OPTIMIZED CONSTRAINTS FOR THE LINEARIZED GEOACOUSTIC INVERSE PROBLEM

A Dissertation in
Acoustics

by
Megan S. Ballard

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The dissertation of Megan S. Ballard was reviewed and approved* by the following:

Kyle M. Becker  
Assistant Professor of Acoustics  
Dissertation Advisor  
Chair of Committee

David Bradley  
Professor of Acoustics

Charles Holland  
Professor of Acoustics

Tim Kane  
Professor of Electrical Engineering

David Knobles  
Research Scientist, University of Texas at Austin  
Special Member

Anthony A. Atchley  
Professor of Acoustics  
Chair of the Graduate Program in Acoustics

*Signatures are on file in the Graduate School.
Abstract

This work is concerned with developing inverse methods to estimate sound speed profiles of the water column and seabed in the shallow-water environment. The approaches are based on a perturbative technique which uses estimates of horizontal wave numbers to determine sound speed profiles. This technique is appropriate for the highly variable shallow-water environment as horizontal wave numbers can be measured semi-locally (1-2 km aperture) and their values are a direct measurement of the local environmental properties. In all previous applications of this technique, this inversion scheme has been used to estimate sound speed in the seabed at a number of discrete depths while assuming water column properties are known.

Compared to past applications of perturbative inversion, the accuracy of the solution is improved by implementing more suitable constraints to address issues of stability and uniqueness associated with solving the ill-posed inverse problem. One such improvement uses qualitative regularization which allows for discontinuities in the sediment sound speed profile to be resolved. The resulting solution is a more accurate representation of the layered structure of the seabed. Another advancement makes reliable inversion for water column properties possible. This is accomplished using approximate equality constraints to address the solution’s instability in portions of the waveguide for which the wave number data are insensitive. A further enhancement uses a synthesis of qualitative regularization and approximate equality constraints to provide a mechanism to efficiently invert for sediment and water column sound speed profiles simultaneously. This technique allows for rapid assessment of the water column and the seabed properties in a single step.
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Introduction and Overview

1.1 Motivation

In the underwater environment, acoustic waves play the same essential role as radio waves in air. Acoustic waves are used to image underwater features, communicate information, and to measure oceanic properties [1]. Military applications of acoustics include active sonars which work by transmitting a signal and receiving echoes from a target and passive sonars which are designed to intercept noises radiated by a target vessel. Civilian uses of acoustics involve bathymetry sounders to measure water depth, fishery sounders to locate fish shoals, sidescan sonars to image the seabed, acoustic communication systems to transmit messages and data, and positioning systems to track submersibles [2].

One of the main differences between electromagnetic waves in air and acoustic waves in water resides in constraints brought about by the propagation medium [2]. High-resolution environmental data are required for accurate predictions of acoustic propagation and scattering in shallow water [3]. Additionally, the property that most heavily influences acoustic propagation is sound speed [4]. For the case of the seabed, it is not only difficult to explicitly measure sound speed directly, but time and financial constraints can make it prohibitive to survey any region extensively. Although water column sound speed profiles may be estimated from measurements of conductivity, temperature, and depth (CTD) with more ease, they can be highly variable both spatially and temporally so that a single point measurement is often inadequate. Thus, inversion methods which are non-invasive
and which can rapidly and efficiently estimate environmental properties are very desirable.

1.2 Objectives

This work is concerned with developing inverse methods to estimate sound speed profiles of the water column and seabed using acoustic signals. These methods are designed to manage complications caused by the high spatial and temporal variability of the shallow-water environment as well as address the task of decoupling of the seabed and water column inverse problems.

The approach used in this work is based on a perturbative technique which was developed to use estimates of horizontal wave numbers to determine sound speed in the seabed [5]. This technique is appropriate for the highly variable shallow-water environment as horizontal wave numbers can be measured semi-locally (1-2 km aperture) and their values are a direct measurement of the local environmental parameters [6, 7, 8]. The output of this inversion scheme is sound speed in the sediment at a number of discrete depths. In previous applications of this technique, water column properties have been assumed to be known.

Compared to past applications of perturbative inversion, the accuracy of the solution is improved by implementing more suitable constraints to address issues of stability and uniqueness associated with solving the ill-posed inverse problem. One such improvement allows for discontinuities in the sediment sound speed profile to be resolved which provides for a better representation of the layered structure of the seabed. Another improvement makes reliable inversion for water column properties possible by using a priori knowledge of the background environment; this is a new capability for perturbative inversion. A further enhancement provides a mechanism to efficiently invert for sediment and water column sound speed profiles simultaneously. This technique allows for rapid assessment of the water column and the seabed properties in a single step.

Horizontal wave numbers suggest a way to separate the sediment inversion problem from that of the water column [9, 10]. Low-order wave numbers, which correspond to modes that propagate at shallow angles, are more sensitive to properties of the water column. On the other hand, high-order modes propagate at
steeper angles and penetrate deeper into the seabed. By carrying out the water column sound speed inversion first using low-order wave number inputs, an accurate estimate of water column properties can be obtained even when the sediment sound speed profile is not well known. Then sediment sound speed profile can be estimated using the inverted water column properties in the inversion. This two step process provides an estimate of environment properties throughout the waveguide.

1.3 Background

The nonlinear inverse problem has been treated by Monte Carlo and locally-linear techniques. While these techniques do provide statistical information about the solution (i.e., mean and variance), each suffers from inherent limitations in their approach to nonlinear problems. Monte Carlo techniques are expensive to compute and do not contribute to an intuitive interpretation of the problem; locally-linear techniques are limited by the multimodal objective landscape of nonlinear problems. In spite of the challenges, successful estimation of geoacoustic parameters has been achieved using both approaches. Monte Carlo methods have made use of sophisticated global search techniques such as simulated annealing (SA) [11, 12, 13, 14, 15] and genetic algorithms (GA) [16, 17, 18, 19] to efficiently search the multimodal parameter space. Locally linear techniques avoid local minimum solutions by relying on prior knowledge of the environment or by trying a number of starting profiles to ensure convergence.

Monte Carlo methods are generally designed as matched-field inversions that exploit the spatial coherence in the acoustic field. When using these methods, the seafloor is typically parameterized as to two or three layers over a half space, each with its own sound speed, density, and attenuation. Water column properties may or may not be included as parameters sought by the inversion scheme. These methods are very effective when parameters in the seafloor are constant over the area of the experiment. When the environment is range-dependent and several sediment profiles are needed to describe the bottom, the size of the parameter space is multiplied and the computational time is increased exponentially. In spite of these difficulties, Monte Carlo methods have been used on range-dependent
data with some, but limited success [20, 21, 22]. Additionally, nonlinear tracking algorithms have recently been applied to the inversion problem [23]. Although this method is promising, the computational burden is still very high.

On the other hand, locally-linear methods typically make use of data derived from field measurements. In the case of the perturbative inversion technique used in this work, these data are the horizontal wave numbers which can be used to characterize the local environment. This method has been applied to experimental data from a number of range-dependent environments [24, 25, 26, 27]. This method is focused on inverting for sound speed only and does not provide estimates of additional properties. Supplementary perturbative schemes are available for determining attenuation and density of the seabed [5]. Since the profile is not parameterized by these perturbative schemes, user error caused by over/under parameterization of the model space is eliminated.

In many inversion schemes, it is assumed that the either the properties the water column or of the seabed are known and the inversion is carried out for the unknown quantities alone. For example, it is common practice to perform the inversion solely for geoacoustic properties of the seabed [12, 28, 29, 24, 25, 26, 27]. In this case, a single water column sound speed profile may be accepted as truth based a sparse spatial and/or temporal sampling of the environment. If knowledge of the water column sound speed profile is poor, the effects of such an assumption can lead to significant errors in the estimation of the sediment properties [30, 31]. To address this deficiency, water column properties have been included as parameters in matched-field inversions, typically as empirical orthogonal function (EOF) coefficients [32, 33, 34, 35, 15]. However, although this technique has been successful for parameter estimation in environments where the water column sound speed profile can be approximated as range invariant, it does not address the issue of the increased size of the parameter search space for environments with range-dependent water column properties.

1.4 Approach

The approach to estimating the geoacoustic parameters conceived in this thesis was motivated by data from the “Shallow Water ’06” (SW06) experiment [36] which
took place on the New Jersey shelf area of the North Atlantic in the summer of 2006. Beginning in the 1960’s, this site has served as a natural laboratory for seabed studies in marine geology [37] and more recently shallow water acoustics. Past work shows this environment is characterized by a high degree of spatial variability of the sediment sound speed structure. Additionally, measurements collected during SW06 indicate evidence of significant fluctuations of the water column sound speed field.

The range dependence of the seabed on the New Jersey shelf is demonstrated by a wide range of results from a number of recent acoustic experiments. Compressed High Intensity Radar Pulse (CHIRP) reflection data inversions from the Shallow-Water Acoustics in a Random Medium (SWARM) experiment were used to create a complete three dimensional statistical representation of the bottom inhomogeneities [38]. Near surface sound speeds measured by the In situ Sound Speed Attenuation Probe (ISSAP) in the Geoclutter area were found to vary by 100 m/s over scales of 1 km [39]. In addition, local reflection coefficient inversions, yielding profiles to depths considered in this work, performed at a number of sites as part of the Boundary Characterization Experiment Series [40], indicated large spatial variability in the underlying sediments [41]. The stratigraphy of the seabed is also known to vary on the New Jersey shelf. Previous data from the Geoclutter area [5], combined with CHIRP seismic surveys taken during SW06, formed the basis for a detailed stratigraphic model of the subbottom in the area. A dominant feature of this seafloor model is the “R” reflector, a prominent shallow sub-surface seismic reflector that is subparallel to the seafloor on the middle and outer shelf [42, 43, 44].

The New Jersey shelf is a dynamically complex environment, where cool, fresh continental shelf water interacts with warmer, more saline water of the continental slope. The sharp transition in both properties, which occurs near the 100 meter isobath is known as the shelfbreak front [45]. Intrusions of slope water onto the continental slope are an important source of water column variability. During the SW06 experiment, water column properties were measured by a fleet of autonomous Webb gliders [46]. Based on these data, slope water eddies were identified as the main forcing mechanism for surface and pynocline intrusions. Slope water eddies are 50-100 km across and can vary significantly in just a few days.
Along shelf wind was played an important role for driving bottom intrusions by controlling the location of the foot of the front. The sub-pycnocline intrusions frequently took the form of shoreward extending arms that appeared rooted in the bottom intrusion. These intrusions were often located in the mid water column and were highly variable, having a cross-shelf extent of up to 20 km and thickness of 15 meters.

To address the high spatial variability of the New Jersey shelf environment, range-dependent estimates of the wave number spectrum were used to track the changes in water column and seabed properties. State-of-the-art wave number estimation techniques were applied to maximize wave number and range resolution. A number of the most successful wave number estimation techniques are described in Appendix A. The discussion begins in with a review of the Hankel Transform, which is defined for a point source in an axisymmetric environment. The explanation continues by summarizing several methods which are applicable to range-dependent environments. The Short-Time Fourier Transform (STFT), which utilizes a fixed aperture sliding window, is introduced for weakly range-dependent environments. Use of the sliding window is extended to more strongly range-dependent environments by implementing autoregressive (AR) techniques which make it possible to reduce the length of the estimation aperture. However, it is still not possible to resolve discontinuous changes in wave number data because the environment is assumed to be range-independent within the estimation aperture. By using the Kalman filter to update estimates of the AR coefficients, sudden changes in the environment can be tracked. An alternative method, the Wigner-Ville Distribution (WVD), which does not require use of a sliding window is also presented. Although it may not be suitable for the strongly range-dependent environments handled by AR methods, it does not suffer from the same drawbacks as the non-linear estimator.

The perturbative inverse algorithm involves solving a problem that is both ill-posed and underdetermined. For this reason, issues such as stability and uniqueness must be addressed. In past work, these concerns had been dealt with by adding a constraint to the solution such that the smoothest sound speed profile is chosen. However, owing to geological processes, sediments are often better described by layers having distinct properties and are not well represented by a
smooth profile. A new way to constrain the solution which emphasizes the layered structure of the seabed is accomplished using qualitative regularization [47]. This method makes use of a priori information about the depth of the “R” reflector and other sediment layers to resolve discontinuities in the sediment sound speed profile.

Estimation of the seabed properties using perturbative inversion requires a priori knowledge of the water column sound speed profile. Given the highly dynamic nature of the water column on the New Jersey shelf, limited direct measurements are likely to be inadequate to model the water column properties at all points along an acoustic propagation path. To address the need for detailed information about the range-dependence of the water column, perturbative inversion is applied to estimate the water column sound speed profile. However, the wave number data are insufficient to determine water column properties in some portions of the waveguide. This is particularly true near the sea surface where pressure release boundary combined with the downward refracting sound speed gradient present during summer months on the New Jersey shelf causes the acoustic field to have very little energy above the thermocline. Since the wave number data are insensitive to this portion of the waveguide, the solution deviates wildly from true values. A different type of error occurs near the seafloor where poor assumptions about the unknown seabed properties are aliased into the solution for the water column sound speed profile. Both of these issues are addressed by application of approximate equality constraints which force the solution to be close to likely values at prescribed locations [48]. This method works well for the water column inverse problem considered in this work because, in this area on the continental shelf, most of the variability occurs near the thermocline and sound speed values near the sea surface and seafloor are relatively stable [35].

Although use of the horizontal wave number data does afford some opportunity to isolate the water column sound speed inverse problem from that of the seabed, the two problems are not completely separable. Poor knowledge of the water column will have adverse affects on the solution for sediment sound speed profile and vice-versa. To address this shortcoming, a joint inversion technique was developed. This method uses a synthesis of qualitative regularization and approximate equality constraints to determine sound speed in the water column
and seabed simultaneously. The solution to this inverse problem is a single sound speed profile which uses qualitative regularization to specify a discontinuity in the sound speed profile at the seafloor as well as additional discontinuities within the seabed. Approximate equality constraints are applied to portions of the waveguide for which the wave number data are insensitive.

1.5 Outline

Chapter 2 reviews the forward problem of acoustic propagation in the shallow-water environment in the context of normal modes. The majority of the chapter is focused on an analysis of the Pekeris waveguide. The effects of changing the sound speed of the seabed and water column are studied to provide an intuitive understanding of the discrete horizontal wave numbers as they pertain to the perturbative inversion scheme.

Chapter 3 is concerned with the inverse problem. Perturbative inversion, the method upon which the work contained in this thesis is based, is reviewed. Qualitative regularization and approximate equality constraints are introduced as optimal methods for constraining the ill-posed inverse problem. The joint inverse scheme to determine water column and sediment sound speed profiles simultaneously is also presented.

The utility of qualitative regularization to constrain the sediment inverse problem is appraised in chapter 4. Using synthetic data, the algorithm’s robustness to inaccurate data, poor knowledge of the water sound speed profile, and faulty \textit{a priori} information concerning sediment layer depths are examined. Then the technique is applied to data collected during the SW06 experiment to create a three-dimensional model for sediment sound speed covering a 90 sq. km area.

The application of approximate equality constraints for constraining the water column inverse problem is evaluated in chapter 5. An analysis using synthetic data is presented to evaluate the algorithm’s robustness to inaccurate data, poor knowledge of seabed properties, and incorrect assumptions about the value of sound speed at the sea surface and seafloor. After that the method is applied to data from the SW06 experiment to estimate the range-dependent water column sound speed field.
The joint inverse scheme is assessed in chapter 6. Using synthetic data, the robustness of algorithm to inaccurate data is evaluated. The stability and resolution of the solution to the joint inverse problem are compared to that of the separate inverse problems. Then the technique is applied to data from the SW06 experiment.

A summary of the work is contained in chapter 7. Suggestions for future work are also offered.
Chapter 2

The Forward Problem

2.1 Introduction

In acoustics, the “forward problem” is concerned with calculating the acoustic field for a known environment and source conditions. There are essentially five types of models to calculate sound propagation in the sea: fast field program, normal mode, ray, parabolic equation, and direct finite difference or finite element [49]. This chapter reviews the forward problem in the context of normal modes. The discussion begins with an explanation of why normal modes are an appropriate choice for modeling sound propagation in the shallow-water waveguide. Then a generalized derivation using the spectral integral representation of the pressure field (Green’s function) is presented. The Green’s function characterizes the shallow-water waveguide and contains all information necessary to solve the forward problem [7]. The rest of the chapter presents an analysis of a simplified shallow-water environment: the Pekeris waveguide. The objective of this analysis is to provide an intuitive understanding of the discrete horizontal wave numbers as they pertain to the perturbative inversion scheme.

2.2 Normal Modes

At low frequencies, normal modes are the most efficient way to model acoustic propagation in shallow-water [50]. By considering length scales associated with the
environment and the acoustic wavelength, the following equation can be written

\[ R\lambda = 2H^2 \]  \hspace{1cm} (2.1)

where \( R \) is range, \( H \) is water depth, and \( \lambda = \frac{\omega}{c} \) is wavelength. Equation 2.1 specifies a division: for \( R\lambda < 2H^2 \) there are fewer rays than modes and it is convenient to use the ray approach; for \( R\lambda > 2H^2 \) there are more rays than modes and the mode concept is more appropriate. For the range of frequencies and environmental geometries studied in this thesis, the mode approach is deemed most applicable.

### 2.3 Derivation

The problem considered is that of calculating the response to a point source in a stratified fluid medium. An example of such an environment is shown in figure 2.1 where the properties of the water column and seabed are arbitrary functions of depth but are independent of range. Within each layer the solution is governed by the wave equation. Assuming harmonic time dependence of the source, i.e. \( s(t) = e^{-i\omega t} \), results in the Helmholtz equation. The inhomogeneous Helmholtz equation in cylindrical coordinates, under the assumption of cylindrical symmetry is given by

\[
\frac{\rho(z)}{r} \frac{\delta}{\delta r} + \rho(z) \frac{\delta}{\delta z} \left[ \frac{1}{\rho(z)} \right] \frac{\delta p(r,z)}{\delta z} + k^2 p(r,z) = -2\frac{\delta r}{r} \delta(z-z_0) \hspace{1cm} (2.2)
\]

where \( p(r,z) \) is pressure, \( \rho(z) \) is density, \( k(z) = \frac{\omega}{c(z)} \) is the total acoustic wave number, \( z_0 \) is the depth of the source, and \( z \) is the depth of the receiver.

The solution to equation 2.2 can be found by taking the spectral integral representation of the solution, closing the contour, and calculating the integral as a sum of residuals \([49]\). The spectral integral representation resulting from applying the zero-order Hankel transfer to equation 2.2 is given by

\[
p(r,z) = \frac{1}{2} \int_{-\infty}^{\infty} G(z, z_0; k_r)H_0^{(1)}(k_r r) k_r dk_r \hspace{1cm} (2.3)
\]

where \( H_0^{(1)} \) is the zero order Hankel function of the first kind and the Green’s
function $G(z, z_0; k_r)$ satisfies

$$
\rho(z) \left[ \frac{1}{\rho(z)} G'(z) \right]' + \left[ \frac{\omega^2}{c^2(z)} - k_r \right] G(z) = -\frac{\delta(z - z_0)}{2\pi} \tag{2.4a}
$$

$$
f^T(k_r^2) G(0) + g^T(k_r^2) \frac{dG(0)}{dz} = 0 \tag{2.4b}
$$

$$
f^B(k_r^2) G(D) + g^B(k_r^2) \frac{dG(D)}{dz} = 0, \tag{2.4c}
$$

where primes denote differentiation with respect to $z$. The top and bottom boundary conditions involve functions $f^{T,B}$ and $g^{T,B}$ representing an angle dependent impedance.

The modal solution is approximate because the horizontal wave numbers are determined by a choice of the branch cut in evaluation of the integral shown by equation 2.3. A common choice is the EJP branch cut, named after Ewing, Jardetzky, and Press [51], which divides the wave number spectrum into three spectral domains: the continuous spectrum, the discrete spectrum, and the evanescent spectrum. The discussion here is limited to the discrete spectrum for which the horizontal wave numbers are bounded by the bottom wave number $k_b = \frac{w}{c_b}$ and the water wave number $k_w = \frac{w}{c_w}$ such that $k_b < k_{rn} < k_w$. The EJP branch cut with the location of eigenvalues for the Pekeris problem (discussed in the next section)
Figure 2.2. The EPJ branch cut with the locations of eigenvalues for the Pekeris problem.

is shown in figure 2.2.

2.4 Pekeris Waveguide

In the rest of this chapter, determination of the eigenvalues and eigenfunctions is explained for the Pekeris waveguide which is a particular case of the shallow-water waveguide shown in figure 2.1, such that the sound speed and density in the water column and in the bottom have constant values and the sound speed in the bottom is greater than the sound speed in the water column, i.e. \( c_w(z) = c_w, \rho_w(z) = \rho_w, \)
\( c_b(z) = c_b, \rho_b(z) = \rho_b, \) and \( c_b > c_w. \) First, the solution for the Pekeris waveguide is described in general and then the effects of changing the sound speed in the bottom and in the water column are examined.

In this section an alternative approach is taken to solve equation 2.2. The procedure begins by assuming a separable solution, i.e. \( p(r, z) = R(r)Z(z). \) This leads to the following expression for pressure under the condition \( k_{rn}r >> 1:\)

\[
p(r, z) \approx \frac{i}{\rho(z)} \sqrt{\frac{2\pi}{r}} e^{-i\pi/4} \sum_{n=1}^{\infty} Z_n(z_0)Z_n(z) \frac{e^{ik_{rn}r}}{\sqrt{k_{rn}}} \tag{2.5}
\]
where $Z_n(z)$ are the depth dependent eigenfunctions with eigenvalues $k_{rn}$.

To gain insight into equation 2.5, we begin by evaluating the depth dependent eigenfunctions. The depth separated modal equation is given as

$$\frac{d^2 Z(z)}{d^2 z} + k_z^2 Z = 0 \quad (2.6)$$

where $k_z$ is the vertical wave number. Application of the boundary conditions at the top and bottom of the water column results in the characteristic equation:

$$1 - R_s R_b e^{2ik_z D} = 0 \quad (2.7)$$

where $R_s$ and $R_b$ represent plane wave reflection coefficients at the surface and bottom. As indicated in figure 2.1, the boundary condition at the surface is approximated as a pressure release boundary, $R_s = -1$. The boundary condition at the seafloor depends on the sediment properties and it can be expressed in terms of the complex reflection coefficient $R_b = |R_b| e^{i\Phi}$ where

$$|R_b| = \frac{\rho_b \cos \theta - i \rho_w \sqrt{\sin^2 \theta - \frac{c_w^2}{c_b^2}}}{\rho_b \cos \theta + i \rho_w \sqrt{\sin^2 \theta - \frac{c_w^2}{c_b^2}}} \quad (2.8)$$

$$\Phi = -2 \tan^{-1} \left[ \frac{\rho_w \sqrt{\sin^2 (\theta - \frac{c_w^2}{c_b^2})}}{\rho_b \cos \theta} \right] \quad (2.9)$$

where $\theta$ is the incident angle measured with respect to the vertical.

Substituting these expressions into equation 2.7, noting that $|R_b| = 1$ for angles greater than the critical angle $\sin \theta_c = c_w/c_b$, and using the relation $e^{i2\pi n} = 1$, an expression for $k_{zn}$ can be found:

$$k_{zn} = \frac{(n - 1/2)\pi - \Phi_n/2}{D} \quad \text{for } n = 1, 2, 3... \quad (2.10)$$

For the Pekeris waveguide, the eigenfunctions are sine functions in the water column and exponentially decay below the seafloor. The expression for pressure is given by
\( p(r, z) = \begin{cases} 
\frac{2\pi e^{\frac{i}{2} N}}{\rho} \sum_{n=1}^{N} A_n^2 \sin(k_{zn} z_0) \sin(k_{zn} z) e^{-ik_{zn} r} \sqrt{k_{zn}} & \text{for } 0 \leq z \leq D, \\
\frac{2\pi e^{\frac{i}{2} N}}{\rho} \sum_{n=1}^{N} A_n^2 \sin(k_{zn} z_0) \sin(k_{zn} D) e^{-k_{bn}(z-D) \sqrt{k_{zn}}} & \text{for } z > D 
\end{cases} \) 

(2.11)

where \( k_{zn}^2 = k^2(z) - k_{zn}^2(z) \), \( k_{bn} \) is the vertical wave number in the bottom, and \( A_n \) is given by

\[
A_n = \sqrt{2} \left[ \frac{1}{\rho_w} \left( D - \frac{\sin(2k_{zn} D)}{2k_{zn}} \right) + \frac{1}{\rho_b} \frac{\sin^2(k_{zn} D)}{k_{bn}} \right]^{-\frac{1}{2}}. 
\]

(2.12)

Normal modes can be thought of as plane waves that propagate along the waveguide at angles associated with each mode. The angles are determined by the horizontal wave number so that higher order wave numbers propagate at decreasing angles with respect to the vertical: \( \sin \theta_n = \frac{k_{zn}}{k} \). The number of modes that propagate in a waveguide corresponds to modes that propagate at angles greater than the critical angle \( \theta_n > \theta_c \). The number of modes propagating in a given waveguide is calculated by

\[
n_{\text{prop}} < \frac{2D}{\lambda} \sqrt{1 - \frac{c_w^2}{c_b^2}} + \frac{\Phi_n}{\Phi_{\text{cr}}} + \frac{1}{2} \quad \text{.} 
\]

(2.13)

Calculation of the number of propagating modes for an excitation frequency of 100 Hz is demonstrated for the Pekeris waveguide with \( c_w = 1500 \text{m/s}, c_b = 1600 \text{m/s}, \rho_w = 1.0 \text{g/cm}^3, \rho_b = 2.0 \text{g/cm}^3, \) and \( D = 100 \text{m}. \) Figure 2.3 shows the magnitude and phase of the complex reflection coefficient with angles associated with the propagating modes indicated by the dots. All propagating modes correspond to angles greater than the critical angle for which \( |R_b| = 1 \). According to equation 2.13, \( n_{\text{prop}} = 5 \). For this example, the angle associated with mode five is very close to the critical angle.

### 2.5 Group Speed

Modal group velocity dispersion curves provide additional insight about the normal mode solution. Group velocity dispersion data contains valuable information about
Figure 2.3. The magnitude and phase of the complex reflection coefficient for the Pekeris waveguide with $c_w = 1500 m/s$ and $c_b = 1600 m/s$. Angles associated with propagating modes are indicated by the dots.

The shallow-water waveguide and has been used in numerous inversion schemes [5, 52, 53, 54, 55]. Group velocity is a measure of the energy transport of a particular mode, i.e. $v_{gn} = d\omega/dk_{rn}$. For the Pekeris waveguide, group velocity is given by

$$v_{gn} = \frac{k_{rn}}{c_w} \left[ \frac{D(k_{rn}^2 - k_b^2)}{\cos^2[D\sqrt{k_b^2 - k_{rn}^2}]} + \frac{\rho_b}{\rho_w} (k_{rn}^2 - k_b^2) \right] + \frac{\rho_b k_b}{\rho_w c_w} (k_{rn}^2 - k_b^2)$$

The limiting behavior the group speed curves at the cutoff frequency $\omega_c$ and infinite frequency $\omega_\infty$ is

$$\lim_{\omega \to \omega_c} v_{gn} = c_b$$

$$\lim_{\omega \to \omega_\infty} v_{gn} = c_w$$

An additional characteristic of the group velocity dispersion curve is the minimum value it takes after the cutoff frequency known as the Airy Phase. In time
Figure 2.4. Mode shapes for the Pekeris waveguide with $D = 100m$, $c_w = 1500m/s$, $\rho_w = 1.0g/cm^3$, $c_{b1} = 1800m/s$ (solid blue line), $c_{b2} = 1600m/s$ (dashed red line), $\rho_b = 2.0g/cm^3$, and a frequency of 100 Hz. The solid black line represents the water depth.

domain solutions, this feature causes the tail of a transient modal arrival [49].

2.6 Effect of Sediment Sound Speed

The effect of sediment sound speed on normal modes can be understood by studying a set examples. Consider two waveguides, both 100 meters deep with $c_w = 1500m/s$, $\rho_w = 1.0g/cm^3$, and $\rho_b = 2.0g/cm^3$. Let the sound speed in the bottom of the first waveguide be $c_{b1} = 1800m/s$ and let the sound speed in the bottom of the second be $c_{b2} = 1600m/s$. Figure 2.4 shows mode shapes calculated for a frequency of 100 Hz and the corresponding horizontal wave number values are listed in table 2.1. One of the most notable features of the plot is the similarity in the shapes of the modes. In fact, the shapes of mode one calculated for the two environments are almost indistinguishable. Likewise, the values of the horizontal wave numbers as shown in table 2.1 are nearly equal. The fact that large changes
Table 2.1. Horizontal wave numbers and e-folding depths for two different bottoms.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Horizontal Wave Number</th>
<th>E-Folding Depth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1800m/s</td>
<td>1600m/s</td>
</tr>
<tr>
<td>1</td>
<td>0.4179 m⁻¹</td>
<td>0.4180 m⁻¹</td>
</tr>
<tr>
<td>2</td>
<td>0.4148 m⁻¹</td>
<td>0.4151 m⁻¹</td>
</tr>
<tr>
<td>3</td>
<td>0.4097 m⁻¹</td>
<td>0.4102 m⁻¹</td>
</tr>
<tr>
<td>4</td>
<td>0.4022 m⁻¹</td>
<td>0.4031 m⁻¹</td>
</tr>
<tr>
<td>5</td>
<td>0.3923 m⁻¹</td>
<td>0.3939 m⁻¹</td>
</tr>
<tr>
<td>6</td>
<td>0.3797 m⁻¹</td>
<td>6.69 m</td>
</tr>
<tr>
<td>7</td>
<td>0.3643 m⁻¹</td>
<td>9.60 m</td>
</tr>
</tbody>
</table>

in the sound speed of the seabed can have little effect on the modal eigenvalues has lead to the application of high resolution estimators to accurately estimate horizontal wave number values.

The most distinct differences in the mode shapes are seen in the higher order modes, i.e. those modes with steeper propagation angles. Thus, it is evident that the bottom is most clearly characterized by the higher order modes. This observation is quantified by examining the e-folding depths of the modes, which is the depth at which the modal amplitude decays to \(1/e\) its value at the seafloor. Table 2.1 shows the e-folding depths for the modes plotted in figure 2.4. The data show that the higher order modes penetrate deeper into the sediment. In particular, mode five for \(c_{b2} = 1600\text{m/s}\) has a significantly deeper penetration depth. This is because it propagates at angles very close to the critical angle as shown by figure 2.3.

The data in table 2.1 indicates that the slower bottom produces modes with greater e-folding depths. Therefore, it is possible to estimate sound speed profiles deeper into the seabed in waveguides with slower bottoms. A further difference between the two waveguides is the number of modes that propagate: seven for the fast bottom and only five for the slow bottom. This results from equation 2.13 which shows the number of propagating modes decreases as the index of refraction \(\frac{c_w}{c_b}\) approaches one. Consequently, the environment with the fast bottom provides more data to use in the inversion scheme.

Group speed dispersion curves for the two Pekeris waveguides are plotted in
Figure 2.5. Group speed dispersion curves for the Pekeris waveguide with $D = 100\,\text{m}$, $c_w = 1500\,\text{m/s}$, $\rho_w = 1.0\,\text{g/cm}^3$, $c_{b1} = 1800\,\text{m/s}$ (solid blue line), $c_{b2} = 1600\,\text{m/s}$, (dashed red line) $\rho_b = 2.0\,\text{g/cm}^3$.

Figure 2.5. Consistent with the earlier examples, at 100 Hz there are seven and five modes propagating for the 1800 m/s and the 1600 m/s bottoms respectively. For the slower bottom, the fifth mode is near cutoff at this frequency. This is congruous with the observation from figure 2.3 that the angle associated with mode five is close to the critical angle.

The limiting behavior of the group speed dispersion curves at infinite frequency is clearly visible in the figure. Another important feature is the local minimum that occurs shortly after the cutoff frequency. This minimum is much deeper for the faster bottom. For this case, the modes propagate at steeper angles resulting in slower transfer of radial energy. Consequently, greater modal dispersion is observed for the environment with the faster bottom.

An alternative expression for group speed presented by Chapman and Ellis provides a more intuitive understanding of group speed [56]. This expression is applicable to all waveguides whose properties are well modeled by fluid layers (as opposed to equation 2.14 which was derived specifically for the Pekeris waveguide).
Group speed is calculated by

$$v_{gn} = \frac{1}{c_n} \left( \int_0^\infty dz \frac{1}{\rho(z)} Z_n^2(z) \left/ \int_0^\infty dz \frac{1}{\rho(z)c^2(z)} Z_n^2(z) \right. \right)$$

(2.16)

where $c_n = \omega / k_{rn}$ is the phase speed. Equation 2.16 involves the depth dependent eigenfunctions weighted by the density and sound speed profiles. These weighting coefficients indicate the percent that the water column and bottom each contribute to the integral and provide insight into features of the group speed dispersion curves.

Equation 2.16 was used to calculate the group speed dispersion curve for mode two for the waveguides with the fast and slow bottoms. The percentage contribution of the water column and seabed to the group speed dispersion curve was calculated by evaluating the integrals for each portion of the waveguide separately and comparing them to the total integral to get a ratio. This calculation was carried out for several frequencies and the results are shown in figure 2.6. For both waveguides, the first frequency for the calculation corresponds to the minimum value of the group speed dispersion curve. It is clear from the figure that the water column is the dominant influence to the integral at all frequencies. This is expected as most of the modal energy is trapped within the water column. Another observation gleaned from figure 2.6 is that the bottom contributes most to the group velocity integral near the cutoff frequency and its contribution decreases as frequency increases. This agrees with the previous conclusion drawn from the narrow band data that higher order wave numbers are more sensitive to bottom properties. This is because at a particular frequency the group speeds for the higher order modes will be closest to cutoff. Comparison of the integral contributions for different bottoms shows that the group speed calculated for the slower bottom involves a higher contribution from the seabed. This too reinforces an earlier conclusion, i.e. wave number data from slower bottoms contains more information about the seabed.
Figure 2.6. (a) Group speed dispersion curves for mode two with the frequencies for the percent contribution calculation indicated by a star. (b) Percent contribution to group speed for the environment with bottom sound speed of 1800 m/s (c) and 1600 m/s.
2.7 Effect of Water Column Sound Speed

Changes in horizontal wave numbers due to water column sound speed changes can be orders of magnitude larger than the same change in sound speed of the bottom [30]. This statement is illustrated by the example shown in figure 2.7 which shows the percent change in horizontal wave number values due to a 200 m/s decrease in sediment sound speed and 10 m/s decrease in water column sound speed. This example is based on the Pekeris waveguide with the fast bottom so that the decrease in sound speed of the bottom effectively compares the to same two Pekeris waveguides previously discussed. Clearly, the water column sound speed has the dominant effect on wave number values, especially for the low-order modes.

To understand the dominant influence of the water column properties on horizontal wave number values, recall that the discrete spectrum is bounded by the bottom wave number $k_b$ and the water wave number $k_w$ such that $k_b < k_{rn} < k_w$. Formerly, it was observed that increasing $c_b$ lessened the lower bound of the spectrum, allowing more modes to propagate. On the other hand, decreasing $c_w$ increases the upper bound of the spectrum, effectively shifting the wave number spectrum upward. Thus, a change in sediment properties and a change in water column properties have distinctly different effects on the discrete wave number spectrum.

2.8 Effect of Bottom Attenuation, Density, and Shear Speed

The perturbative inverse scheme used in this work is focused on estimating sound speed only. Therefore, when carrying out the inversion, values of attenuation, density, and shear speed of the bottom are assumed to be known. It is often necessary to make assumptions about these properties given little a priori knowledge of their true values. In most cases, poor knowledge of these properties will not seriously affect the solution to the inverse problem because the property that most heavily influences acoustic propagation is sound speed [4]. Moreover, it is well known that acoustic propagation is less sensitive to density [57] and that the effect of low shear
Sensitivity to environmental parameters can only be established on a case-by-case basis [5]. To this end, the effect of changing attenuation, density, and shear speed of the bottom on the value of the horizontal wave numbers is considered for the Pekeris waveguide with the fast bottom from the previous examples. For reference, these effects are compared to the effect of changing the sound speed of the bottom. The properties used to construct the bottoms for each calculation of the horizontal wave numbers are summarized in table 2.2. In all cases, the sound speed and density of the water column were 1500 m/s and 1 g/cm$^3$.

The percent change in the value of the first five horizontal wave numbers due to a 200 m/s change in sediment sound speed, a 1.0 dB/λ change in attenuation, a 0.3 g/cm$^3$ change in density, and 250 m/s change in shear speed are shown in figure 2.8. For all parameters, the high-order modes which penetrate the most deeply into the seabed are most affected the changes as indicated by the greatest percent change in the wave number values. As expected, sound speed has the greatest affect on the wave number values. The effect of changing attenuation by 1.0 dB/λ is almost undetectable with mode five changing by only 0.0008%. The effects of changing density and shear speed are also small compared to the effect resulting
Table 2.2. Properties of the environments used to study the effects of attenuation, density, and shear speed of the bottom on horizontal wave numbers.

<table>
<thead>
<tr>
<th>Environments</th>
<th>Sound Speed</th>
<th>Attenuation</th>
<th>Density</th>
<th>Shear</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original Environment</td>
<td>1800 m/s</td>
<td>0.0 dB/λ</td>
<td>2.0 g/cm³</td>
<td>0 m/s</td>
</tr>
<tr>
<td>Change in Sound Speed</td>
<td>1600 m/s</td>
<td>0.0 dB/λ</td>
<td>2.0 g/cm³</td>
<td>0 m/s</td>
</tr>
<tr>
<td>Change in Attenuation</td>
<td>1800 m/s</td>
<td>1.0 dB/λ</td>
<td>2.0 g/cm³</td>
<td>0 m/s</td>
</tr>
<tr>
<td>Change in Density</td>
<td>1800 m/s</td>
<td>0.0 dB/λ</td>
<td>1.7 g/cm³</td>
<td>0 m/s</td>
</tr>
<tr>
<td>Change in Shear Speed</td>
<td>1800 m/s</td>
<td>0.0 dB/λ</td>
<td>2.0 g/cm³</td>
<td>250 m/s</td>
</tr>
</tbody>
</table>

Figure 2.8. Percent change in horizontal wave number values due to a 200 m/s change in sediment sound speed (blue), a 1.0 dB/λ change in attenuation (cyan), a 0.3 g/cm³ change in density (gold), and 250 m/s change in shear speed (maroon).

from the change in sound speed.

2.9 Effect of Excitation Frequency

Although not an environmental parameter, excitation frequency also has a significant effect on the horizontal wave number data. Whereas the analysis of the effects of the sediment and water column sound speed were studied to understand the inputs to the inverse problem, knowledge of the influence of excitation frequency can be used to optimally design an experiment. Frequencies can be chosen
Table 2.3. Horizontal wave numbers and e-folding depths for two different frequencies.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Horizontal Wave Number</th>
<th>E-Folding Depth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>100 Hz</td>
<td>50 Hz</td>
</tr>
<tr>
<td>1</td>
<td>0.4179 m⁻¹</td>
<td>0.2077 m⁻¹</td>
</tr>
<tr>
<td>2</td>
<td>0.4148 m⁻¹</td>
<td>0.2022 m⁻¹</td>
</tr>
<tr>
<td>3</td>
<td>0.4097 m⁻¹</td>
<td>0.1923 m⁻¹</td>
</tr>
<tr>
<td>4</td>
<td>0.4022 m⁻¹</td>
<td>0.1774 m⁻¹</td>
</tr>
<tr>
<td>5</td>
<td>0.3923 m⁻¹</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.3797 m⁻¹</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.3643 m⁻¹</td>
<td></td>
</tr>
</tbody>
</table>

to maximize penetration into the seabed or to determine fine scale fluctuations of the sound speed profile. Additionally, some frequencies are more appropriate for use in the water column inverse problem while others are better suited for the seabed inverse problem.

The effect of frequency on modal sensitivity to the seabed has already been eluded to in the discussion of group speed dispersion curves. As shown by figure 2.6(b) and (c), for a particular mode, the seabed contributes less to the calculation for group speed at higher frequencies. This point is affirmed by comparing the e-folding depths of the modal eigenvalues at two different frequencies. Using the Pekeris waveguide with the fast bottom from the previous examples, penetration in to the seabed is compared for modes corresponding to excitation frequencies of 50 and 100 Hz in table 2.3. The data shows that lower excitation frequencies produce modes which penetrate significantly deeper into the seabed.

The compliment to this argument is also true. By considering figure 2.6 it is shown that, for a particular mode, the water column contributes more to the calculation for group speed at higher frequencies. This can be understood from the narrowband wave number data as well: the wave numbers corresponding to higher excitation frequencies propagate at larger angles with respect to the vertical. For the 50 and 100 Hz data, the first wave numbers propagate at angles of 1.4418 and 1.5016 radians, respectively.

On the other hand, use of higher frequency modal data makes it possible to resolve smaller features in the sound speed profile. The shorter wavelengths are
Table 2.4. Summary: effects of environmental parameters on normal modes.

<table>
<thead>
<tr>
<th>Increase in Parameter</th>
<th>Horizontal Wave Number</th>
<th>No. of Modes</th>
<th>E-Folding Depth of $v_{gn}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>↑ Bottom Sound Speed</td>
<td>↓</td>
<td>↑</td>
<td>↓</td>
</tr>
<tr>
<td>↑ Water Sound Speed</td>
<td>↓</td>
<td>↓</td>
<td>↑</td>
</tr>
</tbody>
</table>

more sensitive to detailed structure. Additionally, higher excitation frequencies produce more modes, providing more data for the inverse problem.

The choice of excitation frequency involves a tradeoff between seabed penetration and model resolution or compromise between modal sensitivity to water column and seabed properties. Typically, experiments are designed to make use of a range of frequencies: low frequencies to characterize deeper sediments and higher frequencies data to determine the small-scale structure of the seabed.

### 2.10 Summary

Table 2.4 summarizes the effects of the water column and sediment sound speed on horizontal wave number values, number of propagating modes, e-folding depth, and spread of the group velocity dispersion curves for the case of the Pekeris waveguide. The double arrows indicate which factors have the strongest effect, i.e. the value of the horizontal wave numbers is most influenced by the water column sound speed.

The relationships summarized in the table provide a basis for an understanding of how horizontal wave numbers are influenced by environmental properties. This knowledge can be used when making decisions involved in performing the inversion. Such determinations include selection of the starting profile for the perturbative process which can be surmised from the number of horizontal wave numbers. Additionally, the maximum depth to include in the seabed inversion can be inferred from the e-folding depth. Most importantly, the fact that the values of the horizontal wave numbers are dominated by the water column sound speed profile indicates that water column properties must be well known before attempting inversion for seabed properties.

The analysis of the effects of sound speed in the seabed and water column
on normal modes was carried out to provide a basic understanding of the wave number data. However, it should be noted that the relations developed in this chapter were derived for the Pekeris waveguide which is a very restricted case of the shallow-water environment. More complicated seabeds will have modes that do not simply decay exponentially below the seafloor; typically, higher order modes will have significant fluctuations within the seabed. Additionally, the group speed dispersion curves for such environments are more complex, having features associated with energy trapped within layers of the seabed. For example, Potty and Miller provide an analysis of modal arrival time data for environments with one and two layers of sediment in [52]. In addition, Non-uniform water column sound speed profiles will cause a non-uniform shift to the horizontal wave number values. As a result, low order modes, which propagate at shallower angles, refract more in the water column. Therefore, these modes are more influenced the minimum sound speed in the water column. On the other hand, high order modes, which propagate at steeper angles are more influenced by the depth averaged water column sound speed profile [10]. In later chapters, the relationships developed for the Pekeris waveguide are extended to more complicated environments.
Chapter 3

The Inverse Problem

3.1 Introduction

In chapter 2, the effects of environmental parameters on normal modes were described. This chapter presents the method by which measurements of the horizontal wave numbers are used to estimate environmental parameters, particularly sound speed in the water column and in the seabed as a function of depth. In the first part of the chapter, the perturbative inversion algorithm is reviewed. Obtaining the solution involves solving an ill-posed problem and the corresponding stability and uniqueness issues are addressed. New ways of constraining the inverse problem to obtain a more accurate solution for sound speed in the seabed and water column are introduced. Methods for calculating resolution and variance are also presented.

3.2 Perturbative Inversion

As indicated in chapter 1, this work makes use of the locally-linear inversion technique developed by Rajan, Frisk, and Lynch [5]. The method is perturbative, meaning that the solution is found by computing a small change to a background profile. Additionally, the method is iterative: after a perturbation is calculated, the new profile becomes the background and the process is repeated. Iteration ends when the mismatch between the measured data and the data produced by
the model has been reduced to an acceptable level.

Derivation of the perturbative inversion algorithm begins with the depth separated normal mode equation:

\[
\frac{1}{\rho(z)} \frac{d^2 Z_n}{dz^2} + \frac{d}{dz} \left( \frac{1}{\rho(z)} \right) \frac{d Z_n}{dz} + \frac{1}{\rho(z)} \left[ q(z) - k_{rn}^2 \right] Z_n = 0 \tag{3.1}
\]

where \( q(z) = k^2(z) = \frac{\omega^2}{c^2(z)} \).

A perturbation is made to the waveguide \( q(z) \rightarrow \Delta q(z) \), which causes a perturbation in the other terms \( Z_n(z) \rightarrow \Delta Z_n(z) \) and \( k_{rn} \rightarrow \Delta k_{rn} \).

Making use of the orthogonality of the eigenfunctions leads to:

\[
\Delta k_{rn} = \frac{1}{k_{rn}^0} \int_0^\infty \rho^{-1} |Z_n^0(z)|^2 k^2(z) \Delta c(z) \frac{\Delta c(z)}{c_0(z)} dz \tag{3.2}
\]

where \( k_{rn}^0 \) and \( Z_n^0(z) \) are the horizontal wave numbers and depth dependent eigenfunctions of the background environment.

This equation can be written in the form of a Fredholm integral of the first kind:

\[
d_i = \int_0^\infty m(z) G_i(z) dz \quad \text{for } i = 1, ..., N \tag{3.3}
\]

where \( d_i \) is the perturbation to the \( i \)th wave number measurement, \( G_i(z) \) is the forward model or kernel, \( m(z) \) is the perturbation to the sound speed profile, and \( N \) is the number of modes. Note that while \( d_i \) represents a number of discrete measurements, the model parameters \( m(z) \) are a continuous function of depth. Solution of a continuous inverse problem such as this requires estimates of the model parameters at an infinite number of depths. As a result, the problem is inherently underdetermined [58].

Equation 3.3 can be used to invert for either sediment or water column sound speed profile. This is done by setting the limits of the integral to the depths of interest. For example, to invert for seabed properties, the limits of integration should be set from the seafloor to a chosen inversion depth. On the other hand, estimates of water column sound speed can be obtained by limiting the integral between the sea surface and the seafloor. To invert for properties of both the
water column and the bottom simultaneously, the integral limits should be set to encompass depths from the sea surface to the chosen inversion depth below the seafloor.

The continuous inverse problem is approximated as a discrete problem by assuming the sound speed speed profile can be represented by a finite number of points. After discretization, equation 3.3 is expressed in matrix form as

\[ \mathbf{d} = \mathbf{Gm} \]  

(3.4)

where \( \mathbf{d} \) is an \([N \times 1]\) vector representing the data, \( \mathbf{G} \) is an \([N \times M]\) matrix representing the forward model or kernel, \( \mathbf{m} \) is an \([M \times 1]\) vector representing the model parameters.

### 3.3 Stability and Uniqueness

Equation 3.4 represents an ill-posed inverse problem. In an ill-posed inverse problem, small errors on the data can create large deviations in the solution. Additionally, there may be infinitely many least squares solutions to such a problem. These two issues are commonly known as stability and uniqueness. There are a number of methods available in the literature for solving ill-posed inverse problems. The most prevalent techniques are singular value decomposition (SVD) and Tikhonov regularization (or simply regularization) [59, 58]. Both of these methods have been successfully applied to the geoacoustic perturbative inverse problem [5, 24, 25, 60].

In the remainder of this section, Tikhonov regularization will be reviewed followed by the introduction of other non-traditional methods that are more suitable for handling stability and uniqueness issues particular the types of inverse problems of interest. Application of both Tikhonov regularization and SVD result in a smoothed version of the true sound speed profile. However, owing to geological processes, sediments are often better described by layers having distinct properties and are not well represented by a smooth profile. To address this shortcoming of the conventional methods, qualitative regularization is applied to the sediment sound speed inverse problem making it possible to incorporate discontinuities in the sound speed profile [47]. This method relies on \textit{a priori} knowledge of the lo-
cation of sound speed discontinuities to produce a superior solution. On the other hand, water column sound speed profiles can be well approximated as smooth functions of depth and, therefore, are well served by the traditional smoothness constraint. However, the smoothness constraint alone is not enough to stabilize the solution. By further constraining the inversion using knowledge of the time- or range-average sound speed profile, more reliable inversion results can be obtained. This approach, which provides an improved solution compared to conventional methods by incorporating a priori information, is known as approximate equality constraints [48]. Finally, it is possible to simultaneously estimate both water column and seabed properties. This is accomplished by a synthesis of qualitative regularization and approximate equality constraints.

3.3.1 Tikhonov Regularization

For all the techniques listed above, the solution is selected from all possible models that satisfying equation 3.4 by choosing one that has some characteristic of the expected solution. In the case of Tikhonov regularization [59, 58], the flattest or smoothest solution is chosen. This is done by finding a solution that satisfies both the data (equation 3.5a) and a smoothness constraint (equation 3.5b):

\[ Gm = d \]  \hspace{1cm} (3.5a)
\[ Lm = 0 \]  \hspace{1cm} (3.5b)

where \( L \) is a discrete version of the differential operator \( \frac{d^n}{dz^n} \) where \( n = 1 \) favors the flattest solution and \( n = 2 \) favors the smoothest solution.

Equation 3.5a must be solved simultaneously with equation 3.5b. Assigning a weighting matrix \( W \) to the constraint, the solution can be found from:

\[
\begin{pmatrix}
G \\
WL
\end{pmatrix} m =
\begin{pmatrix}
d \\
o
\end{pmatrix}.
\]  \hspace{1cm} (3.6)

Let \( W^T W \) be equal to \( \lambda^2 I \), where \( I \) is the identity matrix and the Lagrange multiplier \( \lambda \) is chosen to conform with expectations of a smooth profile. Then the
least squares solution to equation 3.6 is given by:

\[ \hat{m} = (G^T G + \lambda^2 L^T L)^{-1} G^T d. \]  (3.7)

### 3.3.2 Qualitative Regularization

When there is some prior insight about the solution, this information can be included in the inversion to obtain a more accurate result. For the case of qualitative regularization this \textit{a priori} information comes in the form of location of sound speed discontinuities. Such information can be gleaned from seismic reflection measurements by converting two way travel time data to depth by assuming a value for sound speed. For an example of such seismic reflection measurements, see figure 4.13. In many cases, this information is readily available as seismic surveys are often part of large scale ocean acoustic experiments.

In qualitative regularization, the derivative operator \( L \) from the original regularization problem is replaced by user defined operator \( L_q \) [47]. As before, the solution is found by minimizing the residual (equation 3.8a) under the condition that the semi-norm (equation 3.8b) is also minimized:

\[ Gm = d \]  (3.8a)
\[ L_q m = 0 \]  (3.8b)

where the user defined operator \( L_q \) is given by

\[ L_q = L(I - \sum_{i=1}^{r} q_i q_i^T) \]  (3.9)

where the set of vectors \( \{q_i\}_{i=1}^{r} \) is an orthogonal basis for the subspace \( Q \) which contains all models that have discontinuities at the prescribed locations.

To understand the construction of the subspace \( Q \), consider the following example. Suppose there is a sound speed profile representing the first 24 m below the seafloor discretized in depth such that \( \Delta z = 2 \) m. Assume there is \textit{a priori} knowledge of a discontinuity in the sound speed profile at a depth of 9 m below the seafloor (between the 5th and 6th indices). In this case, the subspace \( Q \) is defined
Figure 3.1. Comparison of inversion results using (a) Tikhonov regularization and (b) qualitative regularization to stabilize the solution.

In this way, qualitative regularization removes the penalty imposed by regularization by allowing for a discretized Heaviside step function (or jump discontinuity) at the specified location. The jump may be positive, negative, or remain constant. There is no constraint on the size or direction of the discontinuity.

The solution to the inverse problem using qualitative regularization is given by:

\[
\hat{m} = (G^T G + \lambda^2 L_q^T L_q)^{-1} G^T d. 
\] (3.11)

For a known problem, inversion results obtained using Tikhonov and qualitative regularization are compared in figure 3.1. The sediment sound speed profile for this environment was taken from the Geoacoustic Inversion Techniques (GAIT) workshop 2003 [20]. Source frequencies of 75 and 100 Hz were used in the inversion, producing a total of 18 propagating modes. Exact wave number data were
used as input into the inversion scheme. For both inversions, the starting environment consisted of the same isovelocity sediment profile. Other specifics about the problem such as the value of the Lagrange multiplier and discretization of the sound speed profile are given in chapter 4 where this problem is presented in detail. In the figure, the true sound speed profile is shown by the solid black line and the inversion result is shown by the dashed red line. As shown by figure 3.1 (a), application of Tikhonov regularization results in a smoothed version of the true sound speed profile. On the other hand, by admitting discontinuities into the solution, qualitative regularization is better able to approximate the true sound speed profile as shown in figure 3.1 (b).

### 3.3.3 Approximate Equality Constraints

The water column sound speed inverse problem is also served by inclusion of pre-existing knowledge of the solution. In this case, *a priori* information in the form of the mean (range or time averaged) sound speed profile is used to stabilize the solution and chose a unique answer. Knowledge of the mean sound speed profile in the water column may be gleaned from temporally and spatially relevant measurements made during an experiment or from historical data. This type of information is not typically available for the seabed. Knowledge of the mean water column sound speed profile is included in the inversion using approximate equality constraints [48]. According to this technique, the solution is estimated using two types of constraints: a relative equality constraint which requires the elements of the solution vector be close to each other and an absolute equality constraint which requires the solution vector be close to a prescribed vector. The goal is to find a solution that satisfies both the data (equation 3.12a) and the constraints (equations 3.12b and 3.12c):

\[
Gm = d \quad (3.12a)
\]

\[
Rm = \rho \quad (3.12b)
\]

\[
Am = \alpha \quad (3.12c)
\]
where $R$ is an $[L \times M]$ matrix and $\rho$ is an $[L \times 1]$ vector that specify the relative constraints and $A$ is an $[H \times M]$ matrix and $\alpha$ is an $[H \times 1]$ vector that specify the absolute constraints.

In the same way the solution was obtained for Tikhonov regularization, equations 3.12a - 3.12c must be solved simultaneously:

$$
\begin{pmatrix}
G \\
W_R R \\
W_A A
\end{pmatrix}
m = 
\begin{pmatrix}
d \\
W_R \rho \\
W_A \alpha
\end{pmatrix}
$$

(3.13)

where $W_R$ and $W_A$ are weighting matrices for the relative and absolute constraints.

Assume the weighting matrices $W_R^T W_R$ and $W_A^T W_A$ are equal to $\lambda_1^2 I$ and $\lambda_2^2 I$. Then the least squares solution to equation 3.13 is given by:

$$
\hat{m} = (G^T G + \lambda_1^2 R^T R + \lambda_2^2 A^T A)^{-1} (G^T d + \lambda_1^2 R^T \rho + \lambda_2^2 A^T \alpha)
$$

(3.14)

Choice of the values of $\lambda_1$ and $\lambda_2$ must be decided by the user. In cases for which there are large uncertainties associated with the wave number estimates, larger values of the Lagrange multipliers should be chosen so that the solution will be well constrained. On the other hand, if the solution is expected to have significant fluctuations with depth, smaller values for $\lambda_1$ should be applied. If the solution is expected to deviate from the a priori information, smaller values for $\lambda_2$ are appropriate.

The relative equality constraint (Eqn. 3.12b) is a generalization of the smoothness constraint (Eqn. 3.5b). By defining $R = L$ where $L$ represents the discrete version of the second differential operator, and defining $\rho = 0$, the solution will be a smooth transition between the points specified by the absolute equality constraint.

The absolute equality constraint requires the solution be close to specified values at designated points. For the water column inverse problem, the solution is improved by the addition of the absolute equality constraint near both the sea surface and the seafloor. The pressure release boundary at the air-water interface causes the eigenfunctions to have very little energy near the sea surface. This effect is exaggerated in many shallow water environments by a downward refracting sound speed gradient. This type of water column sound speed profile is prevalent
at mid-latitudes during the summer months. The absolute equality constraint at
the sea surface mitigates this problem by forcing the solution to be close to likely
values. The solution is also improved by application of the absolute equality con-
straint near the seafloor, but for a different reason. Often, when the inversion
for water column sound speed is performed, the seabed properties are not well
known. This leads to errors in the estimated water column sound speed profile,
especially near the seafloor. Application of the absolute equality constraint near
the seafloor decreases the effects of using an erroneous seabed sound speed
profile in construction of the background environment. Other authors have expe-
rienced solutions which deviate from true values at the sea surface [55] and at the
seafloor [22] using global search methods. These errors are consistent with results
from perturbative inversion obtained by this author using only the relative equality
constraint.

To address inaccuracies in these portions of the waveguide, the matrix \( A \) spec-
ifies that the two shallowest and two deepest points of the sound speed profile be
constrained. It is necessary to constrain at least two consecutive points to estab-
lish the trajectory of the profile. The majority of the water column, particularly
the region near the thermocline where most of the variability is expected to oc-
cur, is not affected by the absolute equality constraint. The vector \( \alpha \) is chosen
to be a vector of zeros. This condition does not allow for perturbations from the
background profile at the specified points.

Substituting the assigned values for \( R \), \( \rho \), and \( \alpha \) into equation 3.14 provides a
more simplified version of the equation:

\[
\hat{m} = (G^T G + \lambda_1^2 L^T L + \lambda_2^2 A^T A)^{-1} G^T d.
\]  

(3.15)

Figure 3.2 shows estimates of the water column sound speed profiles obtained
using perturbative inversion applied with and without the absolute equality con-
straint. The inputs into the inversion were exact wave number data from 125 and
175 Hz. In both plots, the true sound speed profile is shown by the solid black
line and the estimate is shown by the red dashed line. The true water column
sound speed profile is based on measurements taken during the SW06 experiment.
The starting profile was taken to be the mean of measurements at a fixed location
over a 37 day period. The sediment sound speed profile of the background was in
error of 50 m/s compared to the true environment, that is the environment used to construct the data vector. As a result of this erroneous input, the solution obtained without the absolute equality constraint, shown in figure 3.2(a), deviates near the seafloor. Additionally, this solution also suffers near the sea surface where information content of the data is inadequate to estimate the sound speed. On the other hand, the solution obtained using both relative and absolute equality constraints, shown in figure 3.2(b), accurately estimates the sound speed profile at all depths.

### 3.3.4 Combined Constraints

Qualitative regularization and approximate equality constraints can be combined to allow for simultaneous inversion of both water column and sediment sound speed profiles. The synthesis of the approaches is accomplished by replacing the derivative operator $L$ of equation 3.15 with user defined operator $L_q$ which is given by equation 3.9. For this application, $L_q$ is chosen to specify a discontinuity in the

Figure 3.2. Comparison of inversion results for the water column sound speed profile (a) using only relative equality constraint and (b) using relative and absolute equality constraints to stabilize the solution.
sound speed profile at the seafloor. The definition of $L_q$ can be defined to include layers in the seabed as well. Thus, the solution to the joint water column and sediment inverse problem is given by:

$$\hat{m} = (G^T G + \lambda_1^2 L_q^T L_q + \lambda_2^2 A^T A)^{-1} G^T d. \tag{3.16}$$

The principle benefit of solving for water column and sediment sound speed profiles simultaneously is that the effect of erroneous inputs is avoided. Inaccuracies enter the inverse problem through construction of the background environment used in the perturbative scheme. For example, when inverting for the sediment sound speed profile independently, the water column sound speed profile is assumed to be known. If the assumed water column sound speed profile is not equal to that of the true environment, the solution for sediment sound speed will compensate for this error. This result is well known and it has been documented [30, 61]. Likewise, if incorrect parameters are used for the seabed when determining the water column sound speed profile independently, the solution for the water column will suffer.

An example of the joint water column-seabed inverse problem is shown in figure 3.3. This solution approach is not compared with any other as simultaneous inversion of water column and sediment sound speeds profiles using perturbative inversion was not previously available. Instead, the starting profile is shown to illustrate the inversion scheme’s ability to concurrently solve for both water column and sediment properties. In the figure, the true sound speed profile is shown by the solid black line, the starting profile is shown by the green dotted line, and the estimate is shown by the dashed red line. Both the water column and sediment sound speed profiles are well recovered using this inversion technique.

3.4 Resolution

Depth averaging results from solving the Fredholm integral of equation 3.3. In this problem, the model parameters are a continuous function of depth and must be determined using a finite amount of data. A continuous inverse problem such as this can be viewed as the limit of a discrete problem where the number of model parameters becomes infinite, and as such it is inherently underdetermined [58].
Figure 3.3. Simultaneous inversion of sediment and water column sound speed profiles. The true sound speed profile is shown by the solid black line, the starting profile is shown by the green dotted line, and the estimate is shown by the dashed red line.

Given the finite amount of data, it is unreasonable to model \( m(z) \) exactly at a specific point \( z = z_0 \). All determinations of the model parameters must be made in terms of local averages. According to the method of Backus and Gilbert [62], an estimate of the model parameters is given by

\[
\hat{m}(z_0) = \int_{0}^{D} R(z, z_0) m(z) dz
\]

(3.17)

where \( R(z, z_0) \) is the averaging kernel and \( \hat{m}(z_0) \) is the estimate of \( m \) at \( z = z_0 \). If \( R(z, z_0) \) is a delta function, the model will be recovered exactly. Thus, the goal is to construct the most peaked averaging kernel possible for a given set of measurements. A detailed derivation of the averaging kernel is provided in [62] and a summary is given in [5]. Descriptions of Backus-Gilbert theory are also available in standard inverse textbooks [58, 59, 63].

The resulting expression for the averaging kernel in discrete form is given by

\[
R = G^T (GG^T)^{-1} G.
\]

(3.18)

Resolution length describes how well the averaging kernel resembles an identity
matrix [60, 26]. It is defined by
\[
R_{L,i} = \frac{\sum_{j=1}^{N} R_{ij}^2}{R_{ii}} \Delta z.
\] (3.19)

Resolution length is a measure of the spread of the averaging kernel: higher values of the resolution length indicate the solution is vertically averaged to a greater extent.

Resolution length is calculated directly from the forward model and it only describes the resolving power of the data. It does not account for data errors or a priori information. However, both qualitative regularization and approximate equality constraints address issues of stability and uniqueness by balancing information from the data against information from the constraint. As a result, the solution is vertically averaged to a higher degree than specified by the resolution matrix. Thus, the resolution calculation represents the best case for vertical averaging. Nevertheless, it represents a useful measure of how well it can be expected to resolve sound speed profiles in the seabed and water column.

### 3.5 Variance

The complete solution of the inverse problem requires not only the estimates of the model parameter values, but also a measure of the uncertainty of the estimates. Although it is straightforward to calculate the variance of a linear inverse problem directly from the kernel, for this application this was not possible because the kernel was too poorly conditioned. Instead, estimates of the uncertainty of the inversion results can be determined by Monte Carlo methods [59, 64]. The technique is applied by simulating a collection of noisy data vectors and then examining the statistics of the resulting models:
\[
d + n_i = Gm_i.
\] (3.20)

From this ensemble of models, the empirical estimate of the covariance matrix can be obtained:
\[ \text{Cov} = \frac{D^T D}{N} \]  

(3.21)

where \( D = m_i^T - \bar{m}^T \), \( \bar{m} \) is the mean of the model estimates, and \( N \) is the number of models.

Error bars are defined as one standard deviation of the model estimates:

\[ \sigma_m = \sqrt{\text{diag}(\text{Cov})}. \]  

(3.22)

### 3.6 Summary and Conclusion

In this chapter, the perturbative inverse technique relating measurements of horizontal wave numbers to sound speed in the water column and seabed was reviewed. Qualitative regularization was introduced as a new way to deal with issues of stability and uniqueness associated with solving the ill-posed sediment inverse problem. This technique uses *a priori* information about seabed stratigraphy to resolve discontinuities in the sediment sound speed profile.

Qualitative regularization differs from other methods which have been used to treat the perturbative inverse problem. Souza [65] and Pool [66] made use of functional differentiation to form basis functions to discretize the integral. These authors attempt to describe the environment in terms of a finite number of parameters, for example sound speed at the seafloor and a linear gradient. Then the task is to invert for these two parameters. The advantage of this method is that the number of model parameters has been significantly reduced. However, selection of an appropriate basis for a particular problem requires detailed knowledge of the problem as well as the behavior of the basis functions. A poorly selected basis may not adequately approximate the solution, resulting in an estimated model that is very wrong [59]. Furthermore, functional differentiation frequently causes divergent solutions [67]. Qualitative regularization is not a discretization technique. Contrary to functional differentiation which may under parameterize the solution, qualitative regularization increases the degrees of freedom of the solution by removing the requirement of at flat/smooth solution by allowing discontinuities at specified points.

A new application of perturbative inversion was presented to estimate the water
column sound speed profile. For this case, the wave number data alone are often insufficient to determine sound speed in portions of the water column. This deficiency was addressed by application of approximate equality constraints. By using 
\textit{a priori} information, the solution is forced to be close to likely values near the sea surface and seafloor where water column properties are relatively stable. Near the thermocline, where most of the variability is expected to occur, the solution is not constrained by the \textit{a priori} information.

Finally, a combination of qualitative regularization and approximate equality constraints made it possible to estimate water column and sediment sound speed profiles simultaneously. This technique allows for rapid assessment of the water column and the seabed properties in a single step. The primary advantage of this approach eliminates inaccuracies caused by erroneous environmental inputs.
Chapter 4

Sediment Sound Speed Inversion: Application of Qualitative Regularization

4.1 Introduction

In this chapter, practical aspects of applying perturbative inversion for determination of sound speed in the seabed are presented. Qualitative regularization is applied to stabilize the solution making it possible to resolve discontinuities in the sound speed profile. As detailed in chapter 3, this method uses on prior knowledge of the location of sound speed discontinuities to produce a solution superior to that of conventional methods.

This chapter is divided into two parts. The first section is concerned with evaluating the robustness of the algorithm to inaccurate data and erroneous inputs. Synthetic data is used so that the effects of these deficiencies can be considered separately and the results can be compared to a known solution. In the second part of the chapter, the method is applied to data from the Shallow Water 2006 (SW06) experiment. This section addresses each of the steps in the procedure for determining sediment sound speed profiles, beginning with horizontal wave number estimation and concluding with an assessment of the results.
4.2 Evaluation of the Algorithm

Initially, a sample application using exact data and a known input model is considered. This allows for an appraisal of the capability of the inversion algorithm under ideal conditions. This example will provide a complete description of the inverse problem, giving attention to such topics as discretizing the continuous integral, choosing a starting profile, and assessing convergence of the solution. After the utility the inversion under ideal conditions has been established, consideration is given to the algorithm’s robustness to inaccurate wave number estimates, poor knowledge of the water column properties, and incorrect inputs for the discontinuities in the sound speed profile.

For the purpose of this evaluation, the inversion algorithm is applied to data from the GAIT 2003 Workshop [20]. The environmental parameters are taken from test case 1. The sediment sound speed profile for this environment is shown by the solid blue line in figure 4.1 and sound speed values are listed table 4.1 under the column titled “true profile”. The water column sound speed profile was downward refracting according to

\[ c_w(z) = 1495.0 - 0.04z \]  \hspace{1cm} (4.1)

where \( z \) is depth in meters.

The data used for the inversion are wave numbers at four frequencies: 50, 75, 125, and 175 Hz for which the waveguide supports 5, 8, 13, and 18 propagating modes respectively.

4.2.1 Example Application Using Idealized Inputs

In this section, the ability of the inversion algorithm to accurately estimate sediment sound speed profiles is assessed within the context of an example. This application utilizes exact data and known environmental inputs. These conditions allow for examination of the inversion algorithm itself which is inexact due to the linearization of the perturbed Helmholtz equation. Within the framework of this example, guidelines are set for making decisions about mesh size, setting up an appropriate background environment, and evaluating convergence of the algorithm.
The first topic addressed is discretization of the sediment sound speed profile. For this example, the seabed was discretized into half meter increments to a depth of 40 meters below the sea floor. The relatively small discretization was chosen so that the discontinuities in the sound speed profile could be distinctly resolved. Although using a coarser mesh typically results in a better conditioned system of linear equations, it also reduces the resolution of the solution. This relation between mesh size and resolution is known as “regularization by discretization” [59]. The effect of using smaller depth increments can be balanced by choosing greater values for the Lagrange multiplier $\lambda$. The maximum depth for inversion was chosen according to the e-folding depth of the deepest penetrating mode. This is typically the highest order mode of the lowest frequency. Knowing the true model parameters values, it is possible to make this calculation. In this complex environment, the high-order modes do not decay exponentially with depth and modal penetration into the seabed is much deeper than a Pekeris waveguide of the same depth with a sediment sound speed of 2060 m/s, which is the sound speed of the basement of the true environment. At depths greater than the inversion depth, the bottom is assumed to be a half space with sound speed equal to that of the deepest inversion result.

The starting profile of constant sound speed of 1675 m/s is shown by the black dotted line in figure 4.1. The starting profile was chosen to be as simple as possible, containing no a priori knowledge of the environment. In perturbative inversion schemes, choice of the starting profile can influence the solution. Linear methods are sensitive to gradient information and converge to the nearest local minimum [4]. Often a poor choice of the starting profile will lead to a divergent solution, which can be immediately discarded. Then the inversion can be tried again with a new starting profile. Many times it is necessary to try several starting profiles and then choose the solution that best fits the data.

Many times there is some prior knowledge of the environment that can be used to choose the starting profile. For an example, in a procedure developed by NAVOCEANO, parameter estimates and bounds for a matched-field search are determined using empirical relationships to describe geoacoustic parameters as function of grain size and depth [68].

A global search may also be useful to assist in the choice of the starting profile.
There is a precedent of using nonlinear search methods to come close to the solution and then applying linear methods to refine the result. For example, Taroudakis and Markaki proposed a scheme where the reference environment is defined using matched field processing and then a linearized model phase inversion scheme is applied [55]. Many other global search methods acknowledge the power of linear methods near the true solution and include a linear step within the Monte Carlo search [11, 15, 16].

Alternatively, the inversion algorithm can initially be run using Tikhonov regularization which does not require any \textit{a priori} information. Since the method does not allow for discontinuities in the sound speed profile, there are fewer degrees of freedom and the inversion is more likely to converge. These results can be used to identify a trend in the solution, i.e. a sound speed profile that increases with depth or a profile with a duct. In all cases, linearization implies that the solution to the inverse problem is close to the background profile. Although it is possible for the inversion to converge to the true model parameters when the starting profile is far from the solution, the problem becomes increasingly difficult when the data are inaccurate or when the sediment sound speed profile is exceedingly complicated.

In figure 4.1, the inversion result obtained after 20 iterations is shown by the dashed red line. This result was obtained using perturbation inversion using qualitative regularization with the depths of the sound speed discontinuities given as \textit{a priori} information. The value of the Lagrange multiplier $\lambda^2$ was chosen to be $10^4$ so that the resulting sound speed profile is very smooth within each layer. The solution is a good approximation to the true sound speed profile; the depth average error between the true profile and the solution is less than 7 m/s. Moreover, the layered features of the true sound speed profile are clearly represented by the solution.

Now, consider the smaller features of the inversion result, i.e. the negative gradient in the shallow layer, the point where the solution deviates from truth near the top of the second layer, and the decreasing trend in the deepest layer. These characteristics of the solution are artifacts of regularization. Qualitative regularization insures a smooth solution with break points, such that the solution would be smooth if the break points were removed. Thus, the gradient in the first layer is continued in the second layer. The Lagrange multiplier $\lambda$ controls the
Figure 4.1. Sediment sound speed for Test Case I from the GAIT 2003 Workshop. The true sound speed profile is shown by the blue solid line, the starting profile for the inversion is shown by the black dotted line, and the inversion result is shown by the red dashed line. Depth is measured from the sea surface.

smoothness of the solution; it can be viewed as a dial that adjusts the trade-off between the inversion’s sensitivity to the data and to the a priori information. By increasing the value of $\lambda$ the smoothness of the solution can be increased so that the sound speed within each layer has a constant gradient. For this problem, using this starting profile, using greater values of the Lagrange multiplier allowed too little information from the data to be used in the inversion and, consequently, the algorithm diverged. The superfluous features are a product of the iterative process; they are incorporated into the result when the solution is far from the true model parameter values.

The residual for the solution is plotted in Figure 4.2. Residual is defined as the difference between the measured data $d_i^{\text{obs}}$ and the data predicted by the inversion result $d_i^{\text{pre}}$:

$$\text{Res} = \sum_{i=1}^{N} d_i^{\text{obs}} - d_i^{\text{pre}} \quad (4.2)$$

where $N$ is the length of the longer data vector. As stated in chapter 3, the residual
is calculated for each iteration and perturbation to the background profile ends when the mismatch between the data vectors has been reduced to an acceptable level. Following from the definition of $N$, the residual will be much greater if the measured data vector and the data vector produced by the model are not the same length. As indicated in chapter 2, the number of eigenvalues is a clear indicator of bottom properties and the definition of the residual has been chosen to exploit this information.

Examination of the residual shows that the correct number of wave numbers has been reached after six iterations. Moreover, percent change in residual has decreased to less than 5% after 13 iterations. Clearly, the algorithm has converged after 20 iterations. In fact, between the 13th and 20th iterations, there is less than 5 m/s of change in sound speed profile at any depth.

Interestingly, the residual increases between the second and fourth iterations. The increase in the residual can be understood by examining the sound speed profiles resulting from different iterations. Profiles from selected iterations are shown in figure 4.3. The zero-th iteration corresponds to the isovelocity background of the starting profile. The first iteration correctly estimates the increase in sound
Figure 4.3. Selected iterations for the perturbative inversion to obtain the result shown in figure 4.1. The zero-th iteration corresponds to the isovelocity background of the starting profile. Depth is measured from the sea surface.

speed with depth at the first discontinuity, but wrongly indicates a decrease in sound speed at the second discontinuity. The next iterations improve the solution for both layers above the second discontinuity; however, in the bottom layer the incorrect trend of a decrease in sound speed is continued. This causes the inversion result to support fewer modes compared to the previous iteration and the residual is increased. After the first three iterations, the solution is well matched above the second sound speed discontinuity and the fourth iteration begins to correct the sound speed in the lowest layer. After this point the algorithm converges to the correct solution.

In this example, the initial divergence of the solution could have been avoided given a better choice of the starting profile. Selection of a more complicated starting profile which supports more modes than the isovelocity background would provide more data to use in the inversion. Moreover, when the starting profile contains the same trend as the true profile, the inversion is likely to converge more quickly. On the other hand, if the starting profile has distinctly different characteristics than the true profile, it can cause a number of the modes to be poorly matched, leading to a divergent solution.
Figure 4.4. Both sets of plots show sediment sound speed profiles on the left and the corresponding shape of mode seven for 125 Hz on the right. The (a) Layered Starting Profile and the (b) Alternate Starting Profile and their corresponding mode shapes are shown by the red dotted lines. The true sound speed profile and corresponding mode shapes are shown by the solid blue lines. Depth is measured from the sea surface.

To understand this concept, consider the mode shapes resulting from two different starting profiles. Let the first profile be known as the Layered Starting Profile and let it contain the same increasing pattern of the true sound speed profile, such that the sound speed in the shallowest layer is 1600 m/s and increases with depth in each layer at a rate of 50 m/s. This starting profile is not “close” to the solution; it differs from the true profile by almost 100 m/s in the shallowest layer and more than 350 m/s in the deepest layer. Also consider a second sound speed profile characterized by a sound speed duct in the second layer, such that the sound speeds in each layer are 1600, 1550, and 1700 m/s. Let this profile be referred to as the Alternate Starting Profile. Both starting profiles are shown with the true sound speed profile in the left plots of figure 4.4(a) and (b). Sound speed in each layer of the three profiles is summarized in table 4.1. To the right of the sound speed profiles in figure 4.4, the shape of mode seven in the seabed for 125 Hz is plotted for the true environment and for the starting environment. As shown in figure 4.4(a), the shape of mode seven from the Layered Starting Profile is very similar to that of the true profile. Conversely, the Alternate Starting Profile causes
Table 4.1. Sound speed profiles for the mode shape construction example.

<table>
<thead>
<tr>
<th>Thickness [m]</th>
<th>True Profile Top [m/s]</th>
<th>Bottom [m/s]</th>
<th>Layered Starting Profile Top [m/s]</th>
<th>Bottom [m/s]</th>
<th>Alternate Starting Profile Top [m/s]</th>
<th>Bottom [m/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1</td>
<td>1508.2</td>
<td>1508.2</td>
<td>1600.0</td>
<td>1600.0</td>
<td>1600.0</td>
<td>1600.0</td>
</tr>
<tr>
<td>2.3</td>
<td>1726.3</td>
<td>1737.9</td>
<td>1650.0</td>
<td>1650.0</td>
<td>1550.0</td>
<td>1550.0</td>
</tr>
<tr>
<td>4.1</td>
<td>1737.9</td>
<td>1750.9</td>
<td>1650.0</td>
<td>1650.0</td>
<td>1550.0</td>
<td>1550.0</td>
</tr>
<tr>
<td>6.0</td>
<td>1750.9</td>
<td>1762.1</td>
<td>1650.0</td>
<td>1650.0</td>
<td>1550.0</td>
<td>1550.0</td>
</tr>
<tr>
<td>5.4</td>
<td>1762.1</td>
<td>1774.0</td>
<td>1650.0</td>
<td>1650.0</td>
<td>1550.0</td>
<td>1550.0</td>
</tr>
<tr>
<td>4.2</td>
<td>2060.0</td>
<td>2060.0</td>
<td>1700.0</td>
<td>1700.0</td>
<td>1700.0</td>
<td>1700.0</td>
</tr>
</tbody>
</table>

the shape of mode seven to be very different than that of the true profile. This disparity of the mode shapes can cause divergence of the inversion algorithm. It is evident from this example that the values of the starting profile do not have to be very close to the true profile in order for the shapes of the modes to be similar enough to insure convergence. However, it is important that the starting profile and the true profile contain the same trend.

4.2.2 Robustness to Inaccurate Data

In acoustic experiments, wave number data are not measured directly; rather, they must be estimated from the complex pressure field. The estimation process introduces uncertainties to the data. Regardless of the procedure used for wave number estimation, there will be some inaccuracies in the estimates. Reasons for the inaccuracies include low acoustic signal to noise ratio (SNR), indeterminate Doppler shifts, and environmental range dependence. A significant amount of research has been carried out to identify estimators that mitigate the effect these factors have on the data. Ohta examined the asymptotic Hankel transform [25]; Becker appraised the AR estimator [8]; Souza provided uncertainty bounds for the Kalman filter [65]; the accuracy of the WVD was analyzed by Matsumoto for range-independent environments [69] and an analytical expression for WVD in range-dependent environments is provided in appendix A of this thesis. Accepting the inaccuracies of the data, the goal of this work is to optimize the solution to
the inverse problem.

Even when the wave numbers are poorly estimated, accurate inversion results can still be obtained using qualitative regularization. For inaccurate data, the value of the Lagrange multiplier $\lambda$ should be increased. This places less emphasis on the data and higher weight on the a priori information. When using qualitative regularization, the prominent effect of inaccurate data is that it causes the solution to diverge when the starting profile is far from the true model parameters. A convergent solution is likely to be highly accurate, especially near the sea floor. The data are less sensitive to deeper sediments and the inaccurate inputs most affect the deeper inversion results.

Figure 4.5 shows error bars corresponding to a collection of solutions resulting from 100 realizations of wave numbers with zero-mean Gaussian distributed added noise with standard deviation $\sigma_N = 10^{-3}$. This magnitude of data error is consistent with the error associated with wave numbers estimated from measurements taken during the SW06 experiment. For this example, the Layered Starting Profile of section 4.2.1 was used. Choice of this starting profile does not require any additional a priori information, since the layer depths are already required by the qualitative regularization algorithm. The value of the Lagrange multiplier was increased by an order of magnitude compared to the exact data case, $\lambda^2 = 10^5$. For this example, convergence was achieved 97% of the time. For comparison, using the isovelocity background shown in figure 4.1 caused the solution to diverge 47% of the time. As shown in the figure, inversion results are highly accurate in the shallower layers. As expected, the sound speed of the deeper sediments are less accurately estimated.

To increase the inversion’s robustness to noisy inputs, more data should be included in the inversion scheme. When considered together, wave number inputs from multiple frequencies decrease the effects of inaccurate data.

### 4.2.3 Robustness to Inaccurate Water Column Properties

Typically, water column properties are measured during an acoustic experiment. However, these measurements often provide a limited sampling of the environment. For example, a ship’s CTD sensor characterizes the water column at a particular
Figure 4.5. Error bars corresponding to solutions resulting from inversions using 100 realizations of wave number data with zero mean Gaussian distributed added noise with $\sigma_N = 10^{-3}$ are indicated by area encompassed by the cyan region. The true sound speed profile is shown by the solid blue line. Depth is measured from the sea surface.

Time for a single location. Since many ocean environments exemplify significant temporal and spatial variability, they are not well represented by a single measurement.

If the wrong water column sound speed profile is assumed when inverting for seabed properties, the solution will be adversely affected [9]. The inversion suffers most near the sea floor; the results for the deeper sediments are not significantly effected. This is because the low-order wave numbers which are most affected by the water column mismatch are also most sensitive to the shallow sediments. On the other hand, the high order wave numbers which penetrate more deeply into the seabed are less "contaminated" by the incorrect water column input and so inversion results for the deeper sediments remain relatively accurate.

The effects of an erroneous water column sound speed profile are illustrated for exact data. In this example, the Layered Starting Profile was used. Additionally, it was again necessary to use the higher value for the Lagrange multiplier $\lambda^2 = 10^5$ because use of the wrong sound speed profile for the background environment can be interpreted as a source of noise on the data [61]. Figure 4.6 shows inversion
Figure 4.6. Inversion results when the wrong water column sound speed profile is used in the background environment. The true environment had a linear sound speed profile defined according to equation 4.1; the background sound speed profiles assumed (a) an isovelocity sound speed profile of 1491 m/s, (b) an isovelocity sound speed profile of 1495 m/s. Depth is measured from the sea surface.

results obtained using two different isovelocity water column sound speed profiles in the background environment instead of the true sound speed profile. In figure 4.6(a) the background assumes the water column sound speed profile is a constant 1491 m/s; in figure 4.6(b) the water column is assumed to be a constant 1495 m/s. These are the minimum and maximum sound speeds of the true water column sound speed profile which was defined according to equation 4.1. When the water column sound speed of the background environment is slower than the depth averaged sound speed of the true environment (1493 m/s) the inversion result compensates by estimating a faster sound speed in the top layer of the seabed. The inverse is also true: when background environment contains a faster average water column than that of the true environment, the inversion underestimates sound speed in the shallowest sediment layer.

This example is evidence of the claim that changes in horizontal wave numbers due to the water column fluctuations can be orders of magnitude larger than the same change in sound speed of the bottom [6]. In this example, the mean water column sound speed profile of the background environment was increased (or de-
creased) 2 m/s over that of the true environment. As a result, the inversion under-
(or over-) estimated sound speed in the top layer of sediment by close to 50 m/s.

In this example, knowledge of neither the maximum nor the minimum water
column sound speed was sufficient to obtain accurate inversion results. Recall
that the discrete wave number spectrum is bounded by the bottom wave number
\( k_b = \omega / c_{b_{\text{max}}} \) and the water wave number \( k_w = \omega / c_{w_{\text{min}}} \) where \( c_{b_{\text{max}}} \) is the max-
umimum sound speed in the seabed and \( c_{w_{\text{min}}} \) is the minimum sound speed in the
water column, such that \( k_b < k_n < k_w \). Thus, \( c_{w_{\text{min}}} \) sets the upper bound for
the spectrum. Low-order modes are refracted into the region of minimum sound
speed and are heavily influenced by the value of \( c_{w_{\text{min}}} \). However, high-order modes
propagate at steeper angles and are not as severely refracted by water column
fluctuations. Therefore, the depth averaged water column sound speed profile is
more important information than either the maximum or minimum wave column
sound speed. Thus, accurate inversion results can be obtained when the depth
averaged water column sound speed profile is known. When the background water
column sound speed profile has the same depth averaged value as the true profile,
the wave numbers calculated for the background will only slightly differ from the
wave numbers of the true environment [9]. While this is equivalent to introducing
a source of noise on the wave number estimates, it will not introduce a bias as
occurs when the depth averaged sound speed is not matched. As demonstrated
in the previous section, accurate inversion results can be obtained when the wave
numbers are inexact; however, when the wave numbers background environment
are offset from the measured data, the inversion results can deviate significantly
from true values.

### 4.2.4 Robustness to Inaccurate Location of Sound Speed Discontinuities

When the incorrect inputs for the sound speed discontinuities are used in the inver-
sion, the solution will reflect this erroneous information. Qualitative regularization
is based on a user defined operator which is not updated as part of the perturbative
process. In addition to having discontinuities in the wrong locations, the solution
will also differ from the true profile by having different sound speed values in each
layer. To understand this consequence, recall that the perturbative inversion algorithm, shown by equation 3.2, is a continuous inverse problem. Consequently, all determinations of the model function must be made in terms of local averages. Therefore, when the wrong locations for the sound speed discontinuities are used as input into the inversion, the result will be a layered profile with sound speed in each layer being the weighted average of the true sound speed values integrated over the depths encompassed by the layer. The weighting is determined by the modal eigenfunctions which are often more sensitive to the shallower sediments.

Figure 4.7 illustrates the effect of specifying the incorrect locations for the shallow and deep sound speed discontinuities. For both cases, the starting environment used the true sound speed profile for the water column and the Layered Starting Profile for the bottom. The appropriate value of the Lagrange multiplier $\lambda^2$ was determined to be $10^5$. As expected, sound speeds of the inversion results in the layers affected by the incorrect layering information were weighted averages of the true sound speeds for the regions included by the layers.

In the first example, shown in figure 4.7(a), the input for the shallow discontinuity was input 2.9 meters deeper than the true location. As a result, the solution overestimated the true sound speed of the first layer by 85 m/s and underestimated the shallow portion of the second layer by 144 m/s. Surprisingly, these large deviations from the true values are not signified by a high mismatch of the wave number data. The residual for the misidentification of the shallow discontinuity was very low: $\text{Res} = 3.3e − 5$. This is significantly higher than the residual for the exact data case with the correct discontinuity location inputs of section 4.2.1, which was determined to be $\text{Res} = 8.4e − 8$. However, the difference is not enough to distinguish it from the true model parameters in a realistic application where the wave number data will not be perfectly accurate. Consider the average residual resulting from the inexact data inversion of section 4.2.2 which was $\text{Res} = 4.4e − 5$. Comparing the residual from the incorrect discontinuity input to the average residual from inaccurate data shows that the effect of a bad discontinuity input is effectively below the noise floor of realistic data. Therefore, the residual cannot be used to determine the accuracy of the discontinuity input.

The second example, shown in figure 4.7(b), used an input for the deep sound speed discontinuity that was 5.4 meters shallower than the true value.
Figure 4.7. Inversion results obtained using the wrong inputs for the discontinuities in the sound speed profile. The plots show wrong inputs for (a) the shallow discontinuity and (b) the deep discontinuity. Depth is measured from the sea surface.

Subsequently, the inversion result was in error of 198 m/s compared to the deeper portion of the middle layer and in error of 92 m/s below the discontinuity. Since the solution had a lower value for the maximum sound speed in the seabed than the true environment, the lower bound of the spectrum $k_b = \omega/c_{b_{\text{max}}}$ was increased causing the resulting environment to support fewer modes. Consequently, the residual for this example is much greater: $\text{Res} = 1.9$.

4.2.5 Summary and Conclusions

The capability of perturbative inversion using qualitative regularization to determine sound speed in the seabed was assessed. Using exact data and correct environmental inputs, it was shown that the inversion will converge even when the simplest starting profile is used. It was explained that it is advantageous to select a fine discretization in order to distinguish sharp discontinuities in the sound speed profile. Considerable attention was given to the importance of selecting a favorable starting profile. Through an example, it was shown that, provided the starting profile contains the same trend as the true profile, the algorithm is likely to converge. Several suggestions were given to assist in choosing the best starting
profile when no prior knowledge of seabed properties is available.

When the data are inexact, accurate inversion results can still be obtained. However, because the problem is poorly conditioned, it is necessary to place a higher weight on the constraints. As a result, the inversion is less sensitive to the data making the algorithm more likely to diverge. Convergence is improved when an appropriate starting profile is selected. Inexact data cause the solution to suffer most for deeper sediments because few modes penetrate into this portion of the seabed.

It was shown that inaccurate inputs for the water column sound speed profile will degrade the solution to the seabed inverse problem. The solution suffers most near the seafloor with the solution for deeper sediments being relatively unaffected. In the example considered, the inaccuracies of the background water column sound speed profile were small. When the depth averaged sound speed of the water column deviates from that of the true profile, the wave number data will be significantly biased. As a result, convergence of the inversion algorithm is unlikely. Thus, it is very important to have good knowledge of the depth averaged water column sound speed profile in order to estimate seabed properties.

Finally, it was shown that correct locations for discontinuities in the sound speed profile are necessary to obtain accurate inversion results. Erroneous inputs will cause sound speed values to deviate from true values to compensate for the incorrect layer depth. This correlation between sediment sound speed and layer thickness has been documented in the context of matched-field inversion [14]. As a result of this correlation, it is not possible to determine erroneous layer depth inputs have been used based on the data mismatch. However, if the values obtained by the inversion differ significantly from the sound speeds used to calculate the layer depths from CHIRP seismic data, there is an opportunity to improve the solution by recalculating the layer depths using the sound speeds from the inversion result and then repeating the inversion. This process can be iterated until there is agreement between the inversion result and the sound speeds used to calculate the layer depths.
4.3 Application to Data from the SW06 Experiment

4.3.1 Introduction

As described in chapter 1, the New Jersey Shelf area of the North Atlantic is characterized by highly variable water column and seafloor properties. In this section, additional information concerning the geological structure of the seafloor is provided and results from previous experiments are summarized. Emphasis is placed on experiments that utilize mode theory to understand acoustic phenomena.

Amongst previous studies of the geological structure of the New Jersey shelf, cores collected during the Atlantic Margin Coring Project (AMCOR) provided valuable information about geophysical parameters of the seafloor [70]. In addition, a large portion of the middle and outer shelf was mapped using multibeam backscatter and bathymetry data as part of the STRATA FORMation on Margins (STRATAFORM) experiments [71]. Previous data from the Geoclutter area [5], combined with CHIRP seismic surveys taken during SW06, formed the basis for a detailed stratigraphic model of the subbottom in the area. A dominant feature of this seafloor model is the “R” reflector, a prominent shallow sub-surface seismic reflector that is subparallel to the seafloor on the middle and outer shelf [42, 43, 44].

To exploit knowledge gained from the geologic studies, normal mode theory has been used to predict signals for two different acoustic experiments conducted in the vicinity of AMCOR borehole 6010. In an experiment conducted near the Hudson Canyon area, a low-frequency sound source was towed while broadcasting pure tones and the resulting wave number spectrum analyzed [72]. The observation of a “double mode” was attributed to a duct in the sediment sound speed profile. In the Acoustic Characterization Test II (ACT II) experiment, modal dispersion from SUS charges was investigated [73]. The complicated dispersion structure observed during ACT II was attributed to features of the water column sound speed profile [74].
4.3.2 Description of Experiment

4.3.2.1 Experiment Design

The acoustic data for SW06 were recorded on the Shark array, a 48-channel horizontal/vertical line array [75] located at 39°1.254′ N, 73°2.982′ W. The system was designed to record data continuously at a sample rate of 9765.625 Hz. Data from 16 channels making up a vertical line array (VLA) spanning the water column between 13.5 and 78 meters depth were used in the analysis. Broadcasting continuous tones at 50, 75, 125 and 175 Hz, a low-frequency J-15-3 acoustic source was towed by the R/V Endeavor out and back along radials with respect to the VLA. Ship tracks oriented along, across, and oblique to the shelf are shown by solid lines in figure 4.8.

Data for the along shelf track were collected on August 4 between the hours of 12:00 and 20:30 Greenwich Mean Time (GMT). The R/V Endeavor traversed a 5 km long track parallel to the shelf break between 39°4.452′ N, 73°0.498′ W and 39°2.070′ N, 73°2.706′ W. The ship made five sets of repeated runs along the track at speeds of 2, 4, 6, 8, and 10 knots with each set of in- and outgoing runs performed at the same speed.

The measurement procedure was repeated for the across and oblique shelf tracks. The across shelf data were recorded between the hours of 10:10 and 15:00 GMT on August 5. The ship track, which was perpendicular to the shelf break, spanned 5 km between 39°0.258′ N, 73°0.198′ W and 39°1.518′ N, 73°2.406′ W. The ship made three sets of repeated runs along this track at speeds of 3, 5, and 7 knots.

The oblique shelf data were collected between the hours of 10:20 and 17:40 GMT on August 6. This ship track was on a 45° angle with respect to the along and across shelf tracks with endpoints at 39°1.266′ N, 72°58.896′ W and 39°1.248′ N, 73°1.938′ W and a length of approximately 5 km. The ship made four sets of repeated runs along the oblique shelf track at speeds of 2, 4, 6, and 8 knots.

Varying ship speed resulted in the source depth being different, ranging from 10 to 25 m, for each set of runs. Source depth was measured by a self contained temperature and depth data logger mounted on the source housing. For each run pair, source depth was equal and very stable with very little change in depth over
Figure 4.8. Bathymetry contours shown for the site of the SW06 experiment. The black solid lines indicate ship tracks for the towed cw signals and used in the inversion. The location of the Shark VLA is shown by the black star.

the course of an out and back run. Source depth determines the extent to which each of the propagating modes is excited. For example, consider the modes shapes shown in figure 4.23. A source depth of 25 m corresponds to an anti-node for mode two, meaning it will be highly excited, and corresponds to a node for mode four, meaning that it will not be excited at all. Other modes are excited to a moderate degree.

4.3.2.2 Acoustic Measurements

Transmission loss (TL) was calculated from the raw data measured by the Shark VLA. Details of the data processing are provided in appendix B. TL from the fourth run of the oblique shelf track is plotted as a function of range and depth in figures 4.9 through 4.12 for the four frequencies broadcast by the source. In each of the figures, a distinct modal interference pattern can be observed. The simple three mode interference pattern corresponding to the 50 Hz signal can be identified in figure 4.9. The higher frequencies show increasingly complex modal interference patterns.
Figure 4.9. Transmission loss measured at the Shark VLA for a 50 Hz signal as a function of source range and receiver depth.

Figure 4.10. Transmission loss measured at the Shark VLA for a 75 Hz signal as a function of source range and receiver depth.
Figure 4.11. Transmission loss measured at the Shark VLA for a 125 Hz signal as a function of source range and receiver depth.

Figure 4.12. Transmission loss measured at the Shark VLA for a 175 Hz signal as a function of source range and receiver depth.
4.3.2.3 CHIRP Seismic Measurements

The area surrounding the axis of the SW06 experiment was extensively surveyed in 2001-2002 using CHIRP sonar [76]. A grid of closely spaced tracks provided the basis for a stratigraphic model for this part of the shelf. Additional measurements were taken during the SW06 experiment to augment the earlier work with longer, more widely spaced tracks covering a larger area [77].

An example of CHIRP seismic data from the SW06 site is shown in figure 4.13. The “R” reflector, a prominent feature in the data, varies with depth on the shelf, and in the area of this experiment is covered with about 20 meters of sediment known as the outer shelf wedge [78],[79]. Initial deposition of the outer shelf wedge consists of finely laminate dipping layers of alternating sandy clay and clay [43]. The erose boundary between the upper and lower units was caused by a major erosional episode of indeterminate origin [78],[80]. Cores into the upper unit, collected in 2007 [77], primarily sampled clay, with occasional sand lenses. Fluvial channels filled with denser material are also known to be present in this area, but these do not affect any of the propagation paths in this experiment [78]. Our study area is 8-10 km seaward of the AMCOR 6010 core site [81]. However, unlike that site, which is located over a Holocene sand ridge [43], our study area is largely devoid of any Holocene sand [43] other than a very thin (∼20 cm) surficial layer [44].
4.3.2.4 Oceanographic Measurements

Sound speed in the water column was measured at both the source and receiver locations. The Shark VLA was equipped with temperature sensors spanning the water column and a single pressure sensor. Salinity was spatially interpolated from an environmental mooring (WHOI mooring SW30, located at 39°07.175' N, 73°16.640' W [75]) by fitting the temperature data to the T-S curve (temperature vs. salinity curve) [82]. A cluster of 16 nearby environmental moorings [75] provided oceanographic data to monitor the direction and speed of nonlinear internal waves.

Sound speed was measured from the ship using a towed CTD chain\(^1\) [83]. The towed chain was comprised of CTD sensors spaced three meters apart that were inductively coupled to a jacketed wire rope. To maximize depth coverage, the system was terminated with a depressor\(^2\). Twenty-five of the system’s 48 sensors were used during this part of the experiment. The sampling frequency was set to 0.5 Hz to provide fine horizontal spatial sampling of the water column. The CTD chain data were merged with the ship’s Global Positioning System (GPS) time and location data to interpolate the measurements onto a range grid. Sound speed was calculated from the measurements using Mackenzie’s nine term sound speed equation [84]. A further description of the CTD chain instrumentation and of processing of the data is found in appendix C.

The spatial sound speed measurements made by the chain were combined with the time averaged sound speed measurements recorded at the Shark VLA to provide a continuous measurement of the sound speed field along entire propagation paths. Significant variability of the water column was observed during SW06. An example of the range-dependent water column measurements is illustrated by an outgoing track from the along-shelf data shown in figure 4.14 when a region of lower sound speed was observed at the far end of the ship track.

\(^1\)ADM Elektronik

\(^2\)Yellow Springs Instruments (YSI) type-167 Vehicles for Instrumentation (V-Fin)
4.3.3 Analysis of data

4.3.3.1 Horizontal wave number estimation

Horizontal wave numbers were estimated for each out-and-back run along the three primary tracks shown in figure 4.8. For any particular run, wave numbers associated with each mode were estimated by the mean value obtained from all 16 channels of the Shark VLA taken individually. Doppler effects were removed by averaging wave number estimates from the ingoing and outgoing runs made at the same tow speeds [85].

Range-dependent wave number estimation was accomplished using a sliding-window autoregressive estimator with an aperture of 2 km weighted by a Hann window. Aperture length was chosen to maximize range resolution while maintaining enough wave number resolution to resolve the peaks of the Green’s function. An overlap of 95% was chosen to obtain a smooth evolution of the modal eigenvalues. The high rate of overlap addresses difficulties in picking the peak locations from the spectrum due to spurious peaks and/or peak broadening which are encountered in the case of very short apertures and/or very noisy signals [8]. Wave
numbers as a function of range, estimated from a single channel of 125 Hz pressure data collected along the oblique shelf track, are shown in figure 4.15. For this run, the ship was moving away from the VLA at a speed of 4 kts and the source and receiver depths were 32.3 m and 16.0 m, respectively. While estimates of the low-order modes appear almost constant, range dependence is clearly visible in the high-order modes. Low-order modes propagate at shallower angles and are more sensitive to water column properties. Their nearly constant value is consistent with the relatively benign water column measurements for this run. On the other hand, high-order modes propagate at steeper angles and penetrate more deeply into the seabed. Thus, the range dependence of the high-order modes indicates variability of seabed properties. The measurements show the greatest range dependence in mode seven: at the beginning of the track it has a value of $0.472 \, m^{-1}$ and after 3.5 km its value decreases to $0.467 \, m^{-1}$ near 5 km.

To aid in associating wave numbers estimated by the autoregressive estimator with the proper mode number, modal eigenfunctions were also estimated from the data. Mode shapes were determined by applying the Hankel transform to
Figure 4.16. Mode shapes estimated from the fourth run of the oblique shelf track from 125 Hz pressure data when the ship is moving away from the VLA at a speed of 4 kts.

the full 5 km aperture of pressure data for all VLA receivers and plotting the resulting spectrums versus depth. Mode shapes for the oblique shelf track run at 4 knots are shown in figure 4.16. Peaks corresponding to the high order modes were spread due to range dependence in the waveguide. Mode six (0.478 m\(^{-1}\)) and mode seven (0.472 m\(^{-1}\)) appear to have the same shape in the water column. This phenomenon has been reported previously [72] and its cause will be discussed in light of the inversion results.

Additional challenges were encountered when making wave number estimates for the along- and across-shelf tracks. For the case of the along-shelf data, water-column variability caused significant migrations of the closely spaced low-order modes. Specifically, the presence of a cold water mass at the far end of the track caused the value of mode two to increase so that its value was nearly equal to that of mode one at the beginning of the track where the water below the thermocline was warmer. Figure 4.14 shows the spatial sound speed field in the water column derived from the towed CTD chain data, with the cold water mass indicated by the region of slower sound speeds occurring after about 4 km. In order to account for
the wave number fluctuations and resolve the low-order wave numbers along this track, it was necessary to increase the aperture to 4 km. Wave number estimation for the across-shelf track was complicated by the ship track not being perfectly aligned along a radial with respect to the Shark VLA (see figure 4.8). The consequence of this off-radial motion was a relative source acceleration resulting in large non-constant Doppler frequency and wave number shifts in the data near the VLA. As a result, wave numbers could not be accurately estimated for ranges closer than 1.8 km.

4.3.3.2 Range-dependent inversion results

Inversion results were obtained using the perturbative method with the location of layer depths given as a priori information for qualitative regularization. As described, layer depths were estimated using CHIRP seismic reflection data with a depth resolution of 10-40 cm, where optimal resolution is approximately 10 cm [86]. Layer depths were calculated from CHIRP seismic travel time data using an assumed sound speed value for each layer. Sound speed values for this calculation were obtained from an initial range-independent inversion effort [64]. The layer depths estimated this way as a function of range are shown by the solid lines in figures 4.17, 4.18, and 4.19 for the oblique-, across-, and along-shelf tracks.

The initial sound speed values used in the layer depth calculation differ from the final inversion results by 22 m/s in the upper unit and 177 m/s in the lower unit. The error in the calculated layer thickness $\Delta H = H_{true} - H_{est}$ is related to error in the sound speed $\Delta S = S_{true} - S_{est}$ by:

$$\frac{\Delta H}{H_{true}} = \frac{\Delta S}{S_{true}}.$$  \hspace{1cm} (4.3)

According to equation (4.3), the percent error in the depth calculation is 1.4% for the erose boundary and 11.2% for the “R” reflector. At its thickest point, the width of the upper unit is 19.9 meters. Therefore, the maximum error in the location of the erose boundary is approximately 0.2 meters. The error in the depth of the “R” reflector is much potentially greater. At its thickest point, the lower unit has a width of 15.4 meters. At this location, this translates to the estimated depth of the “R” reflector being 1.7 meters too shallow.
Figure 4.17. Layer depths estimated from chirp seismic data for the oblique shelf track (solid line). Inputs to the inversion algorithm are digitized onto a discrete depth/range grid (dots).

The sand layer was not directly imaged by the CHIRP data. Rather, its presence was inferred primarily from a seaward decrease in the acoustic penetration of the CHIRP acoustic energy into the seafloor. Additionally, when regularized inversion (which does not require \textit{a priori} information about sediment structure) was applied to the wave number data from the region where the sand ridge was present, inversion results indicated faster sound speeds near the seafloor.

For use in the inversion algorithm, the stratigraphy data were discretized in both range and depth. The range discretization was set by the aperture used for wave number estimation. At each range on the resulting grid, the inversion algorithm was run with a starting model based on local water column measurements and input data based on the local wave number estimates. The depths of the seafloor and subbottom layer interfaces were averaged over overlapping apertures of length 2 km (or 4 km), the same apertures used for wave number estimation. Aperture length is indicated in the figures. Inversions were performed to a depth of 30 meters on a 1 meter depth grid. The \textit{a priori} layer depths, where jump discontinuities in the output sound speed profiles were allowed, are indicated on the discretized range grid by dots in the figures.
Figure 4.18. Layer depths estimated from chirp seismic data for the across shelf track (solid line). Inputs to the inversion algorithm are digitized onto a discrete depth/range grid (dots).

Figure 4.19. Layer depths estimated from chirp seismic data for the along shelf track (solid line). Inputs to the inversion algorithm are digitized onto a discrete depth/range grid (dots).
The ensemble of inversion results at each range, with depth measured from the air-sea interface, along the oblique shelf track is shown in figure 4.20. These results were obtained using horizontal wave numbers as input data to the inversion algorithm obtained at each range interval by integrating over the prescribed data aperture. As a result, local range-dependencies in the data over this interval, owing to changes in bathymetry or ocean and seabed variability, can be considered as a source of noise in the estimates. However, variability in the individual wave number estimates between each range-step is small and does not inhibit the ability to track gross changes in waveguide properties with range. This was particularly true for the relatively flat bathymetry and benign water column sound speed profile of the oblique shelf track. As shown in the figure, the range-dependent features present in the CHIRP seismic data are well tracked by the inversion result. This is indicated by the consistency of values obtained for sound speed in each sediment layer determined by the inversion algorithm at each range step. An exception occurred around 3.5 km in range where the lower layered unit abruptly ends and the wave number estimates became excessively noisy, leading to inconsistencies in the inversion results at that range. Nevertheless, the use of qualitative regularization at each range-step allowed for resolution of large changes in the sound speed profile at the vertical locations corresponding to the erose boundary and “R” reflector of approximately 85 m/s and 140 m/s respectively.

Across shelf inversion results, shown in figure 4.21, were found to be similar to those obtained for the oblique-shelf track. Compared to the previous results, the across-shelf track inversion was complicated by range-dependencies in the form of greater bathymetric changes, combined with a steeper rise in the “R” reflector. The higher frequency variations of the erose boundary were effectively averaged out by the 2 km aperture used for wave number estimation. However, range-dependent features, such as the dip in bathymetry, which were on the order of 1 km, were more problematic: they are too large to be averaged over but too short to be considered range-independent within the aperture. Sound speed in the sand layer was difficult to estimate because the sand layer is thin compared to a wavelength and the wave number data are not very sensitive to it. Therefore, the sound speed variation of the sand layer is not physical; it is caused by an inability to resolve its properties given this data set.
Figure 4.20. Range ensemble of sediment sound speed inversion results for the oblique shelf track.

Figure 4.21. Range ensemble of sediment sound speed inversion results for the across-shelf track.
As stated, it was necessary to use a longer aperture for wave number estimation for the along-shelf data. Therefore, the along-shelf inversion results are range averaged to a higher degree than those of the oblique or across shelf tracks. However, seismic data indicated that layer thicknesses were fairly constant over the course of the along-shelf track allowing for consistent inversion results to still be obtained. These results are shown in figure 4.22. Although the upper diffusely reflective unit is much narrower and the lower layered unit is wider, similar sound speed values to those found for the across- and oblique-shelf tracks were obtained.

The inversion results from all three tracks are characterized by a low-speed layer between the erose boundary and the “R” reflector. This duct, bounded both above and below by regions of higher sound speeds, can have a considerable effect on normal mode propagation and dispersion. For particular frequencies and modes, anti-resonant points occur which are located at depths bounding the duct [87]. Within the low-speed layer, the mode functions are strongly amplified, or resonant, relative to the amplitude in surrounding layers. This effect is illustrated by mode shapes calculated at a frequency 125 Hz for a three-layer model based on the inversion results and shown by the solid lines in figure 4.23. In the figure, the
The dashed lines represent the depth of the seabed, the erose boundary, and R reflector. The result clearly shows mode seven to be resonant in the low-speed layer. Above the duct, mode seven has a much lower amplitude and has a shape similar to that of mode six within the water column. As previously discussed, this was observed in mode shapes estimated from data for this environment and shown in figure 4.16.

Further insight into the resonant mode can be gained from studying its group speed dispersion curve, shown in figure 4.24(a). In the figure, the green dashed lines indicate sound speed in the low-speed layer and in the water column. The group speed dispersion curve has a local maximum at the sound speed of the duct and asymptotically approaches the sound speed in the water column as frequency goes to infinity. In figure 4.24(b), the contribution each portion of the waveguide makes to the group speed calculated according to equation 2.16 is shown for several frequencies. At 125 Hz, near the frequency of resonance, the contribution to the group speed calculation is most influenced by the low-speed layer. At all other

Figure 4.23. Modes shapes calculated for a frequency of 125 Hz for an environment representative of the inversion results and measured water column sound speed profile. The dashed lines represent the depth of the seabed, the erose boundary, and R reflector.
Figure 4.24. (a) Group speed dispersion curve for mode seven, green dashed lines indicate sound speed in the low-speed layer and in the water column. (b) Contribution each portion of the waveguide makes to the group speed calculation at several frequencies.

frequencies, the contribution is dominated by the water column.

The increased sensitivity of mode seven to subbottom features is observed in the data by the abrupt change in its value with range as shown in figure 4.15. This range-dependence is an indication of the truncation of the lower layered unit, or duct. This result is consistent with past work, where it was shown that by considering the aperture length used for wave number estimation, the AR estimator can be used to identify abrupt changes in sediment composition [88]. In light of this consideration, along the oblique-shelf track both the modal data shown in figure 4.15 and CHIRP seismic data shown in figure 4.17 indicate that a change in sediment composition occurs near a range of 4.2 km.
Measurement and identification of sediment trapped modes can be difficult due to their low amplitude in the water column. However, they have a large influence on the inversion results. Using a subset of low order, higher amplitude modes may seem advantageous because the wave number estimates will have less error and, therefore, produce a more stable solution to the inverse problem. However, if a low-speed layer is present, it will not be resolved by using only the low-order modal information in the inversion. Modes with wave numbers greater than that of the resonant mode are not as sensitive to the deeper sediments and differences between wave number values for sediments with and without the low-speed layer are negligible. The consequence of using only low-order modes in an inversion is a solution that does not fit high-order mode data. An example of such stable, but non-unique solutions are the range-independent inversion results from this data set presented in [64]. For the range-independent case, it was difficult to identify the high-order modes and a truncated set of modes was used in the inversion. As a rule, low-order modes are easier to estimate accurately, but high-order modes contain more information about bottom properties. As a result, the choice of modes to use in an inversion is a trade-off: stability of the solution comes at the cost of uniqueness.

4.3.3.3 Range and depth averaging of the solution

Sound speed values obtained by the inversion are both range and depth averaged over the true sediment sound speed values. Range averaging results from the horizontal wave number estimation procedure. As stated, a 2 km (or 4 km) long sliding window was used to estimate the horizontal wave numbers. Consequently, the wave number estimates and the inversion results at a particular range are horizontally averaged over the aperture.

Depth averaging results from the inversion algorithm itself. As with all continuous inverse problems, determination of the model function must be made in terms of local averages. Resolution length expresses the extent the solution is averaged at each depth. Resolution length is calculated according to equation 3.19. Resolution length vs. depth is shown in figure 4.25 for the oblique shelf track at a range of 3 km. As shown in the figure, resolution length increases with depth. This is expected as the modes are less sensitive to the deeper sediment.
4.3.3.4 Uncertainty Estimates

Estimates of the uncertainty of the inversion results were determined using Monte Carlo error propagation as described in section 3.5. To generate the noisy data vectors for error analysis, statistics were computed from the wave number estimates. Multiple estimates of the wave numbers were available at each range as the data were recorded on 16 channels for as many as ten repeated source tows over a given track. This resulted in up to 160 estimates of each individual wave number. For this analysis, the mean wave number estimate was assumed to be exact and 100 noisy wave number vectors were created by adding zero-mean Gaussian distributed noise with standard deviation $\sigma_d$ to the noise-free data vector, where $\sigma_d$ was calculated from the collection of wave number estimates. Mean wave number estimates and their standard deviations at each range for the across shelf track are shown in figure 4.26. The higher order wave numbers have significantly greater variance. These data are more difficult to estimate due to their lower amplitude. According to the Monte Carlo method, the inversion was carried out for each realization of the noisy data vectors. One hundred realizations was deemed to be sufficient for convergence of the model uncertainty as the uncertainty estimate changed by less
than 2% between 90 and 100 models. Average uncertainties, given by one standard deviation from the mean, for the primary sediment layers are shown in table 4.2.

The smallest uncertainty was determined for the sound speed estimate of the upper unit. This is expected since the low-order modes, which are easiest to estimate, are most sensitive to this layer. The lower-layered unit had a larger uncertainty than either the upper unit or the sediment below. The larger uncertainty was consistent with sediment trapped modes being more difficult to estimate and also there being fewer of them. The sediment below the “R” reflector is deeper than the e-folding depth for most of the low-order modes, and the uncertainty was greater than for the shallower upper unit. Sound speed in the sand layer was not well determined. For the frequencies considered, the sand layer is much thinner than a wavelength and modes are insensitive to it.

An important consideration is the effect of uncertainty in the layer depth inputs on the inversion results. The consequences of using faulty a priori information were described in section 4.2.4. Another example representative of the SW06 environment is considered here. Figure 4.27(a) shows the true sound speed profile, the solution obtained using faulty a priori information, and an alternate profile which has the correct sound speeds within each layer, but incorrect layer depths. These profiles are shown by the solid blue line, black dots, and green dotted line, respec-
Figure 4.27. The consequences of using faulty \textit{a priori} information. (a) The true sound speed profile, the solution obtained using faulty \textit{a priori} information, and an alternate profile which has the correct sound speeds within each layer, but uses incorrect layer depths. (b) The shape of mode 8 for 125 Hz corresponding the true profile, the solution, and the alternate profile.

tively. The corresponding shapes of mode seven for 125 Hz are shown in figure 4.27(b). As discussed in section 4.3.3.2, this is a resonant mode [56] which contains a significant portion of its energy within the low-speed layer. The solution to the inverse problem has a narrower duct as dictated by the \textit{a priori} information. To compensate for its reduced size, its low sound speed is exaggerated. The inversion result also differs from the true profile in the bottom layer. Because the perturbative algorithm involves solving a continuous inverse problem, all determinations of the model must be made in terms of local averages. As a result, the sound speed in the third layer is a weighted average of the true sound speed values integrated over the depths encompassed by the layer with the weighting determined by the modal eigenfunctions. Mode shapes corresponding to the true profile and the solution agree well. On the other hand, the alternate profile, which has the same sounds speeds within each layer, produces a mode with a significantly different shape.

This example demonstrates the type of error that can be expected for the given inverse problem. In the example, the error in the layer depths was exaggerated for the purpose of illustration. The magnitude of the bias depends on the stratigraphy
of the individual tracks. For the across and oblique shelf tracks, for which the lower unit is narrower, the effect is dominated by the resonant modes and the bias is largest in the low-speed layer. Calculations indicate the error in the solution could be as great as 7 m/s. On the other hand, for the along shelf track, which is characterized by a thicker lower unit, the bias is principally controlled by the averaging effect and the error is greatest below the "R" reflector. Due to faulty a priori information, the solution for sound speed below the 'R’ reflector could be as much as 18 m/s too slow. However, these calculations can only be used as a guideline as the exact values of true layer depths and sound speeds remain unknown.

4.3.3.5 Three dimensional model

A three dimensional model of the SW06 experimental region was constructed using a combination of inversion results and CHIRP seismic data. Based on these data, a model was specified comprised of layers having constant sound speeds. The sound speed values were determined from the average inversion result for each
layer as determined using data from all three tracks, with values given in table 4.2. Given the consistency of the inversion results amongst tracks, this is believed to be a good assumption. Spatial variability of the model was attributed solely to changes in the layer depths and thicknesses as determined by the chirp data as shown in figure 4.28. The three dimensional model can be used to provide a realization of the sediment sound speed field along any path by considering a vertical slice through the model. For example, the range-dependent structure of the sediment sound speed field according to the three dimensional model is shown for the oblique, across, and along shelf tracks in figures 4.29, 4.30, and 4.31. This approach allowed for continuity in the resulting model over regions of the environment where CHIRP data were collected but wave numbers were not estimated for inversion, i.e., between the tracks. This approach of interpolation between local measurements using seismic reflection data to obtain a regional range-dependent model with high resolution has been suggested previously [3, 68]. To demonstrate the effectiveness of this approach, in section 4.3.4 the three-dimensional model described here is used to predict signals along propagation paths that differed from those used in the inversion.

### 4.3.4 Evaluation of Results

#### 4.3.4.1 Comparison to core data

Prior to SW06, several cores were collected within a few kilometers of the location of the Shark VLA [89, 90]. Additional cores were collected in 2006 [77]. A variety of different technologies were used for the collection of samples including gravity

**Table 4.2. Sediment Sound Speed**

<table>
<thead>
<tr>
<th>Layer</th>
<th>Inversion Result</th>
<th>Direct Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper Unit</td>
<td>1670 ± 12 m/s</td>
<td>1653 ± 34 m/s*</td>
</tr>
<tr>
<td>Lower Unit</td>
<td>1585 ± 19 m/s</td>
<td>1638 ± 56 m/s*</td>
</tr>
<tr>
<td>Below R</td>
<td>1725 ± 15 m/s</td>
<td>1850 m/s*</td>
</tr>
<tr>
<td>Sand Layer</td>
<td>1740 ± 37 m/s</td>
<td>1760 ± 20 m/s**</td>
</tr>
</tbody>
</table>

*Core (230 kHz)
**ISSAP (65 kHz)
Figure 4.29. The range-dependent structure of the sediment sound speed field according to the three dimensional model for the oblique shelf track.

Figure 4.30. The range-dependent structure of the sediment sound speed field according to the three dimensional model for the across shelf track.
cores, vibracores, and hydraulic piston cores. Compressional sound speeds were measured in cores using a 230 kHz logger, and upper and lower outer shelf wedge unit values shown in table 4.2 represent averages and standard deviations of these measurements. Because the composition of the sediment layers is assumed to be uniform over the surveyed area, the standard deviation of the estimates is equivalent to the uncertainty of the measurements.

Core samples of the upper unit were easiest to obtain because of its proximity to the seafloor. The sound speed estimate for the lower-layered unit was based primarily on a single core taken from a location about 30 km northeast of the Shark VLA where the layer is shallower. This sediment was immediately above the “R” reflector and is expected to be similar to the sediment of the lower unit studied near the VLA. The core measurements for sound speed below the “R” reflector are very erratic due to poor coupling between the sediments and the core liner. The value in the table is consistent with the largest values measured. Due to the limited sample size, uncertainty estimates were not calculated for this layer. Cores were not available from the sand layer within the survey region. Average
and standard deviation values shown for the sand layer in Table 4.2 are derived from ISSAP seafloor measurements of sandy seafloors in the general vicinity [44].

Overall, there is good agreement between the sediment sound speeds obtained by the perturbative inversion and those estimated from the core data and ISSAP measurements. The inversion results were all within one standard deviation of the values surmised from the direct measurements. An exception to this is the sediment below the “R” reflector for which there was not enough core data available to determine standard deviation of the measurement. As stated, there is significant uncertainty associated with this value. The difference between the inversion results and core data may also be explained by the use of faulty a priori information about layer depths. The core data show evidence of the sound speed duct, although its presence is more prominent in the inversion results. Compared to the core data, the inversion results appear to over-estimate the sound speed in the upper unit and under-estimate it in the lower unit. This type of compensation can be attributed to the well known correlation between geoacoustic parameters [14].

4.3.4.2 Prediction of transmission loss

In previous modal experiments, the quality of the inversion results was evaluated by matching mode shape predictions constructed using the model to mode shapes estimated from the data [72, 91]. However, this approach, based on comparing a prediction and measurement at one location in the waveguide is not effective for evaluating a 3D model. Instead, we examine the ability of the three-dimensional sediment model to predict the coherent transmission loss measured as a function of range during the experiment. This approach effectively evaluates the match of the range-dependent eigenvalues as well as their amplitudes. Predicted transmission loss was calculated using the normal mode code Kraken and assuming density and attenuation values of 2.0 g/cm$^3$ and 0.05 dB/λ. The effect of the assumed density and attenuation values on the predicted transmission loss was shown to be small by a sensitivity study. Experimental and predicted transmission loss are shown in Figs. 4.32 and 4.33 for 50 Hz and 125 Hz data for the first run of the oblique-shelf track. The data are shown for 4 depths spanning 13 m to 78 m on the Shark VLA. At both 50 Hz and 125 Hz, the modal interference pattern predicted by the model is in excellent agreement with the measured data. Similar agreement
Figure 4.32. Measured 50 Hz transmission loss recorded on Shark VLA with prediction from 3D sediment model.

between measurements and predictions were obtained for all the tracks.

To quantify the agreement demonstrated in the figures, incoherent correlation was used to assess the ability of the model to predict the acoustic field [26]. Let $\hat{P}$ and $P$ be the measured and predicted fields. The correlation between the measured and predicted fields over an interval $r_{\text{min}}$ to $R$ is calculated by

$$
cor(R) = \frac{\langle P(r)\hat{P}^*(r) \rangle}{\left[ \langle P(r)P^*(r) \rangle \langle \hat{P}(r)\hat{P}^*(r) \rangle \right]^{1/2}} \tag{4.4}
$$
where the operator \( \langle \cdot \rangle \) is defined

\[
\langle QP \rangle = \frac{1}{R - r_{\min}} \int_{r_{\min}}^{R} P(r)Q(r)dr.
\] (4.5)

Figure 4.33. Measured 125 Hz transmission loss recorded on Shark VLA with prediction from 3D sediment model.

Correlation values were averaged over the multiple runs along the different tracks and are plotted vs. depth for each frequency in figure 4.34. Correlation at all depths is generally very good, particularly at the lower frequencies. However, correlation degrades with increased frequency where the modal interference pattern
is more complicated and difficult to predict. Nevertheless, as shown in figure 4.33, qualitatively good agreement between the measured data and the prediction is still obtained. Correlation at all frequencies for the along-shelf track was slightly lower than for the other two tracks. The reduction in horizontal resolution for this case resulting from the use of a 4 km aperture for wave number estimation may be the cause of the lower correlation.

### 4.3.4.3 Prediction of other signals

In this section, the three-dimensional sediment model is used to predict acoustic signals for comparison to data not used for inversion. During SW06, several other acoustic experiments were conducted where signals propagated over the region characterized by the inversion results presented in this paper. Two types of data are considered: towed cw data along a path overlapping the oblique shelf track and linear modulated frequency (LFM) pulse originating from a stationary source located 15 km NE of the Shark VLA.

The cw data considered were collected on August 28, 2006, several weeks after the data used for the inversion. The signal was recorded on the Marine Physical Laboratory (MPL) VLA consisting of 16 receivers spaced every 3.75 m with a total
Figure 4.35. Locations of sources and receivers during the SW06 experiment. The solid lines indicate ship tracks for the towed cw signals and used in the inversion. The dashed line and star show locations of signals used for validating the inversion result. The Shark VLA and MPL VLA are shown by the diamond and square.

aperture of 56.25 m. The two uppermost sensors were contaminated by mechanical noise and could not be used. The \textit{R/V Knorr} was used to tow a low-frequency source at a constant depth of 30 meters at a speed of 5 kts. The ship track is shown by the dotted line in figure 4.35. Also shown in the figure is the receiver location, which was to the NE of the Shark VLA. The geometry created by this source and receiver was wholly contained within the domain of the 3D inversion result.

Coherent TL recorded on the MPL VLA at 4 depths, for this ship track, are plotted with predictions from the sediment model in Figs. 4.36 and 4.37 at frequencies of 53 and 103 Hz. Qualitatively, the agreement between model and prediction is excellent at 53 Hz, and slightly reduced at 103 Hz. It is probable that the degradation at 103 Hz is caused by water-column variability. At the time of the experiment, small non-linear internal waves were recorded by environmental sensors on the Shark VLA. However, water column properties were not measured at the locations of the \textit{R/V Knorr} or at the MPL VLA. Despite this deficiency, correlation between the measured and predicted fields were greater than 0.95 and 0.90.
Predictions based on the sediment model were also compared to LFM data recorded on August 5, 2006. The LFM signal was a 0.5 second pulse with 250 Hz bandwidth, centered at 165 Hz. The pulse was broadcast from a source suspended by the R/V Endeavor and recorded at the Shark VLA. Multiple pings were transmitted at a given location to allow averaging for improved SNR. As shown in figure 4.35, the source was approximately 15 km NE of the Shark VLA, outside the region characterized by the inversion results. For prediction, sediment sound
Figure 4.37. Measured 103 Hz transmission loss recorded on MPL VLA with prediction from 3D sediment model.

Speed values were spatially interpolated to the location of the source based on seismic data. Figure 4.38 shows modal arrival times estimated from an average of ten pulses with predictions of the first nine mode arrival times also plotted. Qualitatively, there is very good agreement between the first seven predicted arrivals and the arrivals distinguishable in the data. In particular, the mode four arrival is very weak in the data which agrees with the predicted low amplitude for mode four based on the source and receiver depths.
4.3.5 Conclusion

A three dimensional model of sediment sound speed for a 90 sq. km area on the New Jersey shelf was constructed by application of a new inversion technique. This approach is based on a combination of CHIRP seismic measurements and a perturbative inversion scheme using horizontal wave number estimates. In this two-step process, CHIRP seismic measurements were used to locate the depths of discontinuities in the sound speed profile. This information was then used in a perturbative inversion algorithm employing qualitative regularization. The benefits of using qualitative regularization are twofold: it does a superior job of dealing with issues of stability and uniqueness associated with solving an ill-posed problem and it provides a mechanism for resolving the layered structure of the seabed.

The experiment was of the highest quality and made accurate estimation of bottom properties possible. Repeated runs along the tracks allowed for efficient removal of Doppler effects and drove down error in the wave number estimates. Furthermore, use of the towed CTD chain provided a direct measurement of the water column and made it possible to isolate the effects of water column variability from sediment variability.

Horizontal wave numbers proved be a powerful indicator of sediment range.
dependence. This was illustrated by the 125 Hz data from the oblique-shelf track. It was shown that mode seven experienced a resonance in which a significant portion of energy was trapped in a low-speed sediment layer. Between 3.5 km and 4 km, wave number estimates indicated an abrupt change in mode seven. This behavior was corroborated with CHIRP seismic data that showed the low-speed layer in which mode seven was trapped pinching out in that range interval.

Several methods were used to evaluate the inversion results. The estimated sound speed for each of the model’s sediment layers was within one standard deviation of the sound speed values estimated from core data. Moreover, model accuracy was established by the ability to predict acoustic signals recorded over a variety of propagation paths and oceanographic conditions.
Chapter 5

Water Column Sound Speed Inversion: Application of Approximate Equality Constraints

5.1 Introduction

This chapter mirrors the previous one; attention is given to many of the same issues when applying perturbative inversion for estimating water column sound speed as were addressed for estimating sediment sound speed. In this work, approximate equality constraints are applied to the inverse problem making it possible to obtain accurate estimates of the water column sound speed profile. For this problem, the wave number data alone are insufficient to solve the inverse problem because depth dependent eigenfunctions are not sensitive to certain portions of the waveguide. As detailed in chapter 3, this method relies on prior knowledge of the mean water column sound speed profile to constrain the solution. The mean profile may be estimated from measurements taken during the acoustic experiment or from historical data.

This chapter consists of two main sections. The first section is concerned with evaluating the robustness of the inversion algorithm. Synthetic data are used so that the effects of common errors can be considered separately and the results can be compared to a known solution. In the second part of the chapter, the method
is applied to data from the SW06 experiment. The inversion results are compared to *in situ* measurements from the towed CTD chain.

### 5.2 Evaluation of the Algorithm

To begin the evaluation of the perturbative inversion algorithm, a sample application using exact data and known inputs is considered. This allows for assessment of the capability of the inversion algorithm under ideal conditions. This example will provide a complete description of the inverse problem. Attention is given to such topics as discretizing the continuous integral and choosing a starting profile. After this has been established, consideration is given to the algorithm’s robustness to inaccurate wave number estimates and poor knowledge of the seabed properties.

For the purpose of this evaluation, perturbative inversion using approximate equality constraints is applied to realistic sound speed profiles taken from the SW06 experiment. During the experiment, water column properties were measured by numerous moorings located throughout the experiment site. Sound speed profiles at the location of the Shark VLA were calculated by merging temperature measurements recorded at the Shark VLA with conductivity measurements from nearby mooring SW30 [75]. Data for the upper five meters of the water column were collected by instruments on the ASIS (Air-Sea Interaction Spar) buoy [92]. Details of this calculation are contained in [82]. Figure 5.1 shows sound speed profiles calculated for the location of the Shark VLA from August 3 through September 9, 2006.

These measurements are evidence of significant water column variability over the 37 day period. Fine scale features such as internal waves are superimposed over larger features like the diurnal tide. Intra-seasonal variability is also evident by the gradual deepening of the thermocline as the stratification becomes more diffuse. These trends were also measured by a fleet of Webb Slocum Electronic Gliders from Rutgers University and Oregon State University during SW06 [46]. According to the glider data set, in July, the mean depth of the thermocline was 15 meters. As the summer progressed, its depth reached 20 meters in August, and 25-30 meters in September [46]. The deepening of the thermocline and reduction of stratification were partially caused by storm events on the New Jersey shelf which
Figure 5.1. Sound speed profiles at the location of the Shark VLA for the time period of August 3 through September 9, 2006.

helped to mix the water column, decreasing the temperature near the surface [93]. Hurricane Ernesto, which passed over the region in early September, contributed to this mixing.

For the purpose of studying the inversion algorithm, two extreme profiles are considered. These profiles are shown in figure 5.2(a) and (b) by the solid blue lines. Each profile was averaged over a three hour period from data recorded on the Shark VLA. The time-averaging effectively removed smaller transient features from the data that are beyond the spatial resolution of the inversion algorithm. The first profile was recorded on August 12, when the thermocline was shallow and had a strong gradient. The slowest sound speeds below the thermocline were recorded are during this time period. The second profile, recorded two weeks later on August 26, consisted of a depressed thermocline and a weaker gradient. Some of the fastest depth-averaged sound speeds occurred during this time. These profiles have drastically different effects on an acoustic signal: the profile from August 12 is characterized by a sound speed minimum below the thermocline, which acts as a duct, trapping acoustic energy. On the other hand, the second profile has
Figure 5.2. Selected sound speed profiles recorded at the Shark VLA averaged over a three hour period on (a) August 12 and (b) August 26 shown by the solid black lines. Average sound speed for the entire 37 day deployment of the VLA is also shown on both plots by the dotted magenta line.

Figure 5.2. Selected sound speed profiles recorded at the Shark VLA averaged over a three hour period on (a) August 12 and (b) August 26 shown by the solid black lines. Average sound speed for the entire 37 day deployment of the VLA is also shown on both plots by the dotted magenta line.

A downward refracting trend causing more acoustic energy to interact with the bottom. In figure 5.2, the two extreme profiles are shown with mean for the 37 day period, plotted by the magenta dotted line. The mean profile was between the limits set by the two extremes: its thermocline occurred at a medium depth and it had a moderate gradient. The sound speed below the thermocline had a slightly increasing trend. This profile suggested neither the strong ducting nor the strong downward refraction seen for the two extreme profiles.

The inversion will be carried out for each of the two extreme profiles shown in figure 5.2 using the 37 day average profile for the background environment. This will show the inversion’s ability to resolve diverse profiles using a starting profile that is not biased toward a particular solution. The data used for the inversion are wave numbers at four frequencies: 50, 75, 125, and 175 Hz for which the waveguide supports two, three, five, and six propagating modes respectively. For the purpose of the inversion, the bottom is assumed to be a half space with a sound speed of
1600 m/s. This bottom is used in both the simulation of the “measured” wave number data and in the background environment used for the inversion.

5.2.1 Example Application Using Idealized Inputs

In this section, the ability of perturbative inversion using approximate equality constraints to estimate water column sound speed profiles is assessed within the context of an example. This application utilizes exact data and known environmental inputs. These conditions allow for examination of the inversion algorithm itself which is inexact due to the linearization of the perturbed Helmholtz equation. Within the framework of this example, guidelines are set for making decisions about mesh size and setting up an appropriate background environment.

The discretization for the water column sound speed profile was chosen to be relatively fine, such that \( \Delta z = 1/2 \) meter. A fine mesh is desired to minimize the extent that the absolute equality constraints influence the solution. If the discretization is chosen to be too large, the solution could become over constrained. As described in chapter 3, the absolute equality constraints were applied to two consecutive points at both the top and the bottom of the water column sound speed profile in order to establish the trajectory of the solution.

The inversion results are shown by the red dashed line in figure 5.3(a) for the August 12 profile and in figure 5.3(b) for the August 26 profile. In both plots, the true sound speed profile is shown by the solid blue line. For each of the inversions, the same values of the Lagrange multipliers were chosen: \( \lambda_1^2 = \lambda_2^2 = 1e4 \) which control the relative and absolute equality constraints respectively. In both cases, the solution well captures the main features of the true profiles. However, there are some deficiencies. The solution for the August 12 profile contains some non-physical fluctuations below 60 meters depth. Although a stronger relative equality constraint could have eliminated this error, it would have also made it impossible to resolve the gradient of the thermocline. Another type of error occurs for the August 26 data: the profile is mismatched near the sea floor. At this depth, the absolute equality constraint is inconsistent with the true sound speed profile (see figure 5.2(b)). Recall, the absolute equality constraint does not allow perturbations from the background at the specified locations. Thus, the inaccurate input is observed
Figure 5.3. Inversion results for the sound speed profiles from (a) August 12 and (b) August 26. In both plots, the true profiles are shown by the solid blue line, the estimate is shown by the red dashed line.

Despite these imperfections, the depth-averaged error for both solutions is on the order of one meter per second. Moreover, the depth-averaged sound speeds of the solutions match those of the true profiles almost exactly. As displayed in table 5.1, the depth-averaged water column sound speeds for the true profiles and for the solutions are essentially equal. This result is of particular interest to the scientist intending to perform inversion for seabed properties following the estimation of the water column sound speed profile. According to section 4.2.3, good knowledge of the depth-averaged water column sound speed profile is a necessary condition for successful inversion of the sediment sound speed profile. If inversion for the water column sound speed profile was not available, one might chose to use the water column sound speed profile from the 37 day average in construction of the background environment for the sediment inversion. As indicated in table 5.1, the depth-averaged value of the background profile differs from the depth-averaged values of the true profiles by more than 5 m/s for August 12 and more than 12
Table 5.1. Depth-averaged water column sound speeds.

<table>
<thead>
<tr>
<th>Sound Speed Profile</th>
<th>August 12</th>
<th>August 26</th>
</tr>
</thead>
<tbody>
<tr>
<td>True Environment</td>
<td>1496.8852 m/s</td>
<td>1514.6659 m/s</td>
</tr>
<tr>
<td>Inversion Result</td>
<td>1496.8860 m/s</td>
<td>1514.6856 m/s</td>
</tr>
<tr>
<td>Background</td>
<td>1502.0827 m/s</td>
<td>1502.0827 m/s</td>
</tr>
</tbody>
</table>

m/s for August 26. Based on the work in presented in section 4.2.3, this would most certainly cause large errors in the sediment inversion. Furthermore, the magnitude of the difference in water column sound speed would cause the wave numbers to be extremely biased which would likely result in divergence of the inversion algorithm. Thus, even if there is only interest in determination of seabed properties, in many cases it is necessary to carry out the inversion for water column sound speed first.

5.2.2 Robustness to Inaccurate Data

Per the discussion in section 4.2.2, in practice, inaccuracies are introduced into the wave number data through the estimation process. To analyze the effect of inexact data on the solution, 100 realizations of the wave number data with zero mean Gaussian distributed added noise with standard deviation $\sigma_N = 10^{-3}$ were simulated. The inversion was carried out for each realization of data and the resulting models were examined. Error bars, defined as the standard deviation of the resulting models calculated at each depth, are shown in figure 5.4(a) and (b) for the August 12 and 26 profiles. The magnitude of data error considered in this example is consistent with the error associated wave numbers estimated from measurements taken during the SW06 experiment.

To address the inaccuracies of the data, it was necessary to place higher weight on the constraints. Because the value of the Lagrange multiplier responsible for weighting the absolute equality constraint $\lambda_2$ is already high enough to restrict any perturbations at the specified points, only $\lambda_1$ is increased. Thus, the value of the Lagrange multiplier for the relative equality constraint $\lambda_1^2$ was increased to 1e6. This choice of $\lambda_1$ results in a smoother sound speed profile than the exact data example. Consequently, it is not possible to resolve the abrupt change in the sound speed near the top of the thermocline present in the August 12 profile.
Figure 5.4. Error bars corresponding to solutions resulting from inversions using 100 realizations of wave number data with zero mean Gaussian distributed added noise with $\sigma_N = 10^{-3}$ are indicated by area encompassed by the cyan region for data from (a) August 12 and (b) August 26. In both plots, the true sound speed profile is shown by the solid blue line.

The effect of increased smoothing takes a different form for the August 26 data. Because the perturbations are smooth, irregular features of the background profile are transferred to the solution. In this case, the perturbations are featureless curves which are added to the rough background profile. This explains the error present near ten meters depth in the August 26 solution.

The inaccurate data have the greatest effect on the solutions near the thermocline. For the August 12 profile the errors bars shown in figure 5.4(a) have their greatest width of 5.8 m/s at a depth of 20.5 meters. The large sound speed gradient at the thermocline is a difficult feature for the inversion to reproduce. On the other hand, the August 26 profile is characterized by a deeper thermocline with a more gradual change in sound speed. Consequently, the greatest width of the errors bars for this profile is narrower having a maximum value of 4.0 m/s at a depth of 24.0 meters.
5.2.3 Robustness to Inaccurate Bottom Properties

When carrying out the inversion for the water column sound speed profile, it is unlikely there will be satisfactory knowledge of the bottom properties. This circumstance frequently occurs because it is necessary to have sufficient knowledge of the water column properties before attempting to estimate the sediment sound speed profile. As demonstrated in section 4.2.3, small deviations from the true water column sound speed profile can lead to large errors in the solution for sediment sound speed profiles. However, it is possible to obtain reasonable estimates for the water column sound speed profile even when the seabed properties are poorly known.

In the same way that poor assumptions about the water column used in the background model adversely affected the solution to the sediment sound speed inverse problem, incorrect bottom parameters will degrade the solution for the water column sound speed profile, but to a lesser degree. As before, the error from incorrect environmental inputs are aliased into the solution. However, considerable improvement over the mean water column sound speed profile can still be achieved because the values of the horizontal wave numbers are dominantly influenced by water column properties. For this evaluation, the sound speed of the bottom used to construct the background model was incorrectly assumed to be 100 m/s faster than the true value, so that the sediment sound speed profile of the background environment was a constant 1700 m/s.

The solutions obtained using the erroneous sediment sound speed are shown in figure 5.5(a) and (b) for the August 12 and 26 data, respectively. Since use of an incorrect sound speed for the bottom in the background model can be viewed as a source of added noise on the data [61], it is necessary to increase weight on the relative equality constraint; in this set of examples, the value of $\lambda_1^2$ was increased to $1e6$ for the August 12 profile and to $1e5$ for the August 26 profile. As before, increasing $\lambda_1$ makes the abrupt change in the sound speed profile of the August 12 data unresolvable. Additionally, the increased sound speed of the bottom causes the thermocline to shift upwards. Also, sound speed below the thermocline is underestimated. Together, these effects are primarily responsible for lowering the depth-averaged sound speed of the solution. Since $\lambda_1$ was not increased as much for the August 26 data, the solution is able to better resolve the features of the
Figure 5.5. Inversion results for the sound speed profiles from (a) August 12 and (b) August 26 when the incorrect inputs are used for the seabed in the background environment. In both plots, the true profiles are shown by the solid blue line, the estimate is shown by the red dashed line.

true profile. However, the thermocline is pushed shallower for this case as well, causing a decrease in the depth-averaged sound speed of the solution.

It is possible to improve upon the solutions shown in figure 5.5 by considering a subset of wave number data. As detailed in chapter 2, low-order wave numbers are more sensitive to water column properties and the high-order wave numbers are more influenced by the bottom. Moreover, low frequencies, which produce modes that penetrate more deeply into the seabed, are more affected by bottom properties. Thus, low-frequency data and high-order wave numbers are more “contaminated” by the incorrect sediment sound speed of the background environment. Hence, low-order wave numbers from higher frequencies can be used to obtain a more accurate solution when seabed properties are poorly known. To understand which modes are best suited to use in such cases, the contributions of the seabed and water column to the group speed dispersion curves can be used as a tool. This calculation was described in chapter 2 and illustrated by a set of examples.
Figure 5.6. Inversion results obtained using a subset of low-order higher frequency wave number data for the sound speed profiles from (a) August 12 and (b) August 26 when incorrect assumptions are made concerning the seabed of the background environment. In both plots, the true profiles are shown by the solid blue line, the estimate is shown by the red dashed line.

The improved solutions are shown in figure 5.6. In calculating these solutions, the first two out of five and first three out of six wave numbers were used from the 125 and 175 Hz data, respectively. Wave numbers from the 50 and 75 Hz data, which produced two and three modes respectively, were not considered. These low frequency data are strongly influenced by the incorrect sediment sound speed of the background environment. Moreover, since these frequencies produce so few propagating modes, there is limited ability to separate the effects of the water column from those of the seabed. Although these new results exhibit the same failings as the original solutions, the magnitude of these errors is significantly reduced. Furthermore, the depth-averaged sound speeds of the new solutions are closer to those of the true sound speed profiles. The improved matches are detailed in table 5.2. Although these solutions do not equal those of the inversion performed
Table 5.2. Effect of sediment sound speed on depth-averaged water column sound speeds.

<table>
<thead>
<tr>
<th>Depth-Averaged Sound Speed</th>
<th>August 12</th>
<th>August 26</th>
</tr>
</thead>
<tbody>
<tr>
<td>True Profile</td>
<td>1496.8852 m/s</td>
<td>1514.6659 m/s</td>
</tr>
<tr>
<td>Original Solution</td>
<td>1494.9463 m/s</td>
<td>1512.3158 m/s</td>
</tr>
<tr>
<td>Improved Solution</td>
<td>1495.4031 m/s</td>
<td>1513.3007 m/s</td>
</tr>
</tbody>
</table>

using accurate inputs for the seabed properties, significant improvement has been achieved over the starting profile by using a subset of data.

5.2.4 Summary and Conclusions

Practical aspects of estimating the water column sound speed profile were presented. It was shown that perturbative inversion can be used to achieve highly accurate estimates of the water column sound speed profile when approximate equality constraints are applied. Even though the inversion result and the true profile differed by about 1 m/s on average, the depth-averaged sounds speeds were essentially identical. This is an interesting result, since it was found in section 4.2.3 that accurate sediment inversions can be performed when knowledge of the depth-averaged water column sound speed profile is good.

However, when the wave number data are inaccurate, it is necessary to increase the value of the Lagrange multiplier $\lambda_1$ which makes it difficult to distinguish abrupt changes in the sound speed profile. This problem can be remedied by choosing a starting profile closer to the solution. In the set of examples considered here, the starting profile was taken as the average profile from data recorded over a 37 day period. During this extended time period, seasonal changes in the sound speed profile can be observed. By using a starting profile based on local measurements recorded at times near the acoustic experiment, it is likely the inversion results will be improved.

Finally, the effect of using an incorrect sound speed for the seabed of the background environment was considered. It was shown that the solution compensates for the inaccurate environmental parameters, i.e. when the sediment sound speed is assumed to be too fast, the solution will balance the effect of the error by making
the depth-averaged water column sound speed too slow. The impact of this error on the solution was mitigated by using a subset of low-order, higher frequency wave number data. In the example considered, inversion results were significantly improved. The enhancement can be expected to be even greater in realistic environments with more complicated seabeds. When the bottom is not an acoustic half space, many of the high-order modes will not decay exponentially with depth and thus contain more faulty information from the erroneous seabed parameters.

5.3 Application to Data from the SW06 Experiment

5.3.1 Introduction

The waters of the New Jersey shelf experience a seasonal cycle of stratification and mixing [93]. During the fall, vertical gradients of temperature and salinity are relatively weak. The salinity difference across the shelf is high, with low-salinity waters restricted to near the coast. Salinity increases in the offshore direction. During the winter, temperatures are lower and also vary across the shelf, with colder water close to the coast and warmer water near the shelfbreak. During this time, temperature increases with depth, especially in the offshore region. In spring and summer, the area influenced by low-salinity water is much larger, with the freshwater plume from the Hudson River reaching almost 100 km from the coast. Surface waters are also considerably warmer, and a strong thermocline is formed. The Mid Atlantic Bight (MAB) cold-pool, a well known feature present during the summer on the New Jersey shelf, is found in the offshore region at depth. It is defined by water colder than 8°C [94]. During SW06, its core was located at the 55 meter isobath with a width of 20 km and a vertical extent to just below the thermocline [46]. By the end of the summer, the vertical stratification was considerably reduced with the freshwater plume again confined to the coast [93].

During the summer stratified season, three distinct water masses can be identified based on their temperature and salinity: surface shelf water, bottom shelf water, and slope water [46]. The surface shelf water is warm and fresh while the bottom shelf water is much colder and saltier. The slope water is warm and salty,
a combination termed “spicy”. The term spice refers to spatial variations in the temperature and salinity of seawater whose effects on density cancel each other. It is defined as a state variable that is perpendicular to potential density in the potential temperature and salinity plane [95]. Spice is used to distinguish regions where the water is well mixed from regions where the water is inhomogeneous.

Using data from the Rutgers gliders, Gong et. al. [46] identified four types of slope water intrusions: surface intrusions, pycnocline intrusions, sub-pycnocline intrusions, and bottom intrusions. The intrusion types were differentiated by density, spiciness, and depth. Sub-pycnocline intrusions are perhaps the most interesting intrusion type acoustically. Surface and pycnocline intrusions are generally confined to the upper portion of the water column. These features do not contribute significantly to acoustic variability because acoustic energy is refracted away from this portion of the water column. Although bottom intrusions have a significant effect on acoustic signals, they are persistent and stationary. Thus, they are not an important cause of variability. Conversely, sub-pycnocline intrusions are often located in the mid water column and are highly variable. Formation of sub-pycnocline intrusions is possibly associated with detachment of the bottom boundary layer. Detachment of the bottom boundary layer has been observed to occur frequently during the summer on the New Jersey shelf [45].

5.3.2 Experiment Description

The acoustic data for the SW06 experiment were collected over a three day period in early August. The R/V Endeavor was used to tow a low frequency source, making repeated runs along radials with respect to the Shark VLA. These radials were orientated along, across, and oblique to the shelfbreak. The ship traversed a different radial track on each of the three days. For this work, data from a single run of each track will be considered. The dates and times for each of the runs are listed in table 5.3. A more complete description of the SW06 experiment is provided in section 4.3.2.

Each of these three data sets displays distinct characteristics owing to the unique state of the environment. During SW06, water column properties were measured by a cluster of moorings immediately adjacent to the location of the
Table 5.3. Dates and times of the runs considered in the analysis.

<table>
<thead>
<tr>
<th>Ship Track</th>
<th>Date</th>
<th>Julian Day in 2006</th>
<th>Start Time (GMT)</th>
<th>End Time (GMT)</th>
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</thead>
<tbody>
<tr>
<td>Along Shelf</td>
<td>Aug. 4</td>
<td>216</td>
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<td>16:31</td>
</tr>
<tr>
<td>Across Shelf</td>
<td>Aug. 5</td>
<td>217</td>
<td>13:00</td>
<td>13:31</td>
</tr>
<tr>
<td>Oblique Shelf</td>
<td>Aug. 6</td>
<td>218</td>
<td>16:41</td>
<td>17:28</td>
</tr>
</tbody>
</table>

acoustic experiment [75]. The majority of these moorings were equipped with thermistors at 14, 25, and 40 meters depth. Water column properties were also measured by the CTD chain which was towed from the R/V Endeavor. This instrument is described in appendix C. By examining the measurements taken below the thermocline a basic understanding of the gross range dependence of the water column can be established. Temperature data at a depth of 40 meters from the moorings is plotted with the CTD chain measurements in figure 5.7 for each of the tracks. The measurements from the moorings are time-averaged over the duration of the chain measurement.

Temperature measurements for the along shelf track recorded on Julian Day (JD) 216, shown in figure 5.7(a), indicate warmer water present at the southwestern part of the ship track and at the northeast moorings. Significant variability was recorded along the ship track as evidenced by the comparatively colder water over its northern portion. More moderate temperatures were recorded at moorings located to the southwest. Because the higher temperature measurements are surrounded by measurements of cooler water, they are believed to be caused by a localized feature.

Mooring data recorded on JD 217 show a trend of increasing temperature to the south. These measurements are shown in figure 5.7(b). However, the chain data, which extends to the southeast of the mooring cluster, does not follow this pattern. Temperatures are much colder over the entire ship track. Only the nearest mooring to the northwest displays similar temperature readings.

The mooring data recorded during the oblique shelf track on JD 218, shown in figure 5.7(c), do not show evidence of any significant variability over the entire cluster. Although the mooring measurements suggest the water column is well mixed during this time period, the CTD chain measurements indicate water column
Figure 5.7. Temperature in degrees Celsius recorded at a depth of 40 meters by the SW06 moorings and the towed CTD chain for the (a) along shelf, (b) across shelf, and (c) oblique shelf tracks.
temperatures were considerably cooler over the ship’s track.

All three data sets show that measurements from the mooring cluster were not adequate to infer water column properties along the ship’s tracks. In this highly range-dependent environment, the CTD chain has clearly been proven to be a valuable tool. However, high resolution spatial measurements of this type are often not available. Consequently, methods to estimate range-dependent water column sound profiles are advantageous.

5.3.2.1 Water Column Measurements

Sound speed fields estimated from the towed CTD chain measurements are shown for the along, across, and oblique shelf tracks in figures 5.8, 5.9, and 5.10. In all three cases, there were no measurements recorded deeper than 60 meters. The data aperture was limited both by the vertical extent of the chain and by the deepest working sensor. It was not possible obtain measurements nearer to the seafloor since clearance must be left for the depressor.

The sound speed field of the along shelf track shown in figure 5.8 indicates significant range dependence. For ranges less than 4 km, a collection of sub-pycnocline intrusions are responsible for increasing sound speed. This observation is consistent with the conclusion draw in section 5.3.1 based on the mooring cluster temperature measurements that the higher temperature recordings at the southwest portion of the CTD chain measurement are caused by localized feature(s). Measurements from the far end of the ship track show colder bottom shelf water present directly under the thermocline indicated by slower sounds speeds. In this portion of the water column, the thermocline has a much stronger gradient. Repeated measurements of water column properties over the along shelf track indicated the range-dependent features to be relatively stationary for an hour before and after the measurements shown here. Thus, the assumption of a “frozen ocean” is considered a fair approximation. This assumption is invoked for the prediction of the the acoustic fields in section 5.3.4.2.

Data from both the across and oblique shelf tracks shown in figures 5.9 and 5.10 display similar characteristics. In both cases, the measured sound speed fields display minimal range dependence. Additionally, they are characterized by much lower sound speeds than observed at the majority of the nearby moorings as indi-
Figure 5.8. Water column sound speed measured by the towed CTD chain for the eighth run of the along shelf track.

cated in section 5.3.1. These slow sound speeds result from the presence of cold bottom shelf water. Both tracks exhibited a minimum sound speed around 9.5°C which indicates these waters are not part of the MAB cold pool [94]. Although these temperatures are significantly colder than those reported by the surrounding moorings, there is much colder water elsewhere on the New Jersey shelf. Positioning the experiment further up shelf would likely have resulted in additional water column variability due to interaction with the MAB cold pool which was situated near the 55 m isobath [46].

Sound speeds calculated from measurements taken at the Shark VLA are another important measurement. These data provide a measure of sound speed at the location of the acoustic receivers. Recall from section 5.2, these sound speed profiles are compiled from measurements at the Shark VLA and nearby mooring SW30 and the ASIS buoy. Sound speed calculated from measurements recorded for the location of the Shark VLA during the duration of the along, across, and oblique shelf tracks are shown in figures 5.11, 5.12, and 5.13. Although all of these profiles were recorded at the same location, the measurements are separated
Figure 5.9. Water column sound speed measured by the towed CTD chain for the fourth run of the across shelf track.

Figure 5.10. Water column sound speed measured by the towed CTD chain for the fourth run of the oblique shelf track.
Figure 5.11. Water column sound speed measured at the Shark VLA over the duration of the along shelf track.

temporally by almost 24 hours and they all display distinct characteristics.

Measurements recorded at the Shark VLA during the course of the along shelf track shown in figure 5.11 reveal a sound speed structure similar to that measured at the near end of the ship track. This agreement was expected based on observations of figure 5.7(a). In fact, a specific feature observed in the chain data can also be seen in the Shark VLA measurements. Both data sets show a subpynocline intrusion at a depth of 28 meters. This intrusion persists at the Shark VLA location for a period of five hours. It can also be observed at mooring SW30, which is located 1.5 km west-northwest of the Shark VLA and is equipped with environmental sensors spanning the water column.

Figure 5.12 shows sound speed recorded at the Shark VLA over the duration the across shelf track. These measurements reveal that the water column was much faster at the Shark VLA than the waters measured over the across shelf track. The range-averaged minimum sound speed recorded at the CTD chain was 1496 m/s. This is 12 m/s slower than the time-averaged minimum sound speed recorded at the Shark VLA. The faster sound speeds at the Shark VLA are likely caused by a
Figure 5.12. Water column sound speed measured at the Shark VLA over the duration of the across shelf track.

large intrusion of warm, salty slope water. As shown in figure 5.7(b), this intrusion appears to come from the southwest. It is not too surprising the intrusion did not reach the location of the ship track as the intrusion was not a persistent feature at the Shark VLA. Cooler waters were recorded at the Shark VLA in the time period immediately following the measurements shown here.

The sound speeds recorded during the oblique shelf track displayed an interesting structure. As shown by figure 5.13, there was an intrusion in the center of the water column, potentially creating two separate sound speed ducts. This feature persisted for more than three hours at the Shark VLA. However, its spatial extent is limited as it is not observed at either the mooring SW30 or by the CTD chain.

5.3.3 Analysis of Data

5.3.3.1 Horizontal wave number estimation

Range-dependent wave number estimation was accomplished using a sliding window autoregressive estimator with an aperture of 1600 meters weighted by a Hann
Figure 5.13. Water column sound speed measured at the Shark VLA over the duration of the oblique shelf track.

window with 95% overlap. The reduced aperture size was chosen to maximize spatial resolution of the range-dependent wave number evolution. This is a smaller aperture than was applied in chapter 4, particularly for the case of the along shelf track for which an aperture of 4 km was applied. Due to the uncertainty principle, by shortening the data aperture, wave number resolution is sacrificed for increased spatial resolution. The limited wave number resolution particularly complicates the estimation of the closely spaced low-order modes. In chapter 4, this difficulty was overcome by using a larger data aperture; in this work, we take advantage of the mode shapes. An accurate wave number estimate for a closely spaced low-order mode can be obtained by examining data from receiver depths such that the surrounding modes are not detected by the estimator due to their very low amplitude. However, the reduced aperture length increases the uncertainty of the estimates in two ways: the estimates themselves have a higher variance and there are fewer repeated measurements of the low-order wave number data.

For example, consider wave numbers estimated from the along shelf track shown in figure 5.14. By considering a receiver from a mid-water column depth of 47.25
meters, where mode two is in a null, an accurate estimate of the first wave number is obtained. This is shown by figure 5.14(a). To estimate the second wave number, data from a receiver on the seafloor was used. At this depth, mode one has very low amplitude and is not detected by the auto regressive estimator. Thus, an estimate of mode two is acquired as shown in figure 5.14(b). The locations of modal nulls are illustrated by figure 5.15 which shows mode shapes corresponding to the range-dependent wave number estimates displayed in figure 5.14. Mode shapes were estimated by calculating the Hankel transform using the full 5 km aperture of data for all channels and plotting the wave number spectrums versus depth. The peak of mode one is difficult to identify in this plot due to range dependence of the water column. However, the null of mode two is clearly visible at a depth of 47.25 meters as well as the low energy of mode one near the seafloor.

This procedure for wavenumber estimation was carried out for the across and oblique shelf tracks as well. For all three tracks, it was not possible to estimate wave number values for mode two from the 175 Hz pressure field given the 1600 meter aperture. For this frequency, modes one and three were successfully estimated when mode two was in a null.

Because the inversion is being applied to determine a single realization of the water column sound speed field, the procedure for removing Doppler shifts discussed in chapter 4 must be modified. Previously, wave number estimates from each run were averaged with estimates from the corresponding run with the same speed and in the opposite direction. This procedure was appropriate for the sediment inversion because the seabed features are not expected to change over the course of the experiment. However, due to the temporal variability of the water column this averaging procedure would cause the loss variability specific to an individual track. Instead, Doppler shifts were removed from the estimates by calculating the range-averaged wave number value for the particular track and its corresponding incoming track at the same speed. Then these mean values were used to calculate the Doppler shift and the correction was applied to the range-dependent wave number estimates. Whereas the original method calculated a range-dependent correction to remove Doppler shifts, the technique used here determines a range-averaged correction.

For the along shelf track, range dependence of the water column is most clearly
observed from the evolution of mode one shown in figure 5.14(a). Its value increases with range from 0.521 m$^{-1}$ at near ranges to 0.525 m$^{-1}$ at ranges furthest from the VLA. This magnitude of range dependence seen in mode one can only be attributed to variability of the water column. The increased value of mode one indicates slower sound speed in the water column. Recall from chapter 2, the discrete wave number spectrum is bounded by the bottom wave number $k_b = \omega / c_{b_{\text{max}}}$ and the water wave number $k_w = \omega / c_{w_{\text{min}}}$, where $c_{b_{\text{max}}}$ and $c_{w_{\text{min}}}$ are the maximum sound speed of the sediment and the minimum sound speed of the water column, such
Figure 5.15. Mode shapes estimated by calculating the Hankel transform for the full 5 km aperture of data for all channels and plotting vs. depth the eighth run of the along shelf track using 125 Hz pressure data.

that $k_b < k_n < k_w$. Decreasing the minimum sound speed of the water column increases the upper bound of the spectrum, effectively shifting the wave number spectrum upward. The low-order wave numbers are more sensitive to the water column properties because they propagate at shallower angles and refract more in the water column. Thus, these modes are more sensitive to minimums in the sound speed profile. The higher order modes do not experience the magnitude of the shift of the first wavenumber because they travel at steeper angles. These modes are more influenced by the depth-averaged sound speed of the water column.

Compared to the along shelf data, the low-order wave numbers from the oblique and across shelf tracks displayed less variability. This leads one to expect the water column properties are relatively range-independent along these tracks.

Wave number estimation for the across-shelf track was complicated by the ship’s off-radial motion. Consequently, there was a relative source acceleration resulting in non-constant Doppler shift. This effect is most detrimental at ranges near the VLA and, as a result, wave numbers could not be accurately estimated
for ranges closer than 1.8 km. Although the approximation of radial motion is better at ranges further from the VLA, all the data are contaminated by this error to some extent. The effect of this error was mitigated by the method used to remove Doppler shift discussed in chapter 4 as the corrections were range-dependent. The method applied to remove Doppler shift in this chapter conserves the wave number variations of the individual run, including the error caused by off-radial ship motion.

5.3.3.2 Range-dependent inversion results

Inversion results were obtained using perturbative inversion with approximate equality constraints. As with all perturbative methods, the solution is assumed to be “close” to the starting profile. Thus the choice of the starting profile is an important step in applying the inversion algorithm. The measurements at the Shark VLA are both spatially and temporally relevant; they are recorded at the time of the experiment and are within a few kilometers of the ship’s track. These facts make the sound speed calculated from measurements recorded at the Shark VLA the logical choice for the starting profile. Therefore, the starting profiles were constructed from the time-averaged Shark VLA measurements shown in figures 5.8, 5.9, and 5.10. Each of these profiles was then refined to remove small scale features. As described in section 5.2.2, the relative equality constraint requires the perturbations to be smooth. Thus, any rough features in the starting profile will be aliased into the solution. Sound speed values of the starting profile at the top and bottom of the waveguide play an important role in the inversion. Since the absolute equality constraint does not allow perturbations at these points, the values of the starting profile will appear in the solution.

As shown in section 5.2.3, the solution to the water column inverse problem will be degraded if incorrect values are used to describe the sound speed of the bottom. In this work, the inversions are carried out using the three dimensional sediment model for the seabed created in chapter 4 to construct the background environments. Since the three dimensional model provides the best estimate available for seabed properties, its use in the inversion minimizes the error resulting from incorrect environmental inputs. Note that this is contrary to the usual procedure. Typically, detailed measurements of the water column such as those from the
towed CTD chain are not available. Since good knowledge of the water column is
needed in order to invert for seabed properties, it is usually necessary to carry out
inversion for water column sound speed profiles first. However, since the low-order
modes utilized here do not penetrate deeply into the seabed the use of detailed
bottom structure is not expected to have a significant effect on the solution.

The ensemble of inversion results at each range of the along shelf track is shown
in figure 5.16. The results clearly show the trend of decreasing sound speed with
range predicted from of the spatial evolution of mode one as shown in figure 5.14.
The results exhibit an intrusion centered at a depth of 48 meters and range of 3
km that appears to be responsible for raising the sound speed of the water column
at ranges less than 4 km.

The across shelf inversion results, shown in figure 5.17, show low sound speed
values extending from the thermocline to the foot of the front. The sharp increase
in sound speed near the sea floor is atypical of sound speed profiles found on the
New Jersey shelf. At far ranges, the inversion results show an increase in sound
speed: the extreme minimum in sound speed below the thermocline disappears and
the foot of the front extends upward to shallower waters. This range dependence is possibly an artifact of the error in the wave number data caused by non-radial ship motion. The radial projection of ship speed is greater at ranges further from the VLA resulting in a greater downward shift of the spectrum. From the inverse relationship between upper bound of the spectrum $k_w$ and minimum sound speed in the water column $c_{w_{\text{min}}}$, it is known that lower wave number values correspond to faster water column sound speeds.

Figure 5.18 shows the oblique shelf inversion results. These results exhibit a more typical structure to sound speed profiles: there is a distinct sound speed minimum below the thermocline and a gradual increase in sound speed with depth. For the most part, the inversion results indicate the water column properties are range-independent. However, there are some small scale fluctuations around a depth of 60 meters. These are likely due to small inaccuracies in the wave number data and are not believed to be caused by range-dependent features in the environment.
5.3.3.3 Range- and depth-averaging of the solution

Sound speed values obtained by the inversion are both range- and depth-averaged over the true water column sound speed values. Range-averaging results from the horizontal wave number estimation procedure. As stated, a 1600 meter long sliding window was used to estimate the horizontal wave numbers. Depth-averaging results from the inversion algorithm itself. As with all continuous inverse problems, determination of the model function must be made in terms of local averages. Resolution length, calculated according to equation 3.19, expresses the extent the solution is averaged at each depth.

Resolution length was calculated at each range for all three tracks. For these calculations, the forward model $G$ was composed of eigenfunctions used in the the inversion, i.e. the first four modes from 125 Hz and modes one, three, and four from 175 Hz. For each of the tracks, resolution was fairly constant over all ranges. Range-averaged resolution length versus depth is shown in figure 5.19(a). For reference, the range-averaged inversion results are shown in 5.19(b). All three

![Inversion Result](image.png)

**Figure 5.18.** Range ensemble of water column sound speed inversion results for the oblique shelf track.
tracks show resolution lengths with similar characteristics. Resolution length is about 10 meters over most of the water column. Above the thermocline, resolution length increases sharply. The data contain little information about this portion of the water column because these very shallow depths are above the turning point of the modes used in the inversion. Additionally, all three tracks show an increase in resolution length near the seafloor because several of the modes used in the inversion have low amplitude near the seafloor.

5.3.3.4 Uncertainty estimates

Estimates of the uncertainty of the inversion results were determined by Monte Carlo methods [59, 64]. The technique is applied by simulating a collection of noisy data vectors and then examining the statistics of the resulting models. As discussed in section 5.2.2, greater values for the Lagrange multiplier controlling smoothness $\lambda_1$ should be applied for data that has greater uncertainty. However, application of greater values $\lambda_1$ mean less information from the data is utilized in
the inversion and, as a result, features of the true sound speed profile cannot be resolved. In order to be consistent with the level of resolution used to produce the inversion results, the same value of $\lambda_1$ was used. Due to the greater uncertainty of the data, the models resulting from the Monte Carlo procedure contain more fluctuations than the original solutions. To generate the noisy data vectors for error analysis, statistics were computed from wave number estimates in the same manner as section 4.3.3.4. Using the data uncertainty values, 100 realizations of data vectors were constructed for each range. The resulting model uncertainty depends on data uncertainty and varies with depth.

Depth-averaged model uncertainty is plotted in figure 5.20(a) with data uncertainty shown in figure 5.20(b). For the across shelf track, there is a clear correlation between model uncertainty and data uncertainty. The correlations are less clear for the along and oblique shelf data. Moreover, low data uncertainty does not guarantee low model data uncertainty. For example, the oblique shelf track has the greatest data uncertainty but the lowest model uncertainty for ranges less than 3 km. This inconsistency could be caused by a bias in the mean wave number estimate. In the application of Monte Carlo error propagation, we have assumed the mean wave number estimate is equal to the true wave number. If this assumption is violated, model uncertainty is likely to be greater than expected.

The depth dependence of model uncertainty is shown by figure 5.21(a). For reference, the range-averaged inversion results are shown in 5.21(b). All three tracks have zero uncertainty at the surface and very low uncertainty at the seafloor. (Each range has zero uncertainty at the seafloor, but since the depth is not constant, the range-averaged uncertainty is non-zero.) At these depths, the models are all constrained by the absolute equality constraints. All three tracks display relatively high uncertainty between 10 and 20 meters depth which is associated with uncertainty of the depth of the thermocline. The along shelf track has a maximum uncertainty near 28 meters depth. This is caused by the inability of the models to consistently capture the large sound speed gradient caused by the mass of colder water. All the tracks show a local maximum in uncertainty at mid-water depths between 40 and 50 meters. This is an effect of the higher data uncertainty which causes fluctuations in the solution. All the tracks show a second maximum model uncertainty below 60 meters depth. This corresponds to increased resolution.
Figure 5.20. (a) Depth-averaged model uncertainty and (b) data uncertainty for the along, across, and oblique shelf tracks.

lengths (figure 5.19) at these depths which indicates the forward models contain less information about this part of the water column.

Overall, the average uncertainty of the solutions is 1.2 m/s indicating that the inversion results are highly precise. However, this is not a measure of the inversion’s accuracy. This uncertainty estimate is defined as the variance of the solutions obtained using inaccurate data; it is not a measure of error between the solution and the true sound speed profile.

5.3.4 Evaluation of results

5.3.4.1 Comparison to CTD chain measurements

In this section, the inversion results are compared to the CTD chain measurements described in section 5.3.2.1. To quantify this comparison, three metrics are used. The first is examination of the range and depth dependent differences between the inversion results and the chain measurements. In the second metric, the depths of the thermoclines are compared. The last indicator considers the depth-averaged
Figure 5.21. (a) Range-averaged model uncertainty and (b) range-averaged inversion results for the along, across, and oblique shelf tracks.

To make the range and depth dependent comparison of the inversion results to the CTD chain measurements, the CTD chain measurements were averaged over the effective aperture of the inversion results. Then the inversion results were subtracted from the range-averaged CTD chain measurements. These differences are plotted in figures 5.22, 5.23, and 5.24 for the along, across, and oblique shelf tracks.

The depth of the thermocline is also used as to quantify the accuracy of the inversion results. Thermocline depth was estimated as the midpoint of the sound speed gradient occurring between sound speed values of 1505 and 1520 m/s. Time-averaged thermocline depths for the Shark VLA measurements (starting profile) and range-averaged thermocline depths for the CTD chain measurements and inversion results for each of the tracks are shown in table 5.4. As indicated in the table, the thermocline depths estimated from the CTD chain measurements are routinely less than that of the Shark VLA measurements. This is believed to be true for the case of the across shelf track for which the presence of warmer waters
Figure 5.22. Difference between the range-averaged CTD chain measurement and the inversion result for the along shelf track.

Figure 5.23. Difference between the range-averaged CTD chain measurement and the inversion result for the across shelf track.
Figure 5.24. Difference between the range-averaged CTD chain measurement and the inversion result for the oblique shelf track.

Table 5.4. Estimated thermocline depth.

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<th>Chain Measurement</th>
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<td>Oblique Shelf</td>
<td>16.27 m</td>
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<td>15.43 m</td>
</tr>
</tbody>
</table>

at the Shark VLA caused the thermocline to be located much deeper. However, thermocline depth is not expected to vary significantly between the location of the Shark VLA and the CTD chain measurement for the along or oblique shelf tracks. This difference suggests an error in the sound speed profiles calculated using the CTD chain data. As discussed in section C, the high failure rate of the pressure sensors in combination with poorly assigned calibration coefficients makes the calculated depths of the CTD chain sensors questionable. Inaccuracies in the CTD chain data must be taken into account when making comparisons to the inversion results.
A third indicator of inversion accuracy is the depth-averaged sound speed. As discussed in section 5.2.1, when the environmental inputs are correct, the depth-averaged sound speed of the solution should match that of the true sound speed profile. This is expected to be true even when the inversion result fails to reproduce specific features of the sound speed profile. Moreover, knowledge of the depth-averaged sound speed is important for performing sediment inversions. Depth-averaged sound speed profiles for the CTD chain data were calculated by linearly interpolating sound speed from the deepest measurement to the seafloor.

Comparison of the along shelf inversion result (figure 5.16) to the CTD chain measurement (figure 5.8) shows the inversion result captures the larger features measured by the towed CTD chain. At ranges closer than 4.5 km, the inversion results show faster sound speed and the presence of colder bottom shelf water at the far end of the track is clearly identifiable from the decreased sound speed in that region. However, the small scale features measured by the towed CTD chain are not accurately reproduced by the inversion result. The differences between
the CTD chain measurement and the inversion result can be observed from figure 5.22. In this figure, red indicates regions where the measurement is faster than the inversion result and blue indicates regions where the inversion result is faster than the CTD chain measurement. The red area present at ranges less than 4 km at 28 meters depth evidences the inversion algorithm’s inability to detect the intrusion directly below the thermocline. A second, deeper intrusion, also observed at near ranges, is present in both the chain data and the inversion result. However, its size and amplitude are overestimated by the inversion. This is shown by the blue region centered at at 45 meters depth at a range of 2.9 km. At ranges greater than 4 km, the pattern is reversed; there is a blue region centered at 20 meters depth above a red region centered at 50 meters depth. This is caused by a failure of the inversion to resolve to sharp change in sound speed at the thermocline. Although the inversion does capture the presence of the low sound speed region, it is at a deeper depth than the chain measurement. Moreover, the inversion result has a slightly deeper thermocline than the chain measurement as shown in table 5.4. The disagreement in values can be attributed to inaccuracies in the CTD chain measurement. The depth-averaged sound speed of the inversion results and the chain measurement are compared in figure 5.25(a). Both the data sets show the same trend of decreasing sound speed with range. The depth-averaged sound speed of the inversion result is slightly lower than that of the chain measurement. There is high confidence in the inversion result because at near ranges, its depth-averaged sound speed is in agreement with that of the Shark VLA. As discussed in sections 5.3.1 and 5.3.2.1, water column properties at the Shark VLA and at southwest portion of the ship track are believed to be very similar. Therefore, the difference in depth-averaged sound sound speed between the inversion results and CTD chain measurements is likely due to inaccuracies in the CTD chain data.

The difference plot of the across shelf track plotted in figure 5.23 shows a large red region above 18 meters depth which persists through the entire track. This mismatch is a direct result of a poorly chosen starting profile. The starting profile, which was determined from measurements at the Shark VLA, had a very weak sound speed gradient compared to the CTD chain measurement. Because the inversion algorithm requires perturbations to be smooth, the strong gradient of the true environment could not be reproduced by the inversion. The poorly estimated
depth of the thermocline as indicated in table 5.4, is also a consequence of the poorly chosen starting profile coupled with the highly weighted smoothness constraint. In an effort to match the sound speed minimum of the true environments located at 30 meters, the thermocline of the background is tilted upward as well as stretched by the perturbations. The blue region centered at 30 meters depth shows that the value of the sound speed minimum is slightly underestimated. This is an exaggerated case of what occurred for the synthetic data shown in figure 5.4 for which the inversion result also failed to estimate the strong gradient at the thermocline present in the true profile. Furthermore, it is likely there are errors associated with CTD chain measurement which would the depth of the thermocline specified in table 5.4 to be too shallow. As a result of this error, the true thermocline depth presumably differs more from the thermocline depth of the inversion result than indicated by the values in the table. The red region beginning between 40 to 45 meters depth and extending to the deepest CTD chain measurements indicates a trend of increasing sound speed with depth measured by the CTD chain that does not exist in the inversion result. The depth-averaged sound speed of the inversion result is considerably slower than the chain measurement as shown by figure 5.25(b). This large deviation is primarily caused by differences in the sound speed profiles below the thermocline. The inversion results indicate the water column has very low sound speed values from the thermocline to about 10 meters above the seafloor where there is a sharp increase in sound speed. Conversely, the CTD chain measurement indicates a gradual increase in sound speed beginning at 30 meters. This trend is continued by the linearly interpolation from the deepest chain measurement to the seafloor. The sound speed profiles of the inversion result are considered atypical and could be caused by a bias in the wave number data resulting from non-radial ship motion. Despite these errors, the depth-averaged sound speed of the inversion result is an improvement over the depth-averaged sound speed of the starting profile plotted as the Shark VLA measurement.

For the oblique shelf track, a qualitative comparison of the inversion result (figure 5.18) and the chain measurement (figure 5.10) show good agreement. Both the solution and the CTD chain measurement show a generally range-independent water column with a minimum sound speed near 30 meters depth. However, the difference between the chain measurement and the inversion result shown in figure
5.24 do show some disagreement. The chain measurement indicates slower sound speeds everywhere shallower than 30 meters. This was caused by a disagreement in the thermocline depth which is located slightly deeper in the inversion result, as shown in table 5.4. It is possible to attribute this disagreement to inaccuracies in the CTD chain data. The regions of red centered at 40 meters are caused by a quick increase in sound speed with depth directly below the sound speed minimum that is not resolved by the inversion algorithm. Everywhere else, the difference plot is white, indicating very good agreement between the CTD chain measurement and the inversion result. There is also very good agreement in the depth-averaged sound speeds as shown by figure 5.25(c).

5.3.4.2 Prediction of transmission loss

In this section, the ability of the inversion results to predict transmission loss of measured signals from the SW06 experiment is evaluated. Incoherent correlation, calculated according to equation 4.4, is used as a measure of how well the inversion results predict the measured signals. These correlation values were compared to those from predictions made using the CTD chain measurements and the Shark VLA measurements. For all predictions, the three-dimensional sediment model from chapter 4 is used to construct the seabed and bathymetry. The Shark VLA data was time-averaged and the resulting range-independent water column sound speed profile was used for the predictions. Predictions made using the inversion results and CTD chain measurements were constructed by interpolating the sound speed profile from the Shark VLA measurement to the nearest range of inverted or measured data. Correlation values were averaged over depth and are plotted for all four frequencies in figure 5.26 for each of the ship tracks.

In general, the data show the expected results: the predictions made using the CTD chain data give the highest correlations, followed by the inversion results and then the Shark VLA measurements. The CTD chain data best predict the measured transmission loss because these data have the finest spatial resolution, set by the sensor spacing of three meters and sampling rate of one second. The inversion results contain the mesoscale variability of the water column, but spatial resolution is limited to 1600 meters horizontally and 10 meters vertically due to range and depth-averaging of the solution. Finally, the Shark VLA measurements
Figure 5.26. Correlations of measured signals to model predictions using water column properties specified by the inversion results, CTD chain measurements, and Shark VLA measurements for the (a) along, (b) across, and (c) oblique shelf tracks.
are the poorest predictor of transmission loss as they do not contain any of the range-dependent water column fluctuations. Moreover, in some cases, this spatially separated measurement is a clear misrepresentation of the water column, particularly for the across shelf track for which the measurements at the Shark VLA indicate a much faster water column sound speed profile compared to the rest of the propagation path. Another expected trend shown by the data in figure 5.26 is that spread of correlation values increases with frequency. Higher frequency pressure fields are made up of modes which travel at shallower angles and, therefore, are more sensitive to water column properties.

However, there are some inconsistencies in the trends described above. In particular, for the oblique shelf track, the chain measurement does not always provide a better prediction than the inversion result. For this case, the CTD chain measurements and the inversion results show very similar representations of the water column sound speed field. It is not possible to conclude from the correlation data which representation of the water column is most accurate. Additionally, the 175 Hz data fail to follow the expected trend for the along and across shelf data. For the along shelf data, the prediction from the inversion result has a smaller correlation value than the Shark VLA measurement. In this instance, the inversion result fails to capture small scale features of the water column, particularly the intrusion directly beneath the thermocline which is present in both the Shark VLA and CTD chain measurements. For the across shelf data, the prediction made using the CTD chain data is lower than either the prediction made using the inversion results or the Shark VLA measurement. This is unexpected based on the water column inputs alone. However, the transmission loss predictions depend on the environment’s bathymetry and bottom properties as well. It is suspected that the shorter wavelengths of the higher frequency data could be sensitive to some small, range-dependent feature present in the seafloor of the true environment which is not depicted by the simplicity of the three dimensional sediment model. The combination of the inaccurate seafloor and incorrect water column could give a higher correlation value due to local minimums in the multimodal parameter space.

The effect of water column variability on transmission loss was isolated by a study using simulated data. For this analysis, transmission loss was calculated for
the SW06 environment constructed using to the towed CTD chain measurements from the along shelf track (figure 5.8) for frequencies ranging from 10 Hz to 10 kHz. Then inversion results based on the synthetic data were created using wave numbers from 125 and 175 Hz. By calculating the inversion for a known environment, the solution is not contaminated by incorrect environmental inputs. Furthermore, the synthetic SW06 environment is well understood; the actual SW06 environment contained uncertainties resulting from ambiguities in the CTD chain measurements and assumptions about the seabed. Next, transmission loss was predicted using the range-dependent inversion results and the range-independent Shark VLA measurement. These predictions were compared to the “true” transmission loss by taking the difference between the pressure fields. This difference was range-averaged over an aperture of 250 meters to remove modulations caused by the modal interference pattern. These calculations were carried out for all channels on the VLA and averaged over depth. The resulting transmission loss error caused by using the range-independent VLA measurement or the range- and depth-averaged inversion results is shown in figure 5.27.

As shown by figure 5.27(a), ignoring the range dependence of the water column caused significant error in the prediction, including as much as 4 dB of error for frequencies less than 100 Hz. Using the range-dependent inversion results, predictions at low frequencies were greatly improved: predictions are accurate to within 1 dB below 100 Hz. Between 100 Hz and 1 KHz, predictions made using the inversion results were still more accurate than the predictions made using the range-independent Shark VLA data, but prediction accuracy decreases with frequency. For frequencies above 1 KHz, prediction error will be on the order of 5 - 6 dB regardless the sound speed field used. Since the inversion results were constructed using 125 and 175 Hz data, they are depth-averaged to the same degree discussed previously. By using higher frequency data in the inversion, resolution of the inversion results can be increased, extending prediction capabilities to higher frequencies.
Figure 5.27. Range- and depth-averaged error in transmission loss predictions using (a) the range-independent VLA measurement (b) the range-dependent inversion results to construct the prediction.

5.3.5 Conclusion

Estimates of range-dependent water column sound speed profiles were obtained for three distinct data sets. This is the first documented application of perturbative inversion to estimate sound speed of the water column. In this work, approximate equality constraints made use of pre-existing knowledge of the sound speed profiles based on measurements taken at the Shark VLA to constrain the problem in regions of the water column where the data alone were insufficient to estimate the solution.

Horizontal wave numbers proved to be a powerful indicator of water column range dependence. This was illustrated by the 125 Hz wave number data from the along shelf track. It was shown that mode one experienced significant variability from which the presence of low sound speed at ranges greater than 4km was predicted. This result was corroborated by measurements from the towed CTD chain.

Accuracy of the inversion results was evaluated by comparison to in situ measurements from the towed CTD chain. Although the smallest features observed in the CTD chain measurements are beyond the resolution of the inversion algo-
algorithm, the gross range dependence of the water column is well represented in the solutions. Furthermore, the importance of knowledge of range-dependent water column sound speed fields was demonstrated by prediction of transmission loss of measured signals. Typically, better predictions were obtained using the inversion results than using the measurements taken at the Shark VLA.

In this chapter, extensive use was made of measurements from the towed CTD chain. While the data collected by this instrument is invaluable to the ocean scientist, there are many challenges encountered in the operation of the equipment and processing of the data. Deployment of the chain is a labor intensive process which requires specialized equipment and involves maneuvering the heavy depressor. Because of this, the CTD chain cannot be deployed in rough weather. Additionally, during the SW06 experiment, the jacket on the wire cable had to be patched numerous times. While these repairs were being made to the system, measurements could not be taken. Processing of the data was complicated by the high failure rate of the sensors. During the SW06 experiment, the pressure sensors were particularly unreliable, making it difficult to estimate cable shape. Consequently, there is a low level of confidence about the depth of important features such as the thermocline.
Simultaneous Sediment and Water Column Sound Speed Inversion: Application of the Joint Inverse Scheme

6.1 Introduction

A joint inversion scheme for simultaneously estimating water column and sediment sound speed profiles is evaluated in this chapter. The technique is a synthesis of the methods applied to the separate inverse problems: qualitative regularization is used to resolve the discontinuity in the sound speed profile at the seafloor as well as additional discontinuities in the seabed and approximate equality constraints are used to constrain the solution in portions of the water column for which the data alone is insufficient. This method was introduced in section 3.3.4 where the solution is given by equation 3.16. The primary advantage of jointly inverting for water column and sediment sound speed profiles at the same time is that poor assumptions about the background profile are not aliased into the solution. This can be the case when fixing the water column (or sediment) sound speed profile which can introduce errors into the problem and lead to inaccuracies in the solution in the sediment (or water column) sound speed profile.
This chapter is divided into two parts. The first section is concerned with assessing the accuracy of the algorithm under ideal conditions and evaluating robustness to inaccurate data. Synthetic data are used so that accuracy of the result can be determined by comparison to a known solution. The stability and resolution of the solution to the joint inversion scheme are compared to that of the separate inversion schemes. In the second part of the chapter, the method is applied to data from the SW06 experiment. The solution obtained using the joint inversion scheme is compared to the results presented in chapters 4 and 5.

6.2 Evaluation of the Algorithm

In this part of the chapter, the joint inversion algorithm is evaluated using synthetic data. The data for this analysis was constructed from an environment composed of seabed parameters from TC1 of the GAIT workshop [20] and the August 12 water column properties measured at the Shark VLA during the SW06 experiment. That is, the environment is a combination of the sediment sound speed profile used for the evaluation of the inversion algorithm in chapter 4 and the water column sound speed profile used in chapter 5. The resulting sound speed profile is shown by the solid blue line in figure 6.1. Individual plots of the seabed and water column sound speed profiles can be found in figures 4.1 and 5.2(a).

Horizontal wave numbers from 50, 75, 125, and 175 Hz for which the waveguide supports four, six, 10, and 15 propagating modes respectively were used in the inversions. The environment considered here supports fewer modes compared to the environment used in chapter 4 because the waveguide is shallower. There are more modes than the example presented in chapter 5 due to the more complicated structure of the seabed and faster maximum sound speed.

6.2.1 Example Application Using Idealized Inputs

In this section, the capability of the inversion algorithm to simultaneously estimate water column and sediment sound speed profiles under ideal conditions is assessed. This application utilizes exact data to allow for examination of the inversion algorithm itself which is inexact due to the linearization of the perturbed Helmholtz
The same discretization of the sound speed profile used in the previous chapters is also applied here, i.e. $\Delta z = 1/2$ meter. The operator $L_q$ was defined to specify discontinuities in the sound speed profile at 78.0 m, 82.1 m, and 102.4 m corresponding to the depth of the seafloor and two subbottom sediment layer interfaces. The starting profile, shown by the black dotted line in figure 6.1, is a composite of the starting profiles used in the evaluation of the separate sediment and water column inversion schemes presented in chapters 4 and 5. The starting profile for the water column is made up of the 37-day average from the Shark VLA measurements. For the seabed, the Layered Starting Profile is used. This is a departure from the analysis presented in chapter 4, which used an isovelocity profile for the initial example using exact data. For the joint inversion, it is necessary to use the Layered Starting Profile in order to be “close” enough to true model values for the solution to converge. Compared to the separate sediment inversion scheme, mode shapes of the starting profile are more dissimilar to those of the true environment because the true water column (or seabed) sound speed profile is not part of the background environment. The value of the Lagrange multiplier for the relative equality constraint was chosen to be $\lambda_1^2 = 1e5$ to conform with expectations of a smooth solution. The Lagrange multiplier for the absolute equality constraint was selected to be $\lambda_2^2 = 1e4$ so that perturbations at the top and bottom of the water column were restricted. Convergence was reached after 13 iterations when the residual (given by equation 4.2) changed by less than 5% between iterations.

The solution to the joint inverse problem is shown by the red dashed line in figure 6.1. The inversion result well captures the main features of both the water column and sediment sound speed profiles. Additionally, the solution is as accurate as inversion results obtained by the separate inversion schemes using correct environmental inputs. To confirm this statement the inversion was performed separately for the water column and sediment sound speed profiles using the same environment with the same wave number inputs, the same values for the Lagrange multipliers, and the same starting profiles. The background environments included the true sound speed values for the portion of the waveguide not being estimated, i.e. true values for water column sound speed were used when estimating sediment sound speed and true values for sediment sound speed were used when estimating
Figure 6.1. Solution to the simultaneous inverse problem. The true sound speed profile is shown by the blue solid line, the starting profile for the inversion is shown by the black dotted line, and the inversion result is shown by the red dashed line.

The environment considered here is well-suited for application of the joint inversion technique. However, due to the nature of the constraints, the solution cannot readily represent environments which contain a strong gradient in the sound speed.
Table 6.1. Inversion Error

<table>
<thead>
<tr>
<th>Inversion Scheme</th>
<th>Metric</th>
<th>Joint</th>
<th>Separate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ave. Error</td>
<td>5.01 m/s</td>
<td>6.55 m/s</td>
<td></td>
</tr>
<tr>
<td>Seabed Ave. Error</td>
<td>1.45 m/s</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Water Column Ave. Error</td>
<td>1.45 m/s</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Diff. Ave. SSP</td>
<td>0.12 m/s</td>
<td>0.09 m/s</td>
<td></td>
</tr>
</tbody>
</table>

profile of the seabed. This limitation results from the constraints. According to qualitative regularization, discontinuities are resolved by allowing a discrete Heaviside step function at specified points. Thus, the shape of the profile is continued above and below breaks in the sound speed profile. Because water column sound speeds vary over a much smaller range than sediment sound speeds, water column profiles have a much smaller gradient. The gradient of the solution to the joint inversion scheme is controlled by the water column since wave numbers values are most influenced by this portion of the waveguide. As a result, the joint inversion scheme produces solutions with small sound speed gradients. Hence, it is not possible to resolve large sound speed gradients in the seabed. If the joint inversion scheme is applied to such an environment, the resulting solution can be expected to provide an accurate representation of sound speed in the water column but not the seabed. The sound speed profile in the seabed will be made up of nearly constant values representing a weighted average of sound speed of the shallow sediments. Although the seabed is poorly characterized by this solution, it provides a good starting profile for application of the separate inversion scheme which can then be used to improve the estimate.

6.2.2 Robustness to Inaccurate Data

As discussed in sections 4.2.2 and 5.2.2, in practice, inaccuracies are introduced into the data through the wave number estimation process. To analyze the effect of inexact data on the solution, 100 realizations of the wave number data with zero mean Gaussian distributed added noise with $\sigma_N = 10^{-3}$ were simulated. The inversion was carried out for each realization of data and the resulting models were examined. Error bars, defined as the standard deviation of the resulting
models calculated at each depth, are shown in figure 6.2. The magnitude of data error considered in this example is consistent with the error associated with wave numbers estimated from measurements taken during the SW06 experiment.

The width of the error bars calculated for the joint inversion scheme shown in figure 6.2 are plotted by the solid blue line in figure 6.3(a) for the water column and in figure 6.3(b) for the seabed. The red dashed and dotted green lines in the plots indicate the width of error bars calculated for the separate seabed and water column inversion schemes. Error bars for separate inversion schemes were calculated for the same environment as the joint scheme using the same collection of noisy data vectors, the same values for the Lagrange multipliers, and the same starting profiles. When inverting for the sediment (or water column) sound speed profile independently, the water column (or sediment) sound speed profile of the true environment was used. The inaccurate data have the same effects on the solution to the joint inversion scheme as observed for the separate inversion schemes. In both cases, the widest error bars correspond to the deepest layer of the seabed. This is because there are few modes which penetrate to this depth. The shallowest sediment layer has wider error bars than the middle layer because it is narrow with respect to the wavelengths of the acoustic data. The portion of the solution pertaining to the water column has the widest error bars near the thermocline where there is a large gradient in the sound speed profile.

Examination of the error bar widths allows for comparison of the stability of the joint and separate inversion schemes. For the water column, both inversion schemes appear to have the same level of stability; on average, the widths of the errors bars differ by less than 0.02%. However, for the seabed, the joint inversion scheme is characterized by narrower error bars at all depths. The most notable difference is in the deepest layer where error bars corresponding to the separate inversion scheme are almost three times wider. Another indicator of the stability of the inverse scheme is the number of divergent solutions. Given the 100 realizations of noisy data vectors, the separate inverse scheme produced 10 divergent solutions whereas the joint inverse scheme produced only two.

Further insight about the stability of the inverse problem can be gained from examination of the kernel $G$. A measure of instability of the solution is the condi-
Figure 6.2. Error bars corresponding to solutions resulting from inversions using 100 realizations of wave number data with zero mean Gaussian distributed added noise with $\sigma_N = 10^{-3}$ are indicated by area encompassed by the cyan region. The true sound speed profile is shown by the solid blue line.

The condition number is defined as

$$\text{cond}(G) = \frac{s_1}{s_k}$$

(6.1)

where $s_1$ and $s_k$ are the largest and smallest singular values of $G$. Singular values were calculated using singular value decomposition (SVD). A condition number near one indicates a well-conditioned matrix. If $G$ is less than full rank, the condition number is effectively infinite. Condition numbers for the separate seabed, water column, and joint inversion schemes are shown in table 6.2.

The rank of $G$ is another indicator of stability of the solution to the inverse problem [59]. A matrix is said to be rank-deficient if there is a clear distinction between non-zero and zero singular values. Numerically computed singular values often include some values that are extremely small. Using these small singular values in the inversion can amplify data inaccuracies. To increase the stability of such
Figure 6.3. Error bar width for the joint inversion results (solid blue line) and separate inversion results (red dashed line) for the (a) water column and (b) seabed.

Table 6.2. Inversion Scheme Stability

<table>
<thead>
<tr>
<th>Inversion Scheme</th>
<th>Condition Number</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seabed</td>
<td>$1.247e12$</td>
<td>29</td>
</tr>
<tr>
<td>Water Column</td>
<td>$9.087e7$</td>
<td>26</td>
</tr>
<tr>
<td>Joint</td>
<td>$1.253e7$</td>
<td>11</td>
</tr>
</tbody>
</table>

a problem, truncated singular value decomposition (TSVD) can be applied. This method truncates the singular values below some threshold. This procedure results in a solution with lower resolution because some of the data has been thrown out. The methods applied in this thesis are based on regularization which effectively handles the instability of the inverse problem by adding small values to the diagonal of the matrix inverse. The magnitude of the added amount is controlled by the Lagrange multiplier. Using higher values of the Lagrange multiplier increases the stability of the solution at the cost of resolution. Thus, the value of the Lagrange multiplier in applications of regularization and the number of singular values to
include in TSVD have similar effects on the solution. Likewise, the rank of \( G \) is an indicator of inversion stability regardless of the method used to constrain the solution.

For the inverse problems considered here, the distinction between non-zero and zero singular values is not clearly identifiable. As shown in figure 6.4, there is a gradual transition from large singular values to extremely small singular values. Although all three inversion schemes show this trend, the seabed inversion scheme clearly has the fewest non-zero singular values, indicated by the very low values shown by the dotted green line after index 11. The water column and joint inversion schemes have many more non-zero singular values, as shown by the red dashed and solid blue lines which do not appear to reach near zero values until indexes 26 and 29, respectively.

Because the same value for the Lagrange multiplier is applied for all three cases, stability of the matrix inverse can be concluded based on \( G \) alone. Table 6.2 summaries the condition number and rank of \( G \) calculated for each of the inversion schemes. The very high values of the condition number indicate the all three matrices are rank-deficient with the seabed inversion being more ill-conditioned.
than the water column and joint inversion schemes. Moreover, the low rank of $G$ for the seabed inversion scheme is a further indication of the instability of the solution. These results, together with the greater error bar widths and higher divergence rate of the seabed inversion scheme indicate that the joint inversion scheme is more stable than the separate seabed inversion scheme. Comparison of the joint and the water column inversion schemes show they are approximately equally stable.

The superior stability of the joint inversion scheme comes at the cost of resolution. Resolution length, calculated according to equation 3.19, is plotted for each of the inversion schemes in figure 6.5. For the water column, resolution length is 30% greater for the joint inversion scheme compared to the separate inversion scheme. In the seabed, the difference is even more substantial: the average resolution length of the joint inversion scheme is twice as long as the separate inversion scheme.
6.2.3 Summary and Conclusions

Simultaneous inversion of water column and sediment sound speed profiles was demonstrated using a joint inversion scheme composed of a synthesis of qualitative regularization and approximate equality constraints. For exact data, this method was shown to be as accurate as inversion techniques presented in chapters 4 and 5 for determining sediment and water column sound speed profiles separately. However, due to the nature of the constraints, the joint inversion scheme is incapable of determining strong sound speed gradients in the seabed.

Horizontal wave numbers are never perfectly known and so robustness of the inversion scheme to inaccurate data is a more practical test. Compared to the separate inversion schemes, the joint scheme provides a more stable solution. This is especially true for the portion of the sound speed profile involving the seabed. Inversion stability was quantified by examining the error bar width and divergence rate of solutions resulting from 100 realizations of inaccurate data as well as by the condition number and rank of the kernel. However, there is a trade-off between stability and resolution of the inversion schemes. For, while the joint scheme provides a more stable solution to the inverse problem, the separate schemes supply solutions with higher resolution. This result suggests a process: use of the joint inversion scheme to obtain a rough estimate of the sound speed profiles when there is little a priori knowledge of environmental properties. Then apply the separate inversion schemes to refine the solution.

6.3 Application to Data from the SW06 Experiment

In this section, the joint inversion scheme is applied to data from the SW06 experiment. The highly variable nature of this shallow water environment was described in chapter 1. In chapter 4, range dependence of the seabed was attributed to the variable thickness of sediment layers. Fluctuations in the water column sound speed profile, studied in chapter 5, were associated with intrusions of warm, salty slope water onto the continental shelf.
6.3.1 Experiment Description

In this chapter, acoustic data from the SW06 experiment collected on August 6, 2006 is considered. The analysis focuses on the fourth run along the oblique shelf track. The location of the ship track and Shark VLA where the signals were recorded are shown in figure 4.35. A more complete description of the SW06 experiment is provided in section 4.3.2.

6.3.2 Analysis of Data

6.3.2.1 Horizontal wave number estimation

The horizontal wave number estimates obtained and presented in chapter 4 from the 125 and 175 Hz pressure fields (shown in figure 4.15 for 125 Hz) are used an input for the joint inversion scheme. Recall, for these estimates, Doppler effects were removed by averaging wave number estimates from the ingoing and outgoing runs made at the same tow speeds [85]. For this analysis, wave number estimates from the third and fourth runs along the track are used. Towed CTD chain measurements indicated that water column properties remained constant over the time period the data were collected. Therefore, the averaging procedure does not cause significant loss of water column variability specific to an individual track. In addition to providing an effective method of removing Doppler shifts, use of acoustic data from multiple runs makes it possible to obtain accurate wave number estimates of the low amplitude high-order modes. These data are necessary for determining properties of the seabed.

6.3.2.2 Range-dependent inversion results

Inversion results were obtained using the joint inversion scheme with the solution given according to equation 3.16. The locations of the seafloor and subbottom sediment layer depths were given as a priori information and used to construct the user defined operator \( L_q \). These depths were estimated using chirp seismic travel time data and are shown in figure 4.17. The water column sound speed profile measured at the Shark VLA was given as apriori information for the approximate equality constraints to restrict perturbations at the sea surface and the seafloor.
This measurement of the water column sound speed profile is shown in figure 5.13.

The discretization of the sound speed profile was chosen to be relatively small, such that \( \Delta z = 1/2 \) meter. A fine mesh is desired to minimize the extent that the absolute equality constraints influence the solution and to resolve sharp discontinuities in the sound speed profile. The values of \( \lambda_1^2 \) and \( \lambda_2^2 \) were chosen to be \( 5 \times 10^4 \) and \( 1 \times 10^3 \) to specify the smoothness of the solution and to restrict perturbations from the background at points defined by the absolute equality constraint.

The inversion results are shown in figure 6.6(a) for the water column and 6.6(b) for the seabed. The results have been divided onto separate plots so that a different color scale could be used for each portion of the solution. The inversion results obtained using the joint inversion scheme are similar to solutions obtained using the separate inversion schemes. These results were shown in figure 5.18 for the water column and figure 4.20 for the seabed.

As shown in the figure, the water column properties are nearly range-independent, although there does appear to be slightly slower sound speeds at the far end of the track. The range invariance of the results was expected from nearly constant values of the low-order wave numbers. The range-dependent features present in the chirp seismic data are well tracked by the inversion result. This is indicated by the consistency of values obtained for sound speed in each sediment layer determined by the inversion algorithm at each range step. An exception occurs in the upper unit around 4.0 km in range where the lower layered unit abruptly ends and the wave number estimates became excessively noisy, leading to inconsistencies in the inversion results at that range. The solution below the “R” reflector indicates very high sound speeds in excess of 2000 m/s for ranges less than 4 km. These results are not expected to be physical, but rather result from a limitation of the algorithm given this particular data set.

6.3.2.3 Range- and depth- averaging of the solution

Sound speed values obtained by the inversion are both range- and depth-averaged over the true sediment sound speed values. Range-averaging results from the horizontal wave number estimation procedure. A 2000 meter long sliding window was used to estimate the horizontal wave numbers. Consequently, the inversion results are range-averaged over this aperture. Depth-averaging results from the inversion
Figure 6.6. Inversion results obtained using the joint inversion scheme for sound speed in the (a) water column and (b) seabed.

algorithm itself. As with all continuous inverse problems, determination of the model function must be made in terms of local averages. Resolution length, calculated according to equation 3.19, expresses the extent the solution is vertically averaged at each depth.

Resolution length is shown for two different ranges in figure 6.7(a) with the
inversion results for the same ranges shown in figure 6.7(b). These ranges are representative of the inversion results when the low speed layer is and is not present. In both cases, resolution length for the majority of the water column is about four meters. Above the thermocline, resolution length is very large indicating the data does not contain much information about this portion of the waveguide. This deficiency was also observed for the separate inversion scheme. The approximate equality constraints were applied to address this insufficiency of the data. Resolution length for the upper layers of sediment is approximately seven meters. Resolution length below the “R” reflector is very large indicating the data does not contain sufficient information about the sediment at these depths. Therefore, the very high sound speed values obtained for this portion of the waveguide are suspect.
Estimates of the uncertainty of the inversion results were determined by Monte Carlo methods [59, 64]. The technique is applied by simulating a collection of noisy data vectors and then examining the statistics of the resulting models. To generate the noisy data vectors for error analysis, statistics were computed from wave number estimates as documented in section 4.3.3.4.

Range-dependent uncertainty estimates for the inversion results are shown for the water column in figure 6.8(a). The greatest uncertainty occurs near the high sound speed gradient of the thermocline with a maximum value of 1.5 m/s for most ranges. Both the location and magnitude of the uncertainty are consistent with the uncertainty calculated for the separate water column inversion scheme. The water column sound speed field shows higher uncertainty near 4 km. Due to the high variability of the seabed in this part of the waveguide, the high-order wave numbers are not well estimated at these ranges. The increased model uncertainty is a direct result of the higher data uncertainty. The model uncertainty calculated for the separate water column inversion scheme did not display this range dependence because only low-order wave number data were used.

As indicated by figure 6.8(b), the smallest uncertainties in the seabed correspond to the upper unit and the lower layered unit. Low uncertainty in the first two layers of the seabed is expected because the low-order modes, which are easiest to estimate, are most sensitive to this portion of the seabed. On the other hand, the sediment below the “R” reflector has much greater uncertainty. Sound speed estimates in this portion of the waveguide are ambiguous because not many modes penetrate to this depth. The greatest uncertainty corresponds to sediments at ranges less than 4.2 km for which the inversion estimated very high sound speed values. At ranges greater than 4.2 km, the sediment below the “R” reflector has a smaller level of uncertainty and the inversion results reported more reasonable values. Sound speed in the sand layer was not well determined. This is because the thickness of the sand ridge is significantly less than the resolution of the model. The average uncertainty for each layer is listed in the second column of table 6.4.
Figure 6.8. Uncertainty estimates calculated using Monte Carlo error propagation and the joint inverse scheme for the (a) water column and (b) seabed.

6.3.3 Evaluation of results

6.3.3.1 Comparison to CTD chain measurements

In this section, the inversion results are compared to the CTD chain measurements described in section 5.3.2.1. To quantify this comparison, the same three metrics
applied in chapter 5 are used. The first metric is an examination of the range- and depth-dependent differences between the inversion results and the CTD chain measurements. In the second metric, depths of the thermoclines are compared. The last indicator considers the depth-averaged sound speed. These metrics are described in detail in section 5.3.4.1.

A qualitative comparison of the inversion result (figure 6.6(a)) and the chain measurement (figure 5.10) show good agreement. Both the solution and the CTD chain measurement show a generally range-independent water column with a minimum in the sound speed profile near 30 meters depth. However, the difference between the CTD chain measurement and the inversion result shown in figure 6.9 does show evidence of some disparity. The CTD chain measurement indicates slower sound speeds over much of the extent of the measurement. An exception to this trend occurs near a range of 4 km where the inversion results have greater uncertainty due to the increased variance of the data. Part of the reason for the increased sound speed of the inversion result is that the thermocline depth is located slightly deeper in the inversion result, as indicated in table 6.3. The inconsistency
in thermocline depth can be attributed to inaccuracies in the CTD chain measurements which suffer from a high failure rate of the pressure sensors. There is very good agreement in the depth-averaged sound speeds as shown by figure 6.10.

The separate inversion scheme provided very similar results to the solution obtained using the joint scheme as indicated by all three metrics. This is shown by the similarities in the plots which show the difference between the CTD chain measurements and inversion results, i.e. figures 5.24 and 6.9. As indicated in table 6.3, the mean thermocline depths calculated from the solutions differ by less than 10 cm. Also, the depth averaged water column speeds are approximately equivalent.

### Table 6.3. Estimated thermocline depth.

<table>
<thead>
<tr>
<th>Inversion Result</th>
<th>Shark VLA Measurement</th>
<th>Chain Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Joint Scheme</td>
<td>16.19 m</td>
<td>16.28 m</td>
</tr>
<tr>
<td>Separate Scheme</td>
<td>16.27 m</td>
<td>15.43 m</td>
</tr>
</tbody>
</table>

6.3.3.2 Comparison to core data

In chapter 4, a model comprised of layers having constant sound speeds was constructed from the inversion results. In the same manner, sound speed in each layer is assumed to be the average value obtained by the range-dependent solutions to the joint inversion scheme, ignoring the high variance region near 4 km. Sound speed below the “R” reflector was taken from inversion results corresponding to ranges greater than 4 km. These results are preferred because the sound speed values in this region were consistently estimated within the expected range and the uncertainty analysis indicated these results are less ambiguous. The resulting sound speed values are given in the second column of table 6.4.

Several cores were collected preceding SW06 as part of the Geoclutter program within a few kilometers of the location of the Shark VLA [89, 90]. Additional cores were taken after SW06 as part of an ONR/Industry program. The direct measurements of the sand layer are derived from ISSAP sea floor measurements of sandy sea floors in the general vicinity [44]. Sound speeds estimated from both the cores and the ISSAP measurements are shown in the last column of table 6.4.
A detailed description of these measurements is presented in section 4.3.4.

Overall, there is fairly good agreement between the sediment sound speeds obtained using perturbative inversion and those estimated from the core data and IS-SAP measurements. The inversion result for the upper unit is within one standard deviation of the value surmised from the direct measurement. Both the inversion result and the core data also show evidence of the sound speed duct, although its presence is more prominent in the inversion results. Compared to the core data, the inversion results appear to under-estimate the sound speed in the seabed. One possible cause for this discrepancy is over-estimating the sound speed in the water column. However, this is not believed to be the case since comparison to direct measurements indicate inversion results for the water column are highly accurate.

Comparison to the solutions obtained using the separate inversion scheme provide further insight. As indicated in table 6.4, compared to the solution obtained from the separate inversion scheme, the solution obtained from the joint inversion
Table 6.4. Sediment Sound Speed

<table>
<thead>
<tr>
<th>Sediment Layer</th>
<th>Inversion Result Joint Scheme</th>
<th>Inversion Result Separate Scheme</th>
<th>Direct Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper Unit</td>
<td>1625 ± 15 m/s</td>
<td>1670 ± 12 m/s</td>
<td>1653 ± 34 m/s**</td>
</tr>
<tr>
<td>Lower Unit</td>
<td>1570 ± 20 m/s</td>
<td>1585 ± 19 m/s</td>
<td>1638 ± 56 m/s**</td>
</tr>
<tr>
<td>Below R</td>
<td>1790 ± 81 m/s*</td>
<td>1725 ± 15 m/s</td>
<td>1850 m/s**</td>
</tr>
<tr>
<td>Sand Layer</td>
<td>1645 ± 33 m/s</td>
<td>1740 ± 37 m/s</td>
<td>1760 ± 20 m/s***</td>
</tr>
</tbody>
</table>

* Based on inversion results for ranges greater than 4.2 km
** Core (230 kHz)
*** ISSAP (65 kHz)

scheme shows lower sound speeds above the “R” reflector and higher sound speeds below. Because the joint inversion scheme contained little information about the waveguide below the “R” reflector, the solution has suspiciously high sound speeds values for this portion of the waveguide. It is possible that the inversion results for the layers above “R” reflector balance by being artificially too low. This type of compensation can be attributed to the well-known correlation between geoacoustic parameters [14]. Additionally, as discussed in section 4.3.3.4, faulty a priori information about sediment layer depths is another source of error.

The comparison between sediment inversion results from the joint and separate inversion schemes is complicated by the use different water column sound speed profiles in each application. Due to the ambiguity of the pressure sensors the representation of the water column used for the separate inversion scheme had a shallower thermocline which caused the depth-averaged sound speed profile to be about 1.5 m/s slower than the values for listed the CTD chain measurement listed in table 6.3. As described in section 4.2.3, using artificially low values for the water column sound speed profile will cause the inversion to over-estimate sound speed in the seabed, especially in the shallow sediments. Hence, this is also a possible cause of discrepancy in the inversion results obtained for the separate and joint inversion schemes.
6.3.3.3 Prediction of transmission loss

In this section, the ability of the inversion results to predict transmission loss of measured signals from the SW06 experiment is evaluated. Incoherent correlation, calculated according to equation 4.4, is used as a measure of how well the inversion results predict the measured signals. Correlation values as a function of depth for all four frequencies are plotted by the solid blue line in figure 6.11. For comparison, correlation values calculated from predictions made using water column and seabed sound speed profiles estimated by the separate inversion schemes are shown by the dotted red line. Although different versions of the water column sound speed field were used to calculate the transmission loss, the predictions are dominated by the properties of the seabed at these frequencies.

As shown by the figure, fields calculated using sound speed profiles from the joint and separate inversion schemes both give good predictions of the measured signals. On average, the correlations differ by less than 2%. However, at frequencies
below 175 Hz, the fields calculated using sound speed profiles from the separate inversion schemes consistently produced a better match to the measured data.

6.3.4 Conclusion

The joint inversion scheme for simultaneously estimating water column and seabed sound speed profiles was applied to data from the SW06 experiment. For this application, the solution was obtained using no *a priori* knowledge of the local water column or seabed properties. This is a clear advantage over the separate sediment inversion scheme which relied on measurements of water column properties from the towed CTD chain.

The inversion results revealed sound speed profiles in the water column and seabed that were somewhat similar to those obtained using the separate inversion schemes. Differences occurred in the seabed sound speed profile: the solution obtained using the joint inversion scheme produced faster sound speeds below the “R” reflector. These faster sound speeds are questionable because the kernel contains little information about these deeper sediments. This was indicated by low model resolution corresponding to this part of the waveguide. Moreover, the solution below the “R” reflector was characterized by high uncertainty, making the inversion result for this portion of the waveguide ambiguous.
Chapter 7

Conclusion

7.1 Summary

Three new methods for constraining the perturbative inverse problem have been presented. These techniques allow for more accurate estimation of sound speed profiles in both the water column and seabed than was formerly possible. Each of the constraints was described in chapter 3. Then the applications were evaluated using both synthetic and experimental data in chapters 4, 5, and 6.

The first of these constraints, qualitative regularization, was introduced to account for discontinuities in the sediment sound speed profile. This technique is an improvement over conventional methods such as Tikhonov regularization and SVD which approximate the layered structure of the seabed by a smooth sound speed profile. Results obtained using qualitative regularization were shown to be robust to inaccurate data, especially for sediments near the seafloor. For inexact inputs, inversion results were degraded below the e-folding depth of the low-order modes because the data are less sensitive to the deeper sediments. It was shown that it is necessary to have good knowledge of the water column sound speed profile in order to obtain an accurate solution for seabed properties. This result is expected since it was shown in chapter 2 that wave number values are more sensitive to the sound speed in the water column. When detailed knowledge of the water column sound speed profile is not available, accurate inversion results can still be obtained if the depth averaged water column sound speed is known.

An obvious limitation of qualitative regularization is the requirement of a pri-
ori knowledge of the location of sediment layer depths as this information about may be lacking for many smaller experiments taking place in less studied areas. Additionally, since the location of sound speed discontinuities is not updated as part of the perturbative process, incorrect inputs for the sediment layer depths will be reflected in the solution. The values of sound speed in each layer will differ from the true model parameters in order to compensate for incorrect layer depths.

In chapter 5, approximate equality constraints were applied to estimate water column sound speed profiles. This is considered a novel approach as there is no previous documented application of perturbative inverse used to estimate water column sound speed profiles. To obtain an accurate representation of the true sound speed profile, an additional constraint is imposed on the regularized inverse problem to restrict perturbations in portions of the waveguide for which the data are insufficient. This method was shown to be robust to inaccurate data, providing accurate solutions with a very high rate of convergence. Since low-order wave numbers are relatively insensitive to properties of the seabed, accurate inversion results can be obtained using a subset of low-order wave number data when the properties of the bottom are unknown.

As with qualitative regularization, approximate equality constraints provide an improved solution to the inverse problem by taking a priori knowledge of the environment into account when performing the inversion. In this case, the a priori information is the value of the solution at specified points, typically near the sea surface and seafloor. If the a priori information is incorrect, the solution will have erroneous values at the specified points and sound speed in the rest of the water column may deviate from the true parameter values to compensate. If the a priori information is suspect, it can be given less weight in the inversion. However, if the approximate equality constraints have been applied because it has been established that the data are not sensitive to these portions of the waveguide, giving less weight to the constraint will not improve the solution.

Simultaneous inversion of water column and sediment sound speed profiles was obtained using a combination of qualitative regularization and approximate equality constraints. Qualitative regularization was used to resolve the discontinuity in the sound speed profile at the seafloor as well as additional discontinuities in the seabed and approximate equality constraints were applied to constrain the so-
solution in portions of the water column for which the data alone are insufficient.
The primary advantage of jointly inverting for water column and sediment sound speed profiles at the same time is that poor assumptions about the “known” profile are not aliased into the solution. That is, assuming an incorrect sound speed profile for the water column does not result in an erroneous solution for the seabed properties and vice-versa. The solution to the joint inverse problem was shown to be more stable than the solution obtained using either of the individual inversion schemes. However, the increased stability comes at the cost of the resolution of the solution. This trade-off between solution stability and resolution suggests a two step process: apply the joint inversion scheme first to obtain a stable solution when little is known about the environmental properties. Then refine the solution by applying the separate inversion schemes using the previous results to construct the background environment to get a high resolution estimate of the sound speed profiles.

7.2 Future Work

Although qualitative regularization has been shown to be a powerful method for obtaining accurate solutions for the sound speed profile in the seabed, its requirement of a priori knowledge of the depths of sediment layers restricts its application. Although this information is often readily available in large scale experiments which typically include a chirp seismic survey of the area, this data may be lacking for many smaller experiments. Therefore, it would be useful to have an alternative method for determining this information. Optimally, this method should make use of the same narrow band tonals used for the inversion scheme in order to eliminate the need for additional measurements. While there are numerous techniques available capable for Fredholm integrals with discontinuous solutions, the challenge is to find such a method that is well-suited for the highly ill-conditioned geoacoustic inverse problem.

A straight forward extension of this work is to apply the constraints described in this thesis to linearized inversion schemes involving other data types, particularly group speed dispersion measurements [5, 53]. For this data type, inclusion of the water column sound speed profile in the inversion scheme is especially important.
because water column properties are rarely measured over the entire propagation path of a pulsed signal.

Although the methods presented in this thesis concentrated on a local linear inversion technique, nonlinear matched-field inversion techniques have been more widely applied. Matched-field techniques are often applied for inversion of seabed properties alone, using a finite number of measured water column sound speed profiles to approximate range-invariant water column properties over the propagation path. However, when the measurements are not temporally or spatially co-located with the acoustic signals, the measured data may not provide an accurate representation of true water column properties [12, 28, 29]. In these cases, the conventional practice of using individual sound speed profiles measured at a few sites and times is not effective. While it is possible to include the water column sound speed profile as inversion parameters [35, 32, 33], this procedure can have the effect of widening the posterior probability distributions (PPD) making the inversion results ambiguous [96]. Furthermore, when water column properties vary over the propagation path, the assumption of range invariance fails and inversion results are degraded [97]. To address the need for knowledge of water column properties along the propagation path, perturbative inversion using approximate equality constraints can be applied. Then the nonlinear matched-field inversion can be carried out in a straightforward manner by assuming range-invariant seabed properties. The inclusion of range-dependent water column properties and the assumption of a range-invariant seabed is reasonable because water column fluctuations can occur over a smaller scale than seabed variability [31, 35]. There are several reasons to believe this approach would be successful. First, horizontal wave numbers are a clear indicator of environment variability. Secondly, the wave number data are well suited for water column inversion: low-order wave numbers which are more sensitive to water column properties are easier to estimate due to their greater amplitude. Additionally, it is easier to correct low-order modes for Doppler effects when there is no a priori knowledge of the environment since the group speed of these modes will be in the asymptotic portion of the group speed dispersion curve well away from the Airy Phase [98]. Finally, the water column inverse problem is generally relatively well-conditioned with a high rate of convergence. On the other hand, perturbative inversion for seabed properties can be complicated by insuffi-
cient data since high-order modes can have very low amplitude and Doppler effects can be difficult to remove. Furthermore, the inversion algorithm can suffer from a high rate of divergence when a poor starting profile is chosen. On the other hand, nonlinear matched-field inversion techniques which make use of pressure field data do not suffer from these limitations. Also these methods provide estimates of additional parameters such as density and attenuation. Thus, this hybrid approach combines strengths of both perturbative and matched-field inversion.

A related area of research is development of improved methods for estimating range-dependent wave numbers. Although several powerful estimation techniques are described in appendix A, each has limitations. Accurate estimation of wave number values in strongly range-dependent waveguides would make it possible to estimate seabed properties in more complex environments such as the shelf slope. Given this information, perturbative inversion could be applied to reveal distinct features such as buried channels in the seabed and internal waves in the water column.
Appendix A

Horizontal Wave Number Estimation

A.1 Introduction

Effective employment of the perturbative inversion scheme requires accurate input data in the form of $k_r$. Consequently, a number of techniques for estimating horizontal wave numbers have been developed. The research has concentrated on identifying estimators to track wave number evolution in range-dependent environments. A number of the most successful techniques are described here. The discussion begins in section A.2 with the Hankel transform, which is defined for a point source in an axisymmetric environment. The explanation continues by summarizing several methods which are applicable to range-dependent environments. The short-time Fourier transform (STFT), which utilizes a fixed aperture sliding window, is introduced for weakly range-dependent environments in section A.3. Use of the sliding window is extended to more strongly range-dependent environments in section A.4 by implementing autoregressive (AR) techniques which makes it possible to reduce the length of the data estimation aperture. However, it is still not possible to resolve discontinuous changes in wave number data because the environment is assumed to be range-independent within the estimation aperture. By using a Kalman filter to update estimates of the AR coefficients, abrupt changes in the environment can be tracked. This method is described in section A.5. The Wigner-Ville Distribution (WVD) which does not require use of a sliding window is presented in section A.6. Although it may not be suitable for the strongly range-dependent environments handled by AR methods, it does not
suffer from the same drawbacks as the non-linear estimator.

Each of the estimation techniques is demonstrated by applying them to the same pressure data. Accuracy of each technique is evaluated according to its ability to track the evolution of mode five. Mode five was chosen because it was tracked by all four estimation techniques. The error of the wave number estimates is defined as the standard deviation between the estimate and the true wave number value:

$$\sigma_E = \left[ \sum_{n=0}^{N} (k_{5}^{est}(\hat{r}_n) - k_{5}^{true}(\hat{r}_n))^2 \right]^{\frac{1}{2}}. \quad (A.1)$$

where $\hat{r}_n$ is the range corresponding to the $n^{th}$ wave number estimate and $N$ is the number of range-dependent estimates.

The pressure data used for the evaluation of the estimators was computed for a 50 Hz source in the environment described by test case 2 (TC2) from the Geoacoustic Inversion Techniques (GAIT) 2003 workshop [20]. The water column sound speed profile of this environment was downward refracting according to

$$c_w(z) = 1495.0 - 0.04z \quad (A.2)$$

where $z$ is depth in meters. The sound speed profile of the bottom follows a generally increasing trend with depth; details are available in [20]. Range dependence enters the problem through bathymetry. As shown in figure A.1, the environment consists of a flat, range-independent region followed by a sloping region. The slope represents the shelf break.

Transmission loss for a source at 20 meters depth calculated for a depth of 70 meters is shown in figure A.2. The pressure field is sampled every 5 meters in range. This environment supports five wave numbers in the shallow portion of the waveguide.

The true values of the modal eigenvalues are plotted as a function of range in figure A.3(a). Wave number values are constant throughout the range-independent portion of the waveguide and increase over the sloping bottom. Modal amplitude provides additional insight into the ability of the different techniques to estimate wave number values. Wave numbers for modes with very low amplitude are more difficult to estimate. Normalized modal amplitude values, plotted in figure A.3(b),
Figure A.1. Range-dependent bathymetry used to construct the environment considered for wave number estimation. This environment is modeled after TC2 of the GAIT 2003 workshop.

Figure A.2. Transmission loss from a 50 Hz source at a depth of 20 meters recorded at a depth of 70 meters.
Figure A.3. (a) Modal wave numbers and (b) modal amplitude as a function of range for the TC2 environment.

Figure A.3. (a) Modal wave numbers and (b) modal amplitude as a function of range for the TC2 environment.

Indicate that mode two had the lowest amplitude at almost all ranges. Normalized modal amplitude changes as a function of range because as the waveguide deepens, the mode shapes are stretched but the receiver depth remains constant. The amplitudes of modes two and four both have zero amplitude in the range-dependent part of the environment. The nulls in mode shapes correspond to the 70 meter receiver depth for mode two and four at ranges of 3.6 km and 4 km, respectively.
Figure A.4. The wave number spectrum calculated using the Hankel transform (solid blue line). True wave numbers from the range-independent portion of the waveguide (red dotted lines).

A.2 The Hankel Transform

A Hankel transform pair can be defined to relate the pressure field due to a point source in an axisymmetric environment to its transfer function [49]. By applying the far field approximation, this relation is given by:

\[
p(r; z, z_0) \approx \frac{e^{-i \frac{\pi}{4}}}{\sqrt{2 \pi r}} \int_{-\infty}^{\infty} g(k_r; z, z_0) \sqrt{k_r} e^{ik_r r} dk_r \quad (A.3a)
\]

\[
g(k_r; z, z_0) \approx \frac{e^{i \frac{\pi}{4}}}{\sqrt{2 \pi k_r}} \int_{-\infty}^{\infty} p(r; z, z_0) \sqrt{r} e^{-ik_r r} dr \quad (A.3b)
\]

where \(p\) is acoustic pressure and \(g\) is the Green’s function. Pressure is a function of range \(r\), source depth \(z_0\), and receiver depth \(z\). The Green’s function, also known as the wave number spectrum, has poles corresponding to the locations of the horizontal wave numbers \(k_r\).

The Green’s function was estimated for the pressure field described above ac-
according to equation A.3b. The resulting wave number spectrum is shown by the solid blue line in figure A.4. The large peaks correspond to the propagating modes in the range-independent portion of the waveguide. These peaks agree well with the true wave numbers for the same portion of the waveguide, plotted by the dotted red lines. As expected from figure A.3(b), mode two is the most difficult to identify because of its low amplitude. Although the smaller peaks are primarily sidelobes, wave numbers from the range-dependent part of the waveguide also contribute to this portion of the spectrum. Because of their low amplitude, these peaks are impossible to identify.

**A.3 Short-Time Fourier Transform**

Short-time Fourier transform (STFT) techniques are applied for range-dependent wave number estimation [25]. This method is synonymous with use of the spectrogram for non-stationary signals. A range-dependent estimate of the Green’s function according to the STFT is given by

$$g(k_r; \hat{r}, z, z_0) = \frac{e^{i \frac{\pi}{4} \sqrt{2}}}{2\pi k_r} \int_{-\infty}^{\infty} w_L(r; \hat{r}) p(r; z, z_0) \sqrt{r} e^{-ik_v r} dr$$  \hspace{1cm} (A.4)

where $w_L$ is a window of length $L$, centered at $\hat{r}$. Range-dependent estimates of the Green’s function are obtained by sliding the window $w_L$ across the pressure field. This method is appropriate for weakly range-dependent environments as the environmental properties within the window are approximated as range-independent.

The peaks of the STFT estimate of the range-dependent Green’s function are shown by the black dots in figure A.5. A 1250 meter aperture with 95% overlap was used to calculate the estimates. This window length was chosen to track the modal evolution of the fifth wave number. The short aperture makes it impossible to distinguish the peaks of the first two wave numbers which are very closely spaced. This tradeoff between spatial resolution and wave number resolution is the well-known uncertainty principle.

Values for the higher order wave numbers are estimated throughout the waveguide. Even though mode four has zero amplitude at 4 km, an estimate for the fourth wave number is still obtained because the 1250 meter aperture always en-
Figure A.5. The range-dependent wave number spectrum calculated using the STFT (black dots). True wave numbers are overlaid (solid lines).

compasses portions of the pressure field for which mode four has high enough amplitude to be detected.

The wave number estimation error of mode five for the STFT estimate was $E = 1.166e-3$ m$^{-1}$. This error is of the same order of magnitude as the simulated noisy wave number data used to evaluate the robustness of the inversion algorithm in chapters 4 and 5. However, as only three of five wave numbers were estimated, the data may not be sufficient for use in the inversion scheme given this level of noise.

### A.4 Autoregressive Estimator

Application of nonlinear spectral estimation techniques such as auto regression (AR) make it possible to shorten the window length while retaining wave number resolution [8]. Using the notation $r_n = r_o + n\Delta r$; $y(n) = p(r_n; z)\sqrt{r_n}$; where $r_o$ is some initial range, the $p$-order AR model is given by:
\[ y(n) = a_1(n) + \ldots + a_p(n)y(n-p) + v(n) \]
\[ = a(n)^T \varphi(n) + v(n) \quad (A.5) \]

where \( v_n \) is a white noise sequence with variance \( \sigma_v \), \( a_n = [a_1(n)\ldots a_p(n)]^T \) is the vector of AR coefficients, \( \varphi(n) = [y(n-1)\ldots y(n-p)]^T \) is the vector of \( p \) past signal samples. The AR spectrum is estimated from the AR coefficients \( a^T \) as:

\[ P_{AR}(k_r, r_n) = \frac{\sigma_v^2}{\|1 - a_1(n)s^{-1} - \ldots - a_p(n)s^{-p}\|^2_{s=\exp(i\Delta r k_r)}}. \quad (A.6) \]

This expression is used to estimate the local spectrum associated with ranges defined by a sliding window. \( P_{AR} \) is obtained by estimating the coefficients \( a^T \) and noise variance \( \sigma_v \), and then evaluating the denominator of equation A.6 by application of a FFT. Zero padding the signal by increasing the number of FFT points increases the estimator’s performance.

The AR spectrum provides for higher resolution estimates, but does not conserve the amplitude of the spectrum. Since the perturbative inversion scheme requires only estimates of the eigenvalues (and not their amplitudes) the AR estimator is effective for this application.

The peaks of AR estimate of the Green’s function are shown by the black dots in figure A.6. The spectrum was calculated using a 500 m aperture with 90% overlap. The order of the estimator was chosen to be one third the number of samples as suggested by Becker [8]. All five modes are very accurately identified within the range-independent region of the environment. However, there are some inaccuracies in the range-dependent portion of the waveguide. Since the assumption of a range-independent environment within the aperture is violated, accuracy of the estimates is degraded. The AR estimator is most successfully applied in environments that are well approximated by having range-independent regions; for example, consider [88]. Systematic eigenvalue change, as for a sloping bottom, degrades the performance of the AR estimator, a result previously observed in [99]. Mode two is not estimated in the range-dependent region due its low amplitude in combination with its nonstationary and proximity to mode one.

Compared the the STFT transform estimate of the wave number spectrum,
the AR estimate is a significant improvement. The range aperture is reduced by more than a factor of two and, as a result, the range dependence of the waveguide is much better represented. Furthermore, the wave number estimates are more accurate: the estimation error for mode five has been reduced to $\sigma_E = 7.282e - 4$ m$^{-1}$. Additionally, the AR estimator was able to detect all five modes in the range-independent portion of the waveguide and four of the modes in the range-dependent region.

**A.5 Range Varying AR Estimator**

The range-varying AR model is an extension of the conventional, stationary process model, where the AR coefficients are sequentially estimated at each range step using a Kalman filter [65]. For spectrum peak identification, the zeros of the denominator of equation A.6 are estimated directly using an adaptive filter with variable forgetting factor (VFF).
Figure A.7. The Kalman filter architecture: state equation (blue), measurement equation (red), reproduction of the system (green), and correction step (purple).

For the Kalman filter application, equation A.5 is interpreted as the measurement equation, relating the measured quantity \( y(n) \) to the system state \( a \). In addition, the system state is assumed to evolve in range according to a state equation. A simple Gaussian random walk model is assumed for the state equation:

\[
a(n) = a(n-1) + w(n)
\]

where \( w(n) \) is a white Gaussian noise vector with variance \( \sigma^2 \).

Kalman filter implementation is illustrated by the flow chart in figure A.7. The state equation and measurement equation are shown in blue and red, respectively, and the block \( T \) represents a time delay. The task is to estimate the system state so that the effects of the noise \( w(n) \) and \( v(n) \) are minimized. This is done by reproducing the system architecture to get an initial estimate of \( a(n) \) which is designated as \( \hat{a}^-(n) \). This part of the procedure is diagrammed in the figure in green. This estimate of the system state will be updated to account for the noise. The final step of the Kalman filter is shown in purple. In this step, a correction is calculated based on the difference between the measured data \( y(n) \) and the
Figure A.8. The range-dependent wave number spectrum calculated using the range-varying AR estimator (black dots). True wave numbers are overlaid (solid lines).

estimated data $\hat{y}(n)$. This correction provides a new estimate for $\hat{a}^{-}(n)$ called $\hat{a}(n)$.

Souza proposes numerous ways to optimize the Kalman filter performance. Among his suggestions is competitive smoothing which involves using several different realizations of the noise variance. Another suggestion is downsampling the data and shifting the spectrum which allows for much lower orders of the AR estimator to be used increasing the efficiency of the algorithm. Details of these techniques are described in [65].

The peaks of the range-varying AR estimate of the Green’s function are shown by the black dots in figure A.8. The order of the AR estimator was 12 and the range grid was down sampled by a factor of 10 so that $\Delta r = 50$. A suite of Kalman filters competed to obtain the results. Like the stationary AR estimator, the range-varying AR estimates all five modes in the range-independent region of the waveguide and modes one, three, and five in the range-dependent portion. As before, mode two is not detected in the range-dependent portion of the waveguide. The value of mode four is not tracked as it moves through the null. Although
this maybe seen as a deficiency because an estimate of mode four is not obtained for some ranges, these results are really a more accurate depiction of the range dependence of mode four. Compared to the stationary AR estimator, accuracy of the estimate of mode five is about the same; the estimate error is $\sigma_E = 7.245e - 4$ m$^{-1}$.

As shown in figure A.8, the estimator is less accurate in the range-dependent portion of the waveguide. This degradation is characteristic of the Kalman filter. In general, sequential estimators can identify nonstationary parameters that drift slowly, have infrequent abrupt changes, or a combination of these two behaviors. Most adaptive identification methods fail with fast varying parameters [65]. The range varying AR estimator produces superior results when applied to data from environments which cause the eigenvalues to change abruptly. An example of such an environment is TC3 of the GAIT 2003 workshop; this environment is characterized by an intrusion of a different type of sediment [20]. When applied to this data, range varying AR produced highly accurate estimates [65].

### A.6 Wigner-Ville Distribution

The Wigner-Ville Distribution is a joint distribution that describes the energy density or intensity of a signal simultaneously in time and frequency [100], [101], [102]. Such a distribution can be used and manipulated in the same manner as any joint density function. For example, the distribution can be used to tell what fraction of energy is contained within a certain range of frequencies and times. Moreover, tracking the peaks of the distribution indicates how frequency components evolve over time.

Ideally, the time-frequency distribution of a multicomponent signal is desired to be composed of narrow peaks that track the time dependent changes of frequency components over time. Instantaneous frequencies, defined as the derivative of the signal’s phase, would correspond to these peaks. In order to formulate such a distribution, consider the following example [102]. Let $\rho_z$ be a function of frequency $f$ and represent the spectrum of the signal $z(t)$. It is reasonable to assume $\rho_z$ to be the Fourier Transform of some function $K_z$ related to the signal. Thus, the time-frequency representation can be written as:
\[ \rho_z(t, f) = \mathcal{F}_{\tau \to f} \{ K_z(t, \tau) \} \] (A.8)

Now the task is to find a suitable expression of \( K_z \). For simplicity, consider the unit amplitude monocomponent FM signal:

\[ z(t) = e^{i\phi(t)} \] (A.9)

whose instantaneous frequency is given by:

\[ f_i(t) = \frac{\phi'(t)}{2\pi}. \] (A.10)

We would like the time-frequency distribution of \( z(t) \) at any given time to be a unit delta function at the instantaneous frequency; that is, we want

\[ \rho_z(t, f) = \delta(f - f_i(t)) \] (A.11)

Substituting this into A.8 and taking the inverse Fourier transform, we obtain:

\[ K_z(t, \tau) = \mathcal{F}_{\tau \leftarrow f}^{-1} \{ \delta(f - f_i(t)) \} = e^{i2\pi f_i(t) \tau} = e^{i\phi'(t) \tau} \] (A.12)

Because \( \phi'(t) \) is not directly available, we approximate it using the central finite difference theorem:

\[ \phi'(t) \approx \frac{1}{\tau} [\phi(t + \tau) - \phi(t - \tau)] \] (A.13)

Substituting eqn. A.13 into A.12 and using A.9 gives the signal kernel:

\[ K_z = e^{i\phi(t+\tau/2)} e^{-i\phi(t-\tau/2)} = z(t + \tau) z^*(t - \tau) \] (A.14)

Finally, substitution of A.14 into A.8 gives the Wigner-Ville Distribution:
The Wigner-Ville Distribution provides an estimate of both instantaneous frequency and instantaneous amplitude. However, there are some disadvantages. Because it is bilinear in the signal rather than linear, it suffers from spurious features called artifacts or cross-terms which appear midway between the true signal components. The effect of these cross-terms can be mitigated by windowing the lag. For a given time, the WVD weights equally all times of the future and past. Similarly, for a given frequency, it weights equally all frequencies below and above the given frequency. Windowing is introduced to emphasize the signal around time $t$ and frequency $f$. The windowed version is known as the Smoothed Pseudo Wigner-Ville Distribution [103]:

$$SPWVD(t, f) = \int_{-\infty}^{\infty} h(\tau) \int_{-\infty}^{\infty} g(s-t)z(s + \frac{1}{2}\tau)z^*(s - \frac{1}{2}\tau)dse^{-j2\pi f\tau}d\tau \quad (A.16)$$

where $g$ is the time smoothing window, with $G(f)$ equal to the Fourier transform of $g(t)$ and $G(0) = 1$; $h$ is the frequency smoothing window and $h(0) = 1$.

However, even with windowing, the cross-terms are still present in multicomponent signals with closely spaced frequencies. This is of importance to the wave number estimation problem because at high frequencies wave numbers will be closely spaced and these artifacts make interpretation of the Wigner-Ville transform difficult. Windowing improves readability at the sacrifice of satisfying the marginals and broadening frequency widths of the auto-terms. For smoothed versions of the WVD, a larger smoothing kernel reduces the cross-terms, but yields a poorer localization of the signal components. Where the limitations of the STFT can be described as a trade-off between time resolution and frequency resolution, the WDV’s limitations are described as a trade-off between the joint time-frequency resolution and level of the cross-terms.
The Wigner-Ville Distribution estimate of the Greens function is given by:

\[ g^2(k_r, r; z, z_0) = \frac{1}{2\pi k_r} \int_{-\infty}^{\infty} q(r + \tau/2)q^*(r - \tau/2)e^{ik_r\tau}d\tau \]  

(A.17)

where \( q(r; z, z_0) = p(r; z, z_0)\sqrt{r} \).

Note that A.17 gives the amplitude of the green’s function squared. This is because the signal enters the transform twice. This expression differs from the one given in Matsumoto’s paper [69] where the WVD by is scaled by \( \frac{e^{i\pi/4}}{\sqrt{2\pi k_r}} \) instead of \( \frac{1}{2\pi k_r} \).

Interpretation of the output of the WVD is examined for range-dependent environments using the adiabatic approximation for pressure:

\[ p(r; z, z_0) = e^{i\pi/4} \sqrt{\frac{2\pi}{r}} \sum_n \frac{1}{\sqrt{k_n}} u_n(0, z_0)u_n(r, z)e^{i \int k_n(r)dr}. \]  

(A.18)

Substituting the adiabatic approximation for pressure into equation A.17 gives:

\[ g^2(k_r, r; z, z_0) = \]

\[ \frac{1}{k_r} \sum_n |u_n(0, z_0)|^2 \int_{-\infty}^{\infty} \frac{u_n(r + \tau/2, z)u_n^*(r - \tau/2, z)}{\sqrt{k_n(r + \tau/2)k_n(r - \tau/2)}} e^{i \int_{r-\tau/2}^{r+\tau/2} k_n(r)dr} e^{ik_r\tau}d\tau. \]  

(A.19)

Since \( u_n(r, z) \) and \( k_n(r) \) are weak functions of \( r \) under the adiabatic approximation, they are not dominant terms in the shifting the peaks of the Green’s function. The phase term \( i \int_{r-\tau/2}^{r+\tau/2} k_n(r)dr \) is the most important factor. This term depends on the lag \( \tau/2 \) which is windowed in both the range and wave number domains in the case of the Smoothed-Pseudo WVD. In the limit of strong smoothing \( r \) approaches \( \tau/2 \), and \( i \int_{r-\tau/2}^{r+\tau/2} k_n(r)dr \) approaches \( i \int_0^\tau k_n(r)dr \). This is the adiabatic approximation for the wave number exactly. However, there is a limit to how much one should smooth the WVD. The expense of stronger smoothing is resolution of the distribution. Too much smoothing spreads the peaks of green’s function over each other so that they are no longer readable.

The SPWVD estimate of the peaks wave number spectrum are shown in fig-
Figure A.9. The range-dependent wave number spectrum calculated using the WVD (black dots). True wave numbers are overlaid (solid lines).

Even though windowing was applied to both the range domain and the wave number domain, cross-terms are still present between true wave number values. The cross-terms are most prominent between the closely spaced low order modes, compromising the estimate of mode one and making mode two unidentifiable. Cross-terms are present between the high-order modes as well, but the wave number estimates are not seriously affected. Estimates for all the modes are affected by edge affects near the beginning and end of the aperture. In the range-dependent portion of the waveguide, only the third and fifth wave numbers are well estimated. The zero amplitude of mode four near 4 km is evident as there is no estimate of mode four corresponding to this range. The range dependence of mode five very well tracked by the SPWVD; its estimation error is the lowest of all the estimators: $\sigma_E = 2.151 e - 4$ m$^{-1}$.

The WVD excels in the estimation of systematically changing values. The estimator does poorly when faced with abrupt changes in wave number values. This is evident in the example shown: greatest error in the estimate of mode five occurs where the environment transitions from flat to sloping.
Table A.1. Accuracy of the wave number estimation techniques to estimate mode five.

<table>
<thead>
<tr>
<th>Estimator</th>
<th>$\sigma_E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>STFT</td>
<td>$1.166e - 3 \text{ m}^{-1}$</td>
</tr>
<tr>
<td>Stationary AR</td>
<td>$7.282e - 4 \text{ m}^{-1}$</td>
</tr>
<tr>
<td>Range Varying AR</td>
<td>$7.245e - 4 \text{ m}^{-1}$</td>
</tr>
<tr>
<td>WVD</td>
<td>$2.151e - 4 \text{ m}^{-1}$</td>
</tr>
</tbody>
</table>

A.7 Conclusion

Four different techniques for tracking modal evolution in range-dependent environments have been presented. Each method has its own advantages and drawbacks which were illustrated through application to an example. The STFT has the advantage of being a linear estimator: its output is the easiest to interpret and amplitude of the modes is conserved. However, the STFT provided the least accurate estimate of mode five as shown in table A.1. Due to its limited spatial resolution, the STFT technique is best suited for environments that are weakly range-dependent. For the example considered, significant improvement was achieved by application of the AR estimator. This enhancement is quantified by the number of modes tracked as well as the accuracy of the estimate of mode five. Because the AR estimator models signals as stationary within the aperture, it works bests in environments that are well modeled by range-independent regions. The range varying AR estimator is appropriate for a wider range of environments because it utilizes a sequential estimator which can identify nonstationary parameters that drift slowly, have infrequent abrupt changes, or a combination of these two behaviors. Both the stationary AR estimator and the range varying AR estimator perform poorly for the type range dependency examined here. As shown by table A.1, the most accurate estimates of the fifth wave number were obtained using the WVD. This technique is well-suited for estimating deterministic changes in wave number values. However, due to its bilinear nature, the WVD suffers from cross-terms which degraded the estimates of the closely spaced low order modes.

There is no single “best” estimator for tracking modal evolution. Following from the analysis provided in this appendix, particular estimators are better suited
for certain types of environmental variability. Thus, the most accurate wave number estimates can be obtained by choosing the most appropriate estimator.

The summary provided here is not an exhaustive list of all the wave number estimation techniques available. Moreover, new techniques are continuing to be investigated. For example, use of Particle filters [104], which have been applied for estimating modal arrivals from broadband signals, could potentially improve the current capacity to estimate range-dependent wave number values. The more accurately wave number evolution can be estimated, the better environmental range dependence can be tracked. This allows for improved inversion results which leads to better predictions of acoustic signals.
Appendix B

Data Processing of Acoustic Data from SW06

B.1 Introduction

This appendix describes the processing of the acoustic data from the SW06 experiment. Details of the data collection and instrumentation were described in section 4.3.2. The discussion provided here describes the procedures for downsampling the data, basebanding the narrowband signals, merging the acoustic data with the GPS data, and estimating the horizontal wave numbers. In addition, the signal to noise ration (SNR) of the narrowband signals is evaluated.

B.2 Downsampling the data

Downsampling is the process of reducing the sampling rate of a signal. This was done to reduce the size of the data. The original sampling rate of 9765.625 Hz was decreased by a factor of 50 to produce a new sampling rate of 195.3125 Hz. Downsampling was accomplished by reading in a 2048 second data record, lowpass filtering the signal to eliminate frequency components greater than the new sampling frequency, and then selecting every 50th measurement. During this process, raw data were converted to voltages. This procedure was then repeated with the following data records.
Figure B.1. Power spectral density function of data from the third run of the along shelf track recorded on channel six of the Shark VLA.

### B.3 Signal to Noise Ratio

Signal to noise ratio (SNR) was calculated from the downsamed time series. SNR was calculated according to

\[
\text{SNR} = 10 \log_{10} \frac{P_{\text{signal}}}{P_{\text{noise}}}
\]  

(B.1)

where \( P \) is the average power. The average power of the signal and noise were estimated from power spectral density function (PSD). The PSD calculated for data from the third run of the along shelf track recorded on channel six of the Shark VLA is shown in figure B.1. This data set corresponds to the 36 minute time period the ship took to traverse the track. At a sampling rate of 195.3125 Hz, this corresponds to an FFT length of 419728. As shown by the figure, all measured signals had very high SNR, near 50 dB.

Doppler shift, caused by motion of the ship, is evident in the figure. At the time these data were recorded, the ship was moving with an average speed of 2.05
m/s toward the VLA. This caused a upward shift in the received frequencies. The effect of a moving source on the acoustic field in shallow-water can be calculated according to [98].

B.4 Baseband the Narrow Band Signals

After downsampling to reduce the data set to a manageable size, the next step was to isolate the narrow band signals. This was done by shifting the signal components of interest to zero by multiplying by a complex exponential $e^{i2\pi f_0 t}$, where $i = \sqrt{-1}$, $f_0$ is the frequency corresponding to the signal components of interest, and $t$ is time. Then the data were lowpass filtered using an finite impulse response (FIR) filter. For the lower frequency (50 and 75 Hz) data, filters with 0.25 Hz bandwidth were applied. The frequency response of such a filter is shown in figure B.2. For the higher frequencies (125 and 175 Hz), 0.50 Hz bandwidth FIR filters were used. Figure B.3 shows an example of narrow band data after basebanding.
Figure B.3. (a) Transmission Loss and (b) unwrapped phase for 50 Hz signal calculated by basebanding data from the third run of the along shelf track recorded on channel six of the Shark VLA.

B.5 Merge Acoustic Data with GPS Data

Ship location was monitored using four GPS receivers on the *R/V Endeavor* which each had a sampling rate of 1 Hz. The position of the source was calculated based on its relative location to these measurements. Latitude and longitude were converted to distance from the Shark VLA using a geodesic earth model [105]. Pressure data was merged with range data by interpolating the respective time vectors. To illustrate this process, the same transmission loss data plotted as a function of time in figure B.3 are plotted as a function of range in figure B.4. Figure B.4 is a mirror image of B.3 because the ship is moving toward the VLA.

B.6 Horizontal Wave Number Estimation

In order to apply the FFT to estimate the Green’s function, it is necessary to put the complex pressure data on an even range grid. This was accomplished
Figure B.4. a) Transmission Loss and (b) unwrapped phase for 50 Hz signal merged with GPS data from the third run of the along shelf track recorded on channel six of the Shark VLA.

by interpolating the magnitude and phase of the pressure signal separately. The range grid should be chosen fine enough to resolve the first wave number, which can be approximated from the upper bound of the discrete wave number spectrum

\[ k_w = \frac{\omega}{c_{w_{\text{min}}}}. \]

Using \( k_{r_{\text{max}}} = 2\pi/\Delta r \) where \( k_{r_{\text{max}}} \) is the highest resolvable wave number, the range grid discretization should be chosen so that \( \Delta r > \frac{c_{w_{\text{min}}}}{f} \) where \( f \) is the excitation frequency. The resolution of the wave number spectrum is limited by the range aperture, such that \( \Delta k_r = 2\pi/R \) where \( R \) is the range aperture.

Wave numbers estimated using the far field approximation of the Hankel transform (equation A.3b) for the 50 Hz data from the third run of the along shelf track recorded on channel six of the Shark VLA are shown in figure B.5. For this calculation, \( \Delta r \) was chosen to be 30 meters and the inherent wave number resolution was 0.0014 m\(^{-1}\). The spectrum plotted in the figure is zero padded by a factor of 25 to increase the resolution.
Figure B.5. Wave numbers estimated from 50 Hz pressure data from the third run of the along shelf track recorded on channel six of the Shark VLA.
Data Processing of CTD Chain Data from SW06

C.1 Introduction

The towed CTD chain\(^1\) was developed to address the need for a manageable instrument that measures the ocean structure in more detail than distributed vertical profiles [83]. The design of the chain makes use of inductive coupling of sensor fins to a coated steel wire to produce a mechanically simple chain of CTD sensors. This system allows for real time assessment of oceanographic measurements on board the tow ship.

C.2 System Description

The system architecture is illustrated in figure C.1. Sensor fins are mounted on the tow cable by a pair of plastic collars. The tow cable is connected to the deck unit at the upper end and to an electrode at the lower end. The current loop from the electrode to the deck unit is closed by sea water. The data set of every sensor is transmitted within a data cycle initiated by a deck unit. No battery is needed in any underwater unit; energy to drive the sensor electronics is supplied by the deck unit, which periodically switches between send and receive. A six foot, 250

\(^1\)ADM Elektronik (www.adm-elektronik.com)
Figure C.1. System architecture of the towed CTD chain. The sensor fins are mounted on the tow cable which is connected to the deck unit at the upper end and to an electrode at the lower end. The current loop from the electrode to the deck unit is closed by sea water.

lb. depressor\(^2\) attached to the end of the tow cable balances the lifting force caused by the chain’s resistance to flow.

The CTD chain utilizes molded sensor fins which can withstand water pressure at up to 1000 m depth, as well as the necessary rough handling on board a ship in heavy weather conditions. A single sensor fin with its labeled components is shown in figure C.2. The raw data returned from the sensors are 16 bit numbers. They are converted into conductivity, temperature, and pressure with calibration coefficients supplied by the manufacturer. Conductivity and pressure are converted to salinity and depth in post processing. These values are used to calculate sound speed by applying Mackenzie’s nine term sound speed equation [84].

C.3 System Performance

During the SW06 experiment, the sensor fins were mounted on the tow cable spaced three meters apart. Twenty-five sensors were in the water so that the ver-

\(^2\)Yellow Springs Instruments (YSI) type-167 Vehicles for Instrumentation (V-Fin)
tical sampling aperture was 75 meters. However, when the ship is moving, the drag pulls the chain upward so that the aperture is reduced to 50 to 65 meters, depending on ship speed. Sensors were queried every second for all three parameters: pressure, temperature, and conductivity. However, many of the sensors did not report consistently and some sensors did not work at all over the entire course of the experiment. Typically, there was data from less than 20% of the pressure sensors. The temperature and conductivity sensors reported more reliably, providing measurements about 65% of the time.

C.4 Data Interpolation

The missing pressure data was filled in using the known sensor spacing and fitting a cable shape the available data. A second-order polynomial curve fit was used to estimate cable shape. Examples of estimated cable shapes are shown for data from each of the three ship tracks in figure C.3. In the figure, pressure measured by the CTDs is shown by the red circles and the estimated cable shape is shown by the blue line with dots indicating the estimated sensor locations. The cable shape for the along shelf track shown in figure C.3(a), resulted from a ship speed of 8 knots (~4 m/s). The cable shape for the across and oblique shelf tracks shown in

Figure C.2. A single sensor fin with labeled components.
Pressure data recorded by the CTD chain at a single instance in time (red circles) and estimated cable shape (blue line with dots at estimated sensor locations) are shown for the (a) along shelf, (b) across shelf, and (c) oblique shelf tracks. The x-axis is distance behind the tow point.

Figures C.3(a) and (b), have a larger vertical aperture, a consequence of the slower tow speed of 4 knots (~2 m/s). In all three plots, the data show some outliers. Two potential causes for these outliers have been identified: erroneous calibration coefficients and displacement of sensors from their original positions on the cable. These errors contribute some ambiguity of the CTD chain measurements.

For temperature and conductivity data, missing values were filled in by vertically and horizontally interpolating between measured values to produce a continuous spatial measurement. When several consecutive sensors fail for an extended period, this procedure can cause aliasing of features normally resolvable by the chain’s configuration.
Bibliography


Vita
Megan S. Ballard

Education
B.S. Ocean Engineering, Florida Atlantic University, 2005.

Experience
Research Assistant, Pennsylvania State University, State College, Pennsylvania, August 2005-present.

Professional Activities
Underwater Acoustics Technical Committee Member, Student Committee Member, Acoustical Society of America, 2007-present.
Secretary, Central Pennsylvania Chapter of the Acoustical Society of America, 2006.
President, Society of Naval Architects and Marine Engineers, Florida Atlantic University, 2004-2005.

Honors & Awards
Simowitz Citation, Acoustics Program at Pennsylvania State University, April 2009.
National Defense Industrial Association Undersea Systems Division Fellowship Award, January 2006.
Kenneth R. Williams Leadership Award, Florida Atlantic University, May 2005.
Charles Stephan Award, Florida Atlantic University, May 2005.
Scholar for the College of Engineering, FAU Honors Convocation, Florida Atlantic University, April 2004.
Society of Naval Architects and Marine Engineers Scholarship, Florida Atlantic University, 2004.
University STEP Scholarship, National Science Foundation, Florida Atlantic University, 2002-2004.