MODELING AND SIMULATION OF ELECTROMAGNETIC BAND GAP STRUCTURES AND METAMATERIALS

A Dissertation in
Electrical Engineering
by
Kyungho Yoo

© 2010 Kyungho Yoo

Submitted in Partial Fulfillment
of the Requirements
for the Degree of

Doctor of Philosophy

December 2010
The dissertation of Kyungho Yoo was reviewed and approved* by the following:

Raj Mittra
Professor of Electrical Engineering
Dissertation Advisor
Chair of Committee

James K. Breakall
Professor of Electrical Engineering

Anthony J. Ferraro
Professor Emeritus of Electrical Engineering

Michael T. Lanagan
Professor of Engineering Science and Mechanics

W. Kenneth Jenkins
Professor of Electrical Engineering
Head of the Department of Electrical Engineering

*Signatures are on file in the Graduate School
ABSTRACT

Electromagnetic Band Gap (EBG) structures, engineered to achieve desired transmission and reflection characteristics in specific frequency bands, have recently attracted considerable attention due to growing interest in improving antenna performance. Almost simultaneously, metamaterials (MTMs) became popular because they have unusual features, not readily available in nature, promised to make possible new applications for microwave circuits and antenna composites. EBGs and MTMs are typically created by using periodic inclusions of metallic or dielectric material embedded in a homogeneous background medium. Due to their unique bandgap features, EBG structures can be regarded as a special type of MTM. In fact, these two terms, EBG and MTM, are sometimes used interchangeably. Typical examples of their applications for enhancing antenna performance include high-directivity antennas, low-profile antennas, high impedance surfaces, and dichroic surfaces.

In this dissertation, we focus primarily on three aspects. First, we develop a novel and systematic approach to enhancing the directivity of an antenna covered by an EBG and MTM and show that high directivity is achieved at the resonant frequency of the structure. We also illustrate the application of the above procedure by designing directivity-enhanced planar antennas, for instance, dipole and microstrip patch antenna arrays used as exciters for a Fabry–Perot cavity with a dielectric slab as the superstrate.

Second, we describe two novel techniques involving a combination of the Dipole Moment (DM) method and the Characteristic Basis Function Method (CBFM) to model periodic structures of EBGs and MTMs. We show that combining the above two methods leads to a relatively small matrix, often only 2×2 or 3×3 in size, without running into ill-conditioning problems for many typical elements.
Third, we develop guidelines for using the Equivalent Medium Approach (EMA) to the study of metamaterial structures. We examine the concept of effective material parameters by completing the retrieval process of these parameters according to the inversion approach and identify some fundamental problem areas encountered when applying the algorithm to a slab of artificial dielectrics. We then use a Gaussian beam to excite this structure in order to examine the direction of the beam’s wave propagation in such a medium.
# TABLE OF CONTENTS

LIST OF FIGURES ................................................................................................................. vii

LIST OF TABLES ................................................................................................................... xii

ACKNOWLEDGEMENTS ..................................................................................................... xiii

Chapter 1  Introduction ............................................................................................................ 1
  1.1 Overview .................................................................................................................... 1
  1.2 Research Objectives ................................................................................................... 5

Chapter 2  A Systematic Approach to Enhancing the Directivity of Antennas via the Use of EBG Superstrates ................................................................. 10
  2.1 Introduction ................................................................................................................ 10
  2.2 Fabry-Perot Cavity ..................................................................................................... 13
  2.3 Proposed Composite Structures ................................................................................. 15
     2.3.1 Octagonal Aperture FSS Superstrate-Antenna Composite .............................. 16
     2.3.2 Strip FSS Superstrate–Antenna Composite ..................................................... 28
     2.3.3 Dielectric Superstrate–Antenna Composite .................................................... 31
     2.3.4 Performance comparison of three superstrates ................................................ 36
  2.4 Antenna Array ............................................................................................................ 43
     2.4.1 Dipole Antenna Array ..................................................................................... 43
     2.4.2 MPA Antenna Array ....................................................................................... 46
  2.5 Conclusions ................................................................................................................ 50

Chapter 3  A Novel Technique for the Analysis of Periodic Structures Including EBGs ....... 52
  3.1 Introduction ................................................................................................................ 52
  3.2 Dipole Moment Method ............................................................................................. 54
     3.2.1 Introduction to the Dipole Moment (DM) Formulation ................................. 54
     3.2.2 Numerical Result for PEC rod ................................................................. 59
  3.3 Characteristic Basis Function Method ....................................................................... 63
     3.3.1 Introduction to Characteristic Basis Function (CBF) Formulation ............... 63
     3.3.2 Analytical expression for the fields radiated by a piecewise sinusoidal ...... 65
     3.3.3 Reflection and Transmission Coefficients for Periodic Array ..................... 70
  3.4 Numerical Examples .................................................................................................. 70
     3.4.1 Dipole Array .................................................................................................... 71
     3.4.2 Tripole Array ................................................................................................... 75
     3.4.3 Square Loop Array .......................................................................................... 84
     3.4.4 Split-Ring Array .............................................................................................. 89
  3.5 Conclusions ................................................................................................................ 92

Chapter 4  Electromagnetic Characteristics of Metamaterials Using Rigorous Numerical Modeling ................................................................. 93
LIST OF FIGURES

Figure.2.1: EBG–Antenna composite. ................................................................. 12
Figure.2.2: Candidates for antenna superstrates. ............................................. 12
Figure.2.3: Schematic view of the Fabry–Perot interferometer. ........................ 14
Figure.2.4: Fabry–Perot cavity formed by the superstrate and the ground plane. 15
Figure.2.5: (a) Geometry of the 7 × 7 octagonal aperture–FSS composite; and (b) One cell. ............................................................... 17
Figure.2.6: Unit cell of the octagonal aperture–FSS composite. .......................... 17
Figure.2.7: Transmission coefficient of the FSS unit cell with one layer and two layers. 18
Figure.2.8: Electric field (Ex) distribution of the structure of Fig. 2.6 at (a) 33.6 GHz; and (b) 45.1 GHz. ................................................................. 19
Figure.2.9: Directivity of the FSS–antenna composite with patch antennas operating at two different frequencies. .................................................... 20
Figure.2.10: Electric field (Ex) distribution of magnitude and phase on the center plane between the superstrate and the ground plane at (a) 34.5 GHz; and (b) 45.1 GHz. ............... 21
Figure.2.11: Electric field (Ex) distribution of magnitude and phase on the YZ plane along the centerline at (a) 34.5 GHz; and (b) 45.1 GHz. ......................... 22
Figure.2.12: Geometry of a 15 × 15 octagonal aperture–FSS composite. ............... 23
Figure.2.13: Electric field (Ex) distribution of magnitude and phase on the ① and ② planes of the structure in Fig. 2.12 at 33.6 GHz..................... 24
Figure.2.14: Directivity of the FSS–antenna composite with two different sizes of FSS (7 × 7 and 15 × 15 elements). ........................................ 24
Figure.2.15: Comparison of radiation patterns of FSS–antenna composite with two different sizes of FSS (7 × 7 at 34.5 GHz and 15 × 15 at 33.5 GHz) at (a) φ = 0°; and (b) φ = 90°. ...................................................... 25
Figure.2.16: (a) Geometry of a 7 × 7 FSS and a substrate with an oblique angle of plane wave incidence; and (b) Side view (XZ cross-section). ................................... 26
Figure 2.17: Electric field (Ex) distribution of magnitude and phase on the ① plane of the structure in Fig. 2.16 as a function of theta.

Figure 2.18: Comparison of the angular response of the FSS–composite directivity and the cavity field of Fig. 2.17.

Figure 2.19: (a) Geometry of a strip–FSS composite; and (b) One cell of the FSS superstrate.

Figure 2.20: Electric field (Ex) distribution of magnitude and phase on the ① and ② planes of the FSS composite at 34.4 GHz.

Figure 2.21: Directivity of the FSS–antenna composite.

Figure 2.22: Dielectric slab–antenna composite.

Figure 2.23: (a) One cell of the dielectric superstrate; (b) Transmission coefficient of the dielectric slab unit cell with one layer and two layers; and (c) Electric field (Ex) distribution of a unit cell with two layers, one at 32.2 and one at 42.5 GHz.

Figure 2.24: Electric field (Ex) distribution of magnitude and phase on the ① and ② planes of the FSS composite at 34 GHz.

Figure 2.25: Directivity of the dielectric slab–antenna composite.

Figure 2.26: MPA with different superstrates: (a) Dielectric slab; and (b) FSS.

Figure 2.27: MPA with a DNG superstrate. Dipole parameters: length = 5 mm; width = 0.5 mm; - 1 mm. SRR parameters: side length = 3 mm; strip width = 0.25 mm; gap = 0.5 mm. Distance between the SRR and dipole is 0.25 mm.

Figure 2.28: Response of SRR + dipole combination: (a) Effective permeability; and (b) Effective permittivity (shaded areas represent non-physical regions).

Figure 2.29: Comparison of directivities realized by using dielectric, FSS, and DNG superstrates.

Figure 2.30: E-field distributions along lines 1 and 2 on the aperture plane.

Figure 2.31: Dielectric slab–Antenna composite excited with a dipole.

Figure 2.32: (a) Directivity of the composite; and (b) return loss of a dipole source.

Figure 2.33: (a) Dielectric slab–antenna composite excited with three dipole sources; and (b) Their S11 characteristics.

Figure 2.34: (a) Directivity; and (b) Radiation pattern at 31 GHz of the composite.
Figure 2.35: (a) Case 1: 25-patch array without superstrate; (b) Case 2: 25-patch array with one-layer superstrate; (c) Case 3: 25 patch array with two-layer superstrate; and (d) Case 4: 25-patch array with one-layer high permittivity superstrate.

Figure 2.36: Directivity comparison of the four cases.

Figure 2.37: Radiation patterns of case 2 and case 3.

Figure 2.38: Return loss of case 3 and of case 4.

Figure 3.1: Infinite doubly periodic structure with arbitrary geometrical shapes.

Figure 3.2: Dipole Moment Concept.

Figure 3.3: (a) PEC rod; (b) using DMs; and (c) using DMs and macro-basis functions.

Figure 3.4: Current magnitude flowing along the wire.

Figure 3.5: (a) Top half of bent wire; and (b) Bottom half of bent wire.

Figure 3.6: Geometry of thin-wire element.

Figure 3.7: (a) FSS composed of infinite doubly periodic dipoles; and (b) Dipole FSS element.

Figure 3.8: (a) Current maximum on the center element of the dipole array for the periodicity of $0.7\lambda$ in the normal incidence case; and (b) Prony Estimation.

Figure 3.9: Reflection Coefficient of the dipole array for the periodicity of $0.7\lambda$.

Figure 3.10: (a) FSS composed of infinite doubly periodic tripoles; and (b) Tripole FSS element.

Figure 3.11: Characteristic Basis Functions of the tripole element.

Figure 3.12: Current magnitudes for the three tripole branches.

Figure 3.13: (a) Current maximum on the center element of the tripole array for the periodicity of $0.5\lambda$ of the normal incidence case; and (b) Prony Estimation.

Figure 3.14: Reflection Coefficient of the tripole array, for the periodicity of (a) $0.5\lambda$; and (b) $0.7\lambda$.

Figure 3.15: FSS composed of infinite doubly periodic tripoles rotated to 45° clockwise around the z axis.

Figure 3.16: Definition of Euler angles ($\phi$, $\theta$, $\psi$).
Figure 3.17: Reflection Coefficient of the tripole array rotated to 45° clockwise around the z axis, for the periodicity of (a) 0.5λ; and (b) 0.7λ.

Figure 3.18: FSS composed of infinite doubly periodic square loops.

Figure 3.19: Characteristic Basis Functions of the square loop element.

Figure 3.20: Current magnitude and phase for different polarization angles of the incident at 1 GHz (a) z-polarized; (b) y-polarized; and (c) diagonally polarized.

Figure 3.21: Reflection coefficient of the square loop array for the periodicity of 0.5λ.

Figure 3.22: Reflection Coefficient of the square loop array for the periodicity of 0.5λ rotated to θ clockwise around the z axis (a) θ = 45; and (b) θ = 90.

Figure 3.23: Reflection Coefficient of the split-ring array for the periodicity of 0.5λ rotated to θ clockwise around the z axis (a) θ = 0; (b) θ = 45; and (c) θ = 90.

Figure 4.1: Applications of metamaterials: (a) Superlensing effect by a DNG slab; (b) MPA with a DNG superstrate; (c) Dipole encased in a spherical shell of ENG material; and (d) Cylindrical cloak.

Figure 4.2: Inversion approach for calculating the effective ε and μ of a metamaterial slab.

Figure 4.3: Geometry of split rings and wires.

Figure 4.4: Reflection and transmission coefficients of the unit cell of split-rings and wires: (a) magnitude; and (b) phase.

Figure 4.5: Effective material parameters: gray area represents the non-physical region where no solutions can be found to satisfy both ε″ and μ″ ≤ 0.

Figure 4.6: The magnitudes (Left) and phases (Right) of Ez at 15 GHz on the XY plane (H-plane) for three different configurations at normal incidence (the two dashed lines indicate the planar interfaces of the slab).

Figure 4.7: Phase comparison of Ez along the x-axis at 0.1λ above the top surface of the slabs.

Figure 4.8: The magnitudes (Left) and phases (Right) of Ez at 15 GHz on the YZ plane (E-plane) for three different configurations at oblique TMy incidence (30° off-normal) (the two dashed lines represent the planar interfaces of the slab).

Figure 4.9: Geometry of a multi-layer holey dielectric plate.

Figure 4.10: Reflection and transmission coefficients of the unit cell of holy dielectric plates: (a) magnitude; and (b) phase.
Figure 4.11: Effective material parameters: gray area represents the non-physical region where no solutions can be found to satisfy both $\varepsilon''$ and $\mu'' \leq 0$.

Figure 4.12: The magnitudes (Left) and phases (Right) of Ez at 30 GHz on the XY plane (H-plane) for three different configurations at normal incidence. The two dashed lines indicate the planar interfaces of the slab.

Figure 4.13: Phase comparison of Ez along x-axis at $0.1\lambda$ above the top surface of the slabs.

Figure 4.14: The magnitudes (Left) and phases (Right) of Ez at 30 GHz on the YZ plane (E-plane) for three different configurations at oblique TMy incidence (30° off-normal) (the two dashed lines represent the planar interfaces of the slab).

Figure 4.15: Geometry of cross-wires.

Figure 4.16: Reflection and transmission coefficients of the unit cell of cross-wires: (a) magnitude; and (b) phase.

Figure 4.17: Effective material parameters: gray area represents the non-physical region where no solutions can be found to satisfy both $\varepsilon''$ and $\mu'' \leq 0$.

Figure 4.18: The magnitudes (Left) and phases (Right) of Ez at 36 GHz on the XY plane (H-plane) for three different configurations at normal incidence (the two dashed lines represent the planar interfaces of the slab).

Figure 4.19: The magnitudes (Left) and phases (Right) of Ez at 36 GHz on the YZ plane (E-plane) for three different configurations at oblique TMy incidence (30° off-normal) (the two dashed lines represent the planar interfaces of the slab).

Figure 4.20: Geometry of the fishnet.

Figure 4.21: Reflection and transmission coefficients of the unit cell of fishnets: (a) magnitude; and (b) phase.

Figure 4.22: Effective material parameters.

Figure 4.23: The magnitudes (Left) and phases (Right) of Ex at 32 GHz on the XY plane (E-plane) for three different configurations at normal incidence (the two dashed lines represent the planar interfaces of the slab).

Figure 4.24: The magnitude (Left) and phase (Right) of Ex at 32 GHz on the YZ plane (H-plane) for three different configurations at oblique TEy incidence (30° off-normal) (the two dashed lines represent the planar interfaces of the slab).
LIST OF TABLES

Table 3-1: Comparison of the total simulation times using the CBF-Closed Form code and the commercial packages for the dipole array .......................................................... 74

Table 3-2: Comparison of the total simulation times using the CBF-Closed Form code and the commercial packages for the tripole array ......................................................... 84

Table 3-3: Comparison of the total simulation times using the CBF-Closed Form code and the commercial package for the square loop array ........................................................... 89
I would like to dedicate this dissertation to my family. My mom, Jungsook, and my brother, Changwook, have always supported me and prayed sincerely that I would be successful. Without their deep concern and encouragement, I would never have made it through some of the hard times of my journey towards completing my Ph.D. degree.

I would like to express my hearty thanks to Dr. Raj Mittra for his instruction and support throughout my academic program. Dr. Mittra, a great master in the electromagnetic field, is my mentor and my role model. It was a great honor for me to study this most fascinating and rewarding field under his supervision. I shall also always treasure my precious memories of the great times I spent in his company at State College’s Korean and Indian restaurants.

I would like to express my sincere gratitude to all my committee members, Dr. James K. Breakall, Dr. Anthony J. Ferraro and Dr. Michael T. Lanagan, for the time and effort they have put into reviewing this work. I am also indebted to my friends and colleagues at the Electromagnetic Communication Laboratory (ECL) at Penn State, Dr. Jonathan N. Bringuier, Kadappan (Kip) Panayappan, Nikhil (Nik) Mehta, Arash Rashidi, Dr. Wenhau Yu, Yongjun Liu, Xialong (Bob) Yang, Dr. Kai Du, Dr. Lai-Ching (Kit) Ma, Dr. Neng-Tien Huang, Dr. Nader Farahat, Jing (Irene) Qu, and Yanfei (Tracie) Li. Without their technical support and valuable contributions, I would never have been able to complete this dissertation.
Chapter 1

Introduction

1.1 Overview

Electromagnetic Band Gap (EBG) structures [1] have attracted considerable attention in recent years, as evidenced by the numerous studies that have created a considerable body of knowledge on the subject. EBGs are engineered to realize desired transmission and reflection characteristics in specific frequency bands. They are typically fabricated by using periodic inclusions of metallic or dielectric material embedded in a homogeneous background material. Though primarily known as Photonic Band Gap (PBG) structures [2] that originated from research into solid-state physics and optics, EBGs are now finding a wide variety of applications as components in microwave and millimeter devices, as well as in antennas. Since the bandgap feature was first realized and experimentally demonstrated with periodic dielectric structures in the early 1990s, EBG designs have flourished, and a wide variety of materials and geometries, i.e., arrays of dielectric rod structures or dielectric woodpile structures [3], have been investigated.

At approximately the same time, another term, *metamaterials*, [4-5] also appeared and became popular among physics and electromagnetic field communities. The ancient Greek prefix *meta* (meaning *beyond*) has been used to describe composite materials with unusual features not readily available in nature. It is worth pointing out that metamaterials (MTMs)—also called *artificial dielectrics*—are realized by periodic inclusions in the same way as EBGs are; yet, the former provides many interesting wave phenomena such as the negative refraction and focusing effect. MTMs are frequently designed to exhibit DNG (double negative: both permittivity $\varepsilon$ and permeability $\mu$ are negative) properties in the frequency range of interest. However, for other
frequency regions, they may exhibit DPS (double positive: both $\varepsilon$ and $\mu$ are positive), ENG ($\varepsilon$ negative), or MNG ($\mu$ negative) properties. Due to their unique bandgap features, EBG structures can be regarded as a special type of metamaterial. In fact, these two terms, EBG and MTM, are sometimes used interchangeably.

To characterize and design EBG structures, various modeling and simulation methods have been implemented. The lumped LC model was first proposed in [6] to explain the bandgap feature of the EBG structure as an LC resonant circuit. This is a fairly simple model, but it does not guarantee accurate results, as it relies on the approximated inductance $L$ and the capacitance $C$. The periodic transmission line model [7] is another popular technique used in EBG characterizations. It consists of a common transmission line represented by a series inductance $L_R$ and a shunt capacitance $C_R$, extended by a series capacitance $C_L$ and a shunt inductance $L_L$ (all per unit length). The dispersion curve can be readily obtained by applying the Floquet spatial harmonic expansion for the cascaded transmission line, which provides information on the slow-wave, leaky-wave, backward-wave, and bandgap regions. However, this method gives rise to difficulties in the process of determining the equivalent medium characteristics of the EBG structure. Another option, though, is a new and promising technique called the plane wave spectral (PWS) expansion approach [8-9], which has been used to analyze the scattering problems of EBG structures. It begins by expanding the aperture electric fields and magnetic fields of the structure in terms of a spectrum of plane waves. The structure’s electromagnetic responses are then calculated for each of these spectral components and superimposed to construct the final solution. This approach has been found to be computationally efficient, and it can readily handle very large EBG structures.

Although numerical methods present considerable challenges, e.g., inefficiency in modeling of complex structures with multi-scale features, they are still widely used in the analysis and design of EBG materials because they can analyze general EBG structures with
arbitrary configurations. The most commonly used computational electromagnetic (CEM) techniques are the Method of Moments (MoM) [10-11], the Finite Element Method (FEM) [12], and the Finite Difference Time Domain (FDTD) Method [13-15]. The MoM algorithm is the most efficient for analyzing large structures involving only metallic conductors or homogeneous dielectrics. However, using it to solve inhomogeneous dielectric or complex geometry presents great challenges. In addition, the conventional MoM has difficulty managing the low-frequency breakdown problem introduced by the dominance of the scalar potential term over the vector potential as the frequency approaches zero. In contrast to MoM, FEM handles complex geometries and inhomogeneous materials with relative ease. However, when the electrical size of the geometry of interest is large, FEM requires the solution of a large sparse matrix, which can make the solver computationally infeasible. While both MoM and FEM are frequency–domain methods, time–domain techniques such as the FDTD or FIT (Finite Integration Technique) have also been widely utilized to perform broadband analysis of EBGs. The 3-D full-wave FDTD solver is a good and versatile tool capable of handling complex, large-scale, and arbitrarily inhomogeneous structures. However, an FDTD modeling of multi-scale structures or elements with fine features can place a very heavy burden on both the CPU time and memory.

With the increasing complexity of EBG geometries and challenges in EBG properties, optimization schemes such as the Genetic Algorithm (GA) [16] and Particle Swarm Optimization (PSO) [17] are being used to design optimum EBG structures.

The astonishing growth of the fields based on the use of metamaterials has also created electromagnetic modeling problems on a grand scale, even though their physical dimensions are small compared to the wavelength corresponding to the operation frequency. Traditional circuit theories, which rely on the assumption of low-frequency operation and negligible mutual coupling effects between components, may no longer be valid for analyzing artificially structured metamaterials, which are inherently inhomogeneous. It is, therefore, necessary to search for ways
to systematically characterize the electromagnetic properties of metamaterials. The ability to homogenize MTMs is very important from the point of view of reducing the complexity of the problem during the process of simulation. As reported in the literature, some of the approaches to obtaining the effective parameters of MTMs from numerical data include averaging methods based on effective medium theory [18], phase velocity calculations from time-domain simulations [19], and inversion techniques from the reflection and transmission coefficients of the slab illuminated by a normally incident plane wave [20-21]. The first two approaches utilize the field behavior inside the composite, but the last one relies entirely on knowledge of the scattering data of the structure to extract the effective parameters. An important attribute of the inversion approach is that it makes no physical assumptions about the particular nature of a structure; instead, this approach focuses on outside properties such as scattering problems.

Because of their unique electromagnetic properties, EBGs and MTMs have led to a wide range of applications in antennas and microwave circuits. Here, our primary interest lies in their applications for the performance enhancement of antennas. Typical examples of applications can be classified into four categories:

1. High-directivity antenna using a Fabry–Perot-type cavity formed by an EBG or MTM superstrate and a ground plane underneath [22-24]: Several types of configurations can be used to design these antennas, such as Frequency Selective Surfaces (FSSs), dielectric slabs, and DNG lenses. Furthermore, EBG superstrates for dual-band directivity enhancement and electronic beam steering are realized in [25-26].

2. Low-profile antenna by replacing the ground plane underneath the antenna with an Artificial Magnetic Conductor (AMC) [27-28]: Due to the in-phase reflection feature, the radiation efficiency of the antenna near an EBG or MTM ground plane can be significantly improved. These surfaces have also been optimized to realize better performance in multi-band
and wideband. Typical configurations include wire antennas such as dipole antennas, monopole antennas, and curl antennas.

(3) High Impedance Surface (HIS) composed of two-dimensional EBG or MTM structures used as microstrip antenna substrates to suppress the surface wave [29]: The bandgap feature of the structures is useful for enhancing the antenna’s gain and efficiency, while the backlobes are reduced.

(4) Dichroic surface of reflector antennas in the frequency reuse system [30]: An EBG or MTM structure can be used as a subreflector in a dual-band reflector antenna, where it acts as a spatial filter that is transparent in one frequency band and opaque in the other.

In addition, it is worth mentioning the interesting and novel applications of metamaterials only. One is a superlens with DNG properties capable of focusing images superior to those produced by, for example, the conventional dielectric lens of diffraction-limited systems. The other is the electromagnetic cloak, which coats a target with metamaterials such that it becomes invisible to the interrogating wave using unusual transmission characteristics.

Besides antenna applications, EBG structures have also found numerous applications in microwave circuit designs. These include directional couplers [31], power dividers/combiners [32], and harmonic filters [33], just to name a few.

1.2 Research Objectives

Among some of the notable applications of EBGs and MTMs described in the preceding section, we first focus our attention primarily on achieving a high-directivity antenna. Because their prominent property is the ability to totally reflect or totally transmit in the neighborhood of the element resonance, EBGs and MTMs are usually used in high-performance radomes for radars and communication antennas. However, they have found new applications as superstrates
designed to enhance the directivity of planar antennas without sacrificing the low-profile characteristics of the same. Unlike planar array antennas, which require a complex feed network that introduces undesired losses, this antenna–superstrate composite achieves a relatively high gain and can be designed to achieve a competitive bandwidth, accompanied by low sidelobe levels, without the use of an elaborate feed network. Most of the previous studies on antenna–superstrate composites have focused on computing the frequency response of related periodic and infinite structures. They have also discussed the performance of composites only operating in the transmit-mode [34-35]. However, no guidelines have been provided for systematically increasing the size of the effective aperture of the structure, which would, in turn, lead to enhanced directivity.

The first research objective of this dissertation is to develop a novel and systematic approach to enhancing the directivity of an antenna–superstrate composite. Specifically, this study uses a microstrip patch antenna and a dipole antenna covered by an FSS, a dielectric slab, and a Double Negative (DNG) superstrate, which form a Fabry–Perot Resonator (FPR) configuration. The method is based on initially simulating the composite operating in the receive-mode and then, mapping the field distribution in the Fabry–Perot (FP) type cavity region formed by the superstrate and the ground plane. We show that the high directivity of the composite is achieved at the resonant frequency of the structure. Additionally, we use the Finite Difference Time Domain (FDTD) method to correlate the results for the frequency response of an associated two-layer structure as formed by the superstrate and its image through the ground plane. Finally, we illustrate the application of the above procedure by designing directivity-enhanced planar antennas, for instance, dipole and microstrip patch antenna arrays used as exciters for an FP cavity.

Thanks to the fast development of computational electromagnetics, a number of commercial EM tools, HFSS, CST, FEKO, and GEMS, are available for analyzing the periodic
structures of EBGs and MTMs. However, all these codes can run into difficulties when dealing with periodic elements with complex geometries having multi-scale features, as they can place a very heavy burden on both the CPU time and memory [36-40]. This, in turn, can limit our ability to deal with elements that have complex and arbitrary geometries and consist of arbitrarily inhomogeneous media. It is important, therefore, to develop EM simulation techniques that reduce run-times sufficiently, so as to make them more suitable as design tools than the existing codes are.

The second research objective of this effort is to describe a new and general-purpose technique that is (i) capable of handling periodic structures that have elements with arbitrary geometrical shapes and material properties; and (ii) both memory- and time-efficient. The formulation is based on a combination of the Dipole Moment (DM) method, introduced recently, and the Characteristic Basis Function Method (CBFM), which reduces the size of the matrix equation to be solved via the use of macro-basis functions that are physics-based and tailored for the geometry being analyzed. Unlike the conventional MoM, the proposed formulation employs closed-form expressions to compute the fields radiated by the CBFs, and no Green’s functions are used, periodic or otherwise. We show that combining the above two methods leads to a relatively small matrix, often only 2×2 or 3×3 in size for many typical elements, such as tripoles and rings. A number of numerical examples are included in the thesis to illustrate the computational efficiency as well as versatility of the proposed approach.

The last research objective is to develop guidelines for using the Equivalent Medium Approach (EMA). This is done by examining a number of commonly used metamaterial elements in the context of analyzing various structures such as planar antennas covered with superstrates and/or backed by AMCs. The use of effective medium descriptions for artificially structured media, including metamaterials, is certainly convenient, since such a description is very appealing for interpreting the physical characteristics of the artificial materials; specifically, such
descriptions relate the $\varepsilon$ and $\mu$ of the materials to those of equivalent homogeneous media with the same permittivities and permeabilities. However, the use of the effective medium parameters often oversimplifies the EM properties of metamaterials, so much so that these parameters may fail to accurately predict the performance of the antenna–metamaterial composites designed for real-world applications. To critically examine the question of whether the performances predicted by the effective medium approach are in accordance with those realized on the basis of rigorous numerical simulations, we investigate the EM response of a finite artificial DNG slab, illuminated by a Gaussian beam either normally or from an oblique angle. We show, through several examples, that studies of this type of beam propagation are very helpful in developing an understanding of the scope and limitations of homogenizing MTM slabs.

This dissertation is organized into five chapters and one appendix. Chapter 1 introduces the background and basic properties of EBG and MTM structures. It also summarizes the analysis methods, antenna applications, and research objectives.

Chapter 2 presents a novel and systematic approach to designing high-directivity antenna systems, whereby a microstrip patch antenna (MPA) covered by a planar EBG or MTM constitutes the design. A Fabry–Perot design that utilizes an MPA array is illustrated to create an essentially uniform distribution in the aperture above the superstrate.

Chapter 3 presents two novel techniques involving a combination of the Dipole Moment (DM) method and the Characteristic Basis Function Method (CBFM), for modeling periodic structures. Numerical examples are included to illustrate the efficiency as well as versatility of the method.

Chapter 4 introduces the equivalent effective medium description of metamaterials to describe the characteristics of such media. The propagation of the Gaussian beams both in the interior and exterior regions of the MTM slab, is examined to gain a good understanding not only of the behaviors of the fields but also the limitations of homogenizing metamaterials.
Chapter 5 presents a summary and the conclusions of this research. It also discusses some avenues for future research.

The Appendix provides a discussion of Prony’s method.
Chapter 2

A Systematic Approach to Enhancing the Directivity of Antennas via the use of EBG Superstrates

2.1 Introduction

Endowed with desirable electromagnetic properties that have not been observed in natural materials, Electromagnetic Band Gap (EBG) structures have, in recent years, attracted significant attention among researchers in the microwave area. Though known as Photonic Band Gap (PBG) structures that originated from research into solid-state physics and optics, they are now finding a wide variety of applications as components in microwave and millimeter devices, as well as in antennas. An important property of any EBG material is its ability to prevent the propagation of EM fields at frequencies that fall within its bandgap and to transmit the wave through the material with little or no attenuation within its passband. This property of EBGs makes them suitable candidates for high-performance frequency-selective radomes. However, because they are designed to be essentially transparent at the operating frequency, use of such a radome does not typically enhance the directivity of the antenna; that is, it does not increase the size of the effective aperture of the antenna, which is necessary to enhance its directivity.

Recently, however, EBGs have found new applications as superstrates for planar antennas designed to enhance their directivity without sacrificing their low-profile characteristics. Unlike planar array antennas, which require a complex feed network that introduces undesired losses, this antenna–superstrate composite achieves a relatively high gain, and can be designed to
achieve a competitive bandwidth, accompanied by low sidelobe levels, without the use of an elaborate feed network.

Lee et al. [25] have presented guidelines for achieving a substantial increase in the directivity of antenna composites that consist of a thin two-layer Frequency Selective Surface (FSS) placed over a microstrip patch antenna using a unit cell simulation approach. Their approach to directivity enhancement was based on a deliberate introduction of defect modes—achieved by breaking the periodicity in the vertical direction—of the two-layer superstrate FSS, consisting of the superstrate and its image through the ground plane of the patch antenna. However, for a single-layer FSS superstrate, the defect mode interpretation is perhaps not a very pertinent way to explain the directivity-enhancement phenomenon of the composite structure.

This chapter presents a novel and systematic approach to enhancing the directivity of an antenna–EBG composite; specifically, it uses a microstrip patch antenna covered by an FSS, a dielectric slab, or a Double Negative (DNG) superstrate, as shown in Figs. 2.1 and 2.2, which forms a Fabry–Perot Resonator (FPR) configuration. The method is based on initially simulating the composite operating in the receive-mode and then mapping the field distribution in the type of Fabry–Perot (FP) cavity region formed by the superstrate and the ground plane. Additionally, we use the Finite Difference Time Domain (FDTD) method to correlate the results for the frequency response of an associated two-layer structure as formed by the superstrate and its image through the ground plane. Finally, we illustrate the application of the above procedure by designing directivity-enhanced planar antennas, for instance, dipole and microstrip patch antennas array used as exciters for an FP cavity. Our objective is to realize the most efficient aperture possible, and we describe a systematic procedure for achieving this goal.
Typical patch antenna setting

Improved setting

Fig. 2.1: EBG–Antenna composite.

Fig. 2.2: Candidates for antenna superstrates.
2.2 Fabry–Perot Cavity

Before discussing the Fabry-Perot Resonator (FPR) antennas and their capabilities, it is worth briefly commenting on the origin of the term “Fabry–Perot (FP)”. A multiple-beam interferometer was first constructed by Charles Fabry and Alfred Perot in the late 1800s and was of considerable interest for use in optics. Besides being a spectroscopic device with extremely high resolving power, it serves as the basic laser resonant cavity. In principle, the device consists of two parallel highly reflective surfaces separated by some distance $d$ as shown Fig. 2.3. The reflecting surfaces are each formed by a very thin silver (or aluminum) film, or by multi-layer dielectric films, deposited on the surface of glass plates. Incident from the left on the first mirror, the lights are transmitted and reflected back, after which they undergo additional reflections within the gap. When the difference in optical path length between the rays transmitted at successive reflections is such that the emerging waves are in the same phase, constructive interference occurs and the interferometer achieves its maximum transmission. If this condition does not hold, the interference between successive emerging rays is destructive and the transmission is relatively low. Then the transmitted rays are collected by a lens and brought to a focus on a screen.
This theory can also be used to explain how the FPR antenna works. Fig. 2.4 shows the geometry of the proposed FPR antenna, which consists of a primary radiator that is backed with a metal ground plane and covered by a Partially Reflective Surface (PRS). The radiator excites one of the cavity’s resonant modes, and when the spacing between these plates is approximately $\lambda/2$, where $\lambda$ is the free-space wavelength at the operating frequency, the forward radiation can be enhanced. If the energy leakage through the surface is too high, the build-up of the required field distribution in the resonator does not occur as well as it does for a superstrate with a higher reflectivity. On the other hand, if the leakage is too low, the antenna becomes less efficient because of losses in the resonator. Hence, a superstrate with appropriate reflectivity should be selected to strike a balance between a desired improvement in directivity and an increase in resonator losses. We note that the directivity and half-power fractional bandwidth of an antenna composite comprising the FPR are primarily determined not by the focusing behavior of the
superstrate but by these parameters: the reflection coefficient of the superstrate; its angular response; the distance of the cover from the ground plane, etc. In the next section, we will characterize the performance of an antenna composite with different types of superstrates.

Fig. 2.4: Fabry–Perot cavity formed by the superstrate and the ground plane.

2.3. Proposed Composite Structures

Most of the previous studies on antenna–EBG composites have focused on computing the frequency response of related periodic and infinite structures. They have also discussed the performance of composites operating in the transmit-mode. However, no guidelines have been provided for systematically increasing the size of the effective aperture of the structure, which would, in turn, lead to enhanced directivity.

In this section, we briefly investigate the frequency response of an EBG structure by using the numerically efficient Periodic Boundary Condition–Finite Difference Time Domain (PBC–FDTD) technique. We next analyze the receive-mode properties of a Fabry-Perot type cavity formed by an EBG superstrate and a ground plane underneath (as described in Sections 2.1
and 2.2), to determine the operating frequency of this cavity. Following this, we design a Microstrip Patch Antenna (MPA) to be inserted into this cavity, and then evaluate the transmit-mode characteristics of a finite, moderate-size antenna–EBG composite excited by the MPA, thereby validating our design methodology. For the analysis and design of the antenna composite, all simulations in this work have been performed with the General Electromagnetics Simulator (GEMS) code, a 3-D parallel FDTD solver capable of efficiently handling complex, large-scale, and arbitrarily inhomogeneous structures with a large number of unknowns.

2.3.1 Octagonal Aperture FSS Superstrate-Antenna Composite

Unit Cell Characteristics of the FSS Superstrate

We now proceed to characterize the infinite doubly-periodic version of the superstrate by examining its transmission and reflection properties as we vary the frequency. We begin our discussion by considering an FSS screen formed by using octagonal-shaped apertures. The geometry of the composite with a 7×7 FSS superstrate is shown in Fig. 2.5. The dimensions of the superstrate are 17.4 mm (2λ) × 17.4 mm (2λ), and it is located at a height of 4.2 mm (0.5λ) above the ground plane, where λ is the free-space wavelength at 34.4 GHz. Next, we analyze the unit cell consisting of the FSS superstrate and its image through the ground plane, with dielectric layers εr equal to 3.2 and 2.8 (see Fig. 2.6). The unit cell structure is illuminated from the top by a normally incident x-polarized plane wave. Its strength is 1 V/m. The four sides of the unit cell are assigned Periodic Boundary Conditions (PBCs), while Perfectly Matched Layers (PMLs) are used to terminate the computational domain at the top and bottom of the cell. This method of unit cell simulation is computationally efficient for examining the infinite doubly-periodic version. It also offers a quick way to predict the resonant frequencies of the Fabry–Perot structure.
Fig. 2.5: (a) Geometry of the $7 \times 7$ octagonal aperture–FSS composite; and (b) One cell.

Fig. 2.6: Unit cell of the octagonal aperture–FSS composite.
The transmission coefficient of the FSS unit cell is plotted in Fig. 2.7, which shows that the maximum transmissions occur at four frequencies: 33.6, 39.9, 45.1, and 53.7 GHz.

Our next step is to retain only those frequencies for which the horizontal components of the electric field distributions exhibit a minimum at the location where the ground plane would be inserted in a real structure. As we can see from Fig. 2.8, the response of the transmission coefficient occurs at the first and third peaks in the frequency, namely 33.6 and 45.1 GHz.

![Fig. 2.7: Transmission coefficient of the FSS unit cell with one layer and two layers.](image_url)
Performance of the FSS–Antenna Composite

Next, we design patch antennas at two different frequencies, namely 33.6 and 45.1 GHz, for which maximum transmissions occur in the two-layer unit cell simulation. We observe from Fig. 2.9 that the directivity of the FSS antenna composite (12.8 dBi at 34.4 GHz and 12.9 dBi at 45.0 GHz) is enhanced by about 6 dB over the patch-antenna-only case. These frequencies are very close to those for which the unit cell structure of Fig. 2.6 shows a transmission maximum. There is a slight frequency shift of 0.8 GHz at the lower frequency, because of the truncation effect of the superstrate, which is less pronounced at the higher frequency.
Receive-Mode Characteristics: Cavity Field Distribution

Most of the previous studies on this subject have focused on the performance of antenna–EBG composites operating in the transmit-mode. However, in this study, we first examine the cavity field characteristics inside these composites when it is operating in receive-mode. To this end, we investigate a $7 \times 7$ FSS superstrate placed above a substrate and a ground plane, but without the microstrip patch. The structure is excited by a plane wave from the top, and the strongest field distributions in the region between the superstrate and the ground plane are presented at two frequencies, namely 34.5 and 45.1 GHz, as shown in Fig. 2.10. The magnitude of the field decreases smoothly with distance from the middle of the cavity, and its phase is close to uniform over the entire area. Other field distributions on the YZ plane along the centerline are shown at the same frequencies in Fig. 2.11. We note that the fields are concentrated inside the
cavity and that the highest field intensity occurs near the center of the cavity. In addition, the maximum directivity of the antenna–FSS composite is realized at frequencies at which the field intensity is highest inside the cavity operating in receive-mode.

Fig. 2.10: Electric field (Ex) distribution of magnitude and phase on the center plane between the superstrate and the ground plane at (a) 34.5 GHz; and (b) 45.1 GHz.
Fig. 2.11: Electric field (Ex) distribution of magnitude and phase on the YZ plane along the centerline at (a) 34.5 GHz; and (b) 45.1 GHz.

Performance Comparison of 7 × 7 and 15 × 15 Elements in the FSS–Antenna Composite

Next, we consider a 15 × 15 octagonal aperture antenna–FSS composite, as shown in Fig. 2.12. For this case, the strongest field distributions occur at 33.6 GHz, both at the middle of the cavity region and along the centerline in the YZ plane, as shown in Fig. 2.13. With the increase in the size of the FSS superstrate and the number of elements, we see a stronger correlation between
the frequency at which the field intensity is highest inside the cavity and the transmission of the unit cell is maximum.

Fig. 2.12: Geometry of a $15 \times 15$ octagonal aperture–FSS composite.
① on the center plane between superstrate and ground plane (XY plane)

Fig. 2.13: Electric field (Ex) distribution of magnitude and phase on the ① and ② planes of the structure in Fig. 2.12 at 33.6 GHz.

② on YZ plane along the center line

Fig. 2.14 compares the directivities of the antenna–FSS composites wherein the microstrip patch antennas have been designed to operate at the frequency at which the cavity fields are maximum, and the FSSs have 7 × 7 and 15 × 15 elements. The maximum directivity of 15.9 dBi is obtained for the 15 × 15 FSS composite at 33.5 GHz, which is the same frequency at which the
transmission peak occurs in the unit cell, and the maximum cavity field exists in the receive-mode.

Fig. 2.14: Directivity of the FSS–antenna composite with two different sizes of FSS (7 × 7 and 15 × 15 elements).

Fig. 2.15 shows a comparison of the radiation patterns of two FSS–antenna composites at $\phi = 0^\circ$ and $\phi = 90^\circ$. We note that a higher front-to-back ratio is obtained for the larger FSS superstrate, which radiates more efficiently along the zenith.

**Oblique Angle of Incidence**

It is desirable to have low sidelobe levels to reduce the unwanted radiation from an antenna. To study the sidelobe behavior, we investigate the fields inside the cavity for oblique incidence, as shown in Fig. 2.16. The magnitude and phase of the fields inside the cavity region
are presented in Fig. 2.17 as functions of $\theta$. We note the change in the field pattern of the cavity and observe that the highest field intensity decreases with an increase in $\theta$.

![Fig. 2.15: Comparison of radiation patterns of FSS–antenna composite with two different sizes of FSS (7 \times 7 at 34.5 GHz and 15 \times 15 at 33.5 GHz) at (a) $\phi = 0^\circ$; and (b) $\phi = 90^\circ$.](image)

![Fig. 2.16: (a) Geometry of a 7 \times 7 FSS and a substrate with an oblique angle of plane wave incidence; and (b) Side view (XZ cross-section).](image)
Fig. 2.17: Electric field (Ex) distribution of magnitude and phase on the ① plane of the structure in Fig. 2.16 as a function of theta.
In Fig. 2.18, the directivity of the FSS composite and the cavity field magnitude in the receive-mode are normalized and plotted in dB. We observe that the normalized cavity field distributions for oblique incident angles show a trend similar to that of the sidelobe level of the FSS composite. This type of study is helpful in determining the FSS elements that would provide an improved antenna pattern for this type of composite structure.

![Graph showing comparison of directivity and cavity field magnitude](image)

Fig. 2.18: Comparison of the angular response of the FSS–composite directivity and the cavity field of Fig. 2.17.

2.3.2 Strip FSS Superstrate–Antenna Composite

Next, we investigate another example of an FSS resonator–antenna composite. This example consists of a $4 \times 12$ array of x-directed strips located at a distance of $0.5\lambda$ above the ground plane of the patch antenna. The FSS unit cell structure is a rectangular lattice where $a =$
4.7 mm, \( b = 1.5 \) mm, where “\( a \)” is the periodicity of the cell in the \( x \)-direction and “\( b \)” is the periodicity in the \( y \)-direction, and the strips have a length of 4.3 mm and a width of 0.35 mm (see Fig. 2.19). After performing the receive and transmit-mode analyses, as before, we reconfirm the fact that the maximum directivity of the FSS–antenna composite is achieved at the frequency of 34.4 GHz, where the field intensity is highest inside the cavity consisting of the FSS superstrate and the ground plane operating in the receive-mode. Figs. 2.20 and 2.21 illustrate this relationship.

Fig. 2.19: (a) Geometry of a strip–FSS composite; and (b) One cell of the FSS superstrate.
① on the center plane between superstrate and ground plane (XY plane)

② on YZ plane along the center line

Fig. 2.20: Electric field (Ex) distribution of magnitude and phase on the ① and ② planes of the FSS composite at 34.4 GHz.
2.3.3 Dielectric Superstrate–Antenna Composite

Next, we characterize the performance of an antenna composite with a dielectric slab as a superstrate, which is much simpler to fabricate than the FSS covers we have studied thus far. We investigate the performance of a dielectric slab as a superstrate, whose relative permittivity is 9.8 (for Alumina). The dielectric slab’s dimensions are 17.4 mm ($2\lambda$) $\times$ 17.4 mm ($2\lambda$), and it is located at 4.4 mm ($0.5\lambda$) above the ground plane, where $\lambda$ is the free-space wavelength at 34 GHz (see Fig. 2.22).
Fig. 2.22: Dielectric slab–antenna composite.

We analyze the unit cell consisting of the dielectric superstrate and its image through the ground plane. The transmission coefficient of the unit cell is plotted in Fig. 2.23(b), which shows that maximum transmissions occur at two frequencies, namely 32.2 and 42.5 GHz. As we can see from Fig. 2.23(c), the magnitude of the tangential field at the first peak frequency inside the model is minimum at the plane of symmetry, which is where the ground would be located in the original structure. This prompts us to choose it as the operating frequency.
Next, we investigate the receive-mode behavior at the frequency selected. In order to do so, we use a dielectric slab without a microstrip patch that is located above the ground plane covered by the substrate. The structure is excited by a plane wave from the top. Fig. 2.24 displays the strongest field distributions in the region, which are between the superstrate and the ground.
plane at 34 GHz. We note that the fields are concentrated inside the cavity and that the highest field intensity occurs near the center of the cavity.

① on the center plane between superstrate and ground plane (XY plane)

② on YZ plane along the center line

Fig. 2.24: Electric field (Ex) distribution of magnitude and phase on the ① and ② planes of the FSS composite at 34 GHz.
The maximum directivity of the antenna composite is realized at the frequency where the field intensity is highest inside the cavity operating in the receive-mode (see Fig. 2.25). Because it is based on the infinite doubly-periodic structure, the two-layer unit cell of transmission maximum shows a frequency shift of 1.8 GHz. This frequency shift is expected to decrease with the increase in the size of the dielectric slab. We note that the radiation bandwidth (based on 3 dB below maximum criterion of the directivity) is approximately 16.7%, which is 8.4% better than that of the two EBG–FSS superstrate-antenna composites. The increase in the bandwidth can be primarily attributed to the difference in the reflection coefficient and the angular response of the cover, determined by the dielectric constant and its thickness as well as the distance of the superstrate of the Fabry–Perot cavity. The dielectric superstrate has the desired reflection characteristics, and it acts as a layer of Partially Reflective Surface (PRS) to form the cavity. In order to achieve high directivity, it is desirable to use a highly reflective surface with a reflection phase close to $\pi$ as a superstrate. We observe that with appropriate parameters a dielectric slab can produce directivity enhancement similar to that obtained with EBG and FSS covers. In addition, such a dielectric slab is much easier to fabricate than an EBG.

![Graph of Directivity vs Frequency](image)

Fig. 2.25: Directivity of the dielectric slab–antenna composite.
2.3.4 Performance comparison of three superstrates

The previous sections have presented the use of superstrates to improve the performance of microstrip patch antennas (MPAs) used to excite the FPRs. Different types of superstrates have been proposed for obtaining gain enhancement, including EBGs, FSSs and plain dielectric slabs [41]. Of these, the EBG and FSS types exhibit similar transmission and reflection characteristics, but the latter is the thinner of the two. The dielectric slab is typically low loss and is relatively easy to procure. However, provided that they have similar reflectivities, EBGs and FSSs and dielectric slabs offer comparable levels of directivity enhancement when used as a superstrate for an MPA. Now we consider a structure with DNG (double negative) characteristics as an alternate configuration for the superstrate.

The first attempt to explore the concept of artificial materials appears to date back to the late 19th century when in 1898 Jagadish Chandra Bose conducted the first microwave experiment on twisted structures—geometries that are known as artificial chiral elements by today’s terminology [73]. In 1914, Karl F. Lindman worked on artificial chiral media by embedding a collection of randomly oriented small wire helices in a host medium [74]. In the 1940s, W. E. Kock [75] made microwave lenses (metallic delay lenses) by arranging conducting spheres, disks, ellipsoids, and spheroids periodically and effectively tailoring the effective refractive index of the artificial media [76]. The concept of DNG was first introduced by V. G. Veselago [42], in an article that traces back to 1968. Many years later, several researchers proposed a design for an artificial medium consisting of split-ring resonators (SRRs) and thin dipoles that exhibited negative values of effective permittivity $\varepsilon$ and permeability $\mu$. Since Veselago had argued that a DNG slab would have superlensing characteristics, several authors proposed using DNGs as superstrates to enhance the directivity of small antennas by focusing at infinity the radiation emanating from them.
In this section, we present the simulation results of MPA–superstrate composites, with a view to comparing the levels of directivity enhancement achieved by using a dielectric slab, an FSS, and a DNG superstrate. We also provide information regarding an aperture E-field distribution on a surface located at 0.25\(\lambda_0\) (\(\lambda_0\) is the free-space wavelength at 15 GHz) above the superstrate. We do this to determine why the composites that produce more nearly uniform amplitude and phase distribution in the aperture than do others produce higher directivity. It is evident that more uniform the E-field distribution, both in terms of magnitude and phase, the higher the directivity will be.

We begin by designing a dielectric slab and an FSS, shown in Fig. 2.26, whose reflectivities at 15 GHz are 81.3% and 81.5%, respectively. We next analyze the receive-mode properties of an FP-type cavity formed by the EBG superstrates above and a ground plane underneath, to determine the distance between them. We find that the optimal distance between the lower edge of the dielectric slab superstrate and the ground plane is 0.515\(\lambda_0\), when the thickness of the slab is 0.25\(\lambda_g\) (\(\lambda_g\) is the guided wavelength in the slab). Next, we conduct the same experiment with the FSS superstrate, consisting of a set of PEC strips and find that the optimum distance between the ground plane and the FSS superstrate is 0.56\(\lambda_0\). The dimensions of the MPA are width \(w = 6.09\) mm and length \(l = 3.8\) mm. Also, the FSS superstrate is mounted on a substrate that is 1 mm thick and has a dielectric constant of 4.4.

Next, we turn to the case of an MPA with a DNG slab as its superstrate. Fig. 2.27 shows its geometry [43]. Once again we begin the investigation of the DNG superstrate by simulating the composite in the receive-mode and set the distance between the slab and the ground plane at \(~0.5\lambda_0\). We use its transmission and reflection coefficients to extract the effective permittivity \(\varepsilon_r\) and permeability \(\mu_r\) by using a technique described in [19]. The results are shown in Fig 2.28, and
we note that between 14.3 and 16.8 GHz, the real parts of both \( \varepsilon_r \) and \( \mu_r \) are negative for the DNG slab.

**Fig. 2.26:** MPA with different superstrates: (a) Dielectric slab; and (b) FSS.

**Fig. 2.27:** MPA with a DNG superstrate. Dipole parameters: length \( l = 5 \) mm; width \( w = 0.5 \) mm; \( b = 1 \) mm. SRR parameters: side length \( p = 3 \) mm; strip width \( a = 0.25 \) mm; gap \( g = 0.5 \) mm. Distance \( d \) between the SRR and dipole is 0.25 mm.
Next, we compare the directivities realized by the plain dielectric, FSS, and DNG superstrates when placed above an MPA with each of the composites designed for optimal performance. Fig. 2.29 shows that the DNG superstrate achieves the lowest directivity of the three, and that the other two perform comparably. Specifically, when the superstrate is \(2.5 \lambda_0 \times 2.5 \lambda_0\), the directivities of the dielectric slab, FSS, and DNG superstrates at 15 GHz are 16.08, 15.77, and 10.2 dBi, respectively. Note that a larger superstrate does not appreciably change the directivity, because the aperture distribution is tapered at the edges. We should also mention that the directivity of a uniformly illuminated aperture of the same size is 18.95 dBi; this offers a reference for computing the aperture efficiencies, which are 51.6%, 48.1%, and 13.3%, for the dielectric, FSS, and DNG slabs, respectively. Fig. 2.30 shows the magnitude and phase distributions along the center line of the observation plane, which is around \(0.25\lambda_0\) away from the superstrate. We can see that the DNG superstrate does not create as uniform an E-field distribution as do the dielectric and FSS superstrates, and this is consistent with the directivity each achieves. Thus, we should follow the FP theory and choose a slab with high reflectivity as the superstrate for the MPA to realize a high directivity.
Fig. 2.28: Response of SRR + dipole combination: (a) Effective permittivity; and (b) Effective permeability (shaded areas represent non-physical regions).
Fig. 2.29: Comparison of directivities realized by using dielectric, FSS, and DNG superstrates.

It is widely held that the superlensing effect produced by a DNG slab can enhance the directivity of a relatively small antenna by focusing the energy emanating from it to infinity. The present study does indeed confirm that when a DNG superstrate is used above an MPA, directivity improves. However, we also show that an even greater enhancement can be achieved by using a plain dielectric slab or by using an FSS. This is because the physical mechanism, which contributes to the directivity enhancement, is not the focusing effect of the superstrate but the resonance phenomenon in the FPR created by the highly reflecting superstrate placed above the ground plane. Since we have decided to give the DNG slab $\varepsilon$ and $\mu$ values that are both negative at the design frequencies—in order to achieve a focusing effect in accordance with Veselago’s theory—the slab is more transparent than it is reflective. This, in turn, makes the
resonance of the FP cavity less efficient than if the DNG were replaced either by a dielectric slab or an FSS superstrate, which is designed to have a higher reflectivity than that of the DNG.

Fig. 2.30: E-field distributions along lines 1 and 2 on the aperture plane.
2.4 Antenna Array

2.4.1 Dipole Antenna Array

The directivity of an antenna as well as its radiation bandwidth not only depends on the choice of the EBG material for the superstrate but also on the way it is excited. We will now study the second characteristic, that is, the bandwidth of the antenna. FDTD simulations show that the EBG antenna, excited by a single source, has a relatively small bandwidth, especially when its gain is high. If we replace a single source with a dipole array at the center of the cavity—where the field intensity is highest—we find that both the directivity, as well as its radiation bandwidth, are affected and that the performance of the antenna depends on the number of sources and their spacing. Let us consider a dielectric slab as a superstrate with a permittivity of 9.8 and dimensions of 34.8 mm ($4\lambda$) × 34.8 mm ($4\lambda$). We locate it at 4.5 mm (0.52$\lambda$) above the ground plane, where $\lambda$ is the free-space wavelength at 34 GHz (see Fig. 2.31). When the EBG antenna is excited with a single source at the center inside the cavity, the maximum directivity of 18 dBi is obtained at 32 GHz, and the return loss ($S_{11}$) at the frequency is around -10 dB, as Fig. 2.32 shows.

Fig. 2.31: Dielectric slab–Antenna composite excited with a dipole.
The directivity of the EBG–antenna composite can be directly related to the size of its effective radiating aperture. The array is much larger in size than when a single source is used, and this, in turn, enables the array to enhance directivity more than a single feed would, albeit with a larger superstrate, as the effective aperture size is determined by the number of sources and their spacing. Further, it is possible to obtain both a higher directivity and a better return loss ($S_{11}$) by using an array feed, given that it has much wider spacing than that of a conventional array and that this spacing leads to reduced coupling between the dipoles. However, we find that such a configuration with wide spacing between the array elements, can lead to higher sidelobes. Thus, we choose a spacing of $1.34\lambda$ as a compromise and also taper the weights of the excitations to achieve relatively low sidelobes. Note that the composite operates in the FP mode, which is different from the case in free-space; therefore, even though the element spacing of the array is greater than $1\lambda$, grating lobes are not generated. The geometry of the composite excited with the three dipole sources and their $S_{11}$ characteristics are shown in Fig. 2.33.
Next, we experiment with different excitations of voltage magnitudes and phase delays for the dipole sources in order to increase the aperture efficiency of the composite. We choose one dipole source in the middle of the cavity and excite it with a voltage of 1V. For the other two dipoles, we use an excitation voltage of 0.92 V and a phase delay of 17.3°, located 1.34λ away from the center, where we have calculated these optimal values from a set of FDTD simulations. The maximum directivity of 20.2 dBi is obtained for the composite at 31 GHz, and the radiation patterns are presented in Fig. 2.34 for this frequency. We note that the antenna directivity and the radiation bandwidth (22.9%) can be increased significantly with just a few sources without compromising the sidelobe characteristics.
2.4.2 MPA Antenna Array

In this section, we present an FP design that utilizes an array consisting of 25 MPAs covered by a $14\lambda \times 14\lambda$ simple dielectric layer to realize a directivity level of 30 dBi or better, where $\lambda$ is the free-space wavelength at 15 GHz (see Fig. 2.35). First, we design a thinned array of microstrip patches with a separation distance of $2.5\lambda$ between the elements. Though such spacing would normally produce grating lobes when a superstrate cover is not present, this does not occur when the array is placed in the FP resonator. Second, we position a dielectric superstrate at $0.495\lambda$ above the ground plane of the array. We do this, not only to suppress the grating lobes, but also to realize the maximum directivity at 15 GHz, given that the value of the relative permittivity of the cover is moderate: $\varepsilon_r = 9.8$. For the third and final step, we introduce an additional dielectric superstrate above the original one to further enhance the directivity. The separation distance between the lower and upper superstrates is chosen to be 3.6 mm, and the relative permittivity of the upper superstrate is set to $\varepsilon_r = 4.4$. In order to compare the
performance of our design with that proposed in [44], we also simulate an FP structure with a single-layer dielectric superstrate with a relative permittivity $\varepsilon_r$ of 43, which is considerably higher than that in the design proposed by the present study.

Fig. 2.36 compares the directivities of the array with no cover (case 1) with those of arrays with a single-layer superstrate (case 2), a double-layer superstrate (case 3), and a single layer with a superstrate (case 4) whose dielectric constant is 43. We find that the directivity of an array with a one-layer superstrate (case 2) with only a moderate permittivity of $\varepsilon_r = 9.8$ can reach the 30 dBi gain value at its peak, which is 10 dB higher than that of the array-only case (case 1). Next, we realize a directivity of 32 dBi by using a two-layer superstrate structure (case 3), even though the dielectric constant of the second superstrate is only 4.4. Note that the directivity of the two-layer superstrate case is comparable to that realized with a much larger value of $\varepsilon_r$, namely 43. The radiation patterns, presented in Fig. 2.37, show that the sidelobes of case 3 are lower than those in case 2. Next, in Fig. 2.38 we compare the bandwidth of the one- and two-layer superstrates. We observe from this figure that the bandwidth for case 4 is only slightly lower than that of case 3, implying that the overall performance of the two-layer case is better. We also compute the aperture efficiency of this geometry, whose size is $14\lambda \times 14\lambda$. The maximum achievable directivity for this aperture is 33.91 dBi, which corresponds to a uniform distribution. We note that the achieved directivity for the two-layer case is 32.36 dBi with the present design, indicating an aperture efficiency of 69.9%, which is quite high.
Fig. 2.35: (a) Case 1: 25-patch array without superstrate; (b) Case 2: 25-patch array with one-layer superstrate; (c) Case 3: 25 patch array with two-layer superstrate; and (d) Case 4: 25-patch array with one-layer high permittivity superstrate.
Fig. 2.36: Directivity comparison of the four cases.

Fig. 2.37: Radiation patterns of case 2 and case 3: (a) E-plane (b) H-plane.
Electromagnetic Band Gap (EBG) structures that are engineered to achieve desired transmission and reflection characteristics in specific frequency bands have long been studied in the microwave regime for a wide variety of applications [45-47]. These characteristics make EBGs suitable candidates for high-performance frequency-selective radomes. However, the use of such an EBG does not typically enhance the directivity of the antenna, since the EBG is designed to be essentially transparent at the operating frequency; and, hence, it does not increase the size of the effective aperture of the antenna. Recently, however, FSSs have found new applications as superstrates, whose primary function is to enhance the directivity of a dipole or a microstrip patch antenna. Most previous works on antenna–EBG composites have focused on the performance of the composites operating in the transmit-mode. However, in this chapter, we
presented a systematic approach for designing a high-directivity EBG resonator antenna utilizing different types of superstrates: an FSS, a dielectric slab, and a planar DNG lens. This method is based on a receive-mode analysis, an examination of the field distribution inside the FP-type cavity formed by an EBG superstrate above and a ground plane underneath. We have found that its behavior strongly correlates with that of the composite.

Researchers have argued that a superstrate with Double Negative (DNG) characteristics that focus the energy emanating from a small antenna, e.g., from a microstrip patch to infinity, can enhance the directivity of an antenna. However, we observe that although the use of a DNG superstrate offers some improvement over an MPA, this enhancement is relatively moderate. In fact, better performance can be achieved with an FSS, or even by using a plain dielectric superstrate. This is because the physical mechanism that contributes to directivity enhancement is not the focusing effect of the superstrate; instead, the enhancement is the work of the resonance phenomenon in the FPR created by the highly reflecting superstrate placed above the ground plane.

Our FP-type design, which involves a microstrip array covered by a dielectric slab as the superstrate, realizes a directivity of 30 dBi when the superstrate is excited by an array of 25 microstrip patches. We showed that the directivity of the array can be enhanced by an additional 2 dB by using a two-layer superstrate. The key to improving the directivity of the antenna–superstrate composite is to create an essentially uniform distribution in the aperture above the superstrate by exciting the dominant mode of the resonator efficiently with the MPA array. We also presented the radiation patterns for the two cases to show that the sidelobe level for the two-layer case is lower than that for the one-layer version.
Chapter 3

A Novel Technique for the Analysis of Periodic Structures Including EBGs

3.1 Introduction

In recent years, Electromagnetic Band Gap (EBG) structures consisting of periodic arrays of metallic or dielectric elements have been widely investigated for use in antenna designs. EBGs have many applications, though they are usually used in high-performance radomes for radars and communication antennas. Some notable applications include using EBGs in Frequency Selective Surfaces (FSS) as superstrates to improve the directivity of antennas and as substrates to suppress surface waves. These surfaces typically manifest total reflection or transmission in the neighborhood of the element resonance. Currently, a number of commercial CEM tools, HFSS, CST, FEKO and GEMS, are available for analyzing periodic structures. However, all these codes can run into difficulties when dealing with periodic elements with complex geometries having multi-scale features, since they can place a very heavy burden on both the CPU time and memory. This, in turn, can limit our ability to deal with elements that have complex and arbitrary geometries and consist of arbitrarily inhomogeneous media. It is important, therefore, to develop EM simulation techniques that reduce run-times sufficiently, so as to make them more suitable as design tools than are the existing codes. Furthermore, though many of the commonly used EBG elements are planar, we will use EBG elements that are three-dimensional, as this will provide increased design flexibility and our method of analysis should be able to handle such cases easily.

The most widely used full-wave technique for modeling periodic structures is based on the Method of Moments (MoM) algorithm. It is well-known that this algorithm is the most efficient for analyzing large structures involving only metallic conductors or homogeneous
dielectrics. The first step in this formulation involves replacing the original scatterer with equivalent conduction or polarization currents, for PEC or dielectric bodies, respectively. Next, we represent these currents in terms of a suitable set of basis functions. Then, we apply the boundary condition on the tangential E-field—derived from the induced currents using the free-space Green’s function—with a set of testing functions, via the Galerkin procedure, for instance. However, using the algorithm to solve the inhomogeneous dielectric or complex geometry presents great challenges: such problems require a volume formulation, making them very expensive computationally. In addition, the conventional MoM has difficulty managing the low-frequency breakdown problem introduced by the dominance of the scalar potential term over the vector potential as the frequency approaches zero.

In this chapter, we propose a new and general-purpose technique that is (i) capable of handling periodic structures that have elements with arbitrary geometrical shapes and material properties, as shown schematically in Fig. 3.1; and is (ii) both memory- and time-efficient. Although the method is quite general, we consider, for the sake of illustration, several types of FSS elements comprising wire dipoles, wire tripoles and wire square loops with and without a gap. We note that some of the existing FSS codes, including commercial ones, often have difficulty dealing with this type of geometry, especially when the wire diameter is small. In fact, some codes are unable to handle the geometries and model them only as strips. The proposed method, however, is capable of modeling not only the strip or the wire structure, but also the cases where the element has an arbitrary orientation and the FSS is not planar. One of the strong points of the proposed method is that it reduces the matrix size to only 2 or 3, by using a combination of the newly developed Dipole Moment (DM) and the Characteristic Basis Function Method (CBFM). Unlike the conventional Method of Moments (MoM), the present formulation employs closed-form expressions to compute the fields radiated by the CBFs, and no Green’s
functions are used, periodic or otherwise. We demonstrate, via several examples, the efficiency as well as versatility of the method in this chapter.

![Infinite doubly periodic structure with arbitrary geometrical shapes.](image)

**Fig. 3.1:** Infinite doubly periodic structure with arbitrary geometrical shapes.

### 3.2 Dipole Moment Method

#### 3.2.1 Introduction to the Dipole Moment (DM) Formulation

Before proceeding directly to the dipole moment concept, it is worth briefly commenting on the conventional Green’s function-based MoM formulation. The equation constituting the starting point for solving the Electric Field Integral Equation (EFIE) is given in potential form by

\[
E = -j\omega\mathbf{A} - \nabla \phi
\]  

(3.1)
where $\mathbf{A}$ is the magnetic vector potential and $\phi$ is the electric scalar potential. This equation can be expressed by using the Lorentz gauge condition as

$$\mathbf{E} = -j\omega \mathbf{A} + \frac{1}{j\omega\varepsilon}\nabla(\nabla \cdot \mathbf{A})$$  \hspace{1cm} (3.2)$$

where $\mathbf{A}(\mathbf{r}) = \frac{\mu}{4\pi} \int_{S'} \mathbf{J}(\mathbf{r}') \epsilon e^{-j|\mathbf{r}-\mathbf{r}'|} d\mathbf{r}'$ is the vector potential and $G(\mathbf{r}, \mathbf{r}') = e^{-j|\mathbf{r}-\mathbf{r}'|}/|\mathbf{r}-\mathbf{r}'|$ is the free-space Green’s function. With some known basis function, $f_n(\mathbf{r}')$, the unknown current can be expanded as

$$\mathbf{J}(\mathbf{r}') = \sum_{n=1}^{N} w_n f_n(\mathbf{r}')$$  \hspace{1cm} (3.3)$$

where $w_n$ is the unknown coefficient of the basis function to be determined. This expansion is applied to the same number of field points as that of expansion terms. Therefore, it transforms the integral equation into a set of simultaneous algebraic equations in the unknown coefficients by imposing the appropriate boundary conditions, depending on the geometry and the material parameters of the medium. If we concentrate on a PEC, the boundary condition is applied on the surface of the object where the total tangential electric field is zero. This leads to the following EFIE:

$$-\mathbf{E}'(\mathbf{r}) = \frac{j}{\omega\varepsilon} \left[ \int_{S} k^2 \mathbf{J}(\mathbf{r}') G(\mathbf{r}, \mathbf{r}') ds' - \nabla' \cdot \mathbf{J}(\mathbf{r}') G(\mathbf{r}, \mathbf{r}') ds' \right]$$  \hspace{1cm} (3.4)$$

However, the Green’s function used in (3.4) has a singularity when the locations of the source and the observation points coalesce. As a result, special treatments are needed to handle the
diagonal terms and those close to the diagonal. Additionally, for low frequencies, the MoM formulation based on the commonly used basis functions, such as one introduced by Rao, Wilton and Glisson [48], called RWG, becomes unstable. This is because of the disparity in the relative weights of the scalar and vector potential terms that contribute to the electric field derived from the induced currents. Specifically, the scalar potential terms play a dominant role in the conventional MoM formulation based on the use of RWGs.

As an alternative formulation, we propose a novel approach, based on using the dipole-moment (DM) type of basis functions in the context of the Method of Moments (MoM). We will, therefore, demonstrate that the problem of scattering by objects of arbitrary shape with small features can be solved in an accurate and numerically efficient way. The DM approach begins by representing the given object with an array of electrically small spheres, whose electromagnetic behavior can be conveniently represented by electric and magnetic dipole moments. In common with the conventional MoM, the weight coefficients of the dipole moments are obtained by solving a matrix equation, derived by imposing the boundary conditions on conducting objects, or by matching the polarization currents in the dielectric region. The boundary conditions read:

\begin{align}
\text{On a perfect electric conductor: } & \mathbf{E}^i + \mathbf{E}^s = 0 \quad (3.5) \\
\text{In a dielectric region: } & \varepsilon_o(\varepsilon_r - 1)(\mathbf{E}^i + \mathbf{E}^s) = F(I) \quad (3.6)
\end{align}

where \( \mathbf{E}^i \) is the incident field, \( \mathbf{E}^s \) is the scattered field, \( F \) is the consistency factor, and \( I \) is the condensed dipole moment. For dielectrics where the fields exist inside the sphere, we are obligated to find an equivalent volume representation of it. We assume the final distributed moment to be a constant times the condensed value. We call this constant denoted by \( F \) as the
consistency factor. $F$ is derived by analytically solving the problem of a dielectric sphere with a small radius. This factor is given by

$$F = \frac{-3j}{4\pi\omega a^3} \quad (3.7)$$

Once the weight coefficients have been determined, they are used to calculate the scattered field by superimposing the fields radiated by each dipole.

We begin our discussion by considering the problem of plane wave scattering by a small PEC sphere of radius $a$ illuminated by an incident wave traveling in the positive $y$-direction, whose electric field is polarized along the $z$-axis. As stated earlier, the fields from the small scatterer can be characterized by a superposition of those from a $z$-directed electric dipole, $DM_E$ plus a $x$-directed magnetic dipole, and $DM_M$ with appropriate weights. For a PEC sphere,

$$DM_E = E_0 \frac{4\pi j}{\eta k^2} (ka)^3, \quad DM_M = E_0 \frac{2\pi}{jk^2} (ka)^3 \quad (3.8)$$

where $E_0$ is the magnitude of the incident electric field and $k$ is the wave number in the medium, as shown in Fig. 3.2. It is well-known that the dipole moment representation of a scatterer generates the same far fields as those scattered by the original object. However, the question of whether or not these dipole moment representations are as valid in the near-field region of the scatterer as in the far-field region was not addressed in the literature. Recently, it has been proven analytically in [36] that, for a sphere whose radius is small, the near-field behavior of the DM representation is quasi-static in nature and these dipole moment fields exactly match the original ones scattered by the sphere, all the way up to its surface. We are able to establish this by comparing the scattered electric fields obtained from the Mie series representations with those
produced by the dipole moments. The magnetic dipole moment contribution in the near-field region is negligible for all the field components, as the electric dipole moment alone completely represents this field for all $\theta$ and $\phi$. It then becomes clear that a PEC sphere can be completely described by a single electric dipole moment representation—not only in the far-field region but also in the near-field region.

In general, a single sphere will have a dipole moment in the direction coinciding with the polarization of the incident field. However, if we were to cascade these spheres in such a way that they provide a conducting path through which the current will flow, the dipole-moments will all be oriented along this direction. This will render consistent the direction of the dipole-moments calculated with the direction of the current path through the network of cascaded spheres that comprise the conducting structure.

Similarly, when a dielectric sphere is illuminated by a plane wave of amplitude $E_0$, the resulting scattered fields can be represented in terms of electric and magnetic dipole moments. For a dielectric sphere,

$$DM_E = E_0 \frac{4\pi}{\eta k^2} (ka)^3 \frac{\varepsilon_r - 1}{\varepsilon_r + 2}, \quad DM_M = E_0 \frac{2\pi}{jk^2} (ka)^3 \frac{\mu_r - 1}{\mu_r + 2}$$ \hspace{1cm} (3.9)

Note that the magnetic dipole vanishes if the dielectric is nonmagnetic, that is, if $\mu_r = 1$.

General numerical approaches that use conventional integral equation methods can be computationally expensive for certain geometries. However, the physics-based DM approach, introduced herein, is not only totally free of the singularity problem, but it is valid, universally, over the entire frequency range, including in the quasi-static regime. This universal validity accrues from the absence of Green’s functions and the absence of the vector and scalar potentials used for the conventional MoM formulation. Another salutary feature of the DM approach is its
ability to handle thin and lossy structures, whether they be metallic, dielectric-type, or even combinations of these. We have found that the DM formulation can handle these types of objects with ease, without running into ill-conditioning problems. And, this is so even for very thin wire-like or surface-type structures, which lead to ill-conditioned MoM matrices when these problems are formulated in the conventional manner.

3.2.2 Numerical Result for PEC rod

To illustrate the DM approach, we consider a PEC rod, whose length and radius are $\lambda/2$ and $\lambda/400$, respectively, where $\lambda$ is the free-space wavelength at 1 GHz, as shown in Fig. 3.3(a).

![Fig. 3.2: Dipole Moment Concept.](image)

$$DM_E = E_0 \frac{4\pi j}{\eta k^2} (ka)^3 \quad DM_M = E_0 \frac{2\pi}{jk^2} (ka)^3$$
It is illuminated by an Ez-polarized, normally incident plane wave with an amplitude of 1 V/m traveling in the negative x-direction.

Fig. 3.3: (a) PEC rod; (b) using DMs; and (c) using DMs and macro-basis functions.

When formulating a problem that involves only PEC objects, our first step is to represent the original scatterer by using a collection of PEC spheres. Next, we replace these spheres with their corresponding DMs, and then evaluate the electric fields generated by these DMs. The rod is discretized into N=400 multiple spheres whose diameter is the same as that of the wire (See Fig. 3.3(b)). Using the dipole moment equivalence for the scattered field, we can write that the electric dipole moments are z-oriented, such that
where \( w_n \) is the weighting factor for the sphere \( n \). We apply the boundary condition at the surface of each sphere (e.g., \( x=a \)), and the weighting functions are obtained from

\[
\begin{align*}
\mathbf{Z}[\mathbf{V}] &= \mathbf{V} \\
\Rightarrow \mathbf{V} &= \mathbf{Z}^{-1} \mathbf{I}
\end{align*}
\]  

(3.13)

where \( \mathbf{Z} \) is the \( N \times N \) impedance matrix, \( \mathbf{I} \) is the \( N \times 1 \) current distribution coefficient matrix to be determined, and \( \mathbf{V} \) is the \( N \times 1 \) incident field matrix on the sphere. Finally, current flowing on the rod is computed by dividing dipole moments obtained above with the spacing between the centers of two adjacent spheres.

Next, in order to reduce the number of unknowns, we consider an approach that combines the DM algorithm with the use of macro-basis functions (See Fig. 3.3(c)). For this method, a set of DMs associated with the spheres that serve as building blocks are aggregated to form a suitable set of macro-basis functions. We then evaluate the electric fields generated by these macro-basis functions and compute the reactions between them and the testing functions, which are also the same as the basis functions (Galerkin method), used to generate the elements of the MoM matrix. The right-hand side of this matrix is obtained by applying the boundary condition on the total tangential E-Field by testing it with the same functions as those used to generate the matrix elements. This method considerably reduces the number of unknowns to be solved, specifically to
19, via the use of triangle macro-basis functions. As shown in Fig. 3.4, we plot the magnitudes of the currents, induced on a PEC rod illuminated by a plane wave, using the DM/macro-basis function approach and a commercial MoM package. We observe that the comparison between them is very good for this simple example. We, therefore, note that the DM approach is considerably more general than is the MOM approach, in that the former can handle arbitrary materials, including lossy dielectrics and plasmonic materials with little or no change in its formulation.

![Fig. 3.4: Current magnitude flowing along the wire.](image)

However, although the DM method is accurate and captures all the physics, it is not the most efficient algorithm from a numerical point of view. This is because the number of spheres needed to represent a three-dimensional object can grow very rapidly and, hence, the associate matrix can become rather large. For instance, the diameter of the spheres used to represent a thin-
wire scatterer must be the same as that of the wire. Hence, for the example shown in Fig. 3.3, the number of constituent spheres needed to form the wire can be quite large, even when the wire is relatively short in length. The number of unknowns can be significantly reduced, however, by using macro-basis functions, which can render this approach comparable to that used in the conventional MoM formulation. We have shown that it is relatively easy to choose these macro-basis functions, and that we can conveniently express the fields generated by these functions in closed forms (see section 3.3.2).

The formulation of the dielectric scattering problem essentially follows the same line as in the case of PEC objects, in that we again represent the original scatterer as a collection of small-size dielectric spheres. As before, we replace these spheres with their corresponding DMs, which are then used to form a set of macro-basis functions. At this point, instead of generating the MoM matrix by imposing a boundary condition as in the PEC case, we generate the MOM matrix by applying a consistency condition on the tangential E-Field, presented in (3.6).

3.3 Characteristic Basis Function Method

3.3.1 Introduction to Characteristic Basis Function (CBF) Formulation

Next, we describe how the DM approach can be combined with the Characteristic Basis Function Method (CBFM) to improve the computational efficiency of the method and to significantly reduce even further the matrix size to be solved [49-50]. We begin by stating that the distribution of the current on the FSS elements, when illuminated by a plane wave, depends on their shape and electrical length. As a result, excitation of the higher-order modes, in addition to the fundamental mode, becomes inevitable when the incident angle moves from normal to the FSS or the incident frequency changes. This suggests that the induced currents can be
approximated with a set of piece-wise sinusoidal-basis functions and can serve as CBFs. This method is especially attractive when modeling electrically large geometries or periodic structures. The initial step in determining the CBFs is to illuminate the object with a number of plane waves that impinge upon the object at $\theta$ and $\phi$ angles at certain intervals, for the two orthogonal polarizations, and at a certain number of frequencies around the actual frequency of operation. These simulations can easily be conducted by using the DM/macro-basis function approach in the same way as in 3.2.2. The induced currents due to the plane wave illuminations are good candidates as CBFs. Because the linear dependency among the CBFs can lead to ill-conditioning, we apply a Singular Value Decomposition (SVD) to the currents, which yields the following factorized representation:

$$J = USV^T$$ (3.14)

where $S$ is a diagonal matrix, of the same dimension as $J$, with nonnegative singular values in decreasing order, and $U$ and $V$ are unitary matrices whose inverse is equal to their conjugate transpose. Singular values above a certain threshold are only retained to remove the redundant CBFs and to generate the final CBFs. Our next step is to linearly combine the final CBFs for the solution of the current. We then evaluate the electric fields expressed in closed form for this assumed current distribution and use the Galerkin procedure, which employs the CBFs as both basis and testing functions, to arrive at the reduced matrix system. We show that combining the above methods leads to a relatively small matrix, often only $2 \times 2$ or $3 \times 3$ in size for many typical elements, such as tripoles, rings, and spirals. Hence, this combination of methods is numerically very efficient. It is worth noting here that, once generated, the same basis functions can be used to represent the induced current regardless of incident angles and frequencies.
3.3.2 Analytical expression for the fields radiated by a piecewise sinusoidal

As stated, the current for the wire structures can be approximately represented by piecewise sinusoidal-basis functions. This suggests that CBFs as entire-domain basis functions can be used to represent the current distribution on these antennas, regardless of the incident angle of the illuminating field or excitation. In this section, we show that the electric fields generated by these functions can be expressed in closed forms without using Green’s functions.

First, let us consider the top part of the bent wire, as shown in Fig. 3.5(a). We can assume that the center of the wire is at the origin of the global coordinate system. Let \( \mathbf{u}_1 \) be the polarization unit vector of the current flowing along the wire and \( \mathbf{v}_1 \) be the unit vector perpendicular to \( \mathbf{u}_1 \) lying on the plane containing \( \mathbf{u}_1 \) and \( \mathbf{R} \). The co-ordinates of the point \( P \) are \((v_1, \phi, u_1)\) in cylindrical co-ordinates (normally expressed as \((\rho, \phi, z)\)). In this figure, the following relations will hold:

\[
R = \sqrt{(u_1 - h)^2 + v_1^2}
\]
\[
R_1 = \sqrt{(u_1 - H)^2 + v_1^2}
\]
\[
r = \sqrt{u_1^2 + v_1^2}
\]

Assuming a sinusoidal distribution of current, the wire current will be

\[
I = \begin{cases} 
I_n \sin \beta (H - h) & \text{if } h > 0 \\
0 & \text{otherwise}
\end{cases}
\]
The expression for vector potential at point P will be

\[
A_{m} = \frac{\mu l_{m}}{4\pi} \int_{0}^{H} \frac{\sin \beta (H-h) e^{-jH}}{R} dh
\]

\[
= \frac{\mu l_{m}}{8\pi j} \left[ e^{jH} \int_{0}^{H} \frac{e^{-j\beta(R+h)}}{R} dh - e^{-jH} \int_{0}^{H} \frac{e^{-j\beta(R-h)}}{R} dh \right]
\]  

(3.15)

Fig. 3.5: (a) Top half of bent wire; and (b) Bottom half of bent wire.

In cylindrical co-ordinates, the magnetic field strength at point P will be given by

\[
\mu H_{\phi} = (\nabla \times \overrightarrow{A})_{\phi} = -\frac{\partial A_{m}}{\partial v_{1}}
\]
\[ H_\phi = -\frac{I_m}{8\pi j} \left[ e^{j\beta(R+h)} \frac{\partial}{\partial R} - e^{-j\beta(R-h)} \frac{\partial}{\partial R} \right] \]  

Consider the first term only,

\[ \int_0^H \frac{\partial}{\partial R} \left( \frac{e^{-j\beta(R+h)}}{R} \right) dh = \int_0^H \left[ -jB_v e^{-j\beta(R+h)} \frac{\partial}{\partial h} - \frac{v_1 e^{-j\beta(R+h)}}{R^2} \right] dh 
\]

\[ = \left[ \frac{v_1 e^{-j\beta(R+h)}}{R(R+h-z)} \right]_{h=0}^{h=H} 
\]

\[ = \frac{1}{v_1} \left[ (1 - \frac{H - u_1}{R_1}) e^{-j\beta(R_1 - H)} - (1 + \frac{u_1}{r}) e^{-j\beta} \right] \]

Similarly, the second term is

\[ \int_0^H \frac{\partial}{\partial R} \left( \frac{e^{-j\beta(R-h)}}{R} \right) dh = \frac{1}{v_1} \left[ (1 + \frac{H - u_1}{R_1}) e^{-j\beta(R_1 + H)} - (1 - \frac{u_1}{r}) e^{-j\beta} \right] \]

When these two terms are added together, the magnetic field strength can be obtained:

\[ H_\phi = -\frac{I_m}{4\pi j} \left[ \frac{e^{-j\beta R_1}}{v_1} - \frac{e^{-j\beta}}{v_1} \cos(\beta H) - j\beta \sin(\beta H) \frac{e^{-j\beta}}{v_1r} \right] \]  

The electric field can be obtained from the magnetic field by recalling that in free space

\[ \overline{E} = \frac{1}{j\omega \varepsilon} (\nabla \times \overline{H}) \]
Substituting the expression for $H_\phi$ in these equations gives

$$E_n = \frac{1}{j \omega v_1} \frac{\partial}{\partial v_1} (v_1 H_\phi), \quad E_\nu = -\frac{1}{j \omega} \frac{\partial}{\partial z} (H_\phi)$$

Similarly, when the same procedure is applied to the bottom part of the bent wire, as shown in Fig. 3.5(b), the electric field will be

$$E_{\nu_1} = -j 30 I_m \left[ \frac{e^{-j p R_1}}{R_1} - \cos(\beta H) \frac{e^{-j p r}}{r} - j u_1 \sin(\beta H) e^{-j p r} \left( \frac{1}{r^2} + \frac{1}{j \beta r^3} \right) \right]$$

$$E_{\nu_2} = \frac{j 30}{v_1} I_m \left[ (u_1 - H) \frac{e^{-j p R_1}}{R_1} - u_1 \cos(\beta H) \frac{e^{-j p r}}{r} - \frac{j \sin(\beta H)}{\beta r^3} e^{-j p r} (r f u_1^2 + j v_1^2) \right]$$

From the expressions (3.18) to (3.21), we can also retrieve the analytical formula for the field computation in the case of a straight wire in [51], setting $u_1 = u_2 = z$ and $v_1 = v_2 = \rho$ (see Fig. 3.6).

$$E_z = -j 30 I_m \left[ \frac{e^{-j p R_1}}{R_1} + \frac{e^{-j p R_2}}{R_2} - 2 \cos(\beta H) \frac{e^{-j p r}}{r} \right]$$
\[ E_\rho = \frac{j30}{\rho} I_m \left[ (z - H) \frac{e^{-j\beta r}}{R_1} + (z + H) \frac{e^{-j\beta r}}{R_2} - 2z \cos(\beta H) \frac{e^{-j\beta}}{r} \right] \] (3.23)

\[ E_x = E_\rho \cos \phi = E_\rho \frac{x}{\rho} = \frac{x}{\sqrt{x^2 + y^2}} \] (3.24)

\[ E_y = E_\rho \sin \phi = E_\rho \frac{y}{\rho} = \frac{y}{\sqrt{x^2 + y^2}} \] (3.25)

Fig. 3.6: Geometry of thin-wire element.
3.3.3 Reflection and Transmission Coefficients for Periodic Array

Typically, we are interested in the Reflection (R) and Transmission (T) characteristics of EBGs that are periodic structures. Before presenting the numerical analysis of these periodic structures in the following section, we will first outline the procedure, based on a combination of DM and CBFM, for calculating the R and T coefficients. The matrix equation is first constructed for the isolated element in order to extract the Characteristic Basis Functions (CBFs). The above matrix is then generalized for the truncated periodic case without using additional unknowns by invoking the Floquet’s theorem for periodic structures, which dictates that the current distribution in the different cells would be identical to each other, apart from a phase shift, dictated by the angle of incidence of the plane wave. As a consequence, it is only necessary to consider the fields in a single unit cell and apply the boundary condition on the tangential E-field in this cell. The next step is to extrapolate the solution for the truncated structure by using Prony’s method to derive the current distribution for the infinite, doubly periodic, array. Finally, the last step is to use the extrapolated solution for the induced current to compute the reflection coefficient for the infinite array problem by using the Reciprocity Principle. As stated earlier, the techniques presented herein for solving the induced current, as well as for computing the reflection coefficient are very general; they are, therefore, applicable to arbitrary geometries that are generally three-dimensional in nature.

3.4 Numerical Examples

In this section, we use the method presented in the earlier sections to analyze four examples that involve infinite doubly periodic structures: (i) wire dipoles; (ii) wire tripoles; (iii) wire square loops; and, (iv) wire split-rings [52-54].
3.4.1 Dipole Array

Dipole arrays are often used for studying FSSs because their geometry is simple and, hence, they are relatively straightforward to analyze. Additionally they can also form the constituents of complex element geometries. For example, the square loop array is formed by the superposition of four linear segments or dipoles spaced (horizontally and vertically) from the origin of the unit cell. Dipoles are single resonant elements and are polarization dependent. The latter feature limits their use in high-performance applications [52-54]. As a simple example, as an array, an infinite doubly periodic dipole with its unit cell is shown in Fig. 3.7. Its element length and radius are $\lambda/2$ and $\lambda/100$, respectively, arranged on identical square lattices, $D_y = D_z = 0.7\lambda$, where $\lambda$ is the free space wavelength at 1 GHz. A formulation based on a combination of the Characteristic Basis Function Method (CBFM) and the closed-form representations of the scattered fields of $E_z$ using (3.22) is employed for the problem at hand. We find that only a $1\times1$ matrix adequately represents the induced current with sufficient accuracy due to the presence of only one half-sinusoid CBF in each of its elements. The matrix equation used is

$$w\mathbf{E}_b = \mathbf{E}_{in}$$  \hspace{1cm} (3.26)

where $w$ is the weight coefficient of the CBF, $\mathbf{E}_b$ is the scattered E-field generated by the CBF of the array elements, and $\mathbf{E}_{in}$ is the incident plane wave, 1 V/m in amplitude on the center element.
After obtaining the currents for the truncated periodic case, we can apply Prony’s method [55] to extrapolate the solution for these current distributions for the infinite doubly periodic case. Prony’s method is a powerful tool that estimates the current behavior of the array as a weighted linear combination of complex exponentials given by

\[
F(m) = \sum_{k=1}^{N} C_k e^{A_k m}
\]  

(3.27)

where \(C_k\) and \(A_k\) minimize error in the least-square sense. A graphical illustration of how we use Prony’s method to address this problem is shown in Fig. 3.8.
Fig. 3.8: (a) Current maximum on the center element of the dipole array for the periodicity of 0.7λ in the normal incidence case; and (b) Prony Estimation.

Next, we invoke the reciprocity theorem to argue that the reaction of the array current residing on a dipole element, and the electric field due to a dipole source located in the far field can be used to compute the radiated field from the current. Based on this, we compute the reflection coefficient as follows: We calculate the ratio of the reaction of the incident plane wave on the induced current on the element of the array, to that of the same plane wave incident upon a PEC plate, which fills the entire unit cell, and the induced current on which can be readily estimated by using Physical Optics. The relevant equations for this computation are as follows:

\[
\frac{\int_{\text{on the wire}} E_{\text{pw}} \cdot J_{\text{wire}} \, dl}{\int_{\text{entire unit cell}} E_{\text{pw}} \cdot J_{\text{pec}} \, ds} = \frac{\sum E_{\text{in}} \times \{\text{mean}(I_e)\} \times \text{length}}{E_{\text{in}} \times (2 \frac{E_{\text{in}}}{\eta}) \times \text{period}^2}
\]

The reflection coefficients of the FSSs computed in this way are plotted in Fig. 3.9 as a function of the frequency, where these results are also compared with those generated by using FEKO and a legacy code. It is worth pointing out that the legacy code is unable to handle wire structures, and
it, therefore, models the wires as strips. Thus, we should, expect some differences in the frequency-response characteristics of the two configurations, with the thin-wire element exhibiting a narrower bandwidth. Fig. 3.9 confirms that this is indeed the case. Table 3-1 compares the simulation time for the resources involved. From the table below, we can learn that the proposed technique certainly supersedes its commercial counterparts.

Fig. 3.9: Reflection Coefficient of the dipole array for the periodicity of $0.7\lambda$.

Table 3-1: Comparison of the total simulation times using the CBF-Closed Form code and the commercial packages for the dipole array.

<table>
<thead>
<tr>
<th></th>
<th>CBF-Closed Form</th>
<th>Commercial Packages (FEKO, FSS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Computer Memory</td>
<td>3 GB</td>
<td>3GB</td>
</tr>
<tr>
<td>Run Time</td>
<td>1 min. 20 sec.</td>
<td>11 min.</td>
</tr>
</tbody>
</table>
3.4.2 Tripole Array

As noted earlier, the linear dipole is restricted in its applications because of strong polarization dependence of its response to the incident electromagnetic waves. On the other hand, due to their geometry, the tripoles are inherently depolarizing elements, and they can be packed tightly to reduce the inter-element spacing, thereby enhancing the bandwidth. This characteristic renders the tripoles suitable for packing into the curved surfaces of radomes. Low cross-polarization of the scattered fields can be achieved by altering the direction of the incident field with respect to the array lattice geometry [52-54].

The tripole element consists of three linear dipoles arranged at an angle of 120° with each other and connected at one end, as shown in Fig. 3.10. The length and radius of the element are $\lambda/4$ and $\lambda/100$, respectively, where $\lambda$ is the free space wavelength at 1 GHz, arranged on identical square lattices, $D_y = D_z = 0.5\lambda$ and $0.7\lambda$. With the advantage of having only one current basis function, namely a quarter-sinusoid (see Fig. 3.11), in each of the three arms, it is convenient to calculate only the transformation of one arm when we compute the scattered fields radiated by the CBFs using (3.18) to (3.21). If $(v,u)$ are the rotated coordinates at an angle $\phi$ with respect to the $(y,z)$ axes, then the $y$ and $z$ electric field components of the transformation can be derived by a simple geometrical projection given by

$$
\begin{bmatrix}
\cos \phi & \sin \phi \\
-\sin \phi & \cos \phi
\end{bmatrix}
$$

(3.29)

These are needed to calculate the tangential E-field to impose the boundary condition on the center element of the array. The matrix equation to be solved is constructed as

$$
W_1 E_{b_1\text{ branch}_i} + W_2 E_{b_2\text{ branch}_i} + W_3 E_{b_3\text{ branch}_i} = E_{\text{in\ branch}_i} \quad i = 1, 2, 3
$$

(3.30)
where $w_1$, $w_2$, and $w_3$ are the weight coefficients of the CBFs of the three arms; $E_{_{b_1\_branch\_i}}$, $E_{_{b_2\_branch\_i}}$, and $E_{_{b_3\_branch\_i}}$ are the scattered E-fields radiated by the CBFs of each arm of the array elements; and $E_{_{in\_branch\_i}}$ is the incident field at a branch $i$ on the center element. The matrix size is finally reduced to $3 \times 3$ by using the testing functions, which are also the same as the basis functions (Galerkin method).

![Fig. 3.10: (a) FSS composed of infinite doubly periodic tripoles; and (b) Tripole FSS element.](image-url)
For a normally incident plane wave excitation for the isolated element at 1 GHz, Fig. 3.12 compares the current magnitudes for the three tripole branches against the length from the CBF-Closed Form analysis to those generated by using FEKO. Excellent agreement can be observed, which verifies the validity of this analysis method. We note that the current magnitude in branch 1 equals the sum of the currents flowing in the other two branches at the junction, and we note that the y-directed currents ($I_y$) in branches 2 and 3 are $180^\circ$ out-of-phase with each other.
Fig. 3.12: Current magnitudes for the three tripole branches.
The Prony algorithm is applied to the solution for the truncated structure for two different periodicities, namely $0.5\lambda$ and $0.7\lambda$, for the normal incidence case, and the resulting current distribution for the infinite array, as an example for the periodicity of $0.5\lambda$ is shown in Fig. 3.13. The reflection coefficients of the FSSs, computed in the same way as before are shown in Fig. 3.14 as a function of the frequency, where these results are compared with those obtained by using commercial codes. It can be seen that there is little difference in the coefficients derived from either the CBF-Closed Form or FEKO.

![Fig. 3.13: (a) Current maximum on the center element of the tripole array for the periodicity of 0.5\lambda of the normal incidence case; and (b) Prony Estimation.](image)
Fig. 3.14: Reflection Coefficient of the tripole array, for the periodicity of (a) $0.5\lambda$; and (b) $0.7\lambda$. 
Next, we model a case in which each element of the array is rotated to 45° clockwise around the z axis, and the FSS is no longer planar (See Fig. 3.15). We use Euler Angles [56] to represent the transformed field components from 3-dimensional rotations. It is well-known that any rotation between different coordinate systems can be expressed in terms of three successive rotations using the Euler angles ($\phi$, $\theta$, $\psi$), as shown in Fig. 3.16. The combined effect of these three rotations is given by this transformation matrix:

$$
\begin{bmatrix}
\cos \psi \cos \phi - \cos \theta \sin \phi \sin \psi & \cos \psi \sin \phi + \cos \theta \cos \phi \sin \psi & \sin \psi \sin \theta \\
-\sin \psi \cos \phi - \cos \theta \sin \phi \cos \psi & -\sin \psi \sin \phi + \cos \theta \cos \phi \cos \psi & \cos \psi \sin \theta \\
\sin \theta \sin \phi & -\sin \theta \cos \phi & \cos \theta 
\end{bmatrix}
$$

(3.31)

Finally, the reflection coefficients of the FSS are computed for the periodicities of $0.5\lambda$ and $0.7\lambda$, as shown in Fig. 3.17, in the same way as before. Table 3-2 compares the simulation time for the resources involved.

Fig. 3.15: FSS composed of infinite doubly periodic tripoles rotated to 45° clockwise around the z axis.
Fig. 3.16: Definition of Euler angles ($\phi, \theta, \psi$).
Fig. 3.17: Reflection Coefficient of the tripole array rotated to 45° clockwise around the z axis, for the periodicity of (a) 0.5\( \lambda \); and (b) 0.7\( \lambda \).
Table 3-2: Comparison of the total simulation times using the CBF-Closed Form code and the commercial packages for the tripole array.

<table>
<thead>
<tr>
<th></th>
<th>CBF-Closed Form</th>
<th>Commercial Packages (FEKO, HFSS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Computer Memory</td>
<td>3 GB</td>
<td>3 GB</td>
</tr>
<tr>
<td>Run Time</td>
<td>1 min. 20 sec.</td>
<td>15 min. 30 sec.</td>
</tr>
</tbody>
</table>

3.4.3 Square Loop Array

The square loop geometry is commonly employed as an FSS element. It has a simple frequency response consisting of a transmission band followed by a reflection band at which the elements resonate. The advantage of using square loop element to design FSS screens is that the symmetrical nature of the loop geometry makes it suitable for both horizontal and vertical polarizations [52-54]. Fig. 3.18 shows an FSS comprising of a doubly-periodic array of loops with a periodicity of 0.5λ, where λ is the wavelength at 1 GHz. The element length and radius of the FSS are λ/4 and λ/100, respectively. The procedure for arriving at the solution of the induced current and the reflection coefficient is the same as that used in preceding sections. The generated CBFs with the number of plane wave illuminations for the isolated element are presented in Fig. 3.19, which are sine and cosine functions. The matrix equation to be solved is given by

\[ w_1 E_{b_1} + w_2 E_{b_2} = E_{in} \]  

(3.32)

where \( w_1 \) and \( w_2 \) are the weight coefficients of the CBFs, and \( E_{b_1} = [E_{1,b_1}, \ E_{2,b_1}, \ E_{3,b_1}, \ E_{4,b_1}] \) and \( E_{b_2} = [E_{1,b_2}, \ E_{2,b_2}, \ E_{3,b_2}, \ E_{4,b_2}] \) are the scattered E-fields radiated by the CBFs of the array.
elements and $\mathbf{E}_{in} = [E_{in1} E_{in2} E_{in3} E_{in4}]$ is the incident field on the center element. The matrix size is finally reduced to $2 \times 2$ by using the testing functions, which are also the same as the basis functions. Fig. 3.20 depicts the magnitude and phase of the current for the isolated element for different polarization angles of the incident at 1 GHz. Good agreement between the currents from the CBF-Closed Form and FEKO is obtained. The reflection coefficient of the FSS is computed for the periodicity of $0.5\lambda$ as a function of the frequency, as shown in Fig. 3.21, where this result is compared with that generated by FEKO.

![Fig. 3.18: FSS composed of infinite doubly periodic square loops.](image1)

![Fig. 3.19: Characteristic Basis Functions of the square loop element.](image2)
Fig. 3.20: Current magnitude and phase for different polarization angles of the incident at 1 GHz (a) z-polarized; and (b) y-polarized.
Fig. 3.20: Current magnitude and phase for different polarization angles of the incident at 1 GHz (c) diagonally polarized.

Fig. 3.21: Reflection coefficient of the square loop array for the periodicity of 0.5λ.
Next, when each element of the array is rotated to 45° and 90° clockwise around the z axis, the computed reflection coefficients are presented in Fig. 3.22. Table 3-3 compares the simulation time for the resources involved.

Fig. 3.22: Reflection Coefficient of the square loop array for the periodicity of 0.5λ rotated to φ clockwise around the z axis (a) φ = 45°; and (b) φ = 90°.
Table 3-3: Comparison of the total simulation times using the CBF-Closed Form code and the commercial package for the square loop array.

<table>
<thead>
<tr>
<th></th>
<th>CBF-Closed Form</th>
<th>Commercial Package (FEKO)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Computer Memory</td>
<td>3 GB</td>
<td>3GB</td>
</tr>
<tr>
<td>Run Time</td>
<td>1 min. 20 sec.</td>
<td>15 min. 30 sec.</td>
</tr>
</tbody>
</table>

### 3.4.4 Split-Ring Array

To analyze this array, we again use the usual matrix notation,

\[
[Z][I] = [V]
\]  \hspace{1cm} (3.33)

where \([Z]\) is the \(N \times N\) impedance matrix, \([I]\) is the \(N \times 1\) current distribution coefficient matrix to be determined, and \([V]\) is the \(N \times 1\) incident field matrix. In the matrix equation (3.33), \([I]\) is now expanded by using the set of CBFs, generated previously by using the process described earlier. We write:

\[
[I] = [I^{CBF}][w] = \sum_{n=1}^{M} w_n I_n^{CBF}
\]  \hspace{1cm} (3.34)

where \(M\) is the total number of CBFs, \([w]\) is the coefficient vector of dimension \(M \times 1\) and \([I^{CBF}]\) is the matrix form of CBFs of dimension \(N \times M\). By inserting (3.34) into (3.33), we get
Then by using the transpose of $[\mathbf{1}^{\text{CBF}}]$ as the testing function, we obtain the final form of new, reduced matrix equation, which reads

$$
[Z][\mathbf{1}^{\text{CBF}}][\mathbf{w}] = [\mathbf{V}]
$$

(3.35)

or, in a simpler form

$$
[Z^{\text{CBF}}]^{\top}[Z][\mathbf{1}^{\text{CBF}}][\mathbf{w}] = [\mathbf{1}^{\text{CBF}}]^{\top}[\mathbf{V}]
$$

(3.36)

where $[Z^{\text{CBF}}] = [\mathbf{1}^{\text{CBF}}]^{\top}[Z][\mathbf{1}^{\text{CBF}}]$ and $[V^{\text{CBF}}] = [\mathbf{1}^{\text{CBF}}]^{\top}[\mathbf{V}]$. Typically $M$, the dimension of the reduced matrix, is much smaller than that of the original matrix equation ($N$), and the reduced matrix equation can be solved directly. Once the coefficients of the reduced matrix equation have been obtained, the solution for the current is obtained from a weighted linear combination of these CBFs using the equation (3.34). Having discussed the derivation of the reduced matrix equation, we will go on to apply it to the analysis of split-ring array [57].

The element length and radius of the array are $\lambda/4$ and $\lambda/100$, respectively. And its split length is $\lambda/40$, which is oriented along the y-direction. We generate 4 CBFs with the number of plane wave illuminations for the isolated element. Therefore, according to the procedure above, the matrix size is finally reduced to 4 by using the testing functions, which are also the same as the basis functions. The reflection coefficients of the FSS are computed for the periodicity of $0.5\lambda$. 
as a function of the frequency, as shown in Fig. 3.23, when each element of the array is rotated to 0°, 45° and 90° clockwise around the z axis, where these results are compared with those generated by FEKO.

Fig. 3.23: Reflection Coefficient of the split-ring array for the periodicity of 0.5λ rotated to φ clockwise around the z axis (a) φ = 0; (b) φ = 45; and (c) φ = 90.
3.5 Conclusions

General numerical approaches that use conventional integral equation methods can be computationally expensive for analyzing complex and arbitrary geometries. It is important, therefore, to develop EM simulation techniques that reduce run-times sufficiently, so as to make them more than are existing codes for use suitable as design tools.

In this chapter, we presented two novel techniques involving a combination of the Dipole Moment (DM) method and the Characteristic Basis Function Method (CBFM) to formulating the EM-scattering problems of periodic structures. In the MoM version of the new approach, the DM formulation is used to generate the matrix equation, instead of the traditional Green’s function method. Therefore, this formulation neither suffers from the singularity problem associated with the Green’s function method, nor does it experience any difficulties at low frequencies, unlike the conventional MoM formulations. However, although the DM method is accurate and captures all the physics, it is not the most efficient algorithm from a numerical point of view. This is because the number of spheres needed to represent a three-dimensional object can grow very rapidly and, hence, the associate matrix can become rather large. To mitigate this problem, we considered an approach that combines the DM algorithm with the Characteristic Basis Function Method (CBFM), in which the macro basis functions called the characteristic basis functions (CBFs) are used to generate a relatively small-sized matrix even for large problems. The proposed formulation employs closed-form expressions to compute the fields radiated by the CBFs, and no Green’s functions are used. We have shown, for instance, that for a whole host of element shapes the matrix size that we need to deal with remains as small as just 2 or 3.
Chapter 4

Electromagnetic Characteristics of Metamaterials Using Rigorous Numerical Modeling

4.1 Introduction

In recent years, metamaterials (MTMs) have been the subject of considerable attention in the scientific community: their unusual features, which are not readily available in nature, promise to make possible new applications for microwave circuits and antenna composites. MTMs are typically created by using periodic inclusions, which often have resonant properties, and embedding them in a background dielectric medium. Depending on the electromagnetic properties they exhibit, MTMs are referred to by various names in the literature, including (i) Double Negative (DNG) materials, which have both negative permittivity $\varepsilon$ and permeability $\mu$; (ii) Left-Handed (LH) materials, inside which the electric field $E$, magnetic field $H$, and wave vector $k$ form a left-handed system; and (iii) Negative Refractive Index (NRI) materials, which have a negative refractive index. It is worth pointing out that metamaterials, also called artificial dielectrics, are frequently designed to exhibit DNG (double negative) properties in the frequency range of interest. However, for other frequency regions, they may exhibit DPS (double positive), ENG ($\varepsilon$ negative), or MNG ($\mu$ negative) properties. Due to their unique bandgap features, the Electromagnetic Band Gap (EBG) structures that we studied in Chapters 2 and 3 can be regarded as a special type of metamaterials (the two terms, namely EBG and MTM, are sometimes used interchangeably).

Effective medium descriptions are widely used to describe the characteristics of artificially structured media, including metamaterials. This is certainly convenient, since such a
description is very appealing for interpreting the physical characteristics of the artificial materials; specifically, such descriptions relate the \( \varepsilon \) and \( \mu \) of the materials to those of equivalent homogeneous media with the same permittivities and permeabilities. First, essentially via the artifice of homogenization, an equivalent effective medium must be found that is capable of replacing the original configurations of the antenna–metamaterial composites. Once this has been achieved, analyzing the composites becomes relatively straightforward using conventional techniques and simulation tools. Overall, then, this approach is both memory- and time-efficient. In fact, some studies have offered some interesting and novel applications based on the effective medium characteristics of metamaterials. Four such applications that have featured prominently in recent publications (see Fig. 4.1) are as follows:

(i) Achieving superlensing by using DNG media that magnify evanescent waves

(ii) Using a metamaterial superstrate or metamaterial substrate to enhance the performance of planar antennas

(iii) Using an ENG shell to enhance the performance of small antennas

(iv) Creating electromagnetic cloaks
We now return to the Equivalent Medium Approach (EMA) issue. In doing so, we assume that (i) we have carried out the important step of replacing the original metamaterial medium with its equivalent medium, and (ii) we have proceeded to the next step of predicting the performance of the composite that integrate this medium with an antenna to achieve certain performance characteristics. We then pose two important questions: (i) How well does the equivalent medium represent the original structure that it replaces and how accurate is this representation? (ii) If we are to be confident in our response to this question, what methods should we use to determine our answer?

Fig. 4.1: Applications of metamaterials: (a) Superlensing effect by a DNG slab; (b) MPA with a DNG superstrate; (c) Dipole encased in a spherical shell of ENG material; and (d) Cylindrical cloak.
In this chapter, we examine the issue of homogenizing metamaterials when modeling practical devices that are based on using these materials for various applications, such as focusing, directivity enhancements of antennas, size reduction, and cloaking. The element geometries that we study in this section are as follows: (i) split-rings and wires; (ii) holey dielectric plates; (iii) cross-wires; and (iv) fishnets. Next, we investigate these configurations with the objective of comparing the performances predicted by the effective medium approach with those actually realized through rigorous numerical simulations. To gain a good understanding of the behaviors of the fields inside an MTM slab, it is useful to track the propagation of the Gaussian beams both in the interior and exterior regions of the slab. Finally, we discuss the feasibility of employing the effective parameters to model the antenna–metamaterial composites, specifically in connection with the four applications listed. All simulations in this work have been carried out using GEMS, a 3-D parallel FDTD solver capable of handling a large number of DoFs (upwards of $10^{9}$).

### 4.2 Equivalent Medium Approach

Analytical approaches to characterizing artificially structured metamaterials, which are inherently inhomogeneous, are very important inasmuch as they offer simple physical insight into the nature of the electromagnetic response of a composite. With the astonishing growth of the field of DNG metamaterials, a variety of studies have sought ways to systematically characterize the electromagnetic properties of these materials. The literature reports the following approaches to obtaining the effective parameters from numerical data: averaging methods [18] in which the local electromagnetic fields of a structure are calculated by direct integration of Maxwell’s equations, and an averaging procedure is applied to define the macroscopic fields and material parameters; phase velocity calculations [19] from time-domain simulations; and inversion techniques [20-21] from the reflection and transmission coefficients of the slab illuminated by a
normally incident plane wave. The first two approaches utilize the field behavior inside the composite, but the inversion approach relies entirely on knowledge of the scattering data (reflection and transmission) of the structure to extract the effective parameters. An important attribute of the inversion approach is that it makes no physical assumptions about the particular nature of a structure; instead, this approach focuses on outside properties such as scattering problems. For this reason, the inversion method is the most popular methodology for characterizing the design of metamaterials.

Next, we briefly review the inversion procedure for the retrieval of effective material parameters. Following this, we apply this approach to processing the scattering parameters generated from the FDTD simulations obtained by using the PBC/FDTD technique. The simulations are carried out for a number of different element geometries, namely split-rings and wires, holey dielectric plates, cross-wires and fishnets.

It should be noted, first of all, that the homogenization is carried out by illuminating the given metamaterial slab with a normally incident plane wave, and measuring (or computing) the reflection and transmission coefficients of the slab as functions of frequency (see Fig. 4.2).

Fig. 4.2: Inversion approach for calculating the effective $\varepsilon$ and $\mu$ of a metamaterial slab.
The scattering parameters of a homogeneous slab of thickness $d$ are given by

$$S_{11} = \frac{\Gamma(1 - e^{-j2nk_0d})}{1 - \Gamma^2 e^{-j2nk_0d}}$$  \hspace{1cm} (4.1)$$

$$S_{21} = \frac{(1 - \Gamma^2) e^{-jnk_0d}}{1 - \Gamma^2 e^{-j2nk_0d}}$$  \hspace{1cm} (4.2)$$

where $\Gamma = \frac{Z - 1}{Z + 1}$ is the reflection coefficient at the boundary between the two media, $Z$ is the normalized wave impedance, $n$ is the refractive index of the slab, and $k_0$ is the free-space wave number. We then appeal to the equations developed by Kong et al. [19] to extract the material parameters. The refractive index as well as the wave impedance can be calculated by inverting (4.1) and (4.2) to get

$$Z = \pm \sqrt{\frac{(1 + S_{11})^2 - S_{21}^2}{(1 - S_{11})^2 - S_{21}^2}}$$  \hspace{1cm} (4.3)$$

For passive media, the real part of the wave impedance $Z$ should always be positive, thus restricting the sign on the right-hand side of equation (4.3). The intermediate variable $Y$ is given by
\[ Y = e^{-j\kappa_0 d} = X \pm j\sqrt{1-X^2} \quad (4.4) \]

\[ X = \frac{1}{S_{21}} (1 - S_{11}^2 + S_{21}^2) \quad (4.5) \]

The refractive index \( n \) can be calculated from (4.4) and (4.5) by using the expression

\[
n = \frac{1}{k_0 d} \{[ln(e^{-j\kappa_0 d})]^* + 2m\pi] + j[ln(e^{-j\kappa_0 d})]^* \} \quad (4.6)
\]

\[
= \frac{1}{k_0 d} \{[ln(Y)]^* + 2m\pi] + j[ln(Y)]^* \}
\]

where \( m \), an integer related to the branch index of \( n' \) (real part), is related to the thickness of the slab. \( n'' \) (imaginary part) is selected in accordance with the principle of causality; i.e., \( n'' < 0 \) when a time dependence in \( e^{j\omega t} \) is assumed for electromagnetic field propagation. Once the refractive index and the wave impedance have been found, the effective permittivity and permeability can be obtained from

\[ \varepsilon_{eff} = \frac{n}{Z} \quad (4.7) \]

\[ \mu_{eff} = nZ \quad (4.8) \]

As seen in (4.3), (4.4), and (4.6), the expressions contain complex multi-value functions. Hence, a difficulty arises: it is not an easy task to select the correct branch from the multiple sets of possible solutions. The procedure for extracting the effective medium parameters from the S-parameters and the selection rules for choosing the correct branches are discussed in detail in [43].
In particular, several important aspects of the inversion procedure have been investigated, including determining the boundary of the homogenized slab, selecting the sign of the roots when solving the impedance $Z$, and choosing the correct branch of the real part of the refractive index $n$. A modified inversion approach that employs the Small-Phase-Small-Loss (SPSL) solution was also proposed to assist in the selection of the correct root for all the solutions that remained after the non-physical ones had been eliminated in the inversion method.

Next, we examine the question of whether the effective medium approach to designing these metamaterial-based devices leads us to designs that perform in accordance with the effective medium theory’s predictions. The EM response of a finite, artificial DNG slab illuminated by a Gaussian beam either normally or from an oblique angle is studied by using the parallel FDTD solver. We show, through several examples, that studies of this type of beam propagation are very helpful in developing an understanding of the scope and limitations of homogenizing MTM slabs [58-66].

4.2.1 Split-rings and wires

Pendry et al. [4-5] have proposed the possibility that a DNG slab with a negative refractive index can recover the evanescent waves at the image location and, hence, can achieve a perfect lens that would not be limited by the diffraction limit associated with conventional lenses. In 2001, Smith et al. [67] used the concepts in [4-5] to design a device that experimentally realized the first DNG metamaterial composed of thin wires and split-ring resonators (SRRs). Their design uses an array of metallic wires and SRRs to realize a negative permittivity and a negative permeability, respectively. And, in this combination structure, the refractive index is found also to be unambiguously negative at frequencies where both the recovered $\varepsilon$ and $\mu$ are simultaneously negative.
In this section, we first extract the effective parameters of a well-known DNG slab consisting of thin wires and SRRs that together exhibit DNG behavior at certain frequencies. We then use a Gaussian beam to excite this structure in order to examine the direction of the beam’s wave propagation in such a medium.

The metamaterial slab considered in this investigation for extracting its effective parameters is an infinite doubly periodic array of single layer. The SRRs and the wires are made of perfect electric conductors (PEC), and their geometries are shown in Fig. 4.3. The configuration of the unit cell is similar to that used in the first DNG structure as realized by Smith et al. [67], except that the present study uses the somewhat simpler configuration of a single split-ring as opposed to a double ring. The SRR is a square ring, 3 mm in height and 0.25 mm in width, with a gap of 0.5 mm wide in the center of its side arm. The thin wire, located 0.25 mm away from the SRR, has a width of 0.5 mm, and the wires on the same row of the array are connected to each other so that they form a long wire. The array has a periodicity of 2.25 mm and 5 mm along the x- and z-directions, respectively.

![Geometry of split rings and wires.](image)

Fig. 4.3: Geometry of split rings and wires.
Figs. 4.4 (a) and (b) show the magnitude and phase of the reflection and transmission coefficients from 10 to 20 GHz obtained from a periodic-FDTD code that employs a plane wave illumination with the thin wires placed parallel to the electric field and SRRs aligned normal to the magnetic field. For this combined structure, a 3-dB passband is observed, starting at 14.8 and going up to 20 GHz. The refractive index $n$, the impedance $Z$, and the effective material parameters $\varepsilon$ and $\mu$ of the medium all are extracted by using the inversion approach described earlier. They are plotted in Figs. 4.5 (a)–(d), with the gray area representing the non-physical region in which the criteria $\varepsilon''$ and $\mu'' \leq 0$ are violated. The medium exhibits a DNG behavior in the frequency range of 14.4 to 17.0 GHz.

![Magnitude of Reflection (R) and Transmission (T) Coefficients](image1)

![Phase of Reflection (R) and Transmission (T) Coefficients](image2)

(a) (b)

Fig. 4.4: Reflection and transmission coefficients of the unit cell of split-rings and wires: (a) magnitude; and (b) phase.
Fig. 4.5: Effective material parameters: gray area represents the non-physical region where no solutions can be found to satisfy both $\varepsilon$ and $\mu$.

Next, we investigate the scattering and propagation characteristics of a moderate-size, finite DNG array when it is illuminated by a localized Gaussian beam incident either normally or
from an oblique angle. The array consists of inclusions identical to those we have studied previously. The dimensions of the slab are 50 mm ($2.5\lambda$) $\times$ 50 mm ($2.5\lambda$), where the thickness of the slab is 10 mm ($0.5\lambda$) and where $\lambda$ is the free-space wavelength at 15 GHz, as shown in Fig. 4.3. First, the slab is illuminated with a normally incident Gaussian beam of Ez-polarization whose beam waist is 10 mm. Its strength is 1 V/m. The results of the simulation are presented in Fig. 4.6, where the magnitudes and phases of Ez at 15 GHz on the XY plane (H-plane) are shown for three cases, namely (i) free-space; (ii) a dielectric slab with a permittivity of 6, the same dimensions as the DNG slab; and (iii) a DNG slab consisting of thin wires and SRRs. From the previous results obtained from the effective medium approach, we note that the array is totally transmitting at 15 GHz and that it has a refractive index close to -1. The two dashed lines in the figure represent the planar interfaces of the slab. From the plots for the magnitudes in this figure, it is evident that the magnitude decays smoothly in the free-space region. Inside the dielectric slab, we clearly see an interference pattern formed by the standing waves that are caused by the reflection at the two dielectric/air interfaces. On the other hand, the distribution inside the artificial DNG array is dominated by strong fields arising from the discontinuities presented by the inclusions, and no clear interference patterns can be seen in these field plots. We also note that the beamwidth radiating from the DNG slab becomes narrower than does the beamwidth in either the free-space or the dielectric slab case.

However, the phase distribution clearly demonstrates the backward wave nature of the phase velocity, which is one of the signature characteristics of the DNG materials. A positive phase velocity is indicated by the decreasing trend of phase along the propagating direction, i.e., the positive y-direction (see Fig. 4.6), and vice versa for the negative phase velocity. From the phase plots of the figures, a positive phase velocity can be clearly observed in the free-space region. Inside the dielectric slab, the phase velocity is found to be smaller than that in the free-space, which is indicated by the smaller spacing between the phase fronts, though it is still
positive. By tracking the phase inside the artificial DNG slab, we find a backward wave in the slab, which is indicative of negative refraction.

Fig. 4.6: The magnitudes (Left) and phases (Right) of Ez at 15 GHz on the XY plane (H-plane) for three different configurations at normal incidence (the two dashed lines indicate the planar interfaces of the slab).
To better display the differences between the field variations in the three cases, we plot, in Fig. 4.7, the phase distributions on the centerline of the aperture, which is 0.1λ above the top surface of the slabs that cut through the aperture in the middle. We note from this figure that the phase distribution above the DNG slab exhibits less of a departure from a uniform distribution than does either the free-space or the dielectric slab case. This suggests that the DNG slab can be used to enhance the directivity of small antennas and we already investigated the case in Chapter 2.

![Fig. 4.7: Phase comparison of Ez along the x-axis at 0.1λ above the top surface of the slabs.](image)

Another unusual phenomenon associated with negative refraction is that when the rays enter a DNG slab from free-space, or when they leave a DNG slab to enter free-space, at oblique incidence, they bend towards the same side of the normal at the interface (Snell’s law can still be applied to negative values of the refractive index found inside a DNG medium). Fig. 4.8 plots the magnitudes and phases of Ez at 15 GHz on the YZ plane (E-plane), which is also the plane of
incidence, for the three different configurations illustrated in Fig. 4.6. As shown in Fig. 4.8, the incident beam propagates along the positive y-direction with decreasing z. A rough estimate of the beam direction can be made by tracking the beam maximum as it propagates through various media. When the beam first enters the dielectric slab, the beam first bends slightly towards the normal, and then bends away from the normal when it exits the slab, which is an indication of positive refraction of the medium (see the ray picture in Fig. 4.8). For the artificial DNG slab, though, the beam is unable to define a continuous path inside the slab because of the strong irregularity of fields in the inhomogeneously filled slab. Nonetheless, it is still evident that the beam through the DNG slab does not bend towards the same side of the normal when it enters the slab, as we would expect with negative refraction. This is because the beam lies not only at the bottom half of the figure all through the slab but also when it exits the slab in the free-space region. Our simulation results indicate that the propagation behavior of the Gaussian beam does not exactly follow the negative refraction behavior predicted by the effective medium theory.
Fig. 4.8: The magnitudes (Left) and phases (Right) of Ez at 15 GHz on the YZ plane (E-plane) for three different configurations at oblique TM$_y$ incidence (30° off-normal) (the two dashed lines represent the planar interfaces of the slab).
4.2.2 Holey dielectric plates

After the double-negative property was first experimentally observed in a periodic artificial structure combining SRRs with thin wires, most subsequent studies on metamaterials were based on such structures. However, Holloway [68] proposed that these structures are not necessary to the design of a negative refraction material, which, in principle, can be realized by using purely dielectric inclusions in a background dielectric medium. This, in turn, opened up the possibility of synthesizing negative refractive index metamaterials much more simply than had previously been proposed. A holey dielectric slab is one of the candidates that could provide a negative refractive index metamaterial derived from the information of phase and group velocities of the dispersion diagram. The structure was examined using a point-source excitation, and the measurement results were presented in [69]: the results showed that this structure images a point source placed at one side of the slab to the other side of the same, at a frequency of 31.5 GHz, at which a homogenized model of the medium exhibits DNG characteristics.

Fig. 4.9 shows the geometry of the holey dielectric plate used for this study. The dimensions of the structure are 30 mm (3\(\lambda\)) x 30 mm (3\(\lambda\)), when the thickness of the slab is 11.9 mm (1.2\(\lambda\)), and its dielectric constant is 20, where \(\lambda\) is the free-space wavelength at 30 GHz. The diameter of the holes is \(d=1.6\) mm and the triangular lattice constant is \(p=2.3\) mm. Fig. 4.10 shows the magnitude and phase of the reflection and transmission coefficients of an infinite doubly periodic array of a single layer from 25 to 35 GHz obtained from a periodic-FDTD code.
Fig. 4.9: Geometry of a multi-layer holey dielectric plate.

Fig. 4.10: Reflection and transmission coefficients of the unit cell of holy dielectric plates: (a) magnitude; and (b) phase.
The extracted parameters for the holey dielectric plate, obtained by using the inversion approach described earlier, are shown in Fig. 4.11, with the grey area representing the non-physical region in which the criteria $\varepsilon''$ and $\mu'' \leq 0$ are violated. The medium exhibits a DNG behavior in the frequency range of 29.7 to 32.5 GHz.

Next, we investigate the scattering and propagation characteristics of a Gaussian beam impinging on a finite holey plate. First, the slab is illuminated with a normally incident Gaussian beam of Ez-polarization whose beam waist is 5 mm. The results of the simulation are presented in Fig. 4.12, where the magnitude and phase of Ez at 30 GHz on the XY plane (H-plane) are shown for three cases, namely (i) free-space; (ii) a dielectric slab with a permittivity of 20, the same as that of the holey dielectric slab; and (iii) a holey plate. From the previous results of the effective medium approach, we note that the array is totally transmitting at 30GHz and that it has a refractive index close to -1. The two dashed lines in the figure represent the planar interfaces of the slab. By tracking the phase inside the holey plate, we establish that the plate’s periodic structure has excited a backward wave. We note further that the existence of this wave indicates that negative refraction has taken place.
Fig. 4.11: Effective material parameters: gray area represents the non-physical region where no solutions can be found to satisfy both $\epsilon^*$ and $\mu^* \leq 0$. 

(a) Wave impedance $Z$

(b) Refractive index $n$

(c) Effective electric permittivity $\epsilon$

(d) Effective magnetic permeability $\mu$
Fig. 4.12: The magnitudes (Left) and phases (Right) of Ez at 30 GHz on the XY plane (H-plane) for three different configurations at normal incidence. The two dashed lines indicate the planar interfaces of the slab.

Fig. 4.13 shows the phase distributions on the centerline of the aperture, which is 0.1λ above the top surface of the slabs that cut through the aperture in the middle. We observe that the phase distribution of the DNG slab is considerably more uniform over the entire aperture than is the phase distributions of the free-space or the dielectric slab case.
Fig. 4.13: Phase comparison of $E_z$ along x-axis at $0.1\lambda$ above the top surface of the slabs.

The magnitudes and phases of $E_z$ at 30 GHz on the YZ plane (E-plane) when the plane is illuminated by a Gaussian beam incident at a $30^\circ$ angle are plotted in Fig 4.14, for the same configurations as studied previously. We note the following: (i) the beam energy propagates in a direction that is close to the normal within the artificial DNG slab, (ii) the phase propagates backward within the slab, and (iii) the normal of the phase does not bend towards the same side of the normal of the slab at the interface between the DNG medium and the free-space.
Fig. 4.14: The magnitudes (Left) and phases (Right) of Ez at 30 GHz on the YZ plane (E-plane) for three different configurations at oblique TM\(_2\) incidence (30° off-normal) (the two dashed lines represent the planar interfaces of the slab).
4.2.3 Cross-wires

It has been theoretically demonstrated that a micro-structured material formed by an array of densely packed crossed metallic wires can be used to enhance the near-field, thereby achieving sub-wavelength imaging in the optical range [70]. Because of the anomalous interaction between the cross-wires, micro-structured materials are characterized by an anomalously high refraction index that supports strongly confined guided modes with very short propagation wavelengths.

The schematic of the cross-wire structure, shown in Fig. 4.15, is based on the design of the microwave range. An array of crossed metallic wires tilted by $\pm 45^\circ$ with respect to the interfaces is embedded in free-space with a thickness of $L=0.2\lambda$, where $\lambda$ is the free-space wavelength at 36 GHz. The wires are contained in planes parallel to the YZ plane, and each wire mesh is arranged in a square lattice with a lattice constant of $a=0.16$ mm. The wires in the adjacent planes are orthogonal and spaced at intervals of $a/2=0.08$ mm. The extracted parameters of the cross-wire element are shown in Figs. 4.16–4.17. To further investigate the scattering characteristics of the cross-wire slab, we use a Gaussian beam to excite the structure, and examine how the wave propagates in both the inside and outside regions of the slab. The results are shown in Figs. 4.18–4.19.
Fig. 4.15: Geometry of cross-wires.

- \( \lambda \) : wavelength at 36GHz

Fig. 4.16: Reflection and transmission coefficients of the unit cell of cross-wires: (a) magnitude; and (b) phase.
Fig. 4.17: Effective material parameters: gray area represents the non-physical region where no solutions can be found to satisfy both $\varepsilon^*$ and $\mu^* \leq 0$. 

(a) Wave impedance $Z$

(b) Refractive index $n$

(c) Effective electric permittivity $\varepsilon$

(d) Effective magnetic permeability $\mu$
Fig. 4.18: The magnitudes (Left) and phases (Right) of Ez at 36 GHz on the XY plane (H-plane) for three different configurations at normal incidence (the two dashed lines represent the planar interfaces of the slab).
Fig. 4.19: The magnitudes (Left) and phases (Right) of Ez at 36 GHz on the YZ plane (E-plane) for three different configurations at oblique TM$_{z}$ incidence (30° off-normal) (the two dashed lines represent the planar interfaces of the slab).
4.2.4 Fishnets

The Metal–Dielectric–Metal fishnet structure is presently one of the most promising designs for realizing negative index resonances in the microwave and optical ranges, associated with the inductance and capacitance. The Metal–Dielectric–Metal fishnet structure is capable of realizing the necessary inductance and capacitance conveniently because it is simple to construct and is able to produce LH behavior for a wave normally incident to the layered structure. The magnetic response of the structure is ascribed to the excitation of a virtual current loop between the strips parallel to the incident magnetic field $H$. If these currents are strong enough, they can counteract the incident $H$ field and give rise to a resonance exhibiting negative permeability. Further, the symmetric nature of the fishnet structure also makes it polarization-insensitive. The unit cell investigated herein is shown in Fig. 4.20, which also defines all the relevant geometrical parameters. The property of the metal is treated as PEC, and a dielectric permittivity of 1.9 is considered. The periodicity in the XZ plane is fixed at 6 mm, and the light propagates along the y-direction with Ex-polarization.

The extracted parameters for the fishnet element are shown in Figs. 4.21–4.22. Also, Figs. 4.23–4.24 show examples of Gaussian beams propagating through a finite MTM slab. We observe that two or more beams emerge from different positions at the surface of the slab and travel in different directions for an oblique incident case, as shown in Fig. 4.24. This indicates that the behavior of the physical structure is different from that predicted by the effective medium approach.
Fig. 4.20: Geometry of the fishnet.

Fig. 4.21: Reflection and transmission coefficients of the unit cell of fishnets: (a) magnitude; and (b) phase.
(a) Wave impedance $Z$

(b) Refractive index $n$

(c) Effective electric permittivity $\varepsilon$

(d) Effective magnetic permeability $\mu$

Fig. 4.22: Effective material parameters.
Fig. 4.23: The magnitudes (Left) and phases (Right) of Ex at 32 GHz on the XY plane (E-plane) for three different configurations at normal incidence (the two dashed lines represent the planar interfaces of the slab).
Fig. 4.24: The magnitude (Left) and phase (Right) of Ex at 32 GHz on the YZ plane (H-plane) for three different configurations at oblique TE sub y incidence (30° off-normal) (the two dashed lines represent the planar interfaces of the slab).
4.2.5 Limitations of Homogenization of Metamaterials

In the previous sections, we have carried out a number of case studies on the effective medium representation in order to characterize the metamaterials. We note, first of all, that the homogenization is achieved by illuminating the given metamaterial slab with a normally incident plane wave and measuring (or computing) the reflection and transmission coefficients of the slab as functions of frequency. The material parameters are then extracted through the process of fitting these measured data by assigning $\varepsilon$ and $\mu$ values that provide a match to the above data to the material slab. However, this method suffers from an inherent multiple-branch ambiguity problem arising from the multi-value nature of the logarithmic and square root functions in the expressions for the refractive index $n$ and the impedance $Z$. In order to overcome this problem, the strategy is to first eliminate the non-physical solutions using a set of rules based on, for example, the causality and passivity conditions of the medium, and then to assume that the only surviving solution is the correct one.

Even when we are able to extract the $\varepsilon$ and $\mu$ values that satisfy the physical conditions imposed on the behavior of the medium, there is no guarantee that these values won’t change as we make the slab thicker by using a multi-layered structure, or that they will remain invariant as we change the angle of incidence. And yet, we also find that there are a number of situations in which $\varepsilon$ and $\mu$ become highly dependent on both the thickness and the incident angle [43]. Therefore, it is important to understand the parameter regimes of the geometries, as well as the shapes of the MTM elements; specifically, we must explore the parameter regimes and the MTM shapes for which we can extract scalar values of $\varepsilon$ and $\mu$, as opposed to the tensor representations of the same, which would suggest that the media are anisotropic in nature.

The issue of thickness dependence is a little more complex to address. This is because it implies close coupling between the various layers of a multi-layer MTM slab to the extent that the
higher-order Floquet harmonics that are invariably excited in the longitudinal direction of the multi-layer slab are not negligible; therefore, the material cannot be represented solely in terms of the dominant Floquet mode inside the slab. In light of this, it is important to recognize that the ray picture, which implies refraction in a single direction, does not adequately describe the wave propagation in the periodic structure. In addition, as illustrated through the previous examples, we run into a fundamental problem: the extracted parameters $\varepsilon_{\text{eff}}$ and $\mu_{\text{eff}}$ are not always stable functions of frequency; instead, they often display rapid variations as we move through the resonance regions of the inclusions. And yet, it is in these regions that they exhibit DNG properties, which we frequently seek for novel applications of MTMs.

Numerical examples are presented to illustrate the scattering of a Gaussian beam from a homogenous dielectric slab embedded in a background medium and from a metamaterial slab containing several types of inclusions likewise embedded in a background medium. For the oblique incident case, none of the results indicates that the beam bends towards the same side of the normal at the interface between the DNG medium and the free-space; this is so, even though the phase clearly demonstrates a backward phase velocity inside the slab at the frequency at which the slab shows a negative refraction. This suggests that the effective medium approach, though it matches the external parameters such as the reflection and transmission coefficients, cannot often explain the field behaviors within the slab correctly. It also suggests that these behaviors cannot be deduced from the result of the simulation for the infinite periodic array illuminated by a plane wave. In view of this, we recommend, instead, to use rigorous simulation to study the characteristics of metamaterials, albeit at an increased computational cost.
4.3 Use of effective material parameters for real-world applications

As stated, this study is primarily interested in using metamaterials to improve the performance of antennas. For many such applications, the ideal conditions under which to derive the material parameters for the metamaterials/slab have not been satisfied. In this section, we discuss the extent to which it may still be possible to apply the equivalent medium approach to solving the modeling problem associated with these structures, specifically in connection with the four applications we listed.

Many investigators have attempted to create a metamaterial lens with DNG properties capable of focusing images superior to those produced by, for example, the conventional dielectric lens of diffraction-limited systems. Attempts to realize this “perfect” lens, however, encounter severe practical limitations. First, since the DNG materials must be synthesized artificially, typically by using periodic inclusions in a background medium, the resulting metamaterial lens is inherently anisotropic. The source to be imaged—say a point source or a small electric (magnetic) dipole—radiates a spectrum of plane waves covering a wide angular range; thus, unlike an ideal, homogeneous DNG slab, the slab in this configuration does not react equally well to these constituent plane waves. Another important point is that the effective parameters of the slab are typically determined through measuring the reflection and transmission properties of a plane wave, which is normally incident on a thin slab of the material. However, given the slab’s anisotropic nature, its effective $\varepsilon$ and $\mu$ so obtained do not in general correctly predict its performance at oblique angles. Furthermore, the effective parameters fail to predict the characteristics of slabs with larger thicknesses that are constructed of multiple layers of a thin single-layer slab, for which the effective parameters were initially determined. The reason the multi-layer slab properties cannot be predicted correctly from that of a thin single-layer structure is that the thicker structure is quasi-periodic in the longitudinal (i.e., y) direction and, in general,
supports higher-order Floquet modes in the above direction that are totally neglected in the simplistic single-mode description associated with the equivalent material parameters. Studies by numerous authors have all shown that the observed behaviors of artificially synthesized DNG lenses are not intuitively obvious. Our observations in the study lead us to conclude that the behavior of the field propagation can be considerably different in a DNG slab than that predicted by the effective medium theory, and the same may be true for its focusing characteristics.

Next, we move to another application, namely the performance enhancement of planar antennas using a metamaterial superstrate or a metamaterial substrate. A number of authors have argued that the superlensing effect produced by a DNG superstrate can enhance the directivity of a relatively small antenna by focusing the antenna’s energy to infinity and they stay clear of the use of either the DPS or ENG superstrate, because the former would not have any focusing properties, and the latter would block the transmission through it. However, it has been convincingly demonstrated that higher directivity can indeed be achieved through placing a DPS and/or an ENG superstrate above the microstrip patch antennas, and that the levels achieved can be comparable to that of an array with the same-size aperture. This is because the focusing effect of the superstrate is not the physical mechanism responsible for the directivity enhancement; it is the resonance of the Fabry–Perot resonator, formed by the ground plane underneath and the highly reflecting superstrate above, which plays this role. The important point is this: although similar performance can be achieved using a DNG superstrate, it has not been demonstrated that there is an advantage to be gained from using this type of superstrate over an FSS or a simple dielectric slab. In fact, the DNG slab is usually thicker and lossier than a simple FSS-type superstrate, which not only provides a superior performance, but is also easier to fabricate. Furthermore, DNG slabs have a narrow band and may be cumbersome to manufacture. Thus, in any case, the effective medium approach, though it simplifies the analysis, leads to less-than-optimal designs. Incidentally, the same can be said about comparisons between the DNG
substrate and the EBG substrate, the latter of which is often placed below a planar antenna to enhance its bandwidth and suppress the propagation of surface waves along the substrate: no clear advantage has been shown to accrue from replacing an EBG slab with a DNG slab.

It has been argued that a small antenna encased in a spherical ENG shell can provide high directivity, even exceeding 50 dBi, possibly by compensating for the positive reactive energy of the antenna with the negative energy of the shell. We might think that we can overcome the limitations of the small antenna by covering it with an ENG medium, but this would be incorrect. This is because the effective medium approach is based on the reflection and transmission characteristics of a slab when illuminated by a normally incident plane wave; it cannot correctly predict the response of an ENG shell equally well for all angles of incidence. Furthermore, even if we rigorously analyze the shell configuration by computing the electric field inside the shell region, and then use it to evaluate the energy, we will find that the stored electric energy is positive, which contradicts the prediction of effective medium theory. The reason is that the true $\varepsilon$ of the medium inside the shell, in which the wires are embedded, is obviously positive; hence, the reactive energy must be positive everywhere in the shell region. Thus, the small antenna enclosed by a shell would achieve neither the desired matching nor the directivity enhancement that we expected the ENG metamaterial shell to deliver based on the predictions from the effective medium approach.

Recently, the concept of metamaterial cloaking to produce invisibility has attracted a great deal of attention from the scientific community. The objective of cloaking is to coat a target with metamaterials such that it becomes invisible to the interrogating wave. The perfect cloak ensures that for any incident field, the electromagnetic scattered field vanishes in the free-space external to the cloaking shell, as though the object was totally transparent to the incident wave. However, the main roadblocks in realizing a cloak for practical radar and antenna application is
that the cloak is inherently narrowband, and it only works for a normal case and for a single polarization of the incident wave.

The use of the effective medium parameters, through several examples described above, often oversimplifies the EM properties of metamaterials, so much so that these parameters may fail to accurately predict the true performance of the antenna–metamaterial composites designed for real-world applications. Therefore, this observation leads us to conclude that although simplified theories may be useful for initial design of MTMs, only rigorous simulations can predict their true characteristics accurately [71].

4.4 Conclusions

In this chapter, we first analyzed the EM response of an infinite doubly periodic DNG slab consisting of arrays of a combination of split-rings and wires, holey dielectric plates, and cross-wires and fishnets, by using the PBC/FDTD technique. Next, we examined the concept of effective material parameters by completing the retrieval process of these parameters according to one of the most widely used techniques, namely, the inversion approach. However, since this method is inherently based on a curve-fitting type of procedure, we have identified some fundamental problem areas encountered when applying the algorithm to a slab of artificial dielectrics. These are;

(i) The effective parameters are not always stable functions of frequency.

(ii) Whenever either |S_{11}| or |S_{21}| is small, the parameters are very sensitive to the inaccuracies (noise) in the S-parameters.

(iii) The parameters do not always remain unchanged as we change the thickness of the slab using additional layers.

(ii) The parameters change with the angle of incidence of the interrogating field.
(iii) No physically acceptable solutions exist in certain frequency bands.

The rigorous simulation results of the physical structure using a localized Gaussian beam incident reveal a number of important discrepancies, when compared with those predicted by using the effective medium approach. In fact, they point to the need to model the real structure to obtain reliable results, which may elude us when we use simplified models. On the basis of these studies, we derived certain guidelines that help us understand not only the scope but also the limitations of the board and medium approaches to real-world applications of metamaterials. With this in mind, we have developed a 3-D parallel FDTD solver [72] capable of handling a large number of DoFs (upward of 10E+9). This general-purpose field solver is necessary, as such computing power is essential if we are to accurately predict the performance of many of the antenna–metamaterial composites, which the simplified models often cannot describe correctly.
Chapter 5

Conclusion and Future Work

This research study focused on three primary objectives, namely those of (i) developing a novel and systematic approach to enhancing the directivity of antennas covered by an EBG and MTM; (ii) describing two novel techniques—involving a combination of the Dipole Moment (DM) method and the Characteristic Basis Function Method (CBFM)—for modeling periodic structures such as EBGs and MTMs; and (iii) developing guidelines for using the Equivalent Medium Approach (EMA) to study metamaterial structures.

In Chapter 2, we presented a systematic approach for designing a high-directivity resonator antenna utilizing different types of superstrates: an FSS, a dielectric slab, and a planar DNG lens. This method is based on a receive-mode analysis, an examination of the field distribution inside the FP-type cavity formed by a superstrate above and a ground plane underneath. We found that the antenna–superstrate composite achieves maximum directivity at the frequency where field intensity is highest inside the cavity, which consists of the superstrate and the ground plane, operating in the receive-mode. We also observed that although a DNG superstrate offers some improvement to an MPA, this enhancement is relatively moderate in comparison with the improvement that an EBG superstrate is capable of effecting. This is because the focusing effect of the superstrate does not contribute to directivity enhancement; instead, the enhancement is the work of the resonance phenomenon in the FPR created by the highly reflecting superstrate placed above the ground plane. We presented an FP-type design, which involves a microstrip array covered by a dielectric slab as the superstrate. We also showed that the directivity of the array can be enhanced by an additional 2 dB by using a two-layer superstrate.
In Chapter 3, we developed the Dipole Moment (DM) approach and the Characteristic Basis Function Method (CBFM) for the purpose of analyzing periodic structures. The resulting technique, based on a combination of two methods, is both memory- and time-efficient, and is applicable to arbitrarily shaped elements, whose material properties can also be arbitrary. First, we described a novel approach, based on using the dipole-moment (DM) type of basis functions in the context of the Method of Moments (MoM), which is not only totally free of the singularity problem, but is valid, universally, over the entire frequency range, including the quasi-static regime. Next, we showed how the DM and CBFM approaches can be combined to improve the computational efficiency of the method and to further significantly reduce the matrix size to be solved. We also showed that the electric fields generated by these high-level macro-basis functions can be expressed in closed forms without using Green’s functions. And, we included several numerical examples to illustrate the computational efficiency as well as versatility of the proposed method.

In Chapter 4, we examined the concept of effective medium parameters by completing the retrieval process of these parameters according to one of the most widely used techniques, namely, the inversion approach. However, as a curve-fitting type of procedure is intrinsic to this method, we encountered some fundamental problem areas in applying the algorithm to a slab of artificial dielectrics. We closely examined the propagation of the Gaussian beams, both in the interior and exterior regions of the MTM slabs, in order to establish whether their performances as realized in the rigorous numerical simulations were in accordance with those predicted by the effective medium approach. We found that the effective medium approach, though it matches the external parameters such as the reflection and transmission coefficients, does not often predict the true response of the MTMs. Therefore, we concluded that although simplified theories may be useful for the initial design of MTMs, only rigorous simulations can predict their true characteristics accurately.
In regard to designing high-directivity antenna systems, we see an opportunity to further enhance the superstrate–antenna composite’s directivity by increasing its aperture efficiency. It should be pointed out that simply increasing the physical size of the superstrate does not increase its effective aperture, as the aperture distribution of an FP resonator, excited by an MPA, has a natural edge taper. Therefore, possibilities include employing a dielectric lens or graded-index dielectric slab to render the phase of field distribution above the superstrate to be close to uniform. Additionally, another suggestion would be replacing the ground plane underneath the antenna with an AMC realized with an EBG or MTM structure, thereby reducing the height of the antenna system.

The DM approach presented in Chapter 3 formulates a problem that involves only wire types of PEC elements. The formulation of the dielectric objects essentially follows the same line as the PEC objects do, with two exceptions. These are: (i) The dielectric object is replaced by an array of electrically small spheres, whose electromagnetic behavior can be represented by three independent dipole moments, one for each principal direction; and (ii) The MoM matrix is generated by matching the polarization currents at the center of each sphere. This method applies equally well to dielectric objects in an accurate and numerically efficient way. However, further work is required to demonstrate that the dipole moment concept can be extended to handle elements that combine PEC and dielectric bodies, and, therefore, have arbitrary geometrical shapes and material properties. In addition, the DM approach has been (i) hybridized with the FDTD approach in order to simulate several practical antennas, e.g., dipole, monopole, and helix antennas, involving objects with fine features [36]; and (ii) applied to modeling metamaterials by our colleagues at Penn State’s Electromagnetic Communication Laboratory (ECL).

Metamaterials generally have shortcomings, specifically high losses, narrow bandwidth, and anisotropic behavior. And, though, none of these are included in idealized descriptions of these materials, it would be worthwhile to establish which, if any, metamaterials either do not
display these characteristics or can be applied in such a way as to overcome them. Perhaps developing a DNG structure by using dielectric inclusions in a background dielectric medium would be a good candidate. It would be interesting to follow the procedures described in Chapter 4 to determine if the performances predicted by the effective medium approach are in accord with those actually realized through rigorous numerical simulations. Furthermore, an important goal of such further research would be to develop a systematic approach to synthesizing metamaterials with the specified electric and magnetic characteristics, whose parameters are dictated by the optimization algorithm.
Bibliography


[34] Li Yanfei, Mittra, R., Lu Guizhen, and Kyungho Yoo, “Realization of high directivity enhancement by using an array of microstrip patch antennas (MPAs) covered by a dielectric superstrate,” Antennas & Propagation Conference, pp. 493-495, 2009


[64] Mittra, R., “To Use or Not to Use the Effective Medium Approach for Designing Performance-Enhanced Small Antennas - That is the Question,” Antenna Technology: Small Antennas and Novel Metamaterials, iWAT, pp. 55-58, 2008


Appendix

Prony’s Method

In Chapter 3, we found it necessary to extrapolate the solution for the current distributions for the infinite doubly periodic case. Prony’s method is a powerful tool that estimates the current behavior of the array as a weighted linear combination of a damped complex exponential. The essence of Prony’s method is that it separates the problem into two parts: first, the exponents must be found, and then the coefficients likewise. The algorithm of derivation of Prony’s method is discussed in detail in [55]. We are assuming that the desired function \( f(x) \) can be written in the form:

\[
f(x) = A_0 e^{\alpha_0 x} + A_1 e^{\alpha_1 x} + \cdots + A_{k-1} e^{\alpha_{k-1} x}\quad (A.1)
\]

for unknown values of the coefficients \( A_j \), and exponents \( \alpha_i \ (i, j = 0, 1, \ldots, k-1) \). The data points are sampled at \( x = x_j \ (j = 1, 2, \ldots, n) \), which are equally spaced. And, we can choose \( x = j - 1 \) without loss of generality. Prony observed that each of the exponential terms

\[
e^{\alpha_i j} = (e^{\alpha_i})^j = \rho_i^j \quad i = 0, 1, \ldots, k - 1 \quad (A.2)
\]

satisfies some homogeneous \( k \)th-order linear difference equation

\[
y(j + k) + C_{k-1} y(j + k - 1) + C_{k-2} y(j + k - 2) + \cdots + C_0 y(j) = 0 \quad (A.3)
\]

with constant coefficients. The characteristic equation of this difference equation is
\[ \rho^k + C_{k-1}\rho^{k-1} + C_{k-2}\rho^{k-2} + \cdots + C_0 = 0 \quad (A.4) \]

and has the roots \( \rho_i \). Since each exponential term satisfies (A.3), then any linear combination of these terms will also satisfy (A.3). In particular, the original function of (A.1) satisfies (A.3), that is,

\[ f(j+k) + C_{k-1}f(j+k-1) + C_{k-2}f(j+k-2) + \cdots + C_0 f(j) = 0 \quad j = 1, 2, \ldots, n-k \quad (A.5) \]

We need \( n = 2k \) samples of data for obtaining the unique solution for the assumed form of the function, which has \( 2k \) unknowns. In this case, we have exactly \( k \) equations; therefore, we can determine that \( C_i (i = 0, 1, \ldots, k-1) \). If we are to find the \( C_i \) then the persymmetric determinant \( \Delta = \left| f(j+k) \right| \) cannot be zero. Once we know the \( C_i \), the roots \( \rho_i \) is determined by the equation (A.4), which in turn gives the exponents \( \alpha_i = \ln(\rho_i) \). Finally, we can use the first \( k \) of the defining equations of (A.1) to determine the coefficients \( A_j \).
Kyungho Yoo received his B.S. degree magna cum laude from the Department of Electrical and Computer Engineering at Hanyang University, Seoul, in 2002. Through college coursework, he learned basic scientific principles and discovered how to apply them to engineering problems. He dedicated himself to engineering, and his ability in this subject won him numerous scholarships and awards. As a graduate student at Seoul National University’s Department of Electrical Engineering, he concentrated on determining the small-signal equivalent circuit model and the noise parameters of HBTs and on developing a fully integrated LC–VCO to achieve low-phase noise specifications with very low power consumption using the extracted characteristics of the devices. Having completed his M.S. degree in 2004, he began his doctoral studies in electrical engineering at the Pennsylvania State University in 2005, where he held a research assistantship at the Electromagnetic Communication Laboratory under the direction of his advisor and mentor, Professor Raj Mittra. His research interests are in the area of EBG–antenna and MTM–antenna composite design and modeling, and computational electromagnetics with emphasis on DM–CBFM techniques.