SCALABILITY OF THE FORCE OF A NORMALLY IMPACTING VORTEX RING ON A PLANAR SURFACE

A Thesis in
Aerospace Engineering

by
Michael R. McErlean

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The thesis of Michael R. McErlean was reviewed and approved* by the following:

Michael H. Krane  
Research Associate, Applied Research Laboratory  
Thesis Co-Advisor

Arnold A. Fontaine  
Assistant Director, Applied Research Laboratory  
Thesis Co-Advisor

Cengiz Camci  
Professor of Aerospace Engineering

Dennis K. McLaughlin  
Professor of Aerospace Engineering

George A. Lesieutre  
Professor of Aerospace Engineering  
Department Head, Aerospace Engineering

*Signatures are on file in the Graduate School.
Abstract

This thesis addresses one of the issues in developing a non-lethal vortex ring weapon. Such a weapon would take advantage of the transfer of momentum from a vortex ring to a target and the encapsulation of fluid within the vorticity core. By transferring its momentum, a vortex ring can knock over or capsize a target in air and water while carrying a chemical agent such as tear gas or marker dye to the target. Vortex rings of sufficient strength to accomplish these goals have been generated previously, but the relation between the initial strength of the vortex ring and the force that would be imparted to a target is unknown.

This study hopes to provide the means for enabling a large scale, effective vortex ring weapon by deriving a scaling relation for the vortex strength and measuring the impact in a smaller, laboratory setting. The analytical expression for the impact force of a vortex ring in a normal impact on a planar surface is derived using dimensional analysis of the ring properties. The relation is experimentally verified by measuring the impact force for vortex rings for a range of initial vortex ring strengths. The normal impact of a vortex ring with a planar wall has been examined for the motion of the vortex on the wall and the vorticity generated. However, the force generated by a vortex ring on a wall has never been measured experimentally.

Experiments measuring the impact force, pressure on the surface, and motion of the vortex rings were conducted in a large glass tank full of quiescent water with the vortex rings generated by a piston-cylinder assembly. A planar wall was placed in the path of the vortex rings between 0.45 to 1 m from the nozzle. The planar surface was on a pivoting assembly with the force transducer anchoring the assembly above the water line. A pressure transducer was mounted flush with the wall. An in situ calibration was performed on the measurement system using an impact hammer to ensure an accurate measure of the vortex ring impact force. The time evolution of the force and pressure on the wall was measured for four different ring strengths and at three distances from the nozzle.
Concurrent to the force and pressure measurements, the motion of the vortex rings before and during impact was recorded with high-speed digital video camera (HSV). The high-speed images were analyzed to determine the speed and size of the rings and determine their changes during an impact. The flowfield of the vortex rings during an impact was measured with digital particle image velocimetry (DPIV) to determine the ring’s size, speed, and strength. The measurements of the vortex rings obtained using HSV and DPIV combined with the force and pressure measurements were used to confirm the underlying assumptions made in deriving the analytical expression for force scaling before being applied to the relation itself.
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1. Introduction

1.1 Motivation

This thesis addresses issues that arise in designing a non-lethal vortex ring weapon. A vortex ring from such a weapon would be used to either impart a force to a target or to deliver an incapacitating agent. The impact force of a vortex ring could be used in situations where the intent of the target is unknown or lethal force is unnecessary. Two examples are underwater harbor defense and crowd control on land. For harbor defense, the force of impact of a vortex ring weapon would disorient underwater divers and disrupt small watercraft. As a crowd control device, the vortex ring would target individual agitators and knock them down without hitting any bystanders. In both underwater and air applications, the vortex ring could also be used to transport chemical incapacitating agents or markers.

The applications of a vortex ring weapon mentioned above would require different amounts of force and this thesis focuses on the impact force delivered by a vortex ring to a target. In order to construct a non-lethal vortex ring weapon, the following two questions are treated:

1. What level of impact force a vortex ring will impart to a target in a normal impact on a planar surface?
2. How does the impact force depend on vortex ring strength and momentum?

The impact force of the vortex ring needs to be measured in order to determine if it is adequate for use in the proposed situations. The parameters of the vortex ring that will enable scaling of the impact force will be examined.

1.2 Background

1.2.1 Vortex Rings

Vortex rings are formed by ejecting fluid from a circular opening. Shear stresses along the walls before the exit cause vorticity to form in the boundary layer. Upon exiting the opening, a cylindrical sheet of vorticity forms and rolls up into a vortex ring (Didden, 1979) as shown in Figure 1.1. As more fluid is ejected, it continues to roll up into the vortex ring until the ring is large enough that the self-induced velocity from the vorticity within the core causes it to break away from the orifice.

The vortex ring is composed of a core containing the vorticity and the fluid within the vortex bubble. A Rankine vortex pair is shown in Figure 1.2 where the streamlines around the vortex are plotted for velocity vectors in the vortex frame of reference. The vortex bubble is the streamline that surrounds the core and is where the streamline along the centerline stagnates as shown as the black line in Figure 1.2 (Saffman, 1992). The fluid within the vortex bubble is comprised of both ejected fluid and entrained fluid (Krueger and Gharib, 2003). Ejected fluid is that which exited the opening and contains the vorticity. Entrained fluid is the ambient fluid outside that opening that was pulled into the ring as it formed. The entrained and ejected fluids compose the total mass of the vortex ring.

1.2.2 Vortex Ring Generation

Vortex rings can be easily generated by pushing a slug of fluid through a nozzle. The shape of the nozzle affects the ring formation and there are two characteristic types: an
orifice or a tube. The orifice can either be an opening in a wall or a small opening in a larger pipe shown in Figure 1.3(a) and (b). Both orifice types have a wall present on the outside that the tube does not have in Figure 1.3(c). Even though the internal geometry of the orifice in Figure 1.3(b) and the pipe in Figure 1.3(c) are the same, the presence of the wall outside of the opening will cause differences in the vortex rings. Didden (1979) concluded that the wall causes lower circulation in the rings. The numerical simulations
of James and Madnia (1996) and Rosenfeld et al. (1998) found that the total circulation in the flow stays the same, but the circulation of the ring is indeed lower. As the ring is forming, a secondary vortex of opposite sign from the primary vortex ring is formed along the walls. The secondary vortex lowers the circulation of the primary ring and the impulse in the field. As the ring moves further from the wall the effect of the secondary vortex diminishes and the impulse in the field increases.

The circulation of a vortex ring is a measure of the strength of the ring and can be predicted by the slug model. The slug model assumes that the boundary layer along the wall is thin so that velocity profile across the opening is uniform (Shariff and Leonard, 1992). The vorticity flux across the \((x, y)\) plane of the opening is

\[
\frac{d\Gamma}{dt} = \int \omega \phi u dy \approx \int -\frac{\partial u}{\partial y} u_x dy \approx \frac{1}{2} U_p^2(t) \tag{1.1}
\]

where \(\omega \phi\) is the azimuthal component of vorticity, \(u\) is the axial velocity component, and \(U_p\) is the piston speed. For a specified piston velocity \(U_p(t)\) moving during a time \(t_p\), a jet of length \(L\), defined as

\[
L = \int_0^{t_p} U_p(t) dt, \tag{1.2}
\]
is ejected. Integrating Equation 1.1 during the piston motion yields

\[ \Gamma = \frac{1}{2} \int_{0}^{t_p} U_p^2(t) \, dt. \]  

(1.3)

Substituting Equation 1.2 into 1.3 gives

\[ \Gamma = \frac{1}{2} L U_p \]  

(1.4)

which is the slug model giving the vortex ring circulation in terms of the piston velocity, \( U_p \), and fluid length, \( L \) (Glezer, 1988).

While simple, the slug model does not accurately predict the circulation of a vortex ring. Many studies performed by Didden (1979), James and Madnia (1996), Weigand and Gharib (1997), Gharib et al. (1998), Rosenfeld et al. (1998) and others have found that the slug model under-predicts the circulation of the vortex ring by as much as 40%. The discrepancies result from a non-uniform velocity profile across the exit, the presence of a \( \frac{\partial v}{\partial x} \) contribution to the vortex flux, and the type of nozzle geometry. The boundary layer inside the nozzle contributes to the non-uniform velocity profile while over-pressure outside the nozzle exit contributes to both the velocity profile and the \( \frac{\partial v}{\partial x} \) portion of the vortex flux (Krueger, 2005).

The slug model in Equation 1.4 implies that the circulation of vortex rings can be increased infinitely as long as more fluid is ejected at a higher velocity. However, Gharib et al. (1998) found a limit, named the formation number, at which the rings attain their maximum circulation for a given set of generation conditions. The formation number is comprised of the piston stroke length \( L \), which in their model is the same as the length of the fluid column ejected, and the piston diameter \( D \), which is the same as the nozzle diameter. Gharib et al. (1998) found that at \( L/D \gtrsim 4 \) the excess fluid ejected was not entrained into the vortex ring and was not contributing to the ring circulation. Instead, the left over fluid would form a trailing jet behind the ring. As the formation number increased beyond \( L/D \approx 4 \), the trailing jet would overtake and destabilize the vortex
The existence of the formation number is due to the Kelvin-Benjamin variational principle where a vortex ring must have the maximum energy (Kelvin, 1875). The vortex ring separates from the trailing jet at pinch-off when the generator is no longer able to deliver energy greater than the energy of the forming vortex ring (Mohseni and Gharib, 1998). The kinematic model of Shusser and Gharib (2000) shows that the ring pinches off when the velocity of the steady translating vortex ring due to its vorticity is equal to the piston velocity. Both the numerical simulations of Rosenfeld et al. (1998) and the experimental measurements of Gharib et al. (1998) found that the orifice geometries have a lower formation number than the tube geometries due to the lowered circulation of the rings.

Because the formation number depends on the energy of the piston relative to the energy of the ring, two methods to form rings with a higher formation number are to increase the piston energy or decrease the vortex ring energy during formation. The time variations of most piston velocity programs are impulsive or trapezoidal, so that the piston energy increases at a slower rate than the energy of the vortex ring. Krueger and Gharib (2003) manipulated the piston energy by using the negative sloping and positive sloping velocity programs shown in Figure 1.4. The negative sloping programs had a fast acceleration to maximum velocity and slow deceleration whereas the positive sloping had the opposite. The positive sloping velocity profiles had an increase in piston energy during the vortex ring formation and shifted vortex ring pinch off by +1 L/D compared to the negative sloping profiles. This was numerically modeled by Rosenfeld et al. (1998) and derived analytically by Shusser et al. (2006) with similar results. Instead of changing the piston velocity program, Dabiri and Gharib (2005) adjusted the nozzle area during formation. A decrease in nozzle area during formation increases the energy delivered by the piston due to an increase in jet ejection speed to give higher formation numbers.

The other method to alter the formation number is to decrease the ring energy.
Krueger et al. (2003) measured the formation number when there was a uniform flow around the nozzle moving in the same direction as the ejected slug. The co-flow decreases the vorticity in the shear layer and convects the ring away from the nozzle. This allows less time for the ring to form which had the effect of increasing the ring energy, so, as the co-flow speed increased, the formation number decreased. Dabiri and Gharib (2004) used the same setup as Krueger et al. (2003) but reversed the direction of the flow around the nozzle so that it created a uniform velocity against the slug ejection. The counter-flow results in a decrease in vortex ring energy which delays pinch-off. The vortex ring is unable to convect away from the nozzle so more fluid is entrained into the ring increasing its circulation. The excess circulation was not immediately shed and instead showed a slower decay due to viscosity than rings where there was no counter-flow.

Once a vortex ring is formed, it moves under its own induced velocity from the circulation in the cores. Thus the propagation speed of a vortex ring is related to the ring strength and size. The speed of a thin core ring with a uniform vorticity is given
by Kelvin (1867) as

\[ U_R = \frac{\Gamma}{4\pi R} \left[ \log \frac{4R}{d_c} - \frac{1}{4} \right] \]  

(1.5)

where \( \Gamma \) is the circulation, \( R \) is half the distance between ring cores, and \( d_c \) is the core diameter. Even for a viscous vortex core with a Gaussian distribution of vorticity, the ring speed maintains the \( \Gamma \) and \( R \) relationship in the equation derived by Saffman (1992):

\[ U_v(t) = \frac{\Gamma}{4\pi R} \left[ \log \frac{4}{\tau} - 0.558 + O(\tau \log \tau) \right] \]  

(1.6)

where

\[ \tau = \frac{\sqrt{8\nu t}}{R}. \]  

(1.7)

Viscosity causes the speed of the ring to slow down over time as the core diameter grows, but the general relationship between \( U_R, \Gamma, \) and \( D_R \) still exists.

1.2.3 Vortex Ring Impact

When a vortex ring approaches a planar surface it undergoes three phases of motion. The first is a free-traveling phase where it is unaffected by the wall. The second is a vortex stretching and slowing phase as it interacts with the wall. The third is rebound off the wall. The three phases can be distinguished by the entrosphy, or vortex potential energy, of the ring (Chu et al., 1993) and the stretching phase starts around one ring diameter from the surface (Walker et al., 1987). The stretching and rebound are caused by viscous effects producing vorticity on the boundary layer of the plate. As the vortex ring gets closer to the surface, the induced vorticity is outside the radius of the ring, stretching it, until a secondary vortex forms causing the ring to rebound (Lim et al., 1991).

The motion of a vortex ring has been studied by several groups, but rarely has the pressure or force on the wall been looked at. Chu et al. (1995) measured the wall pressure at a single point during a vortex ring interacting with a planar wall and ran a numerical
Figure 1.5. Figure 7(b) from Chu et al. (1995) showing the change in stagnation point pressure over time. The experimental results are the markers and the simulations are the lines.

simulation of a single vortex, not a vortex ring, interacting with a wall. They located their pressure measurements to be in the center of the ring where the stagnation streamline occurred. The experimental and numerical results for the normalized stagnation point pressure are shown in Figure 1.5. The pressures are normalized by their maximum values. The experimentally measured rings were \( Re_d = 830 \) and 1000 and the numerical simulation used the equivalent of \( Re = 500 \) and 1500. Measuring the vortex ring speed and diameter at a specified distance from the nozzle, they found the experimental Reynolds number, \( Re_d = \frac{U d}{\nu} \), where \( U \) is the translational speed of the vortex ring and \( d \) is the diameter. For the numerical studies, the Reynolds number was determined from the initial vorticity distribution, \( Re = \frac{\Gamma_0}{\nu} \), to match the experimental vortex rings. As the vortex ring nears the wall, the stagnation point pressure increases. They found that the pressure reaches its maximum when the vortex ring is in its early stages of stretching. The experimental measures and the numerical simulation agreed on the time scale of the pressure increase and the shape, but differed after the maximum was reached. The vortex rings had a much slower decline in wall pressure than the simulation due to three dimensional effects the simulation did not account for.

To get a radial distribution of pressure along the wall, Chu et al. (1995) integrated
Figure 1.6. Figure 7(a) from Chu et al. (1995) showing the radial surface pressure distribution for the two numerical simulations. \( r = 1 \) corresponds to the initial position of the vortex.

The momentum equation to obtain

\[
p|_{z=0} = \int_{0}^{\infty} \left( \frac{\partial u_z}{\partial t} - r u_r \omega + \frac{1}{Re} \frac{1}{r} \frac{\partial}{\partial r} (r \omega) \right) dz. \tag{1.8}
\]

and applied Equation 1.8 to their numerical results for a single vortex. The radial pressure distributions along the wall from Chu et al. (1995) are shown in Figure 1.6. The vortex starts at \( r = 1 \), and as it approaches the wall, the vortex moves outward as shown in Figure 1.7. The minimum surface pressure is always located downstream of the center of the primary vortex. The variation in pressure between \( 1 < r < 2 \) is due to boundary layer separation and the formation of the secondary vortices seen in Figure 1.7. The surface pressure was similar to the changes in the surface signature of a vortex ring impinging on a normal free surface from Chu et al. (1993) shown in Figure 1.8. Vortex rings were fired upward toward the top of the tank and the surface deformation was measured using high speed video.

The pressure in the whole field of a vortex ring impacting a wall was calculated by Naguib and Koochesfahani (2004) by measuring the velocity field using molecular tagging velocimetry and applying the solution to Poisson’s equation. Naguib and Koochesfahani (2004) used slightly stronger rings than Chu et al. (1995) with \( Re_d = 1860 \) or \( Re = 4500 \).

The radial distribution of the coefficient of pressure \( C_p = \frac{p - p_{\infty}}{0.5 \rho U_0^2} \), where \( p_{\infty} \) is the pressure
Figure 1.7. Figure 4 from Chu et al. (1995) showing vortex location and vorticity contours.
Figure 1.8. Figure 7 from Chu et al. (1993) showing the deformation of a free surface when a vortex ring of $Re_d = 1000$ approaches. $x$ is the radial distance from the ring centerline.

far from the wall and $U_0$ is the initial convection speed of the vortex ring, is shown in Figure 1.9. The non-dimensional radial distance, $r^* = r/D_0$ where $r$ is the distance from the centerline and $D_0$ is the initial diameter of the vortex ring. An $r^* = 0.5$ in Figure
1.9 corresponds to $r = 1$ in Figure 1.6. Naguib and Koochesfahani (2004) found that the minimum pressure occurred at the radial location of the core and the overall maximum pressure was along the stagnation point streamline. However, once regions of positive pressure develop outside the primary vortex, the waviness in the wall pressure observed by Chu et al. (1995) is present. Naguib and Koochesfahani (2004) do not report the pressure at the stagnation point over time, but the trend during the interaction in the time frame reported in Figure 1.9 can be found. The $C_p$ at $r^* = 0$, shows that the pressure is at its maximum when the core is located at $r^* = 0.58$, or the beginning of the vortex stretching phase. The pressure at the stagnation point decreases in the next two intervals which is similar to the behavior shown in Figure 1.5 at $t = 12, 16, 20$.

In addition to computing the radial pressure distribution of the impact of a vortex ring, Chu et al. (1995) derived an expression for the surface force

$$F = -\int 2\pi u_r \omega dr dz. \quad (1.9)$$

Using the pressure distribution along the wall from their numerical study, the results were integrated to determine the force shown in Figure 1.10. The overall force is small, and Chu et al. (1995) found that there were many large positive and negative contributions shown in Figure 1.11. The primary and secondary vortices seen in Figure 1.7 both cause positive and negative forces on the wall but the primary vortex had a net positive force while the secondary vortex had a net negative force of similar magnitude. The resulting total force is small due to the positive and negative forces canceling each other out.

### 1.2.4 Vortex Ring Weapon

Lucey and Jasper (1998) proposed that a non-lethal vortex ring weapon would take advantage of the ability of a vortex ring to transfer momentum to a target at range and to encapsulate an agent that would be released on impact. The vortex rings would be used to hit single targets with impact and concussion pulses near the human resonance
Figure 1.9. Figure 9 from Naguib and Koochesfahani (2004) showing the pressure field and pressure distribution along the surface at different times during a vortex ring impact with a planar wall. Re = 4500.
Figure 1.10. Figure 8 from Chu et al. (1995) showing the overall surface force integrated from the surface pressure.

Figure 1.11. Figure 10 from Chu et al. (1995) showing the force contributions to the total force. $F_{pp}$ is positive contribution, $F_{pn}$ is the negative contribution, and $F_p$ is the total force of the primary vortex ring. $F_{ip}$ is the positive contribution, $F_{in}$ is the negative contribution, and $F_i$ is the total force of the induced flow.

frequency (3-15 Hz) that would cause physical discomfort if the person did not leave the area. Or the ring would also carry skunk perfume to a target to similarly force people to leave the area. The numerically predicted impact force of a vortex ring by Chu et al. (1995) would suggest that vortex rings would be unsuitable for use as a weapon due to the impractically small force they impart. However, Lucey (2000) tested a full size vortex
ring weapon that utilized a 40-mm grenade launcher fitted with a supersonic nozzle. The generation impulse was provided by a practice grenade round containing only propellant rather than a piston like the laboratory studies.

The vortex rings generated by the 40-mm grenade launcher were 2 ft in diameter and moving at 160 ft/s at a distance of 120 ft downrange of the generator. Lucey (2000) did not measure the initial velocity of the vortex rings or relation of the supersonic impulse to the ring velocity, so the initial Reynolds number of the rings is unknown. At the downstream location the vortex rings have a \( Re_d \approx 2.0 \times 10^6 \) based on the data provided.

In order to measure the kinetic energy of the high Reynolds number rings, Lucey (2000) measured the vortex spin, vortex momentum, and impact strength. The spin was measured by the acoustic signature of the ring as it propagated downstream. The momentum was measured by placing a target of varying weights in the path of the vortex ring and determining the maximum weight that the ring could knock over. The vortex rings knocked over a 150 lb mannequin at 30 ft, a 125 lb mannequin at 40 ft, and a 75 lb mannequin at 50 ft. The impact strength was measured by placing a grid of clothespins in the ring path and measuring how far they were thrown after being dislodged. The best displacement achieved was 80 ft by the clothespins.

The work of Lucey and Jasper (1998) and Lucey (2000) was focused on designing and optimizing the supersonic generation of the vortex rings. They did not make any measurements of the ring properties that govern the vortex ring interaction with the targets.

1.3 Objectives

The pressure and force studies of Chu et al. (1995) and Naguib and Koochesfahani (2004) used low Reynolds number rings that are unsuitable as a weapon while Lucey (2000) showed that a high Reynolds number ring is capable of imparting a sizable force.
This thesis seeks to apply the accurate vortex ring and wall measurement methodology of Chu et al. (1995) and Naguib and Koochesfahani (2004) to strong high Reynolds number rings of the type used by Lucey (2000). First, an analytical expression for the magnitude of the impact force in terms of the ring size and strength is developed. Then, the full ring motion of high Reynolds number rings ($Re = 10^4 - 10^5$) is measured and compared to the impact force and pressure measured on the wall.
Scaling Law for Vortex Ring Impact

2.1 Problem Statement

The normal impact of a vortex ring with a planar surface is illustrated in Figure 2.1. The vortex ring is shown as the blue sphere surrounding the green core of vorticity. The core has a circulation of $\Gamma$ and is moving at a self induced speed of $U_R$. The ring diameter, $D_R$, is the distance between the core centers through a slice of the ring as shown. As the vortex ring approaches the surface, the ring flattens as it expands. The change in momentum of the ring is caused by the presence of the wall and next section derives an expression for the force, $F(t)$, the ring exerts on the wall.

2.2 Derivation of Scaling Law

An expression for the impact force of the vortex ring on a planar wall is derived by examining the change in kinetic energy of the mass of fluid contained within the ring. A schematic of the problem is shown in Figure 2.1. A simplified dimensional analysis is used rather than a full analytical expression in order to determine the ring parameters on which the impact force is dependent. Once the scaling relation is established, the impact force will be measured and compared to the prediction.
Beginning with Newton’s Second Law of Motion for the volume of fluid:

\[ F = \rho V_R \frac{\partial U_R}{\partial t}. \] (2.1)

where \( \rho \) is the density of the fluid, \( V_R \) is the volume, and \( U_R \) is the speed of the volume of fluid. For a vortex ring, \( V_R \) is all the fluid contained within the vortex bubble shown in blue in Figure 2.1 and \( U_R \) is the axial speed of the vortex ring.

The ring diameter, \( D_R \), is the distance between the core center in a two dimensional slice of a vortex ring. The vortex bubble is approximately spherical and on the same order of magnitude as \( D_R \), so the volume of the fluid contained in the vortex ring is estimated as

\[ V_R = \frac{4}{3} \pi R^3 \propto R^3. \] (2.2)

where \( R = D_R/2 \) and the constants have also been removed.

From Equation 1.5 and 1.6, \( U_R \) has the same relation to \( \Gamma \) and \( R \) regardless of the vorticity distribution. The bracketed terms from Equation 1.5 are condensed into a
constant to give

\[ U_R = \frac{C \Gamma}{R} \propto \frac{\Gamma}{R}. \]

(2.3)

As the ring approaches the plate, its speed will slow down. However, Equation 2.3 does not explicitly include the change, so a suitable time scale is estimated. If the ring only consisted of the sphere of fluid in the absence of any vorticity, the time it would take that sphere to pass a point is on the same order of magnitude as it would take the sphere to be flattened against a surface, so the impact time, \( t_{imp} \), is

\[ t_{imp} = \frac{D_R}{U_R} = \frac{2R}{U_R} \propto \frac{R}{U_R}. \]

(2.4)

And substituting the relation for the ring speed from Equation 2.3 into Equation 2.4 gives

\[ t \approx t_{imp} \propto R \frac{\Gamma}{\Gamma} \propto \frac{R^2}{\Gamma}. \]

(2.5)

Combining Equations 2.2, 2.3, and 2.5 into Equation 2.1 yields

\[ F \propto \rho R^3 \frac{\Gamma}{R^2} \propto \rho \Gamma^2. \]

(2.6)

For any ring the impact force would scale as the circulation squared multiplied by the density of the fluid.

### 2.3 Approach

In order to determine the validity of the scaling law in Equation 2.6, measurements of a vortex ring interacting in a normal impact with a planar surface were taken. The existing vortex ring facility in the Applied Research Laboratory at the Pennsylvania State University was used with some modifications. The facility was designed to generate long persistence, high energy vortex rings. The generator can produce vortex rings with a Reynolds number over 100,000 that travel up to 100 ring diameters before impacting the
far wall of the tank.

The following chapters detail the experiment designed to measure the necessary quantities in the scaling law as well as those used in the steps in deriving the law. Measurements of the vortex ring before and during an impact were taken as well the effects on the planar surface during the same time period. The ring motion is captured using digital High-speed Video (HSV) and the instantaneous fluid velocity by Digital Particle Image Velocimetry (DPIV). Processing of the images will allow extraction of the ring radius, velocity, and circulation before and during the impact. The impact force and wall pressure at the stagnation point will be directly measured by placing a planar surface connected to a force sensor in the path of the vortex rings with a pressure sensor mounted in the surface.

Chapter 3 describes the modifications made to the vortex ring facility as well as the calibration and measurement procedures for all the instruments. The cases studied and the results of the HSV and DPIV measurements with the corresponding impact force and surface pressure are presented in Chapter 4. A discussion of the results obtained and comparisons of the force and pressure to previous work are also included. The final conclusions are in the last chapter with suggestions on how to expand upon the work presented.
Methodology

3.1 Experimental Setup

The existing vortex ring facility in the Applied Research Laboratory at the Pennsylvania State University was modified to be able to measure the impact force and pressure of a ring on a vertical plate while capturing DPIV and high-speed video of the ring before and during impact. The facility consisted of a tank, a vortex ring generator, and a moveable instrument gantry system as shown in Figure 3.1. The glass tank measured 274 cm long by 91 cm wide and by 91 cm tall and was filled with water. The vortex ring generator was a piston located outside one end of the tank with only the nozzle and some pipe inside protruding through a gasket into the tank. Attached to the vortex ring generator were a linear encoder and an accelerometer to characterize the piston motion. A digital high-speed video camera and a DPIV camera were mounted on the instrument rail outside the tank to record the ring motion. The impact plate was suspended from the instrument support gantry into the tank and fitted with a force and pressure sensor to measure the ring impact.
3.1.1 Vortex Ring Generator

The vortex ring generator was a piston-cylinder design shown in Figure 3.2. The cylinder was constructed of nylon 23 cm long with a total piston traverse range of 20.3 cm. The piston was constructed of several pieces of acrylic bolted to the shaft. A rubber O-ring and a Teflon O-ring sealed the piston against the cylinder. The generator sat outside the tank and an anodized aluminum pipe connected the cylinder to the tank. The inner diameter of the cylinder was 62.7 mm and the pipe was shaped to reduce the inner diameter to 31.75 mm before entering the tank. The aluminum pipe extended 15 cm into the tank and was threaded on the end to allow different nozzles to be attached.

The piston was driven by a linear electromagnetic motor attached to the shaft with a Kollmorgen SERVOSTAR CD motor controller. The motor controller used feedback provided by an RSF Electronics MSA 6706 linear encoder mounted on top of the electromagnetic rail and connected to the piston shaft. A control program written in MATLAB
Figure 3.3. MATLAB interface of Runservo control program which controlled the piston velocity, acceleration, deceleration, and increment settings as well as recording the feedback from the linear encoder.

Interfaced with the motor controller using an RS-232 serial connection. The program allowed the user to input the piston velocity, acceleration, deceleration, and increment distance as shown in Figure 3.3. Other options included setting a pulse generator delay time for use in the synchronization described in Section 3.1.7 and saving the waveforms measured by the linear encoder.

The nozzle used for this study is shown in 3.4. The nozzle was designed in two sections. The inner section attached to the aluminum pipe and the outer section screwed onto the inner section. The outer section reduced the inner diameter from the pipe diameter of 31.75 mm to 25.4 mm at the exit. A gap before the contraction was left for the injection of visualization elements. The jet was fed by a reservoir between the two nozzle sections and a porous resistance element provided a uniform jet velocity around
Figure 3.4. Schematic of the dye-injection nozzle. The dye reservoir and jet injection are created by the interstice between the inner (tan) portion and the outer (gray) portion of the nozzle.

the circumference of the nozzle. An O-ring sealed the reservoir from the surrounding external fluid.

The jet injection system is illustrated in Figure 3.5. A tank pressurized to 6 psi containing either a water/fluorescein dye mixture or a water/PIV seed mixture was attached to the nozzle reservoir through four taps. A solenoid valve was used to control the injection. When the valve was open, the mixture would be uniformly forced into the nozzle because of the porous resistance element. When the valve was closed, the porous resistance element prevented the mixture from leaking out of the reservoir.

The face of the nozzle was flat, and, to provide a continuous surface beyond the extent of the nozzle, a sheet of clear plastic was fitted around it. The sheet was 61 cm wide by 100 cm tall and was attached to the overhanging instrument gantry.

Additional measurements of the piston were made using an accelerometer mounted to the end of the piston shaft. These were separate from the linear encoder and used in the synchronization described in Section 3.1.7 and the processing described in Section 3.2.
3.1.2 Instrument Support Gantry

The tank was all glass and had no supports or mounting points, so an external instrument support gantry was used for the cameras and sensors. The gantry consisted of two rails on either side of the tank and the support structure was built on top of them as shown in Figure 3.6. The whole structure could slide back and forth on the rails. The gantry supported breadboards on either side of the tank that the high-speed or DPIV cameras were mounted on, as well as the force impact assembly described in Section 3.1.3 and the nozzle baffle. To reduce movement in the gantry from room vibrations or piston motion, it was diagonally braced to the rails and to the building support beams.

Two different support gantries were constructed during the course of the study. Both are shown in Figure 3.6. The original gantry was about 65 cm wide and needed to be moved along the rails whenever the impact assembly needed to be placed further away from the nozzle. The second version incorporated the original gantry and extended it to be 2/3 the length of the tank. A wider range of locations could be chosen for the impact plate assembly without having to reconfigure everything. The second gantry was also stiffer than the first and permitted better force measurements of the ring impact. The first gantry was used for all the high-speed video measurements and the second gantry
Figure 3.6. Instrument support gantry. Original gantry is shown in blue and held the impact plate assembly, the nozzle baffle (not shown), and the camera breadboards. The extension is shown in red and only supported the impact plate assembly.

was used for all the DPIV measurements.

3.1.3 Impact Plate Assembly

The impact plate assembly consisted of a plate, a frame to suspend it into the tank from the overhanging support gantry, a force sensor, and a pressure sensor as shown in Figure 3.7. The impact plate was a 20 cm wide by 25 cm tall piece of acrylic stiffened by two crossing acrylic strips cemented to the back. At the center of the cross, a PCB 105M147 pressure sensor was mounted with its tip flush with the plate. The impact plate was attached to a lightweight aluminum frame that was connected to a hinge above the waterline and anchored by a PCB 208C01 force sensor as shown in Figure 3.8. The impact assembly was designed so that when a vortex ring hit the plate, the whole assembly would pivot about the hinge and created a mechanical gain on the measured force. The center of the plate was 53 cm below the hinge and the force sensor was 6.5 cm above the hinge which resulted in a mechanical gain of 8. The impact assembly was
Figure 3.7. Plate assembly showing the location of the impact plate, pressure sensor, and force sensor.

Figure 3.8. Detail of impact assembly connection showing the force sensor, hinge, and bracing, positioned vertically so that the center of the plate with the pressure sensor was along the centerline of the nozzle.
3.1.4 WaveBook Data Acquisition

The accelerometer on the piston and the force and pressure sensors on the impact plate assembly were connected to a WaveBook Data Acquisition System. The accelerometer and pressure sensor were powered by a PCB 480B21 three channel signal conditioner and the force sensor was powered by a PCB 280E09 signal conditioner. During the first set of measurements with the high-speed video, a 12-bit IOtech WaveBook 512 system was used with an external bank of Krohn-Hite 3916 anti-aliasing filters. When the second set of measurements were made with DPIV of the rings, a 16-bit IOtech WaveBook-516 with a built in WBK-13A low-pass filter card was used. The data acquisition was externally triggered by a pulse generator described in Section 3.1.7.

3.1.5 High-speed Digital Video

The motion of the vortex rings before and during impact with the plate was recorded with a TSI PIV00354 PowerView digital B&W camera with a resolution of 1280 by 1024 pixels. The camera was mounted on a vertical traverse stage on one of the breadboards outside of the tank. A Sigma 28-mm lens created a field of view of approximately 40 cm giving a spatial resolution of 0.3 mm/pixel. The camera frame rate, exposure, digital gain, intensity offset, and number of frames recorded was controlled by EPIX X-CAP.

The vortex rings were visualized by injecting a fluorescein dye mixture into the nozzle prior to piston motion. The dye was injected 5 s before the piston initiation to prevent interference in the formation of the vortex ring. The tank was illuminated by a halogen lamp positioned at the end of the tank opposite the piston. The clear acrylic of the impact plate allowed the light to shine through. The tank was shrouded with dark cloth to give a high contrast to the rings on the camera images. The HSV camera system was externally triggered by a pulse generator described in Section 3.1.7.
3.1.6 Digital Particle Image Velocimetry

The DPIV system consisted of a New Wave Research Gemini Nd:YAG pulsed laser and a TSI PIV-CAM 13-8 digital PIV camera with a resolution of 1280 by 1024 pixels. The laser was positioned on an angled table above the vortex generator and mounted on a traverse stage. The traverse stage allowed alignment of the laser with the center of the impact plate. The camera was mounted on a vertical traverse stage on one of the breadboards to the side of the tank. A Nikon Nikkor 28-mm lens provided a field of view of approximately 15 cm wide with an image resolution of 0.12 mm/pixel.

A second harmonic generator on the Nd:YAG laser doubled the frequency to give 532 nm light that was focused using a cylindrical lens to give a vertical sheet of light. The laser shone through the clear plastic baffle around the nozzle. A 532 nm filter was placed over the camera lens to block out other light sources.

Control for the PIV camera and laser was provided by a computer running Insight 3G and a LaserPulse 610034 Synchronizer. The PIV system was triggered externally by a pulse generator described in Section 3.1.7.

3.1.7 Experimental Synchronization and Control

The vortex ring generator, WaveBook DAQ system, high-speed digital video camera, and the DPIV system were all independent. For synchronization between all the components, two BNC pulse generators were used. The vortex ring generator could only be triggered by the computer running the MATLAB control program, so that computer was used to trigger all the other devices.

When the high-speed video was being recorded, the control setup shown in Figure 3.9 was used. The RS-232 connection to the piston motor controller carried the initiation signal as well as returning the waveform data from the linear encoder. A two channel BNC-565 pulse generator was connected to the computer via USB. One channel of the pulse generator was connected to the solenoid valve controlling the dye injection, the other went to the EXT/IN of an eight channel BNC-555 pulse generator. One channel
from the BNC-555 generator was connected to the external trigger of WaveBook and the other to a TTL to serial converter needed to trigger the high-speed camera.

The dye needed to be injected before the piston initiation. A time was entered into the Pulse Generator Delay field in the MATLAB control program seen in Figure 3.3. This did not control the delay in triggering the BNC generator through the USB. Instead, it controlled the delay to piston initiation. When the piston was given the command to move, the BNC-565 generator was triggered, then the piston was initiated after a delay $\Delta t_1$. The solenoid valve was triggered the same time as the BNC-565 generator, meanwhile there was a delay before the BNC-555 generator was triggered. The WaveBook DAQ and the high-speed camera were triggered after a delay $\Delta t_2$ which was set the same as $\Delta t_1$ (5000 ms) so that everything started recording at piston initiation. Any small discrepancies between the start of recording and piston initiation were measured by the accelerometer and corrected in the processing described in Section 3.2.9.

Though the nozzle was capable of injecting PIV seed, the feature was unused during the DPIV measurements. The control system was simplified as shown in Figure 3.10. The trigger delay for the piston was removed and the BNC-555 generator was no longer needed. WaveBook was set to record simultaneous to piston initiation on no delay, while the DPIV synchronizer was on a delay $\Delta t$. This delay was adjusted to capture the vortex rings in different locations of impact and the different times used are listed in Section 4.2.2.

3.2 Measurement and Calibration Procedures

3.2.1 Piston Waveform

The piston waveform was measured by the linear encoder. The MATLAB piston control program samples the position and velocity from the linear encoder at 1000 Hz with 1024 samples and it is saved after the piston finishes moving. The MATLAB program calculates the acceleration of the piston based on the measured velocity and
Figure 3.10. Experimental control for DPIV measurements.

The sample rate $\Delta t$ and displays the position, velocity, and acceleration of the piston on the three graphs in Figure 3.3.

The flow conditions at the nozzle were needed to compare the waveforms to previous studies and the modeling equations, but the encoder measured the piston waveform. Both the jet velocity of the fluid, $U_j$, and the length of the slug ejected from the nozzle, $L$, can be calculated from the piston velocity, $U_p$. The fluid was incompressible, the
volumetric flow rate was constant, and the velocity of the fluid at the nozzle exit is related to the velocity of the piston by the ratio of their areas:

\[ U_j = \frac{D_p^2 U_p}{D^2} = \frac{63.7^2}{25.4^2} U_p = 6.0935 U_p. \] (3.1)

Instead of converting the piston stroke length to the equivalent \( L \), \( L \) was found from integrating the jet velocity:

\[ L = \int_0^t U_j dt. \] (3.2)

The calculations for \( U_j \) and \( L \) were performed using a MATLAB code \textit{Pistonwaveforms.m} described in Appendix A.1 which was separate from the piston control program. The program created a waveform for each realization and computed an average waveform from the mean \( U_j \) and \( L \). In creating the average waveform, the program accounted for any changes in the piston initiation delay. The maximum jet velocity, \( U_{j,\text{max}} \), the stroke ratio of the maximum velocity, \( (L/D)_{\text{max}} \), and the total ejected fluid length, \( L \) were saved for each realization. Then the mean, standard deviation, minimum, and maximum were calculated for each quantity.

### 3.2.2 High-speed Digital Video Calibration

In order to calculate the ring motion recorded by the high-speed video camera, the images needed to be calibrated and corrected for any distortion. The calibration found the resolution of the image in mm/pixels, while the distortion correction accounted for the camera not being perpendicular to the ring’s plane of travel. Both the calibration and correction are done using a MATLAB code \textit{Imagesequencecalibration.m} described in Appendix A.2.

First, a single image of a reference target was taken as seen in Figure 3.11a. The target had white dots a known distance apart. The four points in the corners marked by an (*) were selected to create the calibration and transform matrix. The image was transformed to give the one shown in Figure 3.11b. The transformed image’s real
3.2.3 High-speed Digital Video Processing

The high-speed video sequences were saved as individual images and were analyzed using the MATLAB code *Imagesequenceprocessor.m* which is described more in Appendix A.3. The calibration information and transformation matrix from *Imagesequence-calibration.m* was imported and used on the images before the vortex ring was isolated from the rest of the image.

The first step in isolating the vortex ring was the subtraction of a background image. The background image was taken from the beginning of the image sequence before the ring had entered the frame and is shown in Figure 3.12a. A frame showing the vortex ring just before impact is shown in Figure 3.12b. After the background subtraction the result is shown in Figure 3.12c. Most of the non-ring elements in the image were removed.

Figure 3.11. Example of image distortion correction. a) Original image. b) Transformed image. The points denoted as starts are the reference points selected in the original image and their location after the transformation.
by this process, but the dye left in the wake of the ring is still visible.

Next, the image was converted into a black and white binary image shown in Figure 3.12d. The pixel intensity threshold for the conversion was determined using a built-in MATLAB algorithm named that minimized variance of the black and white pixels. The vortex ring was separated from the dye left in the wake except for a few small spots. A median averaging filter was applied to the image to remove the spots and smooth the ring boundary with the result seen in Figure 3.12e.

To determine which of the remaining white areas was the vortex ring, each region was checked for size and intensity. Most of the time, the ring was the largest region, but if there were multiple large regions due to excess dye in the wake, the ring was the one with the highest average intensity. The boundary of the region identified as the vortex ring is marked with a red line in Figure 3.12f.

Once located, the size, position, and speed of the vortex ring could be calculated. The shape of the ring was approximated as an ellipse. Then the major axis, minor axis, and centroid of the ellipse were determined by finding the points marked in Figure 3.13. The major axis of the ellipse was the measure of the ring diameter while the minor axis was the not through the center but closer to the ends. Figure fig:imageprocess shows that excess dye near the center of the ring can cause it to be fatter than at the top and bottom. To get a better estimate of the core diameter, a distance 10% of the ring diameter in the ends was chosen as the minor axis measure. The approximate core centroid coordinates \((x_1, y_1)\) and \((x_2, y_2)\) were estimated along those lines. Then the centroid of the ring was found by taking the mean of the core centroids and is plotted as the blue circle in 3.12g.

The speed of the vortex ring, \(U_R\), was calculated from the locations of the ring centroid across adjacent image frames. The \((x, y)\) coordinates of the centroid were smoothed with a 4\(^{th}\) order moving average filter and the velocity was computed using a 4\(^{th}\) order central difference of the smoothed positions:

\[
U_R(f) = \frac{x_c(f - 2) - 8x_c(f - 1) + 8x_c(f + 1) - x_c(f + 2)}{12\Delta t}
\]  

(3.3)
Figure 3.12. Sequence showing the image processing steps. 

a) background image, b) raw image with ring, c) ring after background subtraction, d) conversion to binary image, e) after filtering/smoothing, f) identification of ring, g) background subtracted ring with boundaries and centroid.
where \( f \) is the current frame number.

The ring size, position, and speed was all computed in units of pixels and needed to be converted to the experimental dimensions using the calibration factors. To locate the ring in relation to the impact plate the pressure sensor was chosen as the reference point.

### 3.2.4 DPIV Calibration

The same reference target that was used in the HSV calibration was used for the DPIV calibration. The TSI Insight 3G has a built-in tool for image calibration. The vertical and horizontal directions were calibrated individually by selecting two points in the calibration image and inputting their real distance. The calibrations were saved in the program for use in the DPIV processing described in Section 3.2.5. No image transformation or distortion correction was applied to the DPIV images because the camera was perpendicular to the laser sheet and the path of the vortex ring.
Table 3.1. Summary of PIV processor and post-processor settings used in TSI Insight 3G.

<table>
<thead>
<tr>
<th>Stage</th>
<th>Component</th>
<th>Setting</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Processor</strong></td>
<td>Grid Engine</td>
<td>Recursive Nyquist (with Pass Validation)</td>
</tr>
<tr>
<td></td>
<td>Starting Spot Dimensions</td>
<td>64 x 64</td>
</tr>
<tr>
<td></td>
<td>Final Spot Dimensions</td>
<td>16 x 16</td>
</tr>
<tr>
<td></td>
<td>Correlation Engine</td>
<td>Hart Correlator (0.4 Displacement Range)</td>
</tr>
<tr>
<td></td>
<td>Peak Engine</td>
<td>Bilinear</td>
</tr>
<tr>
<td><strong>Post-Processor</strong></td>
<td>Vector Validation</td>
<td>Median Test (3x3 Neighborhood, 0.6 Tolerance)</td>
</tr>
<tr>
<td></td>
<td>Vector Conditioning</td>
<td>Local Mean (3x3 Neighborhood, Recursive Filling)</td>
</tr>
<tr>
<td></td>
<td>Vector Smoothing</td>
<td>Gaussian Mean (3x3 Neighborhood)</td>
</tr>
</tbody>
</table>

3.2.5 DPIV Processing

The processing of the PIV image pairs was done using TSI Insight 3G which results in a velocity vector file used in the analysis described in Section 3.2.6. The processing pipeline was split into pre-processing, processing, and post-processing. The only pre-processing performed was to place a mask over the region occupied by the impact plate to prevent spurious vectors due to reflected images of particles near the plate. The processing stage components were the Grid Engine, the Correlation Engine, and the Peak Engine. The Grid Engine divided the images into small sections. The Correlation Engine computed the correlation map that showed the correlation between the particle images in two consecutive frames over different displacement values. It determined the most likely location of a particle in the second frame by the highest correlation peak. The Peak Engine found the displacement vector for the region to sub-pixel accuracy by mathematically extrapolating the maximum correlation peak. The post-processing included Vector Validation to remove bad vectors and Vector Conditioning to fill holes and to smooth the entire vector field. For all trials, the same processor and post-processor settings were used and are listed in Table 3.1.

3.2.6 DPIV Vector Analysis

The velocity vector files created by Insight 3G were analyzed by a MATLAB code PIVAverage.m described in Appendix A.4 to determine the locations of the ring cores and calculated the ring diameter, velocity, and circulation. The code was tested and
developed using an analytical vector field generated by a pair of Lamb-Oseen vortices as documented in Appendix B. The methods of finding the core location, velocity, and circulation using DPIV are based on the techniques of Weigand and Gharib (1997) and an analysis of those methods compared to other ones is presented in Appendix B.

The velocity vector files had a resolution of 80 by 64 vectors as shown in Figure 3.14. The vorticity for the whole field was computed and is shown in Figure 3.15. The cores could be located by calculating the weighted average of the positive and negative vorticity. The distance between the core centroids is the ring diameter. Individual core velocities were found by looking at horizontal and vertical slices through the cores. An example of a vertical slice through the core showing the horizontal velocity is shown in Figure 3.16. The core velocity was estimated as the mean of the sum of the maximum velocity on either side of the core because the velocity change was approximately linear through the center. This was done in both the horizontal and vertical directions.

The velocity of each core was averaged together to give a total ring velocity. The ring
Figure 3.15. Vorticity contours computed from the vector field for a vortex ring when no impact plate is present. The cores are the only significant source of vorticity in the field.

Figure 3.16. Vertical slices through both cores that show the horizontal velocity along the slices. The velocity has the jump because the core centeroids are at two different $x$-coordinates.
velocity was subtracted from all the velocity vectors to place them in the vortex frame of reference as shown in Figure 3.17. When the velocity vectors in the vortex frame are overlaid on top of the vorticity contours in Figure 3.18 the point of zero velocity is the center of the core with the maximum vorticity. The circulation of each core was calculated from a square path integral around the cores using the vortex frame velocity. The circulation was computed for different region sizes and the asymptotic value of circulation was taken for each core as shown in Figure 3.19. The core diameter was the smallest region to give the circulation value and is contained by the black box shown in Figure 3.18.

3.2.7 Force Sensor Calibration

An in situ calibration of the force sensor was performed by finding the transfer function of the impact plate assembly and accounting for the mechanical gain. Two sources of the measured force from a vortex ring impact were identified to be the vortex
Figure 3.18. Vorticity contour of the lower core overlaid with the vortex frame of reference velocity vectors. The black box shows the extent of the core found from the asymptotic value of circulation.

Figure 3.19. Calculated circulation as a function of region size for both cores. The dashed line shows the asymptotic value of circulation and the radius of the core.
ring and vibrations from the piston generator transmitted through the support gantry. The transfer function of the assembly was found by striking it with a PCB 086C02 impact hammer with a soft plastic tip and measuring the force at the sensor. We also found that vibrations from the piston initiation were transmitted directly through the platform everything rested on to the gantry holding the force sensor. A second transfer function between the piston and the force sensor in the absence of vortex ring impacting was found by using the accelerometer mounted on the piston to measure the piston motion and comparing it to the measured force.

The general method of finding the transfer functions is to apply a measured input and record the output. The input and output are transformed to the frequency spectrum by a Fourier transform. For a single input, single output system, \( \hat{O} = \hat{H} \hat{I} \) where \( \hat{O} \) is the transformed output, \( \hat{I} \) is the transformed input, and \( \hat{H} \) is the transfer function. When both the input and output are known, the transfer function can be solved for to give

\[
\hat{H} = \frac{\hat{O} \hat{I}^*}{\hat{I} \hat{H}^*},
\]  

which has also been multiplied by the complex conjugate of the input. The data for the transfer functions was processed using the MATLAB code SystemTransferFunction.m described in Appendix A.5.

The system is modeled as shown in Figure 3.20 with two sources contributing to the measured force. The vortex ring impact force is multiplied by the mechanical gain, \( M \). Each source independently causes the impact plate to move so the measured force is the sum of the two sources and their respective transfer functions:

\[
\hat{F} = \hat{H}_V \hat{V} + \hat{H}_P \hat{A}
\]  

The measured force, \( \hat{F} \), is the sum of the vortex ring impact force, \( \hat{V} \), times the transfer function of the impact assembly, \( \hat{H}_V \), plus the motion of the piston measured by the accelerometer, \( \hat{A} \), times the transfer function of the gantry, \( \hat{H}_P \). Both transfer functions
are found by isolating each input and solving for the transfer function as shown in Equation 3.4.

To find $\hat{V}$, a vortex ring impact was approximated by hitting the impact plate assembly with the impact hammer. However, if struck impulsively, higher frequency modes were excited in the plate assembly whereas the ring impacts were slower and lower in frequency and resulted in a poor transfer function. Instead, the hammer was placed against the plate and pushed before quickly releasing and the result is shown in Figure 3.21. The resulting transfer function in shown in Figure 3.22 along with the auto-spectra for the hammer input and the measured output. Only up to 50 Hz is shown even though up to 400 Hz was measured because the system only had low frequency responses.

The calibration for the HSV measurements was done by placing the hammer against a plate above the waterline which was only a quarter of the distance to the impact plate. To obtain a better transfer function during the DPIV measurements, the hammer was waterproofed by placing it in plastic bags. The hammer could now hit the impact plate in the same location as a vortex ring, and the same push and release procedure was now used underwater.

To find $\hat{H}_p$, the piston needed to generate vortex rings but they were angled away from the impact plate by placing a plastic sheet between the plate and the nozzle. The measured force due to the piston motion is shown in Figure 3.23. The transfer function and the auto-spectra are shown in Figure 3.24 up to 400 Hz because the piston vibrations occurred at a much higher frequency than the impact assembly transfer function.
Figure 3.21. Force sensor response to impact hammer excitation on the impact assembly.

Figure 3.22. Magnitude of impact plate assembly transfer function ($\tilde{H}_T$) and auto-spectra of the force sensor ($S_{ff}$) and impact hammer ($S_{hh}$). Only up to 50 Hz is shown, but measured up to 400 Hz.
Figure 3.23. Force sensor response to piston motion when a ring is prevented from impacting the plate.

Figure 3.24. Magnitude of piston-force sensor transfer function ($\hat{H}_P$) and auto-spectra of the force sensor ($S_{ff}$) and accelerometer ($S_{aa}$).
A transfer function for the impact assembly had to be found at each location of the impact assembly due to small changes in the bracing of the structure. A transfer function for the piston had to be found at each plate location as well as for each piston waveform used.

3.2.8 Vortex Ring Impact Force Deconvolution

Using the transfer functions from Section 3.2.7, a MATLAB code VortexRingImpactForce.m, described in Appendix A.6, applied them to the measured force and piston acceleration to find the vortex ring impact force. For the system model in Equation 3.5, \( \hat{V} \) is unknown during a vortex ring impact. The equation needs to be rearranged. The transfer function of the impact assembly, \( \hat{H}_T \) is also going to be redefined for the inverted direction, or

\[
\hat{H}_T = \frac{I \hat{O}^*}{\hat{O} \hat{O}^*}
\] (3.6)

using the same measured input and output. The impact assembly transfer function, \( \hat{H}_T \), only operates on the force of the ring impact through the impact assembly, so when Equation 3.5 is rearranged, the force due to the piston vibration needs to be subtracted from the total measured force. The difference is then multiplied by the moment arm ratio and the impact assembly transfer function:

\[
\hat{V} = \hat{H}_T M \left( \hat{F} - \hat{H}_P \hat{A} \right)
\] (3.7)

This gives the frequency response of the vortex ring impact force. The force is converted to a time series using the Inverse Fourier Transform.

The raw data for a vortex ring impact from the force sensor and accelerometer are shown in Figure 3.25. It appears as a combination of the piston motion shown in Figure 3.23 and the hammer excitation in Figure 3.21. When the measured force is multiplied by the calibration of the force sensor only, the result is the blue time series in Figure 3.26. When the accelerometer measurements are multiplied by \( H_P \), it gives the red
time series. The piston motion causes most of the oscillations prior to the large force maximum and when the piston motion is subtracted from the total measured force, the resulting force is shown as the green time series. The next step is to multiply by $M$ and $H_T$ to give the force due to the vortex ring shown as the black time series. The result is a single impact peak with some small oscillations.

When the previous steps are viewed in the frequency spectrum as shown in Figure 3.27 the majority of the noise occurs at frequencies greater than 9 Hz and there is a peak at 4.75 Hz corresponding to the natural frequency of the impact plate assembly oscillations. A 5th order, 4 Hz low-pass Butterworth filter shown in Figure 3.28 was applied to the final vortex ring force so that the peak could be clearly obtained. The frequency spectrum of the vortex ring force before and after filtering is shown in Figure 3.29.
Figure 3.26. Stages of the impact force deconvolution. The force from piston is $H_P$ applied to the accelerometer, and the final ring force is after the full deconvolution and moment arm correction.

Figure 3.27. Steps in force deconvolution shown in the frequency domain to illustrate the natural resonance frequency of the system.
Figure 3.28. Frequency response of a 5th order 4 Hz Butterworth low-pass filter applied to deconvolved force.

Figure 3.29. Frequency response of vortex ring impact force before and after application of a 5th order 4 Hz Butterworth low-pass filter.
3.2.9 Synchronization and Averaging

Both force sensor and the pressure sensor had a different sampling rate than what the high-speed video was recorded at. In addition, there was a delay between when the trigger signal was sent to the piston and WaveBook, and when the piston motion initiated as shown in Figure 3.30. In the MATLAB code `VortexRingImpactForce.m` that found the deconvolved vortex ring impact force, the time delay between the trigger and piston motion was accounted for by setting the piston motion to be zero in the accelerometer, pressure, and force measurements. A second MATLAB code `ImpactPlateProperties.m` described in Appendix A.7 used the delay time found in `VortexRingImpactForce.m` to remove the delay from the high-speed video data so that it could be synced to the WaveBook data. The final step in the code was to average the measured force, impact force, pressure, acceleration, ring position, ring diameter, ring speed, and core diameter from all the realizations.
3.2.10 Experimental Procedure

The flowchart in Figure 3.31 illustrates the steps needed in order to find the impact force and pressure of a vortex ring and the ring motion from high-speed video. The transfer functions of the impact plate assembly and piston motion were found individually and processed using `SystemTransferFunction.m`. The transfer functions were then imported into `VortexRingImpactForce.m` along with the measurements made during a vortex ring impact. Separately, a calibration of the HSV image was done using `Imagesequencecalibration.m` and used to process the image sequences in `Imagesequenceprocessor.m`. The results from `VortexRingImpactForce.m` and `Imagesequenceprocessor.m` were combined and averaged in `ImpactPlateProperties.m`.

The flowcharts in Figure 3.32 show the steps in acquiring the piston waveform data and the DPIV data. The information from the linear encoder was processed using `Pistonwaveforms.m` to average and convert the realizations to conditions at the nozzle.
Figure 3.32. Flowchart showing steps in waveform and DPIV data acquisition and processing.

A calibration of the DPIV image was performed in Insight 3G before the processing of the DPIV image pairs. The vector files were imported into $PIVAverage.m$ where the ring properties were computed and averaged.
4.1 Cases Studied

The goal of this study was to determine the force and stagnation point pressure during a vortex ring impact on a planar surface and relate them to the behavior of the ring before and during impact. The impact force and pressure were measured directly and the ring motion captured with high speed video and DPIV. We expected the impact force and pressure of the vortex ring to be strong functions of the vortex ring circulation.

The circulation the vortex rings was controlled by through the generator and propagation distance. Different piston waveforms generated vortex rings with differing initial circulation, and, by positioning the impact plate at various distances from the vortex ring generator, the circulation of a viscous vortex ring decreased over time (Shariff and Leonard, 1992). The vortex ring impact was measured at two impact plate locations $x/D = 20$ and 40 for three piston waveforms $L/D = 6$, 10, and 20 using HSV with corresponding impact force and pressure measurements. The DPIV measurements with impact force and pressure were done with the impact plate at $x/D = 18$ with three piston waveforms $L/D = 4$, 6, and 10. Measurements using $L/D = 4$ and 6 waveforms for HSV and $L/D = 4$, 6, and 10 waveforms for DPIV were also done of rings when no impact plate was present. Forty realizations were done for each HSV case of the ring
impacting the plate. Sixty realizations were used for the no-plate HSV measurements and for all the DPIV cases.

The differences in the piston waveforms and impact plate locations between the HSV and DPIV measurements were due to the changes in the support gantry. A stiffening bar in the second generation gantry prevented the impact plate from being positioned at $x/D = 20$ and the more rigid structure allowed better force measurements of the weaker rings generated by the $L/D = 4$ waveform.

As described in Section 3.1, the maximum speed, acceleration, deceleration, and stroke length of the piston could be controlled. However, due to physical limitations on the electromagnetic drive, the maximum could not be reached in the incremental distance specified. In order to get different piston speeds, the stroke length was altered while leaving the other settings constant. This produced the negative sloping piston waveforms. For all test cases, the piston velocity was set to 2540 mm/s and the acceleration and deceleration were both 203,200 mm/s. The piston stroke ratios, $L/D$, are based on the diameter of the nozzle and not the piston. The linear encoder on the piston measured the displacement and velocity of the piston, which were used to find maximum jet velocity, $U_{j,max}$, using Equation 3.1 and $L/D$ using Equation 3.2 at the nozzle for the waveforms listed in Table 4.1. The Reynolds number based on the maximum jet velocity

$$Re_m = \frac{U_{j,max}D}{\nu} \quad (4.1)$$

was computed and is shown in Table 4.1. The $Re_m$ varied between 30,000 for $L/D = 4$ to 122,000 for $L/D = 20$, giving a wide range of high Reynolds number rings.

The mean of all the waveforms used are shown in Figure 4.1 with the jet velocity plotted against the stroke ratio. The shape of the waveform was negative sloping in the terminology of Krueger and Gharib (2003) because the piston accelerated quickly to its maximum velocity and decelerated through the majority of the motion. Krueger and Gharib (2003) found this waveform to give to a stable vortex ring, most likely because
Table 4.1. Measured piston waveform parameters for the impact studies. $L/D$, $U_{j,max}$, and $Re_m$ are for conditions at the nozzle exit.

<table>
<thead>
<tr>
<th>Case</th>
<th>$x/D$</th>
<th>$L/D$</th>
<th>$L/D_{meas}$</th>
<th>$U_{j,max}$ (mm/s)</th>
<th>$(L/D)_{max}$</th>
<th>$Re_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>HSV</td>
<td>20</td>
<td>6</td>
<td>5.94</td>
<td>1914.9</td>
<td>1.02</td>
<td>48,445</td>
</tr>
<tr>
<td>HSV</td>
<td>20</td>
<td>10</td>
<td>9.88</td>
<td>3060.0</td>
<td>2.07</td>
<td>77,415</td>
</tr>
<tr>
<td>HSV</td>
<td>20</td>
<td>20</td>
<td>19.79</td>
<td>4806.8</td>
<td>6.21</td>
<td>121,606</td>
</tr>
<tr>
<td>HSV</td>
<td>40</td>
<td>6</td>
<td>5.92</td>
<td>1905.4</td>
<td>1.00</td>
<td>48,204</td>
</tr>
<tr>
<td>HSV</td>
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<td>9.74</td>
<td>3071.0</td>
<td>2.07</td>
<td>77,692</td>
</tr>
<tr>
<td>HSV</td>
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<td>20</td>
<td>19.82</td>
<td>4803.5</td>
<td>6.29</td>
<td>121,524</td>
</tr>
<tr>
<td>HSV</td>
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<td>6</td>
<td>4.01</td>
<td>1195.5</td>
<td>0.94</td>
<td>30,243</td>
</tr>
<tr>
<td>HSV</td>
<td>$\infty$</td>
<td>6</td>
<td>6.08</td>
<td>1813.0</td>
<td>1.25</td>
<td>45,866</td>
</tr>
<tr>
<td>HSV</td>
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<td>4</td>
<td>4.02</td>
<td>1511.1</td>
<td>1.00</td>
<td>38,229</td>
</tr>
<tr>
<td>HSV</td>
<td>18</td>
<td>6</td>
<td>6.09</td>
<td>1847.7</td>
<td>1.33</td>
<td>46,744</td>
</tr>
<tr>
<td>HSV</td>
<td>18</td>
<td>10</td>
<td>10.11</td>
<td>2377.6</td>
<td>2.31</td>
<td>60,149</td>
</tr>
<tr>
<td>HSV</td>
<td>$\infty$</td>
<td>4</td>
<td>4.01</td>
<td>1660.5</td>
<td>1.05</td>
<td>42,009</td>
</tr>
<tr>
<td>HSV</td>
<td>$\infty$</td>
<td>6</td>
<td>6.08</td>
<td>2103.5</td>
<td>1.50</td>
<td>53,215</td>
</tr>
<tr>
<td>HSV</td>
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<td>10.12</td>
<td>2263.1</td>
<td>1.89</td>
<td>57,253</td>
</tr>
</tbody>
</table>

Figure 4.1. Jet velocity of the piston waveforms used in the HSV and DPIV studies. NP indicates when no impact plate is present.

The piston velocity peak occurred prior to vortex ring pinch-off at $L/D = 4$ and the trailing jet moves more slowly than the motion that generated the ring.

The range of formation number values reported by Gharib et al. (1998) is 3.6 to 4.5
Figure 4.2. Image of a well-formed (left) and a destabilized (right) ring generated by the \( L/D = 20 \) waveform.

and is dependent on several factors including nozzle geometry and the piston waveform shape. The flat nozzle face lowers the circulation of the rings than those compared to a straight nozzle because of the oppositely signed vorticity generated along the wall during formation (Didden, 1979). Krueger and Gharib (2003) determined that a negative sloping piston velocity profile, such as the current ones, will lower the formation number. In order to determine the formation number of a particular setup, the circulation of vortex rings forming at the nozzle would need to be measured. These measurements were not taken in the present study, but the formation number is assumed to lie within the reported range, most likely in the lower end due to the factors listed.

The stroke ratios of the maximum piston velocity, \((L/D)_{max}\) was found using the piston waveforms and are shown Table 4.1. For the negative sloping waveform, \((L/D)_{max}\) should be below the lowest value of 3.6 reported by Gharib et al. (1998), however, for the L/D = 20 waveform, \((L/D)_{max} > 6.2\). Using this waveform, the additional fluid not in the ring was still accelerating after the ring formed, and it was observed to sometimes destabilize the ring as shown in Figure 4.2. With the finer measurements allowed by the improved gantry, the L/D = 20 waveform was replaced with an L/D = 4 waveform.

Even though the piston controller settings were kept the same, the piston waveform
changed between the HSV trials with the plate to the HSV measurements with no plate and all the DPIV measurements. After the HSV measurements with the impact plate, the piston control computer was replaced before the HSV measurements with no plate. Before the DPIV measurements, the nylon cylinder had become warped due to wear, heating, and water absorption, and the seal around the piston was leaking. The warping could not be corrected, but a larger O-ring was placed around the piston to prevent leaking and a new controller unit had to be installed. After the control computer was changed, the maximum velocity was 5% lower and occurred 25% later in all the waveforms with the same controller settings as seen in Figure 4.1 when comparing HSV \( L/D = 6 \) to HSV NP \( L/D = 6 \). After the O-ring and controller were replaced, the maximum speed of the \( L/D = 4 \) waveform increased by 25% when comparing HSV NP \( L/D = 4 \) to PIV \( L/D = 4 \) in Figure 4.1. The maximum speed of the \( L/D = 6 \) waveform remained the same between HSV NP \( L/D = 6 \) and PIV \( L/D = 6 \) as seen in Figure 4.1.

The force sensor, pressure sensor, and accelerometer were recorded through WaveBook at 400 Hz during the HSV measurements and 800 Hz during the DPIV measurements. For an anti-aliasing filter, the HSV measurements used Krohn-Hite filters set to an elliptic low-pass of 200 Hz, while the DPIV measurements used the built-in WaveBook elliptic low-pass filter at 400 Hz. The minimum settings for the WaveBook anti-aliasing filter could not be set any lower than 400 Hz forcing the higher sampling rate. During the HSV measurements, a total record length of 32 s was used with the impact occurring at approximately 16 s. The long time before impact was susceptible to external vibrations in the room interfering with the measurements during what should be a stationary phase, so the DPIV measurements were recorded for 16 s total with only 1 s before piston initiation. The record length was determined by the time the pressure signal needed to return to its undisturbed level.

The high speed video camera recorded at 200 frames per second with a 5 ms exposure for all runs. The camera lens fstop was 2.8 and an intensity offset of 50 was set in XCAP with a digital gain of x2. The number of frames depended on the speed of the ring and
the distance of the plate from the nozzle. The total frames recorded varied between 150 and 550 for the different waveforms.

Three unique DPIV measurements were performed during the vortex ring propagation and impact. The framing rate of the DPIV system was 30 Hz and was too slow to capture more than one frame of the vortex ring as it impacted. The DPIV system was placed on a delay after piston initiation to capture the vortex ring at approximately 100, 50, and 0 mm from the target. The acquisition delay time was controlled using the BNC-555 waveform generator, which had a programmable delay between the piston generation and the trigger to the DPIV system. The camera exposure was 255 µs and the $\Delta t$ between laser pulses was adjusted for different ring strengths to optimize the particle displacements across the large spatial gradients expected within the vortex ring core. The slower rings required a longer $\Delta t$ while faster rings needed a shorter delay. The longest $\Delta t$ was for the $L/D = 4$ waveform ($\Delta t = 290 \mu s$) and the shortest for the $L/D = 10$ waveform ($\Delta t = 150 \mu s$). The attenuator on the lasers was set to between 250 and 330 to ensure adequate brightness while minimizing overall laser light energy to reduce glare and damage to the cameras. The attenuator setting depended on the seeding conditions.

4.2 Impact Study Results

The results of the measurements the vortex ring impacts are presented in two sections. First, the time-resolved measurements of vortex ring motion and the deformation are presented in concert with measurements of the impact force and stagnation point wall pressure. These measurements are used to correlate the phases of vortex ring motion to the observed waveforms of wall pressure and impact force. Second, the measurements of the vortex ring captured at three specific time delays relative to the generation pulse using DPIV are presented along with impact pressure and force measurements in order to correlate the vortex ring circulation to the impact force.
4.2.1 Time Resolved Vortex Motion Measurements

Still images for the HSV captures at the $x/D = 20$ location are shown in Figure 4.3 for a ring generated by the $L/D = 6$ waveform with a $Re_m = 48,000$. The ring is moving from left to right. When the ring nears the impact plate, the diameter grows dramatically. After impact, the ring core becomes unstable and the dye is released. Rings for the $L/D = 10$ and $L/D = 20$ waveforms at the same impact plate location are shown in Figure 4.4 and Figure 4.5 with $Re_m = 77,000$ and $121,000$ respectively. The behavior of these rings is similar to that of the $Re_m = 48,000$ ring, but occurs over a much faster time scale.

The ring behavior during impact is analyzed from the high-speed video records using the processing methods discussed in Section 3.2.3. First, a single representative impact case is shown. Then, a similar presentation of the other cases is given to show the degree to which the results from the first case are representative over the parameter range studied.

The ensemble averaged ring speed $U_R$, for $Re_m = 48,000$ is shown in Figure 4.6 with the initiation of piston motion corresponding to 0 s. The ensemble averaged ring diameter $D_R$, impact force $F$, and impact pressure $p$, for $Re_m = 48,000$ are shown in Figure 4.7 through Figure 4.9. The standard deviation from the ensemble average is small and consistent throughout the whole time series. The only major departure is the standard deviation of the pressure after the negative dip.

Both the velocity and diameter in Figure 4.6 and Figure 4.7 show two distinct phases. The velocity decreased while the ring diameter barely increased from the time period 0.15 s to 0.7 s. At 0.7 s, both underwent a rapid change with the ring decelerating to zero and the diameter growing by 60%. The deconvolved impact force, shown in Figure 4.8, displayed only a single peak, reaching it at about 0.7 s. The shape of the pressure waveform shown in Figure 4.9 is more complex than the force, reaching a positive peak at approximately 0.8 s, followed by a rapid decline to a negative peak at approximately 1.8 s. The pressure then relaxed back to zero by approximately 6 s after generation. The
Figure 4.3. Stills from high speed video of vortex ring impact at $x/D = 20$ for $Re_m = 48,000$. 

final, rapid changes in vortex ring speed and diameter occurred in less than 0.1 s, much faster than the corresponding changes in the pressure and impact force.

Figure 4.10 shows the relationship between vortex ring motion and the impact pressure and force. All of the quantities are normalized by their maximum values. The dash-dot lines designate the instants at which the force and pressure maxima occur. The correlation between vortex ring motion during approach and impact and the effect on the wall is clearly seen. In particular, there appears to be a coincidence between the abrupt
change in vortex ring motion at the beginning of its collision with the impact plate and the occurrence of the force maximum. Similarly, the maximum value of impact pressure appears to coincide with the instant that the vortex ring axial speed reached zero and the ring reached its maximum diameter. In addition, it can be observed that the wall pressure began to increase appreciably from zero only just prior to the occurrence of the maximum force. As a result, the time interval between the pressure and force peaks provides a convenient measure of the time over which the collision occurs.

Figure 4.4. Stills from high speed video of vortex ring impact at $x/D = 20$ for $Re_m = 77,000$. 
Other observations can also be made to correlate the evolution of the vortex ring with the impact force and pressure. The impact force began to increase from zero well in advance of the impact proper, and started to decrease only as the ring began to rapidly distort during impact. This suggests that the impact force is not simply a response to
Figure 4.6. Ensemble averaged axial ring velocity for impact plate at $x/D = 20$. $Re_m = 48,000$. Dashed lines show mean ± the standard deviation.

Figure 4.7. Ensemble averaged ring diameter for impact plate at $x/D = 20$. $Re_m = 48,000$. Dashed lines show mean ± the standard deviation.
Figure 4.8. Ensemble averaged impact force for impact plate at $x/D = 20$. $Re_m = 48,000$. Dashed lines show mean ± the standard deviation.

Figure 4.9. Ensemble averaged impact zone pressure for impact plate at $x/D = 20$. $Re_m = 48,000$. Dashed lines show mean ± the standard deviation.
Figure 4.10. Ensemble averaged ring diameter, axial ring speed, impact force, and impact zone pressure normalized by their respective maxima for impact plate at \( x/D = 20 \). \( Re_m = 48,000 \). Dash dot lines mark the time of maximum force and pressure.

The deceleration of the ring during the final, most rapid phase of the motion, but also involves how the wall distorts the vortex ring flowfield far upstream during its approach.

The wall pressure behavior may be explained by first noting that it departs from zero only when the ring began its rapid axial deceleration and diameter increase. When the pressure sensor is inside the ring diameter, the pressure is essentially a stagnation pressure corresponding to the distortion of the vortex ring flowfield during the final phase of collision. This pressure reaches its maximum when the forward motion of the ring stopped. However, because the ring had not fully broken up, there was residual motion above the sensor, which gives rise to a rapid drop in pressure to a negative value. The negative pressure slowly decays to zero, persisting during the slow dissipation of the remnants of the ring as it cross-diffused with the vorticity of opposite sign generated at the wall during the final phase of collision.

Also of interest is the distance between the vortex ring centroid and the impact plate
Figure 4.11. Ensemble averaged ring diameter and axial speed normalized by their respective maxima vs their distance from the impact plate for impact plate at $x/D = 20$. $Re_m = 48,000$. Maximum force at 42 mm. Maximum pressure at 3 mm.

when the maximum force and pressure peaks occur. Figure 4.11 shows $D_R$ and $U_R$ at $x/D = 20$, for $Re_m = 48,000$ as a function of their distance from the plate with $F_{max}$ and $p_{max}$ marked. The maximum force occurred when the ring centroid was 41.9 mm from the plate, and the maximum pressure occurred at 2.8 mm from the plate. The ring cores never hit the plate due to the ring momentum changing from axial to radial; 2.8 mm was the point of their closest approach before the dye was released. The average $D_R$ before impact is 46.7 mm, so the maximum force occurs at $x_f/D_R = 0.90$, where $x_f$ is the location of the centroid corresponding to the maximum force.

The differences in the force and pressure waveforms suggest that they are associated with different phases of the ring motion. The maximum force occurred when the ring was a distance from the target approximately equal to its diameter. At the time of the maximum force, the ring velocity was still decreasing slowly and the diameter was fairly constant. However, the maximum pressure did not occur until the ring had reached
Table 4.2. Mean and ensemble averaged of maximum impact force and pressure during impact. $x_f$ and $x_p$ are the distance from the plate the ring is for $F_{\text{max}}$ and $p_{\text{max}}$. $t_{\text{int}}$ is the measured interaction time between the force and pressure maxima.

<table>
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<tr>
<th>$x/D$</th>
<th>$\text{Re}_m$</th>
<th>$F_{\text{max}}$ (N)</th>
<th>$p_{\text{max}}$ (kPa)</th>
<th>$x_f$ (mm)</th>
<th>$x_p$ (mm)</th>
<th>$t_{\text{int}}$ (ms)</th>
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Table 4.3. Mean and ensemble averaged ring diameter and speed before impact from HSV. $t_{\text{imp}}$ is the impact time estimated from ring diameter and speed.

<table>
<thead>
<tr>
<th>$x/D$</th>
<th>$\text{Re}_m$</th>
<th>$D_R$ (mm)</th>
<th>$U_R$ (mm/s)</th>
<th>$t_{\text{imp}}$</th>
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<td>327.91</td>
<td>145.6</td>
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</tr>
<tr>
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<td>114.5</td>
<td>0.845</td>
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<td>643.59</td>
<td>122.0</td>
<td>0.614</td>
</tr>
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</table>

its closest approach and the horizontal velocity had been reduced to zero. From the dimensional analysis in Section 2.2, the impact time of the vortex ring with a surface was estimated as $t_{\text{imp}} = D_R/U_R$. Because the maximum force occurred when the ring was nearly $D_R$ from the target, and the pressure occurred at the closest point, the time between $F_{\text{max}}$ and $p_{\text{max}}$ is used to calculate the interaction time $t_{\text{int}}$. The interaction time is shown in Table 4.2 and the impact time estimated from the average $D_R$ and $U_R$ before impact is shown in Table 4.3. The estimated time is smaller than the actual interaction time because the ring slows down as it nears the plate, which the model does not account for, and is shorter by about 25%.

Plots of the combined results for the $\text{Re}_m = 77,000$ and 121,000 rings at $x/D = 20$ are shown in Figure 4.12 and Figure 4.13, and the results for $\text{Re}_m = 48,000$ at $x/D = 40$ are shown in Figure 4.14. The velocity and diameter as functions of distance from the plate for the three cases are shown in Figure 4.15 through Figure 4.17. The impact behavior of the vortex rings at $x/D = 40$ for $\text{Re}_m = 77,000$ and 121,000 was too varied to be ensemble averaged. Instead, the mean of the recorded values are reported in Table
Figure 4.12. Ensemble averaged ring diameter, axial ring speed, impact force, and impact zone pressure normalized by their respective maxima for impact plate at $x/D = 20$. $Re_m = 77,000$. Dash dot lines mark the time of maximum force and pressure.

4.2 and Table 4.3 along with the impact values from ensemble averaged time series of the other conditions. For all tested cases, the ring velocity and diameter exhibited two phases with the demarcation in each occurring simultaneously. The demarcation was always at the same time as $F_{max}$. The maximum pressure always occurred after the maximum force, and at the time the axial velocity $U_R$ had reached zero.

The scaling relation between $U_R$ and $\Gamma$ in Equation 2.3 indicates that

$$\Gamma \approx DR U_R.$$  \hspace{1cm} (4.2)

Without precise measurements of $\Gamma$, the $U_R$ and $D_R$ from the HSV can be used to look at the trends in the impact force and pressure. As $U_R$ and $D_R$ increased due to the higher piston velocity from the larger stroke ratios, the impact force and pressure increased at a given location. However, the effects of viscosity and time on the speed and size of a vortex ring are apparent when the impact values for $x/D = 40$ are compared to those
Figure 4.13. Ensemble averaged ring diameter, axial ring speed, impact force, and impact zone pressure normalized by their respective maxima for impact plate at $x/D = 20$. $Re_m = 121,000$. Dash dot lines mark the time of maximum force and pressure.

at $x/D = 20$. $D_R$ grew slightly whereas $U_R$ decreased by approximately half. Taken together, the changes in $D_R$ and $U_R$ result in a maximum impact force at $x/D = 40$ that was 50% of the force at $x/D = 20$. The change in pressure was not the same for the different strength rings. Between $Re_m = 48,000$ rings, the pressure only decreased slightly, but for the $Re_m = 121,000$ rings the pressure decreased by over 50%.

The distance of the vortex ring centroid from the plate at the time of the maximum force, $x_f$, and pressure, $x_p$, increased as $U_R$ increased. Because the faster rings also have a larger diameter, the ratios of the distance from the plate of $x_f$ and $x_p$ over the ring diameter, $D_R$, are listed in Table 4.3. At both plate locations, $F_{max}$ occurred at the highest $x_f$ for $Re_m = 77,000$. At $x/D = 20$, $x_f$ was nearly equal to the ring diameter. It was slightly closer for $Re_m = 48,000$ and the closest for $Re_m = 121,000$. At $x/D = 20$, for $Re_m = 48,000$, the ring cores reached the closest at 3 mm while the other two stroke ratios were over 4 mm. At $x/D = 40$, the rings from all three stroke ratios were between
Figure 4.14. Ensemble averaged ring diameter, axial ring speed, impact force, and impact zone pressure normalized by their respective maxima for impact plate at $x/D = 40$. $Re_m = 48,000$. Dash dot lines mark the time of maximum force and pressure.

8 to 8.5 mm from the plate when the maximum pressured was measured at the wall.

The measured interaction time and estimated impact times for all cases are shown in Table 4.2 and Table 4.3. At $x/D = 20$, as the speed of the rings increased, the interaction time shortened even though the ring diameter was larger as well. When the same rings hit the impact plate at $x/D = 40$, the ring diameter had grown slightly, but the ring speed was approximately half of what it was when the impact plate was closer to the nozzle. Together these cause the interaction and impact times to increase. Because the faster rings decrease in velocity and expand more quickly than the slower rings, there is a larger corresponding increase in tint for the $Re_m = 121,000$ rings than the $Re_m = 30,000$ rings.

To determine if the plate has an effect on vortex rings beyond one diameter from the surface, the trajectories and diameters of rings with and without the impact plate are shown in Figure 4.18 and Figure 4.19 as a function of distance from the nozzle. The
Figure 4.15. Ensemble averaged ring diameter and axial speed normalized by their respective maxima vs their distance from the impact plate for impact plate at $x/D = 20$. $Re_m = 77,000$. Maximum force at 52 mm. Maximum pressure at 4 mm.

scatter in the trajectory of the rings for $Re_m = 77,000$ and 121,000 at $x/D = 40$ was too large to ensemble average them. A no-plate case for the $L/D = 20$ waveform was not recorded because it had already been determined that a stable ring was not always formed and was replaced by an $L/D = 4$ waveform. The trajectory and diameters of the rings generated by the $L/D = 6$ stroke ratio have a similar slope and size with and without the plate. The change in the generator characteristics described in Section 4.1 is why the $Re_m = 45,000$ rings are faster without the plate present than the $Re_m = 48,000$ rings with the impact plate. Both $Re_m = 30,000$ and the $L/D = 6$ stroke ratio rings have a gently decreasing $U_R$, while the higher Reynolds number rings exhibit a more rapid decrease in speed, though still linear.
Figure 4.16. Ensemble averaged ring diameter and axial speed normalized by their respective maxima vs their distance from the impact plate for impact plate at \( x/D = 20 \). \( Re_m = 121,000 \). Maximum force at 56 mm. Maximum pressure at 5 mm.

Figure 4.17. Ensemble averaged ring diameter and axial speed normalized by their respective maxima vs their distance from the impact plate for impact plate at \( x/D = 40 \). \( Re_m = 48,000 \). Maximum force at 39 mm. Maximum pressure at 8 mm.
Figure 4.18. Ensemble averaged $U_R$ for both no-plate and with impact plate at $x/D = 20$ and 40.

Figure 4.19. Ensemble averaged $D_R$ for both no-plate and with impact plate at $x/D = 20$ and 40.
4.2.2 Velocity Flowfield Measurements

The velocity vector fields for rings with $Re_m = 38,229$ generated by the $L/D = 4$ waveform with the impact plate at $x/D = 18$ are shown in Figure 4.20. The impact plate is located at 155 mm and the three images represent three separate rings. Only one location of the ring could be captured during a single impact, so the DPIV system was on a delay trigger. The three time delays for the $Re_m = 38,229$ ring were 950, 1060, and 1170 ms and were based on the simultaneous trigger sent to the piston and the DPIV system. The initiation delay of the piston was not subtracted for the DPIV delay like it was during the HSV measurements, but it was still removed from the force and pressure measurements. The piston initiation delay is still removed from the force and pressure measurements. There is little change in the velocity field of the ring between the 950 and 1060 ms delays. Only when the ring was close to the impact plate at the 1170 ms delay, is the distortion evident.

The vorticity contours for $Re_m = 38,229$ rings were computed using the method described in Section 3.2.6 and are also shown in Figure 4.20. For the 950 and 1060 ms delays, the cores are circular in shape, but at the 1170 ms delay, the cores distorted with the ring and became more diffuse.

The velocity vector fields and vorticity contours for rings with $Re_m = 46,744$ from the $L/D = 6$ waveform are shown in Figure 4.21 and for rings with $Re_m = 60,149$ from the $L/D = 10$ waveforms are shown in Figure 4.22. These waveforms had a higher jet velocity than the $L/D = 4$ waveform and needed shorter time delays to capture the ring in the same distances from the plate. The three time delays for $Re_m = 46,744$ were 830, 900, and 960 ms, and they were 750, 800, and 860 ms for $Re_m = 60,149$.

The core centroids are plotted in Figure 4.23 to Figure 4.25 as a function of distance from the plate, $x_c$, to compare the trajectory of the rings with and without the plate present. The scatter in the position of the rings is a combination of small changes in the jet velocity of the generator and some variability in the piston initiation after trigger. At the first two time delays, which correspond to distances from the plate of approximately
Figure 4.20. Velocity vector field and vorticity contours at 950, 1060, and 1170 ms time delays for impact plate at $x/D = 18$ (150 mm). $Re_m = 38,229$.

$x_c = 50$ and 120 mm, the ring diameters for all three ring strengths were the same between the plate and no-plate cases. The ring growth was not accelerated by the plate until within 20 mm of the plate.

Using the velocity field, the ring diameter $D_R$, ring speed $U_R$, ring circulation $\Gamma$, and
Figure 4.21. Velocity vector field and vorticity contours at 830, 900, and 960 ms time delays for impact plate at \(x/D = 18\) (150 mm). \(Re_m = 46,744\).

Core diameter \(d_c\) were calculated according to the methods presented in Section 3.2.6. The average values of the ring parameters measured by DPIV are shown in Table 4.4 for when the impact plate is present and in Table 4.5 for when it is not. Between the first and second time delays for all conditions, there are only small differences in the measured
properties. The speed and circulation decreased slightly, while the ring diameter grew a small amount. The major difference occurs at the third time delay when the collision with the plate forced the momentum of the ring to divert from axial to radial. The collision caused the axial speed and circulation to decrease and the diameter to increase.
Figure 4.23. Vortex ring core locations for Re \(_m = 42,000\) rings at each delay time with and without impact plate. Open shapes are for the impact plate present at 0 mm.

Figure 4.24. Vortex ring core locations for Re \(_m = 53,000\) rings at each delay time with and without impact plate. Open shapes are for the impact plate present at 0 mm.
Figure 4.25. Vortex ring core locations for $R_{\text{em}} = 57,000$ rings at each delay time with and without impact plate. Open shapes are for the impact plate present at 0 mm.

Table 4.4. Mean DPIV measurements of vortex ring with impact plate present.

<table>
<thead>
<tr>
<th>$L/D$</th>
<th>Delay</th>
<th>$D_R$ (mm)</th>
<th>$U_R$ (mm/s)</th>
<th>$\Gamma$ (m$^2$/s)</th>
<th>$d_c$ (mm)</th>
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Table 4.5. Mean DPIV measurements of vortex ring without impact plate present.

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Table 4.6. Maximum force, pressure, and interaction time for DPIV. $t_{int}$ is the measured interaction time between the force and pressure maxima.

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<tr>
<td></td>
<td>T3</td>
<td>0.169</td>
<td>17.487</td>
<td>0.1476</td>
</tr>
<tr>
<td>6</td>
<td>T1</td>
<td>0.342</td>
<td>22.611</td>
<td>0.1346</td>
</tr>
<tr>
<td></td>
<td>T2</td>
<td>0.353</td>
<td>21.525</td>
<td>0.1349</td>
</tr>
<tr>
<td></td>
<td>T3</td>
<td>0.362</td>
<td>22.170</td>
<td>0.1339</td>
</tr>
<tr>
<td>10</td>
<td>T1</td>
<td>0.908</td>
<td>28.091</td>
<td>0.1127</td>
</tr>
<tr>
<td></td>
<td>T2</td>
<td>0.919</td>
<td>27.619</td>
<td>0.1107</td>
</tr>
<tr>
<td></td>
<td>T3</td>
<td>0.944</td>
<td>27.575</td>
<td>0.1123</td>
</tr>
</tbody>
</table>

The large difference in velocity and circulation in the rings generated by the $L/D = 6$ and $L/D = 10$ waveforms was caused by changes in the piston waveform. From Table 4.1 and Figure 4.1, the maximum piston velocity of the no-plate case was greater than the plate case as well as having a larger $(L/D)_{max}$. Even though $(L/D)_{max}$ occurred before $L/D = 4$, the extra time allowed more fluid to roll up in the ring, increasing its circulation. With a similar maximum piston velocity, rings of different strengths were generated between the plate and no-plate cases for the $L/D = 10$ waveform. The change in ring speed and delay time for the second delay of the $L/D = 4$ waveform with no-plate was due to the piston controller freezing. After rebooting, the piston initiation delay was smaller and the maximum piston velocity increased by 7.4%. The two factors resulted in a shorter delay time being needed for the second and third time delays of rings generated by $L/D = 4$ as well as those of the $L/D = 6$ and $L/D = 10$ waveforms.

The force and pressure of the impact at $x/D = 18$ for the three ring strengths are shown in Figure 4.26 to Figure 4.28. The waveforms of the impact force and pressure are essentially identical to those made in the earlier measurements at $x/D = 20$ and $x/D = 40$ and show the same relationship between the time of the maximum force and pressure. The pressure signals shown in Figure 4.26 to Figure 4.28 do not possess the ripple around the maximum pressure due to the stiffening of the support frame. The maximum values of the impact force and wall pressure are shown in Table 4.6 along with the measured values of the interaction time.
Figure 4.26. Ensemble averaged force and pressure time series normalized by their maximum values for Reₘ = 38,229. Dash dot lines mark $F_{max}$ and $p_{max}$.

Figure 4.27. Ensemble averaged force and pressure time series normalized by their maximum values for Reₘ = 46,744. Dash dot lines mark $F_{max}$ and $p_{max}$. 
As the piston velocity increased, the measured force and pressure increased and the interaction time shortened which are consistent with the earlier measurements. The rings have a higher velocity and circulation when the piston velocity increases, however it needs to be tested if the force is scaling as predicted by Equation 2.6. The maximum impact force is plotted against $\rho \Gamma^2$ in Figure 4.29. The assumptions in the scaling law are based on a ring without the presence of wall. Data from the DPIV velocity field estimates was taken at the first time delay because the ring had a similar speed, diameter, and circulation for both when the plate was and was not present. A linear regression analysis of all the values of $\rho \Gamma^2$ and $F_{max}$ was done in Microsoft Excel. The regression line had an equation of

$$y = 0.0984x + 0.0469$$  \hspace{1cm} (4.3)

with an $R^2 = 0.988$ and is shown in Figure 4.29. The maximum impact force of a vortex ring scales as $\Gamma^2$ for the studied range of ring strengths, as predicted by Equation 2.6.
4.3 Evaluation of Results

4.3.1 Comparison of Flow Behavior

The impact force and pressure waveforms were recorded for vortex rings impacting a plate at three different distances from the nozzle for four different piston waveforms. The relationship between the impact force and pressure is shown in Figure 4.10, Figure 4.12, Figure 4.13, Figure 4.14, Figure 4.26, Figure 4.27 and Figure 4.28. For all the different locations and waveforms, the maximum force occurred at the same time at which the pressure began to rise. For the impact force and pressure measured during the high-speed video recording, the maximum force occurred at the demarcation marked by the sudden changes in $D_R$ and $U_R$. The pressure reached its maximum value at the time when $U_R$ had decreased to zero and later than the force maximum.

Comparing the values of the maximum impact force and pressure at each plate location in Table 4.2 and Table 4.6, both decreased as $x/D$ increased. The circulation of
the vortex rings decreased over time as the core lost vorticity and was evident in the measured values in Table 4.4 and Table 4.5. As the circulation decreased, the impact force and pressure decreased as well.

The decrease in maximum force between the three plate locations cannot be examined more closely for two reasons. The first is the change in the piston generation characteristics after equipment was replaced. The second is the improved measurements taken with DPIV due to the stiffening of the frame and the method of obtaining the transfer function (see Section 3.2.7). The magnitudes of the transfer functions for the three plate locations are shown in Figure 4.30. The resonance peaks that appeared at 4.75 Hz in Figure 3.22 are now the troughs because Figure 4.30 is showing the inverted transfer functions that were used to deconvolve the force. The transfer function at $x/D = 18$ had a magnitude almost 10 dB higher for much of the range shown which resulted in a higher force transmission when applied.

The pressure waveforms have a different shape during the HSV measurements than
the DPIV ones. During the HSV trials, the pressure sensor was located approximately 15 mm above the centerline of the nozzle and the ring centers. With ring radii from about 22 to 34 mm, the offset caused the pressure sensor to be closer to the low pressure ring core rather than the stagnation point and lowered the wall pressure reading. This would lower the maximum pressure measured at \( x/D = 20 \) and \( x/D = 40 \) compared to \( x/D = 18 \) more than the increase in distance would. Before the DPIV trials, the impact plate was remounted to be lower and put the pressure sensor in line with the ring center. The remounting also removed the ripple in the pressure readings.

When the interaction times at \( x/D = 18 \) are compared to those measured at \( x/D = 20 \), they are slower for the similar waveforms even though the impact plate is closer to nozzle. When the impact plate was at \( x/D = 18 \), the \( L/D = 10 \) waveform had a \( Re_m = 60,149 \) compared to a \( Re_m = 77,415 \) when the impact plate was at \( x/D = 20 \). This resulted in \( U_R = 868 \text{ mm/s} \) for the \( Re_m = 60,149 \) ring instead of a \( U_R = 955 \text{ mm/s} \), so the interaction time is longer by 0.027 s. The \( L/D = 6 \) piston waveform at both locations was similar in shape and the rings had similar speeds. However, \( t_{int} \) at \( x/D = 18 \) was 0.0213 s longer than at \( x/D = 20 \). With the change in the location of the pressure sensor relative to the ring impact and the improvements in measuring the force, the resulting force and pressure peaks shifted slightly in relation to each other. The differences in the interaction times between the impact plate locations are a combination of changes in the ring generation and the experimental setup.

4.3.2 Comparison of Results to Previous Studies

With one or two exceptions, the impact behavior of turbulent vortex rings, observed here, is qualitatively the same as impacts of laminar vortex rings observed by Chu et al. (1993), Chu et al. (1995), and Naguib and Koochesfahani (2004). From the high-speed video, two of the three phases of the vortex ring impact described by Chu et al. (1993) were observed. The free-travel phase displayed a similar trend in the velocity and diameter as a ring without the plate present. And the stretching and deceleration of the
second phase were clearly present.

From the measurements of the distance from the plate, the demarcation between phases occurred roughly one ring diameter from the plate for the upstream plate condition similar to the observation of Walker et al. (1987). For the target located farthest from the vortex ring generator, the demarcation occurred less than a ring diameter from the plate. In both of our instances, the ring diameter is over-estimated using the HSV. The ring diameter is defined as the core to core distance, but the HSV could only measure the end to end distance of the ring, so the actual ratio would be closer to one ring diameter.

The experimental and numerical results for the stagnation point pressure of Chu et al. (1995) are shown in Figure 1.5. The maximum reported pressure is 0.080 mm H$_2$O or 0.785 Pa which was considerably less than the maximum pressure of our weakest ring at 18 Pa, but the present rings are between 18 and 45 times stronger than the rings used by Chu et al. (1995). To compare the shape and time of the stagnation point waveform, our pressure measurements from the DPIV trials were normalized as shown in Figure 4.31. They show the same trend as those shown in Figure 1.5 where the Re$_d$ $\approx$ 37,000 ring that hit earlier had a broader peak than Re$_d$ $\approx$ 22,000 and Re$_d$ $\approx$ 15,000. The rise time of the Re$_d$ $\approx$ 830 and Re$_d$ $\approx$ 1000 rings in Figure 1.5 were both about 12 s whereas the Re$_d$ $\approx$ 15,000, Re$_d$ $\approx$ 22,000, and Re$_d$ $\approx$ 37,000 in Figure 4.31 all had a rise time of about 0.5 s. The increase in pressure, which scales as $\rho U R^2$, and decrease in rise time, which scales as $D_R/U_R$, is due to the increase in the ring convection speed. The large negative drop in pressure that occurred after the maximum in the present study was not observed by either Chu et al. (1995) or Naguib and Koochesfahani (2004).

Comparing the force waveform shown in Figure 1.10 to the stagnation point pressure waveform shown in Figure 1.5, the maximum force occurred 4 s after the maximum pressure. In the numerical simulation of Chu et al. (1995) shown in Figure 1.7, the maximum pressure in Figure 1.5 occurred at the time the ring had halted its axial motion, and, in their experimental study, they determined that the pressure maximum occurred
in the early stages of vortex stretching. This differs from the current study in which the maximum force occurred at the beginning of the vortex stretching before the maximum pressure. The maximum pressure did still correspond to the axial speed of the vortex ring reaching zero.

Using the correlation in Equation 4.3, the force of impact of a vortex ring with a strength and size of those generated by Lucey (2000) can be estimated. Lucey (2000) did not report the circulation of their rings at any of the downstream distances, but the circulation was estimated using \( \Gamma = 2\pi dU \) where \( d = 2 \text{ ft} \) and \( U = 160 \text{ ft/s} \) given at 120 ft downstream. This yields \( \Gamma = 2010 \text{ ft}^2/\text{s} \) or 186 \( \text{m}^2/\text{s} \). With \( \Gamma \) and the density of air, Equation 4.3 estimates \( F = 1208 \text{ N} \) or 272 lb. This is higher than the maximum measured force of 150 lb by Lucey (2000) at 30 ft downstream. The correlation in Equation 4.3 does not account for any changes in circulation due to the distance from the vortex generator as well as only using an estimate of the vortex ring circulation from Lucey (2000).
Summary

This thesis describes a study of the impact force of a vortex ring during a normal collision with a planar wall. A scaling law relating the impact force to ring circulation was derived from simple theoretical considerations. An experimental study was conducted to confirm the predicted relationship using a range of ring strengths.

In this experiment, vortex rings were generated so that they would collide with a plate instrumented to measure impact force and wall pressure at the center of the impact region. The measured force was composed of the impact of the vortex ring, the oscillation of the plate, and vibrations due to piston initiation. The three components were separated by finding the appropriate transfer functions by performing an in situ calibration and applying the transfer functions to the measured force signal.

The motion of the vortex ring tagged with fluorescein dye was captured using high-speed video. The HSV images were analyzed using a code developed in MATLAB to characterize the ring in terms of size, speed, and trajectory. More accurate measurements of the vortex ring flowfield at three instants during impact were made using DPIV. The vector fields from the DPIV were analyzed in another MATLAB code to determine the size, speed, and circulation of the vortex ring. The measurements were first examined to correlate the behavior of the vortex ring as it approaches and collides with the plate with the force and wall pressure waveforms. Then, the validity of the scaling law was
tested as well as some of the important assumptions underlying it.

5.1 Principle Observations and Conclusions

High-speed video sequences of the ring motion were used to construct time series of the vortex ring axial speed and diameter. These measurements showed two regimes of motion. The first regime was a slow, linear decrease in velocity and a small increase in ring diameter. The second regime was a rapid decrease in velocity to zero and a rapid growth in ring diameter. A comparison of the waveforms of vortex ring axial speed and diameter to the impact force and wall pressure show that these two regimes correlated to specific instances in the force and pressure. The maximum impact force occurred concurrent to the regime change of the ring motion. The maximum pressure was found to occur at a later time when the axial speed of the ring had reached zero.

The ring was approximately one ring diameter from the plate when the maximum force occurred. As the rings became larger and slower at the longer nozzle-plate distances, the distance of the ring from the wall at the maximum force decreased. Given that the maximum force essentially coincides with the beginning of the collision, and the maximum wall pressure essentially coincides with the end of the collision, the time between these two peaks could be used to define a collision time scale. This measured interaction time was found to be longer than the estimated time in the scaling argument, but the estimated time does not include the deceleration of the ring as it approached the wall. However, the measured time was on the same order of magnitude as the estimated time and is proportional to it.

The circulation of the vortex rings during impact was compared to the circulation of rings when there was no plate present. It was found that the rings measured at the furthest distance from the wall exhibited minimal plate interference due to being an adequate distance from the plate. The maximum impact force was shown to be proportional to the square of the vortex ring circulation \(R^2 = 0.988\). By deriving
and experimentally testing a scaling law for the vortex ring impact force in a normal impact on a planar surface, the force can be effectively scaled for other applications using the appropriate vortex ring circulation. This was shown by predicting the force using estimates of the circulation of the vortex rings used in a previous study. The correlation overestimated the force of impact compared to the experimental study, but this study only looked at a limited set of cases so further study will be needed.

5.2 Future Work

This study analyzed the relation between the vortex ring strength and the impact force it imparts onto a planar surface during a normal impact. Refinements to the present study, as well as studying other aspects of the collision, should be considered. The impact force was shown to decrease from the nozzle as the ring diameter increased and the axial speed decreased. However, DPIV measurements of the vortex flowfield were only performed at one location, so the circulation should be measured for impacts at different plate locations to ensure that the scaling law holds up for longer distances. In addition, a high speed DPIV system that is capable of continually recording the vortex ring impact would capture the vortex ring flowfield at the time of the maximum force and pressure so that the behavior could be better understood.

To expand on the present work, the impact force at a greater range of Reynolds numbers should be considered. This would require some changes in the piston generator and nozzle to allow formation of higher Reynolds number rings without the destabilization present in the current generator. If the generator was modified to produce stronger rings, then the efficiency of the generator impulse to the impulse of the vortex ring can be studied. The efficiency study could also encapsulate the ratio of the impact force to the momentum of the vortex ring before impact to determine how much ring momentum is diverted in the axial direction.

And finally, if vortex rings were to be used as a non-lethal weapon, both watercraft
and people have non-planar surfaces, so vortex ring collisions in other orientations should be studied. The impact force in the scaling law is due to the deceleration of the ring by re-direction of its axial momentum. If the vortex ring impacts on an angle during an oblique collision or on a curved surface, the force would be proportional to the normal component of axial ring momentum, and the interaction time will be altered. We expect the difference in impact force in such cases compared to the normal planar case should depend more strongly on the normal component of momentum than on the change in interaction time.
A.1 Piston Waveform Processing Code

The code *Pistonwaveforms.m* was written to convert the measured waveforms at the piston, to their equivalents at the nozzle. The saved spreadsheets from *RunServo.m* were selected by first choosing the directory they were located in, and then selecting the files themselves. The diameters of the piston and nozzle needed to be typed in, but were the same for every case. The variable `save` determined if the nozzle equivalent data would be saved. The output filenames and directory need to be written in for each case. The *RunServo.m* spreadsheets were saved incrementally with their name based on the data and *Pistonwaveforms.m* split them into their individual cases.

The code loops through each waveform reading in the data from the spreadsheets. The $\Delta t$ was constant and found from the difference between two points in the time array. The jet velocity at the nozzle was found from the piston velocity by multiplying the piston velocity by the ratio of the piston area to the nozzle area. The nozzle was the only exit from the piston so their velocities were related by their areas (Equation 3.1). The nozzle diameter was 25.4 mm and the piston diameter was 62.7 mm.

The length of the piston increment and the length of the fluid ejected were found by integrating the velocity. The integrated $L_p$ was compared to the measured piston
location and was found to be the same. Two types of integration were done for the
length of the ejected fluid. The first using the command \texttt{cumtrapz} gave the \(L_j\) at each
time step while \texttt{trapz} gave the overall \(L\). The incremental \(L_j\) replaced \(t\) for comparisons
between different waveform types and \(L\) was the reported ejected fluid length.

The maximum jet velocity was found along with the \(L_j\) at which it occurred. Two
average velocities were calculated using \(\bar{U}_j = 1/t \int_0^t U_j \, dt\). The first was the average of
the entire piston motion while the second was only from rest until the maximum velocity
was reached. The piston waveforms recorded about 1 s of data, but the piston motion
occurred for about half of that. The time that the piston comes to rest was found by
comparing each value of the jet velocity to the mean of the next three values. If they
were equal, then the piston was considered to have stopped. The time of the maximum
velocity was already known.

A new spreadsheet was saved with a run specific name and directory containing
the original waveform as measured by the linear encoder plus the jet velocity and jet
length. The process was repeated for every case before moving on to the next part.
The jet velocity and length were plotted verses time as a check of their waveforms. The
old computer and software would sometimes not properly save the waveform so those
instances were noted before moving on to the next section. The improper waveform
numbers were written in as the values of \texttt{bad} and those runs would be replaced with
\texttt{NaN}. By replacing with \texttt{NaN}, they could be ignored when taking the mean and statistical
quantities without changing their run numbers. The average jet velocity and length were
found with the standard deviations. Then using the values for all the cases, the average,
standard deviation, minimum, and maximum were found for \(U_j\), the \(L_j\) of the maximum
jet velocity, the total \(L\), the average using the total time \(U_j\), and the average using the
time to maximum velocity \(U_{j,max}\). Then a spreadsheet was saved for the average nozzle
properties and for the average waveform.

1 \begin{verbatim}

2 \texttt{Pistonwaveforms.m}
3 \texttt{1/13/2009}
4 \texttt{Michael McErlean}
\end{verbatim}
% This code reads in the piston waveform files saved by RunServo.m in
% either .xls or .csv format. It computes the flow properties at the
% nozzle and saves each waveform, the maximum velocity, and an average of
% all the waveforms.

% Data Input. Dialog box appears to select directory and waveform files
pathname=uigetdir;
files=dir(pathname);
selection=listdlg('ListString',{files.name});
datafile={files.name};
selectedfiles=datafile(selection);
datafilename=cellstr(selectedfiles);
run=length(datafilename);
% The Diameter of the piston and nozzle in mm.
Dp=62.7;
Dn=25.4;
rho=998.2;
LD='4';
Height='NP';
Date='27Oct2009';
% Data Output. The output files need to be renamed for case. A directory
% is created within the waveform directory for the processed files.
save='on';
output=[pathname,'\VRP_','Height','_',LD,'_Waveforms_',Date,'.csv'];
output2=[pathname,'\VRP_','Height','_',LD,'_AverageWaveforms_',Date,...
'_''.csv'];
runs=1:1:run;
mkdir(pathname,['\VRP_','Height','_',LD,'_Waveforms_',Date]);
indir=[pathname,'\','\VRP_','Height','_',LD,'_Waveforms_',Date];
head1='Time(s),Xp(mm),Up(mm/s),Ap(mm/s/s),Lj(mm),Uj(mm/s)';
head2='Run,Ujmax(mm/s),L/Dmax,L(mm),Ujbar(mm/s),Ujbarmax(mm/s)';
head3='Time(s),L/D,L/Dstd,L/D+std,L/D-std,Uj(mm/s),Ujstd,Uj+std,Uj-std';
for tally = 1:run
    filename=char(datafilename(tally));
    wavefile=[pathname,'\', filename];
    wavedata=importdata(wavefile);
    time=wavedata.data(:,1);
    Xp=wavedata.data(:,2);
    Up=wavedata.data(:,3);
    Acp=wavedata.data(:,4);
    dt=time(2)-time(1);
    % Nozzle jet velocity from piston velocity and area ratio.
    Uj(tally,:)=Up*(Dp^2/Dn^2)*25.4; % mm/s
    % L at piston integrated from piston velocity
    Lp(tally,:)=cumtrapz(Up)*dt; % mm
    % L at nozzle integrated from nozzle velocity
    Lj(tally,:)=cumtrapz(Uj(tally,:))*dt; % mm
\% Total L at nozzle
\nL(tally) = trapz(Uj(tally,:)) \times dt; \text{ mm}
\\%
\% Maximum jet velocity and L/D of maximum velocity
\nUjmax(tally) = max(Uj(tally,:)); \text{ mm/s}
ind = find(Ujmax(tally) == Uj(tally,:), 1);
LDmax(tally) = Lj(tally, ind) / 25.4;
\%
\% Finds time that piston motion has ended
\nfor jh = 1:length(time)
    if jh + 3 <= length(Xp) && Lj(tally, jh) == mean(Lj(tally, jh + 1:jh + 3))
        Tend(tally) = time(jh);
        break
    else
        Tend(tally) = max(time);
    end
end
\%
\% Average velocity for total piston motion
\nubar(tally) = (1/Tend(tally)) \times \text{trapz}(Uj(tally, 1:jh)) \times dt; \text{ mm/s}
\%
\% Average velocity from rest to maximum jet velocity
\nubarmax(tally) = (1/time(ind)) \times \text{trapz}(Uj(tally, 1:ind)) \times dt; \text{ mm/s}
\%
\% Save the piston and nozzle parameters on a single spreadsheet
\nif strcmp(save, 'on')
    indout = [indir, '\VORTEX_', Date, '_R', num2str(tally, '%0.3d'), '.csv'];
    fib = fopen(indout, 'w');
    fprintf(fib, '%s\r', head1);
    fprintf(fib, '%g,%g,%g,%g,%g,%g\r', [time'; Xp'; Up'; Acp'; ...
    Lj(tally,:); Uj(tally,:)]);
    fclose(fib);
end
\%
\% Plot Uj and Lj to check for aberrant waveforms.
\nfigure(1)
plot(time, Uj)
\%
figure(2)
plot(time, Lj)
pause
\%
\% List the run numbers of bad waveforms to replace with NaN.
\nbad = [10];
\nUj(bad,:) = NaN;
\nL(bad) = NaN;
\nLj(bad,:) = NaN;
\nUjmax(bad) = NaN;
\nLDmax(bad) = NaN;
\nubar(bad) = NaN;
\nubarmax(bad) = NaN;
\%
\% Find Average ignoring NaN.
\nUjmean = nanmean(Uj);
\nUjstd = nanstd(Uj);
\begin{verbatim}
Ujplus = Ujmean + Ujstd;
Ujminus = Ujmean - Ujstd;
Lplus = Lmean + Lstd;
Lminus = Lmean - Lstd;
statout = [nanmean(Ujmax), nanstd(Ujmax), min(Ujmax), max(Ujmax);
    nanmean(LDmax), nanstd(LDmax), min(LDmax), max(LDmax);
    nanmean(L), nanstd(L), min(L), max(L);
    nanmean(ubar), nanstd(ubar), min(ubar), max(ubar);
    nanmean(ubarmax), nanstd(ubarmax), min(ubarmax), max(ubarmax)];

%% % Save average waveforms and properties
if strcmp(save,'on')
    singleout = [runs, 0, 0, 0, 0; Ujmax, statout(1,:);...
                 LDmax, statout(2,:); L, statout(3,:); ubar, statout(4,:);...
                 ubarmax, statout(5,:)];
    fid = fopen(output,'w');
    fprintf(fid,'%s',head2);
    fprintf(fid,'%g,%g,%g,%g,%g', singleout);
    fclose(fid);

    avgout = [time'; Lmean; Lstd; Lplus; Lminus; Ujmean; Ujstd; Ujplus; Ujminus];
    fic = fopen(output2,'w');
    fprintf(fic,'%s',head3);
    fprintf(fic,'%g,%g,%g,%g,%g,%g', avgout);
    fclose(fic);
end
\end{verbatim}

A.2 High Speed Digital Video Calibration Code

The MATLAB code \textit{Imagesequencecalibration.m} was used to calculate the spatial calibration of the high speed camera images and correct any perspective distortion of the images. Both the calibration and the transform were saved for use in the high speed video image processing code \textit{Imagesequenceprocessor.m} described in Appendix A.3. The code required a single calibration image that contained references a known distance apart.

The calibration image was selected using the file selection UI. The dimensions of the image were found to be used later in the calibration. When the image appears five points were chosen to divide the image into four smaller zones. The points were selected using the command \textit{getpts}. By left clicking, five points could be found, right clicking would immediately exit the command. The five points were the guidelines for the sectioning
and needed to be chosen in the correct order which was center, left side, right side, top, and bottom. The points do not need to be exact since the calibration reference points were found after the image was divided.

Once the image was divided, each quadrant was displayed individually after being zoomed to 3x. A dialog box appeared asking for the coordinates of each point. These were the real dimensions, and by default were set to the size of the calibration target. If another reference or only part of the target was visible, then different coordinates would need to be entered. MATLAB numbers image arrays from the top left corner, so to keep the transformed image in the same orientation as the original, the top left corner needs to be (0,0). Once the correct coordinates were entered, getpts was called again to select the reference point. The corners of the target were used to give the widest range of points.

With the real and pixel coordinates of the input points, MATLAB calculated the transformation matrix \( TFORM \) by using \( cp2tform \) which inferred a spatial transformation from pairs of control points. The \textit{projective} transformation type was selected because the image contained parallel lines that appeared tilted. The calibration image was transformed by this matrix and the coordinates of the maximum and minimum \( x \) and \( y \) values for new image were recorded. The maxima and minima were divided by the number of pixels in each dimension to get the calibration factors. The calibration in each direction should be approximately equal because the transform was creating an image on a square grid. The transformation matrix \( TFORM \) and the calibration factors \( \text{calFactors} \) were saved as MATLAB data structures to be used by the high speed video processing code \textit{Imagesequenceprocessor.m} (Appendix A.3).

```
1 % Imagesequencecalibration.m
2  % 9/22/2008
3  % Michael McErlean
4  % This code takes a single input image with points a known distance apart
5  % to calculate a perspective correction and calibration
6  % Calibration Image
7  [calname, pathnamecal] = uigetfile('*.tif');
8  calfilename = [pathnamecal, calname];
```
calimage = imread(calfilename);
[jmax,imax] = size(calimage);
dsave='on';
xy=[0,0;0,0;0,0;0,0];

% Points on the calibration target are selected to divide into smaller
% zones. (1) Center, (2) Left Side, (3) Right Side, (4) Top, (5) Bottom
figure(1)
imshow(calimage)

[jt_junk, it_junk] = getpts;

% The Image is partitioned using the 5 points selected and zoomed in on the
% regions. For each region a single point is chosen with the real
% coordinates of the point. By default the coordinates are set at the size
% of the target and need to be adjusted for other sizes. Points are
% selected by left mouseclicking and hitting enter or just right mouse
% clicking.

% Upper left corner of calibration of target
prompt = {'|*********************************************************|',...'
          |            |','Physical location of mark, X,Y','|     |'};
figure(2)
imshow(calimage(round(jt_junk(4)):round(jt_junk(1)),...
          round(it_junk(2)):round(it_junk(1))),'InitialMagnification',300)
prompt2 = 'Anchor Point Select';
num_lines = 3;
xdef='0';
ydef='0';
def={'Select Upper Left Anchor Point',...'
     ',...'
     xdef,ydef};
answer = inputdlg(prompt,prompt2,num_lines,def);
xy(1,1) = str2num(char(answer(3,1)));
xy(1,2) = str2num(char(answer(4,1)));

[itl,jtl] = getpts;
itl=itl+round(it_junk(2))-1;
jtl=jtl+round(jt_junk(4))-1;

% Upper right corner of calibration target
imshow(calimage(round(jt_junk(4)):round(jt_junk(1)),...
          round(it_junk(1)):round(it_junk(3))),'InitialMagnification',300)
prompt2 = 'Anchor Point Select';
um_lines = 3;
xdef='180';
ydef='0';
def={'Select Upper Right Anchor Point',...'
     ',...'
     xdef,ydef};
answer = inputdlg(prompt,prompt2,num_lines,def);
xy(2,1) = str2num(char(answer(3,1)));
xy(2,2) = str2num(char(answer(4,1)));

[itr,jtr] = getpts;
itr=itr+round(it_junk(1))-1;
itr=itr+round(jt_junk(4))-1;
% Bottom right corner of calibration target
figure(2)
imshow(calimage(round(jt_junk(1)):round(jt_junk(5)),...  
    round(it_junk(1)):round(it_junk(3))),'InitialMagnification',300)
prompt2= 'Anchor Point Select';  
num_lines = 3;  
odef='180';  
ydef='180';  
def={'Select Lower Right Anchor Point',...  
      '' ,...  
      xdef,ydef};  
answer = inputdlg(prompt,prompt2,num_lines,def);
xy(3,1) = str2num(char(answer(3,1)));  
xy(3,2) = str2num(char(answer(4,1)));  
[ilr,jlr]=getpts;  
ilr=ilr+round(it_junk(1))-1;  
jlr=jlr+round(jt_junk(1))-1;
% Bottom left corner of calibration target
figure(2)
imshow(calimage(round(jt_junk(1)):round(jt_junk(5)),...  
    round(it_junk(2)):round(it_junk(1))),'InitialMagnification',300)
prompt2= 'Anchor Point Select';  
num_lines = 3;  
odef='0';  
ydef='180';  
def={'Select Lower Left Anchor Point',...  
      '' ,...  
      xdef,ydef};  
answer = inputdlg(prompt,prompt2,num_lines,def);
xy(4,1) = str2num(char(answer(3,1)));  
xy(4,2) = str2num(char(answer(4,1)));  
[ill,jll]=getpts;  
ill=ill+round(it_junk(2))-1;  
jll=jll+round(jt_junk(1))-1;
% The pixel and real coordinates of the four reference points are combined  
% into two arrays and used to calculate the transformation matrix. The  
% transformation is applied to the image and the calibration found from the  
% transformed dimensions.
input_points=[itl,jtl; itr,jtr; ilr,jlr; ill,jll];  
base_points=double(xy);  
TFORM=cp2tform(input_points,base_points,'projective');  
[CImGrTf,ydt,xdt]=imtransform(calimage,TFORM,'Size',[jmax imax]);  
CalFact_x=(xdt(2)-xdt(1))/jmax;  
CalFact_y=(ydt(2)-ydt(1))/imax;  
calFactors=[CalFact_x,CalFact_y];  
% Saving the Calibration transform
if strcmp(dsave,'on')  
    pathnamevecs = pathnamecal;  
    basename = 'VR_NP_Location1';  
    WriteCalImageFile=[pathnamevecs,basename,'_tran.jpg'];
The high speed video images were saved as a sequence of .tif images in a single file. The image sequences were processed with Imagesequenceprocessor.m using the transformation and calibration from Imagesequencecalibration.m (Appendix A.2). The images were processed to obtain the location, diameter, and velocity of the vortex ring.

The first section of the code read in the image sequence files, the calibration files, and required some inputted data. The framerate and framestart were entered in before the code was run. The framerate was the frames per second that the videos were recorded at and framestart was the frame at which the code should start looking for the ring. The frames containing the vortex rings were determined by opening a sample image sequence and finding the first frame where the ring was visible and giving a 10 frame buffer. Designating a starting frame number reduced the computation time because empty frames were not analyzed.

A reference point was chosen in the transformed images that was at a known location, such as the pressure sensor tip. The coordinates realx and realy could either be the distance from the nozzle or made the origin for the transformed coordinates. The pixel location of the reference point Xref and Yref was found by selecting the point on the transformed background image. The selection of the reference point was not automated so the code needed to be executed to display a transformed image and stopped once the
point was found. The image could be zoomed in to improve accuracy and did not affect the coordinates of the point.

The next part read in the image sequence files by displaying two dialogue boxes. The first was used to pick the directory of the files, while the second chose the files themselves. The transformation and calibration were saved as single files, so they were chosen with their own dialogue boxes. The transformation and calibration were loaded in, and the remainder of the first section found the number of image sequences from the selected files, the $\Delta t$ from the framerate, and created a directory to save the files.

The next section read in and processed the image files by reconstructing each filename. The command `imfinfo` created a structure with information about each image. The size of the structure was used to determine the number of images in the sequence. The background image was found for each sequence by using the first image in the sequence. The dimensions of the background image were found and would be used for the size of subsequent images in the sequence. The transformation $TFORM$ was applied to the background image using `imtransform`. The transformed background image was used to find the reference point and would be subtracted from all the images starting with `framestart`. Finally, a time array based on the framerate was generated to synchronize the images to the WaveBook data. Before analyzing each sequence, arrays for all the output variables were set equal to zero to increase the processing speed.

The code then tested each image in the range `framestart` to `framecount` for the vortex ring. After each image was loaded, the transformation was applied. Then the background image was subtracted using `imsubtract` that did a pixel by pixel intensity subtraction of every point in the image. If a background image pixel was more intense than one in the current image, the subtraction would result in an intensity of 0. Most of the bright areas of the image outside of the ring were removed at this step.

The boundaries of the ring were traced with `bwboundaries`, but the grayscale image needed to be converted to a black and white binary image first. The conversion was done using `im2bw` which uses an intensity threshold above which the pixels were white
and below were black. The intensity threshold was determined using `graythresh` which automatically finds the level that minimizes variance between black and white pixels.

After the conversion to a binary image, white regions outside of the ring appeared. These were from dye or reflections from the plate. The ring would also appear jagged. To smooth the ring and remove the small spots, an averaging filter was applied to the whole image. The command `medfilt2` was a two-dimensional median filter that uses a 7 by 7 neighborhood to compute the median and replace the center pixel. After the noise spots were removed and the ring smoothed, all the boundaries between black and white regions were found using `bwboundaries`. The coordinate pairs of each region boundary would be stored as a separate entry in a structured array. To determine which region was the ring, the next section checked each one for size and average intensity.

The regions were analyzed individually. First, their size was found by the number points on the boundary. To exclude small areas that were large enough to survive the filtering, but still too small to be the vortex ring, a minimum size of 150 points was needed for intensity of the region to be analyzed. Two blank images were created that would be used to isolate the region being analyzed. The first was a binary image on which the coordinates of the boundary being analyzed were traced. The region was filled in with `imfill` and the coordinates of the boundary and filled region were found. Then the second image was a grayscale on which the intensities of the background subtracted ring image corresponding to the points just found were imported. The average intensity of the region was found by taking the sum of the intensity of the second image and dividing by the number of filled points.

Once the average intensities for all the large regions were found, then the size and intensities were compared. The ring should have been the largest region and was checked to see if it was the brightest. If it was then, that region was identified as the ring. If another region was large enough and bright enough to be the ring, then it was the ring.

Once the region of the ring was identified, the location and size of the ring could be computed. Another blank image was created on which the only the ring region boundary
was copied. The top and bottom points of the ring were found using the \textit{min} and \textit{max} commands on the boundary coordinates. The rings did not come to a clean point like diagram shows in Figure 3.13, so the average of the $x$-coordinate of all the minima and maxima were used to define the top and bottom. The average was rounded to an integer pixel value to correspond to a point on the image. The ring was approximated as an ellipse, and the major axis was the distance between the top and bottom points on the boundary. Then, two points 10\% of the major axis in from each side were found. From those two points, the front and back sides of the boundary were located. The locations of the front and back ring boundaries were averaged to get the approximate coordinates of the ring cores ($x_1, y_1$) and ($x_2, y_2$). The centroid of the ring was the mean of the cores at ($x_c, y_c$). The minor axis that approximated the core diameter was found along the horizontal at ($x_1, y_1$) and ($x_2, y_2$) and the two values were averaged to give a single value for the ring. The tilt angle between the two core locations was found using the four quadrant tangent function \textit{atan2}.

The position of the ring centroid was converted to a real location using the calibration factors and the distance from the pixel reference point. The distance between the coordinates of the centroid and the reference point were found, then multiplied by the calibration factors and added to the real location. The two axes were multiplied by the calibration factors only. The velocity of the ring was found from the centroid positions across frames after all the images in the sequence were analyzed. To reduce the effect of jitter in the centroid position on the velocity, the centroid locations were smoothed with a fourth order moving average filter. To reduce the noise further, the fourth order central difference in Equation 3.3 was used to find the velocity.

After the velocity was calculated, a spreadsheet was saved for the sequence with the time, real positions of the vortex, the major axis length, the tilt angle, ring velocity, and minor axis length. The spreadsheets would be analyzed using the MATLAB codes \textit{VortexRingImpactForce.m} and \textit{ImpactPlateProperties.m} in Appendices A.6 and A.7.
%, % 1/16/2009
% Michael McErlean
% Image processing code for highspeed video. Requires calibration files
% from Imagesequencecalibration.m
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% User Input Section
framerate=200; % Camera framerate
framestart=15; % First frame to analyze for a ring

realx=496.0620; % Horizontal distance of reference point from nozzle
realy=11.43; % Vertical distance of reference point from nozzle
Xref=611; % Horizontal pixel location of reference point
Yref=208; % Vertical pixel location of reference point
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% This part opens dialogue boxes to choose the image sequences and the
% calibration files

% Image Sequence Files
prompt1= 'select dir with Image Sequences';
pathnameimages = uigetdir(‘’,prompt1);
files = dir(pathnameimages);
imagefiles = {files.name};
selection1 = listdlg(‘ListString’,imagefiles);
selectedfiles = imagefiles(selection1);
imagefilenames = cellstr(selectedfiles); % List of files to be used

% Transformation and Calibration Files
[TFFilename,TFPath]=uigetfile(‘*.mat’,’Select TFORM file’,pathnameimages);
TFFile=[TFPath,TFFilename];
TFORMall=load(TFFile);
TFORM=TFORMall.TFORM;
[CalFilename,CalPath]=uigetfile(‘*.mat’,’Select CalFactor file’,TFPath);
CalFile=[CalPath,CalFilename];
calFactors_Struct=load(CalFile);
calFactors=calFactors_Struct.calFactors;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
run = length(imagefilenames); % Number of image sequences
dt = 1/framerate; % time between frames based on framerate
% Output files
mkdir(pathnameimages,’Analysis’)
save=’on’;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% Main part of the code where tally increments through all the selected
% image files
for tally=1:run
% Build filenames of the image sequences
filename=[pathnameimages ‘\’ char(imagefilenames(tally))];
framenum=imfinfo(filename);
framecount=length(framenum); % number of frames in the sequence
bkgdim=imread(filename,1); % background image
[X,Y]=size(bkgdim); % Dimensions of images
% Apply transformation to background
106

BkgdImTr=imtransform(bkgdim,TFORM,'Size',[X Y]);

time=0:dt:(framecount-1)*dt; % Create a time array for image sequence

clear bkgdim framenum

% Initialize arrays to improve code speed
average_intens = zeros(1,framecount);
xcentreal = zeros(1,framecount);
ycentreal = zeros(1,framecount);
mycentvel_x = zeros(1,framecount);
mycentvel_y = zeros(1,framecount);
majoraxis = zeros(1,framecount);
dcore = zeros(1,framecount);
myangle = zeros(1,framecount);
mycent = zeros(framecount,2);

% For each sequence cycle through all the images beginning at
% framestart.
for frame = framestart:framecount
    % Read in images one frame at a time
    [image,map]=imread(filename,frame);
    % Import calibration transform and apply to image
    ImGrTf=imtransform(image,TFORM,'Size',[X Y]);

    % Remove background from image with imsubtract. Imsubtract does a
    % point by point subtraction of the intensity values.
    ring = imsubtract(ImGrTf,BkgdImTr);

    % Convert to a black and white binary image. The second argument of
    % im2bw controls the intensity threshold above which is made white
    % in the binary image.
    level=graythresh(ring);
    BWimage=im2bw(ring,level);

    % Filter BW image to remove small spots by averaging over
    % surrounding pixels.
    BWfiltered=medfilt2(BWimage,[7,7]);

    % Trace boundaries in BW image. The boundary coordinates are saved
    % in cell array B and the binary image is saved as K.
    [B,K]=bwboundaries(BWfiltered,'noholes');

    clear image ImGrTf BWimage

% Identify ring in image based on size and intensity analysis
bound=0;
prince=0;
sizering=0;
% check to see if a boundary was found
if ~isempty(B)
    for regnum=1:length(B) % cycle through all the white zones
        inbound=B{regnum}; % create an array from the cells
        sizering(regnum)=length(inbound); % spot size
        % Check the intensity if the spot size is large enough.
        if sizering(regnum)>150 % arbitrary spot size
            ringboundi=logical(zeros(X,Y)); % create a black image

% End script
intensi=uint8(zeros(X,Y)); % create a black image

% For the coordinates of the boundary, plot on the new black image ringboundi
for coord=1:length(inbound)
    ringboundi(inbound(coord,1),inbound(coord,2))=1;
end

% Use imfill to create a filled region
ringboundi_filled=imfill(ringboundi,'holes');

% Find coordinates of all the white pixels on ringboundi
[d,g] = find(ringboundi_filled == 1);

% Use the coordinates found by d and g to create the array intensi with the intensity values from the background subtracted image for the region.
for pointi=1:length(d)
    intensi(d(pointi),g(pointi))=ring(d(pointi),g(pointi));
end

% Sum up the total intensity and divide by region size to get the average intensity.
average_intensi(regnum)=sum(sum(intensi))/length(d);

% Define bound to be the boundary of the ring if it meets the criteria below
if (prince(1)==prince2) && (length(B{prince(1)})>150)
    bound=B{prince(1)};
elseif (prince(1)~=prince2) && (length(B{prince2})>150)
    bound=B{prince2};
elseif (prince(1)~=prince2) && (length(B{prince2})>150)
    bound=B{prince(1)};
end

% Create a BW image with the ring boundary
ringbound=logical(zeros(X,Y));
for h=1:length(bound)
    ringbound(bound(h,1),bound(h,2))=1;
end

% Identify distinct points on the ring to find centroid
ymin=max(bound(:,1)); % Top point of ring
ymax=min(bound(:,1)); % Bottom point of ring
ypoint1=find(bound(:,1)==ymin); % Coordinates of top
ypoint2=find(bound(:,1)==ymax); % Coordinates of bottom

% In case the ring does not come to a clean point, xpoint11, xpoint12, xpoint21, and xpoint 22 find any
other points that have the same y-coordinate as ypoint1 and ypoint2.

% xpoint1 and xpoint2 are averages of the points found above. Round keeps everything an integer pixel value.

xpoint1=(min(xpoint11,xpoint12))+
    round(abs(xpoint11-xpoint12)/2);

% To define the front and back edges of the ring, first 10% of the ring diameter is found. Then points 10% in from ypoint1 and ypoint2 are found.

dy=round(0.1*(ymin-ymax)); % 10% of diameter

% The x-coordinates of the points found the same way as the points on the edges of the ring

The axes, angle, and the centroids are all converted to real coordinates by using the calibration factors.

The centroids are smoothed to eliminate some jitter

% Fourth-order central difference is used on the centroids with the dt
% taken from the framerate
for v=3:framecount-2
    mycentvel_x(v)=(smoothcent(v-2,1)-8*smoothcent(v-1,1)+...
                    8*smoothcent(v+1,1)-smoothcent(v+2,1))/(12*dt);
    mycentvel_y(v)=(smoothcent(v-2,2)-8*smoothcent(v-1,2)+...
                    8*smoothcent(v+1,2)-smoothcent(v+2,2))/(12*dt);
end

if strcmp(save,'on')
    output=[pathnameimages,'\Analysis\',strrep(imagefilenames{tally},...
                          '.tif','.csv')];
    header='Time(s),XR(mm),YR(mm),DR(mm),Theta(deg),UR(mm/s),VR(mm/s),dc(mm)';
    dataout=[time;xcentreal;ycentreal;majoraxis;myangle;mycentvel_x;...
             mycentvel_y;dcore];
    fid=fopen(output,'w');
    fprintf(fid,'%s
',header);
    fprintf(fid,'%g,%g,%g,%g,%g,%g,%g,%g
',dataout);
    fclose(fid);
end

clear bkgdimage ImGrTf imageKernel image ring BWimage BWfiltered B K
clear ringbound ringbound_filled intens total_intens
clear time xcentreal ycentreal majoraxis myangle mycentvel_x

clear mycentvel_y dcore_intens
end

A.4 DPIV Vector Analysis Code

The processing of the DPIV images in Insight 3G resulted in .vec files that were
imported into MATLAB with PIVaverage.m and analyzed to obtain the ring diameter,
core diameter, ring speed, and core circulation. The .vec files contained five columns
of data. The first two were the $x$- and $y$-coordinates of the vector grid, the second two
were the $u$- and $v$-velocities, while the fifth was the Vector Choice Code (CHC). The
CHC contained information about the quality of the vector and the different codes used
are listed in Table A.1. For the processing settings used, only three of the codes were
common. All of the code vectors were a 5 because of the vector smoothing. Bad vectors
were indicated by a -1 and there were very few of those due to the interpolation. The
final code used was a -2 for the grid points that were covered by the mask such as where
the impact plate was. The mask did not cause the vector grid to be smaller, it only
made the values equal to zero.
Table A.1. Description of Vector Choice Codes used by Insight 3G.

<table>
<thead>
<tr>
<th>CHC</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Highest correlation peak used for vector</td>
</tr>
<tr>
<td>2</td>
<td>2nd highest correlation peak used for vector</td>
</tr>
<tr>
<td>3</td>
<td>3rd highest correlation peak used for vector</td>
</tr>
<tr>
<td>4</td>
<td>Interpolated vector</td>
</tr>
<tr>
<td>5</td>
<td>Smoothed Measured Vector; was a 1, 2, or 3 before smoothing</td>
</tr>
<tr>
<td>0</td>
<td>Temporally Blank. Did not pass validation and is awaiting interpolation</td>
</tr>
<tr>
<td>-1</td>
<td>SNR Fail. Vector was removed for not passing SNR validation</td>
</tr>
<tr>
<td>-2</td>
<td>Removed point. Point has been disabled.</td>
</tr>
<tr>
<td>-3</td>
<td>Bounds, 2-D vector is out of image bounds after spot offset</td>
</tr>
<tr>
<td>-4</td>
<td>Vector has not been set</td>
</tr>
</tbody>
</table>

The vector files were opened using the select directory and then files UI. The tolerance of the asymptotic value of circulation was set with tol. A separate directory was created for the processed spreadsheets. All of the data arrays were initialized with NaN instead of zeros because the NaN was ignored for averaging. Then each vector file was opened individually.

The columns of data are converted into an individual 2-D array for each one to correspond to their actual orientation and the velocity vectors. Bad vectors marked by a CHC of -1 were replaced with a mean of the two nearest neighbors in each direction. The vectors needed to be replaced because holes or large jumps in velocity caused the vorticity to be incorrect when it was calculated later. The vectors marked with a -2 were used to determine the boundaries of the vector field. Vectors that would be outside of the boundaries would have zero velocity and cause a large vorticity on the boundary. The $\Delta x$ and $\Delta y$ spacing of the vector grid was computed by taking the difference between two points.

For a 2-D velocity field, the vorticity is only in the third orthogonal direction and is given by

$$\omega = \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right).$$

(A.1)

The derivatives of the velocity field at every point were found using a second order central difference in the subfunction vorticity. The vorticity was not computed for vectors that were outside of the boundary defined by the CHC. The vorticity along the edges of the...
grid were noisy from velocity vectors missing correlation peaks so the vorticity was set equal to zero for the first three grid points on the edges.

The locations of the two vortex core centers were found based on their vorticity. A reference point at the center of the vector field was chosen and the weighted average of vorticity was found relative to the reference point:

\[
x_{cv} = \frac{\sum_{i=1}^{M} |\omega_i| (x_i - x_{ref})}{\sum_{i=1}^{M} |\omega_i|}, \quad y_{cv} = \frac{\sum_{i=1}^{N} |\omega_i| (y_i - y_{ref})}{\sum_{i=1}^{N} |\omega_i|}.
\]  

(A.2)

A threshold level of vorticity was set to exclude noise. The minimum and maximum values of vorticity were found, one core was the minimum, and the other was the maximum. Only vorticity values that were greater than 40% their respective maximum were used in the weighting. The weighted vorticity for each point was found that met the criteria and the total vorticity was summed. When the weighted values were divided by the totals, the distance to each core from the reference point was found. The distance was converted to the grid reference frame to give the centroids of the cores. The ring diameter was the distance between the core centroids. The weighted average of vorticity gave an exact centroid, so the nearest grid point was found for further computations.

The propagation velocity of the ring was found by taking a cross section of velocity through the core center both in \(x\) and \(y\). By taking a slice through in the \(y\) direction and plotting \(u\) for points along the slice, the velocity had a maximum and minimum on either side of the core with a nearly linear change in the center. Taking a slice through in the \(x\) direction and using \(v\) showed a similar profile, though of smaller magnitude. The velocity at the core center could be estimated by taking the average of the minimum and maximum. For an overall ring propagation speed, the velocity from both cores was averaged together. To reduce the effect of a noisy vector on the velocity profile, three slices were taken across the core in each direction and the results were averaged. The
propagation velocity was subtracted from the vector field to put them in the vortex frame and reference and used to calculate the circulation of the cores.

The circulation of both cores can be found using either the vorticity or the velocity

\[
\Gamma = \int \oint_S \vec{\omega} \cdot d\vec{S} = \oint_C \vec{v} \cdot d\vec{s}. \tag{A.3}
\]

The closed loop that defines region \(S\) can be any one that encloses all the vorticity of the core. Each core was integrated separately using the area integral of vorticity and the path integral of velocity. Both integrals were taken as a comparison however the path integral of velocity was the value of circulation reported. To take the integral, a square region was defined with its center at the reference point closest to the ring core centroid. The vorticity integral was taken numerically by multiplying each vorticity value by \(\Delta x \Delta y\). The values of vorticity on the edges of the region required a smaller area and the function \(\text{areacorrector}\) created a matrix of areas to multiply each vorticity by. The matrix had the full sized areas in the center, half sized on the straight edges, and one-quarter sized on the corners. The velocity integral was taken numerically by integrating the tangential velocities on the region bounding using \(\text{trapz}\) and summing all four sides. After computing both integrals, the region sized was increased. The region initially began as a 3 by 3 grid and by successively enlarging the region the calculated circulation increase until the region was large enough to contain entire vorticity of the core.

When the entire core was contained, the calculated circulation leveled off and stayed constant to within the noise. The asymptotic value of circulation was found for both integration methods. The asymptote was determined by comparing a value of circulation with the mean of the next three values. If the percent difference was less than that specified by \(\text{tol}\) then that was taken as the circulation of the core. The region size of the circulation asymptote was used as the diameter of the ring core.

1 \% PIVaverage3.m
% Code to import .vec files. The vortex ring cores are located using the
% weight average of vorticity and the diameter is the distance between the
% cores. The ring velocity is found from the average of slices through the
% cores, and the circulation is calculated from the integral of velocity
% around a square path. The diameter of the core is determined from the
% asymptote of the circulation.

global dx dy imax jmax

prompt1='select Vector file directory';
pathnamevecs=uigetdir(' ',prompt1);
Files=dir(pathnamevecs);
VecFiles={Files.name};
prompt2='Select Vector files to process';
selection=listdlg('PromptString',prompt2,'ListString', VecFiles);
selectedFiles=VecFiles(selection);
vecfilenames=cellstr(selectedFiles);
save='on'; % Save conditions. off doesn't save, on saves
tol=0.01; % circulation tolerance
rho=998.2; % Density of water at 20 C

% Create a new directory for processed spreadsheets
if strcmp(save,'on')
    [output,path]=uiputfile('*.csv','Save file name',pathnamevecs);
end

% Initialize arrays. NaN is used in place of zero to prevent averaging of
% those values which don't exist.
NumFiles=length(vecfilenames);
u3=zeros(imax,jmax,NumFiles);
v3=zeros(imax,jmax,NumFiles);
x3=zeros(imax,jmax,NumFiles);
y3=zeros(imax,jmax,NumFiles);
TSICnt=zeros(imax,jmax,NumFiles);
cvx(1:NumFiles)=NaN;
cvy(1:NumFiles)=NaN;
cvxu(1:NumFiles)=NaN;
cvyu(1:NumFiles)=NaN;
DR(1:NumFiles)=NaN;
TA(1:NumFiles)=NaN;
Ucore(1:NumFiles)=NaN;
Vcore(1:NumFiles)=NaN;
BotX(1:NumFiles)=NaN;
BotY(1:NumFiles)=NaN;
TopX(1:NumFiles)=NaN;
TopY(1:NumFiles)=NaN;
Circvortl(1:NumFiles)=NaN;
Circvell(1:NumFiles)=NaN;
Circvortu(1:NumFiles)=NaN;
Circvelu(1:NumFiles)=NaN;
dvell(1:NumFiles)=NaN;
dvortl(1:NumFiles)=NaN;
dvelu(1:NumFiles)=NaN;
dvr(1:NumFiles)=NaN;
Iimpl(1:NumFiles)=NaN;
Iimpu(1:NumFiles)=NaN;
Iimp(1:NumFiles)=NaN;
Iche(1:NumFiles)=NaN;


h=waitbar(0,'0% Complete','Name','Vector Processing Progress');
baddie=1;

for Inum=39%1:NumFiles
    filename=char(vecfilenames(Inum));
    VecFile=[pathnamevecs,'\',filename];
    vecdata=importdata(VecFile);
    % Dimensions of PIV vector files
    imax=find(max(vecdata.data(:,1))==vecdata.data(:,1),1);
    jmax=length(vecdata.data)/imax;
    % VecFile saves the data as a five column array.
    % Column 1 is x-coordinates
    % Column 2 is y-coordinates
    % Column 3 is x-velocity
    % Column 4 is y-velocity
    % Column 5 is vector choice code
    for j=1:jmax
        for i=1:imax
            ind=(j-1)*imax+i;
            x3(i,j,Inum)=vecdata.data(ind,1); % in mm
            y3(i,j,Inum)=vecdata.data(ind,2); % in mm
            v3(i,j,Inum)=vecdata.data(ind,4); % m/s
            u3(i,j,Inum)=vecdata.data(ind,3); % m/s
            TSICnt(i,j,Inum)=vecdata.data(ind,5);
        end
    end
    % Create 2-D arrays to simplify calculations
    x=x3(:,:,Inum); % in mm
    y=y3(:,:,Inum); % in mm
    u=u3(:,:,Inum); % m/s
    v=v3(:,:,Inum); % m/s
    CntM=TSICnt(:,:,Inum);
    % The vectors with CHC = -3 or -1 cause large gradients when the
    % derivatives are taken for the vorticity. To prevent this, the bad
    % vectors are replaced with an average of surrounding vectors. For
    % areas of multiple bad vectors the mean is found iteratively. Not a
    % final method.
    [CntIndI,CntIndJ]=find(CntM==-1);
    for bv=1:length(CntIndI)
        oldi=CntIndI(bv);
        oldj=CntIndJ(bv);
        if oldi>2 && oldi<imax-2 && oldj>2 && oldj<jmax-2
            u(oldi,oldj)=(u(oldi-2,oldj)+u(oldi-1,oldj)+u(oldi+1,oldj)+u(oldi+2,oldj))/4;
        end
    end

\[
\begin{align*}
    &+u(\text{oldi}, \text{oldj}+1) + u(\text{oldi}, \text{oldj}) + u(\text{oldi}, \text{oldj}+2)/8; \\
    &v(\text{oldi}, \text{oldj}) = (v(\text{oldi}+2, \text{oldj}) + v(\text{oldi}+1, \text{oldj}) + v(\text{oldi}, \text{oldj})) \\
    &+ v(\text{oldi}+2, \text{oldj}) + v(\text{oldi}, \text{oldj}+1) + v(\text{oldi}, \text{oldj}+1)/8; \\
    &\text{for } \text{cv} = \text{b} + \text{length}(\text{CntIndI}) \\
    &\text{newi} = \text{CntIndI}(\text{cv}) \\
    &\text{newj} = \text{CntIndJ}(\text{cv}) \\
    &\text{if } (x(\text{newi}, \text{newj}) - x(\text{oldi}, \text{oldj}) == 1 && y(\text{oldi}, \text{oldj}) == y(\text{newi}, \text{newj})) \\
    &\quad || (y(\text{newi}, \text{newj}) - y(\text{oldi}, \text{oldj}) == 1 && x(\text{oldi}, \text{oldj}) == x(\text{newi}, \text{newj})) \\
    &\quad u(\text{newi}, \text{newj}) = (u(\text{newi}+2, \text{newj}) + u(\text{newi}+1, \text{newj}) + u(\text{newi}+1, \text{newj}) + u(\text{newi}+2, \text{newj}) + u(\text{newi}+1, \text{newj}) + u(\text{newi}, \text{newj})) / 8; \\
    &\quad v(\text{newi}, \text{newj}) = (v(\text{newi}+2, \text{newj}) + v(\text{newi}+1, \text{newj}) + v(\text{newi}+1, \text{newj}) + v(\text{newi}+2, \text{newj}) + v(\text{newi}+1, \text{newj}) + v(\text{newi}, \text{newj})) / 8; \\
    &\quad \text{end} \\
    &\text{end} \\
    &u(\text{oldi}, \text{oldj}) = (u(\text{oldi}+2, \text{oldj}) + u(\text{oldi}+1, \text{oldj}) + u(\text{oldi}+1, \text{oldj}) + u(\text{oldi}, \text{oldj}+2)/8; \\
    &v(\text{oldi}, \text{oldj}) = (v(\text{oldi}+2, \text{oldj}) + v(\text{oldi}+1, \text{oldj}) + v(\text{oldi}+1, \text{oldj}) + v(\text{oldi}+2, \text{oldj}) + v(\text{oldi}+1, \text{oldj}) + v(\text{oldi}, \text{oldj}+2)/8; \\
    &\text{end} \\
    &\text{end} \\
    &\text{The edge of the masked region is found and set as the limit} \\
    &\text{Jbound} = \text{find}(\text{CntM}(1, :)==-2, 1); \\
    &\text{Ibound} = \text{find}(\text{CntM}(:, 1)==-2, 1); \\
    &\text{The distance between points in x and y are found.} \\
    &\text{dx} = \text{abs}(x(1, 1)-x(2, 1)); \quad \text{in mm} \\
    &\text{dy} = \text{abs}(y(1, 1)-y(1, 2)); \quad \text{in mm} \\
    &\text{vorticity is computed using vorticity.m and limits found above} \\
    &[\text{vort}] = \text{vorticity}(\text{u}, \text{v}, \text{Ibound}, \text{Jbound}); \\
    &\text{The coordinates of the vortex center are found using the weighted} \\
    &\text{average of vorticity} \\
    &\text{The reference point is chosen as the center of the vector field} \\
    &\text{xref} = x(\text{round}(\text{imax}/2), \text{round}(\text{jmax}/2)); \\
    &\text{yref} = y(\text{round}(\text{imax}/2), \text{round}(\text{jmax}/2)); \\
    &\text{Initialize the arrays} \\
    &\text{totvortl} = 0.; \quad \text{total vorticity in the lower core} \\
    &\text{totvortu} = 0.; \quad \text{total vorticity in the upper core} \\
    &\text{wrxl} = \text{zeros}(\text{imax}, \text{jmax}); \quad \text{weighted value of lower core vorticity in x} \\
    &\text{wryl} = \text{zeros}(\text{imax}, \text{jmax}); \quad \text{weighted value of lower core vorticity in y} \\
    &\text{wrxu} = \text{zeros}(\text{imax}, \text{jmax}); \quad \text{weighted value of upper core vorticity in x} \\
    &\text{wryu} = \text{zeros}(\text{imax}, \text{jmax}); \quad \text{weighted value of upper core vorticity in y} \\
    &\text{For every point in the vector field, the weighted value of vorticity} \\
    &\text{is found if it is greater than 40\% of the peak value and the} \\
    &\text{vorticity at the point is added to the total.} \\
    &\text{for } i = 1: \text{imax} \\
    &\quad \text{for } j = 1: \text{jmax} \\
    &\end{align*}
\]
if vort (i, j) < 0.6*min(min(vort)) && j > jmax/2 && i > imax/2
  rxl = x(i, j) - xref;
  ryl = y(i, j) - yref;
  wrxl(i, j) = abs(vort(i, j)) * rxl;
  wryl(i, j) = abs(vort(i, j)) * ryl;
  totvortl = totvortl + abs(vort(i, j));
elseif vort(i, j) > 0.6*max(max(vort)) && j < jmax/2 && i > imax/2
  rxu = x(i, j) - xref;
  ruy = y(i, j) - yref;
  wrxu(i, j) = abs(vort(i, j)) * rxu;
  wryu(i, j) = abs(vort(i, j)) * ruy;
  totvortu = totvortu + abs(vort(i, j));
end
end

% Locations of the centroids of vorticity.

% lower core x
cvxl(Inum) = sum(sum(wrxl))/totvortl + xref;
% lower core y
cvyl(Inum) = sum(sum(wryl))/totvortl + yref;
% upper core x
cvxu(Inum) = sum(sum(wrxu))/totvortu + xref;
% upper core y
cvyu(Inum) = sum(sum(wryu))/totvortu + yref;

% Ring diameter as the distance between the centroids.
DR(Inum) = sqrt((cvyl(Inum) - cvyu(Inum))^2 + (cvxl(Inum) - cvxu(Inum))^2);

% Tilt angle between the core centroids.
TA(Inum) = atan2((cvyl(Inum) - cvyu(Inum)), (cvxl(Inum) - cvxu(Inum)));

% The nearest grid point to the centroids are found using dsearch.m
% lower core x
vMnI = dsearchn(x(:,1), cvxl(Inum));
% lower core y
vMnJ = dsearchn(yp(:,1), cvyl(Inum));
% upper core x
vMxI = dsearchn(x(:,1), cvxu(Inum));
% upper core y
vMxJ = dsearchn(yp(:,1), cvyu(Inum));

% lower core x grid point
BotX(Inum) = x(vMnI, vMnJ);
% lower core y grid point
BotY(Inum) = y(vMnI, vMnJ);
% upper core x grid point
TopX(Inum) = x(vMxI, vMxJ);
% upper core y grid point
TopY(Inum) = y(vMxI, vMxJ);

% The core velocity is found by taking the average of the maximum
% velocity on either side of the core. The velocity of both cores are
% averaged to get a ring velocity.

if vMnI == 1 || vMxI == 1
  Ucorel(Inum) = NaN;
  Ucoreu(Inum) = NaN;
  Vcorel(Inum) = NaN;
  Vcoreu(Inum) = NaN;
else
  sweep = 2;
  for jh = 1:2*sweep+1
    umin1(jh) = min(u(vMnI+jh-sweep-1, vMnJ-round((vMnJ-vMxJ)/2):jmax));
    umax1(jh) = max(u(vMnI+jh-sweep-1, vMnJ-round((vMnJ-vMxJ)/2):jmax));
    umin2(jh) = min(u(vMxI+jh-sweep-1, vMxJ+round((vMnJ-vMxJ)/2)));
    umax2(jh) = max(u(vMxI+jh-sweep-1, vMxJ+round((vMnJ-vMxJ)/2)));
    vmin1(jh) = min(v(:, vMnJ+jh-sweep-1));
    vmin2(jh) = min(v(:, vMxJ+jh-sweep-1));
    vmax1(jh) = max(v(:, vMnJ+jh-sweep-1));
    vmax2(jh) = max(v(:, vMxJ+jh-sweep-1));
  end
end
Ucorel(Inum)=mean2([umin1;umax1]);
Ucoreu(Inum)=mean2([umin2;umax2]);
Vcorel(Inum)=mean2([vmin1;vmax1]);
Vcoreu(Inum)=mean2([vmin2;vmax2]);

end

Ucore(Inum)=mean([Ucorel(Inum),Ucoreu(Inum)]);
Vcore(Inum)=mean([Vcorel(Inum),Vcoreu(Inum)]);

% The core velocity is subtracted from the entire vector field to give
% the velocity in the ring reference frame.
u2=u-Ucore(Inum);
v2=v-Vcore(Inum);

% The circulation is found by integrating the vorticity over a region
% and by the integral of the velocity along the boundary of the region.
% These are calculated to successively larger region sizes. The
% function areacorrector.m is used to account for the corners and
% boundaries of the region not being the same area as the central ones.
% The regions are limited by the edges of the zero mask.
Circsl(1:round(jmax/2))=NaN;
Circvl(1:round(jmax/2))=NaN;
Circsu(1:round(jmax/2))=NaN;
Circvu(1:round(jmax/2))=NaN;
for l=1:round(jmax/2)
    if vMnI+l<=imax-2 && vMnJ+l<=jmax-2 && vMnI-l>=3 && vMnJ-l>=3
        S=areacorrector(l,l);
        Circsl(l)=abs(sum(sum(vort(vMnI-l:vMnI+1,...
            vMnJ-l:vMnJ+1).*S))/1000^2);
        Circvl(l)=abs(sum(trapz(v2(vMnI+l,vMnJ-l:vMnJ+l))*dy...
            +trapz(u2(vMnI-l:vMnI+1,vMnJ+1))*dx...
            -trapz(v2(vMnI-l,vMnJ-l:vMnJ+1))*dy...
            -trapz(u2(vMnI-l:vMnI+1,vMnJ-l))*dx)/1000);
        clear S
    end
    if vMxI+l<=imax-2 && vMxJ+l<=jmax-2 && vMxI-l>=3 && vMxJ-l>=3
        S=areacorrector(l,l);
        Circsu(l)=abs(sum(sum(vort(vMxI-l:vMxI+1,...
            vMxJ-l:vMxJ+1).*S))/1000^2);
        Circvu(l)=abs(sum(trapz(v2(vMxI+l,vMxJ-l:vMxJ+l))*dy...
            +trapz(u2(vMxI-l:vMxI+1,vMxJ+1))*dx...
            -trapz(v2(vMxI-l,vMxJ-l:vMxJ+1))*dy...
            -trapz(u2(vMxI-l:vMxI+1,vMxJ-l))*dx)/1000);
        clear S
    end
end

% The circulation of the ring is determined from where it no longer
% changes. The current circulation is compared to the average of the
% next three values and if the difference is less than tol, it is used.
% The velocity of the core is taken at the same index and a diameter
% based on the integration region is also found.
[Circvortl(Inum),mvortl,dvortl(Inum)]=circulation(Circsl,tol);
if vMnI˜=1 && vMxI˜=1 && ~isnan(mvell) && ~isnan(mvelu)
    ringcentx=round(mean([vMxI,vMnI]));
    ringcenty=round(mean([vMxJ,vMnJ]));
    for ix1=vMnI-mvell:vMnI+mvell
        for jy1=ringcenty:vMnJ+mvell
            r5=(y(ix1,jy1)-y(ringcentx,ringcenty))/1e3;
            Itempl(ix1+1-vMnI+mvell,jy1+1-ringcenty)=...
                vort(ix1,jy1)*r5^2*dx*dy/1e6;
            Etempl(ix1+1-vMnI+mvell,jy1+1-ringcenty)=...
                r5*u2(ix1,jy1)^2*dx*dy/1e6;
        end
    end
    for ix2=vMxI-mvelu:vMxI+mvelu
        for jy2=vMxJ-mvelu:ringcenty
            r6=(y(ix2,jy2)-y(ringcentx,ringcenty))/1e3;
            Itempu(ix2+1-vMxI+mvelu,jy2+1-vMxJ+mvelu)=...
                vort(ix2,jy2)*r6^2*dx*dy/1e6;
            Etempu(ix2+1-vMxI+mvelu,jy2+1-vMxJ+mvelu)=...
                r6*u2(ix1,jy1)^2*dx*dy/1e6;
        end
    end
    Iimpl(Inum)=abs(rho*pi*(sum(sum(Itempl))));
    Iimpu(Inum)=abs(rho*pi*(sum(sum(Itempu))));
    Iimp(Inum)=mean([Iimpl(Inum),Iimpu(Inum)]);
    Iche(Inum)=rho*pi*(DR(Inum))^2*mean([Circvell(Inum),...]
        Circvelu(Inum)])/4e6;
    El(Inum)=abs(rho*pi*(sum(sum(Etempl))));
    Eu(Inum)=abs(rho*pi*(sum(sum(Etempu))));
    Ering(Inum)=mean([El(Inum),Eu(Inum)]);
    alpha(Inum)=Ering(Inum)/(Iimp(Inum)^(.5)*mean([Circvell(Inum),...]
        Circvelu(Inum)]^(3/2));
else
    Iimpl(Inum)=NaN;
    Iimpu(Inum)=NaN;
    Iimp(Inum)=NaN;
    Iche(Inum)=NaN;
end

clear Itempl Itempu Etempl Etempu
waitbar(Inum/NumFiles,h,[num2str(round(Inum/NumFiles*100)),'% Complete']);
end
delete(h)

if strcmp(save,'on')
    outfile=[path,'\',output];
    header1='Run,Xcvl,Ycvl,Xcvu,Ycvu,DR,TA,Ulower,Uupper,Ucore,Vlower,';
    header2='Vupper,Vcore,Circvell,Circvortl,Circvelu,Circvortu,dvell,';
    header3='dvortl,dvelu,dvortu,Iimpl,Iimpu,Iimp,Iest';
    runs=1:NumFiles;
    dataout=[runs;cvxl;cvyl;cvxu;cvyu;DR;TA;Ucorel;Ucoreu;Ucore;Vcorel;...
% Function to calculate vorticity of a vector field
function [vort] = vorticity(u, v, ib, jb)
    global imax jmax dx dy
dudy = zeros(imax, jmax);
dvdx = zeros(imax, jmax);
    dudy(1:3,:) = 0.0;
dudy(:,1:9) = 0.0;
dudy(ib-2:ib,:) = 0.0;
dudy(:,jb-9:jb) = 0.0;
dvdx(1:3,:) = 0.0;
dvdx(:,1:9) = 0.0;
dvdx(ib-2:ib,:) = 0.0;
dvdx(:,jb-9:jb) = 0.0;
    vort = (dvdx - dudy);
end
% Function to give correct area of integration
function S = areacorrector(m, l)
global dx dy
S(1:2*m+1, 1:2*l+1) = dx*dy;
S(:,1) = .5*dx*dy;
S(:,2*m+1) = .5*dx*dy;
S(1,:) = .5*dx*dy;
S(2*m+1,:) = .5*dx*dy;
S(1,1) = .25*dx*dy;
S(2*m+1,1) = .25*dx*dy;
S(1,2*m+1) = .25*dx*dy;
S(2*m+1,2*m+1) = .25*dx*dy;
% Function to determine the asymptotic circulation
function [circ,mout,dia]=circulation(circin,tol)
    global dx
    for m=2:length(circin)-3
        circinmean=(circin(m+1)+circin(m+2)+circin(m+3))/3;
        if abs((circinmean-circin(m))/circin(m))<=tol
            circ=circin(m);
            mout=m;
            dia=2*m*dx;
            break
        else 
            circ=NaN;
            mout=NaN;
            dia=NaN;
        end
    end
end

A.5 Force Sensor Calibration Code

The transfer functions between the piston and the force sensor and of the impact assembly were found using the same MATLAB code SystemTransferFunction.m. The code had the option of calculating only one transfer function or both with hamtf and pistf. The impact assembly transfer function would only be found if hamtf was set to on and the piston-force sensor transfer function was found when pistf was set to on. To find the transfer functions, the calibration data was selected with a UI. The sampling frequency, sample time, sensor calibrations, and the moment arms of the impact assembly needed to be inputted manually.

The transfer functions were calculated separately but the procedure for both was the same. Each file was read in one at a time and the data displayed. The plot was to check for extra hits with the hammer, external vibrations, or incorrect piston motion. If the data was clean, then a dialogue box appeared asking to use it. Only the runs where Yes was selected would be used to compute the transfer function.

For the impact assembly, the hammer and force sensor were multiplied by their calibrations. The hammer had a 10x gain on the signal conditioner that was also accounted
for. The calibrated force data was multiplied by the moment arm to get the actual force on the system. The DC offset of the hammer and force sensor was found by taking the mean of the data pre-trigger and subtracting it from the whole series. A Hann window was applied to the data before the FFT was taken, so the hammer impulse needed to be in the center of the time series. If the time series was triggered so that the impulse occurred in the middle, this step was ignored. For non-centered triggered time series, a new longer time series that was twice the length of the original was formed by padding the beginning and ends of the original series with zeros to place the impulse in the center.

The FFT of the centered time series was computed using `transform` which multiplied the series by a Hann window beforehand. The FFT of each case was saved before the transfer function was found. The transfer function was the cross-spectrum of the force sensor and hammer over the auto-spectrum of the hammer. The mean of all the transfer functions was taken. The transfer function was saved to a spreadsheet to be used in `VortexRingImpactForce.m`. The spreadsheet could not interpret imaginary numbers so the transfer function was split into its real and imaginary parts and recombined in the next code.

The piston-force sensor transfer function followed a similar procedure, only the calibrations of the accelerometer and force sensor were not needed. The piston transfer function was saved to a separate spreadsheet than the impact assembly transfer function.

```matlab
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% SystemTransferFunction.m
% 10/29/2008
% Michael McErlean
% This uses the hammer calibration and the piston calibration data to find
% the transfer functions for the two components. One file is generated for
% the Hammer Calibration, while the piston calibrations are for each L/D.
% The imported data is graphed for review. The figures are on pause so that
% the data can be scrutinized, and when unpause a dialog box appears
% asking if the data should be kept. If it is, it is processed, otherwise a
% note is made that the data was not used.
% Switch if only one transfer function needs to be found
hamtf='on';
pistf='on';
```
% Hammer Calibration Input Files
if strcmp(hamtf,'on')
    prompt1='Select Hammer Calibration File Directory';
    pathnameham=uigetdir('',prompt1);
    hamfilelist=dir(pathnameham);
    hamfiles={hamfilelist.name};
    prompt2='Select Hammer Calibration Files to Process';
    hamselection=listdlg('PromptString',prompt2,'ListString', hamfiles);
    selectedhamfiles=hamfiles(hamselection);
    hamfilenames=cellstr(selectedhamfiles);
    hamnum=length(hamfilenames);
end
%
% Piston Calibration Input Files
if strcmp(pistf,'on')
    LD=10;
    prompt3=['Select Piston Calibration File Directory for L/D=',num2str(LD)];
    pispath=uigetdir(' ',prompt3);
    pisfilelist=dir(pispath);
    pisfiles={pisfilelist.name};
    prompt4=['Select Piston Calibration Files to Process for L/D=',num2str(LD)];
    pisselection=listdlg('PromptString',prompt4,'ListString', pisfiles);
    selectedpisfiles=pisfiles(pisselection);
    pisfilenames(1:length(pisselection))=cellstr(selectedpisfiles);
    pisnum=length(pisfilenames);
    pathnamepis=cellstr(pispath);
end
%
% Output
datasave='on';
%
% System Variables
fs=800; % Sampling frequency
dt=1/fs; % dt based on sampling frequency
T=16.; % Sample time of ring impacts
TT=T*fs; % Total number of Samples
hamcal=0.01173; % Hammer Calibration in V/N
sencal=0.1147; % Force sensor calibration in V/N
L1=65; % Moment arm from force sensor to hinge
L2=530; % Moment arm from hammer hit location to hinge
moment=L1/L2; % Ratio of moment arms
%
% Target Transfer Function
if strcmp(hamtf,'on')
    hamgood=1.;
    hambad=NaN;
    for ham=1:hamnum
        hamcalname=char(hamfilenames(ham));
        hamfile=[pathnameham,'\',hamcalname];
        hamdata=importdata(hamfile);
        hammer=hamdata.data(:,2); % Column in hamdata.data with hammer
hamforce=hamdata.data(:,3); % Column in hamdata.data with force sensor

% Construct time and frequency based on data length
datalength=length(hamforce);
t=-1:dt:(datalength-dt)/fs-1;

% freq definition depends on the length of the pretrigger
freq=fs*[0:TT/2,-TT/2+1:-1]/(TT);
freq=fs*[0:TT,-TT+1:-1]/(2*TT);

% Plots data to determine if it should be used. Code Pauses on graph so you can zoom in and adjust to get a better view. When satisfied press enter to bring up the dialog box to use the data.
figure(1)
plot(t,hammer,'b',t,hamforce,'r')
title(hamcalname,'Interpreter','none')
xlabel('Time (sec)')
ylabel('Voltage (V)')
legend('Impact Hammer','Force Sensor')
xlim([0 5])
axes('XMinorTick','on')
grid on
pause
button=questdlg('Use Data?');

if strcmp(button,'Yes')
hammercal=hammer/(10*hamcal); % 10 is from gain.
hamforcecal=moment*hamforce/sencal; % Input force with moment arm
hammax=find(max(hammercal)==hammercal,1); % Time of maximum force
hz=mean(hammercal(1:.5/dt)); % Finds DC offset
hammercaladj=hammercal-hz; % Removes DC offset

% Creates a new series with impulse in center and zeros padding
hammercal2=[zeros(TT-hammax,1);hammercaladj;zeros(hammax+(TT-datalength),1)]; % Non-centered data
hammercal2=hammercaladj; % Use for triggered centered data
hfz=mean(hamforcecal(1:.5/dt)); % Repeat for hammer.

hamforceadj=hamforcecal-hfz;

hamforce2=hamforceadj; % Use for triggered centered data
% Use for non centered data
hamforce2=[zeros(TT-hammax,1);hamforceadj;zeros(hammax+... (TT-datalength),1)];

% The FFT is taken with transform that also windows the data
DFThammer(hamgood,:)=transform(hammercal2,dt);
DFThamforce(hamgood,:)=transform(hamforce2,dt);
hamgood=hamgood+1.; % Increment counter
else
if isnan(hambad)
hambad=ham;
else hambad=[hambad,ham];
end
end

Shh=DFThammer.*conj(DFThammer); % Auto-Spectrum of Hammer
Sfh=DFThamforce.*conj(DFThammer); % Cross-Spectrum
Shf=DFThammer.*conj(DFThamforce);
Sff=DFThamforce.*conj(DFThamforce);
Hhamforce=mean(Shf./Sff); % Transfer function as a mean of all.

% Data output formatting
headerham1='Frequency (Hz),Hammer-Force Sensor Transfer Function(Real),';
headerham2='Hammer-Force Sensor Transfer Function(Imag)';
Hhamforcereal=real(Hhamforce);
Hhamforceimag=imag(Hhamforce);
hamout=[freq;Hhamforcereal;Hhamforceimag];
if strcmp(datasave,'on')
    [hamoutfilename,hamoutfilepath]=uiputfile('*.csv','...
    'Save Hammer-Force Sensor Transfer Function as',...
    [pathnameham,'\','Hamforce_TF.csv']);
    hamoutfile=[hamoutfilepath,hamoutfilename];
    fic=fopen(hamoutfile,'w');
    fprintf(fic,'%s\r',headerham1,headerham2);
    fprintf(fic,'%g,%g,%g\r',hamout);
    fclose(fic);
end
end

%%
if strcmp(pistf,'on')
% _________________________________________________________________________
% Piston Transfer Function
pissgood=1.;
pisbad=NaN;
for pis=1:pisnum
    piscalname=char(pisfilenames(pis));
    pisfile=[char(pathnamepis),'\',piscalname];
    pisdata=importdata(pisfile);
    pisforce=pisdata.data(:,3);
    accelerometer=pisdata.data(:,4);
    datalength=length(pisforce);
t=1:dt:(datalength-dt)/fs-1;
    freq definition depends on triggering
    freq=fs*[0:TT/2,-TT/2+1:-1]/(TT);
    freq=fs*[0:TT,-TT+1:-1]/(2*TT);
    % Plot data and determine if time series are good
    figure(2)
    plot(t-15,accelerometer,'b',t-15,pisforce,'r')
    title(piscalname,'Interpreter','none')
    legend('Accelerometer','Force Sensor')
    legend('Accelerometer','Force Sensor')
    xlabel('Time (s)')
    ylabel('Voltage (V)')
    xlim([0 3])
    grid on
    pause
    button=questdlg('Use Data?');
    if strcmp(button,'Yes')
        % Create new time series with the impulse in the center and zeros
        % padding

acmin = find(min(min(accelerometer)) == accelerometer, 1);
aczero = mean(accelerometer(1:.5/dt));
acceladj = accelerometer - aczero;
accel2 = acceladj; % Use for trigger centered data
accel2 = [zeros(TT-acmin,1);acceladj;zeros(acmin+(TT-datalength),1)];
pisforcecal = pisforce / sencal;
pfzero = mean(pisforcecal(1:.5/dt));
pfadj = pisforcecal - pfzero;
pisforce2 = pfadj; % Use for trigger centered data
pisforce2 = [zeros(TT-acmin,1);pfadj;zeros(acmin+(TT-datalength),1)];

% The FFT is taken with transform that also windows the data
DFTpisforce(pisgood,:) = transform(pisforce2, dt);
DFTaccelerometer(pisgood,:) = transform(accel2, dt);
pisgood = pisgood + 1;
else
  if isnan(pisbad)
    pisbad = pis;
  else pisbad = [pisbad, pis];
  end
end

% Auto-spectrum of accelerometer
Saa = DFTaccelerometer .* conj(DFTaccelerometer);
Sfa = DFTpisforce .* conj(DFTaccelerometer);
Sff = DFTpisforce .* conj(DFTpisforce);

Hpisforce = mean(Sfa ./ Saa);

headerpis1 = 'Frequency (Hz),Piston-Force Sensor Transfer Function(Real),'
headerpis2 = 'Piston-Force Sensor Transfer Function(Imag)'
Hpisforcereal = real(Hpisforce);
Hpisforceimag = imag(Hpisforce);
pisout = [freq; Hpisforcereal; Hpisforceimag];
if strcmp(datasave,'on')
  [pisoutfilename, pisoutfilepath] = uiputfile('*.csv', ...
  ['Save Piston-Force Sensor Transfer Function for L/D=', ...
  num2str(LD)], [char(pathnamepis), '\', ...]
  'Pisforce_TF_LD', num2str(LD)])
  pisoutfile = [pisoutfilepath, pisoutfilename];
  fib = fopen(pisoutfile, 'w');
  fprintf(fib, '%s\r', headerpis1, headerpis2);
  fprintf(fib, '%g,%g,%g\r', pisout);
  fclose(fib);
end

function DFTsignal = transform(input, dt)
  hw = hann(length(input));
  signalout = input .* hw;
  DFTsignal = fft(signalout) * dt;
end
A.6 Vortex Ring Impact Force Deconvolution Code

The impact force of the vortex ring was deconvolved from the measured force using the MATLAB code `VortexRingImpactForce.m`. The code needed the measured WaveBook data, the transfer functions saved from `SystemTransferFunction.m`, and the processed high speed video data. In addition to calculating the impact force, the code found the delay time between piston initiation and motion and subtracted the delay from the time arrays of both the WaveBook and HSV data. The output was a spreadsheet containing all the HSV, WaveBook, and deconvolved force synchronized to the piston motion in a new subfolder. The input files were all selected from a UI and the sampling frequencies, sensor calibrations, and moment arms were inputted manually.

The WaveBook DAQ data was read in first and the length was found. The combined data set saved at the end needed all the arrays to be the same length so arrays for the HSV as long as the DAQ data were created by filling with NaN. Then the HSV data was only read into the beginning of the arrays. The timestamp from WaveBook did not save in a format that could be easily read in, so a time array was created based on total length and pre-trigger time.

Next, the WaveBook data had its DC offset removed by finding the mean of the pre-trigger data and the calibration was applied to the force sensor. A Hann window was applied to the data before the FFT was taken, so the ring impact needed to be in the center of the window. The impact time of the ring was not easily distinguishable on the measured force, so the maximum pressure was used as the center of the time series. All of the WaveBook data was moved into new arrays twice as long as the original and padded with zeros. Then the FFT was taken of the shifted data using `transform` which applied a Hann window beforehand.

To find the moment arm for the ring impact, the position of the ring before impact was found. The pressure sensor at the center of the impact plate was set as the origin during the high speed video processing so the distance of the ring from the plate was
known. Using the index of the maximum force, the time that it occurred was found, and the index of the HSV time closest to the DAQ time was found using `dsearchn`. The indices of when the ring centroid was between 75 to 100 mm from the location of the centroid when the maximum pressure occurred where found. Then the mean $y$-centroid in that range was calculated for the height of the ring. The height of the ring above or below the pressure sensor was added to the distance of the pressure sensor from the hinge to give the moment arm for that ring.

The impact force was deconvolved with `VRdeconv` using the transformed force sensor data, the transformed accelerometer data, the piston transfer function, the impact assembly transfer function, the $\Delta t$ of the DAQ based on the sampling frequency, and the moment arm of the ring from the HSV.

The piston delay time was found from the accelerometer by using `diff` on the whole series. The new series was the simple difference between two points in the array and was one data point shorter. The minimum in the difference occurred during the initial piston acceleration. Moving backwards along the series from the minimum, the time of motion was determined from where the difference became greater than -.005 or the difference between a point and the next point backwards was .005 or less. The trigger pulse occurred at zero, so the time of motion was the same as the delay time and subtracted from both the DAQ time array and the HSV time array. Each piston delay was different and found individually.

A spreadsheet containing the original WaveBook data, with the HSV data extended to be the same size, the deconvolved force, and the piston delay corrected time arrays were all saved for each run to be used in `ImpactPlateProperties.m`.

```plaintext
1 % VortexRingImpactForce.m
2 % 10/29/2008
3 % Michael McErlean
4 % This code reads in the WaveBook DAQ files, piston transfer function,
5 % impact assembly transfer function, and HSV files. The impact force of
6 % the vortex ring is calculated using the transfer functions and the
7 % location of the impact is found from the HSV. The delay time between
8 % initiation and motion is found and subtracted from the time arrays. A
9 % spreadsheet containing all of the combined data is saved.
```
% DAQ Input Files
LD = 10;
prompt1 = ['Select Force Sensor File Directory for L/D=', num2str(LD)];
senpath = uigetdir(' ', prompt1);
senfilelist = dir(senpath);
senfiles = {senfilelist.name};
prompt2 = ['Select Force Sensor Files to Process for L/D=', num2str(LD)];
senselection = listdlg('PromptString', prompt2, 'ListString', senfiles);
selectedsenfiles = senfiles(senselection);

% Piston Transfer Function
[pffilename, pathnamepf] = uigetfile('*.csv',
    ['Select Piston Transfer Function File for L/D=', num2str(LD)]);
pffile = [pathnamepf, pffilename];
pfdata = importdata(pffile);
Hpisforcereal = pfdata.data(:, 2);
Hpisforceimag = pfdata.data(:, 3);
Hpisforce = Hpisforcereal + 1i * Hpisforceimag;

% Impact Assembly Transfer Function File
[ttfilename, pathnamett] = uigetfile('*.csv',
    ['Select Impact Transfer Function File for L/D=', num2str(LD)]);
ttdata = importdata(ttfile);
Hhamforcereal = ttdata.data(:, 2);
Hhamforceimag = ttdata.data(:, 3);
Hhamforce = Hhamforcereal + 1i * Hhamforceimag;

% HSV Files
hsv = 'on';
if strcmp(hsv, 'on')
prompt3 = ['Select HSV File Directory for L/D=', num2str(LD)];
hsvp = uigetdir(' ', prompt3);
hsvfilelist = dir(hsvp);
hsvfiles = {hsvfilelist.name};
prompt4 = ['Select HSV Files to Process for L/D=', num2str(LD)];
hsvselection = listdlg('PromptString', prompt4, 'ListString', hsvfiles);
selectedhsvfiles = hsvfiles(hsvselection);

% Data Acquisition Parameters
fsdaq = 800; % Sampling frequency
dt = 1/fsdaq;
sencal = 0.1147; % Force sensor calibration in V/N
prescal = 0.006697; % Pressure sensor calibration in V/kPa
L1 = 65; % Vertical distance between force sensor and hinge (mm)
Py = 530.; % Distance from hinge to pressure sensor (mm)
fr = 200; % HSV frame rate
save='on';
headerhsv1='Timehsv(s),Xcent(mm),Ycent(mm),DR(mm),U(mm/s),V(mm/s),dc(mm),';
headerhsv2='Timedaq(s),Trig(V),Sforce(N),Pres(kPa),Accel(V),Rforce(N)';
headerdaq='Timedaq(s),Trig(V),Sforce(N),Pres(kPa),Accel(V),Rforce(N)';
mkdir(char(pathnamesen),'Deconvolved Data');
idir=[char(pathnamesen),'/','Deconvolved Data'];
for tally=1:sennum
    senname=char(senfilenames(tally));
    senfile=[char(pathnamesen),'/',senname];
    daqdata=importdata(senfile);
    trigger(tally,:)=daqdata.data(:,2);
    senforce(tally,:)=daqdata.data(:,3);
    pressure(tally,:)=daqdata.data(:,5);
    accel(tally,:)=daqdata.data(:,4);
    DAQ=size(daqdata.data,1);
    Yimp=0;
    datalength=length(trigger);
    timedaq=-1:dt:(16-dt);
    if strcmp(hsv,'on')
        % Create arrays for the HSV data as long as the DAQ data
        timehsv(tally,1:DAQ)=NaN;
        xcent(tally,1:DAQ)=NaN;
        ycent(tally,1:DAQ)=NaN;
        DR(tally,1:DAQ)=NaN;
        xvel(tally,1:DAQ)=NaN;
        yvel(tally,1:DAQ)=NaN;
        dc(tally,1:DAQ)=NaN;
        hsvname=char(hsvfilenames(tally));
        hsvfile=[char(pathnamehsv),'/',hsvname];
        hsvdata=importdata(hsvfile);
        HSVl=size(hsvdata.data,1);
        timehsv(tally,1:HSVl)=hsvdata.data(:,1);
        xcent(tally,1:HSVl)=hsvdata.data(:,2);
        ycent(tally,1:HSVl)=hsvdata.data(:,3);
        DR(tally,1:HSVl)=hsvdata.data(:,4);
        xvel(tally,1:HSVl)=hsvdata.data(:,6);
        yvel(tally,1:HSVl)=hsvdata.data(:,7);
        dc(tally,1:HSVl)=hsvdata.data(:,8);
        timedaq=-16:dt:(16-dt);
        % The DC offset is corrected in the Wavebook data by subtracting the
        % mean before triggering.
        % The data is shifted to be windowed in the transform functions. The
        % pressure and trigger are shifted as well to keep the time correct.
        acz=mean(accel(tally,1:.5/dt));
        accadj=accel(tally,:)-acz;
        senforcecal=senforce(tally,:)/sencal;
        senfz=mean(senforcecal(1:.5/dt));
        senfadj=senforcecal-senfz;
pz=mean(pressure(tally,1:.5/dt)/prescal);
padj=pressure(tally,:)/prescal-pz;
pressureshift(tally,:)=padj;
senforceshift(tally,:)=senfadj;
accelshift(tally,:)=acadj;
triggershift(tally,:)=trigger(tally,:);
timedaqshift(tally,:)=timedaq;
% The index of the maximum pressure peak is found to use as the point
% around which the Hann window is centered.
pmax=find(max(padj)==padj,1);
pressureshift(tally,:)=zeros(1,datalength-pmax),padj,zeros(1,pmax);
% A rough estimate of the impact time is found to take the average of
% the y-centroid before impact
rh=dsearchn(timehsv(tally,:)',timedaq(pmax));
xh1=dsearchn(xcent(tally,:)',xcent(tally,rh)+100);
xh2=dsearchn(xcent(tally,:)',xcent(tally,rh)+75);
Yimp=nanmean(ycent(tally,xh1:xh2));
if isnan(Yimp)
  Yimp=0;
else
  acz=mean(accel(tally,1:.5/dt));
  acadj=accel(tally,:)-acz;
  senforcecal=senforce(tally,:)/sencal;
  senfz=mean(senforcecal(1:.5/dt));
  senfadj=senforcecal-senfz;
  pz=mean(pressure(tally,1:.5/dt)/prescal);
  padj=pressure(tally,:)/prescal-pz;
  pmax=find(max(padj)==padj,1);
  pressureshift(tally,:)=zeros(1,datalength-pmax),padj,zeros(1,pmax);
  senforceshift(tally,:)=zeros(1,datalength-pmax),sendifz,...
  zeros(1,pmax);
  accelshift(tally,:)=zeros(1,datalength-pmax),acadj,zeros(1,pmax));
  triggershift(tally,:)=zeros(1,datalength-pmax),trigger(tally,:),...
  zeros(1,pmax));
  timedaqshift(tally,:)=timedaq(1)-(datalength-pmax)*dt:dt:15+pmax*dt-dt;
end
L2=Py-Yimp;
marm=L1/L2; % Ratio of moment arms
% The impact force is found using the functions transform and VRdeconv.
DFTaccel=transform(accelshift(tally,:)',dt);
DFTsenforce=transform(senforceshift(tally,:)',dt);
DA(tally,:)=DFTaccel;
DS(tally,:)=DFTsenforce;
ringforceshift(tally,:)=VRdeconv(DFTsenforce,DFTaccel,Hpisforce,...
Hhamforce,dt,marm);
% Piston initiation is found by locating the maximum acceleration
% caused by the motion, and find the point that is 2% of the max. Then
% the index of that point is recorded.
diffaccel=diff(accelshift(tally,:));
acind=12000-1+find(min(accelshift(tally,12000:12500))==...
accelshift(tally,12000:12500),1);
acindi=acindi-1;
for sw=acindi(1)-1:-1:1
    if abs(diffaccel(sw)-diffaccel(sw-1))<=0.01 &
        accelshift(tally,sw+1)>=-.03 &
        diffaccel(sw)>=-.008
        aczero(tally)=sw+1;
        break
    end
end

% The index of the trigger pulse is found and the difference between
% the piston initiation and the trigger is calculated and the time
% difference is subtracted off of the original time arrays.
tpind(tally)=find(triggershift(tally,:)>2.5,1);
tdif(tally)=aczero(tally)-tpind(tally);

% The time arrays are adjusted for the time difference between
% triggering and initiation.
if strcmp(hsv,'on')
timehsvshift(tally,:)=timehsv(tally,:)-tdif(tally)/fsdaq;
end
timaqshift2(tally,:)=timedaqshift(tally,:)-tdif(tally)/fsdaq;

if strcmp(save,'on')
    if strcmp(hsv,'on')
iout=[idir,'\',hsvname];
idataout=[timehsvshift(tally,:);xcent(tally,:);ycent(tally,:);...  
        DR(tally,:);xvel(tally,:);yvel(tally,:);dc(tally,:);...  
        timedaqshift2(tally,:);triggershift(tally,:);...
        senforceshift(tally,:);pressureshift(tally,:);....  
        accelshift(tally,:);ringforceshift(tally,:)];
    fid=fopen(iout,'w');
    fprintf(fid,'%s\r',headerhsv1,headerhsv2);
    fprintf(fid,'%g,%g,%g,%g,%g,%g,%g,%g,%g,%g,%g,%g\r',idataout);
    fclose(fid);
    else
        iout=[idir,'\',strrep(senname,'.TXT','.csv')];
idataout=[timedaqshift2(tally,:);triggershift(tally,:);...
        senforceshift(tally,:);pressureshift(tally,:);....
        accelshift(tally,:);ringforceshift(tally,:)];
    fid=fopen(iout,'w');
    fprintf(fid,'%s\r',headerdaq);
    fprintf(fid,'%7f,%g,%g,%g,%g\r',idataout);
    fclose(fid);
    end
end

freq=fsdaq*[0:DAQ,-DAQ+1:-1]/(2*DAQ);
vpis=Hpisforce.*DFTaccel;
vp=ifft(vpis/dt);
vf=Hhamforce.*(marm*(DFTsenforce-vpis));
vx=(DFTsenforce-vpis);
vft=ifft(vf/dt);
vxt=ifft(vx/dt);
[b,a]=butter(5,4/400);
vfft=filtfilt(b,a,vft);
vfftx=fft(vfft)*dt;
end

% Function finds the FFT of the time series input. The DC-offset is first removed by averaging the first 12 seconds of data.
function DFTsignal=transform(input,dt)
    hw=hann(length(input));
    signalout=input.*hw;
    DFTsignal=fft(signalout)*dt;
end

% Using the FFT of the inputs and outputs along with the transfer function, the time series of the impact force is found.
function VRforce=VRdeconv(DFTforce,DFTacc,Hpis,Hham,dt,moment)
    vpiscon=DFTacc.*Hpis;
    vforcefx=(DFTforce-vpiscon)./Hham;
    VRforce=moment*ifft(vforcefx/dt);
end

A.7 Synchronization and Averaging Code

The combined data was averaged in the MATLAB code ImpactPlateProperties.m. The files were opened in a UI and no other data needed to be inputted. The averaging function in MATLAB could take the mean down the columns of the arrays, so the data arrays needed to be lined up so that a point in the array corresponded to the same time. The array indices closest to 0 were found for both the DAQ time and the HSV time using dsearchn. These were found for all the runs before finding the index with the lowest value.

New data arrays of identical size to the originals were created with zeros. Then the indices of all the original data arrays were shifted to match the lowest index value. The HSV DAQ arrays shifts were computed separately because of the different sampling frequencies used. Then the maximum force, maximum pressure, the impact time, the rise time of the force and pressure, the fall time of the force and pressure, and the time of the maximum force and pressure were found.

The force and pressure data was noisy so 5th order low-pass Butterworth filters were
used on them. The force was filtered at 4 Hz while the pressure was filtered at 30 Hz. Then the process of finding the maximum force, maximum pressure, impact time, rise time, fall time, and time of the impacts was repeated for the filtered data.

Next, the ring diameter and velocity before impact as well as the distance from the plate of the maximum force and pressure were found. The index of the time of the maximum force was used as the starting point and the velocity and diameter of the 20 points prior to that were averaged together. The HSV sampling rate was lower than the DAQ rate so the piston motion could have begun between frames. In those cases the distance of the ring from the plate was linearly interpolated between the two closest frames. The same calculations were done for the filtered data as well.

The arrays bad and bad2 were used to remove runs from being averaged. Runs were included in bad when the accelerometer gave no clear signal for when the piston motion began. Zero was added to bad to allow proper code execution. Even when the accelerometer gave a clear signal, if the piston delay time could not be properly measured and those runs were included in bad2. The runs were checked by plotting the shifted values of acceleration. Zero could not be included in bad2 because there was no zero array index in MATLAB. The zeros from the shifting and the data for the bad runs were replaced with NaN before the averages were found. The average values of all the important parameters of the unfiltered and filtered data were saved on spreadsheets. An average waveform of force, pressure, velocity, diameter, and location for unfiltered and filtered was also saved.
prompt2='Select deconvolved impact force files to process';
hsvelection=listdlg('PromptString',prompt2,'ListString',hFiles);
hsvelectedFiles=hFiles(hsvelection);
hsvfilenames=cellstr(hsvelectedFiles);
HS=length(hsvfilenames);
 hsv='on'; % turn on for HSV data

% Create headers for the saved data
headhsv1='Timehsv(s),xcent(mm),ycent(mm),DR(mm),xvel(mm/s),yvel(mm/s),';
headhsv2='dc(mm),Timedaq(s),trigger(V),senforce(V),pressure(V),accel(V),';
headhsv3='ringforce(F),'
headdaq='Timedaq(s),trigger(V),senforce(V),press(V),accel(V),ringforce(N),';
headtem1='Run,Max force(N),Max Pressure(kPa),Impact time(s),';
headtem2='Fx(mm),Px(mm),average DR(mm),average UR(mm/s),';
headtemdaq='Run,Max force(N),Max Pressure(kPa),Impact time(s),Impulse(Ns),';
save='on';

for tally=1:HS
    hsvfilename=char(hsvfilenames(tally));
    hsvfile=[hsvname,'\',hsvfilename];
    hsvdata=importdata(hsvfile);
    if strcmp(hsv,'on')
        % Create arrays for all the data
        timehsv(tally,:)=hsvdata.data(:,1); % Time from HSV
        xcent(tally,:)=hsvdata.data(:,2); % x-centroid of ring from HSV
        ycent(tally,:)=hsvdata.data(:,3); % y-centroid of ring from HSV
        DR(tally,:)=hsvdata.data(:,4); % ring diameter from HSV
        xvel(tally,:)=hsvdata.data(:,5); % ring speed in x direction from HSV
        yvel(tally,:)=hsvdata.data(:,6); % ring speed in y direction from HSV
        dc(tally,:)=hsvdata.data(:,7); % core diameter from HSV
        timedaq(tally,:)=hsvdata.data(:,8); % time from Wavebook DAQ
        trigger(tally,:)=hsvdata.data(:,9); % trigger time series
        senforce(tally,:)=hsvdata.data(:,10); % force sensor time series
        pressure(tally,:)=hsvdata.data(:,11); % pressure sensor time series
        accel(tally,:)=hsvdata.data(:,12); % accelerometer time series
        ringforce(tally,:)=hsvdata.data(:,13); % deconvolved force time series
        thsvint(tally)=dsearchn(timehsv(tally,:),0);
    else
        % For use when there is no HSV data
        timehsv(tally,:)=hsvdata.data(:,1);
        trigger(tally,:)=hsvdata.data(:,2);
        senforce(tally,:)=hsvdata.data(:,3);
        pressure(tally,:)=hsvdata.data(:,4);
        accel(tally,:)=hsvdata.data(:,5);
        ringforce(tally,:)=hsvdata.data(:,6);
        tdaqint(tally)=dsearchn(timedaq(tally,:),0);
    end
end
pause

% Temporal Averaging
% list of bad files found from processing
bad2=[2,10,16,35,39];
tdaqint2=tdaqint;
% replace time indices of bad files
tdaqint2(bad2)=NaN;
tdaqintmin=min(tdaqint2);
DAQ=size(timedaq,2);
% initialize arrays
timedaq3=zeros(HS,DAQ);
trigger3=zeros(HS,DAQ);
senforce3=zeros(HS,DAQ);
pressure3=zeros(HS,DAQ);
accel3=zeros(HS,DAQ);
ringforce3=zeros(HS,DAQ);
I=zeros(1,HS);
If=zeros(1,HS);
if strcmp(hsv,'on')
    thsvint2=thsvint;
    thsvint2(bad2)=NaN;
    thsvintmin=min(thsvint2);
timehsv3=zeros(HS,DAQ);
xcent3=zeros(HS,DAQ);
ycent3=zeros(HS,DAQ);
DR3=zeros(HS,DAQ);
xvel3=zeros(HS,DAQ);
yvel3=zeros(HS,DAQ);
dc3=zeros(HS,DAQ);
end
% add 0 to prevent crash
bad=[bad2,0];
% Loop through the data
for gally=1:HS
    if bad(bady)==gally
        bady=bady+1;
    else
        timedaq3(gally,1:DAQ-(tdaqint2(gally)-tdaqintmin))=timedaq(gally,...
        tdaqint2(gally)-tdaqintmin+1:DAQ);
        trigger3(gally,1:DAQ-(tdaqint2(gally)-tdaqintmin))=trigger(gally,...
        tdaqint2(gally)-tdaqintmin+1:DAQ);
        senforce3(gally,1:DAQ-(tdaqint2(gally)-tdaqintmin))=senforce(gally,...
        tdaqint2(gally)-tdaqintmin+1:DAQ);
        pressure3(gally,1:DAQ-(tdaqint2(gally)-tdaqintmin))=pressure(gally,...
        tdaqint2(gally)-tdaqintmin+1:DAQ);
        accel3(gally,1:DAQ-(tdaqint2(gally)-tdaqintmin))=accel(gally,...
        tdaqint2(gally)-tdaqintmin+1:DAQ);
        ringforce3(gally,1:DAQ-(tdaqint2(gally)-tdaqintmin))=ringforce(gally,...
        tdaqint2(gally)-tdaqintmin+1:DAQ);
        timedaqt=timedaq3;
timedaqt(timedaqt==0)=NaN;
ints=dsearchn(timedaqt(gally,:),0);
dt=(timedaqt(gally,2)-timedaqt(gally,1));
125  mf3(gally)=max(ringforce3(gally,12500:DAQ)); % Maximum force
126  imf3(gally)=12500-1+find(mf3(gally)==ringforce3(gally,12500:DAQ),1);
127  mp3(gally)=max(pressure3(gally,:)); % Maximum pressure
128  imp3(gally)=find(mp3(gally)==pressure3(gally,:),1);
129  % time between max force and pressure
130  timp3(gally)=timedaq3(gally,imp3(gally))-timedaq3(gally,imf3(gally));
131  % Rise time force
132  trisef3(gally)=timedaq3(gally,imf3(gally))-timedaq3(gally,1+imf3(gally)-...
133  % Fall time force
134  tfallf3(gally)=timedaq3(gally,imf3(gally)-1+find(ringforce3(gally,...
135  % Rise time Pressure
136  trisep3(gally)=timedaq3(gally,imp3(gally))-timedaq3(gally,1+imp3(gally)-...
137  % Fall time pressure
138  tfallp3(gally)=timedaq3(gally,find(min(pressure3(gally,:))==...
139  % Design filters for force and pressure data
140  [b,a]=butter(5,4/400); % Force filter
141  [d,c]=butter(5,30/400); % Pressure filter
142  rf=ringforce3(gally,:); % filtered force
143  pf=pressure3(gally,:); % filtered pressure
144  rff3(gally,:)=filtfilt(b,a,rf); % Filtered force
145  pff3(gally,:)=filtfilt(d,c,pf); % Filtered Pressure
146  mff3(gally)=max(rff3(gally,:)); % Maximum filtered force
147  impf3(gally)=find(mff3(gally)==rff3(gally,:),1);
148  mfp3(gally)=max(pff3(gally,:)); % Maximum filtered pressure
149  impf3(gally)=find(mfp3(gally)==pff3(gally,:),1);
150  % time between max filtered force and filtered pressure
151  timpf3(gally)=timedaq3(gally,impf3(gally))-timedaq3(gally,imf3(gally));
152  % Rise time filtered force
153  trisef3(gally)=timedaq3(gally,imf3(gally))-timedaq3(gally,1+...
154  % Fall time filtered force
155  tfallff3(gally)=timedaq3(gally,imf3(gally)-1+find(rff3(gally,...
156  % Rise time filtered Pressure
157  trisepf3(gally)=timedaq3(gally,impf3(gally))-timedaq3(gally,1+...
158  % Fall time filtered pressure
159  tfallpf3(gally)=timedaq3(gally,find(min(pff3(gally,:))==...
160  % Imp2= cumulative integral of ringforce3
161  Imp2=cumtrapz(ringforce3(gally,ints:imp3))*dt;
162  I(gally)=trapz(ringforce3(gally,ints:imp3))*dt;
163  Imp(gally)=max(Imp2);
164  Imp2= cumtrapz(rff3(gally,ints:impf3(gally)))*dt;
165  Impf(gally)=max(Imp2);
166  If(gally)= trapz(rff3(gally,ints:impf3(gally)))*dt;
167  if strcmp(hsv,'on')
168    timehsv3(gally,1:DAQ-(thsvint2(gally)-thsvintmin))=timehsv(gally,...
\[
\text{thsvint2(gally) - thsvintmin+1:DAQ;}
\]
\[
\text{xcent2(gally,1:DAQ-(thsvint2(gally) - thsvintmin)) = xcent(gally,...}
\]
\[
\text{thsvint2(gally) - thsvintmin+1:DAQ;}
\]
\[
\text{ycent2(gally,1:DAQ-(thsvint2(gally) - thsvintmin)) = ycent(gally,...}
\]
\[
\text{thsvint2(gally) - thsvintmin+1:DAQ;}
\]
\[
\text{DR2(gally,1:DAQ-(thsvint2(gally) - thsvintmin)) = DR(gally,...}
\]
\[
\text{xvel2(gally,1:DAQ-(thsvint2(gally) - thsvintmin)) = xvel(gally,...}
\]
\[
\text{thsvint2(gally) - thsvintmin+1:DAQ;}
\]
\[
\text{yvel2(gally,1:DAQ-(thsvint2(gally) - thsvintmin)) = yvel(gally,...}
\]
\[
\text{thsvint2(gally) - thsvintmin+1:DAQ;}
\]
\[
\text{dc2(gally,1:DAQ-(thsvint2(gally) - thsvintmin)) = dc(gally,...}
\]
\[
\text{thsvint2(gally) - thsvintmin+1:DAQ;}
\]
\[
\%
\text{Distance from plate at maximum force}
\]
\[
\text{[xts(gally), xd(gally)] = dsearchn(timehsv3(gally,:),',...}
\]
\[
\text{timedaq3(gally,round(imf3(gally)))};
\]
\[
\%
\text{Distance from plate of maximum pressure}
\]
\[
\text{xtf(gally) = dsearchn(timehsv3(gally,:),',timedaq3(gally,...}
\]
\[
\text{round(imp3(gally)));}
\]
\[
\%
\text{Distance from plate of max filtered force}
\]
\[
\text{[xtsf(gally), xdf(gally)] = dsearchn(timehsv3(gally,:),',...}
\]
\[
\text{timedaq3(gally,round(imff3(gally)));}
\]
\[
\%
\text{Distance from plate of max filtered pressure}
\]
\[
\text{xtf(gally) = dsearchn(timehsv3(gally,:),',timedaq3(gally,...}
\]
\[
\text{round(impf3(gally)));}
\]
\[
\%
\text{The velocity and diameter are found from the 20 points before the max}
\]
\[
\%
\text{force}
\]
\[
\text{if xts(gally)>20}
\]
\[
\text{if xd(gally) <> 0}
\]
\[
\text{upi(gally) = nanmean(xvel3(gally,xts(gally)-5:xts(gally))};
\]
\[
\text{DRi(gally) = nanmean(DR3(gally,xts(gally)-5:xts(gally))};
\]
\[
\text{xi(gally) = nanmean([xcent3(gally,xts(gally)),xcent3(gally,...}
\]
\[
\text{xts(gally)+1])};
\]
\[
\text{xf(gally) = nanmean([xcent3(gally,xtf(gally)),xcent3(gally,...}
\]
\[
\text{xtf(gally)+1])];
\]
\[
\text{else}
\]
\[
\text{upi(gally) = nanmean(xvel3(gally,xts(gally)-5:xts(gally))};
\]
\[
\text{DRi(gally) = nanmean(DR3(gally,xts(gally)-5:xts(gally))};
\]
\[
\text{xi(gally) = xcent3(gally,xts(gally))};
\]
\[
\text{xf(gally) = xcent3(gally,xtf(gally))};
\]
\[
\text{end}
\]
\[
\text{else}
\]
\[
\text{upi(gally) = NaN;}
\]
\[
\text{DRi(gally) = NaN;}
\]
\[
\text{xi(gally) = NaN;}
\]
\[
\text{xf(gally) = NaN;}
\]
\[
\text{end}
\]
\[
\text{if xtsf(gally)>20}
\]
\[
\text{if xdf(gally) <> 0}
\]
\[
\text{upif(gally) = nanmean(xvel3(gally,xtsf(gally)-5:xtsf(gally))};
\]
\[
\text{DRif(gally) = nanmean(DR3(gally,xtsf(gally)-5:xtsf(gally))};
\]
xif(gally)=nanmean([xcent3(gally,xtsf(gally)),xcent3(gally,...
xtsf(gally)+1]));
xff(gally)=nanmean([xcent3(gally,xtff(gally)),xcent3(gally,...
xtff(gally)+1]));
else
upif(gally)=nanmean(xvel3(gally,xtsf(gally)-5:xtsf(gally)));
DRif(gally)=nanmean(DR3(gally,xtsf(gally)-5:xtsf(gally)));
xif(gally)=xcent3(gally,xtsf(gally));
xff(gally)=xcent3(gally,xtff(gally));
end
else
upif(gally)=NaN;
DRif(gally)=NaN;
xif(gally)=NaN;
xff(gally)=NaN;
end
end
% Replace any 0 with NaN
timedaq3(timedaq3==0)=NaN;
trigger3(trigger3==0)=NaN;
senforce3(senforce3==0)=NaN;
pressure3(pressure3==0)=NaN;
accel3(accel3==0)=NaN;
ringforce3(ringforce3==0)=NaN;
I(I==0)=NaN;
If(If==0)=NaN;
timedaq3(bad2,:)=NaN;
trigger3(bad2,:)=NaN;
senforce3(bad2,:)=NaN;
pressure3(bad2,:)=NaN;
accel3(bad2,:)=NaN;
ringforce3(bad2,:)=NaN;
rff3(bad2,:)=NaN;
pff3(bad2,:)=NaN;
% Find average time series
timedaq3mean=nanmean(timedaq3);
trigger3mean=nanmean(trigger3);
senforce3mean=nanmean(senforce3);
pressure3mean=nanmean(pressure3);
accel3mean=nanmean(accel3);
ringforce3mean=nanmean(ringforce3);
% Find average values
rff=nanmean(rff3);
rffstd=nanstd(rff3);
pff=nanmean(pff3);
pffstd=nanstd(pff3);
% Replace any 0 with NaN
mf3(bad2)=NaN;
imf3(bad2)=NaN;
mp3(bad2)=NaN;
imp3(bad2)=NaN;
timp3(bad2)=NaN;
trisef3(bad2)=NaN;
tfallf3(bad2)=NaN;
trisep3(bad2)=NaN;
tfallp3(bad2)=NaN;
mff3(bad2)=NaN;
mpf3(bad2)=NaN;
imff3(bad2)=NaN;
impf3(bad2)=NaN;
timpf3(bad2)=NaN;

% Find average values
mf3mean=nanmean(mf3);
imf3mean=nanmean(imf3);
mp3mean=nanmean(mp3);
imp3mean=nanmean(imp3);
timp3mean=nanmean(timp3);
trisef3mean=nanmean(trisef3);
tfallf3mean=nanmean(tfallf3);
trisep3mean=nanmean(trisep3);
tfallp3mean=nanmean(tfallp3);
mff3mean=nanmean(mff3);
imff3mean=nanmean(imff3);
mpf3mean=nanmean(mpf3);
impf3mean=nanmean(impf3);
timpf3mean=nanmean(timpf3);
trisef3mean=nanmean(trisef3);
tfallf3mean=nanmean(tfallf3);
trisep3mean=nanmean(trisep3);
tfallp3mean=nanmean(tfallp3);
mff3mean=nanmean(mff3);
imff3mean=nanmean(imff3);
mpf3mean=nanmean(mpf3);
impf3mean=nanmean(impf3);
timpf3mean=nanmean(timpf3);
trisef3mean=nanmean(trisef3);
tfallf3mean=nanmean(tfallf3);
trisep3mean=nanmean(trisep3);
tfallp3mean=nanmean(tfallp3);
mff3mean=nanmean(mff3);
imff3mean=nanmean(imff3);
mpf3mean=nanmean(mpf3);
impf3mean=nanmean(impf3);
timpf3mean=nanmean(timpf3);

if strcmp(hsv,'on')
    % zeros are replaced with NaN for averaging
    timehsv3(timehsv3==0)=NaN;
xcent3(xcent3==0)=NaN;
DR3(DR3==0)=NaN;
xvel3(xvel3==0)=NaN;
yvel3(yvel3==0)=NaN;
dc3(dc3==0)=NaN;
    % Bad runs are replaced with NaN
    timehsv3(bad2,:)=NaN;
xcent3(bad2,:)=NaN;
ycent3(bad2,:)=NaN;
DR3(bad2,:)=NaN;
xvel3(bad2,:)=NaN;
yvel3(bad2,:)=NaN;
dc3(bad2,:)=NaN;
    % Mean values ignoring NaN are calculated
    timehsv3mean=nanmean(timehsv3);
xcent3mean=nanmean(xcent3);
ycent3mean=nanmean(ycent3);
DR3mean=nanmean(DR3);
xvel3mean=nanmean(xvel3);
yvel3mean=nanmean(yvel3);
dc3mean=nanmean(dc3);
% Standard deviations ignoring NaN
DR3std=nanstd(DR3);
vel3std=nanstd(xvel3);

xts(bad2)=NaN;
xtf(bad2)=NaN;
upi(bad2)=NaN;
DRi(bad2)=NaN;
xi(bad2)=NaN;
xf(bad2)=NaN;
upif(bad2)=NaN;
DRif(bad2)=NaN;
xif(bad2)=NaN;
xff(bad2)=NaN;

xtsmean=nanmean(xts);
xtfmean=nanmean(xtf);
end

if strcmp(save,'on')
if strcmp(hsv,'on')

tpaoutm=['char(hsvname),','\','TEM_Average.csv'];
tpadataoutm=['timehsv3mean;xcent3mean;ycent3mean;DR3mean;xvel3mean;...'
yvel3mean;dc3mean;timedaq3mean;trigger3mean;senforce3mean;...'
pressure3mean;accel3mean;ringforce3mean];
 fid=fopen(tpaoutm,'w');
 fprintf(fid,'%s
',headtem1,headtem2);'
 fid=fopen(tpamoutm,'w');
 fprintf(fid,'%s
',headhsv1,headhsv2,headhsv3);
 fid=fopen(timoutm,'w');
 fprintf(fid,'%s
',headtemp1,headtemp2);

 else

end
tpaoutm=[char(hsvname),’\’,’TEM_Average.csv’];

tpadataoutm=[timedaq3mean;trigger3mean;senforce3mean;...
  pressure3mean;accel3mean;ringforce3mean];

fid=fopen(tpaoutm,’w’);
fprintf(fid,’%s\r’,headdaq);
fprintf(fid,’%g,%g,%g,%g,%g,%g\r’,tpadataoutm);
fclose(fid);

tiout=[char(hsvname),’\’,’TEM_Totals.csv’];

tidataoutm=[1:HS;mf3;mp3;timp3;I];

fie=fopen(tiout,’w’);
fprintf(fie,’%s\r’,headtemdaq);
fprintf(fie,’%g,%g,%g,%g,%g\r’,tidataoutm);
fclose(fie);

tpamoutm=[char(hsvname),’\’,’TEM_Average_Filt.csv’];

tpamdataoutm=[timedaq3mean;trigger3mean;senforce3mean;...
  pff;accel3mean;rff];

fid=fopen(tpamoutm,’w’);
fprintf(fid,’%s\r’,headdaq);
fprintf(fid,’%g,%g,%g,%g,%g,%g\r’,tpamdataoutm);
fclose(fid);

timout=[char(hsvname),’\’,’TEM_Totals_Filt.csv’];

timdataoutm=[1:HS;mff3;mpf3;timpf3;If];

fie=fopen(timout,’w’);
fprintf(fie,’%s\r’,headtemdaq);
fprintf(fie,’%g,%g,%g,%g,%g,%g\r’,timdataoutm);
fclose(fie);

end

end
Appendix B

Analytical Cases for DPIV Analysis

The analysis of the DPIV vector fields was developed using the MATLAB code PIVvortextViscous.m. The model case was a pair of counter-rotating Lamb-Oseen vortices placed a fixed distance apart. The Lamb-Oseen vortex was chosen because the velocity and vorticity distribution approximated a vortex ring. The velocity field induced by the cores was solved for on an orthogonal grid similar to the DPIV vector grid. Multiple techniques for measuring the core velocity and circulation were tested using the normal velocity distribution before testing less ideal cases where noise was introduced into the vector field or the core centers were offset from grid points.

B.1 Velocity Field

The radial velocity distribution for a Lamb-Oseen vortex is

\[ v_\theta = \frac{\Gamma_0}{2\pi r} \left( 1 - e^{-r^2/r_0^2} \right). \]  \hspace{1cm} (B.1)

where \( \Gamma_0 = \omega_0 \pi r^2 \), \( r_0 \) is the radius of the vortex, and \( r \) is the distance measured from the center of the vortex. The radial vorticity distribution is

\[ \omega = \frac{\Gamma_0}{\pi r_0^2} \left( e^{-r^2/r_0^2} \right). \]  \hspace{1cm} (B.2)
Figure B.1. Radial vorticity and velocity. The vorticity is normalized by its maximum value, the velocity is normalized by the vorticity and the core diameter.

The normalized radial distribution of the velocity and vorticity is shown in Figure B.1. The velocity reached its maximum when the vorticity is about 30% of its peak value at the center of the core. The vorticity reached 1% of its maximum value at a non-dimensional radius of 2.14. The velocity for the whole field was the sum of the contributions from the individual cores so for every grid point the velocity was found as a function of the distance from the core center. The ring was given dimensions of $r_0 = 7$, $\Gamma_0 = \pm 1$, and $D_R = 40$. A vertical slice through the ring showing the horizontal velocity contributions from each core as well as the total the velocity is shown in Figure B.2. A horizontal slice showing the vertical velocity of each core is shown in Figure B.3. The total velocity field is shown in Figure B.4.

To generate a less ideal case, noise was introduced by using the MATLAB function `randn` to create a two dimensional array of random values for every grid point with a mean of zero. The noise values were scaled by the maximum value of the noise to be between 0 and 1. The noise level was multiplied by a factor of 0.05 and added to all of
Figure B.2. Horizontal velocity profile through two Lamb-Oseen vortices. The vortex centers are marked by the black dashed lines. Blue is the velocity contribution from the lower core, red is the velocity contribution from the upper core, and green is the combined velocity.

The real vortex rings would not be likely to have their core centroids correspond exactly to a point in the vector grid, so the centroid of one core was offset. The ring diameter was kept constant, but the centroid of the upper core was placed at a different \( x \)-coordinate than the lower core. The \( x \)-coordinate was moved in 0.25 \( x \) increments up to a \( .75 \Delta x \) offset. The \( y \)-coordinate of the upper centroid varied slightly from the constant diameter.

### B.2 Core Centroids

Multiple methods of finding the core center were tested. The first two use the vorticity. The radial vorticity in Figure B.1 shows that the vorticity is at its maximum at
Figure B.3. Vertical velocity profile through two Lamb-Oseen vortices. The vortex centers are marked by the dashed black line. Blue is the velocity contribution from the lower core, red is the velocity contribution from the upper core, and green is the combined velocity.

Figure B.4. Velocity field of Lamb-Oseen vortex pair
the center of the core and makes an excellent indicator for the core location. For a 2-D velocity field, the vorticity is only in the third orthogonal direction and given by

$$\omega = \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right).$$

The derivatives of the velocity field at every point were found using a second order central difference,

$$\frac{dv}{dx} = \frac{v(i+1,j) - v(i-1,j)}{2\Delta x}, \quad \frac{du}{dy} = \frac{u(i,j+1) - u(i,j-1)}{2\Delta x}$$

and the vorticity was found from the difference and shown in Figure B.7. The two methods were to find the point of maximum vorticity in each core or to find the weighted average of vorticity across the whole field.

With two vortex cores, one core would be the source of negative vorticity, the other
Figure B.6. Noisy vertical velocity profile through two Lamb-Oseen vortices. The vortex centers are marked by the dashed black line. Blue is the velocity contribution from the lower core, red is the velocity contribution from the upper core, and green is the combined velocity. The noise is only added to the combined velocity.

Figure B.7. Computed vorticity of Lamb-Oseen vortex pair.
the source of positive vorticity as in Figure B.7. The coordinates of the minimum and maximum vorticities were found using the *find* command to return their array values.

The weighted average of vorticity was computed using the vorticity values for the whole field and multiplying by the distance they were from a reference point and dividing by the total vorticity:

\[
x_{cv} = \frac{\sum_{j=1}^{M} \sum_{i=1}^{N} |\omega_{i,j}| (x_i - x_{ref})}{\sum_{j=1}^{M} \sum_{i=1}^{N} |\omega_{i,j}|}, \quad y_{cv} = \frac{\sum_{i=1}^{N} \sum_{j=1}^{M} |\omega_{i,j}| (y_j - y_{ref})}{\sum_{i=1}^{N} \sum_{j=1}^{M} |\omega_{i,j}|}.
\] (B.5)

The reference point was chosen to be at the center of the field and to locate each core a weighted average was computed for the positive vorticity and the negative vorticity. To exclude noise, the average only used vorticity values that were greater than 25% of the maximum value.

The next two methods for finding the core centroid used the velocity field after the vorticity methods were used as a preliminary guess of the centroid. The centroid of the core could not be found from the velocity field directly. The velocity at the center was zero for a single vortex, but for two vortices, the point of zero velocity changed depending on the location of the second vortex. Though the velocity field changes, the vorticity field within a core stays the same because the cores were far enough apart to not be overlapping their vorticity. Locating the core center initially by the vorticity was the only reliable method.

The velocity vectors were converted to the vortex frame of reference by finding the velocity of the core as described in Appendix B.3. Multiple vertical and horizontal slices were taken through the cores. The maximum \(u\)-velocity was found on the vertical slices and the \(v\)-velocity was found on the horizontal slices. The maximum velocities as distances from the centroid found from the vorticity are shown in Figure B.8. The highest velocity occurred in the slice containing the core center in both directions.

If the core center is off a grid point as the X is in Figure B.9, then the maximum
velocities will generally give the wrong location of the centroid. Only grid points are calculated so the nearest grid point is used as a reference point. In Figure B.9 the true maximum velocities are shown as the blue arrows, however when the slices are taken, the red arrows will be the highest found because the blue arrows are off the grid. This results in the circled point \((x_c, y_c)\) being the centroid from the maximum velocity.

Another velocity method uses the velocity field in the vortex frame of reference shown in Figure B.10. The average velocity was found in the square region used for the circulation integration in Appendix B.4. The region was offset from the vorticity centroid in both \(x\) and \(y\) and traversed horizontally and vertically as shown in Figure B.11. The average velocity changes as the region moves, and for shifts through the vorticity centroid, the velocity changes as shown in Figure B.12. When the region moves horizontally, the \(u\)-velocity does not change much. However, there is a maximum velocity when the region is centered on the core centroid. The same maximum would occur in the \(v\)-velocity for a vertical shift if it was not zero. The core centers were found from

\[
\text{Figure B.8. Velocity as a percentage of maximum at different distances from core center}
\]
Figure B.9. An off-grid core center location with the locations of the true maximum tangential velocities shown as the blue arrows and the locations of the measured tangential velocities shown as the red arrows.

the locations of the maximum velocities.

B.3 Core Velocity

Three methods of finding the core velocity were used. The core velocity needed to be found because it was used as the ring velocity. By subtracting the ring velocity from all the velocity vectors, the velocity field could be put in the vortex frame of reference. Once in the vortex frame of reference the centroid could be found as in Appendix B.2 or the circulation as in Appendix B.4.

The velocity profiles in Figure B.2 and Figure B.3 show that there is a maximum and minimum velocity on either side of the core and the velocity change through the core is approximately linear. The core centroid is at the center of the linear region as shown by the dotted black lines in Figure B.2 and Figure B.3. The velocity at the centroid was approximated by taking the mean of the minimum and maximum on either side of it. In case of a noisy vector, the mean of a slice through the centroid along with three slices on either side were used. The mean velocity through the different slices changed by less
Figure B.10. Velocity field in vortex frame of reference created by subtracting the vortex speed from all the individual velocity vectors.

Figure B.11. Moving region used to find the core centroid. The core centroid from the vorticity is \((x_c, y_c)\). The region begins at the black square and is moved to the right by one unit. When it reaches the lower blue dotted square, it starts back on the left and is traversed vertically by one unit. The process is repeated until the region reaches the upper blue square.
than 1% from the centroid as shown in Figure B.13.

The square region used for the circulation calculation in Appendix B.4 was also used to find the velocity. The region size that corresponded to the asymptotic circulation determined the region size for the velocity. The limit of this method is to take the mean velocity of the entire field. However, the ring would need to be placed in the center with a symmetric velocity field on all sides which would be unlikely in a real DPIV image. In the absence of a symmetric field, the integration region was used.

As shown in Appendix B.2, if the integration region was not centered on the core centroid the mean velocity would be different. The velocity at every location of the moving regions is shown in Figure B.14 and Figure B.15. To find the correct velocity, slices through the centroids as in Figure B.12 were used. The maximum that corresponded to the core center was also the velocity of the core.

The final velocity method used the value of the velocity vector at the core centroid coordinates. However, this single vector was prone to noise and for core centroids that
were off a grid point, the velocity at the nearest grid point was found. The nearest point could result in a velocity vector that was pointed back against the propagation direction of the core. For the off-grid, as well as the on-grid as a check, the velocity at the centroid was also found using the MATLAB command interp2 with a spline fit. The spline used the eight nearest points and their derivatives to determine the value.

### B.4 Core Circulation

The two equivalent methods of determining the circulation of a vortex are to integrate the vorticity over an area or the tangential velocity along a path containing the vorticity:

\[
\Gamma = \oint_{C} \mathbf{v} \cdot ds = \iint_{S} \omega \cdot \mathbf{n} dS. \tag{B.6}
\]
Figure B.14. Mean horizontal velocity for each region location.

Figure B.15. Mean vertical velocity for each region location.
Figure B.16. Grid showing $s$ as the black square with sides of length $2l$. The blue squares show the area, $\Delta x \Delta y$, of each grid point (black dot) and $(x_c, y_c)$ is the center of the vortex.

For simplicity, both methods used the same square region centered on the core centroid. The circulation was calculated on the smallest square region possible, a 3 by 3 square, and recalculated for regions increasing in size until the maximum. The maximum region size was determined by the size of the velocity field.

The circulation from vorticity was found by multiplying the value of vorticity at each grid point by its area. In Figure B.16, the grid points are shown as the black dots and the square region is the black line connecting the outer dots. The area of each grid point is shown as the blue squares and is equal to $\Delta x \Delta y$. For the grid points on the region boundary, the black line bisects the squares on the sides, and quarters the corner squares. The function $area\text{corrector}$ multiplied each vorticity point by the appropriate area. Then the resulting values were summed to give the circulation for that region size. The circulation is plotted for half a square side in Figure B.17 for different region sizes.

### B.5 Core Diameter

Even though the core radius, $r_0$, in the analytical expression corresponds to the distance of the maximum velocity, the core diameter was calculated from the extent of the vorticity. The diameter was the size of the core needed to contain all the vorticity. From Figure B.1 the vorticity never reaches zero, but the asymptotic circulation occurred
for a region that contained almost all of the vorticity. The region size corresponding to the asymptotic value was used for the core diameter. In Figure B.17, the asymptotic value occurred at a region size of 14, which was only half of the length of the square, so the core diameter was 29. As a visual check, the square region is plotted over top of the vorticity contours and velocity vectors in Figure B.18.

**B.6 Measurement Comparisons**

The measurement techniques described in Appendix B.2 to B.5 were tested on a clean analytical case, on a case with noise added, for a range of upper core centered not on the grid, and a combination of the noisy and off-grid cases. The estimates of the core centroid for the upper and lower core are shown in Table B.1 and Table B.2. The real coordinates were the ones used in the analytical expression. The second column in the centroid found from the location of the maximum vorticity. This method can only resolve
Figure B.18. Vorticity contours overlaid with vortex frame of reference velocity vectors. The (*) are the core centroids and the black squares are the region of the asymptotic circulation.

onto the grid, so it did not work for the offset cases. The third column is the weighted average of vorticity. This method found the cores much more accurately and in between grid points. As the core centroid was moved off the grid, the weighted average was accurate to within $0.04\Delta x$. Compared to the maximum vorticity, the weighted average was closer on the offset and noisy case, as well as being able to resolve sub-grid spacing. The velocity methods also only resolved to the grid spacing. The centroid from the maximum velocity was off for the lower core because of the reliance on a single vector. For the upper core it did better, but, as the core was offset, the method found the $x$ centroid to be further away. The moving regions were more consistent to the maximum vorticity centroid than the maximum velocity, but they could still only resolve to the grid points. The weighted average of vorticity was chosen as the measure to use on the DPIV images.

The results of the horizontal velocity measurements are shown in Table B.3 and Table B.4. The horizontal velocity was more significant than the vertical velocity so only the
Table B.1. Comparisons of centroid of lower core found from the maximum vorticity \( \omega \), the centroid of vorticity \( cv \), the maximum velocity \( (u_{\text{max}}, v_{\text{max}}) \), and the moving region \( mr \).

<table>
<thead>
<tr>
<th>Case</th>
<th>Real</th>
<th>( x_{\omega, \text{max}}, y_{\omega, \text{max}} )</th>
<th>( x_{cv}, y_{cv} )</th>
<th>( x_{u, \text{max}}, y_{u, \text{max}} )</th>
<th>( x_{mr}, y_{mr} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clean</td>
<td>80.00, 45.00</td>
<td>80.00, 45.00</td>
<td>80.00, 45.00</td>
<td>80.00, 45.00</td>
<td>80.00, 45.00</td>
</tr>
<tr>
<td>Noisy</td>
<td>80.00, 45.00</td>
<td>80.00, 45.00</td>
<td>80.03, 44.98</td>
<td>80.00, 43.00</td>
<td>80.00, 45.00</td>
</tr>
<tr>
<td>Offset 0.25</td>
<td>80.00, 45.00</td>
<td>80.00, 45.00</td>
<td>80.00, 45.00</td>
<td>80.00, 45.00</td>
<td>80.00, 45.00</td>
</tr>
<tr>
<td>Offset 0.50</td>
<td>80.00, 45.00</td>
<td>80.00, 45.00</td>
<td>80.00, 45.00</td>
<td>80.00, 45.00</td>
<td>81.00, 45.00</td>
</tr>
<tr>
<td>Offset 0.75</td>
<td>80.00, 45.00</td>
<td>80.00, 45.00</td>
<td>80.00, 45.00</td>
<td>80.00, 45.00</td>
<td>81.00, 45.00</td>
</tr>
<tr>
<td>Offset 0.50/Noisy</td>
<td>80.00, 45.00</td>
<td>80.00, 45.00</td>
<td>79.00, 45.00</td>
<td>80.00, 44.97</td>
<td>81.00, 45.00</td>
</tr>
</tbody>
</table>

Table B.2. Comparisons of centroid of upper core found from the maximum vorticity \( \omega \), the centroid of vorticity \( cv \), the maximum velocity \( (u_{\text{max}}, v_{\text{max}}) \), and the moving region \( mr \).

<table>
<thead>
<tr>
<th>Case</th>
<th>Real</th>
<th>( x_{\omega, \text{max}}, y_{\omega, \text{max}} )</th>
<th>( x_{cv}, y_{cv} )</th>
<th>( x_{u, \text{max}}, y_{u, \text{max}} )</th>
<th>( x_{mr}, y_{mr} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clean</td>
<td>80.00, 85.00</td>
<td>80.00, 85.00</td>
<td>80.00, 85.00</td>
<td>80.00, 85.00</td>
<td>80.00, 85.00</td>
</tr>
<tr>
<td>Noisy</td>
<td>80.00, 85.00</td>
<td>80.00, 85.00</td>
<td>80.05, 85.02</td>
<td>80.00, 85.00</td>
<td>80.00, 85.00</td>
</tr>
<tr>
<td>Offset 0.25</td>
<td>80.25, 85.00</td>
<td>80.00, 85.00</td>
<td>80.21, 85.00</td>
<td>80.00, 85.00</td>
<td>80.00, 85.00</td>
</tr>
<tr>
<td>Offset 0.50</td>
<td>80.50, 85.00</td>
<td>80.00, 85.00</td>
<td>80.50, 85.00</td>
<td>81.00, 85.00</td>
<td>81.00, 85.00</td>
</tr>
<tr>
<td>Offset 0.75</td>
<td>80.75, 85.00</td>
<td>81.00, 85.00</td>
<td>80.79, 85.00</td>
<td>82.00, 85.00</td>
<td>81.00, 85.00</td>
</tr>
<tr>
<td>Offset 0.50/Noisy</td>
<td>80.50, 85.00</td>
<td>80.00, 85.00</td>
<td>80.53, 85.04</td>
<td>80.00, 85.00</td>
<td>80.00, 85.00</td>
</tr>
</tbody>
</table>

Horizontal velocity is shown. The real velocity was found for a point one ring diameter from a core. The \( u \)-velocity decreased slightly as the ring became more tilted as the upper core was offset. The second column is the average of slices through the core. They were consistently greater than the real core velocity by between 3-6%. The velocity from the circulation region and the moving region were usually the same because the region was centered correctly on the centroid. They differed when the moving region found a different centroid than the maximum vorticity. Both differ from the real velocity by less than 1%, but also corresponded to the wrong centroid. The velocity at the centroid was the velocity at the grid point closest to the centroid found by the weighted average of vorticity. The centroid velocity was generally the same as the real velocity, but as seen in the noisy case for the lower core, it was very susceptible to a noisy vector. And as the centroid became offset, the velocity estimate became worse. The interpolation used the exact coordinates of the weighted average of vorticity centroid to find a velocity and was more accurate for the offset cases. However, like the velocity at the centroid, it was affected by noisy vectors. The average methods, though less accurate, were more consistent across the test range. The mean velocity based on slices was chosen for the
Table B.3. Comparisons of velocity of lower core found from vertical slices, an average of the whole core, the maximum from the moving region, at the centroid from the weighted average of vorticity, and using the spline interpolation.

<table>
<thead>
<tr>
<th>Case</th>
<th>Real</th>
<th>Mean Slice</th>
<th>Core Region</th>
<th>MR</th>
<th>Centroid</th>
<th>Spline</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clean</td>
<td>0.6125</td>
<td>0.6371</td>
<td>0.6097</td>
<td>0.6097</td>
<td>0.6125</td>
<td>0.6125</td>
</tr>
<tr>
<td>Noisy</td>
<td>0.6125</td>
<td>0.6421</td>
<td>0.6099</td>
<td>0.6099</td>
<td>0.5813</td>
<td>0.5714</td>
</tr>
<tr>
<td>Offset 0.25</td>
<td>0.6125</td>
<td>0.6371</td>
<td>0.6097</td>
<td>0.6097</td>
<td>0.6125</td>
<td>0.6125</td>
</tr>
<tr>
<td>Offset 0.50</td>
<td>0.6125</td>
<td>0.6371</td>
<td>0.6097</td>
<td>0.6097</td>
<td>0.6125</td>
<td>0.6125</td>
</tr>
<tr>
<td>Offset 0.75</td>
<td>0.6124</td>
<td>0.6369</td>
<td>0.6096</td>
<td>0.6098</td>
<td>0.6124</td>
<td>0.6124</td>
</tr>
<tr>
<td>Offset 0.50/Noisy</td>
<td>0.6125</td>
<td>0.6487</td>
<td>0.6108</td>
<td>0.6108</td>
<td>0.6059</td>
<td>0.5923</td>
</tr>
</tbody>
</table>

Table B.4. Comparisons of velocity of upper core found from vertical slices, an average of the whole core, the maximum from the moving region, at the centroid from the weighted average of vorticity, and using the spline interpolation.

<table>
<thead>
<tr>
<th>Case</th>
<th>Real</th>
<th>Mean Slice</th>
<th>Core Region</th>
<th>MR</th>
<th>Centroid</th>
<th>Spline</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clean</td>
<td>0.6125</td>
<td>0.6371</td>
<td>0.6097</td>
<td>0.6097</td>
<td>0.6125</td>
<td>0.6125</td>
</tr>
<tr>
<td>Noisy</td>
<td>0.6125</td>
<td>0.6306</td>
<td>0.6095</td>
<td>0.6095</td>
<td>0.6165</td>
<td>0.6070</td>
</tr>
<tr>
<td>Offset 0.25</td>
<td>0.6125</td>
<td>0.6371</td>
<td>0.6097</td>
<td>0.6097</td>
<td>0.6121</td>
<td>0.6123</td>
</tr>
<tr>
<td>Offset 0.50</td>
<td>0.6125</td>
<td>0.6372</td>
<td>0.6095</td>
<td>0.6095</td>
<td>0.6109</td>
<td>0.6166</td>
</tr>
<tr>
<td>Offset 0.75</td>
<td>0.6124</td>
<td>0.6366</td>
<td>0.6087</td>
<td>0.6091</td>
<td>0.6086</td>
<td>0.6107</td>
</tr>
<tr>
<td>Offset 0.50/Noisy</td>
<td>0.6125</td>
<td>0.6377</td>
<td>0.6083</td>
<td>0.6089</td>
<td>0.6092</td>
<td>0.5988</td>
</tr>
</tbody>
</table>

Table B.5. Comparisons of circulation of lower core found from the path integral of velocity and the area integral of vorticity.

<table>
<thead>
<tr>
<th>Case</th>
<th>Real</th>
<th>$\Gamma_{vel}$</th>
<th>$\Gamma_{vort}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clean</td>
<td>153.938</td>
<td>152.453</td>
<td>152.466</td>
</tr>
<tr>
<td>Noisy</td>
<td>153.998</td>
<td>153.001</td>
<td>152.651</td>
</tr>
<tr>
<td>Offset 0.25</td>
<td>153.938</td>
<td>152.453</td>
<td>152.466</td>
</tr>
<tr>
<td>Offset 0.50</td>
<td>153.938</td>
<td>152.453</td>
<td>152.466</td>
</tr>
<tr>
<td>Offset 0.75</td>
<td>153.938</td>
<td>152.453</td>
<td>152.466</td>
</tr>
<tr>
<td>Offset 0.50/Noisy</td>
<td>153.938</td>
<td>152.596</td>
<td>152.522</td>
</tr>
</tbody>
</table>

DPIV because the region methods were completely dependent on the core centroid being correct, whereas the slices were not.

The two circulation measurements are mathematically equal and are compared to the analytical value of the circulation in Table B.5 and Table B.6. Both underestimated the circulation because of the cut-off by the asymptote, but only by less than 1%. Through both noise and the offset, the circulation of both cores was calculated to be very close to the clean case. The integration of the tangential velocity was chosen for the DPIV images because the vorticity requires an extra numerical step.

The core diameter was the same for all cases at 28 for both circulation measurements.
Table B.6. Comparisons of circulation of upper core found from the path integral of velocity and the area integral of vorticity.

<table>
<thead>
<tr>
<th>Case</th>
<th>Real</th>
<th>$\Gamma_{vel}$</th>
<th>$\Gamma_{vort}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clean</td>
<td>153.938</td>
<td>152.453</td>
<td>152.466</td>
</tr>
<tr>
<td>Noisy</td>
<td>153.938</td>
<td>152.791</td>
<td>152.451</td>
</tr>
<tr>
<td>Offset 0.25</td>
<td>153.938</td>
<td>152.445</td>
<td>152.458</td>
</tr>
<tr>
<td>Offset 0.50</td>
<td>153.938</td>
<td>152.421</td>
<td>152.432</td>
</tr>
<tr>
<td>Offset 0.75</td>
<td>153.938</td>
<td>152.445</td>
<td>152.457</td>
</tr>
<tr>
<td>Offset 0.50/Noisy</td>
<td>153.938</td>
<td>152.758</td>
<td>152.653</td>
</tr>
</tbody>
</table>

The region size of the asymptotic circulation for the tangential velocity integration was used for the DPIV.


