INVESTIGATION OF LASER DOPPLER ANEMOMETRY IN DEVELOPING
A VELOCITY-BASED MEASUREMENT TECHNIQUE

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ABSTRACT

Acoustic properties, such as the characteristic impedance and the complex propagation constant, of porous materials have been traditionally characterized based on pressure-based measurement techniques using microphones. Although the microphone techniques have evolved since their introduction, the most general form of the microphone technique employs two microphones in characterizing the acoustic field for one continuous medium. The shortcomings of determining the acoustic field based on only two microphones can be overcome by using numerous microphones. However, the use of a number of microphones requires a careful and intricate calibration procedure.

This dissertation uses laser Doppler anemometry (LDA) to establish a new measurement technique which can resolve issues that microphone techniques have: First, it is based on a single sensor, thus the calibration is unnecessary when only overall ratio of the acoustic field is required for the characterization of a system. This includes the measurements of the characteristic impedance and the complex propagation constant of a system. Second, it can handle multiple positional measurements without calibrating the signal at each position. Third, it can measure three dimensional components of velocity even in a system with a complex geometry. Fourth, it has a flexible adaptability which is not restricted to a certain type of apparatus only if the apparatus is transparent. LDA is known to possess several disadvantages, such as the requirement of a transparent apparatus, high cost, and necessity of seeding particles.

The technique based on LDA combined with a curvefitting algorithm is validated through measurements on three systems. First, the complex propagation constant of the
air is measured in a rigidly terminated cylindrical pipe which has very low dissipation. Second, the radiation impedance of an open-ended pipe is measured. These two parameters can be characterized by the ratio of acoustic field measured at multiple locations. Third, the power dissipated in a variable RLC load is measured. The three experiments validate the LDA technique proposed.

The utility of the LDA method is then extended to the measurement of the complex propagation constant of the air inside a 100 ppi reticulated vitreous carbon (RVC) sample. Compared to measurements in the available studies, the measurement with the 100 ppi RVC sample supports the LDA technique in that it can achieve a low uncertainty in the determined quantity. This dissertation concludes with using the LDA technique for modal decomposition of the plane wave mode and the (1,1) mode that are driven simultaneously. This modal decomposition suggests that the LDA technique surpasses microphone-based techniques, because they are unable to determine the acoustic field based on an acoustic model with unconfined propagation constants for each modal component.
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LIST OF SYMBOLS

English Letters

- \(a\): Inner radius of the pipe (m)
- \(A\): Magnitude of complex pressure amplitude coefficient for the incident plane wave (Pa)
- \(\hat{A}, \hat{A}_0\): Complex pressure amplitude coefficient for the incident plane wave (Pa)
- \(\hat{A}_{11}\): Complex pressure amplitude coefficient for the incident wave of the (1,1) mode (Pa)
- \(b\): Coefficient of a series
- \(\hat{B}, \hat{B}_0\): Complex pressure amplitude coefficient for the reflected plane wave (Pa)
- \(\hat{B}_{11}\): Complex pressure amplitude coefficient for the reflected wave of the (1,1) mode (Pa)
- \(c_0\): Speed of sound in free space (m/s)
- \(c_p\): Specific heat per unit mass at constant pressure (J/kg·K)
- \(C\): Compliance (m³/Pa)
- \(\hat{C}\): Complex pressure amplitude coefficient for the incident plane wave in a porous sample (Pa)
- \(d\): Thickness of a porous sample (m) or half of the distance between the projected two laser beams (m)
\( \hat{D} \) Complex pressure amplitude coefficient for the reflected plane wave in a porous sample (Pa)

\( \hat{e} \) The dissipation per unit of surface area of the resonator (or a pipe) (W/m²)

\( \hat{E} \) Complex pressure amplitude coefficient for the incident plane wave between a porous sample and the termination of the impedance tube (Pa)

\( f \) Acoustic frequency (Hz)

\( f_m \) Orthogonal DINI series coefficients in Eq. (2.51)

\( f_{ml} \) Cut-on frequency of the \((m,l)\) mode in a pipe (Hz)

\( f_c \) Thermoviscous \( f \)-function due to thermal effect in a pipe

\( f_{c,s} \) Thermoviscous \( f \)-function due to thermal effect in a porous sample

\( f_v \) Thermoviscous \( f \)-function due to viscosity in a pipe

\( f_{v,s} \) Thermoviscous \( f \)-function due to viscosity in a porous sample

\( \hat{F} \) Complex pressure amplitude coefficient for the reflected plane wave between a porous sample and the termination of the impedance tube (Pa)

\( F_B \) Frequency shifted by Bragg cell (Hz)

\( g_m \) Orthogonal DINI series coefficients in Eq. (2.50)

\( h_c \) Thermoviscous \( h \)-function due to thermal effect in a pipe

\( h_v \) Thermoviscous \( h \)-function due to viscosity in a pipe

\( H_n^{(2)} \) The \( n \)th order Hankel function of the second kind

\( H_1 \) The first order Struve function

\( I[n] \) Number of samples at \( n \)th harmonic
Axial component of acoustic intensity (Pa\cdot m/s)

\[ j = \sqrt{-1} \]

\[ j_{ml}' \] The \( l \)th zero of the \( m \)th order Bessel function of the first kind

\[ J_n \] The \( n \)th order Bessel function of the first kind

\[ \hat{k} \] Complex propagation constant of the air inside a pipe (1/m)

\[ \hat{k}_s \] Complex propagation constant of the air inside a porous sample (1/m)

\[ k \] Real part of complex propagation constant of the air inside a pipe (1/m)

\[ k_0 \] Wave number \( \omega / c_0 \) (1/m)

\[ k_{ml} \] Wave number in the radial direction for the \((m,l)\) mode (\( = j_{ml}' / r_0 \)) (1/m)

\[ k_z \] Real part of the complex propagation constant of the air in a sample (1/m)

\[ k_z \] Propagation constant in the axial direction for the \((m,l)\) mode (1/m)

\[ L \] Length of apparatus (m)

\[ l \] Distance between the intersecting point of laser beams and the wall (m) or radial mode number for the \((m,l)\) mode

\[ m \] Angular mode number for the \((m,l)\) mode

\[ O \] Intersection position of laser beams

\[ \hat{p} \] Complex-valued first order of the acoustic pressure (Pa)

\[ p_1 \] Real-valued first order of the acoustic pressure (Pa)

\[ \hat{p}_0 \] Complex acoustic pressure measured at \( z = 0 \) (Pa)

\[ \hat{p}_c \] Complex acoustic pressure measured in the volume (Pa)
$\hat{p}_I$  Complex acoustic pressure for an incident plane wave (Pa)

$\hat{p}_R$  Complex acoustic pressure for a reflected plane wave (Pa)

$P$  Total pressure (Pa)

$P_o$  Equilibrium pressure (Pa)

$Pr$  Prandtl number ($= \mu c_p / \kappa$)

$r$  Radial position (m)

$r_o$  Inner radius of a pipe ($= a$) (m)

$r_i$  Outer radius of a pipe (m)

$R$  Gas constant (J/kg·K)

$\hat{R}$  Complex reflection coefficient

$s$  Total entropy per unit mass (J/kg·K) or Sample standard deviation

$s$  Complex-valued first order of the perturbed entropy per unit mass (J/kg·K)

$s_o$  Equilibrium entropy per unit mass (J/kg·K)

$s_i$  Real-valued first order of the perturbed entropy per unit mass (J/kg·K)

$S$  Cross sectional area of a cylindrical pipe $\pi r_o^2$ (m$^2$)

$t$  Time (s)

$tr$  Transit time (s)

$T$  Total temperature (K) or Period of sinusoidal signal (s)

$\hat{T}$  Complex-valued first order of the perturbed temperature (K)

$T_o$  Equilibrium temperature (K)

$T_i$  Real-valued first order of the perturbed temperature (K)
\( u \)  Three dimensional first order acoustic particle velocity of fluid (m/s)
\( \hat{u} \)  The first order axial component of complex acoustic particle velocity of fluid on the center axis (m/s)
\( u_i \)  Magnitude of axial velocity measured at the \( i \)th position (m/s)
\( \hat{u}_i \)  Scaled axial component of velocity (m/s)
\( u_z \)  The first order axial component of velocity (m/s)
\( u_r \)  The first order radial component of velocity (m/s)
\( u_\theta \)  The first order angular component of velocity (m/s)
\( \dot{\mathbf{U}} \)  The first order complex volume velocity of fluid (m\(^3\)/s)
\( V \)  Volume of the RLC load (m\(^3\))
\( \dot{W} \)  Acoustic power (W)
\( y[n] \)  Discrete time signal in the \( n \)th bin
\( y(t) \)  Continuous time signal at \( t \)
\( y_s(t) \)  Synthesized time signal at \( t \)
\( Y[k] \)  Fourier coefficient of the \( k \)th harmonic
\( z \)  Axial position (m)
\( \hat{z} \)  Complex specific acoustic impedance of the termination for an incident plane wave (Pa\cdot s/m)
\( \hat{Z}_{\text{rad}} \)  Complex radiation impedance (N\cdot s/m)
\( \hat{Z}_{\text{air}} \)  Complex acoustic impedance of the air for an incident plane wave (Pa\cdot s/m\(^3\))
\( \hat{Z}_s \) Complex acoustic impedance of the air inside a porous sample for an incident plane wave (Pa·s/m³)

**Greek Letters**

\( \alpha \) Absorption coefficient for the plane wave mode (Np/m)

\( \alpha_{m,l} \) Absorption coefficient for the \((m,l)\) mode (Np/m)

\( \alpha_s \) Absorption coefficient of the air in a sample (Np/m)

\( \gamma \) Ratio of isobaric to isochoric specific heats

\( \delta_f \) Fringe spacing (m)

\( \delta_{f,\theta} \) Effective fringe spacing for the oblique velocity (m)

\( \delta_\kappa \) Thermal penetration depth \( (= \sqrt{2\kappa/(\omega\rho_0 c_p)} \) (m)

\( \delta_\nu \) Viscous penetration depth \( (= \sqrt{2\mu/(\omega\rho_0)} \) (m)

\( \eta \) Arbitrary real constant in Eq. (2.35)

\( \theta \) Angular position (rad) or angle between two laser beams (rad)

\( \theta \) Arbitrary real constant in Eq. (2.35) (rad)

\( \theta_i \) Phase of axial velocity at the \( i \)th position (rad)

\( \vartheta \) Angle between the normal direction to the fringes and the oblique velocity (rad)

\( \kappa \) Thermal conductivity (W/m·K)

\( \lambda \) Acoustic wavelength (m) or optical wavelength of the laser beams (m)

\( \mu \) Dynamic viscosity (kg/m·s)
\( \mu_B \)  Bulk viscosity (kg/m\( \cdot \)s)

\( \rho \)  Total density of fluid (kg/m\(^3\))

\( \dot{\rho} \)  Complex first order perturbed density of fluid (kg/m\(^3\))

\( \rho_0 \)  Equilibrium density of fluid (kg/m\(^3\))

\( \Phi \)  Viscous dissipation function (1/s\(^2\))

\( \varphi_i \)  Angle of incidence for the plane wave (rad)

\( \phi[k] \)  Phase of Fourier coefficient for the \( k \)th harmonic (rad)

\( \sigma \)  Standard deviation

\( \sigma_0 \)  Static flow resistivity (Pa\( \cdot \)s/m\(^2\))

\( \omega \)  Angular frequency (\( = 2\pi f \)) (rad/s)

**Subscripts**

\( I \)  Region I

\( II \)  Region II

\( III \)  Region III

\( \text{FIXED} \) Measurement at a fixed location

\( i \)  The \( i \)th measurement or the \( i \)th position

\( \text{LOAD} \) Measurement in the RLC load

\( m \)  Measured value

**Special Symbols**
\( \Re \{ \} \) Real part of

\( \Im \{ \} \) Imaginary part of

* Complex conjugate

\| \| Magnitude of

\( \angle \) Phase of (rad)

\( \wedge \) Complex

→ Vector

\( \partial \) Partial derivative

\( \nabla \) Gradient operator

\( \nabla^2 \) Laplacian
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Chapter 1

Introduction

1.1 Motivation and goals

Although many phenomena are well understood theoretically, there are also still many complex systems that no analytic study has fully explained. Instead of an analytic approach, these phenomena can be explored and identified through experiment. This situation has inspired the author to develop a new measurement technique that can provide both accuracy and precision in determining the characteristics of an unknown phenomenon experimentally. A good example of such technique that has been used in acoustics is the two-microphone technique, which has been involved in measuring power flow and characterizing the acoustical properties (e.g., the characteristic impedance and the complex propagation constant) of a porous material in an impedance tube. However, one of the main concerns with this technique is to predict the pressure and velocity based on the information attained with the limited number of microphones. In addition, the flexibility in choosing the microphone positions may be eliminated after the apparatus, such as an impedance tube, is constructed.

In order to overcome these disadvantages, the author proposed a new technique based on velocity measurements using Laser Doppler Anemometry (LDA). Compared to the microphone techniques, this new technique has several superior features. First, it can measure velocity at any position in the apparatus. Second, the number of measurement
positions is easily increased without modifying the structure of the apparatus. Third, it can capture the three dimensional information of velocity. All these features allow for adjusting the measurement plan when it is considered inadequate for the experiment. There are, of course, some disadvantages of the LDA technique. These include the cost of the LDA, the need for an optically transparent test section, the need for seeding particles, and the need for a periodic driving signal. The purpose of this dissertation is 1) to explore the capabilities of and validate the proposed LDA technique, 2) use it to measure the acoustical characteristics of a porous material, and 3) investigate the technique’s utility for modal decomposition. Fulfillment of these will result in development of a velocity-based measurement technique using LDA. The developed technique is aimed not only to determine the power flow in a complex geometry, but to supply simple applications with better accuracy and precision than the conventional microphone techniques do. Subsequently, proceeding with the applications selected, the dissertation will serve a complete and practical guidance in using LDA/BSA system.

1.2 Background

There are many applications involving the use of an impedance tube, but most of them fall into the category in which the pressure and velocity are necessarily predicted at a certain position. For instance, to determine the acoustical properties of a porous sample, two or more microphones are used to estimate the pressure and velocity on the two surfaces of the sample. Measuring the radiation impedance of the tube necessitates determining the reflection coefficient, which can be expressed by the pressure and
velocity at the termination. Likewise, the acoustic power at a position is acquired from the integration of acoustic intensity, which is proportional to the product of the pressure and the velocity at the position. Regardless, all the three applications are based on the linear Euler equation to estimate the acoustic velocity from the pressure gradient.

In such applications exemplified above, the typical two-microphone technique has played a great role. (Note that the four-microphone technique by Song and Bolton [1] is a variation of this two-microphone technique, which is implemented in either side of a porous sample.) However, it may not be ideal to depend on the velocity and pressure which are extracted from the measurements performed at two positions, because noise can be introduced to the measured values. It is also reported that the measurement is not credible if the spacing between two microphones is an integer multiple of half of a wavelength. [2]

These disadvantages of the two-microphone method can be overcome by using multiple microphones. As shown in Fig. 1.1, Jones et al. [3] have used ninety-five microphones to characterize the impedance of porous materials in their impedance tube. Although it was stated that this setup was built as an improvement over their previous setup, the setup shown in Fig. 1.1 can be substantially affected by the calibration of ninety-five microphones even when only overall ratio of measured values is of interest. The measurements of the characteristic impedance and the reflection coefficient of a porous material are considered as this type of experiment. In this situation, the use of a single sensor can avoid the effect of calibration. However, as reported by Jones et al. [3], the use of a single microphone would cost a significant amount of time in removing and replacing the microphone in sensor fixtures.
Thus, the previously suggested technique based on LDA is designed to address these shortcomings of the two-microphone method as well as the example shown in Fig. 1.1. The experiments that determine the acoustic field of an apparatus usually with the microphone techniques will be executed with the measurement of velocity at multiple positions using LDA.

1.3 Outline

This dissertation is organized as follows. In Chapter 2, the theoretical bases for the acoustical systems investigated experimentally are provided. Following this chapter, the setup and the apparatus of each experiment that is conducted in this dissertation are illustrated in Chapter 3. The basic principles of LDA are also introduced in Chapter 3. These two chapters are designed to reduce unnecessary redundancy that may occur in later chapters. Chapter 4 discusses the measurement of the complex propagation constant
of the air inside a rigidly terminated cylindrical pipe with the LDA technique. This chapter concludes with the introduction of a new method of estimating the pressure from the measured velocity. Next, the LDA technique is applied not only to determining radiation impedance of an open ended cylindrical pipe with non-zero wall thickness, but also to measuring power dissipated in a variable RLC load in Chapter 5. The measured value is compared with the theoretical prediction based on the Wiener-Hopf technique in the first part, whereas it is compared with both the direct estimation based on a lumped parameter model and the two-microphone method in the second part. Chapters 4 and 5 serve the purpose of validating the LDA technique. Following this validation, Chapter 6 discusses the measurement of the complex propagation constant of the air inside of a reticulated vitreous carbon (RVC) sample, which does not have a concrete theoretical basis, using the LDA technique. The LDA technique is used to modally decompose an acoustic field composed of the plane wave and the (1,1) cross mode in Chapter 7. Finally, the dissertation is concluded by Chapter 8 that encompasses the overall discussions and possible future works.

1.4 Previous Work

1.4.1 General microphone-related research

Many studies have been published in determining acoustical properties of a material sample, especially a porous sample, using the microphone-based measurement techniques.
Commonly, these techniques use Euler’s equation to estimate the axial velocity from the measured pressure with microphones.

The first notable work on this subject was presented by Utsuno et al. [4]. Their method was based on two microphones with a sample supported by two different types of terminations with the excitation of a random signal in the apparatus. Though this method could extract the acoustic impedance and the absorption coefficient of samples, it was reported to be problematic when the sample has a high flow resistivity.

Champoux and Stinson [5] improved the idea of two cavities employed by Utsuno et al. [4] with three microphones in an impedance tube. Despite the actual number of microphones, this technique practically utilized one microphone to measure the pressure at multiple locations by moving it in the axial direction. This guaranteed that the number of measurement positions was freely selectable. Compared to the method of Utsuno et al.’s, their technique was successfully able to extend the frequency range for samples having high flow resistivity with a better precision.

Slightly preceding Champoux and Stinson’s method, a five-microphone method was developed by McIntosh et al. [6]. In its description, the method could measure the complex propagation constant, the characteristic impedance, and the flow resistance of a sample situated between the first and the second microphone. However, high flow resistance materials were not tested with their method. Hence, the degree of accuracy that this method could achieve was not provided, although the configuration of the technique had similarity with the method suggested by Champoux and Stinson. [5]

Cheung et al. [7] presented a microphone technique similar to that developed by Champoux and Stinson. They also used a microphone that was movable axially. However,
this method was based on a linear least mean squared fit which could be easily implemented, while Champoux and Stinson’s was rather based on an iterative nonlinear curvefitting. Unfortunately, they did not demonstrate the experimental result of their new technique against other microphone-based techniques in their paper.

Interesting research was performed by Jang and Ih [2] regarding the impact of the microphone spacing and the number of microphones on the results of microphone-based techniques. Their work suggested two things. First, the microphone spacing should be equal rather than random between every two adjacent microphones. Second, the number of microphones should be increased, in order to guarantee better precision in estimating the reflection coefficient. They confirmed these with the result of their experiment in measuring the reflection coefficient of an open ended pipe without flange.

Later, Song and Bolton [1] developed a four-microphone technique. In their publication, they proposed a transfer matrix model for the sample situated between the second and the third microphone. Based on this model, they demonstrated that the sample’s property could be determined with the pressure measured at four positions. Even though this technique is a variant of the two-microphone technique which is implemented in either side of a porous sample, this technique did not depend on a specific type of termination that was necessary for any previous work published. Either side of a sample being considered, this technique did not provide any improvement over the two-microphone method.
1.4.2 General LDA-related researches

Laser Doppler anemometry (LDA) was first introduced to acoustics by Taylor. [8] Its most common applications have been in the area of thermoacoustics and acoustic streaming. However, it has not been used to any extent for the types of applications discussed in this dissertation.

Using both microphone and LDA method, Bailliet et al. [9] measured the acoustic power flow at a stationary position in a thermoacoustic resonator. In order to achieve precision in the measured acoustic power within the frequency range chosen in their research (100 Hz < \( f \) < 600 Hz), it was required to calibrate the phase between microphone and LDA. In the similar experiment conducted by Yazaki and Tominaga [10], the measured power was reported as a function of axial position at a single frequency (\( f \approx 149 \) Hz). As opposed to the work by Bailliet et al., no indication of phase calibration was shown, presumably because the frequency selected was in the low side of Bailliet et al.’s frequency range. At low frequencies, the agreement between the predicted and the measured values was fairly good even for Bailliet et al.’s work as well.

Another type of experiment that has been performed is the measurement of the acoustic streaming velocity. Using LDA and a Burst Spectrum Analyzer (BSA), Thompson and Atchley [11] reported that the axial velocity and the acoustic streaming velocity can be simultaneously measured. In their work, the detection of the acoustic streaming velocity was based on the correction of bias that was present in the acoustic streaming velocity. Moreau et al. [12] also successfully measured the acoustic streaming velocity with a LDA/BSA system. In consequence, their work confirmed what Thompson
and Atchley were able to observe. However, it was suggested that the use of bias correction is not needed if a large number of samples compared to Thompson and Atchley’s work is used.

### 1.4.3 Complex propagation constant of the air inside the pipe

Kirchhoff [13] provided the first theory in the propagation of a plane wave inside a pipe by including the effect of the viscosity as well as that of the thermal conductivity simultaneously. Although the solution of his theory, called *Kirchhoff’s formulae*, was given for ‘wide’ tube with a cylindrical cross section, the general solution should be obtained through a complicated numerical computation of the solution. Contrary to Kirchhoff’s approach, Zwikker and Kosten [14] proposed an approximate theory by treating the thermal conductivity and the viscosity separately, noting that the two mechanisms of loss did not appear simultaneously in the equations derived by Kirchhoff.

The solutions of Kirchhoff’s theory were derived by Weston [15] for various regimes of tube diameter based on a proper approximation for each diameter regime, which was defined by Kirchhoff in terms of the frequency and tube diameter. In addition, Weston extensively discussed the validity of Kirchhoff’s formulae. Later, Stinson [16] suggested that Kirchhoff’s theory can be approximated to that of Zwikker and Kosten’s when the frequency $f < 10^6$ Hz and the radius $r_o > 10^{-5}$ m.

Another theory of wave propagation in a pipe was presented by Rott. [17] In his analysis, the effect of the temperature gradient was included. However, its effect could be neglected in a pipe with large $r_o / \delta_k$, where $\delta_k$ is the thermal penetration depth. Hence,
the propagation constant of this situation was the same as that from the assumption that the temperature inside the pipe was constant. His expression for the propagation constant involved the thermoviscous $f$-function, which depends upon the shape of the cross section. Since the thermoviscous $f$-functions were available for a rich variety of the cross sectional shapes as shown in Swift’s text [18], it has been possible to obtain the propagation constant for the corresponding shapes.

Measuring pressure at multiple locations, Champoux and Stinson [5] predicted the pressure as a function of axial position for the entire axial range of their apparatus. However, the fit of data was based on the propagation constant theoretically predicted from Kirchhoff’s theory, instead of obtaining the propagation constant from the fit of the data. Wilson et al. [19] performed a preliminary test for the propagation of plane waves in a water-filled tube with measurements of pressure at multiple positions. However, the result was not given for their test in detail.

Tashiro et al. [20] experimentally measured the thermal $f$-function by calibrating thermocouples in a cylindrical tube based on Rott’s theory for a narrow pipe with the ratio that $r_0 / \delta_k = 5.4 \ (f = 15 \text{ Hz}, r_0 = 1.05 \text{ cm})$. Although their research showed that the measured thermal $f$-function agreed with the theoretical prediction to within 5%, it was not discussed whether the viscous $f$-function agreed with the theoretical prediction which also affects the propagation constant according to Rott’s theory.

The propagation constant in narrow tubes was measured for the ratio $r_0 / \delta_k < 15$ by Yazaki et al. [21] Their experiment was based on the measurement of the pressure in very long, narrow tubes ($L > 40 \text{ m}, r_0 \leq 2 \text{ mm}$) where the absorption coefficient of the
air allowed only a traveling wave to propagate in the tubes. Although the implementation of the apparatus was outstanding, this technique was not applicable to the frequencies with the ratio $r_0/\delta_k > 100$.

### 1.4.4 Radiation impedance of an open ended pipe

The radiation impedance of an open ended pipe in an infinite flange is commonly discussed in the literature in the form of the end correction of the pipe. The first study of that without flange was reported by Lord Rayleigh. [13] Due to the absence of a theoretical basis to solve the problem at that moment, Lord Rayleigh determined that the end correction of the pipe should be $0.6r_0$ based on both his own experimental observation and the work by Bosanquet [13], when the wavelength was greater than the diameter of the pipe. However, the frequency dependence of this end correction was almost unknown.

The complete theory on the end correction of an unflanged open-ended pipe with zero-wall thickness was founded by Levine and Schwinger [22] using Green’s function. In their publication, they showed that the end correction was a function of $k_0a$, where $k_0$ is the acoustic wave number, proportional to frequency, and $a$ is the radius of the pipe.

The study with more complexity was published by Jones. [23] In his work, the scattering from the semi-infinite long rigid cylindrical rod was calculated based on the Wiener-Hopf technique, which allows for decomposing a function using two analytical complex functions defined in each domain. Although its discussion was not about a cylindrical pipe, both the approach using Laplace transform and the calculation of the split function give a profound basis for later works. Those works include the sound
Based on Jones’ theoretical foundation, Ando presented the analytic solution for the radiation impedance of an open ended pipe with non-zero wall thickness. His solution suggested a substantial difference from the solution of Levine and Schwinger’s when the pipe wall gets thicker.

### 1.4.5 Power dissipation in an arbitrary termination

As discussed by Fahy [26], there have been many works that attempted to estimate the power flow at a certain position in a pipe. In those works, microphones have been widely accepted as sensors for this purpose in general.

The first notable work on this subject was reported by Seybert. [27] His method was based on the cross-spectrum and auto-spectrum of the signals from two microphones, which are situated at two different locations, in a waveguide driven by a random signal added to the mean flow. Since the measurement was based on the random signal, this method could determine the power very fast. However, no effect of the thermoviscous boundary layers was taken into account by his work.

Another method of measuring power flow was proposed by Fusco et al. [28] based on not only Euler’s equation to estimate the axial velocity in the midpoint of two microphones, but the arithmetic mean to estimate the pressure at that position. Contrary to the method proposed by Seybert, their method measured the power in the center location of two microphones, as well as included the effect of the thermoacoustic
boundary layers present in the waveguide. The measurements were performed at various levels of power dissipation using a variable RLC load based on a lumped parameter model. The result of their experiment showed a good agreement with the prediction from the thermoviscous boundary layer theory by Swift. [29]

While Fusco et al.’s method could precisely determine the power in a waveguide with high $r_0 / \delta_c$, it was suggested by Biwa et al. [30] that the method was unable to produce the same precision in a waveguide with the ratio $r_0 / \delta_c \sim 1$. In their work, Biwa et al. substituted the full thermoviscous $f$-functions into both the propagation constant and the equation for calculating the power flow, instead of the boundary layer approximation used by Fusco et al. Although the improvement in precision was evident over the method of Fusco et al.’s, no significant difference was present between two methods in the form of the physical implementation.

1.4.6 Acoustical properties of reticulated vitreous carbon (RVC)

In thermoacoustics, porous materials play an important role in constructing a stack or a regenerator depending on the size of pores. From a standpoint of thermoacoustics, the performance of a porous material can be defined by the thermoviscous $f$-functions which are mostly determined by the shape of pores. [18] Many theories have been established to acquire an analytical solution corresponding to each shape of pores. In practice, the shapes are, however, based on simple geometry such as cylindrical pores, parallel plates, and so on.
More recently, a porous material called reticulated vitreous carbon (RVC) began to draw an attention from thermoacoustics. [31] It is reported that RVC possesses several advantages, such as easy machinability, relatively low price, and high melting temperature, so that it can serve as a stack. However, due to its irregular pore geometry, its acoustical properties are less well known both experimentally and theoretically.

The first investigation into the acoustical properties of RVC was the measurement of the thermoviscous $f$-functions performed by Petculescu and Wilen [32], based on a lumped parameter model. In their results, a scaling factor, which was not observed for a simple geometry, was present between the two thermoviscous $f$-functions $f_c$ and $f_k$.

Muehleisen et al. [33] provided an empirical model for RVC’s characteristic impedance as well as the complex propagation constant of the air inside RVC using the four-microphone method suggested by Song and Bolton. [1] In order to avoid the necessary calibration for four individual microphones, an interchanging method was used in their work. Although two studies presented different quantities, the thermoviscous $f$-functions measured by Petculescu and Wilen can be easily converted into the values measured by Muehleisen et al., if Rott’s theory is valid for the interior of RVC.

1.4.7 Axial propagation constant for non-planar wave mode

It is theoretically understood that the propagation constant for plane waves in a pipe is complex to account for the loss mechanism due to the thermoviscous boundary layers, and its validity will be shown experimentally later. However, for the non-planar wave
modes, not only is a solid explanation of the propagation constant absent, but also no
direct empirical verification has been made.

In the early 1950’s, Beatty [34] published the first theoretical work that discussed
the complex axial propagation constant for non-planar wave modes. Based on the wall
admittance expressions given by Morse [35], the expression for the complex axial prop-
agation constant was derived for both rectangular and cylindrical waveguides. Based on
his theory, the absorption coefficient of any non-planar wave mode approaches infinity at
the cut-on frequency of that mode. However, no experiment was performed to support his
theory.

Approximately thirty-five years after Beatty’s work, Hudde [36] proposed another
theory. His work suggested that the expressions derived by Beatty were incorrect,
because only the first order of Taylor’s series expansion was utilized near the cut-on
frequency. He also suggested that the zeros of the Bessel’s functions should be complex,
because of the complex wall admittance given by Morse. [35] The correction of those
resulted in distinctive expressions from Beatty’s. For a main distinction, he claimed that
no discontinuity was present in the axial propagation constant at each cut-on frequency.
The experimental test performed through measuring the reflection coefficient success-
fully showed the result that supported his claim. However, the result indicated that the
absorption coefficient in his theory was underestimated, as well.

Based on Beatty’s theory, Hamilton and TenCate [37] reported the importance of
the absorption coefficient for non-planar wave modes in their experiment of nonlinear
effects in a rectangular waveguide. Though they suspected that the absorption coefficient
given by Beatty was underestimated, no further investigation was provided.
According to the experiment done by Hudde and by Hamilton and TenCate, the absorption coefficient caused by non-planar wave mode certainly existed, regardless of its exact value. Hence, its effect would exist on calculating acoustic power flow, as well as on measuring acoustical properties of a porous sample in an apparatus, when non-planar wave mode is present in the apparatus. However, it is surprising that the effect of the complex axial propagation constant is still ignored by some researchers. [38,39]

For example, Schultz et al. [38] used a method of modal decomposition to extract each modal components corresponding to its mode based on the real-valued axial propagation constant. As indicated in their study, their model was restricted to a lossless system. This indicates that the basic assumption used in developing their method was invalid because it did not reflect the true situation of an impedance tube.
Chapter 2
Theory

2.1 Introduction

This chapter provides the theoretical base to support the experiments and the analyses discussed later. For efficiency, the chapter is divided into two parts: The first part is mainly focused on the derivation of general equations, such as the equations for both components of two dimensional acoustic velocity based on pressure, the equation of continuity, and Helmholtz equation. Its topic also includes a discussion of the curvefitting algorithms used to extract the needed parameters from the measurements of acoustic velocity at multiple positions. The second part of this chapter is dedicated to developing specific equations based on the physical models of the particular experiments that were conducted. This chapter aims to consolidate all required theories in order to avoid unnecessary redundancy in later chapters.

2.2 Acoustic theory

In this section, the mathematical expressions and acoustic principles are developed for the several applications investigated in this research. The list of applications covered comprises 1) the determination of complex propagation constant of the air inside a cylindrical pipe, 2) the measurement of radiation impedance in an open ended pipe, 3) the determination of power flow in a pipe terminated with an RLC load, 4) the measurement
of the complex propagation constant of the air inside a sample of reticulated vitreous carbon (RVC), and 5) the determination of complex axial propagation constant of the air inside a pipe for the first non-planar wave mode. The derived theoretical foundation will be joined by the curvefitting algorithm required in analyzing the velocity data taken to serve a purpose of developing a velocity-based measurement technique using LDA.

2.2.1 Basic equations

The expressions for the axial velocity and the pressure under the plane wave mode assumption in a cylindrical pipe are derived in this section. They will be used in predicting pressure and axial velocity based on the method of analysis discussed in this dissertation, as well as in the actual calculation of the desired quantity for each application. The derivation presented here closely follows those of Waxler [40] and Hamilton et al. [41] As such, the details contained here are abbreviated. As in their works, \( e^{j\omega t} \) time dependence, where \( \omega \) is the angular frequency, is assumed in the following derivation. It is also assumed that the equilibrium temperature \( T_0 \), density of gas \( \rho_0 \), dynamic viscosity \( \mu \), bulk viscosity \( \mu_b \), and thermal conductivity \( \kappa \) are independent of position. Additionally, a steady state is assumed.

Based on these assumptions, three fundamental equations, the equation of continuity, the Navier-Stokes equation, and the equation of heat transfer, are written as

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0, \tag{2.1}
\]
\[ \rho \left( \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla P + \mu \nabla^2 \mathbf{u} + \left( \frac{1}{3} \mu + \mu_B \right) \nabla (\nabla \cdot \mathbf{u}), \quad (2.2) \]

\[ \rho T \left( \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \right) s = \kappa \nabla^2 T + \mu \Phi + \mu_B (\nabla \cdot \mathbf{u})^2, \quad (2.3) \]

where \( \Phi \) is the viscous dissipation function defined in a cylindrical coordinate system as [42],

\[ \Phi = 2 \left[ \left( \frac{\partial u_r}{\partial r} \right)^2 + \left( \frac{1}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r} \right)^2 + \left( \frac{\partial u_z}{\partial z} \right)^2 \right] + \left[ \frac{r}{r} \frac{\partial (u_\theta)}{\partial r} + \frac{1}{r} \frac{\partial u_r}{\partial \theta} \right]^2 \]

\[ + \left[ \frac{1}{r} \frac{\partial u_z}{\partial \theta} + \frac{\partial u_\phi}{\partial z} \right]^2 + \left[ \frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r} \right]^2 - \frac{2}{3} (\nabla \cdot \mathbf{u})^2, \quad (2.4) \]

and where velocity \( \mathbf{u} \) can be expressed by means of vector decomposition as

\[ \mathbf{u} = u_r \mathbf{e}_r + u_\theta \mathbf{e}_\theta + u_z \mathbf{e}_z. \quad (2.5) \]

In Eq. (2.5), use was made of the fact that \( \mathbf{e}_r, \mathbf{e}_\theta, \) and \( \mathbf{e}_z \) are unit vectors in each coordinate.

Because nonlinear effects are not a concern of the study, Eq. (2.1) can be linearly approximated to contain only most significant terms for acoustic signals caused by small perturbations. Based on the definition that

\[ P = P_0 + \Re \{ \hat{p} e^{j\omega t} \}, \quad p_i = \Re \{ \hat{p} e^{j\omega t} \}, \quad (2.6) \]

\[ u_r = \Re \{ \hat{u}_r e^{j\omega t} \}, \quad u_z = \Re \{ \hat{u}_z e^{j\omega t} \}, \quad (2.7) \]

\[ T = T_0 + \Re \{ \hat{T} e^{j\omega t} \}, \quad T_1 = \Re \{ \hat{T} e^{j\omega t} \}, \quad (2.8) \]

\[ s = s_0 + \Re \{ \hat{s} e^{j\omega t} \}, \quad s_1 = \Re \{ \hat{s} e^{j\omega t} \}, \quad (2.9) \]

and

\[ \rho = \rho_0 + \Re \{ \hat{\rho} e^{j\omega t} \}, \quad (2.10) \]
where $s$ and $s_0$ are the entropy and the equilibrium entropy per unit mass, and where the terms in curly brackets are the most significant terms originating from the perturbation for each quantity, it follows that
\[
\frac{\partial \hat{p}}{\partial t} + \rho_0 \left( \frac{\partial \hat{u}_z}{\partial z} + \frac{1}{r} \frac{\partial (r \hat{u}_r)}{\partial r} \right) = 0. \tag{2.11}
\]
Provided that the air inside a pipe acts as an ideal gas, it satisfies the equation of state which is written as
\[
P = \rho RT. \tag{2.12}
\]
Taking only the first order terms among the perturbed acoustic signals from Eq. (2.12) leads to
\[
\hat{p} = R \left( \rho_0 \hat{T} + \hat{p}_0 \right). \tag{2.13}
\]
Since the pressure at the equilibrium state can be expressed by the density of air and the temperature at the equilibrium as
\[
P_0 = \rho_0 RT_0, \tag{2.14}
\]
Eq. (2.13) can be rewritten as the ratio of the perturbed signal to the value at the equilibrium as
\[
\frac{\hat{p}}{\rho_0} = \frac{\hat{T}}{T_0} + \frac{\hat{p}_0}{\rho_0}. \tag{2.15}
\]
Substitution of Eq. (2.15) into Eq. (2.11) yields
\[
\frac{j \omega}{P_0} \hat{p} - \frac{j \omega}{T_0} \hat{T} + \frac{\partial \hat{u}_z}{\partial z} + \frac{1}{r} \frac{\partial (r \hat{u}_r)}{\partial r} = 0. \tag{2.16}
\]
When only the plane wave mode exists in a pipe, the length scale of the acoustic variations is on the order of thermal penetration depth from the wall in radial direction, and on the order of acoustic wavelength in axial direction. As such, \( \partial / \partial r \sim 1 / \delta_k \), \( \partial / \partial z \sim 1 / \lambda \), and \( |\hat{u}_z / \hat{u}_r| \sim \lambda / \delta_k \). Taking these ratios into account enables \( \hat{c} \hat{p} / \partial r \) to be neglected, and leads to the following linear approximation of Eq. (2.2)

\[
\hat{u}_z + \frac{1}{j \omega \rho_0} \frac{d\hat{p}}{dz} \approx \frac{\mu}{j \omega r} \frac{\partial}{\partial r} \left( r \frac{\partial \hat{u}_z}{\partial r} \right).
\] (2.17)

Turning attention now to Eq. (2.3), it should be noted that the first term on the right hand side is larger than the second and the third terms which are quadratic in acoustic velocity. Hence, Eq. (2.3) can be simplified by taking only the first order terms among the perturbed signals as

\[
\rho_0 T_0 \left( \frac{\partial s_1}{\partial t} + \hat{u} \cdot \nabla s_0 \right) = \kappa \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T_1}{\partial r} \right),
\] (2.18)

where the ideal gas relation \( s_0 = c_p \ln T_0 - R \ln P_0 + \text{const.} \). Here, the gradient of the equilibrium entropy \( \nabla s_0 \) becomes negligible because \( \nabla T_0 = 0 \) by assumption. Therefore, Eq. (2.18) can be rewritten as

\[
\rho_0 T_0 \left( \frac{\partial s_1}{\partial t} \right) = \kappa \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T_1}{\partial r} \right).
\] (2.19)

In order to express Eq. (2.19) without an entropy term, the derivative of entropy with respect to time is expressed as follows:

\[
\frac{\partial s_1}{\partial t} = \left( \frac{\partial s}{\partial T} \right)_p \left( \frac{\partial T}{\partial t} \right)_p - \left( \frac{\partial T}{\partial P} \right)_p \left( \frac{\partial P}{\partial t} \right)_p.
\] (2.20)
Under adiabatic conditions, \(\frac{T}{T_0} = \left(\frac{P}{P_0}\right)^{(r-1)\gamma}\), where \(\gamma\) is the ratio of isobaric to isochoric specific heats. Using this expression along with the definition of constant-pressure specific heat \(c_p = T_0 \left(\frac{ds}{dT}\right)_p\), Eq. (2.19) can be rewritten as

\[
\hat{T} = -\frac{1}{j \omega \rho_0 c_p} \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \hat{T}}{\partial r} \right) + \frac{(\gamma-1) T_0}{\gamma P_0} \hat{p}.
\]  

(2.21)

Because \(\hat{p}\) is a function of only axial position, Eqs. (2.17) and (2.21) can be solved through the separation of variables for \(\hat{u}_z\) and \(\hat{T}\), the solutions being

\[
\hat{u}_z = \frac{j}{\rho_0 \omega} \left[1 - h_v(r)\right] \frac{d\hat{p}}{dz},
\]

(2.22)

\[
\hat{T} = \frac{1}{\rho_0 c_p} \left[1 - h_c(r)\right] \hat{p}.
\]

(2.23)

Here \(h_v(r)\) and \(h_c(r)\) are the thermoviscous \(h\)-functions defined [18] as

\[
h_{v,c}(r) = J_0 \left[\left(j-1\right) r / \delta_{v,c}\right] / J_0 \left[(j-1) r_0 / \delta_{v,c}\right]
\]

(2.24)

for a cylindrical pore. Here \(J_0\) is the zeroth order Bessel function of the first kind, the thermal penetration depth \(\delta_c = \sqrt{2 \kappa / (\omega \rho_0 c_p)}\), and the viscous penetration depth \(\delta_v = \sqrt{2 \mu / (\omega \rho_0)}\), respectively.

Integrating Eq. (2.16) over the entire cross sectional area of the pipe and substituting Eqs. (2.22) and (2.23) into the resulting equation, the equation of continuity can be expressed as

\[
\hat{p} = \frac{j \rho_0 c_p^2}{\omega} \left[1 + (\gamma-1) f_c\right] \left[1 - h_c(r)\right] \frac{\partial \hat{u}_z}{\partial z},
\]

(2.25)
where \( f_{\nu,\kappa} \) is the thermoviscous f-function defined as
\[
f_{\nu,\kappa} = \frac{\int_0^{2\pi} \int_0^\infty h_{\nu,\kappa}(r)rdrd\theta}{\pi r_0^2}
\]
for a cylindrical pore, and where \( c_0 \) is the speed of sound.

Substitution of Eq. (2.22) into Eq. (2.25) yields
\[
\frac{d^2 \hat{p}}{dz^2} + \hat{k}^2 \hat{p} = 0,
\]
where \( \hat{k} = \frac{\omega}{c_0} \sqrt{\frac{1+(\gamma-1)f_\nu}{1-f_\nu}} \) is the complex propagation constant for the plane wave mode.

Equation (2.26) is the Helmholtz equation as well as a special type of Rott’s wave equation when the gradient of the equilibrium temperature is zero. [18]

Meanwhile, the integration of Eq. (2.16) over the cross sectional area of the pipe only up to \( r \) instead of \( r_0 \) and substitution of Eq. (2.25) into the resulting equation yields
\[
\hat{u}_r(r,z) = \frac{1}{2} \frac{j\hat{k}^2}{\rho_0 \omega} f_\nu r_0 R_r(r) \hat{p},
\]
where
\[
R_r(r) = \frac{r}{r_0} \frac{J_1[(j-1)r / \delta_\nu]}{J_1[(j-1)r_0 / \delta_\nu]} + \frac{\gamma-1}{1+(\gamma-1)f_\nu} f_\nu \left( \frac{r}{r_0} - \frac{J_1[(j-1)r / \delta_\nu]}{J_1[(j-1)r_0 / \delta_\nu]} \right).
\]

### 2.2.2 Standing wave tube

The Helmholtz equation Eq. (2.26) has two homogeneous solutions. As shown in Fig. 2.1, the linear combination of these solutions can represent the total pressure in terms of the counter-propagating plane wave model as
\[
\hat{p}(z) = \hat{A} e^{-j\hat{k}z} + \hat{B} e^{j\hat{k}z},
\]
\[
= \hat{A} e^{-j(k-j\omega)z} + \hat{B} e^{j(k-j\omega)z},
\]
where $\hat{A}$ and $\hat{B}$ are complex pressure amplitude coefficients determined by the curve-fitting as described in the later section, and where $k$ and $\alpha$ are the real and imaginary part of the complex propagation constant $\hat{k}$. Inserting Eq. (2.28) into Eq. (2.22) yields the axial component of the velocity,

$$\hat{u}_z (r,z) = \frac{j}{\omega \rho_0} [1 - h_v (r)] \left\{ - j\hat{k} \left( \hat{A} e^{-j\hat{k}z} - \hat{B} e^{j\hat{k}z} \right) \right\},$$

(2.29)

$$= \frac{\hat{k}}{\omega \rho_0} [1 - h_v (r)] \left( \hat{A} e^{-j\hat{k}z} - \hat{B} e^{j\hat{k}z} \right).$$

Figure 2.1. Diagram of a driven standing wave tube terminated at $z = 0$. Regardless of the termination, under the plane wave assumption, the pressure field can be expressed as superposition of two counter-propagating plane waves. The incident wave and the reflected wave are denoted as $\hat{p}_i$ and $\hat{p}_R$, respectively. $\hat{A}$ and $\hat{B}$ are the complex amplitude coefficient for each wave component.

As can be seen from Eq. (2.29), the radial dependence is due entirely to the $h_v (r)$ term. Based on Eq. (2.29), the axial velocity on the center axis can be expressed,

$$\hat{u} (z) \equiv \hat{u}_z (0,z),$$

$$= \frac{\hat{k}}{\omega \rho_0} \left( \hat{A} e^{-j\hat{k}z} - \hat{B} e^{j\hat{k}z} \right),$$

(2.30)

where $\hat{u} (z)$ is defined as the axial velocity on the center axis.
2.2.3 Curvefitting algorithm

A key element to this dissertation is to determine the pressure and axial velocity for the entire axial range, based on the limited number of velocity measurements. In order to accomplish this goal, two curvefitting algorithms are used. The first algorithm used is based on pressure measurement at two positions, while the second algorithm is based on the velocity measurements at multiple positions. Since this dissertation is focused on the development of a velocity-based measurement technique, the axial velocity is usually the measured quantity, in the description of the second algorithm. In the following discussion, two conditions are assumed: first, the region to which the curvefitting algorithms are applied is a homogeneous medium in a uniform cross sectional pipe, and second, the position where this region ends is situated at \( z = 0 \). These assumptions will simplify the discussion.

As discussed previously, under the plane wave approximation, the total pressure can be expressed as the superposition of two counter-propagating plane waves as in Eq. (2.28)

\[
\hat{p}(z) = \hat{A}e^{-j\hat{k}z} + \hat{B}e^{j\hat{k}z}.
\]  

The coefficients \( \hat{A} \) and \( \hat{B} \) can be determined if the pressure is known at two points, \( z_1 \) and \( z_2 \). Specifically,

\[
\begin{bmatrix}
\hat{A} \\
\hat{B}
\end{bmatrix} =
\begin{bmatrix}
e^{-j\hat{k}z_1} & e^{j\hat{k}z_1} \\
e^{-j\hat{k}z_2} & e^{j\hat{k}z_2}
\end{bmatrix}^{-1}
\begin{bmatrix}
\hat{p}(z_1) \\
\hat{p}(z_2)
\end{bmatrix},
\]  

where \( \hat{k} \) is assumed to be known.
Equation (2.31) is the basis of the two-microphone technique which is frequently used in determining the reflection coefficient off the surface of a sample. Given $\hat{A}$ and $\hat{B}$ from Eq. (2.31), the total pressure can be estimated from Eq. (2.28). This scheme is specifically used when the two-microphone method is tested against the LDA-based technique in this study.

A strength of the LDA technique is that it readily provides a method for measuring the acoustic field at multiple locations rather than two. Given that axial velocity at more than two positions, a more sophisticated curvefitting scheme is required than extracting $\hat{A}$ and $\hat{B}$ in Eq. (2.31). At first sight, the measurement at multiple positions not only brings a nuisance, but also necessitates a complex algorithm. However, the measurement at multiple positions is reported to improve the quality of the curvefitting, especially when the separation of two adjacent positions is the same for all data points. [2] Here the data, which are collected at equidistantly separated positions, are fitted to the model equation with the curvefitting algorithm based on a linear least mean squared method for parameters $\hat{A}$ and $\hat{B}$. It follows that the problem becomes to find $\hat{A}$ and $\hat{B}$ which satisfy the expression

$$\min \left[ \sum_{i=1}^{N} \left| \hat{u}_i - \frac{k}{\rho_0 \omega} \left( \hat{A} e^{-j\hat{k}z_i} - \hat{B} e^{j\hat{k}z_i} \right) \right|^2 \right], \quad (2.32)$$

where $\hat{u}_i$ is the complex axial velocity measured at $z = z_i$, $N$ is the number of measurement positions. Note that Eq. (2.32) is optimized when the partial derivative of the sum of
squares of residuals with respect to each parameter (i.e., \( \Re\{\hat{A}\} \), \( \Im\{\hat{A}\} \), \( \Re\{\hat{B}\} \), and \( \Im\{\hat{B}\} \)) is equivalent to zero.

Decomposed in real and imaginary part, hence, the coefficients \( \hat{A} \) and \( \hat{B} \) can be expressed as

\[
\begin{bmatrix}
\Re\{\hat{A}\} \\
\Im\{\hat{A}\} \\
\Re\{\hat{B}\} \\
\Im\{\hat{B}\}
\end{bmatrix} = \frac{\hat{k}}{\rho_0 \omega} \begin{bmatrix}
\sum_{i=1}^{N} \exp(-2\alpha z_i) & 0 & -\sum_{i=1}^{N} \cos(2kz_i) & \sum_{i=1}^{N} \sin(2kz_i) \\
0 & \sum_{i=1}^{N} \exp(-2\alpha z_i) & -\sum_{i=1}^{N} \sin(2kz_i) & -\sum_{i=1}^{N} \cos(2kz_i) \\
-\sum_{i=1}^{N} \cos(2kz_i) & -\sum_{i=1}^{N} \sin(2kz_i) & \sum_{i=1}^{N} \exp(2\alpha z_i) & 0 \\
\sum_{i=1}^{N} \sin(2kz_i) & -\sum_{i=1}^{N} \cos(2kz_i) & 0 & \sum_{i=1}^{N} \exp(2\alpha z_i)
\end{bmatrix}^{-1}
\]

\[
\begin{bmatrix}
\sum_{i=1}^{N} u_i \exp(-\alpha z_i) \cos(\theta_i + kz_i) \\
\sum_{i=1}^{N} u_i \exp(-\alpha z_i) \sin(\theta_i + kz_i) \\
-\sum_{i=1}^{N} u_i \exp(\alpha z_i) \cos(\theta_i - kz_i) \\
-\sum_{i=1}^{N} u_i \exp(\alpha z_i) \sin(\theta_i - kz_i)
\end{bmatrix},
\]

where \( \exp(x) \) denotes the exponential function \( e^x \), and substitution is made that \( \hat{u}_i = u_i e^{i\theta_i} \). \( u_i \) and \( \theta_i \) are the magnitude and the phase of the axial velocity measured at \( z = z_i \).

This curvefitting algorithm gives a result without need of initial estimation for the four parameters; \( \Re\{\hat{A}\} \), \( \Im\{\hat{A}\} \), \( \Re\{\hat{B}\} \), and \( \Im\{\hat{B}\} \). However, this method curvefitting is
not constrained in a physical sense, i.e., the magnitude of the reflection coefficient sometimes becomes greater than unity. This issue is especially important when the magnitude of the reflection coefficient is close to unity. In order to resolve this issue, use was made of the fact that the reflection coefficient at \( z = 0 \) can be expressed as,

\[
\hat{R} = \frac{\hat{B}}{\hat{A}},
\]

\[
= e^{-(\eta + j\theta)},
\]

where \( \eta \) and \( \theta \) are arbitrary real constants. It follows that \( |\hat{R}| \leq 1 \) when \( \eta \geq 0 \) is imposed.

Substitution of Eq. (2.34) into Eq. (2.30) leads to

\[
\hat{u}(z) = [(k - j\alpha) / \omega \rho_0] A e^{j\phi} e^{-j\pi/2} e^{-\eta^2/2} 2 j \sin[\theta / 2 - j\eta / 2 - (k - j\alpha)z],
\]

where the substitution is made that \( \hat{A} = A e^{j\phi} \), where \( A \) and \( \phi \) are magnitude and phase of \( \hat{A} \). However, the four unknown parameters become \( A, \phi, \eta, \) and \( \theta \) as a result.

After Eq. (2.35) is substituted into Eq. (2.32), the second-pass curvefitting algorithm is implemented by use of MATLAB’s \textit{lsqnonlin} function which has an option in setting the lower and the upper bound for the curvefitting algorithm. With the condition \( \eta \geq 0 \) imposed in the lower bound of the option, the constraint \( |\hat{R}| \leq 1 \) is automatically satisfied. Because the initial guesses are required in the second-pass curvefitting algorithm by \textit{lsqnonlin} function, they are provided by the first-pass curvefitting algorithm expressed by Eq. (2.33). Hence, the curvefitting algorithm based on the measurement of axial velocity becomes a two-pass curvefitting algorithm.

Normally, the number of unknown parameters determined by the curvefitting algorithm is four, as described above, except in two applications. In Chapter 4, where the
complex propagation constant of the air inside a cylindrical pipe with rigid walls is measured, the number of unknown parameters becomes six which comprises $A$, $\phi$, $\theta$, $\eta$, $k$, and $\alpha$. This application must use the two-pass curvefitting algorithm because two additional unknown parameters $k$ and $\alpha$ in each exponent cause the curvefitting to become nonlinear, regardless of the constraint on the reflection coefficient. Because a nonlinear least mean squared fit needs an initial guess for the parameters, the first-pass curvefitting algorithm in Chapter 4 is necessary to estimate the $A$, $\phi$, $\theta$, and $\eta$ for the second-pass curvefitting based on theoretically predicted $k$ and $\alpha$. In Chapter 7, where the axial propagation constant in the first non-planar wave mode is discussed, a more sophisticated scheme based on the two-pass curvefitting algorithm is applied. It will be explicitly described in Chapter 7.

### 2.2.4 Acoustic intensity and acoustic power

Equations (2.28) and (2.29) provide the information necessary to determine the intensity flow and the power flow in the pipe. Specifically, the axial component of the acoustic intensity as a function of position along the pipe is,

$$ I_z (r, z) = \frac{1}{2} \Re \{ \hat{p} \hat{u}^* \}, $$

$$ = \frac{1}{2} \Re \left\{ \left( \hat{A} e^{-j\hat{z}z} + \hat{B} e^{j\hat{z}z} \right) \left( \frac{k}{\omega \rho_0} [1 - h_v(r)] \left( \hat{A} e^{-j\hat{z}z} - \hat{B} e^{j\hat{z}z} \right) \right)^* \right\}. \quad (2.36) $$

Integrating both sides of Eq. (2.36) over the cross sectional area of the pipe gives the acoustic power,
\[
\dot{W}(z) = \frac{1}{2} \Re \left\{ \left( \hat{A}e^{-j\tilde{k}z} + \hat{B}e^{j\tilde{k}z} \right) \left( \frac{k}{\omega \rho_0} S \left[ 1 - f_z \right] \left( \hat{A}e^{-j\tilde{k}z} - \hat{B}e^{j\tilde{k}z} \right) \right)^* \right\}, \tag{2.37}
\]

where \( S \) is the cross sectional area of the cylindrical pipe.

As seen in Eq. (2.37), the acoustic power, \( \dot{W}(z) \), is a function only of axial position. Figure 2.2 shows the normalized acoustic power as a function of non-dimensional axial position in comparison with the normalized axial acoustic intensity scaled by the cross sectional area in a rigidly terminated pipe. As illustrated in Fig. 2.2, the acoustic power and the axial acoustic intensity exhibit different trends as functions of axial position. This phenomenon results from the fact that the thermoviscous boundary layers exist near walls. Therefore, the acoustic power in a pipe cannot be obtained simply by multiplication of the cross sectional area and the axial acoustic intensity.
Figure 2.2. Figure (a) illustrates the normalized acoustic power and the normalized axial component of the on-axis intensity multiplied by the cross sectional area as a function of non-dimensional axial position in a rigidly terminated cylindrical pipe. Note that the two lines follow different trends as a function of non-dimensional axial position. Figure (b) shows the corresponding magnitude of normalized pressure as a function of non-dimensional axial position.

It is interesting to note that the slope at a point on the curves of acoustic power or axial intensity changes as a function of axial position. If the derivative of the acoustic
power, \( \dot{W}(z) \), is taken with respect to \( z \), the result will represent the slope at a point on the curve of the acoustic power. Swift [29] denotes the dissipation per unit surface area of the resonator as \( \dot{e} \). The unit surface area in this case is \( 2\pi r_0 dz \). Taking the derivative of \( \dot{W}(z) \) as given by Eq. (2.37) with respect to \( z \) and dividing it by \( 2\pi r_0 dz \) yields,

\[
\dot{e} = -\frac{1}{2\pi r_0} \frac{d\dot{W}(z)}{dz} = \frac{1}{4} \frac{|\dot{p}|^2}{\rho c_0^2} \delta_\kappa (\gamma - 1) \omega + \frac{1}{4} \rho_0 |\dot{u}|^2 \delta_\epsilon \omega.
\] (2.38)

In a rigidly terminated pipe, the pressure amplitude is at its maximum where the axial velocity is at its minimum, and the pressure amplitude is minimized where the axial velocity is maximized. In low loss systems, it can be assumed that the axial velocity is very nearly zero at a pressure antinode, and that pressure is almost zero at a velocity antinode to a good approximation. As a consequence, the dissipation per unit surface area near each antinode can be written as

\[
\dot{e}_{PA} = \frac{1}{4} \frac{|\dot{p}|^2}{\rho_0 c_0^2} \delta_\kappa (\gamma - 1) \omega, \quad \text{(2.39)}
\]

\[
\dot{e}_{PN} = \frac{1}{4} \rho_0 |\dot{u}|^2 \delta_\epsilon \omega, \quad \text{(2.40)}
\]

where subscripts \( PN \) and \( PA \) are a pressure node and a pressure antinode. Here \( \dot{e}_{PA} \) is mainly from the dissipation due to the thermal loss, and \( \dot{e}_{PN} \) the dissipation due to the viscous loss, respectively.
Provided that the velocity antinode and the pressure antinode are adjacent, it is also true that the ratio of $|\hat{p} / \hat{u}|$ is approximately $\rho_0 c_0$ in low loss systems with good precision. It follows that the ratio of the slopes at the pressure and velocity antinodes is

$$\frac{\hat{e}_{pr}}{\hat{e}_{PN}} = \left( \frac{\hat{p} / \hat{u}}{\rho_0 c_0} \right)^2 \frac{\delta_x (\gamma - 1)}{\delta_v}$$

$$= \frac{1}{\sqrt{\text{Pr}}} (\gamma - 1) = 0.478 \text{ (Air)},$$

(2.41)

where $\text{Pr}$ is the Prandtl number. Equation (2.41) suggests that the dissipation based on viscous loss is greater than the dissipation due to thermal loss, and that the magnitude of the slope changes as a function of axial position. Although a rigidly terminated pipe is used to interpret Eq. (2.38) in this section, the equation is valid for a pipe with the condition $r_0 / \delta_{v,\kappa} \gg 1$, regardless of the kind of the termination.

### 2.2.5 Acoustic power in an arbitrarily terminated pipe

In the previous section, the acoustic power in a rigidly terminated pipe is discussed. Here, instead of a rigid termination, an arbitrary termination is applied to the end of the pipe. Since Eq. (2.37) is valid regardless of the termination, it is obvious that the acoustic power dissipation along the pipe up to the termination is affected only by the thermoviscous boundary layers, while the dissipation at the termination is mainly due to the acoustic power dissipated in the arbitrary termination.

Figure 2.3 illustrates the distribution of acoustic power as a function of axial position in an arbitrarily terminated pipe. For example, $\hat{W}(z_i)$ includes the power dissi-
pated through the thermoviscous boundary layer losses along the pipe from \( z_i \) to 0 plus the power dissipated by the load at \( z = 0 \). In this study, a variable RLC load \([28,43]\) is attached at \( z = 0 \).

\[ \text{Figure 2.3. Acoustic power } \dot{W}(z) \text{ is shown as a function of axial position } z. \text{ A dissipative load is located at } z = 0. \text{ The acoustic power dissipated in the load is } \dot{W}(0). \]

It follows, from Eq. (2.37), that the acoustic power dissipated in the RLC load is

\[ \dot{W}_{\text{LOAD}} = \dot{W}(0) \]
\[ = \frac{1}{2} \Re \left\{ \left( \hat{A} + \hat{B} \right) \left( \frac{\hat{k}}{\omega \rho_0} S \left[ 1 - f_c \right] \left( \hat{A} - \hat{B} \right) \right)^* \right\}. \]  

(2.42)

As can be seen from Eq. (2.42), \( \hat{A} \) and \( \hat{B} \) are the only parameters required to know the acoustic power dissipated in the RLC load, since the other quantities are known or can be calculated from the thermoviscous boundary layer theory.
The power dissipated in the RLC load can also be calculated based on a lumped parameter model, assuming that the wavelength is much larger than any dimension of the RLC load, as in Fig. 2.4.

\[
\begin{align*}
\hat{p}_0 & \quad R \quad j\omega L \quad \hat{p}_C \\
\frac{1}{j\omega C} & \quad j\omega C \hat{p}_C
\end{align*}
\]

Figure 2.4. The equivalent circuit diagram of a variable RLC load in acoustic domain is based on a lumped parameter model. The volume velocity through the circuit is \( j\omega C \hat{p}_C \) as indicated in the figure. As long as the circuit is in the form of either RC or RLC, the dissipated power can be calculated from \( \hat{p}_0 \) and \( \hat{p}_C \).

The resulting amount of power dissipated can be expressed as [28,29,43]

\[
\dot{W}_{\text{LOAD}} = \frac{\omega C}{2} \Im\left\{ \hat{p}_0 \hat{p}_C^* \right\},
\]

\[
= \frac{\omega V}{2\gamma P_0} \Im\left\{ \hat{p}_0 \hat{p}_C^* \right\},
\]

where \( C \) and \( V \) are the compliance and volume of the load. \( \hat{p}_0 \) and \( \hat{p}_C \) represent the pressure measured at \( z = 0 \) and in the volume, respectively. Comparing the dissipated power calculated according to Eq. (2.42) with that from Eq. (2.43) provides a check of the LDA technique.
2.2.6 Radiation impedance of an open ended pipe

In acoustics, several types of impedances are used, such as specific acoustic impedance, acoustic impedance, and radiation impedance. Although each terminology is different from the others, all impedances are proportional to each other. The great feature of the concept of impedance is to obtain pressure-related quantities from velocity-related quantities, and vice versa. For example, the radiation impedance, defined as the ratio of force to axial velocity at an open-ended termination, allows for extracting force from axial velocity, or axial velocity from force. Hence, the acoustic power dissipated through the termination can be determined by the radiation impedance and the force (or the axial velocity) at the termination.

Among many conditions for termination of the pipe, theories for a rigid termination and an open termination are well established. For instance, the specific acoustic impedance, defined by the ratio of pressure to acoustic velocity, for a rigid termination is given as [44],

\[
\frac{1}{Z} = \frac{e^{j\pi/4}}{\rho_0 c_0} \sqrt{\frac{\omega \mu}{\rho_0 c_0^2}} \left[ \sin^2 \varphi_i + \frac{\gamma - 1}{\sqrt{Pr}} \right],
\]

where $\varphi_i$ is the angle of incidence relative to the normal direction to the surface of reflection. Because the termination is placed at one end of the pipe in this study, the direction of wave propagation is normal to the termination. Accordingly, Eq. (2.44) reduces to

\[
\frac{1}{Z} = \frac{e^{j\pi/4}}{\rho_0 c_0} \sqrt{\frac{\omega \mu}{\rho_0 c_0^2}} \frac{\gamma - 1}{\sqrt{Pr}}.
\]
As for an open ended pipe, the radiation impedance for two conditions are widely given in textbooks \([44,45]\) under the constraint \(k_0r_0 \ll 1\) as,

\[
\dot{Z}_{\text{rad}} = \rho_0 c_0 S \left[ \frac{1}{2}(k_0r_0)^2 + j\frac{8}{3\pi}k_0r_0 \right], \quad \text{(flanged pipe)} \tag{2.46}
\]

\[
\dot{Z}_{\text{rad}} = \rho_0 c_0 S \left[ \frac{1}{4}(k_0r_0)^2 + j0.6k_0r_0 \right], \quad \text{(unflanged pipe)} \tag{2.47}
\]

where \(k_0 = \omega / c_0\). Although these two equations look simple and similar in their forms, they are different in a few aspects. First, Eq. (2.46) is independent of the pipe wall thickness, while Eq. (2.47) assumes zero wall thickness. Second, Eq. (2.46) can be easily modified for the case \(k_0r_0 \geq 1\) by using the first order of Bessel function \(J_1\) and the Struve function \(H_1\) as follows

\[
\dot{Z}_{\text{rad}} = \rho_0 c_0 S \left[ 1 - \frac{J_1(2k_0r_0)}{k_0r_0} + j\frac{H_1(2k_0r_0)}{k_0r_0} \right], \tag{2.48}
\]

where \(H_1(2k_0r_0) = (k_0r_0)^2 \sum_{m=0}^{\infty} \frac{(-1)^m (k_0r_0)^{2m}}{\Gamma(m + 3/2)\Gamma(m + 5/2)}\) based on the Gamma function \(\Gamma\), whereas Eq. (2.47) should be reevaluated through a set of integral equations based on either Green’s function method \([22]\) or the Wiener-Hopf technique \([23]\). The case of non-zero wall thickness is more complicated than the unflanged pipe with zero wall thickness shown in Eq. (2.47), but it is successfully analyzed by Ando based on the Wiener-Hopf technique. \([24]\) Here and in this dissertation, however, not only is the discussion of the Wiener-Hopf technique in detail beyond the scope of the dissertation, it is too lengthy to be contained. Therefore, it is omitted, despite use of this technique.
Figure 2.5 shows the diagram of a semi-infinite cylindrical pipe with non-zero wall thickness. Although the concept of velocity potential is used in Ando’s work, pressure is used in this section because pressure is proportional to velocity potential with $e^{j\omega t}$ time harmonic assumed.

![Diagram of a semi-infinite cylindrical pipe with non-zero wall thickness](image)

Figure 2.5. A diagram of a semi-infinite cylindrical pipe with non-zero wall thickness ($r_1 - r_0$) in Ando’s work. The incident wave comes from $z = -\infty$, and is reflected at $z = 0$.

Provided that non-dimensional pressure $e^{-jk_0z}$ is incident from the direction $z = -\infty$, the total pressure including the pressure reflected off the open end can be approximated as

$$
\hat{p}_1(r, z) = e^{-jk_0z} + \hat{R}e^{jk_0z} + \sum_{\mu=1}^{N} b_{\mu} \frac{J_0\left(j_{0\mu}'r / r_0\right)e^{z[j_{0\mu}'r - k_0^2r_0^2]^{1/2}}}{J_0\left(j_{0\mu}'\right)},
$$

(2.49)

where $j_{0\mu}'$ is the $\mu$th zero that satisfies $J_0\left(j_{0\mu}'\right) = 0$.

For convenience, the entire domain is divided into three regions, and the corresponding pressure fields are $\hat{p}_1\left( r < r_0, z < 0 \right)$, $\hat{p}_2\left( r < r_1, z > 0 \right)$, and $\hat{p}_3\left( r > r_1 \right)$, where $\hat{p}_1$, $\hat{p}_2$, and $\hat{p}_3$ satisfy the Helmholtz equation. The relevant boundary conditions are
1) \( \frac{\partial \hat{p}_2}{\partial r} \bigg|_{r=r_1} = 0 \) for \( z < 0 \),

2) \( \frac{\partial \hat{p}_2}{\partial z} \bigg|_{z=0} = 0 \) for \( r_0 < r < r_1 \),

3) \( \frac{\partial \hat{p}_2}{\partial r} \bigg|_{r=r_1} = \frac{\partial \hat{p}_2}{\partial r} \bigg|_{r=r_1} \) for all \( z \neq 0 \),

4) \( \hat{p}_1 = \hat{p}_2, \ \frac{\partial \hat{p}_1}{\partial z} = \frac{\partial \hat{p}_2}{\partial z} \) for \( z = 0, \ r < r_0 \),

5) \( \hat{p}_2 = \hat{p}_3 \) for \( z > 0, \ r = r_1 \),

6) \( \hat{p}_2 \sim \text{const.} \times e^{-jk_0(z \cos \theta)} \) as \( z \to \infty, \ r < r_1 \),

and \( \hat{p}_3 \sim \text{const.} \times e^{-jk_0(r^2 + z^2)^{1/2}} / (r^2 + z^2)^{1/2} \) as \( (r^2 + z^2)^{1/2} \to \infty, \ r > r_1 \).

Meanwhile, the initial values for \( \hat{p}_2 \) can be expressed as

\[
\hat{p}_2 (r, +0) = \sum_{m=0}^{\infty} g_m \frac{J_0 \left( j_{0m}^r / r_1 \right)}{J_0 \left( j_{0m}^r \right)},
\]

and

\[
\frac{\partial \hat{p}_2}{\partial z} \bigg|_{z=0} = \sum_{m=0}^{\infty} f_m \frac{J_0 \left( j_{0m}^r / r_1 \right)}{J_0 \left( j_{0m}^r \right)}.
\]

With all things combined, the equations for coefficients to obtain the reflection coefficient \( \hat{R} \) are

\[
(1 + \hat{R}) = g_0 + \sum_{m=1}^{\infty} \frac{2J_1 \left( j_{0m}^r / r_1 \right)}{j_{0m}^r / r_1 \cdot J_0 \left( j_{0m}^r \right)} g_m,
\]

\[
b_\mu = \sum_{m=1}^{\infty} \frac{2j_{0m}^r / r_1}{j_{0m}^r / r_1^2 - j_{0m}^{r \mu}} \frac{J_1 \left( j_{0m}^r / r_1 \right)}{J_0 \left( j_{0m}^r \right)} g_m,
\]
\[ f_m = \begin{cases} 
 jk_0 \left( \hat{R} - 1 \right) \left( \frac{r_0}{r_1} \right)^2, & \text{for } m = 0 \\
 jk_0 \left( \hat{R} - 1 \right) \left( \frac{r_0}{r_1} \right)^2 \frac{2J_1 \left( \frac{j_{0m} r_0}{r_1} \right)}{J_1 \left( \frac{j_0 r_0}{r_1} \right)} \sum_{\mu=1}^{N} \left( \frac{j_{0\mu}^2}{r_0^2} - k_0^2 \right)^{1/2} \left( \frac{r_0}{r_1} \right)^2 \\
 \times \frac{2j_{0m} r_0}{r_1} \frac{J_1 \left( \frac{j_{0m} r_0}{r_1} \right)}{J_0 \left( \frac{j_{0m} r_0}{r_1} \right)} b_\mu, & \text{otherwise} 
\end{cases} \]  

(2.54)

\[ \alpha_m = \begin{cases} 
 \frac{1}{2} \left( 1 - \hat{R} \right) \left( \frac{r_0}{r_1} \right)^2, & \text{for } m = 0 \\
 \frac{jk_0 + \chi_m}{4j k_0} \frac{K_p \left( \chi_m \right)}{K_p \left( jk_0 \right)} f_m, & \text{for } m \geq 1 
\end{cases} \]  

(2.55)

\[ \beta_m = \begin{cases} 
 -\frac{1}{2} \left( 1 + \hat{R} \right), & \text{for } m = 0 \\
 -\frac{jk_0 + \chi_m}{4j k_0} \frac{K_p \left( \chi_m \right)}{K_p \left( jk_0 \right)} g_m, & \text{for } m \geq 1 
\end{cases} \]  

(2.56)

and

\[ \sum_{m=0}^{N} \frac{\beta_m}{\chi_m + \chi_n} + \frac{\left( 2 \chi_n \right)^2}{\left( \chi_m + \chi_n \right)^2} \sum_{m=0}^{N} \frac{\alpha_m}{2 \chi_n} = - \frac{\left( 2 \chi_n \right)^2}{\left( \chi_m + \chi_n \right)^2} \sum_{m=0}^{N} \frac{\alpha_m}{2 \chi_n} = 0, \quad \text{for } n = 0, 1, 2, ..., N \]  

(2.57)

where \( K(S) = -j\pi J_0'(\chi_1 r_1)H_0^{(2)}(\chi_1 r_1) = K_p \left( S \right) / K_N \left( S \right) \), and \( \chi_n = \left[ \left( j_{0n} / r_1 \right)^2 - k_0^2 \right]^{1/2} \).

From the truncation of the summation after \( m = N \) for \( g_m \), it follows that the number of variables and the number of equations are equally \( 5N + 6 \). Thus, the variables can be determined. In this study, \( N \) is chosen to be two according to Ando’s paper. [24] The actual solution of equations is found by Mathematica®. The computation of the split
function $K_p(S)$, which is required preliminarily, is based on the algorithm developed by Turing. [23]

### 2.2.7 Impedance tube

In Chapter 6, the complex propagation constant of the air inside a porous material called reticulated vitreous carbon (RVC) is measured, based on the counter-propagating plane wave model as is assumed in earlier applications.

Figure 2.6 illustrates a porous sample with thickness of $d$ is situated in a standing wave pipe. The entry face is located at $z = 0$. The following theory is based on the assumption that all the quantities, such as pressure, volume velocity, and the acoustical properties of the gas in the impedance tube, are known, while those inside of the porous sample are not.

![Figure 2.6. A driven impedance pipe with a porous sample with thickness $d$ is illustrated in this schematic diagram. As can be seen, the face nearer to the driver is located at $z = 0$. Three separate regions are denoted as $I$, $II$, and $III$.](image)

Based on Rott’s theory, the pressure and the volume velocity for each region can be expressed as

$$\hat{P}_I(z) = A e^{-jkz} + B e^{jkz},$$

(2.58)
\[ \hat{U}_I (z) = \left( \hat{A}e^{-jk_Iz} - \hat{B}e^{jk_Iz} \right) / \hat{Z}_{air}, \]  
(2.59)

\[ \hat{p}_{II} (z) = \hat{C}e^{-jk_{II}z} + \hat{D}e^{jk_{II}z}, \]  
(2.60)

\[ \hat{U}_{II} (z) = \left( \hat{C}e^{-jk_{II}z} - \hat{D}e^{jk_{II}z} \right) / \hat{Z}_s, \]  
(2.61)

\[ \hat{p}_{III} (z) = \hat{E}e^{-jk_{III}z} + \hat{F}e^{jk_{III}z}, \]  
(2.62)

\[ \hat{U}_{III} (z) = \left( \hat{E}e^{-jk_{III}z} - \hat{F}e^{jk_{III}z} \right) / \hat{Z}_{air}, \]  
(2.63)

where \( \hat{k}_s \) and \( \hat{Z}_s \) are the complex propagation constant and the acoustic impedance of the sample, and where \( \hat{Z}_{air} \) is the acoustic impedance of air. It is assumed that the pressure in the sample is a function only of axial position, and that no near-field effect is present in the proximity of the faces of the sample.

Combined with the boundary conditions that pressure and the volume velocity are continuous at any axial position, the equations to be solved for \( \hat{C}, \hat{D}, \hat{k}_s, \) and \( \hat{Z}_s \) are written as

\[ \hat{p}_I (0) = \hat{A} + \hat{B}, \]  
(2.64)

\[ = \hat{C} + \hat{D}, \]

\[ \hat{U}_I (0) = \left( \hat{A} - \hat{B} \right) / \hat{Z}_{air}, \]  
(2.65)

\[ = \left( \hat{C} - \hat{D} \right) / \hat{Z}_s, \]

\[ \hat{p}_{III} (d) = \hat{E}e^{-jk_{III}d} + \hat{F}e^{jk_{III}d}, \]  
(2.66)

\[ = \hat{C}e^{-jk_{III}d} + \hat{D}e^{jk_{III}d}, \]

and
\[ \hat{U}_{mm}(d) = \left( \hat{E} e^{-j\hat{k}_d} - \hat{F} e^{j\hat{k}_d} \right) / \hat{Z}_{air}, \]  
\[ = \left( \hat{C} e^{-j\hat{k}_d} - \hat{D} e^{j\hat{k}_d} \right) / \hat{Z}_s. \]  

Using these equations, the pressure and volume velocity at \( z = d \) can be related to those at \( z = 0 \), according to

\[ \hat{p}_{mm}(d) = \cos(\hat{k}_d d) \hat{p}_i(0) - j \hat{Z}_s \sin(\hat{k}_d d) \hat{U}_i(0), \]  
\[ \hat{U}_{mm}(d) = \left[ -j \sin(\hat{k}_d d) / \hat{Z}_s \right] \hat{p}_i(0) + \cos(\hat{k}_d d) \hat{U}_i(0). \]  

Eliminating \( \hat{Z}_s \) in Eqs. (2.68) and (2.69) yields the expression for the complex propagation constant of the air in the sample \( \hat{k}_s \) as

\[ \hat{k}_s = \frac{1}{d} \cos^{-1} \left( \frac{\hat{U}_i(0) \hat{p}_i(0) + \hat{U}_{mm}(d) \hat{p}_{mm}(d)}{\hat{U}_i(0) \hat{p}_{mm}(d) + \hat{U}_{mm}(d) \hat{p}_i(0)} \right). \]  

Equation (2.70) is used to determine the complex propagation constant of the air in an RVC sample in Chapter 6. It should be noted that the axial velocity and the pressure on both faces of the sample are extrapolated from discrete measurements using a curvefitting algorithm as discussed previously. Because of uncertainty in specifying where the faces of the sample are located (due to the porous nature of the material), direct measurement of pressure and axial velocity on faces of the sample is difficult.

2.2.8 Non-planar wave mode in a cylindrical pipe

The loss mechanism in plane wave mode is sufficiently described by Rott’s wave equation which can be derived as shown previously. However, no such solid theory,
which is verified experimentally, exists for non-planar wave modes. Nonetheless, theories proposed until now are briefly introduced here.

The Helmholtz equation in a cylindrical cavity is

$$\nabla^2 \hat{p} + k^2 \hat{p} = 0,$$

(2.71)

where $\nabla^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$ in a cylindrical coordinate system. Assuming rigid boundary conditions, the general solution for a wave of the $(m,l)$ mode in the cavity can be expressed as [44,45]

$$\hat{p}_{ml} = \hat{A}_{ml} J_m(k_{ml}r) \cos(m\theta) e^{-jk_{ml}z} + \hat{B}_{ml} J_m(k_{ml}r) \cos(m\theta) e^{jk_{ml}z},$$

(2.72)

where $\hat{A}_{ml}$ and $\hat{B}_{ml}$ are the complex amplitude coefficients for the incident wave and the reflected wave for the $(m,l)$ mode, respectively, and where $k_z = \sqrt{(\omega/c)^2 - k_{ml}^2}$ and $k_{ml}$ satisfies $J_m'(k_{ml}r_0) = 0$. Similar to the analysis of the plane wave mode, here the pressure field for each $(m,l)$ mode is represented by two counter-propagating waves in an arbitrarily terminated cylindrical pipe, as shown in Eq. (2.72).

Now including the plane wave and $(1,1)$ modal components, when the source frequency is greater than the cut-on frequency of the $(1,1)$ mode, the total pressure field can be written as

$$\hat{p} = \hat{A}_{00} e^{-jk_z} + \hat{B}_{00} e^{jk_z} + \hat{A}_{11} J_1(k_{11}r) \cos(\theta) e^{-jk_{11}z} + \hat{B}_{11} J_1(k_{11}r) \cos(\theta) e^{jk_{11}z}. $$

(2.73)

As can be seen in Eq. (2.73), the pressure of the $(1,1)$ mode involves the lossless axial propagation constant $k_z$. However, a few studies [34,36] suggest that the axial propagation constant for non-planar wave mode should be complex so that the absorptive
mechanism can be implied, as opposed to the real \( k_z \), when non-planar wave mode propagates. Derivation of the expression for calculating the lossy axial propagation constant utilizes the wall admittance for a higher mode excitation given by Morse. [35]

When the \((m,l)\) mode is activated, the expression for the absorption coefficient \( \alpha_{ml} \) for the \((m,l)\) mode proposed by Beatty [34] is

\[
\alpha_{ml} = \left[ 1 - \left( \frac{f_{ml}}{f} \right)^2 \right]^{-1/2} \left\{ 1 + \frac{\gamma - 1}{\sqrt{Pr}} \right\} \left[ 1 - \left( \frac{m}{f_{ml}^\prime} \right)^2 \right] \left( \frac{f_{ml}}{f} \right)^2 \omega \delta \frac{1}{2 \nu \omega r_0 c_0}, \tag{2.74} \right.
\]

where \( f_{ml} \) and \( f \) are the cut-on frequency of the \((m,l)\) mode and the source frequency, and where \( f_{ml}^\prime \) is the \( l \)th zero that satisfies \( J_m(f_{ml}^\prime) = 0 \).

Using a different approach, Hudde [36] claims that the absorption coefficient is

\[
\alpha_{ml} = -\mathcal{Z} \{ \hat{k}_{ml} \} = \mathcal{Z} \left\{ j \left( \frac{f_{ml}^\prime}{r_0^2} - \frac{\omega^2}{c_0^2} + \frac{2j\omega\hat{Y}_{ml}}{c_0 r_0 \left[ 1 - \left( \frac{m}{f_{ml}^\prime} \right)^2 \right]} \right) \right\} \right)^{1/2}, \tag{2.75} \]

where \( \hat{Y}_{ml} = \frac{\omega \delta}{2c_0} (1 + j) \left( \frac{\gamma - 1}{\sqrt{Pr}} + 1 - \left( \frac{f_{ml}}{f} \right)^2 \right) \left[ 1 - \left( \frac{m}{f_{ml}^\prime} \right)^2 \right] \) and \( \hat{k}_{ml} \) is the complex axial propagation constant for the \((m,l)\) mode. According to Hudde, the absorption coefficient is continuous at the cut-on frequency of each mode, while that proposed by Beatty is discontinuous. However, it is a quite improper comparison, because Beatty’s solution is more focused on the absorption coefficient when a specific mode becomes activated while Hudde’s solution concerns the specific modal component even when it is evanescent. Since evanescent waves do not transfer power, the point that Hudde made is irrelevant to the loss mechanism in non-planar wave modes that this study is interested in.
In fact, if only activated modal components are being considered, both solutions based on Eqs. (2.74) and (2.75) are discontinuous at each cut-on frequency. Furthermore, in his own research [36], Hudde reported that Eq. (2.75) underestimated the absorption coefficient for higher modes. Thus, nothing has been proven correct pertaining to the complex axial propagation constant in higher modes.

In this study, only the first non-planar wave mode (i.e., (1,1) mode) is investigated to verify if the axial propagation constant measured is a complex value, because the lossless axial propagation constant is still used in some research. [38,39] However, the pursuit of finding the discontinuity near the cut-on frequency is not attempted because of its difficulty.
Chapter 3

Experimental setup

3.1 Introduction

The setup of apparatus and equipment used in this research are discussed in this chapter. The discussion is divided into three sections. The first section will mainly discuss the laser Doppler anemometer (LDA) and burst spectrum analyzer (BSA) system. Included are the theory and the signal processing required to use LDA/BSA system. In the second section, the apparatus and experimental procedure for each experiment performed is discussed. The calibration of the microphone is also covered. The third section discusses the chain of the data acquisition system. The connection among all the instruments is also briefly described in this section.

3.2 Laser Doppler anemometry (LDA)/burst spectrum analyzer (BSA)

This section is devoted to the discussion of LDA/BSA system from the standpoint of the basic operational principles as well as the signal processing method to attain the velocity from the raw data taken by LDA/BSA system.
3.2.1 Theory of laser Doppler anemometry

The fundamental principle upon which Laser Doppler anemometry (LDA) is based is the Doppler effect. Although its operation can be explained by Doppler effect, it is not easy to understand how LDA operates from this perspective. Instead, the fringe model can be an adequate theory to illustrate the basic operational mechanism of LDA, because no derivation of equations is involved.

Basically, two coherent laser beams emitted from the LDA lens intersect, and generate interference patterns within the volume where two laser beams cross, as shown in Fig. 3.1. This volume is often called fringe volume or measurement volume, and the each plane of interference pattern is known as a fringe. When a particle scattering light passes in the direction normal to fringes through the fringe volume, the scattered light is detected by a photodetector in a LDA probe. The intensity of the scattered is modulated by the frequency called Doppler frequency equivalent to the particle velocity divided by fringe separation, which is known from crossing angle of two laser beams and the wavelength of laser beams.

Figure 3.1. The intersection of two coherent laser beams with the angle $\theta$ and the wavelength $\lambda$ is illustrated. The interference of two laser beams forms fringes indicated by the bold horizontal lines within the intersection which is called the fringe volume or the measurement volume. Note that the direction of measured component of velocity is always normal to the fringes.
However, the direction of particle velocity cannot be determined because the receiver is unable to distinguish a negative frequency from a positive frequency, if the LDA sends out two laser beams at a same frequency. This ambiguity in the direction of particle velocity is resolved by allowing the frequency of one laser beam to be shifted by $F_B$ (i.e., 40 MHz in this study) relative to the other beam using a Bragg cell. [46] While a Doppler frequency larger than $F_B$ indicates a particle velocity in the positive direction, a Doppler frequency smaller than $F_B$ indicates a particle velocity in the negative direction.

The Burst Spectrum Analyzer (BSA) performs the signal processing required to recover the velocity from the LDA signal. The BSA detects and analyzes the voltage transmitted from the photo-multiplier when a seeding particle triggers the generation of a signal burst while passing through the fringe volume. Next, the voltage is band-pass filtered, frequency demodulated, and band-pass filtered to produce a signal corresponding to the velocity detected by LDA. The signal, then, is sampled and digitized to be prepared for discrete Fourier transform (DFT), which allows for spectrum analysis in frequency domain. Based on the idea that the spectral peak occurs at the Doppler frequency, a method of curvefitting, such as three points parabolic curvefitting, is utilized to find a location of the true peak.

3.2.2 Calibration of LDA

As described previously, the Doppler frequency is the quantity of interest measured by LDA. Thus, the calibration of LDA is necessary to recover the velocity information from this frequency. Because the previous section suggests that the Doppler frequency detected
by LDA is equal to the velocity divided by the fringe spacing, the determination of the fringe spacing results in the calibration of LDA.

As discussed, two laser beams have the frequency difference of \( F_b \) equivalent to 40 MHz. However, this difference can be neglected, compared to the original frequency, i.e., \( 4.75 \times 10^8 \) MHz, of the laser beam. Thus, the wavelength of the laser beams \( \lambda \) can be considered to remain unchanged. It follows that the calibration factor of LDA only depends on \( \sin(\theta/2) \), where \( \theta \) is the crossing angle of two laser beams. As shown in Fig. 3.2, laser beams being projected on a wall far from the fringe volume, the angle \( \theta \) can be estimated from the measurement of the distances \( l \) and \( d \). In practice, the uncertainty in calibrating LDA is approximately 0.3%, based on the uncertainty in the measurement of \( l \) and \( d \).
Figure 3.2. A diagram for the calibration of LDA is illustrated. Simple trigonometry reveals that the fringe (horizontal dashed lines) spacing \( \delta_f = \frac{\lambda}{2\sin(\theta/2)} \) for the velocity normal to the fringes. Assuming \( \theta/2 \ll 1 \), it follows that \( \sin(\theta/2) = \tan(\theta/2) = d/l \), where \( d \) is half of the distance between two laser beams on the wall, and where \( l \) is the distance between the intersecting point of the laser beams, \( O \), and the wall.

If the velocity component has an angle of \( \vartheta \) relative to the direction normal to the fringes, the effective fringe spacing for this oblique velocity component can be expressed as

\[
\delta_{f,\vartheta} = \delta_f / \cos(\vartheta).
\]  

(3.1)

Based on Eq. (3.1), it follows that the LDA is insensitive to the velocity which is parallel to the fringes, because infinitesimal \( \cos(\vartheta) \) causes the effective fringe spacing to become infinite. Therefore, when fringes are set normal to the axial component of velocity, the
velocity field detected by the LDA/BSA system is one dimensional, even though the axial and the radial component of velocity are present in the pipe under the plane wave mode.

### 3.2.3 Fourier averaging (FA) method

The instantaneous velocity detected by LDA/BSA system is a discrete time signal containing velocity of individual seeding particle when it triggers LDA/BSA system by passing through the measurement volume. The arrival of each seeding particle is random and discrete in time. If it is assumed that the signal to be measured originates from a monofrequency source, the individual velocity sample detected by the LDA/BSA system can be wrapped onto a single period of the source signal. The resulting signal will have the same fundamental frequency as the source frequency. Figure 3.3 illustrates this process well.
Figure 3.3. Figure (a) shows the Doppler frequency of each seeding particle as a function of time as it is measured. On the other hand, Fig. (b) illustrates the same quantity as a function of non-dimensional time by wrapping individual velocity sample onto a single period of the source signal.

Based on the periodic source signal, the frequency components of the resulting acoustic field will be at multiples of the fundamental frequency, and the time signal detected by LDA/BSA can be expressed as superposition of components at those harmonic frequencies. A technique called Fourier Averaging (FA) is used to obtain the
magnitude as well as the phase of the time signal. This method was first developed by Sonnenberger et al. [47], and has been reported as a powerful tool to estimate the each harmonic term except the dc term by Thompson and Atchley [11]. Because the dc term is not used in this dissertation, the FA method is commonly used for both pressure and velocity.

The first step of the FA method is to divide a time signal into a certain number of phase bins with approximately the same width of the recording interval of the LDA/BSA system. Then, neighboring particles within a single bin are considered to be events of the random process, and the mean value of their individual velocities is assumed to be the velocity at the bin. This process can be expressed in an equation as

\[
y[n] = \frac{1}{I[n]} \sum_{i=1}^{I[n]} y_n[i]
\]

\[
y = \frac{T}{N} n, \quad \text{for } n = 0, 1, 2, ..., N - 1
\]

where \(I[n]\) is the number of samples, \(y[n]\) the mean velocity, \(y_n[i]\) the velocity of the \(i\)th sample in the \(n\)th bin, and \(N\) the number of bins, respectively. For this analysis, the discrete time signal \(y[n]\) is regarded as the sampled value of the continuous time signal \(y(t)\) at \(t = nT / N\), where \(T\) is the period of \(y(t)\).

Definition of discrete Fourier transform allows the velocity information in the time domain to be converted into the velocity information in the frequency domain as follows,
\[ Y[k] = \frac{2}{N} \sum_{n=0}^{N-1} y[n] e^{-\frac{2\pi kn}{N}}, \]
\[ = \frac{2}{N} \sum_{n=0}^{N-1} y(n) \left( \frac{T}{N} \right) e^{-\frac{2\pi kn}{N}}, \]
\[ = |Y[k]| e^{j\phi[k]}, \quad \text{for } k \geq 1 \]
\[ Y[0] = \frac{1}{N} \sum_{n=0}^{N-1} y[n], \quad \text{(3.4)} \]
\[ y[n] = \Re \left\{ \sum_{k=0}^{N-1} Y[k] e^{\frac{2\pi kn}{N}} \right\}, \quad \text{(3.5)} \]

where \( Y[k] \) and \( \phi[k] \) are the Fourier coefficient and the phase of the \( k \)th harmonic component. By inserting Eq. (3.3) into Eq. (3.5), the continuous time signal can be synthesized as

\[ y_s(t) = \Re \left\{ \sum_{k=0}^{N-1} Y[k] e^{j\phi[k]} e^{j\omega t} \right\}, \]
\[ = \sum_{k=0}^{N-1} |Y[k]| \cos(\omega k t + \phi[k]), \quad \text{(3.6)} \]
\[ \approx y(t). \]

The synthesized signal \( y_s(t) \) is compared with the original signal \( y(t) \), which was previously wrapped onto a single period, to provide a check if the Fourier Averaging method is applied correctly. However, instead of performing all the analyses based on the time signal reconstructed from the Fourier coefficients, the concept of phase-vector (i.e., phasor) is used to simplify analyses of data in this study.

In some measurements, two LDA/BSA systems were used because of the limited number of sensors available in the laboratory. In these situations, a control signal from one LDA/BSA was used to synchronize the other. It was observed that an application of
the FA method described above caused an unexplained problem. From an empirical perspective, it was also observed that using *transit time* weighting in the FA method produced better results. The transit time, often called residence time, is defined as the time spent at the value of velocity. It is known to be inversely proportional to velocity.

Based on the transit time weighted mean value of velocities in each bin width, it follows that a simple modification on Eq. (3.2) leads to the expression as [46],

\[
y[n] = \left( \sum_{i=1}^{I[n]} y_n[i]tr_n[i] \right) \left/ \sum_{i=1}^{I[n]} tr_n[i] \right.
\]

\[= y \left( \frac{T}{N} \right)^n, \quad \text{for } n = 0, 1, 2, \ldots, N - 1
\]

where \( tr_n[i] \) is the transit time of the \( i \)th velocity sample in the \( n \)th bin width.

Figure 3.4 shows the synthesized velocities based on the transit time weighted FA method against the ordinary FA method, when two LDA/BSA systems are operational. The synthesized velocity based on the FA method is considered abnormal near \( t/T = 0 \) in Fig. 3.4.(b). Therefore, the use of the transit time weighted FA method is adequate for the signal processing of the velocity based on two LDA/BSA systems.
Figure 3.4. Figures (a) and (b) exemplify the velocity wrapped onto a normalized period after raw data is measured by the first LDA and the second LDA, respectively. Note that the difference between two methods is observed near $t/T = 0$ in Fig. (b).

3.2.4 Uncertainty of LDA

In the previous section, the uncertainty of calibrating LDA in magnitude is approximately 0.3%. In addition to this uncertainty, two other uncertainties are involved in the measurement of velocity using LDA. The first uncertainty is of the measured Doppler frequency in magnitude, while the second uncertainty is of calibrating phase of velocity. The first uncertainty is independent of the uncertainty caused by the calibration of LDA, because it
is present when the Doppler frequency is not converted into velocity. In general, this uncertainty is approximately 0.5 mm/s when it is represented in velocity. Thus, it would be dominantly large compared to the uncertainty of calibrating LDA in magnitude, when the measured velocity is less than 10.0 cm/s.

The second uncertainty in phase calibration is considered to originate from the record interval in LDA/BSA system. When phase is expressed into time, this uncertainty appears to be a function of the record interval, regardless of acoustic frequency. Thus, if the uncertainty is represented in phase, it becomes proportional to the acoustic frequency. Here this uncertainty is assumed to be approximately 5% of the record interval, based on empirical observation, because no specific information regarding this type of uncertainty is available.

### 3.2.5 Seeding particles

As previously discussed, seeding particles are necessary to measure velocity through detecting their motion with LDA. Thus, seeding particles should be light so that they can track the motion of fluid. In this study, seeding particles are of smoke generated from a piece of burnt rope that is blend of cotton and polyester. The diameter of each seeding particle was reported to be approximately 1 µm by Thompson. [48] The seeding particles generated from this smoke are introduced to the apparatus using a siphon pump which is attached to a galvanized metal funnel.
3.3 Acoustic apparatus

In this section, the acoustic apparatus used in each application is described in detail. The acoustic apparatus mainly consists of the pipe, the compression driver, the microphones, and any sub part which can holds microphones, excluding the LDA/BSA systems discussed previously and the data acquisition system which will be covered later. The procedure of each experiment follows the description of the corresponding acoustic apparatus.

3.3.1 Rigidly terminated pipe

Figures 3.5 and 3.6 illustrate the setup and the apparatus configured for the experiment of Chapter 4 which discusses the complex propagation constant of the air inside a cylindrical pipe. Assuming an inner radius of 17 mm and nominal temperature of 23 °C in the laboratory, the cut-on frequency of the first non-planar wave mode is situated at approximately 5.95 kHz. The out-of-roundness of the pipe is specified as less than 0.7% of the outer diameter of the pipe by the manufacturer, when the outer diameter of the pipe is less than 180 mm. [49] The outer diameter of this pipe is 38 mm.

Figure 3.5 shows the setup photographically, while Fig. 3.6 illustrates a diagram of the overall setup for this experiment. The cylindrical pipe (E) is mainly made of borosilicate glass, to provide the transparency required for LDA measurements of velocity. As shown in Fig. 3.6.(b), a single LDA probe is positioned along the cylindrical pipe. The LDA probe is mounted on an automated mechanical positioning system (Daedal 404300XRMS) in order to measure the axial velocity as a function of axial position. The
accuracy of the positioner is to within 30 µm, which is much smaller than the uncertainty in the measurement with a ruler. The pressure at its antinodal position is monitored by an Endevco pressure sensor 8510B-5 (S/N: 10060) mounted in the stainless-steel end cap. Because the acoustic amplitude can drift over the course of a measurement, the pressure is used as a reference for correcting this drift for the measured axial velocity. The location of the LDA probe is chosen not to be too close to the compression driver in order to avoid the compression driver’s near-field effect, which may affect the validity of the counter-propagating plane wave model in this study.
Figure 3.5. Figure (a) shows a photo of the main pipe where axial velocity is measured. The LDA probe mounted on the mechanical positioner is also shown. The driver section which is attached to the main pipe is shown in Fig. (b).
Figure 3.6. Figure (a) shows the frontal view of the apparatus used for the determination of complex propagation constant of the air inside the apparatus. A compression driver (University Sound 1928BT, A) wired to the output of the power amplifier is connected to the driver section, which comprises two stainless-steel pipes (B and C) and a stainless-steel reducing adapter (D). While one end of a 150 cm long cylindrical pipe (E) made of borosilicate glass is connected to section D as a main pipe section to be excited, the other end is terminated by a stainless-steel end cap where an Endevco pressure sensor is mounted to monitor the pressure during the experiment. All the joints between pieces of apparatus are made by using flanges and quick-connect clamps including apparatus for later experiments. Figure (b) shows the top view of the apparatus. As shown, one LDA is operational to measure axial velocity as a function of axial position.

The data is collected in the following procedure. The seeding particles are introduced into the main pipe connected to the driver section through the farther end of section E from the driver section. The pipe is then terminated by the stainless-steel end
cap. Because the seeding particles are injected only once during the experiment, a sufficient amount is required. The sound field is turned up for at least several seconds after the injection to ensure that the seeding particles are distributed evenly in the pipe. With the sound field activated, the axial velocity is measured over a period of time ranging from 15 seconds to 30 seconds, depending on the number of events captured by the photodetector. In general, the number of events should be at least five-thousand. However, in the case of measuring velocity with less than 5 cm/s, it is ensured that the number is at least ten-thousand. Once the data is stored, the LDA probe is relocated to the next axial location, and the same procedure is repeated until the measurements are finished for all of the axial positions. When the data collection is completed, the sound field is disengaged from the apparatus.

3.3.2 Open-ended pipe

As shown in Fig. 3.7, an almost identical configuration of the pipe to that in the previous experiment is used in the first part of Chapter 5, where the radiation impedance of an open ended pipe is measured. Contrary to the previous pipe, however, the open ended side of the pipe remains unflanged, because the shape of the flange may introduce a complex near-field effect at the open termination. For this experiment, one LDA/BSA system is used to measure the axial velocity as a function of axial position on the center axis of the pipe, while an Endevco pressure sensor 8510B-5 (S/N: 10060) is mounted in the fixture fabricated near the compression driver to monitor the pressure for a reference signal. However, the axial position of the LDA probe is not too close to the termination
lest the LDA probe should detect the scattering which is not accounted for by the counter propagating plane wave model. This setup is shown photographically in Fig. 3.8.

Figure 3.7. The top view of the apparatus used for the measurement of radiation impedance is illustrated. While many features are identical to those of the apparatus shown in Fig. 3.6, section E in this apparatus is unflanged.

Figure 3.8. A photo of the unflanged open-ended pipe. As can be seen, the movable LDA probe is located along the pipe.

The experimental procedure is described as follows. After the smoke particles are introduced inside the pipe, the sound field is driven in the pipe by the compression driver (University Sound 1829BT). Because the pipe is open, the experiment should be performed more swiftly than in other experiments in order to avoid losing the seeding
particles through the open end. At each of sixteen axial locations, the axial velocity is collected as a function of axial position for approximately 15~30 seconds, using a single LDA/BSA system. The pressure is also monitored by an Endevco pressure sensor in the fixture prepared near the driver. When the data collection is completed, the acoustic field is deactivated.

3.3.3 Pipe terminated by a variable RLC load

In the second part of Chapter 5, the pipe is terminated by an RLC load to dissipate power sufficiently enough for the LDA technique to measure against the two other methods, such as the two-microphone method and the direct estimation based on a lumped parameter model. This setup is designed to dissipate more power than the very small dissipation through an open end or a rigid termination, to validate the LDA-based measurement technique for determining power flow. Although the theoretical prediction is unavailable for the power dissipated in this RLC load, the comparison among three experimental methods can provide a check of the LDA technique, showing a good match among the three methods.

As in previous experiments, the overall length of the pipe is kept the same. However, a 150 cm long cylindrical glass pipe is cut into three pieces. The center piece is replaced by a PVC section as illustrated in Fig. 3.9.(e). The inner radii of all three sections are 17.0 mm to replicate a continuous cylindrical pipe with a uniform cross section after they are united. The junctions among three individual sections (E,F, and G) are made using quick-connect clamps and flanges installed on both sides of each section.
Section G is terminated by a variable RLC load in which power is dissipated. In order to simulate the two-microphone method, two Endevco pressure sensors (S/N: 10060 and B15Y) connected to the input channels of pre-amplifiers are mounted in the PVC section (F), as seen in Fig. 3.9.(e). The RLC load also has two microphone holes for direct estimation of power dissipated based on a lumped parameter model. In the implementation of the LDA technique for this experiment, two LDA/BSA systems are utilized. Both LDA probes are located on the center axis of the pipe along section E. The LDA probe nearer to section F is moving axially, whereas the other probe is set stationary to provide the reference signal which will be used to scale the axial velocity measured as a function of axial position.

Figure 3.10 shows the photographs of the apparatus that is fully assembled. Figure 3.10.(a) illustrates the configuration for the LDA technique and the two-microphone method, since the two Endevco pressure sensors are situated in the PVC section. Figure 3.10.(b) shows the setup for the direct estimation of the power dissipated in the RLC load, based on the fact that two pressure sensors are relocated in the load, as discussed above.
Figure 3.9. Figures (a) and (b) show the top view and the interior view of the PVC section marked as section F in Fig. (e). As shown in Fig. (a), the microphone spacing is approximately 5 cm. The RLC load is shown lengthwise in Fig. (c). Two black screws indicate the positions where two microphones will be located for the direct measurement of the power dissipated based on a lumped parameter model. Figure (d) shows the RLC load and the wire mesh screens (mesh #: 40, mesh opening: 381 μm, open area: 36%, and diameter: 19.0 mm) seen from the end cap side. The top view of the apparatus setup is illustrated in Fig. (e).
Figure 3.10. Figure (a) shows the configuration of the apparatus when the LDA technique and the two-microphone method are operational. The two LDA probes are shown in Fig. (a). Figure (b) shows the experimental setup when the direct estimation of power is attempted based on a lumped parameter model.

Figure 3.11 shows the schematic diagram for the interior of the RLC load used in this experiment. The wire mesh screens that provide acoustic resistance are stacked inside of section A. As shown in Fig. 3.9.(d), those wire mesh screens are flush mounted at the tip of section A. The resistance of this load is easily adjusted by changing the number of layers of stacked wire mesh screens. As seen in Fig. 3.11, because of the complexity introduced by the abrupt transition in the cross sectional area between sections B and D, the RLC load cannot be accurately modeled with DeltaE. [50]
The experiment is performed as follows. After the three sections comprised by the main pipe are united, two Endevco pressure sensors are mounted in the microphone fixtures of the PVC section. This enables the experiment to be performed with the two-microphone method simultaneously with the LDA technique. The seeding particles generated from a burnt rope piece, which is made of cotton and polyester, are introduced to the joined pipe using a siphon pump, before the variable RLC load is attached. Since the seeding particles are introduced once, they should be sufficiently dense when injected to last the duration of the measurement. At the initial location, axial velocities and pressures are recorded for 15 seconds to 30 seconds with the start signal given by the master BSA. Once the data is stored, the moving LDA is relocated to the next axial position to be ready for the next measurement at that position. The same procedure is undertaken until the measurement is finished at the last axial location. Sixteen axial positions are used for the LDA technique. To estimate power dissipated in the load

Figure 3.11. A schematic diagram of the RLC load is shown. Its resistance value is adjusted by changing the number of wire mesh screen layers stacked in section A.
directly, two Endevco pressure sensors are relocated in microphone holes in the load, as indicated in Fig. 3.11. Pressures in the load and axial velocities at the sixteenth axial location are recorded. They are then denoted as the measurement at the seventeenth location. With the completion of measurements, the monofrequency sound is turned off. The same experiment is repeated with different number of mesh screen layers at 349 Hz and 417 Hz.

### 3.3.4 RVC sample in an arbitrarily terminated pipe

A microphone-based impedance tube has been widely used to characterize a porous sample. In this section, the LDA technique is used instead of the microphone technique.

Figure 3.12 shows a 119 cm long cylindrical pipe with the inner radius of 17.0 mm that is used in Chapter 6, where the complex propagation constant of the air inside an RVC sample is measured. The RVC sample is wrapped with several layers of Teflon tape in order to avoid providing the air with any major passage around the sample. The sample is then located in position by pushing it with a soft sponge-ended PVC rod to avoid any crack on the sample. Another identical RVC sample is situated near the stainless-steel end cap terminating the pipe. This sample is installed to avoid the rigid boundary condition that could be provided by the stainless-steel end cap. To measure axial velocity in the pipe, one LDA/BSA system is used, along with the use of an Endevco pressure sensor 8510B-5 (S/N: 10060) to monitor the pressure at a fixed position prepared in the wall of the pipe. Figure 3.13 is a photograph of the apparatus illustrated in Fig. 3.12.
Figure 3.12. The top view of the apparatus used in Chapter 6 is shown. As can be seen, the setup is almost identical to the previous setup shown in Fig. 3.6 except a couple of aspects; first, the length of the main pipe (E) is 119 cm, and second, an Endevco pressure sensor that monitors the pressure at a fixed location is not mounted in the end cap. Although it is not illustrated in detail, this pressure sensor connected to a pre-amplifier (Stanford Research SR560) is mounted in the leftmost fixture near the stainless-steel end cap to monitor the pressure for each measurement of axial velocity. Two darker rectangular blocks indicate the RVC samples installed in the pipe. While the one farther from section A is placed to reduce the standing wave ratio, the other one is installed to determine the complex propagation constant of the air inside that.

Figure 3.13. A photo of the apparatus used to characterize an RVC sample. As shown, the white object in the pipe indicates the RVC sample. The other RVC sample is hidden in the flange fitting at the termination of the pipe. Although the mounted pressure sensor is not shown in this figure, the black screw mounted at the leftmost location indicates the position where the pressure sensor is situated in the experiment.
The experiment proceeds as follows. First, plentiful seeding particles are introduced to the pipe for both sides of the sample situated farther from the end cap, in order to avoid additional injection during the experiment. The pipe is then terminated by another RVC sample and the stainless-steel end cap. For even distribution of seeding particles, the sound field is driven in the pipe several seconds after the injection of seeding particles. As many as eighteen measurement positions are selected as for each side of the RVC sample to be characterized. For a reference signal, pressure data is collected using an Endevco piezoresistive transducer (S/N: 10060) mounted in the microphone hole nearest to the stainless-steel end cap. Depending on the number of velocity samples detected by the LDA at each axial position, the duration of each measurement lasts for 15 seconds to 30 seconds. As soon as the measurement of axial velocity at all of the axial positions is completed, the sound source is turned off.

3.3.5 Rigidly terminated pipe with a larger radius

Lastly, the axial propagation constant is investigated in the (1,1) mode of the pipe. Since the cut-on frequency of the (1,1) mode in the pipe used for previous experiments is too high for the compression drivers to provide a stable acoustic signal during the experiment, another cylindrical pipe with a larger radius is used in this experiment.

Figure 3.14 shows a 122 cm long cylindrical pipe that is used. The inner radius of the pipe is 23.3 mm to lower the cut-on frequency of the first non-planar wave mode down to 4.35 kHz at which the compression driver (JBL 2426H) manages to generate a sufficiently large acoustic signal. Because its outer diameter is approximately 50.7 mm,
the out-of-roundness for this pipe is also less than 0.7% of that. As before, the pipe is made of borosilicate glass. One end of the pipe is terminated by a PVC end cap that mounts an Endevco pressure sensor 8510B-5 (S/N: 10060) at an off-centered location via a flange fitting, while the other end is connected to a compression driver. The throat of the driver is blocked halfway by a piece of cardboard box to drive the (1,1) mode more effectively. In order to be operated within the L-shaped box inside section B, the axially moving LDA is mounted on a second linear mechanical positioning system (Daedal 404150XRMS) in a direction perpendicular to the original linear mechanical positioning system. The other LDA probe measures the reference velocity at a fixed location nearer to the compression driver. The detailed experimental procedure will be described in Chapter 7. Figure 3.15 shows photographs of this setup.

Figure 3.14. A diagram of the apparatus used in investigating the (1,1) mode in a cylindrical pipe. A 122 cm long cylindrical pipe with inner radius of 23.3 mm (B) is directly connected to the compression driver (A). The L-shaped box in section B indicates the region where the moving LDA probe collects the axial velocity as a function of axial position at a fixed radial position, and as a function of radial position at a fixed axial position.
Figure 3.15. Figure (a) shows a photo of the apparatus illustrated in Fig. 3.14 is shown. A length of rope is used to tie the pipe to the guiding rail because of the weight of the driver. A blocked compression driver is shown in Fig. (b).
3.3.6 Calibration of Endevco pressure sensor

The calibration of Endevco pressure sensors is critical when the measurement of the power dissipated in a variable RLC load is based on the two-microphone method. However, it may not be important when the pressure value is used to provide the reference signal for some applications. Regardless, the calibration of these sensors is performed.

In order to calibrate the Endevco pressure sensors, two methods can be utilized. The first method is to calibrate the sensor with Bruel & Kjaer type 4228 pistonphone (S/N: 2350128) and a custom-made adapter to the sensor. The pressure sensor output signal is measured with a dynamic signal analyzer (Stanford Research SR785). Since the signal generated by the pistonphone has its peak amplitude approximately at 251 Hz, the averaging method based on the root mean square value is utilized with the flattop window. This setup helps the voltage measured to achieve better precision with the dynamic signal analyzer used for the calibration. This first method can serve as a way of calibrating the magnitude response of the Endevco pressure sensors. The calibration chart suggests that the uncertainty in the calibration of the pistonphone should be less than approximately 0.09 dB with 99% of confidence level which corresponds to $3\sigma$. This uncertainty in relative magnitude of the pistonphone results in 0.35% of uncertainty in the calibration of magnitude with 67% of confidence level corresponding to $\sigma$. The determined magnitude response is assumed to be flat in the frequency range of interest.

The second method of calibration is to perform a comparison calibration between two Endevco pressure sensors mounted in a stainless-steel end cap. This method can be
used only under plane wave assumption in the pipe. The pressure is measured simultaneously by two Endevco sensors mounted in the end cap. The transfer function between two sensors can be measured in the magnitude and phase by the dynamic signal analyzer. The magnitude of the transfer function is then used to calibrate the magnitude response of one Endevco pressure sensor (S/N: B15Y) relative to the Endevco pressure sensor (S/N: 10060) calibrated by use of Bruel & Kjaer type 4228 pistonphone. The phase of the transfer function is used to estimate the sensors’ phase difference which is crucial in the measurement of power.

The calibration factor of the Endevco pressure sensor (S/N: 10060) is \(1.318 \times 10^5\) Pa/V, while that of the other pressure sensor (S/N: B15Y) is \(1.205 \times 10^5\) Pa/V. The uncertainty of calibration is approximately 0.35% for both pressure sensors, based on the uncertainty in the pistonphone. The uncertainty in the measured voltage by a dynamic signal analyzer is negligible compared to that.

### 3.4 Data acquisition system

As described in the previous sections, this research utilizes both the LDA/BSA system and the acoustic apparatus at the same time. Since the measurement of velocity is independent of the measurement of pressure in the time domain, the entire system must be synchronized in order to provide coherent data. In order to achieve the synchronization and control efficiently, the instruments are connected as shown below. For more detailed description of connections, the author strongly recommends Thompson’s dissertation. [48]
As shown in Fig. 3.16, the entire data acquisition system is synchronized with a trigger generator (HP 8111A). A 1 MHz reference signal is fed by the trigger generator into the external reference input of a function generator (HP 3324A), the Burst Spectrum Analyzers (Dantec 57N11), and the trigger input of a data acquisition card (National Instruments AT-MIO-16E-10) via a connector box (National Instruments SCB-68). The function generator produces a sinusoidal output signal which is amplified by a power amplifier (Crown K2) with a proper gain. The output of the power amplifier drives a compression loudspeaker attached to one end of a pipe required to contain a monofrequency sound field.

Figure 3.16. The schematic diagram of equipment used in this study is illustrated. The sinusoid produced by the function generator is fed into the power amplifier. The power amplifier amplifies the input signal for a compression driver generating the sound field in the apparatus which is not shown here. The input channel of each pre-amplifier (i.e., Preamps #1 and #2) is connected to a microphone, and its output channel is fed into the connector box in which the sinusoid generated by the function generator and the trigger signal generated by the trigger generator are shared with both the Burst Spectrum Analyzer (BSA) and the function generator. The computer controls the LDA/BSA systems, the data acquisition via the connector box, and the positioning system for locating a LDA probe. The dynamic signal analyzer can monitor the signal produced by the function generator as well as the signal detected by one of the microphones.
Each Endevco piezoresistive pressure transducer (Endevco 8510B-5) is connected to a pre-amplifier (Stanford Research SR560) with a low pass filter whose cut-off frequency is 30 kHz to ensure that none of the harmonic content in the sound field is discarded. The output of each pre-amplifier is connected to an input channel of the data acquisition card through the connector box. The output from a pre-amplifier is monitored with a dynamic signal analyzer (Stanford Research SR785) in real time to ensure that the acoustic field in the apparatus does not change substantially during the experiment, depending on the application. The ambient temperature and the atmospheric pressure are monitored by a thermometer and a barometer available in the laboratory.

The velocity of a seeding particle passing through the fringe volume formed by two intersecting laser beams emitted from a LDA probe is detected by the photodetector of a LDA. To measure the velocity as a function of axial location, a LDA probe is usually mounted on an automated mechanical linear positioning system (Daedal 404300XRMS) which is controlled by the computer using Compumotor Motion Planner software. A second LDA probe is located at a fixed position to monitor the axial velocity during the experiment, depending on the application. The output of each photodetector is sent to a BSA, which performs the signal processing necessary to extract the velocity data based on the zero-crossing time of the acoustic signal fed by the function generator. This study utilizes Dantec BSA Flow Software to operate and control the BSA/LDA system. The BSA Flow Software collects the pressure data from the pressure sensors connected to the connector box and the velocity data from the LDA system. The collected data are exported into plain text files, and undergo either the transit time weighted FA method or
the FA method to express the axial velocity and the pressure in several harmonic terms as described in the previous section.
Chapter 4

Determination of complex propagation constant and acoustic power in a rigidly terminated pipe

4.1 Introduction

The purpose of this chapter is to determine the complex propagation constant of the air inside a rigidly terminated cylindrical pipe as the first step to establish and investigate the LDA technique. In order to reach the goal, the axial velocity at multiple locations in an air-filled cylindrical pipe is measured using laser Doppler anemometry (LDA). It will be shown that the complex propagation constant, determined by the two-pass curvefitting algorithm of the data to the axial velocity equation developed in Chapter 2, agrees with the prediction based on thermoviscous boundary layer theory given in Swift’s text [18] to within 1\% for both real and imaginary parts on average. A numerical simulation will be introduced as a tool to check the effect of uncertainty in the measured velocity on the measured complex propagation constant.

4.2 Procedures

The experiments comprise two individual parts. The first part is to extract the complex propagation constant using two-pass curvefitting of axial velocity measurements to determine the unknown parameters in Eq. (2.35),

\[
\hat{u}(z) = \left[ \frac{(k-j\alpha)}{\omega \rho_0} \right] A e^{i\theta} e^{-j\theta/2} e^{-\eta/2} \sin \left[ \theta / 2 - j \frac{\eta}{2} - (k-j\alpha)z \right], \quad (2.35)
\]
where $A, \phi, \theta, \eta, k,$ and $\alpha$ are six unknown parameters to be determined. The second part is to check the effect of uncertainty in the measured axial velocity with the LDA/BSA system on the measured complex propagation constant through a numerical simulation.

As discussed in Chapter 3, in the first part of the experiment, the measurements of axial velocity are performed in the main pipe ($r_0 = 17$ mm, $L = 1.5$ m) with one LDA probe. At the same time, a reference signal is monitored by an Endevco pressure sensor 8510B-5 (S/N: 10060) mounted in a stainless-steel end cap terminating the pipe.

The number of axial locations is chosen to be at least thirty-two, and the separation between two adjacent locations, though frequency-dependent, is set to be constant. Because the axial range of the mechanical positioner is limited to 31 cm, the lower frequency bound of 395.6 Hz corresponds to the traversed axial range of $0.36\lambda$, while the upper frequency bound of 1742 Hz corresponds to the traversed axial range of $1.57\lambda$. The upper bound of this frequency range is chosen to avoid measuring small values of axial velocity, since the compression driver used does not effectively excite the apparatus over the upper limit for a long period of time. Within the bounds, most of the experiments are performed with the axial range of about half of a wavelength. However, it is important to ensure that the LDA should actually measure axial velocity on the center axis within the entire axial range, because Eq. (2.35) is based on the assumption that the thermoviscous $h$-function in Eq. (2.29) is negligible on the center axis. As a reminder, Eq. (2.29) is
\[ \dot{u}_z(r,z) = \frac{k}{\omega \rho_0} [1 - h_\nu(r)] (\hat{A} e^{-jxz} - \hat{B} e^{jxz}). \] (2.29)

Figure 4.1 illustrates the magnitude and phase of \([1 - h_\nu(r)]\) as a function of normalized radial position at \(f = 396\) Hz. Because the peak of the magnitude is approximately located at \(2\delta_\nu\) from the wall when \([1 - h_\nu(r)]\) is regarded as a function of radial position, the lowest frequency has the largest thermal penetration depth. It follows that the effect of the function \([1 - h_\nu(r)]\) on the measured axial velocity is the greatest at the lowest frequency in the frequency range chosen in the experiment. As can be seen in Fig. 4.1, however, no such detrimental influence is expected by the possible dislocation of the LDA probe, i.e., 2 mm, from the center axis, as long as the dislocation remains small. Furthermore, even though the trajectory of the LDA probe is not perfectly parallel to the center axis of the pipe, it will have little impact on the axial velocity measured as long as the angle between the center axis of the pipe and the trajectory of the LDA probe is insignificant. Thus, Eq. (2.35) is valid to a good approximation for all frequencies chosen for this experiment.
Figure 4.1. Figures (a) and (c) show the magnitude and phase of the radial-dependent function \([1 - h_r(r)]\) as a function of normalized radial position at \(f = 396\) Hz. Assuming that the radial position of the LDA probe is within 2 mm from the center axis, Figs. (b) and (d) show the magnitude and phase of \([1 - h_r(r)]\) as a function of normalized radial position for the maximum possible error in the radial position of the LDA probe. Note that the magnitude error due to \([1 - h_r(r)]\) is within 0.01% from unity, and that phase error is on the order of \(10^{-58}\).

In order to reduce the effect of the variation in the acoustic field due to the time consumed by the whole set of measurements, the axial velocity can be scaled as

\[
\hat{u}_s(z_i) = \hat{u}(z_i) \left| \hat{p}_{\text{END}}(1) / \hat{p}_{\text{END}}(i) \right|
\]

(4.1)
where \( \hat{u}_s \), \( \hat{u} \), \( \hat{p}_{\text{END}} \), \( z_i \), and \( i \) are the scaled axial velocity, the measured axial velocity with the LDA probe, the measured pressure with the pressure sensor mounted in the end cap, the \( i \)th axial position, and the \( i \)th measurement, respectively. The axial position and the scaled axial velocity are then treated as the independent and the dependent variables for the two-pass curvefitting algorithm. In the analysis, the scaled axial velocity is usually treated as the axial velocity. For the purpose of curvefitting, a rigid boundary condition is not assumed, and thus the resulting number of unknown parameters is six, as described previously.

In the second part, the same experimental parameters as in the first part are used for numerical simulation. Preliminarily, the ideal axial velocity is determined based on both the boundary conditions and the thermoviscous boundary layer theory. In order to implement uncertainties into the ideal axial velocity, it is assumed that the magnitude of axial velocity is larger than 15.0 cm/s to render the magnitude uncertainty approximately 0.3%. Along with this assumption, the phase uncertainty is assumed to be approximately 5.0% of the time interval used, i.e., 10.667 \( \mu \text{s} \), as discussed in Chapter 3. Based on these uncertainties, Gaussian-distributed uncertainties are then added to both the magnitude and phase of the ideal axial velocity according to the uncertainty in measuring velocity using the LDA/BSA system. Since the complex propagation constant of the air inside a cylindrical pipe is known analytically, first-pass curvefitting is used to estimate the complex pressure amplitude coefficients \( \hat{A} \) and \( \hat{B} \) based on the theoretical value. The estimated \( \hat{A} \) and \( \hat{B} \) from the first-pass will be used as the initial guesses for the second-pass curvefitting, which practically determines the complex propagation constant of the
air inside the pipe through adjusting all six parameters until finding the best solution to the fit. However, no assumption will be imposed on the second-pass curvefitting except the constraint $|\hat{R}| \leq 1$. This simulation is continued until the mean value and the standard deviation of the determined parameters reach convergence. The number of simulations chosen is two-thousand, and the convergence in values is attained fairly quickly. Finally, the mean value of the complex propagation constant determined based on the numerical simulation is compared with the predicted value based on the thermoviscous boundary layer theory.

### 4.3 Results

Fitted to Eq. (2.35), the axial velocity field measured with a LDA/BSA system is used to determine the axial velocity expected on the center axis for the entire range of axial position. In the following examples shown in Figs 4.2 and 4.3, data measured at different frequencies are presented. Even though it is observed that the uncertainty in the phase of the velocity is proportional to frequency, the purpose of this choice is not to illustrate this phenomenon, but to show examples of good and bad phase data.

Figure 4.2 shows an example of the magnitude and phase of axial velocity measured at 721.5 Hz as a function of axial position by LDA and the fitted results. As can be seen in Fig. 4.2, the measured magnitude and phase generally tend to lie on the fitted curve. However, Fig. 4.2.(d) shows a discrepancy in phase between the measured velocity and the fitted curve near a velocity nodal position. The magnitude does not exhibit such discrepancy in Fig. 4.2.(b). In fact, the root mean square deviation from the fit is 1.2
mm/s, corresponding to 0.1% of the root mean square value, for the magnitude and 1.19 degrees for the phase, when data near velocity nodal positions are excluded. This deviation of phase is much smaller than 5.42 degrees from the case that all data points are considered, whereas the deviation in magnitude remains the same. Thus, Fig. 4.2 suggests that the error is more evident in phase than in magnitude, especially when the measurement is performed near a velocity nodal position.

Figure 4.2. Figure (a) shows the magnitude of axial velocity measured (blue circles) as a function of axial position at 721.5 Hz by LDA and the curve based on the two-pass curvefitting with the data (black dashed line), while Fig. (c) shows the phase of the data and the fitted curve as a function of axial position. Note that Fig. (d) closely illustrates the large discrepancy in phase between the measured quantity and the fitted curve near a velocity nodal position, while the magnitude of the measured axial velocity tracks the tendency shown by the fitted curve in Fig. (b).

---

1 Because these values of deviation seem large compared to the uncertainty of LDA in Doppler frequency and phase, it is possible that another source of uncertainty may be introduced. However, the exact reason for this is unknown.
Data for 1198 Hz is shown in Fig. 4.3. In this case, the phase data exhibits a more visible deviation from the fitted curve. It is interesting to note that the magnitude is relatively uncontaminated by noise, even while the phase substantially displays the discrepancy between the fitted curve and the measured data regardless of location. This tendency is supported by the root mean square deviation of the measured axial velocity from the fit. It is 1.2 mm/s, which corresponds to 0.25% of the root mean square value, for the magnitude and 3.38 degrees for the phase when the data near velocity nodal positions are excluded. This deviation becomes 1.2 mm/s for the magnitude and 4.52 degrees for the phase when all the data points are considered. Since the deviation is relatively unaffected by inclusion of positions near velocity nodes, the phase in this data is considered noisy in general. As can be seen in Fig. 4.3, the phase data cannot perfectly lie on any fitted curve, because of its noisy behavior. The data presented in Figs. 4.2 and 4.3 may show the possibility that the phase is more susceptible to noise than the magnitude in operating the LDA/BSA system. Hence, the phase of axial velocity is hereafter disregarded in the second-pass curvefitting procedure regardless of the application performed, although there might be a possibility to acquire a set of clean phase data. This scheme is validated qualitatively based on the result of the first part of the experiment.
Figure 4.3. Figure (a) shows the magnitude of the measured axial velocity (blue circles) as a function of axial position at 1198 Hz and its fitted curve (black dashed line) based on the two-pass curvefitting algorithm. Figure (b) shows the phase of the measured axial velocity and the fitted curve as a function of axial position. As opposed to Fig. 4.2, it is evident that noise is more introduced to the phase of axial velocity.

This idea of discarding the measured phase requires the second-pass curvefitting algorithm to be modified. As discussed in Chapter 2, the first- and second-pass of the curvefitting algorithm are based on the Eq. (2.32) which is

$$
\min \left[ \sum_{i=1}^{N} |\hat{u}_i - \hat{u}(z_i)|^2 \right],
$$

(2.32)
where \( \hat{u}(z_i) \) is the axial velocity at \( z = z_i \) based on Eq. (2.35). Discarding the phase in Eq. (2.32) yields

\[
\min \left[ \sum_{i=1}^{N} \| \hat{u}_i - \hat{u}(z_i) \|^2 \right].
\]  

(4.2)

It should be noted that the first-pass curvefitting algorithm is not altered by discarding the phase in the second-pass curvefitting algorithm, because the first-pass curvefitting algorithm is necessary for generating initial guesses of unknown parameters in the second-pass curvefitting algorithm, as discussed previously.

Figure 4.4 illustrates the complex propagation constant of the air inside the pipe by means of the real and imaginary part (or absorption coefficient) as a function of frequency. Compared to the theoretical value, the measured values using the two-pass curvefitting algorithm based on two Eqs. (2.32) and (4.2) are presented. Because each experiment is performed at different temperature and atmospheric pressure, the mean value of temperature (23.2 °C with sample standard deviation \( s = 5.32\% \)) and that of atmospheric pressure (9.75×10\(^4\) Pa, \( s = 0.73\% \)) are used to provide the theoretical values in Fig. 4.4. Another reason for using the mean value of temperature and that of atmospheric pressure is that it is difficult to scale the axial velocity measured at each condition into the axial velocity expected at the mean value of temperature and of atmospheric pressure. Based on this strategy, the measured complex propagation constant is obtained as the ratio of the measured value to the theoretical value at each temperature and atmospheric pressure where each experiment is performed. The ratio is then multiplied to the theoretical value calculated at the mean value of temperature and that of atmospheric pressure to represent the measured value in Fig. 4.4. As can be seen, the real
parts of the measured complex propagation constants based on both expressions deviate little from the prediction. However, while the measured absorption coefficient based on Eq. (4.2) exhibits some amount of error around the theoretical value, the quantity based on Eq. (2.32) clearly presents a substantial amount of error. To illustrate the amount of error, the relative error can be calculated. This relative error is the error in the measured quantity relative to the theoretical quantity. Qualitatively, the worst relative error of the real part is -0.36% for the measurement based on Eq. (4.2), whereas it is -8.65% for the measurement based on Eq. (2.32). Likewise, the worst relative error of the absorption coefficient is +4.29% for the measurement based on Eq. (4.2), while it is -100% for the measurement based on Eq. (2.32). Meanwhile, the root mean square error for the real part is 0.17% for the measurement based on Eq. (4.2) and 1.75% for that based on Eq. (2.32). Insofar as the absorption coefficient is concerned, it is 2.00% for the measurement based on Eq. (4.2) and 53.5% for that based on Eq. (2.32). Thus, Fig. 4.4 suggests that the second-pass curvefitting based on only the magnitude of axial velocity performs better than that based on both magnitude and phase. Therefore, the second-pass curvefitting algorithm is always based on Eq. (4.2), hereafter.
Figure 4.4. Figure (a) shows the real part of the complex propagation constant of the air inside the apparatus based on the predicted values from the thermoviscous boundary layer theory for a circular pore (solid line), the measured values using the LDA and the modified second-pass curvefitting algorithm given by Eq. (4.2) (blue circles), and the measured values using the LDA and the unmodified second-pass curvefitting algorithm given by Eq. (2.32) as a function of frequency (red squares). In Fig. (b), the corresponding absorption coefficient is shown as a function of frequency for all quantities.

Figure 4.5 illustrates the relative error of the measured complex propagation constant compared to the theoretical value as a function of frequency in both real part and
absorption coefficient. In Fig. 4.5, only the measured value based on Eq. (4.2) is shown, because inclusion of the measured value based on Eq. (2.32) will increase the range of ordinate. Since the better performance of Eq. (4.2) is already shown in Fig. 4.4, this exclusion is fairly reasonable. Although the measured absorption coefficient is fairly close to the theoretical value, the measured real part shows better agreement with the theoretical value. As previously discussed, the root mean square error of the measured value based on Eq. (4.2) is 0.17\% for the real part, and 2.00\% for the absorption coefficient, respectively. The larger error in the absorption coefficient may result from the difficulty in determining the relatively small absorption coefficient caused by the weakly absorptive mechanism. Contrary to that, in a low loss system such as the apparatus used in this study, the real part can be approximated to $\frac{2\pi}{\lambda}$, where $\lambda$ is the wavelength in a free space. Because of this, the sine function in Eq. (2.35), which the curvefitting algorithm relies on, can detect the real part easily if the traversed axial range of the LDA probe is larger than half of a wavelength in a standing wave pipe. Since the number of experiments performed is small, the result may not be fully assessed from a statistical standpoint. Nonetheless, along with the root mean square error, the mean value of the relative error for the real part and the absorption coefficient is -0.11\% and +0.26\%, which indicates that the complex propagation constant of the air inside the pipe is well determined using this LDA technique.
Figure 4.5. The relative error of the measured value is shown in the real part and the absorption coefficient in Figs. (a) and (b), respectively. Note that the range of ordinate in Fig. (a) is within 0.01, while the range of that in Fig. (b) is within 0.1.

Figure 4.6 shows the complex propagation constant determined by the numerical simulation as a function of the traversed axial range divided by the wavelength. For comparison, the previously measured complex propagation constant is also presented as a function of the traversed axial range divided by the wavelength. As described in the
previous section, each run of the simulation is based on the two-pass curvefitting algorithm which is applied to the artificially generated noisy axial velocity data. The entire numerical simulation comprises two-thousand runs of each simulation.

The numerical simulation is performed at the lowest \( f = 395.6 \) Hz and the highest frequency \( f = 1742 \) Hz of the experimental frequencies. Based on the assumption as to the phase uncertainty in the measured velocity with LDA, the lowest frequency is expected to show the narrowest uncertainty range, while the highest frequency is expected to present the widest uncertainty range. However, the predicted uncertainty of simulation at the lowest frequency is sometimes larger than the predicted uncertainty of simulation at the highest frequency, when the traversed axial range is less than half of a wavelength. This may either indicate that the two-pass curvefitting algorithm performs poorly below that boundary, or result from the fact that the constraint that \( \left| \hat{R} \right| \leq 1 \) is used. Actually, the second-pass curvefitting algorithm using that constraint tends to force the reflection coefficient to become unity when the magnitude of the reflection coefficient is very close to unity. Since the stainless-steel end cap used is assumed to provide the rigid boundary condition with the apparatus, there is possibility that this artifact is present in the measured and simulated values. As can be seen in Fig. 4.6.(b), the biased mean value from the numerical simulation may be caused by this artifact, because this biased mean value disappears when the numerical simulation is performed in a non-rigidly terminated pipe. However, the mean value of absorption coefficient based on the simulation appears to converge, and the uncertainty decreases as the normalized traversed axial range approaches and exceeds one. Contrary to the absorption coefficient, it is suggested that
the real part of the complex propagation constant is fairly well determined from the numerical simulation for even small abscissa values, and that the mean value rapidly converges to the real part of the theoretical complex propagation constant given by the thermoviscous boundary layer theory. The uncertainty for the real part is by far smaller than that of the absorption coefficient.

Figure 4.6 also shows that the measured absorption constant is well within the expected bounds of uncertainty based on the numerical simulation, whereas the real part of complex propagation constant appear to exceed the range of uncertainty. Although the reason for the excessive real part is unknown, overall agreement of the real part of the complex propagation constant with the ideal value suggests that the quality of the measured real part is good. Hence, Fig. 4.6 implies that the LDA technique with the modified two-pass curvefitting algorithm determines the complex propagation constant of the air in a cylindrical pipe quite well based on experiments performed in this dissertation. According to the numerical simulation, Fig. 4.6 also suggests that the LDA technique may determine even better if the ratio of the traversed axial range to the wavelength is sufficiently larger than one. Furthermore, this LDA technique can be applied to the system with an unknown complex propagation constant which needs to be determined, although the scheme in the use of the LDA technique would be more sophisticated.
Figure 4.6. Figures (a) and (b) show the real part and the absorption coefficient of the complex propagation constant of the air inside the pipe as a function of the ratio of the traversed axial range to the wavelength for both the measured and predicted values. The predicted value is based on the numerical simulation performed on artificially generated noisy axial velocity. Note that both mean value and standard deviation are presented for the numerical simulation.
4.4 Further discussion

In the previous section, it is shown that the LDA technique successfully determines the complex propagation constant of the air in a standing wave pipe to within 2.00% of the root mean square error for the absorption coefficient, and to within 0.17% of that for the real part of the complex propagation constant.

In this section, the method is applied to determining the power flow as a function of axial position in the rigidly terminated pipe. A key element of this method is extracting the acoustic pressure from measurements of the axial velocity. Based on the extracted pressure and the measured axial velocity, the power flow can be calculated as a function of axial position. This power flow is then compared to the value from the simulation based on DeltaE [50], which is widely used to simulate a rich variety of thermoacoustic applications. Rather than model the compression driver in DeltaE, the pressure measured with the pressure sensor mounted in the end cap is used as one of targeted values for the simulation, along with the default rigid boundary condition in DeltaE.

Analogous to using Euler’s equation to obtain the axial velocity from the measured pressure, the equation of continuity based on the two assumptions, such as the linear approximation and the plane wave mode assumption, can be used to extract the pressure from the measured axial velocity. As shown in Eq. (2.25),

$$\hat{p} = \frac{j\rho_0 v_0^2}{\omega} \frac{(1 - f_v)}{[1 + (\gamma - 1) f_x][1 - h_v(r)]} \frac{\hat{c}u_z}{\hat{c}z},$$  \hspace{1cm} (2.25)

the equation of continuity involves only the axial velocity to estimate the pressure, thus it is obvious that the contaminant in the measured axial velocity with LDA would affect the
pressure as well. It is discussed, however, that the LDA probe is insensitive to the velocity normal to the fringes. Therefore, the measured axial velocity with the LDA technique can be considered uncontaminated by the radial velocity. Even though it is common to utilize Euler’s equation to estimate the axial velocity in the midpoint of two microphones, there are in fact two options. First, the axial velocity is estimated in the midpoint of two microphones as is used by many. Second, the axial velocity is estimated from the pressure field reconstructed for the entire axial range from a curvefitting of the measured pressure at two positions to a model equation. These two schemes are demonstrated in this section.

Analogous to the first option, the first scheme estimates the pressure in the center of two axial positions based on the finite difference equations as,

\[
\begin{align*}
  z \left[ n + \frac{1}{2} \right] &= \frac{1}{2} \left\{ z[n] + z[n+1] \right\}, \\
  \hat{u} \left[ n + \frac{1}{2} \right] &= \frac{1}{2} \left\{ \hat{u}[n] + \hat{u}[n+1] \right\}, \\
  \hat{p} \left[ n + \frac{1}{2} \right] &= j \rho_0 c_0^2 \frac{1 - f_v}{\omega \left( \gamma - 1 \right) f_v \frac{\hat{u}[n+1] - \hat{u}[n]}{z[n+1] - z[n]}},
\end{align*}
\]

where \( z, n, \) and \( n + 1/2 \) are axial position, the \( n \)th measurement location, and the midpoint of the \( n \)th and the \( n+1 \)th measurement locations. For the second scheme analogous to the second option, the pressure is computed as follows. First, the curvefitting algorithm is applied to the measured axial velocity. Next, the equation of continuity is used on the rebuilt axial velocity for the entire axial range to estimate the pressure for the entire axial range. The major difference between two schemes is that the first scheme relies on the actual data, while the second scheme uses the fitted curve of the actual data to Eq. (2.35).
Thus, the first scheme is denoted as ‘local difference method,’ whereas the second scheme is denoted as ‘curvefitting-oriented method.’

The comparison between two methods in estimating the pressure is presented in Fig. 4.7. Although the data for Fig. 4.7 is the same used in Fig. 4.2, there is more jitter in the phase of the pressure based on the local difference method than in the measured axial velocity. The pressure based on the curvefitting-oriented method appears to be the best-fit to the pressure data based on the local difference method. It is interesting to note that the magnitude of the pressure estimated by the local difference method suffers from a large amount of errors around the pressure antinodal positions which correspond to the nodal positions of the axial velocity. This poor performance of the local difference method may result from the fact that the separation of two adjacent positions is not optimal for this method. Indeed, Jang and Ih [2] suggested that the uncertainty in measurements would be minimized when the separation is equivalent to a quarter of a wavelength. In this study, this condition was not satisfied due to the limited range of the mechanical positioner used. However, this result alternatively suggests that the curvefitting-oriented method is preferable for estimating the pressure, specifically near the pressure antinodal positions, in situations that an apparatus is not easily modifiable.
Figure 4.7. Figures (a) and (b) show the pressure estimated from the axial velocity. While the local difference method (blue circles) is based on Eqs. (4.3), (4.4), and (4.5), the curvefitting-oriented method (black solid line) is based on Eq. (2.25) with the curve-fitted axial velocity. Figures are generated from the same data set used in Fig. 4.2.

Figure 4.8 shows the acoustic power in a rigidly terminated pipe plotted as a function of axial position, based on three methods, such as the local difference method, the curvefitting-oriented method, and DeltaE. Calculation of power is based on Eq. (2.37) can be expressed as,

$$\hat{W}(z) = \frac{1}{2} \Re \left\{ \hat{\rho} \left( S \left[ 1 - f_c \right] \hat{u} \right) \right\}. \quad (2.37)$$

Thus, the power based on the local difference method is calculated from combining the pressure obtained from this method with the measured axial velocity, while the power...
Based on the curvefitting-oriented method is calculated from the curve-fitted axial velocity and the pressure based on this axial velocity. For DeltaE, the volume velocity and pressure are exported into text files to compute the power.

As discussed in Chapter 2, the main mechanism for the loss along the pipe is the thermoviscous boundary layers. As can be seen in Fig. 4.8, the acoustic power calculated from the curvefitting-oriented method is closer to the prediction given by the simulated result of DeltaE, although some amount of systematic error is present. This systematic error may result from two facts. First, the curvefitting-oriented method uses the measured complex propagation constant, while the DeltaE simulation uses the theoretical complex propagation constant. Second, the DeltaE simulation model attains a much larger acoustic impedance at the termination which achieves a much smaller axial velocity at the rigid termination than the curvefitting-oriented method. By default, DeltaE tends to pursue the zero admittance, which corresponds to the infinite acoustic impedance, for a rigid boundary condition. Unlike the curvefitting-oriented method, the acoustic power values based on the local difference method are scattered everywhere from negative values to positive values. This result may be caused by the slight phase jitter in the pressure shown in Fig. 4.7, and shows the importance of the phase information in the measurement of acoustic power, especially for the system that dissipates a small amount of acoustic power. As discussed previously, this result is also affected by the separation of two adjacent positions, because the measurements of power flow performed by Fusco et al. [28] were successful when the minimum separation was 7.14% of the wavelength chosen. For the data shown in Fig. 4.8, the separation is less than 1.7% of the wavelength.
Figure 4.8. The acoustic power is illustrated as a function of axial position using three methods. The local difference method is based on the actual data taken and the equation of continuity, while the curvefitting-oriented method relies on the fitted curve to the actual data taken and the equation of continuity. The DeltaE simulation model is based on the measured pressure with an Endevco pressure sensor mounted in the end cap.

For a close look at the power calculated from DeltaE and the curvefitting-oriented method, the power is investigated through the power dissipation per unit surface area \( \dot{e} \) in Eq. (2.38) expressed as,

\[
\dot{e} = \frac{1}{4} \frac{|\hat{n}|^2}{\rho c_0^2} \delta_k (\gamma - 1) \omega + \frac{1}{4} \frac{\rho_0^2}{|\hat{n}|^2} \delta_k \omega. \tag{2.38}
\]

Based on Eq. (2.38), Fig. 4.9 illustrates the power dissipation per unit surface area as a function of axial position for both the DeltaE simulation and the curvefitting-oriented method. As shown in Fig. 4.9, the value from the curvefitting-oriented method is quite
close to that from the DeltaE simulation. However, one plot is slightly misaligned in axial position, compared to the other plot. This phenomenon may result from the same causes for the difference in the acoustic power between the curvefitting-oriented method and the DeltaE simulation, as discussed in Fig. 4.8.

Figure 4.9. Power dissipation per unit surface area is plotted as a function of axial position, based on the DeltaE simulation and the curvefitting-oriented method. Note that two curves are slightly out of phase.

Implemented into the target impedance of the DeltaE simulation, the measured acoustic impedance with the LDA technique produces the results shown in Figs. 4.10 and 4.11. In Fig. 4.10, the measured values based on the local difference method are not presented, because the curvefitting-oriented method has clearly shown a better performance than that. Figure 4.10 shows that the correction of the target impedance in the
DeltaE simulation results in an almost perfect agreement between the DeltaE simulation and the curvefitting-oriented method. The slight difference shown is caused by the fact that the curvefitting-oriented method uses the measured complex propagation constant while the DeltaE simulation uses the value given by the thermoviscous boundary layer theory.

Figure 4.11 also illustrates the power dissipation per unit surface area as a function of axial position after the correction of impedance. As can be seen in Fig. 4.11, the correction not only resolves the slight misalignment visible in Fig. 4.9, but also achieves a very good agreement to within 0.8% between the DeltaE simulation and the curvefitting-oriented method in the axial range shown in Fig. 4.11.

![Graph showing acoustic power as a function of axial position](image-url)

Figure 4.10. Acoustic power shown as a function of axial position in Fig. 4.8 is illustrated after the correction of the target impedance in the DeltaE simulation. The slight difference between two curves is due to the use of the measured propagation constant for the curvefitting-oriented method.
Figure 4.11. Power dissipation per unit surface area is shown as a function of axial position as in Fig. 4.9. Figure 4.11 illustrates a great agreement between two methods after the correction of the target impedance in the DeltaE simulation. The discrepancy between two methods is less than only 0.8%.

4.5 Conclusion

In this chapter, the two-pass curvefitting algorithm is modified to employ only the magnitude of axial velocity measured with LDA in the second-pass curvefitting. The LDA technique with the modified curvefitting algorithm shows that it can detect the complex propagation constant of the air inside a cylindrical pipe with fairly good precision of -0.11% and +0.26% on average for the real and imaginary part of the complex propagation constant of the air inside the pipe. The root mean square error of the complex propagation constant is 0.17% for the real part and 2.00% for the imaginary part.
It is also shown that the uncertainty in the measurement performed is within the bounds of the uncertainty predicted by the numerical simulation implemented with assumed uncertainties in both magnitude and phase of LDA. Based on the experiment and the numerical simulation, the complex propagation constant of the air inside the pipe can be considered to follow the theoretical prediction given by Rott’s theory.

Later, two methods are introduced to estimate the pressure from the measured axial velocity. The curvefitting-oriented method provides not only less noisy pressure than the local difference method, but an extremely good result in estimating the acoustic power. This shows that the LDA technique has a flexibility that allows it to be adapted to any measurement situation without modifying an apparatus accordingly, especially when the wavelength is much larger compared to the spacing of two adjacent positions. The result is also supported by the DeltaE simulation. The agreement between the DeltaE simulation and the curvefitting-oriented method is reasonably good with a small discrepancy when DeltaE’s default setting is used, and it is almost perfect when the measured impedance of the termination with the LDA technique is implemented into the DeltaE simulation.

This experiment is a good example of experiment that the LDA technique is suitable for, because it is not only the experiment that cannot be handled by any variant of the two-microphone method, but also it needs only overall ratio of the acoustic field in the apparatus. In this type of experiment, the LDA technique is one of the best measurement techniques, because the uncertainty due to the calibration of sensors can be avoided.
Chapter 5

Validation of the LDA technique through the determination of radiation impedance of an open pipe and power dissipated in a variable RLC load

5.1 Introduction

The purpose of this chapter is to further validate the LDA technique. To accomplish this goal, the technique was applied not only to the measurement of the radiation impedance of an open ended pipe, but also to that of the power dissipated in an RLC load. In the first part of the experiment, the LDA technique is tested against the analytical solutions based on the Wiener-Hopf technique. [22-24] In the second part of the experiment, the power dissipated in a variable RLC load is measured using three different methods: the LDA technique, the two-microphone method, and the direct evaluation based on a lumped parameter model.

5.2 Procedures

The main purpose of the experiment performed in this chapter is to validate the LDA technique proposed in the previous chapter. This validation may result in using this technique as an alternative method of determining the power flow in a system. To simplify the validation, the LDA technique is tested against two systems which are well understood from the standpoints of both experiment and theory. As a starting point, the radiation impedance of an open-ended cylindrical pipe with non-zero wall thickness is
examined. The ability of determining the radiation impedance in a low-dissipation system would be an indicator showing the accuracy and the precision of the LDA technique. The second experiment is performed in a cylindrical pipe terminated by a variable RLC load, based on a lumped parameter model. Rather than the rigidly terminated pipe in Chapter 4 and the open-ended pipe in the first part of the experiment in which the dissipated power is low, the RLC load provides values of dissipated power up to tens of milliwatts. The detailed description of both systems is provided in Chapter 3.

In the first part of the experiment, for the purpose of minimizing the effect of elapsed time in the acoustic field during measurements, the measured axial velocity with the LDA probe is scaled as,

$$\hat{u}_s(z_i) = \hat{u}(z_i) \left| \hat{p}_{\text{FIXED}}(1) / \hat{p}_{\text{FIXED}}(i) \right|, \quad (5.1)$$

where $\hat{u}_s$, $\hat{u}$, $\hat{p}_{\text{FIXED}}$, $z_i$, and $i$ are the scaled axial velocity, the measured axial velocity, the measured pressure at a fixed location, the $i$th axial position, and the $i$th measurement, respectively.

The resulting scaled axial velocity is fitted to the model equation (i.e., Eq. (2.35)) by the two-pass curvefitting algorithm. The number of unknown parameters is selected to be four, because it is assumed that the complex propagation constant is that predicted by the thermoviscous boundary layer theory. Assuming that the reduced number of unknown parameters requires less data to achieve an acceptable result, the number of data points taken is reduced to sixteen from thirty-two.
In the second part of the experiment, the power dissipated in a variable RLC load is measured with the LDA technique, compared to the value predicted by a lumped parameter model as well as that measured with the two-microphone method.

As described previously, because the number of pressure sensors is limited in the laboratory, two LDA/BSA systems are utilized to implement the LDA technique. Based on this, in order to minimize the effect of time consumed during the experiment, the measured data are scaled with the axial velocity $\hat{u}_{\text{FIXED}}$, which is measured with the stationary LDA probe. Expressed into equations, this process can be written as,

$$
\begin{bmatrix}
\hat{u}_s(z_i) \\
\hat{p}_1(i) \\
\hat{p}_2(i)
\end{bmatrix} =
\begin{bmatrix}
\hat{u}_{\text{FIXED}}(1)/\hat{u}_{\text{FIXED}}(i) & \hat{p}_{1,m}(i) \\
\hat{p}_{2,m}(i)
\end{bmatrix},
$$

(5.2)

where the quantities in the brackets on the left sides and on the right sides of Eqs. (5.2) and (5.3) are the scaled and the measured values, respectively. Additionally, 1, 17, and $i$ in parentheses are the first, the seventeenth, and the $i$th measurement, respectively. $\hat{p}_1$ and $\hat{p}_2$ are two measured pressure values for the two-microphone method, while $\hat{p}_0$ and $\hat{p}_C$ are the measured pressure values at the termination and in the volume of the RLC load for a direct estimation. The subscript $m$ on the right sides indicates the measured values.

Once all quantities are scaled, the axial location $z_i$ and the scaled axial velocity $\hat{u}_s$ are the independent and dependent variables for the two-pass curvefitting algorithm. The resulting coefficients $\hat{A}$ and $\hat{B}$ are inserted into Eq. (2.42),
\[
\dot{W}_{LOAD} = \frac{1}{2} \Re \left\{ \left( A + \tilde{B} \right) \left( \frac{k}{\omega \rho_0} S [1 - f_v \left( \tilde{A} - \tilde{B} \right)] \right) \right\},
\]

(2.42)

in order to estimate the power dissipated in the RLC load.

For the two-microphone method, the pressures \( \hat{p}_1 \) and \( \hat{p}_2 \) are used as two dependent values corresponding to the axial locations of the first microphone and of the second microphone, based on Eq. (2.31),

\[
\begin{bmatrix}
\hat{A} \\
\hat{B}
\end{bmatrix} =
\begin{bmatrix}
e^{-j\alpha z_1} \quad e^{j\alpha z_2}
\end{bmatrix}^{-1}
\begin{bmatrix}
\hat{p}_1(z_1) \\
\hat{p}_2(z_2)
\end{bmatrix}.
\]

(2.31)

Equation (2.31) is the basis of the two-microphone method, as described in Chapter 2. The coefficients \( \hat{A} \) and \( \hat{B} \) are then substituted into Eq. (2.42) to yield the value of power dissipated in the RLC load based on the two-microphone method.

Based on \( \hat{p}_b \) and \( \hat{p}_c \), the power dissipated in the RLC load is directly calculated from Eq. (2.43), which is based on a lumped parameter model,

\[
\dot{W}_{LOAD} = \frac{\omega V}{2 \gamma P_0} \Im \left\{ \hat{p}_b \hat{p}_c^* \right\}.
\]

(2.43)

In order to validate the LDA technique for a wide range in the amount of power dissipated, the second part of the experiment is performed at two different frequencies, 349 Hz and 417 Hz, and with the three different numbers of layers of wire mesh screen.
5.3 Results

5.3.1 Experiment in an open-ended pipe

The determination of the radiation impedance at the open termination is performed from 108 Hz to 2887 Hz. Higher frequencies than 2887 Hz are not tested, because of the difficulty in measuring the small axial velocity generated by the compression driver used in this experiment. Based on the previous studies [22-24], frequency is converted into non-dimensional frequency involving the inner radius of the pipe. As a consequence, 108 Hz corresponds to the non-dimensional frequency \( k_o a = 0.033 \), and 2887 Hz corresponds to \( k_o a = 0.895 \) in the pipe used. To be compared with the analytic solutions, the radiation impedance is normalized by \( \rho_o c_o S \), where \( S \) is the cross-sectional area of the pipe, at the temperature and atmospheric pressure of each measurement. This normalization conveniently leads to expressing the radiation impedance in a non-dimensional form.

Figure 5.1 shows the real and imaginary parts of the normalized radiation impedance measured by the LDA technique, compared to three analytical solutions. They are the solution based on the infinite baffle, the solution of an unflanged pipe with zero-wall thickness based on Levine and Schwinger’s publication, and the solution of an unflanged pipe with non-zero wall thickness based on Jones’ and Ando’s publications. Because Ando’s study is based on Jones’ work, they are grouped together. Among the three analytic solutions, the two solutions with unflanged pipes use the Wiener-Hopf technique. As shown in Fig. 5.1, the result of the experiment is placed on the curves predicted by Ando and Jones’s and Levine and Schwinger’s work. It is clearly apart from the flanged
pipe in an infinite baffle whose normalized radiation impedance is the largest compared to two other theoretical solutions. In Fig. 5.1, the discrepancy between Ando and Jones’s prediction and Levine and Schwinger’s prediction is little noticeable below $k_0a = 1.000$ for both the real and imaginary parts of the normalized radiation impedance. This small difference is caused by the ratio of the inner radius to the outer radius of the pipe chosen in this experiment. If the pipe had a smaller ratio than the current pipe’s ratio of 0.895, the difference would be more noticeable than for the current experiment. [51,52]
Figure 5.1. Figures (a) and (b) illustrate the real part and the imaginary part of the normalized radiation impedance determined by the LDA technique. The figures also contain the predicted values based on three analytic solutions, such as the infinite baffle, Ando and Jones’s work, and Levine and Schwinger’s work. The abscissa values for the experiment and Ando and Jones’s prediction are scaled to force the non-dimensional frequency to be incorporated with inner radius $a$. 
Figure 5.2 shows the error in measured values relative to the predicted values based on the analytic solution of Ando and Jones’s as a function of non-dimensional frequency $k_\omega a$. As can be seen, the worst relative error for the real part reaches 250%, while it reaches approximately 40% for the imaginary part. Those values are specifically located at low $k_\omega a$ values. As shown in Fig. 5.1, this large error results from the fact that the predicted values of the normalized radiation impedance are quite small at those $k_\omega a$ values. Under these circumstances, a slight error in the measure value causes a large relative error in the measured value compared to the analytic solution. However, the measured values at $k_\omega a$, higher than approximately 0.25, exhibit reasonable amount of relative error in the measured values. This suggests that the quality of the measured values could have been improved if the experiment had been performed at $k_\omega a$ higher than 0.25 in both the real and imaginary parts of the normalized radiation impedance.

Figure 5.2 also suggests that the average error of the result relative to the analytic solution of Ando and Jones’s is 1.57% for the real part, and is 4.35% for the imaginary part. Although the relative errors are large at low $k_\omega a$, their effects on the averaged error are cancelled out, specifically for the real part of the normalized radiation impedance. The quality of the result, specifically at low $k_\omega a$, may be affected by the artifact introduced by the constraint that $|\hat{R}| \leq 1$, as discussed in the previous chapter, because the magnitude of the reflection coefficient would be very close to unity. In addition, Ando’s analysis may have room for improvement if the concept of thermoviscous boundary
layers is implemented in the study, in lieu of using the same ideal wave number $k_0$ for both the free space and the interior of the pipe.

Figure 5.2. Figures (a) and (b) show the relative error of measured value compared to the predicted value based on Ando and Jones’s analytic solution as a function of non-dimensional frequency $k_0a$. As can be seen, the relative error is large especially at low $k_0a$ smaller than 0.2. Those values contribute large error to the measured values overall.
5.3.2 Experiment in a pipe terminated with the RLC load

In the second experiment, the power is intentionally dissipated by terminating the pipe with a variable RLC load. Through measuring axial velocity as a function of axial location with the LDA technique, the power dissipated in the load is calculated, while the two-microphone method estimates the same power based on the pressure measurement at two axial positions. Though the measured quantities, i.e., axial velocity and pressure, are different, the two techniques use Eq. (2.42) in common to determine the power dissipated in the load. The dissipated power is also calculated by using Eq. (2.43) which is established on a lumped parameter model.

Figure 5.3.(a) shows the power dissipation measured with the LDA technique \( \dot{W}_{\text{LDA}} \) as a function of the power dissipation measured with a two-microphone method \( \dot{W}_{\text{MICS}} \), while Fig. 5.3.(b) illustrates \( \dot{W}_{\text{LDA}} \) as a function of the power dissipation estimated with a direct measurement \( \dot{W}_{\text{LOAD}} \) based on Eq. (2.43). The measured amount of the power dissipated in the load ranges from less than 1 mW up to 37 mW. Because uncertainties of measurements are present for abscissa and ordinate, a simple linear least squared fit [53] is inappropriate in the analysis. Instead, a linear weighted total least square fit [54] is applied to the data, suggesting that \( \dot{W}_{\text{LDA}} = 0.995 \dot{W}_{\text{MICS}} \) in Fig. 5.3.(a). Based on Eq. (2.42) and the uncertainty of calibration with pressure sensors, the numerical simulation used in Chapter 4 expects that the uncertainty of \( \dot{W}_{\text{MICS}} \) is approximately 0.7%. The uncertainty of \( \dot{W}_{\text{LDA}} \) is also about 0.2% based on the uncertainty inherent to the LDA/BSA system in both magnitude and phase of the measured axial
velocity. This reduced uncertainty can be expected from the fact that the number of the measurement positions in the LDA technique is sixteen while the two-microphone method relies on the measurements at two positions. The propagation of error being considered, the difference between $\dot{W}_{LDA}$ and $\dot{W}_{2MICS}$ based on the fit is within the expected uncertainty of the measurement. Figure 5.3.(b) illustrates another linear weighted total least squared fit which indicates that $\dot{W}_{LDA} = 0.997\dot{W}_{LOAD}$ based on the data taken. The uncertainty in $\dot{W}_{LOAD}$ is overwhelmed by the uncertainty in determining the size of the volume $V$ with the precision of approximately 6.9% in Eq. (2.43) due to the complexity of the inner shape of the load. Hence, the discrepancy shown by the fit between $\dot{W}_{LDA}$ and $\dot{W}_{LOAD}$ is also well within the uncertainty of the measurement. Since the measurements based on three different methods provide the same quantity, the result suggests that the LDA technique is successfully validated by the widely used two other methods. Additionally, this result implies that the calibration of microphones and LDAs’ fringe spacing is performed well, because the LDA technique and the two-microphone method use only LDAs and microphones, respectively. From a different perspective, for all conditions of this experiment, the magnitude of the second harmonic is less than 1% of the magnitude of the fundamental frequency component based on the axial velocity data taken, and therefore, the departure of slopes in two fitted equations from unity is not caused by a nonlinear acoustic phenomenon.
Figure 5.3. In Fig. (a), the power measured with the LDA technique is shown as a function of the power measured with the two-microphone method. The dissipated power estimated with the LDA technique is illustrated as a function of the power predicted directly from the lumped parameter model in Fig. (b). In both figures, the fitted lines are based on the weighted total least squared method, because the uncertainty of the measurement exists for both abscissa and ordinate. The suggested fits are $\hat{W}_{\text{LDA}} = 0.995\hat{W}_{\text{MICS}}$ for Fig. (a), and $\hat{W}_{\text{LDA}} = 0.997\hat{W}_{\text{LOAD}}$ for Fig. (b). The uncertainty in $\hat{W}_{\text{LDA}}$, $\hat{W}_{\text{MICS}}$, and $\hat{W}_{\text{LOAD}}$ are approximately 0.2%, 0.7%, and 6.9%, respectively.
Figure 5.4 illustrates the deviation of the two ratios, such as \( \frac{\dot{W}_{MICS}}{\dot{W}_{LDA}} \) and \( \frac{\dot{W}_{LOAD}}{\dot{W}_{LDA}} \), as a function of \( \dot{W}_{LDA} \). The values shown in Fig. 5.4.(a) are clearly larger than unity, while those shown in Fig. 5.4.(b) are rather distributed around unity. Although the tendency shown in Fig. 5.4.(a) may appear as a systematic error, the root mean square error is 0.7%, which suggests that the deviation from unity is within the uncertainty of measurements with the LDA technique and the two-microphone method. Similarly, Fig. 5.4.(b) suggests that the root mean square error is 1.8%. Since the uncertainty in determining the volume of the RLC load is 6.9%, the deviation shown in Fig. 5.4.(b) is also within the uncertainty of measurements with the LDA technique and the direct estimation based on a lumped parameter model. However, the amount of the dissipated power being considered, the agreement among three methods is fairly good.

Since the two-microphone method developed by Fusco et al. [28] is well known for its robustness in estimating the power dissipation, which includes the amount caused by the thermoviscous boundary layers, the results shown in Figs. 5.3 and 5.4 suggest that the LDA technique is a reliable method to measure the power.
Figure 5.4. Figure (a) shows the ratio of the measured power with the two-microphone method ($\tilde{W}_{2\text{MICS}}$) to that with the LDA technique ($\tilde{W}_{\text{LDA}}$) as a function of $\tilde{W}_{\text{LDA}}$. Figure (b) illustrates the ratio of the measured power from the direct estimation ($\tilde{W}_{\text{LOAD}}$) to $\tilde{W}_{\text{LDA}}$ as a function of $\tilde{W}_{\text{LDA}}$. The root mean square error is 0.7% for Fig. (a), while it is 1.8% for Fig. (b). The uncertainty in $\tilde{W}_{\text{LDA}}$, $\tilde{W}_{2\text{MICS}}$, and $\tilde{W}_{\text{LOAD}}$ are approximately 0.2%, 0.7%, and 6.9%, respectively.

\[ \frac{\tilde{W}_{2\text{MICS}}}{\tilde{W}_{\text{LDA}}} \]
\[ \frac{\tilde{W}_{\text{LOAD}}}{\tilde{W}_{\text{LDA}}} \]
5.4 Further discussion

In previous sections, the LDA technique is validated through the measurement of the radiation impedance of an open-ended cylindrical pipe and that of power dissipated in an RLC load terminating a cylindrical pipe. While the LDA technique has been tested against the analytic solutions available for an open-ended pipe, it has been tested against the two different methods for the power dissipation. Included are the two-microphone method and the direct estimation based on a lumped parameter model in those two methods.

In this section, the reference signal, that is used to scale the measured axial velocity in order to minimize the effect of time elapsed during the measurements, is further discussed. As described in Chapter 3, some applications use an Endevco pressure sensor, while the others use another LDA/BSA system. For example, until this point, the measurement of the power dissipated in the RLC load is the first application that fits in the latter category. Although the use of another LDA is required due to the limited number of Endevco pressure sensors, the switch between an Endevco pressure sensor and another LDA/BSA system has to be justified.

In order to accomplish the justification, the data taken for the second part of the experiment shown in the previous section is used. Because two pressure values are always available when the LDA technique is operational, the measured axial velocity can be scaled by either pressure at a fixed location or axial velocity at another fixed location. The signal that scales the measured axial velocity is referred to as the reference signal.
Since a close look is taken only at the LDA technique, the first row of Eq. (5.2) is used to establish the equation for scaling the measured axial velocity by a reference pressure. Based on the first row of Eq. (5.2), the scaled axial velocity is expressed as,

$$\hat{u}_s (z_i) = \left| \hat{p}_{\text{FIXED}} (1) / \hat{p}_{\text{FIXED}} (i) \right| \hat{u} (z_i),$$

(5.4)

where $\hat{p}_{\text{FIXED}}$ is the measured reference pressure with one of two Endevco pressure sensors that are used to implement the two-microphone method. In selecting the reference pressure, the pressure value with the larger magnitude is chosen to reduce the uncertainty of the measured value in voltage.

Based on this scheme, one method uses the measured axial velocity, and the other method uses the measured pressure as the reference signal. These two methods are tested in the measurement of the power dissipated in the RLC load.

Figure 5.5 shows that the power dissipation measured with the LDA technique scaled by the measured pressure ($\hat{W}_{\text{LDA/MIC}}$) as a function of the power dissipation measured with the LDA technique scaled by the measured axial velocity ($\hat{W}_{\text{LDA/LDA}}$). As discussed, since uncertainties of measurements are present in both abscissa and ordinate, a linear weighted total least square fit is applied to the data. The resulting fit suggests that $\hat{W}_{\text{LDA/MIC}} = 1.002 \hat{W}_{\text{LDA/LDA}}$ based on the data in Fig. 5.5. Since the same technique with a different reference signal is used for both abscissa and ordinate, the same amount of uncertainty is assumed. Based on the uncertainty present in the LDA/BSA system in both magnitude and phase, the uncertainty is approximately 0.2%. The uncertainty in calibrating microphone is not involved in this consideration because the pressure captured by the microphone is only used to scale the axial velocity. The propagation of error being
considered, the difference between two methods based on the fit is within the expected uncertainty of the measurement. Thus, the effect of switching the reference signal, though present, is not dramatic in the measured power dissipation.

Figure 5.5. The power measured with the LDA technique scaled by the reference pressure \( (\hat{W}_{LDA/MIC}) \) is plotted as a function of the power measured with the LDA technique scaled by the reference axial velocity \( (\hat{W}_{LDA/LDA}) \). The fit to the data suggests that \( \hat{W}_{LDA/MIC} = 1.002 \hat{W}_{LDA/LDA} \). The uncertainty of measurement in both abscissa and ordinate is 0.2%.

### 5.5 Conclusion

The LDA technique has been applied to determining the radiation impedance of an open ended pipe. Compared to the theoretical prediction proposed by Ando and Jones, the radiation impedance based on the experiment is determined with the precision of 1.57% for the real part, and 4.35% for the imaginary part on average. If a pipe with small value of the ratio of the inner radius to the outer radius were chosen, each analytic solution
could be discernable. [51,52] Thus, the measured data could be used to distinguish between the two analytic solutions. [24]

Through the measurements of the power dissipated in a variable RLC load, the results suggest that the dissipated power measured by the LDA technique agrees with two other methods to within 0.5% overall. The root mean square error of the ratio of the measured power with either of the two methods, such as the two-microphone method and the direct estimation, to that with the LDA technique also implies that the deviation of the ratio from the unity is within the uncertainty in the measurements. Thus, the LDA technique has been validated as an alternative method to determine power flow in an apparatus, based on the comparison with other widely used methods, such as the two-microphone method developed by Fusco et al. [28] and the direct estimation based on a lumped parameter model.

Based on the LDA technique, the pressure and axial velocity fields need not be reconstructed based on values at only two positions, which the two-microphone method relies on. Additionally, Fig. 5.5 suggests that the kind of the reference signal has a small effect on the measurements, based on the slope of the fit and the uncertainty inherent to LDA/BSA system.
Chapter 6

Determination of the complex propagation constant of the air inside a reticulated vitreous carbon sample

6.1 Introduction

In the previous chapters, the LDA technique has been proposed and validated in well known systems, such as a rigidly terminated pipe, an open-ended pipe, and an arbitrarily terminated pipe based on a lumped parameter model. The purpose of this chapter is to investigate and examine a porous material called reticulated vitreous carbon (RVC) composed of a random geometry of pores. Because of this, RVC is considered as an unknown system to be explored. Specifically, the LDA technique is applied to determining the complex propagation constant of the air inside a RVC sample. The measured complex propagation constant will be compared with the estimated values based on measurements provided in previous studies by Petculescu and Wilen [32], and by Muehleisen et al. [33]

6.2 Procedures

RVC is usually categorized by ‘pores per inch’ (ppi) number which represents the inverse of each pore’s effective radius which is half of the pore diameter. In this study, the LDA technique is applied to a 100 ppi RVC sample whose effective radius is 254 μm, in order to measure the complex propagation constant of the air inside the sample. Figure 6.1
shows a microscopic picture of a 100 ppi RVC sample. As can be seen, a pore is not only ran-dom in its size, but also irregular in its shape. Despite its random geometry, this study assumes that the air in the RVC sample behaves as a homogeneous medium. It is also assumed that the counter-propagating plane wave model is valid in the RVC sample.

Figure. 6.1. A microscopic photo of a 100 ppi RVC sample (This photo is used based on the permission given by Linde Clark and Dr. Steven L. Garrett)

As described in Chapter 3, the experiment of this chapter uses a non-rigidly terminated cylindrical pipe. This condition is accomplished by placing a 26.2 mm thick 100 ppi RVC sample right next to the stainless-steel end cap. Accordingly, the termination comprises an RVC sample and a stainless-steel end cap. Thus, the artifact, which is introduced by the constraint that $|\hat{R}| \leq 1$, would not affect the acoustic field in the pipe. To be more specific, this constraint is observed to modify the reflection coefficient, when the magnitude of the reflection coefficient is very close to unity. Avoidance of the artifact will lead to a better estimation of the acoustic field for each side of the sample,
specifically for the region between the sample and the termination, than in a rigidly terminated pipe. Consequently, the four values $\hat{p}_I (0),$ $\hat{p}_{III} (d),$ $\hat{U}_I (0),$ and $\hat{U}_{III} (d)$ on the surfaces of the sample can be determined better based on the counter-propagating plane wave model. This leads to an accurate estimation of the complex propagation constant of the air inside the RVC sample based on Eq. (2.70),

$$\dot{k} = \frac{1}{d} \cos^{-1} \left( \frac{\hat{U}_I (0) \hat{p}_I (0) + \hat{U}_{III} (d) \hat{p}_{III} (d)}{\hat{U}_I (0) \hat{p}_{III} (d) + \hat{U}_{III} (d) \hat{p}_I (0)} \right).$$

(2.70)

In this pipe, the entry face of the RVC sample, which is to be characterized, is situated approximately 23.2 cm away from the closest position of the termination, as illustrated in Chapter 3. The measurement frequencies chosen are 250 Hz, 500 Hz, 750 Hz, and 1000 Hz. As before, to minimize the effect of the elapsed time in the acoustic field during measurements, the measured axial velocity with the LDA probe is scaled according to

$$\hat{u}_S (z) = \hat{u} (z) \left| \frac{\hat{p}_{FIXED} (1)}{\hat{p}_{FIXED} (i)} \right|,$$

(6.1)

where $\hat{u}_S , \hat{u} , \hat{p}_{FIXED} , z_i , \text{ and } i$ represent the scaled axial velocity, the measured axial velocity, the pressure measured at a fixed location, the $i$th axial position, and the $i$th measurement, respectively.

The two-pass curvefitting algorithm is then applied to the scaled axial velocity $\hat{u}_S$ to estimate the axial velocity field in the pipe on either side of the RVC sample. Based on the thermoviscous boundary layer theory, the resulting axial velocity is converted into the corresponding volume velocity. Consequently, the pressure and volume velocity on both faces of the sample can be obtained from the fitted coefficients for the sound fields on
either side of the sample. These values are \( \hat{\dot{p}}_I (0), \hat{\dot{p}}_{III} (d), \hat{\dot{U}}_I (0), \) and \( \hat{\dot{U}}_{III} (d) \) that were previously discussed. Using Eq. (2.70), the complex propagation constant \( \hat{k}_s \) can be computed. The resulting quantity should be normalized by the ideal wave number \( \omega / c_0 \), according to the notation used by Muehleisen et al. [33]

However, because it is extremely time-consuming to collect velocity at thirty-six positions to evaluate a single value of \( \hat{k}_s \) and its uncertainty, the method described above was not applied to each set of data collected at thirty-six positions. Instead, the value of \( \hat{k}_s \) for a 100 ppi RVC sample is determined in the following way. The axial velocity was measured three times at each of thirty-six positions. The arithmetic mean of the three axial velocities at each position is then calculated. To estimate the uncertainty in \( \hat{k}_s \), it is assumed that this mean value at each position is the mean value of the population at that position. It is also assumed that the uncertainty in the magnitude and phase of the measured velocity using LDA is equivalent to 0.5 mm/s and 5.0% of the time interval (i.e., 10.667 \( \mu s \)), respectively. Note that the magnitude of velocity is not affected by the uncertainty inherent to calibration of LDA, because detected Doppler frequencies do not need to be calibrated when the pursued quantity can be obtained from the ratio of measured values. This is always true when only one sensor is used to estimate a reflection coefficient, a complex propagation constant of a continuous fluid medium, and any kind of impedance. Hence, it is appropriate to apply Gaussian-distributed noises based on these uncertainties to the mean value at each of thirty-six positions. A resulting data at thirty-six positions is considered as a realization. This realized data set is curvefitted to determine one realization of \( \hat{k}_s \). This process is repeated for a total of two thousand
realizations, from which the mean and standard deviation of the real and imaginary parts of $\hat{k}$ are determined.

Muehleisen et al. [33] executed experiments to measure the complex propagation constant and the characteristic impedance of the air inside several RVC samples with different ppi number including 100 ppi. The measured complex propagation constant of the air inside samples being considered, it is normalized by the wave number of air in free space. This normalized quantity is then fitted to the power law of the form, which can be written as

$$\frac{\hat{k} c_{0}}{\omega} = 1 + a_{k} C_{n}^{b_{k}} + j c_{k} C_{n}^{d_{k}},$$

(6.2)
as a function of non-dimensional frequency $C_{n} = \rho_{0} f / \sigma_{0}$ to estimate four unknown parameters $a_{k}, b_{k}, c_{k},$ and $d_{k}$. Here $\sigma_{0}$ is the static flow resistivity of an RVC sample.

Based on the value of $\sigma_{0}$ for the corresponding RVC sample, it was attempted to model the tested RVC samples by a single form of Eq. (6.2). They proposed this fitted curve as the empirical model for the complex propagation constant of the air inside the RVC samples. However, a close look of their results (see Fig. 8 in Ref. [33]) reveals that there is a discrepancy between the measured and the empirical model values, when the 100 ppi RVC sample is considered. Thus, instead of using the empirical model, the actual measured data is used in this study. To extract the measured data from their study, both values, such as the measured and the empirical model value, shown in the result are estimated by counting the number of pixels between the data point and both abscissa and ordinate. This task was accurately performed using any of graphical software that tells the exact
coordinate values of the position where the mouse cursor is located. A curvefit of the extracted empirical model data shown in the result to the empirical model expressed as Eq. (6.2) suggests that the density of air is approximately that of what was expected in their laboratory. This confirms that the method of extraction is reliable. Moreover, a curvefit of the measured values of Muehleisen et al.’s to the form of Eq. (6.2) provides the four unknown parameters $a_k$, $b_k$, $c_k$, and $d_k$ for the measured values based on the obtained density of air. As a consequence, the determined parameters based on the measured values and Eq. (6.2) form a model equation for the measured values. Substituted into the non-dimensional frequency $C_n$ of the model equation for the measured values, the density of air in this laboratory will supply this study with the normalized complex propagation constant of the air inside of 100 ppi RVC based on Muehleisen et al.’s study.

Additionally, the thermoviscous $f$-functions, denoted as $f_{v,s}$ and $f_{k,s}$ here in this dissertation, of a 20 ppi RVC sample were measured as a function of non-dimensional frequency (i.e., $\sqrt{2r_0/\delta_{v,k}}$) by Petculescu and Wilen [32]. In their study, the pore radius was estimated from the inverse of the ppi number. Based on the ppi number (i.e., 100 ppi) and the frequencies chosen in this dissertation, the values of $\sqrt{2r_0/\delta_{v,k}}$ can be obtained. These values are used in finding the same abscissa values in their result (see Fig. 7.(a) in Ref. [32]) to extract the measured $f_{v,s}$ and $f_{k,s}$ at the corresponding $\sqrt{2r_0/\delta_{v,k}}$ for this study. The method of extracting those values was also based on any graphical software that tells the exact coordinate values of the mouse cursor location. The extracted values of $f_{v,s}$ and $f_{k,s}$ are then substituted into the equation written as
for the purpose of evaluating the normalized complex propagation constant of the air inside a 100 ppi RVC sample. In Eq. (6.3), $k_s$ and $\alpha_s$ are the real part and the absorption coefficient of the air inside the 100 ppi RVC sample. However, to accomplish this task, it is assumed that the 20 ppi RVC sample exhibits the same behavior as that of 100 ppi when both of them have the same value of $\sqrt{2r_0/\delta_{\nu,\kappa}}$.

Clark [55] also reported measurements with RVC samples. In that study, various methods of determining hydraulic radius of porous samples were discussed. One of the methods was based on the measurement of the thermoviscous $f$-functions. Although the method was able to measure both $f_{\nu,s}$ and $f_{\kappa,s}$ for a rich variety of samples, those are unavailable specifically for the 100 ppi RVC sample. Thus, the result of her study is not used in this study.

### 6.3 Results

Figures 6.2.(a) and (b) show an example of volume velocity profile normalized by the cross sectional area of the pipe as a function of axial position in both magnitude and phase at 750 Hz. As discussed previously, in a low loss system such as a rigidly terminated pipe, the axial velocity (or volume velocity) is very nearly zero at a pressure antinode, and vice versa to a good approximation. Hence, Fig. 6.2.(a) suggests that the
termination is neither of an open-ended pipe nor of a rigidly terminated pipe, based on the magnitude of volume velocity at the termination. This indicates that a non-rigid termination is successfully implemented with a combination of an RVC sample and a stainless-steel end cap.

Figure 6.2. Figures (a) and (b) illustrate the magnitude and phase of the volume velocity normalized by the cross sectional area of the pipe at 750 Hz as a function of axial position, when a 100 ppi RVC sample to be determined is located at \( z = 0 \) cm and the lossy termination is at approximately \( z = 23.2 \) cm. The red curve shown indicates the region where the target RVC sample is located.

Figure 6.3.(a) shows the estimated magnitude of reflection coefficient (i.e., \( |\hat{R}| \)) at the termination and its uncertainty as a function of frequency. As described previously,
this result is obtained from the simulation based on the measured values and the assumed uncertainties. As shown in Fig. 6.3.(a), $|\hat{R}|$ clearly deviates from unity for all frequencies chosen in this study. The relative uncertainty, defined here as the standard deviation divided by the mean value, in the reflection coefficient is illustrated as a function of frequency in Fig. 6.3.(b). From its definition, the illustrated relative uncertainty suggests the value with 67% of confidence level, which corresponds to $\sigma$. With this confidence level, the relative uncertainty is less than 0.15% at the frequency where $|\hat{R}|$ is the largest. Based on this value, the relative uncertainty with 99.99% of confidence level, which corresponds to $4\sigma$, is less than 0.60%. Since the mean value of $|\hat{R}|$ is approximately 0.96 at that frequency, $|\hat{R}|$ is definitely smaller than unity. Applying the same analysis to the values at other frequencies, it is revealed that $|\hat{R}|$ deviates from unity. It is interesting to note that the relative uncertainty decreases with increase of frequency except at 250 Hz, as shown in Fig. 6.3.(b). At 250 Hz, the traversed axial range by LDA is approximately 7.4% of a wavelength, while it is 29.6% of a wavelength at 1000 Hz for either side of the sample. This increase in the traversed axial range of LDA may play a role in reducing the relative uncertainty as discussed previously in Chapter 4. Though the traversed axial range of 250 Hz is the smallest compared to other frequencies, it is guessed that the exceptional behavior at that frequency may be caused by two factors. First, the smallest phase uncertainty is expected in the measured velocity with LDA at 250 Hz. Second, the smallest magnitude of measured velocity at 250 Hz is approximately 7.14 cm/s, while that of measured velocity at the other frequencies are below 5.00 cm/s. This second factor
suggests that the effect of the uncertainty in Doppler frequency on the magnitude of velocity is reflected in the least at 250 Hz compared to that at the other frequencies. However, what truly affected this behavior is still unknown. Regardless, the dissipative termination is designed well, and the artifact from a rigid boundary condition is avoided.

Figure 6.3. Figure (a) shows the magnitude of the reflection coefficient at the termination and its uncertainty as a function of frequency, based on the measurement and the simulation. Due to the unrecognizable size of error bar in Fig. (a), the relative uncertainty is plotted as a function of frequency in Fig. (b).

The real and imaginary parts of the complex propagation constant of the air inside a 100 ppi RVC sample normalized by the ideal wave number \( k_0 \) are illustrated as a func-
tion of frequency in Fig. 6.4. As can be seen, the determined values are compared with these based on the measurements by Muehleisen et al. [33]\(^2\) and the measurement of thermoviscous \(f\)-functions by Petculescu and Wilen. [32]

![Graph](image)

Figure 6.4. Determined by the LDA technique and simulation, the normalized real part and the normalized absorption coefficient of the complex propagation constant of the air inside a 100 ppi RVC sample are illustrated as a function of frequency in Figs. (a) and (b). They are also compared to the same quantities based on the four-microphone method by Muehleisen et al. [33] and the measurements of thermoviscous \(f\)-functions by Petculescu and Wilen. [32]

Figure 6.5 shows a closer view of the determined quantities based on three studies as a function of frequency. However, it is illustrated in terms of the relative difference, which is defined here as the ratio of the difference between one value and a reference value to the reference value. In this discussion, the reference value is the mean value of

\(^2\)Muehleisen et al. did not measure data at 250 Hz.
the real and imaginary parts of $\hat{k}_x$. While the real part agrees better with Petculescu and Wilen’s result with the relative difference of 2.05%, the absorption coefficient agrees better with Muehleisen et al.’s result with relative difference of -3.79% on average. As indicated, the three values for the 100 ppi RVC sample are similar. The difference may be resulting from the fact that some of experimental conditions for each experiment are different. First, the experiment performed in this dissertation uses an RVC sample manufactured by the different company from two other studies. Second, Petculescu and Wilen’s study is based on the 20 ppi RVC sample, while two other experiments use the 100 ppi RVC sample. Although it is previously assumed that the 20 ppi RVC sample exhibits the same behavior as that of 100 ppi when both of them have the same value of $\sqrt{2r_0/\delta_{v,\kappa}}$ in order to use the result of Petculescu and Wilen’s study [32], the effect of using samples with different ppi number is well illustrated in Muehleisen et al.’s research. [33] In Muehleisen et al.’s publication, it is shown that the generalized empirical model of RVC samples tends to track the measured quantity with less precision for some samples. Based on these two conditions, the overall condition is different for each study. Accordingly, the discrepancy present in the complex propagation constant of the air inside the 100 ppi RVC sample is not surprising.

Indicated by the error bars, the uncertainty of the real and imaginary parts of the measured $\hat{k}_x$ in this study is also illustrated as a function of frequency in Fig. 6.5. As can be seen, the largest uncertainty is 2.04% and 3.75% for the real and imaginary part, respectively, at 250 Hz. In general, the uncertainty is below 0.94% and 1.77% for the real

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3 This study used the sample provided by Ultramet, while the two other studies used samples from Energy Research and Generation, Inc.
and imaginary part, respectively, at the other frequencies. The traversed axial range of the LDA probe being considered at 250 Hz, the amount of uncertainties is quite small.

Figure 6.5. The determined value and error bar of the complex propagation constant of the air inside a 100 ppi RVC sample are presented as a function of frequency in both non-dimensional real part (Fig. (a)) and absorption coefficient (Fig. (b)). Presented are the values based on Petculescu and Wilen’s (2001) and Muehleisen et al.’s (2005) as well, relative to the value of this study. The size of the error bars indicates the uncertainty corresponding to $\sigma$ with the confidence level of 67%.

6.4 Conclusion

The complex propagation constant of the air inside a 100 ppi RVC sample is determined, based on the LDA technique and the simulation. The normalized complex propagation constant is experimentally determined with the uncertainty of less than 2.04% for real
part and less than 3.75% for absorption coefficient, respectively, relative to the measured quantity. This quantity is also situated between the predicted values based on two other studies. [32,33] Although the comparison with the previous publications suggests that the measurement of the current experiment tends to propose different values, it is not abnormal because the experimental conditions and samples are not identical for all three experiments. The result also suggests that the complex propagation constant of the air inside a 100 ppi RVC sample is considered to be determined quite well.
Chapter 7

Axial propagation constant in the first non-planar wave mode

7.1 Introduction

Until this chapter, only the plane wave mode in a cylindrical pipe is studied to investigate the LDA technique proposed in this dissertation. It has been shown that the LDA technique based on the counter-propagating plane wave model is reliable for characterizing an unknown one-dimensional system, such as an RVC sample under the plane wave mode assumption. The purpose of this chapter is to test the utility of the LDA technique in non-planar wave situations. Specifically, the axial propagation constant is determined for the (1,1) mode in a cylindrical pipe. The result will show the importance of the complex axial propagation constant for the (1,1) mode. It will also demonstrate the ability to reconstruct the radial dependence of the axial velocity.

7.2 Procedures

As described in Chapter 2, there are few dedicated studies [34,36] about the axial propagation constant for non-planar wave modes in a waveguide, especially pertaining to the loss mechanism along the axial direction. Although these studies have shown that the axial propagation constant should be complex, the axial propagation constant is still treated as a real value in some studies. [38,39] In this study, the reconstructed axial
velocity as a function of radial position will demonstrate that the axial propagation constant for non-planar wave mode should be complex.

In order to reconstruct the axial velocity as a function of radial position, the following strategy is employed. The total pressure in the (1,1) mode is

\[
\hat{p} = A_{00}e^{-jk_zz} + B_{00}e^{jk_zz} + \hat{A}_{11}J_1(k_{11}r)\cos(\theta)e^{-jk_zz} + \hat{B}_{11}J_1(k_{11}r)\cos(\theta)e^{jk_zz}, \tag{2.73}
\]

where \( \hat{A}_{00}, \hat{B}_{00}, \hat{A}_{11}, \) and \( \hat{B}_{11} \) are the complex amplitude coefficient for the incident plane wave, for the reflected plane wave, for the incident (1,1) mode wave, and for the reflected (1,1) mode wave, respectively, and where \( \hat{k}, k_{11}, \) and \( k_z \) are the complex propagation constant for the plane wave mode, the radial wave number which can be expressed as \( k_{11} = 1.8412/r_0 \) for the (1,1) mode, and the axial propagation constant for the (1,1) mode.

Based on Eq. (2.73), the axial velocity is obtained using Eq. (2.22) written as

\[
\hat{u}_z = \frac{j}{\rho_0\omega_0} \left[ 1 - h_v(r) \right] \frac{dp}{dz}. \tag{2.22}
\]

As a reminder, Eq. (2.22) is based on the plane wave mode assumption. Specifically, with this assumption, the relationship that \( \partial / \partial r \sim 1/\delta_x \), \( \partial / \partial z \sim 1/\lambda \), and \( |\hat{u}_z / \hat{u}_r| \sim \lambda / \delta_x \) could be used to assume that the variation of pressure in the radial direction is negligible. This assumption is crucial in derivation of Eq. (2.22). It is still valid for the plane wave mode coexisting with the (1,1) mode. However, this relationship is not valid for the (1,1) mode, because the length scale of the acoustic variation in the radial direction becomes on the order of the pipe radius for the (1,1) mode based on Eq. (2.73). The issue of invalidity in Eq. (2.22) for the (1,1) mode would be more important when the radial position of interest is within several penetration depths of the pipe wall. However,
because velocity is never measured within this distance from the pipe wall in this study, the effect of \([1 - h_\nu (r)]\) would not be substantial. Therefore, Eq. (2.22) is still used, although the assumption that supports Eq. (2.22) is violated for the (1,1) mode.

The resulting axial velocity \(\hat{u}_z (r, \theta, z)\) is expressed as,

\[
\hat{u}_z (r, \theta, z) = \hat{k} / (\rho_0 \omega) \left[ \hat{A}_{00} e^{-jkz} - \hat{B}_{00} e^{jkz} \right] + J_1 (k_1 r) \cos (\theta) k_z / (\rho_0 \omega) \left[ \hat{A}_{11} e^{-jk_z z} - \hat{B}_{11} e^{jk_z z} \right].
\] (7.1)

Up to this point, the axial velocity has been measured on the axis of the pipe (i.e., \(r = 0\)). The axial velocity is not measured on the center axis in this study. In Eq. (7.1), the term \([1 - h_\nu (r)]\) is absorbed into four coefficients \(\hat{A}_{00}, \hat{B}_{00}, \hat{A}_{11}, \) and \(\hat{B}_{11}\). Since the selected radial position is not within several viscous penetration depths of the wall in this study, the plane wave components \(\hat{A}_{00}\) and \(\hat{B}_{00}\) are not affected by the term \([1 - h_\nu (r)]\). Thus, its effect on the measurement can be neglected.

To determine \(\hat{u}_z\) at any radial and axial position, all the parameters including \(k_z\) of Eq. (7.1) should be known. These can be determined by a modal decomposition based on the measurement of \(\hat{u}_z\) as a function of \(z\) at \(r = r_2\), where \(r_2\) is a non-zero value. This measured \(\hat{u}_z\) is denoted as \(\hat{u}_{z,m} (r_2, z)\). The modal decomposition is performed by curve-fitting \(\hat{u}_{z,m} (r_2, z)\) to \(\hat{u}_z (r_2, \theta, z)\), which is acquired from Eq. (7.1) in the form

\[
\hat{u}_z (r_2, \theta, z) = \hat{k} / (\rho_0 \omega) \left[ \hat{A}_{00} e^{-jkz} - \hat{B}_{00} e^{jkz} \right] + J_1 (k_1 r_2) \cos (\theta) k_z / (\rho_0 \omega) \left[ \hat{A}_{11} e^{-jk_z z} - \hat{B}_{11} e^{jk_z z} \right].
\] (7.2)
As a result of the modal decomposition, not only can the coefficients $\hat{A}_{00}$ and $\hat{B}_{00}$ due to the plane wave mode existing with the (1,1) mode be determined, but the two coefficients $J_1(k_1r_2)\cos(\theta)k_z/(\rho_0\omega)\hat{A}_{11}$ and $J_1(k_1r_2)\cos(\theta)k_z/(\rho_0\omega)\hat{B}_{11}$ can be determined. Since the Bessel function of the first kind is the radial dependence of both pressure and axial velocity for the (1,1) mode, $\cos(\theta)k_z/(\rho_0\omega)\hat{A}_{11}$ and $\cos(\theta)k_z/(\rho_0\omega)\hat{B}_{11}$ can be determined based on known $J_1(k_1r_2)$. Since all four coefficients are determined, Eq. (7.1) can construct the axial velocity as a function of $r$ at $z = z_0$. The resulting velocity, denoted as $\hat{u}_z(r, \theta, z_0)$, is expressed as

$$\hat{u}_z(r, \theta, z_0) = \hat{k} / (\rho_0\omega) \left[ \hat{A}_{00}e^{-j\hat{z}_0} - \hat{B}_{00}e^{j\hat{z}_0} \right] + J_1(k_1r)\cos(\theta)k_z/(\rho_0\omega) \left[ \hat{A}_{11}e^{-j\hat{z}_0} - \hat{B}_{11}e^{j\hat{z}_0} \right],$$

(7.3)

based solely on the measurement of $\hat{u}_{z,m}(r_2, z)$. Providing a check of $\hat{u}_z(r, \theta, z_0)$ that is constructed based on Eq. (7.3), axial velocity is also measured as a function of $r$ at $z = z_0$. The measured axial velocity is denoted as $\hat{u}_{z,m}(r, z_0)$.

According to the scheme described above, the experiment is performed with a cylindrical pipe with approximately 23.3 mm of the inner radius to drive the (1,1) mode at a lower frequency than in the pipes used previously. As shown in the strategy, much care should be taken to ensure that the axes of all the systems, which includes the axially moving LDA probe, are parallel to each other, because the radial dependence of pressure and axial velocity cannot be neglected for the (1,1) mode. Through careful adjustments,
the resulting angle between the axis of the moving LDA probe and the axis of the pipe is approximately 0.06 degrees which correspond to 1 mm of offset in the radial direction along 1 m of length in the axial direction. Because the maximum axial range of the moveable LDA probe is 31 cm, this degree of misalignment may be considered to be fine. The center axis is also located through a careful calibration. First, the closest external point of the glass wall is found by focusing the measurement volume of the LDA. Two laser beams being projected on a piece of Teflon tape attached to the outside of the glass wall closer to the LDA probe, the radial position of the mechanical positioner is found when the size of the focal point on the tape is minimized. This radial position of the mechanical positioner is denoted as 0. The farthest external point of the glass wall is found by projecting two laser beams on a Teflon tape attached to the outside of the farther glass wall from the LDA probe. However, refraction of laser beams will occur in this case, because laser beams pass through two glass walls between the Teflon tape and the LDA probe. Because the index of refraction is 1.517 for the borosilicate glass, the focal point through a glass wall will move farther from the LDA probe than the focal point through air. If the length of path difference is denoted as \( d_r \), due to refraction of laser beams in a single glass wall, the total length of path difference is \( 2d_r \). Thus, when the focal point is formed on the Teflon tape, the true radial position of the mechanical positioner is \( 2r_i - 2d_r \), where \( r_i \) is the outer radius of the cylindrical pipe used. Based on this value, the mechanical positioner is relocated at \( r_i - d_r \) in radial position. Since a single glass wall is present between the focal point and the LDA probe, the refraction causes the focal point to be located at \( r_i \) in radial position. The resulting radial position of the focal point is located in
the midpoint of two external points of the glass walls. Because this calibration relies on focusing the focal point on a piece of Teflon tape at both external points, the uncertainty in locating the center axis is approximately on the order of the length of the measurement volume, which is 630 µm.

The seeding particles are introduced into the pipe, before the pipe is terminated by a PVC end cap which mounts an Endevco pressure sensor 8510B-5 (S/N: 10060) at off-center location to monitor the pressure for the (1,1) mode in real time. A halfway blocked compression driver (JBL 2426H) delivers the monofrequency sound to the pipe at a frequency which is approximately 7% larger than the cut-on frequency of the (1,1) mode of the cylindrical pipe. As discussed in Chapter 3, two LDA/BSA systems are used in this study. With this setup, a detailed shape of the Bessel function of the first kind is captured, through measuring \( \hat{u}_{z,m}(r, z_0) \) for nineteen radial positions with the increment of 2 mm. The fixed axial position \( z_0 \) is chosen near a pressure antinodal position of the plane wave mode, in order to minimize its effect in \( \hat{u}_{z,m}(r, z_0) \). For a modal decomposition, \( \hat{u}_{z,m}(r_2, z) \) is measured at nineteen axial positions with the increment of 1 cm. When \( \hat{u}_{z,m}(r_2, z) \) is measured at the final position (i.e., the thirty-eighth position) is completed, the source signal is turned off.

The measured axial velocity is then scaled as,

\[
\hat{u}_s(r_i, \theta, z_i) = \hat{u}_z(r_i, \theta, z_i) \left| \frac{\hat{u}_{z,m}(i)}{\hat{u}_{z,m}(1)} \right|,
\]  

(7.4)
where $\hat{u}_s$, $\hat{u}_z$, $\hat{u}_{\text{FIXED}}(i)$, $z_i$, and $r_i$ are the scaled axial velocity, the measured axial velocity measured with the moving LDA probe, the $i$th axial velocity measured with the stationary LDA probe, the $i$th axial position, and the $i$th radial position, respectively.

The scaled axial velocity $\hat{u}_s$ from the measurement for a modal decomposition is fitted to Eq. (7.2). The number of unknown parameters depends on the kind of axial propagation constant for the (1,1) mode. In order to reduce the number of unknown parameters, it is assumed that the complex propagation constant $\hat{k}$ for the plane wave mode is known. The number of unknown parameters then becomes ten with complex $k_z$, because each of $\hat{A}_{00}$, $\hat{B}_{00}$, $\hat{A}_{11}$, $\hat{B}_{11}$, and $k_z$ have real and imaginary parts to be determined. However, if the axial propagation constant $k_z$ is assumed to be real, $k_z$ is calculated from the definition written as,

$$k_z = \sqrt{(\omega / c_0)^2 - k_{11}^2}. \quad (7.5)$$

Hence, the number of unknown parameters is eight, because each of $\hat{A}_{00}$, $\hat{B}_{00}$, $\hat{A}_{11}$, and $\hat{B}_{11}$ has its real and imaginary parts. Based on Eq. (7.3) with the determined parameters, the axial velocity as a function of radial position at a fixed axial position is computed, as discussed. The two reconstructed axial velocities are then compared with the measured $\hat{u}_{z,m}(r, z_0)$. 
7.3 Results

Compared to other experiments in the previous chapters, this experiment is extremely difficult, because the cut-on frequency is easily shifted by the change of temperature in the laboratory. It is also observed that there are less smoke particles available near the wall of the pipe than on the axis. Because of these factors, the collection of data has to be completed within a short amount of time with a single introduction of seeding particles to the pipe.

The $\hat{u}_{z,m}(r, z)$ is shown in Fig. 7.1 as a function of axial position, compared to the fits of the data to Eq. (7.2) based on two models of $k_z$ for the (1,1) mode. As illustrated, little difference is shown between two fitted curves within the axial range where $\hat{u}_{z,m}(r, z)$ is measured. This may suggest that the plane wave mode components are dominant over the (1,1) mode components. As such, the fit indicating the total axial velocity may not be substantially influenced by the additional absorption coefficient in $k_z$ of the (1,1) mode. Nonetheless, the fit including a loss mechanism for the (1,1) mode agrees slightly better with the measured data than the fit based on a lossless mechanism for the (1,1) mode. However, Fig. 7.1 suggests that the expected acoustic field can be quite different between two models of $k_z$, if the axial position of interest is not within the axial range where the axial velocity is measured. This suggests that the appropriate model of $k_z$ may be crucial in estimating the reflection coefficient and the characteristic impedance of a porous sample in a non-planar mode, because the sample cannot be situated within the axial range of the measurements.
Figure 7.1. The magnitude and phase of the fits are provided in Figs. (a) and (b), based on the complex $k_z$ and the real $k_z$. The discrepancy between two fits is little noticeable within the traversed axial range of the LDA probe. However, the difference is conspicuous as the axial position approaches the termination in the figures. The axial velocity is measured as a function of axial position at $r = 17.5$ mm. Compared to the measured axial velocity (blue dots), the fit based on complex $k_z$ (black squares) agrees slightly better with the measured axial velocity than the fit based on real $k_z$ (red circles).
Figure 7.2 shows the same modal decomposition of the measured \( \hat{u}_{z,m}(r_2, z) \), based on the same fits shown in Fig. 7.1. The axial range shown in Fig. 7.2 is chosen to indicate the axial range where the measurement is performed. While the magnitude and phase of the plane wave modal components (\( \hat{u}_{z,\text{plane}} \)) are illustrated as a function of \( z \) in Figs. 7.2.(a) and (b), respectively, those of the (1,1) modal components (\( \hat{u}_{z,11} \)) are illustrated as a function of \( z \) in Figs. 7.2.(c) and (d). As estimated previously, it is clear that the plane wave modal components are dominant over the (1,1) modal components. Because of this, Figs. 7.2.(a) and (b) show no dramatic difference in the plane wave modal components between two models of \( k_z \). However, a slight difference is predicted in the magnitude and phase of the (1,1) modal components between two models in Figs. 7.2.(c) and (d).
Figure 7.2. Based on two models for the axial propagation constant for the (1,1) mode, the modal decomposition is applied to the measured axial velocity as a function of axial position at a fixed radial position. The magnitude and phase of the plane wave modal components ($\hat{u}_{z,\text{plane}}$) are shown as a function of axial position in Figs. (a) and (b), respectively. The magnitude and phase of the corresponding (1,1) modal components ($\hat{u}_{z,11}$) are shown as a function of axial position in Figs. (c) and (d), respectively. $\hat{u}_{z,m}(r,z)$ is measured at $r_2 = 17.5$ mm. Note that only Figs. (c) and (d) show visible differences between two models.
Figure 7.3 shows the magnitude and phase of the two reconstructed \( \hat{u}_z(r, \theta, z_0) \), where \( z_0 = 22.01 \) cm from the end cap, based on the modal decomposition and the Bessel function of the first kind. Compared to these reconstructed \( \hat{u}_z(r, \theta, z_0) \), the measured \( \hat{u}_{z,m}(r, z_0) \) is also presented. It is clear that the \( \hat{u}_z(r, \theta, z_0) \) based on the complex \( k_z \) agrees with \( \hat{u}_{z,m}(r, z_0) \) to within the better precision than the \( \hat{u}_z(r, \theta, z_0) \) based on the real \( k_z \). Although discrepancies are present near \( r = 0 \) in both magnitude and phase between the \( \hat{u}_z(r, \theta, z_0) \) based on the complex \( k_z \) and the \( \hat{u}_{z,m}(r, z_0) \), it is not surprising because several factors may affect the quality of measurements. First, the modal decomposition of this study determined ten unknown parameters from the curvefitting of data at nineteen positions to Eq. (7.2). In fact, the ratio of the number of data to the number of unknown parameters for this modal decomposition is smaller than that for determining the complex propagation constant of the air in a cylindrical pipe under the plane wave mode in Chapter 4. This condition being considered, the error in each parameter may be accumulated in \( \hat{u}_z(r, \theta, z_0) \). Second, based on the measured \( \hat{u}_{z,m}(r, z_0) \) near \( r = 0 \), their values may be affected by the uncertainty in Doppler frequency, because the magnitude of \( \hat{u}_{z,m}(r, z_0) \) is less than 1 cm/s near \( r = 0 \). Since this uncertainty is approximately 0.5 mm/s, its influence can be significant. Finally, because the uncertainty in locating the center axis is on the order of the length of the measurement volume, this uncertainty may also account for some amount of discrepancy shown in Fig. 7.3. Regardless, Fig. 7.3 indicates that the axial propagation constant \( k_z \) for the (1,1) mode should be complex.
Figure 7.3. The magnitude and phase of the reconstructed axial velocity are shown as a function of radial position at a fixed axial position in Figs. (a) and (b), respectively. Compared to two reconstructed $\hat{u}_z(r, \theta, z_0)$, $\hat{u}_{z,m}(r, z_0)$ is also presented. Despite some discrepancies, the model based on complex $k_z$ agrees with $\hat{u}_{z,m}(r, z_0)$ to within the better precision than the model based on real $k_z$. 

[Graph (a) showing magnitude of $|\hat{u}_z|$ versus radial position]

[Graph (b) showing phase $\Delta \hat{u}_z$ versus radial position]
7.4 Conclusion

In this chapter, the axial velocity is measured not only as a function of radial position at a fixed axial position, but as a function of axial position at a fixed radial position with the LDA/BSA system. Fitted to Eq. (7.2) based on two models of axial propagation constant for the (1,1) mode, the measured data shows a slightly better agreement with the model of complex $k_z$ than with the model of real $k_z$. Moreover, the disagreement between the two fits near the termination suggests that the experiment in a non-planar mode could be affected by the model for the axial propagation constant, when the axial position of interest is not within the axial range that is covered by sensors.

Based on this modal decomposition and the Bessel function of the first kind, the axial velocity at a fixed axial position could be synthesized as a function of radial position. Compared to the two synthesized data, the measured data agrees better with synthesized data based on the complex $k_z$ for the (1,1) mode than the other data based on real $k_z$ for the (1,1) mode. This result suggests not only that axial velocity at any axial position is constructed as a function of radial position using this method of the modal decomposition, but also that some studies which discussed the non-planar wave mode based on real $k_z$ should be modified. [38,39] Overall, the axial propagation constant $k_z$ for a non-planar wave mode should be complex.

This result demonstrates that the LDA technique can be used to identify the acoustic field based on an acoustic model with unconfined propagation constants for each modal component. This type of experiment cannot be handled properly by any of prevailing microphone techniques, such as the two-microphone method and the four-
microphone method, because they require a priori knowledge of the propagation constant for each modal component. Although multiple microphone technique shown by Jones et al. [3] may obtain a similar result, it would require a significant calibration that is not necessary for this LDA technique.
8.1 Summary

This dissertation presents an analysis of the potential for laser Doppler anemometry to reconstruct one and two dimensional acoustic fields and to measure power flow in systems with very low to low levels of dissipation. Traditionally, microphone methods are used for these types of measurements, usually based on a counter-propagating plane wave model. According to this method, microphones are employed to measure pressure at several locations, and to predict the pressure and axial velocity for the entire axial range of the pipe. As opposed to this approach, a velocity-based measurement technique based on the LDA/BSA system was used in this dissertation.

In order to maintain the simplicity of the counter-propagating plane wave model, several strategies were devised. First, for the purpose of suppressing nonlinear effect, the second harmonic term of the velocity was maintained to be less than 1.0% of the velocity at the fundamental frequency at any axial position. Furthermore, a reference signal was employed to scale the measured axial velocity to reduce the effect of drift over the duration of the experiment. The type of the reference signal utilized was either pressure or axial velocity measured at a fixed position.

In addition, a two-pass curvefitting algorithm was proposed to utilize the full potential of the LDA technique. While a first-pass curvefitting algorithm used both
magnitude and phase of axial velocity to estimate the approximate acoustic field in the pipe, the second-pass curvefitting algorithm used only its magnitude to reduce the uncertainty in the measurement. Moreover, the magnitude of the reflection coefficient was constrained to be no greater than unity, in order to implement a physical sense in the technique. Associated with the axial velocity measured at multiple axial positions, this algorithm was designed to provide an improvement over the prevailing four-microphone method. [1]

The LDA technique developed was assessed in a variety of known systems, such as a rigidly terminated, an open-ended, and an arbitrarily terminated cylindrical pipe. Later, the LDA technique was applied to the unknown systems which included the complex propagation constant of the air in a RVC sample and the axial propagation constant for the (1,1) mode in a cylindrical pipe.

8.1.1 Rigidly terminated pipe

The complex propagation constant of the air inside a rigidly terminated cylindrical pipe was determined, based on the measured axial velocity at not less than thirty-two different axial positions with LDA. Additionally, a curvefitting algorithm was used to determine that quantity from the measured velocity. This curvefitting algorithm employed a constraint that the magnitude of the reflection coefficient should be no larger than unity.

The complex propagation constant for such a system is well established. An intentionally large ratio of the pipe radius to the thermal penetration depth was chosen because it provides for very low dissipation. Hence, it was considered to be a good test
for the LDA technique. Nonetheless, the measured complex propagation constant agreed well with the prediction from the thermoviscous boundary layer theory [18] to within the root mean square error of 0.17% for the real part, and to within that of 2.00% for the absorption coefficient. This result suggested that the LDA technique is validated through the measuring the complex propagation constant of the air in a known system with very low dissipation. It also implied that this technique can determine the complex propagation constant in an unknown system with a similar approach to this.

To understand the effect of uncertainties inherent to an LDA/BSA system on the measured quantity, a numerical simulation was used. This simulation was performed as follows. Based on the rigid termination, the noise-free axial velocity was constructed at each of thirty-two axial positions. Uncertainties then were applied to both magnitude and phase of the predetermined velocity at each location. These uncertainties were based on Gaussian distribution with the standard deviation of 0.3% for the magnitude and of 5.0% of the time interval for the phase. The realization of data from the procedure above was curvefitted to determine the complex propagation constant. This process was repeated two thousand times to provide the mean value and the standard deviation of the determined values. The result of this simulation showed that the uncertainty in the measured values was within the boundary suggested by this simulation to some extent. In this simulation, however, it was observed that the constraint that was imposed on the reflection coefficient introduced a significant amount of bias to the mean values of the determined values, because the reflection coefficient magnitude of a rigid termination that was very close to unity.
Two methods of reconstructing the pressure were also presented. Both methods make use of the equation of continuity. The two methods are: 1) applying a two-point finite difference version of the one-dimensional equation of continuity to adjacent pairs of velocity measurements to approximate pressure and 2) fitting the entire set of velocity measurements to the counter-propagating plane wave model to extract the parameters needed to reconstruct the pressure field. The results of both methods were then used to calculate the power flow in the pipe. The power flow was compared to that predicted from a DeltaE simulation. The curvefitting approach agreed well with the simulation. The two-point measurement approach showed considerable scatter. To some extent, this unrealistic result of the local difference method was expected from the noise present in magnitude and phase for both pressure and axial velocity. This could have been avoided if the spacing of two adjacent positions were chosen more carefully. In fact, Fusco et al. [28] reported that the measured power flow based on their method was in a good agreement with the theoretical prediction in the situation that the microphone spacing was at least 7.14% of a wavelength. Such spacing was not allowed by the linear mechanical positioner that was used in this study. As such, this study used the spacing which was less than 1.7% of a wavelength. Despite the limitation in the positioner, however, the result of the power flow in this study indicated that the LDA technique has a flexibility that allows it to be adapted to any measurement situation without modifying an apparatus accordingly, especially when the wavelength is much larger compared to the spacing of two adjacent positions. In such a situation as of this study, the curvefitting-oriented method can perform better than the local difference method.
This experiment exemplifies the ability of the LDA technique, because it only requires the knowledge of overall ratio of the acoustic field inside the pipe. In this situation, the calibration of the sensor is not necessary for the LDA technique. Furthermore, any of prevalent microphone techniques, such as the two-microphone method and the four-microphone method, are unsuitable for this experiment, because of the number of unknown parameters that need to be determined.

### 8.1.2 The pipes terminated with an open end and with an RLC load

Measurements in the rigidly terminated pipe validated the LDA technique in low loss systems. The next step was to move to systems with greater dissipation. However, the loss needed to be independently verifiable. Two terminations were chosen: 1) an open-ended pipe and 2) a pipe terminated with an RLC load.

In the open-ended pipe, the LDA technique was tested in determining the radiation impedance against three theoretical predictions: one based on the oscillatory piston in an infinite baffle [44,45], the other two based on the Wiener-Hopf technique [22-24]. One of these two predictions used the standard zero wall thickness calculation, whereas the second assumed non-zero thickness. As implemented, however, there was little difference in these two predictions. Regardless, the measured radiation impedance tracked the prediction according to Ando and Jones’s studies [23,24], while it was discerned from the prediction for the oscillatory piston in an infinite baffle. The average error relative to the prediction was 1.57% for the real part, and 4.35% for the imaginary part, respectively.
For a further validation, a variable RLC load was built based on a lumped parameter model. Attached to the cylindrical pipe, the RLC load could dissipate power up to tens of milliwatts. The dissipated power was measured with three techniques: 1) the LDA technique, 2) the two-microphone method [28], and 3) the direct estimation based on a lumped parameter model described in the texts [18,28,29,43]. Although a purely theoretical prediction was unavailable for the RLC load due to its complex inner shape, the comparison of the results with two other methods can validate the LDA technique when all of three methods present comparable quantities. In fact, the result showed that the LDA technique results agreed with the quantities based on the two other methods to within 0.5%. The disagreement was much less than the uncertainty of the measurement caused by the uncertainty in calibration of the microphones as well as the fringe spacing of LDA. This result suggested that the LDA technique is validated through the comparison with well established microphone-based measurement methods.

Based on the data with the RLC load, the effect of change in the reference signal was investigated. The result showed that only a 0.2% difference existed between the pressure reference signal and the velocity reference signal. According to the propagation of error, this discrepancy was well within the uncertainty of the measurement with the LDA technique. Thus, the effect of the change in the reference signal on the measurement was neglected.
8.1.3 Reticulated vitreous carbon

Having validated the LDA technique in theoretically well-understood systems, the method was applied to characterizing a more complicated system. To this end, the complex propagation constant of the air inside a 100 ppi RVC sample was measured as a function of frequency. Because the constraint on the magnitude of the reflection coefficient can introduce some unwanted artifacts, a rigid boundary condition was considered undesirable. Hence, in order to avoid a rigid boundary condition, another same RVC sample was located right next to the stainless-steel end cap to introduce additional dissipation.

The propagation constant was determined as follows. The axial velocity was measured three times at each of thirty-six axial locations on either side of the sample under test. Uncertainties were then applied to both magnitude and phase of the arithmetic mean value of the three axial velocities at each of thirty-six positions. The added uncertainties were based on Gaussian-distributed noise with the uncertainty in Doppler frequency as well as in phase. This formed a realization of data with which the complex propagation constant of the air inside a 100 ppi RVC sample was determined through curvefitting. This process was repeated two thousand times to obtain the mean and the standard deviation of the quantity.

Compared to two previous studies that discussed RVC samples [32,33], the result revealed that the complex propagation constant of the air inside 100 ppi RVC sample of this study was situated between the quantities based on those determinations. The discrepancy among three values is not unexpected, because the conditions of each exper-
iment were somewhat different. However, it was encouraging that the uncertainty in the
determined quantity by the LDA technique was less than 2.04% for the real part and less
than 3.75% for the imaginary part at all of the four frequencies. This is a promising
aspect of the LDA technique, implying that it can suppress the uncertainty in determining
a quantity when it is combined with both the two-pass curvefitting algorithm and the
measurement at multiple positions. Although the time consumed by the LDA technique is
substantially more than by the four-microphone method, the small uncertainty may play
an important role in exploring an unknown system.

8.1.4 Non-planar wave mode

Up to this point, only one-dimensional systems have been investigated. As a final vali-
dation, a two-dimensional sound field is reconstructed from a set of one-dimensional
measurements. This study specifically scrutinized the (1,1) mode of a cylindrical pipe.
Specifically, the radial dependence of the axial velocity was reconstructed from meas-
urements of axial velocity as a function of axial position, demonstrating a method of
modal decomposition.

For the purpose of comparison, two models were proposed. One was the real axial
propagation constant as defined, while the other model was a complex axial propagation
constant. Applied to the measured axial velocity as a function of axial position at a fixed
non-zero radial position, the two-pass curvefitting algorithm determined eight or ten un-
known parameters, depending on the kind of model. This modal decomposition suggested
that the measured axial velocity agreed slightly better with the fit to the model assuming
a complex axial propagation constant than with that assuming a real axial propagation constant. Though indiscernible in the axial range of measurement, the two fits indicated that the disagreement of those fits can be important when the target point is not within the axial range of measurement.

Axial velocity based on each of the two models was constructed as a function of radial position at a fixed axial location. The two constructed velocities were compared with the axial velocity measured as a function of radial position at that axial location. The comparison among the measured and the constructed velocities clearly showed a difference. This suggests that the LDA technique can identify the acoustic field based on a rather unrestricted form of acoustic model without the calibration of sensor involved, whereas any of prevalent microphone techniques are unable to do.

### 8.2 Future work

The LDA technique was proposed and validated through the application to well understood systems. Furthermore, it was applied to two unknown systems to be characterized. All the results shown in the dissertation were promising. The theoretical predictions as well as the measurement with the two-microphone method indicated that the LDA technique can achieve both the result with quality and the applicability to many systems. However, it would be desirable to use this technique in investigating more complex acoustic fields. For example, although the acoustic streaming velocity had been identified with LDA at low frequencies in some studies [11,12], no measurement of acoustic
streaming velocity at high frequencies has been reported yet, especially when a non-planar wave mode is present in the pipe.

Another candidate for future work is to develop a way of performing the signal processing of the raw data taken by the LDA, instead of relying on a BSA. This is suggested because of the symptom that the BSA appears to fail to handle the data fully at high data rates, which is crucial for measurements at high frequency.

The work reported here has demonstrated the utility of the LDA method for modal decomposition. More work can be done. In particular, measuring the power flow in more complicated, multi-dimensional propagation situations could be a potential area of application.
Appendix A

Sample code of the curvefitting algorithm

% This code exemplifies the use of the curvefitting algorithm devised in this dissertation.

% Assumption:
% The row vector of scaled axial velocity: us
% The row vector of axial positions: x
% Based on counter-propagating model, the on-axis (scaled) axial velocity can be expressed as,
% us = A1*exp(Vj*kc*x)VB1*exp(j*kc*x), (A.1)
% where A1 and B1 are the coefficients for the incident and reflected plane wave. kc is the complex propagation constant of the air inside the pipe.
% w and rho are the angular frequency and the density of air.
% Compared with Eq. (2.30), Eq. (A.1) does not include the (kc/rho*w)
% This was considered unnecessary in the curvefitting, because the measured values are velocities directly from the LDA/BSA system, not from estimation based on pressure measurements.
% Neglecting the (kc/rho*w) will require this term to be included when the pressure needs to be estimated.

% The first-pass curvefitting algorithm
% This algorithm is based on the linear algebra which allows for evaluating the solutions of Eq. (2.32) using the fact that the partial derivative of each element with respect to every parameter is zero.
% The following code will yield the same result as Eq. (2.33).

for ii = 1:length(x)
    Xij(ii,:) = [exp(Vj*kc*x(ii)) j*exp(Vj*kc*x(ii)) ... 
                 Vj*exp(j*kc*x(ii)) Vj*exp(j*kc*x(ii))];
end
Y = us(1:length(x)).';
beta = ((Xij.'*conj(Xij)+Xij'*Xij)^-1)*(Xij.'*conj(Y)+Xij'*Y);
A1 = beta(1,)+j*beta(2,);
B1 = beta(3,)+j*beta(4,);
R1 = B1/A1;

% The second-pass curvefitting algorithm
% Based on the definition that 
% $R_1 = B_1/A_1 = \exp(-\eta+j*\theta)$ and assumption that the number of unknown 
% parameters is four (i.e., $k_c$ is known), 
% the second-pass curvefitting using only magnitude of velocity is 
% expressed as follows

\[
z = @(t)\text{abs}(\text{abs}(t(1)\cdot\exp(j*t(2)))\cdot\exp(-j\cdot t(3)/2) \ldots 
  \cdot\exp(-t(4)/2\cdot j\cdot \sin(t(3))/2-j\cdot t(4)/2-k_c\cdot x))\cdot\text{abs}(|u_1|));
\]
\[
t_0 = [\text{abs}(A_1)\ \text{angle}(A_1)\ -\text{angle}(R_1)\ -\log(\text{abs}(R_1))];
\]

% Note that 'kc' can be considered unknown as in Chapter 4. 
% $t_0$ is the initial guess for the second-pass curvefitting. 
% The third and fourth element in the $t_0$ is theta and eta, respectively. 
% $R_1$ is the reflection coefficient from the first-pass.

options = optimset('Display','off','TolX',1e-12,'TolFun',1e-12,... 
  'MaxFunEvals',1e4,'MaxIter',1e4);

% To provide a room for each parameters, 
% 1) 50% of the magnitude for the incident plane wave based on $A_1$ is 
% allowed for the second-pass. 
% 2) phase terms are allowed to choose in the full range. (from $\pi$ to $\pi$) 
% 3) the lower limit of 'eta' is 0 in order to impose the constraint that 
% $|R| \leq 1$.

[output,fval] = lsqnonlin(z,t0,[0 -pi 0],[abs(AA)*1.5 pi pi Inf],options);
A2 = output(1)*\exp(j*output(2)); 
B2 = A2*(\exp(-j*output(3))\cdot\exp(-output(4)));

u2_fit = A2*\exp(-j*k_c*x)-B2*\exp(j*k_c*x);
% $u_2$ fit is the final result of the curvefitting based on this algorithm.
Appendix B

DeltaE model in Chapter 4

The DeltaE model used in Chapter 4 is provided here. (Originally, it was a DeltaE model, but using it in DeltaEC caused the model to be changed into a DeltaEC model. Regardless, they are compatible.) This code is specifically based on the modified impedance for the rigid termination to examine the slight misalignment between the measurement and the DeltaE simulation.

```
TITLE   Sample model for DeltaE simulation used in Chapter 4
!->C:\DeltaE\Kiwon\duct_721.5Hz_correction.out
!Created@15:59:30 04-Nov-2009 with DeltaEC version 6.1b3 under win32, using Win 6.1.7600 () under Python DeltaEC.
BEGIN    Initial
9.6850E+04 a Mean P Pa
721.50   b Freq   Hz
294.15   c TBeg   K
415.60   d |p| Pa  G
22.931   e Ph(p) deg G
3.0317E-04 f |U| m^3/s G
0.0000   g Ph(U) deg
air      Gas type
SURFACE  First end
sameas   2a a Area  m^2  415.60 A |p| Pa
         22.931 B Ph(p) deg
         3.0272E-04 C |U| m^3/s
         -3.6117E-02 D Ph(U) deg
         5.8021E-02 E Htot  W
ideal    Solid type  5.7919E-02 F Edot W
DUCT     First Pipe
1.7738E-03 a Area  m^2  139.44 A |p| Pa
```
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### RPN Monitoring

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Bibliography


Ki Won Jung was born on May 24, 1974 in Seoul, Korea. From the early years, he had expressed his deep interest in mathematics, but not in physics. However, when he became a senior of the high school, he met a physics teacher who provided a turning point that made him fond of physics. After the graduation, he went to Seoul National University in order to study a variety of fields in Electrical Engineering. During the college years, he had enjoyed challenging problems, and never stopped studying a new subject. Among many fields, he was specifically attracted to acoustics. To quench the thirst, he sought for a way to study acoustics further and deeply.

As a solution, he went to the Graduate Program in Acoustics at the Pennsylvania State University. In that program, he met Dr. Anthony Atchley and earned an opportunity in working under his supervision. Due to the endless support and inspiration of his advisor, he undertook the task of developing a new measurement technique based on Laser Doppler Anemometry. At the same time, he has served as the teaching assistant whose duty is to manage the teaching laboratory of the Graduate Program in Acoustics. Being the teaching assistant, he assisted not only Dr. Steven Garrett and Dr. Thomas Gabrielson to provide a lab class which dealt with many measurement techniques in acoustics, but also students to acquire what they needed from the laboratory.

During the years in State College, he became the husband of Ji-Hye Kim, whom he had met while both of them were undergraduate students of the same university in Korea. Currently, he is expecting the graduation after completing his research.