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BI-CRITERIA OPTIMIZATION MODEL FOR A FOUR STAGE CENTRALIZED SUPPLY CHAIN

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ABSTRACT

The aim of this thesis is to develop a multi-criteria multi-period linear integer programming model and solution methods for a centralized four-stage supply chain. In most supply chain optimization models, the decisions on transportation and inventory have traditionally been considered as separate entities to avoid complex mathematical formulations. This thesis incorporates the transportation and inventory decisions in the same model. The model helps in identifying the optimal modes of transport to be used between each stage of the supply chain, the optimal inventory levels to be maintained at each stage thereby minimizing the overall cost incurred in the supply chain. While minimizing the overall supply chain cost, the responsiveness of the supply chain is also maximized by modeling it as a bi-criteria optimization problem.

In the proposed model, multiple modes of transport are used between the different stages of the supply chain with an incremental quantity discount cost structure for the freight rates. For inventory, there is a company owned warehouse with a finite storage capacity and a leased warehouse with infinite capacity if the capacity at the company owned warehouse is exceeded. The leased warehouse has an incremental quantity discount cost structure for the storage space. There is only one product flowing in the supply chain. The multi-criteria optimization model is solved using non preemptive goal programming by giving weights to the two objectives and also by preemptive goal programming assigning different priorities to the objectives. The model identifies the right amount of quantity to be shipped by the appropriate mode of transport at each

supply chain stage, the right amount of inventory to be stored at the warehouses (company owned and leased) and the accumulated backorder at the various stages (if any) in order to minimize the total supply chain cost and maximize the customer responsiveness.

The model is illustrated with simulated data for two scenarios, one for prioritizing the cost over the customer responsiveness and another for prioritizing customer responsiveness over cost. The two scenarios are compared based on the shipping pattern, inventory storage and the number of backorders.

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CHAPTER 1

INTRODUCTION

The supply chain management (SCM) literature presents several variations on the same theme when defining a supply chain. One of the most comprehensive definition would be "Supply chain management is a set of approaches utilized to efficiently integrate suppliers, manufacturers, warehouses and stores so that merchandise is produced and distributed at the right quantities to the right locations at the right time in order to minimize system wide costs while satisfying service level requirements" as defined by Simchi-Levi [1].

A supply chain can be either *centralized* or *decentralized*. The supply chain that has a single decision maker to determine its optimal policies throughout the supply chain is called a centralized supply chain. In a centralized supply chain, the company owns its distribution network which includes the plants and the modes of transport. In a decentralized supply chain, the distribution network is owned by different companies. There are multiple decision makers that make decisions independent of the other members and often the objectives of one member conflict with the objectives of another. We consider a centralized supply chain in this thesis.

There are four drivers identified in the literature [2] to measure the supply chain performance. They include *facilities*, *inventory*, *transportation* and *information*. Facilities are the places in the supply chain where products are manufactured or assembled and the place where raw materials or products are stored and distributed. The facility location, size, design and the equipment installed in these facilities play important roles on the *efficiency* and *responsiveness* of the supply chain. For example, having several warehouses in close proximity to the retailers could increase the responsiveness of the supply chain but decrease the efficiency (measured in terms of cost) as costs of handling many warehouses increases. On the other hand, having less number of warehouses at strategic location to the several retailers could decrease the responsiveness but increase the efficiency of the supply chain. Inventory is raw materials, finished products and work in progress or in-transit products within the supply chain. The inventory policies adopted by the company can have a significant effect on the efficiency and responsiveness of the supply chain. For example, a retailer can store large amounts of inventory and make its supply chain more responsive than being efficient by replenishing the product from the storage once it sells out from the shelf. Reducing the inventory stored will make the supply chain more efficient but less responsive. Transportation is the moving of the products or raw materials from one facility to another in the supply chain. There are several modes of transport such as air, ship or truck and several possible routes that could be adopted which in turn determines the efficiency and responsiveness of the supply chain. For example, an online book store could use a courier service such as UPS or FedEx to ship the books to the customer. This increases the costs involved but decreases the time in which the book reaches the customer. Using a ground transportation service could increase the efficiency, but it limits the responsiveness. Information is the most significant driver as it has an effect on all the other drivers aforementioned. Information is the flow of data across all the entities of the supply chain and this data is used for analysis to make the supply chain more responsive and efficient. For example, information on customer demand can help a company to become more efficient by producing only what is needed. The forecasted customer demand can make the company more responsive by being able to produce in advance for an order and meet the customer demands immediately.

There is always a tradeoff between efficiency and responsiveness with respect to all the supply chain drivers. This research focuses on two of the supply chain drivers namely inventory and transportation and studies the tradeoff between efficiency and responsiveness using multi-criteria optimization techniques. Most of the companies consider transportation and inventory as separate entities for the sake of convenience and model their supply chain accordingly. These two drivers are the major contributors to the supply chain costs and minimizing them will result in the overall increase in the supply chain profitability. This has not been done in the past since it involves complex mathematical formulations which often are non-linear in nature and are difficult to solve. Only recently some attempt has been made in this direction where both transportation costs and inventory holding costs are considered together to minimize the overall expenditure in the supply chain. Even fewer models are found in the literature for incorporating both cost and customer responsiveness since it involves dealing with multi-criteria optimization.

Multi-criteria optimization

To solve the problem with conflicting criteria we use the multiple criteria optimization techniques. We begin by defining some terminology and the different methods of solving such problems [3].

A multi criteria mathematical programming problem can be represented as

$$\textbf{Max } f_1(x)$$

$$\textbf{Max } f_2(x)$$

$$\dots$$

$$\textbf{Max } f_K(x)$$

Subject to

$$g_i(x) \leq 0 \quad \text{for } i = 1, \dots, m$$

$$x \geq 0$$

where,

f_1, f_2, $f_3 \dots f_K$ are the conflicting objectives and $g_i(x)$ are the set of constraints.

Definitions:

Ideal Solution

Ideal solutions are best values achievable for individual objectives while ignoring other objectives. The problem is not a multi-criteria problem if the ideal solution is achievable.

Efficient Solution

There are solutions called efficient solutions where one can improve an objective only by sacrificing one or more of the other objectives. An efficient solution is also called a non-dominated solution or pareto optimal solution. A set of all efficient solutions is known as the efficient frontier.

Dominated Solution

Dominated solutions are solutions where it is possible to improve one of the objectives without sacrificing on the other objectives.

Best Compromise Solution

The best compromise solution is an efficient solution that maximizes the Decision Maker's (DM) preference function. The preference function is the utility function of the DM and is not explicitly known.

Methods to Solve Multi criteria optimization problems:

Methods for solving the multi criteria problems can be categorized into three types:

1) **All the necessary information required for solving the problem is obtained from the decision maker beforehand:**

The most popular method under this category is *Goal Programming* [3] and the problems that fall into this category are classified as *preemptive* and *non preemptive* goal programming. In non preemptive goal programming, weights are obtained from the Decision Maker(s) by ranking the criterion using different methods such as simple rating, Borda count, Pair wise comparison and Analytic Hierarchy Process [3]. Based on the weights the problem is converted into a single objective optimization problem and solved.

In many cases it is not possible to assign weights to criteria and the Decision Maker (s) may be inconsistent in their responses. It is easier in such cases to determine which criteria are more important to the DM and solve the multi criteria problem in a hierarchical manner such that the goals with the higher priority are satisfied as far as possible before the lower priority goals are considered. These problems are called the pre-emptive goal programming problems. Once the priorities are known, they can be solved using Sang Lee's simplex method [4] or use the more efficient Partitioning Algorithm for Goal Programming (PAGP) developed by Arthur and Ravindran [5].

2) All the efficient solutions are generated without any prior information:

In this category, the DM's preference is not asked but for a given problem, the efficient set is generated and the DM is given the entire efficient set and asked to choose the preferred solution from it. Geoffrion's parametric formulation [6] can be used to generate the entire efficient set. Zeleny's Compromise Programming [7] is another method used to solve these problems.

3) Interactive methods:

The methods that solve the multi-criteria problems through continuous interaction with the DM are known as interactive methods. The first step is to generate a set of efficient solutions and present it to the DM. The DM specifies his preference and based on his response additional efficient solutions are generated thereby reducing the region of interest till we reach the best compromise solution. STEM/STEP method [8], Zionts-Wallenius Method [9] and the Paired Comparison method developed by Sadagopan and Ravindran [10] are examples of some of the interactive methods. An exhaustive study of the interactive methods used to solve multi-criteria problems has been done by Shin and Ravindran [11].

In this thesis a four stage centralized supply chain is considered with multiple suppliers, a manufacturer, a warehouse and multiple retailers. The objective of the model is to minimize overall costs and maximize the customer responsiveness. The results generated from the model help in making decisions regarding the inventory and transportation at each of the four stages of the supply chain. Chapter 2 consists of literature review on topics relevant to the research work such as performance measures, inventory control, transportation cost functions and multi-criteria optimization. Chapter 3 presents the problem description and the model formulation. The model is illustrated with an example problem and the results are analyzed in Chapter 4. The conclusions and the scope for future work are presented in Chapter 5.

CHAPTER 2

LITERATURE REVIEW

This chapter discusses the relevant work from the literature before moving into the model description and formulation.

Performance Measures

There is a large number of performance measures used to characterize the supply chain and hence the selection of relevant performance measures for the system under consideration becomes a tedious job. Benita Beamon [12] in her work mentions that there are two performance measures that have been consistently used for supply chain models which include cost and a combination of cost and customer responsiveness. The author identifies certain measures such as customer satisfaction, information flow and flexibility, the qualitative nature of which makes it difficult to be incorporated as quantitative models. The strengths and weaknesses of the performance measures used in a model can be evaluated using criteria such as inclusiveness, universality, measurability and consistency. The use of a single performance measure, say cost is non-inclusive as minimizing cost could lead to increase in customer response time. The strategic goals of a company often point to many performance measures and each measure can be quantified using several methods. The performance measures have been classified into three types namely resource, output and flexibility by the author. Resources include total cost, distribution costs, manufacturing cost, inventory cost and return on investment and the objective is to minimize the utilization of the resources. Output measures include customer responsiveness, quality and quantity of final product produced and they can be numerically measured using number of items produced, time required to produce and number of on-time deliveries. On a broader scale the output performance measures are made up of sales, profit, fill rate, on-time deliveries, backorder, customer response time, manufacturing lead time, shipping errors and customer complaints. Flexibility which is rarely part of the performance measurement can help measure the supply chain's ability to adapt to changes and fluctuations such as increase in orders or placement of orders that require a quicker delivery than usual.

Gunasekaran et al. [13] emphasize the need for performance measures for efficient management of the supply chain though the number of performance measures should be limited to a few good measures to maintain a balanced approach to meet the goals of the company. The authors develop a framework for measuring the performance that is classified at the strategic, tactical and operational levels in supply chains and the measures include both financial (e.g. Cost) and non-financial (e.g. Customer responsiveness) performance measures. This framework of performance measures mainly deals with planned order procedures, supplier relationship, production, delivery, customer service and inventory and logistics cost.

Inventory Control and Transportation decisions

With cost and customer responsiveness identified as the two major performance measures, cost can be quantified using transportation and inventory. The results of the model that will be presented in this thesis will help in making the transportation and inventory decisions for the supply chain considered. A survey of the relevant work in the literature that has been done on modeling inventory control and transportation decisions will help the formulation for the problem under consideration in this work.

The paper by Newhart et al. [14] discusses on the solution to a problem faced by Bethlehem Steel Corporation which emphasizes the need for proper inventory control in the industry. The manufacturing plant is located in Maryland whereas the growing market for their product is in the south-western United States. In order to provide excellent customer service while minimizing the investment made on inventory the company chooses between four alternatives which is a combination of two strategies one being building new facilities near the markets which involves huge capital and the other is to stock inventory which increases inventory holding costs. A mathematical formulation for a consolidated coil size problem is introduced followed by determining inventory levels at each point in the supply chain namely pipeline or in-transit inventory, work in process inventory, interdependencies of stages and safety stock inventory. The best alternative is determined by using the model developed to obtain results for all four alternatives. Even though the model is not a bi-criteria optimization problem, it is a classic example of considering both cost and responsiveness while making supply chain decisions.

Cohen and Lee [15] consider a network that connects suppliers of raw materials, manufacturers of both intermediate and finished products, warehouses, distribution channels and customers who are geographically dispersed. The resource deployment decisions involve the design of the network which includes both the location and capacity of all the sites and the material flow management within this network. The value-added chain model formulated by the authors has increased flexibility as it can respond to different scenarios and adjust the sourcing, production and distribution plans. Cohen and Moon [16] extend Cohen and Lee [15] by using a constrained cost minimization problem called PILOT to study the effect of supply chain costs on network facility configuration, plant production charter and distribution policy. PILOT accepts as input the fixed costs, transportation costs and the production costs and provides us output on opening/closing of manufacturing and distribution centers, order quantities and production quantities for all locations and the amount of products to be shipped between all the locations. The results obtained from the analysis of using a particular class of supply chain design problems indicate that there are a number of factors that dominate the supply chain costs under various scenarios but the transportation costs have a great impact across the supply chain and on the overall manufacturing strategy.

Bookbinder and Chen [17] have formulated and solved for a multi-criteria trade-off in a two echelon serial inventory/distribution system consisting of a warehouse and a retailer. They solve the problem for both the deterministic and stochastic demand cases. For the deterministic demand case the two objectives are the annual inventory cost which is a sum of the total holding costs and the ordering costs at both the stages and the annual transportation costs. The order quantity at the warehouse is assumed to be an integer multiple of the order quantity at the retailer. For the probabilistic demand case they construct a non-linear integer programming model with customer service as the third objective. The customer service is quantified in terms of the expected number of stockout occasions per year.

Difillipo [18] solves a multi-criteria optimization problem with three criteria, namely, capital invested in inventory, frequency of orders and transportation cost. He considers a single warehouse multiple retailer supply chain with deterministic demand and fixed lead time and solves it for both the decentralized and the centralized cases. The author incorporates a freight rate function that best emulates the freight rates into both the cases. Under the decentralized case, the most preferred solution to the supply chain problem is solved for both the fixed order policy and the non-stationary order policy. Under the centralized case, common retailer replenishment time and different retailer replenishment time are the two policies for which the most preferred solution is obtained. The policies under each case were compared with single objective problems using marginal costs because the multi-criteria solutions uses weights and are unique to a given problem. It is a well know fact that the centralized supply chain will perform better than the decentralized supply chain. The example problem solved shows that the decentralized fixed order policy dominates both the policies under the centralized case and hence contradicting the well known result. This result is attributed to the freight rate structure.

Ajay Natarajan [19] considers a three stage supply chain with a single manufacturer, single warehouse and multiple retailers with one product flowing through this decentralized supply chain. The manufacturer is assumed to have infinite capacity and the objective is to obtain the ordering policies for all the retailers and the warehouse under different scenarios. In the first section a single objective cost model is developed assuming that the retailer faces deterministic demand and that there is instantaneous replenishment at both the warehouse and the retailer. A control policy is developed by the author that helps the warehouse better manage its inventory and group the retailers into clusters according to their importance to the warehouse. In the next section a multi-criteria supply chain model is developed with three criteria namely capital invested in inventory, annual number of orders placed and the annual transportation costs. The transportation costs are modeled using full truck load at the warehouse and less than full truck load at the retailer. Continuous review policies are followed at each location, demand is deterministic at the retailer and the lead times are fixed. The weights assigned to the criterion are changed to generate the efficient solutions and value path method is used to visualize the tradeoff information that assists the DM (Decision Maker) to choose the most preferred solution at each location. In the last section the multi-criteria model is extended to a case where both demand and lead time are random. Fill rate which is defined as the proportion of demand met from the available inventory is added as the fourth criteria to measure the customer satisfaction. The solution to the problem is similar to the previous section where the weighted objective method is used in conjunction with the value path method to enable the DM to choose the most preferred solution.

Gaur and Ravindran [20] in their paper on inventory aggregation or risk pooling emphasize the fact that minimizing cost is not the sole priority for supply chains anymore and that maintaining a high level of customer responsiveness is an equally important objective. They develop a bi-criteria non-linear stochastic integer programming model that determines the best possible distribution network system which meets the customer demands while minimizing costs and maximizing the customer response time. The cost function is calculated as a sum of inventory holding cost, facility cost or cost needed to setup warehouses, operating cost or the cost needed to run these warehouses and the transportation costs. Responsiveness is calculated in terms of product miles which is the product of the number of units shipped to the retailers and the distance traveled by these units, the minimization of which maximizes the responsiveness. A two stage optimization algorithm is developed by the authors to solve this problem which reduces the number of alternatives needed to be ranked by the DM.

Inventory decisions made without taking into consideration the transportation costs would fail to take advantage of the economies of scale in shipping. This fact is emphasized in the paper by Baumol and Vinod [21] and they are first to address this issue. Their inventory theoretic approach involves a trade-off among freight rates, speed (average transit time), dependability (variance of transit time) and losses en-route to choose the optimal mode of transport. A slow mode of transport which is considered economical, delays the recipient from acquiring his product. There are no disadvantages if shipments are made at regular intervals which imply that the transit time does not affect the receipt of products. It is here that the inventory theory brings into light the advantages of speed. The freight in-transit is analogous to the work in process inventory and hence a slower mode of transport results in larger in-transit inventory. In case of an unanticipated demand, a special order is placed and the shipment being delayed en route could lead to a loss of order and customer satisfaction. As the uncertainty and in-transit time increases it is necessary to increase the safety stocks to overcome such contingencies which in turn increase the inventory holding costs. Initially a deterministic model is developed which provides the optimal order and cost. In order to present the impact on safety stock, demand and lead time are considered as random variables with demand following a Poisson distribution and lead time following a normal distribution.

The work of Baumol and Vinod [21] is further extended by Constable and Whybark [22] by including backorders in the inventory theoretic model. They discuss the interaction of transportation and inventory decisions and determine the transportation alternative and the inventory parameters that lead to the minimal transportation and inventory cost. A computationally complex enumeration procedure is developed which is an iterative approach used to decrease the value of the reorder point till the cost function begins to increase. This procedure is repeated for all the transportation alternatives and the least cost alternative is finally selected. The lead time was assumed to be normal for the heuristic method they developed to determine the inventory parameters for a particular alternative. The transportation alternatives were evaluated based on the inventory parameter and the least cost alternative was chosen. The heuristics were applied to an example and a company data and the results were close to the actual solution. The solutions favored the lowest cost transportation alternative and hence the authors conclude that the transportation cost ratio can be used a rational basis for separating the inventory and transportation decisions.

Swenseth and Godfrey [23] analyze the effectiveness of five continuous freight rate functions and compared them to one another or against actual rates to determine how well they emulate actual freight rates for 40 different routes. They used the minimum squared difference (MSD) test to select the best function. The functions that were considered in this study are constant, proportional, inverse, adjusted inverse and exponential functions.

The proportional function dominates the other functions because of its simplicity and its ability to accurately match the actual freight rate function but there is no consideration of its effect on the logistics decisions. The exponential function is the most complex and can underestimate or overestimate the freight rates. The inverse function performs better when high shipping weights are used emulating the Truck Load (TL) rates whereas the adjusted inverse emulates the Less than Truck Load (LTL) rates. They also concluded that a combination of the inverse and the adjusted inverse function would perform better than the proportional function [24]. The inverse and the adjusted inverse functions were incorporated into the logistics cost function to study their impact on the purchasing decisions. The procedure developed by the authors can be used to determine the minimum weight at which a shipment can be over declared as a TL shipment. Regression was used to predict the over declared point significant at a level of 0.0001 and explained approximately 75% of the variance in the actual over declared points. The procedure then provides the best solution for the parameters considered and it is proven that the model was correct 99% of the time was shipments that should have been over declared and for the shipments that should not have been over declared it was correct 75% of the time. This shows that the freight rate functions can be incorporated into the inventory replenishment decisions without compromising the accuracy of the decision and without adding undue complexity into the decision process.

Kuzdrall and Britney [25] proposed a total setup lot sizing model (T-S) that determines the critical interval containing the least cost quantity given the supplier's price schedule. They proposed a new algorithm for locating the optimal order quantity when inventory costs are proportional to purchase price and quantity discounts are quoted and the algorithm is based on the supplier's formulated price schedule. The T-S model developed by the authors reduces the computations required compare with the traditional approaches which were tedious. The first step in their approach is to use the least squares linear regression of the supplier's price schedule to determine the supplier's fixed and variable cost components. The fixed and the variable costs are used with the buyer's ordering costs to determine the EOQ and the breakpoints for the interval surrounding the EOQ are calculated using the condition of Inequality to evaluate the appropriate order quantity. The authors describe this method as "convenient alternative to the aimless searching of traditional approaches" which "reduces the computations required to find the least-total-cost quantity". Their claim to superiority of the T-S model to the classical search methods is examined by Rubin et al. [26]. They describe a modified version to both the classical approach and the T-S model along with its algorithm. The comparison of the two methods was based on the computational efficiency of the models as well as the ability of the model to locate the globally optimal order quantity. From their findings they concluded that the T-S algorithm does not provide the optimal order quantity for all scenarios although it did so for several examples. The algorithm requires computational less effort than the classical approach only when applied to very large problems. The authors claim that the modified classical algorithm presented by them is an efficient algorithm which provides the optimal order quantity even though it is a brute force approach. Kuzdrall et al. [27] state that the T-S model performs poorly but only on poorly formulated price schedules and claim that these poor results should not be considered as a drawback as it stems from bad data.

Two algorithms for optimal inventory and transportation decisions for freight discounts based on the inventory theoretic model formulation was developed by Tersine et al. [28]. In their model the freight rates are structured with weight based discounts for both the all weight freight discount and the incremental freight rate discount cases. The objective of the model is to determine the optimal replenishment order size with the assumption that management strategy is to minimize the long term costs. The additional assumptions made in this model are single product with independent demand, deterministic demand and lead time, replenishment rate is infinite and no stockouts are allowed. Optimal lot sizing algorithms were developed for both the freight discount cases and can be applied for both single and multiple modes of transportation scenarios. The algorithm helps in choosing the lowest cost alternative. With organizations moving to becoming more responsive to customers at a lower cost, decision support systems such as the one developed by the authors are good investments for removing cost out of operations and using the resources more effectively.

Similar to Baumol and Vinod [21], Russell and Krajewski [29] consider the effect of transportation cost on the optimal order quantity, the difference being the latter has included freight rate discounts into their proposed model. They present a simple analytical procedure that minimizes the total purchase costs while computing the optimal order quantity that reflect both the transportation economies and the quantity discounts. A total purchase cost expression is presented which properly takes into consideration the actual freight rate structure after solving for a series of indifference points in that particular freight rate schedule. The optimal purchase order quantity would be one of the following four possibilities: (1) economic order quantity (EOQ); (2) a purchase price breakpoint which is in excess of the EOQ; (3) a transportation rate breakpoint which is in excess of the EOQ; (4) a modified EOQ which provides an over declared shipment which is in excess of the EOQ. The algorithm presented by the authors scan through the four possibilities in a systematic fashion to result in the optimal purchase order quantity. The important applications of this paper in the industry is to over declare the shipment to the next breakpoint when the shipment is less than the rate breakpoint but greater than the cost indifference point between two adjacent rates in order to take advantage of the lower cost associated to the entire shipment in a freight rate discount structure.

Qu et al. [30] provide an integrated approach where decisions for both inventory and transportation are made simultaneously. The supply chain considered for this research work is a centralized warehouse and several geographically dispersed suppliers with multiple items and stochastic demand. The objective of the model is to determine the inventory control rules and routing patterns that makes it possible for the centralized warehouse to meet its demand at minimum long-run total cost per unit time. The total costs considered in this model consists of transportation cost which is the sum of dispatching, stopover and routing costs and inventory costs which is made up of ordering, holding and backorder costs. The solution method assumes a modified periodic review inventory policy and developed with a traveling salesman component. The solution method decomposes the model into the inventory problem and the transportation problem and solves by iterating between the two problems. The method is tested by varying the number of items in the supply chain and the authors conclude that the model works better when the number of items in the system is large given the number of suppliers.

Tyworth and Zeng [31] present a sensitivity analysis approach to estimate the effect of carrier transit time performance on logistics cost and service. They emphasize the joint determination of an appropriate mode of transport and an optimal inventory control policy in supply chain management. Their methodology relaxed the assumption that shipping cost is a linear function of the order quantity unlike the traditional inventory transportation models. The lead time is assumed to be made up of transit time which is a discrete random variable and the fixed order processing time. The demand was assumed to follow the gamma distribution and fill rate is used as a service measure. The gamma distribution assumption illustrates the flexibility of the method. They recommend the use of power function for modeling transportation costs. The methodology is best suited for the transportation of a service-sensitive, independent demand inventory item controlled by a continuous review inventory system.

Multi-criteria Optimization

The literature on performance measures such as cost and customer responsiveness, inventory control and transportation decisions have been discussed so far. Incorporating these into the multi-criteria optimization model forms one part of the research and solving this model forms the other. The literature presented below discuss on methods to solve multi-criteria optimization problems.

Masud and Ravindran [3] in their chapter on multi-criteria decision making, present an overview of multi-criteria optimization techniques. The chapter discusses the different methods for computing the criteria weights. The methods used to solve the multi-criteria problems with finite and infinite number of alternatives are discussed. The use of goal programming, compromise programming and other interactive methods are presented followed by a brief section on the applications of multi-criteria decision making models. Evans [32] provides an overview of multi-criteria mathematical programming and some techniques used to solve it. The author specifies that the algorithms involve a combination of three types of approaches namely prior, progressive and a posteriori articulation of preferences. There have been work done to compare the approaches for solving the multi-criteria problems in the literature but the results remain inconclusive. This is because of the applications of these approaches being very few. The work concludes by saying that the best approach for a particular problem depends on the characteristics of the problem situation.

The two major approaches to solve multi-criteria problems involve prior articulation and progressive articulation of preference information. Paired Comparison Method (PCM) which is a progressive articulation (interactive) optimization approach is compared to a prior articulation (utility measurement) approach by Klein et al. [33]. They conclude that interactive procedures result in more satisfactory solutions and are considered easier to use. However, the utility measurement methods gave better insight into the problem structure. The conclusions are based on a study of a quality control problem using management students as decision makers. Ravindran and Shin [11] present an exhaustive survey of the interactive methods for solving continuous multi-criteria optimization problems and their applications. They classify the techniques found in the literature as feasible region reduction methods, feasible direction methods, criterion weight space methods, tradeoff cutting plane methods, Lagrange multiplier methods, visual interactive methods, branch and bound methods, relaxation methods, sequential methods and scalarizing function methods.

Lee [4] presents a modified simplex procedure to solve goal programming problems. It is an algorithmic procedure that involves an iterative process to achieve the optimal solution through progressive operations. Goal programming uses preemptive priority factors in the objective function and hence the simplex criterion is expressed as a matrix rather than a single row like in linear programming. The optimal column is selected by choosing the highest priority level over the lower levels of priority and the variable in that column will enter the basis. The variable that leaves the basis is determined followed by the new basic feasible solution. Finally the goal attainment is analyzed and the value of the simplex criterion will indicate if the optimal solution has been reached.

An efficient algorithm to solve the linear goal programming problem has been developed by Arthur and Ravindran [34]. The efficiency of the algorithm is achieved by the use of constraint partitioning and variable elimination. The algorithm takes advantage of the definition of the ordinal priority factors in the goal programming objective function to partition the goal constraints of the problem. The definition of the ordinal priority factors implies that the higher priority goals must be optimized first before the lower priority goals are even considered. Hence a sequence of smaller sub-problems is solved by using the solution of the higher priority problem as the starting solution to the lower priority problem. The algorithm proceeds until no alternate optimum exists for one of the sub-problems or until all the priorities have been included in the procedure. The fewer constraints involved in the sub-problem leads to a smaller basis and this helps the partitioning algorithm to find the optimal solution is lesser time compared to the other methods. The comparisons with the Lee's simplex method result in the partitioning algorithm taking as little as 12% and a maximum of 60% of Lee's time and consistently provide better results for larger problems.

Sadagopan and Ravindran [10] develop an efficient interactive method called the Paired Comparison Method to solve bi-criteria optimization problems. The assumption for this method is that the DM's preference function is a quasi concave function. This method works by finding the best and the worst values for a reference objective. Between these two extreme points, two intermediate efficient points is generated by solving two single objective optimization problems. These two intermediate solutions are presented to the DM and asked for a preference between them. The DM's preference eliminates a portion of the efficient frontier and reduces the search space. This process is repeated until it eventually converges to a best compromise solution. In addition to the bi-criteria problems, Sadagopan and Ravindran [35] have developed interactive methods to solve the multi-criteria non-linear programming problems as well.

Mysore [36] considers a three stage centralized supply chain with multiple modes of transport among all the stages over a multiple period time horizon. The option of a leased warehouse facility is considered in case the company owned warehouse exceeds its limited capacity. Quantity discount models are incorporated into the leased warehouse cost function and the transportation cost function. The work is a single objective optimization model that maximizes profit, which is the difference in the revenue generated and the sum of transportation, inventory and backorder costs. A comparison of two scenarios with a low dollar value item in one and a high dollar value item in the other is done by the author to observe the transportation and inventory decisions made by the model. There are striking differences identified by the author between the two scenarios with the low dollar value item having more inventory stocked, less backorders and fewer shipments made between the manufacturer and the warehouse whereas the high dollar value item having less inventory stocked, more backorders and frequent shipment using faster modes of transport between the manufacturer and the warehouse. Though the results are consistent with the trends in a real world supply chain, there are opportunities for extending this work to make it more general. This thesis extends Mysore's work by adding the supplier stage into the supply chain and also solving a bi-criteria optimization model considering both cost and customer responsiveness as the objectives of the model.

CHAPTER 3

PROBLEM DESCRIPTION AND FORMULATION

3.1 PROBLEM DESCRIPTION

We consider a multi-period four stage centralized supply chain with multiple suppliers, a manufacturer, a warehouse and multiple retailers (See Figure 3-1). There are different modes of transport between the supplier and the manufacturer, the manufacturer and the warehouse and between the warehouse and the retailer. The shipper can choose from different modes of transport such as air, rail, ocean and truck for shipping either the raw materials or the finished products between stages. Each mode of transport has its freight rate structures and the lead times associated with them. Only one product flows along the supply chain over a multiple period time horizon. Each product is made up of 'N' raw materials which are made available by 'N' different suppliers, i.e., each raw material is associated with one supplier. Once the retailer places an order, the raw materials are shipped from the suppliers to the manufacturer. The manufacturer waits till it receives all the raw materials needed to produce the finished product. The manufacturer uses the raw materials to produce the finished products and ships them to the warehouse. The finished products can be stored at a company owned warehouse which has a finite storage space. If the capacity of the company owned warehouse is exceeded the company has an option of leasing storage space from outside. The leased warehouse has quantity discount rates associated with storing the finished products. The warehouse then ships the finished products to the different retailers depending on the order received from each retailer. The objectives of the model are (1) to minimize the overall cost which includes the transportation cost and inventory holding cost and (2) maximize the responsiveness of the supply chain over the planning horizon.

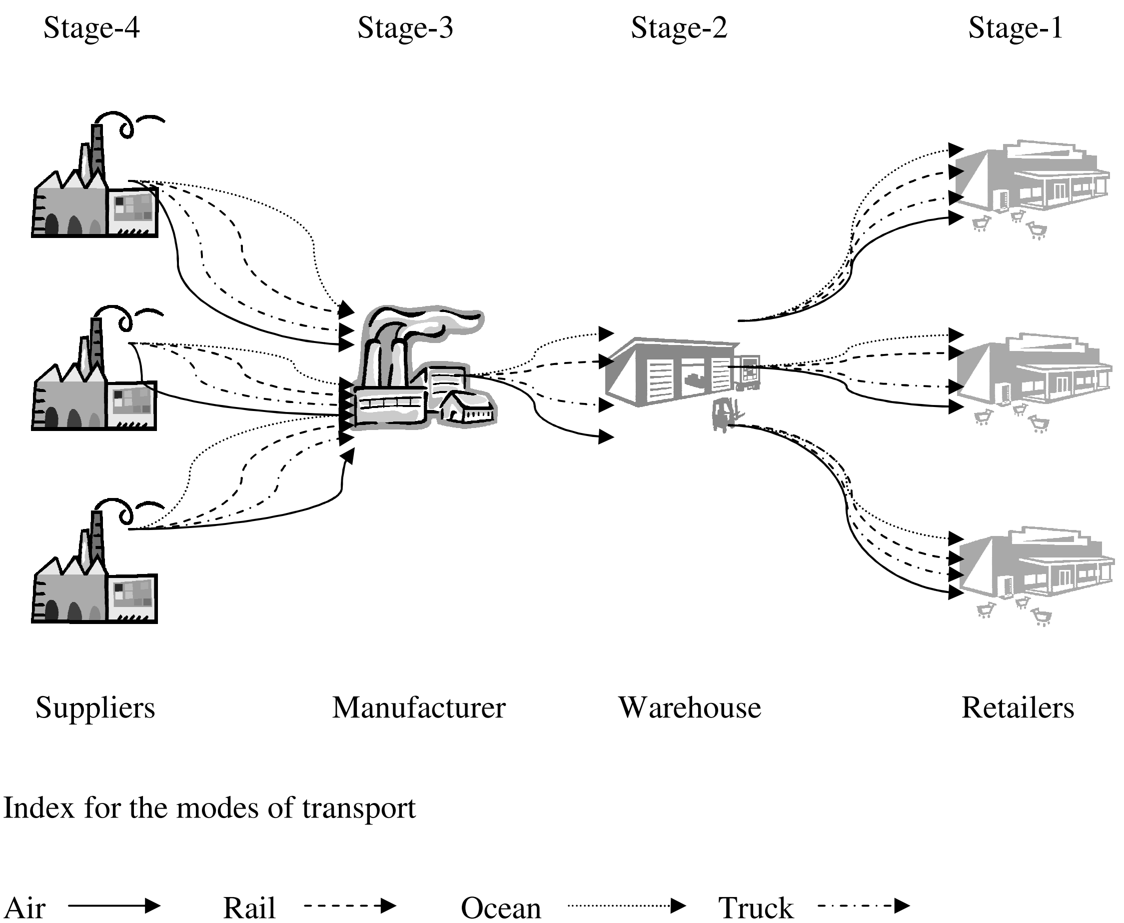

Figure **3-1**: Four stage centralized supply chain

The novel part of this work is integrating the transportation and inventory problem and solving this integrated model for both minimizing cost and maximizing customer responsiveness. We solve this bi-criteria problem by the preemptive goal program by assigning priorities to the two criteria and interchanging them. The problem is also solved using non pre-emptive goal program by means of assigning weights to each criteria. The solutions are obtained by incorporating deviation variables to the criteria and minimizing them. The solutions are then compared among themselves to identify the changes in the transportation and inventory decisions.

3.2 ASSUMPTIONS

- We consider a four-stage centralized supply chain with several suppliers, a manufacturer, a warehouse and several retailers.
- There are multiple time periods, $t = 1, 2, \ldots, T$.
- There is only one product which flows in the supply chain.
- The product is made up of 'N' number of raw materials.
- Each raw material is obtained from one supplier. If there are 'N' number of raw materials then there are 'N' number of suppliers.
- There are several modes of transport between the supplier and the manufacturer, the manufacturer and the warehouse and the warehouse and the retailer to choose from (e.g. Rail, air, truck and ocean).
- Transportation cost and the delivery time vary among the different modes.
- The incremental quantity discount model is used to structure the cost ranges for each mode of transport.
- The company owns a finite storage space at the warehousing facility.
- The company can lease additional storage space at the warehouse stage to store finished products.
- The incremental quantity discount model will be used for calculating costs at the leased storage space.

- The shipments arriving from the manufacturer are first stored in the company owned warehouse. The leased warehouse space is utilized once the company owned warehouse space is completely filled.
- The shipments to the retailer from the warehouse are first done from the leased warehouse before utilizing the products from the company owned warehouse.
- Inventory holding cost for the raw material that is being shipped from the supplier to the manufacturer begins only when the raw material reaches the manufacturer.
- Inventory holding cost for the product that is being shipped from the manufacturer to the warehouse begins only when the product reaches the warehouse.
- Inventory holding cost is calculated based on ending inventory position at each time period.
- Inventory holding cost varies for the manufacturer and the warehouse.
- There is no provision for holding inventory at the retailer.
- No backorder is possible at the supplier.
- All the demand must be satisfied by the end of the planning time horizon.

3.3 Model Formulation

The model that is developed for this problem is a mixed integer linear programming model. The non-linear cost functions that represent quantity discounts for the leased warehouse cost structure and the freight cost structure are transformed to linear functions using binary variables. A detailed description of all the variables and constraints is presented in this section.

The index sets used in the model formulation are:

t = time periods (1, 2, …, T)

n = number of raw materials (1, 2, …, N)

j = number of time periods needed to manufacture a product from raw materials

i = modes of transport from supplier to manufacturer (also represents lead time)

k = index for the transportation cost ranges from the supplier to the manufacturer

h = modes of transport from manufacturer to warehouse (also represents lead time)

l = index for the transportation cost ranges from the manufacturer to the warehouse

q = modes of transport from warehouse to retailer (also represents lead time)

s = index for the transportation cost ranges from the warehouse to the retailer

r = number of retailers (1, 2, …, R)

p = index corresponding to the cost bracket in the leased warehouse.

The time periods can be expressed as days, weeks or months specific to the problem that the model is used for. The transit time of all the modes of transport is an integer multiple of this base time period.

The decision variables that will be used in this model are:

$X_{n\,i\,t}$ = quantity of raw material 'n', shipped from supplier 'n' to the manufacturer using transportation mode 'i' in time period 't'

$Y_{h\,t}$ = quantity of products shipped from the manufacturer to the warehouse using transportation mode 'h' in time period 't'

$Z_{r\,q\,t}$ = quantity of products shipped from the warehouse to retailer 'r' using transportation mode 'q' in time period 't'

$TRM_{n\,t}$ = total quantity of raw material ‘n’ at the manufacturer in time period ‘t’

PM_t = total number of finished products at the manufacturer in time period ‘t’

$QTSM_{n\,i\,t\,k}$ = quantity of raw material ‘n’ shipped from supplier ‘n’ to the manufacturer using transportation mode ‘i’ in time period ‘t’ and cost bracket ‘k’

$QTMW_{h\,t\,l}$ = quantity of finished products shipped from the manufacturer to the warehouse using transportation mode ‘h’ in time period ‘t’ and cost bracket ‘l’

$QTWR_{r\,q\,t\,s}$ = quantity of products shipped from the warehouse to retailer ‘r’ using transportation mode ‘q’ in time period ‘t’ and cost bracket ‘s’

$WINV_t$ = total inventory at the company owned warehouse at the end of time period ‘t’

$TLWINV_t$ = total inventory at the leased warehouse at the end of time period ‘t’

$LWINV_{p\,t}$ = total inventory at the leased warehouse in cost range ‘p’ and time period ‘t’

MBO_t = cumulative backorders at the manufacturer at the end of time period ‘t’

WBO_t = cumulative backorders at the warehouse at the end of time period ‘t’

$RBO_{r\,t}$ = cumulative backorders at the retailer ‘r’ at the end of time period ‘t’

WD_t = demand from the warehouse in time period ‘t’

$\delta_{n\,i\,t\,k}$ = binary variable {0,1} indicating whether or not shipment is made from supplier ‘n’ to the manufacturer using transportation mode ‘i’ in time period ‘t’ and cost bracket ‘k’

$\theta_{h\,t\,l}$ = binary variable {0,1} indicating whether or not shipment is made from the manufacturer to the warehouse using transportation mode 'h' in time period 't' and cost bracket 'l'

$\Delta_{r\,q\,t\,s}$ = binary variable {0,1} indicating whether or not shipment is made from the warehouse to retailer 'r' using transportation mode 'q' in time period 't' and cost bracket 's'

$\alpha_{p\,t}$ = binary variable {0,1} indicating whether or not warehouse spaced is leased using cost bracket 'p' in time period 't'

The given data includes:

a_n = fraction of raw material 'n' needed to produce one finished product

$MCAP$ = production capacity of the manufacturer (same for all time periods)

$WINVCAP$ = inventory holding capacity at the company owned warehouse

$SMMAX_i$ = maximum quantity of raw materials that can be shipped from supplier to the manufacturer using transportation mode 'i' (same capacity for all time periods)

$SCAP_n$ = production capacity of the supplier 'n' to provide the raw material 'n' (same for all time periods)

$MWMAX_h$ = maximum quantity of products that can be shipped from the manufacturer to the warehouse using transportation mode 'h' (same capacity for all time periods)

$WRMAX_q$ = maximum quantity of products that can be shipped from the warehouse to the retailers using transportation mode 'q' (same capacity for all time periods)

$CD_{r\,t}$ = demand at the retailer 'r' in time period 't'

$INVCR_n$ = inventory holding cost of raw material 'n' at the manufacturer per unit per time period

$WINVC$ = inventory holding cost at the company owned warehouse per unit per time period

$LWINVC_p$ = inventory holding cost at the leased warehouse per unit per time period in the cost bracket 'p'

$TC_{i\,k}$ = transportation cost from supplier to the manufacturer using transportation mode 'i' and cost bracket 'k'

$TCM_{h\,l}$ = transportation cost from the manufacturer to the warehouse using transportation mode 'h' and cost bracket 'l'

$TCC_{q\,s}$ = transportation cost from the warehouse to the retailers using transportation mode 'q' and cost bracket 's'

3.3.1 Leased Space and Freight Rate Cost Function

The 'Incremental' quantity discount structure is used to model the leased space and freight rate cost functions. This quantity discount structure is different than the 'All-Units' quantity discount structure. The incremental quantity discount structure is one, where a discounted cost is associated with the portion of the order that satisfies that particular cost bracket (See Figure 3-2). A cost of 'C_1' is charged per unit when the quantity lies between '0' and 'k_1' and a cost of 'C_2' is charged when the quantity lies between 'k_1' and 'k_2'. The variable 'x' is the quantity used and the function f(x) is the total cost of the quantity being used. The total cost function can be written as, f(x) = C_1*(min(x, k_1)) + C_2*(min(max(0, x-k_1), k_2) + .. (etc.). This cost structure with the min and max functions can be implemented by using binary variables. For example, take a quantity 'x' between 'k_1' and 'k_2'. The total cost function, f(x) = C_1* k_1 + C_2*(x-k_1).

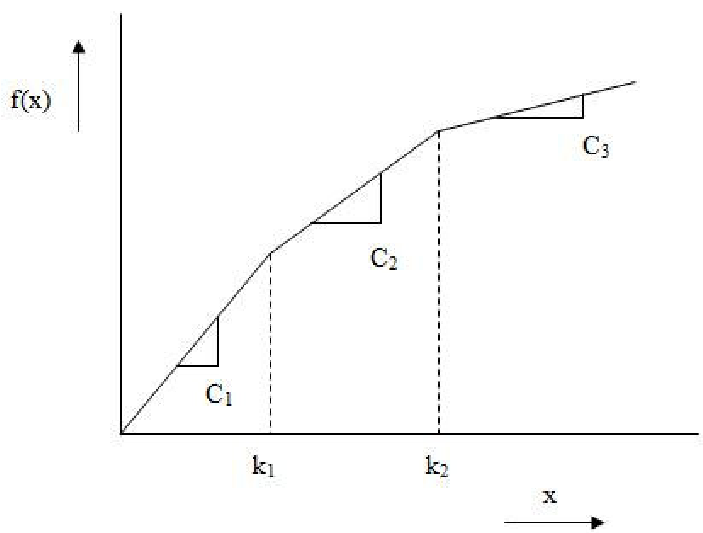

Figure **3-2**: Incremental quantity discount structure

The cost structure of the transportation quantity between the suppliers and the manufacturer is presented in Table 3-1. The first value in the cost bracket 'k', $\beta_{i\,1}$ is zero. Suppose the quantity shipped (X_i) lies between $\beta_{i\,k}$ to $\beta_{i\,k+1}$, then the total cost for the transportation mode 'i' will be $\beta_{i\,2} * TC_{i\,1} + \beta_{i\,3} * TC_{i\,2} + \cdots + \beta_{i\,k} * TC_{i\,k-1} + (X_i - \beta_{i\,k}) * TC_{i\,k}$. There are different cost ranges ('k') for the different modes of transport ('i'). For example, shipment by truck might consist of two cost ranges, namely, Truck Load (TL) and Less than Truck Load (LTL) whereas the shipment by air might consist of three cost ranges. The same transportation cost structure is assumed for all suppliers.

Mode	Cost range	Quantity bracket	Price per unit ($)
i	k = 1,2,…	$\beta_{i\,k}\ to\ \beta_{i\,k+1}$	$TC_{i\,k}$

Table 3-1: Cost structure of transportation quantity between supplier and manufacturer

The cost structure of the transportation quantity between the manufacturer and the warehouse is presented in Table 3-2. Suppose the quantity shipped lies between $\gamma_{h\,l}$ to $\gamma_{h\,l+1}$, a cost of $TCM_{h\,l}$ is charged per unit for all the products in that bracket.

Mode	Cost range	Quantity bracket	Price per unit ($)
h	l = 1,2,…	$\gamma_{h\,l}\ to\ \gamma_{h\,l+1}$	$TCM_{h\,l}$

Table 3-2: Cost structure of transportation quantity between manufacturer and warehouse

The cost structure of the transportation quantity between the warehouse and the retailer is presented in Table 3-3. Suppose the quantity shipped lies between $\tau_{q\,s}$ to $\tau_{q\,s+1}$, a cost of $TCC_{q\,s}$ is charged per unit for all the products in that bracket. The transportation cost structure is assumed to be the same for all retailers.

Mode	Cost range	Quantity bracket	Price per unit ($)
q	s = 1,2,…	$\tau_{q\,s}\ to\ \tau_{q\,s+1}$	$TCC_{q\,s}$

Table 3-3: Cost structure of transportation quantity between warehouse and retailer

The leased warehouse space has a cost structure which is presented in Table 3-4. The cost of the quantity stored in the leased space per unit is $LWINVC_p$ when the amount of quantity stored lies in the S_p to S_{p+1} quantity range.

Cost range	Quantity bracket	Price per unit ($)	Total cost ($)
p = 1,2,…	$S_p\ to\ S_{p+1}$	$LWINVC_p$	$LWINV_{p\,t} * LWINVC_p$

Table 3-4: Cost structure of leased warehouse space

The binary variables, such as $\delta_{n\,i\,t\,k}$, $\theta_{h\,t\,l}$, $\Delta_{r\,q\,t\,s}$ and $\alpha_{p\,t}$, are used in the above mentioned quantity discount structures respectively to convert the non-linear function into a linear function. It is essential to convert non-linear functions to linear functions, because the solution of a non-linear integer program is usually more difficult and time consuming.

3.3.2 Cost Function and Constraints at each Stage

The cost function includes the transportation cost and inventory holding cost that occur at each stage. The constraints at each stage include but not limited to the inventory balance constraints and the demand constraints.

3.3.2.1 MANUFACTURER

The raw materials arrive at different time intervals depending on the mode of the transport used to ship it from the supplier to the manufacturer. The materials that arrive in time period 't' are shipped by the supplier at time period 't-i' (where 'i' is the mode of transport and equal to the number of time periods it takes to arrive when sent by that mode). After the raw materials arrive at the manufacturer, all the raw materials are used in the defined ratio to manufacture the product. It takes 'j' time periods to manufacture a product from the raw materials. The raw materials in the manufacturing facility are considered as work in process inventory and have costs associated with it. The finished products are immediately shipped to the warehouse for storage. The manufacturing facility is not equipped with a storage facility as we have installed a company owned warehouse for this purpose.

Costs associated with the Manufacturer

The transportation cost of the raw materials from the supplier to the manufacturer is given by,

$$\sum_{t} \sum_{n} \sum_{i} \sum_{k} QTSM_{n\,i\,t\,k} * TC_{i\,k}$$

Let 'j' time periods be required for manufacturing the product from the raw materials. Hence the raw materials are held as work in process inventory for duration of 'j' time periods. It is assumed that the raw materials that are shipped from the supplier to the manufacturer are sent for manufacturing immediately after it reaches the manufacturing facility. Then the inventory holding costs at time period ‘t’ is given by,

$$\sum_{n} [j * TRM_{n\,t} * INVCR_{n}]$$

The total inventory holding costs at the manufacturer for the entire planning time horizon is,

$$\sum_{t} \sum_{n} [j * TRM_{n\,t} * INVCR_{n}]$$

Constraints at the Manufacturer

There is a continuous flow of raw materials from the suppliers to the manufacturer in each time period. Since transportation mode ‘i’ takes ‘i’ time periods to deliver the shipment, the amount of raw material ‘n’ shipped from the supplier ‘n’ using mode ‘i’ in time period ‘t-i’ reaches the manufacturer at time period ‘t’. The total inflow of raw material ‘n’ at the manufacturer from the supplier ‘n’ in time period ‘t’ is,

$$X_{n\,1\,t-1} + X_{n\,2\,t-2} + \cdots + X_{n\,i\,t-i} \qquad \forall\, n, t \text{ and } t \geq i + 1$$

Hence, the total amount of raw material ‘n’ at the manufacturer in time period ‘t’ is,

$$TRM_{n\,t} = X_{n\,1\,t-1} + X_{n\,2\,t-2} + \cdots + X_{n\,i\,t-i} \qquad \forall\, n, t \text{ and } t \geq i + 1$$

The raw materials have to be present in the ratio $a_1, a_2, \ldots, a_n$ in order to produce one final product. For example, if there are three raw materials to produce the product in the ratio 2:3:5, we need 200, 300 and 500 of the three raw materials respectively in order to produce 100 products. We assume that the raw materials are sent for manufacturing, immediately after it reaches the manufacturing facility, i.e., that there is no storage (JIT manufacturing environment). In order for this assumption to work, the raw materials that arrive from the supplier must be able to produce some positive integer number of finished products. From the above example, a shipment of 200, 300 and 500 number of raw materials arriving at the manufacturer in a time period ‘t’ will help produce 100 products. The following constraints ensure that this assumption is incorporated.

$$TRM_{1\,t} = a_1 * PM_{t+j}$$

$$TRM_{2\,t} = a_2 * PM_{t+j}$$

……………………

……………………

$$TRM_{n\,t} = a_n * PM_{t+j}$$

Since it takes ‘j’ time periods to manufacture the products from the raw material, $PM_{t+j}$is used in the above equation. This signifies that the raw materials that arrive at the manufacturer in time period ‘t’ will amount to the products that will be available at time period ‘t+j’.

The manufacturer has a production capacity associated with it. At any time period the manufacturer cannot produce more than its production limit which is represented by the maximum production capacity constraint.

$$PM_t \leq MCAP \qquad \forall\, t$$

The transportation cost from the supplier to the manufacturer is subject to the following constraints in order to make the model linear.

$$QTSM_{n\,i\,t\,k} \leq (\beta_{i\,k+1} - \beta_{i\,k}) * \delta_{n\,i\,t\,k} \qquad \forall\, n, i, t, k$$

$$QTSM_{n\,i\,t\,k} \geq (\beta_{i\,k+1} - \beta_{i\,k}) * \delta_{n\,i\,t\,k+1} \qquad \forall\, n, i, t, k$$

$$\sum_{i} X_{n\,i\,t} = \sum_{i} \sum_{k} QTSM_{n\,i\,t\,k} \qquad \forall\, n, t$$

$$QTSM_{n\,i\,t\,k} \geq 0 \qquad \forall\, n, i, t, k$$

$$X_{n\,i\,t} \geq 0 \qquad \forall\, n, i, t, k$$

$$\delta_{n\,i\,t\,k} \in (0,1) \qquad \forall\, n, i, t, k$$

The quantity shipped from the supplier to the manufacturer has a maximum shipment constraint for the quantity shipped using transportation mode ‘i’ for each time period.

$$X_{n\,i\,t} \leq SMMAX_i \qquad \forall\, n, i, t$$

The suppliers have a maximum capacity constraint and hence limit the quantity that is being shipped to the manufacturer.

$$\sum_{i} X_{n\,i\,t} \leq SCAP_n \qquad \forall\, n, t$$

The manufacturer does not have a storage facility to store the finished products. All the products manufactured at any time period are shipped immediately to the warehouse in the same time period for storage. There are several modes of transport 'h' that could be used depending on the customer demand. Hence the inventory balance constraint at the manufacturer is given by,

$$PM_t = \sum_h Y_{h\,t} \qquad \forall\, t$$

The manufacturer tries to meet the demand from the warehouse and we assume that backorders are allowed at this stage. Hence the demand constraint at the manufacturer is,

$$\sum_h Y_{h\,t-h} + MBO_t = WD_t + MBO_{t-1} \qquad \forall\, t$$

where the demand from the warehouse is represented by,

$$WD_t = \sum_r CD_{r\,t} + WBO_{t-1} \qquad \forall\, t$$

The total inflow of products at the warehouse from the manufacturer in time period 't' is,

$$\sum_h Y_{h\,t} = \sum_h \sum_l QTMW_{h\,t\,l} \qquad \forall\, t$$

The transportation cost from the manufacturer to the warehouse is subject to the following constraints in order to make the model linear.

$$QTMW_{h\,t\,l} \le (\gamma_{h\,l+1} - \gamma_{h\,l}) * \theta_{h\,t\,l} \qquad \forall\, h, t, l$$

$$QTMW_{h\,t\,l} \geq (\gamma_{h\,l+1} - \gamma_{h\,l}) * \theta_{h\,t\,l+1} \quad \forall\, h, t, l$$

$$QTMW_{h\,t\,l} \geq 0 \quad \forall\, h, t, l$$

$$Y_{h\,t} \geq 0 \quad \forall\, h, t$$

$$\theta_{h\,t\,l} \in (0,1) \quad \forall\, h, t, l$$

There is constraint on the maximum quantity that could be shipped using transportation mode ‘h’ in each time period.

$$Y_{h\,t} \leq MWMAX_h \quad \forall\, h, t$$

3.3.2.2 WAREHOUSE

The products are shipped from the manufacturer to the warehouse for storage. There is a company owned warehouse that has a limited capacity. A leased warehouse can be used once the company owned warehouse has reached its maximum capacity. The products that are shipped to the customers are first shipped from the leased warehouse before using the products stored in the company owned warehouse. There are ‘R’ retailers in the supply chain and the warehouse has to ship the products to each retailer depending on the demand at that retailer. There are several modes of transportation that can be used while shipping the products.

Costs associated with the Warehouse

The transportation cost of the products shipped from the manufacturer to the warehouse is given by,

$$\sum_{t} \sum_{h} \sum_{l} QTMW_{h\,t\,l} * TCM_{h\,l}$$

The inventory holding cost at the warehouse stage includes the cost of holding inventory at the company owned warehouse and the cost involved at the leased warehouse. The leased warehouse has a quantity discount price structure as discussed previously. The inventory holding cost at the leased warehouse is given by,

$$\sum_{t} \sum_{p} [LWINV_{p\,t} * LWINVC_{p}]$$

The inventory holding cost at the company owned warehouse is,

$$\sum_{t} [WINV_{t} * WINVC]$$

The total inventory cost at the warehouse stage is the sum of the above mentioned costs.

$$\sum_{t} [WINV_{t} * WINVC] + \sum_{t} \sum_{p} [LWINV_{p\,t} * LWINVC_{p}]$$

Constraints at the Warehouse

The total quantity of products arriving at the warehouse by the various modes of transportation at time period ‘t’ is,

$$Y_{1\,t-1} + Y_{2\,t-2} + \cdots + Y_{h\,t-h} \qquad \forall\, h, t \text{ and } t \geq h+1$$

The inventory balance constraint at the warehouse has to take into consideration the inventory at both the company warehouse and the leased warehouse. Hence the inventory balance constraint is computed as,

$$\sum_{h} Y_{h\,t-h} + WINV_{t-1} + TLWINV_{t-1}$$

$$= \sum_{r} \sum_{q} Z_{r\,q\,t} + WINV_t + TLWINV_t \qquad \forall\, t \;\; and\; t \geq h+1$$

There is a capacity constraint on the space available at the company owned warehouse. It is assumed that the leased warehouse has infinite space and hence no capacity constraint is associated with it.

$$WINV_t \leq WINVCAP \qquad \forall\, t$$

The cost for the leased warehouse space is subject to the following constraints in order to make it linear. This is similar to the constraints for the transportation cost as both these costs follow the incremental quantity cost structure.

$$LWINV_{p\,t} \leq \left(S_{p+1} - S_p\right) * \alpha_{p\,t} \qquad \forall\, p, t$$

$$LWINV_{p\,t} \geq \left(S_{p+1} - S_p\right) * \alpha_{p+1\,t} \qquad \forall\, p, t$$

$$TLWINV_t = \sum_{p} LWINV_{p\,t} \qquad \forall\, t$$

$$LWINV_{p\,t} \geq 0 \qquad \forall\, p, t$$

$$TLWINV_t \geq 0 \qquad \forall\, t$$

$$\alpha_{p\,t} \in (0,1) \qquad \forall\, p, t$$

Since backorders are allowed at this stage, the demand constraint at the warehouse at time ‘t’ is given as,

$$\sum_{r} \sum_{q} Z_{r\,q\,t-q} + WBO_t = \sum_{r} CD_{r\,t} + WBO_{t-1} \qquad \forall\, t$$

The quantity of products that is demanded by the warehouse from the manufacturer at each time period is the sum of the retailer demand and the cumulative backorder that need to be filled at that time period.

$$WD_t = \sum_{r} CD_{r\,t} + WBO_{t-1} \qquad \forall\, t$$

3.3.2.3 RETAILER

There are ‘R’ retailers present in the supply chain. The products are shipped from the warehouse to the retailer ‘r’ depending on the demand at each of the retailers.

Costs associated with the Retailer

The transportation cost of the products shipped from the warehouse to the retailer is given by,

$$\sum_{t} \sum_{r} \sum_{q} \sum_{s} QTWR_{r\,q\,t\,s} * TCC_{q\,s}$$

Constraints at the Retailer

The total quantity of products arriving at retailer ‘r’ by the various modes of transportation at time period ‘t’ is,

$$Z_{r\,1\,t-1} + Z_{r\,2\,t-2} + \cdots + Z_{r\,q\,t-q} \qquad \forall\, r, t \text{ and } t \geq q+1$$

The transportation cost from the warehouse to the retailer is subject to the following constraints in order to make the model linear.

$$QTWR_{r\,q\,t\,s} \leq \left(\tau_{q\,s+1} - \tau_{q\,s}\right) * \Delta_{r\,q\,t\,s} \qquad \forall\, r, q, t, s$$

$$QTWR_{r\,q\,t\,s} \geq \left(\tau_{q\,s+1} - \tau_{q\,s}\right) * \Delta_{r\,q\,t\,s+1} \qquad \forall\, r, q, t, s$$

$$\sum_{q} Z_{r\,q\,t} = \sum_{q} \sum_{s} QTWR_{r\,q\,t\,s} \qquad \forall\, r, t$$

$$QTWR_{r\,q\,t\,s} \geq 0 \qquad \forall\, r, q, t, s$$

$$Z_{r\,q\,t} \geq 0 \qquad \forall\, r, q, t$$

$$\Delta_{r\,q\,t\,s} \in (0,1) \qquad \forall\, r, q, t, s$$

The demand at the retailers is obtained prior to the optimization and is considered as given data. The demand at the retailer ‘r’ in each time period ‘t’ is,

$$CD_{r\,t} \qquad \forall\, r, t$$

Since backorders are possible at the retailers, the total demand at retailer ‘r’ in time period ‘t’ is,

$$CD_{r\,t} + RBO_{r\,t-1} \qquad \forall\, r, t$$

The demand constraint at the retailer ‘r’ in time period ‘t’ is given by,

$$Z_{r\,1\,t-1} + Z_{r\,2\,t-2} + \cdots + Z_{r\,q\,t-q} + RBO_{r\,t} = CD_{r\,t} + RBO_{r\,t-1} \qquad \forall\, r, t \text{ and } t \geq q+1$$

There is constraint on the maximum quantity that could be shipped using mode ‘q’ which is,

$$\sum_{r} Z_{r\,q\,t} \leq WRMAX_q \qquad \forall\, q, t$$

We assume that backorders are not allowed at the end of the planning time horizon, i.e. all orders are fulfilled by the end of the final time period.

$$RBO_{r\,T} = 0 \qquad \forall\, r$$

3.3.3 OBJECTIVE FUNCTION

The model we have formulated is a bi-criteria optimization problem. The first objective is to minimize the total cost of the centralized supply chain. The second objective is to maximize the customer responsiveness measured in terms of the accumulated backorders (unmet demands) in the supply chain.

Objective – 1: Minimize the total cost of the supply chain

The total cost of the supply chain is the sum of the transportation costs and the inventory holding costs at each stage of the supply chain.

Total transportation cost of the supply chain is,

$$\sum_{t} \sum_{n} \sum_{i} \sum_{k} QTSM_{n\,i\,t\,k} * TC_{i\,k} + \sum_{t} \sum_{h} \sum_{l} QTMW_{h\,t\,l} * TCM_{h\,l} + \sum_{t} \sum_{r} \sum_{q} \sum_{s} QTWR_{r\,q\,t\,s} * TCC_{q\,s}$$

Total inventory holding cost of the supply chain is,

$$\sum_{t} \sum_{n} [j * TRM_{n\,t} * INVCR_{n}] + \sum_{t} [WINV_{t} * WINVC] + \sum_{t} \sum_{p} [LWINV_{p\,t} * LWINVC_{p}]$$

Minimizing the total supply chain cost function which is the sum of the total transportation cost and the total inventory holding cost is the first objective.

Minimize

$$OBJ_1 = \sum_t \sum_n \sum_i \sum_k QTSM_{n\,i\,t\,k} * TC_{i\,k} + \sum_t \sum_h \sum_l QTMW_{h\,t\,l} * TCM_{h\,l} + \sum_t \sum_r \sum_q \sum_s QTWR_{r\,q\,t\,s} * TCC_{q\,s} + \sum_t \sum_n [j * TRM_{n\,t} * INVCR_n] + \sum_t [WINV_t * WINVC] + \sum_t \sum_p [LWINV_{p\,t} * LWINVC_p]$$

Objective – 2: Maximize the customer responsiveness

The customer responsiveness is quantified by using the total quantity of backorders that occur through the entire planning time horizon. Minimizing the total number of backorders will make the model meet the demand at every stage on time. Hence *minimizing the total number of backorders will maximize the customer responsiveness*. It is assumed that backorders are allowed at the manufacturer, the warehouse and the retailers. Since backorders occur in each time period, the backorders at every time period has to be minimized.

Total number of backorders at the manufacturer at each time period 't' is,

$$MBO_t = WD_t + MBO_{t-1} - \sum_h Y_{h\,t-h} \qquad \forall\, t$$

Total number of backorders at the warehouse at each time period ‘t’ is,

$$WBO_t = \sum_r CD_{r\,t} + WBO_{t-1} - \sum_r \sum_q Z_{r\,q\,t-q} \qquad \forall\, t$$

Total number of backorder at the retailer ‘r’ at each time period ‘t’ is,

$$RBO_{r\,t} = CD_{r\,t} + RBO_{r\,t-1} + \sum_q Z_{r\,q\,t-q} \qquad \forall\, r, t$$

Maximizing the customer responsiveness is obtained by minimizing the total number of backorders at the manufacturer, the warehouse and the retailers at each time period.

Minimize

$$OBJ_2 = \sum_t WBO_t + \sum_t MBO_t + \sum_r \sum_t RBO_{r\,t}$$

Bi-criteria optimization problem

The bi-criteria optimization problem for the four stage centralized supply chain is:

$$Minimize\ OBJ_1$$

$$= \sum_t \sum_n \sum_i \sum_k QTSM_{n\,i\,t\,k} * TC_{i\,k}$$

$$+ \sum_t \sum_h \sum_l QTMW_{h\,t\,l} * TCM_{h\,l}$$

$$+ \sum_t \sum_r \sum_q \sum_s QTWR_{r\,q\,t\,s} * TCC_{q\,s}$$

$$+ \sum_t \sum_n [j * TRM_{n\,t} * INVCR_n] + \sum_t [WINV_t * WINVC]$$

$$+ \sum_t \sum_p [LWINV_{p\,t} * LWINVC_p]$$

$$Minimize\ OBJ_2$$

$$= \sum_t WBO_t + \sum_t MBO_t + \sum_r \sum_t RBO_{r\,t}$$

subject to all the constraints at each stage of the supply chain mentioned previously

3.4 Solution Procedure

The problem in this research work is a *multi-criteria mathematical programming problem* (MCMP). In most MCMP problems, solution that simultaneously maximizes all the objectives does not exist as the objectives conflict with one another. One way to solve multi-criteria problems is the *goal programming approach* which is a practical method for handling multi-criteria problems [37]. Goal programming uses completely pre-specified preferences of the decision maker in solving the MCMP problems. In this approach, target levels for achievement are obtained for all the objectives. These target values are treated as goals to aspire for (goal constraint). They may or may not be achievable. The goal programming approach attempts to find an optimal solution that comes as close as possible to the assigned targets in the order of specified priorities. The goal programming model of an MCMP problem [3] is given by,

$$Minimize\ Z = \sum_{i=1}^{k} (w_i^{+} d_i^{+} + w_i^{-} d_i^{-})$$

$$f_i(x) + d_i^{-} - d_i^{+} = b_i \quad for\ i = 1, \dots, k$$

$$g_j(x) \le 0 \quad for\ j = 1, \dots, m$$

$$x_j, d_i^{-}, d_i^{+} \ge 0 \quad for\ all\ i\ and\ j$$

The decision maker specifies the acceptable level of achievement (b_i) for each criterion f_i and specify a weight w_i (ordinal or cardinal) to be associated with the deviation between f_i and b_i. The deviation variables are represented by d_i^{-} and d_i^{+} where d_i^{-}

represents the under achievement and d_i^+ represents the over achievement of the ith goal. The objective function minimizes the weighted sum of the deviation variables. The system of equations represents the goal constraints relating the decision variables to the goals or targets for each criterion. The set of weights could take two forms: (1) Cardinal or pre-specified weights and (2) Ordinal or pre-emptive priorities.

The goal programming problem that uses cardinal or pre-specified weights is called *non pre-emptive goal program* and the goal programming problem that uses ordinal or pre-emptive priorities is called *pre-emptive goal program*. According to how much one objective is preferred to the other, pre-specified weights are assigned in non pre-emptive goal programming, which are specific values representing the decision maker's trade-off among the objectives. Here $(w_1, w_2, \ldots, w_k)$ denote the weights of the objective functions respectively. These weights are used to reduce the goal programming problem to a single objective optimization problem. The units of measurement differ between the objectives and hence the objective functions have to be scaled for this method to be effective. We use the *Ideal solutions* to scale the objective values. Ideal solutions are defined as the best values achievable for individual objectives while ignoring other objectives.

Pre-emptive goal programming requires the decision maker to provide a priority to each objective. Here $(P_1, P_2, \ldots, P_k)$ denote the priorities assigned to the objective functions respectively. In pre-emptive goal programming, higher order goals must be satisfied before lower order goals are even considered. Hence P_1 is satisfied first, and then improvements to P_2 are sought, without degrading the achievement on P_1. It is essentially a sequence of single objective optimization problems.

To solve the pre-emptive and the non pre-emptive goal programming problems, the following steps will be used.

1. Calculate the ideal solutions for each objective by solving the problem as a single objective optimization problem ignoring the other objectives.
2. By solving the single objective optimization problems we can find the upper and lower bounds for the objective values. The target values can be set within these bounds. Since both the objectives are minimization objectives in our example, we set some percentage increase from the lower bounds/ideal solutions as our target values on both the objectives making sure that the target value does not exceed the upper bound.
3. Solve the pre-emptive goal program by using the pre-emptive priorities. For the non pre-emptive goal program, scale the objective values by using the ideal values and solve using the pre-specified weights.

Step – 1: Ideal solution

The ideal solution for each objective is obtained by solving each objective as a single objective optimization problem.

Objective – 1: Minimize the total cost of the supply chain ignoring customer responsiveness

$Minimize\ OBJ_1$

$$= \sum_t \sum_n \sum_i \sum_k QTSM_{n\,i\,t\,k} * TC_{i\,k}$$

$$+ \sum_t \sum_h \sum_l QTMW_{h\,t\,l} * TCM_{h\,l}$$

$$+ \sum_t \sum_r \sum_q \sum_s QTWR_{r\,q\,t\,s} * TCC_{q\,s}$$

$$+ \sum_t \sum_n [j * TRM_{n\,t} * INVCR_n] + \sum_t [WINV_t * WINVC]$$

$$+ \sum_t \sum_p [LWINV_{p\,t} * LWINVC_p]$$

subject to the constraints at each stage of the supply chain as listed in Appendix A

Let $Ideal_1$ be the ideal solution for the first objective.

Objective – 2: Minimize the total backorders (Maximize the customer responsiveness) ignoring the total cost of the supply chain

$Minimize\ OBJ_2$

$$= \sum_t WBO_t + \sum_t MBO_t + \sum_r \sum_t RBO_{r\,t}$$

subject to the constraints at each stage of the supply chain as listed in Appendix A

Let $Ideal_2$ be the ideal solution for the second objective.

Step – 2: Target values

Target value for objective 1 is

$$Target_1 = \frac{(100 + incr_1)}{100} * Ideal_1$$

where $incr_1$ is the percentage increase from the ideal solution for objective 1. If $incr_1 = 10\%$, then $Target_1 = \frac{(100+10)}{100} * Ideal_1$. Hence $Target_1 = 1.1 * Ideal_1$.

Similarly, target value for objective 2 is

$$Target_2 = \frac{(100 + incr_2)}{100} * Ideal_2$$

where $incr_2$ is the percentage increase from the ideal solution for objective 2.

Step – 3: Solution by Goal Programming

<u>Method 1: Pre-emptive Goal Program</u>

Goal constraint for the total supply chain cost,

$$OBJ_1 + d_1^- - d_1^+ = Target_1$$

Goal constraint for the total backorders,

$$OBJ_2 + d_2^- - d_2^+ = Target_2$$

The objective function of the goal program,

Case-1: Prioritizing total cost of the supply chain over the customer responsiveness

$$Minimize\ P_1(d_1^+) + P_2(d_2^+)$$

Case-2: Prioritizing customer responsiveness over the total cost of the supply chain

$$Minimize\ P_1(d_2^+) + P_2(d_1^+)$$

<u>Method 2: Non Pre-emptive Goal Program</u>

The objective and the target values are scaled using the ideal values so that the deviation variables are normalized to the same magnitude. The weights when added to the scaled values will achieve the necessary goal.

Goal constraint for the total supply chain cost,

$$\frac{OBJ_1}{Ideal_1} + d_1^- - d_1^+ = \frac{Target_1}{Ideal_1}$$

Goal constraint for the total backorders,

$$\frac{OBJ_2}{Ideal_2} + d_2^- - d_2^+ = \frac{Target_2}{Ideal_2}$$

The objective function,

$$Minimize\ w_1(d_1^+) + w_2(d_2^+)$$

Note that under pre-emptive goal program (method 1) we have to solve two linear integer programs sequentially. Under non pre-emptive goal program (method 2) only one linear integer program is solved. The pros and cons of the pre-emptive and the non pre-emptive goal programming approaches are tabulated in Table 3.5.

	Pre-emptive Goal Program	Non Pre-emptive Goal Program
Pros	The pre-emptive priorities or the preference can be obtained easily from the Decision Maker	The use of pre-specified weights allows for trade-offs
	No scaling is required	The problem is converted to a single objective optimization problem and hence easier to solve
Cons	It is a sequential optimization problem	Scaling is needed as the units of measurement might differ among the objectives
	Does not allow trade-offs because we need to satisfy the first priority before moving to the second priority	It is difficult and time consuming to obtain the weights
		Linear utility function is used

Table 3.5: Pros and Cons of Goal Programming approaches

CHAPTER 4

ILLUSTRATIVE EXAMPLE AND MODEL ANALYSIS

4.1 ILLUSTRATIVE EXAMPLE

The aim of the model discussed in Chapter 3 is to aid in making the inventory and transportation decisions for a four stage centralized supply chain while minimizing costs and maximizing the customer responsiveness. In this section we illustrate the model using simulated data. We try to identify the right amount of quantity to be shipped by the appropriate mode of transport, the right amount of inventory to be stored at the warehouses and hence the number of backorders accumulated depending on two scenarios, one for prioritizing the cost over the customer responsiveness and another for prioritizing customer responsiveness over cost.

Due to the difficulty of obtaining real time data from the companies, the data used in this example problem is a combination of assumed values and values obtained from the work of Mysore [36]. The data is organized as the follows.

The centralized supply chain configuration is

- 3 suppliers, 1 manufacturer, 1 warehouse, 2 retailers
- 1 product, 3 raw materials in the ratio 2:3:5 (a_1= 2, a_2= 3, a_3= 5,)
- 4 modes of transport between suppliers and the manufacturer, the manufacturer and the warehouse and the warehouse and retailers namely air (lead time =1 time period), truck (lead time =2 time periods), rail (lead time =3 time periods) and ship (lead time =4 time periods)

- Finite company owned warehouse space and a leased warehouse space if needed
- It takes two time periods to manufacture the product from the raw materials at the manufacturer
- Time horizon is 24 time periods

The index sets used in the model formulation are:

t = time periods (t = 1, 2, …, 24)

n = number of raw materials (n = 1, 2, 3)

j = number of time periods needed to manufacture a product from raw materials (j = 2)

i = modes of transport from supplier to manufacturer (also represents lead time) (i = air, truck, rail, ship)

k = index for the transportation cost ranges from the supplier to the manufacturer (k = 1, 2, 3)

h = modes of transport from manufacturer to warehouse (also represents lead time) (h = air, truck, rail, ship)

l = index for the transportation cost ranges from the manufacturer to the warehouse (l = 1, 2, 3)

q = modes of transport from warehouse to retailer (also represents lead time) (q = air, truck, rail, ship)

s = index for the transportation cost ranges from the warehouse to the retailer (s = 1, 2, 3)

r = number of retailers (r = 1, 2)

p = index corresponding to the cost bracket in the leased warehouse (p = 1, 2, 3)

The demand at the retailers for 24 time periods is assumed to be deterministic.

Time	1	2	3	4	5	6	7	8	9	10	11	12
Retailer 1	3000	2000	1000	1000	1500	1500	750	1000	1000	2000	4000	5000
Retailer 2	3500	2500	1500	1500	2000	2000	1250	1500	1500	2000	4500	5500

Time	13	14	15	16	17	18	19	20	21	22	23	24
Retailer 1	3000	2000	1000	1000	1500	1500	750	1000	1000	2000	4000	5000
Retailer 2	3500	2500	1500	1500	2000	2000	1250	1500	1500	2500	4500	5500

Table 4.1: Demand data for retailers

There are four different modes of transportation between the suppliers and the manufacturer, transporting three different raw materials and their transportation costs follow the quantity discount cost structure with price breaks as shown below.

Mode (i)	Time taken (in periods)	Cost bracket (k)	Quantity range ($\beta_{i\,k} + 1$ to $\beta_{i\,k+1}$)	Cost (in \$) $TC_{i\,k}$
Air	1	1	1 - 5000	0.7
		2	5001 - 20000	0.5
Truck	2	1	1 - 10000	0.4
		2	10001 - 50000	0.3
		3	50001 - 100000	0.2
Rail	3	1	1 - 10000	0.3
		2	10001 - 75000	0.2
		3	75001 - 150000	0.1
Ship	4	1	1 - 20000	0.2
		2	20001 - 200000	0.1

Table 4.2: Cost structure data of transportation quantity between supplier and manufacturer

The transportation costs between the manufacturer and the warehouse while transporting the product follows the quantity discount cost structure with price breaks as shown below.

Mode (h)	Time taken (in periods)	Cost bracket (l)	Quantity range ($\gamma_{h\,l} + 1$ to $\gamma_{h\,l+1}$)	Cost (in \$) $TCM_{h\,l}$
Air	1	1	1 - 2000	0.9
		2	2001 - 10000	0.7
Truck	2	1	1 - 5000	0.5
		2	5001 - 25000	0.4
Rail	3	1	1 - 5000	0.3
		2	5001 - 25000	0.2
		3	25001 - 100000	0.1
Ship	4	1	1 - 20000	0.2
		2	20001 - 200000	0.1

Table 4.3: Cost structure data of transportation quantity between manufacturer and warehouse

The warehouse ships the product to two different retailers and the transportation costs between the warehouse and the two retailers follows the quantity discount cost structure with price breaks as shown in the following table.

Mode (q)	Time taken (in periods)	Cost bracket (s)	Quantity range ($\tau_{q\,s} + 1$ to $\tau_{q\,s+1}$)	Cost (in $) $TCC_{q\,s}$
Air	1	1	1 - 2000	0.9
		2	2001 - 10000	0.7
Truck	2	1	1 - 5000	0.5
		2	5001 - 25000	0.4
Rail	3	1	1 - 5000	0.3
		2	5001 - 25000	0.2
		3	25001 - 100000	0.1
Ship	4	1	1 - 20000	0.2
		2	20001 - 200000	0.1

Table 4.4: Cost structure data of transportation quantity between warehouse and retailer

The maximum quantity that can be shipped between the different stages of the supply chain using each transportation mode (i, h, q) is tabulated below.

Mode (i)	Quantity ($SMMAX_i$)	Mode (h)	Quantity ($MWMAX_h$)	Mode (q)	Quantity ($WRMAX_q$)
Air	20000	Air	10000	Air	10000
Truck	100000	Truck	25000	Truck	25000
Rail	150000	Rail	100000	Rail	100000
Ship	200000	Ship	200000	Ship	200000

Table 4.5: Maximum quantity that can be shipped using all transportation modes

The production capacity of the suppliers to produce the respective raw materials for all time periods is tabulated as follows.

Raw material (n)	Quantity ($SCAP_n$)
1	300000
2	600000
3	700000

Table 4.6: Production capacity for raw materials at the suppliers

The production capacity at the manufacturer for all time periods, $MCAP$ = 100000

The inventory holding cost of each raw material at the manufacturer is given below.

Raw material (n)	Cost (in $) ($INVCR_n$)
1	0.005
2	0.0025
3	0.0015

Table 4.7: Inventory holding cost of raw material at the manufacturer

The capacity of the company owned warehouse, $WINVCAP$ = 10000

The inventory holding cost at the company owned warehouse, $WINVC$ = $0.01

The leased warehouse capacity also follows a quantity discount cost structure with price breaks as represented in the table below.

Quantity range ($S_p + 1$ to S_{p+1})	Cost bracket index (p)	Cost per unit in the index (in $) $LWINVC_p$
1 - 10000	1	.10
10001 - 60000	2	.09
60001 - 500000	3	.07

Table 4.8: Capacity and cost structure data of the leased warehouse

The above data is used to solve the model formulated in Chapter 3. The solution procedure discussed in the previous chapter is used to solve the model using pre-emptive and non pre-emptive goal programming.

4.2 MATHEMATICAL MODEL

The bi-criteria optimization problem for the four stage centralized supply chain is:

$$Minimize\ OBJ_1 = Total\ Cost\ of\ the\ Supply\ Chain$$

$$= \sum_t \sum_n \sum_i \sum_k QTSM_{n\,i\,t\,k} * TC_{i\,k}$$

$$+ \sum_t \sum_h \sum_l QTMW_{h\,t\,l} * TCM_{h\,l}$$

$$+ \sum_t \sum_r \sum_q \sum_s QTWR_{r\,q\,t\,s} * TCC_{q\,s}$$

$$+ \sum_t \sum_n [j * TRM_{n\,t} * INVCR_n] + \sum_t [WINV_t * WINVC]$$

$$+ \sum_t \sum_p [LWINV_{p\,t} * LWINVC_p]$$

$$Minimize\ OBJ_2$$

$$= Total\ number\ of\ backorders\ accumulated\ in\ the\ supply\ chain$$

$$= \sum_t WBO_t + \sum_t MBO_t + \sum_r \sum_t RBO_{r\,t}$$

subject to the constraints at each stage of the supply chain discussed in Chapter 3. They are also listed in Appendix A.

4.3 Solution Method

Step – 1: Find the ideal solution

The ideal solution for each objective is obtained by solving each objective as a single objective optimization problem ignoring the other objective.

Objective – 1: Minimize the total cost of the supply chain ignoring customer responsiveness

$$Minimize\ OBJ_1$$

subject to the constraints at each stage of the supply chain as listed in Appendix A

The total cost of the supply chain = $260,925

The total number of backorders accumulated through all the time periods and all the stages of the supply chain = 2,889,000

The ideal solution for the first objective, $Ideal_1$ = 260,925

Objective – 2: Minimize the total backorders (Maximize the customer responsiveness) ignoring the total cost of the supply chain

$$Minimize\ OBJ_2$$

subject to the constraints at each stage of the supply chain as listed in Appendix A

The total cost of the supply chain = $521,787

The total number of backorders accumulated through all the time periods and all the stages of the supply chain = 434,500

The ideal solution for the second objective, $Ideal_2$ = 434,500

Step – 2: Set the target values

The two criteria for the problem are the total cost and the total number of backorders. The criteria values obtained for each of the two single objective optimization problems are tabulated below.

	Total cost (in $)	Total number of backorders (units)
Minimize Total cost of the supply chain (OBJ_1)	260,925	2,889,000
Maximize Customer responsiveness (OBJ_2)	521,787	434,500

Table 4.9: Criteria values for the single objective optimization problem

The upper bound and the lower bound values of the total cost is 521,787 and 260,925 respectively. There is a 99.98% difference between the two values. The upper bound and the lower bound values of the total number of backorders is 2,889,000 and 434,500 respectively. There is a 564.90% difference between the two values.

Target value for objective 1 is

$$Target_1 = \frac{(100 + incr_1)}{100} * Ideal_1$$

where $incr_1$ is the percentage increase from the ideal solution for objective 1. For illustrative purposes we assume $incr_1$ = 5%. Therefore,

$$Target_1 = \frac{(100 + 5)}{100} * Ideal_1 = 1.05 * Ideal_1$$

$$Target_1 = 1.05 * 260{,}925 = 273{,}971$$

Target value for objective 2 is

$$Target_2 = \frac{(100 + incr_2)}{100} * Ideal_2$$

where $incr_2$ is the percentage increase from the ideal solution for objective 2. We assume $incr_2$ = 5%. Therefore,

$$Target_2 = \frac{(100 + 5)}{100} * Ideal_2 = 1.05 * Ideal_2$$

$$Target_2 = 1.05 * 434{,}500 = 456{,}225$$

Step – 3: Solve using goal programming

<u>Method 1: Pre-emptive Goal Program</u>

Goal constraint for the total supply chain cost,

$$OBJ_1 + d_1^- - d_1^+ = Target_1$$

$$\therefore\ OBJ_1 + d_1^- - d_1^+ = 273{,}971$$

Goal constraint for the total backorders,

$$OBJ_2 + d_2^- - d_2^+ = Target_2$$

$$\therefore\ OBJ_2 + d_2^- - d_2^+ = 456{,}225$$

The objective function,

Case-1: Prioritizing total cost of the supply chain over the customer responsiveness

$$Minimize\ P_1(d_1^+) + P_2(d_2^+)$$

Case-2: Prioritizing customer responsiveness over the total cost of the supply chain

$$Minimize\ P_1(d_2^+) + P_2(d_1^+)$$

Method 2: Non Pre-emptive Goal Program

The objective value and the target value are scaled using the ideal values so that the deviation variables are normalized to the same magnitude.

Goal constraint for the total supply chain cost,

$$\frac{OBJ_1}{Ideal_1} + d_1^- - d_1^+ = \frac{Target_1}{Ideal_1}$$

$$\therefore\ \frac{OBJ_1}{260{,}925} + d_1^- - d_1^+ = \frac{273{,}971}{260{,}925}$$

$$\therefore\ \frac{OBJ_1}{260{,}925} + d_1^- - d_1^+ = 1.05$$

Goal constraint for the total backorders,

$$\frac{OBJ_2}{Ideal_2} + d_2^- - d_2^+ = \frac{Target_2}{Ideal_2}$$

$$\therefore \ \frac{OBJ_2}{434{,}500} + d_2^- - d_2^+ = \frac{456{,}225}{434{,}500}$$

$$\therefore \ \frac{OBJ_2}{434{,}500} + d_2^- - d_2^+ = 1.05$$

The objective function,

$$Minimize\ w_1(d_1^+) + w_2(d_2^+)$$

One of the above mentioned objective functions will be used along with the goal constraints and the real constraints at each stage of the supply chain to solve the pre-emptive or the non pre-emptive goal programming problem.

The model statistics for this example problem gives an idea of the size of the problem we are solving.

6419 constraints

4425 continuous variables

1800 binary variables

4.4 Pre-emptive Goal Programming Solutions

The problem is solved using GAMS (General Algebraic Modeling System) to get the output that shows the amount of raw materials and products shipped using the different transportation modes, the amount of inventory stored at the warehouses and the backorders that are accumulated at each stage of the supply chain. The results are tabulated and the shipping and storage patterns are discussed.

The execution time of the pre-emptive goal programming model using GAMS/CPLEX (version 11.01) on a 2.6 GHz AMD Opteron Processor with 32 GB of ECC RAM is 0.056 seconds.

4.4.1 CASE-1: PRIORITIZING TOTAL COST OF THE SUPPLY CHAIN OVER THE CUSTOMER RESPONSIVENESS

Quantity shipped from each supplier to the manufacturer

Supplier (n)	Mode (i)	Time period (t)	Cost bracket (k)	Bracket capacity	Units shipped	Variable ($X_{n\,i\,t}$)
1	Air	1	1	5000	5000	$X_{1\,Air\,1}$ = 8000
			2	15000	3000	
2	Truck	1	1	10000	10000	$X_{2\,Truck\,1}$ = 72000
			2	40000	40000	
			3	50000	22000	
3	Rail	1	1	10000	10000	$X_{3\,Rail\,1}$ = 60000
			2	65000	50000	
2	Ship	3	1	20000	20000	$X_{2\,Ship\,3}$ = 54000
			2	180000	34000	

Table 4.10: Summary of the quantity shipped from supplier to manufacturer using the different transportation modes

Quantity shipped from the manufacturer to the warehouse

Mode (h)	Time period (t)	Cost bracket (l)	Bracket capacity	Units shipped	Variable ($Y_{h\,t}$)
Air	4	1	2000	2000	$Y_{Air\ 4}$ = 4000
		2	8000	2000	
Truck	6	1	5000	5000	$Y_{Truck\ 6}$ = 15000
		2	20000	10000	
Rail	20	1	5000	2500	$Y_{Rail\ 20}$ = 2500
Ship	17	1	20000	3250	$Y_{Ship\ 17}$ = 3250

Table 4.11: Summary of the quantity shipped from manufacturer to warehouse using the different transportation modes

Quantity shipped from the warehouse to each retailer

Retailer (r)	Mode (q)	Time period (t)	Cost bracket (s)	Bracket capacity	Units shipped	Variable ($Z_{r\,q\,t}$)
1	Air	6	1	2000	2000	$Z_{1\ Air\ 6}$ = 10000
			2	8000	8000	
2	Truck	7	1	5000	3250	$Z_{2\ Truck\ 7}$ = 3250
2	Rail	7	1	5000	2000	$Z_{2\ Rail\ 7}$ = 2000
1	Ship	9	1	20000	3000	$Z_{1\ Ship\ 9}$ = 3000

Table 4.12: Summary of the quantity shipped from warehouse to retailer using the different transportation modes

The company owned warehouse holds an inventory of 10000 products from time periods 8 through 19.

Amount of inventory stored at the leased warehouse

Time period (t)	Cost bracket (p)	Bracket capacity	Quantity stored ($LWINV_{p\,t}$)
10	1	10000	$LWINV_{1\,10}$ = 10000
	2	50000	$LWINV_{2\,10}$ = 5780
17	1	10000	$LWINV_{1\,17}$ = 7000
18	1	10000	$LWINV_{1\,18}$ = 5000

Table 4.13: Summary of the quantity stored in the leased warehouse

Cumulative backorders at each stage of the supply chain

Time period (t)	Backorders
2	MBO_2 = 6500
3	MBO_3 = 17500
6	WBO_6 = 19000
7	WBO_7 = 11000
19	$RBO_{1\,9}$ = 750
20	$RBO_{1\,20}$ = 1750

Table 4.14: Summary of the backorders accumulated at each stage of the supply chain

Total cost of the supply chain and the total number of backorders

Total cost (in $)	Total number of backorders (units)
273971	495440

Table 4.15: Objective values for Case - 1

Discussion of Results of Case-1 (Prioritizing Cost over Responsiveness)

(i) Shipping and Storage patterns in Case-1

The values in the Table 4.10 represent the quantity shipped from the supplier ‘n’ to the manufacturer in time period ‘t’ and transportation mode ‘i’. For example, in time period ‘1’ a quantity of 8000 is shipped using air from supplier ‘1’. The air transportation mode takes one time period to reach the manufacturer and hence the quantity of 8000 reaches the manufacturer in time period ‘2’. It takes two time periods for the manufacturer to produce the products from the raw materials and the products become available only in time period ‘4’. Hence the first shipment from the manufacturer to the warehouse is at time period ‘4’. The values in Table 4.11 summarize the quantity shipped from the manufacturer to the warehouse in time period ‘t’ and transportation mode ‘h’ and the values in Table 4.12 summarize the quantity shipped from the warehouse to the retailer ‘r’ in time period ‘t’ and transportation mode ‘q’. If need be, the products shipped to the warehouse are stored in the company owned warehouse. Once the maximum capacity limit is reached at the company owned warehouse, the products are stored at the leased warehouse, the values of which are tabulated in Table 4.13. The delay in shipping and manufacturing the products leads to the accumulation of backorders at each stage of the supply chain which are summarized in Table 4.14. For instance, a shipment arriving in time period ‘5’ to the warehouse is immediately shipped to retailer ‘2’ without storing in the warehouse in order to meet the retailer demand and reduce the backorders accumulated. The complete solution for the problem solved using Case-1 is presented in Table A.1 through Table A.18 in Appendix B.

(ii) Objective Values in Case-1

In this case we prioritize the total cost of the supply chain over the customer responsiveness and the objective values that we have obtained have been mentioned in Table 4.15. The target value for the total cost, $Target_1$ = \$273,971 and the target value for the total number of backorders, $Target_2$ = 456,225 was set prior to solving this pre-emptive goal program. Since we prioritized the total cost (OBJ_1) over the customer responsiveness (OBJ_2) we were able to achieve the $Target_1$ value but deviated from the $Target_2$ value by 8.60%.

4.4.2 Case-2: Prioritizing customer responsiveness over the total cost of the supply chain

Quantity shipped from each supplier to the manufacturer

Supplier (n)	Mode (i)	Time period (t)	Cost bracket (k)	Bracket capacity	Units shipped	Variable ($X_{n\,i\,t}$)
1	Air	1	1	5000	5000	$X_{1\,Air\,1}$ = 8000
			2	15000	3000	
2	Truck	1	1	10000	10000	$X_{2\,Truck\,1}$ = 72000
			2	40000	40000	
			3	50000	22000	
3	Rail	1	1	10000	10000	$X_{3\,Rail\,1}$ = 60000
			2	65000	50000	
3	Ship	2	1	20000	20000	$X_{3\,Ship\,2}$ = 60000
			2	180000	40000	

Table 4.16: Summary of the quantity shipped from supplier to manufacturer using the different transportation modes

Quantity shipped from the manufacturer to the warehouse

Mode (h)	Time period (t)	Cost bracket (l)	Bracket capacity	Units shipped	Variable ($Y_{h\,t}$)
Air	4	1	2000	2000	$Y_{Air\ 4} =$ 4000
		2	8000	2000	
Truck	6	1	5000	5000	$Y_{Truck\ 6} =$ 23000
		2	20000	18000	
Rail	20	1	5000	2500	$Y_{Rail\ 20} =$ 2500
Ship	17	1	20000	3250	$Y_{Ship\ 17} =$ 3250

Table 4.17: Summary of the quantity shipped from manufacturer to warehouse using the different transportation modes

Quantity shipped from the warehouse to each retailer

Retailer (r)	Mode (q)	Time period (t)	Cost bracket (s)	Bracket capacity	Units shipped	Variable ($Z_{r\,q\,t}$)
1	Air	6	1	2000	2000	$Z_{1\ Air\ 6} =$ 10000
			2	8000	8000	
2	Truck	7	1	5000	3250	$Z_{2\ Truck\ 7} =$ 3250
2	Rail	7	1	5000	2000	$Z_{2\ Rail\ 7} =$ 2000
1	Ship	14	1	20000	1500	$Z_{1\ Ship\ 14} =$ 1500

Table 4.18: Summary of the quantity shipped from warehouse to retailer using the different transportation modes

The company owned warehouse holds an inventory of 10000 products from time periods 8 through 18 and 4000 products in time period 19.

Amount of inventory stored at the leased warehouse

Time period (t)	Cost bracket (p)	Bracket capacity	Quantity stored ($LWINV_{p\,t}$)
8	1	10000	$LWINV_{1\,8}$ = 7500
9	1	10000	$LWINV_{1\,9}$ = 10000
	2	50000	$LWINV_{2\,9}$ = 1000
18	1	10000	$LWINV_{1\,18}$ = 5000

Table 4.19: Summary of the quantity stored in the leased warehouse

Cumulative backorders at each stage of the supply chain

Time period (t)	Backorders (MBO_t)
2	MBO_2 = 6500
3	MBO_3 = 17500
4	WBO_4 = 16000
5	WBO_5 = 19500
20	$RBO_{2\,20}$ = 1500
22	$RBO_{2\,22}$ = 2500

Table 4.20: Summary of the backorders accumulated at each stage of the supply chain

Total cost of the supply chain and the total number of backorders

Total cost (in $)	Total number of backorders (units)
277,892	456,225

Table 4.21: Objective values for Case-2

Discussion of Results of Case-2 (Prioritizing Responsiveness over Cost)

(i) Shipping and Storage patterns in Case-2

The values in Table 4.16 represent the quantity shipped from the supplier ‘n’ to the manufacturer in time period ‘t’ and transportation mode ‘i’. For example, in time period ‘1’ a quantity of 72000 is shipped using truck from supplier ‘2’. The truck transportation mode takes two time periods to reach the manufacturer and hence the quantity of 8000 reaches the manufacturer in time period ‘3’. It takes two time periods for the manufacturer to produce the products from the raw materials and the products become available only in time period ‘5’. The values in Table 4.17 summarize the quantity shipped from the manufacturer to the warehouse in time period ‘t’ and transportation mode ‘h’ and the values in Table 4.18 summarize the quantity shipped from the warehouse to the retailer ‘r’ in time period ‘t’ and transportation mode ‘q’. The products shipped to the warehouse are stored in the company owned warehouse or shipped to the retailer ‘r’ immediately depending on the demand from that retailer. Once the maximum capacity limit is reached at the company owned warehouse, the products are stored at the leased warehouse, the values of which are summarized in Table 4.19. The delay in shipping and manufacturing the products leads to the accumulation of backorders at each stage of the supply chain which are summarized in the Table 4.20. The complete solution for the problem solved using Case-2 is presented in Table A.19 through Table A.36 in Appendix B.

(ii) Objective Values in Case-2

In this case we prioritize the customer responsiveness over the total cost of the supply chain and the objective values that we have obtained have been mentioned in Table 4.21. The target value for the total cost, $Target_1$ = \$273,971 and the target value for the total number of backorders, $Target_2$ = 456,225 was set prior to solving this pre-emptive goal program. Since we prioritized the customer responsiveness (OBJ_2) over the total cost (OBJ_1) we were able to achieve the $Target_2$ value but deviated from the $Target_1$ value by 1.43%.

4.4.3 COMPARISON OF THE RESULTS OBTAINED BY CASE-1 AND CASE-2 (PRE-EMPTIVE GOAL PROGRAMMING)

The two cases that we are dealing with in this problem are:

Case-1: Prioritizing total cost of the supply chain over the customer responsiveness

Case-2: Prioritizing customer responsiveness over the total cost of the supply chain

When the shipping patterns of the two cases are compared there is a striking difference in the manner in which the quantities are shipped using the four modes of transportation. The charts in the Figure 4-1 through Figure 4-4 represent the cumulative quantity shipped from the supplier 'n' to the manufacturer using each transportation mode during the entire planning horizon. Air is the quickest transportation mode as it reaches the manufacturer in one time period followed by truck (2 time periods), rail (3 time periods) and ship (4 time periods).

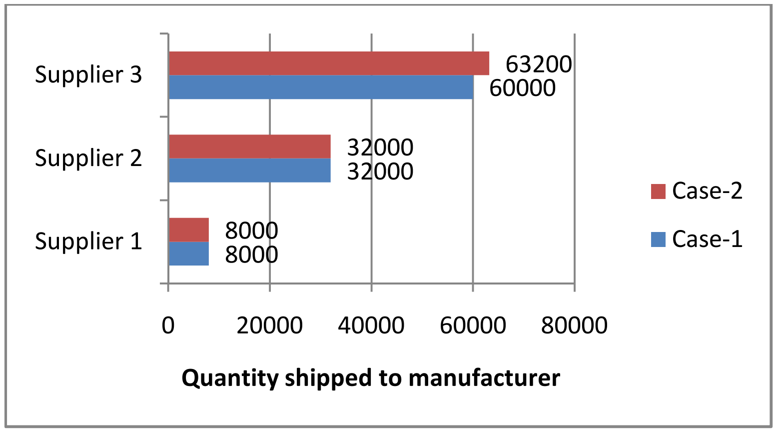

Figure 4-1: Quantity shipped from supplier to manufacturer using Air transportation mode

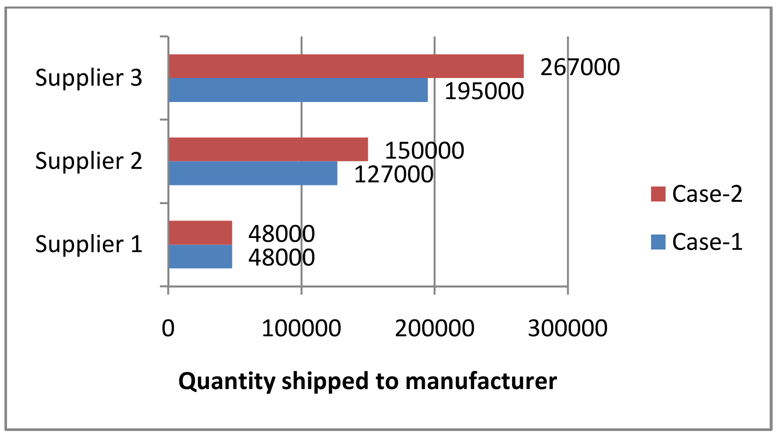

Figure 4-2: Quantity shipped from supplier to manufacturer using Truck transportation mode

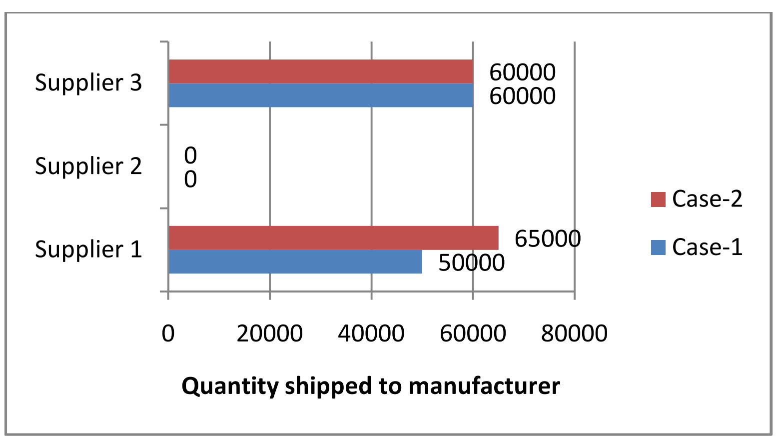

Figure 4-3: Quantity shipped from supplier to manufacturer using Rail transportation mode

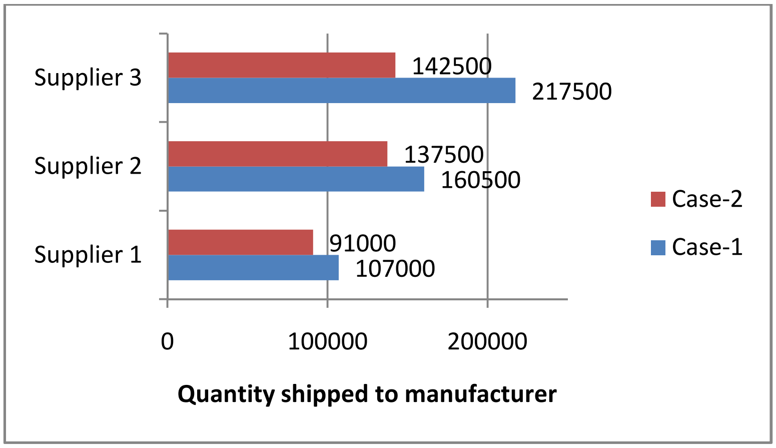

Figure 4-4: Quantity shipped from supplier to manufacturer using Ship transportation mode

When we prioritize the customer responsiveness over the total cost of the supply chain (Case-2), we can see that more quantities are shipped using air, truck and rail. When we prioritize the total cost of the supply chain over the customer responsiveness (Case-1), we can see that more quantities are shipped using ship which is the slowest but the cheapest mode of transport compared to the other three transportation modes. A similar pattern is followed for the quantities shipped from the manufacturer to the warehouse (Figure 4-5).

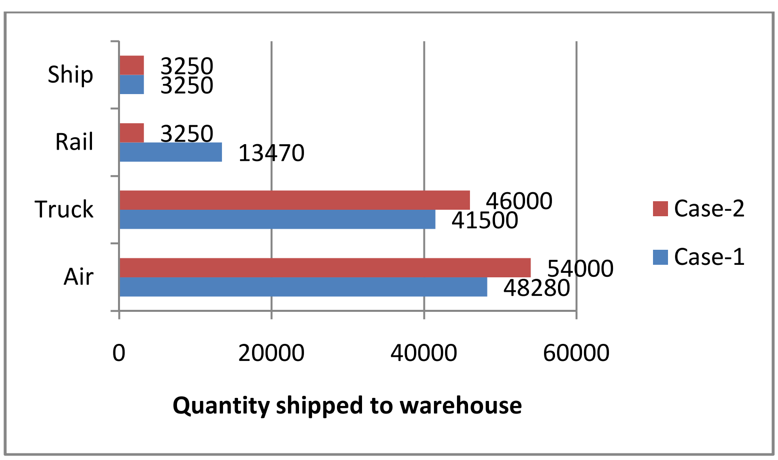

Figure 4-5: Quantity shipped from manufacturer to warehouse using all transportation modes

While implementing Case-2, air and truck transportation modes are used more to transport products when compared to Case-1. From the manufacturer to warehouse, rail is used more often when implementing Case-1. The shipping pattern that we observe in this example is in accord to the definition of customer responsiveness which dictates the product to be available as and when there is a demand at a particular stage of the supply chain.

Even though we identify such a pattern between the suppliers and the manufacturer and the manufacturer and the warehouse, there is no difference in the shipping pattern when the product is shipped from the warehouse to the retailers. The same amount of quantity is shipped using each transportation mode for both the cases. This can be explained by the fact that more inventory is stored in the leased warehouse for Case-2 when compared to Case-1. The retailer demand is satisfied immediately by using the inventory in the warehouse.

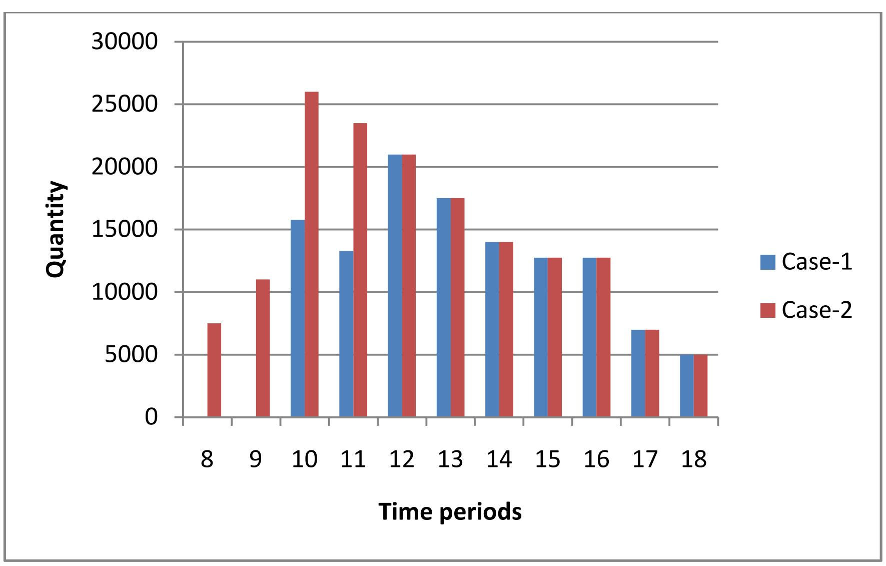

Figure 4-6: Leased warehouse inventory

The chart in Figure 4-6 represents the inventory stored in the leased warehouse from time periods 8 to 18. The figure shows that inventory is stored beginning time period '8' for Case-2 whereas inventory starts to accumulate only from time period '10' for Case-1. We can observe that more inventory is stored for Case-2 compared to Case-1.

4.5 NON PRE-EMPTIVE GOAL PROGRAMMING SOLUTIONS

The execution time of the non pre-emptive goal programming model using GAMS/CPLEX (version 11.01) on a 2.6 GHz AMD Opteron Processor with 32 GB of ECC RAM is 0.114 seconds.

In the non pre-emptive goal programming problem weights are assigned to the deviation variables such that the sum of the weights is equal to one ($w_1 + w_2 = 1$). The variation in the total cost and the total number of backorders is observed according to the weights assigned (Table 4.22).

Weight on the deviation variable associated with the total cost (w_1)	Weight on the deviation variable associated with the total number of backorders (w_2)	Total cost (OBJ_1) (in $)	Total number of backorders (OBJ_2) (units)
0.0	1.0	512,978	456,225
0.1	0.9	300,197	456,225
0.2	0.8	277,892	456,225
0.8	0.2	277,892	456,225
0.9	0.1	273,971	551,500
1.0	0.0	268,972	2,351,250

Table 4.22: Weights and objective values for non pre-emptive goal programming problem

From the objective values listed in Table 4.22, we observe that when the weight on one of the objective is 1.0 and the other objective is 0.0, the objective with the maximum weight reaches the target value set for it. For example when w_1 = 0.0 and w_2 = 1.0, OBJ_2 = 456,225 which equals $Target_2$. Similarly when w_1 = 1.0 and w_2 = 0.0, OBJ_1 = 268,972 which is greater than $Target_2$= 273,971 by 1.85%.

Since both the objective functions are minimization functions we see a decreasing function of the objective values with increasing value of their weights. This is illustrated in Figure 4-7 for the total cost and Figure 4-8 for the total number of backorders. We see that the total cost is the largest when the weight assigned to it is 0.0 and it decreases when the weight is increased to 0.1 and 0.2 from Figure 4-7. The cost remains the same from 0.2 to 0.8 and decreases when the weight is 0.9. The total cost is minimal when the weight assigned to it is 1.0. Similarly, we find the total number of backorders to be highest when the weight assigned to it is 0.0. It decreases when the weight assigned is 0.9 and 0.8 but remains the same throughout.

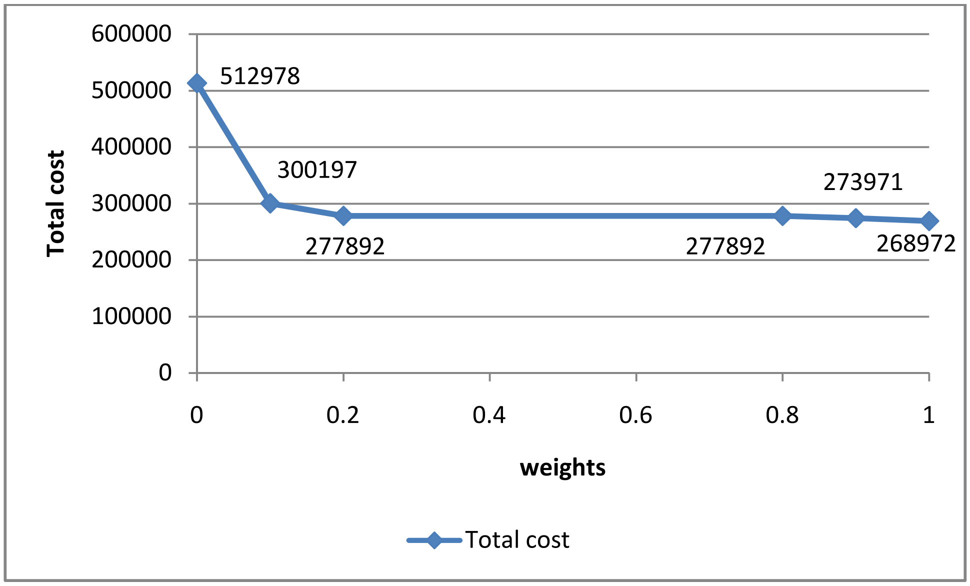

Figure 4-7: Variation in Total Cost according to the weights assigned

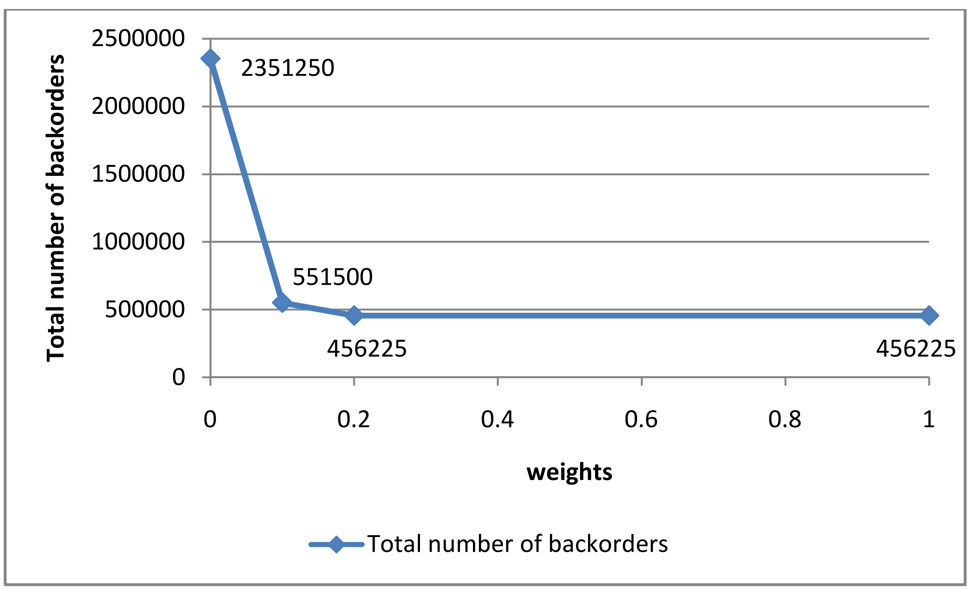

Figure 4-8: Variation in Total number of Backorders according to the weights assigned

From Table 4.22 we can see that the solution with the weights w_1 = 0.8 and w_2 = 0.2 dominates other solutions with $w_1 <$ 0.8. Hence we choose the solutions with the weights w_1 = 0.8, 0.9 and 1.0 and w_2 = 0.2, 0.1 and 0.0 respectively for our discussion. The three cases that we consider are:

Case-3: Weight on the total cost of the supply chain, w_1 = 0.8 and weight on the total number of backorders accumulated in the supply chain, w_2 = 0.2

Case-4: Weight on the total cost of the supply chain, w_1 = 0.9 and weight on the total number of backorders accumulated in the supply chain, w_2 = 0.1

Case-5: Weight on the total cost of the supply chain, w_1 = 1.0 and weight on the total number of backorders accumulated in the supply chain, w_2 = 0.0

The charts in Figure 4-9 through Figure 4-12 represent the cumulative quantity shipped from supplier 'n' to the manufacturer using each transportation mode during the entire planning horizon. When there is some weight on the customer responsiveness (Case-3, Case-4), we can see that more quantities are shipped using air and truck. When the weight on the total cost of the supply chain is 1.0 and the weight on the customer responsiveness is 0.0 (Case-5), we can see that the quantities shipped using ship, which is the slowest but the cheapest mode of transport, is almost tripled compared to the usage of the same mode in the other two cases (Figure 4-12).

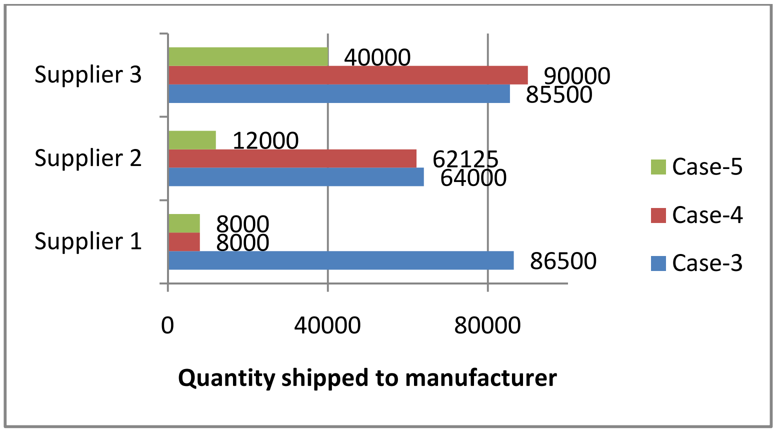

Figure 4-9: Quantity shipped from supplier to manufacturer using Air transportation mode

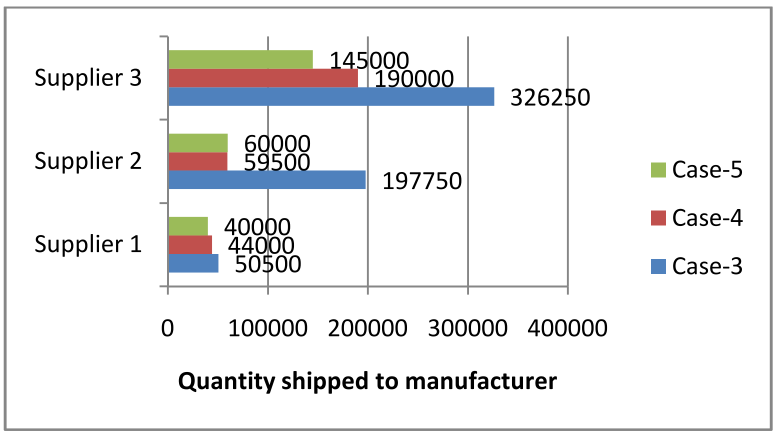

Figure 4-10: Quantity shipped from supplier to manufacturer using Truck transportation mode

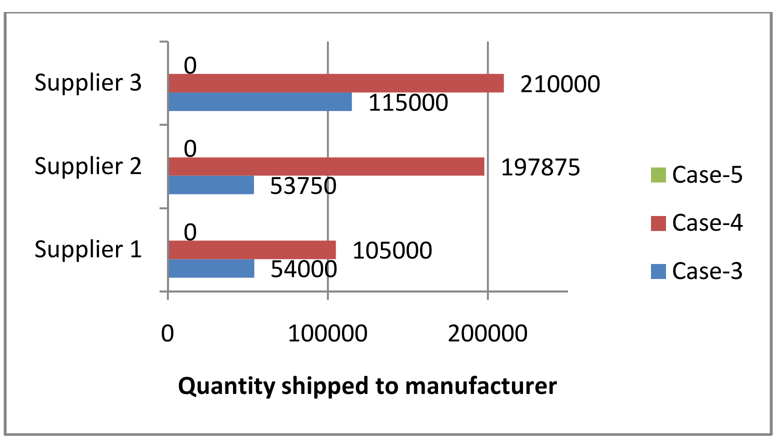

Figure 4-11: Quantity shipped from supplier to manufacturer using Rail transportation mode

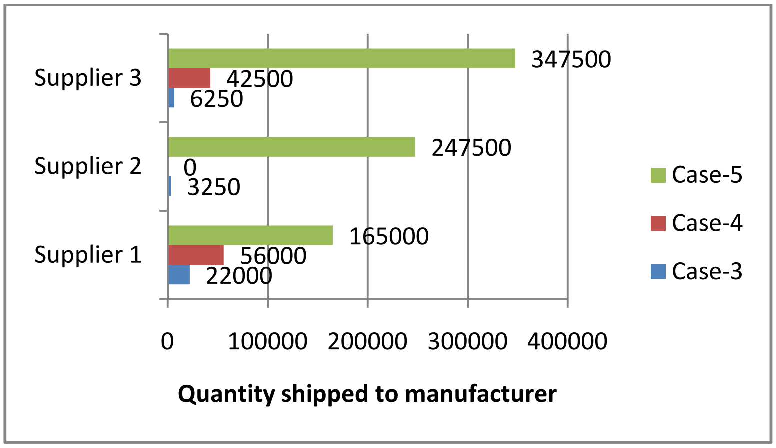

Figure 4-12: Quantity shipped from supplier to manufacturer using Ship transportation mode

Figure 4-13: Quantity shipped from manufacturer to warehouse using all transportation modes

The shipping pattern for the quantities shipped from the manufacturer to the warehouse (Figure 4-13) follows a similar pattern. As we go from Case-3 to Case-5, we see that the amount of products shipped using air and truck decreases whereas the amount of products shipped by rail and ship increases. This indicates that fewer products are shipped using faster modes of transport and stored in the warehouses as the weight on the customer responsiveness decreases. Since the products arrive at the warehouse using slower modes of transport, the products have to be shipped to the retailers using faster modes of transport in order to meet the demand as soon as possible. This is shown in Figure 4-14 which displays the shipping pattern for all three cases for the quantity shipped from the warehouse to the retailers.

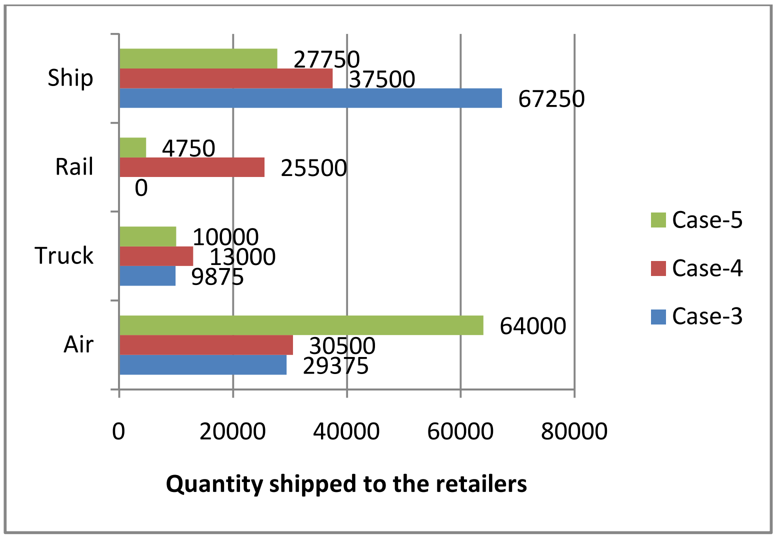

Figure 4-14: Quantity shipped from warehouse to retailers using all transportation modes

The leased warehouse inventory decreases as we increase the weight on the total cost of the supply chain. The leased warehouse is not used for Case-5 when the weight on total cost of the supply chain is 1.0. The quantity stored in the company owned warehouse is also less for Case-5 compared to the other two cases. Figure 4-15 showcases the pattern of the inventory stored in the leased warehouse.

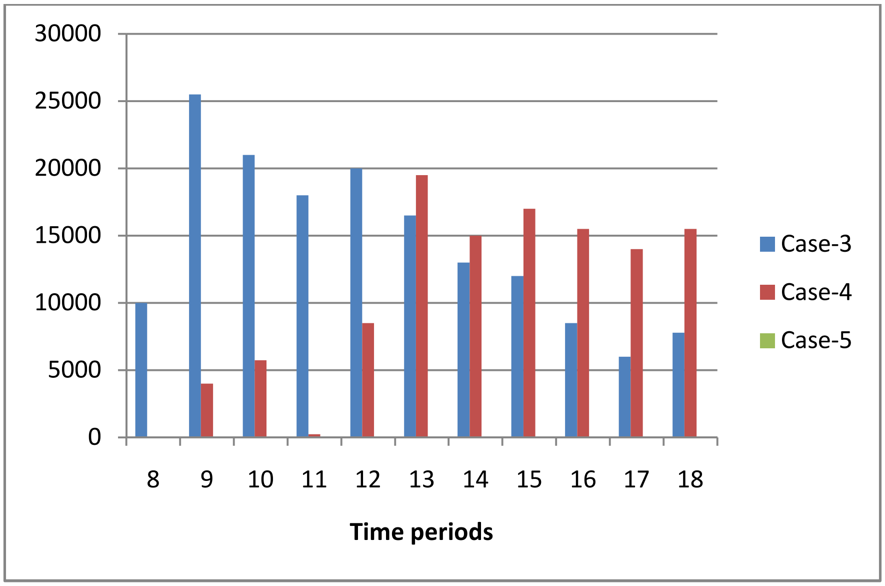

Figure 4-15: Leased warehouse inventory

The shipping pattern and the inventory storage pattern vary similar to the results discussed in the pre-emptive goal program. If the total number of backorders is closer/equal to its target value then the customer responsiveness is maximized. Hence more products are shipped through faster modes of transportation and there is more number of products stored as inventory in the warehouse. If the total cost is closer/equal to its target value then more products are shipped through cheaper modes of transportation and relatively less number of products is stored as inventory in the warehouse.

4.6 MANAGERIAL IMPLICATIONS OF GOAL PROGRAMMING SOLUTIONS

In this section we will compare the results of the pre-emptive and non pre-emptive goal programming solutions. This comparison will help a manager choose the best solution for implementation. We use both the solutions from the pre-emptive goal programming problem and consider three of the solutions from the non pre-emptive goal programming problem. The five different solutions will be compared with respect to the objective values, namely, the total cost of the supply chain and the total number of backorders. The shipping pattern and the storage pattern will also be used for comparing these solutions. The five scenarios that we consider for the comparison are:

Case-1: Pre-emptive Goal Program, where we prioritize the total cost of the supply chain over the customer responsiveness

Case-2: Pre-emptive Goal Program, where we prioritize the customer responsiveness over the total cost of the supply chain

Case-3: Non pre-emptive Goal Program, with weight on the total cost, w_1 = 0.8 and weight on the total number of backorders, w_2 = 0.2

Case-4: Non pre-emptive Goal Program, with weight on the total cost, w_1 = 0.9 and weight on the total number of backorders, w_2 = 0.1

Case-5: Non pre-emptive Goal Program, with weight on the total cost, w_1 = 1.0 and weight on the total number of backorders, w_2 = 0.0

The target value for the total cost of the supply chain, $Target_1$ = \$273,971 and the target value for the total number of backorders accumulated in the supply chain, $Target_2$ = 456,225.

	Case-1 Pre-emptive Goal Program Total cost over the customer responsiveness	Case-2 Pre-emptive Goal Program Customer responsiveness over the total cost	Case-3 Non Pre-emptive Goal Program w_1 = 0.8 w_2 = 0.2	Case-4 Non Pre-emptive Goal Program w_1 = 0.9 w_2 = 0.1	Case-5 Non Pre-emptive Goal Program w_1 = 1.0 w_2 = 0.0
Total cost of the supply chain (in $)	273,971	277,892	277,892	273,971	268,972
Total number of backorders accumulated in the supply chain	495,440	456,225	456,225	551,500	2,351,250
Is the target on total cost of the supply chain satisfied?	Yes	No. It fails to meet the target by 1.43%.	No. It fails to meet the target by 1.43%.	Yes	Yes. It exceeds the target by 1.82%.
Is the target on total number of backorders accumulated in the supply chain satisfied?	No. It fails to meet the target by 8.60%.	Yes	Yes	No. It fails to meet the target by 20.88%.	No. It fails to meet the target by 415.37%.

Table 4.23: Comparison on Targets and Objective values for all the scenarios

Table 4.23 lists the total cost of the supply chain and the total number of backorders accumulated in the supply chain for each case. It also specifies if the target values have been achieved and if not, the percentage deviation from the target values. Table 4.24 summarizes the quantity shipped among all the stages in the supply chain using the different modes of transportation. This helps the manager identify the shipping pattern and choose the solution that best suits his/her goal to be achieved.

	Case-1 Pre-emptive Goal Program Total cost over the customer responsiveness	Case-2 Pre-emptive Goal Program Customer responsiveness over the total cost	Case-3 Non Pre-emptive Goal Program w_1 = 0.8 w_2 = 0.2	Case-4 Non Pre-emptive Goal Program w_1 = 0.9 w_2 = 0.1	Case-5 Non Pre-emptive Goal Program w_1 = 1.0 w_2 = 0.0
Quantity shipped from supplier 1 to the manufacturer	Air – 8000 Truck – 48000 Rail – 50000 Ship – 107000	Air – 8000 Truck – 48000 Rail – 65000 Ship – 91000	Air – 86500 Truck – 50500 Rail – 54000 Ship – 22000	Air – 8000 Truck – 44000 Rail – 105000 Ship – 56000	Air – 8000 Truck – 40000 Rail – 0 Ship – 165000
Quantity shipped from supplier 2 to the manufacturer	Air – 32000 Truck – 127000 Rail – 0 Ship – 160500	Air – 32000 Truck – 150000 Rail – 0 Ship – 137500	Air – 64000 Truck – 197750 Rail – 53750 Ship – 3250	Air – 62125 Truck – 59500 Rail – 197875 Ship – 0	Air – 12000 Truck – 60000 Rail – 0 Ship – 247500
Quantity shipped from supplier 3 to the manufacturer	Air – 60000 Truck – 195000 Rail – 60000 Ship – 217500	Air – 63200 Truck – 267000 Rail – 60000 Ship – 142500	Air – 85500 Truck – 326250 Rail – 115000 Ship – 6250	Air – 90000 Truck – 190000 Rail – 210000 Ship – 42500	Air – 40000 Truck – 145000 Rail – 0 Ship – 347500
Quantity shipped from the manufacturer to the warehouse	Air – 48280 Truck – 41500 Rail – 13470 Ship – 3250	Air – 54000 Truck – 46000 Rail – 3250 Ship – 3250	Air – 49625 Truck – 54875 Rail – 1000 Ship – 1000	Air – 46500 Truck – 37000 Rail – 190000 Ship – 4000	Air – 21000 Truck – 22500 Rail – 20500 Ship – 42500
Quantity shipped from the warehouse to retailer 1	Air – 15250 Truck – 0 Rail – 6250 Ship – 26000	Air – 15250 Truck – 0 Rail – 6250 Ship – 26000	Air – 11750 Truck – 3000 Rail – 0 Ship – 32750	Air – 14500 Truck – 3750 Rail – 11500 Ship – 17750	Air – 36250 Truck – 3500 Rail – 750 Ship – 7000
Quantity shipped from the warehouse to retailer 2	Air – 14000 Truck – 3250 Rail – 2750 Ship – 39000	Air – 14000 Truck – 3250 Rail – 2750 Ship – 39000	Air – 17625 Truck – 6875 Rail – 0 Ship – 34500	Air – 16000 Truck – 9250 Rail – 14000 Ship – 19750	Air – 27750 Truck – 6500 Rail – 4000 Ship – 20750

Table 4.24: Comparison on the shipping pattern for all the scenarios

	Case-1 Pre-emptive Goal Program Total cost over the customer responsiveness	Case-2 Pre-emptive Goal Program Customer responsiveness over the total cost	Case-3 Non Pre-emptive Goal Program w_1 = 0.8 w_2 = 0.2	Case-4 Non Pre-emptive Goal Program w_1 = 0.9 w_2 = 0.1	Case-5 Non Pre-emptive Goal Program w_1 = 1.0 w_2 = 0.0
Inventory stored in the company owned warehouse	120000	114000	114000	139000	44750
Is the leased warehouse used to store inventory?	Yes. It is used from time period 10 through 18.	Yes. It is used from time period 8 through 18.	Yes. It is used from time period 8 through 18.	Yes. It is used from time period 9 through 18.	No
Inventory stored in the leased warehouse	119060	158300	158275	115000	0

Table 4.25: Comparison on the inventory storage pattern for all the scenarios

Table 4.25 illustrates on the pattern of inventory in each case. It specifies if the leased warehouse is used and if so, the time period through which it is used.

The Table 4.23 through Table 4.25 and Figure 4-1 through Figure 4-15 helps a manager to identify the major changes in the solutions using Goal Programming approach. They clearly summarize the shipping pattern and the inventory storage pattern which helps to identify the transportation and inventory decisions.

CHAPTER 5

CONCLUSIONS

Supply chains often deal with conflicting criteria, two such conflicting criteria are cost and customer responsiveness. When the cost is minimized it is called an *efficient supply chain* and when the customer responsiveness is maximized it is called a *responsive supply chain*. There is always a tradeoff between efficiency and responsiveness with respect to all the supply chain drivers. This research focused on two of the supply chain drivers, namely, inventory and transportation and used multi-criteria optimization techniques to study the tradeoff between efficiency and responsiveness. Most of the companies consider transportation and inventory as separate decisions for the sake of simplicity and optimize their supply chain accordingly. These two drivers are the major contributors to the supply chain costs and minimizing them will result in the overall increase in the supply chain profitability. Fewer models are found in the literature for incorporating both cost and customer responsiveness in the optimization models since it involves dealing with multi-criteria optimization.

In this thesis, we considered a centralized four stage supply chain with multiple suppliers, a manufacturer, a warehouse and multiple retailers. We developed a multi-criteria multi-period linear integer programming model for optimizing such a centralized four-stage supply chain and discussed solution methods to solve this problem. The objective of the bi-criteria optimization model was to minimize the overall cost of the supply chain and maximize the customer responsiveness (which is quantified by the total number of backorders to be minimized). The bi-criteria optimization problem was solved using both non preemptive goal programming by giving weights to the two objectives and by preemptive goal programming assigning different priorities to the objectives. The goal programming approach required the ideal solution for both the objective values, i.e., the minimum cost of the supply chain and the minimum number of backorders. The ideal values were obtained by solving the single objective optimization problem ignoring the other objective. The target values were set for each objective based on the ideal values. The model solution identified optimal modes of transport to be used between each stage of the supply chain, the optimal inventory levels to be maintained at each stage and the backorder levels at each stage in order to minimize the total supply chain cost and maximize the customer responsiveness.

The model was illustrated using an example problem with simulated data. The problem was solved using GAMS (General Algebraic Modeling System) which is a high level modeling system for mathematical programming and optimization that can handle large size real world problems. The results were analyzed for the pre-emptive goal programming approach by prioritizing the total cost of the supply chain over the customer responsiveness in one case and by prioritizing the customer responsiveness over the total cost of the supply chain in another case. While assigning different weights in the non pre-emptive goal programming approach, the results were analyzed from the efficient or the non dominated set. The analysis was done by means of comparing the shipping patterns, the inventory storage patterns and whether the objective values reach the set target values for each objective.

The results indicate that when the total cost of the supply chain is prioritized/assigned higher weights over the customer responsiveness, there are more quantities shipped using the slower modes of transport between the suppliers and the manufacturer and the manufacturer and the warehouse. The inventory stored at the company owned and the leased warehouses are less and hence more quantities are shipped using faster modes of transport between the warehouse and the retailers in order to meet all the retailer demands before the end of the planning time horizon. This is in contrast to the results in the case when the customer responsiveness is prioritized/assigned more weights over the total cost of the supply chain. In this case more quantities are shipped using the faster modes of transport between the suppliers and the manufacturer and the manufacturer and the warehouse. As a result more quantities are stored at the company owned and the leased warehouses and products are shipped to the retailers from the warehouse using the slower modes of transport to meet their demand. With enough products stored at the warehouse, the products are shipped to the retailers when they need it and hence reduce the accumulation of backorders as much as possible.

The goal programming solutions have significant managerial implications. They identify the right amount of quantity to be shipped by the appropriate mode of transport at each supply chain stage, the right amount of inventory to be stored at the company owned and leased warehouses and the accumulated backorder at the various stages (if any) in order to minimize the total supply chain cost and maximize the customer responsiveness. They help the manager with the transportation and inventory decisions. The manager can prioritize either the total cost of the supply chain or the customer responsiveness in the pre-emptive goal programming approach. The manager is provided with the efficient set in case of non pre-emptive goal programming approach and hence can choose the weights on the total cost of the supply chain and the customer responsiveness depending on his/her needs or the goals he/she tries to achieve for his/her company.

Some of the possible extensions of this thesis could be to model customer responsiveness using other methodology from the literature such as lead time, stockout probability and fill rate and comparing it with our model to find which works best for a centralized supply chain. The model studied the case where demand was deterministic. A further extension could be to study the case of stochastic demand and stochastic transportation time which are more realistic. In reality a supply chain could have multiple manufacturers and warehouses, the inclusion of which would enhance the model. A model that involves multiple products in the supply chain would also be a significant extension of this thesis. Further researchers could use this model and compare the results by using the other multiple criteria optimization approaches such as Chebyshev's goal programming, Fuzzy goal programming, Compromise programming and interactive methods.

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APPENDIX A

CONSTRAINTS OF THE MATHEMATICAL MODEL

The real constraints have been grouped together as follows,

Constraints at the manufacturer

$$TRM_{n\,t} = X_{n\,1\,t-1} + X_{n\,2\,t-2} + \cdots + X_{n\,i\,t-i} \qquad \forall\, n, t \text{ and } t \geq i+1$$

$$TRM_{n\,t} = a_n * PM_{t+j} \qquad \forall\, n, t$$

$$PM_t \leq MCAP \qquad \forall\, t$$

$$QTSM_{n\,i\,t\,k} \leq (\beta_{i\,k+1} - \beta_{i\,k}) * \delta_{n\,i\,t\,k} \qquad \forall\, n, i, t, k$$

$$QTSM_{n\,i\,t\,k} \geq (\beta_{i\,k+1} - \beta_{i\,k}) * \delta_{n\,i\,t\,k+1} \qquad \forall\, n, i, t, k$$

$$\sum_i X_{n\,i\,t} = \sum_i \sum_k QTSM_{n\,i\,t\,k} \qquad \forall\, n, t$$

$$QTSM_{n\,i\,t\,k} \geq 0 \qquad \forall\, n, i, t, k$$

$$X_{n\,i\,t} \geq 0 \qquad \forall\, n, i, t, k$$

$$\delta_{n\,i\,t\,k} \in (0,1) \qquad \forall\, n, i, t, k$$

$$X_{n\,i\,t} \leq SMMAX_i \qquad \forall\, n, i, t$$

$$\sum_i X_{n\,i\,t} \leq SCAP_n \qquad \forall\, n, t$$

$$PM_t = \sum_h Y_{h\,t} \qquad \forall\, t$$

$$TRM_{n\,t} \leq TRINVCAP_n \qquad \forall\, n, t$$

$$\sum_h Y_{h\,t-h} + MBO_t = WD_t + MBO_{t-1} \qquad \forall\, t$$

$$WD_t = \sum_r CD_{r\,t} + WBO_{t-1} \qquad \forall\, t$$

$$\sum_h Y_{h\,t} = \sum_h \sum_l QTMW_{h\,t\,l} \qquad \forall\, t$$

$$QTMW_{h\,t\,l} \leq (\gamma_{h\,l+1} - \gamma_{h\,l}) * \theta_{h\,t\,l} \qquad \forall\, h, t, l$$

$$QTMW_{h\,t\,l} \geq (\gamma_{h\,l+1} - \gamma_{h\,l}) * \theta_{h\,t\,l+1} \qquad \forall\, h, t, l$$

$$QTMW_{h\,t\,l} \geq 0 \qquad \forall\, h, t, l$$

$$Y_{h\,t} \geq 0 \qquad \forall\, h, t$$

$$\theta_{h\,t\,l} \in (0,1) \qquad \forall\, h, t, l$$

$$Y_{h\,t} \leq MWMAX_h \qquad \forall\, h, t$$

Constraints at the warehouse

$$\sum_h Y_{h\,t-h} + WINV_{t-1} + TLWINV_{t-1}$$

$$= \sum_r \sum_q Z_{r\,q\,t} + WINV_t + TLWINV_t \qquad \forall\, t \;\; and \; t \geq h + 1$$

$$WINV_t \leq WINVCAP \qquad \forall\, t$$

$$LWINV_{p\,t} \leq \left(S_{p+1} - S_p\right) * \alpha_{p\,t} \qquad \forall\, p, t$$

$$LWINV_{p\,t} \geq \left(S_{p+1} - S_p\right) * \alpha_{p+1\,t} \qquad \forall\, p, t$$

$$TLWINV_t = \sum_p LWINV_{p\,t} \qquad \forall\, t$$

$$LWINV_{p\,t} \geq 0 \qquad \forall\, p, t$$

$$TLWINV_t \geq 0 \qquad \forall\, t$$

$$\alpha_{p\,t} \in (0,1) \qquad \forall\, p, t$$

$$\sum_r \quad \sum_q Z_{r\,q\,t-q} + WBO_t = \sum_r CD_{r\,t} + WBO_{t-1} \qquad \forall\, t$$

Constraints at the retailers

$$QTWR_{r\,q\,t\,s} \leq \left(\tau_{q\,s+1} - \tau_{q\,s}\right) * \Delta_{r\,q\,t\,s} \qquad \forall\, r, q, t, s$$

$$QTWR_{r\,q\,t\,s} \geq \left(\tau_{q\,s+1} - \tau_{q\,s}\right) * \Delta_{r\,q\,t\,s+1} \qquad \forall\, r, q, t, s$$

$$\sum_q Z_{r\,q\,t} = \sum_q \quad \sum_s QTWR_{r\,q\,t\,s} \qquad \forall\, r, t$$

$$QTWR_{r\,q\,t\,s} \geq 0 \qquad \forall\, r, q, t, s$$

$$Z_{r\,q\,t} \geq 0 \qquad \forall\, r, q, t$$

$$\Delta_{r\,q\,t\,s} \in (0,1) \qquad \forall\, r, q, t, s$$

$$Z_{r\,1\,t-1} + Z_{r\,2\,t-2} + \cdots + Z_{r\,q\,t-q} + RBO_{r\,t} = CD_{r\,t} + RBO_{r\,t-1} \qquad \forall\, r, t \; and \; t \geq q+1$$

$$\sum_{r} Z_{r\,q\,t} \leq WRMAX_{q} \qquad \forall\, q, t$$

$$RBO_{r\,T} = 0 \qquad \forall\, r$$

APPENDIX B

RESULTS FROM SOLVING THE EXAMPLE PROBLEM

Case-1: Prioritizing total cost of the supply chain over the customer responsiveness

The transportation and inventory decisions that result from the optimization model are summarized.

Quantity shipped from each supplier to the manufacturer

Mode of transportation - Air

Supplier (n)	Mode (i)	Time period (t)	Cost bracket (k)	Bracket capacity	Units shipped	Variable ($X_{n\,i\,t}$)
1	Air	1	1	5000	5000	$X_{1\,Air\,1}=$ 8000
			2	15000	3000	
2	Air	1	1	5000	5000	$X_{2\,Air\,1}=$ 12000
			2	15000	7000	
			2	15000	15000	
2	Air	3	1	5000	5000	$X_{2\,Air\,3}=$ 20000
			2	15000	15000	
3	Air	1	1	5000	5000	$X_{3\,Air\,1}=$ 20000
			2	15000	15000	
3	Air	2	1	5000	5000	$X_{3\,Air\,2}=$ 20000
			2	15000	15000	
3	Air	4	1	5000	5000	$X_{3\,Air\,4}=$ 20000
			2	15000	15000	

Table A.1: Quantity shipped from supplier to manufacturer using ‘Air’ transportation

Mode of transportation - Truck

Supplier (n)	Mode (i)	Time period (t)	Cost bracket (k)	Bracket capacity	Units shipped	Variable ($X_{n\,i\,t}$)
1	Truck	1	1	10000	10000	$X_{1\,Truck\,1}$ = 48000
			2	40000	38000	
2	Truck	1	1	10000	10000	$X_{2\,Truck\,1}$ = 72000
			2	40000	40000	
			3	50000	22000	
2	Truck	2	1	10000	10000	$X_{2\,Truck\,2}$ = 55000
			2	40000	40000	
			3	50000	5000	
3	Truck	1	1	10000	10000	$X_{3\,Truck\,1}$ = 100000
			2	40000	40000	
			3	50000	50000	
3	Truck	2	1	10000	10000	$X_{3\,Truck\,2}$ = 65000
			2	40000	40000	
			3	60000	15000	
3	Truck	3	1	10000	10000	$X_{3\,Truck\,3}$ = 30000
			2	40000	20000	

Table A.2: Quantity shipped from supplier to manufacturer using ‘Truck’ transportation

Mode of transportation - Rail

Supplier (n)	Mode (i)	Time period (t)	Cost bracket (k)	Bracket capacity	Units shipped	Variable ($X_{n\,i\,t}$)
1	Rail	1	1	10000	10000	$X_{1\,Rail\,1}$ = 50000
			2	65000	40000	
3	Rail	1	1	10000	10000	$X_{3\,Rail\,1}$ = 60000
			2	65000	50000	

Table A.3: Quantity shipped from supplier to manufacturer using ‘Rail’ transportation

Mode of transportation - Ship

Supplier (n)	Mode (i)	Time period (t)	Cost bracket (k)	Bracket capacity	Units shipped	Variable ($X_{n\,i\,t}$)
1	Ship	1	1	20000	20000	$X_{1\,Ship\,1}$ = 20000
1	Ship	2	1	20000	20000	$X_{1\,Ship\,2}$ = 38000
			2	180000	18000	
1	Ship	3	1	20000	20000	$X_{1\,Ship\,3}$ = 36000
			2	180000	16000	
1	Ship	11	1	20000	8000	$X_{1\,Ship\,11}$ = 8000
1	Ship	14	1	20000	5000	$X_{1\,Ship\,14}$ = 5000
2	Ship	1	1	20000	20000	$X_{2\,Ship\,1}$ = 30000
			2	180000	10000	
2	Ship	2	1	20000	20000	$X_{2\,Ship\,2}$ = 57000
			2	180000	37000	
2	Ship	3	1	20000	20000	$X_{2\,Ship\,3}$ = 54000
			2	180000	34000	
2	Ship	11	1	20000	12000	$X_{2\,Ship\,11}$ = 12000
2	Ship	14	1	20000	7500	$X_{2\,Ship\,14}$ = 7500
3	Ship	2	1	20000	20000	$X_{3\,Ship\,2}$ = 95000
			2	180000	75000	
3	Ship	3	1	20000	20000	$X_{3\,Ship\,3}$ = 90000
			2	180000	70000	
3	Ship	11	1	20000	20000	$X_{3\,Ship\,11}$ = 20000
3	Ship	14	1	20000	12500	$X_{3\,Ship\,14}$ = 12500

Table A.4: Quantity shipped from supplier to manufacturer using 'Ship' transportation

Quantity shipped from the manufacturer to the warehouse

Mode of transportation - Air

Mode (h)	Time period (t)	Cost bracket (l)	Bracket capacity	Units shipped	Variable ($Y_{h\ t}$)
Air	4	1	2000	2000	$Y_{Air\ 4}$ = 4000
		2	8000	2000	
Air	5	1	2000	2000	$Y_{Air\ 5}$ = 10000
		2	8000	8000	
Air	6	1	2000	2000	$Y_{Air\ 6}$ = 10000
		2	8000	8000	
Air	7	1	2000	2000	$Y_{Air\ 7}$ = 10000
		2	8000	8000	
Air	8	1	2000	2000	$Y_{Air\ 8}$ = 6500
		2	8000	4500	
Air	9	1	2000	2000	$Y_{Air\ 9}$ = 7780
		2	8000	5780	

Table A.5: Quantity shipped from manufacturer to warehouse using 'Air' transportation

Mode of transportation - Truck

Mode (h)	Time period (t)	Cost bracket (l)	Bracket capacity	Units shipped	Variable ($Y_{h\ t}$)
Truck	5	1	5000	5000	$Y_{Truck\ 5}$ = 14000
		2	20000	9000	
Truck	6	1	5000	5000	$Y_{Truck\ 6}$ = 15000
		2	20000	10000	
Truck	8	1	5000	5000	$Y_{Truck\ 8}$ = 12500
		2	20000	7500	

Table A.6: Quantity shipped from manufacturer to warehouse using 'Truck' transportation

Mode of transportation - Rail

Mode (h)	Time period (t)	Cost bracket (l)	Bracket capacity	Units shipped	Variable ($Y_{h\,t}$)
Rail	9	1	5000	5000	$Y_{Rail\ 9}$ = 10220
		2	20000	5220	
Rail	17	1	5000	750	$Y_{Rail\ 17}$ = 750
Rail	20	1	5000	2500	$Y_{Rail\ 20}$ = 2500

Table A.7: Quantity shipped from manufacturer to warehouse using 'Rail' transportation

Mode of transportation - Ship

Mode (h)	Time period (t)	Cost bracket (l)	Bracket capacity	Units shipped	Variable ($Y_{h\,t}$)
Ship	17	1	20000	3250	$Y_{Ship\ 17}$ = 3250

Table A.8: Quantity shipped from manufacturer to warehouse using 'Ship' transportation

Quantity shipped from the warehouse to each retailer

Mode of transportation - Air

Retailer (r)	Mode (q)	Time period (t)	Cost bracket (s)	Bracket capacity	Units shipped	Variable ($Z_{r\,q\,t}$)
1	Air	6	1	2000	2000	$Z_{1\ Air\ 6}$ = 10000
			2	8000	8000	
1	Air	8	1	2000	2000	$Z_{1\ Air\ 8}$ = 2750
			2	8000	750	
1	Air	23	1	2000	2000	$Z_{1\ Air\ 23}$ = 2500
			2	8000	500	
2	Air	5	1	2000	2000	$Z_{2\ Air\ 5}$ = 4000
			2	8000	2000	
2	Air	7	1	2000	2000	$Z_{2\ Air\ 7}$ = 10000
			2	8000	8000	

Table A.9: Quantity shipped from warehouse to retailer using 'Air' transportation

Mode of transportation - Truck

Retailer (r)	Mode (q)	Time period (t)	Cost bracket (s)	Bracket capacity	Units shipped	Variable ($Z_{r\,q\,t}$)
2	Truck	7	1	5000	3250	$Z_{2\,Truck\,7}$ = 3250

Table A.10: Quantity shipped from warehouse to retailer using 'Truck' transportation

Mode of transportation - Rail

Retailer (r)	Mode (q)	Time period (t)	Cost bracket (s)	Bracket capacity	Units shipped	Variable ($Z_{r\,q\,t}$)
1	Rail	7	1	5000	2000	$Z_{1\,Rail\,7}$ = 2000
1	Rail	8	1	5000	1750	$Z_{1\,Rail\,8}$ = 1750
1	Rail	21	1	5000	2500	$Z_{1\,Rail\,21}$ = 2500
2	Rail	7	1	5000	2000	$Z_{2\,Rail\,7}$ = 2000
2	Rail	21	1	5000	750	$Z_{2\,Rail\,21}$ = 750

Table A.11: Quantity shipped from warehouse to retailer using 'Rail' transportation

Mode of transportation - Ship

Retailer (r)	Mode (q)	Time period (t)	Cost bracket (s)	Bracket capacity	Units shipped	Variable ($Z_{r\,q\,t}$)
1	Ship	7	1	20000	2250	$Z_{1\,Ship\,7}$ = 2250
1	Ship	8	1	20000	5000	$Z_{1\,Ship\,8}$ = 5000
1	Ship	9	1	20000	3000	$Z_{1\,Ship\,9}$ = 3000
1	Ship	10	1	20000	2000	$Z_{1\,Ship\,10}$ = 2000
1	Ship	11	1	20000	1000	$Z_{1\,Ship\,11}$ = 1000

Retailer (r)	Mode (q)	Time period (t)	Cost bracket (s)	Bracket capacity	Units shipped	Variable ($Z_{r\,q\,t}$)
1	Ship	12	1	20000	1000	$Z_{1\,Ship\,12} =$ 1000
1	Ship	13	1	20000	1500	$Z_{1\,Ship\,13} =$ 1500
1	Ship	14	1	20000	1500	$Z_{1\,Ship\,14} =$ 1500
1	Ship	17	1	20000	2750	$Z_{1\,Ship\,17} =$ 2750
1	Ship	18	1	20000	2000	$Z_{1\,Ship\,18} =$ 2000
1	Ship	19	1	20000	4000	$Z_{1\,Ship\,19} =$ 4000
2	Ship	7	1	20000	4500	$Z_{2\,Ship\,7} =$ 4500
2	Ship	8	1	20000	5500	$Z_{2\,Ship\,8} =$ 5500
2	Ship	9	1	20000	3500	$Z_{2\,Ship\,9} =$ 3500
2	Ship	10	1	20000	2500	$Z_{2\,Ship\,10} =$ 2500
2	Ship	11	1	20000	1500	$Z_{2\,Ship\,11} =$ 1500
2	Ship	12	1	20000	1500	$Z_{2\,Ship\,12} =$ 1500
2	Ship	13	1	20000	2000	$Z_{2\,Ship\,13} =$ 2000
2	Ship	14	1	20000	2000	$Z_{2\,Ship\,14} =$ 2000
2	Ship	15	1	20000	1250	$Z_{2\,Ship\,15} =$ 1250
2	Ship	17	1	20000	3000	$Z_{2\,Ship\,17} =$ 3000
2	Ship	19	1	20000	7000	$Z_{2\,Ship\,19} =$ 7000
2	Ship	20	1	20000	4750	$Z_{2\,Ship\,20} =$ 4750

Table A.12: Quantity shipped from warehouse to retailer using ‘Ship’ transportation

Amount of inventory stored at the company owned warehouse

Time period (t)	Quantity stored ($WINV_t$)
8	$WINV_8$ = 10000
9	$WINV_9$ = 10000
10	$WINV_{10}$ = 10000
11	$WINV_{11}$ = 10000
12	$WINV_{12}$ = 10000
13	$WINV_{13}$ = 10000
14	$WINV_{14}$ = 10000
15	$WINV_{15}$ = 10000
16	$WINV_{16}$ = 10000
17	$WINV_{17}$ = 10000
18	$WINV_{18}$ = 10000
19	$WINV_{19}$ = 10000

Table A.13: Quantity stored in the company owned warehouse

Amount of inventory stored at the leased warehouse

Time period (t)	Cost bracket (p)	Bracket capacity	Quantity stored ($LWINV_{p\,t}$)
10	1	10000	$LWINV_{1\,10}$ = 10000
	2	50000	$LWINV_{2\,10}$ = 5780
11	1	10000	$LWINV_{1\,11}$ = 10000
	2	50000	$LWINV_{2\,11}$ = 3280
12	1	10000	$LWINV_{1\,12}$ = 10000
	2	50000	$LWINV_{2\,12}$ = 11000
13	1	10000	$LWINV_{1\,13}$ = 10000
	2	50000	$LWINV_{2\,13}$ = 7500
14	1	10000	$LWINV_{1\,14}$ = 10000
	2	50000	$LWINV_{2\,14}$ = 4000
15	1	10000	$LWINV_{1\,15}$ = 10000
	2	50000	$LWINV_{2\,15}$ = 2750
16	1	10000	$LWINV_{1\,16}$ = 10000
	2	50000	$LWINV_{2\,16}$ = 2750
17	1	10000	$LWINV_{1\,17}$ = 7000
18	1	10000	$LWINV_{1\,18}$ = 5000

Table A.14: Quantity stored in the leased warehouse

Cumulative backorders at the manufacturer

Time period (t)	Backorders (MBO_t)
2	MBO_2 = 6500
3	MBO_3 = 17500
4	MBO_4 = 31000
5	MBO_5 = 43000
6	MBO_6 = 52500
7	MBO_7 = 47500
8	MBO_8 = 33500
9	MBO_9 = 30500
10	MBO_{10} = 10220
11	MBO_{11} = 10220

Table A.15: Backorders accumulated at the manufacturer

Cumulative backorders at the warehouse

Time period (t)	Backorders (WBO_t)
1	WBO_1 = 6500
2	WBO_2 = 11000
3	WBO_3 = 13500
4	WBO_4 = 16000
5	WBO_5 = 19500
6	WBO_6 = 19000
7	WBO_7 = 11000
8	WBO_8 = 3500
19	WBO_{19} = 750
20	WBO_{20} = 3250
22	WBO_{22} = 2500

Table A.16: Backorders accumulated at the warehouse

Cumulative backorders at each retailer

Retailer (r)	Time period (t)	Backorders $(RBO_{r\,t})$
1	1	$RBO_{1\,1}$ = 3000
1	2	$RBO_{1\,2}$ = 5000
1	3	$RBO_{1\,3}$ = 6000
1	4	$RBO_{1\,4}$ = 7000
1	5	$RBO_{1\,5}$ = 8500
1	6	$RBO_{1\,6}$ = 10000
1	7	$RBO_{1\,7}$ = 750
1	8	$RBO_{1\,8}$ = 1750
1	19	$RBO_{1\,9}$ = 750
1	20	$RBO_{1\,20}$ = 1750
2	1	$RBO_{2\,1}$ = 3500
2	2	$RBO_{2\,2}$ = 6000
2	3	$RBO_{2\,3}$ = 7500
2	4	$RBO_{2\,4}$ = 9000
2	5	$RBO_{2\,5}$ = 11000
2	6	$RBO_{2\,6}$ = 9000
2	7	$RBO_{2\,7}$ = 10250
2	8	$RBO_{2\,8}$ = 1750
2	20	$RBO_{2\,20}$ = 1500
2	22	$RBO_{2\,22}$ = 2500

Table A.17: Backorders accumulated at the retailer

Total cost of the supply chain and the total number of backorders

Total cost (in $)	Total number of backorders (units)
273971	495440

Table A.18: Objective values for Case - 1

Case-2: Prioritizing customer responsiveness over the total cost of the supply chain

The transportation and inventory decisions that result from the optimization model are summarized.

Quantity shipped from each supplier to the manufacturer

Mode of transportation - Air

Supplier (n)	Mode (i)	Time period (t)	Cost bracket (k)	Bracket capacity	Units shipped	Variable ($X_{n\,i\,t}$)
1	Air	1	1	5000	5000	$X_{1\,Air\,1} =$ 8000
			2	15000	3000	
2	Air	1	1	5000	5000	$X_{2\,Air\,1} =$ 12000
			2	15000	7000	
2	Air	3	1	5000	5000	$X_{2\,Air\,3} =$ 20000
			2	15000	15000	
3	Air	1	1	5000	5000	$X_{3\,Air\,1} =$ 20000
			2	15000	15000	
3	Air	2	1	5000	5000	$X_{3\,Air\,2} =$ 20000
			2	15000	15000	
3	Air	3	1	5000	3200	$X_{3\,Air\,3} =$ 3200
3	Air	4	1	5000	5000	$X_{3\,Air\,4} =$ 20000
			2	15000	15000	

Table A.19: Quantity shipped from supplier to manufacturer using ‘Air’ transportation

Mode of transportation - Truck

Supplier (n)	Mode (i)	Time period (t)	Cost bracket (k)	Bracket capacity	Units shipped	Variable ($X_{n\,i\,t}$)
1	Truck	1	1	10000	10000	$X_{1\,Truck\,1}$ = 48000
			2	40000	38000	
2	Truck	1	1	10000	10000	$X_{2\,Truck\,1}$ = 72000
			2	40000	40000	
			3	50000	22000	
2	Truck	2	1	10000	10000	$X_{2\,Truck\,2}$ = 78000
			2	40000	40000	
			3	50000	28000	
3	Truck	1	1	10000	10000	$X_{3\,Truck\,1}$ = 100000
			2	40000	40000	
			3	50000	60000	
3	Truck	2	1	10000	10000	$X_{3\,Truck\,2}$ = 100000
			2	40000	40000	
			3	50000	50000	
3	Truck	3	1	10000	10000	$X_{3\,Truck\,3}$ = 30000
			2	40000	20000	
3	Truck	4	1	10000	10000	$X_{3\,Truck\,4}$ = 37000
			2	40000	27000	

Table A.20: Quantity shipped from supplier to manufacturer using ‘Truck’ transportation

Mode of transportation - Rail

Supplier (n)	Mode (i)	Time period (t)	Cost bracket (k)	Bracket capacity	Units shipped	Variable ($X_{n\,i\,t}$)
1	Rail	1	1	10000	10000	$X_{1\,Rail\,1}$ = 65000
			2	65000	55000	
3	Rail	1	1	10000	10000	$X_{3\,Rail\,1}$ = 60000
			2	65000	50000	

Table A.21: Quantity shipped from supplier to manufacturer using ‘Rail’ transportation

Mode of transportation - Ship

Supplier (n)	Mode (i)	Time period (t)	Cost bracket (k)	Bracket capacity	Units shipped	Variable ($X_{n\,i\,t}$)
1	Ship	1	1	20000	20000	$X_{1\,Ship\,1}$ = 20000
1	Ship	2	1	20000	20000	$X_{1\,Ship\,2}$ = 38000
			2	180000	18000	
1	Ship	3	1	20000	20000	$X_{1\,Ship\,3}$ = 20000
1	Ship	11	1	20000	8000	$X_{1\,Ship\,11}$ = 8000
1	Ship	14	1	20000	5000	$X_{1\,Ship\,14}$ = 5000
2	Ship	1	1	20000	20000	$X_{2\,Ship\,1}$ = 30000
			2	180000	10000	
2	Ship	2	1	20000	20000	$X_{2\,Ship\,2}$ = 58000
			2	180000	38000	
2	Ship	3	1	20000	20000	$X_{2\,Ship\,3}$ = 30000
			2	180000	10000	
2	Ship	11	1	20000	12000	$X_{2\,Ship\,12}$ = 12000
2	Ship	14	1	20000	7500	$X_{2\,Ship\,14}$ = 7500
3	Ship	2	1	20000	20000	$X_{3\,Ship\,2}$ = 60000
			2	180000	40000	
3	Ship	3	1	20000	20000	$X_{3\,Ship\,3}$ = 50000
			2	180000	30000	
3	Ship	11	1	20000	20000	$X_{2\,Ship\,11}$ = 20000
3	Ship	14	1	20000	12500	$X_{3\,Ship\,14}$ = 12500

Table A.22: Quantity shipped from supplier to manufacturer using 'Ship' transportation

Quantity shipped from the manufacturer to the warehouse

Mode of transportation - Air

Mode (h)	Time period (t)	Cost bracket (l)	Bracket capacity	Units shipped	Variable ($Y_{h\,t}$)
Air	4	1	2000	2000	$Y_{Air\ 4} =$ 4000
		2	8000	2000	
Air	5	1	2000	2000	$Y_{Air\ 5} =$ 10000
		2	8000	8000	
Air	6	1	2000	2000	$Y_{Air\ 6} =$ 10000
		2	8000	8000	
Air	7	1	2000	2000	$Y_{Air\ 7} =$ 10000
		2	8000	8000	
Air	8	1	2000	2000	$Y_{Air\ 8} =$ 10000
		2	8000	8000	
Air	9	1	2000	2000	$Y_{Air\ 9} =$ 10000
		2	8000	8000	

Table A.23: Quantity shipped from manufacturer to warehouse using ‘Air’ transportation

Mode of transportation - Truck

Mode (h)	Time period (t)	Cost bracket (l)	Bracket capacity	Units shipped	Variable ($Y_{h\,t}$)
Truck	5	1	5000	5000	$Y_{Truck\ 5} =$ 14000
		2	20000	9000	
Truck	6	1	5000	5000	$Y_{Truck\ 6} =$ 23000
		2	20000	18000	
Truck	8	1	5000	5000	$Y_{Truck\ 8} =$ 9000
		2	20000	4000	

Table A.24: Quantity shipped from manufacturer to warehouse using ‘Truck’ transportation

Mode of transportation - Rail

Mode (h)	Time period (t)	Cost bracket (l)	Bracket capacity	Units shipped	Variable ($Y_{h\,t}$)
Rail	17	1	5000	750	$Y_{Rail\ 17}$ = 750
Rail	20	1	5000	2500	$Y_{Rail\ 20}$ = 2500

Table A.25: Quantity shipped from manufacturer to warehouse using 'Rail' transportation

Mode of transportation - Ship

Mode (h)	Time period (t)	Cost bracket (l)	Bracket capacity	Units shipped	Variable ($Y_{h\,t}$)
Ship	17	1	20000	3250	$Y_{Ship\ 17}$ = 3250

Table A.26: Quantity shipped from manufacturer to warehouse using 'Ship' transportation

Quantity shipped from the warehouse to each retailer

Mode of transportation - Air

Retailer (r)	Mode (q)	Time period (t)	Cost bracket (s)	Bracket capacity	Units shipped	Variable ($Z_{r\,q\,t}$)
1	Air	6	1	2000	2000	$Z_{1\ Air\ 6}$ = 10000
			2	8000	8000	
1	Air	8	1	2000	2000	$Z_{1\ Air\ 8}$ = 2750
			2	8000	750	
1	Air	23	1	2000	2000	$Z_{1\ Air\ 23}$ = 2500
			2	8000	500	
2	Air	5	1	2000	2000	$Z_{2\ Air\ 5}$ = 4000
			2	8000	4000	
2	Air	7	1	2000	2000	$Z_{2\ Air\ 7}$ = 10000
			2	8000	8000	

Table A.27: Quantity shipped from warehouse to retailer using 'Air' transportation

Mode of transportation - Truck

Retailer (r)	Mode (q)	Time period (t)	Cost bracket (s)	Bracket capacity	Units shipped	Variable ($Z_{r\,q\,t}$)
2	Truck	7	1	5000	3250	$Z_{2\,Truck\,7} =$ 3250

Table A.28: Quantity shipped from warehouse to retailer using 'Truck' transportation

Mode of transportation - Rail

Retailer (r)	Mode (q)	Time period (t)	Cost bracket (s)	Bracket capacity	Units shipped	Variable ($Z_{r\,q\,t}$)
1	Rail	7	1	5000	2000	$Z_{1\,Rail\,7} =$ 2000
1	Rail	8	1	5000	1750	$Z_{1\,Rail\,8} =$ 1750
1	Rail	21	1	5000	2500	$Z_{1\,Rail\,21} =$ 2500
2	Rail	7	1	5000	2000	$Z_{2\,Rail\,7} =$ 2000
2	Rail	21	1	5000	750	$Z_{2\,Rail\,21} =$ 750

Table A.29: Quantity shipped from warehouse to retailer using 'Rail' transportation

Mode of transportation - Ship

Retailer (r)	Mode (q)	Time period (t)	Cost bracket (s)	Bracket capacity	Units shipped	Variable ($Z_{r\,q\,t}$)
1	Ship	7	1	20000	2250	$Z_{1\,Ship\,7} =$ 2250
1	Ship	8	1	20000	5000	$Z_{1\,Ship\,8} =$ 5000
1	Ship	9	1	20000	3000	$Z_{1\,Ship\,9} =$ 3000
1	Ship	10	1	20000	2000	$Z_{1\,Ship\,10} =$ 2000
1	Ship	11	1	20000	1000	$Z_{1\,Ship\,11} =$ 1000
1	Ship	12	1	20000	1000	$Z_{1\,Ship\,12} =$ 1000
1	Ship	13	1	20000	1500	$Z_{1\,Ship\,13} =$ 1500
1	Ship	14	1	20000	1500	$Z_{1\,Ship\,14} =$ 1500
1	Ship	17	1	20000	2750	$Z_{1\,Ship\,17} =$ 2750
1	Ship	18	1	20000	2000	$Z_{1\,Ship\,18} =$ 2000
1	Ship	19	1	20000	4000	$Z_{1\,Ship\,19} =$ 4000
2	Ship	7	1	20000	4500	$Z_{2\,Ship\,7} =$ 4500
2	Ship	8	1	20000	5500	$Z_{2\,Ship\,8} =$ 5500
2	Ship	9	1	20000	3500	$Z_{2\,Ship\,9} =$ 3500
2	Ship	10	1	20000	2500	$Z_{2\,Ship\,10} =$ 2500
2	Ship	11	1	20000	1500	$Z_{2\,Ship\,11} =$ 1500
2	Ship	12	1	20000	1500	$Z_{2\,Ship\,12} =$ 1500
2	Ship	13	1	20000	2000	$Z_{2\,Ship\,13} =$ 2000
2	Ship	14	1	20000	2000	$Z_{2\,Ship\,14} =$ 2000

Retailer (r)	Mode (q)	Time period (t)	Cost bracket (s)	Bracket capacity	Units shipped	Variable ($Z_{r\,q\,t}$)
2	Ship	15	1	20000	1250	$Z_{2\,Ship\,15}$ = 1250
2	Ship	17	1	20000	3000	$Z_{2\,Ship\,17}$ = 3000
2	Ship	18	1	20000	2500	$Z_{2\,Ship\,18}$ = 2500
2	Ship	19	1	20000	7000	$Z_{2\,Ship\,19}$ = 7000
2	Ship	20	1	20000	4750	$Z_{2\,Ship\,20}$ = 4750

Table A.30: Quantity shipped from warehouse to retailer using 'Ship' transportation

Amount of inventory stored at the company owned warehouse

Time period (t)	Quantity stored ($WINV_t$)
8	$WINV_8$ = 10000
9	$WINV_9$ = 10000
10	$WINV_{10}$ = 10000
11	$WINV_{11}$ = 10000
12	$WINV_{12}$ = 10000
13	$WINV_{13}$ = 10000
14	$WINV_{14}$ = 10000
15	$WINV_{15}$ = 10000
16	$WINV_{16}$ = 10000
17	$WINV_{17}$ = 10000
18	$WINV_{18}$ = 10000
19	$WINV_{19}$ = 4000

Table A.31: Quantity stored in the company owned warehouse

Amount of inventory stored at the leased warehouse

Time period (t)	Cost bracket (p)	Bracket capacity	Quantity stored ($LWINV_{p\,t}$)
8	1	10000	$LWINV_{1\,8}$ = 7500
9	1	10000	$LWINV_{1\,9}$ = 10000
	2	50000	$LWINV_{2\,9}$ = 1000
10	1	10000	$LWINV_{1\,10}$ = 10000
	2	50000	$LWINV_{2\,10}$ = 16000
11	1	10000	$LWINV_{1\,11}$ = 10000
	2	50000	$LWINV_{2\,11}$ = 13500
12	1	10000	$LWINV_{1\,12}$ = 10000
	2	50000	$LWINV_{2\,12}$ = 11000
13	1	10000	$LWINV_{1\,13}$ = 10000
	2	50000	$LWINV_{2\,13}$ = 7500
14	1	10000	$LWINV_{1\,14}$ = 10000
	2	50000	$LWINV_{2\,14}$ = 4000
15	1	10000	$LWINV_{1\,15}$ = 10000
	2	50000	$LWINV_{2\,15}$ = 2750
16	1	10000	$LWINV_{1\,16}$ = 10000
	2	50000	$LWINV_{2\,16}$ = 2750
17	1	10000	$LWINV_{1\,17}$ = 7000
18	1	10000	$LWINV_{1\,18}$ = 5000

Table A.32: Quantity stored in the leased warehouse

Cumulative backorders at the manufacturer

Time period (t)	Backorders (MBO_t)
2	MBO_2 = 6500
3	MBO_3 = 17500
4	MBO_4 = 31000
5	MBO_5 = 43000
6	MBO_6 = 52500
7	MBO_7 = 47500
8	MBO_8 = 26000
9	MBO_9 = 19000

Table A.33: Backorders accumulated at the manufacturer

Cumulative backorders at the warehouse

Time period (t)	Backorders (WBO_t)
1	WBO_1 = 6500
2	WBO_2 = 11000
3	WBO_3 = 13500
4	WBO_4 = 16000
5	WBO_5 = 19500
6	WBO_6 = 19000
7	WBO_7 = 11000
8	WBO_8 = 3500
19	WBO_{19} = 750
20	WBO_{20} = 3250
22	WBO_{22} = 2500

Table A.34: Backorders accumulated at the warehouse

Cumulative backorders at each retailer

Retailer (r)	Time period (t)	Backorders $(RBO_{r\,t})$
1	1	$RBO_{1\,1}$ = 3000
1	2	$RBO_{1\,2}$ = 5000
1	3	$RBO_{1\,3}$ = 6000
1	4	$RBO_{1\,4}$ = 7000
1	5	$RBO_{1\,5}$ = 8500
1	6	$RBO_{1\,6}$ = 10000
1	7	$RBO_{1\,7}$ = 750
1	8	$RBO_{1\,8}$ = 1750
1	19	$RBO_{1\,19}$ = 750
1	20	$RBO_{1\,20}$ = 1750
2	1	$RBO_{2\,1}$ = 3500
2	2	$RBO_{2\,2}$ = 6000
2	3	$RBO_{2\,3}$ = 7500
2	4	$RBO_{2\,4}$ = 9000
2	5	$RBO_{2\,5}$ = 11000
2	6	$RBO_{2\,6}$ = 9000
2	7	$RBO_{2\,7}$ = 10250
2	8	$RBO_{2\,8}$ = 1750
2	20	$RBO_{2\,20}$ = 1500
2	22	$RBO_{2\,22}$ = 2500

Table A.35: Backorders accumulated at the retailer

Total cost of the supply chain and the total number of backorders

Total cost (in $)	Total number of backorders (units)
277892	456225

Table A.36: Objective values for Case-2