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ABSTRACT

This thesis contains three essays on the formation of social networks. The first essay focuses on citation networks while the second one proposes a general model of network formation. The third essay is an empirical investigation of a person's kinship network.

Chapter 1: Citations and the Diffusion of Knowledge: An Economic Analysis (with Kalyan Chatterjee)

Citation patterns in many academic disciplines have displayed a pattern of connections similar to those observed in many other different real-world contexts, such as links on the world-wide web. The various models that have been proposed to generate such networks, generically called "preferential attachment models", rely solely on random link formation and copying and do not take into account rational choice among authors in an academic community, which would consider the competition for citations and ensuing professional success. In this paper we construct such a model with rational agents to understand some aspects of citation patterns and knowledge diffusion in a specific academic field .. We show that rivalry or competition in citations might be an obstacle to diffusion, depending on behavioral rules specific to the field. Increased heterogeneity in the quality of papers reduces this effect. After considering models with complete information, we analyse models with private information about quality of one's own paper and use the framework to consider the interaction of this process with acquaintance networks and strategic entry. Superimposing the citation process on an acquaintance network yields patterns different from preferential attachment. Strategic entry leads to cascades of papers. Though we might have ex-ante efficiency in some equilibria, ex-post efficiency is not guaranteed. Ex post efficiency cannot be guaranteed since it is always possible in equilibrium that a good paper "dies" and a worse one survives, but ex ante efficiency is sometimes attainable.

Chapter 2: Competing to be a 'Star': A model of sequential network formation

This paper studies a sequential-move game of network formation with capacity constrained agents. The sequential nature of the game allows us to focus on the the role played by foresight in the formation of particular network configurations. We find that the equilibrium network structure depends crucially on the rate of decreasing returns (decay) in the payoffs. With homogeneous agents and a capacity of one link, the equilibrium network is a fully connected graph for extreme levels of decay. The architectures for very low and very high levels of decay are a complete star and a graph similar to the hub-and-spoke architecture, respectively. For intermediate levels of decay, however, the network configuration might have more than one component. This occurs because agents have the incentive to reject a link in some subgame in order to become central in the network at some future date. With heterogeneous agents, the equilibrium network for both very high and very low levels of decay is a star with a high ability agent as the center. But it is possible to have structures where the high ability agent is kept isolated by the rest of the society comprising of low types. This depends on the decay factor and the difference in the abilities of agents. The equilibrium is, for most values of decay, inefficient. Hence, we can possibly explain the existence of multiple small groups in various social settings even when a single cohesive network would be more efficient.

Chapter 3: Family Embeddedness: An Empirical Investigation

Using the Netherlands Kinship Panel Data, we investigate the relationship between an individual's embeddedness in family ties, effort exerted on the job and social attitude. In particular, we want to explore whether stronger familial ties are associated with any negative impact on personal incentives or on society. We find that agents with a higher level of family ties at birth maintain a higher level of ties at a later stage. They also have a higher involvement in volunteer work and are more tolerant of migrants. Moreover, the level of inherited ties affect a person's current work effort positively. This is possible due to peer effects or due to a correlation with the occupational choice of individuals. Other factors which influence both ties and effort have an opposite effect on the two. Hence, the level of initial embeddedness in family, by itself, does not have negative effects on social attitude or effort. However, there possibly exists a negative relationship between current ties and effort due to family obligations or time constraints. We also find that negative social attitudes could be nurtured by higher income or wealth.

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Lastly, I want to dedicate this thesis to my parents and to my uncle for years of love, understanding and support.

Chapter 1

Citations and the Diffusion of Knowledge: An Economic Analysis

1 Introduction

The aim of this paper is to model aspects of the process by which rational agents engaged in research use and cite earlier related work in their field. Citations constitute visible evidence of the diffusion of ideas and are therefore important in studying the influence or impact of particular pieces of research. It is also possible to think of citations as directed links in a network, so the nature of the diffusion of ideas also endogenously generates a network whose properties can be studied.

It is natural to think of citations in academic research, since the academic enterprise is one with which we, engaged in it, are intimately familiar. Academic administrators often ask for citation counts as evidence of the impact of an individual's research and this might translate into salary raises or external offers. Being cited by one's peers also gives us pleasure and not being cited, when one should have been, is frequently cause for discomfort. Citations therefore have real consequences for an academic's utility.

We can also think of citations of patents in industrial research and development in a similar way. An individual firm might have a project that could be facilitated by using someone else's idea. The firm could choose, however, to try to avoid having to pay royalty fees for using the patent and develop its own original solution to its problem. Such a new product or process could itself generate fees from future entrants to the field.

We shall focus on academics for convenience, though sometimes keeping the industrial R&D context in mind could help motivate some of the assumptions.

As stated earlier, we shall not consider every aspect of the decision on whether to cite a preceding paper. In particular, we abstract from issues relating to repeated interactions, where reciprocity could play an important role. The basic tradeoff that we shall examine is that between investigating and using an existing good idea to simplify one's own task versus expending costly effort to come up with one's own solution to generate high future payoffs from others with similar projects.

The context we wish to model might be thought of as a research field progressing by the solution of many related small problems. The person who solves one of these problems puts in effort and gets an expected value commensurate with the effort put in from her proposed solution to the problem. However, if other papers are known to exist in related areas, an individual might read one or more of them to obtain ideas that would simplify her task thereby reducing the cost of effort and increasing the value of the solution to her. Any ideas from other papers so used would need to be cited. (This is an assumption, but probably a good one for the vast majority of researchers; it is certainly something that firms engaged in R&D have to do, to avoid lawsuits) However, by citing a related paper, the individual who cites signals to other future entrants that the cited paper has proved useful and therefore directs other researchers to it rather than the academic's own work, whose usefulness to future entrants is uncertain. Note that in much of this paper, we assume that there is *perfect information about who has done relevant work*, though whether the relevant work is useful is unknown before someone investigates it. Survey papers and textbooks often garner citations because they themselves cite large numbers of other papers and books and serve to dispel lack of information about earlier research. With perfect information, this motive for citing (or for writing textbooks) is absent.

There appear to be relatively few papers dealing with the endogenous formation of citation networks. One exception is the paper by Mikhail V. Simkin and Vwani P. Roychowdhury ([23]), which relies on random copying of citations in previous papers.[1] In this model, an author randomly chooses some previous paper that appears related and randomly copies some proportion of the references in that paper. The randomness generates increasing returns; the more one is cited the more often one will be cited. However, here the number of citations of a paper is independent of that paper's characteristics, which appears to suggest that administrators counting citations are irrational. In the R&D context, random citation could lead to a high

[1] [9] has a sentence suggesting copying of citations might be rational: "Signalling by third parties: The latter, when deciding whom or what to cite, may be concerned to demonstrate that they are conversant with the reputational ranking of people in a specific area of science."

volume of lawsuits for patent infringement, though Simkin and RoyChowdhury do not intend their paper to apply to this.

As a contrast to our view of the sequence of related small research problems as constituting a field, the paper by Paul David [10] considers different scientists pondering the truth or falsity of some major proposition or theory. They become aware of the opinions of individuals they are connected to; since these opinions contain information, Bayesian updating leads to their adopting the opinions of the majority in their neighbourhood; the neighbourhood structure is *given*. The ultimate disposition of the theory is then found by using techniques from probability theory, namely the voter model discussed in Rick T. Durrett (1988). An individual cites the opinions of others in his neighbourhood as justification for his own opinion. This explanation has something in common with lawyers citing precedent and case law to justify a particular argument. One can interpret the neighbourhood here as coming from a social network. In a later section, we consider the interaction between the process we model and the existing social network to see how the latter constrains the former in situations where information about past work flows only through the social network (as in David's model).

Citation patterns have been used in empirical exploration of academic research communities in physics by Sidney Redner([21]); C. Lee Giles and Isaac G. Councill ([14]) have used acknowledgements to trace a similar network of influence. These various networks display some form of a "power law" structure in aggregate; that is, the degrees of nodes in the network follow a power law distribution with a small number of highly connected nodes[2]

One early empirical discussion of citations is contained in Derek S. Price [20]. He looked at the patterns from 1862 to 1961 across many fields and did extensive analysis of the empirical regularities of the network of scientific papers. Price calculated that on average there were seven new papers a year for every hundred papers in that field, and each new paper contained about fifteen references. Therefore, on average, each paper is cited once per year. However,he found that in *any given year*, about 35% of existing papers are not cited at all and 49% are cited only once. Of the others,

[2]The "power law" states that the probability that a randomly selected node in a network has r links is $r^{-\gamma}$. The parameter γ has been estimated to be between 2 and 4.1 for different networks such as the web or the network of citations.

the percentage of papers cited n times falls off rapidly with n (in the order of $n^{2.5}$). The data appears to fit the hypothesis, according to Price, that about 10 percent of existing papers "die" each year. Also citations for a paper tend to occur in "capricious bursts".

The findings of Price regarding the rapidly decreasing proportion of papers with higher citations has the same flavour as the "power law" mentioned earlier. The models constructed to generate networks following such a power law, such as the preferential attachment models of Albert-Laszlo Barabasi and Reka Albert ([4]) and Bela Bollobas and Oliver Riordan ([5]), all rely on some exogenously specified process by which links are formed in the network.

In these models new nodes are born each period and each of them links with a existing one randomly but with a probability that is proportional to the number of links the node already has. This results in a well-defined stochastic process and we can calculate the properties of the network generated. Here, the older nodes would tend to have more links than newer ones and the process implies that there is a tendency of cumulation, which is similar to the observed cumulation of citations to a small set of papers.

In our model we will try to address the '*why*' of the preferential attachment model in the specific context of academic citations by pinning down the possible economic motivations at work.[3]

Our paper also relates to the literature of the spread of technology and information among agents in a community.[4]The results we obtain illustrate the effects of competition and strategic considerations on the diffusion of useful ideas. For efficient dissemination of ideas, existing papers should be investigated immediately to see if they have ideas that are broadly useful. To the extent that competition delays such investigation, it creates some inefficiency. In some variants of our model, such inefficiency occurs. We briefly summarize the qualitative features of our results below.

In the model with complete information we find an irreducible multiplicity of

[3]The nature of citations bears some resemblance to that discussion of "cumulative advantage" presented by Robert K. Merton in the 'Matthew effect' papers. Small (2004) makes this connection clear-"When a paper is cited, other authors can see that it is and this heightens their interest in the paper and their likelihood of citing it as well at some later date. In this sense, citation acts like an expert referral."

[4]See [11] for economic models of such social dynamics and [6] for a discussion of social learning.

equilibria. However, the multiplicity is caused by different agents using different rules to identify past papers to investigate. If all agents use the same rule, corresponding to norms in different fields,[5]we obtain a single equilibrium for each rule. The different rules have different implications for how efficiently knowledge diffuses and give rise to different patterns of investigation and citation. In particular, the norm that specifies citing only the most recent paper in an area leads to inefficient delay and the pattern of the expected number of citations oscillates with the age of a paper, whilst for the other norms, earlier entrants (the pioneers) should expect to be cited more often. These results are with a finite number of entrants. With an infinite horizon, the unique stationary mixed strategy equilibrium does not display the oscillating pattern

We then assume each agent has a better idea about the usefulness of her own paper than other agents. The private information leads to a combination of behavioural norms-randomising among uncited individuals initially and then choosing the most recent previous entrant.

We then consider strategic entry if the agents all have their ideas simultaneously. We find there exists a "signalling" equilibrium, in which earlier entry implies higher average quality. Once entry occurs, there is a cascade of related papers.

Finally, we constrain the directed citation graph by an undirected acquaintance network. Now we relax the assumption that the existence of all previous papers is known and assume instead that one learns about papers that acquaintances have written or ones they have cited.[6]This gives rise to the closest analogue in our paper to the preferential attachment models. The probability a paper is cited then depends on two factors, one (the number of previous citations) arising from the (superposed) network structure and the other (which is a probability itself and hence less than 1) arising from the strategic incentives of players. The process is sublinear and therefore does not give a power law (see Fan Chung, Shirin Handjani and Doug Jungreis (2003)).

We make several strong assumptions in our model, though it is not clear that relaxing them would give any new insights. The two strongest ones are: (i) Once a previous piece of work is found useful by someone, it will be found useful by everyone following and if it is not useful, it remains not useful; (ii) Only one citation is allowed

[5]hese are: citing the most recent paper or the oldest one or citing all available papers with equal probability

[6]This does not seem that far-fetched though whether reciprocity is at play here is hard to tell.

per new paper. We discuss relaxing the first in the extensions section (section 7). The second implicitly takes into account the time spent in investigating past work or past patents. One could think of it as choosing to refer to one directly useful paper and referring to survey articles or textbooks for the others.

The rest of the paper is organized as follows. In Section 2 we describe the basic model while Section 3 deals with the analysis.In section 4 we introduce a model with private information about types. In Section 5 we discuss strategic entry decisions. Section 6 deals with social networks. Section 7 concludes with discussions on possible efficiency issues and on introducing heterogeneous quality . All detailed proofs are relegated to the appendices.

2 The benchmark model: single entrant per period and two qualities

The set of players is denoted by N={1,2,...k,...,n}. Players are ex ante identical. In each period, one player enters; the order of entry is predetermined. We shall denote by Player k the individual who enters and writes a paper in period k. Agent k can write on his own or use some existing paper, 1,2,...$k-1$ before publishing (or "entering"). A paper k can be "useful" or "not useful"[7]. If useful, the paper gives a value[8] $v > 0$ to any player $k+1, ...N$ who cites it. The payoff to the paper being cited is w for each citation it gets. We assume $v > w$. If not useful, the value is 0 and the paper is not cited. The prior probability that any paper k is useful is p_0.Paper k being useful is independent of the usefulness of the sequence of papers $1, 2...k-1$. Any entrant first observes the state of citations C_{ik}, the number of citations received by paper i till period k. We assume $C_{ik} \geq 1$,that is, writing a paper is, by convention, a citation. Any paper with $C_{ik} \geq 2$ is revealed to be useful. After entry each agent updates his beliefs about each agent/paper. Then he decides whether to incur cost c to *Investigate*(read) some $j = \{1, 2, ..., k-1\}$ or *not* to *investigate* at all (NI). We assume $p_0 v > c$. If investigated, it is revealed whether $j's$ paper is useful or not.

[7]This simplification is made for analytical tractability-clearly papers can be useful to different degrees.

[8]This could be interpreted as the additional value obtained from a useful paper.

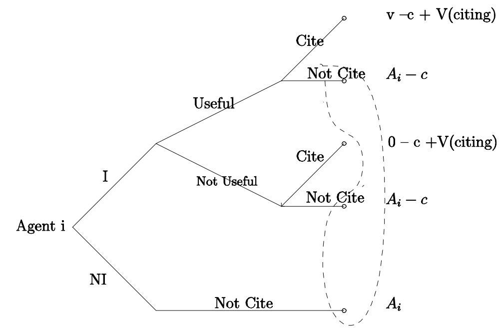

Figure 1: Original Game

If useful, agent k decides whether to use (and cite) it or not. At the time agent k enters, he observes the identities of all previous entrants and the citations each has, including the virtual self-citation. But k does not observe the *actions* prior entrants have taken with respect to reading or not reading previous papers. Therefore k is unable to distinguish between the histories where Player i $(1 < i < k)$ chooses NI and where she chooses I but does not cite (because, perhaps, the investigated paper was not useful). By choosing to cite(use the paper of a previous agent) agent k gets an immediate payoff of v and some future payoff depending on whether he is ever cited, which again depends on the state C_{k+1} at $t = k + 1$. By choosing NI or not cite, even after Investigation, he has to write on his own and gets a lower current payoff, which is normalised to 0, and an expected payoff depending on the state next period. After $k's$ decision, the next agent enters, observes state $C_{i,k+1}$ and takes decisions as specified in the previous steps. All agents have the same discount factor $\delta \in (0, 1)$.

We can represent the actions of any player i in a schematic tree(Fig 1) where A_i is the expected future benefit from not citing any previous player.

Note that after the uncertainty is resolved, the paper either gives a payoff of v or 0. As long as the cost of reading c is positive, an agent i would always use(cite) $j's$ paper if he found it useful after reading (otherwise it would have been optimal not to read it). Also, if not useful i has no choice but to write on his own as if he had not read any other paper in the first place. So, this game can be reduced to an equivalent

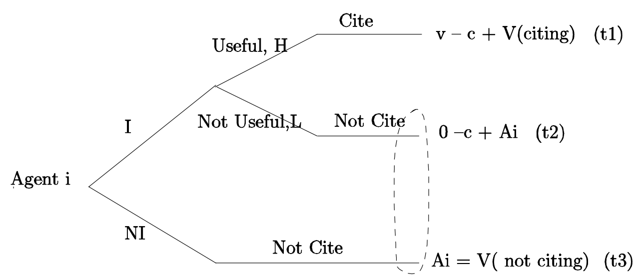

Figure 2: Reduced Game

game, represented by Fig 2.

Note that both the nodes t2 and t3 involve no citation and are in the same information set. Hence, if the equilibrium strategy of i is to I and he deviates, then $i+1$ observes no citation and believes that i is at node t2, when he is actually at t3 and so not all deviations are detected.

Let us now consider the assumptions made in the specification of the model. The payoff v can be interpreted as a private benefit an agent gets from writing a paper. The payoff from a paper when it uses another's idea (v) is higher than when it is written solely by the agent, due to the fact that the agent writing a paper entirely on his own has to put in a much higher effort to achieve the same quality than when he is supplementing his idea with that of another agent. Hence the net benefit of writing a paper(not taking the cost into account) is lower if the agent writes by himself. The payoff w is the benefit from the fame and other associated consequences an agent gets from being cited.

The assumption made for analytical tractability is that a paper is either always useful (high quality) or never useful (low quality). First, note that the evolution of the probabilities is now very simple. Following one success of an investigation, the paper will be revealed to be of high quality and belief about its usefulness goes up from p_0 to 1. On the other hand, one failure does the opposite. If it is known that a paper was investigated but not cited, it is revealed that the paper is of low quality and it is never investigated again by any agent i.e. $p_s = 1$ and $p_f = 0$. In other words, we have perfect signals regarding the quality of the papers investigated. As for papers not investigated, the belief remains at the prior p_0.[9]

The trade-off involved here is between investigation of some $j's$ paper with potential current benefit v associated with lower future benefit (because by citing the agent i is signalling that $j's$ idea is useful) or no investigation (and hence no citation for sure), which has lower current payoff but a higher expected future payoff (since i does not give anything away about j). We want to focus on the equilibrium pattern of investigation and citation.

We also assume a version of private uncertainty in the basic model in that a particular agent does not know the quality of his own paper, that is, his belief about the probability that his paper would be useful to somebody is also p_0. This is sometimes reasonable in the context of academic papers since the quality of a paper is determined by the judgement of one's peers. Also, this implies that no agent knows his own type (high or low), which helps us to abstract initially from signalling motives.[10]

[9] In the basic model, the probability that a paper is useful in any one instance, given it is of high quality, is given by h and, given it is of low quality is given by l. For the sake of tractability we took $h = 1, l = 0$, which implied that once a paper is found useful (or not useful), it remains so for all future readers. If we relax one of the equalities then the tradeoff remains the same. For example, if $l = 0, h < 1$ then one citation of a paper would reveal that it is of high quality as in the basic model. A non-citation however does not reveal for sure that it will never be useful but the probability of its usefulness goes below the prior. Hence an agent would not investigate that paper. Similarly, for $l > 0, h = 1$ one non-citation reveals that it must be a low-quality paper and the probability of it being useful later is less than the prior whereas a citation increases the proabability of another sucess for that paper.

For general h and l, i.e. $h > l > 0$, we would get a non-degenerate distribution of citations. Bayesian updating would then imply that the prior p_0 goes up following one success but not to 1. Generally, it is increasing in the number of successes and decreasing in failures. For a given (h, l), agents follow the Bayesian updating rule and there will be a certain number of failures, x_f, of a paper after which the belief about its usefulness goes below p_0. So, we might observe patterns of the following sort: a paper is investigated for the first time and if it is a failure the belief goes below p_0 and it is not chosen again. If it is a success it gets a citation, the belief increases to $p_1 > p_0$ and it is chosen again. This second investigation might be a failure and hence the belief goes down to $p_2 < p_1$. However, p_2 might still be greater than p_0, in which case, it is chosen again. Consecutive failures will take the belief below p_0. Once that happens this paper would not be chosen again and papers whose quality is at the prior level will be chosen and the same process followed. Hence, we might observe a group of papers with more than one citation, though the second paper was chosen only when the first cited paper had enough consecutive failures. In fact, we can also make h and l dependent on the number of citations of a paper. For example, when a paper has been used and cited say m times, the probability that there is anything useful remaining in it decreases, which implies that both h and l are decreasing in the number of citations a paper gets. This will also result in a distribution of citations instead of a spike for only one paper .

[10] We do discuss possible effects of signalling in Sections 5-8

2.1 Strategies, Payoffs and Equilibrium

We define strategies for the N players and the equilibrium of the game.

Strategies*: Let the set of information sets where agent k has to move be IS_k and let z be an element of IS_k.A strategy s_k for the agent k is his choice of an element from the set $S^k = \{NI/I_0, I_1, I_2, ..., I_{k-1}\}$ at each z, given his entire set of beliefs $\mu^k = (\mu_1, \mu_2, ...\mu_{k-1})$ about the probability of usefulness of all earlier entrants at each z.Thus a strategy $s_k = \{(I_i^z)\}_{z=1}^{Z}$, where $|IS_k| = Z$ and $i = 0, 1, ..., k-1$.*

Agents choose their strategies to maximize their expected payoffs, where the expectation is with respect to the beliefs μ.

Equilibrium*: A Perfect Bayes Equilibrium is a N-tuple of strategies for all N players $< s_1^*, s_2^*, ...s_N^* >$ such that s_k^* is a best response to s_{-k}^* at every information set of Player k,given beliefs μ,which are derived from the prior p_0 and the history of play using Bayes' theorem, wherever possible.*

To derive the beliefs, note that, at any history h_t which has a citation other than a self-citation, it is revealed that the paper was investigated and is of high quality and will guarantee v if investigated. All papers with any citation will have this feature. Other papers without any citation can belong to one of the two groups: a) the paper has been investigated but not found useful, in which case $p = 0$ for sure; or b)the paper has not been investigated in which case the ex-ante probability of it yielding v is $p_0 < 1$. So, if an agent wants to choose I at any node, he will choose the paper with a citation since the expected payoff from so choosing is the highest. It follows that once there is a revelation of a high quality paper k at time t, that paper will be investigated (and cited) by all agents from time $t+1$. This is true since the person investigating paper k at time t, must have done so because the expected payoff from investigation is more than that from not investigating a paper with usefulness probability p_0. Given this, investigation of a surely high-quality paper must have a higher expected payoff for agent $t+1$ onwards. Moreover, once a paper is cited, any paper without a citation (including the current entrant's) ceases to be competitive and the trade-off disappears. So, for any history with a citation of paper k', the equilibrium strategies of subsequent entrants will be investigation of paper k' and consequent citation for all periods hence. The trade-off between present and future benefits mentioned is relevant only after a history with no citations.

Let h'_t denote any history with no successes (or equivalently citations). We have to specify equilibrium strategies (NI or I_k) for each agent k after such a history h'_t. We define an *equilibrium string* for this purpose.

An Equilibrium String is a N-dimensional array where the k^{th} element is the equilibrium decision of the k^{th} agent from set S_k after a history of no citations. Since the trade-off between current and future benefits kicks in at these histories, we have to figure out what the *equilibrium string* is, which along with the decision $s'_k = I_j$ *whenever* $\exists j$ *with* $C_j > 1$ and the belief μ, will be the Perfect Bayesian Equilibrium.

To completely specify the equilibrium we have to specify the belief μ_{ik} each agent k has about i, $i = 1, 2, ...k - 1$. On the equilibrium path, μ is derived by Bayesian updating. Information regarding the paper is revealed following one citation and $\mu_{ik}(\mathrm{c}_i > 1) = 1$ while $\mu_{ik}(C_i = 1) = p_0$ if i was investigated with probability 0 in equilibrium. If i was investigated with some positive probability r in equilibrium, then $\mu_{ik}(C_i = 1) = (1 - r)p_0 < p_0$. Histories off the equilibrium path involve deviations that *are* revealed to be such. If some i deviates from I_j to NI , this is not observable by $k > i$.If instead, i deviated from NI to citing some j not cited before, then this deviation might be observed if j is useful. The out-of-equilibrium belief here can naturally be set at $\mu_j = 1$. If the deviation is not revealed to $k > i$ then $\mu_{kj}(C_j = 1)$ remains at p_0 (this is on the equilibrium path).

3 Equilibrium Analysis

We first give a trivial lemma for the updated priors.

Lemma 1 *If some previous agent is investigated with positive probability, then in states of no citation, the belief regarding his usefulness is less than the prior* p_0.

After history h_t with some $C_{jt} > 1$, the equilibrium behaviour is also trivial and given by:

Lemma 2 *After any history* h_t *with* $C_{jt} > 1$ *for some* $j = 1, 2...t$, *agent* $t+1$ *chooses to Investigate* $j^* = \arg\max C_{jt}$ *in equilibrium.*

Proof. We prove this by backward induction. Suppose at time t, agent t enters and observes $C_i > 1$(wlog).Consider the last agent, N. He chooses to investigate $\widehat{j} =$ argmax $C_j, j < N$, since he only cares about the current benefit. Now, suppose agents $\tau, \tau+1, ..., N$ follow this strategy. We need to show Player $\tau - 1$ also follows this strategy. (Note:If there is a $C_j > 1, j < \tau$, then argmax $C_j, j < \tau$ is same as argmax $C_j, j < \tau - 1$, since no agent except agent τ can cite $\tau - 1$, so this player cannot have more than a self-citation). He knows that τ will choose $\widehat{j}$ and hence, the expected future benefit of $\tau - 1$ is 0. Given some $C_{\widehat{j}} > 1$, $\tau - 1$ obtains a payoff $v - c$ from investigating $\widehat{j}$, $p_0 v - c$ from investigating j with $C_j = 1$ and 0 from not investigating. If there are multiple j with $C_j > 1$, $\tau - 1$ chooses one of them randomly. (This last case is off the equilibrium path.) Thus the hypothesis holds for all t. ■

Given the preceding lemmas, we will now focus on characterizing the equilibrium decisions of agents after observing history $h_t^{'}$ (i.e. with $C_j = 1, \forall j \leq t$). Before the characterization of an equilibrium string, we give some examples for purposes of exposition. Let the total number of agents be N=6. Throughout we follow a common tie-breaking rule: If an agent is indifferent between I and NI, he chooses to investigate.

Example 1 *The equilibrium string is [NI, NI, I_2, I_1, I_3 , I_4] for some parameter values.*

To see whether this can be an equilibrium for some parameter values, we have to check whether all six no-deviation conditions can be satisfied simultaneously. Note that 1 gets future payoff only when 3's investigation of 2 is a failure, which has a probability of $1 - p_0$ *and 4's subsequent investigation of 1 is useful (probability* p_0*). The condition for 1 is irrelevant here since he has no choice effectively. His future payoff is always higher than the current one which is* 0.

$$0 < (1 - p_0)p_0(\delta^3 w + \delta^4 w + \delta^5 w) = (1 - p_0)p_0\delta^3 w(1 + \delta + \delta^2)$$

The condition for 2 however is that the current net payoff be lower than the expected future payoff i.e.

$$v - \frac{c}{p_0} < p_0 \delta w(1 + \delta + \delta^2 + \delta^3)$$

Now if 3 deviates, no citation is observed for 2. Hence no future player would investigate 2, but 4 investigates 1. 3 would be investigated by 5 only if 4's investigation *is not useful(probability $1-p_0$). So, 3's expected future payoff from deviating is $(1-p_0)p_0\delta^2 w(1+\delta) = Q(say)$ while his payoff from investigating is $p_0 v + (1-p_0)Q - c$. the condition needed for 3 not to deviate from I_2 is*

$$v - \frac{c}{p_0} \geq (1-p_0)p_0\delta^2 w(1+\delta)$$

For 4, the condition is

$$v - \frac{c}{p_0} \geq (1-p_0)p_0\delta^2 w$$

while for 5 and 6 it is simply $v - \frac{c}{p_0} > 0$.

So, the parameter values needed to sustain the specified equilibrium string should satisfy

$$L' = (1-p_0)p_0\delta^2 w(1+\delta) \leq v - \frac{c}{p_0} < (1-p_0)p_0\delta^3 w(1+\delta+\delta^2) = H' \qquad (1)$$

We see this is possible for δ high enough.

Example 2 *Now let us consider the equilibrium string [NI, I_1, NI, I_3, I_4, I_5]. We will check if there exists some values of parameters such that no one deviates from this equilibrium.*

Conditions needed for this to be an equilibrium are:

1: $0 < p_0\delta w(1+\delta+\delta^2+\delta^3+\delta^4)$
2: $v - \frac{c}{p_0} \geq 0$
3: $v - \frac{c}{p_0} < p_0\delta w(1+\delta+\delta^2)$
4: $v - \frac{c}{p_0} \geq p_0\delta w(1+\delta)$
5: $v - \frac{c}{p_0} \geq p_0\delta w$
6: $v - \frac{c}{p_0} \geq 0$
Hence the condition needed is

$$L = p_0\delta w(1+\delta) \leq v - \frac{c}{p_0} < p_0\delta w(1+\delta+\delta^2) = H \qquad (2)$$

So we see from these two examples that both the strings can be equilibrium strings depending on whether the values of parameters satisfy the respective conditions. Now we can in fact show that the two ranges of $v - \frac{c}{p_0}$: [L,H] and [L',H'] may not be disjoint.

If they are not, then given that parameters satisfy (1), we cannot be sure that the string is as in Example 1. So, for some parameter ranges, both (1) and (2) might be satisfied and hence there can be multiple equilibria.

Hence we see that there is an irreducible multiplicity of equilibria, pure as well as mixed and we cannot make any precise predictions regarding the pattern of investigations by agents. Note that, crucial to this multiplicity is the behaviour of agents when indifferent between investigating two or more agents. However, if we impose some rules (corresponding perhaps to social norms in the fields concerned-see the next subsection) on how agents behave if they are indifferent, we can obtain partial characterisations of equilibrium behaviour. We now turn to these.

3.1 Behavioural Assumptions

In case some entrant is indifferent investigating among a set of agents, he can choose any one or mix between them. We impose some simple behavioural rules in these cases. Suppose entrant k is indifferent between agents $1, 2, ...k-3$. Some of the simpler rules could be (1) k chooses the earliest i.e. 1 to investigate or (2) k chooses the most recent agent i.e. $k-3$ or (3) he mixes between all of them with equal probability. Hence we focus on two types of pure strategies and one completely mixed strategy. These could be thought of as extreme cases of some regularities observed in practice. Price [20] observed that different subjects can be categorised into two classes: classical or ephemeral. Subjects like Physics and Engineering are *ephemeral* i.e. recent papers tend to be cited more often while Geology, Mathematics are *classical* ; they cite more of the older papers. Some subjects, however, show no clear trend. We take our cue from these observations and analyse the game with these three behavioural assumptions.

Any new entrant:

BA1: If indifferent among r agents, investigates the earliest among them.

BA2: If indifferent among r agents, investigates the most recent agent.

BA3: If indifferent, mixes among all r agents with equal probability.

Now, we focus on each of these at a time and characterise the equilibrium string corresponding to each. The proofs are relegated to the Appendix.

Behavioural Assumption 1: *Any new entrant, if indifferent between r agents, investigates the earliest among them.*

Proposition 1 *In any equilibrium string, for $N > 4$, $\exists K^* \leq \frac{N-4}{2}$ s.t.$\forall k \leq K^*$,the k^{th} entry is NI and $\forall k > K^*$, the k^{th} entry is I. The exact value of K^* depends on the parameter values v, δ, p_0, w.*

Proof. *The proof involves 2 steps. Step 1 shows that number of agents choosing NI is less than that choosing I, in equilibrium. This is because we consider pure strategies i.e. k agents investigating implies exactly k agents are investigated. So, no more than k agents would like to choose NI in order to be investigated. In fact, it can be shown that number of agents choosing NI is less than $\frac{N-4}{2}$, given our assumptions on parameters.*

The next step involves showing that there will be no gaps. Suppose an agent i finds it profitable to choose NI and let his payoff from doing so be U_i. Then the agent j preceding him must also find it profitable to choose NI since $U_j > U_i$. This increase is due to two things: one, i will be investigated later than j conditional on j being not useful and hence the unconditional payoff is lower; second, conditional on being useful, j would get one extra citation than i would potentially get on account of being an earlier entrant. Hence j would also choose NI and so would any agent entering before i.(See Appendix 1 for details) ■

Remark 1 *This implies that there will be no gaps in equilibrium. That is, if, say, agent 5 is the first one to investigate and his investigation is not useful so that agent 6 observes no citations, then in equilibrium, it cannot be that agent 6 chooses NI and writes on his own. K^* is the entrant who first starts investigating in equilibrium. i.e. the first player for whom the expected current benefit outweighs the prospect of future benefits from being cited. This agent is ready to forgo possible payoffs of w from each future citer to get the current benefit v. Players before him, i.e. those who choose to write on their own, do not want to investigate some earlier agent and cite him, since in that case, they would never be cited.*

Remark 2 *Also, $K^* \leq \frac{N-4}{2}$ implies that the total number of agents not investigating is strictly less than those investigating, in equilibrium. The number of citations though would depend on the outcome of those investigations and is bounded above by the* *number of agents investigating. The exact value of agent K^* depends on the parameter values. For given w, p_0, δ the higher the v the higher the incentive for earlier players to Investigate and not wait, since the future payoff relative to v is not high enough. So, higher the v, the lower the K^* i.e. investigations start earlier.*

Behavioural Assumption 2: *Any new entrant, if indifferent between r agents, investigates the most recent among them.*

Proposition 2 *In any equilibrium string, $\exists \overline{K}, s.t. \forall i \leq \overline{K}, \forall j \leq \overline{K}-1$, i: $NI \Rightarrow i+1 : I$ and $j : I \Rightarrow j+1 : NI$ and $\forall i > \overline{K}$, $i : I$ .The value of $\overline{K}$ depends on parameter values and, for fixed w, p_0, δ, is decreasing in v.*

Proof. *First note that two consecutive NI is not an equilibrium since the earlier agent will not be investigated and hence would deviate to I. So, a NI must be followed by a I. Also, two consecutive Is preceded and followed by NIs cannot be an equilibrium either. Suppose $i, i+1$ chooses I and agents $i-1$ and $i+2$ choose NI.In this case, agent i finds it profitable to investigate which implies that $U_i(I) > U_i(NI)$. Now for agent $i+2$, $U_{i+2}(NI) < U_i(NI)$ since there are less number of agents entering after him.Since utility from investigating is same for all player and equal to $v - \frac{c}{p_0}$, given agent i is choosing optimally, agent $i+2$ should deviate since $U_{i+2}(NI) < U_i(NI) < U_i(I)$. Ruling out these patterns leaves the specified pattern as an equilibrium depending on parameter values. (Appendix 1 for details)* ■

Behavioural Assumption 3: *Any new entrant, if indifferent between r agents, investigates them with equal probability $\frac{1}{r}$.*

Proposition 3 *In any equilibrium string, $\exists \widetilde{K} \leq 2, s.t. \forall k < \widetilde{K}$ the k^{th} entry is NI and $\forall k \geq \widetilde{K}$, the k^{th} entry is I . In fact, $\widetilde{K} = 2$.*

Proof. Note that if $i, i+1$ chooses I and $i-1$ chooses NI, then only i mixes (by Lemma 1).

Let the $i_1^{th}, i_2^{th}, ...i_k^{th}, i_{k+1}^{th}, i_{k+2}^{th}, ...N^{th}$ agents be the ones choosing I in equilibrium; $i_1 < i_2 < .. < i_k$.Hence i_k is the agent after which there is no agent choosing NI in equilibrium and i_1 is the first agent to choose I. From the conditions for no unilateral deviation by agents i_k and $i_k - 1$, the parameters should satisfy the following condition:

$$p_0 w\delta(1+\delta+...+\delta^{N-i_k-1}) \leq v - \frac{c}{p_0} < \frac{1}{i_k - i_{k-1}} p_0 w\delta(1+\delta+...\delta^{N-i_k})$$

the necessary condition for this to hold is

$$or, \ (1+\delta+...+\delta^{N-i_k-1})(i_k - i_{k-1}) < (1+\delta+...\delta^{N-i_k})$$

which can hold only if $i_k - i_{k-1} = 1$. Hence everyone after i_{k-1} investigates. We can redefine i_k now and do the same exercise which implies that there can be no gaps in equilibrium.

Next we can show that the number of NIs, in fact, cannot be more than 2. (See Appendix 1 for details). ■

Remark 3 *In this case we see that investigations start very early, as implied by the value of $\widetilde{K}$. Whatever be the value of v relative to the other parameters, only the first entrant waits(since he has no choice) and investigation starts from the second agent. This equilibrium entails the maximum number of investigations and expected citations.The intuition behind the early investigations is that by the strategy of randomising among agents when indifferent, each early entrant, say t faces elimination from the race even when some other agent is investigated. If the investigated agent's idea was useful, then t will obviously get no future citations. But even when the idea is not useful, the new entrant would not investigate t since the probability that t was investigated is positive. This significantly reduces incentives to write on one's own and try to get future citations-payoffs resulting in investigation starting early in the process.*

Now, we can compare these different equilibria in terms of when investigations start. BA3 obviously involves early investigations. We can compare K^* and $\overline{K}$ for given parameter values and this will tell us which type of behaviour entails early investigations and hence early revelation of a high-quality (always useful) paper . We

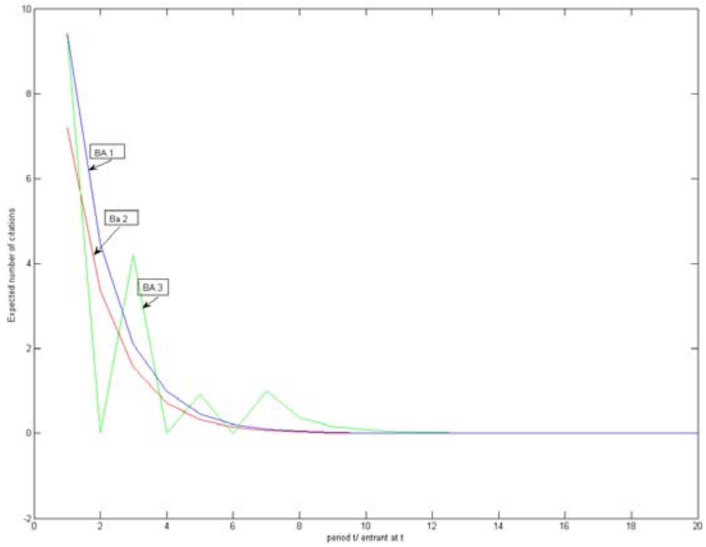

Figure 3: Expected number of citations

can show that for any set of parameter values, BA1 induces earlier revelation of the quality of papers compared to BA2. i.e. $\widetilde{K} \leq K^* < \overline{K}$ for any given parameters.

Proposition 4 *For any given set of parameters,* (v, p_0, w, δ)*,* $\widetilde{K} \leq K^* < \overline{K}$.

Proof. To show this we need to fix K^* at a particular value, say Y. This corresponds to some range of parameter values, set υ, say. Now each $\overline{K}$ corresponds to some set of parameter values, say ω. For a $\overline{K} \leq K^*$, call the set ω_l. We can show that ω_l does not intersect υ. Hence, given $K^* = Y$ i.e. given that parameters lie in υ, they cannot belong to ω_l which implies that $\overline{K} \nleq K^*$.(See Appendix 1 for details). ■

Following the characteristics of the equilibria outlined in this section, we plot the expected discounted number of citations for each entrant under the different behavioural assumptions. The parameter values for Fig 3 are $\delta = 0.99, N = 20, p_0 = 0.5$ and v, w, c such that $K^* = 4, \overline{K} = 7$.

3.2 The infinite horizon model

In the previous section we considered a finite number of agents making decisions of investigating and citing. One natural question that arises is what happens when there is no known bound on the number of agents. In this section, we modify the basic model by considering an infinite horizon game where one agent enters in each period. The rest of the game is as before. After entry an agent observes the state of citations for the existing agents and updates his priors regarding the usefulness of a paper. Then he decides whether or not to investigate one of the existing papers. We will focus on the stationary equilibria of this game where the strategy depends only on the citations observed.

To characterize the equilibrium stationary strategy, one needs to define the history at any time t, h_t. At any time t, there could be two types of histories: (i) $h_t^1 = h_t(C_\tau > 1$, for some $\tau < t)$ or (ii) $h_t^2 = h_t(C_\tau = 1, \forall \tau < t)$, i.e. a history with one or more citations or one with no citations (apart from self-citations). The stationary strategy of agent t can be denoted by $s_t = \{s_t^1, s_t^2\} = \{s(h_t^1), s(h_t^2)\}$. The next proposition characterizes the equilibrium stationary strategy, s^*.

Proposition 5 *The unique stationary equilibrium is (i)* $s^* = \{I, I\}$ *for all* t *if* $\frac{p_0 \delta w}{1-\delta} < v - \frac{c}{p_0}$ *and (ii)* $s^* = \{I, \sigma(\lambda)\}$ *if* $\frac{p_0 \delta w}{1-\delta} > v - \frac{c}{p_0}$ *where* $\sigma(\lambda)$ *denotes the mixed strategy with* λ *being the probability of investigating.*

Proof. The proof proceeds by first characterizing the equilibrium strategy for $h_t = h_t^1$. From the previous analysis it is obvious that $s^*(h_t^1) = I$. Next, note that $s^*(h_t^2) \neq NI$. Suppose it is. Therefore, when agent t observes no citation, then he chooses NI. This in turn implies that $h_{t+1} = h_{t+1}^2$ and since $s^*(h^2) = NI$, agent $t+1$ chooses NI. Hence, agent t has no current or future payoff and is better off deviating to investigating some $\tau < t$ and getting an expected payoff of at least $p_0 v - c > 0$.

The next claim is that $s^*(h^2) = I$ is an equilibrium for some parameter values. Note that the given equilibrium strategy implies that agent 2 investigates 1, 3 investigates 2 at history h^2 and so on. This also implies that at history h_t^2 agent t knows that the investigations of agents 2 through $t-1$ have been unsuccessful and hence the probability of usefulness of agents 1 through $t-2$ is zero. Hence agent t would investigate agent $t-1$. Note that this also implies that in the event that $t+1$ faces history h_{t+2}^2, he will investigate agent t. Therefore, the expected payoff of t from I

can be written as

$$\begin{aligned} E\pi_t(I) &= p_0 v - c + (1-p_0)p_0(\delta w + \delta^2 w + ...) \\ &= p_0 v + (1-p_0)\frac{p_0 \delta w}{1-\delta} - c \end{aligned}$$

If t deviates to NI, he gets

$$\begin{aligned} & p_0(\delta w + \delta^2 w + ...) \\ = & \frac{p_0 \delta w}{1-\delta} \end{aligned}$$

So, if $v - \frac{c}{p_0} > \frac{p_0 \delta w}{1-\delta}$, then agent t chooses I. Hence for this set of parameter values the unique stationary equilibrium is to investigate the immediately preceding entrant with probability 1.

If $v - \frac{c}{p_0} \leq \frac{p_0 \delta w}{1-\delta}$, then the pure strategy $s^* = \{I, I\}$ is not an equilibrium. Let the mixed strategy σ be the following: When $h_t = h_t^2$, investigate the previous entrant with probability λ and choose NI with probability $1-\lambda$. For σ to be an equilibrium it must be the case that

$$\begin{aligned} E\pi_t(I) &= E\pi_t(NI) \\ & \quad or \\ p_0 v - c + (1-p_0)\lambda p_0 w(\delta + \delta^2 + ...) &= \lambda p_0 w(\delta + \delta^2 + ...) \end{aligned}$$

This holds for $\lambda = \frac{1-\delta}{p_0 w \delta}[v - \frac{c}{p_0}] \in (0,1)$ for $v - \frac{c}{p_0} \leq \frac{p_0 \delta w}{1-\delta}$. ■

Thus, each agent would choose to investigate with some probability λ until the first time the investigation is successful, after which everyone would cite the successful paper. Thus there would be a probabilistic "monopoly"with the ex ante probability that the t^{th} entrant is the monopolist being $(1-\lambda p_0)^{t-2}\lambda p_o$.

4 Private Information

In this section we consider the case where each player receives a private signal about the 'quality' of his paper before taking any decision[11]. We represent this signal as an agent-specific probability of success and denote it by p_i for the i^{th} agent with $E(p_i) = p_0$. Let the distribution of p_i be denoted by $F(.)$ and let p_i be i.i.d across agents. Now, consider the basic model with one entrant each period; if useful a paper yields the same value v. The cost of investigation is c, as before. Let r_{t+1} be the probability of agent $i+1$ (entering at $t+1$) investigating any of his predecessor(which can be anything $\in [0,1]$). The i^{th} agent's decision between investigating and not at any time will depend on his type.

At any time t, he can choose one of the two and get the corresponding payoff.

Investigate: $p_0 v + (1-p_0)\delta p_i r_{t+1} W_{t+1} - c$ where $W_{t+1} = (w + \delta w + ... + \delta^{N-i-1} w)$, which represents future payoffs from being cited.

Not Investigate: $\delta p_i r_{t+1} W_{t+1}$

Therefore NI is chosen when

$$p_0 v + (1-p_0)\delta p_i r_{t+1} W_{t+1} - c < \delta p_i r_{t+1} W_{t+1}$$

$$or,\ v - \frac{c}{p_0} < \delta p_i r_{t+1} W_{t+1}$$

$$or,\ p_i > \frac{v - \frac{c}{p_0}}{\delta r_{t+1} W_{t+1}} = p_t^*. \tag{10}$$

Then the probability that i investigates a predecessor is $F(p_t^*) = r_t$.Hence for every period there is some cutoff type p_t^* such that all $p_i < p_t^*$ entering at time t will investigate some predecessor.

Let the cutoff levels of the types for each period be represented by $p^* = (p_1^*, p_2^*, .., p_{N-1}^*, p_N^*)$. This sequence also defines the sequence of $r^{*\prime}s$ by the relation $F(p_t^*) = r_t\ \forall t$.If there are N time periods (or equivalently, N entrants), $r_N = 1$. Likewise, depending on the values of v, w, p_o, δ, c, all entrants from some $k+1 \leq N$ onwards will investigate with

[11] Baliga and Sjostrom ([2]) design a mechanism for self-assessment and peer review (in a different context) based on a similar assumption.

probability 1, and k is the last time period for which $r_k < 1$. The rest of the sequence is defined recursively by $\frac{v-\frac{c}{p_0}}{\delta F(p^*_{t+1})W_{t+1}} = p^*_t$.

Now given this sequence we can argue that in equilibrium, if an agent i investigates a predecessor, it has to be $i-1$ whom he investigates. This is a direct consequence of Lemma 1. Conditional on observing no citations, the probability that $i-1$ is of type p_i is $F(p_i)$, so that ex-ante probability of $i-1$ being useful is $E(p_i) = p_0$. Some agent $j < i-1$, on the other hand has been investigated by some agent $p_k < p^*_j$ which occurs with positive probability. Hence by Lemma 1, the probability of agent j being useful $< p_0$. So, if i investigates at all, he will investigate $i-1$. This gives some some justification for the behavioural assumption 2 that we imposed in the main analysis. (Unfortunately, this was the least efficient one.)

One odd feature of this setup is that the sequence of cutoffs alternates in size-a high probability of citing next period leads to a lower probability today, other things being equal, and vice versa.

5 Private Information with Strategic Entry

In this section we further extend the model to include entry as a strategic decision. If agents know their type and are free to choose when to enter, then we might observe some sorting regarding timing of entry. Each agent receives a signal about his type p_i before the start of the game. He, agent i, has two decisions to make at every time period t; to 'enter , E' or 'Wait one period, W'. If he chooses E, he then chooses to I(nvestigate) or NI, as before. If he chooses W, then at time $t+1$, he again has the same choices. All agents make these choices simultaneously. So, the decisions are functions of their types (and of course, the history at any time t) only.

Let the distribution of p_i be i.i.d uniform [0,1].[12]Let T be the fixed number of periods and $T > 2$.

Proposition 6 *Let N, w, v be such that $(N-1)w > v$.Then there exists an equilibrium described by the cutoffs α_j, $1 > \alpha_1 > \alpha_2 > ...\alpha_{T-1} > 0$ with $\alpha_t = \alpha_1^t$ such that:*

[12]This is without much loss of generality and saves on notation.

1. *If $p_i \geq \alpha_1$, Player i enters in period 1. If there is at least one entrant in period 1, all other players enter in period 2.*

2. *If there are no entrants for periods t=1,2,..$\tau < T-1$, Player i enters if $p_i \geq \alpha_{\tau+1}$.If there is at least one entrant in any period τ, all other players enter in the following period.*

3. *The players who (simultaneously) enter first do not investigate. Those who enter in the following period investigate.*

4. *If the first entry occurs at period τ, any player who has not entered by period $\tau+1$ does so in $\tau+2$.*

5. *In period T, everyone who has not entered, enters.*

Proof. Note first that the histories in this game are characterised by the identities of the players who enter in each period. The *state* of the game is given by $(a_t, k_\tau, \tau \leq t)$ where the distribution of p_i after period t is uniform $[0, a_t]$ and k_τ is the number who have entered at period τ. We shall limit ourselves to strategies that depend only on the state and not on the identities of the players who have entered at different periods.The only out-of-equilibrium moves we need to consider are given by point 4 above. The effect of such moves on beliefs is irrelevant for the equilibrium strategies.

Suppose no player has entered by period τ. Then the belief $a_\tau = \alpha_\tau$. The probability that any player will enter is then $\gamma_{\tau+1} = \frac{\alpha_\tau - \alpha_{\tau+1}}{\alpha_\tau}$.[13] Let $m_{\tau+1} := 1 - \gamma_{\tau+1}$. Also, let $\widehat{p}_{\tau+1}$ be the highest expected probability (entrants at different times might have different probabilities of being useful) at time $\tau+1$ that any of the earlier entrants is useful. Consider Player i in period τ, conditional on no previous entry. If he enters and k others out of $N-1$ enter then his expected payoff conditional on k is given by:

$$p_i \delta \frac{N-k-1}{k+1} w. \tag{11}$$

Here we are assuming each of the $k+1$ initial entrants is investigated by the others in the following period with equal probability (a version of BA3 but possibly the only reasonable assumption here). If Player i is found useful by anyone of these others

[13] If $\tau = 0$, i.e. we are referring to the first period, we adopt the convention that $\alpha_0 = 1$.

who investigates him, he will be found useful by all the others who also choose him to investigate. This expression (11) is decreasing in k.

The unconditional expected payoff is therefore:

$$p_i \delta w E_k(\frac{N-k-1}{k+1} \mid m_{\tau+1}).$$

Since the term to the right of the expectation operator is decreasing in k, the expectation is decreasing in the probability of entry (by first-order stochastic dominance) and therefore increasing in $m_{\tau+1}$. If Player i chooses to wait, his expected payoff, by the equilibrium strategies, is

$$(m_{\tau+1})^{N-1} p_i \delta^2 w E_k(\frac{N-k-1}{k+1} \mid m_{\tau+2}) + (1-m_{\tau+1}^{N-1})\delta(\widehat{p}_{\tau+1} v - c), \tag{12}$$

These last two expressions are equal for $p_i = \alpha_{\tau+1}$.Also, by Bayes' Theorem, $m_{t+1} = \frac{\alpha_{t+1}}{\alpha_t}$. The conditional probability $\widehat{p}_{\tau+1} = \frac{\alpha_t + \alpha_{t+1}}{2}$. The cutoff $\alpha_{\tau+1}$ is defined by the following equality:

$$\alpha_{\tau+1} \delta w E_k(\frac{N-k-1}{k+1} \quad \mid \quad m_{\tau+1}) = (m_{\tau+1})^{N-1} \alpha_{\tau+1} \delta^2 w E_k(\frac{N-k-1}{k+1} \mid m_{\tau+2})$$
$$+(1-m_{\tau+1}^{N-1})\delta(\widehat{p}_{\tau+1} v - c)$$

If $\tau = T-1$, i.e. the current period is the last, $m_T = 0$. (All remaining players enter and everyone gets 0.) Suppose m_t is defined for $t = T, T-1, ...\tau+2$ and suppose the belief in period $\tau+1$ is that p_i is uniformly distributed between $[0,a_\tau]$. We now show there exists a $\alpha_{\tau+1} \epsilon (0,1)$ such that a player i will enter if and only if $p_i \epsilon [\alpha_{\tau+1}, a_\tau]$.

Suppose, in the above equation, $\alpha_{\tau+1} = 0$. Then $m_{\tau+1} = \frac{0}{a_\tau} = 0$. Also, $\widehat{p}_{\tau+1} = \frac{a_\tau}{2}$. The left-hand side of the expression above is 0 and the right-hand side is positive, so the LHS<RHS. Now put $\alpha_{\tau+1} = a_\tau$. Now the LHS is greater than the RHS, ($\delta < 1$ even if $m_{\tau+2} = 1$ and stochastic dominance give us this inequality). But both the LHS and the RHS are continuous in $\alpha_{\tau+1}$. Therefore the LHS=the RHS for some interior value of $\alpha_{\tau+1}$.It is clear that if $p_i = \alpha_{\tau+1}$ is indifferent between entering and waiting, every $p_i > \alpha_{\tau+1}$ will enter

Therefore, in equilibrium, $a_t = \alpha_t$ for all t.[14] ■

Example 3 *Let the distribution of p_i be i.i.d uniform [0,1]. Let T be infinite.Also, let N, w, v be such that $(N-1)w > v$, and let $c = 0$.Then there exists an equilibrium described by the cutoffs α_j, $1 > \alpha_1 > \alpha_2 > ...\alpha_{T-1} > 0$ with $\alpha_t = \alpha_1^t$, which satisfies the conditions of the previous proposition.*[15]

Proof. This proceeds in the same way as the proof of the proposition except we are able to show that m_t is a constant if $c = 0$. ■

Now, we argue that non-monotonic equilibria are not possible.

Proposition 7 *There cannot be an equilibrium in which there exist α and $\alpha', \alpha < \alpha'$(say), such that all players with $p_i > \alpha'$ and some with $p_i < \alpha$ enter in period 1, while players with $\alpha \leq p_i \leq \alpha'$ enter in period 2 with other players entering after period 2.*[16]

Proof. See Appendix 2. ■

Note that we have discussed $t = 1, 2$ and $t > 2$. There could be equilibria in which no one moves before some $\tau > 1$. This could be sustained by a belief that anyone who enters before period τ has $p_i = 0$ with probability 1. This seems an unreasonable belief in that earlier entrants should have higher probabilities of usefulness, since by being early entrants they are giving up the advantages of using other people's work. We assume therefore that players who enter earlier than τ have probability 1 of being useful. This destroys equilibria with delay driven by beliefs.

All this is still not sufficient to claim uniqueness because the proof of the existence of the α_t did not claim the sequence was unique.

This result is similar to some results in the endogenous timing literature. Brian Rogers (2005) and Jianbo Zhang (1997) also find that in an environment with private signals about a state of the world, when actions and timing of actions of agents are made endogenous, the agent with the most precise signals acts first and all other

[14] We are not claiming the sequence α_t is unique.

[15] That is, once someone enters in a period, everyone else enters in the next period. The infinite horizon is needed to keep m_t constant.

[16] Using period 1 in the statement is without loss of generality-we can replace it by "period τ such that there has been no entry up to $\tau - 1$."

agents mimic his actions immediately. Thus there is an information cascade (with possible initial delay). The information structure of their models differs from ours. Apart from there being no imperfect monitoring in their model, the main difference is that there is no competition, in the papers of Jianbo Zhang and Brian Rogers, among agents entering at the same time . However, even with different setups and information structures, the results have a qualitative resemblance.

6 Citations in a Random Social Network

In this section, we consider how the citation network interacts with the structure of social acquaintance. One obvious way in which social connections influence citations is that it is easier to learn of the existence of a paper through one's colleagues and friends. This might account for the frequency with which some colleagues cite each other, though there might be other issues involved there as well.

Whilst this is certainly less important now than it was in the past because of the easy accessibility of new work on the internet, it still plays a major role in pointing us to papers that reduce our search costs. Of course, another way would be to consider people who write in a given field and check whether a particular person (a "star") has worked on the specific topic, even if he or she is not an acquaintance. We shall briefly consider a "star" network later.

Here the graph of social links is assumed to be random. The agent has an existing social network in which each of the $N-1$ possible links is open with probability q and edges are open and closed independently of each other. The probability that the agent is completely isolated is therefore $(1-q)^{N-1} = 1-\rho$, say.[17]. The process then continues as follows:

1. Each period, one randomly chosen agent enters. (That is, any given agent has a probability $\frac{1}{N}$ of entering at any position in the order.)

2. Agent k upon entering realises his type p_k which is the probability that k is useful. One of the social links to the $N-1$ other agents is then activated randomly. Since each link is just as likely to be activated and just as likely to

[17] Recall that in Price's work, 1% of the entering agents were completely isolated.

be open as any other, the probability that any given other agent will be chosen is $\frac{1}{N-1}$.

3. Agent k observes the agent he is linked to has or has not entered; if agent k is linked to agent j and agent j has cited some j', then k observes this and any citations j might have received.

4. Agent k then decides whether or not to investigate j or j', just as in the basic model, if j has entered.

5. If investigation of j is successful, a citation for j results. Agents get their payoffs just as in the basic model and the game reaches the next period.

There are therefore four possible states of information for k. He is completely isolated (h_0), linked to j who has not entered (h_{ne}), linked to j who has entered and has not cited anyone (h_j) or has cited j' ($h_{jj'}$). The number of citations that j has received is also observable and is denoted by C_j. A strategy for k would specify whether to investigate and whether to cite if found useful for each history. Note that k can infer something about his relative position in the order of entry based on the state of information for the last three states (h_{ne} shifts the probability attached to possible entry times towards earlier periods and h_j and $h_{jj'}$ with $C_j > 1$ towards later ones). The later the entry the lower the possible future benefits from being cited, so the incentive to hold out is lowest in the last two types of states.

Since we have two kinds of networks here, let us denote the two by g and g^c. The social network is denoted by g and the directed graph of citations is g^c. Note that g is formed randomly while g^c is formed by strategic decisions, which depend on the subgraph of g at each period. We say $ij \in g$, if they are linked socially and $ij \in g^c$ if i has cited j.[18]

Now we analyze $i's$ decision to investigate or not.

Suppose agent k enters at period k and the network at that time is given by the pair (g, g_c^k).Suppose, the number of agents who have entered is $k-1$. Agent $i's$ type is given by p_i which is distributed with cdf $F(.)$ with $E(p_i) = p_0$.[19] We consider the

[18] Note that g^c is a directed graph and hence, $ij \in g^c$ is not the same as $ji \in g^c$.
[19] The distribution is absolutely continuous.

different cases that might arise. Let $W^k(C_j)$ denote the expected future payoff of the $k^{th} - period$ entrant, as calculated by the entrant (conditional on his being useful for sure) when his (social network) neighbour j has C_j citations,

Case I (occurs with probability $1-\rho$): i remains isolated. In this case, i has no option but to write on his own.

Case II: Agent k is connected to j, s.t. $C_j = 1$ and $\nexists \ell, s.t. j\ell \in g_c^t$ (i.e. j has not cited anyone). Here, k has two options:

$Investigate$ j and get $p_0 v - c + (1-p_0)p_i W_0^k$

$Not\ Investigate$ and get $p_i W_0^k$ where $W_0^k = W^k(C_j = 1)$

So, i will investigate if $\quad p_0 v - c + (1-p_0)p_i W_0^k \geq p_i W_0^k \quad or,\ p_i \leq \frac{v - \frac{c}{p_0}}{W_0^k}$

The probability that the new entrant investigates is denoted by $r = F(\frac{v-\frac{c}{p_0}}{W_0^k})$

So, the probability that j (with $C_j = 1$) gets a citation is $\quad \frac{\rho}{N-1} F(\frac{v-\frac{c}{p_0}}{W_0^k})p_j$

Case III: Agent i connects to j with $C_j > 1$ and/or $\exists \ell$ s.t. $j\ell \in g_c^t$.

First note that, if $\exists \ell$ s.t. $j\ell \in g_c^t$, then ℓ must be the earlier entrant. So, the first agent $\widehat{j}$ connecting to j observes ℓ with $C_\ell > 1$ and $C_j = 1$. Agent $\widehat{j}'s$ decision is to

i) Investigate $j : p_0 v - c + (1-p_0)p_{\widehat{j}} W_0^\tau$

ii) Investigate $\ell : v - c$

iii) Not Investigate: $p_{\widehat{j}} W_0^{\widehat{j}}$

So, if $\widehat{j}$ does investigate, he will investigate agent ℓ and not j.(To see this, note that if $p_0 v - c + (1-p_0)p_{\widehat{j}} W_0^{\widehat{j}} \geq p_{\widehat{j}} W_0^{\widehat{j}}$, i.e.$v - \frac{c}{p_0} > p_{\widehat{j}} W_0^{\widehat{j}}$, then it is true that $v - c > v - \frac{c}{p_0} > p_{\widehat{j}} W_0^{\widehat{j}}$). This implies that $C_j > 1$ and $\exists k$ s.t. $j\ell \in g_c^t$ cannot hold together in equilibrium. Either $C_j > 1$ **or** $\exists \ell$ s.t. $j\ell \in g_c^t$ but not both. If $j\ell \in g_c^t$ for some ℓ, then the probability of j getting a citation $= 0$.[20] We look at the case where $C_j > 1$. In such a situation, the new entrant i has two options again with the payoffs as follows:.

$Investigate$ j: $v - c$

$Not\ Investigate : p_i W^k(C_j > 1)$

So, the new entrant will investigate if $p_i \leq \frac{v-c}{W^k(C_j>1)}$, which implies that the

[20] Of course this could occur off the equilibrium path. In this case, the new entrant believes that all those who have cited J made mistakes and cites the person j has cited. (This is an assumption on beliefs but a natural one.)

probability of getting another citation for j, given $C_j > 1$ is

$$\Pr(Citation_j \mid k) = \frac{\rho}{N-1} F(\frac{v-c}{W^k(C_j > 1)}) \tag{A}$$

Note that W^k is k's future payoff if he does not investigate any agent. Agent k cannot observe the whole graph (g, g_c) but only the one he is connected to and anyone this person has cited. So, k has some expectation of the period of his entry, which determines W^k. Observation of a higher C_j implies that more people have entered and hence k has entered relatively late. This in turn implies that the W^k is low. Note that Player $k+\tau$, $\tau \geq 1$,if she links to k and no one else has cited k, will be in information state h_j and will not assign a high probability to being late in the game. Therefore $k+\tau$ will cite with a relatively low probability independent of the value of C_j. Therefore C_j does not affect the probability of citation for k but does affect the expected number of citations, conditional on being cited. So, a higher C_j implies a lower $W^k(C_j)$.[21] From expression A, we see that this implies a higher $r = F(\frac{v-c}{W^k(C_j)})$ and a higher $\Pr(\text{Citation}_j)$.

However, this is the probability of citation if k links directly with j. If k links to someone who has linked to j, the probability k cites j is independent of C_j and is lower. Note therefore that the probability of j being cited with $C_j > 1$ earlier citations is proportional to $C_j F(.) < C_j$. We write this as a proposition.

Proposition 8 *Suppose $C_j(t)$ denotes the number of citations j has received at time t and $C_j(t) > 1$. Then*

$$\begin{aligned} C_j(t+1) &= C_j(t) + 1 \textit{ with probability } \varphi(C_j, t) \\ &= C_j(t) \textit{ otherwise.} \end{aligned}$$

Here

$$\varphi(C_j, t) = \frac{\rho}{N-1}[F(\frac{v-c}{W^t(C_j > 1)}) + (C_j - 1)F(\frac{v-c}{W_0^t})]$$

Proof. See preceding discussion. ■

[21] Player i has a link with a player who has cited Player j who has C_j citations. Therefore Player i has been preceded by at least $C_j + 1$ agents.

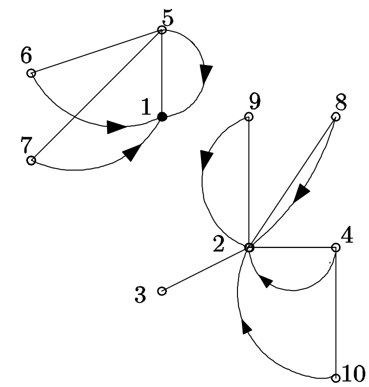

Figure 4: Co-evolution of citation and social networks

Remark 4 *The probability of an additional citation is therefore increasing in the number of citations as in the preferential attachment models, but unlike these models is not directly proportional to the number of existing citations. The probability has two factors, one arising from the social network and the other from the strategic/competitive motives of the players. The sublinear nature of the dynamic does not give a power law (as Chung et al have shown).*

We have earlier referred to a star network. Suppose the network consists of "stars", who might be connected to each other, and "planets", who revolve around particular stars. In this case, if a star enters early, his paper will be likely to receive wide dissemination. However, an idea generated from a planet can only diffuse if a star decides to cite it. This can only happen if a star enters relatively late, so has no incentive to seek her own citations. Thus ideas generated from the peripheries take an inefficiently long period of time to diffuse.

We now give an example where the strategic aspect of network formation is absent, so as to give a flavour of the effect of the acquaintance network on citations.

Example 4 *Let N=10. Say, 1 and 2 entered sequentially but no social link was formed. So,* $C_1 = C_2 = 1$ *at period 3. 3 enters and forms a link with 2, say. Suppose 3 chooses NI. So, after period 3 the network is:* $<$*1(1) 2(1)–3(1)* $>$

The numbers in the parentheses are the citations for each agent. Now 4 enters. Suppose he links to 2 and investigates 2. Also suppose it is useful. Then the updated network is

<1(1) 2(2)—3(1) 4(1) >.Now say 5 links to 1 and investigates and cites 1. 6 and 7 link to 5 and he can observe $C_1 > 1$ and hence cites 1 again. Hence at the end of period 7, 1 has 3 citations and is linked to 5,6,7 in g or g^c. Say, 8,9, and 10 all link to 2 or 4. In either case they can observe $C_2 > 1$ and hence would cite 2. So, the final graph $< g^c(g) >$ we end up with looks like figure 4. We could have the same g^c with multiple g's. Figure 4 includes only one of the possible g which is consistent with the g^c.

7 Private information and heterogeneous quality

In this subsection we discuss heterogeneity in the qualities of papers with private information. We explore whether greater heterogeneity in the quality of papers will speed up or delay the revelation of information about quality. An individual's paper might be of quality 0 (not useful for related papers) with probability $1-p_i$;useful with a value $\underline{v}$, with probability $p_i(1-q)$, or with value $\overline{v}$, with probability p_iq.We still maintain that $h = 1, l = 0$ i.e. once useful(non-useful), a paper is always useful(non-useful).The quantity p_i is, as in the preceding section, private information for Player i and is drawn independently for each i from a commonly known, absolutely continuous distribution on [0,1], with $E(p_i) = p_0$. We assume

$i)$ $\underline{v} > w$

$ii) v = q\overline{v} + (1-q)\underline{v}$

$iii) p_0 v > c$ (from the basic model)

Therefore the prior probability of a paper yielding $\overline{v}$ is p_0q and that of it yielding $\underline{v}$ is $p_0(1-q)$; while the probability of a paper not being useful or of value 0 is $(1-p_0)$ as before. The difference comes from the fact that if both $\overline{v}$ and $\underline{v}$ types are cited if used, a citation does not partition the two useful types though it does separate them from a non-useful paper. So, there is partial information revelation, i.e. citations are not perfect signals any more. In this setup, an agent might choose to incur cost c and investigate a paper if the ex-ante net payoff from doing so is high enough. After reading, the information is revealed and depending on the quality, he might or might not cite that paper. We consider N entrants in a fixed order.

We now introduce some notation prior to outlining the result. Let r_k be the probability that Player k will investigate one of his predecessors. (For $k = N$, the probability is 1). We consider a $k < N$, again such that there have been no citations upto $k's$ entry. If Player k does not investigate or investigates and does not cite, her expected value to others will be updated, since the players with higher values of p_k will be more likely not to investigate. (If $p_k = 1$, the player knows his paper is useful and therefore has a high expected future payoff from being cited). Suppose her probability of being investigated is r_{k+1}[22] and her expected payoff, if cited is W_{k+1}. Note that if Player k is not found useful, she is not cited, but if she is found useful, she is cited with probability r'_{k+1} if her value is $\overline{v}$ and with probability r''_{k+1} if her value is $\underline{v}$, where $r'_{k+1} \geq r''_{k+1}$.If cited once, her expected value to future entrants is at least $q\overline{v} + (1-q)\underline{v}$, which is always greater than the expected value of someone who has never been cited. However, if she is cited again, her expected future value increases and if she is not cited it decreases. This changes the investigation decision for future entrants. This is all encapsulated in W_{k+1}. Thus r_k, W_k are well-defined (by backward induction) for all k.

Proposition 9 *Suppose $(\overline{v} - \underline{v})p_0q > c$, where c is the cost of investigation. Then, for every k, given no citations before k, there will exist cutoff values, $\alpha_1 > \alpha_2 > \alpha_3$* [23]*, such that Player k will not cite if $p_k \geq \alpha_1$, will not investigate if $p_k \geq \alpha_2$, will investigate and cite only $\overline{v}$ if $p_k \epsilon [\alpha_3, \alpha_2]$ and will investigate and cite both $\overline{v}, \underline{v}$ if $p_k \epsilon [0, \alpha_3]$.*

Proof. Given investigation a player with private information p_k will not cite $\overline{v}$, if $p_k r_{k+1} \delta W_{k+1} \geq \overline{v}$.This gives $\alpha_1 = \frac{\overline{v}}{r_{k+1}\delta W_{k+1}}$. Clearly, someone who is not going to cite even $\overline{v}$ would never investigate. Similarly, if $p_k \epsilon [\frac{\underline{v}}{r_{k+1}\delta W_{k+1}}, \frac{\overline{v}}{r_{k+1}\delta W_{k+1}}]$, Player k would cite only $\overline{v}$.For lower values of p_k, she would cite both positive values. Let $\frac{\underline{v}}{r_{k+1}\delta W_{k+1}} = \alpha_3$.

Consider now the player who would, if she investigates and finds a positive value, cite only $\overline{v}$. Her choice would be not to investigate if $p_k r_{k+1} \delta W_{k+1} \geq p_0 q \overline{v} + (1 - p_0 q) p_k r_{k+1} \delta W_{k+1} - c$,or , $p_k \geq \frac{\overline{v}}{r_{k+1}\delta W_{k+1}} - \frac{c}{r_{k+1}\delta W_{k+1} p_0 q} = \alpha_2 < \alpha_1$.For $\alpha_2 > \alpha_3$,$(\overline{v} - \underline{v})p_0 q > c$ must be satisfied (and conversely). We now check the investigation decision

[22] Once again, the probability of citation will be shown to be positive in every period, so lemma 1 will in fact ensure that only the most recent paper is investigated, in the absence of a citation.

[23] These cutoffs depend on k; this dependence is suppressed for notational convenience.

for a player who would cite both $\overline{v}$ and $\underline{v}$, if he investigates. Such a person would not investigate if $p_k \geq \frac{q\overline{v}+(1-q)\underline{v}}{r_{k+1}\delta W_{k+1}} - \frac{c}{r_{k+1}\delta W_{k+1}p_0}$.

We check the difference between the right-hand side of the last expression and α_3.This difference is $\frac{q\overline{v}+(1-q)\underline{v}}{r_{k+1}\delta W_{k+1}} - \frac{c}{r_{k+1}\delta W_{k+1}p_0} - \frac{\underline{v}}{r_{k+1}\delta W_{k+1}}$

$= \frac{1}{p_0 r_{k+1}\delta W_{k+1}}(p_0 q\overline{v} + p_0\underline{v} - p_0 q\underline{v} - p_0\underline{v} - c)$

$= \frac{1}{p_0 r_{k+1}\delta W_{k+1}}(p_0 q(\overline{v} - \underline{v}) - c) > 0.$

This implies that the player with $p_k \leq \alpha_3$ would always investigate, because as p_k rises, she would shift first to citing only $\overline{v}$ (after investigating) before choosing not to investigate. ■

Note that, by lemma 1, in equilibrium, Player k, if he cites, will cite $k-1$ if no previous papers have citations other than self-citations. A paper that has received a citation will be chosen by any future entrant who wishes to investigate (and the cutoffs in the proposition will change to reflect the new expected value, obtained by Bayesian updating). However, someone investigating who cites only $\overline{v}$ papers might discover the cited paper is $\underline{v}$ and not cite it. Every non-citation will decrease the expected value of the paper and it is possible this will go below the prior, in which case the most recent paper will again start to be investigated. Thus it is possible that several papers will obtain citations and then die out and be replaced by others. As a cited paper adds citations, it will, of course, become more popular. As it accumulates non-citations, the entrants who would wish to cite only $\overline{v}$ papers might switch more to not investigate, so the information content of more non-citations would diminish. This also depends of course on how close to the end of the game the field is, because every type of agent has an incentive to investigate at the end of the game. We can therefore conclude that, with heterogeneous quality, (i) a higher quality paper has a higher probability of being cited and a higher expected number of citations; (ii) with positive probability a lower quality paper will be cited first and obtain citations, while a higher quality paper from a later entrant "dies"; (iii) some papers might enjoy a vogue and then be replaced by other more recent ones.

8 Efficiency

This paper is an attempt to model observed patterns of citation as a result of strategic choice by rational agents and its implication on diffusion of knowledge. The irreducible multiplicity of equilibria makes determinate predictions difficult. But we can rank these equilibria with respect to a certain notion of ex-ante efficiency. Efficiency here relates to the idea that the earlier the investigations start, the earlier the information about the quality of paper is revealed, probabilistically. Hence potentially, the benefits of a good paper are available earlier. Consider a social planner who wants to maximise the sum of expected payoffs of all N agents. Say, the planner specifes that in states with no citations, agents 1,2,...$i-1$ would not investigate and all agents $i, i+1, ...N$ investigate. Also, whenever a paper gets a citation, it gets cited by all entrants thereafter. This allocation of decisions entails a payoff of $v+w-c$ for each citation ($v-c$ to the one citing, w to the one cited). If agent i starts investigating, agents 1,2,...$i-1$ get no benefit nor do they incur any cost. So, the sum of expected payoffs of N agents is

$$\begin{aligned} U &= \Pr(i's \text{ investigation is a success})[(v+w)(N-i-1)] \\ &\quad + \Pr(i's \text{ investigation failure}) \Pr(i+1's \text{ investigation success})[(v+w)(N-i-2)] \\ &\quad + ... \\ &\quad + \Pr(\text{investigations of } i, i+1, ..N-1 \text{ failures}) \Pr(N's \text{ investigationn success})[v+w] \\ &\quad -(N-i-1)c \end{aligned}$$

The expression is strictly decreasing in i, for small c (since the last term involves the term $+ic$). So, a social planner would set $i=2$ to maximize U. Hence we see that earlier investigations entail higher aggregate payoffs. The equilibrium in Proposition 3 , therefore, is efficient, both ex-ante and ex-post. In fact, when citations are perfect signals, as in the basic model, there is no difference between ex-ante and ex-post efficiency.

With multiple qualities of papers, the ex-ante probability that the better paper is cited and known is higher than the probability of the worse paper being known.[24]

[24] Compare this with the David-Simkin-Roychowdhury explanation, where there is no expected

Note that existence of equilibria mentioned in section 4(with agent i choosing to cite both types and $i+1$ choosing to cite only the high type) implies that ex-post, it might be the case that the low-quality paper gets citations before a high quality one and potentially better papers do not get known. So, there are equilibria that are ex-post inefficient.

9 Conclusion

We have looked at a specific stylised model on the effect of rivalry on the diffusion of useful ideas. Whilst we have focused on academic citations, the model can be interpreted without too much difficulty as one of firms engaged in R&D deciding whether to use existing patents or to work around them.

Our basic findings are: (i) In a complete information model, the rule by which a new entrant chooses to cite the work of earlier entrants among whom she is indifferent determines the equilibrium. The most efficient case for dissemination of ideas is the rule by which the new entrant chooses randomly. The structure of the equilibrium often, but not always, has a cutoff entrant such that all who enter earlier decide not to investigate earlier work and those who enter later do. (ii) WIth private information and entrants deciding when to enter, the equilibrium structure is monotonic in that players who believe their own ideas to be relatively good enter early and there is then a cascade, similar to information cascades in the literature. (iii) With citations superimposed on a simple social network (so that individuals find out about other earlier work by direct acquaintance or simple word of mouth), the dynamic of citations is shown to follow sublinear preferential attachment. (iv) In no case, do we get a "power law". We get either a monopoly or, with sublinear preferential attachment, something involving the product of a power law with some other factor.

Our findings can also be related to the literature on diffusion of technology and social norms, which point out that "local externalities" like conformity can be a possible obstacle to the spread of optimal technology. Papers by Munshi [19] and Banerjee-Duflo [3] deal with specific externalities. In our model, it is the rivalry or

difference in quality between highly cited and less cited papers, a somewhat counter-intuitive conclusion that would probably have some academic administrators worried.

competition among agents that becomes the hindrance to speedy diffusion. This is in contrast to existing literature where agents are usually assumed to be non-strategic. (Diffusion in the case of partially rational players who do not compete with each other is addressed in a different context by [1] and for non-rational players by [7].)

We can place our work in the network literature (citations being directed links), but the flavour is different from many papers in that literature, since we do not rely solely on exogenous randomness or on built-in network externalities. In the model without private information, our result might be considered too extreme in that there is one randomly chosen centre in a star network. In order to match the data, we need to include other factors, which contribute to the decision of citation. In our paper, private information about quality contributes substantially in matching model results with the qualitative features of the data. Additional considerations arising from repeated interactions and asymmetries in initial social connectedness among agents might induce completely different strategic considerations. For example, the presence of cliques or clusters in citation networks suggest that in a repeated game framework, (some) citations might occur in the hope of getting favours returned. In fact, it is most likely that both the competition (discussed in our paper) and co-operation effects work together to determine actual citation networks.

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Appendix A Proofs of propositions, Section 3.1

Behavioural Assumption 1: *Any new entrant, if indifferent between r agents, investigates the earliest among them.*

Proposition 1: *In any equilibrium string, $\exists K^* \leq \frac{N-4}{2}$ s.t.$\forall k \leq K^*$,the k^{th} entry is NI and $\forall k > K^*$, the k^{th} entry is I. The exact value of K^* depends on the parameter values v, δ, p_0, w.*

Proof. We will prove this in three steps. First, let us number the agents from the end,i.e. the last agent is number 1. Let, i be the position(from the end) of the first entry of NI in a string i.e. no agent $j < i$ chooses NI. Also, let there be k_I entries of I and k_{NI} entries of NI after $i \Rightarrow k_I + k_{NI} = N - i$.

Step 1: We show that number of entries NI $\leq$ number of I's in an equilibrium string. Note that k_I agents among $N - i$ are investigating. Imposition of A1 restricts us to pure strategies$\Rightarrow$ k_I agents among $N - i$ are investigated. $\Rightarrow$ k_{NI} agents are not investigated. Now consider the decision of i. He knows that k_{NI} agents are not yet investigated. Given A1, this implies that the next k_{NI} agents will not choose to read i. He can only hope to get investigated by the agent numbered $(i - 1 - k_{NI})$, that too, conditional on the fact that all investigations done by agents $i - 1$ to $i - k_{NI}$ are failures. Now by definition of i, he is the first(from end) to choose NI. i.e. agent $i - 1, i - 2, ..., 2, 1$ all chose I. The no-deviation condition for i implies that v must be lower than his expected future payoff(A). Expected future payoff from NI =

$$A = (1 - p_0)^{k_{NI}} p_0(\delta^{k_{NI}+1} w + \delta^{k_{NI}+2} w + ... + \delta^{i-1} w) \tag{1'}$$

Payoff from deviating to I =

$$p_0 v + (1 - p_0)A \quad - c \tag{2'}$$

No profitable deviation from NI requires

$$v - \frac{c}{p_0} < A = (1 - p_0)^{k_{NI}} p_0(\delta^{k_{NI}+1} w + \delta^{k_{NI}+2} w + ... + \delta^{i-1} w) \tag{3'}$$

The first term in A is for the condition that investigation by agents $i-1, ...1-k_{NI}$ are failures so that i is investigated, which happens with probability $(1-p_0)^{k_{NI}}$. Recall that the quantity p_0 is the probability that i is found useful. Also, $\delta^{k_{NI}}(\delta w + ... + \delta^{i-1-k_{NI}}w)$ is the discounted sum of payoffs if found useful. Now, note that if there are fewer than $k_{NI}+1$ agents following i, then i is never investigated and hence i deviates from NI. But by definition i is the first to choose NI in equilibrium. This implies

$$i - 1 \geq k_{NI} + 1 \tag{4'}$$

Total number of agents $= N = (N-i)+i = (k_I + k_{NI}) + 1 + (i-1) = (k_{NI}+1) + (k_I + i - 1)$. Therefore the number of agents choosing NI$= k_{NI}$ before i and i himself $= k_{NI}+1$ and the number of agents choosing I=$k_I + i - 1 \geq i - 1 \geq k_{NI}+1$ (by equation 4'). Hence in any equilibrium string, No. of I entries $\geq$ No. of NI entries.

Step 2: Now, we will prove that in fact, $i \geq k_{NI}+4$. The total number of NI$=$ $k_{NI}+1 = k, say$.The total number of I$=$ $N-k$ of which k are at the end. From equation (4') , $i \geq k_{NI}+2 = k+1$. Let $i = k+1$. He hopes to get investigated only by the last agent . His expected future payoff from NI is $(1-p_0)^{k-1}p_0\delta^k w < w < v - \frac{c}{p_0}$ i.e. i cannot choose NI which implies the $k+1^{th}$ entry in the equilibrium string is a I .

Now, let $i = k+2$. The next $k-1$ out of $k+1$ agents would not choose i due to A1. They would choose to investigate some agent $j > i$.Therefore i can get investigated by the second last agent. Hence, his expected payoff is $(1-p_0)^{k-1}p_0\delta^k w\,(1+\delta) = A'$.For $i = k+2$, we need $A' > v - \frac{c}{p_0}, or\;\; (1-p_0)^{k-1}p_0\delta^k w\,(1+\delta) > v - \frac{c}{p_0} > w$. So, the necessary condition for such a case to exist is $(1-p_0)^{k-1}p_0\delta^k w\,(1+\delta) > w$ or $(1-p_0)^{k-1}p_0\delta^k\,(1+\delta) > 1$ or $(1-p_0)^{k-1}p_0 > \frac{1}{\delta^k(1+\delta)}$. For this to be satisfied, we need $(1-p_0)^{k-1}p_0 \geq \frac{1}{2}$. To see the last condition note that $\frac{1}{\delta^k(1+\delta)}$ is decreasing in δ and reaches a minimum at 1/2 whereas the maximum value of $(1-p_0)^{k-1}p_0 < \frac{1}{4}$. Therefore $v - \frac{c}{p_0} < A'$ is not possible $\Rightarrow i > k+2$. Now, we check the same condition for $i = k+3$ and $i = k+4$. The necessary conditions for these to happen are $(1-p_0)^{k-1}p_0 \geq \frac{1}{3}$ and $(1-p_0)^{k-1}p_0 \geq \frac{1}{4}$ respectively, neither of which is possible . Hence, $i \geq k+5$.

Step 3: Suppose there exists an equilibrium string with gaps and with k agents out of N choosing NI. Define i as before ,i.e. the first agent (from the end) choosing NI. Let the agent $i+1$ choose I. Given s_{-i}, the condition for no-deviation for i from

NI is

$$v - \frac{c}{p_0} < (1-p_0)^{k-1} p_0 \delta^k w(1+\delta+...+\delta^{i-k-1}).$$

Similarly given $s_{-(i+1)}$ the no-deviation condition for $i+1$ is

$$v - \frac{c}{p_0} \geq (1-p_0)^{k-2} p_0 \delta^k w(1+\delta+...+\delta^{i-k})$$

So the necessary condition for the two to hold simultaneously is

$$(1-p_0)^{k-2} p_0 \delta^k w(1+\delta+...+\delta^{i-k}) < (1-p_0)^{k-1} p_0 \delta^k w(1+\delta+...+\delta^{i-k-1})$$

or

$$(1+\delta+...+\delta^{i-k}) < (1-p_0)(1+\delta+...+\delta^{i-k-1}) \tag{5'}$$

But

$$(1-p_0)(1+\delta+...+\delta^{i-k-1}) < (1+\delta+...+\delta^{i-k-1}) < (1+\delta+...+\delta^{i-k})$$

So, (5') cannot hold. So, given the definition of i, $i+1$ must also choose NI. We can apply the same logic to any $j > i$ choosing I and will arrive at a contradiction. Reversing the numbering of agents, we conclude that there cannot be any gaps i.e. $\exists K^*$ such that $\forall j \leq K^*$ choose NI and $\forall j > K^*$ choose I. So, the number of agents choosing NI is K^* and choosing I is $N - K^*$. We also know that at least the last $K^* + 4$ agents have to choose I. So, $N - K^* \geq K^* + 4 \Rightarrow K^* \leq \frac{N-4}{2}$. The exact value of K^* is given by the following condition:

$$(1-p_0)^{K^*} p_0 \delta^{K^*} w(1+\delta+...+\delta^{N-2K^*-1}) \leq v - \frac{c}{p_0} < (1-p_0)^{K^*-1} p_0 \delta^{K^*} w(1+\delta+...+\delta^{N-2K^*}) \tag{6'}$$

■

Behavioural Assumption 2: *Any new entrant, if indifferent between r agents, investigates the most recent among them.*

Proposition 2: *In any equilibrium string, $\exists \overline{K}, s.t. \forall i \leq \overline{K}, \forall j \leq \overline{K} - 1$, i: $NI \Rightarrow i+1 : I$ and $j : I \Rightarrow j+1 : NI$ and $\forall i > \overline{K}$, $i : I$ .The value of $\overline{K}$ depends on parameter values and, for fixed w, p_0, δ, is decreasing in v.*

Proof. First note that there cannot be 2 or more consecutive NI in any equilibrium string. Suppose not. Let i and $i+1$ both choose NI, with $i+2$ choosing I. Then $i+2$ is indifferent between investigating i and $i+1$. By BA2, he chooses $i+1$. This implies that i has future payoff of zero no matter what he does. Hence, i will deviate from NI.

Next we will show that $\nexists$ 2 consecutive I entries preceded and followed by NI in the array of the equilibrium string, i.e. $\nexists$ a sequence $i, i+1, i+2, i+3$ such that i and $i+3$ choose NI and $i+1, i+2$ choose I. By way of contradiction, suppose there is. Since $i, i+3$ chooses NI, by the first argument, $i-1$ and $i+4$ choose I in equilibrium. We will now put down the no-deviation conditions for each of the agents i to $i+3$.

$i \rightarrow NI \Rightarrow v - \frac{c}{p_0} < A_1 = p_0 w(\delta + \delta^2 + ... + \delta^{N-i})$
$i+1 \rightarrow I \Rightarrow v - \frac{c}{p_0} \geq B = p_0 w(\delta + \delta^2 + ... + \delta^{N-i-1})$
$i+2 \rightarrow I \Rightarrow v - \frac{c}{p_0} \geq 0$
$i+3 \rightarrow NI \Rightarrow v - \frac{c}{p_0} < A_3 = p_0 w(\delta + \delta^2 + ... + \delta^{N-i-3})$

Note that $A_3 < A_1$. Hence the condition required is $B \leq v - \frac{c}{p_0} \leq A_3$. which is impossible since $B > A_3$. We can conclude that in equilibrium if $\exists$ *some* i, s.t. $i, i+1$ choose I,then $s_k^* = I \ \forall k > i+1$. Otherwise, the string has to be characterised by alternating patterns ie. if $i \rightarrow NI$, then $i+1 \rightarrow I$ and if any $j \rightarrow I$, then $j+1 \rightarrow NI$. More generally, $\exists \overline{K}$ such that $\forall i < \overline{K}, \forall j < \overline{K} - 1, i \rightarrow NI \Rightarrow i+1 \rightarrow I$ and $j \rightarrow I \Rightarrow j \rightarrow NI$ and $\forall i \geq \overline{K}, i \rightarrow I$. The no-deviation condition for each agent is as follows: $i \rightarrow NI \Rightarrow v - \frac{c}{p_0} < A_1 = p_0 w(\delta + \delta^2 + ... + \delta^{N-i})$
$1 \rightarrow I \Rightarrow v - \frac{c}{p_0} \geq B = 0$
$i+2 \rightarrow NI \Rightarrow v - \frac{c}{p_0} < A_2 = p_0 w(\delta + \delta^2 + ... + \delta^{N-i-2})$
$3 \rightarrow I \Rightarrow v - \frac{c}{p_0} \geq 0$
$4 \rightarrow NI \Rightarrow v - \frac{c}{p_0} < A_4 = p_0 w(\delta + \delta^2 + ... + \delta^{N-i-4})$ ...and so on. The lower v, more of these conditions are satisfied i.e. $v - \frac{c}{p_0} < A_y$ is true for higher values of y(since the sequence $A_1, A_2...$ is decreasing). Hence the alternating pattern can go on for longer and $\overline{K}$ is higher. ■

Behavioural Assumption 3: *Any new entrant, if indifferent between r agents, investigates them with equal probability $\frac{1}{r}$.*

Proposition 3: *In any equilibrium string, $\exists \widetilde{K}, s.t. \forall k < \widetilde{K}$ the k^{th} entry is NI and $\forall k \geq \widetilde{K}$, the k^{th} entry is I . In fact, $\widetilde{K} = 2$.*

Proof. First note that if any two entries $i, i+1$ are I, with $i-1$ being NI, then only i mixes. Player $i+1$ uses pure strategy of I_i, since the belief about all other past entrants' usefulness is less than p_0(by Lemma 1). Now, we characterise the pattern in equilibrium.

Let the $i_1^{th}, i_2^{th}, ...i_k^{th}, i_{k+1}^{th}, i_{k+2}^{th}, ...N^{th}$ agents be the ones choosing I in equilibrium; $i_1 < i_2 < .. < i_k$. Hence by definition, i_1 is the first one to choose I and everyone after agent i_k investigates some agent. We know that the last two agents would always choose I . Hence the agent i_k can be N-1 or smaller. Here, there are $i_1 - 1$ agents before i_1 who have not been investigated.Agent i_1 is indifferent between them and reads each of their papers with equal probability, $\frac{1}{i_1-1}$. Similarly, i_2 investigates each of $i_1, i_1+1, ..., i_2-1$ with probability $\frac{1}{i_2-i_1}$ and so on. Given this equilibrium, we can derive the updated beliefs of each agent whenever the state of no citations is reached and hence calculate the no-deviation (unilateral) condition for each agent.

$1 : 0 < \frac{1}{i_1-1} p_0 w \delta^{i_1-1}(1+\delta+\delta^2+...\delta^{N-i_1})$

$2 : v - \frac{c}{p_0} < \frac{1}{i_1-1} p_0 w \delta^{i_1-2}(1+\delta+\delta^2+...\delta^{N-i_1})$

.

.

$i_1 - 1 : v - \frac{c}{p_0} < \frac{1}{i_1-1} p_0 w \delta(1+\delta+\delta^2+...\delta^{N-i_1})$

$i_1 : v - \frac{c}{p_0} \geq \frac{1}{i_2-i_1} p_0 w \delta^{i_2-i_1}(1+\delta+\delta^2+...+\delta^{N-i_2})$

$i_1 + 1 : v - \frac{c}{p_0} < \frac{1}{i_2-i_1} p_0 w \delta^{i_2-i_1-1}(1+\delta+\delta^2+...+\delta^{N-i_2})$

.

.

.

$i_k - 1 : v - \frac{c}{p_0} < \frac{1}{i_k-i_{k-1}} p_0 w \delta(1+\delta+...\delta^{N-i_k})$

$i_k : v - \frac{c}{p_0} \geq p_0 w \delta(1+\delta+...+\delta^{N-i_k-1})$

$i_k + 1 : v - \frac{c}{p_0} \geq p_0 w \delta(1+\delta+...+\delta^{N-i_k-2})$

.

.

$N : v > 0$

Take the 2 equations for $i_k - 1$ and i_k.

$$p_0 w\delta(1+\delta+...+\delta^{N-i_k-1}) \leq v - \frac{c}{p_0} < \frac{1}{i_k - i_{k-1}} p_0 w\delta(1+\delta+...\delta^{N-i_k})$$

A necessary condition for this to hold is

$$p_0 w\delta(1+\delta+...+\delta^{N-i_k-1}) < \frac{1}{i_k - i_{k-1}} p_0 w\delta(1+\delta+...\delta^{N-i_k})$$

$$or,\ (1+\delta+...+\delta^{N-i_k-1}) < \frac{1}{i_k - i_{k-1}} (1+\delta+...\delta^{N-i_k})$$

Call the LHS, A. Then the previous expressions can be rewritten as:

$$A(i_k - i_{k-1}) < A + \delta^{N-i_k}$$

$$A(i_k - i_{k-1} - 1) < \delta^{N-i_k}$$

But $A > \delta^{N-i_k}$. So this can hold only if

$$i_k - i_{k-1} = 1$$

,which again implies that there is no gap between i_k and i_{k-1}.[25] Hence everyone after agent i_{k-1} investigates.

Similarly we can write out the new set of conditions where agents $i_1, i_2, ..., i_{k-2}, i_{k-1}, i_{k-1}+1, i_{k-1}+2, ..., N$ choose I and compare the conditions for agents i_{k-1} and i_{k-2}. We would arrive at a contradiction if $i_{k-1} - i_{k-2} > 1$. Hence in any equilibrium string there cannot be gaps. Such a string must be of the form $[NI, NI, NI.....NI, I, I, I....I]$, where the I starts at period $\widetilde{K}$.

Now we can go on further and find the value of $\widetilde{K}$. We know that all the agents before $\widetilde{K}$ were not investigated. Hence $\widetilde{K}$ investigates each of them with probability $\frac{1}{\widetilde{K}-1}$.If his investigation is not useful, $\widetilde{K}+1$ investigates $\widetilde{K}$. The no-deviation

[25] Note the difference between i_{k-1} and $i_k - 1$.

conditions for agents $\widetilde{K}-1$ and $\widetilde{K}$ are

$$p_0\delta w(1+\delta+\delta^2+...+\delta^{N-\widetilde{K}-1}) \leq v-\frac{c}{p_0} < \frac{1}{\widetilde{K}-1}p_0\delta w(1+\delta+\delta^2+...+\delta^{N-\widetilde{K}})$$

The necessary condition again is

$$(1+\delta+\delta^2+...+\delta^{N-\widetilde{K}-1}) < \frac{1}{\widetilde{K}-1}(1+\delta+\delta^2+...+\delta^{N-\widetilde{K}})$$

$$or,\ \widetilde{K}-1 < \frac{1+\delta+\delta^2+...+\delta^{N-\widetilde{K}}}{1+\delta+\delta^2+...+\delta^{N-\widetilde{K}-1}} = 1+\frac{\delta^{N-\widetilde{K}}}{1+\delta+\delta^2+...+\delta^{N-\widetilde{K}-1}} < 2$$

$$i.e.\ \widetilde{K} < 3$$

Since the first agent has no one to investigate his only choice is NI. So, $\widetilde{K} < 3$ implies that investigation would start from agent 2 and no later. ■

Proposition 4: *For any given set of parameters,* (v, p_0, w, δ)*,* $\widetilde{K} \leq K^* < \overline{K}$.

Proof. From Proposition 3, we know that $\widetilde{K} = 2$. By way of contradiction, we assume $K^* \geq \overline{K}$ and show that set of values of the parameters that satisfy this inequality is empty.

>From Proposition 1, we know that given a K^*, the parameters should satisfy equation (6') [See Appendix].

$$\begin{aligned} L_{K^*} \quad : & \quad = (1-p_0)^{K^*}p_0\delta^{K^*}w(1+\delta+...+\delta^{N-2K^*-1}) \\ \leq & \quad v-\frac{c}{p_0} < (1-p_0)^{K^*-1}p_0\delta^{K^*}w(1+\delta+...+\delta^{N-2K^*}) = H_{K^*}. \end{aligned} \tag{3}$$

>From Proposition 2, given a $\overline{K}$, the conditions to be satisfied are

$$L_{\overline{K}} := p_0\delta w(1+\delta+...+\delta^{N-2-\overline{K}}) \leq v-\frac{c}{p_0} < p_0\delta w(1+\delta+....+\delta^{N-1-\overline{K}}) = H_{\overline{K}} \tag{4}$$

Now fix the value of $K^* = Y \geq 2$. Therefore parameters satisfy (3).

Now, we want to check whether $\overline{K}$ can be $\geq Y$.

Let $\overline{K} = Y$.

$$
\begin{aligned}
H_{K^*=Y} &= (1-p_0)^{Y\ -1} p_0 \delta^Y w(1+\delta+...+\delta^{N-2Y}) \\
&< p_0 \delta^Y w(1+\delta+...+\delta^{N-2Y}) \\
&< p_0 \delta w(1+\delta+...+\delta^{N-2Y}) \\
&= p_0 \delta w(1+\delta+...+\delta^{N-Y-Y}) \\
&= p_0 \delta w(1+\delta+...+\delta^{N-Y-\overline{K}}) \\
&\leq p_0 \delta w(1+\delta+...+\delta^{N-2-\overline{K}}) = L_{\overline{K}}|_{\overline{K}=Y}
\end{aligned}
$$

Given the fixed value of $K^* = Y$, $v - \frac{c}{p_0} < H_{K^*} < L_{\overline{K}}|_{\overline{K}=Y}$. Hence $v - \frac{c}{p_0}$ does not lie in the range $[L_{\overline{K}}, H_{\overline{K}}]|_{\overline{K}=Y}$.

Hence given (3), $\overline{K.} \neq Y$.

Also note that $L_{\overline{K}}$ is decreasing in $\overline{K.}$ which implies that for values of $\overline{K} < Y, H_{K^*} < L_{\overline{K}}$. and hence (3) and (4) cannot hold together. So, given that parameter values satisfy (3), which corresponds to a K^*, $\overline{K} > K^*$. ■

Appendix B: Proofs for Extensions

Proposition 8: *There cannot be an equilibrium in which there exist α and α', $\alpha < \alpha'$(say), such that all players with $p_i > \alpha'$ and some with $p_i < \alpha$ enter in period 1, while players with $\alpha \leq p_i \leq \alpha'$ enter in period 2 with other players entering after period 2.*[26]

Proof. Suppose, there exists $\alpha, \alpha', \alpha''$, with $\alpha > \alpha' > \alpha''$ such that $p_i > \alpha$ and $\alpha'' < p_i < \alpha'$ enter in period 1, $\alpha' < p_i < \alpha$ enter in period 2 and $p_i < \alpha''$ enter after period 2. We will show that there will be a profitable deviation for some agent. Note that, after observing the state in each period up to and including t the probabilities of usefulness for any entrant in period $s \leq t$ are updated to $\widehat{p}^{s}_{t+1}$.

Case I : $\widehat{p}^{2}_{t+1} < \widehat{p}^{1}_{t+1}$ In this case the agents entering after period 2 investigate and get $\widehat{p}^{1}_{t+1}v - c$. But if $\widehat{p}^{2}_{t+1} < \widehat{p}^{1}_{t+1}$, these agents would prefer to enter at period 1 and get a less discounted payoff

Case II: $\widehat{p}^{1}_{t+1} = \widehat{p}^{1}_{t+1}$ In this case again, agents entering after 2 would like to enter in period 2 and get $\widehat{p}^{1}_{2}v - c$ at period 2 .

Case III: $\widehat{p}^{2}_{t+1} > \widehat{p}^{1}_{t+1}$ This is the only case when agents entering after 2 would not want to enter earlier. We consider the following three subcases:

a) All $\alpha' \leq p_i \leq \alpha$ enter at t = 2 and Investigate: If it is optimal for these agents to do so it implies that

Utility from investigating $= X = \widehat{p}^{1}_{2}v - c + (1 - \widehat{p}^{1}_{2})p_i E_2 W_2 >$ *utility from entering at time* $1 = p_i E_1 W_1 = Z$ where $E_t W_t$ is the expected future payoff from being cited after entering in period t.[27] Agents at t=1 are potentially cited by agents at 2 and later while agents at t=2 are potentially cited by agents after 2 only. The decision is represented in Figure 5 a and 5b. Fig. 5a shows when function X is steeper than Z and 5b is the opposite. We see that in both cases if some $\alpha' \leq p_i \leq \alpha$ prefers X, it must be the case that $p_i < \alpha'$ also prefers X. Therefore, if it is optimal for agents entering at 2 to investigate and not enter in period 1, then agents $\alpha'' \leq p_i \leq \alpha'$ cannot find it profitable to enter at period 1.

b) All $\alpha' \leq p_i \leq \alpha$ enter at t = 2 and choose Not Investigate: In this case, agents

[26] Using period 1 in the statement is without loss of generality-we can replace it by "period τ such that there has been no entry up to $\tau - 1$."

[27] The inequality could be weak for a boundary type of p_i.

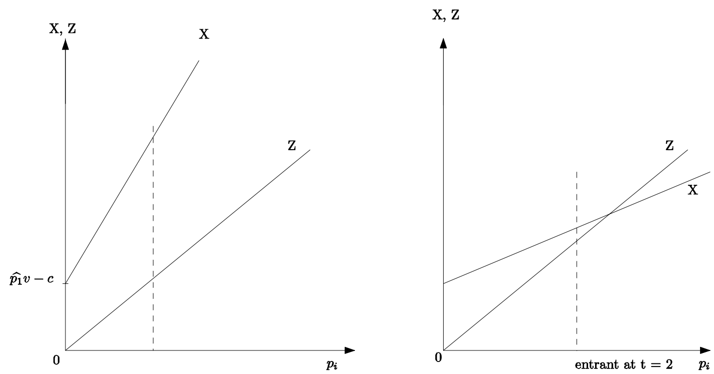

Figure 5: Proposition 8, Case a)

at t>2 investigate entrants in period 2 only because of their higher probability of being useful. So, an agent entering at t=1 should deviate and wait to enter in period 2.

c) Some agents entering at t=2 investigate while some choose Not Investigate. Since later entrants are not able to tell, given no citation, whether entrants in period 2 investigated or not, there will be two revisions of probability. The probability of usefulness of period 1 entrants will drop to $\widehat{p}^{1'} < \widehat{p}^1$(from Lemma 1). The probability of usefulness of period 2 entrants will go up, i.**e.** $\widehat{p}^{2'} > \widehat{p}^2$, given that within the set of period 2 entrants the ones with higher p_i choose Not Investigate (and hence have a higher probability of not citing). Given the candidate equilibrium strategies for other players, the choice between investigating or not for players who have entered at t=2 entails comparison of $\widehat{p}_2^1 v - c + (1 - \widehat{p}_2^1) p_i EW_2$, where EW' is the expected payoff from entrants at t>2 investigating second period entrants, and $p_i EW_1$. This gives a cutoff p_i such that all $p_i > Y^*$ will choose NI, while others choose I. Suppose such a Y^* exists. So, we have $\alpha'' \leq p_i \leq \alpha'$ enter in period 1, $\alpha' \leq p_i \leq Y^*$ enter in period 2 and investigate. Thus there is an agent with $p_i = \alpha'$ who is indifferent between the two. The payoff from entering in period 1 $= p_i E_1 W_1$ (denoted by A) and that from entering at t=2 and investigating is $\widehat{p}_2^1 v - c + (1 - \widehat{p}_2^1) p_i EW_2$ (B). We need to compare these two payoffs as functions of p_i. Two cases are possible: i) The slope of A is less than slope of B ii) The slope of A is greater than that of B. The two cases are represented in Fig 6 a,b. In (i) the two are not equal at any p_i. So, this

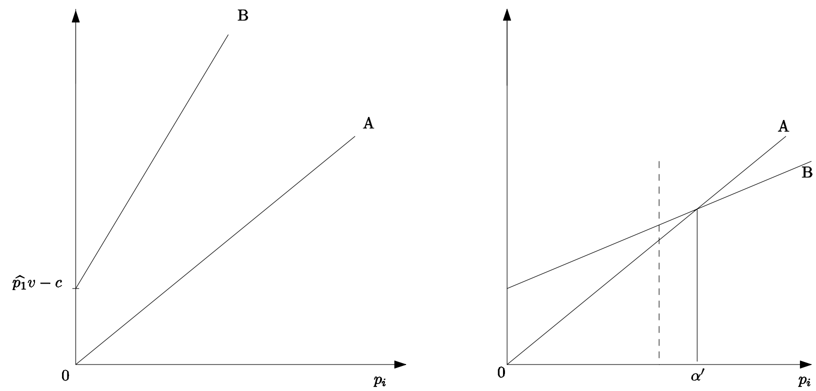

Figure 6: Proposition 8, Case b)

equilibrium is not possible. In (ii), they intersect at α'(say). Then from the graph we can see that any $p_i > \alpha'$ will prefer A to B. i.e. will prefer entering in period 1. So, for $i, \alpha' < p_i < Y^*$, entering in period 2 and investigating cannot be an equilibrium strategy. ■

Chapter 2

Competing to be a 'Star': A model of sequential network formation

1 Introduction

Interpersonal networks, be it within one's kin, among neighbours or among colleagues, have been found to have a significant and persistent impact on economic and social outcomes, both at the individual and the aggregate level. For example, on the aggregate level, it influences adoption of new technologies (Conley and Udry 2001) and sociological interactions such as social norms, status attainment and ethnic segregation[1]. In the context of firm strategy, in the presence of a consumer network, firms can target few central agents order to promote their product at minimum cost[2](Feick and Price 1987; Ellison and Fudenberg 1995). On the individual level, there is substantial evidence that one's position in a network affects performance and central agents are better off in most environments. For example, in an informal network, a person with more friends is more likely to get information about relevant job vacancies or have a higher chance of getting a job through referrals (Granovetter 1974, Montgomery 1991, Holzer 1987); a well-connected person gets more credit since his network can be used as sanctions against default (McMillan and Woodruff 1999, Fafchamps and Lund 2001, Banerjee and Munshi 2003). Similar advantages are observed in formal networks like trade networks (Lazerson 1993, Nishiguchi 1994) and R&D alliances among firms (Powell 1996, Delapierre and Mytelka 1998).

The above evidence suggests that there are incentives for agents to compete in order to become central in a network. This paper studies such incentives through a model of network formation with forward looking agents. In our model, networks are formed through deliberate decisions of individuals who consider all the current and future costs and benefits of establishing links with each other. Moreover, the benefits

[1]There exists a large literature in sociology on issues of family and kinship networks, ethnic networks and their effects on income, labour market participation, social cohesion and ethnic integration. See, among others, Coleman (1988), Burt (1992). See Wasserman and Faust (1994) for an introductory book on networks in sociology.

[2]These consumers are called 'market mavens' in the marketing literature.

from a network consists of both direct benefits from one's neighbours and indirect benefits via the network of one's neighbours. We model the benefit an agent gets from others in the network as depending on the distance between agents in the network and the cohesion level of the society, represented through the concept of 'decay'. To fix ideas, think of a highly cohesive society where agents enjoy high levels of trust with each other. In network terminology, this implies a lower level of decay along the network because in such an environment the benefit from an indirect friend is only slightly less than that from a direct friend. Any positive level of decay, however, drives a wedge between the payoffs of a central and not-so-central agent and creates the incentive to compete for centrality. Hence, in a political environment the agents are politicians forming ties with other such agents. A well-connected politician would hope to be the leader of the group and have a higher payoff. In a pharmaceutical industry, firms form research collaborations with each other and those with higher number of collaborators have higher potentially gain from higher know-how. The magnitude of these gains, however, depend on the spillover effects particular to the industry.

To model forward looking behaviour, we propose a sequential-move game in which a finite number of agents make link proposals. Initially, all the agents are isolated. In the first period, one agent is randomly chosen to be the proposer and the game begins. Thereon, each agent has the option of proposing a link at each period by choosing one agent to link to. If an agent does propose a link, he has to bear a cost c. Following a link proposal at any stage, the responder accepts or rejects the link. Whatever be the decision, the responder becomes the proposer in the next period and so on. The game ends when all agents have had the opportunity to propose a link and benefits are exchanged.

We show that when agents are farsighted, then, whether they compete to become the central agent or not, crucially depends on the rate of decay. This is because any positive level of decay drives a wedge between payoffs of a central and not-so-central agent since the former has a high proportion of direct connections. In our basic model with homogeneous agents, we find that the unique subgame-perfect equilibrium network architecture is a single connected component whenever the rate of decay is very high or very low. In particular, it is a star when the extent of decay is close to

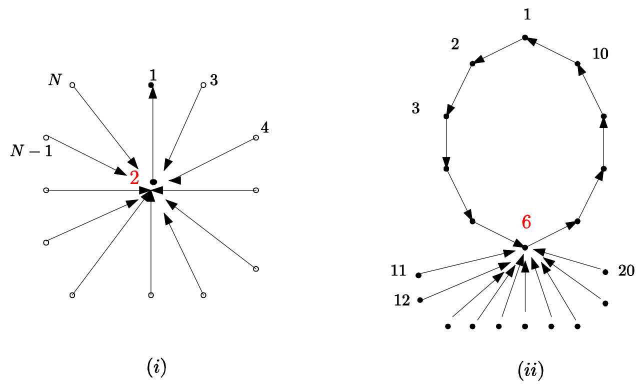

Figure 1: Equilibrium networks

zero, i.e. one agent is the centre and all other agents are directly connected to him. On the other hand, when there are very strong decreasing returns (or high decay) the equilibrium network resembles a hub-and-spoke architecture. Figure 1 demonstrates the equilibrium structures for the extreme values of decay.

The star network involves only one agent as the centre. The intuition for this result is that when there is very little decay, the payoff from being a peripheral agent is not too low compared to a that of a central agent and the incentive to compete is weak. Hence, connecting to the central agents provides maximal acess to indirect benefits. However, when the decay is high, the competition is strong and the earlier agents form a 'hub' in the form of a wheel or circle while later agents connect to one person in the hub. Hence, it explains the endogeneous emergence of highly central agents or hubs with peripheral agents. This network of R&D collaborations among pharmaceutical firms have been found to have a similar structure.

Moreover, we show that for some intermediate levels of decay the equilibrium may not be connected and the network can consist of isolated groups of agents. We observe such group formation in various political situtations such as the formation of multiple parties with the same ideologies in India and other multiparty systems. To give a specific example, politician Ram Manohar Lohia formed his own Socialist Party with few others rather than join the existing Praja Socialist Party (which had essentially the same idealogies) and share centrality with multiple leaders. As stated in an article in New York Times, the rebels in Darfur lack a strong leadership due to the power struggle amongst the top leaders which has resulted in factionalisation even though the formation of a single powerful group would arguably be beneficial to the movement. In more recent times, sociologist Robert Putnam while commenting on structural changes in American society notes that over the past three decades there has been a decrease in general trustworthiness and a simultaneous increase in the number of civic associations, each with a smaller membership. For a certain range of values of the decay parameter, our model implies that such phenomena can be explained in terms of the incentives of later agents to reject links in order to have a higher probability of becoming the centre himself. Similar incentives might also work among social groups of young adolescents or teenagers.

Equilibrium networks, in general, are inefficient, except for when the level of decay is very low. With heterogeneous types of agents, the range of values of decay that support the efficient structure as the equilibrium increases. The high ability agent becomes the centre of the star. However, it is possible, for some parameter values, to have the high ability agents isolated from a group of low ability agents. This happens when the difference in abilities or the extent of heterogeneity is moderate and the decay rate is at some intermediate level.

One feature of the model is that each agent has a capacity of one on the number of link proposals. The underlying premise is that forming links takes up resources in terms of time, money or effort. The capacity constraint is reasonable since link formation in a network is only one among many sources of economic benefits. Agents participating, say, in the labour market, can only invest upto a certain proportion of their time in their social networks due to opportunity costs of social investment. In the extension, we provide an example to show how relaxing the assumption would affect the network configuration.

The literature in economics on network formation follows two main strands. The first strand of literature on network formation follows Jackson and Wolinsky (1996)(henceforth JW). Their work is closely related to the literature on coalition formation in cooperative games (Myerson1977, Aumann and Myerson 1988) but the value of the network depends on its exact structure. JW focus on individual incentives to form or sever links and highlight the conflict between pairwise stable networks and socially efficient ones[3]. Bala and Goyal (2000, henceforth BG),on the other hand, model network formation as a non cooperative game by introducing one-sided link costs. BG propose a one-shot simultaneous move game where homogeneous agents form links with each other at some exogenous cost, which is borne entirely by the intiator. They consider both one-way (only the initiator getting the benefit) and two-way (benefits accruing to both agents in the link) flow of benefits with and without decay[4] and characterise the set of strict Nash networks for the different specifications[5]. Hojman and Szeidl (2004) use the model of BG to focus on the role of decay in network formation and highlight periphery-sponsorship as a robust feature of the equilibrium network in such settings. They however limit the access to the benefits from connectedness to a finite distance in the network.

All these papers analyse network formation as a simultaneous-move game and hence are myopic and static in nature. Watts (2001a) proposes an infinite horizon dynamic model of network formation where a finite set of agents decide to form and sever links to maximise their *myopic* payoff. Watts (2001b) focuses on a specific model with forward looking agents that results in circle networks. Deroian (2006) analyses a finite time sequential move game and shows a non-monotonic relationship between the level of cost and the formation of a complete graph (specifically, a star). In these models an agent can form as many links as she wants. There is no capacity constraint. Another point to note is that the formation of a link is unilateral i.e. it does not require consent from the agent being linked to. Of course, for a static, myopic game there is no reason for not accepting a link. But, when agents are forward looking an agent might reject a link for strategic reasons. The exact nature of such strategic concerns will be clear later.

[3]For an excellent survey of the literature on various approaches to network formation see Jackson (2003). For papers using the notion of pairwise stability, refer to Dutta and Mutuswami (1997), Dutta, B., A. van den Nouweland and S. Tijs (1998), Johnson and Gilles (2000).

[4]i.e. the benefit exchanged between two agents may or may not decline (decay) with the distance between them.

[5]Following BG there have been a number of attempts to extend this basic model. Galeotti et al (2005) and Galeotti (2006) extend their model to include heterogeneity in values and costs in both one-way and two-way flow of benefits. The heterogeneity in these models is not partner specific, i.e. the values and costs depend on the agent's own type only. Kannan et al (2007) follow BG and introduce costs of indirect links. Hence, the benefits from one's indirect contacts (friends e.g.) are not free, as was the case in BG. They look at different cost specifications and analyse how the cost structure affects the network.

The remainder of the paper proceeds as follows. Section 2 describes the basic model followed by the analysis in Section 3. Section 4 discusses welfare and efficiency issues. Sections 5 and 6 present extensions and examples. Section 7 discusses results and concludes. All proofs are relegated to appendices A and B.

2 Model

This section introduces the basic model of network formation. We study a sequential move game in which N players decide to form links with each other. We chose a finite game, in which everyone makes one proposal, to bring out the dynamic aspects of the problem in a simple yet interesting environment. Each link represents social connections between the two players whereby they exchange some benefits. All the links together define a network or a graph where the agents are represented as nodes and the links as the edges of the graph. From now on, we will use the words graph and network interchangeably.

2.1 The Environment

$N = \{1', 2', ...n'\}$ is the set of agents forming a network. The network at time 0 is empty i.e. each agent is isolated. Agents are budget constrained. We assume that all agents have the same contraint and each can propose only one link. An agent is randomly chosen to be the first proposer and is relabeled agent 1. He proposes a link to some agent j'. The responder accepts or rejects the link and becomes the next proposer and is relabeled agent 2. In general, the agent that proposes at time period t is relabeled as agent t. If t proposes to $k' \neq t$, k' rejects or accepts. If k' had already proposed, then an agent is randomly chosen from the set of agents who have not proposed any link. Otherwise k' is the next proposer and is relabelled as agent $t+1$. Links once formed are not allowed to be severed. The proposals stop once every agent has had the chance to propose[6]. Hence the game continues for N periods. If i

[6] Note that if everyone can make only one link, then in a model where agents have another chance to propose after the first round and there is discounting, links will be added only if some agent abstained in the first round, which in equilibrium would possibly be only agent 1. A model without discounting, however, might involve some delay due to the incentive to free-ride.

initiates a link and j accepts, they are linked and it is denoted by $g_{ij} = 1$, which is represented by a directed arrow from i to j. This incorporates the information that i is the one who proposed and j accepted. The collection of all such links make a graph g. Let g_{t-1} denote the graph at time t before agent t makes a decision. At N, g_N is formed and payoffs are realised. The sequential nature of our model is similar to that of the multiperson bargaining literature, the 'offer' here being a social link. Similar to a bargaining situation, different orders of play could be relevant. For example, it could be a pre-determined order as in Shaked (1986) or random proposers like Okada (1996). The order of play considered here is similar to that of Selten (1981) and Chatterjee et al (1993).

We also define a *path* between agents. There exists a path connecting i and j if $g_{ij} = 1$ or $g_{ji} = 1$, i.e. i and j are directly connected or $\exists j_1, j_2, ...j_K$ such that all of the following holds: i) g_{ij_1} or $g_{j_1 i} = 1$ (ii), $g_{j_K j}$ or $g_{j j_K} = 1$, (iii) $g_{j_k j_{k-1}}$ or $g_{j_{k-1} j_k} = 1$ for all $k = 2, 3, ...K$

Also define $N_i^d(g) = \{j \neq i : g_{ij} = 1 \text{ or } \ g_{ji} = 1\}$, i.e. the set of agents to whom i is directly linked and $N_i(g) = \{j : \text{there exists a path between } i \text{ and } j\}$

2.2 The Strategy

At any time t, the agent i who moves could have two decisions to make. First, he accepts or rejects if he has been proposed to at $t - 1$ which is denoted by action $a_t \in \{A, R\}$. If he is isolated, then he has no such decision to make which is denoted by ϕ. The second decision is that of initiating a link to some agent $l_t \in N \backslash \{i\}$ or abstaining, denoted by ϕ_l. The history at any time t, is g_{t-1}, the graph formed till stage $t - 1$ and the history of acceptance/rejections, i.e. history $h_t = \{a_\tau, l_\tau\}_{\tau=0}^{t-1}$.

Hence, the *strategy* of an agent i who moves at time t is a mapping from the history at time t to the action set. We represent the strategy by $s_i : h_t \rightarrow \{A, R, \phi\} X \{\{\phi_l\} U N \backslash \{i\}\}$.

2.3 The Payoffs

A link in a network represents the channel of exchange of both direct and indirect benefits between agents. In particular, the benefit exchanged between agents depend on the distance between them in the network. Formally, define $d(i, j, g)$, the geodesic distance between i and j as the length of the shortest path between the two in g. If i and j are directly linked, $d(i,j,g) = 1$. By convention, if two agents are not connected then $d(i,j,g) = \infty$. Given the distance between two agents i and j, the value of the benefits exchanged between them is $p_{ij}(g).u_{ij} = p^{d(i,j,g)-1}u_{ij}$ where $p \in (0,1)$ and u_{ij} is the value of the benefit. The formation of a link is costly and the cost c is borne entirely by the initiator of the link. This assumption tries to capture the fact that there are significant asymmetries in effort even in relationships that require reciprocal investment.

Given a network g, the total payoff to agent i in g , can be written as

$$\pi_i(g) = \sum_{j \neq i} \{p^{d(i,j,g)-1}(u_{ij}) - cI(g_{ij} = 1)\}$$

where u_{ij} is the value of the benefit i gets from j. Since agents can form only one link, the link formation cost is c, if i initiates a link and 0 otherwise. We also assume that $u_{ij} = u = 1$ for all i, j i.e. agents are homogeneous and give a direct benefit of 1.

In the payoff structure defined above, the benefits exchanged between two agents decrease with the distance between them at a geometric rate p. One interpretation of this decay rate p is the probability of meeting indirect connections. Consider, for example, friendship networks. A direct link between i and j represents a direct friendship. If, on the other hand, i is a (direct) friend of k who is also a friend of j, but i and j are not directly linked, then i is indirectly connected to of j and $d(i,j,g) = 2$. We could think of situations where benefits are exchanged only when two agents meet and that direct friends meet each other with probability 1. An agent i, however, meets a friend's friend with probability p and a friend's friend's friend with probability p^2 and so on. Hence the ex[ected benefit i gets from j is $p.u$. Alternatively, in most social environments, it is the case that agents get benefits, for sure, from their direct friends while the benefits from a friend's friend are obtained only with some probability. This probability depends on characteristics of the society like levels of trust and social activity which are indicators of the social cohesion of a community. Hence, p is, in some sense, a measure of social cohesion of a society. This geometric decay structure is less general than that of Hojman and Szeidl (2004) who consider a general form of decreasing returns. They, however, restrict the benefits from indirect connections to be positive only upto a fixed finite distance in the network after which it is zero.

Before we proceed to the analysis we introduce some standard network architectures. Given a graph g, a set $C \subset N$ is called a *component* of g if for a pair of agents i and j in C, $j \in N_i(g)$ and there is no strict superset C' of C for which this is true. A component is *minimal* if C is no longer a component if $g_{ij} = 1$ is replaced by $g_{ij} = 0$. A network g is said to be *connected* if it has a unique component. A network is an *empty network* g^e if $N_i(g) = \{i\}$ and is a *complete network* g^c is $N_i(g) = N \backslash \{i\}$ for all $i \in N$. A *wheel network,* g^w(*or* just W) is one where agents are arranged as $i_1, i_2, ..., i_n$ with $g_{i_1 i_2} = g_{i_2 i_3} = ... = g_{i_n i_1} = 1$ and there are no other links. A *star network* g^s has a central agent i such that $N_j^d(g) = \{i\}$ for all $j \in N \backslash \{i\}$ and there are no other links. A wheel with l agents is denoted by W_l. A star is *centre sponsored* if $g_{ij} = 1$ and $g_{ji} = 0$ for all j and *periphery-sponsored* if $g_{ij} = 0$ and $g_{ji} = 1$ for all j in g^s. A *mixed star* is a combination of the two. We call a graph a *wheel with local stars* if the graph contains both a wheel and a star. Let us denote it by WS_k, when the wheel has k agents and the local star has the $N - k$ spoke agents. A connected acyclic component with exactly one path is called a *chain.* Let us denote it by Ch_l, where the subindex is the length of the chain. These architectures are illustrated in Figure 2.

3 Analysis

The network formation game of our model is a finite horizon problem and the equilibrium concept is that of subgame perfect equilibrium. We assume that when a proposer of a link is indifferent between multiple agents, he chooses one randomly and with equal probability.

Assumption (A1): When indifferent between a set of agents to link to, the proposer randomises between them with equal probability.

Note that given **A1** rejections cannot occur on the equilibrium path since no agent i would incur the cost of intitating a link which will be rejected and would yield no myopic benefit[7]. Hence, the relevant history is the current graph (which influences the

[7] However, this could still be beneficial if this would lead some future agent to choose agent

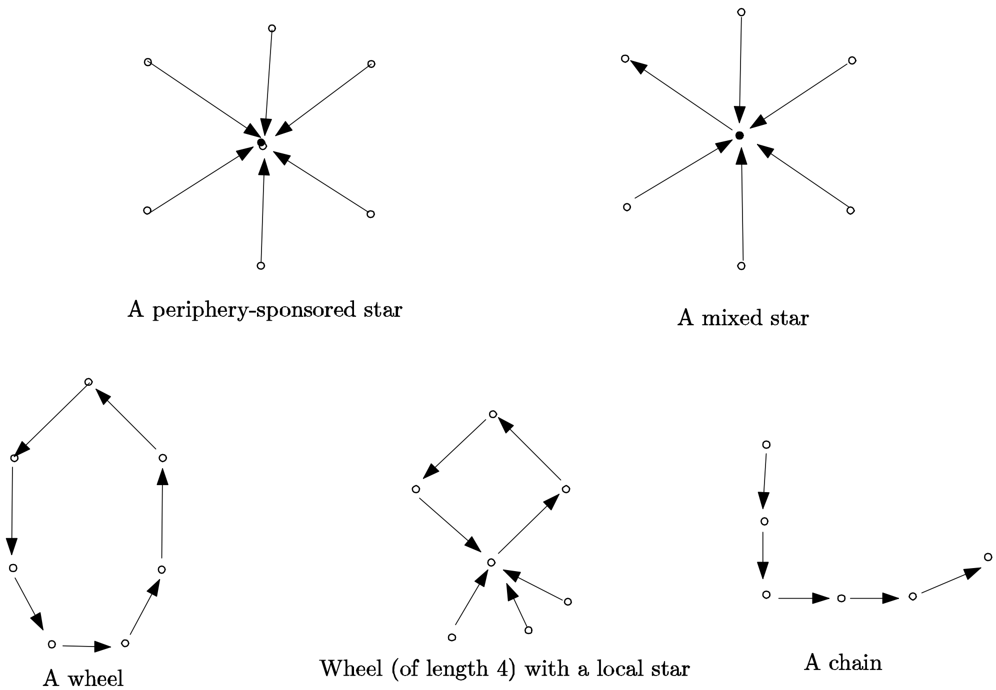

Figure 2: Network architectures

myopic payoff) and not the past actions of acceptance and rejection. This also allows us to abstract from possible reputation issues involved in rejection. Hereafter, we confine ourselves to strategies that depend only on the graph formed till the current stage.

We also assume that if indifferent between accepting or rejecting a link, an agent always accepts the link. This, however would not be the case for generic values of parameters.

A2: Accept if indifferent between accepting or rejecting a link proposal.

Note that since $c < u = 1$ agents always get net positive benefit from meeting some agent. The responder of time $t-1$ is the proposer at time t which gives the responder some power. The agents in this model have perfect foresight and realise the implications of their strategies on future action by players. We use backward induction to analyse the actions of each player.

Before we proceed let us introduce some more notations.

$\pi_t(g_{t-1})$: Total payoff agent t is assured to get given the graph g_{t-1}.

$\pi_t^{myo}(j, g_{t-1})$: Total payoff t receives from agents $k < t$, after $\{tj\}$ link is formed and is equal to $\sum_{j<t} p_{tj}(g_{t-1} + \{tj\}).1$. This is the myopic payoff since t does not take into account the payoff from meeting the future entrants.

$\pi_t^f(j, g_{t-1})$: Payoff t receives from all future entrants if he links to j. It is determined by the equilibrium strategies of future players given that the state at $t+1$ will be the graph $g_t = g_{t-1} + \{tj\}$

$v_t(g_{t-1}) = \arg\max_{j<t} \pi_t^{myo}(j, g_{t-1})$

$\Pi_t(j, g_{t-1})$: Total payoff to agent t from linking to j, when the graph is g_{t-1}. Note that Π is determined by the equilibrium strategy of all other agents. Note that $\Pi_t(j, g_{t-1}) = \pi_t^{myo}(j, g_{t-1}) + \pi_t^f(j, g_{t-1})$.

Remark 1: $\pi_t(g_{t-1}) = \sum_{j<t} p_{tj}(g_{t-1}).1 = 0$ if t is isolated and > 0 if t is linked. Also, we distinguish between $v_t(g_{t-1})$ when t is isolated and when he is not since the agent v that maximises $t's$ payoff in the 2 cases is potentially different. Let the

i (whose proposal was rejected earlier) with higher probability. This cannot happen since (i) if indifferent, the future agent randomises and does not choose i with higher probability and (ii) if not indifferent, then agent i would have been chosen even if he had not made a rejected offer (for his "centrality").

optimal agent in the case when t is isolated and when he is not be $v_t^{iso}(g_{t-1})$ and $v_t^{con}(g_{t-1})$ respectively. By defining $\pi_t^{myo}(j, g_{t-1})$ and $\pi_t^f(j, g_{t-1})$ separately we will try to disentangle the two types of incentives an agent has when he considers linking to an agent j : (i) t wants to maximise his payoff from meeting all earlier entrants. but (ii) $t's$ choice would change the graph g_t and future entrants would link according to their equilibrium strategies which affects $t's$ payoff from agents $k > t$ through $d(t, k, g_n)$.

Remark 2: The payoffs are functions of p_{tj} which in turn is a function of distances between agents $d(t, j, g)$. In particular, the final payoff to agent t will be functions of $d(t, j, g_n)$ where g_n is determined by the equilibrium strategy profile of all n agents. So, when deciding whom to link to, t considers both current and future payoffs and links to a j such that $\Pi_t(j, g_{t-1}) = \pi_t^{myo}(j, g_{t-1}) + \pi_t^f(j, g_{t-1})$ is maximised.

Before we state the main propositions of this section, note that due to the assumption of $u > 0$, no agent would reject a proposal unless the graph is such that following a rejection of a link he has a positive probability of being chosen as the centre of a star by the following agents. In other words, if the graph is such that this incentive is not at work, an agent would accept a proposed link since he is not incurring any cost and the net benefit from the link is $u = 1$ which is positive. We first give an example to illustrate the incentives of agent t when the graph is g_{t-1}.

Example 1 *Suppose, $N = 8$. Also, suppose the graph at beginning of $t = 3$ is as follows: 1 and 2 are linked. 3 is isolated and have to propose a link. Note that 3 is indifferent between 1 and 2. Agent 3 has the following options:*

(i) 3 could link to 1 (or equivalently 2)

(ii) 3 could link to some other agent 4 (after the renumbering)

If 3 links to 1, then note that the proposer of $t = 4$ is isolated. Also, suppose that the parameters are such that following this subgame, the equilibrium strategy of all future players is to link to 1. In this case, a complete star with 1 as the centre is formed and 3 is a peripheral agent. The payoff to 3 can be written as $\pi_3(l_3 = 1) = 1 - c + 6p$. Alternatively, suppose 3 links to some new agent j' (who is then renumbered as 4) and j' or 4 accepts. Also, suppose 4 abstains and hence the proposer at $t = 5$ is isolated and his equilibrium strategy is to link to one of 1,2,3,4[8]*. Moreover, let the equilibrium strategy profile be such that all future agents link to 5's choice. In this*

[8] 5 is indifferent between the first four agents and chooses each with probability $1/4$.

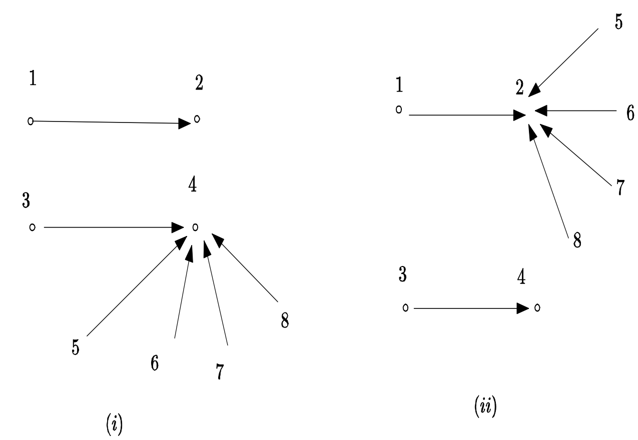

Figure 3: Example 1

case, the network formed is as depicted in fig 3 where the centre of the star could be any one of 1,2,3 and 4. In this case, 3 becomes the centre with probability $1/4$ *and with probability* $1/4$*.he is a peripheral node in a star of* $N-2$ *agents. Also, with probability* $1/2$*, agent 5 links to 1 or 2 and agent 3 is isolated from the star and hence gets a much lower payoff. The expected payoff of agent 3 can be written as* $\pi_3(l_3=4) = 1-c+\frac{1}{4}[4]+\frac{1}{4}[4p] = 1-c+1+p$*. Hence linking to 4 makes 3 disconnected from 1 and 2 but makes him central with some probability while linking to 1 maximises 3's myopic payoff but lowers his probability of being the central agent to 0 and hence his future payoff. Which payoff is higher and hence which incentive domiantes depends on the value of* p*. In this case,* p *has to be higher than* $\frac{1}{5}$ *for 3 to give up future payoff considerations and link to 1. Hence, if* $p > 1/5$ *then 3 will link to 1 or 2. Now, if 1 abstains at* $t=1$ *then at* $t=2$*, the proposer is isolated. We can analyse,as above, agent 2's incentive to link to 1 or some other agent 3. The payoffs for 2 can be written as* $\pi_2(l_2=1) = 1-c+\frac{1}{2}[6]+\frac{1}{2}[6p]$ *and the maximum payoff if 2 links to 3 is* $\pi_2(l_2=3) = 1-c+\frac{1}{2}[5]+\frac{1}{2}[5p]$*. Hence 2 would always choose 1, if* $p > 1/5$*, since in that case, 3 would choose one of 1 or 2. Therefore, at* $t=1$*, the proposer would abstain and save* c *since 2 would choose 1. So, for* $p > 1/5$*, the equilibrium will be a complete star: one where 7 agents would be directly linked to 1 agent, the centre.*

The following Lemmas gives some properties of the equilibrium network configuration.

Denote the graph at the beginning of time t, i.e. before t makes a decision, by g_{t-1}.

Lemma 1 *If $t \in C' \subseteq g_{t-1}$, then C' must be a chain.*

Proof: Suppose not. Let agents $\{k_0, ...t\ \} \in C'$. Since C' is not a chain there must be an agent $k_1, k_0 < k_1 < t$ who proposed to agent $j < k_1$. Let k_1 be the first such agent. Hence k_1+1 is isolated at time k_1+1 and since $k_1+1 \in C'$, he connects to $j < k_1 + 1$ in equilibrium which implies that $k_1 + 2$ is isolated and so on. Continuing the argument it implies that t is isolated at the beginning of period t. Therefore, $t \notin C'$ and we have a contradiction. Hence C' must be a chain.

Let there be $l + 1$ agents in the chain i.e.the length of the chain is l. (distance between t and the farthest agent $\in C'$).

Lemma 2 *If, for any isolated $t, s^*_t = j < t$, then $j = v^{iso}_t(g_{t-1})$ i.e. if an isolated agent t is linking to some agent j who has already moved, then he will choose the myopic payoff maximiser.*

Proof: See Appendix A.

The intuition is that when any agent t chooses an agent $j < t$, he is making j more central and hence making himself non-central at the same time. This implies that the incentive to be central if not at work. This implies a lower future payoff and it is best for agent to maximise his myopic payoff by connecting to v^{iso}_t.

Lemma 3 *If $W_l, W_{l'} \in g_{t-1}, l > l'$, then agent t if isolated will not choose $j \in W_{l'}$.*

Proof: This obtains directly from comparing the payoffs of an isolated agent. Suppose, t is isolated and $s^*_t = j < t$. Then from Lemma 2, we know that $j = v^{iso}_t$. His myopic payoff from connecting to an agent in a W_l is

$$\pi_t(s_t = j \in W_l) = [1 + 2p + 2p^2 + ... + 2p^{\frac{l-1}{2}}]$$

Since the payoff is increasing in the size of the wheel l, the lemma follows.

Lemma 4 *If agents $\{k+1, k+2, ..., k+l_1\}$ form a wheel, then agents $\{k+l_1+1, ..., k+l_1+l_2\}$ will not form a wheel if $l_1 > l_2$.*

Lemma 4 says that at any time t, if $W_{l_1} \in g_{t-1}$, then agents $j \geq t$ will not form a wheel of length $l_2 < l_1$.

Proof: Suppose not. Let $l_1 \geq l_2 + 1$ and let agents $\{k+1, k+2, ..., k+l_1\}$ form W_{l_1} and agents $\{k+l_1+1, ..., k+l_1+l_2\}$ form W_{l_2}. Also let x be the first agent such that $s_x^* = j < x$. By Lemma 3, we know that $s_x^* = j \in g_{x-1} \backslash W_{l_2}$. This implies that

$$\pi_{j \in W_{l_2}}(s_j^* = j+1) = \pi_j^{myo} + \pi_j^f = 2 + 2p + 2p^2 + ... + 2p^{\frac{l_2-3}{2}} + [0]$$

Now consider agent $k + l_1 + l_2$. This agent is connected to a chain $\{k + l_1 + 1, ...\}$. He can link to $k + l_1 + 1$ and form W_{l_2} or link to some $j \in W_{l_1}$. His payoff from $s^* = j \in W_{l_1}$ is

$$\begin{aligned}
\pi_{k+l_1+l_2}(s^* &= j \in W_{l_1}) = 1 + p + p^2 + ... + p^{l_2-2} + 1 + 2p + 2p^2 + ... + 2p^{\frac{l_1-1}{2}} + \pi^f \\
&\geq p + p^2 + ... + p^{l_2-2} + 2 + 2p + 2p^2 + ... + 2p^{\frac{l_1-3}{2}} + 2p^{\frac{l_1-1}{2}} \\
&> 2 + 2p + 2p^2 + ... + 2p^{\frac{l_1-3}{2}} + 2p^{\frac{l_1-1}{2}} \\
&> 2 + 2p + 2p^2 + ... + 2p^{\frac{l_1-3}{2}} \\
&> 2 + 2p + 2p^2 + ... + 2p^{\frac{l_2-3}{2}} = \pi_{j \in W_{l_2}}(s_j^* = j+1)
\end{aligned}$$

Hence, $k + l_1 + l_2$ deviates to link to some $j \in W_{l_1}$.

Lemma 5 *For any t, $\nexists W_{l_1}, W_{l_2} \in g_t$ with $l_1 = l_2$.*

Proof: Suppose not. Say $\exists 2$ wheels of length y and no wheel of length of length $l > y$. Suppose, wlog, given a g_{t-1} agents $t+1, ...t+2y$ formed the two wheels and agents $j \geq t + 2y + 1$ chooses some agent $j' < j$. By Lemma 3, we know agent $j = t + 2y + 1$ is indifferent between agents $t + 1, ...t + 2y$. Therefore the payoff of $t \in W_y$ is

$$\pi_{t \in W_y}(.) = 2 + 2p + ... + 2p^{\frac{y-3}{2}} + \frac{N-t-2y}{2y}[1 + 2p + ... + 2p^{\frac{y-1}{2}}]$$

Now, given g_{t-1}, if the two wheels merge and forms W_{2y}, then $j = t + 2y + 1$ chooses $t \in W_{2y}$. In this case the payoff of agent t is

$$\begin{aligned}\pi_{t\in W_{2y}}(.) &= 2 + 2p + ... + 2p^{\frac{2y-3}{2}} + \frac{N-t-2y}{2y}[1 + 2p + ... + 2p^{\frac{2y-1}{2}}] \\ &> \pi_{t\in W_y}\end{aligned}$$

Hence in equilibrium, all agents belonging to the two wheels of size y are better off by offering to the next agent and accepting offers to form a wheel of size $2y$.

3.1 Special Cases

We first consider extreme values of the decay factor p and characterize the equilibrium networks in those cases.

Proposition 1 *There exists a $p^* < 1$ such that for $p \in [p^*, 1]$, the unique subgame-perfect equilibrium structure is a complete star.*

Proof. See Appendix A. ■

Proposition 1 characterises the subgame perfect equilibrium network structure for p high enough. When p is very high, the loss in benefit from an indirect source is not too much. In this case, the payoff from being a direct neighbour of the central agent is not too different from that of the central agent himself. The gain in payoff and hence, the incentive to become the central agent (i.e. centre) is not strong. It is still true, though, that as long as $p < 1$ an agent would prefer to be the centre. But in order to do so he has to compete with the agent before him and this might be risky. Consider Example 1. Agent 3 could link to 4 making sure that some future agent, say 5, will be indifferent between 1,2,3, and 4 but with probability 1/2, agent 3 is isolated from $N-2$ agents who form the star and 3 gets a much lower payoff. When $p \simeq 1$ this loss is too high and hence 3 would prefer linking to 1 or 2 even though he will be a peripheral node.

We use backward induction to show first, that given any graph at time t an isolated agent would either abstain or choose some agent $j < t$ who had already moved. This implies that no agent starting from time 1 would choose to link to an agent who has not moved yet. Hence agent 2 either abstains or links to 1. The same holds for agent

3,4,5,....n. Also, an agent will abstain provided all agents before him have abstained and the cost c is higher than their expected gain from not linking. This is so because the payoff from being the centre of a star is higher than any other position. The expected gain from abstaining will be positive only when there is positive probability of being the centre of a star which obtains only when agent t is symmetric with all agent $k < t$ with respect to his links. If agent $k < t$ have links, then an isolated agent t has zero probability of being the centre of a star if he abstains. Hence, he would not abstain and will choose some $j < t$.

The next proposition characterizes the equilibrium network structure for p low enough.

Proposition 2 *There exists a p^{**} such that for $p < p^{**}$, the unique equilibrium network structure is a wheel of length $L < N$ with a local star where $L = \lfloor \frac{N+1}{2} \rfloor$.*

Proof. See Appendix A ■

The intuition for this result is that when $p \approx 0$, the difference in payoffs from being the centre and the peripheral node is very high. For a low p, each agent has the incentive to compete with already connected agents and hence become the centre with positive probability by forming a wheel. This incentive decreases for agents moving later since the payoff from becoming the centre decreases. This is so because the number of future entrants decreases. For p close to 0,however, even agent $N-1$ has such an incentive, provided the play reaches such a subgame. This subgame will not, however, be reached on the equilibrium path. The proof of proposition 2 identifies the condition under which agent $N-1$ would have such an incentive. It goes on to use Lemma 4 and 5 to argue that in equilibrium agent $N/2$ would link to 1 thereby forming a wheel of length $N/2$.

Note that for p close to 1, we get a periphery sponsored or a mixed star with equal probability. Even in the mixed star, only one link is sponsored by the centre. This is consistent with Hojman and Szeidl (2004) who highlight periphery sponsorship as a robust feature in models with decay. Goyal and Vega-Redondo (2007) refers to this structure as the 'hybrid cycle-star'. They present a simultaneous move model of network formation without decay where agents compete for intermediation rents. However, in equilibrium, the hybrid cycle-star is ruled out due to deviations by agents wanting lesser number of intermediaries and hence, more rent. This architecture for $p \simeq 0$ is also interesting since it is similar to the hub-and-spoke architecture which is observed in real-life networks of biotech firms.

3.2 General p

For general levels of decay, all the afore mentioned incentives will be at play and to various degrees. Note that as p increases, the agents at the end of the play will have less incentive to form a wheel since expected gain from being the centre decreases as p increases. This in turn will increase the potential future payoff from being the centre for the earlier players. This increase in incentives might result in some link rejection at the beginning of the order since the rejector expects to form a smaller wheel (hence probability of being centre higher) with larger number of agents as spokes. This increase in such incentives has to be high enough to compensate for the loss in myopic payoff from a smaller wheel. When p is high enough, however, the number of agents at the end of play who do not have incentive to form a wheel increases and in the limit as $p \to 1$, even agent 2 has no such incentive and the star obtains as the equilibrium structure.

Note that the magnitude of the two opposing incentives of maximising current payoff by connecting to the most connected agent and maximising future payoff for a single agent depends on the value of p. It also depends on the history and the number of agents.yet to move. Either one of the incentives might dominate for an arbitrary value of p. The following example, however, shows that the equilibrium network is not necessarily connected, i.e. agents might form disconnected groups. This is so because an agent might have the incentive to reject a link in some subgame if it increases his probability of being the centre. The rejection does not occur on the equilibrium path since no agent would propose a link if it would be rejected.

Example 2 *Possibility of two components in equilibrium.*

Consider the equilibrium as depicted in figure 4. In equilibrium, 2 is rejecting 1's link and forming a W_3 with 3 and 4 while agents 5-8 choose one agent in the wheel. Now for this to be equilibrium p has to be such that no agent t has the incentive to deviate given the graph at time t. Using backward induction, this translates into the

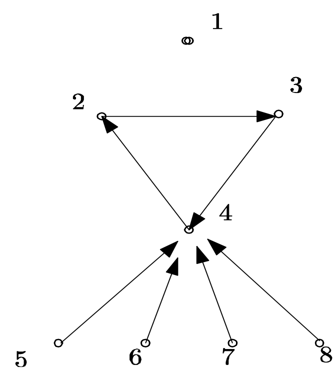

Figure 4: Two Components

following conditions:

(i) For $t > 5$,*given* g_{t-1}, *agent* t *does not deviate*

(ii) Given g_4, *agent* $5, 6, 7$ *does not form* W_3

(iii) Given g_4, *agent* $5, 6, 7, 8$ *does not form* W_4.

(iv) If 2 *accepts* 1*'s link and* $1, 2, 3$ *forms* W_3, *then either*

a) $4, 5, 6$ *forms* W_3 *or b)* $4, 5, 6, 7$ *forms* W_4.

(v) 2 prefers W_3 *with 3,4 to* W_4 *with 1,3,4.*

Condition (i) is self-explanatory. Condition (iii) holds for all p *since the last agent* 8 *will not complete the wheel. Lemma 4 implies that condition (ii) must hold since if 5,6,7 does form* W_3, *then agent* 4 *will not close the first wheel but will propose to* 5. *Now, by way of contradiction, suppose condition (iv) does not hold. In that case, if 1,2,3 form* W_3, *future agents cannot form another competing wheel since some agent before 8 will deviate. Since this implies that 2 shares the probability of being centre with 2 others AND gets an extra agent (4) as a spoke, 2 is better off accepting 1's link. Condition (iv) (either one of (a) or (b)) ensures no such deviation by 2. Even if condition (iv) holds, agent 2 has yet another possible deviation of accepting 1's link and forming* W_4 *with 1,3 and 4. Condition (v) says that such a deviation is not profitable since 2 prefers rejecting 1's link to form the smaller wheel (and have a* $\frac{1}{3}$ *probability of being the centre instead of* $\frac{1}{4}$*).*

The conditions imply bounds on p. *The weakest necessary conditions on* p *are given by:*

(i) $0 < p < 1$

(ii) $p > 1/4$ *or* $p > 1/22$

(iii) $0 < p < 1$

(iv).a) $p < \frac{1}{13}$; *b)* $p(1+p) < 1$ *and* $14p - p^2 < 1$

(v) $p + 3p^2 < 1$

Any one condition of (ii) and (iv) along with (v) would qualify for equilibrium conditions. It can be verified that any $p \in [\frac{1}{22}, \frac{1}{13}]$ *satisfies (v). Hence the given network is an equilibrium for all such p. Also note that given the mixing rule A1 and the tie-breaking rule A2, there is a unique best response at any subgame which ensures that equilibrium is unique for a given set of parameters. So, the given structure is the unique equilibrium for* $N = 8, p \in [\frac{1}{22}, \frac{1}{13}]$.

The conditions in Example 2, however, need not be satisfied for a N higher or lower. For example, with p fixed at 0.3, the conditions do not hold for $N = 20$ or $N = 6$ and this is not the equilibrium network. Also, it is instructive to see that the equilibrium conditions for a network with 2 agents in the first component involves different constraints. For example, let us consider a similar example but with $N = 9$ where agents 1 and 2 form the first component with the others forming the same architecture as the 2nd component of example 1. In this case, the necessary and sufficient conditions are $p < \frac{1}{13}, p > \frac{1}{22}, 10p + 15p^2 < 1$. In this case, for all $p \in [\frac{1}{22}, \frac{1}{13}]$ and $N = 9$ the equilibrium is 1-2; 3-4-5 in a W_3, 6-9 as spokes.

Proposition 3 *Let there be* $M \geq 1$ *components* $(C_1, C_2, ..., C_M)$ *in the equilibrium structure, formed in that specific order. Let the number of agents in each component be* $n_1, n_2, ..., n_M$ *respectively with* $\sum_{y=1}^{M} n_m = N$. *Then,*

(P1) Each component C_m *is a wheel with a local star* $C_m = W_m + S_m$ *with* $\#W_m \leq \#S_m$.

(P2) For $m > m'$, $\#W_m > n_{m'}$

P1 says that any equilibrium component must be a wheel with a local star with the number of agents in the wheel weakly smaller than the number of spoke agents. Note that the wheel size may be 1, which is a star. This obtains when p$\simeq$ 1, in which case we get $M = 1$ with only 1 agent in the wheel. For $p \simeq 0$, $M = 1$ with

$\#W_M = N/2 = \#S_M$. For $M > 1$, it must be the case that on some off-equilibrium path some agent k would reject a link. This rejector would have some incentive to do so only when in the subgame following a rejection (and for a strategy that leads to the equivalent graph) he has a positive probability of being chosen as the centre for future agents. This will be the case only when the rejector k along with agents $\tau > k$ form a wheel of some size d. Also, if the number of agents connecting as spoke agents are smaller than number of agents in the wheel, the expected payoff of k is low. In such a case, k could deviate to accept the link and form a wheel of the same size d one period earlier.

The first step of proof of P1 is to show that the last component must be a wheel with a local star (Lemma 6). Then using backward induction, it proceeds to show that the second last component should also be a WS and so on (Lemma 7).

Lemma 6 *Suppose there are M components in equilibrium. Let them be $C_1, C_2, ...C_M$, formed in that specific order. The last component C_M is a wheel with a local star.*

Proof: The proof is by contradiction. If the last component is not a wheel with star then it implies that the agent who started this component, say, t is not chosen as the centre with a positive probability at any subgame following his move. Note that if t is the agent who started the last component then his strategy is to link to $t+1$. Lemma 6 shows that if t cannot be the centre with a positive probability then the same is true for any isolated agent moving in any subsequent period. Hence the best strategy for such an agent $j > t$ is not to form a new component. This in turn would imply that some agent $k > t$ who belong to the chain starting from t would benefit from linking to t, thereby forming a wheel. Sunsequent agent would then link to someone in the wheel. (See Appendix A for a formal proof).

Lemma 7 *Suppose there are $M > 1$ components in equilibrium. Let them be $C_1, C_2, ...C_M$, formed in that specific order with $C_M = WS$. Then C_{M-1} is also a wheel with local star.*

The proof of Lemma 7 is very similar to that of Lemma 6.

P2 says that the number of agents forming the wheel of each component must be greater than the total number of agents in the previous component. This is due to

Lemma 2 and 3. By P1, each components is a wheel with a star. Now, in order for this to form, the first spoke agent in C_m, say k_{m1}, must find it profitable to connect to an agent $k' \in W_m$ and not to the centre of C_{m-1}, say k''. Now if $W_j \subset C_j$, then there are agents in C_j connected as spoke agents, which implies that distance to these spoke agents through k'' is the smallest possible i.e. 1. If k_{m1} connects to $k' \in W_m$, then the number of agents in W_m has to be large enough so that it gives a higher payoff than the payoff from k''. (If not, then the first spoke agent connecting to W_m will deviate to connecting to the central agent of C_{m-1} since the myopic payoff in that case is higher). A necessary condition for that is $n_j < \#W_m$. Note that lower the p, the greater the required difference between $\#W_m$ and n_j. In the extreme case of $p \simeq 0$, the difference is so high that two components are not possible.

Corollary: Each subsequent components are increasing in size; i.e.if $m > m'$, then $n_m > n_{m'}$

P1 implies $W_m \leq n_m$. Combining with P2, we get $n_{m'} < \#W_m \leq n_m$, for $m' < m$.

4 Welfare and Inequality

In this section we focus on the efficient network architectures for different values of the decay factor. An *efficient* network architecture is defined as one that maximises the sum of payoffs of all agents. Note that when there is decay i.e.$p < 1$, increasing distances between agent reduce their payoffs. Hence, the efficient structure should minimize the distances between agents.

Proposition 4 *The efficient network architecture is (i) a star if* $p > \frac{2-c}{2}$ *and (ii) a wheel of length 3 with a local star if* $p < \frac{2-c}{2}$.

Proof. (See Appendix A). ■

Note that when $c < 1$, all agents must be connected (in a single component) in an efficient network since the net benefit from connecting an isolated agent to some other agent is always positive. Now, N agents have to be connected with at least $N-1$ links, Also, given the capacity, the maximum number of total links in a network with N agents is N where all agents initiate a link. Note that a single component with N

agents connected with N links must have the WS architecture. The proof proceeds to show that the aggregate payoff in a WS_k architecture is higher than that in a WS_{k+1} architecture for $k \geq 3$ which implies that among networks with N links, the efficient one is WS_3. This involves a cost of Nc. On the other hand, among networks with $N-1$ links, the star network minimizes distances between agents through the centre who has the most links. Any redistribution of links from the center to a peripheral nodes would only increase the distances between all the agents in the network and hence lower aggregate payoff. Note that the star involves a total cost of $(N-1)c$ and is the efficient one among networks with $N-1$ links. Also note that the condition $p < \frac{2-c}{2}$ can be rewritten as $c < 2-2p$. The two expressions represent the gain $(2-2p)$ and the cost c of an additional link between any two peripheral nodes. If $p < \frac{2-c}{2}$, WS_3 obtains as the efficient structure since it does not increase the distances between agents (as compared to a star) but gives a positive net benefit to the two peripheral agents connected by the N^{th} link.

This also shows the tension between efficiency and the equilibrium. The conflict disappears for $p \simeq 1$, particularly $p > \widetilde{p} = Max\{\frac{2-c}{2}, p^*\}$, since the equilibrium is a star which is also efficient. But for lower values of p, the equilibrium is inefficient as seen from the equilibrium for $p \simeq 0$. The equilibrium might also be effcient for a range of p below $\widetilde{p}$. This happens if there is a $p < \frac{2-c}{2}$, for which the equilibrium structure is WS_3. In general, the relationship between equilibrium efficiency and p is non-monotonic.

Another issue that arises when thinking about welfare is that of inequality among the agents in the network. The individual payoffs in a network depends both on the structure and the decay factor. When the structure is a star, it is true that the centre is asymmetric with all other players because of his direct links but the extreme inequality in connections does not translate to an equivalent inequality in payoffs because this architecture is formed when p is close to 1. In this case, the maximum possible payoffs to the centre and a peripheral player are $(N-1)$ and $1+(N-2)p-c$ respectively. Hence the highest possible inequality, measured by the difference, is of the order of the cost level c. For small p however, the inequality is high. Consider the example with $N = 8$ and $p \simeq 0$. The equilibrium network, in this case, is a wheel of length 4 with agents 5-8 as spoke agents. In this case, the highest payoff is obtained by the centre of the local star and is equal to $6 + p$ while the the spoke agent get the least payoff of $1 + 5p + p^2$. For $p \simeq 0$, the maximum difference is of the order of $5 - \varepsilon$ which is far greater than c. As with the connectedness of the network, the inequality in payoffs is not monotonic in p. Using Example 2, we see that the least payoff, in this case, is zero (for the isolated agent) and the maximum payoff is $6 - c$ obtained by agent 4. Hence the inequality in this case is greater than the case of $p \simeq 0$. Note that the first agent is worst off in this example followed by the end agents. The middle-ranked agents get the most payoff. Hence, the value of decay not only determines the network structure but also the relative payoff advantages of agents moving at different stages of the game.

5 Heterogeneous Agents

In most networks agents are not likely to be symmetric and identical. Individuals differ in their ability to provide favours as well as in the time they spend on social relationships. In this section we consider the first type of heterogeneity by making the benefit an agent provides to the network, individual-specific. In particular, we consider two types of agents according to whether they provide a high benefit θ_h or a low one θ_l. The high (low) type could represent those agents in an ethnic network who are (not so) well-placed in the labour market. Examples include a manager of a local bank who can help in obtaining credit and a labourer who can lend a hand in farm-work for a friend. Introducing heterogeneity, therefore, brings forth the issue of whether to connect to an isolated high type agent or a well-connected low type agent. This of course, would depend on the difference $\theta_h - \theta_l$. Also, the proportion of high types is likely to matter since low types may not be succeed to become well-connected in presence of a high proportion of θ_h-agents. In this section, we analyse the same game when there is a single high type in the population. We also assume $\theta_l = 1 < \theta_h$.

Proposition 5 *With one θ_h in the population and $p \simeq 1$, the equilibrium network structure is a complete star or a WS_3 with the θ_h as the centre if $\theta_h > 1 + p$.*

Proof. See Appendix B. ■

The intuition for this result is similar to that of Proposition 1 with, the incentive to compete to become the central agent, even lower in this case due to the presence of θ_h. This is so firstly because a direct connection to the high type is worth more than to a low type or an indirect connection to θ_h. Moreover, in presence of a high type, it is harder for a low type to compete, since, given the value of θ_h, the relative number of direct links a low type needs to become the central agent at any stage is very large. The two effects work in lowering the incentive to compete and for $p \simeq 1$, the incentive to directly connect to θ_h dominates and a θ_h*centred star* obtains. The proof, however, needs to consider all possible subgames that can arise and is lengthy. Readers can see Appendix B for the detailed analysis of relevant subgames.

When the rate of decay is very high, however, the network is starkly different from the one in the homogeneous case. In fact, the equilibrium architecture is again, a complete star with θ_h as the centre. The intuition is quite simple. When there is a very high level of decay, a high type agent, even if isolated, is valued more than a highly connected low type since indirect connections give a benefit close to zero. Agent N would always connect to θ_h even when the latter is isolated and low types proposing in earlier stages know they would not be chosen as centres. Hence, earlier agents maximise their myopic payoffs by connecting to the high type agent resulting in a $\theta_h - centered$ star.

Proposition 6 *When $p < \widetilde{p}$, the equilibrium network is a complete $\theta_h -$centered star.*

For intermediate levels of the decay factor, it is possible for low types to form a group to compete with the high type agent who is kept isolated. This incentive depends on the values of p and θ_h.Intuitively, the value of θ_h should not be too high for a low type to compete. A not-so-obvious observation is that low types might not succeed to keep the high type isolated even when θ_h is too low. This is because when θ_h is very low, some connected low type agent l might deviate to connect to θ_h and become the central agent himself. This cannot happen with a high θ_h since in the case of a deviation, it is the high type who becomes the central agent. For example, with $N = 8$, and $p = 0.2$ if $\theta_h \in [1.24, 1.866]$, then the equilibrium network is such that the high ability agent θ_h remains isolated with the seven low type forming a separate component similar to Example 2. If θ_h is higher than 1.866 the opportunity cost of keeping him isolated is too high for some agent and he deviates. This effect unravels and all agents connect to θ_h. Surprisingly, θ_h cannot remain isolated even when θ_h is lower than 1.24. This is because at the subgame where 2 connects to 3 and 3 to 4, say, agent 4 will have the incentive to link to θ_h, because in that case, agent 5 onwards would link to agent 4! We need θ_h to be low for a low type agent to be the centre in the presence of a connected high type. Hence, when θ_h is too low, agent 4 would deviate and hence agent 2 will too.

6 Robustness

6.1 Random Proposers

One restrictive feature of our model is the specific order of moves of players. In particular the fact that period t's responder becomes next period's proposer does have some implications on the equilibrium architecture. This order reduces the possible types of histories that agent t faces at any stage t and simplifies the analysis for $p \simeq 1$ and $p \simeq 0$. It is also responsible for possible rejection by a responder in a subgame (leading to 2 components in equilibrium) since the order gives some power to the responder by making him the proposer of the in the next stage. If the order of proposers is made completely random at each stage, then for $p \simeq 1$ however, our result will not change and a complete star network would form with either the 1st or 2nd mover as the centre since it is still the case that the incentive to become the centre is very small for each agent.

Proposition 7 *With the order of proposers completely random in each period, the equilibrium network is a complete star for $p \simeq 1$.*

Proof. The proof is similar to the proof of proposition 1. The difference is that the payoff functions in this case are expected payoffs, the expectation taken with respect to the random order. The payoffs in proposition 1 are valid for a unique realisation of the order which chooses the responder of a period as the next proposer. For all other realisations of draws for proposers each period, the payoff functions are modified but the inequalities still hold. We will point out the differences in the payoffs and show that the inequalities hold in each case. See Appendix B for the details. ■

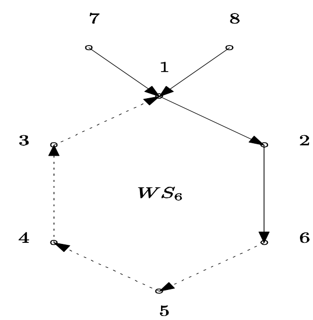

Figure 5: Random proposers, $p \simeq 0$

The difficulty with random proposers arises in analysing the case for lower p. For $p \simeq 0$, the order considered in the model facilitates formation of the wheel since there is no randomness in the future play at any stage. With random proposers, the responder of stage t might not be the proposer at $t+1$ in which case the wheel of size $N/2$ might not be formed. We have an example with 8 agents which shows that when p is very low, then with random proposers, the outcome is not deterministic. With positive probability the architecture could be any of $WS_{N/2}$ and $WS_{k>N/2}$. For example, it could be as in Fig 5 with positive probability. From Fig 5, notice that 2 proposed to an agent who accepted the link but did not get the chance to propose in period 3. Some other isolated agent was chosen as the proposer and since the incentive to form a wheel dominates for $p \simeq 0$, agent 3 linked to 1 and 4 linked to 3 and so on. The agent 2 proposed to was chosen as the proposer in period 6 and she closed the wheel so that agents 7 and 8 are spoke agents. The architecture could also not have a wheel if an agent connected in the chain gets to propose only in the last period. For a general p, the order of random proposers would take away power from the responder since he might be chosen to propose only at the end of play. In this case, we might not see rejections in any subgame and a single equilibrium component unlike example 1. See Appendix B for the details of the example (Example 4).

6.2 Increased Capacity

A second feature of our model is the capacity constraint, which by itself is reasonable in most social environments. For example, consider a community of agents of the same ethnic group where agents participate in the formal labour market as well as in their ethnic network. They earn a market wage and get some non-market benefits from their ethnic network. The benefits from their network could be information about better job prospects, a favour done by a friend or simply dinner invitations from neighbours. In order to form friends, however, a person has to put effort in terms of time or money. Given the agent's participation in the labour market, he probably would not want to spend too much time on such effort and hence have a capacity on the number of network links he himself initiates. Of course, different agents might have different opportunity costs and hence invest different amounts in social networking. This heterogeneity in capacity might be potentially related to heterogeneities in agents' abilities. Agents could be of two types as in section 5 and a high(low) type agent with a well placed job could earn more(less) in the labour market and also provide higher(lower) benefits to his friends in the network. In the basic model, the capacity is a single link, which is albeit, an extreme one. This restriction, though not realistic, does allow us to get clear patterns and architectures for some decay factors. The extreme level of constraint is particularly important to reinforce the strategic concerns when agents are non-myopic. With an increase in capacity agents still face the same tradeoff but the conflict between the current and the future reduces. For example, if we remove the constraint altogether i.e. agents can initiate any number of links, then for $c < 1$ the complete network obtains in equilibrium with the earlier agents having the advantage of some level of free-riding. We present here an example which shows how the network structure is modified when the capacity is increased to two and p is close to 0.

Example 3 *Let $N = 10$ and $p \simeq 0$. Let the capacity be two links, each at a cost c. Suppose the graph at the end of $t = 4$, g_4 is as depicted in Fig 6. Denote the structure by GW_4 (a generalised wheel). Hence agents 1-4 are symmetric with 3 direct links each. Since p is close to 0, we know that agents 5-9 have the incentive to compete with 1-4 in order to be chosen by the last agent 10. First note that we cannot have a component of 5 agents where all five are symmetric. Hence if agents following 4 want to compete they can do so only by forming another component exactly like g_4. Suppose agents 5-7 have such an incentive and hence propose links such that the graph is g_7 in Fig 6. Agent 8, in this case, can propose to 5 and 6 to form another GW_4 or*

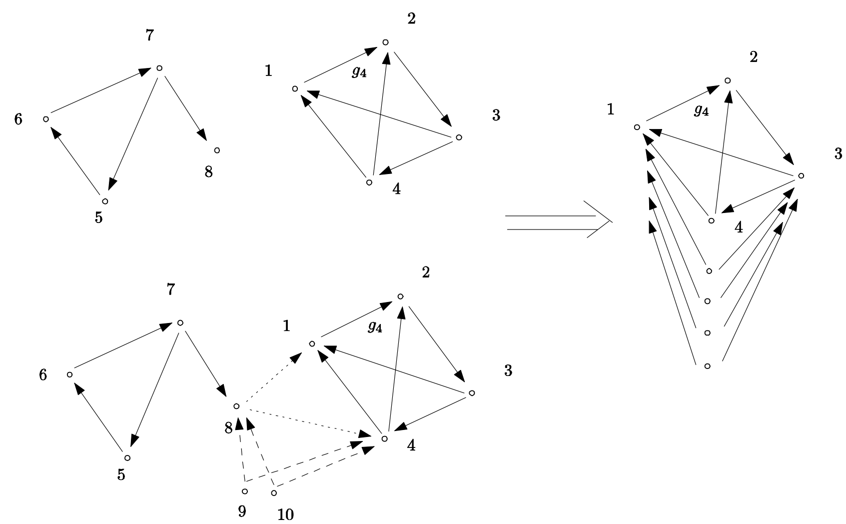

Figure 6: Smaller 'hub' for $p \simeq 0$

link to any two agents from among 1,2,3,4. The payoff to 8 from the former choice is $3-2c+\frac{2}{8}(1+3p)$*. To obtain the payoff of 8 from linking to1 and 4 (say) we need to know the equilibrium outcome following such a strategy. If 8 links to 1 and 4, then it can be verified that for the isolated agent 9 the strategy of linking to 8 and either one of 1 and 4.strictly dominates all other strategies. Hence, agent 8 is definitely chosen by 9 and 10. This implies that the payoff to 8 from linking to 1 and 4 is* $5+4p-2c$ *which is strictly greater than* $3-2c+\frac{2}{8}(1+3p)$*. Hence agent 8 facing* g_7 *will not form a* GW_4*. This implies that 5-8 cannot compete and form another* GW_4 *and by backward induction, agent 7,6,5 will deviate and choose two of 1,,2,3,4 at the state* g_4*. Hence, agents 1-4 form* GW_4 *and agents 5-10 connect to 2 of the first four agents as centres and themselves remain peripheral. So, when* $p \simeq 0$*, an increased capacity results in a configuration with a smaller and denser 'hub' with higher spokes as compared to a capacity of one.*

The network structure for p close to 1 is easier to characterize. When decay is low, then incentive to become central is very weak and the proof of Proposition 1 goes through in exactly the same way. Any isolated agent in this case would want to maximise his myopic payoff and connect to the most central agent of the current stage. This would lead to the formation of a generalised star with two centres. The

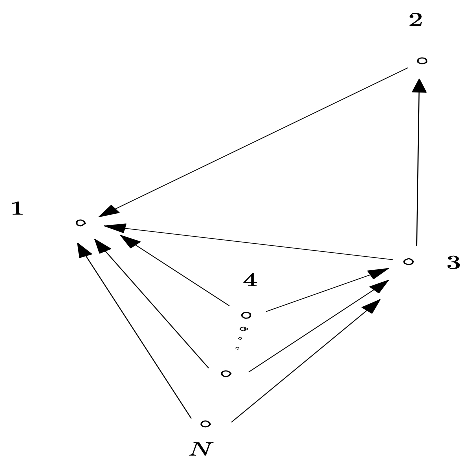

Figure 7: A star with two centres

network is depicted in Fig 7. The initial agents would enjoy some level of free-rising in the sense of not using full capacity. For example, with $N = 10$ and $p \simeq 1$, agent 1 would abstain followed by agent 2 who would propose only one link to 1. At $t = 3$, agent 3 would propose to both 1 and 2 and hence at $t = 4,$ agents 1,2 and 3 would be symmetric. The isolated agent 4 would choose two of these three agents since the incentive to compete is weak. So would all other agents resulting in the generalised star.

7 Discussion and Conclusion

This paper focuses on the network architectures that arise in equilibrium when agents are farsighted and capacity constrained and there are decreasing returns (represented by the decay factor p). Our first result of a mixed or periphery-sponsored star for p close to 1 is consistent with the results of Bala and Goyal (2000), Galeotti et al. (2006) and Kannan et al (2007) who show the emergence of mixed and periphery sponsored stars in models with no or a small amount of decay. Most of the work in the literature do not have a characterisation for a general p with the possible exception of Hojman and Szeidl (2004) who highlight the emergence of a periphery-sponsored star in a one-shot game whenever there is decay. The result of a wheel with local star for p close to 0 has some similarity to the flower networks in the one-way flow model with decay in BG (2000). Galeotti (2006) also finds a wheel with a (multiple) centre-sponsored star(s) as the equilibrium network for some parameters in a one-way flow model with heterogeneous values and costs and without decay. The higher value agents belong to the wheel and become the centres of the stars. The formation of wheels is similar to Watts (2001b) who proposes a dynamic model of network formation and shows that circle networks may be formed by a strategy similar to trigger strategy. For a general level of p we show that there might be two components in equilibrium. The reason for this is that an agent wants a positive probability of being the centre and also share this probability with the least number of agents possible. This incentive to share with less agents might cause a rejection of a link in some (off-equilibrium) subgame. Watts (2001a) also points out in a discussion that one might expect rejections in some off-equilibrium path because of the forward looking behaviour of agents. This rejection decreases the probability of the formation of a star when agents are not myopic in her model since the payoff from being a central agent is lower than being non-central and hence no agent wants to be the centre. This, however, is just a conjecture in Watts (2001a). The other papers abstain from this issue of consent altogether by looking at one-shot games of network formation. Another point to note is that, in addition to a standard disadvantage from being too late in the order, there is also the possibility of disadvantages of being too early, as seen in Example 2.

In conclusion, we stress that the types of networks that form under different economic and social environments is important since the exact structure of these networks significantly affect economic outcomes. We find that the shapes of equilibrium networks depend crucially on the rate of decay in the payoff function. With a low decay a complete star network is formed while with high levels of decay the equilibrium architecture involves a wheel with a local star. This incidentally resembles the hub-and-spoke architecture observed in studies of R&D firms and social groups. For intermediate levels of decay, we have an example where some agents might be isolated from a bigger component, which can explain segregation among ex-ante homogeneous agents into different groups.

A promising area of further work is to analyse the effect of a changing marginal cost of link formation when agents have a capacity of r links, $1 < r < N-1$. One could also analyse a form of heterogeneity encompassing both one's value in the network and one's capacity. For example, suppose an agent is of some particular ability θ which is also the value of benefits he provides to the network. One situation could be that a high θ agent has a higher wage in the labour market and hence a higher opportunity cost of investing in social links. This implies that the optimal level of social links that a high type intiates would be less than a low type. Hence the heterogeneity in innate ability determines both one's value in the network and his capacity which are, in this case, negatively related. The other possibility is that a high type agent is overall more efficient in the market and hence has more time available for his social network which implies a positive correlation between one's level of human capital and his social investment. This would be useful to explore what kind of networks emerge with both types of heterogeneity among agents. It could possibly throw some light on why agents in different societies with different norms and cohesion invest differently in their social connections.

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Appendix A: Proofs for section 3

Lemma 2 *If, for any isolated $t, s_t^* = j^* < t$, then $j = v_t^{iso}(g_{t-1})$ i.e. if an isolated agent t is linking to some agent j^* who has already moved, then he will choose the myopic payoff maximiser.*

Proof: Note that the statement is true for agent N. Since N is the last agent, if isolated he chooses an agent $j = \arg\max_{j \neq N} \Pi(j^*, g_{N-1}) = \arg\max_{j \neq N} \pi^{myo}(j, g_{N-1}) = v_N^{iso}(g_{N-1})$. Suppose that the statement is true for all agent $k > t$.

Also assume that it is not true for agent t and $s_t^* = j^* < t, j^* \neq v_t^{iso}(g_{t-1})$. This implies that $\Pi_t(j^*, g_{t-1}) > \Pi_t(v_t^{iso}, g_{t-1})$. But we know that by definition, $\pi_t^{myo}(j^*, g_{t-1}) < \pi_t^{myo}(v_t^{iso}, g_{t-1})$. Therefore, it must be the case that $\pi_t^f(j^*, g_{t-1}) > \pi_t^f(v_t^{iso}, g_{t-1}) \geq 0$. Also since t is isolated, we know that from $t+1$ onwards he is not the myopic payoff maximiser and hence cannot be the centre. Hence, $\pi_t^f(j^*, g_{t-1}) \leq xp$, where x is the number of future agents who link to this component. Note that since, $\pi_t^f(j^*, g_{t-1}) > 0$, there must be some agent $j > t$ who connects to the component g_{t-1}. Suppose, x such future agents connects to j. Hence, $\pi_t^f(j^*, g_{t-1}) = xp$. Take k to be the first of such future agents. Since k connects to $j^* < k$ it must be that $\Pi(j^*) > \Pi(j' > k)$ and $j^* = v_k^{iso}$. Now, suppose agent t links to $v_t^{iso}(g_{t-1})$ instead. Then the additional link to $v_t^{iso}(g_{t-1})$ ensures that $v_t^{iso}(g_{t-1}) = v_{t+1}^{iso}(g_t)$. In particular consider agent k. Since he is the first agent to connect to the component g_{t-1}, and he connects to the myopic payoff maximiser in the component, k must connect to $v_t^{iso}(g_{t-1}) = v_{t+1}^{iso}(g_t)$. Hence the future payoff to agent t, $\pi_t^f(v_t^{iso}, g_{t-1})$ is xp which is not less than $\pi_t^f(j^*, g_{t-1})$. Hence, $\Pi_t(j^*, g_{t-1}) > \Pi_t(v_t^{iso}, g_{t-1})$ cannot hold.

Hence the statement is also true for agent t.

Proposition 1 *There exists a p^* such that for $p \in [p^*, 1]$,the unique subgame-perfect equilibrium structure is a (complete or incomplete) star.*

Proof: *Step 1:* We claim that any agent $t > 1$, if isolated chooses v_t^{iso}.

This statement is obviously true for $t = N$.

Suppose that the statement is true for agent $\tau \in \{k+1, k+2, ..., N\}$. We will prove it true for agent $\tau = k$.

Agent k has two options:

i) $s_k = k + 1$

ii) $s_k = j < k$

Note that if $s_k = j < k$, then $s_k = v_k^{iso}$ since in that case the myopic payoff π_k^{myo} is maximised and $v_k^{iso} = v_\tau^{iso}, \tau > k$ and hence all future agent choose v_k^{iso}. This implies that the future payoff is $p(N-k)$ which is the maximum possible future payoff given that k is isolated. (i.e. if k chooses some $j < k$, then $k \neq v_{k+1}^{iso}$ and $\pi^f \leq (N-k)p$). We can write the payoff from (ii) explicitly as

$$\pi_k(v_k^{iso}) = V_{k-1} + p(N-k)$$

where $V_{k-1} = 1 + [a]$ denotes the maximum total payoff from agents $1, 2, .., k-1$ (i.e. through v_k^{iso}).

Now the payoff from (i) depends on the equilibrium strategy of the players starting this subgame. Denote the subgame by G_k.

Case I: On the equilibrium path, let j_0 be defined as the first player such that $s_{j_o} = j < k; j_0 \in \{k+1, k+2, ..., N\}$[9]

Hence, $j_0 + 1$ is isolated and chooses $v_{j_0+1}^{iso} \neq k$. In this case

$$\begin{aligned}
\pi_k(k+1) &= 1 + p + p^2 + ... + p^{j_0-k-1} + p^{j_0-k}V_{k-1} + (N-j_0)p^{d'} \\
&= 1 + p + p^2 + ... + p^{j_0-k-1} + p^{j_0-k}(1 + [a]) + (N-j_0)p^{d'} \\
&= 1 + (p + p^2 + ... + p^{j_0-k-1} + p^{j_0-k}) + p^{j_0-k}[a] + (N-j_0)p^{d'} \\
&< 1 + (j_0 - k)p + [a] + (N-j_0)p = \pi_k(v_k^{iso})
\end{aligned}$$

where $d' = d(v_{j_0+1}^{iso}, k, g)$.

QED

Case II: As before, define j_1 as the first player who chooses some agent j', $k < j'$ $< j_1 - 1$ on the equilibrium path. Since, $j' \neq k$, $k \neq v_{j_1+1}^{iso}$. The maximum payoff for k is when all agents $\{j_1+1, ..., N\}$ connect to some agent j'' such that $d(k, j'') = d' = 1$. In this case,

$$\pi_k(k+1) = 1 + r_1 p + r_2 p^2 + ... + r_q p^q$$

where $1 + r_1 + r_2 + ... + r_q = N - k$

[9] j_0 exists since agent N would link to some player $j < N$.

Now, it is immediate that

$$\begin{aligned}
\pi_k(k+1) &= 1 + r_1 p + r_2 p^2 + ... + r_q p^q \\
&\leq 1 + (N-k-1)p \\
&< 1 + (N-k)p \\
&\leq 1 + [a] + (N-k)p \\
&= \pi_k(v_k^{iso})
\end{aligned}$$

Case III : Suppose agent $k+l$ chooses k, hence forming a wheel. The best situation for k is when the probability of being chosen by agents $\tau > k+l$ is positive. Let us focus on that situation. In equilibrium the wheel contains $(l+1)$ agents and $N-k-l$ agents choose $j \in W_l$ randomly.

Then

$$\pi_k(k+1) = [2 + 2p + ...\frac{l}{2} \text{ terms}] + \frac{N-k-l}{l+1}(1 + p + p + p^2 + p^2 + ...l+1 \text{ terms})$$

Recall that

$$\pi_k(v_k^{iso}) = 1 + [a] + p(N-k)$$

Now, both payoffs are monotone increasing in p.

For $p = 0, \pi_k(k+1) > \pi_k(v_k^{iso})$

For $p = 1, \pi_k(k+1) = N-k < 1 + [a]|_{p=1} + N - k = \pi_k(v_k^{iso})$

Hence there exists a $p_k^* < 1$, such that for $p > p_k^*$, $\pi_k(v_k^{iso}) > \pi_k(k+1)$ and hence $s_k^* = v_k^{iso}$.

Since this is for a general k, we can conclude that for $p > p^* = Max\{p_N^*, p_{N-1}^*, ..., p_2^*\}$, any isolated agent $t > 1$ would connect to v_t^{iso}.

Step 2: We argue that agent 1 would abstain so that 2 is isolated and 2 chooses 1 by Step 1.

Given a p high enough, 1 knows that if 1 abstains then 2 is isolated and chooses 1. In that subgame, 3 is isolated and chooses one of $\{1, 2\}$. Hence the expected payoff of 1 from the strategy ϕ is

$$E\pi_1(\phi) = \frac{1}{2}[N - 1 + 1 + (N - 1)p]$$

If 1 chooses 2, then following that subgame, any equilibrium network involves one of the following

(i) 2 abstains.

(ii) 2 chooses 3.

In case (i), 2 and 1 are symmetric and hence equal probability of being chosen as the centre (decided by the choice of 3). The payoff of 1 in this case if

$$\frac{1}{2}[N - 1 + 1 + (N - 1)p] - c = E\pi_1(\phi) - c$$

In case (ii), 3 could abstain, choose 1 or choose 4. If 3 abstains then 2 becomes the most central agent and 1 the peripheral node for sure. Agent 1's payoff is therefore $1+(N-1)p-c$ which is less than $E\pi_1(\phi)$. If 3 chooses 1, then 4 is indifferent between 1, 2 and 3. In this case 1's probability of being centre is lower at 1/3 and thus gets a lower payoff, that too at a cost. The same argument can be applied when 3 links to 4.

Hence, 1's payoff by the strategy $s_1(2) < s_1(\phi)$. So, 1 abstains and 2 links to 1 followed by agent 3 who chooses one of 1 or 2 randomly.

Proposition 2: *There exists a p^{**} such that for $p \leq p^{**}$, the unique equilibrium network structure is a wheel of length $L < N$ with a local star. Also, $L = N/2$ or $(N+1)/2$ if N is even or odd respectively.*

Proof: Consider a subgame where agent $N-1$ is connected and hence by Lemma 1 belong to a chain of length l denoted by Ch_l. Suppose there is another component C'' with $C'' \cap Ch_l = \phi$. Let the maximum value of C'' to any agent connecting to C'' be denoted by $V(C'')$. This will depend on the number of agents in C'' and its architecture.In general it is of the form $r_1p + r_2p^2 + ... + r_qp^q$ where $r_1 + r_2 + ... + r_q = \#C''$.

We know by Lemma 4 and 5 that if $W_{l'} \in g_{N-l-1}, l' > l$, then $N-1$ will not choose to form a wheel since probability of being chosen by N is 0. Now say $l' \leq l$.

Hence we focus on subgames such that, if $s_{N-1} = N - l$,i.e. $N-1$ forms a W_l, then $s_N^* = j \in W_l$. Hence, $N-1$ might form the wheel for the precise reason that N will link to $N-1$ with positive probability.

Now, among this type of subgames $N-1$ has the following options:

i) $s = N - l(wheel)$

ii) $s = j \notin \{N, N-l\}$

iii) $s = N$

Step 1: We rule out strategy (iii) for p low enough.

Suppose, $s_{N-1} = N$. In the subgame following $s_{N-1} = N$, agent N has 2 options

a) $s_N = j \in C_l$

b) $s_N = j \in C''$

Note that if N chooses $j \in C_l$, he will choose j=argmax$_{j\in C_l}$ $\pi_N^{myo} = j^*$. Similarly, $b)$ implies $j = j^{**} \in C''$.

Now, if (a) $\succ$ (b)$\Rightarrow \pi_N(a) > \pi_N(b)$ i.e.

$$\begin{aligned} \pi_N(a) &= \pi_N(j^*) = 2 + 2p + ... + 2p^{\frac{l+1}{3}} + [p + p^2 + ... + p^{\frac{l+1}{3}}] \\ &> \pi_N(b) = [1 + p + p^2 + ... + p^{l-1}] + V(C'') \end{aligned} \tag{@}$$

In this case,

$$\pi_{N-1}(iii) = 2 + 2p + ... + 2p^{\frac{l+1}{3}} + [p^2 + p^3 + ... + p^{\frac{l+4}{3}}]$$

Suppose $N-1$ deviates to $s_{N-1} = j^*$. Now in this (deviation) subgame, say, g^d, N is isolated and can choose between j^* and j^{**}.

It is simple to verify that $\pi_N(j^*, g^d) > \pi_N(j^{**}, g^d)$ given @.

$[\pi_N(j^*, g^d) = 1 + [p + p^2 + ... + p^{\frac{l+1}{3}}] + [2p + 2p^2 + ... + 2p^{\frac{l+1}{3}-1} + p^{\frac{l+1}{3}}]$

$= 2 + 2p + ... + 2p^{\frac{l+1}{3}} + [p + p^2 + ... + p^{\frac{l+1}{3}}] - 1 - p^{\frac{l+1}{3}}$

$> [1 + p + p^2 + ... + p^{l-1}] + V(C'') - 1 - p^{\frac{l+1}{3}}$

$= V(C'') + p - p^{\frac{l+1}{3}} + [p^2 + ... + p^{l-1}]$

$> V(C'') = \pi_N(j^{**}, g^d)]$

Therefore

$$\begin{aligned}\pi_{N-1}(j^*) &= [2+2p+...+2p^{\frac{l+1}{3}-1}+p^{\frac{l+1}{3}}]+[p+p^2+...+p^{\frac{l+1}{3}}]+\pi^f \\ &= [2+2p+...+2p^{\frac{l+1}{3}}]-p^{\frac{l+1}{3}}+[p+p^2+p^3+...+p^{\frac{l+1}{3}}]+p \\ &> [2+2p+...+2p^{\frac{l+1}{3}}]+[p-p^{\frac{l+1}{3}}]+[p^2+p^3+...+p^{\frac{l+4}{3}}] \\ &> 2+2p+...+2p^{\frac{l+1}{3}}+[p^2+p^3+...+p^{\frac{l+4}{3}}] \\ &= \pi_{N-1}(iii)\end{aligned}$$

Hence, $N-1$ will deviate and $s_{N-1}\neq N$

If (b) $\succ$(a), then

$$\pi_{N-1}(iii)=[1+p+p^2+...+p^{l-2}]+1+pV(C'') \qquad \text{(P(iii))}$$

If $N-1$ chooses $s=N-l, i.e.(i)$

$$\pi_{N-1}(i)=2+2p+...+2p^{\frac{l-2}{2}}+\frac{1}{l}[1+2p+...+2p^{\frac{l}{2}}] \qquad \text{(P(i))}$$

Now compare P(i) and P(iii).Both are monotone in p. At $p=1, P(iii)>P(i)$. At $p=0, P(iii)<P(i)$.

So, there exists a $\widetilde{p}_{N-1}>0, s.t. for\ p<\widetilde{p}_{N-1}$, $s_{N-1}=N$ is dominated by $s_{N-1}=N-l(s_w)$.

Step 2: To rule out strategy (ii).

Consider the strategies $s=s_w$ and $s=j<N-l$.

The payoffs to $N-1$ from the two strategies are:

$\pi_{N-1}(W_l)=2+2p+...+2p^{\frac{l-3}{2}}+\frac{1}{l}(1+2p+2p^2+...+2p^{\frac{l-1}{2}})$

$\pi_{N-1}(j<N-l)=1+p+p^2+...+p^{l-2}+1+k_1p+k_2p^2+...+k_rp^r$

$\pi_{N-1}(W_l)>\pi_{N-1}(j<N-l)$

$\Leftrightarrow 2+2p+...+2p^{\frac{l-3}{2}}+\frac{1}{l}(1+2p+2p^2+...+2p^{\frac{l-1}{2}})>1+p+p^2+...+p^{l-2}+1+k_1p+k_2p^2+...+k_rp^r$

$\Leftrightarrow 2+2p+...+2p^{\frac{l-3}{2}}+\frac{1}{l}(1+2p+2p^2+...+2p^{\frac{l-1}{2}})>2+p+p^2+...+p^{l-2}+k_1p+k_2p^2+...+k_rp^r$

Both LHS and RHS are monotone in p.

Also for $p = 0$, LHS > RHS

and for $p = 1$, RHS $> l = LHS$

Hence $\exists$a $\widehat{p}_{N-1} \in (0, 1)$, such that $LHS > RHS$ for $p < \widehat{p}_{N-1}$ and agent $N-1$ prefers to form the wheel.

Now consider $s = s_w$ and $s = j_0 \in intC_l$.

If $s_{N-1} = j \in C_l$ and N chooses $j \in V(C'')$ in this subgame, the maximum $N-1$ can get is

$$\pi_{N-1}(j_0) = 2 + 2p + ... + 2p^{\frac{l}{3}-1} + [p + p^2 + ... + p^{\frac{l}{3}}]$$

which is $< \pi(s_w)$ for all p.

If N does not choose $j \in V(C'')$, then he chooses j_0 and

$$\pi_{N-1}(j_0) = 2 + 2p + ... + 2p^{\frac{l}{3}-1} + [p + p^2 + ... + p^{\frac{l}{3}}] + p$$

which is $< \pi(s_w)$ for any $p < 1$.

So, $s_{N-1} = s_w$ strictly dominates $s = s' \neq s_w$ for $p < p_{N-1} = Min\{\widehat{p}_{N-1}, \widetilde{p}_{N-1}\}$.[10]

For agent $N-2$ similarly we can get the threshold value of p such that for any $p < \widehat{p}_{N-2}$, agent $N-2$ would prefer to form the wheel. It is fairly simple to show that $\widehat{p}_{N-1} \leq \widehat{p}_{N-2} \leq .. \leq \widehat{p}_{N-l}$.

Hence for $p < \widehat{p}_{N-1}$, an agent at time t, given graph g_{t-1} would prefer to form a wheel if the probability of being chosen by any future agent is positive. This implies that if agent t completes a wheel of length l and the number of remaining agents $N - t \geq l + 2$, then agents $\{t+1, t+2, ..., t+l+1\}$ would form a wheel of length $l+1$.Hence in equilibrium no agent t would complete W_l if $N - t \geq l + 2$. Let the number of remaining agents be x_t for agent t. Hence if t forms a W_l, the upper bound for x_t for getting any future payoff is l. Hence

$$\begin{aligned}
\pi_t(W_l) &= 2 + 2p + ... + 2p^{\frac{l-3}{2}} + \frac{x_t}{l}(1 + 2p + ... + 2p^{\frac{l-1}{2}}) \\
&= \frac{x_t}{l} + [2 + 2p + ... + 2p^{\frac{l-3}{2}}]\frac{px_t}{l} \\
&\leq 1 + [2 + 2p + ... + 2p^{\frac{l-3}{2}}]p
\end{aligned}$$

[10] If the graph g_{N-1} is such that there are two distinct components C'', C''' with $C'' \cap Ch_l = \phi, C''' \cap Ch_l = \phi$, then the more valuable component would matter and the same proof applies.

Note that the payoff $\pi_t(W_l)$ of any agent belonging to W_l is increasing in x but may increase of decrease with l. It is in fact increasing in the ratio $\frac{x}{l}$. The agents want to maximise $\pi_t(W_l)$ with $\frac{x}{l} \leq 1$. At the optimum, $\frac{x}{l}$ should be the maximum i.e. 1. Fixing $\frac{x}{l} = 1$, the payoff is increasing in l. This implies that agents would want to form the largest wheel possible with $x = l$. This is possible only when $N/2$ agents form a wheel with $N/2$ agents remaining who link to an agent in W randomly to form a local star.

(it is easy to see that no agent will reject any link to create 2 components. E.g. say N=20. Consider the subgame where2 proposes to 3. 3 could accept and go on to form a wheel of 10 with last 10 agents as spokes. or 3 could reject in order to form a smaller wheel. But for p low enough, if 3-11 forms a wheel of 9, agents 12-20 become spokes but the max value of $\frac{x}{l} = 1$ which is same as accepting 2 and forming W_{10}. But rejecting 2, implies loss of payoff from 2 (and possibly 1) for any $p > 0$. Hence agents will form the largest wheel possible (i.e. with $x \leq l$) in equilibrium. It can also be shown that W_{10} entails a higher payoff than W_{11}, since in that case $\frac{x}{l} < 1$ and the loss due to the fall in $\frac{x}{l}$ is higher than the gain due to the extra agent in the wheel for any $p > 0$).

Proof of Proposition 3

Lemma 6 Suppose there are M components in equilibrium. Let them be $C_1, C_2, ...C_M$, formed in that specific order. The last component C_M is a wheel with a local star.

Proof: Suppose not. Let t be the first agent in C_M. If C_M is not a wheel with a local star, it must be that agent t proposes to $t+1$ but no agent $j > t$ links back to t. Hence, the probability of t being centre is zero. Suppose that, in equilibrium agent $t+k$ links back to some agent $t+r$, $r < k$. We claim that this is not possible. We use the following steps:

Step 1: *FIrst we will argue that if agent t cannot be the centre in the subgame following the link {t,t+1}, then an isolated agent $j > t$ cannot be the centre in the subgame following the strategy $l_j = j+1$.*

Proof: **Step 1 A**: If agent t cannot be the centre in the subgame following $l_t = t+1$ it is because some agent $t+k$ prefers to link to $t+r(say), r > 0, r < k-1$ and agents $t+k+1$ onwards link to $t+r$[11]. In this case, $t+k$ is the neighbour of

[11] Let SG_1 be the subgame faced by $t+k+1$ on this path.

the centre with probability 1. Let this structure lead to a wheel of $w+1$ agents and a tail of d agents with x agents as spoke agents, $x = N-(t+k)$ Agent $t+k$ gets a payoff of

$$[2+2p+.....w\ terms]+[p+p^2+...dterms]+xp$$

Alternatively, $t+k$ could have linked to t and the highest payoff $t+k$ could have achieved is when he had a positive probability of being centre of x future players. The payoff in that case would be

$$2+2p+......(w+d)terms+\frac{x}{w+d}(1+2p+2p^2+...(w+d+1)term)$$

If in equilibrium, t is not centre, then $t+k$ chooses $t+r$ and hence it must be the case that

$$\begin{aligned}
& [2+2p+.....w\ terms]+[p+p^2+...d\ terms]+xp \\
> \ & 2+2p+......(w+d)terms+ \\
& \frac{x}{w+d}(1+2p+2p^2+...(w+d+1)term)
\end{aligned}$$

$$\begin{aligned}
& or, x[\frac{1}{w+d}(1+2p+2p^2+...(w+d+1)term)-p] && \text{(E)} \\
< \ & [2+2p+.....w\ terms]+ && \text{(1)} \\
& [p+p^2+...d\ terms]\ -[2+2p+......(w+d)terms]
\end{aligned}$$

In this equilibrium, the payoff of $t+k-1$ is

$$[2+2p+.....w\ terms]+p\{[p+p^2+...dterms]+xp\}$$

Step 1B Now consider a deviation by $t+k-1$ to $t+r$. In this case, the myopic payoff is more and equal to

$$[2+2p+.....w-1terms]+[p+p^2+...dterms]$$

To figure out the future payoff of $t+k-1$, we need to know what happens in this subgame.

Claim 1B: We claim that in this subgame where $t+k$ is isolated, $t+k$ would link to $t+r$.

Suppose not. Suppose the following holds:

$t+k$ links to $t+k+1$ and $t+k$ is the centre with some probbability.

First note that agents t to $t+k-1$ have formed a component with $w+d$ agents and agent $t+r$ is the central agent in this architecture.

Hence if agent $t+k$ wants to be the centre with positive probability, the component must be a wheel and it must be large enough to be more valuable than $t+r$. By Lemma 4 and 5, this implies that there has to be at least $w+d+1$ agents in the wheel. Let there be $w+d+1$ agents in the wheel. Note that this implies that number of spoke agents $= N-(t+k+w+d) = x' < x = N-(t+k)$.

Suppose, k'' is the agents closing the wheel with $t+k$.The payoff to agent k'' by forming a wheel is

$$\begin{aligned} & \pi_{k''}(t+k) \\ = \; & 2+2p+...(w+d)terms + \frac{x'}{w+d}(1+2p+....(w+d+1)terms) \end{aligned}$$

If agent k'' links to $t+k+r$ instead, then, all agents $k''+1$ onwards link to $t+k+r$[12]. Then payoff to k'' is

$$\pi_{k''}(t+r) = [2+2p+....w \text{ terms}] + [p+p^2+...d\ terms] + x'p$$

Now it can be verified that $\pi_{k''}(t+r) > \pi_{k''}(t+k)$ given the inequality E.

$$E\ is \quad : \tag{2}$$

$$x[\frac{1}{w+d}(1+2p+2p^2+...(w+d+1)term) - p] \quad < \quad [2+2p+.....w\ terms]+ \tag{3}$$

$$[p+p^2+...d\ terms] - [2+2p+......(w+d)terms] \quad = \quad RHS \tag{4}$$

[12]This is so because, if facing SG_1, the isolated agent $t+k+1$ links to the central agent $t+r$, then in this subgame at time $k'' > t+k+1$, the isolated agent $k''+1$ would link to the corresponding central agent $t+k+r$.

$$\begin{aligned} or, \frac{1}{w+d}(1+2p+2p^2+...(w+d+1)term)-p &< \frac{1}{x}[RHS] \\ &< \frac{1}{x'}[RHS] \end{aligned}$$

$$\begin{aligned} & or, x'[\frac{1}{w+d}(1+2p+2p^2+...(w+d+1)term)-p] \\ < \ & RHS = [2+2p+.....w\ terms]+[p+p^2+...d\ terms] \\ & -[2+2p+......(w+d)terms] \end{aligned}$$

Rearranging we get,

$$\begin{aligned} \pi_{k''}(t+k) &= [2+2p+......(w+d)terms]+x'.\frac{1}{w+d}(1+2p+2p^2+...(w+d+1)term) \\ &< [2+2p+.....w\ terms]+[p+p^2+...d\ terms]+x'p = \pi_{k''}(t+r) \end{aligned}$$

Q.E.D.

Hence we showed that an isolated agent $j > t$ cannot be the centre if t is not the centre.

Step 2 : *We show that if agent $t+k-1$ deviates to link to $t+r$, then all agents $j \geq t+k$ links to $t+r$.*

Proof: If agent $t+k-1$ links to $t+r$, then $t+k$ is isolated and hence, by step 1 cannot be the centre. If $t+k$ is not the centre then, the best structure for him is when he is the neighbour of the centre. If $t+k$ links to $t+k+1$, then the best possible situation for $t+k$ is if $t+k+1$ becomes the centre. Hence the payoff to $t+k$ is the payoff from $(N-(t+k))$ agents and is of the form

$$1+p(2+2p+...w'terms)+x''p$$

If, $t+k$ linked to $t+r$, we know that $n-(t+k)$ agents would link to $t+r$ Hence payoff of $t+k$ from future agents is $p(N-t-k)$. Also, $t+k$ gets a direct payoff of 1 from $t+r$ and indirect payoff from all agents in the set $S = \{t, t+1, ..., t+k-1\} \backslash \{t+r\}$.

Hence the total payoff is $1+p(N-t-k)+\pi(S) > 1+p(2+2p+...w'terms)+x''p$. Therefore, agent $t+k$ would link to $t+r$ in the subgame where $t+k-1$ deviated to

$t + r$.

Step 3: Agent $t + k - 1$ would deviate to link to $t + r$.

Proof: Given Steps 1 and 2, we can see that $t + k - 1$ has a weakly higher payoff if he deviates to $t + r$. This is because, this strategy weakly reduces his distance from agents $t, t + 1, ..t + k - 2, t + k$ and strictly decreases his distance from all agents $j > t + k$.

Since this holds for any structure with a wheel and a tail (of any length), we claim that the component C_M has to be such that the 1st isolated agent who starts the component .i.e. t has positive probability of being the centre. This would be the case when C_M is a wheel with a .local star.

Lemma 7 Suppose there are $M > 1$ components in equilibrium. Let them be $C_1, C_2, ...C_M$, formed in that specific order with $C_M = WS$. Then C_{M-1} is also a wheel with local star.

Proof: Note that $M > 1$ implies that the number of components in C_{M-1} is positive. Let agents $t' + 1$ to $t' + n$ belong to C_{M-1} and agents $t' + n + 1$ to N belong to C_M. Let $t = t' + n + 1$. Following steps similar to Lemma 6, suppose that C_{M-1} is not a wheel with star, i.e. t' has 0 probability of chosen as the centre. Let $t' + r'$ be the agent who is the centre in this component. Let $t' + k' - 1$ be the agent who chooses $t' + r', r' < k' - 1$ and agents $t' + k'$ to $t' + n$ chooses $t' + r'$. Since agent t onwards belong to a different components it implies that $t' + n$ does not propose to t. This must be due to either one of the following reason:

(i) Agent $t' + n$ would get a higher payoff from linking to t but t does not accept such a link

(ii) Agent $t' + n$ gets a higher total payoff from linking to $t' + r'$ (We will show that (ii) cannot hold)

Suppose case (i) is true. In this case, agent t would reject links because it is not optimal to do so, i.e. if t does accept then he has to form a bigger wheel which reduces the probability of being the centre (it must be the case that t cannot form a wheel of same size one period earlier, because if he could then he would have done so in order to get a higher number of spoke agents). Hence agent $t' + n$ links to $t' + r'$. Now suppose, as in step 2 of Lemma 6, agent $t' + k' - 2$ deviates to $t' + r'$. In this subgame, agent $t' + k' - 1$ is isolated and by the logic of Step 1 and 2 (lemma 6) we argue that the optimal strategy of $t' + k' - 1$ is to link to $t' + r'$.Hence such deviation would be profitable for $t' + k' - 2$. Hence the component C_{M-1} has to be a wheel with local star.

For case (ii), the logic is simpler. We will show that case (ii) is not possible.

Suppose, it is the case that agent $t' + n + 1$ would accept if $t' + n$ did propose to him. Hence agents $t' + n$ to N would form the last component which would be a wheel with local star. Hence the expected payoffs of $t' + n$ and $t' + n + 1$ are the same. Now, denote the payoff of $t' + n$ from proposing to $t' + n + 1$ and the latter accepting as $\pi_{t'+n}(t' + n + 1|A)$. Since $t' + n$ chooses $t' + r'$ in equilibrium, it implies that $\pi^*_{t'+n}(t' + r') \geq \pi_{t'+n}(t' + n + 1|A)$. Also, the fact that $t' + n + 1$ accepts the link implies that $\pi_{t'+n+1} = \pi_{t'+n}(t' + n + 1|A) \geq \pi_{t'+n+1}(reject\ t' + n$ and link to $t' + n + 2) = \pi^*_{t'+n+1}$.

We will show that $\pi^*_{t'+n}(t' + r') \geq \pi_{t'+n}(t' + n + 1|A)$ is not possible hence ruling out the possibility of case (ii). To see this, let the component formed by agents t' to $t' + n - 1$ be denoted as $C' = C_{M-1} \backslash \{t' + n\}$. Also note that $\pi^*_{t'+n}(t' + r')$ be written as the sum of payoff from $t' + r'$ and that from all other agents when $t' + n$ connects to $t' + r'$. 'i.e. $\pi^*_{t'+n}(t' + r') = 1 + \pi(C' \backslash \{t' + r'\})$. In the other case when $t\prime + n + 1$ accepts the offer of $t' + n$, let the agents form a wheel with $z + 1$ agents (say) with a local star with x spoke agents with $x \geq z$. The payoff of $t' + n$, $\pi_{t'+n}(t' + n + 1|A)$ is of the form

$$\begin{aligned} & (2 + 2p + 2p^2 + ...z \text{ terms}) + \frac{x}{z}(1 + 2p + ...z + 1\ terms) \\ = \; & \pi(W_z) + \frac{x}{z}(1 + 2p + ...z + 1\ terms) \end{aligned}$$

We know from Lemma 2and 3 that size of W_z has to be large enough such that the first spoke agent of C_M will connect to $j \in W_z$ and not $t' + r'$. This implies that

$$1 + p\pi(W_z) > 1 + \pi(C' \backslash \{t' + r'\})$$

But

$$\begin{aligned}
\pi_{t'+n}(t'+n+1|A) &= \pi(W_z)+\frac{x}{z}(1+2p+...z+1terms)\\
&\geq \pi(W_z)+(1+2p+...z+1terms),\ \text{since } x\geq z\\
&> 1+\pi(W_z)\\
&> 1+p\pi(W_z)\\
&> 1+\pi(C'\backslash\{t'+r'\})=\pi^*_{t'+n}(t'+r') \qquad \text{Q.E.D.}
\end{aligned}$$

Hence an isolated agent $t'+n$ will prefer belonging to the wheel with $t'+n+1$, is such a link is accepted. Hence, if agent $t'+n$ links to $t'+r$ in equilibrium, it must be because agent $t'+n+1$ would reject a link from $t'+n$ as in case (i).

Proposition 4: *The efficient network architecture is (i) a star if* $p>\frac{2-c}{2}$ *and (ii) a wheel of length 3 with a local star if* $p<\frac{2-c}{2}$.

Proof: First note that due to the capacity constraints of agents, the maximum number of links in any architecture is N. Also, for $c<1$, each agent would prefer forming a link unless it is redundant (e.g. linking to a person who has already made the link with the agent).

Step 1: The first step of the proof is a direct result from Jackson and Wolinsky (1996)((Proposition 1 section 3.1.1) which shows that for any c,p the star is the efficient structure among all networks with $N-1$ links. Step 2 of the proof shows that WS_3 is the efficient structure among all networks with N links.

Step 2: We show that for any p, the aggregate payoff in a connected network with a wheel of length $k, k>3$ is less than that in a wheel of length 3 with a local star. For this purpose, we compare the aggregate payoffs for a WS_k and WS_{k+1} for any $k>3$. Let us assume that k is odd.

Note that the payoff to an agent belonging to the wheel from other agents in the wheel is $(2+2p+2p^2+...+2p^{\frac{k-3}{2}})=z_w(say)$.

Also, the payoff to any spoke agent from the agents in the wheel is $(1+2p+2p^2+...+2p^{\frac{k-1}{2}})=z_s(say)$.

The other part of the payoffs are those from the spoke agents. The payoff of each spoke agent from other spoke agents is $(N-k-1)p$. The payoff of a wheel agent from a spoke agent varies according to his position in the wheel. The aggregate payoff of the wheel agents (except the sentre) from the spoke agents can be calculated as

$$2[(N-k)p+(N-k)p^2+...+(N-k)p^{\frac{k-1}{2}}]=2z'$$

while the payoff of the centre from the spoke agents is simply $(N-k)$. Hence the aggregate payoff $P(WS_k)$ is

$$\begin{aligned}
P(WS_k) &= k(2+2p+2p^2+...+2p^{\frac{k-3}{2}}) \\
&\quad +(N-k)(1+2p+...+2p^{\frac{k-1}{2}}) \\
&\quad +(N-k)(N-k-1)p \\
&\quad +2[(N-k)p+(N-k)p^2+...+(N-k)p^{\frac{k-1}{2}}] \\
&\quad +(N-k) \\
&= kz_w+(N-k)z_s+2z'+(N-k)(N-k-1)p+(N-k)
\end{aligned}$$

Similarly, we can write out the aggregate payoff from WS_{k+1} as

$$\begin{aligned}
P(WS_{k+1}) &= (k+1)(2+2p+...+2p^{\frac{k-3}{2}}+p^{\frac{k-1}{2}}) \\
&\quad +(N-k-1)(1+2p+...+2p^{\frac{k-1}{2}}+p^{\frac{k+1}{2}}) \\
&\quad +(N-k-1)(N-k-2)p \\
&\quad +2(N-k-1)[p+p^2+...p^{\frac{k-1}{2}}]+(N-k-1)p^{\frac{k+1}{2}} \\
&\quad +(N-k-1)
\end{aligned}$$

We can show that $P(WS_k)-P(WS_{k+1})>0$ for any p, N and $k>3$.
A similar exercise can be done for k even.
Step 3: Comparing the payoffs from the star and the WS_3, we see that

Star $\succ_{eff}$ WS_3 iff

$$\begin{aligned}
P(star) &> P(WS_3) \\
2(N-1)+(N-1)(N-2)p-(N-1)c &> (N-1)+2(2+(N-3)p) \\
&\quad +(N-3)(1+(N-2)p)-Nc \\
(N-1)+(N-1)(N-2)p+c &> 4+2Np-6p+N-3 \\
&\quad +(N-3)(N-2)p \\
c &> 2+2Np-6p-(N-2)2p \\
c &> 2-2p \\
p &> \frac{2-c}{2}
\end{aligned}$$

Appendix B: Extensions

Proposition 5: *With one $h - type$ in the population and $p \simeq 1$, the equilibrium network structure is a complete star or a WS_3 with the θ_h as the centre if $\theta_h > 1+p$.*

Proof. The proof obtains from a series of lemmas. ■

Lemma B.1: Consider a history h_t such that θ_h has moved and $g_t \neq g^e$. Then an (low type) isolated agent at t would choose v_t^{iso} $for\ p \simeq 1$.

Proof: The proof proceeds as that of Proposition 1 with the term V_{k-1} weakly larger due to the presence of θ_h. Hence, $\pi_k(v_k^{iso})$ is larger for any p and any isolated agent at $t > 1$ will choose v_t^{iso} to maximise the myopic payoff for $p \simeq 1$. This implies that the threshold value of p for agent k is weakly lower. i.e.$p > \widetilde{p}_k, \pi_k(v_k^{iso}) > \pi_k(k+1)$ and $\widetilde{p}_k < p_k^*$ for any k. This in turn implies that $\widetilde{p} = Max\{\widetilde{p}_N, \widetilde{p}_{N-1}, ..., \widetilde{p}_2\} < p^* = Max\{p_N^*, p_{N-1}^*, ..., p_2^*\}$.

Lemma B.2: Consider a subgame where an isolated low-type agent at $t-1, t > 2$, links to the high type agent. Then at time t, θ_h will link to some agent $j < t-1$.

Proof: Let the component with agents $1, 2, ..., t-2$ be denoted by $\widetilde{C}$. Let the payoff from linking to $j \in \widetilde{C}$ be denoted by $V(\widetilde{C}, j) = 1 + [a(j)]$.Define $j^* = \arg\max_{j \in \widetilde{C}} V(\widetilde{C}, j)$. We could be in one of the following 3 cases, given this subgraph $g_t = \widetilde{C} + \{\theta_{l,t-1}\theta_{ht}\}$.

Case I: $s_{ht} = j^*$ and $v_{t+1}^{iso}(g_t + \{\theta_{ht}j^*\} = \theta_h$

In this case, θ_h is maximising myopic payoff $V(\widetilde{C}, j)$ and future payoff $\pi^f = N - t$. No other strategy s_{ht} would increase his payoffs. Hence, if g_t is such that $v_{t+1}^{iso}(g_t + \{\theta_{ht}j^*\} = \theta_h$, $s_{ht}^* = j^*$.

Case IIa: $s_{ht} = j^*$ and $v_{t+1}^{iso}(g_t + \{\theta_{ht}j^*\} = j^*$

In this case, $\pi_h(s_{ht} = j^*) = 1 + V(\widetilde{C}, j^*) + (N-t)p = 2 + [a(j^*)] + (N-t)p$.

Now, if θ_h deviates to link to some agent $j > t$ then in the subgame that follows there are additional agents towards $\widetilde{C}$ with respect to θ_h. If j^* is more central than θ_h at the subgame with graph $g_t + \{\theta_{ht}j^*$, then for the graph $g = g_t + \{\theta_{ht}\theta_{l,t+1}\} + \{\theta_{l,t+1}, \theta_{l,t+2}\} ... + \{\theta_{l,t+l}, j^*\}$, θ_h cannot be the central agent at any time $\tau > t+1$.

Hence the maximum payoff for $s_h = j \nless t$ is

$$\begin{aligned}\pi_h(s_{ht} &= t+1) = 1 + 1 + pV(\widetilde{C}, j^*) + (N-t-1)p \\ &= 2 + p + p[a(j^*)] - p + (N-t)p \\ &< \pi_h(s_{ht} = j^*)\end{aligned}$$

Hence, θ_{ht} will not link to $j \nless t-1$.

(Note that Case IIa is impossible if $\widetilde{C}$ contains no link since $\theta_h > 1$).

Case IIb: Suppose it is the case that if $s_{ht} = j^*$, then $v_{t+1}^{iso}(g_t + \{\theta_{ht} j^*\} = j^*$. Also suppose, $\exists j^0 \neq j^*, j^0 \in \widetilde{C}$ such that if $s_{ht} = j^0$, then $v_{t+1}^{iso}(g_t + \{\theta_{ht} j^0\} = \theta_h$.and θ_{ht} prefers j^0 to $j^*, i.e.$

$$1 + 1 + [a(j^0)] + (N-t) > 1 + 1 + [a(j^*)] + (N-t)p$$

Hence,

$$\pi_h(s_{ht} = j^0) = 1 + 1 + [a(j^0)] + (N-t)$$

If θ_h links to some agent $j \nless t-1$, then in the subgame that follows, either he is still the centre but only one link further away from the component $\widetilde{C}$ or he is not the centre anymore and also further away from $\widetilde{C}$ which implies a strictly lower payoff for θ_{ht}.

Hence θ_{ht} will link to some agent $j \in \{1, 2, ..., t-1\}$.

Lemma B.3: Consider a subgame where θ_h has not moved. Then at time t an isolated low type agent θ_{lt} will link to $j \in L = \{1, 2, ...t-1, \theta_h\}$.

Proof: Suppose at time t, an isolated low-type agent is selected to move. Let the subgraph at time t be $\widetilde{C}$ and the value of $\widetilde{C}$ through agent j is $V(\widetilde{C}, j) = 1 + [a(j)]$. As before, define $j^* = \arg\max_{j \in \widetilde{C}} V(\widetilde{C}, j)$. Also, note that if θ_{lt} links to θ_h, we have the 3 situations as in Lemma B.2. In each case, θ_h links to some agent $j < t$ and θ_{lt} cannot be the centre.

In case I, $\pi_l(s_{lt} = \theta_h) = \theta_h + p(1 + [a(j^*)]) + (N-t)p$. Alternatively, linking to an agent outside set L, will not make θ_{lt} the centre and increases his distance from other players. The associated payoff is

$$\pi_l(s_{lt} = j \notin L) = 1 + p + p^2 + ..p^{l-2} + p^{l-1}(1 + [a(j^*)]) + p^{l'}(N - t - l) + p^d\theta_h$$

where l is the chain length that follows the subgame,$l \geq 1; d$ i sthe distance to $\theta_h, d > 1; l'$ is the distance to the central agent to which isolated end agents connect to,$l' \geq 1$. It can easily be shown that for any $\theta_h > 1$, this payoff is strictly less than $\pi_l(s_{lt} = \theta_h)$.

However, if $V(\widetilde{C}, j) = 1$, then θ_{lt} could link to $j < t$ and have some positive probability of being the centre, depending on the value of θ_h. Let the payoff in that case be denoted by $\pi_l(s_{lt} = j < t)$. If $\pi_l(s_{lt} = j < t) < \pi_l(s_{lt} = \theta_h)$, then $s_{lt} = \theta_h$ dominates strategies $s'_{lt} \in \{j < t, j \notin L\}$. If $\pi_l(s_{lt} = j < t) > \pi_l(s_{lt} = \theta_h)$, then the previous analysis implies that $\pi_l(s_{lt} = j < t) > \pi_l(s_{lt} = j \notin L)$. Hence, $s_{lt} = j \notin L$ is always dominated by $s_{lt} = j \in L$.

Similarly, we can show that for cases IIa and IIb too, $\pi_l(s_{lt} = j \notin L) < \pi_l(s_{lt} = \theta_h)$.

Lemma B.4: Consider a subgame where θ_h has not moved and g_t is such that $V(\widetilde{C}, j^*) \geq 1 + p$.Then at time t an isolated low type agent θ_{lt} will link to θ_h if $\theta_h > V(\widetilde{C}, j^*)$.

Proof: For the first part we need to show that if $\theta_h > V(\widetilde{C}, j^*)$, then $v^{iso}_{t+2}(g) = \theta_h$ for $g = g_t + \{\theta_{lt}\theta_h\} + \{\theta_h j^*\}$ and in that case, the optimal choice for θ_{lt} is $s^*_{lt} = \theta_h$.Suppose $\theta_h > V(\widetilde{C}, j^*)$. Also suppose, $g_{t+1} = g_t + \{\theta_{lt}\theta_h\} + \{\theta_h j^*\}$. Now, consider an isolated agent $t + 1$ who is chosen to propose a link. For $t + 1$,

$$\pi^{myo}_{t+1}(\theta_h) = \theta_h + p + p(1 + [a(j^*)])$$

and

$$\pi^{myo}_{t+1}(j^*) = (1 + [a(j^*)]) + p\theta_h + p^2$$

Hence $\pi^{myo}_{t+1}(\theta_h) > \pi^{myo}_{t+1}(j^*)$ according as

$$\theta_h + p + p(1 + [a(j^*)]) > (1 + [a(j^*)]) + p\theta_h + p^2$$

$$\Leftrightarrow (1 - p)\theta_h > (1 - p)(1 + [a(j^*)]) - (1 - p)p$$

$$\Leftrightarrow \theta_h > V(\widetilde{C}, j^*) - p$$

which holds. Hence, $t+1$ chooses θ_h, for $p \simeq 1$.In this case, payoff to θ_{lt} is

$$\pi_{lt}(\theta_h) = \theta_h + pV(\widetilde{C}, j^*) + (N - t - 1)p$$

If θ_{lt} deviates to j^*, the maximum possible payoff is when all future players link to j^* and is

$$\begin{aligned} \pi_{lt}(j^*) &= p\theta_h + V(\widetilde{C}, j^*) + (N - t - 1)p \\ &< \pi_{lt}(\theta_h) \text{ for } \theta_h > V(\widetilde{C}, j^*). \end{aligned}$$

Hence, θ_{lt} will link to θ_h if $\theta_h > V(\widetilde{C}, j^*)$.

Proof of Proposition 5 (contd)

Consider agent 1. If agent 1 is the high type then given Lemma B.1, the optimal strategy is for θ_h to abstain and agent 2 connects to θ_h, agent 3 connects to θ_h and so on.

Suppose agent 1 is a θ_l. He has 3 choices:

$i) s_1 = \phi$

$ii) s_1 = \theta_l$

$iii) s_1 = \theta_h$

If $s_1 = \theta_h$, then given Lemma B.1, the optimal strategy for θ_h is to abstain since then agent 3 will link to θ_h and the equilibrium structure is a complete star with θ_h as the center. Hence, $\pi_1(s_1 = \theta_h) = \theta_h + (N-2)p - c$.

If $s_1 = \phi$, then with probability $\frac{1}{N-1}$, θ_h is chosen as the 2nd proposer and we are in a case similar to Case I of Lemma B.2. Hence it is optimal for him to link to 1, since from period 3 agents would link to θ_h and $\pi_1 = \theta_h + (N-2)p$.With probability $\frac{N-2}{N-1}$ however, some θ_l is chosen as the proposer. Now, $\theta_{l,2}$ can link to θ_h who by Case I, Lemma B.2, will choose $\theta_{l,1}$ and the network will be a $\theta_h - star$. In this case,

$$\pi_{l,2} = \theta_h + (N-2)p - c$$

$\theta_{l,2}$ can also link to 1. In this case, with probability $\frac{N-3}{N-2}$, the 3rd agent is θ_l. By

Lemma B.4, for $\theta_h > 1+p$, θ_{l3} links to θ_h. By Lemma B.2, θ_h links to 1 or 2 (chooses randomly) and from period 5 onwards agents link to θ_h. The payoff to 2, therefore, is

$$\pi_2|_{agent3=\theta_l} = 1 - c + \frac{1+p}{2}(\theta_h + (N-3)p)$$

With $\frac{1}{N-2}$ probability the 3rd agent is θ_h. Since $\theta_h > 1+p$, the high type knows that even if $s_{h3} = 1/2$, $v_4^{iso} = \theta_h$. Hence, θ_{h3} links to 1 or 2 and agent 4 onwards links to $v_4^{iso} = \theta_h$. In this case, $\pi_2|_{agent3=\theta_h} = \pi_2|_{agent3=\theta_l} = 1 - c + \frac{1+p}{2}(\theta_h + (N-3)p)$.

Hence the payoff to $\theta_{l,2}$ from $s_2 = 1$ is

$$\pi_{l,2}(s_2 = 1) = 1 - c + \frac{1+p}{2}(\theta_h + (N-3)p)$$

Hence if $s_1 = \phi$, then $\theta_h \succ_2 1 \; iff$

$$\theta_h + (N-2)p - c > 1 - c + \frac{1+p}{2}(\theta_h + (N-3)p)$$

$$or, 2\theta_h + 2(N-2)p > 2 + (1+p)(\theta_h + (N-3)p)$$

$$or, (1-p)\theta_h > 2 + p(N-3-2N+4) + (N-3)p^2$$

$$\begin{aligned} or, (1-p)\theta_h &> 2 - p(N-1) + (N-3)p^2 \\ &= 2 - 2p^2 + (N-1)(p^2 - p) \\ &= 2(1-p)(1+p) - (1-p)p(N-1) \\ &= (1-p)(2 - p(N-3)) \end{aligned}$$

$$or, \theta_h > 2 - p(N-3)$$

which holds for $p \simeq 1$. Hence if $s_1 = \phi$, 2 chooses θ_h who in turn chooses agent 1. Hence,

$$\pi_1(s_1 = \phi) = \theta_h + (N-2)p$$

If $s_1 = \theta_l$, then in this subgame, 2 can choose ϕ or to link to some other agent.

If $s_2 = \phi$, then the next agent, if θ_l chooses θ_h and if θ_h, he chooses one of 1 or 2. In either case, θ_h becomes the central agent for all future agents and $\pi_1(\theta_l) = 1 - c + \frac{1+p}{2}(\theta_h + (N-3)p) < \pi_1(\phi)$.

It could be the case that $s_2 = \theta_h$, In this case θ_h links to 1 and becomes the centre and hence, $\pi_1(\theta_l) = 1 - c + \theta_h + (N-3)p$. For $p > 1-c$, $\pi_1(\phi) > \pi_1(\theta_l)$ and vice versa. If $s_2 = \theta_l$, then on the path we could have either of the 2 cases. One where some agent $j > 2$ links to θ_h who then would link to $k > 1$ and $\pi_1(\theta_l) = 1 + r_1 p + r_2 p^2 + ... + r_q p^q + p^d \theta_h + (N-l-1)p^{d+1} < \theta_h + (N-2)p = \pi_1(\phi)$. In the other case, suppose l low types form a wheel so that probability of centre is positive. Hence, $\pi_1(\theta_l) = (2+2p+...(l-1)terms) + \frac{1}{l}(1+2p+...lterms)(\theta_h + N - l - 1)$. Its is easily shown that for $p = 1$, $\pi_1(\theta_l) = \theta_h + (N-2) - c < \pi_1(\phi)|_{p=1}$ and the inequality holds for $p \simeq 1$.

Hence, for $p \simeq 1$, agent 1 would either abstain and hence $\theta_h - star$ obtains for any $\theta_h > 1 + p$ or agent 1 links to 2 who then links to θ_h, hence forming a WS_3 with the centre being the high type agent.

Example 4

Let $N = 8$. Consider the subgame at the beginning of period 4 where agents $1, 2, 3, 4'$ are connected in a chain. With the particular order we have in our model, $4'$ is the next proposer for sure and and hence proposes to 1 to complete the wheel. But suppose the proposer chosen at $t = 4$ is some other (isolated) agent j', then the wheel cannot be completed. In this case, j' can connect to 1, can connect to 4 or connect to some central agent (2 or 3 in this case). For $p \simeq 0$, j' s optimal choice would be agent 1. To see this consider the following subgame G. $G = \{12\} + \{23\} + \{34'\} + \{41\} + \{54\} + \{65\}$. Hence there are two periods remaining and 2 agents, $4'$ and $8'$ left to propose. With probability $1/2$ agent $4'$ is chosen as the proposer. He has 3 options.

(i) $s = 6$

(ii) $s = v^{con}$

(iii) $s = 8'$.

If agent $4'$ who is now renumbered as 7 chooses 6, a wheel of length 7 is formed and the last agent $8' \equiv 8$ links to one of them with equal probability. Hence, $\pi(s =$

$6) = 2 + 2p + 2p^2 + \frac{1}{7}(1 + 2p + ... + 2p^3)$.

If $s = j = v^{con}$, then the last agent chooses j and $\pi(s = v^{con}) = 1 + r_1 p + r_2 p^2 + .. + r_q p^q$

If $s = 8'$, then agent $8'$ in the last period chooses v^{con} and $\pi(s = 8') = 2 + r'_1 p + ... + r'_q p^q$

For $p \simeq 0$, strategy $s = 6$ dominates the others and $4'$ will choose 6.

After period 6 however, with probability 1/2 agent $8'$ is chosen as the proposer at $t = 7$. In that case, 7 knows that the last agent $4'$ will choose v^{con} and hence agent 7 cannot be the centre (since he is one the extreme end agents). Hence 7 chooses v^{iso}.

Now, at $t = 6$, when an isolated agent is chosen and faces the graph $G - \{65\}$, his payoff from linking to 5 is

$$\pi_6(s = 5, G) = \frac{1}{2}[2 + 2p + 2p^2 + \frac{1}{7}(1 + 2p + ... + 2p^3)] + \frac{1}{2}[1 + r_1 p + ...r_q p^q]$$

His payoff from $s = v^{iso}$ is

$$\pi_6(v^{iso}, G) = 1 + r'_1 p + ... + r'_q p^q$$

For $p \simeq 0$, $\pi_6(s = 5, G) > \pi_6(v^{iso}, G)$ and agent 6 links to agent 5.

The same incentive works for any isolated agent chosen as the proposer. But the exact network formed depends on the realisations of the orders of prposers. If, at some stage, the responder at stage t is not chosen as the proposer at any $\tau, t < \tau < N$, then the structure contains no wheel. If the responder of stage t is chosen as the proposer as any time before the last period, then a wheel forms and the remaining agents link to one of the agents in the wheel. The wheel size in this case is weakly bigger since the responder at time 4, could be chosen to propose at time 6, say, resulting in a wheel of size 6.

Proposition 7 *With the order of proposers completely random in each period, the equilibrium network is a complete star for* $p \simeq 1$.

Proof The Proof is similar to the proof of proposition 1. The difference is that the payoff functions in this case are expected payoffs, the expectation taken with respect to the random order. The payoffs in proposition 1 are valid for a unique realisation of the order which chooses the responder of a period as the next proposer. For all other realisations of draws for proposers each period, the payoff functions are modified but the inequalities still hold. We will point out the differences in the payoffs and show that the inequalities hold in each case. Denote a particular realisation of the order by r and the set of all possible orders by R.

Step 1: Any agent $t > 1$, if isolated chooses v_t^{iso}.

This statement is true for the last agent moving, $t = N$. Now consider k with the statement true for all $\tau > k$.

Consider the payoff for strategy $s_k = k + 1$.

For Case I, the payoff is $E\pi_k(k+1) = \sum_{r \in R} \pi_k(k+1|order\ r). \Pr ob(order\ r)$.

Now, $\pi_k(k+1)$ for each order r is of the form

$$1 + p + p^2 + ... + p^{j_0-k-y-1} + p^{j_0-k-y}(1 + [a] + p * y) + +(N - j_o)p^{d'}$$

where the order r is such that y agents between k and j_0 were isolated when chosen as proposers and chose v_k^{iso} since statement if true for all $\tau > k$. It is easy to see that

$$\begin{aligned} & \pi_k(k+1|order\ r) \\ = \ & 1 + p + p^2 + ... + p^{j_0-k-y-1} + p^{j_0-k-y}(1 + [a] + p * y) + +(N - j_o)p^{d'} \\ < \ & 1 + (j_0 - k - y)p + [a] + py + (N - j_0)p \\ = \ & 1 + [a] + (N - k)p = \pi_k(v_k^{iso}) \end{aligned}$$

Hence, $E\pi_k(k+1) < \pi_k(v_k^{iso})$. Case II is similar.

For Case III, due to the random order, the wheel length that can form following $s_k = k+1$, is random and hence the payoff is $E\pi_k(k+1)$, the expection taken over the length l of the wheel which in turn depends on the order r. Note again that expected number of agents in the wheel $El \leq l$ since with positive probability a positive number y of isolated agent is chosen as the proposer and he chooses v^{iso} which implies that he cannot belong to the wheel component. (It could also be the case that no wheel forms). Hence

$$E\pi_k(k+1) = [2 + 2p + ..\frac{El}{2} terms] + E[\frac{N-k-l-y}{l+1}(1 + p + ..l + 1 terms)]$$

As before, $\pi_k(v_k^{iso}) = 1 + [a] + (N-k)p$

For $p = 1, E\pi_k(k+1) = N - k - y \leq N - k < 1 + [a]|_{p=1} + (N-k) = \pi_k(v_k^{iso})|_{p=1}$

By similar logic, for $p \simeq 1, E\pi_k(k+1) < \pi_k(v_k^{iso})$ and hence $s_k^* = v_k^{iso}$.

The rest of the proof is exactly same as that of proposition 1.

Chapter 3

Family Embeddedness: An Empirical Investigation

1 Introduction

Since the studies of Loury (1977) and Coleman (1988), social capital has been mentioned and studied in a variety of contexts across various fields in the social sciences. The recent surge in studies on social capital followed Putnam's work on American society (1993). Like other forms of capital, the term 'social capital' has a broad set of interpretations. It varies from club-memberships to trust in government to the network of family and friends[1]. In effect, social capital implies some form of externality arising from social interactions between agents. For example, a person's network of friends gives him better information regarding jobs or provides financial credit when required. It is important to stress, however, that the externality is not necessarily positive. It has been pointed out by Coleman (1988), Dasgupta (2005), Glaeser (2002), among others, that social capital can also create negative social worth. The social capital particular to a network, while benefitting the 'insiders' can have negative effects on 'outsiders'[2]. Communatarian relationships can be impediments to personal advancements too. As Dasgupta (2005, pp 13), while talking about kinship ties, says:

"..there is a functional side to kinship ties. ..offers a way to pool individual risks. However, there is a bad side of the coin..they dilute personal incentives to invest for prosperity because of kinship obligations.". Austen-Smith and Fryer (2005), in fact, proposes a model which relies on this inhibitive nature of community ties to explain

[1] Coleman (1988) defined social capital as "social organisations" facilitating "the achievment of goals that could not be achieved in its absence or could be achieved only at a higher cost". Putnam et al (1993) defines it as "trust, norms and networks that can improve the efficiency of the society". Ostrom (2000) claims that social capital is the "shared knowledge, understandings, norms, rules and expectations" about how people interact in recurrent interactions. Lin (2001) describes it as "resources embedded in social networks and accessed and used by actors for actions"

[2] Gambetta (1990) shows how Sicilian mafia families trust only family members and not outsiders. High 'insider' trust also allows the mafia to function more efficiently and imposes a negative externality on society

inner-city behaviour[3]. It is, therefore, best to interpret social capital as a network of relationships which may have positive and negative externalities on agents inside and outside the network.

In this paper we focus on a specific form of social capital, namely, the level of family ties and explore its relationship with two things: first, the personal incentives to succeed in one's job; second, one's attitude towards 'outsiders' i.e. people outside the family network. We use the term family ties to refer to the strength of ties with the extended family or alternatively, a person's embeddedness in his extended family. Embeddedness is a multidimensional concept and can be measured by the frequency and quality of interaction with family members, feeling of closeness with family or assurance of monetary or non-monetary help from family members. Moreover, the level of family ties of a person has both inherited and acquired components. When young, a person has a certain level of ties which is given to him by birth. However, he might choose to strengthen, break or change these ties at a later stage[4]. This choice can be affected by family circumstances, societal norms, his ability and/or his belief about his income potential. A person also chooses the level of effort to exert in his career and his attitude towards agents outside the kin. The effort exerted on the job is representative of how much importance one gives to personal advancement and is driven by personal incentives while social attitude reflects his behaviour outside of his 'network', i.e. family. In fact, there might be a systematic relationship between these choices. For example, if having strong family ties necessitates extending considerable monetary help to family members, then it might dilute personal incentives to earn more and hence reduce work effort. The negative correlation could also be due to time constraints when maintaining ties requires spending a lot of time with family. On the other hand, if it is the case that an agent has altruistic utility from helping family members monetarily then a higher effort and higher income increases the marginal utility of ties and he would maintain a higher level of current ties. Similarly, the values imbibed in a person when young can cause him to choose a higher level of involvement in both society and family. It is equally possible that an individual's embeddededness

[3] In the model, the family punishes agents acquiring high education since it is interpreted as an attempt to move away from family.

[4] Note that we use the word 'family ties' for ties with the extended family and not just immediate family.

in his kin network nurtures strong 'insider' attitudes and at the same time, negative attitudes towards people outside the kin. If so, strong ties though beneficial to the individual and his kin, imposes a negative externality on society and is not necessarily desirable.

Our empirical analysis of the choices of family ties, work effort and social attitude is carried out using data from the Netherlands Kinship Panel Study (NKPS). This dataset has detailed questions aimed at capturing the strength of family ties along with detailed information on work, attitude about different social and family values. A special feature of the dataset is a set of questions regarding family interaction when the respondent was young along with the current level of interaction. This feature allows us to distinguish between inherited or initial ties and those chosen at a later stage. We refer to the latter as acquired ties or current ties. We construct measures of the levels of initial and current family ties using individual responses to questions about the frequency of interaction among family members and/or his opinion about the strength of ties within his family. To capture personal motivation to succeed, we use effort exerted on the job which we proxy by the actual number of hours a person works at his job. We represent social attitude by two different proxies: the level of volunteering activity and attitude towards migrants. Our empirical model involves regressing the three choice variables on the level of inherited ties and other exogenous factors using the Seemingly Unrelated Regression (SUR) method. The set of exogenous factors include demographic variables and other factors that affects one or all of the three variables.

We find that the level of inherited family ties affects current ties and the extent of volunteering. This effect is positive for both i.e. agents born with a higher level of ties have more ties at a later stage in life and are also more involved in volunteering activity. The level of inherited ties, however, does not affect current work effort. Work effort is mainly determined by occupation, income, sex and age. Moreover, whether one's parents are divorced or not influences current ties negatively and work effort positively. This may be due to a lower amount of time spent with family following parents' divorce and consequently, a higher availability of time for work. It might also be preference-driven. Parents' divorce presumably decreases the cost of severing ties and results in a lower level of current ties. If it is the case that strong ties with the family dilutes personal incentives to work, then the lower current ties would increase the marginal utility of effort resulting in higher work effort. We also find life-cycle effects on work effort and family ties. Work effort increases upto a certain age and then falls. The effect of age is the opposite for family ties. Females maintain a higher level of current ties and exert less effort on their jobs. Overall, most of the regressors have opposite signs for effort and current ties. This suggests a negative relationship between the two. Alesina and Guiliano (2007) also find that in countries with stronger family ties market participation is lower, particularly for women. To summarise, we find that in the Netherlands stronger familial ties when young have positive effects on the level of ties maintained a later stage and also on volunteering for a social cause and tolerance towards migrants. Agents with stronger current ties however, exert less work effort. These results are, of course, specific to the society we are looking at. Casual observation suggests that the tradeoff between family ties and effort is likely to be stronger in developing countries since the reliance on family is higher. The relationship between strong kin ties and good social attitude in these countries, however, is not so obvious.

This paper is related to the literature on social capital. One strand of work in this field studies the effect of social capital on various outcomes, both at the aggregate and at the individual level[5]. The measures of social capital typically used vary from trust levels to memberships in associations and are taken as exogenous. For example, a few studies (Furstenburg and Hughes 1995, Mcneal 1999, Sandefur, Meier and Hernandez 1999) take the number of family moves as a measure of social capital and study its effect on academic performance, college enrollment etc. But the number of moves itself is possibly endogenous and determined by the parents' embeddedness in the society. Families with high levels of social capital might be less likely to move and hence such studies might suffer from endogeneity problems. Studies which take club memberships as a measure of social capital also suffer from the same problem since the decision of being a member of a club is endogenous. We address this problem

[5] Knack and Keefer (1997), LaPorta et al (1997) conduct a cross-country analysis of the effect of trust on corruption levels and growth differences. Individual-level studies include Fafchamps and Lund (2003), Fafchamps and Minten (1999), Narayan and Pritchett (1999) which focus on effect of social capital on some income measure. For an excellent survey of the literature on social capital, see Durlauf (2002).

explicitly by investigating the choices of family ties, work effort and social attitude together and not looking at social attitude or effort in isolation[6].

This paper also relates to studies that analyse the determinants of social capital in some specific form. Rahn and Rudolph (2005) focus on the determinants of trust in local governments; Sampson, Morenoff and Earls (1999) analyse factors that affect behaviour among neighbours. Others have analysed the determinants of social capital in the form of carpooling (Charles and Kline 2002), volunteering (Costa and Kahn 2003a) and citizenship (DiPasquale and Glaeser 1999). One feature of these studies is that the forms of social capital considered are inherently positive in nature. By focusing on volunteering, carpooling etc they have completely ignored the possibility of the 'negative worth' of social capital. In this paper, we focus on social capital in the form of kin networks which could have positive or negative effects on work effort and social attitude. More specifically, our paper relates to studies by sociologists and some economists on the socioeconomic role of family. For example, LaFerrara (2003) shows how extended family ties substitute for credit markets or provide risk-sharing[7]. Alesina and Guiliano (2007) conducts a cross-country analysis of the effect of family ties on home production, household labour allocation, fertility and mobility.

The paper is organised as follows. Section 2 presents the empirical strategy followed by a discussion of the data in Section 3. Section 4 presents the estimation results. Section 5 concludes.

2 Family ties, work effort and social attitude

As mentioned before, the relationship between a person's social capital, his personal incentive to succeed in the market and his social attitude could be positive or negative. We measure personal incentive through the amount of effort exerted in the job. A person is born with a certain level of social capital which, in this case, is the level of inherited family ties. Let this be denoted by f_0. The person chooses a new level of ties, f_1 along with work effort e and social attitude sa. If an agent chooses to sever some of his initial ties then he bears some psychological cost proportional to the level

[6] Most of these studies look at the effect on income. In our case, we look at work effort which is expected to be positively correlated with income, controlling for factors like occupation.

[7] For more, see La Porta et al (1999), Prez-Gonzales (2004), Bentolilla and Ichino (2006).

of severance. This cost might depend on the level of f_0. A higher f_0 implies that the person will find it harder to sever ties. This cost can also be affected by other factors like parents' divorce or family values in the society. If a person is born in a highly cohesive society which values family ties a lot, then the cost of the same level of severance will be higher. On the other hand, if parents' are divorced, then this cost is likely to be lower and hence the level of f_1 lower. This lower level of f_1 can, in turn, change one's personal incentive to work and his social attitude. The decisions could also be influenced by other factors like wage and social values. Hence, an agent makes three decisions (f_1, e, sa) based on his individual and family characteristics and past history.

Our model for the decisions of individual i is as follows:

$$e_i = \alpha_0 + \alpha_1 f_{0i} + \alpha_2 X_i + \varepsilon_{1i} \tag{1}$$

$$f_{1i} = \beta_0 + \beta_1 f_{0i} + \beta_2 X_i + \varepsilon_{2i} \tag{2}$$

$$sa_i = \gamma_0 + \gamma_1 f_{0i} + \gamma_2 X_i + \varepsilon_{3i} \tag{3}$$

where X_i is the set of exogenous variables. We assume that the errors $\varepsilon_{ji}, j = 1, 2, 3$ are correlated for each i but independent across i. The set X_i includes demographic and other factors that influence any one the dependent variables. For example, it could contain variables that capture a person's earning potential and affect work hours; social values imbibed when a person is young, which influences his involvement in society at a later stage. We estimate the system of equations (1)-(3) as a Seemingly Unrelated Regressions (SUR) model. This allows for the errors to be correlated across the equations in an unrestricted way. Errors in each equation includes factors which influence a person's choice of ties, effort and attitude but are unobserved. In fact, the same observables can be part of the errors of each equation. For example, a person's motivation level or peer influence within the family could affect both work effort and current ties. Similarly, both effort and social attitude is influenced by how a person feels about personal gain vis-à-vis doing something for society. Allowing for such correlation gives efficient estimates and correct variances which are used in inference.

In the next section, we describe the dataset used in the analysis along with the empirical proxies.

3 The NKPS Data

One reason most of the existing work has not focused on the choice of social capital at the individual level is the lack of detailed information regarding such ties. The existing studies have analysed the determinants of social capital but have not distinguished between the level of social capital one is endowed with i.e. inherited and that which is a choice or acquired. In this paper, we can distinguish between the two by the use of a unique dataset, the Netherlands Kinship Panel Study (NKPS) collected by the Netherlands Interdisciplinary Demographic Institute. The sample covers individuals between age 18 and 79 living in the Netherlands. The respondents were chosen through random sampling of addresses from the population registers. The sample size is 8161. The dataset is aimed at collecting detailed information regarding interaction with family and friends over three waves. The second wave of the study is in the process of being completed. We use only the first wave (conducted in 2002/03) in this analysis. In addition to questions about the current family ties (e.g. going on vacations with extended family, going to family reunions, opinion about the strength of family ties), it also contains information about interaction with family at age 15 which is a potential proxy for inherited ties. In addition, it also contains other demographic, individual and family level variables to capture other types of heterogeneity in the population. The data was collected both by interviews with the respondent and through self-completion questionnaires. We will use relevant information from both. For the purpose of our analysis, we select respondents between age 18 and 60 who are employed The sample size in this group is 4986.

3.1 Empirical Proxies

3.1.1 Family Ties

To represent family ties we need proxies for two types of ties: 1. Inherited ties f_0 and 2. Acquired ties f_1. To measure f_0, we construct a multidimensional measure, F_0 by using five variables that reflect the extent of interaction with family at age 15. The questions that we use are a & b) "Did you ever go and stay with your mother's (father's) family when you were young?" c&d) "Did members of your mother's (father's) family ever come and stay with you when you were young?" e) "Did you ever go on holiday with relatives (other than your immediate family) in those years?". The responses are one of "never', 'occasionally' or 'frequently' and are coded 1-3. We attach equal weights to each of these questions. To deal with missing values of one variable, we take only a subset of the questions and adjust the weight accordingly. This index reflects the level of embeddedness the respondent had with the extended family when he/she was age 15. We think that this is a good proxy for initial ties f_0 since presumably, a child of age 15 does not take independent decisions regarding visits to or vacations with family. To assess the quality of the index , we calculate the Cronbach's α which is 0.61 implying that there is sufficient consistency between the variables[8]

In order to measure the level of current ties f_1, we use multiple proxies. The first proxy $F1_1$ is constructed from responses to the question "Have you gone for a holiday with extended family in the past twelve months?". The responses were either "never", "once or twice" or "several times" and were coded 1-3 respectively. Since the question specifically asks for responses in the context of the last 1 year, it is possible that such a proxy is not capturing current level of embeddedness for the respondent. Certain circumstances specific to the past year may prevent or reduce the ease of staying with family for some respondents. To account for such cases we use another proxy $F2_1$. The second proxy $F2_1$ is a multidimensional measure but reflecting the respondent's opinion about solidarity in his family. The respondents were asked whether they agreed or disagreed and if so, strongly or not, with statements: "The ties between members of my extended family are tightly knit" and "Should I need help, I can always turn to my family". The Cronbach's α for this index is 0.63. We use the logarithms of f_0 and f_1 for our analysis.

3.1.2 Effort

To represent the effort e we use the actual number of hours worked in a week. This is constructed from the responses to the question "How many hours a week on average do you actually work?". We use the logarithm of work-hours in the analysis.

The data also has variables capturing the type of employment which is either Paid employment, Self employment or Help in family business. We combine the last two

[8] Cronbach's α is an average measure calculated from the correlation coefficients between each pair of variables in an index.

into the same category of Self-employed.

Table 1: Inherited and Acquired Family Ties

f_0	Components	f_1	Components
$F1_0$	i)Stay with mom's family	$F1_1$	Holiday with family in last 1 year?
	ii)Mom's family stay with you		
	iii)Stay with dad's family?		
	iv)Dad's family stay w you?		
	v)Holiday with relatives?		
		$F2_1$	(i) Ties b/w family strong
			(ii) Can always turn to family

3.1.3 Social Attitude

Social attitude sa can be captured in multiple ways. In particular, we want to analyse an individual's attitude towards people outside his family. It could be his behaviour with neighbours, attitude toward ethnic integration, involvement in volunteer work etc. In this paper, we proxy social attitude by the level of volunteering (v)[9]. The self-completion questionnaire asks the respondent several questions regarding his involvement in various activities. The question asked is "Did you participate in any of the following activities in your free time in the past 12 months? If so, about how often?" along with the following list "A Sports; B. Participate in school association, parent-teacher association or other activity for school ; C Provide unpaid help to sick or handicapped acquaintances or neighbors (not family); D Volunteer work for association, church or other organization (not for school); E Visit neighbors, have neighbors visit you; F Cultural activities, such as theatre, concert or museum; G Going out to a restaurant, café, movie or party". The responses to the questions were one of the following choices: a. Not at all; b. 1-2 times; c. 3-11 times; d. 12 times or more; and were coded 1 through 4. Of these, some activities pertain to private consumption while some are related to one's civic engagement. We use the response to C and D above to construct our measure of volunteering.

[9] We also use attitude towards migrants as another proxy later.

3.1.4 Other controls

In order to capture individual-specific heterogeneity, we use standard controls in our regressions. We use demographic variables like age, sex, marital status along with variables like employment type and whether parents are divorced or not. The variable 'sex' takes the value 1 for female and 0 for male; 'marstat' representing marital status is 1 for married, 0 otherwise; 'pardivorce' is 1 if parents were divorced, 0 if not. We also include dummies for household size in the set of controls. We create dummies, one each for each household size starting from one to seven.

We also include a proxy for hourly earnings as a control, since this affects the incentive to put more or less effort. We use a direct measure of income as a proxy. This measure, though, has a high possibility of measurement error. It has been documented that people tend to understate their income. Moreover, for agents without a fixed pay every month there is likely to be an error in recalling or averaging his income. We do, however, use homeownership as an alternative measure of income to check for the robustness of our results. Table 2 presents the summary statistics of some of the variables used. Table 3 includes some simple correlations between variables. We see that F_0 is positively correlated with current ties F_1 while it's correlation with work-hours, although small, is negative. Volunteering has a negative correlation with work effort and positive ones with F_0 and F_1.

3.2 Data Limitations

As with most survey data, this dataset suffers from certain limitations. As mentioned before, the reported income level of respondents potentially suffers from measurement error. Another limitation is the presence of unobserved heterogeneity. For example, a person's motivation level and aspirations affect his behaviour with regard to work effort and/or family ties. This characteristic of the individual is probably determined by his innate ability or if he observes someone in his family or neighbourhood with similar ability and highly successful. The problem of innate ability is usual in most studies. Unfortunately, we cannot control for peer effects due to lack of suitable data. Another problem might be that number of hours worked does not represent all of the market effort. Agents might be engaging in professional networking which affects their market success but is not taken into account in work hours. Here we assume that work hours reflect an agent's market activity to a large extent. There might also be a selection problem if agents choose particular occupations and the level of family ties systematically.

Table 2: Summary Statistics

	Mean	Std. D	Min	Max
age	40.732	10.893	18	60
sex	0.583	0.493	0	1
marstat	0.539	0.498	0	1
pardivorce	0.123	0.328	0	1
household size	2.508	1.359	1	13
$F1_0$	0.489	0.270	0	1.1
$F1_1$	0.219	0.351	0	1.1
$F2_1$	-.739	0.376	-1.6	0
$effort$	3.401	0.551	0	4.49
volunteering v	1.637	.976	1	4
income	7.066	1.015	0	11.69

Table 3: Correlations

Correlations	F_0	$F2_1$	work-hours
$F1_1$	0.116		
$F2_1$	0.173	1	
work-hours	-.046	-.042	1
volunteering	.084	.033	-0.128

4 Estimation Results

We present the results of the SUR estimation of our model as represented by equations (1)-(3) in Table 4[10]. The Breusch-Pagan test of independence[11] gives a $p - value$ of 0.002 and the hypothesis that the errors from the three equations are independent is

[10] The proxy used for f_1 in Table 4 is the family solidarity proxy $F2_1$. The results for $F1_1$ are similar with minor changes in the significance levels of some variables.

[11] This test computes the correlation matrix of the errors and tests the hypothesis that the correlations are jointly zero.

rejected. In the set X_i we include age, age-squared, sex and marital status. We also include income, household size dummies, employment type and parents' divorce. The variable 'Income' acts as a proxy of hourly earnings which influences the incentive to exert effort. We also include a variable reflecting the respondent's parents' experience at war. We include the latter since a person's experience at war probably influences how he values social work and this is imbibed in his children through the upbringing, even if partially.

The first three columns of Table 4 report the coefficients for each decision variable. For current ties, we find that both initial ties and whether parents are divorced or not, are significant. A higher level of initial ties implies a higher level of current ties while parent's divorce affects current ties negatively. The former could be because of a higher psychological cost of severing ties if one has been brought up in a tighly-knit family. Moreover, family ties possibly provide some benefit, monetary or otherwise, and severing ties implies forgoing those benefits. The negative effect of parent's divorce is not surprising. Ties with the extended family operates mainly through one's parents and a divorce between the parents reduces the strength of the initial ties at an interim stage. This in turn reduces the psychological cost of severing ties further. Older individuals have a lower level of ties. Note that we use the proxy $F2_1$ in these regressions. The negative sign might be because older generations' family consists predominantly of the younger generation and the former feels that there is less family solidarity relative to their youth. The other determinants of current ties are sex and parents' war experience. Women maintain a higher level of ties. Income is significant only at a 20% confidence level and the effect is positive.

With regard to work effort, we find that the coefficient of f_0 is positive and significant. This could be possible due to multiple reasons. One is that higher f_0 facilitates peer effects since the respondent observes family members more closely. This peer effects might motivate him to work. Alternatively, a higher f_0 could result in a higher sense of responsibility for family which in turn affects his effort. Yet another reason might be through the selection effect. Initial level of embeddedness and the choice of occupation at a later stage could be related and it is the occupation that affects work effort. Since we have not controlled for the occupation, the significance of f_0 could be the due to this correlation with occupational choice. The other variables determining effort is age, sex, marital status, income and parents' divorce. The coefficient of income is as expected and positive since an agent with higher earnings has a higher incentive to put effort. We also find some life cycle effects on work effort. It increases with age upto a point and then falls. Moreover, an individual is likely to put more effort if parents' are divorced. This might be because of a lower family interaction and the consequent increase in the time available for work.

The third column reports the results of the regression for the extent of involvement in volunteer work. We find that the level of volunteering is mainly determined by initial ties, parents' war experience, sex and income. Agents whose parents' life was highly influenced by war are more involved in volunteering. This is possibly because values taught by the parents when the respondents were young differ with this experience. But note that this variable is possibly correlated with age of the respondent. Parents who were affected by the war would be of the age group 75 or above and hence their children are likely to be 40 or more years of age. To check that the significance of the war experience is not due to its correlation with age, we run the same regressions without the war-experience variable. This is included in the 4th column of Table 4. We find that age does become significant now but only at the 10% level. Hence, part of the effect of war-experience might be due to its correlation with age but not entirely. Parents' war-experience does influence the level of civic engagement. The level of initial ties f_0 is also significant and higher f_0 has a positive effect on civic engagement. Higher income agents, however, are less involved in volunteering.

Note that initial ties f_0 affect both current family ties and volunteering positively. That higher initial embeddedness in family would imply a higher f_1 is not surprising. But stronger initial family ties also imply a better social attitude. This might be because stronger ties when young help develop certain social values that affect an individual's level of volunteering. This is in contrast to Alesina and Guiliano (2007) who find that societies with strong family ties harbour a strong "inside" attitude. Stronger ties are associated with a lower level of trust and a lower inclination to accept new ideas.

Table 4: Seemingly Unrelated Regression

	family ties	effort	volunteering	volunteering
f_0	.262093***	.027459*	.198405***	.18043***
	(.000)	(.075)	(.000)	(.000)
age	-.030861***	.053876***	.010965	.018813*
	(.001)	(.000)	(.296)	(.074)
age squared	.0002865**	-.000684***	-.000085	-.00096
	(.015)	(.000)	(.853)	(.454)
sex	.097614***	-.328629***	.238562***	.25135***
	(.002)	(.000)	(.000)	(.000)
marital status	.009941	-.028994	.042322*	.02603
	(.656)	(.013)	(.083)	(.249)
parent's divorce	-.26342***	.052290***	.018527	.02502
	(.000)	(.011)	(.667)	(.539)
income	.017059	.208657***	-.028183**	.01956
	(.208)	(.000)	(.057)	(.205)
self-employed	-.001205	.008627	.013027	.03238
	(.208)	(.722)	(.798)	(.501)
war experience	-.05589***	.005821	.0328504**	
	(.000)	(.394)	(.022)	
No. of observations	N = 3724			

-p-values reported in the parentheses

-*** significant at 1% level, ** at 5% level, * at 10% level

- includes household dummies

With regard to family ties and work effort, all regressors except f_0 and income have opposite signs for the two variables. The variable parents' divorce is significant in both equations. Parents' divorce reduces current ties and increases work effort. There is possibly no reason for parent's divorce to directly increase the incentives to work. However, as parents' divorce decreases f_1, work effort could increase due to increased availability of time. It could also be preference-driven. For example, a factor like parents' divorce that decreases the cost of severing ties would result in a lower level of current ties. It will be accompanied by a higher level of effort if a lower f_1 increases the marginal utility of effort. We would expect that to be the case when stronger ties with the family dilutes personal incentives to work due to financial obligation towards family. The effect of age on the two variables is also opposite. Work effort increases with age up to a point while family ties fall. Females maintain a higher level of family ties and exert less effort.

Note that one of the right hand side variables is the variable 'self-employed'. This variable could possibly be endogenous if agents systematically choose to be self employed and maintain higher level of family ties. This might happen because self employment involves a higher risk relative to paid employment and stronger ties might provide higher financial support to pool the risk. To test for the endogeneity of self-employed, we instrument it with father's employment type and estimate the IVSUR coefficients of our model. The variable takes value 1 when father is self-employed. Tsukahara (2007) in a study with Japanese data finds that the variable 'father's occupation' is highly significant in the employment choice of respondents[12]. The Hausman test for endogeneity yields a *p-value* of 0.30 and we cannot reject the hypothesis that the SUR and IV SUR estimates are similar. Hence, self-employed is not endogenous. The results are included in Table 5.

4.1 Robustness

Since the reports of income might not be so reliable, we also carry out the analysis with homeownership as a proxy for income (See Table 6). The coefficients of the other exogenous variables have the same sign as in the Table 4. Homeownership is significant in all the three equations. Owning a home increases effort, increases f_1 and reduces volunteering v. Hence richer individuals are less involved in volunteering. This might be because a higher level of income or wealth increases the access to social interactions like memberships in elite clubs and other forms of entertainment and in the process, active volunteering is given a low priority. It is also true, however, that richer individuals can make donations without actively participating in social work.

In order to investigate whether higher wealth does have a negative impact on social attitude, we carry out our analysis with a different proxy for social attitude.

[12] Using our data, a regression of employment choice on age, sex, marital status, f_0, father's occupation and father's education yields highly significant coefficients for father's occupation.

We use a question regarding the respondent's attitude toward intermarriage with migrants as another dimension of social attitude. The four largest migrant groups in Netherlands are the Turks, Moroccans, Surinamese and Dutch Antilleans.The specific questions asked is: "What are your views about your children's choice of partner? Would it bother you if one of your children decided to marry someone of Turkish/Moroccan/Surinamese/Antillean descent? (The question is NOT about the actual situation)". The options for the answers were: Bother me a lot; Bother me a little; neutral; Not bother me; Not bother me at all, and were coded 1-5. We use this variable to create the proxy 'Attitude towards migrants' represented by a. A higher value of this variable reflects a higher tolerance and openness towards migrants and hence a higher a. Unlike volunteering, this proxy should not suffer from the problem mentioned in the previous paragraph.

Table 7 includes the results of our estimation for (f_1, e, a). As before, f_0 is significant for f_1 and a with a positive coefficient in both. The coefficients of homeownership are also of the same sign as those of Table 10. Respondents belonging to a higher income group are less open to intermarriage. This could be because the average income of migrants are much lower and this aversion to intermarriage is not so much an aversion to migrants per se as to a lower income[13]. Hence, it seems that negative social attitudes outside one's kin arise not due to higher kinship ties but rather due to higher wealth or income. Initial ties have a positive impact on social attitude.

5 Conclusion

In this paper we investigate how an individual's social capital in the form of his embeddedness in his kin network affects his attitude towards market activity and his attitude towards society in general. The fact that strong ties within one's network, in addition to benefits, can have negative impacts on both personal incentives and on individuals outside the network, has been stressed by sociologists (Coleman) as well as economists (Dasgupta, Glaeser). In modelling the decisions of agents, we treat the choices of family ties, market effort and social attitude as endogenous since these

[13] Table 8 reports some comparisons between the migrant and the native populations. The average incoem for migrants are indeed much lower than that of the main sample.

decisions are often taken at the same time. Part of one's ties, however, are inherited and hence exogenous. The use of the unique dataset, the Netherlands Kinship Panel Data, allows us to distinguish between inherited and acquired ties. First, we find that embeddedness in one's family does not have a negative impact on one's social attitude. In fact, higher initial ties imply a higher level of family ties and a higher level of volunteering activity at a later stage in life. With regard to work effort, we find that one's inherited ties affect work effort positively . However, most of the other factors affect effort in a direction opposite to the level of current ties. This maybe due to the inhibitive nature of family ties or simply due to time constraints.

It would be interesting to conduct the same investigation with data from a developing country and compare it with this study. The lack of well-developed credit markets in such countries make familial ties more valuable in terms of financial assistance by alleviating effects of negative shocks. Casual observation suggests that these ties also hinder mobility and efficient allocation of human capital. Whether it imposes any negative externality on society is not so clear. We leave this study for future work.

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Appendix A: Regression tables

Table 5: IVSUR with father's occupation

	family ties	work effort	volunteering
f_0	.27042***	.00727	.18523***
	(.000)	(.624)	(.000)
age	-.02684***	.028165***	.018136*
	(.006)	(.000)	(.089)
age-squared	.000269**	-.000423***	-.000113
	(.023)	(.000)	(.384)
sex	.06543***	-.471215***	.255065***
	(.007)	(.000)	(.000)
marital status	.00422	-.02368**	.03197
	(.839)	(.035)	(.161)
parent's divorce	-.26133*** (.000)	.03589* (.077)	.026116(.526)
father's employment	.0000847	.0514	.01285
	(.594)	(.110)	(.694)
income	-.00133 (.597)	.002917** (.031)	.00625 (.819)
household size	.01821** (.050)	-.066359*** (.000)	.00309 (.762)
war experience	-.05617*** (.000)	.005853 (.371)	-.034277*** (.010)
No. of observations	4224		

-p-values reported in the parentheses

-*** significant at 1% level, ** significant at 5% level, * significant at 10% level

- includes household dummies

Table 6: Effect of Homeownership

	family ties	effort	volunteering
f_0	.27357***	.02613*	.19344***
	(.000)	(.097)	(.000)
age	-.03104***	.06486***	.01504
	(.001)	(.000)	(.329)
age squared	.00029***	-.00081***	-.00006
	(.011)	(.000)	(.993)
sex	.07384***	-.45716***	.25510***
	(.003)	(.000)	(.000)
marital status	-.0084	-.02029*	.03627
	(.684)	(.087)	(.035)
parents' divorce	-.24385***	.03378*	.00911
	(.000)	(.090)	(.722)
homeowner	.10342***	.07439***	-.07239***
	(.008)	(.000)	(.008)
self-employed	-.01931	.0159	.03599
	(.668)	(.509)	(.403)
war experience	-.05826***	.00850	.02846**
	(.000)	(.243)	(.021)
No. of observations	4349		

p-values reported in the parentheses

-*** significant at 1% level, ** significant at 5% level, * significant at 10% level

- includes household dummies

Table 7: Attitude towards Migrants

	family ties	effort	attitude a
f_0	.26019***	-.02109	.14980***
	(.000)	(.179)	(.000)
age	-.02909***	.06963***	.03806***
	(.002)	(.000)	(.002)
age squared	.00026**	-.00075***	-.00052***
	(.046)	(.000)	(.001)
sex	.09633***	-.48071***	.02893
	(.002)	(.000)	(.373)
marital status	-.00784	-.04428***	-.11767***
	(.627)	(.000)	(.000)
parents' divorce	-.277975***	.0409**	.08705**
	(.000)	(.013)	(.043)
homeowner	.10252***	.08219**	-.0652*
	(.000)	(.047)	(.069)
self-employed	-.02341	.00375	-.05294
	(.541)	(.865)	(.288)
father's education	-.00475	-.00454**	.02351***
	(.254)	(.054)	(.000)
No. of observations	N = 4501		

p-values reported in the parentheses

-*** significant at 1% level, ** significant at 5% level, * significant at 10% level

- includes household dummies

Appendix B: Migrants and Natives

The NKPS dataset also includes a separate sample for the four largest migrant groups in Netherlands, namely the Turks, Moroccans, Surinamese and Dutch Antilleans. Unfortunately, we cannot carry out the same analysis for this group due to unavailability of crucial variables. But it would be interesting to see how these migrant groups differ in terms of their family ties and work effort relative to the native Dutch population. All the ethinic groups in the migrant sample are from non-western cultures with traditionally stronger family ties. Alesina (2007) in his cross-country study of family ties classifies all these 4 groups as having very strong family ties whereas Netherlands belongs to a lower level of family ties on average. Hence we would expect some systematic difference between the two samples. However, it should be noted that the respondents of the migrant sample are not in their home countries. They are either 1st generation or 2nd generation migrants and presumably most of their family members do not live in Netherlands. Due to the geographical distance from their families, it is natural that their interaction with extended family would be biased downwards to a considerable extent. Hence, we consider interaction between family members only if they live in Netherlands. We do not differentiate between different locations in Netherlands since the travel costs are not too high within the country. Table 8 gives the average level of interaction with family members among migrants and natives. The variable 'mother contact' and 'sibling contact' gives the number of times in a week respondents are in face-to-face contact with their mother and siblings, respectively. We find that the frequency is, on average, higher for the migrant sample. It is possible that mothers or siblings of migrant respondents are more likely to stay in the same household or same neighbourhood and hence face-to-face contact is higher. But the variable is still higher for migrants when we use 'sibling phone' which is the the frequency of contact with sibling via phone or email. Table 8 also includes the average correlation between effort and family solidarity. The value is much more negative for the migrants.

Table 8: Comparison : Migrants and Dutch

Variables	Main Sample	Migrants
mother contact (mean)	4.729	5.436
sibling contact (mean)	3.617	4.495
sibling phone (mean)	3.543	4.65
e (mean)	33.70	36.30
corr $(e, F2_1)$	-0.041	-0.128
monthly income	7.066	0.536

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- David Grow Scholarship, Summer 2005

TEACHING EXPERIENCE:

- Instructor: Intermediate Microeconomics (1 semester)
- Teaching Assistant: Intermediate Microeconomics (3 semesters), Economics of Corporation (2semesters), Game Theory (PhD level, 3 semesters)

RESEARCH EXPERIENCE:

- Research Assistant to Kalyan Chatterjee, Summer 2005 and 2007

REFERENCES:

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