GRANULAR MATERIALS UNDER VIBRATION AND THERMAL CYCLES

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by

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ABSTRACT

We report flow rate measurement of granular materials from a lab size silo with and without sinusoidal vibration, and the flows from a jammed container under mechanical shocks. We also report the investigation of fragility in granular materials using controlled cyclic temperature variation, or thermal cycling that induces microscopic changes in the size of the grains and the container.

When placed under sinusoidal vibration, the flow rate or flux from an unjammed container decreases with the peak velocity of the vibration, and becomes a constant at the highest peak velocities. The flux under vibration follows a 5/2 power scaling rule to corrected orifice diameter, the same scaling rule that is also observed in the absence of vibration. Under vibration, granular flux is no greater than the flux without vibration. Density dilution of granular packs under vibration is likely the cause for such reduced flux, and can be described by a model based on energy balance at the vibrating boundary. The eventual saturation of flux at the highest peak velocities signifies a possible transition from granular fluid to granular gas, as the density decreases and inter-grain interaction changes.

Brief flows can be initiated from a jammed container using mechanical impacts. The number of grains flowing out of the container as well as the duration of these flows follows an almost exponential decay distribution. The probability that a flow can be initiated by an impact increases with impact intensity and ratio the diameters of the orifice and the grain. The possible container size and filling depth dependence are also discussed.
For the thermal cycling measurement, data show that the packing fraction of granular samples increases under thermal cycles regardless of the relative thermal expansions of the grains or the container. A heavy intruder, when passing a density threshold, sinks in a granular pile under thermal cycles. The results show that the bulk property of granular materials can be impacted by microscopic changes orders or magnitudes less, and without the input of mechanical energy. We believe both the packing fraction relaxation and the intruder displacement in thermally cycled granular systems demonstrate the fragility of disordered granular media, which can be defined as the “inability to elastically support some infinitesimal loads”.
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Chapter 1
Introduction to Granular Media

1.1 An Overview of the thesis

Granular material can be defined as a conglomeration of discrete solid particles with typical sizes larger than 100 μm. The interactions between the dry grains are usually only repulsive elastic forces and frictional forces. When liquids are involved, there are also attractive interactions between the grains. In this dissertation, we limit our discussion to only dry granular materials.

Granular materials are ubiquitous, from the rings of Saturn, to the sand castles on the beach, and to the salt and pepper on the dining table. In fact granular media are among the mostly used materials in human activities, second only to water. Many industries such as agriculture, food, construction, and pharmaceuticals utilize granular materials on a large scale. From the physics point of view, granular media are complex systems governed by simple interactions. This makes them ideal testing grounds for novel theories as well as a field of exciting discoveries. [1][2]

This dissertation is focused on granular materials under physical agitation including mechanical vibration and temperature changes. This chapter gives an overview of granular media and previous research relevant to the work in this dissertation. Chapter
Two describes the experimental techniques used in the study of granular flows under sinusoidal vibration, and presents our data and conclusions. Chapter Three and Four follow a similar structure with a discussion of granular flows under mechanical shocks and granular fragility, which can be defined as the “inability to elastically support some infinitesimal loads”, under thermal cycling, respectively.

1.2 Granular materials - science and applications

Granular materials can exhibit properties similar to an ordinary solid, liquid or gas. Granular media can support a heavy weight with minimal deformation, just like a solid [3]. When sufficient shearing is applied, granular media start to flow like a liquid. Granular materials conform to the shape of the container when filled, also like a liquid. When sufficiently disturbed, grains start to fly, colliding with each other and container walls, like gas molecules.

Curiously, however, established theories of solid, liquid or gas state of matter cannot well describe granular systems [4]. Grains in a granular pile are not positioned in repeated lattices, and there are no attractive forces between dry grains as in molecules in solid materials. The closest analog of static granular materials in solid state physics is probably glass, in which molecules are frozen into a random configuration.[5] Granular media cannot be treated as conventional fluids either. When a granular pile is tilted
beyond a certain angle, grains start to flow down the pile surface, creating an avalanche.

Such flows are different from conventional fluid flows, as only the few layers of 

Figure 1-1

grains near the surface actually move, while the bulk of the grains remain unaffected, as 
shown in Fig. 1-1 [1][6]. Even the simplest problem such as bottom pressure in a bucket 
full of grains often defies conventional hydraulic analysis. The pressure at the bottom of a 

Figure 1-1: Granular flows down a slope. Only the few layers close to the surface actually move. (This figure is reproduced from Reference [1], copyright 1996 by the American Physical Society)

grain silo is often less than the ratio between the total mass of the grains and the area of 
the silo bottom. A detailed analysis shows that it is the friction from the wall that is 
taking a great portion of the grain weight from the silo bottom. Pressure accumulates
when the pile goes deeper, and the frictional force from the wall increase accordingly. At very large depths, the pressure at the bottom saturates when the wall friction counters any increased depth of the grains. This is known as Janssen effect. [7][8][9] More interestingly, the pressure at the bottom of a sand cone has a minimum right below the tip of the cone, where the depth of the sand is the greatest. [10][11][12] Granular gases are also different from ordinary gases. A granular medium is a dissipative system in which all the collisions between granular grains are inelastic. The dissipation is greatly enhanced by the large number of grains and the resulting high collision rate.[13] One example is that a single glass ball can rebound several times before coming to a rest when dropped, while a whole bag of the same glass balls can hardly rebound even once. The dissipation in granular gas is so great that granular gas freezes almost immediately when external energy source is turned off. A granular “temperature” can be defined from the kinetic energy of the grains as $T \propto \langle mv^2 \rangle$, where $m$ is the mass of individual grains, and $v$ is the velocity of the grains. Different from ordinary gases, the kinetic energy is not evenly distributed when two types of grains are mixed [14], and the velocity distribution of granular gases does not follow the Gaussian distribution that one would find in conventional gases [15][16][17].

On the application side, granular materials are widely used in human activities. In the agriculture, the soil in which crops grow, fertilizers, and the harvested grains are all in granular forms. Many solid fertilizers are mixtures of different particles. Uniform mixing of these particles is important for the fertilizer to function best, as segregation of these
particles can occur at vibration or shearing during the manufacturing or transportation. Farmers often use outdoor silos to store grains. These silos and the grains within are under constant diurnal temperature variation, i.e. they are heated by the sun during the day, and cool down during the night. The silo wall and the grains respond to the temperature variation by expanding and contacting. The grains inside the silo become more compacted during the process, and gradually build up pressure at the silo wall[18][19][21][20]. For silos that are not frequently used, such pressure build-up can pose a potential danger to the structure of the silo, in some cases even leading to the collapse of the storage silo [22].

Other industries such as construction, mining, and pharmaceutical industry also handle granular materials on a large scale. Concrete, for example, is essentially a mixture of gravels and cement. High strength concretes are needed to construct high-rise buildings or large projects like a hydraulic dam. This is realized by mixing gravels and powders of different sizes to fill the empty volume between large gravels. Many empirical standards have been established to evaluate concretes in applications, but little is known about the physics behind this important construction material. In the mining industry, transportation of ore or coal often utilizes hoppers, and jamming is a constant problem in hopper operations. This can be especially disruptive when jamming occurs in hoppers that handle large quantities of materials. Despite its regular occurrence and significance to economic activities, the physics of jamming of granular flows is still poorly understood.
1.3 Granular Fragility

The simplest example of granular materials is a cone shaped pile of sand. Each grain in the pile is in a static equilibrium state as a result of forces from contacting grains, and no obvious ordering of the arrangement of the grains can be found. The surface of a sand cone has a consistent and well define slope. This slope cannot be at any arbitrary angle. There is a maximum angle known as the “angle of repose” at which the pile can remain at rest without causing an avalanche. When undisturbed, the pile can remain in the initial state for a long period of time. The system is “jammed”, which is defined as being unable to explore phase space spontaneously [24]. But such stability is fragile, when slightly tilted the cone surface will start to flow until the angle of the surface reaches the angle of repose again [23]. When vibrated, the cone quickly flattens out like a drop of water [25].

Even though the interaction between individual grains are relatively easy to investigate, an analytical description of a static sand pile remains daunting, mainly due to the large number of grains involved. Each grain must be individually balanced, and there is no long range order to the grain arrangement to help simplify the calculation. The problem is further complicated by the frictional force between the grains that can vary from 0 to $F\mu_s$ and at two opposite directions, where the $F$ is the normal force and $\mu_s$ is the coefficient of static friction. Thus mechanically, there are numerous different configurations in which a static pile can form [26]. Classical statistical mechanics that deals with systems with a large number of possible configurations has difficulty
describing a granular system because a static granular system does not explore those different possible configurations even though it is sometimes energetically favorable to do so. One example is shown in Fig.1-2; a vacancy can exist in a cannon ball stack due to frictions between the cannon balls. Even though it is energetically favorable to fill the vacancy with a cannon ball above it, this will not happen unless the stack is sufficiently disturbed.

Figure 1-2

![Vacancy in a cannon ball stack. The stack is not able to fill the vacancy spontaneously, even if it would be energetically favorable to do so.](image)

A mean-field approach such as continuum media theory of granular materials is also problematic, despite the seemingly complete randomness of the configurations. Granular systems maintain a high level of inhomogeneity and are history-sensitive [27]. This is best demonstrated in the distribution of stress in a granular pack when an external load is applied. Fig. 1-3 shows the propagation of the stress in a two dimensional granular system. It can be clearly seen that the external load is not evenly distributed to the disks.
under it. The weight is carried by a small fraction of the disks while the majority of the disks are not affected. Those weight-carrying disks form extending lines commonly known as “force chains” [28]. Another example that differentiates the granular media from traditional continuous media is the transmission of sound. Sound transmission in granular media is highly sensitive to local configurations. Sound transmits from one point to another through the contacts between the two points and can take different paths [29]. These paths are susceptible to tiny perturbations. There are reports that a 1 degree Celsius change of temperature in a single glass bead can substantially change the sound transmission in granular media [30].

Figure 1-3

![Force chains in a two dimensional system, as shown in photoelastic image of force distributions in a pile of disks. Bright region indicates large local force, blue region indicates weak local force. a. force distribution of a 2-D pile without external loading, b. force distribution of a 2-D pile with external loading. (This figure is reproduced from Reference [93], copyright 2001 by the American Physical Society)](image)

Inhomogeneity leads to fragility in granular materials. Granular fragility can be defined as the inability to “withstand some infinitesimal external loading” [31]. Take the force chains for example; a small change in a contact in the force network can propagate through the whole network. The loss of a contact in the force chain can lead to the
imbalance of neighboring grains, and cause local reorganization, which can lead to the change of equilibrium state of grains further away \[32\][33][34]. Similar fragility can be found in large electricity grid. The failure of a few major hubs in the grid can lead to the overloading of connected sub-stations, and cause a black-out in a large area. [35]. Granular fragility seems just the opposite to the jamming of granular materials. Jamming enables the granular system to resist certain external loading such as stress. The interlocking of grains makes it very difficult to compress granular systems [3]. But a jammed granular system is susceptible to some perturbations, such as vibration or deformation of grains. The study of granular fragility triggered by microscopic deformation under control temperature changes is discussed in Chapter 4.

1.4 Granular Material under Vibration

Governed by Newtonian mechanics, a granular system can remain in its settled state until disturbed. Vibration is one of the most commonly used ways to disturb a granular system. In industry for example, vibration hoppers and conveyer belts are used to handle or transport granular material. In scientific research, mechanical vibration provides a tool to perturb otherwise unresponsive granular systems.

Vibration can change the packing fraction of granular media [36][37]. Packing fraction is defined as the fraction of volume occupied by the grains rather than by empty space. Due to the jamming in granular systems, the configuration of grains does not
always minimize the total energy of the system. This is reflected in the relatively low packing fraction. For monodisperse spheres the highest possible packing fraction is 74% with face center cubic (FCC) or hexagonal close packing (HCP) lattices. Granular systems are rarely arranged in an ordered lattice, so the packing fractions are much lower than that of the FCC structure. For randomly packed spheres, the packing fraction can vary from 57-64% [38]. Random packing fraction higher than 64% is possible for grains with greater aspect ratio, for example ellipsoids [39]. Under vibration, the packing fraction of a loosely packed granular medium increases with the number of vibrations applied, and eventually saturate at the close random packing limit of 64% [36]. The mechanical energy injected by external vibrations briefly frees the grains from the confinement of neighboring grains and allows the system to explore a small window of possible configurations. Under some conditions, this process is also reversible, i.e., vibration can also decrease the packing fraction of a highly packed granular sample [40].

When mixtures of different grains are involved, vibration often leads to segregation in mixed granular media [41][42][43]. Segregation or demixing can occur based on different size, density or mechanical properties [41] [44] [45] [46]. One example is that a large intruder tends to rise toward to the top of a granular pile when vibrated. This is also known as the “Brazil nut effect” – referring to the phenomenon that large Brazil nuts often appear at the top of a mixture of nuts. From a physics point of view, this may be counter-intuitive as the system ends up in a state with higher energy when given a chance to settle. Under some conditions, the reverse can also happen, i.e., a large intruder can sink in a shaken granular medium [42][47][48]. Another kind of demixing of grains can
occur between grains of different density. Under vibration or shearing, grains of similar density will gradually gather together and the system stratifies [44]. This is also very puzzling as the system evolves into a more orderly state and therefore with lower entropy. In industrial applications, segregation of granular materials is used in the selection of minerals. It also poses a great challenge to processes that require thorough mixing of granulated ingredients, e. g., preparing batches for pills of medicine in pharmaceutical industry.

Percolation and convection have been proposed as possible mechanisms of granular segregation. [48][49][50][51]. Percolation theory suggest that small grains falls through the gap between large grains when vibrated and push the larger grains to the surface. For example, sesame seeds can easily fall through a pile of soy beans. The convection theory suggests that the intruders in vibrated granular media follow the flow patterns of the grains in which they are buried [51]. Under vibration, grains near the container wall tend to move downward due to the friction from the wall, while grains closer to the center tend to move upward [52][53][54][55]. Buried intruders follow the center flow to the surface, and remain on top as the sheer size of the intruders prevents them from following the thin layer of downward flow near the container wall. This can be clearly observed in Fig. 1-4.

Figure 1-4
An example of granular convection is given the Ref [56]. A French automobile company planned to use sacrificial polystyrene model in the casting of engine modules. The planned method is illustrated in Fig. 1-5. A polystyrene model is buried in a box of casting sand. The box is then vibrated to compact the sand. Molten metal is then poured into the casting box; the polystyrene model evaporates instantly, leaving only the metal to form the engine module. In real operation, however, this method resulted in large number of slightly deformed rejects. Further investigation revealed that it is the convection of the casting sand during the vibration of the box that bends the plastic model beyond the tolerance of the design.

Figure 1-5

Figure 1-4: Convection roll and intruder displacement in vibrated granular medium. Black dots are tracer particles. The intruder rides on the upward convection flow to the surface, the flow near the container wall moves downward (Figures are reproduced from Reference [50], copyright 1993 by the American Physical Society)
Granular medium enters a stable, but non-equilibrium state when placed under continuous vibrations. Vibrating of thin layers of granular particles can result in pattern formation. The patterns (striation, squares and hexagons) evolves with the peak acceleration of the vibration, and the wave length changes with vibrating frequency, as shown in Fig. 1-6. The packing fraction or the bulk density of granular media decreases as a result of constant influx of mechanical energy. Grains start to move relative to each other when the peak acceleration of the vibration is slightly higher than 1 g, where g is the gravitational acceleration (9.8 m/s²). This is known as the “fluidization” of granular materials. Fluidization of soil can be very damaging during earthquakes. Most of the building collapse during earthquakes is due to the fluidization of soil at the foundations.

Figure 1-6: Convection of casting deforms the model of engine module. Molten metal is poured into polystyrene model to form engine modules, the convection of the casting sand during vibration deforms the model, and resulted in rejected parts. (Figures are reproduced from Reference [55], copyright 1999 by Spinger, New York)
When the vibration intensity is low, the grains move while still in contact with neighboring grains, and elastic force and frictional force are the main interaction between the grains. Relative motion of grains dilutes the bulk density of the system, even when the grains are still in contact with another. This is known as “Reynolds’ dilantancy principle”[60]. An example is demonstrated in the diagram in Fig. 1-7. In a simple two dimensional model, when one row of grains starts to move against the row below it, the top row must first rise to overcome the curvature of the grains from the other row, thus
decreases the bulk density of the sample. As the intensity of vibration increases the grains gain enough energy to escape the confinement of neighboring grains and start to fly, interacting with other grains through inelastic collisions. The granular pack enters a state similar to that of a molecular gas, only that the collisions in granular gases are highly inelastic, with much lower density than the static grains.

Figure 1-7

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Figure 1-7: Raynolds’ dilantancy. To move relative to the lower row, the upper row must first rise to overcome the curvature of the lower row, thus diluting the bulk density.
2.1 Experimental Techniques

The study of granular flows under sinusoidal vibration, or salt shaker experiment, investigates the discharge rate of granular materials from a lab-size silo with a hole at the center of the silo bottom under controlled sinusoidal vibrations. Granular flux from a container has analogs both in the ordinary act of shaking salt from a salt shaker and also in the important statistical mechanics problem of fluid flow through a hole. The vibration was provided by a commercially available electro-magnetic shaker VTS-150 manufactured by Vibrational Test Systems. The mass of the discharged beads were measured by a digital scale (Schimadzu, BW4200H) and a force sensor (Cooper Instrument, LFS-210). Glass beads (Jaygo Inc.) and regular sand was used in the experiment.

The experimental setup is illustrated in Fig. 2-1. The VTS shaker is mounted on a 46 by 46 cm square aluminum plate with a thickness of 1.27 cm. Rubber pads are placed between the aluminum plate and the floor to cushion the vibration during the experiment. A head expander was mounted on the shaker through four screws. The expander is threaded on the top surface in a well planned pattern. This allows the connection between the shaker and different upper structures. The vessel from which beads flow out during
the experiment consists of an aluminum cylinder and a bottom plate with patterned holes. The inner diameter of the cylinder is 15 cm with a wall thickness of 2 mm. A circular edge of 3.8 cm wide is welded at the lower end of the cylinder. Multiple threaded holes were bored on the edge for connection to the bottom plates, the expander and control sensors. The bottom plates are circular aluminum plates of 0.32 cm thick and 22 cm in diameter. Circular holes of different diameters and patterns are opened on the plate so that the beads can flow through during the experiment. The vessel is not directly mounted on the expander in order to allow a free flow of beads during the experiment. Instead, four aluminum rods with threads at both ends (1.27 cm diameter, 15 cm long) are used to connect the vessel to the expander. A collector is suspended in the space between the bottom of the vessel and the expander to collect the beads flowing out of the vessel. The collector is isolated from vibration for accurate measuring of grain mass. Bubble wrap is placed at the bottom of the collector to reduce rebounding of beads. A digital scale and a force sensor are connected to the collector to measure the mass of exiting beads.

Figure 2-1
Figure 2-1: a. Experimental setup of the shaker experiment. The shaker applies vibration to the vessel, the grains flowing into the collector suspended below the vessel. b. Shaker control unit. The shaker utilizes feed-back control from the accelerometers on the vessel. The signal is analyzed by the control module. Corrected signals are amplified to drive the shaker.
The control unit is shown in Fig. 2-1b. The shaker consists of a field coil and an armature coil. The field coil provides a magnetic field in which the armature coil can move. The direction and the magnitude of the force exerted on the vessel are determined by the direction and magnitude of the current in the armature coil. During the experiment, the shaker works on a feedback-control mode. Two or three accelerometers (Dytran, 3030B4) are mounted on the rim of the vessel. The signals from these accelerometers are sent to a control module and compared to the pre-set vibration parameters. The control module then generates adjusted signals to the shaker. The signals from the control module are very weak and need to be amplified in order to drive the shaker. We use a CE2000 amplifier made by Crown International. The control module adjusts the acceleration of the vibration by varying the current in the armature coil. Since the current in the armature coil only affects the force the shaker generates, knowledge of the mass of the system being driven is needed for accurate control. This is obtained during the initialization process of the shaker. During the initialization, a burst of vibrations is applied to the system and the resultant accelerations are used to determine the mass of the system.

The controlling software of the shaker provides three different vibration modes – sinusoidal, random and shock vibrations. We use sinusoidal vibration in the study of this chapter. The software allows one to set up a sequence of vibrations in computer. The most used sequence in this study is a frequency sweep, in which the acceleration is kept constant while the frequency varies from one value to another. The shaker works stably at
each frequency for a few minutes when the data are acquired before move on to the next frequency.

We were concerned about the horizontal vibration of the vessel. Extreme care was taken to ensure that all the horizontal surfaces, including shaker base, head expander, connecting rods, and the vessel top, were well leveled so that there are not horizontal components from the shaker vibrations. The horizontal vibrations were experimentally measured using a tri-axial accelerometer (Dytran, 3053B2). A tri-axial accelerometer is capable of simultaneously measuring accelerations in three perpendicular directions. The results of the vibrations in horizontal plane and the corresponding vertical vibrations are plotted in Fig. 2-2. Acceleration in the horizontal plane is very small at high frequencies, but increases at lower shaker frequencies. This is probably due to the violent motion of the blocks of beads in the vessel during low frequency runs. The magnitude of horizontal acceleration is, however, always less than 1 g, where g is the gravitational acceleration, and can be considered insignificant to the affect the flow results [65][66].

Figure 2-2
Horizontal acceleration of the vessel at different frequencies. Empty vessel. Horizontal vibration is noticeable at low frequencies, but still less than 1 g.
2.2 Data acquisition

The digital scale and the force sensor measured the mass of the exiting beads landed in the collector. The digital scale was connected to a PC via RS-232 serial port. The data rate is 1 sample/second, and the resolution of the scale is 0.01 grams. The scale was zeroed before data acquisition so that only the mass of the beads in the collector was recorded. The force sensor was suspended below the scale and was able to measure the mass of the beads in the collector. The signal from the forces sensor was amplified by an in-line amplifier (Cooper Instruments, DCM490) before being fed into a data acquisition board (I/OTech, Daqboard2000). The data acquisition rate was 20 Hz with a conversion factor of 22.03 N/V. The scale data are digital, but at a lower sampling rate. The force sensor data are noisier, but at a much higher rate, which is useful to detect fast variation in the data.

During the experiment, the shaker started to vibrate with an empty vessel at a preset acceleration and frequency. The vessel was then filled with a pre-weighed amount of glass beads (600-3000 grams), and a granular flow followed immediately. The filling method was simply pouring beads into the vessel by hand. It took about 10 seconds for the beads in the vessel and the flow to stabilize. Waiting for this time erased any variation that could be introduced by the filling process. The digital scale and the force cell began to record the mass of the exiting beads in the collector when a stable flow was established. In a frequency sweep run, during which the peak accelerations of the vibrations were kept
constant, data were acquired at each frequency for 120 second before moving to the next frequency. Data at each frequency were saved into individual files for data analysis.

The mass of the beads in the vessel was constantly monitored, and refills were performed when it fell below 80 % of the initial value. There were two reasons to refill the vessel during the experiment. One was to keep the experimental condition consistent between runs. The flow rate can be significantly affected when the mass of the beads in the bucket is too low as will be examined later in this Chapter. The other was to maintain the accurate control of the shaker. As mentioned before, the shaker works on a feedback control that depends on a constant mass of the system being driven, the total mass of the structure including head expander, vessel etc. is 5.3 kg. Large deviations of the vessel mass reduced the quality of vibration or even stop the shaker if the software was not able to keep the parameters within an acceptable range. To minimize the effect on the data quality, refills were usually performed during the transition between frequencies.

2.3 Data analysis

The primary quantity of interest is the discharge rate or flux of granular media from a vibrating vessel. The flux was obtained by calculating the time derivative of the data collected during the measurement. The flux from the scale data was calculated directly using a 3-point derivative algorithm. The data from the force sensor were smoothed by averaging neighboring 10 points before the derivative can be calculated.
The flux was a constant at a given frequency and peak acceleration, and the mass recorded by the scale and the force sensor agree with each other very well as shown in Fig. 2-3. No fine structure of the flux such as phase dependence of the vibration is revealed in the force sensor data. We conclude, after detailed comparison of the result from the two instruments, that only one data source is needed to characterize the granular flux in this study. The data from the digital scale were chosen because of the relatively low volume and convenience to analyze by a personal computer. Unless otherwise specified, the flux data presented in this chapter were acquired using the digital scale. The flux from an individual measurement is the average of the calculated flux values during the 120 seconds measurements.

Figure 2-3
2.4 Experimental Results

We first investigated the granular flux from the cylindrical vessel in the absence of vibration, termed “DC flux”. The bottom plate used a single center hole pattern. DC flux increases rapidly with the diameter of orifice, and is independent of the pile depth. The results are plotted in Fig. 2.4. The DC flux results agree with the Beverloo equation of granular fluxes from a static container [67].

Eq. 2.1
Where $C$ is a dimensionless constant, $\rho_b$ is the bulk density of the grains in the container, $g$ is the gravitational acceleration, $D$ is the diameter of the exit orifice, $k$ is also a dimensionless constant, and $d$ is the diameter of the grains. When the diameter ratio between the hole and the bead is less than 3, DC flux cannot be obtained due to the jamming near the orifice.

Figure 2-4

Granular flows from a static container have been extensively studied in the last century, mainly in the engineering community[67][68][69][70][71][72], and Beverloo
equation is one of the most used. The Beverloo equation can be understood from simple dimensional analysis. Granular flux from a container is the result of continuous collapse of granular arches near the orifice. So unlike the exit velocity of fluids which increases with the depth of the fluid, the exit velocity of granular flow depends on the dimension of the arch from which the grains fall. The dimension of the collapse region can be characterized by the size of the opening. A simple picture is that the beads break and then experience free fall from a dome with a height proportional to the hole diameter $D$ as shown in Fig. 2-5. The mass flow of granular materials from a static container are associated with the bulk density near the orifice $\rho_B$, exit velocity of the beads $((Dg)^{1/2})$, and the cross-section area of the exit ($D^2$). The constant $k$ is a correction factor to account for the jamming of flows at a non-zero hole diameter.

Figure 2-5
Granular flux under vibration, which we term AC flux, was investigated for a wide range of parameters, including peak acceleration, frequency, hole diameter, and grain diameter. Most of the data presented in this thesis were acquired using the simplest pattern, which consists of a single hole located at the center of the vessel bottom. Other patterns were also explored and will be discussed later in the section. AC flux is acquired during discrete sweeps of frequency steps, in which the peak acceleration is kept constant while the vibration frequency moves from one value to the next. Parameter space includes peak acceleration, vibration frequency, hole diameter, and grain diameter. An initial mass of 600 - 3000 grams was used and the mass of the beads in the vessel is kept above 80% of the initial weight. Greater initial mass is used for holes of greater diameter.

Figure 2-5: Granular flow from a static container: Beverloo equation. Grains experience fall from a dome with a height of $H$, which is proportional to the diameter of the orifice $D$. The exit velocity is then proportional to $\sqrt{gD}$. 

Granular flux under vibration, which we term AC flux, was investigated for a wide range of parameters, including peak acceleration, frequency, hole diameter, and grain diameter. Most of the data presented in this thesis were acquired using the simplest pattern, which consists of a single hole located at the center of the vessel bottom. Other patterns were also explored and will be discussed later in the section. AC flux is acquired during discrete sweeps of frequency steps, in which the peak acceleration is kept constant while the vibration frequency moves from one value to the next. Parameter space includes peak acceleration, vibration frequency, hole diameter, and grain diameter. An initial mass of 600 - 3000 grams was used and the mass of the beads in the vessel is kept above 80% of the initial weight. Greater initial mass is used for holes of greater diameter.
for the practical purpose of continuous measurement without refill in a period of 120 second. The flux dependence of the filling depth will also be discussed later in the section.

A typical frequency sweep result is plotted in Fig. 2-5. At a given acceleration, the flux increases with vibration frequency and flattens out at the highest frequencies. Higher peak accelerations result in lower flux value at low frequencies, while flux values for different peak accelerations collapse at the highest frequencies. The same trend is observed in frequency sweeps of different hole diameters, and bead diameters. The flux is normalized by the DC flux.

Figure 2-6

---

Figure 2-6: Granular flux under frequency sweeps. The flux is normalized by the DC flux of 6.35 mm hole diameter, which is 4.95 gram/second. The flux increases with vibration frequency and saturates at DC flux.
The magnitude of the AC flux varies greatly for different hole diameters as shown in Fig. 2.6. When normalized by the DC flux of the corresponding hole diameters, the frequency sweep data from different hole diameters collapse, suggesting that AC flux follows the same scaling to the hole size as the DC flux does. Another interesting feature in the normalized AC flux data is that the flux under vibration is no greater than the DC flux, i.e., vibration does not facilitate the flow of grains. Flux under vibration is about 1/3 of the DC flux at the lowest frequencies, then increases with vibration frequency and saturates at DC flux at the highest frequencies. This is somewhat counter to the usage of a salt-shaker, in which more vibration is needed to move the salt grains out of the container. This is because the holes on a real salt-shaker are typically too small to allow the free flow of salt grains without vibration.

Figure 2-7
Figure 2-7: Hole size dependence of granular flux under vibration.  a. Granular flux under vibration from different hole diameters; peak acceleration 4g. Log scale is used in the y-axis for comparison.  b. Normalized flux of frequency sweeps from different hole diameter.
DC flux ceases to exist when the diameter ratio between the exit hole and the grains is less than 3. The AC flux, however, can still be measured for hole diameter down to 1.7 mm (D/d ratio at 1.9). The granular flux under vibration with an initially jammed configuration is plotted in Fig. 2-7. The flux still follows the rise-to-saturate trend of the AC flux from unjammed container, i.e. flux increases with vibration frequency and flattens out at the highest frequencies, but for the smallest hole diameter, large error bars make it very difficult to interpret the data. The large error bars are the result of the flux of exiting beads being very small, close to the time and mass resolution of the scale. The jam-breaking mechanism of granular flows is studied in Chapter 3.
Figure 2-8: AC flux from jammed vessels. 0.9 mm glass beads, a. AC flux from 2.51 mm hole diameter, b. AC flux from 1.7 mm hole diameter. DC flux are not available for these two hole diameters. The large fluctuation in plot b is due to the resolution limit of the digital scale.
The effect of hole patterns on the bottom plate was also investigated. We found that as long as the distance between the holes is large compared to the diameter of individual holes, the granular flux from multi-hole pattern is simply the flux from a single hole times the total number of holes on the plate. The mass of the grains remaining in the vessel can influence the granular flux under vibration when it becomes too low. Fig.2-8 shows the flux as a function of mass remained in the vessel. The data show that the flux is independent or only weakly dependent on the grain mass in the vessel until the mass is less than about 400 grams. Granular flux declines significantly when the grain mass is below the 400 gram level, down to zero as the beads are depleted in the bucket.

Figure 2-9

![Graph showing granular flux as a function of mass in the vessel. Initial grain mass 2000 grams, 0.9 mm glass beads, hole diameter 6.35 mm. The flux remains a constant until the mass in the vessels drops below 400 grams.]
The temperature during the experiment varied between 22 and 26 °C. All data were acquired with a relative humidity of 30±10%, and no significant effect of humidity was observed. Comparison of the flux results from fresh beads and beads that have experienced several vibration runs shows no significant differences.

2.5 Discussions

Intuitively, one might expect that granular flux under vibration to be a dynamic process that involves the velocity distribution of the grains; even phase dependence of the vibration. The collapse of the normalized flux from different hole diameters, however, shows that the same scaling rule applies to flux under vibration. As mentioned above, the 5/2 power scaling rule to the exit diameter describes the continuous collapse of the arch structure near the orifice. The validity of the scaling rule suggests that the vibration does not change this picture and the average exiting velocity of the grains. Our results also agree with a previous study on granular flow from a hopper [73]. It is curious that the geometrical scaling in the static case is still valid under vibration, since no stable arches can be formed under vibration. The arch, in a general sense, is the transition region from a high packing density to a low packing density near the orifice [74]. The continuous granular flow from a container is a process of dynamical jamming, i.e. the geometry of the grains and the exit prevents the grains from flowing out freely. Grains become less mobile as they approach the exit before falling out of the container. This primarily depends on the interaction between the grains near the exit, and the presence of vibration.
is not essential. By plotting the normalized flux data, we are able to isolate the contribution of the vibration to the granular flux, and seek the essential vibration parameter that controls the AC flux.

The frequency sweep data in Fig. 2-5 clearly show that neither the vibration frequency nor the peak acceleration is the only parameter that determines granular flux under vibration. The flux is plotted against different physical quantities including peak velocity and vibration amplitude in Fig. 2-9. When plotted as a function of the peak velocity of the vibration, data from different frequency sweeps collapse to a single curve. This suggests that the peak velocity is the essential parameter of granular flux under vibration. The peak velocity plot also reveals an interesting characteristic that is not shown in the frequency dependence plot, that the flux becomes flat after decreasing to about 1/3 of DC flux for a wide range of velocity values.

Figure 2-10
Knowing that the granular flux under vibration follows the same geometrical scaling rule as in the static case, and the peak velocity of the vibration is the essential dynamical parameter that controls the AC flux, we can start to formulate an equation that describes granular flux under vibration. We first write the equation in two terms, one is the geometrical term and the other is the vibration term.

\[ \Phi(v_{\text{max}}, D, d) = \Theta(v_{\text{max}})g^{\frac{1}{2}}(D - kd)^{\frac{3}{2}} \]

Where, \( v_{\text{max}} \) is the peak velocity of the vibration. Comparison with the Beverloo equation reveals that \( \Theta(v_{\text{max}}) \) should have the dimension of density and represent the density dilution of granular layers near the orifice. The effective gravity grains experience may also change during vibration, e.g. the grains experience higher gravitational acceleration when the plate is accelerating upward, and lower gravitational acceleration when the

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**Figure 2-10**: Normalized flux as function of vibration parameters. Left, granular flux as a function of vibration amplitude (A), Right, granular flux as a function of peak velocity (Aω). \( \Gamma \) is the normalized acceleration \( \Gamma = A\omega^2/g \)
plate is accelerating downward. But this variation of effective acceleration averages out during repeated cycles, so does the exit velocity [73]. When the granular pile is fluidized under vibration, the bulk density of the packing decreases [13][62]. This density dilution is not uniform in the pile. The density near the vibrating surface is lower than the density of the upper layers. The density near the free surface is almost the same as the bulk density at rest[61][62][75][76][77][78][79][80]. This dense block of granular beads floating on a low density region is also known as “granular Leidenfrost effect” [64] (The Leidenfrost effect is a phenomenon in which a water droplet floating on its own vapor when placed on a hot surface). The flux is determined only by the density in the vicinity of the orifice, or the density of the bottom layers.

To establish a quantitative relation between the density dilution and the peak velocity of the vibration, we consider the energy balance at the bottom plate. The vibrating plate provides a constant influx of mechanical energy that excites the grains in the container. Due to the highly dissipative nature of inter-grain collisions, and the confinement of the packing, only the grains in the bottom layers can gain enough energy to move significantly, resulting in the dilution of bulk density. Under stable conditions, an energy balance must be established at the boundary. We avoid using a model based on hydrodynamics theories used in many of the previous studies of density dilution because of the experimental fact that the flux from a vibration container is independent of filling depth for a wide range of grain mass in the container as shown in Fig. 2-8. Most hydrodynamics theories are pressure sensitive, and therefore depth dependent.[62][61][76]
The kinetic energy impacted by the vibrating plate is dictated by the peak velocity of the vibration. We use the gap between neighboring grains, $s$, to describe the density dilution of the grain pack, $s$ is 0 when there is no vibration. Certain amount of energy is needed for the grains to separate $s$. Without the knowledge of the specific of the relation between $s$ and the energy input from the vibration, we assume the simplest linear relation with a dimensionless parameter $\alpha$.

\[
\alpha_{msg} = m v_{\text{max}}^2 \tag{2.2}
\]

where $m$ is the mass of a grain, $g$ is the gravitational acceleration, and $v_{\text{max}}$ is the peak velocity of the vibrating plate. Eq. 2.2 describes the energy balance at the bottom plate, under vibration $v_{\text{max}}$, the grains expand a gas of $s$ determined by $\alpha$. A uniform vibration of the grain pack cannot be assumed. The vibration of grains far from the vibrating plate is difficult to assess as the interaction between grains is much more complicated than the interaction between the grains and the vibrating plate. It should be noted that such linear approach would require $s$ to be small compared to the dimension of the grains, as the interaction will change as the gap between grains increases. When $s$ is small we can ignore the geometric factors, and the bulk density near the bottom of the vessel can be written as

\[
\rho_B = \rho_0 (1 + \frac{s}{d})^{-3} \tag{2.3}
\]

Plug in to Eq. 2.1 we have the granular flux under vibration
validity of Eq. 2.4 is tested by fitting the equation to the normalized flux data as a function of peak velocity. With only one dimensionless parameter, the equation fits the data very well at low peak velocities. It well describes the sharpest transition of granular flux under vibration in experimental result. Similar fits were also obtained using beads of 2 mm and 3 mm diameters. The resultant $\alpha$ value is proportional to the bead diameter as shown in the inset of Fig. 2-10. This indicates that more energy is needed to move larger particles.

At high peak velocity, the fitted curve starts to deviate from the flux data. This is not surprising however since Eq. 2.4 is derived on the assumption that $s<<d$. At the point of where the fitted curve starts to deviate from the actual data the $s/d$ is about 0.4.

Figure 2-11
The constant flux at the highest peak velocities suggests that the bulk density near the bottom of the plate is independent of the energy input from the vibrating plate. We speculate that this indicates the beginning of a transition from granular fluid to granular gas at the bottom layers. When the bulk of the granular pile is fluidized, the beads start to move relative to each other. The gaps between the beads are small compared to the

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**Figure 2-11**: Comparison between experimental data and Eq. 2.4 (blue line). $\Gamma$ is the normalized acceleration $\Gamma = A\omega^2/g$. Inset: fitted parameter $\alpha$ at different bead diameters. $\alpha$ increases linearly with bead diameter (red line). Eq. 2.4 describes the sharp transition of granular flux under vibration, but deviates from the experimental data at the largest peak velocities, where the flux becomes a constant
diameter of the beads. Inter-particle friction is the main mechanism of energy dissipation. The energy loss is proportional to the relative displacement as described by equation 2.2. When the distance between particles further increases, beads near the bottom begin to behave like gas molecules. Inelastic collisions between particles become the main mechanism of the energy dissipation that depends on the collision rate and restitution coefficient of the grains. [81][82]. When the peak velocity of the bottom of the vessel increases, the granular temperature increases. This leads to higher collision rate and lower restitution coefficient [83][84]. Both the collision rate increase and restitution coefficient decrease will serve to balance the increased energy input and thus keep the granular gas density constant.

**2.6 Conclusions**

The main conclusions of our study are the following. Vibration does not facilitate granular flows from an unjammed container. The geometrical scaling of granular flux observed in static cases is found still valid under vibration. The data clearly show, over a wide range of parameters, that the peak velocity of the vibration applied is the essential parameter in granular flux under vibration. The low flow rate under high intensity vibrations confirms the density inversion predicted in previous theoretical and numerical studies. The constant flux at the highest peak velocity reflects a change of grain interactions in the diluted granular media.
3.1 Experimental Technique

Jamming of a hopper or a pipe is a very common phenomenon in industry and everyday life. Jamming is often due to the formation of arch structures near the orifice, and when the diameter ratio of the orifice and the beads is less than 3\[85][86][87], the container orifice will be jammed and spontaneous granular flows under gravity cease to occur. Mechanical energy input, such as impact from a heavy hammer or vibration, is needed to break the jamming and to initiate flow, which usually last for a brief period of time before another clog is formed near the orifice. Despite its frequent occurrence and economic significance, the understanding of jamming of granular flow and how to prevent or efficiently break jams is still poor. In this study, we employ a pneumatic cylinder to generate impacts to break the jamming in a vessel filled with glass beads, and investigate the jam-breaking mechanism and the statistical characteristics of granular flows from a jammed container. This study is also referred as “shock experiment” in this thesis.
The experimental setup of the shock experiment is illustrated in Fig. 3-1. Similar to the shaker experiment, we use a cylindrical aluminum vessel with replaceable bottom plates as the container of glass beads. The inner diameter of the vessel is 20 cm with the wall 0.64 cm thick. An accelerometer is mounted on the edge of the vessel to characterize the impacts. The bottom plates are made of 0.64 cm aluminum plate with a hole at the center. A counter-sink is bored on this center hole so that the beads do not jam once they exit the surface of the container bottom. The vessel is mounted on two linear bearings (VXB, KIT2063) to ensure one dimensional motion. The vessel is supported by a horizontal wooden board covered with thick foam to absorb the shocks generated by the vessel when it lands. To further reduce rebound of the vessel during impacts, an extension damper (AVM, P5906) is mounted on the vessel. The damper does not impede

Figure 3-1: Experimental setup of the shock experiment. Pneumatic cylinder pushes the hammer head to impact with the vessel. Exiting beads are detected by the capacitor. Left: cross-section view of vessel bottom plate, a countersink is used to prevent exiting beads from jamming.
the upward motion of the vessel, but damps out the downward motion so that the vessel lands slowly onto the supporting board. The hammer that generates the impacts is positioned below the vessel, aligned with the center hole. The hammer consists of a gas cylinder and a metal hammer head. The gas cylinder is made of stainless steel and with a piston diameter of 7.6 cm (Clippard, UDR-48-8). The piston separates the cylinder into two chambers. The motion of the piston is controlled by inflating one chamber with pressured gas while releasing the gas in the other chamber. To achieve maximum impact velocity, a quick-release valve is connected to the upper chamber of the cylinder to facilitate the upward motion of the piston. A head expander is mounted on the rod, and a hammer head made of aluminum or brass is mounted on the head expander. The hammer head is a hollow metal cylinder with an outer diameter of 7.6 cm and inner diameter of 5 cm. The length of the hammer head is 8.9 cm. The hammer head is connected to the head expander through eight 10-32 screws. The assembled hammer head is shown in Fig. 3-2 a. Two skewed holes are bored on the head expander to allow the beads to flow down to a digital scale through two plastic pipes. A thin rubber ring is mounted on the top surface of the hammer head to cushion the impact with the vessel bottom. Another accelerometer is mounted on the lower surface of the head expander to characterize the motion of the hammer head. Signals from both the vessel accelerometer and the hammer head accelerometer can be logged by a digital oscilloscope (HP, 54602B) and saved on a computer. Lead bricks are placed against the bottom of the gas cylinder to mitigate recoils during operation. The whole system is mounted on a metal frame, and placed on rubber sheets for shock absorption.
Exiting beads are detected by a parallel plate capacitor and a digital scale. The design of the capacitor is illustrated in Fig. 3.2b. The capacitor, also termed sensor, is made of a square PVC pipe with two pieces of copper foil positioned on the center of two inner walls facing each other. The pipe is 7.6 cm long with 2.54 cm at each side, and the foils are 2.29 cm wide and 5 cm long each. The thickness of the wall of the PVC pipe is 1.2 mm. The capacitance of the capacitor is 0.1077 pF, and may vary slightly from day to day. The capacitor is fixed to the bottom plate through a connection block so there is no relative motion between the vessel and the capacitor. The capacitor is carefully shielded from environmental noise. Two coaxial cables are connected to the foils in the pipe, and a high precision capacitor bridge (Andeen-Hagerling, AH2700A) measures the capacitance of the sensor during experiment. The results are sent to a PC via a data acquisition board (Keithley, KUSB-3100).

Figure 3-2

Figure 3-2: Hammer head assembly (left) and the capacitor (right). The hammer head assembly consists of head expander and a hollow metal rod, beads fall out of the vessel are directed to a scale through two skewed holes in the head expander. The capacitor is carefully shielded from environmental noise.
During the experiment, the vessel is filled to 12.7 cm deep (8000 grams) with glass bead with an average diameter of 2 mm. The vessel is then hit by the hammer for 50 to 100 times so that the packing fraction remains stable during actual data acquisition. After the working pressure and the timing of the cylinder is properly set, the control circuit is switched to automatic mode, in which a triggering signal of a selected duration is sent to the control valve at a constant interval (30 seconds to 2 minutes). The number of impacts is around 500 and is logged by a counter. The accelerations of the vessel and the hammer heads are captured by a digital oscilloscope and the results are saved on a computer.

An impact is generated when the hammer head rams against the vessel bottom. The motion of the hammer head is controlled by the working pressure and the inflation time of the cylinder. The gas circuit is shown in Fig. 3-3. When a 12 volt signal is generated from the control circuit (the diagram and the function of the control circuit is in the Appendices) the first stage solenoid valve (Clippard, MME-43WES) activates and generates a pressure difference in the second stage air-pilot valve (Clippard, MMA-43WAA), which then activates the second stage valve. This connects the lower chamber of the cylinder to the pressured gas tank, and pushes the piston upward. At the same time the upper chamber is connected to the atmosphere, and the gas in it is released. When the 12 volt signal is turned off, the first stage valve returns to original position and the pressure difference in the second stage valve reverses. The upper chamber of the cylinder is inflated with pressured gas, while the gas in the lower chamber of the cylinder is
released. The piston moves downward. Two valves are used in order to increase the range of impacts the hammer can deliver. The solenoid valve requires a minimum pressure of 20 psi, and can not generate impacts less than 2 g peak acceleration. The air-pilot valve can work at any working pressure, but can not be directly activated with electronic signals. Every time an activation signal is sent to the valves, a counter registers the signal and logs the total number of impacts.

Figure 3-3

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Figure 3-3: Gas circuit of the shock experiment. Red lines are high pressure gas lines (80 psi), blue lines are low pressure gas lines (<45 psi). Solenoid valve controls the air-pilot that inflates the cylinder.

The accelerometer mounted on the hammer head recorded the motion of the hammer. Figure 3-4 shows a profile of the motion of the hammer head during the
experiment. The hammer head experiences a sudden upward acceleration and rapidly reaches a terminal velocity long before the actual impact. The inflation time of the upper chamber is critical to the control of impacts. If the inflation time is too short, the hammer head can not reach the vessel. If the inflation time is too long, the hammer carries the bucket even after the impact and often result in multiple impacts. The inflation time is controlled by varying the value of a resistor in a RC circuit in the control circuit. An ideal impact occurs when the hammer head reaches the bottom of the vessel near the highest point of its motion, and then starts to move downward right after impact. This can be achieved by monitoring the acceleration of the vessel while adjusting the inflation time until a clean impact peak is obtained.

Figure 3-4
The intensity of the impacts, i.e. the peak acceleration, is controlled by the working pressure in the cylinder and the mass of the vessel. The working pressure is set by a regulator. The hammer head moves at higher velocity at higher working pressures and the resultant peak acceleration of the impact is also higher, as shown in Fig. 3-5a. A lighter vessel (with fewer beads) has greater peak acceleration than a heavier vessel (with more beads) under the same working pressure in the cylinder. Low vessel weight also leads to less consistent impacts as shown in Fig. 3-5b.

Figure 3-4: Motion of the hammer head. The accelerometer is at the bottom of the hammer head, a negative sign indicates upward acceleration. Different stages of motion of the hammer head can be clearly seen.
The system is able to generate consistent impacts when the working parameters are properly set. The impact intensity can be controlled by the working pressure in the cylinder. Some typical results are shown in Fig. 3-6. Repeatable and well-controlled impacts are essential to studying statistical characteristics of jammed granular flows.

Figure 3-6: Impact control. a. peak vessel acceleration as a function of cylinder working pressure; b. vessel acceleration with different vessel weight. Error bars are the standard deviations of individual measurements. The peak acceleration of the vessel is controlled by the working pressure in the cylinder and the vessel mass.
3.2 Data acquisition

Four streams of data were acquired during the shock experiment. They were the capacitance of the capacitor, the mass of the exiting grains from the digital scale, and the acceleration of the vessel and of the hammer head. The capacitance of the capacitor was measured by the high precision capacitance bridge. The result can be presented in either digital or analog format. The digital results were recorded via RS-232 serial port, and the analog results were acquired using a data acquisition board. The bridge worked at a frequency of 1000 Hz with a voltage of 15 volts. The sampling rate of the digital signal was 20 Hz, and the sampling rate at which the Daqboard can convert the analog signal can reach up to 10,000 Hz.

Figure 3-6: Impact consistency  a. acceleration of the vessel, over 550 impacts; b. the averaged curves of vessel acceleration at different peak accelerations. A small second peak can be seen for the impacts with the highest peak acceleration, this is due to the damping from the extension damper.
Digital output measures the capacitance of the sensor by balancing a series of capacitors. It takes certain amount of time to balance the bridge, and this limits the reaction time and the sampling rate of digital output. The digital capacitance results were used when the diameter ratio \( R \) \( (D_{\text{orifice}}/d_{\text{bead}}) \) was large. The number of grains flow out between two adjacent jamming events is large and the duration of these flows are also long compared to the sampling rate of the digital output. When the diameter ratio was small, only a few grains can pass through when a flow is triggered by the impact. The duration of the flows is also shorter than that from large diameter ratios. For example, the time it takes for a single bead to pass through the sensor area is a little less than 0.1 second. The digital output of the capacitance bridge, with a frequency of 20 Hz, cannot efficiently characterize such small flows. Analog output is needed to provide faster data rate. When the analog output was used, the bridge worked at “deviation mode”. Deviation mode is an off-balance measurement. The capacitance of the sensor is first estimated, and this estimation is set as a reference point. Any deviation from this reference value will generate a voltage signal that is proportional to the amount of deviation. This signal is the analog output of the capacitance bridge. Deviation mode and analog output are especially useful in this study since we concern more about the change of capacitance than the actual value of capacitance. When deviation mode was used, the reference point is set to be the capacitance of the sensor when no bead passes through. The range in which the capacitance changes can be estimated from trial runs, and set up in the bridge settings. The deviation mode is able to capture fast changing signals, but the noise level of the analog signal was much higher than that of the digital signal. We used the 0.1077 pF as
the reference point with a range of 0.008 pF. The conversion factor between the analog output and capacitance was 2500 V/pF.

The triggering signal from the control circuit was also logged by the Daqboard when the analog capacitance signal was being acquired. The beads exiting the vessel under impacts pass through the hammer head and the plastic pipes connected to the bottom of the hammer, and land in a container placed on top of the digital scale. The resolution of the scale is 0.01 grams and the sampling rate is about 1/sec. The results from the scale were also logged by a computer. The signals from the accelerometers were recorded by an oscilloscope, which was triggered by the activation signal from the control circuit. The result was sent to an Excel worksheet and saved at the end of the run.

3.3 Data analysis

Data analysis of the shock experiment includes flow characterization and impacts characterization. Flow characterization identifies individual flow events and quantifies the size and duration of each flow event. A slightly different approach was used in the flow characterization when the digital or analog signal from the capacitance bridge is analyzed. The impact characterization extracts information about the dynamic parameters of the impacts such as peak acceleration, impact duration and impact velocity.
3.3.1 Flow characterization

The digital capacitance signal was used in large diameter ratio ($D_{\text{orifice}}/d_{\text{bead}}$) runs. The duration of the flow is long and the data rate of the digital output was sufficient to capture the flows. The noise level of digital signal was very low. The beginning of a flow event is identified when the signal passes a preset threshold level above the baseline. The end of a flow event is identified when the signal returns to baseline and remains constant. The constant impact interval during the data acquisition is used to identify impacts that fail to initiate a flow. When no flow is found during a period of time more than 1.5 times the impact interval, a “no flow” or “zero flow” is also identified for the impact during that period of time. Once a flow event is found, the duration of the flow is calculated as the time difference between the beginning and the end of the flow event. The capacitance integral is numerically determined against the baseline (only the change of capacitance from the baseline value accounts for the passage of beads). For the scale data, a flow event consists of a sudden jump of the mass reading from the scale. “No flow” events can also be identified using similar algorithm as for the digital capacitance data. The flow duration results from the scale data are not reliable due to the low sampling rate and the delay in flow measurement by the scale.

The results from the digital capacitance and scale data are a set of flow or zero-flow events with the flow size and duration determined. The capacitance integral can not be analytically converted to the mass of the beads. We found that when the flow line-up from the capacitance bridge and the scale are plotted against each other, a good linear
relation can be seen for a wide range of magnitude as shown in Fig. 3-7. This provides a conversion factor to convert the capacitance integral to actual mass of the beads.

Figure 3-7

![Graph showing the correlation between mass of beads and capacitance integral. The red line is the linear fit between the scale data and the capacitance data, and the slope of the fit yields the conversion factor.]

The fast data rate of the analog output comes with increased noise level and the practical issue of large data volume. The noise level is comparable to the signal level of a single bead passing through the sensor, and a typical run of 500 impacts generates a raw data file from 60 megabytes to 300 megabytes, making it difficult for an ordinary PC to efficiently process. The raw data are first truncated to keep only 10 – 20 second of data
after the impact, since flows initiated by impacts only last a very short period of time.

This is realized by using the triggering signal in the same data file to mark the initiation of an impact and keeping 10-20 seconds worth of data after that point. Then the truncated data are smoothed by averaging neighboring data points. This decreases the noise, which tends to be at higher frequencies. The real signal from the glass beads will not be lost by the smoothing process because the duration of a flow event is much greater than the smoothing window. Fig 3-8 shows the comparison between the raw data and the smoothed data of a single bead passage.

Figure 3-8
Truncated and smoothed data are then analyzed by a computer program similar to the program used to analyze the digital signal described above. The conversion factor is generated differently than that in the digital signal processing. When the diameter ratio is small, the digital scale can not differentiate flows of a few beads from environmental fluctuations such as air movement, and correlation of flow events from the capacitance bridge and the scale are usually not attainable. In this case, the conversion factor is

Figure 3-8: Smoothing of analog capacitance signal from a single bead passage. 20-point average algorithm is used to smooth the signal
calculated as the ratio between the total capacitance integral and the total mass of the exiting beads during the whole run, which usually include hundreds of impacts.

### 3.3.2 Impact characterization

The accelerometers mounted on the vessel and the hammer head provide information on the motion of the two moving components in the experiment. The signals from accelerometers are in voltages which can be converted to actual acceleration by a conversion factor of 10 mV/g, where g is the gravitational acceleration (9.8 m/sec²).

The acceleration data are first aligned to examine the consistency of impact during the experiments. The vessel and hammer head acceleration are aligned separately at the highest peak. Then the peak values are averaged and a standard deviation is calculated. The velocity profile of the hammer head and the vessel can also be derived from the acceleration record. Further integral to displacement is, however, not usable due to accumulated numerical error.

### 3.4 Results from shock experiment

The scope of this study is to investigate the flow characteristics under a jammed configuration, both statistical and dynamical. When a jammed structure collapses under impact, the beads flow out of the vessel until another jamming occurs. This is a random
process, and there is no clear correlation between the impact intensity and the flow characteristics on the level of individual impact [86][87], i.e. a strong impact does not necessarily induce a large flow or a flow at all, and a weak impact can also trigger a flow of considerable magnitude. Fundamentally, the flow must be considered statistically, depending on the specific geometry for the grains to form the jam near the orifice.

3.4.1 Impact acceleration and velocity

Consistent impacts are delivered by the pneumatic hammer in the system. Fig. 3-6 shows the aligned acceleration results from a typical 500-impact run. The duration of the impact is 2 ms. The standard deviation of the peak acceleration is less than 1 g, a very narrow spread for such fast exchange of momentum. When the impact intensity is low, the foam absorbed the momentum of the landing vessel, and the acceleration profile is rather flat after the main impact peak. When the impact intensity is high, however, the extension damper slowly lowers the vessel down to its original position. This is shown on the vessel acceleration as a section of upward acceleration that decline linearly.

Fig. 3-9 shows the hammer acceleration and the hammer velocity integrated from the acceleration. The acceleration of the hammer head is very consistent before the impact. The quality of the data during and after the impact is poor and not usable to further derive the velocity result. The acceleration stage occurs during a very brief period of time after the lower chamber is inflated, and the hammer head moves at a relatively constant velocity until impact. The velocity of the vessel is numerically integrated from
the acceleration result. The peak velocity of the vessel is proportional to the peak acceleration as the shape and duration of the impacts are similar to different peak accelerations. Higher working pressures result in higher impact velocity, therefore higher peak acceleration of the vessel. One possible dynamic parameter other than the peak vessel acceleration is the take-off velocity of the grains. The take-off velocity is the velocity when the vessel moves at \( -g \). This is the point when the grains in the vessel take off from the bottom of the vessel before land again on the bottom plate. The take-off velocity is also proportional to the peak acceleration as shown in Fig. 3-10

Figure 3-9
Figure 3-9: Motion of the hammer. a) the hammer acceleration; b) the hammer velocity. The red bold line is the average of multiple curves. The scattering of the impact velocity integral is due to the small initial offset in the acceleration data, shown in the initial tilting of the velocity curves. The velocity of the hammer after the impact is not reliable due to the insufficient resolution of the accelerometer during the impact. The result is the large spread of velocity tail at the end of the hammer motion.
3.4.2 Flow size, duration and flux distribution

Under jammed situation, a flow may or may not occur after an impact. In the case that a flow is initiated, it is often only a brief time before the next jamming obstructs the orifice. We measured the number of beads flowing out of the vessel between two adjacent jamming events or flow size, the duration of the flows, and the statistical features of these flows.
The flow sizes of individual flow events are averaged according to the diameter ratio under different peak accelerations. The average flow size increases rapidly with hole diameter as shown in Fig. 3.11a. Similar to the flow size, the average flow duration also increases with the diameter ratio, but at a much slower rate than the average flow size as shown in Fig. 3.11b.

The flow size distribution is plotted in Fig. 3-12a. The flow size probability decreases exponentially with the flow size. Comparisons between different diameter ratios can be made by normalizing flows by the average flow sizes of the corresponding diameter ratio. The normalized flow size distributions follow the same exponential decay trend, and the result agrees quantitatively with previous reports on granular flows from a jammed container, in which compressed air jets were employed to break the jamming [86]. Whether or not the flow size distribution depends on the impact intensity is not conclusive due to the relatively small number of impacts applied at this stage of the study. The distribution of the flow duration is also different from that of the flow size. An obvious peak appears near half of the average flow duration on the histogram plot of the flow duration normalized by the average duration of corresponding diameter ratio as seen in Fig. 3-12b.

Figure 3-11
Figure 3-11: Average flow size and flow duration under impacts. Each point is the average over all the flows identified at different peak vessel accelerations.

Figure 3-12
3.4.3 Flow probability

Another quantity of interest in this study is the probability that an impact can initiate a flow from a jammed container. Flow probability is defined as the ratio between the number of flows initiated and the number of impacts applied. The flow probability result is plotted in Fig. 3-13. For the diameter ratios examined in this study, the flow probability increases with impact intensity, in this case, the peak acceleration of the vessel. The flow probability increases linearly with the peak acceleration until it reaches 100% flow probability. Smaller diameter ratios depress the flow probability under the
same impact intensity. This is demonstrated in the different slopes in the flow probability vs. impact intensity plot of different diameter ratios.

Figure 3-13

**Figure 3-13**: Flow probability under impact. The flow probability increases with peak acceleration before reaching 100%. Large diameter ratio results in steeper slope. The dashed lines are the linear fit of R = 3 (blue), and R = 1.9 (brown) data.
3.5 Discussions

The jamming of flows is caused by the forming of an arch or dome structures that obstructs the orifice. Such structures are often very robust to vertical loading, as one can see from some ancient architecture. Dilution of the bottom layers [88], rather than the sudden vertical loading, is likely the cause of the arch destruction. Breaking the jamming and the flows between two jams are two distinctive processes and will be discussed separately below.

An arch structure is held together by the lateral forces at the ends of the arch [56] [89]. Mechanical shocks dilute the bottom layers, and the lateral forces relax, which makes the arch structure unstable [88]. Breaking an arch structure requires removing at least one of the beads in the arch. The more beads are in the arch, the greater the chance that one of the beads will be knocked out of the arch. The length of an arch is determined by the diameter ratio between the orifice and the grain. The longer the arch, the more beads are at the “risk” of being knocked off the structure, and the probability that a flow is initiated increases. This trend is reflected on the different slopes of the flow probability plot. Large diameter ratios show a steeper slope than smaller diameter ratios.

The flow statistics, the average flow size, flow distribution and flow duration depends on the probability a new arch structure can be formed near the orifice after the destruction of the last one. The result of the flow probability agrees with previous study
in [87] that the flow size distribution follows an exponential function. The distribution of the flow duration distribution suggests that the flow rate is not uniform during the falling of grains even for the same diameter ratio. For example, the flow rate of a few discrete grains is less than that of a continuous flow.

3.6 Current and Future Work

As discussed in Chapter 1, the pressure at the bottom of a vessel filled with granulates deviates from the hydraulic pressure as the depth of the grains becomes comparable to the diameter of the container. The data above were acquired using a wide container (20 cm in diameter). The depth of the grains in the container (12.7 cm) is less than the diameter of the vessel. This is apparently still in the hydraulic pressure regime. Test runs were performed to examine the effect of depth of the granular bed on the flow probability under impacts. The results are plotted in Fig. 3-14. It seems that the depth of the pile, therefore the pressure at the container bottom, can significantly influence the granular flows from a jammed container. It is more difficult to initiate a flow from a container filled with deeper layers of grains than with shallower layers. Even though this per se is an interesting subject to investigate, we want to focus on fewer variables, and concentrate on the dynamical parameters at this stage of the study. When the depth of the grains is several times greater than the container diameter, the pressure at the bottom becomes a constant. This is realized by using vessels of smaller diameters.
We used brass cylinders with inner diameters 5.08 cm and 8.89 cm. The brass cylinder is bolted to the bottom of the large vessel in a co-centric arrangement. The cylinders were filled to 25 cm deep so that the bottom pressure is in the constant regime. The flow data were acquired the same way as the described in section 3.1. The preliminary data are shown in Fig. 3-15. The flow probability from a smaller cylinder is significantly lower than the flow probability using a wide vessel. This is probably due to the increased pressure near the orifice.

Figure 3-14: Flow probability at different filling depths. Vessel diameter, 20 cm, diameter of the exit hole, 4 mm, and the beads diameter 2 mm. The filling depth apparently affects the probability to initiate a flow under impact.
Aftershocks of the vessel were observed when small cylinders were used as shown in Fig. 3-16. This is because of the non-uniform mass distribution at the bottom. More mass is concentrated at the center of the plate where the cylinder is positioned. The bottom plate experiences fast membrane vibration (like a trampoline) after the initial impact. These aftershocks are not desirable as they complicate the characterization of the impacts. Filling the space between the vessel and the brass cylinder with steel spheres can significantly damp out the aftershocks.

Figure 3-15: Flow probability with different vessel diameters. Red symbols are the results from 5.5 mm hole diameters, and the blue symbols are the results from 4 mm hole diameter. The flow probability is much lower when vessels with smaller diameters are used.

Aftershocks of the vessel were observed when small cylinders were used as shown in Fig. 3-16.
Future work will focus on the flow probability at different impact intensities, container size and depth, as well as the possible dependence of the flow sizes on the impact intensity. There are concerns that the acceleration recorded by the accelerometers on the rim of the vessel could be the sound pulse traveling through the aluminum vessel instead of the motion of the vessel itself. Even though both can be used to characterize the intensity of the impact, the mechanism of the unjamming process could be very different. Tests need to be performed to determine the nature of the signal, for example,
by comparing signal at different parts of the vessel. Impacts from sides and above can also be investigated with slight modification of the current setup.

3.7 Conclusions

This project is not completed at the time this dissertation is written. Yet the preliminary data allow some reasonable speculations. The statistics of flow size we found confirms previous reports on the flows from a jammed container that flow size follows an exponential decay. The different characteristics of the flow duration suggest that there might be a transition between small and large flows. The diameter ratio dictates the number of grains in an arch structure, and the impact intensity controls the dilution of granular density near the orifice. Both contribute to the destruction of an arch structure. Future study will investigate that flow probability in different container diameters, and the possible dependence of the flows on impact intensity.
Chapter 4

Granular fragility under thermal cycles

Granular materials have typically been considered as athermal due to the huge energy difference between the thermal energy and the energy needed to move a grain. For example, the energy needed to move a 1 mm glass beads will require a temperature of $10^{14}$ K. We will demonstrate in this Chapter, that despite the energy barrier, temperature changes in granular systems can affect the structure of the granular system by exploiting the fragility in jammed granular media. Randomly packed granular material is frozen into a state far from equilibrium due to jamming, and highly inhomogeneous. A small portion of the grains bear the majority of the load in a granular pack. Any changes at these force-bearing grains can significantly affect the structure of the system, and thus make it fragile to some perturbations such as vibration or deformation of grains from temperature changes. Vibration allows the system to briefly explore a small window of configurations, and can result in higher packing fractions, but typically does not allow the grains to reach equilibrium. This chapter reports two experiments that explore the small size changes induced by temperature variations and the impact on the bulk properties of the granular ensemble.
4.1 Packing fraction relaxation under thermal cycles

We studied the change of packing fraction relaxation under thermal cycles, which is termed in this dissertation as “thermal packing”. Packing fraction is defined as the fraction of volume occupied by the solid grains rather than empty space. For randomly packed spherical grains the packing fraction can vary between 57% and 64%. The highest possible packing fraction for non-random packing is 74% at FCC lattice. Granular packs and containers expand and contract when the ambient temperature is increased and then decreased to initial temperature, which is referred in this thesis as “thermal cycling”. Such change in sizes, even though microscopic, can change the packing fraction of randomly packed grains.

4.1.1 Experimental Techniques

The experimental setup of the thermal packing experiment is illustrated in Fig. 4-1. Cylindrical containers with graduation marks are filled with granular beads to a well controlled initial packing fraction and depth. The samples are then placed into a convection oven (Thermal Electron, Precision 6530) and gradually heated to a preset temperature (termed cycle temperature in this dissertation). The samples are kept in the oven long enough to ensure consistent temperature through the sample volume before being gradually cooled down to the initial ambient temperature. The temperature measurement is discussed in details below. The packing fraction of the sample is measured after the sample is completely cooled down.

Figure 4-1
4.1.1.1 Temperature control

The temperature of the oven was controlled by an internal thermostat which can be adjusted by a control knob on the oven. The heating element was in the bottom of the oven and was turned on and off until the target temperature was reached. The position of the knob does not directly correspond to the final temperature in the oven. The oven temperature needs to be calibrated by a digital thermometer to set the correct cycle temperature. This was achieved by constantly adjusting the control knob until the oven temperature stably varied around the target temperature. Once the oven was calibrated, the internal thermostat showed very good repeatability, and the actual cycle temperature varied less than 2 °C around the target temperature. Calibration was needed every time a new cycle temperature was used.

Figure 4-1: Experimental setup of thermal packing experiment. The plot on the oven is the oven temperature during thermal cycling. The dashed line indicate the change of the sample volume, thus the packing fraction of the sample before and after thermal cycling.
The temperature was continuously logged during the experiment. Digital thermometers (Sper Scientific, 800009) with flexible probes were connected to a computer via RS-232 port. Temperature was read every 10 seconds and the results were saved as part of the experimental record. We also used temperature logging to determine the correct heating time for the sample to reach thermal equilibrium, and the cooling time to a complete cooled down to the room temperature. Trial runs were performed by simultaneously logging temperatures from two probes. One probe was suspended in the air in the oven, and the other probe was buried in the center of the container filled with beads. The oven was then kept on long enough until the temperature readings from the two probes were the same. The same testing was performed for the cooling process and the result is shown in Fig. 4-2b. During actual experiment only the temperature of the air in the oven was logged since the packing of the grains will be affected if a probe is placed in sample.

Figure 4-2
One concern in this experiment was the possible fire hazard from an overheating oven. It usually takes more than 10 hours for the sample to reach thermal equilibrium as shown in Fig. 4-2a. The timing of the thermal cycles was controlled by a digital timer switch, and the ovens are often operated unattended. Precautions were taken to prevent the accidental overheating of the ovens. A limiter was installed on the control knob of the oven so that the cycle temperature will not accidentally be set too high. An overheating protection circuit was installed to shut down the ovens in the case of overheating. The circuit diagram and the working mechanism of the overheating protection circuit are listed in the Appendices.

Figure 4-2: a. Oven temperature as a function of time (polystyrene beads, cycle temperature 76 °C); b. Temperature inside and outside the sample (glass beads, cycle temperature 112 °C), red line, oven air temperature, black line, temperature at the center of the pile.
4.1.1.2 Sample Preparation

The samples in thermal packing experiment were cylindrical containers filled with grains to a pre-set packing fraction. Spherical beads made of soda lime glass, polystyrene, steel and High Density Polyethylene (HDPE) were used in the experiment. The containers are made of either borosilica glass or polymethylpentene plastic (PMP). The grains size varies from 0.5 mm to 3 mm, and the diameter of the containers ranges from 14 mm to 100 mm. To obtain relatively low and consistent initial packing fraction in the samples, we employed a “gradual filling” technique that steadily raises a funnel filled with grains from the bottom of the empty container. Some early samples were prepared by hand while most of the data presented in this thesis were prepared by a filling machine. The diagram of the filling machine is shown in Fig. 4-3. An empty cylinder was placed on a digital scale. The funnel sprout was positioned at the center of the container bottom and filled with a pre-weight amount of grains. The stepper motor then slowly lifts the funnel out of the container until all the grains settle in the cylinder. A plastic block with a circular opening was used to prevent the sprout from swinging during the ascent. The mass of the grains in the cylinder was then read from the scale, and the top surface of the pack of grains was flattened using a soft brush. Funnels of different capacities and sprout diameters were used in preparing samples with different container volume and grain size to ensure a smooth filling. Static charge accumulation was observed when using polystyrene beads, and metal tubes were used as the sprout of the funnel to drain the charges. The resulting packing fraction of the samples are $58.9 \pm 0.1 \%$ from small grains $(d < 2 \text{ mm})$ and $58.9 \pm 0.2 \%$ for grains with diameters of 2 mm or greater, when
the funnel moves at 1 mm per second. The error bars for different grain diameters are due to the flatness of the free surface, large grains result in a rougher surface that prevents accurate reading of the sample volume. The coefficients of thermal expansion of the materials used in the experiment are listed in Table 4.1.

Table 4-1: Coefficient of Thermal Expansion of materials used in the experiment

<table>
<thead>
<tr>
<th>Granular media</th>
<th>Coefficient of thermal expansion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soda lime glass beads</td>
<td>$9 \times 10^{-6}$/K</td>
</tr>
<tr>
<td>Steel shots</td>
<td>$12 \times 10^{-6}$/K</td>
</tr>
<tr>
<td>Polystyrene beads</td>
<td>$70 \times 10^{-6}$/K</td>
</tr>
<tr>
<td>HDPE plastic beads</td>
<td>$110 \times 10^{-6}$/K</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Container material</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>PMP plastic</td>
<td>$117 \times 10^{-6}$/K</td>
</tr>
<tr>
<td>Borosilica glass</td>
<td>$3 \times 10^{-6}$/K</td>
</tr>
</tbody>
</table>
The packing fraction of the samples is calculated using Eq. 4.1

Eq. 4.1

\[
P = \frac{m}{\rho V}
\]

Where \( P \) is the packing fraction, \( m \) is the mass of the beads in the cylinder; \( \rho \) is the density of the bead material, and \( V \) is the volume reading from the graduations of the cylinder. With the density of the grain material known and the high accuracy of the digital scale in the mass measurement (resolution 0.01 gram), the precise determination
of the sample volume is particularly important. The sample volume was read from the graduations on the cylinders. The resolution of volume reading depends on the grain size. It is more difficult to determine the volume of a pack consists of large grains than small grains. This is reflected in the different error bars in the initial packing fractions of different grain sizes.

New cylinders, especially those made of plastic, decrease in volume when initially subjected to thermal cycles. This is due to the tension built up in the cylinders during manufacturing. Thermal cycles release the tension and slightly reduce the size of the cylinders, as compared to their initial sizes. Tension release is completed after 2 or 3 initial thermal cycles, and the volume of the containers stabilizes. This initial volume reduction of container must be taken into account when calculating the packing fraction of the sample. We introduce a correction factor for each container. The correction factor is defined as the ratio between the reading volume and the actual volume of the cylinder:

\[
CF = \frac{V_{\text{read}}}{V_{\text{water}}}.
\]

The actual packing fraction is calculated with

\[
P_{\text{corrected}} = CF \times \frac{m}{\rho V_{\text{reading}}}
\]

Experimentally, the correction factor was determined by filling the cylinders with water to a certain level and measured the weight of the water in it. The weight of the water was then converted to the actual volume that corresponds to the reading on the cylinder. Three different filling levels were chosen for each cylinder, and the correction
factor was the average of these three results. Correction factor ranges from 0.994 to 1.016 for different cylinders.

After the samples were prepared, they were gently moved into ovens for thermal cycling. The containers were placed on a wire rack covered with Teflon sheets to prevent inhomogeneous heating of the sample. Great care was taken to prevent the shaking of the sample during the experiment. Trial runs with no thermal cycling were performed to ensure that vibration from environment or sample movement had no measurable effects. We also measured the volume of the containers after thermal cycling to make sure that no deformation occurs from possible pressure accumulation at the container wall.

4.1.2 Results from thermal packing experiment

For a single thermal cycle, the packing fraction of the samples increases after thermal cycling regardless of the thermal properties of the beads or the container, and the change of packing fraction increases with cycle temperature. Fig. 4-4a shows the change of packing fraction as a function of cycle temperature in different systems. The magnitude of packing fraction change in different systems can be drastically different. For example, in the glass beads/PMP container system, a small change of temperature (~5 °C) can induce considerable packing fraction increase, while very small changes of packing fraction is observed in the glass beads/glass container system even at the highest
cycle temperatures. It seems that the thermal property of the system may influence the result of thermal packing. This is demonstrated when the change of packing fraction is plotted against the fractional changes. Fig. 4.4 b, c., where the data from low thermal expansion coefficients systems align with systems with high thermal expansion coefficients at lower cycle temperatures. The relative change is quantified as the difference between the fractional size change of the two materials ($\Delta T \times |(\text{CTE}_{\text{beads}} - \text{CTE}_{\text{container}})|$), where $\Delta T$ is the change of temperature and $\text{CTE}_{\text{beads}}$ and $\text{CTE}_{\text{container}}$ are the coefficients of thermal expansion of the grains and the container respectively. We also define a combined change as the sum of the fractional changes of both the grains and the container ($\Delta T \times |(\text{CTE}_{\text{beads}} + \text{CTE}_{\text{container}})|$).

Figure 4-4
Figure 4-4: a. Change of packing fraction as a function of temperature in different systems; the initial packing fraction is 58.9 ± 0.2; b. Change of packing fraction as a function of relative thermal changes; c. Change of packing fraction as a function of combined thermal changes. Each point is the average value of at least 12 individual measurements, and the error bars represent the standard deviation of those measurements.
The sample size dependence was examined by using cylinders of different diameters and filling the cylinders to different depths. The results are plotted in Fig.4-5. No apparent container diameter dependence or was filling-depth dependence was observed when the cylinder diameter was varied by a decade, or the depth was varied by a factor of three. The error bars at smaller cylinder diameter or low filling depth are greater simply because of the relatively low resolution when lower volume of beads is used. For this case, more runs were performed to obtain an accurate average value. These results show that the packing fraction relaxation is a bulk effect rather than a boundary effect. In fact, the change in container diameter is less than one diameter of the beads for most cycle temperatures except the highest. Fig. 4-6 shows the change of packing fraction using different diameter grains. There is no noticeable correlation between the change of packing fraction and the bead sizes.

Figure 4-5
Figure 4-5: Sample size dependence of packing fraction relaxation under thermal cycles. Single cycle with cycle temperature at 107 °C, PMP plastic cylinder and glass beads of 0.5 mm in diameter.

Figure 4-6: Change of packing fraction under thermal cycles using beads of different sizes. Glass beads in PMP cylinders.
When subjected to multiple successive thermal cycles, the packing fraction continues to rise, while the increment after each cycle becomes progressively less. Multi-cycle measurements were performed in systems of glass grains in PMP cylinder and polystyrene grains in glass cylinder systems with results shown in Fig. 4-7. The packing fraction vs cycle numbers can be well fitted to a double relaxation function

\[ y(t) = y_0 + A_1 e^{-t/\tau_1} + A_2 e^{-t/\tau_2} \]

with two distinctive time constants, suggesting there might be two different relaxation mechanisms in this phenomenon, which will be discussed in section 4.3. No reasonable single exponential decay fit can be obtained primarily due to the sharp increase of packing fraction at the first few cycles.

Figure 4-7
Packing fraction relaxation in systems with different thermal expansion coefficients reveals how the thermal properties of the grains and the containers affect the change of granular packing fraction under thermal cycles. Two extreme cases are particularly interesting, the glass beads in PMP containers and the polystyrene beads in glass container. Curiously, on the temperature dependence plot, the results from these two systems seem to be on top of each other, even though the thermal properties of these systems are different. The fitted parameters to the double relaxation function $y(t) = y_0 + A_1 e^{-x/t_1} + A_2 e^{-x/t_2}$ are $A_1 = 1.81 \pm 0.12$, $t_1 = 1.74 \pm 0.26$, $A_2 = 3.21 \pm 0.11$, $t_2 = 57.96 \pm 6.54$ for glass in PMP at 107 ºC (blue), $A_1 = 0.90 \pm 0.07$, $t_1 = 2.72 \pm 0.48$, $A_2 = 4.11 \pm 0.06$, $t_2 = 131.79 \pm 10.18$ for glass in PMP at 41 ºC (red), and $A_1 = -0.90 \pm 0.07$, $t_1 = 2.72 \pm 0.48$, $A_2 = 4.12 \pm 0.06$, $t_2 = 132 \pm 10$ for polystyrene in glass (brown).

Figure 4-7: Packing fraction relaxation under multiple cycles. Solid lines are the fitted to double relaxation function $y(t) = y_0 + A_1 e^{-x/t_1} + A_2 e^{-x/t_2}$ with $y_0 = 64$ and the fitted parameters are $A_1 = 1.81 \pm 0.12$, $t_1 = 1.74 \pm 0.26$, $A_2 = 3.21 \pm 0.11$, $t_2 = 57.96 \pm 6.54$ for glass in PMP at 107 ºC (blue), $A_1 = 0.90 \pm 0.07$, $t_1 = 2.72 \pm 0.48$, $A_2 = 4.11 \pm 0.06$, $t_2 = 131.79 \pm 10.18$ for glass in PMP at 41 ºC (red), and $A_1 = -0.90 \pm 0.07$, $t_1 = 2.72 \pm 0.48$, $A_2 = 4.12 \pm 0.06$, $t_2 = 132 \pm 10$ for polystyrene in glass (brown).
two systems are quite different. If the small size changes during thermal cycling are the essential parameter that leads to the packing fraction relaxation, then one would expect that the change of packing fraction scales to the dimensional changes of the system during thermal cycles. The plots in Fig. 4-4 b and c clearly suggest otherwise. No universal collapse of data is observed between different systems. The lack of a scaling rule in the relaxation of granular packing fractions under thermal cycles suggests that there are other factors besides the thermal expansion coefficient that can influence the packing fraction relaxation of granular system under thermal cycles, which will be discussed later this Chapter.

For most of the materials used in the thermal packing experiment, the thermal expansion coefficients are well documented except for HDPE. The available literature only lists a range of thermal expansion for HDPE. We used a high precision capacitance bridge to determine the exact thermal expansion coefficient of the HDPE grains used in this study. The set-up of the measurement is illustrated in Fig. 4-8. Two rectangular copper plates of 12.4 cm by 14.6 cm were used. Three 1.27 mm diameter holes were bored on each plate. 3 mm HDPE beads were placed between the two copper plates, and anchored in those three pairs of holes. This separates the two copper plates by a small gap and forms a simple parallel plate capacitor. Two contacts are made on the outer surface of the plates for capacitance measurement. A temperature probe is also attached to one of the plates to measure the temperature of the capacitor. The capacitor is then placed into the oven, and slowly heated up. The temperature and the capacitance between the plates are closely monitored and recorded.
The capacitance of a parallel plate capacitor is proportional to the plate area and inversely proportional to the distance between the plates.

Eq. 4.3

\[ C \propto \frac{S}{d} \]  

Where \( S \) is the area of the plates and \( d \) is the gap between the plates. Now consider a small temperature change \( \delta T \) that causes the expansion of both the plates and the grain, and we have

Eq. 4.4

\[
S(\delta T) = S_0(1 + \delta T \alpha_{\text{copper}}) \\
d(\delta T) = d_0(1 + \delta T \alpha_{\text{HDPE}})
\]

Where \( \alpha_{\text{copper}} \) and \( \alpha_{\text{HDPE}} \) are the linear thermal expansion coefficients of the copper and the HDPE grains respectively. And the resulting capacitance is

Eq. 4.5

\[
C(\delta T) = C_0 \left( \frac{1 + \delta T \alpha_{\text{copper}}}{1 + \delta T \alpha_{\text{HDPE}}} \right)
\]

With the thermal expansion coefficient of the copper known \((17 \times 10^{-6}/\text{K})\), the thermal expansion coefficient of the HDPE grains can be calculated using capacitance data at different temperatures. The measured thermal expansion coefficient is
110 × 10⁶/K, within and close to the upper bound of the literature values.

Figure 4-8

Figure 4-8: Measuring the thermal expansion coefficient of HDPE grains. The expansion of the beads increases the distance, and thus changes the capacitance between the two copper plates.

4.2 Intruder displacement in granular piles under thermal cycles

Granular packing fraction can be changed either by mechanical vibration or thermal cycling. Even though these two processes are quite different, they both exploit the fragility of the granular packing. Another well-known phenomenon in granular
materials under vibration is the segregation of grains of different sizes [51] [41] – large intruders move to the surface when vibrated. We systematically studied this displacement of an intruder in thermally cycled granular samples and examined the possible equivalence to “Brazil nut effect” by thermal perturbation.

4.2.1 Experimental Techniques

Figure 4-9

Figure 4-9: Experimental setup of intruder experiment. The plot on the oven is the oven temperature during thermal cycling. The dashed lines indicate the position changes of the intruder and the free surface of the pile.
The experimental setup of the intruder experiment is shown in Fig. 4-9. Spherical intruders are buried into granular piles and placed in an oven for thermal cycling. The oven and the thermal cycling process are discussed in thermal packing experiment. The intruder’s position is indicated by a thin stiff wire attached to the top of the sphere. The beads used in the experiment are primarily 1 mm polystyrene beads. The containers are borosilica glass beakers with a diameter of 10 cm and depth of 16 cm. Glass beads and plastic containers were also used. The intruders are made of Lucite, Teflon, brass and aluminum with diameter of the intruders varying from 0.32 cm to 5 cm. The position of both the intruder and the free surface of the granular pile were measured before and after thermal cycles.

The sample filling apparatus in the thermal packing experiment is used in the sample preparation of the intruder experiment. The preparation procedures are shown in Fig. 4.10. Slightly different methods were used for intruders with different density. Heavy (high density) intruders that would sink into the pile under their own weight were slowly lowered into the pile from the top surface until it reached an equilibrium position. Light or low density intruders do not sink into the granular pile spontaneously. For these intruders, the beaker was first filled to a preset depth before placing the intruder on the top surface. A second filling was performed to completely bury the intruder in the pile. The sample preparation was completed by flattening the top surface of the pile using a soft brush. The initial position of the intruder was about 2-3 layers (2-3 mm) below the pile surface.

Figure 4-10
The position of the buried intruder is indicated by a stiff wire attached to the top of the sphere. The wire is made of stainless steel and is 0.4 mm in diameter and 15 – 20 cm long. The wire was attached to the spheres through a set screw or a short threaded rod for easy switch and repair. Density of hollow spheres can be changed by filling with tiny metal particles.

After the spheres were positioned in the grains, the samples were photographed against a fixed ruler with a minimum division of 1 mm. Positions of both the intruder and the granular free surface were photographed. The scheme of photographing is shown in

Figure 4-10: Position intruders in granular pile. Left: a heavy intruder is slowly lowered into the pile from the free surface. Right: a light intruder is first placed at the top of the pile, and then buried by a second filling.
Fig. 4-11. When the position of the intruder was measured, the ruler was positioned on the rim of the beaker, and the wire sticking out was perpendicular to the graduations on the ruler. When the position of the pile surface was measured, the ruler was positioned on the bench surface on which the beakers were placed, parallel to the beaker wall. Parameters such as intruder, container, grains, and cycle temperature were written on a small white board and photographed together with the sample and the ruler. Great care was taken to make sure that the samples were photographed at the same position relative to the camera.

Figure 4-11

The samples were then placed into the oven for the same thermal cycling as described in thermal packing experiment. When polystyrene beads were used, the cycle temperature was set to below the softening temperature of the polystyrene (80°C) so that

Figure 4-11: Photographing the samples a. Photographing the intruder position; b. Photographing the pile surface position. A lab jack is used to position the camera on the same level at the positions being photographed.
there was no plastic deformation of the grains during thermal cycling. The possible change of the surface condition of the grains as a result of repeated thermal cycling was examined by using fresh grains and grains that have been repeatedly heated and cooled. No significant difference was observed.

The samples were photographed again after being thermally cycled. The position of the intruder and the pile surface in reference to the ruler can be read to 1/10 mm from the digital photograph. Both the intruder and the pile surface may move after the thermal cycle. The difference between the positions before and after the thermal cycle is the absolute displacement of the intruder $d_{\text{intruder}}$ or the surface $d_{\text{surface}}$, and the relative displacement between the intruder and the pile surface is determined as $d_{\text{relative}} = d_{\text{intruder}} - d_{\text{surface}}$. A negative sign is used to indicate a downward motion. Each result is an average of at least 12 measurements except for the multiple cycle runs, which are the averages of six measurements. The error bars are the standard deviation of the individual measurements.

4.2.2 Results from intruder experiment

When polystyrene grains and borosilica beaker are used, brass intruders sink relative to the grain pack after thermal cycles as shown in Fig. 4-12, and the relative displacement increases linearly to the cycle temperature. Brass intruders with diameter of
1.9 cm sink less than 2.54 cm diameter brass intruders. No significant relative displacement of Lucite or Teflon intruders (2.54 cm in diameter) is observed. Within the temperature range of the ovens (< 170º C), no significant relative displacement is observed when brass intruders are placed in glass grain packs in a PMP container, or Teflon intruder placed in steel grain packs in a glass container. The results are shown in Fig. 4.13

Figure 4-12

Figure 4-12: Temperature dependence of intruder displacement in polystyrene pile in a borosilica beaker. Heavy intruders (brass) sink after thermal cycling, and the displacement increases with cycle temperature. Light intruders (Teflon) do not show relative displacement to the pile.

Figure 4-13
The density dependence was examined by using 3.81 diameter hollow aluminum spheres filled with metal particles. No significant displacement is observed for low density intruders. The intruder starts to move, and the relative displacement increases with intruder density, after the density passes a threshold between 2 and 3 g/cc as shown in Fig. 4-14 a. The effect of the intruder size was studied by using brass intruder of different diameters. Similar to the density dependence, the relative displacement is zero at the lowest intruder size, and then increases with the intruder diameter as shown in Fig. 4-14b. Even though aluminum and brass intruders were used in density dependence and intruder size dependence study respectively, the difference in intruder materials does not seem to affect the result as the thermal expansion coefficients of brass and aluminum are very close, and much less than that of the grains around the intruder.
Figure 4-14: a, Relative displacement as function of intruder density; b, Relative displacement as a function of intruder size. Polystyrene pile in borosilica beaker, c, Pressure dependence of relative displacement from different intruders. cycle temperature 76 °C. The density is varied by filling hollow aluminum spheres. The pressure is calculated as the ratio between the intruder weight and the cross-section area.
When placed under multiple successive thermal cycles, intruders continue to sink in the grain pack, while the incremental displacement becomes progressively smaller with the number of cycles. Fig. 4-15 a shows the relative displacement of 2.54 cm diameter brass intruder in polystyrene grain packs. The result is the average of 6 measurements. Under single thermal cycle, the intruder also sinks less when buried deeper from the surface of the pile than when buried close to the free surface as shown in Fig. 4.15 b. due to the fewer number of grains below the intruder

Figure 4-15

Figure 4-15: a. Brass intruder displacement under multiple cycles. The solid line is the fitting result to $y_0 + A_1 e^{-x/\tau_1} + A_2 e^{-x/\tau_2}$ and the fitted parameters are $y_0 = -3.3$, $A_1 = 0.97$, $\tau_1 = 0.87$, $A_2 = 2.34$, and $\tau_2 = 18.03$. b. Intruder displacement when buried at different depth. In both plots, polystyrene beads and glass beakers are used with a cycle temperature of 76 °C.
4.3 Discussion: Granular fragility under temperature variation

The packing fraction relaxation and the displacement of an intruder object in thermally cycled granular piles are both demonstrations of fragility of jammed granular media. Unlike many physical systems that can explore the phase space and evolve spontaneously to an equilibrium state, a granular system is locked into the state as soon as the grains settle. Such configurations are usually not the favored in terms of energy. Once a granular pack is prepared, it would stay at the same configuration for a rather long period of time until disturbed by external energy. This inability to move from one state to another without external interference is known as jamming, as discussed in Chapter 1.

Jammed systems can be very robust against static loading as anyone standing on a sand beach would notice. Instead of sinking into the beach like one would in water, the weight of a person is distributed through a complex network of contacting grains, or force chains. Forcing granular materials into higher packing fraction by static compression is extremely difficult [3]. However, this resilience in jammed granular materials comes with the fragility of the system. The fragility in a granular system is defined as the inability to “withstand some infinitesimal perturbations” [31]. The force chains that support the structure of a granular pile are disordered, and highly dependent on local contacts. Such force chains can only support longitudinal loading along the direction of the contacts while very susceptible to transverse force components. Particle deformation caused by temperature variation changes the line of contacts and transverse loadings may occur as
shown in Fig.4-16. This breaks the force chains, and cascades through the contact networks, leading to greater reorganizations in the bulk of the pile [32][33][34][90]. Energy “locked” in the granular packing due to the jamming is released during these reorganizations, even though the thermal energy per se is not enough to move grains inside the packing.

Figure 4-16

Figure 4-16: Grain deformation allows transverse components to arise, resulting breaking the force chains and greater reorganization. (Figure is reproduced from Reference [31], copyright 1998 by the American Physical Society)

On the temperature dependence plot, Fig.4-4 a, the packing fraction does not increase strictly linearly with the temperature. A steeper slope can be observed at the lowest cycle temperature. This result, together with the double relaxation decay of the packing fractions in multiple cycle measurements, suggests that there might be two competing mechanisms in the packing fraction relaxations. This can be understood qualitatively using Mehta’s general theory on the packing fraction relaxation of granular systems [91]. These two mechanisms can be categorized as block relaxation and single grain relaxation. Block relaxation is dominant at low cycle temperatures when the
The lack of a scaling rule of packing fraction relaxation on the thermal deformations suggests that there are other factors that can influence the packing fraction relaxation. One of possible factor is the friction in the system. This includes the inter-grain friction and the friction between the grains and the container. Friction plays an important role in the stability of granular packing. For example, the pressure saturates to a finite value in deep containers filled with grains due to the frictional support from the container wall. Generally, the frictional forces between grains helps granular media resist external perturbations. Systems with different frictions with respond differently under the same perturbation, e.g. thermal deformation of the grains [92][92]. This makes it very difficult experimentally to isolate the contributions from different thermal expansion coefficients and different friction properties in the system.

We believe that the intruder displacement in thermally cycled granular piles is essentially the packing fraction relaxation under an overload. An intruder introduces extra load to the force bearing grains in the pile [93][94], and thus introduces more energy to
release in the case of a broken force chain. Under thermal cycles, the grains under the influence of the intruder weight experience greater packing fraction relaxation, and the intruder sinks in the granular pile. The relative displacement is determined by thermal deformation of the grains, therefore the cycle temperature, and the number of grains that support the intruder. The number of grains bearing the weight of the intruder depends on the weight of the intruder. The intruder weight can only penetrate a limited distance due to the jamming nature of the granular packing. The heavier the intruder, the more grains are affected and the deeper the intruder will sink under thermal cycles as shown in Fig. 4.14. With fewer grains under them, intruders buried deeper, moves much less than shallowly buried intruders. When placed under multiple thermal cycles, the grains become more packed. This makes it progressively difficult for the grains to reorganize, and the intruder displacement becomes incrementally less. The double relaxation fit of the intruder displacement further demonstrates that the intruder displacement in thermally cycled granular pile is indeed the result of packing fraction relaxation under an overload.

Another interesting result observed in the intruder experiment is the zero relative displacement of light intruders under thermal cycles. We believe that this is due to the “elasticity”[92] of the granular packing. Granular packing can show a certain level of elasticity resulting from the inter-grain friction in granular piles. This enables the granular pile to support a light static load without structural reorganizations. The granular medium behaves like a continuous elastic solid, and the intruder position relative to the grains is not affected by thermal cycling. Intruders start to move under thermal cycles when the weight exceeds the elastic threshold.
4.4 Conclusions

The observed packing fraction relaxation and the displacement of an intruder in thermally cycled granular packs contradict conventional argument that granular media are athermal. We demonstrate for the first time that the bulk properties of the granular packing can be changed by moderate temperature variation, without the injection of mechanical energy. The slow and highly controllable thermal process provides a new tool in the investigation of granular fragility rooted from the jammed nature of randomly packed granular media.


60. O. Reynolds, “On the dilatancy of media composed of rigid particles in contact”, *Phil. Mag. Ser.* 5 50, 469 (1885)


Appendix A

Overheating protection of the ovens

This appendix introduces the circuit and the working mechanism of the overheating protection employed in the thermal cycles experiments. The circuit diagram is in Fig. A-1.

The circuit utilizes an operational amplifier to trigger the relays that connect the oven to the power sources. An operational amplifier has two input ends, one is non-inverting (+), and the other is inverting (-). The output of an ideal open-loop OP-AMP is 
\[ V_{\text{out}} = (V_+ - V_-) \times G_{\text{openloop}}, \]
where \( G_{\text{openloop}} \) is the gain which is often very large. In real application, however, the output can only be the most positive or most negative voltage level the power source can supply. An OP-AMP without negative feedback can be used as a comparator. When using a 5-volt power source, the output voltage is +5v when \( V_+ > V_- \), and zero when \( V_+ < V_- \).

The oven temperature is measured by a thermocouple probe (Type K). A thermocouple probe consists of a loop of two different metals. Two junctions are created at the points these two metals meet. A voltage difference is generated when there is temperature difference between the two junctions. If the temperature of one junction is known, the temperature at the other junction can be calculated. This voltage signal is very small, and often need to be amplified. AD595 chip is an amplifier specifically designed to amplify the signal from a type K thermo couple. It converts the voltage from the thermo couple to a signal proportional to the Celsius temperature.
During experiment, the probe is placed in the oven. The amplified signal is 1/100 of Celsius temperature, i.e. 24 °C corresponds to 0.024 v. This signal is fed into the non-inverting input of the OP-AMP, and compared to the signal from the inverting end, which can be set by adjusting a potentiometer. The voltage at the potentiometer sets the tripping point of the circuit. When the oven temperature is low, the signal at non-inverting input is less than the signal at the inverting input, and the output signal is zero. This keeps the 5-volt and 120-volt relay closed so the oven remains connected to the wall outlet. When the oven temperature rise above the tripping point, the non-inverting signal is higher than the inverting signal and the output voltage flips to 5 v. This opens up the relays and disconnects the oven from electricity power. To prevent the oven from being reconnected to power source automatically when the oven temperature drops, the output of the OP-AMP is fed back to the non-inverting end. This keeps the output at 5v, and the oven off even when the oven temperature drops below the tripping point. One can reconnect the oven to power source by connecting the non-inverting end to the ground.

This designed can also be slightly modified be switching the non-inverting and inverting inputs, and using relays that are normally closed

Figure A-1
Figure A-1: Overheating circuit
Appendix B

Hammer control circuit

This appendix introduces the circuit and working mechanism of the hammer control circuit. The circuit generates a square signal of adjustable width that corresponds to the inflation time of the pneumatic cylinder. The circuit diagram is in Fig. B-2.

The circuit is built around a 555 timer chip. 555 timer is one of the most popular and versatile IC. The block diagram of the IC is shown in Fig. B-1. The simplified logic is when the voltage at pin 6 (THR) is greater then 2Vcc/3, the output assumes a low state (zero); when the voltage at pin 2 (TRG) is lower than Vcc/3, the output assumes a high state (Vcc). At stable state, pin 6 and pin 2 assume high state, and the output is zero. The timer is triggered by introducing a low pulse to pin 2. This is realized by connecting the capacitor at pin 2 to ground. This brings the voltage at pin 2 below Vcc/3, and the output assumes a high state. The voltage at pin 6 is lowered due to the low pulse. The capacitor at pin 6 starts to charge and the voltage increases exponentially at a time constant of RC. When the voltage at pin 6 is greater than 2Vcc/3, the output flips and assumes a low state. The time for the output signal to flip back to stable state depends on the time constant of the RC circuit at pin 6, and therefore adjustable.

Figure B-1
The high state at pin 3 saturates a transistor that activates a relay. The relay connects the 12-volt DC power source to the BNC connector that is connected to the solenoid valve that controls the gas cylinder. A diode is placed between the output and the ground to discharge inducted voltage from the coil of the solenoid valve.

The timer can be triggered either manually by using a switch or automatically by using a repeat cycle relay. The repeat cycle relay allows one to trigger the timer at a preset time interval. Each time the timer is triggered, a counter registers the number of pulses, thus keeping the number of impacts the hammer generated.

Figure B-1: Block diagram of 555 timer (copyright Fairchild Semiconductor)
Figure B-2: Circuit Diagram of the hammer control
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