CHARACTERIZATION OF BLASTS FROM LABORATORY-SCALE
COMPOSITE EXPLOSIVE CHARGES

A Dissertation in
Mechanical Engineering

by

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Though multitudes of experiments have been conducted on explosive material characterization, none have properly addressed the effects of a booster charge, which is used to surpass the high-energy threshold required for detonation of an insensitive explosive material. The added pressure effects produced by the booster have long been disregarded. To fill this gap, the present research addresses the booster charge effects when employed in a composite-charge arrangement at the laboratory scale, i.e. with explosive charges in the gram-range. The composite charge consists of a well-characterized spherical booster charge surrounded by a concentric, spherical “candidate material” shell charge. By way of composite-charge explosive characterization, the radial-TNT equivalent of the candidate explosive material is determined through the removal of the known booster effects. Laboratory-scale air-blast explosive tests are conducted using digital high-speed shadowgraph visualization to measure the radial propagation of the explosively-driven shock wave as a function of time. An ultra-high-speed digital camera is also incorporated to measure the radial propagation rate in close proximity to the explosive charge itself. Profiles of peak shock wave pressure vs. shock wave radius are determined from the temporal history of the shock wave propagation. Initial experiments are performed with both booster and shell charges made of characterized pentaerythritol tetranitrate (PETN), in order to determine the booster relationship within a composite-charge arrangement. Using peak shock wave pressure vs. shock wave radius profiles, a procedure is developed to remove the booster effects from the composite-charge signature, yielding the sole effects of the shell material, as if it
were detonated alone. Method verification is performed by comparing results to previous homogeneous explosive material characterization. A second booster removal procedure is developed based upon the explosive impulse rather than peak shock wave pressure. Temporal pressure histories of the shock wave pressure decay are measured using piezoelectric pressure transducers. Explosive impulse is calculated from these pressure histories through knowledge of the peak shock wave pressure, the positive pressure duration, and the waveform parameter describing the pressure decay. Composite explosive charges are numerically modeled to verify the proposed peak shock wave pressure and explosive impulse booster removal procedures. Finally, this composite-charge characterization procedure is extended to the characterization of smokeless powder (SP), cyclotrimethylene trinitramine (RDX), and cyclotetramethylene tetranitramine (HMX). The results demonstrate the ability to successfully test and characterize insensitive explosive materials, requiring a booster charge for detonation, at the laboratory scale by way of composite charges.
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Chapter 1

Introduction

1.1 Research Motivation

Explosive characterization has long depended upon various methods to assess explosive damage potential. The characterization of explosive materials allows scaling calculations to be performed for technical design considerations in the prediction of blast waves, crater sizes, structural responses, etc. The resulting properties from an explosion are used to determine explosive damage potential and thus characterize the explosive of interest.

The fundamental outcome from the detonation of an explosive material is the generation of a shock wave. Explosive damage potential is contained within this shock wave, due to the discontinuous jump in property magnitudes downstream of the shock wave, and the velocity of the gas trailing the shock wave. The peak shock wave pressure, velocity of the trailing gas, and the ensuing explosive impulse produced are the main underlying shock wave properties that cause damage. By experimentally measuring these shock wave properties, explosives are characterized based upon a trinitrotoluene, or TNT, equivalent. The TNT equivalent relates the shock wave properties produced by an explosive of interest to those produced by TNT, the previously-established baseline standard.
Explosive materials possess detonation properties which are unique to an individual explosive. These properties: detonation velocity, detonation pressure, and heat of detonation, result in different rates of energy release from one explosive to the next, ultimately determining the detonation performance of an explosive [1, 2]. Subsequently, this results in dissimilar shock wave speeds as a function of shock wave radius. Thus, the previous universally-accepted explosive material characterization of a single TNT equivalent yields an inadequate, oversimplified characterization. Rather, a variable TNT equivalent with respect to shock wave radius is required for proper explosive material characterization.

Due to the substantial cost of the traditionally-conducted full-scale field test, recent interest has developed in the characterization of explosive materials at the laboratory scale, i.e. gram-range explosive testing. These laboratory-scale experiments offer a more physical explosive characterization procedure than the full-scale experiments. Additionally, the wealth of information acquired and minimal cost incurred offer an attractive alternative to full-scale experiments.

The research capability for explosive material air-blast characterization at the laboratory-scale has been demonstrated using high-speed digital cameras with traditional optical imaging techniques in controlled atmosphere environments [3, 4]. Using optical imaging diagnostics, the radial profile of the shock wave as a function of time is measured. These measurements are subsequently converted to shock wave Mach number as a function of shock wave radius. From this, peak shock wave pressure vs. shock wave radius is calculated, thus permitting the calculation of a radially-dependent TNT explosive equivalent profile.
Presently, a multitude of tests exist for the characterization of an explosive material. The inconsistency amongst these tests is largely due to the physical differences between one test and the next. One such variable is the requirement of a booster charge (e.g. detonator) for the detonation of an insensitive explosive material. The effects produced by the booster charge have, to this point, been disregarded, since the booster mass usually only represents a small percentage of the total charge mass. However, if the booster mass is comparable to that of the main charge mass, booster effects need to be accounted for to ensure a proper explosive-material characterization.

The present research will expand laboratory-scale air-blast research to insensitive explosive materials that require a booster charge for detonation. To ensure detonation at the laboratory-scale, the ratio of booster mass to main charge mass becomes significant; thus, quantification of the booster charge effects is necessary. Using a composite explosive-charge arrangement, air-blast characterization of an insensitive candidate explosive material will be demonstrated. A procedure will be developed for removing the booster charge effects from the composite-charge characterization, yielding the sole effects produced by the candidate material if detonated alone.

The research into composite explosive-charges at the laboratory scale offers a more complete characterization of insensitive explosive materials by understanding the physics of the shock wave propagation and quantifying the effects of the booster charge. Combined, these result in a more physical representation of explosive TNT equivalent. Additionally, laboratory-scale research offers improved cost efficiency and safety when compared to traditional full-scale tests.
1.2 Literature Review

1.2.1 Laboratory-scale characterization

Small-scale-air-blast research with charge masses in the kilogram range and much smaller have become an attractive alternative to traditional full-scale experiments. Laboratory-scale research is conducted in controlled environments that provide experimental test reproducibility at a reduced cost and improved safety of handling using low-mass charges [3-8]. Laboratory-scale research also provides improved instrumentation through optical imaging techniques, such as schlieren, shadowgraph, and interferometry for explosive shock wave visualization and measurement.

Laboratory-scale optical imaging of explosively-driven shock waves was improved upon by Reichenbach and Kuhl using a shadowgraph system [9] based on original work by Schardin [10]. Later, tracking of the shock wave was demonstrated by Kleine et al. and Hargather et al. for the characterization of explosives in the milligram and gram range [3, 4]. Kleine and Hargather used schlieren and focused shadowgraph techniques to optically track the shock wave radius as a function of time. These radial shock wave history profiles led to a more physical characterization procedure in comparison to the traditional full-scale pointwise pressure transducer measurements. Furthermore, Hargather et al. demonstrated the ability to accurately scale the laboratory experimental results to full-scale, or kilogram, charge masses through the previously-defined explosive scaling laws [4].
1.2.2 Explosive scaling laws

The first blast-wave scaling laws were developed during World War I by Hopkinson in Great Britain and later Cranz in Germany [11, 12]. The blast wave resulting from a point explosion was later developed by Taylor and Sedov [13, 14]. In their analyses, Hopkinson and Cranz concluded that individual explosives of similar geometry, detonated in the same atmosphere, produced identical shock-wave-specific properties at equivalent scaled distances, according to the explosive charge linear dimension, i.e. its diameter. Thus, for a spherical charge geometry, the explosive charge diameter is proportional to the cube root of the explosive charge mass $W$ and therefore, the charge energy $E$. Thus, the relationship between radial shock wave dimensions $R_1$ and $R_2$ at which identical shock-wave-specific properties are produced by charge masses $W_1$ and $W_2$ are related according to the cube-root ratio of the explosive charge masses or energies $E_1$ and $E_2$, Equation (1-1).

\[
\frac{R_1}{R_2} = \left(\frac{W_1}{W_2}\right)^{1/3} = \left(\frac{E_1}{E_2}\right)^{1/3}
\]

(1-1)

Further research at various elevation and ambient temperature conditions led Sachs [15] to improve upon these laws, accounting for deviations from normal atmospheric pressure and temperature conditions, Equation (1-2). Sachs developed the relationship between radial shock wave dimensions $R_1$ and $R_2$ at which identical shock-wave-specific properties are produced by two charge masses exploded in different
atmospheres with pressures $P_{01}$ and $P_{02}$, Equation (1-2). Shock wave time of arrival is additionally scaled by the square-root ratio of atmospheric temperatures $T_{01}$ and $T_{02}$ according to Equation (1-3).

$$\frac{R_1}{R_2} = \left(\frac{W_1}{W_2}\right)^{1/3} \left(\frac{P_{02}}{P_{01}}\right)^{1/3} = \left(\frac{E_1}{E_2}\right)^{1/3} \left(\frac{P_{02}}{P_{01}}\right)^{1/3} \tag{1-2}$$

$$\frac{t_1}{t_2} = \left(\frac{W_1}{W_2}\right)^{1/3} \left(\frac{P_{02}}{P_{01}}\right)^{1/3} \left(\frac{T_{02}}{T_{01}}\right)^{1/2} = \left(\frac{E_1}{E_2}\right)^{1/3} \left(\frac{P_{02}}{P_{01}}\right)^{1/3} \left(\frac{T_{02}}{T_{01}}\right)^{1/2} \tag{1-3}$$

Thus, the scaling process reduces the number of explicit parameters required in the description of shock wave properties by removing deviations in charge mass and atmospheric conditions. Through decades of testing, these scaling laws have been verified by multiple investigators over a wide range of masses, ranging from $10^{-7}$ to $10^6$ kg [3, 4, 16].

Kleine et al. developed scaling variables $S$ and $c$ to scale shock wave radius and time data to a standard charge mass $W_{std}$ and Normal Temperature and Pressure conditions ($T_{NTP} = 288.16K$ and $P_{NTP} = 101.325kPa$) [3]. Shock wave radii $R$ were scaled by scaling parameter $S$ (Equation (1-4)) to account for variations in charge mass $W$ and atmospheric pressure $P_o$. Shock wave times of arrival $t$ were scaled by scaling parameters $S$ and $c$ (Equations (1-4) and (1-5)), to account for charge mass variability and atmospheric temperature and pressure conditions, $W$, $T_o$, and $P_o$ respectively. Scaled
shock wave radii and shock wave time of arrival are represented by Equations (1-6)-(1-7), where variables with subscript “s” are scaled values.

\[
S = \left( \frac{W}{W_{std}} \frac{P_{NTP}}{P_0} \right)^{1/3} \tag{1-4}
\]

\[
c = \left( \frac{T_0}{T_{NTP}} \right)^{1/2} \tag{1-5}
\]

\[
R_s = \frac{R}{S} \tag{1-6}
\]

\[
t_s = \frac{ct}{S} \tag{1-7}
\]

As previously stated, Kleine et al. and Hargather et al. conducted experimental explosive research by optically measuring the explosively-driven shock wave radius as a function of time [3, 4]. For ease of comparison, data were first scaled to a standard charge mass and atmosphere. Data were then compiled and fit to a shock wave radius vs. time curvefit developed by Dewey [17], Equation (1-8). Equation (1-8) is adapted to fit individual explosive material data, detonated in various atmospheres, through use of curvefit coefficients \(A, B, C,\) and \(D\) and atmospheric sound speed \(a_0.\) Atmospheric sound speed is calculated using air’s ratio of specific heats \(\gamma,\) gas constant \(R_a,\) and atmospheric temperature \(T,\) Equation (1-9).
Although an empirical correlation, Equation (1-8) does satisfy the appropriate physical condition as \( t \to \infty \). This is represented by setting curvefit coefficient \( B = 1 \), ensuring that the shock wave velocity approaches the atmospheric speed of sound \( a_0 \). However, the equation lacks a physically-based initial condition at \( t = 0 \).

From Equation (1-8), calculation of the shock wave Mach number \( M_s \) is permissible. First, the derivative of the scaled shock wave radius with respect to scaled time \( t_s \) is calculated, yielding shock wave velocity as a function of scaled time, Equation (1-10). The shock wave velocity is then divided by the atmospheric speed of sound \( a_0 \), yielding the shock wave Mach number as a function of scaled time, Equation (1-11). As a result, a curve representing the shock wave Mach number as a function of shock wave radius is acquired. From the shock wave Mach number, all other properties downstream of the shock wave are able to be determined, such as: incident and reflected peak shock wave pressures, temperature, density, positive pressure duration, and ultimately, explosive damage potential, or TNT equivalent.

\[
R_s = A + Ba_0 t_s + C \ln(1 + a_0 t_s) + D \sqrt{\ln(1 + a_0 t_s)} \quad (1-8)
\]

\[
a_0 = \sqrt{\frac{\gamma R_a T}{\gamma R_a T}} \quad (1-9)
\]

\[
\frac{dR_s}{dt_s} = Ba_0 + \frac{Ca_0}{1 + a_0 t_s} + \frac{Da_0}{2(1 + a_0 t_s)\sqrt{\ln(1 + a_0 t_s)}} \quad (1-10)
\]
These downstream shock wave properties are determined through use of the Rankine-Hugoniot theory [17]. Using conservation of mass, momentum, and energy in addition to the perfect gas equation of state, downstream shock wave properties are related to the quiescent upstream atmospheric properties through the shock wave Mach number. Assuming perfect-gas behavior of air, i.e. constant specific heats with $\gamma = 1.4$, the peak shock wave pressure $P_s$ is determined from the shock wave Mach number using the Rankine-Hugoniot relationship, Equation (1-12) [17]. Here, perfect gas behavior is a reasonable assumption, since resulting imperfect gas corrections for the pressure ratio across a normal shock wave differ by less than 6% from perfect gas results for shock wave Mach numbers below 10 [18].

From the peak shock wave pressure as a function of shock wave radius profile, calculation of a TNT mass required to produce an equivalent peak shock wave pressure at an identical shock wave radius is possible. Consequently, this yields a radially-dependent TNT equivalent characterization, see Section 1.2.3 [19]. The aforementioned procedure demonstrates that laboratory-scale, experimentally-measured, explosively-driven shock wave radius history profiles permit a more physically-based characterization than that produced by a collection of individual full-scale pressure transducer measurements.
1.2.3 TNT Equivalent

Contradictory conclusions have previously been reached regarding the shock wave radial history profile produced in air. Studies by Formby concluded that, at certain distances from an explosive charge, the profile shape remained approximately invariant for multiple explosives [20]. Other investigators have found the shape to vary between individual explosives, and these to differ from that of TNT [3, 4, 21, 22]. As a result, a direct comparison of explosives is required since explosive materials have inherently-different detonation properties that result in varying energy release rates.

This is accomplished by relating explosives to the baseline standard, TNT [16]. A “TNT equivalent” is calculated by relating the radially-propagating shock wave properties resulting from the detonation of a given explosive to those produced by TNT [23, 24]. The fundamental shock wave properties measured for determining TNT equivalent are the peak shock wave pressure and explosive impulse [25].

TNT was chosen as the baseline reference because it is the most widely used explosive and has been well-documented [16]. Additionally, TNT was chosen for its precise scalability over a wide range of masses. Experiments by Dewey demonstrated the scalability of TNT over four orders of magnitude, i.e. 25 to $10^5$kg [26]. However TNT, which lacks a sufficient supply of oxygen (~74%), is actually not the “ideal” explosive of choice. This oxygen-deficiency causes afterburning of the explosive with atmospheric

$$\frac{P_s}{P_{NTP}} = \frac{7(M_s^2 - 1)}{6} + 1$$

(1-12)
oxygen, resulting in a post-detonation combustion energy release [26]. As a result, the afterburning increases the positive pressure duration at greater radial shock wave distances, not characteristic of an “ideal” explosive that contains an ample oxygen supply required for complete detonation. Furthermore, detonation of cast TNT (\( \rho = 1.63 \text{g/cm}^3 \)) in unconfined masses less than 5kg becomes unreliable [16].

1.2.4 Characterization tests

Due to the multitude of characterization tests that have been conducted, in addition to the variable conditions under which these tests are run, numerous different TNT equivalents have been reported for a given explosive material [23]. The most common equivalent tests include: plate dent, ballistic mortar, trauzl, sand crush and air blast. Each of these tests takes an empirically-measured explosive property and calculates the mass of TNT that produces an equivalent property at an equivalent shock wave radius [23]. However, these tests lack a firm scientific basis of the measurements made, yielding at best only a single TNT equivalent.

In the air-blast characterization test, the TNT equivalent is calculated by determining the mass of TNT required to produce an equivalent peak shock wave pressure or explosive impulse at an equivalent radial shock wave distance as that produced by the explosive charge of interest [23]. Experimental data are gathered by measuring the peak shock wave pressure and its decay at a limited number of radial standoff locations using piezoelectric pressure gages. In addition to the lack of sufficient data, a drawback of the air-blast test is the required steel case containment of the
explosive material; the mass ratio of steel case to explosive charge being approximately unity [19]. As a result, explosive energy is lost in the fragmentation of the case, yielding inaccurate TNT equivalents at only a limited number of radial shock wave positions.

Recent experiments have progressively reduced the inherent uncertainty in air-blast testing by eliminating the standard steel casing. Formby and Wharton implemented a thin-walled, frangible-plastic shell for explosive confinement of kg-range charges which was shown to have no effect on shock wave intensity [20, 27]. Kleine et al. and Hargather et al. removed all confinement and simply used bare charges for laboratory-scale experiments [3, 4].

1.2.5 Explosive Impulse

Due to the vast array of explosives currently available, a TNT equivalent based solely on peak shock wave pressure is inadequate for complete explosive characterization. For example, two explosives exhibiting the same peak shock wave pressure may possess dissimilar explosive impulses; e.g. high explosives versus thermobaric explosives. The peak shock wave pressure and explosive impulse are the fundamental shock wave properties that cause structural damage [25]. For this reason, Held states that both peak shock wave pressure and explosive impulse should be used in determining the TNT equivalent of an explosive material [25].

Accordingly, to complete the present explosive material characterization, both the peak shock wave pressure and its temporal decay are required to calculate explosive impulse. An “ideal” explosive pressure trace, at a fixed radial-standoff distance from the
explosive charge center, is illustrated in Figure 1-1. The trace contains both a positive phase (pressures above atmospheric pressure $P_0$) and a negative phase (pressures below atmospheric pressure). The positive pressure duration is defined from peak shock wave pressure $P_s$, at the shock wave time of arrival $t_a$, through its decay down to atmospheric pressure at time $t_a + T^+$. The negative pressure duration is defined from the end of the positive pressure duration until pressure returns to atmospheric pressure at time $t_a + T^+ + T^-$. 

The positive phase of explosive impulse $I$ is defined as the integral with respect to time of the positive pressure phase, Equation (1-13) [28]. Thus, explosive impulse is experimentally-determined by measuring the peak shock wave pressure and its decay as a function of time at a specified radial-standoff distance. Only the positive phase of impulse will be considered for the present research, as this is generally orders of magnitude larger than the negative phase [28].

$$I = \int_{t_a}^{t_a + T^+} [P(t) - P_0]dt$$  \hspace{1cm} (1-13)
Figure 1-1: Ideal peak shock wave pressure decay.
A deterrent of laboratory-scale experiments is the inability to accurately measure peak shock wave pressure $P_s$ due to the discontinuous jump from atmospheric pressure to peak shock wave pressure, the signature of a shock wave [3, 5]. Full-scale experiments also lack this ability [28-30], but additional error results at the laboratory-scale due to the inherent spatial and temporal averaging effects of the pressure transducer [5]. These averaging effects result from the analogous characteristic time and length scales of the explosion and the pressure transducer.

Pressure transducers in principle measure static or stagnation peak shock wave pressure, depending on the orientation of the transducer to the flow, either parallel or perpendicular, respectively. Side-on (static) peak shock wave pressure measurements are susceptible to both temporal and spatial averaging of the pressure transducer. Temporal averaging is attributed to the finite response time of the pressure transducer to the discontinuous jump in pressure from atmospheric to peak shock wave pressure. Spatial averaging results from both the shock wave traversing the finite area of the transducer face and from shock wave diffraction effects [5]. Face-on (stagnation) peak shock wave pressure measurements are only susceptible to temporal averaging. Additionally, the magnitude of the temporal averaging effect has been shown to lessen with increased radial standoff distances due to the decrease in shock wave strength [5]. Thus, face-on stagnation pressure measurements are the preferred method for measuring the peak shock wave pressure and its decay at the laboratory-scale.

The inability to accurately measure the peak shock wave pressure and its decay hamper explosive impulse characterization compared to the peak shock wave pressure characterization procedure previously-outlined, see [29, 30] [3, 5]. Additionally, the
discontinuous jump in pressure resulting from the shock wave causes excitation of the eigenfrequencies of the pressure transducer, resulting in “noise” being superimposed on the pressure trace [5]. These effects are able to be eliminated through filtering of the signal in the frequency domain, but the results still fail to yield a clean, smooth, ideal pressure trace as shown in Figure 1-1. To more easily compare these data, multiple smoothing curves have been developed by previous authors based upon empirical data. These curves represent the peak shock wave pressure and its decay as a function of time [28, 30-33].

Of the proposed relationships, the modified Friedlander equation is the generally-accepted form, Equation (1-14); where pressures and time are as previously defined, $T^+$ is the positive pressure duration, and $\alpha$ is referred to as the waveform parameter [5, 6, 20, 26, 27, 34]. Once regarded as an empirically-based relationship, Thornhill proved that the modified Friedlander equation is actually a useful approximation to the derived analytical solution developed from the theory of far-field blasts [35].

$$P(t) = P_0 + P_s \left(1 - \frac{t}{T^+}\right) \exp\left(-\frac{\alpha t}{T^+}\right)$$  \hspace{1cm} (1-14)

Ethridge developed a smoothing procedure for shock wave pressure signals by using linear approximations to semi-logarithmic plots of the pressure signal [30]. This procedure arose from the analyses of nuclear explosion data by Porzel and Schmidt [36]. Ethridge built upon their extensive data analysis to suggest a universal method for handling shock wave pressure signals.
Accordingly, a procedure was developed for estimating the actual peak shock wave pressure from the pressure signal using a linear approximation to the logarithmic plot of pressure versus time [29, 30]. Through subsequent extrapolation of the linear approximation, an approximate value of peak shock wave pressure is calculated. It should be noted though, that the method applied by Ethridge is purely subjective, as the experimentalist defines what portion of the pressure signal is approximately linear, which can lead to additional experimental error.

Similarly, the positive pressure duration is approximated by plotting pressure vs. the logarithm of time. Fitting a linear curve to the data and extrapolating, the experimentalist is able to determine the approximate end time of the positive pressure phase [30]. Here again, an approximation is made as to what part of the pressure signal is linear, leading to additional error in determining the final explosive impulse.

Kinney et al. hypothesized that knowledge of the shock wave Mach number vs. shock wave radius profile permitted the calculation of a theoretical positive pressure duration vs. shock wave radius profile [29]. Hargather et al. applied this theoretical procedure to experimental positive pressure duration results for 1g PETN and TATP charges; a reasonable correlation was found to exist with the exception that an initial condition was unknown, thus requiring further analysis [4].

Lastly, Ethridge developed a procedure for fitting the waveform parameter \( \alpha \) to the two curvefit approximations for peak shock wave pressure and positive pressure duration. A second method developed by Kinney et al. [29] and later modified by Ismail et al. [6], approximates the waveform parameter \( \alpha \) through mathematical formulation. Further details will be discussed in Section 3.8. Accordingly, this completes the
determination of the modified Friedlander equation variables, permitting a smoothed-curve approximation to a typically-noisy experimental pressure signal.

### 1.2.6 Double-base propellants

Double-base propellants, e.g. smokeless power (SP), have received minimal research attention concerning their detonation characteristics. Though typically burning via deflagration, double-base propellants boast the ability to detonate as a vigorous high explosive [37, 38]. It is known that the combustion velocity of SP increases with an increase in the pressure under which combustion occurs [39]; thus, through the use of a sufficiently-powerful booster, detonation of SP is achievable. Okada et al. achieved steady-state detonation of SP with the aid of a C-4 booster, as compared to a deflagration-to-detonation transition (DDT) observed with a black powder fuse-head-igniter booster [40].

Double-base propellants exhibit extremely high critical pressures required for detonation, similar to liquid high explosives [41]. The Chapman-Jouguet pressure (see Section 2.3), or critical pressure, for a 4.0\text{cm} diameter double-base-propellant charge was found to vary between 6.0 – 9.0\text{GPa}. Additionally, critical pressures were found to increase with decreasing charge diameters [41, 42].

The TNT equivalent of SP has been established using two different tests: sand crush and crater volume [29, 43]. The sand crush test determined SP to have a TNT equivalent of 0.93 while the crater volume method established an equivalent of 0.68. This vast difference emphasizes that the experiments conducted are crude engineering
approximations compared to the highly-physics-based radially-dependent TNT equivalent procedure outlined using optical shock wave tracking.

1.2.7 Composite explosive-charges

The present research focuses on the characterization of insensitive explosive materials through the use of composite explosive-charges. Minimal previous research was found using composite charges. The use of booster charges and explosive trains at the full scale are the main cases where two homogeneous explosives exist in a composite arrangement. In these arrangements, the booster effects are typically ignored or are crudely accounted for, see [17, 20, 26, 27, 34, 43].

Previously, the booster charge has been quantified in one of three ways: 1) ignore its effects; i.e., when the booster charge mass is less than 5% of the total charge mass, 2) add the booster charge mass to the total charge mass, and 3) subtract a calculated equivalent weight of booster from the results through peak shock wave pressure calculations. The first two options are extremely crude, in that all effects produced by the booster charge are ignored. The third option, the most practical solution to date, accounts for the booster charge effects by calculating a peak shock wave pressure produced by the booster and subtracting it from the total peak shock wave pressure produced by the composite charge at an equivalent shock wave radius. Even so, it should be noted that the third option is only the application of a hypothesis, unsupported by any experimental or numerical data [44]. Kuhl et al. documented an increase in peak shock wave pressure from a composite arrangement of PETN and TNT, but failed to remove the effects of the
PETN booster (one-third of the total charge mass) from the composite-charge results [45].

1.2.8 Research Goals

The goal of the present research is to expand the scientific knowledge of explosive-material characterization by addressing the pre-existing lack of understanding pertaining to booster charge effects when utilized in the detonation of insensitive explosive materials. Additionally, it is desired to determine a more physically-based initial condition for the radial propagation of the shock wave resulting from a high-explosive charge. Laboratory-scale experiments will be conducted using optical imaging techniques, scaling laws, and pressure transducers to gather scientific data resulting from the detonation of explosive materials in a composite-charge arrangement.

Principle explosive properties of the explosively-driven shock wave resulting from laboratory-scale gram-range explosive charges will be measured experimentally using ultra-high-speed and high-speed shadowgraph visualization and piezoelectric pressure transducers. Procedures will be developed for the characterization of candidate explosive materials through the use of composite explosive-charges consisting of a well-characterized spherical booster charge surrounded by a candidate-explosive shell charge.

The radially-dependent TNT equivalent of candidate explosive materials, if detonated alone, will be experimentally determined by removing the effects of the booster charge from the composite-charge signature based upon peak shock wave pressure and explosive impulse at the laboratory scale. Numerical modeling of
composite explosive-charge detonations will compliment the experimental data by checking the results. The prescribed procedure will be applied to the characterization of Alliant Bullseye® smokeless powder (SP) and the insensitive secondary high explosives cyclotrimethylene trinitramine (RDX) and cyclooctamethylene tetranitramine (HMX). If successful, this research will substantially extend the validity and utility of laboratory-scale explosive testing.
Chapter 2

Experimental Methods

2.1 Explosive materials

Pentaerythritol tetranitrate (PETN) was the only secondary explosive material synthesized in the laboratory. All other explosive materials: smokeless powder (SP), cyclotrimethylene trinitramine (RDX) and cyclotetramethylene tetranitramine (HMX), were purchased from suppliers. Synthesis instructions for the PETN were obtained from an internet website [46] and verified by reference to Urbanski [47]. The smokeless powder was purchased from Alliant Powder Company, and the RDX and HMX were purchased from Holston Defense Corporation. The RDX and HMX were in the form of large crystalline structures that required re-crystallization for manufacturing gram-range explosive-charges.

2.1.1 PETN synthesis

PETN synthesis required standard laboratory glassware and reactants including: 98% concentrated sulphuric acid (H$_2$SO$_4$), 70% concentrated nitric acid (HNO$_3$), acetone (C$_3$H$_6$O), pentaerythritol (C$_5$H$_{12}$O$_4$), sodium bicarbonate (NaHCO$_3$), distilled water, and dry ice. Synthesis was performed under a chemical fume hood. The procedure outlined below yielded a batch of approximately 80g of PETN.
First, a cold acetone bath \((T < -248K)\) was prepared using \(\sim 4l\) of acetone and dry ice. A 400\(ml\) beaker was placed in the bath into which 140\(ml\) of concentrated nitric acid and 100\(ml\) of concentrated sulphuric acid were combined. The temperature of the solution was monitored with a thermometer. Once the solution temperature reached \(T < 258K\), the pentaerythritol was added in 8\(g\) increments. The solution was continuously stirred upon adding the pentaerythritol and its temperature constantly monitored, ensuring that it remained below 263\(K\). A total of 40\(g\) of pentaerythritol were added to the solution. The solution was constantly stirred for an additional 10\(min\) after the entire 40\(g\) of pentaerythritol was added.

Next, the reaction beaker containing the solution was placed in a hot water bath with a temperature held between 318\(K\) and 323\(K\). The solution was constantly stirred for 25\(min\) while remaining in the hot water bath. After 25\(min\), the solution was poured into 1.8\(l\) of distilled water \((T \approx 283K)\). The solution then remained unperturbed, allowing the precipitate to settle to the bottom. The precipitate was filtered off from the solution and subsequently neutralized by alternating between rinsing with cold distilled water \((T \approx 283K)\) and hot sodium bicarbonate solution \((T \approx 318K)\), which was prepared from 5\(g\) sodium bicarbonate and 500\(ml\) of distilled water. The precipitate was rinsed until neutralized, indicated by its failure to foam when rinsed with the hot sodium bicarbonate solution.

Once neutralized, the precipitate was added to 900\(ml\) of hot acetone \((T = 323K)\) while constantly-stirring. The pure PETN dissolved in the hot acetone while impurities remained in precipitate form. This precipitate was filtered off while pouring the hot acetone solution into 2.5\(l\) of distilled water at \(T \approx 283K\). Pure PETN precipitated out of
the hot acetone solution upon entering the distilled water. Finally, the PETN precipitate was filtered from the acetone/distilled water solution and allowed to dry for 24 hrs. An x-ray diffraction (XRD) analysis performed at the National Institute of Standards and Technology (NIST) determined the final product purity to be 95% PETN [48].

2.1.2 RDX re-crystallization

To allow manufacturing of gram-range explosive charges, it was necessary to re-crystallize the large purchased RDX crystals into smaller crystals. A solution of RDX and hot acetone was created according to the solubility of RDX [49]. The present solution consisted of 10.3 g of large-crystal RDX and 100 g of room-temperature acetone. The solution was constantly stirred while heated on a hot plate to a temperature of \( T \approx 313K \) until all of the RDX crystals dissolved in the acetone. Once dissolved, the beaker was plunged into an acetone ice bath \( (T < 263K) \) while vigorously stirring the solution. Small-crystal RDX subsequently precipitated out of the acetone solution. After reaching the ice bath temperature, the precipitate was filtered off from the acetone solution and was dried for ~6 hrs. As RDX is still soluble in acetone at this temperature, the remaining acetone was evaporated from the solution to collect the remaining RDX. The re-crystallization process was then repeated. An XRD analysis performed at the NIST determined the re-crystallized product purity to be 93% RDX [48]. This agreed with the purchased RDX data sheet which stated the material to be 94% RDX.
2.1.3 HMX re-crystallization

The large HMX crystals that were purchased also required re-crystallization before manufacturing gram-range explosive-charges. A solution of HMX and $\gamma$-butyrolactone ($C_4H_6O_2$) was prepared according to Heinemeyer [50]. The present solution consisted of 10g of large crystal HMX and 50ml of $\gamma$-butyrolactone heated to a temperature of 303K. Using a burette, the solution was slowly dripped into 500ml of toluene ($C_7H_8$) held between 281K and 288K. Small HMX crystals precipitated out of the solution upon mixing with the toluene. The precipitate was filtered off from the toluene/$\gamma$-butyrolactone solution and washed with room-temperature distilled water. The re-crystallized HMX was filtered off from the distilled water and allowed to dry for 24hrs. The re-crystallized yield was calculated to be 90%, which agreed with Heinemeyer [50]. An XRD analysis performed at the NIST determined the final product purity to be 98% HMX [48]. This also agreed with the purchased HMX data sheet which stated the material to be 98% HMX.

2.2 Explosive-charge construction

A reproducible method of explosive-charge construction was required to properly characterize composite explosives. This is because the present scaling laws of Hopkinson, Cranz, and Sachs do not account for the individual scaling of explosives in a composite arrangement. Thus, it was necessary to control the variability between charge masses. Previous work by Kleine et al. and Hargather et al. made use of cylindrical explosive-charges that required a specific detonation orientation due to resulting 3-
dimensional shock asymmetry [3, 4]. The use of a cylindrical booster charge in a composite-charge arrangement is possible if a cylindrical shell charge is used, but would still require the need for proper optical-axis orientation. Rather, a concentric, spherical composite-charge was developed to eliminate the orientation requirement and to avoid shock asymmetry, Figure 2-1. Additionally, the concentric, spherical arrangement facilitates a more uniform detonation throughout the composite charge; i.e., transferring of the detonation wave through the booster charge and into the shell charge.

![Figure 2-1: Concentric, spherical composite-charge arrangement; (White) Central, spherical booster charge and (Gray) Concentric, spherical shell charge.](image)

Two types of die sets were developed to produce hemispherical charges of equivalent mass and density. One set produced a hemispherical booster charge and the second a hemispherical shell. As illustrated in Figure 2-2, the female components contain a hemispherical cavity of known depth, 20mm, and diameter (multiple diameter sets were manufactured to develop varying mass charges). As shown in Figure 2-2, the
difference in booster- and shell-charge die sets lies in the male component. The male end of the hemispherical booster charge die is flat whereas the male end of the hemispherical shell charge contains a concentric external hemisphere of booster-charge diameter. This external hemisphere creates the proper internal hemispherical core in the shell charge to accept the booster charge. A specified mass of explosive was measured and poured into the cavities, along with several drops of acetone (~0.5 ml/g of explosive). The male and female components were subsequently pressed together using a press requiring average-human strength, compressing the explosive material, and producing a hemispherical booster charge or hemispherical shell charge of known mass and density. Scanning electron microscope images were taken at the NIST to ensure uniform density of the hemispherical charges [48]. Final hemispherical charges are shown in Figure 2-3.

Figure 2-2: Hemispherical die sets; (Left) 12.7 mm diameter booster and (Right) 15.9 mm outer diameter shell with 12.7 mm inner diameter.
Three types of explosive charges were developed during the present research: a booster charge, a high-explosive (HE) composite charge, and a smokeless powder (SP) composite charge. The booster charge consisted of a centrally-positioned 0.15\( mm \) copper wire surrounded by two hemispherical booster charges glued together with a paste of nitrocellulose and acetone. The nitrocellulose and acetone paste accounted for less than 2\% of the final charge mass. The copper wire served as an exploding bridgewire (EBW) initiated through the use of a capacitor discharge unit (see Section 2.3 for detailed explanation).

The HE composite charge consisted of a booster charge surrounded by two hemispherical shell charges. The hemispherical shell charges were assembled around the booster charge and were glued together with the nitrocellulose/acetone paste. Lastly, for the SP composite charge, plastic wrap was placed in a hemispherical mold with a centrally-located booster charge. The EBW ends were fed through the saran wrap, which served as a method of containment for the SP. A specified mass of SP was poured.
around the booster charge, with the saran wrap subsequently being wrapped approximately spherically by hand, Figure 2-4.

![Figure 2-4: Spherical explosive charges; Left to right: Booster charge, HE composite charge and SP composite charge.](image)

### 2.3 Explosive charge detonation

The present research investigated the air-blast characterization of secondary explosives. The sequence of events leading up to and through the propagation of the explosively-developed shock wave is a complex phenomenon. These events include: the generation of a shock wave for explosive initiation, the propagation of the detonation wave, and the propagation of the explosively-driven shock wave in air. Herein, a detailed description of these events, starting from the detonation of the exploding bridgewire (EBW), is given.

Secondary explosives require the impulse of a shock wave for detonation initiation [38]. This shock was generated and introduced into the explosive material
through the use of an EBW. The EBW used here was the previously-defined 0.15mm copper wire positioned centrally across and between the two hemispheres of the spherical booster charge. A capacitor discharge unit (CDU) was used to explode the EBW by supplying a sufficiently-large current at a very fast rise rate, on the order of \(~200A/\mu s\) [51].

A complex sequence of events occurs as a result of the rapid discharge of energy into the EBW. First, the EBW heats up to near its melting temperature, causing the wire resistance to rise due to thermal effects. Almost instantaneously, the EBW melts to its liquid state. At this point, inertia effects of the wire prevent expansion of the melted copper. The EBW continues to heat past its melting point, resulting in a phase change from liquid to vapor. Once vaporized, its resistance rises rapidly, causing a drop in the current flowing through the wire. At this instant in time, the wire resistance and voltage across it are at their maximums. The time at which this voltage peak occurs is referred to as the “burst time”; the corresponding current is referred to as the “burst current”. As an excess of current is still passing through the vaporized wire, the metal vapor continues to heat. Finally, the vaporized material overcomes its inertia and explosively expands causing a shock wave to develop, which is transferred into the explosive material, ultimately resulting in detonation of the explosive charge. The total time interval of events from the EBW melting through its vaporization is only a few tens of nanoseconds [52].

The minimum burst current of an EBW required to detonate a given explosive material is a function of the explosive-charge density (i.e. the percentage of air-filled voids), the explosive particle size, and specific surface area of these particles. In
addition, the minimum burst current is a function of the bridgewire parameters, i.e. the bridgewire material and its wire diameter and length. To achieve detonation, the critical energy fluence of the explosive material must be exceeded by the introduced shock wave from the EBW, where the shock wave pressure and its duration are functions of the burst current [52]. Thus, the bridgewire used and the electrical discharge applied by the CDU must be tailored to meet or exceed the characteristic minimum burst current unique to the explosive material of interest.

Detonation of the explosive material will result by exceeding its critical energy fluence, a function of the minimum burst current. The simplest theory for modeling the detonation of an explosive material is known as the “ZND model” after Zeldovich, von Neumann and Döring [52]. The model requires that several assumptions be made, which include: 1) the flow is one-dimensional, 2) the detonation front is a “jump” or discontinuity, 3) reaction-product gases leaving the detonation front are in chemical and thermodynamic equilibrium and the chemical reaction is completed, 4) the chemical reaction-zone length is zero, 5) the detonation velocity is constant, and 6) after leaving the detonation front, the reaction products may be time-dependent and are affected by the surrounding system [52].

Once introduced into the explosive material, the EBW shock wave acts to compress the particles of the explosive. Each particle is compressed and heated by the shock wave, triggering its chemical reaction. Gas pressure from the chemical reaction energy released supports the shock front, driving it through the material towards full steady-state detonation, while acting to compress and heat subsequent explosive particles. As these gaseous products expand, a rarefaction moves forward into the shock from
behind. The shock wave pressure and velocity adjust to steady-state detonation values independent of the initiation source, but dependent on the initial density of the explosive material. At this time, the shock front, chemical reaction, and leading edge of the rarefaction are in equilibrium [52]. The detonation velocity of explosive materials may vary anywhere from 1.5 km/s for blasting explosives, to an excess of 8 km/s for military explosives [24]. Thus, total chemical reaction time for a 1g charge of secondary explosive material at a nominal density of 1.5 g/cm$^3$ is on the order of 1μs.

The previously-mentioned detonation jump from unreacted-explosive to completely-reacted gaseous-detonation-products is best visualized by plotting Hugoniot curves of the two states on the $P$-$v$ plane, i.e. gage pressure as a function of specific volume, $v = 1/\rho$, Figure 2-5.
Figure 2-5: $P-v$ Hugoniots of the reaction and detonation products.
The Hugoniot curve represents all possible equilibrium states at which a given material can exist [52]. The initial state of the unreacted explosive is denoted as point (1) and the fully-shocked state as point (3). Point (3) is referred to as the von Neumann spike and is defined as the state that brings on the chemical reaction of the material [52]. The line connecting these states is designated the Rayleigh line, representing a change in energy without friction. The Rayleigh line is drawn from point (1) and tangent to the detonation products Hugoniot to determine point (3). Point (2) represents the Chapman-Jouguet (CJ) point, defined as the steady-state detonation condition where the Rayleigh line is tangent to the detonation-products Hugoniot. The CJ point represents the only location where a single state can exist for the detonation products due to tangency of the Rayleigh line. Thus, the passing of the shock wave through the unreacted material causes a jump in conditions from (1) to (3) and then back down to (2). This is best represented on a pressure vs. radial distance diagram, Figure 2-6.
Figure 2-6: Pressure vs. radial distance diagram of detonation wave $D$ moving left-to-right.
Figure 2-6 shows the unreacted-material pressure jump up to the von Neumann spike followed by its decay to the CJ point, where the material is converted to the detonation products. After the CJ point, the gaseous products expand along the Taylor rarefaction wave. Due to the reaction zone being extremely narrow and the chemical reaction occurring so fast, the energy involved in the von Neumann spike is ignored as being inconsequential to the total energy of the fully-reacted detonation products [52]. This coincides with the ZND model’s assumption that the reaction zone length is zero.

Once the detonation wave has propagated through the entire spherical explosive-charge, gaseous-expansion of the detonation products results in the formation of a spherical shock wave in the surrounding atmosphere. The complex air-blast physics resulting from a detonation are best illustrated on a radius vs. time diagram, Figure 2-7.
As shown, the compressed detonation gases are expanded through an expansion fan composed of spherical inward-moving rarefaction waves, simplified as $R$. The contact surface $C$ separates these expanding gases from shock-compressed air behind the primary shock wave $S_1$. The primary shock wave $S_1$ is initially driven by the expansion of the contact surface, but the contact surface exhibits a continually-decreasing velocity with radial propagation. Once the shock wave and the contact surface separate, at any particular radial distance, the shock compressed air expands, causing pressure to decay with time. As a result, expansion waves move throughout the flowfield, overtaking the primary shock wave, resulting in its attenuation [53].
The expansion fan, centered at the explosive-charge radius, develops as a result of the combustion-gases expansion. At the head of this fan is the initially-weak, inward-facing secondary shock $S_2$ propagating radially-outward due to the strongly-expanding combustion gases [53, 54]. Eventually, the secondary shock comes to rest and begins moving inward towards the explosive charge origin, strengthening due to its interaction with the expansion fan and finally reflecting from the explosion origin with greatly-increased strength [54, 55]. The presence of a secondary shock wave is a result of the excessive explosive-gases expansion which, if not present, results in the gas pressure being too low when compared with the adjacent shock-compressed air. Thus, it creates an increase in the gas pressure, ensuring continuity at the boundary [54].

After reflecting from the origin, the secondary shock wave propagates radially-outward through the flow, interacting with the contact surface and continuing its propagation behind the primary shock wave. The interaction of the secondary shock and the contact surface produces an expansion fan due to the shock passing from the dense, expanding, combustion gases to the shock-heated air outside the contact surface; a tertiary shock $S_3$ is produced as a result. These events continue to repeat themselves as additional reflections and rarefactions occur at the contact surface, though they may be very weak.

Figure 2-8 is an experimental composite radius vs. time diagram of the detonation of an explosive charge. The figure illustrates the propagation of the primary shock wave, secondary shock wave, and the expanding gaseous products. For a 1g charge, the total time from formation of the primary shock wave to its propagation through the radial-range of experimental measurements (i.e. $\sim 0.25m$) takes $\sim 0.25ms$. In 0.5ms both the
detonation reaction and the initial interaction with the surrounding atmosphere are completed for a 1g explosive charge.

Figure 2-8: Experimental radius vs. time wave diagram of an explosion.
2.4 Optical visualization methods

Explosive charge detonations were observed using a focused Z-type shadowgraph system to optically track the shock wave radius as a function of time. This was similar to the setups used by Kleine et al. and Hargather et al., who used schlieren and focused shadowgraph systems [3, 4]. Z-type focused shadowgraphy is identical to Z-type schlieren, with the exception that the shadowgraph technique does not have a knife edge cutoff [56]. For the present research, focused shadowgraphy was chosen as it is robust and adequate for shock wave imaging, whereas schlieren was unnecessary.

Though the focused shadowgraph system has theoretically zero sensitivity when sharply focused, the inherently-strong refractive index gradient produced by a shock wave of significant strength still allows a visualization [56]. The shadowgraph system, Figure 2-9, consisted of twin 0.76m diameter f/5 parabolic mirrors with a 200W ozone-free xenon arc lamp light source (Newport/Oriel Instruments). Shock wave radius vs. time data were recorded with Photron APX-RS, Photron Fastcam SA-1, and DRS Ultra8 high-speed digital cameras capable of 210,000, 675,000, and 100,000,000 frames/s, respectively. A sample shadowgram is exemplified in Figure 2-10.
Figure 2-10 represents the radial shock wave distance $R$ as the abscissa taken at an instant in time. The shock wave is denoted by the outer-edge dark band of pixels furthest from explosion center $X$, due to the parallel light being refracted towards the high-density inner region trailing the shock, represented by the light band of pixels on the inner-edge of the shock wave. In this image, the secondary shock wave is also visible and its location is labeled.

Resulting shadowgrams were post-processed and analyzed using a MATLAB code to determine the radial pixel location of the shock wave from the explosive charge.
center as a function of time, as was done by Hargather [4]. In addition to the shock wave radius history profile, a composite shock wave radius vs. time diagram was generated by concatenating the center row of pixels from each shadowgram. An example “streak” image of this type was shown in Figure 2-8.

Radial pixel distances were converted to dimensional lengths by way of a calibration object. A calibration image of a standardized-length object was taken at the conclusion of experiments each day. The calibration object, an aluminum plate of length 0.4890m, was imaged while positioned perpendicular to the optical path in the test section of the shadowgraph system. A dimensional pixel length was then calculated from the calibration image.

2.5 Peak shock wave pressure and its decay

Explosive charges were detonated at a specified radial-standoff distance from a piezoelectric pressure transducer to measure the peak shock wave pressure and its decay as a function of time. Temporal shock wave pressure histories were measured using a Kistler 603B1 quartz pressure transducer (Kistler Instrument Corp.). Acceleration of the transducer, as a result of the shock impingement, is compensated using a high frequency response, and is capable of measuring a maximum pressure of 103MPa with a 2μs response time. The transducer was connected to a Kistler 5010B1 dual-mode charge amplifier with a low noise Kistler 1631C5 transmission cable. Pressure decay histories were captured using a Tektronix TDS-2024 oscilloscope at a sampling rate between 2.5-10MHz.
Simultaneous shadowgrams were gathered during each explosive charge pressure test using the high-speed Photron Fastcam SA-1 digital camera and the focused Z-shadowgraph system. These radial shock wave propagation data were captured at 300,000 frames/s and 0.37 μs exposure, yielding a frame size of 704x16 pixels. The resulting shadowgrams aided in identifying causes for non-ideal pressure traces recorded by the pressure transducer, e.g. irregular shock front, pressure transducer/particle impact or non-perpendicular shock wave impingement on the pressure gage. Non-ideal shock wave pressure vs. time histories were eliminated when irregularities were able to be identified from the shadowgrams.

The pressure transducer was flush-mounted in a 0.25 m square aluminum impingement plate with a Kistler 223A mounting adapter. The impingement plate was secured to an optics table in the test section of the Z-shadowgraph system and fastened to a weighted structure for rigidity of the pressure transducer, Figure 2-11. As shown, the impingement plate was oriented perpendicular to the radial shock wave propagation direction so as to measure face-on, or stagnation, shock wave pressure histories per the results and suggestion of Rahman et al [5].
For the given experimental setup, i.e. stagnation pressure measurements, the pressure transducer measured the reflected shock wave pressure history $P_r(t)$, due to the shock wave having reflected off of the impingement-plate surface. To compare results with the previous optical measurements, the resulting data were converted to incident shock wave pressure histories $P_s(t)$, using atmospheric pressure $P_0$ and assuming perfect gas behavior ($\gamma = 1.4$) for air according to Held [57], Equation (2-15).

\[
\frac{P_r}{P_s} = 2 + \frac{6(P_s / P_0)}{(P_s / P_0) + 7}
\]  

(2-15)

Explosive charges were positioned at an approximate specified radial-standoff distance from the impingement plate, whereupon a laser level was used to optically align
the explosive charge center with the pressure transducer face center. The radial-standoff distance of the explosive charge from the impingement plate was imaged for calibration purposes. Following the completion of experiments, a calibration image was obtained. The calibration object, a known length aluminum plate (0.1950 m), was imaged while positioned perpendicular to the optical path in the test section of the shadowgraph system. As before, this was used during post-processing methods to convert pixel length to a dimensional length for determining an accurate radial-standoff distance between the explosive charge center and the impingement plate.
Chapter 3

Experimental Results

3.1 Explosive shock wave sphericity

Shock sphericity was first verified to ensure uniform spherical detonation was occurring for the spherical charge configuration, thus eliminating the need for charge orientation. To verify sphericity, an Edgerton-type retroreflective shadowgraph system was set up perpendicular to the present Z-type focused shadowgraph system, Figure 3-1 [8]. A second high-speed digital camera (Photron APX-RS) recorded simultaneous perpendicular shadowgrams of the propagating shock wave resulting from non-oriented booster and composite charges at 30,000 frames/s. The resulting shock waves were determined to be spherical and nearly-identical in radii at equivalent times for all configurations, i.e. a booster charge, a HE composite-charge, and a SP composite charge. An example set of images is shown in Figure 3-2.
Figure 3-1: Edgerton shadowgraph set up perpendicular to the focused Z-shadowgraph.

Figure 3-2: Perpendicular shadowgrams illustrating shock sphericity; (Top) Focused Z-shadowgram and (Bottom) Edgerton-type retroreflective shadowgram (not focused).
3.2 Homogeneous charge characterization

Explosive materials to be used for booster and shell charges were first individually characterized. Explosive characterization was performed for two densities of PETN: $\rho = 0.90\text{g/cm}^3$ and $\rho = 1.34\text{g/cm}^3$. The lower-density PETN was chosen as this was found to be the minimum density attainable that still permitted mechanically-reproducible charges to be manufactured. Additionally, it was desired that the low-density charge be approximately equal to the standard-density charge used in detonators, i.e. $\rho = 0.88\text{g/cm}^3$ [49]. The higher-density PETN represents the maximum density attainable that permitted mechanically-reproducible charges to be manufactured. A theoretical maximum-density charge was attempted, $\rho = 1.77\text{g/cm}^3$, but was unattainable due to charge die limitations.

Two different diameter hemispherical charge dies were used for manufacturing different-mass explosive charges at each PETN density to re-illustrate the ability to scale explosive charges to a standard charge mass at the laboratory scale [4]. PETN charge masses ranged from $0.418 - 0.972\text{g} \pm 0.005\text{g}$ for the low-density PETN and $0.871 - 1.492\text{g} \pm 0.005\text{g}$ for the high-density PETN, Table 3-1. Shock wave radius vs. time data were taken by exploding 20 charges of each density. Several of these charges were eliminated due to shock wave irregularities, e.g. the presence of unburned particles, resulting in 16 data sets for each charge density.

Resulting shock wave radius vs. time data were scaled using Equations (1-4) – (1-7) of Section 1.2.2. Deviations in atmospheric temperature and pressure from NTP conditions were recorded on the day of each test. Although, atmospheric pressure
deviations were present, they were found to be small and their result insignificant in the scaling of shock wave radii and times. Thus, in the present research, only atmospheric temperature variations were accounted for in the scaling process. Shock wave radii and times were scaled to a standard charge mass of $W_{std} = 1g$ and atmospheric temperature of $T_{NTP} = 288.16K$. The resulting data sets are shown in Figure 3-3 and Figure 3-4.

Table 3-1: Homogeneous PETN explosive-charge masses.

<table>
<thead>
<tr>
<th>Charge Number</th>
<th>Mass [g]</th>
<th>Charge Number</th>
<th>Mass [g]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.955</td>
<td>1</td>
<td>1.451</td>
</tr>
<tr>
<td>2</td>
<td>0.972</td>
<td>2</td>
<td>1.492</td>
</tr>
<tr>
<td>3</td>
<td>0.961</td>
<td>3</td>
<td>1.467</td>
</tr>
<tr>
<td>4</td>
<td>0.965</td>
<td>4</td>
<td>1.432</td>
</tr>
<tr>
<td>5</td>
<td>0.952</td>
<td>5</td>
<td>1.449</td>
</tr>
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<td>6</td>
<td>0.971</td>
<td>6</td>
<td>1.443</td>
</tr>
<tr>
<td>7</td>
<td>0.433</td>
<td>7</td>
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</tr>
<tr>
<td>8</td>
<td>0.445</td>
<td>8</td>
<td>1.486</td>
</tr>
<tr>
<td>9</td>
<td>0.424</td>
<td>9</td>
<td>0.910</td>
</tr>
<tr>
<td>10</td>
<td>0.423</td>
<td>10</td>
<td>0.902</td>
</tr>
<tr>
<td>11</td>
<td>0.422</td>
<td>11</td>
<td>0.910</td>
</tr>
<tr>
<td>12</td>
<td>0.428</td>
<td>12</td>
<td>0.911</td>
</tr>
<tr>
<td>13</td>
<td>0.421</td>
<td>13</td>
<td>0.898</td>
</tr>
<tr>
<td>14</td>
<td>0.426</td>
<td>14</td>
<td>0.906</td>
</tr>
<tr>
<td>15</td>
<td>0.422</td>
<td>15</td>
<td>0.895</td>
</tr>
<tr>
<td>16</td>
<td>0.418</td>
<td>16</td>
<td>0.871</td>
</tr>
</tbody>
</table>
Figure 3-3: PETN ($\rho = 0.90g/cm^3$) shock wave radius versus time data.
Figure 3-4: PETN ($\rho = 1.34\text{g/cm}^3$) shock wave radius versus time data.
For ease of comparing explosive data, shock wave radius vs. time data were fit to Dewey’s equation, Equation (1-8) of Section 1.2.2. For both PETN densities tested here, coefficients $A$, $B$, $C$, and $D$ were calculated by way of a linear least-squares regression calculation, yielding an equation of best fit for each shock wave radius vs. time profile.

As previously mentioned, Dewey’s equation does not possess a physically-based initial condition of the shock wave radius vs. time data. For the present data, it is impossible to determine the exact time of explosive charge initiation due to time resolution limitations of the camera. Additionally, the shadowgrams do not illustrate the actual charge initiation, but rather the propagation of the shock wave as it transfers from the solid explosive charge into the surrounding quiescent air. Therefore, the time of initiation is only determinable within one time step at the present frame rate of 210,000 frames/s, i.e. $5\mu s$.

The argument can be made that, due to the number of charges detonated, statistically the time of initiation is approximately one-half the camera interframe time step. Secondly, an argument can be made that the initial shock wave radius is approximately equal to the explosive-charge radius at $t = 0s$; a result of the orders-of-magnitude-difference between explosive material detonation time (nanoseconds) and shock wave propagation time (milliseconds).

Curvefit coefficients for both PETN densities were recalculated using the above two assumptions. Firstly, curvefit coefficients were calculated after subtracting one-half time step off the measured data (the statistical argument). Secondly, curvefit coefficients were calculated by shifting the previously-calculated curvefit to enforce the initial condition of the shock wave radius equality with the explosive-charge radius. For both
densities of PETN, these two resulting curvefits yielded nearly-identical data representations. Thus, the statistics of the data validate the initial-condition hypothesis that the shock wave radius is approximately equal to the explosive-charge radius. The results of the low-density PETN are given as an example in Figure 3-5 and final curvefit coefficients are given in Table 3-2.

As determined, even with shifting the initially-calculated curvefit to the data, curvefit coefficient $A$ is still not equal to the explosive-charge radius. It was attempted to force curvefit coefficient $A$ to equal the explosive charge radius, however, this resulted in a poor correlation to the data. This is due to Dewey’s equation lacking a true physical basis and the lack of available data for explosive-charge shock wave radii less than 5 cm. Further analysis and explanation will be given in Section 3.3.

Table 3-2: Equation (1-8) homogeneous 1g PETN explosive curvefit coefficients.

<table>
<thead>
<tr>
<th>Explosive</th>
<th>$A$</th>
<th>$B$</th>
<th>$C$</th>
<th>$D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PETN @ $\rho = 0.90 g/cm^3$</td>
<td>-0.00768</td>
<td>1.00000</td>
<td>-0.87394</td>
<td>0.80721</td>
</tr>
<tr>
<td>PETN @ $\rho = 1.34 g/cm^3$</td>
<td>-0.01204</td>
<td>1.00000</td>
<td>-1.13330</td>
<td>0.89543</td>
</tr>
</tbody>
</table>
Figure 3-5: PETN ($\rho = 0.90\text{g/cm}^3$) shock wave radius versus time curve using assumptions of shifting the data (statistical argument) and shifting the curvefit.
Shock wave Mach number $M_s$ was determined as a function of shock wave radius using Equations (1-8) – (1-11) from Section 1.2.2. Plots of shock wave Mach number as a function of shock wave radius are shown for a 1g PETN charge at NTP for each PETN density, Figure 3-6. The experimental error for the shock wave Mach number data was determined from errors in the shock wave radius pixel distance, the calibration image pixel distance, and the atmospheric temperature uncertainty according to Moffat [58]. As illustrated, shock wave Mach numbers up to 6 were measured. Figure 3-6 reveals that changing the packing density produces slightly different shock wave Mach number vs. shock wave radius profiles for the same explosive, PETN.

The Rankine-Hugoniot relationship for perfect air, Equation (1-12), was implemented to determine the ratio of peak shock wave pressure $P_s$ to normal atmospheric pressure $P_{NTP}$ from the shock wave Mach number. Using the resulting peak shock wave pressure vs. shock wave radius profile, a radially-dependent TNT equivalent was determined by calculating the mass of TNT required to produce an identical peak shock wave pressure at an equivalent shock wave radius.

The data were compared to the reference spherical free-air burst TNT standard established by Kingery and Bulmash, Equations (3-16) and (3-17) where $T$ is the common logarithm of the shock wave radial distance in meters and $Y$ is the common logarithm of the peak shock wave pressure in Pascals [59]. As shown in Figure 3-7, by varying the density of PETN, a different characterization results. This can be attributed to the rate of explosive energy release, which is dependent upon the explosive material detonation properties. These detonation properties include: detonation velocity, detonation pressure, heat of detonation, etc. Thus, varying the density of the explosive
charge yields a different air-blast characterization as a result of different rates of explosive energy release [49]. TNT equivalent error bars are representative of the uncertainty in peak shock wave pressure as determined from the shock wave Mach number error.

The TNT equivalent for both densities of PETN declines at larger scaled radii $R_s$ because PETN is a more-nearly-ideal explosive (10% oxygen-deficient compared to 74% for TNT [60]). That is to say, the PETN produces a stronger initial shock wave Mach number than TNT, causing larger entropy changes in the process, and ultimately resulting in less energy at larger shock wave radii [16].

$$U = -0.214362789151 + 1.35034249993T$$

$$Y = 2.611368669 - 1.69012801396U + 0.00804973591951U^2 + 0.336743114941U^3 - 0.00516226351334U^4 - 0.0809228619888U^5 - 0.00478507266747U^6 + 0.00793030472242U^7 + 0.0007684469735U^8 + 1$$
Figure 3-6: Shock wave Mach number vs. shock wave radius for two densities of 1g PETN.
Figure 3-7: Radially-dependent TNT equivalent of a scaled homogeneous 1g PETN explosive charge.
3.3 Initial shock wave Mach number

It was hypothesized that the initial shock wave Mach number may be related to the detonation velocity of the explosive material, thus yielding a physically-based initial condition for representation of the radial shock wave propagation. The Photron APX-RS camera was unable to resolve the shock wave radial history profile for shock wave radii less than 5\textit{cm} due to its time resolution limitations and the strong shock wave Mach numbers present at this range, i.e. $M_s > 6$. Accordingly, the previously-measured shock wave radius vs. time profiles were insufficient to properly address the non-physical initial condition of Equation (1-8).

Resolution limitations were determined by calculating the total time required for the shock wave to propagate from the initial explosive-charge radius to a radius of 5\textit{cm}. This total time was calculated using the following assumptions: 1) the initial shock wave velocity is equal to the explosive material detonation velocity, i.e. $M_s = 15.14$ for PETN at $\rho = 0.90g/cm^3$ in air [49], and 2) the shock wave Mach number decays linearly as a function of shock wave radius from the initial explosive-charge radius to a shock wave radius of 5\textit{cm}. The total time was calculated using the linear decay approximation profile between the initial condition and the known shock wave Mach number of $M_s = 5.75$ at a shock wave radius of 5\textit{cm}, Figure 3-6. The total time was determined to be 15\textit{$\mu$s}. Therefore, a maximum of three shock wave radius data points are able to be measured from the shock wave emergence at the explosive-charge radius to a shock wave radius of 5\textit{cm}, when imaging at 210,000\textit{frames/s} with the APX-RS camera. These three points are considered to be insufficient data for the purpose at hand.
Thus, a different camera was used to measure the explosively-driven shock wave at radii less than 5cm. A DRS Ultra 8 ultra-high-speed digital camera (DRS Data and Imaging Systems) was used in place of the current Photron camera in the focused Z-shadowgraph system. The Ultra 8 camera is capable of capturing 8 independently-controllable frames of 520x520 pixel resolution at recording rates up to 100,000,000 frames/s, and exposure times down to 10 ns. The Ultra 8, unlike the Photron APX-RS, is only capable of capturing 8 images per explosive test. Thus, accurate triggering of the camera and knowledge of the total time for the shock wave to propagate from the initial explosive-charge radius to a shock wave radius of 5cm was required.

Accurate camera triggering was accomplished using the photodiode sensor of a commercial electronic flash unit (Calumet Photographic). The photodiode output was connected to the DRS camera input. To consistently and accurately capture the explosive event, visible-spectrum light emitted from the explosive fireball was used for triggering the photodiode. When triggered, the photodiode transmits a small voltage signal to the camera input, triggering the camera to begin capturing images. An EBW was first exploded to verify that the power supply would produce an adequate voltage signal with minimum time delay to trigger the camera; this was found to be the case.

The explosively-driven shock waves resulting from the two characterized densities of PETN (ρ = 0.90 g/cm³ and ρ = 1.34 g/cm³) were measured using the Ultra 8 camera to research the initial-condition shock wave Mach number hypothesis. Three different charge mass arrangements were manufactured using the low-density PETN to measure the initial shock wave Mach number and verify scaling-law validity in proximity to the fireball, whereas only a single charge mass arrangement was manufactured using
the high-density PETN for measuring the initial shock wave Mach number. Exploded charges are tabulated in Table 3-3 and Table 3-4.

The Ultra 8’s independently-controllable time delay between frames was determined from the previously-outlined procedure of assuming a linear decay of the shock wave Mach number between the explosive-charge radius and a shock wave radius of 5cm. Time delays were calculated using the detonation velocities of the PETN at the two densities tested and the measured shock wave Mach number at a shock wave radius of 5cm, from the previous characterization. Times were established so as to capture the explosively-driven shock wave at approximately 0.5cm intervals (i.e. a total of ten data points whereas only three were possible with the APX-RS camera).

The initial-frame time delay between explosive-material detonation and camera triggering was determined from baseline EBW tests. A 15μs delay was determined to exist between camera triggering and explosion of the EBW. That is to say, the camera was triggered by the commercial flash unit first and explosion of the EBW occurred 15μs later. It is possible that the commercial flash unit was sending a trigger signal at the time of CDU discharge. However, this time delay was determined to be consistent from conducting multiple EBW tests and was therefore used in determining the total initial-frame time delay.

Explosive-charge detonation times were determined from the known explosive-charge radius and explosive-material detonation velocity. The detonation time was calculated to be approximately 1μs for all explosive-charge arrangements. Thus, the total initial-frame time delay between camera triggering and when the shock wave is expected to emerge from the explosive-charge was calculated to be 16μs. An example sequence of
images captured by the DRS Ultra 8 camera is shown in Figure 3-8 with imaging and timing sequences labeled.

Table 3-3: Homogeneous PETN ($\rho = 0.90g/cm^3$) explosive-charge masses.

<table>
<thead>
<tr>
<th>Charge</th>
<th>Arrangement 1 [g]</th>
<th>Arrangement 2 [g]</th>
<th>Arrangement 3 [g]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.434</td>
<td>0.650</td>
<td>0.966</td>
</tr>
<tr>
<td>2</td>
<td>0.413</td>
<td>0.630</td>
<td>0.944</td>
</tr>
<tr>
<td>3</td>
<td>0.418</td>
<td>0.645</td>
<td>0.972</td>
</tr>
<tr>
<td>4</td>
<td>0.428</td>
<td>0.644</td>
<td>0.963</td>
</tr>
<tr>
<td>5</td>
<td>0.394</td>
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</tr>
<tr>
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<td>0.431</td>
<td>0.634</td>
<td>0.914</td>
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<tr>
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<td>0.641</td>
<td>0.961</td>
</tr>
<tr>
<td>8</td>
<td>0.444</td>
<td>0.653</td>
<td>0.965</td>
</tr>
<tr>
<td>9</td>
<td>0.405</td>
<td>0.653</td>
<td>0.970</td>
</tr>
</tbody>
</table>
Table 3-4: Homogeneous PETN ($\rho = 1.34g/cm^3$) explosive-charge masses.

**Arrangement 4**

<table>
<thead>
<tr>
<th>Charge</th>
<th>Mass [g]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.853</td>
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<tr>
<td>2</td>
<td>0.888</td>
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<tr>
<td>9</td>
<td>0.866</td>
</tr>
</tbody>
</table>

Figure 3-8: Focused Z-shadowgrams using the DRS Ultra 8 camera (Imaging sequence order labeled).
Shock wave radius vs. time data were measured in this manner for the exploded charges of Table 3-3 and Table 3-4. For each of the four arrangements, shock wave radius vs. time data were scaled to a standard charge mass of $W_{std} = 1\, g$ and atmospheric temperature of $T_{NTP} = 288.16\, K$. The scaled data from each explosive-charge arrangement were fit to Dewey’s equation, Equation (1-8) in Section 1.2.2. Shock wave radius vs. time curves of the three explosive-charge arrangements of PETN at $\rho = 0.90\, g/cm^3$ were compared to the previously-characterized PETN, and are shown in Figure 3-9. The initial explosive-charge radius is also plotted at $t = 0$. Error bars are represented by the size of the data symbols. The data were found to scale well as shown by Figure 3-9 while also agreeing with the previously-characterized PETN profile. The results demonstrate the validity of the scaling laws in the immediate proximity to the fireball.

As shown by Figure 3-9, a small finite time delay (~0.5 $\mu$s) exists between the true zero location of the measured shock wave radius data and the calculated zero point. However, the hypothesized shock wave Mach number initial condition is not dependent upon this delay as it is only dependent upon the shock wave radius.

The shock wave Mach numbers were calculated using Equation (1-11) of Section 1.2.2 and are plotted against shock wave radii in Figure 3-10 for the low-density PETN and Figure 3-11 for the high-density PETN. Measurement error was governed by the 1 $\text{pixel}$ uncertainty in locating the proper radial shock wave pixel distance, which is represented by the error bars. The profiles were found to agree well with the previously-characterized PETN profile as illustrated by the overlap of data.

The hypothesized initial condition, however, did not agree with the profiles, i.e. that the initial shock wave Mach number was equivalent to the explosive material
detonation velocity. Both experimental shock wave Mach number profiles predict higher shock wave Mach numbers at the explosive-charge radius than the explosive-material detonation velocity. Nevertheless, the results are still physical, as the corresponding peak shock wave pressures of 71.1 MPa and 68 MPa (calculated assuming air to behave as an imperfect gas [62]) are less than the explosive-material detonation pressures (7.1 GPa and 16.4 GPa, for the low- and high-density PETN respectively [49]). As shown by Figure 3-10 and Figure 3-11, a higher shock wave Mach number was determined for the low-density PETN at the explosive charge radius. However, this is a result of the large experimental error present at the explosive charge radius. Thus, the results are valid and demonstrate that the hypothesis was incorrect and that more experiments are necessary to determine the proper initial condition. Future work is recommended in Section 5.2.

Remaining shock wave radius vs. time profiles are calculated for shock wave radii greater than 5 cm, as other explosive-materials were unable to be characterized with the ultra-high-speed camera. Therefore, shock wave radius vs. time curvefit coefficients are fit to the measured Photron data for shock wave radii greater than 5 cm. These curvefits are subsequently shifted and recalculated to enforce the previously-established explosive-charge radius approximation of Section 3.2. Lastly, peak shock wave pressures are calculated from shock wave Mach numbers via Equation (1-12), which assumed air to behave as a perfect gas. This is still a valid assumption as data at shock wave radii larger than 5 cm correspond to shock wave Mach numbers smaller than 10, see Section 1.2.2.
Figure 3-9: 1g PETN ($\rho = 0.90\text{g/cm}^3$) shock wave radius versus time.
Figure 3-10: 1g PETN (ρ = 0.90g/cm³) shock wave Mach number versus radius.
Figure 3-11: 1g PETN ($\rho = 1.34g/cm^3$) shock wave Mach number versus radius.
As shown by Figure 3-10 and Figure 3-11, shock wave Mach numbers approaching 27 are calculated by Equation (1-11). The calculation of such strong shock wave Mach numbers was verified from the presence of visible ionization behind the shock wave for Mach numbers greater than 7 [63]. A second sequence of Ultra 8 images is shown in Figure 3-12, illustrating the ionization of air as a result of the strong shock wave. Ionization is indicated by the visibly-intense light present immediately behind the shock wave, whereas visibly-intense light resulting from the bridgewire and fireball are located towards the center of the shock wave. The presence of ionization can clearly be seen in images 4 and 5, and is faintly present in images 6 and 7 of Figure 3-12. Thus, the presence of ionization supports the shock wave Mach number profiles for Figure 3-10 and Figure 3-11.

To additionally verify the accuracy of these profiles, shock wave Mach numbers were calculated using the shock wave radius vs. time data with a second-order-accurate central finite differencing scheme, Equation (3-18). For images \( n = 4 – 7 \) of Figure 3-12, the shock wave Mach number \( M_s^n \) is calculated from the previous image shock wave radius \( R_{n-1} \), the subsequent shock wave radius \( R_{n+1} \), the time delay between those images \( \Delta t \), and atmospheric sound speed \( a_0 \). For images 4 – 7, shock wave Mach numbers were calculated to be 14.8, 13.6, 12.4, and 10.2 respectively. Thus, measurement of such strong shock wave Mach numbers is verified.

\[
M_s^n = \frac{1}{a_0} \frac{dR}{dt} = \frac{R_{n+1} - R_{n-1}}{2a_0\Delta t} \quad (3-18)
\]
3.4 High explosive composite-charge characterization

3.4.1 Homogeneous composite-charge PETN

After characterizing two differently-behaving explosives, the characterization of the booster charge effects in a composite charge arrangement became possible. Composite charges were first constructed with both booster and shell of PETN at the same density, \( \rho = 1.34 \text{g/cm}^3 \). This was to verify that an identical-explosive composite charge yielded the same results as an equivalent-mass homogeneous charge. A single composite charge arrangement was manufactured. The composite charge arrangement

Figure 3-12: Focused Z-shadowgrams using the DRS Ultra 8 camera illustrating the presence of ionization (Imaging sequence labeled).
consisted of nine charges with an average booster mass of 0.854\,g \pm 1\% and an average shell mass of 1.788\,g \pm 0.5\%. Charge masses detonated for characterization are presented in Table 3-5. Composite charges were exploded and shock wave radii vs. time data measured. Measured data were scaled to the standard charge mass of $W_{\text{std}} = 1\,g$ and atmospheric temperature of $T_{\text{NTP}} = 288.16\,K$. Data were found to scale well compared to the previously-characterized high-density PETN profile, demonstrating that an identical-explosive composite charge yielded the same results as an equivalent-mass homogeneous charge, Figure 3-13.

Table 3-5: Equivalent-density PETN ($\rho = 1.34\,g/cm^3$) composite charges detonated for characterization.

<table>
<thead>
<tr>
<th>Charge</th>
<th>Booster mass [g]</th>
<th>Shell mass [g]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.855</td>
<td>1.792</td>
</tr>
<tr>
<td>2</td>
<td>0.853</td>
<td>1.791</td>
</tr>
<tr>
<td>3</td>
<td>0.859</td>
<td>1.792</td>
</tr>
<tr>
<td>4</td>
<td>0.853</td>
<td>1.791</td>
</tr>
<tr>
<td>5</td>
<td>0.844</td>
<td>1.785</td>
</tr>
<tr>
<td>6</td>
<td>0.855</td>
<td>1.781</td>
</tr>
<tr>
<td>7</td>
<td>0.856</td>
<td>1.786</td>
</tr>
<tr>
<td>8</td>
<td>0.854</td>
<td>1.791</td>
</tr>
<tr>
<td>9</td>
<td>0.856</td>
<td>1.786</td>
</tr>
</tbody>
</table>
Figure 3-13: Equivalent-density PETN composite charge scaled shock wave radius vs. scaled time data compared to the previously-characterized high-density PETN profile.
3.4.2 Heterogeneous composite-charge PETN

Next, composite charges of previously-characterized unequal-density PETN were constructed in order to develop a procedure for the removal of the booster charge effects. Two composite-charge mass configurations were constructed, each consisting of a lower-density PETN booster, $\rho = 0.90g/cm^3$, and a higher-density PETN shell, $\rho = 1.34g/cm^3$, Table 3-6 and Table 3-7. The first arrangement consisted of ten charges with an average booster mass of $0.426g \pm 2\%$ and an average shell mass of $0.734g \pm 1\%$, while the second arrangement consisted of nine charges with an average booster mass of $0.634g \pm 2\%$ and an average shell mass of $1.788g \pm 0.5\%$.

Table 3-6: 1.160g PETN composite charges detonated for characterization.

<table>
<thead>
<tr>
<th>Charge</th>
<th>Booster mass ($\rho = 0.90g/cm^3$) [g]</th>
<th>Shell mass ($\rho = 1.34g/cm^3$) [g]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.425</td>
<td>0.738</td>
</tr>
<tr>
<td>2</td>
<td>0.426</td>
<td>0.738</td>
</tr>
<tr>
<td>3</td>
<td>0.427</td>
<td>0.735</td>
</tr>
<tr>
<td>4</td>
<td>0.418</td>
<td>0.737</td>
</tr>
<tr>
<td>5</td>
<td>0.417</td>
<td>0.738</td>
</tr>
<tr>
<td>6</td>
<td>0.420</td>
<td>0.730</td>
</tr>
<tr>
<td>7</td>
<td>0.430</td>
<td>0.736</td>
</tr>
<tr>
<td>8</td>
<td>0.431</td>
<td>0.728</td>
</tr>
<tr>
<td>9</td>
<td>0.430</td>
<td>0.722</td>
</tr>
<tr>
<td>10</td>
<td>0.433</td>
<td>0.734</td>
</tr>
</tbody>
</table>
Shock wave radii as a function of time data were measured for the two composite charge arrangements. Composite charge time data were scaled to account for atmospheric temperature variations according to Equation (3-19), where $c$ is calculated from Equation (1-5) in Section 1.2.2. Thus, all time data were scaled to normal atmospheric temperature standard. Profiles were developed for each arrangement by determining the coefficients to Equation (1-8) of Section 1.2.2. Shock wave radius vs. time profiles of the composite charges and their individual components of Table 3-6 and Table 3-7 are shown in Figure 3-14 and Figure 3-15. Experimental measurement error is represented by the size of the data symbols, which accounts for the error in determining the radial pixel position, total calibration object pixel length, and atmospheric temperature error.

Table 3-7: 2.422g PETN composite charges detonated for characterization.

<table>
<thead>
<tr>
<th>Charge</th>
<th>Booster mass ($\rho = 0.90g/cm^3$)</th>
<th>Shell mass ($\rho = 1.34g/cm^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.626</td>
<td>1.787</td>
</tr>
<tr>
<td>2</td>
<td>0.628</td>
<td>1.788</td>
</tr>
<tr>
<td>3</td>
<td>0.634</td>
<td>1.785</td>
</tr>
<tr>
<td>4</td>
<td>0.626</td>
<td>1.791</td>
</tr>
<tr>
<td>5</td>
<td>0.630</td>
<td>1.782</td>
</tr>
<tr>
<td>6</td>
<td>0.638</td>
<td>1.792</td>
</tr>
<tr>
<td>7</td>
<td>0.642</td>
<td>1.787</td>
</tr>
<tr>
<td>8</td>
<td>0.638</td>
<td>1.794</td>
</tr>
<tr>
<td>9</td>
<td>0.646</td>
<td>1.787</td>
</tr>
</tbody>
</table>
For each composite charge arrangement, booster and shell shock wave radius vs. time profiles were constructed through scaling of the previously-characterized PETN profiles, Section 3.2. With the individual shock wave radius history profiles of the booster, shell, and composite charges known, it is desired to develop a procedure to remove the effects of the booster charge from the composite charge signature, yielding the sole effects of the shell charge, if detonated alone.

\[ t_c = c t \]  

(3-19)
Figure 3-14: Shock wave radius vs. time profiles for the individual components of the PETN composite charge arrangement of Table 3-6.
Figure 3-15: Shock wave radius vs. time profiles for the individual components of the PETN composite charge arrangement of Table 3-7.
3.5 Booster effects removal procedure

From the explosive scaling laws, it is known that the explosions from two unequal-mass charges will produce identical shock waves at unequal radial distances that are proportional to the cube roots of the charge masses. It is also known that all quantities with dimensions of pressure remain unchanged in the scaling [28]. In other words, the peak shock wave pressure $P_s$ is the same for any explosive charge mass $W$, so long as it is scaled to the same radial shock wave distance $R_s$, Equation (3-20). Thus, a straightforward relationship is developed between peak shock wave pressure ratio and scaled shock wave radius, such that, the peak shock wave pressure ratio is proportional to the cube root of the explosive charge mass $W$ within a proportionality constant $k$, Equation (3-21). The constant has dimensions of $m$ for dimensional consistency. Physically, $k$ accounts for the variation in detonation chemistry between individual explosive materials. That is to say, it satisfies the difference in shock wave propagation rate as a result of the variation in contact surface propagation rate for unlike explosive materials.

\[
R_s = R \left[ \frac{W_{std}}{W} \right]^{1/3}
\]  

(3-20)

\[
\frac{P_s}{P_{NTP}} = k \left[ \frac{W}{W_{std}} \right]^{1/3}
\]  

(3-21)
In keeping with the scaling laws, i.e. scaling the shock wave radius $R$, Equation (3-21) is rearranged to represent shock wave radius $R$ as a function of the peak shock wave pressure ratio, explosive charge mass $W$, standard explosive charge mass $W_{std}$, and constant $k$, Equation (3-22). By writing Equation (3-22) in terms of the individual charges of the booster 1, shell 2, and composite 3, a system of equations is produced, Equations (3-23)-(3-25). Physically, $R_1$, $R_2$, and $R_3$ represent the shock wave radii produced by the individual detonations of the booster mass alone, shell mass alone, and composite-charge mass, respectively.

It is known that the composite-charge mass $W_3$ is equal to the sum of the booster charge mass $W_1$ and shell charge mass $W_2$. Therefore, Equation (3-25) is able to be rewritten in terms of the booster and shell charge masses, Equation (3-26). Equations (3-23) and (3-24) are subsequently solved for the booster and shell charge masses, yielding Equations (3-27) and (3-28). Substituting Equations (3-27) and (3-28) into Equation (3-26) allows the composite-charge shock wave radius to be written in terms of its individual properties and the booster and shell properties, Equation (3-29).

At different shock wave radii, the individual booster charge, shell charge, and composite charge will all possess the same peak shock wave pressure, see Figure 3-16. Taking this as the case, i.e. $P_{s1} = P_{s2} = P_{s3}$, the shell shock wave radius $R_2$ is solved for from Equation (3-29), allowing it to be written as a function of some composite charge shock wave radius less some booster charge shock wave radius when taken at equivalent peak shock wave pressures, Equation (3-30).
\[ R = k \frac{P_{NTP}}{P_s} \left[ \frac{W}{W_{std}} \right]^{1/3} \]  
\[ R_1 = k_1 \frac{P_{NTP}}{P_{s1}} \left[ \frac{W_1}{W_{std}} \right]^{1/3} \]  
\[ R_2 = k_2 \frac{P_{NTP}}{P_{s2}} \left[ \frac{W_2}{W_{std}} \right]^{1/3} \]  
\[ R_3 = k_3 \frac{P_{NTP}}{P_{s3}} \left[ \frac{W_3}{W_{std}} \right]^{1/3} \]  
\[ R_3 = k_3 \frac{P_{NTP}}{P_{s3}} \left[ \frac{W_1 + W_2}{W_{std}} \right]^{1/3} \]  
\[ W_1 = \left[ \frac{R_1 P_{s1} W_{\text{std}}^{1/3}}{k_1 P_{NTP}} \right]^3 \]  
\[ W_2 = \left[ \frac{R_2 P_{s2} W_{\text{std}}^{1/3}}{k_2 P_{NTP}} \right]^3 \]
\[ R_3 = \frac{k_3 P_{NTP}}{P_{s3} W_{std}^{1/3}} \left[ \left( \frac{R_1 P_{s1} W_{std}}{k_1 P_{NTP}} \right)^{1/3} + \left( \frac{R_2 P_{s2} W_{std}}{k_2 P_{NTP}} \right)^{1/3} \right]^{1/3} \quad (3-29) \]

\[ R_2 = \left[ \left( \frac{k_2}{k_3} R_3 \right)^3 - \left( \frac{k_2}{k_1} R_1 \right)^3 \right]^{1/3} \quad (3-30) \]

For the present composite charge arrangements of PETN, all values in Equation (3-30) are known for the individual booster, shell, and composite charges. A MATLAB code was written according to Equation (3-30) to calculate the shell shock wave radius \( R_2 \) from the booster and composite shock wave radii at equivalent peak shock wave pressures.
Figure 3-16: Schematic profiles of peak shock wave pressure vs. shock wave radius for the booster 1, shell 2, and composite 3 charges. Locations of peak shock wave pressures and shock wave radii variables from Equation (3-30) are marked.
Shock wave radius vs. time profiles were input into the program for the booster, shell, and composite charges. Using these profiles with Equations (1-8) and (1-11) of Section 1.2.2, the program calculates the booster, shell, and composite charge times at which an equivalent shock wave Mach number (and therefore, peak shock wave pressure) is produced; a user-specified number of shock wave Mach numbers are calculated for determining the individual charge profiles.

Shock wave Mach numbers were distributed between a calculated maximum and minimum shock wave Mach number according to the composite charge of interest. The maximum shock wave Mach number is determined from the booster charge profile, as the booster charge exhibits the maximum shock wave Mach number (and therefore, peak shock wave pressure) that all three individual charges will produce, allowing a calculation via Equation (3-30). The minimum shock wave Mach number is determined from the composite charge profile as the composite charge produces the minimum shock wave Mach number that all three individual charges also produce.

Booster, shell, and composite charge shock wave radii are then determined from the equivalent shock wave Mach number calculated times using Equation (1-8) of Section 1.2.2, and peak shock wave pressures are calculated from the shock wave Mach numbers using the Rankine-Hugoniot relationship for perfect air, Equation (1-12) of Section 1.2.2. Thus, a unique peak shock wave pressure vs. shock wave radius profile is developed for the individual charges. These profiles are unique, in that, the individual booster charge, shell charge, and composite charge are all represented by the same number of user-specified points located at equivalent peak shock wave pressures. For ease of presenting results, the peak shock wave pressure was divided by normal atmospheric pressure,
yielding the peak shock wave pressure ratio. Figure 3-17 shows the individual peak shock wave pressure ratio vs. shock wave radius profiles for the 1.160g PETN composite charge arrangement of Table 3-6.

Lastly, shell shock wave radii $R_2$ were calculated from these profiles via Equation (3-30) for both of the PETN composite charge arrangements. Results were found to agree well with the previously-characterized homogeneous-charge PETN profiles as illustrated by Figure 3-18 and Figure 3-19; results yielded shell shock wave radii with absolute average errors of 1.6% and 0.7% compared to the previously-characterized PETN explosive shock wave radii.
Figure 3-17: Peak shock wave pressure ratio vs. shock wave radius profiles for the individual components of the composite charge arrangement of Table 3-6.
Figure 3-18: 0.734g PETN peak shock wave pressure ratio vs. scaled shock wave radius profiles for a homogenous charge and the calculated shell charge of Table 3-6.
Figure 3-19: 1.788g PETN peak shock wave pressure ratio vs. scaled shock wave radius profiles for a homogenous charge and the calculated shell charge of Table 3-7.
For a not-yet-characterized shell material, however, the shell constant $k_2$ is unknown. Plotting constant $k$ against shock wave Mach number, Figure 3-20, for the individual charge components of the first PETN composite charge arrangement (Table 3-6), reveals that $k_2$ is approximately equal to $k_3$. This is as expected because the proportionality constant accounts for the variation in contact surface propagation rates. The detonation of a composite charge would therefore be influenced by the outermost explosive material, here being the shell. Thus, the composite charge proportionality constant is approximately equal to the shell constant. Utilizing this approximation permits the calculation of a shell shock wave radius $R_2$ by way of Equation (3-31) for a known mass of un-characterized explosive material in the form of the shell component in a composite charge that contains a previously-characterized booster. Thus, a laboratory-scale physically-based characterization is possible for insensitive explosive materials by way of a composite charge arrangement.

$$R_2 = \left[ R_3^3 - \left( \frac{k_3}{k_1} R_1 \right)^3 \right]^{1/3} \quad (3-31)$$
Figure 3-20: PETN $k$ vs. shock wave Mach number profiles for the individual components of the composite charge arrangement of Table 3-6.
3.6 Insensitive explosive material characterization

At theoretical maximum density, RDX (cyclotrimethylene trinitramine) and HMX (cyclotetramethylene tetranitramine) are ~2.5 times less sensitive to detonation initiation than PETN [60]; thus, due to capacitor limitations, a booster charge is required for detonation. The theoretical maximum density charge of RDX and HMX is $\rho = 1.82 \text{ g/cm}^3$ and $\rho = 1.96 \text{ g/cm}^3$ (HMX $\beta$-modification) [60], but due to charge die limitations, manufacturing of charges at these densities was unattainable. Thus, for both RDX and HMX, two composite charge arrangements were constructed which consisted of a PETN booster at $\rho = 0.90 \text{ g/cm}^3$ and either an RDX or HMX shell at $\rho = 1.34 \text{ g/cm}^3$. The first RDX/PETN arrangement consisted of ten charges with an average PETN booster mass of 0.047g ±4% and an average RDX shell mass of 0.855g ±1%. The second RDX/PETN arrangement consisted of eleven charges with an average PETN booster mass of 0.427g ±3% and an average RDX shell mass of 0.745g ±1%. The first HMX/PETN arrangement consisted of nine charges with an average PETN booster mass of 0.048g ±5% and an average HMX shell mass of 0.839g ±2%. The second HMX/PETN arrangement consisted of ten charges with an average PETN booster mass of 0.428g ±2% and an average HMX shell mass of 0.745g ±1%. Tabulated masses for these charges are presented in Table 3-8 through Table 3-11.
Table 3-8: RDX/PETN composite charge arrangement 1 detonated for characterization.

<table>
<thead>
<tr>
<th>Charge</th>
<th>Booster mass [g]</th>
<th>Shell mass [g]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.049</td>
<td>0.852</td>
</tr>
<tr>
<td>2</td>
<td>0.048</td>
<td>0.854</td>
</tr>
<tr>
<td>3</td>
<td>0.049</td>
<td>0.857</td>
</tr>
<tr>
<td>4</td>
<td>0.048</td>
<td>0.851</td>
</tr>
<tr>
<td>5</td>
<td>0.045</td>
<td>0.856</td>
</tr>
<tr>
<td>6</td>
<td>0.045</td>
<td>0.860</td>
</tr>
<tr>
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<td>0.848</td>
</tr>
<tr>
<td>8</td>
<td>0.045</td>
<td>0.854</td>
</tr>
<tr>
<td>9</td>
<td>0.048</td>
<td>0.860</td>
</tr>
<tr>
<td>10</td>
<td>0.047</td>
<td>0.859</td>
</tr>
</tbody>
</table>

Table 3-9: RDX/PETN composite charge arrangement 2 detonated for characterization.

<table>
<thead>
<tr>
<th>Charge</th>
<th>Booster mass [g]</th>
<th>Shell mass [g]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.438</td>
<td>0.741</td>
</tr>
<tr>
<td>2</td>
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</tr>
<tr>
<td>3</td>
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<td>0.740</td>
</tr>
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</tr>
<tr>
<td>5</td>
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</tr>
<tr>
<td>6</td>
<td>0.429</td>
<td>0.750</td>
</tr>
<tr>
<td>7</td>
<td>0.430</td>
<td>0.745</td>
</tr>
<tr>
<td>8</td>
<td>0.428</td>
<td>0.747</td>
</tr>
<tr>
<td>9</td>
<td>0.426</td>
<td>0.750</td>
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<tr>
<td>10</td>
<td>0.428</td>
<td>0.746</td>
</tr>
<tr>
<td>11</td>
<td>0.427</td>
<td>0.748</td>
</tr>
</tbody>
</table>
Table 3-10: HMX/PETN composite charge arrangement 1 detonated for characterization.

<table>
<thead>
<tr>
<th>Charge</th>
<th>Booster mass [g]</th>
<th>Shell mass [g]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.050</td>
<td>0.827</td>
</tr>
<tr>
<td>2</td>
<td>0.050</td>
<td>0.821</td>
</tr>
<tr>
<td>3</td>
<td>0.048</td>
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</tr>
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<td>0.047</td>
<td>0.845</td>
</tr>
<tr>
<td>5</td>
<td>0.048</td>
<td>0.836</td>
</tr>
<tr>
<td>6</td>
<td>0.050</td>
<td>0.849</td>
</tr>
<tr>
<td>7</td>
<td>0.047</td>
<td>0.840</td>
</tr>
<tr>
<td>8</td>
<td>0.045</td>
<td>0.840</td>
</tr>
<tr>
<td>9</td>
<td>0.046</td>
<td>0.850</td>
</tr>
</tbody>
</table>

Table 3-11: HMX/PETN composite charge arrangement 2 detonated for characterization.

<table>
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<tr>
<th>Charge</th>
<th>Booster mass [g]</th>
<th>Shell mass [g]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.423</td>
<td>0.741</td>
</tr>
<tr>
<td>2</td>
<td>0.425</td>
<td>0.738</td>
</tr>
<tr>
<td>3</td>
<td>0.430</td>
<td>0.742</td>
</tr>
<tr>
<td>4</td>
<td>0.430</td>
<td>0.750</td>
</tr>
<tr>
<td>5</td>
<td>0.430</td>
<td>0.742</td>
</tr>
<tr>
<td>6</td>
<td>0.437</td>
<td>0.741</td>
</tr>
<tr>
<td>7</td>
<td>0.424</td>
<td>0.750</td>
</tr>
<tr>
<td>8</td>
<td>0.427</td>
<td>0.743</td>
</tr>
<tr>
<td>9</td>
<td>0.421</td>
<td>0.754</td>
</tr>
<tr>
<td>10</td>
<td>0.430</td>
<td>0.753</td>
</tr>
</tbody>
</table>
These 40 composite charges were exploded and shock wave radius vs. time data were gathered for each of the four composite-charge arrangements. As before, time data were scaled to normal atmospheric temperature to account for atmospheric temperature variations according to Equation (3-19). Profiles were developed for each arrangement by determining the coefficients to Equation (1-8) of Section 1.2.2. Plots of shock wave radius vs. time for the composite-charges and booster charges of Table 3-8 through Table 3-11 are shown in Figure 3-21 through Figure 3-24. Booster shock wave radius vs. time profiles were constructed through scaling of the previously-characterized homogeneous PETN, Section 3.2. Experimental measurement error is represented by the size of the data symbols.

Shell shock wave radii $R_2$ were calculated via Equation (3-31) for the four composite charge arrangements using the measured shock wave radius vs. time profile as previously-outlined in Section 3.5. The calculated shell shock wave radii were subsequently scaled to the standard mass $W_{std} = 1g$ for each explosive arrangement and results compared.

Figure 3-25 shows the peak shock wave pressure ratio vs. scaled shock wave radius results for a 1g RDX charge, if detonated alone, from the calculated shell charges of the two composite charge arrangements. As shown, the two composite charge arrangements yielded nearly-identical peak shock wave pressure ratio vs. scaled shock wave radius profiles. Figure 3-26 illustrates the peak shock wave pressure ratio vs. scaled shock wave radii for a 1g HMX charge from the calculated shell charges of the two composite charge arrangements. As shown, the resulting peak shock wave pressure
vs. scaled shock wave radius profiles of the calculated HMX shell charge, if detonated alone, also yielded nearly-identical results.
Figure 3-21: RDX/PETN composite charge arrangement 1 shock wave radius vs. time profiles for composite charge and booster of Table 3-8.
Figure 3-22: RDX/PETN composite charge arrangement 2 shock wave radius vs. time profiles for composite charge and booster of Table 3-9.
Figure 3-23: HMX/PETN composite charge arrangement 1 shock wave radius vs. time profiles for composite charge and booster of Table 3-10.
Figure 3-24: HMX/PETN composite charge arrangement 2 shock wave radius vs. time profiles for composite charge and booster of Table 3-11.
Figure 3-25: Scaled homogeneous 1g RDX charge peak shock wave pressure ratio vs. scaled shock wave radius profiles for the calculated shell charges of Table 3-8 and Table 3-9.
Figure 3-26: Scaled homoegenous 1g HMX charge peak shock wave pressure ratio vs. scaled shock wave radius profiles for the calculated shell charges of Table 3-10 and Table 3-11.
For comparing experimental data, the calculation of coefficients to Dewey’s shock wave radius vs. time profile for the calculated 1g charges of RDX and HMX was not possible. This is because only peak shock wave pressure (or shock wave Mach number) vs. shock wave radius data result from Equation (3-31), not shock wave radius vs. time. Additionally, as an initial shock wave Mach number was unable to be determined, Section 3.3, an empirically-based curvefit is required.

To compare these data, Kleine proposed an empirically-based relationship for scaled shock wave radius as a function of peak shock wave pressure, Equation (3-32) [3]. Coefficients $E$, $F$, $G$, and $H$ of Equation (3-32) are adjusted to adapt to various explosive peak shock wave pressure vs. shock wave radius profiles. Equation (3-32) was rearranged to represent the shock wave radius as a function of shock wave Mach number, Equation (3-33), which permits the calculation of all downstream properties of the shock wave; a more useful representation of the data for comparison.

\[
\ln R_s = E + F \ln P_s + G \ln(P_s)^2 + H \ln(P_s)^3 \quad (3-32)
\]

\[
\ln R_s = E + F \ln M + G \ln(M)^2 + H \ln(M)^3 \quad (3-33)
\]

Shock wave Mach numbers were calculated from peak shock wave pressures using the Rankine-Hugoniot relationship from Section 1.2.2 for the 1g RDX and 1g HMX profiles. A linear least squares-regression calculation was then performed to determine coefficients $E$, $F$, $G$, and $H$, for the 1g RDX and 1g HMX data. The resulting
coefficients are given in Table 3-12. Lastly, the radially-dependent TNT equivalent of 1g RDX and 1g HMX were calculated from the peak shock wave pressure vs. scaled shock wave radius profiles, Figure 3-27 and Figure 3-28.

Table 3-12: Equation (3-33) coefficients for 1g RDX and 1g HMX.

<table>
<thead>
<tr>
<th>Explosive</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>RDX @ ( \rho = 1.34, g/cm^3 )</td>
<td>-1.01729</td>
<td>-1.10358</td>
<td>0.09349</td>
<td>-0.04053</td>
</tr>
<tr>
<td>HMX @ ( \rho = 1.34, g/cm^3 )</td>
<td>-1.05200</td>
<td>-1.02770</td>
<td>0.04046</td>
<td>-0.03059</td>
</tr>
</tbody>
</table>

The TNT equivalents of RDX and HMX are quite similar as shown in Figure 3-27 and Figure 3-28. This is attributed to the nearly-identical explosive detonation properties of RDX and HMX. As previously-mentioned, the detonation performance of an explosive material is determined by the explosive material’s detonation velocity \( V_D \), detonation pressure \( P_{CI} \), and heat of detonation \( Q_{det} \) [1, 2]. Accordingly, the detonation properties of RDX and HMX at a density of \( \rho = 1.34\, g/cm^3 \) are nearly-identical, Table 3-13; thus, similar radially-dependent TNT equivalents result.

Additionally, the TNT equivalent of RDX and HMX exhibit the same radial-trend as did PETN, i.e. an increase to a maximum equivalent value followed by a decay throughout remaining radii. This is a result of RDX and HMX being more nearly-ideal explosives than TNT (both -21.6% oxygen-deficient [60]). Figure 3-27 and Figure 3-28 also illustrate that RDX and HMX generate stronger initial shock wave Mach numbers than TNT, causing larger entropy changes and resulting in weaker shock wave Mach numbers at larger shock wave radii due to less available energy [16].
The characterization procedure for insensitive secondary explosives RDX and HMX demonstrates the utility of laboratory-scale research to properly characterize insensitive explosive materials that require a booster charge for detonation. The results demonstrate the ability to experimentally-determine a more physically-based characterization for insensitive explosive materials by removing the effects produced by the booster charge.

Table 3-13: Explosive material detonation properties for RDX and HMX @ $\rho = 1.34 g/cm^3$.

<table>
<thead>
<tr>
<th>Explosive</th>
<th>$V_D$ [64] [km/s]</th>
<th>$P_{CJ}$ [1] [GPa]</th>
<th>$Q_{det}$ [2] [kJ/kg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>RDX</td>
<td>7.12</td>
<td>18.78</td>
<td>6322</td>
</tr>
<tr>
<td>HMX</td>
<td>7.10</td>
<td>18.74</td>
<td>6197</td>
</tr>
</tbody>
</table>
Figure 3-27: Radially-dependent TNT equivalent of a scaled homogeneous 1g RDX explosive charge.
Figure 3-28: Radially-dependent TNT equivalent of a scaled homogeneous 1g HMX explosive charge.
3.7 Smokeless powder characterization

From preliminary experiments it was determined that the smokeless powder (SP) had to be loosely-packed around the booster charge in order to achieve detonation. Thus, due to charge manufacturing constraints discussed in Section 2.2, the SP density was held approximately constant. Three composite charge arrangements consisting of a PETN booster, $\rho = 0.90g/cm^3$, and SP shell, $\rho \approx 0.90g/cm^3$, were constructed. The first SP/PETN arrangement consisted of five charges with an average PETN booster mass of $0.466g \pm 3\%$ and a SP shell mass of $1g \pm 1\%$. The second SP/PETN arrangement consisted of thirteen charges with an average PETN booster mass of $0.661g \pm 2\%$ and a SP shell mass of $1g \pm 1\%$. The third SP/PETN arrangement consisted of twelve charges with an average PETN booster mass of $0.972g \pm 1\%$ and a SP shell mass of $2g \pm 1\%$. Tabulated charge masses are presented in Table 3-14 through Table 3-16.

Shock wave radius vs. time history data were gathered for the explosively-driven shock wave from the 34 composite charges. Time data were scaled to normal atmospheric temperature to account for atmospheric temperature variations according to Equation (3-19). A shock wave radius vs. time profile was developed for each of the three arrangements by determining the coefficients to Equation (1-8) of Section 1.2.2. Shock wave radius vs. time profiles are plotted for the three composite charge arrangements and their booster charges in Figure 3-29 through Figure 3-31. Booster shock wave radius vs. time profiles were constructed through scaling of the previously-characterized homogeneous PETN profiles, Section 3.2. Experimental measurement error is represented by the size of the data symbols.
Table 3-14: SP/PETN composite charge arrangement 1 detonated for characterization.

<table>
<thead>
<tr>
<th>Charge</th>
<th>Booster mass [g]</th>
<th>Shell mass [g]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.465</td>
<td>1.000</td>
</tr>
<tr>
<td>2</td>
<td>0.478</td>
<td>1.000</td>
</tr>
<tr>
<td>3</td>
<td>0.466</td>
<td>1.000</td>
</tr>
<tr>
<td>4</td>
<td>0.454</td>
<td>1.000</td>
</tr>
<tr>
<td>5</td>
<td>0.466</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Table 3-15: SP/PETN composite charge arrangement 2 detonated for characterization.

<table>
<thead>
<tr>
<th>Charge</th>
<th>Booster mass [g]</th>
<th>Shell mass [g]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.666</td>
<td>1.000</td>
</tr>
<tr>
<td>2</td>
<td>0.656</td>
<td>1.000</td>
</tr>
<tr>
<td>3</td>
<td>0.668</td>
<td>1.000</td>
</tr>
<tr>
<td>4</td>
<td>0.664</td>
<td>1.000</td>
</tr>
<tr>
<td>5</td>
<td>0.657</td>
<td>1.000</td>
</tr>
<tr>
<td>6</td>
<td>0.666</td>
<td>1.000</td>
</tr>
<tr>
<td>7</td>
<td>0.667</td>
<td>1.000</td>
</tr>
<tr>
<td>8</td>
<td>0.658</td>
<td>1.000</td>
</tr>
<tr>
<td>9</td>
<td>0.647</td>
<td>1.000</td>
</tr>
<tr>
<td>10</td>
<td>0.664</td>
<td>1.000</td>
</tr>
<tr>
<td>11</td>
<td>0.656</td>
<td>1.000</td>
</tr>
<tr>
<td>12</td>
<td>0.658</td>
<td>1.000</td>
</tr>
<tr>
<td>13</td>
<td>0.662</td>
<td>1.000</td>
</tr>
</tbody>
</table>
Using Equation (3-31), SP shell shock wave radii were calculated and scaled to the standard mass $W_{std} = 1\text{g}$. As shown in Figure 3-32, the resulting three SP peak shock wave pressure vs. shock wave radius curves collapse essentially to a single curve, further demonstrating the ability to successfully test and characterize insensitive explosive materials at the laboratory scale. Coefficients of Equation (3-33) were calculated for 1g homogeneous SP charge and are given in Table 3-17. The TNT equivalent of 1g SP was calculated according to its peak shock wave pressure vs. shock wave radius profile, and is shown in Figure 3-33. The TNT equivalent of SP remains relatively constant up to a scaled shock wave radius of 0.10m, followed by a decline over the remaining radii.

Table 3-16: SP/PETN composite charge arrangement 3 detonated for characterization.

<table>
<thead>
<tr>
<th>Charge</th>
<th>Booster mass [g]</th>
<th>Shell mass [g]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.970</td>
<td>2.000</td>
</tr>
<tr>
<td>2</td>
<td>0.970</td>
<td>2.000</td>
</tr>
<tr>
<td>3</td>
<td>0.974</td>
<td>2.000</td>
</tr>
<tr>
<td>4</td>
<td>0.974</td>
<td>2.000</td>
</tr>
<tr>
<td>5</td>
<td>0.977</td>
<td>2.000</td>
</tr>
<tr>
<td>6</td>
<td>0.961</td>
<td>2.000</td>
</tr>
<tr>
<td>7</td>
<td>0.969</td>
<td>2.000</td>
</tr>
<tr>
<td>8</td>
<td>0.970</td>
<td>2.000</td>
</tr>
<tr>
<td>9</td>
<td>0.973</td>
<td>2.000</td>
</tr>
<tr>
<td>10</td>
<td>0.969</td>
<td>2.000</td>
</tr>
<tr>
<td>11</td>
<td>0.980</td>
<td>2.000</td>
</tr>
<tr>
<td>12</td>
<td>0.971</td>
<td>2.000</td>
</tr>
</tbody>
</table>
Again, this is similar to PETN, RDX and HMX, which is a result of SP being a more nearly-ideal explosive as it contains an ample oxygen supply.

Table 3-17: Equation (3-33) coefficients for 1g SP.

<table>
<thead>
<tr>
<th>Explosive</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>SP @ ρ ≈ 0.90g/cm³</td>
<td>-1.27240</td>
<td>-0.93489</td>
<td>0.02118</td>
<td>-0.03231</td>
</tr>
</tbody>
</table>
Figure 3-29: SP/PETN composite charge arrangement 1 shock wave radius vs. time profiles for composite charge and booster of Table 3-14.
Figure 3-30: SP/PETN composite charge arrangement 2 shock wave radius vs. time profiles for composite charge and booster of Table 3-15.
Figure 3-31: SP/PETN composite charge arrangement 3 shock wave radius vs. time profiles for composite charge and booster of Table 3-16.
Figure 3-32: Scaled homogeneous 1g SP charge peak shock wave pressure ratio vs. scaled shock wave radius profiles for the calculated shell charges of Table 3-14, Table 3-15, and Table 3-16.
Figure 3-33: Radially-dependent TNT equivalent of a scaled homogeneous $1 \text{g SP}$ explosive charge.
3.8 Explosive impulse characterization

3.8.1 Booster removal procedure

Similar to peak shock wave pressure, explosive impulse $I$ remains unchanged in the scaling procedure. From the explosive scaling laws, it is known that the explosive impulses produced by two unequal-mass charges will be identical at unequal radial standoff distances that are proportional to the cube roots of the charge masses [28]. That is to say, explosive impulse $I$ is the same for a particular explosive material at any explosive charge mass $W$, so long as it is scaled to the same radial shock wave distance $R_s$, Equation (3-34). The constant $j$ has dimensions of $Pa \cdot s \cdot m$ for dimensional consistency.

$$I = j \left[ \frac{W}{W_{std}} \right]^{1/3}$$  \hspace{1cm} (3-34)

Thus, similar to Equation (3-21) of Section 3.4.2, a procedure is developed here to relate the booster charge, shell charge, and composite charge radial standoff distances at equivalent impulses. As with the previous derivation of Section 3.5, Equation (3-34) is rearranged to represent shock wave radius $R$ as a function of explosive impulse $I$, explosive charge mass $W$, standard explosive charge mass $W_{std}$, and constant $j$, Equation (3-35). By writing Equation (3-35) in terms of the individual charges of the booster $R_1$, shell $R_2$, and composite $R_3$, a system of equations is produced, Equations (3-36) – (3-38).
$R_j, R_2,$ and $R_3$ represent the shock wave radii produced by the individual detonations of the booster mass alone, shell mass alone, and composite-charge mass, respectively.

\[
R = \frac{j}{I} \left[ \frac{W}{W_{std}} \right]^{1/3} \tag{3-35}
\]

\[
R_1 = \frac{j_1}{I_1} \left[ \frac{W_1}{W_{std}} \right]^{1/3} \tag{3-36}
\]

\[
R_2 = \frac{j_2}{I_2} \left[ \frac{W_2}{W_{std}} \right]^{1/3} \tag{3-37}
\]

\[
R_3 = \frac{j_3}{I_3} \left[ \frac{W_3}{W_{std}} \right]^{1/3} \tag{3-38}
\]

It is known that the composite-charge mass $W_3$ is equal to the sum of the booster charge mass $W_1$ and shell charge mass $W_2$. Therefore, Equation (3-38) can be rewritten in terms of the booster and shell charge masses, Equation (3-39). Equations (3-36) and (3-37) are subsequently solved for the booster and shell charge masses, yielding Equations (3-40) and (3-41). Substituting Equations (3-40) and (3-41) into Equation (3-
39) allows the composite-charge shock wave radius to be written in terms of its properties and the booster and shell properties, Equation (3-42).

At different shock wave radii, the individual booster, shell, and composite charges will each deliver the same explosive impulse, see Figure 3-34. Taking this as the case, i.e. $I_1 = I_2 = I_3$, the shell shock wave radius $R_2$ is found from Equation (3-42), allowing it to be written as a function of some composite charge shock wave radius $R_3$ less some booster charge shock wave radius $R_1$ when taken at equivalent explosive impulses, Equation (3-43).

$$R_3 = \frac{j_3}{I_3} \left[ \frac{W_1 + W_2}{W_{std}} \right]^{1/3}$$  \hspace{1cm} (3-39)

$$W_1 = \left[ \frac{R_1 I_1 W_{std}^{1/3}}{j_1} \right]^3$$  \hspace{1cm} (3-40)

$$W_2 = \left[ \frac{R_2 I_2 W_{std}^{1/3}}{j_2} \right]^3$$  \hspace{1cm} (3-41)

$$R_3 = \frac{j_3}{I_3 W_{std}^{1/3}} \left[ \left( \frac{R_1 I_1 W_{std}^{1/3}}{j_1} \right)^3 + \left( \frac{R_2 I_2 W_{std}^{1/3}}{j_2} \right)^3 \right]^{1/3}$$  \hspace{1cm} (3-42)
Figure 3-34: Schematic profiles of explosive impulse vs. shock wave radius for the booster 1, shell 2, and composite 3 charges. Locations of explosive impulses and shock wave radii variables from Equation (3-43) are marked.
\[
R_2 = \left[ \left( \frac{j_2}{j_3} R_3 \right)^3 - \left( \frac{j_2}{j_1} R_1 \right)^3 \right]^{1/3} \quad I_1 = I_2 = I_3
\]  

(3-43)

3.8.2 Homogeneous charge characterization

Materials to be used for booster and shell charges were first individually characterized. As before, explosive characterization was performed for two densities of PETN: \( \rho = 0.90\text{g/cm}^3 \) and \( \rho = 1.34\text{g/cm}^3 \). Charges used for calculating explosive impulse are shown in Table 3-18 and Table 3-19. Peak shock wave pressure and its decay were measured for each charge throughout a range of radial standoff distances. Radial standoff distances were scaled to the standard mass of \( W_{\text{std}} = 1\text{g} \) according to Section 1.2.2; as before variations in atmospheric pressure were ignored due to their insignificance with respect to the results.
Table 3-18: Homogeneous PETN ($\rho = 0.90 g/cm^3$) explosive-charge masses.

<table>
<thead>
<tr>
<th>Charge</th>
<th>Mass [g]</th>
<th>Radial standoff [m]</th>
<th>Scaled radial standoff [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.438</td>
<td>0.150</td>
<td>0.198</td>
</tr>
<tr>
<td>2</td>
<td>0.446</td>
<td>0.149</td>
<td>0.195</td>
</tr>
<tr>
<td>3</td>
<td>0.426</td>
<td>0.120</td>
<td>0.160</td>
</tr>
<tr>
<td>4</td>
<td>0.427</td>
<td>0.119</td>
<td>0.157</td>
</tr>
<tr>
<td>5</td>
<td>0.430</td>
<td>0.135</td>
<td>0.178</td>
</tr>
<tr>
<td>6</td>
<td>0.428</td>
<td>0.134</td>
<td>0.178</td>
</tr>
<tr>
<td>7</td>
<td>0.425</td>
<td>0.165</td>
<td>0.219</td>
</tr>
<tr>
<td>8</td>
<td>0.425</td>
<td>0.164</td>
<td>0.218</td>
</tr>
<tr>
<td>9</td>
<td>0.427</td>
<td>0.164</td>
<td>0.218</td>
</tr>
<tr>
<td>10</td>
<td>0.420</td>
<td>0.104</td>
<td>0.139</td>
</tr>
<tr>
<td>11</td>
<td>0.429</td>
<td>0.104</td>
<td>0.138</td>
</tr>
<tr>
<td>12</td>
<td>0.437</td>
<td>0.127</td>
<td>0.167</td>
</tr>
<tr>
<td>13</td>
<td>0.446</td>
<td>0.144</td>
<td>0.188</td>
</tr>
<tr>
<td>14</td>
<td>0.440</td>
<td>0.159</td>
<td>0.209</td>
</tr>
<tr>
<td>15</td>
<td>0.401</td>
<td>0.177</td>
<td>0.240</td>
</tr>
<tr>
<td>16</td>
<td>0.651</td>
<td>0.061</td>
<td>0.070</td>
</tr>
<tr>
<td>17</td>
<td>0.652</td>
<td>0.060</td>
<td>0.069</td>
</tr>
<tr>
<td>18</td>
<td>0.666</td>
<td>0.069</td>
<td>0.079</td>
</tr>
<tr>
<td>19</td>
<td>0.666</td>
<td>0.069</td>
<td>0.079</td>
</tr>
<tr>
<td>20</td>
<td>0.648</td>
<td>0.078</td>
<td>0.090</td>
</tr>
<tr>
<td>21</td>
<td>0.430</td>
<td>0.067</td>
<td>0.089</td>
</tr>
<tr>
<td>22</td>
<td>0.423</td>
<td>0.074</td>
<td>0.098</td>
</tr>
</tbody>
</table>
Table 3-19: Homogeneous PETN ($\rho = 1.34\,g/cm^3$) explosive-charge masses.

<table>
<thead>
<tr>
<th>Charge</th>
<th>Mass [g]</th>
<th>Radial standoff [m]</th>
<th>Scaled radial standoff [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.917</td>
<td>0.095</td>
<td>0.098</td>
</tr>
<tr>
<td>2</td>
<td>0.918</td>
<td>0.096</td>
<td>0.099</td>
</tr>
<tr>
<td>3</td>
<td>0.937</td>
<td>0.096</td>
<td>0.098</td>
</tr>
<tr>
<td>4</td>
<td>0.897</td>
<td>0.072</td>
<td>0.074</td>
</tr>
<tr>
<td>5</td>
<td>0.900</td>
<td>0.073</td>
<td>0.075</td>
</tr>
<tr>
<td>6</td>
<td>0.881</td>
<td>0.070</td>
<td>0.072</td>
</tr>
<tr>
<td>7</td>
<td>0.883</td>
<td>0.068</td>
<td>0.070</td>
</tr>
<tr>
<td>8</td>
<td>0.900</td>
<td>0.075</td>
<td>0.078</td>
</tr>
<tr>
<td>9</td>
<td>0.899</td>
<td>0.076</td>
<td>0.078</td>
</tr>
<tr>
<td>10</td>
<td>0.907</td>
<td>0.087</td>
<td>0.089</td>
</tr>
<tr>
<td>11</td>
<td>0.909</td>
<td>0.095</td>
<td>0.098</td>
</tr>
<tr>
<td>12</td>
<td>0.913</td>
<td>0.096</td>
<td>0.099</td>
</tr>
<tr>
<td>13</td>
<td>0.912</td>
<td>0.094</td>
<td>0.097</td>
</tr>
</tbody>
</table>

Peak shock wave pressure and its decay were measured by a piezoelectric pressure transducer using the prescribed setup of Section 2.5. Simultaneous focused shadowgrams were taken to aid in identifying pressure-signal irregularities that resulted from a non-perpendicular shock wave impact or unburned explosive particle impact upon the pressure transducer face; several tests were eliminated as a result. An example shadowgram and resulting shock wave pressure trace are shown in Figure 3-35 and Figure 3-36, respectively.

As shown by Figure 3-36, a small time delay exists between shock wave time of arrival (start of the pressure rise) and the time of peak shock wave pressure. This delay is
a result of the pressure transducer’s finite response time to the discontinuous jump signature of the shock wave. For the present research, the shock wave time of arrival, i.e. the beginning of the pressure rise, will be used for calculating explosive impulse measurements.

Figure 3-35: Focused shadowgram of the explosively-driven shock wave about to impinge on the pressure transducer. The explosive charge center is indicated by X.
Figure 3-36: Experimental face-on pressure trace of the peak shock wave pressure and its decay for a 0.440g low-density homogeneous PETN charge at 0.159m radial standoff.
3.8.3 Explosive impulse calculation

For ease of comparison, data for the peak shock wave pressure and its decay were fit to the smoothed-curve modified Friedlander equation, Equation (3-44). The modified Friedlander equation is a three-parameter equation that represents the peak shock wave pressure and its decay as a function of time. Its parameters are the peak shock wave pressure $P_s$, positive pressure duration $T^+$, and waveform parameter $\alpha$, which represents the pressure signal decay down to atmospheric pressure. Parameters are typically fit to the data through the methods outlined in Section 1.2.5. However, a new method is proposed here for fitting the modified Friedlander equation to experimental pressure traces.

$$P(t) = P_s \left(1 - \frac{t}{T^+}\right)^{-\alpha} e^{\frac{-\alpha t}{T^+}}$$  \hspace{1cm} (3-44)

Experimentally-measured pressure histories were first converted from reflected pressures to incident pressures using Equation (2-15) of Section 2.5. The positive pressure duration $T^+$ was calculated by fitting a linear curve to the end of the positive pressure history when plotted as pressure vs. the logarithm of time, as outlined by Ethridge [30]. Note that this is a purely-subjective procedure in which the experimentalist defines what portion of the pressure trace is considered to be “linear”. The procedure was nevertheless performed on the data sets resulting from the exploded charges of Table 3-18 and Table 3-19.
Positive pressure durations were also calculated based upon the “theoretical suggestion” of Kinney and Graham [29]. Kinney and Graham hypothesized that the end of the positive pressure phase, or the “zero-point”, will propagate at the speed of sound based upon the static gas temperature downstream of the shock wave. Thus, the positive pressure duration is determinable from knowledge of the shock wave Mach number vs. shock wave radius profile. The following derivation along these lines uses the data resulting from the low-density PETN charges of Table 3-18.

A finite-difference scheme was used to radially propagate the theoretical zero-point using the shock wave Mach number vs. radius profile determined in Section 3.2. The static temperature $T_2$ immediately behind the shock wave was calculated using the normal shock relation and knowledge of the upstream ambient temperature $T_{NTP}$, Equation (3-45). From this, the ambient speed of sound downstream of the shock wave was calculated as a function of shock wave radius using Equation (1-9) of Section 1.2.2.

$$\frac{T_2}{T_{NTP}} = \left[ 1 + \frac{2\gamma}{\gamma + 1} \left( M^2 - 1 \right) \left[ \frac{2 + (\gamma - 1)M^2}{(\gamma + 1)M^2} \right] \right]$$

(3-45)

To begin the finite-differencing scheme, a proper initial condition is required to calculate the theoretical positive pressure duration. For the low-density PETN, multiple measurements were taken in a small-range of radial-standoff distances. From these data, an initial positive pressure duration, or location of the zero-point, was determined. This
zero-point was subsequently propagated throughout the flow field from knowledge of the ambient sound speed immediately following the shock wave as a function of radius.

Figure 3-37 illustrates the “theoretical” zero-point propagation compared to the durations calculated using the semi-logarithmic procedure of Ethridge [30]. As shown, the theoretical duration is in excellent agreement with the experimental data. Therefore, the use of the “theoretical” positive pressure duration reduces the error associated with the calculation of explosive impulse by eliminating the semi-logarithmic approximation procedure. Using the theoretical calculation of a positive pressure duration also permits the analysis of data sets where the end of the positive pressure phase was not clearly defined due to the presence of noise. Henceforth, positive pressure durations for the present research will be calculated “theoretically” according to Kinney and Graham [29].

An experimental shock wave radius vs. time plot of a 1g homogeneous PETN charge is shown in Figure 3-38, along with the radial propagation location of the theoretically-calculated zero-point. Also plotted is a theoretical pressure trace that illustrates the resulting peak shock wave pressure immediately trailing the shock wave and its decay back to atmospheric pressure through the theoretical zero-point.
Figure 3-37: Positive pressure durations as functions of scaled shock wave radius for 1g explosive charges of low-density, homogeneous PETN ($\rho = 0.90\text{g/cm}^3$).
Ethridge outlined a procedure for determining the peak shock wave pressure using a linear approximation to the plot of the logarithm of pressure as a function of time [30]. This procedure arose from the previous work of Porzel and Schmidt who considered the decay of blast parameters to be exponential in character [36].

Estimation of the waveform parameter α was built upon by Ismail and Murray [6] from the analysis by Kinney and Graham [29]. However, both of these procedures
introduce error into the calculation of explosive impulse, since they use approximations to the un-measurable initial peak shock wave pressure. Rather, a new procedure was implemented to determine peak shock wave pressure and the waveform parameter by performing a nonlinear regression calculation of the modified Friedlander equation to the data. Thus, the modified Friedlander equation was fit to the actual experimental data rather than using the approximations of Ethridge and Ismail and Murray.

The theoretical positive pressure duration $T^+$ was used in determining the peak shock wave pressure $P_s$ and waveform parameter $\alpha$. It was not possible to perform a nonlinear regression of all three parameters for the modified Friedlander equation due to the orders-of-magnitude-difference between the peak shock wave pressure and positive pressure duration. This orders-of-magnitude-difference results in a large condition number for the matrix, yielding an unsolvable, ill-conditioned matrix. Thus, nonlinear regression was only used for the peak shock wave pressure and the waveform parameter.

Figure 3-39 illustrates an example modified Friedlander curve plotted on top of the experimental pressure trace of a 0.440g low-density, homogeneous PETN charge at 0.159m radial standoff. As shown, the nonlinear regression procedure produces an excellent fit to the experimental data. A semi-logarithmic plot of Figure 3-39 is given in Figure 3-40, which illustrates that the initial pressure decay is approximately a straight-line decay as previously predicted by Porzel and Schmidt [36]. Thus, the use of a “theoretical” positive pressure duration and the nonlinear regression calculation of the peak shock wave pressure and waveform parameter eliminates the previous subjective procedures.
Figure 3-39: Incident peak shock wave pressure and its decay for a 0.440g low-density PETN charge at 0.159m radial standoff and its modified Friedlander curve.
Figure 3-40: Logarithm of incident peak shock wave pressure ratio and its decay for a 0.440g low-density, homogeneous PETN charge at 0.159m radial standoff distance, compared with modified Friedlander curve.
Pressure vs. time histories were measured at radial standoff distances ranging from 0.07 – 0.24 m for the low-density PETN charges of Table 3-18, and from 0.07 – 0.10 m for the high-density PETN charges of Table 3-19. The limited radial-standoff distances of the high-density PETN were chosen because this is where the high- and low-density PETN were found to possess significantly-different shock wave Mach numbers from their previous characterization in Section 3.2.

The calculated modified Friedlander peak shock wave pressure is plotted vs. scaled shock wave radius in Figure 3-41 for the low-density, homogeneous PETN charges of Table 3-18 and in Figure 3-42 for the high-density, homogeneous PETN charges of Table 3-19. Additionally, the peak shock wave pressure determined from the optical characterization of Section 3.2 is plotted. As shown in Figure 3-41, the calculated modified Friedlander peak shock wave pressures from the experimental pressure traces are in good agreement with the optical characterization profile for shock wave radii larger than 0.09 m. However, a disagreement exists for shock wave radii less than 0.09 m for both the low- and high-density PETN, as shown in Figure 3-41 and Figure 3-42.

This is a result of the pressure transducer “overshooting”, or over-compensating, the pressure signal due to the strong shock wave that is present at small radii from the explosion center. This can be corrected by critically damping the pressure transducer as suggested by Kinney and Graham [29], however, this increases the response time of the transducer, causing a rounded pressure-signal rise rather than the discontinuous jump that is signature of a shock wave. Ultimately, this results in the loss of valuable initial pressure rise and decay data. Lastly, excessive noise was present in the experimental
pressure traces within this radial standoff region, thus requiring additional data analysis considerations.
Figure 3-41: Peak shock wave pressure ratio vs. scaled radius from the optical characterization and the modified Friedlander curve for PETN @ $\rho = 0.90 \text{g/cm}^3$. 
Figure 3-42: Peak shock wave pressure ratio vs. scaled radius from the optical characterization and the modified Friedlander curve for PETN @ $\rho = 1.34\, \text{g/cm}^3$. 
Explosive impulse was determined after calculating the three modified Friedlander equation parameters. Explosive impulse is calculated through integration of the modified Friedlander equation over the positive phase of the pressure trace with respect to time, Equation (3-46). The explosive impulse (Equation (3-47)) was calculated for the low- and high-density, homogeneous PETN charges of Table 3-18 and Table 3-19.

The resulting explosive impulse vs. scaled shock wave radius profiles are plotted in Figure 3-43. Error bars are representative of the error in measuring peak shock wave pressures, and positive pressure durations. As shown, there is no measurable difference in explosive impulse between the low- and high-density PETN charges. Thus, an identical radially-dependent explosive impulse profile results for both the booster and shell charges, making it impossible to verify the booster removal procedure (Equation (3-43)) with these two explosives.

\[
I = \int_{t_a}^{t_a+T^+} P(t) dt
\]  
\[
I = P_s T^+ \left[ \frac{1}{\alpha} - \frac{1}{\alpha^2} \left( 1 - e^{-\alpha} \right) \right]
\]
Figure 3-43: Calculated explosive impulse vs. scaled shock wave radius for scaled homogeneous 1g PETN charges.
To properly verify Equation (3-43), the booster and shell explosives must possess significant differences in their explosive impulse vs. shock wave radius profiles as shown by curves 1 and 2 in Figure 3-34. A different, reproducible, explosive-booster material is required to accomplish this task due to the inherently-large error associated with measuring the explosive impulse, and the fact that most secondary explosives possess similar TNT equivalents. Reproducibility is required, as this explosive must first be individually characterized as a booster charge and then implemented in a composite charge arrangement.

Several attempts were made at producing a booster charge with such properties. A pressed mixture of PETN and sugar at a density of $\rho = 0.90\text{g/cm}^3$ was attempted, but found to be un-detonable due to the high-mass percentage of sugar required to produce a significantly-different TNT equivalent. Other methods explored included using a thicker EBW or a pressurized glass sphere to produce the desired difference in the explosive booster properties. The thicker EBW required an excessively-large current for detonation and was therefore a non-practical solution. A pressurized glass sphere was a more reasonable solution, but it was concluded that the reproducibility of spheres that produced near-identical burst properties was unattainable.

Therefore, it is suggested that a primary explosive be used as the booster charge in a composite charge arrangement for proper validation of Equation (3-43). It is known that primary explosives possess smaller TNT equivalents than secondary explosives while also yielding reproducible results [3, 4]. Thus, a composite charge consisting of a primary explosive booster charge and a secondary explosive shell charge would satisfy the above requirements, i.e. a significant difference between the booster and shell charge
explosive impulse vs. shock wave radius profiles. This particular composite charge arrangement was unable to be tested at present, due to the unavailability of a suitable primary explosive material and is therefore left open for future work. However, the explosive impulse booster removal procedure is computationally investigated in Section 4.3.
Chapter 4

Numerical Modeling

4.1 Computational software

Experimental results were used to check computational modeling of explosive charge detonations using the strong shock physics code CTH (Sandia National Laboratories) [65]. The CTH code is capable of modeling multi-dimensional, multi-material, large deformation, strong-shock physics by solving the finite volume analogs of the Lagrangian equations of motion using an Eulerian differencing scheme [65]. Thus, a spherical 1g explosive-charge CTH model is developed for determining the radial propagation rate of the resulting primary shock wave, as well as other explosion features.

The detonation of an explosive charge is modeled using the conservation of mass, momentum and energy and an equation of state (EOS). Here, the Jones-Wilkins-Lee (JWL) EOS is used for the detonation products, Equation (4-48); other EOS were available, however, the JWL EOS coefficients have been determined for a larger number of explosives. Equation (4-48) represents the resulting pressure of the detonation products \( P \) as a function of the volume ratio \( V \) at constant entropy, i.e. the ratio of the volume of the detonation products to the volume of the un-detonated explosive [66]. It therefore determines the detonation products state immediately following the completion of the chemical reaction by modeling the expansion of the detonation products from their initial high-pressure and high-density state through their eventual decay back to normal
atmospheric pressure and density [67]. Individual explosives are modeled using the JWL EOS through coefficients $A$, $B$, $C$, $R_1$, $R_2$, and $\omega$. Thus, the JWL EOS is an empirical mathematical curvefit that is used to describe the pressure-volume relationship of the gaseous detonation products [67].

However, it is known that the JWL EOS is insufficient to properly model free-air explosive events. It satisfies the mechanics of the flow (i.e. pressure and volume), but fails to account for the thermodynamic aspects of the system. Additionally, it is an ill-posed EOS for modeling free-air blasts, since the coefficients are determined from cylinder expansion tests where a specified mass of explosive is confined by a metal cylinder and detonated. The resulting expansion rate of the metal cylinder is used to approximate the expansion of the detonation product gases [68]. Coefficients are therefore calculated based upon the expansion rate of the cylinder. For modeling free-air explosive events, this method will produce inaccurate coefficients due to improper boundary conditions imposed by the metal cylinder. However, even with its shortcomings, the JWL EOS was used for the present modeling due to the lack of a more proper EOS.

$$P_s = A \exp(-R_1V) + B \exp(-R_2V) + CV^{-(\omega+1)}$$

(4-48)
4.2 Homogeneous charge characterization

4.2.1 Peak shock wave pressure profile

Spherical explosive charges were modeled in CTH as 1-dimensional spherical elements. The computational domain length was $R = 0.25m$ long, corresponding to the experimental measurement domain length, and meshed with 4500 evenly-spaced elements between $0 – 0.25m$. A Richardson-analysis study was initially performed to determine the proper mesh size using mesh sizes of 500, 1500, and 4500 elements [69]. The 4500 element mesh was found to produce $< 1\%$ error from the extrapolated exact solution, and was therefore used in all modeling cases. The domain was bounded by a symmetry boundary condition imposed at $R = 0m$ and a sound-speed-based absorbing boundary condition at the end of the computational domain, i.e. $R = 0.25m$. This allowed mass to flow into and out of the computational domain while approximating an infinite or semi-infinite medium [65].

Downstream shock wave property solutions were measured using “tracers” at specified fixed locations throughout the domain. The tracers measured the static pressure history downstream of the shock wave produced from the detonation of the explosive. A total of 36 tracers were added to the domain, with 21 tracers positioned at radial intervals of $0.0025m$ between $0.05 – 0.10m$ and 15 tracers positioned at intervals of $0.01m$ between $0.11 – 0.25m$. This tracer spacing was determined to produce smooth profiles of peak shock wave pressure as a function of shock wave radius.

The computational domain consisted of atmospheric air at temperature $T_{NTP}$ and pressure $P_{NTP}$. 1g explosive charges of PETN ($\rho = 0.88g/cm^3$), HMX ($\rho = 1.81g/cm^3$),
and TNT ($\rho = 1.63 \text{g/cm}^3$) were individually modeled and characterized. For each explosive material, an explosive charge was positioned with its center at $R = 0 \text{m}$. The explosive charge was subsequently detonated using the CTH programmed high-explosives burn model (HEBURN), which generates a detonation wave within the explosive material at a specified time $t$ and location $R$; all charges were centrally detonated, i.e. at $R = 0 \text{m}$, at time $t = 0 \text{s}$.

Modeling was performed for a total time of 0.35 ms, allowing the shock wave to propagate through the entire computational domain, which was determined from the previous experimental data. Pressures were collected at time step intervals of $\Delta t = 1\text{E-7s}$. The peak shock wave pressure ratio vs. shock wave radius profile was subsequently determined from the tracer outputs.

Profiles from the 1g PETN, HMX, and TNT charges are plotted in Figure 4-1 – Figure 4-3, respectively. Experimental peak shock wave pressure ratios vs. shock wave radius profiles are also plotted in Figure 4-1 and Figure 4-2 for PETN and HMX, and the 1g TNT standard of Kingery and Bulmash [59] is shown in Figure 4-3 for TNT.

As shown, the computational results are in reasonable but not perfect agreement with the present experimental results for PETN and HMX. This difference results from the use by CTH of the JWL EOS for both PETN and HMX. As previously stated, the JWL model is based upon cylinder expansion data, whereas the experimental results are for free-air blast tests. Additionally, a larger difference is evident between the experimental and computational results for HMX, which can be attributed to the difference in densities of HMX tested and modeled, i.e., $\rho = 1.34 \text{g/cm}^3$ (experimental) and $\rho = 1.81 \text{g/cm}^3$ (computational). This was due to the limited availability of JWL
coefficients for certain explosives at certain densities. A small discrepancy is also present for TNT results.
Figure 4-1: Peak shock wave pressure vs. shock wave radius profiles for the detonation of a spherical 1g PETN explosive charge.
Figure 4-2: Peak shock wave pressure vs. shock wave radius profiles for the detonation of a spherical 1g HMX explosive charge.
Figure 4-3: Peak shock wave pressure vs. shock wave radius profiles for the detonation of a spherical 1g TNT explosive charge.
4.2.2 Explosive impulse profile

The positive pressure duration $T^+$ was calculated from the pressure histories at the fixed tracer locations for the 1g PETN charge at $\rho = 0.88g/cm^3$. The positive pressure duration was defined by the time from the start of the pressure rise, through its subsequent decay back to atmospheric pressure. The resulting computational duration profile is compared to the previous theoretical positive pressure duration vs. shock wave radius profile in Figure 4-4.

A difference in positive pressure duration exists between the theoretical calculation from Section 3.8.3 and the CTH results. The positive pressure duration trend is approximately the same at shock wave radii larger than 0.15$m$, appearing to be off only by a proportionality constant, whereas a large difference in both trend and duration is present at smaller shock wave radii. This non-physical behavior at small shock wave radii was also found by previous authors [70, 71] (see Hargather [71] for a detailed explanation).

Positive pressure durations were also calculated for the homogeneous 1g HMX and 1g TNT charges from their respective pressure history profiles. Figure 4-5 illustrates the positive pressure duration as a function of shock wave radius profiles for all three charges modeled. As shown, all three explosives produce near-identical positive pressure duration profiles in addition to the same non-physical behavior at shock wave radii less than 0.08$m$. Thus, for the present research, CTH solutions will only be considered at shock wave radii larger than 0.08$m$.
Figure 4-4: “Theoretical” and CTH positive pressure durations as a function of scaled radius for a spherical 1g PETN charge.
Figure 4-5: CTH positive pressure durations vs. scaled radius for spherical 1g charges of PETN, HMX, and TNT.
Using the calculated positive pressure duration and the tracer pressure histories, the explosive impulse was subsequently calculated by integrating the area under the positive pressure history curve from the start of the pressure rise through its decay back to atmospheric pressure. The calculated explosive impulse profile for PETN is plotted in Figure 4-6 and is compared to the previous experimental results for the low-density PETN ($\rho = 0.90\text{g/cm}^3$). As shown, the CTH model under-predicts the explosive impulse compared to the experimental results from Section 3.8.3. This is as expected, since CTH under-predicted the positive pressure duration, while its peak shock wave pressure vs. radius profile was in reasonable agreement with the present experimental results. However, similar to the positive pressure duration profiles, the explosive impulse vs. shock wave radius profiles from the CTH calculation and the present experimental results exhibit the same trend in their decay, differing only by a constant of proportionality.

Explosive impulse vs. shock wave radius profiles were calculated for the 1g HMX and 1g TNT charges. Solutions are plotted in Figure 4-7 along with the explosive impulse profile for PETN. The profiles for all three explosives exhibit a smooth decay for shock wave radii larger than 0.08m. This is as expected, since it is outside the region where the non-physical behavior occurred in the positive pressure duration profiles shown earlier. Additionally, the HMX and PETN produced near-identical explosive impulse solutions as shown by Figure 4-7.
Figure 4-6: Experimental ($\rho = 0.90\text{g/cm}^3$) and CTH ($\rho = 0.88\text{g/cm}^3$) explosive impulse vs. scaled radius for a spherical 1g PETN charge.
Figure 4-7: CTH explosive impulse vs. scaled radius for a spherical 1g charges of PETN, HMX, and TNT.
4.3 Composite charge characterization

A composite charge consisting of a 1g HMX booster charge and 1g TNT shell charge was modeled in CTH to verify the peak shock wave pressure and explosive impulse booster removal procedures of Chapter 3. HMX and TNT were chosen as they exhibit the largest difference in their peak shock wave pressure and explosive impulse vs. shock wave radius solutions from the CTH results (the HMX and PETN results possessed nearly-identical peak shock wave pressure and explosive impulse vs. shock wave radius profiles, which was insufficient for verifying the booster removal procedure).

A 2g composite charge consisting of a 1g HMX booster and 1g TNT shell was modeled using the same computational domain outlined in Section 4.2. The HMX booster charge was centrally detonated at time $t = 0s$ using the CTH programmed burn (HEBURN). The TNT, however, was detonated using the history-variable-reactive burn (HVRB) program in CTH. The HVRB models shock-induced initiation and detonation wave propagation in heterogeneous explosives [65]. Thus, detonation of the TNT is initiated when the detonation wave from the HMX reaches the HMX-TNT interface.

Peak shock wave pressure history solutions were collected at the 36 tracer locations for the 2g composite charge. The resulting peak shock wave pressure vs. shock wave radius solution is plotted in Figure 4-8 along with the 1g HMX and 1g TNT solutions from Figure 4-2 and Figure 4-3, respectively. Equation (3-30), which assumed that constant $k_2$ was known, was first used to subtract the known booster charge peak shock wave pressure effects from the composite charge signature. The calculated 1g TNT shell charge profile is plotted in Figure 4-9 and compared with the 1g homogeneous
TNT charge solution. As shown, the calculated shell charge shock wave radii are nearly identical to the homogeneous charge shock wave radii solution.

Shell charge shock wave radii were also calculated using Equation (3-31), which assumed the shell charge constant $k_2$ was equal to the composite charge constant $k_3$. The resulting peak shock wave pressure vs. shock wave radius profile is plotted in Figure 4-10 compared with the 1g homogeneous TNT charge solution. As shown, the shock wave radii match well with the homogeneous charge shock wave radii solution, therefore numerically verifying the booster removal procedure based upon the radial shock wave position of the booster charge, shell charge, and composite charge at equivalent peak shock pressures.
Figure 4-8: Peak shock wave pressure vs. shock wave radius profiles for a 2g composite charge composed of a 1g HMX booster charge and 1g TNT shell charge.
Figure 4-9: Peak shock wave pressure vs. shock wave radius profiles for a 1g TNT homogeneous charge and the calculated 1g TNT shell charge from Equation (3-30).
Figure 4-10: Peak shock wave pressure vs. shock wave radius profiles for a 1g TNT homogeneous charge and the calculated 1g TNT shell charge from Equation (3-31).
After calculating the positive pressure duration of the composite charge, its explosive impulse vs. shock wave radius solution was next determined. This is plotted in Figure 4-11 along with the explosive impulse solutions of the homogeneous 1g HMX and 1g TNT charges. Here, unlike the experimental results, a difference in the explosive impulse vs. shock wave radius solutions resulted for the booster, shell, and composite charges. This is because the composite charge of HMX and TNT can be simulated in CTH, but cannot be tested experimentally with current methods. Therefore, the hypothesized explosive impulse booster removal procedure, Equation (3-43), is able to be tested for the present composite charge arrangement.

Shell shock wave radii were calculated from Equation (3-43) using the known properties of the booster and composite charge solutions in addition to the known shell constant $j_2$. The resulting explosive impulse vs. shock wave radius profile is plotted in Figure 4-12. As shown, Equation (3-43) returns a nearly-identical explosive impulse vs. shock wave radius solution to that simulated by CTH. This numerically verifies the presently-proposed composite-charge booster removal procedure based upon explosive impulse profiles.

However, unlike the case of the peak shock wave pressure booster removal procedure, here an assumption is unable to be made concerning equality of proportionality constants in the numerical solution. Thus, the shell shock wave radius is indeterminate from knowledge of only the booster and composite charge explosive impulse vs. shock wave radius signatures. However, this is according to numerical solutions that have been shown to conflict with experimental measurements, e.g. positive pressure duration and explosive impulse profiles. Therefore, further experimental
research on composite charge arrangements is necessary to properly verify the explosive impulse booster removal procedure without \textit{a priori} knowledge of the shell charge properties. Also, if the JWL EOS coefficients were known for free-air explosions of the present explosives, then CTH solutions could be run that are expected to match the current experimental results.
Figure 4-11: Explosive impulse vs. shock wave radius for the individual components of the 2g composite charge modeled (i.e., 1g HMX booster, 1g TNT shell and 2g composite).
Figure 4-12: Explosive impulse vs. shock wave radius profiles for a 1g TNT homogeneous charge and the calculated 1g TNT shell charge from Equation (3-43).
4.4 Computational wave diagram

To gain a more comprehensive understanding of the CTH solution, a computational wave diagram of the detonation solution was developed. Previous authors have used extensive numerical modeling procedures to generate interferograms, schlieren, and shadowgrams of 3-dimensional flowfields, i.e. modeling the deflection of computational light rays [72]. Here, however, a 1-dimensional flowfield is analyzed, thus allowing a more simplistic approach. The shadowgraph optical method responds to the Laplacian of the index of refraction, which can be linearly related to the flowfield density through the Gladstone-Dale law [56]. Thus, a computational shadowgram can be developed by taking the Laplacian of the 1-dimensional radial-density field solution, i.e., \( \partial^2 \rho / \partial x^2 \).

A homogeneous 1g PETN charge was modeled with CTH to determine the radial-density field solution. The previous computational domain was decreased to 0.20m in length and fitted with 200 evenly-distributed tracers at radial standoff distances between 0 – 0.20m. The density field solution was gathered at time steps of \( \Delta t = 1E-7s \) for a total time of \( t = 5E-5s \). Thus, a computational radius vs. time wave diagram is developed.

Figure 4-13 illustrates the CTH solution as a shadowgraphic wave diagram. As shown, the computational wave diagram reveals the complex flow solution produced from the detonation of PETN, similar to the previous experimental wave diagram of Figure 2.8. Here, however, additional flow features described in Section 2.3 are discernible in the computational image. As shown, the location of the contact surface, which contains the gaseous detonation products, is unmistakable. Its location is indicated
by the reflection of the secondary shock wave from it as described in Section 2.3, and by Brode [73], Glass [74], and Boyer [53]. Additionally, the multiple reflections at $R = 0m$ are visible which result in the secondary and tertiary shock waves.

For the 1g PETN charge, the contact surface was found to expand to a maximum radial distance of 0.10m. Up to this radial location, large discrepancies between experimental and numerical results were found to exist. Thus, improper modeling of the contact surface motion is possibly causing the previously-mentioned non-physical flow solutions from CTH. This would again indicate that the JWL EOS is too simplified for proper modeling of a free-air detonation.

The CTH solution also predicts that a significant tertiary shock wave results from the detonation of PETN, as shown in Figure 4-13. However, a tertiary shock wave was never witnessed in any of the multitude of experimental PETN tests done for this dissertation. In the experimental tests, though, PETN was detonated along a line using the EBW, rather than at a point as modeled in CTH. Thus, it is plausible that a high-degree of symmetry is required throughout the detonation process for the development of a tertiary shock wave. Otherwise, though, good correlation was found to exist between the computational wave diagram and those measured experimentally.
Figure 4-13: Computational shadowgraphic wave diagram of the detonation of a spherical 1g PETN charge.
Chapter 5

Summary and Recommendations for Future Research

5.1 Summary

The present research has demonstrated the utility of laboratory-scale explosive characterization using high-speed digital cameras and traditional optical imaging techniques. It has supplemented the current literature with a more complete characterization procedure for insensitive explosive materials by understanding the physics of the shock wave propagation, and by quantifying the effects produced by a booster charge.

Thus, laboratory-scale air-blast characterization of explosive materials has shown the traditional full-scale procedure (which depends upon pointwise measurements) to be inadequate for characterization of explosive materials, notably those that are insensitive. This is a result of the pointwise-measurement data acquisition approach and the necessary booster charge, which has, to this point, been ignored. The present research addressed the previously-unavailable knowledge of the booster charge effects, in a composite charge arrangement, to develop a better characterization procedure for insensitive explosive materials at the laboratory scale.

Two different densities of homogeneous PETN were initially characterized prior to conducting the composite charge research. Peak shock wave pressure vs. shock wave radius and subsequent TNT equivalent as a function of shock wave radius profiles were
developed from the radial shock wave propagation measurements for each density of PETN. The variation in explosive charge density produced different TNT equivalent profiles for the same explosive material due to the detonation velocity dependence on the explosive charge density. This characterization of homogeneous PETN was used to develop a booster removal procedure from a composite charge composed of known booster- and shell-charge explosive materials.

Due to temporal resolution limitations of the primary high-speed camera, additional experiments were conducted with an ultra-high-speed camera to measure shock wave radial-propagation rates in close proximity to the explosive charge surface. The experiments were conducted in an effort to determine a physical initial condition to Dewey’s shock wave radius vs. time curvefit [17]. However, the measured shock wave Mach numbers were found to be larger than the hypothesized detonation-velocity Mach number in air. Further experiments are thus required to investigate initial condition shock wave Mach numbers. These experiments should be conducted with several different explosives having a large variation in their detonation velocities.

Spherically-concentric composite-explosive charges were developed for the characterization of insensitive explosive materials that require a booster charge for detonation. The composite charge consisted of a spherical booster charge surrounded by a spherical, concentric shell charge. This geometry was chosen to promote a more uniform, spherical detonation of the composite charge, as opposed to previous work conducted with cylindrical charges that required a specific orientation. The spherically-concentric composite-charge geometry was detonated in the common test region of two perpendicular shadowgraph systems for simultaneous imaging of the shock wave. These
perpendicular images verified that a spherically-uniform shock wave was produced by a spherical, concentric composite charge.

Composite charges were subsequently constructed from the two previously-characterized PETN densities and exploded for characterization of the composite charge radial peak shock wave pressure signature. Using the peak shock wave pressure signatures of the booster charge, shell charge, and composite charge, a procedure was developed to determine a shell charge shock wave radius from the known booster and composite charge characterizations. The results from two different composite charge mass arrangements were shown to agree well with the previous homogeneous charge characterizations. This booster removal procedure was used to characterize RDX, HMX, and smokeless powder, all of which were shown to conform to the procedure. Additionally, a composite charge of HMX and TNT was numerically modeled and shown also to conform to the booster removal procedure.

A second booster removal procedure was developed for the characterization of insensitive explosive materials based upon the explosive impulse produced by the individual components of the composite charge. Similar to the peak shock wave pressure procedure, the explosive impulse effects produced by the characterized booster charge are removed from the composite charge signature, yielding the sole effects of the shell charge. However, due to the inherent experimental error associated with measuring explosive impulse and the similarities in explosive impulse profiles of the booster and shell, the procedure was unable to be experimentally verified. It was, however, verified numerically for cases where the shell charge properties were known \textit{a priori}. 
An assumption could not be made to relate the proportionality constant of the shell charge to the composite charge for the numerical explosive impulse verification. However, the JWL equation of state used to model the explosive event is insufficient and ill-represented for proper modeling of free-air explosive blasts. That is, the equation is insufficient as it only satisfies the mechanics of the flow (pressure and volume), failing to account for the thermodynamic effects (temperature and entropy) [75]. Additionally, the JWL EOS coefficients are determined from cylinder expansion tests wherein a specified mass of explosive is confined by a metal cylinder and detonated, as opposed to the current unconfined free-air explosive blasts. The JWL coefficients are therefore based upon the expansion rate of the metal cylinder, yielding inaccurate coefficients for free-air explosive blasts due to improper boundary conditions. Thus, additional experimental research is necessary to properly validate the proposed explosive impulse procedure.

5.2 Future Research

Previous research by Lutzky et al. linked the peak shock wave pressure, or shock wave Mach number, to the explosive fireball pressure [61]. Measurements of the fireball pressure and the shock wave Mach number in close proximity to the charge radius may yield a more physical initial shock wave propagation condition. However, proper measurement of the fireball pressure is difficult, as it would require a transducer with near-zero response time, while also withstanding the temperatures produced in close proximity to the charge radius. If properly measured, though, a correlation could be
developed between the known explosive material detonation velocity, the fireball pressure and the resulting shock wave Mach number at the explosive charge radius.

As previously indicated, future research should focus on explosive impulse characterization of composite charge arrangements composed of a primary explosive booster charge and secondary explosive shell charge. The use of a primary explosive booster charge would yield a measurable radial shock wave difference between its explosive impulse and an equivalent impulse produced by a secondary explosive. Thus, the hypothesized procedure for removing the effects of the booster charge from the composite charge signature could be verified experimentally. Computational verification was already performed in this thesis.

In calculating explosive impulse, future research should focus on the waveform parameter of the pressure decay. As shown by Figure 1-1, a maximum in curvature exists at the absolute negative pressure location of the shock wave pressure trace, indicating that its visualization might be possible through optical means. Calculation of the waveform parameter is therefore possible from knowledge of the minimum pressure location, the positive pressure duration, and the peak shock wave pressure. Thus, the peak shock wave pressure, positive pressure duration, and waveform parameter may all be determinable from the shadowgrams, ultimately allowing the explosive impulse to be calculated. This would eliminate the need for piezoelectric pressure transducers, thus reducing the experimental error in explosive impulse calculations that result from spatial and temporal averaging of the pressure transducer and the typically-noisy signals that are obtained.

The present research analyzed the spherical free-air relationship of a booster charge in a composite charge arrangement. To better relate laboratory-scale experiments
to full-scale experiments, future research should extend the developed procedures to the characterization of hemispherical composite charges on a ground plane. This would permit a more straightforward comparison between laboratory-scale and full-scale research, as the full-scale tests are typically limited to a hemispherical arrangement due to the physical size of the explosive charge. Additionally, as shown by Kingery and Bulmash [59], the resulting shock wave propagation rate and its properties vary between hemispherical and spherical charge arrangements, a result of the added shock strength from the reflected ground-plane shock for a hemispherical charge.

The utility of composite-charge laboratory-scale research should also be extended to the enhanced-blast explosives (EBEs) arena, i.e. the thermobarics. Currently, there are minimal data available concerning the explosive effects of a thermobaric explosive. Thus, the demonstrated booster removal procedure could be applied to other un-characterized explosive formulations, such as ammonium nitrate fuel-oil (ANFO), or peroxide-based, liquid homemade explosives (HMEs). Initial research was conducted to demonstrate the ability to detonate ANFO at the laboratory scale through the use of a composite charge arrangement. A sizeable increase in shock wave strength was measured for the composite charge compared to the known booster charge properties, demonstrating the ability to detonate ANFO at the laboratory scale. Thus, the utility of the composite-charge booster-removal procedures developed in the present research should be extended for the characterization of these EBEs and HMEs.

Lastly, new JWL equation of state coefficients should be determined for the free-air blast resulting from the detonation of a spherical charge. As previously stated, the present JWL coefficients are inadequate for modeling anything but the detonation of an
encased explosive charge. Thus, future work should address this, and determine new coefficients for a free-air blast. A shadowgraph system is one possible instrument for determining the detonation product’s pressure-volume relationship from a free-air blast. Initial work was performed by Itoh et al. [76] using shadowgraphy for the visualization of underwater explosions from spherical and cylindrical charges. As shown above, determining the location of the contact surface in free-air blasts is also possible. Thus, this method should be extended to free-air blasts resulting from spherical explosive charges. This would yield the proper JWL coefficients necessary for modeling explosively-driven shock waves.
References


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EDUCATION


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PUBLICATIONS


M. M. Biss, G. S. Settles (in press), “On the use of composite charges to determine insensitive explosive material properties at the laboratory-scale,” *Propellants, Explosives, Pyrotechnics*.

M. M. Biss, G. S. Settles (in preparation), “Laboratory-scale procedure to determine the explosive impulse of insensitive materials,” *Propellants, Explosives, Pyrotechnics*.

PRESENTATIONS

“Laboratory-scale experiments to determine explosive properties using concentric composite charges,” 62nd Annual Meeting of the American Physical Society: Division of Fluid Dynamics, November 22nd-24th, 2009, Minneapolis, Minnesota, presented.


UNITED STATES PATENT APPLICATIONS


