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RESOURCE MANAGEMENT

FOR WIRELESS AD HOC NETWORKS

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by

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Abstract

Wireless ad hoc networking is an emerging technology for next generation wireless communication systems due to its ability of providing communication without the need for infrastructure. In such networks, the system resources are limited: the terminals are constrained in energy, bandwidth, storage, and processing capabilities. As a result of the decentralized and possibly large scale network structure, resource management becomes a prominent design issue for wireless ad hoc networks. In this dissertation, we investigate resource management, with an emphasis on power, for various ad hoc networks including wireless sensor networks, parallel relay networks and multiuser two-way relay networks. We aim at providing efficient communication strategies that will fully utilize the limited resources to improve the Quality of Service (QoS) that ad hoc networks can offer.

We first study efficient scheduling for a delay-constrained wireless sensor network. We consider a two-tiered model of clustered sensors, whose data collection includes intra- and inter-cluster communications. We first find the optimum schedule with polynomial complexity for intra-cluster communications, which minimizes the total power of all sensors while maintaining a short term average throughput at each sensor. Next, we show that the proposed scheduling protocol provides a near-optimum solution for inter-cluster communications where multi-antenna technique is employed.

Next, we consider resource management for two types of relay-assisted ad hoc networks: parallel relay networks where the communication from a source to a destination is assisted by a number of intermediate relay nodes over orthogonal channels; multiuser

two-way relay networks where multiple pairs of users exchange information with their pre-assigned partners via the help of a single relay node. Both relay networks in consideration are the building blocks of the future ad hoc networks.

For the parallel relay networks, we first propose a distributed decision mechanism for each relay to make decisions on whether to forward the source data. Next, we identify the optimum power allocation strategy, based on limited channel state information, that minimizes the total transmit power while providing a target signal-to-noise ratio (SNR) at the destination with an outage probability constraint. In addition, We consider two simpler distributed power allocation models, where the source does not contribute to the relay selection in the first one, and single relay is employed in the second one.

For the multiuser two-way relay networks, frequency/time/code division multiple access (F/T/CDMA) techniques can be employed to support multiuser communications. When F/TDMA is employed to provide orthogonal channels for each user pair, we investigate the optimum relay power allocation problem for various relay forwarding mechanisms to maximize the arbitrary weighted sum rate of all users. When CDMA is employed which results in an interference limited system, we design the multiuser two-way relaying scheme that allows each pair of partners to share a common CDMA spreading signature, and solve the interference management problem by constructing the iterative power control and receiver updates that converge to the corresponding unique optimum.

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Chapter 1

Introduction

With the rapidly increasing demand for efficient and reliable communications among mobile wireless terminals, wireless ad hoc networks have attracted attention due to their infrastructure-less nature [1–3]. A wireless ad hoc network consists of a set of nodes that are organized and maintained in a distributed manner. The interests in such networks arise from their well-known advantages, such as quick deployment, self-configuration, self-healing, scalability, flexibility and robustness. With this inherent flexible structure, ad hoc networks appear appealing for various applications such as control systems, home applications, distributed data collection and exchange, communications in emergency as well as military services [4, 5].

The impairments of the wireless channel, such as path loss, shadowing and multipath fading, as well as the significant resource constraints such as limited battery energy, communication bandwidth, memory and computational capacity, create serious design challenges on wireless ad hoc networks [6]. A significant effort has been devoted to the study of *resource management* for ad hoc networks at various layers of the network stack. These efforts include power allocation, bandwidth allocation, scheduling, routing as well as design of new transmission strategies; all aiming at taking full advantage of the network resources to combat the channel impairments, so that to maximize the Quality of Service (QoS) and/or the lifetime of the networks [7].

Among various resources, transmit power is a fundamental one in wireless ad hoc networks, due to the small size and the limited battery life of the scattered terminals. The main objective of many communication protocols proposed for wireless ad hoc networks is to minimize the power consumption and maintain the overall connection quality, while performing their specific tasks [5, 8, 9]. Our aim in this dissertation is to concentrate on power as the fundamental resource and look for practically implementable algorithms for utilizing this resource as well as related radio resources in various ad hoc network settings, including wireless sensor networks, parallel relay networks and multiuser two-way relay networks, the later two of which are the building blocks of the ad hoc networks in the near future.

1.1 Wireless Sensor Networks

A wireless sensor network consists of a number of sensors deployed across a geographical area for various implementation purposes [5]. Each sensor has wireless communication capability and some level of intelligence for signal processing and networking of the data. The features of wireless sensor networks include collaborative signal processing involving data querying from the end users and data fusion from multiple sensors [10], fairness among sensors [11] and strict time requirements in some applications of emergency detection [12].

Scheduling is a way of managing resources, and plays an important role in efficient data collection and network lifetime maximization [13, 14]. As code division multiple access (CDMA) technology has recently been applied to wireless sensor networks to support applications with high bandwidth and strict latency requirements [15,16], careful coordination of transmissions is needed for CDMA wireless sensor networks as well, with emphasis on battery efficiency and delay requirement. Scheduling for CDMA systems have been studied intensively [17, 18], however, existing protocols cannot be directly applied to CDMA wireless sensor networks since fairness in terms of the throughput of each terminal is not considered. This motivates us to find efficient schedules that will provide not only the efficient reliable communication but also a short term average throughput guarantee at each sensor for CDMA wireless sensor networks.

In the first part of this dissertation, we investigate the efficient scheduling and the resulted power allocation problem for a delay-constrained CDMA wireless sensor network, which is modeled as a two-tiered network whose data collection consists of intra- and inter-cluster phases. Our aim is to schedule the sensor transmissions into a given number of time slots, such that the total transmission power is minimized while the system QoS requirement, specifically, the signal-to-interference-plus-noise ratio (SINR) target, is satisfied. We first show that the scheduling problem at the intra-cluster level is polynomially solvable by a shortest path algorithm. Next, we propose a near-optimum scheduling solution with polynomial complexity for the inter-cluster communications where multiple antennas are employed to provide transmit diversity.

1.2 Parallel Relay Networks

In the past decade, considerable research effort has focused on exploiting space diversity by using multiple antennas at the transmitters and the receivers. It has been shown that space diversity can significantly increase spectral efficiency and mitigate the effects of fading [19, 20]. However, practical issues limit the implementation of multiple antennas on the terminals in wireless ad hoc networks. Relay-assisted transmission schemes have thus become a prominent candidate for combating the wireless channel in ad hoc networks, due to their potential of providing powerful benefits of space diversity without the need of physical antenna arrays [21]. An intensive research effort has been dedicated to investigating the transmission strategies and the performance of cooperative relaying techniques [22–29]. References so far have shown that, relay assistance can provide performance improvement in terms of outage behavior [24, 25], achievable rate region [22, 23, 28, 29], and error probability [26, 27, 30, 31].

Power efficiency is a critical design consideration for relay-assisted wireless ad hoc networks. To that end, as part of resource management, choosing the appropriate relays to forward the source data, as well as allocating the power among all terminals, become important design issues. Optimum power allocation strategies for relay networks are studied up-to-date for several network structures and relay transmission schemes [30, 32–38]. These proposed power management schemes result in power efficient transmissions that achieve attractive performance. However, the implementations of these algorithms require either the destination or the source to have full side information, such as the channel state information (CSI) of all communication links and the topology of the whole network. Such centralized solutions may not be implementable in wireless ad hoc networks, especially in the networks with a large number of terminals where collecting the detailed status information from individual terminals will result in unacceptable large overhead and delay.

Motivated by the advantages of the relay-assisted transmissions and the decentralized nature of the wireless ad hoc networks, in the second part of this dissertation, we aim to design the *distributed* resource management strategies for relay-assisted wireless ad hoc networks. Specifically, we study a two-hop parallel relay network where there is a source-destination pair and a set of decode-and-forward relay nodes in between. Relay nodes devote all of their resources to assist the traffic between the source and the destination. The source and the relay nodes operate in orthogonal channels. We assume that only partial CSI is available at the source and the relay nodes, which is more practical in wireless ad hoc networks. We first propose a distributed decision mechanism for each relay node to individually make a decision on whether to forward the source data. Secondly, We solve the distributed power allocation problem that aims at minimizing the expected value of the total transmit power while providing the target signal-to-noise ratio (SNR) at the destination with an outage probability constraint. In addition, we consider two special cases with simpler implementation, namely the *passive source model* where the source does not contribute to the relay selection process, and the *single relay model* where one relay node is selected to forward the source data based on limited CSI. For each case, we optimize the respective relevant parameters.

1.3 Multiuser Two-Way Relay Networks

While the relay-assisted communication has recently become a promising technique in supporting wireless ad hoc networks, it may incur an inherent loss in spectral efficiency due to the half-duplex constraint at the relay node in practical systems [22–24], i.e., the transmission via the relay node may cost additional resources in time or frequency domain because it is practically difficult for the relay node to receive and transmit simultaneously.

When there are two users a and b wishing to exchange independent information with each other via the relay node r, the traditional relay-assisted transmission needs four phases to accomplish the communication, i.e., $a \rightarrow r$, $b \rightarrow r$, $r \rightarrow a$ and $r \rightarrow b$. Enabled by the recently emerged network coding techniques [39], the spectral efficiency can be significantly improved through *two-way* (or *bidirectional*) relaying [40–43]. Specifically, the transmissions in the third and fourth phases can be combined into a single transmission by taking advantage of the broadcast nature of the wireless communications and the bi-directional communication structure. Knowing the side information which is its own transmitted signal, each user is able to recover its partner's information from the common signal broadcasted by the relay. A number of protocols for two-way relay channels have been proposed that outperform the traditional four-phase relaying communications in terms of achievable rates and power efficiency [44–53].

While most work on two-way relaying has focused on single pair of partners, two-way relaying is naturally expected to improve the system performance in multiuser scenarios, especially in ad hoc networks. Wireless ad hoc networks of the near future are most likely to consist of many nodes wishing to exchange information, potentially having to share intermediate relays. To that end, in the third part of this dissertation, we propose a *multiuser two-way relay network* where a single relay node assists communications between multiple user pairs. To support multiple users, several multi access techniques can be employed including frequency, time or code division multiple access (F/T/CDMA). While F/TDMA enable communication over orthogonal channels with the expense of bandwidth or delay, CDMA with non-orthogonal spreading signatures results in an interference-limited system. We investigate the resource management in both scenarios.

When F/TDMA techniques are employed, the relay's resources, most notably, its power, need to be appropriately distributed among users whose data exchange it shall aid. Therefore, for a variety of existing two-way relaying schemes, we investigate the problem of optimally allocating the relay's power among all the user pairs it assists such that an arbitrary weighted sum rate of all users is maximized. We show that each problem can be converted to one or a set of convex problems, and can be solved via convex optimization techniques as well as an iterative algorithm we develop to show how the power allocation is affected by the channel gain of different users and the amount of the available relay power. The resulted weighted sum rates with all arbitrary weights allow us to trace the boundary of the achievable rate region of the multiuser two-way relay network with various relaying strategies.

When CDMA is employed, our focus is in establishing the resource sharing in terms of CDMA spreading sequences and the consequent interference management problem. *Interference management* herein refers to the reduction and control of the interference experienced by each end user via careful choice of the *relaying scheme* as well as the *transmit and receive strategies*, so as to optimize system performance. We first propose that each pair of users share a common spreading signature, and design a jointly demodulate-and-XOR-forward (JD-XOR-F) relaying scheme where all users transmit to the relay in phase one, followed by the relay jointly demodulates and generates an estimate of the XORed symbol to broadcast for each user pair in phase two. Next, we investigate the interference management problem via joint power control and receiver optimization for each phase, and construct the corresponding iterative algorithms, each converging to its unique optimum. The proposed JD-XOR-F relaying with interference management is observed to provide significant power savings and double the system user capacity, i.e., the maximum number of users that can be simultaneously supported, over the designs with a "one-way" communication perspective.

1.4 Dissertation Road Map

First, we consider the efficient scheduling for delay-constrained CDMA wireless sensor networks in Chapter 2. Next, we investigate the distributed power allocation for parallel relay networks in Chapter 3. Third, we proposed the multiuser two-way relay network, examine the relay power allocation problem when F/TDMA is employed in Chapter 4, and study the interference management when CDMA is employed in Chapter 5. Finally, we conclude the dissertation and present future directions in Chapter 6.

Chapter 2

Efficient Scheduling for Delay-Constrained CDMA Wireless Sensor Networks

2.1 Introduction

Efficient transmission strategies are of great interest in wireless sensor networks (WSNs) due to the limited battery resources of the sensor nodes [10]. Scheduling plays an important role in efficient data collection and network lifetime maximization by coordinating the sensor data transmissions in WSNs [13,14]. As code division multiple access (CDMA) technology has recently been applied to WSNs to support applications with high bandwidth and strict latency requirements [15, 16], careful coordination of transmissions are needed for CDMA WSNs as well with emphasis on battery efficiency and delay requirement. Given the fact that in many WSNs, fairness among sensor nodes is a critical design issue [11], existing scheduling protocols for CDMA systems [17,18] cannot be directly applied to CDMA WSNs since fairness in terms of the throughput of each node is not considered. This motivates us to find the efficient schedule that will provide not only the efficient reliable communication but also a short term average throughput guarantee at each sensor for CDMA WSNs.

In this chapter, we investigate efficient scheduling and the resulted power allocation problem for a delay-constrained CDMA WSN, which is modeled as a two-tiered network shown in Figure 2.1. The tiered network structure is preferred especially in

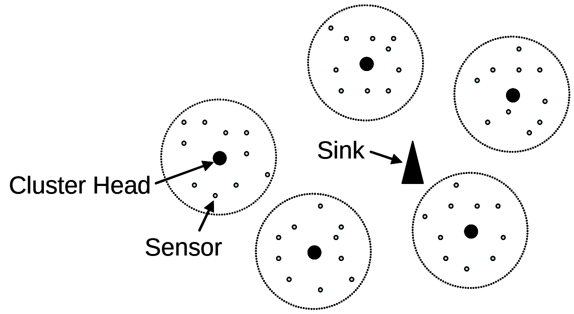

Figure 2.1. A two-tiered wireless sensor network model.

large-scale WSNs due to the advantages such as simpler logic functions on sensor nodes, easier management of the network, and longer system lifetime [54, 55]. The data collection includes two phases, *intra-cluster* collection at each cluster head (CH) from sensors belonging to that cluster, and *inter-cluster* collection at the sink from all CHs. Specifically, we consider a multi-rate CDMA WSN facilitated by the aid of multiple codes. Multiple codes belonging to each node become *virtual* nodes, and will create interference for each other if they transmit at the same time. Our aim is to schedule the virtual nodes into a given number of time slots with equal duration, such that the total transmit power is minimized, while the signal-to-interference-plus-noise ratio (SINR) target is satisfied at all CHs and the sink.

The scheduling problem looks similar to the bin packing problem which is NP-complete [56], fortunately, the specifics of the intra-cluster communications enables it polynomially solvable by a shortest path algorithm. Next, we investigate the scheduling problem for inter-cluster communications when each CH employs the Alamouti scheme to achieve the transmit diversity (TD). We show that the proposed scheduling strategy with polynomial complexity provides a near-optimal solution in such case. We observe that considerable power savings can be obtained by the proposed schemes with respect to the time division multiple access (TDMA)-type scheduling scheme, which schedules nodes in a round robin fashion, i.e., one node transmitting in one slot.

The organization of this chapter is as follows. The system model is described in Section 2.2. We develop the power efficient scheduling algorithms for intra-cluster and inter-cluster communications in Section 2.3 and 2.4, respectively. Numerical results are presented in Section 2.5, and Section 2.6 concludes the chapter.

2.2 Two-Tiered CDMA WSN Model and Assumptions

We consider a WSN consisting of a data sink and K_c clusters. Each cluster includes K sensor nodes equipped with single antenna due to the size and cost limitations, and a CH which is the device of larger size and more power that can be equipped with two antennas and apply Alamouti space-time coding [57] to achieve transmit diversity. We assume passive clusters waiting for data queries from the sink, which is a common approach [58]. When the clusters are triggered by a query, the data collection starts, and all nodes are synchronized by the trigger signal from the sink. It involves two consecutive phases Ph_1 and Ph_2, consisting of a frame of n and m time slots, respectively. All intra-cluster communications simultaneously happen in Ph_1, when each CH collects data from sensors belonging to the same cluster. By the end of Ph_1, CHs complete the local sensor data aggregation and processing. Next, the inter-cluster communications proceed in Ph_2, when CHs transmit the processed data to the sink. We assume all slots have equal duration.

We consider a multi-rate CDMA WSN where each node (sensor node as well as CH) may change its transmission rate by the number of codes it uses in each slot, but maintains the required average rate in a frame. Multiple codes are considered as *virtual* nodes, and interfere with each other if they transmit in the same slot. The spreading codes are assumed to be randomly generated signature sequences.

We assume all channels are quasi-static with flat fading, i.e., the fading coefficients remain constant during a frame. Given the fact that in many applications, each cluster would be deployed at a strategic location, we can safely assume that the clusters are sufficiently far away from each other. Thus, rather than considering a schedule over multiple clusters, we assume that the inter-cluster interference is included in the noise term and concentrate on each cluster. We also assume that the power levels of sensor nodes are much lower as compared to CHs, a reasonable assumption in light of the fact that CHs are considered to be able to communicate over longer distances to the sink, and this enables us to consider each tier separately. Having this two-tiered WSN model, we next address the efficient scheduling problem for both intra- and inter-cluster communications.

2.3 Scheduling for Intra-Cluster Communications

In this section, we investigate the scheduling protocol for the communications in one cluster, which would be implemented in each cluster in Ph_1.

2.3.1 Problem Formulation

We consider a cluster where K sensor nodes communicate with a CH. Let g_i denote the channel fading coefficient of the ith sensor node to the CH, for $i = 1, ..., K$, and σ^2 denote the variance of the additive white Gaussian noise (AWGN) term at the CH. The CH employs matched filters to decode the sensors' data from the received signals. The SINR of the kth virtual sensor of node i in time slot l is defined as

$$SINR_{i_k l} = \frac{N p_{i_k l} |g_i|^2}{(\sum_{j=1}^{K} \sum_{j_k=1}^{K_{jl}} p_{j_k l} |g_j|^2 - p_{i_k l} |g_i|^2) + I} \tag{2.1}$$

where $p_{j_k l}$ denotes the transmit power of the kth virtual sensor of the jth node in the lth slot, K_{jl} denotes the number of virtual sensors of node j in slot l, for $j = 1, ..., K$ and $l = 1, ..., n$, N denotes the processing gain, and $I = N\sigma^2$. The average throughput of node i, R_i, during the frame of n slots is

$$R_i = \sum_{l=1}^{n} \frac{K_{il} \times R_{\text{base}}}{n} \tag{2.2}$$

where $R_{\text{base}} = W/N$ is the rate of a virtual node in one slot, and W is the spreading bandwidth.

We aim to minimize the total power expenditure of all sensors belonging to this cluster in n slots, while satisfying the received SINR target, $SINR_{\text{target}}$, for each virtual node in each slot and the short term throughput requirement, $R_{i_{\text{target}}}$ for node i, $i =$

$1, ..., K$. The optimization problem can be expressed as

$$\min_{\{K_{il}, p_{i_k l}\}} \quad \sum_{l=1}^{n} \sum_{i=1}^{K} \sum_{i_k=1}^{K_{il}} p_{i_k l} \tag{2.3}$$

$$\text{s. t.} \quad SINR_{i_k l} \geq SINR_{\text{target}}, \quad \forall i_k, l \text{ such that } p_{i_k l} > 0 \tag{2.4}$$

$$R_i = R_{i_{\text{target}}}, \quad \forall i \tag{2.5}$$

$$p_{i_k l} \geq 0, \quad \forall i_k, l. \tag{2.6}$$

We note that the optimum received power for each virtual sensor is achieved when the SINR constraint in (2.4) is satisfied with equality [59]. Thus, the optimum received power for each virtual sensor in slot l is

$$q_l^* = \frac{I\gamma}{(1+\gamma) - |s_l|\gamma} \tag{2.7}$$

where $\gamma = SINR_{\text{target}}/N$, s_l denotes the set of virtual sensors scheduled in slot l, and $|s_l| = \sum_{i=1}^{K} K_{il}$. Note that the maximum number of virtual sensors in a slot is limited by $\lfloor (1+\gamma)/\gamma \rfloor$ due to the fact that $q_l^* \geq 0$.

Given the relation between the optimum transmit and received power, $p_{i_k l}^* |g_i|^2 = q_l^*$, the problem in (2.3)–(2.6) can be rewritten as

$$\min_{\{K_{il}\}} \quad \sum_{l=1}^{n} q_l^* \sum_{i=1}^{K} \frac{K_{il}}{|g_i|^2} \tag{2.8}$$

$$\text{s. t.} \quad \sum_{l=1}^{n} K_{il} = \frac{nR_{i_{\text{target}}}}{R_{\text{base}}} \quad \forall i. \tag{2.9}$$

The problem in (2.8)–(2.9) is to find K_{il}, the number of virtual sensors of node i in time slot l, for $i = 1, ..., K$ and $l = 1, ..., n$, to minimize the total transmit power in n slots, while node i has $T_i = \frac{nR_{i_{\text{target}}}}{R_{\text{base}}}$ virtual sensors in n slots.

2.3.2 Optimum Schedule

In this section, we provide the solution to the optimization problem in (2.8)–(2.9). First, we have two observations which give the structure of the optimum scheduling policy.

OBSERVATION 2.1. *The optimum policy always schedules a virtual sensor with a lower channel gain to a slot with a lighter load, i.e., a slot with fewer virtual sensors.*

To see the validity of observation 2.1, we suppose that two virtual sensors j and i are scheduled to slot 1 and 2, respectively, with $|g_i|^2 > |g_j|^2$, and $|s_1| > |s_2|$. If we exchange i and j between the two slots, all the virtual sensors except i and j remain the same transmit power level, since q_1^* and q_2^* remain the same. However, the sum of the transmit power of i and j is decreased, i.e., $\frac{q_1^*}{|g_i|^2} + \frac{q_2^*}{|g_j|^2} < \frac{q_1^*}{|g_j|^2} + \frac{q_2^*}{|g_i|^2}$. Hence, the total transmit power is decreased.

Observation 2.1 provides a valuable clue as to the structure of the optimum schedule. Note that, the collection of virtual sensor sets resulting from *any* scheduling policy can be reordered as $\{s_1, s_2, ..., s_n\}$, such that $|s_1| \leq |s_2| \leq ... \leq |s_n|$, $|s_l| \neq 0$ for $l \in \{1, ..., n\}$, and $\sum_{l=1}^{n} |s_l| = T$. This reordering of virtual sensor sets does not change the total transmit power. Therefore, we only need to find the optimum solution with the reordered virtual sensor sets. Next, we have the following observation.

Observation 2.2. *For any given group of reordered virtual sensor sets, the optimum scheduling order of T virtual sensors is in the order of increasing channel gain, i.e.,*

$$\underbrace{|g_K|^2, ..., |g_K|^2}_{T_K}, \underbrace{|g_{K-1}|^2, ..., |g_{K-1}|^2}_{T_{K-1}}, ..., \underbrace{|g_1|^2, ..., |g_1|^2}_{T_1} \tag{2.10}$$

where $|g_K|^2 \leq |g_{K-1}|^2 \leq ... \leq |g_1|^2$.

Note that the scheduling order in (2.10) satisfies Observation 2.1. Hence, the optimization problem in (2.8)–(2.9) is to find the best group of reordered virtual sensor sets such that the sum of the total transmit power is minimized, given the optimum scheduling order as in (2.10). By appropriate transformation, this problem can be formulated as a graph partitioning problem with polynomial complexity, as described next.

The reordered virtual sensor sets in (2.10) constitute a string $G = (V, E)$ with vertices $V = \{v_1, v_2, ..., v_T\}$ and edges $E = \{(v_1, v_2), ..., (v_{T-1}, v_T)\}$, by sequentially mapping each virtual sensor to the vertex along the string from the left to the right. Given the string G, the virtual sensor sets $\{s_1, s_2, ..., s_n\}$ represent the partition of the set of vertices V into n subsets, with each subset s_l consisting of a set of connected vertices. The cost of a virtual sensor set s_l is $q_l^* \sum_{i \in s_l} \frac{1}{|g_i|^2}$. Therefore, the optimization problem in (2.8)–(2.9) is equivalent to finding a feasible n-partition such that the total cost is minimized, i.e.,

$$\min_{\{s_1, ..., s_n\}} \quad \sum_{l=1}^{n} q_l^* \sum_{i \in s_l} \frac{1}{|g_i|^2} \tag{2.11}$$

$$\text{s. t.} \quad |s_1| \leq |s_2| \leq ... \leq |s_n|. \tag{2.12}$$

We note that, although optimum partitioning an arbitrary graph with an arbitrary cost function is NP-hard, optimum partitioning a string with a separable cost function can be achieved in polynomial time, i.e., the problem in (2.11)–(2.12) is reduced to a shortest path problem with complexity $O(nT^2)$ [60]. The solution is described in the following.

We construct a network from the string G which represents the ordered virtual sensors. The nodes that lie between the origin-destination pair are given by the set

$$\{(i,j) : 1 \leq i \leq n-1; i \leq j \leq T-n-1\}. \tag{2.13}$$

An edge is placed from node $(i1, j1)$ to node $(i2, j2)$ if $i2 = i1+1$ and $j2 > j1$. Otherwise, there is no edge between $(i1, j1)$ and $(i2, j2)$. There is a one-to-one mapping between the cost function of a feasible partition in a string, and that of a path in the network constructed from the string. For node (i, j), i and j denote the index of the time slot and the index of the virtual sensor, respectively. The cost of a path between nodes $(l-1, t)$ and $(l, t+x)$ is the transmit power cost of the virtual sensor set s_l, i.e., $q_l^* \sum_{i \in s_l} \frac{1}{|g_i|^2}$, where $x = |s_l|$. Note that the optimum policy should satisfy $|s_1| \leq |s_2| \leq ... \leq |s_n|$, and $|s_l| \leq \lfloor (1+\gamma)/\gamma \rfloor$ for any l. If a path violates any of these two constraints, the cost of the path is set infinite, i.e., the path is infeasible. In Figure 2.2, we present a network constructed from a 5-vertex string with 3 partition sets, i.e., $T = 5$ and $n = 3$. Next, a shortest path from the origin to the destination with minimum cost is obtained by a shortest path algorithm such as Dijkstra's algorithm with complexity $O(nT^2)$. The resulting optimum partition $\{s_1, s_2, ..., s_n\}$ provides the optimum schedule with K_{il}, for $i = 1, ..., K$and $l = 1, ..., n$.

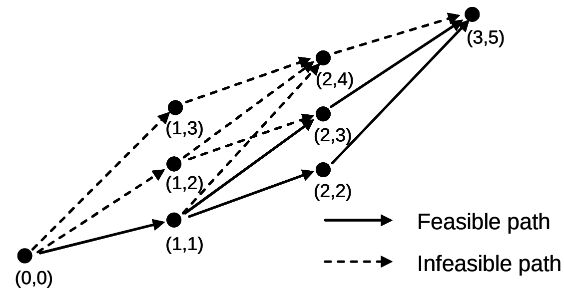

Figure 2.2. A network constructed by a 3-partitioning of a 5-vertex string.

2.4 Scheduling for Inter-Cluster Communications

In this section, we investigate the efficient scheduler for the inter-cluster communications in phase Ph_2. As assumed in Section 2.2, when the CHs are nodes of larger size that have more complicated hardware and more processing capacity, it is feasible to have each CH be equipped with two transmit antennas and employ Alamouti scheme [57] to exploit the transmit diversity.

2.4.1 Transmission Scheme of CHs with TD

We first present the transmission scheme of the CHs employing the Alamouti scheme. Contrary to [61] which studies the space-time spreading scheme for systems with orthogonal spreading codes, we assume here a CDMA WSN with non-orthogonal spreading codes. We consider the WSN with K_c CHs, and m time slots in Ph_2. The ith CH cooperatively communicates zero-mean independent signals s_{i1} and s_{i2} from two antennas with the sink in two time slots. The transmission scheme of CH i is shown in Figure 2.3. We assume that both antennas of CH i have the same transmit power level,

		slot m_1	slot m_2
CH i	Antenna 1 (g_{i1})	$s_{i1}c_i$	$-s_{i2}^{*}c_i$
	Antenna 2 (g_{i2})	$s_{i2}c_i$	$s_{i1}^{*}c_i$

Figure 2.3. Transmission scheme of CH i with Alamouti scheme.

i.e., $p_{i1} = p_{i2} = p_i$, so that the total power is $2p_i$. The channel fading coefficients of the antenna 1 and 2 of CH i are denoted by g_{i1} and g_{i2}, respectively, and c_i denotes the randomly generated spreading code of CH i, for $i = 1, ..., K_c$. We next investigate the efficient scheduling protocol for inter-cluster communications.

2.4.2 Scheduling for CHs Transmissions

In this section, we provide the solution to the problem that schedules the transmissions of K_c CHs with TD into m time slots. We define the simultaneous transmissions of the signals s_{i1} and s_{i2} from CH i's antenna 1 and 2 as a *super transmission* TX_i, and the simultaneous transmissions of the conjugate signals, i.e., s_{i1}^* and $-s_{i2}^*$ as a *super transmission* TX_i^*, $i \in \{1, ..., K_c\}$. Since each CH has two super transmissions, each taking one slot, there are $2K_c$ super transmissions to be scheduled into m slots. We have two observations showing that with some scheduling constraints, the optimum scheduling protocol proposed in Section 2.3 is readily applicable to the scheduling problem for the inter-cluster transmissions.

Observation 2.3. *The super transmission of CH i, TX_i, should not be scheduled into the same time slot with the super transmission TX_j^* of CH j, for $i, j \in \{1, ..., K_c\}$.*

It is easily seen that if TX_i, the transmissions of s_{i1} and s_{i2}, are scheduled in the same time slot with TX_j^*, the transmissions of s_{j1}^* and $-s_{j2}^*$, for $i, j \in \{1, ..., K_c\}$, the decoder structure of the Alamouti scheme cannot decouple either s_{i1} and s_{i2}, or s_{j1} and s_{j2}, and therefore cannot successfully recover the data at the sink. Thus, any schedule violates Observation 2.3 should be avoided.

For any scheduler consistent with Observation 2.3, let CH i be the target CH, then the other CH j $(j \neq i)$ has four possible schedule schemes as shown in Figure 2.4:

- Case 1: $(T_j T_i)$ and $(T_j^* T_i^*)$;
- Case 2: $(T_j T_i)$ and $(T_j^* | T_i^*)$;
- Case 3: $(T_j | T_i)$ and $(T_j^* T_i^*)$;
- Case 4: $(T_j | T_i)$ and $(T_j^* | T_i^*)$;

where (XY) means that super transmissions X and Y are scheduled in the same time slot, and $(X|Y)$ means that X and Y are scheduled in different time slots. It is easily shown that the SINR of CH i can be written as

$$SINR_{s_{i1}} = SINR_{s_{i2}} = \frac{N|g_i|^2 p_i}{\left\{A_i \sum_{j \in \mathcal{F}_i} |g_j|^2 p_j + B_i \sum_{j \in \mathcal{L}_i} |g_j|^2 p_j\right\} + I} \tag{2.14}$$

where $|g_i|^2 = |g_{i1}|^2 + |g_{i2}|^2$, $|g_j|^2 = |g_{j1}|^2 + |g_{j2}|^2$, $A_i = |g_{i1}|^2 / |g_i|^2$, $B_i = |g_{i2}|^2 / |g_i|^2$, and $A_i + B_i = 1$. $\mathcal{F}_i$ denotes the set of CHs whose TX_j are scheduled in the same time slot as TX_i of CH i, and $\mathcal{L}_i$ denotes the set of CHs whose TX_j^* are scheduled in the same time slot as TX_i^* of CH i.

We note that in the SINR expression given in (2.14), we lose the form given in Section 2.3, and the scheduler with polynomial complexity is no longer guaranteed. However, when $\mathcal{F}_i = \mathcal{L}_i$, the SINR in (2.14) is reduced to

$$SINR_{s_{i1}} = SINR_{s_{i2}} = \frac{N|g_i|^2 p_i}{\sum_{j \in \mathcal{F}_i} |g_j|^2 p_j + I}. \quad (2.15)$$

Note that (2.15) is in the same form as (2.1). Thus, we have the following observation.

Observation 2.4. *The problem of scheduling $2K_c$ super transmissions into m time slots for inter-cluster communications is equivalent to scheduling K_c sensor nodes into $m/2$ time slots*[1] *for intra-cluster communications, given that each CH is considered as a single-antenna node with the equivalent channel gain $|g_i|^2 = |g_{i1}|^2 + |g_{i2}|^2$, and the two time slots it takes are bounded together into one.*

With Observation 2.4, we note that the scheduling protocol proposed in Section 2.3 achieves a near-optimum schedule to the inter-cluster transmissions, by excluding the scheduling schemes of case 2 and 3 in Figure 2.4. It significantly reduces the computational cost of finding the optimum schedule with the modest performance penalty, shown in Section 2.5 by numerical results. Note that same results can be directly applied to the multi-rate CDMA CHs as well.

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[1] *It is assumed that m is an even integer, i.e., the delay requirements can accommodate up to one wasted slot if necessary.*

	other slot	slot m_1	slot m_2	other slot
CH i		T_i	T_i^*	
CH j: case 1		T_j	T_j^*	
CH j: case 2		T_j		T_j^*
CH j: case 3	T_j		T_j^*	
CH j: case 4	T_j			T_j^*

Figure 2.4. Possible scheduling schemes of CH j with Alamouti scheme.

2.5 Numerical Results

In this section, we present numerical results related to the performance of the efficient scheduling protocols[2]. We consider a two-tiered CDMA WSN consisting of 5 clusters distributed without overlapping in a circle area with radium $1000m$, and a sink located in the center. Each cluster includes 20 sensor nodes distributed in a circle area with radium $100m$, and a CH in the center. The spreading bandwidth is $W = 1.228MHz$, and the processing gain is $N = 128$, equivalently, $R_{\text{base}} = 9.6Kbps$. The duration of Ph_1 and Ph_2 is 5 and 10 time slots, respectively. The fading coefficient of sensor i, g_i, is modeled as independent complex Gaussian with variance $\sigma_{g_i}^2 = C/d_i^{\alpha}$, where d_i denotes the distance between sensor i and its CH. We assume that the two antennas of CH i have the same distance to the sink, denoted by d_{CH_i}, and therefore the fading coefficients of the two antennas are independent complex Gaussian with the same variance, i.e., $\sigma_{g_{i1}}^2 = \sigma_{g_{i2}}^2 = C/d_{CH_i}^{\alpha}$. The path-loss exponent is denoted by α, and C is a constant.

[2]Figures and tables of this section are located at the end of this chapter.

The values $\alpha = 3$, $C = 7 \times 10^{-4}$, and $SINR_{\text{target}} = 7dB$ are used throughout our simulations. The AWGN variance is assumed to be 10^{-13}.

Simulation results are presented to demonstrate the performance of the proposed scheduling protocols, compared with the TDMA-type schedule, in which node i transmits with rate $nR_{i_{\text{target}}}$, and all nodes transmit in a round robin fashion, i.e., only one node transmits in one time slot. Specifically, we plot the total transmit power versus the average throughput requirement at each node.

We first compare the performance of different schedules for inter-cluster communications. Figure 2.5 shows the total transmit power for a common average throughput requirement at each sensor node as $R_{i_{\text{target}}} = \{1.92Kbps, 3.84Kbps, 5.76Kbps,\ 9.6Kbps\}$, for $i = 1, ..., 20$. We observe that a substantial amount of power is saved by employing the optimum schedule, with respect to the TDMA-type schedule. As the average throughput requirement increases, i.e., the sensors loads get heavier, the gap between the performance of the optimum schedule and that of the TDMA-type schedule increases. This result clearly indicates the benefit of the optimum schedule for a loaded CDMA WSN.

We also investigate the performance of the schedule for inter-cluster communications. We first compare the optimum and the near-optimum schedule for a WSN consisting of $K = \{4, 5, 6\}$ CHs, each taking 2 slots to transmit and having the average throughput requirement $R_{i_{\text{target}}} = 9.6Kbps$. The transmission frame consists of 6 slots. Table 2.1 shows that the near-optimum schedule incurs less than 10% performance penalty while significantly reduces the computational complexity of the optimum schedule, which is achieved by exhaustive search. Next, we consider the larger system with

5 CHs and 10 slots. The common average throughput requirements for the CHs are $R_{i_{\text{target}}} = \{3.84Kbps, 7.68Kbps, 11.52Kbps, 15.36Kbps, 19.2Kbps\}$, for $i = 1, ..., 5$. For the case without TD, we assume that each CH is equipped with single antenna and no TD is exploited. Comparing the performance with and without TD as shown in Figure 2.6, we observe that a large amount of power is saved by TD. At the same time, more power is saved by the near-optimum schedule with respect to the TDMA-type schedule.

2.6 Conclusion

In this chapter, we have considered efficient scheduling strategies for delay constrained multi-rate CDMA WSNs. Short term average throughput requirements are imposed to maintain an average throughput in addition to the QoS requirements (SINR target) for each node. It is assumed that multiple data rates are provided by means of multiple spreading codes, each of which is treated as a virtual node and interferes with each other when transmitting simultaneously. We have provided the efficient scheduling algorithm with polynomial complexity, which is the optimum and near-optimum solution to the intra-cluster and inter-cluster communications, respectively. The numerical results demonstrate that significant power savings is achieved by the proposed scheduling protocols.

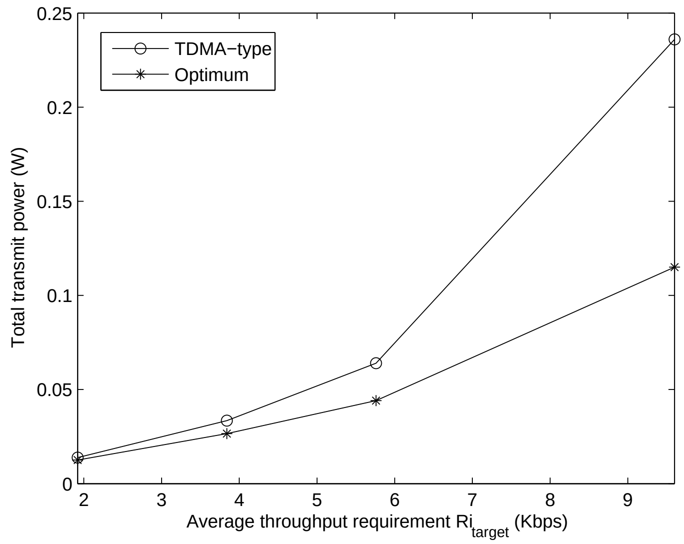

Figure 2.5. Total power consumption of intra-cluster communications.

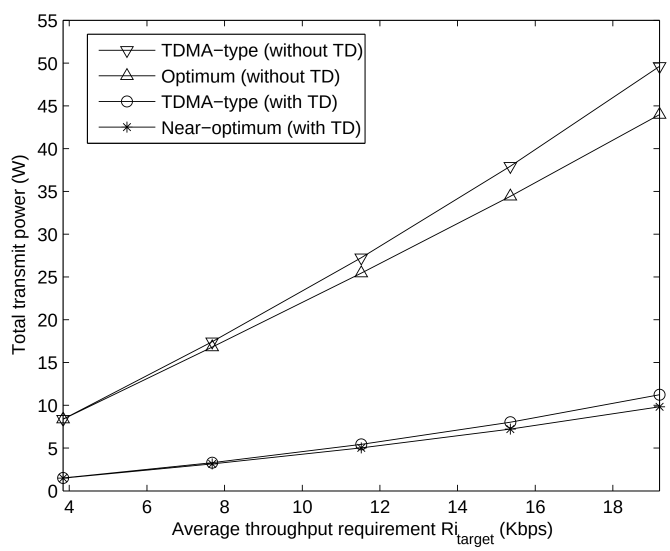

Figure 2.6. Total power consumption of inter-cluster communications.

Table 2.1. Comparison of the optimum and near-optimum schedule on total transmit power.

Number of CHs	Optimum (W)	Near-optimum (W)	Penalty
4	0.5516	0.5738	4.03%
5	0.6915	0.7386	6.82%
6	0.8307	0.8993	8.26%

Chapter 3

Distributed Power Allocation Strategies for Parallel Relay Networks

3.1 Introduction

Relay-assisted transmission schemes for wireless networks are continuing to flourish due to their potential of providing the benefits of space diversity without the need for physical antenna arrays [21]. Among the earliest work on cooperative networks are references [22–24]. A cooperative diversity model is proposed in [22] and [23], in which two users act as partners and cooperatively communicate with a common destination, each transmitting its own bit in the first time interval and the estimated bit of its partner in the second time interval. In [24], several low-complexity cooperative protocols are proposed and studied, including fixed relaying, selection relaying and incremental relaying, in which the relay node can either amplify-and-forward (AF) or decode-and-forward (DF) the signal it receives. In [25], networks consisting of more than two users that employ the space-time coding to achieve the cooperative diversity are considered. Coded cooperation schemes are discussed in [26] and [27], where a user transmits part of its partner's codeword as well. References [28] and [29] investigate the capacity of relay networks of arbitrary size. References so far have shown that, relay nodes can provide performance improvement in terms of outage behavior [24, 25], achievable rate region [22, 23, 28, 29], and error probability [26, 27, 30, 31].

Power efficiency is a critical design consideration for wireless networks such as ad-hoc and sensor networks, due to the limited transmission power of the (relay and the source) nodes. To that end, choosing the appropriate relays to forward the source data, as well as the transmit power levels of all the nodes become important design issues. Optimum power allocation strategies for relay networks are studied up-to-date for several structures and relay transmission schemes. Three-node models are discussed in [32] and [33], while multi-hop relay networks are studied in [34–36]. Relay forwarding strategies for both AF and DF parallel relay channels in wideband regime are proposed in [37]. Recent works also discuss relay selection algorithms for networks with multiple relays. Optimum relay selection strategies for several models are identified in [30, 37, 38]. Recently proposed practical relay selection strategies include pre-select one relay [62], best-select relay [62], blind-selection-algorithm [63], informed-selection-algorithm [63], and cooperative relay selection [64]. All of these proposed methods result in power efficient transmission strategies. However, the common theme is that, the implementations of these algorithms require either the destination or the source to have substantial information about the network, such as the channel state information (CSI) of all communication channels, received signal-to-noise ratio (SNR) at every node, the topology of the network, etc. Such centralized power allocation/relay selection schemes may be infeasible to implement due to the substantial feedback requirements, overhead and delay they may introduce.

To overcome the obstacles of a centralized architecture, several heuristic approaches have been proposed in [65], for multiuser networks with coded cooperation. In this work, users select cooperation partners based on a priority list in a distributed manner. Although the proposed algorithms are advantageous due to their ease of implementation, their performance depends on the fading conditions, and the randomness in the channel may prevent the protocols from providing full diversity. In [66], an SNR threshold method is proposed for the relay node to make a decision on whether to forward the source data in a three-node model. Since there is only one relay node in the considered system, relay selection is not an issue. Reference [67] provides a relay selection algorithm based on instantaneous channel measurements done by each relay node locally. For the purpose of reducing the communication among relays, a flag packet is broadcasted by the selected relay to notify the other relays of the result.

In this chapter, we investigate *optimum distributed power allocation* strategies for decode-and-forward parallel relay networks, in which only partial CSI is accessible at the source and the relay nodes. We first propose a distributed decision mechanism for each relay node to individually make a decision on whether to forward the source data. In contrast to the SNR based decision protocol presented in [66], in our proposed decision mechanism, the relay makes its decision not only by considering its received SNR, but also by comparing its relay-to-destination channel gain with a given threshold, and no feedback from the destination is needed. The overall overhead is further reduced as compared to the method proposed in [67] since the distributed decision mechanism does not require communication among relays. Secondly, given such a relay decision scheme, and considering an outage occurs whenever the SNR at the destination is lower than the required value (target), we formulate the distributed power allocation problem that aims to minimize the expected value of the total transmit power while providing the target SNR at the destination with an outage probability constraint. We identify the solution of this problem, that consists of the optimum value of the source power, and the corresponding relay decision threshold based on the partial CSI available at the source. The extra power the distributed power allocation mechanism needs as compared to the optimum centralized power allocation mechanism, i.e., the *additional power expenditure*, is examined to observe the tradeoff between the outage probability and the additional power expenditure.

We next consider two special cases with simpler implementation, namely the *passive source model* where the source does not contribute to the relay selection process, and the *single relay model* where one relay node is selected to forward the source data based on limited CSI. For each case, we optimize the respective relevant parameters. Our results demonstrate that considerable power savings can be obtained by our proposed distributed relay selection and power allocation schemes with respect to random relay selection.

The organization of this chapter is as follows. In Section 3.2, the system model is described. The distributed power allocation problem is formulated and the optimum solution is given in Section 3.3. In Section 3.4, we investigate the passive source model and the single relay model. Numerical results supporting the theoretical analysis are presented in Section 3.5, and Section 3.6 concludes the chapter.

3.2 System Model and Background

We consider a relay network consisting of a source-destination pair and N relay nodes employing decode-and-forward. We assume that the relay nodes operate in pre-assigned orthogonal channels, e.g. in non-overlapping time/frequency slots, or using

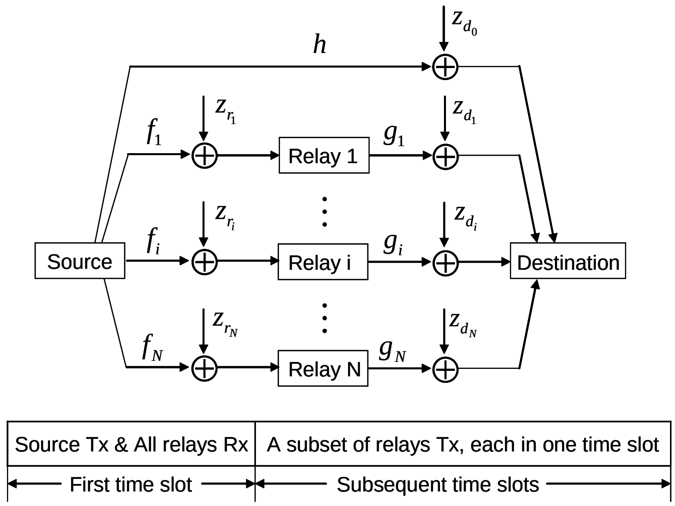

Figure 3.1. System model: parallel relay network.

orthogonal signatures. The source is assumed to transmit in a time slot prior to (and non-overlapping with) the relays. Let f_i and g_i denote the fading coefficients of the source-to-relay and relay-to-destination channels for the ith relay node, for $i = 1, ..., N$. The fading coefficient of the source-to-destination link is denoted by h. We assume that each channel is flat fading, and f_i, g_i and h are all independent realizations of zero mean complex Gaussian random variables with variances $\sigma^2_{f_i}$, $\sigma^2_{g_i}$ and σ^2_h per dimension, respectively.

Without loss of generality, we will assume that we have a time slotted system in the sequel. The system model is shown in Figure 3.1. In the first time slot, the source broadcasts X_o with power P_s. The destination observes y_{d_0}:

$$y_{d_0} = \sqrt{P_s} h X_o + z_{d_0} \tag{3.1}$$

and the ith relay observes y_{r_i}:

$$y_{r_i} = \sqrt{P_s} f_i X_o + z_{r_i} \quad \text{for} \quad i = 1, ..., N \tag{3.2}$$

where z_{d_0} and $\{z_{r_i}\}_{i=1}^{N}$ are additive white Gaussian noise (AWGN) terms at the destination and the relays, respectively. Assume without loss of generality that they are of variance 1/2 per dimension. The ith relay node is said to be *reliable* and can correctly decode X_o when its received SNR, SNR_{r_i}, satisfies

$$SNR_{r_i} = P_s \left| f_i \right|^2 \geq SNR_{target} \tag{3.3}$$

where SNR_{target} is the given decodability constraint. In the subsequent time slots following the first one, the relays that belong to the set of reliable relays, A_R, can decode and forward the source data to the destination, each in its assigned time slot. Throughout this chapter, we assume that the reliable relays simply regenerate the source data X_o [24,33,35]. The signal received at the destination from the reliable relay i is

$$y_{d_i} = \sqrt{P_i} g_i X_o + z_{d_i}, \qquad i \in A_R \tag{3.4}$$

where P_i is the transmit power of the ith relay node, and z_{d_i} is the AWGN term at the ith relay-to-destination channel. The destination combines signals received from the reliable relay nodes and the direct link with a maximum ratio combiner (MRC), and the resulting SNR at the destination is

$$SNR_d = P_s |h|^2 + \sum_{i \in A_R} P_i |g_i|^2 . \tag{3.5}$$

We consider that the destination can correctly receive the source data whenever $SNR_d \geq SNR_{target}$.

Given this system model, the power allocation problem for regenerative DF relay networks with parallel relays can be posed as

$$\min_{Ps, \{P_i\}} \quad P_s + \textstyle\sum_{i \in A_R} P_i \tag{3.6}$$

$$\text{s. t.} \quad P_s |h|^2 + \textstyle\sum_{i \in A_R} P_i |g_i|^2 \geq SNR_{target} \tag{3.7}$$

$$P_s |f_i|^2 \geq SNR_{target} \quad \text{for each } i \in A_R. \tag{3.8}$$

We note that the resulting power allocation strategy may prevent some reliable relays from participating simply by assigning zero power to those relays.

The optimum power allocation strategy for DF relay networks using different code books at the relays is identified in [37]. This strategy, re-stated below for the benefit of the reader, is easily seen to be the optimum *centralized* power allocation strategy for regenerative DF relay networks as well,

$$P_s^* = \frac{SNR_{target}}{\left|f_{k^*}\right|^2} \tag{3.9}$$

$$P_i^* = \begin{cases} \left(\frac{SNR_{target}-|h|^2 SNR_{target}/\left|f_{k^*}\right|^2}{\left|g_{k^*}\right|^2}\right)^+, & i = k^* \\ 0, \text{otherwise} \end{cases} \tag{3.10}$$

$$k^* = \arg\min_{\{k \in A_E\}} \left[\frac{1}{|f_k|^2} + \frac{1}{|g_k|^2} - \frac{|h|^2}{|f_k|^2 |g_k|^2}\right] \tag{3.11}$$

where $(\cdot)^+ = \max(0, \cdot)$. In (3.11), the set A_E denotes the set of efficient relays such that the transmission through the relay is more power efficient than the direct transmission, i.e.,

$$A_E = \{i|(|f_i|^2 \geq |h|^2) \cap (|g_i|^2 \geq |h|^2), i = 1..N\}. \tag{3.12}$$

Observe that when the source power is assigned as in (3.9), the relay node k^*, chosen according to (3.11), is the only relay node with received SNR equal to SNR_{target}. Thus, each relay node can decide whether it is the intended relay node by simply checking its received SNR. When the SNR contribution of the relay node, $SNR_{target} - |h|^2 SNR_{target}/\left|f_{k^*}\right|^2$, is indicated explicitly by the source, the intended relay node can calculate its required transmit power as in (3.10) and forward X_o to the destination. Alternatively, the source can broadcast the selected relay and the optimum power level in a side channel.

A moment's thought reveals that to implement the strategy given by (3.9)-(3.11), the full CSI, i.e., $\{f_i, g_i\}_{i=1}^N$ and h, at the source node, and the individual CSI, i.e.,

$\{f_i, g_i\}$, at relay node i are needed. Although (3.9)-(3.11) provides the most power efficient DF relay transmission strategy, its centralized nature, i.e., the fact that it requires the channel estimate of each link and the feedback of this information to the source, may render its implementation impractical. As such, distributed strategies are needed. In the following, we devise efficient distributed power allocation strategies.

3.3 Distributed Power Allocation

Our aim in this chapter is to find power allocation schemes that *do not require a centralized mechanism*, and utilize the limited available CSI at each node. In practice, it is feasible that the channels are estimated by training before the actual data transmission, when each node operates in TDMA mode. When the source transmits the training bits, all relay nodes can simultaneously estimate their source-to-relay fading coefficients $\{f_i\}_{i=1}^N$ due to the broadcast nature of the wireless medium. Similarly, when the relay node i transmits the training bits, the source-to-relay coefficient f_i can be estimated at the source. However, for $\{g_i\}_{i=1}^N$ to be available at the source, the feedback from the destination for each realization is required, which may be impractical. Thus, we investigate distributed power allocation schemes when the source has the realizations $\{f_i\}_{i=1}^N$ and h, and only the statistics of $\{g_i\}$. The relay nodes are assumed to have their individual CSI, i.e., f_i and g_i for relay i, $i = 1, ..., N$.

3.3.1 Distributed Decision Mechanism

We first derive a distributed decision mechanism with the model assumptions given above. Since the source has only the statistical description instead of the realizations $\{g_i\}_{i=1}^N$, the optimum centralized power allocation indicated by (3.9)-(3.11) cannot be implemented by the source. Also, while it is clear that for a fixed source power, the best strategy is transmitting through the reliable relay node that has the highest relay-to-destination channel gain, this mechanism requires a comparison of all $\{g_i\}_{i=1}^N$. The distributed nature of the strategy requires that each relay should make its decision relying only on its individual CSI. Since each relay can easily determine whether it is a reliable relay by using its SNR value, i.e., its individual CSI, we propose that the ith reliable relay decides it will be a forwarding node when its channel gain to the destination satisfies

$$|g_i|^2 \geq \gamma \tag{3.13}$$

where γ is a given threshold value. Relay i then forwards the decoded signal with *sufficient* power. That is, we have

$$P_i^* = SNR'_{target}/|g_i|^2 \tag{3.14}$$

where $SNR'_{target} = (SNR_{target} - P_s|h|^2)^+$ denotes the SNR contribution from the relay[1].

[1] The values of γ and SNR'_{target} are assumed to be broadcasted by the source on a side channel.

We note that such a distributed decision mechanism includes the probability that more than one relay will transmit. Similarly, we note that with any $\gamma > 0$, the scheme results in a nonzero probability that none of the relay nodes satisfies (3.13), and hence a nonzero outage probability Prob($SNR_d < SNR_{target}$). As such, the source should determine the optimum source power and the corresponding threshold γ by considering the realizations of $\{f_i\}$ and the randomness in $\{g_i\}$, to meet a system given specification, i.e., an outage probability requirement.

3.3.2 Source Power Allocation and Threshold Decision

Given the above described strategy, we now investigate how the source should decide the value of its transmit power P_s and the relay decision threshold γ, to satisfy the target SNR, SNR_{target}, at the destination with a target outage probability, ρ_{target}.

From the source's point of view, the relay transmit powers are random variables with known statistics because the realizations $\{g_i\}_{i=1}^{N}$ are not available at the source. We have the pdf of $X_i = |\mathrm{g_i}|^2$ as

$$p_{X_i}(x_i) = \frac{1}{2\sigma_{g_i}^2} \exp\left(-\frac{x_i}{2\sigma_{g_i}^2}\right), \quad \text{for} \quad i = 1, ..., N \tag{3.15}$$

where $\mathrm{g_i}$ is a zero mean complex Gaussian random variable with variance $\sigma^2_{g_i}$ per dimension. We consider the expected value of the transmit power of relay i

$$E[P_i] = \int_{\gamma}^{\infty} \frac{SNR'_{target}}{x_i} p_{X_i}(x_i) dx_i \tag{3.16}$$

$$= \int_{\gamma}^{\infty} \frac{SNR'_{target}}{2\sigma^2_{g_i} x_i} \exp(-\frac{x_i}{2\sigma^2_{g_i}}) dx_i. \tag{3.17}$$

The distributed power allocation problem can then be expressed as

$$\min_{\gamma, P_s} \quad P_s + \textstyle\sum_{i \in A_R(P_s)} E[P_i] \tag{3.18}$$

$$\text{s. t.} \quad \mathrm{Prob}(SNR_d \leq SNR_{target}) \leq \rho_{target} \tag{3.19}$$

$$P_s \, |f_i|^2 \geq SNR_{target} \ \ \forall \; i \in A_R \tag{3.20}$$

where we explicitly state the dependency of the set of reliable relay A_R on P_s. Observe that the deterministic quality-of-service guarantee in (3.7) is replaced by the probabilistic constraint (3.19). The following theorem provides the optimum solution:

Theorem 3.1. *The optimum source power, P_s^{**}, can only be one of the $(M+1)$ discrete values in the set*

$$\{\frac{SNR_{target}}{|f_1|^2}, ..., \frac{SNR_{target}}{|f_M|^2}, \frac{SNR_{target}}{|h|^2}\} \tag{3.21}$$

where we reorder the indices of the relay nodes such that $|f_1|^2 > |f_2|^2 > ... > |f_M|^2 > |h|^2 > |f_{M+1}|^2 ... > |f_N|^2$, i.e., $\frac{SNR_{target}}{|f_1|^2} < \frac{SNR_{target}}{|f_2|^2} < ... < \frac{SNR_{target}}{|f_M|^2} <$

$\frac{SNR_{target}}{|h|^2} < \frac{SNR_{target}}{|f_{M+1}|^2} < ... < \frac{SNR_{target}}{|f_N|^2}$. [2] *For each possible* P_s^{**} *value, there exist a corresponding reliable relay set* A_R^{**}*, and a unique optimum threshold value,* γ^{**}.

Proof: See Appendix 3.7.1. □

The source should simply compare $(M+1)$ possible $E[P_{total}]$ values and decide the best (P_s^{**}, γ^{**}) pair. Note that when the expected value of the total transmit power is higher than that with direct transmission, the source will prefer to transmit directly to the source[3].

The cost of the lack of full CSI at the source, i.e., the cost of using the distributed relay decision mechanism, is an additional power expenditure. Let P_{total}^{**} and P_{total}^{*} denote the total power of the proposed optimum distributed power allocation scheme, and that of the optimum centralized allocation scheme which is the sum of the source power P_s^* and the relay power P_i^* given in (3.9)-(3.11), respectively. The expected value of the additional power expenditure is:

$$E[P_{add}] = E[P_{total}^{**}] - E[P_{total}^{*}] \tag{3.22}$$

$$= P_s^{**} + \sum_{i \in A_R^{**}} \int_{\gamma^{**}}^{\infty} \frac{SNR'_{target}}{2\sigma_{g_i}^2 x} \exp(-x/2\sigma_{g_i}^2) dx - E[P_{total}^{*}]. \tag{3.23}$$

We observe that in (3.58), ρ_{target} is an increasing function of γ, while in (3.23), $E[P_{add}]$ is a decreasing function of γ. Thus, there exists a tradeoff between the outage probability

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[2] $P_s = \frac{SNR_{target}}{|h|^2}$ *is the largest candidate of the source power. With this power level, source can reach the destination via the direct link and relay-assisted transmission is not needed.*

[3] The source would communicate this decision via the side channel.

and the additional power expenditure: reducing the target outage probability will require more additional power. While designing the power allocation strategy, a reasonable target outage probability should be chosen in accordance with this tradeoff.

3.4 Simpler Schemes

The optimum distributed power allocation strategy still requires the realizations of $\{f_i\}_{i=1}^N$ and h, i.e., the CSI of the source-to-relay and the direct links, available at the source. It also requires the source to update the threshold γ^{**} and the source power P_s^{**} at each time when these channel coefficients change. Due to further limitations on the availability of this CSI and for implementation complexity, we may opt for even simpler schemes. In this context, we next consider two special cases, namely the passive source model and the single relay model. For both cases, we have the previous assumption that each relay has its individual CSI, i.e., f_i and g_i for relay i, $i = 1, ..., N$. Below are the brief descriptions of the two models.

- *Passive source model:* We assume that the source only has the statistics of *all* communication channels, and does not participate in the relay selection process at all. For this model, we fix the source power P_s, and the relay decision threshold γ, and employ the same distributed decision mechanism as proposed in 3.3.1.

- *Single relay model:* We assume that the source has CSI of the direct and the source-to-relay links, i.e., $\{f_i\}_{i=1}^N$ and h, and the statistics of the relay-to-destination links $\{g_i\}$. We have the source select *one* assisting relay node to satisfy the system requirements on received SNR and the outage probability.

3.4.1 Passive Source Model

In practice, we may have situations where the source does not have the realizations of any of the channels, but has access only to the statistical descriptions of them. It may also be the case that the source may not be able to do computationally expensive operations, e.g., due to hardware constraints in sensor or RFID networks. We term such source nodes, *passive*. Considering these practical issues, in this section, we investigate the distributed power allocation for the passive source model.

Since each relay has its individual CSI, we can apply the same distributed decision mechanism as proposed in Section 3.3.1. However, a passive source cannot optimize its power P_s or γ based on channel realizations; they should be found off-line based on the statistical descriptions of the channel and kept fixed for all realizations. Note that, different from Section 3.3, in this case, we may end up having no reliable relay if the fixed source power value is too small.

Let us now develop the criterion on how to choose the source power P_s and the threshold γ by considering the outage probability and the additional power expenditure jointly. The outage probability of the direct link is given by

$$d_{out} = \text{Prob}\{P_s\,|\text{h}|^2 < SNR_{target}\} = 1 - \exp\left(-\frac{SNR_{target}}{P_s \cdot 2\sigma_h^2}\right). \tag{3.24}$$

For clarity of exposition, let us define a_i as the probability that the ith relay is a reliable relay, b_i as the probability that the ith relay satisfies (3.13), and c_i as the probability that the ith relay is in set A_C, which denotes the set of relays that satisfy both (3.3) and (3.13). We have

$$a_i = \text{Prob}\{i \in A_R\} = \text{Prob}\{P_s\,|\mathrm{f_i}|^2 \geq SNR_{target}\} = \exp\left(-\frac{SNR_{target}}{P_s \cdot 2\sigma^2_{f_i}}\right) \tag{3.25}$$

$$b_i = \text{Prob}\{|\mathrm{g_i}|^2 \geq \gamma\} = \exp\left(-\frac{\gamma}{2\sigma^2_{g_i}}\right) \tag{3.26}$$

$$c_i = \text{Prob}\{i \in A_C\} = a_i \cdot b_i \tag{3.27}$$

where $\mathrm{f_i}$ and $\mathrm{g_i}$ are zero mean complex Gaussian random variables with variances $\sigma^2_{f_i}$ and $\sigma^2_{g_i}$ per dimension, respectively. The overall outage probability becomes

$$\rho_{outage} = \text{Prob}\{A_C = \emptyset\} \cdot d_{out} = \prod_{i=1}^{N} \text{Prob}\{i \notin A_C\} d_{out} = \prod_{i=1}^{N} [1 - c_i] \cdot d_{out}. \tag{3.28}$$

Observe in (3.28) that ρ_{outage} is a function of the source transmit power, P_s and the threshold γ. To choose the (P_s, γ) pair that satisfies (3.28), we make two observations. The first one is

$$\rho_{outage} \geq \prod_{i=1}^{N} [1 - a_i]\, d_{out} \tag{3.29}$$

where equality occurs when $\gamma = 0$, that is when *all* reliable relays forward the source data. Thus, to achieve a target outage probability, ρ_{target}, there exists a minimum source power P_s, that provides the target outage probability with $\gamma = 0$. Note that when P_s is chosen close to this minimum value, the corresponding γ factor will be close to 0, resulting in many relays transmitting. This may result in unnecessarily large extra power expenditure and care must be exercised to choose the correct pair. Secondly, we observe

$$\rho_{outage} \geq \prod_{i=1}^{N} [1 - b_i] \, d_{out}. \tag{3.30}$$

Thus, for a given P_s value, γ should be strictly less than some threshold to provide a target outage probability.

When we consider a special case where $d_{out} \approx 1$, i.e., the direct link is not reliable, and $\{\mathrm{f_i}\}_{i=1}^{N}$ and $\{\mathrm{g_i}\}_{i=1}^{N}$ are i.i.d., we have

$$\rho_{outage} \approx (1 - \exp{(-\frac{SNR_{target}}{2P_s\sigma_f^2} - \frac{\gamma}{2\sigma_g^2}))}^N \tag{3.31}$$

and (P_s, γ) pair that aims to achieve an outage probability ρ_{target} should satisfy

$$\frac{SNR_{target}}{2P_s\sigma_f^2} + \frac{\gamma}{2\sigma_g^2} \approx -ln(1 - (\rho_{target})^{1/N}) \tag{3.32}$$

Since the relays employ the distributed decision mechanism proposed in 3.3.1, there exists a nonzero probability that additional relay nodes besides the best relay decide to forward the source data. In this case, additional power is expended. For a realization of $|\mathrm{g_i}|^2$, $x_i = |g_i|^2 \geq \gamma$, the probability that relay i makes a forwarding decision even though it is not the best relay in set A_R, $W_i(x_i)$, can be expressed as

$$W_i(x_i) = \text{Prob}(\text{Wrong forwarding decision by relay } i | x_i \geq \gamma) \tag{3.33}$$

$$= \text{Prob}\Big\{(i \in A_R) \cap (\exists j \in A_R \text{ and } j \neq i, \text{ such that } X_j > x_i \geq \gamma)\Big\} \tag{3.34}$$

$$= \text{Prob}\{i \in A_R\} \cdot \text{Prob}\Big\{\exists j \in A_R \text{ and } j \neq i, \text{ such that } X_j > x_i \geq \gamma)\Big\} \tag{3.35}$$

$$= \text{Prob}\{i \in A_R\} \cdot (1 - \text{Prob}\{\forall j \in [1, ..., N] \text{ and } j \neq i, (j \notin A_R)$$

$$\cup \Big((j \in A_R) \cap (X_j < x_i)\Big)\Big\}\Big) \tag{3.36}$$

$$= \text{Prob}\{i \in A_R\} \cdot \Bigg(1 - \prod_{j=1, i \neq j}^{N} (\text{Prob}\{j \notin A_R\}$$

$$+\text{Prob}\{j \in A_R\} \cdot \text{Prob}\{X_j < x_i\}\Big)\Big) \tag{3.37}$$

$$= a_i \cdot \Bigg(1 - \prod_{j=1, i \neq j}^{N} \Bigg((1 - a_j) + a_j \cdot (1 - \exp(-\frac{x_i}{2\sigma_{g_j}^2}))\Bigg)\Bigg) \tag{3.38}$$

$$= \exp(-\frac{SNR_{target}}{P_s 2\sigma_{f_i}^2}) \cdot \Bigg(1 - \prod_{j=1, i \neq j}^{N} \Bigg(1 - \exp(-\frac{SNR_{target}}{P_s 2\sigma_{f_j}^2}) \cdot \exp(-\frac{x_i}{2\sigma_{g_j}^2})\Bigg)\Bigg). \tag{3.39}$$

If relay i makes a wrong forwarding decision, it will transmit with power value SNR'_{target}/x_i. In essence, the power of relay i is wasted, since the relay with the highest relay-to-destination channel gain in A_R also transmits the source data to the destination reliably but with a lower power. We have the expected value of the wasted power of relay i, $E[P_{waste_i}]$ as

$$E[P_{waste_i}] = \int_{\gamma}^{\infty} W_i(x_i) \frac{SNR'_{target}}{x_i} p_{X_i}(x_i) dx_i. \tag{3.40}$$

The expected value of the additional power expenditure of all relays is[4]

$$E[P_{add_{Relay}}] = \sum_{i=1}^{N} E[P_{waste_i}]. \tag{3.41}$$

Observe that in (3.28), ρ_{outage} is an increasing function of γ when other parameters are fixed, while in (3.41), the expected value of the additional power expenditure is a decreasing function of γ. There exists a tradeoff between the outage probability and the additional relay power expenditure. A reasonable pair of the source power and the threshold γ should be chosen by considering both the tradeoff and the properties of the (P_s, γ) pair in (3.28), (3.29) and (3.30).

3.4.2 Single Relay Model

The distributed power allocation schemes proposed up to this point in general result in multiple relays transmitting to the destination, causing additional power expenditure. In this section, we investigate the case where only one relay node selected by the source is allowed to transmit. In contrast to the centralized solution in (3.9)-(3.11), however, we consider that the source has limited CSI. In particular, we re-emphasize that, only the statistical descriptions of the relay-to-destination channels are available at the source. Adopting the single relay model, we will see that the task of finding the threshold value for the relay forwarding decisions can be substantially simplified as compared to the optimum distributed strategy.

---

[4]Note that $E[P_{waste_i}] = 0$ if i is an unreliable relay or the best reliable relay.

When relay k is selected, the source transmits with just enough power $P_s = SNR_{target}/|f_k|^2$ to make relay k a reliable relay. So, the source-to-relay link does not have outage. However, since relay k will forward the decoded source data only when its channel gain to the destination satisfies $|g_k|^2 \geq \tau_k$, we may have an outage on the relay-to-destination link. Observe that, if relay k decides to forward the data it will do so with power $P_k = SNR'_{target}/|g_k|^2$.

Therefore, to satisfy the outage constraint ρ_{target}, the relay-to-destination gain threshold, τ_k should satisfy

$$\int_{\tau_k}^{\infty} p_{X_k}(x_k)d(x_k) = \int_{\tau_k}^{\infty} \frac{1}{2\sigma_{g_k}^2} \exp\left(-\frac{x_k}{2\sigma_{g_k}^2}\right) dx_k = 1 - \rho_{target}. \tag{3.42}$$

Thus, we have

$$\tau_k = -2\sigma_{g_k}^2 \ln(1 - \rho_{target}). \tag{3.43}$$

The expected value of the transmit power of the relay node is

$$E[P_k] = \int_{\tau_k}^{\infty} \frac{SNR'_{target}}{x_k} p_{X_k}(x_k)dx_k \tag{3.44}$$

$$= \frac{\int_{\tau}^{\infty} \frac{SNR'_{target}}{x_k} \exp(-x_k/2)dx_k}{2\sigma_{g_k}^2} = \frac{SNR'_{target}K(\tau)}{2\sigma_{g_k}^2} \tag{3.45}$$

where

$$K(\tau) = \int_{\tau}^{\infty} \frac{1}{x_k} \exp(-x_k/2)dx_k \tag{3.46}$$

and $\tau = -2\ln(1-\rho_{target})$. We observe that $E[P_k]$ inversely proportional to the variance of the fading coefficient, $\sigma^2_{g_k}$.

The optimum power allocation problem in this case becomes

$$\min_{P_s,k} \quad P_s + E[P_k] \tag{3.47}$$

$$\text{s. t.} \quad \text{Prob}(SNR_d \leq SNR_{target}) \leq \rho_{target} \tag{3.48}$$

$$P_s\,|f_k|^2 \geq SNR_{target}. \tag{3.49}$$

Theorem 3.1 is valid for (3.47)-(3.49) as well, i.e., the optimum source power P_s^{**}, has to be one of the $(M+1)$ possibilities. The proof follows the same steps with the total power expression (3.18) replaced by (3.47), i.e., $\sum_{i\in A_R^{\dagger}} \int_{\gamma^{\dagger}}^{\infty} \frac{1}{2\sigma^2_{g_i} x_i} \exp(\frac{-x_i}{2\sigma^2_{g_i}})dx_i$ should be replaced by $\frac{K(\tau)}{2\sigma^2_{g_k}}$.

The optimum solution can be expressed as

$$P_s^{**} = SNR_{target}/\left|f_{k^{**}}\right|^2 \tag{3.50}$$

$$k^{**} = \arg \min_{|h|^2<2\sigma^2_{g_k}/K(\tau)} \frac{1}{|f_k|^2} + \frac{K(\tau)}{2\sigma^2_{g_k}}\left(1-\frac{|h|^2}{|f_k|^2}\right)^+. \tag{3.51}$$

(3.50)-(3.51) result in only the relay selected by the source, k^{**}, satisfying SNR target. Thus, each relay can decide whether it is the selected node by examining its own received SNR.

From (3.43) and (3.45), we note the tradeoff between the outage probability and the additional power expenditure in this scheme as well. We also note that the relay threshold τ_k is a scaled version of $\sigma^2_{g_k}$ for each relay i. The complexity for calculating the relay threshold at the source is thus significantly less compared to that of the optimum distributed power allocation scheme derived in Section 3.3, making the model and the corresponding strategy given in this section attractive from a practical stand point. However, we note that, with this scheme, since *exactly* one relay will be reliable, additional power may be needed as compared to the optimum distributed strategy to satisfy the same outage requirement.

3.5 Numerical Results

In this section, we present numerical results related to the performance of the proposed distributed power allocation schemes[5]. We consider a relay network consisting of a source and a destination 100 m apart, and $N = 15$ relay nodes that are distributed in a 50×50 m^2 square area, as shown in Figure 3.2. We consider the fading model as in [24], i.e., the variance of the channel gain is proportional to the distance between nodes. Thus, we have $\sigma_{f_i}^2 = C/d_{SR_i}^{\alpha}$, $\sigma_{g_i}^2 = C/d_{R_iD}^{\alpha}$ and $\sigma_h^2 = C/d_{SD}^{\alpha}$, where d_{AB} is the distance between node A and B, and S, D and R_i denote the source, the destination and the ith relay node, respectively. The path-loss exponent is denoted by α. C is a constant that is expressed as $C = G_t G_r \lambda^2/(4\pi)^2 L$, where G_t is the transmitter antenna gain, G_r is the receiver antenna gain, λ is the wavelength, and L is the system loss factor not related to propagation ($L \geq 1$). The values $\alpha = 3$, $G_t = G_r = 1$, $\lambda = 1/3$ m (carrier frequency $f = 900$ MHz), $L = 1$, are used throughout the simulations. The AWGN variances on all communication links are assumed to be 10^{-10}. We set $SNR_{target} = 10$ as the system SNR requirement.

[5]Figures of this section are located at the end of this chapter.

Simulation results are presented to demonstrate the performance of the proposed power allocation strategies. Specifically, we plot $E[P_{total}]$, the expected value of the total power expended versus ρ_{outage}, the target outage probability. Note that in the theoretical analysis, there is no outage in the optimum centralized power allocation (OCPA) in Section 3.2, since the source and the relay can always adjust their transmit power to satisfy the SNR requirement at the destination. For a fair comparison, we define that an outage occurs for OCPA when the total transmit power is higher than a given power constraint. This is reasonable since if there is no maximum power constraint, the expected value of the transmit power goes infinite to achieve a zero outage probability on a fading channel.

We first compare the performance among the proposed optimum distributed power allocation (ODPA) scheme, the OCPA scheme, and the random relay selection (RRS) scheme, in which the source randomly selects one out of all relays with equal probability to forward the source data. We observe in Figure 3.3 that a substantial amount of power is saved by employing ODPA, with respect to RRS. The power savings is more pronounced for low outage probability values. As expected, an additional power expenditure, which is the penalty of lack of full CSI, is introduced by ODPA. We observe that the additional power expenditure decreases as the outage probability increases, which is expected from the discussion on the tradeoff between the outage probability and the additional power expenditure in Section 3.3.

We also compare all of the proposed distributed power allocation schemes in Figure 3.3. As expected, we observe that the best performing scheme is ODPA. Passive source model (PSM) and single relay model (SRM) both have some performance loss due to the fact that, for PSM the values of the source power P_s and the threshold γ are fixed; for SRM only one relay node is used for forwarding transmission. However, the two special cases still outperform RRS by considering the limited available CSI for power allocation, and they simplify the optimization process of ODPA and facilitate the implementations. Thus, PSM and SRM may be preferred when computational complexity is at a premium. When $\rho_{outage} = 0.05$, approximately, 80%, 77% and 67% power is saved by ODPA, SRM and PSM with respect to RRS, respectively.

Figure 3.4 remarks that the performance of the system with PSM depends strongly on the value of the source power (which is fixed). For low outage probability values, a high source power is favorable since it reduces the SNR contribution from the relay nodes, and hence the transmit power of the relay nodes. On the other hand, for high outage probability values, the source power becomes a lower bound for the total power. Thus, a low source power is preferred in this case.

We also investigate the effect of the direct link on the performance. Figure 3.5 and Figure 3.6 show the effect of the direct link SNR contribution on PSM and SRM, respectively. It is observed that a small amount of power savings is obtained when the direct link is considered. This amount vanishes as the quality of the direct link decreases. With this observation, when the direct link has a poor channel quality, the transmitting relay i can forward the signal with power $SNR_{target}/\left|g_i\right|^2$ instead of $SNR'_{target}/\left|g_i\right|^2$ without a significant performance loss. Employing such a strategy has the advantage that, the direct link, h, is not required for calculating SNR'_{target}, and thus the amount of feedback from the destination is reduced.

In addition, to show the power efficiency advantage of the relay-assisted transmission scheme ODPA, we compare the performances of ODPA and the direct transmission scheme where the signal is transmitted from the source to the destination via the direct link only. To show that ODPA benefits more general networks than the one we considered in Figure 3.2 where the direct link distance is larger than that of any source-to-relay or relay-to-destination link, we now consider that the destination's position is randomly chosen in the area of $X \times Y = [20, 100]m \times [-50, 50]m$ for each realization, while the source and relay nodes remain in the same position as in Figure 3.2. In Figure 3.7, we plot the expected value of total power expenditure, $E[P_{total}]$, versus the target outage probability, ρ_{outage}, for ODPA and the direct transmission scheme. We observe that in the absence of the relays, direct transmission scheme requires a relatively high power expenditure to achieve the same outage probability as compared to ODPA. It is observed that the proposed relay-assisted transmission scheme provides significant performance gain in terms of power efficiency upon the direct transmission. This is intuitively pleasing since the relay selection and power allocation algorithms in the proposed scheme guarantee that the more power efficient way is always selected out of the relay-assisted transmission and the direct transmission for each channel realization.

3.6 Conclusion

In this chapter, we addressed the distributed power allocation problem for parallel relay networks. Given the partial CSI available at the source and the relay nodes, we proposed a distributed relay decision mechanism and developed the optimum distributed power allocation scheme. By optimizing the relay selection strategy and power allocation, the optimum distributed power allocation strategy performs close to the optimum centralized scheme. We have also considered two simple distributed power allocation strategies, the passive source model and the single relay model. Both schemes have significantly less computational complexity requirements at the source with a modest sacrifice in performance. Our main result is that by using distributed power allocation and partial CSI, we can develop power efficient transmission schemes, reducing the amount of control traffic overhead for relay-assisted communications.

3.7 Appendix

3.7.1 Proof of Theorem 3.1

Assume that $P_s = SNR_{target}/|f_i|^2$ and there exist a reliable relay set $A_R^{\dagger}$ containing R_i relay nodes and a corresponding threshold value $\gamma^{\dagger}$. Then, the expected value of the total power is

$$E[P_{total}] = P_s + \sum_{i \in A_R^{\dagger}} \int_{\gamma^{\dagger}}^{\infty} \frac{(SNR_{target} - P_s|h|^2)^+}{2\sigma_{g_i}^2 x_i} \exp(\frac{-x_i}{2\sigma_{g_i}^2}) dx_i. \tag{3.52}$$

We consider the set of transmitting relays as a super relay node whose effective channel gain to the destination is $\left|g_{eff}\right|^2$. Thus, the expected value of the total power can be expressed as

$$E[P_{total}] = P_s + \frac{(SNR_{target} - P_s|h|^2)^+}{\left|g_{eff}\right|^2} \tag{3.53}$$

where

$$\left|g_{eff}\right|^2 = \frac{1}{\sum_{i \in A_R^\dagger} \int_{\gamma^\dagger}^{\infty} \frac{1}{2\sigma_{g_i}^2 x_i} \exp(-x_i/2\sigma_{g_i}^2) dx_i}. \tag{3.54}$$

The direct transmission is more power efficient than the relay-assisted transmission when the channel gain of the direct link, $|h|^2$, is greater than the effective channel gain of the relay-to-destination links, $\left|g_{eff}\right|^2$, i.e.,

$$|h|^2 > \left|g_{eff}\right|^2. \tag{3.55}$$

In this case, the optimum source power is $P_s^{**} = SNR_{target}/|h|^2$.

On the other hand, the relay transmission is preferred when

$$|h|^2 < \left|g_{eff}\right|^2. \tag{3.56}$$

We note that the derivative of $E[P_{total}]$ with respect to P_s is

$$\frac{\partial E[P_{total}]}{\partial P_s} = 1 - \frac{|h|^2}{\left|g_{eff}\right|^2} \tag{3.57}$$

and (3.56) implies $\frac{\partial E[P_{total}]}{\partial P_s} > 0$, which means increasing P_s beyond $SNR_{target}/|f_i|^2$ until the value $SNR_{target}/|f_{i+1}|^2$ for $i = 1, \ldots, M$ does not change $A_R^\dagger$ but increases the expected value of the total power $E[P_{total}]$. Thus, the optimum source power P_s^{**} can be only one of the (M+1) discrete values in the set given by (3.21).

For $P_s = SNR_{target}/|f_i|^2$, one of the candidates of the optimum source power, and its corresponding reliable set $A_R^\dagger$, when γ increases, the expected value of the total power decreases, while the outage probability increases. Therefore, threshold $\gamma^\dagger$ should be chosen as the value that satisfies the outage probability with equality, i.e.,

$$\prod_{i \in A_R^\dagger} (1 - \int_{\gamma^\dagger}^{\infty} \frac{1}{2\sigma_{g_i}^2} \exp(-\frac{x_i}{2\sigma_{g_i}^2}) dx_i) = \rho_{target}. \tag{3.58}$$

It can be further reduced to

$$\prod_{i \in A_R^\dagger} (1 - \exp(-\frac{\gamma^\dagger}{2\sigma_{g_i}^2})) = \rho_{target}. \tag{3.59}$$

Let $\sigma_{g_{min}}^2 = \min\{\sigma_{g_i}^2, i \in A_R^\dagger\}$ and $\sigma_{g_{max}}^2 = \max\{\sigma_{g_i}^2, i \in A_R^\dagger\}$, we have

$$(1 - \exp(-\frac{\gamma^\dagger}{2\sigma_{g_{max}}^2}))^{|A_R^\dagger|} \leq \prod_{i \in A_R^\dagger} (1 - \exp(-\frac{\gamma^\dagger}{2\sigma_{g_i}^2})) \leq (1 - \exp(-\frac{\gamma^\dagger}{2\sigma_{g_{min}}^2}))^{|A_R^\dagger|}. \tag{3.60}$$

Therefore, $\gamma^\dagger$ is bounded as

$$\gamma_{min}^\dagger \leq \gamma^\dagger \leq \gamma_{max}^\dagger \tag{3.61}$$

where $\gamma_{min}^\dagger = -\ln(1 - (\rho_{target})^{\frac{1}{|A_R^\dagger|}}) \cdot 2\sigma_{g_{min}}^2$ and $\gamma_{max}^\dagger = -\ln(1 - (\rho_{target})^{\frac{1}{|A_R^\dagger|}}) \cdot 2\sigma_{g_{max}}^2$. The value of $\gamma^\dagger$ can be obtained by a search in the range $[\gamma_{min}^\dagger, \gamma_{max}^\dagger]$ numerically.

Note that for $P_s = SNR_{target}/|h|^2$, i.e., when the source can reach the destination via the direct link, $\gamma^\dagger = \infty$ to prevent any redundant relay transmission and power consumption.

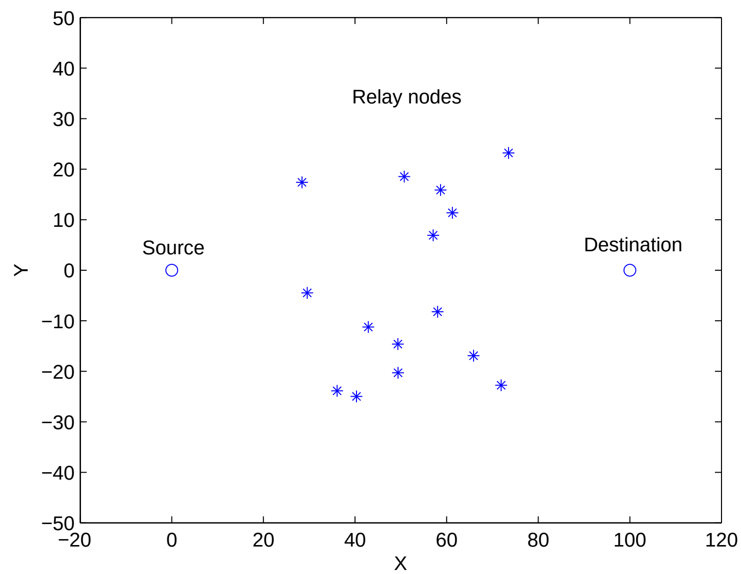

Figure 3.2. System set-up for the simulation.

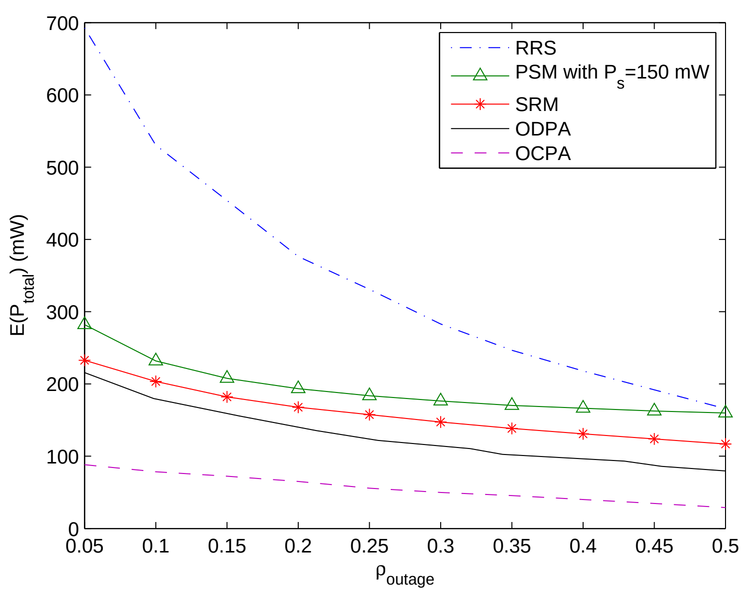

Figure 3.3. $E[P_{total}]$ vs ρ_{outage} for different power allocation schemes.

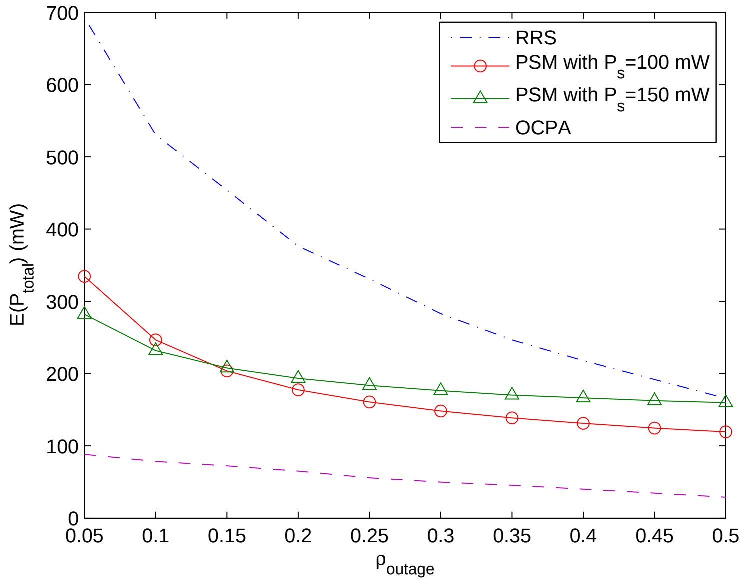

Figure 3.4. $E[P_{total}]$ vs ρ_{outage} for the passive source model (PSM).

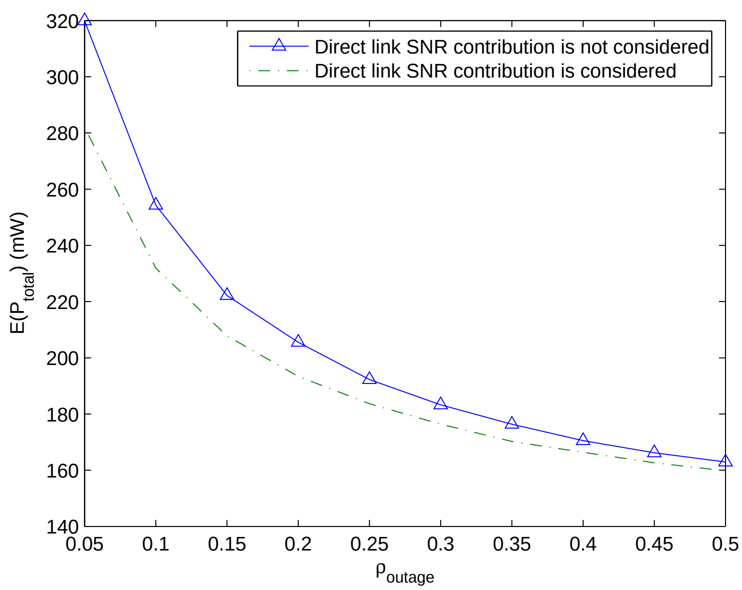

Figure 3.5. Effect of the direct link SNR contribution on the passive source model (PSM) ($P_s = 150\ mW$).

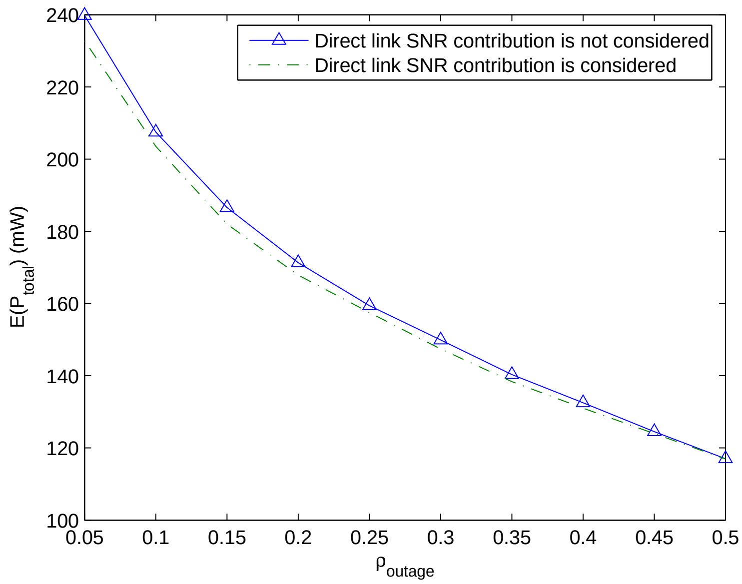

Figure 3.6. Effect of the direct link SNR contribution on the single relay model (SRM).

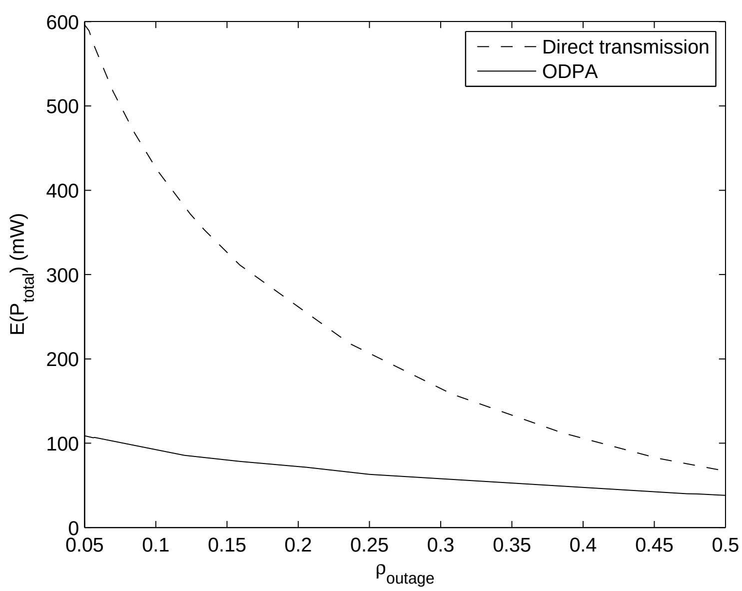

Figure 3.7. Comparison of the relay-assisted transmission scheme ODPA and the direct transmission scheme.

Chapter 4

Multiuser Two-Way Relaying with F/TDMA: Optimum Relay Power Allocation

4.1 Introduction

Recent demand on wireless ad hoc and peer-to-peer networking [3,68] has prompted interest towards the design of relay-assisted communication protocols. To this end, two-way relay networks, where nodes exchange information via the help of intermediate relay node(s) and make explicit use of the bi-directional nature of communication, have attracted recent attention.

A number of different protocols have been proposed for the two-way relay channel. These protocols aim to improve upon the traditional four-phase relaying in terms of throughput, achievable rates and power efficiency [40, 41, 43, 45, 46, 48–50, 69–71]. In the three-phase protocols proposed in [40, 41, 46], the two end nodes transmit sequentially to a relay which broadcasts the XORed version of the two nodes' symbols in the third phase after decoding them. In contrast, in the two-phase protocols, the nodes simultaneously transmit to the relay node in the first phase, and the relay, in the second phase, can amplify and forward [43, 48, 49], or compress and forward [52, 53] the received signal, or decode and forward an XORed or superposed symbol of both parties [43, 46, 50].

The two-way relay network model is extended to scenarios where nodes are equipped with multiple antennas as well as the systems with multiple pairs of communicating partners and/or multiple relay nodes. Transceiver design for MIMO two-way relaying is studied in [72–75]. For the networks consisting of one user pair assisted by multiple relay nodes, relay selection problem is studied in [76], and distributed space-time coding at relays is considered in [77]. Two-way relaying for multiple user pairs is considered in [43, 78], where there are a sufficiently large number of relay nodes, or antennas, such that the overall channels of all user pairs are orthogonalized by zero-forcing. Two-way amplify-and-forward relaying over OFDM is investigated in [79] where tone permutation and power allocation algorithms are found to maximize the sum rate of the two partners.

Wireless networks of the near future are most likely to consist of many nodes wishing to exchange information, potentially having to share intermediate relays. To that end, we consider the communication scenario where a *single* intermediate relay node assists multiple user pairs, henceforth termed, *multiuser two-way relaying*. In such a multiuser two-way communication system, using orthogonal channels to support multiple users is a valid choice under various scenarios and availability of bandwidth, and it can be accomplished by means of frequency or time division multiple access (F/TDMA). In this scenario, the relay's resources, most notably, its power, need to be appropriately shared between the pairs whose data exchange it shall aid. Therefore, in this chapter we address the problem of optimally allocating relay's power among all the user pairs it assists such that an arbitrary weighted sum rate of all users is maximized, for a variety of two-way relaying schemes including decode-and-forward (DF), amplify-and-forward (AF) and compress-and-forward (CF).

In order to characterize all resource allocation policies that would produce sum rate maximizing policies with different user priorities, we consider the weighted sum rate of all the communicating nodes as our performance metric. We formulate and solve the power allocation problem as one or a set of convex problems for each relaying scheme. Since the closed-form solution for the power allocation problem does not exist, we develop an iterative algorithm, which can be applied to all relaying schemes, to show how the relay power allocation is affected by the channel gains of different users as well as the amount of the available relay power. We demonstrate the performance of the optimum relay power allocation and the comparison among different two-way relaying schemes with numerical examples, and observe that the optimum power allocation achieves a significant performance improvement upon equal power allocation.

The remainder of the chapter is organized as follows. In Section 4.2, we present the system model, introduce the various two-way relaying mechanisms, and formulate the relay power allocation problem. We solve the power allocation problem for various relaying schemes in Section 4.3, 4.4 and 4.5, respectively. The aforementioned iterative algorithm is developed in Section 4.3 as well. Numerical results are presented in Section 4.6. Section 4.7 concludes this chapter.

4.2 System Description and Problem Formulation

We consider an F/TDMA multiuser two-way relay network shown in Figure 5.1, which consists of K pairs of users and an intermediate relay node. User a_i and b_i ($i \in [1, K]$) are a pair of pre-assigned partners who wish to communicate with each other. The relay node r is willing to devote its resources to assist the communications of all the user pairs. We assume that the users and the relay node are half-duplex and each is equipped with one antenna.

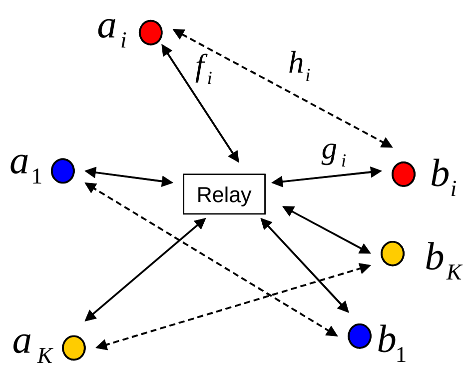

Figure 4.1. System model: F/TDMA multiuser two-way relay network.

4.2.1 Multiuser Two-way Relaying Schemes over Orthogonal Channels

We study both three- and two-phase decode-and-forward type relaying schemes and two-phase AF and CF relaying schemes. Each user pair is assigned an orthogonal channel in each phase of equal duration, by means of non-overlapping time/frequency slots with equal time duration/bandwidth[1]. The channel assignments of various multiuser two-way relaying schemes are shown in Figure 4.2. We assume reciprocal channels and denote f_i, g_i and h_i the channel coefficients of the links between a_i and the relay, b_i and the relay, and a_i and b_i on the ith channel, and assume all channels stay constant for the duration of the communication. Without loss of generality, at each receiver in different phase, we assume i.i.d. additive white Gaussian noise $n_{rx,p} \sim \mathcal{CN}(0, 1)$, where

[1]We assume equal time/frequency resource for each channel for the purpose of a clear exposition of the power allocation problem as well as for the sake of analyzing a practical setting that can potentially win its way into next generation standards. A more general "theoretical" resource allocation problem can jointly optimize the time/bandwidth as well as the power, at the expense of added complexity.

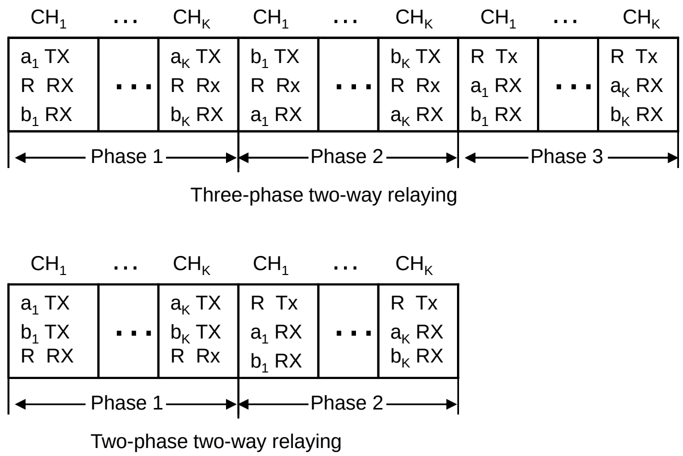

Figure 4.2. Channel assignment of various multiuser two-way relaying schemes.

$rx \in \{a_i, b_i, r_i\}$ denotes the receiver on the ith channel and $p \in \{1, 2, 3\}$ denotes the phase. In the following, we briefly describe the relaying schemes in consideration.

- **Three-phase Decode-and-Forward**: In three-phase protocols, user a_i transmits, and its partner b_i and the relay listen in the ith channel in phase one. The signals received at the relay and at the user b_i are[2]:

$$y_{r_i,1} = f_i \sqrt{P_{a_i}} x_{a_i} + n_{r_i,1}, \quad y_{b_i,1} = h_i \sqrt{P_{a_i}} x_{a_i} + n_{b_i,1}. \tag{4.1}$$

[2]The communication is based on transmission and reception of codewords in terms of signal sequences, however, due to the assumption of the memoryless channel, we consider single-letter formulation for simplicity.

Similarly, user b_i transmits in the second phase, and the signals received at the relay and at the user a_i on the ith channel are:

$$y_{r_i,2} = g_i\sqrt{P_{b_i}}x_{b_i} + n_{r_i,2}, \quad y_{a_i,2} = h_i\sqrt{P_{b_i}}x_{b_i} + n_{a_i,2} \tag{4.2}$$

where $\sqrt{P_{a_i}}x_{a_i}$ and $\sqrt{P_{b_i}}x_{b_i}$ denote the transmitted signals of users a_i and b_i with transmit power P_{a_i} and P_{b_i}, and x_{a_i} and x_{b_i} are drawn from Gaussian codebooks. Upon receiving the signals, the relay decodes the messages m_{a_i} of a_i and m_{b_i} of b_i. To help forward the messages in phase three, it can employ decode-and-superposition-forward (DSF), i.e., re-encode the messages individually and transmit

$$t_{r_i} = \sqrt{P_{r_{a_i}}}x_{r_{a_i}} + \sqrt{P_{r_{b_i}}}x_{r_{b_i}} \tag{4.3}$$

or employ decode-and-XOR-forward (DXF) [46], i.e., encode the message $m_i = m_{a_i} \oplus m_{b_i}$ and transmit

$$t_{r_i} = \sqrt{P_{r_i}}x_{r_i} \tag{4.4}$$

where $P_{r_{a_i}}$, $P_{r_{b_i}}$ and P_{r_i} denote the corresponding relay transmit power, and $x_{r_{a_i}}$, $x_{r_{b_i}}$ and x_{r_i} are drawn from Gaussian codebooks. The received signals at the users a_i and b_i, $i = 1, ..., K$, in phase three are

$$y_{a_i,3} = f_i t_{r_i} + n_{a_i,3}, \quad y_{b_i,3} = g_i t_{r_i} + n_{b_i,3}. \tag{4.5}$$

- **Two-phase Decode-and-Forward**: Different from the three-phase schemes, in the two-phase protocols, two users a_i and b_i transmit simultaneously on the ith channel in phase one, and the received signal at the relay is:

$$y_{r_i,1} = f_i\sqrt{P_{a_i}}x_{a_i} + g_i\sqrt{P_{b_i}}x_{b_i} + n_{r_i,1}. \tag{4.6}$$

Since users are half-duplex nodes, they do not hear from each other on the direct link in phase one. In the second phase, the relay can employ DSF or DXF relaying [43, 46, 76] after decoding m_{a_i} and m_{b_i}, same as in the three-phase schemes.

- **Amplify-and-Forward**: In phase two, the relay can simply amplify and forward (AF) the received signal from phase one [43, 48, 49], which results in

$$t_{r_i} = \alpha_i y_{r_i,1} \tag{4.7}$$

transmitted by the relay, where α_i is a scalar such that the relay's transmit power for the ith user pair is P_{r_i}, i.e.,

$$\alpha_i = \sqrt{\frac{P_{r_i}}{P_{a_i}|f_i|^2 + P_{b_i}|g_i|^2 + 1}}. \tag{4.8}$$

- **Compress-and-Forward**: Similar to AF, the relay node does not need to decode the users' signals perfectly in phase one. It can compress the received signal using Wyner-Ziv lossy source coding and broadcast the quantized version $\hat{x}_{r_i}$ with power

P_{r_i} [52,53], i.e.,

$$t_{r_i} = \sqrt{P_{r_i}}\hat{x}_{r_i}. \tag{4.9}$$

At the end of the last phase, each user can subtract its self-interference from the common signal broadcasted by the relay and decode its partner's message.

4.2.2 The Relay Power Allocation Problem

Next, we formulate the relay power allocation problem for the multiuser two-way relay network. The optimum power allocation problem is posed as allocating the relay power to different user pairs such that an arbitrary weighted sum rate of all users is maximized:

$$\max_{\{P_{r_i}\}_{i=1}^K} \quad \sum_{i=1}^K (w_{ab_i} R_{ab_i} + w_{ba_i} R_{ba_i}) \tag{4.10}$$

$$\text{s. t.} \quad \sum_{i=1}^K P_{r_i} \leq P_{r,total} \tag{4.11}$$

$$\{R_{ab_i}, R_{ba_i}\} \in R_* \tag{4.12}$$

where $P_{r,total}$ denotes the total relay power constraint[3], R_{ab_i} and R_{ba_i} denote the rate from user a_i to b_i and that from b_i to a_i, respectively, and R_* is the achievable rate region of one of the relaying schemes described in Section 4.2.1. These achievable rate regions will be presented in the following sections. For DSF schemes the optimization variables are $\{P_{r_{a_i}}, P_{r_{b_i}}\}$ instead of $\{P_{r_i}\}$ since we further have to distribute the power per pair to the two superposed codewords. The non-negative weights $\{w_{ab_i}, w_{ba_i}\}$ are to

[3] In TDMA, the total power constraint is converted from the average power constraint $\overline{P}_{r,total}$ due to the assumption of equal time slot duration.

indicate the priority of the traffic amongst different directions and pairs, a larger weight indicating priority. The resulting weighted sums for all $\{w_{ab_i}, w_{ba_i}\}$ clearly allow us to trace the boundary of the achievable rate region.

4.3 Decode-and-Superposition-Forward (DSF) Relaying

4.3.1 Three-phase DSF (3pDSF)

In the three-phase DSF relaying scheme, the users can hear directly from their partners in the first two phases. Hence, when the direct link has higher channel gain than the user-to-relay link, i.e., $|f_i|^2 \leq |h_i|^2$ or $|g_i|^2 \leq |h_i|^2$, the direct transmission achieves higher rate than the relay-assisted transmission. In this case, the relay should not allocate any power to assist such a user since it will not increase the rate no matter how much power it spends. We define sets S_a and S_b where $S_a = \{i||f_i|^2 > |h_i|^2\}$ and $S_b = \{i||g_i|^2 > |h_i|^2\}$, i.e., relay-assisted transmission can increase R_{ab_i} and R_{ba_i} for user pairs in S_a and S_b, respectively. The achievable rate region for 3pDSF relaying when different codebooks are used at the relay and the users is:

$$R_{ab_i} \leq \begin{cases} \frac{1}{3K}\min(C(P_{a_i}|f_i|^2), C(P_{a_i}|h_i|^2) + C(P_{r_{a_i}}|g_i|^2)), \forall i \in S_a; \\ \frac{1}{3K}C(P_{a_i}|h_i|^2), \text{otherwise} \end{cases} \tag{4.13}$$

$$R_{ba_i} \leq \begin{cases} \frac{1}{3K}\min(C(P_{b_i}|g_i|^2), C(P_{b_i}|h_i|^2) + C(P_{r_{b_i}}|f_i|^2)), \forall i \in S_b; \\ \frac{1}{3K}C(P_{b_i}|h_i|^2), \text{otherwise} \end{cases} \tag{4.14}$$

where $C(x) = \log(1 + x)$.

Accordingly, the equivalent power allocation problem can be expressed as:

$$\max_{\{P_{r_{a_i}},\, P_{r_{b_i}}\}_{i=1}^{K}} \quad \sum_{i\in S_a} \frac{w_{ab_i}}{3K} C(P_{r_{a_i}}|g_i|^2) + \sum_{i\in S_b} \frac{w_{ba_i}}{3K} C(P_{r_{b_i}}|f_i|^2) \tag{4.15}$$

$$\text{s. t.} \qquad \textstyle\sum_{i=1}^{K} P_{r_{a_i}} + P_{r_{b_i}} \leq P_{r,total} \tag{4.16}$$

$$C(P_{a_i}|h_i|^2) + C(P_{r_{a_i}}|g_i|^2) \leq C(P_{a_i}|f_i|^2),\ \forall i \in S_a \tag{4.17}$$

$$C(P_{b_i}|h_i|^2) + C(P_{r_{b_i}}|f_i|^2) \leq C(P_{b_i}|g_i|^2),\ \forall i \in S_b \tag{4.18}$$

$$P_{r_{i_a}}, P_{r_{i_b}} \geq 0,\ \forall i; \quad P_{r_{a_i}} = 0, \forall i \notin S_a; \quad P_{r_{b_i}} = 0, \forall i \notin S_b. \tag{4.19}$$

For simplicity, we have removed all the additive constant terms in the objective function. Note that 3pDSF can be considered an extension of the "one-way relaying philosophy", which superposes the codewords to both directions in phase three. The traffic in the two directions can be considered independently, and this problem has the same form as that of the one-way nonregenerative decode-and-forward relaying in [80]. Therefore, the optimum relay power allocation has the similar form of the modified water-filling solution as in [80]:

$$P_{r_{a_i}} = \min\left((\frac{w_{ab_i}}{\mu_0} - \frac{1}{|g_i|^2})^+, P^{max}_{r_{a_i},3p} \right), \forall i \in S_a \tag{4.20}$$

$$P_{r_{b_i}} = \min\left((\frac{w_{ba_i}}{\mu_0} - \frac{1}{|f_i|^2})^+, P^{max}_{r_{b_i},3p} \right), \forall i \in S_b \tag{4.21}$$

$$P_{r_{a_i}} = 0, \forall i \notin S_a; \quad P_{r_{b_i}} = 0, \forall i \notin S_b \tag{4.22}$$

where $P_{r_{a_i},3p}^{max} = \frac{P_{a_i}(|f_i|^2 - |h_i|^2)}{|g_i|^2(1+P_{a_i}|h_i|^2)}$, $P_{r_{b_i},3p}^{max} = \frac{P_{b_i}(|g_i|^2 - |h_i|^2)}{|f_i|^2(1+P_{b_i}|h_i|^2)}$ result from the constraints (4.17) and (4.18) with equality, μ_0 is the Lagrangian multiplier that satisfies the constraint (4.16) with equality, and $(.)^+ = \max(., 0)$. Note that the water level is a function of the weight coefficients.

4.3.2 Two-phase DSF (2pDSF)

The achievable rate region using 2pDSF relaying is described as [43, 76]:

$$R_{ab_i} \leq \frac{1}{2K} \min(C(P_{a_i}|f_i|^2), C(P_{r_{a_i}}|g_i|^2)), \forall i \tag{4.23}$$

$$R_{ba_i} \leq \frac{1}{2K} \min(C(P_{b_i}|g_i|^2), C(P_{r_{b_i}}|f_i|^2)), \forall i \tag{4.24}$$

$$R_{ab_i} + R_{ba_i} \leq \frac{1}{2K} C(P_{a_i}|f_i|^2 + P_{b_i}|g_i|^2), \forall i \tag{4.25}$$

where the first term of the RHS of (4.23) and (4.24) and the RHS of (4.25) are due to the multiple access channel (MAC) constraint in phase one, and the second term of the RHS of (4.23) and (4.24) are due to the broadcast channel (BC) constraint in phase two.

The equivalent power allocation problem can be expressed as:

$$\max_{\{P_{r_{a_i}},\, P_{r_{b_i}}\}_{i=1}^{K}} \quad \sum_{i=1}^{K} \frac{w_{ab_i}}{2K} C(P_{r_{a_i}}|g_i|^2) + \frac{w_{ba_i}}{2K} C(P_{r_{b_i}}|f_i|^2) \tag{4.26}$$

$$\text{s. t.} \quad \textstyle\sum_{i=1}^{K} P_{r_{a_i}} + P_{r_{b_i}} \leq P_{r,total} \tag{4.27}$$

$$C(P_{r_{a_i}}|g_i|^2) \leq C(P_{a_i}|f_i|^2), \forall i \tag{4.28}$$

$$C(P_{r_{b_i}}|f_i|^2) \leq C(P_{b_i}|g_i|^2), \forall i \tag{4.29}$$

$$C(P_{r_{a_i}}|g_i|^2) + C(P_{r_{b_i}}|f_i|^2) \leq C(P_{a_i}|f_i|^2 + P_{b_i}|g_i|^2), \forall i \tag{4.30}$$

$$P_{r_{a_i}} \geq 0, P_{r_{b_i}} \geq 0, \forall i. \tag{4.31}$$

Constraint (4.30) is a nonconvex set [81] over $(P_{r_{a_i}}, P_{r_{b_i}})$. Fortunately, a simple change of variables overcomes this hardship. Let $x_{a_i} = C(P_{r_{a_i}}|g_i|^2), x_{b_i} = C(P_{r_{b_i}}|f_i|^2)$, and the problem becomes

$$\max_{\{x_{a_i},\, x_{b_i}\}_{i=1}^{K}} \quad \tfrac{1}{2K} \textstyle\sum_{i=1}^{K} w_{ab_i} x_{a_i} + w_{ba_i} x_{b_i} \tag{4.32}$$

$$\text{s. t.} \quad \textstyle\sum_{i=1}^{K} (e^{x_{a_i}} - 1)/|g_i|^2 + (e^{x_{b_i}} - 1)/|f_i|^2 \leq P_{r,total} \tag{4.33}$$

$$x_{a_i} \leq c_{a_i} = C(P_{a_i}|f_i|^2), \forall i \tag{4.34}$$

$$x_{b_i} \leq c_{b_i} = C(P_{b_i}|g_i|^2), \forall i \tag{4.35}$$

$$x_{a_i} + x_{b_i} \leq c_i = C(P_{a_i}|f_i|^2 + P_{b_i}|g_i|^2), \forall i \tag{4.36}$$

$$x_{a_i} \geq 0, x_{b_i} \geq 0, \forall i. \tag{4.37}$$

For simplicity, we let $f(\mathbf{x}) = -\sum_{i=1}^{K}(w_{ab_i} x_{a_i} + w_{ba_i} x_{b_i})$ be the objective function where $\mathbf{x} = [x_1 \ x_2 \ \ldots \ x_K]^T$ with $x_i = [x_{a_i} \ x_{b_i}]^T$, $\mathbf{q}(\mathbf{x}) \leq \mathbf{0}$ be the vector of the inequality constraints where $\mathbf{q}(\mathbf{x})=[q_0(\mathbf{x})\ q_{a_1}(\mathbf{x})\ q_{b_1}(\mathbf{x})\ q_1(\mathbf{x})\ ...\ q_{a_K}(\mathbf{x})\ q_{b_K}(\mathbf{x})\ q_K(\mathbf{x})]^T$ and $q_0(\mathbf{x})=\sum_{i=1}^{K}(e^{x_{a_i}}-1)/|g_i|^2+(e^{x_{b_i}}-1)/|f_i|^2-P_{r,total}$, $q_{a_i}(\mathbf{x})=x_{a_i}-c_{a_i}$, $q_{b_i}(\mathbf{x})=x_{b_i}-c_{b_i}$, $q_i(\mathbf{x})=x_{a_i}+x_{b_i}-c_i$, $c_{a_i}=C(P_{a_i}|f_i|^2)$, $c_{b_i}=C(P_{b_i}|g_i|^2)$ and $c_i=C(P_{a_i}|f_i|^2+P_{b_i}|g_i|^2)$. Thus, the problem can be compactly rewritten as

$$\min_{\mathbf{x}\geq\mathbf{0}} \quad f(\mathbf{x}) \tag{4.38}$$

$$\text{s. t.} \quad \mathbf{q}(\mathbf{x}) \leq \mathbf{0}. \tag{4.39}$$

It can be verified that the above problem is convex [81], i.e., $\mathbf{f}$ is a convex function and the constraint $\mathbf{q} \leq \mathbf{0}$ and $\mathbf{x} \geq \mathbf{0}$ define a convex set, and hence has a unique global optimum. Thus, the optimum solution can be found via convex optimization techniques [81].

Since the closed-form solution for the power allocation problem does not exist, we next develop an iterative algorithm to show how the power allocation is affected by the channel gains of different users as well as the available relay power. Let $\mu = [\mu_0\ \mu_{a_1}\ \mu_{b_1}\ \mu_1\ ...\ \mu_{a_K}\ \mu_{b_K}\ \mu_K]^T$ be the vector of Lagrangian multipliers corresponding to the inequality constraints in $\mathbf{q}(\mathbf{x})$. The Lagrangian dual problem is

$$\max \quad \theta(\mu) \tag{4.40}$$

$$\text{s. t.} \quad \mu \geq \mathbf{0} \tag{4.41}$$

where $\theta(\mu) = \min_{\mathbf{x}\geq\mathbf{0}} f(\mathbf{x})+\mu^T\mathbf{q}(\mathbf{x})$. It can be verified that $f(\mathbf{x})$ and $\mathbf{q}(\mathbf{x})$ are both convex and there exists an $\hat{\mathbf{x}} \geq 0$ such that $\mathbf{q}(\hat{\mathbf{x}}) < 0$. Therefore, the primal and dual problems satisfy strong duality [81, Ch. 6.2], i.e., there is no duality gap. In particular, we can use subgradient method [81] to iteratively solve the Lagrangian dual problem, which also finds the optimum solution to the primal problem. The iterative algorithm so developed is presented in Table 4.1[4].

Remark 4.1. *In the first iteration ($n = 1$), we set $\mu_0 = 0$ which means we temporarily remove the total power constraint, and hence the problem becomes a linear programming problem.*

Remark 4.2. *Updating x_{a_i}, x_{b_i} in the nth iteration for $n > 1$ is equivalent to updating the relay power allocation as*

$$P_{r_{a_i}}(n) = \left(\frac{1}{\mu_0(n)}(w_{ab_i} - \mu_{a_i}(n) - \mu_i(n)) - \frac{1}{|g_i|^2}\right)^+ \tag{4.42}$$

$$P_{r_{b_i}}(n) = \left(\frac{1}{\mu_0(n)}(w_{ba_i} - \mu_{b_i}(n) - \mu_i(n)) - \frac{1}{|f_i|^2}\right)^+. \tag{4.43}$$

We observe that, the power allocation $P_{r_{a_i}}$ ($P_{r_{b_i}}$) is a modified water-filling solution, with a base level at $\frac{1}{|g_i|^2}$ ($\frac{1}{|f_i|^2}$). The water level depends on μ_0, μ_{a_i} (μ_{b_i}) and μ_i, and we use the subgradient method to update μ_0 and all $\{\mu_i, \mu_{a_i}, \mu_{b_i}\}$.

Note that the iterative algorithm can be used to solve other convex problems emerging in the following sections by properly replacing the corresponding objective function and constraints.

[4] Table 4.1 is located at the end of this chapter.

4.4 Decode-and-XOR-Forward (DXF) Relaying

Unlike the DSF scheme where the portions of the relay power for forwarding the messages to two partners individually control the rates on two directions, in DXF relaying, the relay forwards a single XORed message with power P_{r_i} for the ith pair of users, and consequently, P_{r_i} simultaneously controls the rates on both directions. The achievable rate region of the three-phase and two-phase DXF relaying schemes can be obtained by replacing both $P_{r_{a_i}}$ and $P_{r_{b_i}}$ by P_{r_i} in (4.13)-(4.14), and in (4.23)-(4.25), respectively [46].

4.4.1 Three-phase DXF (3pDXF)

For 3pDXF relaying, we have the following observations:

- Similar as in 3pDSF two-way relaying, the relay-assisted transmission only helps users in S_a or S_b and hence no relay power is allocated to users in $\overline{S_a \cup S_b}$, recalling that $S_a = \{i||f_i|^2 > |h_i|^2\}$ and $S_b = \{i||g_i|^2 > |h_i|^2\}$.

- Assigning more relay power to the ith user pair in $S_{ab} = S_a \cap S_b$ increases R_{ab_i} until $P_{r_i} \geq P^{max}_{r_{a_i},3p}$, and also increases R_{ba_i} until $P_{r_i} \geq P^{max}_{r_{b_i},3p}$, where $P^{max}_{r_{a_i},3p}$ and $P^{max}_{r_{b_i},3p}$ are given in Section 4.3.1. Therefore, we further partition S_{ab} as S_{ab1} and S_{ab2}, where $S_{ab1} = \{i|i \in S_{ab} \text{ and } P^{max}_{r_{a_i},3p} \geq P^{max}_{r_{b_i},3p}\}$, and $S_{ab2} = S_{ab} \backslash S_{ab1}$. In set S_{ab1}, increasing P_{r_i} beyond $P^{max}_{r_{b_i},3p}$ but below $P^{max}_{r_{a_i},3p}$ will increase the data rate R_{ab_i} but not R_{ba_i} since it has reached the upper bound. Similarly, in set S_{ab2}, increasing P_{r_i} beyond $P^{max}_{r_{a_i},3p}$ but below $P^{max}_{r_{b_i},3p}$ will increase R_{ba_i} but not R_{ab_i}.

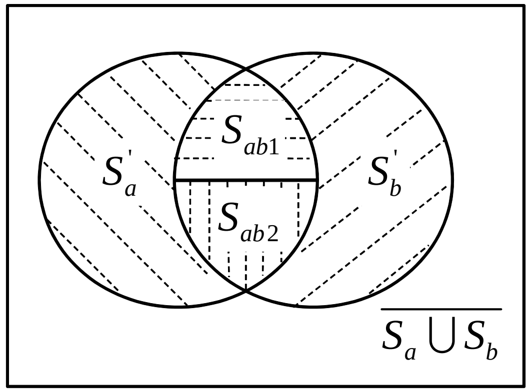

Figure 4.3. User partition for 3pDXF relaying.

Figure 4.3 illustrates the above partition of the users. The achievable rates of the users in different sets are as follows:

- when $i \in S_{ab1}$ or $i \in S_{ab2}$,

$$R_{ab_i} = \frac{1}{3K} \min\left(C(P_{a_i}|f_i|^2), C(P_{a_i}|h_i|^2) + C(P_{r_i}|g_i|^2)\right) \tag{4.44}$$

$$R_{ba_i} = \frac{1}{3K} \min\left(C(P_{b_i}|g_i|^2), C(P_{b_i}|h_i|^2) + C(P_{r_i}|f_i|^2)\right) \tag{4.45}$$

- when $i \in S'_a$

$$R_{ab_i} = \frac{1}{3K} \min\left(C(P_{a_i}|f_i|^2), C(P_{a_i}|h_i|^2) + C(P_{r_i}|g_i|^2)\right) \tag{4.46}$$

$$R_{ba_i} = \frac{1}{3K} C(P_{b_i}|h_i|^2) \tag{4.47}$$

- when $i \in S'_b$

$$R_{ab_i} = \frac{1}{3K} C(P_{a_i} |h_i|^2) \tag{4.48}$$

$$R_{ba_i} = \frac{1}{3K} \min\Big(C(P_{b_i}|g_i|^2), C(P_{b_i}|h_i|^2) + C(P_{r_i}|f_i|^2)\Big) \tag{4.49}$$

- when $i \in \overline{S_a \cup S_b}$

$$R_{ab_i} = \frac{1}{3K} C(P_{a_i} |h_i|^2) \tag{4.50}$$

$$R_{ba_i} = \frac{1}{3K} C(P_{b_i} |h_i|^2) \tag{4.51}$$

Thus, the relay power allocation problem is equivalent to:

$$\max_{\{P_{r_i},\hat{P}_{r_i}\}_{i=1}^K} \sum_{i \in S_{ab1}} \left(\frac{w_{ab_i}}{3K} C(P_{r_i}|g_i|^2) + \frac{w_{ba_i}}{3K} C(\hat{P}_{r_i}|f_i|^2)\right) + \sum_{i \in S'_a} \frac{w_{ab_i}}{3K} C(P_{r_i}|g_i|^2)$$
$$+ \sum_{i \in S_{ab2}} \left(\frac{w_{ab_i}}{3K} C(\hat{P}_{r_i}|g_i|^2) + \frac{w_{ba_i}}{3K} C(P_{r_i}|f_i|^2)\right) + \sum_{i \in S'_b} \frac{w_{ba_i}}{3K} C(P_{r_i}|f_i|^2) \tag{4.52}$$

$$\text{s. t.} \quad \sum_{i=1}^{K} P_{r_i} \le P_{r,total}; \quad \hat{P}_{r_i} \ge 0, P_{r_i} \ge 0, \forall i; \tag{4.53}$$

$$\hat{P}_{r_i} \le P_{r_i}, \hat{P}_{r_i} \le P^{max}_{r_{b_i},3p}, P_{r_i} \le P^{max}_{r_{a_i},3p}, \text{ for } i \in S_{ab1} \tag{4.54}$$

$$\hat{P}_{r_i} \le P_{r_i}, \hat{P}_{r_i} \le P^{max}_{r_{a_i},3p}, P_{r_i} \le P^{max}_{r_{b_i},3p}, \text{ for } i \in S_{ab2} \tag{4.55}$$

$$P_{r_i} \le P^{max}_{r_{a_i},3p}, \text{ for } i \in S'_a; \quad P_{r_i} \le P^{max}_{r_{b_i},3p}, \text{ for } i \in S'_b \tag{4.56}$$

$$P_{r_i} = 0, \text{ for } i \in \overline{S_a \cup S_b}; \quad \hat{P}_{r_i} = 0, \quad \text{for } i \in S'_a \cup S'_b \cup \overline{S_a \cup S_b}. \tag{4.57}$$

For simplicity, we have removed all the additive constant terms in the objective function. The new variables $\{\hat{P}_{r_i}\}$ in the problem formulation, which are not the actual power allocation, are introduced to ensure that the upper bound of R_{ba_i} for $i \in S_{ab1}$ and that of R_{ab_i} for $i \in S_{ab2}$ are not violated in the problem formulation. The problem is convex, consisting of concave objective function and linear constraints, and the global optimum can be found. Note that $\hat{P}_{r_i}$=min$(P_{r_i}, P^{max}_{r_{b_i},3p})$ for i∈S_{ab1} and $\hat{P}_{r_i}$=min$(P_{r_i}, P^{max}_{r_{a_i},3p})$ for i∈S_{ab2} in the optimum solution.

4.4.2 Two-phase DXF (2pDXF)

We first define the thresholds $P^{max}_{r_{a_i},2p} = P_{a_i}|f_i|^2/|g_i|^2$ and $P^{max}_{r_{b_i},2p} = P_{b_i}|g_i|^2/|f_i|^2$. We note that the relay-assisted transmission can potentially increase R_{ab_i} when $P_{r_i} \leq P^{max}_{r_{a_i},2p}$, and increase R_{ba_i} when $P_{r_i} \leq P^{max}_{r_{b_i},2p}$, under the sum rate constraint $R_{ab_i} + R_{ba_i} \leq C(P_{a_i}|f_i|^2 + P_{b_i}|g_i|^2)/2K$. The equivalent sum rate constraints for various relay powers are as follows:

$$C(P_{r_i}|g_i|^2) + C(P_{b_i}|g_i|^2) \leq C(P_{a_i}|f_i|^2 + P_{b_i}|g_i|^2), \text{ if } P^{max}_{r_{b_i},2p} \leq P_{r_i} \leq P^{max}_{r_{a_i},2p} \quad (4.58)$$

$$C(P_{a_i}|f_i|^2) + C(P_{r_i}|f_i|^2) \leq C(P_{a_i}|f_i|^2 + P_{b_i}|g_i|^2), \text{ if } P^{max}_{r_{a_i},2p} \leq P_{r_i} \leq P^{max}_{r_{b_i},2p} \quad (4.59)$$

$$C(P_{r_i}|g_i|^2)+C(P_{r_i}|f_i|^2) \leq C(P_{a_i}|f_i|^2+P_{b_i}|g_i|^2), \text{ if } P_{r_i} < \min(P^{max}_{r_{a_i},2p}, P^{max}_{r_{b_i},2p}) \quad (4.60)$$

or, equivalently, we have $P_{r_i} \leq P^{max}_{r_i,2p}$ with

$$P^{max}_{r_i,2p} = \begin{cases} P^{max1}_{r_i,2p} = \frac{P_{a_i}|f_i|^2}{|g_i|^2(1+P_{b_i}|g_i|^2)}, \text{ if } P^{max}_{r_{b_i},2p} \leq P_{r_i} \leq P^{max}_{r_{a_i},2p} \\ P^{max2}_{r_i,2p} = \frac{P_{b_i}|g_i|^2}{|f_i|^2(1+P_{a_i}|f_i|^2)}, \text{ if } P^{max}_{r_{a_i},2p} \leq P_{r_i} \leq P^{max}_{r_{b_i},2p} \\ P^{max3}_{r_i,2p}, \text{ if } P_{r_i} < \min(P^{max}_{r_{a_i},2p}, P^{max}_{r_{b_i},2p}) \end{cases} \tag{4.61}$$

where $P^{max3}_{r_i,2p} = \frac{-(|g_i|^2+|f_i|^2)+\sqrt{(|g_i|^2+|f_i|^2)^2+4|g_i|^2|f_i|^2(P_{a_i}|f_i|^2+P_{b_i}|g_i|^2)}}{2|g_i|^2|f_i|^2}$.

Next, we partition all user pairs as sets $\tilde{S}_{ab}$, $\tilde{S}_a$ and $\tilde{S}_b$, where

$$\tilde{S}_{ab} = \{i | P^{max3}_{r_i,2p} \leq \min(P^{max}_{r_{a_i},2p}, P^{max}_{r_{b_i},2p})\} \tag{4.62}$$

$$\tilde{S}_a = \{i | P^{max3}_{r_i,2p} > \min(P^{max}_{r_{a_i},2p}, P^{max}_{r_{b_i},2p}) = P^{max}_{r_{b_i},2p}\} \tag{4.63}$$

$$\tilde{S}_b = \{i | P^{max3}_{r_i,2p} > \min(P^{max}_{r_{a_i},2p}, P^{max}_{r_{b_i},2p}) = P^{max}_{r_{a_i},2p}\}. \tag{4.64}$$

Let us assume arbitrary subsets $\tilde{S}_{a1} \subseteq \tilde{S}_a$ and $\tilde{S}_{b1} \subseteq \tilde{S}_b$, and define $\tilde{S}_{a2} = \tilde{S}_a \backslash \tilde{S}_{a1}$ and $\tilde{S}_{b2} = \tilde{S}_b \backslash \tilde{S}_{b1}$. Figure 4.4 illustrates the user partition. The achievable rates of the users in different sets are as follows:

- when $i \in \tilde{S}_{ab}$ or $i \in \tilde{S}_{a1}$ or $i \in \tilde{S}_{b1}$

$$R_{ab_i} = \frac{1}{2K} \min\left(C(P_{a_i}|f_i|^2), C(P_{r_i}|g_i|^2)\right) \tag{4.65}$$

$$R_{ba_i} = \frac{1}{2K} \min\left(C(P_{b_i}|g_i|^2), C(P_{r_i}|f_i|^2)\right) \tag{4.66}$$

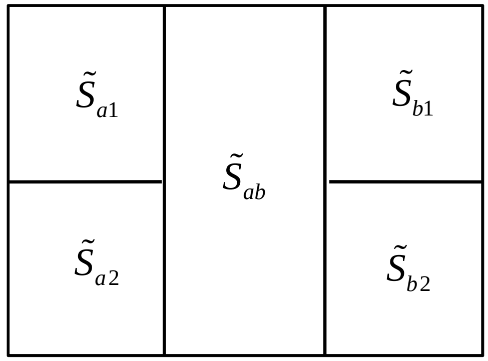

Figure 4.4. User partition for 2pDXF relaying.

- when $i \in \tilde{S}_{a2}$

$$R_{ab_i} = \frac{1}{2K} \min \left(C(P_{a_i}|f_i|^2), C(P_{r_i}|g_i|^2) \right) \tag{4.67}$$

$$R_{ba_i} = \frac{1}{2K} C(P_{b_i}|g_i|^2) \tag{4.68}$$

- when $i \in \tilde{S}_{b2}$

$$R_{ab_i} = \frac{1}{2K} C(P_{a_i}|f_i|^2) \tag{4.69}$$

$$R_{ba_i} = \frac{1}{2K} \min \left(C(P_{b_i}|g_i|^2), C(P_{r_i}|f_i|^2) \right) \tag{4.70}$$

Thus, the relay power allocation problem is equivalent to:

$$\max_{\{P_{r_i}\},\tilde{S}_{a1},\tilde{S}_{b1}} \sum_{i\in\tilde{S}_{ab}\cup\tilde{S}_{a1}\cup\tilde{S}_{b1}} \frac{w_{ab_i}}{2K}C(P_{r_i}|g_i|^2)+\frac{w_{ba_i}}{2K}C(P_{r_i}|f_i|^2)$$

$$+\sum_{i\in\tilde{S}_{a2}}\frac{w_{ab_i}}{2K}C(P_{r_i}|g_i|^2)+\sum_{i\in\tilde{S}_{b2}}\frac{w_{ba_i}}{2K}C(P_{r_i}|f_i|^2) \tag{4.71}$$

$$\text{s. t.} \quad \sum_{i=1}^{K} P_{r_i} \le P_{r,total} \tag{4.72}$$

$$0\le P_{r_i}\le P_{r_i,2p}^{max3}, \forall i\in\tilde{S}_{ab};\ 0\le P_{r_i}\le P_{r_{b_i},2p}^{max}, \forall i\in\tilde{S}_{a1};\ 0\le P_{r_i}\le P_{r_{a_i},2p}^{max}, \forall i\in\tilde{S}_{b1} \tag{4.73}$$

$$P_{r_{b_i},2p}^{max} < P_{r_i} \le P_{r_i,2p}^{max1}, \forall i\in\tilde{S}_{a2};\quad P_{r_{a_i},2p}^{max} < P_{r_i} \le P_{r_i,2p}^{max2}, \forall i\in\tilde{S}_{b2} \tag{4.74}$$

$$\tilde{S}_{a1}\subseteq\tilde{S}_a,\quad \tilde{S}_{b1}\subseteq\tilde{S}_b. \tag{4.75}$$

Note that for fixed $\tilde{S}_{a1}$ and $\tilde{S}_{b1}$, the above problem is convex. The optimum solution can be found by comparing the solutions corresponding to different $(\tilde{S}_{a1},\tilde{S}_{b1})$.

4.5 Amplify-and-Forward (AF) and Compress-and-Forward (CF) Relaying

The achievable rate region for AF [43] and CF relaying [53] [52] are given as:

$$AF: \quad R_{ab_i} \le \frac{1}{2K}C\Big(\frac{P_{r_i}|f_i|^2|g_i|^2P_{a_i}}{P_{r_i}|g_i|^2+P_{a_i}|f_i|^2+P_{b_i}|g_i|^2+1}\Big), \forall i \tag{4.76}$$

$$R_{ba_i} \le \frac{1}{2K}C\Big(\frac{P_{r_i}|f_i|^2|g_i|^2P_{b_i}}{P_{r_i}|f_i|^2+P_{a_i}|f_i|^2+P_{b_i}|g_i|^2+1}\Big), \forall i \tag{4.77}$$

$$CF: \quad R_{ab_i} \le \frac{1}{2K}C\Big(\frac{P_{r_i}|f_i|^2|g_i|^2P_{a_i}}{P_{r_i}|g_i|^2+P_{a_i}|f_i|^2+1}\Big), \forall i \tag{4.78}$$

$$R_{ba_i} \le \frac{1}{2K}C\Big(\frac{P_{r_i}|f_i|^2|g_i|^2P_{b_i}}{P_{r_i}|f_i|^2+P_{b_i}|g_i|^2+1}\Big), \forall i. \tag{4.79}$$

The relay power allocation problem for AF and CF relaying can thus be expressed as:

$$\max_{\{P_{r_i}\}} \quad f_{AF/CF}(P_{r_1}, P_{r_2}, ...P_{r_K}) \tag{4.80}$$

$$\text{s. t.} \quad \textstyle\sum_{i=1}^{K} P_{r_i} \leq P_{r,total}, \quad P_{r_i} \geq 0, \forall i \tag{4.81}$$

where

$$\begin{aligned} f_{AF/CF} = & \textstyle\sum_{i=1}^{K} \frac{w_{ab_i}}{2K} C(\frac{P_{r_i}|f_i|^2|g_i|^2 P_{a_i}}{P_{r_i}|g_i|^2+P_{a_i}|f_i|^2+P_{b_i}|g_i|^2+1}) \\ & +\frac{w_{ba_i}}{2K} C(\frac{P_{r_i}|f_i|^2|g_i|^2 P_{b_i}}{P_{r_i}|f_i|^2+P_{a_i}|f_i|^2+P_{b_i}|g_i|^2+1}) \end{aligned} \tag{4.82}$$

for AF relaying, and

$$f_{AF/CF} = \sum_{i=1}^{K} \frac{w_{ab_i}}{2K} C(\frac{P_{r_i}|f_i|^2|g_i|^2 P_{a_i}}{P_{r_i}|g_i|^2 + P_{a_i}|f_i|^2 + 1}) + \frac{w_{ba_i}}{2K} C(\frac{P_{r_i}|f_i|^2|g_i|^2 P_{b_i}}{P_{r_i}|f_i|^2 + P_{b_i}|g_i|^2 + 1}) \tag{4.83}$$

for CF relaying. Again, the problem is convex since the objective function is a concave function with respect to $\{P_{r_i}\}$ for both AF and CF relaying. Unlike the decode-and-forward relaying schemes, the AF scheme does not have decodability constrains. In addition, for both cases the objective function is an increasing function of P_{r_i} and approaches to the upper bound $\sum_{i=1}^{K} \frac{w_{ab_i}}{2K} C(P_{a_i}|f_i|^2) + \frac{w_{ab_i}}{2K} C(P_{b_i}|g_i|^2)$ as P_{r_i} goes to infinity. Comparing the achievable rate region of CF with that of AF, we observe that CF has the same numerator but a smaller denominator on the RHS of (4.78) and (4.79). Therefore, with the same power allocation, CF achieves higher rates than AF.

4.6 Numerical Results

In this section, we present numerical results to demonstrate the performance of the optimum power allocation for various multiuser two-way relaying schemes[5].

We first consider a network with 3 user pairs and one relay node[6], as shown in Figure 4.5. The users are located at $\{(-90m, 90m), (90m, 90m), (-150m, 0), (50m, 0), (-70m, -70m), (120m, -120m)\}$ and the relay is located at the origin. The channel gains $\{|f_i|^2, |g_i|^2, |h_i|^2\}$ are inversely proportional to the fourth power of the corresponding distances between the nodes. We set the users' transmit powers to $P_{a_i} = P_{b_i} = 0.1\ Watts$, and assume all AWGN terms have variance 10^{-10}. Setting $w_{ab_i} = w_{ba_i} = 1$ for all i, we investigate the rate and the sum rate of users achieved by the optimum relay power allocation with different multiuser two-way relaying schemes, for a range of total relay power levels. In Figure 4.6-4.9 we plot the individual rate or sum rate of the user pairs achieved by DSF, DXF, AF and CF relaying with optimum relay power allocation. Figure 4.10 and 4.11 present the corresponding power allocation for DXF and CF schemes respectively, which reflect the similar observations obtained from Figure 4.8 and 4.9.

In Figure 4.6 and 4.7 for DSF relaying, we observe that when the relay power is relatively low, the relay first allocates power to assist users a_2, b_3, a_1 and b_1 since they have the best relay-to-partner links, equivalently, the lowest base levels in the modified water-filling solution. Note that for the whole range of relay power shown in the figures, a_2 and b_3 have already reached their rate upper bounds. As the available relay power

[5] Figures of this section are located at the end of this chapter.

[6] The circles plotted in Figure 4.5 are to show the different distances from the users to the relay node.

becomes higher, the relay starts to help users b_2 and a_3 who have lower relay-to-partner links. Eventually all users arrive their rate upper bounds.

In Figure 4.8 for DXF relaying, we observe that, as the total relay power increases, the power allocated to a user pair can stay at some non-zero value before it increases for the second time. At that power level, one partner of the user pair reaches its rate upper bound while the other one has not. This is due to the asymmetric channels of the pair of users, such as user pairs a_2 and b_2, a_3 and b_3.

In Figure 4.9 for AF and CF relaying, we observe that the sum rate of each user pair converges to its maximum point gradually. CF scheme always outperforms AF scheme as expected, especially when the relay power is low.

In Figure 4.12, we compare the sum rate of all users achieved by different relaying schemes. The sum rate achieved by direct transmission is also included in the figure. We observe that different relaying schemes outperform one another for different range of relay power. When the relay has a low power budget, three-phase schemes outperform two-phase ones due to the dominating contribution from the direct links. As the relay power increases, two-phase schemes may become better when the relay-assisted transmissions dominate the rates since the pre-log factor is 1/2 for two-phase schemes while it is 1/3 for three-phase schemes. As observed in Figure 4.6-4.9, while the relay power keeps increasing, all schemes eventually reach (DSF/DXF) or approach (AF/CF) their upper bounds. We also note that the DXF schemes outperform the corresponding DSF schemes until they reach their upper bounds, because forwarding an XORed message to both directions is more power efficient than forwarding individual messages. From the above observations, we conclude that, given a relay power budget, we can always choose the relaying scheme (DXF/CF) and the corresponding power allocation algorithm to obtain the highest weighted sum rate.

Next, we compare the average performance of the optimum and equal power allocation, with the latter equally distributing the relay power among all the assisted users. We generate 100 networks with the relay at the origin and 3 user pairs randomly distributed in the area of $[-250m, 250m]^2$. For each network, we generate 1000 Rayleigh fading realizations, i.e., the channel coefficients follow the Rayleigh distribution with the variances following the path-loss model. For every realization, at different relay power, we find the highest sum rate achieved by one of the considered relaying schemes with the optimum and the equal power allocation, respectively. The results are averaged over all fading and network topology realizations and presented in Figure 4.13. We observe that the optimum power allocation achieves a significant sum rate performance gain upon the equal power allocation, especially when the relay power is low.

4.7 Conclusion

In this chapter, we have considered a multiuser two-way relay network where multiple pairs of partners exchange information via a shared intermediate relay node over orthogonal channels. For a variety of two-way relaying protocols including DF, AF and CF, we have formulated and solved the optimum relay power allocation problem that maximizes an arbitrary weighted sum of rates in the network, thereby tracing the boundary of the achievable rate region for each relaying scheme. We have shown and compared the performance of different two-way relaying schemes with optimum power allocation, and demonstrated their significant performance gain over equal power allocation. We have thus provided design guidelines for F/TDMA multiuser two-way relay networks to fully utilize the available relay resources in terms of power.

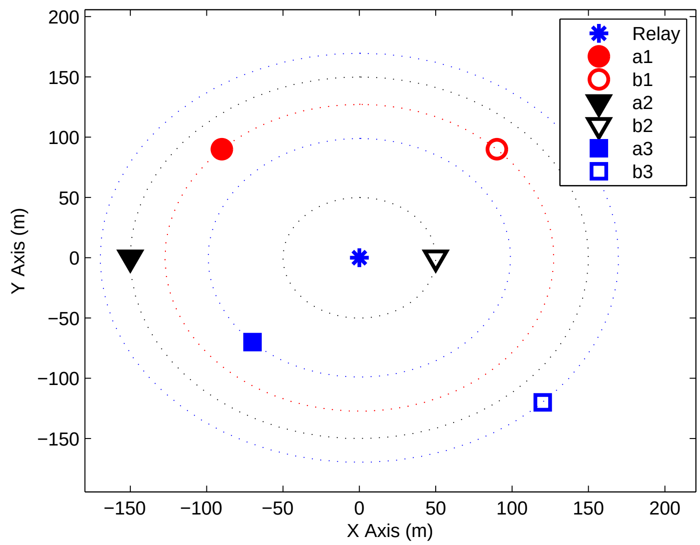

Figure 4.5. A multiuser two-way relay network realization.

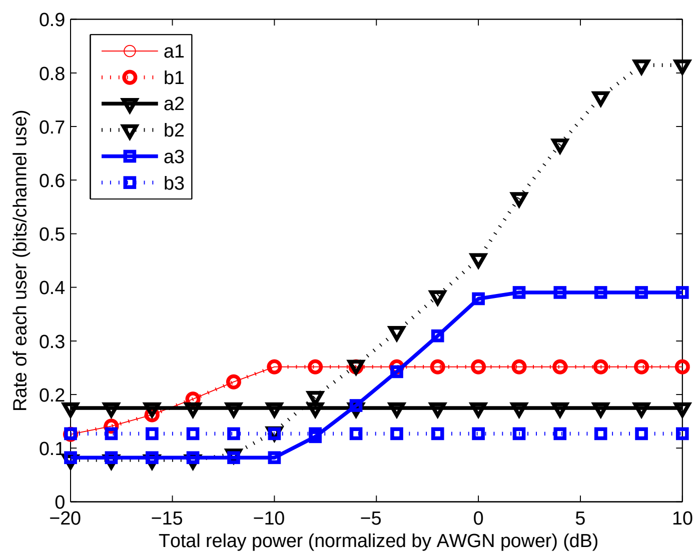

Figure 4.6. Achievable rates: three-phase DSF relaying.

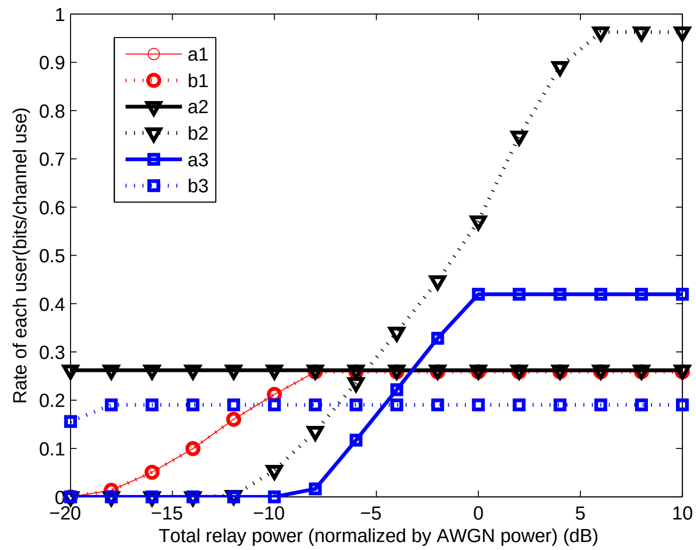

Figure 4.7. Achievable rates: two-phase DSF relaying.

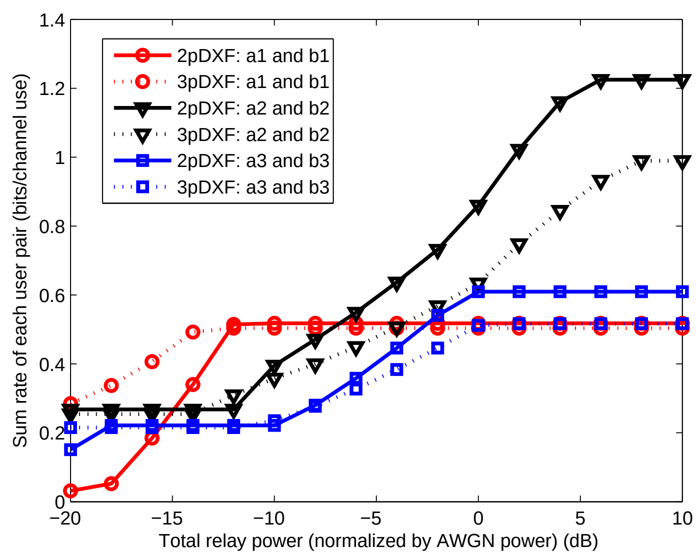

Figure 4.8. Achievable rates: three-phase and two-phase DXF relaying.

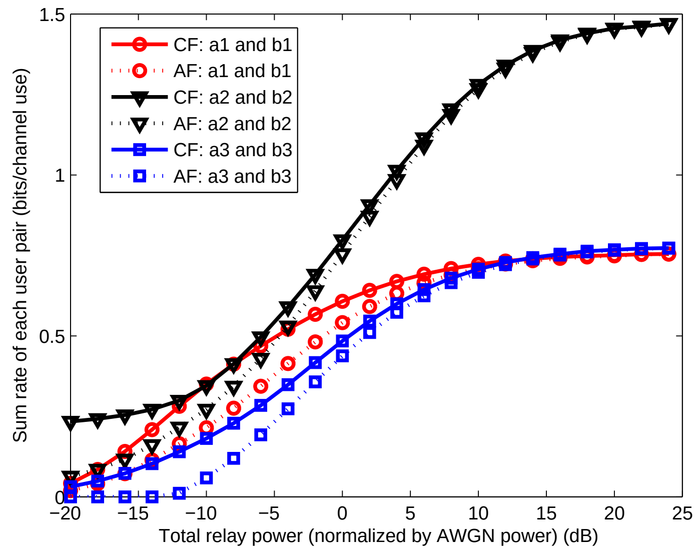

Figure 4.9. Achievable rates: AF and CF relaying.

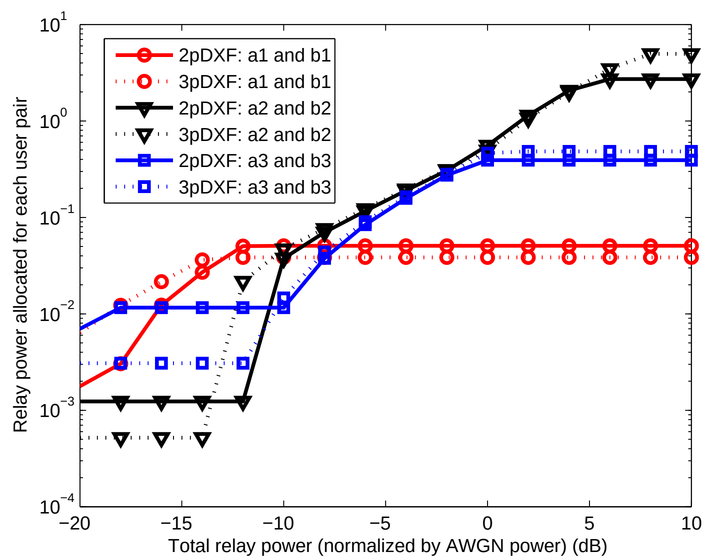

Figure 4.10. Relay power allocation: DXF relaying.

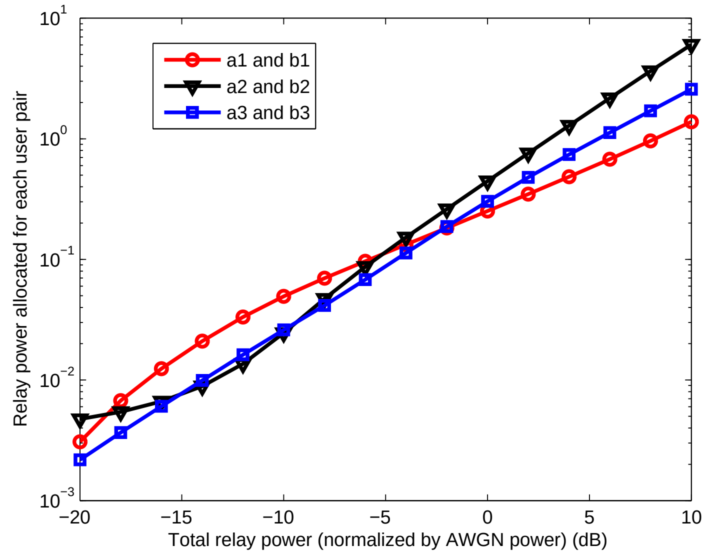

Figure 4.11. Relay power allocation: CF relaying.

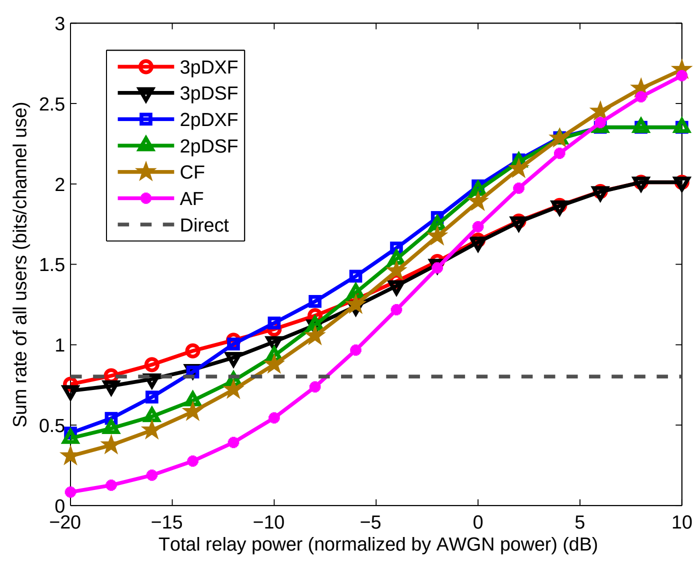

Figure 4.12. Comparison of various two-way relaying schemes with optimum power allocation.

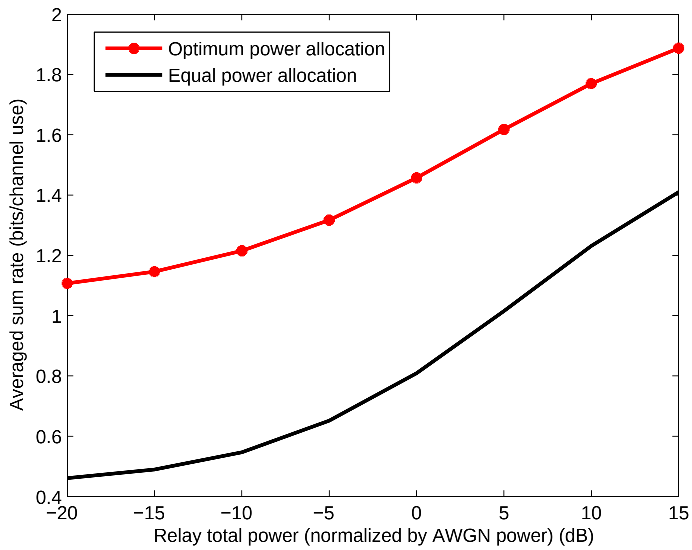

Figure 4.13. Comparison of optimum power allocation and equal power allocation.

Table 4.1. Iterative algorithm solving power allocation problem for 2pDSF relaying

Initialization Step Set the maximum allowed number of iteration $n_{max} \in \mathbb{N}^+$, stop criterions $\epsilon_1, \epsilon_2, \epsilon_3 > 0$. Set the start iteration index $n = 1$. Select a start solution $\mu(1) = 0$. Let the current optimum objective value be $\theta_{best} = -\infty$, the current optimum value for $f(\mathbf{x})$ be $f_{best} = \infty$.

Main Step

Step 1): Start with $\mu(n)$, solve $\max_{\mathbf{x}\geq 0} f(\mathbf{x}) + (\mu(n))^T \mathbf{q}(\mathbf{x})$

Specifically, for 2pDSF:
If $n = 1$,
If $w_{ab_i} \geq w_{ba_i}$,
let $x_{a_i} = c_{a_i}, x_{b_i} = c_i - c_{a_i}$;
else
let $x_{a_i} = c_i - c_{b_i}, x_{b_i} = c_{b_i}$;
end;
else
If $-w_{ab_i} + \mu_{a_i}(n) + \mu_i(n) < 0$,
let $x_{a_i} = (\log \frac{|g_i|^2}{\mu_0(n)}(w_{ab_i} - \mu_{a_i}(n) - \mu_i(n)))^+$;
else
let $x_{a_i} = 0$;
end;
If $-w_{ba_i} + \mu_{b_i}(n) + \mu_i(n) < 0$,
let $x_{b_i} = (\log \frac{|f_i|^2}{\mu_0(n)}(w_{ba_i} - \mu_{b_i}(n) - \mu_i(n)))^+$;
else
let $x_{b_i} = 0$;
end;
end;
Let the solution be $\mathbf{x}(n)$; Go to Step 2;

Step 2): Calculate $\theta(\mu(n)) = f(\mathbf{x}(n)) + (\mu(n))^T \mathbf{q}(\mathbf{x}(n))$;
If $\theta(\mu(n)) > \theta_{best}$,
let $\theta_{best} = \theta(\mu(n))$ and $\mu_{best} = \mu(n)$;
end; Go to Step 3;

Step 3): Calculate $\mathbf{q}(\mathbf{x}(n))$;
If $\mathbf{q}(\mathbf{x}(n)) \leq 0$,
If $f(\mathbf{x}(n)) \leq f_{best}$,
let $f_{best} = f(\mathbf{x}(n))$ and $x_{best} = \mathbf{x}(n)$;
end;
end; Go to Step 4;

Step 4): If $|\frac{f_{best} - \theta_{best}}{\theta_{best}}| < \epsilon_1$,
Stop;
end; Go to Step 5;

Step 5): Let subgradient $\xi(n) = \mathbf{q}(\mathbf{x}(n))$;
If $||\xi(n)|| < \epsilon_2$,
Stop;
end;
Let $\bar{\mu}(n+1) = \mu(n) + \lambda(n)\frac{\xi(n)}{||\xi(n)||}$;
(Note: the update step size $\lambda(n)$ is set using block halving method [81, Ch. 8.9].)
Let $\mu(n+1) = (\bar{\mu}(n+1))^+$;
If $||\mu(n+1) - \mu(n)|| < \epsilon_3$,
Stop;
end; Go to Step 6;

Step 6): Replace n by $n+1$;
If $n \leq n_{max}$,
Go to Step 1;
else
Stop;
end;

Chapter 5

Multiuser Two-Way Relaying with CDMA: Detection and Interference Management Strategies

5.1 Introduction

Two-way relaying [40, 41], where the intermediate relay(s) help communicating nodes exchange information, has recently emerged as a means to facilitate relay-assisted cooperation in ad hoc and peer-to-peer wireless networks. A variety of two-way relaying protocols have been proposed, relying on decode-and-forward (DF) [40, 41, 43, 46], amplify-and-forward (AF) [43, 48, 49] and compress-and-forward relaying (CF) [52, 53], and have shown significant improvement on spectral efficiency upon one-way relaying.

Two-way relaying for multiple users via a sufficiently large number of relay nodes or relay antennas is considered in [43, 78]. In Chapter 4, we have proposed a multiuser two-way relay network with multiple user pairs assisted by a shared relay, to accommodate the scenario when the number of relay nodes is limited and users need to share the relay(s). We have studied the optimum relay power allocation problem for this system when multiuser communications take place on orthogonal channels by means of frequency or time division multiple access (F/TDMA). While the employment of F/TDMA avoids multiuser interference and simplifies the receiver's design, the requirement of orthogonal channels for all user pairs may be undesired when bandwidth efficiency or delay is a concern. In such a scenario, a reasonable choice to support multiple users is code division multiple access (CDMA) whose merits and limitations are well understood in the context of one-way cellular communications as well as some ad hoc settings [6, 82].

It is well known that CDMA systems are interference limited, and can provide reliable communication for a desired number of users at a quality of service (QoS) level and processing gain. Multiuser two-way relaying systems employing CDMA are naturally interference limited as well: random locations and different surroundings of communicating nodes result in different radio propagation on the communication channels, affecting greatly the level of interference experienced by each user in the system, and hence the QoS of the communications. Therefore, *interference management* is a key design issue for the CDMA multiuser two-way relay systems and is a focus of this chapter: via careful choice of the *relaying scheme* as well as the *transmit and receive strategies*, we aim to reduce and control the interference experienced by each user such that the system QoS requirements are met with minimum power expenditure at all users and the relay.

In this chapter, we first design the multiuser two-way relaying strategy for the system at hand. We observe that, the bi-directional nature of communications allows each pair of users to share a common spreading signature, which, when used with the appropriate detection and relaying strategy, significantly reduces the multiple access interference (MAI). The communication is carried out in two phases: all users transmit to the relay simultaneously in phase one, and the relay jointly demodulates and generates an estimate of the XORed symbol to broadcast for each user pair in phase two. This relaying scheme is referred to as *jointly demodulate-and-XOR forward* (JD-XOR-F) in the sequel.

Next, we derive the decision rules and the corresponding bit error rates (BERs) at the relay and at the end users when the JD-XOR-F relaying is employed. We further consider interference management for this multiuser two-way relay network via joint power control and receiver optimization for each phase, and construct iterative algorithms, each of which is shown to converge to the corresponding unique optimum. We validate our theoretical findings by showing numerical examples where the proposed JD-XOR-F relaying with interference management is observed to provide significant power savings and improve the user capacity over the designs with a "one-way" communication perspective.

The remainder of the chapter is organized as follows. In Section 5.2, we introduce the system model and propose the JD-XOR-F relaying. We derive the decision rule and the BER for phase one in Section 5.3, and investigate the power control and receiver optimization problem for phase one in Section 5.4. Phase two is studied in Section 5.5 and 5.6. Numerical results are presented in Section 5.7. We discuss the system-wide optimization in Section 5.8. Section 5.9 concludes the chapter.

5.2 System Model and JD-XOR-F Relaying

We consider the multiuser two-way relay network shown in Figure 5.1, which consists of $2K + 1$ nodes: K pairs of users and an intermediate relay node. User i_1 and i_2 ($i \in [1, K]$) are a pair of partners who wish to communicate with each other via the relay node 0. Each node wishes to communicate with one pre-assigned partner and sees the remaining transmissions as interference. We assume all nodes are half-duplex and there is no direct link between partners. The information exchange is done in two

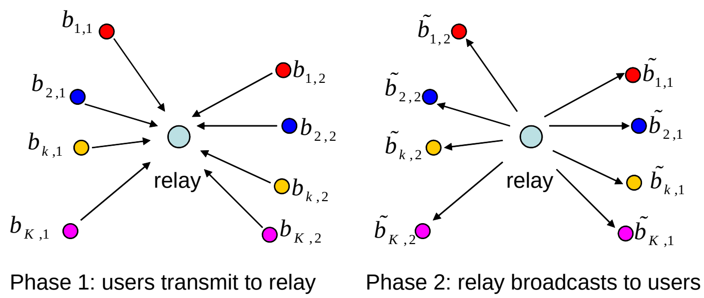

Figure 5.1. System model: CDMA multiuser two-way relay network.

phases. The first phase is for the transmissions from all the users to the relay, and the second phase is dedicated to the transmission from the relay to all the users.

To accommodate the communications of multiple pairs of users simultaneously, direct sequence (DS)-CDMA is employed. For clarity of exposition, we assume a synchronous[1] DS-CDMA system with coherent detection and real-valued signal processing. We assume that non-orthogonal signatures are employed and the spreading gain is N. If CDMA is employed in the traditional sense, i.e., for "one-way" communication, each user's symbol needs to be spread by a distinct signature, $2K$ signatures are needed, and the number of interfering users is $2K-1$. On the other hand, in two-way communications, each user can recover its partner's symbol from a common signal broadcasted from the relay, utilizing the side information that is *its own symbol*. Therefore, we propose that in the multiuser two-way relay network, each pair of users transmit with the same

[1]The synchronous model can be extended to asynchronous systems with time offset or phase offset when the channel state information is known at the related receivers.

signature waveform. This way, only K signatures are needed, and the multiple access interference (MAI) present in the system can be (potentially) reduced.

Considering the bi-directional communication structure, we propose the *jointly demodulate-and-XOR forward* (JD-XOR-F) relaying scheme as follows. The relay node receives the signals superimposed from all users in the first phase, employs a linear multiuser detector to jointly demodulate and decide for each pair whether the two partners have sent the same symbol. Once this decision is made, the relay can generate the XORed symbol to transmit in the second phase for that user pair, since the XOR operation of two binary symbols is equivalent to indicating whether they are the same. Receiving this from the relay, each user can employ a linear multiuser detector to suppress interference contributed by users other than its partner and itself, and then recover its partner's symbol by an XOR operation of the received XORed symbol with the symbol it transmitted in the first phase.

5.3 Phase One: Decision Rule and the BER

In phase one, all users transmit their symbols to the relay simultaneously. The ith pair, users i_1 and i_2, use their common signature waveform $s_i(t)$ to spread their symbols $b_{i,1}$ and $b_{i,2}$, with transmit power $p_{i,1}$ and $p_{i,2}$, respectively. The index t indicates time. The channel gain from user i_m to the relay is denoted by $h_{i,m}$ for $i = 1, ..., K$ and $m = 1, 2$. Reciprocal channels are assumed, i.e., the channel gain from the relay to user i_m is $h_{i,m}$ as well. The channel gains stay constant for the duration of the communication. The received signal at the relay is given by

$$r_0(t) = \sum_{i=1}^{K} (\sqrt{p_{i,1}h_{i,1}}b_{i,1} + \sqrt{p_{i,2}h_{i,2}}b_{i,2})s_i(t) + n_0(t) \tag{5.1}$$

where $n_0(t)$ denotes the additive white Gaussian noise (AWGN) at the relay. We assume that $b_{i,1}, b_{i,2} \in \{-1, +1\}$ with equal probability. The discrete-time equivalent received signal at the output of the chip matched filter is

$$\mathbf{r}_0 = \sum_{i=1}^{K} (\sqrt{p_{i,1}h_{i,1}}b_{i,1} + \sqrt{p_{i,2}h_{i,2}}b_{i,2})\mathbf{s}_i + \mathbf{n}_0 \tag{5.2}$$

where $\mathbf{s}_i$ denotes the unit norm spreading sequence, $\mathbf{n}_0$ is the zero-mean Gaussian random vector with $E[\mathbf{n}_0\mathbf{n}_0^\mathsf{T}] = \sigma_{n_0}^2\mathbf{I}_N$, $(\cdot)^\mathsf{T}$ denotes the transpose operation, and $\mathbf{I}_N$ denotes the $N \times N$ identity matrix. In the sequel, we will use this discrete-time representation.

Upon receiving $\mathbf{r}_0$, the relay employs linear filter $\mathbf{c}_i$ to obtain $y_i = \mathbf{c}_i^\mathsf{T}\mathbf{r}_0$ as

$$\begin{aligned} y_i &= (\sqrt{p_{i,1}h_{i,1}}b_{i,1} + \sqrt{p_{i,2}h_{i,2}}b_{i,2})\mathbf{c}_i^\mathsf{T}\mathbf{s}_i \\ &\quad + \underbrace{\sum_{j\neq i}^{K} (\sqrt{p_{j,1}h_{j,1}}b_{j,1} + \sqrt{p_{j,2}h_{j,2}}b_{j,2})\mathbf{c}_i^\mathsf{T}\mathbf{s}_j + \mathbf{c}_i^\mathsf{T}\mathbf{n}_0}_{N_i} && (5.3) \\ &= \sqrt{q_{i,1}}b_{i,1} + \sqrt{q_{i,2}}b_{i,2} + N_i && (5.4) \end{aligned}$$

where $q_{i,1} = p_{i,1}h_{i,1}(\mathbf{c}_i^\mathsf{T}\mathbf{s}_i)^2$ and $q_{i,2} = p_{i,2}h_{i,2}(\mathbf{c}_i^\mathsf{T}\mathbf{s}_i)^2$ denote the received power of user i_1 and i_2 at the output of the filter, and N_i denotes the interference plus noise term. Let $\sigma_i^2 = \sum_{j\neq i}(p_{j,1}h_{j,1}+p_{j,2}h_{j,2})(\mathbf{c}_i^\mathsf{T}\mathbf{s}_j)^2+\sigma_{n_0}^2\mathbf{c}_i^\mathsf{T}\mathbf{c}_i$ be the variance of N_i. We approximate

N_i with a Gaussian. This makes the detection for each user pair and the BER analysis tractable. As we will observe in Section 5.7, the analytical results we obtain in the following match with the simulation results.

The relay uses y_i to make a decision in favor of one of two hypotheses:

$$H_{i0} : b_{i,1} = b_{i,2} \tag{5.5}$$

$$H_{i1} : b_{i,1} \neq b_{i,2}. \tag{5.6}$$

Notice that this is tantamount to determining $\hat{b}_i$, the estimate of $b_i = b_{i,1} \oplus b_{i,2}$, where $\oplus$ represents XOR, the bitwise exclusive operation[2]. Specifically, we have $\hat{b}_i = -1$ when the relay decides $b_{i,1}$ and $b_{i,2}$ are same (H_{i0}) and $\hat{b}_i = 1$ if they have the opposite sign (H_{i1}). The maximum aposteriori probability (MAP) decision rule can be expressed as

$$\frac{\Pr(y_i|b_{i,1} = b_{i,2}) \Pr(b_{i,1} = b_{i,2})}{\Pr(y_i|b_{i,1} \neq b_{i,2}) \Pr(b_{i,1} \neq b_{i,2})} \underset{\hat{b}_i=1}{\overset{\hat{b}_i=-1}{\gtrless}} 1. \tag{5.7}$$

With the Gaussian assumption on N_i, this is equivalent to the following:

$$e^{-\frac{(y_i-(-\sqrt{q_{i,1}}-\sqrt{q_{i,2}}))^2}{2\sigma_i^2}} + e^{-\frac{(y_i-(\sqrt{q_{i,1}}+\sqrt{q_{i,2}}))^2}{2\sigma_i^2}} \underset{\hat{b}_i=1}{\overset{\hat{b}_i=-1}{\gtrless}}$$
$$e^{-\frac{(y_i-(-\sqrt{q_{i,1}}+\sqrt{q_{i,2}}))^2}{2\sigma_i^2}} + e^{-\frac{(y_i-(\sqrt{q_{i,1}}-\sqrt{q_{i,2}}))^2}{2\sigma_i^2}}. \tag{5.8}$$

[2] "0" is mapped to "$\hat{b}_i = -1$" and "1" to "$\hat{b}_i = 1$".

Based on (5.8), we find that the optimum decision rule is:

$$\hat{b}_i = \begin{cases} 1, & \text{when} \quad y_i \in R^{opt} = \{y_i | -y_{th}^{opt} < y_i < y_{th}^{opt}\}; \\ -1, & \text{otherwise.} \end{cases} \tag{5.9}$$

where the decision threshold y_{th}^{opt} is the positive root of the following equation:

$$1 + e^{\frac{2y_{th}^{opt}(\sqrt{q_{i,1}}+\sqrt{q_{i,2}})}{\sigma_i^2}} = \left(e^{\frac{2y_{th}^{opt}\sqrt{q_{i,1}}}{\sigma_i^2}} + e^{\frac{2y_{th}^{opt}\sqrt{q_{i,2}}}{\sigma_i^2}} \right) e^{\frac{2\sqrt{q_{i,1}}\sqrt{q_{i,2}}}{\sigma_i^2}} \tag{5.10}$$

which can be shown to have two real roots with identical absolute values. Under this decision rule, $Pe1_i^{opt}$, the probability that the relay makes an incorrect decision on $\hat{b}_i$, is obtained as

$$\begin{aligned} Pe1_i^{opt} &= \frac{1}{2}\left(Q\left(\frac{-y_{th}^{opt} + \sqrt{q_{i,1}} + \sqrt{q_{i,2}}}{\sigma_i} \right) + Q\left(\frac{-y_{th}^{opt} - \sqrt{q_{i,1}} - \sqrt{q_{i,2}}}{\sigma_i} \right) \right. \\ & \left. + Q\left(\frac{y_{th}^{opt} - \sqrt{q_{i,1}} + \sqrt{q_{i,2}}}{\sigma_i} \right) + Q\left(\frac{y_{th}^{opt} + \sqrt{q_{i,1}} - \sqrt{q_{i,2}}}{\sigma_i} \right) - 1 \right) \end{aligned} \tag{5.11}$$

where $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$. Since the above decision rule does not have a closed-form expression for the decision threshold in general[3], it may bring implementation difficulties in practice and the evaluation of the BER may become intractable.

[3] When a pair of users have equal received power level, the analytical expression for the decision threshold y_{th}^{opt} is found as given in [50, 83].

Alternatively, we can consider a four-hypothesis testing model with $H_{i(0,0)}$, $H_{i(0,1)}$, $H_{i(1,0)}$, $H_{i(1,1)}$ corresponding to $(b_{i,1}, b_{i,2})$ =$(-1,-1)$, $(-1,1)$, $(1,-1)$ and $(1,1)$, respectively. The relay first uses the MAP rule to detect $(\hat{b}_{i,1}, \hat{b}_{i,2})$, i.e.,

$$(\hat{b}_{i,1}, \hat{b}_{i,2}) = \underset{(\hat{b}_{i,1},\hat{b}_{i,2})\in\{(-1,-1),(1,-1),(-1,1),(1,1)\}}{\arg\max} \Pr\left(y_i|(\hat{b}_{i,1}, \hat{b}_{i,2})\right)\Pr\left(\hat{b}_{i,1}, \hat{b}_{i,2}\right) \tag{5.12}$$

and then generates $\hat{b}_i$ as $\hat{b}_i = \hat{b}_{i,1} \oplus \hat{b}_{i,2}$. While this decision rule has a potential performance loss[4] as compared to the one based on the two-hypothesis model as our aim is to estimate the XORed quantity rather than the individual bits, the performance loss is negligible as we will observe in Section 5.7, and it greatly simplifies the decision rule to be:

$$\hat{b}_i = \begin{cases} 1, \quad \text{when} \quad y_i \in R = \{y_i| - y_{th} < y_i < y_{th}\}; \\ -1, \quad \text{otherwise.} \end{cases} \tag{5.13}$$

with

$$y_{th} = \begin{cases} \sqrt{q_{i,1}}, \quad \text{when} \quad q_{i,1} \geq q_{i,2}; \\ \sqrt{q_{i,2}}, \quad \text{otherwise.} \end{cases} \tag{5.14}$$

Under this decision rule, $Pe1_i$, the probability of incorrectly determining $\hat{b}_i$ is

$$Pe1_i = \begin{cases} Q(\sqrt{\eta_{i,2}}) + \frac{1}{2}Q(2\sqrt{\eta_{i,1}} - \sqrt{\eta_{i,2}}) - \frac{1}{2}Q(2\sqrt{\eta_{i,1}} + \sqrt{\eta_{i,2}}), \quad \text{when} \quad \eta_{i,1} \geq \eta_{i,2} \\ Q(\sqrt{\eta_{i,1}}) + \frac{1}{2}Q(2\sqrt{\eta_{i,2}} - \sqrt{\eta_{i,1}}) - \frac{1}{2}Q(2\sqrt{\eta_{i,2}} + \sqrt{\eta_{i,1}}), \quad \text{when} \quad \eta_{i,1} < \eta_{i,2} \end{cases} \tag{5.15}$$

[4]The four-hypothesis approach has the same performance as the two-hypothesis one in the absence of noise.

where $\eta_{i,1} = q_{i,1}/\sigma_i^2$ and $\eta_{i,2} = q_{i,2}/\sigma_i^2$ are the received SIR of user i_1 and i_2 at the relay.

Note that having $\eta_{i,1}$=0 or $\eta_{i,2}$=0 leads to $Pe1_i$=0.5, which is not a desired situation in communication systems. In the sequel, we will exclude this case. We then have the following lemma.

LEMMA 5.1. *The error probability* $Pe1_i(\eta_{i,1}, \eta_{i,2})$ *is a quasiconvex function of* $(\eta_{i,1}, \eta_{i,2})$ *on both* R_1 *and* R_2*, where* $R_1 = \{(\eta_{i,1}, \eta_{i,2}) | \eta_{i,1} > 0,\ \eta_{i,2} > 0,\ \text{and}\ \eta_{i,1} \geq \eta_{i,2}\}$*, and* $R_2 = \{(\eta_{i,1}, \eta_{i,2}) | \eta_{i,1} > 0,\ \eta_{i,2} > 0,\ \text{and}\ \eta_{i,1} < \eta_{i,2}\}$.

Proof: See Appendix 5.10.1. □

5.4 Phase One: Power Control and Receiver Optimization

Having determined the BER performance for given transmit power levels of all users and the receivers at the relay, let us now turn to optimum interference management in phase one. Specifically, we will seek to expend the minimum total transmit power while satisfying the BER requirement on estimating each XORed symbol by jointly optimizing the powers and the receivers.

$$\min_{\{p_{i,1}, p_{i,2}, \mathbf{c}_i\}} \quad \textstyle\sum_{i=1}^{K} (p_{i,1} + p_{i,2}) \tag{5.16}$$

$$\text{s. t.} \quad Pe1_i \leq \rho 1_i \tag{5.17}$$

$$p_{i,1} \geq 0, \quad p_{i,2} \geq 0, \quad \mathbf{c}_i \in \mathbb{R}^N, \quad \forall i \tag{5.18}$$

where $\rho 1_i$ is the corresponding system QoS requirement which is given.

At the outset, one might be tempted to think that a direct application of the existing power control algorithms for (one-way) CDMA systems [59,84,85] would work for the two-way system. In reality, the very advantage that the bi-directional communication presents, i.e., the use of a common signature per pair in conjunction with JD-XOR-F relaying, is the reason why a direct application does not work. In particular, the probability that the relay estimates an incorrect XORed symbol of a pair of users becomes a function of the received signal-to-interference ratio (SIR) of both partners, and we no longer have a one-to-one mapping between the error probability and the partners' received SIRs. To tackle this difficulty, employing the four-hypothesis decision rule found in Section 5.3 and the property of the corresponding BER in Lemma 5.1, we first convert each BER constraint in (5.17) to its equivalent received SIR constraint by solving a convex problem, and then implement a suitable iterative power control and receiver optimization algorithm whose convergence to the optimum is guaranteed.

First, let us fix the power levels of all users but the ith pair, $\{p_{j,1}\}_{j \neq i}$ and $\{p_{j,2}\}_{j \neq i}$, and the linear filter $\mathbf{c}_i$. The optimization problem in (5.16)-(5.18) then reduces to the following optimization problem over the transmit power of the ith user pair only with fixed $\sigma_i^2/(\mathbf{c}_i^\mathsf{T}\mathbf{s}_i)^2$,

$$\min_{(\eta_{i,1},\eta_{i,2})} \quad g(\eta_{i,1},\eta_{i,2}) = (\frac{\eta_{i,1}}{h_{i,1}} + \frac{\eta_{i,2}}{h_{i,2}}) \cdot \frac{\sigma_i^2}{(\mathbf{c}_i^\mathsf{T}\mathbf{s}_i)^2} \tag{5.19}$$

$$\text{s. t.} \qquad Pe1_i \leq \rho1_i \tag{5.20}$$

$$\eta_{i,1} > 0, \quad \eta_{i,2} > 0. \tag{5.21}$$

Note that since $Pe1_i$ is expressed as a function of $(\eta_{i,1}, \eta_{i,2})$ in (5.15), we replace the variables $p_{i,1}$ and $p_{i,2}$ in (5.16) by $\eta_{i,1}$ and $\eta_{i,2}$, using the relationship $p_{i,1} = \frac{\eta_{i,1}\sigma_i^2}{h_{i,1}(\mathbf{c}_i^\intercal \mathbf{s}_i)^2}$ and $p_{i,2} = \frac{\eta_{i,2}\sigma_i^2}{h_{i,2}(\mathbf{c}_i^\intercal \mathbf{s}_i)^2}$, and let $g(\eta_{i,1}, \eta_{i,2})$ denote the objective function.

Let the feasible set of the problem in (5.19)-(5.21) be S, which can be partitioned into two sets, $S_1 = \{(\eta_{i,1}, \eta_{i,2}) | (\eta_{i,1}, \eta_{i,2}) \in S, \text{ and } \eta_{i,1} \geq \eta_{i,2}\}$ and $S_2 = \{(\eta_{i,1}, \eta_{i,2}) | (\eta_{i,1}, \eta_{i,2}) \in S, \text{ and } \eta_{i,1} < \eta_{i,2}\}$. Since it is proved in Lemma 5.1 that $Pe1_i$ is quasiconvex on R_1 and R_2, the corresponding lower level sets S_1 and S_2 are both convex sets [81, Ch. 3]. Therefore, replacing S by S_1 and S_2 respectively in problem (5.19)-(5.21) leads to two convex problems, each having a unique optimum solution, $(\eta^\dagger_{i,1}, \eta^\dagger_{i,2})$ and $(\eta^{\dagger\dagger}_{i,1}, \eta^{\dagger\dagger}_{i,2})$, respectively, due to the fact that each problem minimizes a linear objective function g over a convex feasible set. Thus, the optimum solution of problem (5.19)-(5.21), $(\eta^\star_{i,1}, \eta^\star_{i,2})$, can be obtained as

$$\begin{cases} \eta^\star_{i,1} = \eta^\dagger_{i,1}, \eta^\star_{i,2} = \eta^\dagger_{i,2}, \text{ when } g(\eta^\dagger_{i,1}, \eta^\dagger_{i,2}) \leq g(\eta^{\dagger\dagger}_{i,1}, \eta^{\dagger\dagger}_{i,2}) \\ \eta^\star_{i,1} = \eta^{\dagger\dagger}_{i,1}, \eta^\star_{i,2} = \eta^{\dagger\dagger}_{i,2}, \text{ otherwise.} \end{cases} \tag{5.22}$$

We note that minimizing g is equivalent to minimizing $\frac{\eta_{i,1}}{h_{i,1}} + \frac{\eta_{i,2}}{h_{i,2}}$ since $\sigma_i^2/(\mathbf{c}_i^\intercal \mathbf{s}_i)^2$ is fixed. Therefore, $(\eta^\star_{i,1}, \eta^\star_{i,2})$ depends only on $\{\rho 1_i, h_{i,1}, h_{i,2}\}$.

Now, we define two quantities γ_i and α_i as

$$\gamma_i = \frac{\eta^\star_{i,1}}{h_{i,1}} + \frac{\eta^\star_{i,2}}{h_{i,2}} \tag{5.23}$$

$$\alpha_i = \frac{p^\star_{i,1}}{p^\star_{i,2}} = \frac{\eta^\star_{i,1}}{h_{i,1}} \cdot \frac{h_{i,2}}{\eta^\star_{i,2}} \tag{5.24}$$

where $(\eta^{\star}_{i,1}, \eta^{\star}_{i,2})$ is the optimum solution of the problem in (5.19)-(5.21). Note that γ_i and α_i depend only on $\{\rho 1_i, h_{i,1}, h_{i,2}\}$. We have the following proposition.

PROPOSITION 5.1. *The constraint (5.17) is satisfied by the following conditions on the transmit power pair* $(p_{i,1}, p_{i,2})$*:*

$$p_{i,1} + p_{i,2} \geq \gamma_i \frac{\sigma_i^2}{(\mathbf{c}_i^{\mathsf{T}} \mathbf{s}_i)^2} \quad \text{and} \quad \frac{p_{i,1}}{p_{i,2}} = \alpha_i. \tag{5.25}$$

Equivalently, (5.25) is a sufficient condition for $Pe1_i \leq \rho 1_i$*, with the minimum total transmit power requirement on the* i*th user pair.*

Proof: See Appendix 5.10.2. □

Thus, after we solve the optimization problem (5.19)-(5.21) for the ith user pair, we can replace (5.17) by (5.25) and rewrite the optimization problem in (5.16)-(5.18) as

$$\min_{\{p_{i,1}, p_{i,2}, \mathbf{c}_i\}} \quad \textstyle\sum_{i=1}^{K} (p_{i,1} + p_{i,2}) \tag{5.26}$$

$$\text{s. t.} \quad p_{i,1} + p_{i,2} \geq \gamma_i \frac{\sigma_i^2}{(\mathbf{c}_i^{\mathsf{T}} \mathbf{s}_i)^2} \quad \text{and} \quad \frac{p_{i,1}}{p_{i,2}} = \alpha_i \tag{5.27}$$

$$p_{i,1} > 0, \quad p_{i,2} > 0, \quad \mathbf{c}_i \in \mathbb{R}^N, \quad \forall i. \tag{5.28}$$

Letting $p_{i,1} = \alpha_i p_{i,2}$, $h_i' = \alpha_i h_{i,1} + h_{i,2}$, $\gamma_i' = \frac{\gamma_i}{1+\alpha_i}$, and recalling that $\sigma_i^2 = \sum_{j \neq i}(p_{j,1}h_{j,1} + p_{j,2}h_{j,2})(\mathbf{c}_i^\mathsf{T}\mathbf{s}_j)^2 + \sigma_{n_0}^2 \mathbf{c}_i^\mathsf{T}\mathbf{c}_i$, the above optimization problem is equivalent to

$$\min_{\{p_{i,2}\}} \quad \textstyle\sum_{i=1}^{K}(1+\alpha_i)p_{i,2} \tag{5.29}$$

$$\text{s. t.} \quad p_{i,2} \geq \gamma_i' \min_{\mathbf{c}_i \in \mathbb{R}^N} \frac{\sum_{j \neq i} h_j' p_{j,2}(\mathbf{c}_i^\mathsf{T}\mathbf{s}_j)^2 + \sigma_{n_0}^2 \mathbf{c}_i^\mathsf{T}\mathbf{c}_i}{(\mathbf{c}_i^\mathsf{T}\mathbf{s}_i)^2} \tag{5.30}$$

$$p_{i,2} > 0 \quad \forall i. \tag{5.31}$$

Note that, we have moved the receiver optimization to the constraint (5.30) [84], which optimizes the receiver of each user pair for a fixed power vector, and its solution is the well-known maximum SIR (and minimum mean-squared error (MMSE)) filter given by

$$\begin{aligned} \mathbf{c}_i^\star &= \left(\sum_{j=1}^{K} h_j' p_{j,2}\mathbf{s}_j\mathbf{s}_j^\mathsf{T} + \sigma_{n_0}^2 \mathbf{I}\right)^{-1} h_i' p_{i,2}\mathbf{s}_i & (5.32) \\ &= \left(\sum_{j=1}^{K} (p_{j,1}h_{j,1} + p_{j,2}h_{j,2})\mathbf{s}_j\mathbf{s}_j^\mathsf{T} + \sigma_{n_0}^2 \mathbf{I}\right)^{-1} (p_{i,1}h_{i,1} + p_{i,2}h_{i,2})\, \mathbf{s}_i. & (5.33) \end{aligned}$$

We can now define

$$I_i(\mathbf{p}_2, \mathbf{c}_i) = \gamma_i' \frac{\sum_{j \neq i} h_j' p_{j,2}(\mathbf{c}_i^\mathsf{T}\mathbf{s}_j)^2 + \sigma_{n_0}^2 \mathbf{c}_i^\mathsf{T}\mathbf{c}_i}{(\mathbf{c}_i^\mathsf{T}\mathbf{s}_i)^2} \tag{5.34}$$

$$U_i(\mathbf{p}_2) = \min_{\mathbf{c}_i \in \mathbb{R}^N} I_i(\mathbf{p}_2, \mathbf{c}_i) \tag{5.35}$$

and the iterative power control algorithm

$$\mathbf{p}_2(n+1) = \mathbf{U}(\mathbf{p}_2(n)) \tag{5.36}$$

where $\mathbf{p}_2 = [p_{1,2}, ..., p_{K,2}]^\mathsf{T}$, $\mathbf{U}(\mathbf{p_2}) = [U_1(\mathbf{p_2}), ..., U_K(\mathbf{p_2})]^\mathsf{T}$, and n is the iteration index. It is worth emphasizing that h_i' and γ_i' are calculated prior to the iterative updates, using γ_i and α_i in (5.23) and (5.24), which are obtained by solving the problem in (5.19)-(5.21) for the ith user pair. As shown in [84,85], $\mathbf{U}(\mathbf{p}_2)$ is a standard interference function for all $\mathbf{p}_2 \geq 0$, i.e., it satisfies the three properties [59]:

- Positivity: $\mathbf{U}(\mathbf{p}_2) > 0$
- Monotonicity: If $\mathbf{p}_2 \geq \mathbf{p}_2'$ then $\mathbf{U}(\mathbf{p}_2) \geq \mathbf{U}(\mathbf{p}_2')$
- Scalability: For all $\mu > 1$, $\mu\mathbf{U}(\mathbf{p}_2) > \mathbf{U}(\mu\mathbf{p}_2)$

Therefore, the power control algorithm (5.36), which includes an optimization of the linear receivers, converges to the minimum total transmit power solution of the optimization problem in (5.16)-(5.18), where the linear receivers converge to the corresponding MMSE receivers at the optimum. Recall that $p_{i,1} = \alpha_i p_{i,2}$. Note that the implementation of this algorithm requires that the pair of partners know each other's channel gain, but not those of the interferers.

5.5 Phase Two: Decision Rule and the BER

In phase two, the relay spreads $\hat{b}_i$ with $\mathbf{s}_i$, for $i = 1, ..., K$, and broadcasts

$$x_0 = \sum_{i=1}^{K} \sqrt{p_{0,i}}\hat{b}_i\mathbf{s}_i \tag{5.37}$$

to all users, where $p_{0,i}$ is the transmit power to broadcast $\hat{b}_i$ at the relay. The received signal at user i_m in the second phase is

$$\mathbf{r}_{i,m} = \sqrt{h_{i,m}} \left(\sum_{j=1}^{K} \sqrt{p_{0,j}} \hat{b}_j \mathbf{s}_j \right) + \mathbf{n}_{i,m}, \quad i = 1, ..., K, \; m = 1, 2 \tag{5.38}$$

where $\mathbf{n}_{i,m}$ is the AWGN vector with covariance matrix $\sigma^2_{n_{i,m}} \mathbf{I}_N$. User i_m employs $\mathbf{c}_{i,m}$:

$$y_{i,m} = \mathbf{c}^{\mathsf{T}}_{i,m} \mathbf{r}_{i,m} = \sqrt{p_{0,i} h_{i,m}} (\mathbf{c}^{\mathsf{T}}_{i,m} \mathbf{s}_i) \hat{b}_i + \sum_{j \neq i} \sqrt{p_{0,j} h_{i,m}} (\mathbf{c}^{\mathsf{T}}_{i,m} \mathbf{s}_j) \hat{b}_j + \mathbf{c}^{\mathsf{T}}_{i,m} \mathbf{n}_{i,m}. \tag{5.39}$$

The received SIR is given by

$$SIR_{i,m} = \frac{p_{0,i} h_{i,m} (\mathbf{c}^{\mathsf{T}}_{i,m} \mathbf{s}_i)^2}{\sum_{j \neq i} p_{0,j} h_{i,m} (\mathbf{c}^{\mathsf{T}}_{i,m} \mathbf{s}_j)^2 + \sigma^2_{n_{i,m}} \mathbf{c}^{\mathsf{T}}_{i,m} \mathbf{c}_{i,m}}. \tag{5.40}$$

Since $\hat{b}_i$ is binary with equal probability, user i_m can obtain $\hat{b}_{i,m}$, the hard decision estimate of $\hat{b}_i$, as $\hat{b}_{i,m} = 1$ if $y_{i,m} > 0$ and $\hat{b}_{i,m} = -1$ otherwise. Similar as in phase one, we approximate the interference plus noise term with a Gaussian. This way, the error probability of recovering $\hat{b}_i$ at user i_m can be approximated as

$$Pe2_{i,m} \approx Q(\sqrt{SIR_{i,m}}). \tag{5.41}$$

Next, user i_m performs an XOR operation on $\hat{b}_{i,m}$ and its own symbol $b_{i,m}$ to recover its partner's symbol.

5.6 Phase Two: Power Control and Receiver Optimization

The joint power control and receiver optimization problem in phase two can be formulated as

$$\min_{\{p_{0,i},\mathbf{c}_{i,1},\mathbf{c}_{i,2}\}} \quad \textstyle\sum_{i=1}^{K} p_{0,i} \tag{5.42}$$

$$\text{s. t.} \quad Pe2_{i,1} \le \rho 2_{i,1}, \quad Pe2_{i,2} \le \rho 2_{i,2} \tag{5.43}$$

$$p_{0,i} \ge 0, \quad \mathbf{c}_{i,1} \in \mathbb{R}^N, \quad \mathbf{c}_{i,2} \in \mathbb{R}^N, \quad \forall i \tag{5.44}$$

where $\rho 2_{i,1}$ and $\rho 2_{i,2}$ are the corresponding system QoS requirements at the ith pair of users, which are given.

From the one-to-one mapping in (5.41), we note that the error probability requirement $Pe2_{i,m} \le \rho 2_{i,m}$ is equivalent to the SIR requirement $SIR_{i,m} \ge \gamma_{i,m}$ with $\gamma_{i,m}$ satisfying $\rho 2_{i,m} = Q(\sqrt{\gamma_{i,m}})$. Therefore, we have

$$p_{0,i} \ge V_{i,m}(\mathbf{p}_0) = \min_{\mathbf{c}_{i,m} \in \mathbb{R}^N} I_{i,m}, \quad i = 1, ..., K, \ m = 1, 2 \tag{5.45}$$

where $\mathbf{p}_0 = [p_{0,1}, ..., p_{0,K}]^\mathsf{T}$ and

$$I_{i,m} = \frac{\gamma_{i,m}}{h_{i,m}} \cdot \frac{\sum_{j \ne i} p_{0,j} h_{i,m} (\mathbf{c}_{i,m}^\mathsf{T} \mathbf{s}_j)^2 + \sigma_{n_{i,m}}^2 \mathbf{c}_{i,m}^\mathsf{T} \mathbf{c}_{i,m}}{(\mathbf{c}_{i,m}^\mathsf{T} \mathbf{s}_i)^2}. \tag{5.46}$$

The solution of the optimization problem on the right hand side of (5.45) for fixed power levels is, once again, the MMSE filter that is given by:

$$\mathbf{c}^{\star}_{i,m} = \left(\sum_{j=1}^{K} h_{j,m} p_{0,j} \mathbf{s}_j \mathbf{s}_j^{\mathsf{T}} + \sigma^2_{n_{i,m}} \mathbf{I}\right)^{-1} \sqrt{h_{i,m} p_{0,i}} \mathbf{s}_i. \tag{5.47}$$

The optimization problem in (5.42)-(5.44) is equivalent to

$$\min_{\mathbf{p}_0} \quad \textstyle\sum_{i=1}^{K} p_{0,i} \tag{5.48}$$

$$\text{s. t.} \quad p_{0,i} \geq V_i(\mathbf{p}_0) = \max(V_{i,1}(\mathbf{p}_0), V_{i,2}(\mathbf{p}_0)), \quad \forall i. \tag{5.49}$$

We note that $V_{i,1}(\mathbf{p}_0)$ and $V_{i,2}(\mathbf{p}_0)$ are in the same form as the interference function defined in [84], which is proved to be standard. Also, we know by [59, Theorem 5] that the maximum of two standard interference functions is standard. Thus, $V_i(\mathbf{p}_0)$ is standard. We define the power control algorithm for the second phase as

$$\mathbf{p}_0(n+1) = \mathbf{V}(\mathbf{p}_0(n)) \tag{5.50}$$

where $\mathbf{V}(\mathbf{p}_0) = [V_1(\mathbf{p}_0), ..., V_K(\mathbf{p}_0)]^{\mathsf{T}}$. The standard interference function $\mathbf{V}(\mathbf{p}_0(n))$ guarantees that the power control algorithm in (5.50) converges to the minimum total transmit power solution of the optimization problem in (5.42)-(5.44). Similar as in phase one, updating the relay power and the receivers for the ith pair in phase two only requires the information of the channel gains of that pair, but none of the interferers.

5.7 Numerical Results

In this section, we present numerical results related to the performance of the proposed multiuser two-way relay system with JD-XOR-F scheme, and compare it with the one-way CDMA relaying, where all the users transmit to the relay with distinct signatures in the first phase, and the relay estimates the symbol for each user, and spreads and forwards it to its corresponding partner with the partner's signature in the second phase[5].

In Figure 5.2, we first present the performance comparison between the two-hypothesis and the four-hypothesis decision rule at the relay in (5.9) and (5.13), respectively, for one pair of users. For different QoS requirement of the first phase, $\rho 1_1$, ranging from 0.01 to 0.1, we search the minimum total transmit power of the users for 1000 network topology realizations using the two- and four-hypothesis decision rules, according to (5.11) and (5.15), respectively, and average the total transmit power over all realizations. In each realization, the users are randomly distributed on a disk with the relay at the center, and the distances between the relay and users are between 100m-1000m following a uniform distribution. All channel gains follow the path-loss model, i.e., $h_{1,m} = d_{1,m}^{-a}$ where $d_{1,m}$ is the distance between user 1_m and the relay, for $m = 1, 2$. The path-loss exponent $a = 4$ and the AWGN variance 10^{-13} are used in the simulations[6]. In Figure 5.2, we observe that the performance loss of the four-hypothesis decision rule compared with the two-hypothesis one on the average total transmit power is negligible.

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[5] Figures of this section are located at the end of this chapter.

[6] With these settings, a user located 500m away from the relay with transmit power 0.0625 Watts has received SNR 10dB at the relay.

Next, we demonstrate the performance gain on end-to-end BER and user capacity of the proposed two-way JD-XOR-F relaying scheme, when all users have equal received power. In Figure 5.3, we plot the end-to-end BER of a user receiving its partner's symbol in the multiuser two-way relay system using different relaying schemes: JD-XOR-F relaying, one-way CDMA relaying, and amplify-and-forward (AF) relaying. With the two-way AF relaying, upon receiving the signals from users in phase one, the relay amplifies and broadcasts the received signal in phase two, where the amplifying scalar is chosen to satisfy the relay transmit power constraint. In phase two, each user first subtracts its self-interference from the signal received from the relay, and then recovers its partner's symbol with a linear MMSE receiver. We consider a system with K pairs of users where $K \in [3, 11]$ and spreading gain $N = 20$. We set the distance between each user and the relay to 500m, and use the same path-loss model and AWGN variance 10^{-13} as in Figure 5.2. The transmit power of each user in phase one is set to 0.0625 Watts for all schemes, so that the received SNR of each user symbol at the relay is 10dB. To ensure a fair comparison between the different schemes, the relay transmit power in phase two is set to 0.0625 Watts for each XORed symbol in JD-XOR-F scheme, 0.0625 Watts for each user symbol in one-way CDMA, and $2K \times 0.0625$ Watts as the total relay power in the AF scheme. The BER is averaged over 100 sets of randomly generated sequences, and the BER of each set of sequences is averaged over 1000 realizations of the AWGN channels. We also calculate the approximate BER using the corresponding Q-function expressions, denoted by the subscript "approx." in Figure 5.3. We observe that the BER approximations match with the simulation results.

We observe in Figure 5.3 that, the proposed JD-XOR-F scheme provides a remarkable BER performance gain upon one-way CDMA scheme as a pair of users share a common signature. The AF scheme outperforms one-way CDMA scheme as K increases beyond 6, since the AF scheme acts as one-hop communication with an enlarged noise term which no longer dominates the BER when MAI becomes larger. The JD-XOR-F scheme outperforms the AF scheme due to the fact that the former one reduces the MAI by transmitting an XORed symbol from the relay for each user pair in the second phase.

In Figure 5.4, we compare the maximum number of pairs, K_{max}, that can be simultaneously supported in the multiuser two-way relay system with JD-XOR-F relaying and one-way CDMA relaying. We set the received SNR levels large enough (40dB) so that the maximum number of supported users of the one-way CDMA scheme approaches the user capacity achieved by optimal sequences and power control without power constraint [86]. For processing gain $N \in \{24, 28, 32, 36, 40\}$, we find the K_{max} that achieves an end-to-end BER≤ 0.01. We observe that the multiuser two-way relay system with JD-XOR-F scheme can support almost *twice* of the number of user pairs that can be supported with the one-way CDMA scheme. This is expected as the two-way relaying utilizes the bi-directional communication structure and needs only half number of the signatures as compared to the one-way CDMA relaying. It significantly reduces the MAI experienced in the system by jointly demodulating and XOR combining the symbols of each user pair at the relay.

Next, we demonstrate the performance gain of the proposed joint power control and receiver optimization algorithm for the multiuser two-way relay system. Specifically, we plot the updated total power with respect to the number of iterations for different iterative power control algorithms for the JD-XOR-F relaying and the one-way CDMA relaying. In the results presented in Figures 5.5-5.7, all users are randomly distributed in the same area and follow the same path-loss model as in the settings of Figure 5.2. Both the system topology and the signatures are generated once and then fixed for all the simulations. The variance of all AWGN terms is 10^{-13}. The system BER requirements for the multiuser two-way JD-XOR-F relaying scheme in both the first and the second phases are $\rho1_i = \rho2_{i,1} = \rho2_{i,2} = 0.0189$. The BER requirements of receiving each user's symbol at the relay in phase one and at the users in phase two in the one-way CDMA relaying are set to 0.0189 as well. This way, the two systems achieve the same end-to-end BER at 0.037 for a fair comparison. The spreading gain is $N = 20$, and the number of user pairs is $K = 11$ in Figures 5.5-5.6 and $K = 13$ in Figure 5.7, i.e., the number of users is 22 and 26 respectively.

In Figure 5.5, we compare the total user transmit power of three different power control algorithms in the first phase of the multiuser two-way JD-XOR-F relaying: the optimum power control (Optimum), the power control with equal received power (Eq-RX-Power) and with equal transmit power (Eq-TX-Power) of each of the two partners. As expected, the optimum power control achieves the minimum total user power consumption. While both Eq-TX-Power and Eq-RX-Power consume more power than the optimum power control, we observe that Eq-RX-Power algorithm may not be too far off as compared to the optimum one. When the ith pair of partners have equal received power at the relay, the BER expression in (5.15) can be simplified to $Pe1_i = \frac{3}{2}Q(\sqrt{\eta_i}) - \frac{1}{2}Q(3\sqrt{\eta_i})$, where $\eta_i = \eta_{i,1} = \eta_{i,2}$ is the equal received SIR of both partners. It can be verified that $Pe1_i$ is a strictly decreasing function of η_i for $\eta_i > 0$. Therefore, the BER constraint in (5.17) can be replaced by the equivalent SIR constraint $\eta_i \geq \eta_i^\star$ where $\eta_i^\star$ satisfies $\frac{3}{2}Q(\sqrt{\eta_i^\star}) - \frac{1}{2}Q(3\sqrt{\eta_i^\star}) = \rho 1_i$, and the power control problem for phase one in (5.16)-(5.18) can be solved directly by the iterative power control algorithm. Therefore, by applying the equal received power constraint, the implementation of the algorithm can be simplified with a moderate sacrifice in total power expenditure.

Figure 5.6 presents the comparison between the multiuser two-way JD-XOR-F relaying and the one-way CDMA relaying, on the total user power expended for phase one, and on the total relay power for phase two, respectively. A large power savings of the JD-XOR-F relaying upon the one-way CDMA relaying is presented in both phases. This is due to the fact that the relay jointly demodulates and generates the estimate of the XORed symbol for each user pair in the first phase, and transmits one binary symbol for each user pair in the second phase, and hence in both phases the interference is significantly reduced.

In Figure 5.7, the first phase power control of an overloaded system is considered, i.e., $N = 20$ and $K = 13$. For the one-way CDMA relaying scheme, the user capacity [86], i.e., the maximum number of users that can be supported by optimal interference management, is 24. As expected, we observe in Figure 5.7 that the power control problem with the one-way CDMA scheme is infeasible since the number of users is $26 > 24$, i.e., there are no transmit power values that satisfy the QoS requirements. On the other hand, the multiuser two-way JD-XOR-F relaying scheme has a feasible power control solution in phase one since it only needs half number of the signatures needed in the one-way CDMA scheme. This shows the considerable benefit on user capacity of considering the two-way communication structure.

5.8 Discussion on System-Wide Optimization

The system-wide optimization problem, which jointly optimizes the transmit power levels and the receivers at all users and the relay while satisfying the end-to-end BER constraints, is formulated as

$$\min_{\{p_{i1},p_{i2},p_{0i},\mathbf{c}_i,\mathbf{c}_{i1},\mathbf{c}_{i2}\}} \quad \sum_{i=1}^{K}(p_{i1}+p_{i2}+p_{0i}) \tag{5.51}$$

$$\text{s. t.} \quad Pe1_i(1-Pe2_{i1})+(1-Pe1_i)Pe2_{i1} \le \rho_{i12} \tag{5.52}$$

$$Pe1_i(1-Pe2_{i2})+(1-Pe1_i)Pe2_{i2} \le \rho_{i21} \tag{5.53}$$

$$p_{i1} \ge 0, \quad p_{i2} \ge 0, \quad p_{0i} \ge 0 \tag{5.54}$$

$$\mathbf{c}_i \in \mathbb{R}^N, \quad \mathbf{c}_{i1} \in \mathbb{R}^N, \quad \mathbf{c}_{i2} \in \mathbb{R}^N, \quad \forall i. \tag{5.55}$$

Recall that $Pe1_i$ is the probability of incorrectly determining $\hat{b}_i$ at the relay, and $Pe2_{im}$, $m = 1, 2$ is the error probability of recovering $\hat{b}_i$ at user im. ρ_{i12} and ρ_{i21} are the system QoS requirements on the end-to-end error probabilities of recovering the partner's symbol at user $i1$ and $i2$. We note that, for each fixed $Pe1_i = \rho 1_i$ and $0 < \rho 1_i < \min(\rho_{i12}, \rho_{i21})$, the constraints (5.52) and (5.53) become

$$Pe2_{i1} \le \frac{\rho_{i12} - \rho 1_i}{1 - 2\rho 1_i} \tag{5.56}$$

$$Pe2_{i2} \le \frac{\rho_{i21} - \rho 1_i}{1 - 2\rho 1_i}. \tag{5.57}$$

This way, the optimization problem is equivalent to

$$\min_{\{0<\rho 1_i<\min(\rho_{i12},\rho_{i21})\}} \quad \min_{\{p_{i1},p_{i2},p_{0i},\mathbf{c}_i,\mathbf{c}_{i1},\mathbf{c}_{i2}\}} \quad \sum_{i=1}^{K}(p_{i1}+p_{i2}+p_{0i}) \tag{5.58}$$

$$\text{s. t.} \quad Pe1_i = \rho 1_i \tag{5.59}$$

$$Pe2_{i1} \leq \frac{\rho_{i12}-\rho 1_i}{1-2\rho 1_i} \tag{5.60}$$

$$Pe2_{i2} \leq \frac{\rho_{i21}-\rho 1_i}{1-2\rho 1_i} \tag{5.61}$$

$$p_{i1} \geq 0, \quad p_{i2} \geq 0, \quad p_{0i} \geq 0 \tag{5.62}$$

$$\mathbf{c}_i \in \mathbb{R}^N,\ \mathbf{c}_{i1} \in \mathbb{R}^N,\ \mathbf{c}_{i2} \in \mathbb{R}^N,\ \forall i. \tag{5.63}$$

Since $\{Pe1_i\}_{i=1}^K$ depend only on the users transmit power $\{p_{i1}, p_{i2}\}_{i=1}^K$ and the filters $\{\mathbf{c}_i\}_{i=1}^K$ in phase one, and $\{Pe2_{i1}, Pe2_{i2}\}_{i=1}^K$ depend only on the relay transmit power $\{p_{0i}\}_{i=1}^K$ and filters $\{\mathbf{c}_{i1}, \mathbf{c}_{i2}\}_{i=1}^K$ in phase two, the inner minimization problem for fixed $\{\rho 1_i\}_{i=1}^K$ can be decoupled into two problems as

$$\min_{\{p_{i1},p_{i2},\mathbf{c}_i\}} \quad \textstyle\sum_{i=1}^K (p_{i1}+p_{i2}) \tag{5.64}$$

$$\text{s. t.} \quad Pe1_i = \rho 1_i \tag{5.65}$$

$$p_{i1} \geq 0, \quad p_{i2} \geq 0, \quad \mathbf{c}_i \in \mathbb{R}^N, \quad \forall i \tag{5.66}$$

and

$$\min_{\{p_{0i}, \mathbf{c}_{i1}, \mathbf{c}_{i2}\}} \quad \textstyle\sum_{i=1}^{K} p_{0i} \tag{5.67}$$

$$\text{s. t.} \quad Pe2_{i1} \leq \tfrac{\rho_{i12} - \rho 1_i}{1 - 2\rho 1_i} \tag{5.68}$$

$$Pe2_{i2} \leq \tfrac{\rho_{i21} - \rho 1_i}{1 - 2\rho 1_i} \tag{5.69}$$

$$p_{0i} \geq 0, \quad \mathbf{c}_{i1} \in \mathbb{R}^N, \quad \mathbf{c}_{i2} \in \mathbb{R}^N, \quad \forall i. \tag{5.70}$$

We note that the problem in (5.64)-(5.66) has the same solution as that in (5.16)-(5.18) since (5.16)-(5.18) is shown to be a convex problem, and the problem in (5.67)-(5.70) is equivalent to that in (5.42)-(5.44) if we set $\rho 2_{i1} = \frac{\rho_{i12} - \rho 1_i}{1 - 2\rho 1_i}$ and $\rho 2_{i2} = \frac{\rho_{i21} - \rho 1_i}{1 - 2\rho 1_i}$ in (5.43). Therefore, problems (5.64)-(5.66) and (5.67)-(5.70) can be solved by the iterative algorithms proposed in Section 5.4 and 5.6 respectively.

The outer minimization over $\{\rho 1_i\}$ has K variables to optimize, which needs K-dimension search. Seeking a feasible near-optimum solution, we fix $\{\rho 1_i\}$ to some reasonable values. We observe that $\{\rho 1_i\}$ being close to 0 or close to $\min(\rho_{i12}, \rho_{i21})$ results in high power expenditure either in phase one or phase two. Therefore, we choose $\rho 1_i$ such that

$$\rho 1_i = \min\left(\frac{\rho_{i12} - \rho 1_i}{1 - 2\rho 1_i}, \frac{\rho_{i21} - \rho 1_i}{1 - 2\rho 1_i}\right) \tag{5.71}$$

i.e., we balance the BER performances over the two phases for the direction with a smaller end-to-end BER requirement. We simulate for a system with $K = 5$ user pairs and spreading gain $N = 5$. We randomly generate three realizations, T_1, T_2 and T_3, of the network topology using the same settings as in Figure 5.5-5.7. The distances

Table 5.1. Three realizations of the network topology (distance unit: meter).

	(d_{11}, d_{12})	(d_{21}, d_{22})	(d_{31}, d_{32})	(d_{41}, d_{42})	(d_{51}, d_{52})
T_1	$(100.5, 878.9)$	$(651.3, 991.0)$	$(574.9, 531.6)$	$(821.2, 305.1)$	$(548.3, 910.8)$
T_2	$(473.8, 374.5)$	$(886.9, 113.5)$	$(791.2, 973.8)$	$(991.1, 890.0)$	$(494.8, 548.5)$
T_3	$(424.3, 593.7)$	$(335.6, 637.6)$	$(144.4, 614.0)$	$(730.8, 966.1)$	$(775.5, 766.0)$

Table 5.2. Percentage of the extra power penalty of the near-optimum solution as compared to the optimum one on system-wide optimization.

$\rho_{i12} = \rho_{i21}$	0.01	0.02	0.055	0.07	0.1
T_1	0.16%	0.22%	0.33%	0.46%	0.58%
T_2	1.52%	1.95%	2.91%	3.11%	3.90%
T_3	1.17%	1.51%	2.34%	2.35%	3.26%

between users and the relay, i.e., (d_{i1}, d_{i2}) for $i = 1, ..., 5$, in each realization are listed in Table 5.1. For each network realization, we set the end-to-end BER constraints as $\rho_{i12} = \rho_{i21} = \{0.01, 0.02, 0.055, 0.07, 0.1\}$, and search the optimum solution and the near-optimum solution for each case. In Table 5.2, we show the extra power penalty of the near-optimum solution as compared to the optimum one. We observe that the near-optimum solution incurs a slight power penalty while significantly reduces the computational complexity of the optimum solution, which is achieved by exhaustive search.

5.9 Conclusion

In this chapter, we have considered an interference limited multiuser two-way relay network where multiple pairs of partners exchange information via a shared intermediate relay node, using CDMA. We have proposed a two-phase communication scenario where each pair wishing to exchange information shares a CDMA signature in the uplink, and the relay performs digital network coding per pair in the downlink. We have proposed the jointly demodulate-and-XOR forward relaying scheme, constructed iterative power control and linear multiuser detection algorithms that converge to their optimum solution, and showed that the design choices made considering the bi-directional nature of communication lead to significant power savings. We remark that interesting and challenging issues remain to be discovered in multiuser two-way relay networks including employing nonlinear multiuser detection and channel coding, CDMA signature design, partner selection and channel estimation.

5.10 Appendix

5.10.1 Proof of Lemma 5.1

First, we prove that $Pe1_i(\eta_{i,1}, \eta_{i,2})$ is quasiconvex on R_1. Let $f = Pe1_i$, $x_1 = \eta_{i,1}$, $x_2 = \eta_{i,2}$ and $\mathbf{x} = [x_1, x_2]^\mathsf{T}$ for simplicity. A sufficient condition for f to be quasiconvex on R_1 is that for each $\mathbf{x} \in R_1$, $\det(B_n(\mathbf{x})) < 0$ for $n = 1, 2$ [81], where $\det(\cdot)$ is the determinant of a matrix, and B_n denotes the nth submatrix of the bordered Hessian of f, i.e.,

$$B_1 = \begin{bmatrix} f_{11} & f_1 \\ f_1 & 0 \end{bmatrix} \quad \text{and} \quad B_2 = \begin{bmatrix} f_{11} & f_{12} & f_1 \\ f_{21} & f_{22} & f_2 \\ f_1 & f_2 & 0 \end{bmatrix} \tag{5.72}$$

where $f_m = \dfrac{\partial f}{\partial x_m}$ and $f_{mn} = \dfrac{\partial^2 f}{\partial x_m \partial x_n}$ with $n, m \in \{1,2\}$. From (5.15), f can be expressed as

$$\begin{aligned} f(\mathbf{x}) &= \int_{\sqrt{x_2}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt + \frac{1}{2} \int_{2\sqrt{x_1}-\sqrt{x_2}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt \\ &\quad - \frac{1}{2} \int_{2\sqrt{x_1}+\sqrt{x_2}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt. \end{aligned} \tag{5.73}$$

The first and second order derivatives in B_1 and B_2 are found as follows. Note that all the following inequalities are held only for $R_1 = \{\mathbf{x} | x_1 > 0,\ x_2 > 0, \text{ and } x_1 \geq x_2\}$.

$$f_1 = -\frac{\sqrt{2}}{4\sqrt{\pi}\sqrt{x_1}} (e^{-\frac{1}{2}(2\sqrt{x_1}-\sqrt{x_2})^2} - e^{-\frac{1}{2}(2\sqrt{x_1}+\sqrt{x_2})^2}) < 0 \tag{5.74}$$

$$f_2 = \frac{\sqrt{2}}{8\sqrt{\pi}\sqrt{x_2}} e^{-\frac{1}{2}x_2} (-2 + e^{-2x_1+2\sqrt{x_1 x_2}} + e^{-2x_1-2\sqrt{x_1 x_2}}) < 0. \tag{5.75}$$

Inequality (5.74) is because $(2\sqrt{x_1}-\sqrt{x_2})^2 < (2\sqrt{x_1}+\sqrt{x_2})^2$. Inequality (5.75) is because $e^{-2x_1+2\sqrt{x_1 x_2}} < 1$ and $e^{-2x_1-2\sqrt{x_1 x_2}} < 1$.

$$\begin{aligned} & f_{21} = f_{12} \\ & = -\frac{\sqrt{2}}{8} \frac{(2\sqrt{x_1}-\sqrt{x_2})}{\sqrt{\pi x_1 x_2}} e^{-\frac{1}{2}(2\sqrt{x_1}-\sqrt{x_2})^2} - \frac{\sqrt{2}}{8} \frac{(2\sqrt{x_1}+\sqrt{x_2})}{\sqrt{\pi x_1 x_2}} e^{-\frac{1}{2}(2\sqrt{x_1}+\sqrt{x_2})^2} < 0 \end{aligned} \tag{5.76}$$

$$f_{22} = \underbrace{\frac{\sqrt{2}}{8\sqrt{\pi x_2}} e^{-\frac{x_2}{2}}}_{A} \underbrace{+ \frac{\sqrt{2}}{8\sqrt{\pi}(x_2)^{3/2}} e^{-\frac{x_2}{2}}}_{B}$$

$$\underbrace{+ \frac{\sqrt{2}}{16\sqrt{\pi} x_2} (2\sqrt{x_1} - \sqrt{x_2}) e^{-\frac{1}{2}(2\sqrt{x_1}-\sqrt{x_2})^2}}_{C} \underbrace{- \frac{\sqrt{2}}{16\sqrt{\pi}(x_2)^{3/2}} e^{-\frac{1}{2}(2\sqrt{x_1}-\sqrt{x_2})^2}}_{D}$$

$$\underbrace{-\frac{\sqrt{2}}{16\sqrt{\pi}x_2}(2\sqrt{x_1}+\sqrt{x_2})e^{-\frac{1}{2}(2\sqrt{x_1}+\sqrt{x_2})^2}}_{E}\underbrace{-\frac{\sqrt{2}}{16\sqrt{\pi}(x_2)^{3/2}}e^{-\frac{1}{2}(2\sqrt{x_1}+\sqrt{x_2})^2}}_{F}. \quad (5.77)$$

In the above we find that $A+C+E>0$ and $B+D+F>0$, leading to $f_{22}>0$.

$$f_{11}=\frac{\sqrt{2}e^{-2x_1-\frac{x_2}{2}}}{8\sqrt{\pi}(x_1)^{3/2}}\underbrace{\left((4x_1-2\sqrt{x_1x_2}+1)e^{2\sqrt{x_1x_2}}-(4x_1+2\sqrt{x_1x_2}+1)e^{-2\sqrt{x_1x_2}}\right)}_{G} \quad (5.78)$$

$$\frac{\partial G}{\partial x_1}=(4+4\sqrt{x_1x_2}-2x_2)\left(e^{2\sqrt{x_1x_2}}-e^{-2\sqrt{x_1x_2}}\right)+8\sqrt{x_1x_2}e^{-2\sqrt{x_1x_2}}>0 \quad (5.79)$$

$$\frac{\partial G}{\partial x_2}=\left(4x_1\frac{\sqrt{x_1}}{\sqrt{x_2}}-2x_1\right)e^{2\sqrt{x_1x_2}}+\left(4x_1\frac{\sqrt{x_1}}{\sqrt{x_2}}+2x_1\right)e^{-2\sqrt{x_1x_2}}>0. \quad (5.80)$$

Inequalities (5.79) and (5.80) are due to $x_1 \geq x_2$, $x_1>0$ and $x_2>0$. Therefore, $G(x_1,x_2)$ is a strictly increasing function of x_1 for fixed x_2, and a strictly increasing function of x_2 for fixed x_1 as well. Hence, $G(x_1,x_2)>\lim_{x_1\to 0, x_2\to 0} G(x_1,x_2)=0$. Thus, $f_{11}>0$.

Since $f_1<0$, $f_2<0$, $f_{11}>0$, $f_{22}>0$, and $f_{12}=f_{21}<0$, the determinants of the submatrices of the bordered Hessian are both negative, i.e.,

$$\det(B_1)=-(f_{11})^2<0; \quad \det(B_2)=-(f_1)^2f_{22}+2f_1f_2f_{12}-(f_2)^2f_{11}<0. \quad (5.81)$$

Therefore, $f(\mathbf{x})$, equivalently, $Pe1_i(\eta_{i,1},\eta_{i,2})$, is quasiconvex on R_1.

We note that for the case when $\eta_{i,1}<\eta_{i,2}$, $Pe1_i$ can be obtained by switching $\eta_{i,1}$ and $\eta_{i,2}$ in the case of $\eta_{i,1}\geq\eta_{i,2}$. Therefore, by switching x_1 and x_2 in the above proof for R_1, we can obtain the first and second order derivatives and show that the determinants of the submatrices of the bordered Hessian are negative. Thus, $Pe1_i(\eta_{i,1}, \eta_{i,2})$, is quasiconvex on R_2 as well.

5.10.2 Proof of Proposition 5.1

First, we convert the transmit power condition in (5.25) to its equivalent received SIR condition

$$\frac{\eta_{i,1}}{h_{i,1}} + \frac{\eta_{i,2}}{h_{i,2}} \geq \gamma_i \quad \text{and} \quad \frac{\eta_{i,1}}{\eta_{i,2}} = \frac{\eta^\star_{i,1}}{\eta^\star_{i,2}}. \tag{5.82}$$

Next, we show that (5.82) is sufficient for $Pe1_i \leq \rho 1_i$, with the minimum total transmit power requirement on the ith user pair. Let $\eta_{i,1} = \eta_{i,2}\frac{\eta^\star_{i,1}}{\eta^\star_{i,2}}$, from (5.82) we have

$$\frac{\eta_{i,2}}{\eta^\star_{i,2}} \left(\frac{\eta^\star_{i,1}}{h_{i,1}} + \frac{\eta^\star_{i,2}}{h_{i,2}} \right) \geq \gamma_i. \tag{5.83}$$

Since $\frac{\eta^\star_{i,1}}{h_{i,1}} + \frac{\eta^\star_{i,2}}{h_{i,2}} = \gamma_i$ as defined in (5.23), we have $\eta_{i,2} \geq \eta^\star_{i,2}$ and $\eta_{i,1} \geq \eta^\star_{i,1}$. As we show in Appendix 5.10.1 that $\frac{\partial Pe1_i}{\partial \eta_{i,1}} < 0$ and $\frac{\partial Pe1_i}{\partial \eta_{i,2}} < 0$, $Pe(\eta_{i,1}, \eta_{i,2})$ is a decreasing function of $\eta_{i,1}$ or $\eta_{i,2}$. Therefore, $Pe1_i(\eta_{i,1}, \eta_{i,2}) \leq Pe1_i(\eta^\star_{i,1}, \eta^\star_{i,2}) \leq \rho 1_i$. Thus, (5.82) is a sufficient condition for $Pe1_i \leq \rho 1_i$. When $\eta_{i,1} = \eta^\star_{i,1}$ and $\eta_{i,2} = \eta^\star_{i,2}$, it achieves the minimum total transmit power $\gamma_i \frac{\sigma_i^2}{(\mathbf{c}_i^\top \mathbf{s}_i)^2}$ that satisfies the BER condition, as it is the optimum solution to the problem in (5.19)-(5.21).

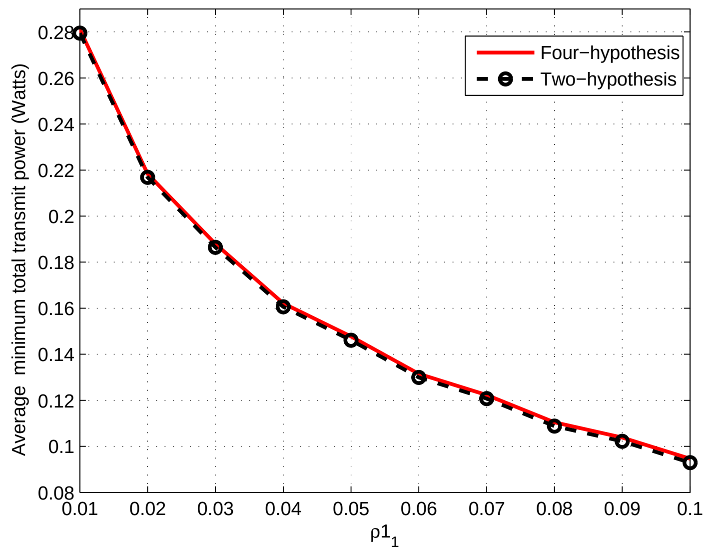

Figure 5.2. Comparison of the two- and four-hypothesis decision rules for JD-XOR-F scheme at the relay in phase one.

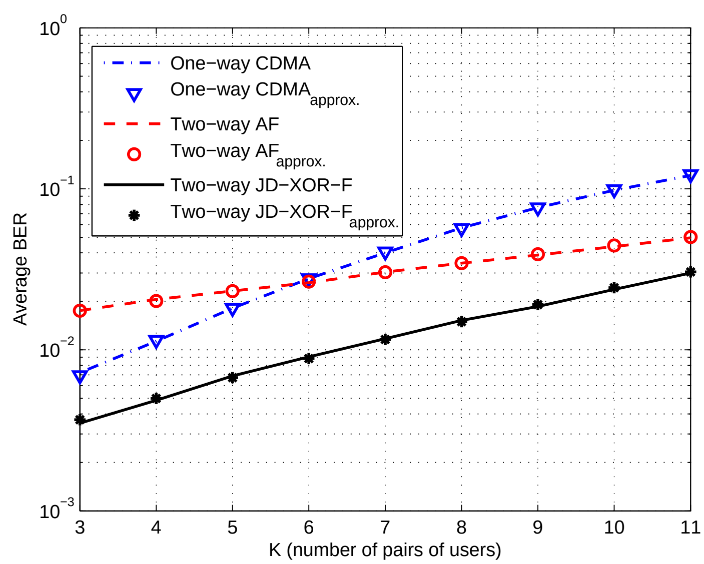

Figure 5.3. End-to-end BER performance of various relaying schemes.

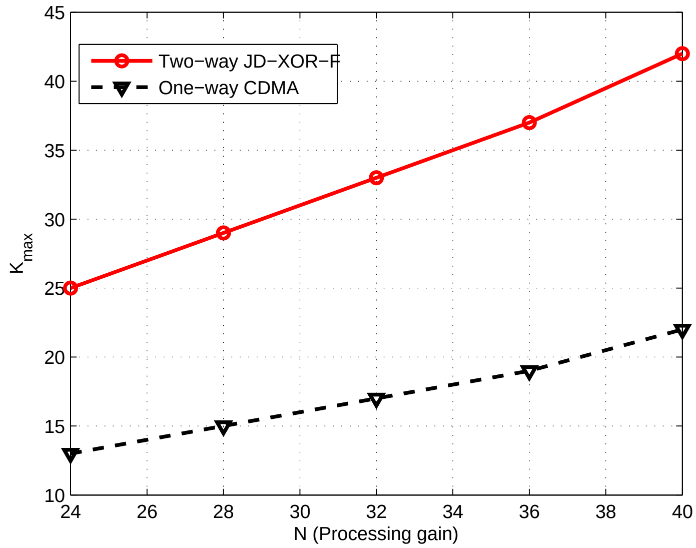

Figure 5.4. Maximum number of pairs of users that can be supported with end-to-end BER ≤ 0.01.

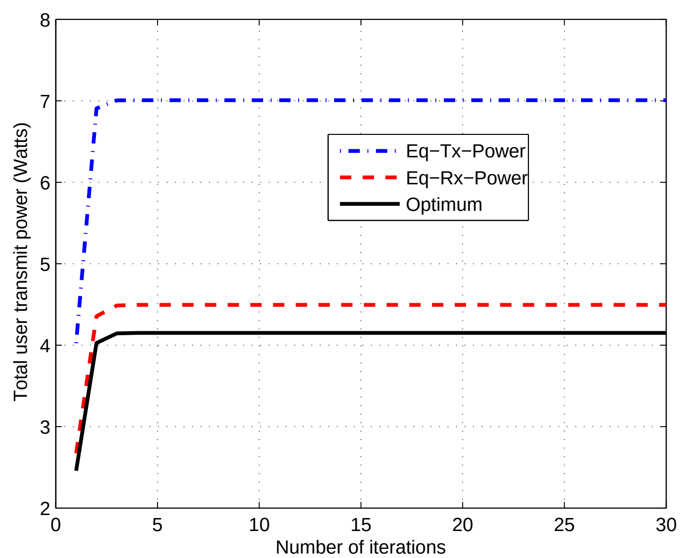

Figure 5.5. Comparison of the total user transmit power among different power control algorithms in phase one of the two-way JD-XOR-F relaying.

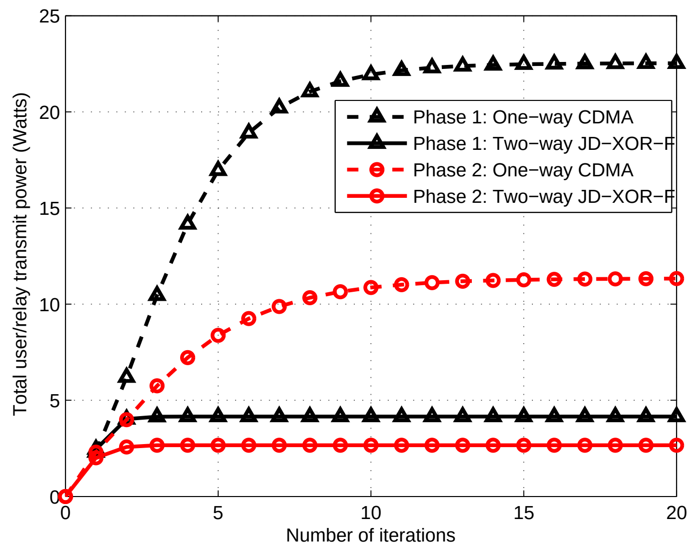

Figure 5.6. Comparison of the total user/relay transmit power between two-way JD-XOR-F relaying and one-way CDMA relaying in phase one/two.

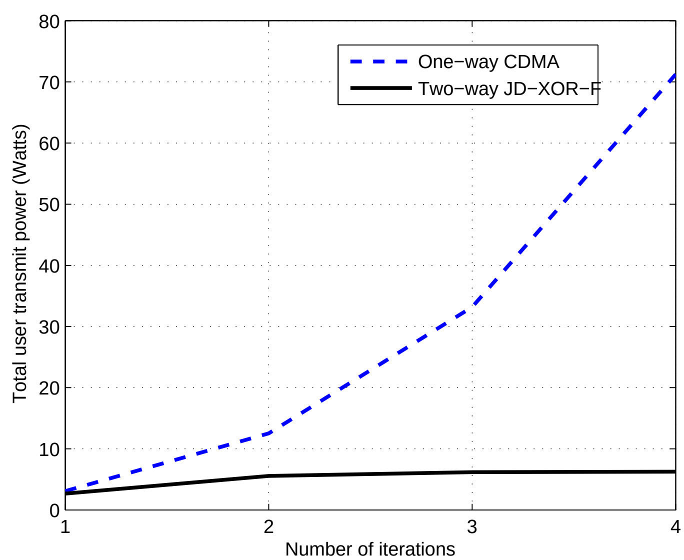

Figure 5.7. Comparison of the total user transmit power between two-way JD-XOR-F relaying and one-way CDMA relaying in phase one in an overloaded system.

Chapter 6

Conclusion

6.1 Dissertation Summary

In this dissertation, we studied resource management for various wireless ad hoc networks. Specifically, we considered wireless sensor networks and relay-assisted ad hoc networks with different network topologies and transmission schemes. In each case, we investigate the appropriate resource management problems in accordance with the requirements and the needs dictated by the application, and the nature of the particular ad hoc network.

First, we considered efficient scheduling strategies for delay-constrained wireless sensor networks. Focusing on the short term average throughput requirements as well as the QoS requirements for each sensor node and cluster head, we found the schedule with polynomial complexity that minimizes the total sensor transmit power. The schedule protocol is shown to be the optimum and near-optimum solution to the intra-cluster and inter-cluster communications, respectively, and achieves significant power savings in both cases.

Next, we investigated the distributed power allocation problem for parallel relay networks. Given the partial CSI available at the source and the relay nodes, we proposed a distributed relay decision mechanism and developed the optimum power allocation scheme. By optimizing the relay selection strategy and power allocation, we showed that the optimum distributed power allocation strategy performs close to the optimum centralized scheme. In addition, we considered two simpler distributed power allocation strategies which have significantly less computational complexity requirements at the source with a modest sacrifice in performance.

Later, we considered bi-directional relaying in multiuser scenario, namely, multiuser two-way relay networks. For the network where communication occurs on orthogonal channels by means of F/TDMA techniques, we investigated the optimum relay power allocation problem for a variety of existing two-way relaying protocols. By maximizing the arbitrary weighted sum rate, we enabled the way to trace the boundary of the achievable rate region for each relaying scheme.

Finally, we studied the multiuser two-way relay network supported by CDMA. We proposed the resource sharing method in terms of CDMA spreading signatures, which enables each pair of users share a common signature without introducing interference to each other. We further investigated the interference management by constructing iterative power control and multiuser detection algorithms that converge to their optimum solutions. We showed that the design choices made considering the bi-directional nature of communication lead to significant system-wide power savings

Most of the results presented in this dissertation were presented at various academic conferences, [83, 87–90], and published as journal publications, [91–93].

6.2 Future Research

Throughout this dissertation, we have investigated resource management, with an emphasis on transmit power, for various ad hoc networks. There are other types of power expenditure and other types of wireless resources such as bandwidth and time, whose amount is typically limited in wireless communications. A more general resource allocation problem would be jointly allocating diverse radio resources in an efficient manner. In addition, distributed resource management is always of great interest for more practical applications due to the decentralized nature of wireless ad hoc networks.

Moreover, while the ad hoc networks considered in this dissertation are all static, future ad hoc networks are expected to provide services to mobile users. Mobile ad hoc networks (MANETs) [94] bring new design challenges, such as dynamic network topology, time-varying channels, Doppler shift, to resource allocation due to the mobility of the nodes. Therefore, efficient and dynamic resource management for MANET is an interesting topic for future research.

Finally, in this dissertation, we have focused on two building blocks of the future ad hoc networks, namely, (one-way) parallel relay networks and multiuser two-way relay networks. Future ad hoc networks can be more complicated. For example, instead of information exchange between a pair of pre-assigned partners, a group of nodes may want to share information among each other via the help of the relay node(s) [95]. Resource management for ad hoc networks that consist of multiple users and multiple relays communicating in multi-way remains to be investigated to maximize the resource efficiency of the entire system.

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