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ACI 318 CODE PROVISIONS FOR DEFLECTION CONTROL OF TWO-WAY CONCRETE SLABS

A Thesis in

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by

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Abstract

The objective of the study conducted was to identify and address concerns with ACI 318 section 9.5, the deflection specifications for two-way slabs. Its scope includes proposing the inclusion of two new equations: to compute effective moment of inertia by Bischoff (2005) and Bischoff and Scanlon (2007) and to calculate minimum thickness required for a two-way slab by Scanlon and Choi (1999) respectively.

For the purpose of studying the merits of using a proposed moment of inertia by Bischoff, as against the existing ACI specified equation, an experimental plan was developed to observe the effect of percentage of reinforcement on the moment resisting capacity and deflection of the section. Parametric studies were conducted to study the effect of percentage reinforcement on the effective moment of inertia of the section by proposed and ACI methods. Studies were also carried out to analyze the effect of various factors including live load, sustained load, span length, long-term multiplier, etc on the minimum thickness requirements of a two-way slab, unsupported by drop panels. Effects of inclusion of boundary condition factor in the proposed h_{min} equation were studied and justified using SAP2000 software.

The results of the experiments and parametric studies were analyzed using the MS-EXCEL software and a characteristic behavior pattern was developed over the range of parameters for each study. The behavior pattern thus obtained was then used to rationalize the use of proposed equations.

Finally a detailed analysis is carried out on the studies and experiments and conclusions were drawn. These conclusions are further formatted to satisfy the ACI code language, since they are to be implemented upon approval by the presiding committee, as a modification/addition to the existing deflection control provisions of ACI 318 section 9.5, concerning two-way slabs.

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Δ - deflection

E_c - modulus of elasticity of concrete, 57000 $\sqrt{(f'_c)}$

E_s - modulus of elasticity of steel

f'_c - concrete strength

f_r - modulus of rupture = $7.5\sqrt{f_c'}$

I - moment of inertia (including effects of cracking)

I_{cr} - cracked moment of inertia of the section

I_g - gross moment of inertia of the section

k_Δ - span-to-depth ratio

k_{SS} - factor to account for distribution of total static moment to column and middle strips

k_{DP} - factor to account for increased stiffness in column strips due to the presence of drop panels.

ℓ - span length

ℓ_n - clear span length of the member

m – factor used in Bischoff's equation = 2

M_a- maximum moment on the section due to service loads at the stage where deflection is computed

M_{cr} - moment of cracking

w - total uniformly distributed load

$W_{\ell(ADD)}$ - variable portion of live load

W_{sdl} - superimposed dead load

W_{sus} - sustained load on the member which is sum of the slab self weight, superimposed dead load (W_{sdl}) and percentage of live load (here, 10%) sustained

y_t - depth from the bottom of the section to the neutral axis

ε_{sh} - shrinkage strain

λ - long- term multiplier

ρ - reinforcement percentage

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Chapter 1

INTRODUCTION

1.1 Background:

A serviceability limit defines the performance criterion for serviceability and corresponds to conditions beyond which specified service requirements resulting from the planned use are no longer met. In limit state design, a structure fails its serviceability if the criteria of the serviceability limit state are not met during the specified service life and with the required reliability.

A structure that fails serviceability would have exceeded a defined limit for one of the following properties:

- Deflection
- Vibration
- Local deformation

The exact limit states for different purposes are defined by government or regulatory agencies. For concrete buildings, ACI 318 provides the necessary guidelines and requirements. Concerns have been raised within ACI Committee 318 that current provisions for deflection control of two-way slab systems, flat plates and flat slabs in particular, need to be re-evaluated due to reported cases of excessive slab deflections in structures designed to ACI 318 requirements.

1.2 **Objective and Scope:**

The objective of this study is to evaluate current ACI 318 provisions for deflection control of two-way slab systems and provide recommendations for revisions as needed. Particular attention is paid to the effective moment of inertia for calculation of immediate deflection and the use of minimum thickness provisions for controlling deflection of flat plates and flat slabs. This objective is achieved within the following scope:

1. Review of literature on deflection control of two-way slabs including current code provisions.
2. Review of deflection calculation procedures for immediate and time-dependent deflections.
3. Review of minimum thickness provisions.
4. Development of spread sheets for deflection and minimum thickness computation.
5. Parametric study to identify required changes and limitations to deflection-control provisions of ACI 318-08.
6. Development of recommendations for ACI building code provisions for two-way slab systems.

1.3 **Review of Existing Code Provisions:**

In the structural design of concrete buildings, one of the most important serviceability criteria is deflection control. The introduction for ACI 318-08 code for building construction using structural concrete states, "A building code states only the minimum requirements necessary to provide for public health and safety. The Code is based on this principle." A structure might possess adequate margin of safety against collapse but may have deflections too large to be acceptable. The ACI Code approaches deflection control in a two-tier procedure, by specifying the following:

1. Minimum thickness criteria
2. Limits on calculated deflection

In this section, these approaches are discussed, analyzed and research concerning these issues is reviewed. Minimum thickness requirements for one-way slabs and non-prestressed beams are provided by Table 9.5(a) of ACI 318-08, where as for two-way slabs, they are enlisted under Table 9.5(c).

<u>Table 9.5(a):</u> Minimum thickness of one-way non-prestressed construction

In this Table, span length is defined as clear span plus depth of member but not exceeding the distance between centers of supports. This Table comes with a precondition that it can be used for only those members NOT supporting or attached to partitions or other construction likely to be damaged by large deflections. Those members supporting partitions or likely to be damaged by large deflections are to be assessed based on calculated deflection and the deflection limits defined in Table 9.5(b).

Table 1-1: Minimum thickness of non-prestressed beams or one-way slabs unless deflections are calculated (reproduced with permission from ACI 318-08)

	Minimum thickness, h			
	Simply supported	One end continuous	Both ends continuous	Cantilever
Member	Members not supporting or attached to partitions or other construction likely to be damaged by large deflections			
Solid one-way slabs	$\ell/20$	$\ell/24$	$\ell/28$	$\ell/10$
Beams or ribbed one-way slabs	$\ell/16$	$\ell/18.5$	$\ell/21$	$\ell/8$

Notes:
Values given shall be used directly for members with normalweight concrete and Grade 60 reinforcement. For other conditions, the values shall be modified as follows:
a) For lightweight concrete having equilibrium density, w_c, in the range of 90 to 115 lb/ft^3, the values shall be multiplied by $(1.65 - 0.005w_c)$ but not less than 1.09.
b) For f_y other than 60,000 psi, the values shall be multiplied by $(0.4 + f_y/100{,}000)$.

Table 9.5(b): Permissible computed deflections

This Table (Table 1-2) specifies limits on calculated deflections for members under both immediate application of live load and for members subjected to sustained time-dependent load. Limits are specified for immediate deflection under live load and for time-dependent deflections occurring after installation of non-structural elements. For the scope of this study, no changes are proposed for the current limits.

Table 1-2: Maximum Permissible Computed Deflections (reproduced with permission from ACI 318-08)

Type of member	Deflection to be considered	Deflection limitation
Flat roofs not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to live load L	$\ell/180$*
Floors not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to live load L	$\ell/360$
Roof or floor construction supporting or attached to nonstructural elements likely to be damaged by large deflections	That part of the total deflection occurring after attachment of nonstructural elements (sum of the long-term deflection due to all sustained loads and the immediate deflection due to any additional live load)†	$\ell/480$‡
Roof or floor construction supporting or attached to nonstructural elements not likely to be damaged by large deflections		$\ell/240$§

*Limit not intended to safeguard against ponding. Ponding should be checked by suitable calculations of deflection, including added deflections due to ponded water, and considering long-term effects of all sustained loads, camber, construction tolerances, and reliability of provisions for drainage.
†Long-term deflection shall be determined in accordance with 9.5.2.5 or 9.5.4.3, but may be reduced by amount of deflection calculated to occur before attachment of nonstructural elements. This amount shall be determined on basis of accepted engineering data relating to time-deflection characteristics of members similar to those being considered.
‡Limit may be exceeded if adequate measures are taken to prevent damage to supported or attached elements.
§Limit shall not be greater than tolerance provided for nonstructural elements. Limit may be exceeded if camber is provided so that total deflection minus camber does not exceed limit.

Table 9.5(c): Minimum thickness for two-way non-prestressed construction

This Table specifies the minimum thickness requirements for two way slab construction, along with equations 9-11 and 9-12. As opposed to Table 9.5(a), there is no restriction on applying these rules to members supporting or not supporting partitions. The commentary contains a cautionary note that these limits do not apply to design conditions falling outside the domain of previous of experience. However, no guidance is provided on the range of design conditions covered.

Table 1-3: Minimum thickness of slabs without interior beams §9.5 of ACI 318-08 for two-way construction (reproduced with permission from ACI 318-08)

	Without drop panels‡			With drop panels‡		
	Exterior panels		Interior panels	Exterior panels		Interior panels
f_y, psi†	Without edge beams	With edge beams§		Without edge beams	With edge beams§	
40,000	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$	$\ell_n/36$	$\ell_n/40$	$\ell_n/40$
60,000	$\ell_n/30$	$\ell_n/33$	$\ell_n/33$	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$
75,000	$\ell_n/28$	$\ell_n/31$	$\ell_n/31$	$\ell_n/31$	$\ell_n/34$	$\ell_n/34$

*For two-way construction, ℓ_n is the length of clear span in the long direction, measured face-to-face of supports in slabs without beams and face-to-face of beams or other supports in other cases.

†For f_y between the values given in the table, minimum thickness shall be determined by linear interpolation.

‡Drop panels as defined in 13.2.5.

§Slabs with beams between columns along exterior edges. The value of α_f for the edge beam shall not be less than 0.8.

The following are concerns about the current code provisions, as discussed in Scanlon et al (2001).

1. The expected deflection associated with a thickness based on minimum thickness provisions is unknown.
2. Different levels of conservatism between one way and two way slab provisions.

3. Different definitions of span length for one-way slabs, flat plates, beams and two-way slabs.
4. Minimum thickness values are independent of concrete strength.
5. Minimum thickness values are also independent of design loads.
6. Effect of shrinkage restraint stress on cracking is not explicitly considered.
7. Effect of construction loads are not accounted for.
8. The range of span lengths for which the provisions are applicable is not specified.
9. For two-way construction, ACI 318-08 does not provide any restriction on the application of the defined provisions other than a statement in the commentary that "These limits apply to only the domain of previous experience in loads, environment, materials, boundary conditions and spans". However no guidance is provided related to the range of previous experience.

Scanlon and Choi et. al (1999) recommended an alternative approaches to define minimum required slab thicknesses for one-way construction, including effects of design loads, span length, loading and time-dependent deflection. Scanlon and Lee (2007) refined this proposed minimum thickness equation and extended its use to two-way slab systems. Bischoff (2005) and Bischoff and Scanlon (2007) demonstrated that the effective moment of inertia equation adopted by ACI 318 is not valid for members such as slabs with low reinforcement ratios and proposed an alternative formulation that is valid for all reinforcement ranges. Scanlon and Bischoff (2008) also showed that effect of shrinkage restraint and construction loads should also be considered in deflection calculations.

Chapter 2

CODE PROVISIONS FOR DEFLECTION CALCULATIONS

In concrete structures, deflections are divided into two categories-immediate and long-term. ACI 318 provides limits and methods to compute both kinds of deflection. Immediate deflections are those that occur as soon as the load is applied to the structure. They are computed by usual methods or formulae for elastic deflections as provided by structural analysis. They may be used either with a constant value of $E_c * I_g$ (stiffness) along the length of the member for uncracked members or by more exact means of calculation for members cracked at one or more sections or if the member depth varies along the span. Long term deflections include not only the effect of immediate loading but also of shrinkage and creep.

2.1 Immediate Deflections

ACI 318 provides limits for immediate deflections in Table 9.5(b). Live load deflection limits are $\ell/180$ for roof members and $\ell/360$ for floor members. Immediate deflections are also needed for long-term deflections because; time-dependent deflections are normally calculated as a multiple of immediate deflection.

Immediate deflection, Δ can be calculated by the standard beam deflection equation as:

$$\Delta = \frac{k\, w\ell^4}{384EI} \tag{2-1}$$

Where,

k = boundary condition factor

w= uniformly distributed load

ℓ= span length

E= young's modulus of concrete

I = moment of Inertia (including effects of cracking)

2.1.1 **Effective Moment of Inertia**:

Concrete cracks as load applied on it increases. The cracked region of concrete in tension zone becomes unproductive in resisting the moments and forces experienced by the section though the steel reinforcement still is effective. Hence, an effective moment of inertia that accounts for the effect of cracking is used for estimating immediate deflection of a cracked flexural member

The current ACI 318 Code adopts the effective moment of inertia, I_e equation proposed by Branson (1965) :

$$I_e = \langle\frac{M_{cr}}{M_a}\rangle^3 I_g + \left[1 - \langle\frac{M_{cr}}{M_a}\rangle^3\right] I_{cr} \quad (2\text{-}2)$$

Where,

M_{cr} = cracking moment $= \dfrac{f_r * I_g}{y_t}$

f_r = modulus of rupture = $7.5\sqrt{f_c'}$

I_g = gross moment of inertia of the section

y_t = depth from the bottom of the section to the neutral axis

M_a = maximum moment on the section due to service loads at the stage where deflection is computed

I_{cr} = moment of inertia of the cracked section

An alternative equation proposed by Bischoff (2005) and Bischoff and Scanlon (2007) is given by:

$$I_e = \frac{I_{cr}}{\left[1 - \left(\frac{M_{cr}}{M_a}\right)^m \left(1 - \frac{I_{cr}}{I_g}\right)\right]} \quad (2\text{-}3)$$

Where, $m = 2$

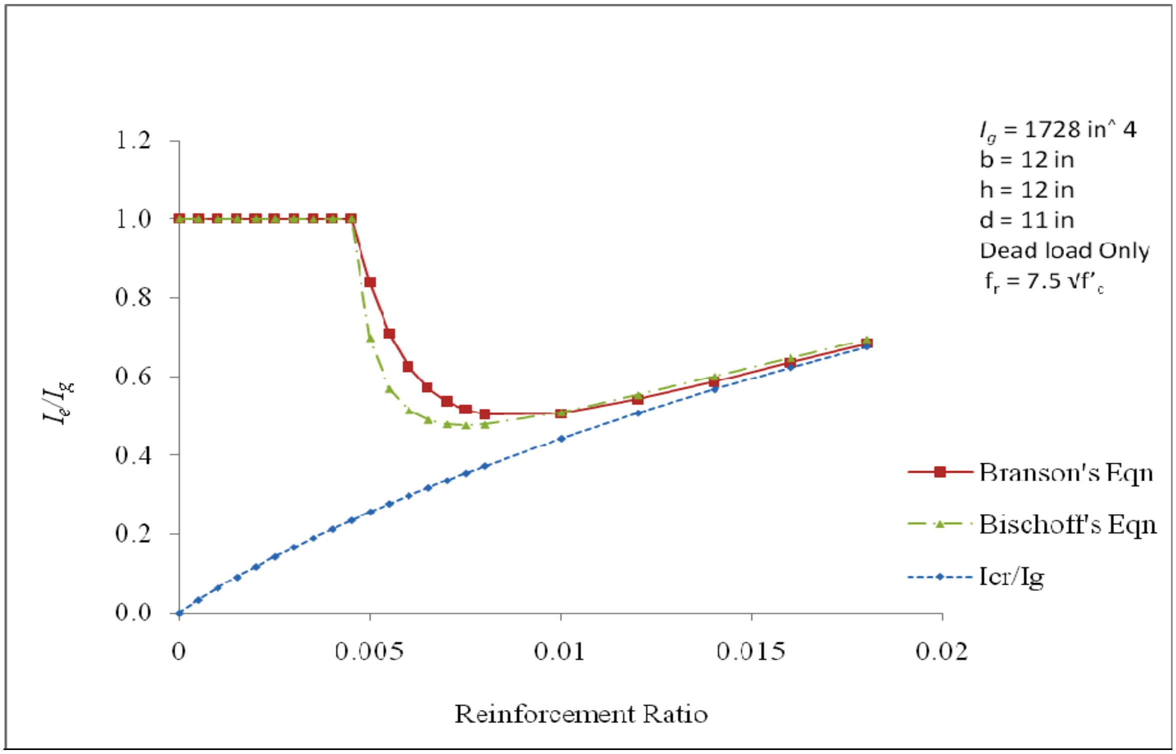

Fig 2-1: Plots for ratio of moment of inertia vs. reinforcement ratio for Branson and Bischoff's equation

Fig 2-1 shows a comparison of the ratio of effective moment of inertia to gross moment of inertia by Branson's and Bischoff's equations as provided in equations 2-1 and 2-2 respectively. They are the values obtained for a rupture modulus value of f_r, for dead load (self weight only). Ratio of M_a/M_u is 0.38. , where,

Ultimate Moment, M_u = *1.2 Dead Load Moment + 1.6 Live Load Moment* (2-4)

Actual Moment, M_a = *1.0 Dead Load Moment* (2-5)

The ratio I_{cr} / I_g is also shown.

Beyond ρ= 0.01 both curves converge with the I_{cr}/I_g plot, verifying the validity of both equations beyond this range. In the range ρ = (0.2%, 0.8%). Bischoff's equation produces lower MI ratios in the above range of reinforcement percentage (ρ). This would lead to increased deflections predicted for an applied load. Scanlon and Bischoff (2008) demonstrated that equation (2-3) is valid for all reinforcement ratios while Branson's equation provides poor correlation for members with low reinforcement such as typical slab systems.

2.1.2 Shrinkage and Restraint Stress:

A known contributor to the cracking of a concrete slab is shrinkage restraint stress. These stresses decrease the cracking moment M_{cr} of the member under applied loads by reducing the effective tensile strength or modulus of rupture of the concrete. In other words

$$f_{re} = f_r - f_{res} \tag{2-5}$$

Where, the rupture modulus of concrete $f_r = 7.5\sqrt{f_c'}$ (psi) is reduced by the restraint stress f_{res}.

A value of $4\sqrt{f_c'}$ (psi) or approximately half the code specified value was proposed for the reduced effective modulus of rupture by Scanlon and Murray (1982). Good correlation between calculated and reported mean field measured deflections (Jokinen and Scanlon 1985; Graham and Scanlon 1986) was obtained based on the proposed value implemented in Branson's equation. Scanlon and Bischoff (2007) proposed the following expression to calculate the restraint stress, f_{res} along with Bischoff's effective moment of inertia equation.

$$f_{res} = 2.5 * \sqrt{f'_c} \quad (2\text{-}6)$$

$$f_r = 7.5 * \sqrt{f'_c} \quad (2\text{-}7)$$

Substituting equations 2-7, and 2-6 in equation 2-5, we have f_{re}

$$f_{re} = 5 * \sqrt{f'_c} \quad (2\text{-}8)$$

This results in a decreased cracking moment,

$$M'_{cr} = \frac{f_{re} y_t}{I_g} = \frac{f_{re} M_{cr}}{f_r} = 5M_{cr}/\,7.5 = 2M_{cr}/3$$

Where

M'_{cr} = reduced cracking moment to account for shrinkage restraint stresses.

Structural members with lower reinforcement ratio are more sensitive to these shrinkage restraint effects than those with higher reinforcement ratio (ρ >0.8%). Fig 2-2 showing I_e/I_g vs. reinforcement ratio using Bischoff's equation, demonstrates the sensitivity of I_e to load level, effective modulus of rupture and reinforcement ratio in the lower range of reinforcement ratio (0.002 to 0.006).

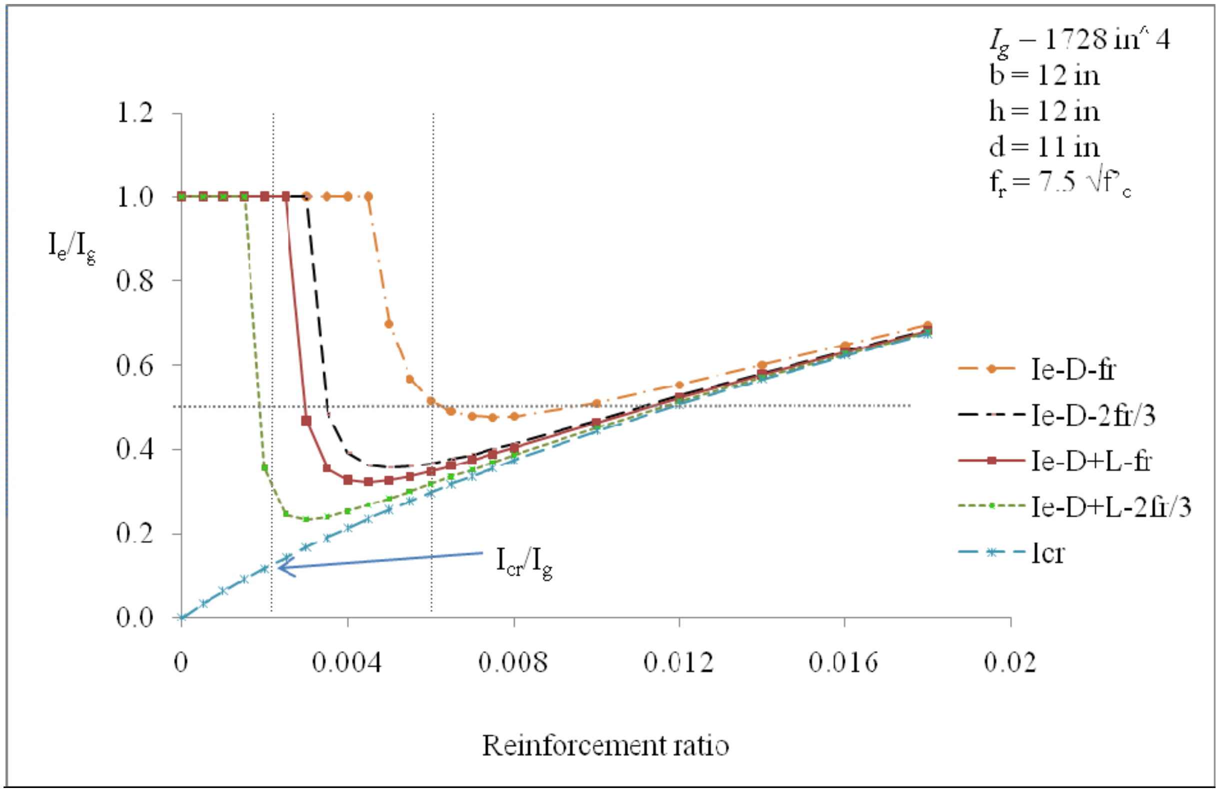

Fig 2-2: Plots for ratio of moment of Inertia vs. reinforcement ratio for Bischoff's equation. For different modulus of rupture (f_r) and combination of loads. *D*(dead load) , *L* (Live load).

2.1.3 Construction Loading (M_a):

Buildings are often subjected to loads before the materials used for construction attain their full specified strengths. Grundy and Kabaila (1963), have emphasized the fact that shoring and re-shoring sequence have a significant effect on the magnitude of these loads. Others (e.g. Graham and Scanlon, 1986) have also demonstrated the effect of construction loads on the long-term serviceability of floor systems.

Hence, a study combining the effects of shrinkage restraint and loading history on the long-term and immediate deflections is carried out. Afore mentioned study plots are shown in Fig 2-2. The cases considered are as follows:

1. I_e/I_g for a dead load only, with a M_a/M_u ratio of 0.38, for rupture modulii of f_r and $2f_r/3$
2. I_e/I_g for a combination of dead plus live load, with a M_a/M_u ratio of 0.66, for rupture modulii of f_r and $2f_r/3$
3. I_{cr}/I_g, which is independent of loading and modulus of rupture for a given strength of concrete.

The sensitivity of the equation for computing I_e (Bischoff's method) to these factors is discussed in Scanlon and Bischoff (2008). They also recommended that I_e be calculated based on dead and live load because construction loads at early age often approach the full design load.

Under full service load, all plots converge asymptotically towards the line representing I_{cr}, beyond $\rho = 0.008$. This occurs due to I_e approaching I_{cr}, as M_a/M_{cr} increases with reinforcement ratio. I_e also becomes insensitive to loading or reinforcement ratio as ρ exceeds 0.012. At lower reinforcemnt ranges, the graphs diverge and again converge at M_a=M_{cr}, thus rendering I_e highly dependent on the loading and rupture modulus. Thus, these plots clearly indicate the influence of considering live load also in computing the applied to ultimate moment ratio, M_a/M_u and a reduced modulus of rupture in computing the immediate and long-term deflections(calcluated as a factor of immediate deflections).

2.1.4 <u>Comparison with Test Data:</u>

Tests were conducted on two simply supported slabs, for further validation of Bischoff's equation to compute effective moment of inertia and immediate deflections. The two slab specimens contain 2#5 (ρ= 0.5%) and 2 #8 (ρ = 1.2%) bars respectively. Shear stirrup reinforcement using #3 bars, provided for the section with 2#8 bars to prevent a shear failure. Top reinforcement is absent in the middle third clear span (6'-4") of the slab section in both specimens. The concrete cross-section is a 12" x 12" square and total length is 20 feet. The test specimens are represented in the Figs 1(a), and 1(b). The beam is placed on supports, such that clear span between supports is 19 ft, hence allowing for 6 inches overhang on the roller and knife-edge supports. A W12 x 21 spreader beam was used to apply the load at two points, 6'-4" apart. The spreader beam is 8'-4" long end to end, with 1 foot overhangs. The actuator of the compression testing machine applies the load onto the spreader beam and this load is transferred to the concrete beam at its middle thirds as shown in Fig 2-5.

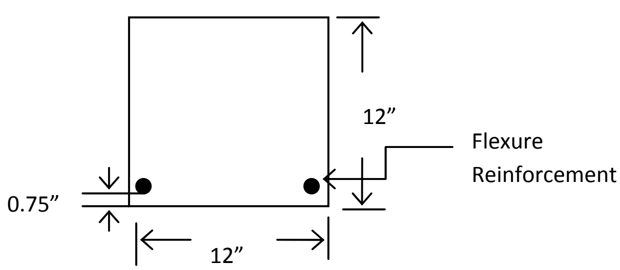

Fig 2-3 Slab section with 2# 5 Bars

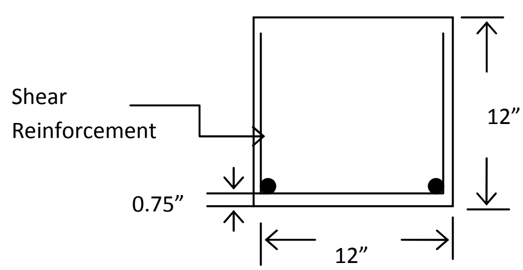

Fig 2-4 Slab section with 2#8 Bars

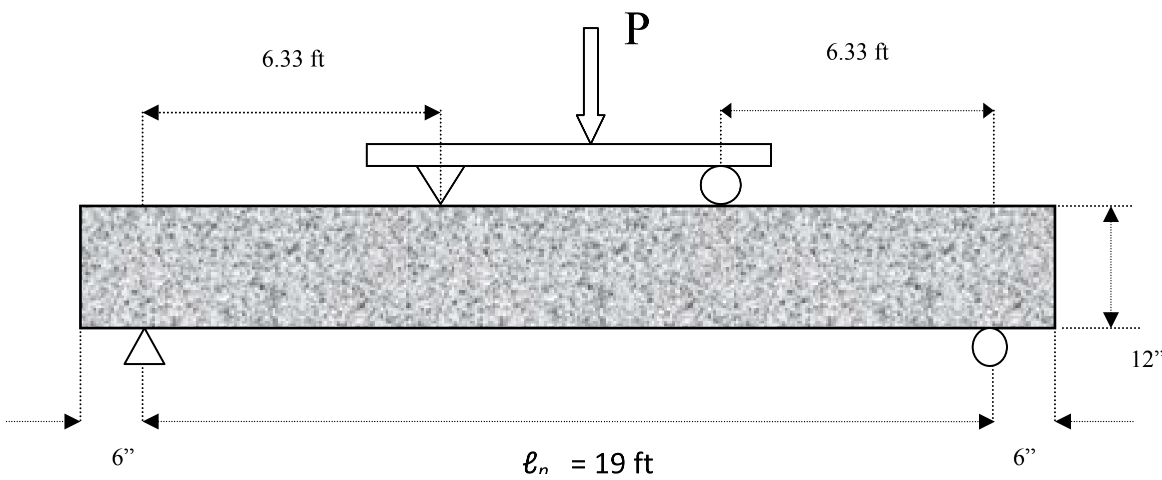

Fig 2-5: Testing set-up. P is the load applied

Mid-span deflections due to applied load were measured using a linear potentiometer. Fig 2-6 is a picture of the crack pattern on a2 #8 beam under applied load. Moment versus deflection graphs are plotted for three cases: experimental results, theoretical results based on Branson’s (ACI 318) and Bischoff’s equation for two different reinforcement cases as used in the experiments. Dead load deflection due to self weight was calculated and added to the measured deflections to obtain deflection under dead plus applied live load. Fig 2-7 and Fig 2-8 show that until the cracking moment, experimental deflections agree well with the theoretical values. Beyond the cracking moment, Branson’s model underestimates the immediate deflection, while Bischoff’s model conservatively approximates the deflection at the design service load level.

Fig 2-6: 2#8 beam crack pattern under applied load

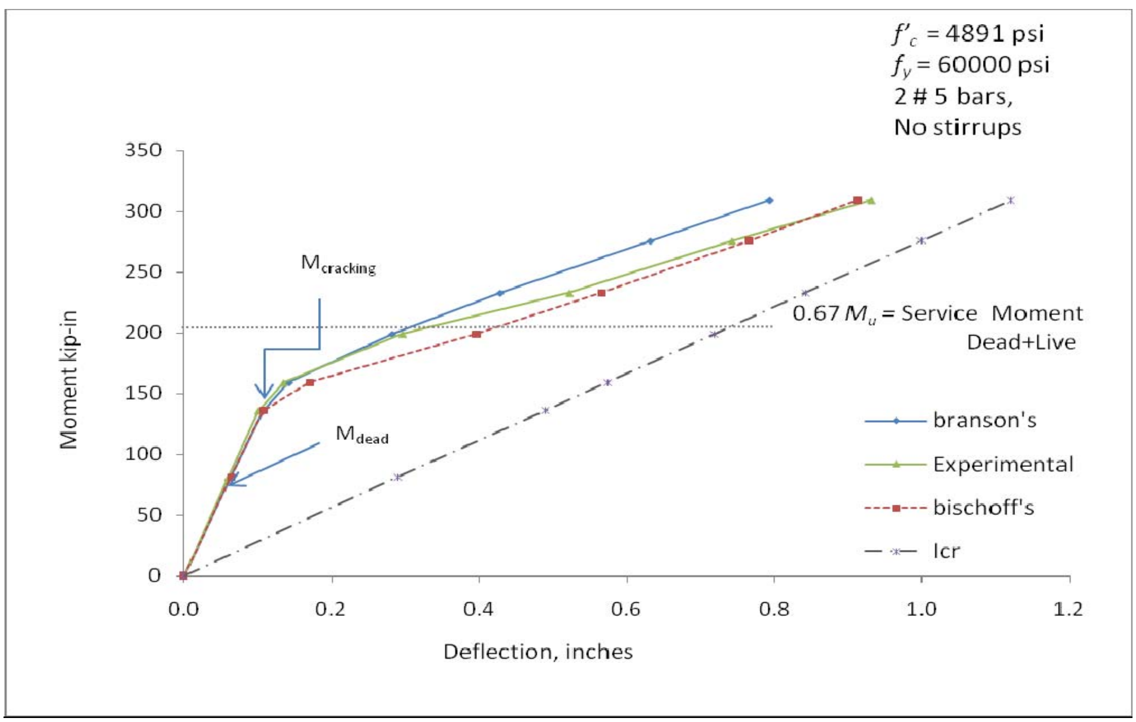

Fig 2-7 :Moment vs. Deflection curves for experimental & theoretical values for ρ = 0.5%

It asserts the fact observed by Bischoff and Scanlon (2007) that for lower reinforcement ratios (ρ <= 0.8%), Bischoff's model for computing effective moment of inertia (I_e), predicts the expected deflection closer to the realistic deformations. These observations are valid in the service load range when the steel reinforcement is in the elastic range.

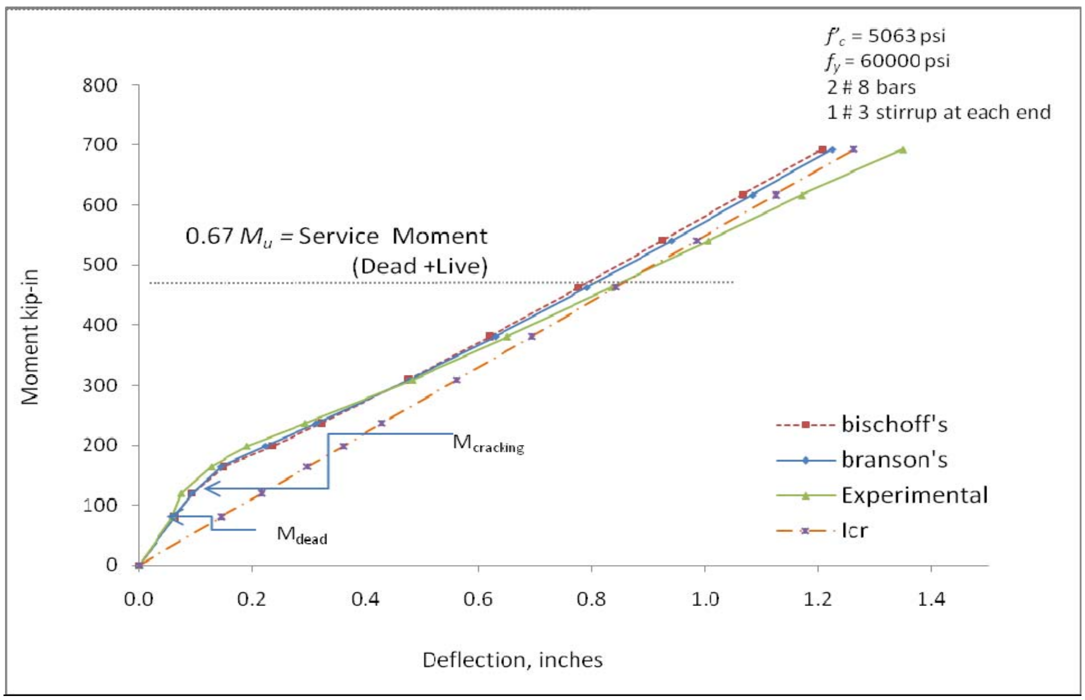

Fig 2-8 :Moment vs. Deflection curves for experimental and theoretical values for ρ=1.2%

For specimens with larger reinforcement ratio (ρ > 0.8%), both the theoretical methods (Branson's and Bischoff's) adequately predict the deflections beyond the cracking moment. Common slab reinforcement ratios are usually in the range 0.2% - 0.6%. Hence, adopting Bischoff's equation for members with afore mentioned ranges of reinforcement, proves to be efficient in effectively predicting the deflections.

2.2 <u>Time-Dependent Deflections</u>

Time has a significant effect on the material properties of concrete. The superimposed loads also vary with time. Time-dependent deflections therefore need to be considered for deflection control. The following is a summary of factors effecting time-dependent deflections.

2.2.1 <u>Long-term Multiplier, λ_Δ:</u>

As stated by ACI 318 commentary," Since available data on long-term deflections of slabs are too limited to justify more elaborate procedures, the additional long-term deflection for two-way construction is required to be computed using the multipliers." Additional long-term deflection resulting from creep and shrinkage of flexural members (normal weight or lightweight concrete) shall be determined by multiplying the immediate deflection caused by the sustained load by the factor λ_Δ. The deflection computed in accordance with this section is the additional long-term deflection occurring after installation of non-structural elements such as partitions, due to the dead load and that portion of the live load considered to be sustained for a sufficient period to cause significant time-dependent deflections. Long-term multiplier is applied to the immediate deflection due to sustained load (usually taken to be 10 to 25% of the total live load) multiplied by longtime multiplier and instantaneous live and dead load.

The long-term multiplier, λ_Δ is defined by the equation,

$$\lambda_\Delta = \frac{\xi}{1 + 50\rho'} \tag{2-6}$$

Where, ρ' shall be the value of reinforcement at mid-span for simple and continuous spans, and at support for cantilevers. It shall be permitted to assume ξ, the time-dependent factor for sustained loads, to be equal to:

5 years or more...2.0

12 months...1.4

6 months...1.2

3 months...1.0

Other values of ξ for a time period less than 5 years may be obtained from Fig 2-9.

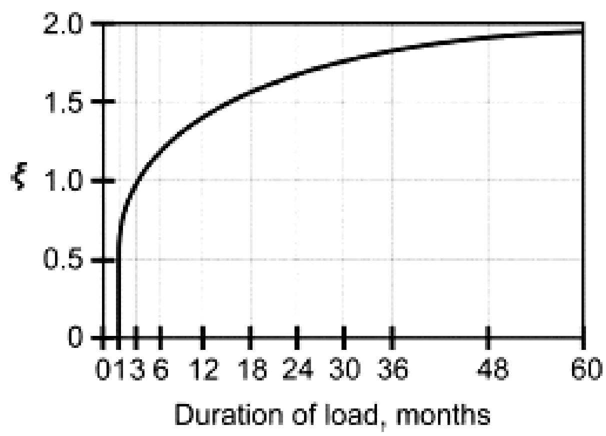

Fig 2-9: Multipliers for long-term deflections (used with permission from ACI 318-08)

Chapter 3

MINIMUM THICKNESS OF TWO-WAY SLABS

Table 9.5(c) as replicated in Table 1-3, sets the minimum thickness requirements for two-way slab construction. It deals with the exterior and interior panel thickness values for different steel yield strengths for slabs with and without drop panels, with different edge constraints. Slabs without drop panels have lesser stiffness values compared to those with drop panels. Hence, these constitute our major focus of interest. The Table does not address the influence of strength of concrete, loading values, end restraints, span ranges, etc to which it is applicable, but for a commentary statement that "the minimum thicknesses in Table 9.5(c) are those that have been developed through the years. Slabs conforming to those limits have not resulted in systematic problems related to stiffness for short- and long-term loads. These limits apply to only the domain of previous experience in loads, environment, materials, boundary conditions, and spans". However, the commentary does not provide guidance regarding the range of previous experience covered.

3.1 Minimum Thickness equation

The critical deflection limit is usually associated with time-dependent deflection occurring after installation of non-structural elements. A new equation of computing minimum thickness of two –way slabs has been proposed by Scanlon and Lee (2006) for non-prestressed beams and slabs. This equation can be derived based on rearranging the standard beam deflection and applying factors to account for two-way action.

From equation (2-1),

$$\Delta = \frac{k\,w\ell^4}{384EI}$$

To compute the deflection that occurs after installation of non-structural elements, the equation can be written in the form:

$$\Delta = \frac{k\,w_{\ell(ADD)}\ell^4}{384E_cI_e} + \lambda\frac{k\,w_{sus}\ell^4}{384E_cI_e} \qquad (3\text{-}1)$$

Where,

Δ = total incremental deflection which is a sum of time-dependent deflections due to sustained load plus immediate deflection due to portion of live load not considered to be sustained

λ = long-term deflection multiplier

k = deflection coefficient depending on support condition

= 5 for simply supported

= 1.4 for both ends continuous

= 2 for one end continuous

= 48 for fixed cantilever

W_{sus} = sustained load; sum of super-imposed dead load, self weight of member and sustained portion of live load

$W_{\ell(ADD)}$ = variable portion of live load

ℓ = span length

I_e = Effective moment of inertia

E_c = Elastic modulus of concrete

equation 3-1 can be rewritten as:

$$\Delta = \frac{k\,(w_{\ell(ADD)} + \lambda w_{sus})\ell^4}{384 E_c I_e} \tag{3-2}$$

By applying factors to account for two-way action and the effect of drop panels, deflection of column and middle strip deflections can be obtained from equation (3-2)

So equation (3-2) can now be written as:

$$\Delta = \frac{k\ell^4 k_{SS}\,(w_{\ell(ADD)} + \lambda w_{sus})}{384 E_c k_{DP} I_e} \tag{3-3}$$

Where, k_{SS} is a factor to account for distribution of total static moment to column and middle strips and k_{DP} is a factor to account for increased stiffness in column strips due to the presence of drop panels. Scanlon and Lee (2007) proposed the following values,

k_{SS} = 1.35 (column strip)

k_{SS} = 0.65 (middle strip)

k_{DP} = 1.35

Expressing I_e in terms of I_g, using a factor, α

$I_e = \alpha * I_g$ and $I_g = bh^3/12$. Substituting in equation 3-3, we obtain:

$$\Delta = 12\frac{k\ell^4 k_{SS}(w_{\ell(ADD)} + \lambda w_{sus})}{384 E_c k_{DP} \alpha b h^3}$$

Rearranging the equation provides an expression for span to depth ratio, ℓ/h:

$$K_\Delta = \frac{\ell}{h} = \left[\left(\frac{\Delta_{inc}}{\ell}\right)_{allow} \frac{32 E_c k_{DP} b \alpha}{k\, k_{SS}(w_{\ell(ADD)} + \lambda w_{sus})} \right]^{\frac{1}{3}} \tag{3-4}$$

$(\Delta_{inc})_{allow}$ = required incremental deflection limit taken from Table 9.5(b) of ACI 318

For interior panels, k = 1.4 for exterior panels, k = 2

Scanlon and Lee (2007) proposed the following expressions for flat plates based on equation (3-4) with α = 0.52.

a) $K_{\Delta} = \left[\langle \frac{\Delta_{inc}}{\ell_n} \rangle \frac{1200 E_c}{(\lambda_{\Delta} w_{sus} + w_{Ladd} \rangle} \right]^{\frac{1}{3}}$ for slabs without interior beams, without drop panels, for interior panels and exterior panels with edge beams having α_f, aspect ratio not less than 0.8

b) $K_{\Delta} = 0.9 \left[\langle \frac{\Delta_{inc}}{\ell_n} \rangle \frac{1200 E_c}{(\lambda_{\Delta} w_{sus} + w_{Ladd} \rangle} \right]^{\frac{1}{3}}$ for exterior panels without edge beams

c) $K_{\Delta} = 1.1 \left[\langle \frac{\Delta_{inc}}{\ell_n} \rangle \frac{1200 E_c}{(\lambda_{\Delta} w_{sus} + w_{Ladd} \rangle} \right]^{\frac{1}{3}}$ for slabs without interior beams with drop panels for interior panels and exterior panels with edge beams having α_f not less than 0.8.

d) $K_{\Delta} = \left[\langle \frac{\Delta_{inc}}{\ell_n} \rangle \frac{1200 E_c}{(\lambda_{\Delta} w_{sus} + w_{Ladd} \rangle} \right]^{\frac{1}{3}}$ for exterior panels without edge beams

Where,

k_{Δ} = span-to-depth ratio at the deflection

w_{add}= additional live load (uniformly distributed)

w_{sus} = sustained load (uniformly distributed)

Δ_{inc} = deflection

ℓ_n = clear span length

E_c= young's modulus of concrete

$\lambda_\Delta = \dfrac{\xi}{1+50\rho'}$ where

ξ= time dependent factor, equal to:

5 years or more….2.0

12 months ……….1.4

6 months ……......1.2

3 months………… 1.0

ρ' = reinforcement at mid-span for simply supported and continuous members and at supports for cantilevers

Based on k_{DP} = 0.52, span to depth ratios for flat slabs with drop panels are equal to the equivalent values for flat plates increased by 10 %.

An excel spreadsheet has been developed (refer: Appendix) to compute and compare the minimum thicknesses for the above cited conditions as against the ACI 318 specified limits in Table 9.5 (c).

3.2 Boundary Condition Factor

Slabs are most generally constructed integrally with the columns (which act as supports). The nature of the columns is to provide additional flexural rigidity to the slabs. The amount to which columns can provide this additional resistance varies between that of a simply supported slab case to that of a continuously fixed column-slab unit. The effect of boundary conditions plays a vital role in determining the deflections of the slabs.

Hence SAP 2000 models of the two extreme cases are carried out to evaluate the values proposed by Scanlon and Lee (2007) as follows:

- A three-span slab section continuous over supports not integrally built but simple supported as shown in Fig 3.2 (a).
- A three span slab section integrally built with columns as shown in Fig 3.2(b): Two different values of moment of inertia for slab are used.

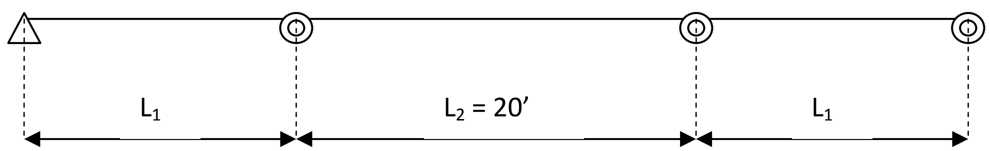

Fig 3.1: Slab section point supported. I = 512 in^4

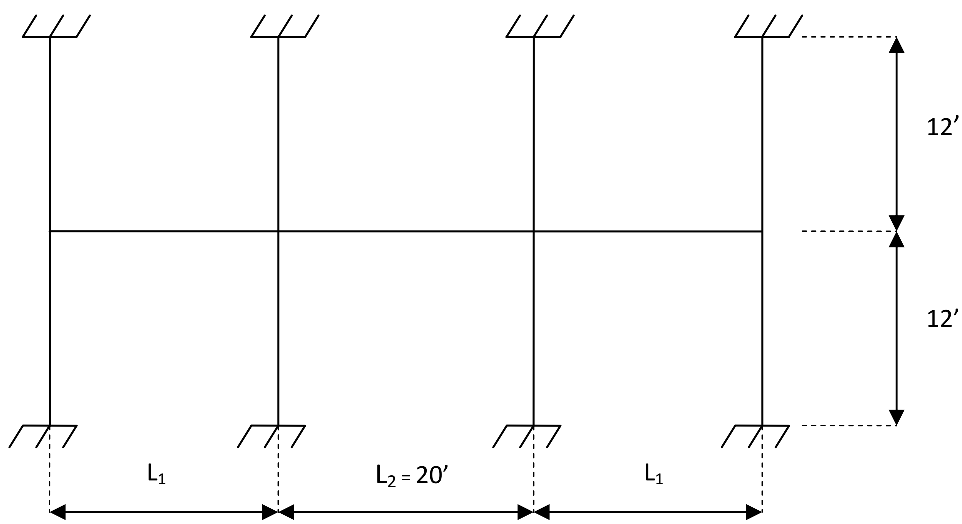

Fig 3.2: Slab integrally built with columns I = 10245 in^4 and 5120 in^ 4

The center-to-center span length was 20’ for interior span and varied for the exterior span. Slab depth was taken as 8 inches, and the column dimensions were 16” x 16”. Strength of concrete is 4000 psi, and total dead load (inclusive of self-weight) on the slab is 1000 psf. Column height is 12 feet both above and below the slab. These models were analyzed in SAP 2000 and the results were obtained for various span ratios in terms of deflections. Deflection, Δ was computed as a factor of:

$$\Delta = \frac{\alpha w \ell^4}{384EI} \qquad (3\text{-}1)$$

Where,

α = boundary condition factor

w= total uniform load

ℓ= span length

E= young’s modulus of concrete

I = Moment of Inertia of slab cross section

The value for moment of inertia of slab, for the column-slab case was taken to be 0.5I and I respectively. Alpha varies from 1 for fixed spans to a maximum of 5 for simply supported spans from structural analysis computations. For various span ratios and the boundary conditions, the alpha variations are presented in the following graphs. The graph represent sections with slabs supported on pin-supported connections as slab, with ratio of slab moment of inertia to column moment of inertia equal to 0.53, as slab-column-0.53 and that equal to1 as slab-column-1, respectively.

Graphs for alpha vs. span ratios are plotted for both interior spans and exterior spans. The area of interest is deflection in the direction of gravity. Hence, only those span ratios for which alpha values are positive are of concern. It is observed that alpha values do not exceed 5 and for the case where columns are present, interior spans have alpha values less than 1. This is due to the effect of loading in adjacent spans, which in turn reduce the downward deflection of the span considered.

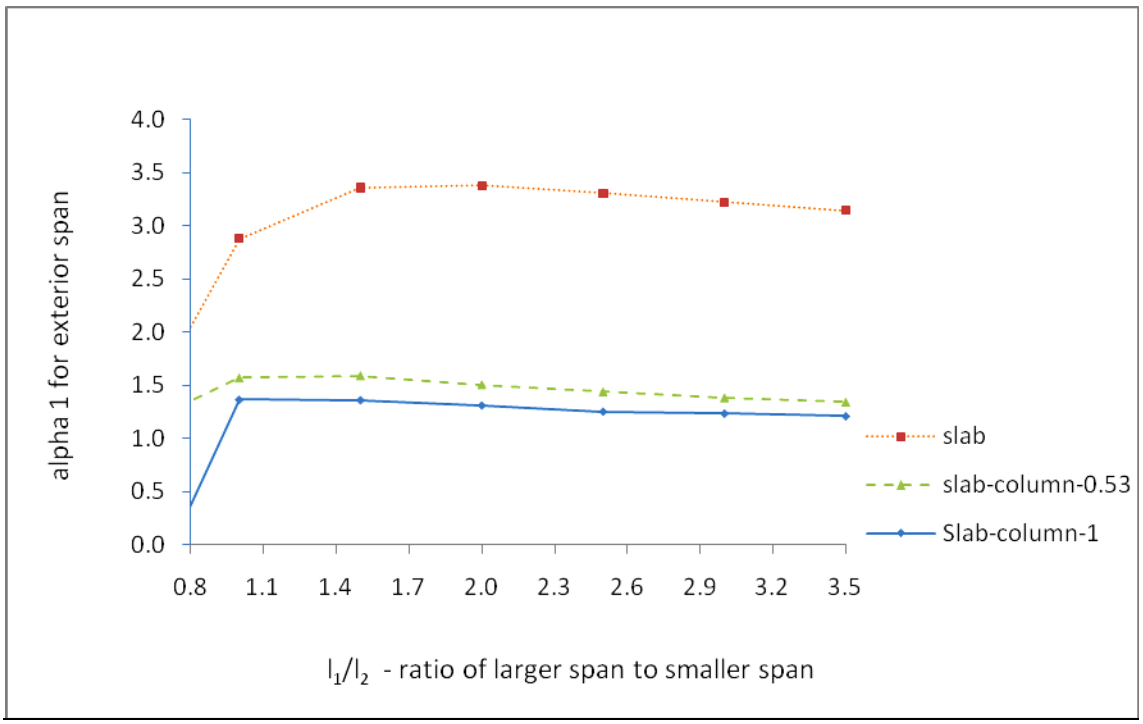

Fig 3.3: Alpha as a factor of deflection for exterior span vs. span ratio

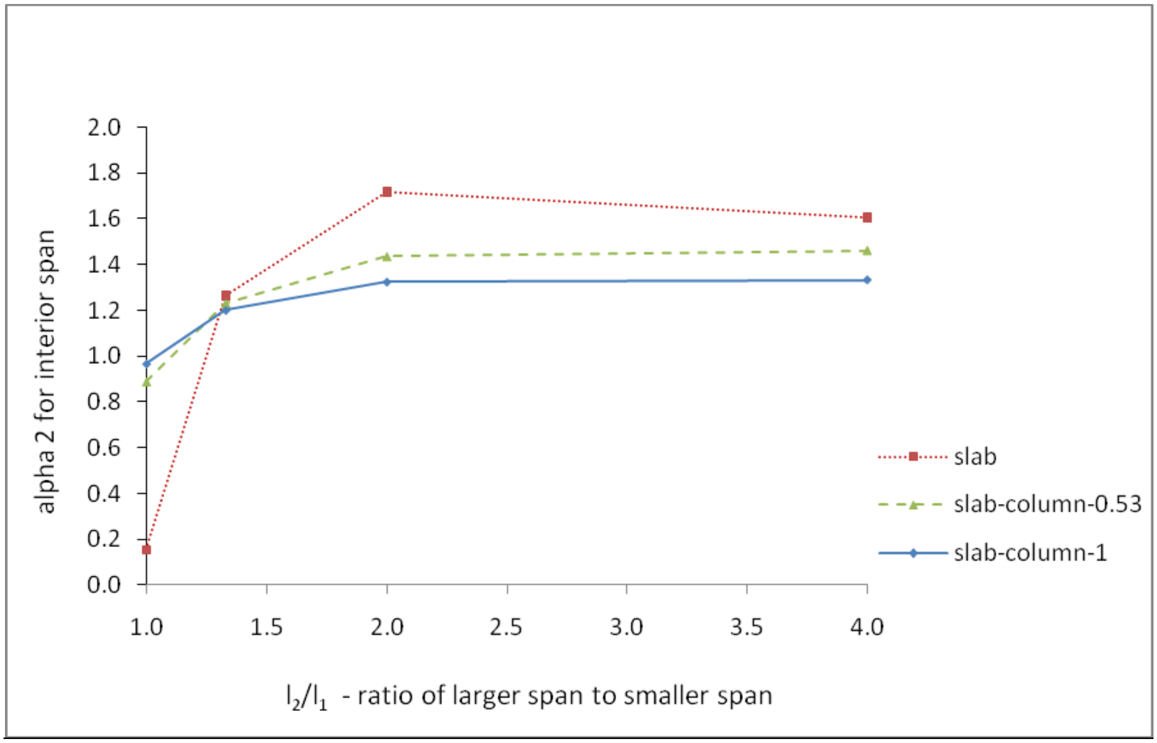

Fig 3.4: Alpha for interior span vs. span ratio

Usual construction practices cast slabs monolithically with columns and beams. Completely fixed ends and simply supported cases are the two extreme cases conditions for a column-slab connection. Actual boundary conditions occur between these two extremes. As suggested in the proposed h_{min} equation, a value of two for exterior span seems to be conservative from Fig 3.3 and a value of 1.4 for interior spans seems to be a good approximation as shown in Fig 3.4 for column supported slabs

Chapter 4

PARAMETRIC STUDY OF FACTORS EFFECTING MINIMUM THICKNESS OF TWO-WAY SLABS

4.1 Range of Variables Considered

The primary objective of the parametric study described in this chapter dealing with minimum thickness is to use the equations proposed by Scanlon and Lee (2007) as mentioned in section 2.3, to determine changes needed for Table 9.5(c) and to determine limitations on applicability of the Table beyond which the minimum thickness equation or deflection calculation should be used to satisfy deflection control requirements. The variables considered in the study are tabulated in Table 4.1.

These parametric studies are run for two limits of deflection, specified under Table 9.5(b) of ACI 318-08:

- Roof or floor construction supporting or attached to nonstructural elements likely to be damaged by large deflection --> $\ell_n/480$
- Roof or floor construction supporting or attached to nonstructural elements not likely to be damaged by large deflections --> $\ell_n/240$

Parametric studies are conducted for five different variables, as listed in the Table 4-1. The various parameters are listed in the first column, and their values are listed in the second column against the constant parameters listed in the third column for each case studied. Plots for each analysis case are presented for interior and exterior spans, corresponding to the two limits of deflections. The values were selected to cover the range of design values typically (Bondy 2005) found in design practice. The constants used for each study are specified in the plots. Finally, a comprehensive summary is presented at the end of the section.

Table 4-1: Variable Ranges Considered in the parametric study

Parameter Studied	**Variable**	**Constants**
Uniform Superimposed Dead Load	W_{sdl} (psf)= 20, 50, 100, 200	W_{live} = 70 psf λ_{Δ} =2 f'_c = 4 ski Case 1: ℓ_n = 20 ft Case 2: ℓ_n = 30 ft
Uniform Live Load	W_{live} (psf) = 40, 70, 100, 150	W_{sdl} = 20 psf λ_{Δ} =2 f'_c = 4 ski Case 1: ℓ_n = 20 ft Case 2: ℓ_n = 30 ft
Clear Span	ℓ_n (ft) = 10,20,30,40	W_{live} = 70 psf λ_{Δ} =2 f'_c = 4 ski Case 1: W_{sdl} = 20 psf Case 2: W_{sdl} = 100 psf

		Case 3: W_{sdl} = 200 psf
..contd Parameter Studied	**Variable**	**Constants**
Long-term Multiplier	λ_{Δ} = 2,3	W_{live} = 70 psf W_{sdl} = 20 psf f'_c = 4 ski ℓ_n = 20 ft
Concrete Strength	f'_c (ksi) = 3, 4, 5, 6	W_{live} = 70 psf W_{sdl} = 20 psf λ_{Δ} =2 Case 1: ℓ_n = 20 ft Case 2: ℓ_n = 30 ft

4.2 Results

In this section, the results of the parametric study are presented considering each parameter in turn. The legend in the plots is explained as follows:

1. Int- hmin : span vs. depth ratio, K_{Δ} as computed using h_{min} equation for interior two-way slab panels

2. Int-ACI : span vs. depth ratio, K_{Δ} as computed using Table 9.5(c) of ACI 318 for interior two-way slab panels

3. Ext-hmin : span vs. depth ratio, K_{Δ} as computed using h_{min} equation for interior two-way slab panels

4. Ext-ACI : span vs. depth ratio, K_Δ as computed using Table 9.5(c) of ACI 318 for interior two-way slab panels

4. 2.1 Superimposed Dead Load, W_{sdl}:

Two cases are considered: case 1 for ℓ_n = 20 ft and case 2 for ℓ_n = 30 ft. Results are presented in terms of the factor K_Δ $(=\ell_n / h)$ vs. the value of the parameter considered.

Case 1: To study the influence of superimposed dead load on the minimum thickness requirements of two way slabs, W_{sdl} was varied from 20 psf to 200 psf. It is observed that for deflection limit of ℓ_n /240, the limits specified in Table 9.5(c), give a reasonable estimate for minimum thickness values of two-way slabs up to a W_{sdl} value of 100 psf as shown in Figs 4-1, and 4-2.

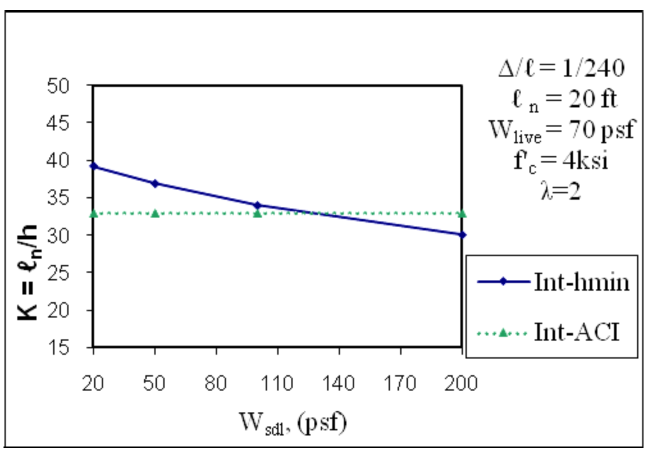

Fig 4-1: Span-to-depth ratio vs. superimposed dead load for interior panels without drop panels for deflection limit of ℓ_n/240 for a span length of 20 ft.

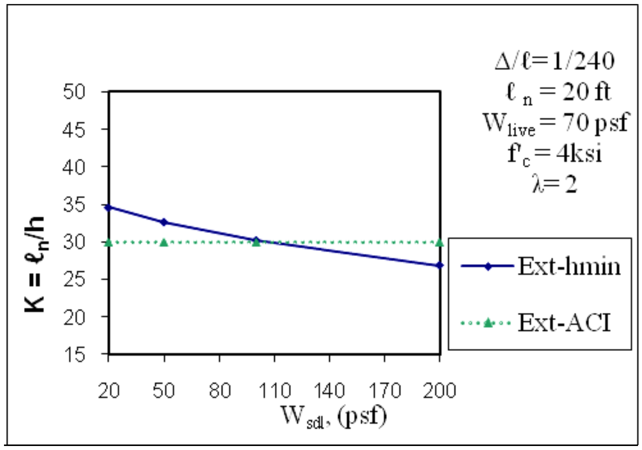

Fig 4-2: Span-to-depth ratio vs. superimposed dead load for exterior panels without drop panels for deflection limit of $\ell_n/240$ for a span length of 20 ft

The above calculations are repeated for a deflection limit of $\ell_n/480$ and it is observed that ACI specified limits underestimate minimum thickness requirements for both interior and exterior spans of slabs wthout drop panels. From Figs, 4-3, and 4-4 it is evident that $\ell_n/33$, does not provide an adequate thickness limit. A value of $\ell_n/30$, which would be a 10% increase from the existing value, as the minimum thickness would be a reasonable thresold for minimum depth for emembers with W_{sdl} values upto 20 psf.

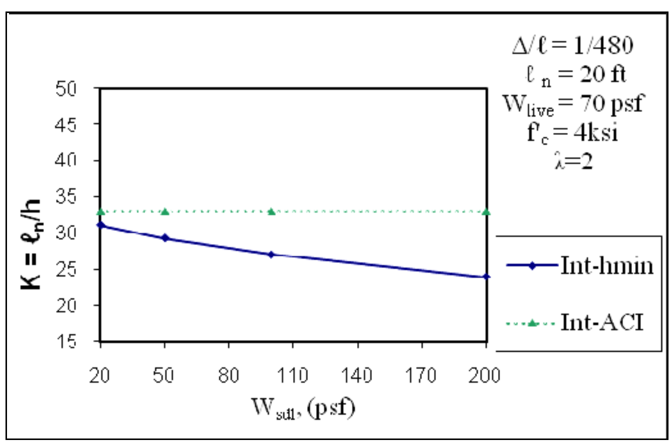

Fig 4-3: Span-to-depth ratio vs. superimposed dead load for interior panels without drop panels for deflection limit of $\ell_n/480$ for a span length of 20 ft

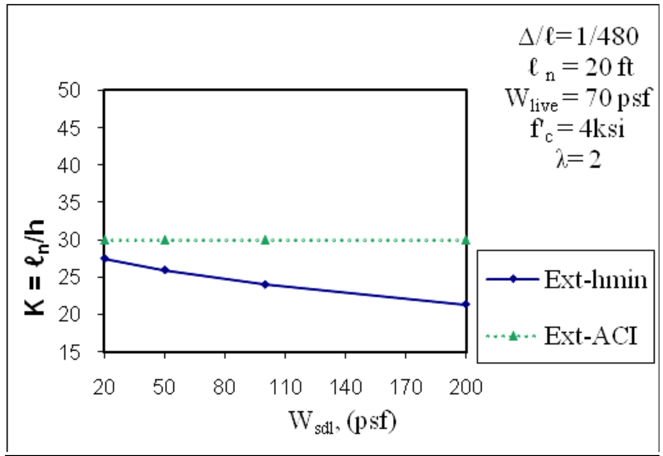

Fig 4-4: Span-to-depth ratio vs. superimposed dead load for exterior panels without drop panels for deflection limit of ℓ_n//480 for a span length of 20 ft

<u>Case 2:</u> This analysis is carried out similar to the one made previously but the clear span length is 30 feet instead of 20 ft. Table 4-1 lists the different values of W_{sdl} used along with the constants in the study. Figs 4-5 and 4-6 show that for W_{sdl} upto 60 psf, ACI provided minimum thickness values are satisfactory for deflection limit of ℓ_n/240. But for deflection limit of ℓ_n/480, the limits provided by ACI are much lower than required.

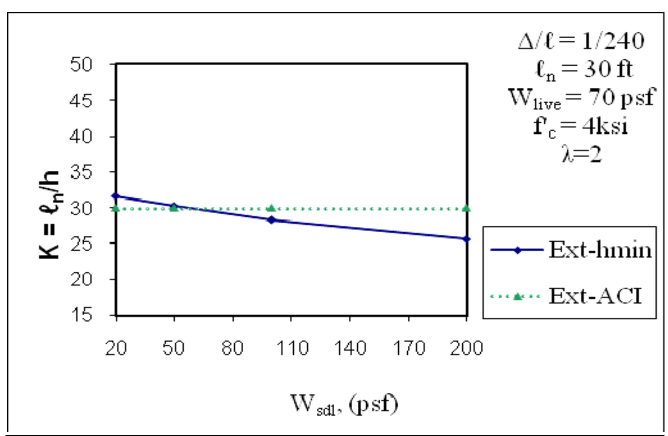

Fig 4-5: Span-to-depth ratio vs. superimposed dead load for interior panels without drop panels for deflection limit of ℓ_n/240 for a span length of 30 ft

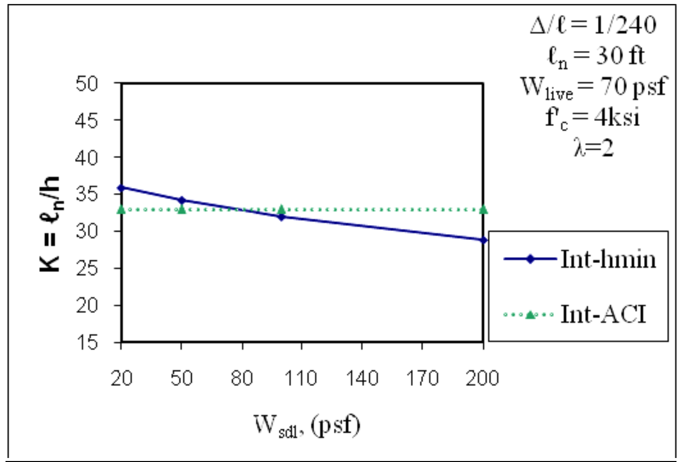

Fig 4-6: Span-to-depth ratio vs. superimposed dead load for exterior panels without drop panels for deflection limit of ℓ_n /240 with span length of 30 ft

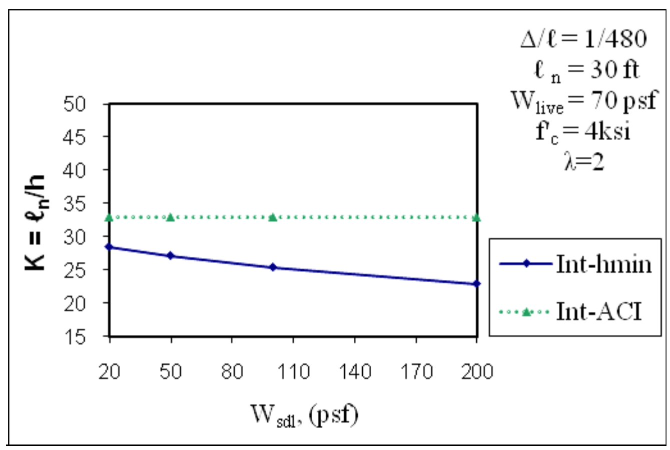

Fig 4-7: Span-to-depth ratio vs. superimposed dead load for interior panels without drop panels for deflection limit of ℓ_n /480 for a span length of 30 ft

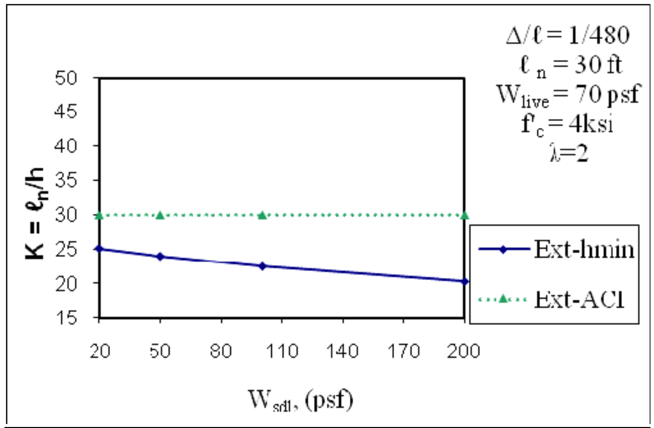

Fig 4-8: Span-to-depth ratio vs. superimposed dead load for exterior panels without drop panels for deflection limit of ℓ_n /480 for a span length of 30 ft

4.2.2 Live Load, W_{live}:

The second parameter studied is the live load. Sustained live load is assumed to be 10% of the total live load. The studies are conducted for two cases:

Case 1: For a span length of 20 ft, slab sections are satisfactorily dimensioned by ACI under the ℓ_n /240 deflection limit as shown in Figs 4-9 and 4-10. However for ℓ_n /480 (Figs 4-11 and 4-12), a 10 % increase in interior span thickness is required which would be satisfactory for live loads up to 70 psf.

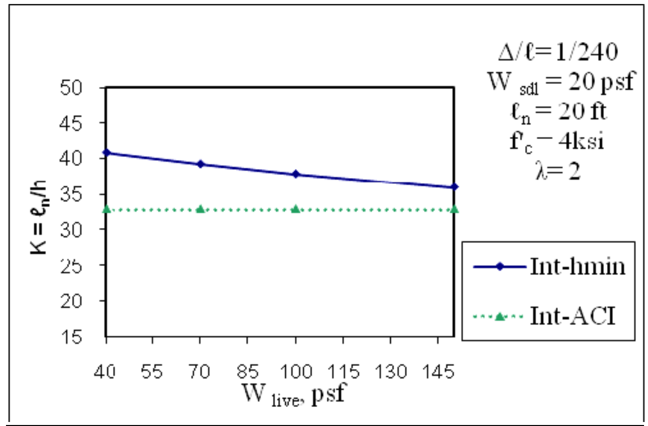

Fig 4-9: Span-to-depth ratio vs. live load for interior panels without drop panels for deflection limit of ℓ_n /240 with span length of 20 ft

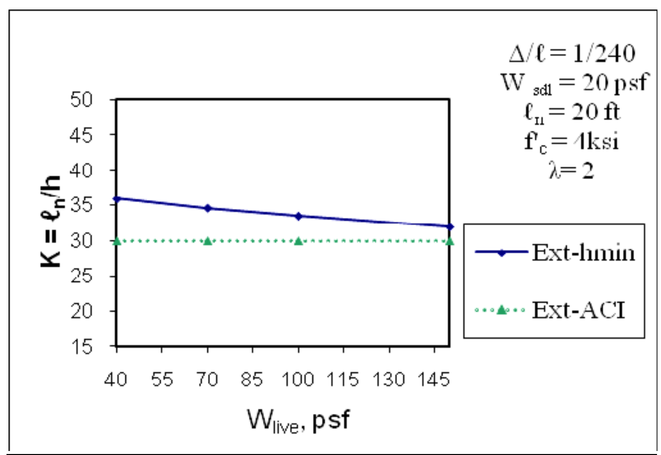

Fig 4-10: Span-to-depth ratio vs. live load for exterior panels without drop panels for deflection limit of ℓ_n /240 with span length of 20 ft

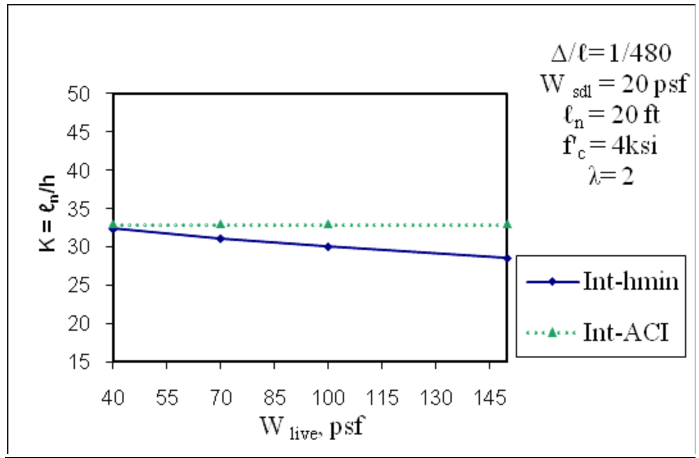

Fig 4-11: Span-to-depth ratio vs. live load for interior panels without drop panels for deflection limit of ℓ_n /480 with span length of 20 ft

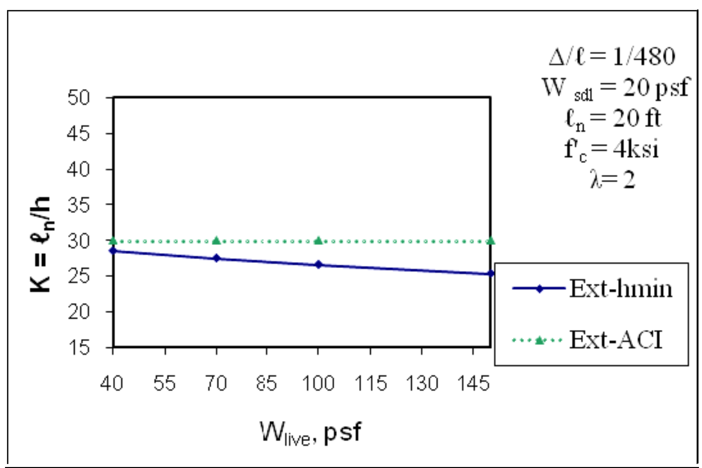

Fig 4-12: Span-to-depth ratio vs. live load for exterior panels without drop panels for deflection limit of ℓ_n /480 with span length of 20 ft

Case 2: For a span length of 30 ft, slab sections are satisfactorily dimensioned by ACI under the ℓ_n /240 deflection limit as shown in Figs 4-13 and 4-14. However for ℓ_n /480 (

Figs 4-15 and 4-16), a 15 to 20 % increase in interior span thickness would be required with live load limited to 70 psf.

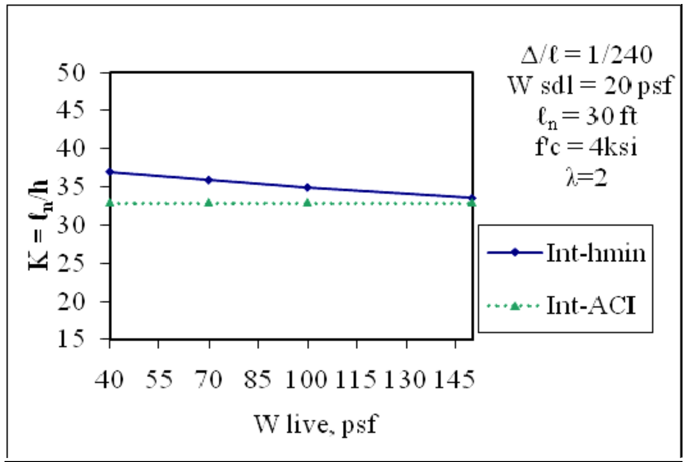

Fig 4-13: Span-to-depth ratio vs. live load for interior panels without drop panels for deflection limit of ℓ_n /240 with span length of 30 ft

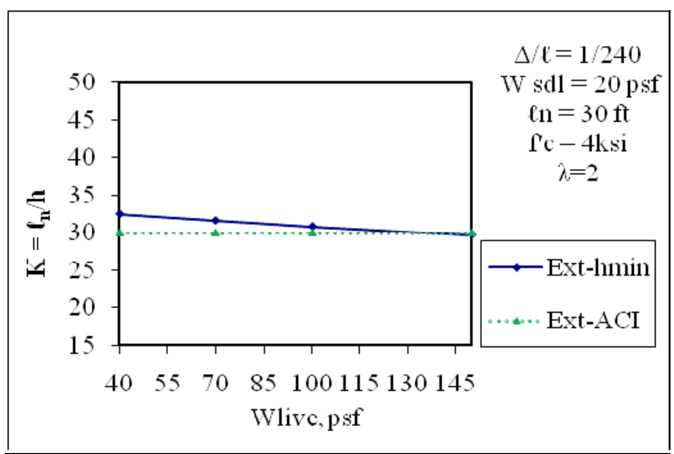

Fig 4-14: Span-to-depth ratio vs. live load for exterior panels without drop panels for deflection limit of ℓ_n /240 with span length of 30 ft

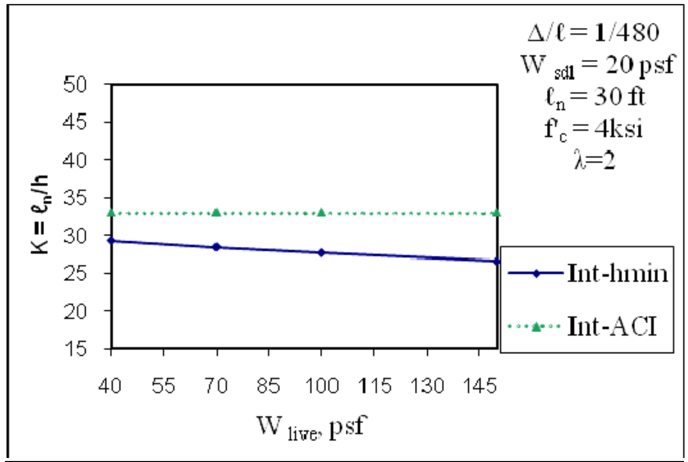

Fig 4-15: Span-to-depth ratio vs. live load for interior panels without drop panels for deflection limit of ℓ_n /480 with span length of 30 ft

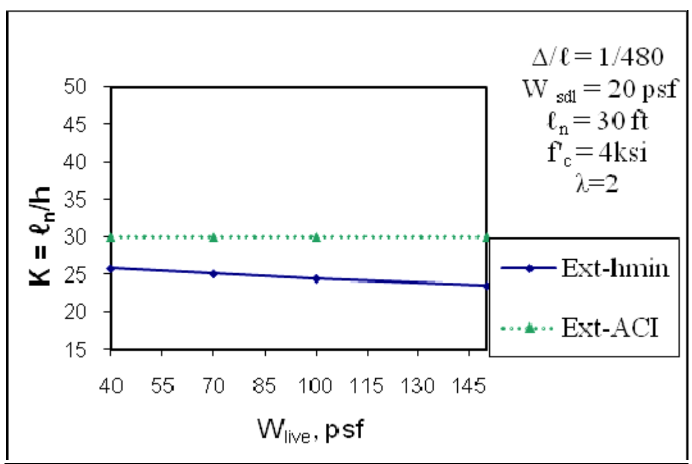

Fig 4-16: Span-to-depth ratio vs. live load for exterior panels without drop panels for deflection limit of ℓ_n /480 with span length of 30 ft

4.2.3 Span Length, ℓ_n :

Case 1: For a superimposed load of 20 psf, slab sections are satisfactorily dimensioned by ACI under the ℓ_n /240 deflection limit as shown in Figs 4-17 and 4-18. However for ℓ_n /480 (Figs 4-19 and 4-20), a 10 % increase in interior span thickness would be required for live loads up to 70 psf when superimposed dead load is 20 psf. This value is at the lower end of typical range of values. Hence, two more cases are studied for which the superimposed load values lie at the middle and at the higher extreme end.

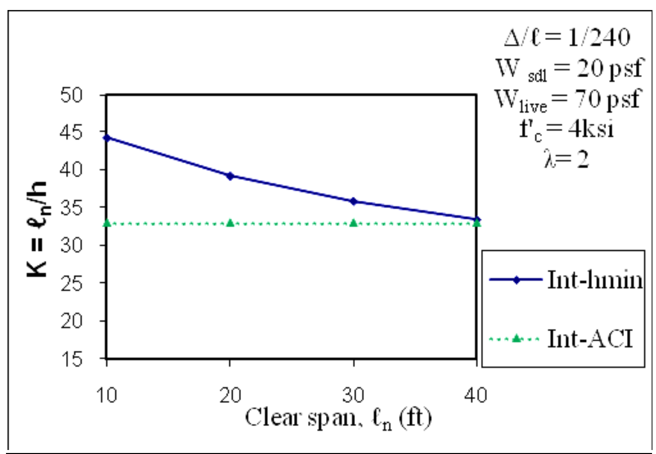

Fig 4-17: Span-to-depth ratio vs. clear span length for interior panels without drop panels for deflection limit of ℓ_n /240 with superimposed dead load of 20 psf

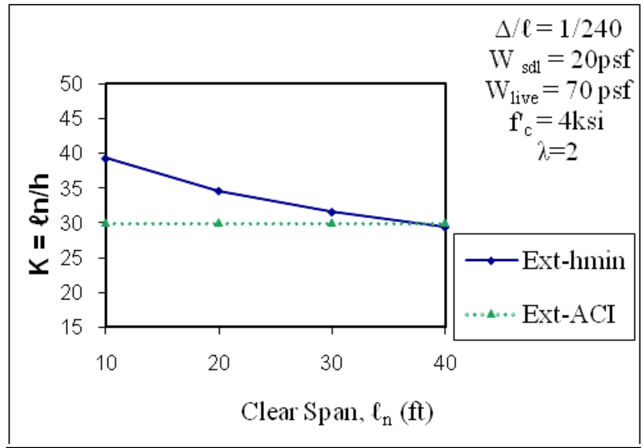

Fig 4-18: Span-to-depth ratio vs. clear span length for exterior panels without drop panels for deflection limit of ℓ_n /240 with superimposed dead load of 20 psf

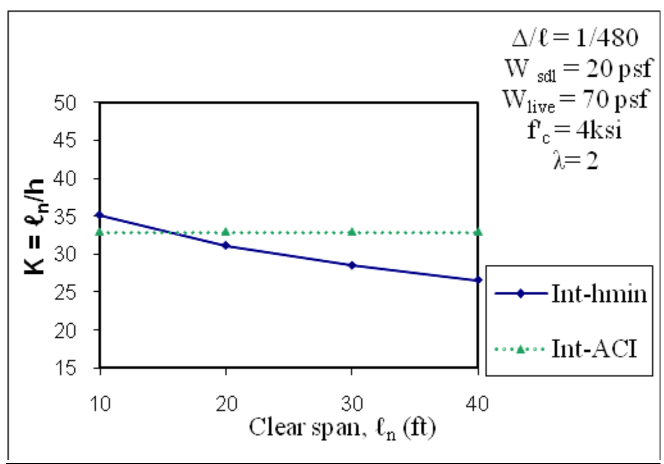

Fig 4-19: Span-to-depth ratio vs. clear span for interior panels without drop panels for deflection limit of ℓ_n /480 with superimposed dead load of 20 psf

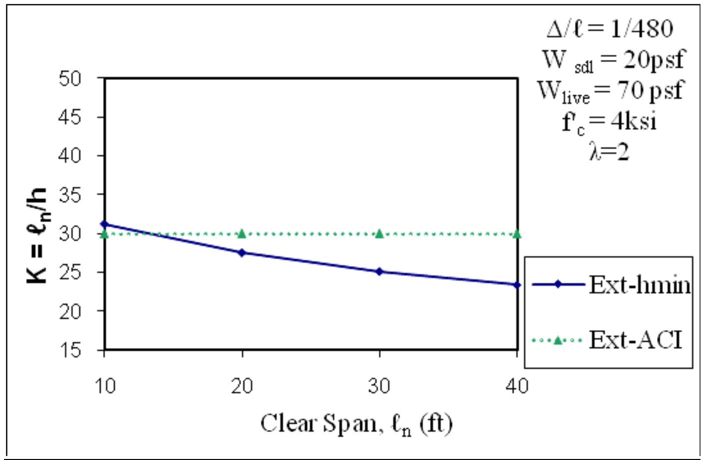

Fig 4-20: Span-to-depth ratio vs. clear span for exterior panels without drop panels for deflection limit of ℓ_n /480 with superimposed dead load of 20 psf

<u>Case 2:</u>

For a superimposed load of 100 psf, the same parametric study is carried out. Slab sections are satisfactorily dimensioned by ACI under the ℓ_n /240 deflection limit for span length up to 20ft as shown in Figs 4-21 and 4-22. However for ℓ_n /480 (Figs 4-23 and 4-24), a 10 % increase in interior span thickness does not provide adequate thickness.

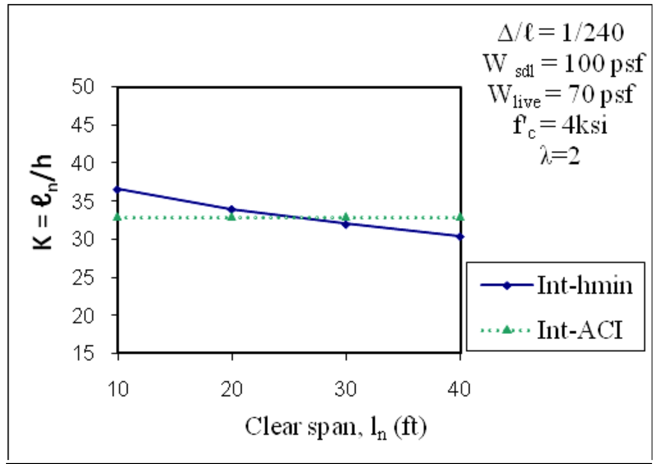

Fig 4-21: Span-to-depth ratio vs. clear span for interior panels without drop panels for deflection limit of ℓ_n /240 with superimposed dead load of 100 psf

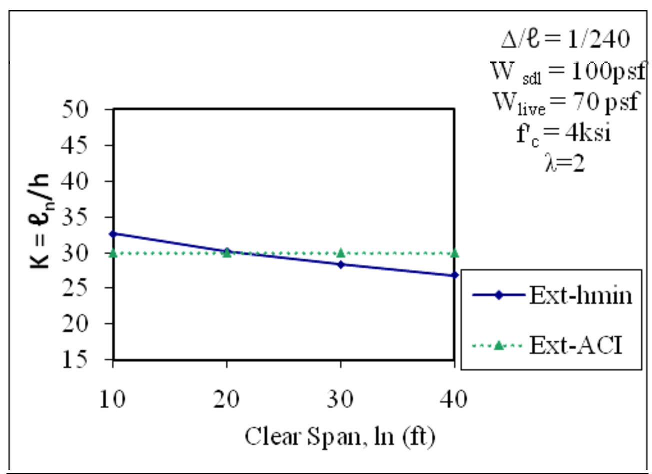

Fig 4-22: Span-to-depth ratio vs. clear span for exterior panels without drop panels for deflection limit of ℓ_n /240 with superimposed dead load of 100 psf

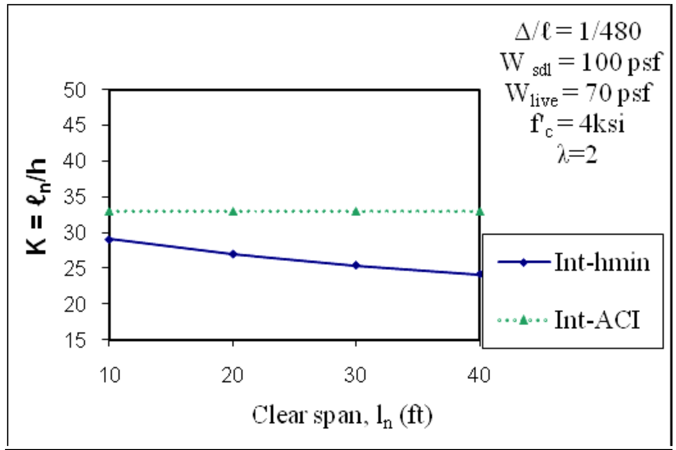

Fig 4-23: Span-to-depth ratio vs. clear span for exterior panels without drop panels for deflection limit of ℓ_n /480 with superimposed dead load of 100 psf

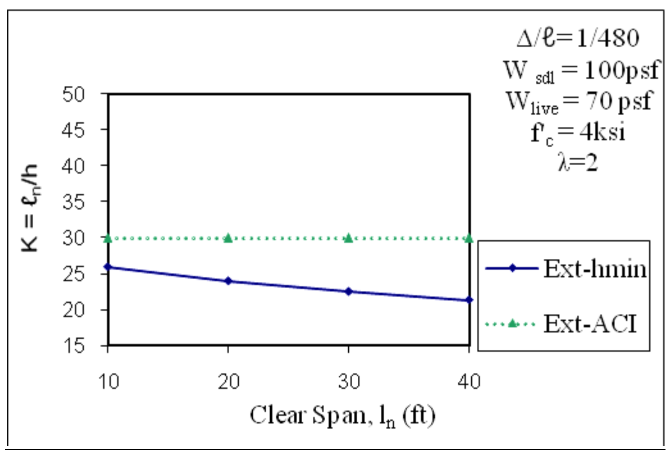

Fig 4-24: Span-to-depth ratio vs. clear span for exterior panels without drop panels for deflection limit of ℓ_n /480 with superimposed dead load of 100 psf

Case 3: For a high value of superimposed load, 200 psf, slab sections are not satisfactorily dimensioned by ACI under the ℓ_n /240 deflection limit for span lengths greater than 10 to 15 ft as shown in Figs 4-25 and 4-26. Figs 4027 and 4-28 indicate the situation for a deflection limit of ℓ_n /480 which is clearly more severe.

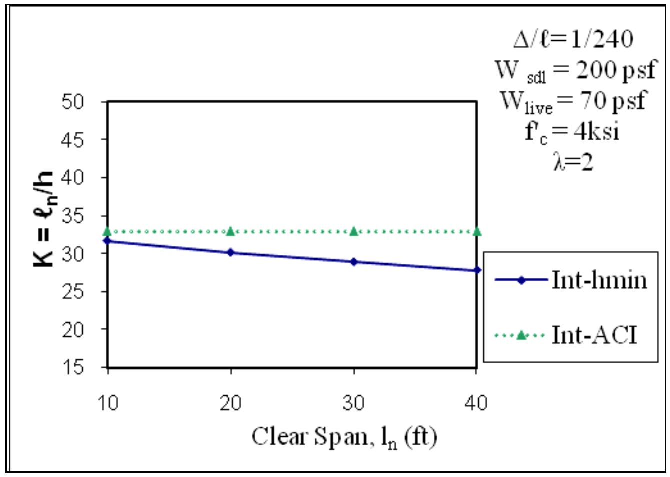

Fig 4-25: Span-to-depth ratio vs. clear span for interior panels without drop panels for deflection limit of ℓ_n /240 with superimposed dead load of 200 psf

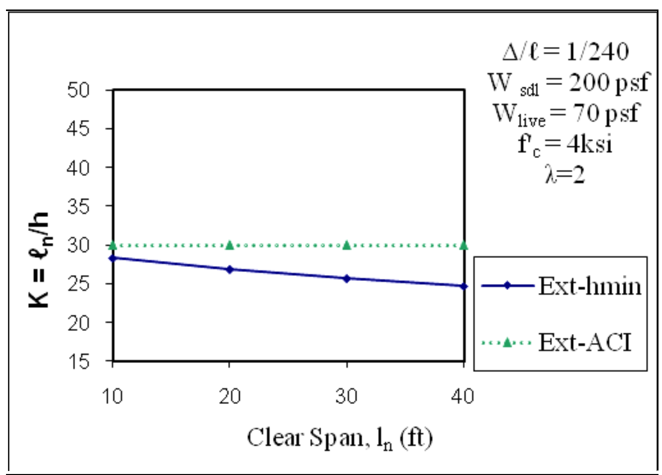

Fig 4-26: Span-to-depth ratio vs. clear span for exterior panels without drop panels for deflection limit of ℓ_n /240 with superimposed dead load of 200 psf

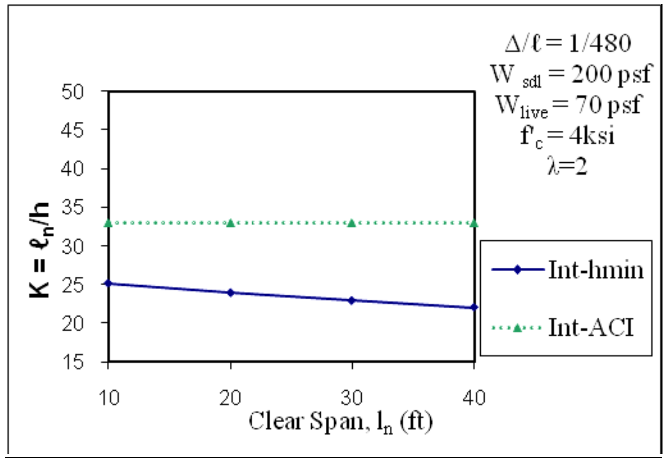

Fig 4-27: Span-to-depth ratio vs. clear span for interior panels without drop panels for deflection limit of ℓ_n /480 with superimposed dead load of 200 psf

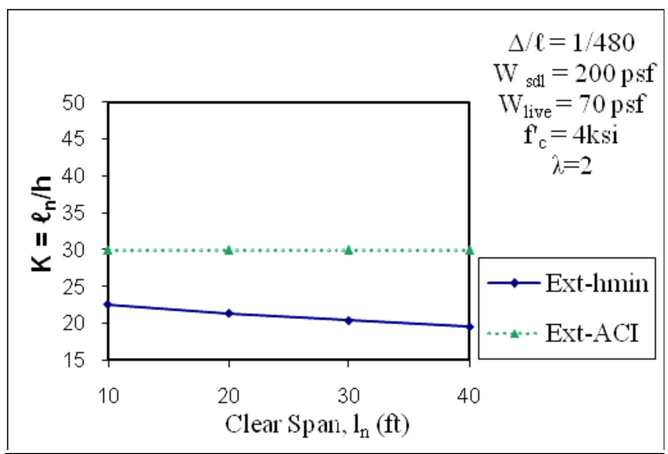

Fig 4-28: Span-to-depth ratio vs. clear span for exterior panels without drop panels for deflection limit of ℓ_n /480 with superimposed dead load of 200 psf

4.2.4 Long-term Multiplier, λ:

For the long-term multiplier, only one case was run and it is observed from the Figs 4-29 to 4-31 that for ℓ_n /240, ACI minimum thickness is adequate for a long-term mulplier value limit up to 3.0

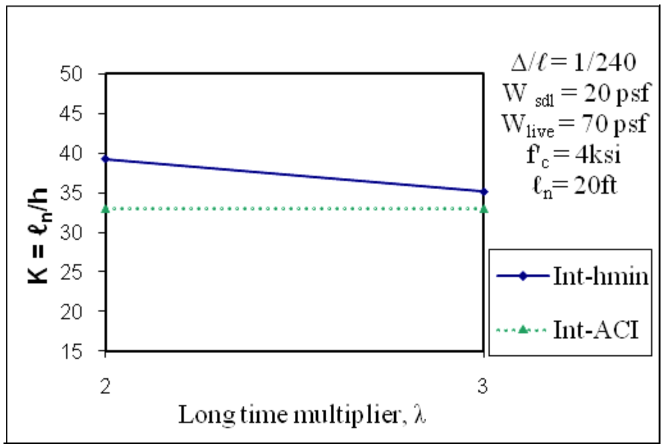

Fig 4-29: Span-to-depth ratio vs. long-term multiplier for interior panels without drop panels for deflection limit of ℓ_n /240 with superimposed dead load of 20 psf

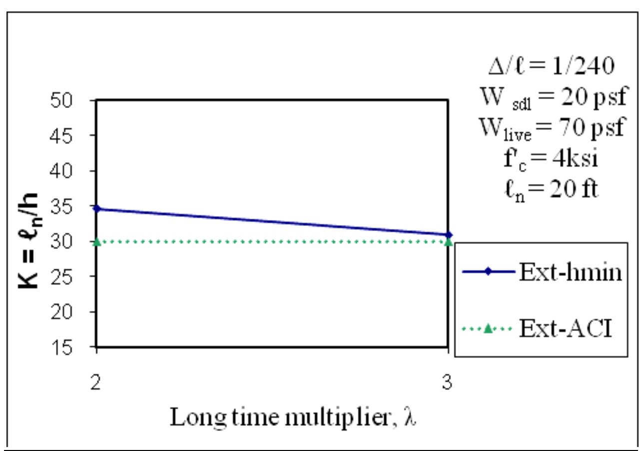

Fig 4-30: Span-to-depth ratio vs. long-term multiplier for exterior panels without drop panels for deflection limit of ℓ_n /240 with superimposed dead load of 20 psf

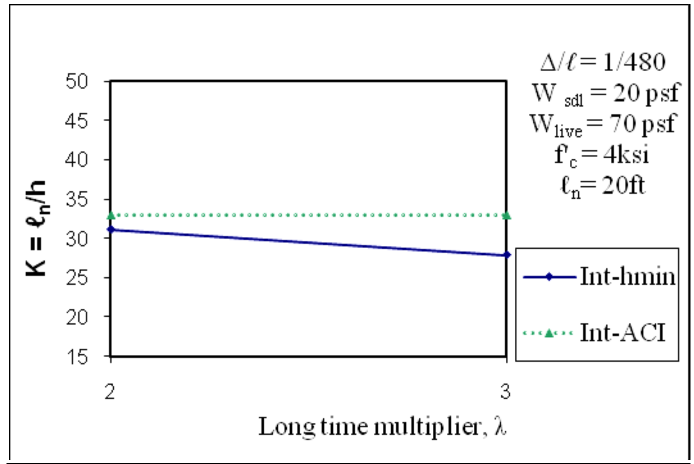

Fig 4-31: Span-to-depth ratio vs. long-term multiplier for interior panels without drop panels for deflection limit of ℓ_n /480 with superimposed dead load of 20 psf

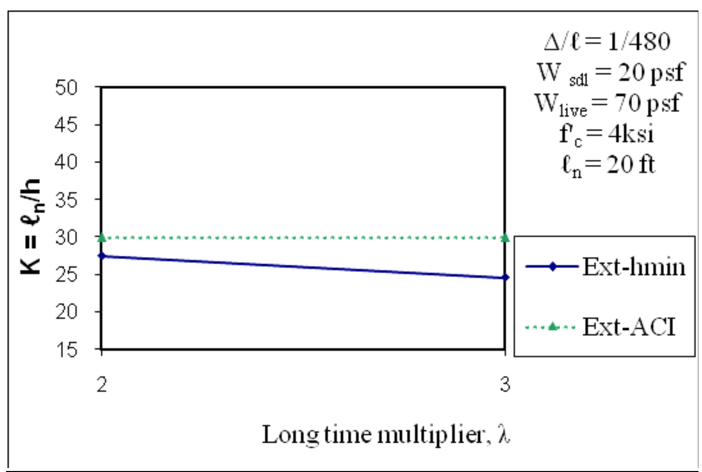

Fig 4-32: Span-to-depth ratio vs. long-term multiplier for exterior panels without drop panels for deflection limit of ℓ_n /480 with superimposed dead load of 20 psf

4.2.5 <u>**Concrete Strength, *f'c*:**</u>

As concrete's strength increases, so does the modulus of elasticty. Increase in concrete strength decreases the minimum thickness required. Figs 4-33 and 4-34 show that ACI minimum thickness values are adequate for concrete strengths greater than 3000 psi for the conditions indicated with a deflection limit of ℓ_n / 240. For the ℓ_n /480 limit, the concrete strength has to be incraesed to 6000 psi or more for the same conditions.

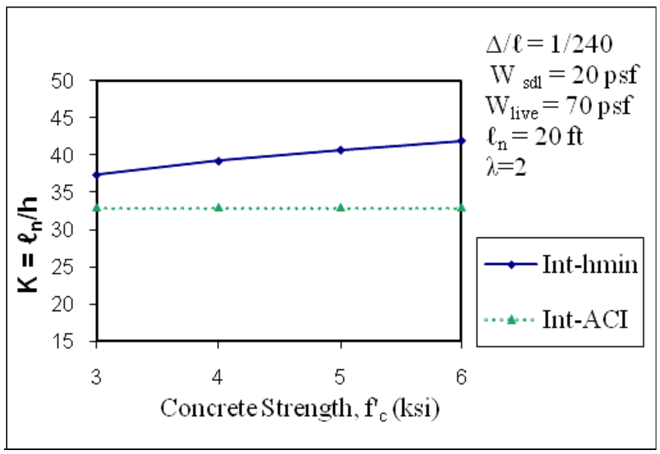

Fig 4-33: Span-to-depth ratio vs. concrete strength for interior panels without drop panels for deflection limit of ℓ_n /240 with span length of 20 ft

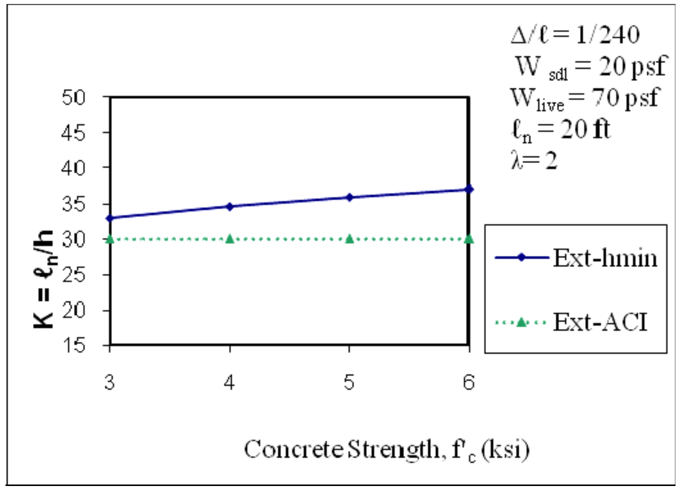

Fig 4-34: Span-to-depth ratio vs. concrete strength for exterior panels without drop panels for deflection limit of ℓ_n /240 with span length of 20 ft

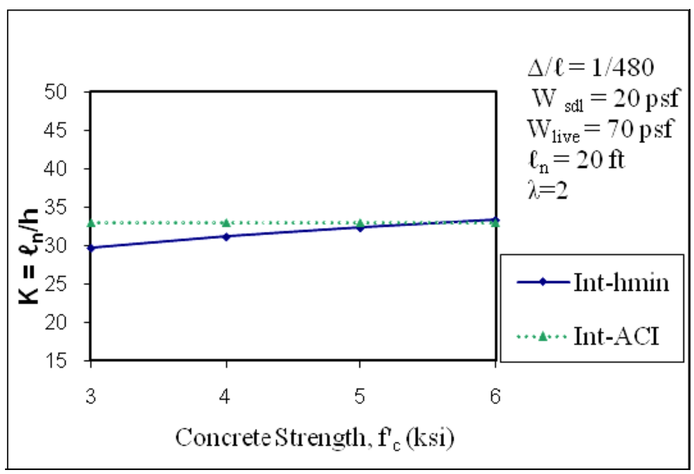

Fig 4-35: Span-to-depth ratio vs. concrete strength for interior panels without drop panels for deflection limit of ℓ_n /480 with span length of 20 ft

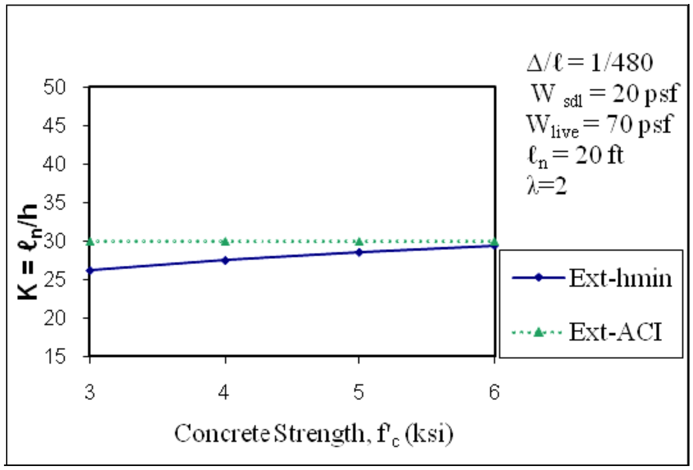

Fig 4-36: Span-to-depth ratio vs. concrete strength for exterior panels without drop panels for deflection limit of ℓ_n /480 with span length of 20 ft

Case 2: This study was done for a span of 30 ft. From Figs 4-37 and 4-38 it is observed that the ℓ_n /240 requirement is still met for the longer span. However for thr ℓ_n/480 limit, the thickness is not adequate at a span of 30 ft.

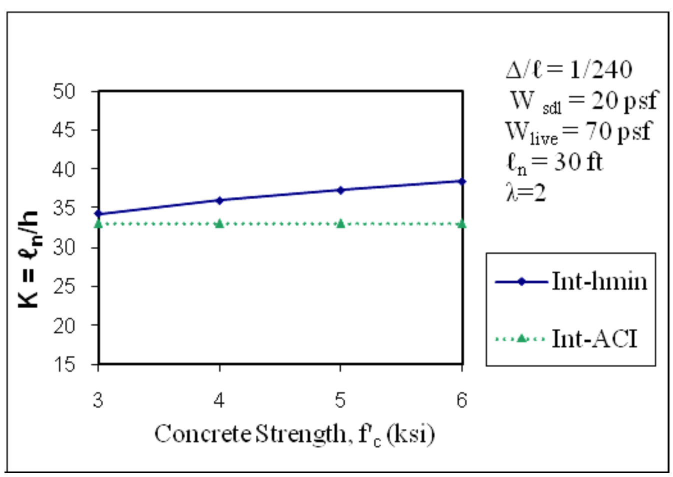

Fig 4-37: Span-to-depth ratio vs. concrete strength for interior panels without drop panels for deflection limit of ℓ_n /240 with span length of 30 ft

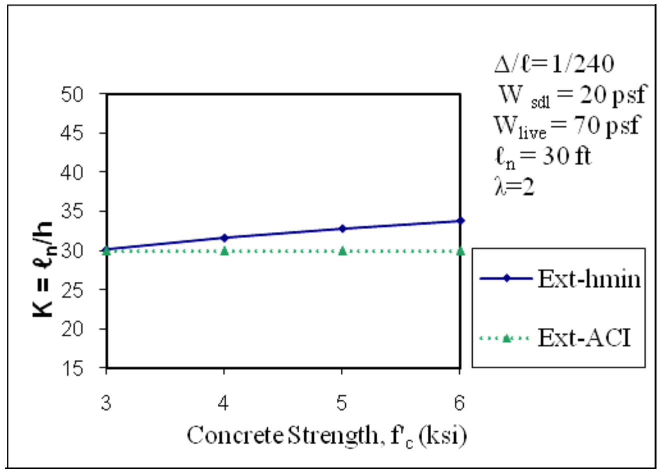

Fig 4-38: Span-to-depth ratio vs. concrete strength for exterior panels without drop panels for deflection limit of ℓ_n /240 with span length of 30 ft

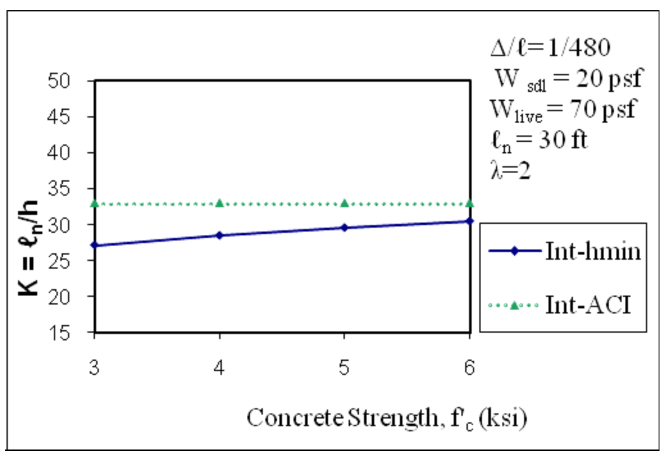

Fig 4-39: Span-to-depth ratio vs. concrete strength for interior panels without drop panels for deflection limit of ℓ_n /480 with span length of 30 ft

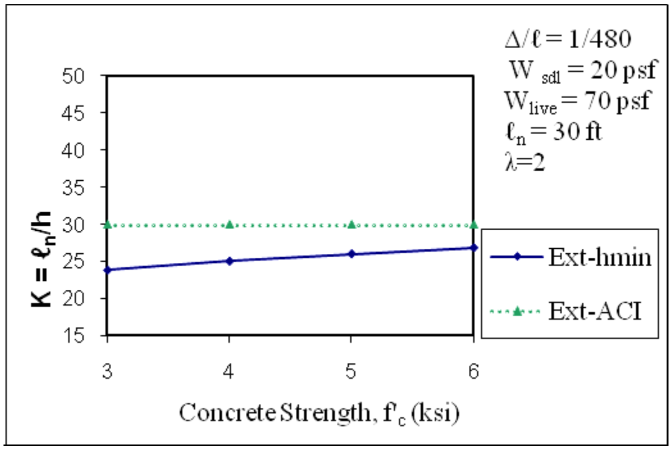

Fig 4-40: Span-to-depth ratio vs. concrete strength for exterior panels without drop panels for deflection limit of ℓ_n /480 with span length of 30 ft

<u>Summary:</u>

Results of the parametric study indicate that for typical building applications including residential and office occupancies the minimum thickness values given in Table 9.5(c) are adequate for slabs not supporting or attached to non-structural elements likely to be damaged by large deflections(ℓ_n /240 limit). However in most cases, the types of occupancies do involve non-structural elements likely to be damaged by large deflections and for these cases limitations should be placed on the application of Table 9.5 (c). For flat plates, an increase of 10% in thickness should be adequate under tha following conditions:

a) Longer clear span not greater than 20 ft

b) Superimposed dead load not greater than 20 psf

c) Live load not greater than 70 psf

d) Concrete compressive strength not less than 3000 psi

For design conditions falling outside this range, it is recommended that slab thickness be determined based on the equation proposed by Scanlon and Lee (2006) or justified based on calculation taking into account the effects of cracking, creep and shrinkage including the effects of construction loading and shrinkage restraint stresses as recommended by Scanlon and Bischoff (2008).

These recommendations are based on consideration of deflections that occur after installation of non-structural elements. An evaluation should be made of the proposed shoring/reshoring procedure to assure that deflections occuring prior to installation of non-structural elements are not excessive for the intended use of the structure (Motter, 2009).

Chapter 5

RECOMMENDATIONS FOR CODE REVISION

Based on the calculation of effective moment of inertia equation and results of parametric study on slab minimum thickness, the following recommendations are made for ACI 318:

- After the concrete section cracks, the moment of inertia should be computed using the formula, $I_e = \dfrac{I_{cr}}{\left[1-\left(\dfrac{M_{cr}}{M_a}\right)^m\left(1-\dfrac{I_{cr}}{I_g}\right)\right]}$

  Where, $m = 2$

  M_a = the moment due to both dead and live loads

  M_{cr} = the cracking moment $= f'_r * \dfrac{y_t}{I_g}$

  Where,

  $f'_r = 5 * \text{sqrt}\,(f'_c)$

- Proposed modifications to Table 9.5(c) refer summary in Chapter 4 (pp 57-58) of this thesis.
- For slabs not meeting the requirements for Table 9.5(c), minimum thickness may be obtained from:

  $h_{\min} = \dfrac{\ell_n}{K_\Delta}$, where

a) $K_\Delta = \left[\langle \frac{\Delta_{inc}}{\ell_n} \rangle \frac{1200 E_c}{(\lambda_\Delta w_{sus} + w_{Ladd} \rangle} \right]^{\frac{1}{3}}$ for slabs without interior beams, without drop panels, for interior panels and exterior panels with edge beams having α_f, aspect ratio not less than 0.8

b) $K_\Delta = 0.9 \left[\langle \frac{\Delta_{inc}}{\ell_n} \rangle \frac{1200 E_c}{(\lambda_\Delta w_{sus} + w_{Ladd} \rangle} \right]^{\frac{1}{3}}$ for exterior panels without edge beams

c) $K_\Delta = 1.1 \left[\langle \frac{\Delta_{inc}}{\ell_n} \rangle \frac{1200 E_c}{(\lambda_\Delta w_{sus} + w_{Ladd} \rangle} \right]^{\frac{1}{3}}$ for slabs without interior beams with drop panels for interior panels and exterior panels with edge beams having α_f not less than 0.8.

d) $K_\Delta = \left[\langle \frac{\Delta_{inc}}{\ell_n} \rangle \frac{1200 E_c}{(\lambda_\Delta w_{sus} + w_{Ladd} \rangle} \right]^{\frac{1}{3}}$ for exterior panels without edge beams

- Alternatively, slab thickness should be determined by calculation considering the effects of superimposed dead load, live load, span length, strength of concrete and long-term multiplier

Chapter 6

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

Summary:

This study was carried out with the idea to identify the shortcomings in section 9.5 of ACI 318 building code concerning deflections and minimum thickness requirements of two-way slabs, and to propose changes. The study is a result of extensive literature search, parametric studies and verifiable experimental results. Factors investigated include the influence of reinforcement ratio on effective moment of inertia, shrinkage restraint stresses in the computation of final deflections and experiments were conducted to verify these factors. Studies were also conducted on the inclusion of boundary condition factor, slabs, effect of loads, span lengths, concrete strength, and long-term multiplier on the deflection, in turn minimum thickness of flat plates and slabs. Analysis was conducted on the results obtained and recommendations were proposed to ACI 318 for revising the section 9.5 concerning minimum thickness requirements of two-way slabs and effective moment of inertia which would affect computation of final slab deflection.

Conclusions:

Bischoff's equation to compute effective moment of inertia is effective for all ranges of reinforcement unlike the existing Branson's equation, which is adopted by ACI and fails to accurately predict the deflections for low reinforcement ratio.

1. Restraint stresses, cracking and creep must be taken into account, which in turn effect the rupture modulus and hence the deflection characteristics of two-way slab members

2. Table 9.5(b) and Table 9.5(c) together do not provide enough thickness requirements for two-way slabs and hence must be either modified or presented with an alternate solution as recommended.

3. Restrictions on loading intensities, span lengths, concrete strength must be levied so as to have a far better estimate of the cracked section properties and to resist deflections, both long-term and short-term.

Recommendations:

Recommendations for changes to the ACI Building Code (ACI 318) are provided in Chapter 5 of this report.

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APPENDIX

- Table A: Excel spread sheet to compute Effective moment of inertia using Branson's and Bischoff's equation for various rupture modulus ratios

Simply supported beam			
GIVEN		check if it is OK to continue "1" if OK "0" if not	Remarks
enter steel yield strength ski, Fy	60		
enter concrete strength ksi,Fc	4		
enter factor of safety	0.9		
enter j	0.9		
enter span length in feet	20		
Enter beam width in inches	12		
Enter beam depth in inches	12		
live load to dead load ratio	0.7		
enter unit weight of concrete kips/ft^3	0.15		
enter support conditions at:			
A-roller			
B-pinned			
□	0.85		Changes with % reinforcement and Fc
Es, Psi	29000000		
Ec,Psi	3604997		
Flexure			
input Ma/Mu ratio	0.659		
Input or Calculate "d" in inches	11		
Gross Moment of inertia Ig	1728		
Corresponding percentage of reinforcement	0.018		
Area of steel required (sq.in) As	2.38		
a ,inches	3.49	1	
C	4.53		
Moment Capacity of Section k-ft	109.9		
Ma k-ft	72.4		

Deflection:			
fr = fr			
fr ,psi	474.3		
Mcr,K-ft	12.4		
Mu, K-ft	72.4	1	Mu>Mcr, section is cracked at service loads.
Mcr/Ma	0.171		
fr=fr/2			
fr ,psi	237.2		
Mcr,K-ft	6.2		
Mu, K-ft	72.4	1	Mu>Mcr, section is cracked at service loads.
Mcr/Ma	0.086		
fr=2fr/3			
fr ,psi	316.2		
Mcr,K-ft	8.3		
Mu, K-ft	72.4	1	Mu>Mcr, section is cracked at service loads.
Mcr/Ma	0.114		
n=Es/Ec	8.00		
ρ=As/(bw*d)	0.018		
ρ*n	0.144		
K	0.412		
Calculating Icr :			
Compression zone area, sq.in	54.3		
Y bar for compression zone area, inches	2.3		
I for compression area, in^4	92.8		
Steel area, sq.in	2.4		
Y bar for steel area, inches	6.5		
Icr	1167.5		
Icr/Ig	0.68		
Ie,Branson's fr=fr	1170.4	1170.4	1170.4
Ie/Ig Branson's fr=fr	0.677		

Ie,Branson's	fr=fr/2	1168.4	1168.4	1168.4
Ie/Ig Branson's fr=fr/2		0.676		
M		2		
Ie,Bischoff's fr=fr		1178.8	1178.8	1178.8
Ie/Ig Bischoff's fr=fr		0.682		
Ie,Bischoff's	fr=fr/2	1170.3	1170.3	1170.3
Ie/Ig Bischoff's fr=fr/2		0.677		
Ie,Bischoff's	fr=2fr/3	1172.5	1172.5	1172.5
Ie/Ig Bischoff's fr=2fr/3		0.679		

This spread sheet is modified to generate data for dead load only case, slab sections with different reinforcement ratios, reinforcement areas. The data so generated is plotted using MS-Excel.

- Table B: Excel Spreadsheet to compute h_{min} equation for various boundary conditions of Two-way slabs. Subject to the following conditions:

Factor	**Limit**	**Remarks**
β	=2	maximum
For, α_{fm}	<= 0.2	β =0.2
For, α_{fm}	= (0.2,2]	h_{min} >= 5 in
For, α_{fm}	>2.0	h_{min} >=5 in

Table C: Table to enter constants and variables for parametric study of h_{min} equation

ENTER		
Factor	**Factor Value**	**Remarks**
span length, L	31	in feet
enter column dimension along the span direction	12	in inches
ℓn	30	span length - column dimension in feet
ℓn short	30	clear span in short direction
$\Delta/\ell n$	0.0021	from the Table 9.5(b) ACI 318-08
f_y	60000	in psi
f'_c	4000	in psi for beam
f'_c	4000	in psi for slab
W_{sdl}	200	sustained load in psf
α	0.1	percentage of live load
W_{live}	70	In psf
W_{add}	63	$W_{live}(1-\alpha)$; Additional live load in psf
λ_D	2	5 years or more
β	1	ratio of long side clear span to short side clear span

The spreadsheet presented below shows the minimum thickness calculated for each case as listed in the remarks column, against the ACI specified limits. Parametric studies are conducted by inputting variables in Table C. Then the plots are drawn for the exterior and interior panels for two limits of deflection as mentioned in Chapter 4 of this thesis and comparison is made with the existing ACI code 318's limitations. The remarks column includes the units of the terms, boundary conditions of the cases studied.

Table D: H_{min} equation compared against ACI 318-08 specifications, as in Table 1-3 of this thesis

Calculate Factor	W dead, psf	Factor Value	Remarks	ACI Table 9.5© limits	Minimum thickness required (inches)
Ec		3604996.53	in psi, Young's Modulus of Concrete		
K	136.36	22.91	Slabs without interior beams, without drop panels, for interior and exterior panels with edge beams having ***af*** not less than 0.8		
h calc		15.72	-----------------in inches	ℓn/33	10.91
K	150.00	20.37	Exterior panels without edge beams		
h calc		17.67	-----------------in inches	ℓn/30	12.00
K	125.00	25.20	Slabs without interior beams, with drop panels, for interior and exterior panels with edge beams having ***af*** not less than 0.8		
h calc		14.29	---------------- in inches	ℓn/36	10.00
K	136.36	22.91	for exterior panels without edge beams		
h calc		15.72	----------------- in inches	ℓn/33	10.91