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ABSTRACT

CHAPTER 1: Informational Control and Organizational Design

This essay focuses on organizational issues of allocating authority between an uninformed principal and an informed expert. We show that the established result of Dessein (2002) that delegating decisions to a perfectly informed expert is generally better than communication is reversed if the principal can restrict the precision of the expert's information (without learning its content). We demonstrate that these organizational forms—informational control and delegation—can be either complements or substitutes, depending on the principal's ability to affect the expert's discretion about the set of allowed policies.

CHAPTER 2: Dynamic Information Revelation in Cheap Talk

This essay investigates a multi-stage version of Crawford and Sobel's (1982) communication game in which the principal can affect the quality of the expert's private information at each stage (without learning its content). We construct a mechanism of dynamic updating of expert's information, which refines the expert's information step-by-step, preserving truth-telling communication at every stage. This allows the principal to reveal approximately full information in a large sub-interval of the state space. As a result, the payoff efficiency in multi-stage communication relative to one-stage communication and other organizational forms rises without a bound as the bias in preferences falls.

CHAPTER 3: Information Revelation in Competitive Markets

This essay analyzes a market with multiple sellers and differentiated products. We investigate the sellers' incentives to reveal product relevant information that affects the buyer's private valuations. The main finding is that when the number of sellers reaches some critical (but finite) number, this results in the unique symmetric equilibrium with full disclosure of information by all sellers. Thus, unlike the results by Lewis and Sappington (1994) and Johnson and Myatt (2006) for monopoly, which state that the monopolist reveals either full information or no information, competition refines the seller's dichotomic decisions to a single extreme only. Also, we show that the market efficiency is always bounded away from full efficiency, but the magnitude of inefficiency converges to zero at a high rate as competition intensifies.

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To my wife.

Chapter 1. Informational Control and Organizational Design

1.1 Introduction

Situations in which principals do not have enough information and need to consult experts before implementing a policy can be found almost everywhere. Investors consult investment bankers about the value of securities, headquarters consult managers before making corporate decisions, and politicians consult advisors on special subjects. However, the benefits of communication are often impaired by a conflict of interest. If the parties' interests differ, the expert may want to strategically misrepresent information in an attempt to manipulate a principal's decision.[1]

A potentially effective solution to this communication problem is to delegate authority to the expert herself and get the benefits of her informational advantage. Then, even though the expert's decision is biased, the trade-off between the loss of authority in delegation and the loss of information in communication often favors the former (Dessein, 2002). However, many companies today still centralize authority at the upper level of the hierarchy.[2] In this chapter, we provide an argument in favor of such organizational form by analyzing the benefits of another instrument, which is sometimes available to the principal—controlling the quality of the expert's private information without learning its content (hereafter, informational control).

Generally, informational control represents a situation in which the expert conducts an experiment and reports its outcome to the principal, whereas the principal determines the precision of the measurement device and makes a decision. In many situations, the principal can directly affect the quality of the expert's information. For instance, top managers restrict access to decision-relevant information for their subordinates. A typical example is the electric and electronics business Emerson, which 'looks like a company in which an organization man would feel at home. "Planning and control are central to the way Emerson works," according to the head Charles Knight. Communication, says Mr. Knight, is kept to a minimum: "Our planning and control cycle provides ample opportunity to communicate the most important business

[1] In a survey by McKinsey & Company (2007), 36% of respondents (executives) say that managers hide, restrict, or misrepresent information at least "somewhat" frequently when requesting funds.

[2] According to the Boston Consulting Group, centralization is still the most common type of organization. Moreover, companies with decentralized decision making and accountability have sometimes opted to centralize their structure. For example, Nestlé, a Swiss food and drinks group, initially was decentralized. This "was seen as the best way to cater to local taste and to establish emotional links with clients in far-flung places". Nevertheless, it recently centralized control over specific businesses and consolidated the management of its factories in individual countries into regions, even though the company's performance strongly depended on local preferences of consumers, which are known better by local management (*The Economist*, Aug. 5th 2004).

issues...we don't burden our system with non-essential communications and information' (*The Economist,* Jan. 21st 2006). Other similar examples can be found in medicine.[3]

In general, loss of information in communication implies that the expert possesses too much information relative to the amount that she is ready to reveal to the principal. Thus, some information is not essential for the principal's decisions because it is never discovered. However, properly restricting the expert's information fosters her incentives to reveal it truthfully. That is, the principal faces a trade-off between the precision of the expert's primary information and her incentive to convey it in further communication. As we demonstrate below, he prefers to restrict the expert's information to only decision-relevant information, which can be fully revealed, and to make this information as precise as possible.

As shown by Fischer and Stocken (2001) for some special cases, restricting the amount of information available to the expert can be better for the principal.[4] It is natural to ask whether restricting the availability of the expert's information does better than a more effective organizational form—full or partial delegation—and how the two modes of modifying the communication game interact with each other. This chapter addresses these questions. The key point of this chapter is a race between generalized versions of two instruments: informational control and delegation.

Our major contributions are as follows. First, we demonstrate that controlling the expert's information before communication is generally beneficial for the principal compared to both communication with the perfectly informed expert and delegation. In particular, informational control results in a higher payoff for the principal than optimal delegation if the bias in the preferences of the players is not very large relative to the principal's prior uncertainty about the unknown information. For the leading uniform-quadratic specification, informational control is

[3]Typically, doctors give advice on treatments of health problems, given results of the medical tests that determine a condition of a patient. However, this involves a conflict of interest, since the doctors' payment frequently depends on a chosen method of treatment. Clearly, a surgeon stands to get considerably higher payoffs by recommending surgery than by prescribing a special therapy or a drug. As a result, doctors often recommend unnecessary procedures. A study on surgery has found that for some procedures, the percentage of operations that were unwarranted was more than 50% (Consumer Reports on Health, 1998). Incidentally, many medical tests return results in a discrete form rather than by indicating, say, the exact concentration of some variable (a chemical, bacteria, etc.) Consider, for instance, a litmus paper test for pH measurement, or a variety of diagnostic tests, which return a "yes/no" answer, depending on whether a variable exceeds the cut-off level. This limited availability of information turns out to be a positive factor in mitigating the communication problem. Also, by approving the standard techniques and types of the tests, the FDA can change the quality of information available to doctors.

[4]Fischer and Stocken (2001) consider a particular uniform-quadratic case of the model and specific values of the bias in the players' interests. We generalize this result by showing that the principal can elicit more information from the expert in a wider class of environments as soon as communication is informative.

payoff superior to optimal delegation if and only if informative communication is feasible.

Second, if the principal wants to restrict the expert's information and delegate decision making afterwards, then the efficiency of a combined mechanism is purely determined by the principal's ability to affect the set of allowed policies. If the principal cannot restrict this set, then a combination of informational control and delegation cannot improve the more efficient instrument. In contrast, if the principal is able to affect the expert's discretion about decisions, then an optimal combination of these tools performs strictly better than any separate instrument. This result indicates that informational control is a universal incentive device that can effectively complement other mechanisms.

These findings have two implications which directly address the questions examined by Dessein (2002), who compares the performance of full delegation versus Crawford–Sobel (1982) communication. First, Dessein establishes that "the principal prefers to delegate control to a better informed agent rather than to communicate with this agent as long as the incentive conflict is not too large relative to the principal's uncertainty about the environment." This chapter shows that the first result is reversed as soon as the principal has the power to influence the precision of the expert's information. That is, as the divergence in preferences decreases, the principal prefers controlling the expert's information and keeping authority rather than delegating decision making. The argument in favor of communication becomes even stronger if one notices that full delegation is a special case of the restricted delegation. In the latter case, the principal can delegate decision rights only partially by restricting policies that can be chosen by the expert to prevent her from implementing, for example, extreme actions. Moreover, recent papers by Goltsman et al. (2007) and Kovac and Mylovanov (2006) show that restricted delegation is an optimal mechanism in a general space of arbitration mechanisms in which players interact through a neutral mediator, who collects information from the expert and gives enforceable recommendations to the principal. Thus, informational control allows the principal to gain from the expert's information and to keep full control over decisions without transferring it to the expert in delegation or a third party in arbitration.

Second, comparing communication versus full delegation, Dessein (2002) argues that "delegation is more likely when the amount of private information of the agent is large," where a measure of the informational advantage of the agent is the variance of the state. Since the agent's information is endogenous in our model, the variance of the state represents a measure of potentially available information of the agent. Given this measure, we demonstrate that the large variance extends the principal's opportunities to affect the agent's information without changing her incentives to reveal all observed information truthfully. Therefore, if the principal chooses the optimal quality of the agent's private information, then communication with the agent dominates full delegation to the agent with any quality of private information.[5] In other words, delegation is less likely when the amount of potentially available information of the agent is large.

Expert's information \ Organizational form	*Communication (no delegation)*	*Full delegation*	*Optimal delegation*
Perfect information			
Restricted information			

Figure 1: Payoff relations between instruments for small conflict in preferences

Figure 1 compares the performance of three organizational forms for different qualities of an expert's information (arrows represent the payoff dominance). The first form is pure communication, when the principal requests an expert's advice, but can use the obtained information to make an arbitrary decision. In the second case, the principal completely delegates authority to the expert or commits to comply with any expert's recommendation. Finally, the third form is optimal delegation, which imposes the policy restrictions on the set of expert's actions that maximize the expected payoff of the principal.[6]

As this figure shows, there is only one situation in which the principal prefers not to limit the precision of the expert's information. This is the case of full delegation, where the principal cannot affect the expert's discretion about the set of delegated decisions, which implies full loss of authority. However, even in this scenario, the principal can gain from influencing the expert's information and purely communicating with her afterwards.

Since our work contributes to the literature by comparing the benefits of different organizational forms, it is related to two areas of the existing studies: that which deals with various aspects of endogenous information in communication, and that which focuses on delegation.

[5] This results in a situation in which full delegation dominates communication in the case of the perfectly informed agent, but performs worse in the case of the imperfectly informed one.

[6] For a detailed discussion of the optimal delegation, see Alonso and Matouschek (2007).

With respect to the former topic, the first analysis of strategic communication is attributed to a seminal paper by Crawford and Sobel (1982). They introduce a model of the interaction between a perfectly informed expert and an uninformed principal whose payoffs depend on a random state of nature. After a private observation of the state, the expert sends a costless message to the principal, who implements an action. Crawford and Sobel (1982) show that full information revelation is never possible unless the players' interests are perfectly aligned. In addition, when a conflict of interest grows, the quality of the disclosed information falls, eventually resulting in the *babbling* equilibrium with no useful information conveyed.

The fact that the imperfect quality of an expert's information can be beneficial to the principal was first demonstrated by Fischer and Stocken (2001), who considered the uniform-quadratic setup of the Crawford and Sobel (hereafter, CS) model. They, however, restrict the set of possible biases in the players' preferences, introduced by Crawford and Sobel, to that of the discrete form, and analyze pure-strategy equilibria only.[7] Their main result is that the optimal structure of the information partition is uniform of a finite size, that is, equally spaced. This is not a general feature of the model for other values of the bias. In this chapter, we characterize the optimal partitional structure for an arbitrary bias. In general, non-uniform partitions can result in a higher expected payoff to the principal than delegation. This outcome, however, cannot be achieved with uniform partitions only.

Austen-Smith (1994) considers strategic communication with costly information acquisition. In particular, the expert can observe the state at some privately known cost. Also, the expert is able to prove that she has acquired information, but not the fact that she is uninformed. Intuitively, positive costs of information acquisition decrease the expert's incentives to acquire it and, as a result, the average quality of her information. However, introducing partial verifiability of the quality of the expert's information extends the range of biases for which informative communication is possible. In contrast, there are no such verifiability issues in our case since the principal determines the expert's information structure directly.

The issue of the endogenous quality of information has attracted a lot of attention in recent years. Lewis and Sappington (1994) consider the monopoly market in which a seller can allow buyers to acquire private information about the product. They demonstrate that the optimal policy of the seller is either to let buyers acquire perfect information or learn nothing. Ivanov (2007) applies that approach to a competitive setting with multiple sellers and differentiated

[7] In particular, they consider the bias $b = 1/2N$, where N is an integer.

products. In this case, when the market becomes sufficiently competitive, the result is full disclosure of information. Ganuza and Penalva (2006) extend the framework of Lewis and Sappington (1994) to an auction setting, in which the seller can supply costly information to the bidders. The result is that the auctioneer provides less than the efficient level of information. However, both the socially efficient and the auctioneer's optimal choice of precision increase with the number of bidders, and both converge as the number of bidders goes to infinity. In the area of mechanism design, Bergemann and Pesendorfer (2006) consider an auction in which the seller determines the precision of the valuations for each bidder and to whom to sell at what price. That is, the seller specifies the information structure for each bidder without learning their private signals. In this case, the information structures in the optimal auction are coarse and represented by the finite number of monotone partitions.[8]

Alternatively, there is an established literature on delegation or communication with commitment, where the principal commits to rubber-stamp any agent's recommendations if they belong to the specified delegation set. Dessein (2002) studies the benefits of the special forms of delegation—full delegation, delegating control to a biased intermediary, and delegation with a veto-power—and compares them with the benefits of pure communication. Holmström (1977), Melumad and Shibano (1991), and Alonso and Matouschek (2007) investigate the optimal restrictions on the set of delegated policies, which maximize the principal's expected payoff. Goltsman et al. (2007) and Kovac and Mylovanov (2006) extend that setup by introducing a disinterested arbiter between the parties, or, equivalently, by allowing the principal to commit to lotteries over actions as functions of the expert's reports. Whereas these studies consider the information structure of the expert as exogenous, this work connects the endogenous quality of information with delegating control over decisions to the expert.

Bester and Strausz (2001) and Krishna and Morgan (2005) analyze a different instrument to improve the quality of the conveyed information in communication—monetary transfers from the principal to the expert as the functions of messages. Bester and Strausz (2001) extend the revelation principle to the finite type environment in which the principal can commit only to some dimensions of the whole set of decisions. They show that any incentive efficient outcome (i.e., that which provides equilibrium payoffs on the Pareto frontier) is payoff-equivalent to the equilibrium outcome in some direct mechanism. Krishna and Morgan (2005) extend this result to

[8] An interesting property of the optimal structure is that the partitions are asymmetric across bidders even for symmetric distributions of the object's values.

the infinite type space and characterize the optimal contracts under both perfect and imperfect commitment. They demonstrate that the gains from contracting are highest for moderate values of the bias in preferences. Similar to these studies, we establish a model-specific revelation principle which narrows the set of the optimal information structures to only those in which the expert reveals all available information.

1.2 Examples

We start with the uniform-quadratic variant of the communication model introduced by Crawford and Sobel (1982). Two players, the uninformed principal (or the receiver) and the better informed expert (or the sender), communicate on some state of nature. The state is represented by a random variable θ, uniformly distributed on the unit interval. We will refer to the expert as "she" and the principal as "he". The expert sends a costless message m to the principal, who then implements an action a, which affects the payoffs of both players. The players' utility functions are quadratic:

$$U_R(a,\theta) = -(a-\theta)^2, \text{ and } U_S(a,b,\theta) = -(a-b-\theta)^2, \tag{1}$$

where parameter $b > 0$ reflects the bias in the players' interests.

Suppose first that the expert is *perfectly* informed about the state. Crawford and Sobel (1982) demonstrate that all of the equilibria are characterized by finite monotone partitions. That is, for any bias there are at most $N^{CS}(b)$ intervals on the state space so that the expert sends one message for each interval, which is associated with a corresponding action.[9] Also, there are exactly $N^{CS}(b)$ equilibria with $1, 2, ..., N^{CS}(b)$ intervals, where the equilibrium with $N^{CS}(b)$ intervals is Pareto superior to all other equilibria.

Example 1. Let the bias be $b = \frac{1}{5}$. In the most informative equilibrium, the expert sends a "low" message if the state is less than $\frac{1}{10}$, and a "high" message otherwise. Thus, if the principal receives a high message (which occurs with the probability $\frac{9}{10}$), his prior information is updated insignificantly. A low message is more informative, but the probability of receiving it is just $\frac{1}{10}$. The reason is that the principal knows the expert's motives to exaggerate information and tries to correct his actions correspondingly. As a result, if the principal gets a low message, he infers

[9] Formally, Crawford and Sobel (1982) define equilibrium strategies in a slightly different way to avoid probability zero messages. They require $m(\theta)$ to be uniformly distributed on $[w_k, w_{k+1}]$, if $\theta \in (w_k, w_{k+1})$, and $a(m) = E[\theta|\theta \in W_k]$ for all $m \in (w_k, w_{k+1})$.

that the expert's type has to be very low, whereas a higher message is more expected and thus is not very informative. That is, communication is effective for low states only. This results in the principal's expected payoff $U_R^{CS} \simeq -\frac{1}{16}$, which only slightly exceeds his payoff $-\frac{1}{12}$ in the case of no communication.

However, if the principal controls the expert's information in a such way that the expert observes only whether the state is higher or lower than $\frac{1}{2}$, then there is an equilibrium in which the expert truthfully reveals her information. This increases the principal's expected utility to $-\frac{1}{48}$. Moreover, there is an equilibrium with three messages—for a state less than $\frac{1}{5}$, between $\frac{1}{5}$ and $\frac{4}{5}$, and higher than $\frac{4}{5}$—which provides the expected payoff $U_R \simeq -\frac{1}{52}$. A finer information structure violates the expert's incentives to communicate truthfully, which results in the distortion of information and lowers the principal's payoffs.[10]

The intuition for this result is that the preferences of a less-informed expert become closer to those of the principal. In the CS case, the partitional structure is determined by marginal types who are indifferent between two consequent actions. In the above example, it is a single type $\theta_1 = \frac{1}{10}$. Technically, because of the expert's positive bias, the next higher action has to be far from the marginal type. Given the principal's decisions in response to received information (the conditional means of the state in the intervals), this is possible only if the next interval is sufficiently large. However, if the expert cannot distinguish among different states in a lower interval, this decreases her incentives to induce a higher action, because for all states in the interval the lower action is strictly better. That is, information imperfection replaces the marginal CS type by the mean type in the lower interval. As a result, finer partitions can be supported as equilibria than in the CS case.

Moreover, the beneficial effect of controlling the expert's information is so powerful that this organizational form can bring higher payoffs to the principal than delegation, as demonstrated in the example below.

Example 2. Let the bias be $b = \frac{1}{5}$. As shown above, the most informative equilibrium in the CS communication provides the principal's ex-ante payoff of approximately $-\frac{1}{16}$. However, if the principal delegates authority completely, that is, without restrictions on the set of expert's feasible policies, then for any state θ, the expert implements her optimal policy $\theta + b$, which has

[10] Like Crawford and Sobel (1982), we use the term "finer" informally, implying a partition with a larger number of elements.

a constant bias b relative to the principal's optimal policy θ. That is, both ex-post and ex-ante utilities to the principal are $-b^2 = -\frac{1}{25}$. The optimal delegation set $[0, \frac{4}{5}]$ brings the expected payoff $-b^2 + \frac{4}{3}b^3 \simeq -\frac{1}{34}$. Therefore, the principal's expected payoff in delegation is higher than that in CS communication. However, it is lower than his payoff $-\frac{1}{52}$ in the case of communication with an imperfectly informed expert.

In this context it is important to note that full delegation is not necessarily optimal in the space of all delegation sets, that is, sets of actions that can be delegated to the expert. Because of the expert's preferences toward higher actions, for high states she favors decisions that are never optimal for the principal. Excluding these extreme actions from the delegation set forces the expert to implement the highest possible action for the high states, which is close to the principal's optimal policies. Holmström (1977) proves that the optimal delegation set for the uniform-quadratic settings is the single interval $\left[0, \max\left\{1-b, \frac{1}{2}\right\}\right]$. However, the rest of this chapter will show that even optimal delegation performs worse than communication with the imperfectly informed expert.

1.3 The Model

Consider the standard setup of the CS model, in which the principal takes control of the quality of the expert's information about the state without observing its content. The key extension of our model is a preliminary stage in which the principal specifies the expert's information structure at zero cost.

Information structure. Before the expert privately observes a signal s about an unknown state of nature θ, the principal defines an expert's information structure, which is common knowledge. An **information structure** is defined by a triplet $\langle \mathcal{S}, \mathcal{M}, F(s|\theta) \rangle$, where $\mathcal{S}$ and $\mathcal{M}$ are measurable spaces of expert's signals and messages, respectively, and $F(s|\theta)$ is a conditional distribution function of a signal s for a given state θ. The state is assumed to be drawn from a twice differentiable distribution $F(\theta)$ with a density $f(\theta)$, supported on the unit interval Θ.

After observing a signal, the expert estimates a true valuation of the state, which is given by

$$\omega_s = E[\theta|s] = \int_{\Theta} \theta dF(\theta|s),$$

where $F(\theta|s)$ is a conditional distribution of the state θ given a signal s.

Thus, every information structure generates a prior distribution of posterior mean valuations

$$G(\omega) = \int_{\{s:\omega_s \leq \omega\}} dF(s),$$

where $F(s)$ is a marginal distribution of the joint distribution $F(s,\theta) = F(\theta)F(s|\theta)$.

We say that the information structure is **discrete** if $G(\omega)$ is supported on a finite set Ω.

Preferences. Throughout this chapter, we focus on the class of the quadratic utility functions (1) as the standard preferences, used in the related literature.[11] That is, the principal's payoff function $U_R(a,\theta)$ has a unique maximum for the action $a = \theta$ and the expert's payoff function $U_S(a,b,\theta)$ has a maximum for $a = \theta + b$. Thus, the specification of the model with respect to a distribution of the state and the preferences is identical to that in Krishna and Morgan (2001b).

Even though we restrict our attention to the case of quadratic preferences, we show that this assumption is not crucial for general results and can be relaxed later when we consider the role of risk aversion of the players.

The timing of the game. The game is played as follows. First, the principal specifies an information structure. Second, a realization of the state occurs, and the expert privately observes a signal. Then, the expert transmits a costless message to the principal. In general, the expert may mix over messages. After receiving the message, the principal updates his beliefs about the state and implements an action that determines the players' payoffs.

1.3.1 Equilibrium

Given an information structure $\langle \mathcal{S}, \mathcal{M}, F(s|\theta) \rangle$, a perfect Bayesian equilibrium (hereafter, equilibrium) consists of a signaling strategy $\sigma : \mathcal{S} \to \Delta\mathcal{M}$, which specifies a probability distribution $\sigma(m|s)$ over the space of messages for each signal; the principal's action rule $a : \mathcal{M} \to \mathcal{R}$; and a belief function $G : \mathcal{M} \to \Delta\Theta$, which specifies a probability distribution over Θ for each message m, including messages that are not sent in the equilibrium.[12] The belief function is constructed on the basis of Bayes' rule where applicable.

[11] See, for example, Blume et al. (2007), Goltsman et al. (2007), Krishna and Morgan (2001a, 2001b, 2004, 2005), Melumad and Shibano (1991), Ottaviani and Squintani (2006).

[12] For all messages $m \notin M$, we define the receiver's beliefs in such a way that he interprets them as some $m_0 \in M$.

The action rule $a(m)$ maximizes the principal's utility given the belief function[13]

$$U_R(a|m) = E_\theta [U_R(a,\theta)|m] = \int\limits_\Theta U_R(a,\theta)\, dG(\theta|m).$$

Given the action rule $a(m)$, the signaling strategy maximizes the expert's utility

$$U_S(a,b|s) = E_\theta [U_S(a,b,\theta)|s] = \int\limits_\Theta U_S(a,b,\theta)\, dF(\theta|s).$$

That is, the signaling strategy $\sigma(m|s)$ must satisfy

$$\text{if } \bar{m} \in \operatorname{supp} \sigma(.|s), \text{ then } \bar{m} \in \underset{m\in\mathcal{M}}{\arg\max}\, U_S(a(m), b|s), \text{ and} \tag{2}$$
$$\int\limits_{\mathcal{M}} \sigma(m|s)\, dm = 1, s \in \mathcal{S}.$$

Let $\mathcal{M}(\bar{a}) = \{m : a(m) = \bar{a}\}$. We say that an action $\bar{a}$ is **induced** by a signal s, if $\int\limits_{\mathcal{M}(\bar{a})} \sigma(m|s)\, dm > 0$, and is **purely induced** if $\int\limits_{\mathcal{M}(\bar{a})} \sigma(m|s)\, dm = 1$.

The principal's expected utility is

$$U_R = \int\limits_{\mathcal{M}} U_R(a(m)|m)\, dF(m),$$

where $F(m)$ is a distribution of the expert's messages.

The following subsection provides the general analysis of the model.

1.4 Equilibrium Characterization

Before we proceed to the general analysis, it is helpful to outline the class of optimal information structures and characterize the key properties of the players' strategies in optimal equilibria. We start with an observation that, in our environment, information takes a simple form. In particular, the players' preferences over policies are purely determined by the posterior values of the state, which represent expert's types. Then, we demonstrate that the inefficiency of the interaction with the fully informed expert is driven not only by the bias in players' interests, but also by the uncountability of generated valuations. The excessive number of posterior values

[13] Due to the strict concavity of the principal's utility function over actions, he never mixes between actions.

gives the expert unlimited opportunities to distort her information in the attempt to imitate a different type, which undermines the principal's belief in the expert's truth telling. Restricting the expert's information to a finite number of types substantially restricts such possibility. This forces all remaining types to reveal information truthfully and improves the efficiency of the communication between the parties as a whole. In fact, the optimal information structure does not provide the expert any decision relevant information that she will not reveal to the principal.

1.4.1 Transformation of utilities

Given any signal, the expert's utility function can be additively separated into two components:

$$\begin{aligned} U_S(a,b|s) &= -\int_\Theta (a-b-\theta)^2\, dF(\theta|s) = -(a-b-\omega_s)^2 - \int_\Theta (\theta-\omega_s)^2\, dF(\theta|s) \\ &= U_S(a,b,\omega_s) - D_s, \end{aligned} \tag{3}$$

where $D_s = \int_\Theta (\theta-\omega_s)^2\, dF(\theta|s)$ is the conditional residual variance of the state. It represents the informational losses of the expert, which always exist whenever the expert does not know the state precisely.

Similarly, the principal's utility function $U_R(a|s)$ can be written as

$$U_R(a|s) = U_R(a,\omega_s) - D_s. \tag{4}$$

From (3) and (4), it follows that given any expert's information, the players' preferences over actions are purely determined by the posterior valuations.[14] In addition, new utility functions (3) and (4) inherit all important properties of the initial ones (1): the strict concavity over actions, the single-crossing, and the symmetry with respect to optimal actions $a^S(\omega_s) = \omega_s + b$ and $a^R(\omega_s) = \omega_s$. This also implies the no-crossing property: $a^S(\omega_s) > a^R(\omega_s)$, $\forall\omega \in \Omega$. Since the posterior valuation ω_s is the only decision relevant information for both players, we denote it as a **type** of the expert.

Given the preliminaries above, we can apply the same technique as that developed in Lemma 1 in Crawford and Sobel (1982) to show that the number of induced actions in any equilibrium is finite. All proofs can be found in the Appendix.

[14] That is, $U_S(a,b|s) \geq U_S(a',b|s)$ if and only if $U_S(a,b,\omega_s) \geq U_S(a',b,\omega_s)$, and $U_R(a|s) \geq U_R(a'|s)$ if and only if $U_R(a,\omega_s) \geq U_R(a',\omega_s)$.

Lemma 1 *In any equilibrium, the set of induced actions $\mathcal{A}$ is finite. Further, the distance between any two actions is not less than* $2b$.

If the information structure is finite, then the number of actions is finite also, due to the fact that the strict concavity of the utility function (3) over actions guarantees that the expert of each type induces at most two actions. However, this lemma demonstrates that the finiteness of the number of actions comes from the bias in the players' interests rather than from the cardinality of the type space. Thus, an increase in the fineness of the information structure through generating a large number of posterior valuations does not eventually bring further informational benefits to the principal, since the expert chooses among a finite set of actions. Instead, this introduces additional incentive-compatibility constraints for each type. As a result, for a substantially fine information structure, the expert's signaling strategy is no longer invertible, which leads to losses in conveyed information and the principal's payoff. The next subsubsection formalizes this logic.

1.4.2 Discrete information structures

The following two subsections focus on characterizing the class of the optimal information structures and equilibria. We begin the analysis with the proof that an arbitrary information structure that generates a non-finite set of mean valuations is payoff equivalent to some discrete information structure. Since the set of induced actions is finite, the principal can collapse the finite number of the subsets of types that purely induce these actions. Because the number of types that induce two actions is finite as well, and each type induces the same action as in the initial equilibrium, a new equilibrium with a modified information structure does not change the principal's expected payoff.

Moreover, if there is an uncountable interval of mean values that induces distinct actions, then such an information structure is never optimal. The reason is that the information structure inherits the main source of the inefficiency of CS communication. Basically, this interval contains an expert's type, which is indifferent between two actions. Thus, a positive mass of types above the indifferent type purely induces a higher action. Because of the expert's positive bias, this action is more distant from the principal's optimal action than a lower one. The following lemma demonstrates that the principal can eliminate this inefficiency by linking these types to a lower action and get a strictly higher expected payoff without violating the incentive-compatibility constraints.

Lemma 2 *Any information structure is payoff equivalent to a discrete one. Also, if a type space includes an interval with a continuous density that induces two actions, then such information structure is strictly payoff inferior to some discrete one.*

We remarked already that the principal can always modify any information structure to a payoff equivalent discrete one. The transformation of the information structure for the second part of the lemma above requires three steps. First, we derive all types that induce two actions and modify their signaling strategies by attributing probability one to a lower action. Given a new strategy, an expert of each type purely induces some action. Second, consider the interval with a positive density that induces distinct actions, and extract a small subinterval above the indifferent type. Then, link this subinterval to the lower action by switching from purely inducing a higher action to a lower one. Finally, collapse all types that induce identical actions. The resulting information structure sustains a truth-telling equilibrium, which provides a strictly higher expected payoff than that in the initial equilibrium.

Though the lemma above narrows the set of optimal information structures to finite ones, it leaves much freedom in terms of the principal's payoff as a function of an expert's strategy. To address this issue, we need to formulate a model-specific revelation principle, which is described in the next subsubsection.

1.4.3 Equilibrium selection: revelation principle

The absence of the principal's ability to commit to actions results in the failure of the standard revelation principle, which restricts the set of all equilibrium outcomes to that of incentive-compatible equilibria, in which the expert conveys all information that she possesses. Two examples of contracting with imperfect commitment are due to Bester and Strausz (2001) and Krishna and Morgan (2005). In both cases, the expert of a binary type transmits three messages in equilibrium. No direct mechanism can replicate these equilibria in terms of induced actions and outcomes.[15]

Nevertheless, in our setup without monetary transfers, among the set of all possible equilibria with an arbitrary information structure, we can focus only on **incentive-compatible information structures**, which sustain a truth-telling, or **incentive-compatible, equilibrium**.

[15] The positive result of Bester and Strausz (2001) is that for a finite set of states any incentive-efficient mechanism (i.e., the one which provides equilibrium payoffs on the Pareto frontier) is payoff-equivalent to some direct mechanism. Similarly, Krishna and Morgan (2005) demonstrate that in the case of a continuum of types, any equilibrium outcome of an indirect mechanism can be replicated in a direct mechanism.

In particular, we show that any equilibrium is payoff inferior to some incentive-compatible equilibrium with a possibly different information structure. To prove this, we split the problem into two parts. First, we establish that no information structure can sustain an indirect equilibrium in which the cardinality of the action space exceeds that of the type space. Second, for any equilibrium, the principal can construct an incentive-compatible information structure that provides a (weakly) superior expected payoff in the incentive-compatible equilibrium.

Lemma 3 *Any equilibrium is direct, that is, the cardinality of the action set does not exceed that of the type space.*

The result above is proved by contradiction. Formally, in any indirect equilibrium there must be a type which induces two actions so that the higher action is induced by this type only. Because of the principal's best response, the higher action coincides with this type, and is less than the expert's optimal action. Hence, a lower action is even more distant from the expert's optimal one. The concavity of the expert's utility function implies that the expert cannot be indifferent between the induced actions.[16]

Now, we can constitute a model-specific revelation principle, which states that any optimal equilibrium payoff can be replicated with an incentive-compatible information structure.

Lemma 4 *Any equilibrium is payoff inferior to some incentive-compatible equilibrium.*

The superior incentive-compatible equilibrium is constructed in two steps. First, we derive all types that play mixed strategies and assign probability one to a lower action. Since for an arbitrary expert's type, a lower action is closer to the principal's optimal policy, a new signaling strategy provides higher ex-post payoffs to the principal. However, this argument is incomplete because of the different incentive-compatibility constraints of the expert, which stem from the fact that the principal adjusts his best response to a new signaling strategy. To resolve this issue, the principal has to collapse all types that induce identical actions. As a result, we obtain an information structure, which is incentive-compatible and payoff superior to the initial one.[17]

[16] In Krishna and Morgan's (2005) example of an indirect equilibrium, the main incentive for an expert of the higher type to induce a lower action is a higher payment for sending lower messages, which is sufficient compensation for a less desirable policy implemented afterwards. The lack of such transfers in our setup narrows the set of equilibria.

[17] Hereafter, by saying that the information structure is incentive-compatible and provides a payoff to the principal, we imply a payoff in the incentive-compatible equilibrium.

The main implication of this result is that the principal never wants to provide the expert with information that would still be her private knowledge after she sends a report to the principal. However, the cost of truth-telling is the expert's informational losses, which the principal needs to minimize. In other words, the principal has to compromise between the losses of the expert's primary information and those in further communication. The next subsection estimates the efficiency of this trade-off.

1.5 The Value of Controlling Information

How powerful a tool is informational control? That is, when does the principal benefit from restricting the expert information? How does the value of controlling information depend on the degree of commitment? In this subsection, we compare the efficiency of informational control versus two main forms of interaction between the principal and the perfectly informed expert: CS communication and optimal delegation. In the next subsections, we analyze the interaction between the quality of the expert's information and the degree of commitment.

For these purposes, it is important to note that, although the results in the previous subsection help characterize the class of the optimal information structures and equilibria, they do not provide much information about the principal's payoff as a function of the information structure. Thus, before the formal analysis, it is helpful to specify a class of the information structures which shares the properties of the optimal information structures, provides a simple characterization of the principal's expected payoff, and is not difficult to implement.

All of these requirements are satisfied by the simplest form of discrete information structures: **partitional**, in which the expert observes an interval of states that contains the true state, but not the state itself. That is, the state space is partitioned into a finite number N of intervals $\{\Theta_k\}_{k=0}^{N-1} = (\theta_k, \theta_{k+1}]_{k=0}^{N-1}$. Equivalently, a partition can be described by a strictly increasing sequence $\{\theta_k\}_{k=0}^{N}$ of its boundary points, or a positive sequence of interval lengths $\{\Delta\theta_k\}_{k=0}^{N-1}$, where $\Delta\theta_k = \theta_{k+1} - \theta_k$.[18] Also, it generates a discrete type space $\Omega = \{\omega_k\}_{k=0}^{N-1}$, where each type $\omega_k = E[\theta | \theta \in \Theta_k]$ is generated with a probability $P(\omega_k) = F(\theta_{k+1}) - F(\theta_k)$. Since the expert observes only an element of the partition Θ_k, but cannot distinguish among different states in

[18]Notice that the described partitional information structure assumes the monotonicity of partitions. That is, $\theta \in \Theta_k, \theta' \in \Theta_j, j > k$ implies that $\theta < \theta'$. A monotone form of a partitional information structure is chosen for convenience. Notice that all characterized equilibria in the CS model have a monotone partitional structure. Thus, the monotonicity is useful for comparing, for example, the distribution of informational losses in the CS case and that in our model through comparing the number and lengths of the intervals in the information partitions.

an interval, her information is imperfect and determined by the informational losses

$$D_k = \frac{1}{P(\omega_k)} \int_{\theta_k}^{\theta_{k+1}} (\theta - \omega_k)^2 \, dF(\theta) .$$

Intuitively, the principal cannot benefit from introducing noise in the expert's information in an arbitrary way. For example, a natural "truth-or-noise" information structure in which, with some probability, the expert observes a true state, and with a complement probability, observes an indistinguishable random draw from the same distribution, results in an inferior payoff compared to that in CS communication.[19] As we demonstrated above, the optimal information structure must be of a discrete form. Though there are multiple ways to generate a discrete structure, not all of them can be easily implemented. In general, the principal has to specify a continuum of conditional probability distributions that map each state of nature into a set of mean valuations, which can be a difficult task. In contrast, a partitional structure requires only attributing information to an element of a category.

Implementation of the information structure. As previously mentioned, informational control represents a situation in which the expert conducts an experiment and submit a report to the principal, whereas the principal is responsible for determining the precision of the measurement device and making an action. Consider, for example, the management information systems (MIS), which are broadly used in many organizations. These are typically computer-based systems that collect, process, and convey information, which is used to support managers' current decisions and provide reports to the principals.[20]

[19] The proof is available upon request. For additional information on the "truth-or-noise" information structures, see Johnson and Myatt (2006).

[20] Most organizations are structured along functional lines, and the typical systems are financial MIS, which provides financial information to all financial managers within an organization including the chief financial officer (CFO); or marketing MIS, which supports managerial activity in the area of product development, distribution, pricing decisions, promotional effectiveness, and sales forecasting (*Encyclopedia of Business*, by The Gale Group, Inc.) However, financial or marketing departments are not the highest hierarchical levels of companies. For instance, CFOs report directly to the chief executive officers (CEO) and are responsible for analyzing and reviewing financial data, reporting financial performance, preparing budgets and monitoring expenditures and costs. The CFO is required to present this information to the board of directors at regular intervals and provide this information to shareholders and regulatory bodies such as the Securities and Exchange Commission (Investopedia.com). However, according to a survey by McKinsey & Company (2007), more than 80% of respondents say that CEOs play an important role in investment decisions regarding R&D, capacity expansion, or withdrawing funds from underperforming projects. Hence, the executives have to consult the associated division managers before making decisions about, say, fulfilling current or future financial needs or production quantities. The importance of this communication is highlighted by an example of the Enron, where the CFO was the key figure behind concealing massive losses and misleading investors, which eventually resulted in the collapse of the whole company.

However, the structure and the quality of informativeness of MIS are determined by top management. Also, it is an easy task to design and program the system so that it represents information in a discrete and partitional form, by rounding information to the required unit of measurement or associating it with an element of a category. In the simplest case, it returns a "yes/no" answer, which is similar to Example 1 above. For instance, it is known that the interests of managers have systematic biases relative to those of the companies.[21] Thus, if a manager of a chain store has to prepare a report for headquarters about local market conditions that can affect the headquarters decision about his store, then an outcome of the information system (e.g., the analysis of consumers' expenditures or a market survey) regarding the size of a local market can take a partitional form: the system returns "large" if the size exceeds some cut-off level, and "small" otherwise.

There is another factor that is important for our choice of information structures. In particular, focusing on the partitional setup does not affect qualitative results about the efficiency of informational control. Using a simple characterization of the players' expected payoffs, we show below that informational control with monotone partitions dominates optimal delegation if the players' interests do not differ significantly. In contrast, if they are far apart, then any incentive-compatible information structure generates no more than two types. We show that any such structure is inferior to a partitional structure, which is, however, not necessarily better than optimal delegation. Moreover, in the uniform-quadratic case, this assumption does not play any role in identifying scenarios in which informational control is more beneficial than delegation. This follows from the fact that informational control dominates optimal delegation *if and only if* there exists a partitional structure that is superior to optimal delegation. Thus, expanding the class of information structures cannot qualitatively improve the result by showing the unconditional dominance of informational control over delegation.

Given these preliminaries, the next subsubsection examines the performance of informational control versus CS communication.

1.5.1 Informational control versus CS communication

In this part of the chapter, we establish the dominance of informational control over CS communication. The only condition necessary to guarantee this result is that the divergence in

[21] See, for example, Alonso, Dessein, and Matouschek (2007) and Dessein (2002).

the preferences must not be extremely large, that is, informative CS communication is feasible.

Theorem 1 *If CS communication is informative, then informational control is payoff superior to CS communication.*

In general, any informative CS partition is characterized by an unequal distribution of the informational losses across the state space. Intuitively, the expert with the preferences toward high states of nature has incentives to exaggerate information, proportionally to the level of her bias. Thus, the expert's message that the state is high is more expected by the principal than that the state is low. This weakens the principal's trust to the messages about high states and makes them less informative relative to those about low states. As a result, the quality of communication decreases monotonically in the value of the state of nature. This intuition is reflected in unequal sizes of the associated partition elements in any CS equilibrium.

The argument above crucially relies on the fact that the expert has virtually unlimited possibilities to distort her information by claiming a different state of nature since the set of possible states is infinite. However, the principal can limit this possibility by specifying a discrete information structure in which the expert's information is determined by only a few points. Such an information structure makes slight exaggerating impossible. Since the principal knows the set of expert's valuations, he would not believe that the expert's information is somewhere between these points. On the other hand, substantial exaggerating by reporting another possible value is very costly, as it induces an undesirable action that is far from the expert's first best decision. This does not leave the expert any choice but to communicate truthfully. Thus, if the informational losses of the expert's primary information are lower than those in communication with a perfectly informed expert, then the overall effect of informational control is positive.

The formalization of this intuition is as follows. The principal can take any CS partition as a basic one and locally modify it in such a way that the variance in the intervals' lengths becomes smaller. Note that for almost all states in each interval of a CS equilibrium, the associated action is strictly preferred to all other actions. Therefore, it is strictly preferred by a mean type of the interval. That is, a local modification of the partition does not affect the incentives of mean types. However, a more effective distribution of the informational losses relative to those in CS interaction results in a strictly higher principal's payoff.

1.5.2 Informational control versus optimal delegation

Delegation is broadly considered as an alternative to communication. Instead of relying on the expert's non-verifiable information, the principal can delegate his power to the expert and gain from her superior information.[22] However, the informational benefits are mitigated by losses due to a bias in the expert's decisions. Nevertheless, in a variety of situations, the aggregate effect leads to an ex-ante Pareto improvement compared to communication with a fully informed expert. Another useful feature of delegation is its ease of implementation: generally, there are no costs of empowering the expert with a right to carry out policies. Due to these factors, many firms push decision rights down in the hierarchy.[23]

Moreover, delegation can be a solution to the optimal mechanism in a general class of stochastic mechanisms in which the principal commits to a lottery over actions as a function of the expert's report. In other words, this is a class of arbitration mechanisms in which the parties interact through a disinterested mediator who collects information from the expert and uses it to give (stochastic) recommendations about decisions to the principal. In addition, the mediator's choice is binding, since the principal commits to agree to a mediator's advice. In a recent paper, Kovac and Mylovanov (2006) show that for the quadratic preferences and a small bias in players' interests, the optimal arbitration mechanism is restricted delegation with a delegation set, which consists of a single interval.

Despite the seemingly obvious benefits of delegation, however, a surprising number of companies today still have the centralized structure. In fact, it remains the most popular organizational form. Moreover, companies that do decentralize decision making and accountability often centralize it again when they run into trouble.[24] At least two factors may contribute to explaining the popularity of centralized control. First, there is a commitment problem. It is not clear through what means the principal can commit to rubberstamp the expert's decisions. Given the fact that the expert's action reveals her information precisely, the attractiveness of the first-best decision makes it difficult for the principal to get rid of the incentives to overrule the previous choice. Since the expert has reasons to anticipate such behavior, he never reveals full information, which collapses all the benefits of delegation. Second, if the principal

[22] See, for example, Alonso and Matouschek (2005), Dessein (2002), Holmström (1997), and Melumad and Shibano (1991).

[23] For examples, see Dessein (2002).

[24] For example, Motorola had a decentralized structure by the mid-1990s. However, then the company's mobile-phone business was growing so fast that decentralization made it impossible to control. In 1998, the company repatriated control to the headquarters (*The Economist,* Jan. 21st 2006).

optimizes over the set of delegated decisions, then the optimal delegation mechanism may take a complicated form, making the clear description of the optimal contract between the parties involved essentially difficult.[25]

In addition, the example of Emerson, mentioned above, illustrates that keeping control over decisions is generally not independent from keeping control over information. Thus, when comparing the performance of different organizational forms, we have to consider the possibility that the principal may restrict the expert's information.

Technically, delegation and informational control utilize different factors for payoff improvement. Delegation allows the principal to acquire benefits from the expert's informational advantage at a cost of losing control over decisions, whereas controlling the expert's knowledge restricts her possibilities to misrepresent information at a cost of a lower quality of the expert's primary information. Thus, at first glance, there seems to be no clear intuition about which effect is generally stronger. The following result shows that informational control performs better than optimal delegation, when the players' interests are not too far apart. This finding is in stark contrast to that of Dessein (2002), who demonstrates that delegation is more likely than communication when the players' preferences are close.

Theorem 2 *There exists a bias $\bar{b}$ such that for all biases below $\bar{b}$, informational control is payoff superior to optimal delegation.*

The intuition behind this result can be explained in two steps. First, compare informational control with full delegation. By Lemma 1, incentive-compatibility requires that the distance between any two actions is at least $2b$. In any truth-telling equilibrium, however, the action space coincides with the set of generated types. This implies that, as the bias declines, the distance between adjacent mean values, and as a result, the size of the intervals in the finest incentive-compatible partition, falls also. That is, the variation of the density on each partition's element decreases with a size of this element. Hence, all mean values converge to the middle points of the associated intervals. As a result, the size of the intervals in the finest incentive-compatible partition converges to $2b$ *regardless* of the distribution of states, and the principal's expected losses in the most informative equilibrium tend to $-\frac{1}{12}(2b)^2 = -\frac{b^2}{3}$. These are expected losses in the case of a uniform distribution, which are three times lower than those in full delegation, $-b^2$.

[25] Indeed, the optimal delegation mechanism can be stochastic (Kovac and Mylovanov, 2006). That is, the contract has to specify all lotteries over decisions as a function of each possible expert's message.

Second, Kovac and Mylovanov (2006) show that when the bias falls, the principal's incentives to restrict the set of expert's decisions decrease as well. This is reflected in the behavior of the optimal delegation set, which converges to the entire state space. That is, the benefits of restricted delegation become negligible relative to that of full delegation, which implies that the previous argument still holds.

Note that, instead of considering a fixed distribution of the state and a variable bias, we can fix the bias and change the scale of the distribution.[26] Then, using the variance of the state as a measure of the informational advantage of the expert (see Dessein, 2002), a larger informational advantage results in finer incentive-compatible information structures. That is, even though the principal's uncertainty about the state grows, he obtains more possibilities to modify the initial distribution without breaking truth-telling communication, which eventually results in the payoff dominance over delegation.

1.6 The Uniform-Quadratic Case

In this subsection, we focus on the particular uniform-quadratic setup of the model, which has been a central framework for a significant part of the related economic and political science literature. This case with a uniform distribution of states and quadratic preferences is known for its flexibility to modifications of the basic CS model and the possibility to obtain closed-form solutions in various applications.[27]

Investigating the uniform-quadratic case allows us to sharpen the previous results. First, we obtain an explicit characterization of the optimal partition. Second, we employ the exact characterization of the optimal delegation set for an arbitrary value of the bias. Combining these components, we demonstrate that controlling the expert's information performs strictly better than optimal delegation if and only if informative communication is feasible.

1.6.1 Optimal information structure

In this subsubsection, we characterize the optimal information partition. By the above argument about the discrete nature of the optimal information structure, it is always bounded away from perfect information as long as the player's interests are imperfectly matched. Further, the

[26] That is, we can consider a family of random variables $\theta_L = L\theta$, parametrized by the scale factor L.

[27] See, for example, Blume et al. (2007), Gilligan and Krehbiel (1987, 1989), Krishna and Morgan (2001a, 2004, 2005), Melumad and Shibano (1991), Ottaviani and Squintani (2006).

structure of the optimal partition substantially differs from the CS partitions in a few aspects. First, the cardinality of the optimal partition grows much faster as the bias in preferences tends to zero. Second, the optimal partition allocates informational losses more efficiently, but not necessarily uniformly, across the state space. Finally, the Pareto superior partition does not necessarily have the largest number of elements, among all incentive-compatible ones.

Since, by Lemma 4, we can restrict attention to incentive-compatible equilibria only, one can observe that the expert's incentives have a simple form. In particular, an expert of any type ω_k prefers to induce an action a_k instead of a_{k+1} if and only if

$$\omega_k + b \leq \frac{a_k + a_{k+1}}{2}, \forall k \tag{5}$$

and never induces an action $a < a_k$, since the principal's best response implies $a < a_k = \omega_k < \omega_k + b$, and the strict concavity of the utility function results in $U_S(a, b|\omega_k) < U_S(a_k, b|\omega_k)$. In addition, plugging $a_k = \omega_k = \frac{\theta_k + \theta_{k+1}}{2}$ into (5) results in

$$\theta_{k+2} - \theta_k = \Delta\theta_{k+1} + \Delta\theta_k \geq 4b, k = 0, 1, ..., N-2. \tag{6}$$

The family of inequalities (6) determines the incentive-compatibility (IC) constraints. These conditions are an analogue of the CS no-arbitrage conditions $\theta_{k+2} = 2\theta_{k+1} - \theta_k + 4b$, which can be rewritten as

$$\Delta\theta_{k+1} = \Delta\theta_k + 4b.$$

Comparing the last two expressions, one can make a few observations. First, constraints (6) are less restrictive, which implies that the principal can specify a finer information structure in the informational control model than in the CS setup.[28] Second, the CS arbitrage condition implies that the length of any interval in a CS partition must exceed that of the previous interval by $4b$. In contrast, (6) excludes the CS inefficiency of communication for high values of the state.

To find the optimal incentive-compatible partition, we first determine the maximal size of the incentive-compatible partition $N(b)$. It can be shown that

$$N(b) = \lfloor \frac{1}{4b} \rfloor + \lceil \frac{1}{4b} \rceil, \tag{7}$$

[28] Actually, any CS partition satisfies (6).

where $\lfloor x \rfloor$ is the largest integer smaller than or equal to x, and $\lceil x \rceil$ is the lowest integer bigger or equal to x.

Notice that for $b > \frac{1}{4}$, informative communication is not feasible. However, for $b = \frac{1}{4}$, the finest partition is a uniform two-element one, where we say that a partition is **uniform of size** N if it consists of N intervals of the same lengths, that is, $\Delta\theta_k = \frac{1}{N}, \forall k$. Thus, communication is informative in contrast to the CS case. The next proposition describes the structure of the optimal partition for an arbitrary bias in preferences.

Lemma 5 *For any b, there exists $b_c \in (\frac{1}{2c}, \frac{1}{2(c-1)})$, where $c = N(b)$, such that if $b > b_c$, then the optimal partition is uniform of size $c-1$. For $b \leq b_c$, the optimal partition is one of size c such that: 1) if $\frac{1}{2c} < b \leq b_c$, then the IC constraints are binding for all $\{\theta_k\}_{k=0}^{N(b)}$, and 2) if $\frac{1}{2(c+1)} < b \leq \frac{1}{2c}$, then the optimal partition is uniform.*

The stark difference of the optimal information partition as compared to endogenous CS partitions is that the principal does not always prefer the partition with the highest number of elements. Basically, the optimal partition highlights a trade-off between two different structures. One of them is uniform, so that it efficiently shares the informational losses of the risk averse principal, whereas the other benefits him because of the possibility of better responding to expert's messages through a higher number of actions. Notice that the latter structure is never optimal in the model of Fischer and Stocken (2001) due to a special choice of the bias in their model. The cut-off levels b_c are exactly the biases, at which these information structures are payoff equivalent.[29]

The structure of the optimal partition demonstrates that the cardinality of the optimal partition and the distribution of the informational losses in it are both crucial factors that determine a payoff dominance of informational control over delegation. The cardinality of the finest partition grows as $1/b$ in the information control model relative to $1/\sqrt{b}$ in the CS case. As a result, the principal is able to respond to changes in the state more sensitively. In addition, the variance in the lengths of the partition elements is essentially smaller than that in CS case, since the incentive-compatibility constraints (6) impose fewer restrictions on the

[29] For instance, for the bias $b = \frac{1}{5}$, we have $N(\frac{1}{5}) = 3$ and the cutoff level $b_3 = 0.202$. The principal's expected payoffs under the three-element partition with the binding IC constraints $\{0, 0.2, 0.8, 1\}$ and the two-element uniform partition $\left\{0, \frac{1}{2}, 1\right\}$ are $-\frac{1}{52}$ and $-\frac{1}{48}$, respectively. However, as the bias grows to 0.22, the IC constraints make the finest incentive-compatible partition less uniform, so it becomes $\{0, 0.12, 0.88, 1\}$, which decreases the payoff to approximately $-\frac{1}{27}$. In contrast, this change in the bias has no effect on the uniform partition of a smaller size, which is still incentive-compatible.

functional relationship between lengths of different intervals. Hence, informational losses do not vary significantly with the value of the state.

1.6.2 Informational control versus optimal delegation

In the uniform-quadratic specification, the optimal delegation set consists of a single interval $[0, 1 - \min\{b, 1/2\}]$ (Holmström, 1977). Moreover, it is an optimal solution among a more general class of arbitration mechanisms for any level of bias (Goltsman et al., 2007; Kovac and Mylovanov, 2006). Then, combining the exact solutions for optimal delegation and the partitional structure results in a simple criterion that allows us to outline all scenarios, when informational control is more attractive than delegation. In particular, a necessary and sufficient condition for this is the feasibility of informative communication.

Theorem 3 *In the class of all information structures, informational control is payoff superior to optimal delegation if and only if informative communication is feasible.*

The intuition behind this result relies on the same background as used in Theorem 2. However, using the solutions to the optimal delegation set and information partition, we can show that the critical level of the bias $\bar{b} = 1/4$ is exactly the one, which sustains informative communication. In contrast, if the bias exceeds $\bar{b}$, then there is no an information structure with informative communication, which brings the principal only the "uninformed" payoff given his prior information. Moreover, expanding the class of information structures has no effect on the outcome of communication. Intuitively, the simplest feasible informative interaction involves just two mean values that are associated with two actions. However, among all information structures that generate two posterior valuations, the two-element partition has an important feature: it maximizes the distance between the values. As a result, shifting a higher action too far from the optimal point of a low type expert forces her to communicate truthfully, because of higher punishment for claiming a higher value. That is, for a large bias in the preferences, interaction is informative if and only if it is informative with the partitional information structure.

Thus, as soon as there is a scope for informative communication, the principal is better off controlling the expert's information than delegating authority to the expert. Figure 2 demonstrates the principal's expected payoff under the optimal partition in informational control, optimal delegation, and CS communication.[30]

[30] A specific feature of our model is that the principal's expected payoff is discontinuous in the bias due to the "regime changing" effect. When the bias falls, this effect takes place at points $b = \frac{1}{2N}$, where N is an even integer.

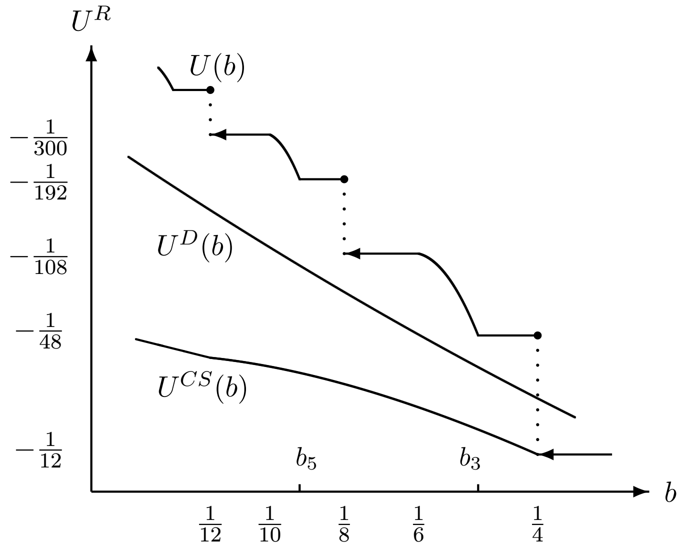

Figure 2: Payoffs in informational control, optimal delegation, and CS communication

The above discussion raises a natural question of whether the result is driven by the specific quadratic preferences of the players or it can be replicated in a more general specification. The next subsubsection illustrates that extending the class of utility functions generally does not change the result.

1.6.3 The role of risk aversion

This part examines the robustness of the previous results to changes in the players' risk aversion. These results generally hold whenever the difference in players' interests is not too large relative to the principal's uncertainty about the environment.

To isolate the effects of risk-aversion consider the symmetric form of the players' preferences similar to that used by Dessein (2002):

$$U_R(a, \theta) = U(|a - \theta|), \tag{8}$$

where $U(.)$ is twice differentiable, $U'(0) \leq 0$, and $U''(x) < 0$.[31] If $U'(0) = 0$, we additionally

At these points, the maximal size of the incentive-compatible partitions changes from $N-1$ to N and the uniform partition of this size becomes incentive-compatible. As a result, the optimal partition switches from the uniform of size $N-1$ to the uniform partition of size N, and the expected payoff jumps from $-\frac{1}{12(N-1)^2}$ to $-\frac{1}{12N^2}$. In contrast, the incentive-compatibility of a partition of an odd size does not guarantee that the uniform partition of the same size is incentive-compatible. Thus, the optimal partition of an odd size can be non-uniform and provide an expected payoff, which is continuous in b. Hence, the switch between a uniform partition of an even size $N-1$ to a non-uniform partition of an odd size N at points b_N is not accompanied by a discontinuous change in payoffs.

[31] Formally, Dessein (2002) also specifies normalization components in the players' utility functions, which do not affect the results.

require $U'''(.)$ to be continuous in the neighborhood of 0.

Similarly, the expert's utility function $U_S(a, b, \theta)$ is

$$U_S(a, b, \theta) = V(|a - b - \theta|), \tag{9}$$

where $V'(x) \leq 0$ and $V''(x) < 0$. For future references we will refer to (8) and (9) as **symmetric preferences**.[32]

Given a partitional information structure, the players' type-relevant utility functions $U_S(a, b|s)$ and $U_R(a|s)$ are concave in a and symmetric around the optimal policies $\omega_s + b$ and ω_s, respectively. This implies that the principal's action rule in an incentive-compatible equilibrium is the same as for quadratic preferences, and the expert's incentives are not affected either.[33] That is, communication is informative only if $b \leq 1/4$. Similarly, the arbitrage condition in the CS model is not affected, which implies that CS equilibria are invariant to this modification in preferences. Based on these observations, the dominance of informational control over CS communication can be proved straightforwardly.

Theorem 4 *If the state is uniformly distributed and preferences are symmetric, then informational control is payoff superior to informative CS communication.*

Since IC constraints are not affected by the change in preferences, the optimal information partition preserves all positive properties of that in the uniform-quadratic case, such as a finer structure and efficient sharing of informational losses. In particular, given any CS partition, the uniform partition of the same size is incentive-compatible in our model and provides strictly higher expected payoff.

Before we compare the principal's payoffs in informational control with that in delegation, notice that the optimal delegation set is still of a form $[0, 1 - \min\{b, 1/2\}]$ by Proposition 3 in Alonso and Matouschek (2007). Then we can replicate the result of Theorem 2: informational control performs better than the optimal delegation, when the bias in the players' interests is not too large.

[32] Krishna and Morgan (2004) consider a special case of such preferences, namely, $U_R(a, \theta) = -|a - \theta|^\rho$ and $U_S(a, b, \theta) = -|a - b - \theta|^\rho$, where $\rho \geq 1$.

[33] In addition, the optimal partition is the same as that determined by Lemma 5 up to the values of the switching points b_c between uniform partitions of size $c - 1$ to non-uniform partitions of size c.

Theorem 5 *If the state is uniformly distributed and the preferences are symmetric, then there exists a bias $\bar{b}$ such that for all $b < \bar{b}$, informational control is payoff superior to optimal delegation.*

This result is weaker than Theorem 3 for the case of the quadratic preferences since it does not guarantee that the controlling information performs better than delegation whenever informative communication is feasible. Basically, this result cannot be strengthened because of the risk aversion of the principal.[34] In communication, an induced action is unbiased on average, but there is a chance of making a wrong action (if a state is close to a boundary of a partition element). This increases informational losses for highly concave utility functions. Delegation, however, provides a permanent bias in the expert's decision, which can be more preferable by a very risk averse principal. Nevertheless, when the bias decreases, the optimal information structure becomes sufficiently fine to reduce the variance between optimal and induced actions, which results in better performance of informational control over delegation.

Thus, if the bias is moderate, then the relationship between the principal's payoffs in different organizational forms strongly depends on the structure of the information partition. For instance, for $b = \frac{3}{17}$ and $U(|a-\theta|) = -|a-\theta|^4$, the optimal information structure is the three-element partition $\left\{0, \frac{5}{17}, \frac{12}{17}, 1\right\}$. It provides the expected payoff -2.03×10^{-4}, which exceeds that in optimal delegation -6.96×10^{-4}. However, restricting information structures to only uniform partitions gives a lower payoff -7.81×10^{-4}, because the three-element uniform partition is not incentive-compatible, whereas two-element partitions are too coarse.

1.6.4 Delegation to an Imperfectly Informed Expert

This part investigates the situation, in which the principal can use both of the analyzed instruments—delegating control over decisions and restricting the quality of the expert's information—if a combined effect from utilizing them is positive. For example, a top manager can restrict the employee's access to information and delegate a task afterwards. Moreover, he can determine the set of policies from which the employee is allowed to choose.

The analysis above shows that delegation can outperform informational control, if the players' interests significantly diverge or the principal is highly risk averse. However, even in this case, the

[34] Consider the principal's utility function $U_2(|a-\theta|) = -|a-\theta|^7$, and the bias $b = 0.126$. Then the optimal partition in the informational control model is the uniform three-element one. It is informative and provides expected payoff $U_R \simeq -4.5 \cdot 10^{-7}$. However, optimal delegation gives $U_R^D \simeq -3.9 \cdot 10^{-7}$, which is superior to that in informational control.

principal may have incentives to deteriorate the expert's information. Our main finding is that the total effect from using both instruments simultaneously purely depends on the principal's ability to restrict the set of delegated policies. An example below illustrates this argument.

Example 3. Consider the uniform-quadratic setup with the bias $b = \frac{1}{5}$. If the expert is perfectly informed, then the principal's payoffs in the cases of full and optimal delegation are $-b^2 = -\frac{1}{25}$ and $-b^2 + \frac{4}{3}b^3 = -\frac{1}{34}$, respectively. Thus, the principal's losses from choosing the delegation set optimally decrease by 36%. Remember that CS communication and informational control provide the payoffs $-\frac{1}{16}$ and $-\frac{1}{52}$, respectively.

On the other hand, if the expert's information structure is a three-element partition $\left\{0, \frac{3}{10}, \frac{7}{10}, 1\right\}$, then full delegation brings the principal a payoff of approximately $-\frac{1}{20}$. In contrast, the three-action delegation set $\{0.17, 0.53, 0.87\}$ results in a payoff of $-\frac{1}{96}$. That is, the principal's losses fall by almost five times.

1.6.5 Full delegation

The example above illustrates that in the case of full delegation, the perfectly informed expert performs better than the imperfectly informed one. This is true in general. If there are no restrictions on the delegation set, then controlling the expert's information before delegating authority is always detrimental. Intuitively, given any precision of the expert's information, full delegation implies that the bias between the expert's decision and the principal's optimal policy will be the same as in the case of the perfectly informed expert. In addition to this bias, reducing the quality of information only introduces extra losses of information.

The immediate corollary of this fact is that a combination of informational control and full delegation cannot improve the better instrument of these two. For the quadratic preferences, the above logic is shaped into the following lemma.

Lemma 6 *A combination of informational control and full delegation is payoff inferior to the more efficient instrument.*

That is, in the principal's choice about controlling the expert's information and transferring decision rights simultaneously, he has to use only one of these organizational forms. A question about which form is a better one, given the degree of a conflict between the players, is not difficult. From the previous results, it is clear that the second instrument can be more beneficial only if the bias is sufficiently large, but not extreme. If the players' interests are closely aligned, then informational control dominates optimal delegation, which is superior to unrestricted delegation. Moreover, if the preferences are far apart, then very biased expert's decisions in delegation cannot improve an uninformed payoff. Therefore, the principal gains from delegating rights to the perfectly informed expert only if the players' interests are moderately close.[35]

To summarize, the results above put some restrictions on the area of applications of informational control. If the principal cannot restrict the expert's discretion because of limited monitoring or enforcement possibilities, then he should never decrease the precision of the expert's information before delegating authority to her. In contrast, having the ability to affect the set of delegated decisions completely reverses the effect of interaction of these two incentive instruments.

1.6.6 Restricted delegation

Given the finding above, controlling the expert's information before delegating power to her can be beneficial only if the principal is able to restrict the set of expert's decisions. In this case, controlling both information and the delegation set cannot perform worse than pure communication with the imperfectly informed expert, since the principal can always specify an information structure and a set of actions as those in a communication equilibrium. However, this observation raises two natural questions. Can a combination of the instruments bring a strictly higher payoff than any separate organizational form? If yes, then under what circumstances is it profitable to combine these tools?

The answer to the first question is not obvious because of the following argument. Along all lines of the analysis of informational control, we relied on the derived model-specific revelation principle, which states that the optimal payoff is provided in a truth-telling equilibrium. Moreover, if a positive measure of the expert's types mix over decision-relevant information, then such equilibrium is strictly inferior to some incentive-compatible one. However, in incentive-compatible equilibria, commitment is useless, since the expert reveals her information without any commitment on the part of the principal. Thus, it is not clear how commitment to actions can improve the principal's payoff in such a situation, where there seems to be no value of commitment.

The above intuition misses one issue. Even if it is true that commitment to actions cannot

[35] For example, in the uniform-quadratic framework, full delegation dominates informational control if and only if $b \in \left(1/4, 1/\sqrt{12}\right) = (0.25, 0.289)$.

increase the efficiency of interaction for a given incentive-compatible information structure, it can expand the set of such structures. This is because the principal has more possibilities for specifying the expert's information and aligning it with associated actions. Therefore, if the space of the information structures that are incentive-compatible with commitment, contains a payoff superior equilibrium, a combination of the discussed tools can benefit the principal.

Thus, to answer the above questions, it is helpful to characterize general properties of the optimal mechanism, which consists of the information structure, the delegation set, and the action rule that maps the expert's messages into a set of actions. For these purposes, we focus on the tractable uniform-quadratic case with partitional information structures and restrict attention to deterministic mechanisms, in which the principal makes a particular decision after receiving a message from the expert. Given these specifications, the following lemma outlines the features of the optimal combination of informational control and delegation.

Lemma 7 *Any mechanism is payoff inferior to the mechanism, in which each expert's type induces a separate action. Also, the optimal mechanism has the following properties: given the expert's information that* $\theta \in [\theta_k, \theta_{k+1}]$*, the induced action is* $a_k \in \left[\frac{\theta_k+\theta_k}{2}, \theta_{k+1}\right], \forall k$*; no expert's type induces her optimal action;* $\Delta\theta_{k+2} = \Delta\theta_k$ *for all* k*; and the number of actions* $N \leq \lfloor \frac{1}{b} \rfloor + 1$.

In general, the information structure of the optimal mechanism inherits the major features of the information structure in the communication game. First, the cardinality of the type space is bounded from above by $1/b$. Thus, even though commitment to actions can generate finer incentive-compatible structures than those in the communication game, the cardinality of the information structures has the same order as a function of the bias in preferences. Intuitively, a very fine information structure requires a large number of actions in the delegation set.[36] This creates difficulties with separating actions apart from each other to satisfy the expert's incentive-compatibility constraints. Second, the relationship between the interval sizes $\Delta\theta_{k+2} = \Delta\theta_k, \forall k$ implies that the expert's informational losses are distributed more or less uniformly across the state space. Finally, given the expert's information that the state is in some interval, the associated action belongs to this interval as well. Equivalently, each action is the first-best decision for some state in the interval.

In contrast, comparing the combined mechanism to the case of optimal delegation with the fully informed expert, one can see that the delegation set and the action rule substantially

[36] This is due the fact that the expert of each type induces a separate action. Otherwise, if several types induce the same action, the principal can collapse them into one type.

differ from those in the case of the perfectly informed expert. First, each mechanism is either payoff equivalent or inferior to the mechanism, in which each expert's type induces a distinct action. Second, all actions are located between the optimal actions of the expert and those of the principal. Thus, there is no expert's type which is allowed to choose her optimal action. Such behavior is consistent with the above argument for the case of informational control with full delegation.

Relying on the properties of the optimal mechanism, it is straightforward to prove the following result.

Theorem 6 *For $b < \frac{1}{2}$, a combination of informational control and restricted delegation is strictly payoff superior to each separate instrument.*

In a combined mechanism, the principal faces a trade-off between providing the expert with more information to reduce her informational losses and creating incentives for the expert of each type to take an action sufficiently close to the principal's optimal policy. Thus, even though the principal may not react to the expert's information optimally, a more flexible choice over actions allows him to specify a finer information structure than that in communication, and, hence, reduce the expert's informational losses. This is a main factor that determines the efficiency of the combination of the incentive tools relative to each separate instrument. In particular, it dominates informational control with communication because of a better informed expert. On the other hand, it dominates delegation to the fully informed expert, since the expert always makes a decision, which is closer to the principal's optimal action. In the case of the fully informed expert, this is true for high states only.

1.7 Concluding Remarks

The main contribution of this chapter is as follows: if the principal is able to control the precision of the expert's information (without knowing its content), he can do better than by optimally delegating decisions to the expert. This finding reverses the result about the payoff dominance of delegation over pure communication. This might be one of the factors that explain the fact that, despite seemingly clear benefits of delegation, many companies do not decentralize decision-making, and even often recentralise their structures after decentralization.

We deliberately did not address the case, in which the person who determines the quality of the expert's information is the expert herself, because the answer is straightforward. As demonstrated by Crawford and Sobel (1982) for the leading uniform-quadratic example, the principal's expected utility is equal to the residual variance of the state in any communication equilibrium, since the principal's decisions are, on average, unbiased. As a result, the expert's expected utility differs from that of the principal by a constant term.[37] This argument holds for any communication equilibrium unconditionally on the quality of the expert's information. Thus, if there is a credible mechanism of the expert's commitment to the precision of information, in which the expert commits "not to know too much" or her competence of the subject is verifiable, then the expert's choice of the optimal information partition will be the same.

Another issue which we left behind, is a comparison of controlling information to other organizational forms such as delegating authority to a biased intermediary or delegation with a veto power, when the principal has a choice between only two decisions: recommended by the expert and some default option. These institutions are special cases of restricted delegation, which implies that they cannot perform more effectively than optimal delegation. Therefore, as soon as controlling information is preferred by the principal to optimal delegation, it is strictly preferred to all discussed forms of interaction.

A natural question to ask is whether informational control is a universal tool. In other words, can it work jointly with other incentive devices, such as monetary transfers between the players, or communicating through a mediator? A partial answer to these questions is provided by the analysis above, which shows that a combination of informational control and restricted delegation is sufficiently powerful to strictly dominate each separate instrument. Also, there is another subtlety which is worth observing. The main benefit of informational control is limiting the expert's possibilities to distort her information by providing less information to her. In general, this factor can interfere with other instruments that extract more information from the expert, since informational control damages the expert's primary information. Nevertheless, other incentive devices have a common feature that can be used as a complement to informational control. In particular, they provide the expert incentives to convey the information that she is reluctant to reveal, that is, when the state is low, for example, by paying for reporting such information. However, they are less efficient for high states. In contrast, the efficiency of informational control does no depend on the value of the state, since the informational losses are shared approximately equally across the state space. Thus, the optimal combination of informational control and other tools should provide more information to the expert for low

[37] In particular, $U_S = U_R - b^2$.

states along with incentives to reveal that information, and less information without additional incentives for high states. For example, applying this intuition to informational control and monetary transfers shows that a combination works better than each tool.[38] Another positive factor that would play a role in an optimal combination is the different efficiency of separate instruments as a function of the divergence in players' interests. For instance, informational control is effective for small values of the bias, but loses the efficiency if the bias becomes very large. In contrast, an optimal combination of delegation and monetary transfers provides insignificant benefits over delegation, if the bias is small. However, it is especially effective for extreme biases (Krishna and Morgan, 2005), when delegation itself is useless and cannot improve a single decision of the uninformed principal.

An important aspect of the considered model is the number of equilibria that significantly exceeds that in Crawford and Sobel communication. In addition to pure-strategy equilibria, there exist multiple mixed-strategy equilibria even with the same information structure. Thus, we need to take care about ranking equilibria in terms of the principal's expected payoff. However, despite the fact that all mixed-strategy equilibria are payoff inferior to pure-strategy ones, they can still be superior to equilibria in the CS model and delegation.

[38] Consider the uniform-quadratic setup with the bias 0.22. The principal determines the information structure that reveals perfect information to the expert if the state is below $z = 0.03$, and partitions the rest of the state space $[z, 1]$ into two uniform intervals. Also, given the expert's message that the state is $\theta \leq z$, the principal pays the transfer $T(\theta) = \frac{153}{2500} - \frac{11}{25}\theta$. Using the analysis by Krishna and Morgan (2005), this transfer scheme results in full separation for states below z. Also, the information structure is incentive-compatible for states above z. Moreover, the combination of the two instruments results in the principal's expected payoff $-1/48.4$, which is higher than that $-1/48$ in the case of informational control and $-1/18.4$ in the case of communication with transfers only.

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Chapter 2. Dynamic Information Revelation in Cheap Talk

2.1 Introduction

This chapter focuses on the standard problem of communication between an expert who has some private information and an uninformed principal who has the authority to make decisions that affect both players' payoffs. In general, communication between the involved parties is characterized by two features. The first feature is a conflict of interest. The divergence in preferences creates the incentives for the expert to strategically misrepresent information in her favor, which results in losses of disclosed information. The second feature is the imperfect primary information of the expert. Even the most knowledgeable expert may have only noisy or insufficient information.

However, while the difference in players' preferences is generally exogenous, the quality of the expert's information can sometimes be endogenized by the principal. Moreover, if communication is conducted through multiple stages, the principal may have the possibility to affect the precision of the expert's information without learning its content before every round of communication, whereas the expert can update her report afterwards.[39] In other words, the principal can ask for more information rather than make the decision at the end of the first stage.

Our major contribution is that we show how the principal can use these instruments—controlling the quality of the expert's private information (hereafter, informational control) and dynamic interaction—to effectively extract all expert's information about an unknown state of nature in each round of communication. As a result, he is able to obtain (almost) full information over a large interval of states, which converges to the whole state space as the divergence in preferences tends to zero. An immediate implication of this finding is that the principal's payoffs in multi-stage interaction relative to one-stage communication rise infinitely as the bias in preferences falls. This results becomes even stronger, given that informational control with one-stage communication is payoff superior to many existing mechanisms, such as delegation, in which the principal delegates decision making to the expert, or mediation, in which parties communicate through a disinterested mediator.[40]

In particular, the revealing mechanism is similar to *sequential sampling*, that is, testing

[39] That is, the expert's information is still completely private at each moment of time.

[40] For a detailed discussion of delegation, see, for example, Dessein (2002). The problem of optimal mediation is analyzed by Goltsman et al. (2007). For comparison of the performance of these organizational forms to one-stage information control, see Ivanov (2007).

a hypothesis when a sample size is not fixed (see, for instance, Feldman and Fox, 1991). For example, if a test turns out to be negative for the current sample, then the hypothesis is rejected, and no other analysis is conducted. Otherwise, if the test's outcome is positive, then the sample size increases and a better test is being conducted.[41] In our situation, multi-stage interaction can be considered as follows: the principal allows the expert to conduct a sequence of tests, and requests a new report about each result. The key feature of this sequence is that, from stage to stage, tests become less accurate for high values of the unknown state of nature and more accurate for low values. We show that such updating of the expert's information is consistent with her positively biased interests, which results in a truth-telling communication in all rounds of communication. Moreover, the principal bases his decision given only the information at the stage when the expert's report on the test is positive.

As an economic example, it is well recognized that the interests of managers (experts) are biased relative to those of shareholders (principals), which creates losses in efficiency. According to Jensen (1986), managers have incentives to cause their firms to grow beyond the optimal size or undertake excessive and/or low-return investment projects. On the other hand, if the outcome of a risky project depends on the amount of investment, then a lot of information has to be collected prior making to a decision. In this case, data analysis is often delegated to a special company (e.g., marketing, financial, or geological) that collects and analyzes the data, which is then used by the manager to give a recommendation for the optimal size of investment. Thus, if the collection of data is outsourced and the information is updated over time, then the manager is unable to obtain all the information at once and at each moment in time her information is incomplete.[42] Given this, shareholders can request an additional report from the manager after each period of time and make a decision only after all reports have been submitted. This chapter shows that such a combination of the dynamic updating information by the expert and updating reports by the principal results in a Pareto improvement compared to both the static and dynamic cases when the expert knows all available information from the beginning. Moreover, the benefits of multistage communication relative to the one-stage increase at a higher rate when the divergence in the players' preferences falls.

[41] For instance, a variety of medical tests have a form of sequential sampling. If, say, a strip pregnancy test shows a negative result, then most people interpret this result as final. However, if it is positive, then a professional conducts a more precise test.

[42] The expert's information can be updated exogenously when new data and estimates become available over time, or it can be organized endogenously through a contract between the initial and outsourcing companies, given when the latter releases its data step by step.

Our analysis is based on a simple extension of the classical model of Crawford and Sobel (1982), which incorporates the discussed features: communication through multiple stages and imperfect information of the expert, the quality of which is controlled by the principal at every stage. In particular, we introduce a mechanism of information updating through which the expert truthfully discloses all available information at each stage of communication. As a result, if the state exceeds some cut-off level, it can be revealed with an arbitrary precision when the number of communication stages is sufficiently large. The result relies on the following intuition: step-by-step updating of the expert's information at every stage can be organized in such a way that the expert has a possibility of inducing only those actions that are either optimal for the principal or substantially different from the expert's ideal policy, given her current information.

The first analysis of strategic communication is attributed to Crawford and Sobel (1982) in their seminal paper. They introduce a model of a perfectly informed expert and an uninformed principal whose payoffs depend on a random state of nature. After privately observing the true state, the expert sends a costless message to the principal. The principal can use this information to implement an action, which determines both parties' payoffs. Crawford and Sobel (hereafter, CS) show that full information revelation is never possible unless the players' interests perfectly match. In addition, when a conflict of interest arises, the quality of the disclosed information falls, eventually resulting in an equilibrium with no useful information conveyed.

Crawford and Sobel's characterization of the equilibria is predicated upon two assumptions. First, the expert is perfectly informed about the realization of the state of nature. Second, the communication process consists of one stage only. There is an established literature that deals with relaxing both of these assumptions.

Fischer and Stocken (2001) first recognized the fact that the quality of information of the principal is not monotone in that of the expert. Ivanov (2007) extends this result by demonstrating that restricting the expert's information generally performs better than delegating authority to the expert whenever informative CS communication is feasible. Austen-Smith (1994) considers the situation in which the expert can observe the state at some privately known cost. In addition, the expert is able to prove the fact of information acquisition, but not the fact that she is uninformed. Since information is costly, this decreases the expert's incentives to acquire it and, as a result, the average quality of her information. However, introduction of the partial verifiability of the expert's knowledge extends the range of biases, for which informative communication is possible.

Another approach to disclosing more information from the expert is to organize communication through multiple stages. Aumann and Hart (2003) consider two-person games with two-sided cheap talk in which one side is better informed than the other, and the players can communicate without time constraints. They completely characterize the equilibria and demonstrate that the set of equilibrium outcomes can be significantly expanded. Their general analysis is restricted to the class of games with incomplete information with discrete types and a bimatrix structure of players' strategies and payoffs. Krishna and Morgan (2004) investigate multi-stage communication with the active participation of the principal in the communication process. In particular, players interact through *face-to-face* meetings, that is, the expert and the decision maker meet face to face in the first stage and simultaneously send messages to each other. In the second stage, the expert can be allowed to send a further message conditional on the outcome of the first meeting. Krishna and Morgan demonstrate that with only two stages there exists an equilibrium that almost always ex-ante Pareto dominates all of the CS equilibria. Moreover, it is possible to sustain informative multi-stage communication even if the bias in the players' interests is so large that no informative equilibria exist in the CS model.[43] Our setup differs from the environments considered by Aumann and Hart (2003) and Krishna and Morgan (2004) in a crucial aspect. In these works, the quality of the expert's information is exogenously determined before communication starts, and is not affected by the players' interaction. In our model, the expert's information is being updated over time. This modification results in two effects, which suppress the expert's incentives to distort her information. First, the future informational benefits can enforce the expert to reveal more information in a current stage, since the process of informational update depends on the expert's messages. Second, the principal can use the expert's messages in previous stages to partially verify the expert's current report.

Battaglini (2004) considers a model with multi-dimensional signals and multiple imperfectly informed experts. He shows that when experts have different preferences, the number of experts is large, and the principal has a limited ability to commit, then it is possible to construct an equilibrium in which the quality of extracted information is arbitrarily close to complete information.[44] While the discussed studies in the literature separate the effects of the expert's informativeness and multi-stage communication, this chapter combines the discussed approaches

[43] These equilibria have a non-monotonic structure, that is, a sender of a high type can be associated with a lower action.

[44] The assumption of the principal's limited ability to commit can be omitted, if the game is played through an arbitrary, but finite number of periods, where a new state of nature and new experts' signals are drawn in each period.

and demonstrates their complementarity.

2.2 The Model

We focus on the particular uniform-quadratic setup of the CS model, which has been a central specification for a large part of the related literature. This case with a uniform distribution of states and quadratic preferences is known for its tractability and the possibility to obtain closed-form solutions in various modifications of the basic CS model.[45]

2.2.1 The Crawford-Sobel model

The expert privately knows the state of nature θ, which is uniformly distributed on the unit interval, whereas the principal has authority to make a decision a that affects both players. To make an action, the principal can take into account the expert's costless message m about the state. The players' preferences are represented by the payoff functions

$$U(a, \theta) = -(a - \theta)^2$$

for the principal and

$$V(a, b, \theta) = -(a - \theta - b)^2 \tag{10}$$

for the expert, where a bias parameter $b > 0$ reflects the divergence in the players' interests.

Crawford and Sobel (1982) prove that all equilibria have the form of the finite monotone partitions. That is, for any bias b, the state space is partitioned into at most $N(b)$ intervals, which are determined by

$$N(b) = \lceil -\frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{2}{b}} \rceil,$$

where $\lceil x \rceil$ is the smallest integer larger than or equal to x.

Then, for large values of the bias $b \geq 1/4$, we have $N(b) = 1$, which implies that informative communication is not feasible. For $b < 1/4$, there are exactly $N(b) > 1$ equilibria with $1, 2, ..., N(b)$ intervals so that the expert sends one message for all states within the interval

[45] See, for example, Blume et al. (2007), Gilligan and Krehbiel (1987, 1989), Goltsman et al. (2007), Krishna and Morgan (2001a, 2004, 2005), Melumad and Shibano (1991), Ottaviani and Squintani (2006).

$W_k = [\theta_k, \theta_{k+1}]$, which is associated with a corresponding action[46]

$$a_k = E[\theta | \theta \in W_k] = \frac{\theta_k + \theta_{k+1}}{2}.$$

Crawford and Sobel (1982) show that the players' payoffs are monotone in the number of partition elements, that is, the equilibrium with $N(b)$ intervals is Pareto superior to all other equilibria. On the other hand, the perfectly precise information of the expert is not optimal for the players' payoffs, which is the focus of the next subsection.

2.2.2 Static informational control

As demonstrated by Fischer and Stocken (2001) for the bias $b = 1/2n$, where n in an integer, and extended by Ivanov (2007) for any value of the bias, the principal can improve communication by controlling the quality of the expert's primary information. In particular, the principal partitions the state space $\Theta = [0, 1]$ into a collection of intervals $W_k = [\theta_k, \theta_{k+1}]$, $k = 0, 1, ..., n-1$. Then, the expert privately observes the element of the partition that contains the state. Thus, she cannot distinguish among states in the same subinterval. In all other components, the environment is the same as the standard CS model. However, this modification of the expert's information structure improves her incentives to communicate truthfully, which leads to the higher principal's expected payoff. The following example illustrates how a less informed expert can result in a better informed principal.

Example 1. Let the bias $b = 3/14$. Then, the most informative CS equilibrium is characterized by two messages. Namely, the expert sends a "low" message if the state is below $\theta_1 = 1/14$, and a "high" message otherwise. This communication is not very informative, since the principal receives a "high" message with a probability of $13/14$ that updates his prior information insignificantly. As a result, the gains from communication with the perfectly informative expert are small (the principal's expected payoff is approximately $-1/15$ versus $-1/12$ in the case of no communication).

However, if the expert knows only whether the state is lower or higher than $1/2$, then there is an equilibrium, in which she reveals this information truthfully, so that the principal's expected payoff rises to $-1/48$. That is, even though communication with the imperfectly informed expert

[46] Formally, Crawford and Sobel (1982) require that $m(\theta)$ is uniformly distributed on $[\theta_k, \theta_{k+1}]$, if $\theta \in (\theta_k, \theta_{k+1})$, and $a(m) = E[\theta | \theta \in W_k]$ for all $m \in (\theta_k, \theta_{k+1})$. This, however, does not affect the principal's beliefs and actions.

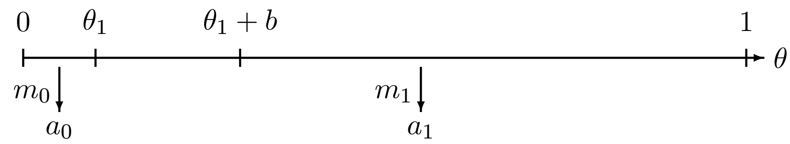

Figure 3: Communication in CS model

is less informative for very small values of the state, its overall effect on the principal's expected payoff is positive.

Intuitively, the preferences of the less informed expert are more closely aligned with those of the principal. In the CS case, displayed in Fig.1, the partitional structure is determined by the experts of marginal types who are indifferent between two consecutive actions. In the above example, it is the single type θ_1, which is quite low because the expert has the incentives to exaggerate her information to manipulate the principal's decision. That is, the indifferent type must be closer to the lower action, which implies that communication can be informative only for low states.

On the other hand, if the expert cannot distinguish among different states in a lower interval, this shifts her preferences toward the lower action, since for all states in the interval (except this marginal type) the lower action is strictly better. In other words, the partitional information structure replaces the marginal CS type by the mean type in the lower interval. As a result, the principal can specify a finer information structure than in the CS case, without breaking the expert's incentives to communicate truthfully.

In general, communication between players may not be restricted to a single stage, because the principal can affect the precision of the expert's information in every round, and request a new report afterwards. The next subsection investigates the benefits of this instrument.

2.2.3 Dynamic informational control

Consider a situation in which the expert is allowed to acquire additional information between rounds of communication and send a new report afterwards. In this environment, we derive our central result, which states that by proper updating of the expert's information from stage to stage, the principal can learn approximately full information in a large interval of the state space due to the expert's truth-telling communication in all stages. In addition, this interval converges to the whole state space when the divergence in the players' preferences converges to zero.

Before proceeding with dynamic information updating, consider first the multi-stage version of the CS model, which implies that the expert knows all information at the beginning. Krishna and Morgan (2004) illustrate that simply extending the model to the multi-stage case does not improve communication, since the set of equilibrium outcomes is identical to that in the one-stage communication game. Because the expert knows all the information before the communication starts, she sends the sequence of messages that induces the most preferable action. As a result, the principal infers the same information about the state as in the one-stage case. Thus, the set of induced actions is also not affected, and any equilibrium in the multi-stage game is payoff equivalent to that in the one-stage game. This argument can be directly reapplied to the case of the imperfectly informed expert without information updating. In contrast, if the expert's information is even slightly updated at every stage, the outcome of the multi-stage communication differs significantly from the one-stage case.

To introduce such updating into the model, the principal specifies a **communication schedule**: a family of intervals $\{W_k^s\}_{k=0,s=1}^{n_s-1,T}$, where n_s intervals $\{W_k^s\}_{k=0}^{n_s-1}$ form a partition of the state space at each round of communication $s = 1, ..., T < \infty$. Once chosen, a communication schedule becomes common knowledge.

In every stage s, the expert observes an index i_s of the partition's element $W_{i_s}^s$ that contains the state, which is assumed the same in all periods. That is, the imprecision of the expert's information about the state is determined by a measure of the set $M_s = \bigcap\limits_{\tau=1}^{s} W_{i_\tau}^{\tau}$. Then, the expert transmits a message $m_s \in \Delta M$ to the principal. The expert's signaling strategy σ is a mapping from the space of all sequences $\{i_s\}_{s=1}^T$ to a probability distribution over the message set $\underset{s=1,...,T}{\times} M$. After receiving a sequence of messages $\{m_s\}_{s=1}^T$, the principal updates his posterior beliefs about the state and implements an action a.

Then, if the expert knows that the state is in the set W_k, her payoff function is represented by

$$V\left(a, b|W_k\right) = E_\theta\left[V\left(a, b, \theta\right)|\theta \in W_k\right] = \frac{1}{P\left(W_k\right)} \int\limits_{W_k} V\left(a, b, \theta\right) d\theta,$$

where $P\left(W_k\right) = \Pr\left(\theta \in W_k\right)$. From (10), $V\left(a, b|W_k\right)$ can be expressed as

$$V\left(a, b|W_k\right) = V\left(a, b, \omega_k\right) - D\left(W_k\right),$$

where $\omega_k = E\left[\theta|\theta \in W_k\right]$ is the mean of the set and $D\left(W_k\right) = \frac{1}{P(W_k)} \int\limits_{W_k} \left(\theta - \omega_k\right)^2 d\theta$ is the residual variance of the state, which represents the informational losses of the expert. Similarly,

$$U(a|W_k) = U(a, \omega_k) - D(W_k).$$

An example below illustrates how just a two-stage interaction can reveal more information compared to a one-stage communication.

Example 2. Let the bias $b = 3/14$. In the one-stage game, the most informative communication is reached under the two-element partition $\{0, 1/2, 1\}$, which provides the expected payoff to the principal $-1/48$ (Ivanov, 2007). Fig. 2 displays communication in two rounds with the communication schedule

$$W_0^1 = [0, \theta_1] = [0, 6/7], \; W_1^1 = [6/7, 1],$$
$$W_0^2 = [0, \theta_2] = [0, 3/7], \; W_1^2 = [3/7, 1].$$

The principal's action rule is determined by

$$a(m_0, m_0) = a_{00} = 3/14, \; a(m_0, m_1) = a_{01} = 9/14, \text{ and } a(m_1, .) = a_1 = 13/14.$$

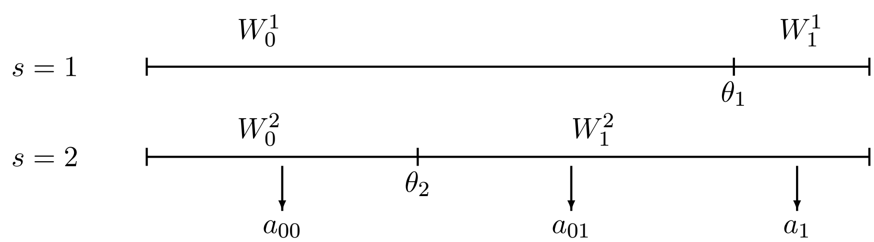

Figure 4: Two-stage communication

Suppose that at the first stage the expert observes $i_1 = 1$, which implies that the state is in the interval $[6/7, 1]$. Given this information, the expert's payoff function $V\left(a, b|W_1^1\right)$ is maximized at $a' = 8/7$. In addition, the sender infers that her current information will not be updated in the next stage since W_1^1 belongs to W_1^2. Then, the message m_1 induces the action a_1 unconditionally on the expert's report in the next round. An interpretation of this action rule is that communication stops as the principal receives this message. In contrast, the message m_0 can induce two actions, a_{00} and a_{01}, depending on the message in the next stage. Since $V\left(., b|W_1^1\right)$ is increasing for all actions less than a' and $\max\{a_{00}, a_{01}\} < a_1 < a'$, then the expert strictly prefers the message m_1 to m_0.

If $i_1 = 0$, then the state is in the interval $[0, 6/7]$. Moreover, the expert infers that her current information will be updated in the next stage. If the expert lies by sending the message m_1, the principal immediately implements the action a_1, which provides the expected payoff to the expert $V\left(a_1, b|W_0^1\right) = -1/7$. In contrast, the truthful reporting in this and the next stages results in a higher expected payoff

$$E[V\left(a\left(m_0, m_k\right), b, \theta\right)|W_0^1\right) = \frac{1}{\theta_1}\left(\int_0^{\theta_2} V\left(a_{00}, b, \theta\right) d\theta + \int_{\theta_2}^{\theta_1} V\left(a_{01}, b, \theta\right) d\theta\right) = -\frac{3}{49}.$$

Thus, the expert still has no incentives to distort information.

In the second round, first let $i_2 = 1$. If the expert observed $i_1 = 1$ in the first stage, then the analysis above shows that she induced the action a_1 at that stage by sending the message m_1. If $i_1 = 0$, then the expert infers that the state is in the set $W_0^1 \cap W_1^2 = [3/7, 6/7]$, and her optimal action becomes $a'' = (\theta_2 + \theta_1)/2 + b = 6/7$. Then, only actions a_{01} and a_{00} are feasible in the second round, conditional on truth-telling in the first stage. Because $a_{00} < a_{01} < a''$, it follows that sending message m_1 is strictly preferable to m_0.

If $i_2 = 0$, then the expert deduces that the state is in the interval $[0, 3/7]$. Given this information, the expert is indifferent between the feasible actions a_{00} and a_{01}, since they are equidistant from the expert's optimal policy $a''' = \theta_2/2 + b = 3/7$. Thus, the expert still cannot deviate from revealing her information. Finally, it can be easily seen that induced actions are the principal's best-response to the expert's signaling strategy.

The expected payoff of the principal in the two-stage interaction is approximately $-1/75$, which significantly exceeds that in the most informative one-stage equilibrium.

The discussion above raises a natural question—whether the principal can replicate this information structure in the one-stage game through specifying the partition $\{0, 3/7, 6/7, 1\}$ and reacting to the expert's messages as if they are honest. Then, the truthful communication fails. The reason is that if the expert infers that the state is in the interval $[3/7, 6/7]$, she prefers to send the message m_1 and induce the action a_1 instead of a_{01}, since it is closer to her optimal action. In contrast, in the two-stage game the expert can obtain this information only in the second round. However, at that time the action a_1 is not feasible, which does not leave her any choice but to provide the truth and induce the action a_{01}. On the other hand, if the state belongs to the set $[3/7, 6/7]$, then the expert can observe only the interval $[0, 6/7]$ in the first period. Given such imprecise information, distorting information in the first period is strictly dominated by the strategy of being truthful at this and all future stages. The following subsection applies this logic to the general case.

2.3 The Revealing Mechanism

In this subsection, we present our major result for the multi-stage communication. Namely, we construct the communication schedule, through which the principal can reveal (almost) all information in the interval $[4b, 1]$ and partial information in the interval $[0, 4b]$. In particular, the principal's decisions in the revealing equilibrium are of the trigger form. That is, at any stage, some messages serve as the "trigger" signals that induce the actions unconditionally on the expert's future messages. Through sending other messages, the expert has the possibility of inducing more than one action, but only those actions that are no "trigger" in the previous stages. Thus, as the expert's information is being updated, the set of feasible actions shrinks. In other words, at each moment, the expert faces the trade-off between the bigger choice over actions at this stage or the informational benefits in the future.[47] The key property of the communication mechanism is that the trigger actions are beneficial only if the sender's information becomes precise at the current stage. Then, the expert provides this information to the principal by sending the trigger message, which significantly improves the principal's information as well. In contrast, if the available information of the risk-averse expert is not precise, then tomorrow's informational gains exceed the possibility of inducing more actions today, which results in sending a different signal.

We start the construction by restricting attention to two-element partitions $W_0^s = [0, \theta_s]$ and $W_1^s = [\theta_s; 1]$ at each stage $s = 1, ..., T$. Equivalently, such communication schedule is determined by a sequence of the boundary points $\{\theta_s\}_{s=0}^{T+1}$, where we let $\theta_0 = 1$ and $\theta_{T+1} = 0$. In addition, consider a *decreasing* communication schedule, that is, such schedule that a sequence $\{\theta_s\}_{s=0}^{T+1}$ is decreasing.

In general, any decreasing communication schedule is characterized by two properties, which are crucial for our analysis. First, the expert's information in each round is either very precise or very imprecise. As soon as the expert observes the higher interval $[\theta_s, 1]$, she deduces that

[47] Otherwise, if all actions were available at the last stage, the game would be equivalent to the one-stage game with the finest information structure, which consists of the collection of intersections of the information structures across all stages $\left\{ \bigcap_{s=1,\ldots,T,} W_{k_s}^s \right\}_{k_1,\ldots,k_T}$. This would result in less information revealed because of the larger number of the incentive-compatibility constraints.

the state is in the set $W_0^{s-1} \cap W_1^s = [\theta_s, \theta_{s-1}]$, that substantially updates her information. In contrast, observing the lower interval updates the expert's information insignificantly. Namely, the set of possible states shrinks from $[0, \theta_{s-1}]$ to $[0, \theta_s]$. Second, if the expert's information is precise, she knows that her current information will not be improved in future rounds. In contrast, if her information is vague, then it will surely be updated in the future.

Given a particular state θ, define $\tilde{s} = \min \{s : i_s = 1\}$ to be the first round, in which the expert observes a higher interval. If $i_s = 0$ for all s, then put $\tilde{s} = T + 1$. For a decreasing communication schedule, we have $i_s = 1$ if $s \geq \tilde{s}$, since $W_1^s \subset W_1^{s+1}$ for all s. That is, the expert's information is not updated after the stage $\tilde{s}$. Thus, the space of all sequences $\{i_s\}_{s=1}^T$ consists of $T + 1$ non-decreasing sequences $I_0 = \{0\}_{s=1}^T$ and $I_k = \{\{i_s\}_{s=1}^T : i_s = 1 \text{ for } s \geq k,$ $i_s = 0 \text{ for } s < k\}$, $k = 1, ..., T$.

Then, consider a decreasing communication schedule, depicted in Fig. 3, such that

$$\theta_{T-1} \geq 4b,\ 0 < \theta_T < \theta_{T-1},\ \theta_0 = 1, \text{ and } \theta_{T+1} = 0. \tag{11}$$

This implies that the sequence $\{\omega_s\}_{s=1}^{T+1}$, where $\omega_s = \frac{\theta_s + \theta_{s-1}}{2}$, $s = 1, ..., T + 1$, is decreasing.

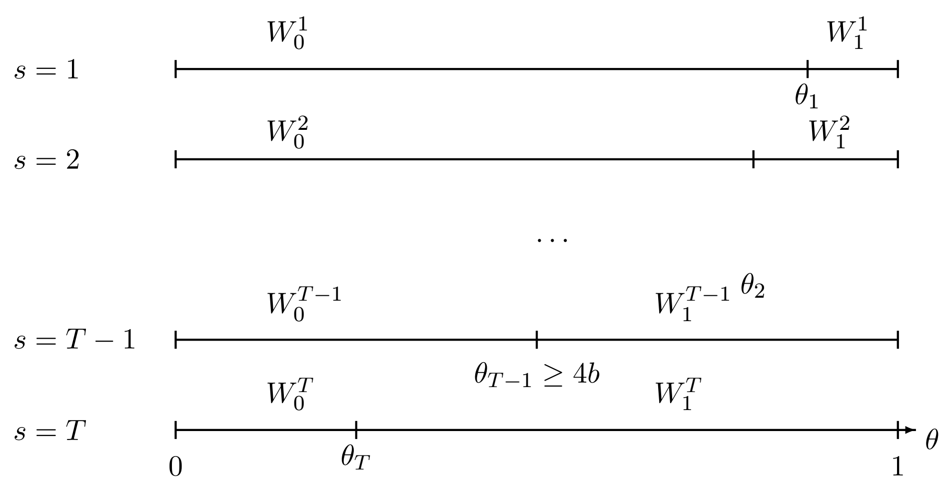

Figure 5: A decreasing communication schedule

Define $\Theta_i = [\theta_i; \theta_{i-1}]$, $i = 1, ..., T+1$. At the end of the communication process, the principal's beliefs about θ can be expressed as $\mu(\Theta_i | \{m_s\}_{s=1}^T)$, which denotes a belief that θ is uniformly distributed on Θ_i with the probability $\mu(\Theta_i | .)$.

Given this setup, the main result is characterized by the following theorem. All proofs can be found in the Appendix.

Theorem 1 *For any decreasing communication schedule $\{\theta_s\}_{s=0}^{T+1}$, which satisfies (11), there exists an equilibrium such that:*

1) the expert reports truthfully at each stage, so that $m_s\{I_s\} = i_s$, $s = 1, ..., T$,

2) the principal implements an action $a\left(\{m_s\}_{s=1}^{T}\right) = \omega_j$, where $j = T+1$, if $m_s = 0$ for all s, and $j = \min\{s : m_s = 1\}$ otherwise, and

3) beliefs are consistent, so that $\mu(\Theta_j | \{m_s\}_{s=1}^{T}) = 1$ and $\mu(\Theta_i | \{m_s\}_{s=1}^{T}) = 0$, $i \neq j$.[48]

Thus, if there are no exogenous time limits on communication, the principal can disclose the information about any state in the interval $[4b, 1]$ as precisely as he wants.

Corollary 1 *By choosing a decreasing communication schedule $\{\theta_s\}_{s=1}^{T}$ such that $\theta_{T-1} = 4b$, $0 < \theta_T < 4b$, and $\max_{s=1,...,T-1} |\theta_s - \theta_{s-1}| \to 0$ as $T \to \infty$, the principal discloses approximately full information in the interval $[4b, 1]$ in the above equilibrium.*

By slight updating information at all stages except the last two, the expert seems to get a small piece of information at every stage. This argument is true only partially, however. If the expert observes $i_{s-1} = 0$ in stage $s-1$ and $i_s = 1$ in stage s, then she infers that the state belongs to the set $W_0^{s-1} \cap W_1^s = [\theta_s, \theta_{s-1}]$. Hence, her information about the state becomes very precise. The intuition here is that given this updated information and the principal's beliefs, the expert's best *feasible* action is the one that will be implemented after revealing her information truthfully. In contrast, if the expert observes the lower interval, her information is still vague and will be improved in the future. Due to the combination of the risk-aversion and the imprecise information, the expert's expected payoff from sending the message m_1 (and inducing the action $a = \omega_s$) is low and strictly dominated by providing truthful information at this and all future stages. The condition crucial for this result is that, given $i_s = 0$, the quality of the expert's information must be sufficiently imperfect, which is achieved by choosing coarse partitions in the last two stages ($\theta_{T-1} \geq 4b$).

2.4 The Value of Multi-stage Communication

The central question of this subsection is how valuable for the principal to control the process of learning information by the expert over time? In other words, what are the payoff benefits in

[48] *Notice that the principal's posterior beliefs are consistent for both equilibrium and out-of-equilibrium messages of the expert. That is, if the expert sends, say, a sequence $\{m_1, m_0\}$, which corresponds to the event "$\theta \in [\theta_1, 1] \cap [0, \theta_0] = \varnothing$", the principal unambiguously interprets this information as if the state is in $[\theta_1, 1]$.*

dynamic interaction versus those in one-stage communication and other known organizational forms. Before answering this question, it is useful to calculate the upper limit for the principal's expected payoff, which is reached as the number of communication stages increases infinitively.

2.4.1 The limit of disclosed information

When $\max_{s=1,\dots,T-1} |\theta_s - \theta_{s-1}| \to 0$ as $T \to \infty$, approximately full information is revealed in the interval $[\theta_{T-1}, 1]$. Thus, the principal's expected utility in the described equilibrium is determined by boundary points θ_{T-1} and θ_T in the last two stages:

$$U^{\lim}(\theta_{T-1}, \theta_T) = -\sum_{\tau=T-1}^{T} \int_{\theta_{\tau+1}}^{\theta_\tau} \left(\frac{\theta_{\tau+1} + \theta_\tau}{2} - \theta \right)^2 d\theta = -\frac{1}{12}\theta_T^3 - \frac{1}{12}(\theta_{T-1} - \theta_T)^3.$$

Given constraint (11), $U^{\lim}(\theta_{T-1}, \theta_T)$ is maximized at $\theta_{T-1} = 4b$ and $\theta_T = 2b$, which results in the limiting expected payoff

$$U^{\lim} = -\frac{4}{3}b^3.$$

The full disclosure of information in the interval $[4b, 1]$ requires infinitely many stages of communication. Given the principal's utility U^T in the game with T stages, a relative difference between U^T and $U^{\lim}$, that is,

$$\varepsilon = \left| \frac{U^{\lim} - U^T}{U^{\lim}} \right|,$$

can serve as the measure of imperfection of disclosed information.

Referring to the above example of $b = 3/14$, the limit of the principal's expected payoff in the multi-stage equilibrium is approximately $-1/76$. However, Example 2 demonstrates that only two stages of communication provide the expected payoff $-1/75$, so that $\varepsilon = \left| \frac{U^{\lim} - U^2}{U^{\lim}} \right| \simeq 2\%$. In general, the number of communication stages T, which guaranties that the inefficiency does no exceed ε, increases as $\varepsilon^{-1/2}$.[49] Thus, to decrease ε, say, from 4% to 1%, the number of rounds of communication must be doubled.

[49] Since conveyed information in the multi-stage equilibrium under a communication schedule $\{\theta_s\}_{s=0}^{T+1}$ is equivalent to the truthful communication in the one-stage game with the partition $\{\theta_s\}_{s=0}^{T+1}$, the most informative communication schedule is such that $\theta_{T-1} = 4b$, $\theta_T = 2b$, and $\theta_s = 1 - \frac{1-4b}{T-1}s$, $s = 0, \dots, T-2$, which results in the receiver's expected utility $U^T = U^{\lim} - \frac{1}{12}\frac{(1-4b)^3}{(T-1)^2}$. From this expression, the result follows immediately.

2.4.2 The efficiency of multi-stage communication

Given the expression for the limiting utility of the principal, we can compare the performance of the multi-state communication versus the one-shot game. As shown by Ivanov (2007), the incentive-compatibility (IC) constraints in the one-stage game can be written as

$$\Delta\theta_{k+1} + \Delta\theta_k \geq 4b, \, \forall k, \tag{12}$$

where $\Delta\theta_k = \theta_{k+1} - \theta_k$ is the length of the interval W_k in the information partition. In addition, the feasibility constraint requires $\sum_k \Delta\theta_k = 1$. From (12) and (11), there exist informative equilibria in both one-stage and multi-stage versions of the game if $b < 1/4$. The optimal partition, which maximizes the principal's expected payoff, satisfies the IC constraints (12) and provides the expected payoff

$$U = -\sum_{k=1}^{n(b)} \int_{\theta_k}^{\theta_{k+1}} \left(\frac{\theta_k + \theta_{k+1}}{2} - \theta \right)^2 d\theta = -\frac{1}{12} \sum_{k=1}^{n(b)} \Delta\theta_k^3.$$

In the multi-stage communication, however, the information cannot be fully revealed only if the state is smaller than $4b$, since $\Delta\theta_T + \Delta\theta_{T-1} \geq 4b$ for the last two stages only, and the distance between the cut-off points in other stages can be chosen arbitrarily small. Hence, it directly follows that

Theorem 2 *If communication is informative, then there exists an equilibrium in the multi-stage communication game, which is Pareto superior to all equilibria in the model of one-stage communication.*

The implication of this result is that if the bias is not very large, so that there exists an informative equilibrium the multi-stage game, then multi-stage informational control is payoff superior to such organizational forms as optimal delegation (to the fully informed expert) and communication through a disinterested mediator, since these mechanisms are dominated by a one-stage informational control (Ivanov, 2007).[50] Moreover, as the bias in players' preferences b tends to zero, the number of intervals $n(b)$ in the optimal partition in the one-stage

[50] Due to Goltsman et al. (2007), optimal delegation is a solution to an optimal arbitration mechanism, in which players communicate through an neutral arbiter, who can enforce his decision. In addition, mediation is a special case of arbitration, and hence, is payoff inferior to optimal delegation.

communication grows as $1/2b$, and the length of intervals $\Delta\theta_k$ decreases as $2b$.[51] Thus, the principal's expected payoff grows as $-n(b)\frac{\Delta\theta_k^3}{12}$ or $-\frac{b^2}{3}$. In contrast, the principal's expected utility in multi-stage interaction, which is characterized by the residual variance of the state in the interval $[0, 4b]$, multiplied by the probability that the state is in this interval, increases as $-\frac{4}{3}b^3$. As a result,

Theorem 3 *The performance of multi-stage communication relative to one-stage communication rises without a bound as the bias in preferences falls.*

Even though dynamic updating information is a powerful tool that allows the principal to extract a lot of information from the expert, the following subsection demonstrates that there is a possibility for reveal even more information if the principal can partially commit to decisions in some stages. However, in contrast to a common argument for one-stage games that delegation allows the principal to benefit from the expert's informational advantage, our result relies on a different argument.

2.5 Extensions and Discussion

In this part, we discuss two issues that address to different aspects of principal's commitment. First, we investigate the possibility for the principal to commit to actions. Second, we consider the situation, in which the principal cannot commit to a particular communication schedule before communication starts.

2.5.1 Commitment to actions

In the previous analysis, the principal extracted information from the expert without any commitment to decisions. However, the principal can partially delegate authority over actions to the expert in some stages or, equivalently, rubber-stamp any expert's recommendation, if it belongs to some set of allowed decisions. Here, we introduce an extension of the model to a combination of communication in some stages with partial delegation in others. We show that such a combination extends the set of biases, for which informative communication is feasible, to $b < 3/5$. As a result, the interval, in which approximately full information can be revealed, expands from $[4b, 1]$ to $\left[\frac{5}{3}b, 1\right]$, which increases the principal's limiting expected payoff from $-\frac{4}{3}b^3$ to $-b^3$.

[51] See Lemma 5 in Ivanov (2007).

In particular, we modify the setup as follows. The principal specifies a communication schedule such that

$$\{\theta_s\}_{s=1}^{T} \text{ is decreasing, } \theta_T > \frac{5}{3}b, \text{ and} \tag{13}$$
$$\text{the } \textit{full} \text{ information in the interval } [0, \theta_T] \text{ after stage } T.$$

Also, he implements an action $a\left(\{m_s\}_{s=1}^{T}\right) = \frac{\theta_j + \theta_{j-1}}{2}$, where $j = \min\{s : m_s = 1\}$. If the expert sends the message $m_s = 0$ in all stages except the last one, then in the last stage the principal delegates authority to the expert. The authority is restricted since the expert is allowed to choose policies only from the delegation set $[0, \theta_T]$. That is, if the state is in the delegation set and the expert truthfully reveals her information in stages $s = 1, ..., T$, then in the last stage $T + 1$ the expert knows the state precisely and can implement any policy from this interval. Using the same approach as in Theorem 1, we obtain the following result:

Lemma 1 *For any communication schedule which satisfies (13), there exists an equilibrium such that the expert truthfully reveals information in all communication stages.*

Thus, approximately full information can be revealed in the interval $\left[\frac{5}{3}b, 1\right]$, which increases the principal's expected utility to $-b^3$. This implies that it is possible to sustain informative communication even when the bias is extremely large (up to 3/5) so that no informative communication is achievable in the cheap-talk game. Nevertheless, given that the uniformed decision provides the payoff $-1/12$ to the principal, the considered mechanism is beneficial only if $b < (1/12)^{1/3} \simeq 0.437$. The main reason for this is that, as the bias in preferences rises, informational benefits in communication stages fall, since the interval $\left[\frac{5}{3}b, 1\right]$ shrinks. In addition, losses due of the biased decision of the expert in the last stage grow and, eventually, exceed informational gains.

In this context it is interesting to note that partial commitment raises benefits the principal not because of the expert's informational advantage, but through a different channel. If the state $\theta < \theta_T$, then it will be imperfectly revealed in the last stage of the multi-stage cheap-talk game. However, communication in the last stage is equivalent to the one-shot communication with the imperfectly informed expert, which performs better than delegation.[52] Mainly, the possibility of implementing the expert's favorite policy at the last stage plays a role of an attractive "carrot",

[52] See Theorem 2 in Ivanov (2006).

which enforces her incentives to communicate truthfully in all previous stages and, thus, provides the overall gain over the pure communication.

2.5.2 Commitment to communication schedule

In the above settings, the principal specifies the communication schedule at the beginning of the communication process, so that in each round, the sender can predict whether her current information will be updated in the future or not. Thus, it is natural to ask whether the result about information disclosure still holds if the principal cannot commit to the initially specified communication schedule, even though at some period before the last stage he believes that there will be no improvement in the expert's information.[53]

The answer to this question is positive. More precisely, any revealing equilibrium with a predetermined communication schedule can be replicated in the game in which the principal can change the quality of the expert's information at any moment. The intuition for this result is most easily gained from the following observations. First, in any communication game, there exists the babbling equilibrium with no information revealed. Second, in the revealing equilibrium, the parties' beliefs about the state are the same at any stage, since the expert truthfully reports all available information.[54] Finally, in any equilibrium with no commitment to the quality of information, the expert knows the equilibrium communication schedule as soon as it is deterministic.

Given these preliminaries, we can modify the out-of-equilibrium beliefs and strategies in the game with no predetermined communication schedule as follows: if the principal deviates from the predetermined communication schedule at some stage, then the sender starts babbling in the current and all future stages. The principal's best-response to this is to ignore the sender's messages from now on and to base his decision only on the information in the previous periods. Given this rule, the babbling strategy is the sender's best-response, since all her messages between today and the final stage are ignored. However, the principal's expected payoff due to deviating cannot exceed the equilibrium one, since the strategy of ignoring the future expert's messages was initially feasible.

[53] I am thankful to Kalyan Chatterjee and Dirk Bergemann for asking this question.

[54] In other words, any subgame from the period s becomes equivalent to the communication game with $T-s$ stages and the state space, determined by the available information at this period.

2.6 Conclusion

We have demonstrated that through communication with an imperfectly informed expert in multiple rounds, where the principal controls the precision of the expert's information in every round without learning its content, the principal can elicit almost all information for a large interval of the state space. This results in an ex-ante Pareto-improvement compared to one-stage communication. Moreover, as the divergence in players' preferences decreases, the relative performance of multi-stage interaction versus one-stage game rises infinitely.

In general, updating the expert's information and many stages of communication result in multiple equilibria, compared to the CS case, because there exist, for example, mixed-strategy equilibria. Also, there exist other less informative babbling and semi-babbling equilibria such that the sender does not reveal information in some stages of the communication process. Finally, through sending the same sequences of messages for different histories of updated information, the expert can generate the principal's beliefs of the non-monotone form and, as a result, associate higher states with a lower action.[55]

A possible argument against the suggested mechanism is that it can be difficult to implement in practice, since information systems in organizations are usually rigid. However, even if the expert's access to information is determined only once, new information can arrive over time exogenously so that the principal may request new reports. Second, the collecting of information can be delegated to a third party, which commits to releasing available information to the expert step-by-step, according to, say, a contract between this party and the principal.

[55] The non-monotonicity of beliefs implies that at the end of the communication process, the principal would believe that the state belongs to, say, $[0, \theta_1]$ or $[\theta_2, \theta_3]$, where $\theta_2 > \theta_1$.

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Chapter 3. Information Revelation in Competitive Markets

3.1 Introduction

There are multiple situations, which can serve as vivid illustrations of the strategic information disclosure in a market with several goods, where sellers strategically reveal information about their products to potential buyers. In general, examples of voluntary revelation of information by sellers can be found almost everywhere and take a variety of different forms. Consider, for instance, movie trailers, informative commercials on TV, screen shots of computer games, free trial versions of products, etc. That is, in addition to providing a good, the supplier serves as a significant source of information about it, especially if the product is new or highly sophisticated.

Despite the fact that information about products is controlled by suppliers, it is typically more precisely assessed by consumers, who know better how closely the specific characteristics of the product match their preferences. In other words, an important feature of information is that it often reflects a valuation of the product by the particular buyer, but does not characterize the product quality. For example, valuations of movies or computer games vary substantially across different consumers. Similarly, some characteristics of cars cannot be measured on the quality scale. For instance, heavy cars are generally safer, but they consume more gas. Thus, by releasing the product relevant information, the supplier affects the consumers' private values, but he cannot precisely predict its impact on the buyers' willingness to purchase the product. That is, revealing information creates a lottery over different types of buyers, depending on whether the properties of the product match their needs or not. In general, if the buyer privately updates her valuation after obtaining information from the seller, then the role of information is dual: it helps the seller to segment the market and, at the same time, it provides the informational rent to consumers. In addition to this trade-off, if the market is not monopolistic, then each seller must take into account the role of information in winning competition against rivals, which is also affected by their information disclosure decisions.

These observations raise several questions that form the central focus of our chapter. How much information is released by sellers in oligopolistic markets? How are the sellers' disclosure and pricing policies affected by the intensity of competition? What happens to the structure of the market if it becomes more competitive? Is it possible to reach full efficiency in the market, and if not, what is the source and the magnitude of inefficiency?

In this chapter, we address the above questions. For these purposes, we analyze a price-setting model of competition with a representative consumer and multiple sellers, who offer distinct and substitutable products. Each seller determines the precision of the signal about his product, which is privately observed by the consumer, and sets the price of his product. The consumer wants to purchase an indivisible unit of any good.[56] After observing signals and prices from all sellers, the consumer rationally estimates valuations of the products and purchases the product that brings her the highest expected payoff net of the price. The model is equivalent to common values situations with multiple consumers, when either different consumers observe the same signal or a continuum of consumers observe conditionally independent realizations of the signals. Also, the model is equally applicable to other situations such as the job search, where employers play a role of sellers, who possess information about characteristic of their jobs and compete for a candidate by offering the wage.

Our results can be summarized as follows. First, we identify scenarios, in which all sellers in the market fully reveal available information to the buyer. In particular, we demonstrate that if the market becomes sufficiently competitive, this results in full disclosure of information by all suppliers. Second, we show that full information revelation does not guarantee the full market efficiency. However, as competition intensifies, the magnitude of the inefficiency converges to zero at the rate, which is faster than exponential. Thus, even though our environment differs from the standard model of Bertrand competition, the market structure converges to the fully revealing competition with the unique symmetric price that tends to the marginal cost as the number of sellers goes up. Finally, in order to apply the results derived in the chapter to a given market, we provide criteria to see whether competition is sufficiently tense to ensure full information disclosure.

Our framework is closely related to the paper by Lewis and Sappington (1994), who first investigate the incentives of a *monopolist* to reveal the product relevant information. They consider the situation, in which the monopolist controls the quality of the buyer's information by choosing the probabilities, with which the buyer receives an informative signal or pure noise. Lewis and Sappington (1994) demonstrate that the trade-off between two different effects of information disclosure—segmenting the market and endowing the buyer with the informational rent—usually strictly favors one of them. That is, the optimal policy of the seller is either to disclose full information or reveal nothing.[57] We depart from Lewis and Sappington (1994) in

[56] For example, the products can be viewed as different brands, and the buyer needs just one unit of any brand.
[57] This trade-off crucially depends on the willingness of the average consumer to purchase the product and on

that we allow for *multiple* sellers and products. In addition, we allow each seller to determine the quality of information about his product, which is supplied to the buyer. By analyzing the role of competition among sellers, we show that this additional factor magnifies the benefits of segmenting the market relative to the costs of providing the informational rent to the buyer. As a result, only the perfectly informative structure survives as competition becomes sufficiently fierce.

Johnson and Myatt (2006) consider the problem of the monopolist with a wider class of buyer's information structures. In particular, they introduce the *rotation order* to rank the quality of the buyer's information, which we also use in this chapter. Johnson and Myatt (2006) characterize the sufficiently mild conditions that guarantee the monopolist's preference for extreme qualities of information. They also analyze the case of Cournot oligopoly and study the effects of increased competition on the relationship between demand dispersion (which is given exogenously) and firm profitability. Our model differs from their in two respects. First, we consider the price-setting model similar to Shaked and Sutton (1982, 1983) and Moscarini and Ottaviani (2001), in which sellers produce distinct and substitutable products. Second, in our model each seller is able to strategically affect the distribution of the buyer's valuations and, hence, the demand dispersion for his product by changing the precision of information, which is observed by the buyer. A combination of these differences allows to get insight into strategic interactions between sellers through varying the quality of their information. As a result, our findings are in stark contrast with those by Johnson and Myatt (2006). In particular, it is the case in their model that "if a firm dislikes any local increase in dispersion, that firm will continue to dislike increased dispersion when the number of competitors rises," which is exactly the opposite to the results we obtain. As a starting point, we consider the situation, in which the monopolist dislikes any dispersion in demand. However, when competition intensifies, all sellers eventually prefer the highest possible dispersion in demand.

Moscarini and Ottaviani (2001) investigate price competition in the duopoly market with private information of the buyer. They analyze the influence of prior common and buyer's private information on pricing decisions of the sellers. Damiano and Li (2007) extend the model of Moscarini and Ottaviani (2001) by allowing the sellers to control the precision of the buyer's

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the variance of buyers' valuations. The demand function with a high variance of buyers' values increases the attractiveness of segmenting the market by releasing information and serving only high-value consumers. In a recent paper, Saak (2006) demonstrates that if the monopolist can control the precision of the buyer's private valuations in an arbitrary way, then he prefers to let the buyer know only whether her valuation is above or below the unit production cost.

private information about their products. These models differ from ours in two regards. First, we do not limit the number of sellers by two. As a result, we are able to estimate the relationship between the intensity of competition and the sellers' disclosure and pricing policies. Second, the state space in the mentioned works is binary, as is the signal space for the buyer. Together, these imply that full information disclosure is possible even with two sellers (Damiano and Li, 2007), a property that does *not* generally hold with a richer signal space. Moreover, even a bigger number of sellers does not necessarily result in the perfectly informative structures. We demonstrate that the particular properties of the distribution of states crucially affect the number of sellers that guarantees full disclosure.

The issue of the endogenous quality of information has also acquired attention in the auction design literature. Bergemann and Pesendorfer (2007) consider an auction, in which the seller maximizes the expected revenue by specifying the information structure and the price for each bidder. They show that the bidders' information structures in the optimal auction are coarse, that is, the perfectly informative information structures are never optimal. However, as the number of bidders goes up, the optimal information structures converges to the fully informative ones. Ganuza and Penalva (2006) consider the environment, in which the auctioneer can choose the accuracy of the bidders' information about their private values at some cost. The quality of information is costly, identical for all bidders, and ordered according to a special criterion. In this case, the auctioneer provides less than the efficient level of information. However, both the socially efficient and the auctioneer's optimal choice of precision increase with the number of bidders, and both converge as the number of bidders increases infinitely. Contrary to these findings in the auction setting, we demonstrate that in the market setup, full information revelation can be reached with a finite number of players. Nevertheless, the fact that the buyer obtains perfectly precise information does not imply that the market is fully efficient. In particular, the source of inefficiency is the price level, which is symmetric across sellers and bounded away from the unit production cost.

Finally, our work relates to, but is separated from a large strand of the literature that is concerned with environments, in which the products are characterized by some *quality*, which is known to the seller(s).[58] This difference in the nature of information crucially affects the seller's

[58] The first papers that investigated the seller's incentives to reveal the product's quality are due to Grossman and Hart (1980), Grossman (1981), and Milgrom (1981). Recent papers on the quality disclosure include, for example, Levin, Peck and Ye (2005), who consider costly information signaling with horizontally differentiated products under duopoly and monopoly. Cheong and Kim (2004) examine the effect of competition on the firms'

motives to change the buyer's valuations via information disclosure. In particular, the sellers' incentives to let buyers know the information about the product *quality* and that of consumer's *private value* can be exactly opposite, even in the simplest case of a single seller.[59]

3.2 The Model

Consider a market with a finite number N of sellers, who compete for a single representative consumer (she) through selling N differentiated and indivisible products, where each seller (he) produces one product. The consumer makes a mutually exclusive purchase among these substitutable goods in the sense that she either buys exactly one unit from one of the sellers or makes no purchase.

The consumer's private valuations of the products $\{v_i\}_{i=1}^{N}$ are drawn independently from a distribution $G(v)$, which has a positive and differentiable density $g(v)$, supported on $[0, 1]$. All consumer's valuations are net of the sellers' values, which are identical for all sellers. However, each seller has full control over information about his product, which implies that the buyer does not possess any private information prior to interactions with the sellers. That is, all products are ex-ante identical with expected values $v^e = E[v]$.

Sellers compete for a buyer over two dimensions. First, a seller i offers a price p_i for his product. Second, he *covertly* decides on how much information about his product to reveal to the buyer.[60] In particular, a seller i chooses the quality η_i of the signal s_{η_i} about the product characteristics, which is privately observed by the buyer. For example, if the signal $s_{\eta_i} = v_i + \varepsilon_i$ is a sum of the true value v_i and the noisy component ε_i, then the quality of the signal can be represented by $\eta_i = 1/\sigma_{\varepsilon_i}^2$, where $\sigma_{\varepsilon_i}^2$ is the variance of noise. As a matter of notation, we use s_i^η

incentives to disclose quality when information disclosure is costly. Dye and Sridhar (1995) investigate the role of competition in disclosure of information about the expected profitability of company's cash flows. Other studies consider extended information structures, in which sellers can have information about products of their competitors (Board, 2006) or when each market participant possesses some private information (Daughety and Reinganum, 2007). Daughety and Reinganum (2007, 2008) investigate markets, in which the product quality may be signaled via prices. Stivers (2004) considers oligopolistic competition with vertically differentiated products, when buyers can be unaware of the existence of that information.

[59] For example, consider a random variable v distributed uniformly over the unit interval. If v represents the product quality for consumers, then the seller (whose value is assumed to be zero) would disclose it almost surely. If he does not disclose it, then he cannot charge more than the expected value $E[v] = 1/2$. However, if $v > E[v]$, then the seller would reveal this information. Thus, the seller could potentially hide information only if $v \leq E[v]$, which, in turn, would decrease the consumer's expected value. Repeating this argument iteratively results in the full information disclosure for any positive value. On the other hand, if v represents the buyer's private value, then revealing information and setting the optimal monopoly price $p_M = 1/2$ brings the seller expected payoff $1/2 \times 1/2 = 1/4$, since the buyer would buy the product only if $v > p_M$. In contrast, the seller can extract the total surplus $1/2$ by not revealing information and setting the price at the expected value.

[60] That is, the seller's decisions are not observed by its competitors.

instead of s_{η_i}.

After observing all signals s_i^η and prices p_i, $i = 1, 2, ..., N$, all decision relevant information for the buyer is contained in the posterior valuations of the products $\omega_i^\eta = \omega\left(s_i^\eta\right) = E\left[v|s_i^\eta\right]$. Thus, she buys the product j that gives her the highest non-negative expected payoff $\omega\left(s_j^\eta\right) - p_j$. Otherwise, the buyer does not make a purchase.

3.2.1 Information structure.

For simplicity, suppose that the signal quality $\eta_i \in [0, 1], \forall i$, where $\eta_i = 0$ and $\eta_i = 1$ imply the perfectly uninformative and perfectly informative signals, respectively. By choosing the signal quality, a seller i affects the distribution function of posterior values of his product

$$G_{\eta_i}(\omega) = \int\limits_{\left\{s_i^\eta : \omega\left(s_i^\eta\right) \leq \omega\right\}} dF_{\eta_i}\left(s_i^\eta\right),$$

where $F_{\eta_i}(.)$ is the marginal distribution of the signal s_i^η.

We assume that $G_{\eta_i}(\omega)$ is twice continuously differentiable in η_i and ω, and has a positive and differential density on the support $[\underline{\omega}_i^\eta, \bar{\omega}_i^\eta]$. To rank the family of distributions $G_{\eta_i}(\omega_i)$ with respect to the quality of signals, notice first that by partially revealing or distorting information, a seller cannot shift the buyer's taste toward his product, since the average valuation of a product is the same:

$$E\left[\omega_i^\eta\right] = E_{s_i^\eta}\left[E\left[v|s_i^\eta\right]\right] = E[v] = v^e, \forall \eta_i. \tag{14}$$

However, by changing the signal quality η_i, the seller affects the spread of the buyer's posterior valuations. In order to capture the effect of a change in the spread of $G_{\eta_i}(\omega)$ due to varying the signal's quality, we apply the *rotation order* introduced recently by Johnson and Myatt (2006) for the monopolistic setup.[61]

Definition 1 *The family of distributions $G_\eta(\omega)$ is rotation-ordered if, for each η, there exists a rotation point ω'_η, such that $\omega \gtreqless \omega'_\eta \iff \frac{\partial G_\eta(\omega)}{\partial \eta} \lesseqgtr 0$.*[62]

[61] Johnson and Myatt (2006) provide two examples of the information structures that can be ranked according to this order. The first example is the "truth-or-noise" technology that returns a signal, which is equal to the true value with a probability η or a random indistinguishable draw from $G(v)$ with a probability $1 - \eta$. In the other example, the value v is drawn from the normal distribution $N\left(\mu, \sigma^2\right)$. However, the consumer observes the conditionally unbiased signal x from the distribution $N\left(v, \frac{1}{\eta^2}\right)$.

[62] *Formally, Johnson and Myatt (2006) determine the rotation order as $\omega \gtreqless \omega'_\theta \iff \frac{\partial G_\theta(\omega)}{\partial \theta} \gtreqless 0$. However, in*

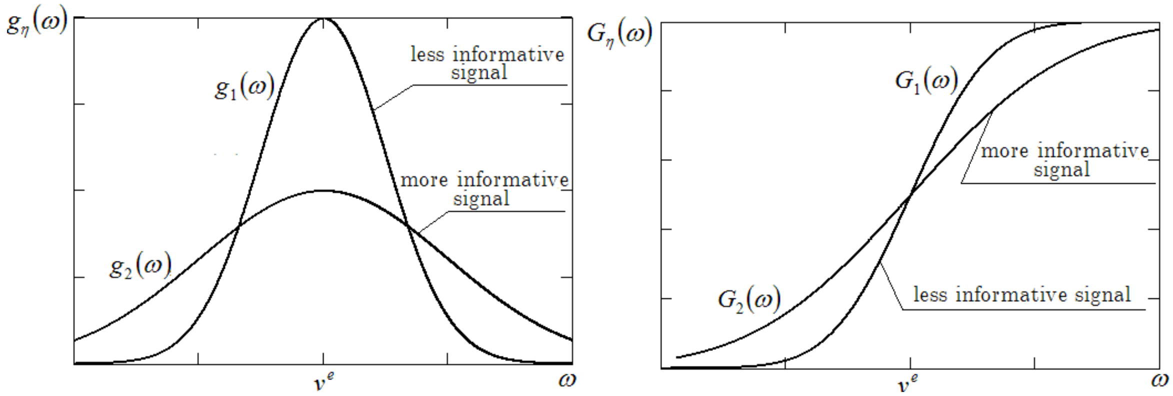

Figure 6: Concentration of the density $g_\eta(\omega)$ Rotation of the distribution $G_\eta(\omega)$

The main idea behind this ordering is that a less informative signal has a smaller influence on updating the buyer's prior information and, hence, stochastically shifts her posterior valuations toward prior expected value. More precisely, suppose first that the buyer observes a completely informative signal $s = v$. Then, the ex-post value of the product $\omega(s)$ is equal to the value of the signal s, and the distribution of posterior valuations coincides with the distribution of the prior values, or $G_\eta(\omega) \equiv G(\omega)$. However, if the signal contains a noisy component, the buyer does not purely rely on the value of the signal when she estimates the product valuation. Moreover, she corrects for this noise by taking into account the prior information. Since prior valuation of the product is characterized by its expected value, the buyer's posterior valuations stochastically shift toward v^e. That is, the density of posterior valuations $g_\eta(\omega)$ becomes more concentrated around v^e relative to the case of the more informative information structure. In general, this implies that the slope of the distribution $G_\eta(\omega)$ locally rotates around some point within the support of the distribution (see Fig. 6).

Rotating distributions implies that any two distribution functions $G_\eta(\omega)$ and $G_{\eta'}(\omega)$ intersect only once. Equivalently, for a fixed η, the function $\frac{\partial G_\eta(\omega)}{\partial \eta}$ possesses the single-crossing property, that is, it intersects the horizontal axis only once (from above) for all $\omega \in (0,1)$. Also, since $\eta = 0$ implies that the buyer does not get any useful information, so that her valuation is v^e, it follows that $\omega_0' = v^e$.

Before starting the analysis, we first consider a few motivating examples that demonstrate

their paper, an increase in θ results in a less informative signal s^θ, which is opposite to our case. Thus, these definitions are equivalent, if we put $\eta = 1 - \theta$.

the role of competition in the sellers' decisions regarding information disclosure and highlight the general intuition behind the main results below.

3.3 Examples

In three examples below, we look at the suppliers' incentives to reveal information about their products in markets with different magnitudes of competition. By considering situations with one, two, and three sellers, we demonstrate that the sellers' incentives to reveal information increase monotonically as competition becomes more tense, and change from one extreme to another in terms of the quality of provided information. In particular, the examples will show that the monopolist never reveals any useful information. In the case of duopoly, the sellers reveal information partially. Finally, the market with three sellers results in full information revelation.

For simplicity, we assume that the buyer's valuations are distributed uniformly on the unit interval. Also, the quality of signals is binary, i.e., the buyer either learns a product's value precisely or gets a fully uninformative signal.

First, consider the case of the monopolist. If the seller discloses information, then in any incentive-compatible mechanism, the seller cannot extract more than the virtual value of a good $v - \frac{1-G(v)}{g(v)}$, which is less than the true value v (Myerson, 1981). Hence, the seller's expected profit is strictly below the expected value of the good v^e. In contrast, not revealing information deprives the buyer of any informational rent, which allows the seller to get all social surplus v^e by setting the price $p_M = v^e$.

Introducing the second seller in the market dramatically affects the sellers' decisions about information disclosure. To see this, suppose first that both sellers reveal information about their products. Then, there exists a unique symmetric equilibrium price $p = \sqrt{2} - 1 \simeq 0.414$, which results in the expected profit $\pi \simeq 0.172$ for each seller. In contrast, if one of the sellers does not disclose information about his product and charges the same price, his expected profit increases to 0.207. Thus, full information revelation is not a part of the equilibrium strategy.

However, there is no equilibrium, in which both sellers hide information. By contradiction, if both sellers do not disclose information, then their products are ex-ante identical to the buyer. That is, the market transforms into the classic Bertrand competition with the unique equilibrium price at zero, which results in the zero expected profits for both sellers. However, each seller can guarantee a positive expected profit by revealing information and charging a small positive price

p_ε. In this case, the buyer will prefer the product of this seller if $v - p_\varepsilon > v^e$, which occurs with a positive probability.

Finally, consider a market with three sellers. In this case, there exists an equilibrium such that all sellers reveal information, charge the price $p \simeq 0.322$, and receive the expected profit $\pi \simeq 0.104$. In contrast to the previous situation, hiding information by any seller and adjusting the price optimally results in a lower expected profit of 0.082.

To explain the intuition behind these examples, we start with a simple case of the duopolistic market. Suppose that both sellers reveal information. This market has several important features. First, revealing information by sellers implies that the buyer's posterior valuations are more dispersed across products compared to the case of non-disclosure. In other words, the fully revealing market is characterized by substantial product differentiation that relaxes competition. A combination of product differentiation and a small number of competitors provides significant market power to each seller. This is reflected in a high probability that the buyer would prefer the product of a particular seller to those of competitors.[63] At the same time, it allows sellers to set high prices. However, to sell the product, its valuation must be also above the price. Since market power enforces sellers to set high prices, the sellers' benefits are damaged by the fact that the buyer is fully informed about products, so that she does not make a purchase if the product's value is below its price. We refer to this as the *informational rent effect*. For example, for the seller 1 this effect is graphically represented by area C in the left part of Fig. 7.

Given these observations, each seller may increase the overall probability of selling the product by not revealing information and leaving the buyer with the expected valuation regarding his product. For example, for seller 1, the new probability is determined by the mass of points in area A' in the right part of Fig. 7. The reason is that a small number of competitors implies that, with a sufficiently high probability, the buyer would still prefer the product of this seller if the net value of the competing product is below the net expected value. At the same time, hiding information eliminates the risk of not selling the product if its true value is below the price. In other words, it removes the informational rent effect, while keeping product differentiation. Together, these two factors enforce sellers to hide information, which breaks full information revelation in the market.

However, if competition becomes fiercer, then choosing the non-revealing policy by some

[63] For, say, seller 1, this probability is determined by the mass of points in the area above line $v_2 = v_1 - p_1 + p_2$ in the left part of Fig. 7.

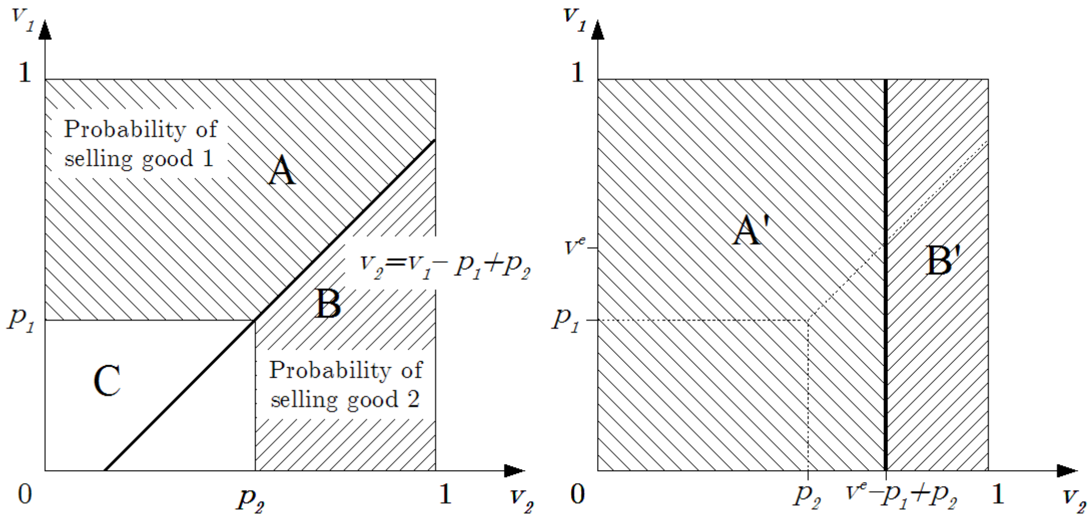

Figure 7: Disclosure of information by seller 1 Non-disclosure of information by seller 1

seller significantly reduces the chance of selling his product. To see this, notice first that the market with multiple products is equivalent to a duopolistic market, which consists of the product of this seller and the most valuable product offered by competitors. In this environment, the expected value of seller 1's product is much lower than that of the best competitor's. Under these circumstances, even though not revealing information expands the set of values that results in selling seller 1's product (area A' is bigger than A), the total mass of values is mainly concentrated in the complement area B'. That is, hiding information results in a loss of the possibility of serving the high-value consumer, who would prefer seller 1's product to all others (area $A \cap B'$). In contrast, revealing information and, thus, increasing the variance of ex-post values of the product is the only possibility to attract the buyer with a high realization of the product's value and sell the product. In this situation, the role of the buyers' informational rent is minor.

The above intuition can be reinterpreted for the case of a continuum of buyers as follows. A fully revealing market is characterized by high segmentation, since each seller serves only the share of high-value consumers who prefer his product to those of competitors. If competition is not fierce, then each seller controls a large segment of the market. Moreover, this segment does not change significantly if some seller decides to hide information about his product. The reason is that a large share of consumers always prefer the product with the expected value to others with known valuations. In other words, the trade-off between the benefits of segmenting the market and the losses due to providing the informational rent to the buyer is essentially not affected by the presence of competitors. Thus, each seller has incentives to hide information without loosing his market segment.

However, tense competition essentially influences the above trade-off via the market segmentation. In particular, hiding information results in a loss of the most attractive share of buyers, who would prefer this product to all others if they were informed about it. In addition, the share of consumers who prefer the product with the expected value to all others, significantly shrinks because of a variety of other products. That is, hiding information cuts a large piece of the seller's segment of the market in favor of the rivals. Thus, lack of information moves the product into a "low value" niche of the market relative to the best product among competing ones.

While considering strategic interaction among the sellers in the example above, one can see that the informational component in the sellers' decisions cannot be isolated from the prices. The reason is that prices play a dual role. First, they influence market segmentation by changing the net value of each product relative to those offered by competitors. Second, they determine the magnitude of the informational rent effect.[64] Thus, in order to isolate the pricing effects, we start the general analysis by investigating the seller's decisions over this dimension.

3.4 Pricing

In this subsection, we analyze the sellers' pricing policies. Since our primary goal is to focus on the existence of fully revealing equilibria, we start with a situation in which sellers disclose all available information.[65] In this case, we characterize the regularity condition that guarantees the existence of the unique symmetric price across sellers. In general, this condition is not restrictive and holds for a large class of standard distributions.

3.4.1 Monopoly

First, consider the case of the monopolist, who sells the product to the buyer with a valuation $v \sim G[0,1]$. Let p_M be the monopoly price, i.e., it is the solution to the profit-maximization problem

$$\max_{x} \ \pi(x) = \max_{x} (1 - G(x))\, x \tag{15}$$

[64] Graphically, a change in prices shifts the "market segmenting" line $v_2 = v_1 - p_1 + p_2$ in Fig. 7. In addition, a change in prices affects low-value areas below them (e.g., area C), which reflects the informational rent effect.

[65] If sellers disclose information completely, then $G_{\eta_i}(\omega) \triangleq G(\omega)$ for all i.

The first-order condition for this problem is

$$1 - G(x) - xg(x) = (1 - G(x))(1 - x\lambda(x)) = 0,$$

where $\lambda(x) = \frac{g(x)}{1-G(x)}$ is the hazard rate function of the distribution $G(.)$. Thus, the monopoly price must satisfy

$$p_M \lambda(p_M) = 1. \tag{16}$$

To guarantee that the monopoly profit is quasi-concave in x, so that the monopoly price is uniquely determined and characterized by (16), we impose the following regularity condition on the distribution of posterior values.

Condition 1 *The density function $g(v)$ is log-concave.*

Given this condition, the distribution possesses two useful properties: the increasing hazard rate (IHR) property (or, equivalently, the log-concavity of the survival function $1 - G(.)$) and the log-concavity of the distribution function $G(.)$.

Lemma 1 *(Bagnoli and Bergstrom, 2005) If $g(.)$ is continuously differentiable and log-concave on (a, b), then $G(.)$ is log-concave on (a, b), and the hazard rate $\lambda(.)$ is monotone increasing on (a, b).*

Even though Condition 1 is generally more restrictive than the IHR property, it holds for a large class of standard distributions in the subspace of parameters that imply the IHR property (e.g., beta, gamma, Weibull, power, and other distributions).[66]

In addition, we require that

Condition 2 *$G_\eta(\omega)$ has an increasing hazard rate $\lambda_\eta(\omega)$ for all η.*

Notice that the log-concavity of the density for an arbitrary η is not required for our purposes, since we are mainly interested in the behavior of $G_\eta(\omega)$ around the fully informative quality of information. However, the increasing hazard rate of $G_\eta(\omega)$ is the standard condition that guarantees the uniqueness of the monopoly price, when the buyer's valuations are distributed according to this distribution. In the case of competition, we show that this property implies that the price in the competitive market is lower than the monopoly price.

[66] For other examples, see Bagnoli and Bergstrom (2005).

3.4.2 Competition

If the number of sellers exceeds one, the IHR property may be insufficient to prove the existence of a symmetric price p_N^S. In order to resolve this issue, we employ Condition 1. In particular, consider the problem of, say, seller 1, who sets the price x, given that the other sellers set the price p. In this case, the expected profit of seller 1 is

$$\pi(x,p) = P(x,p)\,x,$$

where $P(.,.)$ is the probability of selling the product. It is determined by

$$P(x,p) = G_Y(p)\,(1 - G(x)) + \int_p^1 (1 - G(y - p + x))\, dG_Y(y)\,, \tag{17}$$

where $G_Y(y) = G^{N-1}(y)$ is the distribution of the maximal value $y = \max\{v_2, ..., v_N\}$ across the products of competitors $j = 2, ..., N$.

Intuitively, the first component in (17) reflects the fact that seller 1 may become the monopolist if the values of all other products are below p, which occurs with the probability $G_Y(p)$. With the complement probability, seller 1 has to compete against the most valuable product offered by competitors. Hence, he sells the product only if $v - x \geq y - p$, or $v \geq y - p + x$. Taking the average over all possible values of y gives the expected profit.

Notice that the seller's expected profit is a convex combination of the quasi-concave functions, which is generally not quasi-concave. However, the following lemma demonstrates that the regularity condition resolves this problem. In particular, Condition 1 guarantees that there is a unique symmetric equilibrium price in the fully revealing market, and outlines some properties of this price.[67]

Lemma 2 *For any number of sellers N, there exists a unique symmetric equilibrium price p_N^S, which is (1) less than the monopoly price, (2) strictly positive, and (3) converges to zero as N increases.*

The proof of the existence of the price consists of four steps. First, the log-concavity of $g(.)$ implies that the density $g_Y(y)$ is also log-concave. Second, it also implies that the function

[67] With minor abuse, we call a price an *equilibrium* one, if it constitutes an equilibrium for a given quality of information revealed.

$\min\{1-G(x), 1-G(y-p+x)\}$ is log-concave in (x,y). Third, we can represent $P(x,p)$ as

$$P(x,p) = \int_0^1 \min\{1-G(x), 1-G(y-p+x)\}\, g_Y(y)\, dy.$$

Finally, using Prékopa's theorem on the preservation of the log-concavity by integration over a convex set gives the desired result.

In order to characterize the asymptotic properties of the price, we demonstrate that it has the rate of convergence $1/N$ as the number of sellers grows. Moreover, the rate of convergence does not depend on the distribution of consumer's values.

An important implication of the above lemma is that fiercer competition reduces the magnitude of the informational rent effect due to two factors. First, a lower price directly reduces the chance of not selling a product to the buyer if its value is low. Second, before making a decision about a purchase, the buyer always prefers the product, which gives her the highest net value, and buys this product if this value is non-negative. In the case of a symmetric price, it is equivalent to preferring the product with the highest value. However, the distribution function of the best product shifts toward higher values as competition becomes tenser, which additionally decreases the probability of not selling the product to the informed buyer. This is reminiscent of the "winner curse" effect in the auction theory. Since a seller of a particular product cannot observe the buyer's signal, then the fact that his product wins against competing ones raises the product's expected value.[68]

Given these preliminaries, we are ready to explore the sellers' decisions about the quality of provided information.

3.5 Information Disclosure

In this subsection, we investigate the sellers' incentive to reveal information and provide the main results about information disclosure. The examples above demonstrate that if the number of sellers in the market is small, then full information disclosure is not feasible. The reason is that in the market with few suppliers the magnitude of the informational rent effect is unambiguously bigger. First, the fully revealing market is characterized by a high equilibrium price, which decreases the chance of selling the product. Second, each seller cannot sell his product if one of

[68] Graphically, this means that for a given area C in Fig. 2, the mass of points in the area falls as the number of sellers goes up.

the following events occurs: the product's value is below the price or the buyer's payoff is lower than that from another product. Thus, if one seller, for instance, hides information completely, then the buyer will value his product at the mean. This eliminates the risk of the first event without raising significantly the risk of the second event. As a result, the chance of selling the product increases.

However, if there are many competitors in the market, then the above logic does not work. Generating the expected valuation by hiding information gives a small chance of selling the product, since there is a high probability that the maximal valuation across other products is large.[69] That is, the probability of the second event increases enormously. Moreover, tense competition pushes the prices down, which leaves even less freedom for a potential deviator to attract the consumer by offering her a lower price. In other words, it is better to reveal the information in a hope that the value will be high.

Before formalizing the intuition above, we characterize first the necessary conditions for full revelation. These conditions allow us to get insight into the main properties of the distributions of consumer's values, which determine the trade-off between the benefits of segmenting the market and providing the informational rent to the buyer. All proofs can be found in the Appendix.

Lemma 3 *If $G^{N-1}(v^e) \geq \frac{1}{N}\max\left\{\frac{p_M}{v^e}, 1\right\}$, there is no fully revealing symmetric equilibrium in the market with N sellers.*

The above lemma relates the properties of distribution of values to the magnitude of the informational rent effect. In general, given a fixed number of sellers, the condition in the lemma is likely to hold if the distribution of values shifts toward low values. Intuitively, if the value is likely to be low, then not revealing any information increases the chance of selling the product to the buyer, since it generates a deterministic valuation as compared to the random value v.[70] At the same time, there are no incentives to reveal information in a hope that the value of product will exceed the best competing product, since the values of competing products are also likely to be low. In contrast, if the density is skewed to the right, then the magnitude of these two factors becomes smaller. As a results, it is possible to obtain the fully revealing market with two

[69] Instead of hiding information completely, revealing information only partially forms a distribution of expected valuations that is concentrated around the mean more than the prior distribution of values. Thus, the argument is still true.

[70] Consider, for instance, the truncated exponential distribution with the density $g(v) = \frac{\alpha\exp(-\alpha v)}{1-\exp(-\alpha)}$, $v \in [0,1]$, parameterized by α. Then, for $\alpha = -2$, the condition in Lemma 3 is satisfied only for the monopolistic market. However, for $\alpha = 8$, the density shifts towards zero, and even three sellers are not enough guarantee full disclosure.

sellers only.

Also, for a given distribution, as the number of sellers increases, the condition in Lemma 3 is violated for sure, since the left-hand side of the inequality declines at the exponential rate, whereas the right-hand side has the rate of convergence $1/N$. This implies that fierce competition reduces the sellers' incentives to hide information.

The interpretation of the lemma above may be seen more clearly in the duopoly market.

Corollary 2 *If v^e is no less than both the median value and the monopoly price, then there is no fully revealing symmetric equilibrium in the market with two sellers.*

Corollary 1 implies that for the duopoly market, there is no fully revealing equilibrium for all *symmetric* distributions. For this class of distributions, the expected value is equal to the median. Moreover, it can be shown that Condition 1 guarantees that the monopoly price is below the expected value.

Lemma 4 *If $g(v)$ is symmetric, then there is no fully revealing equilibrium in the market with two sellers.*

Intuitively, for any distribution, the mass of points below the mean value can serve as an approximate measure of the informational rent effect. At the same time, the mass of points above the mean measures the attractiveness of the competing product in the case of hiding information. This is because by not disclosing information, the seller guarantees that the valuation of his product is equal to the mean value. Given the fact that the competitor reveals information, the mass of points above the mean determines the probability of the competing product having a higher value than that of seller 1. Following this logic, the above result demonstrates that if weights of these two factors are the same, the informational rent effect on the sellers' incentives dominates that of competition with the other seller. In other words, by hiding information, the probability of selling the product to an uninformed consumer dominates the probability of losing her to the seller with a better product.

The discussions above lead to the following major result.

Theorem 1 *There is a critical number of sellers N' such that for all $N \geq N'$, there exists a unique symmetric equilibrium, in which all sellers disclose full information.*

In addition to the intuition for the examples above, a decrease in the sellers' incentives to hide the information in response to fiercer competition can be explained from a different angle.

Consider the monopoly that sells the product to a buyer, who has an outside option. If the value of this option is small, then the monopolist has no incentives to disclose information, since it just endows the buyer with the informational rent. However, when the outside option becomes more attractive (in particular, if it exceeds the mean value), the monopolist has to reveal his information in a hope that the buyer would like his product more than the outside option. Now, if we consider a particular seller in the competitive market, then the maximal value across the products of competitors is the buyer's outside option. In other words, all sellers play a role of a buyer's outside option for each other. Thus, as competition becomes tenser, the value of an outside option (stochastically) grows, thereby, enforcing the sellers to reveal full information.[71]

3.6 Information and Market Efficiency

In this subsection, we relate the quality of information that is endogenously determined by the market to the market efficiency. The market efficiency is measured by the relative difference between the total surplus, i.e., a sum of the consumer surplus and the sellers' expected profits, and social welfare measured by the expected value of the best product in the market. Full efficiency can be reached if we introduce the central planner, who makes all sellers reveal full information and set the price at the unit cost level.

In the case of the monopoly, equilibrium is socially optimal. Even though the expected payoff of the buyer is zero, the monopolist's profit is equal to the expected value of the good, which coincides with social welfare.

If the number of sellers exceeds one, the market is not efficient. First, if competition is not tense, so that sellers do not reveal information precisely, then the buyer loses because of the possibility of not buying the most valuable product. Second, if the market is highly competitive and sellers disclose all information, the product differentiation allows the sellers to sustain a positive price for an arbitrary number of competitors. This, however, creates inefficiency, since there is a chance that the buyer would not purchase the best product, if its value is below the price. This raises a natural question of how fast the market converges to the social optimum as

[71] Technically, by decreasing the quality of information about his product, the seller rotates the distribution of expected valuations so that the density becomes concentrated around the mean. This decreases the degree of differentiation among products. In particular, it reduces the chance of selling the object by making the tail of the distribution $G_\eta(.)$ thinner. Since the product has to compete against a product with a value Y, which is likely to be high, the chance of selling the product falls. Notice that this intuition holds true, even when all other competitors disclose information only partially. In this case, it is still profitable for any seller to reveal information completely, which guarantees the uniqueness of the symmetric equilibrium.

the number of sellers grows. The following lemma addresses this issue. In particular, we show that the welfare losses due to the decision of an unsatisfied buyer to leave the market decrease faster than exponentially.

Lemma 5 *As the number of sellers goes up, the inefficiency of the market converges to zero at the rate* $1/N^N$.

Thus, the magnitude of inefficiency goes down at a very fast rate as the number of the sellers grows. This is because of two factors. First, the higher number of sellers implies the highest value across products stochastically grows. Second, competition decreases the market price, which tends to the marginal cost at the rate $1/N$.

Finally, consider the asymptotic properties of the market structure. An implication of the above result is that more severe competition results in the convergence of the market structure to the standard Bertrand competition not only in terms of the symmetry of the price and the perfect quality of buyer's information, but also in terms of the efficiency, even though the model's settings are essentially different from the classic setup.

3.7 Conclusion

In this chapter, we provide a possible explanation of different behavior of firms with respect to unraveling information that can affect buyers' private valuations of products. We demonstrate that the firms' incentives to provide such information crucially depend on the fierceness of competition in the market. Starting with the analysis of the non-disclosure case of a monopoly, we show that firms never reveal full information in low-competitive markets, and eventually disclose full information as the market becomes sufficiently competitive. Thus, full information revelation is an endogenously determined attribute of sufficiently competitive markets only. This result demonstrates that competition refines the extreme choices of the monopolist, which prefers to reveal either full information or no information (Lewis and Sappington, 1994; Johnson and Myatt, 2006).

Second, as competitive becomes fiercer, the market structure becomes similar to Bertrand competition. That is, even though all products are ex-ante different, sellers charge the symmetric price that converges to the marginal cost as the number of sellers goes up. Since there are no informational losses, the price is the only source of inefficiency, the magnitude of which, however, virtually disappears as competition intensifies.

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Appendix A

Proofs to Chapter 1.

Denote by $S(\omega) = \{s : \omega_s = \omega\}$ the set and by $F(s|\omega) = F(s|s \in S(\omega))$ the distribution of signals that generate a type ω. Also, let $U_S(a, b|\omega)$ be the utility function of an expert's type ω:

$$U_S(a,b|\omega) = \int\limits_{S(\omega)} U(a,b|s)\, dF(s|\omega) = \frac{\int\limits_{S(\omega)} U(a,b|s)\, dF(s)}{\int\limits_{S(\omega)} dF(s)} \tag{18}$$
$$= U_S(a,b,\omega) - D_\omega,$$

where $D_\omega = \frac{\int_{S(\omega)} D_s dF(s)}{\int_{S(\omega)} dF(s)}$ are informational losses of a type ω. Similarly, define $U_R(a|\omega) = U_S(a,0|\omega)$.

Proof of Lemma 1. Let a and a' be two induced actions, where $a' > a$. Consider types ω and ω', which induce corresponding actions, that is, $U_S(a,b|\omega) \geq U_S(a',b|\omega)$ and $U_S(a',b|\omega') \geq U_S(a,b|\omega')$. Then, it follows from (18) that $U_S(a,b,\omega) \geq U_S(a',b,\omega)$ and $U_S(a',b,\omega') \geq U_S(a,b,\omega')$.

The single-crossing property of the expert's utility function $\frac{d^2}{dad\theta}U_S(a,b,\theta) > 0$ implies that there exists a state $\theta \in (\omega_s, \omega_s')$ such that $U_S(a',b,\theta) = U_S(a,b,\theta)$. Also, this property leads to (i) $a < a^S(\theta) < a'$, where $a^S(\theta) = \theta + b$, (ii) a is not induced by any type $\omega > \theta$, and (iii) a' is not induced by any type $\omega < \theta$. The last two properties along with the single-crossing property of $U_R(a,\theta)$ imply $a \leq a^R(\theta) = \theta \leq a'$.

In addition, the symmetry of $U_S(a,b,\theta)$ with respect to $a^S(\theta)$ implies that $a' - \theta - b = \theta + b - a$, or $a^S(\theta) = \theta + b = \frac{a+a'}{2}$. This means that both $a^S(\theta)$ and $a^R(\theta)$ belong to the interval $[a, \frac{a+a'}{2}]$. Since $a^S(\theta) - a^R(\theta) = b$, it follows that $\frac{a+a'}{2} - a \geq b$, or $a' - a \geq 2b$. To complete the proof, notice that the set of induced actions is bounded by $a^R(0)$ and $a^R(1)$. ■

Denote by $\sigma_a(\omega)$ the probability of inducing an action a by the expert of type ω. Since the set of actions $\mathcal{A}$ is finite, we can put $\mathcal{A} = \{a_i\}_{i=1}^N$, where $a_{i+1} > a_i, \forall i$. For simplicity, we use $\sigma_i(\omega)$ for the probability of inducing an action a_i by the type ω. That is,

$$\sigma_i(\omega) = \int\limits_{S(\omega)} \int\limits_{\mathcal{M}(a_i)} \sigma(m|s)\, dm dF(s|\omega).$$

Given a message m, the principal's best response is

$$a(m) = E[E[\theta|s]|m] = E[\omega_s|m]. \tag{19}$$

Then, the principal's expected payoff can be represented as

$$U_R = \sum_{i=1}^{N} \int_{\Omega} \sigma_i(\omega) U_R(a_i|\omega) dG(\omega).$$

The following lemma outlines the properties of $\{\sigma_i(\omega)\}_{i=1}^N$.

Lemma 6 *In any equilibrium, $\{\sigma_i(\omega)\}_{i=1}^N$ satisfies the following conditions:*

(A) $\sigma_i(\omega) > 0$ implies $\sigma_j(\omega) = 0$, for all $j < i-1$ and $j > i+1$,

(B) the expert of the highest type $\bar{\omega}$ purely induces the highest action,

(C) $\sigma_i(\omega') > 0$ implies that $\sigma_j(\omega) = 0$, for all $j < i, \omega > \omega'$, and $j > i, \omega < \omega'$,

(D) $\sigma_i(\omega') > 0, \sigma_i(\omega'') > 0$ imply $\sigma_i(\omega) = 1, \omega \in (\omega', \omega'')$, and

(E) if $\sigma_i(\omega') > 0, \sigma_{i+1}(\omega') > 0$, then there exists $\omega'' > \omega'$ such that $\sigma_{i+1}(\omega'') > 0$ and $\sigma_{i+1}(\omega) = 1, \omega \in (\omega', \omega'')$.

Proof (A) This property follows from the strict concavity of $U_S(a, b|\omega)$ in a. By contradiction, let $\sigma_i(\omega) > 0$ and $\sigma_j(\omega) > 0$, where $j > i+1$, for some $\omega \in \Omega$. This implies that $U_S(a_i, b|\omega) = U_S(a_j, b|\omega) \geq U_S(a, b|\omega), \forall a \in \mathcal{A}$. Since a_{i+1} can be represented as a convex combination of a_i and a_j, $a_{i+1} = \lambda a_i + (1-\lambda) a_j$ for some $\lambda \in (0,1)$, this results in a contradiction: $U_S(a_{i+1}, b|\omega) > \lambda U_S(a_i, b|\omega) + (1-\lambda) U_S(a_j, b|\omega) = U_S(a_i, b|\omega)$.

(B) From (19), the highest induced action $a_N \leq \bar{\omega} < \bar{\omega} + b$. Since $U_S(a, b, \omega)$ is strictly increasing in a, for all $a \leq \omega + b$, the type $\bar{\omega}$ cannot be indifferent between $\bar{a}$ and any $a < \bar{a}$.

(C) By contradiction, let $\sigma_i(\omega') > 0$ and $\sigma_j(\omega) > 0$ for some $j < i, \omega > \omega'$. Then, $\sigma_i(\omega') > 0$ implies $U(a_i, b, \omega') \geq U_S(a_j, b, \omega')$ and $\sigma_j(\omega) > 0$ implies $U(a_j, b, \omega) \geq U(a_i, b, \omega)$. Combining these inequalities results in $U(a_j, b, \omega') - U(a_i, b, \omega') \leq 0 \leq U(a_j, b, \omega) - U(a_i, b, \omega)$, which contradicts the single-crossing property $U(a_i, b, \omega) - U(a_j, b, \omega) > U(a_i, b, \omega') - U(a_j, b, \omega')$.

(D) For any $\omega \in (\omega', \omega'')$, applying property (C) to both $\sigma_i(\omega')$ and $\sigma_i(\omega'')$ implies that $\sigma_j(\omega) = 0$ for all $j \neq i$ that gives the desired result.

(E) By contradiction, let $\sigma_i(\omega') > 0, \sigma_{i+1}(\omega') > 0$, and $\sigma_{i+1}(\omega) = 0$ for all $\omega > \omega'$. Since ω' is indifferent between a_i and a_{i+1}, then $\omega' + b < a_{i+1}$. Property (C) for $\sigma_i(\omega')$ implies $\sigma_{i+1}(\omega) = 0$ for all $\omega < \omega'$. That is, $\sigma_{i+1}(\omega) = 0$ for all $\omega \neq \omega'$. Hence, a_{i+1} is induced by the type ω' only, which means $a_{i+1} = \omega'$. This results in a contradiction $a_{i+1} = \omega' > \omega' + b$. Thus, there exists a type $\omega'' > \omega'$ that induces an action a_{i+1}, or, equivalently, $\sigma_{i+1}(\omega'') > 0$. Then, property (D) implies that $\sigma_{i+1}(\omega) = 1, \omega \in (\omega', \omega'')$.

■

The first condition states that the expert of each type can mix between two adjacent actions only. The second condition requires the expert of the highest type to purely induce the highest action. The third condition is the single-crossing property, which implies that if an expert of some type induces an action, then no expert of a higher type induces a lower action, and vice versa. Condition (D) argues that the set of types that induce the same action is an interval. Finally, the last condition states that if some type induces two actions, then there exists a higher type that induces the higher action also.

Using the lemma above, the expert's signaling strategy can be described by a pair of functions $\{j(\omega), \sigma_\omega\}, \omega \in \Omega$, which means that a type ω induces a lower action $a_{j(\omega)}$ with a probability σ_ω. With a complement probability, this type induces $a_{j(\omega)+1}$.

Proof of Lemma 2. In any equilibrium with an information structure $\langle \mathcal{S}, \mathcal{M}, F(\theta|s) \rangle$ and an action set $\mathcal{A} = \{a_i\}_{i=1}^N$, denote $\mathcal{W} \subset \Omega$ the set of types that induces two actions. That is, $\mathcal{W} \subset \{y_i\}_{i=1}^{N-1}$, where $y_i = \frac{a_i + a_{i+1}}{2} - b, i = 1, ..., N-1$, and $y_i \in \mathcal{W}$ implies $\sigma_{y_i} \in (0,1)$.

Split the set $\Omega \backslash \mathcal{W}$ into a finite collection of subsets $\{\mathcal{W}_i\}_{i=1}^N$ of types that purely induce an action a_i.[72] By property (C) of Lemma 6, these sets are strictly monotone in the sense that $i > j$ implies $\omega' > \omega$ for all $\omega' \in \mathcal{W}_i, \omega \in \mathcal{W}_j$.

Modify the initial information structure as follows. For each action a_i, collapse all types $\omega \in \mathcal{W}_i$ into a single type

$$\omega_i = \frac{\int\limits_{\mathcal{W}_i} \omega dG(\omega)}{\int\limits_{\mathcal{W}_i} dG(\omega)} \tag{20}$$

by, for instance, collapsing all signals $s \in \bigcup\limits_{\omega \in \mathcal{W}_i} S(\omega)$ that generates these types. Thus, a type ω_i is generated with a probability $P(\omega_i) = \int\limits_{\mathcal{W}_i} dG(\omega)$.

Notice that $\mathcal{W} \cap \{\omega_i\}_{i=1}^N = \varnothing$. Since $y_i \in \mathcal{W}$ induces actions a_i and a_{i+1}, then by property (C) of Lemma 6, we have $\omega > y_i$ for all $\omega \in \mathcal{W}_j, j \geq i+1$. Hence, $\omega_j = E[\omega | \omega \in \mathcal{W}_j] > y_i, j \geq i+1$. Similarly, $\omega_j = E[\omega | \omega \in \mathcal{W}_j] < y_i, j \leq i$, since $\omega < y_i$ for all $\omega \in \mathcal{W}_j, j \leq i$.

Given that the expert's strategy on a modified type space is not affected by the modification in the information structure, the principal's best response is not affected either.[73]

Since each ω_i is bounded by the set $\mathcal{W}_i$, property (D) of Lemma 6 implies that $\sigma_i(\omega_i) = 1, \forall i$. Similarly, all types $y_i \in \mathcal{W}$ cannot beneficially deviate from mixing between a_i and a_{i+1}. Finally, the

[72] Notice that $\mathcal{W}_i$ can be empty for some a_i, but not for all actions, since the expert of the highest type purely induces the highest action.

[73] That is, if each type $\omega_i, i = 1, ..., N$, purely induces an action a_i, and each type $y_i \in \mathcal{W}$ induces a_i and a_{i+1} with the same probabilities as in the initial equilibrium.

principal's expected payoff U_R' is equal to that in the initial equilibrium, since

$$U_R = \int_{\Omega} \sigma_\omega U_R\left(a_{j(\omega)}|\omega\right) + (1-\sigma_\omega)\, U_R\left(a_{j(\omega)+1}|\omega\right) dG(\omega)$$
$$= \sum_{i=1}^{N-1} P(y_i)\left(\sigma_{y_i} U_R(a_i|y_i) + (1-\sigma_{y_i})\, U_R(a_{i+1}|y_i)\right) + \sum_{i=1}^{N} \int_{\mathcal{W}_i} U_R(a_i|\omega)\, dG(\omega)$$
$$= \sum_{i=1}^{N-1} P(y_i)\left(\sigma_{y_i} U_R(a_i|y_i) + (1-\sigma_{y_i})\, U_R(a_{i+1}|y_i)\right) + \sum_{i=1}^{N} P(\omega_i)\, U_R(a_i|\omega_i) = U_R',$$

where $P(\omega) = \int_{S(\omega)} dF(s)$ is a probability of generating a type ω.

Now, we prove first Lemma 3 and Lemma 4, and use them to compete the second part of the this lemma. ■

Proof of Lemma 3. By contradiction, suppose that there exists an equilibrium, in which the finite number of induced actions $\#\mathcal{A}$ exceeds the number of types $\#\Omega$. This implies that there exists a type $y_i \in \mathcal{W}$, such that $\sigma_{i+1}(\omega) = 0$ for all $\omega > y_i$. However, this violates property (E) of Lemma **6**, which completes the proof. ■

Now, we identify the necessary and sufficient conditions for an information structure to be incentive-compatible.

A type ω_i prefers an action a_i to $a_{i+1} > a_i$ if and only if a_i is closer to her optimal action $\omega_i + b$; that is, if

$$\omega_i + b \leq \frac{a_i + a_{i+1}}{2}, i = 1, ..., N-1. \tag{21}$$

Since the principal's best response is $a_i = \omega_i$ and $a_{i+1} = \omega_{i+1}$, (21) can be transformed into

$$\omega_{i+1} - \omega_i \geq 2b, i = 1, ..., N-1. \tag{22}$$

Notice that the expert of a type ω_i never induces an action $a < a_i$, since $a < a_i = \omega_i < \omega_i + b$ means $U_R(a|\omega_i) < U_R(a_i|\omega_i)$. That is, (22) is sufficient to guarantee that an information structure is incentive-compatible.

Also, consider a type ω, which induces two actions, a_i and a_{i+1}. That is, $\omega + b = \frac{a_i + a_{i+1}}{2} < a_{i+1}$, or

$$\left|a_{i+1} - \omega\right| = a_{i+1} - \omega = \omega - a_i + 2b > |a_i - \omega|,$$

where the last inequality follows from the fact that $a_{i+1} - \omega > a_i - \omega$ and $\omega - a_i + 2b > \omega - a_i$.[74]

Because $U_R(a, \omega) = U(|a - \omega|)$, it follows that $U_R(a_i, \omega) - U_R(a_{i+1}, \omega) > 0$. Thus,

$$U_R(a_i|\omega) - U_R(a_{i+1}|\omega) > 0, \tag{23}$$

and

$$U_R(a_i|\omega) > \sigma_\omega U_R(a_i|\omega) + (1 - \sigma_\omega) U_R(a_{i+1}|\omega), \tag{24}$$

since $U'(x) \leq 0$ and $U''(x) < 0$, $x \geq 0$ result in $U'(x) < 0, x > 0$.

Proof of Lemma 4. Let $\mathcal{W}$ be the set of types that induce two actions. For each $i \in \#\mathcal{A}$, denote by Ω_i the set of types that induces an action a_i, that is, $\omega \in \Omega_i$ implies $\sigma_i(\omega) > 0$. By property (D) of Lemma 6, each Ω_i is an interval that includes $\mathcal{W}_i$. Also, if there exist $y_{i-1}, y_i \in \mathcal{W}$, then y_{i-1} and y_i are the lower and the upper boundary points of Ω_i, respectively. Thus, two intervals Ω_i and Ω_{i+1} have at most one common point $y_i \in \mathcal{W}$. Then, the principal's best response can be described by

$$a_i = \frac{\left(1 - \sigma_{y_{i-1}}\right) P(y_{i-1}) y_{i-1} + P(\omega_i) \omega_i + \sigma_{y_i} P(y_i) y_i}{\left(1 - \sigma_{y_{i-1}}\right) P(y_{i-1}) + P(\omega_i) + \sigma_{y_i} P(y_i)}, \forall i, \tag{25}$$

where ω_i is determined by (20).

Consider a modified expert's signaling strategy $\{\sigma_i^o(\omega)\}_{i=1}^N$, which is derived from $\{\sigma_i(\omega)\}_{i=1}^N$ as follows. For any type $y_i \in \mathcal{W}$, assign probability one to a lower action; that is, $\sigma_{y_i}^o = 1$. Therefore, $\mathcal{W}_i^o = \mathcal{W}_i \cup y_{i+1}$ is a new set of types that purely induce an action a_i. Denote by $\{a_i^o\}_{i=1}^N$ the action set, adjusted to this signaling strategy.

Given $\{\sigma_i^o(\omega)\}_{i=1}^N$ and $\{a_i^o\}_{i=1}^N$, the principal's expected payoff is

$$\begin{aligned}
U_R^o &= \sum_{i=1}^{N-1} P(y_i) U_R(a_i^o|y_i) + \sum_{i=1}^{N} \int_{\mathcal{W}_i} U_R(a_i^o|\omega) \, dG(\omega) \\
&\geq \sum_{i=1}^{N-1} P(y_i) U_R(a_i|y_i) + \sum_{i=1}^{N} \int_{\mathcal{W}_i} U_R(a_i|\omega) \, dG(\omega) \\
&\geq \sum_{i=1}^{N-1} P(y_i) \left(\sigma_{y_i} U_R(a_i|y_i) + (1 - \sigma_{y_i}) U_R(a_{i+1}|y_i)\right) \\
&\quad + \sum_{i=1}^{N} \int_{\mathcal{W}_i} U_R(a_i|\omega) \, dG(\omega) = U_R,
\end{aligned}$$

where the first inequality follows from the fact that $\{a_i^o\}_{i=1}^N$ is a best response to $\{\sigma_i^o(\omega)\}_{i=1}^N$, and the second inequality follows from (24) by summing across $y_i \in \mathcal{W}$.

[74] Formally, we need to consider the case $\omega - a_i \geq 0$ only, since an action a_i is not induced by any type above ω. However, the above argument does not depend on the allocation of a_i and ω.

Finally, modify the information structure by collapsing each $\mathcal{W}_i^o$ into a single type

$$\omega_i^o = \frac{\int\limits_{\mathcal{W}_i^o} \omega dG(\omega)}{\int\limits_{\mathcal{W}_i^o} dG(\omega)} = \frac{P(\omega_i)\,\omega_i + P(y_i)\,y_i}{P(\omega_i) + P(y_i)}, \forall i,$$

and consider a new expert's strategy, such that a type ω_i^o purely induces a_i^o. This modification of the information structure and an expert's strategy preserves inducing an action a_i^o by all $\omega \in \mathcal{W}_i^o$. That is, the principal's best response is the same and is determined by $a_i^o = \omega_i^o, \forall i$. Thus, the principal's expected payoff is U_R^o.

We complete the proof by showing that each new type $\omega_i^o, i = 1, ..., N$ cannot beneficially deviate from inducing a_i^o. First, notice that $a_i^o \geq a_i, \forall i$. By construction, $a_i^o = a_i|_{\sigma_{y_{i-1}}=1, \sigma_{y_i}=1}$. Taking the derivative of a_i with respect to $\sigma_{y_{i-1}}$ results in

$$\frac{\partial a_i}{\partial \sigma_{y_{i-1}}} = P(y_{i-1}) \frac{(\omega_i - y_{i-1})\,P(\omega_i) + (y_i - y_{i-1})\,\sigma_{y_i} P(y_i)}{\left(\left(1 - \sigma_{y_{i-1}}\right) P(y_{i-1}) + P(\omega_i) + \sigma_{y_i} P(y_i)\right)^2} \geq 0,$$

since $y_{i-1} < \omega_i < y_i$ by the same argument as used in the proof of Lemma 2. Similarly, $\frac{\partial a_i}{\partial \sigma_{y_i}} \geq 0$, which implies that $a_i^o \geq a_i, \forall i$.

For any i, consider the highest type $\bar{\omega}_i$ that weakly prefers a_i to a_{i+1}. Since inducing an action a_i by a type $\omega \in \Omega_i$ means that it is weakly preferred to a_{i+1}, we have $\bar{\omega}_i \geq \sup \Omega_i = \sup \mathcal{W}_i^o$.[75] Thus,

$$\omega_i^o + b \leq \bar{\omega}_i + b \leq \frac{a_i + a_{i+1}}{2} \leq \frac{a_i^o + a_{i+1}^o}{2} = \frac{\omega_i^o + \omega_{i+1}^o}{2},$$

which means that (22) holds for $\{\omega_i^o\}_{i=1}^N$. ■

Proof of Lemma 2 (cont.) Suppose that there exists an interval of types $\mathcal{W}'$ with a positive and continuous density $g(\omega)$, which induces two actions, a_k and a_{k+1}.[76] That is, there is a type ω_s, such that all types in $\mathcal{W}'$ below ω_s purely induce an action a_k, and all types in $\mathcal{W}'$ above ω_s purely induce a_{k+1}.

First, transform this equilibrium into an incentive-compatible one, using the same technique as that developed in the proof of Lemma 4. That is, for all types $y \in \mathcal{W}$, attribute 1 to the probability of inducing a lower action. Then, collapse each set of types $\mathcal{W}_i^o$ that purely induce an action a_i, into a single type ω_i^o. The resulting information structure is incentive-compatible and provides a payoff U_R^o to the principal that is (weakly) superior to that in the initial equilibrium. Denote by $\{a_i^o\}_{i=1}^N$ the action set in the incentive-compatible equilibrium.

Now, we modify the initial information structure as follows. As previously, put $\sigma_y = 1, y \in \mathcal{W}$. In addition, consider a type $\omega_\delta = \omega_s + \delta$, where $\delta \downarrow 0$, and put $\sigma_\omega = 1, \omega \in \mathcal{W}_\delta = [\omega_s, \omega_\delta]$. Finally, collapse

[75] Notice that $\bar{\omega}_i$ may be out of Ω_i, if $\bar{\omega}_i$ is indifferent between a_i and a_{i+1}, and purely induces a_{i+1}.
[76] An example of such an information structure is CS communication.

all types $\mathcal{W}_i^\delta$ that induce an action a_i. Thus, for $\delta > 0$, we reallocate a positive mass of types from ω_{k+1}^o to ω_k^o. We prove now that the resulting information structure is incentive-compatible, and provides an expected payoff $U_R^\delta > U_R^o$.

Let $\left\{a_i^\delta\right\}_{i=1}^N$ be the principal's best response to the truth-telling strategy $\sigma_i\left(\omega_i^\delta\right) = 1, \forall i$. That is,

$$a_i^\delta = \omega_i^\delta = \frac{\int\limits_{\mathcal{W}_i^\delta} \omega dG(\omega)}{\int\limits_{\mathcal{W}_i^\delta} dG(\omega)}, \forall i,$$

where $\mathcal{W}_i^\delta = \mathcal{W}_i^o$, and, thus, $a_i^\delta = a_i^o$ for all $i \neq k, k+1$.

The principal's expected payoff is

$$U_R^\delta = \int\limits_{\mathcal{W}_k^\delta} U_R\left(a_k^\delta|\omega\right) dG(\omega) + \int\limits_{\mathcal{W}_{k+1}^\delta} U_R\left(a_{k+1}^\delta|\omega\right) dG(\omega) + \int\limits_{\Omega\setminus\left\{\mathcal{W}_k^\delta \cup \mathcal{W}_{k+1}^\delta\right\}} U_R\left(a_{j(\omega)}^o|\omega\right) dG(\omega).$$

The continuity of $g(\omega)$ around ω_s guarantees that $\frac{\partial a_k^\delta}{\partial \delta}$ and $\frac{\partial a_{k+1}^\delta}{\partial \delta}$ are continuous in δ around 0. Since ω_δ is the upper bound of $\mathcal{W}_k^\delta$ and the lower bound of $\mathcal{W}_{k+1}^\delta$, we have

$$\begin{aligned}
\frac{d}{d\delta} U_R^\delta|_{\delta=0} &= U_R\left(a_k^o, \omega_s\right) g(\omega_s) - U_R\left(a_{k+1}^o, \omega_s\right) g(\omega_s) \\
&+ \int\limits_{\mathcal{W}_k^o} \frac{\partial}{\partial a} U_R\left(a_k^o, \omega\right) \frac{\partial a_k^\delta}{\partial \delta}|_{\delta=0} dG(\omega) + \int\limits_{\mathcal{W}_{k+1}^o} \frac{\partial}{\partial a} U_R\left(a_{k+1}^o, \omega\right) \frac{\partial a_{k+1}^\delta}{\partial \delta}|_{\delta=0} dG(\omega) \\
&= \left(U_R\left(a_k^o, \omega_s\right) - U_R\left(a_{k+1}^o, \omega_s\right)\right) g(\omega_s) \\
&+ \frac{\partial a_k^\delta}{\partial \delta}|_{\delta=0} \int\limits_{\mathcal{W}_k^o} \frac{\partial}{\partial a} U_R\left(a_k^o, \omega\right) dG(\omega) + \frac{\partial a_{k+1}^\delta}{\partial \delta}|_{\delta=0} \int\limits_{\mathcal{W}_{k+1}^o} \frac{\partial}{\partial a} U_R\left(a_{k+1}^o, \omega\right) dG(\omega).
\end{aligned} \tag{26}$$

By the principal's best response, $\int\limits_{\mathcal{W}_k^o} \frac{\partial}{\partial a} U_R\left(a_k^o, \omega\right) dG(\omega) = 0, \forall k$. That is, the last two components in (26) are equal to 0. Since type ω_s is indifferent between a_k and a_{k+1}, then $a_{k+1} - \omega_s = \omega_s - a_k + 2b$. Since $a_k^o \geq a_k$ and $a_{k+1}^o \geq a_{k+1}$, we have

$$\left|a_{k+1}^o - \omega_s\right| = a_{k+1}^o - \omega_s \geq a_{k+1} - \omega_s = \omega_s - a_k + 2b > \omega_s - a_k^o + 2b > \left|a_k^o - \omega_s\right|,$$

and $\frac{d}{d\delta} U_R^\delta|_{\delta=0} > 0$.

To complete the proof, we need to show that the truth-telling strategy $\sigma_i\left(\omega_i^\delta\right) = 1, \forall i$, is incentive-compatible. That is, it is sufficient to show that

$$\omega_{i+1}^\delta - \omega_i^\delta \geq 2b, \forall i. \tag{27}$$

By construction,

$$\omega_k^\delta = \frac{P(\omega_k^o)\,\omega_k^o + P(\omega_\delta)\,\omega_\delta}{P(\omega_k^o) + P(\omega_\delta)} \text{ and } \omega_{k+1}^\delta = \frac{P(\omega_{k+1}^o)\,\omega_{k+1}^o - P(\omega_\delta)\,\omega_\delta}{P(\omega_k^o) - P(\omega_\delta)},$$

where $\omega_\delta = \dfrac{\int_{\omega_s}^{\omega_s+\delta} \omega g(\omega) d\omega}{\int_{\omega_s}^{\omega_s+\delta} g(\omega) d\omega}$ and $P(\omega_\delta) = \int_{\omega_s}^{\omega_s+\delta} g(\omega)\, d\omega$.

Since $\omega_k^o < \omega_\delta < \omega_{k+1}^o$, and $P(\omega_\delta) > 0$, we have $\omega_k^\delta > \omega_k^o$ and $\omega_{k+1}^\delta > \omega_{k+1}^o$.

Now, for $i < k-1$ and $i > k+1$, (27) is satisfied, since $\omega_i^\delta = \omega_i^o$ and (22) holds for $\{\omega_i^o\}_{i=1}^N$. For $i = k-1$, we have

$$\omega_k^\delta - \omega_{k-1}^\delta > \omega_k^o - \omega_{k-1}^\delta = \omega_k^o - \omega_{k-1}^o \geq 2b.$$

For $i = k$, since $g(\omega) > 0$ around ω_s, then $\mathcal{W}_k^o$ is not a singleton. Hence, $\omega_s = \sup \mathcal{W}_k^o > \omega_k^o$. Then,

$$\omega_k^o + b < \omega_s + b = \frac{a_k + a_{k+1}}{2} \leq \frac{a_k^o + a_{k+1}^o}{2} = \frac{\omega_k^o + \omega_{k+1}^o}{2}$$

means that $\omega_{k+1}^o - \omega_k^o > 2b$. Since ω_k^δ and ω_{k+1}^δ are continuous in δ around 0, then $\omega_{k+1}^\delta - \omega_k^\delta > 2b$, for $\delta \downarrow 0$.

Similarly, $\mathcal{W}_{k+1}^o$ is not a singleton. That is, if $\bar{\omega}_{k+1}$ is the highest type that prefers a_{k+1} to a_{k+2}, then $\bar{\omega}_{k+1} > \omega_{k+1}^o$. Hence,

$$\omega_{k+1}^o + b < \bar{\omega}_{k+1} + b \leq \frac{a_{k+1} + a_{k+2}}{2} \leq \frac{a_{k+1}^o + a_{k+2}^o}{2} = \frac{\omega_{k+1}^o + \omega_{k+2}^o}{2},$$

and $\omega_{k+2}^o - \omega_{k+1}^o > 2b$. Since $\omega_{k+2}^\delta = \omega_{k+2}^o$, and ω_{k+1}^δ is continuous in δ around 0, then $\omega_{k+2}^\delta - \omega_{k+1}^\delta > 2b$, for $\delta \downarrow 0$. ■

Proof of Theorem 1. Consider any CS equilibrium, which contains more than one partition element. In this case, the arbitrage condition for a type θ_{k+1}, which is indifferent between actions a_k and a_{k+1}, is

$$\theta_{k+1} + b - a_k = a_{k+1} - \theta_{k+1} - b, \tag{28}$$

where $a_k$is determined by the principal's best response:

$$a_k = \omega_k = E[\theta | \theta \in (\theta_k, \theta_{k+1}]] = \frac{1}{F(\theta_{k+1}) - F(\theta_k)} \int_{\theta_k}^{\theta_{k+1}} \theta dF(\theta). \tag{29}$$

Then, (28) can be expressed as $\omega_{k+1} - \omega_k = 2(\theta_{k+1} - \omega_k) + 2b > 2b, \forall k$, since $f(\theta) > 0, \theta \in \Theta$, implies $\theta_{k+1} > a_k = E[\theta | \theta \in (\theta_k, \theta_{k+1}]]$. Thus, any CS partition $\{\theta_k\}_{k=0}^{N}$ generates the information structure $\{\omega_k\}_{k=0}^{N-1}$ that satisfies (22). Moreover, (22) are satisfied for all $\theta'_k \downarrow \theta_k, k = 1, ..., N$, since each ω_k is continuous in θ_k and θ_{k+1}.

The principal's expected payoff in the incentive-compatible equilibrium is

$$U_R = \sum_{k=0}^{N-1} U_R^k = \sum_{k=0}^{N-1} \int_{\theta_k}^{\theta_{k+1}} U_R(a_k, \theta) \, dF(\theta). \tag{30}$$

Then,

$$\begin{aligned} \frac{dU_R}{d\theta_1} &= U_R(a_0, \theta_1) f(\theta_1) - U_R(a_1, \theta_1) f(\theta_1) \\ &+ \frac{\partial a_0}{\partial \theta_1} \int_0^{\theta_1} \frac{\partial}{\partial a} U_R(a_0, \theta) \, dF(\theta) + \frac{\partial a_1}{\partial \theta_1} \int_{\theta_1}^{\theta_2} \frac{\partial}{\partial a} U_R(a_1, \theta) \, dF(\theta). \end{aligned}$$

From (29), the last two terms in the expression above are equal to 0. Also, (23) implies

$$\frac{dU_R}{d\theta_1} = (U_R(a_0, \theta_1) - U_R(a_1, \theta_1)) f(\theta_1) > 0.$$

Thus, the partition $\{0, \theta'_1, \theta_2, ..., 1\}$, where $\theta'_1 \downarrow \theta_1$, is incentive-compatible and provides a strictly higher expected payoff.[77] ■

Proof of Theorem 2. First, notice that $f(\theta)$, $f'(\theta)$, and $\frac{f'(\theta)}{f(\theta)}$ are bounded, since $f(\theta)$ and $f'(\theta)$ are continuous, and $f(\theta) > 0$ for all $\theta \in \Theta$. That is, $f(\theta) \leq \bar{f}$ and $|f'(\theta)| \leq \eta$.

If $\Delta\theta_k = \theta_{k+1} - \theta_k$ is the length of a partition's element Θ_k, then the principal's best response (29) can be represented by Taylor's formula around θ_k as

$$a_k(\Delta\theta_k) = \theta_k + \frac{1}{2}\Delta\theta_k + \mu\left(\tilde{\theta}_k\right)\Delta\theta_k^2,$$

where $\tilde{\theta}_k \in [\theta_k, \theta_{k+1}]$, and $\mu\left(\tilde{\theta}_k\right) = \frac{1}{12}\frac{f'\left(\tilde{\theta}_k\right)}{f\left(\tilde{\theta}_k\right)}$. Similarly, we can represent a_{k-1} around θ_k as

$$a_{k-1}(\Delta\theta_{k-1}) = \theta_k - \frac{1}{2}\Delta\theta_{k-1} + \mu\left(\tilde{\theta}_{k-1}\right)\Delta\theta_{k-1}^2,$$

where $\tilde{\theta}_{k-1} \in [\theta_{k-1}, \theta_k]$. Then, (22) becomes

$$\Delta\theta_{k-1} + \Delta\theta_k + 2\mu\left(\tilde{\theta}_k\right)\Delta\theta_k^2 - 2\mu\left(\tilde{\theta}_{k-1}\right)\Delta\theta_{k-1}^2 \geq 4b. \tag{31}$$

[77] This argument can be reapplied to all boundary points θ_k, $0 < k < N - 1$.

Taking the length of the partition's element $\Delta\theta_k = \Delta = c_b b$, where $c_b \in (2,3)$ is such that $c_b bN = 1$ for some integer N, transforms (31) into

$$(2c_b - 4)\, b + 2c_b^2 b^2 \left(\mu\left(\tilde{\theta}_k\right) - \mu\left(\tilde{\theta}_{k-1}\right)\right) \geq 0,$$

which is satisfied for $b \downarrow 0$, since $\mu(\theta)$ is bounded.

Because $f(\theta) \leq f(\theta_k) + \eta(\theta - \theta_k)$, the sum's element U_R^k in (30) is bounded from below by

$$\begin{aligned}
U_R^k &= -\int_{\theta_k}^{\theta_{k+1}} (a_k - \theta)^2 f(\theta)\, d\theta = -\int_{\theta_k}^{\theta_{k+1}} \left(\theta_k + \frac{1}{2}\Delta + \mu\left(\tilde{\theta}_k\right)\Delta^2 - \theta\right)^2 f(\theta)\, d\theta \\
&\geq -\int_{\theta_k}^{\theta_{k+1}} \left(\theta_k + \frac{1}{2}\Delta + \mu\left(\tilde{\theta}_k\right)\Delta^2 - \theta\right)^2 \left(f(\theta_k) + \eta(\theta - \theta_k)\right) d\theta \\
&= -\frac{1}{12} f(\theta_k)\Delta^3 - \frac{1}{24}\eta\Delta^4 - \left(\mu\left(\tilde{\theta}_k\right)^2 f(\theta_k) - \frac{1}{6}\eta\mu\left(\tilde{\theta}_k\right)\right)\Delta^5 - \frac{1}{2}\eta\mu\left(\tilde{\theta}_k\right)^2\Delta^6 \\
&= -\frac{1}{12} f(\theta_k)\Delta^3 + O\left(\Delta^4\right) = -\frac{1}{12} f(\theta_k)\, c_b^3 b^3 + O\left(b^4\right),
\end{aligned}$$

where $O(x)$ has an order x.

Kovac and Mylovanov (2006, Proposition 4) show that when b is small, the optimal mechanism is delegation, restricted on an interval $[\alpha_b, \beta_b] \subset [b, 1+b]$, where $\alpha_b \underset{b\to 0}{\to} 0$ and $\beta_b \underset{b\to 0}{\to} 1$.[78]

The principal's expected payoff in the optimal delegation is

$$\begin{aligned}
U^D &= \int_0^{\alpha_b - b} U_R(\alpha_b, \theta)\, dF(\theta) + \int_{\alpha_b - b}^{\beta_b - b} U_R(\theta + b, \theta)\, dF(\theta) + \int_{\beta_b - b}^{1} U_R(\beta_b, \theta)\, dF(\theta) \\
&= \int_0^1 U_R(\theta + b, \theta)\, dF(\theta) + \int_0^{\alpha_b - b} U_R(\alpha_b, \theta) - U_R(\theta + b, \theta)\, dF(\theta) \\
&+ \int_{\beta_b - b}^{1} U_R(\beta_b, \theta) - U_R(\theta + b, \theta)\, dF(\theta) \\
&= -b^2 + \int_0^{\alpha_b - b} b^2 - (\alpha_b - \theta)^2\, dF(\theta) + \int_{\beta_b - b}^{1} b^2 - (\beta_b - \theta)^2\, dF(\theta).
\end{aligned}$$

[78] In their setup, the expert's bias is 0. Hence, to guarantee that the expert's bias is positive relative to the principal's one, we put the principal's bias $-b$. Proposition 4 states that if $b \to 0$, then the optimal mechanism is delegation on the interval $[\alpha_0, \beta_0] \subset [0,1]$, where $\alpha_0 \to 0$ and $\beta_0 \to 1$. Shifting actions by b adjusts their coordinates to our model. Hence, $[\alpha_b, \beta_b] \subset [b, 1+b]$. Then, $\lim_{b\to 0} \alpha_b = \lim_{b\to 0} \alpha_0 = 0$, and $\lim_{b\to 0} \beta_b = \lim_{b\to 0} \beta_0 = 1$.

Then, it follows that $\alpha_b = b$. Since $\phi(\theta) = b^2 - (\theta - \alpha_b)^2$ is increasing in θ for $\theta \leq \alpha_b$, it leads to $\phi(\theta) < \phi(\alpha_b - b) = 0$, if $\theta < \alpha_b - b$. Hence, $\varphi(\alpha_b) = \int\limits_0^{\alpha_b - b} b^2 - (\alpha_b - \theta)^2 \, dF(\theta) < 0 = \varphi(b)$, $\alpha_b > b$. Thus,

$$U^D = -b^2 + \int\limits_{\beta_b - b}^{1} b^2 - (\beta_b - \theta)^2 \, dF(\theta) \leq -b^2 + b^2 (1 + b - \beta_b) \bar{f} = -b^2 (1 - \varepsilon_b),$$

where $\varepsilon_b = (1 + b - \beta_b) \bar{f}$ is an upper bound on the gains from the restricted delegation relative to the full delegation.

This results in $U_R^D \leq -b^2 (1 - \varepsilon_b) \sum\limits_{k=0}^{N-1} (F(\theta_{k+1}) - F(\theta_k)) = \sum\limits_{k=0}^{N-1} U_D^k$, where

$$\begin{aligned} U_D^k &= -b^2 (1 - \varepsilon_b) (F(\theta_{k+1}) - F(\theta_k)) \leq -b^2 (1 - \varepsilon_b) \left(f(\theta_k) \Delta - \eta \Delta^2 \right) \\ &\leq -(1 - \varepsilon_b) \left(f(\theta_k) c_b b^3 - \eta c_b^2 b^4 \right) = -(1 - \varepsilon_b) f(\theta_k) c_b b^3 + O\left(b^4\right), \forall k. \end{aligned}$$

Since $c_b \in (2, 3)$ and $\varepsilon_b \to 0$ as $b \downarrow 0$, it follows that

$$U_R^k - U_D^k \geq f(\theta_k) \left(1 - \varepsilon_b - \frac{c_b^2}{12} \right) c_b b^3 + O\left(b^4\right) > 0, \forall k.$$

Finally, summing across all $k = 0, ..., N-1$, results in $U_R > U_R^D$. ■

Lemma 7 *In the uniform-quadratic case, if the uniform partition of size N is incentive-compatible, then it is payoff superior to all partitions of the same size.*

Proof The expected utility of the principal in an incentive-compatible equilibrium is

$$\begin{aligned} U_R &= -\sum_{k=0}^{N-1} \int\limits_{\theta_k}^{\theta_{k+1}} (a_k - \theta)^2 d\theta = \sum_{k=0}^{N-1} P(\omega_k) \left(U_R(\omega_k, \omega_k) - D_k \right) = \\ &= -\sum_{k=0}^{N-1} P(\omega_k) D_k = -\sum_{k=0}^{N-1} \frac{\Delta\theta_k^3}{12} = \sum_{k=0}^{N-1} \phi(\Delta\theta_k), \end{aligned} \tag{32}$$

where $\Delta\theta_k = \theta_{k+1} - \theta_k > 0$ and $\phi(x) = -\frac{1}{12} x^3$.

Clearly, $\phi(x)$ is strictly concave for $x > 0$ and $\sum\limits_{k=0}^{N-1} \Delta\theta_k = 1$. For the uniform partition of size N, we have $\Delta\theta'_k = \frac{1}{N}$ for all k. For any other partition of the same size, the Jensen's inequality results in

$$U_R = \sum_{k=0}^{N-1} \phi(\Delta\theta_k) < N\phi\left(\frac{1}{N} \sum_{k=0}^{N-1} \Delta\theta_k \right) = N\phi\left(\frac{1}{N} \right) = \sum_{k=0}^{N-1} \phi(\Delta\theta'_k) = U'_R.$$

■

Lemma 8 *If a partition of size N is incentive-compatible, then the uniform partition of size $N-1$ is incentive-compatible as well.*

Proof A sufficient condition for the uniform partition $\{\theta'_k\}_{k=0}^{N-1}$ to be incentive-compatible is $\theta'_{j+2} - \theta'_j = \frac{j+2}{N-1} - \frac{j}{N-1} = \frac{2}{N-1} \geq 4b$. Since a partition $\{\theta_k\}_{k=0}^{N}$ is incentive-compatible, we have $\theta_N = 1 \geq \theta_{N-2} + 4b \geq ... \geq \theta_1 + \frac{N-1}{2}4b \geq \frac{N-1}{2}4b$ for odd N, and $\theta_N = 1 \geq \theta_{N-2} + 4b \geq ... \geq \theta_0 + \frac{N-1}{2}4b = \frac{N-1}{2}4b$ for even N. In both cases, we obtain $\frac{2}{N-1} \geq 4b$. ■

Lemma 9 *Among all partitions of an odd size N such that $\frac{1}{2N} < b \leq \frac{1}{2(N-1)}$, the highest expected payoff in the incentive-compatible equilibrium is reached under the partition with all binding IC constraints (6).*

Proof We prove the lemma using Karamata's inequality.[79] Let sequences $\{x_k\}_{k=1}^{N}$ and $\{y_k\}_{k=1}^{N}$ be non-increasing, that is, $x_1 \geq x_2 \geq ... \geq x_N$ and $y_1 \geq y_2 \geq ... \geq y_N$. If all the following conditions are satisfied: $x_1 \geq y_1$, $x_1 + x_2 \geq y_1 + y_2$, $x_1 + x_2 + x_3 \geq y_1 + y_2 + y_3$, ..., $x_1 + x_2 + ... + x_{N-1} \geq y_1 + y_2 + ... + y_{N-1}$, and $x_1 + x_2 + ... + x_N = y_1 + y_2 + ... + y_N$, then we say that $\{x_k\}_{k=1}^{N}$ **majorizes** $\{y_k\}_{k=1}^{N}$. The Karamata's inequality states that if $\{x_k\}_{k=1}^{N}$ majorizes $\{y_k\}_{k=1}^{N}$, and a function $\phi(x)$ is continuous and concave, then $\sum_{k=1}^{N} \phi(x_k) \leq \sum_{k=1}^{N} \phi(y_k)$.

From (32), the principal's expected payoff in the incentive-compatible equilibrium is $U_R\left(\{\theta_k\}_{k=0}^{N}\right) = \sum_{k=0}^{N-1} \phi(\Delta\theta_k)$, where $\Delta\theta_k = \theta_{k+1} - \theta_k > 0$, and $\phi(x) = -\frac{1}{12}x^3$, which is continuous and strictly concave for $x > 0$.

Consider the partition $\{y_k\}_{k=0}^{N}$, for which the IC constraints are binding, hence, $y_k = 2kb$ for even k, and $y_k = 1 - 2b(N-k)$ for odd k. We need to show that if $\frac{1}{2N} < b \leq \frac{1}{2(N-1)}$, then $U_R\left(\{y_k\}_{k=0}^{N}\right) \geq U_R\left(\{\theta_k\}_{k=0}^{N}\right)$ for any partition $\{\theta_k\}_{k=0}^{N}$, which satisfies (6).

For the sequence $\{y_k\}_{k=0}^{N}$, we have $\Delta y_k = y_{k+1} - y_k = 1 - 2b(N-k-1) - 2bk = 1 - 2b(N-1)$ for even k. The condition $b < \frac{1}{2(N-1)}$ implies $\Delta y_k > 0$. Similarly, we have $\Delta y_k = 4b - \Delta y_{k-1} = 2b(N+1) - 1$ for odd k, and $b > \frac{1}{2N} > \frac{1}{2(N+1)}$ implies $\Delta y_k > 0$. In addition, for odd k, we obtain $\Delta y_k - \Delta y_{k-1} = 2(2bN-1) > 0$. Thus, by permuting $\{\Delta y_k\}_{k=0}^{N-1}$, we obtain a non-increasing sequence $\{Y_k\}_{k=1}^{N} = \{Y_1, Y_2, ..., Y_{\frac{N-1}{2}}, Y_{\frac{N+1}{2}}, ..., Y_N\}$, where $Y_k = 2b(N+1) - 1$ for $k \in S_1 = 1, 2, ..., \frac{N-1}{2}$, and $Y_k = 1 - 2b(N-1)$ for $k \in S_2 = \frac{N+1}{2}, ..., N$. Note that S_1 has one element less than S_2, since N is odd. Also, (6) implies $Y_k + Y_j = 4b$, $k \in S_1$, $j \in S_2$.

Now, consider a sequence $\{\theta_k\}_{k=0}^{N}$, which satisfies (6). We need to show that a non-increasing permutation $\{X_k\}_{k=1}^{N}$ of $\{\Delta\theta_k\}_{k=0}^{N-1}$ majorizes $\{Y_k\}_{k=1}^{N}$.

First, for even k, we have $\theta_k \geq \theta_{k-2} + 4b \geq ... \geq \theta_0 + \frac{k}{2}4b = 2kb = y_k$. Similarly, for odd k, we have $\theta_k \leq y_k$. Therefore, $\Delta\theta_k = \theta_{k+1} - \theta_k \geq y_{k+1} - y_k = \Delta y_k$ for odd k and $\Delta\theta_k \leq \Delta y_k$ for

[79] See, for example, Hardy, Littlewood and Polya (1988).

even k. Thus, a non-increasing permutation $\{X_k\}_{k=1}^{N}$ of $\{\Delta\theta_k\}_{k=0}^{N-1}$ can be represented as $\{X_k\}_{k=1}^{N} = \{X_1, X_2, ..., X_{\frac{N-1}{2}}, X_{\frac{N+1}{2}}, ..., X_N\}$, where $X_j \geq Y_k$ for all $j, k \in S_1$, and $X_j \leq Y_k$ for all $j, k \in S_2$. This means $\sum\limits_{k \in S_1'} X_k \geq \sum\limits_{k \in S_1'} Y_k$ for any $S_1' \subset S_1$ and $\sum\limits_{k \in S_2'} X_k \leq \sum\limits_{k \in S_2'} Y_k$ for any $S_2' \subset S_2$.

Also, the IC constraints require that for any $k \in \tilde{S}_2 = S_2 - \{N\} = \frac{N+1}{2}, ..., N-1$, there must exist $q(k) \in S_1$ such that $X_{q(k)} + X_k \geq 4b$, which we define as follows. Let i_N be the index of the smallest element $\Delta\theta_{i_N}$ of the sequence $\{\Delta\theta_k\}_{k=0}^{N-1}$, which implies $\Delta\theta_{i_N} = X_N$. Then, for all X_k, $k \in \tilde{S}_2$, if $X_k = \Delta\theta_i$, then $X_{q(k)} = \Delta\theta_{i+1}$ for $i < i_N$ and $X_{q(k)} = \Delta\theta_{i-1}$ for $i > i_N$. Note that $k \neq k'$ for $k, k' \in S_2$, implies $q(k) \neq q(k')$.

Clearly, $X_1 \geq Y_1$, $X_1 + X_2 \geq Y_1 + Y_2, ...,$ $X_1 + ... + X_{\frac{N-1}{2}} \geq Y_1 + ... + Y_{\frac{N-1}{2}}$. Also, we obtain

$$X_1 + ... + X_{\frac{N-1}{2}} + X_{\frac{N+1}{2}} = \sum_{k \in S_1 - q\left(\frac{N+1}{2}\right)} X_k + X_{q\left(\frac{N+1}{2}\right)} + X_{\frac{N+1}{2}} \geq \sum_{k \in S_1 - q\left(\frac{N+1}{2}\right)} X_k + 4b$$
$$\geq \sum_{k \in S_1 - q\left(\frac{N+1}{2}\right)} Y_k + 4b = \sum_{k \in S_1 - q\left(\frac{N+1}{2}\right)} Y_k + Y_{q\left(\frac{N+1}{2}\right)} + Y_{\frac{N+1}{2}} = Y_1 + ... + Y_{\frac{N-1}{2}} + Y_{\frac{N+1}{2}}.$$

This argument can be reapplied iteratively for all $k \in \tilde{S}_2$. Since $\sum\limits_{k=1}^{N} X_k = \sum\limits_{k=1}^{N} Y_k = 1$, this completes the proof. ■

Proof of Lemma 5. We can rewrite $N(b)$ as follows: if $\frac{1}{2(c+1)} < b < \frac{1}{2(c-1)}$ for some odd c, then $N(b) = c$, otherwise, for $b = \frac{1}{2(c-1)}$, we have $N(b) = c - 1$. Then, by Lemma (8), the uniform partition of size $c - 1 = N(b) - 1$ is incentive-compatible, and provides the expected payoff

$$U_R^{c-1} = -\sum_{k=0}^{c-2} \frac{(\theta_{k+1} - \theta_k)^3}{12} = -\sum_{k=0}^{c-2} \frac{1}{12(c-1)^3} = -\frac{1}{12(c-1)^2}.$$

By Lemma 7, this partition is payoff superior to all partitions of the same size. In addition, it is superior to partitions of a smaller size.

Now, consider two cases: $\frac{1}{2(c+1)} < b \leq \frac{1}{2c}$ and $\frac{1}{2c} < b \leq \frac{1}{2(c-1)}$. In the first case, the uniform partition of size c is incentive-compatible, thus, it is optimal and brings the expected payoff $U_R = -\frac{1}{12c^2}$. In the second case, Lemma (9) implies that among all partitions of size $c = N(b)$, the superior partition is that with the binding IC constraints (6). It provides the expected payoff

$$U_R^c = -\frac{1}{12}\left(4b^2\left(c^2 - 1\right)(4bc - 3) + 1\right). \tag{33}$$

For $b = \frac{1}{2c}$, we obtain $U_R^c = -\frac{1}{12c^2}$, which is equal to the expected payoff with the uniform partition of size c. For $b = \frac{1}{2(c-1)}$, we obtain $U_R^c = -\frac{1}{3(c-1)^2} = -\frac{1}{12\left(\frac{c-1}{2}\right)^2}$, which is equal to the expected payoff with the uniform partition of size $\frac{c-1}{2}$.

Notice that $N(b) = c$ for all $b \in \left(\frac{1}{2c}, \frac{1}{2(c-1)}\right)$. Taking the derivative of (33) with respect to b gives

$$\frac{\partial}{\partial b} U_R^c(b) = -2b\left(c^2-1\right)(2bc-1),$$

which is negative for $b > \frac{1}{2c}$. Finally, $U_R^c\left(\frac{1}{2c}\right) = -\frac{1}{12c^2} > U_R^{c-1} = -\frac{1}{12(c-1)^2} > -\frac{1}{3(c-1)^2} = U_R^c\left(\frac{1}{2(c-1)}\right)$ implies that there exists a unique $b_c \in (\frac{1}{2c}, \frac{1}{2(c-1)})$, such that $U_R^c(b_c) = U_R^{c-1}$. ■

Now, we prove the following claim.

Claim 1 *Any incentive-compatible information structure that generates a two-point type space is payoff inferior to some partitional information structure.*

Proof For a two-point type space $\Omega = \{\omega_1, \omega_2\}$, denote $\rho_1(\theta) = \int\limits_{S(\omega_1)} dF(s|\theta)$ the probability of generating a mean value ω_1 by a state θ. With a complement probability $\rho_2(\theta) = 1 - \rho_1(\theta)$, θ generates ω_2. Given $\rho_1(\theta)$, the values of ω_1 and ω_2 are given by

$$\omega_1 = \frac{\int\limits_{\Theta} \theta\rho_1(\theta)\, dF(\theta)}{\int\limits_{\Theta} \rho_1(\theta)\, dF(\theta)} \text{ and } \omega_2 = \frac{\int\limits_{\Theta} \theta\rho_2(\theta)\, dF(\theta)}{\int\limits_{\Theta} \rho_2(\theta)\, dF(\theta)}.$$

Denote by $S_1(\theta) = \int\limits_0^\theta \rho_1(t)\, dF(t)$and $S_2(\theta) = \int\limits_0^\theta \rho_2(t)\, dF(t) = F(\theta) - S_1(\theta)$. Since $\rho_1(t) \in [0,1]$, $S_1(\theta)$ is non-decreasing in θ, $S_1(0) = 0$, and $S_1(\theta) \leq F(\theta)$. Thus,

$$\omega_1 = \frac{\int\limits_{\Theta} \theta\rho_1(\theta)\, dF(\theta)}{\int\limits_{\Theta} \rho_1(\theta)\, dF(\theta)} = \frac{\int\limits_0^1 \theta\rho_1(\theta)\, dF(\theta)}{S_1(1)} = \frac{\theta S_1(\theta)\,|_0^1 - \int\limits_0^1 S_1(\theta)\, d\theta}{S_1(1)} = 1 - \frac{\int\limits_0^1 S_1(\theta)\, d\theta}{S_1(1)}.$$

Similarly,

$$\omega_2 = 1 - \frac{\int\limits_0^1 S_2(\theta)\, d\theta}{S_2(1)} = 1 - \frac{\int\limits_0^1 F(\theta) - S_1(\theta)\, d\theta}{1 - S_1(1)}. \tag{34}$$

Then,

$$\omega_2-\omega_1=\frac{\int_0^1 S_1(\theta)\,d\theta}{S_1(1)}-\frac{\int_0^1 F(\theta)-S_1(\theta)\,d\theta}{1-S_1(1)}$$
$$=\frac{(1-S_1(1))\int_0^1 S_1(\theta)\,d\theta-S_1(1)\int_0^1 F(\theta)-S_1(\theta)\,d\theta}{S_1(1)(1-S_1(1))} \tag{35}$$
$$=\frac{\int_0^1 S_1(\theta)\,d\theta-S_1(1)\int_0^1 F(\theta)\,d\theta}{S_1(1)(1-S_1(1))},$$

where $S_1(1)\in(0,1)$, since each type is generated by a positive mass of states.

The principal's expected payoff in the incentive-compatible equilibrium is

$$U=-\sum_{i=1}^{2}\int_{\Theta}\rho_i(\theta)(\theta-\omega_i)^2\,dF(\theta)$$
$$=-\sum_{i=1}^{2}\omega_i^2\int_{\Theta}\rho_i(\theta)\,dF(\theta)+2\sum_{i=1}^{2}\omega_i\int_{\Theta}\theta\rho_i(\theta)\,dF(\theta)-\int_{\Theta}\sum_{i=1}^{2}\rho_i(\theta)\,\theta^2 dF(\theta)$$
$$=\sum_{i=1}^{2}\omega_i^2\int_{\Theta}\rho_i(\theta)\,dF(\theta)-\int_{\Theta}\theta^2 dF(\theta)=\sum_{i=1}^{2}\omega_i^2 S_i(1)-\int_{\Theta}\theta^2 dF(\theta).$$

Now, consider a two-element partition $\{0,\theta_1,1\}$ which generates $\Omega^o=\{\omega_1^o,\omega_2^o\}$. That is, $\rho_1^o(\theta)=1$, if $\theta\le\theta_1$, and $\rho_1^o(\theta)=0$ otherwise. Choose θ_1, such that $S_1^o(1)=\int_0^{\theta_1} dF(\theta)=F(\theta_1)=S_1(1)$. From (35), we have

$$\omega_2^o-\omega_1^o=\frac{\int_0^1 S_1^o(\theta)\,d\theta-S_1(1)\int_0^1 F(\theta)\,d\theta}{S_1(1)(1-S_1(1))}$$
$$\ge\frac{\int_0^1 S_1(\theta)\,d\theta-S_1(1)\int_0^1 F(\theta)\,d\theta}{S_1(1)(1-S_1(1))}=\omega_2-\omega, \tag{36}$$

where the inequality holds, because $S_1^o(\theta)=\min\{F(\theta),S_1(1)\}\ge S_1(\theta)$. That is, (36) implies that Ω^o is incentive-compatible. It provides the expected payoff

$$U^o=\sum_{i=1}^{2}(\omega_i^o)^2 S_i(1)-\int_{\Theta}\theta^2 dF(\theta).$$

From (34), $S_2(1)(\omega_2^o-\omega_2)=\int_0^1 S_1^o(\theta)\,d\theta-\int_0^1 S_1(\theta)\,d\theta\ge 0$. Also, since

$\sum_{i=1}^{2} \omega_i^o S_i(1) = \sum_{i=1}^{2} \omega_i S_i(1) = E(\theta)$, and $\phi(x) = x^2$ is convex, then the weighted Karamata's inequality implies $\sum_{i=1}^{2} (\omega_i^o)^2 S_i(1) \geq \sum_{i=1}^{2} (\omega_i)^2 S_i(1)$, or $U^o \geq U$. ■

Proof of Theorem 3. Informative communication is feasible, if $b \leq \frac{1}{4}$. Melumad and Shibano (1991) prove that the optimal delegation set is the interval $[0, 1 - \min\{b, 1/2\}]$. For $b \leq 1/2$, the expert's actions are

$$a^S(\theta) = \min\{\theta + b, 1 - b\}, \tag{37}$$

which results in

$$U_R^D(b) = \int_0^1 U_R\left(a^S(\theta), \theta\right) d\theta = -\int_0^{1-2b} (\theta + b - \theta)^2 d\theta - \int_{1-2b}^{1} (1 - b - \theta)^2 d\theta = -b^2 + \frac{4}{3}b^3, \tag{38}$$

and $U_R^D(b) = -\frac{1}{12}, b > 1/2$.

From Lemma (8), a uniform partition of size $N(b) - 1 = \lfloor \frac{1}{4b} \rfloor + \lceil \frac{1}{4b} \rceil - 1 = 2\lfloor \frac{1}{4b} \rfloor$ is incentive-compatible and provides the expected payoff $U_R(b) = -\frac{1}{12 \times \left(2\lfloor \frac{1}{4b} \rfloor\right)^2} = -\frac{1}{48 \times \lfloor \frac{1}{4b} \rfloor^2}$. Since $\lfloor \frac{1}{4b} \rfloor \geq \frac{1}{4b} - 1$, we have $U_R(b) \geq -\frac{1}{48\left(\frac{1}{4b} - 1\right)^2} = -\frac{b^2}{3(1-4b)^2}$, and

$$U_R(b) - U_R^D(b) \geq -\frac{b^2}{3(1-4b)^2} + b^2 - \frac{4}{3}b^3 = \frac{2}{3}b^2 \frac{1 - 14b + 40b^2 - 32b^3}{(1-4b)^2}.$$

The function $A(b) = 1 - 14b + 40b^2 - 32b^3$ has three roots. Only one of them, namely, $b_0 = \frac{1}{8}\left(3 - \sqrt{5}\right) \simeq \frac{1}{11}$ is in the interval $\left[0, \frac{1}{4}\right]$. Since $A(0) = 1$, it follows that $U_R - U_R^D > 0$ for all $b < b_0$. For $b \in [b_0, \frac{1}{4}]$, consider three cases. If $b \in [\frac{1}{6}, \frac{1}{4}]$, then the uniform partition of size 2 is incentive-compatible, and provides the expected payoff $U_R = -\frac{1}{48}$. Then, $D(b) = U_R(b) - U_R^D(b) = -\frac{1}{48} + b^2 - \frac{4}{3}b^3$. Since $D'(b) > 0$ for $b < \frac{1}{2}$, and $D\left(\frac{1}{6}\right) = \frac{1}{1296}$, this implies $U_R(b) - U_R^D(b) > 0$ for all $b \in [b_0, \frac{1}{4}]$. For $b \in [\frac{1}{8}, \frac{1}{6})$, the uniform three-element partition is incentive-compatible, and brings the expected payoff $-\frac{1}{108}$. Then, $D(b) = U_R(b) - U_R^D(b) = -\frac{1}{108} + b^2 - \frac{4}{3}b^3 > 0$ for all $b \in [\frac{1}{8}, \frac{1}{6})$, since $D\left(\frac{1}{8}\right) = \frac{13}{3456}$. Finally, for $b \in [b_0, \frac{1}{8})$, the uniform 4-element partition is incentive-compatible, which results in the payoff $U_R = -\frac{1}{192}$. Using the same technique as for $b \geq \frac{1}{6}$, it gives $D(b) > D\left(\frac{1}{12}\right) = \frac{5}{5184}$, which completes the sufficient part of the proof.

To prove the necessary part, we show that for $b > \frac{1}{4}$, there is no information structure that sustains informative communication. First, there is no incentive-compatible information structure with more than two types. By contradiction, if $\omega_1 < \omega_2 < \omega_3 \in \Omega \subset [0, 1]$, then (22) results in $\omega_3 - \omega_1 = \omega_3 - \omega_2 + \omega_2 - \omega_1 \geq 4b > 1$. In addition, (6) means that there is no incentive-compatible partition. Thus, by Claim 1, there is no two-element incentive-compatible information structure, so that informational control provides the uninformed payoff $-\frac{1}{12}$, whereas the optimal delegation provides the payoff $\max\left\{-\frac{1}{12}, -b^2 + \frac{4}{3}b^3\right\} \geq -\frac{1}{12}$, this completes the second part of the proof. ■

Proof of Theorem 4. For the symmetric preferences, the CS arbitrage condition and the IC constraints (6) do not change. Given $b \leq 1/4$ and any informative CS equilibrium with an $N-$element partition $\{\Delta\theta_k\}_{k=1}^{N}$, the uniform partition of size N is incentive-compatible in the informational control model. This is because

$$\begin{aligned} 1 &= \sum_{k=1}^{N} \Delta\theta_k = \sum_{k=1}^{N} \left(\Delta\theta_1 + 4b\left(k-1\right)\right) \geq \Delta\theta_1 N + 4bN - 4b \\ &> 2bN + 2bN - 4b \geq 2bN, \end{aligned}$$

or $\Delta\theta_{k+1} + \Delta\theta_k = 2\frac{1}{N} > 4b$.

The principal's expected payoff in the CS equilibrium is

$$U_R^{CS} = \sum_{k=1}^{N} \int_{\theta_k}^{\theta_{k+1}} U\left(\left|\frac{\theta_k + \theta_{k+1}}{2} - \theta\right|\right) d\theta = 2\sum_{k=1}^{N} \int_{0}^{\frac{\theta_{k+1}-\theta_k}{2}} U\left(t\right) dt = \sum_{k=1}^{N} \upsilon\left(\Delta\theta_k\right),$$

where $\upsilon(x) = 2\int_0^{\frac{x}{2}} U\left(t\right) dt$. Then, for $x > 0$, we have $\upsilon'\left(x\right) = U\left(\frac{x}{2}\right)$, and $\upsilon''\left(x\right) = \frac{1}{2}U'\left(\frac{x}{2}\right) < 0$. Informational control with the uniform partition of size N results in the payoff

$$\begin{aligned} U_R &= \sum_{k=1}^{N} \int_{\theta_k}^{\theta_{k+1}} U\left(\left|\frac{\theta_k + \theta_{k+1}}{2} - \theta\right|\right) d\theta = 2\sum_{k=1}^{N} \int_{0}^{\frac{\theta_{k+1}-\theta_k}{2}} U\left(t\right) dt \\ &= 2N \int_{0}^{\frac{1}{2N}} U\left(t\right) dt = N\upsilon\left(\frac{1}{N}\right) = N\upsilon\left(\frac{1}{N}\sum_{k=1}^{N} \Delta\theta_k\right). \end{aligned} \tag{39}$$

Since $\upsilon\left(x\right)$ is strictly concave and $\Delta\theta_{k+1} = \Delta\theta_k + 4b > \Delta\theta_k$, then Jensen's inequality implies $\upsilon\left(\frac{1}{N}\sum_{k=1}^{N} \Delta\theta_k\right) > \frac{1}{N}\sum_{k=1}^{N} \upsilon\left(\Delta\theta_k\right)$ or $U_R > U_R^{CS}$. ■

Proof of Theorem 5. If the preferences are symmetric, then, by Proposition 3 in Alonso and Matouschek (2007), the optimal delegation set is the same as for quadratic preferences, hence, it is the interval $[0, 1 - \min\{b, 1/2\}]$. Similarly, the expert's choice over actions is determined by (37). This results in the principal's ex-ante payoff

$$U_R^D = \int_0^1 U\left(\left|a^S(\theta) - \theta\right|\right) d\theta = \int_0^{1-2b} U(b)\, d\theta + \int_{1-2b}^1 U(|1-b-\theta|)\, d\theta$$
$$= U(b)(1-2b) + 2\int_0^b U(\theta)\, d\theta.$$

Now, consider the informational control. The partition of size $N(b) = \lfloor \frac{1}{4b} \rfloor + \lceil \frac{1}{4b} \rceil$ is incentive-compatible. From Lemma 8, the uniform partition of size $c = N(b) - 1 = 2\lfloor \frac{1}{4b} \rfloor \geq \frac{1}{2b} - 1$ is incentive-compatible also, and provides the principal's expected payoff

$$U_R(c) = c \int_{-\frac{1}{2c}}^{\frac{1}{2c}} U(\theta)\, d\theta = 2c \int_0^{\frac{1}{2c}} U(\theta)\, d\theta = E\left[U(\theta) | \theta < \frac{1}{2c}\right].$$

Since $U(.)$ is decreasing, it follows that U_R is increasing in c. Then,

$$U_R(c) \geq U_R\left(\frac{1}{2b} - 1\right) = 2\left(\frac{1}{2b} - 1\right) \int_0^{\frac{1}{2\left(\frac{1}{2b}-1\right)}} U(\theta)\, d\theta = \frac{1-2b}{b} \int_0^{\frac{b}{1-2b}} U(\theta)\, d\theta.$$

Thus, $\left(U_R - U_R^D\right) \frac{b}{1-2b} \geq \int_0^{\frac{b}{1-2b}} U(\theta)\, d\theta - U(b)\, b - \frac{2b}{1-2b} \int_0^b U(\theta)\, d\theta = \phi(b)$. Clearly, $\phi(0) = 0$. Taking a derivative of $\phi(b)$ with respect to b gives

$$\phi'(b) = U\left(\frac{b}{1-2b}\right) \frac{1}{(1-2b)^2} - U'(b)\, b - U(b) - \frac{2}{(1-2b)^2} \int_0^b U(\theta)\, d\theta - \frac{2b}{1-2b} U(b).$$

From the last expression, $\phi'(0) = 0$. Taking the second derivative results in $\phi''(0) = -U'(0) \geq 0$. If $U'(0) < 0$, then by Taylor's formula $\phi(b) = \phi(0) + \phi'(0)\, b + \frac{1}{2}\phi''\left(\tilde{b}\right) b^2 = \frac{1}{2}\phi''\left(\tilde{b}\right) b^2$, where $\tilde{b} \in [0, b]$. Since $\phi''(0) > 0$ and $\phi''(b)$ is continuous, then there exists b^* such that $\phi''(b) > 0$, and hence, $\phi(b) > 0$ for all $b \in (0, b^*)$. If $U'(0) = 0$, then $\phi''(0) = 0$. Taking the third derivative gives $\phi'''(0) = -2U''(0) > 0$. By Taylor's formula, $\phi(b) = \phi(0) + \phi'(0)\, b + \frac{1}{2}\phi''(0)\, b^2 + \frac{1}{6}\phi'''(b^*)\, b^3 = \frac{1}{6}\phi'''(b^*)\, b^3$, where $b^* \in [0, b]$. Since $\phi'''(0) > 0$ and $\phi'''(b)$ is continuous, then $\phi(b) > 0$ for all b in the neighborhood of 0. ■

Proof of Lemma 6. If the expert receives a signal s, then she implements the action

$$a_s' = \arg\max_a - \int_\Theta U_S(a, b, \theta)\, dF(\theta | s) = \omega_s + b.$$

This results in the principal's payoff

$$U_R(s) = \int_{\Theta} U_R(a'_s, \theta)\, dF(\theta|s).$$

By Jensen's inequality,

$$U_R(s) = E_\theta[U_R(a'_s, \theta)|s] < U_R(a'_s, E[\theta|s]) = U_R(\omega_s + b, \omega_s) = -b^2,$$

where the right part is the expected payoff of the principal in the case of the perfectly informed expert. To complete the proof, it is sufficient to integrate over $\mathcal{S}$ according to a distribution $F(s)$. ■

For the type space Ω and the delegation set $\mathcal{A}$, the incentive-compatible choice $a_S(\omega_k)$ of the expert's type $\omega_k = \frac{\theta_k + \theta_{k+1}}{2}$ over actions $a \in \mathcal{A}$ is determined by (21). Without loss of generality, we can restrict attention to the class of mechanisms, in which each expert's type induces a separate action in the optimal mechanism. To demonstrate this, suppose that a type $\omega \in \mathcal{W}$ mixes between actions a' and $a'' > a'$, that is, $U_S(a'', b|\omega) = U_S(a', b|\omega)$, we assign 1 to the probability of inducing a lower action. Then, by (23), it follows that $U_S(a'', b|\omega) - U_S(a', b|\omega) > 0$. Reapplying this argument to all mixing types, we obtain a pure-strategy action rule, which is payoff superior to the initial one. Thus, the cardinality of the optimal delegation set does not exceed that of the type set. However, if the number of types strictly exceeds that of actions, then the principal can collapse all types that induce identical actions, without affecting the expert's incentives for the initial and the modified types. Hence, in the payoff superior mechanism, the number of types is equal to the number of actions.

Also, property (C) of Lemma 6 guarantees that induced actions are monotone in types, that is, the delegation set $\mathcal{A} = \{a_k\}_{k=0}^{N-1} = \{a_S(\omega_k)\}_{k=0}^{N-1}$ is a strictly increasing sequence. Thus, the expert of a type ω_k prefers an action a_k to all $a \in \mathcal{A} \setminus \{a_k\}$ if and only if

$$a_k + a_{k+1} \geq \theta_k + \theta_{k+1} + 2b \text{ and } a_k + a_{k-1} \leq \theta_k + \theta_{k+1} + 2b, \ \forall k. \tag{40}$$

Now, consider two sequences, $\{\Delta_k\}_{k=0}^{N-1} = \{a_k - \theta_k\}_{k=0}^{N-1}$ and $\{\xi_k\}_{k=0}^{N-1} = \{\theta_{k+1} - a_k\}_{k=0}^{N-1}$. Note that $\theta_{k+1} - \theta_k > 0$ results in

$$\Delta_k + \xi_k > 0, \tag{41}$$

and $a_{k+1} - a_k > 0$ results in

$$\Delta_{k+1} + \xi_k > 0. \tag{42}$$

In addition, the condition $\sum_{k=0}^{N-1}(\theta_{k+1}-\theta_k)=1$ implies

$$\sum_{k=0}^{N-1}\Delta_k+\sum_{k=0}^{N-1}\xi_k=1. \tag{43}$$

Conversely, any sequences, that satisfy the properties above, determine the information structure $\{\theta_k\}_{k=0}^{N-1}$ and the delegation set $\{a_k\}_{k=0}^{N-1}$ by $\theta_{k+1}=\theta_k+\Delta_k+\xi_k$ and $a_k=\theta_k+\Delta_k$, where $\theta_0=0$.

Then, we can rewrite (40) as

$$\Delta_k+\Delta_{k+1}\geq 2b, \text{ and} \tag{44}$$

$$\xi_{k-1}+\xi_k+2b\geq 0. \tag{45}$$

Similarly, the expected utility of the principal can be expressed as

$$U_R=-\sum_{k=0}^{N-1}\int_{\theta_k}^{\theta_{k+1}}(a_k-\theta)^2d\theta=-\frac{1}{3}\sum_{k=0}^{N-1}\left[(a_k-\theta_k)^3+(\theta_{k+1}-a_k)^3\right]=-\frac{1}{3}\sum_{k=0}^{N-1}\left(\Delta_k^3+\xi_k^3\right).$$

The optimal communication structure and the delegation set are the solution to the problem:

$$\max_{\{\Delta_k\}_{k=0}^{N-1},\{\xi_k\}_{k=0}^{N-1}}U_R\left(\{\Delta_k\}_{k=0}^{N-1},\{\xi_k\}_{k=0}^{N-1}\right)=\max_{\{\Delta_k\}_{k=0}^{N-1},\{\xi_k\}_{k=0}^{N-1}}-\frac{1}{3}\sum_{k=0}^{N-1}\left(\Delta_k^3+\xi_k^3\right), \tag{46}$$

given constraints (41)–(45).

Proof of Lemma 7. We have already proved that each expert's type induces a separate action. To prove the other statements, we prove first several facts.

F_1) Constraints (45) are never binding. By contradiction, let $\xi_k+\xi_{k+1}+2b=0$ for some k. First, $\Delta_{k+1}+\xi_k>0$, $\Delta_{k+1}+\xi_{k+1}>0$ and $\xi_k+\xi_{k+1}=-2b<0$ imply that $\min\{\xi_k,\xi_{k+1}\}<0$ and $\Delta_{k+1}>\max\{|\xi_k|,|\xi_{k+1}|\}$.

If $\xi_{k+1}<0$, then put $\tilde{\Delta}_{k+1}=\Delta_{k+1}-\delta$ and $\tilde{\xi}_{k+1}=\xi_{k+1}+\delta$, where, here and below, $\delta\downarrow 0$. This results in the higher expected payoff to the principal, since

$$\begin{aligned}3\Delta U&=3U_R\left(\left\{\tilde{\Delta}_k\right\}_{k=0}^{N-1},\left\{\tilde{\xi}_k\right\}_{k=0}^{N-1}\right)-3U_R\left(\{\Delta_k\}_{k=0}^{N-1},\{\xi_k\}_{k=0}^{N-1}\right)\\&=-\tilde{\Delta}_{k+1}^3-\tilde{\xi}_{k+1}^3+\Delta_{k+1}^3+\xi_{k+1}^3=-\left(\Delta_{k+1}-\delta\right)^3-\left(\xi_{k+1}+\delta\right)^3+\Delta_{k+1}^3+\xi_{k+1}^3\\&=3\delta\left(\Delta_{k+1}^2-\xi_{k+1}^2-\delta\Delta_{k+1}-\delta\xi_{k+1}\right)>0.\end{aligned}$$

Since $\tilde{\xi}_{k+1}>\xi_{k+1}$ and $\tilde{\xi}_{k+1}+\tilde{\Delta}_{k+1}=\xi_{k+1}+\Delta_{k+1}$, the constraints (43) and (45) hold.

Similarly, if $\xi_k < 0$, then put $\tilde{\Delta}_{k+1} = \Delta_{k+1} - \delta$ and $\tilde{\xi}_k = \xi_k + \delta$. This results in

$$3\Delta U = -\tilde{\Delta}_{k+1}^3 - \tilde{\xi}_k^3 + \Delta_{k+1}^3 + \xi_k^3 = 3\delta\left(\Delta_{k+1}^2 - \xi_k^2 - \delta\Delta_{k+1} - \delta\xi_k\right) > 0.$$

Since $\tilde{\xi}_k > \xi_k$ and $\tilde{\xi}_k + \tilde{\Delta}_{k+1} = \xi_k + \Delta_{k+1}$, the constraints (43) and (45) hold.

To show that (44) is satisfied also, notice first that $\Delta_k + \Delta_{k+1} > 2b$. Otherwise, if $\Delta_k + \Delta_{k+1} = 2b$, then $\xi_k + \xi_{k+1} + \Delta_k + \Delta_{k+1} = (\xi_k + \Delta_k) + (\xi_{k+1} + \Delta_{k+1}) = 0$, which contradicts (41). In addition, $\Delta_{k+2} + \xi_{k+1} > 0$, $\Delta_{k+1} + \xi_k > 0$, and $\xi_k + \xi_{k+1} + 2b = 0$ lead to $\Delta_{k+2} + \Delta_{k+1} + \xi_k + \xi_{k+1} + 2b > 2b$ or $\Delta_{k+2} + \Delta_{k+1} > 2b$. Thus, constraints (41), (42), and (45) are not affected by $\delta-$perturbations in Δ_k or $\xi_k, \forall k$.

F_2) $\Delta_k > 0, \forall k$. By contradiction, let $\Delta_k \leq 0$ for some k. Then, $\Delta_k + \xi_k > 0$ implies $\xi_k > |\Delta_k|$. Put $\tilde{\Delta}_k = \Delta_k + \delta$ and $\tilde{\xi}_k = \xi_k - \delta$. Then,

$$3\Delta U = -\tilde{\Delta}_k^3 - \tilde{\xi}_k^3 + \Delta_k^3 + \xi_k^3 = 3\delta\left(\xi_k^2 - \Delta_k^2 - \delta\Delta_k - \delta\xi_k\right) > 0,$$

and (44) holds, because $\tilde{\Delta}_k > \Delta_k$.

F_3) $|\xi_k| = \xi, \forall k$. If there are ξ_j and ξ_k, such that $|\xi_j| > |\xi_k|$, then put $\tilde{\xi}_k = \xi_k + \delta$ and $\tilde{\xi}_j = \xi_j - \delta$. Then,

$$3\Delta U = -\tilde{\xi}_k^3 - \tilde{\xi}_j^3 + \xi_k^3 + \xi_j^3 = 3\delta\left(\xi_j^2 - \xi_k^2 - \delta\xi_j - \delta\xi_k\right) > 0.$$

Thus, we must have $|\xi_k| = \xi, \forall k$.

F_4) $\xi_k \geq 0, \forall k$. First, if $\xi_k = -\xi < 0, \forall k$, then consider the mechanism $\left\{\tilde{\Delta}_k\right\}_{k=0}^{N-2}, \left\{\tilde{\xi}_k\right\}_{k=0}^{N-2}$, which is constructed from the initial ones by eliminating the elements Δ_s and ξ_s, where $\Delta_s = \min_k \{\Delta_k\}$, and putting $\tilde{\xi}_j = \xi_j + \Delta_s + \xi_s = -2\xi + \Delta_s$ for some $j \neq s$. Since $\Delta_s + \xi_s > 0$, it follows that $\tilde{\xi}_j > \xi_j$. This implies that (41) and (45) hold. Also, $\Delta_{s+1} \geq \Delta_s$ results in $\Delta_{s+1} + \tilde{\xi}_{s-1} \geq \Delta_s + \xi_{s-1} > 0$ and $\Delta_{s-1} + \Delta_{s+1} \geq \Delta_{s-1} + \Delta_s \geq 2b$. Thus, (42) and (44) hold. Finally, (43) holds by construction. Then,

$$3\Delta U = -\tilde{\xi}_j^3 + \Delta_s^3 + \xi_s^3 + \xi_j^3 = -\left(-2\xi + \Delta_s\right)^3 + \Delta_s^3 - 2\xi^3 = 6\xi\left(\Delta_s + \xi\right)^2 > 0.$$

Hence, there is j, such that $\xi_j = \xi > 0$. Moreover, if $\xi_k < 0$ for some k, and there are types between j and k, then there must be a type i, such that $\xi_i = -\xi_{i+1}$. If there are no other types, put $i = \min\{j, k\}$.

If $\Delta_{i+1} > \Delta_i$, consider the mechanism $\left\{\tilde{\Delta}_k\right\}_{k=0}^{N-2}, \left\{\tilde{\xi}_k\right\}_{k=0}^{N-2}$, constructed from the initial ones by eliminating the elements Δ_i and ξ_i, and putting $\tilde{\xi}_{i+1} = \xi_{i+1} + \Delta_i + \xi_i = \Delta_i$. Since $\Delta_i + \xi_i > 0$, it follows that $\tilde{\xi}_{i+1} > \xi_{i+1}$. That is, (41) holds. Also, $\xi_{i-1} + \tilde{\xi}_{i+1} + 2b = \xi_{i-1} + \Delta_i + 2b \geq 2b > 0$, which means that (45) holds. In addition, we have $\Delta_{i+2} + \tilde{\xi}_{i+1} = \Delta_{i+2} + \Delta_i > 0$, $\Delta_{i+1} + \xi_{i-1} > \Delta_i + \xi_{i-1} > 0$, and $\Delta_{i-1} + \Delta_{i+1} > \Delta_{i-1} + \Delta_i \geq 2b$. Thus, (42) holds and (44) is not binding for $i-1$. Finally, (43) holds by construction. Thus, we obtain

$$3\Delta U = -\tilde{\xi}_{i+1}^3 + \Delta_i^3 + \xi_i^3 + \xi_{i+1}^3 = -\Delta_i^3 + \Delta_i^3 - \xi^3 + \xi^3 = 0.$$

If $\xi_{i+1} = \xi$, then $\Delta_i + \xi_i > 0$ implies $\tilde{\xi}_{i+1} > \xi$, which contradicts the condition $|\xi_k| = \xi, \forall k$. If $\xi_{i+1} = -\xi, \xi_i = \xi$, and $\Delta_i \neq \xi$, we again obtain $\tilde{\xi}_{i+1} = \Delta_i \neq \xi$. If $\Delta_i = \xi$, then $\Delta_{i+2} + \xi_{i+1} > 0$ requires $\Delta_{i+2} > \xi$, or $\Delta_{i+2} > \Delta_i$. Then, the conditions $\Delta_{i+1} + \Delta_i \geq 2b$, $\Delta_{i+1} + \Delta_{i+2} \geq 2b$, and $\Delta_{i+2} > \Delta_i$ imply that $\Delta_{i+1} + \Delta_{i+2} > 2b$, or (44) is not binding for $i + 1$. Finally, modifying the mechanism $\left\{\tilde{\Delta}_k\right\}_{k=0}^{N-2}, \left\{\tilde{\xi}_k\right\}_{k=0}^{N-2}$ by putting $\Delta'_{i+1} = \Delta_{i+1} - \delta$ and $\xi'_{i+1} = \tilde{\xi}_{i+1} + \delta = \Delta_i + \delta$, and using the fact that $\Delta_{i+1} > \Delta_i$, we obtain the mechanism $\{\Delta'_k\}_{k=0}^{N-2}, \{\xi'_k\}_{k=0}^{N-2}$ that satisfies all necessary constraints and provides the higher payoff to the principal.

Similarly, if $\Delta_{i+1} \leq \Delta_i$, consider the mechanism $\left\{\tilde{\Delta}_k\right\}_{k=0}^{N-2}, \left\{\tilde{\xi}_k\right\}_{k=0}^{N-2}$, constructed from the initial one by eliminating Δ_{i+1} and ξ_{i+1}, and putting $\tilde{\xi}_i = \xi_i + \Delta_{i+1} + \xi_{i+1} = \Delta_{i+1}$, which results in $\Delta U = 0$. Since $\Delta_i + \xi_i > 0$, it follows that $\tilde{\xi}_{i+1} > \xi_{i+1}$. This implies that (41) holds. Also, $\tilde{\xi}_i + \xi_{i+2} + 2b = \Delta_{i+1} + \xi_{i+2} + 2b$. If $\xi_{i+2} > 0$, then $\Delta_{i+1} + \xi_{i+2} + 2b > 0$. If $\xi_{i+2} = -\xi < 0$ and $\xi_{i+1} = -\xi$, it follows from $\xi_{i+1} + \xi_{i+2} + 2b > 0$ that $-2\xi + 2b > 0$, or $\xi < b$. Hence, $\tilde{\xi}_i + \xi_{i+2} + 2b = \Delta_{i+1} - \xi + 2b > \Delta_{i+1} + b > 0$. Finally, if $\xi_{i+2} = -\xi < 0$ and $\xi_{i+1} = \xi > 0$, it follows that $\xi_i = -\xi_{i+1} = -\xi$ and $\tilde{\xi}_i + \xi_{i+2} + 2b = \Delta_{i+1} + \xi_i + 2b > 2b$. Thus, (45) holds. In addition, we have $\Delta_{i+2} + \tilde{\xi}_i = \Delta_{i+2} + \Delta_{i+1} > 0$ and $\Delta_{i+2} + \Delta_i \geq \Delta_{i+2} + \Delta_{i+1} \geq 2b$. Thus, (42) and (44) hold. Finally, (43) holds by construction. However, it contradicts (F_4), since $\Delta_{i+1} + \xi_{i+1} > 0$ and $\Delta_{i+1} + \xi_i > 0$ imply $\tilde{\xi}_i = \Delta_{i+1} > \xi$. Thus, $\left\{\tilde{\Delta}_k\right\}_{k=0}^{N-2}, \left\{\tilde{\xi}_k\right\}_{k=0}^{N-2}$ is feasible, but not optimal.

F_5) $\xi \leq \Delta_s = \min_k \{\Delta_k\}$. If $\xi > \Delta_s$, then consider a modified mechanism, such that $\tilde{\Delta}_s = \Delta_s + \delta$ and $\tilde{\xi}_s = \tilde{\xi} - \delta$, which results in a strictly higher expected payoff.

F_6) $\Delta_{k+2} = \Delta_k, \forall k$. If, say, $\Delta_{k+2} > \Delta_k$, then $\Delta_{k+2} + \Delta_{k+1} > \Delta_k + \Delta_{k+1} \geq 2b$. Then, a modified mechanism, in which $\tilde{\Delta}_k = \Delta_k + \delta$ and $\tilde{\Delta}_{k+2} = \Delta_{k+2} - \delta$ results in a strictly higher payoff to the principal.

Given these facts, $a_k \in (\theta_k, \theta_{k+1}]$ because of (F_2) and (F_4). Also, $\xi \leq \Delta_s$ implies $\theta_{k+1} - a_k \leq a_k - \theta_k, \forall k$ or $a_k \geq \frac{\theta_k + \theta_{k+1}}{2}$.

To prove that there is no a type k, which induces her optimal action $a_k = \frac{\theta_k + \theta_{k+1}}{2} + b$, we show that $a_k < \frac{\theta_k + \theta_{k+1}}{2} + b, \forall k$. By contradiction, let $a_k \geq \frac{\theta_k + \theta_{k+1}}{2} + b$ for some k. It is equivalent to $\Delta_k \geq \xi_k + 2b > \xi_k$, which implies $\Delta_k \geq 2b$. This, along with $\Delta_k > 0, \forall k$, means $\Delta_{k-1} + \Delta_k > 2b$ and $\Delta_k + \Delta_{k+1} > 2b$, that is, (44) is not binding. Then, perturbing the initial mechanism by putting $\tilde{\Delta}_k = \Delta_k - \delta$ and $\tilde{\xi}_k = \xi_k + \delta$, we obtain

$$\begin{aligned} 3\Delta U &= -\tilde{\Delta}_k^3 - \tilde{\xi}_k^3 + \Delta_k^3 + \xi_k^3 = -(\Delta_k - \delta)^3 - (\xi_k + \delta)^3 + \Delta_k^3 + \xi_k^3 \\ &= 3\delta\left(\Delta_k^2 - \xi_k^2 - \delta\Delta_k - \delta\xi_k\right) > 0. \end{aligned}$$

Also, constraints (41)–(43) and (45) hold, since $\tilde{\xi}_k > \xi_k$ and $\tilde{\xi}_{k+1} + \tilde{\Delta}_{k+1} = \xi_{k+1} + \Delta_{k+1}$.

To show that $N \leq \lfloor \frac{1}{b} \rfloor + 1$, notice that (43), (F_2) and (F_4) lead to

$$2\sum_{k=0}^{N-1} \Delta_k + 2\sum_{k=0}^{N-1} \xi_k = \Delta_1 + \Delta_N + \sum_{k=1}^{N-1} (\Delta_k + \Delta_{k+1}) + 2\xi N = 2 \geq 2b\,(N-1)\,,$$

which gives the desired inequality. Finally, since $\Delta_{k+2} = \Delta_k, \forall k$ and $\xi_k = \xi, \forall k$, it follows that $\Delta\theta_{k+2} = \Delta\theta_k, \forall k$. ■

Proof of Theorem 6. First, show that a combination is better than the optimal delegation. By Theorem 3, it is sufficient to show that there exists the information structure and the delegation set, which provide the superior payoff for $b \in \left(\frac{1}{4}, \frac{1}{2}\right)$.

In the case of the perfectly informed expert, the optimal delegation set $[0, 1-b]$ provides the payoff $U_R^{PI} = -b^2 + \frac{4}{3}b^3$. Consider the two-element information structure and the delegation set, such that $\Delta_0 = \Delta_1 = b$ and $\xi_0 = \xi_1 = \frac{1}{2} - b$. The sequences $\{\Delta_0, \Delta_1\}$ and $\{\xi_0, \xi_0\}$ satisfy (41)–(45) and provide the expected payoff

$$U_R = -\frac{1}{3}\left[2b^3 + 2\left(\frac{1-2b}{2}\right)^3\right] = -b^2 + \frac{1}{2}b - \frac{1}{12}.$$

Then, $U_R - U_R^{PI} = \frac{1}{2}b - \frac{4}{3}b^3 - \frac{1}{12} = \frac{1}{12}(1-2b)\left(8b^2 + 4b - 1\right)$. The last term is increasing in b and is equal to $\frac{1}{2}$ for $b = \frac{1}{4}$. Thus, $U_R - U_R^{PI} > 0$ for $b \in \left(\frac{1}{4}, \frac{1}{2}\right)$, which completes the first part of the proof.

Then, in any incentive-compatible communication equilibrium, we have $\Delta_k = \xi_k, \forall k$, because of the principal's best response. Also, any non-uniform partition is not optimal, since ξ_k is different for odd and even k. Thus, it is sufficient to show that any uniform partition in the communication game is not optimal.

Plugging $\Delta\theta_k = \frac{1}{N}, \forall k$, into (6) gives $2bN \leq 1$. Among all uniform partitions that satisfy this condition, the highest payoff $U_R = -\frac{1}{12N^2}$ is reached for $N = \lfloor \frac{1}{2b} \rfloor$. Then, $2b\,(N+1) > 1$, and N is the same for all $b \in (\frac{1}{2(N+1)}, \frac{1}{2N}]$. Consider the mechanism with the information structure and the delegation set of size $N+1$, such that $\Delta_k = b, \xi_k = \frac{1-b(N+1)}{N+1}, \forall k$. This mechanism satisfies (41)–(45), and provides the expected payoff

$$3U_R^D = -\,(N+1)\,b^3 - (N+1)\left(\frac{1-b\,(N+1)}{N+1}\right)^3 = \frac{1}{(N+1)^2}\left(1 - 3b\,(N+1)\,(1-b\,(N+1))\right).$$

This results in

$$3\Delta U = 3\left(U_R^D - U_R\right) = \frac{1}{4N^2} - \frac{1}{(N+1)^2}\left(1 - 3b\,(N+1)\,(1-b\,(N+1))\right).$$

For $b = \frac{1}{2N}$, it follows that $\Delta U = \frac{1}{2}\frac{N-1}{N^2(N+1)^2} > 0$. Since $\Delta U_b' = -\frac{3}{N+1}\left(2b\,(N+1) - 1\right) < 0$, we have $\Delta U > 0$ for all $b \in (\frac{1}{2(N+1)}, \frac{1}{2N}]$, which completes the proof. ■

Appendix B

Proofs to Chapter 2.

Proof of Theorem 1. We prove the statement by induction. That is, we show that the expert cannot benefit by distorting information in any stage, conditional on the truth-telling at all previous stages.

1) For $s = \tilde{s}$, we have $i_s = 1$ and $i_\tau = 0$ for $\tau < s$ (the last condition is omitted for $\tilde{s} = 1$). The expert infers that $\theta \in \Theta_s = [\theta_s, \theta_{s-1}]$, and the optimal action for her becomes $a_S(\Theta_s) = \omega_s + b$. Truth-telling in the previous stages implies $m_\tau = 0$, $\tau < s$. Then, if $m_s = 1$, due to the principal's beliefs, we have $j = s$, and the induced action is $a_s = \omega_s$. On the other hand, if $m_s = 0$, then $j > s$, and any feasible action $a_j = \omega_j < a_s < \omega_s + b$, which results in $V(a_j, b|\Theta_s) < V(a_s, b|\Theta_s)$. Hence, $m_{\tilde{s}} = 1$. Also, for all $s > \tilde{s}$, we have $i_s = 1$. Since $m_{\tilde{s}} = 1$, the induced action is $a_{\tilde{s}} = \omega_{\tilde{s}}$ for any m_s, $s > \tilde{s}$. Thus, the expert still cannot beneficially deviate from $m_s = 1$, $s > \tilde{s}$.

2) For $i_s = 0$, we have $i_\tau = 0$ for $\tau < s$. Given this information, the expert infers that θ belongs to $M_s = [0, \theta_s]$. Assuming $m_\tau = 0$ for $\tau < s$, the message $m_s = 1$ induces the action $a_s = \omega_s = \frac{\theta_s + \theta_{s-1}}{2}$, for any m_τ, $\tau > s$. This brings the expected utility to the expert

$$V(a_s, b|M_s) = \frac{1}{\theta_s}\int_0^{\theta_s} V(a_s, b, \theta)\, d\theta = -\frac{1}{\theta_s}\int_0^{\theta_s}\left(\frac{\theta_s + \theta_{s-1}}{2} - b - \theta\right)^2 d\theta \tag{47}$$
$$= -\left(\frac{\theta_s^2}{12} + \frac{\theta_{s-1}^2}{4} - b\theta_{s-1} + b^2\right).$$

Now, consider the expert's expected payoff from sending $\{i_\tau\}_{\tau=s}^T$. If $s = T$, then θ is in $M_T = [0, \theta_T]$. The message $m_T = 0$ induces the action $a_{T+1} = \omega_{T+1} = \frac{\theta_T}{2}$, and the message $m_T = 1$ induces $a_T = \frac{\theta_T + \theta_{T-1}}{2} \geq \frac{\theta_T + 4b}{2} = \frac{\theta_T}{2} + 2b$. Then, we obtain $a_T + a_{T+1} = \frac{\theta_T + \theta_{T-1}}{2} + \frac{\theta_T}{2} \geq \frac{\theta_T}{2} + 2b + \frac{\theta_T}{2} = \theta_T + 2b$. This results in $\left|\frac{\theta_T}{2} + b - a_{T+1}\right| = \frac{\theta_T}{2} + b - a_{T+1} \leq a_T - \frac{\theta_T}{2} - b = \left|a_T - \frac{\theta_T}{2} - b\right|$, which implies that the action a_{T+1} is closer to the expert's optimal policy $\frac{\theta_T}{2} + b$ than a_T, or $V(a_{T+1}, b|M_T) \geq V(a_T, b|M_T)$.

For $s < T$, the expert's expected payoff from playing $\{i_\tau\}_{\tau=s}^T$ is

$$V\left(a\left(\{i_\tau\}_{\tau=s}^T\right), b|M_s\right) = \frac{1}{\theta_s}\sum_{\tau=s}^{T}\int_{\theta_{\tau+1}}^{\theta_\tau} V(\omega_{\tau+1}, b, \theta)\, d\theta = -\frac{1}{\theta_s}\sum_{\tau=s}^{T}\frac{\Delta\theta_\tau^3}{12} - b^2,$$

where $\Delta\theta_\tau = \theta_\tau - \theta_{\tau+1} > 0$. Then,

$$-\frac{1}{12\theta_s}\sum_{\tau=s}^{T}\Delta\theta_\tau^3 - b^2 > -\frac{1}{12\theta_s}\left(\sum_{\tau=s}^{T}\Delta\theta_\tau\right)^3 - b^2 = -\frac{\theta_s^3}{12\theta_s} - b^2 = -\frac{\theta_s^2}{12} - b^2.$$

In addition, $\theta_{s-1} \geq 4b$ implies

$$\begin{aligned} -\frac{\theta_s^2}{12} - b^2 - V(a_s, b|M_s) &= -\frac{\theta_s^2}{12} - b^2 - (-1)\left(\frac{\theta_s^2}{12} + \frac{\theta_{s-1}^2}{4} - b\theta_{s-1} + b^2\right) \\ &= \frac{1}{4}\theta_{s-1}(\theta_{s-1} - 4b) \geq 0, \end{aligned}$$

which leads to $V\left(a\left(\{i_\tau\}_{\tau=s}^T\right), b|M_s\right) > -\frac{\theta_s^2}{12} - b^2 \geq V(a_s, b|M_s)$. Hence, the expert is worse off by sending $m_s = 1$ instead of $m_s = 0$, $s = 1, ..., T$.

To complete the proof, it easily follows that the described truthful strategy generates beliefs that θ is uniformly distributed on $\Theta_j = [\theta_j; \theta_{j-1}]$, and $a\left(\{m_s\}_{s=1}^T\right) = \omega_j$ is the best-response of the principal. ■

Proof of Lemma 1. The first part of the proof, namely, for $i_s = 1$, is identical to that in the Theorem **1**.

For $i_s = 0$, distorting information by sending the message $m_s = 1$ induces the action $a_s = \omega_s$, which results in the expert's expected payoff $V(a_s, b|M_s)$, determined by (47). In contrast, sending $\{i_\tau\}_{\tau=s}^T$ in communication stages if $\theta \geq \theta_T$ and inducing the action $a(\theta) = \min\{\theta + b, \theta_T\}$ in stage $T+1$ if $\theta < \theta_T$ results in the expected payoff

$$V\left(a\left(\{i_\tau\}_{\tau=s}^T\right), b|M_s\right) = \frac{1}{\theta_s}\sum_{\tau=s}^{T-1}\int_{\theta_{\tau+1}}^{\theta_\tau} V(\omega_{\tau+1}, b, \theta)\, d\theta + \frac{1}{\theta_s}\int_{\max\{\theta_T - b, 0\}}^{\theta_T} V(\theta_T, b, \theta)\, d\theta.$$

Here, the first term is the expert's payoff in communication stages and the second term is the payoff in stage $T+1$, if $\theta \in [\theta_T - b, \theta_T]$. (If $\theta < \theta_T - b$, the expert implements her best policy $\theta + b$, which gives her zero utility). For $\theta_T > \frac{5}{3}b > b$, we can rewrite $V\left(a\left(\{i_\tau\}_{\tau=s}^T\right), b|M_s\right)$ as

$$V\left(a\left(\{i_\tau\}_{\tau=s}^T\right), b|M_s\right) = -\frac{1}{\theta_s}\sum_{\tau=s}^{T-1}\frac{\Delta\theta_\tau^3}{12} - \frac{\theta_s - \theta_T}{\theta_s}b^2 - \frac{1}{3\theta_s}b^3,$$

where $\Delta\theta_\tau = \theta_\tau - \theta_{\tau+1}$.

Then, $\Delta V = V\left(a\left(\{i_\tau\}_{\tau=s}^T\right), b|M_s\right) - V(a_s, b|M_s)$ can be written as

$$\Delta V = -\frac{1}{\theta_s}\sum_{\tau=s}^{T-1}\frac{\Delta\theta_\tau^3}{12} - \frac{\theta_s - \theta_T}{\theta_s}b^2 - \frac{1}{3\theta_s}b^3 + \left(\frac{\theta_s^2}{12} + \frac{\theta_{s-1}^2}{4} - b\theta_{s-1} + b^2\right). \tag{48}$$

As $T \to \infty$ and $\max_{s=1,...,T}|\theta_s - \theta_{s-1}| \to 0$, the first component in (48) disappears and $\theta_{s-1} \to \theta_s$. Thus, ΔV can be represented by

$$\Delta V = \frac{1}{\theta_s}D(\theta_s, \theta_T) + \varepsilon(T),$$

where $\lim_{T\to\infty} \varepsilon(T) = 0$ and

$$\begin{aligned} D(\theta_s, \theta_T) &= -(\theta_s - \theta_T) b^2 - \frac{1}{3} b^3 + \theta_s \left(\frac{\theta_s^2}{3} - b\theta_s + b^2 \right) \\ &= \frac{1}{3}\theta_s^3 - b\theta_s^2 + b^2\theta_T - \frac{1}{3} b^3. \end{aligned}$$

For a given θ_T, $D_1'(\theta_s, \theta_T) = \theta_s(\theta_s - 2b)$. For $\theta_s > 0$, we have $D_1'(\theta_s, \theta_T) \gtrless 0$ conditional on $\theta_s \gtrless 2b$, which implies that $D(\theta_s, \theta_T)$ reaches its minimum at $\theta_s = 2b$. Then, $D(2b, \theta_T) = \frac{1}{3} b^2 (3\theta_T - 5b)$, which is positive for $\theta_T > \frac{5}{3} b$.

Also, given the truth-telling sender's strategy, $a\left(\{m_s\}_{s=1}^T\right) = \omega_j$, where $j = \min\{s : m_s = 1\}$ is the best-response of the principal.

The principal's expected payoff in the constructed equilibrium is

$$\begin{aligned} U_D(\theta_T) &= \int_0^{\theta_T - b} U(\theta + b, \theta)\, d\theta + \int_{\theta_T - b}^{\theta_T} U(\theta_T, \theta)\, d\theta + \sum_{\tau=1}^{T} \int_{\theta_\tau}^{\theta_{\tau-1}} U(\omega_\tau, \theta)\, d\theta \\ &= -(\theta_T - b) b^2 - \frac{1}{3} b^3 - \sum_{\tau=1}^{T} \frac{\Delta\theta_{\tau-1}^3}{12}. \end{aligned}$$

Thus, the principal's limiting expected utility for $\theta_T = \frac{5}{3} b$ is $U_D^{\lim} = -b^3$. ■

4 Appendix C

Proofs to Chapter 3.

In the fully revealing market, the FOC for the seller's problem with respect to price is

$$\Psi(x,p) = \pi_1'(x,p) = P(x,p) + xP'(x,p) = 0.$$

From (17), we obtain

$$\begin{aligned}
\Psi(x,p) &= G_Y(p)\left(1 - G(x) - xg(x)\right) \\
&\quad + \int_p^1 1 - G(y-p+x) - xg(y-p+x)\, dG_Y(y) \\
&= \int_0^1 1 - G(\max\{x, y-p+x\}) - xg(\max\{x, y-p+x\})\, dG_Y(y) \\
&= \int_0^1 \left(1 - G(\max\{x, y-p+x\})\right)\left(1 - x\lambda(\max\{x, y-p+x\})\right) dG_Y(y).
\end{aligned} \tag{49}$$

In the symmetric equilibrium, $x = p$, which implies

$$\Psi(p,p) = 0. \tag{50}$$

Then, we can obtain the following results.

Claim 1 . *$\Psi(0,p) > 0$, $\forall p$.*

Proof

$$\begin{aligned}
\Psi(0,p) &= G_Y(p) + \int_p^1 (1 - G(y-p))\, dG_Y(y) \\
&= G_Y(p) + 1 - G_Y(p) - \int_p^1 G(y-p)\, dG_Y(y) \\
&= 1 - \int_p^1 G(y-p)\, dG_Y(y) > 0.
\end{aligned}$$

■

Claim 2 . *$\Psi(x,p) < 0$ for $x > p_M$, $\forall p$.*

Proof Notice that $\lambda'(x) \geq 0$ implies

$$1 - x\lambda(\max\{x, y - p + x\}) < 1 - p_M\lambda(p_M) = 0, x > p_M,$$

which results in

$$\Psi(x,p) = \int_0^1 (1 - G(\max\{x, y-p+x\}))(1 - x\lambda(\max\{x, y-p+x\}))\, dG_Y(y) < 0,\ x > p_M.$$

■

Claim 3 . *There exists at least one $p_N^S < p_M$, such that $\Psi\left(p_N^S, p_N^S\right) = 0$.*

Proof First,

$$\begin{aligned}\Psi(0,0) &= \int_0^1 (1 - G(y))\, dG_Y(y) = (1 - G(y))\, G_Y(y)\,|_0^1 + \int_0^1 G_Y(y)\, dG(y) \qquad (51)\\ &= \int_0^1 G^{N-1}(y)\, dG(y) = \frac{G^N(y)\,|_0^1}{N} = \frac{1}{N} > 0.\end{aligned}$$

Second, $\Psi(p_M, p_M) < 0$ by Claim 2, which gives the desired result. ■

Claim 4 . *If $\lambda'(x) \geq 0, \forall x$, then p_N^S is unique.*

Proof We prove the statement for a weaker condition, namely, for an arbitrary distribution $F(x)$ instead of $G_Y(x)$, where $F(x)$ has a positive density $f(x)$ and is supported on a bounded interval.

First, $\lambda'(x) = \left(\frac{g(x)}{1-G(x)}\right)' = \frac{g'(x)(1-G(x)) + g(x)^2}{(1-G(x))^2} \geq 0$ gives

$$\frac{g'(x)}{g(x)}x + \lambda(x)\, x \geq 0, \text{ for } x > 0. \qquad (52)$$

Now, rewrite $\Psi(p,p)$ as

$$\begin{aligned}\Psi(p,p) &= F(p)(1 - G(p) - pg(p)) + \int_p^1 (1 - G(y) - pg(y))\, dF(y)\\ &= F(p)(1 - G(p) - pg(p)) + \int_p^1 (1 - G(y) - yg(y) + yg(y) - pg(y))\, dF(y) \qquad (53)\\ &= F(p)(1 - G(p) - pg(p)) + \int_p^1 (1 - G(y) - yg(y))\, dF(y) + \int_p^1 (y-p)\, g(y)\, dF(y).\end{aligned}$$

Then,

$$\frac{d}{dp}\Psi(p,p) = f(p)(1-G(p)-pg(p)) - F(p)(g(p)+pg'(p)+g(p))$$
$$- f(p)(1-G(p)-yg(p)) - \int_p^1 g(y)\,dF(y)$$
$$= -F(p)g(p) - F(p)g(p)\left(p\frac{g'(p)}{g(p)}+1\right) - \int_p^1 g(y)\,dF(y). \tag{54}$$

Since $p_N^S \le p_M$, we have $p_N^S \lambda\left(p_N^S\right) \le p_M \lambda(p_M) = 1$. Then,

$$\frac{g'\left(p_N^S\right)}{g\left(p_N^S\right)}p_N^S + 1 \ge \frac{g'\left(p_N^S\right)}{g\left(p_N^S\right)}p_N^S + \lambda\left(p_N^S\right)p_N^S \ge 0$$

means that $\frac{d}{dp}\Psi\left(p_N^S, p_N^S\right) < 0$. That is, the function $\Psi(p,p)$ can intersect the line $p = 0$ only from above.
■

Claim 5 . *If $\lambda'(x) \ge 0, \forall x$, then $x \le p_M$ for any x, such that $\Psi(x,p) = 0, \forall p$.*

Proof This is a corollary of Claim 2. Since $\Psi(x,p) < 0$ for all $x > p_M$, $\forall p$, the result follows immediately. ■

Proof of Lemma 2. Note first that by Claim 4, there is a unique symmetric price p_N^S, which satisfies the necessary condition (50). To prove that p_N^S is the best response for each seller in the fully revealing market, it is sufficient to show that $P(x,p)$ is log-concave for $x > 0$, because in this case, the function

$$\Psi(x,p) = P(x,p)\,x\left(\frac{1}{x} + \frac{P'(x,p)}{P(x,p)}\right)$$

intersects the $x-$axis only once.

To prove this claim, we employ Prékopa's theorem (Prékopa, 1973) about the preservation of the log-concavity by integration. In particular, let $f(x,y)$ be a function of $n+m$ variables, where x is an $n-$component and y is an $m-$component vector. Suppose that f is log-concave in R^{n+m} and let A be a convex subset of R^m. Then, the function $h(x) = \int_A f(x,y)\,dy$ is log-concave in R^n.

From (17), $P(x,p)$ can be written as

$$P(x,p) = \int_0^1 1 - G(\max\{x, y-p+x\})\, dG_Y(y)$$
$$= \int_0^1 \min\{1-G(x), 1-G(y-p+x)\}\, g_Y(y)\, dy,$$

where $g_Y(y) = G_Y'(y) = (N-1)\, G^{N-2}(y)\, g(y)$. Note that $g(y)$ is log-concave by assumption and $G(y)$ is log-concave by Lemma 1. Thus, $g_Y(y)$ is log-concave, since it is a product of two log-concave functions. In addition, the IHR property of $G(.)$ implies that $1-G(x)$ and $1-G(y-p+x)$ are log-concave in (x,y). Finally, the minimum of log-concave functions is log-concave. This implies that $f(x,y) = \min\{1-G(x), 1-G(y-p+x)\}\, g_Y(y)$ is log-concave in (x,y) and so is $P(x,p)$ as a function of x.

To characterize the asymptotic properties of p_N^S, note first that from (53), we can rewrite $\Psi(p,p)$ as

$$\Psi(p,p) = L_1(p) + L_2(p) + L_3(p),$$

where

$$L_1(p) = G^{N-1}(p)\,(1 - G(p) - pg(p)),$$
$$L_2(p) = \int_p^1 1 - G(y)\, dG^{N-1}(y) = 1 - G^{N-1}(p) - G^N(y)\,|_p^1 + \int_p^1 G^{N-1}(y)\, dG(y)$$
$$= G^N(p) - G^{N-1}(p) + \frac{1-G^N(p)}{N},$$
$$L_3(p) = -p\int_p^1 g(y)\, dG^{N-1}(y) = -p(N-1)\int_p^1 g(y)^2\, G^{N-2}(y)\, dy.$$

Combining all components of $\Psi(p,p)$, we obtain

$$\Psi(p,p) = \frac{1-G^N(p)}{N} - p\xi(p),$$

where

$$\xi(p) = g(p)\, G^{N-1}(p) + (N-1)\int_p^1 g^2(y)\, G^{N-2}(y)\, dy.$$

Then, $\Psi\left(p_N^S, p_N^S\right) = 0$, that is, p_N^S is a solution to the equation

$$p_N^S = \frac{1-G^N\left(p_N^S\right)}{N}\frac{1}{\xi\left(p_N^S\right)}.$$

Note that $\xi(p)$ can be written as

$$\xi(p) = \int_0^1 g(\max\{y, p\})\, dG_Y(y).$$

Then, we have $\xi(p) \geq g_{\inf} = \inf_{\omega \in [0,1]} g(\omega)$ and

$$p_N^S \leq \frac{1 - G^N\left(p_N^S\right)}{N g_{\inf}} < \frac{1}{N g_{\inf}}.$$

By the same argument, $\xi(p) \leq g_{\sup} = \sup_{\omega \in [0,1]} g(\omega)$. Also, for any $b \in (0,1)$, we have $1 - G^N\left(p_N^S\right) > 1 - b$ for sufficiently large N. This leads to

$$p_N^S > \frac{1-b}{N g_{\sup}},$$

which completes the proof. ■

Proof of Lemma 3. By contradiction, suppose that there is a fully revealing symmetric equilibrium, in which the sellers charge the price p. By Claim 5, we have $p \leq p_M$. The profit of each seller is

$$\pi(p,p) = P(p,p)\, p = \frac{1 - G^N(p)}{N} p < \frac{1}{N} p.$$

First, consider the case $p_M \leq v^e$. If, say, seller 1 does not reveal information and charges the price $x_1 \leq v^e$, his profits become

$$\pi^d = G_Y(v^e - x_1 + p)\, x_1, \tag{55}$$

since he sells the product, if $Y - p < v^e - x_1$, or if $Y < v^e - x_1 + p$. For $x_1 = p < p_M \leq v^e$, his expected profit becomes $\pi^d = G_Y(v^e)\, p$. Then, $G_Y(v^e) = G^{N-1}(v^e) \geq \frac{1}{N}$ implies

$$\pi^d = G_Y(v^e)\, p \geq \frac{1}{N} p > \pi(p,p).$$

Second, if $p_M > v^e$, the seller cannot charge the price $x_1 = p$ only if $p > v^e$. Then, for $x_1 = v^e$, his expected profit is

$$\pi^d = G^{N-1}(p)\, v^e > G^{N-1}(v^e)\, v^e \geq \frac{1}{N} p_M > \frac{1}{N} p > \pi(p,p),$$

which means that the seller can beneficially deviate from the revealing policy. ■

Proof of Lemma 4. If $g(.)$ is symmetric on the bounded support $[0,1]$, then $v^e = v^{med} = 1/2$, where v^{med} is the median value $v^{med} = G^{-1}(1/2)$. Thus, it is sufficient to show that $p_M \leq v^e$.

Log-concavity of $g(v)$ implies that it is unimodal, so that it reaches its supremum $g_{\sup} = \sup_x g(x)$ at

1/2. In addition, $g_{\sup} = \sup_x g(x) \geq 1$. Otherwise, we have the contradiction: $\int_0^1 g(v)\,dv \leq \int_0^1 g^{\sup} dv < 1$.

The FOC function for the monopoly's profit $\pi_M(x) = (1 - G(x))\,x$ is

$$\Psi_M(x) = \pi_M'(x) = 1 - G(x) - xg(x) = (1 - G(x))(1 - x\lambda(x)),$$

which intersects the $x-$axis exactly once, because $\mu(x) = x\lambda(x)$ is strictly increasing, $\mu(0) = 0$, and $\lim_{x\to 1} x\lambda(x) = \infty$. Then,

$$\Psi_M(v^e) = 1 - G(v^e) - v^e g(v^e) = \frac{1}{2} - \frac{1}{2}g^{\sup} \leq \frac{1}{2} - \frac{1}{2} = 0,$$

which implies that $p_M \leq v^e$. ■

To proceed to other results, we need to prove the following lemma first.

Lemma 10 *Consider a continuous function $\varphi(v,x), (x,v) \in [0,1]^2$ and a distribution function $G(v)$, such that:*

1) there exists an interval $[a,b] \subset [0,1]$, such that, for any x, $\varphi(v,x) > 0$, for $v \in (a,b)$; and $\varphi(v,x) \geq 0$, for $v \in [b,1]$, and

2) $G(v)$ has a continuous density, which is positive on $[a,b]$.

Then there exists $\bar{N}$, such that $\int_0^1 \varphi(v,x)\,dG^N(v) > 0$ for all x and $N \geq \bar{N}$.

Proof of Lemma 10. Note that $q(v,x) = \varphi(v,x)\,g(v)$ is continuous, that is, there exists $q_1 = -\max_{x\in[0,1],\ v\in[0,a]} |q(v,x)|$.

Take an interval $[c_1,c_2]$, such that $a < c_1 < c_2 < b$ and let $q_2 = \min_{x\in[0,1],\ v\in[c_1,c_2]} q(v,x)$. Clearly, $q_2 > 0$. Now, the integral can be written as

$$\begin{aligned}\int_0^1 \varphi(v,x)\,dG^N(v) &= \int_0^1 \varphi(v,x)\,g(v)\,G^{N-1}(v)\,dv = N\int_0^1 q(v,x)\,G^{N-1}(v)\,dv \\ &= N\int_0^a q(v,x)\,G^{N-1}(v)\,dv + N\int_a^1 q(v,x)\,G^{N-1}(v)\,dv.\end{aligned}$$

Then,

$$I_1(N) = q_1 G^{N-1}(a)\,a < \int_0^a q_1 G^{N-1}(v)\,dv \leq \int_0^a q(v,x)\,G^{N-1}(v)\,dv.$$

Similarly,

$$I_2(N) = q_2 G^{N-1}(c_1)(c_2 - c_1) < \int_{c_1}^{c_2} q_2 G^{N-1}(v)\,dv \leq \int_a^1 q(v,x)\,G^{N-1}(v)\,dv$$

Thus, as N increases, $I_1(N)$ converges to zero at a faster rate than $I_2(N)$. Since $I_2(N) > 0$, this implies $I_1(N) + I_2(N) > 0$ for sufficiently large N, which completes the proof. ∎

Proof of Theorem 1.

Consider the general problem of seller 1, when $N-1$ competitors reveal information and charge price p_N^S. The seller maximizes the profit function

$$R(x,\eta) = x \int_0^1 1 - G_\eta\left(\max\left\{x, y - p_N^S + x\right\}\right) dG^{N-1}(y)$$

with respect to the quality of information η and price x.

Now, we will prove that for an arbitrary $\eta > 0$, the optimal price $x_N^\eta \to 0$ as $N \to \infty$. To prove it, it is sufficient to show that for a fixed $x > 0$, $\Psi_x(x,\eta) = \partial R(x,\eta)/\partial x < 0$ for a sufficiently large N.

First, the expression for $\Psi_x(x,\eta)$ is

$$\Psi_x(x,\eta) = \int_0^1 1 - G_\eta\left(\max\left\{x, y - p_N^S + x\right\}\right) - x g_\eta\left(\max\left\{x, y - p_N^S + x\right\}\right) dG^{N-1}(y).$$

Second, let x_M^η be the price of the monopolist, who faces the demand function $1 - G_\eta(.)$. Since $G_\eta(x)$ has the increasing hazard rate, then, for $x \geq x_M^\eta$ and $y > p_N^S$, we obtain

$$\begin{aligned} &1 - G_\eta\left(\max\left\{x, y - p_N^S + x\right\}\right) - x g_\eta\left(\max\left\{x, y - p_N^S + x\right\}\right) \\ &< 1 - G_\eta\left(x_M^\eta\right) - x_M^\eta g_\eta\left(x_M^\eta\right) = 0. \end{aligned}$$

For $x \geq x_M^\eta$ and $y \leq p_N^S$, we have $\max\left\{x, y - p_N^S + x\right\} = x$ and

$$1 - G_\eta(x) - x g_\eta(x) \leq 1 - G_\eta\left(x_M^\eta\right) - x_M^\eta g_\eta\left(x_M^\eta\right) = 0.$$

Hence, $\Psi_x(x,\eta) < 0$ for $x \geq x_M^\eta$.

Consider the function

$$\varphi_1(y,x) = -\left(1 - G_\eta\left(\max\left\{x, y - p_N^S + x\right\}\right) - x g_\eta\left(\max\left\{x, y - p_N^S + x\right\}\right)\right),$$

which is continuous in y.

If $x < x_M^\eta$, then notice first that $p_N^S \to 0$ as N increases. That is, there exist a sufficiently small $\delta > 0$ and a sufficiently large N_1, such that $y - p_N^S + x > x_M^\eta$ if $y > x_M^\eta - x + \delta$ and $N > N_1$. This implies that $\varphi_1(y,x) > 0$ for $y \in (x_M^\eta - x + \delta, \bar{\omega}^\eta - x)$ and $\varphi_1(y,x) \geq 0$ for $y \in [\bar{\omega}^\eta - x, \bar{\omega}^\eta]$. Thus, all conditions of

Lemma 10 are satisfied that results in $-\Psi_x(x,\eta) > 0$.[80]

Now, consider the FOC function with respect to η:

$$\Psi_\eta(x,\eta) = \frac{\partial R(x,\eta)}{\partial \eta} = -x \int_0^1 \frac{\partial}{\partial \eta} G_\eta\left(\max\left\{x, y - p_N^S + x\right\}\right) dG^{N-1}(y).$$

Notice that both the optimal price x_N^η and p_N^S tend to zero as N grows. Also, we have $\omega'_\eta < \bar{\omega}^\eta, \eta > 0$. Otherwise, if $\omega'_\eta = \bar{\omega}^\eta$, then $\frac{\partial G_\eta(\omega)}{\partial \eta} < 0$ for all $\omega < \bar{\omega}_\eta$. That is, $G_{\eta'}(\omega) > G_\eta(\omega)$ if $\eta' < \eta$, which contradicts (14).

Thus, for an arbitrarily small $\varepsilon > 0$, there is N_2, such that $y - p_N^S + x_N^\eta > \omega'_\eta$ for $y > \omega'_\eta + \varepsilon$ and $N \geq N_2$. This implies that for $x \leq x_{N_2}^\eta$, we have $-\frac{\partial}{\partial \eta} G_\eta\left(y - p_N^S + x\right) > 0$, $y \in \left(\omega'_\eta + \varepsilon, \bar{\omega}^\eta - \varepsilon\right)$ and $-\frac{\partial}{\partial \eta} G_\eta\left(y - p_N^S + x\right) \geq 0$, $y \in [\bar{\omega}^\eta - \varepsilon, \bar{\omega}^\eta]$. Applying Lemma 10 results in $\Psi_\eta(x_N^\eta, \eta) > 0$, $N \geq N_2$, which means that releasing incomplete information reduces the seller's profit.

Finally, $R(x,\eta)$ is continuous in (x,η) that rules out $\eta = 0$ from the set of optimal solutions to the seller's problem. Thus, since no seller can deviate when all competitors reveal information completely, and $x = p_N^S$ is the best response to the market price p_N^S, it follows that the fully revealing market with the symmetric price p_N^S is an equilibrium.

To see that the identified equilibrium is unique in the class of symmetric equilibria as $N \to \infty$, notice that all proofs above still hold if the distribution $G(y)$ is replaced by $G_\eta(y)$ as soon as $G_\eta(y)$ has a positive density on a non-degenerated interval. Thus, for any other symmetric strategy of competitors $\left(\bar{p}_N^S, \bar{\eta}_N\right)$, where $\bar{\eta}_N > 0$, the partial derivatives of the profit function are represented as integrals over $G_{\bar{\eta}}^{N-1}(y)$. That is, even though other sellers do not reveal information completely, one seller can strictly benefit by fully revealing information. In addition, $\bar{\eta}_N = 0$ cannot be an equilibrium, since it implies that all products are ex-ante identical, i.e., both the equilibrium price level $\bar{p}_N^S$ and profits must be zero. This is clearly not an equilibrium, since each seller can guarantee positive expected profits by revealing information completely and charging a sufficiently small price ε. ■

Proof of Lemma 5. By Theorem 1, as the number of sellers N increases, it results in full information disclosure. Thus, the market inefficiency is determined by

$$ME_N = \frac{SW_N - TS_N}{SW_N} = \frac{\int_0^1 v dG^N(v) - \int_{p_N^S}^1 v dG^N(v)}{\int_0^1 v dG^N(v)} = \frac{\int_0^{p_N^S} v dG^N(v)}{\int_0^1 v dG^N(v)},$$

where $TS_N = \int_{p_N^S}^1 v dG^N(v)$ is the total surplus, and $SW_N = \int_0^1 v dG^N(v)$ is the social welfare. Then, we

[80] Since x is fixed, we consider the function $\varphi(v,x)$ in Lemma 10 as a function of a single variable.

have

$$\int_0^{p_N^S} v dG^N(v) = p_N^S G^N\left(p_N^S\right) - \int_0^{p_N^S} G^N(v)\, dv < p_N^S G^N\left(p_N^S\right),$$

and

$$\int_0^1 v dG^N(v) \geq v^e.$$

Expanding $G(v)$ by Taylor's formula results in

$$G\left(p_N^S\right) = g_0 v + \frac{g'(\tilde{p}_N)}{2}\left(p_N^S\right)^2 \leq g_0 p_N^S + \frac{g_s}{2}\left(p_N^S\right)^2,$$

where $g_0 = G(0)$, $\tilde{p}_N \in \left[0, p_N^S\right]$, and $g_s = \sup|g'(v)|$. Thus,

$$ME_N = \frac{p_N^S\left(g_0 p_N^S\right)^N}{v^e} + O\left(\left(p_N^S\right)^2\right),$$

where $O(x)$ has an order x. Since $p_N^S \in \left(\frac{1-b}{N g_{\sup}}, \frac{1}{N g_{\inf}}\right)$, it follows that ME_N converges to zero at the rate $\left(\frac{1}{N}\right)^N$. ■

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