HOT HALO GAS IN NUMERICAL SIMULATIONS OF GALAXY MERGERS

A Dissertation in
Astronomy and Astrophysics
by
Manodeep Sinha

© 2008 Manodeep Sinha

Submitted in Partial Fulfillment
of the Requirements
for the Degree of

Doctor of Philosophy

December 2008
The dissertation of Manodeep Sinha was reviewed and approved* by the following:

Robin Ciardullo  
Professor of Astronomy & Astrophysics  
Dissertation Advisor, Chair of Committee

Pablo Laguna  
Professor of Astronomy & Astrophysics

George Chartas  
Sr. Research Associate of Astronomy & Astrophysics

Caryl Gronwall  
Sr. Research Associate of Astronomy & Astrophysics

Richard W. Robinett  
Professor of Physics

Eric Feigelson  
Professor of Astronomy and Astrophysics  
Asst. Head of the Department

*Signatures are on file in the Graduate School.
Abstract

Galaxy merger simulations have explored the behavior of gas within a galactic disk, yet the dynamics of hot gas within the galaxy halo has been neglected. We report on the results of high-resolution hydrodynamic simulations of colliding galaxies with hot halo gas. We explore a range of mass ratios, gas fractions and orbital configurations to constrain the shocks and the dynamics of the gas within the progenitor halos. We find that:

(i) A strong shock is produced in the galaxy halos before the first passage, increasing the temperature of the gas by almost an order of magnitude to $\sim 10^{6.3}$ K.

(ii) The X-ray luminosity of the shock is strongly dependent on the gas fraction. It is $\gtrsim 10^{39}$ erg/s for gas fractions larger than 10%.

(iii) We find an analytic fit to the maximum X-ray luminosity of the shock as a function of merger parameters. This fit can be used in semi-analytic recipes for galaxy formation to estimate the total X-ray emission from shocks in merging galaxies.

(iv) The hot diffuse gas in the simulation also produces X-ray luminosities as large as $10^{42}$ erg/s. This contributes to the total X-ray background in the Universe.

(v) $\sim 10$-20% of the initial gas mass is unbound from the galaxies for equal-mass mergers, while 3 – 5% of the gas mass is released for the 3:1 and 10:1 mergers. This unbound gas ends up far from the galaxy and can be a feasible mechanism for metal enrichment of the IGM. We use an analytical halo merger tree to estimate the fraction of gas mass lost over the history of the Universe.
# Table of Contents

List of Figures .................................................. vii
List of Tables ................................................. ix
Acknowledgments .................................................. x

Chapter 1
Introduction .................................................... 1
1.1 Motivation .................................................... 1
1.2 Observations of Structure in the Universe ................. 5
1.3 The Mathematical Universe ................................. 10
1.4 Galaxy Formation ........................................... 14
   1.4.1 Dark Matter in Galaxies .............................. 14
   1.4.2 Gravitational Collapse .............................. 15
   1.4.3 Structure Formation in the ΛCDM Universe ...... 16
1.5 Merger Rates and Ratios ................................... 21
   1.5.1 Merger Rates from Analytic Theories ............... 21
1.6 The Inter-galactic Medium .................................. 26
1.7 Overview .................................................... 32

Chapter 2
N-body Techniques & Tools ..................................... 33
2.1 Introduction ................................................ 33
2.2 Collisionless Systems ..................................... 33
2.3 Distribution Function ..................................... 34
2.4 Acceptance-Rejection Technique ......................... 36
2.5 Simulation Software ..................................... 37
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.6</td>
<td>N-body Mechanics</td>
<td>39</td>
</tr>
<tr>
<td>2.7</td>
<td>Smooth Particle Hydrodynamics</td>
<td>43</td>
</tr>
<tr>
<td>2.8</td>
<td>Time Integration</td>
<td>46</td>
</tr>
<tr>
<td>2.9</td>
<td>Overview</td>
<td>47</td>
</tr>
<tr>
<td>Chapter 3</td>
<td>Generating the Galaxy Model</td>
<td>48</td>
</tr>
<tr>
<td>3.1</td>
<td>Dark Matter</td>
<td>48</td>
</tr>
<tr>
<td>3.2</td>
<td>Gas Initialization</td>
<td>51</td>
</tr>
<tr>
<td>3.3</td>
<td>Stability</td>
<td>52</td>
</tr>
<tr>
<td>3.3.1</td>
<td>Softening Length</td>
<td>57</td>
</tr>
<tr>
<td>3.3.2</td>
<td>Artificial Viscosity</td>
<td>58</td>
</tr>
<tr>
<td>3.4</td>
<td>Merger Orbits</td>
<td>58</td>
</tr>
<tr>
<td>3.5</td>
<td>Analysis</td>
<td>63</td>
</tr>
<tr>
<td>3.5.1</td>
<td>X-ray emission</td>
<td>63</td>
</tr>
<tr>
<td>3.5.2</td>
<td>Shock Detection</td>
<td>64</td>
</tr>
<tr>
<td>3.5.3</td>
<td>Unbound particles</td>
<td>65</td>
</tr>
<tr>
<td>3.6</td>
<td>Overview</td>
<td>66</td>
</tr>
<tr>
<td>Chapter 4</td>
<td>Results</td>
<td>67</td>
</tr>
<tr>
<td>4.1</td>
<td>Introduction</td>
<td>67</td>
</tr>
<tr>
<td>4.2</td>
<td>Testing the Artificial Viscosity Parameter</td>
<td>68</td>
</tr>
<tr>
<td>4.3</td>
<td>Merger Simulations</td>
<td>74</td>
</tr>
<tr>
<td>4.3.1</td>
<td>X-ray emission</td>
<td>74</td>
</tr>
<tr>
<td>4.3.2</td>
<td>Unbound Material</td>
<td>107</td>
</tr>
<tr>
<td>4.3.3</td>
<td>Estimating the Unbound Material over the History of the Universe</td>
<td>114</td>
</tr>
<tr>
<td>Chapter 5</td>
<td>Conclusion &amp; Future Work</td>
<td>122</td>
</tr>
<tr>
<td>5.1</td>
<td>X-ray emission</td>
<td>122</td>
</tr>
<tr>
<td>5.2</td>
<td>Unbound gas</td>
<td>124</td>
</tr>
<tr>
<td>5.3</td>
<td>Future Work</td>
<td>125</td>
</tr>
<tr>
<td>Appendix A</td>
<td>Galaxy initialization - a C code</td>
<td>128</td>
</tr>
<tr>
<td>Appendix B</td>
<td>GADGET-2 parameter file</td>
<td>151</td>
</tr>
</tbody>
</table>
List of Figures

1.1 A Chandra view of NGC 5746 overlaid on optical data from DSS . . 3
1.2 The CMB signature as seen by WMAP . . . . . . . . . . . . . . . . 9
1.3 The parameters of the Universe . . . . . . . . . . . . . . . . . . . . 13
1.4 The Millennium Simulation showing structure formation . . . . . . 20
1.5 The evolution of merger rates and merger ratio with redshift . . . . 22
1.6 The number of halos as a function of mass and redshift . . . . . . . 25
1.7 Distribution of metals around galaxies . . . . . . . . . . . . . . . . 30
1.8 The WHIM from simulations with galactic super-winds . . . . . . . 31

2.1 Monte-Carlo estimate of the area of quadrant of an unit circle . . . 37
2.2 Flow-chart for acceptance-rejection technique . . . . . . . . . . . . 38
2.3 Schematic for a Barnes-Hut tree construction . . . . . . . . . . . . 42

3.1 Distribution functions for three NFW halos . . . . . . . . . . . . . . 50
3.2 Iterations for a stable temperature profile . . . . . . . . . . . . . . 54
3.3 Density profiles for the nine isolated galaxies . . . . . . . . . . . . 59
3.4 Temperature profiles for the nine isolated galaxies . . . . . . . . . 60
3.5 Gas density and temperature evolution for different AVP . . . . . . 61

4.1 Gas temperature and density for the merger remnant for equal-mass
merger with 10% gas and $b = 22.8$ kpc . . . . . . . . . . . . . . . . 70
4.2 Evolution of hot gas fraction and total X-ray emission for equal-
mass merger with 10% gas and $b = 22.8$ kpc . . . . . . . . . . . . . 71
4.3 Projected temperatures for an equal-mass merger with 10% gas
fraction and $b = 22.8$ kpc for 3 different artificial viscosities . . . 72
4.4 Evolution of unbound fraction for a equal-mass merger with 10%
gas and $b = 22.8$ kpc . . . . . . . . . . . . . . . . . . . . . . . . . 73
4.5 The evolution of the hot gas fraction in all the equal-mass mergers . 78
4.6 The X-ray luminosity from hot gas and shocked gas in all the equal-
mass mergers . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 79
List of Tables

3.1 Input parameters for the isolated galaxy models ................. 56
3.2 Evolution of isolated galaxies over 1 Gyr .......................... 57
3.3 Table of orbital parameters for all the simulations ............... 62

4.1 This tables shows the properties of the remnant for the 3 simulations with $10^{-4}$, 0.5 & 1.0 artificial viscosity. ............... 69
4.2 Final stage of isolated galaxies after 15 Gyr ......................... 107
4.3 Table for all the simulations showing the fraction of shocked un- bound gas ......................................................... 111
4.4 Properties of the merger remnant for all the simulations ........ 120
First and foremost, I would like to thank my advisor, Kelly Holley-Bockelmann for her guidance through the last two years. Her attention to detail is quite admirable and I have learnt many a thing about numerical simulations through our brain-storming sessions. Due to some rather unfortunate circumstances, she is not officially present on my thesis document but without her this thesis would not have seen the light of day. I would also like to thank Tom Abel, with whom I started working on this project and Mercedes Richards, who kindly supported at some unfortunate circumstances. I thank my thesis committee members, Pablo Laguna, Robin Ciardullo, George Chartas and David Weiss for their involvement in shaping this thesis. I would like to thank Andreas Berlind for sharing the code for generating halo mass functions.

Life in graduate school would have been boring without the colorful group of people that I encountered during my stay at Penn State. I would like to thank Dave, Cate, Miroslav, Britton, John, Avi, Ken, Kim, Helene, Bret, Anand, Judy, Cristian, Nino, Suvrath, Tamara, Jie and most people in the department for making the international grad school experience memorable. My fabulous flat-mates, Kd and Prabhat, were understanding enough to withstand me and I am grateful for that. I am thankful to Vasudha, Ravi and Srilatha for providing me with some amazing food, Sam for getting me those late-night coffees, Nonu for the endless untimely cups of tea, Ritu for being intellectually stimulating, Zeynep, Rambo and Harshini for being ideal coffee buddies, Awnish, Manishi, Laddu, Yogesh, Ankur, Ajeet for all our time spent on the weekend with cards and volleyball, and to Patricia for attempting to get the best astrophysicist out of me. I am also thankful to the people at Vanderbilt and especially to the staff at ACCRE, the computational facility at Vanderbilt, where all the simulations were run.

This thesis would probably not have been possible without the extreme support, both emotional and financial from my friends, Madhu, Chaf, Chors, CJ and Suni. I am really grateful to have their enthusiastic support. Last but not the least, I am thankful to my parents and my elder brother for encouraging me to do whatever I
liked and for always believing in me. My thesis is dedicated to them.
1.1 Motivation

Galaxy mergers play an important part in galaxy evolution since they are one of the main mechanisms by which galaxies grow. In addition, mergers are one of the primary formation routes for elliptical galaxies. Consequently, galaxy mergers have been studied quite extensively both numerically (Holmberg, 1941; Toomre & Toomre, 1972; Barnes & Hernquist, 1992, 1996; Mihos & Hernquist, 1996; Springel & White, 1999; Dubinski et al., 1999; Barnes, 2002) and observationally (Schweizer, 1986; Roberts et al., 2002; Laine et al., 2003; Casasola et al., 2004). The merger process is also accompanied by a strong gas inflow towards the central region (Barnes & Hernquist, 1991; Mihos & Hernquist, 1994; Combes et al., 1994; Sanders & Mirabel, 1996) causing a central starburst accompanied by a possible AGN. However, none of the galaxy merger simulations included hot gas in the galactic halo that is expected from standard galaxy formation theory (White & Rees, 1978; White & Frenk, 1991).

Semi-analytic galaxy formation models have long predicted the existence of an extended reservoir of hot halo gas around spiral galaxies (White & Rees, 1978; White & Frenk, 1991). In that scenario, gas falling into the dark matter potential shock-heats to the virial temperature, $T_{\text{vir}} = 10^6 K \left(\frac{v_{\text{circ}}}{167 \text{ km/s}}\right)^2$ and subsequently cools over a characteristic time-scale, $t_c = kT/n_i \Lambda(T)$, that depends on the temperature $T$, the number density $n_i$ and the cooling function $\Lambda(T)$. This gas cools via thermal bremsstrahlung and atomic line emission processes, with the
majority of the radiation occurring in the soft X-ray band. The expected X-ray luminosity, $L_X$, is a steep function of the circular velocity, $v_{\text{circ}} - L_X \propto v_{\text{circ}}^5$. Since the effect of mass (circular velocity) on this halo X-ray luminosity is so non-linear, any attempts to observe the X-ray signature of this gas would naturally be biased towards very massive halos. The existence of hot gas in massive elliptical galaxies has been shown by X-ray observations of cluster galaxies as well as in isolated ellipticals (see Mathews & Brighenti, 2003, and references therein). The most prominent proof of the gaseous halo comes from the observation of O'Sullivan et al. (2001) showing that for $L_B \gtrsim 3 \times 10^9 L_{B,\odot}$, $L_X \propto L_B^2$, indicating the non-stellar origin of the X-ray luminosity. However, the only significant detection of extra-planar gas in quiescent spiral galaxies was done by Pedersen et al. (2006) (see Fig 1.1), which proved the existence of an extended ($\gtrsim 20$ kpc), hot ($6.5 \times 10^6$ K), faint ($L_X \sim 4.4 \times 10^{39}$ erg/s in 0.3 – 2.0 keV), diffuse X-ray halo in the massive spiral galaxy NGC 5746 ($v_{\text{circ}} = 307$ km/s, $T_{\text{vir}} = 3.6 \times 10^6$ K). In addition, the existence of a rarefied ($n < 10^{-4} - 10^{-5}$) cm$^{-3}$, hot ($T > 10^6$K), extended ($R > 70$ kpc) galactic halo has been shown from the observation of OVI absorption lines in high velocity clouds in the vicinity of the Milky Way (Sembach et al., 2003). The halo gas density expected for a galaxy similar to the Milky Way is $n_H \sim 8 \times 10^{-5}$cm$^{-3}$ at 100 kpc; with observations constraining $n_h \lesssim (1 - 10) \times 10^{-5}$cm$^{-3}$ (Murali, 2000; Snowden et al., 1997). Given the existence of hot halo gas, it is important to include such a component in the simulations of galaxy mergers.

It is not clear how hot halo gas behaves dynamically during a merger and whether it affects the surrounding inter-galactic medium (IGM) and its evolution. Conceptually however, this hot halo gas may play an important role in enriching the IGM. One of the outstanding issues in large scale structure formation is understanding the evolution of the IGM – a major component in the baryon budget of the universe, containing more than 50% of the baryons at $z = 0$ (Fukugita & Peebles, 2004; Danforth & Shull, 2008). The metallicity distribution in the IGM is non-homogeneous with higher (over)density regions containing a larger fraction of the metals (Cen & Ostriker, 1999). Historically, supernova (SN) feedback and starburst assisted super-winds have been invoked to address this issue but the effect of SN feedback alone is about an order of magnitude smaller than the required metal pollution of the IGM (Ferrara et al., 2000). In addition, SN feedback requires...
some other mechanism to mix metals on length scales of the order of inter-galactic distances. Gnedin & Ostriker (1997) suggest galaxy mergers as a viable method to eject metal-enriched gas into the IGM; this process works in addition to regular SN feedback and super-winds. Our set of simulations will also explore whether galaxy mergers can expel gas from the merger remnant and enrich the IGM with metals.

The merger process involves transfer of orbital kinetic energy into the kinetic energy of the stellar component via dynamical friction. The retardation due to dynamical friction on an object of mass $M$ moving through background density $\rho$
with a velocity $v_M$ is given by (Binney & Tremaine, 1987) –

$$\frac{dv_M}{dt} = -\frac{4\pi \ln \Lambda G^2 \rho M}{v_M^3} \left[ \text{erf}(X) - \frac{2X}{\sqrt{\pi}} e^{-X^2} \right] v_M \quad (M \gg m) \quad (1.1)$$

where, $X \equiv v_M/\left(\sqrt{2}\sigma\right)$, $\Lambda = b_{\text{max}} V_0^2 / G (M + m)$, $m$ is the mass, and $\sigma$ is the velocity dispersion of the individual elements constituting the background. $V_0$ is the velocity of the perturber, with mass $M$, at infinite distance from the background. This loss of orbital kinetic energy via dynamical friction is ultimately responsible for the formation of merger remnants.

Another mechanism that facilitates the loss of orbital kinetic energy applies specifically to gas. Bulk motion in the gas is converted into thermal energy of the gas through the formation of shocks. The orbital velocity of the gas during the encounter is $\geq 100$ km/s whereas the sound speed in the gas is $\sim 10$ km/s. Thus, strong shocks with Mach number $> 10$ are produced in the gas. The Rankine-Hugoniot shock jump conditions governing the conditions at the shock front can be re-written in the adiabatic case as

$$\frac{v_2}{v_1} = \frac{\rho_1}{\rho_2} = \frac{\gamma - 1}{\gamma + 1} + \frac{2}{\gamma + 1} \frac{1}{M^2}. \quad (1.2)$$

where $M$ is the Mach number. For strong shocks, $M \gg 1$ and $\rho_2/\rho_1 = 4$. Assuming $\gamma = 5/3$, the preceding equation yields (Ferrara & Salvaterra, 2004)

$$T_{\text{final}} = \frac{3\mu m_H}{16k} v_{\text{initial}}^2 \quad (1.3)$$

where $T_{\text{final}}$ is the gas temperature after the shock, $\mu$ is the mean molecular weight, $m_H$ is the proton mass, $k$ is the Boltzmann constant, and $v_{\text{initial}}$ is approximately equal to the radial velocity prior to the collision. This implies that nearly 50% of the kinetic energy gets converted into thermal energy of the gas particles.

Before we delve straight into the topic, we will review the observations and theories of structure formation in the Universe. In the following sections, we will establish the currently accepted mathematical treatment for the Universe, describe galaxy formation theories, and look at the outstanding issues in the field. We will also motivate the various simulations we perform.
1.2 Observations of Structure in the Universe

We understand the nature of our Universe by making observations and then creating the theories to explain those observations. Modern Cosmology can be linked to the observations of Edwin Hubble with his interpretation that galaxies were distant systems, ‘island universes’, composed of stars and gas, much like our own Milky Way. Through galaxy counts, Hubble found that the number of galaxies increased as the square of the distance, irrespective of the line of sight; this ultimately gave rise to the Cosmological Principle which postulates –

- The Universe is homogeneous on the large-scale – no matter where you are in the Universe, it will appear the same when compared to any other location. For instance, this implies that along one line of sight, we will always encounter the same number of galaxies in the same volume. The homogeneity of our Universe holds when looking at regions of size $\gtrsim 300$ Mpc (Wu, Lahav, & Rees, 1999; Mandolesi, Calzolari, Cortiglioni, Delpino, & Sironi, 1986).

- The Universe is isotropic – the Universe looks the same irrespective of the direction it is being viewed in. From a particular vantage point in the Universe, the number of galaxies counted along any line of sight will result in the same numbers (Wu et al., 1999).

Note that homogeneity does not imply isotropy; a crosswalk appears homogeneous only in the direction perpendicular to the stripes but clearly is not isotropic along any other direction. Similarly, from the top of a hill, the views to all sides may appear isotropic but the view will change from the bottom of the hill. Thus, even though the observer at the top of the hill may see the scenery as an isotropic world, in reality it is not so.

Hubble’s other discovery that all galaxies were moving away from one another, and at a speed increasing linearly with the increasing separation, lead to the idea of the expanding Universe. This is referred as Hubble’s law and relates the velocity of recession, $v_{\text{rec}}$, with the separation, $d$ as:

\[ v = H_0 d, \quad (1.4) \]
where \( H_0 = 72.4 \) km/s/Mpc is the Hubble constant (from the WMAP 5-year data release, Komatsu et al. (2008)). We can immediately make a guess about the age of the Universe; assuming that the Hubble constant was a ‘constant’ in the past, the total time that galaxies have taken to reach the current separation of \( d \) at the velocity of \( v \) is \(- t = d/v = 1/H_0 \). For the accepted value of \( H_0 \), this yields an age of \( \sim 13.5 \) Gyrs, remarkably close to the more accurate value of \( 13.7 \pm 0.1 \) Gyrs obtained from WMAP (Komatsu et al., 2008). When combined with the Cosmological Principle, this increasing recessional velocity of galaxies that is proportional to the separation could be reconciled only if the space-time fabric itself was expanding. Two galaxies, initially separated by some distance \( d_0 \) at time \( t_0 \), will be separated by \( d(t) = a(t) d_0 \) at any time \( t \), where \( a(t) \) is the expansion factor of the Universe. The expansion factor of the Universe controls how fast the Universe is expanding and can only be a function of time in keeping with the Cosmological Principle. The relative rate of recession can then be computed as –

\[
\begin{align*}
  v(t) &= \frac{da(t)}{dt} \\
       &= \frac{\dot{a}}{a} d_0 \\
       &= \frac{\dot{a}}{a} d(t) \\
       &= H(t) d(t).
\end{align*}
\]

Thus, we can equate the Hubble constant with the expansion factor of the Universe and show that the value of the Hubble Constant actually varies with time and can be obtained via the relation \( H(t) = \dot{a}/a \).

In such an expanding Universe, a photon suffers from a particular kind of Doppler shift that always changes the wavelength towards the less energetic range of the spectrum, or a redshift. The redshift, \( z \), that is defined by the ratio of the emitted wavelength of the photon at time \( t \), to the observed wavelength at time, \( t = 0 \), is related to the expansion factor of the Universe as \( z = a_0/a_t - 1 \), where we can assume \( a_0 = 1 \) and the equation reduces to \( a(t) = (1 + z)^{-1} \). For objects that are relatively near-by, a straight-forward relationship holds between the recession velocity and the redshift, \( v = c z \), where \( c \) is the speed of light. Now, the problem

\(^2\)If the expansion factor were to depend on some spatial dimension then the homogeneity and isotropy of the Universe would be violated since some region of the Universe would have a different evolution history than the rest.
that immediately surfaces with the idea of galaxies flying apart is that, going back in time, they get closer and closer. Therefore, just from the observation of galaxies receding currently (and in the past, as seen from the observations of the farthest, and hence the most distant past) one infers that the galaxies started out in in one infinitesimally small region of space-time. This idea provides that basis for the Standard Big Bang Model which postulates that the Universe started out in a cosmic singularity, from a size of the order of Planck length, $\sim 10^{-35}$ meters\(^3\).

For some unexplained reason (so far), they started to go apart, and have been expanding since then. One of the biggest triumphs of the Big Bang Model came with the prediction (Alpher & Herman, 1948) and the subsequent detection of the Cosmic Microwave Background (hereafter, CMB) (Penzias & Wilson, 1965). The CMB reflects the relic photons from a time when matter (in the form of electrons and heavy nuclei) and photons got decoupled; electrons and nuclei started making neutral atoms for the first time, and the photons, now without any electrons to interact with, free-stream through the Universe. The spectrum was predicted to be a black-body with a peak temperature near 3 K; this has been verified in numerous experiments over the last 40 years (Smoot et al., 1992; Mather et al., 1999; Spergel et al., 2003, 2007). The absolute homogeneity and isotropy of CMB is testament to the Cosmological Principle; if the Universe were to not possess either of those qualities, they would show up as irregularities in CMB. However, the CMB uniformity immediately raised other questions –

- If the Universe were perfectly uniform, then there would be no initial density fluctuations, viz., no over-dense or under-dense regions. Given that the only relevant large-scale force (gravity) works to enhance density contrasts, then it is impossible to create any structure. We know that this is not true since we live in a galaxy and can observe billions of other galaxies in the galaxy. Thus, there had to be some ‘anisotropies’ in the CMB. These anisotropies were first detected by CMB experiments in the late 1980’s and were 1 part in $10^5$ – the exact density contrast we would expect from looking at the current density ratios between the critical density and the largest galaxies.

- CMB was too uniform over the entire sky. Now, if a point from one end

\[^3\text{A similar quantity exists for time, viz., Planck time which is the amount of time taken by light to cross a Planck length and is } \sim 10^{-43} \text{ seconds.}\]
of the sky had expanded over the age of Universe, the point on the exact opposite end of sky had done the same and therefore these two points did not have time to communicate with each other (since the light travel-time is twice the age of the Universe between these two points). Therefore, there is no reason for these two points to show the exact same black-body spectrum for CMB. This is the ‘Horizon Problem’ in Cosmology.

- CMB showed that the density of the Universe is very close to the critical density. This is very difficult to achieve over the life-time of the Universe unless the value was exactly equal to 1 at the beginning. This is the ‘Flatness Problem’ in Cosmology.

Both these problems can be reconciled if we assume that the Universe went through a phase of super-luminal expansion where the size increased exponentially within a fraction of a second. This idea, first proposed by Alexei Starobinsky in the late 70’s, and later by Alan Guth in his seminal paper in 1981, is called inflation. Inflation causes the Universe to increase in size by a factor of $\sim 10^{30}$ in a time-interval of $10^{-32}$ seconds! Clearly, this is much much faster than the speed of light, but since no information is propagated, no physical principle gets violated. This super-charged expansion allows for particles at the opposite ends of the sky at present to be in causal contact at the very beginning of the Universe and thus, have identical CMB temperatures. The inflationary scenario also smooths out any fluctuations that were originally present in the space-time; since any 'wrinkle' in space-time will get smoothed out once extended by a factor of $10^{30}$. This is analogous to the situation where the surface of the Earth appears flat to a human on the ground; in the infinitesimal limit, a curved surface is essentially straight. Thus, another natural outcome of the inflationary paradigm is the flatness of the Universe. This fits very spectacularly with the observed homogeneity and isotropy of the CMB (see Fig. 1.2). Since inflation arises out of quantum processes, it leaves a signature on the space-time via density perturbations – seeds for structure formation at later times.
Figure 1.2. The CMB temperature fluctuations from the 5-year WMAP data seen over the full sky. The average temperature is 2.72 K, and the colors represent the tiny temperature fluctuations. Red regions are warmer and blue regions are colder by about $2 \times 10^{-4}$ K. Figure courtesy WMAP team of NASA.
1.3 The Mathematical Universe

The homogeneous, isotropic and expanding Universe can be described only by the Friedmann-Lemaître-Robertson-Walker (hereafter, LFRW) metric –

\[ ds^2 = -dt^2 + a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right], \]

(1.6)

where, \((t, r, \theta, \phi)\) constitute the co-ordinate system and the constant \(k\) determines the shape of the Universe. \(k\) can take on three values, \(-1\) for an open Universe, \(0\) for a flat Universe and \(+1\) for a closed Universe. Many theorists suggest that the fact that the Universe exists today almost necessarily requires a flat Universe with the average density of the Universe equal to a critical value, \(\rho_{\text{crit}}(t) = 3H^2(t)/8\pi G\). A Universe with density much larger than \(\rho_{\text{crit}}\) would have collapsed to an infinitesimal size within a Planck time; similarly, an Universe with much smaller density would have expanded to an infinite (and essentially zero density) within a Planck time. This is just another way of looking at the ‘Flatness Problem’ – an old Universe has to be very close to being flat.

The Einstein field equations of General Relativity for the LFRW metric then reduce to –

\[ \frac{\dot{a}^2}{a} = \frac{8\pi G}{3} \rho + \frac{\Lambda}{3} - \frac{k}{a^2}, \]

(1.7)

\[ \frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3P) + \frac{\Lambda}{3}, \]

(1.8)

where, \(\Lambda\) is the cosmological constant and \(\rho\) and \(P\) are the density and pressure for all the components in the Universe at any particular time. However, we can have three different regimes for a dominant component –

- **Radiation dominated** – the equation of state for radiation (photons) leads to \(P = \rho/3\). The Universe was radiation dominated up to \(z_{\text{eq}} \sim 3500\). In this era, energy density decreases with the expansion factor of the Universe as \(a^{-4}\) corresponding to a factor of \(a^3\) from the increase in volume and another factor of \(a\) for the redshifting associated with the expansion. The scale factor evolves with time as \(a(t) \propto t^{1/2}\).
• **Matter dominated** – the equation of state for a matter-dominated Universe is $P \approx 0$. The energy density of the Universe decreases as $a^{-3}$ from the increase in the volume of $a^3$. The Universe has been matter-dominated from $z_{eq} \sim 3500$ till the present. The scale factor evolves as $a(t) \propto t^{2/3}$.

• **Vacuum-energy dominated** – the equation of state for a Universe dominated by the vacuum-energy, or the cosmological constant, is $P = -\rho$. The energy density of such an Universe does not change with the expansion; implying that the total energy of such an Universe increases as it expands. The scale factor of the Universe evolves exponentially as $a(t) \propto e^{Ht}$. We live in very interesting times where the Universe has changed towards becoming $\Lambda$ dominated.

One important quantity in the evolution is the size of the horizon, $r_H = ct$ at any time $t$ and represents the causally connected region in the Universe. Anything outside the horizon has had no effect on the evolution of the Universe, after inflation ended, since it has never been in causal contact since then.

We can re-write Eqn. 1.7 as –

$$1 = \frac{1}{H_0^2} \left[ \frac{8\pi G}{3} \rho + \frac{\Lambda}{3} - \frac{k}{a^2} \right], \quad \text{where} \quad H_0 = \frac{\dot{a}}{a}. \tag{1.9}$$

Defining a set of density parameters for radiation, matter, vacuum-energy and the geometry as $\Omega_r = \frac{8\pi G}{3H_0^2} \rho_r$, $\Omega_m = \frac{8\pi G}{3H_0^2} \rho_m$, $\Omega_{\Lambda} = \frac{\Lambda}{3H_0^2}$ and $\Omega_k = \frac{k}{H_0^2 a^2}$ respectively, we can see that the above equation reduces to –

$$1 = \Omega_m + \Omega_r + \Omega_{\Lambda} + \Omega_k,$$

$$1 \approx \Omega_m + \Omega_{\Lambda} + \Omega_k \quad \text{where} \quad \Omega_r \sim 0 \text{ now}. \tag{1.10}$$

Thus, at the present time, the sum of the densities for matter, the cosmological constant and the spatial curvature must add up to 1. Given the most likely scenario that we live in a flat Universe with $k = 0$, whence $\Omega_k = 0$, we are left with the relation $\Omega_m + \Omega_{\Lambda} = 1$. The constraints on $\Omega_m$ and $\Omega_{\Lambda}$ are shown in Fig. 1.3. From modeling the CMB (Spergel et al., 2007), observations of galaxy clusters (Allen et al., 2002) and models of Big Bang nucleosynthesis (Copi et al., 1995; Burles & Tytler, 1998), we can arrive at the composition of the Universe. If $\Omega_b$ represents
the baryons, regular matter that makes up what we observe and are made up of, then the primary composition of the Universe turns out to be, $\Omega_b = 0.044 \pm 0.003$, $\Omega_{dm} = 0.214 \pm 0.027$ and $\Omega_\Lambda = 0.742 \pm 0.03$ (from the 5-year WMAP data release) with the total adding up to almost 1. Fig. 1.3 shows the approximate mass budget of the Universe. Thus, it turns out that ordinary matter comprises only 4% of the Universe and $\sim 15\%$ of the total matter content of the Universe. The rest of the matter density of the Universe is made up of theoretical entity called ‘dark matter’. It is called dark matter since it does not radiate nor does it interact with radiation – hence the ‘dark’ part; however it still affects space-time gravitationally and thus behaves in a normal matter-like way. So the matter density of the Universe can ultimately be written as $\Omega_m = \Omega_{dm} + \Omega_b$ where, $\Omega_{dm} = 0.23$. Now we can reconstruct the basic history of the Universe. The Universe started out in the hot phase dictated and for the initial $\sim 3 \times 10^5$ years, the Universe was completely radiation dominated and matter was relativistic. The evolution of baryons and dark matter were governed by the gravity of the photons. As the expansion of the Universe continued, the density of the photons dropped as $a^{-4}$ and at $z_{eq} \sim 3500$, the density of matter became equal to that of radiation. Matter, and in particular dark matter, started to influence the evolution of the Universe. Baryons, were still completely ionized and continued to feel the photon pressure from Thompson scattering. Once the Universe had expanded further, and at $z_{rec} \sim 1100$, most electrons combined with atoms, and the Universe became optically thin to the photons. This is what we see as the CMB photons today; and this event of the neutral atoms forming for the first time in the Universe is called as recombination. In redshift-space, this is also known as the surface of last scattering since this is the last time that the CMB photons were completely scattered in the early Universe.

\footnote{For this purpose, we have ignored the neutrinos and photons that make up a very tiny but finite fraction of the present-day energy-density of the Universe.}
Figure 1.3. Left. The composition of the Universe from the latest WMAP results showing the density composition of the Universe at the present time and at recombination. Photons have a much lower density today because $\Omega_r \propto a^{-4}$. Dark matter was clearly the dominant component in the Universe at recombination and drove the subsequent structure formation. Right. The complementary constraints on $\Omega_\Lambda$ and $\Omega_m$ from the various experiments. Our Universe most likely lies on the $\Omega_\Lambda + \Omega_m = 1$ line signifying a flat geometry.
1.4 Galaxy Formation

1.4.1 Dark Matter in Galaxies

In the previous section we saw that the precise cosmological measurements require an additional matter component, dark matter, to explain the content and formation of the Universe. However, dark matter had long been suspected to exist from the measurements of the Coma cluster, where the individual galaxies were observed to have a typical velocity larger than the velocity dispersion calculated from $\sigma^2 = GM_{\text{lum}}/R$, where $M_{\text{lum}}$ is the mass of the luminous matter (Zwicky, 1933). Zwicky concluded that to account for the velocity dispersion of the galaxies, the mean density of the Coma cluster had to be 400 times greater than the density obtained from merely looking at the luminous matter. This discrepancy can be expressed in the form of the mass-to-light ratio, $M/L$, where prior to the 1930’s, it was assumed that $M/L \sim 1$, viz., all the mass in the galaxies were in the form of optically identifiable stars or gas. Shortly afterwards the discovery of ‘missing mass’ in the Coma cluster, it was found that the Virgo cluster exhibited a similar mass discrepancy (Smith, 1936). Observations of the rotation velocity of M31 using long-slit spectroscopy in the late 1930’s by Babcock (1939) showed a mass-to-light ratio that was much larger than 1. Two papers in the 1970’s, Rubin & Ford (1970) and Roberts & Whitehurst (1975) showed beyond doubt that the rotation curve of M31 did not exhibit a Keplerian drop-off and instead remained constant out to distances $\sim 30$ kpc. These observations definitely proved that the mass at large radii increased for M31 even though the luminosity actually decreased. The reasonable assumption became the presence of dark matter in an extended distribution around galaxies. At present, we see the signature of dark matter on the CMB (see Spergel et al., 2007, and references therein), the rotation curves of galaxies (Vogt et al., 2004), X-ray emitting gas in galaxy clusters (Allen et al., 2002) and weak lensing (Hoekstra, 2007).

The fact that dark matter only interacts with matter and photons via gravity has important ramifications for structure formation. We will explore the effects of having dark matter after we establish the mechanism for structure formation from gravitational instabilities and subsequent collapse.
1.4.2 Gravitational Collapse

Gravity is a force that is always attractive in nature. Thereby, it behaves in a purely capitalistic manner, viz., the rich (over-dense) get richer (denser) and the poor (under-dense) get poorer (more under-dense). This happens because an over-dense region has a stronger local gravity and attracts material from surrounding regions, thus increasing the total mass in the density perturbation itself while emptying out the immediate vicinity. The gravitational collapse was studied by Jeans (1919, 1928) where an initial density fluctuation in the early Universe lead to the formation of galaxies. For an isolated self-gravitating sphere to collapse, gravity must overcome the pressure forces supporting it. Using the Virial Theorem to set an upper limit on the kinetic energy, in the absence of any bulk motion or magnetic fields, we can write:

\[ \langle E_{\text{grav}} \rangle < -2 \langle E_{\text{kin}} \rangle. \]  

(1.11)

For gas with uniform density with total mass \( M \) in a radius \( R \) and temperature \( T \), this becomes

\[ \frac{3GM^2}{5R} > 3NkT, \]  

(1.12)

where \( M = N\mu m_H \), with \( N \) the total number of particles each with mean molecular weight \( \mu m_H \) and \( m_H \) being the mass of an hydrogen atom. Newton’s laws then govern the evolution of such system according to –

\[ \frac{d^2r}{dt^2} = \frac{GM(r)}{r^2}. \]  

(1.13)

After some manipulation, we find a natural length-scale that becomes unstable to gravitational collapse. This is known as the Jeans length and is given by –

\[ L_J = c_s \sqrt{\frac{\pi}{G\rho}}, \]  

(1.14)

where \( c_s \) is the speed of sound in the medium. This is the length-scale over which sound (and hence pressure waves) can travel within the characteristic crossing time of the system. The mass-scale associated with the Jeans length is the Jeans mass and is the total mass enclosed within a sphere of radius Jeans length. An unstable region of size larger than the Jeans length collapses on a free-fall time-
scale, \(t_{\text{ff}} \sim 1/\sqrt{G\rho}\). For dark matter, the speed of sound needs to be replaced by an equivalent velocity dispersion term, \(\sigma\). Note that this entire derivation is valid for a static background and cannot be directly applied to the case of gravitational collapse in an expanding Universe.

### 1.4.3 Structure Formation in the \(\Lambda\)CDM Universe

Consider a small positive spherical density perturbation \(\delta \rho / \rho\) in a flat and expanding Universe. We denote the background with the density, \(\rho_b\) and the scale-factor \(a_b\). Since, the material outside the perturbed region will not affect the evolution of the matter inside the sphere, we can regard this collapsing sphere as a closed Universe with \(k = +1\) embedded in the \(k = 0\) Universe. We can write the following equations for the evolution of the density perturbation and the background

\[
H_b = \frac{8\pi G}{3} \rho_b,
\]

\[
H_p + \frac{1}{a_p^2} = \frac{8\pi G}{3} \rho_p,
\]

where \(H_x\) is the ‘Hubble constant’ for the appropriate region. The expansion of the background Universe will be exactly nullified by the collapsing sphere when \(H_b = H_p\). This results in the relation –

\[
\frac{\rho_p - \rho_b}{\rho_b} = \frac{\delta \rho}{\rho_b}
\propto \frac{1}{\rho_b a_p^2}.
\]

In a radiation dominated Universe, \(\rho_b \propto a_b^{-4}\) and in a matter dominated Universe, \(\rho_b \propto a_b^{-3}\). Using these values and assuming that the density perturbation is small enough such that \(a_p \sim a_b\), we find

\[
\frac{\delta \rho}{\rho} \propto \begin{cases} a^2 & \text{(radiation dominated)} \\ a & \text{(matter dominated)} \end{cases}
\]
However, this equality can not be attained when the Universe is radiation dominated for a perturbed region with size less than $r_H$ since the timescale for the expansion of the Universe, $t_{\text{exp}} \propto (G\rho)^{-1/2} << (G\rho_m)^{-1/2} = t_\text{ff}$. All such perturbations will expand along with the Universe and maintain the value of $\delta \rho/\rho$ as long as the Universe remains radiation dominated. However, once the Universe switches to being matter-dominated (or more precisely, dark matter dominated), the density perturbation will grow as $\delta \rho/\rho \propto a(t)$. Baryons differ from dark matter in that even after $z_{\text{eq}}$, they still feel the photon pressure from Thompson scattering and enter the matter-dominated phase at $z_{\text{rec}}$. By this time, the density contrast in the already collapsing dark matter has increased by $\delta \sim (1 + z_{\text{eq}})/(1 + z_{\text{rec}}) \sim 21 \Omega_m h^2$.

Thus, the growth of structure in the Universe can be summarized as follows –

- $\infty < z < z_{\text{eq}}$ This is the radiation dominated Universe. Perturbations of scale less than the horizon can not grow since the expansion of the Universe occurs on a time-scale shorter than the free-fall time-scale. Perturbations of scale larger than the horizon grow as $a^2$. The Jeans mass stays constant in this regime.

- $z_{\text{eq}} < z < z_{\text{rec}}$ This is the matter dominated era where the dark matter starts collapsing from the density perturbations that are currently present in the Universe. The Jeans mass decreases as $a^{-3/2}$ and $\delta \propto a(t)$. Baryons are still coupled to the photons and continue to expand with the rest of the Universe.

- $z_{\text{rec}} < z \leq 0$ This is still the matter dominated era but the baryons are now cool enough that they no longer are ionised and have ceased to feel the photon pressure. Photons now free-stream through the Universe in the form of the CMB. Baryons feel the enhanced gravity of the pre-collapse dark matter, and essentially immediately collapse into the potential well of the dark matter halo through a process of violent relaxation (Lynden-Bell, 1967). The Jeans mass for the baryons drops by a factor of $\sim 10^9$ right at recombination. Once the over-densities for dark matter and gas are equalized, the density contrast continues to grow as $a(t)$, as expected in a matter-dominated Universe.

Looking at the evolution of the collapsing matter, we see that the over-dense region continues to expand with the rest of the Universe at an ever-decreasing
rate until it completely stops and then starts to collapse. The final halo, once it gets virialized has a characteristic over-density of $\delta_c \sim 180$ and a corresponding mass and radius, $M_{\text{vir}}$ and $R_{\text{vir}}$ respectively. This characteristic over-density will appear in the dark matter density profile that we will be using, where we will adopt a value of $\delta_c = \Delta_{200} = 200$. Two parameters, the over-density and the size of the collapsing region are sufficient to fully describe the evolution of any gravitationally bound halo. Therefore, the final density profiles that the halos will attain will also have to be a two-parameter family. Simulations of large scale structure formation show that such an universal density profile indeed exists for dark matter halos (Navarro et al., 1997). We will introduce the details of this universal dark matter density profile in Chapter 3.

Structure in cold dark matter forms hierarchically; smaller objects collapse first and then merge to produce larger halos. At each redshift there is a characteristic mass that is being assembled. Currently, the massive galaxy clusters with mass $> 10^{14} \, M_\odot$ are in the process of formation. This ‘bottom-up’ scenario for structure formation in a $\Lambda$CDM cosmology can explain the overall properties of galaxies and galaxy clusters over a huge mass range of $10^7 - 10^{15} \, M_\odot$ (Davis et al., 1985; Navarro et al., 1995; Katz et al., 1996; Stoehr et al., 2002; Nagamine et al., 2004; Springel et al., 2005a,c; Croton et al., 2006). However, the formation of luminous galaxies still has to account for the behavior of baryons in the overall potential of the dark matter. White & Rees (1978) proposed that gas gets shock-heated to the virial temperature of the dark matter through a process of violent relaxation (Lynden-Bell, 1967) where $T_{\text{vir}} = \mu m_H v_{\text{circ}}^2 / 2k$ where $v_{\text{circ}} = \sqrt{GM_{\text{vir}} / R_{\text{vir}}}$ is the circular velocity of the halo, $\mu$ is the mean molecular weight of the gas, $m_H$ is the mass of a hydrogen atom and $k$ is the Boltzmann constant. The virial radius of the halo of mass $M_{\text{vir}}$ in a flat Universe is given by –

$$R_{\text{vir}} = \left[ \frac{GM_{\text{vir}}}{100 \, H^2} \right]^{1/3}.$$  \hfill (1.18)

\footnote{It is termed cold because the typical velocities of the dark matter particles are smaller than the peculiar velocity of the galaxies or the Hubble flow.}
The hot gas now cools from $T_{\text{vir}}$ on a time-scale $t_{\text{cool}} = 3\mu m_H kT/n_e^2 \Lambda(T)$, where $\Lambda(T)$ is the cooling function and $n_e$ is the electron density. For primordial gas composed of only hydrogen and helium, $T_{\text{vir}} > 10^4$ K will cause the gas to be completely ionized. If $t_{\text{cool}} \leq t_{\text{ff}}$, then the gas will cool and lose pressure support. In such a case, the gas will sink towards the center of the halo and possibly fragment to form stars. It is clear that $t_{\text{cool}}$ also has to be less than the age of the Universe for the entire process to be completed and result in luminous galaxies that we see today. However, if $t_{\text{cool}} > t_{\text{ff}}$, any radiative losses will change the pressure profile for that gas; under such a case, the gas can only quasi-statically contract towards the center. Also, further mergers with other gaseous dark matter halos, as is warranted in the bottom-up formation scenario, will cause an injection of thermal energy and will heat the halo gas. For gas in very massive halos ($M > 10^{13} M_\odot$), $T_{\text{vir}} > 10^{6.6}$ K is so large that the radiative process is not efficient and such halos will not be able to host luminous galaxies. This is seen in the sharp decrease in the number of observed galaxies at such mass ranges. Numerical simulations with this entire prescription for structure formation can accurately duplicate the number and characteristics of galaxies and galaxy clusters in the Universe for most of the mass range (see Fig 1.4 for a visual representation of structure formation in numerical simulations). Numerical simulations still produce too many small galaxies ($M < 10^7 M_\odot$) (Moore et al., 1999; Klypin et al., 1999); this can probably be resolved by additional baryonic physics (Springel, 2000; Springel et al., 2005b) that becomes important at such low mass scales.
Figure 1.4. A figure showing the formation of a galaxy cluster in the Millenium simulation, one of the biggest N-body simulations of the Universe. The Millenium simulation contains more than $10^{10}$ particles and can resolve structure from scales of several Gpc to $\sim 10$ kpc. The figure shows a projected density field for a 15 Mpc/h thick slice at $z = 0$. Each of the overlaid panels zoom in by factors of 4 and the panel with the highest length resolution shows the formation of a galaxy cluster (brightest region) at the intersection of several filaments.
1.5 Merger Rates and Ratios

Observations of major galaxy mergers show the merger rate of galaxies evolve rapidly, increasing from $\sim 5\text{--}10\%$ at $z \sim 1$ to $\sim 50\%$ at $z \sim 3$ (Conselice et al., 2003). The integration of this merger rate leads to the conclusion that a typical massive galaxy will undergo 4-5 major mergers between $z \sim 3$ and $z \sim 0$ with most of these mergers occurring at $z > 1.5$. Numerical simulations can be used to explore the evolution of the merger rate as well as the merger ratio with $z$. Fakhouri & Ma (2008) analyze the Millenium Simulation to constrain this evolution and Fig 1.5 shows their results on the evolution of the merger rate and the merger ratio with $z$. A major merger for the galaxy scale objects, with the merger ratio, $\xi \geq 1/3$ occurs at the rate of 0.5 per halo per unit redshift. Minor mergers with $\xi \geq 1/10$ are far more common at all redshifts. To have representative mergers in our simulations, we use a massive Milky-Way type galaxy as the primary galaxy. This is the most massive galaxy in our simulations. We sample the major mergers by simulating an equal-mass merger between two such galaxies. We also attempt to get the boundary of the major mergers for a merger ratio, $\xi \sim 1/3$ for our second set of merger ratios. As shown in Fig 1.5, minor mergers are the most frequently occurring mergers in the Universe; we model this by simulating a merger with $\xi \sim 1/10$. This describes the three independent sets of simulations that we will perform to constrain the effect of the merger ratio on the behavior of the hot gas during galaxy mergers. Mergers need an additional specification – the merger orbit. We will describe the parameters for specifying the content of the galaxies (see Sections 3.1 and 3.2), and construction of the merger orbits in Section 3.4.

1.5.1 Merger Rates from Analytic Theories

Current results imply that we live in low-density, $\Lambda$-dominated flat Universe (Riess et al., 1998; Perlmutter et al., 1997; Spergel et al., 2007; Komatsu et al., 2008). In such an Universe, halos form hierarchically, with smaller halos forming early on and merging to form larger structures at later times. This process of halo formation is dictated completely by gravitational processes and can be modeled in a somewhat straightforward manner. The first attempt at this analytical formulation which yields the number density of halos as a function of mass and redshift can be
Figure 1.5. Mean merger rate for each halo per unit time and per unit redshift as a function of the redshift. Various mass scales are plotted but the galaxy scale, plotted with the triangles, is the one relevant to our work. We see that the major merger rate, $\xi \geq 1/3$, is $\sim 0.5$ at $z \sim 0$. The merger rate for $\xi \geq 1/10$ at $z \sim 0$ is $\sim 1.2$. All the rates increase with redshift; minor mergers with $\xi \geq 1/10$ are always the most common mergers occurring. The figure is taken from Fakhouri & Ma (2008).
attributed to Press & Schechter (1974). In the original work, the co-moving number density of halos, \( n(M, z) \) in a mass range, \((M, M + dM)\), at a given redshift, \( z \) is given by:

\[
n(M, z)dM = \sqrt{\frac{2}{\pi}} \frac{\bar{\rho}_0}{M} \frac{d\nu}{dM} \exp\left(-\frac{\nu^2}{2}\right) dM.
\] (1.19)

Here \( \nu = \delta_c / [D(z)\sigma(M)] \), where \( \delta_c \approx 1.69 \) is a constant and the growth factor for linear fluctuations can be taken as \( D(z) = g(z) / [g(0)(1 + z)] \) with

\[
g(z) \approx \frac{5}{2} \Omega_m \left[ \Omega_m^{4/7} - \Omega_\Lambda + (1 + \Omega_m/2)(1 + \Omega_\Lambda/70) \right]^{-1}.
\] (1.20)

Here, \( \sigma(M) \) is the rms density fluctuation according to linear theory for a sphere containing a mass \( M \). Since mass, \( M \) and the radius of the sphere, \( R \), are tied by the relation, \( M = \frac{4}{3}\pi\bar{\rho}_0 R^3 \), where \( \bar{\rho}_0 \) is the mean density of the Universe, one can equivalently express \( \sigma(M) \) as \( \sigma(R) \). Then assuming a Gaussian density field, we can write:

\[
\sigma^2(R) = \frac{1}{2\pi^2} \int_0^{\infty} k^3 P(k) \tilde{W}^2(kR) \frac{dk}{k},
\] (1.21)

where \( \tilde{W} \) is the Fourier transform of the spherical top-hat function and \( P(k) \) is the power spectrum of the density fluctuations. With the above formulation, we can estimate a co-moving number density of halos for any given mass at each redshift. For a more detailed explanation of the derivation of the analytical halo mass function, we refer the reader to Mo & White (2002); Warren et al. (2006).

The previous formulation only accounts for the halo mass function at each redshift but does not constrain the merger rates for the various halos as a function of redshift. To this end, Lacey & Cole (1993); Bower (1991); Bond et al. (1991), developed the extended Press-Schechter formalism which provides the merger history of a halo in the form of a binary merger tree. A binary merger tree assumes that as we step back in time, there are always exactly two progenitors for each halo, i.e., there are no multiple-galaxy mergers. Such analytical merger trees have the advantage that they are computationally much less expensive than a high resolution N-body simulation and thus, can be used to explore and constrain parameter space. In the extended Press-Schechter model, following the formulation of Parkinson et al. (2008), the conditional mass function, \( f(M_1|M_2) \), gives the fraction of mass from a halo with mass, \( M_2 \), at a redshift, \( z_2 \), that was contained in a progenitor halo of
mass, $M_1$, at a previous redshift, $z_1$. Mathematically, this is written as:

$$f(M_1|M_2) d\ln M_1 = \sqrt{2} \frac{\sigma_1^2(\delta_1 - \delta_2)}{\pi [\sigma_1^2 - \sigma_2^2]^{3/2}} \times$$

$$\exp \left[ -\frac{1}{2} \frac{(\delta_1 - \delta_2)^2}{\sigma_1^2 - \sigma_2^2} \right] \left| \frac{d\ln \sigma}{d\ln M_1} \right| d\ln M_1 ,$$

(1.22)

where, $\delta_1$ and $\delta_2$ represent the linear over-densities for collapse at redshifts $z_1$ and $z_2$, $\sigma_1 \equiv \sigma(M_1)$. The derivative of this equation under the limit $z_1 \to z_2$, yields the number of progenitors of mass $M_1$ that make up a halo of mass $M_2$ for a step in redshift space of $dz_1$. This is written as:

$$\frac{dN}{dM_1} = \frac{1}{M_1} \frac{df}{dz_1} \frac{M_2}{M_1} dz_1 \quad (M_1 < M_2).$$

(1.23)

Specifying a minimum mass resolution, $M_{\text{res}}$, then allows us to compute the mean number of progenitors with mass $M_1$, in a mass range, $M_{\text{res}} < M_1 < M_2/2$ via the following equation:

$$N_P = \int_{M_{\text{res}}}^{M_2/2} \frac{dN}{dM_1} dM_1 .$$

(1.24)

The mass fraction of the final halo, $M_2$, that is accreted below the resolution limit, $M_{\text{res}}$, can be estimated from:

$$F = \int_0^{M_{\text{res}}} \frac{dN}{dM_1} \frac{M_1}{M_2} dM_1 .$$

(1.25)

The merger tree can then be constructed given a halo of mass, $M_2$ and a redshift, $z_2$. The limitation of $M_1 < M_2/2$ is imposed to maintain the symmetry required in a binary merger tree, where one fragment of mass $M_1$ fixes the mass of the other fragment as $M_2 - M_1$. The results of the relevant equations are, therefore, considered to be meaningful only under the constraint $M_1 < M_2/2$.

We use the publicly available code from Parkinson et al. (2008) to generate merger trees and the co-moving halo number density in later section of this thesis (see Section 4.3.3) to estimate the total amount of gas liberated from all the mergers that occur throughout the history of the Universe.
Figure 1.6. The co-moving number density of halos greater than a mass, $M$, as a function of redshift. The solid curves show the value of the $\log(M/M_\odot)$. The figure is taken from Mo & White (2002).
1.6 The Inter-galactic Medium

The inter-galactic medium (IGM) is the reservoir of gas located between clusters of galaxies and is distinct from intra-cluster medium (ICM) – gas contained in the cluster potential and heated up to the virial temperature of the cluster, and intra-group medium – gas contained in the combined potential of a galaxy group. The initial evolution of the Universe predicts that the baryons fall into the dark matter potential, and form pancakes, sheets and filaments. Large over-densities, corresponding to galaxy clusters are found at intersections of filaments. The baryons at high-z can be examined through the absorption lines in quasar spectra. An abundance of absorption lines are found originating at varying \( z \), showing the presence of diffuse gas with column density \( N_H \gtrsim 10^{14} / \text{cm}^2 \) (Fransson & Epstein, 1982; Cowie et al., 1995; Wang, 1995). At \( z \sim 3 \), most of the baryonic mass is in the Lyman-\( \alpha \) forest (Fukugita, Hogan, & Peebles, 1998), so called because of the large number of closely spaced lines (in wavelength space) for the absorption of the Lyman-\( \alpha \) photons at rest-frame 1216 Å. The amount of baryons in the Lyman-\( \alpha \) forest systematically decreases with \( z \) and is almost zero at present. It would be reasonable to expect that all the baryons are now part of galaxies, in the form of stars and the inter-stellar medium. However, recent estimates of the mass-fraction of baryons in different components still fail to show the detection of \(~60\%\) of the total baryon content at low-\( z \); with 6\% in stars in galaxies (Fukugita & Peebles, 2004), 2\% in neutral gas (Zwaan et al., 2003), 4\% in galaxy clusters and galaxy groups (Allen et al., 2002) and 29 \( \pm 4\% \) in low-\( z \) Lyman-\( \alpha \) absorbers (Penton et al., 2004; Danforth & Shull, 2008). This would directly imply that the rest of the baryons are in state that has a weakly observable signature. The scientific community arrived at the consensus that the majority of the ‘missing’ gas is located in regions of low over-density, \( \delta \rho / \rho \sim 10 – 100 \) (e.g., Cen & Ostriker, 1999; Davé et al., 1999, 2001; Cen & Ostriker, 2006; Dolag et al., 2006; Davé & Oppenheimer, 2007) with temperatures in the range \( 10^5 – 10^7 \) K (see Fig. 1.8). This is the Warm-Hot Intergalactic Medium or the WHIM and is currently the most elusive astrophysical object (for a recent review see Bregman, 2007). Numerical simulations predict that the WHIM must reside in filaments and relatively low over-density regions with metals having a slightly larger correlation with galaxy
locations (Cen & Ostriker, 1999; Davé et al., 1999; Cen & Ostriker, 2006; Davé & Oppenheimer, 2007, see Fig. 1.7). Numerous efforts have been taken to detect and characterize the WHIM; however, most of the observations to date are not completely convincing and have required some fine-tuning in the fitting procedure for the data to come up with a significant detection (Nicastro et al., 2005; Mathur et al., 2003). Given the temperature and density range of the WHIM, the most important observational signature is the O vi absorption line since its collisionally ionized equilibrium temperature is $\sim 10^{5.45}$ K. The large spatial extent of the WHIM makes it extremely difficult to detect the emission from the WHIM and gets drowned by the radiation from somewhat more compact sources like galaxies, clusters and AGNs. The emission from the WHIM is usually a few orders of magnitude lower than those sources owing to the lower density and temperature. Hydrodynamic simulations predict that the total emission from WHIM makes up about 5-15% of the soft X-ray background (Kravtsov et al., 2002; Phillips et al., 2001) in the 0.5-2.0 keV range. A lot of this energy is expected to be in the form of line emission from species like O vii, O viii, and Fe xvii in colder regions and Fe xxv and Fe xxvi from hotter regions of the WHIM (Fang et al., 2002). The detection of such emission lines from the WHIM would be extremely useful to constrain the temperature, metallicity and density of the WHIM. In turn, this will help us understand feedback processes, viz., AGN feedback, galactic super-winds, and the extent to which they influence the local environment (Pierre et al., 2000).

Though the detection of emission lines from the WHIM remains a controversial and debated topic, the next generation X-ray telescopes like Constellation-X and XEUS should be quite capable of detecting the absorption lines from the WHIM along the line of sight to bright background sources.

The immediate question that arises is how does gas get into the WHIM state? Some form of mass and energy injection is essential to create this hot reservoir of gas; and this form of feedback also has to account for the metal content of the IGM. Numerical simulations of structure formation had to encompass the various baryonic processes that occur along with galaxy formation in order to attempt to solve this puzzle. We know that gas cools inside galaxy halos and forms stars.

\footnote{It is not clear that the WHIM is in ionization equilibrium though most numerical simulations assume as much. E.g., Cen & Fang (2006) claim that the abundance of O vi with non-equilibrium calculations agree better with observations compared to the equilibrium models.}
A number of such stars will go supernovae (SN) and inject metals and kinetic energy into the surroundings. High-mass stars tend to form inside OB-clusters, and if multiple supernovae go off at similar times, then they could feasibly drive a galactic wind and trigger a massive outflow. Another possibility is the accreting super-massive black hole at the center of galaxies; the radiation from the accreting material could power jets and large-scale winds – also capable of powering mass ejection events. Tidal and ram-pressure stripping of the gaseous halos of galaxies could also contribute to the overall mass of the IGM. Clearly, all such energy delivery mechanisms would be more effective in the smaller mass galaxies, e.g., dwarf galaxies. However, numerical simulations show that even though the metal pollution of the IGM can be easily achieved by the SN feedback mechanisms, the amount of gas lost was determined by the mass of the halo (Mac Low & Ferrara, 1999). Galaxies with host halo mass $\gtrsim 10^7$ M$_\odot$ lose < 0.1% of their gas content. Metals, on the other hand, can easily get ejected from even $10^9$ M$_\odot$ halos with an average loss-rate of 70% (Mac Low & Ferrara, 1999). Supermassive black holes (SMBH) go through phases of accretion (Hopkins et al., 2006; Croton et al., 2006) and can expel the gas from their host halos. This is a cyclical process in which the gas is expelled from the host halo, and re-accreted into the galaxy at a later time through smooth accretion, awaiting the SMBH to begin an active phase to be blown out again. This feedback process from the SMBH can be in the form of radiation, thermal, and kinetic pressure. SMBH’s are known to release a significant fraction of their energy in the form of jets (Rees, Begelman, Blandford, & Phinney, 1982; Begelman, Blandford, & Rees, 1984). The interaction of the jet with the ISM of the galaxy causes material to be pushed outwards and often at speeds greater than the escape velocity. Also, in the accretion disk of the SMBH, energy is transported outwards, and unless $2/3$ of the energy is not radiated away, a substantial portion of the mass will gain enough energy to become unbound (Narayan & Yi, 1995; Blandford & Begelman, 1999). In case of radiation pressure, the photons can interact with electrons via Thompson scattering or with the dust particles. The higher effective cross-section of the dust particles make it suitable for imparting sizeable amount of energy from the photon momentum. Since the dust is coupled to the gas, this forms a momentum-driven galactic scale super-wind. Recent numerical simulations using such a formulation have been quite
successful at explaining the nature and evolution of the IGM, and in particular the formation of the WHIM (Aguirre et al., 2001; Theuns et al., 2002; Oppenheimer & Davé, 2006; Cen & Ostriker, 2006; Oppenheimer & Davé, 2008; Bertone et al., 2008). These simulations show that the effect of the super-winds is to push the baryons into regions of lower over-densities and temperatures of $10^5 - 10^7$ K (see Fig. 1.8 for relative compositions of the various baryonic objects). The WHIM, in the presence of the super-winds evolves from a mass-fraction of $< 10\%$ at $z \sim 3$ to $\sim 50\%$ by $z \sim 0$. 
Figure 1.7. Surface density contours on a slice of $50 \times 50 \times 10 \, h^{-3} \, \text{Mpc}^3$ for galaxies (filled pink; at a surface density of 31 times the mean surface density of galaxies), metals (green; at a metallicity of $0.16 \, Z_\odot$) and warm/hot gas with $T = 10^5 - 10^7 \, \text{K}$ (blue; at a surface density of 6.8 times the mean surface density of warm/hot gas). Each respective contour contains 90% of the total mass of the respective component. This figure shows that metals are distributed preferentially around galaxies. The figure is taken from Cen & Ostriker (1999).
Figure 1.8. The figures are taken from Cen & Ostriker (2006) Left The volume fraction of the various gas reservoirs in the galaxy. This is dominated by warm ($T < 10^5$ K) gas. Right. The mass fractions for the different phases of the gas as a function of redshift, $z$. The WHIM contains the most amount of gas for $z < 0.5$. Galactic super-winds reduce the available gas in the WHIM.
1.7 Overview

In this chapter, we establish the importance of galaxy mergers in the context of a ΛCDM cosmology and the existence of hot gas in the halos of galaxies. We will attempt to describe the behavior of this hot gas during the mergers. We construct representative merger ratios that will be used in our simulation while deferring the detailed construction of the simulations for a later chapter. In the next chapter, we will look at all the methods that we will use to create and simulate the galaxy merger.
2.1 Introduction

Our goal is to simulate galaxy mergers consisting of dark matter and gas. To achieve this, we must first create an equilibrium model for different galaxies with gaseous dark matter halos. Then, we must accurately follow the evolution of two such galaxies that are placed in an orbit around each other. Thus, there are two aspects of the modeling: (i). creation of an equilibrium galaxy model, and (ii). correctly computing the gravitational forces to describe the time-evolution of the galaxies as they undergo the violent encounter. In this chapter, we will examine the details of the various N-body techniques and tools that were used to run our merger simulation.

2.2 Collisionless Systems

A galaxy is made up of dark matter, stars and gas that move around under the force of mutual gravitational attraction. The global dynamics of this ensemble of particles is determined by the overall structure and potential of the galaxy, although external perturbations like satellite flybys can trigger long lived global features like bars, spiral arms and warps. Due to the coarse-grainedness of an actual galactic potential, a star will be deflected from its ‘true’ orbit in a smooth potential. A star moving with an average velocity $v$ in a galaxy of radius $R$
containing \( N \) equal-mass stars will have an aggregate velocity change of the order \( v \) in one relaxation time, \( t_{\text{relax}} \), given by:

\[
 t_{\text{relax}} = \frac{N}{8 \ln N} \times \frac{R}{v}.
\] (2.1)

In practice, we compare the relaxation time of a system with its crossing time, \( t_{\text{cross}} = \frac{R}{v} \), to determine whether the star will suffer from significant encounters in a Hubble time. For the \( N = 10^{11} \) stars in a typical galaxy with \( R = 20 \) kpc and \( v = 200 \) km/s, \( t_{\text{cross}} = 10^8 \) years, making \( t_{\text{relax}} = 4.9 \times 10^{16} \) years. So, a typical star will not deviate at all over the period of a Hubble time. Such a system where the timescale for close encounters is much longer than a Hubble time, is called a collisionless system. Stars in the galaxy constitute such a collisionless system. Even if the galaxy were to undergo a merger, it is unlikely for any star to directly encounter another star; hence the stars can be treated as collisionless particles during the course of a merger simulation. N-body models of galaxies, however, are not collisionless systems. The smallest of our galaxies is represented with \( N \sim 10^5 \) particles and has a relaxation time of \( \sim 600 \) Gyrs (see Table 3.1). Fortunately, the galaxies we are simulating undergo a violent merger within 1 Gyr; therefore, we only require galaxies to be stable against two-body interactions over the period of \( \sim 1 \) Gyr.

### 2.3 Distribution Function

The task of building an N-body model requires determining the six phase-space coordinates, i.e., \( (x, y, z, v_x, v_y, v_z) \) at some time \( t_0 \). Given such a description, it is possible to compute the state of a purely collisionless system at any later time by using Newton’s Laws of Motion. This 6-D phase space description is called the distribution function (hereafter, DF), \( f(x, v, t_0) \). The DF gives the number of particles in an infinitesimal spatial volume \( d^3x \) around the position \( x \) and in the infinitesimal velocity volume of \( d^3v \) at the instant \( t_0 \). Thus, a DF is non-negative everywhere. The time-evolution of a collisionless system is governed by
the collisionless Boltzmann equation (CBE) written in its simplest form as:

$$\frac{df(r,v,t)}{dt} = 0.$$  \hspace{1cm} (2.2)

In general, a DF depends on the value of the specific total energy, $E = \Phi(r) + \frac{1}{2}v^2$ and the specific angular momentum, $L = r \times v$; however, it can be shown that the DF for a spherically symmetric density with an isotropic velocity distribution is dependent only on the total energy per unit mass, $E$, i.e., $f(r,v) \equiv \hat{f}(E)$ given by the Eddington inversion:

$$f(E) = \frac{1}{\sqrt{8\pi^2}} \left[ \int_0^E \frac{d^2\rho \ d\Psi}{\sqrt{E - \Psi}} + \frac{1}{\sqrt{E}} \left( \frac{d\rho}{d\Psi} \right)_{\Psi=0} \right] \hspace{1cm} (2.3)$$

where, $E = -E$, $\Psi = -\Phi$ and $\rho$ is the density. For a finite size of a system, the second term is equal to zero and the DF is given by just the first term. Now with a density, $\rho$ and a potential, $\Phi$, we can numerically solve the above equation and derive the numerical DF. We can recover the density from the DF by using the relation:

$$\rho(r) = \int f(r,v) dv.$$  \hspace{1cm} (2.4)

Therefore, given the DF for a system, it is possible to recover the underlying density profile and vice-versa. Likewise, the integral over $d^3r$ yields a velocity profile.

In order to make a galaxy model, one can get the particle positions by directly sampling the density profile using acceptance-rejection technique (see next section); however, obtaining the particle velocities is a bit more involved. For an isolated galaxy in equilibrium, the DF has no dependence on time, i.e., $f(r,v,t) \equiv f(r,v)$. Jeans theorem states that in such a case, any solution to the CBE (Eqn 2.2) can be represented as a function of constants of motion, e.g., angular momentum. If the velocity distribution for the particles in the model is approximated by a 3-D Gaussian, specified by a mean velocity and velocity dispersion along each coordinate axes, then the complete velocity profile can be uniquely determined by computing the constants of motion. Most of the previous numerical work has been done under this simplifying assumption; however, such an approach is approximate and the density and velocity profiles can vary drastically from the intended ones.
(Kazantzidis et al., 2004; Holley-Bockelmann et al., 2005). Therefore, we avoid that method and calculate the distribution function of the potential-density pair given by a density profile and draw position-velocity pairs directly from the DF.

2.4 Acceptance-Rejection Technique

The acceptance-rejection technique is a method of sampling a probability distribution function (hereafter, p.d.f.) by using the fact that a random generator will naturally produce an unbiased sample of random numbers; now, if this sample of random numbers is compared against the p.d.f. and the random numbers greater than the value of the p.d.f. are rejected, then the remaining sample members will naturally reflect the shape of the original p.d.f. This class of methods where a uniform random number generator is used to compute some property or generate a distribution as in this case, is called a Monte-Carlo method. Figure 2.4 shows a Monte-Carlo method to estimate the value of \( \pi \). N-sets of two random numbers are drawn in the range \([0,1]\) and each one is plotted on the figure. The probability that a particular number falls inside the quadrant is proportional to the area of the quadrant. We can directly estimate the probability of a number falling in this quadrant just by counting the number of actual such occurrences, say \( N_1 \) and dividing by the total number of tries (\( N \), in this case). Then we can use \( N_1/N = \text{Area of quadrant}/\text{Area of square} \), to obtain an estimate for \( \pi \). As \( N \) increases, we obtain a better estimate of \( \pi \) since the error of a Monte-Carlo method reduces as \( O(1/\sqrt{N}) \).

The acceptance-rejection technique is a Monte-Carlo method to create a realization of some distribution, \( f(x) \), given another distribution, \( g(x) \) such that \( Ag(x) \geq f(x) \forall x \), where \( A > 1 \). The acceptance-rejection technique is usually used where the form of \( f(x) \) makes it difficult to invert and obtain \( f^{-1}(x) \), making sampling from \( f(x) \) difficult. Therefore, another more easily invertible distribution, \( g(x) \), is used as an limiting value for \( f(x) \), and samples are drawn from \( g^{-1}(x) \) and accepted when an uniform random in \([0,1]\) does not exceed \( f(x)/Ag(x) \). This value of \( x \) drawn from \( g(x) \) is then accepted as a sample of \( f(x) \). The efficiency of the algorithm depends on the value of \( A \); for values of \( A \) closer to 1 the algorithm is more efficient since values of \( x \) sampled from \( g(x) \) get rejected less frequently.
The flow-chart for the acceptance-rejection technique is shown in Fig. 2.4.

\[ \pi \approx 3.000, \; N = 100 \]

**Figure 2.1.** Monte-Carlo estimate of the area of the positive quadrant of a unit circle given by \( x^2 + y^2 = 1 \). Analytically the result is known to be \( \pi/4 \), and numerically it can be approximated by the ratio of the number of points that occur in the shaded region to the total number of points. Here, we show a random sample of \( N = 100 \) \((x, y)\) pairs plotted as the solid circles, and we find an approximate value for \( \pi \) to be 3.0. As the number of tries increase, better convergence is achieved. One of the biggest advantages of a Monte-Carlo method is that the error is always \( \mathcal{O}(1/\sqrt{N}) \) irrespective of the dimensionality of the problem itself. The Monte-Carlo method relates to the acceptance-rejection technique in that the number of solid circles in the shaded region is larger for a smaller value of \( x \) (and a corresponding larger value of \( y = \sqrt{1-x^2} \)). Thus, the actual filled circles in the shaded region for a given value of \( x \) can be used as a representative sample of the probability distribution given by the quadrant.

### 2.5 Simulation Software

We use the parallel, hydrodynamic code GADGET-2 (Springel et al., 2001; Springel, 2005)\(^1\) for all the numerical simulations in this paper. GADGET-2 computes
start

read in desired model

choose random number

reject random number

compare with model

is random number less than model?

NO

YES

are there enough points?

add to model realization

end

Figure 2.2. A flow-chart for the acceptance-rejection algorithm. This is a Monte-Carlo method to create a realization of some distribution, \( f(x) \), given another distribution, \( g(x) \) such that \( A g(x) \geq f(x) \forall x \). This involves generating a uniform random number in \([0,1]\) and a value \( x \) from the distribution \( g(x) \). If the value of the random number is less than the ratio of \( f(x)/A g(x) \), then the value of \( x \) is accepted a sample of \( f(x) \).
the gravitational forces with a hierarchical tree algorithm (Barnes & Hut, 1986) while gas particles receive additional hydrodynamic acceleration as calculated using Smooth Particle Hydrodynamics (Gingold & Monaghan, 1977, hereafter SPH). The code explicitly conserves energy and entropy for the SPH particles and uses adaptive time-steps for the time-evolution. Since this is a first attempt to model the gas behavior during the merger, cooling, star formation and radiative feedback have all been neglected.

2.6 N-body Mechanics

N-body mechanics refers to simulations of dark matter and stars where a group of ‘N’ particles are evolved from an initially specified set of \((\mathbf{r}, \mathbf{v})\) under mutual gravitational forces. The arrangement of the ‘N’ particles are chosen to model a particular situation from planetary rings to the Universe as a whole.

The dominant force on the largest length scales relevant to astrophysics is Newtonian gravity. The set of initial positions and velocities can be evolved using the standard equation of motion under gravity:

\[
a_i = - \sum_{j \neq i} \frac{G m_j |\mathbf{r}_i - \mathbf{r}_j|}{|\mathbf{r}_i - \mathbf{r}_j|^3},
\]

(2.5)

where \(a_i\) represents the net acceleration of the \(i^{th}\) particle from all the other \(j\) particles, each with mass \(m_j\). This equation is valid for each individual particle and leads to a set of \(N\) coupled, non-linear second order differential equations that must be numerically integrated \(\forall N > 2\). However, the force law, written in the above form suffers from a numerical singularity as \(|\mathbf{r}_i - \mathbf{r}_j| \to 0\). In order to retain relative accuracy of the numerical integration, the time-steps for particles with vanishing separation become infinitesimally small, thereby reducing the overall computational efficiency of the simulation. Constant time-stepping could be used to avoid this scenario, but the accumulated errors for such close encounters then becomes large and may lead to unphysical situations such as spurious binary formation. This close encounter scenario is avoided in numerical simulations by

\[\text{GADGET-2 is available for download at http://www.mpa-garching.mpg.de/gadget/}\]
modifying the force law at small separations by adding a ‘softening parameter’, \( \epsilon > 0 \), to the force equation such that the force does not diverge. This modified force law can be written as:

\[
\vec{F}_i = - \sum_{j \neq i} \frac{Gm_im_j|\vec{r}_i - \vec{r}_j|}{(|\vec{r}_i - \vec{r}_j|^2 + \epsilon^2)^{3/2}}.
\] (2.6)

The choice of the softening parameter is highly dependent on the simulation type and the exact object being simulated, but is generally taken to be on the order of the mean inter-particle separation. It has to be noted that any simulation with a finite \( \epsilon \) can not be used to predict results for length scales \( \epsilon \). Thus, the spatial resolution of a gravitational simulation is set by the choice of \( \epsilon \). To make sure that this modification of the small-scale force does not have an effect on the large-scale dynamics, a gravitational kernel is defined so that the force becomes exactly Newtonian at some length scale \( \epsilon \). This is done in GADGET-2 by using a spline kernel (Monaghan & Lattanzio, 1985) \( W(|x|, h = 2.8 \epsilon) \) of the form:

\[
W(r, h) = \frac{8}{\pi h^3} \begin{cases} 
1 - 6 \left( \frac{r}{h} \right)^2 + 6 \left( \frac{r}{h} \right)^3, & 0 \leq \frac{r}{h} \leq \frac{1}{2}, \\
2 \left( 1 - \frac{r}{h} \right)^3, & \frac{1}{2} < \frac{r}{h} \leq 1, \\
0, & \frac{r}{h} > 1.
\end{cases}
\] (2.7)

where, \( r \) is the separation between the particles between which the force is being computed. Even with softening, directly summing the gravitational forces over \( N \) particles involves computing \( \frac{1}{2}N(N-1) \) forces, which is an algorithm of \( O(N^2) \); the processor time per time-step becomes prohibitively large for values of \( N \) required to describe astrophysical systems (e.g., current state of the art simulations use up to \( 10^{10} \) particles, and that number is ever-increasing in keeping with Moore’s Law). The long range nature of gravity makes it difficult to compute the forces accurately and efficiently. One method that is used to achieve the required spatial adaptivity groups particles in a hierarchical multipole expansion, usually referred to as a ‘tree’ algorithm. The specific algorithm used is the Barnes-Hut (BH) octal
tree (Barnes & Hut, 1986), which involves grouping distant particles together such that the total force for each particle can be computed by \( O(\log N) \) interactions. This strategy reduces the overall complexity of the computation to \( O(N \log N) \), and yields an acceptable total run-time for a simulation. The BH tree uses a cubic parent node that contains the entire mass distribution and continues to recursively sub-divide each node into 8 spatially equal ‘daughter cells’ until only one particle is contained within a node (called a ‘leaf’). Each node is then associated with a multipole expansion of all the daughter nodes. The parameter that controls the accuracy of the tree algorithm is the ‘cell opening angle’, or the tolerance, represented by \( \alpha \) in the following equation for a node of mass \( M \) and size \( l \) at a distance \( r \):

\[
\frac{GM}{r^2} \left( \frac{l}{r} \right)^2 \leq \alpha |a|, \tag{2.8}
\]

where \(|a|\) is the total acceleration obtained in the previous time-step. While computing the gravitational force, each node is opened and compared with the set parameter, \( \alpha \), to see if the multipole expansion is accurate enough (this happens if the node is small and distant from the particle for which the force is being computed). If so, the multipole expansion is used, the ‘tree walk’ is terminated; if not, the daughter nodes are opened recursively until the desired accuracy is reached. As such, the accuracy can be increased to match the exact force given by Eqn. (2.6), albeit at a higher computational cost. The cell-opening criteria used in GADGET-2 (refer to Eqn. 2.8) compares an estimate of the total force with the truncation error for each particle-node interaction and limits the absolute error. It has the advantage that the cell opening adaptively adjusts with the acceleration. Such a cell opening criteria can be subject to unbounded errors in the case that a particle is sufficiently close to a node (Salmon & Warren, 1994); to prevent this from occurring an additional criteria is imposed such that a node is always opened if a particle is located inside geometric boundaries 10% larger than the node.
Figure 2.3. Top. A schematic for the Barnes-Hut method of tree construction that is used in GADGET-2. Each cube (node) is recursively divided into 8 smaller cubes until all the smallest cubes contain exactly one particle. Bottom. An illustration of the opening angle for the tree code. If $\theta$ is small as per Eqn 2.8, then the multiple expansion is used. Otherwise the node is opened and similar comparisons are carried out with the daughter cells. Figure is taken from (Wadsley et al., 2004).
2.7 Smooth Particle Hydrodynamics

Gas is difficult to simulate accurately, due to the additional hydrodynamic evolution equations that the gas must follow. In numerical simulations, gas is usually represented as a fluid that must satisfy the continuity equation in addition to Newtonian gravity. Conventional grid-based numerical techniques have extreme memory requirements for 3-D simulations of very large scale regions. This is because each point in space is a part of the mesh which serves to compute spatial derivatives for quantities like velocity, pressure, etc. This means that even empty regions in space must contain grid points, which requires enormous memory if sufficient resolution is to be maintained throughout the simulation box.

Smooth Particle Hydrodynamics is a particle method (Lucy, 1977; Gingold & Monaghan, 1977; Monaghan, 1992) that overcomes the memory constraints by representing the fluid with a finite number of particles that interact hydro-dynamically with each other—characteristic of gas in astrophysical situations. Each particle experiences gravitational forces as well as gas pressure from the particles that are nearby. Ideally, if the particle interacts with every other particle then a complete analytic solution is obtained. In practice, SPH schemes use an interpolating function, a kernel similar to the one used in evaluating softened gravitational forces\(^1\), that has a finite value for small separations between two interacting particles and is zero when the separation is too large compared to a hydrodynamic length scale. This characteristic length scale within which particles interact is called the smoothing length, \(h_i\). The smoothing length is adaptive in most SPH simulations and adjusts to encompass a fixed number of neighbors. Since the particle motion matches the fluid flow, an adaptive smoothing length naturally ensures higher resolution in denser locations.

Any function, \(f(r)\) can be recast in terms of the kernel as:

\[
f_{\text{interp}}(r_i) = \sum_{j \neq i} m_j \frac{f(r_j)}{\rho(r_j)} W(r_i - r_j; h),
\]

(2.9)

where the usual integral has been replaced by a more convenient summation. Replacing \(f(r)\) with \(\rho(r)\), one can immediately see that the density of an SPH particle

\(^1\)In GADGET-2, the kernels for SPH and gravity are identical
is given by:

\[ \rho_i = \sum_{j \neq i} m_j W(\mathbf{r}_i - \mathbf{r}_j; h_i) \]  

(2.10)

The derivative of any function, \( f(r) \), written in terms of the interpolant becomes:

\[ \nabla f_{\text{interp}}(\mathbf{r}_i) = \sum_{j \neq i} m_j \frac{f(r_j)}{\rho(r_j)} \nabla W(\mathbf{r}_i - \mathbf{r}_j; h_i) \]  

(2.11)

Such a formulation ensures that derivatives of numerically ill-behaved quantities can be easily computed without having to resort to special formulations to deal with numerical discontinuities. The derivative of any function then becomes the convolution of the value of the function and the pre-computed analytic derivative of the kernel, an incredible simplification in terms of the numerical overhead. This advantage, however, becomes a problem when it comes to resolving actual physical discontinuities – shocks. SPH techniques will intrinsically smooth out a shock and broaden it over 2-3 smoothing lengths; though observations indicate that the shock front is very sharp. We will discuss the methods used in SPH simulations to realistically capture shocks in a later section.

The conservation of momentum locally requires Newton’s third law be satisfied for each force calculation between two particles. To achieve this, the kernel and its derivatives are symmetrized (Hernquist & Katz, 1989):

\[
W_{ij} = \frac{1}{2} \left[ W_i(\mathbf{r}; h_i) + W_j(\mathbf{r}; h_j) \right]; \mathbf{r} = \mathbf{r}_i - \mathbf{r}_j
\]

(2.12)

\[
\nabla W_{ij} = \frac{1}{2} \left[ \nabla_i W_i(\mathbf{r}_i; h_i) + \nabla_j W_j(\mathbf{r}_j; h_j) \right],
\]

(2.13)

where \( i \) and \( j \) refer to two different particles between which hydrodynamic forces are being computed, \( W_i \) and \( W_j \) are the values of the kernel obtained using the respective smoothing lengths, \( h_i \) and \( h_j \) respectively. The equation of motion in this discretized representation of the gas then becomes:

\[
\frac{d\mathbf{v}_i}{dt} = -\sum_{j=1}^{N_{\text{nb}}} m_j \left[ f_i \frac{P_j}{\rho_i^2} \nabla_i W_{ij}(h_i) + f_j \frac{P_i}{\rho_j^2} \nabla_i W_{ij}(h_j) \right],
\]

(2.14)
where \( f_i = \left(1 + \frac{h_i}{3\rho_i \partial h_i} \right)^{-1} \) and \( P_i \) is the pressure of the \( i^{th} \) gas particle. We explicitly include the fact that the hydrodynamic sum is carried out only over the \( N_{ngb} \) neighbors. If the gas were to be isentropic throughout the course of a simulation, then the above equations are enough to describe the state of the gas at all times. Unfortunately, actual astrophysical situations involve shocks quite frequently which changes the entropy of the system. To handle this additional term, SPH methods use a viscous force and ‘hide’ all the micro-physics of shock generation and dissipation into a term known as artificial viscosity (Monaghan & Gingold, 1983; Monaghan, 1992; Balsara, 1995). This viscous force causes an additional acceleration term to be present in the equation of motion for the gas particles:

\[
\frac{d\mathbf{v}_i}{dt} \bigg|_{\text{visc}} = -\sum_{j=1}^{N_{ngb}} m_j \Pi_{ij} \nabla_i W_{ij},
\]

(2.15)

where \( \Pi_{ij} \) is a viscous term that takes on a non-zero value only for a convergent flow \((v \cdot r < 0)\). This viscous force results in an irreversible transformation of gas kinetic energy into thermal energy with a corresponding rate of entropy generation given by:

\[
\frac{dS_i}{dt} = \frac{1}{2} \frac{\gamma - 1}{\rho_i^{-1}} \sum_{j=1}^{N_{ngb}} m_j \Pi_{ij} v_{ij} \cdot \nabla_i W_{ij},
\]

(2.16)

where \( S_i \) is the entropy for the \( i^{th} \) gas particle, \( \gamma \) is the polytropic index, \( v_{ij} \) is the relative velocity between the \( i^{th} \) and the \( j^{th} \) gas particle and \( \rho_i \) is the density of the \( i^{th} \) particle. In GADGET-2, \( \Pi_{ij} \) is computed from the formula:

\[
\Pi_{ij} = -\frac{\alpha \left[ c_i + c_j - 3w_{ij} \right]}{2 \rho_{ij}} w_{ij},
\]

(2.17)

\[
w_{ij} = \begin{cases} 
 v_{ij} \cdot r_{ij} / |r_{ij}| & \text{if } v_{ij} \cdot r_{ij} < 0, \\
 0 & \text{otherwise},
\end{cases}
\]

(2.18)

where \( c_i \) is the sound speed for the \( i^{th} \) gas particle, \( r_{ij} \) is the relative separation between the \( i^{th} \) and \( j^{th} \) particle, and \( \alpha \) is the artificial viscosity parameter (hereafter, AVP). This ensures that the gas particles receive this additional force only when they approach each other and that the viscous force does not diverge at small
separations, causing the particles to fly apart. Such a formulation of $\Pi_{ij}$ does not diverge and halts the particles but does not allow them to receive an additional acceleration once they are stationary. The SPH time-step is then calculated with:

$$\Delta t_{\text{hydro}} = \frac{C_{\text{Courant}} h_i}{\max_j (c_i + c_j - 3w_{ij})}$$

(2.19)

where $C_{\text{Courant}}$ is a parameter less than 1.0 and the denominator is the maximum found over all the neighbors $j$ for the $i^{th}$ particle. The Courant-Friedrichs-Lewy condition (Courant et al., 1928, 1967), commonly shortened as the Courant condition, is a limitation for ensuring convergence while solving time-dependent differential equations. This condition ensures that the time-step taken in a numerical simulation must be smaller than the time taken by a characteristic signal to cross a characteristic length. For SPH simulations, the relevant quantities are the sound speed, $c_i$ and the smoothing length, $h_i$ for the $i^{th}$ gas particle. $C_{\text{Courant}}$ is taken to be 0.15 in all our simulations.

### 2.8 Time Integration

Gravity naturally leads to large density contrasts in all collapsed objects resulting in a corresponding range in timescales ($\propto 1/\sqrt{\rho}$). If all the particles were to share the same time-step, low density regions would have artificially small time-steps. Adaptive and individual time-steps are, thus, a very essential requirement for all numerical simulations. The most commonly used time integration scheme used is the ‘leapfrog’ – a second-order method. The method gets the name because the velocity and position are evaluated at half time-steps, with the velocity and position ‘leaping’ over each other. Mathematically, for a system with an uniform time-step $dt$, position $x_i$, velocity $v_i$ and acceleration $a_i$ at time $t_i$, the leapfrog method advances the particle with the following algorithm:

$$
\begin{align*}
x_i & = x_{i-1} + v_{i-\frac{dt}{2}} \times \frac{dt}{2}, \\
a_i & = f(x_i), \\
v_{i+\frac{dt}{2}} & = v_{i-\frac{dt}{2}} + a_i \times dt.
\end{align*}
$$

(2.20)
where, $f(x_i)$ is some function that gives the net acceleration based on the position of the particle. Thus, the velocities are defined at $t_{i-1/2}, t_{i+1/2}, t_{i+3/2} \ldots$ while the positions are defined at $t_i, t_{i+1}, t_{i+2} \ldots$

The leapfrog method is symplectic, meaning that it is time-reversible and ensures that the total energy oscillates near the true total energy of the system. For small enough errors in the integrator, all stable orbits in a real astrophysical system will also be stable in the numerical simulation (Quinn et al., 1997).

In GADGET-2 the time-step, $\Delta t_{grav}$, for an individual particle is determined by the formula:

$$
\Delta t_{grav} = \min \left( \Delta t_{max}, \left( \frac{2 \eta \epsilon}{|a|} \right)^{1/2} \right),
$$

(2.21)

where $\Delta t_{max}$ defines the maximum time-step permitted for a particle, $\eta$ is the numerical accuracy parameter, $\epsilon$ is the gravitational softening parameter and $|a|$ is the acceleration of the particle. The time-step for an SPH particle also has to satisfy the Courant condition and is taken as the minimum of $\Delta t_{grav}$ computed above and $\Delta t_{hydro}$ computed in Eqn. 2.19. In practice, GADGET-2 subdivides individual time-steps in powers of two of a global time-step. Particles are allowed to move into a smaller time-step at every iteration but only allowed to attain a larger time-step at every second step if that leads to synchronization with the higher time-step hierarchy.

### 2.9 Overview

In this chapter, we discussed various N-body methods: Eddington inversion for computing the DF; acceptance-rejection technique; tree codes to compute softened gravitational forces in an efficient manner; SPH methods to capture hydrodynamics; and the time-stepping strategies to accurately evolve astrophysical systems. In the following chapters we will see the application of all of these techniques to create equilibrium galaxies and run merger simulations.
3.1 Dark Matter

Large scale cosmological simulations lead to a two-parameter universal dark matter halo density profile as shown by Navarro, Frenk, & White (1997) (hereafter, NFW). The key characteristics are the virial radius, $R_{\text{vir}}$ such that the average density of the halo within that radius is a certain multiple ($\Delta_{200} = 200$) of the critical density of the universe, $\rho_{\text{crit}} = 3H_0^2/8\pi G$, and the concentration parameter, $c = R_{\text{vir}}/r_s$, where $r_s$ is the scale radius of the NFW profile. The density and potential are given as follows:

$$\rho(s) = \frac{\Delta_{200} c^2 g(c) \rho_{\text{crit}}}{3 s (1 + cs)^2},$$

$$g(c) = \frac{1}{\ln(1 + c) - c/(1 + c)},$$

$$M(s) = \frac{g(c)}{g(cs)} M_{\text{vir}},$$

$$\Phi(s) = -g(c) V_{\text{vir}}^2 \ln(1 + cs) s,$$

$$V_{\text{vir}}^2 = \frac{GM_{\text{vir}}}{R_{\text{vir}}},$$

(3.1)
where, $s = r/R_{\text{vir}}$ is a dimensionless measure of the radius, $\rho(s)$ is the density of the NFW dark matter halo, $M(s)$ is the mass enclosed at radius $s$, and $\Phi(s)$ is the gravitational potential. $V_{\text{vir}}$ is the circular velocity at the virial radius and we will use $V_{\text{vir}}$ and $v_{\text{circ}}$ interchangeably. Note that the total mass of a NFW halo is infinite since $\rho$ drops off as $r^{-3}$ for large $r$. The NFW fit is valid only up to $R_{\text{vir}}$ and most numerical works truncate the halo there. However, this implies that particles with highly eccentric trajectories will not be represented in the numerical halo model. We implement an ad-hoc truncation radius for the dark matter (and the gas) at $R_{\text{halo}} = 1.2 \times R_{\text{vir}}$.

To compute the $D_\phi$, we first create a logarithmically spaced array in $s$ with $10^5$ bins. The minima and the maxima of the bins are fixed at $10^{-5}$ and 100 times the virial radius. Thus, we capture the inner cusp of the NFW halo profiles as well as the outlying particles with highly eccentric orbits; this can be seen from the upturn in the $D_\phi$ values for small $E$ in Fig. 3.1. Note that the inner limit is much smaller than our softening length of 0.005 while the outer limit is much larger than the truncation radius of 1.2. This was done to ensure a higher accuracy of the $D_\phi$ itself, even though at present we can not reach such numerical resolution. On this finely-spaced grid, we define the values for $\rho(s)$, $M(s)$ and $\Phi(s)$ and obtain the derivatives in the right hand side of Eqn 2.3 by finite differencing. Eqn. 2.3 is then numerically integrated by using a fourth-order Runge-Kutta integrator to get the final $D_\phi$. Following Lokas & Mamon (2001), we chose our system of units as $G = 1.0$, $M_{\text{vir}} = 1.0$, $R_{\text{vir}} = 2.0$ with the resulting $f(E)$ in units of $\sqrt{8M_{\text{vir}}}/(R_{\text{vir}}V_{\text{vir}})^3$. Once computed, the $D_\phi$ is stored and re-used for initializing galaxies of the same type but different number of halo particles.

Our fiducial galaxy is a Milky-Way type galaxy with a circular velocity of 160 km/s and a concentration parameter $c = 10$ (Bullock et al., 2001). $M_{\text{vir}}$ for this primary galaxy is $1.36 \times 10^{12} M_\odot$ with $R_{\text{vir}} = 228.6$ kpc. The secondary galaxies for our merger simulations are chosen to be one-third and one-tenth of the mass of the primary; this translates approximately to circular velocities of 110 and 74 km/s. To determine the concentrations of such halos, we use the analytic fitting
formula of Bullock et al. (2001):


c = \begin{cases} 
9 \times \left( \frac{M_{\text{vir}}}{M_{\text{typ}}} \right)^{-0.13} & , \frac{M_{\text{vir}}}{M_{\text{typ}}} > 0.2 \\
9 \times \left( \frac{M_{\text{vir}}}{M_{\text{typ}}} \right)^{-0.3} & , \text{otherwise}
\end{cases}

(3.2)

where, $M_{\text{typ}}$ is a typical mass for a given redshift. We assume $M_{\text{typ}} = 3 \times 10^{12} M_{\odot}$ to benchmark it relative to our primary galaxy concentration of $c = 10$. Using this prescription, the concentrations for the two secondary galaxies are 16 and 25 respectively.

Figure 3.1 shows the comparison between the DF from Lokas & Mamon (2001) for an NFW halo with $c = 10$ and the one obtained with our code.

**Figure 3.1.** The computed DF for three NFW halos $c = 10$, 16 and 25 and $V_{\text{vir}} = 160$, 110 and 74 km/s with 10% gas fractions. Overplotted in magenta is the DF from Lokas & Mamon (2001) for a NFW halo with $V_{\text{vir}} = 160$ km/s and $c = 10$. The computed DF agrees very closely and the slight deviation is caused by the additional gas component. Dark matter only halos (not shown here) agree to better than 1% with the curve from Lokas & Mamon (2001).
3.2 Gas Initialization

Now that the dark matter has been initialized, we need to do the same for the gas. Earlier papers have used $\Omega_b/\Omega_{dm} \rho_{dm}$ as the ad-hoc density for the gas for cosmological simulations of structure formation. While that approximation is valid for initial conditions in a cosmological framework where the force field is completely determined by the dark matter (and gas merely falls into the dark matter potential), our simulations are in a fixed space-time and we needed an analytic physically motivated gas density profile. We chose the observationally verified $\beta$-profile used to model gas in galaxy clusters, a more massive analog to our gaseous halos, assuming isothermal nature and hydrostatic equilibrium (Cavaliere & Fusco-Femiano, 1976; Jones & Forman, 1984; Eke et al., 1998) for the gas:

$$\rho_{gas} = \rho_0 (1 + r^2/r_c^2)^{-3\beta/2},$$

$$\beta = \frac{\mu m_H \sigma^2}{3kT}$$

where $\mu m_H$ is the mean molecular weight of the gas, $\sigma$ and $T$ are the velocity dispersion and temperature of the isothermal gas respectively, and $r_c$ is the core radius for the gas. We use $\beta = 2/3$ and a core radius given by $r_c = r_s/3$. Since the density profile is spherically symmetric, we employ the Eddington inversion method to compute the DF for joint dark matter and gas distribution. Pairs of position-velocity are then drawn from this computed DF for both the gas and the dark matter.

A gas distribution requires the temperature of the gas particles as an additional input parameter. The temperature can not be freely assigned since it is determined in a real system by the overall gravitational potential and the density profile of the gas. We assume that the gas is polytropic and follows an equation of state: $P \propto \rho^\gamma$ where $P$ and $\rho$ are the pressure and density of the gas and $\gamma$ is the polytropic index. For the primordial gaseous halos that we are simulating, the most appropriate composition is monoatomic and we chose the corresponding $\gamma = 5/3$. Finally, we use Eqn. 3.6 for hydrostatic equilibrium to compute the temperature of the gas self-consistently. The gas is assumed to be at $T_{vir}$ at $R_{vir}$ with a central density constrained by the total amount of gas in the halo. The gas density and pressure
are given by:

\[
\rho_{\text{gas}} = \rho_0 \left(1 + \frac{r}{r_c}\right)^2, \quad (3.4)
\]

\[
\rho_0 = \frac{f_{\text{gas}} M_{\text{halo}}}{4.0 \pi r_c^3 \left[R_{\text{halo}}/r_c - \tan^{-1}(R_{\text{halo}}/r_c)\right]}, \quad r_c = \frac{R_{\text{vir}}}{3 c} \quad (3.5)
\]

\[
\frac{\partial P}{\partial r} = -\rho_g \frac{\partial \Phi}{\partial r}, \quad T(r = R_{\text{vir}}) = T_{\text{vir}}. \quad (3.6)
\]

Here \(R_{\text{halo}} = 1.2 \times R_{\text{vir}}\) and \(M_{\text{halo}}\) represent the truncation radius and the total mass of the dark matter halo. Once the temperature has been assigned to the gas particles, we calculate the velocity dispersion by assuming virialization. The final gas velocities are drawn from a Gaussian distribution with zero mean velocity and a velocity dispersion given by \(\sigma^2 = 2 k T / (\gamma - 1) \mu m_H\). This completes the full set of input quantities for the initial conditions.

### 3.3 Stability

The input parameters to our galaxy model code are the circular velocity of the halo, the number of halo particles, the gas fraction and the number of gas particles. For an optimal coverage of the parameter space of mass ratio in galaxy mergers, we decided to simulate 1:1, 3:1 and 10:1 mergers where the ‘1’ refers to a Milky-Way type galaxy with circular velocity = 160 km/s; the ‘3’ and the ‘10’ are misnomers – they actually refer to galaxies with one-third and one-tenth the mass compared to the fiducial Milky-Way galaxy. For a galaxy with one-third the mass, the circular velocity is 110 km/s and for one-tenth the mass, the circular velocity is 74 km/s. Since we aim to simulate the mergers of two different mass galaxies, it was imperative to have the same mass per particle between each of the galaxies to prevent spurious numerical relaxation. Thus, the number of particles in the dark matter halo and in the gas in the secondary galaxies are set by the corresponding number in the primary galaxy. We use a fixed number of \(5 \times 10^5\) particles for dark matter and gas in the primary galaxy for all the gas fractions. Since the total halo mass of the primary galaxy is fixed, our halo mass resolution is \(3 \times 10^6\) M\(_\odot\), independent of the gas fraction. The gas mass resolution for the 1%, 10% and 18% gas fractions in the primary galaxy are \(1.5 \times 10^6\), \(1.5 \times 10^7\) and \(2.7 \times 10^7\) M\(_\odot\).
respectively (see Table 3.1). Note that this gas mass resolution is obtained by multiplying the total number of neighbors of a gas particle, 50, by the mass of each gas particle. Thus, the gas mass resolution decreases linearly with the increasing gas fraction, due to the linearly increasing mass of an individual gas particle.

We devised a multi-step process to generate extremely stable initial conditions. Initially, the gas temperature is determined from the hydrostatic equilibrium condition in the joint dark matter and gas potential. Then the gas velocity, i.e., velocity dispersion, were assigned based on virial equilibrium. We find that this assumption is not valid in the inner regions of the galaxy; even in isolation, the gas particles in the centers of the galaxies shock-heat, increase their central temperatures and decrease their velocity dispersion rapidly. Higher artificial viscosity aids this transformation and the core temperatures can increase by as much as a factor of 2 from the originally solved temperature profile. Since we are exploring how gas-rich galaxies behave during galaxy mergers, we did not want to the input temperature profile, \( T(r) \), to vary so much and devised a way to reduce this temperature evolution. We evolved the galaxy model in isolation with AVP= 0.5 with the numerically solved \( T(r) \) to allow the gas particles to attain a new equilibrium \( T(r) \). This is the first iteration on an equilibrium \( T(r) \) for the halo gas. We found that the new \( T(r) \) is larger at the center and turns over at the outer edge with an exponential drop-off near \( R_{\text{vir}} \). We fit this outer temperature profile and apply it as a second iteration on \( T(r) \) for our model (see Fig 3.3 for a comparison of the two iterations). This evolved temperature profile is nearly flat to \( \sim 0.5R_{\text{vir}} \) and is consistent with observations of galaxy clusters showing that the temperature drops off in the outer regions \( (0.2 \lesssim r/R_{\text{vir}} \lesssim 0.6) \) by only 40% from the core temperature (Leccardi & Molendi, 2008; Vikhlinin et al., 2005). Then, we evolve the galaxy with a zero artificial viscosity over a period of \( \sim 1 \) Gyr to get rid of the dynamical irregularities in our \( T(r) \) fit. Since there is absolutely no entropy increase with a zero AVP, our temperature profile essentially remains fixed.

To test the stability of this new galaxy model after iterating the temperature profile, we simulate this evolved galaxy with AVP = 0.5 for a merger time-scale, \( \sim 15 \) Gyr. We find that the change in the overall temperature profile drops to

\[ \text{Since the surface brightness decreases rapidly with radius, the signal in the outer regions is dominated by low photon counts and high background; consequently, the measurements are not as robust in the outskirts of clusters.} \]
Figure 3.2. Iterations for creating a stable temperature profile. The black shows the spherically averaged \( T(r) \) obtained after solving Eqn 3.6. The red shows the \( T(r) \) after an exponential drop-off has been added to the outer region. The dashed line shows the core radius, \( r_c = R_{\text{vir}}/3c \) for the gas and corresponds to the location of the dip in \( T(r) \). The temperature drops from \( 0.7T_{\text{vir}} \) at \( 0.2R_{\text{vir}} \) to \( 0.4T_{\text{vir}} \) at \( 0.6R_{\text{vir}} \) and is consistent with observations of galaxy clusters (Vikhlinin et al., 2005; Leccardi & Molendi, 2008).

\( \sim 10 - 20\% \) and we argue that this is an acceptable level of stability over a Hubble time.

We re-calculate \( M_{\text{vir}}, R_{\text{vir}}, \) and \( L_X \) (see Table 3.2) after iterating the initial conditions. For the primary galaxy, the maximum change in \( R_{\text{vir}} \) is \( \sim 2.5\% \); for the one-third galaxy and the one-tenth galaxy, the corresponding changes are \( \sim 3\% \) and \( \sim 2.6\% \) respectively.

Once the galaxies are initialized, we check for stability by evolving them in isolation. One of the key ways of ascertaining the equilibrium of multi-component systems is by computing the Virial of Clausius, \( VC \) given by:

\[
VC = \sum_i m_i (x_i \times ax_i + y_i \times ay_i + z_i \times az_i),
\]

(3.7)
where $m_i$, $x_i$ and $ax_i$ are the mass, $x$ coordinate of the position, and the acceleration along the X-axis of the $i^{th}$ particle respectively. An equilibrium model for a galaxy should have $VC$ fairly close to 1. The numerical models for our galaxies in the first iteration have a $VC \sim 1.2$, irrespective of the gas fraction or the halo size. Thus, while being close to equilibrium, the galaxies are not exactly in steady state initially. The value of $VC$ for the isolated galaxies with the iteratively-obtained $T(r)$, ranges from 1.12 for the primary to $\sim 1.07$ for both the secondary galaxies. We accept that deviation from the ideal equilibrium as an effect of the order 10% – a reasonable approximation.

To summarize, our initial galaxy models are obtained in the following manner:

- Create galaxy model with the numerical code.
- Evolve with AVP = 0.5. Fit temperature profile with an exponential function.
- Apply the exponential drop-off to the galaxy directly obtained in Step 1.
- Evolve this new galaxy with zero AVP for 1 Gyr.
- Use the evolved galaxy as input galaxy models for the merger simulation.
Table 3.1: This table lists the parameters used to make the 9 isolated galaxy models. A constant physical softening of 1.14 kpc was used for both the gas and dark matter particles in all simulations. The number of particles used to model the two smaller galaxies was chosen such that the mass of each particle (dark matter and gas) was the same as the mass of the corresponding particle for the Primary galaxy.

<table>
<thead>
<tr>
<th>Gas Content</th>
<th>Galaxy Type</th>
<th>Conc.</th>
<th>$v_{\text{circ}}$</th>
<th>$R_{\text{vir}}$</th>
<th>$M_{\text{vir}}$</th>
<th>DM Halo Mass</th>
<th>Tot. DM Particles$^\dagger$</th>
<th>DM Mass Res.$^\dagger$</th>
<th>Gas Mass Res.$^\dagger$</th>
<th>Relaxation time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1% gas</td>
<td>Primary</td>
<td>10.0</td>
<td>160.0</td>
<td>228.6</td>
<td>136.1</td>
<td>150.0</td>
<td>$5.0 \times 10^5$</td>
<td>3.0</td>
<td>1.5</td>
<td>7188.1</td>
</tr>
<tr>
<td></td>
<td>Onethird</td>
<td>16.0</td>
<td>110.0</td>
<td>158.5</td>
<td>45.3</td>
<td>48.0</td>
<td>$1.6 \times 10^5$</td>
<td>3.0</td>
<td>1.5</td>
<td>2242.4</td>
</tr>
<tr>
<td></td>
<td>Onetenth</td>
<td>25.0</td>
<td>74.0</td>
<td>106.1</td>
<td>13.6</td>
<td>14.5</td>
<td>$4.8 \times 10^4$</td>
<td>3.0</td>
<td>1.5</td>
<td>673.8</td>
</tr>
<tr>
<td>10% gas</td>
<td>Primary</td>
<td>10.0</td>
<td>160.0</td>
<td>228.6</td>
<td>136.1</td>
<td>150.0</td>
<td>$5.0 \times 10^5$</td>
<td>3.0</td>
<td>15.0</td>
<td>6657.9</td>
</tr>
<tr>
<td></td>
<td>Onethird</td>
<td>16.0</td>
<td>110.0</td>
<td>158.5</td>
<td>45.3</td>
<td>48.0</td>
<td>$1.6 \times 10^5$</td>
<td>3.0</td>
<td>15.0</td>
<td>2178.6</td>
</tr>
<tr>
<td></td>
<td>Onetenth</td>
<td>25.0</td>
<td>74.0</td>
<td>106.1</td>
<td>13.6</td>
<td>14.5</td>
<td>$4.8 \times 10^4$</td>
<td>3.0</td>
<td>15.0</td>
<td>649.1</td>
</tr>
<tr>
<td>18% gas</td>
<td>Primary</td>
<td>10.0</td>
<td>160.0</td>
<td>228.6</td>
<td>136.1</td>
<td>150.0</td>
<td>$5.0 \times 10^5$</td>
<td>3.0</td>
<td>27.0</td>
<td>6600.1</td>
</tr>
<tr>
<td></td>
<td>Onethird</td>
<td>16.0</td>
<td>110.0</td>
<td>158.5</td>
<td>45.3</td>
<td>48.0</td>
<td>$1.6 \times 10^5$</td>
<td>3.0</td>
<td>27.0</td>
<td>2127.3</td>
</tr>
<tr>
<td></td>
<td>Onetenth</td>
<td>25.0</td>
<td>74.0</td>
<td>106.1</td>
<td>13.6</td>
<td>14.5</td>
<td>$4.8 \times 10^4$</td>
<td>3.0</td>
<td>27.0</td>
<td>639.3</td>
</tr>
</tbody>
</table>

$^\dagger$The number of gas particles is the same as the number of halo particles in each galaxy.
$^\dagger$The total gas mass can be obtained by multiplying the total halo mass with the gas fraction.
$^\dagger$We used a fixed 50 SPH neighbors for all the simulations.
Table 3.2: This table lists some of the physical quantities for the galaxies after they were evolved with zero artificial viscosity for 1 Gyr. The merging galaxies are created by taking two galaxies from this table.

<table>
<thead>
<tr>
<th>Gas Content</th>
<th>Galaxy Type</th>
<th>Unbound gas</th>
<th>$L_X$</th>
<th>$R_{\text{vir}}$</th>
<th>$M_{\text{vir}}$</th>
<th>Hot gas</th>
<th>Virial of Clausius</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>[%]</td>
<td>[10^{40} \text{ erg/s}]</td>
<td>[\text{kpc}]</td>
<td>[10^{10} \text{ M}_\odot]</td>
<td>[%]</td>
<td></td>
</tr>
<tr>
<td>1% gas</td>
<td>Primary</td>
<td>0.36</td>
<td>0.10</td>
<td>224.3</td>
<td>128.9</td>
<td>70.0</td>
<td>1.11</td>
</tr>
<tr>
<td></td>
<td>OneThird</td>
<td>0.50</td>
<td>0.03</td>
<td>154.5</td>
<td>42.1</td>
<td>46.0</td>
<td>1.08</td>
</tr>
<tr>
<td></td>
<td>OneTenth</td>
<td>0.58</td>
<td>0.01</td>
<td>103.9</td>
<td>12.8</td>
<td>21.0</td>
<td>1.07</td>
</tr>
<tr>
<td>10% gas</td>
<td>Primary</td>
<td>0.62</td>
<td>8.87</td>
<td>223.4</td>
<td>126.5</td>
<td>79.0</td>
<td>1.12</td>
</tr>
<tr>
<td></td>
<td>OneThird</td>
<td>0.43</td>
<td>3.17</td>
<td>153.9</td>
<td>41.6</td>
<td>48.0</td>
<td>1.08</td>
</tr>
<tr>
<td></td>
<td>OneTenth</td>
<td>0.56</td>
<td>0.88</td>
<td>103.7</td>
<td>12.8</td>
<td>24.0</td>
<td>1.06</td>
</tr>
<tr>
<td>18% gas</td>
<td>Primary</td>
<td>0.68</td>
<td>26.5</td>
<td>222.9</td>
<td>126.3</td>
<td>77.0</td>
<td>1.12</td>
</tr>
<tr>
<td></td>
<td>OneThird</td>
<td>0.42</td>
<td>10.9</td>
<td>153.6</td>
<td>41.3</td>
<td>50.0</td>
<td>1.08</td>
</tr>
<tr>
<td></td>
<td>OneTenth</td>
<td>0.50</td>
<td>2.75</td>
<td>103.3</td>
<td>12.6</td>
<td>25.0</td>
<td>1.06</td>
</tr>
</tbody>
</table>

3.3.1 Softening Length

Our softening length was chosen using the suggested optimal softening from Power et al. (2003) with $\epsilon_{\text{opt}} \gtrsim 4R_{\text{vir}}/\sqrt{N_{\text{vir}}}$ where $N_{\text{vir}}$ is the number of particles within the virial radius. For our simulations, $\epsilon_{\text{opt}} \approx 0.9$ kpc compares favorably with our chosen $\epsilon = 1.14$ kpc. We also evolved the primary galaxy with 10% gas fraction in isolation with $\epsilon = 0.1$ kpc and 11.4 kpc. For the larger softening, the accumulated force errors were too large and resulted in a different (exploding) behavior for the galaxy; however, the results from the simulation with $\epsilon = 0.1$ kpc were similar to those ran with our standard softening of 1.14 kpc. A smaller softening length, while increasing the accuracy of the force integration, decreases the CPU-efficiency of the computation; the one-tenth softening parameter simulation took much longer to run. Since our fiducial softening parameter of 1.14 kpc was already in the range of optimal softening parameters suggested by Power et al. (2003), we ran all our merger simulations with this softening parameter for both the dark matter and the gas particles in all the simulations. For a merger simulation, even though we ensured that particle mass was constant, the optimal softenings were different for the secondary galaxy. However, because of issues with numerical noise, it is unwise to evolve similar components with different softenings; thus, the softening
parameter for both the dark matter and the gas particles in all our galaxy simulations are the same, viz., $\epsilon = 1.14$ kpc. For the SPH particles, the Jeans mass was always resolved (Truelove et al., 1997; Bate & Burkert, 1997) by at least 3 orders of magnitude throughout the entire duration of all the simulations. In the shocked regions, we have a total of about 5% of the SPH particles present out of which a very small fraction ($\lesssim 2\%$) are identified as shocked particles. The mean smoothing length of the shocked particles is $\sim 25$ kpc. Since a shock gets spread out over 2-3 smoothing lengths, we expect to resolve shocks with sizes $\sim 50$-75 kpc. The average number of SPH particles in the initial shock is 500-1000 and is an order of magnitude larger than the number of neighbors for every SPH particle. An impractically long simulation would be required to obtain better spatial resolution.

### 3.3.2 Artificial Viscosity

Since the AVP determines the strength of the shocks produced in the simulation, we decided to test the effect of a varying AVP. We evolved the primary galaxy with 10% gas fraction in isolation, with 5 different AVP’s, viz., $10^0$, $10^{-1}$, $10^{-2}$, $10^{-3}$ and $10^{-4}$. Note that the recommended range of AVP is $0.1 - 1.0$ for numerical simulations. We find that the difference in the final temperature profile is negligible between the AVP of 0.1 and 1.0; similarly for AVP’s of $10^{-4}$ and 0. However, the average temperature is $20 - 30\%$ larger in the higher AVP case. Figure 3.5 shows the evolution of density, temperature and $L_X$ from these set of simulations. In keeping with the recommended value for AVP, we chose an AVP= 0.5 to run all of our merger simulations. We also ran an equal-mass merger with 10% gas fraction and 0.1 $R_{\text{vir}}$ impact parameter with three different AVP’s of 0.1, 0.5 and 1.0. The results are indistinguishable, implying that a value of AVP= 0.5 is a reasonable choice.

### 3.4 Merger Orbits

Cosmological simulations of large-scale structure formation shows that $\sim 40\%$ of the mergers occur with an eccentricity $e = 1.0 \pm 0.1$ and $\sim 85\%$ have an impact
Figure 3.3. The density profiles for the dark matter and the gas for the nine isolated galaxies. The top, middle and bottom row represents 1%, 10% and 18% gas fractions respectively. The first, second and the third column represents the primary, one-third and the one-tenth galaxies respectively. Red and blue show the initial and final density profiles for both the dark matter and gas. The evolution is minimal for both the gas and dark matter is minimal; however, we see some outflowing features in the dark matter density. This results in a slightly larger galaxy at the end of 1 Gyr. The analytical input profiles shown in magenta (for dark matter) and cyan (for gas). The vertical dashed line is the softening used in the simulation.

parameter $b > 0.1 R_{\text{vir}}$ (Khochfar & Burkert, 2006; Benson, 2005). More than 50% of the mergers occur with $b > 0.5 R_{\text{vir}}$. We keep this larger impact parameters in mind and create three orbits with $b = 0.01$, 0.1 and 0.5 $R_{\text{vir}}$; with two of our three orbits probing the larger impact parameters. We calculate the orbital velocities of the mergers assuming a parabolic trajectory; however, we want to see the effect of a varying total energy in the orbit while getting a merger remnant at the end of the simulation. This is not possible with a parabolic trajectory since the total energy is identically equal to zero. To achieve our goal, we create bound orbits from the parabolic ones by reducing the orbital velocities by a factor of $\sqrt{2}$. The resultant
Figure 3.4. The temperature profile for the gas in all the nine isolated galaxies. The line-types and the plots on each row and column are the same as in Figure 3.3.1. The virial temperatures for each galaxy is also noted in each panel in the top-right. The primary evolution of the gas temperature is the creation of the exponential tail in the temperature values with a ‘knee’ around $\sim 0.5 \, R_{\text{vir}}$.

Elliptical orbits have $e \sim 0.05, 0.2$ and $0.45$. We also check for the effect of the initial velocities, albeit only for the equal-mass mergers by using a set of 9 mergers where the velocities are a factor of $\sqrt{2}$ larger than the parabolic velocities. The resulting encounter is a hyperbolic one with $e > 1$. Only two of these hyperbolic orbits result in a merger remnant; for the 10% and 18% gas fraction case with $b = 0.01 \, R_{\text{vir}}$.

The initial galaxies are separated by $r_{\text{sep}} = 1.5 \times (R_1 + R_2)$. The orbital
Figure 3.5. **Top Left**: The gas (lower curve) and dark matter (upper curve) density profile with the analytical $\beta$ and the NFW profile overlaid as solid lines, for the primary galaxy with gas fraction of 10%. The galaxy is represented by $5 \times 10^5$ dark matter and gas particles each and is evolved for 1 Gyr with an AVP= 0.1. The flattening of the dark matter profile occurs in the inner regions of the halo and corresponds to the softening parameter – represented by the vertical dashed line. **Top Right**: The same plot showing the gas density profile at 1 Gyr for different AVP’s. **Bottom Left**: The evolution of the temperature profile for different AVP’s. The temperature increases by almost a factor of 2 for AVP $> 0.01$. **Bottom Right**: The evolution of the temperature profile for the total X-ray emission via thermal bremsstrahlung from gas particles with $T > 10^{5.2}$K and $\rho < 0.01 \, M_\odot/pc^3$.

Velocities of the center of mass of each galaxy can then be computed from:

\[ \mu = \frac{M_1 M_2}{M_1 + M_2}, \]
\[ L = \mu r_{sep} \times \dot{r}_{sep}, \]
\[ E = \frac{1}{2} \mu r_{sep}^2 - \frac{GM_1 M_2}{r_{sep}}, \]
\[ e = \sqrt{1 + \frac{2EL^2}{\mu (GM_1 M_2)^2}}. \]
where, \(e, E\), and \(L\) are the eccentricity, total energy and total angular momentum of the orbit, and \(M_1, M_2, R_1,\) and \(R_2\) are the virial mass and virial radius of the primary and the secondary galaxies. These equations can then be solved for a parabolic orbit with \(e = 1\). The total velocity obtained is then added to each galaxy in inverse ratio to their masses. Our simulations contain a set of 27 mergers with orbital velocities that are reduced by a factor of \(\sqrt{2}\) obtained from the previous equations. We also ran an additional set of 9 simulations for the 1:1 mergers with initial velocities of the galaxies increased by a factor of \(\sqrt{2}\) – the hyperbolic orbits. The list of the simulations and initial velocities can be found in Table 3.3; the eccentricity, total energy and velocities are obtained from Eqn. 3.8.

**Table 3.3:** The list of all the merger simulations performed. The first three 1:1 mergers for each gas fraction are the hyperbolic encounters, while the rest are all bound elliptical orbits with a fixed amount of energy in the orbit for a given merger ratio. Note that the velocities of the individual galaxies are exactly a factor of 2 larger than the ones in the bound orbits.

<table>
<thead>
<tr>
<th>Gas</th>
<th>Merger Type</th>
<th>b</th>
<th>Eccentricity [kpc]</th>
<th>Orbital Energy [10^{56}\ erg]</th>
<th>Primary Vel. [km/s]</th>
<th>Secondary Vel. [km/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10%</td>
<td>1:1</td>
<td>2.3</td>
<td>2.99</td>
<td>2864.6</td>
<td>137.8</td>
<td>137.8</td>
</tr>
<tr>
<td></td>
<td>1:1</td>
<td>22.8</td>
<td>2.95</td>
<td>2864.6</td>
<td>137.8</td>
<td>137.8</td>
</tr>
<tr>
<td></td>
<td>1:1</td>
<td>114.3</td>
<td>2.78</td>
<td>2864.6</td>
<td>137.8</td>
<td>137.8</td>
</tr>
<tr>
<td></td>
<td>1:1</td>
<td>2.3</td>
<td>0.06</td>
<td>-1432.3</td>
<td>68.9</td>
<td>68.9</td>
</tr>
<tr>
<td></td>
<td>1:1</td>
<td>22.8</td>
<td>0.18</td>
<td>-1432.3</td>
<td>68.9</td>
<td>68.9</td>
</tr>
<tr>
<td></td>
<td>1:1</td>
<td>114.3</td>
<td>0.41</td>
<td>-1432.3</td>
<td>68.9</td>
<td>68.9</td>
</tr>
<tr>
<td></td>
<td>3:1</td>
<td>1.9</td>
<td>0.06</td>
<td>-542.0</td>
<td>29.5</td>
<td>92.2</td>
</tr>
<tr>
<td></td>
<td>3:1</td>
<td>19.4</td>
<td>0.20</td>
<td>-542.0</td>
<td>29.5</td>
<td>92.2</td>
</tr>
<tr>
<td></td>
<td>3:1</td>
<td>96.8</td>
<td>0.44</td>
<td>-542.0</td>
<td>29.5</td>
<td>92.2</td>
</tr>
<tr>
<td></td>
<td>10:1</td>
<td>1.7</td>
<td>0.07</td>
<td>-188.5</td>
<td>10.5</td>
<td>108.9</td>
</tr>
<tr>
<td></td>
<td>10:1</td>
<td>16.7</td>
<td>0.21</td>
<td>-188.5</td>
<td>10.5</td>
<td>108.9</td>
</tr>
<tr>
<td></td>
<td>10:1</td>
<td>83.7</td>
<td>0.48</td>
<td>-188.5</td>
<td>10.5</td>
<td>108.9</td>
</tr>
<tr>
<td>1%</td>
<td>1:1</td>
<td>2.3</td>
<td>2.99</td>
<td>3397.9</td>
<td>143.9</td>
<td>143.9</td>
</tr>
<tr>
<td></td>
<td>1:1</td>
<td>22.8</td>
<td>2.95</td>
<td>3397.9</td>
<td>143.9</td>
<td>143.9</td>
</tr>
<tr>
<td></td>
<td>1:1</td>
<td>114.3</td>
<td>2.78</td>
<td>3397.9</td>
<td>143.9</td>
<td>143.9</td>
</tr>
<tr>
<td></td>
<td>1:1</td>
<td>2.3</td>
<td>0.05</td>
<td>-1698.9</td>
<td>71.9</td>
<td>71.9</td>
</tr>
<tr>
<td></td>
<td>1:1</td>
<td>22.8</td>
<td>0.18</td>
<td>-1698.9</td>
<td>71.9</td>
<td>71.9</td>
</tr>
<tr>
<td></td>
<td>1:1</td>
<td>114.3</td>
<td>0.41</td>
<td>-1698.9</td>
<td>71.9</td>
<td>71.9</td>
</tr>
<tr>
<td></td>
<td>3:1</td>
<td>1.9</td>
<td>0.06</td>
<td>-642.9</td>
<td>30.8</td>
<td>96.3</td>
</tr>
<tr>
<td></td>
<td>3:1</td>
<td>19.4</td>
<td>0.20</td>
<td>-642.9</td>
<td>30.8</td>
<td>96.3</td>
</tr>
<tr>
<td></td>
<td>3:1</td>
<td>96.8</td>
<td>0.44</td>
<td>-642.9</td>
<td>30.8</td>
<td>96.3</td>
</tr>
<tr>
<td></td>
<td>10:1</td>
<td>1.7</td>
<td>0.07</td>
<td>-223.6</td>
<td>10.9</td>
<td>113.6</td>
</tr>
<tr>
<td></td>
<td>10:1</td>
<td>16.7</td>
<td>0.21</td>
<td>-223.6</td>
<td>10.9</td>
<td>113.6</td>
</tr>
<tr>
<td></td>
<td>10:1</td>
<td>83.7</td>
<td>0.48</td>
<td>-223.6</td>
<td>10.9</td>
<td>113.6</td>
</tr>
</tbody>
</table>
3.5 Analysis

3.5.1 X-ray emission

Our analysis assumes that the primary source of X-rays in the galaxy merger is hot, diffuse gas of primordial composition. In real galaxies, a lot of the X-rays are produced by discrete sources like supernova remnants, compact binaries, and accretion events occurring around massive black holes. However, such emission is more likely to occur in the denser central regions of the galaxy (Soifer et al., 1984; Sanders et al., 1986, 1988) and hence, also more likely to be obscured by the larger column density of the intervening gas and dust (Scoville et al., 1986; Sargent et al., 1989, 1987; Hopkins et al., 2005). The hot, diffuse gas should be relatively less affected by in situ absorption. Following (Cox et al., 2006), we choose only those particles that have $T > 10^{5.2}K$ and density $\lesssim 0.01 \, M_\odot \, pc^{-3}$ to identify hot diffuse gas that will radiate in the X-ray band; cooler, denser gas will form stars or cool via atomic line transitions in the optical, infra-red and radio and will not affect the total X-ray luminosity of the galaxy. The SPH particles at $r \approx R_{\text{vir}}$ have the largest smoothing lengths, $\sim 10$ kpc. For the most massive SPH particles (for our 18% gas fraction galaxies), the gas mass resolution is $\sim 2.7 \times 10^7 M_\odot$; this results in a minimum density resolution (assuming a smoothing length of 10 kpc) of $\sim 6.510^{-6} M_\odot / pc^3$. Thus, our threshold density is always well within our resolution limit during our simulations. Throughout this thesis, we will use the term hot gas to denote those gas particles that satisfy the above criteria. For the assumed primordial composition of the gas, it will be completely ionized and emit...
X-rays via thermal bremsstrahlung. Therefore, we estimate the X-ray luminosity as:

\[ L_X = 1.2 \times 10^{-24} (\mu m_p)^{-2} \sum_i m_i \rho_i T_i^{1/2} \text{erg s}^{-1}. \]  

(3.9)

where \( m_p \) is the mass of a proton, \( m_i, \rho_i, T_i \) is the mass, density and temperature of a gas particle respectively.

Note that enriched gas at \( \sim 10^6 \) K will cool primarily through metal line emission, implying that the X-ray emission could well be enhanced in actual colliding galaxies. The emission from the gas is calculated using Eqn. 3.9 assuming the gas, with primordial composition, is completely ionized with a mean molecular weight \( \mu = 0.6 m_p \).

The orbital energy in the gas is \( \gtrsim 10^{58} \) erg; this gets redistributed as the internal kinetic energy of the remnant. The average X-ray luminosity in all the simulations is \( \sim 10^{41} \) erg/s and it would take more than 10 Gyrs to radiate away only the orbital energy. To check the validity of our results under the assumption of inefficient cooling, we compute the cooling times for the hot gas particles and reject the ones that have cooling time less than the dynamical time. As a further safeguard, we also reject the hot gas particles that have a cooling time less than the time between consecutive snapshots (\( \sim 100 \) Myrs for all simulations). After these two step rejection process we recompute the X-ray emission from the hot gas and the shocked gas particles and find that the X-ray emission changes by less than 5% for the hot gas and less than 1% for the shocked gas particles. Thus, our predictions regarding X-ray emission from the hot and the shocked gas are not susceptible to significant changes from radiative losses.

### 3.5.2 Shock Detection

In our simulations, large-scale shocks are developed in the gas. Since our simulations are adiabatic, shocks are the only mechanism that causes increase in the entropy of a gas particle. We take advantage of the conservative-entropy formulation (Springel & Hernquist, 2002) of GADGET-2 (Springel, 2005), and identify the shocked particles based on their rate of change of entropy (calculated with
Each snapshot file in GADGET-2 has an output quantity that gives the rate of change of entropy, $dt_{\text{entropy}}$, for every gas particle. We visually locate the shocked regions in the gas for every merger simulation and plot $dt_{\text{entropy}}$ as a function of the x-coordinate. We then use a threshold value for $dt_{\text{entropy}}$ that uniquely identifies the shocked particles. Since such a threshold value is somewhat ad-hoc, we chose it quite conservatively, i.e., we are likely to miss some shocked particles rather than identify unshocked particles as shocked, a Type-II error for the algorithm. The value of the threshold turns out to be the same for a fixed gas fraction and is $10^{10}$, $3 \times 10^9$ and $2 \times 10^9$ for 1%, 10% and 18% gas fractions respectively. Any gas particle that has a $dt_{\text{entropy}}$ value greater than this threshold is identified as being shocked. We also locate every particle that was ever shocked by analyzing all the snapshots and keeping track of the particle ids. This will be helpful later on in determining whether the gas particles that were unbound at the end of the simulation, got shocked during the merger. In principle, we can iteratively estimate a Mach number, $M_1$, for the shocks by using the formalism of Pfrommer et al. (2006):

$$\left[f_A(M_1) - 1\right] M_1 = \frac{f_h h}{c_1 A_1} \frac{dA_1}{dt},$$

$$f_A(M_1) = \frac{2 \gamma M_1^2 - (\gamma - 1)}{\gamma + 1} \left[\frac{(\gamma - 1) M_1^2 + 2}{(\gamma + 1) M_1^2}\right]^\gamma,$$

where $f_h$ is the number of smoothing lengths, $h$, over which a shock is broadened out due to the SPH method itself, $A_1$ is the entropy of the gas, $c_1$ is the sound speed and $\gamma$ is the polytropic index. The implementation of this method is outside the scope of the project but we will like to use this equation to constrain the behavior of the shocked gas particles as part of a future work.

### 3.5.3 Unbound particles

At the beginning of the simulation, we identify the top 10% of the most bound particles for each galaxy and track them throughout the simulation to identify the center of mass. The average of their positions and velocities gives us an estimate of the center of mass position and velocity during the course of the simulation.
GADGET-2 provides the total potential of each particle and we can compute the total energy of each particle by:

\[ E_i = \Phi_i + \frac{1}{2} \times \left[ (v_x - v_{x,cm})^2 + (v_y - v_{y,cm})^2 + (v_z - v_{z,cm})^2 \right], \]  

(3.11)

where \( v_x \) and \( v_{x,cm} \) is the velocity of the particle and the center of mass of the galaxy respectively, \( \Phi_i \) is the potential obtained from GADGET-2 and \( E_i \) is the total energy of the particle. Particles that have a total energy greater than zero are considered unbound from the system. By defining the unbound material in this fashion, we avoid tagging particles that could be unbound from each galaxy but not unbound from the entire system. In the simulations where we have a merger remnant, we identify the unbound particles at the end of the simulation (when the system has settled down) and track those particles through the duration of the merger.

### 3.6 Overview

In this chapter, we discussed our iterative technique for creating an equilibrium galaxy with gas fractions of 1%, 10% and 18% for halos with \( v_{\text{circ}} = 160, 110 \) and 74 km/s. We also describe our analysis techniques for estimating the X-ray emission, both from regular hot gas and the shocked gas, and the identification of unbound particles in the simulation. For our merger simulations, we place two equilibrium galaxies in different orbits. We will discuss the results of the merger simulations in the next chapter.
Chapter 4

Results

4.1 Introduction

We developed a code to compute the DF for an equilibrium NFW dark matter halo (Navarro et al., 1997) with varying gas fractions with a $\beta$-profile density. As discussed in the previous chapter, we use three kinds of galaxies, each with three gas fractions and place them in an merger orbit; for a total of 36 simulations.

We perform high-resolution numerical simulations to achieve the following:

- Explore the effect of halo gas fraction for a given halo mass on the dynamics and fate of the hot gas in the merger process.

- Study the strength of the merger by changing the impact parameter and the orbital geometry of the collision.

- Explore the effect of the initial gas temperature on the strength of the shock and expected X-ray signature. This would test bi-modal galaxy formation theories (Kereš et al., 2005) that predict smaller galaxies ($\lesssim 10^{11.4} M_\odot$) contain only warm gas ($\lesssim 10^{4.5}$ K). The shocks in colliding hot halos can be used to estimate the initial temperature of the gas, and thereby, constrain the afore-mentioned theory.

In this chapter, we will present the results of our numerical simulations of galaxy mergers. First, we show the simulations where we tested for dependence of the results on the value of AVP; we ran an equal-mass merger 10\% gas fraction on
a parabolic orbit with an impact parameter of $0.1 R_{\text{vir}}$ with three different AVP’s of $10^{-4}$, 0.5 and 1.0. We find that the two simulations with larger AVP’s are essentially identical and we adopt the canonical value of 0.5 for the AVP for all the rest of our 36 simulations. We will then discuss those 36 simulations and broadly discuss the X-ray luminosities, from the hot gas as well as the shocked gas, and the unbound material at the end of the simulations. We will derive an analytic fit for the peak shock X-ray luminosity and the unbound gas mass as a function of relevant physical parameters like the progenitor galaxy masses, impact parameters etc.

### 4.2 Testing the Artificial Viscosity Parameter

Our simulations deal with extensive shocks in the gas. The inherent smoothing over neighbors for the SPH technique makes it difficult to resolve shocks in a simulation. The AVP converts a fraction of the kinetic energy of a convergent flow into thermal energy; this also increases the thermal pressure and acts as a decelerating mechanism. Therefore, the strength of the shocks, measured by the temperatures attained, is somewhat dependent on the value of the AVP. To test the convergence of our simulations, we ran a suite of three equal-mass mergers with 10% gas fraction and a 0.1 $R_{\text{vir}}$ impact parameter with AVP of $10^{-4}$, 0.5 and 1.0 and compared the results. We find that the dark matter distribution in the remnant is identical in all the three simulations.

Expectedly, the gas component behaves completely differently between the low and the high viscosity runs. Shock-heating is almost absent (refer to Fig 4.3) in the low-viscosity run, thereby accounting for the consistent low value of hot gas fraction (Figure 4.2). The gas density profile for the remnant is also lower by an order of magnitude and only matches up to the higher AVP runs close to the virial radius. The lack of a mechanism for dissipating bulk momentum is responsible for this shallower density core in the low AVP case. For the same reasons, the core temperature is also lower by a factor of $\sim 5$ in the low AVP run. In all the cases, the final gas density is shallower than the original analytical cored isothermal density profile. The evolution of total X-ray luminosity shows two major peaks corresponding to the first and the second passage of the galaxies (see Fig 4.2).
The remnant has a X-ray luminosity of $\sim 4 \times 10^{40}$ erg/s and remains constant from 20 Gyr. The peaks in the X-ray luminosity correspond to the first, second passages and the final merging event of the galaxies. The first passage creates the highest luminosity of $\sim 5 \times 10^{41}$ erg/s associated with almost the entire gas mass being in the hot phase (see Fig 4.2). The increase of the hot gas fraction shows the occurrence of strong shocks in the first and second passage of the galaxies and stays fixed at $\sim 0.6$ once the galaxies have merged. The unbound gas fraction for the two larger AVP simulations converge at $\sim 15\%$ at the end of the simulation. The $10^{-4}$ has a larger unbound fraction at $\sim 25\%$; this occurs because the low AVP simulation does not efficiently dissipate kinetic energy (see Fig 4.2).

Table 4.1: A table listing the properties of the remnant for the 3 simulations with AVP of $10^{-4}$, 0.5 and 1.0. The density of the halo is found by making a histogram of the radii using 1000 bins. Then the enclosed mass, and the average density for the halo particles at a given radius was found. The properties of the closest match to the assumed over-density of 200 is shown here.

<table>
<thead>
<tr>
<th>AVP</th>
<th>$R_{\text{vir}}$</th>
<th>$M_{\text{vir}}$</th>
<th>$V_{\text{vir}}$</th>
<th>Gas within $R_{\text{vir}}$</th>
<th>Unb. Gas</th>
<th>Hot Gas</th>
<th>$L_X$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[kpc]</td>
<td>[$10^{10}$M$_\odot$]</td>
<td>[km/s]</td>
<td>[%]</td>
<td>[%]</td>
<td>[%]</td>
<td>[$10^{40}$ erg/s]</td>
</tr>
<tr>
<td>0.5</td>
<td>225.2</td>
<td>130.2</td>
<td>158</td>
<td>43.4</td>
<td>12.6</td>
<td>56.8</td>
<td>4.75</td>
</tr>
<tr>
<td>1.0</td>
<td>226.8</td>
<td>133.0</td>
<td>159</td>
<td>44.2</td>
<td>12.1</td>
<td>57.7</td>
<td>4.81</td>
</tr>
<tr>
<td>$10^{-4}$</td>
<td>221.5</td>
<td>124.0</td>
<td>155</td>
<td>41.4</td>
<td>17.6</td>
<td>5.8</td>
<td>0.21</td>
</tr>
</tbody>
</table>
Figure 4.1. The gas temperature, gas density and halo density profiles for the merger remnant for the equal-mass merger with 10% gas and an impact parameter of 22.8 kpc. Top. The red, blue and the green represent the runs with 0.5, 1.0 and $10^{-4}$ artificial viscosity parameters. The 0.5 and 1.0 parameters give essentially the same results for the temperature profile indicating that convergence has been achieved. The $10^{-4}$ shows a much lower central temperature because of the absence of any effective shock heating during the course of the simulation. Bottom. The gas and halo density profiles for the 3 simulations. Both the gas and halo density profiles show the same values for the merger remnant. Expectedly, the $10^{-4}$ artificial viscosity run has no effect on the halo density profile, but causes a shallower gas density profile due to the lack of shocks and the associated compressions that the gas undergoes. The solid lines in the density plots are the analytical input profiles – black for NFW and cyan for the gas.
Figure 4.2. The evolution of the hot gas fraction and the total X-ray emission for a equal-mass merger with 10% gas fraction and an impact parameter of 22.8 kpc with 3 different artificial viscosities is shown here. The red, blue and the green represent the 0.5, 1.0 and the $10^{-4}$ artificial viscosity respectively. Top. The evolution of the fraction of gas particles with $T > 10^{5.2} K$ and density $\lesssim 0.01 M_\odot /pc^3$. The 0.5 and 1.0 show a higher fraction of hot gas throughout the simulation, consistent with extensive shock-heating, an effect not captured by the simulation with $10^{-4}$ artificial viscosity. The values between the 0.5 and 1.0 are remarkably similar, showing that the simulation is no longer in a regime where it is affected by the precise value of the artificial viscosity parameter. Bottom. The total X-ray emission from the hot gas particles. Thus the hot gas will continue to remain in the hot phase and cool only through X-rays. The X-ray emission shows clear peaks that correspond to the first pass and the final merger event of the galaxies. Such an excess emission would be detectable since with an impact parameter of 22.8 kpc, the disks in case of spiral galaxies will not inter-penetrate and not lead to any source confusion from star-formation and other accretion/feedback processes.
Figure 4.3. The projected temperature for an equal-mass merger with 10% gas and an impact parameter of 22.8 kpc. Each row represents a snapshot from three simulations run with 3 different AVP’s of $10^{-4}$ (leftmost), 0.5 (middle) and 1.0 (right) at a fixed time. A prominent shock is present in the higher viscosity runs and absent in the low one. The maximum temperature in the low AVP case is about an order of magnitude smaller than the other two simulations.
Figure 4.4. Evolution of the unbound gas and dark matter fraction for a equal-mass merger with 10% gas and an impact parameter of 22.8 kpc. The red, blue and the green show the 0.5, 1.0 and $10^{-4}$ AVP simulations. The peaks in the unbound gas fraction correspond to the first and second pericenter passages. It is notable that the first pass produces a larger peak $L_X$ (see Fig. 4.2) while the peak in the unbound fraction occurs at the second pass just prior to the final merger. There is essentially no difference in the unbound gas fraction for the two largest AVP simulations.
4.3 Merger Simulations

In the following sections we will discuss the results from the 36 simulations with different merger ratios, gas fractions, and impact parameters. In particular, we are interested in the effects of the those three parameters on the X-ray emission and the unbound gas. A priori, we expect to find an increasing trend for X-ray emission with increasing progenitor masses – because of the greater $T_{\text{vir}}$ and the larger hot gas fractions in the more massive halos (see Table 4.2). Since we model the X-ray luminosity via thermal bremsstrahlung, we expect an increasing trend for $L_X$ with an increasing gas fraction. A smaller impact parameter implies denser parts of the galaxy inter-penetrate; this leads to stronger shocks and larger hot gas fractions. Therefore we expect the X-ray emission from both the regular hot gas component and the shocked component to increase with a decreasing impact parameter. A larger impulse is delivered during a close encounter with more massive galaxies; therefore, we expect an increasing trend in the unbound gas fractions with increasing galaxy mass and decreasing impact parameter. Thus, both $L_X$ and the unbound gas fraction should behave in an analogous manner. We get confirmation of this assertion when we find a very good linear fit to the peak shock X-ray luminosities and the unbound gas fraction to the impulse approximation of the merger. A detailed analysis of the feasibility of the X-ray detection is beyond the scope of this thesis; however, we will assume a conservative threshold of the X-ray flux as $10^{-15}$ erg/s/cm$^2$ and some representative distance to galaxy pairs, assumed as that of the Virgo cluster, of $\sim 20$ Mpc. This sets a detection threshold in the X-ray luminosity of $5 \times 10^{37}$ erg/s. We will find the hot halo itself exceeds this threshold for all the simulations but the shock X-ray luminosity does not always do so. For the rest of this chapter, we will refer to X-ray luminosities greater than $5 \times 10^{37}$ as detectable.

4.3.1 X-ray emission

The primary mechanism for hot rarefied gas without metals to cool is via thermal bremsstrahlung. As mentioned previously, we limit our calculation of the X-ray luminosity from only hot, $T > 10^{5.2}$ K, and rarefied, $\rho < 0.01 M_\odot$/pc$^3$. Though the isolated galaxies have varying hot gas fractions, during the course of the merger
more than 90% of the gas is in the hot phase. This is caused primarily by the shock heating during the encounter, e.g., the smallest of the secondary galaxies have $\sim 40\%$ in the hot phase while in isolation (see Table 4.2) but that fraction increases to more than 90% during the merger process – similar to the hot gas fraction in the primary galaxy. The total X-ray luminosity of the merger is determined by this hot gas fraction and can be seen from the correlated reduction in the hot gas fraction and the X-ray luminosity (see Fig. 4.5 and Fig. 4.6). The hot gas fraction increases as the secondary galaxy reduces in mass simply because those orbits undergo multiple close passes before the final remnant is produced.

For the equal-mass mergers, the elliptical and the hyperbolic orbits produce a peak $L_X$ of $\sim 10^{40}$ erg/s for the 1% gas fraction. This is from the first peri-center pass of the galaxies and the creation of the strongest compressive forces. Since $L_X$ is proportional to the square of the density, the shocks at this stage create the largest X-ray emission. We do have to keep in mind that even though we are talking about close passages, the average distance between the centers is greater than 20 kpc, greater than the typical disk size in spiral galaxies. $L_X$ in the equal-mass mergers with 10% gas have a peak $L_X$ of $\sim 8 \times 10^{41}$ erg/s occurring at the time of pericenter passage. The 18% gas fraction simulations produce the highest $L_X$ of $\sim 2 \times 10^{42}$ erg/s, since $L_X \propto \rho_{\text{gas}}^2$. The remnant $L_X$ and the peak $L_X$ scales between the simulations with different gas fractions by the square of the ratio of the individual gas fractions. Now, for a given merger type, all the three impact parameter simulations start off with the same initial velocities. Therefore, the dependence of $L_X$ on the impact parameter results from the hydrodynamical evolution during the orbit itself.

The simulations with the smallest impact parameters result in the collision of the densest parts of the galaxy, with resultant shock heating. As the impact parameter increases to 0.5 $R_{\text{vir}}$, the lower density regions of the galaxy inter-penetrate. Since our simulations are adiabatic, the maximum increase in the density can be a factor of 4; the strong dependence of $L_X$ on the gas density results in the lower $L_X$ for the higher impact parameter case – since the compressed gas never reaches the higher densities attained in the shocks occurring in the cases with smaller impact parameters.

The equal-mass mergers create the strongest shocks in our entire suite of sim-
ulations. However, the surprising part is that the strongest temperature enhancements occur after the centers of the galaxies have passed by. At this point the shocked material suddenly sees a lower density material upfront and gets accelerated forward. However, at this stage the material in the far side of the galaxy is still continuing in orbit and has a velocity opposite to that of these accelerated shocked material. When these two fluids collide, the strongest shocks (measured by the rate of entropy change) are produced. This can also be seen from the temperature projections in Fig. 4.15 where the highest temperatures are created by the gas that is thrown forward after the pericenter passage, colliding with the gas in the far side of the galaxy. This is also when the peak shock $L_X$ is created. Consequently, the peak shock $L_X$ occurs some time after the peak $L_X$. The equal-mass mergers with 1%, 10% and 18% gas have a peak shock $L_X$ of $\sim 10^{37}$, $\sim 10^{39}$ and $\sim 3 \times 10^{39}$ erg/s. Like $L_X$, the peak shock $L_X$ shows a strong correlation with the impact parameter for similar reasons.

The 3:1 mergers show a similar pattern for $L_X$ and shock $L_X$ with both the quantities increasing with decreasing impact parameter and increasing gas fraction. The peak $L_X$ for the 1% gas for the 3:1 mergers are $\sim 5 \times 10^{39}$ erg/s for the 0.01 and 0.1 $R_{\text{vir}}$ impact parameters and $\sim 2 \times 10^{39}$ erg/s for the 0.5 $R_{\text{vir}}$. The peak shock $L_X$ for this set of simulations is $5 \times 10^{36}$ erg/s and is too low to be detectable by current instruments. The 3:1 merger with 10% gas fraction has peak $L_X$ of $\sim 4 \times 10^{41}$ erg/s for the 0.01 and 0.1 $R_{\text{vir}}$ impact parameter and $\sim 1.9 \times 10^{41}$ erg/s for the 0.5 $R_{\text{vir}}$ case. The peak shock $L_X$ for this set of mergers are 7.6, 3.4 and $0.5 \times 10^{38}$ erg/s for the 0.01, 0.1 and 0.5 $R_{\text{vir}}$ impact parameters respectively. The 18% gas fraction has peak $L_X$ of $\sim 1.9 \times 10^{42}$, 1.1 $\times 10^{42}$ and $6.2 \times 10^{41}$ erg/s and a peak shock $L_X$ of $2.1 \times 10^{39}$, $1.4 \times 10^{39}$ and $0.2 \times 10^{39}$ erg/s for the 0.01, 0.1 and 0.5 $R_{\text{vir}}$ impact parameters. Though peak shock $L_X$ values are larger than $10^{39}$ erg/s they are still 3 orders of magnitude smaller than the actual $L_X$ signature from the merger event.

The 10:1 mergers follow the trend and for the 1% gas fraction simulations the peak $L_X$ and the peak shock $L_X$ are $\sim 2.0 \times 10^{39}$ and $\sim 10^{36}$ erg/s for all the impact parameters. For the 10% simulations the peak $L_X$ values are 2.7, 1.7 and $1.1 \times 10^{41}$ erg/s for the 0.01, 0.1 and 0.5 $R_{\text{vir}}$ impact parameters while the peak shock $L_X$ values are $\sim 3 \times 10^{37}$ erg/s for all the three impact parameters. For the
18% gas fractions, the peak $L_X$ values are 8.5, 5.5 and $3.6 \times 10^{41}$ erg/s while the peak shock $L_X$ values are 3.5, 1.7 and $0.3 \times 10^{38}$ erg/s for the 0.01, 0.1 and 0.5 $R_{\text{vir}}$ impact parameters respectively. Clearly, the 10:1 mergers do not produce any appreciable X-ray signature from the shock-heating of the hot gas even with the largest gas fractions. See Table 4.4 for detailed data on the $L_X$ and shock $L_X$ for all the simulations.

From our simulations, we see that the expected trends do occur. $L_X$ and shock $L_X$ increase with increasing mass of colliding halos and decreasing impact parameters. Larger initial velocities do result in larger $L_X$; however, if the hot gas fraction drops too much, as it does in the case of some of the hyperbolic mergers, the X-ray emission can get reduced. In light of these trends, the best bet to detect the shock signature of the hot halo gas during mergers would be to locate major mergers between Milky-way type galaxies that are happening in the local universe. Such a system would produce significant X-rays from the hot halo and the shock-heated interface if the galaxies are to be observed in the initial stages of the merger. Spatial resolution will not be an issue since the signature is an extended source; the problem would be the identification of the underlying shock X-ray emission and separate it from the emission from the hot halo particles themselves. $\textit{XMM-Newton}$, with its superior collecting area, would have been the instrument of choice; however, the inferior sensitivity of $\textit{XMM-Newton}$ in the soft X-ray band, where most of the shocked gas in our simulation radiates (the maximum temperature reached in our simulations is $\sim 0.3$ keV), we recommend $\textit{Chandra}$ for the task of the characterization of the hot halo gas. Our threshold for the X-ray flux, $10^{-15}$ erg/s/cm$^2$ are relevant to the $\textit{Chandra}$ on-board instruments.
Figure 4.5. The evolution of the fraction of gas particles with $T > 10^{5.2} K$ and density $\lesssim 0.01 \, M_\odot /pc^3$ – the hot gas, for the equal-mass mergers. The bound mergers are shown in solid line while the hyperbolic ones are shown with dashed lines. Each row represents a fixed gas fraction while the blue, red and the green lines reflect the impact parameters of 0.01, 0.1 and 0.5 $R_{\text{vir}}$ respectively. The hyperbolic orbits have a lower hot gas fraction compared to the elliptical orbits for the entire duration of the merger; this happens because the hyperbolic mergers are unable to extract enough energy from the orbit and convert it into internal energy. The hot gas fraction in the simulation shows almost no dependence on the actual gas fraction of the galaxies.
Figure 4.6. The evolution of \( L_X \) and shock \( L_X \) in all the equal-mass mergers. \( L_X \) is computed for gas particles with \( T > 10^{5.2} K \) and density \( \lesssim 0.01 \text{ M}_{\odot}/\text{pc}^3 \) while shock \( L_X \) is computed only for the shocked gas particles. The bound orbits are shown in solid line while the hyperbolic ones are shown with dashed lines. Each row represents a fixed gas fraction while the blue, red and the green lines reflect the impact parameters of 0.01, 0.1 and 0.5 \( R_{\text{vir}} \) respectively. The peaks in \( L_X \) correspond to pericenter passage or the final merger event while the peaks in shock \( L_X \) are slightly offset in time from the pericenter passage. This happens because the strongest shocks in the simulations are created after the pericenter passage as explained in the text. The hyperbolic 10% and 18% with 0.01 \( R_{\text{vir}} \) impact parameter mergers show a later peak corresponding to the final merger event; no other hyperbolic orbit creates a remnant. \( L_X \) and shock \( L_X \) show a strong dependence on the gas fraction, and specifically for gas fractions \( \gtrsim 10\% \), the peak shock \( L_X \) of \( \gtrsim 10^{39} \text{ erg/s} \) should be detectable by X-ray telescopes.
Figure 4.7. The projection of the temperature for the 18% gas fraction for an equal-mass merger with an impact parameter of 2.3 kpc for the hyperbolic orbit. This particular run is shown because it has the highest X-ray emission, and largest unbound material. The X-ray contours are shown in purple, the unbound material with white contours and the shocked and unbound material with green contours. The shocked and unbound only shows up in the 4th image. The unbound material is identified at the end of the simulation and then traced through the entire run. We can see that most of the unbound material comes from two distinct regions, viz., the far lobes of the galaxy and the shocked front in between the galaxies. The projected X-ray emission also exceeds $10^{38}$ erg/s in the shocked region with a total shocked X-ray emission exceeding $10^{40}$ erg/s as shown in Fig. 4.31.
Figure 4.8. The figures show the projection of the temperature for the 18% gas fraction for an equal-mass merger with an impact parameter of 2.3 kpc for the hyperbolic orbit. This particular run was chosen because of the highest amount of X-ray emission, and largest unbound material. The velocities of a fraction of randomly chosen unbound particles is also plotted; blue represents unshocked unbound material while white shows the shocked unbound material. We see that the unbound material escapes preferentially along the shocked front away from the plane of the merger since this provides the least resistance in terms of gas pressure. Strong shocks are generated once the centers have passed and material is ejected that interacts with the bound material at the other end of the galaxy that is yet to make a close pass (and hence has a velocity vector opposite to that of the unbound material). This is seen in the right image in the middle row. All of the unbound material leaves the vicinity of the galaxy much before they actually merge ($\lesssim 6$ Gyrs).
Figure 4.9. Projected temperatures with contours for L_X for the equal-mass merger simulations with bound orbits. This begins a sequence of images that will follow in the next few pages to give an idea of the overall evolution of the simulation. The minimum temperature has been fixed at 10^{5.2} K since we only use gas hotter than that to calculate the X-ray luminosity. Each row represents a certain gas fraction; top row is 1%, middle row is 10% and the bottom row is 18% gas fraction. Each column represents a fixed impact parameter; left is 2.3 kpc, center is 22.8 kpc and the right is 114.3 kpc. The simulation times are noted on the individual images themselves. The X-ray contours were drawn for fixed values and the 1% gas fraction mergers did not ever get up to the smallest X-ray contour value used – 10^{36.4} erg/s. A strong shock, whose structure depends on the orbital geometry, develops in all the simulations.
Figure 4.10. Projected temperatures and L_X contours for the equal-mass mergers - II.
Figure 4.11. Projected temperatures and L\textsubscript{X} contours for all the equal-mass mergers - III.
Figure 4.12. Projected temperatures and L_X contours for all the equal-mass mergers - IV.
Figure 4.13. Projected temperatures and $L_X$ contours for all the equal-mass mergers - $V$. 
Figure 4.14. Projected temperatures and L_X contours for all the equal-mass mergers - VI. The merger remnant has settled down and is close to equilibrium as evidenced by the symmetry of the X-ray contours.
Figure 4.15. Projected temperature and L$_X$ contours for a 3:1 merger with 18% gas fraction and an impact parameter of 1.9 kpc. A prominent bow shock develops as the secondary galaxy passes through the halo of the primary. Even at 2.2 Gyr, when the centers are separated by $\sim$ 100 kpc, the strong shock persists and can be easily seen in the temperature map. Once the secondary material passes by the primary center, a lot of material is ejected that gets shock-heated by the secondary galaxy as it moves inwards along the orbit. This can be seen in the patch of very bright material in the left-bottom image. The final image at 18.8 Gyr shows the merger remnant completely relaxed with spherical L$_X$ contours. The minimum value of the L$_X$ contour is $10^{36.4}$ erg/s and increases in factors of $10^{0.5}$ erg/s.
Figure 4.16. Projected temperatures with contours for $L_X$ for the 3:1 merger simulations. This begins a sequence of images that will follow in the next few pages to give an idea of the overall evolution of the simulation. The minimum temperature has been fixed at $10^{5.2}$ K since we only use gas hotter than that to calculate the X-ray luminosity. Each row represents a certain gas fraction; top row is 1%, middle row is 10% and the bottom row is 18% gas fraction. Each column represents a fixed impact parameter; left is 1.9 kpc, center is 19.4 kpc and the right is 96.7 kpc. The simulation times are noted on the individual images themselves. The X-ray contours were drawn for fixed values and the 1% gas fraction mergers did not even get up to the smallest X-ray contour value used – $10^{36.4}$ erg/s. A prominent bow shock, whose structure depends on the orbital geometry, develops in all the simulations. The simulation time is shown on the middle image in the top panel.
Figure 4.17. Projected temperatures and L_X contours for the 3:1 mergers - II. A bow shock is visible in all the simulations.
Figure 4.18. Projected temperatures and L_X contours for all the 3:1 mergers - III. The temperature of the bow shock has increased as denser regions of the galaxies collide.
Figure 4.19. Projected temperatures and L_X contours for all the 3:1 mergers - IV
Figure 4.20. Projected temperatures and L_X contours for all the 3:1 mergers - V. The hottest temperatures are attained in the bright patches of outflowing material seen on the side of the secondary galaxy.
Figure 4.21. Projected temperatures and L_X contours for all the 3:1 mergers - VI. The merger remnant has settled and is close to equilibrium as evidenced by the symmetry of the X-ray contours.
Figure 4.22. Projected temperature and L_X contours for a 10:1 merger with 18% gas fraction and an impact parameter of 1.7 kpc. A prominent bow shock develops as the secondary galaxy passes through the halo of the primary. Even at 2.1 Gyr, when the centers are separated by ~100 kpc, the strong shock persists and can be easily seen in the temperature map. Like the 3:1 case in Fig. 4.15, once the secondary galaxy passes the primary center, a lot of material is ejected that gets shock-heated by the material from the secondary galaxy still moving inwards along the orbit. However, in this case this material is not the hottest patch in the simulation as was seen in the 1:1 and 3:1 cases. The final image at 18.8 Gyr shows the merger remnant completely relaxed with spherical L_X contours. The minimum value of the L_X contour is $10^{36.7}$ erg/s and increases in factors of $10^{0.5}$ erg/s.
Figure 4.23. Projected temperatures with contours for $L_X$ for the all the 10:1 merger simulations undertaken. This begins a sequence of images that will follow in the next few pages to give an idea of the overall evolution of the simulation. The minimum temperature has been fixed at $10^{5.2}$ K since we only use gas hotter than that to calculate the X-ray luminosity. Each row represents a certain gas fraction; top row is 1%, middle row is 10% and the bottom row is 18% gas fraction. Each column represents a fixed impact parameter; left is 1.7 kpc, center is 16.7 kpc and the right is 83.7 kpc. The simulation times are noted on the in the middle image in the top row. The X-ray contours were drawn for fixed values and the 1% gas fraction mergers did not ever get up to the smallest X-ray contour value used – $10^{36.7}$ erg/s.
Figure 4.24. Projected temperatures with contours for L_X for all the 10:1 mergers - II.
Figure 4.25. Projected temperatures with contours for $L_X$ for all the 10:1 mergers - III.
Figure 4.26. Projected temperatures with contours for $L_X$ for all the 10:1 mergers - IV.
Figure 4.27. Projected temperatures with contours for L_X for all the 10:1 mergers - V.
Figure 4.28. Projected temperatures with contours for $L_X$ for all the 10:1 mergers - VI.
Figure 4.29. The evolution of the hot gas fraction for all the 36 simulations. The left column contains 1:1 mergers, the middle column contains 3:1 mergers and the right column contains the 10:1 mergers. The top row, middle and the bottom row show simulations with 1%, 10% and 18% gas fractions respectively. The blue, red and the green shows the 0.01, 0.1 & 0.5 $R_{\text{vir}}$ impact parameters respectively. The dashed lines in the left column show the hyperbolic 1:1 mergers. We see that the peaks of the hot gas fractions corresponds to the pericenter passages. The most drastic evolution is seen in the equal-mass mergers where hot gas fraction evolves from \( \sim 80\% \) to \( \sim 50\% \) for 0.01 and 0.1 $R_{\text{vir}}$. For all the other simulations the final hot gas fraction reaches equilibrium at around 80%.
Figure 4.30. The X-ray luminosity versus time for all the simulations. The peak X-ray luminosity increases with increasing gas fractions approximately proportional to $\rho_{\text{gas}}^2$ and occurs at the first closest pass of the two centers. The peak X-ray luminosity also shows an increasing trend with decreasing impact parameter and decreasing secondary galaxy mass, correlated with the amount of shock heating that occurs. The line types and the colors are the same as in Fig 4.29.
Figure 4.31. $L_X$ from the shocks in all the simulations. The shocked particles were determined by setting a threshold for the rate of change of entropy – shocked particles have a much higher value compared to the unshocked particles. The total X-ray emission from all these particles is added up and plotted as a function of time for the duration of the simulation. The shock $L_X$ shows an expected dependence as $\propto f_{\text{gas}}^2$ with the peaks coinciding with the first pericenter passage. With a conservative 10% gas fraction, the equal-mass mergers produce a detectable peak shock $L_X$ of $10^{39}$ erg/s. We note that our estimates are lower limits on the possible $L_X$ since the presence of metals will increase the total amount of emission.
So far we have discussed the production of X-rays in the merger, both from the hot gas in the halo and the shocks. We expect that the X-ray emission will depend on a variety of parameters like the merger ratio, the cumulative mass, the merger orbit and the gas fraction in the simulation. We attempt to combine the effects of all these factors and model the peak shock $L_X$ as a function of the gravitational impulse from the merger. Covington et al. (2008) derive an empirical fitting formula for the impulse motivated by numerical simulations of galaxy mergers. The first term of this parametrized impulse, $\Delta E$, is approximated by:

$$\Delta E \propto \frac{G^2 M_1 M_2}{v_p^2 \left[ r_p^2 + r_p \times R_{vir,1} \right]},$$  \hspace{1cm} (4.1)

where $M_1$ and $M_2$ are the masses of the two galaxies, $v_p$ and $r_p$ are the relative velocity of the galaxies and the pericenter distance as determined from the simulation. The asymmetry in the above expression for the subscripts explicitly shows the assumption that $M_1$ represents the more massive of the two galaxies. We know that $L_X$ depends on square of the density; therefore, it should also scale as the square of the gas fraction. $L_X$ also depends on the square root of the temperature, which in turn depends on square of the relative velocity of the colliding gas (see Eqn. 1.3). Thus, $L_X$ should depend on the relative velocity; we use the velocity at perihelion, $v_p$ determined from the simulation to capture this. Our final equation for the merger strength, $\Delta H$, is then $\Delta E f_{gas}^2 v_p$, and can be written as:

$$\Delta H \propto \frac{G^2 M_1 M_2 f_{gas}^2}{v_p \left[ r_p^2 + r_p \times R_{vir,1} \right]},$$  \hspace{1cm} (4.2)

We then fit the peak shock $L_X$, $L_{\text{peak}}$, as a linear function of $\Delta H$. However, a better fit, with a correlation co-efficient of 0.96, is obtained by using the logarithm of $\Delta H$ to fit the logarithm of $L_{\text{peak}}$. The fit for our set of 36 simulations is:

$$\log_{10} \left[ \frac{L_{\text{peak}}}{10^{38} \text{ erg/s}} \right] = 0.88 \times \log_{10} \Delta H - 1.25.$$  \hspace{1cm} (4.3)

This fit captures the essential of all the various parameters possible in a merger and can be used to estimate the peak shock $L_X$ in a merger. Such a fit is quite useful in modeling the total shock $L_X$ in semi-analytic studies of galaxy formation.
We will come back to this topic in the last chapter as part of a future project.

**Figure 4.32.** The semi-analytic fit for the peak shock luminosity and the merger strength, $\Delta H$ (see Eqn. 4.2). The individual simulations are plotted with the triangles while the fit (see Eqn. 4.3) is plotted with a red line. The correlation for this fit is 0.96 and implies that Eqn. 4.3 is a good linear fit.

\[
\log(y) = 0.88 \log(x) - 1.25
\]

\[\text{corr} = 0.96\]
4.3.2 Unbound Material

As we saw from the three simulations testing the AVP, during the merger gas shock-heats, emit X-rays and possibly some amount gets released from the galaxies. In order to constrain the amount of unbound material resulting from the merger dynamics, we evolve the corresponding initial galaxies in isolation for a similar time-scale. Every galaxy will lose material from two-body relaxation, and by evolving the galaxies in isolation, we can estimate the magnitude of this effect. We deduct the amount of unbound material (column 3 in Table 4.2) from the two isolated galaxies when we compute the final unbound fraction from the merger (see column 4 in Table 4.3).

Table 4.2: This table lists some of the physical quantities for the galaxies after they were evolved with artificial viscosity = 0.5 for \( \geq 10 \) Gyr. We evolved the galaxies to ascertain the amount of material that gets unbound via internal processes; once this fraction is subtracted from the unbound material from the mergers, we can fix the actual amount freed during the merger itself.

<table>
<thead>
<tr>
<th>Gas Content</th>
<th>Galaxy Type</th>
<th>Unbound gas</th>
<th>( L_X ) ( [10^{40} \text{ erg/s}] )</th>
<th>( R_{\text{vir}} ) ( [\text{kpc}] )</th>
<th>( M_{\text{vir}} ) ( [10^{10} \text{ M}_\odot] )</th>
<th>Hot gas</th>
</tr>
</thead>
<tbody>
<tr>
<td>1% gas</td>
<td>Primary †</td>
<td>0.43</td>
<td>0.09</td>
<td>214.8</td>
<td>113.2</td>
<td>93.2</td>
</tr>
<tr>
<td></td>
<td>Onethird</td>
<td>0.69</td>
<td>0.03</td>
<td>149.1</td>
<td>37.8</td>
<td>73.4</td>
</tr>
<tr>
<td></td>
<td>Onetenth</td>
<td>0.84</td>
<td>0.01</td>
<td>100.2</td>
<td>11.5</td>
<td>41.3</td>
</tr>
<tr>
<td>10% gas</td>
<td>Primary ‡</td>
<td>0.60</td>
<td>8.46</td>
<td>212.9</td>
<td>110.2</td>
<td>89.8</td>
</tr>
<tr>
<td></td>
<td>Onethird</td>
<td>0.61</td>
<td>3.38</td>
<td>149.1</td>
<td>37.7</td>
<td>75.9</td>
</tr>
<tr>
<td></td>
<td>Onetenth</td>
<td>0.83</td>
<td>0.94</td>
<td>100.7</td>
<td>11.7</td>
<td>45.3</td>
</tr>
<tr>
<td>18% gas</td>
<td>Primary</td>
<td>0.72</td>
<td>27.5</td>
<td>213.9</td>
<td>112.0</td>
<td>95.2</td>
</tr>
<tr>
<td></td>
<td>Onethird</td>
<td>0.60</td>
<td>11.3</td>
<td>148.6</td>
<td>37.6</td>
<td>79.9</td>
</tr>
<tr>
<td></td>
<td>Onetenth</td>
<td>0.76</td>
<td>3.05</td>
<td>100.7</td>
<td>11.6</td>
<td>48.4</td>
</tr>
</tbody>
</table>

The particles unbound from the entire system are found by computing the total energy of each particle using the potential from GADGET-2 and the velocity of each particle relative to the center of mass velocity. We calculate the position and velocity of the center of mass by the mass-weighted average of the particles located

---

†at 15 Gyrs.
‡at 10 Gyrs.
within the deepest 10% of the potential of the merger remnant. These particles are then located and averaged through all the previous snapshots. We find that most of the unbound material originates from or near the virial radius of the respective galaxies. As the merger progresses, this material begins to escape and lies outside of $3 \times R_{\text{vir}}$ even before the second passage. By the end of the simulation, this unbound gas is far away ($\gtrsim 1$ Mpc) from the remnant (see Fig. 4.3.1); thereby alleviating the problem of transporting metals to inter-galactic distances (Ferrara et al., 2000). We find that $\sim 9\%$ of the gas is unbound along with a similar fraction in the dark matter.

This translates to a gas mass of $\gtrsim 3 \times 10^{10} \, \text{M}_\odot$ from the equal-mass and $\gtrsim 10^{10} \, \text{M}_\odot$ from the 3:1 mergers flowing into the IGM. While our simulation is stopped at 18 Gyrs, most of the physical quantities like density, temperature, and $L_X$ remain unchanged over the last 10 Gyrs. In particular, almost all the unbound material flows outside $3 \, R_{\text{vir}}$ within the first 5-6 Gyrs. Considering that every galaxy has had a major merger in the past, this allows a lot of gas to be liberated into the IGM. Thus galaxy mergers play an important role not only in the evolution of the galaxies and hierarchical structure formation, but they also influence both the mass and the metallicity content of the IGM.

We identify the unbound material at the end of the simulations and track them throughout the duration of the merger. We find that the unbound material comes from two distinct regions – from the near and far lobes of the galaxy along their line of motion. These constitute the regions that are shocked during the course of the merger. The near lobes get shocked before pericenter passage, and can be seen in Figure 4.3.1. These shocked gas particles then leave in directions perpendicular and away from the direction of the motion of the galaxies as they expand into the lower density regions above. The escape is facilitated by the lower density and pressure of the ambient medium above the shocked region and can be seen from the velocity vectors of the unbound particles pointing outwards in Figure 4.3.1. The gas particles from this region escape from the galaxies the earliest. To understand the reason for the ultimate fate of the gas from the far regions of the galaxies, we have to look into the shock process itself. As the galaxies pass through pericenter, the density in front of the shocked material suddenly drops rapidly. This causes the shocked material to receive an acceleration and it is flung forward. However,
the material at the far ends of the galaxy are still moving with the bulk flow. This naturally results in the largest relative velocity and hence, the strongest shocks in the simulation. The original shocked gas interacts with the gas from the far side, and the gas from the far side receives a large impulse and starts to flow outward instead of continuing with the bulk motion of its host galaxy. Such material can be seen as the brightest patches in the temperature images. For the unequal-mass mergers, the impulse affects the lower mass galaxies more, and an asymmetry is introduced whereby the strongly shocked unbound material comes primarily from the smaller galaxy. This can be seen in the images of the temperature projections for the unequal-mass mergers where the brightest patches are towards the side of the smaller galaxies (see Figures 4.15 and 4.22). Although most of the unbound material originates from a radius greater than 0.5 $R_{\text{vir}}$ in the individual galaxies and should have a systematically lower metallicity, they can get enriched during the merger process itself. As the galaxies collide, the gas from the outer regions of one galaxy interacts with the gas originating at smaller radii in the other galaxy.
Figure 4.33. The evolution of the unbound gas fraction for the hyperbolic and the elliptical orbits for the equal-mass mergers. The hyperbolic orbits always have higher unbound gas fractions compared to the elliptical ones. The line types and the colors are the same as in Fig 4.29.
Table 4.3: The first three equal-mass mergers for each gas fraction are the hyperbolic encounters while the rest are all bound elliptical orbits with a fixed amount of energy in the orbit for a given merger ratio. The unbound fraction (Column 4) is obtained by deducting the sum of the unbound gas fractions from the constituent galaxies when evolved in isolated (see Table 4.2) from the unbound fraction determined at the end of the simulation. Column 5 shows the fraction of all the gas particles that are shocked during the simulation. The unbound gas particles are tracked through the snapshots to find where they become unbound or shocked. The fraction of unbound gas particles that get shocked during the simulation is shown in Column 6 while the fraction of the shocked unbound gas particles that are shocked first and then become unbound after-wards is noted in Column 7. Column 7 shows that if an unbound particle is going to get shocked, then it is more likely to have gotten shocked prior to becoming unbound.

<table>
<thead>
<tr>
<th>Gas</th>
<th>Merger Type</th>
<th>b</th>
<th>Unb. frac.</th>
<th>Shocked (all)</th>
<th>Shocked (unb)</th>
<th>Shocked first</th>
</tr>
</thead>
<tbody>
<tr>
<td>1% gas</td>
<td>1:1</td>
<td>2.3</td>
<td>19.2</td>
<td>13.1</td>
<td>49.6</td>
<td>80.8</td>
</tr>
<tr>
<td></td>
<td>1:1</td>
<td>22.8</td>
<td>18.5</td>
<td>11.1</td>
<td>46.7</td>
<td>83.2</td>
</tr>
<tr>
<td></td>
<td>1:1</td>
<td>114.3</td>
<td>11.2</td>
<td>5.9</td>
<td>41.1</td>
<td>88.9</td>
</tr>
<tr>
<td></td>
<td>1:1</td>
<td>2.3</td>
<td>9.4</td>
<td>14.1</td>
<td>84.2</td>
<td>76.6</td>
</tr>
<tr>
<td></td>
<td>1:1</td>
<td>22.8</td>
<td>8.4</td>
<td>13.8</td>
<td>85.8</td>
<td>79.3</td>
</tr>
<tr>
<td></td>
<td>1:1</td>
<td>114.3</td>
<td>5.9</td>
<td>11.9</td>
<td>89.5</td>
<td>76.7</td>
</tr>
<tr>
<td></td>
<td>3:1</td>
<td>1.9</td>
<td>6.7</td>
<td>9.4</td>
<td>69.6</td>
<td>72.2</td>
</tr>
<tr>
<td></td>
<td>3:1</td>
<td>19.4</td>
<td>5.7</td>
<td>9.2</td>
<td>73.1</td>
<td>75.9</td>
</tr>
<tr>
<td></td>
<td>3:1</td>
<td>96.8</td>
<td>4.1</td>
<td>8.0</td>
<td>80.0</td>
<td>77.2</td>
</tr>
<tr>
<td></td>
<td>10:1</td>
<td>1.7</td>
<td>1.2</td>
<td>2.6</td>
<td>60.9</td>
<td>66.1</td>
</tr>
<tr>
<td></td>
<td>10:1</td>
<td>16.7</td>
<td>1.1</td>
<td>2.5</td>
<td>62.6</td>
<td>69.0</td>
</tr>
<tr>
<td></td>
<td>10:1</td>
<td>83.7</td>
<td>1.2</td>
<td>4.0</td>
<td>66.0</td>
<td>75.4</td>
</tr>
<tr>
<td>10% gas</td>
<td>1:1</td>
<td>2.3</td>
<td>23.8</td>
<td>22.5</td>
<td>53.6</td>
<td>78.9</td>
</tr>
<tr>
<td></td>
<td>1:1</td>
<td>22.8</td>
<td>20.0</td>
<td>13.4</td>
<td>53.4</td>
<td>82.3</td>
</tr>
<tr>
<td></td>
<td>1:1</td>
<td>114.3</td>
<td>16.2</td>
<td>8.4</td>
<td>44.8</td>
<td>87.1</td>
</tr>
<tr>
<td></td>
<td>1:1</td>
<td>2.3</td>
<td>9.7</td>
<td>14.6</td>
<td>86.9</td>
<td>59.8</td>
</tr>
<tr>
<td></td>
<td>1:1</td>
<td>22.8</td>
<td>8.9</td>
<td>14.1</td>
<td>86.8</td>
<td>73.0</td>
</tr>
<tr>
<td></td>
<td>1:1</td>
<td>114.3</td>
<td>6.4</td>
<td>12.1</td>
<td>87.7</td>
<td>68.4</td>
</tr>
<tr>
<td></td>
<td>3:1</td>
<td>1.9</td>
<td>6.8</td>
<td>9.1</td>
<td>67.2</td>
<td>70.5</td>
</tr>
<tr>
<td></td>
<td>3:1</td>
<td>19.4</td>
<td>5.5</td>
<td>7.2</td>
<td>63.6</td>
<td>68.2</td>
</tr>
<tr>
<td></td>
<td>3:1</td>
<td>96.8</td>
<td>3.7</td>
<td>6.0</td>
<td>68.5</td>
<td>68.5</td>
</tr>
<tr>
<td></td>
<td>10:1</td>
<td>1.7</td>
<td>0.8</td>
<td>1.4</td>
<td>41.0</td>
<td>65.8</td>
</tr>
<tr>
<td></td>
<td>10:1</td>
<td>16.7</td>
<td>0.9</td>
<td>1.2</td>
<td>36.4</td>
<td>62.9</td>
</tr>
<tr>
<td></td>
<td>10:1</td>
<td>83.7</td>
<td>1.0</td>
<td>2.0</td>
<td>39.6</td>
<td>61.2</td>
</tr>
<tr>
<td>18% gas</td>
<td>1:1</td>
<td>2.3</td>
<td>21.8</td>
<td>19.8</td>
<td>50.7</td>
<td>80.2</td>
</tr>
<tr>
<td></td>
<td>1:1</td>
<td>22.8</td>
<td>17.5</td>
<td>12.2</td>
<td>52.5</td>
<td>81.7</td>
</tr>
<tr>
<td></td>
<td>1:1</td>
<td>114.3</td>
<td>11.1</td>
<td>7.0</td>
<td>50.8</td>
<td>85.5</td>
</tr>
<tr>
<td></td>
<td>1:1</td>
<td>2.3</td>
<td>8.6</td>
<td>13.7</td>
<td>85.0</td>
<td>77.3</td>
</tr>
<tr>
<td></td>
<td>1:1</td>
<td>22.8</td>
<td>8.1</td>
<td>13.6</td>
<td>85.6</td>
<td>76.1</td>
</tr>
<tr>
<td></td>
<td>1:1</td>
<td>114.3</td>
<td>5.6</td>
<td>11.6</td>
<td>88.9</td>
<td>73.9</td>
</tr>
<tr>
<td></td>
<td>3:1</td>
<td>1.9</td>
<td>6.0</td>
<td>8.2</td>
<td>67.8</td>
<td>73.9</td>
</tr>
<tr>
<td></td>
<td>3:1</td>
<td>19.4</td>
<td>5.3</td>
<td>7.8</td>
<td>70.2</td>
<td>72.5</td>
</tr>
<tr>
<td></td>
<td>3:1</td>
<td>96.8</td>
<td>3.9</td>
<td>6.7</td>
<td>77.5</td>
<td>71.8</td>
</tr>
<tr>
<td></td>
<td>10:1</td>
<td>1.7</td>
<td>1.1</td>
<td>2.0</td>
<td>44.7</td>
<td>60.9</td>
</tr>
<tr>
<td></td>
<td>10:1</td>
<td>16.7</td>
<td>1.0</td>
<td>1.9</td>
<td>46.2</td>
<td>61.3</td>
</tr>
<tr>
<td></td>
<td>10:1</td>
<td>83.7</td>
<td>1.1</td>
<td>2.8</td>
<td>54.4</td>
<td>63.6</td>
</tr>
</tbody>
</table>
It is clear that the unbound material is related to the extensive shocks in our simulations. Here, we attempt to disentangle the effects of the dynamics and the hydrodynamics of the gas as it unbinds. We locate the unbound material at the end of the simulation and go back through the snapshots to identify when the gas particle unbinds. At the same time, we note when and if the gas particle is shocked. Table 4.3 shows the results of this process. We find that for equal-mass mergers, galaxies on hyperbolic orbits systematically show that a smaller fraction of the unbound material originates from shocks, compared to galaxies on elliptical orbits. This is possibly due to the fact that galaxies on hyperbolic orbits only undergo one pericenter pass, while galaxies on elliptical orbits experience multiple pericenter passes (and shocks). Elliptical orbits show a strong correlation between the unbound and the shocked particles. For instance, for the equal-mass mergers, \( \sim 85\% \) of all the unbound particles were shocked during the course of the simulation. However, the more interesting fact is that out of this 85\% that was shocked and is unbound at the end of the simulation, most were shocked prior to becoming unbound. We argue that the shocks ultimately unbind the material (see Table 4.3 for the fraction of unbound particles that were shocked first).
Figure 4.34. The evolution of the unbound gas fraction for all the simulations. The equal-mass mergers with the 0.01 & 0.1 \( R_{\text{vir}} \) has ~ 20% unbound gas fraction from the final merger remnant. The 3:1 mergers release about 5-7% gas while the 10:1 mergers unbind the least amount of gas at about 0.5%. The irregular nature of the unbound gas fraction for the 3:1 and the 10:1 mergers is caused by the multiple passages taken by the secondary galaxy.
In a similar spirit as the previous estimation of the peak shock $L_X$, we try to quantify the effect of the orbit on the unbound material. We follow the formulation of Covington et al. (2008) and use the impulse formulated in Eqn. 4.1. A linear regression analysis on $f_{unb}$ and $\Delta E$ and obtain a linear relationship of the form:

$$f_{unb} = 3.14 \times \log_{10} \Delta E - 0.16.$$  

(4.4)

The correlation coefficient for this fit is 0.95 implying that the two quantities are well described by the linear function. Since we started out with a physically motivated basis by equating the impulse with the unbound fraction, this high correlation shows that the fit shows an underlying relation between the impulse and the unbound fraction. Since most of the hyperbolic mergers do not leave a remnant and the 10:1 mergers have only 1% unbound gas, we do not use those simulations to fit this equation. Thus, out of our total 36 simulations, we have used only the 18 1:1 and 3:1 with elliptical orbits to obtain Eqn. 4.4.

### 4.3.3 Estimating the Unbound Material over the History of the Universe

The results of our simulations predict that a fraction of the hot gas escapes from galaxies, with the fraction increasing as the merger becomes more violent. This could result in a significant portion of the total baryon budget once all the mergers in the Universe are accounted for. To estimate the total fraction of gas released by mergers, we construct a series of analytic halo merger trees using a publicly available EPS code from Parkinson et al. (2008). The input parameters to the code are a final parent mass, $M_2$, following the notation in Section 1.5.1, the final redshift, $z_2$ and the mass resolution, $M_{res}$. We explore a range of halo masses; setting the maximum limit as $10^{13}M_\odot$ as representative of the largest galaxies (Nulsen & Bohringer, 1995) and the minimum limit as $10^8M_\odot$. The lower limit is chosen since it has been shown via numerical simulations that such galaxies are quite susceptible to feedback effects from galaxy winds and SNe (Mac Low & Ferrara, 1999; Cen & Ostriker, 2006; Kobayashi et al., 2007) with up to 80% of the baryons getting ejected from galaxies of mass $\lesssim 10^{11}M_\odot$ (Kobayashi et al., 2007) by SNe feedback alone. We use 100 logarithmically spaced bins in mass.
Figure 4.35. The semi-analytic fit (red) for the unbound fraction and the impulse, $\Delta E$ (see Eqn. 4.1). The correlation is 0.95 between $\log_{10} E$ and $f_{\text{unb}}$ showing that a linear relation as given in Eqn. 4.4 is a good fit. We only use the 18 simulations with the 1:1 and 3:1 bound orbits since they leave a merger remnant in all cases and have unbound fractions $> 3\%$. This linear fit can now be extrapolated analytically to find the unbound gas fraction throughout the history of structure formation.

to create a merger tree for a specific halo mass at $z_2 = 6$. To account for the different formation mechanism of an individual halo, we undertake 10 realizations of a fixed halo mass. Thus, overall we create 100 merger trees with different halo masses with 10 realizations of each halo mass and a realistic description of the hierarchical formation scenarios of structure in the Universe. For all the merger trees, we set $M_{\text{res}}$ to be $M_2 \times 10^{-5}$. For the $10^{11}M_\odot$ halos, this value of $M_{\text{res}}$ is close to the mass of an individual dark matter particle in our simulations. We find that this choice of $M_{\text{res}}$ is sufficiently accurate and produces convergent results for the estimation of the unbound gas in our simulations.

Given a merger tree for some fixed halo mass at redshift zero, we locate all mergers at previous redshifts that lead to this halo and keep only the mergers
that have a merger ratio greater than, \( \eta_{\text{min}} = 0.1 \). To calculate an upper limit on the amount of gas mass lost, we assign a gas fraction equal to the universal gas fraction \( = \Omega_b/\Omega_{\text{dm}} \) to all such halos and assume that 10% of the total gas mass gets unbound in that merger. We keep track of all the unbound gas and sum them up to obtain a total unbound gas mass to form this particular halo at redshift zero. We repeat this process for the 10 realizations of the merger tree and this provides the variance in the amount of gas mass lost during the formation of the halo. Taking an average over all the realizations gives us the mean amount of total gas mass loss from the halo. For example, we find that the gas mass lost in making a \( 10^{12} M_{\odot} \) halo can be as much as \( \sim 3 \times 10^{11} M_{\odot} \) (compared to the \( \sim 1.7 \times 10^{11} M_{\odot} \) gas mass at \( z = 0 \) assuming a gas fraction equal to the universal one). The results are shown in Figure 4.36. We find that as much as 15% of the gas mass can get unbound for the more massive halos while \( \sim 5\% \) of the gas gets unbound for the smaller mass halos. In contrast with previous numerical work involving non-gravitational feedback (Kobayashi et al., 2007; Mac Low & Ferrara, 1999) where the effects of mass loss are more severe in smaller halos, this mechanism of unbinding gas from halos via mergers is more effective in unbinding gas from more massive halos. This is also in keeping with observational evidence suggesting that dwarf galaxies are more gas-rich and therefore, could not have suffered a gas blow-out (Kannappan, 2004).

For a more realistic calculation, we assume that initially at redshift 6, all the halos have a fixed gas fraction of \( \Omega_b/\Omega_{\text{dm}} \). Then at each redshift, if a merger occurs, we unbind 10% of the gas mass. If any halo grows by diffuse accretion of matter, we assume that such accretion brings in gas at the universal gas fraction. This gives us the total gas mass of a halo at each redshift. If a new halo appears at some intermediate redshift, we seed it with the universal gas fraction. Thus, the gas mass is tracked through all the halos and all redshifts. Once we have the total amount of gas released in making an individual halo of mass, \( M \) at \( z = 0 \), we can find the total gas released in the universe by convolving with the co-moving number density of those halos (Warren et al., 2006). Note that the halo mass function changes with redshift, so we consider this in our convolution. This yields the total amount of unbound gas in a co-moving \( \text{Mpc}^3 \). We normalise this unbound gas by the baryon mass in all the halos to give us a fraction of unbound gas compared to
the baryonic mass of halos.

The evolution of this unbound gas fraction is shown in Figure 4.37. To test the dependence on the minimum merger ratio, $\eta_{\text{min}}$, we use two values, $\eta_{\text{min}} = 0.1$ and $0.3$. Since there are many more 10:1 mergers in the universe compared to 3:1 (Fakhouri & Ma, 2008), the fractional gas lost increases more rapidly with redshift for $\eta_{\text{min}} = 0.1$, and by $z = 0$, a similar amount of gas exists in the unbound state as in the galaxy halos. Observations show that 60% of the baryons in the universe are missing (Fukugita & Peebles, 2004; Zwaan et al., 2003; Allen et al., 2002) and should be in the WHIM. Previous numerical simulations try to match this by invoking feedback effects from SNe and AGN’s with multiple free parameters. Our results show that it may be possible to populate the WHIM through galaxy interactions with merger ratios greater than $\eta = 0.1$. Since mergers with $\eta = 0.1$ are unlikely to unbind 10% of the total gas mass we consider this an upper limit. This is because the 10:1 mergers occur much more frequently than the other mergers considered. However, the merger rate is relatively flat for $0.3 < \eta \leq 1.0$. Therefore, on an average, it is a reasonable assumption that all mergers with $\eta > 0.3$ release 10% of the gas mass. Considering only these mergers, and without resorting to any additional feedback mechanism, we end up with roughly half of the total mass of the WHIM being released.

As the results of our simulations indicate, the gas mass lost can be substantial, $\sim 20\%$, for a hyperbolic encounter. In this work, we have only accounted for actual merger events and not fly-bys as well. We will investigate the rate of fly-bys during galaxy formation and use that to estimate the gas mass lost more accurately.
Figure 4.36. The fraction of gas mass lost from halos of different mass at redshift zero. 10 realisations were undertaken with 100 logarithmic mass bins in the range of halo masses of $10^8 - 10^{13} M_\odot$. The solid line shows the mean fraction of gas unbound from each halo while the shaded region shows the limits on the total fraction of gas mass lost in our 10 realisations. The gas mass of the halo is computed by assuming an universal gas fraction in the halos at $z = 0$. 
Figure 4.37. The evolution of the fraction of gas mass lost as a function of redshift. The solid and the dashed line shows the evolution for $\eta_{\text{min}} = 0.3$ and 0.1 respectively. The green shaded region shows the $1\sigma$ deviation at each redshift between the 10 realisations. For $\eta_{\text{min}} = 0.1$, the unbound gas mass contains a comparable amount of gas relative to the galaxy halos by $z = 2 - 3$. For $\eta_{\text{min}} = 0.3$, the unbound gas mass is always smaller compared the gas in the halos. Since the galaxy merger rates peak around $z = 2 - 3$, the unbound fraction increases relatively rapidly in that redshift range and by $z = 0$, about 1.2 times more gas is present in the unbound state. This is comparable to the 60% of baryons missing from the universe and can be achieved simply by unbinding through galaxy mergers.
Table 4.4: Properties of the merger remnant for all the simulations.

<table>
<thead>
<tr>
<th>Gas Content</th>
<th>Peak L_X↑</th>
<th>Peak Shock L_X↑</th>
<th>Unb. gas mass</th>
<th>Total unb. mass</th>
<th>Hot Gas</th>
<th>R_vir</th>
<th>Gas within R_vir</th>
<th>DM within R_vir</th>
<th>Remnant L_X</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[10^{40} erg/s]</td>
<td>[10^{38} erg/s]</td>
<td>[10^{10} M⊙]</td>
<td>[10^{10} M⊙]</td>
<td>[%]</td>
<td>[kpc]</td>
<td>[%]</td>
<td>[%]</td>
<td>[10^{40} erg/s]</td>
</tr>
<tr>
<td>1% gas</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.82</td>
<td>0.25</td>
<td>0.57</td>
<td>44.9</td>
<td>51.0</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0.03</td>
</tr>
<tr>
<td>0.54</td>
<td>0.09</td>
<td>0.56</td>
<td>17.5</td>
<td>55.2</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0.04</td>
</tr>
<tr>
<td>0.27</td>
<td>0.005</td>
<td>0.35</td>
<td>9.3</td>
<td>71.8</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0.1</td>
</tr>
<tr>
<td>1.17</td>
<td>0.19</td>
<td>0.29</td>
<td>2.03</td>
<td>80.2</td>
<td>248.1</td>
<td>58.1</td>
<td>58.4</td>
<td>0.13</td>
<td></td>
</tr>
<tr>
<td>0.81</td>
<td>0.15</td>
<td>0.27</td>
<td>1.95</td>
<td>82.1</td>
<td>247.6</td>
<td>59.1</td>
<td>57.8</td>
<td>0.11</td>
<td></td>
</tr>
<tr>
<td>0.42</td>
<td>0.03</td>
<td>0.27</td>
<td>2.6</td>
<td>84.8</td>
<td>246.3</td>
<td>55.2</td>
<td>56.7</td>
<td>0.09</td>
<td></td>
</tr>
<tr>
<td>0.50</td>
<td>0.08</td>
<td>0.14</td>
<td>3.6</td>
<td>84.2</td>
<td>224.4</td>
<td>62.9</td>
<td>65.1</td>
<td>0.07</td>
<td></td>
</tr>
<tr>
<td>0.40</td>
<td>0.04</td>
<td>0.12</td>
<td>2.5</td>
<td>84.6</td>
<td>224.4</td>
<td>62.0</td>
<td>65.2</td>
<td>0.07</td>
<td></td>
</tr>
<tr>
<td>0.20</td>
<td>0.009</td>
<td>0.09</td>
<td>2.9</td>
<td>83.6</td>
<td>222.1</td>
<td>56.8</td>
<td>63.0</td>
<td>0.07</td>
<td></td>
</tr>
<tr>
<td>0.23</td>
<td>0.004</td>
<td>0.03</td>
<td>1.4</td>
<td>90.3</td>
<td>218.4</td>
<td>64.4</td>
<td>72.3</td>
<td>0.07</td>
<td></td>
</tr>
<tr>
<td>0.19</td>
<td>0.008</td>
<td>0.02</td>
<td>2.0</td>
<td>90.7</td>
<td>218.9</td>
<td>68.4</td>
<td>72.8</td>
<td>0.07</td>
<td></td>
</tr>
<tr>
<td>0.13</td>
<td>0.001</td>
<td>0.03</td>
<td>1.6</td>
<td>85.6</td>
<td>218.4</td>
<td>64.4</td>
<td>72.3</td>
<td>0.08</td>
<td></td>
</tr>
</tbody>
</table>

| 10% gas     |           |                 |                |                 |       |       |       |       |             |
| 90.5        | 14.4      | 7.4             | 33.9           | 52.6            | 217.5 | 25.5 | 39.1 | 3.0           |
| 41.7        | 6.0       | 6.1             | 24.5           | 52.9            | –     | –    | –    | –             |
| 23.1        | 0.5       | 3.1             | 12.9           | 59.3            | –     | –    | –    | 9.0           |
| 79.3        | 18.7      | 3.05            | 4.36           | 79.4            | 251.3 | 53.8 | 60.4 | 12.5          |
| 71.9        | 10.2      | 2.84            | 4.16           | 80.9            | 248.6 | 53.3 | 58.3 | 9.5           |
| 34.5        | 1.7       | 2.06            | 3.86           | 83.3            | 246.7 | 49.8 | 57.1 | 8.1           |
| 44.0        | 7.6       | 1.4             | 4.2            | 82.2            | 224.4 | 56.0 | 64.9 | 6.2           |
| 36.0        | 3.4       | 1.2             | 3.3            | 83.6            | 225.3 | 55.3 | 65.8 | 6.8           |
| 18.9        | 0.4       | 0.84            | 2.8            | 84.3            | 223.9 | 52.2 | 64.6 | 5.6           |
| 26.5        | 0.4       | 0.22            | 2.0            | 85.2            | 216.6 | 58.7 | 70.4 | 5.5           |
| 16.4        | 0.2       | 0.22            | 1.4            | 88.4            | 217.1 | 57.7 | 71.0 | 5.8           |
| 11.2        | 0.1       | 0.23            | 1.6            | 84.5            | 215.7 | 55.1 | 69.7 | 6.4           |

Continued on Next Page...
<table>
<thead>
<tr>
<th>Gas Content</th>
<th>Peak $L_X$ $[10^{40} \text{ erg/s}]$</th>
<th>Peak Shock $L_X$ $[10^{38} \text{ erg/s}]$</th>
<th>Unb. gas mass $[10^{10} \text{ M}_\odot]$</th>
<th>Total unb. mass $[10^{10} \text{ M}_\odot]$</th>
<th>Hot Gas [%]</th>
<th>$R_{\text{vir}}$ [kpc]</th>
<th>Gas within $R_{\text{vir}}$ [%]</th>
<th>DM within $R_{\text{vir}}$ [%]</th>
<th>Remnant $L_X$ $[10^{40} \text{ erg/s}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>18% gas</td>
<td>228.2</td>
<td>59.7</td>
<td>11.9</td>
<td>39.4</td>
<td>47.8</td>
<td>223.0</td>
<td>29.1</td>
<td>42.3</td>
<td>10.8</td>
</tr>
<tr>
<td>18% gas</td>
<td>153.2</td>
<td>26.2</td>
<td>9.7</td>
<td>36.2</td>
<td>55.5</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>10.3</td>
</tr>
<tr>
<td>18% gas</td>
<td>78.7</td>
<td>1.8</td>
<td>6.4</td>
<td>14.7</td>
<td>70.8</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>27.6</td>
</tr>
<tr>
<td>18% gas</td>
<td>268.3</td>
<td>30.7</td>
<td>5.0</td>
<td>6.1</td>
<td>81.9</td>
<td>250.4</td>
<td>58.2</td>
<td>59.7</td>
<td>44.0</td>
</tr>
<tr>
<td>18% gas</td>
<td>227.4</td>
<td>26.3</td>
<td>4.7</td>
<td>5.8</td>
<td>83.1</td>
<td>251.8</td>
<td>57.8</td>
<td>60.6</td>
<td>35.0</td>
</tr>
<tr>
<td>18% gas</td>
<td>113.1</td>
<td>7.1</td>
<td>3.4</td>
<td>4.9</td>
<td>85.9</td>
<td>248.6</td>
<td>54.0</td>
<td>58.3</td>
<td>28.2</td>
</tr>
<tr>
<td>18% gas</td>
<td>193.5</td>
<td>20.8</td>
<td>2.3</td>
<td>4.7</td>
<td>85.6</td>
<td>224.8</td>
<td>59.5</td>
<td>65.3</td>
<td>22.2</td>
</tr>
<tr>
<td>18% gas</td>
<td>108.4</td>
<td>14.2</td>
<td>2.1</td>
<td>3.9</td>
<td>86.2</td>
<td>224.4</td>
<td>58.5</td>
<td>65.2</td>
<td>23.2</td>
</tr>
<tr>
<td>18% gas</td>
<td>61.8</td>
<td>1.8</td>
<td>1.6</td>
<td>3.9</td>
<td>86.7</td>
<td>223.9</td>
<td>54.6</td>
<td>64.6</td>
<td>19.6</td>
</tr>
<tr>
<td>18% gas</td>
<td>84.5</td>
<td>3.5</td>
<td>0.39</td>
<td>1.9</td>
<td>91.6</td>
<td>218.4</td>
<td>62.6</td>
<td>72.2</td>
<td>18.4</td>
</tr>
<tr>
<td>18% gas</td>
<td>54.8</td>
<td>1.7</td>
<td>0.38</td>
<td>1.6</td>
<td>90.8</td>
<td>216.6</td>
<td>60.7</td>
<td>70.4</td>
<td>19.3</td>
</tr>
<tr>
<td>18% gas</td>
<td>35.8</td>
<td>0.3</td>
<td>0.45</td>
<td>2.8</td>
<td>89.1</td>
<td>217.5</td>
<td>58.7</td>
<td>71.5</td>
<td>21.4</td>
</tr>
</tbody>
</table>

$L_X$ is calculated by using only the particles that have $T_{\text{gas}} > 10^{5.2}$ K and $\rho < 0.01 \text{ M}_\odot/\text{pc}^3$.

Shock $L_X$ is obtained by adding the $L_X$ for the particles that exceed a threshold for $\frac{dS}{dt}$. The threshold, in code units, is $10^{10}, 3 \times 10^9$ and $2 \times 10^9$ for 1%, 10% and 18% gas fractions.
Conclusion & Future Work

Galaxy mergers are the route by which larger structures grow by assimilating smaller halos. Though the overall gravity is dominated by the dark matter, the observable baryons are the ones that lend themselves to test of structure formation theories. Standard galaxy formation theory posits that gas collapses into a dark matter potential and shock-heats. A fraction then cools down and settles into a disk to form stars; the rest remains hot in the halo. It is imperative to test the effects of this hot halo gas during galaxy mergers. In this thesis, we simulated a suite of galaxy mergers with different mass ratios, gas fractions and orbital parameters to understand how hot halo gas behaves during a merger.

5.1 X-ray emission

In our simulations, we found that the hot halo gas shock-heats to temperatures $\sim 10^{6.3}$ K in the equal-mass mergers and $\sim 10^6$ K in the unequal-mass mergers. This is reflected in a strong temperature jump in the regions between the two colliding galaxies, well before pericenter passage, and persists until the first pass. The strongest shocks, and correspondingly the largest X-ray luminosities due to shocked gas, are created after the pericenter passage. For gas fractions greater than 10%, this shock $L_X$ remains above observable thresholds of $\sim 10^{39}$ erg/s for at least a period of $\sim 300$ Myrs, and thus should be detectable in ongoing mergers with Chandra and XMM-Newton. Compared to the merger timescale, an observable X-ray shock is $\sim 20$ times shorter. During the merger, the hot gas itself also radiates
and can have an X-ray luminosity of $\sim 10^{42}$ erg/s for galaxies with a gas fraction as low as 10%. Herein lies the difficulty of detecting the shocked gas: the shocked gas is 1000 times less luminous than the non-shocked gas. To observe such X-ray shocks from mergers, one would have to look for a signal 3 orders of magnitude smaller than the global X-ray level and the effect only lasts for a timescale that is an order of magnitude shorter than the merger. Merger-induced star formation and AGN activity adds to the confusion by contributing X-rays from accretion disks.

The redeeming feature of our predictions comes from the fact that the production of X-rays occurs even when the galaxy separation is large. Therefore, the ideal procedure to validate our predictions would be to observe close pairs of galaxies, with an undisturbed morphology (indicating that the first pass has not occurred), with current space telescopes like Chandra and XMM-Newton. In such a case, there will be no contamination from merger-associated starbursts and the $L_X$ signature, if observed, could be uniquely described by the shocked halo gas. Our simulations assume that the gas is always heated to the virial temperature in galaxies, irrespective of the galaxy mass; recent numerical simulations show that that may not be the case (Kereš et al., 2005). Smaller mass galaxies, similar to the smallest galaxy we simulated (with $v_{\text{circ}} = 74$ km/s), do not heat above $10^{4.5}$ K. In such a scenario, the expected shock $L_X$ will be much lower. We can use this prediction to constrain the theories of galaxy formation (both semi-analytic and from numerical simulations) against actual observations.

We model the peak shock $L_X$ based on our set of 36 simulations and get an empirical fit of the form –

$$\log_{10} \left[ \frac{L_{\text{peak}}}{10^{38} \text{ erg/s}} \right] = 0.88 \times \log_{10} \Delta H - 1.25. \quad (5.1)$$

Note that we have modeled $L_X$ completely by thermal bremsstrahlung, which is a fairly inefficient radiation mechanism at $\sim 10^{6.2}$ K. The presence of the slightest amount of metals will increase the total emission significantly, possibly by more than an order of magnitude (see Fig 5.1). We will discuss the effect of including metals below. Thus, our predictions about $L_X$ should be considered a lower limit to the likely X-ray emission in actual observations of colliding galaxies.
5.2 Unbound gas

Our simulations show that gas escapes from galaxies during the course of the merger, with most of the gas being outside a $3 \, R_{\text{vir}}$ radius even before the final merger event. For equal-mass mergers, the amount of unbound gas ranges from 10-20% depending on the impact parameter and orbit type. The unequal-mass mergers result in 3-5% unbound gas, with similar fractions coming from each of the merging galaxies. All of this unbound gas, presumably enriched with metals, flows out into the IGM. This process is similar, in principle, to the gravitational heating that has been incorporated for the first time in semi-analytic galaxy formation recipes in a recent paper (Khochfar & Ostriker, 2008). Our simulations provide the empirical fits that can be incorporated into such semi-analytic simulations. Though the amount of gas has a strong dependence on the initial orbital energies, we can see that each merger releases $\gtrsim 2 \times 10^{10} M_\odot$ of material that reaches the IGM.

Given that each galaxy suffers a major merger in its past, the total amount of unbound gas can become significant over the lifetime of the Universe. Observations of the IGM at low redshift show the presence of metals (Danforth & Shull, 2008); cosmological simulations can account for the metals since it only takes trace amounts of metals to leave a large-scale signature. However, about 50% of the baryons are actually present in the IGM (Fukugita & Peebles, 2004) in a temperature-density phase-space that is difficult to probe via observations (Danforth & Shull, 2008). For about a decade numerical simulations of structure formation have been struggling to obtain a prescription that allows such large amounts of gas to escape from galaxies without violating other observational constraints like the star formation rate. Our results show that mergers are an effective way to extract gravitational energy and cause significant mass-loss from galaxies. We use those merger remnants with unbound fractions larger than 3% to obtain a fitting formula for the unbound gas fraction as a function of the merger impulse, $\Delta E$, as follows:

$$f_{\text{unb}} = 3.14 \times \log_{10} \Delta E - 0.16.$$  \hspace{1cm} (5.2)

To estimate the fraction of gas unbound from the galaxy halos via mergers, we create a merger tree using an EPS semi-analytic model. We accurately track
the gas fractions in galaxy halos and in the unbound component and we find that up to 60% of the gas in the universe can be in the unbound state at $z = 0$ for $\eta_{\text{min}} = 0.1$, consistent with the expected mass fraction of the WHIM. This calculation assumes a fixed gas fraction of 10% is unbound from all mergers with $\eta > \eta_{\text{min}}$; this is definitely an over-estimate of the unbinding process during minor mergers, we see that reducing the value of $\eta_{\text{min}}$ to 0.3 correspondingly decreases the fraction to about $\sim 40\%$. So, a conservative value for $\eta_{\text{min}}$ leads to about 40% of the gas mass being unbound from the galaxies. Other feedback mechanisms will work in tandem with this gravitational process and can release more gas, especially from the more numerous smaller halos.

Overall, both X-ray shocks and the unbound gas production are two facets of the same phenomena: the conversion of gravitational potential energy into thermal and kinetic energy. Even though cooling, star formation and other baryonic physics were not included in our simulations, the tremendous amount of available gravitational potential energy and the subsequent conversion of a fraction of it into gas thermal energy continuously heats the gas at a much faster rate than it can cool via any feasible cooling mechanism, at least for the temperatures and densities involved in our simulations.

5.3 Future Work

Cox et al. (2004) showed that it is possible to repopulate the halo with gas (and metals) via mergers of disk galaxies; mass-loaded stellar winds from high mass stars, and AGN activity can also enrich the gas in the halo. We know that the metal content of the gas can strongly enhance the X-ray production, even overshadowing the contribution from thermal bremsstrahlung (see Fig 5.1). For example, for solar metallicity gas at $10^6$ K, the emission for a MEKAL model is 2 orders of magnitude larger than that of thermal bremsstrahlung (see Fig 5.1). As part of a future project, we will include metals in the merging galaxies to determine how this affects the X-ray properties. We expect that the boost from metal line emission will make the emission more detectable, and may allow X-ray shocks to be observed for galaxy pairs with larger separations.

Our simulations were performed with vacuum boundary conditions, but in re-
Figure 5.1. The cooling curves for various ions and models are shown here. The one most relevant to the emission from enriched, diffuse gas would be the MEKAL model, shown in red. The picture is taken from the open source software, cool_curve_create, written by David Strickland.

Galaxy galaxies are continuously evolving objects within the background of an expanding Universe. Pristine gas is falling into the halos, and the halos themselves are growing more massive. Cosmological simulations are required to fully model the mergers within the context of an evolving background. As part of a future project, we will use cosmological simulations with dark matter and gas to explore the phenomena of the shocks in the mergers. In a cosmological context, we will use
Eqn. 3.10 to numerically estimate the Mach numbers for shocks. This will enable us to identify shocks more accurately.

Another aspect of this current work to explore in the future would be to estimate the total emission in X-rays from shocks, and the total amount of unbound gas using the fitting formula we found for the shock $L_X$ and the unbound gas fraction. To do this, we will combine an extended Press-Schechter formalism (Press & Schechter, 1974) with our empirical fits, in a semi-analytic approach. The semi-analytic estimate for the unbound gas fraction over the lifetime of the Universe can be compared against the current gas fractions of galaxies as well as the baryonic content of the IGM. The estimate of the total $L_X$ from mergers would also account for a fraction of the X-ray background in the local Universe.
Appendix A

Galaxy initialization - a C code

```c
#include <stdio.h>
#include <math.h>
#include <stdlib.h>
#include <time.h>
#include <string.h>
/* For random number generators*/
#define N 624
#define N 397
#define MATRIX_A 0x9908b0dfUL /* constant vector a */
#define UPPER_MASK 0x80000000UL /* most significant w-r bits */
#define LOWER_MASK 0x7fffffffUL /* least significant r bits */
static unsigned long mt[N]; /* the array for the state vector */
static int mti=N+1; /* mti==N+1 means mt[N] is not initialized */
double genrand_float(void);
double frand(float lower, float upper);
void init_genrand(unsigned long s);
unsigned long genrand_int32(void);
unsigned long seed;
/* End random number generator requirements*/
#define SQRT(X) (pow(X,0.5))
#define SQR(X) ((X)*(X))
#define CUBE(X) ((X)*((X)*(X)))
#define PI (4.0*atan(1.0))
#define OverDensity (200.0)
#define GAMMA (5.0/3.0)
#define GAMMA_MINUS1 (GAMMA-1.0)
#define BOLTZMANN (1.3806e-16)
#define PROTONMASS (1.6726e-24)

struct particle
{
    float pos[3];
    float vel[3];
} P;

struct global_data_all_processes
{
int TotNumPart,
    TotN_gas,
    TotN_halo,
    TotN_disk,
    TotN_bulge,
    TotN_stars;
int ComovingIntegrationOn;
double Time;

double BoxSize;
double Omega0, OmegaLambda, OmegaBaryon, HubbleParam;
double MassTable[6];
}

struct io_header_1
{
    int npart[6];
    double mass[6];
    double time;
    double redshift;
    int flag_sfr;
    int flag_feedback;
    int npartTotal[6];
    int flag_cooling;
    int num_files;
    double BoxSize;
    double Omega0;
    double OmegaLambda;
    double HubbleParam;
    char fill[256]; /* fills to 256 bytes */
} header1;

double rho(double s);
double pot(double s);
double dpotds(double s);
double potgas(double s);
double dpotgasds(double s);
double gasmass(double s);
void computederiv(double* x,double* y,double* dydx, int NN);
void allocate(void);
void writeicfile(char*);
size_t my_fwrite(void *ptr, size_t size, size_t nmemb, FILE *stream);
double rk4(double xmin,double xmax, double ymin,long nstep,int i,double (*functptr)(double,double,int));
double RHS(double x, double y,int i);
void allocategas(void);
double mforgivens(double s);
double solvemassfors(double n);
double gasmasss(double s);
double gasdm(double E,double* Table,double* Values,int NN);
double ProductLog(double s);
double interpolatedmass(double s);
double interpolatedpotential(double s);
float gaussrand(void);
double getsforgivenmass(double m);
}

static double k,k1,c,gc,rcrit,potedgenfw,potedgegas;
static int NTAB,NGRID;
static int Nhalo=0,Ngas=0,Ndisk=0,Nbulge=0;
double MeanMolecularWeight=0.6,muh;
double* Etab;
double* fEtab;
double* stab;
double* MassTable;
double* rhotab;
double* dpotdstab;  
double* drhopotstab;  
double* d2rhodpsi2tab;  
double* gasrho;  
double* pressure;  
double* gasdPdr;  
double* kT;  
double* InteriorMass;  
float* InternalEnergy;  

double Rvir,Vvir,Mrvir,Mhalo,SpinFactor,Jtotal,Jd,HaloGasFraction,Mgas,Mtot;  
double xmin,xmax,Exin,Emax;  
double CentralGasRho,VirialGasTemp,GasCoreRadius;  

const int MAX_ITERATIONS = 10000;  
const double MAX_EPS = 1e-6;  

double G = 6.672e-8; //CGS Units  
double GRAVITY = 43007.1; //10^10 Msun, 1 kpc, 1km/s; will change to 1.0 later on  
double NNOB1010 = 1.989e43;  
double HO = 7e-2; //km/s/kpc  
double OmegaM = 0.23;  
double OmegaB = 0.04;  
double BaryonFraction ;  
double UnitMass_in_g=UnitTime_in_s=UnitVelocity_in_cm_cm_per_s=UnitLength_in_cm=UnitEnergy;  

double RADIUSFACTOR = 1.2;  
double ds_DF,ds_hydro;  

int main(void) {  
  double fc;  
  SpinFactor = 0.05;  
  BaryonFraction = OmegaB/OmegaM;  
  clock_t c1, c2;  
  char buf[100];  
  double radius, theta, phi;  

  int FixedMassRes=0;  
  double MassResolution=0.0;  
  /* Define convenient physical system of units */  
  UnitVelocity_in_cm_cm_per_s = 1e5; //km/s  
  UnitLength_in_cm = 3.08568e21; //kpc  
  UnitMass_in_g = 1.989e43;  
  UnitTime_in_s = UnitLength_in_cm / UnitVelocity_in_cm_cm_per_s;  
  /* Initialise random number generator */  
  seed = (unsigned long)time(NULL);  
fprintf(stderr,"Seed used for random number generator = %ld \n",seed);  
  init_genrand(seed);  
  
fprintf(stderr,"\n Enter the number of bins to compute DF \n");  
fscanf(stdin,"%d",&NTAB);  
fprintf(stderr,"\n NTAB = %d\n",NTAB);  
fprintf(stderr,"\n Enter the circular velocity at virial radius \n[km/s] \n");  
fscanf(stdin,"%lf",&Vvir);  
  Rvir = Vvir/(10.0*HO);  
  Mvir = pow(Vvir,3)/(10.0*GRAVITY*HO*HO);  
  Tvir = 1e6*SQR(Vvir/167.0);  
}
% Galaxy Model Parameters : Mvir = %g 10^10 solar mass \t Vvir = %g km/s Rvir = %g kpc
Tvir = %g K
fprintf(stderr, " Now switching to Mvir=1.0, Rvir/2=1.0 and G=1.0 in ");
fprintf(stderr, " Enter the concentration parameter [c] for the NFW halo 
(c=5 for galaxy cluster, c= 10 for bright galaxies) ");
fscanf(stdin, " [lf]", &c);
c1 = clock();
UnitLength_in_cm *= (Rvir/2.0);
UnitMass_in_g *= Mvir;
GRAVITY = 1.0;
UnitTime_in_s = sqrt((GRAVITY*pow(UnitLength_in_cm,3.0))/(UnitMass_in_g*G));
UnitVelocity_in_cm_per_s = UnitLength_in_cm/UnitTime_in_s;
UnitEnergy = SQR(UnitVelocity_in_cm_per_s)*UnitMass_in_g;
fprintf(stderr, " SYSTEM OF UNITS -- for GADGET parameter file 
");
fprintf(stderr, " Length = %g cm Mass = %g gm Time = %g seconds Unit Energy = %g G = %g (should be 1) H0 = %g ",
UnitLength_in_cm,UnitMass_in_g,UnitTime_in_s,UnitEnergy,GRAVITY,H0);
Mvir = 1.0;
Rvir = 2.0;
Vvir = sqrt(GRAVITY*Mvir/Rvir);
rhocrit = Mvir/(4.0/3.0*PI*CUBE(Rvir)*OverDensity);
H0 = sqrt(rhocrit*8.0*PI*GRAVITY/3.);
Nhalo = RADIUSFACTOR*Rvir;
min = (double)1.0e-5;
max = (double)100.0;
fprintf(stderr, " minimum mass = %g 
",mforgivens(min));
gc = 1.0/(log(1.0+c) - c/(1.0+c));
k = OverDensity*SQR(c)*gc*rhocrit/3.0;
i = gc*SQ(R(vir));
f = c*(1.0 - 1.0/SQR(1.0 + c) - 2.0*log(1.0 + c)/(1.0 + c))*SQR(gc)/2.0;
Jtotal = SpinFactor*sqrt(2.0*GRAVITY*Mvir*Rvir)*Mvir/sqrt(fc);
Mhalo = mforgivens(RADIUSFACTOR);
Mtot = Mhalo;
fprintf(stderr, "Total halo mass = %g 
",Mhalo);
fprintf(stderr,"Enter 1 if you want fixed mass resolution from previous simulations 
");
fscanf(stdin, " [ld]", &FixedMassRes);
if(FixedMassRes == 1)
{  
  fprintf(stderr,"Enter the mass per halo particle in gm\n\\n");
  fscanf(stdin, " [ld]", &MassResolution);
  Nhalo = ceil(Mhalo*UnitMass_in_g/MassResolution);
  fprintf(stderr,"Number of halo particles used - \ld with mass resolution = %g \n",Nhalo,MassResolution);
  }
else
{
  fprintf(stderr,"Enter the number of halo particles \n");
  fscanf(stdin, " [ld]", &Nhalo);
  fprintf(stderr,"Nhalo = %d\n",Nhalo);
}
}
fprintf(stderr,"Enter the number of grid locations for gas particles \n");
fscanf(stdin, " [ld]", &GRID);
fprintf(stderr,"\n Number of grid points used to compute gas properties = \ld \n",GRID);
/* Allocate memory for the various tables */
allocate();
fprintf(stderr,"\n Allocation ..done \n");
if(GRID > 0)
{
}
fprintf(stderr,"Enter the gas mass fraction in the halo \n");
scanf(stdin,"%lf",&HaloGasFraction);
fprintf(stderr,"Halogen gas mass fraction = %g \n",HaloGasFraction);
Mgas = HaloGasFraction*Mhalo;
if(FixedMassRes == 1)
{
fprintf(stderr,"Enter the mass per gas particle \n");
scanf(stdin,"%lf",&MassResolution);
Ngas = ceill(Mgas*UnitMass_in_g/MassResolution);
fprintf(stderr,"Number of gas particles used - \n with mass resolution = %g \n",Ngas,MassResolution);
}
else
{
fprintf(stderr,"Enter the number of gas particles \n");
scanf(stdin,"%d",&Ngas);
fprintf(stderr,"Ngas = %d \n",Ngas);
}
Mtot = Mhalo + Mgas;
GasCoreRadius = Rvir/(3.0*c);
allocategas();
if(!(P=(struct particle*)malloc((Nhalo+Ngas+1)*sizeof(struct particle))))
{
fprintf(stderr,"failed to allocate memory for 'particle data type' \n");
exit(0);
}
/* Calculate central gas density given rho = rho0/(r^2 + rc^2) */
/* Set central gas temperature to some fraction of Tvir */
double* SmoothkT;
double* gasgrid;
double* dTdr;
const int NumNgb = 50;
int firstngb = 0;
if(Ngas > 0)
{
    if(!(SmoothkT=(double*)malloc((NGRID)*sizeof(double))))
    {
        fprintf(stderr,"failed to allocate memory for SmoothkT \n");
        exit(0);
    }
    if(!(gasgrid=(double*)malloc((NGRID)*sizeof(double))))
    {
        fprintf(stderr,"failed to allocate memory for gas grid \n");
        exit(0);
    }
    if(!(dTdr=(double*)malloc((NGRID)*sizeof(double))))
    {
        fprintf(stderr,"failed to allocate memory for dTdr \n");
        exit(0);
    }
    ds_hydro = (RADIUSFACTOR - smin)/((double)NGRID - 1.0);
    CentralGasRho = Mgas/(4.0*PI*(Rhalo - GasCoreRadius*atan(Rhalo/GasCoreRadius)));//Mgas is contained within Rhalo
VirialGasTemp = 0.35*Tvir;

double gasr = 0.0;
double gasr2 = 0.0;
const int virindex = const int(floor(1.0/ds_hydro));

mumh = MeanMolecularWeight*PROTONMASS/UnitMass_in_g;

for(int i=0;i < NGRID;i++)
{
    gasr = (i*ds_hydro + smin)*Rvir;
gasgrid[i] = gasr;
gasr2 = SQR(gasr);
gasrho[i] = rhogas(i*ds_hydro + smin);
InteriorMass[i] = mforgivens(gasr/Rvir) + gasmass(gasr/Rvir);
gasdPdr[i] = -gasrho[i]*(GRAVITY*InteriorMass[i]/gasr2);
}

kT[virindex] = BOLTZMANN*VirialGasTemp/UnitEnergy;
pressure[virindex] = gasrho[virindex]/mumh*kT[virindex];

//Integrate outwards
for(int i = virindex+1;i<NGRID;i++)
{
    pressure[i] = pressure[i-1] + 0.5*(gasdPdr[i] + gasdPdr[i-1])*ds_hydro*Rvir;
kT[i] = pressure[i]/(gasrho[i]/mumh);
}

//Integrate to the centre
for(int i = virindex-1;i>=0;i--)
{
    pressure[i] = pressure[i+1] - 0.5*(gasdPdr[i+1] + gasdPdr[i])*ds_hydro*Rvir;
kT[i] = pressure[i]/(gasrho[i]/mumh);
}

for(int i=0;i<NGRID;i++)
{
    SmoothkT[i] = 0.0;
    if((i<NumNgb/2)
        firstngb = 0;
    else
        if((i<NumNgb/2) && ((NGRID-i)>NumNgb/2))
            firstngb = i-NumNgb/2;
        if((NGRID - i) <NumNgb/2)
            firstngb = NGRID - NumNgb;
        for(int j=0;j<NumNgb;j++)
            if((firstngb+j)==i)
                SmoothkT[i] += kT[firstngb+j];

    SmoothkT[i]/=NumNgb;
}

computederv(gasgrid,SmoothkT,dTdr,NGRID);
}

/* Don't touch below this */
potedgenfw = 0.0;
potedgenfw = pot(RADIUSFACTOR) - GRAVITY*Mhalo/Rhalo;//Matching the NFW & Newtonian potentials at the truncation radius
/* feel free*/
fprintf(stderr,"Corre to the potential applied = %g k = %g k1 = %g \n",potedgenfw,k,k1);
if (Mgas > 0)
{
    potedgegas = 0.0;
    potedgegas = potgas(RADIUSFACTOR) - GRAVITY*Mgas/Rhalo;
}

du_DF = (log10(smax) - log10(smin))/(NTAB-1.0);
fprintf(stderr,"du_DF = %g \n", du_DF);
Emin = pot(smin);
Emax = 0.0;

/* Read in the distribution function if it has already been calculated. */
int ReadDF = 0;
const int RK4STEPS = NTAB;
double norm = 1.0/(sqrt(8.0)*SQR(PI));

fprintf(stderr," Enter 1 if you want to read in the DF from a file \n");
scanf(stdin,"%d", &ReadDF);

FILE* fp;
char buf1[200], buf2[200], buf3[200];
int location = 0;
if (ReadDF == 1)
{
    /* ask for file name and read in */
    fprintf(stderr,"Enter file name for the DF \n");
    fscanf(stdin, "%s", &buf1);
    fprintf(stderr, "Reading DF from file -- %s \n", &buf1);
    if ((fp = fopen(buf1, "r")) == 0)
    {
        /* read in E,f(E) only */
        while (!feof(fp))
        {
            fgets(buf, 200, fp);
            if (buf[0] == '#')
                continue;
            sscanf(buf, "%s%s%s", &buf1, &buf2, &buf3);
            Etab[location] = atof(buf1);
            fEtab[location] = atof(buf2);
            stab[location] = atof(buf3);
            MassTable[location] = mforgives(stab[location]);
            location++;
        }
        fclose(fp);
    }
    if (location != NTAB)
    {
        fprintf(stderr," Number of lines in the DF file not equal to NTAB ..exiting \n");
        exit(0);
    }
}
     else
    {
        fprintf(stderr," File not found ..exiting \n");
        exit(0);
    }
    potedgenfw = 0.0;
potedgenfw = pot(RADIUSFACTOR) - GRAVITY*Mhalo/Rhalo;
    potedgegas = 0.0;
potedgegas = potgas(RADIUSFACTOR) - GRAVITY*Mgas/Rhalo;
}
else
{
    /* Compute drho/dpsi and then d2drho/dpsi2 */
    for (int i=0; i<NTAB; i++)
    {

stab[i] = smax/pow(10.,(double)i*ds_DF);
Etab[i] = pot(stab[i]) + potgas(stab[i]);
rhotab[i] = rho(stab[i]) + rhogas(stab[i]);
MassTable[i] = mforgivens(stab[i]);
dpotdstab[i] = dpotds(stab[i]) + dpotgads(stab[i]);
}

fprintf(stderr,"\n Initialization for variables \n\n");

comptederiv(Etab,rhotab,drhodpsitab,NTAB); // Etab is just substituting for psitab
comptederiv(Etab,drhodpsitab,d2rhodpsi2tab,NTAB);

/* Compute Distribution Function */
/* Integrate numerically to get f(E) */

fprintf(stderr,"\n just before integration for DF starts\n\n");
fEtab[0] = 0.0;
for(int i=1;i<NTAB;i++)
    fEtab[i] = rk4(stab[0],stab[i],0.0,RK4STEPS,i,&RHS)*norm;
}

/* Find max. of r^2 v^2 f(E) */
double r,v,EE;
double maxr,maxv,maxrrxvdf = 0.0;
double fq;
double vv = sqrt(2.0*c*gc*SQR(Vvir));
r = smin*Rvir;
const int NFORMAXDF = NTAB < 10000 ? NTAB:10000;
double drnorm = (log10(RADIUSFACTOR)-log10(smin))/(NFORMAXDF-1.0);
double dv = 0.0;
for(int i=0;i<NFORMAXDF;i++)
{
    vv = sqrt(2.0*(pot(r/Rvir)+potgas(r/Rvir)));
    dv = vv/(NFORMAXDF-1.0);
    for(int j=0;j<NFORMAXDF;j++)
    {
        v = dv*(double)j;
        EE= pot(r/Rvir) + potgas(r/Rvir) - 0.5*v*v;
        fq = getdf(EE,Etab,fEtab,NTAB);
        if((maxrrxvdf < fq*v*v*r*r) && !(isnan(fq*v*v*r*r))
            {
                maxr = r;
                maxv = v;
                maxrrxvdf = fq*v*v*r*r;
            }
    }
    r = RADIUSFACTOR/pow(10.,(double)i*drnorm)*Rvir;
}

fprintf(stderr,"\nMax value = %g for r = %g and v = %g with vescape = %g with dr = %g\n\n");

/* Now the DF has been computed and is stored in fEtab */
/* Write the DF to a named file */
if(ReadDF == 0)
{
    if(Ngas > 0 )
    {
        if(!(fp = fopen("1e5DFgas.dat", "w")))
        {
            fprintf(stderr,"\n Could not open file for writing out DF\n\n");
            exit(0);
        }
    }
    else
if(!(fp = fopen("1e5DFnogas.dat", "w")))
{
    fprintf(stderr,"\n Could not open file for writing out DF\n");
    exit(0);
}

s = 2.0*RADIUSFACTOR;
while(s > RADIUSFACTOR || s < smin)
{
    s = frand(0.0,Mgas);
    s = solvegasmassfor(s);
}

radius = s*Nvir;
theta = acos(frand(-1.0,1.0));
phi = frand(0.0,2*PI);
P[i+offset].pos[0] = (float)radius * sin( theta ) * cos( phi );
P[i+offset].pos[1] = (float)radius * sin( theta ) * sin( phi );
P[i+offset].pos[2] = (float)radius * cos( theta );

vescape2 = 2.0*(pot(s)+potgas(s));
vescape = sqrt(vescape2);

if(s > 1.0)
    vkludge = 1.0 - 0.15*(s/RADIUSFACTOR);
else
    vkludge = 0.95;

vmax = vkludge*vescape;
/* NOT CORRECT, HAVE TO CHANGE KT to closest in terms of s */
grid2gasloc = (int) floor(s/ds_hydro);
if(grid2gasloc == (NGRID-1))
    grid2gasloc = NGRID-2;
gridgass = (grid2gasloc*ds_hydro + smin);

InternalEnergy[i+offset] = (float)0.5*(((gridgass+ds_hydro-s)*SmoothkT[grid2gasloc] +
(s-gridgass)*SmoothkT[(grid2gasloc+1)]))/(ds_hydro*mumh*GAMMA_MINUS1));

meangasvel = 0.0;
double GasSigmakludge = 1.0;
meangassigma = GasSigmakludge*sqrt(2.0*InternalEnergy[i+offset]);

if((meangasvel + meangassigma) > vescape)
    fprintf(stderr,"\n Gas particles really want to escape within 1 sigma \n");

/* Find a random number generator corresponding to a Gaussian distribution*/
xv = vescape;
while(fabs(xv) > vmax)
    xv = fabs(gaussrand())*meangassigma + meangasvel;
theta = acos(frand(-1.0, 1.0));
phi = frand(0.0, 2.0*PI);

P[i+offset].vel[0] = (float)xv * sin( theta ) * cos( phi );
P[i+offset].vel[1] = (float)xv * sin( theta ) * sin( phi );
P[i+offset].vel[2] = (float)xv * cos( theta );

fprintf(stderr," xv = %g
");

/* Gas has been initialised - now for collisionless */
/* double DFfactor = 1.1; */
/* double pn,p; */
double actualmaxxrv = 0.0;
double actualmaxrvmax=0.0,actualmax=0.0,actualmaxv=0.0;
double minimums = RADIUSFACTOR;
int rejectioncounter =0,zeroposcounter=0;
int actualinitialised =0;
int mncounter=0;

offset = Ngas;
for(int i=1;i<Nhalo;i++)
{ velcounter=0;
s = 2.0*RADIUSFACTOR;
E = -10.0;
p = 1.0;
pn = 0.0;
while(s > RADIUSFACTOR || s < smin || E < 0.0)


\{  
  m = frand(0.0, Mhalo);  
  s = getsforgivenmass(m);  
  radius = s*Rvir;  
  vescape2 = 2.0*(pot(s)+potgas(s));  
  vescape = sqrt(vescape2);  
  if(s > 1.0)  
    vkludge = 1.0 - 0.15*(s/RADIUSFACTOR);  
  else  
    vkludge = 0.95;  
  xv = frand(0.0, vkludge)*vescape;  
  E = pot(s) + potgas(s) - 0.5*SQR(xv);  
\}  

while(p > pn)  
{  
  valcounter++;  
  p = frand(0.0, 1.0);  
 fq = getdf(E, Etab, fETab, NTAB);  
  pm = (radius*radius*xv*xv)*fq/maxxxrvdf;  
  if(pm > 1.0)  
  {  
    fprintf(stderr, "Error in setting halo velocities -- probability greater than 1.0 \n");  
    pn = 1.0;  
  }  
  if(valcounter > MAX_ITERATIONS)  
  {  
    valcounter = 0;  
    rejectioncounter++;  
    s = 2.0*RADIUSFACTOR;  
    E = -10.0;  
    p = 1.0;  
    pn = 0.0;  
    while(s > RADIUSFACTOR || s < smin || E < 0.0)  
    {  
      m = frand(0.0, Mhalo);  
      s = getsforgivenmass(m);  
      radius = s*Rvir;  
      vescape2 = 2.0*(pot(s)+potgas(s));  
      vescape = sqrt(vescape2);  
      if(s > 1.0)  
        vkludge = 1.0 - 0.15*(s/RADIUSFACTOR);  
      else  
        vkludge = 0.95;  
      xv = frand(0.0, vkludge)*vescape;  
      E = pot(s) + potgas(s) - 0.5*SQR(xv);  
    }  
    valcounter = 0;  
    p = frand(0.0, 1.0);  
    fq = getdf(E, Etab, fETab, NTAB);  
    pm = (radius*radius*xv*xv)*fq/maxxxrvdf;  
  }  
  if(p <= pm)  
  {  
    \theta = acos(frand(-1.0, 1.0));  
    phi = frand(0.0, 2*PI);  
    P[i+offset].pos[0] = (float)radius * sin( theta ) * cos( phi );  
    P[i+offset].pos[1] = (float)radius * sin( theta ) * sin( phi );  
    P[i+offset].pos[2] = (float)radius * cos( theta );  
  }  
\}
\[
\theta = \acos(\text{frand}(-1.0, 1.0));
\]
\[
\phi = \text{frand}(0.0, 2.0\times\pi);
\]
\[
P[i+offset].\text{vel}[0] = (\text{float})xv \times \sin(\theta) \times \cos(\phi);
\]
\[
P[i+offset].\text{vel}[1] = (\text{float})xv \times \sin(\theta) \times \sin(\phi);
\]
\[
P[i+offset].\text{vel}[2] = (\text{float})xv \times \cos(\theta);
\]
```
if(pn > actualmaxxrvxv)
{
    actualmax = radius;
    actualmax = xv;
    actualmaxxrvxv = pn;
    actualmax = s;
}
```
```
if(s < minimum)
minimum=s;
```
```
velcounter = 0;
actualinitialised=0;
```
```
if(i==Nhalo/2)
{
    c2=clock();
    fprintf(stderr,\"\n Initialisation done for half the particles -- time taken = \%f seconds \n\",(float) (c2 - c1)/CLOCKS_PER_SEC);
}
```
```
if(P[i+offset].pos[0] == 0.0 || P[i+offset].pos[1] == 0.0 || P[i+offset].pos[2] == 0.0)
{
    fprintf(stderr,\"\n something's wrong -- xpos = 0, radius = %g s = %g \sin(\theta) = %g \sin(\phi)=%g calcd
xpos = %g \sin(\theta) \sin(\phi),radius = s,\sin(\theta),\sin(\phi),radius \times \sin(\theta) \times \cos(\phi)");
    fprintf(stderr,\" i = %d p =%g pn = %g \n\",i,p,pn);
    zeroposcounter++;
}
```
```
fprintf(stderr,\"\n Number of particles re-initialised = %d zero positions = %d total initialisations = %d and nans = %d \n\",
rejectioncounter,zeroposcounter,actualinitialised,nancounter);
```
```
zerovx /=(Nhalo+Ngas);
zerovy /=(Nhalo+Ngas);
zerozv /=(Nhalo+Ngas);
zeroposx /=(Nhalo+Ngas);
zeroposy /=(Nhalo+Ngas);
zeroposz /=(Nhalo+Ngas);
```
```
// Centre the position and velocity on the origin *
```
```
double min = RADIUSFACTOR*Rvir;
for(int i=1;i<Nhalo+Ngas;i++)
{
    P[i].pos[0] = zeroposx;
    P[i].pos[1] = zeroposy;
    P[i].pos[2] = zeroposz;
    P[i].vel[0] = zerovx;
    P[i].vel[1] = zerovy;
    P[i].vel[2] = zerozv;
    r = sqrt(SQR(P[i].pos[0]) + SQR(P[i].pos[1]) + SQR(P[i].pos[2]));
    xv = sqrt(SQR(P[i].vel[0]) + SQR(P[i].vel[1]) + SQR(P[i].vel[2]));
    if((pot(r/Rvir) + potgas(r/Rvir) < 0.0)
    fprintf(stderr,\" Unbound particles with i=%d x=%g y=%g z=%g v = %g pot = %g \n\",i,P[i].pos[0],P[i].pos[1],P[i].pos[2],xv,
    pot(r/Rvir)+potgas(r/Rvir));
    if(r<min)
fprintf(stderr, "\n %g %g %g %g %g %g \n", zeroxpos, zeroypos, zerozpos, zerovx, zerovy, zerovz);
fprintf(stderr, "\n Computational max predicted = %g Actual max obtained after division (should be 1) = %g \n", maxrxxvdf, actualmaxxrxv);
fprintf(stderr, "\n Maxr predicted = %g max r obtained = %g s = %g Max v predicted = %g max v obtained = %g \n", maxxr, actualrmax, actualsmax, maxxv, actualvmax);
fprintf(stderr, "\n Minimum s = %g \n", minimums);
c2=clock();
fprintf(stderr, "\n Collisionless initialisation complete -- total time taken for %d particles = %f seconds \n", Nhalo+Ngas, (float) (c2 - c1)/CLOCKS_PER_SEC);
fprintf(stderr, "Done checking for unbound particles, minimum s = %g \n", rmin/Rvir);
/* Collisionless has been initialised - now for the gas particles */
for(int i = 0; i <6; i++)
   All.MassTable[i] = 0.0;
if(Ngas > 0)
   All.MassTable[0] = (double) Mgas/(double)Ngas;
All.MassTable[1] = (double) Mhalo/(double)Nhalo;
All.TotNumPart = Nhalo + Ngas + Ndisk + Nbulge;
All.TotX_gas = Ngas;
All.TotX_halo = Nhalo;
All.TotX_disk = Ndisk;
All.TotX_bulge = Nbulge;
All.TotX_stars = 0;
All.ComovingIntegrationSm = 0;
All.Time = 0.0;
All.BoxSize = 0.0;
All.DMegaLambda = 0.73;
All.DMegaO = 1.0;
All.HubbleParam = 0.7;
/* Convert to standard units here */
double StandardLength = 3.08568e21; //1 kpc in cm
double StandardMass = 1.989e43; //10^10 Msun
double StandardVel = 1.0e5; //1 km/s
All.MassTable[0] *= (UnitMass_in_g/StandardMass);
All.MassTable[1] *= (UnitMass_in_g/StandardMass);
for(int i=1;i<Ngas+Nhalo;i++)
   {
      P[i].pos[0] *= (UnitLength_in_cm/StandardLength);
P[i].pos[1] *= (UnitLength_in_cm/StandardLength);
P[i].pos[2] *= (UnitLength_in_cm/StandardLength);
P[i].vel[0] *= (UnitVelocity_in_cm_per_s/StandardVel);
P[i].vel[1] *= (UnitVelocity_in_cm_per_s/StandardVel);
P[i].vel[2] *= (UnitVelocity_in_cm_per_s/StandardVel);
      if(i<Ngas)
         InternalEnergy[i] *= SQR(UnitVelocity_in_cm_per_s/StandardVel);
   }
fprintf(stderr, "\n Actual units used are 10^10 msun, 1 kpc and 1 km/s \n");
/* End unit conversion */
fprintf(stderr, "Enter file name for initial conditions -- 
");
scanf(stdin, "%s", buf);
writeicfile(buf);

fprintf(stderr, "done writing to IC file -- %s 
", buf);

/* Free the used memory */
free(stab);
free(rhotab);
free(Etab);
free(Dpotdstab);
free(Drhodpsi2tab);
free(D2rhodpsi2tab);
free(fEtab);
free(MassTable);
free(InteriorMass);

if(Ngas > 0)
{
    free(gasrho);
    free(pressure);
    free(gasdPdr);
    free(xT);
    free(InternalEnergy);
    free(gasgrid);
    free(SmoothkT);
    free(dTdr);
}

free(fp);
free(P);
fprintf(stderr, "freed memory\n");
exit(1);

float gaussrand(void)
{
    float x1, x2, w, y1, y2;
    do {
        x1 = 2.0 * frand(0.0,1.0) - 1.0;
        x2 = 2.0 * frand(0.0,1.0) - 1.0;
        w = x1 * x1 + x2 * x2;
    } while ( w >= 1.0 );
    w = sqrt( (-2.0 * logf( w ) ) / w );
    y1 = x1 * w;
    y2 = x2 * w;
    return y1;
} //end of gaussian random number generator

double solvegasmassfors(double m)
{
    int iteration = 0;
    double sappx=0.0,sappxold=0.0;
    sappxold = 1.0;
    sappx = 0.5;
    if(m == Mgas)
    return Rhalo/Rvir;
    while(iteration++ < MAX_ITERATIONS && fabs(sappx - sappxold) > MAX_EPS)
{ 
    sappxold = sappx; 
    sappx = sappxold - \frac{(gasmass(sappxold) - m)}{(gasdmds(sappxold))}; 
}

if(iteration >= MAX_ITERATIONS) 
    fprintf(stderr, "\n WARNING Failed to converge in solvegasmassfors m = %g s = %g m(s) = %g \n", m, sappx, gasmass(sappx));

if (sappx > Rhalo/Rvir) 
    { 
        sappx = Rhalo/Rvir; 
        fprintf(stderr, "\n WARNING returning a value outside truncation radius \n");
    } 

return sappx;

}

double gasmass(double s)
{
    double r = s*Rvir;
    if(Ngas > 0)
        if(s <= RADIUSFACTOR)
            return \frac{4.0\pi CentralGasRho(r - GasCoreRadius*atan(r/GasCoreRadius)))}{r^2};
        else
            return Ngas;
    else
        return 0.0;
}

double gasdmds(double s)
{
    double r = s*Rvir;
    double dmdr = 4.0*PI*SQR(r)*rhogas(s);
    double drds = Rvir;
    if(Ngas > 0)
        return dmdr*drds;
    else
        return 0.0;
}

double rho(double s)
{
    return \frac{k}{(s+a)^2};
}

double rhogas(double s)
{
    if(Ngas > 0)
        return CentralGasRho/(SQR(s*Rvir) + SQR(GasCoreRadius));
    else
        return 0.0;
}

double pot(double s)
{
    return \frac{k1*log(1.0+c*s)}{s} \ - \ potedge(EW);
}

double interpolatedpotential(double s)
{
    return (pot(s) + potgas(s));
}

double dpotds(double s)
{
    return k1*(c/(s+c*SQR(s)) - log(1.0+c*s)/SQR(s));
}
double potgas(double s)
{
    double r = s*Rvir;
    double rcoverr = GasCoreRadius/r;
    if(Ngas > 0)
    {
        if(s<=RADIUSFACTOR)
            return (4.0*PI*GRAVITY*CentralGasRho*(log(Rhalo/r) - PI*rcoverr/2.0 +
                rcoverr*atan(rcoverr) - log(1.0 + SQR(rcoverr))/2.0) - potedgegas);
        else
            return (GRAVITY*Mgas/r);
    }
    else
        return 0.0;
}

double dpotgasds(double s)
{
    double r = s*Rvir;
    if(Ngas > 0)
    {
        if(s<=RADIUSFACTOR)
            return (-GRAVITY*gasmass(s)/SQR(r));
        else
            return (-GRAVITY*Mgas/SQR(r));
    }
    else
        return 0.0;
}

void computederiv(double* x,double* y,double* dydx, int NR)
{

    /* Taken from IDL distribution deriv.pro */
    double x12,x01,x02;
    long n2 = NR-2;
    for(int i=1;i<NR-1;i++)
    {
        x12 = x[i] - x[i-1];
        x01 = x[i+1] - x[i];
        x02 = x[i+1] - x[i-1];
        dydx[i] = y[i+1]*x12/(x01*x02) + y[i]*(1.0/x12 - 1.0/x01) - y[i-1]*x01/(x02*x12);
    }
    x12 = x[1] - x[2];
    x01 = x[0] - x[1];
    x02 = x[0] - x[2];
    dydx[0] = y[0]*(x01+x02)/(x01*x02) - y[1]*x02/(x01*x12)+ y[2]*x01/(x02*x12);
    x12 = x[n2] - x[n2+1];
    x01 = x[n2-1] - x[n2];
    x02 = x[n2-1] - x[n2+1];
    dydx[NR-1] = -y[NR-3]*x12/(x01*x02) + y[NR-2]*x02/(x01*x12) - y[NR-1]*(x02+x12)/(x02*x12);
}

double getdf(double E,double* Table,double* Values,int NN)
{
    int low = 0,high = NN-1;
    int mid;
    double returnvalue=0.0;
    /* By the code convention, total energy is positive for bound objects*/
    if(E < 0.0)
{ /* fprintf(stderr,"WARNING : Unbound energies with E = %g re turning 0.0 from DF \n",E); */ return 0.0; }
while( low <= high ) {
    mid = ( low + high ) / 2;
    if( Table[mid] == E || (Table[mid-1] < E && Table[mid+1] > E)) {
        if(Table[mid] == E)
            return Values[mid];
        else {
            if(Table[mid] < E) {
                /* Weighted avg. of Table[mid] & Table[mid+1] */
                returnvalue = ((E-Table[mid])*Values[mid+1] + (Table[mid+1]-E)*Values[mid])/(Table[mid+1] - Table[mid]);
                break;
            } else {
                /* Weighted avg. of Table[mid] & Table[mid-1] */
                returnvalue = ((Table[mid]-E)*Values[mid-1] + (E-Table[mid-1])*Values[mid])/(Table[mid] - Table[mid-1]);
                break;
            }
        } else else if( E < Table[mid] )
            high = mid - 1;
        else
            low = mid + 1;
    }
    return returnvalue;
}

double getsforgivenmass(double m)
{
    int low = 0,high = NTAB-1;
    int mid;
    double returnvalue = 0.0;

    if(m < 0.0) {
        fprintf(stderr,"WARNING : NEGATIVE mass \n");
        return 0.0;
    }
    while( low <= high ) {
        mid = ( low + high ) / 2;
        if( MassTable[mid] == m || (MassTable[mid-1] > m && MassTable[mid+1] < m)) {
            if(MassTable[mid] == m)
                return stab[mid];
            else {
                if(MassTable[mid] < m) {
                    returnvalue = ((m-MassTable[mid])*stab[mid+1] + (MassTable[mid+1]-m)*stab[mid])/(MassTable[mid+1] - MassTable[mid]);
                    //Weighted avg. of Table[mid] & Table[mid+1]
                } else {
                    returnvalue = ((MassTable[mid]-m)*stab[mid] + (m-MassTable[mid-1])*stab[mid])/(MassTable[mid] - MassTable[mid-1]);
                    //Weighted avg. of Table[mid-1] & Table[mid]
                } }
        } else {
            high = mid - 1;
        } else {
            low = mid + 1;
        }
    }
    return returnvalue;
}
double mforgiven(double s)
{
    double gcs = 1.0/(log(1.0+c*s) - (c*s)/(1.0+(c*s)));
    return Mvir*gc/gcs;
}

void writeicfile(char* buf)
{
    FILE *fd;
    float dummy[3];
    int i,j,k;
    int blklen,masscount;
    // double a3inv;

    int Npart = All.TotNumPart;
    #define BLKLEN my_fwrite(&blklen, sizeof(blklen), 1, fd);
    if((fd=fopen(buf,"w")))
    {
        header1.npart[0]= header1.npartTotal[0]= All.TotN_gas;

        for(i=0;i<6;i++)
        {
            header1.mass[i]=0;

            for(i=0, masscount=0; i<5; i++)
            {
                header1.mass[i]= All.MassTable[i];
                if(All.MassTable[i]==0 || header1.npart[i]>0)
                { masscount += header1.npart[i];
                }

                if(masscount > 0)
                fprintf(stderr,"ln Apparently all particles of same type dont have the same mass \n"");

                header1.time= All.Time;

                if(All.ComovingIntegrationOn)
header1.redshift=1.0/All.Time - 1.0;
else
    header1.redshift=0;

header1.flag_sfr=0;
header1.flag_feedback=0;
header1.flag_cooling= 0;

header1.num_files= 1;
header1.BoxSize= All.BoxSize;
header1.Omega0= All.Omega0;
header1.OmegaLambda= All.OmegaLambda;
header1.HubbleParam= All.HubbleParam;

fprintf(stderr,"\n WARNING -- HEADER contains Newtonian space non-expanding universe \n");

blklen,sizeof(header1);

my_fwrite(&header1, sizeof(header1), 1, fd);

for(i=1;i<=Npart;i++)
{
    for(k=0;k<3;k++)
        dummy[k]=P[i].pos[k];
    my_fwrite(dummy,sizeof(float),3,fd);
}

for(i=1;i<=Npart;i++)
{
    for(k=0;k<3;k++)
        dummy[k]=P[i].vel[k];
    my_fwrite(dummy,sizeof(float),3,fd);
}

for(i=1;i<=Npart;i++)
    my_fwrite(&i,sizeof(int),1,fd);//id = i

if(Ngas > 0)
{
    blklen=Ngas*sizeof(float);
    for(i=1;i<=Ngas;i++)
    {
        dummy[0]=(float)InternalEnergy[i];
        my_fwrite(dummy,sizeof(float),1,fd);
    }
}

fclose(fd);
else
{
    fprintf(stderr,"Error. Can't write in file "buf", but); exit(0);
}
size_t my_fwrite(void *ptr, size_t size, size_t nmemb, FILE *stream)
{
    size_t nwritten;
    if((nwritten=fwrite(ptr, size, nmemb, stream))!=nmemb)
    {
        printf("I/O error (fwrite) on has occurred.\n");
        fflush(stdout);
        exit(0);
    }
    return nwritten;
}

void allocategas(void)
{

    if(!(gasrho=(double*)malloc(NGRID*sizeof(double))))
    {
        fprintf(stderr,"failed to allocate memory for gas rho.\n");
        exit(0);
    }

    if(!(pressure=(double*)malloc(NGRID*sizeof(double))))
    {
        fprintf(stderr,"failed to allocate memory for gas pressure.\n");
        exit(0);
    }

    if(!(gasdPdr=(double*)malloc(NGRID*sizeof(double))))
    {
        fprintf(stderr,"failed to allocate memory for gas potential derivative.\n");
        exit(0);
    }

    if(!(kT=(double*)malloc(NGRID*sizeof(double))))
    {
        fprintf(stderr,"failed to allocate memory for gas temperature.\n");
        exit(0);
    }

    if(!(InteriorMass=(double*)malloc(NGRID*sizeof(double))))
    {
        fprintf(stderr,"failed to allocate memory for gas temperature.\n");
        exit(0);
    }

    if(!(InternalEnergy=(float*)malloc((Ngas+1)*sizeof(float))))
    {
        fprintf(stderr,"failed to allocate memory for gas temperature.\n");
        exit(0);
    }

    void allocate(void)
    {
        if(!(stab=(double*)malloc(NTAB*sizeof(double))))
        {
            fprintf(stderr,"failed to allocate memory for s table.\n");
            exit(0);
        }
    }
exit(0);
}

if(!(MassTable=(double*)malloc(NTAB*sizeof(double))))
{
  fprintf(stderr,"failed to allocate memory for s table.\n");
  exit(0);
}

if(!(rhotab=(double*)malloc(NTAB*sizeof(double))))
{
  fprintf(stderr,"failed to allocate memory for rho table.\n");
  exit(0);
}

if(!(Etab=(double*)malloc(NTAB*sizeof(double))))
{
  fprintf(stderr,"failed to allocate memory for potential table.\n");
  exit(0);
}

if(!(dpotdstab=(double*)malloc(NTAB*sizeof(double))))
{
  fprintf(stderr,"failed to allocate memory for potential table.\n");
  exit(0);
}

if(!(drhodpsitab=(double*)malloc(NTAB*sizeof(double))))
{
  fprintf(stderr,"failed to allocate memory for drho/dpsi table.\n");
  exit(0);
}

if(!(d2rhodpsi2tab=(double*)malloc(NTAB*sizeof(double))))
{
  fprintf(stderr,"failed to allocate memory for d2rho/dpsi2 table.\n");
  exit(0);
}

if(!(fEtab=(double*)malloc(NTAB*sizeof(double))))
{
  fprintf(stderr,"failed to allocate memory for DF table.\n");
  exit(0);
}

/*
Should not have to touch anything below other than the form for
the function RHS.
*/

double rk4(double xmin,double xmax, double ymin,long nstep,int i,double (*functptr)(double,double,int))
{
  double k1,k2,k3,k4; /* R-K derivatives */
double xx; /* Independent variable */
double dx; /* Discretization scale (dx) */
double x_1, x_2; /* Limits of integration */
double yy; /* Solution */

  x_1 = xmin; /* lower limit of integration */
  x_2 = xmax; /* upper limit of integration */

  /* Initialization */
dx = (x_2-x_1)/nstep; /* step size */
xx = x_1; /* initial point */
yy = ymin;

/* Integrate */
if(xmax == xmin)
    return 0.0;
else
    for(int counter=0;counter<nstep-1;counter++)
    {
        /* First Step */
        k1 = (*functptr)(xx,yy,i) ; /* Derivative of the function at (x,y) */
        k2 = (*functptr)(xx+0.5*dx,yy+0.5*dx*k1,i);
        k3 = (*functptr)(xx+0.5*dx,yy+0.5*dx*k2,i);
        k4 = (*functptr)(xx+dx,yy+dx*k3,i);
        /* Second Step */
        xx += dx ; /* Update x to final step */
        yy += (dx/6.0)*(k1+2.0*k2+2.0*k3+k4) ; /* Update y */
    }
    return yy;
}

/**************************************************
* Function returns r.h.s. of dydx = f(x,y) *
**************************************************/
double RHS(double x, double y,int i)
{
    double xx,d2rhodpsi2;
    int high,low;
    high = (int)floor((log10(smax) - log10(x))/ds_DF) ;
    low = high + 1;
    d2rhodpsi2 = (fabs(x-stab[low])*d2rhodpsi2tab[high] + fabs(stab[high] - x)*d2rhodpsi2tab[low])/(stab[high] - stab[low]);
    xx = d2rhodpsi2/sqrt(Etab[i]-interpolatedpotential(x))*(dpotds(x)+dpotgasds(x));
    if(isnan(xx))
        fprintf(stderr,"NAN WARNING: i = %d s = %g Etab = %g sqrt term = %g potgas = %g
            ",i,x,Etab[i],Etab[i]-interpolatedpotential(x),potgas(x));
    return (xx);
}

//
/* Begin Mersenne Twist algorithm for random number generators*/
void init_genrand(unsigned long s)
{
    mt[0]= s & 0xffffffffUL;
    for (mti=1; mti<N; mti++) {
        mt[mti] =
            (1812433253ULL * (mt[mti-1] ^ (mt[mti-1] >> 30)) + mti);
        /* See Knuth TAOCP Vol2. 3rd Ed. P.106 for multiplier. */
        /* In the previous versions, MERS of the seed affect */
        /* only MERS of the array mt[]. */
        /* */
        /* 2002/01/09 modified by Makoto Matsumoto */
        mt[mti] &= 0xffffffffUL;
        /* for >32 bit machines */
    }
}

unsigned long genrand_int32(void)
{
    unsigned long y;
    static unsigned long mag01[2]={0x0UL, MATRIX_A};
    /* mag01[x] = x * MATRIX_A for x=0,1 */
if (mti >= N) { /* generate N words at one time */
    int kk;

    if (mti == N+1) /* if init_genrand() has not been called, */
        init_genrand(5489UL); /* a default initial seed is used */
    for (kk=0; kk < N-M; kk++) {
        y = (mt[kk] & UPPER_MASK) | (mt[kk+1] & LOWER_MASK);
        mt[kk] = mt[kk+M] ^ (y >> 1) ^ mag01[y & 0x1UL];
    }  
    for (; kk < N-1; kk++) {
        y = (mt[kk] & UPPER_MASK) | (mt[kk+1] & LOWER_MASK);
        mt[kk] = mt[kk+(M-N)] ^ (y >> 1) ^ mag01[y & 0x1UL];
    }
    y = (mt[N-1] & UPPER_MASK) | (mt[0] & LOWER_MASK);
    mt[N-1] = mt[M-1] ^ (y >> 1) ^ mag01[y & 0x1UL];
    mti = 0;
    }

    y = mt[mti++];
    /* Tempering */
    y ^= (y >> 11);
    y ^= (y << 7) & 0x9d2c5680UL;
    y ^= (y << 15) & 0xefc60000UL;
    y ^= (y >> 18);
    return y;
}

/* generates a random number on [0,1]-float-interval */
double genrand_float(void)
{
    return genrand_int32()*(1.0/4294967295.0);  /* divided by 2^32-1 */
}

double frand(float lower, float upper)
{
    return (lower + genrand_float()*(upper-lower));
}

// /* End Mersenne Twist algorithm for random number generators*/
//
Appendix B

GADGET-2 parameter file

InitCondFile /home/sinham/scratch/mergers/gas/gasfractions/0.18/Primary/impactparam/0.01/3RvirPointZeroOneImprim8pergas.dat.intel
OutputDir /home/sinham/scratch/mergers/gas/gasfractions/0.18/Primary/impactparam/0.01/
EnergyFile energy.txt
InfoFile info.txt
TimingsFile timings.txt
CpuFile cpu.txt
RestartFile restart
SnapshotFileBase snapshot
OutputListFilename parameterfiles/output_list.txt
TimeLimitCPU 2.88e+06
ResubmitOn 0
ResubmitCommand my-scriptfile
ICFormat 1
SnapFormat 1
ConovingIntegrationOn 0
TypeOfTimestepCriterion 0
OutputListOn 0
PeriodicBoundariesOn 0
TimeBegin 0
TimeMax 100
Omega0 0
OmegaLambda 0
OmegaBaryon 0
HubbleParam 0
BoxSize 0
TimeBetSnapshot 0.15
TimeOfFirstSnapshot 0.001
CpuTimeBetRestartFile 36000
TimeBetStatistics 0.005
NumFilesPerSnapshot 1
NumFilesWrittenInParallel 1
ErrToInTaccuracy 0.025
CourantFac 0.15
MaxSizeTimeStep 0.1
MinSizeTimeStep 0
ErrToInTheta 0.5
TypeOfOpeningCriterion 1
ErrToInForceAcc 0.005
TreeDomainUpdateFrequency 0.1
DesNumRho 50
MaxNumNghDeviation 2
ArtBulkViscConst 0.5
InitGasTemp 0
MinGasTemp 0
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>PartAllocFactor</td>
<td>2</td>
</tr>
<tr>
<td>TreeAllocFactor</td>
<td>1.5</td>
</tr>
<tr>
<td>BufferSize</td>
<td>25</td>
</tr>
<tr>
<td>UnitLength_in_cm</td>
<td>$3.08568 \times 10^{21}$ $\kappa$pc</td>
</tr>
<tr>
<td>UnitMass_in_g</td>
<td>$1.989 \times 10^{43}$ $10^{10}$ Msun</td>
</tr>
<tr>
<td>UnitVelocity_in_cm_per_s</td>
<td>100000 $1$ km/s</td>
</tr>
<tr>
<td>GravityConstantInternal</td>
<td>0</td>
</tr>
<tr>
<td>MinGasNumFractional</td>
<td>0</td>
</tr>
<tr>
<td>SofteningGas</td>
<td>1.14</td>
</tr>
<tr>
<td>SofteningHalo</td>
<td>1.14</td>
</tr>
<tr>
<td>SofteningDisk</td>
<td>0</td>
</tr>
<tr>
<td>SofteningBulge</td>
<td>0</td>
</tr>
<tr>
<td>SofteningStars</td>
<td>0</td>
</tr>
<tr>
<td>SofteningHndry</td>
<td>0</td>
</tr>
<tr>
<td>SofteningGasMaxPhys</td>
<td>1</td>
</tr>
<tr>
<td>SofteningHaloMaxPhys</td>
<td>1</td>
</tr>
<tr>
<td>SofteningDiskMaxPhys</td>
<td>0</td>
</tr>
<tr>
<td>SofteningBulgeMaxPhys</td>
<td>0</td>
</tr>
<tr>
<td>SofteningStarsMaxPhys</td>
<td>0</td>
</tr>
<tr>
<td>SofteningHndryMaxPhys</td>
<td>0</td>
</tr>
<tr>
<td>MaxRMSDisplacementFac</td>
<td>0.2</td>
</tr>
</tbody>
</table>
Bibliography


Alpher, R. A. & Herman, R. 1948, Nature, 162, 774

Babcock, H. W. 1939, Lick Observatory Bulletin, 19, 41

Balsara, D. S. 1995, Journal of Computational Physics, 121, 357

Barnes, J. & Hut, P. 1986, Nature, 324, 446


Burles, S. & Tytler, D. 1998, Space Science Reviews, 84, 65


Copi, C. J., Schramm, D. N., & Turner, M. S. 1995, Science, 267, 192

Courant, R., Friedrichs, K., & Lewy, H. 1928, Mathematische Annalen, 100, 32

—. 1967, IBM J., 11, 215


Jeans, J. H. 1919, Problems of cosmogony and stellar dynamics (Cambridge, University press, 1919.)
—. 1928, Astronomy and cosmogony (Cambridge [Eng.] The University press, 1928.)


Schweizer, F. 1986, Science, 231, 227


Wadsley, J. W., Stadel, J., & Quinn, T. 2004, New Astronomy, 9, 137


Zwicky, F. 1933, Helvetica Physica Acta, 6, 110
Vita
Manodeep Sinha

Education

**Ph.D.** Astronomy and Astrophysics – The Pennsylvania State University 2002-08
Thesis: Numerical Simulations of Hot Halo Gas in Galaxy Mergers
Advisor: Robin Ciardullo


Experience

Research 08/2002 – Present

*Pennsylvania State University & Vanderbilt University*
Numerical Simulations of Hot Halo Gas in Galaxy Mergers


*Pennsylvania State University*
Taught intro-level Astro 11 for non-majors

Other 01/2002 – 04/2002

Visiting Students Research Program
*Centre for Theoretical Studies,*
*Indian Institute of Technology, Kharagpur*

Publications

**Refereed**

*Hot Halo Gas in Galaxy Mergers.* **Manodeep Sinha** & Kelly Holley-Bockelmann, in prep., 2008

*Bending of Light and Gravitational Signals in Certain On-Brane and Bulk Geometries.* Sayan Kar & **Manodeep Sinha**, 2003, General Relativity & Gravitation, 35, 10