FACTORS THAT INFLUENCE THE LOCK-IN OF FLOW INSTABILITIES WITH STRUCTURAL-ACOUSTIC SYSTEMS

A Dissertation in

Acoustics

by

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ABSTRACT

Lock-in occurs between many different types of flow instabilities and structural-acoustic resonators. Factors that describe the coupling between the fluid and structure have been defined for low flow Mach numbers. This experimental and analytical study shows for which combinations of coupling factors lock-in is possible and for what states it can be achieved. A unified theory has been derived based on describing functions, a nonlinear control theory used to predict limit cycles of oscillation, where a self-sustaining oscillation or lock-in is possible.

A key concept to the lock-in process is the relative source generation versus dissipation. The type of fluid instability source dominates the generation component of the process, so a comparison between a cavity shear layer instability with a relatively stronger source, for example vortex shedding from a bluff body, can be described as a coupling factor. The describing function models capture the primary characteristics of the instability mechanisms and are consistent with Strouhal frequency concepts. The properties of the resonator dominate the dissipation component, so a comparison between an elastic resonator with one that has a similar material to the source, an acoustic resonator, can be described as a coupling factor. The describing function models capture the effect of impedance and damping and are consistent with wake oscillator theory. Also, a comparison between a distributed resonator with one that has a wavelength closer to the source, a discrete resonator, can be
described as a coupling factor. The describing function models capture the effect of a mode shape limited modal mass in comparison to a lumped mass.

In the fluid-elastic cavity lock-in case, the shear layer instability produced by flow over a cavity couples to the elastic structure containing the cavity. This combination of coupling factors has not been studied previously and is the weakest combination under consideration. This type of lock-in was not achieved experimentally. A similar sized cavity, however, was able to achieve lock-in with an acoustic pipe resonator. Fluid-elastic cavity lock-in is unlikely to occur given the critical level of damping that exists for a submerged structure and the relatively weak source strength that a cavity produces. A stronger source such as vortex shedding from a bluff body is able to lock-in to the same resonator as the fluid-elastic cavity case. Thus, the type of instability source and the impedance difference between the source and resonator are dominating coupling factors for lock-in. They represent the generation and dissipation balance that must be met in order to achieve lock-in.

This study shows a strong consistency between the analytical models and experimental results. Describing function modeling is a useful method for better understanding and predicting lock-in, particularly for configurations where synchronization is difficult to achieve experimentally.
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Chapter 1
Introduction

1.1 Overview

The interaction of fluid flow with structures can lead to interesting and sometimes unexpected behaviors in many industrial problems. A wide variety of research in the fields of aeroelasticity and hydrodynamics has generated numerous models for predicting and describing fluid-structure interactions (Naudascher and Rockwell (1994) and Blake (1986)). In the area of flow-induced vibrations, hydrodynamic forces can interact with the elastic motion of a structure, resulting in a fluid-elastic interaction. For example, fluid interacting with a structure may produce vortices which shed at prescribed frequencies. If a shedding frequency coincides with a resonance frequency of an adjoining elastic structure or acoustic fluid volume, as shown in Figure 1-1, the resulting oscillation can reinforce the vorticity, creating a feedback mechanism responsible for producing high energy at resonance. This phenomenon is called lock-in and at low Mach numbers can result in strong narrowband sound radiation and vibration. Lock-in is often undesirable and can promote rapid fatigue failure of structures.

Lock-in can occur between many different kinds of fluid flow instabilities and acoustic or structural resonances. A well studied fluid instability, the
unstable shear layer created from flow over a cavity, produces discrete frequencies associated with flow boundary layer oscillations. This organized vorticity may couple to adjoining or nearby acoustic resonances and to elastic resonances of structures containing the cavity (Rockwell and Naudascher (1978)). An example of this is a cavity shear layer exciting a Helmholtz resonator (e.g., flow across the mouth of a soda bottle). Another commonly studied fluid instability, the separated flow from a bluff body, produces a similar phenomenon but with markedly different characteristics. An example of this is cross-flow shedding from a spring-mounted cylinder (Blevins (1990)) (e.g., wind blowing across a car antenna).

The behavior of lock-in can vary depending on the properties governing not only the fluid instabilities, but also the type of resonator. Resonator characteristics are typically dominated by material properties, damping, and wavelength. Due to the impedance and density differences, a fluid instability will couple to an acoustic resonator of the fluid volume containing the instability more easily than it will to an elastic resonator of the structure exposed to the instability. Also, a fluid instability will couple to a lightly damped, discrete, lumped resonator more easily than a more heavily damped, distributed resonator (e.g., a beam or plate).

The objective of this investigation is to perform a comparative study of the factors associated with lock-in in order to better understand the controlling physics of cavity shear layer instabilities interacting with acoustic and elastic resonators. In particular, very little work in the literature has been found related
to cavity fluid-elastic lock-in. In this case, the shear layer instability produced by a flow over a cavity couples to the elastic structure containing the cavity. Given the difficulty in achieving this type of instability, a systematic evaluation of the factors associated with the fluid instability source and resonator will determine for which combinations of coupling factors lock-in is possible and for what states it can be achieved. The method of describing functions, first introduced by Cremer and Ising (1968), will be expanded to study several forms of lock-in. A review of the concepts and models used to understand the lock-in phenomenon is presented in Section 1.2 followed by a discussion of describing function theory in Section 1.3.
Figure 1-1: Lock-in Process
1.2 Background

This section presents a review of concepts that form the basis of the lock-in phenomenon. Oscillatory and amplification mechanisms used to describe the physics of this phenomenon are captured in several existing types of models. A discussion of the similarities and differences between these models will reveal the most appropriate method for capturing the lock-in coupling factors considered in this study. Then the objectives and approach to the investigation will be formed to improve the understanding of the controlling physics and parameters governing lock-in.

1.2.1 Summary of Flow-Induced Oscillation Mechanisms

The fundamental nature of fluid flows can lead to many classes of flow instabilities. Lock-in due to vortex-induced oscillation is a uniquely different class, distinguishing it from other types of flow instability. Four different types of flow-induced oscillation mechanisms have been identified, including: flutter, galloping, buffeting, and vortex-induced oscillation (Lugt (1996)). Flutter occurs when the interaction between a structure and a flow instability is linear. Galloping occurs when the structural frequency is much lower than that of the vortex shedding and a steady lift force is developed within a stable limit cycle. Buffeting is a turbulence-induced vibration. Vortex-induced oscillation, the subject of this thesis, can lead to lock-in between the natural frequency of a structure, like an elastically mounted cylinder (or bluff body), and the vorticity in
the boundary layer, resulting in large amplitude oscillations. The elastic flexibility or stiffness in the mechanism can also be inherent within the structure itself, as in the modes of a plate. This may occur in cables and bridges, as one example. The degrees of freedom of a structure that couple to an instability can also vary over time and still lock-in. An example of this is a freely falling plate where the motion back and forth as it falls is due to restrictions from gravitational and hydrodynamic forces. Lock-in can be distinguished from other types of instability since it occurs for specific ranges of flow over which coincident frequencies are achieved. At higher flows outside this range, lock out occurs and stability can be reestablished, unlike most other types of instability.

1.2.2 Summary of Shear Flow Fluctuation and Resonant Amplification Mechanisms

When flow separates, as in a jet or a mixing layer, a free shear layer develops and grows, producing oscillatory components (Naudascher (1967)). The growth is initially linear and as it becomes nonlinear, a transfer of energy into higher frequencies occurs. This growth can be defined both spatially and temporally. Small cells of vorticity are associated with the shear layer. As the shear layer grows, the small cells of vorticity cluster together to form a large vortex. If the shear layer impinges on a downstream point, an upstream influence can occur at the shear layer formation point. This influence is nearly instantaneous for low Mach number flows, and can result in a strengthening of the shear layer instability source. Numerous examples of shear layer
impingement exist and have traditionally been categorized by the velocity profile at the separation point (usually a jet or mixing layer) and the shape of the downstream impingement structure. For jet flow, categories include jet-edge (or edge-tone), jet-hole, jet-plate, jet-cylinder, etc. For mixing layer flow, categories include shear-tone, rectangular cavity, and an inclined plate (Naudascher and Rockwell (1994)).

A well studied impingement flow case, the shear layer instability due to a cavity, produces preferred frequencies associated with the hydrodynamic feedback from impingement. This instability occurs without the presence of a resonator, but may act as an excitation source to surrounding resonators. When a resonance is present at a frequency that is close to or coincident with the source frequency and the phase between the resonator and the source is aligned, a lock-in or synchronization can occur.

Three main categories of fluid interaction mechanisms for cavity oscillations have been identified (Rockwell and Naudascher (1978)). Each interaction differs in the feedback path to the fluid source. A fluid-dynamic interaction is defined as the feedback of the shear layer flow from either side of the cavity alone. A fluid-resonant interaction occurs when the shear layer cavity oscillation is coupled to resonant acoustic waves in the cavity. Fluid-resonant interaction is also prevalent for acoustic resonances external to the cavity, such as the fluid in pipes containing the cavity. Both types of fluid-resonant lock-in are shown in Figure 1-2.
A fluid-elastic interaction occurs when a wall of the cavity is able to move and provide a feedback path to the shear layer oscillation. Very limited information exists for this type of fluid-elastic cavity lock-in. In an early investigation (Dunham (1962), Harrington and Dunham (1960)), both water and air were studied in a cavity that initially had rigid walls to achieve a fluid-resonant interaction and then modified with flexible walls to achieve a fluid-elastic interaction. The bottom of the cavity was given a controlled excitation and measurements showed that the pressure fluctuations created within the cavity determined the velocity variations normal to the flow along the cavity. Bland and Skudrzyk (1961) also experimentally studied cavity flow tones with resilient cavity walls. No literature information has been found for lock-in with an elastic resonator external to the cavity. In this case, the structure containing the cavity acts more as a distributed resonator compared to the localized cavity source. Both types of fluid-elastic lock-in are shown in Figure 1-2. In the compliant cavity wall case, the natural frequency of the walls is dominated by the weight of the water loading the structure, rather than the elastic properties of the structure like in the external resonator case. To investigate fluid-elastic coupling, the natural frequency of the resonator would ideally be dominated by the structure alone (along with the displaced water or added mass). One example is an elastic wedge or blade system, where the elasticity or stiffness of the structure interacts with the impinging flow.

Although the characteristics of internal and external resonators are important coupling factors in lock-in, they are not normally considered. If the
resonator is external to the source, the wavelength of the resonator can be much larger than the source instability region. The excitation from the source then produces a response that is limited by the modal mass of the resonator, where the resonator is a distributed system compared to the source. However, if the resonator is internal to the source, the wavelengths of the resonator and source are usually comparable in size. The resonator acts as a discrete mass oscillator which responds more strongly to the excitation than a distributed resonator responds.

Most engineering systems are actually a combination of two or more categories, but are better described in one concentrated area. Much literature exists in the study of fluid-dynamic and fluid-resonant cases, but fluid-elastic cases are more limited and complex due to the coupling of the dynamic characteristics of the structure and the flow. The elastic properties of the structures in the cavity oscillation situations do have an effect on the fluid-dynamic and fluid-resonant categories. By studying the fluid-elastic category in a concentrated manner, models can be developed for predicting fluid-dynamic and fluid-resonant cases where elastic compliance may play a role.
Although elastic interaction with a cavity source has not been widely studied, observations from other types of impingement interactions with elastic interactions may be useful in developing a model for this type of lock-in. One case of interest (listed as an example previously) is a jet followed by a downstream edge, commonly referred to as a jet-edge or edgetone, shown in Figure 1-3. The principle of elastic feedback has been more extensively studied with jets rather than cavities. When a jet impinges on an elastically mounted wedge, the periodicity of the vortex generation provides the frequency interaction.

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for lock-in (Ziada and Rockwell (1982)). The primary vortices distort the wedge position and secondary vortices are created which then feed back on the primary source. The feedback is then strengthened by the wedge oscillation. A similar situation can occur for cavities. The incompressible feedback due to impingement and upstream influence is strengthened when the associated frequency locks into a nearby acoustic or structural resonance frequency, producing a self-sustaining interaction.

Elastic feedback with other types of fluid instability sources has been extensively studied, such as elastically mounted bluff body instability (Blevins (1990)) and blade trailing edge instability (Blake (1986)). These types of fluid sources, however, greatly differ in excitation strength from the impingement cases discussed here, since they are dominated by large separation and gross vortex shedding rather than shear layer instability. Often this type of fluid instability is referred to as a global instability. As a result, oscillations due to
fluid-elastic, rather than fluid-resonant lock-in, can be more violent due to the difference in the energy path created.

1.2.3 Summary of Modeling Approaches for Lock-in

Lock-in is an extremely difficult phenomenon to model due to the nonlinear nature of both the hydrodynamic fluid instability and the resonator feedback mechanism. Several different types of models have been used to describe lock-in. All of the models capture the basic process of lock-in shown in Figure 1-1. The basic premise of lock-in is the excitation of a fluid instability source with a resonator and the feedback of the resonator back onto the source, creating a self-sustained oscillation. In these models, often one component describes the feedforward portion of the process which is dominated by the fluid instability, while another component captures the feedback portion which is dominated by the resonator characteristics. Conditions for which lock-in is achieved vary in each model. The models are similar in concept, while the differences largely depend on the level of fundamental equations or parameters used.

1.2.3.1 Powell Criteria for Lock-in: Importance of Feedback Effectiveness

(Powell (1961, 1995)) created one of the first models to describe lock-in. The effectiveness of the source excitation, transmission, amplification, and other coupling mechanisms are parameterized to find states where stable limit cycles of oscillations occur. The effectiveness for the hydrodynamic feedback of jet-
edge impingement can be described by the following impedance and transfer functions:

source effectiveness \( \eta_s = \frac{F}{u_e} \),

transmission effectiveness \( \eta_t = \frac{p_o}{F} \),

stream disturbance effectiveness \( \eta_d = \frac{u_o}{p_o} \), and

stream disturbance amplification \( q = \frac{u_e}{u_o} \).

The flow parameters shown in Figure 1-4 are pressure \( p_o \), velocity \( u_o \), edge velocity \( u_e \), flow rate \( q \), and dipole force \( F \). For steady-state limit cycle oscillations to exist, the following condition must be met for the gain and phase criteria (complex noted by \( \tilde{\cdot} \))

\[ \tilde{\eta}, \tilde{\eta}, \tilde{\eta}, \tilde{q} = 1. \]

Figure 1-4: Jet-edge Impingement (Powell (1995))

When a resonator oscillation is present, an indirect path from an acoustic resonance may provide feedback in addition to the feedback provided by hydrodynamic impingement, referred to here as the direct path as shown in
Figure 1-5. As the addition of an acoustic resonator forms the indirect path, the limit cycle criteria for lock-in becomes

\[ \tilde{\eta}_s (\tilde{\eta}_r + \tilde{\eta}_d) \tilde{\eta}_d \tilde{q} = 1. \]  

This model demonstrates the use of transfer functions to describe the feedforward and feedback components of the lock-in process. The roles of the coupling factors are apparent in the impedances that form these transfer functions.

**1.2.3.2 Howe Power Model for Lock-in: Importance of Generation and Dissipation**

A more recent model developed in 1998 is called Howe’s power (Howe (1998)). This model predicts lock-in when the acoustic power generated by the
source or derived from the flow exceeds the power dissipated in damping.

Howe’s power is unique in that it is specifically derived for cavity shear layer lock-in as shown in Figure 1-6. The power generation term is given by

$$\Pi = -\rho \int u_{ac} \cdot (\omega \times v) dV$$

where $u_{ac}$ is the particle acoustic velocity vector, $\rho$ is the density, $\omega$ is the vorticity vector, and $v$ is the fluctuating hydrodynamic velocity. These parameters can be reduced conceptually to the vectors depicted in Figure 1-6.

Although fluid-resonant lock-in is more common in industrial applications, this model also applies to fluid-elastic lock-in. The acoustic velocity fluctuation can be used to describe an acoustic or structural vibration (Howe (1998)). This model demonstrates the importance of capturing the roles of source generation versus resonator dissipation in the lock-in process. It is also worth noting that there are other models similar to Howe’s power that use energy-based approaches (e.g., Nelson et al. (1981 and 1983)) to capture this concept.
1.2.3.3 Wake Oscillator Model: Importance of Elastic Instability Concepts

The most common form of lock-in modeling in vortex-induced vibrations is the so-called wake oscillator model. Numerous forms of this model exist (Blevins (1990)). The form commonly used to describe conditions for which instability occurs is an equation of motion governing a body oscillating in flow as

\[ m\ddot{y} + 2ζmω_n\dot{y} + mω_n^2y = F_y(t), \]

where \( m \) is the oscillator mass, \( ζ \) is the oscillator damping, \( ω_n \) is the natural frequency, \( y \) is the displacement of the oscillator, and \( F_y \) is the time-dependent transverse forcing function. For a cylinder in cross flow, a forcing function is a typically used form that captures components in phase with both displacement and velocity

\[ F_y(t) = ρd^2\omega_n(h_a\dot{y} + k_a\ddot{y}), \]

where \( h_a \) is the added mass coefficient and \( k_a \) is the excitation coefficient (Naudascher and Rockwell (1994)), both of which are estimated empirically. A flow instability can occur when \( k_a > S_c \), where \( S_c \) is the Scruton number (or mass-damping parameter) defined as \( 2mζ/ρd^2 \). That is, when the hydrodynamic mass of the cylinder, \( ρd^2 \), is large compared to the oscillator mass (as it is in heavy fluids such as water) and the damping is low, the mass-damping parameter needed for the onset of instability is small. This condition is sometimes referred to in the literature as negative damping and captures the concept of excitation.
versus dissipation similar to that in Howe’s power. For example, with a decrease in system damping, the resonant response peak will sharpen.

More complicated forms of wake oscillator modeling exist which more explicitly include feedback effects. One such model is the Van der Pol model which was successfully used to model trailing-edge lock-in (Blake et al. (1977)).

In general, the area of wake oscillator modeling is a good example of using basic oscillator parameters to capture source and resonator coupling factors which can be used to more accurately describe lock-in.

### 1.2.3.4 Describing Function Model: Importance of Flexibility in Modeling

The final modeling technique presented here is called describing function theory. This type of model was first developed by Cremer and Ising (1968) and then later expanded by Elder (1978) to the form presented here. This method approaches lock-in modeling in a manner similar to Powell’s lock-in effectiveness and transfer functions, and is consistent with the principles of Howe’s power and wake oscillator theory. Like Powell’s model, impedance based functions are used to describe transfer functions for the source and resonator response and feedback. A concept from nonlinear control theory, describing function theory predicts limit cycles of oscillation based on the linear and nonlinear transfer functions. If a limit cycle exists, a self-sustaining oscillation or lock-in is possible.

The describing function method is an alternative to the classical frequency response method for studying a nonlinear system (Slotine and Li (1991)). The
A typical derivation uses the classical Van der Pol equation, where the nonlinear term and linear term, $G$, are put into a transfer function. A quasi-linear approximation is made for the nonlinear term and is defined as $N$. Figure 1-7 illustrates the block diagram for these terms.

![Figure 1-7: Block Diagram for Describing Function Theory](image)

If a sustained lock-in oscillation of amplitude $A$ and frequency $\omega$ exists, then

\[
x = -y, \\
w = Nx, \\
y = Gw, \text{ and} \\
y = GN(-y).
\]  

Because $y \neq 0$, this implies

\[
GN + 1 = 0, \quad \text{and} \\
G = -\frac{1}{N}.
\]

If this system of equations has no solutions, then the nonlinear system does not have limit cycles and lock-in cannot occur. These equations are generally difficult to solve analytically, so they are typically plotted such that the intersection of both sides of Eqn. 1.8 on a real versus imaginary plot are found. The intersections indicate conditions where lock-in can occur.
There have also been recent studies of modeling cavity lock-in by using the method of describing functions (Elder (1978), Elder et al. (1982), Kook and Mongeau (2002), and Mast and Pierce (1995)). In order to study the effect of nonlinearity on an oscillating system, two describing functions can be chosen such that the product is one under conditions for which a limit cycle (lock-in) occurs. The two functions for the oscillator are chosen so that they are reciprocals of each other. For example, a Helmholtz resonator can be modeled with a forward gain function (FGF) as the nonlinear interaction between the oscillation flow in the resonator neck and the unstable mean flow:

\[
\frac{q_o(q_r)}{q_r},
\]

where \(q_o\) is the neck volume velocity and \(q_r\) is the resonator volume velocity as defined for a Helmholtz type cavity in Figure 1-8. The backward gain function (BGF) is the linear effect of the flow disturbances forcing the resonator

\[
\frac{q_r(q_o)}{q_o}.
\]

In the forward gain function, \(q_r\) is given while \(q_o\) is from the backward gain function. The total flow is the sum of the neck and resonator flow. The final describing function equation becomes

\[
\frac{q_o(q_r)}{q_r} \frac{q_r(q_o)}{q_o} = 1,
\]

which is plotted to find the existence of limit cycles where states of lock-in are possible.
The FGF is based on the fluid instability source. For cavity lock-in, this source can be modeled using linear stability theory (Elder (1978)) or a hydrodynamic force approach (Kook and Mongeau (2002)). The hydrodynamic force approach is the basis of the power generation in Howe’s power model and is especially convenient for comparing different types of instability sources since it can be derived for other types of sources, such as gross vortex shedding from a bluff body.

The BGF is a lumped impedance model of the resonator, which captures its mass, stiffness, and damping characteristics. This approach can be rederived for any resonant system, which is especially convenient for comparing different types of resonators.

The describing function models for cavity lock-in with internal acoustic Helmholtz-type resonances have compared well to empirical data in several investigations (Elder et al. (1982)), Kook and Mongeau (2002), and Mast and Pierce (1995)). However, published literature for using this type of modeling for other fluid instability sources and elastic resonators has not been found. Other
investigators who have used the describing function method include Fletcher (1976), Fletcher and Rossing (1998), and Coltman (1968).

Since the method of describing functions can be expanded to capture different types of fluid instabilities and resonators, this method will be adopted for use in this investigation. However, the use of the describing function theory to represent a broad range of lock-in phenomena has not been found in the literature. While this model uses a lumped parameter based approach, it still captures the lock-in fundamental process, the relative source generation versus dissipation for lock-in, and the critical instability states for lock-in. Other models such as Howe’s power model require a large amount of phase-referenced experimental data which makes them impractical except in numerical studies. Also, models like Powell’s require numerous assumptions which would be invalid in a comparative study of coupling factors. In addition, the wake oscillator model is not amenable to a comparative study that includes shear layer instabilities. Therefore, the describing function modeling technique is the most suitable for this investigation. It is applicable to all sources and resonator properties considered in this investigation while capturing the strengths in the other analysis methods.

1.3 Objectives of the Investigation

The objective of this investigation is to perform a systematic evaluation of the controlling factors associated with fluid instability sources and resonators. A unified describing function theory and experimental study will predict which
combinations of coupling factors lock-in is possible and what states it can be achieved.

A review of concepts that form the basis of the lock-in phenomenon has been discussed, along with the many types of lock-in and models which exist. Common coupling factors have been used to distinguish the different types of lock-in and can now be presented in terms of the lock-in process to demonstrate how this investigation will be approached.

A key concept to understanding the lock-in process is the source generation relative to dissipation at lock-in. The type of fluid instability source dominates the source generation component of the process, so a comparison between a cavity shear layer instability with a relatively stronger source, bluff body vortex shedding, can be described with coupling factors. Also, the properties of the resonator dominate the dissipation component, so a comparison between elastic and acoustic resonators can be described with coupling factors. Finally, a comparison between a distributed resonator with one that has a wavelength closer to the source, a discrete resonator, can be described with coupling factors. Systematically, each one of these factors will be independently varied to determine which combination of coupling factors can generate lock-in. The lock-in process is dominated by the relative source strength and resonator damping, which can be evaluated for each combination of factors or lock-in type.

Many combinations of lock-in coupling factors have been studied in the published literature. However, as discussed earlier, the lock-in of a cavity shear layer instability with a distributed elastic resonator has not. Chapter 2 will
present a unified describing function theory that captures the various lock-in coupling factors. This model will be used in conjunction with an experimental study to provide physical insight into why fluid-elastic cavity lock-in is difficult to achieve compared to other types of lock-in. Chapter 3 will discuss the principles of fluid instability and resonator feedback used to design the experimental arrangement.

There are many aspects of lock-in that are not well understood. The results of this investigation will explore the influence and importance of different coupling factors to determine for which conditions lock-in is possible. Chapter 4 will discuss the experimental and analytical results of a fluid-elastic cavity lock-in study and compare them with different combinations of coupling factors to show why this type of lock-in occurs over a narrower range of conditions. In Chapter 5, this investigation will summarize how different types of lock-in are more challenging than others to achieve. Using the results of this study, an improved fundamental understanding of the roles of the coupling factors in generating the lock-in phenomenon will be presented.
Chapter 2

Lock-in Coupling Factor Models

In this investigation, coupling factors that control the lock-in process are explored analytically and experimentally. A systematic evaluation of coupling factors for lock-in has not been found in the literature despite the existence of many individual studies. Numerous types of analytical models exist which attempt to capture the lock-in process for individual cases. A unified model that captures all of the coupling factors is formed in this investigation using the describing function theory presented in Chapter 1. By applying this modeling approach to the lock-in process, a better understanding is formed of why certain cases of lock-in are rarely encountered. This understanding is ultimately confirmed in the experimental portion of this investigation.

This chapter will present the describing function theory approach to lock-in modeling, how a comparative study of lock-in coupling factors will be accomplished, and derives the models necessary to determine why lock-in occurs more readily for certain combinations of coupling factors than others. Since not much is known about fluid-elastic cavity lock-in, this modeling will be used in conjunction with an experimental system to provide physical insight into why fluid-elastic cavity lock-in is difficult to achieve compared to other types of lock-in.
2.1 Modeling Oscillators Using Describing Functions

As initially introduced in Chapter 1, describing function theory is a method used to model nonlinear oscillations and predict states of limit cycle phenomena. This method has been used in the past to model fluid-resonant cases of cavity lock-in. One example is a cavity shear layer locking into a Helmholtz resonator. Figure 2-1 shows the shear layer instability created at the mouth of the resonator interacting with the resonance of the system formed by the neck and volume backing the mouth, commonly known as a Helmholtz resonator. The shear layer instability excites the resonance of the system which then produces a feedback effect from the pressure oscillations onto the shear layer instability, resulting in a self-sustaining oscillation. When the resonator damping is sufficiently small, this lock-in process produces high amplitude narrowband tones.

Figure 2-1: Helmholtz Resonator Lock-in Example. $M$ is the fluid mass in the neck, $C$ is the stiffness of the fluid in the volume, and $R$ is the viscous damping of the resonator.
In order to study the effect of nonlinearity due to lock-in on the oscillating system, two describing functions are chosen such that the product is one under conditions for which a limit cycle occurs. The two functions for the oscillator are chosen such that they are reciprocals of each other. For the example of a Helmholtz resonator, the forward gain function (FGF) is the nonlinear interaction between the oscillation flow in the resonator neck and the unstable mean flow:

\[ \frac{q_o(q_r)}{q_r} , \tag{2.1} \]

where \( q_o \) is the neck volume velocity and is dependent on \( q_r \), the resonator volume velocity, as defined for a Helmholtz type cavity in Figure 2-2. The backward gain function (BGF) is the linear effect of the flow disturbances forcing the resonator

\[ \frac{q_r(q_o)}{q_o} . \tag{2.2} \]

In the forward gain function, \( q_r \) is given while \( q_o \) is given in the backward gain function. The total flow is the sum of the neck and resonator flow. The final describing function equation for lock-in is then

\[ \frac{q_o(q_r)}{q_r} \frac{q_r(q_o)}{q_o} = 1 , \tag{2.3} \]

which is usually plotted to find the existence of limit cycles where states of lock-in are possible.
The BGF includes a lumped impedance model of the resonator, which captures the lumped mass, stiffness, and damping characteristics. Commonly referred to as a jet-drive model, the jet sees the pipe as a parallel-resonant circuit, where the oscillating mass and the compliance interact with different flows (Elder (1973, 1978)). This is modeled as a flow-divider circuit as shown in Figure 2-3 which is expressed by the following equations

\[ q_r = -q_o \left( \frac{z_c}{z_m + z_c} \right), \]

where \( z_c \) is the acoustic impedance (pressure/volume velocity) of the compliance and \( z_m \) is the acoustic impedance of the oscillating mass. The damping of the resonator, as captured in this example by \( R \), includes both the viscous and radiation damping. The flow into the cavity is always modeled as positive, hence \( + q_o \).
The input acoustic impedance of the compliance is given by

\[ z_c = \frac{1}{S^2} \left( \frac{1}{j \omega C} \right) \]

or

\[ z_c = \frac{1}{S^2} \left( \frac{K}{j \omega} \right) \]

where \( S \) is the cross-sectional area of the volume and \( K \) is the stiffness or inverse of the compliance (Blackstock (2000)). The acoustic impedance of the resonator neck includes both the lumped mass and resistance elements as follows

\[ z_m = \frac{1}{S^2} \left( R + j \omega M \right) \]

When combined in the gain function form given in Eqn. 2.4, the BGF for a Helmholtz resonator is

\[ \frac{q_r}{q_o} = \frac{j \frac{K}{\omega}}{R + j \left( \frac{\omega M}{\omega} - \frac{K}{\omega} \right)} \]
Using the following resonance frequency and damping definitions for a single degree of freedom oscillator \( \omega_r = \sqrt{\frac{K}{M}} \) and \( Q = \frac{\sqrt{MK}}{R} \) respectively, the BGF is simplified to

\[
\frac{q_r}{q_o} = \frac{j\omega_r^2}{\omega_o \omega_r + j(\omega^2 - \omega_r^2)}.
\]

This approach for forming the BGF can be rederived for any resonant system, which is especially convenient for comparing different types of resonators.

The FGF represents the fluid instability source. For lock-in due to a shear layer instability, previous studies have used several different methods for modeling this source. One of the original describing function studies modeled the shear layer instability source with linearized stability theory (Elder (1978)). The linearized stability model was first introduced by Michalke (1965) to describe the nonlinear growth rate of a shear layer or mixing layer encountered in cavity flow.

A more recent study modeled the shear layer instability with a hydrodynamic forcing function (Kook and Mongeau (2002)). This approach is derived from power generation in Howe’s power approach, as shown in Section 2.4.2, and is especially convenient for comparing different types of instability sources since it can be derived for other types of sources, such as gross vortex shedding from a bluff body. For the shear layer instability, an external oscillating pressure is derived based on the net circulation of flow in the cavity and models the convection of a line vortex, but accounts for the effect of vortex diffusion.
where $\alpha$ is a vortex concentration factor from 0 to 1 and the Strouhal number is defined as $St_\phi = \omega L_c / U_c$ where $U_c$ is the convection velocity (Kook and Mongeau (2002)). In a similar study by Mast and Pierce (1995), the flow through the neck can be described as

$$q_o = \frac{p_{ext}}{z_m},$$

where the mass impedance of the neck (without the loss terms) is given by the right hand side of Eqn. 2.6, or $M = \rho S \Delta L$ where $\Delta L$ is the height of the neck.

Commonly, $p_{ext}$ is modeled as an oscillating pressure proportional to $\rho U^2$.

Combining Eqns. 2.9 and 2.10, the FGF is

$$\frac{q_o}{q_r} = \frac{S \alpha U^2}{j St_\phi \omega(\Delta L) q_r} e^{-jSt_\phi}.$$

When the BGF and FGF are plotted on a Nyquist plot to satisfy Eqn. 2.3, limit cycles of oscillations can be found in Figure 2-4, where $G_{12}$ is the FGF and $G_{21}$ is the BGF such that the equation

$$G_{12} = \frac{1}{G_{21}}$$

is satisfied. The FGF appears as a spiral due to the time delay, whose domain size depends on $\alpha$. As the strength of the source increases, the FGF domain increases.
The inverse of the BGF appears as a line whose slope depends on $Q$.

The reduced inverse BGF from Eqn. 2.8 is

$$\frac{1}{G_{21}} = -\frac{j\omega}{\omega_r Q} + \left(\frac{\omega}{\omega_r}\right)^2 - 1.$$  \hspace{1cm} 2.13

As the damping of the system decreases (increasing $Q$) and the strength of the source increases, intersections of the two functions are more likely to occur, resulting in limit cycles where states of lock-in are possible. Two different levels of $Q$ are shown in Figure 2-4, where several intersections with the higher level of $Q$ occurs (one intersection example is shown). Thus, this type of stability plot captures the concept of source generation versus dissipation for achieving lock-in (unlike more traditional frequency coincidence plots which will be presented in Chapter 3). The gain functions can be thought of as influence equations, where the controlling parameters for this model are $Q$ and $\omega/\omega_r$. While this modeling approach is not sufficiently accurate for quantitative estimates of lock-in, it can be used to qualitatively study and better understand the lock-in process.
2.2 Describing Function Theory for Comparative Study of Coupling Factors

The lock-in coupling factors under consideration in this study include the type of fluid instability source, the material and physical properties of the resonator, and the relative wavelength between the source and the resonator. Controlling parameters have been found for each coupling factor. For this investigation, a limited combination of coupling factors is explored by independently varying one factor while keeping the others constant. For each
coupling factor, relatively weaker and stronger cases as defined by controlling parameters are considered.

A key concept to the lock-in process is the relative source generation versus dissipation. The type of fluid instability source dominates the source generation component of the process, so a comparison between a cavity shear layer instability with a relatively stronger source, for example a global instability from bluff body separated vortex shedding, can be described as a coupling factor. The properties of the resonator dominate the dissipation component, so a comparison between an elastic resonator with one that has a similar material to the source, an acoustic resonator, can be described as a coupling factor. Also, a comparison between a distributed resonator with one that has a wavelength closer to the source, a discrete resonator, can be described as a coupling factor.

In the fluid-elastic cavity lock-in case, a unique combination of coupling factors occurs that has not been studied previously. This case is a shear layer instability due to a cavity embedded on a distributed elastic resonator, and is the combination of the weakest factors under investigation. When the controlling parameters for each factor are varied to provide a stronger coupling mechanism, a previously studied lock-in case is achieved, as shown in Figure 2-5. When a discrete resonator is combined with the shear layer instability and an elastic resonator, a well known lock-in case referred to as a jet-edge occurs (Ziada and Rockwell (1982)). When a global instability such as separating vortex shedding is combined with a distributed elastic resonator, well known cases of bluff body or trailing edge lock-in are achieved (Blake et al. (1977)). Finally, when a fluid
acoustic resonator is combined with a shear layer instability in a distributed configuration, a previously studied case of the lock-in between a shallow cavity in longitudinal acoustic pipe resonators occurs (Rockwell et al. (2003)).

<table>
<thead>
<tr>
<th>Coupling Factor Combination</th>
<th>Lock-in Case</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Discrete</strong> – <strong>Shear Layer</strong> – <strong>Fluid/Elastic</strong></td>
<td>Jet-edge Upstream Influence Shear Layer Oscillation Vibrating Wedge</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Distributed – <strong>Vortex Shedding</strong> – <strong>Fluid/Elastic</strong></td>
<td>Bluff Body Trailing Edge</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Distributed – <strong>Shear Layer</strong> – <strong>Fluid/Fluid</strong></td>
<td>Shallow Cavity in Pipes</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Distributed – <strong>Shear Layer</strong> – <strong>Fluid/Elastic</strong></td>
<td>None</td>
</tr>
</tbody>
</table>

Figure 2-5: Comparative Cases of Lock-in

Experimentation will reveal if the shear layer instability produced by a cavity embedded on a cantilevered distributed elastic resonator can achieve lock-
in. Describing function models will be used to explore which coupling factor produces the largest contribution in determining whether lock-in can be achieved.

A model has been created for each of the cases shown in Figure 2-6. In order to study the challenges in investigating fluid-elastic cavity lock-in, coupling factors are systematically varied. By varying the coupling factor from elastic to acoustic, the fluid-structure difference between the source and resonator is evaluated. By varying the coupling factor from shear layer instability to global vortex shedding instability, the difference in source coupling is evaluated. By varying the coupling factor from distributed to discrete resonator, the difference in resonator to source wavelength will be evaluated. For each of these combinations, the source strength and system damping is varied.

Thus, the greatest benefit of using a unified describing function theory to understand lock-in is the ability to not only evaluate different combinations of coupling factors, but also to vary the source strength and system damping to evaluate critical states of source generation versus system dissipation in each combination. The schematics in Figure 2-6 represent conceptual interpretations for modeling purposes only. Practicable configurations that are used for experimentation are presented in Chapter 3. Results from the models will be presented along with the experiment in Chapter 4.
2.2.1 Lock-in of Cavity Source with Compliant Wall

One combination that will not be studied is cavity lock-in with a compliant wall. Although this case is the only fluid-elastic cavity lock-in case that has been studied previously, albeit in a limited manner, the coupling between the shear layer instability and the compliance of the wall at the bottom of the cavity, shown in Figure 2-7, is unique. Rather, the purpose of this study is to bound the difference in wavelengths between the instability source and the elastic resonator.

When the resonator wavelength is proportional in magnitude to the source wavelength, coupling can occur in a distributed manner. In the case of lock-in between a cavity shear layer instability and a compliant wall of the cavity, the wall resonance cannot be assumed as lumped compared to the source. Recirculation in the cavity is directly coupled to the compliant wall wave. Since the fluid flow must be implicitly described in great detail to capture the coupling, lumped parameter modeling such as the method of describing functions is not suitable. Numerical studies using computational finite element modeling (Kwon (2006)) have been used for this case.

In the compliant cavity wall case, the natural frequency of the walls is dominated by the mass of the water on the structure, rather than the elastic properties of the structure like in the external resonator case. To investigate fluid-elastic coupling, the natural frequency would ideally be dominated by the structure alone (along with the displaced water or added mass), similar to an
elastic wedge or blade system, where the elasticity or stiffness of the structure interacts with the impinging flow. This configuration is also difficult to study experimentally.

To couple the wall to the shear layer instability, a relatively large flow speed is required to produce a coincident frequency condition which is challenging in a small water tunnel facility. To lower the flow speed required, the natural frequency of the structure should be increased, resulting in a larger size which is in conflict with the need for a smaller inflow cross-section necessary for the increased flow speed.

Also, damping will be a dominating mechanism in water, inhibiting lock-in. Detailed coupled flow and elastic measurements would be necessary. Given these challenges, the study of this type of lock-in is outside the scope of this investigation which is primarily concerned with a comparative study of lock-in coupling factors.
<table>
<thead>
<tr>
<th>Resonator Gain Function $G_{21}$</th>
<th>Source Gain Function $G_{12}$</th>
<th>( \text{Shear Layer} )</th>
<th>( \text{Vortex Shedding} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{Discrete} )</td>
<td>( \text{Elastic} )</td>
<td>( \text{Elastically mounted cavity} )</td>
<td>( \text{Elastically mounted bluff body} )</td>
</tr>
<tr>
<td>( \text{Acoustic} )</td>
<td>( \text{Cavity with Helmholtz resonator} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{Elastic} )</td>
<td>( \text{Cavity on cantilevered beam} )</td>
<td>( \text{Bluff body on cantilevered beam} )</td>
<td></td>
</tr>
<tr>
<td>( \text{Distributed} )</td>
<td>( \text{Acoustic} )</td>
<td>( \text{Cavity in long organ pipes} )</td>
<td>( \text{Bluff body in long organ pipes} )</td>
</tr>
</tbody>
</table>

Figure 2-6: Coupling Factor Configurations

Figure 2-7: Cavity with Compliant Wall
2.3 Derivation of Resonator Backward Gain Functions

The resonator backward gain function models are derived based on impedances. While the Helmholtz resonator model was presented as an example in the preceding section, the other models in Figure 2-6 are complicated by material differences between the source and resonator and the differences in resonance behavior of discrete or lumped systems versus distributed systems. The following equation is the common basis for all of the models:

\[
G_{21} = \frac{q_r}{q_o} = -\left(\frac{z_c}{z_m + z_c}\right), \quad 2.14
\]

where \(z_m\), the acoustic impedance of the oscillating mass, remains the same for each model since it captures the effect of the cavity and \(z_c\) varies depending on the resonator system containing the cavity. A parallel-resonant model like the one shown in Figure 2-3 is still valid since the flow is still split when modified by resonant waves. For the global vortex shedding instability, the oscillating mass is still an accurate model since shedding from a confined bluff body can be modeled by two coupled masses oscillating out of phase. Thus, a single summary of the resonator functions can be formed as shown in Figure 2-8.
### 2.3.1 Discrete Elastic Resonator

The elastic equivalent of an acoustic Helmholtz resonator is an elastically mounted cavity or bluff body. The primary difference between an acoustic and elastic model for a discrete resonator excited by a fluid instability is the fluid-structure difference between the source and the resonator. Another difference is

#### Figure 2-8: Summary of Resonator Models

<table>
<thead>
<tr>
<th>Model Type</th>
<th>Description</th>
<th>Acoustic Equation</th>
<th>Elastic Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cavity or bluff body in long organ pipes</td>
<td>Elastic</td>
<td>( z_m = \frac{\rho_s c_f}{S_s} (j k \Delta L) )</td>
<td>( z_e = \frac{\rho_s c_f}{S_r} \left( \frac{K}{j \omega} + R \right) )</td>
</tr>
<tr>
<td>Cavity or bluff body on cantilevered beam</td>
<td>Elastic</td>
<td>( z_m = \frac{\rho_s c_f}{S_s} (j k \Delta L) )</td>
<td>( z_e = \frac{j M}{\omega^2 S_r^2} - \frac{\phi}{\omega} )</td>
</tr>
<tr>
<td>Cavity with Helmholtz resonator</td>
<td>Acoustic</td>
<td>( z_m = \frac{1}{S^2} \left( R + j \omega M \right) )</td>
<td>( z_e = \frac{1}{S^2} \left( \frac{K}{j \omega} \right) )</td>
</tr>
</tbody>
</table>

\( \phi = \left[ \cosh \left( \frac{\mu \alpha}{L_r} \right) - \cos \left( \frac{\mu \alpha}{L_r} \right) - \sigma \left( \sinh \left( \frac{\mu \alpha}{L_r} \right) - \sin \left( \frac{\mu \alpha}{L_r} \right) \right) \right] \times \left[ \cosh \left( \frac{\mu \alpha}{L_r} \right) - \cos \left( \frac{\mu \alpha}{L_r} \right) - \sigma \left( \sinh \left( \frac{\mu \alpha}{L_r} \right) - \sin \left( \frac{\mu \alpha}{L_r} \right) \right) \right] \)

\( z_e = \frac{\rho_s c_f}{S_r} \left( j \cot k \frac{\pi}{2Q} - \frac{\pi(2n-1)}{4Q} \right) \)

\( z_m = \frac{\rho_s c_f}{S_s} (j k \Delta L) \)

\( z_e = \frac{\rho_s c_f}{S_r} \left( j \tan k \frac{n \pi}{2Q} \right) \)

\( z_e = \frac{\rho_s c_f}{S_r} \left( - j \cot k \frac{\pi(2n-1)}{4Q} \right) \)
that damping is dominated by the mechanical losses in the elastic mounting
mechanism rather than in the acoustic losses in the cavity. Therefore, the elastic
version of the Helmholtz resonator includes the characteristic impedance of the
material, \( \rho_c \), as a scaling factor applied to the original lumped impedance terms in
Eqn. 2.5 (and thus, noted by *):

\[
\begin{align*}
    z_c^* &= \left( \rho_c c_e \right) z_c = \frac{\rho_c c_e}{S_r^2} \left( \frac{K}{j\omega} + R \right), \\
    z_m^* &= \left( \rho_f c_f \right) z_m = \frac{\rho_f c_f}{S_s^2} (j\omega M)
\end{align*}
\]

where the \( e \) and \( f \) subscripts denote elastic and fluid, respectively, and the \( r \) and \( s \)
subscripts denote the resonator and source, respectively. The damping term is
assumed to be dominated by the mechanical losses in the elastic or spring
mounting system rather than source related mechanisms such as radiation or
viscous damping. The resonator gain function is then determined by
substituting Eqn. 2.15 into 2.14 and following the same oscillator reductions used
for the Helmholtz resonator example:

\[
G_{21} = \frac{j\omega_r^2 - \frac{\omega \omega_r}{Q}}{\frac{\omega \omega_r}{Q} + j \left( \frac{\rho_f c_f}{S_r^2} \right) (\omega^2 - j\omega_r^2)} .
\]

Since the gain functions are a ratio of impedance terms, the scaling of the
impedance terms that captures the fluid structure difference between the source
and the resonator falls into a single nondimensional term. This resonator gain function, therefore, can be compared to the others in this study.

The controlling parameters for this model are $Q$, $\omega/\omega_n$, $\rho_f c_f / \rho_e c_e$, and $S_r / S_s$. The graphical interpretation of the inverse resonator gain function and controlling parameters for a discrete elastic resonator is shown in Figure 2-9. As damping decreases (or $Q$ factor increases), the inverse BGF domain gets closer to the center of the Nyquist plot where intersections with the FGF and lock-in are more likely. The origin of the inverse BGF is at (-1, 0), but approaches (0, 0) as $f/f_r$ approaches 1. As the ratio of the characteristic impedance, $\rho c$, of the fluid to the structure decreases, the arc becomes much shorter in length, such that the domain of the inverse BGF is decreased to the limit that lock-in is not likely to occur. Coupling between a source and a resonator is more likely to occur when the density of the two materials are close, such as water and Aluminum as shown in Figure 2-9. When the fluid is air, the inverse BGF is very short and does not produce a sufficiently sizable domain, making an intersection with a FGF and lock-in highly unlikely. This modeling captures the importance of the material density ratio and damping, similar to the mass-damping parameter used in wake oscillator modeling to predict instability. The final controlling parameter, $S_r / S_s$, will be explored in the next section.
2.3.2 Distributed Acoustic Resonator

The primary difference between a discrete and distributed model for an acoustic resonator is the wavelength between the source and the resonator. Therefore, the distributed version of the acoustic resonator includes functions for the distribution of resonance along the length of an organ pipe containing the source. As shown in Figure 2-8, this model has two versions, the first one being a side branch resonator or deep cavity configuration as modeled previously by

Figure 2-9: Nyquist Plot for a Discrete Elastic Resonator; Note the curve for air is extremely short and is highlighted in red
Elder (1978) and the second being a longitudinal resonator with a shallow cavity. Both versions have been investigated experimentally for lock-in (Yang (2005) and Rockwell et al. (2003)). This investigation will be the first to compare the experimental results using a describing function model for both configurations.

The side branch resonator is modeled as a quarter wavelength resonator with open-closed boundary conditions and an oscillating mass representing the cavity as follows

\[ z_c = \frac{\rho f c_f}{S_r} \left( -j \cot k_f L_r + \frac{\pi (2n-1)}{4Q} \right) \]

\[ z_m = \frac{\rho f c_f}{S_s} (jk_f \Delta L) \]

where \( n \) is the mode number, \( k_f \) is the fluid acoustic wavenumber, \( L_r \) is the side branch resonator length, and \( \Delta L \) is the end effect of the cavity which is equivalent to the neck in a Helmholtz resonator. The surface areas of the source, \( S_s \), and resonator, \( S_r \), are identical. The damping term is largely attributed to the acoustic resonator damping mechanisms rather than source related mechanisms. This term was used extensively by Elder (1978) and Cremer and Ising (1968). The amount of absorption, \( \alpha_{abs} \), in the side branch is defined in terms of impedance as

\[ z = \frac{\rho f c_f}{S_r} \left( \alpha_{abs} L_r \right) \]

At mechanical resonance, the quality factor for resonance \( n \) using the half power point method is defined as

\[ Q = \frac{\omega_n}{2c_f \alpha_{abs}} \]
and open-closed acoustic resonant natural modes occur for

\[
\cos(k_j L_r) = 0
\]

or

\[
k_j L_r = \frac{\pi(2n - 1)}{2}.
\]  

Substituting Eqn. 2.19 and 2.20 into 2.18 yields the damping impedance in Eqn. 2.17. The resonator gain function is then

\[
G_{\text{21}} = \frac{j \cot k_j L_r - \frac{\pi(2n - 1)}{4Q}}{\frac{\pi(2n - 1)}{4Q} + j(k_j \Delta L - \cot k_j L_r)}.
\]  

The controlling parameters for this model are \( Q, f/f_r, \) and \( \Delta L/L_r \). The graphical interpretation of a resonator gain function and controlling parameters for a side branch resonator is shown in Figure 2-10. The resulting inverse BGF appears as a circle on a Nyquist plot. As damping decreases, the diameter of the circle increases, which increases the domain similar to that of the arc and line in the previous discrete elastic and Helmholtz resonator examples, respectively. Similar to the previous examples, the domain also starts at approximately (-1, 0) on the plot. Although not shown on this figure, as the resonance frequency decreases, the inverse BGF begins to resemble the arc in the Helmholtz resonator example. The resonator inverse BGF model is consistent with fundamental acoustic theory, distributed resonators approach discrete lumped parameters models at low frequency.

As the ratio of the cavity end effect and the resonator length increases, the diameter of the circle increases, which also increases the domain, as shown in
Figure 2-10. A greater cavity end effect (here increased to 5" (127 mm) from 0.75" (19.1 mm)) produces a stronger coupling since the participation of the oscillating cavity mass in the resonance relative to the side branch resonator length (19" (482.6 mm)) increases, producing a greater likelihood of lock-in. A critical ratio exists for there to be even a possibility of lock-in, as shown by the smaller size ratio (here the end effect decreased to 0.02" (5.1 mm) from 0.75" (19.1 mm)).
Figure 2-10: Nyquist Plot of a Distributed Acoustic Side Branch Resonator Inverse BGF with Varying Q Factor ($\Delta L/L_r = 0.04$, Yang (2005)) and Varying Size Ratio $\Delta L/L_r$ (Q = 500); Based on the following parameters: $L_c = 2"$ (50.8 mm), $\Delta L = 0.75"$ (19.1 mm), $L_r = 19"$ (482.6 mm), $n = 1$. 
The magnitude of the inverse BGF is shown in Figure 2-11, followed by the real and imaginary functions in Figure 2-12. At the natural frequency, $f/fr = 1.0$, the BGF is at a minimum which is a resonance of the system.

Figure 2-11: Magnitude of a Distributed Acoustic Side Branch Resonator Inverse BGF; Based on the following parameters: $L_c = 2''$ (50.8 mm), $\Delta L = 0.75''$ (19.1 mm), $L_r = 19''$ (482.6 mm), $n = 1$. 
Figure 2-12: Real and Imaginary Parts of a Distributed Acoustic Side Branch Resonator Inverse BGF; Based on the following parameters: $L_c = 2''$ (50.8 mm), $\Delta L = 0.75''$ (19.1 mm), $L_r = 19''$ (482.6 mm), $n = 1$. 
As the frequency with respect to the resonance frequency increases, the position along the inverse BGF circle increases clockwise as shown in Figure 2-13. When \( \frac{f}{f_r} = 1.0 \), the domain is on the right hand side of the circle which is in the region where an intersection with the source or FGF and lock-in is possible.

Figure 2-13: Frequency Ratio for a Distributed Acoustic Side Branch Resonator Inverse BGF; Based on the following parameters: \( L_c = 2" \) (50.8 mm), \( \Delta L = 0.75" \) (19.1 mm), \( L_r = 19" \) (482.6 mm), \( n = 1 \).

The longitudinal resonator is similar to the side branch resonator but is modeled as a half wavelength resonator with open-open boundary conditions.
and an oscillating mass representing a shallow cavity whose length is small compared to the pipe containing it. The model differs from the side branch due to not only the resonator boundary conditions but also due to the difference in surface area between the cylindrical cavity source and the resonator pipe:

\[
\begin{align*}
    z_c &= \frac{\rho_f c_f}{S_r} \left( j\tan k_f L_r + \frac{n\pi}{2Q} \right), \\
    z_m &= \frac{\rho_f c_f}{S_s} (jk_f \Delta L)
\end{align*}
\]

which results in a resonator gain function of

\[
G_{21} = \frac{-j\tan k_f L_r - \frac{n\pi}{2Q}}{jk_f \Delta L + \frac{S_s}{S_r} \left( \frac{n\pi}{2Q} + j\tan k_f L_r \right)}.
\]

Again, damping is dominated by the loss in the resonator piping, rather than source mechanisms such as radiation damping. The impedance due to damping has been modified to reflect the boundary conditions of the longitudinal piping resonator which has resonances at \( \sin(k_f L_r) = 0 \).

The controlling parameters for this model are \( Q, ff, \Delta L/L_r, \) and \( S_s/S_r \). The graphical interpretation of the resonator gain function and controlling parameters is shown in Figure 2-14. Similar to the side branch resonator, the domain of the inverse BGF increases as damping decreases, increasing the likelihood of an intersection with a FGF and lock-in. Also similar to the side branch resonator, the domain increases as \( \Delta L/L_r \) increases, as shown in the bottom on Figure 2-14. Since the depth of the cavity in the longitudinal resonator under investigation is
0.5” (12.7 mm), the maximum \( \Delta L/L_r \), under consideration in this study is 0.02 \( (L_r \) is 26.5” (673.1 mm)) (Rockwell et al. (2003b)).

The parameter that is unique to the longitudinal acoustic resonator is \( S_s/S_r \). The cross-sectional area of the cavity mass and the pipe resonator are in two different orientations following the difference in oscillating pressure directions. For the cavity mass, \( S_s \) is \( \pi D L_c \) and the oscillating pressure flows in and out of the cavity which is transverse to the streamwise direction. For the pipe resonator, \( S_r \) is \( \pi D^2/4 \) and the oscillating pressure flows along the streamwise direction. The radiation impedance also changes at the cavity, so the end effect, \( \Delta L \), is scaled by a fourth to roughly approximate the difference between a 90 degree sector and a fully axisymmetric cavity. This difference must be included in order to produce a significant change in the size of the domain on the Nyquist plot. As a result, a fully axisymmetric cavity produces a larger domain on the Nyquist plot than a 90 degree cavity as shown on Figure 2-15, indicating a greater possibility of lock-in.
Figure 2-14: Nyquist Plot of a Distributed Acoustic Longitudinal Resonator Inverse BGF with Varying Q Factor ($\Delta L/L_r = 0.02$, Rockwell et al. (2003)) and Varying Size Ratio $\Delta L/L_r$ ($Q = 500$); Based on the following parameters: $L_c = 2.5''$ (63.5 mm), $\Delta L = 0.5''$ (12.7 mm), $L_r = 26.5''$ (673.1 mm), $n = 2$. 
The magnitude of the inverse BGF for a longitudinal resonator is shown in Figure 2-16, followed by the real and imaginary functions in Figure 2-17. Similar to the side branch resonator, a resonance occurs at \( f/f_r = 1.0 \). As the frequency with respect to the resonance frequency increases, the position along the inverse
BGF circle increases clockwise as shown in Figure 2-18. Similar to the side branch resonator, \( f/f_r \) of 1.0 occurs near zero on the ordinate (despite the origin not starting at (-1, 0)), in the right hand side of the domain increasing the possibility of an intersection with the FGF and lock-in.

Figure 2-16: Magnitude of a Distributed Acoustic Longitudinal Resonator Inverse BGF; Based on the following parameters: \( L_c = 2.5" \) (63.5 mm), \( \Delta L = 0.5" \) (12.7 mm), \( L_r = 26.5" \) (673.1 mm), \( n = 2 \).
Figure 2-17: Real and Imaginary Parts of a Distributed Acoustic Longitudinal Resonator Inverse BGF; Based on the following parameters: $L_c = 2.5''$ (63.5 mm), $\Delta L = 0.5''$ (12.7 mm), $L_r = 26.5''$ (673.1 mm), $n = 2$. 
2.3.3 Distributed Elastic Resonator

The primary difference between an acoustic and elastic model for a distributed resonator excited by a fluid instability is not only the fluid-structure difference between the source and the resonator observed in the discrete
resonator models, but also that the elastic vibration general solution is much more complicated than the one-dimensional general acoustic solution. The distributed elastic resonator under investigation here is a cantilevered beam excited either by a cavity or bluff body vortex shedding source. The beam vibration follows the 4\textsuperscript{th} order bending equation

\[ y(x,t) = (A \cosh k_b x + B \sinh k_b x + C \cos k_b x + D \sin k_b x)e^{j\alpha x}, \]  

where \( y \) is the deflection of the beam, \( x \) is the distance along the beam, \( k_b \) is the bending wavenumber, and \( A, B, C, \) and \( D \) are constants to be determined by the boundary conditions. The general solution to a beam in bending is dispersive which has an effect on the coupling results not seen in the non-dispersive acoustic solutions.

The input impedance of clamped-free beam driven by a point force from the cavity (or bluff body) has been derived similar to Blevins (1990). The equation of motion for a single mode is

\[
\left[ k_n^4 - k_f^4 \right] A \left( \cosh \left( \frac{\mu x}{L_r} \right) - \cos \left( \frac{\mu x}{L_r} \right) - \sigma \left( \sinh \left( \frac{\mu x}{L_r} \right) - \sin \left( \frac{\mu x}{L_r} \right) \right) \right] = \frac{F \delta (x - x_o)}{EI},
\]

where \( A \) is an unknown constant, \( \mu \) and \( \sigma \) are modal parameters, \( F \) is the point force, \( E \) is Young’s modulus, \( L_r \) is the length of the beam, \( I \) is the moment of inertia, and \( \delta \) is the dirac delta function for \( x \), the position along the beam, and \( x_o \), the position where the point force is applied along the beam. For a clamped-free beam, the following orthogonality condition applies
After applying orthogonality, the solution for \( A \) is

\[
\int_0^L y_i y_j \, dx = \begin{cases} 0 & i \neq j \\ L_r & i = j \end{cases}
\]

where

\[
y_i = \cosh \left( \frac{\mu x}{L_r} \right) - \cos \left( \frac{\mu x}{L_r} \right) - \sigma \left( \sinh \left( \frac{\mu x}{L_r} \right) - \sin \left( \frac{\mu x}{L_r} \right) \right).
\]

After applying orthogonality, the solution for \( A \) is

\[
A = \frac{F}{L_r EI} \cosh \left( \frac{\mu x}{L_r} \right) - \cos \left( \frac{\mu x}{L_r} \right) - \sigma \left( \sinh \left( \frac{\mu x}{L_r} \right) - \sin \left( \frac{\mu x}{L_r} \right) \right),
\]

where modal parameters \( \mu \) and \( \sigma \) are 1.875 and 0.734 for the first clamped-free mode. The deflection of the beam is then

\[
y(x) = -\frac{F}{M} \left[ \cosh \left( \frac{\mu x}{L_r} \right) - \cos \left( \frac{\mu x}{L_r} \right) - \sigma \left( \sinh \left( \frac{\mu x}{L_r} \right) - \sin \left( \frac{\mu x}{L_r} \right) \right) \right]^{-1} \left[ \cosh \left( \frac{\mu x}{L_r} \right) - \cos \left( \frac{\mu x}{L_r} \right) - \sigma \left( \sinh \left( \frac{\mu x}{L_r} \right) - \sin \left( \frac{\mu x}{L_r} \right) \right) \right],
\]

where \( M \) is the modal mass of the beam. Dissipation has been included in the resonator impedance in the form of a complex natural frequency as indicated by the ~ symbol:

\[
\tilde{\omega}_n = \frac{\sqrt{2}}{L_r} \left( \frac{EI}{\rho_s S_r} \right)^{1/2} \sqrt{(1 + j \eta)} \approx 1 + j \frac{\eta}{2} \quad \text{(for small damping)},
\]

\[
\tilde{\omega}_n = \frac{\sqrt{2}}{L_r} \left( \frac{EI}{\rho_s S_r} \right)^{1/2} \left( 1 + j \frac{\eta}{2} \right)
\]
where $\eta$ is the loss factor which is $1/Q$.

The mechanical input impedance of the beam is then

$$z_{\text{mech, in}} = \frac{f(t)}{u(t)} = \frac{f(t)}{j\omega v(t)},$$

where the forcing function and deflection are based on the same time harmonic function:

$$z_{\text{mech, in}} = \frac{F e^{j\omega t}}{j\omega v(x)e^{j\omega t}}.$$  

To convert from mechanical impedance (force/velocity) to acoustic impedance (pressure/volume velocity), force and velocity must be applied over the surface area of the effective source region to produce a pressure and volume velocity. The amount of hydrodynamic force that is exerted by the source is from an effective surface area of half of the cavity opening, $S_s/2$. This is an approximation based on experimental imaging of a similar length cavity, where $(\omega \times v)_y$ is concentrated mainly in the upstream half of the cavity with lesser, much smaller regions of concentrations in the downstream half (Oshkai et al. (2005)). The volume velocity should be consistent with that in the source gain function, $q_r$, as this represents the effective flow which is extracted from the mean flow.

Typically the amount of convective flow which impinges the trailing edge of the cavity to produce the shear layer instability is about half of the mean velocity. So $q_r$ can be estimated to be about $\frac{1}{2} U_\infty S_s$. Therefore, converting the mechanical
input impedance of the beam into an acoustic impedance is accomplished by dividing by \( \left( \frac{1}{2} S_s \right)^2 \).

Applying this conversion to Eqn. 2.31 and substituting Eqn 2.28, the resonator impedance at the drive location is

\[
z_c = \frac{jM}{\omega S_s^2} \frac{\omega^2 - \bar{\omega}_n^2}{\phi} \]

\[
\phi = \left[ \cosh \left( \frac{\mu x_o}{L_r} \right) - \cos \left( \frac{\mu x_o}{L_r} \right) - \sigma \left( \sinh \left( \frac{\mu x_o}{L_r} \right) - \sin \left( \frac{\mu x_o}{L_r} \right) \right) \right].
\]

The remaining impedance for the cavity mass is similar to the other models, \( j\omega M/S_s^2 = j\rho_f c_f k_f \Delta L/S_s \), so the resonator gain function is

\[
G_{21} = \frac{-jM}{\omega S_s^2} \frac{\omega^2 - \bar{\omega}_n^2}{\phi} \frac{\rho_f c_f (jk_f \Delta L)}{S_s} + \frac{jM}{\omega S_s^2} \frac{\omega^2 - \bar{\omega}_n^2}{\phi},
\]

where \( x \) was chosen to be at the cavity position, \( x_o \), since that is the position of interest for the input impedance.

The controlling parameters for this model are \( Q \), \( f/f_r \), \( \Delta L/L_r \), \( S_s/S_r \), \( \rho_f/\rho_e \), \( M \), and \( x/L_r \). The graphical interpretation of the resonator gain function and controlling parameters for a distributed elastic resonator is shown in Figure 2-19. Similar to all resonators, the domain of the inverse BGF increases as damping
decreases, increasing the likelihood of an intersection with a FGF and lock-in. Also similar to the distributed acoustic resonator, the domain increases as $\Delta L/L_r$ increases. Also similar, the domain increases as $S_s/S_r$ increases, where a value of 10 matches the fully-axisymmetric cavity from the longitudinal resonator, 2.5 models the cavity embedded in the stinger, and 1.0 models a completely matched case like in the side branch resonator. Similar to the discrete elastic resonator, if the flow interacting with the stinger is air such that the fluid density is much smaller than the elastic density measured by $\rho_f/\rho_e$, lock-in would not be possible due to a completely limited domain.

The parameters that are unique to the distributed elastic resonator are the resonator mass and the location of oscillating cavity or bluff plate mass along the beam. The modal mass, $M$, of the fundamental mode of the cantilevered beam is 0.3. As the resonator mass decreases, the domain size and likelihood of lock-in also increases. These observations are also consistent with the mass-damping concept presented in Section 1.2.3.3. Lock-in is more likely to occur for smaller levels of mass-damping parameter, which is proportional to $m^*\zeta$, where $m^*$ is the oscillator mass normalized by the added mass and $\zeta$ is the critical damping parameter.

The location of the oscillating cavity mass along the length of the beam, $x/L_r$, affects lock-in, since a force at the end of the beam produces more deflection and coupling. Since the cavity is embedded in the stinger and occupies a definitive length to produce a source of substantive vorticity and coincident frequency, the location with respect to resonator length is about 0.75.
For the bluff plate mounted on the end of the stinger, the location with respect to resonator length is maximized to 1.0. Thus, the bluff plate model produces a BGF domain size that is slightly larger than the cavity model, which increases its chance of lock-in, irrespective of the magnitude of oscillating pressure that the source produces. Independent of this effect, the bluff plate model also slightly differs from the cavity version since the source surface area, $S_s$, is formed from circular flow around the square bluff plate, $\frac{\pi L_c^2}{4}$, where $L_c$ is the height of the plate.

The controlling parameters all greatly have an effect on the domain size and likelihood of lock-in. The solid black lines in all Nyquist plots in Figure 2-19 are typical. A summary of all parameters and the controlling physics they represent is presented in Section 2.5. Parameter inputs are based on the experimental design covered in Chapter 3 and the final model results will be discussed in Chapter 4.
Figure 2-19: Nyquist Plot of a Distributed Elastic Resonator Inverse BGF with Varying Parameters: $Q$, $\Delta L/L_r$, $S_s/S_r$, $\rho f/\rho_e$, $M$, and $x/L_r$; Baseline result based on the following parameters: $L_c = 2.25$" (57.2 mm), $\Delta L = 1"$ (25.4 mm), $L_r = 15"$ (381 mm), $S_s = 2.8$ in$^2$ (1.8E3 mm$^2$), $M = 0.494$ lbm (0.224 kg).
The magnitude of the resonator gain function for a distributed elastic resonator is shown in Figure 2-20. Similar to the distributed acoustic resonator, a resonance occurs at $f/f_r = 1.0$. As the frequency with respect to the resonance frequency increases, the position along the inverse BGF circle increases clockwise as shown in Figure 2-21. Similar to the distributed acoustic resonator, $f/f_r = 1.0$ occurs near zero on the ordinate, in the right hand side of the domain increasing the possibility of an intersection with the FGF and lock-in. Similar to the side branch resonator, the origin of the circle appears at (-1, 0).

![Figure 2-20: Magnitude of a Distributed Elastic Resonator Inverse BGF; Based on the following parameters: $L_c = 2.25''$ (57.2mm), $\Delta L = 1''$ (25.4 mm), $L_r = 15''$ (381 mm), $S_s = 2.8$ in$^2$ (1.8E3 mm$^2$), $M = 0.494$ lbm (0.224 kg).]
A summary of all parameters and the controlling physics they represent is presented in Section 2.5. Parameter inputs are based on the experimental design covered in Chapter 3 and the final model results will be discussed in Chapter 4.
2.4 Derivation of Source Forward Gain Functions

Several source forward gain function models have been derived in previous studies, but have only been applied to a cavity shear layer instability. In order to compare the shear layer instability source with a stronger source such as separated vortex shedding from a bluff body, the source gain function model must be extended to include both. Methods of modeling the source that were introduced previously in the Helmholtz resonator example will be expanded in the context of the comparative resonator cases and accompanying experiments in Figure 2-8.

In the Helmholtz resonator example, two different source models were introduced based on previous studies. The first model is based on linearized stability theory (Elder (1978)). The source gain function is derived based on a wave profile assumed in the cavity mouth and analytically formed growth rates of the shear layer instability along the length of the mouth.

The second model is based on the hydrodynamic forcing function generated in the cavity due to the oscillating external pressure and net circulation of flow in the cavity (Kook and Mongeau (2002)). This source gain function is derived based on the empirical pressure response from a cavity. Since the second model is based on a hydrodynamic forcing function, this model will be extended to include vortex shedding from a bluff body. The hydrodynamic model will be the approach used throughout most of this investigation due to its ability to
perform the comparative source study and its reliance on empirical data. The linearized stability model will be used in limited cases in Appendix A.

2.4.1 Cavity Shear Layer Instability Source Model

The primary difference between the cavities contained in the different resonator configurations in Figure 2-8 is the amount of cavity span present to interact with the resonator. The cavity in the discrete elastic and acoustic resonator models is assumed to have quasi-two dimensional flow. Span is not expected to be a factor in coupling with the resonances. The cavity span in the distributed resonator models, however, is limited. In the distributed elastic model, the cavity is embedded on a beam of limited span. This configuration will produce significant three-dimensional effects and weaken the strength of the cavity source. A full span cavity embedded on a beam is not studied for lock-in since this is a stable configuration. In the distributed acoustic models, however, the effect of cavity span can be evaluated since experimental data for both a limited span and full span cavity exists for the longitudinal pipe resonator ((Rockwell et al. (2003b)). The side branch acoustic resonator will provide information related to the effect of the resonator orientation relative to the source. (Unfortunately, this cannot be extended to an elastic resonator application and will remain a topic of future investigation).
By evaluating empirical pressure response data, the vortex concentration factor, \( \alpha \), can be derived from Eqn. 2.9 and used to generate the source gain function from Eqn. 2.11 in terms consistent with the rest of the models as follows:

\[
G_{t2} = \frac{S_\alpha U_r^2}{j\frac{\omega^2 L_c}{U_c} q_r} e^{-\frac{j\omega L_c}{U_c}}. 
\]

A vortex concentration factor of 1 represents a line vortex which is the ideal two-dimensional shear layer instability assumption (Kook and Mongeau (2002)). Any factor between 0 and 1 represents the amount of vortex diffusion in an actual cavity. The pressure response used to determine this factor should be based on the baseline response away from lock-in conditions, since the source gain function should be a function of the source alone and not include the effect of feedback. The baseline data for experiments with turbulent conditions, however, only contain the response of the resonator turbulence itself since it masks the shear layer instability source. Thus, the pressure response at lock-in was used to find the vortex concentration factor, but is suitable only for this qualitative, comparative study. Figure 2-22 shows the vortex concentration factor for both the full span cavity and a 90 degree sector cavity locking into the longitudinal acoustic pipe resonator as a function of \( S_{tL} \) which is the Strouhal frequency of the cavity as a function of cavity length \((jL_c/U)\) (Rockwell et al. (2003b)).
Since the source produces an oscillating pressure, this is the controlling parameter. For this model, the controlling parameter is then the oscillating pressure normalized by the dynamic head, or $\alpha/\text{St}_\phi$ (derived from Eqn. 2.9). Figure 2-23 shows a typical FGF for a cavity shear layer instability using the hydrodynamic source model for various configurations using the $\alpha$'s in Figure 2-22. Since the full span cavity in the longitudinal resonator has the largest strength, the domain of its FGF is the largest, followed by the weaker side branch cavity and 90 degree sector cavity. The domain of this FGF model always starts
at (0, 0). The right hand side of the domain of the inverse BGF, therefore, determines intersections with the FGF.

Figure 2-23: Nyquist Plot for the Cavity Source FGF

Prior to substitution of the impedance terms, the forward gain function is a ratio of volume velocities as shown in Eqn. 2.2. Given the resonator volume velocity, the source volume velocity can be found. The forward gain function represents the nonlinear interaction between the source oscillation and the unstable mean flow, so \( q \), in Eqn. 2.34 represents the amount of flow extracted from the mean to produce the instability. In the longitudinal acoustic pipe resonator cases, \( q \) is the volume velocity inflow to the cavity, \( U_\alpha S_r \). When
applying the 90 degree sector span cavity source model to the cavity embedded in the stinger, \( q_r \) remains the volume velocity inflow to the cavity, but in this case is \( \frac{1}{2} U_\infty S \), based on the convective velocity being approximately half the mean velocity. The effect of the difference in \( q_r \) in the source models is small as shown by Figure 2-24.

![Figure 2-24: Difference in Cavity Source FGF between the Longitudinal Acoustic Pipe Resonator and the Stinger](image)

Although the main controlling parameter for source strength is the oscillating pressure, the remaining terms that form the FGF for both the cavity
and bluff body are also important as shown in Figure 2-25. As the source size
domain, here approximated by $L_c$, increases, the domain of the FGF increases as
expected. When the cavity length normalized by the inflow momentum thickness
is sufficiently large to initiate the lock-in process, strong lock-in occurs
experimentally (Rockwell et al (2003) and Yang (2005)). Parameters of this type,
therefore, control the feedforward part of the lock-in process. The feedback
portion of the lock-in process is captured by resonator response normalized by
the size domain of the source. An example of this is lock-in due to the cross-flow
over a cylinder vortex, where a critical amount of amplitude of the vibration
normalized by the cylinder diameter is used in wake oscillator modeling (Blevins
(1990)). Thus, the source size $L_c$ is an important controlling parameter.

Both the source type, as parameterized by the oscillating pressure, and
the source size, as parameterized by $L_c$, vary the amount of coupling during lock-
in. Other parameters may also be varied in the FGF, but they are not relevant to
the source strength coupling factor and thus are left for future investigation.
2.4.2 Bluff Body Vortex Shedding Source Model

In this investigation, the hydrodynamic model derived by Kook and Mongeau (2002) for the shear layer instability is extended to capture vortex shedding from a bluff body. The acoustic power generated by a cavity shear layer was derived by Howe as discussed previously in Section 1.2.3.2
\[ \Pi = -\rho \int_{V} u_{ac} \cdot (\omega \times v) \, dV \tag{2.35} \]

where \( u_{ac} \) is the acoustic velocity, \( \rho \) is the density, \( \omega \) is the vorticity vector, and \( v \) is the fluctuating hydrodynamic velocity. Assuming a 2D train of vorticies and a shear layer parallel over the cavity, these parameters can be reduced conceptually to the vectors described in Figure 1-6 to \( \omega_3 \) and \( v_1 \).

The transverse force per unit span induced by the vortex on the cavity is then given by

\[
F_{\text{ext}} = \int_{S} -\rho (\omega \times v) \, dS \\
= \int_{S} -\rho (\omega_3 \times v_1) \, dS \\
= -\rho v_1 \int_{S} (\omega_3) \, dS \\
= (-\rho \Gamma v_1) 
\tag{2.36}
\]

where \( \Gamma \) is the amount of circulation in the instability domain. The pressure distribution is then the amount of force extended over the domain length, \( L_c \),

\[
p_{\text{ext},2} = \frac{-F_{\text{ext},2}}{L_c} \\
= \frac{\rho \Gamma U_c}{L_c} 
\tag{2.37}
\]

By empirically measuring the circulation around the bluff body, a force equivalent to Eqn. 2.36 can be generated for the external pressure used in the FGF. The force in the streamwise and transverse direction may be computed from measured data using particle image velocimetry (PIV) as follows

\[
\frac{F}{\rho} (t) = -\sum_{s} (\bar{\omega} \times u)\hat{i} - \sum_{s} (\bar{\omega} \times v)\hat{j} 
\tag{2.38}
\]
where \( \omega \) is vorticity (not to be confused with rotational frequency). Using the mean magnitude of the force computation and assuming a convection time delay or phase based on \( e^{jSt} \) like that in Eqn. 2.34 from Kook and Mongeau (2002), the FGF is obtained by combining Eqns. 2.10, 2.11, 2.37 and 2.38:

\[
G_{12} = \frac{S_s}{j \omega L_c^2 q_r} \left\langle -\sum_s (\omega \times u) j - \sum_s (\omega \times v) j \right\rangle e^{-j\phi_s}.
\]

2.39

The magnitude of the oscillating pressure is the only parameter that really changes in this model. Thus, the controlling parameter for this model is based on the oscillating pressure or Eqn. 2.38. As the force increases, the domain size and likelihood of lock-in increases as shown in Figure 2-26. The domain size is based on the plate height in this model which is an assumption. In reality, the controlling dimension is a function of the wake produced downstream of the plate (Blake (1986)), but this parameter is not available in this investigation. Because the generalized source model has an extra dependence on the size beyond that for the cavity model, the increasing source size produces a decrease in the domain, decreasing that chance for lock-in. This is aphysical and is a consequence of generalizing this model for an equivalent source.
Figure 2-26: Nyquist Plot for the Generalized Source FGF with Varying Parameters
The method of describing functions is valid for this qualitative, comparative study only. To predict absolute states of lock-in, a more refined measurement of force would need to be captured and the measurement domain more clearly defined. But for the purposes of this investigation, this relatively simple extension of the previously existing FGF for the shear layer instability is of practical use. Chapter 3 will present the design of the experiment and discuss the need for a unique bluff body design to compare to the cavity, necessitating empirical measurements for the source function.

2.5 Summary of Controlling Parameters

Controlling parameters have been found that capture the lock-in coupling factors in describing function theory. These parameters are well-founded in the physics of lock-in. All of the various lock-in modeling methods presented in Chapter 1 capture a source generation versus dissipation balance that must be met for lock-in to occur. In the describing function models presented here, the dissipation component is captured by damping in the inverse BGF resonator models, and the generation component is captured by the FGF source models. The degree to which a resonator can be coupled to any available source is based not only on damping but also on mass, as expressed by the mass-damping concept. In the describing function models, mass and damping are captured in
the BGF resonator models by many different parameters depending on the configuration.

Figure 2-27 summarizes the resonator inverse BGF models and controlling parameters. The discrete acoustic Helmholtz resonator model contains the basic parameters for a resonator: damping and frequency ratio. When the model is elastic rather than acoustic, the fluid-structure difference between the source and resonator is evaluated through the addition of the ratio of the densities, $\rho_f/\rho_e$, and their cross-sectional areas, $S_s/S_r$, since an elastically mounted cavity or bluff body must reflect this difference. When the model is distributed rather than discrete, the difference in source to resonator wavelengths is evaluated through the addition of the ratio of the lumped oscillating cavity (or bluff body) mass and the distributed resonator length, $\Delta L/L_r$. When the model is distributed and elastic, all of these parameters are included as well as some unique parameters based on the modal properties of the beam impedance, such as modal mass, $M$, and the location of the oscillating mass along the length of the beam, $x/L_r$.

For each controlling parameter, similar results are attained for each model. As the inverse BGF domain increases, the likelihood of an intersection with the FGF and lock-in increases. Thus, changes in the controlling parameters produce the following results:

- As damping decreases, $Q$ increases and the inverse BGF domain increases, producing a greater likelihood of lock-in.
• As the excitation frequency approaches the resonance frequency \( (ff, = 1) \), the inverse BGF domain approaches \((0, 0)\) which is the center of the FGF domain, increasing the chance of an intersection and producing a greater likelihood of lock-in.

• As the ratio of the oscillating fluid source density and the elastic resonator density increases, \( \rho_f/\rho_e \), the inverse BGF domain increases, producing a greater likelihood of lock-in.

• As the cross-sectional area of the oscillating source and the resonator increases, \( S_s/S_r \), the inverse BGF domain increases, producing a greater likelihood of lock-in. \( S_s \) represents the size available for oscillating pressure flows in and out of the cavity. Coupling between the cavity and the resonator pressure is proportional to \( S_s/S_r \).

• As the ratio of the lumped oscillating cavity (or bluff body) mass and the distributed resonator length, \( \Delta L/L_r \), increases, the inverse BGF domain increases, producing a greater likelihood of lock-in. The participation of the oscillating cavity mass in the resonance relative to the side branch resonator is proportional to \( \Delta L/L_r \).

• As the resonator mass decreases, \( M \), the inverse BGF domain size increases, producing a greater likelihood of lock-in. Lock-in is more likely to occur for smaller levels of mass-damping parameter, which is proportional to \( m^*/\zeta \), where \( m^* \) is the oscillator mass normalized by the added mass and \( \zeta \) is the critical damping parameter.
As the location of the oscillating cavity mass along the length of the cantilevered beam, \( x/L_r \), increases (approaching the free end), the inverse BGF domain size increases, producing a greater likelihood of lock-in. The bluff plate produces a BGF domain size that is slightly larger than the cavity, which increases its chance of lock-in, irrespective of the magnitude of oscillating pressure that the source produces.

<table>
<thead>
<tr>
<th>Cavity or bluff body on cantilevered beam ( Q, ffr, \Delta L/L_r, S_s/S_r, \rho_f/\rho_e, M, \text{ and } x/L_r )</th>
<th>Cavity or bluff body on cantilevered beam ( Q, ffr, \Delta L/L_r, S_s/S_r, \rho_f/\rho_e, M, \text{ and } x/L_r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acoustic</td>
<td>Elastic</td>
</tr>
<tr>
<td>Cavity with Helmholtz resonator ( Q, ffr )</td>
<td>Elastically mounted cavity or bluff body ( Q, ffr, \Delta L/L_r, S_s/S_r, \rho_f/\rho_e )</td>
</tr>
<tr>
<td>( G_{23} = \frac{j \omega^2}{\frac{\omega_0}{Q} + j(\omega^2 - \omega_0^2)} )</td>
<td>( G_{23} = \frac{j \omega^2 - \frac{\omega_0}{Q}}{\frac{\omega_0}{Q} + j} \left( \frac{\rho_c c_r}{S_r} \right)^2 \omega^2 - j \omega^2 )</td>
</tr>
<tr>
<td>Distributed</td>
<td>Distributed</td>
</tr>
<tr>
<td>Longitudinal ( G_{23} = \frac{-j \tan k_j L_r - \frac{n \pi}{2Q}}{j k_j \Delta L + \frac{1}{S_y} \left( \frac{n \pi}{2Q} + j \tan k_j L_r \right)} )</td>
<td>( G_{23} = \frac{-j M \omega^2 - \omega_0^2}{\frac{\omega_0 S_r^2}{4} \phi} \frac{j M \omega^2 - \omega_0^2}{\frac{\omega_0 S_r^2}{4} \phi} )</td>
</tr>
<tr>
<td>( G_{23} = \frac{j \cot k_j L_r - \frac{n(2n-1)}{4Q}}{\pi(2n-1)} + j[k_j \Delta L - \cot k_j L_r] )</td>
<td>( \phi = \left[ \cosh \left( \frac{\mu_c}{L_r} \right) - \cos \left( \frac{\mu_s}{L_r} \right) - \sigma \sinh \left( \frac{\mu_c}{L_r} \right) \sin \left( \frac{\mu_s}{L_r} \right) \right] \cdot \left[ \cosh \left( \frac{\mu_c}{L_r} \right) - \cos \left( \frac{\mu_s}{L_r} \right) - \sigma \sinh \left( \frac{\mu_c}{L_r} \right) \sin \left( \frac{\mu_s}{L_r} \right) \right] )</td>
</tr>
</tbody>
</table>

Figure 2-27: Summary of Resonator BGF Models and Controlling Parameters
The source FGF models and controlling parameters show how the oscillating pressure captures the source generation concept for each model:

\[
G_{12} = \frac{S}{j\omega L_c q_r} \left\{ \frac{\alpha U_\infty^2}{St_\phi} e^{-jSt_\phi} \left[ \sum_s \left( \omega \times u \right) - \sum_s \left( \omega \times v \right) \right] e^{-jSt_\phi} \right\}
\]

As the magnitude of the oscillating pressure increases, the domain of the source FGF increases, producing a greater likelihood of lock-in. Similar results occur for increasing the source size domain as measured by \( L_c \). The domain originates at \((0, 0)\) and spirals outward, intersecting the inverse BGF domain to predict lock-in states.

In Chapter 3, the design of the experimental study will be presented which will further explain the parameter inputs for these models. In Chapter 4, the results of both the experiments and the final models will be discussed to confirm which coupling factors are most important to the lock-in process.
Chapter 3
Experimental Arrangement

To determine why lock-in occurs more readily for certain combinations of coupling factors than others, an experimental system has been designed to test certain coupling factors that control the lock-in process. In Chapter 2, analytical models using the method of describing functions were derived for use in a comparative study of coupling factors. Although experimental data for some combinations of factors exist in the literature, a study of fluid-elastic cavity lock-in and a comprehensive study of multiple coupling factors do not. The fluid-elastic cavity case, where a cavity is coupled to the structure containing it, is the weakest combination of factors under consideration. When each factor is varied to provide a stronger option, a previously studied lock-in case is effectively achieved. To determine which combinations of factors can lead to lock-in, equivalent configurations have been formed to represent the more traditional lock-in cases, as shown in Figure 3-1.
Each configuration shown in the right-hand side of Figure 3-1 is designed to isolate the predominant instability source to either the shear layer fluctuation over the cavity or vortex shedding from the bluff body.

The cavity on a cantilevered beam, referred to in this study as a stinger-cavity system, is comprised of a cavity shear layer instability embedded in a cantilevered beam which acts as a distributed elastic resonator, as shown in Figure 3-2. The clamped end of the stinger is welded to a large block which is
captured into the piping using flanges. The stinger cross-section is rectangular to separate the vertical and horizontal bending modal frequencies. The cavity shear layer oscillation is excited from flow conveyed over it and couples to the stinger bending modes.
Figure 3-2: Stinger Configuration with Cavity
This configuration of source and resonator is designed for ease of testing. Interaction with the low frequency bending modes of the system will occur at lower velocities than with other modes. In addition, coupling between this kind of source and the bending modes will show if the coupling is more dominated by the shear layer vertical velocity fluctuation interacting with the first bending mode (with a source near a transverse vibration antinode, or peak vibration), by the shear layer vorticity interacting with the second bending mode (with the source near a transverse vibration node, or minimum vibration, but peak angular rotation), or some combination of these. This overall configuration enables isolation of the source and resonator to the preferred instability and can be adjusted to avoid other instabilities.

When the distributed resonator is fluid acoustic rather elastic, a pipe containing the cavity acts as a distributed acoustic resonator. The primary difference between an acoustic and elastic resonator is the material difference between the source and the resonator. With an elastic resonator, a fluid-structure coupling will be harder to achieve in the stinger-cavity system than the fluid-fluid coupling occurring in the pipe-cavity system. Strong lock-in was achieved in the pipe-cavity system in a previous investigation (Rockwell et al. (2003)). In the stinger-cavity system, the cavity is limited in span by the width of the stinger. The pipe-cavity system tested sector-shaped cavities of a similar span extent, so this cavity size was used as a basis for the stinger-cavity design in this investigation.
In order to test the effect of different source coupling mechanisms, the stinger-plate system can be modified to incorporate a bluff plate on its trailing edge to produce vortex shedding coupled to the stinger, as shown in Figure 3-3. (In this case, there is no cavity embedded in the stinger.) Lock-in due to vortex shedding from a bluff body is much more frequently encountered than cavity shear layer instabilities. As this thesis will show, this excitation source more readily couples to an elastic structure than a shear layer instability. The shear layer instability is an impingement type of source, whereas bluff body vortex shedding is not. A stationary bluff body could be mounted upstream of the stinger to test for an impingement type of source, but then coupling between the vibration and vortex shedding would be highly dependent on the separation distance and Reynolds number. This is outside of the scope of this investigation, but would be of interest for future study.
The final coupling factor under consideration is the size of the resonator wavelength relative to the source wavelength. An example of a discrete acoustic resonator undergoing lock-in due to a cavity shear layer instability is a Helmholtz resonator. The elastic equivalent of this configuration is a spring-mounted cavity. This configuration proved too difficult to test since the source of excitation could not be isolated to the shear layer instability without interference from turbulent buffeting. Thus, the difference between a discrete and distributed resonator is evaluated through the use of the describing function models derived in Chapter 2. These models are also used to evaluate other combinations of coupling factors to provide a more definitive view of the role of coupling factors independently. Another aspect which will not be studied is the orientation of the source with respect to the resonator. This topic is left for future investigation.
The following sections describe how the experiment is designed for instability isolation, source characteristics, elastic resonator characteristics, and facility systems.

3.1 Instability Description

The stinger configuration enables isolation of the source to the cavity shear layer instability or bluff body vortex shedding and the resonator to the bending modes of the elastic resonances. Avoiding other instabilities and resonators (referred to as interference) is crucial to testing lock-in between a source and an elastic resonance. The strength of the interaction is determined by the amount of modal coupling and the other resonances and/or sources present. Lock-in is measured by the amount of energy created in the interaction that is beyond that of the broadband excitation. When lock-in occurs, the mechanism of the source and resonator becomes self-sustaining. When the vortex formation over the cavity length or bluff body produces separation or a stalled zone, a time delay or phase shift occurs between this area on the stinger and the other areas. The vorticity generated in the separation is considered a time dependent forcing function on the equation of motion. This forcing function is also dependent on the structural motion which results in a net negative damping at the onset of a self-excited oscillation. The only separation or stalled region on the stinger resulting in a significant vorticity should be the cavity or bluff body.
The stinger is a long slender structure which is cantilevered from a tail that provides a clamped condition. In the cavity configuration, other possible sources besides the cavity flow can occur at the nose, along the body, and at the tail. The nose can be involved in a leading edge instability if the flow separates before the cavity. This is avoided through the use of a 5:1 elliptical nose which is ideal for a floating stagnation point (Davis (1980)). In both the cavity and bluff body configurations, trailing edge instability associated with the tail is avoided through a tapering of the geometry.

Resonances in the vicinity of the one of interest would also interfere in the modal coupling. These interferences may come from the support structure and loop themselves. In this test, the facility structural resonances do not interfere with the stinger (as will be shown in the test results in Chapter 4). Acoustic resonances associated with the cavity volume and test section duct have been predicted and are well separated from the stinger resonances. They also have been checked with other possible interference sources in the loop such as the trailing edge flow associated with the tail.

3.2 Source Description

3.2.1 Cavity Shear Layer Instability

The physics of the cavity source should be present regardless of the presence or type of resonator, so the governing physics and parameters
associated with fluid-resonant lock-in testing are used in this test design. A full background of the development of these governing physics and parameters are given in Rockwell et al. (2003). A brief description is given here to justify the design of the test.

When the source is a cavity flow, a hydrodynamic instability occurs which is associated with the free shear layer and vorticity across the cavity, as shown in Figure 3-4. At separation, a free shear layer develops and grows which produces oscillatory components. This growth is initially linear and then becomes nonlinear resulting in a transfer of energy into higher frequencies. This shear layer development can be defined both spatially and in time. Small cells of vorticity are associated with the shear layer. As the shear layer grows, the small cells of vorticity cluster together to form a large vortex. Once the shear layer impinges the trailing edge of the cavity, an upstream influence occurs at the shear layer formation point. This hydrodynamic influence is nearly instantaneous for low Mach number flows, and results in a strengthening of the shear layer instability source. It also results in a selection of preferred frequencies for the source, designated as Strouhal stage frequencies. Within the cavity, a backwards flow of circulation is formed from the trailing edge to the leading edge. This circulation has an effect on the upstream influence and is aided by entrainment. The parameter used to describe the shear layer thickness here is momentum thickness. This hydrodynamic instability is categorized as a free shear layer instability and exists without the presence of an external resonator (Lucas et al. (1997)).
When an acoustic resonator is present at a frequency which is close to or coincident with the Strouhal source frequency and the phase is aligned, lock-in or synchronization can occur. Figure 3-5 shows the frequency dependence for increasing velocity of the resonance frequencies (green lines), which do not change with flow velocity, and the cavity source frequencies (blue lines), which increase linearly with flow velocity. This is a compressible feedback process which amplifies the hydrodynamic instability or incompressible feedback process.

Figure 3-4: Cavity Oscillation Description
The feedforward part of the interaction (excitation of the resonator by the source) is equivalent to a forced vibration of the acoustic resonator by the hydrodynamic fluctuation of the shear layer. The feedback part of the interaction is due to the acoustic fluctuation perturbing the shear layer oscillation. This fluctuation reinforces the shear layer oscillation such that a sustained oscillation may occur. The inclusion of feedback in this process is referred to as lock-in and may result in a flow noise tone. Because of the amount of energy in the resonator and surrounding systems, flow tones often penetrate beyond the resonator and are transferred through fluidborne, airborne and structureborne means.

Considering the cavity source alone, hydrodynamic frequency selection is based on two principles for cavity incompressible feedback. From linearized
stability theory, the instability due to the free shear layer may be amplified over a range of $\beta = \theta/U$ from ~0-0.040, as shown in the shaded area of Figure 3-5. In a cavity, impingement produces a feedback effect based on an instantaneous response of the upstream shear layer due to the pressure produced at the impingement. One form predicts nondimensional source frequencies based on the cavity length and the ratio of vortices’ convective to freestream velocity:

$$fL_c/U = (U_c/U)(n +/- a),$$

as displayed by the blue lines of Figure 3-5. There are numerous unresolved issues with this correlation, mainly because these parameters are empirically based and can vary depending on the experiment.

Amplification frequencies from the combined contribution of the free shear layer and the impingement are unknown. When the vorticity created in the free shear layer clusters into a large vortical structure, the nonlinear growth of the shear layer produces super-harmonics ($n\beta$) and sub-harmonics ($\beta/n$) of the amplification nondimensional frequency. This nonlinear contribution is confirmed in low flow water cavity hydrodynamic experiments (Knisely and Rockwell (1982); Ziada and Rockwell (1982)).

An additional nonlinear contribution may also contaminate the amplification frequencies. When an external resonator is present, strong interaction between the hydrodynamic oscillation frequencies and the external resonator frequencies may occur, producing additional nonlinearity, as shown by the extended circle from the original intersection point in Figure 3-5. This results in a strengthening of the cavity source. Strong interaction depends on the modal coupling between the external resonator and hydrodynamic oscillation.
interactions may occur since there are several source stages and resonator modes which can simultaneously interact. Separating these nonlinear effects may not always be feasible with data alone. Visual confirmation of the nth stage may indicate where nonlinearity due to \( n\beta \) and strong lock-in occur alone. Adequate separation of interactions with velocity may indicate a separation of \( n\beta \) and strong lock-in effects.

The span width of the cavity in this investigation, 1.25" (31.8 mm), is limited by the width of the stinger, 1.5" (38.1 mm). In order to investigate the governing parameters of the lock-in, the cavity depth and length may be modified with inserts. The cavity length is adjustable from a maximum length of 2.25" (57.2 mm) and a depth of 0.5" (12.7 mm).

A source characterization is accomplished using laser Doppler velocimetry (LDV). This type of measurement provides a visualization of the flow over the cavity as well as dynamic single point measurements of the fluctuating velocity. Turbulence statistics such as root-mean-square velocity, intensity, and Reynolds stress can then be generated and plotted against those of other cavity sources and cavity fluid-resonant lock-in investigations for comparison. The LDV results are summarized in Chapter 4 and presented in detail in Appendix B.

3.2.2 Vortex Shedding from Bluff Bodies

When a bluff plate is attached to the stinger, a unique trailing edge configuration is accomplished. This configuration is a flow physics situation that
has not previously been explored. In Figure 3-6, a circulation region occurs just upstream of the plate, followed by a flow confinement and shedding downstream of the plate. This is an interesting time-varying flow restriction which may couple with the downstream shedding.

Pipe Wall

Flow

Vibration

Pipe Wall

Figure 3-6: Flow Schematic of Stinger-Plate Configuration

Since the plate is in a confined turbulent flow, the wake structure is influenced by the confinement and limited by the pipe diameter downstream of separation. The prediction of source frequencies is difficult and must be obtained empirically from measured data. The distance between the vortical cells downstream of the plate can provide an estimated frequency, as will be shown in Chapter 4. Particle image velocimetry (PIV) and high speed video (HSV) measurements will be used to provide for both this measurement in addition to
the force produced by this source for the describing function model. The PIV results are summarized in Chapter 4 and presented in detail in Appendix C.

3.2.3 Instability Frequency Model

In Chapter 2, describing function models for the instability sources, both the cavity shear layer instability and vortex shedding from a bluff body, were derived. Given the descriptions of those instabilities in this chapter, the dependence on the nondimensional source frequency, Strouhal number, can be discussed.

The Strouhal number for a cavity is defined as \((\frac{U_c}{U})(n+/-a)\), as discussed previously in Section 3.2.1. For \(n=1\), velocity ratio usually varies between about 0.4 to 0.6 and the phase delay usually varies between about \(\frac{1}{4}\) and \(\frac{1}{2}\). In Figure 2-24, the Nyquist plot of the forward gain function (FGF) modeling the cavity shows that for a Strouhal number of 0.5, the real part of the FGF is zero. This result is consistent for all multiples of 0.5 for the FGF results as shown in Figure 3-7. Although the FGF is empirically based, all stages of the instability \(n\) fall near zero on the x-axis of a Nyquist stability plot; being near this location on the plot increases the chance of an intersection with the resonator backward gain function and lock-in. The FGF for vortex shedding from a bluff body is also consistent with this result (see Figure 2-26 for Nyquist plot). In general, the FGF source models are consistent with instability theory.
3.3 Elastic Resonator Design

The stinger configuration is modeled as a cantilevered beam with clamped-free boundary conditions. For the cavity excitation condition, the free end is the nose end which is upstream in the flow. The tail end is welded into a large block which is downstream in the flow. The hole is cut out of the large block to allow for flow to pass through, and low profile tabs and a cross-section
matching the stinger are left to provide for the attachment surface. Flanges in the duct clamp the block. The block acts as a filter for extraneous vibrations. Mass, damping, and the structural modes dominate the characteristics of this system as an oscillator.

Damping cannot be well predicted but is a function of the material and attachments in the structure. More importantly, to characterize the lock-in experimentally, damping must be repeatable. Bolted connections were initially used and contributed a significant amount of damping. Thus, welded joints were designed. Since this greatly minimizes the flexibility of the test configuration, cavity placement was not varied. Radiation and hydrodynamic damping due to vibration in water is a significant issue and is evaluated in Chapter 4.

The stinger system is made of aluminum in order to produce a high entrained water to modal mass ratio. Ross defines entrained mass as a function of the densities and source speeds of the fluid and structure and of the relative Mach number (Ross (1976)). The entrained mass is directly proportional to $\frac{\rho_f c_e}{\rho_s c_f}$ in order to take into account the fluid loading. (Note that this ratio is not characteristic impedance, since the density and sound speed are inverted with respect to one another for a given medium.) Aluminum and water produce a higher entrained mass ratio than Stainless Steel and water.

The modes of oscillation for a clamped-free beam shown in Figure 3-8 are unique. The higher order modes are not harmonics of the fundamental mode. When driving the beam to generate a pure tone at its fundamental resonance,
the higher order modes will not be present in the result. This is crucial to the investigation of modal coupling. By being able to separate the response due to different modes, a direct modal interaction can be investigated. In other words, when a cavity source stage and elastic bending mode are driving each other, the energy transfer between the source and resonator will be due to that mode alone. No energy can be lost from the source interacting with a higher order mode.

One disadvantage of a clamped-free beam configuration is that the second mode is about six times higher in resonance frequency than the first mode. In order to investigate the excitation of multiple modes with one source stage, a large velocity range may be needed. This is resolved by testing a flexible configuration in which varying length cavities can be used. Figure 3-9 shows an example of the expected lock-in states for the longest cavity of 2.25”
A Strouhal correlation of \( fL_c/U = 0.6(n-1/4) \) was used to estimate the source lines throughout the investigation.

Both the cavity and bluff body are positioned in the stinger to excite the bending modes of the stinger at frequency coincidence. As shown in Figure 3-2, the cavity is nearest the free end of the stinger such that the fluctuating transverse velocity dominates coupling and the first bending mode response will be high at coincident frequency points. If the fluctuating vorticity from the cavity dominates the coupling, the second bending mode response will be high due to

Figure 3-9: Expected Lock-in Conditions for 2.25" (57.2 mm) Long Cavity with Bending Modes of Stinger; \( n \) is the instability mode
the nodal position of the resonator modal displacement where a moment will dominate vibration. As shown in Figure 3-3, the bluff body is attached to the end of the stinger (without a cavity) to excite the first bending mode, since it is clear that this source produces a fluctuating velocity that couples to the bending modes.

The stinger was initially embedded with light accelerometers potted in four locations along the length in order to measure vibration response. This added too much damping to the system, so the stinger was redesigned without integral instrumentation and a laser vibrometer was used to measure the vibration response.

The modal response of the stinger was measured in air and water. With the stinger-block mounted and installed in the test facility without the test duct piping, modal response was measured using an instrumented hammer and four accelerometers temporarily glued to the stinger along its length in the vertical direction, one in the horizontal direction, and one glued to the block in the third, or axial direction. Damping was measured using the time response decay method. The first bending mode resonates at about 53 Hz in air with a damping of about 0.15-10.18% \( (Q \sim 275-330) \). The second bending mode resonates at about 339 Hz with a damping of 0.17% \( (Q \sim 295) \). The first bending mode resonant frequency in the horizontal direction is well separated from the vertical modes at 150 Hz with a damping of 0.16% \( (Q \sim 310) \). To test the submerged modal response, the accelerometers were removed and the test duct piping was then added and filled. Modal response was measured by impacting the stinger
with a needle inserted through a hole in the piping and a laser vibrometer. As expected, the modal frequencies decrease due to added mass and the damping increases. The first mode resonates at about 35 Hz (decreased from 53 Hz in air) with a damping of 0.7% ($Q \sim 70$) (increased from 0.15-0.18% in air). Higher modes were observed in the data but damping was not measured since the laser vibrometer does not produce sufficiently clean modal response compared to the accelerometer measurements and only one impact location was measured. As discussed previously, the first bending mode is the one under investigation in this study.

Although significant efforts were made to reduce the damping, the level is still high for investigating a flow-induced vibration, but is adequate for the purposes of this comparative study.

3.4 Test Facility Design and Conditions

The test facility is a typical turbulent flow water tunnel, shown in Figure 3-10. Inflow turbulence levels to the stinger are approximately 4%. A 1000 gpm pump in a 4” (101.6 mm) diameter loop provides flow to a 3” (76.2 mm) diameter test duct. Flexible rubber hoses upstream and downstream of the test section are used to isolate from resonances and noise in the facility from the pump and feed piping. The main duct of the test section is 10’ (3.05 m) long circular 3” (76.2 mm) ID cast acrylic duct. The downstream flange is where the stinger is mounted and accessed. A shorter 2’ (609.6 mm) acrylic end is used downstream
of this prior to the exit hose to avoid unsteady end effects on the stinger. The test section is mounted to vibration isolation tables to isolate from facility noise.

The facility acoustics are monitored in various locations with accelerometers and acoustic pressure transducers. Laser Doppler velocimetry (LDV), particle image velocimetry (PIV), and high speed video (HSV) measurements were obtained in this investigation. A waterbox was used to match the index of refraction through the test duct piping.

Since the stinger is mounted with the circular piping surrounding it, the inflow conditions to the cavity are complicated by the flow over the stinger nose and a sudden change in flow area. Surveys using LDV were made throughout this area to ensure the instability was isolated to the shear layer over the cavity. The Reynolds number of the inflow to the stinger is a result of fully developed turbulent pipe flow ($Re_D = UD_{pipe}/\nu$) which is at a maximum of 500,000. Appendix B includes additional measurements including velocity profiles and turbulence statistics (rms fluctuations, intensity, Reynolds stress, and boundary layer momentum thickness).

When the stinger-bluff body configuration was tested, the arrangement was modified to accommodate flipping around the stinger such that the fixed end is upstream and the bluff body is on the trailing edge. The inflow conditions are then further complicated by the change in flow area at the fixed end of the stinger. The flow is then developed along the length of the stinger upstream of the bluff plate. As previously discussed in Section 3.2.2, the inflow to the bluff plate is complicated by the change in flow area and the close confinement of the
test duct piping. The bluff plate is a 1.5" (38.1 mm) square mounted in the center of a 3" (76.2 mm) diameter flow area. Surveys using PIV were made throughout this area to study the unique flow physics produced by this confinement. Appendix C includes measurements of the hydrodynamic force produced as a result of the velocity and vorticity patterns. Chapter 4 also presents more detail of these patterns.

Figure 3-10: Loop Schematic
Chapter 4

Results of Investigation

Experimental and analytical models have been formed to determine why lock-in occurs more readily for certain combinations of coupling factors and which factors are the most important for achieving lock-in. The results of this study will be the first investigation to systematically compare coupling factors.

The coupling factors under consideration in this study include the material and physical properties of the resonator, the type of fluid instability source, and the relative wavelength between the source and the resonator. The properties of the resonator dominate the dissipation component of the lock-in process, so a comparison between an elastic resonator with one that has similar material to the source, a fluid acoustic resonator, will show that the acoustic resonator produces a greater likelihood of lock-in. Larger fluid-structure impedance differences produce weaker coupling.

The type of fluid instability source dominates the source generation component of the lock-in process, so a comparison between a cavity shear layer instability and a bluff body vortex shedding will show that the bluff body produces a stronger oscillating pressure and source domain size coupled to the structure which produces a greater likelihood of lock-in. A comparison between a distributed resonator, where the wavelength is closer to the source, with a discrete resonator, where the wavelength is much larger than the source, will
show that the discrete resonator produces a greater likelihood of lock-in. The
distributed resonator has a limited modal mass in comparison to the lumped
mass of a discrete resonator.

The controlling physics and parameters for each coupling factor are
discussed in the following sections.

4.1 Evaluation of Fluid/Acoustic vs. Structural/Elastic Resonant Behavior

The most commonly studied form of cavity lock-in is the fluid-resonant
case where coupling between a cavity shear layer instability and the resonant
modes of the acoustic cavity volume or surrounding system produces a self-
sustaining oscillation. Experimental results from this case have been used to
design an equivalent fluid-elastic lock-in configuration as presented in Chapter 3.
A cavity supported by long resonator piping (pipes-cavity) was redesigned for a
cavity embedded in a long cantilevered beam (stinger-cavity).

Although strong lock-in was achieved in the pipes-cavity configuration in a
previous study (Rockwell et al. (2001), the following test results show that lock-in
is not achieved in the stinger-cavity configuration. The main difference between
the two cases is the relationship between the source and resonator media. A
fluid-structure interaction is harder to achieve than a fluid-fluid interaction due to
the impedance disparity between the source and resonator properties, as shown
in Chapter 2.
The vibratory response of the stinger-cavity is measured for incrementally changing flow velocity. For example, Figure 4-1 shows the tip acceleration for increasing flow velocity from about 0 to 30 ft/s (9.1 m/s) of the stinger-cavity. Although very clean, repeatable linear vibration is achieved, little interaction between the cavity and stinger is measured at the flow states corresponding to frequency coincidence. Figure 3-8 shows that frequency coincidence between the stinger and cavity is expected to occur at approximately 14.6 ft/s (4.5 m/s). Repeated flowsweeps are acquired for increasing flow (upsweep) and decreasing flow (downsweep) to evaluate for hysteresis and repeatability. For example, Figure 4-2 shows the amplitude of first bending mode response (~35 Hz) for increasing flow velocity.
Figure 4-1: Vibratory Response of the Stinger with Increasing Flow; Measured with Laser Vibrometer Near the Tip of the Stinger; Lock-in Possible at ~15 ft/s (4.6 m/s)
Nonlinear response with the log of flow, where the response is amplified beyond linear, is indicative of a lock-in state where the structural response is providing feedback and strengthening with excitation source. Linear response with respect to the log of flow is due to broadband turbulent excitation of the dynamic system. Response amplitude as a function of flow speed is fit with the relationship: \( dB = C + 10 \log U^n \), where \( dB \) is the resonant response, \( C \) is a constant which corresponds to the y-intercept, and \( n \) is a constant associated with the slope of the logarithmic response with velocity \( U \) in ft/s. This is equivalent to \( 10 \log U^n \) where \( n \) may indicate the type of excitation.

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**Figure 4-2:** Response of Stinger First Bending Mode for Multiple Repeated Runs of Increasing Flow; Filled Symbols Indicate Maximum Response State
In contrast to the stinger-cavity study, in the pipes-cavity study, Figure 4-3 shows how the resonant response of the second acoustic mode is amplified for specific ranges for flow speed, indicating a nonlinear response or feedback between the span-limited cavity source and the acoustic pipe resonator has occurred. Each peak response represents a different stage of the instability. The linear response follows a fit of $n = 3.5$. Lock-in is achieved in this case for both a fully axisymmetric cavity and a 90 degree sector cavity, albeit weaker. As discussed in Chapter 3, the sector cavity size is similar to that in the stinger-cavity case.

![Figure 4-3: Response of 90 Degree Sector Cavity Interacting with Longitudinal Pipe Resonance Taken From Rockwell et al. (2001); Filled Symbols Indicate Maximum Response State](image-url)
For the stinger-cavity data in Figure 4-2, the dashed line curve fits of response for each flowsweep indicate small amounts of amplification above the linear response. Figure 4-4 shows the deviations in acceleration response beyond the curve fits, referred to as strength. There is very little amplification achieved, and thus little interaction between the stinger resonance and the cavity source occurs.

Figure 4-4: Deviation Beyond Linear Response of Cavity Interacting with Stinger First Bending Mode; Filled Symbols Indicate Maximum Response State

In the pipes-cavity case, however, a large amount of amplification or strength occurs for a similar size span-limited cavity as shown in Figure 4-5. In
this case, multiple stages of the instability are amplified. The degree of amplification is dependent on many factors such as which shear layer instability stage is most dominant depending on the length of the cavity, inflow momentum thickness, and the amount of fluid energy and damping. When the lock-in states from Figure 4-5 (as indicated by filled symbols at 37, 63, and 100 ft/s (11.3, 19.2, and 30.5 m/s)) are plotted on a Strouhal plot similar to Figure 3-5, instability stages occurring at Strouhal numbers (with cavity length) of 1.3, 1.8, and 3.1 are shown in Figure 4-6. Typically higher Strouhal numbers occur for larger $L/\theta$.

Recall the hydrodynamic process of a shear layer instability discussed in Section 3.2.1. When an acoustic resonator is present at a frequency coincident with the shear layer instability source, the source provides a force onto the resonator and the resonator feeds back onto the source as shown by the addition of the acoustic process on top of the hydrodynamic process in Figure 4-7. The source and resonator become synchronized to produce a self-sustaining oscillation.
Figure 4-5: Deviation Beyond Linear Response of 90 Degree Sector Cavity Interacting with Longitudinal Pipe Resonance Taken From Rockwell et al. (2001); Filled Symbols Indicate Maximum Response State
Figure 4-6: Lock-in Conditions of 90 Degree Sector Cavity Interacting with Longitudinal Pipe Resonance at Approximately 445 Hz
In the stinger-cavity case, synchronization must take into account the elastic properties of the resonator, which produces a fluctuating deflection rather than a fluctuating acoustic pressure. An equivalent lock-in process is shown in Figure 4-8. In the first half of the synchronization process, energy is extracted from the mean flow into the cavity. In the second half of the synchronization process, energy is dissipated through several possible damping mechanisms, which exist in some amount in any vibrating system. In order to establish fluid-structure synchronization and the self-sustaining lock-in state, a critical amount of

Figure 4-7: Acoustic Resonance Feedback Cycle Overlaying Cavity Oscillation Cycle from Figure 3-4
Energy must be extracted from the flow and incident field to offset the dissipation due to vibration.

**Energy Extraction**

**Energy Dissipation**

Figure 4-8: Synchronization Process for Cavity Interacting with Stinger Bending

Damping occurs though several mechanisms. Damping is due to source radiation, viscosity, hydrodynamic loading, and mechanical systems. In elastic lock-in, damping due to hydrodynamic loading and mechanical systems dominates. In acoustic lock-in, damping due to source radiation and viscosity dominates. The damping due to hydrodynamic loading in a fluid-structure interaction is a function of the hydrodynamic resistance and entrained mass. In the stinger configuration, a certain amount of hydrodynamic damping is unavoidable and difficult to adjust in water. Damping due to mechanical systems is also unavoidable due to material damping and friction in joints and mounting mechanisms. In the pipes configuration, however, the damping due to source
radiation and viscosity is much less limiting, especially if air is the fluid medium. This difference in damping mechanisms allows for lock-in with acoustic resonances to be easier than with more highly damped elastic resonances, without even considering the properties based on impedance difference.

In the stinger-cavity configuration, damping is minimized by use of welded joints. Damping levels are acquired through the use of instrumented impact hammer ring-down measurements. The average damping of the stinger-cavity system dry and wet is \( Q = 250 \) (0.2\% critical damping) and 70 (0.7\% critical damping), respectively. Although this is higher than the pipes-cavity system which is about \( Q = 500 \) (0.1\% critical), this level of damping is not so high as to preclude lock-in in an acoustic resonator.

As flow increases and a possible lock-in state is approached, the damping level actually decreases (\( Q \) increases) when lock-in occurs as shown in Figure 4-9 for the pipes-cavity case. This causes the sharpening of the response at amplification as modeled by the mass-damping and negative damping concepts explained in Section 1.2.3.3. Since the instability at lock-in generates a sufficiently strong excitation to offset dissipation, the damping decreases (\( Q \) increases) at the lock-in state. Figure 4-10 shows the damping level as a function of flow for the stinger-cavity case. As flow increases, the damping level increases (\( Q \) decreases) due to the amount of hydrodynamic damping that occurs. Hydrodynamic damping is a function of hydrodynamic resistance and entrained mass (Blake et al. (1975)). An estimate for hydrodynamic damping from Blake et al. (1975) is shown in the figure. This estimate overestimates
damping. At the expected lock-in point, which is at a velocity of about 14.6 ft/s (4.5 m/s), the source instability does not generate a sufficient amount of excitation to overcome the dissipation from hydrodynamic and other damping mechanisms. Beyond this velocity, hydrodynamic damping then dominates the response which is indicated by a change in slope of the Q factor from about -2.2 to -3.5.

Figure 4-9: Damping Response of 90 Degree Sector Cavity Interacting with Longitudinal Pipe Resonance Taken From Rockwell et al. (2001); Filled Symbols Indicate Maximum Response State
Since the impedance mismatch between the source and resonator and damping are modeled using describing functions as derived in Chapter 2, the stinger-cavity and pipes-cavity cases can be further compared analytically to explore the excitation versus dissipation phenomenon. Fluid-elastic lock-in may not be possible for a cavity source given the large amount of excitation needed to overcome the dissipation in a distributed elastic resonator.

In Figure 4-11, describing functions from Section 2.3.2 are used to analyze the pipes-cavity case. Intersections of the FGF, representing the cavity

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Figure 4-10: Damping Response of Cavity Interacting with Stinger First Bending Mode; Filled Symbols Indicate Maximum Response State; Curve Fits Above and Below Lock-in Speed

Since the impedance mismatch between the source and resonator and damping are modeled using describing functions as derived in Chapter 2, the stinger-cavity and pipes-cavity cases can be further compared analytically to explore the excitation versus dissipation phenomenon. Fluid-elastic lock-in may not be possible for a cavity source given the large amount of excitation needed to overcome the dissipation in a distributed elastic resonator.

In Figure 4-11, describing functions from Section 2.3.2 are used to analyze the pipes-cavity case. Intersections of the FGF, representing the cavity
source, and the inverse BGF, representing the fluid acoustic resonator, show states where lock-in is possible. Table 4-2, at the end of the chapter, contains all of the parameter estimates for each source and resonator model described here. The cavity source is modeled using the limited span 90 degree sector from Section 2.4.1. While the resonator BGF has little uncertainty, the source FGF has significant uncertainty, particularly in the estimate of the vortex concentration parameter, $\alpha$.

Even with this uncertainty in the source model, the describing function method predicts lock-in to occur, but achieves a limited level of accuracy since intersections between the functions did not occur exactly where lock-in occurred experimentally. Lock-in occurred experimentally at Strouhal numbers ($St_L = fL_c/U$) of 1.3, 1.8, and 3.1, whereas modeled intersections occur at 0.6 and 1.0. Note that the model predicts Strouhal numbers close to the correlation in Section 3.2.3. The experimental data in this case is high due to the larger $L/\theta$, which is method does not take into account. This result, however, is sufficiently accurate for this qualitative comparative study as the cavity source model has limited accuracy as discussed in Section 2.4.1. If the full span cavity model is analyzed (bottom of Figure 4-11) (Rockwell et al. (2003)), the intersections between the FGF and the inverse BGF are closer to the experimental lock-in states. Also if the vortex concentration parameter is increased by 25%, the intersections are even closer. Thus, the error is likely to be in the vortex concentration parameter. Appendix A shows a describing function using linearized stability theory as the
basis for a source FGF rather than this method, following previous work in this area (Elder (1978)).

In all pipes-cavity cases, the model accurately predicts that lock-in occurs. The lock-in intersections along the resonator BGF occur near an \( f/f_r \) of 1. The entire region of the inverse BGF in this domain is near the resonance frequency as shown by \( f/f_r \) of 0.99 – 1.01. The coincidence frequencies are the same for each lock-in state as shown by Figure 4-6. If lock-in occurs at a higher mode (which is common as shown by Figure 3-5), the frequencies for the second mode would be on different inverse BGF’s in Figure 4-6 and 4-11. This study is limited to a single mode, however.
In Figure 4-12, the stinger-cavity case is modeled using describing functions from Section 2.3.3. Since the cavity is very similar in size to the pipes-cavity case (2.25" vs. 2.5") and the cavity design does not allow for empirically
measuring the entire cavity volume using PIV, the same source model as the pipes-cavity case is used for this qualitative study. The model predicts that lock-in cannot occur. If the damping levels are decreased, then lock-in could occur at a level of about $Q = 1000$ (0.05% critical). At this low a frequency, this level of damping is usually only possible for structures vibrating in gases such as air, since convecting fluids produce a critical amount of hydrodynamic damping on a structure. Fluid-elastic cavity lock-in in air is not feasible since strong coupling between the fluid and structure requires fluid and structural densities that are sufficiently close. Another method for modeling flow-induced vibrations, wake oscillator theory, also captures this principle.

The mass-damping parameter, $m^*\zeta$, must be sufficiently low to produce lock-in. The structural mass normalized by added mass, $m^*$, must be low, which is accomplished by submerging a relatively light structure in a heavy fluid such as water. The damping must also be low, however, which is hard to accomplish for a submerged structure in water.

In order to achieve lock-in, the excitation coefficient from the source instability must be able to overcome the mass-damping parameter. The cavity shear layer instability is not sufficiently strong to overcome the mass-damping of the stinger. In contrast to an order of $10^{-4}$ expected for lock-in, levels of $m^*\zeta$ for the stinger are only on the order of $10^{-3}$. Resonance characteristics for the stinger were acquired using impact hammer measurements during wet and dry conditions. The resonator modal mass of this mode is 30% of the total mass which is typical of the first bending mode of a cantilevered beam. The entrained
mass of water for this mode is about 0.9 lb or about half of the total mass, as calculated using the resonance frequencies for wet and dry conditions, 54 and 35 Hz, respectively.

When damping is artificially lowered to $Q = 1000$, intersections occur near \( \frac{f}{f_r} = 1.0 \) and \( St_L = 0.5 \), which is ideally the predicted lock-in state for a first stage cavity shear layer instability as discussed in Chapter 3. This level of damping is extremely low for a structure submerged in water and not physically realizable in this configuration. This result is consistent with the mass-damping parameter. A \( m^* \zeta \) on the order of \( 10^{-4} \) corresponds to $Q = 1000$. 
Figure 4-12: Describing Function Model Predictions for the Cavity Interacting with the Stinger First Bending Mode; Lower Plot Shows that Lock-in is More Likely to Occur with Extremely Low Damping
If the source generation levels are increased such that the cavity span was less limited, then the describing function model predicts that even for a damping level of \( Q = 100 \) lock-in may occur. Figure 4-13 shows intersection of the stinger BGF with the full cavity span FGF (from the pipes-cavity model). However, as Chapter 3 discusses, a full cavity span around the stinger would produce a stable configuration since synchronization would not be possible. Note that lock-in is predicted close to \( St_L = 0.92 \) and 1.08 which is near the second stage of a cavity shear layer instability prediction. Higher stages beyond the first may be possible for a longer length cavity with respect to inflow momentum thickness such as this. For a \( Q = 500 \), lock-in is predicted near \( St_L = 1.4 \), which is near the third stage of the cavity shear layer instability prediction. In fact, the full span cavity in the longitudinal pipe resonator locked in at 1.4. Thus, the model predicts that if a stronger cavity source is possible, then lock-in may occur.

This result is also consistent with wake oscillator theory. In order to achieve lock-in, the excitation coefficient from the source instability must be able to overcome the mass-damping parameter. The span-limited cavity shear layer instability is not sufficiently strong to overcome the mass-damping of the stinger, but an axisymmetric full span cavity is strong enough in theory. In reality, however, lock-in would not be possible since synchronization would not be possible with this source and stinger configuration.
Since the describing function model shows that lock-in is possible for a stronger cavity source, a series of modifications were made to the stinger-cavity design. Several different length cavities were tested such that opportunities for interactions between the first and second stages of the cavity oscillation with the first two bending modes of the stinger were tested. A possible instability at the leading edge of the stinger was ruled out. When the stinger is flipped around with respect to the flow velocity condition, the results are comparable to the nominal

Figure 4-13: Describing Function Model Prediction for Full Span Cavity Interacting with the Stinger First Bending Mode
direction. Interference effects from a small varying angle of attack at the nose are not observed. Effects from a possible disturbance at the nose interfering are not observed. LDV measurements show a typical turbulent velocity profile inflow both to the stinger and the cavity. Although a typical shear layer instability occurs at the cavity, LDV measurements show that the levels of Reynolds stress may not be sufficiently strong compared to the inflow levels, for a viable shear layer instability to exist for lock-in. Appendix B discusses the turbulence statistics from the cavity and the effect of vibration on the shear layer instability.

Also, several design additions were made to the cavity to increase the source strength as shown in Figure 4-14. An upstream chamfer was tested to better direct flow into the cavity to enhance impingement on the trailing edge. An upstream aperture was tested to enhance circulation within the cavity. Thin walls were attached to the sides of the stinger to better channel flow over the cavity. The span of both the stinger and cavity were increased by 50% (results shown in this chapter). None of these modifications were able to produce lock-in. High speed video during testing verified that although cavity sources did exist, coupling to the stinger did not.
Varying the controlling parameters for both dissipation and generation show that the describing function models adequately capture the lock-in process. The results show that a fluid-elastic cavity lock-in case may not be possible given the physical constraints of achieving a sufficiently low damping and sufficiently strong cavity source necessary for lock-in. Experimentally, testing the stinger configuration with a stronger source, such as vortex shedding from a bluff body, confirms this, which is presented in the next section.

4.2 Evaluation of Source Instability Coupling Factor

In order to determine if fluid-elastic lock-in is possible using a stronger source, the stinger is evaluated for interaction with vortex shedding from an attached bluff body. By attaching a plate directly to the stinger, the vibration and
vortex shedding are tightly coupled. The stinger is identical to the stinger-cavity case, but without the cavity embedded in it. Figure 3-3 shows the stinger-plate configuration.

In contrast to the linear vibration response for the stinger-cavity case, the stinger-plate case shows clear, repeatable interaction between the stinger and plate source. At specific flow states, the amplitude of the bending response is amplified, indicating coupling and feedback between the plate vortex shedding and the stinger vibration. Figure 4-15 shows the response of the stinger deviating beyond that expected from linear response due to flow turbulence (the average \( n \) in the data curve fit is about 1). Since amplification occurs for only specific velocity states, returning to a linear response during the remaining velocity states where the source does not interact with the resonator, feedback between the source and resonator is achieved. When a self-excited vibration is achieved rather than a lock-in, a critical velocity occurs where the response deviates from that due to response from linear turbulence excitation, but does not return to this state in a repeatable cyclical manner. In other words, an amplification that occurs due to lock-in has a “lock-out” state. Figure 4-16 shows the spectral character of the response increasing with flow. At flows higher than the lock-in state at \( \sim 13 \text{ ft/s} \sim 4 \text{ m/s} \), the amplitude decreases and the stinger vibration becomes more damped as will be shown later.
Figure 4-15: Response of Stinger First Bending Mode Interacting with Bluff Plate for Repeated Multiple Runs of Increasing Flow; Filled Symbols Indicate Maximum Response
The strength of lock-in achieved by the stinger-plate is shown in Figure 4-17. Although these levels are much less than that achieved for fluid-resonant lock-in in the pipes-cavity case, lower strengths are more typical of fluid-elastic lock-in with a distributed resonator as coupling is limited by the deflection of the resonator which is limited by a modal shape. In fluid-elastic lock-in with a discrete resonator, like a more typical elastically-mounted bluff body, lock-in strengths are typically much higher since the vibration deflection is not limited by a mode shape and the coupling is inherently tighter since a single whole body is
moving with the flow. The difference between discrete and distributed resonators and the role they play in controlling lock-in is evaluated in the next section.

Although the strengths are limited, amplification is observed throughout the facility. During the occurrence of strong lock-in, energy levels may increase beyond the response local to the source and resonator due to radiation. Globally increased energy levels at lock-in occurred on surrounding piping flanges, support equipment, and in the piping fluid.

Figure 4-17: Deviation Beyond Linear Response of Bluff Plate Interacting with Stinger First Bending Mode; Filled Symbols Indicate Maximum Response

When fluid-elastic lock-in occurs, the structural resonance frequency may vary slightly. Once the structural resonance is synchronized or locked in with the
source, its frequency increases as the source frequency increases with ascending flow sweep. In Figure 4-18, the resonance frequency increases as flow increases which indicates the presence of synchronization of the resonance with the excitation instability.

The most significant evidence of synchronization is the occurrence of decreased damping at lock-in. During lock-in, negative damping occurs since the amount of excitation offsets dissipation. Figure 4-19 shows a slight decrease in damping (\(Q\) increase) at lock-in. Although this amount is small relative to the decreases observed in the pipes-cavity case, the occurrence of negative
damping occurs consistently for several repeated runs. Above the lock-in speed, hydrodynamic damping increases significantly as shown by the change in slope of the Q factor from about -12 to -36. The hydrodynamic damping estimate from Blake et al. (1975) is consistent with the data beyond the lock-in state.
Figure 4-19: Damping Response of Bluff Plate Interacting with Stinger First Bending Mode; Filled Symbols Indicate Maximum Response
Visually, lock-in is observed using high speed video (HSV) and high speed particle image velocimetry (PIV). The stinger-plate case produces a unique flow instability which was discussed in Section 3.2.2 and is shown in Figure 4-20. The recirculation region just upstream of the plate and the time-varying flow restriction between the plate and test duct piping is coupled to the shedding downstream of the plate. A frequency prediction for this instability is not possible without state of the art computational tools and is also complicated by a large amount of three-dimensional flow observed in HSV. However, the frequency of the instability may be confirmed by recognizing that the distance between vortical cells is on the order of 2” (50.8 mm). For a lock-in frequency of 35 Hz and a convection velocity of 8 ft/s (2.4 m/s) estimated from successive images of HSV, the wavelength of the instability is on the order of 2.75” (69.9 mm) which is shown in Figure 4-20 (green line) to be approximately the width of the instability domain. Thus, frequency coincidence between the stinger and plate shedding is visually verified.
Unfortunately, HSV was not able to capture the organization of instability due to feedback due to excessive 3D flow. Synchronization, although it occurs, could not be observed or studied in this setup, but is a topic of future investigation.

Similar to the previous section, the stinger-plate and stinger-cavity cases can be further compared analytically to explore the excitation versus dissipation phenomenon. The stinger-cavity case did not lock-in experimentally or even analytically with physically realistic damping levels. Only artificially low damping levels were able to attain lock-in. With a stronger source, however, the stinger-
plate case does lock-in. Although the stinger-plate instability is unique, the plate bluff body vortex shedding can be modeled using a hydrodynamic forcing function similar in nature to that of the cavity. Unlike the cavity, however, the forcing function for the plate must be formed empirically.

The force is computed using Equation 2.39 from the measurement volumes shown in Figure 4-21, which is suitable for this qualitative study. In order to perform high quality measurements, the PIV window was limited to the domains shown here. Thus, measurements for the upstream, first downstream, and second downstream positions are from separate times. Since the data is averaged, the non-referenced acquisition has no effect. All measurements were acquired at the lock-in flow state that was achieved in the laser vibrometer portion of the test, 13.2 ft/s (4.0 m/s). Lock-in, however, was not achieved during the PIV portion of the test as test supports isolating the stinger vibration from the facility had to be moved to accommodate the PIV acquisition window. Figure 4-22 shows the vorticity map of each measurement volume. Portions of the upstream and first downstream windows are used in the calculation of the force. Primary and secondary vortical structure is observed downstream of the plate in these windows. No significant structure is observed in the second downstream window, and thus was not used.

Since there is a significant amount of three dimensional flow, the components of force in the streamwise and transverse components are random in nature. No peak excitation frequencies were found. Table 4-1 shows the mean levels of force produced by the flow in the streamwise (i) and transverse (j)
directions. The amplitudes include an adjustment to account for the blocked
portions of the image. For the flow upstream of the plate, a significant amount of
force in both the streamwise and transverse directions occur, due to the
recirculation region and the stinger vibration respectively. For the flow
downstream of the plate, the force is predominantly in the transverse direction
since the primary and secondary vortices in the vortex street as shown in Figure
4-22 produce components of force similar to that of a dipole oriented vertically
with the alternating phase produced by the cells of vorticity.
Figure 4-21: PIV results: Velocity Vectors, Red Boxes Indicate Computation Windows
Figure 4-22: PIV results: Upper – Velocity Vectors, Red Boxes Indicate Computation Window; Lower – Vorticity, Scale of Upstream Position is -3189 to 2578 s\(^{-1}\) and the Scale of the Downstream Positions are -4016 to 3206 s\(^{-1}\)
Table 4-1: Mean Force Computation Results

<table>
<thead>
<tr>
<th></th>
<th>Flow Upstream of Plate (in$^3$/s$^2$)</th>
<th>Flow Downstream of Plate (in$^3$/s$^2$)</th>
<th>Total (in$^3$/s$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{F_x}{\rho} = \langle \sum_i (\vec{\omega} \times \vec{v}) \rangle$ (streamwise)</td>
<td>-8.9E3</td>
<td>198.8</td>
<td>-8.7E3</td>
</tr>
<tr>
<td>$\frac{F_y}{\rho} = \langle \sum_i (\vec{\omega} \times \vec{u}) \rangle$ (transverse)</td>
<td>2.0E4</td>
<td>1.0E4</td>
<td>3.0E4</td>
</tr>
</tbody>
</table>

Although the force appears random in nature due to the large amount of three-dimensional flow, special processing of the PIV measurements was able to confirm that the flow shedding from the plate has a discrete frequency which closely corresponds to the first bending mode of the stinger and lock-in. Global spectral analysis is a method developed by Dr. Rockwell at Lehigh University (Celik et al. (2007)), where a spectral analysis of the flowfield is performed using high-speed PIV measurements. In Figure 4-23, a spectral map shows that the separation at the plate tip has a concentration of transverse velocity at 36.2 Hz. The recirculation cell upstream of the plate does not appear to correlate with the downstream vortex shedding at this frequency, although some random correlation exists. The downstream vorticity also has a concentration of energy at the plate tip at 36.2 Hz. This results in the transverse force having the same concentration. Additional details on global spectral analysis results can be found in Appendix C.
Figure 4-23: Spectral Map of downstream transverse velocity at a discrete frequency of 36.2 Hz (stinger-plate position approximate) (Upper); Spectral Map of downstream transverse force at a discrete frequency of 36.2 Hz (stinger-plate position approximate) (Lower)
In Figure 4-24, the stinger-plate case is modeled using describing functions from Section 2.3.3. Using an average experimental damping of $Q = 300$, the model predicts a lock-in to occur in the same Strouhal number range as that achieved experimentally. Lock-in occurs at a Strouhal number of ~0.35 assuming the controlling dimension is the plate height, 1.5” (38.1 mm). An intersection occurs for a Strouhal number of about 0.4 which is very close considering the amount of uncertainty in the source model. The model also predicts lock-in at a St of 0.6, which corresponds to a velocity of about 7 ft/s (2.1 m/s). This relatively small level of velocity does not likely produce a sufficient amount of energy to achieve lock-in. More detailed quantitative comparisons are not possible, given the limited refinement of this modeling method. However, the describing function model successfully captures lock-in due to the presence of a stronger source, despite the large amount of dissipation or damping in a distributed elastic resonator.
4.3 Evaluation of Resonator Distribution Coupling Factor

The final coupling factor under investigation is the resonator mass distribution relative to the source. When the resonator is external to the source, like in the pipes-cavity and stinger-cavity cases, the wavelength of the resonator can be much larger than the source. The excitation from the source then
produces a response that is limited by the modal mass of the resonator, where
the resonator is a distributed system compared to the source. When the
resonator is internal to the source, like in the Helmholtz-cavity and spring-
mounted cavity cases, the resonator acts as a discrete mass oscillator which
responds more strongly to excitation than the distributed resonator responds. As
discussed in Chapter 3, the difference between a discrete and distributed
resonator is evaluated through the use of describing function models due to the
difficulty encountered in isolating the instability of interest experimentally.

Since both the acoustic and elastic versions of the discrete and distributed
resonators exist, one must evaluate the role of the resonator distribution
combined with the role of the source/resonator fluid-structure impedance
differences. Figure 4-25 shows a composite view of the describing function
model predictions of lock-in for both coupling factors for a cavity source. See
Table 4-2 for the parameter values used to generate this figure. For a discrete
resonator, a cavity Helmholtz resonator locks in for a higher amount of
dissipation more readily than an elastically mounted cavity. This is consistent
with the fluid structure impedance differences evaluated in the preceding section
(results repeated in Figure 4-25). For an acoustic resonator, a cavity Helmholtz
resonator locks in at a comparable amount of dissipation as a cavity side branch
acoustic resonator.

Since the side branch resonator is more coupled to the cavity physically
than the pipes-cavity configuration where the cavity is embedded in the
longitudinal pipes, the side branch resonator model is used to evaluate the
resonator distribution coupling factor. The relative source to resonator cross-sectional area is the controlling parameter that reflects this difference as discussed in Chapter 2. Note that the model for the side branch produces the most accurate lock-in prediction, since the experimental lock-in states line up well with damping a little less than the experimental amount. This may be due to the orientation of the source and resonator oscillation pressure being parallel. The orientation of the source waves relative to the resonator waves is not yet captured by the method of describing functions, and this different source and resonator orientations may produce less accurate results.

For an elastic resonator, an elastically-mounted cavity locks in more readily than the stinger-cavity configuration. Therefore, a discrete resonator produces stronger coupling than a distributed resonator independent of the impedance disparity between the source and resonator mediums. Unlike the other describing function models, the $f/f_r$ for the discrete-elastic inverse BGF model does not approach 1 near where intersections or lock-in occurs. This is a minor inconsistency in the modeling which does not impact the findings of this study, but should be taken into account for future investigations.

The resonator distribution is a weaker or less significant coupling factor than the impedance difference to the lock-in process. The side branch cavity is still able to lock in, whereas the stinger-cavity is not. The impedance difference between the source and resonator dominates the lock-in process.
To evaluate the type of source instability as a coupling factor compared to the resonator distribution and impedance difference, lock-in states are predicted for the same resonator models, but with the bluff plate source rather than the cavity shear layer instability. Figure 4-26 shows a composite view of the model predictions with the plate source. See Table 4-2 for the parameter values used.
to generate this figure. For a distributed resonator, the bluff body mounted in longitudinal acoustic organ pipes locks in for a higher level of damping more readily than the bluff body mounted on the stinger. Damping is not a strong variable in determining lock-in for this configuration due to the dominance of the source generation over the dissipation component of the lock-in process. For an elastic resonator, the elastically mounted bluff body locks in for a higher level of damping more readily than the bluff body on a cantilevered beam.
<table>
<thead>
<tr>
<th></th>
<th>Acoustic</th>
<th>Elastic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><img src="Pipes-Plate" alt="Bluff body in long organ pipes" /></td>
<td><img src="Spring-Plate" alt="Elastically mounted bluff body" /></td>
</tr>
<tr>
<td>Discrete</td>
<td><img src="Stinger-Plate" alt="Bluff body on cantilevered beam" /></td>
<td></td>
</tr>
<tr>
<td></td>
<td><img src="Spring-Plate" alt="Distributed Elastically mounted bluff body" /></td>
<td><img src="Stinger-Plate" alt="Bluff body on cantilevered beam" /></td>
</tr>
</tbody>
</table>

Figure 4-26: Summary of Describing Function Model Predictions for a Bluff Body Source; See Table 4-2 for the parameter values used to generate this figure.
4.4 Summary of Results

The instability source type and the impedance difference between the source and resonator are the most dominant coupling factors for lock-in. These coupling factors represent the generation and dissipation balance that must be met in order to achieve lock-in. Consistent throughout all of the experimental and simulated lock-in studies, damping is the most critical parameter in determining if lock-in occurs. Given this, major observations of the coupling factors role in determining lock-in include the following:

(1) Fluid-elastic cavity lock-in was not achieved experimentally. Analytical modeling shows that it is unlikely to occur given the critical level of damping that exists for most submerged structures and the relatively weak source strength that the cavity produces.

(2) These results are consistent with another method for modeling flow-induced vibrations, wake oscillator theory. The mass-damping parameter, \( m*\zeta \), must be sufficiently low to produce lock-in. The mass normalized by added mass, \( m^* \), must be low, which is accomplished by submerging a relatively light structure in a heavy fluid such as water. The damping must also be low, however, which is hard to accomplish for a submerged structure in water. In order to achieve lock-in, the excitation coefficient from the source instability must be able to overcome the mass-damping parameter. The cavity shear
layer instability is not sufficiently strong to overcome the mass-damping of the stinger.

(3) Experimentally and analytically, a stronger source such as vortex shedding from a bluff body was able to lock-in to the same stinger resonator. Amplified, nonlinear, vibration response and negative damping at the source/resonator frequency coincidence state was observed experimentally. The lock-in prediction from analytical modeling is consistent with this result.

(4) Experimentally and analytically, a fluid-acoustic resonator was able to lock-in to the same cavity source. Analytical models show that the impedance difference between the source and resonator is a controlling factor in determining lock-in. Lock-in from a fluid-structure interaction is more difficult to achieve than from a fluid-fluid interaction. Also, a fluid-structure interaction in the stinger elastic resonator must have lower damping than a fluid-fluid interaction in the organ pipe acoustic resonator for lock-in.

(5) The impedance difference between the source and resonator as a controlling factor for lock-in is also consistent for not only distributed resonators like the stinger and organ pipes, but also discrete resonators like the spring-mounted cavity and Helmholtz resonator which are simulated in this study.

Experimental investigations of vortex-induced vibration are complicated by the difficulty in isolating the instability from facility interference and attaining low damping. Analytical modeling using the describing function method, however, produced results that are qualitatively consistent for all of the combinations of
sources and resonators under investigation. Figure 4-27 is a summary plot of the cases used to explore the coupling factors. (A single damping level is set for ease of viewing). Intersections of the resonator inverse BGF’s with the source FGF’s are increasing likely given: (1) a stronger source, (2) an acoustic resonator which has less impedance difference with the source given a similar fluid, and less significantly (3), a discrete rather than a distributed resonator which is limited by modal mass and mode shape. The consistency in the location of the Strouhal source frequencies in the Nyquist plots, as shown by 0.4 – 0.5 as a reference, are also remarkable between the two different sources and between the modeling and experimental results. This Strouhal number is consistent with the first stage prediction from instability theory.
This study shows a strong consistency between the modeling and experimental observations. The method of describing functions captures the role of the coupling factors under consideration in this study and has been successful in improving the understanding of the lock-in process.
Table 4-2: Parameters for Describing Function Models

<table>
<thead>
<tr>
<th>Distributed Resonators</th>
<th>Acoustic</th>
<th>Elastic</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Longitudinal Pipes</strong></td>
<td><strong>Side Branch</strong></td>
<td><strong>Stinger</strong></td>
</tr>
<tr>
<td>$L_c$, cavity streamwise length (in)</td>
<td>2.5 (63.5 mm)</td>
<td>2 (50.8 mm)</td>
</tr>
<tr>
<td>$\Delta L$, end effect (in)</td>
<td>0.5 (12.7 mm)</td>
<td>0.75 (19.1 mm)</td>
</tr>
<tr>
<td>$L_r$, resonator length (in)</td>
<td>26.5 (673.1 mm)</td>
<td>19 (482.6 mm)</td>
</tr>
<tr>
<td>$D_r$, resonator dimension (in)</td>
<td>(diameter) 1 (25.4 mm)</td>
<td>(span) 1 (25.4 mm)</td>
</tr>
<tr>
<td>$S_s$, cross-sectional area of cavity (in²)</td>
<td>$(\pi D_r L_c)$ 7.85 (5.1E3 mm²)</td>
<td>$(L_r D_r)$ 2 (1.3E3 mm²)</td>
</tr>
<tr>
<td>$S_r$, cross-sectional area of resonator (in²)</td>
<td>$(\pi D_r^2/4)$ 0.79 (5.1E2 mm²)</td>
<td>$(L_r D_r)$ 2 (1.3E3 mm²)</td>
</tr>
<tr>
<td>$n$, acoustic mode number</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>$f_r$, acoustic resonance frequency (Hz)</td>
<td>445</td>
<td>171</td>
</tr>
<tr>
<td>$Q$, quality factor</td>
<td>Test: ~100</td>
<td>Test: 500</td>
</tr>
<tr>
<td>$c_f$, speed of sound in fluid (in/s)</td>
<td>58800 (1.5E6 m/s)</td>
<td>58800 (1.5E6 m/s)</td>
</tr>
<tr>
<td>$c_e$, speed of sound in elastic structure (in/s)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\rho_f$, density of fluid, water (lbm/in³)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\rho_e$, density of elastic structure and added mass, aluminum (lbm/in³)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$E$, Young’s modulus for aluminum (psi)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$I$, moment of Inertia (in⁴)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$x/L$, position where point force applied to beam</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$M$, modal mass of the beam (lbm)</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

**Discrete Resonators**

<table>
<thead>
<tr>
<th></th>
<th>Acoustic Helmholzt</th>
<th>Elastic Spring-mounted</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega/\omega_r$, ratio of frequency to resonance frequency</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$Q$, quality factor</td>
<td>Varied artificially in model</td>
<td>Varied artificially in model</td>
</tr>
<tr>
<td>$c_f$, speed of sound in fluid (in/s)</td>
<td>-</td>
<td>58800 (1.5E6 m/s)</td>
</tr>
<tr>
<td>$c_v$, speed of sound in elastic structure (in/s)</td>
<td>-</td>
<td>212600 (5.4E6 m/s)</td>
</tr>
<tr>
<td>$\rho_f$, density of fluid, water (lbm/in³)</td>
<td>-</td>
<td>0.0361 (999 kg/m³)</td>
</tr>
<tr>
<td>$\rho_e$, density of elastic structure, aluminum (lbm/in³)</td>
<td>-</td>
<td>0.0975 (2700 kg/m³)</td>
</tr>
</tbody>
</table>

**Sources**

<table>
<thead>
<tr>
<th></th>
<th>Full Cavity Span in Longitudinal Resonator</th>
<th>90 Degree Sector Cavity in Longitudinal Resonator</th>
<th>Cavity in Side Branch Resonator</th>
<th>Span-Limited Cavity in Stinger</th>
<th>Bluff Body on Stinger Trailing Edge</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_c$, cavity streamwise length (in)</td>
<td>2.5 (63.5 mm)</td>
<td>2.5 (63.5 mm)</td>
<td>2 (50.8 mm)</td>
<td>2.25 (57.2 mm)</td>
<td>(plate height) 1.5 (38.1 mm)</td>
</tr>
<tr>
<td>$W$, cavity transverse depth (in)</td>
<td>0.5 (12.7 mm)</td>
<td>0.5 (12.7 mm)</td>
<td>19 (482.6 mm)</td>
<td>0.5 (12.7 mm)</td>
<td>-</td>
</tr>
<tr>
<td>$D_r$, resonator dimensions (in)</td>
<td>(diameter) 1 (25.4 mm)</td>
<td>(diameter) 1 (25.4 mm)</td>
<td>(span) 1 (25.4 mm)</td>
<td>(cavity span) 1.25 (31.8 mm)</td>
<td>(span) 1.25 (31.8 mm)</td>
</tr>
<tr>
<td>$S_o$, cross-sectional area of cavity (in²)</td>
<td>$(\pi D_r L_c)/4$ 7.85 (5.1E3 mm²)</td>
<td>$(\pi D_r L_c)/4$ 1.96 (1.3E3 mm²)</td>
<td>$(L_c D_r)/2$ 2.8 (1.8E3 mm²)</td>
<td>$(\pi L_c^2)/4$ 1.8 (1.2E3 mm²)</td>
<td>$(\pi L_c^2)/4$</td>
</tr>
<tr>
<td>$S_r$, cross-sectional area of resonator (in²)</td>
<td>$(\pi D_r^2)/4$ 0.79 (5.1E2 mm²)</td>
<td>$(\pi D_r^2)/4$ 0.79 (5.1E2 mm²)</td>
<td>$(L_c D_r)/2$ 2 (1.3E3 mm²)</td>
<td>$(HW)^{stinger}_{1.5}$ 0.75(1.5)=1.1</td>
<td>$(HW)^{stinger}_{1.5}$ 0.75(1.5)=1.1</td>
</tr>
<tr>
<td>$S_{St}$, Strouhal frequency with cavity length</td>
<td>Test: 1.4, 1.9, 2.8</td>
<td>Test: 1.3, 1.8, 3.1</td>
<td>Test: 0.4, 0.9, 1.4</td>
<td>-</td>
<td>Test: 0.35</td>
</tr>
</tbody>
</table>
$\alpha$, vortex concentration parameter

| $|F/\rho|$, normalized force (in$^3$/s$^2$) | $10^{(-0.62St+0.78)}$ | $10^{(-0.32St-0.84)}$ | $10^{(-0.11St-0.28)}$ | $10^{(-0.32St-0.84)}$ | - |
|----------------------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|

See Table 4-1
Chapter 5
Conclusions and Future Research

Lock-in occurs between many different kinds of fluid flow instabilities and resonators. When a fluid instability source excites a resonator, and feedback from the resonator onto the source occurs, a self-sustaining oscillation or lock-in may result. In this study, lock-in was achieved for specific states where coupling between the source and resonator enabled the energy generation from the instability source to exceed the dissipation or damping. Factors that describe this coupling have been determined by evaluating cases of lock-in that are encountered in the literature. A comparative study of these factors has been done to better understand the controlling physics and parameters associated with lock-in due to cavity shear layer instabilities, bluff body vortex shedding, and acoustic and structural resonators.

In particular, very little work in the literature has been found related to cavity fluid-elastic lock-in. Given the difficulty in achieving this type of lock-in experimentally, a systematic evaluation of coupling factors has determined for which combinations of factors lock-in is possible and for what states it can be achieved.
5.1 Conclusions

A unified theory has been derived to evaluate which coupling factors are most important in determining lock-in. This theory is based on the method of describing functions which is a nonlinear control theory used to predict limit cycles of oscillation, where a self-sustaining oscillation or lock-in is possible. This model is used in conjunction with an experimental study to provide insight into why fluid-elastic lock-in is difficult to achieve compared to other types of lock-in.

The coupling factors under consideration in this study include the type of fluid instability source, the material and physical properties of the resonator, and the relative wavelength between the source and the resonator. For each coupling factor, relatively weaker and stronger cases are considered. These coupling factors represent the generation and dissipation balance that must be met in order to achieve lock-in. Figure 5-1 shows a summary of the cases used to investigate the role of the coupling factors in the lock-in process, along with how various parameters affect lock-in.
The type of fluid instability source dominates the source generation component of the lock-in process, so a comparison between a cavity shear layer instability and a bluff body vortex shedding was completed experimentally and analytically. The bluff body produces a stronger oscillating pressure and source domain size coupled to the structure which produces a greater likelihood of lock-in.

The properties of the resonator dominate the dissipation component of the lock-in process, so a comparison between an elastic resonator with one that has similar material to the source, a fluid acoustic resonator, was completed.
experimentally and analytically. The acoustic resonator produces a greater likelihood of lock-in. Larger fluid-structure impedance differences produce weaker coupling.

A comparison between a distributed resonator, where the wavelength is closer to that of the source, with a discrete resonator, where the wavelength is much larger than the source, shows that the discrete resonator produces a greater likelihood of lock-in. The distributed resonator has a mode shape limited modal mass in comparison to a single moving point and the lumped mass of a discrete resonator.

For each of these coupling factors, controlling parameters were found and evaluated in Chapter 2. The source domain on the Nyquist plot is larger for larger oscillating pressures. The results in Figure 5-1 show that the hydrodynamic force produced by a bluff body attached to a stinger is larger than that produced by a cavity shear layer instability embedded in a stinger. As the resonator domain on the Nyquist plot increases, intersections between the source function and resonator function and the likelihood of lock-in are more likely to occur. These intersections occur near source and resonator frequency coincidence states along the resonator function. Frequency coincidence is a condition for lock-in, as explained in Chapter 3. These intersections also occur near a Strouhal source frequency associated with the instability stage, which is determined by many factors, the size of the source domain being the main one. For both types of instability sources under investigation, the Strouhal source frequencies produce common locations on the Nyquist plot despite the size of the
domains. For example, a Strouhal range of 0.4 – 0.5 is shown for reference in Figure 5-1.

As the resonator coupling factors are varied to produce stronger coupling to the source, intersections are also more likely to occur (given a common damping level). In Figure 5-1, cases representing the resonator coupling factors are shown. Intersections and lock-in are more likely to occur for an acoustic resonator, which has less impedance difference with the source given a similar fluid, and a discrete resonator, rather than a distributed resonator which is limited by modal mass and mode shape.

Consistent in all of the resonator coupling factors, damping is the most critical parameter in determining if lock-in occurs. As damping decreases (Q increases), the resonator function domain increases, resulting in a closer proximity to the source function. As each coupling factor is varied to include a weaker condition, additional controlling parameters are included:

- The primary difference between a discrete and distributed acoustic resonator is the addition of parameters related to the distribution of resonator mass. As the ratio of the lumped oscillating cavity (or bluff body) mass and the distributed resonator length, \( \Delta L / L_r \), increases, the resonator domain increases, producing a greater likelihood of lock-in. The participation of the oscillating cavity mass in the resonance relative to the resonator is proportional to \( \Delta L / L_r \). As the cross-sectional area of the oscillating source and the resonator increases, \( S_s / S_r \), the resonator domain increases, producing a greater likelihood of lock-in. \( S_s \) represents the size
available for oscillating pressure flows in and out of the cavity. Coupling between the cavity and the resonator pressure is proportional to \( S_r/S_c \).

- The primary difference between the acoustic and elastic resonators is the addition of parameters related to the resonator impedance, particularly the material density. As the ratio of the oscillating fluid source density and the elastic resonator density increases, \( \rho_f/\rho_e \), the resonator domain increases, producing a greater likelihood of lock-in.

- The primary difference between a discrete and distributed elastic resonator is the addition of parameters related to the distribution of mass similar to the acoustic resonator, but also an elastic vibration since the beam general solution is much more complicated than the one-dimensional general acoustic solution. As the resonator modal mass, \( M \), decreases, the inverse BGF domain size increases, producing a greater likelihood of lock-in. As the location of the oscillating cavity mass along the length of the cantilevered beam, \( x/L_r \), increases (approaching the free end), the inverse BGF domain size increases, producing a greater likelihood of lock-in.

Maximizing these controlling parameters, as shown in Figure 5-1, allows for the domain size of the source and resonator to increase and intersect. Although some inconsistencies in the controlling parameters exist, the overall consistency between coupling factors is good and provides an understanding of why combinations of strong coupling factors result in lock-in more readily than the weaker ones.
An experimental system was designed to test certain combinations of coupling factors. Chapter 3 discusses the design of a stinger as a distributed elastic resonator to test lock in with either a shear layer instability from a cavity embedded in it or vortex shedding from a bluff body attached to the trailing edge. Both sources interact and couple to the bending modes of the stinger. The cavity is based on one tested in a previous study (Rockwell et al. (2003)) which produced lock-in with a distributed acoustic resonator, or long organ pipe. When coupled to the stinger, this case is referred to the fluid-elastic lock-in.

The fluid-elastic cavity case is the weakest combination of coupling factors under consideration. As discussed in Chapter 4, this type of lock-in was not achieved experimentally. A similar sized cavity, however, was able to achieve lock-in with an acoustic pipe resonator. Fluid-elastic cavity lock-in is unlikely to occur given the critical level of damping that exists for a submerged structure and the relatively weak source strength that cavity produces. Decreasing damping and increasing the source strength analytically shows that lock-in may be possible. Fluid-elastic lock-in, however, is not feasible in air since coupling between the fluid and structure necessitates densities that are sufficiently close. Thus, realistically decreasing damping is not possible. Although a full-span cavity produces much stronger lock-in than a span-limited cavity in an acoustic resonator, a full-span cavity around the stinger would produce a stable configuration since synchronization and initiation of an oscillation would not be possible.
Experimentally and analytically, a stronger source such as vortex shedding from a bluff body is able to lock-in to the same resonator as the fluid-elastic cavity case. Thus, the type of instability source and the impedance difference between the source and resonator are dominating coupling factors for lock-in. They represent the generation and dissipation balance that must be met in order to achieve lock-in.

The results are also consistent with another method for modeling flow-induced vibrations, wake oscillator theory. In order to achieve lock-in, the excitation coefficient from the source instability must be able to overcome the mass-damping parameter from the resonator. A critically low mass-damping is not attained by the stinger compared to a relatively weak cavity source, but the balance is accomplished to produce lock-in with a stronger source from the bluff body vortex shedding. Quantitatively, the describing function model results are very consistent with the mass-damping level for lock-in from other studies.

Vortex-induced vibration experiments are often complicated by the challenging aspects of instability isolation, facility interference, and attaining sufficiently low damping. In this study in particular, the bluff body generated instability is complicated by three-dimensional flow physics and the cavity generated instability is complicated by sizing a sufficient span. Even with these challenges and the resulting adverse effects on the data, the describing function models still sufficiently capture the results. Thus, future use of the unified describing function method developed here may not require rigorous source models to predict lock-in.
Initial evaluations of other source and resonator configurations could be accomplished with rough estimates of the source magnitude and frequencies, which are widely available in the literature. If the possibility of lock-in is considered likely, additional refinement of the source model could be undertaken with assistance of computational tools. If an especially unique source is under consideration, however, an experiment may be necessary, given that the current literature and computational models are all somewhat based on more idealized instability configurations.

In addition to source considerations, describing function models can also be used to bound possible resonators and coupling conditions. As explained in Chapter 1, some engineering systems are actually a combination of two different types of lock-in, fluid-resonant and fluid-elastic. Acoustics waves in the fluid and structure surrounding the source must be taken into account. In either case, this unified method of describing function models is a robust tool for understanding lock-in and particularly useful in scoping future efforts to either ensure or avoid lock-in.

This investigation has shown that some types of lock-in are more challenging than others. An improved fundamental understanding of the roles of coupling factors in generating the lock-in phenomenon has been formed. Analytical modeling using describing functions produced results that are qualitatively consistent for all combinations of source and resonator under investigation. This study also shows a strong consistency between the modeling and experimental observation. Thus, describing function modeling is a useful
and robust method for predicting lock-in. Given these results, the fluid-elastic cavity lock-in case may never be possible given the physical constraints in achieving a sufficiently low damping and a sufficiently strong source necessary for lock-in.

5.2 Suggestions for Future Work

This investigation consisted of analytical modeling and experimental study to generate improved understanding of the lock-in process. Computational studies would greatly add to this understanding.

One such lock-in case that has been numerically simulated is the fluid-elastic cavity configuration with a compliant wall. The coupling between the shear layer instability and the compliance of the wall at the bottom of the cavity is unique and fell outside the scope of this investigation. When the resonator wavelength is proportional to the source wavelength, coupling can occur in a distributed manner. Recirculation in the cavity is directly coupled to the compliant wall wave. Since fluid flow must be implicitly described in great detail, lumped parameter modeling such as the method of describing functions cannot be used. Numerical studies have been used computationally to study this case as described in Chapter 2, but never in comparison to other lock-in cases such as the ones described in this investigation.

Other suggestions for future work are confined to additional variations of the parameters contained in this investigation. Another source that could be
evaluated for elastic and acoustic coupling and lock-in is that of a hole-tone. The plate containing the hole may be compliant and also placed in an acoustic pipe resonator. This combination of coupling factors would include an acoustic and elastic component at the same time to evaluate how synchronization and feedback occurs. Experimentally, this configuration would also be easier to test than the stinger configuration.

The hole-plate-pipe configuration, however, would not enable additional visualization studies beyond the attempts pursued in this investigation. A separate study is recommended to study lock-in synchronization and visualization. Computational simulations, although helpful in visualization, are not sufficient alone for understanding the synchronization process between a shear layer type instability and an elastic resonator.

Concerning the current investigation, several open issues exist. The discrete resonator is artificially simulated using analytical modeling. A spring-mounted cavity was attempted experimentally, but this configuration proved too difficult to test since the source of excitation could not be isolated to the shear layer instability without interference from turbulent buffeting. An experimental investigation of this configuration would be interesting.

Another aspect which should be studied is the orientation of the source with respect to the resonator. This variable is not included in this investigation, but ways to incorporate this into the describing function models should be explored.
Finally, the source modeling currently has a significant amount of uncertainty. While this uncertainty does not appear to adversely affect the lock-in predictions in this study, refinement and improved methods should be explored. If a more accurate source modeling method were to exist, the describing function modeling would be able to distinguish between various impinging flow sources. An expansion of this study capturing these open issues is useful for enhancing the accuracy of lock-in predictions.
Appendix A

Lock-in Coupling Factor Model Using Linearized Stability Theory

The describing function theory approach to modeling lock-in involves the interaction of a forward gain function (FGF) and backward gain function (BGF). The FGF represents the fluid instability source while the BGF represents the impedance of the resonator. For lock-in due to a shear layer instability, numerous methods for modeling the source exist. In Chapter 2, the hydrodynamic forcing function approach was used since it was capable of comparing a shear layer instability source with gross vortex shedding from a bluff body. One of the original describing function studies, however, used linear stability theory to model the shear layer instability (Elder (1978)). This appendix compares the results of using linearized stability theory with the hydrodynamic forcing function approach and its ability to predict lock-in.

The linearized stability model was first introduced by Michalke (1965) to describe the nonlinear growth rate of a shear layer or mixing layer encountered in cavity flow. Linear stability theory is derived from perturbed forms of the Navier-Stokes equations and is used to predict instability frequencies of a disturbance. This derivation involves the Orr-Sommerfeld equation to evaluate the velocity profile in the hydrodynamic instability of two uniform streams and the Rayleigh equation which is the inviscid form of Orr-Sommerfeld. Once the growth of a disturbance is modeled in the Rayleigh equation, the amplification of an instability
frequency can be determined. Michalke (1965) determined the growth disturbance rates for a mixing layer type of flow, which is the velocity profile of a cavity shear layer instability. Other types of flows that have been modeled include jet and wake type velocity profiles. Lucas et al. (1997) includes the derivation and Rayleigh solutions for all types of velocity profiles.

Elder (1978) modeled the FGF after the original approach of Cremer and Ising (1968). Wave profiles across the mouth of the cavity correspond to the phases of oscillation. Estimates for the oscillating cavity flow are largely based on inflection point displacements of the wave at the interface across the mouth of the cavity. Using tables of the complex propagation constant and phase velocity from Michalke (1965), a FGF is derived:

$$G_{12} = \frac{U_o}{U_c} \left[ -\frac{\cos(\gamma)}{\gamma} + j\frac{\sin(\gamma)}{\gamma} \right]$$

and

$$\gamma = 2\pi \frac{fL_c}{U_c},$$

where $U_o$ is the average flow speed at the interface, $L_c$ is the length of the cavity, $f$ is frequency (Hz), and $U_c$ is the convective velocity of the disturbance. Using Michalke (1965), $U_c$ is the product of the nondimensional phase velocity and the freestream velocity inflow to the cavity. The phase velocity is generated from a table that corresponds to the nondimensional instability frequency, $\beta$, which is a function of the velocity profile ($y$ is the transverse to flow direction) which is turbulent for the cases considered in this study

$$\beta = \pi f \left. \frac{dU}{dy} \right|_{turbulent}.$$
Since $U_\infty$ is likely to depend on frequency since full closure of the wave across
the interface is not reached for all conditions, it was assumed to be about 0.75 of
the freestream velocity as an average. The results from Elder (1978) are
repeated to verify that this assumption is valid.

In comparison to the hydrodynamic forcing function (referred to in the
plots as alpha) approach, the linearized stability theory (LST) approach produces
a FGF with a similar sized domain, but with different Strouhal frequency
conditions, as shown in Figure A-1. The Strouhal numbers of 0.4 and 0.5 which
are approximate conditions for the first stage of a cavity shear layer instability are
in the upper right quadrant on the Nyquist plot. Only Strouhal frequencies that
are in the lower quadrants may intersect with the inverse BGF’s of the resonator.
When the inverse BGF is modeled for a side branch resonator from Yang (2005), the Nyquist plot shows intersections with the LST derived FGF, but not close to the Strouhal frequencies that achieved lock-in experimentally. When using the alpha model, however, the intersections do correspond to the Strouhal frequencies (for a damping level smaller than that achieved in testing which is shown). Figure A-2 shows this comparison.
Figure A-2: Describing Function Model Prediction for Side Branch Resonator with LST (upper) and Hydrodynamic Forcing Function (lower); Based on the following parameters: $L_c = 2''$ (50.8 mm), $\Delta L = 0.75''$ (19.1 mm), $L_r = 19''$ (482.6 mm), $n = 1$. 
One major difference exists in the inverse BGF models for acoustic resonators between Elder (1978) and that of Chapter 2. Both models use the following ratio of mass and compliance impedances for the BGF

$$G_{21} = \frac{q_c}{q_o} = -\left(\frac{z_c}{z_m + z_c}\right),$$

but in Elder (1978) damping is only included in $Z_c$ in the denominator. In Chapter 2, damping is included in $Z_c$ for both the numerator and denominator. In modeling elastic resonators such as the cantilevered beam, damping must be included throughout in order to not have a singularity in the solution. Also, damping should be included wherever the resonator compliance is modeled, as this is the dominant mechanism producing dissipation in these models, as first discussed in Section 2.3.2.

Although Elder (1978) chose to simplify damping down to a single term in the inverse BGF, there is little impact to the Nyquist plot results. Figure A-3 shows very little difference in the inverse BGF. There is no difference when $f/f_r = 1.0$, since more difference is seen off resonance as expected. More difference is also seen when the damping is higher ($Q$ decreasing to 500 in Figure A-3 lower), which is not surprising. Given the relatively small amount of damping that occurs in these resonators in order to achieve an intersection with the FGF and lock-in, the difference in modeling damping is negligible compared to other source of uncertainty, such as the FGF.
Figure A-3: Describing Function Model Prediction for a Side Branch Resonator with Different Damping Models; Based on the following parameters: $L_c = 2''$ (50.8 mm), $\Delta L = 0.75''$ (19.1 mm), $L_r = 19''$ (482.6 mm), $n = 1$. 
The hydrodynamic forcing function method used to model the source in Chapter 2 was based on empirical data. Although not as accurate, using LST to model the source FGF is another method that can be used if experimental data does not exist. This is particularly useful to determine if achieving a lock-in state is possible.
Appendix B

Flow Visualization Using Laser Doppler Velocimetry

As discussed in Chapter 4, the stinger with an embedded cavity did not lock-in. The shear layer instability from an embedded cavity is not able to lock-in with a distributed elastic resonator due to several reasons. The cavity source is weak compared to vortex shedding from a bluff body (a similar sized cavity, however, is able to lock-in with an acoustic pipe resonator). Also, too much dissipation in a submerged elastic structure exists for a span-limited cavity source to compensate to produce a self-sustaining oscillation.

The cavity source under investigation is span-limited which produces an unstable configuration when embedded in a stinger oscillating in bending. Laser Doppler Velocimetry (LDV) is a technique to determine the steady and unsteady velocity fields in situ. Previous studies used LDV to evaluate the unsteady flow over the opening of a cavity (Nelson et al. (1981)). In an early stage of this investigation, LDV measurements were acquired across the opening of the stinger-cavity. The stinger-cavity design at the time of the measurements used a cavity with a 0.75" span, rather than the 1.25" span in the final design. The vibratory response of the stinger did not change significantly due to the increased span. The LDV measurements show that a shear layer instability does exist despite the limited span of the cavity. This is also verified in the final design using the flow visualization technique of bubbles and high speed video.
The inflow turbulence to the stinger is typical of fully developed turbulent pipe flow. The inflow turbulence is high compared to the cavity shear layer as shown in Figure B-1. The time-averaged fluctuation amplitude of the inflow and shear layer is evaluated using root-mean-square (rms) velocity fluctuations in the streamwise ($U$) and transverse ($V$) flow directions. Across the mouth of the cavity, the patterns of the turbulence statistics in Figure B-2 are consistent with other investigations (Nelson et al. (1981) and Oshkai et al. (2005)). $U_{\text{rms}}$ increases near the downstream half of the cavity and $V_{\text{rms}}$ is concentrated in the downstream half of the cavity. Overall rms levels decrease downstream of the cavity.

Figure B-1: Steady and unsteady velocity measurements inflow to the stinger-cavity; $U_{\text{mean}}$ is the average flow speed, $U_{\text{rms}}$ is the standard deviation of the flow speed, and TI is the turbulence intensity in %
The amplitudes of the turbulence statistics, however, are high compared to other investigations. Figure B-3 shows a comparison of the stinger-cavity with the cavities investigated by Oshkai et al. (2005), Nelson et al. (1981), and Yang (2005). Particle image velocimetry (PIV) was used to evaluate a similar sized length and depth cavity with fully developed turbulent flow, but fully axisymmetric in the span and at lock-in with an acoustic resonator (Oshkai’s Pipes-Cavity in Figure B-3). In PIV, a snapshot of all of the flow in a measurement frame is acquired in time, while in LDV, select measurement points in the flow are acquired in time. Select measurements at approximately 0.5, 0.75, and 0.95L, where L is the cavity length are shown for limited comparison purposes. Surprisingly, the amplitude of the turbulence intensity of the stinger cavity is higher than Oshkai’s cavity. The orientation of the acoustic resonator has a

**Figure B-2: Steady and unsteady velocity measurements in the stinger-cavity**

<table>
<thead>
<tr>
<th>U_{\text{mean}} (5)</th>
<th>10.8</th>
<th>U_{\text{rms}} (5)</th>
<th>2.76</th>
<th>V_{\text{mean}} (5)</th>
<th>-0.16</th>
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<td>Ti (5)</td>
<td>25.7</td>
<td>V_{\text{rms}} (5)</td>
<td>1.84</td>
<td>V_{\text{rms}/U_{\text{mean}}} (5)</td>
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<td></td>
<td></td>
<td>V_{\text{rms}/U_{\text{mean}}} (5)</td>
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<th>U_{\text{rms}} (6)</th>
<th>2.7</th>
<th>V_{\text{mean}} (6)</th>
<th>-0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ti (6)</td>
<td>24.9</td>
<td>V_{\text{rms}} (6)</td>
<td>1.7</td>
<td>V_{\text{rms}/U_{\text{mean}}} (6)</td>
<td>0.16</td>
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<tr>
<td></td>
<td></td>
<td>V_{\text{rms}/U_{\text{mean}}} (6)</td>
<td>-985</td>
<td></td>
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<th>U_{\text{rms}} (7)</th>
<th>2.9</th>
<th>V_{\text{mean}} (7)</th>
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<td>1.6</td>
<td>V_{\text{rms}/U_{\text{mean}}} (7)</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td></td>
<td>V_{\text{rms}/U_{\text{mean}}} (7)</td>
<td>-1384</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
negligible effect as shown by similar amplitudes for a side branch acoustic resonator. When there is no lock-in, however, the fluctuating velocity amplitudes should be low. The LDV results from a side branch investigation without fully turbulent inflow are also shown, which shows much lower fluctuating velocity amplitudes than the turbulent results, even at lock-in.

Figure B-3: Select turbulence intensity comparisons with other investigations
When the stinger is fixed to prevent it from vibrating or bending (using a fixed strut to the pipe wall), the turbulence intensities fall within levels of the other turbulent investigations as shown in Figure B-4. Therefore, the stinger-cavity vibration without the stinger fixed produces excessive velocity fluctuations beyond that from the inflow. This contribution is equivalent to that from the turbulent studies under lock-in.

Figure B-4: Select turbulence intensity comparisons with other investigations
Although the cavity shear layer instability exists as shown by the flow patterns and is affected by the vibration as shown by the turbulence amplitudes, synchronous coupling between the instability and vibration is still not achieved. One of the major objectives in designing the stinger-cavity, as discussed in Chapter 3, is the need for the instability from the cavity to be isolated from all other flow instabilities in the test. LDV scans throughout the cavity span, and upstream of the cavity and the stinger show that no other instabilities exist.

Reynolds stress, the amount of inertia produced by turbulence compared to laminar flow in the momentum equation, of the cavity shear layer was compared to that of the inflow. Reynolds shear stress (RSS), \( \frac{\langle u'v' \rangle}{U^2} \), is often used to describe the strength of the shear layer instability. Extensive measurements of cavity shear layer instability turbulence statistics were made in Grace et al. (2004) for a cavity at low Mach numbers without lock-in at both laminar and turbulent flow conditions. RSS levels for turbulent flow are significantly higher than laminar flow. Measurements at \( \frac{x}{L} \) positions of 0.1, 0.19, 0.38, 0.58, 0.77, and 0.94 confirm that the RSS levels are at a maximum at \( \frac{x}{L} = 0.94 \) and at the opening interface of the cavity, or \( \frac{y}{L} \) near 0. The RSS levels are over an order of magnitude higher than the inflow levels. In Figure B-5, the RSS levels are shown for the stinger-cavity. The RSS level at the peak measurement location in the cavity, 0.9\( L \), is only about three times that of the inflow levels. The shear layer instability is therefore weak compared to the inflow.
The size of the cavity with respect to the momentum thickness of the shear layer at separation must be sufficiently large to produce a shear layer instability. Momentum thickness is defined as

$$\theta = \int_0^\infty \left( 1 - \frac{u}{U} \right) \frac{u}{U} \, dy,$$  \hspace{1cm} B.1

where $u$ is the streamwise velocity measured from the wall and $U$ is the freestream streamwise velocity. The power law velocity distribution for turbulent flow follows a $1/7^{th}$ profile. Figure B-6 shows the velocity distribution and integrand results, which compare well to the power law fits (measurements near the wall are limited due to the LDV measurement volume). The resulting momentum thickness is about 0.0095" at the center of the cavity span as shown in Table B-1. At the 0.65 and 0.9 positions of the cavity span, the momentum thickness is about 0.015" and 0.0104", respectively. The momentum thickness is about 0.0095" at the center of the cavity span as shown in Table B-1. At the 0.65 and 0.9 positions of the cavity span, the momentum thickness is about 0.015" and 0.0104", respectively. The momentum thickness is about 0.0095" at the center of the cavity span as shown in Table B-1. At the 0.65 and 0.9 positions of the cavity span, the momentum thickness is about 0.015" and 0.0104", respectively. The momentum thickness is about 0.0095" at the center of the cavity span as shown in Table B-1. At the 0.65 and 0.9 positions of the cavity span, the momentum thickness is about 0.015" and 0.0104", respectively.
does not vary greatly through the cavity span. The length and depth of cavity with respect to momentum thickness are both sufficiently large as to not preclude lock-in. As discussed in Chapter 3, the nondimensional frequency or Strouhal number with momentum thickness, $f\theta/U$, for a shear layer instability reaches a peak amplification at corresponding Strouhal number of 0.017 according to linearized stability theory (Lucas et al. (1997), with an amplification range corresponding to 0 to 0.04. At the predicted lock-in state, $f\theta/U$ is within this range.

---

**Figure B-6: Velocity profile at separation point of cavity**
Table B-1: Estimates of Momentum Thickness and Normalized Cavity Sizes and Strouhal Frequency ($W$ is Cavity Depth and $L$ is Cavity Length)

<table>
<thead>
<tr>
<th>Location in Cavity Span (total is 0.75&quot;)</th>
<th>Momentum Thickness, $\theta$ Estimate (mils)</th>
<th>$W/\theta$</th>
<th>$L/\theta$</th>
<th>$f\theta/U$ (at theoretical lock-in point)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Center</td>
<td>9.5</td>
<td>53</td>
<td>237</td>
<td>0.023</td>
</tr>
<tr>
<td>In Between</td>
<td>15</td>
<td>33</td>
<td>150</td>
<td>0.037</td>
</tr>
<tr>
<td>End</td>
<td>10.4</td>
<td>48</td>
<td>216</td>
<td>0.025</td>
</tr>
</tbody>
</table>

Despite the existence of a shear layer instability, the span-limited cavity source in this investigation is weak compared to other investigations as shown by the Reynolds stress relative to inflow levels. The inflow momentum thickness, however, is sufficiently thin for this size cavity to allow lock-in. The momentum thickness can also be used to compare to the vibration displacement during lock-in. In bluff body lock-in of cylinders, the amplitude of vibration is often compared to the cylinder diameter. The vortex shedding produced by the cylinder is on the same order as the cylinder diameter. As the lock-in state is reached, the resonant response of the cylinder displacement with respect to diameter, $A_y/D$, increases dramatically. The span aspect ratio only affects the resonant response when $A_y/D$ is less than that required for full spanwise correlation, about 0.5 (Blevins (1990)). Lock-in is usually achieved for a limited span for $A_y/D$ greater than 0.1 – 0.2.

In a cavity shear layer instability, the dimensional size equivalent to the cylinder diameter is the momentum thickness. The peak displacement of the
stinger with respect to the momentum thickness, \( Ay/\theta \), at resonance at the theoretical lock-in state is on the order of \( 10^{-2} \). In the acoustic lock-in testing with a similar sized cavity (Rockwell at al. (2003)), an equivalent displacement was roughly estimated using Euler's equation, for \( Ay/\theta \) on the order of \( 10^{-1} \) for a fully axisymmetric cavity. The oscillating pressure was an order of magnitude less for the span-limited cavity that locked into the acoustic resonator (Rockwell et al. (2001)), producing \( Ay/\theta \) on the order of \( 10^{-2} \). So although the Reynolds stress measurements show that a weak shear layer instability exists, the displacement of vibration with respect to momentum thickness for the stinger-cavity is similar in magnitude to that of the span-limited cavity that locked into an acoustic resonator, albeit an order of magnitude weaker than that of a cylinder bluff body and a fully axisymmetric cavity.

In summary, flow visualization using LDV is able to show that although lock-in of a cavity embedded on a stinger is not achieved in this investigation, a shear layer instability does exist in the span-limited cavity. No other instabilities in the flow interfere with this instability. The turbulence patterns are similar to those in other investigations. Although Reynolds stress measurements show that the instability is weak, other factors exist which show that it is still sufficiently strong to lock into a resonator. The inflow momentum thickness is sufficiently thin for this size cavity to allow lock-in. The displacement of vibration with respect to momentum thickness for the stinger-cavity is similar in magnitude to that of a similarly sized span-limited cavity that locked into an acoustic resonator.
Appendix C

Global Spectral Analysis

Global spectral analysis is a method used to evaluate the spectral content of a fluctuating flow field. Maps of velocity in the flow field of interest must be acquired using high-speed particle image velocimetry (PIV) or computationally generated using time-accurate CFD. By performing a spectral analysis of the flow field time history, contributions to the energy at defined frequencies or over a cycle of oscillation can be visualized and tracked over distinct flow features. This method has been used extensively by Dr. Rockwell at Lehigh University (Celik et al. (2007), Sever (2005)).

In the current investigation, lock-in is achieved between the vortex shedding from a bluff plate attached to the trailing edge of a cantilevered beam or stinger. As discussed in Chapter 3 and 4, the source produced in this configuration is unique due to the confinement around the plate by the surrounding piping. A recirculation region upstream of the plate and time-varying flow restriction between the plate and test duct piping is coupled to the shedding downstream of the plate. Since an excessive amount of three-dimensional flow occurs, discrete frequencies of the instability are not observed in the PIV-generated force data. The force is computed from the sum of the vorticity and velocity cross-products in the flow:
Force is used to model the instability as a source in a describing function lock-in model. This is in contrast to the more common use of PIV data in lock-in, which is to evaluate the energy production term

\[ \frac{F}{\rho} (t) = - \sum_s (\bar{\omega} \times u) \hat{y} - \sum_s (\bar{\omega} \times v) \hat{y}. \]  

C.1

The same method for analyzing PIV data to observe spectral contributions to the energy can be applied to the force used in this investigation.

Spectral analyses of the unsteady streamwise velocity, transverse velocity, and vorticity are done to clarify the contributions of these components to the streamwise and transverse forces. Since the data was acquired at a 200 Hz sampling rate, spectra for the flowfield maps can be generated for frequencies between 0 and 100 Hz with a resolution of about 0.2 Hz. Movies are generated to quickly find significant flow features at specific frequencies. Select frequencies of interest are shown for the two measurement frames or windows of interest. The upstream window captures the recirculation region upstream of the plate and the initial part of the vortex shedding downstream of the plate. The downstream window captures the entire formation of vortex shedding downstream of the plate.

The only discrete frequency seen in the data is at \( \sim 36.2 \) Hz in the transverse velocity in the downstream window. The spectral map in Figure C-1 shows that the vortex shedding from the plate is the source of this discrete source, which corresponds to the lock-in frequency. The velocity is concentrated
near the separation point of the plate. Unfortunately, lock-in was not observed during the PIV data acquisition since the stinger mounting was modified to allow camera access. Otherwise, the concentration of velocity at 36.2 Hz should be even higher.

Figure C-1: Spectral map of downstream transverse velocity at a discrete frequency of 36.2 Hz (stinger-plate position approximate)

In contrast to the transverse velocity, the streamwise velocity spectral maps show no tones at discrete frequencies. All of the streamwise velocity data
is random as shown by the spread of higher amplitudes for select frequencies in Figure C-2.

In the vorticity of the downstream window, no discrete frequencies are found in surveying all of the spectral maps. The spectral map corresponding to 36.2 Hz in Figure C-3, however, shows that the primary concentration area is near the separation point of the plate.
The force in the streamwise direction is formed from the cross-product of vorticity and transverse velocity. The force in the transverse direction is formed from the cross-product of vorticity and streamwise velocity. In Chapter 4, the transverse force was found to be two orders of magnitude higher than the streamwise in the downstream window, since the stinger and flow are oscillating primarily in the transverse direction. Due to the random nature of the velocities, however, no discrete frequencies in either force component were found. The spectral maps of the forces at 36.2 Hz in Figure C-4 show a concentration near the separation point of the plate. Thus, the global spectral analysis confirms that

Figure C-3: Spectral maps of downstream vorticity at 36.2 Hz (stinger-plate position approximate)
the forces used in the describing function models are due to the vortex shedding of the plate coupled to the stinger vibration at its first bending mode.

Figure C-4: Spectral maps of downstream streamwise force (top) and transverse force (bottom) at 36.2 Hz (stinger-plate position approximate); the magnitude of the transverse image is double that of the streamwise image
The upstream window shows no discrete frequencies in the spectral maps of the velocities, vorticity, or forces. The recirculation region upstream of the plate does not appear to correlate with the downstream vortex shedding at 36.2 Hz frequency found in the downstream transverse velocity spectral map. Figure C-5 shows a random spread out area in amplitude at this frequency.

![Spectral map of upstream transverse velocity at 36.2 Hz (stinger-plate position approximate)](image-url)
Some frequencies were found where the recirculation cell appears to correlate with the downstream vortex shedding. This correlation is weak and occurs at random frequencies, as shown in Figure C-6.

In Chapter 4, PIV images are used in a traditional manner to show patterns of velocity and vorticity. The forces computed from the cross-products of these measurements are random in nature due to the excessive amount of three-dimensional flow. However, through the use of global spectral analysis, the spectral maps of the high-speed PIV image series shown here confirm that the
vortex shedding of the plate has a discrete frequency which closely corresponds to the first bending mode of the stinger and lock-in. Separation at the plate tip has a concentration of transverse velocity at 36.2 Hz. The recirculation cell upstream of the plate does not appear to correlate with the downstream vortex shedding at this frequency, although some random correlation exists. These conclusions could not have been made without the use of global spectral analysis.
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VITA

Kristin Lai-Fook Cody was born on May 13, 1975. She spent her childhood in Rochester, MN, San Francisco, CA, and Lexington, KY. She graduated from Henry Clay High School in 1993. She attended Purdue University in West Lafayette, IN, graduating in 1998 with a B.S. in mechanical engineering. While at Purdue, she completed five semesters of cooperative education at the NASA Johnson Space Center in Houston, TX. Her research mainly focused on robotics in space environments with additional experience in aerothermodynamics and mission operations.

Upon graduation from Purdue, Kristin was accepted into the Lockheed-Martin Engineering Leadership Development Program in Schenectady, NY. Through this work-study program, she attended the Rensselaer Polytechnic Institute and received a M.S. in mechanical engineering in 2001. Her research focused on applied mechanics, in particular flow-induced vibrations. Her thesis was monitored by Dr. Henry Scarton and is entitled, “Oscillations of Flow through Collapsible Tubes.” In 2001, she also graduated from flight school and obtained a private pilot license.

Upon graduation from RPI, Kristin began professionally and academically studying acoustics. In 2001, she enrolled in the distance education program in the Graduate Program in Acoustics at the Pennsylvania State University. She began pursuing a Ph.D. in acoustics soon after, under the guidance of Dr. Stephen Hambric. She has presented a number of papers at professional meetings, mostly in her current research area of fluid structure interactions and flow-induced vibrations. While pursuing her academic studies at Penn State, Kristin married Steven Cody in 2002 and together they have a daughter, Genevieve, born in 2007.

Upon completion of her degree requirements, Kristin plans to continue her career and research in naval acoustics, devote more time to motherhood, and enjoy her first love of acoustics, playing the flute.