A METHOD FOR FOCUSING SOUND IN HARBOR ENVIRONMENTS AT LOW FREQUENCIES: THEORY AND EXPERIMENT

A Dissertation in
Mechanical Engineering

by
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ABSTRACT

A new method to focus low frequency acoustic energy in harbor environments was developed as a means of deterring swimmers with malicious intent. This method requires *a priori* calibration data from the harbor and picks the optimal phases which result in constructive interference at hydrophone locations. The phasing results from this Optimal Phase Search (OPS) method compare favorably with results from classic linear array theory in the experimental test harbor, Coddington Cove. Phasing and sound pressure level (SPL) data from two different source arrays are presented for the harbor at 100 and 200 Hz. An acoustic finite element method (FEM) code has also been developed to predict the sound level in the harbor and assist in understanding the phasing. The code is a simplified FEM code developed in MATLAB using linear shape functions and cuboid elements. The harbor geometry is voxelized and each voxel is turned into an element for the FEM program. The code is validated with numerous simple acoustic enclosures and a more complex underwater wedge. The results from the harbor model are compared to classic spreading theory and experimental data from Coddington Cove. The adaptability of the FEM code is discussed as well as areas of possible future work.
# TABLE OF CONTENTS

LIST OF FIGURES ................................................................................................... vii

LIST OF TABLES ..................................................................................................... xiii

ACKNOWLEDGEMENTS ......................................................................................... xiv

Chapter 1  Introduction .......................................................................................... 1
  1.1 Statement of the Research Problem ............................................................... 1
  1.2 Literature Review ......................................................................................... 2
    1.2.1 Background .......................................................................................... 2
    1.2.2 Underwater Acoustics ......................................................................... 4
      1.2.2.1 Ray Theory .................................................................................. 5
      1.2.2.2 Wavenumber Integration ......................................................... 7
      1.2.2.3 Normal Mode Method .............................................................. 9
      1.2.2.4 Parabolic Equation .................................................................. 10
    1.2.3 Acoustic Finite Elements – A Chronology .......................................... 12

Chapter 2  Simplified Acoustic FEM .................................................................. 20
  2.1 Introduction ................................................................................................... 20
  2.2 FEM Overview ............................................................................................ 21
  2.3 FEM Derivation .......................................................................................... 25

Chapter 3  Validation of the Acoustic FEM Code ............................................. 32
  3.1 Introduction ................................................................................................... 32
  3.2 Basic Enclosures – Eigenvalue Comparisons ............................................. 32
  3.3 Impedance Tube – Eigenvector Comparisons ........................................... 35
  3.4 Transmission Loss for a Point Source ......................................................... 41
  3.5 FEM Validation – Underwater Wedge ....................................................... 45
  3.6 Conclusion ................................................................................................... 51

Chapter 4  Focusing Sound Using an Array of Sources .................................... 53
  4.1 Multiple Sources Background .................................................................... 53
  4.2 Distance Method and Array Theory ........................................................... 57

Chapter 5  Optimal Phase Search Method ......................................................... 63
  5.1 Introduction ................................................................................................... 63
  5.2 Phasor Notation .......................................................................................... 63
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.3 Graphical User Interface</td>
<td>133</td>
</tr>
<tr>
<td>10.4 Acoustic Finite Element Method</td>
<td>134</td>
</tr>
<tr>
<td>10.5 Future Work</td>
<td>135</td>
</tr>
<tr>
<td>Bibliography</td>
<td>137</td>
</tr>
<tr>
<td>Appendix A Matlab Code for Acoustic FEM Program</td>
<td>140</td>
</tr>
<tr>
<td>Appendix B LabVIEW Layouts</td>
<td>156</td>
</tr>
<tr>
<td>Appendix C Source and Hydrophone Specifications</td>
<td>159</td>
</tr>
</tbody>
</table>
LIST OF FIGURES

Figure 1.1: Ray tracing example. Sound speed is 1500m/s at the surface, 1480m/s at 200 m deep, and 1498m/s at 2000m deep. Rays leave source (100m) at horizontal (blue) and +/- 1 degrees (green/red). .................................................. 6

Figure 1.2: Ray tracing example. Sound speed is 1500m/s at the surface, 1480m/s at 200 m deep, and 1498m/s at 2000m deep. Rays leave source (200m) at horizontal (blue) and +/- 1 degrees (green/red). ........................................... 7

Figure 1.3: Wavenumber integration results for 100m deep Pekeris waveguide for 20 Hz point source at 36m. Top is magnitude of depth dependent Green’s function. Bottom is transmission loss................................................................. 8

Figure 1.4: First four normal modes of a zeroth-order waveguide 100m deep, c=1500m/s, 50Hz. Zero depth is at ocean bottom. ................................................................. 10

Figure 1.5: Copied from Jensen et al. [4] [Fig. 6.8. Comparison of PE results with coupled-mode reference solution for upslope propagation in a wedge. (a) Test environment. (b) Split-step result for the Thomson-Chapman equation. (c) Finite-difference result for the Claerbout equation (from Jensen and Ferla [38]).] .................................................................................................................... 11

Figure 2.1: Voxel/Element and node geometry with damping.................................................. 22

Figure 2.2: Acoustic FEM flow chart. Variables: a = length of element divided by 2, c = speed of sound in fluid, ρ = density of fluid, β = damping ratio for boundaries, f = force ........................................................................................................ 24

Figure 2.3: One model acoustic element with node numbering scheme. .................. 30

Figure 3.1: Basic eigenvalue problems. Rigid-walled duct(A), rigid-walled rectangular solid(B), rigid-walled rectangular solid with pressure release top(C). ........................................................................................................ 33

Figure 3.2: Impedance Tube. The left edge consists of a piston vibrating with a prescribed velocity. The right end has a known impedance........................................ 36

Figure 3.3: Impedance tube results. Boundary is a Rigid Wall with β =0. .............. 37

Figure 3.4: Impedance Tube results. Boundary has a ρ-c condition with β =1. ....... 38

Figure 3.5: Impedance Tube results. Boundary has an imaginary impedance with β=0.5 i................................................................................................................... 39
Figure 3.6: Impedance Tube results. Boundary has a complex impedance with $\beta = 0.5 + 0.5i$ .................................................................

Figure 3.7: Impedance Tube results. Boundary has a complex impedance with $\beta = 0.5 + 0.5i$. There are twice as many elements along the length of the tube as compared to Figure 3.6........................................................................

Figure 3.8: Screenshot of ParaView. Quadrant of free space with point source in corner and contoured pressure loss in dB..........................................................

Figure 3.9: Pressure loss in dB along diagonal of quadrant compared with theory ........................................................................

Figure 3.10: Pressure loss in dB along symmetric edge of quadrant compared with theory .................................................................

Figure 3.11: Underwater Wedge modeling approaches – red is source location........

Figure 3.12: Wedge solutions from Acoustic FEM. Source is 100 m deep on black line which is approximately 1 km from tip of wedge. Depth at Source location is 200 m. Pressure release surfaces on top and bottom and $\rho_c$ on the left most side................................................................

Figure 3.13: Wedge results from FEM compared to analytical theory. Source is at 100 m depth excited at 5 Hz, receiver is at 30 m depth..............................

Figure 3.14: Wedge results from FEM compared to analytical theory. Source is at 100 m depth excited at 10 Hz, receiver is at 30 m depth......................

Figure 3.15: Wedge results from FEM compared to analytical theory. Source is at 100 m depth excited at 12 Hz, receiver is at 30 m depth..................

Figure 4.1: Amplitude of acoustic field due to two simple sources with zero phase at 100 Hz in water............................................................

Figure 4.2: Phase of acoustic field due to two simple sources with zero phase at 100 Hz in water.................................................................

Figure 4.3: Pressure amplitude with two sources, phased to focus on the circle at 100 Hz. $\phi = 1.7351$ rads. ..................................................

Figure 4.4: Pressure phase with two sources, phased to focus on the circle at 100 Hz........................................................................

Figure 4.5: Pressure amplitude with a rigid boundary added. ........................................

Figure 4.6: Pressure Phase with boundary........................................................................

Figure 5.1: Phasor representation of acoustic pressure and vector addition of multiple sources. ................................................................. 64

Figure 5.2: Source and receiver located between ocean surface and bottom showing multiple path reflections and phasor representation. ..................... 65

Figure 5.3: Calibration flow chart ........................................................................ 67

Figure 5.4: Anechoic chamber test using phase search algorithm with two loudspeakers at 300 Hz and one microphone in air. ................................. 68

Figure 5.5: Anechoic chamber test using phase search algorithm with two loudspeakers at 1000 Hz and one microphone in air. ................................. 69

Figure 6.1: View of Jacksonville Quarry. Base of operations is in upper left corner with RV. ........................................................................ 71

Figure 6.2: Quarry trip 1. Sources are suspended from PVC pipe floats. .......... 73

Figure 6.3: Linear array from Quarry Trip 2. ...................................................... 74

Figure 6.4: Experimental and Theoretical results from Quarry Trip 2. Inset: Orientation of ropes, hydrophones (circles), and source array (rectangle). ....... 75

Figure 6.5: Arbitrary source locations near quarry wall, marked by yellow buoys. 77

Figure 6.6: Deployment of five-source array (Lubell LL98)............................. 78

Figure 7.1: Aerial view of Coddington Cove from www.mapquest.com. The main part of Newport, RI is to the south on the map. ................................. 81

Figure 7.2: Cargo van base of operations. .......................................................... 82

Figure 7.3: Map of Coddington Cove, Newport, RI. Hydrophone locations are numbered in blue italics, harbor depths in feet in black block numbers, and the sources were at the end of Pier 1 where the blue oval is. .................... 84

Figure 7.4: Three of the four types of sources used at Coddington Cove in 2007. 85

Figure 7.5: Frequency response at the hydrophones for a single J-11 source. ........ 86

Figure 7.6: Array of 3 J-11 sources. ................................................................. 87

Figure 7.7: Phase calibrated for the first and third source in array (second source has zero phase). Dashed lines with solid shapes indicate original depth, solid lines with open shapes indicate new, shallower depth. ................................. 88
Figure 8.14: J15 Source #3 phases, OPS method and array theory, 100 Hz ..........111

Figure 8.15: J15 Source #4 phases, OPS method and array theory, 100 Hz ..........112

Figure 8.16: Comparison of SPL at each hydrophone by focusing with classic array theory and OPS method – J15 sources at 100 Hz........................................113

Figure 8.17: J15 Source #2 phases, OPS method and array theory, 200 Hz ..........114

Figure 8.18: J15 Source #3 phases, OPS method and array theory, 200 Hz ..........115

Figure 8.19: J15 Source #4 phases, OPS method and array theory, 200 Hz ..........116

Figure 8.20: Comparison of SPL at each hydrophone by focusing with classic array theory and OPS method – J15 sources at 200 Hz........................................117

Figure 9.1: Preliminary model of Coddington Cove, view from the floor up, North is to the right, West is to the bottom. This figure is to illustrate the capability of the program, not to infer any results.................................................................121

Figure 9.2: Second iteration of harbor model. Two different source locations near tip. View from the floor up, North is to the right, West is to the bottom ..........122

Figure 9.3: Third iteration of harbor model. Source array focused on hydrophone 3. Transmission Loss is shown in contours. View from the floor up, West is to the left, North is to the bottom. Hydrophones are indicated by the red dots...122

Figure 9.4: Simplified harbor – waveguide model. Side view of harbor. Source is two meters from bottom on right side. Contours show transmission loss........123

Figure 9.5: Numerical approximation of Coddington Cove Reflection Coefficient at 100 Hz.......................................................................................................124

Figure 9.6: Bottom impedance β, as a function of grazing angle at 100 Hz.........125

Figure 9.7: Bottom impedance β as a function of horizontal distance from the source at 100 Hz .................................................................................................126

Figure 9.8: Comparison of FEM with a sloped β boundary condition compared to spherical, cylindrical, and an intermediate spreading at 100 Hz .........................127

Figure 9.9: Transmission loss for the FEM model with new β values, compared to theoretical spherical, cylindrical and an intermediate spreading at 100 Hz. ...129

Figure 9.10: Comparison of FEM with some experimental TL data from 6/5/08 (blue squares). Single HLF1 Source at 100 Hz .................................................130
Figure B.1: LabVIEW Front Panel for Main Firing Program. 156
Figure B.2: Block Diagram for Main Firing Program. 157
Figure B.3: Separate While Loop in Firing Program (portion) 158
LIST OF TABLES

Table 3-1: Slender duct eigenfrequency comparison................................................34
Table 3-2: Rectangular solid eigenfrequency comparison........................................34
Table 3-3: Rectangular solid with pressure release top eigenfrequency comparison............................................................................................................35
Table 6-1: Phasing data for Quarry trip 3, Hydrophone 1..........................................76
Table 6-2: Phase and Hydrophone 1 Pressure Data for Array and J-9 Sources ..........79
Table 7-1: Hydrophone Distances from Source – data gathered acoustically............85
Table 9-1: Some general model sizes and computation times using this FEM code...120
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Chapter 1

Introduction

1.1 Statement of the Research Problem

In the 21st century, the industrialized nations of the world have intensified their interest in developing technologies that may be implemented to protect their people and assets from covert attacks from terrorist organizations. One area of concern is that of ships docked in U.S. harbors that carry important cargo. Even with current surveillance procedures, these ships could fall prey to unnoticed divers carrying explosives. The Office of Naval Research (ONR) initiated an extensive research program to devise nonlethal methods of deterring intruders with destructive intent. The project described herein is part of this larger ONR initiative.

A new technique has been developed that uses low frequency acoustic energy to repel unwanted swimmers in harbor areas containing vulnerable targets. Multiple acoustic sources are phased according to a harbor calibration scheme in such a way to focus acoustic energy at a designated swimmer location. This technique has been tested experimentally with marked success, although only a small portion of the harbor was calibrated at distributed locations for these experiments. Along with the development of an experimental methodology involving the calibration of the harbor acoustics, this dissertation also investigates the possibility of a theoretical formulation of the technique, i.e., an acoustic finite element method (FEM) that allows for the modeling of a harbor
2

and hence its calibration. As long as the bathymetry of the harbor and the impedance of its bounding surfaces (e.g., ships, bottom sediment, piers, etc.) are known, the FEM program can be used to calculate the optimal magnitude and phase of any given number of sources to focus energy and thus allow for the optimization of source placement. Experimental data taken from the harbor at Coddington Cove, Newport, RI, is used to validate the FEM code. The synthesis of experimental methodology and FEM programming is the focus of this thesis.

The remainder of this chapter consists of a literature review on the topics of modeling underwater sound and focuses on the finite element method. Chapters 2 and 3 introduce the FEM derivation used for this project and its validation with benchmark problems. Chapter 4 gives a background on multiple sources and classic beam forming techniques whereas Chapter 5 introduces the new optimal phasing concept which is one of the main contributions of this dissertation. Chapters 6 through 8 show experimental results from a local quarry and both trips to Coddington Cove. Chapter 9 compares the experimental results to FEM results for Coddington Cove. Finally, Chapter 10 summarizes the results and conclusions and looks to future endeavors with this research.

1.2 Literature Review

1.2.1 Background

Since at least the mid 20th century, scientists have studied the effect of extreme frequency and amplitude sound waves on the human body. Many of these studies have
been centered on low frequency (0-100Hz) sound that is often present around heavy machinery. Although several of these studies have been questioned in their methodology and reporting metrics by experts in the field, a general summary of low frequency sound effects can be found in Broner [1]. Depending on the duration, amplitude, and frequency of the sound, a person can become nauseated, get a headache, be unable to concentrate on tasks at hand, and lose sleep. In general, the longer the exposure and the higher the amplitude the more severe the effects become. Also, a frequency range around 20 – 40 Hz showed a higher annoyance level than the 40-100 Hz range. With this knowledge and the information gathered from other concurrent testing, it is evident that a low frequency range at a high amplitude could be implemented in a swimmer deterrent method such as the one proposed in this ONR-funded study. The concurrent experiments conducted with the Naval Submarine Medical Research Laboratory and the University of Rochester focus on the effects of various low frequency ranges on the human body underwater [2]. From these tests a frequency or frequency range will be targeted for use in this thesis investigation. However, since this sound range and amplitude level could also have an adverse effect on marine life, it is imperative that this investigation be conducted at a sound source level that ensures minimal risk of disturbing the habitat of species in the vicinity of any testing.

In order to create an effective deterrent method, the acoustic energy needs to be focused upon the targeted swimmer. For the current project, noise reduction concepts are used to increase and focus acoustic energy. For years, scientists and engineers have been looking for ways to reduce the noise levels in any given environment: automobiles, airplane cabins, headphones, workplaces, etc. This generally can be accomplished
through active or passive means. Active methods include creating a secondary signal out of phase with the primary one in order to negate its effects [3]. It is possible to use the active concepts of controlling noise in reverse, i.e., using constructive interference to increase the total sound pressure level at a specified location. With active noise cancellation, a sound wave that is 180 degrees out of phase with an incoming wave is projected alongside the original wave. This concept can be reversed for the current deterrent method by ensuring that two or more acoustic waves arrive at the specified location with similar phase, thus amplification rather than cancellation will occur.

1.2.2 Underwater Acoustics

The problem then becomes one of how to model underwater acoustic waves when there are a variety of reflective and absorptive structures on the boundaries so that the phase of the transmitted sound at any location is known. The study and modeling of underwater acoustics has been written about extensively over the past decades. Jensen, et al. [4] gives a thorough review of the various methods that have been developed including ray tracing, wavenumber integration, normal modes, parabolic equations and finite element methods. Each of these will be described in brief according to [4] including their shortcomings when it comes to the current investigation.
1.2.2.1 Ray Theory

The use of ray theory emerged from the study of optics and was extended to sound in the mid-twentieth century. The Helmholtz equation is solved for a series solution combining the eikonal equation and the transport equations. The standard simplification is to use only the first term in the ray series leading to a high frequency approximation. Given starting coordinates and angle along with the sound speed profile and depth, a two dimensional approximation of the sound energy propagation can be developed. The phase can be calculated by integrating the eikonal equation with respect to the ray coordinate while the amplitude of the ray can be calculated using the transport equations. Two of the flaws which exist in this method are shadow zones and caustics. Shadow zones are areas where no rays pass and therefore a zero pressure region occurs; caustics occur when a bundle of rays crosses over itself, creating zero area and consequently, infinite intensity. As stated, ray theory is good for high frequencies and a rule of thumb is that the wavelength should be substantially smaller than any physical scale in the problem, e.g., water depth and other bathymetric features. Boundaries are generally treated as reflective with a complex reflection coefficient. Three dimensional ray tracing is possible but computationally costly. Figure 1.1 and Figure 1.2 illustrate simple ray tracing applications.
Figure 1.1: Ray tracing example. Sound speed is 1500m/s at the surface, 1480m/s at 200 m deep, and 1498m/s at 2000m deep. Rays leave source (100m) at horizontal (blue) and +/- 1 degrees (green/red).
1.2.2.2 Wavenumber Integration

The wavenumber integration technique evaluates the depth-separated wave equation directly using numerical quadrature. First introduced by Pekeris in 1948, the technique applied a series of integral transforms to the Helmholtz equation, thus reducing the original four-dimensional PDE to a series of ODE in the depth coordinate, i.e., sound propagates cylindrically in the horizontal direction but like plane waves in the vertical direction. For a small number of layers these equations can be solved analytically within

Figure 1.2: Ray tracing example. Sound speed is 1500 m/s at the surface, 1480 m/s at 200 m deep, and 1498 m/s at 2000 m deep. Rays leave source (200 m) at horizontal (blue) and +/- 1 degrees (green/red).
each layer in terms of unknown amplitudes determined by matching boundary conditions at interfaces. For more layers, the equations must be solved numerically and there are many different solution methods developed over the years with varying degrees of versatility and stability. Wavenumber integration is applicable only to range independent or horizontally stratified environments: all interfaces are plane and parallel with layer properties that are only a function of depth. Figure 1.3 shows results from a standard wavenumber integration problem.

Figure 1.3: Wavenumber integration results for 100m deep Pekeris waveguide for 20 Hz point source at 36m. Top is magnitude of depth dependent Green’s function. Bottom is transmission loss.
1.2.2.3 Normal Mode Method

The normal mode method solves the depth dependent wave equation, similar to wavenumber integration, but unforced. Normal mode codes calculate the eigenmodes and eigenfrequencies of a given water depth. The final solution is then given as a summation of modes, weighted in accordance to the depth of the source. Depending on the frequency of interest, there can be a few or hundreds of modes to include. For range dependent cases, the normal mode solution can be found in segments, and then joined together at the interfaces ensuring continuity, which is also computationally expensive. Often two simplifications are made: (1) the backscattered field is neglected at the interface and (2) coupling between different-order modes is ignored, i.e., all of the energy from the second mode in one section will propagate into the second mode of the next section, also known as the adiabatic approximation. Figure 1.4 shows the first four normal modes for a zeroth order waveguide (rigid bottom, pressure release top).
1.2.2.4 Parabolic Equation

The use of parabolic wave equations dates back to the mid-1940s when Leontovich and Fock used them to model radio wave propagation in the atmosphere but was not used in underwater acoustics until the early 1970s. Since then it has become the most popular technique for solving range dependent propagation problems in ocean acoustics. The starting point for this method is the Helmholtz equation in cylindrical coordinates assuming harmonic point excitation and constant density. It is assumed that
the receiver is in the far field and thus a paraxial, or small angle, approximation is made. Another method is to define operators and instead of the paraxial, a complex square root function is approximated. These estimates tend to have limited accuracy for propagation angles beyond +/- 40 degrees relative to the main propagation direction. There are also phase errors inherent in parabolic equation methods as well as problems with energy conservation, leading to reduced accuracy. Figure 1.5 shows an example of transmission loss using the parabolic equation (PE).

Figure 1.5: Copied from Jensen et al. [4] [Fig. 6.8. Comparison of PE results with coupled-mode reference solution for upslope propagation in a wedge. (a) Test environment. (b) Split-step result for the Thomson-Chapman equation. (c) Finite-difference result for the Claerbout equation (from Jensen and Ferla [38]).]
1.2.2.5 Finite Element Methods

The former methods, although widely used, suffer in certain cases through their generality and the assumptions made. These problematic cases include low and medium frequency backscattering from ocean boundaries. For these cases another method, finite element analysis, can be used. The finite element method discretizes the acoustic environment into three-dimensional building blocks known as elements which are interconnected by nodes. (A related method is known as the finite difference method in which the governing differential equations of a system are discretized, but this method will not be discussed at length.) The pressure inside each of the elements is typically a linear interpolation of the pressure at each of its nodes, but other nonlinear shape functions can be used. The finite element method allows for modeling complex geometries with intricacies dependent only on the resolution of the elements used. A basic rule of thumb is to have at least ten elements per wavelength of the frequency of interest. Because this dissertation involves studying the reverberation of low frequency sound from boundaries, the finite element method was chosen to model the harbor acoustics.

1.2.3 Acoustic Finite Elements – A Chronology

Typically used for mechanical systems, finite elements can be used to solve a number of other engineering problems. For example, using the Navier-Stokes equation for fluid mechanics combined with the Euler equation, one can develop a fluid finite element method to calculate the pressure variations in a fluid system [5]. Using the
Helmholtz equation, one can create an acoustic finite element method to investigate the acoustic properties of a fluid space. The finite element method allows for more complicated geometry and boundary conditions that cannot be solved for analytically.

For acoustic systems, instead of using displacement as a variable, pressure is generally used. This has a large computational benefit in that pressure is a scalar quantity and requires only one unknown per node. Engineers and scientists have been using acoustic finite elements since at least 1965. Gladwell [6] introduced two different approaches to acoustic finite element analysis depending on how the “energies” are expressed: force type and displacement type. He was able to show good agreement in very simple cases using cubic polynomial shape functions.

Another study done in 1972 by Craggs [7] looked at the problem of irregular enclosures in air, specifically the interior of an automobile. Craggs only considered boundary conditions that were either hard, where the gradient of pressure was zero (also known as rigid wall), or soft, where the pressure was zero (also known as pressure release). Any unconstrained condition on the boundaries corresponds to the hard surface case. After investigating several different element shapes with linear shape functions, Craggs concluded that the best overall accuracy was obtained from cuboid elements; they had a better indication of the mode shapes and located the eigenvalues in the correct order with an error that converges as $1/N^2$ where $N$ is the number of elements. Craggs decided that for any enclosure the best scheme would be to fill the majority of the structure with cuboid elements and then fill in the remaining geometry with elements formed by basic tetrahedrons. Craggs also concluded that the mode shapes for these complex structures are beyond description by simple analytical functions and thus a
numeric approach, such as finite elements, is required. The main problem with using the finite element approach with complex structures is matching boundary conditions. Without a good model of the boundary impedances, it is impossible to create an accurate finite element representation. At that time, another limitation of finite element analyses was computer capability; hence Craggs’ three-dimensional model contained only 88 degrees of freedom. Not anticipating the revolution of computational speed that evolved during the next few decades, this computer limitation drove many scientists and engineers to develop ways of minimizing the total number of degrees of freedom for their models.

In 1978 Joppa and Fyfe [8] introduced absorptive boundary impedances in a standard two-dimensional acoustic finite element analysis. Two-dimensional analyses require fewer degrees of freedom, and thus larger problems could be solved. They compared FEM results for an impedance tube with various end effects including a complex impedance, a Helmholtz resonator, an exponential horn, and a permeable membrane with theoretical results. In all cases as long as they had the prerequisite ten elements per wavelength, the impedance results matched well for a variety of frequencies. The only drawback they saw was the amount of computer storage space required for the FEM matrices, although when using sparse matrix solution techniques, more complex geometries could be solved.

Richards and Jha [9] simplified the three-dimensional acoustic finite element approach by reducing a car interior geometry to a two-dimensional shape with a standard depth. This two-dimensional prism allows for fewer degrees of freedom and thus less storage space and quicker calculation times than the standard formulation. The variation
in the prism direction is represented by a sum of Fourier components. Using the Fourier components allows for a decoupling of the matrix equation into a number of smaller matrix equations. Results of a standard rigid-walled rectangular acoustic cavity matched well with theory for a low number of quadratic triangular elements, although a cylinder did not show as much agreement. A scale model of the car interior was tested and experimental resonance frequencies compared well with those calculated with the finite element approach.

Fully three-dimensional acoustic elements were used again by Workman [10] in 1984 to investigate boundary wall impedances. He used eight-noded, six-sided elements with four degrees of freedom at each node (the pressure and the gradient of pressure in each of the Cartesian directions) and Hermitian polynomial shape functions. His code was validated with a uniform rectangular rigid-walled tube with less than 0.3% error for the first twelve modes. His model contained 160 nodes with a total of 640 degrees of freedom. He also compared results for the forced excitation of an impedance tube with a rigid wall and with acoustic material at the other end. Again he showed very good agreement between theory and finite element results for the first twelve eigenmodes.

Murphy and Chin-Bing [11] were one of the first to investigate the use of FE methods in underwater acoustics in 1988. Using a two-dimensional model with linear triangular elements, they compared their FE results with results from the known Lloyd Mirror effect and an ASA benchmark problem of an underwater wedge. The Lloyd Mirror effect is the resulting beam pattern when a source is placed near a pressure release surface. The benchmark wedge problem was of a 25 Hz source propagating upslope with known bottom conditions. Their results for these tests matched theory extremely well.
They introduced two radiation boundary conditions: narrow and wide angle, along with the standard rigid and pressure release. The two radiation conditions result in some error near the boundaries, but in most cases these boundaries are far from the source and the desired measurement location. Two methods are suggested for saving computation storage space (which is one of the main limitations of any FEM code) by using super elements when pressure at certain points is required and marching frames where the pressure at the boundary of one section is transferred to the beginning of the next section.

With the advent of high speed computers with nearly unlimited memory over the last two decades, several commercial programs have come on the market that can do acoustic analysis, one of which is SYSNOISE. These programs may use finite elements, boundary elements, or some combination of the two along with any of the prior discussed methods. Kopuz and Lalor [12] used SYSNOISE’s FEM and boundary element method (BEM) capabilities to model a rectangular cavity. This rectangular cavity represented the passenger compartment of a vehicle and later another rectangular cavity was added representing the boot, or trunk of the vehicle. They compared the results of a direct forced FEM method to FEM modal superposition and collocation to variational BEM. All results compared favorably as long as there were enough modes included in the modal superposition FEM – modes corresponding to two and a half times the highest frequency of interest. They also stated that the FEM method took less time than the BEM method.

Computer advancements over the last two decades have allowed for more complex studies in the FEM arena, including unbounded domains. Finite element methodology typically only works well in bounded domains; however, there are a variety
of ways to implement them in unbounded domains. Harari [13] has a broad summary of some of the methods used in unbounded domains, such as absorbing boundary conditions (ABC), infinite elements, and absorbing layers, including the fairly recent Perfectly Matched Layer (PML). ABCs involve a relation between the unknown solution and its derivatives, such as an impedance condition. Infinite elements utilize different shape functions allowing the pressure of outgoing waves to reduce to zero. PMLs are layers surrounding the finite element configuration which force the outgoing waves to evanesce within a given thickness of the layer. There are numerous articles that exist specifically about these types of methods but they will not be discussed in detail here. The reader is pointed in the directions of the following additional articles for more information on infinite elements [14] and [15] and PML [16]-[20]. Boundary elements and boundary integrals are also used to deal with unbounded domains. Wu et al. [21] used an artificial boundary with a boundary integral to satisfy the Sommerfeld radiation condition: all energy is leaving the space, none is reentering; their results compared well with theory. Again, since this type of method was not implemented in the current research it is left up to the reader to investigate it further. Although these numerical advances have made larger, more complicated problems solvable, there is still a limit on the size of a problem (due to computer memory constraints) and the accuracy of the results (enough elements per wavelength for solution convergence and boundary condition modeling).

To determine the accuracy of FEM code, benchmark problems have been developed for simple geometries. The lack of benchmark problems with analytic solutions when it comes to more complicated structures and complex impedance boundaries makes some FEM verification difficult. Any standard acoustics and vibration
text will have eigenvalues and eigenvector equations for a rigid-walled box or pressure release box, and these are often used as a start to verify whether FEM code is working correctly; however, these do not take into account complex boundaries. The impedance tube, in which a driver at one end of a long, slender tube excites standing waves while a complex impedance is placed at the other end, is typically used for complex impedance verification [8]. Koopmann and Fahnline [22] have a thorough discussion on the impedance tube and its analytical solution.

When it comes to underwater acoustics, another benchmark problem is often used for verification of new numerical methods, the underwater wedge mentioned in [11]. This wedge is sloped so that the sound is either propagating uphill or downhill resulting in two different effects. The effects in the water column consist of a change in modal excitation when certain depths are reached, either cutting on or cutting off of the modes. The number of propagating modes in a zeroth-order waveguide (rigid bottom, pressure release top) is given by

\[ n_{\text{prop}} = \frac{2D}{\lambda} + 0.5 \]  

where \( D \) is the depth of the waveguide and \( \lambda \) is the wavelength of propagating sound. In the wedge, as the depth decreases, fewer modes propagate until eventually all modes evanesce. Jensen et al. have a complete discussion of the wedge problem in their text [4] and also in the papers [23] and [24]. In [23] they look at upslope propagation with a penetrable bottom using the parabolic equation method and gain results comparable to experiment. In [24] they look at upslope and downslope propagation and so-called ‘wedge modes’ in a wedge with different source configurations using the parabolic
equation method. Those results compared well with earlier experimental results.

Buckingham and Tolstoy [25] present an analytical solution to the underwater wedge problem providing the bottom impedance is a pressure release condition. Jensen in [26] examines what size elements are needed on the slope to accurately represent it using finite elements.
Chapter 2
Simplified Acoustic FEM

2.1 Introduction

In the previous chapter, it was stated that when analyzing wave propagation in water with complex boundaries, there are multiple numerical methods available, each with their pros and cons. Normal mode, parabolic equation, wavenumber integration, ray tracing, and finite element methods were the main methods discussed, but these were limited in scope due to the assumptions necessary to make their respective set of equations solvable: ray tracing methods do not work well for lower frequency ranges, normal mode methods do not typically take into account the backscattered acoustic field, wavenumber integration is applicable in horizontally stratified or range independent environments, the parabolic equation has phase errors inherent in the formulation. For low frequency excitations in range-dependent environments where backscattering is important, a finite element approach (FEM) has more positive attributes when compared with the others and thus will be developed in this chapter.

While there are numerous commercial finite element programs on the market, they are relatively expensive (several thousand dollars a year for licenses), especially if only one project requires such a program. The motivation behind part of this dissertation was to write a simplified open source program using basic finite element acoustic equations for use in doing low frequency analysis in semi-enclosed spaces. Running of
the program requires some basic FEM knowledge along with knowledge of MATLAB, which is typically available in most academic and many professional settings already.

### 2.2 FEM Overview

To understand the acoustics of a harbor without doing any experiments, the theoretical wave equation, Eq. 2.1, needs to be solved.

\[ \nabla^2 p = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} \quad 2.1 \]

Due to the complex shape of a harbor, there is no analytical solution for the equation, thus a numerical approach such as the finite element method must be taken. A general computer code was developed using finite element methods to predict the acoustic pressure field in any semi-enclosed space with complex boundaries. This method was chosen for its ease and speed of implementation. While the ten elements per wavelength “rule of thumb” dictated by standard FEM theory still applies, very large spaces such as harbors can be modeled when treating low frequency sound propagation.

The main input for any FEM code is the geometry of the space. This geometry includes node coordinates and the connectivity, or elements which divide the space. For this project, the geometry of a harbor is taken from bathymetry data (e.g., depth, length, width, etc.) and run through a voxelizer program developed at Stanford University and provided as freeware over the internet [27]. A voxel is the smallest distinguishable box-shaped part of a three-dimensional space. The voxelizer program takes a given solid geometry, such as the harbor, and divides it into identical voxels via a process called
“floodfilling,” recreating the shape as truly as the voxel size can replicate it (Figure 2.1). The size of the voxel will directly relate to the size of the final element used in the FEM program, thus the ten elements per wavelength requirement must be met. This follows Craggs’ suggestion of using cuboid elements [7], but neglects areas around the edges that could be filled with differently shaped elements. These areas can be neglected because the wavelength and relative distances are much larger than the voxel size. A Matlab code has been written that takes the voxelized shape and creates node points and an element connectivity matrix. Thus each voxel becomes a finite element.

---

Figure 2.1: Voxel/Element and node geometry with damping.

To devise the finite element code, the fluid was modeled as a “structure” with a distribution of displacements on the “surface.” In these terms the body of water itself is the “structure” while the “surface” consists of any of the boundaries, internal or external to the structure. It is assumed that there is one-way coupling between the surface and the
structure, i.e., the vibration of the surface will affect the pressure in the structure, but the pressure in the structure does not affect the vibration of the surface. This assumption is valid for the high impedance surfaces and sources typically used in underwater experiments. All sources that vibrate will be at the same frequency, although changes in their phases will be permitted.

Boundary conditions are easily satisfied with the new code. The default boundary condition for the border of the domain is rigid-walled, meaning that any edge of the domain will have a pressure gradient of zero. For a pressure release condition, the degrees of freedom on the boundary merely need to be removed from the overall FEM matrices. For more complex boundary conditions, a damping matrix is included in the formulation of the overall FEM matrices. One or more boundary conditions can be included at various locations in the model as required.

One of the main benefits of this method is its universality. The finite elements used are cuboid (cube-shaped) elements with eight degrees of freedom at the corners. Since all of the elements are the same, the finite element matrices are developed for one element and used as a template for all of the others, saving computation time and computer memory. The resulting sparse global matrix reduces the memory required for solving larger problems. This global mobility matrix will be assembled only once for each basic domain, and then altered for changing boundaries. This method is very efficient for problems with finite boundaries as well as problems with semi-infinite boundaries with the use of a $\rho$-$c$ ($rho$-$c$) boundary condition to reduce the overall size of the domain; $\rho$-$c$ boundary conditions absorb all normal and nearly normal incident outgoing waves based on the impedance of a plane wave in the far field. A flow chart for
the FEM program is shown in Figure 2.2. Once the solid geometry is described and voxelized according to the required resolution, the stiffness matrix (coefficients multiplied by the pressure), mass matrix (coefficients multiplied by the second time derivative of pressure), and damping matrix (coefficients multiplied by the first time derivative of pressure) are created. The removal of elements and degrees of freedom to change the geometry, e.g., the presence of ships in a harbor, is completed, the forces are applied in the form of volume velocity sound sources, and then the code solves the system of equations. The removal of elements or change of forces can be repeated as needed, using the same previously formed matrices.

**Figure 2.2:** Acoustic FEM flow chart. Variables: \(a = \) length of element divided by 2, \(c = \) speed of sound in fluid, \(\rho = \) density of fluid, \(\beta = \) damping ratio for boundaries, \(f = \) force
2.3 FEM Derivation

The derivation of the finite element matrices follows a standard derivation [5] using the wave equation, Eq. 2.1, where \( p \) is pressure and \( c \) is the speed of sound in the medium. The weak form of the Helmholtz equation is given by

\[
\int_V \delta p \left( \nabla^2 p - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} \right) dV
\]

where the integral is over a finite volume. Using the properties of divergence it can be shown

\[
\nabla \cdot (\delta p \nabla p) = \nabla \cdot (\delta p) \nabla p + \delta p \nabla^2 p \Rightarrow \\
\delta p \cdot \nabla^2 p = \nabla \cdot (\delta p \nabla p) - \nabla \cdot (\delta p) \nabla p
\]

\[
\int_V \nabla \cdot (\delta p \nabla p) dV = \int_s (\delta p \nabla p) \cdot ndS
\]

and using the Divergence Theorem,

\[
\int_V \nabla \cdot (\delta p \nabla p) dV = \int_s (\delta p \nabla p) \cdot ndS
\]

where \( n \) is the normal direction, Eq. 2.2 can be rewritten in the following form:

\[
\int_s (\delta p \nabla p) \cdot ndS = \int_V \nabla \cdot (\delta p) \nabla p dV - \frac{1}{c^2} \int_V \delta p \frac{\partial^2 p}{\partial t^2} dV = 0
\]

To develop the finite element equations, the pressure is written as a sum of the pressures of an elemental volume multiplied by a shape function. The virtual pressure, \( \delta p \), can be written as the matrix multiplication Eq. 2.7.

\[
p = \{N\}^T \{p_e\} \rightarrow \delta p = \{N\}^T \{\delta p_e\}
\]
The differential operator can be written as follows and can be used to calculate the gradient terms.

\[ L^T = \begin{bmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{bmatrix} \]

\[ \nabla p = \{L\} p = \{L\} \{N\}^T \{p_e\} = \{B\} \{p_e\} \]

\[ \nabla \cdot (\delta p) = \{L\}^T \{N\}^T \{\delta p_e\} \]

The shape functions are constant and therefore do not get differentiated with respect to time.

\[ \frac{\partial^2 p}{\partial t^2} = \{N\}^T \frac{\partial^2}{\partial t^2} \{p_e\} \]

Euler’s Equation can be written using matrix notation where \( n \) is the normal direction and \( u \) is the particle velocity:

\[ \{n\} \cdot \nabla p = -\rho_o \{n\} \frac{\partial^2 \{u\}}{\partial t^2} \]

Eq. 2.6 can then be rewritten as Eq. 2.13

\[- \int_s \rho_o \{\delta p_e\}^T \{N\}^T \{\dot{u}_e\} dS - \int_v \{\delta p_e\}^T \{N\}^T \{L\} \{B\} \{p_e\} dV\]

\[- \frac{1}{c^2} \int_v \{\delta p_e\}^T \{N\}^T \{\ddot{p}_e\} dV = 0\]

which needs to be true for arbitrary \( \delta p \)’s. Rearranging, a system of equations with pressure, its second time derivative and a “force” in terms of acceleration is developed.

\( N \) is the fluid shape function within the acoustic element, where \( N' \) is the structural shape function.
function. The structural shape function is neglected, assuming rigid vibration and one-way coupling between the boundary and the fluid, leading to Eq. 2.14.

\[
\left[ \frac{1}{c^2} \int_N \{N\}^T \{N\}^T dV \right] \{\ddot{p}_e\} + \left[ \int_B \{B\}^T [B] dV \right] \{p_e\} + \\
\left[ \int_S \{N\}^T \{N\}^T dS \right] \{\dot{\bar{u}}_e\} = 0
\]

As in mechanical systems, dissipation can be added by using the first time derivative of the pressure. This leads to the following final FEM equation where \( M, C, \) and \( K \) are similar to the mass, damping, and stiffness matrices used in mechanical systems while \( R \) combined with the acceleration is similar to a force:

\[
\left[ M_e^p \right] \{\ddot{p}_e\} + \left[ C_e^p \right] \{\dot{p}_e\} + \left[ K_e^p \right] \{p_e\} + \rho_0 \left[ R_e \right]^T \{\dot{\bar{u}}_e\} = 0
\]

\[
\left[ M_e^p \right] = \frac{1}{c^2} \int_N \{N\}^T \{N\}^T dV
\]

\[
\left[ K_e^p \right] = \int_B \{B\}^T [B] dV
\]

\[
\left[ C_e^p \right] = \frac{\rho}{c} \int_S \{N\}^T \{N\}^T dS
\]

\[
\left[ R_e \right] = \int_S \{N\}^T \{N\}^T dS
\]
The damping is applied only to boundary surfaces, so $N_s$ is the shape function on the absorbing surface and $S$ is the absorbing boundary surface area. The damping variable, $\beta$, is represented by

$$\beta = \frac{pc}{Z}$$ \hspace{1cm} 2.20

the ratio of the fluid characteristic impedance to the boundary impedance.

Because this analysis is concerned with frequency response, the excitation is assumed harmonic, thus the pressure and velocity can be written as Eq. 2.21 and Eq. 2.22, respectively.

$$\{p_e\} = \{\hat{p}_e\}e^{-i\omega t}$$ \hspace{1cm} 2.21

$$\{u_e\} = \{\hat{u}_e\}e^{-i\omega t}$$ \hspace{1cm} 2.22

The phase of the boundary velocity can also be included as $e^{i\phi}$. When derivatives are taken and substituted into Eq. 2.15 the matrix equation becomes

$$\left( [K^p] - (i\omega)[C^p] - \omega^2 [M^p] \right)\{\hat{p}_e\} = \rho_0 \omega^2 [R_e]^T \{\hat{u}_e e^{i\phi} \}$$ \hspace{1cm} 2.23

and the initial matrices can be combined into one matrix leading to Eq. 2.24.

$$[K^*]\{\hat{p}_e\} = \rho_0 \omega^2 [R_e]^T \{\hat{u}_e \}$$ \hspace{1cm} 2.24
The shape functions used are standard linear ones for FEM analysis as opposed to more complex functions often used in typical FEM. This simplifies the formulation of the code.

\[ N_i = \frac{1 + r_i r}{2} \frac{1 + s_i s}{2} \frac{1 + t_i t}{2} \]

\[
N_1 = \frac{1 - r}{2} \frac{1 - s}{2} \frac{1 - t}{2} \\
N_2 = \frac{1 - r}{2} \frac{1 + s}{2} \frac{1 - t}{2} \\
N_3 = \frac{1 - r}{2} \frac{1 + s}{2} \frac{1 + t}{2} \\
N_4 = \frac{1 - r}{2} \frac{1 + s}{2} \frac{1 + t}{2} \\
N_5 = \frac{1 + r}{2} \frac{1 - s}{2} \frac{1 - t}{2} \\
N_6 = \frac{1 + r}{2} \frac{1 + s}{2} \frac{1 - t}{2} \\
N_7 = \frac{1 + r}{2} \frac{1 + s}{2} \frac{1 + t}{2} \\
N_8 = \frac{1 + r}{2} \frac{1 - s}{2} \frac{1 + t}{2} \\
\]

\[ \{N\} = \begin{bmatrix} N_1 \\ \vdots \\ N_8 \end{bmatrix} \]

In these equations, \( r, s, \) and \( t \) are \(+/-1\) depending on the location of the node with regard to the center of the element in the \( x, y, \) and \( z \) directions, respectively (Figure 2.3).
Using the new shape functions, vector Eq. 2.15 through Eq. 2.19 can be rewritten as Eq. 2.27 through Eq. 2.34.

\[ M_{ij} = \frac{1}{c^2} \int V N_i N_j dV \quad \text{2.27} \]

\[ \left[ K \nu \right] = \int [B]^T [B] dV \quad \text{2.28} \]

\[ [B] = \{L\} \{N\}^T \quad \text{2.29} \]

\[ \{\nabla p\} = [B] [\rho_c] \quad \text{2.30} \]
As stated, this formulation dictates that the standard boundary condition on any surface is rigid. To create a pressure release boundary condition, those degrees of freedom can be removed from the system of equations. For complex impedances on the boundaries, the $\beta$ term is used, relating the characteristic fluid impedance to the impedance of the boundary.
Chapter 3

Validation of the Acoustic FEM Code

3.1 Introduction

The acoustic FEM code has been validated by using basic acoustical enclosures that have analytical solutions. Further, a more challenging validation was completed on an underwater wedge suggested by Jensen, et al. [28] with an analytical solution in Buckingham and Tolstoy [25] that tests the capabilities of the program with more complex geometries. This chapter explains these different validations and illustrates the results from the FEM code. The eigenvalue validation in section 3.2 ensures that the matrices are formed correctly. The eigenvector validation with the impedance tube in section 3.3 ensures that a forced excitation and boundary conditions are programmed correctly. The transmission loss of a point source and the wedge validation in sections 3.4 and 3.5, respectively, validate the capabilities of the FEM program in more complex environments.

3.2 Basic Enclosures – Eigenvalue Comparisons

The initial test of any FEM code is the comparison of computed eigenvalues and eigenvectors with known analytical results [29]. This was done using three different basic geometries: a slender rigid-walled duct, a rigid-walled rectangular solid, and a rigid-walled rectangular solid with a pressure release top (Figure 3.1).
The slender, 4x4x40 m duct was modeled with 2x2x20 voxels. The theoretical eigenvalues for the volume are given by

\[ \omega_{\text{theory}} = \frac{i \pi c}{L} \quad i = 1, 2, 3 \ldots \tag{3.1} \]

where \( c \) is the sound speed in m/s and \( L \) is the length of the duct in m. The results from the FEM and theory are very similar as shown in Table 3-1. According to the ten elements per wavelength rule accurate results should be possible for this model up to a frequency of 471 rad/s. As Table 3-1 shows, the percent difference between analytical and FEA results is less than 1% for those frequencies. The error increases as the frequency increases because the ten elements per wavelength rule is being violated.
Table 3-1: Slender duct eigenfrequency comparison.

<table>
<thead>
<tr>
<th>Analytical ω(rad/s)</th>
<th>FEA ω(rad/s)</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>117.8</td>
<td>117.9</td>
<td>0.08</td>
</tr>
<tr>
<td>235.62</td>
<td>236.6</td>
<td>0.42</td>
</tr>
<tr>
<td>353.43</td>
<td>356.7</td>
<td>0.93</td>
</tr>
<tr>
<td>471.24</td>
<td>479</td>
<td>1.65</td>
</tr>
<tr>
<td>589.05</td>
<td>604.3</td>
<td>2.59</td>
</tr>
<tr>
<td>706.86</td>
<td>733.2</td>
<td>3.73</td>
</tr>
</tbody>
</table>

The 40x20x10 m rectangular solid was 20x10x5 voxels with the theoretical eigenvalues for the volume given by Eq. 3.2,

\[
\omega_{\text{theory}} = \pi c \sqrt{i \left( \frac{L_x}{L_x} \right)^2 + j \left( \frac{L_y}{L_y} \right)^2 + k \left( \frac{L_z}{L_z} \right)^2}
\]

\[i, j, k = 0,1,2,\ldots\]

where \(L_x, L_y,\) and \(L_z\) are the lengths in those respective directions. These results agree well with the FEM model as shown in Table 3-2. Again, FEA and analytical frequencies below 471 rad/s agree within 1%.

Table 3-2: Rectangular solid eigenfrequency comparison.

<table>
<thead>
<tr>
<th>Analytical ω(rad/s)</th>
<th>FEA ω(rad/s)</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>117.81</td>
<td>117.93</td>
<td>0.10</td>
</tr>
<tr>
<td>235.62</td>
<td>236.59</td>
<td>0.41</td>
</tr>
<tr>
<td>263.43</td>
<td>264.35</td>
<td>0.35</td>
</tr>
<tr>
<td>333.22</td>
<td>334.59</td>
<td>0.41</td>
</tr>
<tr>
<td>353.43</td>
<td>356.71</td>
<td>0.93</td>
</tr>
<tr>
<td>424.77</td>
<td>428.04</td>
<td>0.77</td>
</tr>
<tr>
<td>471.24</td>
<td>479.02</td>
<td>1.65</td>
</tr>
</tbody>
</table>

The 40x20x14 m rectangular solid with pressure release top surface was 20x10x7 voxels. The theoretical eigenvalues are given by Eq. 3.2, except \(k=1,3,5\ldots\) due to the pressure release surface. These results also agreed very well and are shown in Table 3-3.
All frequencies analyzed were below the 471 rad/s cut off, thus the FEA and analytical results agree within 1%.

### 3.3 Impedance Tube – Eigenvector Comparisons

To illustrate the accuracy of modeling complex boundary conditions, the simple case of an impedance tube, for which an analytic solution exists, was used with different boundary impedances. An impedance tube is a duct with rigid walls, a piston with prescribed velocity at one end, and a wall of a defined impedance on the other end (Figure 3.2). The derivation of the analytic solution can be found in Koopmann and Fahnline [22], and with a few substitutions becomes:

$$p(x) = \rho c v \left( \frac{i\beta \sin[k(x - L)] + \cos[k(x - L)]}{\beta \cos(ki) - i\sin(ki)} \right)$$

where \(x=0\) is the piston end, \(x=L\) is the impedance end, and \(v\) is the velocity of the piston.

<table>
<thead>
<tr>
<th>Analytical (\omega) (rad/s)</th>
<th>FEA (\omega) (rad/s)</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>168.30</td>
<td>168.65</td>
<td>0.21</td>
</tr>
<tr>
<td>205.44</td>
<td>205.80</td>
<td>0.18</td>
</tr>
<tr>
<td>289.55</td>
<td>290.55</td>
<td>0.35</td>
</tr>
<tr>
<td>312.60</td>
<td>313.57</td>
<td>0.31</td>
</tr>
<tr>
<td>373.30</td>
<td>374.69</td>
<td>0.37</td>
</tr>
<tr>
<td>391.45</td>
<td>394.56</td>
<td>0.79</td>
</tr>
</tbody>
</table>
A Matlab program was developed to calculate the pressure along the center line of the impedance tube using the analytical and FEM methods. Five different boundary conditions were used: rigid wall, pressure release, real impedance, imaginary impedance, and complex impedance. Each element was 1 meter in length with a total length of 40 meters. The results agreed very well with theory, noting that the two models are slightly different. The numerical FEM assumes a three-dimensional space whereas the analytical theory assumes linear, one-dimensional propagation.

Figure 3.2: Impedance Tube. The left edge consists of a piston vibrating with a prescribed velocity. The right end has a known impedance.
Figure 3.3: Impedance tube results. Boundary is a Rigid Wall with $\beta = 0$. 
Figure 3.4: Impedance Tube results. Boundary has a $\rho$-$c$ condition with $\beta = 1$. 
Figure 3.5: Impedance Tube results. Boundary has an imaginary impedance with $\beta=0.5\,i$. 
As shown in Figures 3.5 and 3.6, the agreement between the analytical theory and FEM results decreases when beta has an imaginary part to it. This can be remedied by increasing the number of elements. Figure 3.7 shows the results of doubling the number of elements and reducing the size of each element to 0.5 meters.

Figure 3.6: Impedance Tube results. Boundary has a complex impedance with $\beta = 0.5 + 0.5i$. 

As shown in Figures 3.5 and 3.6, the agreement between the analytical theory and FEM results decreases when beta has an imaginary part to it. This can be remedied by increasing the number of elements. Figure 3.7 shows the results of doubling the number of elements and reducing the size of each element to 0.5 meters.
Three-dimensional visualization software called ParaView [30], available for free over the internet, was used to analyze the FEM results for the previous simple and later more complicated geometries. Matlab code was written to take the pressure results, coordinate matrix, and connectivity matrix and create a .ptk file readable in ParaView. Figure 3.8 shows an example of a point source radiating in free space. The free space is assumed symmetric and a quadrant is modeled with 21 by 21 by 21 elements with one

Figure 3.7: Impedance Tube results. Boundary has a complex impedance with $\beta = 0.5 + 0.5 \, \text{i}$. There are twice as many elements along the length of the tube as compared to Figure 3.6.

3.4 Transmission Loss for a Point Source

Three-dimensional visualization software called ParaView [30], available for free over the internet, was used to analyze the FEM results for the previous simple and later more complicated geometries. Matlab code was written to take the pressure results, coordinate matrix, and connectivity matrix and create a .ptk file readable in ParaView. Figure 3.8 shows an example of a point source radiating in free space. The free space is assumed symmetric and a quadrant is modeled with 21 by 21 by 21 elements with one
quadrant side equal to 21 meters. The symmetric walls were modeled with rigid boundary conditions and the outside walls were given $\rho$-$c$ boundary conditions. A point source at 10 Hz was placed at the corner of the quadrant, i.e., the center of the entire space. The results were compared with theory for the diagonal of the quadrant (Figure 3.9) as well as with the symmetric edge of the quadrant (Figure 3.10). The theory is from Pierce [31] which states

$$ p(r) = -\frac{\rho c k a^2 v}{r(1 - ika)} e^{ik(r - a)} $$

3.4

where $v$ is the surface velocity of a pulsating sphere of radius $a$. These results compare fairly well, especially for the symmetric edge of the quadrant. Discrepancies are due to the proximity of the $\rho$-$c$ boundary conditions to non-normal incident waves, and the discrepancies decrease as the number of elements per wavelength is increased. A method of addressing this discrepancy with absorptive boundaries is discussed in Chapter 9 with a variable $\beta$, but should only occur when the absorptive boundaries are located near to the source or near to the point of desired measurement.
Figure 3.8: Screenshot of ParaView. Quadrant of free space with point source in corner and contoured pressure loss in dB.
Figure 3.9: Pressure loss in dB along diagonal of quadrant compared with theory.
3.5 FEM Validation – Underwater Wedge

The underwater wedge is an accepted challenge to test the capabilities of any type of numerical underwater acoustic method. There are various incarnations of it, but it consists of a sloped bottom surface and a flat water/air interface. The bottom can be penetrable with a certain impedance or pressure release. The water/air interface is typically pressure release as well, but losses can be added. The wedge can be formed as a two-dimensional case (range x and depth z, where y is constant) or an axisymmetric case.

Figure 3.10: Pressure loss in dB along symmetric edge of quadrant compared with theory.
(range r and depth z, where theta is constant)(Figure 3.11). Typically the wedge is treated as two-dimensional (range and depth), and the third dimension (y or theta) is neglected. For a three-dimensional FEM model this third dimension cannot be neglected. The area opposite the wedge, on the other side of the source, is typically neglected as well, which leads to problems with FEM modeling.

![Figure 3.11: Underwater Wedge modeling approaches – red is source location.](image)

Both of these difficulties were dealt with by making the problem two-dimensional (not axisymmetric) so that the third-dimension walls could be modeled as rigid walls. The wedge was extended past the source for a few elements to ensure the $\rho$-c boundary condition on that wall would not interfere with the FEM results. The pressure release surface and bottom model for the wedge was used because there was an analytical solution given in [25] so an easy comparison could be made. The wedge could not be
compared directly to solutions from earlier papers because of the limitations of the FEM program. A smaller wedge with a higher frequency was used, but the analytical solution was validated with the provided wedge dimensions. Figure 3.12 shows the ParaView representations of the wedge being excited at three different frequencies. The source is located at a depth of 100m about 1km from the tip of the wedge. The depth of the wedge at the source location is 200m. The slope of the wedge FEM model was created in accordance with suggestions from Jensen [26] for discrete models which states that the change in distance for each step needs to be less than or equal to a quarter of the excitation wavelength. Modal propagation cutoff is apparent as the depth in the wedge decreases. Figure 3.13, Figure 3.14, and Figure 3.15 show transmission loss comparisons of the FEM results and the analytical results at three different frequencies: 5 Hz, 10 Hz, and 12 Hz (the maximum frequency for accurate FEM resolution). The transmission loss is calculated at a depth of 30m. The results agree well in magnitude and in the location of the pressure minimums.
Figure 3.12: Wedge solutions from Acoustic FEM. Source is 100 m deep on black line which is approximately 1 km from tip of wedge. Depth at Source location is 200 m. Pressure release surfaces on top and bottom and rho-c on the left most side.
Figure 3.13: Wedge results from FEM compared to analytical theory. Source is at 100 m depth excited at 5 Hz, receiver is at 30 m depth.
Figure 3.14: Wedge results from FEM compared to analytical theory. Source is at 100 m depth excited at 10 Hz, receiver is at 30 m depth.
3.6 Conclusion

An acoustic finite element program was created using Matlab and validated with numerous geometries, including the underwater wedge. The results of the FEM compare extremely well with the analytical solution of the wedge. With the validation, this simple FEM code can now be used to model other geometries, including a harbor, which will be the focus of Chapter 9. This code differs from current commercial code in that it is completely open source and uses very simple voxel elements and shape functions. It is
meant for lower frequency excitations in closed or semi-enclosed spaces. The code could be extended or expanded to take into account different shape functions and more complex elements, but that would defeat its simplicity and thus, its straightforward application.

The FEM code described and validated in the last two chapters will be revisited in Chapter 9. It will be used to model the harbor at Coddington Cove and results will be compared to experimental data. The next chapters describe the theory and experiments behind the new focusing technique mentioned in the introduction.
Chapter 4

Focusing Sound Using an Array of Sources

4.1 Multiple Sources Background

One simple harmonic source in free space will propagate spherically according to Eq. 4.1

\[ p(r) = s \frac{e^{-ikr}}{r} \]  \hspace{1cm} 4.1

where \( r \) is the distance from the source, \( s \) is the source strength, \( k \) is the acoustic wavenumber and \( i \) is the imaginary number. As can be seen in the equation the pressure will decay at a rate of \( 1/r \) while the phase of the pressure will decrease as it propagates. Thus, at every point in the acoustic field, a position will have a pressure consisting of amplitude and phase which can also be represented with a complex number.

When a second source is added near the first, the acoustic field created by only the second source will be the same as that created by the first source, assuming the same source strength and initial phasing. Using the principle of superposition, the combined acoustic field can be calculated by adding the pressure from each source at all points in the acoustic field. Eq. 4.2 shows this summation using three sources with the phase of each source represented by the variable \( \phi \) and \( r_i \) is the distance from source \( i \) to the receiver location \( r \).

\[ p(r) = \frac{s_1 e^{i\phi_1} e^{-ikr_1}}{r_1} + \frac{s_2 e^{i\phi_2} e^{-ikr_2}}{r_2} + \frac{s_3 e^{i\phi_3} e^{-ikr_3}}{r_3} \]  \hspace{1cm} 4.2
After substituting $r_1 = r$, $r_2 = r + \Delta r_2$, $r_3 = r + \Delta r_3$, and assuming $\phi_1 = 0$ and $s_1 = s_2 = s_3 = s$, the equation can be rewritten as Eq. 4.3.

$$p(r) = \frac{s}{r} e^{-i\Delta r} + \frac{s}{r + \Delta r_2} e^{-i[k(r + \Delta r_2) - \phi_2]} + \frac{s}{r + \Delta r_3} e^{-i[k(r + \Delta r_3) - \phi_3]} \quad 4.3$$

Figure 4.1 shows the amplitude of the acoustic field after the superposition of two simple sources, each phased at zero. Figure 4.2 shows the phase of the same acoustic field. The phase of each source becomes extremely important in determining where maximum and minimum pressure areas are.
Figure 4.1: Amplitude of acoustic field due to two simple sources with zero phase at 100 Hz in water.
With more than one source, an acoustic beam or beams, typically referred to as lobes, are created where the phases add constructively, creating a maximum pressure, and acoustic nulls where the phases add destructively creating a minimum pressure. In order to get maximum pressure, the pressure due to each source needs to have the same phase, i.e. the arguments of the exponentials need to be equal as shown in Eq. 4.4.

\[ -ikr = -i[k(r + \Delta r_3) - \phi_2] = -i[k(r + \Delta r_3) - \phi_3] \rightarrow \]

\[ \phi_2 = k\Delta r_2 \quad \phi_3 = k\Delta r_3 \]

4.4

Figure 4.2: Phase of acoustic field due to two simple sources with zero phase at 100 Hz in water.
This equation leads to the derivation of a distance method to focus the acoustic lobe, and with some approximations and assumptions leads to the classic linear array theory, both discussed in the next section.

4.2 Distance Method and Array Theory

There are several mutual assumptions made in the following methods. First, the system is assumed to be linear and, thus superposition of sources will hold. Second, the system is assumed to not change rapidly, also known as time invariant. These methods also assume no reflections will interfere with the focusing and that the sources are uncoupled, i.e., one source will not affect the source strength or phase of another.

Focusing multiple sources at a single location involves steering the lobes of the acoustic field so that the main lobe is aiming in the desired direction. One way of doing this is to calculate the difference in time of flight from both sources to the specified focusing location. This difference in time can be used to calculate the difference in distance (Eq. 4.5) and subsequently the phase (Eq. 4.6) to add onto one of the sources to focus the acoustic energy. In the equations, $\Delta d$ is the change in distance in meters, $c$ is the speed of sound in the media in m/s, $\Delta t$ is the change in time of flight in seconds, $\lambda$ is the wavelength of the excitation frequency in meters, and $\phi_r$ is the phase added to the source in radians.

$$\Delta d = c\Delta t$$  \hspace{1cm} 4.5
This can be repeated for any number of sources in free space keeping one source as the reference source at zero phase.

A second method for calculating the required phase to steer the acoustic beam using array theory is described in Kinsler, et al. These phases can be calculated with Eq. 4.7 derived from [29]:

\[
\phi = 2 \pi f \frac{d}{c} \sin \theta_0
\]

where \( \phi \) is the phase angle increment in radians for each consecutive source, \( f \) is the excitation frequency, \( d \) is the distance between sources in the array in meters, and \( c \) is the speed of sound in m/s.

Classic array theory assumes that the sources are in an equally spaced, linear array and that the distance from the array to the source is much further than the distance between sources. With this assumption, an incremental phasing of each source in the array will result in a beam at a particular angle. Instead of focusing energy at a desired location, the energy is focused in a desired direction. For a linear array in free space both methods are comparable, although the distance method is more direct and easier to calculate due to the ability to calculate the distance measurements experimentally even if the sources or receivers are not visible under the water. Figure 4.3 and Figure 4.4 illustrate results of beam steering using the distance method: two sources at (20m, 25m) and (30m, 25m) are phased to focus energy on the circle at (30m, 15m) at 100 Hz.
Figure 4.3: Pressure amplitude with two sources, phased to focus on the circle at 100Hz. Phi=1.7351 rads.
Both of these methods work well in free space, or an open area with few or no obstacles in the acoustic field. As soon as boundaries are introduced they produce reflections which complicate the acoustic field and can destroy the focusing. The addition of one ideal rigid reflective boundary will effectively double the number of apparent sources, although these additional sources have phases which are dependent on the originals. This is based on the method of images from optics, where an image source is placed opposite an original source, equidistant from the boundary with the same or opposite phase depending on whether the boundary is rigid or pressure release, respectively. In more complex geometries, more reflections will cause even more error.

Figure 4.4: Pressure phase with two sources, phased to focus on the circle at 100 Hz.
in an original phasing. Figure 4.5 and Figure 4.6 illustrate what the addition of a boundary does to the acoustic field. The blue line represents the rigid boundary and the sources to the left of the boundary are the image sources.

Figure 4.5: Pressure amplitude with a rigid boundary added.
As more boundaries are added and the boundary impedance changes from the ideal case it becomes more and more difficult to model the reflections. One method investigated in the current project, is to create an acoustic finite element model which will automatically model the boundary conditions. A comparison between experimental and FEM results for a harbor is discussed in Chapter 9. A different, experimental method for focusing the acoustic beam is discussed in Chapter 5.

Figure 4.6: Pressure Phase with boundary.
5.1 Introduction

There are a variety of reasons to be interested in focusing sound in any environment whether it is to send (or receive) information or, as in the case of this dissertation topic, to act as a deterrent. One of the main methods of focusing sound involves a linear array of sources spaced and phased in such a way so as to create a main lobe that is focused in a desired direction (as discussed in Chapter 4). This classic linear array theory is described in detail in Kinsler, et al. [29]. Along with basic linearity and time invariance assumptions, the extra assumptions of this method are (1) the distance to the receiver is much greater than the length of the array, (2) the array is in open space without any noticeable obstructions or boundaries, and (3) the sources of the array have similar transfer functions. Classic linear array theory is only one approach that can be taken with arrays. Urick [32] discusses amplitude shading along with a variety of signal processing methods that are used with acoustic arrays, however only the basic linear array theory is investigated in this study.

5.2 Phasor Notation

To focus sound in a type of environment in which assumptions (1), (2), and (3) cannot be made, such as in a harbor, an alternate method needs to be devised that builds
on a phasor representation of the acoustic pressure. The sound produced by a source can be represented by a magnitude and phase angle: the magnitude decays with distance while the phase oscillates between +/- 180 degrees relative to the source phase as the sound wave propagates. Thus, at any point in a sound field, the pressure due to a single source can be represented by a phasor. Multiple sources can be represented with vector-like addition (Figure 5.1). To maximize the pressure at a given location, the phasors due to each of the sources must have the same phase angle, or be “in phase,” to cause constructive interference.

![Figure 5.1: Phasor representation of acoustic pressure and vector addition of multiple sources.](image)

If a single source is placed between two boundaries (Figure 5.2), multiple path reflections will arrive at a receiver location at different phases, therefore the phasor representation of the pressure is the vector sum of the pressure phasors from the direct path and all reflected paths. Each source in an array creates a similar pressure with different magnitudes and phases, but the phase angle of the pressure phasor can be altered by changing the phase of the source signal. The optimal method used in this dissertation
utilizes this characteristic and the idea of superposition of sources to find the optimal phase for each source to maximize the pressure phasor at a prescribed location.

Another way of approaching the concept is through the use of transfer functions. A transfer function can be defined between the source location and the receiver location. It represents the pressure at the receiver due to an excitation at the source, see Eq. 5.1.

This is what classic array theory uses to calculate their phases. What the optimal phase search (OPS) method does is take a step backwards to the signal going into the sources. Therefore the transfer function is actually the pressure at the receiver location due to the excitation signal sent to the source, thereby taking into account any discrepancies between sources. The OPS method also inherently takes into account any reflections strong enough to show up at the receiver location. It is not limited to the direct path calculation that classic array theory is.

\[
\text{Array Theory: } G = \frac{P_r}{s} \quad \text{(direct path only)}
\]
\[
\text{OPS method: } H = \frac{s}{\text{signal}} \times \frac{P_t}{s} \quad \text{(direct path, reflections, source dynamics)}
\]
5.3 Optimal Method Search Algorithm

This optimal method requires an *a priori* interrogation of the harbor area to find the phases. The search algorithm excites a reference source with a sine wave of zero phase and then increments a second source through 360 degrees of phase while recording pressure at hydrophone locations in the harbor. These hydrophone locations represent the locations on which the sound should be focused. The spacing of the hydrophones should represent the size of a zone or sector which will correspond to one set of phases. After the phase has been cycled through, the second source is turned off and the third is excited with a similar phase incrementing sine wave. This is repeated for all sources, retaining the same reference source. The algorithm is shown in flowchart form in Figure 5.3.
After the data is acquired for each phase angle, an FFT is performed on that small set of data to get a pressure level at each hydrophone location in the harbor. After each source run, a set of data describing the pressure at each hydrophone while the phase changes is investigated, and the phase where the pressure is at its maximum is recorded. Thus, a look up table for each source is created with a phase value to maximize the pressure at each hydrophone location. Due to the variable nature of underwater acoustics, these phase values will change from hour to hour, day to day, month to month,
season to season, etc. The effect of these changes and degree to which they will change has not been fully investigated, due to time constraints.

Prior to testing the algorithm in the field, studies were completed in air in the Penn State Center for Acoustics and Vibration anechoic chamber. These studies were to test the optimal method algorithm to find the phase required to provide maximum pressure at a receiver. Two loudspeakers and one microphone were arbitrarily placed in the chamber and then the algorithm was tested. A plot was created of amplitude versus phase, and the phase corresponding to the maximum amplitude was recorded. The resulting phase curve resembles one cycle of a sine wave with a maximum and minimum pressure within the 360 degree range (Figure 5.4 and Figure 5.5).

Figure 5.4: Anechoic chamber test using phase search algorithm with two loudspeakers at 300 Hz and one microphone in air.
Figure 5.5: Anechoic chamber test using phase search algorithm with two loudspeakers at 1000 Hz and one microphone in air.
6.1 Introduction

To run experiments in underwater acoustics, assuming scaled down modeling is not a feasible option, it is first necessary to find underwater facilities large enough to incorporate the sources and receivers. Ideally these spaces will have low reverberant effects and be large enough to accommodate several acoustic wavelengths within their bounds. For this project, two outdoor facilities were chosen. The first, Jacksonville Quarry, is local to the research institution, the Pennsylvania State University, and was large enough for some small scale studies. The second, Coddington Cove, is a harbor in Narragansett Bay, Newport, RI, and is the site of the final tests of the system. This chapter focuses on the Jacksonville Quarry while later chapters will discuss the two trips to Coddington Cove.

6.2 Jacksonville Quarry

The Jacksonville Quarry (Figure 6.1) is located in Jacksonville, Centre County, PA about 20 miles northeast of the Pennsylvania State University - University Park campus. The former limestone quarry is filled with water and contains an active ecosystem with typical fish, underwater plants, and other animals indigenous to the area. The Quarry is roughly rectangular in shape with sides 100 by 150 meters and a depth up
to 10 meters. Four trips were made out to the Quarry from October 2006 to May 2007. The purpose of these trips was to familiarize the research team, containing mechanical engineers and underwater acousticians, with the testing equipment and to experiment with different methods of calculating the optimum phasing for multiple sources.

6.2.1 Quarry Trip 1 - October 2006

Two Aquasonic (electromagnetic) sources and two hydrophones were used during the first trip in October 2006 to study the Quarry and its acoustic properties. The sources were placed about 20 feet apart along the shoreline (Figure 6.2). Pulse signals (periodic

Figure 6.1: View of Jacksonville Quarry. Base of operations is in upper left corner with RV.
temporally short high frequency signals alternated with a period of silence) were used to
determine the reverberation properties of the Quarry. It was determined that there were
minimal spurious reverberations from the Quarry bottom and sides, and thus the Quarry
would work well for experimental studies. The two sources were then driven in-phase
and 180 degrees out-of-phase leading to a 20 dB drop in pressure at 500 Hz. The sources
were also tested to see if there was a 6 dB increase (doubling) in pressure when the
second source was added to the first in-phase. At 500 Hz it worked well, but at 400 Hz
the second source needed to have its phase changed by about 109 degrees for the same
increase. This reasserted that the source phasing process would be highly frequency
dependent, although more testing needed to be done for confirmation.
The second Quarry trip was taken in November 2006 to test a new Lubell LL98 (piezoceramic) source and investigate the effects of an underwater source array. A linear array of four Lubell sources was built suspending the sources at 2 meters in depth and 1 meter spacing (Figure 6.3). Two hydrophones were placed along tie ropes about three-quarters of the way across the Quarry from the shore. The array equation from [29] was used to calculate the phases required to steer an ideal array. According to [29] the major lobe of a linear array with spacing \( d \) will point in a direction \( \theta_0 \) where
\[ \sin \theta_0 = \frac{c \tau}{d} \] 6.1

with \( \tau \) being the time delay between elements of the array and \( c \) the speed of sound in the medium. The sources were activated at 1 kHz and the calculated phasing was used to compare experimental results at the two hydrophones with theory (Figure 6.4). The experimental data agreed fairly well with theory for the given source/receiver orientation.
6.2.3 Quarry Trip 3 – December 2006

The third Quarry trip, taken in December 2006, was used to study the distance method for phasing three arbitrarily placed Lubell LL98 sources. This method involved calculating the difference in travel time from each source to a given receiver. This distance was then used to calculate a phase by calculating the ratio of the difference in distance compared to a wavelength. The distance method ensures that the acoustic waves
arrive at the receiver in phase when the sources and receivers are in an anechoic environment. This process is described in Chapter 4. The phases were also determined experimentally by manually determining the phase required for a maximum pressure at the receiver. The results are shown in Table 6-1. The agreement between the distance method and the manual phasing was poor. It was determined that when sources and receivers were nearer to reverberant surfaces than previously tested (e.g. the quarry walls Figure 6.5), the distance method would fail to produce accurate phasing, while the manual method should work no matter what the environment. As the distance between sources or receivers and reverberant surfaces decreases, the acoustic wave reflected from the reverberant surfaces becomes more comparable in magnitude, but drastically different in phase to the initial wave, leading to a decreased pressure amplitude. The distance method neglects these reflections and their effect, while the manual method ensures the reflections are taken into account. The change in location of the sources from one part of the quarry to another could also greatly affect the focusing if the bathymetry of the two locations is very different, e.g., shallow water versus deeper water.

Table 6-1: Phasing data for Quarry trip 3, Hydrophone 1.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Source</th>
<th>Manual Phase (rads)</th>
<th>Distance Method Phase (rads)</th>
<th>Difference (rads)</th>
</tr>
</thead>
<tbody>
<tr>
<td>250 Hz</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>4.84</td>
<td>5.64</td>
<td>-1.8</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>5.7</td>
<td>5.9</td>
<td>-0.2</td>
</tr>
<tr>
<td>500 Hz</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.51</td>
<td>5</td>
<td>1.79</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.84</td>
<td>5.51</td>
<td>1.61</td>
</tr>
<tr>
<td>1000 Hz</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>4.87</td>
<td>3.72</td>
<td>1.15</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1.38</td>
<td>4.75</td>
<td>2.91</td>
</tr>
</tbody>
</table>
6.2.4 Quarry Trip 4 – May 2007

In May 2007 the fourth Quarry trip was taken to test out the new phasing algorithm described in Chapter 5 using a five-source array containing Lubell LL98s, spaced 16 inches on center, as well as two J-9 underwater sources (Figure 6.6). To verify the method, the increase in pressure when each source was added with the calibrated phase was compared to ideal theory. Adding a second source should increase the original pressure by 6 dB, a second by 10 dB, etc. up until the addition of the fifth should be about
a 14 dB increase over the original signal. The results showed excellent agreement to theory as shown in Table 6-2. With a reliable method for calculating the phase for the Quarry, it was time to test out the algorithm and equipment in a harbor setting.

Figure 6.6: Deployment of five-source array (Lubell LL98).
6.3 Conclusion

The trips to the Jacksonville Quarry allowed for small-scale validation of two different phasing methods. These phasing methods were determined to be very frequency dependent and were affected by the proximity of reverberant walls. The trips also provided a testing ground for the user interface and source excitation programs created in LabVIEW. These tests demonstrated that the set up was capable of exciting sources and focusing acoustic energy by phasing the sources in preparation for a larger scale test in a harbor environment.

<table>
<thead>
<tr>
<th>Array</th>
<th>Source 1</th>
<th>Source 2</th>
<th>Source 3</th>
<th>Source 4</th>
<th>Source 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>250 Hz Phase</td>
<td>0</td>
<td>6.04</td>
<td>5.84</td>
<td>5.64</td>
<td>0.96</td>
</tr>
<tr>
<td>Cum.</td>
<td>0</td>
<td>6.3</td>
<td>9.6</td>
<td>12.4</td>
<td>14.9</td>
</tr>
<tr>
<td>J-9 Sources</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50 Hz Phase</td>
<td>0</td>
<td></td>
<td></td>
<td>3.14</td>
<td></td>
</tr>
<tr>
<td>Cum.</td>
<td>0</td>
<td></td>
<td></td>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>
Chapter 7

Coddington Cove – July 2007

7.1 Coddington Cove

Coddington Cove was chosen as the model harbor for the experiments due to its proximity to Naval Sea Systems Command/Naval Undersea Warfare Center (NAVSEA/NUWC) and its facilities, the availability of a semi-protected finger pier with power supplied to its end, and the permission to lay hydrophone cables across the Cove and leave them there for the duration of the experiments. The mean water depth in the harbor was 11 meters with surficial sediments characterized by silts, sandy silts, and clay. The trip to Newport, RI, took place on July 7-13, 2007. Coddington Cove is located on the west coast of Newport in Narragansett Bay (Figure 7.1). Two aircraft carriers, the U.S.S. Saratoga and the U.S.S. Forrestal, were docked alongside the test pier during the experiments. The base for operations was a cargo van with computers and amplifiers in it parked at the end of the pier (Figure 7.2).
Figure 7.1: Aerial view of Coddington Cove from www.mapquest.com. The main part of Newport, RI is to the south on the map.
7.2 Experimental Setup

To calibrate the harbor, i.e., interrogate the water with sound and determine the optimal phasing for a multi-source array, eleven hydrophone cables were laid on the harbor floor, radiating out from the tip of the pier suspended by subsurface buoys 2 meters from the bottom. A twelfth hydrophone was somewhat mobile and used in various places around the tip of the pier. Figure 7.3 shows the numbering scheme for the hydrophones as well as their layout; the distances from the source at the base of the pier
to each hydrophone are given in Table 7-1 and were calculated using the time of flight method mentioned in Chapter 4. A GPS was also used to get the latitude and longitude of each of the hydrophones. The sources were suspended by a crane at the tip of Pier 1. Tests were done with several different sources at a variety of frequencies. The sources that were used included a singular J-11 underwater source, an array of three J-11 underwater sources, a single HLF-1 source, a single J-15 source, and a single slotted-cylinder source, see Figure 7.4 (source specifications are in Appendix C). The phasing part of the study used the J-11 (single and array) source data, thus that data will be the focus here.
Figure 7.3: Map of Coddington Cove, Newport, RI. Hydrophone locations are numbered in blue italics, harbor depths in feet in black block numbers, and the sources were at the end of Pier 1 where the blue oval is.
Figure 7.4: Three of the four types of sources used at Coddington Cove in 2007.

Table 7-1: Hydrophone Distances from Source – data gathered acoustically

<table>
<thead>
<tr>
<th>Hydrophone</th>
<th>Distance from Source (m)</th>
<th>Angle with respect to array (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H3</td>
<td>75</td>
<td>90</td>
</tr>
<tr>
<td>H6</td>
<td>83</td>
<td>135</td>
</tr>
<tr>
<td>H9</td>
<td>126</td>
<td>45</td>
</tr>
<tr>
<td>H2</td>
<td>233</td>
<td>90</td>
</tr>
<tr>
<td>H11</td>
<td>243</td>
<td>180</td>
</tr>
<tr>
<td>H5</td>
<td>281</td>
<td>135</td>
</tr>
<tr>
<td>H4</td>
<td>381</td>
<td>135</td>
</tr>
<tr>
<td>H10</td>
<td>390</td>
<td>180</td>
</tr>
<tr>
<td>H7</td>
<td>424</td>
<td>45</td>
</tr>
<tr>
<td>H1</td>
<td>431</td>
<td>90</td>
</tr>
</tbody>
</table>
7.3 Testing Procedures and Results

A single J-11 source was turned on and the frequency response at each hydrophone was investigated as a baseline for transmission loss with the results shown in Figure 7.5. The frequency response was gathered by exciting the source at a single frequency while recording data on all hydrophones. The frequencies tested were 50 through 500 Hz in 50 Hz increments. The dips in pressure at 200 and 450 Hz could be due to a node near the source or receiver at those frequencies.

Figure 7.5: Frequency response at the hydrophones for a single J-11 source.
Next, the array with three JL11 sources was lowered into the water (Figure 7.6). The separation in the array was about 5.5 feet (1.7 m) and was suspended about 5 feet (1.5 m) from the bottom of the 37 feet (11.3 m) deep harbor. The harbor was then calibrated using the same algorithm as described in Chapter 4 at a few prescribed frequencies. The hydrophones were then moved to a shallower depth and the calibration done again. The goal was to see if there was a difference when calibrating at various depths or if it led to a similar phasing. As Figure 7.7 shows, the phase calibrated for different depths does not vary much at lower frequencies. Also, the pressure at each hydrophone when the array was focused on hydrophone 1 shows little difference between the original and shallower depths (Figure 7.8).

Figure 7.6: Array of 3 J-11 sources.
Figure 7.7: Phase calibrated for the first and third source in array (second source has zero phase). Dashed lines with solid shapes indicate original depth, solid lines with open shapes indicate new, shallower depth.
The pressure from a phased array was also compared to the theoretical pressure due to three in-phase sources using the single J-11 source data as a reference. Figure 7.9 shows the comparison between experimentally phased sources, theoretical ideal sources, and the three sources with the same, zero phase. The experimental phasing increases the pressure by anywhere from 5 to 35 dB depending on the location. Also, the pressure produced by the experimental phasing is very close to what theory would predict or even higher in some cases. The phases of each of the sources at certain frequencies on a given “radial” of hydrophones were investigated to see if there was any obvious trend. As evidenced in Figure 7.10, there does not seem to be significant correlation between the phase of the sources and the frequency of excitation, nor between sources on one side of

Figure 7.8: Sound pressure at original height vs. new height.
the array versus the other. It does show, however, that there seems to be less than one radian (60 degrees) difference in phase for hydrophones on the same “radial” at lower frequencies (50 and 100 Hz). Please note that this same “radial” is only assumed, and as observed during the experiment the hydrophones themselves were not in a perfectly straight “radial” from the source due to wind and currents.

Figure 7.9: Pressure comparison between three ideal in-phase collocated sources based on experimental measurements from one source (use experimental data from one source and add 10 dB to results), optimally phased sources (OPS), and three sources with zero phase. The categories are the hydrophone to which the beam is focused with the OPS method and the pressure is read along with the frequency of excitation.
There were several conclusions drawn from the Coddington Cove data. First, the phasing method will work in the harbor environment as desired, matching or exceeding what theory would predict for multiple sources in phase. Second, at the low frequencies investigated, the depth of the hydrophones does not play a large role in the phasing of the sources, and thus acknowledging that there is an obvious gradient of pressure in the vertical direction (zero pressure at the surface to some finite value at the floor), the harbor problem can reduce to two-dimensions when it comes to targeting a location. Third, it

Figure 7.10: Calibrated phase for first and third sources to focus on certain hydrophones along the same radials (7/9, 1/3, 4/6).

7.4 Conclusions

There were several conclusions drawn from the Coddington Cove data. First, the phasing method will work in the harbor environment as desired, matching or exceeding what theory would predict for multiple sources in phase. Second, at the low frequencies investigated, the depth of the hydrophones does not play a large role in the phasing of the sources, and thus acknowledging that there is an obvious gradient of pressure in the vertical direction (zero pressure at the surface to some finite value at the floor), the harbor problem can reduce to two-dimensions when it comes to targeting a location. Third, it
would be difficult to use experience alone to calculate the phases required in such a
complex environment, i.e., there is not an obvious correlation between the phases of each
source at various frequencies and hydrophones. It is possible that this lack of correlation
is due to the small size of the array with respect to the low frequency wavelength. This is
somewhat born out in the subsequent tests at Coddington Cove where a larger-sized array
was used. Finally, it was concluded that a new computational program to recall the
phases from the calibration when needed would be necessary to make this project
useable.

This new program, termed the “firing” code, would need to track a position and
when activated, turn on the sources to the required phases to focus energy on that
position. This code has been programmed in LabVIEW. The firing code will monitor
the incoming position data, eventually from some kind of sonar, and will determine the
sector surrounding the position. These sectors can be imagined as a wedge shape around
the calibration hydrophone location (Figure 7.11). The size of the sectors will depend on
the excitation frequency and the harbor bathymetry. A user will decide whether or not to
activate the sources as a deterrent by clicking the position on the screen. The phasing
will change automatically as the swimmer position changes from sector to sector. The
flow chart in Figure 7.12 illustrates the algorithm for the Firing code.
Figure 7.11: LabVIEW screenshot of Firing Program including sectors.
For the next set of experiments, it will be shown that it is necessary to take measurements throughout the day for several days to gain a better sample for transmission loss data. The limited data from this first trip could lead to faulty conclusions that are due to inconsistencies in the timing of the measurements in such a dynamic environment.

Figure 7.12: Flow chart for Firing Algorithm.
Chapter 8
Coddington Cove – May-June 2008

8.1 Experimental Setup

The second set of experiments was conducted off of the same finger pier in Coddington Cove from May 30 to June 6, 2008. A linear array of 7 acoustic sources was suspended 2 meters from the harbor bottom. The array consisted of four J15-3 sources and three HLF1 sources, all spaced evenly. The arrays were run separately: a four J15-3 source array and a three HLF1 source array. The acoustic field was measured by an array of 11 hydrophones oriented in a cross pattern in the harbor, see Figure 8.1. The hydrophones ranged from 100 meters to 325 meters from the center of the array. Figure 8.2 shows the new LabVIEW Graphical User Interface (GUI) used during this trip. There are lights that change color and blink as the pressure is increased at each hydrophone location and the pressure meters help to visualize the beam pattern.
Figure 8.1: Harbor Hydrophone Locations.
8.2 Testing Procedures

Both sets of sources were suspended from a crane on a pipe used to separate the sources by a specified distance and keep them in alignment (Figure 8.3). The sources were hung so that they alternated, thus creating a four source J15-3 array and a three source HLF1 array on the same pipe. The spacing for the array in these experiments was for 100 Hz ideal spacing which translates to half of an acoustic wavelength, 7.4 meters [29]. The source arrays were lowered into the water until they were about two meters
from the harbor bottom. The sources were numbered from left to right when looking
from the tip of the pier. HLF1 source #1 and source #2 had reference hydrophones
attached one meter from the face of the source as did J15-3 source #2. These were used
to get transmission loss measurements when one source was excited. Transmission loss
is defined as the pressure drop from 1 meter from a source to the point of interest in dB
[4]. The J15-3 sources were excited by equipment purchased by co-investigators from
PSU’s Applied Research Laboratory while the HLF1 sources were excited by equipment
provided by Naval Sea System Command/Naval Undersea Warfare Center
(NAVSEA/NUWC).

Figure 8.3: Crane lifting pipe with arrays of sources.

The procedure for phasing the sources had two parts: (1) harbor interrogation and
(2) swimmer demonstration. The harbor interrogation consisted of the calibration scheme
described in Chapter 5 and illustrated in Figure 5.3. Source #1 (the leftmost source) is
the reference source for each of the arrays unless otherwise noted. The swimmer
demonstration consisted of a computerized “swimmer” who traveled along the line of hydrophones at a prescribed speed. The LabVIEW program treated this virtual swimmer as a real threat: as it approached each hydrophone location, the program recalled the calibrated phases for that hydrophone location and excited the sources accordingly. The classic array theory, discussed in Chapter 4, was also used with the swimmer demonstration for comparison to the Optimal Phased Source (OPS) method.

8.3 Experimental Results

The experimental results are divided into two main sections by the two different source arrays, and then each array has results for 100 and 200 Hz. These results consist of two types: (1) a comparison of the OPS method phasing over several runs with the array theory phasing and (2) a comparison of the sound pressure level (SPL) due to phasing with the OPS method and the classic array theory. The results of the phasing are described in terms of phase, but they could be described in terms of time delay as well. There is a direct correlation between phase and time delay, however since we search for the phase when using the OPS method, all results will be in terms of phase.

8.3.1 HLF1 Source Array

8.3.1.1 100 Hz Testing

From the beginning of the trip until the morning of the last day the HLF sources were mistakenly switched, so that source #3 was excited when the program was intending
source #1 to be excited, thus the phasing values are different from those gathered later in
the day. To illustrate the change in phase over the course of a longer time period, this
earlier data has been included here. The phases calculated by the search algorithm were
saved and the phases for the second and third sources are shown in Figure 8.4 and
Figure 8.5, respectively. Although the data is not exhaustive, there is not a large change
in the phasing between the two days investigated, nor between different times of day, for
most of the hydrophone locations.

![Phasing for HLF1 Source #2: Optimal, 100 Hz, Setup #1](image-url)

Figure 8.4: Optimal phases for HLF1 Array, Second Source, 100 Hz, Setup #1
Once the order of the HLF1 sources was corrected, more data was taken and compared with classic array theory results. Figure 8.6 and Figure 8.7 show the phasing results for source #2 and source #3 using source #1 as a reference compared with the classic array theory as calculated by the LabVIEW program. The classic array theory ideally would be a straight line (assuming the hydrophones were separated by equal angles), but since the phases from the swimmer demonstration were used, there are slight discrepancies from a linear relationship. The phases are similar for the OPS method and classic array theory, although some hydrophones have higher differences, typically hydrophones 6 and 7. The phases agree within an average of 10% +/- 5% for source #2 and 12% +/- 7% for source #3 between the two methods.

Figure 8.5: Optimal phases for HLF1 Array, Third Source, 100 Hz.
Figure 8.6: HLF Source 2, OPS and array theory, 100 Hz.
After each interrogation, a demonstration was run to focus the acoustic energy using the optimal phases. The hydrophone pressures were recorded as described earlier. The sound pressure level (SPL) due to several runs of the HLF sources at 100 Hz is plotted in Figure 8.8. This data was taken over the course of one day. There is a shift in the location of the maximum pressure from hydrophone 3 to hydrophone 4 over the course of the day, but there is still a maximum of only 3 dB change in pressure between runs. The normal to the array, broadside, was directed towards hydrophone 3 and hence, there is a higher amplitude at hydrophones in that area compared with those which are closer to the sources. It becomes more difficult to transmit energy as the focusing angle

Figure 8.7: HLF Source 3 phasing, OPS and array theory, 100 Hz.
gets farther from broadside and closer to end-fire, due to increasing destructive interference and larger differences in distances from each of the sources to the receiver.

Figure 8.8: SPL when Array is focused on each hydrophone using optimal method at 100 Hz.

Figure 8.9 shows a comparison between the OPS method SPL and the classic array theory SPL. The results are within 3 dB of each other at each hydrophone location over the course of a day. This suggests that using either classic array theory or the OPS method under simple harbor conditions will lead to similar focusing results, both in phase and SPL.
8.3.1.2 200Hz Testing

The spacing of the sources was kept the same (7.4 meters) while the frequency was increased to 200 Hz and the calibration and phasing process was repeated. Figure 8.10 and Figure 8.11 show the phasing results from the OPS method and array theory. Again, the phases agree fairly well between the OPS method and the array theory, especially the slope. Figure 8.12 shows the SPL for the HLF array for the OPS method and array theory. Each run comparison between OPS and array theory is within 2 dB of each other, and the change in pressure over all the runs is 6 dB or less.
Figure 8.10: HLF Source #2 phases, OPS method and array theory, 200 Hz.
Figure 8.11: HLF Source #3 phases, OPS method and array theory, 200 Hz
Figure 8.12: Comparison of SPL at each hydrophone by focusing with classic array theory and OPS method – HLF sources at 200 Hz. Array theory has dashed lines.
8.3.2 J15-3 Source Array

8.3.2.1 100 Hz Testing

Figure 8.13, Figure 8.14, and Figure 8.15 show the phasing calculated with the OPS method and classic array theory at 100 Hz. There are some discrepancies around hydrophones 2, 5, and 6. These discrepancies lead to differences in pressure around those hydrophones as seen in Figure 8.16. Without more in-depth data it is difficult to define the source of these discrepancies. They could be due to malfunctioning equipment, environmental factors, or the location of the hydrophones at the edge of the calibrated area.
Figure 8.13: J15 Source #2 phases, OPS method and array theory, 100 Hz
Figure 8.14: J15 Source #3 phases, OPS method and array theory, 100 Hz
Figure 8.15: J15 Source #4 phases, OPS method and array theory, 100 Hz
Figure 8.16: Comparison of SPL at each hydrophone by focusing with classic array theory and OPS method – J15 sources at 100 Hz.
8.3.2.2 200 Hz Testing

Figure 8.17 - Figure 8.19 show the phasing results for the J15-3 sources at 200 Hz. These results have the most discrepancies across all hydrophones, although the general trends seem to agree with theory. The pressure results are shown in Figure 8.20 where the discrepancies appear at the same hydrophones.

Figure 8.17: J15 Source #2 phases, OPS method and array theory, 200 Hz.
Figure 8.18: J15 Source #3 phases, OPS method and array theory, 200 Hz
Figure 8.19: J15 Source #4 phases, OPS method and array theory, 200 Hz.
8.4 Conclusions

The optimal phase search (OPS) method can be used successfully to calculate phases for an array of underwater sources in a harbor environment that result in maximizing the SPL at a prescribed location. Although results are similar to classic array theory results except for the aforementioned discrepancies, the OPS method will take into account any relevant reflections and phase discrepancies between sources which are not addressed with classic array theory. The similarity of the two methods in Coddington Cove reassures the validity of the OPS method when it comes to a location that has very
few obstacles in the directions of propagation. Unlike array theory, the OPS method does not require a linear array, and the sources could be positioned as randomly as necessary for a given harbor area. The OPS method does require the \textit{a priori} interrogation of the harbor, but with the data collected it can be inferred that although the pressure magnitude may change, the phases required to create a maximum pressure remained fairly consistent over the five-day period of this investigation. More studies need to be performed to establish what effects seasonal change, tides, etc. have on the phase stability, but it may be necessary to re-interrogate the harbor only when there are major bathymetry changes such as the presence of a ship in the harbor or a new pier. Also, introducing both methods in an environment where there are reflecting boundaries in the directions of propagation is necessary to validate the accuracy and ability of the OPS method compared with the classic array theory.
Chapter 9
Harbor FEM Validation with Experimental Results

9.1 Introduction

Modeling Coddington Cove with the FEM program requires the definition of the harbor geometry and corresponding boundary conditions. The extent of the harbor needed for the modeling is highly dependent on the excitation frequency investigated and the computational capacity of the available computer. The boundary conditions, and how they are approximated, can vary greatly leading to a wide range of possible solutions. Both of these factors have evolved over the course of this dissertation to create the final model. The last section of this chapter projects the use of this model as a tool in real world situations.

9.2 Model Changes

Initial modeling of the harbor took bathymetry data from oceanographic resources and encompassed the area enclosed by the breakwater to the north and the coast to the south, see Figure 9.1. For very low frequencies (less than 50 Hz) the number of elements was reasonable and the FEM code could run in two hours or less on a high performance computer. When the excitation frequency shifted up to 100 – 200 Hz, the computer system did not have enough memory to solve the system of equations (as the number of
nodes increases, the size of the matrices increases and it requires more memory to store the data as well as inverting the matrices to solve the system of equations) and thus a smaller model was created, containing the pier and just the water in the area of interest (see Figure 9.2). Table 9-1 shows some general computation times for a few different model sizes. At these higher frequencies, there was an insufficient number of elements to model the reflections of the pier, due to the stair-stepping effect of the voxelization. The pier was then shifted so that instead of being on an angle to the orthogonal directions, it was placed along one of the directions, leading to a model that had no aberrant reflections near the pier (see Figure 9.3). Although this model could be used with multiple sources to analyze the focusing capability of the harbor, it was not modeling the bottom impedance well, so a simpler waveguide form of the harbor was modeled, excluding the pier (see Figure 9.4). After the bottom impedance was correctly applied, the fix could be propagated backwards to a more complex model as needed.

Table 9-1: Some general model sizes and computation times using this FEM code.

<table>
<thead>
<tr>
<th>Number of Nodes</th>
<th>Number of Elements</th>
<th>Computation Time</th>
<th>Type of Computer System</th>
</tr>
</thead>
<tbody>
<tr>
<td>150,000</td>
<td>130,000</td>
<td>1.5 Hours</td>
<td>High Performance 2.6 GHz, 8-16 GB RAM</td>
</tr>
<tr>
<td>20,000</td>
<td>19,000</td>
<td>10 Minutes</td>
<td>High Performance 2.6 GHz, 8-16 GB RAM</td>
</tr>
<tr>
<td>13,000</td>
<td>10,000</td>
<td>3 Minutes</td>
<td>Personal 2.8 GHz, 2 GB RAM</td>
</tr>
</tbody>
</table>
Figure 9.1: Preliminary model of Coddington Cove, view from the floor up, North is to the right, West is to the bottom. This figure is to illustrate the capability of the program, not to infer any results.
Figure 9.2: Second iteration of harbor model. Two different source locations near tip. View from the floor up, North is to the right, West is to the bottom.

Figure 9.3: Third iteration of harbor model. Source array focused on hydrophone 3. Transmission Loss is shown in contours. View from the floor up, West is to the left, North is to the bottom. Hydrophones are indicated by the red dots.
9.3 Difficulties with Boundary Conditions

While the scope of the geometrical model is an issue of computer capability, the difficulties with defining boundary conditions are based in interpreting (1) how the FEM code treats the boundary condition and (2) how the properties of the boundary condition are defined. To get a basis for the bottom impedance of the harbor, experimental data from the first Coddington Cove trip was investigated and a numerical model developed consisting of several layers to represent the harbor floor. With this model, a reflection coefficient with respect to grazing angle was calculated that should approximate the actual reflection coefficient of the harbor floor, see Figure 9.5.
Knowing from [4] that the reflection coefficient for a fluid-fluid interface is related to the impedance of each fluid through Eq. 9.1

\[ R = \frac{Z_2 - Z_1}{Z_2 + Z_1} \quad 9.1 \]

where \( Z_2 \) is the impedance for the bottom and \( Z_1 \) is the impedance for the water, along with the definition of \( \beta \), Eq. 9.2 can be derived.

\[ \beta = \frac{1 + R}{1 - R} \quad 9.2 \]

Figure 9.5: Numerical approximation of Coddington Cove Reflection Coefficient at 100 Hz.
Figure 9.6 shows $\beta$ as a function of grazing angle, and then assuming a source depth 2 meters from the harbor floor, Figure 9.7 shows $\beta$ as a function of horizontal distance from the source. It is important to remember that as grazing angle increases the distance from the source is decreased, i.e., a grazing angle of 0 degrees corresponds to an infinite distance away. Note that after about 5 meters from the source, the $\beta$ values become fully imaginary with a positive slope of about 1/6. This agrees with how the FEM is programmed: a large $\beta$ corresponds to a pressure release condition where $R = -1$.

![Figure 9.6: Bottom impedance $\beta$, as a function of grazing angle at 100 Hz.](image-url)
9.4 Revised Boundary Conditions

Two realizations occurred that greatly affected the FEM model. First, a single value of $\beta$ could not be used to represent the boundary as is apparent from Figure 9.7. Secondly, the grazing angle of the incoming direct sound wave had not been taken into account when defining $\beta$ with the normal impedances. When a constant value for $\beta$ (the normal incident value) was placed in the FEM program, the results did not show the typical cylindrical spreading, but a linear reduction in SPL as the distance from the
source was increased. When a sloped value of $\beta$ was used, the results were more realistic as seen in Figure 9.8. Values for $\beta$ were defined in three sections in the smaller model: 0-50 m, 50-100m, and 100-200m. These results were promising, but they still seemed to have a transmission loss that was on the higher end of theory. Note that transmission loss is the difference between the pressure at 1 meter away from a simple source in free space and the pressure at the desired location in the field.

Figure 9.8: Comparison of FEM with a sloped $\beta$ boundary condition compared to spherical, cylindrical, and an intermediate spreading at 100 Hz.

The implementation of $\beta$ in the FEM code was then investigated leading to the conclusion that the incident angle of the sound is not taken into account directly. Kinsler, et al. [29] states that fluid impedance is equal to the characteristic impedance ($\rho c$) divided
by the sine of the grazing angle of the incident wave. This negligence was remedied by multiplying the $\beta$ calculated from the reflection coefficient plots by a weighting factor equivalent to the sine of the grazing angle. Therefore the $\beta$ actually used in the FEM is shown in Eq. 9.3

$$\beta' \equiv \frac{\rho_1 c_1}{Z_2} = \frac{\rho_1 c_1}{1 + R} = \frac{1 - R}{1 + R} \sin \theta_g = \beta \sin \theta_g \tag{9.3}$$

where $\beta$ is defined in Eq. 9.2 and $\theta_g$ is the grazing angle of the direct path from source to bottom location. The FEM code was run using the normal incident $\beta$ values as well as the sloped $\beta$ values. The FEM results agree much better with theoretical cylindrical spreading in both cases as shown in Figure 9.9.
9.5 Comparison with Experimental Results

In the Coddington Cove experiments, sound transmission loss was calculated from the data at several frequencies at each hydrophone location. The distance of each of the hydrophones from the source was used as the horizontal gauge and the results were compared to the FEM model in Figure 9.10. As shown in the figure, the transmission
loss calculated by the FEM program may run a bit high of the average as the distance from the source decreases, although overall the values agree well.

Figure 9.10: Comparison of FEM with some experimental TL data from 6/5/08 (blue squares). Single HLF1 Source at 100 Hz.

9.6 Adaptability and the Inverse Problem

In the case of Coddington Cove, it is evident that the experimental data obtained could be used to approximate a numerical bottom reflection coefficient. However, this data may not be available in all harbors. An inverse problem could be formulated to
achieve this in the following manner. Because the direct-path angle of incidence can be calculated at each voxel position and because the corresponding impedance at each voxel is an angular-dependent impedance value multiplied by the sine of the grazing angle, the FEM program could be used adaptively to solve the inverse problem to determine each portion (at selected voxel locations) of the reflection coefficient. Transmission loss measurements would need to be taken at various distances from the source and an optimization scheme could be used to find the $\beta$ values which best match experimental data. A portion of the reflection coefficient that is closer to zero grazing angle could then be backed out using Eq. 9.2. The portion near zero grazing is the only part that can be calculated because the $\beta$ values have been approximated with the sloped values corresponding to small grazing angles and large distances.

Another method for calculating the normal incidence reflection coefficient would be to use a singular value of $\beta$ with the $\sin(\theta_g)$ multipliers from Eq. 9.3 as shown in Figure 9.9. An optimization scheme could be written to change the base value of the unknown $\beta$ while keeping the multipliers the same, since the grazing angle is dependent only on the physical model. The plot of transmission loss will increase or decrease with $\beta$, thus allowing the adaptive program to match experimental transmission loss data resulting in the normal incidence reflection coefficient.

Although these are options, they are beyond the scope of this dissertation. The concepts are suggested as an opportunity for future research along with the myriad of other inverse methods that can be found in the literature. Since the main concept of this dissertation is focusing energy and not determining the reflection coefficients of the sea floor, that exercise has been left for future work.
Chapter 10

Summary, Conclusions, and Future Work

10.1 Overall Thesis Objectives

There were three main objectives of the work contributing to this thesis: (1) develop a method that will focus low frequency acoustic energy in harbor areas taking into account both natural and manmade environmental changes (such as tides, temperature, season, presence of ships, etc.), (2) develop a Graphical User Interface that will implement the focusing method, (3) develop a method for predicting the acoustic field to better understand the focusing method and to assist in decisions about low frequency harbor acoustics. This concluding chapter discusses each of the objectives and suggests future work using the methods developed here.

10.2 Focusing Method

A new method has been developed to focus low frequency sound in harbor environments using an array of sources. This method, called the Optimal Phase Search (OPS) method, calculates the optimal excitation phases for each source in an array, first by searching available phases experimentally then selecting those that provide constructive interference of all the sources at a given hydrophone location. The OPS method compares favorably with classic array theory methods, but has several advantages. First, the OPS method does not require the sources to be in a linear array nor
to have the same transfer functions. Second, the OPS method takes into account all reflections from bounding surfaces, whereas classic array theory is meant for open spaces. Third, array theory does not take into account changes in sound speeds that occur due to changes of a harbor’s environmental conditions, while the OPS method does. However, the OPS method does require *a priori* acoustic knowledge to calibrate the harbor and the variability of that calibration process with environmental changes remains an unknown parameter.

In the study described herein, the comparison of experimental data using both the OPS method and classic array theory has led to similar results for both the phasing requirements and sound pressure level at field hydrophones in a harbor. Using an array of hydrophones during both of these experiments, it was observed that a definite acoustic beam or lobe was formed and steered. The requirement to have a noticeable acoustic beam would be to have the sources spaced about half an excitation frequency wavelength or more apart. In shallower water, the depth of a hydrophone did not alter the phasing much, thus a two-dimensional focusing system is all that is required.

### 10.3 Graphical User Interface

LabVIEW was used to create a Graphical User Interface (GUI) that would enable a user to implement both the harbor calibration and excite the sources with the required phases to energize a given location. Although it takes time, there is very little human interaction required to calibrate the harbor. Also, with the speed and capabilities of current computers, any phase change can be nearly instantaneous, thus the speed of any
malicious swimmer is inconsequential. The user is required to set the desired source level, and this may require some trial and error for any physical setup, but the end result should be satisfactory. This GUI was demonstrated for the sponsors from ONR and met their requirements in illustrating the OPS method.

10.4 Acoustic Finite Element Method

One of the largest parts of this dissertation has been the challenge of designing an acoustic FEM code from scratch. Developing such an open source code allows for a much greater understanding of the science and math involved in FEM than when numbers are plugged into a “black box” type commercial program. This is especially useful when doing parametric studies and if one of the end goals is optimization. The FEM code was validated with numerous acoustic spaces showing very good agreement with theory.

The purpose of the FEM code was to (1) assist in the placement of sources through an optimization scheme, (2) allow predictions of low frequency sound pressure levels for beam steering in a harbor environment, and (3) be used to adaptively model the harbor environment so that recalculation of phases would not require additional testing. Optimizing source placement via a numerical optimization, although possible with the code, was unrealistic for this harbor. The sources were placed in a given array, and there was no opportunity to move them around, thus that portion of the FEM was left for a future project.
The predictions calculated by the FEM code match very well with experimental data gathered from Coddington Cove. These predictions use a numerical approximation of the reflection coefficient to model the boundary impedance, taking into account the change in the grazing angle as the distance from the source increases. Even though this was shown in a simple harbor model, there should be no difficulties in expanding to a more complex harbor model, assuming the sources are spatially near one another and the computer capability requirements are met.

The adaptability part of this FEM code may be one of the most useful, but more testing is required to make a stronger claim. Using a calculated reflection coefficient allowed for an FEM solution that compared well with experimental data. However, experimental data, i.e., transmission loss, could be used to adapt a value of $\beta$ until the model agrees with the experiment. Thus, without having a reflection coefficient, a model could be made and used to predict transmission loss in the harbor. The two limits of the reflection coefficient could be calculated using the two different adaptive methods discussed in Chapter 9.

10.5 Future Work

Although this dissertation has shown how well the OPS method worked in Coddington Cove, it was not a very complex harbor. There was little ship traffic in the area of interest, and therefore the method was not taxed as much as is possible. The ability of the OPS method should be tested in a more complex environment and compared with array theory to discover any shortcomings with either method.
The sources used in the experiments were limited to a linear array. The ability of the OPS method to work with an arbitrary source placement should be explored. This could be very useful depending on the harbor bathymetry. It is also something that would be impossible with classic array theory. The FEM code could be used to optimize placement of sources by running numerous scenarios of the harbor calibration for different possible source positions. This would be useful prior to any testing to discover how well an installed system could work.

Finally, the adaptive part of the FEM code should be explored more intensely. Different examples of bottom impedances should be used and experimental data compared with FEM results. Then, using an adaptive routine, the FEM code could calculate the reflection coefficient from the experimental data. The accuracy of this should be investigated, but if it proves correct, this method could help characterize a harbor with a few hydrophones rather than a large array.
Bibliography


Appendix A

Matlab Code for Acoustic FEM Program

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%
% Usage: writeunstrVTK(coord,connectivity,pres,vtkfile)
% coord: 3D coordinates of nodes
% connectivity: connectivity matrix
% pres: 3D field data
% vtkfile: filename to be stored.
% notes: Only writes ASCII UNSTRUCTURED_POINTS
%
% Deepak Trivedi, 2007
%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

function [] = writeunstrVTK(coord,connectivity,pres,vtkfile)

fid = fopen(vtkfile,'w');
celldat = [9*ones(size(connectivity(:,1))),connectivity(:,1:2), connectivity(:,4)
connectivity(:,3) connectivity(:,5:6)  connectivity(:,8) connectivity(:,7)]';
sz1 = size(coord);
sz2 = size(connectivity);
numNodes = sz1(1);
numElem = sz2(1);
celltype = 11*ones(numElem,1);

% write header
fprintf(fid, '%s
', '# vtk DataFile Version 2.0');
fprintf(fid, '%s
', 'created by writeunstrVTK (Matlab implementation by Deepak
Trivedi)');
fprintf(fid, '%s
', 'ASCII');
fprintf(fid, '%s
', 'DATASET UNSTRUCTURED_GRID');
fprintf(fid, '%s%d%s
', 'POINTS ', numNodes, ' float');
fprintf(fid, '%se %e %e
', coord');
fprintf(fid, '%s%d%d%d%d%d%d%d%d
', 'CELLS ', numElem, ' ', numElem+prod(size(connectivity)));
fprintf(fid, '%sd %d %d %d %d %d %d %d
', celldat');

fprintf(fid, '%s%d
', 'CELL_TYPES ', numElem);
fprintf(fid, '%d
', celltype);
fprintf(fid, '\%s\%d\n', 'POINT_DATA ', numNodes);
fprintf(fid, '\%s\n', 'SCALARS pressure float 1 ');
fprintf(fid, '\%s\n', 'LOOKUP_TABLE default');
fprintf(fid,"%d\n",pres(:,1));
fclose(fid);

Bathymetryconnect.m
% This code creates the coordinate and connectivity matrices from a % .voxels file

clear all
[header, voxels] = load_voxel_mesh('filename.voxels');
h = 0.025;
cnt = 0;

for i = 1:length(voxels.i)
    E(voxels.i(i)+1, voxels.j(i)+1, voxels.k(i)+1) = 1;
end

sz = size(E);
for i = 1:size(E,1)
    for j = 1:size(E,2)
        for k = 1:size(E,3)
            if E(i, j, k)
                cnt = cnt + 1;
                E(i, j, k) = cnt;
            end
        end
    end
end

element = 0;
nodeID = 0;
connectivity = zeros(length(voxels.i),8);
for i = 1:(max(voxels.i) + 1)
    for j = 1:(max(voxels.j) + 1)
        for k = 1:(max(voxels.k) + 1)
            if ((i==2) & (j==4) & (k==1))
                tofu = 4;
            end
            if E(i, j, k)
element = element + 1;;
elem(i,j,k) = element;

if ((i>1)&&(j>1)&&(k>1)&&(E(iL1,jL1,kL1))
    connectivity(element,1) = connectivity(elem(iL1,jL1,kL1),7);
end

if ((i>1)&&(j>0)&&(k>1)&&(j<(max(voxels.j) + 1))&&(E(iL1,j+1,kL1))
    connectivity(element,1) = connectivity(elem(iL1,j+1,kL1),8);
end

if ((i>1)&&(j>0)&&(k>0)&&(j<(max(voxels.j) + 1))&&(E(iL1,j+1,k))
    connectivity(element,2) = connectivity(elem(iL1,j+1,k),5);
    connectivity(element,3) = connectivity(elem(iL1,j+1,k),8);
end

if ((i>1)&&(k>1)&&(E(iL1,j,kL1))
    connectivity(element,2) = connectivity(elem(iL1,j,kL1),7);
    connectivity(element,1) = connectivity(elem(iL1,j,kL1),8);
end

if ((i>1)&&(E(iL1,j,k)))
    connectivity(element,1) = connectivity(elem(iL1,j,k),5);
    connectivity(element,2) = connectivity(elem(iL1,j,k),6);
    connectivity(element,3) = connectivity(elem(iL1,j,k),7);
    connectivity(element,4) = connectivity(elem(iL1,j,k),8);
end

if ((i>1)&&(j>1)&&(E(iL1,jL1,k))
    connectivity(element,1) = connectivity(elem(iL1,jL1,k),6);
    connectivity(element,4) = connectivity(elem(iL1,jL1,k),7);
end

if ((j>1)&&(k>1)&&(E(i,jL1,kL1))
    connectivity(element,1) = connectivity(elem(i,jL1,kL1),3);
    connectivity(element,5) = connectivity(elem(i,jL1,kL1),7);
end

if ((k>1)&&(E(i,j,kL1))
    connectivity(element,2) = connectivity(elem(i,j,kL1),3);
    connectivity(element,1) = connectivity(elem(i,j,kL1),4);
    connectivity(element,6) = connectivity(elem(i,j,kL1),7);
    connectivity(element,5) = connectivity(elem(i,j,kL1),8);
end
if \((j>1)\&\&(E(i,j-1,k))\)

connectivity(element,1) = connectivity(elem(i,j-1,k),2);
connectivity(element,4) = connectivity(elem(i,j-1,k),3);
connectivity(element,5) = connectivity(elem(i,j-1,k),6);
connectivity(element,8) = connectivity(elem(i,j-1,k),7);
end

if \(\neg\)connectivity(element,1))

nodeID = nodeID + 1;
connectivity(element,1) = nodeID;
node(nodeID).x = 2*h*(iL1) - h;
node(nodeID).y = 2*h*(jL1) - h;
node(nodeID).z = 2*h*(kL1) - h;
end

if \(\neg\)connectivity(element,2))

nodeID = nodeID + 1;
connectivity(element,2) = nodeID;
node(nodeID).x = 2*h*(iL1) - h;
node(nodeID).y = 2*h*(jL1) + h;
node(nodeID).z = 2*h*(kL1) - h;
end

if \(\neg\)connectivity(element,3))

nodeID = nodeID + 1;
connectivity(element,3) = nodeID;
node(nodeID).x = 2*h*(iL1) - h;
node(nodeID).y = 2*h*(jL1) + h;
node(nodeID).z = 2*h*(kL1) + h;
end

if \(\neg\)connectivity(element,4))

nodeID = nodeID + 1;
connectivity(element,4) = nodeID;
node(nodeID).x = 2*h*(iL1) - h;
node(nodeID).y = 2*h*(jL1) - h;
node(nodeID).z = 2*h*(kL1) + h;
end

if \(\neg\)connectivity(element,5))

nodeID = nodeID + 1;
connectivity(element,5) = nodeID;
node(nodeID).x = 2*h*(iL1) + h;
node(nodeID).y = 2*h*(jL1) - h;
node(nodeID).z = 2*h*(kL1) - h;
if (~connectivity(element,6))
    nodeID = nodeID + 1;
    connectivity(element,6) = nodeID;
    node(nodeID).x = 2*h*(iL1) + h;
    node(nodeID).y = 2*h*(jL1) + h;
    node(nodeID).z = 2*h*(kL1) + h;
end

if (~connectivity(element,7))
    nodeID = nodeID + 1;
    connectivity(element,7) = nodeID;
    node(nodeID).x = 2*h*(iL1) + h;
    node(nodeID).y = 2*h*(jL1) + h;
    node(nodeID).z = 2*h*(kL1) + h;
end

if (~connectivity(element,8))
    nodeID = nodeID + 1;
    connectivity(element,8) = nodeID;
    node(nodeID).x = 2*h*(iL1) + h;
    node(nodeID).y = 2*h*(jL1) - h;
    node(nodeID).z = 2*h*(kL1) + h;
end

for ii = 1:nodeID
    coord(ii,:) = [node(ii).x, node(ii).y, node(ii).z];
end
vtkfile = 'filename.vtk'
writeunstrVTK(coord,connectivity,coord(:,1),vtkfile)
save filename_data

% 3D Finite Element Acoustic Using Digitized Domains
%%%% Center for Acoustics and Vibration, Pennsylvania State University
clear all
close all
clc

% INITIALIZE
load filename_data %load .mat file containing geometry data
fname='filename_100Hz.vtk'; %filename that chosen results will be saved into

rho=1000; % density of fluid (kg/m^3)
f=100; % Excitation Freq (Hertz)
% pref=20e-6; % Ref press for air (20 uPa)
pref=1e-6; % Ref press for water (1 uPa)
omega=2*pi*f; % Excitation Freq (rad/s)
i=sqrt(-1); % Imaginary number
c1 = 1500.; % (m/s) speed of sound in fluid
c2 = 1800.;
a = 0.5; % (m) half of the edge length of each hexahedral element

%% defines the size of your overall mesh

% FE-ANALYSIS
%% Note that c can be varied if needed, that is one of the next steps in
%% coding. - ATK
%% ELEMENT STIFFNESS MATRIX

k=[(2*a)/3 -a/6];

KE = [ k(1) 0 k(2) 0 0 k(2) k(2) k(2)
      0 k(1) 0 k(2) k(2) 0 k(2) k(2)
      k(2) 0 k(1) 0 k(2) k(2) 0 k(2)
      0 k(2) 0 k(1) k(2) k(2) k(2) 0
      0 k(2) k(2) k(2) k(1) 0 k(2) 0
      k(2) 0 k(2) k(2) 0 k(1) 0 k(2)
      k(2) k(2) 0 k(2) k(2) 0 k(1) 0
      k(2) k(2) 0 0 k(2) k(2) k(2) k(1)];

%% ELEMENT MASS

m1=(a^3/c1^2)*[(8/27) (4/27) (2/27) (1/27)];
ME1 = [ m1(1) m1(2) m1(3) m1(2) m1(3) m1(4) m1(3) m1(2) m1(1) m1(2) m1(3) m1(4) m1(3) m1(2) m1(1) m1(2) m1(3) m1(4) m1(3) m1(2) m1(1) m1(2) m1(3) m1(4) m1(3) m1(2) m1(1) m1(2) m1(3) m1(4) m1(3) m1(2) m1(1) m1(2) m1(3) m1(4) m1(3) m1(2) m1(1) m1(2) m1(3) m1(4) m1(3) m1(2) m1(1) m1(2) m1(3) m1(4) m1(3) m1(2) m1(1) m1(2) m1(3) m1(4) m1(3) m1(2) m1(1) m1(2) m1(3) m1(4) m1(3) m1(2) m1(1) ];

% m2=(a^3/c2^2)*[(8/27) (4/27) (2/27) (1/27)];
% ME2 = [ m2(1) m2(2) m2(3) m2(2) m2(2) m2(3) m2(4) m2(3)
%        m2(2) m2(1) m2(2) m2(3) m2(3) m2(2) m2(3) m2(4)
%        m2(3) m2(2) m2(1) m2(2) m2(4) m2(3) m2(2) m2(3)
%        m2(2) m2(3) m2(2) m2(1) m2(4) m2(3) m2(2) m2(3)
%        m2(3) m2(2) m2(3) m2(4) m2(2) m2(1) m2(2) m2(3)
%        m2(4) m2(3) m2(2) m2(3) m2(3) m2(2) m2(1) m2(2)
%        m2(3) m2(3) m2(3) m2(2) m2(3) m2(2) m2(1) ];

%%%%%% ELEMENT DAMPING
MATRIX% Note for each Beta value you need a different CE matrix.
%%%% beta=rho*c/Zb where Zb is the complex impedance of the boundary
% beta = 1 is rho*c condition, beta=0 is rigid bc, beta=large is
% pressure release. Standard boundary condition with no beta is rigid

beta=1;
K=omega/c1;
d=((-a^2*beta)/c1)*[(4/9) (2/9) (1/9)];

CE = [d(1) d(2) d(2) d(3)
    d(2) d(1) d(3) d(2)
    d(2) d(3) d(1) d(2)
    d(3) d(2) d(2) d(1)];

% beta1=0.1+0.003*0.4;%0.42;

d1=((-a^2*beta1)/c1)*[(4/9) (2/9) (1/9)];

CE1 = [d1(1) d1(2) d1(2) d1(3)
    d1(2) d1(1) d1(3) d1(2)
\[
\begin{align*}
\text{d1(2)} & \text{ d1(3)} \text{ d1(1)} \text{ d1(2)} \\
\text{d1(3)} & \text{ d1(2)} \text{ d1(2)} \text{ d1(1)}]; \\
\beta_2 & = 0.1 + 0.015 \times 23i; \%0.42; \\
\text{d2} & = (a^2 \beta_2/c1) * [(4/9) (2/9) (1/9)]; \\
\text{CE2} & = [d2(1) \text{ d2(2)} \text{ d2(2)} \text{ d2(3)} \\
& \text{ d2(2)} \text{ d2(1)} \text{ d2(3)} \text{ d2(2)} \\
& \text{ d2(2)} \text{ d2(3)} \text{ d2(1)} \text{ d2(2)} \\
& \text{ d2(3)} \text{ d2(2)} \text{ d2(2)} \text{ d2(1)}]; \\
\beta_3 & = 0.1 + 0.04 \times 11.5i; \%(0.1+0.015)*0.4; \\
\text{d3} & = (a^2 \beta_3/c1) * [(4/9) (2/9) (1/9)]; \\
\text{CE3} & = [d3(1) \text{ d3(2)} \text{ d3(2)} \text{ d3(3)} \\
& \text{ d3(2)} \text{ d3(1)} \text{ d3(3)} \text{ d3(2)} \\
& \text{ d3(2)} \text{ d3(3)} \text{ d3(1)} \text{ d3(2)} \\
& \text{ d3(3)} \text{ d3(2)} \text{ d3(2)} \text{ d3(1)}]; \\
\beta_4 & = 0.1 + 0.1 \times 3.85i; \%(0.2+0.04)*0.4; \\
\text{d4} & = (a^2 \beta_4/c1) * [(4/9) (2/9) (1/9)]; \\
\text{CE4} & = [d4(1) \text{ d4(2)} \text{ d4(2)} \text{ d4(3)} \\
& \text{ d4(2)} \text{ d4(1)} \text{ d4(3)} \text{ d4(2)} \\
& \text{ d4(2)} \text{ d4(3)} \text{ d4(1)} \text{ d4(2)} \\
& \text{ d4(3)} \text{ d4(2)} \text{ d4(2)} \text{ d4(1)}]; \\
\beta_5 & = 0.1 \times (0.5)\%(0.6+0.1)*(0.4); \\
\text{d5} & = (a^2 \beta_5/c1) * [(4/9) (2/9) (1/9)]; \\
\text{CE5} & = [d5(1) \text{ d5(2)} \text{ d5(2)} \text{ d5(3)} \\
& \text{ d5(2)} \text{ d5(1)} \text{ d5(3)} \text{ d5(2)} \\
& \text{ d5(2)} \text{ d5(3)} \text{ d5(1)} \text{ d5(2)} \\
& \text{ d5(3)} \text{ d5(2)} \text{ d5(2)} \text{ d5(1)}]; \\
\end{align*}
\]

%% MASS AND STIFFNESS ASSEMBLY
%% nodeID comes from the connectcoord code - it ends up being the total
%% number of nodes. element is total number of elements.
K = sparse(nodeID, nodeID); % global stiffness matrix
M = sparse(nodeID, nodeID); % global mass matrix
C= sparse(nodeID, nodeID); % global damping matrix
Y= sparse(nodeID, nodeID); % global mobility matrix
F = sparse(nodeID, 1); % global force vector
P = zeros(nodeID, 1); % global pressure vector
damping3 = sparse(nodeID, 1);
damping4 = sparse(nodeID, 1);
damping5 = sparse(nodeID, 1);

for ii=1:element
    % edof = element degrees of freedom, taken from the connectivity matrix
    edof=[connectivity(ii,1);connectivity(ii,2);connectivity(ii,3);connectivity(ii,4);
          connectivity(ii,5);connectivity(ii,6);connectivity(ii,7);connectivity(ii,8)];
    K(edof, edof) = K(edof, edof) + KE;
    % if coord(connectivity(ii,1),2)<0.225
    %     M(edof, edof) = M(edof, edof) + ME2;
    % else
    %     M(edof, edof) = M(edof, edof) + ME1;
    % end
end

%%%%% This first section is for flat boundaries on the positive or negative %
%%%%% Cartesian faces as listed. Uncomment the sections where you want to %
%%%%% apply the boundary condition.

%%%%% Positive Z face
%%%%% Zposnodes finds all nodes on the positive Z face
Zposnodes = find(coord(:,3) == max(coord(:,3)));
Zposelems = []; % Initializes positive Z boundary element matrix

%%%%% Find all element numbers associated with pos Z face
for ii = 1:length(Zposnodes)
    [row, col] = find(connectivity == Zposnodes(ii));
    nodules=ones(length(row),1)*Zposnodes(ii);
    Zposelems = [Zposelems; [row, col, nodules]];
end
Reorganizes the nodes according to element and adds values to the damping matrix, $C$ according to the CE listed

```matlab
for ii=min(Zposelems(:,1)):max(Zposelems(:,1))
    if find(Zposelems(:,1)==ii)>0
        [row,col] = find(Zposelems(:,1)==ii);
        A(1,:)=Zposelems(row(1),2:3);
        A(2,:)=Zposelems(row(2),2:3);
        A(3,:)=Zposelems(row(3),2:3);
        A(4,:)=Zposelems(row(4),2:3);
        AA=sortrows(A,1);
        AAA=AA(:,2)';
        edof= AAA;
        C(edof,edof) = C(edof,edof) + CE;
    end
end
```

%% Negative Z face
%% Znegnodes finds all nodes on the negative Z face
%% Znegnodes = find(coord(:,3) == min(coord(:,3)));
%% Znegelems = [];

%% Find all element numbers associated with neg Z face
%% for ii = 1:length(Znegnodes)
%%     [row,col] = find(connectivity == Znegnodes(ii));
%%     nodules=ones(length(row),1)*Znegnodes(ii);
%%     Znegelems = [Znegelems; [row,col,nodules]];
%% end
%
%
%% Reorganizes the nodes according to element and adds values to the damping matrix, $C$ according to the CE listed
%% for ii=min(Znegelems(:,1)):max(Znegelems(:,1))
%%     if find(Znegelems(:,1)==ii)>0
%%         [row,col] = find(Znegelems(:,1)==ii);
%%         if length(row)==4
%%             A(1,:)=Znegelems(row(1),2:3);
%%             A(2,:)=Znegelems(row(2),2:3);
%%             A(3,:)=Znegelems(row(3),2:3);
%%             A(4,:)=Znegelems(row(4),2:3);
%%             AA=sortrows(A,1);
%%             AAA=AA(:,2)';
%%             edof= AAA;
%%             C(edof,edof) = C(edof,edof) + CE;
%%         end
```
% end
% end
% % %
% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% %
%%% Positive x face
%%% Xposnodes finds all nodes on the positive X face
Xposnodes = find(coord(:,1) == max(coord(:,1)));
Xposelems = [];

for ii = 1:length(Xposnodes)
    [row,col] = find(connectivity == Xposnodes(ii));
    nodules=ones(length(row),1)*Xposnodes(ii);
    Xposelems = [Xposelems; [row,col,nodules]];
end

for ii=min(Xposelems(:,1)):max(Xposelems(:,1))
    if find(Xposelems(:,1)==ii)>0
        [row,col] = find(Xposelems(:,1)==ii);
        if length(row)==4
            A(1,:)=Xposelems(row(1),2:3);
            A(2,:)=Xposelems(row(2),2:3);
            A(3,:)=Xposelems(row(3),2:3);
            A(4,:)=Xposelems(row(4),2:3);
            AA=sortrows(A,1);
            AAA=AA(:,2)';
            edof= AAA;
            C(edof,edof) = C(edof,edof) + CE;
        end
    end
end
% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% Negative X face
%%% Xnegnodes finds all nodes on the negative X face
% Xnegnodes = find(coord(:,1) == min(coord(:,1)));
% Xnegelems = [];
% %
% for ii = 1:length(Xnegnodes)
%    % [row,col] = find(connectivity == Xnegnodes(ii));
%    % nodules=ones(length(row),1)*Xnegnodes(ii);
%    % Xnegelems = [Xnegelems; [row,col,nodules]];
% end
% %
% for ii=min(Xnegelems(:,1)):max(Xnegelems(:,1))
% if find(Xnegelems(:,1)==ii)>0
%     [row,col] = find(Xnegelems(:,1)==ii);
%     A(1,:)=Xnegelems(row(1),2:3);
%     A(2,:)=Xnegelems(row(2),2:3);
%     A(3,:)=Xnegelems(row(3),2:3);
%     A(4,:)=Xnegelems(row(4),2:3);
%     AA=sortrows(A,1);
%     AAA=AA(:,2)';
%     edof= AAA;
%     C(edof,edof) = C(edof,edof) + CE;
% end
% end
% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%Can do similar for positive and negative Y faces
% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%% This second section is for damping on a non uniform surface and can be
%%%% duplicated for other directions if needed, just follow similar suit.
%%%% Comment out portions that are unwanted.
%%%% This method looks at each 'column' of elements and finds the
%%%% minimum or maximum element and applies damping to it
damp5=[];
damp4=[];
damp3=[];

%Irregular surface damping method
xrange=unique(coord(:,1));
yrange=unique(coord(:,2));
zrange=unique(coord(:,3));
%yneg
Ynegnodes=[];
for xx=1:length(xrange)
    for zz=1:length(zrange)
        [row col]=find(coord(:,1)==xrange(xx) & coord(:,3)==zrange(zz)); %Find all nodes in column
        nodie=[row coord(row,:)];
        [row2 col2]=find(nodie(:,3)==min(nodie(:,3))); %Find the minimum node in column
        Ynegnodes=[Ynegnodes; nodie(row2,1)]; %List of all minimum nodes
    end
end

for cnt=1:length(Ynegnodes)
[row, col] = find(connectivity(:,1) == Ynegnodes(cnt)); % Find connectivity of element with node in corner
    if (row)
        edof = [connectivity(row,1) connectivity(row,4) connectivity(row,5) connectivity(row,8)];
        if coord(edof(1),1) < 0.25
            C(edof,edof) = C(edof,edof) + CE5;
            damp5 = [damp5 edof];
        else if coord(edof(1),1) < 2.5
            C(edof,edof) = C(edof,edof) + CE4;
            damp4 = [damp4 edof];
        else if coord(edof(1),1) < 5
            C(edof,edof) = C(edof,edof) + CE3;
        else if coord(edof(1),1) < 10
            C(edof,edof) = C(edof,edof) + CE2;
        else
            C(edof,edof) = C(edof,edof) + CE1;
            damp3 = [damp3 edof];
        end
    end
end

end
% C(edof,edof) = C(edof,edof) + CE5;

% %xneg
% xnegs = []; % for yy = 1:length(yrange)
%    for zz = 1:length(zrange)
%        [row col] = find(coord(:,2) == yrange(yy) & coord(:,3) == zrange(zz)); % Find all
%        nodie = [row coord(row,:)];
%        [row2 col2] = find(nodie(:,2) == min(nodie(:,2))); % Find the minimum node in
%        xnegs = [xnegs; nodie(row2,1)]; % List of all minimum nodes
%    end
% end
%
% % for cnt = 1:length(xnegs)
% [row, col] = find(connectivity(:,1) == xnegs(cnt)); % Find connectivity of element with
% node in corner
% if (row)
% edof=[connectivity(row,1) connectivity(row,2) connectivity(row,3)
connectivity(row,4)];
% C(edof,edof) = C(edof,edof) + CE2; %Build C matrix using chosen CE
% end
% end
%%%%% Similar can be done for other directions by using maximum instead of
%%%%% minimum
%%%%% remove unwanted elements for areas where the fluid does not exist
%%%%% (voids).  Leave commented if not needed.

% erased=[444, 906]; %Element numbers of elements to be removed
% for L=1:length(erased)
%   ii=erased(L);
%   edof=[connectivity(ii,1);connectivity(ii,2);connectivity(ii,3);connectivity(ii,4);
%        connectivity(ii,5);connectivity(ii,6);connectivity(ii,7);connectivity(ii,8)];
%   K(edof,edof) = K(edof,edof) L KE;
%   M(edof,edof) = M(edof,edof) L ME;
%   edofv(:,L)=edof;
% end

%%%%% DEFINE FREE SURFACE BOUNDARY CONDITIONS
%%%%% Pressure release boundary conditions are obtained by removing those
%%%%% degrees of freedom on the boundary surface (surface of the ocean when
%%%%% modeling underwater acoustics)

surfnodes = find(coord(:,2) == max(coord(:,2))); %Find all nodes in the pos y direction

fixeddofs = [surfnodes]; % Contains all pressure release nodes and all nodes contained
%%%%% within groups of removed elements make an empty set ([]); if none
alldofs = [1:nodeID]; % All degrees of freedom (total original nodes)
freedofs = setdiff(alldofs,fixeddofs); % All nodes with a calculable pressure

%%%%% MODAL ANALYSIS
%%%%% This performs a standard eigenvalue analysis using the stiffness and
%%%%% mass matrices, all boundaries are assumed rigid.

% KK=K(freedofs,freedofs);
% MM=M(freedofs,freedofs);
%
% [V,D,flag]=eigs(KK,MM,50,'sm');
% fexp=(D.^0.5)./(2*pi);

%%%%% % CALCULATE FORCING FUNCTION
Forces are placed on nodes and act as monopoles
u0=.001; % uniform displacement in desired direction (m)

Depending on the location of the node for excitation the force
is calculated differently. There are three cases: (1) surrounded by
nodes (2) on the edge of a face (3) on the corner of a face
This difference is due to the relative area effected by the force.
F(node numbers,set number) is the format, typically set number=1

Prototype for first plane velocity excitation.
% Surrounded by nodes: (2*a)^2
% F([26 29 31 33],1)=rho*(omega)^2*u0*(2*a)^2;
% Edge of a face: 2*(a)^2
% F([25 35],1)=rho*(omega)^2*u0*2*a^2;
F([65],1)=rho*(omega)^2*u0*2*a^2;
% Corner of a face: a^2
% F(1,1)=rho*(omega)^2*u0*a^2;

save filename_100Hz_data
% FORMING MOBILITY MATRIX
Y=KL(omega)^2*MLi*omega*C;
Y(fixeddofs,fixeddofs)=0;

% SOLUTION
P(freedofs,:) = Y(freedofs,freedofs) \ F(freedofs,:);
P(fixeddofs,:)= 0;

The following represents post processing of the Pressure vector P to dB
% or relative dB loss
PdB=20*log10(abs(P)/pref);
for ii=1:length(PdB)
  if PdB(ii)==Inf
    PdB(ii)=min(abs(PdB));
  end
end

PdBloss=20*log10(max(abs(P))./abs(P));
for ii=1:length(PdBloss)
  if PdBloss(ii)==Inf
    PdBloss(ii)=90;
  end
end
save filename_100Hz_results
writeunstrVTK(coord,connectivity,PdBloss,fname)  % Writes chosen results to file for viewing in ParaVIEW
Appendix B

LabVIEW Layouts

Figure B.1: LabVIEW Front Panel for Main Firing Program.
Figure B.2: Block Diagram for Main Firing Program
Figure B.3: Separate While Loop in Firing Program (portion)
Appendix C

Source and Hydrophone Specifications

J-9 Moving Coil Source (11.4 cm OD, 28 cm Length)
Range: 40 Hz – 20 kHz, Max. Power Input: 20W above 100 Hz

J-11 Moving Coil Source (22 cm OD, 44 cm Length)
Range: 20 Hz – 12 kHz, Max Power Input: 200W above 100 Hz

J-15(3) Moving Coil Source (73 cm OD, 46 cm Length)
Range: 10 Hz – 600 Hz, Max Power Input: N/A

HLF1 Hydraulic Source – Hydroacoustics, Inc.
Range: ~20 Hz - ~2 kHz, 196 dB at resonance 260 Hz

Lubell LL98 Piezoceramic Source
Range: Drops off below 1kHz, Max Voltage: 30-50 Vrms

Aquasonic AQ339 Moving Coil Source
Range: 10 Hz – 17 kHz, Power Handling (music): 135 Watts continuous

Reson Hydrophones TC4032
Range: 5 Hz – 120 kHz,
Receiving Sensitivity: -170dB re 1V/mPa
VITA

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Publications


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