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INVENTORY DYNAMICS IMPLEMENTATION TO A NETWORK DESIGN MODEL

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Abstract

This research presents an inventory dynamics model that is implemented into an already existing supply chain footprint model for a multinational manufacturing company. The model is a linear programming model that focuses on maximizing the supply chain profit while taking into consideration the inventory levels held in the distribution centers. In a previous study, a footprint model was developed for this company that focused on methods for designing a resilient global supply chain network that considered manufacturing and distribution facilities to service multiple markets. That model determined the manufacturing and conversion facilities within one region and for one product for this company. In order to incorporate the inventory dynamics into the existing footprint model, a linear programming model with a rolling time horizon was developed. Having the facilities fixed, the proposed model is run multiple times in order to determine the optimal inventory strategies for the different distribution centers, as well as to demonstrate how the solution should be implemented and updated for future runs.

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I dedicate this work to my family

Chapter 1. Introduction

In this globalized world, supply chain has evolved from managing a few products in a single country to multiple products in facilities spread throughout the world. Companies get raw materials in one country, manufacture their products in another, and sell them all over the world. In order to manage all of these activities and maintain competitive prices, companies rely on global supply chain management. Depending on the type of product, different types of supply chains can be used to meet different needs. Supply chains can be categorized into two types, *efficient* and *responsive*. An efficient supply chain's main goal is to minimize total costs, or maximize total profits. This type of supply chain works well when used for functional products such as rice, salt and other every day products for which demand is fairly stable. In a responsive supply chain, the main objective is to maximize the customer service level. This supply chain works well for new and innovative products, where the demand is difficult to predict and highly variable. Maximizing the service level means that the supply chain focuses on meeting the demand for the product. Since the demand can highly fluctuate through time, the manufacturer needs to be able to implement changes into its products and its supply chain in order to satisfy the customer's wants and needs. For every company, the supply chain and its primary objective might be different, and it is necessary to find the right balance between responsiveness and efficiency. This balance will vary from company to company depending on their needs and competitive strategy.

In order to improve a supply chain, different drivers of the supply chain must be analyzed. As Chopra and Meindl (2001) mention, the interaction between each driver helps determine the supply chain's performance when it comes to how responsive and efficient it is. The structure of these drivers will determine how a supply chain will fit the requirements of the products being processed through it. These drivers, as defined by Chopra and Meindl (2001), are:

a. Facilities: physical locations in the supply chain network. Depending on the type of facility, the product may be manufactured, assembled or stored.
b. Inventory: amount of product that is stored anywhere in the supply chain network. The products may be finished goods, work in progress or raw material.
c. Transportation: refers to the moving of inventory between two different points in the supply chain.
d. Information: data and analysis concerning any aspect of the supply chain.
e. Sourcing: selection of certain raw materials or parts in the supply chain.
f. Pricing: the cost to the final customer to purchase the good or service made available by the supply chain.

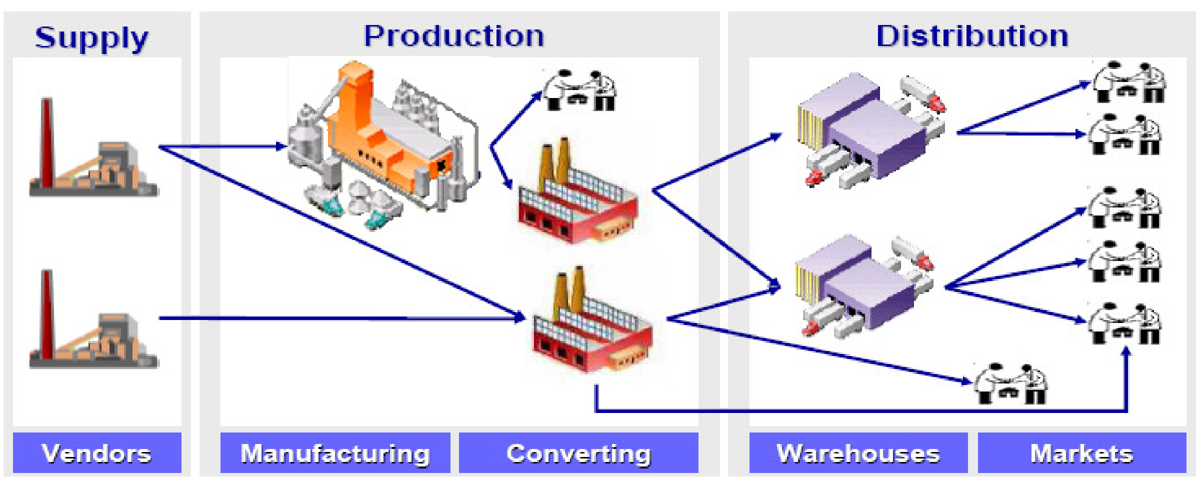

Figure 1 "Supply Chain Network Echelon" (Portillo, 2008)

It can be seen that all of these drivers play a big role in a supply chain and how they are all needed in order for a company to be competitive. <u>This research will focus on developing a</u>

mathematical model that will help determine the optimal inventory policy for a given supply chain. Figure 1 shows the supply chain network echelon that will be used; for the purpose of this research, only the converting, warehouses and markets stages are considered. A linear mathematical programming model is developed with the ultimate goal that it will be implemented into a supply chain that has three different stages: conversion facilities, distribution centers (DC) and the final customer. The model will be run for a multi product supply chain system across several scenarios to assess the impact of the variability in replenishment lead times on the amount of inventory required at each DC. It will also determine the amount of each product to be shipped through the different stages in the supply chain. Included in the model are inventory policies that consider costs, as well as risk factors that might affect the operation of the supply chain.

The model is applied to one of the leading health and hygiene companies in the world. The company employs nearly 53,000 people worldwide and sells over $19 billion a year. With operations in 35 countries, the company's global brands are sold in more than 150 countries and used by approximately 1.3 billion people, holding first or second market positions in the majority of its markets.

In a previous research study, Portillo (2008) developed a supply chain footprint model that allows, at the production line level of detail, to assess whether opening/keeping a location/production line contributes to financial, customer and risk management targets. However, Portillo's approach focused on a strategic level and did not consider the impact of footprint decisions on inventory flows, which are very important in driving costs in a supply chain. Moreover, it is also of interest to assess how footprint decisions affect working capital levels at the company, which was not directly addressed by Portillo's model.

This research consists of incorporating *inventory dynamics* into the original footprint optimization model. Inventory dynamics refers to the amount of inventory held at each facility in order to satisfy the required demand at each time period and the optimal inventory policies applied. The objective of this model will be to minimize the working capital required at the different supply chain stages. In this case, this problem is equivalent to considering the inventory levels along the supply network. Maintaining inventory can be very costly, but it is important to have product on hand in order to avoid stock-outs in cases where demand exceeds expectations. A tradeoff needs to be made where inventory is held at a minimum, but there is still enough to satisfy possible demand levels.

The model consists of an optimization problem with the objective to maximize gross profit taking into consideration the inventory and backorder costs. In order to solve this problem, it is assumed that the manufacturing footprint will remain fixed. In other words, the manufacturing facilities, as well as their capacities, will not vary throughout the modeling and analysis phase. The model will run for a multi-period timeframe and its output will be the production and transportation schedules for all the products throughout the supply chain system.

For a supply chain network, there can be many possible distribution plans. In order to find the optimal plan, the model will be analyzed through several runs. The model will use a rolling time horizon, basis every two time periods with a fixed planning horizon of 8 time periods. Every two time periods, when the model is run, the initial conditions, which consist of the initial inventory levels, expected deliveries and the demand forecasts, will be updated. The solutions obtained for the first two time periods will be implemented, and then the model should be updated and run again.

The model will be implemented into a supply chain system within a region with four different countries in Latin America. This region contains several DCs that currently hold excessive amounts of inventory. In some cases, a DC holds enough inventory to satisfy two months' demand. In order to recommend the amount of inventory at each DC, multiple runs will be made.

Chapter 2 consists of a literature review about the importance of supply chain management, the tradeoffs between costs and service levels, and different methods and applications where inventory control policies have been used.

Chapter 3 provides a description and formulation of the basic inventory dynamics model, as well as a detailed description of the scenarios that will be analyzed.

In Chapter 4, we apply the model to a real supply chain network of a major multinational company. This chapter will demonstrate how the model is used and implemented into the company's planning strategy. Based on the results obtained from the model's multiple runs, the recommended inventory will also be presented and analyzed.

Chapter 5 will consist of concluding remarks and suggestions for future research to extend our work.

Chapter 2. Literature Review

This section will present some background on relevant research that has been done in the past regarding supply chain management, inventory control and methods used to solve nonlinear models that can be useful in solving the inventory dynamics model being proposed.

2.1 Supply Chain Management

In today's competitive world, supply chain management (SCM) has become a very important tool that allows companies to obtain an edge over their competitors. By implementing a good SCM system, companies are able to lower their costs and depending on the type of supply chain, they can be more cost efficient or customer responsive.

Lee and Billington (1992) describe many pitfalls of SCM as well as some opportunities; they mention that "the more complex your network of suppliers, manufacturers, and distributors, the more likely you can gain operational efficiencies by attending to inventory." They define SCM as the coordination of manufacturing, logistics, materials, distribution and transportation functions within a company or group of companies. Davis (1993) shows that in order to have effective supply chain management, a concurrent effort to manage all of the aspects of delivering products to customers has to be taken into consideration. He also states that a supply chain can reduce its inventory by redistributing it efficiently throughout the supply chain. Handfield (1994) considers the logistics perspective of a merchant, and describes the results of a survey of 97 purchasing managers. The survey examined several issues related to different aspects of SCM. The results of the study show that most managers' objective is to improve material flow through the supply chain. As a consequence, information sharing between the different stages is necessary in order to replenish merchandise quickly and deliver it to the customer in smaller and more manageable lot sizes. Farley (1997) shows how SCM's focus is on a company's utilization of their supplier's process and technology to obtain a competitive advantage for their products. Leenders et al. (1997) show that SCM is used by the purchasing department in their efforts to develop more responsive products. They identify SCM as an effective approach to manage the flow of information, materials and services, from raw materials to the final product that reaches the final customer.

Lambert and Cooper (2000), suggest that in order to have successful SCM, cross-functional integration and marketing are critical. They analyze SCM in terms of upstream and downstream relationships depending on how information and goods are being transported through the supply chain network. An upstream relationship means that information or products start in the final stage of the supply chain (typically, the retailer) and travel upstream through the supply chain system until they reach an earlier stage of the supply chain (typically, suppliers). This relationship is often seen with the flow of information regarding purchases, as well as when products are being returned by the customers. A downstream relationship is the opposite; information or products start in the initial stages of the supply chain (usually the raw material supplier, or manufacturer) and travel through the supply chain system until they reach the final stage. This relationship can be seen in a regular production and transportation of goods where they are manufactured, then they move to a warehouse and finally they reach the retailer.

2.2 Inventory Control

During the last few years, researchers have noticed that a very important driver for a supply chain is inventory. Inventory models can be classified under different criteria. Considering the demand, supply and planning horizon; the formulation of the models can change because of different criteria, different assumptions can be made. The classification of these models is as follows:

Based on Demand:

The demand used for modeling can be deterministic or stochastic. A deterministic demand means that the amount of product demanded is a known quantity which can be obtained by using forecasting methods. A stochastic demand introduces uncertainty into the model. Based on past data, a demand distribution should be obtained. Because of the uncertainty that a stochastic demand brings, the modeling can become much more complex because of the use of probability functions.

Based on Supply:

The supply of goods to the customer will depend on the lead time. The lead time can be instantaneous, constant, variable or stochastic. Instantaneous lead time simply means that there is no lead time. The customer will receive the order immediately after it is placed. Similar to the demand, stochastic lead times introduce uncertainty into the model and can complicate it significantly.

Based on Planning Horizon:

The modeling can be done for a single period or for multiple periods (i.e. "multi period"). When using a multi period model, it should be specified whether the time frame will be finite or infinite.

A combination between the three different criteria should be used when modeling inventory control problems. Hadley and Whitin (1963) and Clark (1972) show good reviews and overviews of the concepts of the basic problem of inventory control that were mentioned above.

In order to optimize inventory levels, inventory needs to be reviewed either continuously or periodically. Periodic review occurs when there is a set time period between each time that the inventory is reviewed. By doing this, no constant monitoring is necessary, but there could be scenarios where there is no more inventory before the review is made. Chiang and Gutierrez (1996), Chuang et al. (2004) and Chiang (2008) study different supply chain scenarios where periodic review inventory policies are implemented. Continuous review refers to the case where the inventory levels are being reviewed constantly, in order to know exactly when the inventory reaches a certain point, and an order should be placed. Azoury and Brill (1992), Ghalebsaz-Jeddi et al. (2004) and Tutuncu et al. (2008) use continuous review inventory models in order to optimize inventory levels.

A lot of research has been done for the implementation of different inventory policies in de-centralized supply chains. A decentralized supply chain is one where the different stages are managed by different entities and hence, there is less information sharing compared to a centralized supply chain. Axater (1995, 1998), Svoronos and Zipkin (1998) show different methodologies to set inventory policies in decentralized supply chains. Axsater and Juntti (1996) and Axsater (1997) compare different echelon inventory and installation of reorder point policies for multi-stage supply chains. Cachon and Zipkin (1999) model the evaluation of an inventory policy for products that come in batches. Hence, the supply chain considers one warehouse and several identical retailers. They use a reorder point that needs to be integer and multiple of a fixed batch size.

In a centralized supply chain system, information and products flow differently than in a de-centralized supply chain system. Research has also been done for inventory control in centralized supply chain systems. Marklund (2002) introduces a new replenishment policy for inventory control for a centralized supply chain that consists of one central warehouse that supplies a product to different retailers. The new policy controls the replenishment process at the warehouse. In order to do this, the author assumes that each retailer uses a continuous review (R, Q) policy. He shows that in various cases, when using the proposed inventory control policy, significant savings can be made compared to the (R, Q) policy that is traditionally used in the warehouse.

Zomerdijk and Vries (2003) discuss how it is important to pay attention to other aspects in inventory control in addition to the traditional aspects, for example, order quantities and replenishment strategies. The other important factors that are not commonly considered are the allocation of responsibilities and authorities regarding inventory management, the quality of inventory information and the relevant decision-making processes. The authors take all of these factors, both traditional and nontraditional and integrate them into a framework useful for solving inventory control problems. In order to illustrate the inventory control problem, spare parts that flow through a centralized supply chain of a missionary aviation organization are used and they show how to take into consideration the organizational context of inventories. The authors also show how the understanding of the organizational perspective leads to a more appropriate redesign of the inventory control system.

Wang et al (2003) apply centralized strategies based on model predictive control into the inventory management problem. They determine that, because the good signal information and process knowledge that a centralized supply chain allows, the model is able to coordinate the inventory decisions very quickly and accurately. DiFillipo (2003) examines both a decentralized supply chain system and a centralized supply chain system. The author uses a multi-criteria optimization model under three different objectives: capital invested in inventory, annual number of orders and annual transportation costs. He finds that, even though it goes against intuition, under some scenarios and policies, a decentralized supply chain outperforms a centralized supply chain. Kim et al (2005) propose two adaptive inventory control models for a supply chain system. One model is a centralized model and the other a decentralized model. Both models have as their objective to satisfy a set service level for each retailer. Using a simulation based experiment to compare both models; the authors find that when using a centralized model they are able to achieve the target service level most of the time, whereas in the decentralized model, there were many instances when the service level was not achieved. Ravindran (2008) shows how to specify service constraints for an inventory management model. One of these service constraints is the equation that calculates the safety stock based on a replenishment lead time that follows a normal distribution. This assumption allows a "well-defined convolution of demand over the lead time," and therefore, shows that the demand is also normally distributed. The safety stock equation then is based on the mean lead time, mean demand and their variances.

2.3 Inventory Aggregation

Inventory aggregation has been the focus of many researchers interested in the implementation of inventory dynamics as a means to lowering inventory costs in a supply chain.

Gerchak and He (2003) studied the relations between risk pooling and the variability of demand. Risk pooling involves the aggregation of inventories and/or demand across different locations in the supply chain, in order to diversify the effects of demand variability. They focused on answering a very important question; "do the benefits of risk pooling increase in the variabilities of the original demands?" (Gerchak, 2003) They found that this behavior does not happen for structured models that take into account increased demand variability.

Gaur and Ravindran (2003) developed a bi-criteria model to optimize inventory aggregation. They state that inventory aggregation, also known as risk pooling, is one of the most efficient ways to reduce the level of safety stock in a supply chain. The authors provide a good explanation of what risk pooling is. They mention that risk pooling reduces inventory costs, but it also increases replenishment lead time to the customers, which reduces supply chain responsiveness. Both authors suggest that risk pooling is a good strategy when inventory costs represent a high percentage of the total supply chain costs and when there is high demand uncertainty. When building their model, the authors used a two-stage model in order to find the best distribution plan and safety stocks for a supply chain network. The first stage consisted of finding the best transportation routes in order to minimize costs and the second stage then added the customer responsiveness objective in order to calculate the required safety stock level at each DC based on the given fill rate. Their model focused on finding a balance between cost reduction and maximization of customer responsiveness. Shailesh et al (2005) analyzed the trade-offs between risk pooling and logistics costs. This research was performed on a multi-plant network. The authors wanted to analyze the trade-off between decreasing logistics costs and losing risk-pooling benefits in different plant networks. Even though this research did not focus directly on inventory dynamics, it took into account the importance of risk pooling within a supply chain network. Ozsen et al (2008) created a capacitated warehouse location model with risk pooling. The model takes into account how capacity issues and inventory management at the warehouses are interdependent. Their model consists of an allocation model that incorporates important inventory decisions.

Yang and Schrage (2008) took a different approach on the study of risk pooling. Their research focused on finding conditions that may cause risk pooling to increase inventory. They showed that inventory would not increase if the company did not apply a penalty for shortages, but minimized inventory holding costs subject to a fixed service level. By doing this, the company could be missing an opportunity to increase profits if demand increases but inventory does not.

2.4 Nonlinear Modeling

As mentioned in Chapter 1, safety stock calculations involve dealing with variability in demand and lead times. These are nonlinear terms that lead to nonlinear functions in the model formulation. It can be very complex and time consuming to solve a nonlinear model. If possible, these nonlinear functions should be linearized in order to have an algorithm that is easier to solve and guarantees a global optimum. If linearization is not feasible, there are many different methods that can be used to solve a nonlinear model.

Yokota et al (1995) formulate a model that optimizes the design of a system reliability problem. The formulation of the model becomes a nonlinear integer programming problem with interval coefficients. In order to solve this problem, the authors transform the model into a single objective nonlinear integer programming problem without interval coefficients. This is done by using crossover operators which can be defined as a linear combination of different parameters.

Gill et al (2005) consider problems with smooth nonlinear functions used in the objective and as inequality constraints for modeling. They use sequential quadratic programming in order to solve their model. They solve the nonlinear problem using different quadratic programming sub problems. Each sub problem employs a linearization of the constraints of the original problem and the objective function is a quadratic approximation to the Lagrangian function. Line searches are made in order to "achieve a sufficient decrease in the augmented Lagrangian merit function." Liang et al (2008) solve a constrained nonlinear programming model where they assume that the objective function and the constraints of the model are differentiable. They demonstrate that a sequential quadratic programming algorithm can solve quadratic problems by deriving search directions at every iteration.

Safety stock functions have a square root term; this makes the function non-differentiable and non-smooth. Because of this, the assumptions made by the previous authors would be violated and a different approach needs to be taken in order to solve the problem. Shu et al (2005) formulate a stochastic transportation-inventory network design model that minimizes total network costs by determining the amount of DCs needed as well as their locations, and the optimal assignment of retailers to each DC. They propose a different approach for solving the nonlinear integer programming model. Using an efficient algorithm that solves the pricing problem using a worst-case running time combined with variable fixing techniques the authors are able to solve their model.

Shen and Qi (2007) consider a supply chain model that minimizes total costs and determines the number of DCs required as well as their locations. The model also outputs the amount of safety stock needed at each DC in order to reach a desired service level. Their formulation is a nonlinear integer programming model, which is solved by a Lagrangian relaxation-based algorithm. Instead of using line searches, Shen and Qui relax the problem by using a branch and bound method.

Jung et al (2008) calculate safety stock levels for petrochemical, chemical and pharmaceutical industries. In their formulation they have nonlinear functions. They are able to show that a performance function can be approximated by the convex hull of simple straight lines. By doing this, they are able to linearize their nonlinear terms and simplify the model significantly.

Chapter 3. Model for Inventory Dynamics

This research involves creating and solving an inventory dynamics model that will be incorporated into the already existing supply chain networking design model developed by Portillo (2008). This inventory dynamics model consists of a multiperiod linear programming model. The model's objective is to maximize the total profit of the supply chain taking into account the inventory and backorder costs throughout the DCs of the supply chain network.

Portillo's model will be used to determine the fixed number of manufacturing and warehouse locations throughout the region under study. With the locations fixed, the proposed model can be used to minimize the production, shipping and inventory holding costs while still carrying enough inventory levels to satisfy demand. The model will output the optimum production and inventory levels of each product for the planning horizon based on the company's forecasted demand. The model will be run in a rolling horizon basis, that is, the end of the planning horizon will be shifted two time periods ahead each time. This will ensure that updated forecasts and shipments are included on a continuous basis.

When planning the production of their products, the company's manufacturing facilities use inventory targets as well as demand forecasts. The solutions obtained by this analysis determine these inventory targets and will help the manufacturing facilities to have a better vision of the amount of product that should be produced. The solution will also give each DC a good idea of the amount of inventory that they need to carry each time period; this should decrease inventory holding costs while maintaining a good service level provided by the company.

3.1 Decision Criteria

The mathematical programming model has one objective: the maximization of profit. This objective is a widely used financial objective. It is defined as the net revenue minus the costs. Since the manufacturing and conversion facilities will be fixed, their costs will be known and no additional fixed costs will be included. The variable costs for this model will consist of the production costs, transportation costs, inventory holding costs and backorder costs. The model takes into account variability in the replenishment lead times of each DC.

Since the model will focus on one specific region, the different monetary exchange rates and any political risk that may occur between imports and exports of products between countries will not be considered.

3.2 Model Formulation

The model formulation shown below will be used in order to calculate the optimum amount of inventory of each product that should be held at each DC. The formulation only uses linear constraints and continuous variables, which should allow for the model to be run very efficiently. Figure 2 below shows the different echelons of the supply chain being modeled.

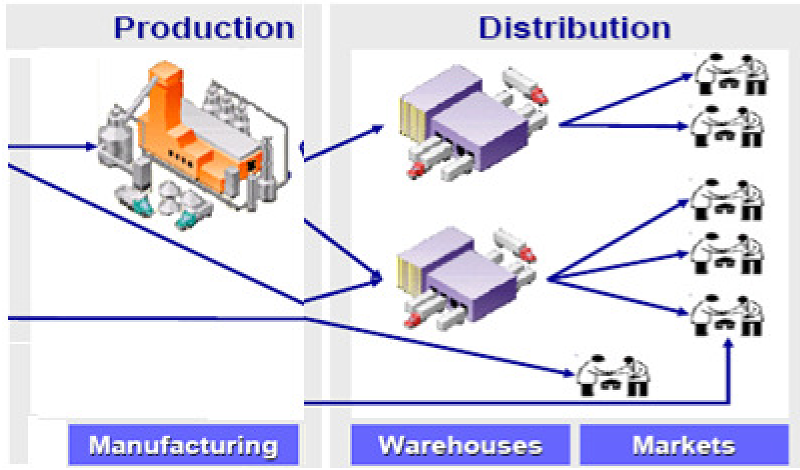

Figure 2 Supply Chain echelons to be modeled

3.2.1 Index

Each echelon of the supply chain is assigned with an index. For this analysis, multiple echelons are part of the supply chain system, and indexes are important for the understanding of the trajectory of each product. Also, since multiple products are considered, they are manufactured in different production lines; both of these are represented by a different index. The indices that will be used are the following:

i, j = Distribution Center

k = Customer

p = Product

t = Time period

3.2.2 Data

These are all of the known parameters that represent financial and operational information.

P_p = Unit selling price of product p

$D_{k,p,t}$ = Demand of product p from customer k during time period t

$PC_{i,p}$ = Unit production and transportation cost of product p shipped from manufacturer to DC i

$TD_{i,j,p}$ = Unit transportation cost of product p shipped from DC i to DC j

$TC_{i,k,p}$ = Unit transportation cost of product p shipped from DC i to customer k

$HC_{i,p}$ = Inventory holding cost per time period per unit of product p stored at DC i (same for all time periods)

$BC_{k,p}$ = Backorder cost per time period per unit of product p for customer k (same for all time periods)

M_p = Production capacity in units of product p

CAP_i = Storage capacity for DC i in pallets

CF_p = Conversion factor from units of product p into pallets (pallets/unit)

$Z^0_{i,p,t}$ = Amount of product p scheduled to be delivered to DC i at time period t (initial conditions)

$INV^0_{i,p}$ = Initial amount of inventory of product p at DC i (initial conditions)

$Y^0_{i,j,p}$ = Amount of product p shipped from DC i to DC j before the planning horizon (initial conditions)

$X^0_{i,k,p}$ = Amount of product p shipped from DC i to customer k before the planning horizon (initial conditions)

$W^0_{k,p}$ = Initial amount of cumulative backorder of product p for customer k (initial condition)

L = Replenishment lead time

Note: We assume L=2 time periods from manufacturer to DC and L=1 time period from DC to DC and from DC to customer

3.2.3 Variables

The following are all continuous variables defined to represent the flow of the products through the supply chain. Some variables are dependent on each other but are important since they will help achieve outputs that are easier to interpret and understand. The relationship between the variables can be seen in the formulation that is given in the next section.

$Z_{i,p,t}$ = Number of units of product p produced and shipped to DC i during time period t

$Y_{i,j,p,t}$ = Number of units of product p shipped from DC i to DC j during time period t

$X_{i,k,p,t}$ = Number of units of product p shipped from DC i to customer k during time period t

$INV_{i,p,t}$ = Total inventory level of product p at DC i at the end of period t

$W_{k,p,t}$ = Cumulative backorder of product p for customer k at the end of period t

The objective of this model is to maximize the total profit of the supply chain network for a given planning horizon. Portillo (2008) created a model for the same supply chain network, but he did not consider the impact of inventory holding costs. The following equations are the components that will form the objective function.

(Total Revenue from Sales)

$$\left(\sum_{p} P_p \sum_{i}\sum_{k}\sum_{t} X_{i,k,p,t}\right) \quad [1]$$

(Production and Shipping Cost from Manufacturer to DC)

$$\left(\sum_{i}\sum_{p}\left(PC_{i,p}\sum_{t} Z_{i,p,t}\right)\right) \quad [2]$$

(Transportation Cost from DC to DC and DC to Customer)

$$\left(\sum_{i}\sum_{p}\left(\sum_{j}\left(TD_{i,j,p}\sum_{t} Y_{i,j,p,t}\right)+\sum_{k}\left(TC_{i,k,p}\sum_{t} X_{i,k,p,t}\right)\right)\right) \quad [3]$$

(Inventory Holding Cost)

$$\left(\sum_{t}\sum_{i}\sum_{p}\left(HC_{i,p}INV_{i,p,t}\right)\right) \quad [4]$$

(Backorder Cost)

$$\left(\sum_{t}\sum_{k}\sum_{p}\left(BC_{k,p}W_{k,p,t}\right)\right) \quad [5]$$

The final objective function is the following:

$$Maximize\ Profit$$

$$= (Total\ Revenue\ from\ Sales)$$

$$-\ (Production\ and\ Shipping\ Cost\ from\ Manufacturer\ to\ DC)$$

$$-\ (Transportation\ Cost\ from\ DC\ to\ DC\ and\ DC\ to\ Customer)$$

$$-\ (Inventory\ Holding\ Cost) -\ (Backorder\ Cost)$$

Subject to the following constraints:

- **Balance Equations:**

The amount of product received from the manufacturer plus the amount of inventory held from the previous time period plus the amount of product received from other DCs has to equal the amount of inventory held at the end of the time period plus the amount of product shipped to other DCs plus the amount of product shipped to the customer. This has to hold true for all DCs and all products during all of the time periods.

These equations are used in order to make sure that the flow that goes into a DC equals the flow that goes out of it. A DC cannot ship more products than what it has available. These constraints make sure that what a DC ships and stores is equal to what it received from conversion facilities plus what it had in inventory from the previous time period.

$$Z_{i,p,t-2} + INV_{i,p,t-1} + \sum_j Y_{j,i,p,t-1} = INV_{i,p,t} + \sum_j Y_{i,j,p,t} + \sum_k X_{i,k,p,t} \quad [6]$$

- $\forall\ i, p, t$

Note: Lead time (L) is 2 for shipment from manufacturer to DC and 1 for DC to DC.

- **Storage Capacity:**

Every DC has a set volume and can only hold so much amount of inventory. The amount of inventory held at the end of each time period at each DC has to be less than or equal to the total storage capacity at each DC.

$$\sum_{p} \left((INV_{i,p,t}) * CF_p \right) \leq CAP_i \qquad [7]$$

- $\forall \ i, t$

Note: (i) CAP_i are in pallets

(ii) CF_p are in pallet/unit for product p

- **Demand Fulfillment:**

The amount of product shipped to each customer has to be equal to that customer's demand, plus the amount of previous backorders, and minus the amount of new backorders at the end of the time period. Note that the lead time for DC to customer is one time period.

$$\sum_{i} X_{i,k,p,t-1} = D_{k,p,t} + W_{k,p,t-1} - W_{k,p,t} \qquad [8]$$

$\forall \ k, p, t$

- **Production Capacity:**

The manufacturer cannot produce more product than what it is capable of producing.

$$\sum_{i} Z_{i,p,t} \leq M_p \qquad [9]$$

$$\forall \ p, t$$

- All other variables are nonnegative

3.2.4 Illustrative Example

3.2.4.1 Data used for the Illustrative Example

For this example, 2 DCs, 2 Products, 3 Customers and 6 time periods will be considered. The data values used are the following:

Production and transportation lead time from Manufacturer to each DC is 2 time periods.

Transportation lead time from DC to the other DC is 1 time period.

Transportation lead time from DC to each Customer is 1 time period.

Production capacity for product 1= 550 units per time period

Production capacity for product 2= 450 units per time period

Storage capacity:

DC 1 = 210 pallets

DC 2 = 140 pallets

Conversion factor from unit of product into pallets of product

Product 1 = 16 units/pallet

Product 2 = 32 units/pallet

Table 1 Unit selling price

	Selling Price
Product 1	$10.00
Product 2	$15.00

Table 2 Unit production and transportation cost from manufacturer to DC

	Product 1	Product 2
DC 1	$0.50	$0.70
DC 2	$0.55	$0.60

Table 3 Unit shipping cost from DC to Customer

		Customer 1	Customer 2	Customer 3
DC 1	Product 1	$0.08	$0.51	$0.20
	Product 2	$0.15	$0.52	$0.31
DC 2	Product 1	$0.79	$0.28	$0.64
	Product 2	$0.83	$0.39	$0.71

Table 4 Unit shipping cost from DC to DC

		DC 1	DC 2
DC 1	Product 1	$0.00	$1.00
	Product 2	$0.00	$1.25
DC 2	Product 1	$1.00	$0.00
	Product 2	$1.25	$0.00

Table 5 Unit inventory holding costs per time period

	Holding Cost	
	Product 1	Product 2
DC 1	$0.73	$0.82
DC 2	$0.67	$0.76

Table 6 Unit backorder cost per time period

	Backorder Cost	
	Product 1	Product 2
Customer 1	$13.87	$15.58
Customer 2	$12.73	$14.44
Customer 3	$13.48	$14.92

Table 7 Customer demand (number of units)

		Demand		
		Customer 1	Customer 2	Customer 3
Product 1	Time 1	152	204	132
	Time 2	173	218	141
	Time 3	128	249	127
	Time 4	134	261	106
	Time 5	157	239	98
	Time 6	193	246	103
Product 2	Time 1	89	52	102
	Time 2	92	39	117
	Time 3	103	37	132
	Time 4	87	46	138
	Time 5	74	57	127
	Time 6	83	71	104

3.2.4.2 Sets

i, j = Distribution Center, i=1, 2 and j=1, 2.

k = Customer, k=1, 2, 3.

p = Product, p=1, 2.

t = Time period, t=1, 2, 3, 4, 5, 6.

3.2.4.3 Decision Variables

$Z_{i,p,t}$ = Number of units of product p produced and shipped to DC i during time period t

$Y_{i,j,p,t}$ = Number of units of product p shipped from DC i to DC j during time period t

$X_{i,k,p,t}$ = Number of units of product p shipped from DC i to customer k during time period t

$INV_{i,p,t}$ = Total inventory level of product p at DC i at the end of period t

$W_{k,p,t}$ = Cumulative backorder of product p for customer k at the end of period t

(Total Revenue from Sales)

$$\left(\sum_{p} P_p \sum_{i} \sum_{k} \sum_{t} X_{i,k,p,t}\right) \qquad [1]$$

$10*(X_{1,1,1,1}+X_{1,1,1,2}+X_{1,1,1,3}+X_{1,1,1,4}+X_{1,1,1,5}+X_{1,1,1,6}+$

$X_{1,2,1,1}+X_{1,2,1,2}+X_{1,2,1,3}+X_{1,2,1,4}+X_{1,1,1,5}+X_{1,2,1,6}+$

$X_{1,3,1,1}+X_{1,3,1,2}+X_{1,3,1,3}+X_{1,3,1,4}+X_{1,3,1,5}+X_{1,3,1,6}+$

$X_{2,1,1,1}+X_{2,1,1,2}+X_{2,1,1,3}+X_{2,1,1,4}+X_{2,1,1,5}+X_{2,1,1,6}+$

$X_{2,2,1,1}+X_{2,2,1,2}+X_{2,2,1,3}+X_{2,2,1,4}+X_{2,1,1,5}+X_{2,2,1,6}+$

$X_{2,3,1,1}+X_{2,3,1,2}+X_{2,3,1,3}+X_{2,3,1,4}+X_{2,3,1,5}+X_{2,3,1,6})+$

$15*(X_{1,1,2,1}+X_{1,1,2,2}+X_{1,1,1,3}+X_{1,1,2,4}+X_{1,1,2,5}+X_{1,1,2,6}+$

$X_{1,2,2,1}+X_{1,2,2,2}+X_{1,2,2,3}+X_{1,2,2,4}+X_{1,1,2,5}+X_{1,2,2,6}+$

$X_{1,3,2,1}+X_{1,3,2,2}+X_{1,3,2,3}+X_{1,3,2,4}+X_{1,3,2,5}+X_{1,3,2,6}+$

$X_{2,1,2,1}+X_{2,1,2,2}+X_{2,1,2,3}+X_{2,1,2,4}+X_{2,1,2,5}+X_{2,1,2,6}+$

$X_{2,2,2,1}+X_{2,2,2,2}+X_{2,2,2,3}+X_{2,2,2,4}+X_{2,1,2,5}+X_{2,2,2,6}+$

$X_{2,3,2,1}+X_{2,3,2,2}+X_{2,3,2,3}+X_{2,3,2,4}+X_{2,3,2,5}+X_{2,3,2,6})$

(Production and Shipping Cost from Manufacturer to DC)

$$\left(\sum_{i}\sum_{p}\left(PC_{i,p}\sum_{t}Z_{i,p,t}\right)\right) \quad [2]$$

$0.5*(Z_{1,1,1}+ Z_{1,1,2}+ Z_{1,1,3}+ Z_{1,1,4}+ Z_{1,1,5}+ Z_{1,1,6}) +$

$0.7*(Z_{1,2,1}+ Z_{1,2,2}+ Z_{1,2,3}+ Z_{1,2,4}+ Z_{1,2,5}+ Z_{1,2,6}) +$

$0.55*(Z_{2,1,1}+ Z_{2,1,2}+ Z_{2,1,3}+ Z_{2,1,4}+ Z_{2,1,5}+ Z_{2,1,6}) +$

$0.6*(Z_{2,2,1}+ Z_{2,2,2}+ Z_{2,2,3}+ Z_{2,2,4}+ Z_{2,2,5}+ Z_{2,2,6})$

(Transportation Cost from DC to DC and DC to Customer)

$$\left(\sum_{i}\sum_{p}\left(\sum_{j}\left(TD_{i,j,p}\sum_{t}Y_{i,j,p,t}\right)+\sum_{k}\left(TC_{i,k,p}\sum_{t}X_{i,k,p,t}\right)\right)\right) \quad [3]$$

$1*(Y_{1,2,1,1}+Y_{1,2,1,2}+Y_{1,2,1,3}+Y_{1,2,1,4}+Y_{1,2,1,5}+Y_{1,2,1,6})+$

$1.25*(Y_{1,2,2,1}+Y_{1,2,2,2}+Y_{1,2,2,3}+Y_{1,2,2,4}+Y_{1,2,2,5}+Y_{1,2,2,6})+$

$1*(Y_{2,2,1,1}+Y_{2,2,1,2}+Y_{2,2,1,3}+Y_{2,2,1,4}+Y_{2,2,1,5}+Y_{2,2,1,6})+$

$1.25*(Y_{2,2,2,1}+Y_{2,2,2,2}+Y_{2,2,2,3}+Y_{2,2,2,4}+Y_{2,2,2,5}+Y_{2,2,2,6})+$

$0.08*(X_{1,1,1,1}+X_{1,1,1,2}+X_{1,1,1,3}+X_{1,1,1,4}+X_{1,1,1,5}+X_{1,1,1,6})+$

$0.15*(X_{1,1,2,1}+X_{1,1,2,2}+X_{1,1,2,3}+X_{1,1,2,4}+X_{1,1,2,5}+X_{1,1,2,6})+$

$0.51*(X_{1,2,1,1}+X_{1,2,1,2}+X_{1,2,1,3}+X_{1,2,1,4}+X_{1,2,1,5}+X_{1,2,1,6})+$

$0.52*(X_{1,2,2,1}+X_{1,2,2,2}+X_{1,2,2,3}+X_{1,2,2,4}+X_{1,2,2,5}+X_{1,2,2,6})+$

$0.20*(X_{1,3,1,1}+X_{1,3,1,2}+X_{1,3,1,3}+X_{1,3,1,4}+X_{1,3,1,5}+X_{1,3,1,6})+$

$0.31*(X_{1,3,2,1}+X_{1,3,2,2}+X_{1,3,2,3}+X_{1,3,2,4}+X_{1,3,2,5}+X_{1,3,2,6})+$

$0.79*(X_{2,1,1,1}+X_{2,1,1,2}+X_{2,1,1,3}+X_{2,1,1,4}+X_{2,1,1,5}+X_{2,1,1,6})+$

$0.83*(X_{2,1,2,1}+X_{2,1,2,2}+X_{2,1,2,3}+X_{2,1,2,4}+X_{2,1,2,5}+X_{2,1,2,6})+$

$0.28*(X_{2,2,1,1}+X_{2,2,1,2}+X_{2,2,1,3}+X_{2,2,1,4}+X_{2,2,1,5}+X_{2,2,1,6})+$

$0.39*(X_{2,2,2,1}+X_{2,2,2,2}+X_{2,2,2,3}+X_{2,2,2,4}+X_{2,2,2,5}+X_{2,2,2,6})+$

$0.64*(X_{2,3,1,1}+X_{2,3,1,2}+X_{2,3,1,3}+X_{2,3,1,4}+X_{2,3,1,5}+X_{2,3,1,6})+$

$0.71*(X_{2,3,2,1}+X_{2,3,2,2}+X_{2,3,2,3}+X_{2,3,2,4}+X_{2,3,2,5}+X_{2,3,2,6})$

(Inventory Holding Cost)

$$\left(\sum_{t}\sum_{i}\sum_{p}\left(HC_{i,p}INV_{i,p,t}\right)\right) \qquad [4]$$

$0.73*(INV_{1,1,1}+ INV_{1,1,2}+ INV_{1,1,3}+ INV_{1,1,4}+ INV_{1,1,5}+ INV_{1,1,6})+$

$0.82*(INV_{1,2,1}+ INV_{1,2,2}+ INV_{1,2,3}+ INV_{1,2,4}+ INV_{1,2,5}+ INV_{1,2,6})+$

$0.67*(INV_{2,1,1}+ INV_{2,1,2}+ INV_{2,1,3}+ INV_{2,1,4}+ INV_{2,1,5}+ INV_{2,1,6})+$

$0.76*(INV_{2,2,1}+ INV_{2,2,2}+ INV_{2,2,3}+ INV_{2,2,4}+ INV_{2,2,5}+ INV_{2,2,6})$

(Backorder Cost)

$$\left(\sum_{t}\sum_{k}\sum_{p}\left(BC_{k,p}W_{k,p,t}\right)\right) \qquad [5]$$

$13.87*(W_{1,1,1}+W_{1,1,2}+ W_{1,1,3}+ W_{1,1,4}+ W_{1,1,5}+ W_{1,1,6})+$

$15.58*(W_{1,2,1}+ W_{1,2,2}+ W_{1,2,3}+ W_{1,2,4}+ W_{1,2,5}+ W_{1,2,6})+$

$12.73*(W_{2,1,1}+ W_{2,1,2}+ W_{2,1,3}+ W_{2,1,4}+ W_{2,1,5}+ W_{2,1,6})+$

$14.44*(W_{2,2,1}+ W_{2,2,2}+ W_{2,2,3}+ W_{2,2,4}+ W_{2,2,5}+ W_{2,2,6})+$

$13.48*(W_{3,1,1}+ W_{3,1,2}+ W_{3,1,3}+ W_{3,1,4}+ W_{3,1,5}+ W_{3,1,6})+$

$14.92*(W_{3,2,1}+ W_{3,2,2}+ W_{3,2,3}+ W_{3,2,4}+ W_{3,2,5}+ W_{3,2,6})$

The final objective function is the following:

$$\begin{aligned} &Maximize\ Profit \\ &\qquad = (Total\ Revenue\ from\ Sales) \\ &\qquad - (Production\ and\ Shipping\ Cost\ from\ Manufacturer\ to\ DC) \\ &\qquad - (Transportation\ Cost\ from\ DC\ to\ DC\ and\ DC\ to\ Customer) \\ &\qquad - (Inventory\ Holding\ Cost) - (Backorder\ Cost) \end{aligned}$$

Subject to the following additional constraints:

- **<u>Balance Equations:</u>** The amount of product received from the manufacturer plus the amount of inventory held from the previous time period plus the amount of product received from other DCs has to equal the amount of inventory held at the end of the time period plus the amount of product shipped to other DCs plus the amount of product shipped to the customer. This has to hold true for all DCs and all products during all of the time periods. (Note: the lead time is 2 two time periods from plant to DC, and one time period from DC to DC and from DC to customer.)

At t=1:

$$Z^0_{i,p,-1} + INV^0_{i,p,0} + \sum_j Y^0_{j,i,p,0} = INV_{i,p,t} + \sum_j Y_{i,j,p,t} + \sum_k X_{i,k,p,t} \qquad [6]$$

$$\forall\ i,p$$

At t=2:

$$Z^0_{i,p,0} + INV_{i,p,t-1} + \sum_j Y_{j,i,p,-1} = INV_{i,p,t} + \sum_j Y_{i,j,p,t} + \sum_k X_{i,k,p,t} \qquad [7]$$

$$\forall\ i,p$$

For any t ≥ 3:

$$Z_{i,p,t-2} + INV_{i,p,t-1} + \sum_j Y_{j,i,p,t-1} = INV_{i,p,t} + \sum_j Y_{i,j,p,t} + \sum_k X_{i,k,p,t} \qquad [8]$$

$$\forall\ i,p,\mathrm{t}$$

Detailed constraints for balance equations are given below:

For DC 1, Product 1:

$Z^0_{1,1,-1}$+$INV^0_{1,1,0}$ + $Y^0_{2,1,1,0}$ = $INV_{1,1,1}$+$Y_{1,2,1,1}$+($X_{1,1,1,1}$+$X_{1,2,1,1}$+$X_{1,3,1,1}$) t=1

Note: Z^0, INV^0 and Y^0 are known constants (initial conditions).

$Z^0_{1,1,0}+INV_{1,1,1} + Y_{2,1,1,1} = INV_{1,1,2}+Y_{1,2,1,2}+(X_{1,1,1,2}+X_{1,2,1,2}+X_{1,3,1,2})$ t=2

$Z_{1,1,1}+INV_{1,1,2} + Y_{2,1,1,2} = INV_{1,1,3}+Y_{1,2,1,3}+(X_{1,1,1,3}+X_{1,2,1,3}+X_{1,3,1,3})$ t=3

$Z_{1,1,2}+INV_{1,1,3} + Y_{2,1,1,3} = INV_{1,1,4}+Y_{1,2,1,4}+(X_{1,1,1,4}+X_{1,2,1,4}+X_{1,3,1,4})$ t=4

$Z_{1,1,3}+INV_{1,1,4} + Y_{2,1,1,4} = INV_{1,1,5}+Y_{1,2,1,5}+(X_{1,1,1,5}+X_{1,2,1,5}+X_{1,3,1,5})$ t=5

$Z_{1,1,4}+INV_{1,1,5} + Y_{2,1,1,5} = INV_{1,1,6}+Y_{1,2,1,6}+(X_{1,1,1,6}+X_{1,2,1,6}+X_{1,3,1,6})$ t=6

For DC 2, Product 1:

$Z^0_{2,1,-1}+INV^0_{2,1,0} + Y^0_{1,2,1,0} = INV_{2,1,1}+Y_{2,1,1,1}+(X_{2,1,1,1}+X_{2,2,1,1}+X_{2,3,1,1})$ t=1

$Z^0_{2,1,0}+INV_{2,1,1} + Y_{1,2,1,1} = INV_{2,1,2}+Y_{2,1,1,2}+(X_{2,1,1,2}+X_{2,2,1,2}+X_{2,3,1,2})$ t=2

$Z_{2,1,1}+INV_{2,1,2} + Y_{1,2,1,2} = INV_{2,1,3}+Y_{2,1,1,3}+(X_{2,1,1,3}+X_{2,2,1,3}+X_{2,3,1,3})$ t=3

$Z_{2,1,2}+INV_{2,1,3} + Y_{1,2,1,3} = INV_{2,1,4}+Y_{2,1,1,4}+(X_{2,1,1,4}+X_{2,2,1,4}+X_{2,3,1,4})$ t=4

$Z_{2,1,3}+INV_{2,1,4} + Y_{1,2,1,4} = INV_{2,1,5}+Y_{2,1,1,5}+(X_{2,1,1,5}+X_{2,2,1,5}+X_{2,3,1,5})$ t=5

$Z_{2,1,4}+INV_{2,1,5} + Y_{1,2,1,5} = INV_{2,1,6}+Y_{2,1,1,6}+(X_{2,1,1,6}+X_{2,2,1,6}+X_{2,3,1,6})$ t=6

For DC 1, Product 2:

$Z^0_{1,2,-1}+INV^0_{1,2,0} + Y^0_{2,1,2,0} = INV_{1,2,1}+Y_{1,2,2,1}+(X_{1,1,2,1}+X_{1,2,2,1}+X_{1,3,2,1})$ t=1

$Z^0_{1,2,0}+INV_{1,2,1} + Y_{2,1,2,1} = INV_{1,2,2}+Y_{1,2,2,2}+(X_{1,1,2,2}+X_{1,2,2,2}+X_{1,3,2,2})$ t=2

$Z_{1,2,1}+INV_{1,2,2} + Y_{2,1,2,2} = INV_{1,2,3}+Y_{1,2,2,3}+(X_{1,1,2,3}+X_{1,2,2,3}+X_{1,3,2,3})$ t=3

$Z_{1,2,2}$+$INV_{1,2,3}$ + $Y_{2,1,2,3}$ = $INV_{1,2,4}$+$Y_{1,2,2,4}$+($X_{1,1,2,4}$+$X_{1,2,2,4}$+$X_{1,3,2,4}$) t=4

$Z_{1,2,3}$+$INV_{1,2,4}$ + $Y_{2,1,2,4}$ = $INV_{1,2,5}$+$Y_{1,2,2,5}$+($X_{1,1,2,5}$+$X_{1,2,2,5}$+$X_{1,3,2,5}$) t=5

$Z_{1,2,4}$+$INV_{1,2,5}$ + $Y_{2,1,2,5}$ = $INV_{1,2,6}$+$Y_{1,2,2,6}$+($X_{1,1,2,6}$+$X_{1,2,2,6}$+$X_{1,3,2,6}$) t=6

For DC 2, Product 2:

$Z^0_{2,2,-1}$+$INV^0_{2,2,0}$ + $Y^0_{1,2,2,0}$ = $INV_{2,2,1}$+$Y_{2,1,2,1}$+($X_{2,1,2,1}$+$X_{2,2,2,1}$+$X_{2,3,2,1}$) t=1

$Z^0_{2,2,0}$+$INV_{2,2,1}$ + $Y_{1,2,2,1}$ = $INV_{2,2,2}$+$Y_{2,1,2,2}$+($X_{2,1,2,2}$+$X_{2,2,2,2}$+$X_{2,3,2,2}$) t=2

$Z_{2,2,1}$+$INV_{2,2,2}$ + $Y_{1,2,2,2}$ = $INV_{2,2,3}$+$Y_{2,1,2,3}$+($X_{2,1,2,3}$+$X_{2,2,2,3}$+$X_{2,3,2,3}$) t=3

$Z_{2,2,2}$+$INV_{2,2,3}$ + $Y_{1,2,2,3}$ = $INV_{2,2,4}$+$Y_{2,1,2,4}$+($X_{2,1,2,4}$+$X_{2,2,2,4}$+$X_{2,3,2,4}$) t=4

$Z_{2,2,3}$+$INV_{2,2,4}$ + $Y_{1,2,2,4}$ = $INV_{2,2,5}$+$Y_{2,1,2,5}$+($X_{2,1,2,5}$+$X_{2,2,2,5}$+$X_{2,3,2,5}$) t=5

$Z_{2,2,4}$+$INV_{2,2,5}$ + $Y_{1,2,2,5}$ = $INV_{2,126}$+$Y_{2,1,2,6}$+($X_{2,1,2,6}$+$X_{2,2,2,6}$+$X_{2,3,2,6}$) t=6

- **Storage Capacity:** The amount of inventory held at the end of each time period at each DC has to be less than or equal to the total storage capacity of that DC.

$$\sum_p \left((INV_{i,p,t}) * CF_p \right) \leq CAP_i \qquad [9]$$

$$\forall \; i, t$$

Note:

$CF_1 = \frac{1}{16} = 0.062 \; pallets \; per \; unit$

$CF_2 = \frac{1}{32} = 0.033\ pallets\ per\ unit$

For DC 1:

$((INV_{1,1,1})*0.062)+(\ (INV_{1,2,1})*0.033) \leq 210$ t=1

$((INV_{1,1,2})*0.062)+(\ (INV_{1,2,2})*0.033) \leq 210$ t =2

$((INV_{1,1,3})*0.062)+(\ (INV^{+}_{1,2,3})*0.033) \leq 210$ t=3

$((INV_{1,1,4})*0.062)+(\ (INV_{1,2,4})*0.033) \leq 210$ t=4

$((INV_{1,1,5})*0.062)+(\ (INV_{1,2,5})*0.033) \leq 210$ t=5

$((INV_{1,1,6})*0.062)+(\ (INV_{1,2,6})*0.033) \leq 210$ t=6

For DC 2:

$((INV_{2,1,1})*0.062)+(\ (INV_{2,2,1})*0.033) \leq 140$ t=1

$((INV_{2,1,2})*0.062)+(\ (INV_{2,2,2})*0.033) \leq 140$ t=2

$((INV_{2,1,3})*0.062)+(\ (INV_{2,2,3})*0.033) \leq 140$ t=3

$((INV_{2,1,4})*0.062)+(\ (INV_{2,2,4})*0.033) \leq 140$ t=4

$((INV_{2,1,5})*0.062)+(\ INV_{2,2,5})*0.033) \leq 140$ t=5

$((INV_{2,1,6})*0.062)+(\ INV_{2,2,6})*0.033) \leq 140$ t=6

- **Demand Fulfillment:** The amount of product shipped to each customer has to be equal to that customer's demand plus the amount of previous backorders minus the amount of new backorders at the end of the time period. Note that the lead time from DC to customer is one time period.

At t=1:

$$\sum_i X^0_{i,k,p,0} = D_{k,p,1} + W^0_{k,p,0} - W_{k,p,1} \qquad [10]$$

$$\forall \ k,p$$

For t ≥ 2:

$$\sum_i X_{i,k,p,t-1} = D_{k,p,t} + W_{k,p,t-1} - W_{k,p,t} \qquad [11]$$

$$\forall \ k,p,t$$

Detailed constraints for demand fulfillment are given below:

Customer 1, Product 1:

$X^0_{1,1,1,0} + X^0_{2,1,1,0} = 152 + W^0_{1,1,0} - W_{1,1,1}$ t=1

Note: X^0 and W^0 are known constants (initial conditions).

$X_{1,1,1,1} + X_{2,1,1,1} = 173 + W_{1,1,1} - W_{1,1,2}$ t=2

$X_{1,1,1,2} + X_{2,1,1,2} = 128 + W_{1,1,2} - W_{1,1,3}$ t=3

$X_{1,1,1,3} + X_{2,1,1,3} = 134 + W_{1,1,3} - W_{1,1,4}$ t=4

$X_{1,1,1,4} + X_{2,1,1,4} = 157 + W_{1,1,4} - W_{1,1,5}$ t=5

$X_{1,1,1,5} + X_{2,1,1,5} = 193 + W_{1,1,5} - W_{1,1,6}$ t=6

Customer 1, Product 2:

$X^0_{1,1,2,0} + X^0_{2,1,2,0} = 89 + W^0_{1,2,0} - W_{1,2,1}$ t=1

$X_{1,1,2,1} + X_{2,1,2,1} = 92 + W_{1,2,1} - W_{1,2,2}$ t=2

$X_{1,1,2,2} + X_{2,1,2,2} = 103 + W_{1,2,2} - W_{1,2,3}$ t=3

$X_{1,1,2,3} + X_{2,1,2,3} = 87 + W_{1,2,3} - W_{1,2,4}$ t=4

$X_{1,1,2,4} + X_{2,1,2,4} = 74 + W_{1,2,4} - W_{1,2,5}$ t=5

$X_{1,1,2,5} + X_{2,1,2,5} = 83 + W_{1,2,5} - W_{1,2,6}$ t=6

Customer 2, Product 1:

$X^0_{1,2,1,0} + X^0_{2,2,1,0} = 204 + W^0_{2,1,0} - W_{2,1,1}$ t=1

$X_{1,2,1,1} + X_{2,2,1,1} = 218 + W_{2,1,1} - W_{2,1,2}$ t=2

$X_{1,2,1,2} + X_{2,2,1,2} = 249 + W_{2,1,2} - W_{2,1,3}$ t=3

$X_{1,2,1,3} + X_{2,2,1,3} = 261 + W_{2,1,3} - W_{2,1,4}$ t=4

$X_{1,2,1,4} + X_{2,2,1,4} = 239 + W_{2,1,4} - W_{2,1,5}$ t=5

$X_{1,2,1,5} + X_{2,2,1,5} = 246 + W_{2,1,5} - W_{2,1,6}$ t=6

Customer 2, Product 2:

$X^0_{1,2,2,0} + X^0_{2,2,2,0} = 52 + W^0_{2,2,0} - W_{2,2,1}$ t=1

$X_{1,2,2,1} + X_{2,2,2,1} = 39 + W_{2,2,1} - W_{2,2,2}$ t=2

$X_{1,2,2,2} + X_{2,2,2,2} = 37 + W_{2,2,2} - W_{2,2,3}$ t=3

$X_{1,2,2,3} + X_{2,2,2,3} = 46 + W_{2,2,3} - W_{2,2,4}$ t=4

$X_{1,2,2,4} + X_{2,2,2,4} = 57 + W_{2,2,4} - W_{2,2,5}$ t=5

$X_{1,2,2,5} + X_{2,2,2,5} = 71 + W_{2,2,5} - W_{2,2,6}$ t=6

Customer 3, Product 1:

$X^0_{1,3,1,0} + X^0_{2,3,1,0} = 132 + W^0_{3,1,0} - W_{3,1,1}$ t=1

$X_{1,3,1,1} + X_{2,3,1,1} = 141 + W_{3,1,1} - W_{3,1,2}$ t=2

$X_{1,3,1,2} + X_{2,3,1,2} = 127 + W_{3,1,2} - W_{3,1,3}$ t=3

$X_{1,3,1,3} + X_{2,3,1,3} = 106 + W_{3,1,3} - W_{3,1,4}$ t=4

$X_{1,3,1,4} + X_{2,3,1,4} = 98 + W_{3,1,4} - W_{3,1,5}$ t=5

$X_{1,3,1,5} + X_{2,3,1,5} = 103 + W_{3,1,5} - W_{3,1,6}$ t=6

Customer 3, Product 2:

$X^0_{1,3,2,0} + X^0_{2,3,2,0} = 102 + W^0_{3,2,0} - W_{3,2,1}$ t=1

$X_{1,3,2,1} + X_{2,3,2,1} = 117 + W_{3,2,1} - W_{3,2,2}$ t=2

$X_{1,3,2,2} + X_{2,3,2,2} = 132 + W_{3,2,2} - W_{3,2,3}$ t=3

$X_{1,3,2,3} + X_{2,3,2,3} = 138 + W_{3,2,3} - W_{3,2,4}$ t=4

$X_{1,3,2,4} + X_{2,3,2,4} = 127 + W_{3,2,4} - W_{3,2,5}$ t=5

$X_{1,3,2,5} + X_{2,3,2,5} = 104 + W_{3,2,5} - W_{3,2,6}$ t=6

- **<u>Production Capacity:</u>** The manufacturer cannot produce more product than what it is capable of producing.

$$\sum_i Z_{i,p,t} \leq M_p \qquad [12]$$

$$\forall \ p, t$$

$Z_{1,1,1} + Z_{2,1,1} \leq 550$

$Z_{1,2,1} + Z_{2,2,1} \leq 450$

$Z_{1,1,2} + Z_{2,1,2} \leq 550$

$Z_{1,2,2} + Z_{2,2,2} \leq 450$

$Z_{1,1,3}+Z_{2,1,3} \leq 550$

$Z_{1,2,3}+Z_{2,2,3} \leq 450$

$Z_{1,1,4}+Z_{2,1,4} \leq 550$

$Z_{1,2,4}+Z_{2,2,4} \leq 450$

$Z_{1,1,5}+Z_{2,1,5} \leq 550$

$Z_{1,2,5}+Z_{2,2,5} \leq 450$

$Z_{1,1,6}+Z_{2,1,6} \leq 550$

$Z_{1,2,6}+Z_{2,2,6} \leq 450$

- All variables are nonnegative

3.2.4.4 Illustrative Example Results

The illustrative example was solved using LINGO. LINGO is an optimization software, and for this example, it used Linear Programming techniques to find the optimum solution. The model had 228 variables and 408 constraints. After 57 iterations, the software found a maximized profit of $41,972.30. There were no backorders or cross-shipment of products between DCs. All customers' demands were satisfied for all time periods.

Initial conditions assumed:

Y^0 and W^0 were all equal to 0.

All Z^0s were equal to 100 units.

Initial inventory levels (INV^0) are given in Table 8.

Table 8 Initial inventory levels (INV^0)

	Product 1	Product 2
DC 1	462	337
DC 2	485	423

Table 9 Amount of product shipped from DC to Customer at time 0 (X^0) and arriving at t=1

		Customer 1	Customer 2	Customer 3
DC 1	Product 1	152	204	132
	Product 2	0	0	0
DC 2	Product 1	0	0	0
	Product 2	89	52	102

Optimal solutions are given in Tables 10, 11 and 12.

Table 10 Inventory levels at the end of each time period

		Product 1	Product 2
DC 1	Time 1	248	228
	Time 2	93	161
	Time 3	0	74
	Time 4	0	0
	Time 5	0	0
	Time 6	0	0
DC 2	Time 1	367	484
	Time 2	218	379
	Time 3	0	295
	Time 4	0	111
	Time 5	0	0
	Time 6	0	0

As it can be seen from Table 10, the inventory levels start high and continue to decrease through time. The reason for this is because there is no uncertainty in this example. It tries to reduce inventory as much as possible because in order to maximize the total profit, the model needs to eliminate all possible costs, including the inventory holding costs. In order to reduce costs, inventory levels at the end of the planning cycle will always be zero unless we impose explicit constraints to maintain minimal inventory.

Table 11 Amount of product produced and shipped from Manufacturer to DC

		Product 1	Product 2
DC 1	Time 1	147	0
	Time 2	255	0
	Time 3	296	83
	Time 4	0	0
	Time 5	0	0
	Time 6	0	0
DC 2	Time 1	43	0
	Time 2	239	0
	Time 3	246	64
	Time 4	0	0
	Time 5	0	0
	Time 6	0	0

It can be noticed in Table 11 that there is no product shipped from the manufacturer to each DC during time period 5 and time period 6. This happens because of the production and shipping lead times between the Manufacturer and each DC. For this example, this lead time was set to two time periods; hence production from the last two time periods would not reach the DC during the planning horizon. Moreover, production during the last two time periods only incur costs on the model, and therefore, the profit would not be maximized.

Table 12 Amount of product shipped from DC to Customer

		Customer 1		Customer 2		Customer 3	
		Product 1	Product 2	Product 1	Product 2	Product 1	Product 2
DC 1	Time 1	173	92	0	0	141	117
	Time 2	128	103	0	0	127	64
	Time 3	134	87	0	0	106	0
	Time 4	157	74	0	0	98	0
	Time 5	193	83	0	0	103	0
	Time 6	0	0	0	0	0	0
DC 2	Time 1	0	0	218	39	0	0
	Time 2	0	0	249	37	0	68
	Time 3	0	0	261	46	0	138
	Time 4	0	0	239	57	0	127
	Time 5	0	0	246	71	0	104
	Time 6	0	0	0	0	0	0

Table 12 shows a similar trend to the one seen in Table 11. The only difference is that the transportation lead time from each DC to each customer was set to one time period for this example. Because of this, there are no shipments made during the last time period. Like before, this would only incur costs on the model and the product would not reach the customers within the planning horizon.

It can be seen in Table 12 that DC 1 ships both products for Customer 1 during all time periods. DC 1 also ships Product 1 to Customer 3 during all periods and all of Product 2 for Customer 3 during the first time period. DC 2 ships both products for Customer 2 during all time periods, as well as Product 2 to Customer 3 from periods 3 to 5. During time period 2, Customer 3's demand for Product 2 is satisfied by both DCs, where both approximately split the demand.

The model will be run on a rolling time horizon basis. This means that the model will be run every two time periods with a planning horizon of 6 time periods. Every two time periods, when the model is run, the initial conditions, which consist of the initial inventory levels and expected deliveries, as well as the demand forecasts will be updated. The reason for this is because as time passes by, the information improves, which allows the forecasts to be more accurate. The model solutions will be implemented only for the first and second time periods; after that, the model will be run again, and the solution for the following time periods will change. This will eliminate the problem of having no inventories at the last time periods as well as having no shipments delivered the last two time periods of the planning horizon.

Chapter 4. Model Results

The model proposed in Chapter 3 was applied to optimize the supply chain of a multinational company that produces and sells consumer goods throughout the world. The company employs nearly 53,000 people and sells over $19 billion a year. Having operations in 35 countries, the company's products are sold in more than 150 countries, holding first or second market share position in most of these.

The analysis was made for a small portion of this company's global supply chain, considering 4 countries that have 3 separate DCs, and 8 customers. The analysis was not made for all of the different products that the company manufactures; only the product category that represents the biggest sales volume in the area being analyzed was used. This product category was composed of 4 different product tiers; each tier represents a product analyzed by the proposed model.

The model analyzes this supply chain for a planning horizon of 8 time periods. Each time period represents 2 weeks, therefore, the model analyzes a 4 month planning horizon. The model will be run on a rolling time horizon basis every two time periods with a fixed planning horizon of 8 time periods. Every two time periods, when the model is run, the initial conditions, which consist of the initial inventory levels, expected deliveries and the demand forecasts, will be updated. The model solutions will be implemented only for the first and second time periods; after that, the initial conditions are updated and the model will be run again. This eliminates the problem seen in the illustrative example where the inventory levels are equal to zero for the last time periods.

Using a rolling time horizon allows the model to be able to react quickly to demand changes and any delays in deliveries. For example, if an order was placed during time period 1, it is expected to arrive during time period 3. In order to update the model for future runs, the amount of product that was ordered would now be the expected orders for the DC for time period 3. If something unexpected happened; such as a road accident that would delay the expected shipment for one whole time period, the model could be updated before running it so that there are no incoming orders for the first time period, but these would now be the expected orders for the second time period. We will discuss in detail how the model should be updated and how to manage these unexpected issues in the next section.

4.1 Model Demonstration

In order to demonstrate how the model works, and how to implement the solutions obtained, we use the same model used for the illustrative example in Chapter 3, but with the company's actual data. Like in the illustrative example, the replenishment lead time from manufacturer to each DC is two time periods, the transportation lead time for products shipped between DCs is one time period, and the transportation lead time for orders shipped from a DC to a customer is one time period. Several runs were be made in order to demonstrate how to modify the model in order to continue using it with the rolling time horizon. With these runs, we also demonstrate how to update the model if something unexpected happens, such as the shipment delay mentioned in the previous paragraph. The demand, initial amount of inventory and expected deliveries used can be found in Tables 13, 14, 15, 16 and 17 below.

Table 13 Customer demand (number of units)

		Time Period							
		1	2	3	4	5	6	7	8
Cust 1	Product 1	1,052,000	1,578,000	1,300,000	1,950,000	1,792,000	2,688,000	1,724,000	2,586,000
	Product 2	876,000	1,314,000	664,000	996,000	648,000	972,000	500,000	750,000
	Product 3	14,504	21,756	13,052	19,578	9,148	13,722	11,856	17,784
	Product 4	16	10	16	9	36	21	38	23
Cust 2	Product 1	88,000	132,000	126,400	189,600	230,400	345,600	246,000	369,000
	Product 2	50,400	75,600	41,600	62,400	52,000	78,000	42,800	64,200
	Product 3	352	211	376	226	380	228	584	876
	Product 4	16	10	9	5	12	7	4	3
Cust 3	Product 1	49,600	74,400	50,400	75,600	94,000	141,000	77,200	115,800
	Product 2	44,800	67,200	30,200	45,300	31,720	47,580	20,000	30,000
	Product 3	252	151	232	139	172	103	250	150
	Product 4	1	1	0	0	143	86	65	39
Cust 4	Product 1	7,120,000	10,680,000	8,760,000	13,140,000	8,760,000	13,140,000	7,800,000	11,700,000
	Product 2	4,360,000	6,540,000	3,680,000	5,520,000	3,280,000	4,920,000	2,160,000	3,240,000
	Product 3	37,320	55,980	31,560	47,340	34,360	51,540	28,800	43,200
	Product 4	608	912	8,640	12,960	12,040	18,060	14,760	22,140
Cust 5	Product 1	22,440	33,660	31,880	47,820	68,000	102,000	40,800	61,200
	Product 2	10,680	16,020	15,720	23,580	17,240	25,860	15,080	22,620
	Product 3	404	606	132	79	63	38	58	35
	Product 4	1	0	4	2	21	12	142	85
Cust 6	Product 1	175,200	262,800	154,800	232,200	220,000	330,000	286,000	429,000
	Product 2	101,200	151,800	56,400	84,600	58,800	88,200	46,400	69,600
	Product 3	528	792	568	852	456	684	484	726
	Product 4	13	8	192	115	112	67	203	122
Cust 7	Product 1	14,840	22,260	66,000	99,000	56,800	85,200	53,200	79,800
	Product 2	8,200	12,300	15,840	23,760	8,480	12,720	12,240	18,360
	Product 3	6	3	90	54	35	21	27	16
	Product 4	7	4	12	7	41	24	54	32
Cust 8	Product 1	178,000	267,000	338,000	507,000	364,800	547,200	222,400	333,600
	Product 2	86,800	130,200	99,600	149,400	66,000	99,000	68,000	102,000
	Product 3	285	171	628	942	468	702	456	684
	Product 4	204	123	140	84	114	69	358	215

Table 14 Customer demand (continued)

		Time Period							
		9	10	11	12	13	14	15	16
Cust 1	Product 1	2,197,500	2,298,875	2,181,938	2,188,672	2,198,833	2,199,268	2,198,144	2,198,099
	Product 2	717,500	734,875	632,813	634,766	637,712	637,839	637,513	637,500
	Product 3	13,128	14,122	15,005	15,052	15,121	15,124	15,117	15,116
	Product 4	29	28	30	30	30	30	30	30
Cust 2	Product 1	297,750	314,588	311,344	312,305	313,755	313,817	313,656	313,650
	Product 2	59,250	61,063	54,169	54,336	54,588	54,599	54,571	54,570
	Product 3	517	551	739	741	745	745	745	745
	Product 4	6	5	3	3	3	3	3	3
Cust 3	Product 1	107,000	110,250	97,706	98,008	98,463	98,482	98,432	98,430
	Product 2	32,325	32,476	25,313	25,391	25,508	25,514	25,501	25,500
	Product 3	168	168	197	196	195	195	195	195
	Product 4	83	68	51	51	51	51	51	51
Cust 4	Product 1	10,350,000	10,747,500	9,871,875	9,902,344	9,948,315	9,950,286	9,945,201	9,944,995
	Product 2	3,400,000	3,430,000	2,733,750	2,742,188	2,754,918	2,755,464	2,754,056	2,753,999
	Product 3	39,475	40,754	36,450	36,563	36,732	36,740	36,721	36,720
	Product 4	16,750	17,928	18,681	18,738	18,825	18,829	18,819	18,819
Cust 5	Product 1	68,000	68,000	51,638	51,797	52,037	52,048	52,021	52,020
	Product 2	20,200	20,940	19,086	19,145	19,233	19,237	19,227	19,227
	Product 3	48	45	46	45	45	45	45	45
	Product 4	65	76	112	112	111	111	111	111
Cust 6	Product 1	316,250	340,313	361,969	363,086	364,772	364,844	364,657	364,650
	Product 2	65,750	67,488	58,725	58,906	59,180	59,191	59,161	59,160
	Product 3	588	620	613	614	617	617	617	617
	Product 4	126	130	160	159	158	158	158	158
Cust 7	Product 1	68,750	71,738	67,331	67,539	67,853	67,866	67,831	67,830
	Product 2	12,950	14,068	15,491	15,539	15,611	15,614	15,606	15,606
	Product 3	25	22	21	21	21	21	21	21
	Product 4	38	37	42	42	42	42	42	42
Cust 8	Product 1	367,000	367,550	281,475	282,344	283,655	283,711	283,566	283,560
	Product 2	83,750	88,188	86,063	86,328	86,729	86,746	86,702	86,700
	Product 3	578	605	577	579	582	582	581	581
	Product 4	189	208	282	281	279	279	280	280

It can be seen in Tables 13 and 14 that the demands for products 1 and 2 are much higher than the demands for products 3 and 4. The reason for this is that products 3 and 4 represent a higher quality tier, and therefore, their prices are much higher which translates into smaller demands.

The initial inventory levels used for the modeling were obtained based on the real amounts of inventory held at each DC at the beginning of time period 1.

Table 15 Initial amount of inventory

	Product 1	Product 2	Product 3	Product 4
DC 1	1,087,510	346,130	4,024	152
DC 2	652,506	692,260	3,353	65
DC 3	543,755	415,356	6,706	0

The expected product to be shipped at the beginning of time period 1were obtained based on the actual amount of product that was on transit, and was expected to arrive at the beginning of the first time period.

Table 16 Expected product to be shipped at the beginning of time period 1

	Product 1	Product 2	Product 3	Product 4
DC 1	3,480,032	1,107,616	12,876	485
DC 2	2,088,019	2,215,232	10,730	208
DC 3	1,740,016	1,329,139	21,460	100

The expected product to be shipped at the beginning of time period 2 were obtained based on the production plan that the company had implemented two time periods before, that should be produced and shipped to the DCs at the beginning of time period 2.

Table 17 Product to be shipped at the beginning of time period 2

	Product 1	Product 2	Product 3	Product 4
DC 1	8,482,578	5,399,628	51,786	694
DC 2	4,567,542	2,907,492	27,885	374
DC 3	1,305,012	830,712	7,967	107

The model consists of linear constraints, linear objective, and continuous variables, which allow the model to be run very quickly. Even though the company's products should be integer valued, because of the high production and sale volume, the solutions obtained can be rounded off to the nearest integers without affecting the quality of the model solution. The proposed model had 1408 variables and 3116 constraints, and took just a few seconds to solve.

4.1.1 Run 1

As mentioned earlier, multiple runs were made in order to demonstrate how the rolling time horizon works, as well as how to implement the solution and update the model for future runs. The ending inventory levels for each period obtained in Run 1 can be found in Table 18.

Table 18 Ending Inventory levels for Run 1

		Time 1	Time 2	Time 3	Time 4	Time 5	Time 6	Time 7	Time 8
DC1	Product 1	0	4,718,936	2,503,736	1,499,000	0	169,461	0	0
	Product 2	0	2,462,441	1,358,741	627,021	0	0	0	0
	Product 3	0	36,728	16,785	7,085	399	399	0	0
	Product 4	616	1,135	1,121	930	816	709	644	0
DC2	Product 1	0	4,229,542	3,722,542	0	0	0	0	0
	Product 2	0	2,807,892	2,658,492	0	0	0	0	0
	Product 3	0	27,257	26,315	0	0	0	0	0
	Product 4	150	384	300	0	0	0	0	0
DC3	Product 1	0	714,332	0	880,000	0	504,939	0	0
	Product 2	0	643,152	0	0	0	0	0	0
	Product 3	0	6,549	0	0	66,639	42,116	47,582	0
	Product 4	88	0	0	0	15,001	15,624	8,644	0

It can be seen from Table 18 that the inventory levels during time period 1 are zero for product 1, 2 and 3. The reason for this is that there were backorders for these products. One reason why the backorders occurred is because of the model's initial conditions. When the production plan was made for orders to arrive in order to be shipped to the customers during time period 1, the forecasted demand might have been different than the one used for the model. This difference can be caused because of information updates that could have changed the demand forecasts. There were 2,286,842 units of product 1, 2,201,387 units of product 2, and 1,398 units of product 3 required by the customer that were not shipped on time. After the first time period, the demand was fully satisfied with no backorders for any product. The service levels obtained during time period 1 can be seen in Table 19.

Table 19 Supply Chain Service Level for time period 1

Product 1	73.71%
Product 2	60.25%
Product 3	97.39%
Product 4	100.00%

The service level obtained for products 1 and 2 during time period 1 was very low, which means that the amount of backorders for these products was very significant. Yet, the model had no impact on the demand fulfillment for this time period. Due to the replenishment lead times, the model will begin having impact starting at time period 3, because of this; the amount of backorders should decrease and the service level should increase on the long run. We should be able to see this in future runs.

It can also be noticed in Table 18 that the inventory levels are higher at the end of time period 2 when comparing them with the other time periods. This happens because of the initial conditions to the model. The inventory levels for this time period depends on the orders that are expected to be received, which was given prior to the modeling. This should change once more runs are made, and the model calculates the required inventory levels.

The inventory levels at the end of time period 8 are all equal zero for all products. The reasons for this is because it is the last time period of the planning horizon, due to the lead times from DC to customer, the DC will not be able sell any items held in inventory in time period 8. Because of this, the model holds no inventory so that no costs are incurred. A similar trend can be found with the amount of units shipped from the manufacturer; these can be seen in Table 20.

Table 20 Number of units shipped from manufacturer to DC during Run 1

		Time 1	Time 2	Time 3	Time 4	Time 5	Time 6	Time 7	Time 8
DC1	Product 1	0	1,111,664	1,675,600	2,216,661	2,901,339	0	0	0
	Product 2	0	0	470,559	562,800	844,200	0	0	0
	Product 3	0	0	7,367	12,690	18,411	0	0	0
	Product 4	0	0	0	0	0	0	0	0
DC2	Product 1	0	4,594,342	547,200	222,400	333,600	0	0	0
	Product 2	0	2,873,892	99,000	68,000	102,000	0	0	0
	Product 3	0	27,725	702	456	684	0	0	0
	Product 4	0	348	69	358	215	0	0	0
DC3	Product 1	13,311,688	9,569,444	12,777,200	8,682,939	11,765,061	0	0	0
	Product 2	5,158,188	3,364,520	5,046,780	2,233,720	3,350,580	0	0	0
	Product 3	42,718	34,914	118,922	4,846	41,977	0	0	0
	Product 4	13,168	12,214	33,164	15,782	15,399	0	0	0

Table 20 shows that most of the units shipped are to DC 3. The reason for this is because of the costs. DC 3 is the newest and largest DC in the system, it is also the closest DC to the manufacturing plant and because of this; the transportation costs to this DC are cheaper than to the others. The amount of product that is shipped to DC 1 and DC 2 are enough for them to satisfy the demands for the customers that are closer to them than to DC 3.

It can be seen that no orders are placed during the last three time periods. The reason for this is the same as with the inventory levels. The only difference is that the replenishment lead time from manufacturer to DC is two time periods, and hence, any order placed during time period 6 would be received by the DC in time period 8, without allowing the DC to ship the products to the customers until after the planning horizon. Hence, the company should not implement the solution output for all 8 time periods, if they do, then they would not have any inventory at the end of period 8 and no orders would arrive for future time periods, which would generate large amounts of backorders. The rolling time horizon is used to avoid that problem; this will be demonstrated in the solutions obtained from the multiple model runs that will be shown.

In order to satisfy the customer demands, some product was shipped between the DCs during time period 2. DC 2 shipped small amounts of product 2 and product 3 to DC 1. DC 3 shipped product 1 and product 2 to DC 2. By doing this, the receiving DCs had enough product to be able to satisfy the demands for future time periods. It is important to remember that the transportation lead times between DCs in this model is one time period, because of this, all of the product shipped between DCs would have arrived at the beginning of time period 3. The transportation lead time between DCs and customers is also one time period, for this reason; any amount of product shipped between DCs during time period 2 will have impact on the demand of period 4.

The company should only implement the model outputs for time periods 1 and 2. This will give them the optimal amount of inventory that should be held, the orders that should be placed to the manufacturer and the amount of product that should be shipped between DCs without generating large backorders. The amount of product shipped from each DC to each customer should also be updated. The transportation lead time between each DC and each customer is one time period, by implementing the shipments between DCs and customers, the initial conditions for future runs will not have to reflect these shipments in the initial inventory levels for each DC. This will simplify the amount of information that needs to be updated during each run since the initial inventory levels will be given by the solutions of the previous run. After the solutions are implemented for the first 2 time periods, the information should be updated and the model should be run again. This will be demonstrated in Run 2.

4.1.2 Run 2

For Run 2, the initial inventory levels, the orders that are scheduled to be received by the DC and the demand forecasts should be updated. The initial inventory levels should be the actual amount of inventory each DC has at the end of time period 2, but for this analysis, the initial inventory levels for Run 2 are the inventory levels that Run 1 suggested for the end of time period 2. The reason for this is that Run 2 now starts at time period 3, and therefore, assuming no demand variability, each DC should end time period 2 with the amount of inventory suggested by the optimal solution of Run 1 (see Table 18).

Table 21 Initial inventory levels for Run 2

DC1	Product 1	4,718,936
	Product 2	2,462,441
	Product 3	36,728
	Product 4	1,135
DC2	Product 1	4,229,542
	Product 2	2,807,892
	Product 3	27,257
	Product 4	384
DC3	Product 1	714,332
	Product 2	643,152
	Product 3	6,549
	Product 4	0

The same holds true for the expected orders to be received from the manufacturer. Because the replenishment lead time is two time periods, the expected amount of product scheduled to be received at the beginning of time 3 is the amount of orders placed during time period 1. The expected amount of product scheduled to be received at the beginning of time 4 is the amount of orders placed during time period 2 shown in the optimal solution of Run 1 (see Table 20).

Table 22 Expected orders to be received by each DC for Run 2

		Time 3	Time 4
DC1	Product 1	0	1,111,664
	Product 2	0	0
	Product 3	0	0
	Product 4	0	0
DC2	Product 1	0	4,594,342
	Product 2	0	2,873,892
	Product 3	0	27,725
	Product 4	0	348
DC3	Product 1	13,311,688	9,569,444
	Product 2	5,158,188	3,364,520
	Product 3	42,718	34,914
	Product 4	13,168	12,214

In the real situation, the initial inventory levels (Table 21) and expected orders (Table 22) will be the actual and updated information. Thus, the model will become more accurate and will be able to predict what the necessary inventory levels required to satisfy future demands. Another parameter that should be updated is the demand forecasts. The solution obtained in Run 1 should have been updated for time periods 1 and 2; because of this, the planning horizon should be shifted, now starting at time period 3, and ending at the end of time period 10, these demands can be found in Tables 13 and 14. The inventory targets obtained after Run 2 can be found in Table 23.

Table 23 Ending Inventory levels for Run 2

		Time 3	Time 4	Time 5	Time6	Time 7	Time 8	Time 9	Time 10
DC1	Product 1	0	1,499,000	0	0	0	0	0	0
	Product 2	0	0	0	0	0	0	0	0
	Product 3	0	0	0	0	0	0	0	0
	Product 4	7,896	114	0	0	22,468	31,761	14,475	0
DC2	Product 1	0	0	0	0	0	0	0	0
	Product 2	0	0	0	0	0	0	0	0
	Product 3	0	0	0	0	0	0	0	0
	Product 4	0	0	0	0	0	0	0	0
DC3	Product 1	0	860,261	0	674,400	0	0	0	0
	Product 2	0	0	0	0	0	0	0	0
	Product 3	0	0	0	0	0	0	0	0
	Product 4	0	0	0	0	0	0	0	0

The output shows that there is no inventory during the last time period, like in the previous run, however, because of the rolling time horizon, now the model suggests that inventory should be carried during time period 8. Another observation that can be made is that the suggested inventory levels for most of the time periods have changed. The reason for this is because the data was updated and now there are different requirements from the model.

It can also be seen that the inventory levels for most products are zero. This happens because each run of the model does not incorporate variability in demand or lead times. The model will adjust and will avoid holding inventory unless it is really necessary. This will not be the situation once the model is implemented by the company. In actual practice, lead times and demands vary frequently, because of this, every run performed by the model will have different demand forecasts, initial inventory levels, and amounts of product in transit that are expected to arrive during the first time periods, which should generate more accurate inventory and ordering results. Table 24 shows the number of units shipped from the manufacturer to each DC obtained for Run 2.

Table 24 Number of units shipped from manufacturer to DC during Run 2

		Time 3	Time 4	Time 5	Time6	Time 7	Time 8	Time 9	Time 10
DC1	Product 1	1,675,600	2,047,200	3,070,800	2,602,250	2,723,713	0	0	0
	Product 2	1,097,580	562,800	844,200	809,075	828,414	0	0	0
	Product 3	14,053	12,690	18,810	13,813	14,841	0	0	0
	Product 4	0	107	22,533	9,411	0	0	0	0
DC2	Product 1	547,200	222,400	333,600	367,000	367,550	0	0	0
	Product 2	99,000	68,000	102,000	83,750	88,118	0	0	0
	Product 3	702	456	684	578	605	0	0	0
	Product 4	69	358	215	189	208	0	0	0
DC3	Product 1	12,796,939	8,854,400	11,595,600	10,803,000	11,803,000	0	0	0
	Product 2	5,046,780	2,233,720	3,350,580	3,498,900	3,532,496	0	0	0
	Product 3	52,283	29,369	43,977	40,136	41,441	0	0	0
	Product 4	18,163	15,159	22,379	16,979	18,171	0	0	0

Similar to the inventory levels, Table 24 now shows orders placed during time periods 6, 7 and 8. Comparable with the solution obtained for Run 1 (see Table 20); there were fewer shipments to DC 1 and DC 2 than to DC 3. Again, the reason for this is the shipping and holding costs; like mentioned previously, DC 3 has the lowest shipping cost to most customers. There were no backorders obtained during this run, which shows how the model reacts quickly when it is updated. Even though some backorders were found in Run 1, once the solution was updated into the initial conditions for Run 2, the model was able to generate solutions that fully satisfy all of the customer's demands. Similar to Run 1, very little shipments between DCs were obtained. The only shipment took place during time period 4, where DC 3 ships product 1 to DC 2.

Like in Run 1, the solutions that should be implemented are the ones obtained for the first two time periods of each run. In this case, the implemented solutions should be those of time periods 3 and 4. The solutions for Run 3 should now generate updated inventory levels as well as orders placed to the manufacturer for the next run.

4.1.3 Run 3

Like in Run 2, the initial conditions should be updated. Again, the initial inventory levels are the ones obtained for time period 4 and the expected orders to be received should be the ones obtained during time periods 3 and 4 found in the Run 2 output. These are shown in Tables 25 and 26.

Table 25 Initial inventory levels for Run 3

DC1	Product 1	1,499,000
	Product 2	0
	Product 3	0
	Product 4	114
DC2	Product 1	0
	Product 2	0
	Product 3	0
	Product 4	0
DC3	Product 1	860,261
	Product 2	0
	Product 3	0
	Product 4	0

Table 26 Orders to be received by each DC for Run 3

		Time 5	Time 6
DC1	Product 1	1,675,600	2,047,200
	Product 2	1,097,580	562,800
	Product 3	14,053	12,690
	Product 4	0	107
DC2	Product 1	547,200	222,400
	Product 2	99,000	68,000
	Product 3	702	456
	Product 4	69	358
DC3	Product 1	12,796,939	8,854,400
	Product 2	5,046,780	2,233,720
	Product 3	52,283	29,369
	Product 4	18,163	15,159

Before starting Run 3, the solution obtained for Run 2 for time periods 3 and 4 should have been implemented. Because of this, the planning horizon for Run 3 should now start at time period 5 and finish at the end of time period 12. Hence, the demands need to be updated as well for this planning horizon. By updating the information we should now see changes in both the inventory levels and the orders placed to the manufacturer during the last time period of Run 2; which is time period 10. The suggested inventory levels are found in Table 27.

Table 27 Ending Inventory levels for Run 3

		Time 5	Time 6	Time 7	Time 8	Time 9	Time 10	Time 11	Time 12
DC1	Product 1	0	0	0	0	0	0	0	0
	Product 2	0	0	0	0	0	0	0	0
	Product 3	0	0	0	0	0	0	0	0
	Product 4	0	0	0	0	0	0	0	0
DC2	Product 1	0	0	0	0	0	0	0	0
	Product 2	0	0	0	0	0	0	0	0
	Product 3	0	0	0	0	0	0	0	0
	Product 4	0	0	0	0	0	0	0	0
DC3	Product 1	1,684,861	0	0	0	0	0	0	0
	Product 2	1,946,580	690,941	0	0	0	0	0	0
	Product 3	16,529	38,599	0	22101	99,894	179,873	63,031	0
	Product 4	0	0	0	0	22,057	47,015	13,799	0

Like in both runs before, there are no inventory levels at the beginning of the last time period. But, like in Run 2, the model has updated and now there are inventory levels at the beginning of time period 10, which were zero in the previous run. A similar thing happens with the orders placed, which is found in Table 28.

Table 28 Number of units shipped from manufacturer to DC during Run 3

		Time 5	Time 6	Time 7	Time 8	Time 9	Time 10	Time 11	Time 12
DC1	Product 1	3,070,800	2,602,250	2,723,713	2,590,988	2,598,985	0	0	0
	Product 2	844,200	809,075	828,414	712,295	714,493	0	0	0
	Product 3	18,810	13,813	14,841	15,941	15,989	0	0	0
	Product 4	65	118	101	84	84	0	0	0
DC2	Product 1	333,600	367,000	367,550	281,475	282,344	0	0	0
	Product 2	102,000	83,750	88,188	86,063	86,328	0	0	0
	Product 3	684	578	605	577	579	0	0	0
	Product 4	215	189	208	282	281	0	0	0
DC3	Product 1	12,270,000	10,803,000	11,227,551	10,352,813	10,384,766	0	0	0
	Product 2	2,659,639	3,498,900	3,532,496	2,827,052	2,835,778	0	0	0
	Product 3	5,378	62,237	119,234	117,109	0	0	0	0
	Product 4	22,379	16,979	40,228	43,953	0	0	0	0

By only doing three runs the model has been able to generate inventory levels and the amount of orders that should be placed for the manufacturer that should satisfy demands for future time periods. As it can be seen in Tables 27 and 28, time periods 6, 7 and 8 now have much different inventory levels than what they did during Run 1 and Run 2. The amount of orders placed by the DCs to the manufacturer have significantly increased during time periods 7 and 8. This happens because of the updated information used for each run, which now indicates that if an order is placed in time period 8, it will be received by a DC and used to satisfy demand. This increase in orders placed are necessary in order to avoid backorders, like in Run 2, this run showed no backorders, and required no shipments between DCs.

4.1.4 Run 4

One more run was made in order to demonstrate how using the rolling time horizon generates updated solutions. After this run occurs, the solution output for the first two time periods should be implemented. These first two time periods represent time periods7 and 8 from the initial run. We can recall that the solution obtained for Run 1 suggested that no inventory should be carried during time periods 7 and 8. The output for Run 4 should have updated, and solution output for this time period should have changed. The initial conditions, initial inventory levels, orders expected to be received and forecasted demands, for this run are updated the same way as they were for the previous runs. The updated inventory levels and expected orders to be received can be found in Tables 29 and 30; the demand forecasts are found in Tables 13 and 14, like in previous runs.

Table 29 Initial inventory levels for Run 4

DC1	Product 1	0
	Product 2	0
	Product 3	0
	Product 4	0
DC2	Product 1	0
	Product 2	0
	Product 3	0
	Product 4	0
DC3	Product 1	0
	Product 2	690,941
	Product 3	38,599
	Product 4	0

Table 30 Orders to be received by each DC for Run 4

		Time 7	Time 8
DC1	Product 1	3,070,800	2,602,250
	Product 2	844,200	809,075
	Product 3	18,810	13,813
	Product 4	65	118
DC2	Product 1	333,600	367,000
	Product 2	102,000	83,750
	Product 3	684	578
	Product 4	215	189
oDC3	Product 1	12,270,000	10,803,000
	Product 2	2,659,639	3,498,900
	Product 3	5,378	62,237
	Product 4	22,379	16,979

The solution outputs for Run 4 are shown in Tables 31 and 32.

Table 31 Ending Inventory levels for Run 4

		Time 7	Time 8	Time 9	Time 10	Time 11	Time 12	Time 13	Time 14
DC1	Product 1	0	0	0	0	0	0	0	0
	Product 2	0	0	0	0	0	0	0	0
	Product 3	0	0	0	0	0	0	0	0
	Product 4	0	0	0	0	0	0	0	0
DC2	Product 1	0	0	0	0	0	0	0	0
	Product 2	0	0	0	0	0	0	0	0
	Product 3	0	0	0	0	0	0	0	0
	Product 4	0	0	0	0	0	0	0	0
DC3	Product 1	0	871,152	0	0	0	0	0	0
	Product 2	0	0	0	0	0	0	0	0
	Product 3	0	0	0	21,512	99,353	179,332	62,642	0
	Product 4	0	3,539	0	0	21,313	46,271	13,387	0

It can be seen that the inventory levels suggested for the beginning of time periods 7 and 8 show differences between Run 1 and Run 4. In both Run 1 and Run 4, the only DC that carries inventory during time periods 7 and 8 is DC 3. Run 1 suggested that inventory of products 3 and 4 should be carried at the beginning of time period 7, but that no inventory was required for time period 8. On the other hand, Run 4 suggests that no inventory is required at the beginning of time period 7, but inventory of products 1 and 4 are required for time period 8. This happens because the information was updated after each run. During Run 1, when time periods 7 and 8 were the final time periods of the planning horizon, the DCs had to hold inventory at the beginning of time period 7 in order to satisfy the demand for time period 8. Since it takes one time period to send product from a DC to a customer, there was no need for any DC to hold inventory at the beginning of time period 8 because it will not be utilized to satisfy demand within the planning horizon. On the other hand, in Run 4, time periods 7 and 8 are the initial time periods. Now the requirements for these time periods have significantly changed, the DCs need to have enough product to satisfy future demands. Any amount of product that the DCs have during these time periods can be shipped to customers within the model's planning horizon. Like in the inventory levels, the amount of orders placed to the manufacturer also changed in Run 4; this can be found in Table 32.

Table 32 Number of units shipped from manufacturer to DC during Run 4

		Time 7	Time 8	Time 9	Time 10	Time 11	Time 12	Time 13	Time 14
DC1	Product 1	2,723,713	2,590,988	2,598,985	2,611,051	2,611,567	0	0	0
	Product 2	828,414	712,295	714,493	717,808	717,952	0	0	0
	Product 3	14,841	15,941	15,989	16,061	16,064	0	0	0
	Product 4	101	84	84	84	84	0	0	0
DC2	Product 1	367,550	281,475	282,344	283,655	283,711	0	0	0
	Product 2	88,188	86,063	86,328	86,729	86,746	0	0	0
	Product 3	605	577	579	582	582	0	0	0
	Product 4	208	282	281	279	279	0	0	0
DC3	Product 1	10,356,399	10,352,813	10,384,766	10,432,977	10,435,044	0	0	0
	Product 2	3,532,496	2,827,052	2,835,778	2,848,942	2,849,506	0	0	0
	Product 3	41,441	58,642	115,084	117,394	0	0	0	0
	Product 4	14,632	18,995	40,364	44,094	0	0	0	0

It can be noticed that in the solution obtained for Run 4, DC 3 places large orders to the manufacturer during time periods 7 and 8. This shows how the model is updated as runs are made. In Run 1, no orders were placed during time periods 7 and 8. If the solutions obtained for the first run would have been fully implemented, the company would not have had the amount of product it needs for time periods 9 and 10, which would generate large amounts of backorders, instead, by implementing the solutions, the model has been able to generate enough inventory levels and orders placed so that the demand is fully satisfied with a service level of 100%. Like in the previous run the model had no backorders and no shipments between DCs.

The rolling time horizon allows the model to give improved suggestions, and eliminate the problem of not placing orders and not having inventory during the last time periods of the model. Yet this is not the only way in which the rolling time horizon improves the model. What would happen if for some reason the expected orders do not arrive during the time period they were supposed to? In order to answer this question Run 4 was used again, but now with shipment delays, this will show how the model is able to incorporate and fix an issue such as this one.

4.1.5 Run 4A (Alternate Run 4)

Run 4A will be used to analyze what would happen if there was an unexpected problem that did not allow the orders to be received by the DCs when they were expected. For example, if there was a flood that forced some roads to be closed for a couple of weeks, the products could not be shipped from the manufacturer to the DCs, and therefore, the orders placed would not be received by the DCs at the beginning of time period 7. Run 4A would be used instead of the previously shown Run 4 to illustrate this problem. Having a rolling time horizon allows the model to update and input new initial conditions that show that no orders will be received at the beginning of time period 7.

The initial inventory levels are updated like before. The difference will be on the expected orders to be received. In the case where the roads were closed for one time period, the model should be updated and the expected amount of product to be received by the DCs at the beginning of time period 7 are equal to zero for all products. The amount of orders scheduled to be received by the DCs at the beginning of time period 8 are now the sum of the orders expected to be received at the beginning of time period 8 plus the amount of orders that were expected to be received at the beginning of time period 7 that are now delayed. For this run, the demand forecasts will remain the same as the ones used for the original Run 4. The updated initial conditions for Run 4A are shown in Tables 33 and 34.

Table 33 Initial inventory levels for Run 4A

DC1	Product 1	0
	Product 2	0
	Product 3	0
	Product 4	0
DC2	Product 1	0
	Product 2	0
	Product 3	0
	Product 4	0
DC3	Product 1	0
	Product 2	690,941
	Product 3	38,599
	Product 4	0

Table 34 Orders to be received by each DC for Run 4A

		Time 7	Time 8
DC1	Product 1	0	5,673,050
	Product 2	0	1,653,275
	Product 3	0	32,623
	Product 4	0	183
DC2	Product 1	0	700,600
	Product 2	0	185,750
	Product 3	0	1,262
	Product 4	0	404
DC3	Product 1	0	23,073,000
	Product 2	0	6,158,539
	Product 3	0	67,615
	Product 4	0	39,358

With the updated information, Run 4A was studied in order to see how the model responds to possible extreme situations. Because of the rolling time horizon the model should be able to adjust and generate new inventory levels and orders that should be placed to the manufacturer. Tables 35 and 36 show the output of Run 4A.

Table 35 Ending Inventory levels for Run 4A

		Time 7	Time 8	Time 9	Time 10	Time 11	Time 12	Time 13	Time 14
DC1	Product 1	0	0	0	0	0	0	0	0
	Product 2	0	0	0	0	0	0	0	0
	Product 3	0	0	0	0	0	0	0	0
	Product 4	0	0	0	0	0	0	0	0
DC2	Product 1	0	0	0	0	0	0	0	0
	Product 2	0	0	0	0	0	0	0	0
	Product 3	0	0	0	0	0	0	0	0
	Product 4	0	0	0	0	0	0	0	0
DC3	Product 1	0	0	0	0	0	0	0	0
	Product 2	0	0	0	0	0	0	0	0
	Product 3	0	0	0	0	70,648	150,628	48,279	0
	Product 4	0	0	0	0	24,321	49,280	14,888	0

It can be seen that the inventory levels are significantly different than the ones obtained in Run 4, when the expected orders were all received on time. There is no inventory held at the beginning of the first four time periods. This is happens because no orders were received, and the DCs need to satisfy the demand with the inventory they have in stock.

By using up the entire inventory the system had during time period 7, the amount of product that was received by the DCs at the beginning of time period 8 was shipped in order to satisfy de demands of time period 9. Since the model started with no amounts of inventory for products 1 and 2, no order of these products was satisfied during time period 8, generating a service level and fill rate of 0% for both these products. The demands generated by customers 2, 3, 5, 6, 7 and 8 for product 2 were completely satisfied during time period 8. The only backorders generated during this time period for product 2 were to customers 1 and 4. Because of the large demand that customer 4 placed, the model was not able to ship any amount of product 2. On the other hand, customer 1 did not receive a fraction of what was demanded (365,839 units). With these backorders, the service level for product 2 during time period 8 was 16.08%. This is a very low service level which was caused because customer 4's demand represented 75% of the total demand for product 2 during time period 8. The fill rate, on the other hand, was not so low; only two orders of product 2 were not completed, generating a 75% fill rate for this product during time period 8. During this time period, some backorders were obtained for product 3. The model was able to fully satisfy the demands placed by customers 1, 2, 3, 5, 6, 7 and 8 during time period 8. A fraction of the demand placed by customer 4 (24,872 units) demand was not satisfied, creating a 57.57% service level for this customer. This generated a total service level of 60.81% for product 3 during time period 8. Like for product 2, the fill rate was significantly higher than the service level. Only one order of product 3 was not satisfied, this generates a fill rate of 87.5% during time period 8.

Even though the service obtained levels for time period 8 should not be acceptable, the scenario used was very pessimistic, and the model was still able to handle the situation well for future time periods. The service levels for products 2 and 3 during time period 8 were very low, but the model was able to adjust and there were no backorders for any product during the rest of the time periods, which allowed the model to increase their inventory level when needed. Like in Run 4, no shipments of products were used between DCs. Table 36 shows the number of units that should be ordered if this scenario would take place.

Table 36 Number of units shipped from manufacturer to DC during Run 4A

		Time 7	Time 8	Time 9	Time 10	Time 11	Time 12	Time 13	Time 14
DC1	Product 1	2,723,713	2,590,988	2,598,985	2,611,051	2,611,567	0	0	0
	Product 2	828,414	712,295	714,493	717,808	717,952	0	0	0
	Product 3	14,841	15,941	15,989	16,061	16,064	0	0	0
	Product 4	101	84	84	84	84	0	0	0
DC2	Product 1	367,550	281,475	282,344	283,655	283,711	0	0	0
	Product 2	88,188	86,063	86,328	86,729	86,746	0	0	0
	Product 3	605	577	579	582	582	0	0	0
	Product 4	208	282	281	279	279	0	0	0
DC3	Product 1	11,227,551	10,352,813	10,384,766	10,432,977	10,435,044	0	0	0
	Product 2	3,532,496	2,827,052	2,835,778	2,848,942	2,849,506	0	0	0
	Product 3	41,441	37,130	107,891	117,395	0	0	0	0
	Product 4	18,171	18,995	43,372	44,095	0	0	0	0

It can be seen in Table 36 that the orders placed to the manufacturer during time period 7 in Run 4A are similar to those placed during time period 7 in Run 4, where the expected orders were received on time. The amount of backorders do not have a big impact on the orders placed. The reason for this is because even though the orders were not received on time, generating the backorders, they were received the following time period with the original orders scheduled to arrive, which allowed the system to satisfy all of the demands and the backorders generated during the previous time period. Even though the orders placed are not exactly the same, the difference is very little. By time period 8, the orders placed are extremely similar to the ones placed in the original Run4, which shows that the model was able to adjust to the situation proposed. This is a big advantage of using a rolling time horizon, most unexpected problems in the delivery of products can be easily updated, and the model will be able to adapt and generate the best possible solution for the company.

4.1.6 Run 4B (Alternate Run 4)

Run 4A showed how the model could adapt when the replenishment lead times vary. Now, with Run 4B we will show how the model adapts to variations in the demand forecasts. This run will analyze a scenario in which the demand forecasts for time periods 8, 9 and 10 increased by 10%. Run 4B would be used instead of the previously shown Run 4 to illustrate this scenario. Having a rolling time horizon allows the model to update and input new initial conditions that reflect the increase in the demand forecasts.

The initial inventory levels and orders that are expected to be received are updated like in previous runs; the only difference is that instead of using the demand forecasts shown in Tables 13 and 14, the model will use the new increased demand forecasts. The demand forecasts that will be used for Run 4B are shown in Table 37.

Table 37 Demand Forecasts used for Run 4B

		Time Period							
		7	8	9	10	11	12	13	14
Cust 1	Product 1	1,724,000	2,844,600	2,417,250	2,528,763	2,181,938	2,188,672	2,198,833	2,199,268
	Product 2	500,000	825,000	789,250	808,363	632,813	634,766	637,712	637,839
	Product 3	11,856	19,562	14,441	15,534	15,005	15,052	15,121	15,124
	Product 4	38	25	32	31	30	30	30	30
Cust 2	Product 1	246,000	405,900	327,525	346,047	311,344	312,305	313,755	313,817
	Product 2	42,800	70,620	65,175	67,169	54,169	54,336	54,588	54,599
	Product 3	584	964	569	606	739	741	745	745
	Product 4	4	3	7	6	3	3	3	3
Cust 3	Product 1	77,200	127,380	117,700	121,275	97,706	98,008	98,463	98,482
	Product 2	20,000	33,000	35,558	35,724	25,313	25,391	25,508	25,514
	Product 3	250	165	185	185	197	196	195	195
	Product 4	65	43	91	75	51	51	51	51
Cust 4	Product 1	7,800,000	12,870,000	11,385,000	11,822,250	9,871,875	9,902,344	9,948,315	9,950,286
	Product 2	2,160,000	3,564,000	3,740,000	3,773,000	2,733,750	2,742,188	2,754,918	2,755,464
	Product 3	28,800	47,520	43,423	44,829	36,450	36,563	36,732	36,740
	Product 4	14,760	24,354	18,425	19,721	18,681	18,738	18,825	18,829
Cust 5	Product 1	40,800	67,320	74,800	74,800	51,638	51,797	52,037	52,048
	Product 2	15,080	24,882	22,220	23,034	19,086	19,145	19,233	19,237
	Product 3	58	39	53	50	46	45	45	45
	Product 4	142	94	72	84	112	112	111	111
Cust 6	Product 1	286,000	471,900	347,875	374,344	361,969	363,086	364,772	364,844
	Product 2	46,400	76,560	72,325	74,237	58,725	58,906	59,180	59,191
	Product 3	484	799	647	682	613	614	617	617
	Product 4	203	134	139	143	160	159	158	158
Cust 7	Product 1	53,200	87,780	75,625	78,912	67,331	67,539	67,853	67,866
	Product 2	12,240	20,196	14,245	15,475	15,491	15,539	15,611	15,614
	Product 3	27	18	28	24	21	21	21	21
	Product 4	54	35	42	41	42	42	42	42
Cust 8	Product 1	222,400	366,960	403,700	404,305	281,475	282,344	283,655	283,711
	Product 2	68,000	112,200	92,125	97,007	86,063	86,328	86,729	86,746
	Product 3	456	752	636	665	577	579	582	582
	Product 4	358	237	208	229	282	281	279	279

It can be seen that the demand forecasts are very similar to the ones showed in Tables 13 and 14, used in the other runs; the only difference is that for time periods 8, 9 and 10 the demand forecasts increased by 10%. We will now see how the model is able to adapt to this change. Table 38 shows the inventory levels that should be held at each DC at the end of each time period.

Table 38 Ending Inventory levels for Run 4B

		Time 7	Time 8	Time 9	Time 10	Time 11	Time 12	Time 13	Time 14
DC1	Product 1	0	0	0	0	0	0	0	0
	Product 2	0	0	0	0	0	0	0	0
	Product 3	0	0	0	0	0	0	0	0
	Product 4	0	0	0	0	0	0	0	0
DC2	Product 1	0	0	0	0	0	0	0	0
	Product 2	0	0	0	0	0	0	0	0
	Product 3	0	0	0	0	0	0	0	0
	Product 4	0	0	0	0	0	333	54	0
DC3	Product 1	0	0	0	0	0	0	0	0
	Product 2	0	0	0	0	0	0	0	0
	Product 3	194	0	0	0	73,665	153,644	49,788	0
	Product 4	0	0	0	0	21,159	21,117	13,253	0

It can be seen that the inventory levels for the first four time periods are extremely low. The reason for this is that because of the demand increase, the amount of product that was expected to arrive at the beginning of time period 7 was not enough to satisfy the demand and maintain inventory. Even though there are only little amounts of products 3 and 4 held in inventory throughout the system, there were not a lot of backorders generated. Most of the backorders generated during time periods 8 and 9 corresponded to the 10% demand increase, which translated to a service level of 90.91% for all products. Only one customer's demand was not completely satisfied during time periods 8 and 9. The customer with the highest backorder cost (customer 4) did not receive the complete quantity of products ordered. The service levels obtained by this customer for all four products are shown in Table 39.

Table 39 Customer 4 Service Level

	Time 8	Time 9
Product 1	87.82%	87.90%
Product 2	87.94%	88.26%
Product 3	86.64%	87.44%
Product 4	90.70%	90.62%

Even though the service levels for each product during each time period as not as high as a company would like, they are acceptable service levels. Because only one customer did not receive all of the demand, the fill rate for each product through the whole system was 87.5% during time periods 8 and 9.

The model was able to adapt to the increased demands, and in time period 11 it seems to start stabilizing the amount of inventory levels without shipping product between DCs throughout the whole planning horizon. The main reason for doing this is the amount of orders placed to the manufacturer, in Table 40 we can see the orders placed during Run 4B.

Table 40 Number of units shipped from manufacturer to DC during Run 4B

		Time 7	Time 8	Time 9	Time 10	Time 11	Time 12	Time 13	Time 14
DC1	Product 1	4,243,715	2,590,988	2,598,985	2,611,051	2,611,567	2,610,232	0	0
	Product 2	1,132,863	712,295	714,493	717,808	717,952	717,585	0	0
	Product 3	20,865	15,941	15,989	16,061	16,064	16,057	0	0
	Product 4	140	84	84	84	84	84	0	0
DC2	Product 1	496,605	281,475	282,344	283,655	283,711	283,566	0	0
	Product 2	122,382	86,063	86,328	86,729	86,746	86,702	0	0
	Product 3	837	577	579	582	582	581	0	0
	Product 4	305	282	281	612	0	226	0	0
DC3	Product 1	15,000,000	10,352,813	10,384,766	10,432,977	10,435,044	0	0	0
	Product 2	4,794,066	2,827,052	2,835,778	2,848,942	2,849,506	0	0	0
	Product 3	56,933	37,130	110,908	117,394	0	0	0	0
	Product 4	25,440	18,995	40,210	19,094	11,276	0	0	0

When comparing the results obtained for the amount of orders placed to the manufacturer between Run 4B and the original Run 4 (see Table 32), it can be seen that the amount of orders placed are very similar. Run 4B orders more amount of product during time period 7 in order to be able to satisfy the inflated demands of time period 10 as well as the backorders generated during previous time periods. Once this happens, the amounts of orders placed stabilize and the solutions are extremely similar, which shows that the model has adapted to the demand increases, and has been able to generate accurate solutions for future time periods.

Like in Run 4A, it can be seen that the model can easily be modified in order to adapt to variability that occurs in the supply chain system. But the two scenarios analyzed are not the only instances for which the model can be modified; the model could also be updated quickly if there is an increase or decrease in the actual demand. If the demand varies, the company will sell a different amount of product than it forecasted. If this happens, the only difference for future runs will be in the initial inventory levels. Instead of using the inventory levels outputted by the model, the actual inventory levels would be inputted to the model. The model will then be able to adapt to this change and output the updated inventory levels that should be carried.

As it has been seen, updating the model to fit into different external conditions is very simple. It does not take a lot of time, and since the model will be used once every month, the time that needs to be spent into updating and running the model is not very significant. The process of updating the model can even be automated for time periods where there are no external factors that would change the initial conditions. The different situations for which the model can quickly adapt in order to generate solutions in order to facilitate fast decision making are:

- Demand changes
- Partial orders received
- Changes in lead times

Chapter 5. Conclusions and Recommendations

In this thesis, a small portion of a company's supply chain system was analyzed in order to make recommendations of the inventory levels that should be carried at each DC. This is a multinational company that manufactures and sells consumer goods throughout the world. The analysis was made for the product category that holds the biggest sales volume in the area analyzed. This product category has four different tiers; each tier represents one product in the analysis. All four products are produced in one manufacturing plant, shipped to one of three DCs, which then have to satisfy eight customer demands, over a planning horizon of eight time periods.

In order to calculate the necessary inventory levels that each DC should carry each time period, a linear programming model was developed. This model maximized the total supply chain profit, taking into consideration revenue from sales, production costs, transportation costs, inventory holding costs and backorder costs. The solution outputted the amount of inventory levels that each DC should carry for each product, the amount of product that should be ordered from the manufacturer during each time period, the amount of product that should be shipped between DCs, and the amount of product that should be shipped from each DC to each customer in order to satisfy their demands.

Even though the model analyzes a planning horizon of eight time periods, only the solution obtained for the first two time periods should be implemented. In order to have solutions for the future time periods, the model works with a rolling time horizon, that is, the time periods of the planning horizon will be shifted two time periods ahead each time the model is run. This will ensure that updated demand forecasts and shipments are included on a continuous basis, and that there is a solution every two time periods. We show that using a rolling time horizon allows the model to update the solution output every time it is run, giving more accurate inventory levels for the initial and ending time periods. We also show that by using a rolling time horizon, many unexpected problems that take place can be easily incorporated into the model, which then adjusts and gives a new solution based on the updated information.

Four runs were made in order to show how to implement the solution output for future runs. Once they were developed, two different scenarios were analyzed in order to see how the model would adjust to some pessimistic situations. The first scenario analyzed a situation in which the shipments expected to be received by each DC were delayed and did not arrive during the first time period. It was shown that even though there were backorders generated, after only two time periods the system had adapted and was fully satisfying all demands. The second scenario analyzed a situation in which all of the demand forecasts for the first four time periods suddenly rose by 10%. Backorders were still generated for this scenario, but again, after only three time periods the model adapted and satisfied all demands.

After completing the analysis, it is recommended that the company review and validate the solutions obtained. Once this is done, they should use the model into their planning strategy in order to implement the solution it provides. If the solutions would have been implemented for the first two time periods, the company would have reduced their inventory levels by approximately 30% of the amount that they had planned to have during the time periods that were analyzed. This is a very significant reduction in supply chain costs which would not have compromised satisfying the forecasted demand.

Once the model is implemented for the product tiers analyzed, the model should be expanded into all of the company's products. The company should also give the model more product detail to analyze. Instead of using product tiers, the information provided to the model should be more specific, analyzing each individual product itself. This would provide more details to the model and its output, without generating more work for the user. Implementing this should not be an issue, it could take more time to gather the initial data required, but after the first run is completed, updating the information should not create any difficulties.

This study analyzed the inventory levels that each DC should carry. Even though different scenarios were used to capture pessimistic scenarios for both demand and replenishment lead times, it does not take into consideration possible demand and lead time variability; which happen every day in practice. Because of this, the company should do a statistical analysis to determine the demand and lead time distributions, means and variances. With this information, the company could generate an efficient safety stock policy for each different product. This could increase the amounts of inventory to be held, as well as the supply chain costs, but would provide protection for every day variability in demand, production and transportation lead times.

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