

The Pennsylvania State University  
The Graduate School

**A STUDY ON THE ANALYSIS OF THE FORMATION OF HIGH  
WATER SATURATION ZONES AROUND WELL PERFORATIONS**

A Dissertation in  
Petroleum and Mineral Engineering  
by  
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# Abstract

The water produced with oil as a result of water coning is a serious problem as it decreases well productivity and increases the cost of operation. The main cause of water coning is the pressure drawdown near the wellbore. Producing a well in an oil reservoir with a bottom water drive will cause the original oil-water contact to rise towards the well in the shape of a cone. As production continues, the height of the cone increases. Once the water reaches the wellbore, the water starts to be produced and water production increases with time while the oil production decreases.

The first part of this thesis is concerned with the analysis of pressure transient data in an oil reservoir with edge water. A *3-D* numerical model is used to generate pressure transient data for a vertical well. The reservoir is considered as a composite reservoir where the inner zone contains the oil phase, and the outer zone contains the water phase. Procedures are proposed to help estimate the distance to the discontinuity under certain conditions. These procedures can be applied using the derivative of the pressure buildup test data if the mobility ratio is less than unity. Some of the parameters that control the accuracy of the proposed analysis procedure are the producing time and the external reservoir radius.

The second part of this study is concerned with the water coning phenomena. Artificial neural networks are developed to predict water saturation around vertical and horizontal wells with a good accuracy in oil reservoirs experiencing a bottom water drive. The data used to develop the neural networks are from a numerical simulation model for reservoirs created using synthetic data. The neural networks have been found to effectively predict the change of water saturation over the time and show the development of the water cone.

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# List of Symbols

- $B$  Formation volume factor, bbl/STB, p. 12
- $C_1$  Constant in the system of equations, p. 10
- $C_2$  Constant in the system of equations, p. 10
- $C_3$  Constant in the system of equations, p. 10
- $C$  Wellbore storage coefficient, bbl/psi, p. 12
- $C_D$  Dimensionless wellbore storage coefficient, p. 9
- $c_t$  total compressibility,  $psi^{-1}$ , p. 11
- $F_s$  Storativity ratio, p. 17
- $h$  Total reservoir thickness,  $ft$ , p. 7
- $h_d$  Depth of the horizontal well from the top of the pay zone,  $ft$ , p. ??
- $h_L$  Length of the horizontal well,  $ft$ , p. ??
- $h_p$  Length of the open-to-flow thickness of the pay zone,  $ft$ , p. 69
- $I$  Modified Bessel Function of the first kind, p. 10
- $K$  Modified Bessel Function of the second kind, p. 10
- $k$  Absolute permeability, md, p. 11
- $k_r$  Permeability in the r-direction in radial system, md, p. 36
- $k_v$  Permeability in the vertical direction in radial system, md, p. 69

$k_x$	Permeability in the x-direction in rectangular system, md, p. 36
$k_y$	Permeability in the u-direction in rectangular system, md, p. 36
$p_i$	Initial reservoir pressure, psi, p. 7
$p_w$	Wellbore pressure, psi, p. 12
$p_{ws}$	Wellbore shut-in pressure, psi, p. 35
$q_L$	Total liquid flow rate, STB/D, p. 69
$r_e$	Reservoir radius, ft, p. 7
$r_{in}$	Distance to the discontinuity in the composite reservoir, ft, p. 2
$r_{inv}$	Radius of investigation, ft, p. 19
$r_{(peak)}$	Radius of investigation calculated using $t_{(peak)}$ , ft, p. 19
$r_w$	Wellbore radius, ft, p. 11
$S$	Skin factor, p. 9
$s$	Laplace parameter, p. 10
$S_{oi}$	Initial oil saturation, p. 36
$S_w$	Water saturation, p. 41
$t$	Time, hours, p. 12
$t_{in}$	Time needed for the pressure transient to reach the discontinuity in a composite reservoir, hours, p. 24
$t_p$	Producing time prior to shut-in, hours, p. 2
$t_{(peak)}$	Time at the peak in pressure derivative curve , hours, p. 19
$\alpha$	Parameter in the system of equation, p. 10
$\Delta t$	Shut-in time, hours, p. 14
$\Delta t_e$	Equivalent time, hours, p. 13
$\eta$	Hydraulic diffusivity, p. 9

- $\lambda$  Mobility ratio, p. 11
- $\mu$  Viscosity, cp, p. 11
- $\rho_o$  Oil Density,  $lb/ft^3$ , p. 81
- $\phi$  Porosity, fraction, p. 7

### **Subscript**

- 1 Inner zone of the composite reservoir, p. 9
- 2 Outer zone of the composite reservoir, p. 9
- D Dimensionless parameter, p. 9

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# Dedication

All Praise be to Allah, Cherisher and Sustainer of Worlds, who bestowed me with life and health to complete this work.

I wish to dedicate this work to my mother and my father for their continuous prayer and patience.

I dedicate this work to my wife for her sacrifices of her health, time, and professional progress to provide me with a comfortable environment. Her patience and encouragement was a drive for me to finish this work.

I dedicate this work to my children, Yousuf, Omar, and Dhuha. They fill my life with happiness and hope.

I dedicate this work to my brothers and sisters for their encouragement and prayers.

# Chapter 1

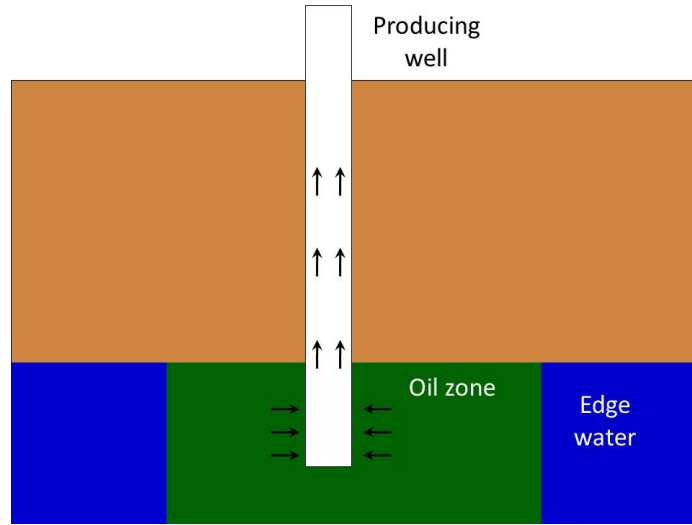
## Introduction

Many hydrocarbon reservoirs contain an active water aquifer. The drilled wells are always completed to produce hydrocarbon. As oil production continues, water starts to appear in the wellbore. This water is undesirable as its presence around the wellbore decreases the well productivity and needs more facilities to be handled, treated and disposed of at the surface resulting in extra investments and operating costs. Production of the hydrocarbon is initiated by the drawdown created at the wellbore, which causes a force that pulls the hydrocarbon towards the wellbore. This force also, with the capillary pressure, brings water in the aquifer upward towards the wellbore. What controls the height of the water is the gravitational force working in the opposite direction. The height of the water cone stops increasing if the upward dynamic flow forces become equal to the downward gravitational forces. The water will be produced once the height of the water reaches the wellbore. By continuing to produce the hydrocarbon with water, formation around the wellbore will be saturated with water in the shape of a cone, a phenomenon that is referred to as water coning.

This study consists of two parts. The first part studied the pressure transient test data for a vertical well in an oil reservoir with edge water (Fig 1.1). The reservoir was considered as a composite reservoir. A reservoir is composite when there is a difference in fluid properties or rock properties in two or more concentric zones in the reservoirs. In this part, the inner zone contains the oil phase, and the outer zone contains the water phase. This study will look into three cases of two-regions

composite reservoirs:

1. Two different absolute permeabilities
2. Two different hydrocarbon liquids (low viscosity oil and high viscosity oil)
3. Two different fluids (oil and water)



**Figure 1.1: Schematic representation of an oil reservoir exposed to edge water encroachment.**

The distance from the well to the interface that separates the two different zones in a composite reservoir is called the distance to the discontinuity ( $r_{in}$ ), and it can be estimated from proposed procedures. The proposed procedures use the pressure derivative buildup data if the mobility ratio is less than unity. The main two factors that affect the accuracy of the estimate are the producing time ( $t_p$ ) and the reservoir radius ( $r_e$ ). These procedures can be used for an oil reservoir with edge water encroachment to monitor the distance to the edge water front over time to help in the future water handling plans.

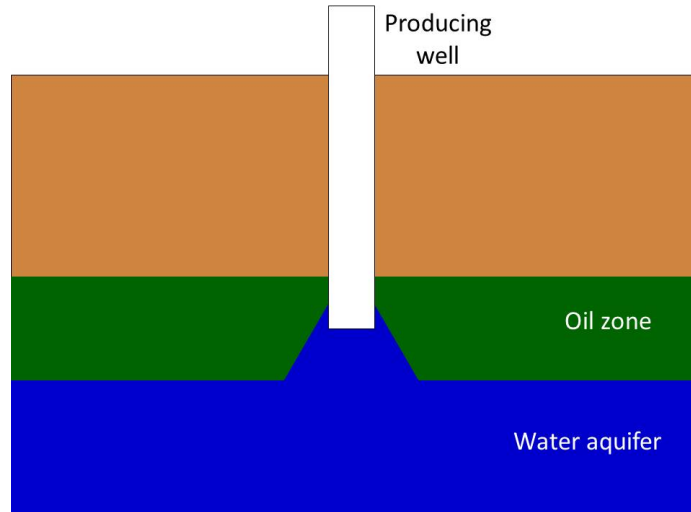
The approach for this problem is done by going through 3 stages. The 1<sup>st</sup> stage is analyzing the pressure transient data for an oil composite reservoir with two different absolute permeabilities. This simple case will assure that the value of  $r_{in}$  will not change during the life of the reservoir and limit the effects on the pressure

transient data with the effect of the discontinuity.

The 2<sup>nd</sup> stage is more complex and involves analyzing a composite reservoir with two different oil viscosities. The value of  $r_{in}$  could change during the life of the reservoir as the produced oil in the inner zone will be displaced by the oil in the outer zone and this could change the value of  $r_{in}$ .

The 3<sup>rd</sup> stage, which is the principal goal of this study, is analyzing a composite reservoir with two different fluids, oil and water. In this stage, the difference is not only for one property of a fluid, but instead, for two different fluids.

The second part analyzes the water coning phenomenon (Fig 1.2). The water coning behavior is very important in hydrocarbon production and the ability to predict its behavior will improve and help in better managing the reservoir. The behavior of the water coning in an oil reservoir was predicted successfully using artificial neural networks by predicting the change of the water saturation distribution in reservoirs over time. The developed neural networks were designed to be used for vertical and horizontal wells with a bottom water aquifer. These developed neural network can be useful in optimizing production to find the optimum perforation interval or the optimum production rate to delay the water production.



**Figure 1.2:** Schematic representation of an oil reservoir experiences water coning.



## Literature Review

### 2.1 Water Coning

The first paper which discussed the water coning phenomenon and its physics was *Muskat and Wyckoff (1935)*. In the paper it was indicated that some of the factors that affect the water coning are production rate and length of perforated interval. *Byrne and Morse (1973)*, *Mungan (1975)* and *Blades and Stright (1975)* performed a numerical study of the effects of various parameters on water coning in vertical wells. *Kuo and DesBrisay (1983)* also studied the effects of various parameters on water coning in vertical wells, and developed correlations to predict critical rate, breakthrough time and watercut after water production. *Yang and Wattenbarger (1991)* studied the water coning effects in vertical and horizontal wells and developed a method to calculate the critical rate, breakthrough time and the water-oil ratio after breakthrough. *Van Golf-Racht and Sonier (1994)* investigated the water coning behavior for a fractured reservoir in a vertical well and studied various parameters and their effects on the water coning.

*Helle and Bhatt (2002)* developed artificial neural networks that predict the underground fluids (water, oil and gas) from the well logs. *Shokir (2004)* presented new artificial neural networks that predict water saturation in shaly formation using the well log data and the core data as the inputs. *Al-Bulushi et al. (2009)* developed artificial neural networks models to predict water saturation from well log data and core data.

## 2.2 Composite Reservoirs

There have been many researchers interested in studying and analyzing composite reservoirs to understand their behavior and the flow performance under these conditions and also to calculate important parameters. *Hurst (1960)* and *Mortada (1960)* studied how the well performance is affected by the interference of two oil reservoirs producing from the same aquifer. *Carter (1966)* presented an analytical solution for a radial composite reservoir with a closed and no-flow outer boundary. He only changed the permeability in the two zones. *Satman et al. (1980)* presented another analytical solution for single phase systems with different fluid properties, two different permeabilities and an infinite reservoir. The wellbore storage and skin factors were taken into consideration. Regarding estimating the size and the distance to discontinuity in a composite reservoir, *Tang (1982)* showed that the intersection method can be used to determine the distance to the discontinuity if the pressure data include two well-defined semi-log straight lines. *Odeh (1969)* generated correlations to be used in trial and error to determine the mobility of the two zones in a radial composite reservoir and the distance to the discontinuity. *Bixel and Van Poollen (1967)* developed a method to calculate the distance to the discontinuity using type curves and then locating the deviation point to calculate the distance. *Loucks and Guerrero (1961)* found that under certain conditions, the permeabilities in both zones and the size of the inner zone can be determined from the pressure drawdown test. *Brown (1985)* presented graphical analysis to determine the mobility of the inner and the outer zones as well as the distance to the discontinuity. In using the semi-log plot to analyze pressure transient data, existing flow regimes need to be identified first. Also it is known that the pressure derivative response is more sensitive than the pressure data [*Bourdet et al. (1983)* and *Tiab and Kumar (1980)*]. *Issaka and Ambastha (1996)* developed a generalized pressure derivative formula for a composite reservoir to identify the flow regimes in four flow geometries (radial, elliptical, linear and spherical). *Olarewaju and Lee (1989)* found that the pressure derivative curves for a composite reservoir show a different response from homogenous reservoirs.

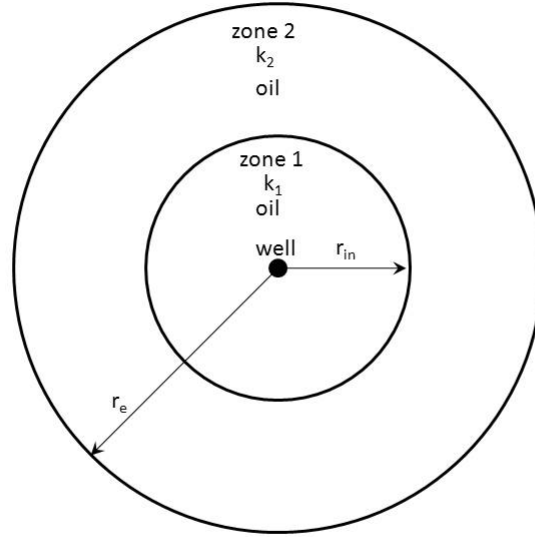
## Composite Reservoir Model

### 3.1 Numerical Model Development

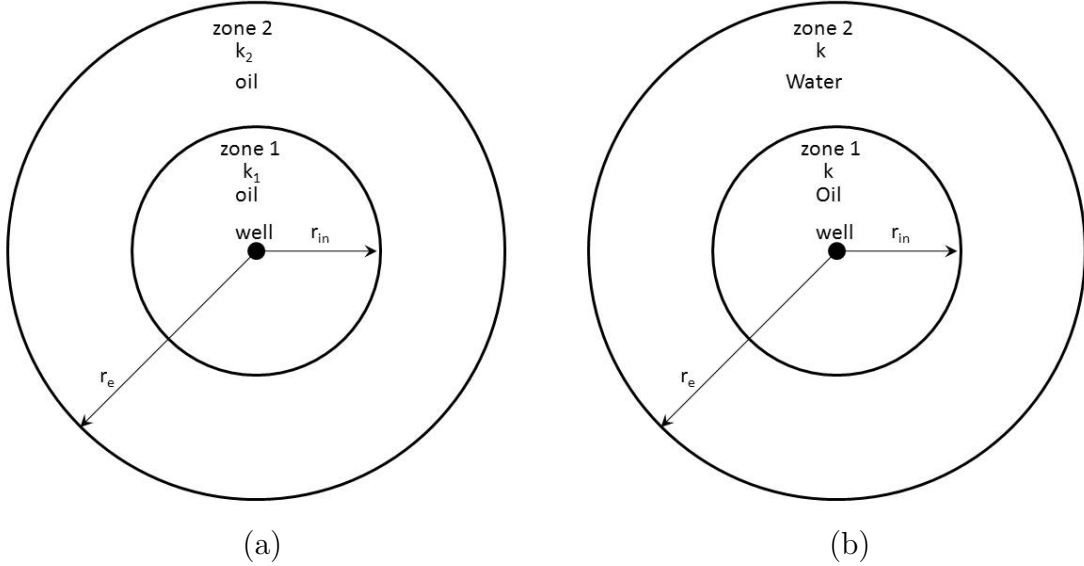
Four 3D numerical models were used for the composite reservoir model study; two of the four models are single phase, oil models, and the other two are two-phase, oil/water models. All of the models are radial models with a vertical well. The radial grid system was used. The reservoir properties are homogeneous and isotropic and for the fluid properties, the reservoir conditions are assumed to be above bubblepoint pressure to ensure that no free gas is present in the reservoir. There is a base case for each model, and then, for each parameter to be studied, its value was changed to produce simulation runs for different values. The study domain used by the numerical model is shown in Fig 3.1 for the composite reservoir study. The model is populated by using homogeneous properties, isotropic permeability, single fluid in some cases and different fluids in other cases in each region. The reservoir is horizontal with uniform thickness. The model is verified against the analytical solution. The reservoir properties are tabulated in Table 3.1.

**Table 3.1: Reservoir properties**

thickness ( $h$ ), ft	50
Porosity ( $\phi$ )	0.25
Initial pressure ( $p_i$ ), psig	5000
Temperature, °F	130
Reservoir radius ( $r_e$ ), ft	6,000

**Figure 3.1: Schematic representation of a two-region radial composite reservoir**

The absolute permeability in the vertical direction ( $z$ -direction) and the angular direction ( $\theta$ -direction) are set equal to zero, so that the only component of the flow in the radial direction is analyzed. The gravitational forces are ignored. The model was run with two different scenarios (Fig 3.2) in order to gradually increase the complexity of the problem.



**Figure 3.2:** Illustration of the reservoir characteristic for: a) First scenario. b) Second scenario

The first scenario involves running the model as a single phase system with two different absolute permeability zones. Analyzing the pressure transient data, a unique behavior that identifies the response at the discontinuity is expected to be observed. The second scenario involved running the model with two different fluids for each zone, oil and water, and the formation absolute permeability is kept the same in both zones. Two different cases are studied in the second scenario; i) oil in the inner zone and water in the outer zone, and ii) water in the inner zone and oil in the outer zone.

### 3.2 Numerical Model Validation

The first step after building the model was to validate it. The verification was undertaken by comparing the numerical model with the analytical solution for a composite reservoir. The analytical solution was developed by *Satman et al. (1980)*. The following assumptions were considered:

- The formation is horizontal, of uniform thickness, and homogeneous.
- The front is of infinitesimal thickness in the radial direction.

- The region behind the front contains only gas, while there is restricted gas flow in the region ahead of the front. However, the mobility of the gas is so much greater than that of the liquid phases that only gas flow needs to be considered.
- Flow is radial, and gravity and capillarity effects are negligible.
- The front can be considered stationary throughout the few hours of the testing period.
- The fluids are slightly compressible.

The diffusivity equations in dimensionless form for the two regions are:

For zone 1,

$$\frac{1}{r_D} \frac{\partial}{\partial r_D} \left[ r_D \frac{\partial P_{D1}}{\partial r_D} \right] = \frac{\partial P_{D1}}{\partial t_D}, 1 \leq r_D \leq R_D \quad (3.1)$$

and for zone 2,

$$\frac{1}{r_D} \frac{\partial}{\partial r_D} \left[ r_D \frac{\partial P_{D2}}{\partial r_D} \right] = \eta \frac{\partial P_{D2}}{\partial t_D}, R_D \leq r_D < \infty. \quad (3.2)$$

Inner boundary conditions are:

$$C_D \frac{dP_{wD}}{dt_D} - \left[ \frac{\partial P_{D1}}{\partial r_D} \right]_{r_D=1} = 1 \quad (3.3)$$

and

$$P_{wD} = P_{D1} - S \left[ \frac{\partial P_{D1}}{\partial r_D} \right]_{r_D=1} = 1. \quad (3.4)$$

Initial conditions are

$$P_{D1}(r_D, 0) = 0; 1 \leq r_D \leq R_D \quad (3.5)$$

and

$$P_{D2}(r_D, 0) = 0; R_D \leq r_D < \infty. \quad (3.6)$$

Conditions at the interface are

$$P_{D1}(R_D, t_D) = P_{D2}(R_D, t_D) \quad (3.7)$$

and

$$\frac{\partial P_{D2}}{\partial r_D} = \lambda \frac{\partial P_{D1}}{\partial r_D}; r_D = R_D, \text{ and } t_D > 0. \quad (3.8)$$

The outer boundary condition is given as:

$$\lim_{r_D \rightarrow \infty} P_{D2}(r_D, t_D) = 0. \quad (3.9)$$

The solution in the Laplace space in terms of the Bessel functions is as follows,

$$\bar{P}_{D1}(r_D, s) = C_1 I_0(r_D \sqrt{s}) + C_2 K_0(r_D \sqrt{s}), 1 \leq r_D \leq R_D \quad (3.10)$$

$$\bar{P}_{D2}(r_D, s) = C_3 K_0(r_D \sqrt{s\eta}), R_D \leq r_D < \infty \quad (3.11)$$

$$\bar{P}_{wD}(s) = C_1 [I_0(\sqrt{s}) - S\sqrt{s}I_1(\sqrt{s})] + C_2 [K_0(\sqrt{s}) + S\sqrt{s}K_1(\sqrt{s})]. \quad (3.12)$$

The constants  $C_1$ ,  $C_2$ , and  $C_3$  are obtained by solving the following system of equations,

$$\alpha_{11}C_1 + \alpha_{12}C_2 = \frac{1}{2} \quad (3.13)$$

$$\alpha_{21}C_1 + \alpha_{22}C_2 + \alpha_{23}C_3 = 0 \quad (3.14)$$

$$\alpha_{31}C_1 + \alpha_{32}C_2 + \alpha_{33}C_3 = 0 \quad (3.15)$$

where

$$\alpha_{11} = C_D s [I_0(\sqrt{s}) - S\sqrt{s}I_1(\sqrt{s})] - \sqrt{s}I_1(\sqrt{s}) \quad (3.16)$$

$$\alpha_{12} = C_D s [K_0(\sqrt{s}) - S\sqrt{s}K_1(\sqrt{s})] - \sqrt{s}k_1(\sqrt{s}) \quad (3.17)$$

$$\alpha_{21} = I_0(R_D\sqrt{s}) \quad (3.18)$$

$$\alpha_{22} = K_0(R_D\sqrt{s}) \quad (3.19)$$

$$\alpha_{23} = -K_0(R_D\sqrt{s}\eta) \quad (3.20)$$

$$\alpha_{31} = \lambda\sqrt{s}I_0(R_D\sqrt{s}) \quad (3.21)$$

$$\alpha_{32} = -\lambda\sqrt{s}K_1(R_D\sqrt{s}) \quad (3.22)$$

$$\alpha_{33} = \sqrt{s}\eta K_1(R_D\sqrt{s}\eta) \quad (3.23)$$

$$\eta = \frac{(k/\phi\mu c_t)_1}{(k/\phi\mu c_t)_2} \quad (3.24)$$

$$\lambda = \frac{(k/\mu)_1}{(k/\mu)_2} \quad (3.25)$$

$$r_D = \frac{r}{r_w} \quad (3.26)$$



$$R_D = \frac{R}{r_w} \quad (3.27)$$

$$t_D = 0.0002637 \times \frac{k_1 t}{\phi_1 \mu_1 c_{t1} r_w^2} \quad (3.28)$$

$$C_D = 5.615 \times \frac{C}{2\pi \phi_1 h c_{t1} r_w^2} \quad (3.29)$$

$$P_{D1} = 0.001127 \times \frac{2\pi k_1 h}{qB\mu_1} (p_i - p_1) \quad (3.30)$$

$$P_{D2} = 0.001127 \times \frac{2\pi k_1 h}{qB\mu_1} (p_i - p_2) \quad (3.31)$$

$$P_{wD} = 0.001127 \times \frac{2\pi k_1 h}{qB\mu_1} (p_i - p_w). \quad (3.32)$$

The solution in Laplace space is then brought to the real space by numerical inversion using the Stehfest algorithm.

Fig 3.3 shows an excellent agreement between the analytical solution and the numerical simulation.

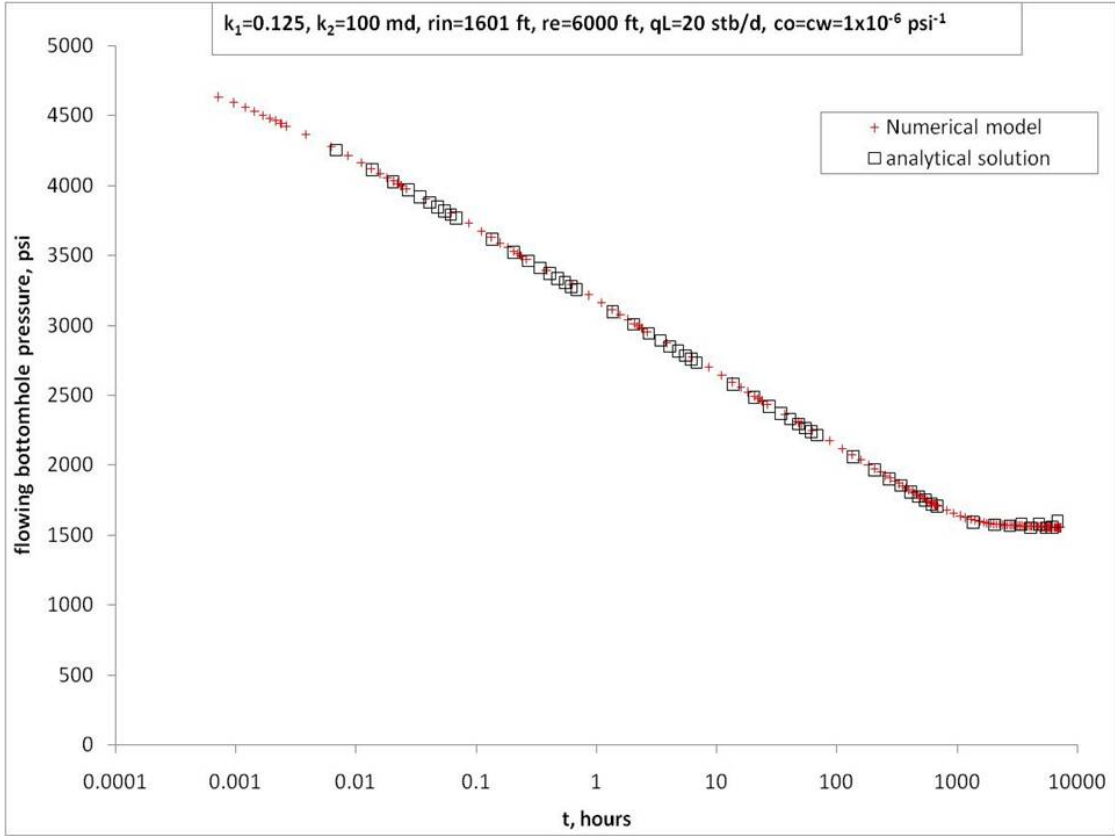


Figure 3.3: Comparison of the numerical model and the analytical solution for a composite reservoir

### 3.3 Results and Discussion

#### 3.3.1 First Scenario

The model is run for two cases. Case one is the study of low permeability in the inner zone, while case two considers high permeability in the inner zone. For all the runs, pressure vs. Horner time ratio and the pressure derivative vs. equivalent time ( $\Delta t_e$ ) were plotted.

The pressure derivative for pressure buildup is calculated using the algorithm suggested by *Bourdet et al. (1989)*,

$$\frac{d\Delta p_{ws}}{d\Delta t_e} = \frac{A + B}{C} \quad (3.33)$$

where

$$A = \left[ \frac{\Delta p_{ws2} - \Delta p_{ws1}}{\log \Delta t_{e2} - \log \Delta t_{e1}} \right] (\log \Delta t_{e3} - \log \Delta t_{e2}) \quad (3.34)$$

$$B = \left[ \frac{\Delta p_{ws3} - \Delta p_{ws2}}{\log \Delta t_{e3} - \log \Delta t_{e2}} \right] (\log \Delta t_{e2} - \log \Delta t_{e1}) \quad (3.35)$$

$$C = (\log \Delta t_{e3} - \log \Delta t_{e1}) \quad (3.36)$$

and  $\Delta t_e$  is the equivalent time, and it is calculated as follows:

$$\Delta t_e = \frac{t_p \Delta t}{t_p + \Delta t} = \frac{\Delta t}{1 + \frac{\Delta t}{t_p}} \quad (3.37)$$

In the above equation,  $t_p$  is the producing time prior to shut-in.

For the drawdown test, the pressure derivative is calculated by replacing  $\Delta p_{ws}$  with  $\Delta p_{wf}$  and replacing  $\Delta t_e$  with  $t$  in Eq 3.33.

Pressure plots and pressure derivative plots for the composite system with different inner and outer zone permeabilities are shown in Fig 3.4 and Fig 3.5, respectively.

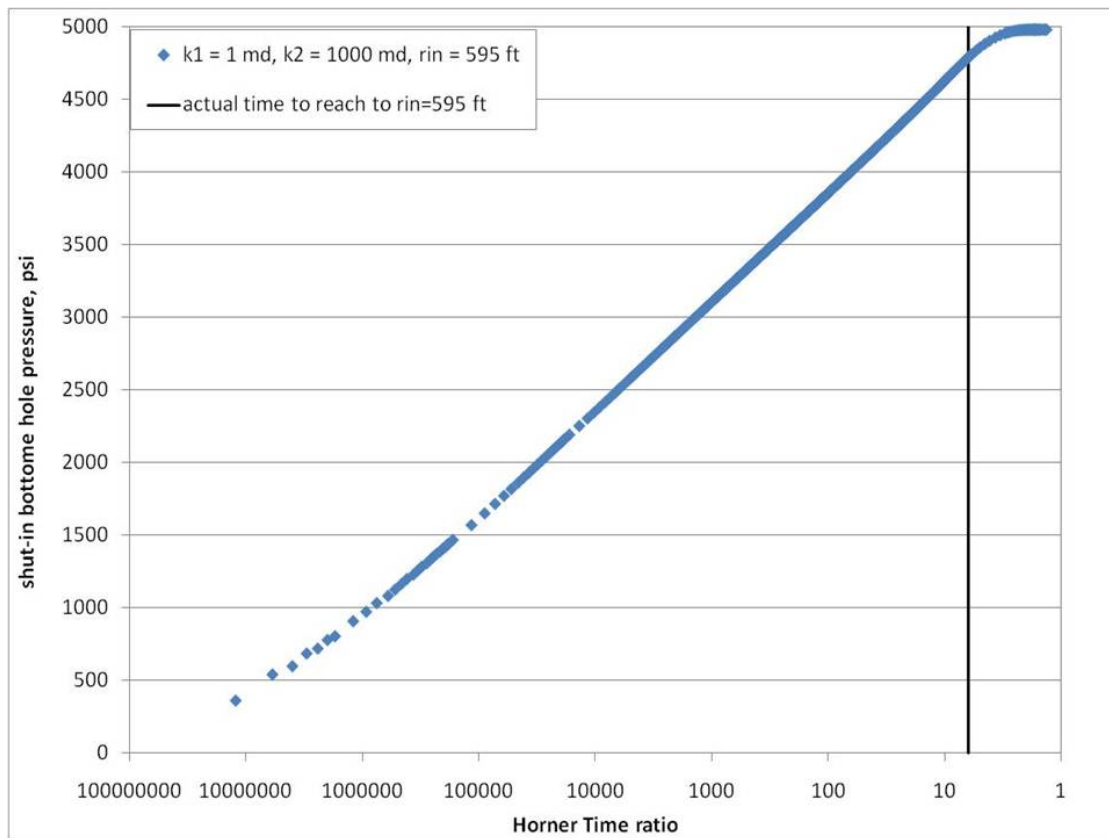
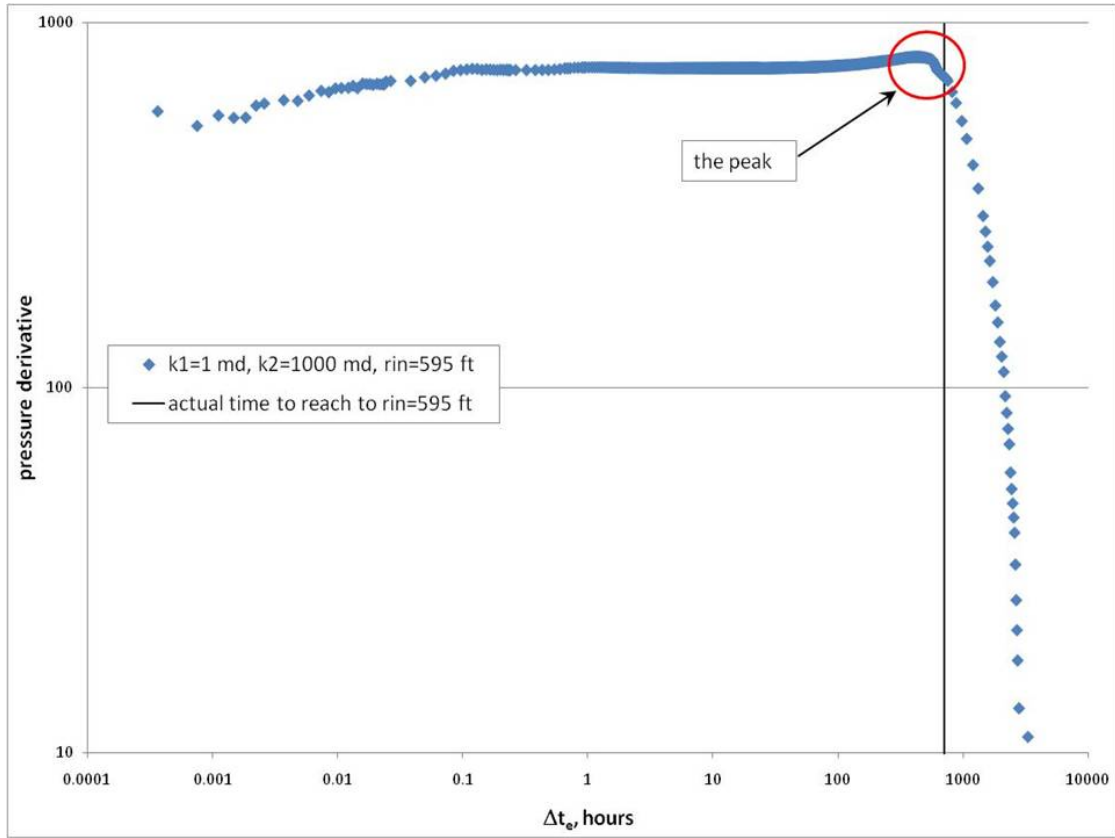


Figure 3.4: Horner plot for a composite reservoir with the smaller permeability in the inner zone



**Figure 3.5: Log-log plot of pressure derivative for a composite reservoir with the smaller permeability in the inner zone**

The plots of the pressure derivative for the buildup data exhibit a peak which is displayed at the end of the horizontal line representing the infinite acting period. This peak appears before the pressure derivative tends towards zero (Fig 3.5). The peak reflects the transition period between the two zones. The reason for the pressure derivative to exhibit a peak is that the pressure transient is traveling from the low mobility zone to the high mobility zone at the discontinuity, and because the flow rate is the same at the discontinuity, the pressure drop in the lower mobility zone needs to increase to deliver the same flow rate that is delivered from the higher mobility zone. On the higher mobility zone, the pressure drop needs to be decreased to deliver the same flow rate which is delivered from the lower mobility zone.

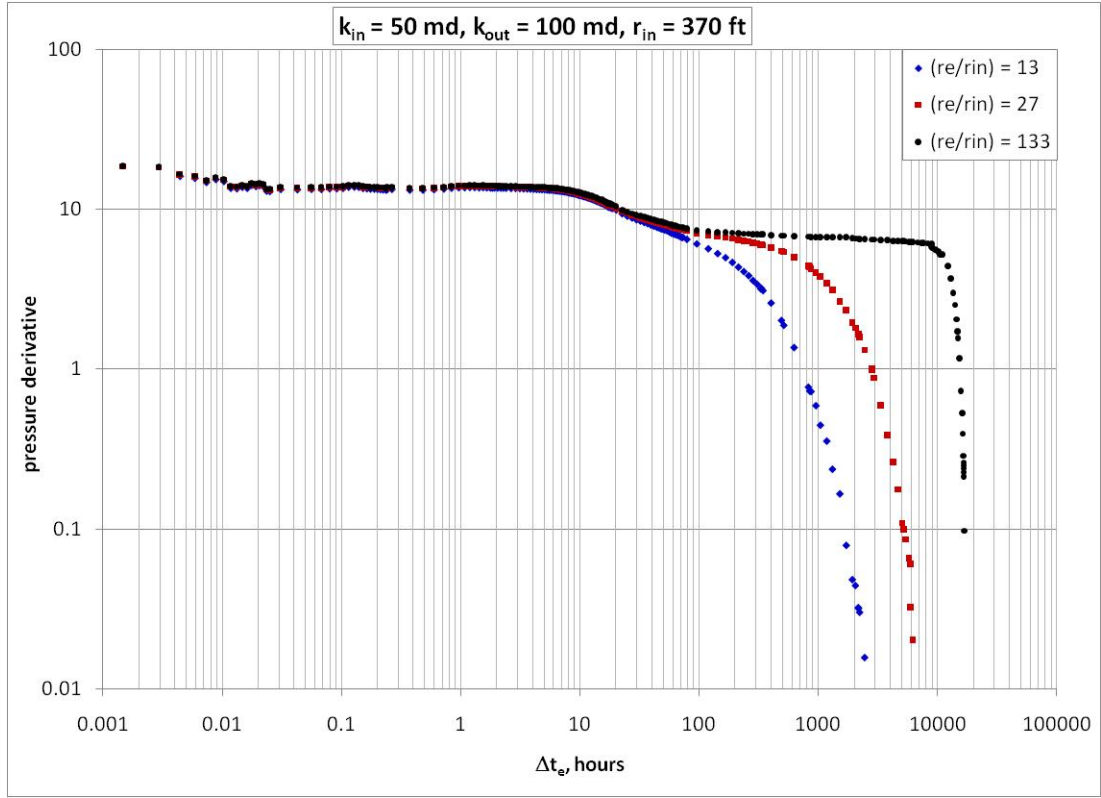
The pressure derivative for a composite system should show two horizontal lines representing characteristics of the two zones, but in Fig 3.5 there is only one horizontal line representing the inner zone. The second horizontal line is not observed or did not develop because of three reasons [*Ambastha and Ramey (1989)*]:

1. The reservoir is not large enough as the  $(r_e/r_{in})$  ratio needs to be very large (see Fig 3.6). *Ambastha and Ramey (1989)* suggested the following condition to observe the second horizontal line in the pressure derivative:

$$\frac{r_e}{r_{in}} > \sqrt{450[1 + \log(F_S)]F_S} \quad (3.38)$$

where  $F_S$  is the storativity ratio defined as

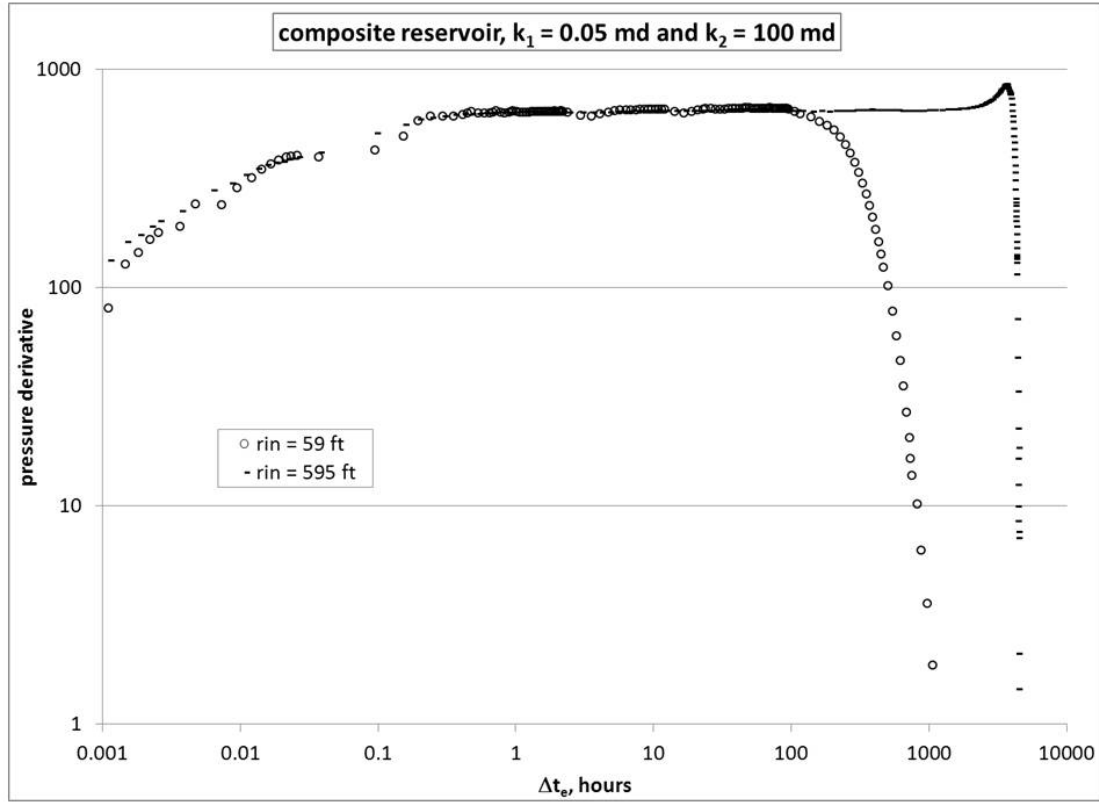
$$F_S = \frac{(\phi c_t)_1}{(\phi c_t)_2}. \quad (3.39)$$



**Figure 3.6: Effect of the composite reservoir's size on the development of the second horizontal line in pressure derivative plot**

2. The outer boundary effects could mask the second horizontal line.
3. The transition period is so long which requires the well test to run for a long enough time to observe the second line.

This peak in many plots takes place at a time which can be used in calculating the distance to the discontinuity. The peak is clearly observed in most cases and in some cases it is not easily recognizable (Fig 3.7).



**Figure 3.7:** Pressure derivative for two cases and the peak is noted explicitly

This study aims at verifying if there is a relation between  $r_{in}$  and the peak. The time at the peak ( $t_{(peak)}$ ) of the pressure derivative is used to calculate the radius of investigation from the following equation:

$$r_{inv} = \sqrt{\frac{k\Delta t}{948\phi\mu c_t}}. \quad (3.40)$$

In order to remove any confusion and to relate the names of the parameters to our study,  $r_{inv}$  will be changed to  $r_{(peak)}$  and  $\Delta t$  to  $t_{(peak)}$ . Then Eq (3.40) will change to

$$r_{(peak)} = \sqrt{\frac{kt_{(peak)}}{948\phi\mu c_t}}. \quad (3.41)$$



$r_{(peak)}$  was compared to  $r_{in}$  by calculating the absolute error percentage as follows:

$$Error(\%) = \frac{|r_{in} - r_{(peak)}|}{r_{in}} \times 100. \quad (3.42)$$

Also, Eq (3.40) was re-arranged to solve for  $t_{in}$ , as follows:

$$t_{in} = 948 \frac{r_{in}^2 \phi \mu c_t}{k}. \quad (3.43)$$

The previous procedures were repeated for different permeability combinations and the results are shown in Table 3.2.

**Table 3.2: The different permeability combinations used with the corresponding percentage error**

	$r_{in}$	$k_1$	$k_2$	$t_{(peak)}$	$r_{(peak)}$	Error
	ft	md	md	Hours	ft	%
1	59	0.01	1,000	528	47.2	20
2	59	0.05	100	109.13	47.98	18.67
3	59	1	100	5.5	48.17	18.40
4	59	1	1,000	5.5	48.17	18.40
5	59	10	1,000	0.18055	27.6	53.22
6	221	0.01	1,000	15,120	252.58	14.30
7	221	0.05	1,000	1,968	203.76	7.80
8	221	0.05	100	1,968	203.76	7.80
9	221	0.05	10	1,968	203.76	7.80
10	221	0.1	100	804	184.18	16.66
11	221	0.1	1,000	804	184.18	16.66
12	221	0.5	100	133.94	168.1	23.94
13	221	0.5	1,000	133.94	168.1	23.94
14	221	1	1,000	71.127	173.24	21.61
15	221	1	100	71.127	173.24	21.61
16	595	0.01	1,000	109,992	681.25	14.50
17	595	0.05	100	19,680	644.35	8.29
18	595	0.05	1,000	19,680	644.35	8.29

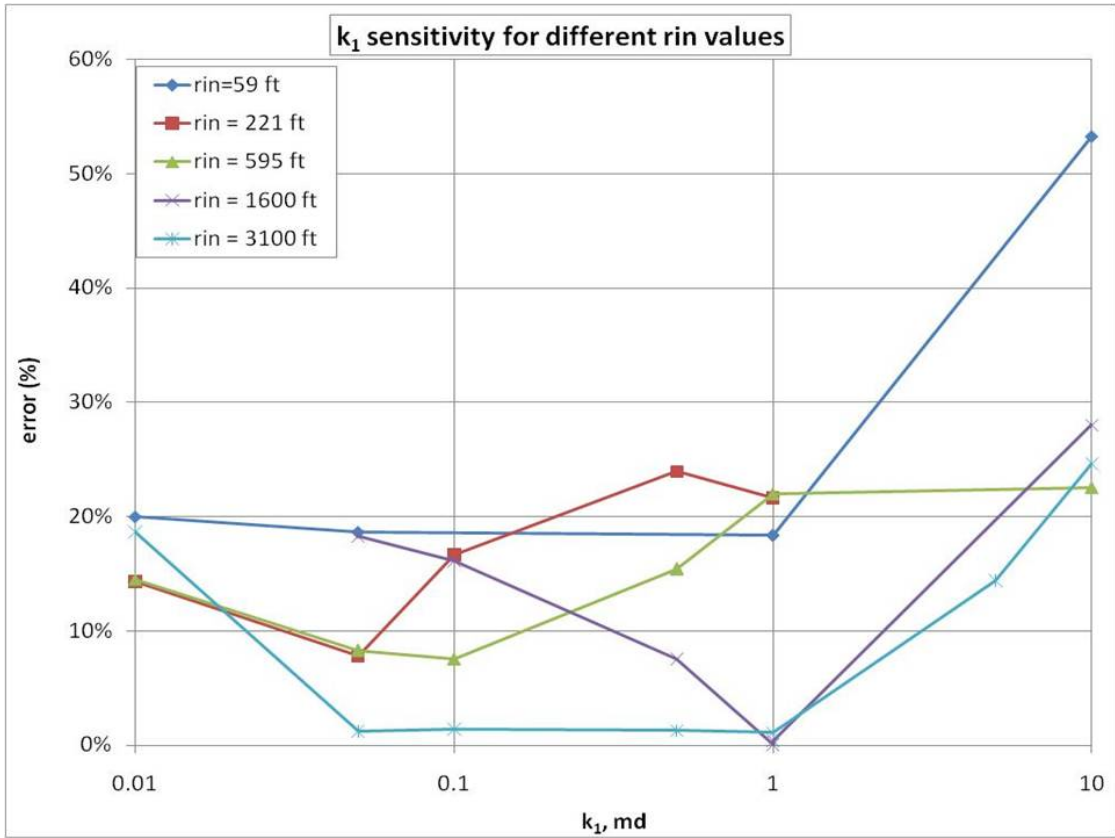
**Table 3.2: (Continued)**

	$r_{in}$	$k_1$	$k_2$	$t_{(peak)}$	$r_{(peak)}$	Error
	ft	md	md	Hours	ft	%
19	595	0.1	1,000	9,696	639.62	7.50
20	595	0.1	100	9,696	639.62	7.50
21	595	0.5	1,000	1,200	503.15	15.44
22	595	0.5	100	1,200	503.15	15.44
23	595	0.5	10	1,200	503.15	15.44
24	595	1	1,000	504	461	22
25	595	1	10	504	461	22
26	595	10	1,000	50.31	460.72	22.57
27	1,600	0.05	100	81,073.7	1,307.83	18.26
28	1,600	0.05	1,000	81,073.7	1,307.83	18.26
29	1,600	0.1	100	81,792	1,857.72	16.11
30	1,600	0.1	1,000	81,792	1,857.72	16.11
31	1,600	0.5	100	14,040	1,721	7.57
32	1,600	0.5	1,000	14,040	1,721	7.57
33	1,600	1	1,000	6,072	1,600.63	1.80
34	1,600	1	100	6,120	1,606.9	1.80
35	1,600	10	1,000	314.4	1,151.8	28
36	3,100	0.01	1,000	3,204,768	3,677.26	18.62
37	3,100	0.05	100	466,704	3,137.85	1.22
38	3,100	0.1	1,000	234,144	3,143.17	1.39
39	3,100	0.5	1,000	44,400	3,060.57	1.27
40	3,100	1	1,000	23,280	3,134.13	1.10
41	3,100	5	1,000	3,336	2,652.9	14.42
42	3,100	10	100	1,296	2,338.45	24.60

Table 3.2 shows that the percentage error for  $r_{in} = 59$  ft is always greater than 18% unlike the other cases with longer distances, where the percentage error sometimes is less than 10%. This observation is attributed to that short distance as the pressure transient travels faster through smaller areas, and the time resolution to

capture the pressure transient data is not fine enough to show the details of the pressure transient behavior, and show the time for the peak accurately.

For all the  $r_{in}$  values, the percentage error goes through a minimum (Fig 3.8). The table also shows that changing  $k_2$  does not affect the error. For  $r_{in} = 3,100$  ft, the error goes through a minimum, but the minimum occurs for more than one value of  $k_1$ . In general, for all  $r_{in}$  values for a given  $t_p$  value, the error goes through a minimum for all the different values of  $k_1$  as displayed in Fig 3.8.



**Figure 3.8:** Generated errors in calculating  $r_{(peak)}$  for different  $r_{in}$  and different  $k_1$  using pressure buildup test data ( $t_p = 184$  days).

For all the previous simulation runs, the buildup test was performed after a producing time ( $t_p$ ) of 184 days (4416 hours). A sensitivity test was run to study the effect of  $t_p$  on the estimated  $t_{in}$ . Fig 3.9 and Fig 3.10 show the results of this sensitivity study.

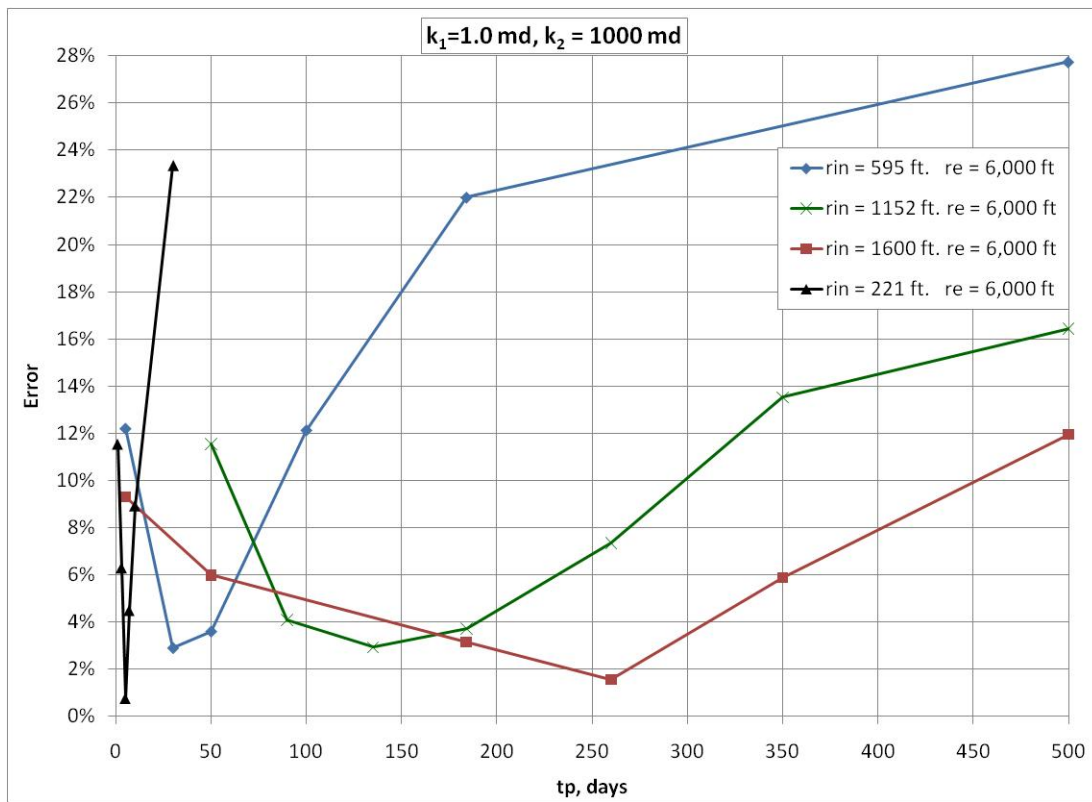
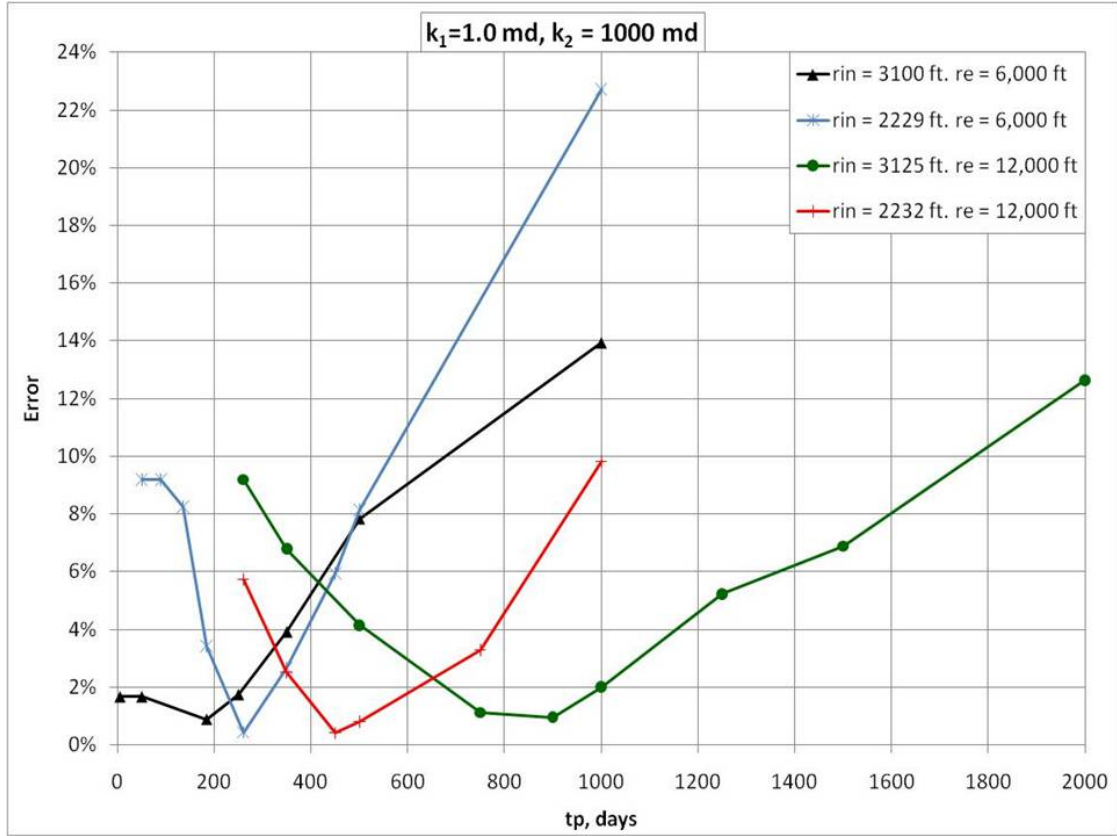


Figure 3.9: Error resulting from changing the  $t_p$  for the buildup test for different  $r_{in}$ .



**Figure 3.10: Error resulting from changing the  $t_p$  for the buildup test for different  $r_{in}$  and  $r_e$ .**

Fig 3.9 shows that the error goes through a minimum at a certain  $t_p$  value. For 221 ft, 595 ft, 1,152 ft, and 1,600 ft, the minimum error occurs at a  $t_p$  value close to the value of  $t_{in}$ .

In Fig 3.10, when  $r_{in}$  is 2,229 ft or 3,100 ft for  $r_e$  equals 6,000 ft, the minimum error is found to be at a time far from  $t_{in}$ . When  $r_e$  was increased to 12,000 ft, the minimum error was at a  $t_p$  value close to  $t_{in}$  (table 3.3). This indicates that the value of  $t_p$  required to achieve the minimum error in calculating  $r_{(peak)}$ , will be close to  $t_{in}$  if the  $(r_e/r_{in})$  ratio is greater than 3.

The effect of  $t_p$  on the estimated  $r_{(peak)}$  is tabulated in Table 3.3.

**Table 3.3: Effect of the reservoir radius ( $r_e$ ) on the minimum error for  $r_{(peak)}$**

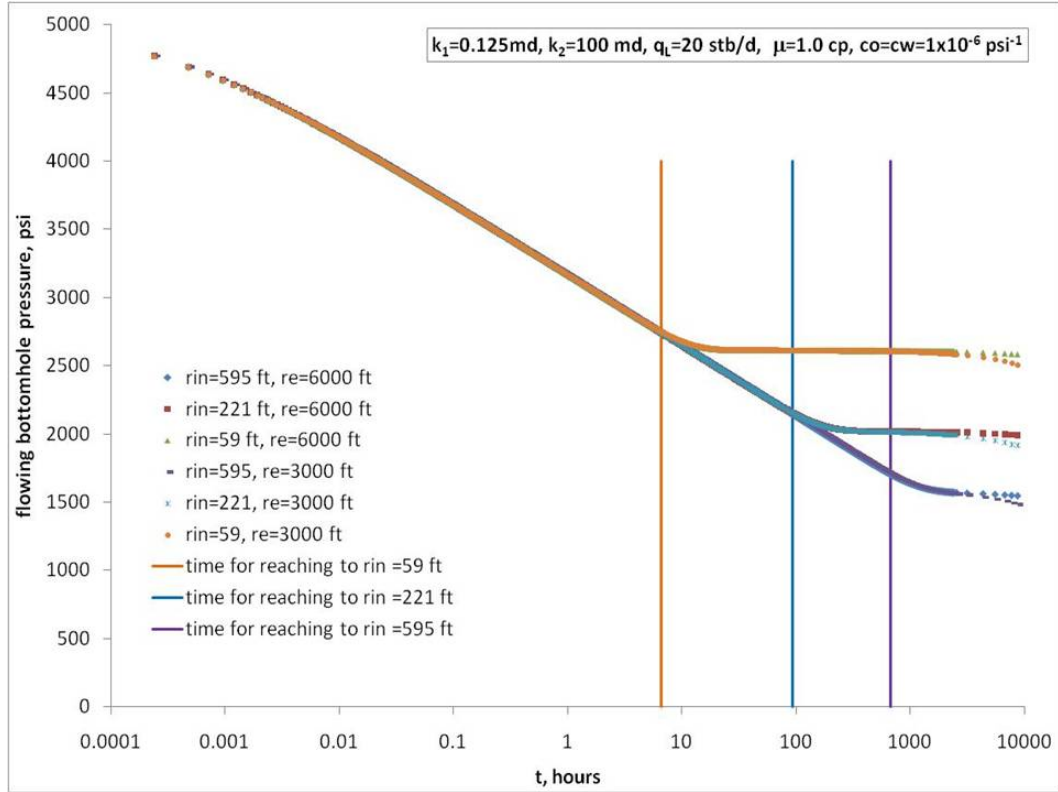
No.	$r_{in}$	$r_e$	$r_e/r_{in}$	$t_{in}$	$t_p$ with minimum error
	ft	ft		days	days
1	221	6,000	27	5	5
2	595	6,000	10	35	30
3	1,152	6,000	5.2	131	135
4	1,600	6,000	3.75	253	260
5	2,229	6,000	2.69	491	260
6	3,100	6,000	1.9	949	184
7	2,232	12,000	5.38	492	450
8	3,125	12,000	3.84	964	900

Note,  $t_{in}$  in Table 3.3 was calculated using Eq (3.43).

The behavior in Fig 3.8 is difficult to be explained, because two parameters are changing in the same time ( $k_1$ , and  $t_p$ ), where for each  $k_1$  value, there is a single value of  $t_p$  that gives a minimum error for estimating  $r_{peak}$ . The behavior observed in Fig 3.9 is due to the change in one parameter,  $t_p$ , and the reason for such a behavior can be related to the distance to where the pressure transient has reached. For  $t_p$  less than or greater than  $t_{in}$ , the pressure transient, during the flow period, moves to a distance less than  $r_{in}$ , and stops there and creates somewhat a boundary layer. The pressure buildup data will then capture this kind of a boundary layer effect as an irregular behavior which will affect the peak by changing the time for its occurrence. If  $t_p$  equals  $t_{in}$ , the pressure drawdown stops at the same distance to the discontinuity, and the response of both effects will be at the same distance and at the same time, which will produce the most accurate estimation for  $r_{peak}$ .

In the pressure drawdown tests for the case when the smaller permeability (or the smaller mobility) is assigned for the inner zone, as shown in Fig 3.11, a unique behavior in the drawdown plots was observed. Two straight lines can be clearly seen for each case. The time when the pressure transient reaches  $r_{in}$  is at the end

of the first line and the beginning of the second line.  $t_{in}$  is indicated by the three vertical lines in the figure for the three cases.



**Figure 3.11: Semi-log plot for pressure drawdown for composite reservoir with the lower permeability in the inner region.**

Fig 3.11 shows 6 cases; The first 3 cases for reservoirs with  $r_e = 6,000$  ft and  $r_{in}$  equals to 595 ft for the 1<sup>st</sup> case, 221 ft for the 2<sup>nd</sup> case and 59 ft for the 3<sup>rd</sup> case. Each of the previous 3 cases were repeated but with  $r_e = 3,000$  ft. The figure shows a match between the cases with equal  $r_{in}$  value, except that after  $t = 1,000$  hours, which shows the effect of the outer boundary for the cases with  $r_e = 3,000$  ft. This proves that the second straight line is related to the outer zone.

The pressure derivative plot (Fig 3.12) shows a horizontal line and at the end of the horizontal line is the time when the pressure response reaches the discontinuity. The vertical lines indicate the time at which the pressure response reaches  $r_{in}$ . Also, the plot shows that the outer boundary has been reached by identifying the unit slope line.

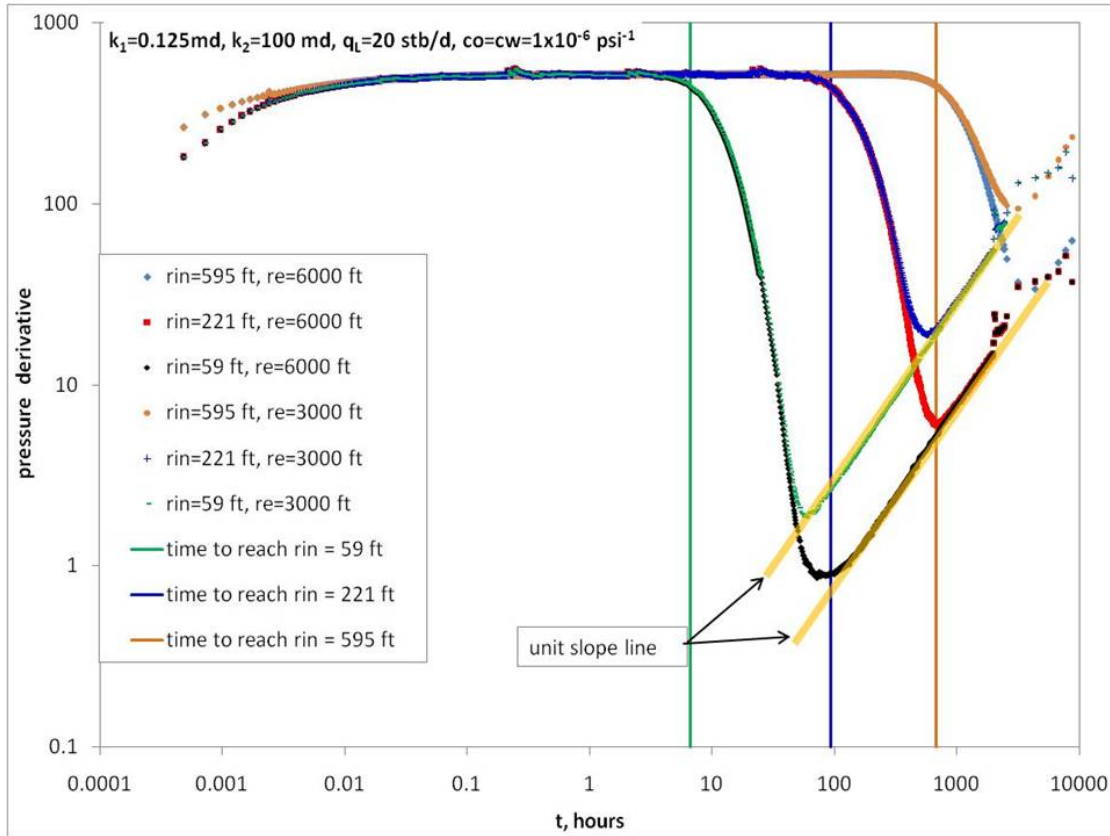


Figure 3.12: Semi-log plot for pressure derivative for composite reservoir with the smaller permeability in the inner region.

### 3.3.2 Second Scenario

For this scenario, single permeability was assigned to the reservoir. The model is run for four cases as follows:

Case (i) : Low viscosity oil is placed in the inner zone and high viscosity oil is placed in the outer zone.

Case (ii) : High viscosity oil is placed in the inner zone and low viscosity oil is placed in the outer zone.

Case (iii) : Water is placed in the inner zone and high viscosity oil is placed in the outer zone.



Case (iv) : High viscosity oil is placed in the inner zone and water is placed in the outer zone.

Properties of the three fluids utilized in this exercise are shown in Table 3.4.

**Table 3.4: Fluid properties for the second scenario.**

	water	Low Viscosity	High Viscosity
FVF (bbl/STB)	1.0	1.0	1.0
Viscosity (cp)	1.0	1.0	800
Compressibility ( $\text{psi}^{-1}$ )	$1 \times 10^{-5}$	$1 \times 10^{-5}$	$1 \times 10^{-5}$
Density ( $\text{lb/ft}^3$ )	62.4	48	48

The plot of the pressure and pressure derivative functions for cases (i) and (ii) are shown in Fig 3.13 and Fig 3.14, respectively.

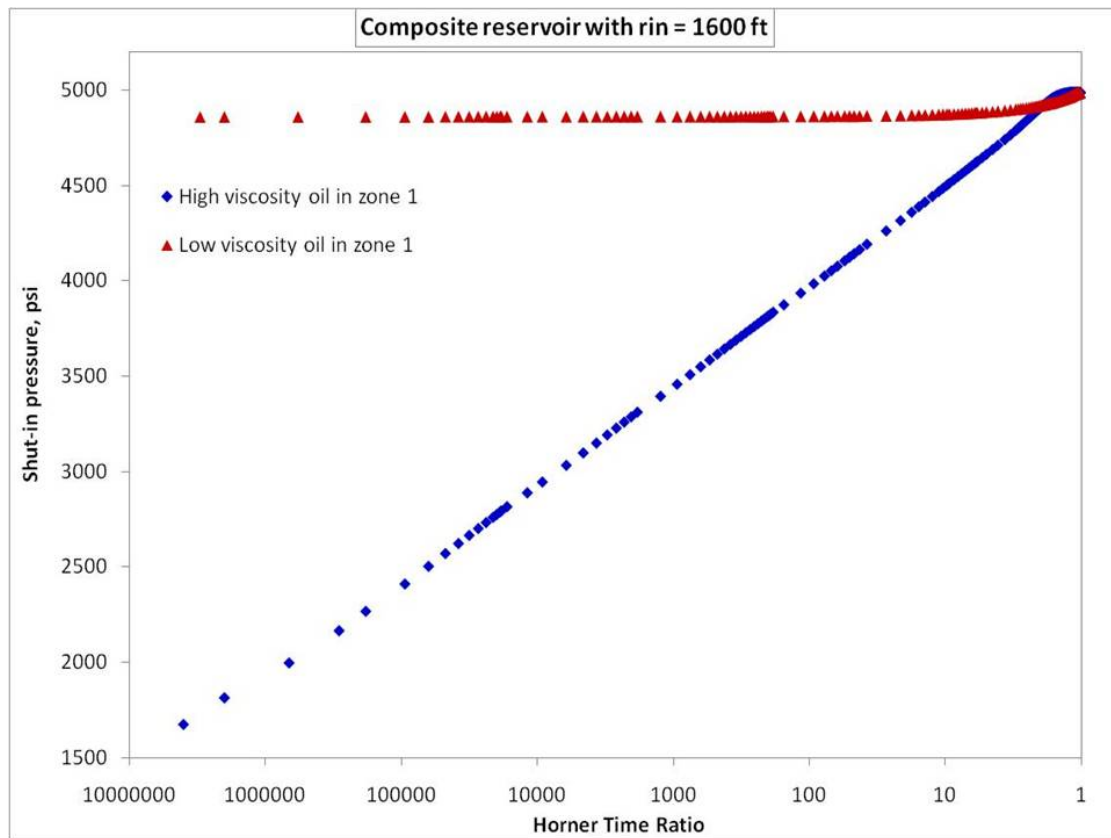
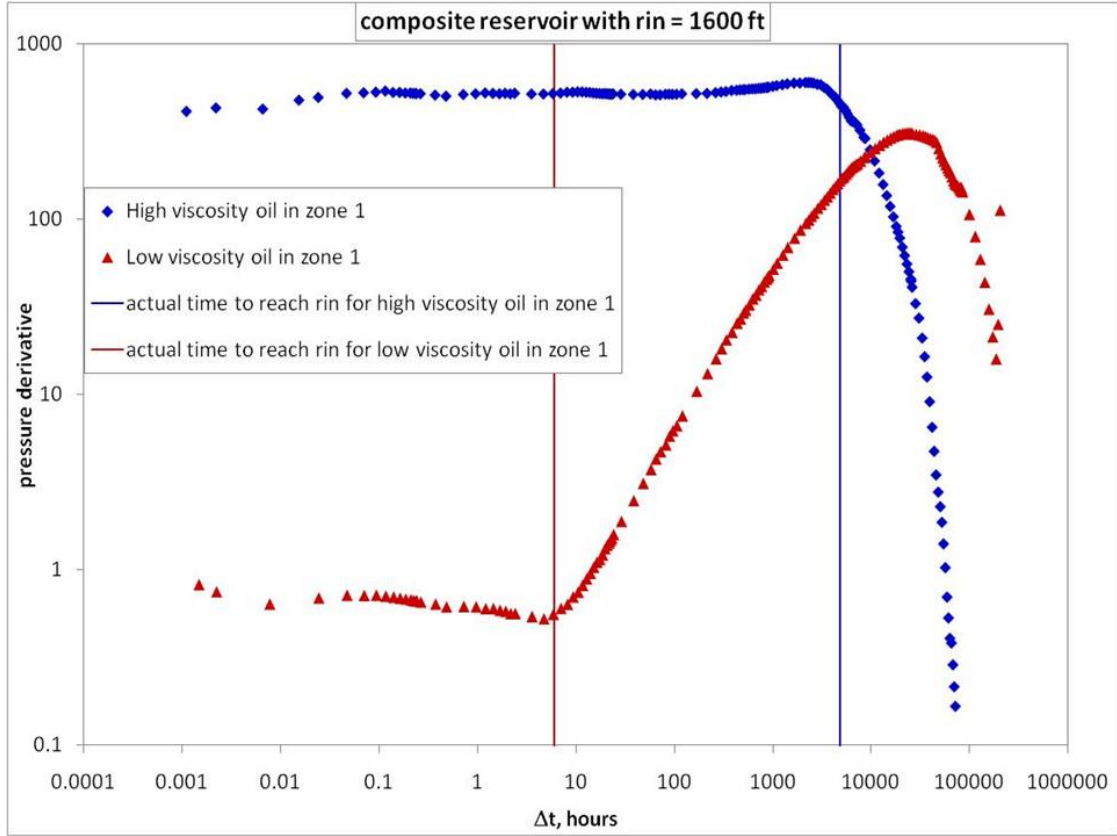


Figure 3.13: Horner plot for the second scenario for cases (i) and (ii).



**Figure 3.14: Pressure derivative plot for the second scenario for cases (i) and (ii).**

For Case (i) (lower viscosity in inner zone), the behavior is different than Case (ii) and the peak is not related to  $r_{in}$  as it can be seen from Fig 3.14 where  $t_{in}$  is 6.07 hours and  $t_{(peak)}$  equals to 24,408 hours. The time indicated by  $t_{(peak)} = 24,408$  hours is more related to the outer boundary.

For Case (ii) (higher viscosity in inner zone), the behavior is similar to the first scenario when the lower permeability was assigned to the inner zone. Both cases have lower mobility in the inner zone.  $t_{(peak)}$  was used to calculate  $r_{(peak)}$  and then the error percentage was calculated and found it to be 14.45%. Numbers of additional cases were run for different permeability and viscosity values, and the results are tabulated in Table 3.5

**Table 3.5: Simulation runs for different values of permeabilities and viscosities for the second scenario (Case (ii)).**

No.	$r_{in}$	$k$	$\mu_1$	$\mu_2$	$t_{(peak)}$	$r_{(peak)}$	Error
	ft	md	cp	cp	hours	ft	(%)
1	1,600	10	800	1	45,984	1,557	2.67
2	1,600	50	800	1	6,960	1,355	15.33
3	1,600	50	800	0.5	6,960	1,355	15.33
4	1,600	100	800	1	3,552	1,369	14.45
5	1,600	100	800	2	3,288	1,317	17.69
6	1,600	200	800	1	1,368	1,201	24.92
7	1,600	500	800	1	440.4	1,078	32.65
8	1,600	1,600	800	1	114.5	983	38.56
9	1,600	50	1,600	1	16,968	1,496	6.51
10	1,600	100	1,600	1	7,680	1,423	11.05
11	1,600	100	1,600	2	6,960	1,355	15.33
12	1,600	200	1,600	2	6,960	1,916	19.75
13	1,600	50	3,000	1	34,400	1,555	2.79
14	1,600	100	3,000	1	18,900	1,630	1.90
15	1,600	500	12,800	1	13,440	1,488	6.98
16	1,600	1,600	12,800	1	3,456	1,350	15.62
17	221	100	800	1	28.15	122	44.87
18	595	100	800	1	274	380	36.11
19	595	100	3,000	1	1,560	468	21.28
20	595	100	8,000	1	5,160	522	12.32
21	595	100	12,800	1	9,000	545	8.46
22	3,100	100	800	1	14,280	2,744	11.47

Note that we observe that as permeability increases, error increases. Another observation related to the viscosity, is that as  $\mu_1$  increases, the error decreases. In other words, as the mobility in the inner zone decreases, the error decreases. Also, as  $r_{in}$  increases, the error decreases. For cases (iii) and (iv), the results were similar to the first two cases as the comparison shown in Fig 3.15 and Fig 3.16.

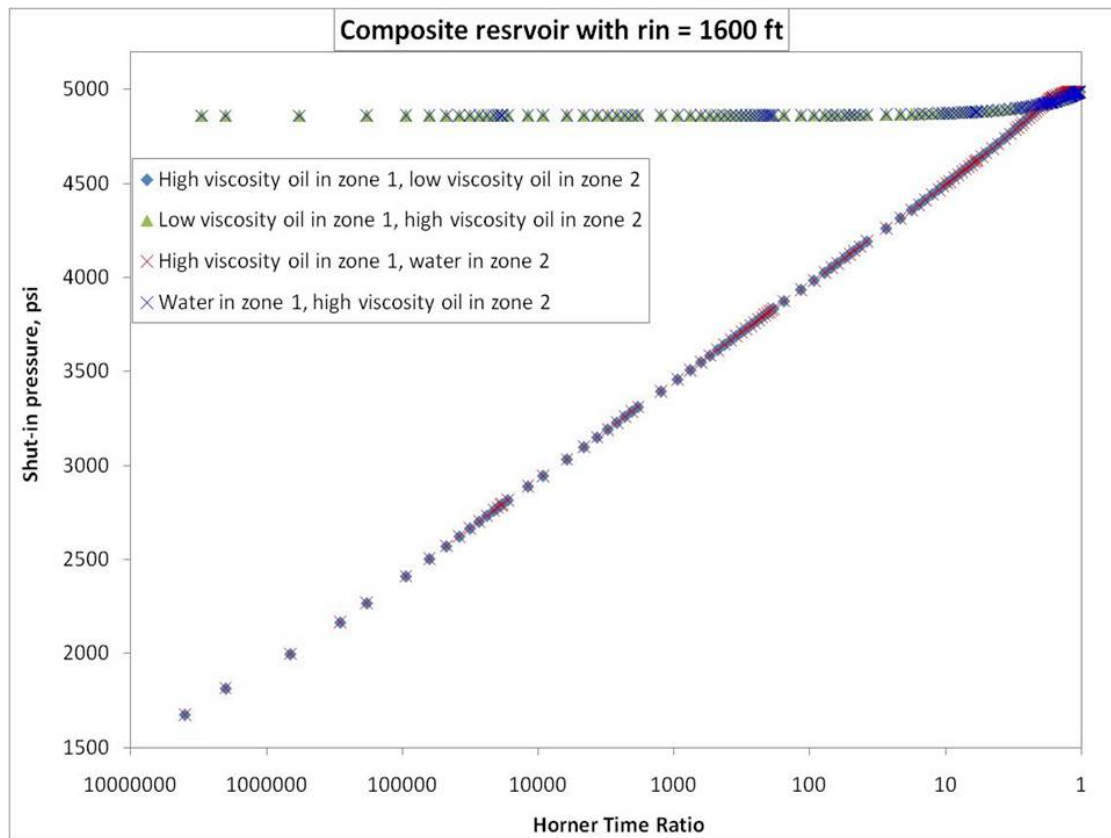
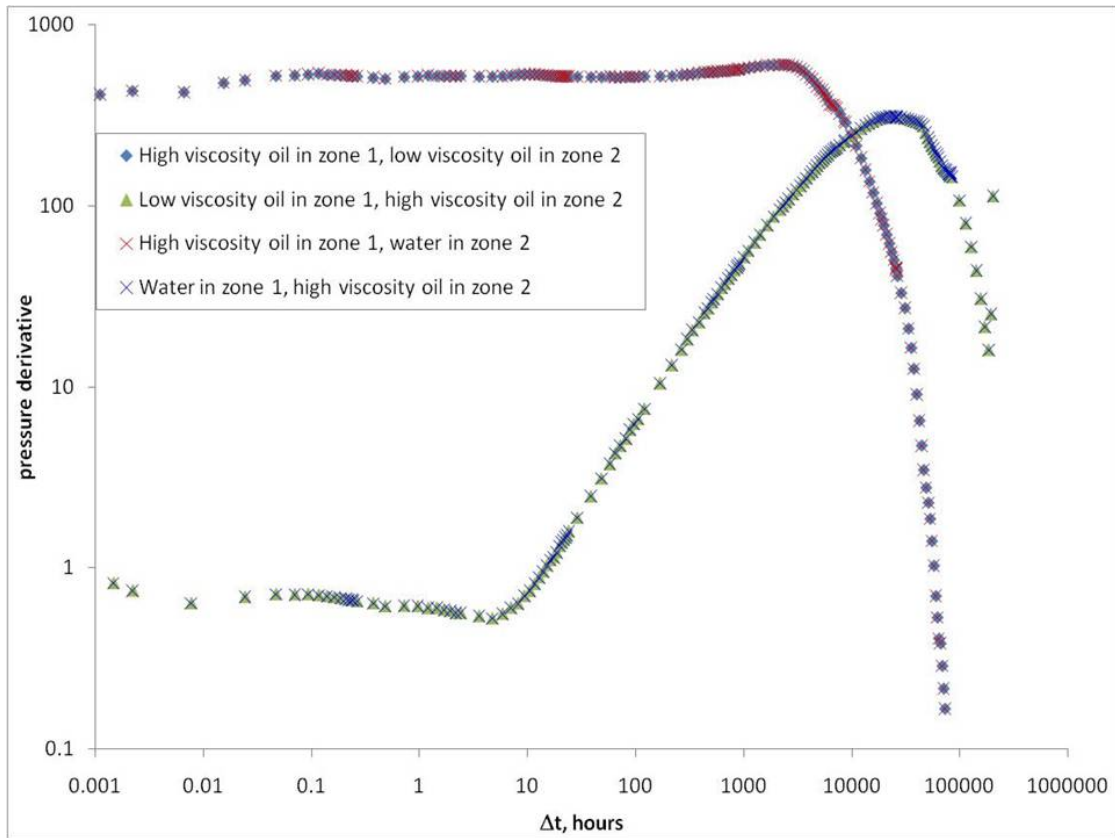


Figure 3.15: Horner plot for the second scenario for all cases.



**Figure 3.16:** Pressure derivative plot for the second scenario for all cases.

Table 3.6 shows additional cases for reservoirs with the high viscosity oil in the inner zone and the water in the outer zone.

**Table 3.6: Simulation runs for different values of permeabilities and viscosities for the second scenario (Case (iv)).**

No.	$r_{in}$	$k$	$\mu_1$	$\mu_2$	$t_{(peak)}$	$r_{(peak)}$	Error
	ft	md	cp	cp	hours	ft	(%)
1	1,600	100	3,000	1	16,008	1,500	6.22
2	1,600	10	800	1	46,944	1,574	1.66
3	1,600	50	1,600	1	17,232	1,507	5.79
4	1,600	500	12,800	1	13,584	1,496	6.48
5	595	100	12,800	1	9,072	547	8.09
6	1,600	100	1,600	1	7,776	1,432	10.5
7	3,100	100	800	1	14,520	2,767	10.73
8	1,600	100	800	1	3,432	1,345	15.91
9	1,600	50	800	1	7,848	1,439	10.09
10	1,600	200	800	1	1,368	1,201	24.92
11	1,600	500	800	1	422	1,055	34.04

The results show that the distance to the discontinuity ( $r_{in}$ ) can be estimated accurately with certain conditions. The three factors affecting the accuracy of this estimations are;  $r_{in}$ ,  $(k/\mu)_1$ , and  $\lambda$ . For the reservoirs with  $r_{in} = 1,600$  ft, the estimation is accurate if the value of  $(k/\mu)_1$  is less than 0.06 and the value of  $\lambda$  is less than 0.00125. But for an accurate estimation for the reservoirs with  $r_{in} = 595$  ft, the value of  $(k/\mu)_1$  should be less than 0.007, which means as  $r_{in}$  decreases, the maximum value of  $(k/\mu)_1$ , which is required for accurate estimation, decreases.

# Water Coning Model

## 4.1 Models Development

### 4.1.1 Numerical Model

Data used to train the artificial neural networks were generated from a numerical reservoir simulation model. Two reservoir numerical models were used: radial and rectangular models. The reservoir properties are homogeneous and isotropic. In terms of fluid properties, the reservoir conditions are assumed to be above bubble-point pressure to ensure that no free gas is present in the reservoir. Furthermore, the capillary pressure was not considered. The reservoirs are horizontal with uniform thicknesses. The radial reservoir model was used to generate data for the vertical well, and the rectangular reservoir model was used to generate data for the horizontal well. The gridding for the radial system was  $r$ - $\theta$ - $z$ , where the number of grids were 30x1x25. The thickness of all the grid blocks are equal, and the radial grids were designed according to the following equation:

$$r_i = r_w \left( \frac{r_e}{r_w} \right)^{(i/30)}. \quad (4.1)$$

The rectangular reservoir gridding was 25x25x15 and  $\Delta x = \Delta y = 200$  ft. The reservoir properties for the radial and the rectangular systems are tabulated in the following tables.



**Table 4.1: Reservoir properties for the radial system.**

Porosity ( $\phi$ )	0.25
$k_r$ , md	500
Reservoir radius ( $r_e$ ), ft	6,000
Oil formation volume factor, RB/STB	1.0
Oil compressibility ( $c_o$ ), $\text{psi}^{-1}$	$1 \times 10^{-6}$
Initial pressure ( $p_i$ ), psi	5,000
Temperature, $^{\circ}\text{F}$	130
$S_{oi}$	1.00

**Table 4.2: Reservoir properties for the rectangular system.**

Porosity ( $\phi$ )	0.25
$k_x$ , md	500
$k_y$ , md	500
Reservoir length, ft	5,000
Reservoir width, ft	5,000
Oil formation volume factor, RB/STB	1.0
Oil compressibility ( $c_o$ ), $\text{psi}^{-1}$	$1 \times 10^{-6}$
Initial pressure ( $p_i$ ), psi	5,000
Temperature, $^{\circ}\text{F}$	130
$S_{oi}$	1.00

For the vertical well, 6 parameters were selected to be changed to create different scenarios. The parameters with their ranges are tabulated in Table 4.3.

**Table 4.3: The selected reservoir properties changed within their ranges for the vertical well study.**

No.	Parameter	Range
1	$\rho_o, lb/ft^3$	48 - 58
2	$\mu_o, cp$	1 - 10
3	$k_v, md$	5 - 500
4	$q_L, STB/Day$	500 - 10,000
5	$h, ft$	25 - 250
6	$h_p, ft$	$0.04h - 0.96h$

The parameters and their ranges for the horizontal well in the rectangular reservoir are tabulated in Table 4.4.

**Table 4.4: The selected reservoir properties changed within their ranges for the horizontal well study.**

No.	Parameter	Range
1	$\rho_o, lb/ft^3$	48 - 58
2	$\mu_o, cp$	1 - 10
3	$k_v, md$	5 - 500
4	$q_L, STB/Day$	500 - 10,000
5	$h, ft$	15 - 240
6	$h_d, ft$	$0.067h - 0.53h$
7	$h_L, ft$	600 - 3400

## 4.1.2 Artificial Neural Networks

### 4.1.2.1 Artificial Neural Networks

Artificial neural networks (ANN) are mathematical networks which are used to simulate our understanding of the biological neural networks in the human brain. When the human brain receives information from the outside environment, it will learn from it, and then make its own decisions or predictions based on the received information. The structure of the ANN is based on the structure of the neural system in the human's brain. The idea of the ANN is to have a network that can change its structure based on information fed from the outside environment. The network will learn from these information and change the structure accordingly.

The human brain contains billions of neurons that process and transmit information by chemical and electrical signals. Each neuron can send signals to many neurons and also can receive signals from many neurons and this is how the neurons are interconnected and form a network.

A simple artificial neural network consists of nodes, which can be called neurons, connected together and forming a network.

The basic structure of an artificial neural network can be explained by explaining a simple neuron model (Fig 4.1) which is the basic element of an artificial neural network.

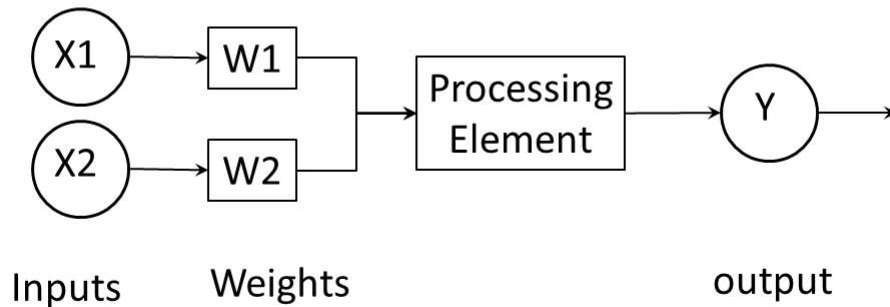


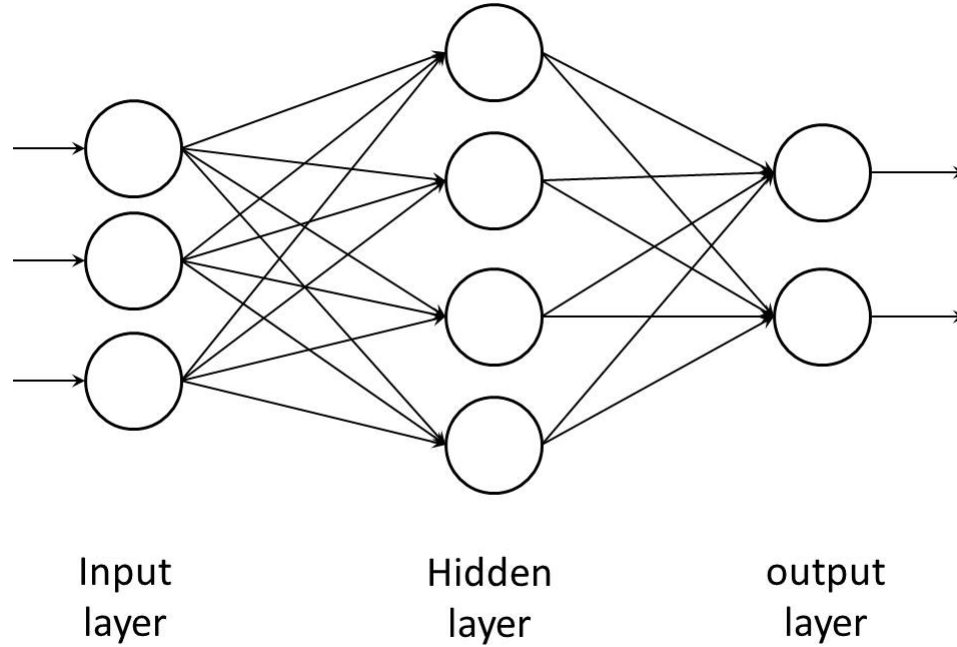
Figure 4.1: A simple model of a neuron.

Each input in Fig 4.1 is multiplied by a weighted factor, which can be adjusted according to the received information. Then, the multiplied inputs with their weights are processed through a processing element which is a transfer function, usually a non-linear function, to produce the output. This process is repeated in a process called "training" to adjust the weights and to enable the network to produce the desired output. At the beginning of the training process, the ANN will not be able to produce the desired outputs accurately, but repeating the training process results in changing the weights of the network to produce more accurate outputs, which indicates that the ANN had learned how to improve its predictions for the outputs, which is called "learning".

There are two main types of neural networks; supervised networks and unsupervised networks. The supervised networks are provided with pairs of inputs and outputs data in the training process. During the training process, the weights will be adjusted to reduce the error between the desired outputs and the neural network outputs. Unlike the supervised networks, only input data is provided for unsupervised networks. The training process of unsupervised networks will try to categorize the input data in a self-organizing behavior or try to find patterns in the input data.

There are many neural network structures, and in the petroleum industry, two types of neural networks are mainly used: 1) Feedforward multilayer perceptron network with back-propagation learning algorithm. This is the most heavily used, and 2) The self-organising network (*Ali (1994)*).

A feedforward multilayer perceptron basic structure consists of an input layer, a hidden layer and an output layer. It's called feedforward because all information flows in one direction from the input layer through the hidden layers to the output layer. Fig 4.2 shows a basic structure of a multilayer neural network with one hidden layer.



**Figure 4.2: Basic structure of a multilayer neural network.**

The backpropagation learning algorithm is an iterative algorithm which tries to minimize the error between the desired output and the neural network output by adjusting the weights of the network.

#### 4.1.2.2 Vertical Well

A total of 233 combinations were generated randomly (Table A.1). Each combination was used to create a reservoir model. All the reservoir models were run using numerical simulation for 10 years. The water saturation data for all the blocks which was generated from the simulation runs was collected and prepared for the artificial neural networks training process. The artificial neural network used for training is a feedforward network, which needs inputs and outputs. The inputs are the 6 parameters in Table A.1 and the outputs are the water saturation values for all the blocks in the reservoir model at the end of each year for 10 years. The 233 reservoirs were divided into three sets; 210 reservoirs for training, 11 reservoirs for validation, and 12 reservoirs for testing. Training and validation data are used in the training of the neural network, and the testing data are only introduced to the network after the end of training process to test the new neural network.

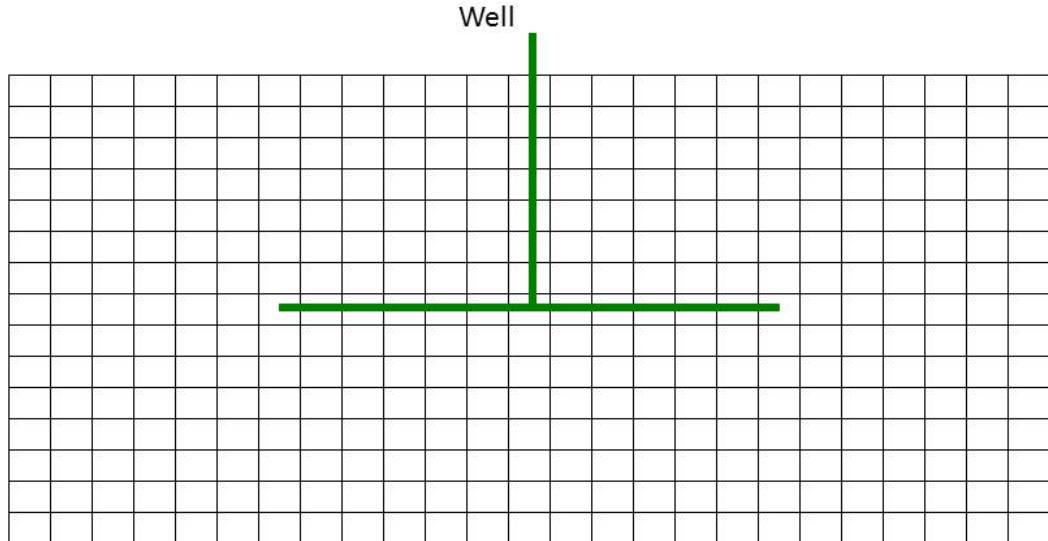
Training the neural network started with including all water saturation values for all the blocks of each reservoir, which will produce a single network to predict the water saturation for the whole reservoir. This did not result in a good network that can predict the water saturation values with good accuracy. The next trial was to reduce the amount of data to simplify the problem for the neural network, and at the same time not produce too many neural networks. The water saturation data considered was only for one year to be used for training; if it works, this means we will have 10 different neural networks for each year. This also didn't result in a good network that can predict water saturation with good accuracy. Therefore, the amount of data was reduced more to simplify the problem by taking the data for only one layer instead of the 25 layers, but this didn't result in a good network. Then again, the data was reduced by taking the data of a single layer and considering only the 16 blocks that are closer to the wellbore. Also, with this simplification, the resulting network was not able to predict the water saturation accurately. Then, again, the number of blocks were reduced from 16 to 6 blocks. This time, a good network was generated, and the absolute error was less than 10% for all the predicted water saturation values. The absolute error is calculated using the following equation,

$$Error = |S_w - S_w(ANN)|. \quad (4.2)$$

After succeeding in designing a good network, the goal now is to increase the complexity of the problem, and reduce the number of the networks needed to predict the water saturation for the reservoir. The complexity was increased by including the data for the 10 years, and not only for one year. The produced network was good. Increasing the number of blocks to 16 blocks was tried, but did not produce a good network. The results are 25 networks, where each network predicts the water saturation for each layer at the end of each year for 10 years.

#### 4.1.2.3 Horizontal Well

A total of 314 combinations were generated randomly (Table A.2), and for each combination, a reservoir was created. The horizontal section was always in the center in the  $j$ -direction for all the reservoirs. This created a symmetry, which will reduce the amount of data to be considered, and in return, will reduce the time needed to train the neural network. Fig 4.3 shows the  $x$ - $z$  plane for the rectangular reservoir system for  $j = 13$ . The water saturation of the blocks in the center column will be included in the output data, and the water saturation for the blocks on both sides will be identical, and either one can be considered.



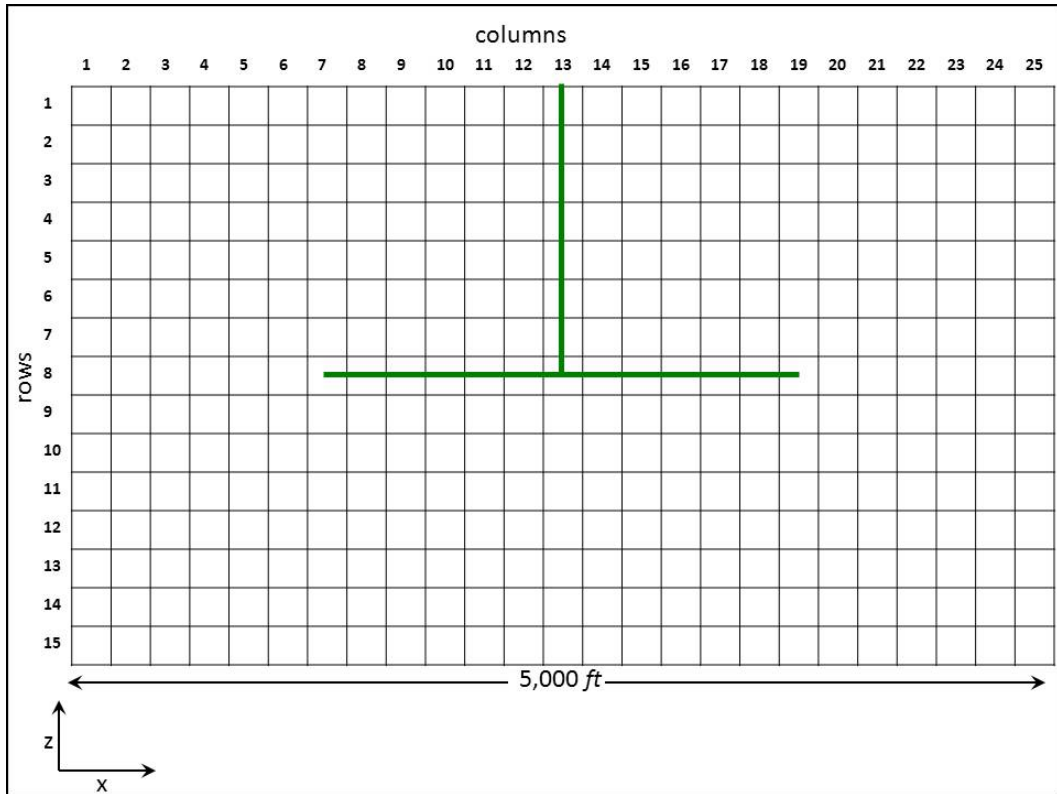
**Figure 4.3:  $x$ - $z$  plane of the rectangular reservoir system for  $j=13$  and showing the horizontal well.**

After running the numerical simulation for all the reservoirs for 10 years, water saturation data was collected and prepared for training the neural network. The data collected was only from the vertical plane which contains the horizontal well (Fig 4.3). This is done to reduce the amount of data, and hence reduce the time for training the neural network.

The water saturation for each block at the end of each year was collected. This was done for all the blocks in Fig 4.4. At the end, we had 375 blocks ( $25 \times 15$ ) to

be considered. Because of the symmetry of the reservoir, we only took the first 13 columns. Training the neural network using the water saturation values for the blocks in a single column, produced 13 different neural networks. All neural networks have the same inputs, but the difference is in the values of the water saturation, where the first neural network used the water saturation values of all the blocks in the first column for the 10 years, and the second neural network used the data for the second column, etc.

The data from the 314 reservoirs were divided into 3 sets: 284 for training, 15 for validation, and 15 for testing. The training and validation set were used in the training process, but the testing set was not introduced to the network during training process. The ANN is considered good when the predicted water saturation of the testing data has an absolute error of less than 10% for all values. Fig 4.4 shows the x-z plane of the reservoir and the numbering of the rows and columns.



**Figure 4.4:** x-z plane of the reservoir for  $j=13$  and showing the horizontal well with numbering of rows and columns.



Three approaches were used to select the best data to train the neural network:

1. Considering the water saturation data for the whole vertical plain (because of the symmetry, half of the plane was considered).
2. Considering the water saturation data for each layer.
3. Considering the water saturation data for each column. (because of the symmetry, only 13 columns were considered out of 25 columns).

Considering columns for training the neural network gave the lowest absolute error.

The first trial to train the neural network was done by using the water saturation data of the blocks in columns 1 - 13. Water saturation values of blocks in columns 14 - 25 were not included because they are identical to the values in columns 1 - 12. No good network was produced. The data then was reduced by selecting the data of a single layer to train a neural network. Starting with the first layer at the top of the reservoir, the error was very small. Moving to the second layer, the error slightly increased, but was still good, because all the predicted values had an absolute error of less than 10%. At Layer 6, some of the blocks showed an error above 10%. Moving to the next layers, the error was increasing until Layer 14. The error for Layer 15 was lower than the previous 5 layers, but not good enough, because many blocks had an absolute error above 10%.

Then, a different approach was followed to see if better results can be produced. Instead of taking the blocks in each layer and use their water saturation values to train a neural network, the blocks in each column were considered. The columns which were considered are columns 1-13 because of the symmetry (see Fig 4.4). Training the neural network started with using the water saturation values in the blocks in Column 13, because it is the most complicated column. The greatest change in water saturation is happening in Column 13. Then moving to Column 12, until Column 1. The average absolute error of Column 1 was the minimum error, and the error increased from Column 2 to Column 13.

The structure of the artificial neural network was selected after trial and error. The network with the lowest error found was the feedforward network. The learning function with lowest error was the gradient descent with momentum weight and bias learning function. The training function with the lowest error was the conjugate gradient backpropagation with Polak-Ribière updates. The transfer functions which showed the lowest error was the hyperbolic tangent sigmoid transfer function.

The neural network structure consists of the input and the output layers and two or more hidden layers. In each layer (input, output and hidden), the number of neurons must be specified. The number of neurons in the input layer are 7, which are the parameters in Table A.2. The number of neurons in the output layer are 150. 150 neurons are required because each column has 15 blocks, and the water saturation value for a single block was taken at the end of each year for 10 years.

Determining the number of hidden layers and neurons in a hidden layer is a trial and error process. Several training processes need to be performed with different numbers of hidden layers and neurons to reach the best structure that gives the lowest average absolute error.

## 4.2 Results and Discussion

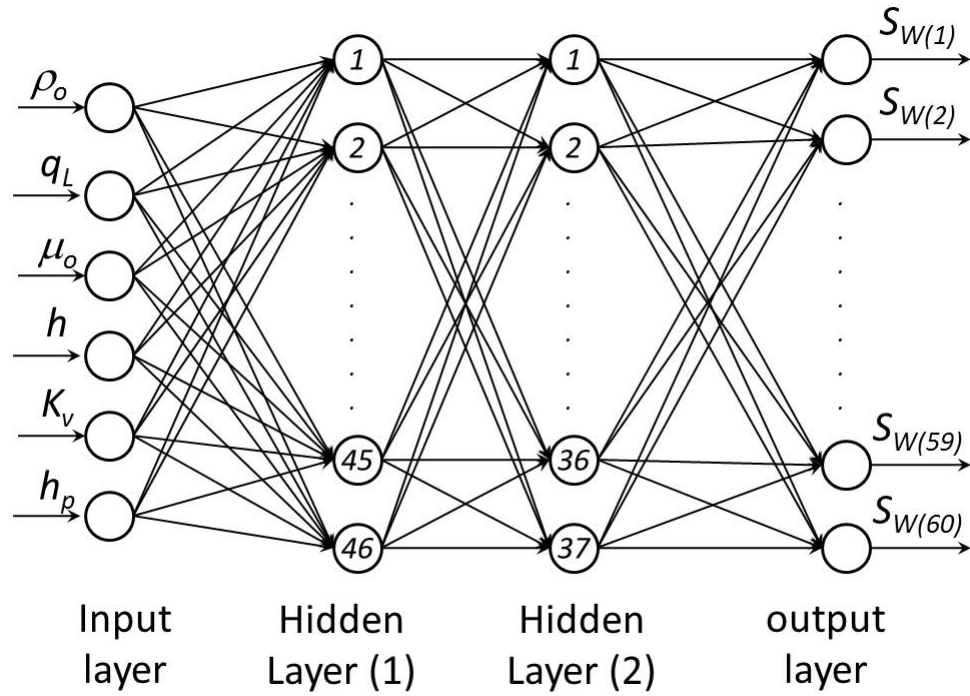
### 4.2.1 Vertical Well

In this case, 25 artificial neural networks were created. They were tested using data from the 12 reservoirs. The average absolute error was less than 10% for the data of the layers for all 12 reservoirs. The structure of all the networks consists of one input layer, one output layer and two hidden layers. For each network, the outputs were the water saturation values for the blocks at the end of the year for the 10 years, and the inputs are 6 parameters, they are:

1. Oil density.
2. total liquid flow rate.

3. Oil viscosity.
4. Reservoir thickness.
5. Vertical permeability.
6. Open interval to the flow.

Fig 4.5 shows the structure of the generated ANN for the first layer. The network has 6 inputs in the input layer, 46 neurons in the first hidden layer, 37 neurons in the second hidden layer, and 60 outputs in the output layer.



**Figure 4.5: ANN structure generated for the first layer for the vertical well.**

Table 4.5 shows the average absolute error for each layer for the 12 reservoirs used for testing the generated neural networks.

**Table 4.5: Average absolute error for the predicted water saturation for the 12 testing reservoirs by ANN for each layer for the vertical well.**

Layer No.	Average absolute Error	Layer No.	Average absolute Error
1	0.07%	14	1.03%
2	0.15%	15	1.15%
3	0.36%	16	1.67%
4	0.76%	17	1.29%
5	0.32%	18	1.52%
6	0.56%	19	1.13%
7	0.73%	20	0.95%
8	0.53%	21	0.97%
9	0.54%	22	1.04%
10	0.49%	23	0.81%
11	0.64%	24	0.80%
12	0.86%	25	0.57%
13	0.97%		

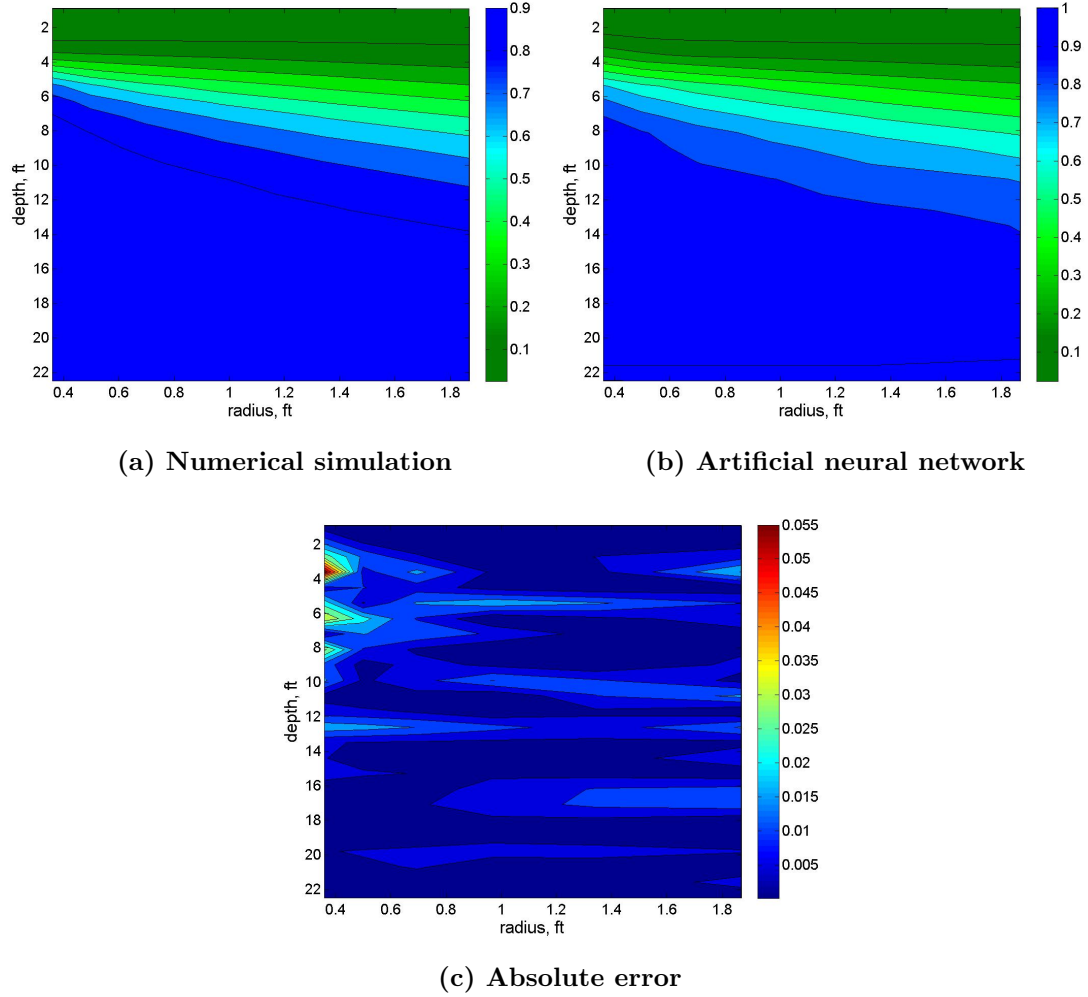
Two reservoirs (reservoir # 230 and #233) were selected, from the 12 reservoirs used to test the generated neural network, to show the behavior of the ANN in predicting the water saturation. Reservoir # 230 has the highest average absolute error, among the 12 tested reservoirs, for the predicted water saturation values, and reservoir # 233 was randomly selected. Table 4.6 shows the input parameters for the two selected reservoirs.

**Table 4.6: input parameters for reservoirs # 230 and # 233.**

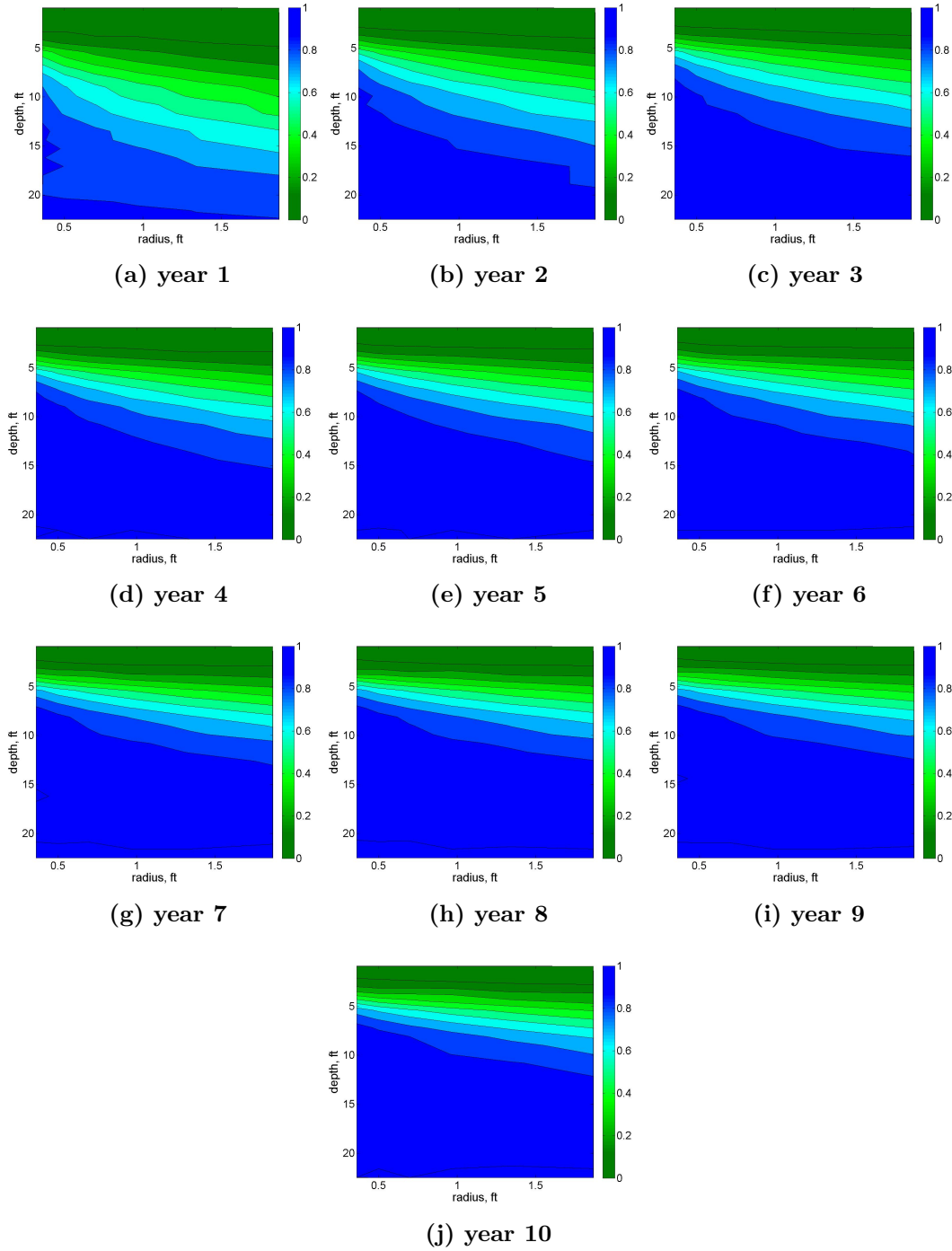
res. #	$\rho_o$ , lb/ft <sup>3</sup>	$q_L$ , STB/D	$\mu_o$ , cp	$k_v$ , md	$h_p$ , ft	$h$ , ft
230	52.9	5,290	8.9	399	3.6	22.5
233	53.05	5,305	9.05	401	4.2	26.25

Fig 4.6 (a) shows the surface map of the water saturation for reservoir # 233 with numerical simulation data. Fig 4.6 (b) shows the same reservoir but with predicted data from ANN. The prediction has a very low error, and the cone shape

is developed very clearly. Fig 4.6 (c) shows the absolute error on a surface map to give a better way of visualizing the error and its location. The highest error is 5.9% and it is in a very small area. Fig 4.7 shows progress of saturation maps for the reservoir # 233 with predicted ANN data over a period of 10 years.



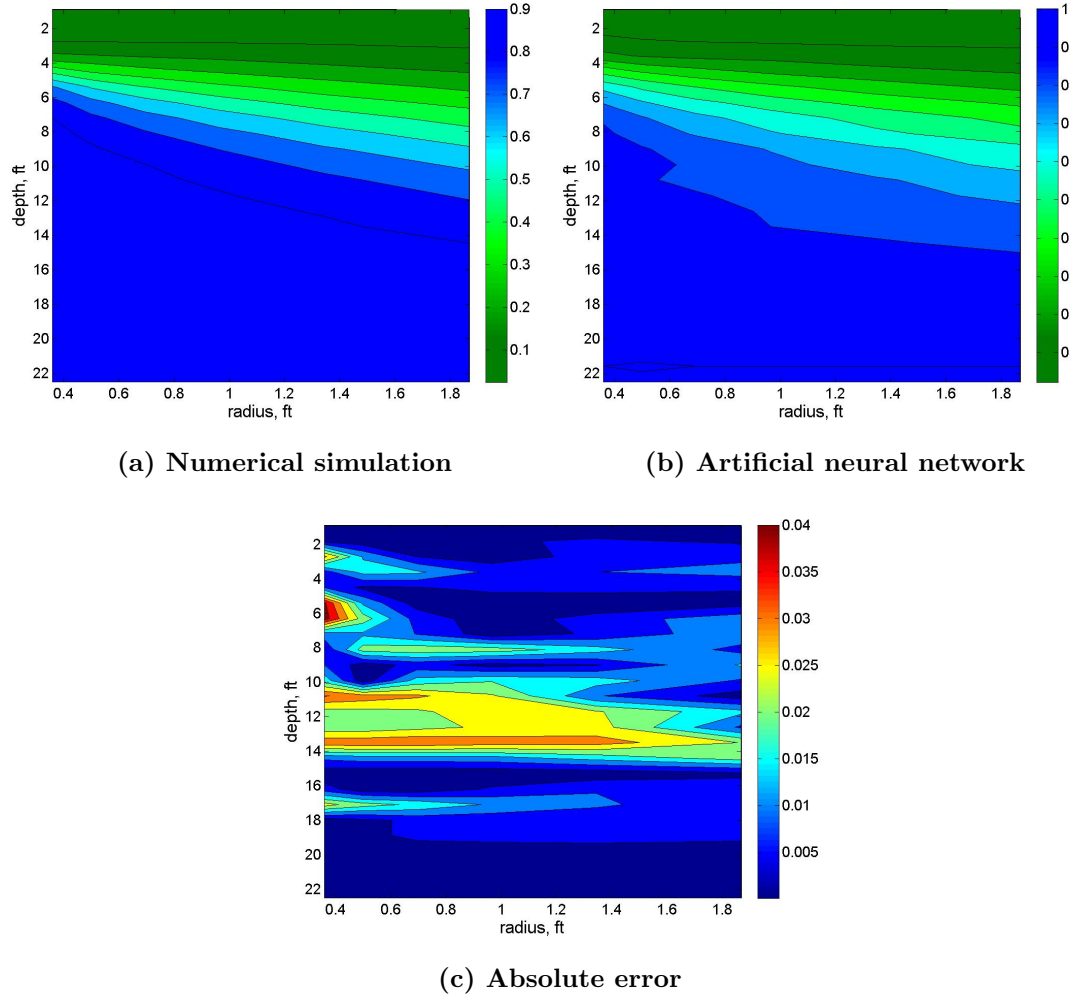
**Figure 4.6:** Surface map of  $S_w$  for reservoir # 233 at the end of the 6th year.



**Figure 4.7: Water saturation map for reservoir # 233 using ANN data.**

Fig 4.8 (a) is for the surface map for reservoir # 230 for the water saturation using the data from the numerical simulation. Fig 4.8 (b) is the surface map for the

same reservoir using data predicted with the ANN. Fig 4.8 (c) shows the absolute error. Prediction for this reservoir has the highest error among the 12 reservoirs used for testing, but the shape of the cone has developed is clearly visible.



**Figure 4.8: Surface map of  $S_w$  for reservoir # 230 at the end of the 5th year.**

The results for the previous two reservoirs show good predictions. The ANN show a good prediction at the areas where the water saturation is not changing, or the change is very small. The error is almost zero at the top of the reservoir, because the water saturation is not changing. Also, at the bottom of the reservoir, the error is very low, because the change of water saturation is very small. The results also show that the high values of error occur around the wellbore and in the areas

of transition saturation, because the saturation is changing at a faster rate than other locations, and it's more challenging for the ANN to predict the change in water saturation over a shorter period of time compared to other locations of the reservoir. In Appendix C, for comparison purposes, two additional case study results are presented for slow and medium moving saturation fronts.

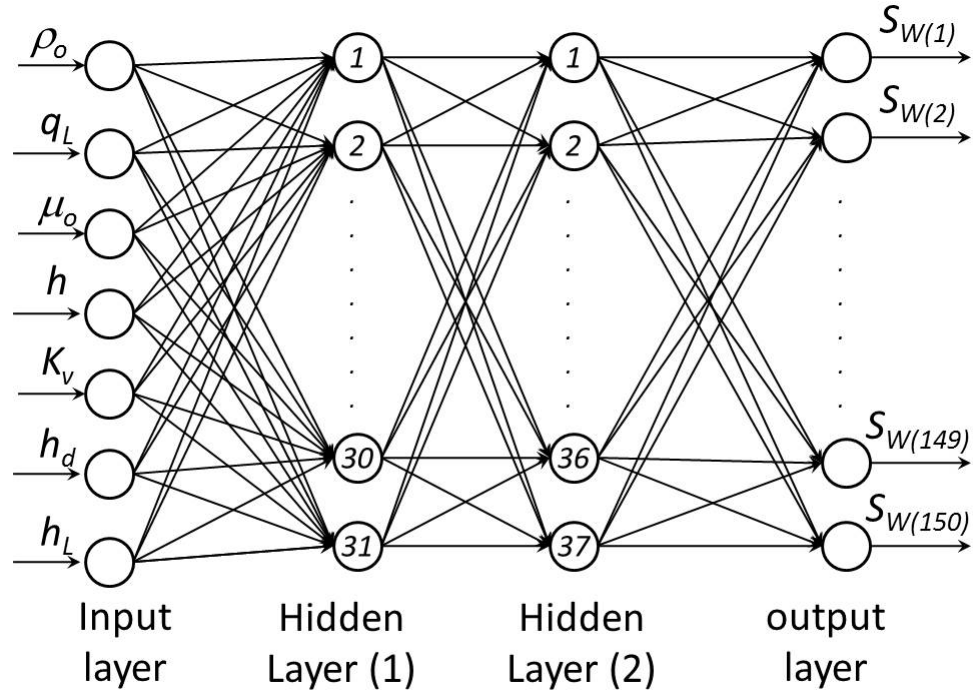
### 4.2.2 Horizontal Well

13 artificial neural networks were generated. Each network predicts the water saturation for each column. The structure of all the networks consists of one input layer, one output layer and 2 or 3 or 4 hidden layers. For the output layer, the outputs were the water saturation values for the blocks at the end of the year for the 10 years. The inputs are 7 parameters, they are:

1. Oil density.
2. Total liquid flow rate.
3. Oil viscosity.
4. Reservoir thickness.
5. Vertical permeability.
6. Depth of the horizontal well.
7. Length of the horizontal well.

Fig 4.9 shows the structure of the generated ANN for the first column. The network has 7 inputs in the input layer, 31 neurons in the first hidden layer, 37 neurons in the second hidden layer, and 150 neurons in the output layer.





**Figure 4.9:** ANN structure generated for the first column for the horizontal well.

Table 4.7 shows the average absolute error for the 15 reservoirs used for testing the generated neural networks.

**Table 4.7: Average absolute error for the predicted water saturation for the 15 testing reservoirs by ANN for each column for the horizontal well.**

Column No.	Average absolute Error
1	0.366%
2	0.343%
3	0.341%
4	0.385%
5	0.467%
6	0.941%
7	1.072%
8	1.916%
9	2.369%
10	2.587%
11	2.403%
12	2.716%
13	1.985%

15 reservoirs were tested with the generated neural networks and two reservoirs were selected to illustrate the results of the neural network predictions. The two selected reservoirs are reservoirs # 8 and # 10 and Table 4.8 shows their input parameters.

**Table 4.8: input parameters for reservoirs # 8 and # 10.**

res. #	$\rho_o, lb/ft^3$	$q_L, STB/D$	$\mu_o, cp$	$k_v, md$	$h, ft$	$h_L, ft$	$h_d, ft$
8	51.766	6,286	9.9006	253	136.47	3,400	40.942
10	50.898	5,319	2.0525	192	210.79	2,600	105.40

Fig 4.10 (a) shows the surface map for reservoir # 8 of water saturation from numerical simulation. The horizontal well is at a depth of 41 ft and the horizontal section is 3,400 ft long extending from 800 ft to 4,200 ft. Fig 4.10 (b) shows the surface map for the same reservoir with water saturation data predicted from ANN.

Fig 4.10 (c) shows the absolute error of the ANN predicted water saturation for reservoir # 8. Fig 4.11 shows progress of saturation maps for the reservoir #8 with predicted ANN data over a period of 10 years.

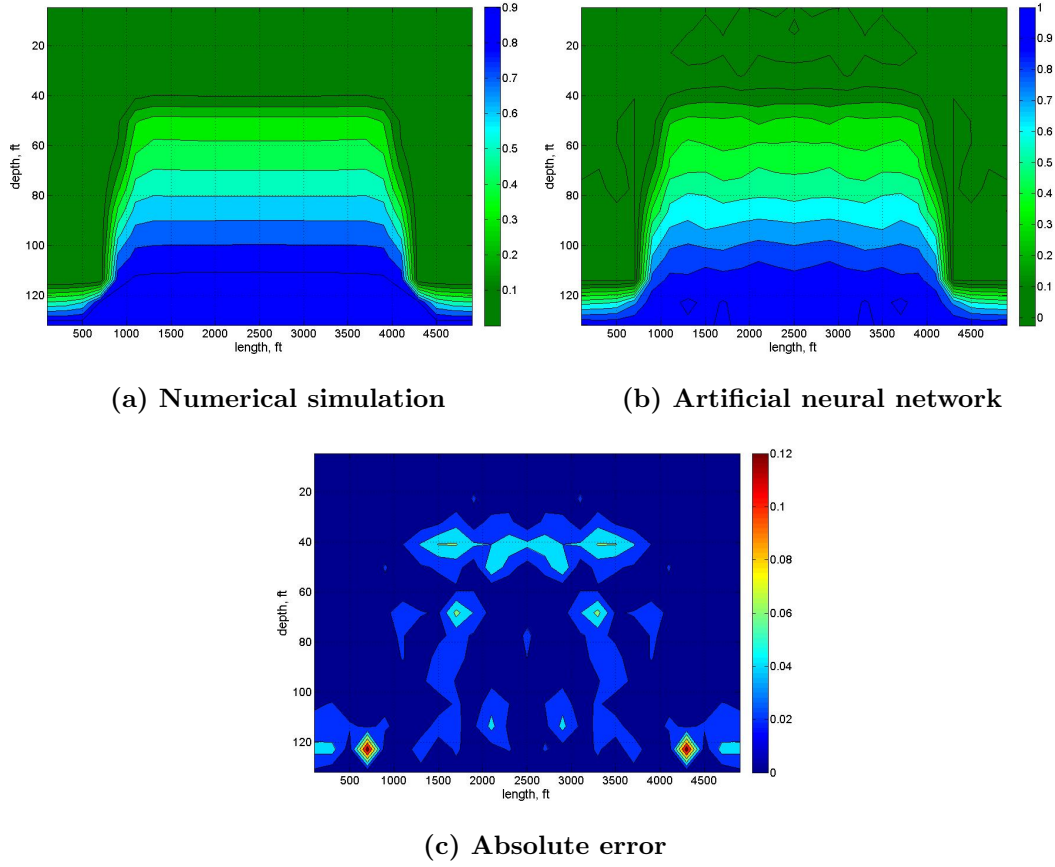


Figure 4.10: Surface map of  $S_w$  for reservoir # 8 at the end of the 10th year.

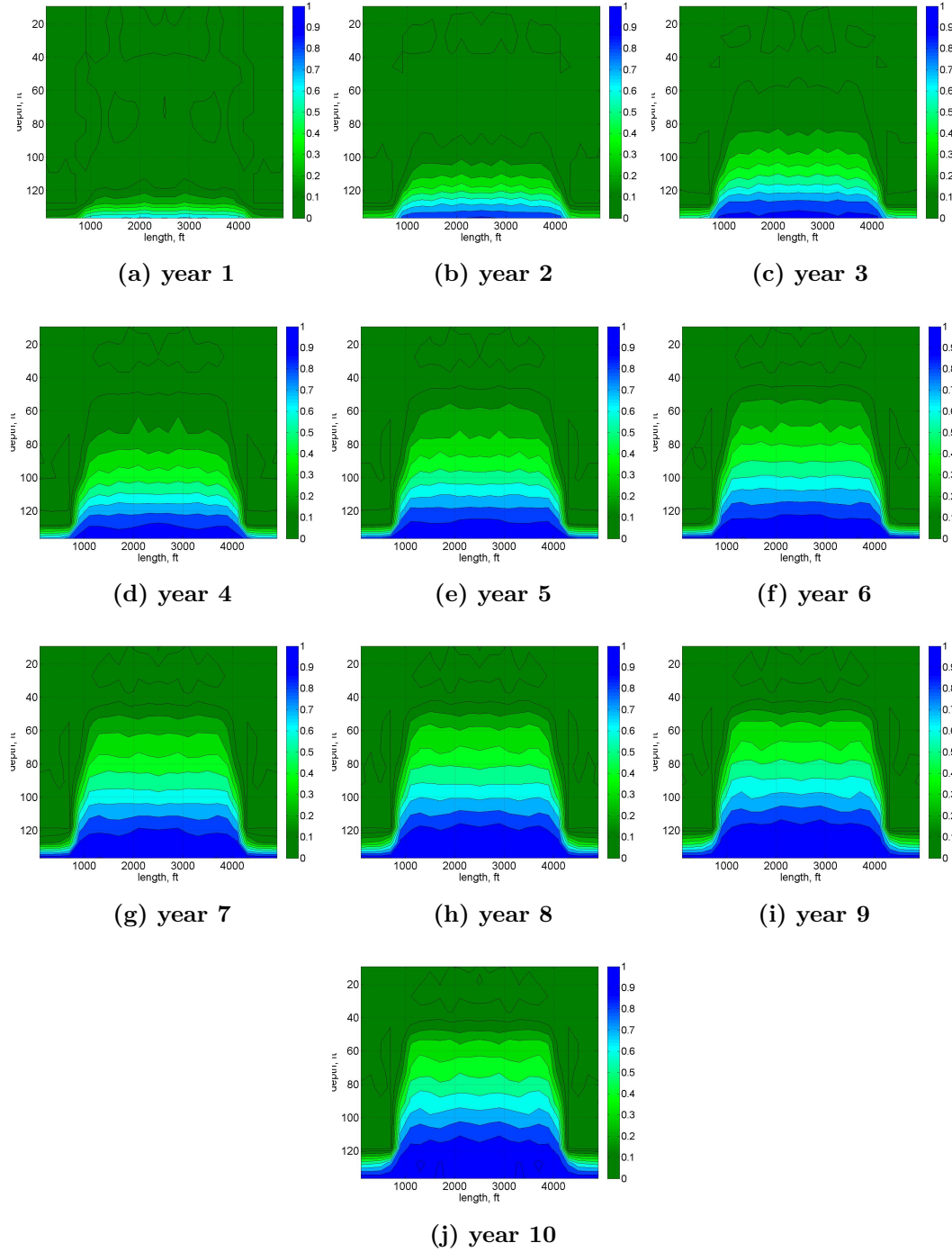
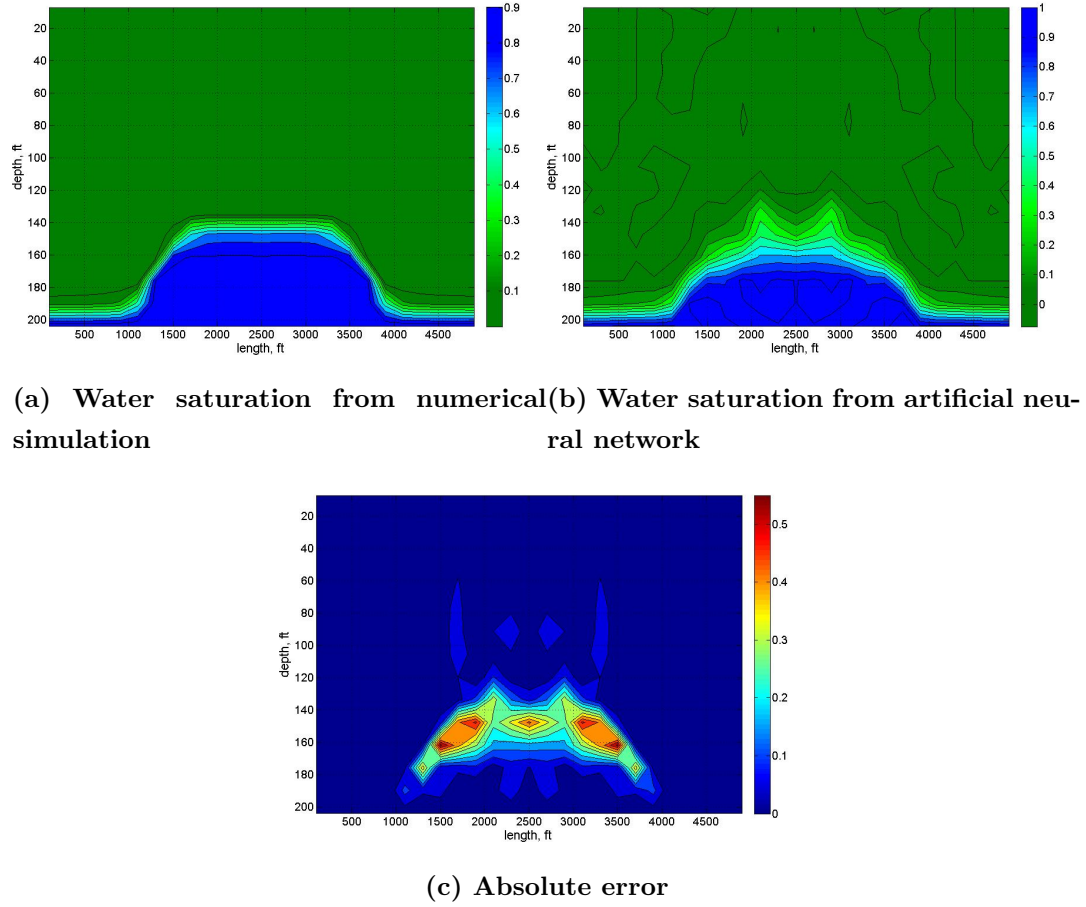


Figure 4.11: Water saturation map for reservoir # 8 using ANN data.

The ANN was able to predict the shape of the cone. There are areas which show an absolute error larger than 10%, but they are at the bottom of the water cone which

is less important (Fig 4.10 (c)). The more important areas are those which show where the water front has reached. The high error zones occurs at the transition zones, similar to results for the vertical wells. The high error occurs at the transition zones because saturation gradients are high over a small area, which creates a greater challenge to the ANN to predict the water saturation values accurately.

The second example to illustrate the ability of the ANN to predict the water coning phenomena is for reservoir # 10. Fig 4.12 (a) shows the surface map of water saturation for the reservoir with the numerical simulation data, and Fig 4.12 (b) shows the surface map for the same reservoir with the ANN predicted data. The ANN was able to predict the cone shape and also to predict the sharp decrease of water saturation at the bottom sides of the cone.



**Figure 4.12: Surface map of  $S_w$  for reservoir # 10 at the end of the 10th year.**

Fig 4.12 (b) shows two identical peaks. This is an overestimate for the water saturation values and this is because the horizontal section of the well, which is off the center, is having more flow than the center section. The ANN was successful in predicting this behavior but the values for water saturation were overestimated. In Appendix D, for comparison purposes, two additional case study results are presented for slow and medium moving saturation fronts.

## Conclusions

This study consisted of two parts. The first part is concerned with the use of the pressure transient data to determine the distance to the discontinuity in an oil reservoir with edge water. The reservoir is considered as a composite reservoir with the oil phase in the inner zone and the water phase in the outer zone. Some of the parameters that affect the determination of the distance to the discontinuity were studied. A procedure is proposed to help estimate the distance to the discontinuity under certain conditions. The procedure can be applied using the pressure derivative for buildup test data if the mobility ratio is less than unity. A peak must be observed in the pressure derivative curve. Looking at the highest value of the pressure derivative in the peak, and then taking the corresponding time for it and using it to calculate the radius of investigation which in this case is the distance to the discontinuity.

For a peak to be observed in a reservoir with  $r_{in} = 1,600$  ft, the mobility of the inner zone has to be very small (less than 0.06), and also the mobility ratio has to be small (preferably less than 0.00125). Considering the previous conditions given for the mobility in the inner zone and the mobility ratio, the procedures will accurately predict the distance to the discontinuity for large distances (1,600 ft or larger), but for shorter distances, the error can be high. For the error to be low for shorter distances, the mobility for the inner zone has to be less than what it should be for  $r_{in} = 1,600$  ft.

The second part is concerned with predicting water saturation distribution in oil reservoirs using artificial neural networks. The developed ANN are for two systems with a bottom water aquifer: vertical well in a radial system, and a horizontal well in a rectangular system. A total of 6 input parameters are needed for the ANN to predict the water saturation distribution for 10 years. The predicted values for the vertical well represent the water saturation distribution around the wellbore at the end of each year for the 10 years. For the horizontal well, the predictions are for the vertical plane that contains the horizontal well.

The examples in this study showed that the accuracy of the prediction is good and matching the numerical simulation results. The high error occurs in the transition zones, because at these zones, the water saturation is changing at a faster rate compared to the water saturation change at zones where the water saturation change is small.

The developed ANN can be used to optimize production, by running the ANN for different scenarios to find the optimum production rate that delay water production and increase oil production. Also, it can be used to select the optimum perforation interval that will increase production of water-free oil and delay water production or reduce it.

This study proposed a solution for water encroachment problems caused by two different boundary conditions. The first part proposed a procedure to estimate the distance to the edge water encroachment front for an oil reservoir. In the second part, an artificial expert system is predicted to understand the water saturation distribution for an oil reservoir with a strong bottom water aquifer. Although, these two proposed solutions are aimed for two different problems, they could be combined to predict water saturation distribution in an oil reservoir that is simultaneously exposed to bottom water drive, and edge water encroachment due to peripheral injection (Fig 5.1).



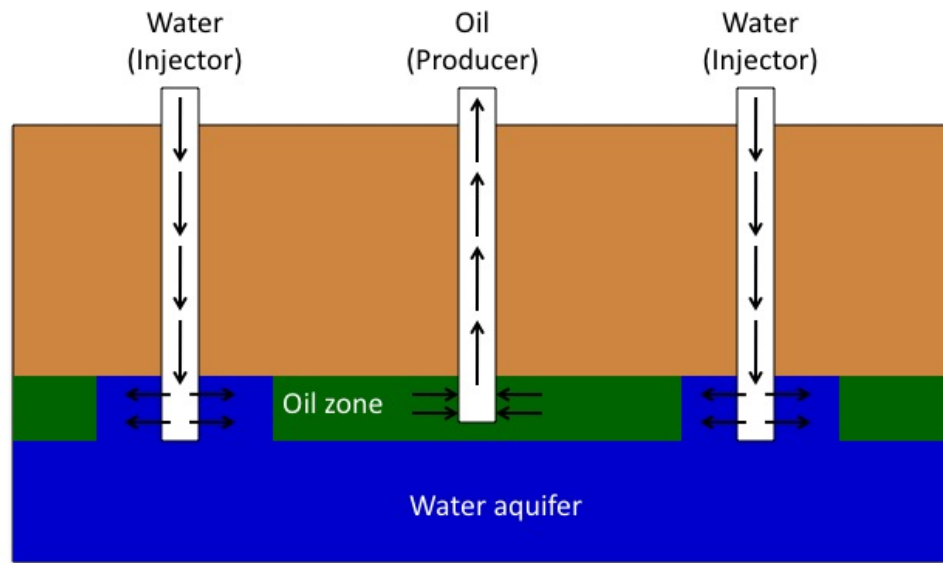


Figure 5.1: Schematic representation of an oil reservoir exposed to bottom water aquifer and edge water encroachment.

# Properties used in designing the simulation models for different scenarios for vertical and horizontal wells

Table A.1: Properties used in designing the simulation models for different scenarios for vertical wells

[illegible]

Table A.1: (Continued)

[illegible]







Table A.1: (Continued)

[illegible]

Table A.1: (Continued)

[illegible]





**Table A.1: (Continued)**

No.	$\rho_o$ , $lb/ft^3$	$q_L$ , STB/D	$\mu_o$ , cp	$k_v$ , md	$h_p$ , ft	$h$ , ft
208	49.95	6,995	7.05	309	134.1	186.2
209	50.05	7,005	7.15	311	135.9	188.7
210	48.95	6,890	6.75	289	102.4	160
211	48.9	6,895	6.7	291	103.2	161.2
212	49.05	6,905	6.85	289	105.6	165
213	49.1	6,910	6.9	291	104.8	163.7
214	53.23	2,190	4.96	119	28.62	39.75
215	53.3	2,195	5.01	121	29.52	41
216	53.43	2,210	5.16	119	32.22	44.75
217	53.36	2,205	5.11	121	31.32	43.5
218	53.03	2,186	4.76	117	25.02	34.75
219	53.08	2,191	4.81	119	25.92	36
220	53.23	2,206	4.96	117	28.62	39.75
221	53.18	2,201	4.91	119	27.72	38.5
222	51.5	7,856	4.18	226	51	75
223	51.55	7,861	4.23	228	51.85	76.25
224	51.7	7,876	4.38	226	54.4	80
225	51.65	7,871	4.33	228	53.55	78.75
226	50.15	5,502	3.2	55	142.5	237.5
227	50.2	5,507	3.25	57	143.2	238.7
228	50.35	5,522	3.4	55	145.5	242.5
229	50.3	5,517	3.35	57	144.7	241.2
230	52.9	5,290	8.9	399	3.6	22.5
231	52.95	5,295	8.95	401	3.8	23.75
232	53.1	5,310	9.1	399	4.4	27.5
233	53.05	5,305	9.05	401	4.2	26.25

Table A.2: Properties used in designing the simulation models for different scenarios for horizontal wells

[illegible]

Table A.2: (Continued)

[illegible]



Table A.2: (Continued)

No.	$\rho_o$ , lb/ft <sup>3</sup>	$q_L$ , STB/D	$\mu_o$ , cp	$k_v$ , md	$h$ , ft	$h_L$ , ft	$h_d$ , ft
85	54.25	9,413	4.39	95	87.81	2,600	32.2
86	48.84	5,208	4.91	127	110.06	1,400	11.01
87	52.35	6,051	5.37	485	139.28	2,600	32.5
88	54.78	7,790	3.66	474	24.32	2,600	10.54
89	54.77	8,216	4.11	454	91.24	3,400	3.04
90	56.94	1,229	2.07	321	85.26	600	19.9
91	51.80	9,421	3.42	401	119.89	1,000	11.99
92	54.51	4,572	2.04	305	132.62	2,200	4.42
93	57.15	8,817	6.43	105	229.9	2,600	68.97
94	55.83	6,179	6.13	359	93.7	1,800	28.11
95	52.26	4,672	4.05	201	74.05	3,000	7.41
96	50.30	4,177	3.00	331	129.84	2,200	38.95
97	57.42	3,505	5.53	124	68.53	1,400	29.7
98	55.79	6,461	6.26	170	150.45	3,000	45.13
99	50.55	9,719	5.64	228	16.25	2,200	2.71
100	56.69	9,014	8.23	322	199.53	3,000	86.46
101	53.39	4,300	6.05	45	175.09	3,000	75.87
102	57.43	5,586	6.32	377	100.02	3,000	16.67
103	55.15	8,518	7.66	374	110.9	2,200	33.27
104	55.98	8,913	8.91	262	118.34	1,800	27.61
105	53.22	9,356	6.64	196	86.99	1,400	26.1
106	54.04	9,290	1.52	292	146.17	3,400	63.34
107	52.63	8,097	1.15	379	180.03	2,600	66.01
108	51.83	9,559	2.22	147	90.96	1,400	21.22
109	48.35	1,583	4.28	208	22.6	1,400	6.78
110	48.92	8,496	9.95	466	147.83	2,600	73.91
111	50.02	2,481	1.35	151	199.83	3,400	86.59
112	48.36	5,470	2.52	293	225.04	3,400	82.51

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Table A.2: (Continued)

No.	$\rho_o$ , lb/ft <sup>3</sup>	$q_L$ , STB/D	$\mu_o$ , cp	$k_v$ , md	$h$ , ft	$h_L$ , ft	$h_d$ , ft
169	57.27	9,442	6.75	311	220.36	600	80.8
170	51.70	5,990	4.30	332	165.63	3,400	27.6
171	50.80	7,534	7.82	374	104.73	600	10.47
172	53.39	6,339	7.59	447	163.18	1,400	16.32
173	56.99	3,040	3.91	438	28.02	1,400	0.93
174	53.50	9,710	9.55	200	57.27	1,400	5.73
175	57.09	1,376	1.26	184	123.55	2,600	12.36
176	53.62	2,844	9.70	485	70.81	3,400	35.41
177	52.33	8,471	9.33	237	185.43	3,000	80.35
178	56.99	9,715	7.78	243	185.87	600	43.37
179	57.69	8,024	8.35	113	24.1	1,400	4.02
180	55.16	2,953	8.49	310	77.22	1,800	28.31
181	55.83	6,634	3.81	265	88.42	1,800	44.21
182	56.59	1,626	9.34	364	163.99	600	49.2
183	48.90	1,342	2.58	485	15.2	1,400	0.51
184	51.71	3,692	9.92	337	196.94	1,400	19.69
185	51.65	9,577	8.63	187	211.62	600	21.16
186	55.39	1,087	5.95	106	175.85	1,000	52.75
187	56.33	5,800	8.35	213	104.25	3,400	38.22
188	52.11	5,937	6.45	39	133.13	1,400	66.56
189	52.77	1,780	9.85	451	144.54	3,000	72.27
190	56.39	1,762	7.53	193	134.15	2,200	31.3
191	54.23	4,969	1.68	472	70.44	1,400	16.44
192	55.70	9,061	2.92	279	118.51	1,800	19.75
193	55.24	5,924	4.84	344	220.4	3,400	80.81
194	53.31	5,570	3.87	149	142.9	2,600	52.4
195	54.03	2,875	6.43	55	165.23	1,400	38.55
196	54.57	4,500	1.75	84	45.47	600	10.61

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**Table A.2: (Continued)**

No.	$\rho_o$ , $lb/ft^3$	$q_L$ , STB/D	$\mu_o$ , cp	$k_v$ , md	$h$ , ft	$h_L$ , ft	$h_d$ , ft
309	52.23	4,960	2.31	228	156	3,400	15.6
310	52.33	4,964	2.41	230	157.35	3,400	15.74
311	50.73	4,683	3.58	74	142.65	3,400	42.8
312	50.83	4,687	3.64	76	144	3,400	43.2
313	51.71	6,281	9.85	252	135.75	3,400	40.73
314	51.8	6,290	9.95	254	137.25	3,400	41.18

## MATLAB code for training the Neural Network

```

clear
close all
clc
format long
load input
load output
P = input';
T = output';
[m,n] = size(P);
[m1,n1] = size(T);
[P2,ps2] = removeconstantrows(P);
%normalising the data
[Pn,ps] = mapminmax(P2,-1,1); gives all values between -1 and 1
[Tn,ts] = mapminmax(T,-1,1); gives all values between -1 and 1
[mi,ni] = size(Pn);
[mo,no] = size(Tn);
%The following five lines specify the data for training, validation and testing
trainInd = [1:81,106:233];
valInd = [82:96];
testInd = [97:105];
    
```

```

[Pn_train,Pn_val,Pn_test,trainInd,valInd,testInd] = divideind(Pn,trainInd,valInd,testInd);
[Tn_train,Tn_val,Tn_test] = divideind(Tn,trainInd,valInd,testInd);
val.T = Tn_val;
val.P = Pn_val;
test.T = Tn_test;
test.P = Pn_test;
%Initiating network parameters
HL1 = 43; state number of neurons in first hidden layer
HL2 = 90; state number of neurons in second hidden layer
net = newff(Pn,Tn,[HL1,HL2],{'tansig'},'traincgp','learngdm','msereg');
%setting training parameters for the network
net.trainParam.goal = 0.0001; accuracy within this range
net.trainParam.epochs = 100; number of iteration sets
net.trainParam.show = 1;
net.trainParam.max_fail = 10000;
%starting training the network
[net,tr] = train(net,Pn_train,Tn_train,[],[],test,val);
%-----
%-----getting data from the trained network -----
%-----
Tn_train_ann = sim(net,Pn_train);
Tn_test_ann = sim(net,Pn_test);
Tn_val_ann = sim(net,Pn_val);
[m_Te,n_Te] = size(Tn_test);
NP_test = 1:n_Te;

%denormalising the data sets obtained
%output reversal
T_train = mapminmax('reverse',Tn_train,ts);
T_test = mapminmax('reverse',Tn_test,ts);
T_train_ann = mapminmax('reverse',Tn_train_ann,ts);
T_test_ann = mapminmax('reverse',Tn_test_ann,ts);
T_val = mapminmax('reverse',Tn_val,ts);

```



```
T_val_ann = mapminmax('reverse',Tn_val_ann,ts);  
%input reversal  
P_train = mapminmax('reverse',Pn_train,ps);  
P_val = mapminmax('reverse',Pn_val,ps);  
P_test = mapminmax('reverse',Pn_test,ps);
```

## **Water Saturation Maps for Two different reservoirs for Vertical Wells Using ANN**

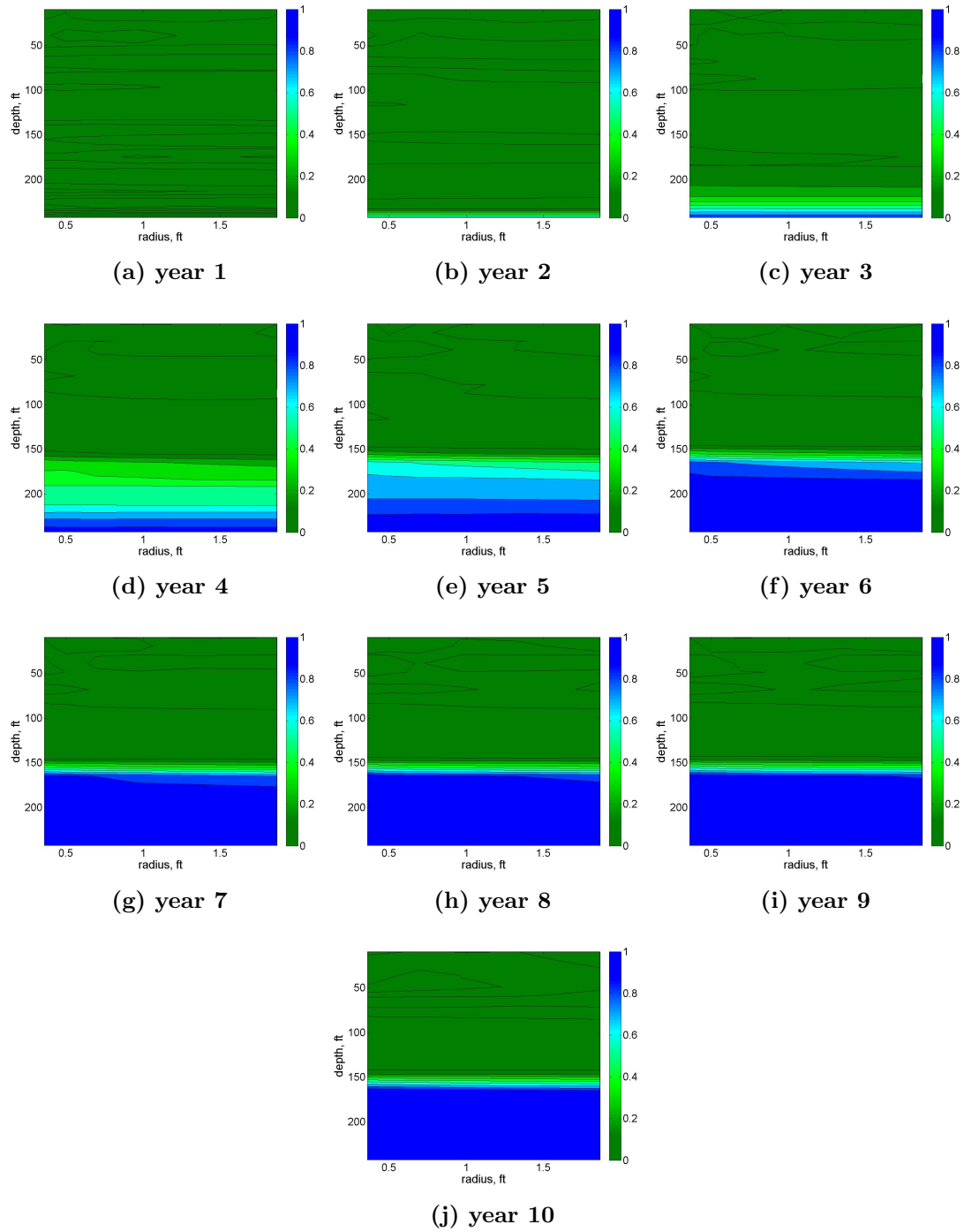


Figure C.1: Water saturation map for reservoir # 228 using predicted ANN data (Note: vertical permeability for this reservoir is 55 md).

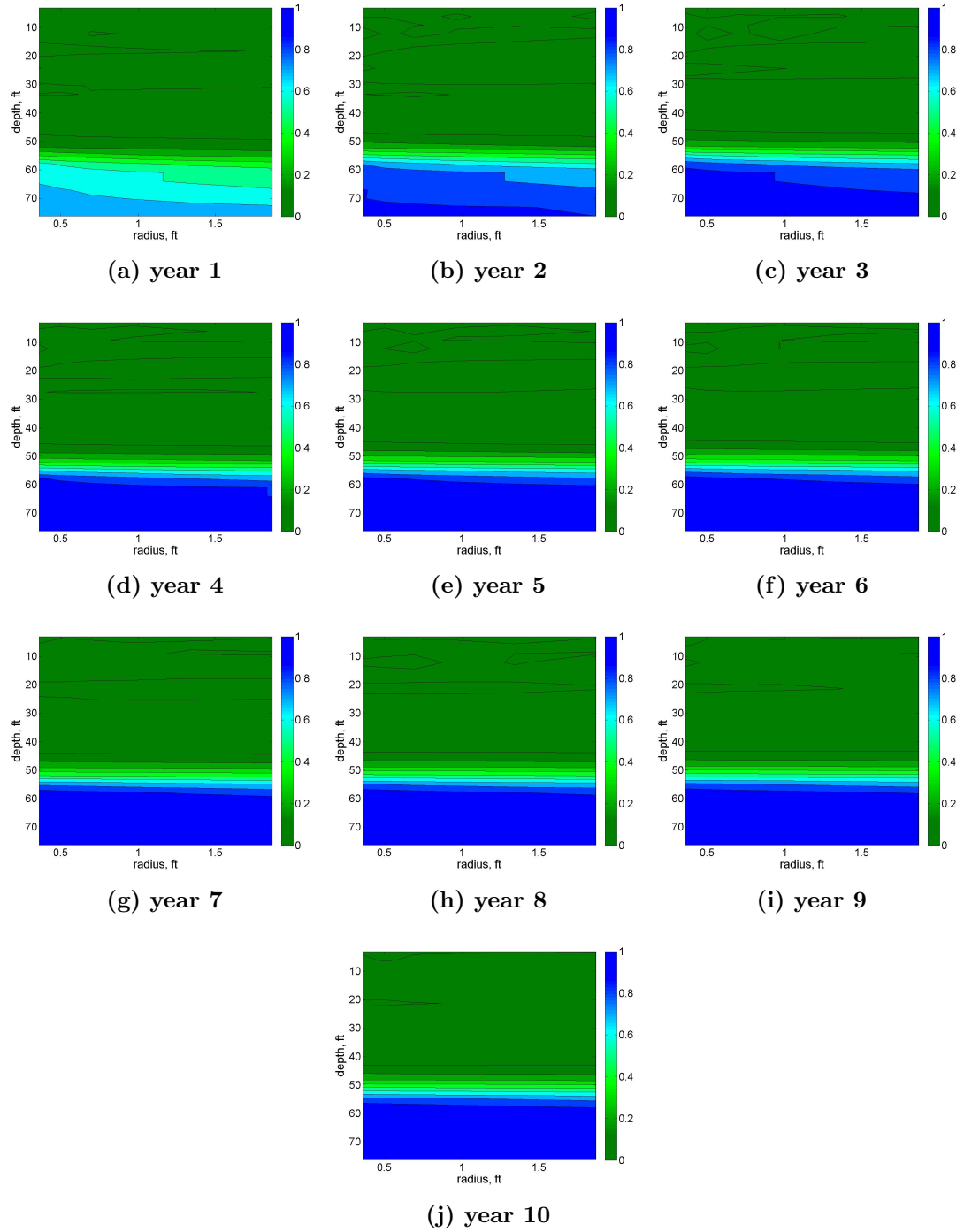


Figure C.2: Water saturation map for reservoir # 223 using predicted ANN data (Note: vertical permeability for this reservoir is 228 md).

## **Water Saturation Maps for Two different reservoirs for Horizontal Wells Using ANN**

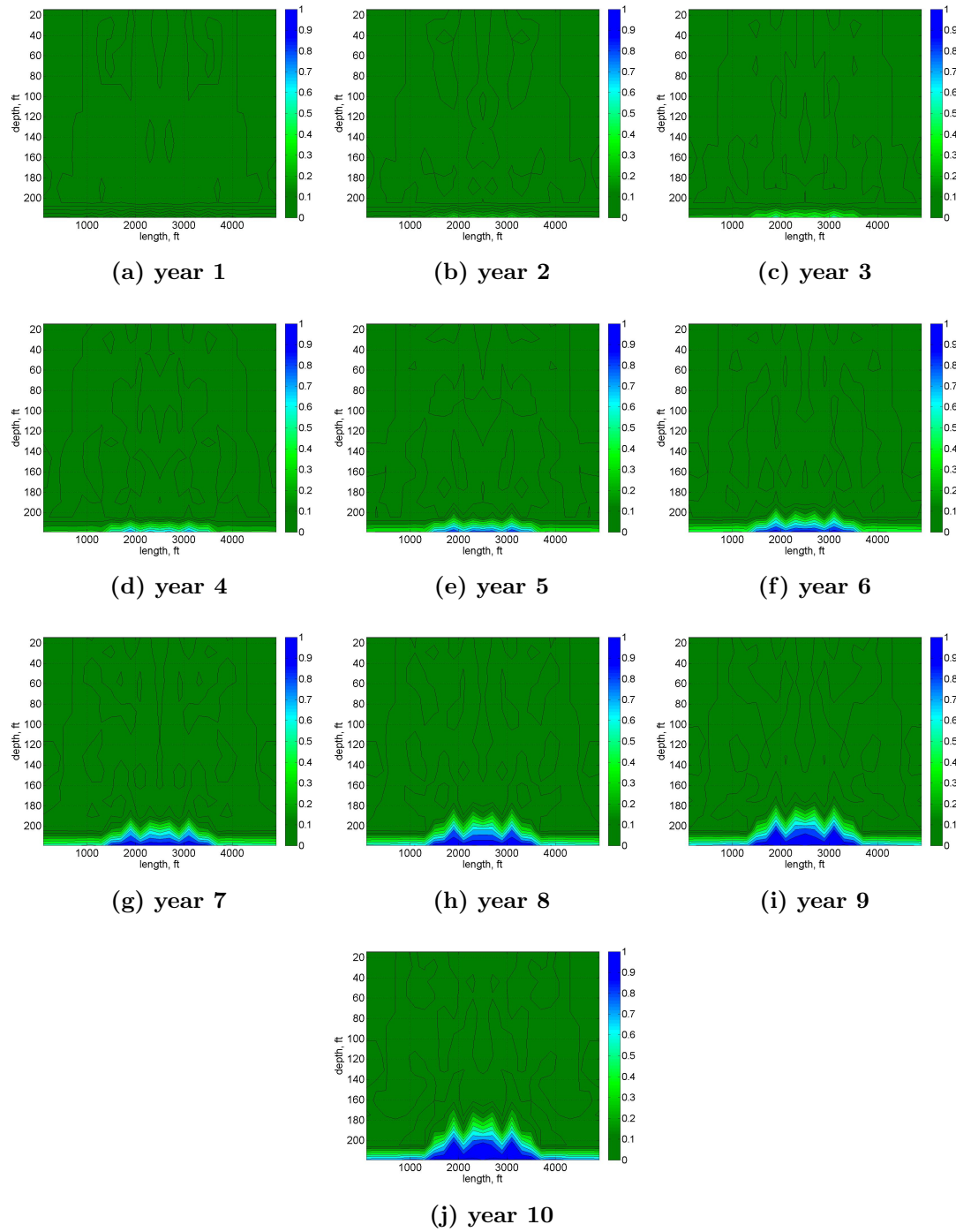


Figure D.1: Water saturation map for reservoir # 11 using predicted ANN data (Note: vertical permeability for this reservoir is 209 md).

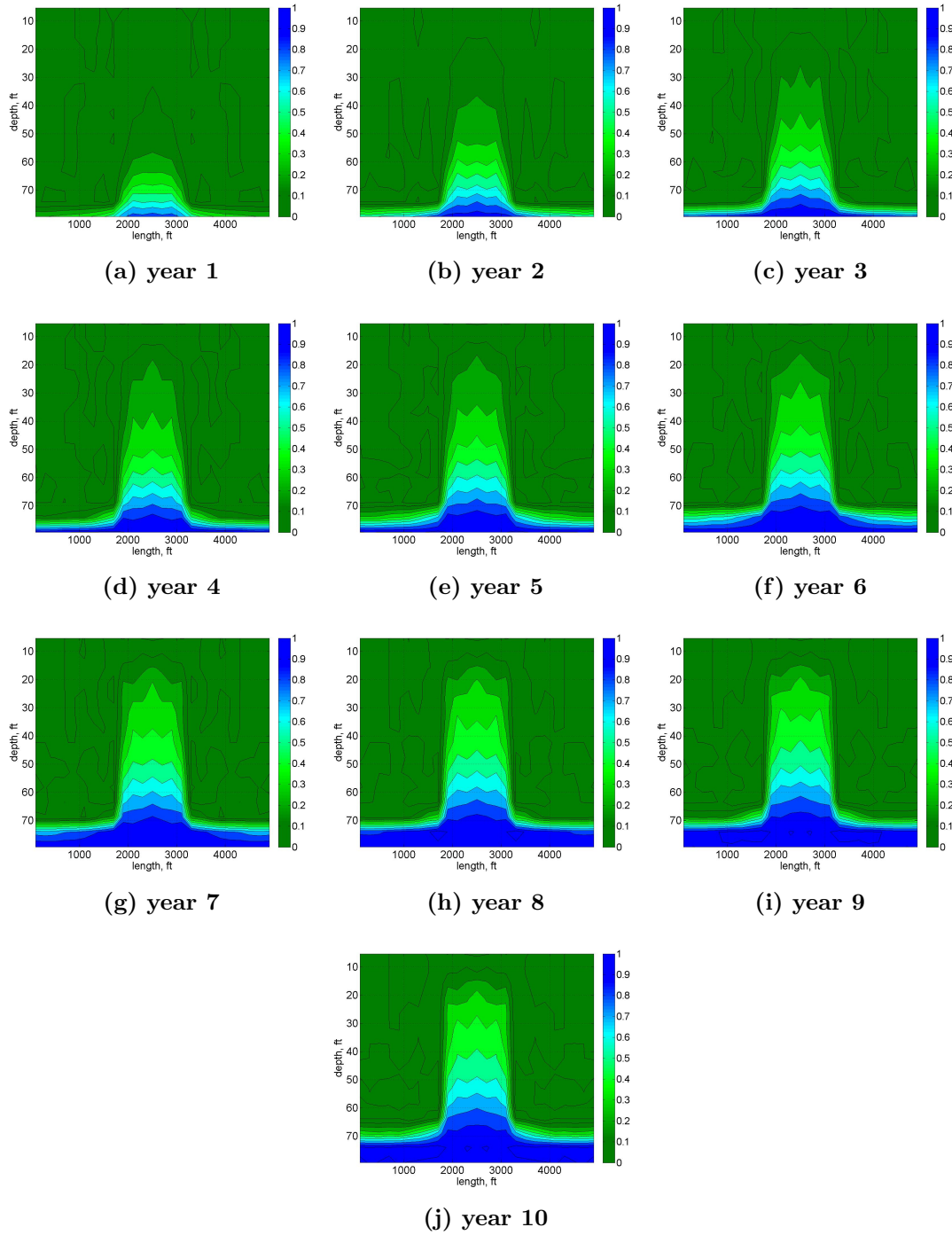


Figure D.2: Water saturation map for reservoir # 7 using predicted ANN data (Note: vertical permeability for this reservoir is 404 md).

# References

Al-Bulushi, N., King, P. R, Blunt, M. J. and Kraaijveld, M.: "Development of Artificial Neural Network Models for Predicting Water Saturation and Fluid Distribution" *Journal of Petroleum Science and Engineering*, 68 (October 2009) 197-208.

Ali, J. K.: *Neural Network: A New Tool for the Petroleum Industry?*, paper SPE 27561, 1949.

Ambastha, A. K. and Ramey, H. J., Jr.: *Thermal Recovery Well Test Design and Interpretation SPEFE* (June 1989) 173-180.

Bixel, H. C. and Poolen, H. K. Van: *Pressure Drawdown and Buildup in the Presence of Radial Discontinuities*, paper SPE 1516, 1967.

Blades, D. N. and Stright, D. H., Jr.: *Predicting High Volume Lift Performance in Wells Coning Water PETSOC-75-04-06-P*, 1975.

Bourdet, D., Whittle, T. M., Douglas, A. A. and Pirard, Y. M.: *A New Set of Type Curves Simplifies Well Test Analysis*, World Oil, 1983.

Bourdet, D., Ayoub, J. A., and Pirard, Y. M.: *Use of Pressure Derivative in Well Test Interpretation SPEFE* (June 1989) 293; *Trans., AIME*, 287.

Brown, L. P.: *Pressure Transient Behavior of the Composite Reservoir*, Paper SPE 14316, 1985.

Byrne, W. B. and Morse, R. A., *The Effects of Various Reservoir and Well Parameters on Water Coning Performance* paper SPE 4287, 1973.

Carter, R. D.: *Pressure Behavior of a Limited Circular Composite Reservoir* 1966. SPE 1621-PA.

Hazebroek, P., Rainbow, H., and Mathews, C.S.: *Pressure Fall-off in Water injection Wells*, paper SPE 925, 1958.

Helle, H.B and Bhatt, A.: "Fluid Saturation from Well Logs using Committee Neural Networks" *Petroleum Geoscience*, 8 (June 2002) 109-118.



- Hurst, W.: Interference between Oil Fields, paper SPE 1335, 1960. Issaka, M. B. and Ambastha, A. K.: A Generalized Pressure Derivative Analysis for Composite Reservoirs, PETSOC 1996-115, 1996.
- Kuo, M. C. T. and DesBrisay, C.L.: A Simplified Method for Water Coning Predictions paper SPE 12067, 1983.
- Loucks, T. L. and Guerrero, E. T.: Pressure Drop in a Composite Reservoir, Paper SPE 19, 1961.
- Mungan, N.: A Theoretical and Experimental Coning Study SPEJ (June 1975) 221-236.
- Mortada, M.: Oilfield Interference in Aquifers of Non-Uniform Properties, Paper SPE 1507, 1960.
- Muskat, M., and Wyckoff, H. D., An Approximate Theory of Water Coning in Oil Production, Trans AIME, 114, 144-63, 1935.
- Odeh, O. S.: Flow Test Analysis for a Well with Radial Discontinuity, Paper SPE 2157, 1969.
- Olarewaju, J. S. and Lee, W. J.: A Comprehensive Application of a Composite Reservoir Model to Pressure-Transient Analysis, Paper SPE 16345, 1989.
- Rowan, G. and Clegg, M. W.: An Approximate Method for Transient Radial Flow, Paper SPE 156-PA, 1962.
- Satman, A. et al.: Interpretation of Injection Well Pressure Transient Data in Thermal Oil Recovery paper SPE 8908, 1980.
- Shokir, E.M.El-M.: "Prediction of the Hydrocarbon Saturation in Low Resistivity Formation via Artificial neural Network", Paper SPE 87001, 2004.
- Stehfest, H.: Numerical Inversion of Laplace Transform: Algorithm 368 Communication of ACM (Jan. 1970) 47-49.
- Tang: Transient Pressure Analysis in Composite Reservoirs, PhD thesis, Stanford University, 1982.
- Tiab, D. and Kumar, A.: Detection and Location of Two Parallel Sealing Faults around a Well, Paper SPE 6056, 1980.

Yang, Weiping, Wattenbarger, R. A. Water Coning Calculations for Vertical and Horizontal Wells, paper SPE 22931, 1991.

# **Vita**

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