STUDENTS’ AND TEACHERS’ REPRESENTATIONAL KNOWLEDGE IN MATHEMATICS

A Thesis in
Educational Psychology
by
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ABSTRACT

This study examines the relation between middle grades students’ ability to translate between multiple representations of fractions and their general mathematics ability and the relation between middle grades teachers’ translation ability and the translation ability of their students. The General Math Ability Assessment and Representational Transfer Test-Fractions developed for this study were completed by 15 fifth grade teachers, 15 sixth grade teachers, 172 fifth grade students, and 153 sixth grade students. Findings show that fifth and sixth grade students’ scores on the General Math Ability Assessment were positively and statistically significantly correlated ($p < .01$) with their scores on the Representational Transfer Test-Fractions. Teachers’ scores on the Representational Transfer Test-Fractions were not statistically significantly associated ($p = .30$) with the Representational Transfer Test-Fractions scores of the students in their class. The nature of students’ and teachers’ performance on translation tasks is also discussed.
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Chapter 1
INTRODUCTION

Fractions have been identified as one of the most important mathematical concepts for students to learn. Ranked as one of the most critical prerequisite skills for upper level mathematics courses such as Algebra, students’ success with fractions is seen as a gateway to increasing the math proficiency of American students (Hoffer, Venkataraman, Hedberg, & Shagle, 2007; National Mathematics Advisory Panel [NMAP], 2008). Yet evidence shows that students of all ages struggle to learn the concept of fractions (Ball, 1993; Behr, Harel, Post, & Lesh, 1992; Lamon, 2001). Surveys by the NMAP (2008), for example, found that 27% of eighth grade students could not correctly shade one-third of a rectangle, 45% could not solve a word problem that required dividing fractions, and 50% could not order fractions from least to greatest. Even teachers fail to demonstrate a deep understanding of fractions (Ma, 1999).

In light of these findings, researchers, educators, and policy makers are suggesting that representations, such as number lines, area models, and other visual representations, should be included in mathematics instruction as a way to increase students’ understanding of fractions. The National Council of Teachers of Mathematics (NCTM, 2000) recommends that students of all ages “create and use representations to organize, record, and communicate mathematical ideas; select, apply, and translate among mathematical representations to solve problems; and use representations to model and interpret physical, social, and mathematical phenomena” (p. 64). A recent report published by the Institute of Education Sciences (IES), which specifically regards the teaching of fractions, explicitly recommends the use of representations in fraction instruction. The report also recognizes that in order for students to
effectively learn from multiple representations of fractions, teachers must understand a variety of representations and how they relate to various fraction concepts (Siegler et al., 2010).

The purpose of this study is to more closely examine the relation between elementary school students’ ability to work with and understand representations and their mathematics ability. We also examine teachers’ understanding of the relationships across different representational forms to consider if a classroom teachers’ knowledge of these representations influences their students’ ability. In this study, both teachers’ and students completed a representational transfer test concerning knowledge of fractions. Participants’ performance on this test is explored. Relations between scores on this measure and students’ mathematical competency are examined along with a testing of differences across classrooms.
Chapter 2

LITERATURE REVIEW

A “representation” is defined as any configuration which can stand for something else (Duval, 2006; Goldin, 2002; Goldin & Kaput, 1996). Representations can be either internal or external to an individual. Internal representations are those mental constructs that are held within the learner’s mind (Dufour-Janvier, Bednarz, & Belanger, 1987; Goldin & Kaput, 1996; Kaput, 1998). External representations, on the other hand, are observable symbols, able to be displayed on a piece of paper, a computer program, or a graphing calculator (Goldin, 2002; Goldin & Kaput, 1996). External representations serve as manifestations of the internal representations of the learner (Behr, Lesh, Post, & Silver, 1983; Janvier, 1987a; Lesh, Post, & Behr, 1987b) as well as models of mathematical “realities” (Ainsworth, 2006; Dufour-Janvier et al., 1987). Examples of external representations include words, numerals, equations, graphs, pictures, diagrams, tables, number lines, manipulative materials, and real-world situations (Duval, 2006; Elia, Panaoura, Eracleous, & Gagatsis, 2007; Goldin, 2002; Goldin & Kaput, 1996; Larkin & Simon, 1987; Post, Behr, Lesh, & Wachsmuth, 1986).

External representations are a fundamental and essential part of the discipline of mathematics (Ball, 1993; Stylianou, 2010). Because of the abstractness of mathematical concepts, the external representations that have been created, shared, and accepted throughout the history of mathematics are the only means for individuals to communicate about unobservable mathematical realities that have no inherent representation of their own (Duval, 2006; Goldin, 1998; Goldin, 2002). Due to this close tie between concepts and their representations, it is sometimes difficult to imagine that mathematical concepts exist outside
of their representations (Niemi, 1996). This, however, is exactly the case, and to truly understand a mathematical concept, one must realize that an external representation is only one way to signify an otherwise unobservable reality and never think of the concept and its representation to be one and the same (Duval, 2002).

Abstraction and Translation

This ability to understand a concept by separating the idea from its representation is known as “abstraction” (Ainsworth, 1999; Goldin, 2002; Post, Behr, & Lesh, 1982) and exposing students to multiple representations of a concept promotes abstraction (Ainsworth, 1999; Kaput, 1998). Put another way, the numerical symbol $\frac{1}{2}$, a rectangle with one of two parts shaded, and a number line marked exactly in between 0 and 1 are three different representations which show the same concept, “half”. Students who recognize the connection between each representation and the concept, but who also appreciate that each of the representations are signifying the same concept and are therefore connected to one another, have a more robust understanding of the concept.

Making connections between representations in such a way that allows learners to move from one representation to another, such as from a geometric figure to a number line or from an equation to a graph, is known as “translation” (Ainsworth, 1999; Janvier, 1987b; Lesh, Behr, & Post, 1987a) or “conversion” (Duval, 2002; Duval, 2006; Elia et al., 2007; Panaoura, Gagatsis, Deli Yianni, & Elia, 2009). Translation requires learners to see that multiple representations are actually representing the same concept by noticing the relations between two or more representations (Ainsworth, 1999; Greer & Harel, 1998).

In practice, students generally translate from one representation, called the “source representation,” to a second representation, called the “target representation” (Janvier,
For example, when a student is given the verbal representation of “one-third” and must generate the numerical representation (i.e., $\frac{1}{3}$), the verbal representation is the source representation and the numerical representation is the target representation. As there are several possible representations of concepts in mathematics (e.g., numerical expression, pictorial representation, verbal description, and number line), to translate from one representation to another can be done directly (e.g., from the word to the number) or indirectly, where a representation that is neither the source nor the target serves as a mediator to help the individual make the translation (Bright, Behr, Post, & Wachsmuth, 1988; Janvier, 1987b). For example, a student who, when asked to translate from a verbal description to a numerical expression, first translates the verbal depiction to a pictorial representation and then to a numerical expression, has performed an indirect translation.

Finally, translations can be both “congruent,” or transparent, and “non-congruent,” or opaque. The similarities of two representations are transparent when there is a one-to-one mapping from the features in the source representation to the features in the target representation. For example, there is a clear relationship between the verbal form “eight-fifths” and the numerical form of the fraction $\frac{8}{5}$, as the word “eight” maps to the number “8” in the numerator and the word “fifths” maps to the number “5” in the denominator. This is an example of a congruent translation. On the other hand, a translation may be between two representations whose contents have very little in common, requiring the learner to perform a manipulation to make the similarities less opaque. For example, the verbal description, “eight parts of wholes that have been divided into five equal parts,” requires the learner to understand conceptually the fraction described by the words before he or she is able to match
it to a target numerical representation. This is an example of a non-congruent representation (Duval, 2002; Duval, 2006).

Translation, Understanding, and Problem Solving

To say that a learner understands a mathematical concept means that learner is able to perform all of the translation processes described above between two or more representations depicting the same concept (Behr et al., 1983; Duval, 2002; Hitt, 1998; Lesh, 1981; Lesh, Landau, & Hamilton, 1983; Post et al., 1982; Post et al., 1986; Post, Cramer, Behr, Lesh, & Harel, 1993). When that learner understands fractions, he or she can see the concept of a certain fraction (e.g., one half) no matter if it is depicted in verbal, numerical, pictorial, or number line form, and that the student can perform arithmetic operations and manipulations with the fraction given in more than one representation (Lesh et al., 1987b).

The theoretical link between translation ability and conceptual understanding has been substantiated by several recent studies measuring students’ problem solving ability and conceptual knowledge. Typically, these studies measure the participants’ translation ability within a mathematical domain, present the participants with a problem solving task or knowledge measure within that domain, and then look for a relationship between the two constructs. Gagatsis and Shiakalli (2004) found that university students’ ability to translate from a source representation of a function to a target representation explained a significant percentage of the variance in their ability to solve problems involving functions. Even (1998) also studied college students’ understanding of functions and found that the ability to translate between different representations of functions was directly related to students’ content knowledge of functions. Through analyzing think aloud protocols that were collected while university students were solving algebra word problems, Cifarelli (1998) determined
that the students who were able to successfully construct a representation from the verbal
word problem and translate that into an algebraic expression were also able to accurately
solve the word problems, see connections between similar problems, anticipate solutions and
potential problems, and create their own word problems.

Similar relationships have been found in studies examining middle school students’
understanding of fractions. Niemi (1996), for example, studied the relationship between fifth
grade students’ translation and problem solving abilities. Translation ability was measured by
a recognition test in which participants were required to circle the symbolic and graphic
representations that were equivalent to the numeric representation of the fraction provided at
the top of the page. Students who performed in the top quartile on the translation task were
also more likely to give correct solutions in a different problem solving task, and were more
likely to justify their solutions using multiple representations, than the students who scored in
the bottom quartile on the translation task. Similarly, Panaoura and colleagues (2009) found a
high, statistically significant correlation between fifth through eighth grade students’
translation ability, measured by a test requiring the identification and generation of
equivalent representations of fraction addition, and their problem-solving ability for fraction
addition.

Benefits of Representations in Problem-Solving

The studies reviewed above clearly indicate a relation between translation ability and
mathematics understanding. However, since the studies are correlational, it is not possible to
conclude the direction of causality between the two constructs. In other words, are students
who best understand better able to use representations, or does the ability to use
representations cause better understanding? The important roles that representations play in
problem solving may lend support to the hypothesis that understanding of representations leads to better mathematics performance.

First, representations can be exploited as a problem-solving strategy (Ainsworth, 1999). For instance, Stylianou and colleagues (Stylianou, 2002; Stylianou & Silver, 2004) found that expert mathematicians often sketch visual representations when problem-solving. They are then able to examine the diagram to gain additional information about the situation that was not obvious from the representation in which the problem was presented. Second, representations may offer students a means to record their internal representations on paper, a process which reduces cognitive load and improves working memory capacity while solving problems (Behr et al., 1983; Izsak, 2008; Larkin & Simon, 1987). Third, the creation of representations can serve as a communication tool between a student and his or her teacher and between a student and his or her classmates (Greeno & Hall, 1997).

The ability to comprehend provided representations is also an advantageous skill for students because different representations afford different information related to a concept, and thus support different inferences (Greeno & Hall, 1997). For example, diagrams are often preferred over sentences because they constrain interpretations that could otherwise be ambiguous (Ainsworth, 1999), their spatial organization reflects the organization of the concept, and the information they contain is often more efficient for computational processes (Larkin & Simon, 1987).

Students’ Difficulties with Translation

Despite the importance of representational knowledge, many students struggle with tasks that require an understanding of multiple representations. Research shows that many students fail to create external representations when solving problems (Gagatsis & Shiakalli,
Researchers agree that students have particular difficulty translating between representations, but the literature shows conflicting evidence regarding the difficulty of different representation types. Some findings indicate that no one representation is generally more problematic for students, and that the difficulty of the translation task depends on the particular source-target pairing (Duval, 2006; Meltzer, 2005). On the other hand, evidence from studies of translation ability with fractions reveals that there are certain representational types that students consistently struggle with more than others (Lesh et al., 1987a; Lesh et al., 1983) and that translational processes involving the number line are hardest for students (Behr et al., 1983; Larson, 1980; Lesh et al., 1983; Niemi, 1996).

Regardless of whether a pattern exists or not, researchers have agreed that students do not automatically translate from one representation to another (Ainsworth, 1999; Ainsworth, Bibby, & Wood, 2002; Bright et al., 1988; Even, 1998; Greer & Harel, 1998). Furthermore, students perform very poorly when given tasks that force translation from one representation to another, such as on a translation test (Ainsworth, Bibby, & Wood, 1998; Ainsworth et al., 2002; Niemi, 1996). Specifically with respect to students’ understanding of fractions, “one result that stands out is the low overall level of performance, which is consistent with results from many studies” (Niemi, 1996; p. 357).

Several explanations have been offered to account for students’ difficulties with translating between representations. One possible explanation is that students do not understand the conventions within a given representation (Ainsworth, 2006; Nistal, Van
Dooren, Glarebout, Elen, & Verschaffel, 2009). This phenomenon was demonstrated in a study by Bright et al. (1988) in which fourth and fifth grade students were instructed to represent and order fractions on a number line. The results of this study showed that many of these students did not understand the partitioning of a number line between whole numbers. Second, the unique features of a specific translation task might contribute to students’ difficulties. For instance, the students’ familiarity with the source and target representations (Ainsworth et al., 2002), the degree of redundancy between the two representations (Ainsworth et al., 1998; Greer & Harel, 1998), and the non-congruence of a translation task (Duval, 2002) all contribute to the task difficulty and subsequent student success.

Third, lack of conceptual understanding within the domain may also explain students’ problems with translation. Recall that the level of understanding in mathematics that leads to translation ability requires learners to abstract a concept from its representation (Ainsworth, 1999; Goldin, 2002; Post et al., 1982). Many students cannot perform this abstraction process, and instead find it impossible to dissociate a concept from its representation (Duval, 2002; Duval, 2006; Elia et al., 2007). In reference to the example offered previously, students who are not able to perform abstraction view the number $\frac{1}{2}$ and a picture with one of two parts of a rectangle shaded as two separate concepts, rather than as two representations of the concept of “half.” The prevalence of this compartmentalization is evidenced by students who solve the same problem using two different representations and arrive at two different answers and do not see a contradiction (Dufour-Janvier et al., 1987), as well as students who respond differently to problems that are conceptually the same but involve different representations (Even & Tirosh, 2002; Gagatsis & Shiakalli, 2004). A lack of deep understanding of a concept inhibits students’ abilities to see underlying similarities between
representations, preventing them from translating accurately (Ainsworth, 2006; Greer & Harel, 1998).

One potential cause of students’ translational “(dis)abilities” (Lesh et al., 1987b, p. 36) that has garnered significant attention from the mathematics education community is that students are not prepared by the education they are receiving to perform translational tasks. The correlation between translation ability and mathematical problem solving ability, as well as the push from national standards, should make the teaching of translation strategies a priority in mathematics education, yet all too often translation is overlooked by mathematics teachers (Duval, 2002; Gagatsis & Shiakalli, 2004; Lesh at el., 1987a).

Reasons for and Evidence of Lack of Instruction in Translation

Teachers Fail to Value Representations

Teachers’ beliefs about the utility and importance of teaching with multiple representations affect the role that representations will play in their instruction (Ball, 1993; Cai, 2005; Drageset, 2010), and the absence of instruction on translating between representations may in part be due to teachers’ beliefs that representations are a superfluous product of mathematics (Stylianou, 2010). Interviews by Stylianou reveal that many middle school teachers view numerical and algebraic representations as more central to mathematics instruction, even going so far as to believe that a numerical representation of a concept is the concept, and that other representations are secondary. Therefore, these teachers believe it is most important for the student to learn numerical and algebraic representations. Other external representations, such as number lines or diagrams, are perceived as something extra to be taught in addition to the already full curriculum. For many of these teachers, there is
just no time in the confines of the curriculum to teach other representations besides the numerical ones.

Some teachers who view alternative representations as extraneous still attempt to incorporate them in some way into their instruction, but to them, representations nevertheless serve a limited purpose. They are seen as a communication tool but not a problem solving tool (Stylianou, 2010). They are an alternative option for students who have a different “learning style” – in other words, visual learners are permitted to learn from a visual representation instead of a symbolic or numerical one (Izsak, 2008; Stylianou, 2010). Representation is seen as an enrichment activity for students who are more mathematically advanced; it is beyond the ability of the typical student (Stylianou, 2010). Finally, representations are oftentimes seen simply as a way to engage students by providing a “fun” manipulative as an introduction to a unit (Ball, 1993; Stylianou, 2010).

Teachers Fail to Understand Representations

The observation that students are not receiving instruction on how to effectively translate between multiple representations may also be accounted for by their teachers’ lack of knowledge. Teachers’ knowledge of content and of pedagogical techniques such as representations has been theoretically and empirically linked to teachers’ instruction and, subsequently, student learning. A teacher who has a deep conceptual understanding of mathematics topics will be able to teach his or her students mathematical concepts, such as the relations between two different representations of a fraction, but a teacher who has limited procedural knowledge will tend to focus his or her instruction on procedures, such as performing operations or simplifying fractions (Eisenhart et al., 1993). Hill and colleagues (2005) found that teachers’ knowledge of pedagogical techniques for teaching mathematics,
as measured by a test called the Content Knowledge for Teaching Mathematics Test, positively predicted their students’ gains in mathematical achievement.

Unfortunately, it seems that neither teachers’ knowledge of mathematics as a discipline nor their knowledge of mathematics pedagogy meet the standard required for effective instruction of representations. Although teachers might believe conceptual understanding is more important than procedural understanding, teachers’ knowledge of concepts is much weaker than their knowledge of mathematical computational procedures, (Eisenhart et al., 1993; Izsak, 2008; Izsak et al., 2010; Ma, 1999). Interviews with teachers in the topic of dividing by a fraction revealed the discrepancy in the depth of mathematical knowledge of Chinese teachers as compared to United States teachers (Ma, 1999).

Specifically, teachers fail to show that they have a deep understanding of multiple representations of fractions. Izsak (2008) extensively observed two middle grade teachers’ lessons of fraction multiplication. Following the lessons, he also interviewed the teachers and the students from their classes. Findings from this study indicate that the teachers did not have a deep understanding of how representations could be used as a pedagogical technique. The teachers did not see a connection between the computational procedure for multiplying fractions and the depiction of that procedure with a visual representation, and they could not adequately address students’ questions involving alternative representations. When asked to verify whether drawings that their students had created in a post-lesson interview with the researcher would work to solve the multiplication problem, neither teacher could reason the answer directly from the students’ drawing. Instead, both teachers computed the correct answer to the multiplication problem and tried to match their calculated answer to the fraction the student had shaded in. The teachers in this case did not think about whether the
student’s drawing was conceptually valid and, as a result, did not consider if this approach could be generalized to all problems. In a larger study with 201 middle school teachers, Izsak and colleagues (2010) found that a relatively high percentage of teachers answered incorrectly to questions of fraction operations that were presented in multiple representations, and many relied on numeric representations as their primary strategy for solving the problems, completely bypassing the need to reason in a different representation.

Importance of Instruction to Students’ Translational Ability

Researchers are focusing on teachers’ instruction with representations because effective instruction has been shown to improve translation ability. Interventions designed to expose children in primary grades to multiple representations and explicitly link one representation to another have resulted in increased translation ability as measured by posttests (Ainsworth et al., 1998; Dufour-Janvier et al., 1987; Lamon, 2001). Students learning under teachers who included active and explicit comparison of representations in their instruction were more likely to utilize multiple representations when problem solving than their peers who received a more traditional type of instruction (Uesaka & Manalo, 2006). Even pre-service teachers, who experienced difficulty interpreting graphical representations of motion functions, deepened their understanding of the concept and improved their ability to reason with multiple representations after being given the opportunity to explore the connection between physical motion and its graph through experimentation and trial-and-error learning (Stylianou, Smith, & Kaput, 2005).

A striking example of the power of instruction comes from a pair of studies by Kohl and Finkelstein (2005, 2006) in which they assessed university students’ performance on physics problems in different representations when they were or were not given a choice of
representation. In the initial study (2005), Kohl and Finkelstein found that students from a “reformed” physics class that offered a distinct style of instruction performed better on the homework and quiz problems and were less affected by whether or not they were given a choice of representational format than their peers who were in a more traditionally taught physics class. These findings led the researchers to speculate that the instruction students received in the reform class helped them to become more skilled at using different representations. In a follow-up study involving observations of lectures and exams, Kohl and Finkelstein (2006) found that the instructors of the reformed classes did, in fact, utilize multiple representations in both their lectures and on exams questions, which they infer led to the students’ “broader set of representational skills” (p. 7) and subsequent higher performance on problems requiring representational knowledge.

Evidence from the literature indicates that teachers’ instruction with multiple representations can have a positive effect on students’ representational understanding. Thus, in fraction education in particular, research has focused on improving teachers’ knowledge of fraction representation and their ability to use this to instruct their students (Siegler et al., 2010; Wu, 2007).

The Present Study

The present study investigates the representational skills of middle school mathematics students and their teachers. The research questions and hypotheses are as follows:

Research Question 1. What is the nature of students’ and teachers’ performance on representational translation tasks?
Based on previous studies with translation tasks, and specifically in the domain of fractions, we expect that students will experience more difficulty with non-congruent translations than with congruent translations. We also anticipate that translation tasks to or from the number line representation will pose a challenge for students.

Research Question 2. What is the relation between students’ translation ability and their general mathematics ability?

Translation ability has been theoretically linked with students’ understanding and has empirically been correlated with students’ problem solving ability. Most of the studies reviewed have found correlations between students’ representational translation ability in one particular area (e.g., fractions, functions, etc.) and their mathematical ability in that same area, although there is some evidence that students’ translation ability may be correlated with general mathematical ability (Lesh, 1981). We test the hypothesis that a student who is able to translate between representations within a particular area has not only abstracted the concepts within that area, but is also able to generalize that ability to other mathematical constructs, so that he or she will be able to use and understand multiple representations when solving problems in any area of mathematics. Therefore, we predict a positive correlation between students’ ability to translate in one area (i.e., fractions) and his or her general mathematical ability across multiple areas of mathematics.

Research Question 3. What is the relation between a teacher’s translation ability and the translation ability of his or her students?

Numerous studies have shown a relation between effective instruction and students’ representational skills. Research has also indicated that teachers’ beliefs about the role of representations in mathematics education and their knowledge of representations influences
the way they use representations in their instruction. We hypothesize that teachers’ translation ability will positively predict the translation ability of their students, as mediated by instruction.
Participants

Participants were recruited from six elementary schools and five middle schools in a large rural and suburban public school district in northern Georgia. The size of the elementary schools included grades K-5 and ranged from 356 to 868 students. The percentage of minority students ranged from 21% to 47%, and the percentage of elementary school students eligible for free or reduced price lunch in 2009 varied from 11% to 58%. The middle schools included grades 6-8 and the population ranged from 426 students to 751 students. The percentage of minority students varied from 20% to 29% in the middle schools, and the percentage of students eligible for free or reduced priced lunch in 2009 ranged from 8% to 47%. A breakdown of these demographics by participating school is shown in Table 1.

Table 1
Demographic Information of Participating Schools

<table>
<thead>
<tr>
<th>School</th>
<th>Population</th>
<th>Percent Minority</th>
<th>Percent Eligible for Free or Reduced Price Lunch</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elementary School A</td>
<td>677</td>
<td>21%</td>
<td>23%</td>
</tr>
<tr>
<td>Elementary School B</td>
<td>810</td>
<td>47%</td>
<td>42%</td>
</tr>
<tr>
<td>Elementary School C</td>
<td>356</td>
<td>25%</td>
<td>54%</td>
</tr>
<tr>
<td>Elementary School D</td>
<td>623</td>
<td>30%</td>
<td>58%</td>
</tr>
<tr>
<td>Elementary School E</td>
<td>500</td>
<td>32%</td>
<td>53%</td>
</tr>
<tr>
<td>Elementary School F</td>
<td>868</td>
<td>25%</td>
<td>11%</td>
</tr>
<tr>
<td>Middle School G</td>
<td>637</td>
<td>27%</td>
<td>28%</td>
</tr>
<tr>
<td>Middle School H</td>
<td>575</td>
<td>20%</td>
<td>19%</td>
</tr>
<tr>
<td>Middle School I</td>
<td>426</td>
<td>21%</td>
<td>47%</td>
</tr>
<tr>
<td>Middle School J</td>
<td>751</td>
<td>29%</td>
<td>36%</td>
</tr>
<tr>
<td>Middle School K</td>
<td>732</td>
<td>20%</td>
<td>8%</td>
</tr>
</tbody>
</table>


Data from 2009

Participants were 15 fifth and 15 sixth grade mathematics teachers and the students from one of their mathematics classes. At the fifth grade level, these mathematics classes
included 14 general education classes and one inclusive education class. At the sixth grade level, 6 gifted education, 7 general education, and 2 combined gifted and general education classes participated. 325 students agreed to participate in this study, with an average of 10.83 students participating from each classroom. Five students (two fifth grade and three sixth grade) were excluded because they failed to answer any items on either test, and six additional cases (three fifth grade and three sixth grade) were excluded from analyses of the Representational Transfer Test-Fractions based on evidence of inadequate completion time.

Table 2 shows demographic information for each teacher, the school in which the teacher taught, and the total number of students that participated from the class.

Table 2
Demographic Information of Participating Teachers

<table>
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*a Participating students

Measures

Teacher and Student Demographic Survey

*Teacher Demographic Survey*

The Teacher Demographic Survey (see Appendix A) was researcher-developed and consists of two parts. The first section gathers information on the teacher’s current teaching practices, such as the number of math classes he or she is currently teaching; and past teaching experience, such as the number of years the participant has taught fifth or sixth grade math. Part two gathers information on the teacher’s educational background, particularly his or her math training.

*Student Demographic Survey*

On the Student Demographic Survey (see Appendix B) student participants provided their name, grade, age, and gender. Each student also identified the classroom teacher, how long the student had been with that teacher, and self-reported his or her current math grade.

General Mathematics Ability Assessment

The General Mathematics Ability Assessment (GMA; see Appendix C) measured students’ general mathematics ability. Each of the 12 multiple-choice GMA questions were aligned with at least one of Georgia’s fifth grade standards and were modeled after study questions published online to help students study for Georgia’s end-of-year state tests. The
competencies tested were number sense, decimal and fraction understanding, area and volume calculation, congruence of geometric figures, basic algebra, and interpretation of graphs.

Correct responses were given a score of 1, and incorrect or missing responses were given a score of 0. The highest possible score on the GMA was 12. The reliability for the entire sample was 0.68. (Reliabilities for the fifth grade sample and the sixth grade sample were comparable, with Cronbach’s $\alpha = 0.70$ and 0.68, respectively.)

Representational Transfer Test-Fractions

The Representational Transfer Test-Fractions (RTT-Fractions; see Appendix D) used in this study was modeled after a measure originally created by Feinn and Kulikowich (1999) and validated by Higley (2009). The original version of this assessment was designed to measure college students’ ability to translate between multiple representations of functions (i.e., verbal, graphical, algorithmic, or tabular). All items on the original test were multiple-choice questions in which the stem of the question was presented using one representation and choice options were presented in an alternative format. One question, for example, would include a stem presented in a graphical form with all answer options given as algorithms.

The RTT-Fractions assessment followed the same format but was revised to measure fifth and sixth grade students’ and teachers’ ability to translate between different representations of fractions. The revisions were made with the Georgia state standards in mind. That is, although students may not have been specifically taught to translate between representations, the skills required to answer each question (e.g., finding equivalent fractions, multiplying fractions, etc.) should have been taught by April of the students’ fifth grade year.
The 24 questions on the RTT-Fractions were presented in multiple-choice format. The representations used on this test were verbal description, numerical expression, number line, and geometric figure. One of these representations was used for the stem of each question and all answer options were presented in one of the alternative forms. Every possible combination of stem-choice representation was included twice on the RTT-Fractions for a total of 24 assessment items.

Efforts were made to ensure that there was a balance in the difficulty of the questions along types of representations, both in the knowledge and operations required to answer the question and the difficulty of the fractions used in the problem. First, in creating the test, each stem-choice representation pair was included in two different questions. One of these questions was a basic matching task. The second question for each pair required higher-order manipulations, such as finding equivalent fractions or performing an operation. Figures 1 and 2 show a matching task item and a manipulation task item where the source representation is a numerical expression and the target representation is a geometric figure.

_Figure 1. Matching Item from the Representational Transfer Test-Fractions_
Second, after the test was written, we examined the distribution of the fractions used in each item to ensure that no systematic bias would be introduced by more heavily weighting one type of representation with less familiar fractions. More familiar and easier to work with fractions were those involving halves, thirds, and quarters. Examples of less familiar and thus, more challenging fractions were numbers such as $\frac{9}{5}$, $\frac{7}{12}$, and $3 \frac{3}{7}$. In the end fraction types (e.g., halves, fifths, ninths, etc.) were relatively evenly distributed across representation type, so as not to confound the representations used in each question with fraction type.

In addition, the RTT-Fractions included both congruent and non-congruent items (Duval, 2002; Duval, 2006). Congruent test items, such as the verbal-numerical example shown in Figure 3, contain a one-to-one mapping from elements of the source representation to the target representation. Non-congruent items require the test taker to manipulate one or
both of the representations before translating. Figure 4 shows an example of a non-congruent verbal-numerical test item.

_Figure 3. Congruent Item from the Representational Transfer Test-Fractions_

#29. One and six-sevenths plus nine-fourths

A. $\frac{16}{7} + \frac{4}{9}$  
B. $1 \frac{6}{7} + 9 \frac{1}{4}$  
C. $\frac{1}{6} + \frac{1}{7} + \frac{9}{4}$  
D. $1 \frac{6}{7} + \frac{9}{4}$

_Figure 4. Non-Congruent Item from the Representational Transfer Test-Fractions_

#20. Twelve parts of wholes that are divided into nine equal parts

A. $12 \frac{1}{9}$  
B. $1 \frac{9}{12}$  
C. $\frac{12}{9}$  
D. $\frac{9}{12}$

In the end, each of the four representational formats (numerical expression, geometric figure, verbal description, and number line) was included in 12 items. Each representation appeared as the source representation (stem) six times and the target representation (answer) six times. Additionally, congruent and non-congruent items were divided evenly between the four representations (i.e., six items containing a number line representation required a congruent translation while the remaining six required a non-congruent translation).

Each correct item was given a score of 1 and each incorrect or missing item was given a score of 0, therefore the highest possible score on this test was 24. The reliability of the RTT-Fractions test for the entire sample of teachers and students was 0.83. (Reliabilities were 0.62 for teachers, 0.79 for the entire student population, 0.77 for fifth grade students, and 0.80 for sixth grade students.)
Procedures

Teachers were first recruited for participation through the school district office. Each teacher then selected one mathematics class to participate in the study and invitation letters were sent home along with informed consent forms. Student assent was obtained when the study instruments were completed.

All materials were sent to participating teachers in May 2011 and all instruments were completed during a single session in the classroom. The Teacher Demographic Survey, General Math Test (GMA), and Representational Transfer Test-Fractions (RTT-Fractions) were given as paper-and-pencil materials. To begin the data collection session, the classroom teacher projected the Student Demographic Survey on an overhead projector and students filled the answers in on a Scantron sheet. Once the demographic survey was complete, students were given a paper copy of the GMA. Although students were told that the test booklet could be used to work out answers to each problem, responses were recorded on the Scantron sheet. Following completion of the GMA, the same procedures were followed for completion of the RTT-Fractions. All three measures took approximately one hour to complete. Teachers completed the Teacher Demographic Survey and the RTT-Fractions on the actual testing materials and these instruments were completed at the same time as the student measures.
Chapter 4

RESULTS

Students’ responses to the demographic survey, General Math Ability Assessment (GMA) and Representational Transfer Test-Fractions (RTT-Fractions) were scored via Scantron. Teachers’ responses to the demographic survey and RTT-Fractions were scored by hand. Data were analyzed using SPSS statistical software.

Five students were excluded from analyses involving the GMA for failing to respond to any items on that test. The final sample size for any analyses involving only the GMA was then 320 students (170 fifth grade students and 150 sixth grade students). Six additional students were excluded from analyses involving the RTT-Fractions because of evidence indicating inadequate completion time. The final sample size for analyses involving the RTT-Fractions was 314 students (166 fifth grade students and 148 sixth grade students).

Performance on Representational Transfer Test-Fractions

The mean RTT-Fractions score for the entire sample of teachers and students was 13.12 (SD = 4.73) out of a possible 24 points. The average score on the RTT-Fractions for all 30 teachers was 20.63 (SD = 2.37). The mode number of items correct was 23, with ten out of 30 teachers earning that score. Fifth grade teachers scored, on average, 20.47 (SD = 2.30) and sixth grade teachers’ mean score was 20.80 (SD = 2.51). There was no effect of grade on RTT-Fractions score for teachers, t(28) = -0.38, p = 0.65.

Teachers’ scores were, not surprisingly, higher than the average score of the students 12.40 (SD = 4.25). The mean score for the fifth grade sample was 11.69 (SD = 3.98) and the mean score for the sixth grade sample was 13.20 (SD = 4.42). There was an effect of students’ grade on RTT-Fractions score, t(312) = -3.19, p = .002.
Table 3 shows a breakdown of the means and standard deviations on each item of the RTT-Fractions for the overall sample, teachers, all students, fifth grade students, and sixth grade students. The type of source-target representational pair (e.g., numerical-geometric, etc.) and congruency of the translation (i.e., congruent or non-congruent) are also indicated.

Table 3  
Performance on RTT-Fractions Items by Source-Target Pair and Congruency

| Item | Source-Target Pair Type | Congruency | Sample  
| n = 344 | Teachers  
| n = 30 | Students  
| n = 314 | Grade 5  
| n = 166 | Grade 6  
| n = 148 |
|---|---|---|---|---|---|---|---|---|---|
| 18 | Num-Geo | cong | .96 (.21) | .97 (.18) | .96 (.21) | .98 (.15) | .93 (.25) | .46 (.24) | .44 (.24) | .43 (.24) |
| 19 | Line-Geo | non-cong | .57 (.50) | .93 (.25) | .53 (.50) | .47 (.50) | .60 (.49) | .49 (.49) | .42 (.50) | .42 (.50) |
| 20 | Verb-Num | non-cong | .31 (.46) | .63 (.49) | .27 (.45) | .25 (.43) | .30 (.46) | .29 (.43) | .29 (.43) | .29 (.43) |
| 21 | Num-Line | cong | .74 (.44) | 1.00 | .72 (.45) | .66 (.47) | .78 (.41) | .31 (.29) | .30 (.29) | .31 (.29) |
| 22 | Geo-Num | non-cong | .20 (.40) | .77 (.43) | .15 (.36) | .16 (.37) | .14 (.35) | .25 (.23) | .25 (.23) | .25 (.23) |
| 23 | Line-Verb | non-cong | .46 (.50) | .90 (.31) | .42 (.49) | .42 (.50) | .42 (.50) | .42 (.50) | .42 (.50) | .42 (.50) |
| 24 | Geo-Verb | cong | .94 (.23) | 1.00 | .94 (.24) | .94 (.24) | .94 (.24) | .94 (.24) | .94 (.24) | .94 (.24) |
| 25 | Verb-Line | cong | .32 (.47) | .93 (.25) | .26 (.44) | .19 (.39) | .34 (.48) | .43 (.50) | .35 (.48) | .35 (.48) |
| 26 | Num-Geo | non-cong | .57 (.50) | .87 (.35) | .54 (.50) | .64 (.48) | .43 (.50) | .25 (.23) | .25 (.23) | .25 (.23) |
| 27 | Line-Verb | cong | .39 (.49) | .93 (.25) | .34 (.47) | .26 (.44) | .43 (.50) | .35 (.48) | .35 (.48) | .35 (.48) |
| 28 | Line-Geo | cong | .30 (.46) | .73 (.45) | .26 (.44) | .18 (.39) | .35 (.48) | .25 (.23) | .25 (.23) | .25 (.23) |
| 29 | Verb-Num | cong | .91 (.28) | 1.00 | .90 (.29) | .91 (.29) | .90 (.30) | .90 (.30) | .90 (.30) | .90 (.30) |
| 30 | Num-Line | cong | .53 (.50) | .97 (.18) | .48 (.50) | .45 (.50) | .53 (.50) | .53 (.50) | .53 (.50) | .53 (.50) |
| 31 | Num-Verb | non-cong | .14 (.35) | .30 (.47) | .13 (.33) | .12 (.33) | .14 (.34) | .14 (.34) | .14 (.34) | .14 (.34) |
| 32 | Geo-Line | non-cong | .69 (.47) | .90 (.31) | .67 (.47) | .61 (.49) | .72 (.45) | .29 (.24) | .29 (.24) | .29 (.24) |
| 33 | Line-Num | non-cong | .45 (.50) | .90 (.31) | .41 (.49) | .35 (.48) | .48 (.50) | .25 (.23) | .25 (.23) | .25 (.23) |
| 34 | Geo-Verb | non-cong | .43 (.50) | .97 (.18) | .38 (.49) | .31 (.46) | .45 (.50) | .25 (.23) | .25 (.23) | .25 (.23) |
| 35 | Verb-Geo | non-cong | .40 (.49) | .50 (.51) | .39 (.49) | .33 (.47) | .45 (.50) | .25 (.23) | .25 (.23) | .25 (.23) |
| 36 | Line-Num | non-cong | .29 (.45) | .77 (.43) | .24 (.43) | .17 (.38) | .32 (.47) | .25 (.23) | .25 (.23) | .25 (.23) |
| 37 | Num-Verb | cong | .91 (.29) | 1.00 | .90 (.30) | .90 (.30) | .90 (.30) | .90 (.30) | .90 (.30) | .90 (.30) |
| 38 | Geo-Num | cong | .92 (.27) | .97 (.18) | .92 (.28) | .93 (.26) | .91 (.29) | .29 (.24) | .29 (.24) | .29 (.24) |
| 39 | Verb-Geo | cong | .48 (.50) | .90 (.31) | .44 (.50) | .37 (.48) | .51 (.50) | .25 (.23) | .25 (.23) | .25 (.23) |
| 40 | Geo-Line | cong | .71 (.46) | .93 (.25) | .69 (.46) | .65 (.48) | .73 (.47) | .25 (.23) | .25 (.23) | .25 (.23) |
| 41 | Verb-Line | non-cong | .52 (.50) | .87 (.35) | .48 (.50) | .46 (.50) | .51 (.50) | .25 (.23) | .25 (.23) | .25 (.23) |

*Note. Num = numerical expression; Geo = geometric figure; Line = number line; Verb = verbal description; cong = congruent; non-cong = non-congruent.*

Table 4 gives the means and standard deviations of teachers’, fifth grade students’, and sixth grade students’ performance on the 12 items requiring congruent translations and
the 12 items requiring non-congruent translations. The means of the non-congruent translations were significantly lower than the means of the congruent test items for teachers ($t(28) = 7.12, p < .001$), fifth grade students ($t(164) = 6.54, p < .001$), and sixth grade students ($t(146) = 8.40, p < .001$), indicating that non-congruent translations were more challenging for all participants.

Table 4  

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Ninety percent or more of students correctly answered items 18, 24, 29, 37, and 38; these were, by far, the translation tasks with which students were most successful. Teachers were also highly successful with these items, with 97% or 100% answering correctly. Not surprisingly, all five of these items required only a congruent translation. The stems and choices of these items contained geometric figures, numerical expressions, or a verbal description. It is noteworthy that no items containing a number line representation are included in this list.

Instead, items that require students to translate to or from a number line representation of a fraction were among those with which students were least successful. There were six items that had a mean of .32 or lower, slightly better than chance; three of these items contained a number line as either the source or target representation in a congruent or non-congruent translation. Two of the six most difficult items required students to make a non-congruent translation with a verbal representation of a fraction. As opposed to the easier, congruent verbal translations, which included one-to-one mappings from words to
numbers, non-congruent verbal translations contain a verbal description of the concept of the fraction or operation. For instance, the phrase “Twelve parts of wholes that are divided into nine equal parts” requires a non-congruent translation to $\frac{12}{9}$, while “twelve-ninths” is the congruent verbal form of $\frac{12}{9}$.

It is not surprising that students performed so poorly on the non-congruent, verbal items because the teachers also struggled with these items. Less than one-third of the teachers correctly answered the same non-congruent number-verbal item that was most difficult for students. A non-congruent verbal-geometric and a non-congruent verbal-number item were the other two most challenging items for teachers. Table 5 shows the means and standard deviations for teachers’, fifth grade students’, and sixth grade students’ performance on the 12 items containing a numerical representation, the 12 items containing a geometric representation, the 12 items containing a verbal representation, and the 12 items containing a number line representation. The means support the findings that number line representations are difficult for students, and non-congruent verbal representations are difficult for both students and teachers.
Table 5
Performance on RTT-Fraction Items by Representation

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</tbody>
</table>

A comparison of sixth grade students’ means to fifth grade students’ means shows that, in general, sixth grade students performed better on both number line and verbal items. Of the nine items that at least ten percent more of the sixth grade students than fifth grade students answered correctly, six were number line items and three were non-congruent verbal items. Since sixth grade students scored significantly higher than fifth grade students on the RTT-Fractions as a whole, it appears that the sixth grade students’ better understanding of the number line and the verbal description of the concept of fractions contributed to their higher score.

Another way to consider the contribution of these items to students’ overall translation ability is to regard the relation between specific types of items and scores on the overall RTT-Fractions test. Table 6 shows the correlations between each source-target representation pair and the total RTT-Fractions score for teachers, fifth grade students, and sixth grade students. For teachers and particularly for students, many of the items that highly
correlate with the total RTT-Fractions score involve the number line. This is another indication of the importance of number line understanding in overall translation ability, as students who answered the number line items correctly were also more likely to answer all other items correctly.

Table 6
Correlations of Source-Target Representation Pairs with Total RTT-Fractions Score

<table>
<thead>
<tr>
<th></th>
<th>Teachers</th>
<th>Grade 5</th>
<th>Grade 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Num-Geo</td>
<td>.54**</td>
<td>.30**</td>
<td>.54</td>
</tr>
<tr>
<td>Num-Verb</td>
<td>.45*</td>
<td>.30**</td>
<td>.32**</td>
</tr>
<tr>
<td>Num-Line</td>
<td>-.03</td>
<td>.68**</td>
<td>.61**</td>
</tr>
<tr>
<td>Geo-Num</td>
<td>.52**</td>
<td>.33**</td>
<td>.30**</td>
</tr>
<tr>
<td>Geo-Verb</td>
<td>.29</td>
<td>.70**</td>
<td>.77**</td>
</tr>
<tr>
<td>Geo-Line</td>
<td>.62</td>
<td>.64**</td>
<td>.60**</td>
</tr>
<tr>
<td>Verb-Num</td>
<td>.24</td>
<td>.44**</td>
<td>.55**</td>
</tr>
<tr>
<td>Verb-Geo</td>
<td>.25</td>
<td>.49**</td>
<td>.43**</td>
</tr>
<tr>
<td>Verb-Line</td>
<td>.35</td>
<td>.63**</td>
<td>.69**</td>
</tr>
<tr>
<td>Line-Num</td>
<td>.54**</td>
<td>.55**</td>
<td>.72**</td>
</tr>
<tr>
<td>Line-Geo</td>
<td>.66**</td>
<td>.43**</td>
<td>.51**</td>
</tr>
<tr>
<td>Line-Verb</td>
<td>.70**</td>
<td>.66**</td>
<td>.65**</td>
</tr>
</tbody>
</table>

Note. Pearson’s $r$ correlations. Num = numerical expression; Geo = geometric figure; Verb = verbal description; Line = number line.

* Correlation is significant at the .05 level.
** Correlation is significant at the .01 level.

Student Translation Ability and General Math Ability

Students’ GMA scores were significantly correlated with their self-reported mathematics grades, Spearman’s $\rho(318) = .40$, $p < .01$. This supports the validity of using the GMA as a measure of students’ general mathematics ability. The overall mean score of the GMA was 8.04 (SD = 2.48) out of a possible 12 points. The mean score for the fifth grade sample was 7.85 (SD = 2.53) and the mean score for the sixth grade sample was 8.26 (SD = 2.41). These two means were not statistically significantly different, $t(318) = -1.49$, $p = .14$. 
The GMA and RTT-Fractions scores for the entire sample of students were moderately strongly correlated, $r(312) = 0.69, p < .001$. The two test scores for the fifth grade sample were also moderately strongly correlated, $r(164) = 0.68, p < .001$, as were the test scores for the sixth grade sample, $r(146) = 0.71, p < .001$. The correlation coefficients were not significantly different between the fifth and sixth grade students, $z = -0.51, p = 0.61$.

Teacher Translation Ability and Student Translation Ability

Before examining the relation between teachers’ RTT-Fractions scores and their students’ RTT-Fractions scores, teachers who had fewer than 10 students participating from their class were separated from teachers who had 10 or more participating students in their class. This was an attempt to ensure that the sample of students in a class was representative of the entire class. Ten was selected as the cutoff for student participants because this number represented approximately half of the entire class. To ensure that the teachers retained for this analysis were comparable to the teachers that were excluded, RTT-Fractions test scores and teaching experience for these two groups of teachers were compared. No differences were found between the groups on these measures. Additionally, the three classes of entirely gifted students, the one class of 50% gifted students, and the one class that contained students with special needs were excluded because, presumably, these students would have relatively higher or lower RTT-Fraction scores regardless of their teachers. Thus, only teachers with general education classes were included. The grade, number of students, and RTT-Fraction scores of the 10 teachers who met all the inclusion criteria are shown in Table 7.
Table 7

<table>
<thead>
<tr>
<th>Teacher ID</th>
<th>Grade</th>
<th>Number of Students&lt;sup&gt;a&lt;/sup&gt;</th>
<th>RTT-Fractions Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>22</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>10</td>
<td>22</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>16</td>
<td>19</td>
</tr>
<tr>
<td>9</td>
<td>5</td>
<td>20</td>
<td>22</td>
</tr>
<tr>
<td>11</td>
<td>5</td>
<td>16</td>
<td>21</td>
</tr>
<tr>
<td>12</td>
<td>5</td>
<td>20</td>
<td>23</td>
</tr>
<tr>
<td>15</td>
<td>6</td>
<td>11</td>
<td>23</td>
</tr>
<tr>
<td>17</td>
<td>6</td>
<td>15</td>
<td>16</td>
</tr>
<tr>
<td>25</td>
<td>6</td>
<td>18</td>
<td>20</td>
</tr>
<tr>
<td>26</td>
<td>6</td>
<td>11</td>
<td>22</td>
</tr>
</tbody>
</table>

<sup>a</sup> n = 161.

To examine whether there was an association between teachers’ representational knowledge and the translation ability of their students, teachers were first grouped based on their RTT-Fractions score. Next, the 161 students’ RTT-Fractions scores were categorized based on their distance from the mean. The scores of these students ranged from 4 to 21, and the average score was 11.58 (SD = 3.80). Therefore, scores of 4, 5, 6, and 7 fell between one and two standard deviations below the mean; scores of 8, 9, 10, and 11 were between one standard deviation below the mean and the mean; scores of 12, 13, 14, and 15 fell between the mean and one standard deviation above the mean; scores of 16, 17, 18, and 19 were between one and two standard deviations above the mean; and scores of 20 and 21 were between two and three standard deviations above the mean. Table 8 shows the frequency of student scores by teacher scores.
Table 8
*Distribution of Students’ RTT-Fractions Scores by Teacher Score*

<table>
<thead>
<tr>
<th>Student Score</th>
<th>23</th>
<th>22</th>
<th>21</th>
<th>20</th>
<th>19</th>
<th>16</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 – 21</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>16 – 19</td>
<td>3</td>
<td>6</td>
<td>1</td>
<td>8</td>
<td>5</td>
<td>1</td>
<td>24</td>
</tr>
<tr>
<td>12 – 15</td>
<td>9</td>
<td>13</td>
<td>6</td>
<td>14</td>
<td>6</td>
<td>3</td>
<td>51</td>
</tr>
<tr>
<td>8 – 11</td>
<td>14</td>
<td>18</td>
<td>7</td>
<td>9</td>
<td>2</td>
<td>7</td>
<td>57</td>
</tr>
<tr>
<td>4 – 7</td>
<td>5</td>
<td>4</td>
<td>2</td>
<td>8</td>
<td>1</td>
<td>4</td>
<td>24</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>32</td>
<td>42</td>
<td>16</td>
<td>40</td>
<td>16</td>
<td>15</td>
<td>161</td>
</tr>
</tbody>
</table>

If there is an association between teachers’ translation ability and their students’ translation ability, then we would expect that teachers with lower RTT-Fractions scores would have a greater percentage of their students who also received lower RTT-Fractions scores; teachers with higher scores would have a greater percentage of students with higher scores. A chi-square test of independence was performed to determine if the association between teachers’ and students’ scores was significant.

There does not appear to be an association between a teacher’s RTT-Fractions score and the RTT-Fractions score of a student in his or her classroom, $\chi^2 (20, N = 161) = 22.78, p = .30$. Thus, teachers’ ability to work with and understand different representations of fractions knowledge does not seem to impact their students’ abilities as shown on the RTT-Fractions test.

**Evidence from Artifacts**

Although students recorded their responses to the RTT-Fractions on a Scantron bubble sheet, they were allowed to show their work on the test booklet, which was collected.
An examination of students’ work shows evidence of some of their misconceptions about representations of fractions as well as the strategies they employ when translating.

Misconceptions

Student work showed several misconceptions about fraction operations. Particularly, some students misunderstood the procedures used when adding fractions and multiplying fractions. Mistakes made by students when adding fractions include adding the numerators and denominators separately and adding the reciprocal. Along similar lines, students showed misconceptions when multiplying fractions. They believed they needed to find like denominators in order to multiply fractions, and some cross-multiplied or multiplied by the reciprocal.

Students’ artifacts also revealed their misconceptions about the number line representation, which could explain their difficulties with the items requiring translation to or from the number line. Figure 5 shows several students’ mis-labeling of the number line in the stem of one of the items on the RTT-Fractions, evidence of their misinterpretation of the partitioning between whole numbers on the number line. In more than one case, students accurately performed the operation indicated in the stem of the question, but failed to select the matching choice in the number line representation, revealing that interpreting the number line posed the higher representational challenge.
Strategies

Students’ work also revealed several strategies that they used in order to translate from the source representation to the target. Many students struggled to automatically translate from one representation to another; instead, these students used other procedures which appeared to be routine, albeit unnecessary, responses to questions with fractions. First, students performed the operations given in the stem, even when it was unnecessary or even utterly ineffective. Figure 6 shows an example of a student who performed an unnecessary operation by adding $\frac{2}{3} + 4$; the correct answer could have just as easily been found by matching $\frac{2}{3}$ and 4 to the number line, therefore the work the student did was unnecessary but could still have helped him or her to find the correct answer. Figure 7, on the other hand, shows how a students’ mechanical response of calculating the answer to the addition expression actually prevents him or her from translating. In essence, the student’s habitual response is a waste of time and cognitive energy.
A second routine response seen from students and teachers was to simplify fractions to their most basic forms. For example, many participants simplified \( \frac{2}{6} \) to \( \frac{1}{3} \) in Item 40, when in actuality, the unsimplified fraction would have created a more congruent translation to the number line, where the correct answer was \( \frac{2}{6} \). That students habitually added and simplified fractions indicates that they are more comfortable and likely more practiced at performing
these procedures than the abstract and conceptual processes required for translation. The fact that many students performed operations that resulted in extra work suggests that they did not assess the fractions shown in the choices before carrying out the procedures.

Other students, however, implemented effective strategies that helped them to successfully translate between representations. While procedures were sometimes unnecessary, performing operations sometimes helped a student to translate. For example, several students performed all of the operations in the choices of Item 22 \((\frac{1}{4} + \frac{1}{3}; 4 - 3; \frac{1}{4} \times 3; \frac{1}{4} \div 3)\). They compared each answer to the shaded region in the geometric figure in the stem and were able to answer the question correctly.

Second, many students manipulated the visual representations in the test booklet in order to translate from one representation to another. Figure 8 shows several students’ drawn additions to the visuals they were provided. The first student extended the number line so it was clear that the number line showed one and one-fourth and not one and one-half. The second student added the fractions in the two geometric figures by actually combining them into one figure. The third student added lines to divide the geometric figure and numbered them to find the fraction equivalent to the one shown.
A final effective strategy used by many participants was to recruit a third representation to mediate the translation process between the source and target representations, thereby changing the task from a direct translation to an indirect translation. Numerical representations were the most common mediating representations used by participants. Students numbered the shaded and unshaded regions of a geometric figure, counted the partitions on a number line, and wrote the numerical form of words. Students also used other representations in their indirect translations. Figure 9 shows the work of a student who translated from the source representation in verbal form by creating a geometric figure and then translating that figure to the target representation, a number line.
Teachers’ Artifacts

The work shown on the teachers’ RTT-Fractions was comparable to that of their students. The most common artifact of teachers’ tests was the use of numerical representations, which they employed when performing operations, simplifying, and labeling other representations. Two teachers also used representations that were neither the stem nor target representations of an item to aid in their translation processes; one teacher used a geometric figure and another teacher used verbal descriptions in her problem-solving process.
Chapter 5

DISCUSSION

Summary of Findings

One of the purposes of this study was to explore students’ and teachers’ understanding of multiple representations of fractions, as assessed by the RTT-Fractions. On average, teachers answered just over 86% of the items correctly, and one-third of all the teachers only missed one item. Fifth grade students answered just under half of the questions correctly, and sixth grade students correctly answered slightly more than half, a significant difference. Other studies have shown a similar effect of age on translation ability (Gagatsis, 2004; Lesh et al., 1987a). The significant difference in performance between fifth and sixth grade students may be due to the fact that sixth grade students had an extra year of experience with fractions, or, with this particular sample, it may be because a larger percentage of sixth grade students were in gifted education than fifth grade students.

An examination of students’ and teachers’ patterns of responses revealed several findings. First, all participants found the non-congruent items to be more challenging than the congruent items. This was expected, as by their nature the connections between representations in non-congruent translations are less obvious; Duval (2006) ventures that, for many, non-congruent translations are the “impassable barrier” to mathematics understanding (p. 123).

Second, both students and their teachers struggled with non-congruent verbal items. These items required participants to understand a verbal description of the conceptual meaning of a particular fraction. Neither teachers nor students had the conceptual understanding necessary to correctly interpret these problems. Rather, both students and
teachers were more comfortable carrying out rote procedures, as evidenced by the artifacts of their tests that show they mechanically performed calculations and simplified fractions, even when it was disadvantageous to do so. Like previous studies have indicated (e.g., Eisenhart et al., 1993; Ma, 1999), teachers have greater procedural knowledge than conceptual knowledge, and as a result, their students learn the same.

Third, the data show the importance of understanding the number line representation for translating between different representations of fractions. Items with a number line representation in the stem or the choices were the ones with which students struggled the most. The small percentage of students who answered the items correctly, as well as students’ notations on the number lines provided on the test, show that many students had misconceptions about how to even interpret the number line itself, let alone translate to or from it. One explanation for students’ widespread struggle with understanding this representation is a unique feature of the number line: it combines a visual representation – the lines and hash marks – with a numerical representation – the labels on the hash marks – and requires students to interpret both simultaneously (Bright et al., 1988). Previous studies also show that students struggle with this particular representation (Behr et al., 1983; Larson, 1980; Lesh et al., 1983; Niemi, 1996).

Nevertheless, the number line is an important representation for students to understand when they are seeking to improve their translation ability between representations of fractions. The high correlations between students’ performance on items containing a number line representation and their overall RTT-Fractions score indicate that a student who is able to understand the number line is likely able to understand the other representations without much difficulty. The findings from this study align with the current suggestions for
mathematics education. Based in part on the work of Wu (2002) and the notable difficulties students face with number line representations of fractions, the 2010 IES practice guide on fractions recommended that number lines be used as the “central representational tool” in teaching fractions (Siegler et al., p. 19).

A second purpose of this study was to examine the relation between students’ translation ability and their general mathematics ability, as measured by the RTT-Fractions and GMA, respectively. Analyses reveal that the correlation is moderately strong ($r = .69$ for the entire sample), meaning that approximately 48% of the variance in RTT-Fractions score can be predicted by GMA score. It seems there is a clear relation between translation ability and mathematics understanding. This is an important finding because, though several other studies have shown a relation between translation ability within a particular area and math ability within that same area (e.g., Cifarelli, 1998; Even, 1998; Gagatsis & Shiakalli, 2004; Niemi, 1996; Panaoura et al., 2009), this study is the first to show the relation between translation ability and general mathematics understanding.

The third purpose of this study was to determine whether teachers’ translation ability, as measured by the RTT-Fractions, influenced the translation ability of their students. Contrary to our hypothesis, there was no association between a teacher’s RTT-Fractions score and the scores of his or her students. Possible reasons for the lack of a significant effect are discussed in the limitations section below. Despite being unexpected, this finding is nevertheless interesting, for it leads to further questions about the relation between teachers’ knowledge of representations and the way they use them in their instruction of fractions, as well as how students come to improve their understanding of fraction representations.
Limitations and Suggestions for Future Research

One limitation of this study is that all of the participants were from one school district in Georgia. Although the elementary and middle schools included in this study represented a range on the continuum of size, economic disadvantage, and population of minority students, this study should be replicated with students from other parts of the country in order to determine the generalizability of these findings.

Another limitation of this study is the small sample of teachers analyzed to determine the relation between teachers’ and students’ translation ability. Although 30 teachers were recruited to participate in the study, half of those teachers’ classes had fewer than ten students that agreed to participate in the study and so were excluded from the analyses. The inclusion of only general education classrooms decreased the sample size to ten teachers. The relation between teachers’ and students’ RTT-Fractions scores should be studied with a larger population of teachers to see if the lack of association is replicated.

Third, we assumed that teachers’ performance on the RTT-Fractions is a proxy for the way they use representations in their classrooms, i.e., teachers who score high on the test are more likely to use representations in their instruction, while teachers who earn a lower score are less likely to incorporate multiple representations in their teaching. It is possible that teachers’ performance on the test has no relation to their instructional practices; indeed, other researchers admit that “teachers may use knowledge when responding to items on survey instruments designed to measure teachers’ knowledge, but not use that knowledge when responding to their students’ thinking during lessons” (Izsak, 2008, p. 140-141). Actually observing the teachers’ mathematics lessons would give a better indication of how the teachers were using representations in their instruction. We believe that observational studies
of fractions lessons would provide valuable data in relating teachers’ performance on the RTT-Fractions with their actual math instruction, and relating teachers’ math instruction with the translation ability of their students.

Finally, the analyses of the relation between students’ general math ability and translation ability is still correlational, begging the question, did students who earned a higher score on the GMA perform better on the RTT-Fractions because they understood math better, or do students who can translate from one representation to another have higher general mathematics ability because representation translation ability causes better math understanding? We hypothesize that increasing students’ representational translation ability would lead to an increase in mathematics understanding, but the nature of our study makes this claim impossible to test.

We suggest that future research attempt to answer this question by developing an intervention to teach students how to better translate from one representation to another. The artifacts on students’ tests show that some students already have strategies that help them succeed at translating. Creating a mediating representation for indirect translation was a method used by several students, and is a strategy recommended by other researchers (e.g., Bright et al., 1988; Lesh, 1981). Student and teacher artifacts also revealed that participants mechanically performed numerical processes like performing an operation or simplifying. Instruction should de-emphasize these rote procedures (Lamon, 2001) yet provide plenty of practice for translation to become as routine a process (Stylianou, 2002; Uesaka & Manalo, 2006; Uesaka, Manalo, & Ichikawa, 2007). It is recommended that translation tasks require students to translate bi-directionally, both from the source representation to the target and from the target representation to the source.
Other suggestions for translation instruction include making explicit how representations relate to one another (Ainsworth et al., 1998; Ainsworth et al., 2002; Bright et al., 1988). Computer programs accomplish this by allowing students to observe how one representation is affected when another representation is manipulated, a process known as dyna-linking (Ainworth, 1999; Ainsworth, 2006; Greer, 2009; Kaput, 1998). Interventions involving dyna-linking in other mathematics topics have been effective at improving fifth and sixth grade students’ translation ability (Ainsworth et al., 1998; Ainsworth et al., 2002). Given the positive relation between performance on number line items and overall translation performance, we suggest that any intervention focus on improving students’ understanding of number lines.

Conclusion

Fraction understanding is an important prerequisite for higher level mathematical learning, and representational knowledge is a key component of that. This study shows that translation ability is positively correlated with students’ general mathematics ability, but unfortunately, many students struggle to understand key fraction representations. Their teachers’ representational knowledge does not explain these students’ translation ability, but perhaps the instruction students are receiving accounts for their performance. Future research should examine this relation and contribute to improving the instruction of multiple representations of fractions.
REFERENCES


Greer (Eds.), *Theories of mathematical learning* (pp. 397-430). Hillsdale, NJ: Erlbaum.


University of California, Berkeley.
Appendix A

TEACHER SURVEY

Name ________________________________________________

Age (please circle):  under 25  25-29  30-39  40-49  50-59  60 or more

Gender (please circle):  female  male

What grade do you teach?  Fifth Grade  Sixth Grade
                         Other _______________________________

How many different students do you teach in a typical day? _______________

Please circle all of the subjects that you teach in a typical day:

Writing  Spelling  Reading  English  Language Arts
Math  Science  Social Studies
Physical Education  Art  Music

Please circle all of the types of classes that you teach in a typical day:

General Education  Special Education  Gifted Education
                         Other _______________________________

How would you classify the class that is taking this test?

General Education  Special Education  Gifted Education
                         Other _______________________________

Do you have teaching certification?  Yes  No

If yes, what are you certified to teach? ________________________________

At the end of this school year, for how many years will you have taught? _______________

How many years will you have taught fifth or sixth grade? _______________
(If you have taught fifth and sixth grade at the same time, please specify.)

How many years will you have taught math? _______________

How many years will you have taught fifth grade math or sixth grade math? _______________
(If you have taught fifth grade math and sixth grade math at the same time, please specify.)
EDUCATIONAL BACKGROUND

Which educational degrees have you obtained?

___ Associate’s degree
   What is that degree in?__________________________________________
   year completed: __________

___ Bachelor’s degree
   What is that degree in?__________________________________________
   year completed: __________

___ Master’s degree
   What is that degree in?__________________________________________
   year completed: __________

___ PhD
   What is that degree in?__________________________________________
   year completed: __________

___ Certificate Program
   What is your certification in?_____________________________________
   year completed: __________

Approximately how many college-level math courses have you taken? __________

Please list them:
____________________________________________________________________
____________________________________________________________________
____________________________________________________________________
____________________________________________________________________
____________________________________________________________________
____________________________________________________________________
____________________________________________________________________
____________________________________________________________________
____________________________________________________________________
____________________________________________________________________
____________________________________________________________________
____________________________________________________________________
____________________________________________________________________
____________________________________________________________________
Appendix B

STUDENT DEMOGRAPHIC SURVEY

In the space at the top marked NAME, please write your name.

In the space at the top marked COURSE, please write your teacher’s name.

Please answer the following questions on your bubble sheet by filling in the correct answer:

1. What grade are you in?
   A. 5th Grade
   B. 6th Grade

2. What gender are you?
   A. Male
   B. Female

3. How old are you?
   A. 9 years old
   B. 10 years old
   C. 11 years old
   D. 12 years old
   E. 13 years old

4. What do you think your math grade is right now?
   A. A (90%-100%)
   B. B (80%-89%)
   C. C (70%-79%)
   D. D (65%-69%)
   E. F (less than 65%)

5. How long have you had this teacher for this class?
   A. I’ve been in this class the entire school year.
   B. I moved into this class before winter break.
   C. I moved into this class after winter break.
   D. I had this same teacher last year.
Appendix C

GENERAL MATHEMATICS ABILITY ASSESSMENT

Directions: Try your best to answer these 12 questions using what you’ve learned about math. You may use the test booklet for scratch work, but please be sure bubble in the correct answer on the Scantron, beginning with number 6. You may NOT use a calculator.

6. Paul is solving a riddle. He needs to find a set of three numbers that contain exactly:
   • one prime number
   • two even numbers
   • one multiple of 6
   • two factors of 24

Which set of numbers is a correct answer to the riddle?

A  { 4, 8, 9 }
B  { 8, 11, 12 }
C  { 3, 14, 18 }
D  { 7, 12, 24 }

7. Which expression is true?

A  0.8 < 0.42 < 0.302
B  0.200 > 0.20 > 0.2
C  0.099 > 0.88 > 0.77
D  0.135 < 0.46 < 0.9

8. Hannah’s chocolate chip cookie recipe calls for \( \frac{1}{4} \) of a cup of sugar and makes 24 cookies. Hannah only wants to make 12 cookies.

How much sugar should Hannah use?

A  \( \frac{1}{2} \) of a cup
B  \( \frac{1}{6} \) of a cup
C  \( \frac{1}{8} \) of a cup
D  \( \frac{1}{12} \) of a cup

9. Nina has \( n \) siblings. The number of siblings that Randy has can be found using the expression:

\[ 3n - 2 \]

If Nina has 2 siblings, how many siblings does Randy have?

A  1
B  3
C  4
D  5
10. Mr. Kramer made a circle graph to show the percentage of his students that like each sport.

**Favorite Sports**

- Baseball
- Basketball
- Soccer
- Football

Which is the BEST estimate for the percentage of students whose favorite sport is baseball?

A  33%
B  45%
C  50%
D  55%

11. Monica and Robert brought apple juice to a school party. Monica brought 13 quarts of apple juice. Robert brought 3 gallons of apple juice.

How many cups of apple juice did they bring all together?
(1 gallon = 4 quarts; 1 quart = 4 cups)

A  25 cups
B  64 cups
C  76 cups
D  100 cups

12. Mrs. Richards wanted to show the change in her class’s average spelling test grade over time.

Which type of graph should she use to show this information BEST?

A  Venn diagram
B  pictrograph
C  line graph
D  circle graph
13. What is the area of this shape?

- A 44 cm²
- B 72 cm²
- C 88 cm²
- D 120 cm²

14. Darryl filled a one-liter container with milk so the container was three-fourths full.

How many milliliters of milk did he pour into the container?

- A 0.75 milliliters
- B 75 milliliters
- C 750 milliliters
- D 75,000 milliliters

15. Angela bought a cage for her pet lizard. The volume of the cage is greater than 700 in.³ but less than 800 in.³.

Which cage did Angela buy? (V = Bh)

- A
  - 10 in.
  - 15 in.
  - 4 in.

- B
  - 10 in.
  - 9 in.
  - 6 in.

- C
  - 9 in.
  - 25 in.
  - 4 in.

- D
  - 10 in.
  - 9 in.
  - 8 in.
16. Which pair of shapes is congruent?

A

A

B

B

C

C

D

D

17. The manager of Brian’s Bookstore made a line graph to show the number of books sold for each season of the year last year.

Which statement is true about the book sales at Brian’s Bookstore last year?

A  Four times as many books were sold in the Summer as in the Spring.
B  More books were sold in the Winter than in the Autumn.
C  300 more books were sold in the Summer than in the Autumn.
D  Half as many books were sold in the Spring as in the Winter.
Appendix D

REPRESENTATIONAL TRANSFER TEST – FRACTIONS

Directions: Try your best to answer these 24 questions about fractions. For each question, match the fraction you see with the same fraction that is shown in the choices. Some of these questions may be unfamiliar to you, and that is okay! Just try your best. You may use the test booklet for scratch work, but be sure to bubble in the correct answer on the Scantron, starting with # 18.

#18. \( \frac{3}{5} \)

A. \[\text{[Diagram A]}\]  
B. \[\text{[Diagram B]}\]  
C. \[\text{[Diagram C]}\]  
D. \[\text{[Diagram D]}\]

#19.

\[\begin{array}{ccc}
0 & 1 & 2 \\
\hline
& & \\
& & \\
& & \\
3 & & \\
\end{array}\]

A. \[\text{[Diagram A]}\]  
B. \[\text{[Diagram B]}\]  
C. \[\text{[Diagram C]}\]  
D. \[\text{[Diagram D]}\]
#20. Twelve parts of wholes that are divided into nine equal parts

A. $12 \frac{1}{9}$  
B. $1 \frac{9}{12}$  
C. $\frac{12}{9}$  
D. $\frac{9}{12}$

#21. $\frac{2}{3} + 4$

A. 

B. 

C. 

D. 
#22. A. \( \frac{1}{4} + \frac{1}{3} \)  
B. \( 4 - 3 \)  
C. \( \frac{1}{4} \times 3 \)  
D. \( \frac{1}{4} \div 3 \)

#23. A. Two and two-thirds times three-fourths.  
B. Three times four.  
C. Four-thirds plus two-thirds.  
D. Four and one-third minus two and two-thirds.
#24. 

A. Six-sevenths.
B. Three and three-sevenths.
C. Six-tenths.
D. Three and three-tenths.


A. 
B. 
C. 
D. 0 1 2 3 4 5 6 7 8 9 10
#26. $\frac{2}{6} \times \frac{4}{5}$

A. ![Grid A]

B. ![Grid B]

C. ![Grid C]

D. ![Grid D]

#27. 0 1 2 3

A. Three-thirds.

B. Nine-thirds.

C. Three and one-third.

D. Three-ninths.
#28. 0 – 1

A. [Diagram of two circles, one whole and one quarter]

B. [Diagram of two triangles, one filled and one unfilled]

C. [Diagram of a rectangle with four sections, one section is different]

D. [Diagram of a square with one section shaded]

#29. One and six-sevenths plus nine-fourths

A. $\frac{16}{7} + \frac{4}{9}$  
B. $1 \frac{6}{7} + 9 \frac{1}{4}$

C. $\frac{1}{6} + \frac{1}{7} + \frac{9}{4}$  
D. $1 \frac{6}{7} + \frac{9}{4}$
#30. \(\frac{38}{3}\)

A. The total of one-ninth combined with two-thirds.

B. The product of one part of nine wholes times two parts of three wholes.

C. Two-thirds of one part of a whole that is divided into nine equal parts.

D. The cross-product multiplicative of one-third and nine-halves.

#31. \(\frac{1}{9} \times \frac{2}{3}\)

A. The total of one-ninth combined with two-thirds.

B. The product of one part of nine wholes times two parts of three wholes.

C. Two-thirds of one part of a whole that is divided into nine equal parts.

D. The cross-product multiplicative of one-third and nine-halves.
A. $\frac{14}{10}$  
B. $\frac{11}{7}$  
C. $1\frac{1}{2}$  
D. $1\frac{7}{10}$
#34. Seven parts of a whole that is divided into twelve equal parts.

A. Eleven-twelfths.
B. Three and one-third.
C. Three halves.
D. Fourteen-fourths.

#35. Seven parts of a whole that is divided into twelve equal parts.

A. [Diagram of shaded parts]
B. [Diagram of shaded parts]
C. [Diagram of shaded parts]
D. [Diagram of shaded parts]
#36.

A. \( \frac{14}{4} + \frac{2}{4} \)  
B. \( \frac{4}{2} - \frac{5}{4} \)  
C. \( \frac{2}{4} \div \frac{3}{4} \)  
D. \( \frac{3}{4} \times \frac{1}{2} \)

#37. \( 3 \frac{1}{2} \)

A. Three times one-half.  
B. Thirty-one-halves.  
C. Three and one-half.  
D. Three parts of one-half.

#38.

A. \( \frac{3}{9} \)  
B. \( \frac{9}{3} \)  
C. \( \frac{3}{6} \)  
D. \( \frac{6}{3} \)
#39. Five-fourths times three.

A.

B.

C.

D.
#40.

A. 0 \hspace{1cm} 1

B. 0 \hspace{1cm} 1

C. 0 \hspace{1cm} 1

D. 0 \hspace{1cm} 1

#41. One-fourth of one-half.

A. 0 \hspace{1cm} 1

B. 0 \hspace{1cm} 1

C. 0 \hspace{1cm} 1

D. 0 \hspace{1cm} 1