DEPOSITION COEFFICIENT, HABIT, AND VENTILATION
INFLUENCES ON CIRRIFORM CLOUD PROPERTIES

A Thesis in
Meteorology
by
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Abstract

Cirriform clouds are atmospheric phenomena of importance, due to their role in global radiative and water vapor budgets. Despite their importance, many of the important influences that define their structure and evolution are still unknown, due to their limited accessibility. We examine the evolution of cirriform cloud properties as they respond to variations three such factors: deposition coefficient, crystal habit, and ventilation.

In an effort to examine surface–kinetic effects on depositional growth of cirrus particles, a parameterization of the deposition coefficient, varying in response to changes in the growth mechanism (2D nucleation or spiral dislocations) and crystal and ambient properties, is incorporated into a microphysical model for spherical crystals. Feedbacks between ice surface and larger–scale properties, such as crystal size and ambient supersaturation, are developed and placed in the context of distinct cirriform cloud types.

Following the analysis on spherical crystals, the effects of crystal habit are explored for their influences on crystal growth. Spheroidal approximations to the primary habits of plates and columns are made, and links between crystal shape and depositional growth rates are determined. An interesting relationship between habit and the evolution of the crystal size distribution is discovered, in which the smallest nucleated crystals are seen to have the most deviation from a spherical shape. This pronouncement of the aspect ratio is found to enhance the growth rates of the crystals to the point where they actually grow larger than the other crystals, altering the structure of the size distribution.

Concluding the analysis of cirriform cloud properties is a study of the effects of ventilation on crystal sublimation in subsaturated air. Two methods for computing ventilation effects, one based on experimental work and the other on theoretical analysis, are incorporated into the model and compared. The fall distances of cirrus particles are analyzed for the two methods of ventilation computation and and the two surface–kinetic growth methods to determine which physical representations lead to the greatest survival of the particles. Crystals growing under 2D
nucleation constraints are found to persist longest, and the applicability of one of the ventilation computations is called into question.
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Chapter 1

Introduction and Motivation

Cirriform clouds play a vital role in the budgets of global energy and water vapor (Fleming and Cox, 1974; Jensen et al., 2007). Due to the limited accessibility for in–situ measurements of these high-altitude clouds, the microphysical drivers behind cirrus clouds rank among the least understood atmospheric processes. Here we examine one microphysical parameter, the deposition coefficient $\alpha$, requires improved understanding due to its significance for crystal growth.

The deposition coefficient $\alpha$ is defined as the fraction of vapor molecules that contact a crystal surface that will actually incorporate into the lattice, causing the crystal to grow. Since $\alpha$ influences the diffusional growth of ice crystals, it also implicitly affects ice concentration and ambient supersaturation. It is assumed that $\alpha$ varies with temperature, supersaturation, and crystal size (Lamb, 1995). Therefore, a feedback exists between the microscale, represented by $\alpha$, and larger–scale cirrus properties such as crystal size, ice content, and vapor availability. However, due to an inadequate physical understanding of $\alpha$ and the desire for computational efficiency, most cloud models ignore $\alpha$ or, at best, approximate it as a constant ranging from 0 (no growth) to 1 (maximum growth). In this study, we seek to relax the constant–$\alpha$ constraint used in models and explore the way $\alpha$ is hypothesized to behave in nature: varying as the ambient atmospheric and crystal properties evolve over time.

In order to explore a non–constant deposition coefficient, the theories and parameterizations of $\alpha$ provided by Nelson and Baker (1996) and Lamb (2000) are used. We extend these ideas to consider evolving ambient conditions, crystal prop-
erties, and vapor transport resistances. Previous use of the Lamb (2000) model considered $\alpha$ as it evolved with supersaturation, for constant representations of vapor transport in the atmosphere (Lamb and Chen, 1995). The parameterization of $\alpha$ has now been expanded to consider vapor transport dependence on diffusive and kinetic influences, processes that vary with changes in the atmosphere. For the first time, the evolution of $\alpha$ is traced as it responds to changes in particle size and shape, ambient temperature and supersaturation, and ventilation effects. This parameterization of $\alpha$ is used in a Lagrangian microphysical parcel model to explore feedbacks of the deposition coefficient with ambient and crystal properties as they evolve over time.

The evolution of the deposition coefficient is explored in a number of situations, which are outlined in the following paragraphs. Following a discussion on theoretical considerations and numerical methods in Chapter 2, $\alpha$ is examined for the two surface-kinetic growth options of new layer formation by 2D nucleation and growth of steps formed by spiral dislocations in Chapter 3. These findings are then placed in the context of cirrostratus and cirrus uncinus clouds. A hypothesis linking the two cirriform cloud types to their microphysical origins is explored, along with sensitivities to such properties as temperature, air motions, and ice concentration.

Next, the effects of crystal shape on the evolution of $\alpha$ are considered in Chapter 4. The model of Lebo (2007), modified for adaptive habits following Chen and Lamb (1994), is used to examine the growth of spheroidal particles that represent the primary habits of plates and columns. A special focus is placed on shapes of particles observed in cirrus clouds, for which new values of shape-controlling parameters have been obtained. Shape effects have been analyzed for their influences on distributions of crystal sizes and the associated microphysical distributions.

Finally, the investigation concludes with a study of the effects of ventilation on falling ice particles and their deposition coefficients, which is found in Chapter 5. Ventilation enhances vapor gradients, and therefore alters the evolution of the deposition coefficient. The methods of Thorpe and Mason (1966), Hall and Pruppacher (1976), and Chen and Lamb (1994) are compared, with a focus placed on the influence of ventilation on the evolution of the deposition coefficient.

Chapter 6 concludes this work with a summary of the analysis performed on the
deposition coefficient. The knowledge gained from the simulations is combined to obtain an improved understanding of the interactions between $\alpha$ and larger-scale particle and ambient properties.
Chapter 2

Theoretical and Numerical Methods

2.1 Crystal Growth by Vapor Diffusion

Recent studies have discussed the existence of large ice crystals in high altitude cirrus clouds (Jensen et al., 2007). The ice particles in these, and other cirriform clouds, grow by vapor diffusion, a process dependent upon conditions at both the ambient and molecular scales. Vapor diffusion is the net transport of vapor molecules from a region of high concentration (far away from the crystal) to a region of lower concentration (at the crystal surface). Crystal growth by vapor diffusion is dependent on several factors. At the ambient scale, the amount of vapor available for uptake by the crystal is described by the ambient supersaturation

\[ s_{amb} = \frac{e}{e_{s,i}} - 1, \quad (2.1) \]

where \( e \) is the vapor pressure and \( e_{s,i} \) is the equilibrium vapor pressure over ice which represents a dynamic equilibrium in which the rate of deposition of vapor (gain) equals the rate of sublimation (loss). In other words, \( e_{s,i} \) represents a state in which there is no net transfer of vapor molecules. The equilibrium vapor pressure decreases with decreasing temperature (Roger and Yau, 1989, p. 16), and supersaturation increases with altitude due to cooling in adiabatic ascent, barring uptake of vapor by any existing particles.
Another ambient contribution to vapor growth is the diffusion coefficient

\[ D_v = 0.211 \left( \frac{T}{273.15 K} \right)^{1.94} \left( \frac{1013.25 \text{mb}}{p} \right). \]  

This parameter is a proportionality constant between the diffusive flux of vapor molecules and gradient of their concentration. The diffusion coefficient is proportional to the speed of the diffusing vapor molecules:

\[ v = \sqrt{\frac{8kT}{\pi m_v}}, \]  

which is dependent upon temperature and the size of the molecules.

Once a quantity of vapor molecules has reached a crystal surface, an event determined by the ambient processes of supersaturation development and vapor diffusion, still another step must be achieved prior to crystal growth: vapor molecules must be incorporated into the lattice. Figure 2.1 displays the vapor profile as a function of distance from a crystal of radius \( r \). The region nearest to the crystal surface is the local, or free–molecular, region defined by vapor jump distance \( \Delta \). Beyond the boundary of \( \Delta \), the continuum approximation can be made in order to assess the ambient region. The vapor profile for a crystal growing by vapor deposition with surface kinetic impedances is

\[ n(x) = n_\infty + (n_i - n_\infty) \frac{r}{x}, \]  

where \( n_\infty \) is the ambient vapor concentration, \( n_i \) is the vapor concentration at the ambient–local interface, and \( n_{eq} \) is the equilibrium vapor concentration. The vapor concentration \( n_i \) is influenced by the ambient vapor concentration, but is not equivalent due to surface kinetic resistances to molecular incorporation. This molecular–scale process is influenced by the ambient properties above, and is represented by the deposition coefficient \( \alpha \).
Figure 2.1. Vapor profile with local and ambient regions indicated. Concentrations: \( n_\infty \) = ambient vapor concentration, \( n_i \) = vapor concentration at ambient–local interface, \( n_{eq} \) = equilibrium vapor concentration. Distances: \( r \) = crystal radius, \( \Delta \) = vapor jump distance, \( x \) = radial distance. \( n(x) = n_\infty + (n_i - n_\infty) \frac{r}{x} \)

2.2 Adaptive Parameterization for a Variable Deposition Coefficient

The deposition coefficient \( \alpha \) is an important, yet insufficiently understood parameter used to describe the role of surface kinetics in the growth of ice crystals. As mentioned in the introduction, \( \alpha \) is the fraction of the vapor molecules impinging on a crystal surface that actually incorporate into the lattice, resulting in crystal growth. While most cloud models ignore surface kinetics or apply \( \alpha \) as a constant, we seek to examine \( \alpha \) as it evolves in response to changes in ambient conditions and crystal properties. Specifically, behavior of \( \alpha \) is examined in the
framework of the two theories that exist to explain crystal growth: spiral dislocations and 2D nucleation. The analysis is then extended to an examination of the feedbacks between crystal microphysics and the large-scale driving forces that produce different types of cirriform clouds, namely cirrostratus and cirrus uncinus.

Before entering into a discussion of the origins of steps on a crystal surface, a brief review of surface kinetics is necessary. Crystal growth is the product of three surface-kinetic processes (Markov, 2003, p. 193): First, vapor molecules are adsorbed onto the crystal surface between steps, which are defined as ledges between crystal layers. Second, the adsorbed vapor molecules (admolecules) diffuse across the surface toward a step. Third, the vapor molecules incorporate into available kink sites along the steps. These processes result in the advancement of steps, and therefore the growth of the crystal. Figure 2.2, from Markov (2003, p. 39), shows the sites on a crystal surface where an admolecule can incorporate. A cubic representation is used for simplicity. The admolecules are differently bound to the crystal surface at each site. For example, admolecule 1 has five bonded surfaces and one lone (unbound) surface. On the other hand, admolecule 5 has one bonded surface and five lone surfaces. With the exception of admolecule 3, detachment of the admolecules leads to a change in the surface energy since the number of exposed surfaces differs from the number of bound surfaces. Admolecule 3 is positioned in what is known as the kink site, a location essential to the theories of crystal growth (Markov, 2003, p. 39).

In reality, deposition varies over each crystalline face, basal and prism (Nelson and Baker, 1996). However, most models do not consider the growth of individual crystal facets. Instead, these models use a bulk deposition coefficient (hereafter, the deposition coefficient), \( \alpha \), which describes the overall constraint on growth due to surface kinetics. The deposition coefficient captures the physics of admolecule incorporation and is included in the mass growth rate equation,

\[
\frac{dm}{dt} = \frac{4\pi Cs_{amb}}{R_eT \epsilon_c D_e} + \frac{L_e}{K_e T} \left( \frac{L_e}{R_eT} - 1 \right),
\] (2.5)
via the modified diffusivity, $D_v^*$,

$$D_v^* = \frac{D_v}{r + \Delta} + \frac{4D_v}{\alpha \pi r^2}, \quad (2.6)$$

following the traditional method as discussed in Pruppacher and Klett (1978, p. 415). Here, $D_v$ is the vapor diffusion coefficient, $r$ is the crystal radius, and $\Delta$ is the vapor jump distance, which characterizes the boundary between conditions ambient and local to the crystal (see Fig. 2.1). All variables in Eq. (2.5) and in this study are defined in Appendix D. Note the behavior of Eq. (2.6): as $\alpha$ decreases, so does $D_v^*$, which then causes $dm/dt$ to decrease. Now that we have seen the placement of the deposition coefficient in the mass growth rate equation of a crystal, we continue with a discussion of deposition theories and the parameterization of $\alpha$. 

**Figure 2.2.** Cubic representation of vapor molecule incorporation. *Figure courtesy of Markov, 2003.*
2.2.1 Surface Kinetic Theories and Local Influences on Crystal Growth

2D Nucleation

Crystals that grow by 2D nucleation are said to be perfect (Burton et al., 1951), that is, their faces are defectless. The growth of a defectless crystal involves the two-dimensional nucleation of a critical embryo of molecules on the crystal surface. Once this critical embryo is formed, new molecules will incorporate along the edges, allowing for rapid crystal growth. The critical embryo is characterized by the critical supersaturation, $s_{\text{crit}}$, which is a transition value that the local supersaturation $s_{\text{local}}$ (a parameter representing the quantity of vapor molecules at the crystal surface; see Fig. 2.1) must achieve prior to the occurrence of significant growth (Lamb, 2000). The critical supersaturation is a function of such microscale properties as the surface diffusion timescale of a vapor molecule and the energy barriers to molecular incorporation. The critical embryo is assumed circular and is characterized by the critical radius, given by Lamb (2000) as

$$r_{\text{crit}} = \frac{\Omega_2 \beta}{\Delta \mu},$$

(2.7)

where $\Omega_2$ is the area of a molecule, $\beta$ is the edge free energy, and $\Delta \mu = kT \ln(s_{\text{local}} + 1)$ is the chemical potential difference between molecules in the vapor and condensed phases.

Spiral Dislocations

Crystals that are able to develop in low supersaturation conditions are characterized by defects on the surfaces (Burton et al., 1951). These defects provide permanent ledges for kink sites, allowing efficient incorporation of vapor molecules into the lattice. In this case, the critical supersaturation needs not to be achieved by the local supersaturation prior to the onset of crystal growth.

The critical radius, defined by 2D nucleation theory, is also equivalent to the minimum radius of curvature found on steps near the emergence of a dislocation on a crystal surface. Growth along the ledges follows a spiral path, and this curvature is limited by supersaturation in such a way that the central spiral is characterized
by the critical radius (Lamb and Scott, 1974). The geometrical sensitivity to
the critical supersaturation of crystals growing according to dislocation theory
produces small variations in crystal growth rate with $s_{\text{crit}}$.

**Incorporating Surface Kinetic Theory (Local Influences) into a Deposi-
tion Coefficient Parameterization**

Since the critical supersaturation $s_{\text{crit}}$ plays an essential role in the formation
of new layers on a crystal surface, it is clear that the deposition coefficient, repre-
senting surface processes, depends upon the local supersaturation $s_{\text{local}}$ relative to
$s_{\text{crit}}$. We recall that crystals growing by 2D nucleation exhibit a large dependence
on $s_{\text{local}}$, whereas dislocation crystals grow only weakly dependent on it. Based on
the work of Burton et al. (1951), Nelson and Baker (1996) developed the following
parameterization of $\alpha$ in terms of the local and critical supersaturation:

$$\alpha = \left( \frac{s_{\text{local}}}{s_{\text{crit}}} \right)^m \tanh \left( \left( \frac{s_{\text{crit}}}{s_{\text{local}}} \right)^m \right).$$  \hspace{1cm} (2.8)

The exponent $m$ is a parameter that modifies the shape of the $\alpha$ curve produced
to emulate the two different growth mechanisms.

Recall that crystals with surface defects efficiently incorporate vapor molecules
into the lattice. This efficiency is represented by the orange curve of the deposition
coefficient in Fig. 2.3, where the deposition coefficient is plotted over a range of
normalized supersaturations $S = s_{\text{local}}/s_{\text{crit}}$. The dislocation $\alpha$ curve is a function
of the available vapor, as represented by the supersaturation, yet it does not require
a certain critical supersaturation value to be achieved prior to the deviation of $\alpha$
from zero. A critical supersaturation, as we will examine, is required only for 2D
nucleation. In order to represent the physics of dislocation growth, a value of unity
is chosen for exponent $m$ in Eq. (2.8).

As previously mentioned, a critical embryo of vapor molecules must nucleate
(2D nucleation) on the smooth crystal surface in order for a defectless crystal to
grow. An exponent choice of $m = 30$ in Eq. (2.9) produces the blue $\alpha$ curve in
Fig. 2.3. Notice that the deposition coefficient hovers near zero for low supersat-
uration ratios $S < 1$. These tiny values of $\alpha$ represent the impedance to crystal
growth caused by the inability to form critical embryos on the surface. In other
words, the critical supersaturation has not been achieved. Once the local supersaturation reaches the critical value, growth occurs rapidly as vapor molecules attach to the ledges of critical embryos and new layers expand.

Due to the difficulties that arise in obtaining data for small–scale properties, such as edge energy $\beta$, the critical supersaturation is an ambiguous parameter. Therefore, in our studies, we examine crystal growth sensitivities to three constant choices of $s_{\text{crit}}$. The first choice is the suggested value of Burton et al. (1951): $s_{\text{crit}} = 0.1$. Since the theoretical investigations of Lamb (2000) and Libbrecht (2003) indicate that $s_{\text{crit}}$ increases with decreasing temperature, we also evaluate $s_{\text{crit}} = 1$ and $s_{\text{crit}} = 10$ since the focus of this study links microscale surface processes to low–temperature cirriform clouds.
2.2.2 Deposition Coefficient Parameterization: Ambient Influences

As a representative of the quantity of vapor molecules at the crystal surface, the local supersaturation $s_{local}$ plays an important role in determining the fraction of molecules incorporating into the lattice, $\alpha$. Physically, a large quantity of vapor molecules existing at the crystal surface (large $s_{local}$) indicates a large probability of molecular incorporation (large $\alpha$). Unfortunately, Eq. (2.8) is useful only if the local supersaturation over the crystal is known; unfortunately this value cannot be measured readily for crystals growing outside of a pure vapor environment (Lamb, 2000). Therefore, Eq. (2.8) is inapplicable in its present state.

The local supersaturation is influenced by environmental and particle conditions, such as ambient temperature and supersaturation and crystal size. As an extension of the work of Nelson and Baker (1996) and Lamb and Chen (1995), Lamb (2000) proposed a modification to Eq. (2.8) to incorporate the physics of a crystal growing in air, making it possible to evaluate $\alpha$:

$$\alpha = \left( \frac{s_{amb}}{s_{crit}(1 + K\alpha)} \right)^m \tanh \left( \frac{s_{crit}(1 + K\alpha)}{s_{amb}} \right)^m. \quad \text{(2.9)}$$

Here, $\alpha$ is a function of the ambient and critical supersaturations, as well as the vapor transport parameter $K$. This equation for $\alpha$ is transcendental and so requires iterative methods to obtain a solution. Vapor transport parameter $K$ is defined as

$$K = \frac{\overline{v_r}}{4D_v} = \frac{\overline{v_r}^2}{4D_v(r + \Delta)} + \frac{1}{\alpha}, \quad \text{(2.10)}$$

where $\overline{v}$ is the mean speed of vapor molecules and $D_v$ is the diffusion coefficient. The two components of the vapor transport parameter represent diffusive (first term) and kinetic (second term) limitations.

Implicit Dependence on Ambient Supersaturation

The amount of vapor reaching the crystal surface (represented by $s_{local}$) is directly related to the amount of ambient available vapor (represented by $s_{amb}$).
Figure 2.4. Implicit $\alpha$ dependence on $s_{amb}$ via $s_{local}$. (a) Deposition coefficient $\alpha$ and (b) local supersaturation are plotted with normalized ambient supersaturation $= s_{amb}/s_{crit}$, for dislocation surface kinetic constraints.

Physically, for identical diffusive conditions, an increase in available ambient vapor will correspond to an increase in vapor molecules crossing the ambient–local interface. Therefore, as expected and depicted in Fig. 2.4, the local supersaturation rises with the ambient supersaturation. Correspondingly, an increase in local supersaturation implies a greater probability of molecular incorporation, a trend captured by the evolution of the deposition coefficient with ambient supersaturation.

Implicit Dependence on Temperature

The deposition coefficient also exhibits an implicit dependence on temperature. This dependence manifests itself within the diffusion coefficient $D_v$ (Eq. (2.2)), and hence the mean speed of vapor molecules $\bar{v}$ (Eq. (2.3)).

Low temperatures hinder diffusion through the speed of vapor molecules toward the crystal. This relationship is represented by the positive feedback between $\alpha$ and $T$, as depicted in Fig. 2.5, which shows deposition coefficient and local supersaturation rising with temperature.
Implicit α dependence on T via $s_{local}$. (a) $\alpha$ and (b) $s_{local}$ varying with normalized ambient supersaturation for cold cirriform temperatures. Cold temperatures hinder diffusion and speed of vapor molecules toward the crystal.

**Implicit Dependence on Crystal Size**

Deposition coefficient $\alpha$ contains an implicit size dependence as well. The vapor profile previously depicted in Fig. 2.1,

$$n(x) = n_{\infty} + (n_i - n_{\infty}) \frac{r}{x},$$

represents the relationship between the ambient and local supersaturation. For a spherical particle of radius $r$, the vapor gradient at the interface between the local and ambient regions is

$$\left. \frac{dn}{dx} \right|_{r+\Delta} = \frac{n_{\infty} - n_i}{r + \Delta}. \quad (2.11)$$

The vapor gradient defined by Eq. (2.11) contains a negative feedback with crystal size. This relationship is depicted in Fig. 2.6, which shows three crystal sizes: small ($r_1$), medium ($r_2$), and large ($r_3$). The smallest crystal, characterized by $r_1$, produces the largest vapor gradient (see Fig. 2.7). In order to represent the enhanced vapor gradient, the value of $\alpha$ for this tiny crystal must be large. The relationships between crystal size and vapor gradient with deposition coefficient are illustrated at left in Fig. 2.6. To summarize, smaller crystals have larger vapor gradients, and by extension larger values of the local supersaturation and
**Figure 2.6.** Size influences on vapor profile: $n(x) = n_\infty + (n_i - n_\infty) \frac{r}{r_i}$. Small particles ($r_1$) have large vapor gradients, and therefore large values of the deposition coefficient.

**Figure 2.7.** Influence of crystal size on vapor gradients.
Figure 2.8. Implicit $\alpha$ dependence on $r$ via $s_{local}$. (a) $\alpha$ and (b) $s_{local}$ varying with normalized ambient supersaturation for crystal radii. Deposition coefficient $\alpha$ is inversely related to crystal size.

deposition coefficient.

The size dependence of the local supersaturation is viewed by considering the vapor pressure at the interface between the local and ambient regions (Chen and Lamb, 1994):

$$e_{local} = \frac{e_{\infty} + \frac{\alpha \pi r}{4D_c} e_{s,i}}{1 + \frac{\alpha \pi r}{4D_c}}. \quad (2.12)$$

The local supersaturation can be obtained by dividing Eq. (2.12) by the equilibrium vapor pressure $e_{s,i}$ and subtracting one:

$$s_{local} = \frac{\frac{e_{\infty}}{e_{s,i}} + \frac{\alpha \pi r}{4D_c}}{1 + \frac{\alpha \pi r}{4D_c}} - 1$$

$$= \frac{s_{amb}}{1 + \frac{\alpha \pi r}{4D_c}}. \quad (2.13)$$

From Eq. (2.13), we see that the ratio of local and ambient supersaturations is inversely dependent on the size $r$ of the crystal:

$$\frac{s_{local}}{s_{amb}} = \frac{1}{1 + \frac{\alpha \pi r}{4D_c}}. \quad (2.14)$$
Hence, smaller crystals will have a higher value of local supersaturation relative to the ambient value, leading to a larger value of the deposition coefficient, as depicted in Fig. 2.8, which shows the largest value of the deposition coefficient and local supersaturation occurring for the smallest crystals.

The previous paragraphs describe the mechanisms behind growth of a particle as a whole. Next, we consider the variation of the deposition coefficient over the crystal surface and how this distribution of $\alpha$ leads to different shapes of crystals.

### 2.3 Deposition and Crystal Habit

In order for various crystal shapes to evolve, the vapor diffusing toward a crystal must be distributed among the different crystal faces: basal and prism. The parameter controlling the amount of deposition on each face is

\[
\Gamma = \frac{\alpha_B}{\alpha_P},
\]

where $\alpha_B$ is the deposition coefficient on the basal face (along the c-axis) and $\alpha_P$ is the deposition coefficient on the prism face (along the a-axis). Experimental values for $\Gamma$ exist for temperatures of $-30^\circ$C and warmer (Chen and Lamb, 1994). Section 2.4 provides estimates of $\Gamma$ at the lower temperatures typical of cirrus clouds.

*Figure 2.9.* Plate-like crystals are approximated as oblate spheroids, while columnar crystals are represented as prolate spheroids. *Figure adapted from Chen and Lamb (1994).*
Following the method of Chen and Lamb (1994), prolate and oblate spheroids are used to represent columnar and plate-like crystals, respectively (Fig. 2.9). These idealized shapes enable us to calculate the capacitance of an ice crystal, using an electrostatic analogy that was developed by Stefan (1873) and Maxwell (1878) to represent the distorted vapor fields surrounding the crystal. The vapor gradient is maximized where surface curvature is the largest. In the case of a columnar crystal, the maximum vapor gradient occurs over the basal faces. For a plate-like crystal, the maximum occurs over the prism faces. Since the sharp edges of ice crystals complicate the boundary conditions of the vapor gradient problem, the spheroidal geometry is an approximation that provides an analytical method for calculating crystal growth while retaining the notion of shape-dependent distortion of the vapor fields.

Chen and Lamb (1994) captured the essence of the above physical phenomena, while retaining computational simplicity. The increase in crystal volume (mass) is calculated for a spherical particle, and then this mass is redistributed over the semi–major $a$ and $c$ axes. This is done by solving the following equations simultaneously:

$$\frac{dV}{dt} = \frac{4\pi Cs_{amb}}{\rho_{ice} \left( \frac{R_v T}{D_v} + \frac{l_s}{K_v T} \left( \frac{l_s}{R_v T} - 1 \right) \right)},$$  \hspace{1cm} (2.16)

$$d \ln(\phi) = \frac{\Gamma - 1}{\Gamma + 2} d \ln(V).$$  \hspace{1cm} (2.17)

where $\phi$ is the aspect ratio, defined as $\phi = c/a$. The growth rate equation is expanded and further explained in Appendix C. The capacitances $C$ for oblate and prolate spheroids are

$$C = \frac{a \epsilon}{\arcsin\epsilon} \hspace{1cm} \text{(oblate)}$$

$$C = \frac{c \epsilon}{\ln((1 + \epsilon) \phi)} \hspace{1cm} \text{(prolate)},$$  \hspace{1cm} (2.18)

where $\epsilon$ is the eccentricity defined as $\sqrt{1 - \phi^{-2}}$ for an oblate spheroid and as $\sqrt{1 - \phi^2}$ for a prolate spheroid. Figure 2.10a shows the capacitance $C$ normalized by the semi–major axis ($a$ for oblate, $c$ for prolate), which we shall call the shape factor, as a function of aspect ratio $\phi$. The shape factor, solely a function of $\phi$, is seen to decrease as $\phi$ deviates from unity. This appears to indicate that
Figure 2.10. Shape factor, capacitance, and semi–axis lengths as functions of aspect ratio, for a constant crystal volume of $10^4 \, \mu m^3$. (a) Shape factor. (b) The capacitance factor $C$. (c) Size contributions to capacitance: $a$ and $c$ axis lengths.

growth would decrease for nonspherical particles. However, as seen in Fig. 2.10b, the inclusion of crystal size in the capacitance factor (Eqs. (2.18)) produces quite the opposite trend. The capacitance now increases as $\phi$ deviates from unity, since for a constant volume, the semi–maximum dimension (seen in Fig. 2.10c) increases as spheroids become more pronounced. Therefore, growth rates increase with non–sphericity.

To use the above method, we must extend $\Gamma$ to lower temperatures, which we turn to next. The values of $\Gamma$ are determined for particle shapes typically found in cirrus clouds.
2.4 Determination of Inherent Growth Ratios

In order to analyze the effects of crystal shape on the development of our modeled cloud, choices of the inherent growth ratio $\Gamma$ appropriate for cirriform clouds must be made. The inherent growth ratio is unknown for temperatures below $-30^\circ C$ (Chen and Lamb, 1994), but ranges of $\Gamma$ can be inferred from observed mass-dimensional relationships (e.g., Mitchell et al., 1990). Idealized shapes are used to compute $\Gamma$ for three types of particles: ice spheres, short columns, and bullets.

Since the typical cirrus cloud contains columnar crystals (Gierens et al., 2003) or bullets (Heymsfield, 1975a), $\Gamma$ must have a value greater than one, representing more mass accommodation on the basal face than on the prism face. A reference $\Gamma$ of one is chosen to represent spherical crystal growth as a means for comparison. This choice for $\Gamma$ may also be the best representation of the polycrystalline bullet rosette, a common crystallographic form found in cirrus clouds (Heymsfield, 1975a).

The inherent growth ratio $\Gamma$ can be extracted from existing ice crystal mass-dimensional relationships. An expression for relating ice particle mass to its maximum dimension, $L$, was used by Mitchell (1988), based on the power-law relationships of Locatelli and Hobbs (1974):

$$m = \xi L^\zeta.$$  \hspace{1cm} (2.19)

Using this expression, $\Gamma$ can be determined for columnar ice particles using the relationship of Chen and Lamb (1994):

$$\zeta = \frac{2}{\Gamma} + 1.$$ \hspace{1cm} (2.20)

The inherent growth ratio chosen to represent short columns is derived from the mass-dimensional relationships of Mitchell et al. (1990), which are based on the geometry of a cylinder, along with the data and relationships of Heymsfield (1972):

$$m = 0.13L^{2.9}.$$ \hspace{1cm} (2.21)

Inserting the exponent $\zeta = 2.9$ into Eq. (2.20) provides a $\Gamma$ of 1.1 for short columns.
Figure 2.11. Bullet crystal approximated by cylindrical and conical geometry

Physically, because this value of \( \Gamma \) is so near to unity, only slightly more vapor is depositing along the c-axis than along the a-axis, producing short columns.

The method of extracting \( \Gamma \) from mass-dimensional relationships is extended to the bullet-shaped crystals, which are commonly found in cirriform clouds (Heymsfield, 1975a). As depicted in Fig. 2.11, the geometry of a bullet is approximated as the geometrical sum of a cylinder and a cone. Mitchell et al. (1990) did not explore a mass-dimensional relationship for bullets, possibly because such a correlation is dependent upon the fraction of the maximum dimension that lies in the cylinder \( x \) and in the cone \( 1 - x \). However, for our purposes, this geometrical approximation provides us with a range of inherent growth ratios that may represent bullets. The mass-dimensional relation for bullets is derived as follows:

The volume of the cylinder is

\[ V_{cyl} = \frac{\pi}{4} D^2 x L, \]  

(2.22)
where $D$ is the diameter of the base of the cylinder. This leads to the mass defined by

$$m_{cyl} = \frac{\pi}{4} D^2 x L \rho_{\text{ice}}.$$

(2.23)

For the conical portion of the approximated crystal, the volume is

$$V_{cone} = \frac{\pi}{12} D^2 (1 - x) L,$$

(2.24)

which results in the mass of the cone:

$$m_{cone} = \frac{\pi}{12} D^2 (1 - x) L \rho_{\text{ice}}.$$

(2.25)

Summing the two masses above provides the overall mass of the approximated bullet:

$$m = \frac{\pi}{12} D^2 L (2x + 1) \rho_{\text{ice}}.$$

(2.26)

Heymsfield (1972) provides the following relationships for bullets:

$$D = \begin{cases} 
0.25 L^{0.7856} & \text{if } L \leq 0.3 \text{mm} \\
0.185 L^{0.532} & \text{if } L > 0.3 \text{mm},
\end{cases}$$

(2.27)

$$\rho_{\text{ice}} = 0.78 L^{-0.0038}, L \geq 0.1 \text{mm}.$$

(2.28)

Incorporating Eqs. (2.27) and (2.28) into Eq. (2.26), the following expressions are obtained:

$$m_{\text{bullet}}(L \leq 0.3 \text{mm}) = \frac{\pi}{12} (2x + 1)(0.049 L^{2.57})$$

(2.29)

$$m_{\text{bullet}}(L > 0.3 \text{mm}) = \frac{\pi}{12} (2x + 1)(0.027 L^{2.06}).$$

(2.30)

The above relationships provide us with values for exponent $\zeta$, and hence $\Gamma$. For bullets with $L \leq 0.3$ mm, $\zeta = 2.57$, and by using Eq. (2.20), $\Gamma = 1.27$. Bullet crystals with $L > 0.3$ mm have a $\zeta = 2.06$, and a corresponding $\Gamma = 1.89$. A $\Gamma$ of 1.25 is chosen to evaluate the short bullets characteristic of cirriform clouds. We use this choice of $\Gamma$ in the parameterization of Chen and Lamb (1994) to simulate the evolution of bullet crystals in cirrus clouds.

We have discussed the inclusion of surface kinetics and shape effects in our
model. To provide further realism, ventilation effects are included and are discussed in the following section.

2.5 Determination of Ventilation Effects

Clouds are not stationary, and sedimenting particles have depositional growth rates that are affected by their motions. Air flow enhances water vapor fluxes to a particle, resulting in more rapid vapor growth than when crystals are stationary. This process is captured by including a ventilation coefficient, \( f \), in the growth rate equation (2.5),

\[
\frac{dm}{dt} = \frac{4\pi C_{s\text{amb}} f}{R_{e}^{2}T} + \frac{I_{s}}{K_{v}^{2}T \left( \frac{I_{e}}{K_{v}^{2}T} - 1 \right)}.
\]  

(2.31)

Two methods for computing \( f \) are incorporated into our vapor growth model for comparative purposes, and are developed in the following paragraphs. The two ventilation methods chosen for analysis involve computations based on experimental work in one case, and theoretical physics in the other.

2.5.1 Ventilation in Terms of the Characteristic Length

The two methods for finding \( f \) are those of Thorpe and Mason (1966) and Hall and Pruppacher (1976). These methods are especially interesting since the computations of Thorpe and Mason (1966) come directly from laboratory measurements, whereas those of Hall and Pruppacher (1976) are mainly rooted in theoretical and numerical analyses. Comparisons such as these are important, as laboratory measurements can be flawed and theory may be based on unrealistic assumptions. Computational methods must be checked against one another to determine areas where improvement is necessary.

Thorpe and Mason (1966) determined the ventilation effect by performing a series of experiments involving evaporation of ice spheres and hexagonal plates over certain ranges of the Reynolds number:

Spheres: \( f = 0.5(1.88 + 0.66(Sc)^{1/3}(Re)^{1/2}) \) \quad 10 < Re < 200

Plates: \( f = 0.65 + 0.44(Sc)^{1/3}(Re_{L})^{1/2} \) \quad 5 < Re_{L} < 100,
where $Re$ is the Reynolds number defined in terms of diameter $2a$, and $Re_L$ is the Reynolds number defined in terms of the characteristic length, which is the ratio of the total particle surface area to the perimeter normal to the flow. The Schmidt number, $Sc$, is the ratio of momentum diffusivity to mass diffusivity, and is in the vicinity of 0.65 for air. The laboratory studies of Thorpe and Mason (1966) have generated some controversy, as a critique from Pitter et al. (1974) accused their methods of being “crude” and “suspect”.

Hall and Pruppacher (1976) correlated available data for ventilation of different particle shapes in terms of the characteristic length:

$$f = 1.00 + 0.14 \left((Sc)^{1/3}(Re_L)^{1/2}\right)^2 \quad (Sc)^{1/3}(Re_L)^{1/2} \leq 1$$

$$f = 0.86 + 0.28(Sc)^{1/3}(Re_L)^{1/2} \quad (Sc)^{1/3}(Re_L)^{1/2} > 1.$$  \hspace{1cm} (2.33)

It is important to take note of a couple of potential discrepancies. First, the nonspherical data used in the relations is entirely generated by numerical and theoretical analyses. Second, the relations are built upon data for spherical and plate-like shapes, such as oblate spheroids, and are assumed to apply to columnar shapes as well.

### 2.5.2 Methods for Computing Ventilation

In order to use the ventilation relations of Thorpe and Mason (1966) and Hall and Pruppacher (1976), a number of other functions must be computed. First and foremost, the Reynolds number for spheres is calculated according to the empirical relationship found in Pruppacher and Klett (1978, p. 323):

$$e^{Re} = B_0 + B_1X + B_2X^2 + \ldots + B_6X^6,$$  \hspace{1cm} (2.34)

where $X$ is defined as $\ln(Be)$. The Best number, $Be$, is an especially useful parameter, as it can be computed entirely from the physical properties of the ice particles and air (Miller and Young, 1979):

$$Be = \frac{8m_g\rho_{air}a^2}{\eta^2A}.$$  \hspace{1cm} (2.35)

Calculating the Reynolds number for a nonspherical particle involves more com-
plexities than it does for the spherical case. In order to use the Thorpe and Mason (1966) relationships, we must determine how well our spheroidal model approximates the Reynolds number for plates. To do this, we consider two functional forms of the Reynolds number in terms of the characteristic length. The relationship for plates is (Miller and Young, 1979):

$$Re_L = \left( \frac{1}{2} + \phi \right) Re. \quad (2.36)$$

The relationship for oblate spheroids is given by Pitter et al. (1974) as

$$Re_L = \frac{1}{2} \left( 1 + \frac{\phi^2}{2\epsilon} \ln \left( \frac{1+\epsilon}{1-\epsilon} \right) \right) Re, \quad (2.37)$$

where $\epsilon$ is the eccentricity of the spheroid. Both of these cases involve the Reynolds number $Re$ in terms of the diameter $2a$. For the case of plates, Miller and Young (1979) reference the empirical relationship of Jayaweera and Cottis (1969):

$$Re = \begin{cases} 
0.0928Be^{0.765} & 20 < Be < 2500 \\
0.593Be^{0.53} & 2500 < Be < 10^5 
\end{cases} \quad (2.38)$$

In the spheroidal case, $Re$ is calculated using Eq. (2.34). For a range of Best numbers, the computations of Reynold number $Re$ for plates and spheroids do not significantly differ, as shown in Figure 2.12.

While $Re$ for plates and spheroids are similar over a large range of $Be$, it is also necessary to examine the limiting behavior of $Re_L$. To go about this, we take the limits of small and large aspect ratios of Eqs. (2.36) and (2.37):

$$\lim_{\phi \to 0} Re_L = \frac{1}{2} Re, \quad \frac{1}{2} Re \quad \lim_{\phi \to 1} Re_L = \frac{3}{2} Re, \quad Re$$

In the limit of a small aspect ratio, Eqs. (2.36) and (2.37) provide an exact match between computations of the Reynolds number in terms of characteristic length for plates and spheroids. Consequently, we can approximate the spheroidal $Re_L$ as the plate–like version to within a small relative error, as long as $\phi$ is small. Table 2.1 shows the absolute differences and relative errors between the $Re_L$ computations
Figure 2.12. Comparison of diameter–defined Reynolds numbers for plates and oblate spheroids. Based on empirical relationships of Jayaweera and Cottis (1969) and Pruppacher and Klett (1978).

For plates and spheroids for chosen aspect ratios averaged over the Best number range of 20–5000. The $Re_L$ absolute differences show that the smaller the value of the aspect ratio, the better the approximation of a plate. For both cases of spheres and spheroids, the terminal fallspeed is calculated according to (Pruppacher and Klett, 1978, p. 324):

$$U_\infty = \frac{\eta Re}{2\rho_{air}a},$$

where the existence of the $a$–axis captures the proportionality of the drag force to the area of the particle normal to the flow.

### 2.5.3 Local Ventilation Effects

The contributions to vapor growth from ventilation as computed by Thorpe and Mason (1966) and Hall and Pruppacher (1976) can be distributed locally over the $a$ and $c$ axes in order to capture the ventilation–enhanced vapor gradients at
the extended edges of a particle (Chen and Lamb, 1994). As discussed in section 2.3, crystal habits (approximated as spheroids) are the results of distributing the increased mass over the $a$ and $c$ axes (Chen and Lamb, 1994). The same principle is applied for the distribution of enhanced vapor growth by ventilation over the semi–major crystal axes. Previously, the effect of shape was determined by the parameter $\Gamma$, the inherent growth ratio. For cases of ventilated ice crystals, $\Gamma$ is modified (Chen and Lamb, 1994):

$$\Gamma^* = \frac{f_c}{f_a} \Gamma,$$

where

$$f_c = b_1 + b_2 \left( Sc^{1/3} Re^{1/2} \right)^\gamma \left( \frac{c}{r} \right)^{1/2},$$

$$f_a = b_1 + b_2 \left( Sc^{1/3} Re^{1/2} \right)^\gamma \left( \frac{a}{r} \right)^{1/2},$$

are the ventilation coefficients for the $c$ and $a$ axes, respectively. Here, the coefficients $b_1$ and $b_2$, and the power $\gamma$ are provided by the spherical ventilation computations of Hall and Pruppacher (1976) and are given in Table 2.2. In this analysis, the spherical ventilation computations of Thorpe and Mason (1966) are tested as well.

### Table 2.1. Reynolds number (characteristic length): plates and spheroids.

<table>
<thead>
<tr>
<th>Aspect Ratio $\phi$</th>
<th>$Re_L$ Plates</th>
<th>$Re_L$ Spheroids</th>
<th>Absolute Difference</th>
<th>Relative Error %</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>33.69</td>
<td>24.11</td>
<td>9.58</td>
<td>28.81</td>
</tr>
<tr>
<td>0.25</td>
<td>25.27</td>
<td>19.79</td>
<td>5.47</td>
<td>22.07</td>
</tr>
<tr>
<td>0.1</td>
<td>20.21</td>
<td>17.99</td>
<td>2.22</td>
<td>11.45</td>
</tr>
</tbody>
</table>

### Table 2.2. Summary of ventilation data.

<table>
<thead>
<tr>
<th>Source</th>
<th>Range</th>
<th>$b_1$</th>
<th>$b_2$</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hall and Pruppacher (1976)</td>
<td>$Sc^{1/3} Re^{1/2} \leq 1$</td>
<td>1.0</td>
<td>0.14</td>
<td>2</td>
</tr>
<tr>
<td>Hall and Pruppacher (1976)</td>
<td>$Sc^{1/3} Re^{1/2} &gt; 1$</td>
<td>0.86</td>
<td>0.28</td>
<td>1</td>
</tr>
<tr>
<td>Thorpe and Mason (1966)</td>
<td>$10 &lt; Re &lt; 200$</td>
<td>0.94</td>
<td>0.33</td>
<td>1</td>
</tr>
</tbody>
</table>
In order to incorporate the ventilation–influenced growth ratio into the model, \( \Gamma \) is simply replaced with \( \Gamma^* \) in Eq. (2.17). The ventilation coefficient \( f \) that goes into the mass growth rate Eq. (2.31) is that for a spherical particle, following Chen and Lamb (1994).

In the previous sections, we have discussed our parameterizations of surface kinetics, habits, and ventilation. In order to understand the feedbacks between these effects and crystal growth within a cloudy environment, these growth methods are incorporated into a microphysical parcel model.

### 2.6 Microphysical Parcel Model

In order to study the effects of deposition on a distribution of ice particles in a cloudy environment, the parameterizations described above were implemented into the Lagrangian microphysical parcel model of Lebo (2007) and Lebo et al. (2008). The parcel model is initialized with a cloud base temperature, pressure, and prescribed vertical velocity profile, and the parcel is initially slightly subsaturated with respect to ice. As the parcel rises, the saturation increases, and once it becomes supersaturated, a gamma distribution of ice crystals with radii ranging from 0.1 to 40 \( \mu \text{m} \) is instantaneously nucleated. The concentration, \( N_i \), is prescribed and does not change throughout the model run. For studies involving falling particles, only one size was used, in order to assess the terminal fallspeed influences on growth.

Cloud parcel evolution is determined by solving a set of \( n \)-differential equations using the Variable Order Differential Equation (VODE) software (Brown et al., 1989). The equations capture the physics of supersaturation development, conservation of mass and energy, ambient pressure changes, and crystal size and shape evolution. The basic parcel equations are presented and described in detail by Lebo et al. (2008) for spherical crystals. The Chen and Lamb (1994) model was recently added by Lebo (personal communication) and is shown here as Eqs. (2.16) and (2.17). A summary of the equations used is presented in Appendix C.

Since the original model developed by Lebo et al. (2008) was designed for simulations of mixed-phase clouds, a number of alterations were made for our studies. First, since the focus of this study is crystal growth in cirriform clouds, all
equations dealing with liquid water are turned off. Similarly, radiative influences on ice growth are beyond the scope of this study and are neglected. The method for computing a variable deposition coefficient, as described in Appendix A, is included in the mass growth rate equation. For simulations including effects of ventilation, the ventilation coefficient is incorporated into the mass growth rate equation via the methods described in section 2.5, and the terminal fallspeed is also included. For tests involving ventilation, the temperature and saturation equations (C.7) and (C.5) have been modified for freely-falling ice crystals. We use the moist adiabatic lapse rate of 7 K/km for changes in temperature, and the saturation ratio is kept constant to mimic the analysis of Hall and Pruppacher (1976).
Chapter 3
Deposition Coefficient Influences

The following sections discuss the general model results for all the tested choices of the deposition coefficient: constant $\alpha$, and $\alpha$ evolving for 2D nucleation and spiral dislocations. These studies are done as sensitivities since some of the parameters in the $\alpha$ relations are not well known. Processes occurring at scales ranging from that of the ambient parcel down to the scale of the deposition mechanisms are examined. Studies are done for parcels of two types: one with a strong vertical velocity, such as would occur in cirrus uncinus, and one with a weak vertical velocity, such as in cirrostratus (Heymsfield, 1975a, 1977). The microphysical simulations are then used to examine the hypothesis that the distinguishing characteristics of cirrus uncinus and cirrostratus clouds are based in their microphysical roots. For instance, since the large crystals of cirrus uncinus clouds produce extensive precipitation tails, it is suspected that ice growth in such clouds is more efficient (larger $\alpha$) than in cirrostratus clouds. Sensitivities of $\alpha$ to updraft velocity, crystal concentration, and ambient temperature are examined within the context of cirrostratus and cirrus uncinus clouds.

We begin our examination of the influence of surface kinetics on modeled cirrus clouds using spherical particles. This is done for two reasons. First, the inclusion of the deposition coefficient in the capacitance model is strictly correct for spheres. Second, neglecting habit variation allows a focus on surface–kinetic dependences alone without the complicating factor of habit change. The following subsections examine the model simulations using constant and variable deposition coefficients. For the variable–$\alpha$ tests, values of the critical supersaturation are required.
cause critical supersaturations are not well known, sensitivities are run over a range of values. These analyses allow us to examine the physical plausibility of the different surface–kinetic growth theories.

Following the general analysis is a focused comparison of the evolution of cirrus properties under each of the two surface–kinetic growth theories: dislocations and 2D nucleation. The section combines the microphysical comparison with a discussion of the hypothesis that diverse cirriform cloud forms arise from distinct surface processes.

### 3.1 Overall Results for Surface–Kinetic Theories

For the initial investigation into ice crystal growth for surface–kinetic growth theories, a parcel of slightly subsaturated (with respect to ice) air is initiated at a temperature of $-30^\circ\text{C}$ and a pressure of 350 mb. The parcel rises at a constant updraft velocity of either 15 cm/s, to represent the slow rising motion that gives rise to cirrostratus (Heymsfield, 1977), or 75 cm/s, to represent a cirrus uncinus updraft (Starr and Cox, 1985). When the parcel becomes supersaturated with respect to ice, a gamma distribution of spherical ice crystals with radii ranging from 0.1 to 40 $\mu$m is instantaneously nucleated. For these model runs, bins containing small ice crystals (a few $\mu$m in radius or less) dominated the concentration. For these general tests, the crystal concentration is set at 100 L$^{-1}$ and ventilation is neglected. The parcel continues to rise to 2 km above cloud base at a constant updraft velocity of 75 cm/s.

Figure 3.1 displays the evolution of crystal radius $r$, ambient supersaturation $s_{amb}$, and deposition coefficient $\alpha$ over a depth of 2 km. While most cloud models do not even include $\alpha$, those that do typically assume that $\alpha$ is constant (Hall and Pruppacher, 1976; Gierens et al., 2003). The black curves display results using a typical cloud model choice of constant $\alpha = 1$. We observe the crystals growing to their maximum potential corresponding to the maximum fraction of molecular incorporation represented by $\alpha = 1$.

In red we see the results using a constant deposition coefficient of $\alpha = 0.006$, a value experimentally determined by Magee et al. (2006). We observe the inhibitions to crystal growth resulting from the limited incorporation of vapor molecules,
**Figure 3.1.** Cirrus property sensitivity to constant (black, $\alpha = 1$; red, $\alpha = 0.006$) and predicted (blue, spiral dislocations, $s_{\text{crit}} = 0.1$) deposition coefficient. Crystal radius (top), ambient supersaturation (middle), and deposition coefficient (bottom) evolve over a depth of 2 km for an updraft velocity of 75 cm/s.
represented by the smaller value of $\alpha$. Also shown are much larger values of the ambient supersaturation, a result produced by the reduced uptake of available vapor by the kinetically–limited ice crystals.

Finally, results using a predicted deposition coefficient are portrayed in blue in Fig. 3.1. Here, the deposition coefficient is evolving according to dislocation theory for a critical supersaturation of 0.1. Figures 3.2 and 3.3 explore the cirrus property trends resulting from predicted $\alpha$ defined by surface–kinetic theory.

Figure 3.2 depicts the paths of $r$, $s_{amb}$, and $\alpha$ for crystals evolving under dislocation constraints. We observe that choices of $s_{crit}$ (0.1, 1, and 10) produce comparable crystal sizes. These similarities come about for a number of reasons. First, the parcel contains spherical crystals, and as we shall examine in detail in section 4.2, linear growth of spherical particles is inversely proportional to size via

$$\frac{dr}{dt} = \frac{1}{r} \rho_{ice} \left( \frac{R_c T}{e_c D_v^*} + \frac{l_s}{K^* T} \left( \frac{l_s}{R_c T} - 1 \right) \right).$$  \hspace{1cm} (3.1)

Second, as previously mentioned, a permanent dislocation existing on a crystal surface implies that crystal growth is relatively insensitive to the critical supersaturation. Third, as we will explore in detail in section 3.2 and Appendix B, large values of $\alpha > 0.01$ imply diffusive limitations. In other words, taking the large $r\alpha$ limit of the modified diffusion parameter $D_v^*$, defined by Eq. (2.6), results in an insensitivity to the surface kinetics represented by $\alpha$:

$$\lim_{ar\to\infty} D_v^* = \frac{D_v(r + \Delta)}{r}. \hspace{1cm} (3.2)$$

The same examination of cirrus property evolution for crystals evolving under 2D nucleation constraints produces drastically different results. Figure 3.3 shows the paths of $r$, $s_{amb}$, and $\alpha$ for the same choices of $s_{crit}$ (0.1, 1, and 10). Here we observe distinct crystal sizes evolving, and diverse ambient supersaturations evolving in response to crystal growth. The crystal radii differ for a number of physical reasons. First, since 2D nucleation requires that $s_{crit}$ is achieved by $s_{local}$ prior to the onset of crystal growth, crystal growth begins at differing altitudes within the cloud (or, in the case of $s_{crit} = 10$, never at all). Also, the values of the deposition coefficient evolved for this case are small ($\alpha < 0.01$), implying kinetic
Figure 3.2. Cirrus property evolution for spiral dislocations, for $s_{\text{crit}} = 0.1$ (blue), $s_{\text{crit}} = 1$ (orange), and $s_{\text{crit}} = 10$ (green). Crystal radius (top), ambient supersaturation (middle), and deposition coefficient (bottom) evolve over a depth of 2 km for an updraft velocity of 75 cm/s.
Figure 3.3. Cirrus property evolution for 2D nucleation, for $s_{\text{crit}} = 0.1$ (blue), $s_{\text{crit}} = 1$ (orange), and $s_{\text{crit}} = 10$ (green). Crystal radius (top), ambient supersaturation (middle), and deposition coefficient (bottom) evolve over a depth of 2 km for an updraft velocity of 75 cm/s.
limitations. In other words, taking the small \( r\alpha \) limit of \( D_v^* \) results in a large sensitivity to surface kinetics, as represented by \( \alpha \):

\[
\lim_{\alpha r \to 0} D_v^* = \frac{\alpha r v}{4} \propto \alpha. \tag{3.3}
\]

Figure 3.3 shows that for a critical supersaturation \( s_{\text{crit}} = 10 \), the onset of crystal growth never occurs, since this value is never reached by \( s_{\text{local}} \). The lack of vapor uptake, and neglect of nucleation in the model, produce unphysically large results for the ambient supersaturation. A critical supersaturation \( s_{\text{crit}} = 1 \) produces similar results: the onset of crystal growth occurs at an altitude high above cloud base, allowing the ambient supersaturation to rise to similarly large values.

A critical supersaturation of 0.1 provides the most realistic results for both surface–kinetic growth theories. These observations are supported by the typical value estimate of Burton et al. (1951), who place the value of \( s_{\text{crit}} \) on the order of 0.1. For these reasons, crystal growth using this choice of critical supersaturation will be the focus of the subsequent analyses.

In summary, one can see that choices of a constant \( \alpha > 0.01 \) compare relatively well with crystal growth using surface dislocations, since the large values of \( \alpha \) in these cases reflect the fact that the dominant vapor transport resistance is caused by diffusive, not kinetic, limitations (see section 3.2 along with Appendix B). A constant value of \( \alpha \) cannot capture the evolution of kinetically-sensitive 2D nucleation growth properties. The next section provides a more in-depth investigation into these findings.

### 3.2 Cirrus Uncinus vs. Cirrostratus:

**Distinctions Resulting from Microphysical Origins?**

All cirriform clouds are composed mainly of ice and characterized by high altitudes and low temperatures. Within this category are a number of cirrus types, most notably cirrus uncinus (Ci) and cirrostratus (Cs). Ci clouds are produced
by strong local updrafts, and they are typically composed of large particles that sediment and are observed as fallstreaks. In contrast, Cs clouds form in response to large-scale motions, and are composed of many tiny ice crystals (Heymsfield, 1975a, 1977). We hypothesize that the distinct crystal sizes observed between the two cirrus cases depend upon the growth processes occurring at the crystal surface.

The model results can be analyzed to see whether or not they support the hypothesis that the size ranges of ice particles in cirriform clouds are the products of surface kinetics. Specifically, do the tiny ice crystals of Cs grow inefficiently (low $\alpha$), whereas the larger crystals of Ci grow efficiently (high $\alpha$)?

Our parcel model allows us to examine crystal growth in response to evolving ambient and surface-kinetic conditions. We must keep in mind that crystal growth is also responsive to ice nucleation, which is not included in the model. Therefore, in the atmosphere, the efficiency with which ice crystals grow cannot be traced solely to the choice of surface-kinetic growth mechanism. Rather, it is the interaction of the crystal growth with larger cloud attributes that cause the distinct characteristics of Cs and Ci clouds. For instance, Fig. 3.4 shows the crystal sizes evolving for an updraft velocity of 15 cm/s and for dislocation and 2D nucleation growth with a critical supersaturation of 0.1. With all other conditions (updraft velocity, ice concentration, etc.) being identical, this analysis allows us to isolate the surface-kinetic dependence of microphysical growth.

The crystal radii displayed in Fig. 3.4 tell us that size sensitivity to growth mechanism is most prominent where crystals form, namely cloud base at 7 km. At the lowest levels of the cloud, differences in crystal radius between the two mechanisms exceed 30 $\mu$m. Sensitivity to growth mechanism decreases as the parcel rises further, since the linear growth of a spherical particle is inversely proportional to its radius (see section 4.2 on page 56).

Although the difference in crystal size near cloud base appears to be significant, it is not as large as the differences between observed cirrus uncinus and cirrostratus crystal sizes. The measurements of Heymsfield (1975a), shown in Table 3.1, provide us with observed ranges of average crystal lengths for five cirrus uncinus cases, two cirrostratus cases, and one case involving a combination of the two. The crystals observed by Heymsfield (1975a) encompass a variety of crystal shapes, whose lengths were averaged to provide the values in Table 3.1. For simplicity, our
Figure 3.4. Crystal radii for growth by dislocations (orange) and 2D nucleation (blue), with critical supersaturations of 0.1. An updraft with a velocity of 15 cm/s lifts 100 L$^{-1}$ crystals for a distance of 2 km.

analysis uses spherical crystals, with the maximum length being the diameter. The maximum difference between sizes evolved via dislocational and 2D nucleational growth in our analysis is, in terms of diameter, about 70 µm. This value is less than half of the minimum size difference (150 µm) between a cirrus uncinus and a cirrostratus case observed by Heymsfield (1975a). A glance at the maximum size difference (550 µm) between the two cloud cases especially drives home the point that in an environment of invariant concentration and dynamics, as in our model, microphysics alone cannot distinguish a cirrus uncinus cloud from a cirrostratus cloud. Further studies of cirrostratus indicate similar findings: McFarquhar and Heymsfield (1997) reported that small ice crystals were primary contributors to the ice concentration in high-altitude layer cirrus clouds observed during the Central Equatorial Pacific Experiment (CEPEX).

Before getting into a discussion of the combination of factors that may distinguish cirriform clouds from one another, let us understand physically why differences in vapor growth alone do not produce large distinctions in crystal sizes for
Table 3.1. Summary of Heymsfield (1975a) cloud measurements.

<table>
<thead>
<tr>
<th>Measurement Date</th>
<th>Cloud Type</th>
<th>Altitude (km)</th>
<th>Temperature Range (°C)</th>
<th>Avg. IWC (g m$^{-3}$)</th>
<th>Total Conc. (cm$^{-3}$)</th>
<th>Avg. Conc. &gt; 0.15 mm (cm$^{-3}$)</th>
<th>Mean Crystal Length (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>07–09–1970</td>
<td>Ci</td>
<td>6.5 – 8.8</td>
<td>-15 to -30</td>
<td>0.24</td>
<td>-</td>
<td>0.019</td>
<td>0.9</td>
</tr>
<tr>
<td>09–19–1971</td>
<td>Ci</td>
<td>8.5 – 9.1</td>
<td>-36 to -43</td>
<td>0.24</td>
<td>-</td>
<td>0.025</td>
<td>0.75</td>
</tr>
<tr>
<td>09–19–1971</td>
<td>Ci</td>
<td>8.9 – 9.35</td>
<td>-40 to -44.5</td>
<td>0.15</td>
<td>-</td>
<td>0.03</td>
<td>0.75</td>
</tr>
<tr>
<td>04–06–1972</td>
<td>Ci</td>
<td>9.6 – 11.</td>
<td>-41.7 to -57.6</td>
<td>0.30</td>
<td>0.40</td>
<td>0.03</td>
<td>0.6</td>
</tr>
<tr>
<td>04–06–1972</td>
<td>Ci</td>
<td>9.8 – 10.1</td>
<td>-43 to -46</td>
<td>0.20</td>
<td>0.20</td>
<td>0.04</td>
<td>0.5</td>
</tr>
<tr>
<td>04–06–1972</td>
<td>Ci to Cs</td>
<td>9.35 – 9.8</td>
<td>-14 to -45</td>
<td>0.075</td>
<td>0.30</td>
<td>0.015</td>
<td>0.5</td>
</tr>
<tr>
<td>03–04–1972</td>
<td>Cs</td>
<td>4.45 – 8.5</td>
<td>-20 to -46</td>
<td>0.026</td>
<td>-</td>
<td>0.008</td>
<td>0.35</td>
</tr>
<tr>
<td>04–04–1972</td>
<td>Cs</td>
<td>7.9 – 8.9</td>
<td>-35 to -44.5</td>
<td>0.025</td>
<td>0.20</td>
<td>0.004</td>
<td>0.35</td>
</tr>
</tbody>
</table>

Note that the deposition coefficient associated with 2D nucleation growth is approximately one order of magnitude smaller than that of dislocation growth, representing a large decrease in growth efficiency. However, as previously noted, the differences in crystal size between each surface–kinetic growth mechanism are not substantially great. How can we reconcile these two seemingly opposing results?

An explanation for this phenomenon can be arrived at by examining the mass growth rate equation in its kinetic and diffusive limits. Appendix B provides the full mass growth rate equation before and after taking limits for small and large products of crystal radius and deposition coefficient.

A particle evolving under diffusion–limited growth ($r\alpha$ is large) will have a mass growth rate proportional to the diffusion coefficient, $D_v$. Since the curves of dislocational modified diffusivity and the diffusion coefficient in Fig. 3.5d follow identical trends, one can clearly see that dislocation growth is close to diffusion–limited. This observation is further supported by Fig. 3.5c, which shows large deposition coefficients for growth by the dislocation mechanism. Recall the constant $\alpha$ results of section 3.1 that showed crystal growth to be independent of $\alpha$ when $\alpha \geq 0.01$, since the dominant resistance to vapor transport is diffusive as opposed to kinetic.
Figure 3.5. Model results for crystals evolving with dislocation (orange) and 2D nucleation (blue) microphysics, with critical supersaturations of 0.1. An updraft velocity of 15 cm/s lifts 100 L⁻¹ over a distance of 2 km. Panel (a) presents crystal radii evolution over height. Panel (b) presents both the ambient (thick) and local (thin) supersaturations. Panel (c) presents the evolution of the deposition coefficient $\alpha$. Panel (d) presents the diffusivity and modified diffusivities for each microphysical option.

In the event of a particle growing while limited by surface kinetics ($r\alpha$ is small), 2D nucleation, the deposition coefficient has values < 0.01. The mass growth rate in this case is proportional to $\alpha$ via the limit of the modified diffusivity (see Appendix B). The blue curves representing 2D nucleation in Figs. 3.5c and 3.5.d portray the modified diffusivity evolving similarly to $\alpha$. Hence, 2D nucleation distinctly corresponds to growth under significant kinetic limitation.
Relating dislocation and 2D nucleation growth to growth under diffusive and kinetic limitations, respectively, provides an improved understanding of the dominant resistances to mass transport, and therefore crystal growth. However, the analysis of limitations lacks the completeness required to explain why the two surface–kinetic theories alone fail to produce the distinct differences in crystal size. The explanation is completed by Fig. 3.5b, which shows the evolutionary curves of the ambient and local ice supersaturations.

For dislocation growth, the local supersaturation over a crystal remains small, due to the existence of permanent defects on the surface of the crystal. These defects provide sites at which a vapor molecule can easily incorporate into the crystal lattice, so the onset of growth is relatively independent of the local supersaturation. The early commencement of crystal growth does not allow for a significant amount of supersaturation production prior to the domination of vapor uptake by the crystal. Thus, the balance between supersaturation production and destruction (seen around 7.5 km) is established at a low supersaturation of around 5%.

The local supersaturation for 2D nucleation is much larger than it is for dislocations, since the critical supersaturation must be reached before a critical embryo will nucleate on the surface. Therefore, the local and ambient supersaturations increase until crystal growth is initiated, and the production–destruction balance is established at a higher value of supersaturation around 20%.

The size similarities between crystals growing under dislocations and 2D nucleation are explained by examining the difference between the ambient and local supersaturation for each case. Note that the difference \(s_{\text{amb}} - s_{\text{local}}\) is approximately 10% for 2D nucleation, while it is a factor of two smaller (5%) for dislocations. The difference is essentially the difference in mass between the two cases. This difference is essentially invariant with altitude above 7.5 km, when a balance is reached between supersaturation production and destruction (see equation (3.4)). The distinction in the supersaturation difference is essentially the difference in the mass of the crystals between 2D nucleation and dislocations. At low altitudes (below 7.5 km), crystal mass is smaller, so that the \(s_{\text{amb}} - s_{\text{local}}\) difference between the two cases shows a significant difference to the growth rates. At higher altitudes, where crystal masses are larger, the supersaturation difference becomes less
influential. Therefore, combined with the principle that crystal growth is inversely proportional to size (see section 4.2), the differences in crystal radius between 2D nucleation and dislocations are expected to be small. In order to understand the distinguishing characteristics of Cs and Ci clouds, we must broaden our comprehension of crystal surface processes to encompass their interactions with other cloud properties.

3.3 Defining Cirriform Clouds: Relationships on All Scales

It has been implied that crystal growth theories alone may not provide the explanation for the small crystals seen in cirrostratus clouds and the large crystals observed in cirrus uncinus clouds. However, one must consider the limitations of the analysis and attempt to place results in a more realistic context in order to discern the roots of cirriform clouds. One of the major limitations with the parcel model is the unrealistic representation of ice nucleation. In a real cloud, ice nucleation depends on a number of factors, most notably homogeneous freezing, the existence and structure of ice nuclei, and the supersaturation (Rogers and Yau, 1989).

Differences in the deposition coefficient alone do not provide exceptionally large crystal size differences when ice concentrations are constant. However, if we add more realism to our model, we can see that vapor growth does in fact have an impact on crystal size and on potentially determining whether a cirrus cloud will be Cs or Ci.

3.3.1 Growth Dependence on Crystal Concentration and Updraft Velocity

In the present framework, we are unable to capture the feedback between crystal growth and the nucleation of new particles. However, we can examine the relationship between microscale processes and crystal sizes as a function of varying temperatures, updraft velocities, and crystal concentrations. Figure 3.6 displays the crystal diameters, ambient supersaturations, and deposition coefficients aver-
Figure 3.6. Ice crystal sizes (top), ambient supersaturations (middle), and deposition coefficients (bottom) as functions of crystal concentration and updraft velocity. The left panel shows results for 2D nucleational growth, the right panel shows results for dislocational growth. Population-averaged values of $D$, $s_{amb}$, and $\alpha$ are averaged over a depth of 2 km. Fallspeed limitations are represented by the white striped area.

Aged first over the distribution, then over the cloud depth of 2 km as functions of updraft velocity (abscissa) and crystal concentration (ordinate) for each growth mechanism. The crystal diameter plots show that the largest crystals result from low concentrations and weak updrafts. The existence of large crystals in low concentrations is to be expected, since little competition for vapor allows for optimal growth.
Somewhat unphysically, a weak updraft is shown to produce large crystals in Fig. 3.6. These large crystals result from the fact that the crystals spend more time in the supersaturated region over the depth of 2 km. To illustrate, Fig. 3.7 depicts crystal sizes, supersaturations, and deposition coefficients plotted over the span of one hour for the updraft velocity extremes of 15 and 75 cm/s. The figure shows the results for 10 $L^{-1}$ crystals evolving under dislocation growth; the 2D nucleational results show similar trends. Crystals ascending at a rate of 75 cm/s grow much larger than their 15 cm/s counterparts for a given amount of time, which helps explain the results presented in Fig. 3.6. The weak updraft results in Fig. 3.6 must be taken cautiously, since low updraft velocities would not suspend the particles long enough for them to grow large. To indicate this, the fallspeed limitations (i.e., the particles which would be precipitated out before reaching large sizes) are indicated by the white striped regions in Fig. 3.6. From this analysis, we observe a dependence of crystal growth on the balance between vertical motions: updraft versus particle fallspeed.

Although the results produce unrealistic diameters at low concentrations and updraft velocities, we can use this discrepancy to highlight the contrast between Cs and Ci clouds. The tiny crystals of Cs appear in high concentrations, and are maintained within the cloud by small updrafts. The large crystals of Ci can only come about through strong updrafts, since weak updrafts would allow crystals to precipitate out before reaching large sizes. The distinction in concentrations between Ci and Cs clouds is not reflected in the Heymsfield (1975a) data presented in Table 3.1. However, we must realize that the few cases (four) for which we have total concentration data is not a representative sample of all cirriform clouds; information is especially lacking for cirrostratus clouds. More importantly, Heymsfield (1975a) mentioned a cirrostratus case in which many small ice crystals were sampled, but no data were taken. The more recent measurements of layered cirrus clouds reported by McFarquhar and Heymsfield (1997) provide records of crystals in larger concentrations, up to the order of 1000 $L^{-1}$. These observations must be taken cautiously, however, as the data set reported included cases of both cirrus anvils and detached cirrus. Additional reports of specific Ci and Cs cases, such as those of Heymsfield (1975a), would be beneficial to simulations of these cloud types.
Figure 3.7. Crystal radius (a), deposition coefficient (b), and ambient (c) and local (d) supersaturations plotted over one hour. A parcel of 10 L$^{-1}$ crystals growing by dislocations is lifted by updrafts of 15 (blue) and 75 (green) cm/s.

The values for ambient supersaturation and deposition coefficient in Fig. 3.6 peak where crystal concentrations are low and updraft velocities are high. These peaks reflect the dependence of $\alpha$ on the ambient supersaturation, which is defined through the vapor budget equation (Lebo, 2007, p. 85):

$$\frac{ds_{amb}}{dt} = (s_{amb} + 1) \left( \left( \frac{glv}{c_pR_vT^2} - \frac{g}{R_{air}T} \right) w - \left( \frac{1}{qv} + \frac{lvl_s}{c_pR_vT^2} \right) \frac{dq_i}{dt} \right), \quad (3.4)$$

where the left term is a production term that represents the increase of super-
saturation due to cooling via parcel ascent. The right term is a destruction term representing vapor loss to growing crystals. The combination of large updraft velocities and low crystal concentrations produces the largest values of supersaturation, which is reflected in the maximum values for $\alpha$.

3.3.2 Growth Dependence on Crystal Concentration and Temperature

Because cirrus clouds reside in environments with different temperatures, we have performed the same analysis as in the previous section over the range $T = -30^\circ\text{C}$ to $-60^\circ\text{C}$. Figure 3.8 shows crystal diameter $D$, $s_{amb}$, and $\alpha$ for given crystals concentrations (ordinate) and cloud base temperatures (abscissa). A constant moderate updraft velocity of 45 cm/s is chosen for the analysis. Crystal growth is seen in Fig. 3.8 to diminish at lower temperatures. Interestingly, the crystal size reduction with temperature decrease cannot be linked to the trend of the deposition coefficient, which increases as temperature decreases, following the trend of the ambient supersaturation. In this case, the reduction in size is brought about because diffusion becomes substantially limited as temperatures decrease, a process described by the trends of the diffusion coefficient $D_v$ and the speed of vapor molecules $\overline{v}$ in Eqs. (2.2) and (2.3).

3.4 Simulated Cs and Ci

Though our microphysical parcel model has dynamic and nucleation limitations that prevent it from emulating every cloud process feedback, it does have one significant advantage over full dynamic bulk cloud models: the ability to trace the evolution of the deposition coefficient as it responds to changes in crystal size, temperature, and supersaturation in a Lagrangian microphysical framework. Because our model is simplified, we simulate the the development of $\alpha$ under imposed Cs and Ci conditions, using the results from the previous sections.

In order to represent a cirrostratus cloud, we begin with the assumption that the initial nucleated crystals are growing inefficiently, so that the available supersaturation is not significantly depleted. Consequently, many crystals would
Figure 3.8. Ice crystal sizes (top), ambient supersaturations (middle), and deposition coefficients (bottom) as functions of crystal concentration and temperature. The left panel shows results for 2D nucleational growth, the right panel shows results for dislocations. Population–averaged values of $D$, $s_{amb}$, and $\alpha$ are averaged over a depth of 2 km, for a constant updraft velocity of 45 cm/s.

nucleate as a result of the inefficiently growing existing crystals. So, we choose a large, constant concentration of $10^4$ L$^{-1}$ for Cs. To represent the small vertical motions in Cs, we use an updraft velocity of 15 cm/s (Heymsfield, 1977). To approximate cirrus uncinus properties, we assume that the original crystals grow efficiently, causing depletion of the available vapor. Since the efficiently growing
crystals would prevent large numbers of nucleated crystals, we opt for a small constant concentration of 100 L$^{-1}$ (Hall and Pruppacher, 1976). A large updraft velocity of 75 cm/s is chosen to mimic observed values (Starr and Cox, 1985). For both the Cs and Ci cases, a cloud base temperature of $-30^\circ$C is specified in order to retain comparative quality.

Figure 3.9 displays the results from the simulated Cs and Ci cases for 2D nucleation and dislocations. For both growth mechanisms, the crystal radius curves portray size differences between the two cases ranging from tens of microns near cloud base to approximately 75 $\mu$m at cloud top. In terms of diameter, the cloud top difference of around 150 $\mu$m fits within the range of the Heymsfield (1975a) observed Ci to Cs crystal size differences (150 – 550 $\mu$m).

The structures of the deposition coefficient curves offer insight into the drastic size differences observed between the Cs and Ci cases. In the Cs case, evolving with inefficient values of $\alpha$, the deposition coefficient spikes once the critical supersaturation has been attained, then falls off so that the average $\alpha$ value is more than two orders of magnitude smaller than that for the efficient Ci case. The deposition coefficient and crystal size remain small, and are placed in the kinetically–limited region of $\alpha < 0.01$, due to the competition for vapor of 10$^4$ L$^{-1}$ crystals.

The Ci case, evolving for efficient values of $\alpha$, produces much larger crystal sizes than the Cs case, as is to be expected. The minimal competition between the 100 L$^{-1}$ crystals, along with the larger updraft velocity, drives the ambient supersaturation to higher values. The deposition coefficient responds to the vapor availability by remaining in the diffusion–limited region of $\alpha > 0.01$.

It is important to note that for the simulated cirrus uncinus and cirrostratus parcels in Fig. 3.9, $w = 15, 75$ cm/s and $N_i = 100$ L$^{-1}$ mimic observed values for these cloud types (Heymsfield, 1977; Starr and Cox, 1985; Hall and Pruppacher, 1976). The simulated Cs particle concentration (10$^4$ L$^{-1}$) is large compared to the results of Heymsfield (1975a). However, the cirrus measurements from CEPEX reported by Heymsfield and McFarquhar (1996) indicate that this value is perhaps not unreasonable.

In order to relate crystal microphysical origins to the evolution of cirrostratus and cirrus uncinus clouds, one must be able to capture feedbacks occurring on all scales, from ambient supersaturation development down to evolution of the
Figure 3.9. Simulated Ci and Cs: evolution of crystal radius (top), ambient ice supersaturation (middle, thick), local ice supersaturation (middle, thin), and deposition coefficient (bottom). Plots a, c, and e evolve under 2D nucleation constraints; plots b, d, and f evolve under dislocation constraints. The Ci curves are simulated by concentrations of $100 \text{ L}^{-1}$ and updraft velocities of 75 cm/s. The Cs curves are simulated by concentrations of $10^4 \text{ L}^{-1}$ and updraft velocities of 15 cm/s. For all cases, cloud base temperature is $-30^\circ\text{C}$. 
deposition coefficient. Currently, cloud models are unable to provide representations of clouds on all scales. We attempt to simulate the feedbacks between the deposition coefficient and larger-scale properties, such as crystal growth and supersaturation evolution, within the nucleation and dynamic limitations of our microphysical model. The analyses suggest that crystal surface kinetics can indeed produce some of the characteristics of the distinct cirriform cloud types that we observe. A model that was able to capture the variations and interactions of cloud microphysics, dynamics, and thermodynamics may be expected to produce the above result.

For now, we can assume that low values of the deposition coefficient would cause high supersaturations and numerous ice particles in a Cs cloud, since the limited mass uptake by the first crystals nucleated will not significantly deplete the available vapor. Eventually, the ice crystal population will become so numerous that the supersaturation will be depleted, resulting in small crystals inefficiently growing with correspondingly small deposition coefficients. Small updraft velocities will also minimize the vapor available to the crystals. In contrast, high values of the deposition coefficient can cause the first crystals nucleated in a Ci cloud to grow very efficiently, thus driving down the available supersaturation and limiting ice nucleation. As strong updraft velocities move the crystals, the supersaturation also increases substantially, allowing for even more efficient growth and large deposition coefficients. The constant-\(\alpha\) simulations of Khvorostyanov et al. (2006) agrees with our hypothesis, in that their model show a small value of \(\alpha (= 0.07)\) corresponding to larger ice concentrations and a large value of \(\alpha (= 1)\) corresponding to smaller ice concentrations, implying a negative feedback between crystal growth and nucleation rates.

As to the significance of step origins to the evolution of either Cs or Ci clouds, for now one can only speculate that the influence, if any, is small. For instance, perhaps the intense ice nucleation in a Cs cloud is brought about because the initial ice crystals are growing inefficiently under 2D nucleation. By contrast, it is possible that the limited ice nucleation characteristic of Ci clouds occurs because the initial crystals are growing efficiently under dislocation growth. Testing this hypothetical projection would require a model with both homogeneous and heterogeneous nucleation. Based on the model results in Fig. 3.9, this thought is
improbable and unlikely, however, as with all ambiguous speculations, should not be disregarded entirely.

Crystal microphysical processes, characterized by the deposition coefficient, are shown to influence the efficiency of vapor uptake by crystals. Put in the appropriate physical context, this efficiency can be linked to the evolution of different types of cirriform clouds. In the next chapter, we expand these ideas to consider the effects of crystal habit on crystal growth efficiency.
Chapter 4

Habit Influences

The results discussed in the previous chapter present the dependence of cirrus cloud properties on surface kinetics for spherical particles. This chapter presents the effects of different crystal shapes on the development of modeled cirriform clouds, with a specific focus on processes occurring on the microscale.

4.1 Shape Effects on Microscale and Particle-Scale Relationships

In order to simulate different spheroidal shapes, we choose values of the inherent growth ratio, $\Gamma$, to represent the primary habits of columns and plates (see Chapter 2). Figure 4.1 shows results for parcel simulations of crystal growth. Shown are crystal size, ambient and local supersaturation, and deposition coefficient for $\Gamma$ values of 1, $1 \pm 0.1$, and $1 \pm 0.25$, with values greater than one representing columns and values less than one representing plates.

Note that the largest particles occur for $\Gamma = 1.25$, and surprisingly, values of the local supersaturation are smallest for this case. This seems counter-intuitive because prior results suggest that the local supersaturation over the face experiencing maximum growth (i.e. the face at the end of the maximum dimension) is large because of the strong vapor gradients (Chen and Lamb, 1994). However, we must keep in mind that the values of local supersaturation are defined for the entire particle. Figure 4.2 is a schematic showing areas of large and small super-
Figure 4.1. Parcel property evolution for different particle shapes characterized by $\Gamma = 1, 1 \pm 0.1,$ and $1 \pm 0.25$. The top panel displays the evolution of the spherical volume equivalent radius over a depth of 2 km for a parcel ascending at 75 cm/s. The middle panel presents the local (thin) and ambient (thick) supersaturation fractions with respect to ice. The bottom panel shows the deposition coefficient. A concentration of 100 L$^{-1}$ crystals evolves under 2D nucleation (critical supersaturation of 0.1) microphysics at an initial temperature and pressure of -30$^\circ$C and 350 mb, respectively.
Figure 4.2. Local supersaturation by crystal area.

saturations for plates and columns. In the case of the column, the basal faces are characterized by small areas and large local supersaturations. Conversely, the prism faces are characterized by large areas and small local supersaturations. The area–averaged local supersaturation can therefore be small. The same is true for plate crystals. The particle–averaged deposition coefficient follows the same trend (Fig. 4.1). We expect that $\alpha$ will be largest at the small areas that extend furthest into the supersaturated region, leading to growth rate enhancement as habits become more pronounced. However, because slower growing faces make up more of the total particle area, the average $\alpha$ should decrease as particle size increases. Indeed, Fig. 4.1 shows that this is the case.

It is also of interest to examine the evolution of the aspect ratio (averaged over the distribution) for each choice of $\Gamma$; the trends are plotted in Fig. 4.3. As expected, $\Gamma = 1$ produces spherical crystals. Similar deviations from $\Gamma = 1$ produce
Figure 4.3. Population–averaged (100 L$^{-1}$) aspect ratio for different crystal shapes. The parcels ascend at 75 cm/s, with an initial temperature and pressure of -30$^\circ$C and 350 mb. Crystals evolve according to 2D nucleation (critical supersaturation of 0.1) microphysics.

similar logarithmic deviations from $\phi = 1$. In order to understand why the growth rates are enhanced for columnar crystals, consider the conceptual model of the growth for plates and columns shown in Fig. 4.4. Assume that the same amount of positive mass change occurs for a plate and a column. As Fig. 4.4 shows, to a rough approximation, the plate is growing in a 2–dimensional fashion, whereas the column essentially grows in one dimension, leading to a larger change of the aspect ratio for columns. We will see that this phenomenon also contributes to the evolution of the size distributions for different particle shapes.
4.2 Effects of Crystal Shape on Size Distribution Evolution

Recall that ice is initiated in the parcel model by nucleating a gamma distribution of ice crystals once the parcel becomes supersaturated with respect to ice. For spherical crystals ($\Gamma = 1$), the crystal spectrum becomes monodisperse in time. This narrowing of the spectrum is due to the fact that the linear growth rate of a spherical particle is

\[
\frac{dr}{dt} = \frac{1}{r} \rho_{\text{ice}} \left( \frac{R_v T}{e_v D_T} + \frac{l_v}{K_v T} \left( \frac{l_v}{R_v T} - 1 \right) \right).
\]

(4.1)

Since linear growth of a spherical crystal is proportional to the inverse of its radius, crystal growth decreases with size. Rapidly growing small crystals will catch up to
slower growing large crystals in time, producing a narrow spectrum of particles.

Nonspherical crystal distributions have quite a different evolution over time. Harrington et al. (1995) discussed the broadening of distributions of pristine needles and plates with time. Figure 4.5 supports this result by comparing the size distributions of three cirrus particle shapes (spheres, short columns, and bullets) as they evolve with height in the model cloud. In all cases, concentrations of 100 L\(^{-1}\) are used with 2D nucleation surface kinetics. Distributions are plotted in terms of the spherical volume equivalent radii of the crystals. Figure 4.5a shows that, indeed, a distribution of spherical crystals (\(\Gamma = 1\)) becomes monodisperse with height. Figures 4.5b and 4.5c show that the distributions of short columns (\(\Gamma = 1.1\)) and bullets (\(\Gamma = 1.25\)) broaden with height, with the broadest spectrum resulting for bullets for the reasons explained in the previous section.

These findings are general for all crystals shapes in that the amount of distribution broadening is related to the deviation of growth ratio \(\Gamma\) from unity. As expected based on our prior discussion (e.g. Fig. 4.1), distributions of columnar crystals display a larger sensitivity to \(\Gamma\) deviation from unity than do distributions of plate-like crystals. To quantify the degree of broadening, Fig. 4.6 displays the distribution standard deviations for \(\Gamma\) values of 1, 1 ± 0.1, and 1 ± 0.25. Standard deviations are computed using:

\[
\sigma = \sqrt{\frac{\sum (r_j - \bar{r})^2}{N - 1}},
\]

(4.2)

where \(r_j\) is the spherical volume equivalent radius of the crystals in each bin, \(N\) is the total number of bins that contain crystals (28 in these cases), and \(\bar{r}\) is the concentration–averaged spherical volume equivalent radius calculated according to

\[
\bar{r} = \frac{\sum r_j n_j}{\sum n_j},
\]

(4.3)

where \(\sum n_j\) is the total crystal concentration (100 L\(^{-1}\) in this case). Again, one sees that spheres (\(\Gamma = 1\)) approach a monodisperse distribution (small \(\sigma\)) with
Figure 4.5. Ice particle distributions for (a) spheres, (b) short columns, and (c) bullets at 50 m, 1 km, and 2 km above cloud base. A concentration of 100 L$^{-1}$ crystals are evolving under 2D nucleation microphysics (with a critical supersaturation of 0.1) at 75 cm/s.
Figure 4.6. Size distribution standard deviations: Distributions of 100 L$^{-1}$ crystals evolve over a depth of 2 km under 2D nucleation (critical supersaturation of 0.1) microphysics. Cloud base temperature and pressure are –30$^\circ$ and 350 mb respectively. Crystals ascend at 75 cm/s.

altitude. Physically, the broadening of the spectra associated with non-spherical particles is due to the physical process discussed in section 4.1. Distributions of prolate spheroids representing columnar crystals have larger standard deviations than their oblate–spheroid counterparts (Fig. 4.6) with the same deviation of $\Gamma$ from unity. As explained above (Fig. 4.4), columns grow approximately in one dimension and so experience larger size increases than do plates.

It is interesting that the standard deviations for nonspherical particles in Fig. 4.6 decrease to a minimum around 7.25 km before increasing, indicating a period of distribution narrowing prior to broadening. This result is perhaps counter–intuitive, but it turns out that this narrowing is due to more rapid initial growth of the smallest crystals. Figures 4.7a and 4.7b show the size and aspect ratio distributions for oblate spheroids ($\Gamma = 0.75$) at different altitudes. The initial crystal distribution is shown in red. The smallest crystals in the distribution (bin 1) occur in a concentration of approximately 0.005 cm$^{-3}$. Bin 1 is indicated throughout with a black
Figure 4.7. Size (a) and aspect ratio (b) distributions at low altitudes for 100 L$^{-1}$ 2D nucleation oblate spheroids ($\Gamma = 0.75$) ascending at 75 cm/s. The black dot indicates bin 1. Since the tiny crystals in bin 1 experience the greatest changes in aspect ratio, their growth rates are enhanced. As time increases, bin 1 eventually contains the largest crystals.

dot. As altitude increases, the distribution first narrows, then broadens. Note that this distribution evolution is due to the fact that the smallest crystals grow more rapidly, eventually over-taking the initially larger crystals. The physical reason for this phenomenon lies in the nonlinear change in aspect ratio for a given change in volume (Chen and Lamb, 1994). The equation relating the aspect ratio ($\phi$) and volume ($V$) is
Figure 4.8. Capacitance (a) and semi-major axis $a$ (b) distributions at low altitudes for 100 L$^{-1}$ 2D nucleation oblate spheroids ($\Gamma = 0.75$) ascending at 75 cm/s. The black dot indicates the position of bin 1. The capacitance of bin 1 increases as the maximum dimension increases.

$$d \ln(\phi) = \frac{\Gamma - 1}{\Gamma + 2} d \ln(V).$$

The relative change of volume is largest when ice particles are very small (Chen, 1992, p. 181), such as those occurring in bin 1. This large relative volume change implies a large change in the aspect ratio for small crystals. As discussed in previous paragraphs, the greater the deviation of the aspect ratio from unity, the greater the mass uptake by the crystal, so the crystals in bin 1 have the largest mass growth rates. The changes in mass (volume) cause $\phi$ to change rapidly, which leads
Figure 4.9. Deposition coefficient distributions at low altitudes for $100 \text{ L}^{-1} \ 2D$ nucleation oblate spheroids ($\Gamma = 0.75$) ascending at 75 cm/s. The black dot indicates bin 1. The deposition coefficient at the lower heights represents the large growth rates of the crystals in bin 1.

to even stronger growth. Hence, by the time the smallest crystals have sizes similar to the initially–larger crystals, their growth rates are much larger. To show this phenomenon, Fig. 4.8 gives the capacitance and maximum dimension $a$ for the same heights as in Fig. 4.7. The capacitance of the smallest crystals in bin 1 increases as crystal habits become more pronounced, and eventually increases beyond the capacitances of the initially larger crystals. We see that the size contribution to the capacitance factor (Fig. 4.8b) dominates over the shape contribution portrayed in Fig. 2.10a in section 2.3, which is shown to decrease as $\phi$ deviates from unity. The large values of the semi–maximum dimension (Fig. 4.8b) dominate, causing the capacitance to increase.

The evolution of the size spectrum has important implications for surface kinetics. Figure 4.9 plots the deposition coefficient distributions at the same heights. One can see that values of $\alpha$ are largest for the smallest crystals (bin 1) until the transition of these crystals to the largest sizes, around 7.225 km. After the transi-
tion, $\alpha$ is smallest in bin 1, presumably due to area–averaging the particles with the greatest aspect ratio deviation from one (section 4.1) and the size–dependence of $\alpha$ (section 2.2).

Crystal shape is seen to have a strong effect on the evolution of the deposition coefficient, and consequently on the distribution of crystals. The next chapter explores the effects of ventilation on different crystal shapes as they fall through regions of subsaturated air.
Chapter 5

Ventilation Influences

The following sections present studies of crystal sublimation enhancement by ventilation. For all studies, particles fall through a region of clear air at a constant value of subsaturation. The temperature evolves according to the moist adiabatic lapse rate (~7 K/km).

5.1 Ventilated Spheres

Recall that we use two methods to compute ventilation in our model: those of Thorpe and Mason (1966) and Hall and Pruppacher (1976). The model results are compared with the observations of Braham and Spyers–Duran (1967), who reported cirrus particle trails with concentrations of 100–1000 $L^{-1}$ surviving a fall distance of 5 km in clear air that was 70% subsaturated with respect to ice. Both Braham and Spyers–Duran (1967) and Hall and Pruppacher (1976) have attempted to numerically reconcile this phenomenon and indicate that ice particle persistence over depths of 5 km is only possible in less subsaturated air. This discrepancy is investigated here using a variety of cases of deposition coefficient evolution and ventilation computations.

In order to perform the sensitivity tests, 100 $L^{-1}$ ice crystals (Braham and Spyers-Duran, 1967) of radius 160 $\mu$m (Hall and Pruppacher, 1976) are initiated at 9 km (cirrus level). The ambient conditions at the cirrus level are obtained from the sounding provided by Braham and Spyers–Duran (1967): $T = -40^\circ$ and $p = 300$ mb. These crystals fall at their terminal velocities, computed according to Eq.
Figure 5.1. Fall distances for spherical ice particles in 70% subsaturated air as a function of growth mechanism and ventilation effect. All 100 L\(^{-1}\) crystals have initial radii of 160 µm. Temperature and pressure at 9 km are -40°C and 300 mb, respectively.

(2.39), through clear air at a constant value of subsaturation, until the point of their complete sublimation.

Figure 5.1 displays the fall distances over time for spherical particles in air 70% subsaturated with respect to ice. Both ventilation methods are plotted, along with a reference case with no ventilation. Each ventilation case uses 2D nucleation or dislocations. Spheres falling with ventilation computed via the method of Thorpe and Mason (1966) produce the shortest fall distances, around 1.25 km. The ventilation coefficient as calculated by Hall and Pruppacher (1976) allows spheres to survive slightly longer than the Thorpe and Mason (1966) spheres. The unphysical case of non–ventilated spheres produces the largest fall distances, around 1.75 to 2 km. These distances are nowhere near the 5 km fall distance observed by Braham and Spyers–Duran (1967) for this amount of subsaturation. Therefore, it seems that Hall and Pruppacher (1976) were correct to suggest that the humidity conditions reported on the day of observation (July 8, 1966) were inaccurate. The microphysical analysis of Jensen et al. (1994) also supports this implied mismeasurement.
Figure 5.2. Properties of the falling spheres of Figure 5.1: (a) radius, (b) ventilation coefficient, (c) deposition coefficient, and (d) local supersaturation.

The reason for the differences in fall distances seen in Fig. 5.1 are found in Fig. 5.2. Here we see that the shortest fall distances (ventilation via Thorpe and Mason, 1966) correspond with the fastest sublimating crystals. The large ventilation coefficients associated with this case enhance the rate of sublimation, as do the larger values of the deposition coefficient. Crystals falling with ventilation effects computed via Hall and Pruppacher persist for longer distances, and are characterized by smaller values of $f$ and $\alpha$. The small ventilation and deposition coefficient values allow for the larger crystals that result and persist over longer periods of time. For the unphysical reference case of no ventilation, crystals persist the longest, since there is no enhancement of the vapor gradients due to air motion. This case has values of the local supersaturation closest to zero, meaning that these crystals are the least locally subsaturated. In response, the deposition coefficients are the smallest, representing the largest impedance to crystal sublimation and
therefore the most preservation of crystal mass.

An interesting discrepancy arises from the ventilation computations of Thorpe and Mason (1966): just prior to the disappearance of the crystals, the ventilation coefficient drops slightly below unity. This is in contrast to the theory of ventilation, which states that air motions around particles enhance mass diffusion (Hall and Pruppacher, 1976). A stationary crystal with no air motions around it should have a ventilation coefficient of unity. This discrepancy may be due to problems with the laboratory techniques as criticized by Pitter et al. (1974). Also, it is suspected that the relationship of Thorpe and Mason (1966) may not be valid for low Reynolds number flows, such as when the fallspeed of the crystal approaches zero.

The analysis also allows us to investigate the crystal surface kinetics and their relationships to the survival distance (Fig. 5.2). Contrary to the results for crystals grown in an updraft, 2D nucleation sublimation produces particles that are larger than their dislocation counterparts. The crystal size evolution is one component of a feedback between the ventilation effect, the deposition coefficient, and the local supersaturation. For 2D nucleation, larger crystals occur since their deposition coefficients are smaller. The low values of $\alpha$ in this subsaturated case indicate an impedance to vapor molecule desorption. Values of the ventilation effect for 2D nucleation are seen to be large, corresponding to the larger crystal sizes and values of the Reynolds number. Typically, large values of ventilation act to sublimate the crystal more rapidly, yet in this case, ventilation effects are balanced by the small deposition coefficient values that reflect inefficient desorption.

The opposite holds true for crystals sublimating by dislocations. These crystals are smaller since the large deposition coefficient values indicate efficient sublimation. The smaller crystal sizes result in smaller ventilation effects, which should result in slower rates of crystal sublimation. Yet again, the greater efficiency of vapor desorption, illustrated in the large values of $\alpha$, causes larger sublimation rates. The important point here is that it is the combination of the ventilation coefficient and the deposition coefficient that determine the sublimation rate of falling crystals.

Figure 5.3 shows the fall distances for the choices of constant subsaturation (70, 50, 30, and 10%) given in Hall and Pruppacher (1976). Both methods for
Figure 5.3. Fall distances for spherical particles in 70%, 50%, 30%, and 10% subsaturated air. Panel (a) presents results using the ventilation relationships of Hall and Pruppacher (1976). Panel (b) presents results using the ventilation relationships of Thorpe and Mason (1966). All 100 L$^{-1}$ crystals have initial radii of 160 µm. Temperature and pressure at 9 km are -40°C and 300 mb, respectively.

computing ventilation (Hall and Pruppacher, 1976; Thorpe and Mason, 1966) are shown. As expected, fall distances increase as the amount of subsaturation decreases. Perhaps most interestingly, we see that crystals evolving under 2D nucleation constraints never sublimate at all in air 10% subsaturated. This situation occurs because the critical subsaturation of 0.1 (10%) has not been exceeded, and correspondingly $\alpha$ remains near zero. In all other cases, the fall distances and
survival times correspond well with those of Hall and Pruppacher (1976).

5.2 Ventilated Oblate Spheroids

We now turn our attention to nonspherical particles. In addition to comparing the methods of Thorpe and Mason (1966) and Hall and Pruppacher (1976) for the growth mechanisms of 2D nucleation and dislocations, we now examine ventilation defined in terms of the characteristic length versus ventilation distributed over the $a$ and $c$ axes. Both Thorpe and Mason (1966) and Hall and Pruppacher (1976) define ventilation in terms of the characteristic length, which is the ratio of the total particle surface area to the perimeter normal to the flow. Chen and Lamb (1994) actually use the computations of Hall and Pruppacher (1976) for a ventilated sphere, and then they distribute the amount of ventilation along the two crystal axes in an effort to mimic greater ventilation along the longest crystal axis (see Section 2.5). These methods are also used here. Namely, we begin with the method of using a characteristic length, and then use the method of Chen and Lamb (1994).

5.2.1 Ventilation in Terms of the Characteristic Length

The results plotted in Figure 5.4 are produced by instantaneously nucleating 100 L$^{-1}$ crystals of the same diameter and thickness used in the Hall and Pruppacher analysis: 800 $\mu$m by 41 $\mu$m. The spheroidal growth computations of Chen and Lamb (1994) are used with a constant growth ratio of $\Gamma = 0.1$ in order to maintain plate-like crystals. As Fig. 5.4 shows, the fall distances of oblate spheroids are much smaller than those for spheres given the same ambient conditions. Our results produce fall distances of a few hundred meters, which are consistent with the results of Hall and Pruppacher (1976). Physically, the fall distances are smaller for the oblate spheroid case since the terminal fallspeed at which the crystals are descending is inversely proportional to the length of the $a$–axis. That is, the larger length of the $a$–axis produces a “parachute effect,” whereby the drag on the spheroid is increased. Also, as previously mentioned, deviations of crystal shapes from spherical act to enhance the vapor gradients, and so oblate spheroids lose
Figure 5.4. Fall distances for oblate spheroids ventilated in terms of the characteristic length. A concentration of 100 L$^{-1}$ crystals have initial diameters and thicknesses of 800 µm by 41 µm. Temperature and pressure at 9 km are -40°C and 300 mb, respectively.

mass more rapidly than do spheres.

On performing initial tests, it was noted that near the point of complete evaporation, the aspect ratio $\phi$ became greater than unity, which is indicative of columnar crystals. This phenomenon is caused by the fixed value of inherent growth ratio $\Gamma$ ($\Gamma$ should vary with temperature). However since we are repeating the experiments of Hall and Pruppacher (1976) for plates in a temperature regime where no data for $\Gamma$ exist ($T < -30^\circ$C), a constant is perhaps the best choice. Also, it is suspected that the growth ratio $\Gamma$ should be influenced by parameters other than temperature, such as the local supersaturation and the current shape of the crystal, since changes in these parameters alter the deposition coefficients over the basal and prism faces. The next paragraph discusses the possible physics behind the aspect ratio evolution for the falling spheroids.

A plate–like crystal falling through subsaturated air with an inherent growth ratio $\Gamma$ less than unity tells us that the deposition coefficient along the $a$–axis is greater than that along the $c$–axis. In other words, mass is being removed from
Figure 5.5. Properties of the oblate spheroids ventilated in terms of the characteristic length: (a) semi-major axis $a$, (b) ventilation coefficient, (c) deposition coefficient, (d) local supersaturation.

The prism faces faster than from the basal faces because the vapor gradients are strongest over the $a$–axes. This process should drive the particle to a spheroidal shape, and the aspect ratio should approach unity. It is suspected that in the atmosphere, once a particle reaches a spherical shape, it should remain that way, since the deposition coefficient should remain the same everywhere on the crystal. This in turn would produce a growth ratio $\Gamma$ of unity. The relationships between $\Gamma$ and parameters such as crystal shape and local supersaturation require future exploration so that questions such as these can be addressed.

Figure 5.5 shows the evolution of the semi–major axis $a$, ventilation coefficient, deposition coefficient, and local supersaturation as the oblate spheroids in Fig. 5.4 sublimate. In Fig. 5.5a, we see a size trend similar to that for spherical particles. For a given altitude, 2D nucleation particles are larger than their dislocation
counterparts, since they are not losing mass to the environment as rapidly. The physical reason for this is shown in Fig. 5.5c, which shows smaller values of $\alpha$ for 2D nucleation than for dislocations. As with the spherical simulations, the ventilation coefficients follow the trend of being larger in the 2D nucleation case due to the larger crystals that exist at a certain altitude. It is interesting that the Thorpe and Mason (1966) method produces ventilation coefficients less than unity (Fig. 5.5b). Again, we suspect that the relationship of Thorpe and Mason (1966) (Eq. (2.32)) may not be valid for small values of the Reynolds number.

### 5.2.2 Ventilation Distributed over Major Axes

We now examine falling oblate spheroids that are ventilated following the method of Chen and Lamb (1994), by which the overall particle ventilation is computed for a sphere and subsequently distributed along the $a$ and $c$ axes. The initial particle, concentration, and ambient conditions are identical to those used in the last section. The fall distances shown in Fig. 5.6 are similar to those produced by ventilation computed in terms of the characteristic length (Fig. 5.4). In all cases, the simulated plates fall only a few hundred meters, and a full discussion of the similarities between the two ventilation methods can be found in the next section.

Figure 5.7 presents three crystal lengths and their corresponding ventilation coefficients. Figures 5.7a and 5.7b shows the spherical volume equivalent radius of the oblate spheroids, and the corresponding particle ventilation coefficient, computed in terms of a sphere. The particle ventilation coefficient is the ventilation coefficient that is incorporated into the overall mass growth rate equation (2.31). Figures 5.7c and 5.7d show the maximum dimension (the $a$–axis) and the ventilation local to that axis. Figures 5.7e and 5.7f show the same for the $c$–axis. The trends between 2D nucleation and dislocation crystals are identical to those mentioned previously for spheres and oblate spheroids ventilated in terms of the characteristic length: 2D nucleation causes larger crystals at a given altitude and therefore larger values of the ventilation coefficient.

An interesting feature of the curves is found in Figures 5.7.d and 5.7.f. For the cases using the ventilation relationships of Hall and Pruppacher (1976), we see that
kinks occur in the ventilation coefficient curves around 8600 m. This location marks the transition between the Hall and Pruppacher (1976) computations for $Sc^{1/3}Re^{1/2} > 1$ and $Sc^{1/3}Re^{1/2} \leq 1$. For computations in terms of the characteristic length, this transition is typically smooth; no kinks are observed in the Hall and Pruppacher computations in Figures 5.2.b, 5.5.b, and 5.7.b. However, the existence of $\sqrt{a/r}$ and $\sqrt{c/r}$ in the local ventilation equations artificially enhances the transition.

### 5.2.3 Comparison of Oblate Spheroid Ventilation Methods

Figures 5.4 and 5.6 show the fall distances of oblate spheroids with ventilation in terms of the characteristic length (Thorpe and Mason, 1966; Hall and Pruppacher, 1976) and those determined for spherical particles and subsequently distributed over the major crystal axes (Chen and Lamb, 1994). The plots show similar trends for each choice of growth mechanism and empirical ventilation relationship. Table 5.1 provides the fall distance for each ventilation method, along with the absolute and relative differences between them.
Table 5.1 confirms that there are only small differences between methods, which indicate the robust nature of the parameterizations of Chen and Lamb (1994). In all comparisons of fall distance with ventilation calculated in terms of the characteristic length, computations using distributed ventilation compared remarkably well, with relative differences below 1%. It is important, however, to consider the absolute differences as well, since a difference of approximately 50 m can be
Figure 5.8. Microphysical properties of falling oblate spheroids from Figure 5.6.

significant to a particle falling only a few hundred meters.

5.3 Summary and Discussion of Ventilation

We have discussed the differences that arise from sublimating particles in a model that neglects and includes ventilation. These differences lead to the conclusion that neglecting ventilation in a microphysical model provides an incomplete
Table 5.1. Comparison of ventilation defined in terms of the characteristic length and ventilation distributed over major crystal axes.

<table>
<thead>
<tr>
<th>Empirical Relationship (Growth Mechanism)</th>
<th>Fall Distance</th>
<th>Fall Distance</th>
<th>Absolute Difference</th>
<th>Relative Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>char. length</td>
<td>axis dist.</td>
<td>(m)</td>
<td>(%)</td>
</tr>
<tr>
<td>Thorpe &amp; Mason (2D Nucleation)</td>
<td>8595.7</td>
<td>8644.4</td>
<td>48.7</td>
<td>0.56</td>
</tr>
<tr>
<td>Thorpe &amp; Mason (Dislocations)</td>
<td>8622.2</td>
<td>8668.5</td>
<td>46.4</td>
<td>0.54</td>
</tr>
<tr>
<td>Hall &amp; Pruppacher (2D Nucleation)</td>
<td>8592.9</td>
<td>8626.4</td>
<td>33.5</td>
<td>0.39</td>
</tr>
<tr>
<td>Hall &amp; Pruppacher (Dislocations)</td>
<td>8619.3</td>
<td>8651.3</td>
<td>32.0</td>
<td>0.37</td>
</tr>
</tbody>
</table>

representation of moving ice particles. It is seen that particles constrained by 2D nucleation will persist over greater distances than their dislocation counterparts, since the small values of the deposition coefficient for 2D nucleation indicates slower sublimation rates.

Our results support the hypothesis of Hall and Pruppacher (1976), who state that the observations by Braham and Spyers-Duran (1967) of ice crystals falling 5 km in 70% subsaturated air are physically unrealistic. The findings also tend to agree with Pitter et al. (1974), who questioned the ventilation relationships of Thorpe and Mason (1966). Since unphysical values of the ventilation coefficient were produced, it is thought that the relationships of Thorpe and Mason may be invalid for low Reynolds number flows, and perhaps for nonspherical ice as well. The relationships of Hall and Pruppacher (1976) are therefore to be preferred.

Two methods for defining ventilation of nonspherical crystals have been explored. For oblate spheroids, the method of computing ventilation in terms of the characteristic length (Thorpe and Mason, 1966; Hall and Pruppacher, 1976) was seen to compare well with the method of computing the ventilation coefficient for a sphere and then redistributing the loss of mass over the major crystal axes (Chen and Lamb, 1994). The methods remain to be compared for columnar crystals, which should be done in future work.
Chapter 6

Summary and Conclusions

The deposition coefficient $\alpha$, representing the molecular-scale processes of ice crystal growth, has traditionally been either ignored or approximated as a constant in cloud models. In this study, the evolution of two cirrus cloud types has been evaluated in a microphysical framework that allows for predictive changes to $\alpha$ and habit. Processes occurring on many scales have been examined in an attempt to determine their relationships with one another. Prognoses of $\alpha$, evolving under different surface-kinetic theories, produce differences in the development of cirrus crystal sizes and supersaturations, especially near cloud base. While these differences in properties illustrate a dependence on the choice of growth mechanism (2D nucleation or dislocations), surface kinetics are not solely responsible for the observed characteristics that distinguish cirriform clouds from one another. Rather, it is the interaction of the surface kinetics with the larger scale dynamical, spatial, and compositional forcings that result in the distinct properties that define cirrostratus and cirrus uncinus clouds.

While the impact of the step origins of the crystals (dislocations or 2D nucleation) on their evolution and that of their surroundings alone does not define the distinguishing characteristics of cirrus clouds, there is much to be gained from a better understanding of the microphysical links to larger-scale cloud properties. For example, while different microphysical growth regimes under identical ambient conditions do not produce the widely disparate particle sizes that characterize cirrus uncinus and cirrostratus clouds, they do produce differences in size greater than 30 $\mu$m at altitudes near cloud base. Since particle size and $\alpha$ determine
the growth rates, there could be important consequences to ice nucleation that were not explored in this work. Also, since ice particle size is one of the factors in the determination of the amount of scattering and absorption of solar and infrared radiation (Fleming and Cox, 1974), a better understanding of microscale ice processes can lead to an improved comprehension and perhaps computation of radiative balances.

It is interesting that the different growth regimes did not produce the distinct cirrus-type characteristics that were hypothesized. An in-depth examination of cloud properties evolving by dislocations and 2D nucleation gives evidence of an analogy to growth under conditions limited by diffusion and kinetics, respectively. Although growing by two different methods, it was seen that the \( s_{\text{amb}} - s_{\text{local}} \) difference between 2D nucleation and dislocation crystals becomes insignificant as the crystals grow. However, since our model cannot capture feedbacks with nucleation and dynamics, it is assumed that our model simply cannot represent all of the causal relations required for Ci and Cs evolution.

The interaction of crystal microphysics with larger-scale processes and nucleation is assumed to be the driver behind the evolution of different cirriform cloud types. It is hypothesized that crystals with small deposition coefficients grow inefficiently (perhaps by 2D nucleation), and therefore maintain a highly supersaturated environment, allowing more crystals to nucleate. The increase in crystal concentration should lead to competition for the available vapor, a phenomenon reflected by small values of \( \alpha \). This chain of processes is hypothesized to produce the tiny crystals of cirrostratus clouds, in conjunction with supersaturation limitations and stabilizing influences provided by weak vertical motions. By contrast, it is hypothesized that crystals with large deposition coefficients grow efficiently (perhaps by dislocations), and therefore rapidly deplete the supersaturated environment, limiting the further nucleation of crystals. The limited competition for available vapor allows for large values of \( \alpha \). This chain of events may produce the large crystals characteristic of cirrus uncinus clouds, in conjunction with the enhancement of supersaturation and instability provided by strong vertical motions. In fact, some recent modeling suggests that this may be the case (Harrington et al., 2008).

The interaction of microscale and particle-scale processes has been examined for oblate and prolate spheroidal particles, shapes representing the primary crystal
habits of plates and columns. Nonspherical shapes are seen to have large impacts on crystal uptake of vapor and on the evolution of the crystal size distribution. The deposition coefficient is seen to become smaller for more pronounced crystal habits. This decrease is thought to result from a combination of vapor uptake (supersaturation depletion) and the fact that $\alpha$ is a particle average in the capacitance model. Nonspherical particles are also seen to have a large influence on the evolving structure of the size distributions, since the mass growth rates are nonlinear functions of the particle dimensions. In fact, size distribution broadening tends to occur, whereby the smallest crystals grow fastest and overtake the initially larger crystals. Comprehension of surface processes would be improved by an understanding of local (along the $a$ and $c$ axes) deposition coefficient evolution.

The fall distances of sublimating ice particles have been examined using the frameworks built by Thorpe and Mason (1966), Hall and Pruppacher (1976), and Chen and Lamb (1994). It is seen that fall distance is a function of the amount of subsaturation, but that it also depends on the deposition coefficient. Smaller values of $\alpha$ (2D nucleation) allow ice particles to persist for longer distances. Larger values of $\alpha$ (dislocations) limit the distance to which a particle can fall prior to complete sublimation. These results are based on the idea that, for a sublimating particle, $\alpha$ represents the fraction of molecules leaving the crystal surface. The empirical ventilation relationships of Thorpe and Mason (1966) and Hall and Pruppacher (1976) were compared for spheres and oblate spheroids, and it is thought that the methods of Thorpe and Mason (1966) are not valid for low Reynolds number flows. Finally, the empirical relationships for oblate spheroids were tested using two computation methods. First, ventilation was computed in terms of the characteristic length. Second, ventilation was calculated for a sphere and then redistributed over the major crystal axes in order to reflect the fact that ventilation enhancement occurs preferentially at the tips of the maximum crystal dimension. These methods were seen to compare well for falling spheroids.

The deposition coefficient is important during spherical growth, nonspherical growth, and sublimation. Most importantly, $\alpha$ has been predicted in these studies, in which it evolves as the ambient and crystal properties change. It is this interaction of the deposition coefficient with larger-scale properties that is thought to produce the distinct types of cirriform clouds that we observe.
Appendix A

Method for Calculating the Variable Deposition Coefficient

The scheme for determining a variable deposition coefficient is an extension of the work of Nelson and Baker (1996) and Lamb (2000), which stems from the theory of Burton et al. (1951). The parameterization of deposition coefficient $\alpha$ is of the form

$$\alpha = \left( \frac{S}{1 + K\alpha} \right)^m \tanh \left( \left( \frac{1 + K\alpha}{S} \right)^m \right),$$

where $S$ is the ratio of ambient and critical supersaturations with respect to ice ($\equiv \frac{s_{amb}}{s_{crit}}$). The exponent $m$ is set to 1 to represent dislocational growth and 30 to represent 2D nucleational growth. Variable $K$ is a vapor transport parameter defined by Lamb and Chen (1995) as

$$K = \frac{\bar{v}r}{4D_v^*},$$

where $\bar{v}$ is the mean speed of vapor molecules, $r$ is the spherical volume equivalent radius of the crystal, and $D_v^*$ is the modified diffusivity according to

$$D_v^* = \frac{D_v}{\frac{r}{r+\Delta} + \frac{4D_v}{3\bar{v}}}.$$  

Here, $D_v$ is the vapor diffusivity and $\Delta$ is the vapor jump distance, set equal to the mean free path of vapor molecules.
Since Eq. (A.1) is a transcendental equation, one must iterate to find deposition coefficient $\alpha$ as a function of other cloud properties. This process is simplified by defining two variables:

$$C_1 \equiv \frac{r}{r + \Delta},$$

$$C_2 \equiv \frac{4}{v'r}.$$

Substituting $C_1$ and $C_2$ into (A.3), and then into (A.2), yields

$$K = (C_2 D_v')^{-1} = \left(\frac{C_2 D_v}{C_1 + \frac{C_2 D_v}{\alpha}}\right)^{-1} = \frac{C_1}{C_2 D_v} + \frac{1}{\alpha}. \quad (A.4)$$

Using (A.4) in (A.1) provides an explicit form of the equation for $\alpha$, which can be iteratively evaluated using Newton’s method:

$$\alpha_{n+1} = \alpha_n \frac{f(\alpha_n)}{f'((\alpha_n)}. \quad (A.5)$$

where $\alpha_n$ is a trial value of $\alpha$ and $f(\alpha)$ is determined by simply rearranging Eq. (A.1) so that $f(\alpha)$ approaches zero as $\alpha$ approaches its exact value:

$$f(\alpha_n) = \left(\frac{S}{1 + \alpha_n K}\right)^m \tanh \left(\left(\frac{1 + \alpha_n K}{S}\right)^m\right) - \alpha_n$$

$$= \left(\frac{S}{2 + \frac{C_1\alpha_n}{C_2 D_v}}\right)^m \tanh \left(\left(\frac{2 + \frac{C_1\alpha_n}{C_2 D_v}}{S}\right)^m\right) - \alpha_n. \quad (A.6)$$

The derivative of $f(\alpha)$ with respect to $\alpha$ is:
\[ f_t(\alpha_n) = \left( \frac{C_1 m}{C_2 D_v S} \right) \left( \frac{S}{2 + \frac{C_1 \alpha_n}{C_2 D_v}} \right)^m \left( \frac{2 + \frac{C_1 \alpha_n}{C_2 D_v}}{S} \right)^{m-1} \text{sech}^2 \left( \frac{2 + \frac{C_1 \alpha_n}{C_2 D_v}}{S} \right)^{m} \]

\[ - \left( \frac{m C_2 D_v S}{C_1 \left( \alpha_n + \frac{2 C_2 D_v}{C_1} \right)^2} \right) \left( \frac{S}{2 + \frac{C_1 \alpha_n}{C_2 D_v}} \right)^{m-1} \text{tanh} \left( \frac{2 + \frac{C_1 \alpha_n}{C_2 D_v}}{S} \right)^{m} - 1. \]

(A.7)

Now that the iterative Eqs. ((A.5)–(A.7)) have been developed, one needs only an initial guess for \( \alpha_0 \) to begin the evaluation. For our model analysis, the ice crystals appear just above cloud base in an environment that is slightly supersaturated with respect to ice. This condition, along with the initial small sizes of the crystals, allows \( \alpha_0 = 0 \) to be a physically sound first guess to begin the iteration. The iteration proceeds until a solution is converged upon, that is, when \( \text{abs}(\alpha_{n+1} - \alpha_n) \) reaches a pre-specified minimum difference of 0.001%. Since we do not expect \( \alpha \) to vary erratically over small timesteps, the converged value of \( \alpha \) is then used as the initial guess for the next timestep.

Larger updraft speeds result in more drastic changes in temperature, supersaturation, and crystal size over a given timestep than do smaller updraft speeds. In the event of a failure to converge, a curve depicting \( \alpha \) as a function of the supersaturation ratio \( S \) is built, as depicted in Figure 2.3. The iterative procedure is initialized with the current temperature, pressure, and spherical equivalent volume crystal radius. The curve is then constructed with an initial guess of \( \alpha = 0 \) at \( S = 0 \) and looped over \( S \) using the converged value of \( \alpha \) as the initial guess for the next \( S \). This procedure continues until the desired value of \( S \) is reached. The corresponding \( \alpha \) is selected from the curve.
Appendix B

Mass Growth Rate Equations for Kinetic and Diffusive Limitations

The mass growth rate equation for an ice particle is:

\[
\frac{dm}{dt} = 4\pi C s_{amb} \frac{R_e T}{R_v^* D_v} + \frac{l_v}{K_T T} \left( \frac{l_v}{R_v T} - 1 \right),
\]  

(B.1)

where \( C \) is the capacitance. The variable of our focus is the modified diffusivity \( D_v^* \), which is defined

\[
D_v^* = \frac{D_v}{r + \Delta} + \frac{4D_v}{\alpha r \bar{v}}.
\]  

(B.2)

Here, \( D_v \) is the diffusion coefficient, \( r \) is the crystal radius, \( \bar{v} \) is the mean speed of vapor molecules, and \( \Delta \) is the vapor jump distance, set equivalent to the mean free path.

Kinetically Limited Growth

In the case of an ice crystal growing under kinetic limitations, the product of the radius and the deposition coefficient is small:

\[
\lim_{\alpha r \to 0} D_v^* = \frac{\alpha r \bar{v}}{4} \propto \alpha.
\]  

(B.3)

With this limitation, the mass growth rate equation becomes
\[
\frac{dm}{dt} = \frac{4\pi Cs_{amb}}{\frac{4R_v T}{\alpha r \tau_e_s} + \frac{l_s}{K_r^T} \left( \frac{l_s}{R_v T} - 1 \right)}, \tag{B.4}
\]

which is seen to be sensitive to the deposition coefficient and not the diffusion coefficient. The mass growth rate is now independent of the diffusion coefficient, \( D_v \).

**Diffusively Limited Growth**

Under the opposite condition of a large product of \( r \) and \( \alpha \), ice crystal growth will be limited by diffusion:

\[
\lim_{\alpha r \to \infty} D_v^* = \frac{D_v (r + \Delta)}{r} \propto D_v. \tag{B.5}
\]

The mass growth rate equation responds to this limitation according to

\[
\frac{dm}{dt} = \frac{4\pi Cs_{amb}}{\frac{R_v T}{D_v e_{s,i}} + \frac{l_s}{K_r^T} \left( \frac{l_s}{R_v T} - 1 \right)}, \tag{B.6}
\]

which is seen to be sensitive to the diffusion coefficient and not the deposition coefficient. In this case, the mass growth rate is independent of \( \alpha \).
Appendix C

Parcel Model Equations

This appendix contains the important equations used in the parcel model analyses of cirriform clouds. These equations portrayed here are identical to the original model of Lebo (2007), with two exceptions: all liquid water and radiative terms have been neglected, as they are not relevant to this analysis. In addition, non-spherical habits have been added. All variables are defined in Appendix D. The following equations are those evaluated by the Variable Order Differential Equation (VODE) solver of Brown et al. (1989).

The volume and shape changes of spheroidal particles are calculated according to the methods of Pruppacher and Klett (1978) and Chen and Lamb (1994), respectively:

\[
\frac{dV}{dt} = \frac{4\pi Cs_{amb}}{\rho_{\text{ice}} \left( \frac{R_v}{D_{v,i}} + \frac{1}{K_f T} \left( \frac{\rho_v}{R_v T} - 1 \right) \right)},
\]

\[
dln(\phi) = \frac{\Gamma - 1}{\Gamma + 2} dln(V).
\]

The capacitances \( C \) for oblate and prolate spheroids are:

\[
\text{Oblate Spheroid} \quad C = \frac{\alpha}{\arcsin \epsilon},
\]

\[
\text{Prolate Spheroid} \quad C = \frac{\epsilon}{\ln((1+\epsilon)\phi)}.
\]

Once crystals the crystals instantaneously nucleate, mass is conserved according
\[
\frac{dq_v}{dt} + \frac{dq_i}{dt} = 0,
\]  
\(\text{Eq. (C.3)}\)

where the ice mixing ratio \(q_i\) is defined as:

\[
\frac{dq_i}{dt} = \sum_j 4\pi N_{i,j} V_j \frac{\rho_{\text{ice}}}{\rho_{\text{air}}} \frac{dV_j}{dt}.
\]  
\(\text{Eq. (C.4)}\)

Once the mixing ratios \(q_v\) and \(q_i\) have been obtained, the ambient supersaturation with respect to liquid evolves according to:

\[
\frac{ds_{\text{amb,l}}}{dt} = (s_{\text{amb,l}} + 1) \left[ \left( \frac{gl_v}{c_p R_v T^2} - \frac{g}{R_{\text{air}} T} \right) w - \left( \frac{1}{q_v} + \frac{l_v l_s}{c_p R_v T^2} \right) \frac{dq_i}{dt} \right],
\]  
\(\text{Eq. (C.5)}\)

and supersaturation with respect to ice is obtained via:

\[
s_{\text{amb}} = \frac{e_{s,l}}{e_{s,i}} (s_{\text{amb,l}} + 1) - 1.
\]  
\(\text{Eq. (C.6)}\)

The ambient temperature evolves via:

\[
\frac{dT}{dt} = -\frac{gw}{c_p} + \left( \frac{l_s}{(1 + q_v)c_p} \right) \frac{dq_i}{dt},
\]  
\(\text{Eq. (C.7)}\)

from which we can arrive at a solution for pressure according to:

\[
\frac{dP}{dt} = -\frac{g P w}{R_{\text{air}} T}.
\]  
\(\text{Eq. (C.8)}\)

For the sublimation of falling particles, Eq. (C.7) is modified to the moist adiabatic lapse rate:

\[
\frac{dT}{dz} = -7K/km.
\]  
\(\text{Eq. (C.9)}\)
Appendix D

Variable Definitions

\( A \)  cross-sectional crystal area normal to flow
\( a \)  length of axis normal to prism faces
\( c \)  length of axis normal to basal faces
\( c_p \) specific heat
\( C \)  capacitance factor
\( D_v \) diffusion coefficient
\( D_v^* \) modified diffusivity
\( e \) vapor pressure
\( e_{s,i} \) equilibrium vapor pressure over ice
\( e_{s,l} \) equilibrium vapor pressure over liquid
\( g \) gravitational acceleration
\( k \) Boltzmann constant
\( K \) vapor transport parameter = \( \bar{v} r/4D_v^* \)
\( K_T^* \) modified thermal conductivity of air
\( l_s \) latent heat of sublimation
\( l_v \) latent heat of vaporization
\( m \) exponent to distinguish \( \alpha \) computation between mechanisms
\( m_i \) ice particle mass
\( n \) vapor concentration
\( n_{eq} \) equilibrium vapor concentration
\( n_i \) interfacial vapor concentration
\( n_\infty \) ambient vapor concentration
p \quad \text{pressure}
q_i \quad \text{ice mixing ratio}
q_v \quad \text{vapor mixing ratio}
r \quad \text{spherical volume equivalent radius}
R_{air} \quad \text{gas constant for dry air}
R_v \quad \text{gas constant for water vapor}
Re \quad \text{Reynolds number in terms of diameter}
Re_L \quad \text{Reynolds number in terms of characteristic length}
s_{amb} \quad \text{ambient supersaturation with respect to ice}
s_{amb,l} \quad \text{ambient supersaturation with respect to liquid}
s_{\text{crit}} \quad \text{critical supersaturation}
s_{\text{local}} \quad \text{local supersaturation}
S \quad \text{ratio of ambient and critical supersaturations}
S_l \quad \text{ratio of local and critical supersaturations}
T \quad \text{temperature}
\bar{v} \quad \text{mean speed of vapor molecules}
V \quad \text{crystal volume}
w \quad \text{updraft velocity}

\alpha \quad \text{deposition coefficient}
\beta \quad \text{edge free energy}
\Gamma \quad \text{inherent growth ratio}
\Delta \quad \text{vapor jump distance}
\zeta \quad \text{mass-dimension relationship power}
\eta \quad \text{dynamic viscosity}
\epsilon \quad \text{eccentricity} = \sqrt{1 - \phi^2} \text{ (oblate)}, \sqrt{1 - \phi^{-2}} \text{ (prolate)}
\mu \quad \text{chemical potential}
\xi \quad \text{mass-dimension relationship coefficient}
\rho_{\text{air}} \quad \text{density of air}
\rho_{\text{ice}} \quad \text{density of ice}
\phi \quad \text{aspect ratio } c/a
\Omega_2 \quad \text{molecular area}
References


