomes when the buyer commits to the announced bias rule and the case when he does not; if the buyer does not
commit, the auction is followed by a bargaining process. Rezende (2009) finds that there are no equilibria in which the winning bid is lower than the maximum cost when the buyer does not commit, and he shows that a property of the equilibrium is that bidders will only lower their bids if they believe they will lose the auction for sure. A similar observation is made by Katok & Wambach (2008), who consider a dynamic buyer-determined auction, and show this type of format enables symmetric bidders to implicitly collude in descending open-bid auctions leading to non-competitive prices.

The results I find for the buyer-determined auction, are different from those in which the buyer does not commit to the result of the auction, in the settings studied by Rezende (2009) and Katok & Wambach (2008). In the model proposed by Rezende (2009), the cost of the bidders has the same distribution, and the asymmetry between the bidders is due to the bias rule. However, in his model, once the buyer does not commit, the bias rule no longer holds and the setting becomes symmetric again, in which case the best strategy is to collude, as is also observed by Katok & Wambach (2008). In my setting, if all suppliers collude at the bid ceiling, the incumbent will win the contract with certainty, therefore an entrant can win only if his bid is below the incumbent’s bid. Since the buyer will allocate the contract to the lowest adjusted bid, once the entrants start competing, the incumbent can be forced to come down from the bid ceiling; hence collusion is an optimal strategy only when the cost of the incumbent is high, and all entrants have the same cost, because in that case all of them will stop at the same bid, which is likely to be close to the bid ceiling.

Zhong & Wu (2006) study bidding behavior in buyer-determined auctions, when the buyer has preferred and non-preferred suppliers due to the existence of non-price attributes. Using data from auctions in the high-tech industry, Zhong & Wu (2006) find that preferred suppliers are more likely to win a contract than non-preferred suppliers, and that final bids from preferred and non-preferred suppliers differ significantly. They argue that these differences are consequences of the non-price attributes, which play a crucial role not only in the buyer’s final decision, but also in the bidding strategies of the suppliers. In line with Zhong & Wu (2006), I find suppliers markup their final bids depending on the distribution of the non-price attributes.
Regarding the bidding strategies in buyer-determined auctions when bidders differ in quality, Haruvy & Jap (2010) use data from buyer-determined auctions in the automotive industry to estimate an econometric model for the bidding strategies. They find that when bidders are differentiated in terms of quality, high quality bidders do not respond to bids from bidders whose bids indicate they might be low quality bidders, but they aggressively respond to the bids of other high quality bidders. In a different setting, Tunca, Wu & Zhong (2010) use data from legal service procurement auctions, to study the trade-off between price and quality in buyer-determined auctions. They find that buyer-determined auctions, in which the buyer’s preferences are uncertain to the suppliers, can achieve price savings without compromising quality. They also observe that the inclusion of non-incumbent suppliers in the auction, benefit the buyer, since it induces competitive behavior from the incumbent. I find that when the buyer runs a buyer-determined auction, whether suppliers (incumbent and entrants) bid more or less aggressively depends on the number of bids the buyer will adjust after the auction, and the difference between their bids and the bids from the competitors.

Wan, Beil & Katok (2010) propose a model in which a buyer wishes to pit an incumbent supplier against a single possibly unqualified entrant in an open-descending auction. They show that in equilibrium, sometimes, the incumbent strategically drops out early to forestall a bidding war, depending on his cost, the entrant’s qualification probability and the qualification cost. They also show it is optimal for the buyer to prevent this by attempting to qualify the entrant before the auction, only when the qualification cost is small and the entrant’s qualification probability is moderate. In their model, once the entrant is qualified, the buyer commits to the result of the auction. When suppliers differ in quality, and the screening process is costly for the buyer, Wan & Beil (2009) propose an optimal auction and post-qualification mechanism for the buyer. That approach bypasses the problem of finding the bidding strategies of the suppliers, because the optimal mechanism can be characterized by a direct mechanism where the suppliers bid truthfully (Myerson 1981). In line with the reports by Millet, Parente, Fizel & Venkataraman (2004) and Jap (2007), in which they find that in practice buyers usually qualify suppliers before the auction, to ensure they are capable of fulfilling the contract, I consider the case where all suppliers are qualified before the auction,
and therefore all of them are eligible to be awarded the contract. Finally, Haruvy & Katok (2010) analyze the effect of price visibility and information transparency on the outcome of a buyer-determined auction. They compare a sealed-bid auction and an open-bid dynamic auction under different information settings, using controlled laboratory experiments. They find the sealed-bid format generates higher buyer surplus than the open-bid dynamic format, and, that the buyer cost is lower when bidders have less information. In my setting, bidders have less information in the buyer-determined auction than in the scoring auction, because they do not know the values of the non-price factors. However, I find the buyer cost is lower in the buyer-determined than in the scoring auction, depending on the cost distribution of the suppliers.

2.2 The model

I consider a setting in which a buyer wishes to allocate a contract between an incumbent and a group of entrants. The entrants are screened before the auction, and therefore all of them are qualified to be awarded the contract. In practice, the buyer can run the auction directly or hire an auctioneer. Examples of auctioneers in this setting are companies such as Ariba or Emptoris that provide buyers with a pool of screened suppliers and the necessary technology and training to host the reverse auction.

If the buyer decides to change his current supplier, he will incur in additional costs such as training or updating inventory lists; consequently, a price bid from the entrant needs to be adjusted by non-price attributes, which represent the advantage the incumbent has over the entrant. I assume these non-price attributes are independent of the production cost of the entrant, and that they have a monetary value for the buyer. To select the winner, the buyer adjusts the bids from the entrants (price bid+non-price attributes) and compares them with the price bid of the incumbent. The lowest adjusted bid determines the winning supplier, who is paid his unadjusted bid, and the buyer assumes the cost of the non-price attributes.

Both auction formats (scoring and buyer-determined), start with the suppliers bidding down from the same bid ceiling and end when no new bids are made. At
any bid level suppliers have to decide to stop bidding or decrease their bid by the minimum decrement, given their competitor’s bids.

We start by considering the setting in which one incumbent competes against one entrant.

### 2.2.1 One incumbent versus one entrant

In the scoring auction, the bids displayed correspond to the adjusted bids (price bid+non-attributes for the entrant, and price bid for the incumbent). When the entrant places a bid, the corresponding non-price attributes are added to it and the bid is displayed, therefore, only the entrant knows the value of his non-price attributes. Since in the scoring auction the winner of the auction is the winner of the contract, and a supplier knows whether he is winning or losing at any given time, it is a standard theory result that the optimal bidding strategy for both suppliers is to bid down in decrements, stopping only when they are winning or they reach their cost.

In the buyer-determined auction, the bids displayed correspond to the price bids. When the bidding is over, the buyer adds the non-price attributes to the bid from the entrant, and compares it to the bid from the incumbent to determine the winner. In this case, the entrant does not know the value of his non-price attributes while bidding in the auction.

Note that in the buyer-determined auction, the entrant has an opportunity of winning the contract only if he wins the auction. Therefore, the entrant should decrease his bid until he is winning or he reaches his cost. If the entrant wins the auction, he wins the contract if the sum of his bid plus the non-price attributes (which are unknown to him), is less than the bid from the incumbent. Consequently, if the entrant is winning the auction, he needs to decide whether to lower his current bid given the bid from the incumbent, which defines a threshold bid denoted by \( \bar{b}_e \).

Regarding the incumbent, note that if he wins the auction he automatically wins the contract. Therefore, the incumbent should stop bidding if he is winning the auction. If he is not winning the auction, he needs to decide whether to lower his current bid given the bid from the entrant, which defines a threshold bid...
denoted by $\tilde{b}_i$.

During the auction, suppliers must decide dynamically whether to decrease their bids by the minimum decrement or stop bidding given the current bid from their competitor. However, before the auction, suppliers can calculate their threshold bids for any given bid from their opponent, write them down, and during the auction make the decision to decrease the bid or stop bidding according to the calculated thresholds (and whether or not they are winning the auction). Note that at any given bid, the information needed to decide between decreasing the bid or stop bidding is revealed during the auction. Therefore, the threshold bids can be characterized at the outset of the auction without losing optimality.

To derive the threshold bids, $\tilde{b}_e, \tilde{b}_i$, let $c_e, b_e$ be the cost and the bid of the entrant, $c_i$ and $b_i$ be the cost and the bid from the incumbent, and $\varepsilon$ be the non-price attributes for the entrant, which follow a commonly known continuous probability distribution $F$, with finite support $[\underline{\varepsilon}, \overline{\varepsilon}]$.

Let me start with the threshold bid for the entrant. Assuming suppliers are risk-neutral, once the entrant is winning he calculates his threshold bid, $\tilde{b}_e$, given the current bid from the incumbent as follows:

$$\max_{c_e \leq b_e} \pi_e = (b_e - c_e) \frac{F(b_i - b_e)}{\text{Prob}(b_e + \varepsilon \leq b_i)}, \quad (2.1)$$

the first order condition yields:

$$b_e^* = c_e + \frac{F(b_i - b_e^*)}{F'(b_i - b_e^*)}, \quad (2.2)$$

$$\tilde{b}_e = \max\{c_e, b_e^*\}. \quad (2.3)$$

where $F' = \frac{dF}{db_e}$. Note that (1) if $b_i$ decreases, $b_e^*$ also decreases (a decrease in $b_i$ cannot result in an increase of $b_e^*$, because that would mean $F$ is non-increasing, which contradicts the properties of cumulative probability distributions), therefore a decrease in the incumbent’s bid, makes the entrant bid more aggressively, and (2) the entrant will markup his cost depending on the difference between the incumbent’s bid and his current bid.

Now let me turn to the threshold bid for the incumbent. If the incumbent is not winning the auction, he calculates his threshold bid, $\tilde{b}_i$, given the bid from the
entrant as follows:

\[
\max_{c_i \leq b_i} \pi_i = (b_i - c_i) \left(1 - F(b_i - b_e)\right) \text{Prob}(b_i \leq b_e + \epsilon),
\]

where \( F' = \frac{dF}{db_i} \). Note that (1) if \( b_e \) decreases, \( b^*_i \) also decreases, therefore a decrease in the entrant’s bid, makes the incumbent bid more aggressively, and (2) the incumbent will markup his cost depending on the difference between the entrant’s bid and his current bid.

Finally, since \( \pi_e \) and \( \pi_i \) are continuous concave functions in a finite support, the existence of an optimal solution is guaranteed.

The resulting bidding strategies, are characterized as follows:

- **Entrant’s bidding strategy**: If the entrant is losing the auction, he should decrease his bid until he is winning or reaches his cost. Once he is winning he should decrease his bid until he reaches his threshold bid, \( \tilde{b}_e \), or his bid is equal to the incumbent’s bid minus \( \epsilon \).

- **Incumbent’s bidding strategy**: If the incumbent is winning the auction, he should stop bidding. If he is losing the auction, he should decrease his bid until he reaches his threshold bid, \( \tilde{b}_i \), or his bid is equal to the entrant’s bid plus \( \epsilon \).

To show the previous bidding strategies correspond to (Bayesian Nash) equilibrium strategies, we need to show they correspond to optimal actions for each supplier at every point during the auction, given the current bids and future competitor’s actions (which are decrease the current bid by the minimum decrement or stop bidding). We start with the entrant’s bidding strategy. If the entrant is losing the auction, is a dominant strategy for him to decrease his bid until he is winning or he reaches his cost, otherwise he loses the auction and the contract with certainty. If the entrant is winning the auction, and his current bid is greater than
max{\(\tilde{b}_e, b_i - \epsilon\)}, he needs to decrease his bid, whether the incumbent stops bidding or decreases his bid afterwards. If the incumbent stops bidding, the optimal stopping bid for the entrant is given by max{\(\tilde{b}_e, b_i - \epsilon\)}, therefore he should decrease his current bid if it is above this stopping point. If the incumbent decreases his current bid, then the entrant needs to update his threshold bid (which will be lower than the current threshold bid) given the new bid from the incumbent. Therefore, if the entrant’s bid is greater than his threshold bid, he should decrease his bid.

Similarly, if the incumbent is losing the auction, and his current bid is greater than max{\(\tilde{b}_i, b_e + \xi\)}, he needs to decrease his bid, whether the entrant stops bidding or decreases his bid. If the entrant stops bidding, the optimal stopping bid for the incumbent is given by the threshold bid. If the entrant decreases his bid, the incumbent needs to update his threshold bid (which will be lower than the current threshold bid), given the new lower bid from the entrant. Therefore, if the incumbent is losing the auction he should decrease his current bid if it is above the threshold. If the incumbent is winning the auction he should stop bidding, since winning the auction, in his case, guarantees he wins the contract.

Given the previous analysis, since in the buyer-determined auction suppliers start bidding down from the same bid ceiling, the entrant is the first to decrease his bid. Then, the incumbent will follow, depending on his threshold bid. Note that after a supplier decreases his bid, the competitor needs to update his threshold bid. Hence, bidders will decrease their bids dynamically until they reach the threshold bids, at which point no new bids will be made and the auction will end.

Whether the scoring or the buyer-determined auction yield the lowest cost for the buyer, depends on the cost of the suppliers and the non-price attributes as follows.

**Proposition 1:** When an risk-neutral incumbent competes against a risk-neutral entrant, given any \(c_i, c_e\), and \(\epsilon, \xi \leq \epsilon \leq \tilde{\epsilon}\), and if the non-price attributes of the entrant follow a continuous probability distribution \(F\), with support \([\xi, \tilde{\epsilon}]\), such that \(\frac{F(b^*_i - b^*_e)}{F(b^*_i - b^*_e)} \geq 0\), then, the cost for the buyer in the scoring auction is less than or equal to the cost in the buyer-determined auction, if \(\epsilon - \frac{(1-F(b^*_i - b^*_e))}{F(b^*_i - b^*_e)} \leq (c_i - c_e) \leq \epsilon + \frac{F(b^*_i - b^*_e)}{F(b^*_i - b^*_e)}\).

The proof consists of two cases: (1) when both threshold bids in the buyer-
determined auction, \( \tilde{b}_i \) and \( \tilde{b}_e \) correspond to \( b_i^* \) and \( b_e^* \), and (2) when one of the threshold bids in the buyer-determined auction corresponds to the supplier’s cost.

We start with case 1. The proof for this part follows 3 steps. First we need to derive the conditions under which the buyer’s cost is less in the scoring auction than in the buyer-determined auction (Step 1), then we need to verify that if those conditions do not hold the buyer-determined auction yields a lower cost for the buyer than the scoring auction (Steps 2 and 3).

Step 1: The conditions for which the buyer’s cost in the scoring auction is less than the buyer’s cost in the buyer-determined auction, when the entrant wins the contract, and when the incumbent wins the contract are as follows. When the entrant wins the contract, the scoring auction yields a lower cost for the buyer if the following condition holds:

\[
 c_i \leq c_e + \frac{F(b_i^* - b_e^*)}{F'(b_i^* - b_e^*)} + \epsilon, \tag{2.9}
\]

\[
 (c_i - c_e) \leq \epsilon + \frac{F(b_i^* - b_e^*)}{F'(b_i^* - b_e^*)}. \tag{2.10}
\]

When the incumbent wins the contract, the scoring auction yields a lower cost for the buyer if the following condition holds:

\[
 c_e + \epsilon \leq c_i + \frac{1 - F(b_i^* - b_e^*)}{F'(b_i^* - b_e^*)}, \tag{2.11}
\]

\[
 (c_i - c_e) \geq \epsilon - \frac{1 - F(b_i^* - b_e^*)}{F'(b_i^* - b_e^*)}. \tag{2.12}
\]

Therefore, when \( \epsilon - \frac{1 - F(b_i^* - b_e^*)}{F'(b_i^* - b_e^*)} \leq (c_i - c_e) \leq \epsilon + \frac{F(b_i^* - b_e^*)}{F'(b_i^* - b_e^*)} \) holds, the scoring auction yields a lower cost than the buyer-determined auction. This corresponds to the first numeral in Proposition 1.

Step 2: If \( (c_i - c_e) \geq \epsilon + \frac{F(b_i^* - b_e^*)}{F'(b_i^* - b_e^*)} \), equation (2.12) automatically holds. This means that the scoring auction yields a lower cost than the buyer-determined auction if the incumbent wins, and a higher cost if the entrant wins. However, \( (c_i - c_e) \geq \epsilon + \frac{F(b_i^* - b_e^*)}{F'(b_i^* - b_e^*)} \) implies that the entrant wins in the scoring auction:

\[
 c_i \geq c_e + \epsilon + \frac{F(b_i^* - b_e^*)}{F'(b_i^* - b_e^*)}, \tag{2.13}
\]
which holds if:
\[
\frac{F(b_i^*-b_e^*)}{F_i^*(b_i^*-b_e^*)} \geq 0. \tag{2.14}
\]

Therefore, if \((c_i - c_e) \geq \epsilon + \frac{F(b_i^*-b_e^*)}{F_i^*(b_i^*-b_e^*)}\) and \(\frac{F(b_i^*-b_e^*)}{F_i^*(b_i^*-b_e^*)} \geq 0\) holds (equation (2.15)), the entrant wins the contract in the scoring auction.

Next, consider case 2: when one of the threshold bids in the buyer-determined auction is equal to the supplier’s cost.

When \(b_e = c_e, b_i^* \leq c_e + \epsilon\), \(^1\) which implies \(c_i \leq c_e + \epsilon - \frac{1-F(b_i^*-b_e^*)}{F_i^*(b_i^*-b_e^*)}\) (note that in turn, this condition is implied by \((c_i - c_e) \leq \epsilon - \frac{1-F(b_i^*-b_e^*)}{F_i^*(b_i^*-b_e^*)}\) in Step 3). Consequently,

\(^1\)Otherwise \(b_e = b_e^*\), which corresponds to case 1.
the incumbent stops bidding at \( c_e + \xi \) and wins the contract. The buyer’s cost in the buyer-determined auction is \( c_e + \xi \); the equivalent cost in the scoring auction is \( c_e + \epsilon \), which is greater or equal to the cost in the buyer-determined auction, since \( \xi \leq \epsilon \).

Similarly, when \( \tilde{b}_i = c_i \), \( b_i^* - b_e^* \geq +\bar{\tau} \), \( F \) which implies \( c_i \geq c_e + \bar{\tau} + \frac{F(b_i^*-b_e^*)}{F_e(b_i^*-b_e^*)} \) (note that in turn, this condition is implied by \( c_i - c_e \geq \epsilon + \frac{F(b_i^*-b_e^*)}{F_e(b_i^*-b_e^*)} \) in Step 2). Consequently, the entrant stops bidding at \( c_i - \bar{\tau} \) and wins the contract.

In this case the buyer’s cost in the buyer determined auction is \( c_i - \epsilon + \epsilon \). The equivalent cost in the scoring auction is \( c_i \), since \( \epsilon \geq \epsilon \), the buyer-determined auction yields a lower cost for the buyer than the scoring auction.

**Lemma 3:** When \( \tilde{b}_e = c_e \) or \( \tilde{b}_i = c_i \), the cost in the scoring auction is greater or equal to the cost in the buyer-determined auction.

Lemmas 1, 2 and 3 combined correspond to the cases when \( (c_i - c_e) \leq \epsilon - \frac{(1-F(b_i^*-b_e^*)}{F_e(b_i^*-b_e^*)} \), or, \( (c_i - c_e) \geq \epsilon + \frac{F(b_i^*-b_e^*)}{F_e(b_i^*-b_e^*)} \), for which the cost for the buyer in the scoring auction is greater or equal than the cost in the buyer determined auction. ■

According to Proposition 1, if the difference in the supplier’s cost goes below or above a threshold, the buyer-determined auction yields a lower cost than the scoring auction. How much lower depends on the distribution of the non-price attributes.

To further investigate this question, consider the case in which \( F \) corresponds to a continuous Uniform distribution in \( [\xi, \bar{\tau}] \). In this case, the threshold bids corresponding to (2.4) and (2.8) are:

\[
\tilde{b}_e = \max \left\{ c_e, \frac{c_e + b_i - \xi}{2} \right\}, \text{ and } \tilde{b}_i = \max \left\{ c_i, \frac{c_i + b_e + \bar{\tau}}{2} \right\}. \tag{2.17}
\]

Since suppliers decrease their bids dynamically depending on their opponent’s bid, we can calculate the threshold bids at which both incumbent and entrant stop bidding, by replacing \( \tilde{b}_e \) in \( \tilde{b}_i \) (and vice versa), to obtain the following thresholds:

\[
\tilde{b}_e = \max \left\{ c_e, \frac{2c_e + c_i + \bar{\tau} - 2\epsilon}{3} \right\}, \text{ and } \tilde{b}_i = \max \left\{ c_i, \frac{2c_i + c_e - \epsilon + 2\tau}{3} \right\}. \tag{2.18}
\]

\(^2\)Otherwise \( \tilde{b}_i = b_i^* \) which corresponds to case 1.
According to Proposition 1 we have the following result:

**Corollary 1:** Suppose the non-price attributes of the entrant follow a continuous Uniform distribution in $[\epsilon, \bar{\epsilon}]$, $0 \leq \epsilon \leq \frac{\bar{\epsilon}}{2}$. Then, given any $c_i$, $c_e$, and $\epsilon$, $\epsilon \leq \epsilon \leq \bar{\epsilon}$:

1. The cost in the scoring auction is less than or equal to the cost in the buyer-determined auction, if $\frac{3\epsilon+\epsilon-2\bar{\epsilon}}{2} \leq (c_i - c_e) \leq \frac{3\epsilon-2\epsilon+\epsilon}{2}$.

2. The cost in the scoring auction is greater than or equal to the cost in the buyer-determined auction, if $(c_i - c_e) \leq \frac{3\epsilon+\epsilon-2\bar{\epsilon}}{2}$, or, $(c_i - c_e) \geq \frac{3\epsilon-2\epsilon+\epsilon}{2}$.

Following the steps of the proof of Proposition 1, we have:

Step 1: When $\frac{3\epsilon+\epsilon-2\bar{\epsilon}}{2} \leq (c_i - c_e) \leq \frac{3\epsilon-2\epsilon+\epsilon}{2}$, the cost in the scoring auction is less than or equal to the cost in the buyer-determined auction.

Step 2: When $(c_i - c_e) \geq \frac{3\epsilon-2\epsilon+\epsilon}{2}$ and $\epsilon \leq \frac{\bar{\epsilon}}{2}$, the entrant wins the contract in both auctions, consequently the cost in the scoring auction is greater or equal than the cost in the buyer-determined auction.

Step 3: When $(c_i - c_e) \leq \frac{3\epsilon-2\epsilon+\epsilon}{2}$ and $\epsilon \leq \frac{\bar{\epsilon}}{2}$, the incumbent wins the contract in both auctions, consequently the cost in the scoring auction is greater or equal than the cost in the buyer-determined auction.

The analysis for case 2 is identical to the one presented in the proof of Proposition 1.

To explore the changes in the gap between the average cost for the buyer in the scoring auction, and the buyer-determined auction, depending on the cost distributions, I simulate the two auction formats considering two cost distributions with finite support, Uniform and Beta. Since the Beta distribution has a lower variance than the Uniform distribution (with the parameters used in the simulations), we can determine if the gap between the cost of the two auction formats, is sensitive to the supplier’s cost distribution. Additionally, the difference between the supplier’s costs, the support of the non-price attributes also defines when the scoring auction yields a lower cost than the buyer-determined auction. Therefore, I consider two supports for the non-price attributes in the simulations, one between 0 and 0.1,
and the other between 0 and 0.2. Tables 2.1 and 2.2 show the specific parameters of the scenarios considered in the simulations.

<table>
<thead>
<tr>
<th>UC1: Uniform cost distributions</th>
<th>BC1: Beta cost distributions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_i$ Uniform [0,1.00],</td>
<td>Beta(4,4),</td>
</tr>
<tr>
<td>$(\mu = 0.50, \sigma^2 = 0.083, c_v = 0.576)$</td>
<td>$(\mu = 0.50, \sigma^2 = 0.028, c_v = 0.335)$</td>
</tr>
<tr>
<td>$c_e$ Uniform [0,0.90],</td>
<td>Beta(4,5),</td>
</tr>
<tr>
<td>$(\mu = 0.45, \sigma^2 = 0.067, c_v = 0.575)$</td>
<td>$(\mu = 0.44, \sigma^2 = 0.025, c_v = 0.359)$</td>
</tr>
<tr>
<td>$\varepsilon$ Uniform [0,0.10],</td>
<td>Uniform [0,0.10],</td>
</tr>
<tr>
<td>$(\mu = 0.05, \sigma^2 = 8 \times 10^{-04}, c_v = 0.566)$</td>
<td>$(\mu = 0.05, \sigma^2 = 8 \times 10^{-04}, c_v = 0.566)$</td>
</tr>
</tbody>
</table>

**Table 2.1.** Scenario 1: non-price attributes $\sim U[0,0.10]$
20

<table>
<thead>
<tr>
<th>UC2: Uniform cost distributions</th>
<th>BC2: Beta cost distributions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_i$</td>
<td>Uniform $[0,1.00]$, Beta(4,4),</td>
</tr>
<tr>
<td>$\mu = 0.50, \sigma^2 = 0.083, c_v = 0.576$</td>
<td>$\mu = 0.50, \sigma^2 = 0.028, c_v = 0.335$</td>
</tr>
<tr>
<td>$c_e$</td>
<td>Uniform $[0,0.80]$, Beta(4,6),</td>
</tr>
<tr>
<td>$\mu = 0.40, \sigma^2 = 0.053, c_v = 0.575$</td>
<td>$\mu = 0.40, \sigma^2 = 0.022, c_v = 0.371$</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Uniform $[0,0.20]$, Uniform $[0,0.20]$,</td>
</tr>
<tr>
<td>$\mu = 0.10, \sigma^2 = 0.003, c_v = 0.548$</td>
<td>$\mu = 0.10, \sigma^2 = 0.003, c_v = 0.548$</td>
</tr>
</tbody>
</table>

Table 2.2. Scenario 2: non-price attributes $\sim U[0,0.20]$}

Note that the expected value of the incumbent’s cost and the sum of the expected value of the entrant’s cost plus the expected value of the non-price attributes, are the same in both scenarios; this set up allows us to compare the differences in the average cost of smaller expected entrant’s costs, versus smaller expected non-price attributes.

The results of 10,000 replications are reported in Table 2.3.

<table>
<thead>
<tr>
<th>UC1</th>
<th>BC1</th>
<th>UC2</th>
<th>BC2</th>
</tr>
</thead>
<tbody>
<tr>
<td>average cost</td>
<td>0.662</td>
<td>0.625</td>
<td>0.591</td>
</tr>
<tr>
<td>(0.051)</td>
<td>(0.046)</td>
<td>(0.019)</td>
<td>(0.017)</td>
</tr>
<tr>
<td>gap</td>
<td>0.036</td>
<td>0.019</td>
<td>0.029</td>
</tr>
<tr>
<td>(0.002)</td>
<td>(0.001)</td>
<td>(0.005)</td>
<td>(0.004)</td>
</tr>
</tbody>
</table>

Table 2.3. Average cost of the buyer and its variance (in parenthesis).

From Table 2.3, we can see that buyer-determined auction yields a lower average cost than the scoring auction in all scenarios. Although the gap between the average cost for the scoring auction and the buyer-determined auction is statistically significant in each setting (the p-values of the corresponding two sided z-test are below 0.01 in all cases), it might not be economically significant.

Before we move on, it is worthwhile mentioning that different combinations of cost distributions (Uniform in $[0,1]$ and Beta) and non-price attributes supports (Uniform in $[0,0.5]$) were simulated. For all cases the buyer-determined auction resulted in a lower average cost than the scoring auction. The gap between the average costs ranged from 0.03 (when the difference between the expected cost of
the suppliers was 0) to 0.06 (when the difference between the expected cost of the suppliers was 0.5). The scenarios reported here correspond to the case where the cost distribution of the entrant (first-order) stochastically dominates that of the incumbent; the non-price attributes represent the advantage the incumbent has over the entrant, given his ongoing relationship with the buyer.

In the next section, I explore the case when the incumbent competes against multiple entrants.

2.2.2 One incumbent versus multiple entrants

Recall that in the scoring auction, once an entrant places a bid his corresponding non-price attribute is added to his bid, and the adjusted bid is displayed (bid+non-price attributes). Therefore each entrant only knows his own price attribute. Since the winner of the auction is also the winner of the contract, the standard theory result holds when the incumbent competes against multiple entrants: all suppliers should bid down in decrements, stopping only when they are winning or reach their cost.

In the buyer-determined auction, all suppliers know the number of bids the buyer will evaluate after the auction ($k$), and also that these bids correspond to the $k$-th lowest entrant’s bids, note that $k = \{1, \ldots, n_e\}$. By assuming $k$ is public information, we can vary its value to see the effect in the cost for the buyer and gain some insight about the structure of the optimal $k$. I also assume that the entrants know which bid corresponds to the incumbent. In practice, the names of the companies might be replaced by usernames to mask the true identity of the suppliers. I use this assumption to structure the model, but as it will be discussed below, the incumbent does not need to engage in competition at the beginning of the auction so his identity can be revealed.

In the buyer-determined auction, the entrant only has an opportunity to win the contract, if (1) his bid is less than the incumbent’s bid and (2) his bid is within the $k$ lowest bids. Therefore, he should decrease his bid until it is less than the incumbent’s bid, and, he is within the $k$-th lowest bidders, or he reaches his cost. Once his bid is less than the incumbent’s bid, and he is in within the $k$-th lowest bids, he needs to determine his threshold bid given the current bids from the other
\(k - 1\) entrants, and the incumbent. Note that given that dynamic, the line that divides the potential winners from the losers is given by the \((k+1)\)-th order statistic from the cost’s distribution of the entrants. When the buyer increases the value of \(k\), an entrant has to bid less aggressively to make the cut. For example, if the costs of the entrants follow a Uniform distribution between 0 and 1, the expected value of the \((k+1)\)-th lowest cost is \(\frac{k+1}{n_c+1}\); therefore for a fixed \(n_c\) increasing the value of \(k\) increases the bid that separates the potential winners from the losers.

When an entrant is within the \(k\) lowest bids and his bid is less than the incumbent’s bid, his threshold bid depends on the bid from the other \(k-1\) entrants bids and the incumbent’s bid. The threshold bid, \(\tilde{b}_{ej}\), can be determined as follows.

Let \(c_{ej}\) and \(b_{ej}\) be the cost and bid from entrant \(j, j \in \{1, \ldots, k\}\), and let \(G\) denote the distribution of the difference between the non-price attributes, \(\varepsilon_j - \varepsilon_l\), which are assume to be i.i.d continuous random variables according to \(F\), with support \([\varepsilon, \bar{\varepsilon}]\).

\[
\max_{c_{ej} \leq b_{ej}} \pi_{ej} = (b_{ej} - c_{ej}) \prod_{l=1, l \neq j}^{k} \frac{\text{Prob}(b_{ej} + \varepsilon_j \leq b_l) \text{Prob}(\varepsilon_j - \varepsilon_l \leq b_{el} - b_{ej})}{\text{Prob}(b_{ej} + \varepsilon_j \leq b_{el} + \varepsilon_l)}, \quad (2.19)
\]

the first order condition yields:

\[
b_{ej}^* = c_{ej} + \frac{F(b_{ej} - b_{ej}^*) \prod_{l=1, l \neq j}^{k} G(b_{el} - b_{ej}^*)}{F(b_{ej} - b_{ej}^*) + \prod_{l=1, l \neq j}^{k} G(b_{el} - b_{ej}^*)}, \quad (2.20)
\]

\[
\tilde{b}_{ej} = \max \{c_{ej}, b_{ej}^*\}. \quad (2.22)
\]

In turn, the threshold bid for the incumbent, \(\tilde{b}_{ik}\) can be determined as follows:

\[
\max_{c_i \leq b_i} \pi_{ik} = (b_i - c_i) \prod_{j=1, b_j > b_{ej}}^{k} \frac{(1 - F(b_i - b_{ej}))}{\text{Prob}(b_i \leq b_{ej} + \varepsilon_j)}, \quad (2.23)
\]

the first order condition yields:

\[
b_{ik}^* = c_i - \frac{\prod_{j=1, b_j > b_{ej}}^{k} (1 - F(b_{ik}^* - b_{ej}))}{d(\prod_{j=1, b_j > b_{ej}}^{k} (1 - F(b_{ik}^* - b_{ej})))/db_{ik}}, \quad (2.24)
\]

\[
\tilde{b}_{ik} = \max \{c_i, b_{ik}^*\}. \quad (2.25)
\]
The calculation of the threshold bids given by equations (2.22) and (2.26) is a very challenging problem (even numerically), because it involves calculating the roots of polynomials of grade $k$.

Using decision analysis as an alternative approach, I assume suppliers (myopically) best-reply to the competitors’ bids they see at any given time. Therefore, their decision of stopping or lowering their bid results from calculating and comparing the expected profit of stopping at his current bid, or, lowering it by the minimum decrement given the current bids from the competitors.

In the scoring auction, the myopic-best reply (MBR) bidding strategies dictate that all suppliers should decrease their bids stopping when they are winning or they reach their cost (which coincide with the equilibrium bidding strategies).

In the buyer-determined auction the MBR bidding strategies are as follows.

- Entrant’s MBR bidding strategy: If the entrant is outside the $k$ lowest bidders, or, the incumbent’s bid is less than his bid, he should decrease his bid until he is within the $k$ lowest bidders and his bid is less than the incumbent’s bid, or he reaches his cost, otherwise he loses with certainty. Once he is within the $k$ lowest bidders and his bid is less than the incumbent’s bid, he should decrease his bid, until the expected profit of lowering his bid by the minimum decrement is less than or equal to the expected profit of stopping at the current bid, given the bids from the other $k - 1$ entrants and the incumbent.

- Incumbent’s MBR bidding strategy: If the incumbent is winning the auction, he should stop bidding since winning the auction guarantees he wins the contract. If he is not winning, he should decrease his bid, until the expected profit of lowering his bid by the minimum decrement is less than or equal to the expected profit of stopping at the current bid, given the bids from the other $k - 1$ entrants.

The threshold bids in the buyer-determined auction can be written as $\bar{b}_{ik} = c_i + \text{incumbent’s markup}$, and $\bar{b}_{ej} = c_{ej} + \text{entrant’s markup}$ (please see equations 2.30 and 2.38). Increasing the number of bids to be adjusted after the auction, $k$, has opposite effects in the bid level at which only $k$ entrants are active, and the markups of the threshold bids. The effects are described in Propositions 2 and 3.
**Proposition 2:** Increasing the value of $k$, makes entrants bid less aggressively to be within the $k$ lowest bids.

Note that the bid level at which there are only $k$ entrants active corresponds to the $k + 1$ order statistic of the entrant’s cost distribution. Therefore, the expected bid level at which there are only $k$ entrants active increases with $k$ (this follows by the definition of order statistics), making the entrants bid less aggressively to be within the $k$ lowest bids. ■

**Proposition 3:** Increasing the value of $k$, decreases the markups of the threshold bids, resulting in more aggressive threshold bids.

The prove this result we use the notation presented in Table 2.4.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_{ej}$</td>
<td>Entrant j’s bid.</td>
</tr>
<tr>
<td>$b_i$</td>
<td>Incumbent’s bid.</td>
</tr>
<tr>
<td>$c_{ej}$</td>
<td>Entrant j’s cost.</td>
</tr>
<tr>
<td>$c_i$</td>
<td>Incumbent’s cost.</td>
</tr>
<tr>
<td>$\varepsilon_j$</td>
<td>Entrant j’s non-price attributes.</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Minimum decrement.</td>
</tr>
<tr>
<td>$F$</td>
<td>Distribution of the non-price attributes.</td>
</tr>
<tr>
<td>$[\varepsilon_j, \varepsilon_j]$</td>
<td>Support of $F$.</td>
</tr>
<tr>
<td>$G$</td>
<td>Distribution of the difference of non-price attributes.</td>
</tr>
<tr>
<td>$[-\varepsilon_j, \varepsilon_j]$</td>
<td>Support of $G$.</td>
</tr>
</tbody>
</table>

**Table 2.4.** Notation for the proof of Proposition 3

At the incumbent’s threshold bid, $\tilde{b}_{ik}$, the following condition must hold:

$$
E[\text{profit of stopping at } b_i] \geq E[\text{profit of lowering } b_i \text{ by } \delta],
$$

$$
(\tilde{b}_{ik} - c_i) \prod_{j=1}^{k} \text{Prob}(\tilde{b}_{ik} \leq b_{e,j} + \varepsilon_j) \geq (\tilde{b}_{ik} - \delta - c_i) \prod_{j=1}^{k} \text{Prob}(\tilde{b}_{ik} - \delta \leq b_{e,j} + \varepsilon_j),
$$

(2.27) 

(2.28)
\[
(\bar{b}_{ik} - c_i) \prod_{j=1}^{k} (1 - F(\bar{b}_{ik} - b_{e,j})) \geq (\bar{b}_{ik} - \delta - c_i) \prod_{j=1}^{k} (1 - F(\bar{b}_{ik} - \delta - b_{e,j})),
\]

(2.29)

Rearranging terms, we obtain:

\[
\bar{b}_{ik} \leq c_i + \delta \frac{\prod_{j=1}^{k} (1 - F(\bar{b}_{ik} - \delta - b_{e,j}))}{\prod_{j=1}^{k} (1 - F(\bar{b}_{ik} - \delta - b_{e,j})) - \prod_{j=1}^{k} (1 - F(\bar{b}_{ik} - b_{e,j}))}.
\]

(2.30)

Next, we need to show that the markup of the incumbent given in (2.30) is non-increasing in \(k\). If the markup is non-increasing in \(k\) the following equation should hold:

\[
\frac{\prod_{j=1}^{k} (1 - F(\bar{b}_{ik} - \delta - b_{e,j}))}{\prod_{j=1}^{k} (1 - F(\bar{b}_{ik} - \delta - b_{e,j})) - \prod_{j=1}^{k} (1 - F(\bar{b}_{ik} - b_{e,j}))} \geq \frac{\prod_{j=1}^{k+1} (1 - F(\bar{b}_{ik} - \delta - b_{e,j}))}{\prod_{j=1}^{k+1} (1 - F(\bar{b}_{ik} - \delta - b_{e,j})) - \prod_{j=1}^{k+1} (1 - F(\bar{b}_{ik} - b_{e,j}))},
\]

(2.31)

\[
\prod_{j=1}^{k} (1 - F(\bar{b}_{ik} - \delta - b_{e,j})) \prod_{j=1}^{k+1} (1 - F(\bar{b}_{ik} - b_{e,j})) \leq \prod_{j=1}^{k+1} (1 - F(\bar{b}_{ik} - \delta - b_{e,j})) \prod_{j=1}^{k} (1 - F(\bar{b}_{ik} - b_{e,j})),
\]

(2.32)

\[
1 - F(\bar{b}_{ik} - b_{e,k+1}) \leq 1 - F(\bar{b}_{ik} - \delta - b_{e,k+1}),
\]

(2.33)

\[
F(\bar{b}_{ik} - b_{e,k+1}) \geq F(\bar{b}_{ik} - \delta - b_{e,k+1}).
\]

(2.34)

Since \(F\) is non-decreasing, equation (2.34) holds, hence the markup for the incumbent is non-increasing in \(k\).
Similarly, for an entrant’s threshold:

\[ \text{LHS} = \text{E}[\text{profit of stopping at } \tilde{b}_{ej}] \geq \text{E}[\text{profit of lowering } \tilde{b}_{ej} \text{ by } \delta] = \text{RHS}. \quad (2.35) \]

\[
\text{LHS} = (\tilde{b}_{ej} - c_{ej}) \prod_{l=1, l \neq j}^{k} \text{Prob}(\tilde{b}_{ej} + \varepsilon_j \leq b_{el} + \varepsilon_l) = (\tilde{b}_{ej} - c_{ej}) \prod_{l=1, l \neq j}^{k} G(b_{el} - \tilde{b}_{ej}),
\]

\[
(2.36)
\]

\[
\text{RHS} = (\tilde{b}_{ej} - \delta - c_{ej}) \prod_{l=1, l \neq j}^{k} \text{Prob}(\tilde{b}_{ej} - \delta + \varepsilon_j \leq b_{el} + \varepsilon_l)
\]

\[
= (\tilde{b}_{ej} - c_{ej}) \prod_{l=1, l \neq j}^{k} G(b_{el} - \tilde{b}_{ej} + \delta).
\]

\[
(2.37)
\]

Rearranging terms we get:

\[
\tilde{b}_{ej} \leq c_{ej} + \delta \frac{\prod_{l=1, l \neq j}^{k} G(b_{el} - \tilde{b}_{ej} + \delta)}{\prod_{l=1, l \neq j}^{k} G(b_{el} - \tilde{b}_{ej} + \delta) - \prod_{l=1, l \neq j}^{k} G(b_{el} - \tilde{b}_{ej})}.
\]

\[
(2.38)
\]

Next, we need to show that the markup given in (2.38) is non-increasing in \( k \).

To ease the notation let \( x_{lj} = b_{el} - \tilde{b}_{ej} \). If the markup is non-decreasing in \( k \) the following equation should hold:

\[
\frac{\prod_{l=1, l \neq j}^{k} G(x_{lj} + \delta)}{\prod_{l=1, l \neq j}^{k} G(x_{lj} + \delta) - \prod_{l=1, l \neq j}^{k} G(x_{lj})} \geq \frac{\prod_{l=1, l \neq j}^{k+1} G(x_{lj} + \delta)}{\prod_{l=1, l \neq j}^{k+1} G(x_{lj} + \delta) - \prod_{l=1, l \neq j}^{k+1} G(x_{lj})},
\]

\[
(2.39)
\]

\[
\frac{1}{\prod_{l=1, l \neq j}^{k} G(x_{lj} + \delta) - \prod_{l=1, l \neq j}^{k} G(x_{lj})} \geq \frac{G(x_{k+1,j} + \delta)}{G(x_{k+1,j} + \delta) - \prod_{l=1, l \neq j}^{k+1} G(x_{lj})},
\]

\[
(2.40)
\]

\[
\prod_{l=1, l \neq j}^{k+1} G(x_{lj} + \delta) - \prod_{l=1, l \neq j}^{k+1} G(x_{lj}) \geq \prod_{l=1, l \neq j}^{k} G(x_{lj} + \delta) - \prod_{l=1, l \neq j}^{k} G(x_{lj})G(x_{k+1,j} + \delta),
\]

\[
(2.41)
\]
\[ G(x_{kj} + \delta) \leq G(x_{k+1,j} + \delta). \]  

Since \( G \) is non-decreasing, (2.42) holds, hence the markup for the entrant is non-decreasing in \( k \). ■

Given the previous observations, when an incumbent competes against multiple entrants, the following questions arise: which auction format is better for the buyer? Does the buyer do better (or worse) by adjusting more bids in a buyer-determined auction?

To compare the results for the average cost for the buyer, between the scoring auction and the buyer-determined auction, I simulate both auction formats. In the simulations, I assume suppliers best-respond to bids they see, and therefore decide if they should stop bidding by calculating the expected profit of stopping at the current bid, and, lowering it by the minimum decrement (\( \delta \)), given the bids they observe from their competitors. The scenarios considered in the simulations correspond to those provided in Tables 2.2 and 2.1, in Subsection 2.2.1. Auctions in the simulations use a minimum decrement equal to 0.001 (\( \delta = 0.001 \)), and the number of entrants varies from 3 to 10 (\( n_e = \{3, 4, 5, 6, 7, 8, 9, 10\} \)).

Figures 2.2, and 2.1 show the results from 10,000 replications of each setting. Each line in each graph corresponds to a particular number of entrants (\( n_e \)), and each point represents the difference between the average cost for the buyer in the buyer-determined auction (BD), when \( k \) takes values between 1 and \( n_e \), minus the average cost for the buyer in the scoring auction (scoring). The buyer’s cost in both cases corresponds to the winning adjusted bid. In each replication, the costs are drawn for all suppliers, and they are used to simulate the buyer-determined auction and the scoring auction. In all figures, a positive difference means the scoring auction yields a lower cost for the buyer than the buyer-determined auction, while a negative difference means the buyer’s cost is higher in the scoring auction.

Note that the lines in Figures 2.1 and 2.2 are non-decreasing, which means the average cost in the buyer determined auction is non-decreasing in \( k \), since the average cost for the buyer in the scoring auction is constant for all values of \( k \). Notice that the differences between the average cost of the scoring auction, and
Figure 2.1. difference=average cost in buyer-determined auction (BD) - average cost in scoring auction (scoring), when the non-price attributes are U[0,0.10].

the average cost of the buyer-determined auction with $k = 1$, are small. Therefore, if calculating the non-price attributes is costly for the buyer, he can run a buyer-determined auction instead of the scoring auction, and set $k = 1$ (please refer to Appendix A for the statistical tests that support the observations in this section).
Figure 2.2. difference=average cost in buyer-determined auction (BD) - average cost in scoring auction (scoring), when the non-price attributes are U[0,0.20].

2.3 Conclusions

I find that when an incumbent competes against one entrant, the buyer-determined auction yields a lower average cost for the buyer than the scoring auction. However, when the number of entrants increases, that changes and the scoring auction yields a lower (or equal) average cost than the buyer-determined auction.
If calculating the non-price attributes is costly for the buyer, running a scoring auction might be prohibitively expensive, in which case the number of bids he can adjust will be dictated entirely by the screening cost. However, in the results of the simulations, differences between the average cost in the scoring auction and the average cost in the buyer-determined auction with $k = 1$ are small (less than 0.02 in absolute value), therefore in this case, the buyer can set $k = 1$ and run a buyer-determined auction instead of a scoring auction.

Given my assumptions there is a variety of extensions to explore, but perhaps the most interesting one, is to understand the bidding strategies when the feedback provided in the auction is only the rank of the bids, instead of their value. This type of feedback is becoming a popular format in procurement auctions, because it provides more information privacy and is less harmful for the supplier-buyer relationships (Jap 2007). In this case, if the buyer runs a scoring auction, the optimal strategy for both entrants and incumbent is still to bid down to cost, since the winner of the auction (the supplier in rank 1) wins the contract. However, when the auction is buyer-determined, the problem is more complicated because bidders do not know by how much they are losing (or winning) the auction. In addition, the information to estimate the difference between the bids depends on the bidding dynamic, and it is different for each supplier.
Chapter 3

An analysis of bidding strategies in procurement auctions with rank feedback

In this chapter I study the bidding strategies of two suppliers competing in a reverse auction with rank feedback. This chapter is part of the paper “A laboratory investigation of rank feedback in procurement auctions” with Wedad Elmaghraby and Elena Katok (Elmaghraby, Katok & Santamaría 2010).

As procurement auctions have expanded to include both domestic and foreign suppliers, practitioners had noted that domestic suppliers would stop bidding if their foreign competitors submit a low bid (Carter et al. 2004), even though the domestic suppliers dominate the foreign suppliers in non-price dimensions. To overcome this anticompetitive effect, suppliers are provided with their bid ranks instead of their bid prices.

To model the bidding strategies, we consider a setting in which a buyer wishes to award a contract to one of two potential suppliers, each of whom could either be a High type (H-type) or Low type (L-type) supplier. A supplier type is defined by his quality and his production cost. The cost for the H-type bidders follows a Uniform distribution between \([h, 100 + h]\) and their quality discount factor is given by \(q_H = h\). The cost for the L-type bidders follow a Uniform distribution between \([0, 100]\) and their quality discount factor is given by \(q_L = 0\). To select a supplier, the buyer runs an open-descending auction, starting from 100 + \(h\), the
feedback provided to the suppliers in the auction corresponds to the rank of their bids (instead of their value), and the bidder with the lowest quality-adjusted bid, \( b_i - q_i \), corresponds to the winning supplier.

We consider the following cases to model the bidding strategies: (1) when bidders have non-overlapping costs (\( h = 100 \)) and the opponent type is known, (2) when bidders have overlapping costs and opponent type is known, and, (3) when bidders have overlapping costs and the opponent type is unknown.

The literature related to procurement auctions overlaps with that of Chapter 1 (please see Section 2.1). The problem of modeling bidding strategies in procurement auctions, when the feedback provided to bidders corresponds to the rank of their bids, is unstudied in the literature (as of May 14th, 2011).

3.1 Bidding strategies for rank feedback with non-overlapping costs and known opponent identity

In this section the cost of the H-type follows a uniform distribution between [100, 200] with a quality discount of 100, and the cost of the L-type follows a uniform distribution between [0, 100].

When the support of the cost for the two types does not overlap, the equilibrium bidding behavior depends on whether a bidder is facing the same type of competitor or not. If a bidder faces an opponent of the same type, he should bid down if he is in rank 2, until either his rank changes to 1, or he bids down to his cost. If a bidder faces a competitor from a different type, the threshold bid corresponds to that of a first-price sealed bid auction. This is because ranks will never change: an H-type is always in rank 2 and an L-type always in rank 1, which makes the bidding problem equivalent to bidding in a first price sealed-bid auction.

To calculate the equilibrium bids, let \( \Pi_i(b, c) \) be the expected profit of bidder \( i \) when he has a cost of \( c \) and submits a bid \( b \), and let \( \phi_i(b) \) denotes the inverse of the equilibrium strategy (assuming it exists). The problem bidder \( i \) has to solve
to calculate the equilibrium bids can be stated as follows:

\[ \Pi_i(b, c) = (b - c) \text{Prob}(b_j > b) \]  
(3.1)

\[ = (b - c)(1 - F_j(\phi_j(b))) \]  
(3.2)

\[ \frac{d\Pi_i}{db} = 1 - F_j(\phi_j(b)) - b f_j(\phi_j(b)) \phi'_j(b) + c f_j(\phi_j(b)) \phi'_j(b) = 0 \]  
(3.3)

\[ \phi'_j(b) = \frac{1 - F_j(\phi_j(b))}{f_j(\phi_j(b))} \frac{1}{(b - \phi_i(b))}. \]  
(3.4)

The solution of the system of differential equations defined above, defines the equilibrium of a reverse “first price” auction between two bidders. If bidders are symmetric, then \( \phi'_j(b) = \phi'_i(b) = \phi'(b) \) and \( F_j(\phi_j(b)) = F_i(\phi_i(b)) = F(\phi(b)) \). For an H-type supplier, \( F(\phi(b)) \) corresponds to a Uniform distribution between 100 and 200. So the system we need to solve reduces to the following differential equation:

\[ \phi'(b) = \frac{200 - \phi(b)}{b - \phi(b)}. \]  
(3.5)

The solution to (3.5) (using the boundary condition \( \phi'(200) = 200 \)) is as follows:

\[ \phi(b) = b + \sqrt{b^2 - 400b + 40000} \]  
(3.6)

\[ = 2b - 200. \]  
(3.7)

From (3.7) we get that the equilibrium strategy in this case is:

\[ b(c) = \frac{c + 200}{2}. \]  
(3.8)

In the case of an L-type supplier, \( F(\phi(b)) \) corresponds to a Uniform distribution between 0 and 100, similar calculations yield the following equilibrium strategy:

\[ b(c) = \frac{c + 100}{2}. \]  
(3.9)
3.2 Bidding strategies for rank feedback with overlapping costs and known opponent identity

When the costs overlap, the bidding strategy - when the two bidders have the same type - is to bid down to cost. The challenge in the analysis comes when two different types compete, because and L-type can update his beliefs over his H-type opponent’s cost as their bids move below 100. As a consequence, an H-type must trade-off decreasing his bid (and increase his changes of winning) with limiting the information flow to his competitor (combined with the possibility that he is currently winning with his rank 2 bid). A consequence of this tradeoff is that the bidders do not have a dominant bidding strategy, which complicates the search for the equilibrium strategies. Therefore, we step back and calculate (myopic) best-response strategies to the rank bidders see at any given time.

When bidders face a different opponent type, if the H-type is in rank 1 and the L-type is in rank 2, the H-type will stop bidding and the L-type will continue lowering his bid as long as his cost is $h$ less than the current bid, or he reaches his threshold. The only setting which remains for the analysis is when the H-type is in rank 2 and the L-type is in rank 1.

To find the threshold bid of low and high types in that setting, define $b_c$ to be the last bid at which the L-type was in rank 2, $c_L$ to be the cost of the L-type and $c_H$ the cost of the H-type.

We start with the threshold for the H-type, which corresponds to the solution of the following problem:

$$\max_{b_H \geq c_H} (b_H - c_H) \text{Prob}(b_H - h < b_L | b_L < b), \quad (3.10)$$

$$\max_{b_H \geq c_H} (b_H - c_H)(1 - \text{Prob}(b_L \leq b_H - h | b_L < b)), \quad (3.11)$$

where $b_L$ is the threshold bid of the L-type, and where the H-type believes the L-type’s bid to be uniformly distributed over $[\alpha_L, b]$. Note that when the H-type is bidding between $[100, 100 + h]$, he does not have additional information regarding the L-type’s cost, and therefore $b = 100$. The first order condition yields the
following solution:

\[ \bar{b}_H = \frac{c_H + b + h}{2} = \frac{c_H + 100 + h}{2}. \]  

(3.12)

Now we turn to the threshold for the L-type. When bidding starts, the L-type needs to go down from the bid ceiling because he cannot let the auction end while his bid is greater than 100, so he will go down to his own space. As long as the H-type threshold is between 100 and 100 + \( h \), the L-type will not have additional information regarding the H-type cost, and therefore \( b_c = 100 + h \). The threshold for the L-type corresponds to the solution to the following problem:

\[
\max_{b_L \geq c_L} (b_L - c_L) \Prob(b_H - h > b_L | b_H < b_c),
\]

(3.13)

\[
\max_{b_L \geq c_L} (b_L - c_L)(1 - \Prob(b_H \leq b_L + h | b_H < b_c)),
\]

(3.14)

where \( b_H \) is the threshold bid of the H-type, and where the L-type believes the H-type’s bid to be uniformly distributed over \([a_H, b_c] \). The first order condition gives:

\[ \widetilde{b}_L = \frac{c_L + b_c - h}{2} = \frac{c_L + 100}{2}. \]  

(3.15)

Summarizing, we have the following results:

\[
\begin{cases} 
\bar{b}_H = \frac{c_H + b + h}{2} & \text{if } b < 100, \\
\bar{b}_H = \frac{c_H + 100 + h}{2} & \text{if } b \geq 100. 
\end{cases}
\]

(3.16)

\[
\begin{cases} 
\bar{b}_L = \max\left\{ \frac{c_L + b_c - h}{2}, c_L \right\} & \text{if } b_c < 100, \\
\bar{b}_L = \frac{c_L + 100}{2} & \text{if } b_c \geq 100. 
\end{cases}
\]

(3.17)

Note that when the type of the competitor is known, and the suppliers are facing an asymmetric setting, the H-type can avoid revealing information regarding his bid to the L-type. Since an L-type cannot let the auction end with a bid greater than 100, at the beginning of the auction he must start bidding down to his threshold immediately. In contrast, knowing his competitor is an L-type, the H-type does not have an incentive to bid down to his threshold immediately (even if he is in rank 2), and will let the L-type lead the way. When \( \bar{b}_H \) is between \([100, 100 + h] \), the L-type will not be able to gather information regarding the
bid from the H-type, and the auction ends with both bidders at their respective thresholds, $\tilde{b}_H$ and $\tilde{b}_L$, corresponding to the first-price sealed-bids. In this case, the L-type will reach his threshold being in rank 1 and the H-type will reach his being in rank 2, and they will not see a change in their ranks.

In contrast, when $\tilde{b}_H$ is between $[h, 100]$, suppliers may see a change in their ranks. When $\tilde{b}_H < \tilde{b}_L$, there is guaranteed to be a change in rank at some bid level at or before reaching $\tilde{b}_L$; at that point both bidders will know the current bid of their opponent, and will be able to bid down as if they were able to see the price bids of their competitors. When $\tilde{b}_L < \tilde{b}_H < 100$, the H-type may bid down to $\tilde{b}_H$ without experiencing a rank change; even if his rank does not change, an H-type’s information set is changing as he decreases his bid, because he will have a new upper bound on the L-type’s cost type (the value of $b$ now corresponding to the H-type’s “new” current bid). Hence, he will (possibly multiple times) update his threshold $\tilde{b}_H$ and adopt a strategy to bid down to it. Note that $\tilde{b}_H = b$, when $b = c_H + h$, so an H-type will not bid lower than $c_H + h$ for all $c_H \in [h, 100 + h]$. If, when $\tilde{b}_L < \tilde{b}_H < 100$, the bidders do experience a change in their ranks, they will be able to identify their opponent’s bid location and bid down as if they were able to see the price bids of his competitors.

3.3 Bidding strategies for rank feedback with overlapping costs and unknown opponent identity

When the bidders do not know the type of their competitor, an H-type in rank 1 should stop bidding, because no matter the type of his competitor he will win. An L-type in rank 2, should decrease his bid until he is in rank 1 or reaches his cost, otherwise he will lose for sure, and, an L-type in rank 1 should decrease his bid only if his current bid minus $h$ is greater than his cost; note that if his competitor is another L-type he does not need to decrease his bid any further to win, and if his competitor is an H-type, he has a chance of winning only if his current bid minus $h$ is greater than his cost.

The threshold for the H-type depends on whether he is currently bidding above
100 or below. If he is bidding above 100, then he has no information regarding the bid of a potential L-type competitor. If he is bidding below 100, then his current bid is an upper bound of the bid of a potential L-type competitor.

We start with the case where the current bid of the H-type, \( b \) is between \( h \) and 100. Let \( b_L \) the threshold bid of an L-type competitor, \( b'_H \) the threshold bid of an H-type competitor, \( p_H \) the conditional probability the competitor is an H-type given \( b \), \( p_L \) the conditional probability the competitor is an L-type, given \( b \), and \( p \) the probability the competitor is an H-type. With those elements, the problem can be formulated as follows:

\[
\max_{b_H \geq 100} (b_H - c_H) \left( \frac{p_L \text{Prob}(b_H - h < b_L | b_L \leq b)}{b_H - c_H} + p_H \times \text{Prob}(b_H < b'_H | b'_H \leq b) \right),
\]

\[
\max_{b_H \geq 100} (b_H - c_H) (p_L (1 - \text{Prob}(b_L \leq b_H - h | b_L \leq b)) + p_H (1 - \text{Prob}(b_H \leq b'_H | b'_H \leq b))),
\]

\[
\max_{b_H \geq 100} (b_H - c_H) \left( p_L \left( \frac{b - b_H + h}{b - \alpha'_L} \right) + p_H \left( \frac{b - b_H}{b - \alpha'_H} \right) \right).
\]

Let \( b_o \) denote the bid of the opponent, so \( p_H \) is defined as follows:

\[
p_H = \text{Prob}(\text{H-type}|b_o \leq b) = \frac{\text{Prob}(b_o \leq b|\text{H-type})\text{Prob}(\text{H-type})}{\text{Prob}(b_o \leq b)}, \quad (3.21)
\]

\[
= \frac{\text{Prob}(b_o \leq b|\text{H-type})\text{Prob}(\text{H-type})}{\text{Prob}(b_o \leq b|\text{H-type})\text{Prob}(\text{H-type}) + \text{Prob}(b_o \leq b|\text{L-type})\text{Prob}(\text{L-type})}, \quad (3.22)
\]

\[
= \left( \frac{b - \alpha'_H}{100 + h - \alpha'_H} \right) p + \left( \frac{b}{100} \right) (1 - p), \quad (3.23)
\]

where \( \alpha'_H \) is the lower bound of the bid from an H-type, which is equal to \( h \). Equivalently, the lower bound of the bid from an L-type, \( \alpha'_L \) is equal to 0. After doing these replacements, the final expressions for \( p_H \) and \( p_L \) are:

\[
p_H = \frac{(b - h)p}{b - h p}, \quad (3.24)
\]
\[
p_L = (1 - p_H) = \frac{b(1 - p)}{b - hp}; \quad \text{(3.25)}
\]
and the first order condition yields the following threshold for the H-type:
\[
\bar{b}_H^N = \frac{c_H + b + h(1 - p)}{2}. \quad \text{(3.26)}
\]

Similarly, when the current bid from the H-type is greater than 100, the problem is as follows:
\[
\max_{b_H \geq c_H} (b_H - c_H) \left( p_L \text{Prob}(b_H - h < b_L | b_L \leq 100) + p_H \times \text{Prob}(b_H < b'_H | b'_H \leq b) \right),
\]
\[
\max_{b_H \leq c_H} (b_H - c_H) \left( p_L \left( \frac{100 - b_H + h}{100 - \alpha'_L} \right) + p_H \left( \frac{b - b_H}{b - \alpha_H} \right) \right), \quad \text{(3.27)}
\]
where \( p_H \) and \( p_L \) are defined as follows:
\[
p_H = \frac{\left( \frac{b - \alpha'_H}{100 + h - \alpha'_H} \right) p}{\left( \frac{b - \alpha'_H}{100 + h - \alpha'_H} \right) p + 1 \times (1 - p)}, \quad \text{(3.29)}
\]
\[
= \frac{(b - h)p}{(b - h)p + 100(1 - p)}, \quad \text{(3.30)}
\]
\[
p_L = \frac{100(1 - p)}{(b - h)p + 100(1 - p)}. \quad \text{(3.31)}
\]
The first order condition yields the following threshold for the H-type in this case:
\[
\bar{b}_H^N = \frac{c_H + (1 - p)(100 + h) + pb}{2}. \quad \text{(3.32)}
\]

Now let us turn to the threshold for the L-type. Define \( b_c \) as the last bid at which the L-type was in rank 2. When \( b_c \leq 100 \). The threshold for the L-type depends on the value of \( b_c \). When \( b_c \geq 100 \), the L-type does not have additional information to update the upper bound on a potential L-type competitor’s bid, he only knows is less than 100. When \( b_c < 100 \), he needs to update his threshold, \( \bar{b}_L^N \) using \( b_c \) as the upper bound of the potential L-type. The problem in this case is
as follows:

\[
\max_{b_L \geq c_L} (b_L - c_L) \left( \frac{q_L \text{Prob}(b_L < b'_L | b'_L \leq b_c) + q_H \times \text{Prob}(b_L < b_H - h | b_H \leq b_c)}{\text{winning versus an L-type}} \right),
\]

\[
\max_{b_L \geq c_L} (b_L - c_L) \left( q_L (1 - \text{Prob}(b'_L \leq b_c | b'_L \leq b_c)) + q_H (1 - \text{Prob}(b_H \leq b_L + h | b_H \leq b_c)) \right),
\]

\[
\max_{b_H \geq c_H} (b_H - c_H) \left( q_L \left( \frac{b_c - b_H + h}{b_c - \alpha'_L} \right) + q_H \left( \frac{b_c - b_H}{b_c - \alpha'_H} \right) \right),
\]

where \(q_H\) and \(q_L\) are the conditional probabilities of the competitor being an H-type and L-type correspondingly, given \(b_c\). They are given by:

\[
q_H = \frac{(b_c - h)p}{b_c - hp},
\]

\[
q_L = \frac{b_c(1 - p)}{b_c - hp}.
\]

The first order condition yields the following threshold:

\[
b^*_N = \frac{c_L + b_c - hp}{2}.
\]

Similarly, when \(b_c \geq 100\), the problem can be formulated as follows:

\[
\max_{b_L \geq c_L} (b_L - c_L) \left( \frac{q_L \text{Prob}(b_L < b'_L | b'_L \leq b_c) + q_H \times \text{Prob}(b_L < b_H - h | b_H \leq b_c)}{\text{winning versus an L-type}} \right),
\]

\[
\max_{b_L \geq c_L} (b_L - c_L) \left( q_L (1 - \text{Prob}(b'_L \leq b_c | b'_L \leq b_c)) + q_H (1 - \text{Prob}(b_H \leq b_L + h | b_H \leq b_c)) \right),
\]

\[
\max_{b_H \geq c_H} (b_H - c_H) \left( q_L \left( \frac{100 - b_H + h}{100 - \alpha'_L} \right) + q_H \left( \frac{b_c - b_H}{b_c - \alpha'_H} \right) \right),
\]

where \(q_H\) and \(q_L\) are the conditional probabilities of the competitor being an H-type
and L-type correspondingly, given \( b_c \). They are given by:

\[ q_H = \frac{(b_c - h)p}{(b_c - h)p + 100(1 - p)}, \tag{3.42} \]
\[ q_L = \frac{100(1 - p)}{(b_c - h)p + 100(1 - p)}. \tag{3.43} \]

The first order condition yields the following threshold:

\[ \tilde{b}_L^N = \frac{c_L + 100(1 - p) + (b_c - h)p}{2}. \tag{3.44} \]

Summarizing, we have the following thresholds:

\[ \tilde{b}_H^N = \begin{cases} \frac{c_H + b + h(1 - p)}{2} & \text{if } b < 100, \\ \frac{c_H + (1 - p)(100 + h) + pb}{2} & \text{if } b \geq 100. \end{cases} \tag{3.45} \]
\[ \tilde{b}_L^N = \begin{cases} \max \left\{ \frac{c_L + b - h p}{2}, c_L \right\} & \text{if } b_c < 100, \\ \max \left\{ \frac{c_L + 100(1 - p) + (b_c - h)p}{2}, c_L \right\} & \text{if } b_c \geq 100. \end{cases} \tag{3.46} \]

When \( p = \frac{1}{2} \), if two L-types compete, they will stop bidding at the maximum cost minus \( \delta \) – this outcome is equivalent to the case when the type of the competitor is known. If two H-types compete, they will stop bidding when they reach the corresponding thresholds – in contrast, when they know the type of the competitor, they bid down to cost. In general, \( \tilde{b}_H^N > \tilde{b}_H \) when \( b < 100 \) and \( \tilde{b}_H^N < \tilde{b}_H \) when \( b \geq 100 \). When an H-type competes against an L-type, bidding stops when they reach the corresponding thresholds – note that \( \tilde{b}_L^N \leq \tilde{b}_L \).

### 3.4 Simulation results

To compare the average cost when the type of the competitor is not known (Rank\(_N\)), and when it is known (Rank), we ran 100,000 replications, setting \( p = \frac{1}{2} \) and \( \delta = 0.001 \). In Rank\(_N\), when an H-type competes with an L-type, and the threshold of the H-type is greater than 100, we assume there is no change in the rank. Therefore, \( b_c = 100 + h \) and \( \tilde{b}_L = \tilde{b}_L^N = \text{first-price sealed-bid} \). Correspondingly, \( \tilde{b}_H = \text{first-price sealed bid} \) and \( \tilde{b}_H^N = \frac{2c_H + 100 + h}{3} \) (this is the value of the bid
for which \( b = \frac{2c_H + b + h + 100}{4} \).

Figure 3.1 shows the results for the average cost for the buyer for different values of \( h \), under Rank, Rank\(_N\) and as a benchmark, the case in which the feedback given corresponds to the price bids, and assuming that bidders know their opponent type (F\(_{\text{info}}\)). Note that in the latter case the strategy for both type of bidders is to bid down in decrements until they are winning -in quality adjusted space- or they reach their cost.

<table>
<thead>
<tr>
<th>( h )</th>
<th>Rank(_N)</th>
<th>Rank</th>
<th>F(_{\text{info}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>66.67</td>
<td>66.69</td>
<td>66.67</td>
</tr>
<tr>
<td>5</td>
<td>67.01</td>
<td>64.04</td>
<td>66.67</td>
</tr>
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<td>67.23</td>
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<td>67.65</td>
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<td>66.67</td>
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</tr>
<tr>
<td>95</td>
<td>66.31</td>
<td>66.67</td>
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<tr>
<td>100</td>
<td>66.31</td>
<td>66.67</td>
<td>66.67</td>
</tr>
</tbody>
</table>

**Figure 3.1.** Average cost for the buyer from 100,000 simulations of the bidding strategies

In the figure, the average cost for the buyer for Rank is flat when \( h \geq 50 \) because in those cases, the threshold for the H-type is always greater than 100, and the resulting final bids corresponds to first-price sealed-bids when bidders of different type compete. Similarly, for Rank\(_N\), when \( h \geq \frac{200}{3} \approx 66.67 \), the threshold for the H-type is always greater than 100, and the resulting final bids correspond to the first-price sealed-bid for the L-type and \( \tilde{b}_H^N = \frac{2c_H + 100 + h}{3} \) for the H-type. In this case the average cost for the buyer is lower for Rank\(_N\), because H-types will bid below their first-price sealed-bid.

### 3.5 Conclusions

The feedback format in a procurement auctions is a key decision in the design of the auction, when suppliers differ in quality and have overlapping costs supports.
The analysis presented here, studies a setting in which suppliers compete on price during a live auction event, but instead of knowing the current bids of their competitors, only the rank of their bid is disclosed. Suppliers in the auction differ in quality (high or low), and the buyer awards the contract to the supplier who submitted the lowest quality adjusted bid. In the model, low quality suppliers have costs that follow a Uniform distribution between \([0, 100]\), and high quality suppliers have costs that follow a Uniform distribution between \([h, 100 + h]\); high suppliers receive a quality discount of \(h\) so their quality adjusted bid corresponds to their price bid minus \(h\).

When the costs of the two types are completely overlapping (\(h = 0\)) or non-overlapping (\(h = 100\)), and the competitor’s type is known, the expected cost for the buyer under rank feedback, and full price feedback is the same.

However, if suppliers know their competitor’s type, but their costs overlap more than 50% (\(0 < h \leq 50\)), rank feedback yields a lower average cost than full price feedback. When the opponent’s type is unknown, and costs overlap more than 44% (\(0 < h \leq 66\)), the average cost of the buyer is less with full feedback than with rank feedback, and, when the costs overlap less than 40% (\(h > 66\)), the average cost with rank feedback is less than with full price feedback.
Chapter 4

An analysis of bidding strategies in sealed-bid combinatorial auctions

In this chapter I analyze the problem faced by bidders in sealed-bid combinatorial auction of two objects, using myopic-best replying bidders. I find that if a bidder bids for the single objects and the package, and his bids are competitive enough to be greater than the corresponding best competitive bids, he should bid only for the set with the highest expected profit. To investigate this prediction with human decision makers, controlled laboratory experiments were run in a simple setting in which the optimal strategy is to bid only for one set. Results from the experiments show that on average subjects failed to realize they should bid only for one set, resulting in significantly lower average profits with respect to the optimal expected profits.

Regarding the characteristics of the optimal bid for a set, I find it is bounded by the optimal bid when (1) the best competitive bid for the package corresponds to the maximum bid for the package, and (2) the best competitive bid for the package corresponds to the sum of the maximum bids for the individual objects.

The literature concerned with the bidding strategies in combinatorial auctions is extensive for the dynamic case, which is mostly related the FCC multi-round combinatorial auctions for spectrum licenses (Hoffman, Menon & van den Heever 2004) and references within (Cramton et al. 2006).
For a first-price sealed-bid combinatorial auction, An, Elmaghraby & Kesnikocak (2005) propose a synergy model to find the bidding strategies, in a setting related to the procurement of transportation. The synergy model takes single item values and pairwise synergies, and calculates the value for combinations of items (bundles), selects the desired packages to bid on, and designs the bidding strategies under different market settings. Results from simulations, suggest that bidders with a large number of objects with positive value, relative to their competitors, should bid with a moderate degree of overlap between the bundles if the competition is low, and they should bid only on their highest valued bundles if the competition is high. I find that bidders should bid for overlapping sets, as long as their bids are not competitive enough to be greater than the corresponding best competitive bids simultaneously, in which case they should bid only for disjoints sets.

Wilenius & Anderson (2007) propose a method to find approximate equilibrium strategies in a first-price, sealed-bid combinatorial auction. The method consists of using best-response iterations, to reach the fixed point that characterizes the equilibrium within some tolerance. They test the method using two strategies. In one strategy, bidders submit bids on all possible combinations, and in the other, bidders bid for one combination and the remaining single items. Given a particular strategy, the method finds a bid function for each combination size. The bid function is independent of exactly which items are included in the combination, because it depends only on the size of the combination, so an increase in the number of objects increases the value of the bid. Wilenius & Anderson (2007) conclude that the resulting combinatorial bid functions increase with the valuation in a similar way as they increase in single-bid strategies. However, they do not discuss the consequences of competing against oneself when bidding for overlapping sets, which are present in the strategies they use to approximate the equilibrium bidding strategies. To understand if a bidder should bid for overlapping sets, I study the bidding strategies in a combinatorial auction of two objects, where bidders can bid for each object and the package.
4.1 The auction

Consider a single-round, first-price, sealed-bid combinatorial auction of two objects \((M = \{\alpha, \beta\})\) between a set of bidders \(N = \{1, \ldots, n\}\). A bidder can bid for all subsets of \(M\), but only one of his bids can win, hence bids are mutually exclusive. This bidding language is known as XOR language (Cramton et al. 2006), and it allows all types of preferences for the objects to be expressed. For example, consider an auction of the two objects, between two bidders. Suppose bidder 1 bids $6 for each object and $13 for the set that contains both objects (the package), and bidder 2 bids $8 for each single object, and zero for the package. In this case, the objects have synergy for bidder 1 and do not have synergy for bidder 2, moreover, bidder 2 only wants to win one of the objects.

A bidder in this combinatorial auction has the following bidding strategies; he can bid for: (1) one object only, (2) each object but not the package, (3) the package only, (4) the package and one object, and (5) the package and both objects separately.

When a bidder bids for the package and something else he can compete against himself. In our previous example, bidder 1 wins one object (he pays $6 for it), bidder 2 wins the other object (he pays $8 for it), and the revenue for the auctioneer is $14. But if bidder 1 bids only for the package, he wins the package (and pays $13 for it). In this case, bidder 1 loses the package because of his bid for the object. Note that this is a desirable outcome for bidder 1, if his profit (defined as his value minus his bid) of winning the object, is greater than his profit of winning the package.

To understand if bidder should avoid bidding for overlapping sets, in the next section I calculate the bidding function for each of the five possible bidding strategies, to compare the expected profit of bidding for disjoint and overlapping sets.

4.2 Bidding strategies

The general framework to derive the bidding strategies is as follows. When the bidder bids for a set, he competes against the best competitive bid for that set. If he bids for \(\{\alpha\}\) his bid has to be greater than his competitors’ maximum bid for
\{\alpha\}, which corresponds to the best competitive bid for \{\alpha\}. When the bidder bids for the package, \{\alpha, \beta\}, the best competitive bid for it can be: (1) his competitors’ maximum bid for the package, or, (2) the sum of his competitors’ maximum bid for \{\alpha\} plus his competitors’ maximum bid for \{\beta\}. In all cases, the bidder’s objective is to find the bid that maximizes his expected profit given a distribution of the best competitive bid.

In the following sections, the optimal bidding strategy is characterized when the bidder bids for the individual objects, the package, and the package and the individual objects.

### 4.2.1 Bidding for the individual objects

When the bidder bids only for the individual objects - since bids are mutually exclusive - he can only win one of the objects but not both, and we can determine the optimal bid for \{\alpha\} and \{\beta\} separately. To ease the exposition, I solve the problem for \{\alpha\}, but similar results hold for the optimal bid for \{\beta\}.

Let \(v_S\) and \(b_S\) be the bidder’s value and bid for the set \(S, S \subseteq M\) correspondingly, and let \(b_{\text{max}, S}\) denote his competitors’ maximum bid for the set \(S, S \subseteq M\), referred as the maximum bid for the set \(S\) from here on. With those elements, the conditions to win \{\alpha\} by bidding \(b_{\alpha}\) are shown in the event tree in Figure 4.1.

When \(b_{\text{max}, \alpha} + b_{\text{max}, \beta} > b_{\text{max}, \alpha \beta}\), the best competitive bid for the package corresponds to the sum of the maximum bids for the objects, we will refer to this case as the best competitive bid for the package being subadditive. In turn, when \(b_{\text{max}, \alpha} + b_{\text{max}, \beta} \leq b_{\text{max}, \alpha \beta}\), the best competitive bid for the package corresponds to the maximum bid for the package, in which case the best competitive bid for the package is superadditive. The conditions to win \{\alpha\} depend on the best competitive bid being subadditive or superadditive, as can be seen from the event tree in Figure 4.1.

To formulate the expected profit maximization problem for the bidder, let \(F_S(b)\) denote the distribution function of the maximum bid for set \(S, S \subseteq M\), and \(\pi_{\alpha}^{\text{sub}}\) the expected profit, when the best competitive bid for the package is subadditive (therefore it corresponds to the sum of the maximum bids for the objects). In this case, the problem the bidder needs to solve to find the bidding strategy can be
Figure 4.1. Event tree when the bidder bids $b_\alpha$

formulated as follows:

$$\max_{0 \leq b_\alpha \leq v_\alpha} \pi^{\text{sub}}_\alpha = (v_\alpha - b_\alpha) \text{Prob}(b_{\max,\alpha} \leq b_\alpha) = (v_\alpha - b_\alpha) F_\alpha(b_\alpha).$$  \hfill (4.1)$$

The first order condition for the optimal bid, $b^{*\text{sub}}_\alpha$, is:

$$b^{*\text{sub}}_\alpha = v_\alpha - \frac{F_\alpha(b^{*\text{sub}}_\alpha)}{F'_\alpha(b^{*\text{sub}}_\alpha)}.$$  \hfill (4.2)$$

When the best competitive bid for the package is superadditive (therefore it corresponds to the maximum bid for the package), the bid for $\{\alpha\}$ combined with the maximum bid for $\{\beta\}$, has to break the maximum bid for the package to be the winning bid, so $b_{\max,\alpha} \leq b_\alpha + b_{\max,\beta}$. Let $G_\Delta(b)$ denote the distribution function of $b_{\max,\alpha} - b_{\max,\beta}$. In this case, $\pi^{\text{super}}_\alpha$ denotes the expected profit for the bidder, and the problem he needs to solve can be formulated as follows:

$$\max_{0 \leq b_\alpha \leq v_\alpha} \pi^{\text{super}}_\alpha = (v_\alpha - b_\alpha) \text{Prob}(b_{\max,\alpha} \leq b_\alpha) \text{Prob}(b_{\max,\alpha} - b_{\max,\beta} \leq b_\alpha),$$

$$= (v_\alpha - b_\alpha) F_\alpha(b_\alpha) G_\Delta(b_\alpha);$$  \hfill (4.3)$$

the corresponding first order condition for the optimal bid, $b^{*\text{super}}_\alpha$, is:

$$b^{*\text{super}}_\alpha = v_\alpha - \frac{G_\Delta(b^{*\text{super}}_\alpha) F_\alpha(b^{*\text{super}}_\alpha)}{G'_\Delta(b^{*\text{super}}_\alpha) F'_\alpha(b^{*\text{super}}_\alpha) + G_\Delta(b^{*\text{super}}_\alpha) F'_\alpha(b^{*\text{super}}_\alpha)}.$$  \hfill (4.4)$$

If the bidder knows with certainty -before submitting his bids- whether the
best competitive bid for the package is subadditive or superadditive, then the optimal bid for \( \{ \alpha \} \) can be calculated according to the previous analysis. But in general that is not the case. Instead, the best competitive bid for the package is superadditive with probability \( p = \text{Prob}(b_{\text{max,} \alpha} + b_{\text{max,} \beta} \leq b_{\text{max,} \alpha \beta}) \). The problem in this case is:

\[
\max_{0 \leq b_\alpha \leq v_\alpha} \pi_\alpha = (v_\alpha - b_\alpha)((1 - p) \times \text{Prob}(b_{\text{max,} \alpha} \leq b_\alpha) + p \times \text{Prob}(b_{\text{max,} \alpha} \leq b_\alpha) \text{Prob}(b_{\text{max,} \alpha \beta} - b_{\text{max,} \beta} \leq b_\alpha)),
\]

\[
= (v_\alpha - b_\alpha) \left( (1 - p)F_\alpha(b_\alpha) + pF_\alpha(b_\alpha)G_\Delta(b_\alpha) \right);
\] (4.5)

and the first order condition for the optimal solution is given by:

\[
b_\alpha^* = v_\alpha - \frac{(1 - p)F_\alpha(b_\alpha^*) + pF_\alpha(b_\alpha^*)G_\Delta(b_\alpha^*)}{F_\alpha'(b_\alpha)((1 - p) + pG_\Delta(b_\alpha)) + pF_\alpha(b_\alpha)G_\Delta'(b_\alpha)}.\] (4.6)

Notice the optimal bid, \( b_\alpha^* \), is bounded by \( b_\alpha^{\text{sub}} \) and \( b_\alpha^{\text{super}} \).

### 4.2.2 Bidding for the package

Now let us explore the case when the bidder bids only for the package. The conditions to win the package by bidding \( b_{\alpha \beta} \) are shown in the event tree in Figure 4.2.

![Figure 4.2. Event tree when the bidder bids \( b_{\alpha \beta} \)](image)

To formulate the expected profit maximization problem, let \( F_{\alpha \beta}(b) \) denote the distribution function of the maximum bid for the package, and \( G_\Sigma(b) \) the distribution function of the sum of the maximum bids for the objects.
When the best competitive bid for the package is subadditive, to win the package, the bid for it has to be greater than the sum of the maximum bids for the single objects. Let $\pi_{\alpha\beta}^{\text{sub}}$ denote the expected profit in this case. The problem the bidder has to solve is as follows:

$$
\max_{0 \leq b_{\alpha\beta} \leq v_{\alpha\beta}} \pi_{\alpha\beta}^{\text{sub}} = (v_{\alpha\beta} - b_{\alpha\beta}) \text{Prob}(b_{\text{max},\alpha} + b_{\text{max},\beta} \leq b_{\alpha\beta}),
$$

$$
= (v_{\alpha\beta} - b_{\alpha\beta}) G_\Sigma(b_{\alpha\beta});
$$

(4.7)

and the first order condition for the optimal bid, $b_{\alpha\beta}^{\text{sub}}$, is:

$$
b_{\alpha\beta}^{\text{sub}} = v_{\alpha\beta} - G_\Sigma(b_{\alpha\beta}^{\text{sub}}),
$$

(4.8)

In turn, when the best competitive bid for the package is superadditive, the bid for the package has to be greater than the maximum bid for the package. Let $\pi_{\alpha\beta}^{\text{super}}$ denote the expected profit in this case. The problem the bidder has to solve is the following:

$$
\max_{0 \leq b_{\alpha\beta} \leq v_{\alpha\beta}} \pi_{\alpha\beta}^{\text{super}} = (v_{\alpha\beta} - b_{\alpha\beta}) \text{Prob}(b_{\text{max,}\alpha\beta} \leq b_{\alpha\beta}),
$$

$$
= (v_{\alpha\beta} - b_{\alpha\beta}) F_{\alpha\beta}(b_{\alpha\beta});
$$

(4.9)

and the first order condition for the optimal bid, $b_{\alpha\beta}^{\text{super}}$, is given by:

$$
b_{\alpha\beta}^{\text{super}} = v_{\alpha\beta} - \frac{F_{\alpha\beta}(b_{\alpha\beta}^{\text{super}})}{F'_\alpha(b_{\alpha\beta}^{\text{super}})}.
$$

(4.10)

Finally, when the bidder does not know with certainty if the best competitive bid for the package is the maximum bid for the package, or, the sum of the maximum bids for the objects, the problem is as follows:

$$
\max_{0 \leq b_{\alpha\beta} \leq v_{\alpha\beta}} \pi_{\alpha\beta} = (v_{\alpha\beta} - b_{\alpha\beta}) \times
$$

$$
((1 - p) \times \text{Prob}(b_{\text{max,}\alpha} + b_{\text{max,}\beta} \leq b_{\alpha\beta}) + p \text{Prob}(b_{\text{max,}\alpha\beta} \leq b_{\alpha\beta})),
$$

$$
= (v_{\alpha\beta} - b_{\alpha\beta}) ((1 - p) G_\Sigma(b_{\alpha\beta}) + p F_{\alpha\beta}(b_{\alpha\beta})).
$$

(4.11)
The corresponding first order condition for the optimal solution is:

\[ b^*_{\alpha\beta} = v_{\alpha\beta} - \frac{(1 - p)G_\Sigma(b^*_{\alpha\beta}) + pF_{\alpha\beta}(b^*_{\alpha\beta})}{(1 - p)G_\Sigma(b^*_{\alpha\beta}) + pF'_{\alpha\beta}(b^*_{\alpha\beta})}. \]  

(4.12)

Notice that \( b^{\text{sub}}_{\alpha\beta} \) and \( b^{\text{super}}_{\alpha\beta} \) provide bounds for the optimal bid for the package, \( b^*_{\alpha\beta} \).

### 4.2.3 Bidding for the package and something else

In the two previous subsections we discussed the bidding strategies when the bidder bids for the package or each object. Here, we turn to the two final cases: (1) when the bidder bids for the package and one object, and (2) when the bidder bids for the package and both objects.

Let me start the analysis with the case when the bidder bids for the package and one object (let that object be \( \{\alpha\} \)). The event tree in Figure 4.3 shows the conditions to win \( \{\alpha\} \) or the package, by bidding \( b_\alpha \) and \( b_{\alpha\beta} \).

![Event tree](image)

**Figure 4.3.** Event tree when the bidder bids \( b_\alpha \) and \( b_{\alpha\beta} \)

Notice that only when the bids, \( b_\alpha \) and \( b_{\alpha\beta} \), are greater than or equal to the
corresponding best competitive bids, $b_{\text{max},\alpha}$ and $b_{\text{max},\alpha\beta}$ (or $b_{\text{max},\alpha} + b_{\text{max},\beta}$), the bidder competes against himself, in which case, he should bid only for the set with the highest profit.

The problem the bidder needs to solve to find the optimal bids for each set can be stated as follows:

$$\max_{0 \leq b_\alpha \leq v_\alpha} \pi = \pi_\alpha^{\text{all}} + \pi_{\alpha\beta}^{\text{all}},$$

(4.13)

where $\pi_\alpha^{\text{all}}$ is defined in equation (4.14), and $\pi_{\alpha\beta}^{\text{all}}$ is defined in equation (4.15).

**Expected profit of bidding only for the single object**

$$\pi_\alpha^{\text{all}} = (v_\alpha - b_\alpha)((1 - p)F_\alpha(b_\alpha) + pG_\Delta(b_\alpha)F_\alpha(b_\alpha))(1 - F_{\alpha\beta}(b_{\alpha\beta}))$$

$$+ (v_\alpha - b_\alpha)p(1 - F_\beta(b_{\alpha\beta} - b_\alpha))F_{\alpha\beta}(b_{\alpha\beta})F_\alpha(b_\alpha).$$

(4.14)

**Expected profit of bidding only for the package**

$$\pi_{\alpha\beta}^{\text{all}} = (v_{\alpha\beta} - b_{\alpha\beta})((1 - p)G_\Sigma(b_{\alpha\beta}) + pF_{\alpha\beta}(b_{\alpha\beta}))(1 - F_\alpha(b_\alpha))$$

$$+ (v_{\alpha\beta} - b_{\alpha\beta})F_\alpha(b_\alpha)F_{\beta}(b_{\alpha\beta} - b_\alpha)(pF_{\alpha\beta}(b_{\alpha\beta}) + (1 - p)).$$

(4.15)

Problem (4.13) can be solved numerically to find the optimal bids for the package and the single object, once the probability distributions for the maximum bids are defined.

When the bidder bids for both objects and the package, the event tree with the conditions to win one object or the package explodes (please see Section B.1 of the Appendix). As before, the bidder competes against himself if simultaneously, the bid for the package is greater than or equal to the best competitive bid for the package, and one or both bids for the objects are greater than or equal to their corresponding maximum bids. In a general setting with $m$ objects, bids for overlapping sets will only compete against each other if they are greater than the corresponding best competitive bids, in which case, the bidder should bid only for the set with the highest profit. Note that when bidders in combinatorial auctions have values for overlapping sets, in order to find the optimal bidding strategy, they need to evaluate and compare the expected profit of bidding for those overlapping sets, with respect of that of bidding for one set only.
In the next section, I assume the distributions of the maximum bids for each object and the package, to calculate the corresponding optimal bids when the bidder bids for one object, the package, and when he bids for one object and the package.

4.3 The Uniform case

In this section I calculate the optimal bids (analytically or numerically) assuming the number of competitors bidding for \{\alpha\}, \{\beta\} and the package is \(n_\alpha\), \(n_\beta\) and \(n_{\alpha\beta}\) correspondingly. Regarding the distributions of the maximum bid for each object, \(b_{\text{max,}\alpha}\) and \(b_{\text{max,}\beta}\), they are given by \(\text{Prob}(b_{\text{max,}\alpha} \leq b) = F_\alpha(b) = b^{n_\alpha}, 0 \leq b \leq 1\), and, \(\text{Prob}(b_{\text{max,}\beta} \leq b) = F_\beta(b) = b^{n_\beta}, 0 \leq b \leq 1\). These distributions correspond to the distribution of the maximum of \(n_\alpha\) and \(n_\beta\) continuous Uniform distributions between 0 and 1 correspondingly. Finally, the distribution of the maximum bid for the package \(b_{\text{max,}\alpha\beta}\) is given by \(\text{Prob}(b_{\text{max,}\alpha\beta} \leq b) = F_{\alpha\beta}(b) = \left(\frac{b}{2k}\right)^{n_{\alpha\beta}}, 0 \leq b \leq 2k, k > 1\). This distribution corresponds to the maximum of \(n_{\alpha\beta}\) continuous Uniform distributions between 0 and 2\(k\), where \(k\) indicates the synergy between the objects. As a consequence, the distributions of \(b_{\text{max,}\alpha\beta} - b_{\text{max,}\beta}, G_\Delta(b),\) and \(b_{\text{max,}\alpha} + b_{\text{max,}\beta}, G_\Sigma(b),\) are derived below.

\(G_\Delta(b)\) is derived as follows:

\[
\text{Prob}(b_{\text{max,}\alpha\beta} - b_{\text{max,}\beta} \leq b) = \frac{1}{(2k)^{n_{\alpha\beta}}} \int_0^1 (b + x)^{n_{\alpha\beta}} n_\beta x^{n_\beta-1} dx = \frac{E[(b + b_{\text{max,}\beta})^{n_{\alpha\beta}}]}{(2k)^{n_{\alpha\beta}}} = \frac{(b + E[b_{\text{max,}\beta}])^{n_{\alpha\beta}}}{(2k)^{n_{\alpha\beta}}} = \frac{1}{(2k)^{n_{\alpha\beta}}} \left( b + \frac{n_\beta}{n_\beta + 1} \right)^{n_{\alpha\beta}}.
\]

(4.16) (4.17) (4.18) (4.19)

Since the maximum bids for the single objects are between 0 and 1, \(0 \leq b \leq 1\).

Next, \(G_\Sigma(b)\) is derived as follows:

\[
\text{Prob}(b_{\text{max,}\alpha} + b_{\text{max,}\beta} \leq b) = \int_0^b \text{Prob}(b_{\text{max,}\alpha} \leq b - x) n_\beta x^{n_\beta-1} dx
\]

(4.20)
\[ = n_\beta \int_0^b (b - x)^{n_\alpha} x^{n_\beta - 1} dx \tag{4.21} \]

By doing a change of variables so \( x = by \), \( dx = bdy \) and by multiplying and dividing by \( \lambda = \frac{\Gamma(n_\alpha + 1) \Gamma(n_\beta)}{\Gamma(n_\alpha + n_\beta + 1)} \), the integral becomes the cumulative function of a Beta distribution:

\[ \text{Prob}(b_{\text{max}, \alpha} + b_{\text{max}, \beta} \leq b) = n_\beta b^{n_\alpha + n_\beta} \lambda \int_0^1 \frac{1}{\lambda} \left( (1 - y)^{n_\alpha} y^{n_\beta - 1} \right) dy \tag{4.22} \]

\[ \int_0^1 \frac{1}{\lambda} (y^{n_\alpha - 1} (1 - y)^{n_\beta}) dy = 1 \tag{4.23} \]

\[ \text{Prob}(b_{\text{max}, \alpha} + b_{\text{max}, \beta} \leq b) = \left( \frac{\Gamma(n_\alpha + 1) \Gamma(n_\beta)}{\Gamma(n_\alpha + n_\beta + 1)} \right) b^{n_\alpha + n_\beta} \tag{4.24} \]

\[ G_\Sigma(b) = \left( \frac{\Gamma(n_\alpha + 1) \Gamma(n_\beta + 1)}{\Gamma(n_\alpha + n_\beta + 1)} \right) b^{n_\alpha + n_\beta}. \tag{4.25} \]

Now, \( G_\Sigma(b) \) is a proper distribution if \( b \leq \left( \frac{\Gamma(n_\alpha + n_\beta + 1)}{\Gamma(n_\alpha + 1) \Gamma(n_\beta + 1)} \right)^{-1} \). So the upper limit of the distribution of the sum of the maximum bids for the single objects, depends on the number of bidders competing for each single object. For example, if \( n_\alpha = n_\beta = 3, \left( \frac{\Gamma(n_\alpha + n_\beta + 1)}{\Gamma(n_\alpha + 1) \Gamma(n_\beta + 1)} \right)^{-1} = 1.6475 \), so if \( b \geq 1.6475, G_\Sigma(b) = 1. \) Only when both \( n_\alpha \) and \( n_\beta \) are large, the upper limit approaches to 2.

### 4.3.1 Results for the bidding strategies

A summary with the results for the optimal bidding strategies, when the bidder bids for one object (\( \{\alpha\} \)), the package (\( \{\alpha, \beta\} \)), and the package and one object (\( \{\alpha, \beta\} \) and \( \{\alpha\} \)), is shown in Table 4.1 (these bidding strategies are the solutions to the corresponding problems in Section 4.2). Recall from Section 4.2, that the optimal bid for set \( S \), \( b^*_S \), when the best competitive bid for the package can be the sum of the maximum bids for the individual objects, or the maximum bid for the package with some probability (unknown case), is bounded by \( b^*_{\text{sub}} \) and \( b^*_{\text{sup}} \), which correspond to the cases when the best competitive bid for the package is the sum of the maximum bids for the objects (subadditive case), and the maximum bid for the package (superadditive case) respectively.

According to the assumed distributions, the best competitive bid for the bundle
is superadditive depending on the number of competitors bidding for the individual objects, the package and the value of \( k \). The probability that the best competitive bid for the bundle is superadditive, \( \text{Prob}(b_{\text{max},\alpha} + b_{\text{max},\beta} \leq b_{\text{max},\alpha\beta}) \), is given by:

\[
p = \int_{0}^{2k} \text{Prob}(b_{\text{max},\alpha} + b_{\text{max},\beta} \leq x) \text{Prob}(b_{\text{max},\alpha\beta} = x) dx, \tag{4.26}
\]

\[
= \int_{0}^{\bar{b}} G_{\Sigma}(x) \times F_{\alpha\beta}'(x) dx + \int_{\bar{b}}^{2k} 1 \times F_{\alpha\beta}'(x) dx, \tag{4.27}
\]

\[
= \int_{0}^{\bar{b}} \left( \frac{\Gamma(n_\alpha + 1)\Gamma(n_\beta + 1)}{\Gamma(n_\alpha + n_\beta + 1)} \right) x^{n_\alpha + n_\beta} \times \left( \frac{n_\alpha^1 x^{n_\alpha - 1}}{(2k)^{n_\alpha\beta}} \right) dx + \int_{\bar{b}}^{2k} \left( \frac{n_\alpha^1 x^{n_\alpha\beta - 1}}{(2k)^{n_\alpha\beta}} \right) dx, \tag{4.28}
\]

\[
= \gamma \times n_\alpha^1 \times \bar{b}^{n_\alpha + n_\beta + n_\alpha\beta} + \left( 1 - \left( \frac{\bar{b}}{2k} \right)^{n_\alpha^1} \right). \tag{4.29}
\]

where \( \gamma = \frac{\Gamma(n_\alpha + 1)\Gamma(n_\beta + 1)}{\Gamma(n_\alpha + n_\beta + 1)}, \) and \( \bar{b} = \left( \frac{\Gamma(n_\alpha + n_\beta + 1)}{\Gamma(n_\alpha + n_\beta + 1)} \right) \frac{1}{n_\alpha + n_\beta}. \)

As an illustration, Figure 4.4 shows the value of \( p \) for different values of \( n_\alpha\beta \), when the number of competitors for the single objects are \( n_\alpha = n_\beta = 3 \) and \( k \) equals 1.2 and 2.

Next, Figure 4.5 shows the optimal bidding strategies when \( n_\alpha = n_\beta = 3, \) \( n_\alpha\beta = 2, k = 1.2, \) and as a consequence \( p = 0.55. \)

Figures 4.5(a) and 4.5(b) show the optimal bids for the package and the object,
Figure 4.4. Probability that the best competitive bid for the package be the maximum bid for the package

\( b^*_{\alpha\beta} \) and \( b^*_\alpha \), and the bounds given by the optimal bids when the best competitive bid is superadditive, \( b^*_{\text{super}} \) and \( b^*_{\alpha\beta\text{super}} \), and when the best competitive bid is subadditive, \( b^*_{\text{sub}} \) and \( b^*_{\alpha\beta\text{sub}} \).

In Figure 4.5(b), the optimal bid when the best competitive bid is subadditive, \( b^*_{\alpha\beta\text{sub}} \), is flat when the value for the package is greater than 2. The reason is that \( G_\Sigma(b) \) takes a value of 1 once the bid is greater than 1.65. Therefore, the maximum bid a bidder would submit in this case is 1.65, which is the optimal bid when the value for the package is greater than 2. If the number of competitors increases, the bid for which \( G_\Sigma(b) \) is equal to 1 also increases, approaching 2 in the limit (note that the sum of the maximum bids for the individual objects cannot be greater than 2).

When the bidder bids for the package and the single object, the optimal bids need to be calculated simultaneously. Figures 4.5(c) and 4.5(d) show \( b^*_{\alpha\beta} \) and \( b^*_\alpha \), given \( v_{\alpha\beta} \) and \( v_\alpha \). When \( v_{\alpha\beta} \geq 2 \), the optimal strategy is to bid only for the package (\( b^*_{\alpha\beta} \approx 1.65 \)), therefore Figure 4.5(c) only shows the optimal bids for \( \{\alpha\} \) when \( v_{\alpha\beta} < 2 \).

4.3.2 Bidding for overlapping sets

From the analysis in Section 4.2, we know that a bidder should avoid bidding for overlapping sets, if his bids can be greater than the corresponding best competitive
bids. If that is not the case, then the question is how much he can gain by bidding for overlapping sets.

Continuing with the previous numerical example \( n_\alpha = n_\beta = 3, n_{\alpha\beta} = 2, k = 1.2, p = 0.55 \), Figure 4.6 compares the expected profit evaluated at the optimal bids: (1) when the bidder bids only for the package (— line), and (2) when the bidder bids for the package and one object (—– line). The distance between the two planes represents the additional expected gains by bidding for the individual object.

**Figure 4.5.** Optimal bids when \( n_\alpha = n_\beta = 3, n_{\alpha\beta} = 2, k = 1.2 \)
From Figure 4.6 we can see that as the value for the package approaches 2, the additional expected gains by bidding for the individual object disappear. The reason is that in this particular setting, once the value for the package is greater than 2, the sum of the maximum individual bids is no longer competitive, since its upper bound is 1.65 (recall that in this case, the optimal bid for the package for values greater than 2 is 1.65). Therefore, there is a very low probability that his bid for the individual object be a winning bid, which in turn results in small additional expected gains by bidding for the individual object. Note that in this particular setting, a bidder has a low risk of competing against himself if he bids for overlapping sets. To see this, suppose the value for the package is greater than 2, and the value for \{\alpha\} is 1. In this case the bid for the package is 1.65 and the bid for \{\alpha\} is 0.65, therefore, only if the maximum bid for \{\beta\} is 1, the bidder will risk losing the package because of his bid for \{\alpha\}.

4.4 Competing against oneself in the lab

According to the previous analysis bidders should avoid bidding for overlapping sets, when their bids for the sets can be greater than the corresponding best competitive bids. To investigate this prediction further, we run controlled laboratory
experiments, in which human subjects participated in a combinatorial auction of two objects (A and B).

In every round subjects faced computerized competitors, and were asked to choose their bids for the package and for the object from a list of bids. The values for the object and the package were the same for each treatment. Given the complexity of combinatorial auctions, we simplified the decision task faced by the subjects by using two objects, providing them with a list of bids, and maintained the values constant during the experiments. The values, bid choices and the bid distributions from the computerized competitors were intentionally chosen so subject’s bids could be greater than the corresponding best competitive bids, placing them in a setting in which the optimal bidding strategy is to bid only for the package.

The description of the experiment protocol, the treatments and the specific parameters used in the experiments are provided below.

Participants were mostly undergraduate students from a variety of majors. They were recruited using a computerized system, offering cash as the only incentive for participation in the sessions. Participants arrived at the computer lab at a pre-specified time and date and were asked to sit in a computer terminal and read the instructions by themselves. Before a session started, a recording of the instructions was played to ensure all participants understood the rules of the game and questions were answered. The experiment software used in all sessions was z-Tree (Fischbacher 2007). At the beginning of each auction, subjects submitted their bids, which were compared to the randomly generated bids from the computerized competitors, and a screen with the winning bid(s) and the corresponding profits was shown. After 40 rounds were completed, earnings from all the auctions were converted to US dollars at a pre-announced conversion rate and paid to participants in private and in cash. All sessions were conducted in the Laboratory for Economics Management and Auctions (LEMA), at the Smeal College of Business in Penn State University in 2011. Average earnings, including a $5 dollar participation fee, were $20 per participant. The total number of participants was 37 (17 in one treatment and 20 in the other one). Sessions lasted 45 minutes on average. The instructions, which contain snapshots of the bidding screen seen by the subjects can be found in Section B.4 of the Appendix.
The distributions of the bids from the computerized competitor for each object, the package and the best competitive bid for the package, are shown in equations (4.30), (4.31) and (4.32).

\[
\text{Prob}(b_{\text{comp},A} = x) = \begin{cases} 
0.8 & \text{if } x = 70, \\
0.2 & \text{if } x = 90.
\end{cases} \quad \text{Prob}(b_{\text{comp},B} = x) = \begin{cases} 
0.2 & \text{if } x = 70, \\
0.8 & \text{if } x = 90.
\end{cases}
\]

(4.30)

\[
\text{Prob}(b_{\text{comp},AB} = x) = \begin{cases} 
0.5 & \text{if } x = 115, \\
0.5 & \text{if } x = 230.
\end{cases}
\]

(4.31)

\[
\text{Prob}(b_{\text{best},AB} = x) = \begin{cases} 
0.08 & \text{if } x = 140, \\
0.34 & \text{if } x = 160, \\
0.08 & \text{if } x = 180, \\
0.50 & \text{if } x = 230.
\end{cases}
\]

(4.32)

The value for the object was set to 110 and the value of the package was set to 260, which remained constant during the 40 rounds. Table 4.2 shows the expected profit (assuming risk neutrality) of the possible combinations of reasonable integer bids for the object and the package, given the experimental parameters. According to those results the optimal bidding strategy is to bid zero for the object, and 161 for the package, thus never competing against oneself. If instead we assume risk averse bidders, using a CRRA utility function \(u(x) = x^{1-\alpha}\), the optimal bidding strategy depends on the coefficient of risk aversion, \(\alpha\): if \(\alpha < 0.23\) the optimal bidding strategy is to bid 161 for the package and zero for the object, if \(0.23 \leq \alpha \leq 0.30\) the optimal strategy is to bid 181 for the package and 0 for the object, and if \(\alpha > 0.30\) the optimal strategy is to bid 231 for the package. Note that under risk aversion, like under risk neutrality, a bidder should never compete against himself (please Tables B.1, B.2 and B.3 in Appendix B.2).
Table 4.2. Expected (risk neutral) profit for bidding strategies in the experiments

<table>
<thead>
<tr>
<th>bid for A</th>
<th>0</th>
<th>141</th>
<th>161</th>
<th>181</th>
<th>231</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>9.52</td>
<td>39.5</td>
<td>29</td>
<td></td>
</tr>
<tr>
<td>71</td>
<td>19.5</td>
<td>23.5</td>
<td>37.5</td>
<td>39.5</td>
<td>29</td>
</tr>
<tr>
<td>91</td>
<td>1.9</td>
<td>9.5</td>
<td>13.5</td>
<td>27.5</td>
<td>29</td>
</tr>
</tbody>
</table>

The treatments are defined as follows.

- **Treatment 71 (T-71):** The value for the object is 110, and the value for the package is 260. The bid choices for the object are \( \{0, 71\} \), and the bid choices for the package are \( \{0, 141, 161, 181, 261\} \). In this treatment, a subject competes against himself if his bid for the object is 71, and his bid for the package is 141 or 161. A total of 17 subjects participated in this treatment.

- **Treatment 91 (T-91):** The value for the object is 110, and the value for the package is 260. The bid choices for the object are \( \{0, 91\} \), and the bid choices for the package are \( \{0, 141, 161, 181, 261\} \). In this treatment, a subject competes against himself if his bid for the object is 91, and his bid for the package is 161 or 181. A total of 20 subjects participated in this treatment.

  Note that in T-91 there is a clear difference in the expected profits between bidding only for the package, and bidding also for the object; whereas in T-71 that difference is smaller. Therefore, the treatments were meant to check the robustness of the results with respect to different parameters.

  In theory, there should not be a difference between T-71 and T-91, and subjects in the experiments should bid 161 for the package and 0 for the object in both cases.

**Hypothesis 1:** (i) The average profit of subjects in T-71 should be equal to 43.68, and (ii) the average profit in T-91 should be equal to 43.19, which correspond to the average profits of bidding according to the optimal strategy.\(^1\)

\(^1\)The optimal average profits were calculated using the actual realizations of the computerized competitors for each subject. Even though they are not equal for both treatments, the difference is not statistically significant.
If however, subjects do not bid according to the theoretical prediction, and instead bid for the object and the package, according to the results from Table 4.2, the average profit in T-71 sessions should be greater than the average profit in T-91 sessions.

**Hypothesis 2:** The average profit in T-71 should be greater than the average profit in T-91.

Table 4.3 shows the results for the average profits (with the corresponding standard deviation in parenthesis) observed in the experiments. We can see that the observed average profits are below the theoretical prediction in both treatments (the $p$-values are less than 0.01 in both cases\(^2\)), and therefore we reject both parts of Hypothesis 1, concluding subjects in the experiments bid more aggressively than they should; which is a well-known effect in sealed-bid auctions run in controlled laboratory experiments (Kagel & Roth 1995).

Comparing the average profit between treatments, I find that in line with Hypothesis 2, the observed average profits are lower in T-91 than in T-71 ($p$-value<0.01).

<table>
<thead>
<tr>
<th></th>
<th>T-71</th>
<th>T-91</th>
</tr>
</thead>
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<tr>
<td>Theoretical prediction</td>
<td>43.68</td>
<td>43.19</td>
</tr>
<tr>
<td>Average profit</td>
<td>34.22</td>
<td>29.54</td>
</tr>
<tr>
<td>(Standard deviation)</td>
<td>(4.86)</td>
<td>(4.58)</td>
</tr>
</tbody>
</table>

Table 4.3. Observed average profits in the experiments

Figures 4.7 and 4.8 show the cumulative distributions of the observed and optimal average profit, when subjects bid for the object and the package, and when they bid only for the package. The difference in the distribution of observed and optimal average profits is not significant only when subjects bid only for the package in T-71, which corresponds to Figure 4.7(b) (the $p$-value of the corresponding Kolmogorov-Smirnov test is 0.163). When subjects bid for the object and the package in T-71 (Figure 4.7(a)), and when they bid for both and only for the package in T-91, the average profit of each subject as the unit of analysis.

\(^2\)The mean tests used in the analysis correspond to two-tails, paired, independent t-tests using the average profit of each subject as the unit of analysis.
package in T-91 (Figures 4.8(a) and 4.8(b)), the difference between the observed and optimal average profits is significant (all $p$-values are less than 0.01).

Figure 4.7. Cumulative distributions of profits (T-71)
To understand the causes of the previous differences between the observed and the optimal average profits, Figure 4.9 shows the percentage of rounds in which a particular bidding strategy was chosen in each of the treatments overall, in the first 20 rounds and in the last 20 rounds. Overall, subjects bid for the object and the package in 47% of the rounds in T-71 and in 42% in T-91. When subjects bid for the object and the package, the best strategies in that scenario are \{71, 181\}.

Figure 4.8. Cumulative distributions of profits (T-91)
and \{71, 161\} under T-71, and, \{91, 181\} and \{91, 231\} under T-91. We can see in Figure 4.9, that those strategies were played in a higher percentage of rounds than the other strategies in which a bid for the object was submitted. Similarly, when subjects bid only for the package the preferred strategies are \{0, 161\} and \{0, 181\} which yield the highest expected profits. These results suggests that once subjects decided to bid for the object and the package, or only for the package, they tended to play the profit maximizing strategies in the corresponding scenario.

![Percentage of rounds for each bidding strategy (T-71)](image1)

(a) T-71

![Percentage of rounds for each bidding strategy (T-91)](image2)

(b) T-91

**Figure 4.9.** Percentage of rounds each bidding strategy was chosen

Next, Figure 4.10 shows the cumulative distribution of the average number of rounds each subject bid for the object and the package. The difference between
the two distributions is not significant ($p$-value=0.864), therefore treatments do not seem to have an impact in the number of rounds subjects bid for the object and the package. This observation is somewhat surprising because in T-91 the incentives -in terms of expected profit- to not bid for the object are greater than in T-71. However, when we look at the percentage of subjects that bid for the object in each round in Figure 4.11, we see a non-increasing trend for both treatments.

Figure 4.10. Cumulative distribution of the average number of rounds subjects bid for the object and the package

Figure 4.11. Percentage of subjects that bid for the object in each round
Therefore, to look for evidence of learning in each treatment, a logistic regression model with random effects was estimated, using the round number as the independent variable, and as a dependent variable, a binary variable that takes a value of 1 if the subject bids for the object in a particular round, and 0 otherwise. The goal here is to determine if the probability subjects bid for the object decreased as rounds progressed, which would provide evidence of learning. The round coefficient in the logit regression of both treatments is negative and significant ($\beta_1 = -0.041$ for T-71 and $\beta_1 = -0.052$ for T-91, both $p$-values $\leq 0.01$). However, the results of estimating a logistic regression for each subject separately, show that the round coefficient is negative and significant ($p$-value $\leq 0.05$) for only 3 subjects in T-71, and for 4 subjects in T-91 (please see Tables B.4 and B.5 in Appendix B.3). When these subjects are removed, the round coefficient in the general logit model is not significant anymore ($p$-value $> 0.13$ for both treatments). Therefore, on average, the evidence of learning is not strong.

The previous analysis suggests that subjects decided how to bid in two (separate) steps. In the first step they decided whether to bid or not for the object, and in the second step, they selected the bidding strategy given their decision in the first step. What subjects bidding for the object failed to realize, specially in T-91, was that bidding only for the package yielded a higher expected profit than bidding for both the object and the package.

4.5 Conclusions

In a combinatorial auction of two objects, in which bids are mutually exclusive, I find that the optimal bidding strategy for any set is bounded by the optimal bidding strategy in two scenarios: (1) when the best competitive bid for the package is the competitors’ maximum bid for the package (superadditive case), and (2) when the best competitive bid for the package is the sum of the competitors’ maximum bids for each object (subadditive case).

Bidders in these auctions should avoid bidding for overlapping sets, if his bids for each set can be greater than the corresponding best competitive bids simultaneously, in which case they should bid only for the set with the highest expected profit. Results from controlled laboratory experiments of a combinatorial auction
of two objects, in which subjects could bid for one of the objects and/or the package that contain the two objects, and the optimal strategy was to bid only for the package, show that on average subjects failed to realize that the optimal strategy was to bid only for the package, and instead bid for the object and the package. Consequently, the realized profits were significantly lower than the optimal profits on average.
Appendix A

Appendix for Chapter 1

A.1 Simulation results

<table>
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<tr>
<th>$n_e$</th>
<th>$k = 1$</th>
<th>$k = 2$</th>
<th>$k = 3$</th>
<th>$k = 4$</th>
<th>$k = 5$</th>
<th>$k = 6$</th>
<th>$k = 7$</th>
<th>$k = 8$</th>
<th>$k = 9$</th>
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<td>(0.033)</td>
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<td>(0.052)</td>
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<tr>
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<td>(0.026)</td>
<td>(0.034)</td>
<td>(0.045)</td>
<td>(0.053)</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$n_e = 5$</td>
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<td>0.363</td>
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Table A.1. Results for the average cost (and its variance) when $c_i \sim U[0, 1]$, $c_e \sim U[0, 0.9]$ and $\varepsilon \sim U[0, 0.1]$. 
Table A.2. $p$-values for the two sided $z$-test between the average cost in the buyer determined auction and the scoring auction, when $c_i \sim U[0,1]$, $c_e \sim U[0,0.9]$ and $\epsilon \sim U[0,0.1]$.

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Table A.3. Results for the average cost (and its variance) when $c_i \sim \text{Beta}[4,4]$, $c_e \sim \text{Beta}[4,5]$ and $\epsilon \sim U[0,0.1]$.

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Table A.4. $p$-values for the two sided $z$-test between the average cost in the buyer determined auction and the scoring auction, when $c_i \sim \text{Beta}[4,4]$, $c_e \sim \text{Beta}[4,5]$ and $\epsilon \sim U[0,0.1]$.
Table A.5. Results for the average cost (and its variance) when $c_i \sim U[0,1]$, $c_e \sim U[0,0.8]$ and $\varepsilon \sim U[0,0.2]$.

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Table A.6. p-values for the two sided z-test between the average cost in the buyer determined auction and the scoring auction, when $c_i \sim U[0,1]$, $c_e \sim U[0,0.8]$ and $\varepsilon \sim U[0,0.2]$.

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<tr>
<td></td>
<td>(0.006)</td>
<td>(0.005)</td>
<td>(0.005)</td>
<td></td>
<td>(0.007)</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>10</td>
<td>0.341</td>
<td>0.341</td>
<td>0.347</td>
<td>0.354</td>
<td>0.362</td>
<td>0.368</td>
<td>0.373</td>
<td>0.377</td>
<td>0.379</td>
<td>0.379</td>
<td>0.328</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.005)</td>
<td>(0.005)</td>
<td></td>
<td>(0.006)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table A.7. Results for the average cost (and its variance) when $c_i \sim \text{Beta}[4,4]$, $c_e \sim \text{Beta}[4,6]$ and $\varepsilon \sim U[0,0.2]$.
<table>
<thead>
<tr>
<th>$n_e$</th>
<th>$k = 1$</th>
<th>$k = 2$</th>
<th>$k = 3$</th>
<th>$k = 4$</th>
<th>$k = 5$</th>
<th>$k = 6$</th>
<th>$k = 7$</th>
<th>$k = 8$</th>
<th>$k = 9$</th>
<th>$k = 10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td></td>
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</tr>
<tr>
<td>8</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
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<tr>
<td>10</td>
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<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Table A.8. $p$-values for the two sided $z$-test between the average cost in the buyer determined auction and the scoring auction, when $c_i \sim \text{Beta}[4,4]$, $c_e \sim \text{Beta}[4,6]$ and $\varepsilon \sim U[0,0.2]$. 
Appendix B

Appendix for Chapter 3

B.1 Event trees

Figure B.1 shows the root branches of the event tree, and Figures B.2, B.3, B.4 and B.5 show the subtrees of part I, part II, part III and part IV correspondingly.

Figure B.1. Event tree when the bidder bids for all
Figure B.2. Event tree when the bidder bids for all - part I

Figure B.3. Event tree when the bidder bids for all - part II
Figure B.4. Event tree when the bidder bids for all - part III

Figure B.5. Event tree when the bidder bids for all - part IV
B.2 CRRA expected profit calculations

<table>
<thead>
<tr>
<th>bid for A</th>
<th>bid for the package</th>
<th>0</th>
<th>141</th>
<th>161</th>
<th>181</th>
<th>231</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>9.2</td>
<td>41.7</td>
<td>41.4</td>
<td>37.1</td>
<td></td>
</tr>
<tr>
<td>71</td>
<td>23.5</td>
<td>26.9</td>
<td>39.2</td>
<td>41.4</td>
<td>37.1</td>
<td></td>
</tr>
<tr>
<td>91</td>
<td>2.6</td>
<td>13.2</td>
<td>16.9</td>
<td>30.1</td>
<td>37.1</td>
<td></td>
</tr>
</tbody>
</table>

Table B.1. Expected CRRA utility function for bidding strategies in the experiments, $\alpha = 0.20$

<table>
<thead>
<tr>
<th>bid for A</th>
<th>bid for the package</th>
<th>0</th>
<th>141</th>
<th>161</th>
<th>181</th>
<th>231</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>9.04</td>
<td>41.7</td>
<td>42.4</td>
<td>42.0</td>
<td></td>
</tr>
<tr>
<td>71</td>
<td>25.9</td>
<td>28.9</td>
<td>40.14</td>
<td>42.4</td>
<td>42.0</td>
<td></td>
</tr>
<tr>
<td>91</td>
<td>3.1</td>
<td>15.6</td>
<td>19.0</td>
<td>31.7</td>
<td>42.0</td>
<td></td>
</tr>
</tbody>
</table>

Table B.2. Expected CRRA utility function for bidding strategies in the experiments, $\alpha = 0.30$

<table>
<thead>
<tr>
<th>bid for A</th>
<th>bid for the package</th>
<th>0</th>
<th>141</th>
<th>161</th>
<th>181</th>
<th>231</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0</td>
<td>8.73</td>
<td>41.8</td>
<td>44.4</td>
<td>53.8</td>
<td></td>
</tr>
<tr>
<td>71</td>
<td>31.2</td>
<td>33.6</td>
<td>42.3</td>
<td>44.4</td>
<td>53.8</td>
<td></td>
</tr>
<tr>
<td>91</td>
<td>4.36</td>
<td>21.8</td>
<td>24.6</td>
<td>35.4</td>
<td>53.8</td>
<td></td>
</tr>
</tbody>
</table>

Table B.3. Expected CRRA utility function for bidding strategies in the experiments, $\alpha = 0.50$

B.3 Individual logistic regression results
<table>
<thead>
<tr>
<th>Subject number</th>
<th>Period coefficient (logit regression)</th>
<th>p-value of coefficient</th>
<th>Number of rounds subject bid for the object</th>
<th>Number of rounds subject competed against himself</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>NA</td>
<td>NA</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>-0.17</td>
<td>0.17</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>-0.14</td>
<td>0.04</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>0.05</td>
<td>0.17</td>
<td>33</td>
<td>16</td>
</tr>
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<td>0.70</td>
<td>39</td>
<td>4</td>
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<td>10</td>
<td>0.06</td>
<td>0.20</td>
<td>35</td>
<td>10</td>
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<tr>
<td>11</td>
<td>-0.43</td>
<td>0.01</td>
<td>23</td>
<td>4</td>
</tr>
<tr>
<td>12</td>
<td>NA</td>
<td>NA</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
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<td>0.17</td>
<td>0.17</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>15</td>
<td>-0.20</td>
<td>0.03</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>16</td>
<td>0.02</td>
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<tr>
<td>17</td>
<td>0.00</td>
<td>0.91</td>
<td>24</td>
<td>10</td>
</tr>
</tbody>
</table>

**Table B.4.** Results for T-71

<table>
<thead>
<tr>
<th>Subject number</th>
<th>Period coefficient (logit regression)</th>
<th>p-value of coefficient</th>
<th>Number of rounds subject bid for the object</th>
<th>Number of rounds subject competed against himself</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.08</td>
<td>0.02</td>
<td>23</td>
<td>18</td>
</tr>
<tr>
<td>2</td>
<td>NA</td>
<td>NA</td>
<td>40</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>-0.08</td>
<td>0.02</td>
<td>17</td>
<td>16</td>
</tr>
<tr>
<td>4</td>
<td>0.45</td>
<td>0.32</td>
<td>39</td>
<td>13</td>
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</tr>
<tr>
<td>9</td>
<td>NA</td>
<td>NA</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>0.00</td>
<td>0.99</td>
<td>27</td>
<td>24</td>
</tr>
<tr>
<td>11</td>
<td>-0.04</td>
<td>0.12</td>
<td>22</td>
<td>18</td>
</tr>
<tr>
<td>12</td>
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<td>0.09</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>13</td>
<td>NA</td>
<td>NA</td>
<td>39</td>
<td>19</td>
</tr>
<tr>
<td>14</td>
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<td>0.05</td>
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<td>3</td>
</tr>
<tr>
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</tr>
<tr>
<td>16</td>
<td>0.00</td>
<td>0.94</td>
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<td>22</td>
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<td>17</td>
<td>-0.16</td>
<td>0.34</td>
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<td>0</td>
</tr>
<tr>
<td>18</td>
<td>NA</td>
<td>NA</td>
<td>40</td>
<td>17</td>
</tr>
<tr>
<td>19</td>
<td>0.00</td>
<td>0.96</td>
<td>7</td>
<td>68</td>
</tr>
<tr>
<td>20</td>
<td>-0.14</td>
<td>0.07</td>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>

**Table B.5.** Results for T-91
B.4 Instructions for experiments
Instructions

The purpose of this session is to study how people make particular decisions. From now until the end of the session, unauthorized communication of any nature between participants is prohibited. During the session you will play a series of games that give you an opportunity to make money, which will be paid to you in cash at the end of the session. In your desk you should have a check-out form and a copy of the consent form. If you have a question at any time, please raise your hand.

How to make money

You are a bidder in an auction of two objects A and B. You can bid for A and for the package that contains A and B. Bids are mutually exclusive, therefore only one of your bids can win. Your value for the package is 260 and your value for A is 110.

In each auction (round), you face three computerized competitors: one competitor bids for A, another one for B and another one for the package. Your bids for A and for the package compete with the competitor’s bid for A, the competitor’s bid for B and the competitor’s bid for the package.

To win A, all of the following statements have to hold simultaneously:
- Your bid for A has to be greater than the competitor’s bid for A.
- The sum of your bid for A plus the competitor’s bid for B, has to be greater than your bid for the package.
- The sum of your bid for A plus the competitor’s bid for B, has to be greater than the competitor’s bid for the package.

To win the package, all of the following statements have to hold simultaneously:
- Your bid for the package has to be greater than the competitor’s bid for the package.
- Your bid for the package has to be greater than the sum of your bid for A plus the competitor’s bid for B.
- Your bid for the package has to be greater than the sum of the competitor’s bid for A plus the competitor’s bid for B.

Ties are broken randomly. If your bids are not winning bids, the winning bids correspond to the maximum between the sum of the competitor’s bid for A plus the competitor’s bid for B, and, the competitor’s bid for the package.

If you win in a round, your Profit is your Value for the set you won (A or the package), minus your Bid for the set you won (A or the package). If you do not win in a round your profit is zero.

In each round, the bids from the computerized competitors will be randomly generated according to the distributions shown in the following tables:
For your convenience, the following table shows the distribution of the best competitive bid for the package coming from your competitors, which can be: the sum of the competitor’s bid for A plus the competitor’s bid for B, or, the competitor’s bid for the package:

<table>
<thead>
<tr>
<th>Competitor’s bid for A</th>
<th>%</th>
<th>Competitor’s bid for B</th>
<th>%</th>
<th>Competitor’s bid for the package</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>80%</td>
<td>70</td>
<td>20%</td>
<td>115</td>
<td>50%</td>
</tr>
<tr>
<td>90</td>
<td>20%</td>
<td>90</td>
<td>80%</td>
<td>230</td>
<td>50%</td>
</tr>
</tbody>
</table>

You will play in 40 rounds. Your value for A and the package will be the same for all rounds (110 and 260 correspondingly). In each round you choose your bid for A (which can be 0 or 71) and for the package (which can be 0, 141, 161, 181 or 231), and the computer will generate the competitor’s bids according to the previous distributions. Remember that you can bid for A and for the package, but only one of your bids can win.

The mechanics of placing a bid

In each round, you will be given the bid choices for A and the package. To place your bids, you select them and then click the “Submit Bids” button:

After you submit your bids, they are compared to the competitor’s bids to determine the winning bids, and the results of the round are displayed. **Once you submit your bids you won’t be able to change them.** In the screen with the results you will see your values, your bids, the competitor’s bids and the winning bids. The winning bids will be displayed using bold numbers. In the figure below, the winning bids correspond to the competitor’s bid for A and the competitor’s bid for B (note that their sum is greater than the competitor’s bid for the package).
How you will be paid
At the end of the session the computer will calculate the total profit you earned in all rounds, and will convert it to US dollars, at an exchange rate of 170 ECU (experimental currency unit) for $1 US dollar. Your dollar earnings will be added to your $5 dollar participation fee and your total earnings for the session will be displayed in your computer screen. Please use this information to complete the check-out form.
Bibliography


Vita
Natalia Santamaría Tobar

Education

**Penn State University** Pennsylvania, USA 2007–Present
Area of Specialization: Supply Chain Management.

**Rutgers University** New Jersey, USA 2005–2007
M.Sc in Operations Research.

**Universidad de Los Andes** Bogotá, Colombia 1996–2003
B.S./M.Sc in Industrial Engineering.

Awards and Honors
Summer Dissertation Award, Smeal College of Business, Penn State (USA), 2010
Fulbright Scholar (Colombia) 2005-2006

Research Experience

**Doctoral Research** Penn State University 2008–Present
Thesis Advisor: Elena Katok
The research involved modeling bidding strategies in reverse and combinatorial auctions.

**Graduate Research** Penn State University 2007–2008
Research Advisor: Michael H. Rothkopf (Dec.)
The research involved modeling bidding strategies in combinatorial auctions.

Teaching Experience

**Teaching Assistant** Penn State University 2009-2010
Courses: Supply Chain Modeling and Demand Fulfillment, offered by the Supply Chain and Information Systems department at the Smeal College of Business.

**Instructor** Universidad de Los Andes, Bogotá, Colombia 2003–2005
Courses: Linear Programming and Probability Models, offered by the Industrial Engineering department.