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ABSTRACT

This study investigated the nature of dyadic interaction between secondary English Language learners engaged in mathematic peer tutoring sessions. An analysis of fifteen ELL expert/novice student tutoring dyads and three mathematics teacher/ ELL novice dyads revealed the importance of questions and wait time in developing novice “tutee” ownership of the mathematical problem solving activities. Under current policy context, mathematics teachers are encouraged to adapt to reform-oriented teaching practices that emphasize discourse and communication, as noted National Council of Teachers of Mathematics Standards published in 1989, and prepare ELLs to participate in high stakes large scale assessments. Often used as gatekeeper to high school graduation, secondary math teachers are not only under immense pressure to adapt their teaching strategies to reflect reform oriented methods, but also under an immense time constraint to prepare all students for graduation. The results of this study are supported by Vygotsky’s theory of cognitive development and can offer secondary mathematics teachers of ELLs practical strategies that reflect reform oriented teaching practices, support active participation, independent problem solving, and vocabulary acquisition, more specifically, ownership of the mathematics problem solving activities.
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CHAPTER ONE

A Mathematical Problem

This study emanates from my personal experience as an ESL teacher in a Pennsylvania public school district. As the only ESL teacher in the school, I worked as a co-teacher of secondary mathematics for newcomer ELLs placed in the mainstream mathematics classroom. The student population of this secondary level co-taught classroom was quite eclectic. It consisted of approximately 85% learning disabled students and 15% of students considered “at risk” of failing math that included a mix of newcomer ELLs and general education students. As I co-taught with two other teachers, a general education math teacher, and special education teacher, I had ample opportunities to observe the strategies they used to teach their students.

During my time spent in this classroom, I noticed that most of the instructional strategies my colleagues used were the same lock step teaching methods used in other traditional math classes. The teacher would present a mathematics problem to the entire class solving it in a step by step fashion and then distribute worksheets filled with problem sets for the students to compute in silence or finish for homework. Although the steps of the problem solving process were demonstrated in detail via visual media (smart board or power point), this mode of teaching did not involve students in co-constructing the mathematics process (i.e. collectively and orally engage in solving the mathematics problem with the teacher), which would allow students to verbally explain the problem solving process using arithmetic vocabulary. This type of instructional practice also did not meet national standards for effective mathematics instruction. For example, NCTM advocates discourse and communication in their standards and principles (NCTM, 2000) and the PSSA mathematics assessments score students on their ability to explain the problem solving process in mathematical terms (see Appendix B). However, this classroom
did not prepare students with the necessary skills to accomplish this goal. Although the students were no longer at risk for failing the mathematics class, students would certainly not be able to accomplish the communication goal of the PSSA test and most likely fail to demonstrate obtainment of grade level mathematical competencies, which would put them in danger of not fulfilling graduation requirements. This is particularly true for ELLs who were all at the “entering” level of English language proficiency (see Appendix A for more detail). When I questioned the teachers why the students did not work in small groups or discuss the problem sets together, the head teacher was concerned about the issue of students completing their own work. He maintained that small group work or co-constructing answers to the problem sets would lead to no evidence (i.e. worksheets) of the students’ own work in which to measure progress. He also alluded to the fact that it would result in bad habits such as cheating or copying from one another. Inspired by his responses, I set out to conduct a study that would dispel this myth and provide evidence that co-construction of knowledge and is more apt to promote learning that leads to student “ownership” (i.e. successful acquisition and demonstration of the vocabulary and problem solving processes) of the mathematic problem solving activity than the demonstration - mimicry - practice sequence currently being used.

School districts throughout the United States are experiencing a rapid increase in the number of English Language Learners (ELLs) entering their schools (National Clearing House for English Language Acquisition, 2008). Representing twenty percent of all school aged students and projected to reach forty percent by 2030 (Thomas and Collier, 2001), ELLs with varying levels of English language proficiency, formal education, and cultural background are making their way into mainstream secondary classes where English is the medium of instruction. Routinely placed in math as their first mainstream subject, ELLs face the daunting task of
learning and demonstrating mathematics content knowledge in a new language while secondary math teachers are charged with teaching mathematics vocabulary and problem solving skills to a growing number of students that do not speak or understand English well (Betne & Henner-Stanchina, 2005; Flynn & Hill, 2005; Irujo, 2007).

The rise in ELL enrollment has drawn attention to the instructional needs of this linguistically and culturally diverse student subgroup, although it is at the secondary level that attention to their educational needs is most urgent (Ruiz-de-Velasco & Fix, 2000). Federal mandates contained in the No Child Left Behind Act (NCLB) require teachers to prepare ELLs to meet academic content standards and demonstrate problem solving skills through mathematical language on large-scale assessments that require a sophisticated level of English language proficiency and cultural knowledge most ELLs have yet to acquire (Abedi, Lord, & Plummer, 1997; Cummins, 1988; National Council of Teachers of Mathematics, 2008). Comprised of specialized, culturally bound vocabulary and symbols, complex grammatical structures, and a style of discourse that includes abstractions and passive voice, the language of math proves to be the primary reason ELLs do not perform well on large-scale assessments (Abedi, 2008; Abedi, Lord, & Plummer, 1997). With initial ELL pass rates on high school exit exams in math thirty to forty percent lower than non-ELL students, many ELLs choose dropping out of high school as an alternative to academic failure (Center on Education Policy, 2005; Fry, 2008; Menken, 2008; National Assessment of Educational Progress, 2010).

**Background**

Employing teaching strategies that support the language of ELLs and encourage active participation in mathematics instruction is necessary to ensure the mathematical development of this diverse student subgroup (National Council of Teachers of Mathematics, 2008).
Unfortunately, most mathematics teachers have not been trained to work with ELLs and feel unprepared to meet the diverse needs of this student subgroup (Abedi, 2008; Lucas, Villegas, Freedson-Gonzalez, 2008; National Center for Educational Statistics, 1999). Unaware of the heterogeneity of the ELL sub-group, many teachers adopt a "one size fits all" when choosing strategies and often employ those used for special education or mainstream English speakers. Such general strategies are neither appropriate nor conducive to the development of the specialized mathematics vocabulary or problem solving skills for ELLs.

Due to the reform efforts of the National Council of Teachers of Mathematics (NCTM), major changes in the teaching practices used in mathematics classrooms throughout the country have begun to change. Implemented through a set of instructional goals outlined in NCTM’s Professional Standards for Teaching Mathematics (1989, 1991, 2000), reform oriented practices required by NCTM advocate discourse in the mathematics classrooms as a means to engage students in mathematical thinking and language development (Cazden, 2001; Fraivillig, 2001). The change from traditional mathematics teaching to reform mathematics teaching is not an easy task as it is unclear to teachers how they can develop the discourse practices that enable them to reach their teaching goals and students to attain the standards needed for academic success (Hufferd-Ackles, Fuson, Sherin, 2004). Although scholars have identified effective instructional practices and developed frameworks for promoting discourse communities in mathematics classrooms (Fraivillig, Murphy, & Fusion, 1999; Hufferd-Ackles, Fuson, Sherin, 2004) little has been done to address the effects of these research-based practices with high school level ELLs (Roberson & Sumerlin, 2005).

The ability to understand and apply mathematical concepts is a necessity for academic success, everyday living, employment, and personal gain. Reluctance to teach all students,
including ELLs, mathematical problem solving skills due to lack of teacher preparation or resources is not only unethical, but against the law (U.S. Department of Education, 2006; National Council of Teachers of Mathematics, 2008). Both NCLB and National Council of Teachers of Mathematics (NCTM) positions on teaching math to ELLs indicate a need for teachers to employ strategies that support the diverse needs of ELLs. In a move to ensure ELLs are properly educated to meet academic content standards, perform adequately on large-scale assessments, and graduate high school, math teachers that employ traditional methods of teaching, such as rote learning and drill, must adapt their methods to meet the unfamiliar, opposing reform oriented approaches that emphasize discourse and communication for teaching math (Garrison & Mora, 2005; NCTM, 2010). ELLs can no longer be denied equal access to mathematical content because their culture, educational experiences, and language are different from the mainstream students.

Considerable evidence indicates a need to identify effective problem solving strategies secondary mathematics math teachers can use to make arithmetic content and vocabulary comprehensible for ELLs to participate in the mathematics content classroom and take ownership and internalize problem solving strategies to become lifelong problem solvers. The increasing number of ELLs entering mainstream secondary math classes, the insufficient preparation of math teachers to teach ELLs, and the negative consequences of high stakes testing are reasons that warrant this study.

**Purpose and Research Questions**

The purpose of this study is to identify interactive discourse strategies secondary mathematic teachers can use in the classroom to develop ELL ownership of the mathematic problem solving activities. More specifically, this study examines how effective tutors use
questions and wait time to increase active participation, independent problem solving, and use of mathematics vocabulary of tutees as they work to solve mathematic problems in small group activities. I conducted an observational study over 4 weeks to determine what types of discourse strategies, specifically tutor questions and wait time during small group activities, affect the use of mathematics vocabulary and problem solving skills of novice ELLs. I used Vygotsky’s concept of Zone of Proximal Development to guide my study and observed discourse patterns that emerged as tutors and tutees collaboratively interacted to solve a set of mathematics problems (approximately 5-10 minutes per episode). In doing so, I documented how different tutor questions and length of wait time contributed to the mathematics learning and vocabulary development, transforming tutees from peripheral to active participants and at times, independent problem solvers of the activity.

My ultimate goal was to observe the connection between tutor questions and mathematical learning of the tutee, therefore, the overarching research questions that guided my study were: How do tutors interact with tutees to develop ownership of mathematical problem solving activity? How do effective tutors interact differently from ineffective tutors? I then developed two sub questions:

a). What types of tutor questions support active participation, independent problem solving, and accurate completion of problem sets as tutees engage in small group mathematic activities?

b). In what way does tutor’s wait time contribute to tutee’s use of arithmetic vocabulary
Significance of the Study

In order for students to be successful participants in the 21st century, they must graduate from high school with skills that enable them to solve problems and communicate mathematically (NCTM, 2010; PDE, 2010). Unfortunately, ELLs are not living up to this lofty goal. Placed in secondary mainstream mathematics classrooms with teachers that have no previous training in working with ELLs, this student subgroup is falling behind their non-ELL peers. Scores of ELLs on large-scale assessments of math, including high school exit exams, are significantly lower than their non-ELL peers and the dropout rate of this student subgroup remains highest among the secondary student population (Cataldi, Laird, & Kewek Ramani, 2009; Kindler, 2002). This is particularly unfortunate in light of the negative outcomes attached to dropping out of high school which include poverty, high rates of unemployment, and incarceration that have a tremendous impact on society as a whole (Moore & Blair, 2008).

Although research indicates a number of reasons ELLs do not perform well in mathematics, the linguistic and cultural complexities of math that prevent access to content instruction and large-scale assessment are the primary causes of academic failure (Abedi & Herman, 2010; Abedi & Lord, 2001). Responsible for providing ELLs with comprehensible content instruction are the content teachers, who according to Téllez & Waxman (2006) are unprepared to accomplish this task. In order for ELLs to achieve in math, it is important that mathematics teachers adopt instructional methods to reach all students, including ELLs so “that all students have the opportunity to develop their mathematical potential, regardless of a lack of proficiency in the language of instruction” (NCTM, 1989, p. 142).

To successfully achieve this goal, teachers must include interactive discourse strategies that develop ELL ownership in the mathematic problem solving activities. Strategies that
increase active participation of ELLs, provide opportunities to demonstrate independent problem solving and acquisition of arithmetic vocabulary support ownership of mathematics. This study seeks to identify discourse strategies, particularly teacher questions and use of wait time, secondary math teachers with or without ESL training, can easily and effectively incorporate into their daily lessons to scaffold mathematics instruction, encourage active participation, and promote mathematics vocabulary and independent learning of ELLs in reform oriented classrooms. Time is of the essence, and stakes are high. Teachers, schools, and students need an immediate solution, not a band aid, but a real solution to an overwhelming problem that will not require lengthy training sessions or tax an already overstretched school budget. This study provides simple, effective strategies that all teachers, particularly secondary math teachers, can implement in the classroom and train their peer tutors (in a relatively short period of time) to use to increase the mathematical learning and language use of ELLs in their classrooms. All students must become the mathematics problem solvers and communicators necessary for 21st century success and it is the teachers’ responsibility to see that this happens. However, as researchers, it is our responsibility to provide teachers theoretical-based strategies that they can use immediately in the classroom.
CHAPTER TWO

Review of Research Literature

Interaction, Mediation, and Development: Sociocultural Approaches to Learning and Development

Social interaction plays an invaluable role in the development of human cognition. According to Vygotsky (1978) and sociocultural theorists, cognitive development is a transformation of elementary or biologically determined mental processes (i.e. vision, hearing, olfactory, tactile, hearing, natural memory, involuntary attention) to higher order sociocultural regulated mental processes that takes place through social interaction and mediation (i.e. logic, planning, problem solving, perception, conceptual thought) (Lantolf & Pavlenko, 1995). Getting into the zone of proximal development is essential for this transformation to occur. To execute self-regulated control of strategic mental processes (internalization) instead of relying on objects (object-regulated functions) or others (other-regulated functions), a child or novice uses mediating artifacts, which include signs, symbols, tools, and language, to function during interaction (Lantolf & Appel, 1998). For example, when children are young, parents often use objects with the intent to direct or mediate the child’s activity. This is an example of object-regulated function in that the actual object is responsible for mediating the child’s actions. As the child cognitively develops, the parent interacts with the child through dialogic speech to help guide the child in completing the task. No longer object-oriented, the child now accomplishes her goal by relying on the guidance of a more capable other and is considered to be other-regulated. Eventually, as the child takes on more responsibility for strategically carrying out the task, self-regulation occurs (i.e. independent strategic functioning) (Lantolf & Appel, 1998). These developmental processes, object-regulated, other-regulated, to self-regulated could not be
accomplished without mediation and social interaction. Mediated interactions are internalized which gives rise to new cognitive functions (Lantolf & Poehner, 2008). This is what enables the child to develop from other-regulation to self-regulation as she internalizes the mediating tools, most often language in the form of speech, used by the parent to guide her in accomplishing the task.

**The More Knowledgeable Other**

The idea of social interaction, mediation, and internalization as loci for cognitive development is fundamental to Vygotsky’s distinction of the Zone of Proximal Development (Lantolf & Thorne, 2006). The Zone of Proximal Development (ZPD) reflects the difference between what a child can do alone, his *actual* level of development, and what he can accomplish under the guidance of a more knowledgeable other or collaboration with peers, his *potential* level of development (Lantolf & Pavlenko, 1998). Identified as an intermental activity through social interaction with the more capable other, this later develops into intramental activity as it is appropriated and internalized by the novice within the ZPD (Lantolf, 1995, pp. 114). Similar to the parent and child episode, through dialogic interaction, the parent enables the child to accomplish tasks she normally could not accomplish alone:

For Vygotsky (1978), social interaction is a mechanism for individual development since in the presence of a more capable other, the novice is drawn in and operates within the space of the expert’s strategic processes for problem solving. More specifically, the dialogically constructed interpsychological event between individuals of unequal abilities is a way for the novice to extend his current competence. During problem solving, the more
experienced individual is often observed to guide, support, and shape actions of the novice who in turn internalizes the expert’s strategic processes (Donato in Lantolf & Appel, 1998).

The metaphor of scaffolding also has its roots embedded in Vygotsky’s theories, particularly in conjunction with the ZPD. Originated in the work of Jerome Bruner (Walqui, 2006), the notion of scaffolding is similar to that of a construction site. The scaffolds are set up in a joint effort to support the learner in accomplishing a task or developing certain skills that he normally could not accomplish on his own. Not a rigid structure, but a fluid dynamic, the scaffold is removed and assembled according to the needs of the learner. As the learner demonstrates internalization of the processes needed to accomplish the task, he gradually takes on more responsibility and the scaffolds can be removed. Should he make a mistake, the scaffolds can be readjusted resulting in a reinforced support structure until they can be dismantled again (Walqui, 2006).

Although Vygotsky maintained that cognitive development occurs through social interaction and mediation through cultural historical tools that include language and the more knowledgeable other, he also maintained that cognitive development is embedded in situated activity (Donato in Lantolf & Appel, 1998). The importance of situated activity and participants of a community of practitioners has been documented through Lave and Wenger’s (1991) investigation into the social organization of learning in which they developed the theory of socially situated cognition and legitimate peripheral participation. This term, legitimate peripheral participation refers to learning as participating in a community of practitioners and ultimately, mastering the knowledge and skills required to move the novice, a peripheral participant to full participation in the sociocultural activities and practices of the community.
(Lave & Wenger, 1991). In order to become an active participant of a mathematics community, one must first learn to talk and write like a mathematician. This can only be accomplished in situations where knowledge is co-constructed through active participation in the social, historical, and cultural contexts of that community (Brown, Collins, & Duiguid, 1989). As the learner participates in the social practices of that community, he adapts to the activities through language and interaction with participants of that community (Mondada & Doehler, 2004).

Studies that focused on human interaction, mediation, internalization, and ZPD for second language learning were embraced by researchers and scholars conducting research in second language acquisition (SLA) from various theoretical perspectives. The cognitivist view that dominated much of the research in the early 80’s and 90’s focused on interaction between native and nonnative speakers of English as an individual cognitive act with little to no emphasis on the social interaction necessities for cognitive development. Scholars including Michael Long (1982; 1983, 1985), Varonis & Gass (1985), Susan Gass (1988), Polio & Gass (1998), conducted multiple studies that identified the effectiveness of interaction in promoting SLA, however, taken from a psycholinguistic, cognitivist perspective, these studies did not identify social interaction as a means for studying SLA but viewed language acquisition as an individual cognitive process. Firth & Wagner (1997) addressed the tensions that were growing between researchers that studied SLA from a sociocultural perspective and those that adhered to the cognitivist perspective which was dominating studies of SLA at the time. In their article, Firth and Wagner (1997) argued for a reconceptualization of SLA research that would broaden the “ontological and empirical parameters of the field” (pp. 285). Disappointed with the attention paid to cognitive theories and methodologies as a means to study discourse and communication
at the expense of the social orientations to language, Firth & Wagner argued for a more balanced approach to research in SLA that emulates a sociolinguistic approach to the study.

Firth and Wagner’s call for a reconceptualization of SLA was well received by many scholars advocating the study of SLA from a sociocultural perspective. Scholars conducting research from a sociocultural perspective emerged in staunch support of Firth and Wagner’s reconceptualization and the number of studies emphasizing social interaction as a means to study SLA broadened.

The emphasis of sociocultural approaches to research provided teachers who advocated social interaction as a means to promote language learning and cognitive development, evidence to justify their teaching practices. Foreign language teachers and ESL teachers began using similar methods to promote language learning through interaction. Rather than focusing on error correction and grammar translation exercises, teachers began to rely on student self-correction and collaborative dialogue as a means to learn a second or other language. Mistakes were welcome as it gave the students an opportunity to practice language and negotiate meaning through interaction in a non-threatening environment.

**Interaction and second language acquisition.**

**Self-correction.** A number of studies that focus on social interaction in SLA have revealed positive results that can influence the language development of ELLs. Studies that focus on Dynamic Assessment (DA) as a means to “simultaneously diagnose and promote learner development by offering learners mediation” have revealed positive results in SLA (Lantolf & Poehner, 2008 p. 273). DA is a way of conceptualizing the ongoing interaction of both in effective teaching and learning (Lantolf & Poehner, 2008). The teacher as the mediator of the interaction employs the role of either an interventionist or an interactionist. An
interventionist approach uses more of a script and targeted interaction, one of the benefits being that results are more quantifiable. An interactionist approach uses a more open-ended mediation and requires cooperative and dialogic mediation (Lantolf & Poehner, 2010). Most current approaches of DA are often product-oriented and supported in the educational field due to outcomes-based education. This method of conventional assessment, followed by targeted intervention, and then a final assessment, often informs a long-term instructional plan when used appropriately (Lantolf & Poehner, 2010). The theoretical approach to DA is more process-oriented, in that the mediator-learner interactions cannot be prescribed beforehand and desired outcomes cannot be fully established beforehand, since the interaction unveils and fosters progress within the learners’ ZPD.

Although most studies on DA have occurred in university settings or under the guidance of a trained linguist, Lantolf & Poehner (2010) describe a teaching situation in a learning lab that centers around an elementary level Spanish teacher (of second language Spanish), who applies Vygotsky’s theory of the Zone of Proximal Development (ZPD) to her role as an interventionist mediator in the classroom. Tracy (the teacher’s pseudonym) prepared an inventory or menu of mediation moves which progressed from implicit to explicit in the corrective feedback. The significance of this scripted component of the lesson has relevant implications for the classroom teacher; if Tracy corrected a student’s error by giving the correct answer, she would possibly deprive the student of learning through co-regulation, and would also deprive herself of some insight into the student’s ZPD in the targeted feature of the language.

Tracy engages the students in two activities, one, a photo cube with Peruvian animals on it and the other activity, a guessing game. In activity one, each student would take a turn rolling the cube, name the animal facing up on the cube, and state two things about that animal. In the
guessing game, students worked in pairs as one student describes an animal while the partner student had to guess which animal the student was describing. Tracy used an inventory of prompts she had generated to mediate the students and encourage them to work within their ZPD. Her inventory of prompts begins with just a pause, meaning that when the student made a performance error, Tracy paused to imply there was an error. This gave the student the opportunity to make progress by self-correcting. If prompt one failed to elicit a response, Tracy proceeded to prompt two, repeating the whole phrase questioningly. Again, the opportunity for self-correction is implicitly presented to the student by the prompt. In prompt three, Tracy would repeat just the part of the sentence with the error. In prompt four, she asked or directly pointed out that something was wrong with the sentence. In prompt five, she pointed out the incorrect word, prompt six offered an either/or question to correct the error, gave the correct answer in prompt seven, and in prompt eight, Tracy explained why the answer she gave was correct. The specific list of prompts varied slightly with different activities, from 6 to 8 prompts. By keeping a systematic log of the prompt-level needed by students to correct their responses, Tracy could see, over time, when students progressed in developing toward independent control over the features of the language. Through her prompts, Tracy was able to support the learners development and “push the development forward” (Lantolf & Poehner, 2010 p. 28).

**Collaborative dialogue.** Studies that focus on collaborative dialogue as a means for SLA and problem solving have been documented in a study conducted by Swain and Lapkin (1998) and a conceptual piece (Swain, 2000) explaining the benefits. Swain (2000) maintains that through collaborative dialogue both language learning and language use co-occur (pp. 97). Positing that interaction in the form of collaborative dialogue provides learners and opportunity to negotiate meaning and reflect on form and meaning of their output, Swain (2000) presents a
strong defense of the benefits collaborative dialogue brings to the second language and content classroom. As the collaborating participants interact, they often notice gaps in their interlanguage which cause them to question their partner or model their peer’s talk in order to overcome their deficiencies (Swain, 2000). As the collaborating participants use language to mediate collective problem solving, the nature of this interaction serves them individually. As they verbalize the solution, they are offered a means to attend and reflect on the language they produce and transform this into a mental representation of the language they will use in the future.

Swain (2000) further expands on the benefits of collaborative dialogue as a mediator of second language acquisition through the results of a study conducted on grade 8 French immersion students (Swain & Lapkin, 1998). As the researchers engaged in examining the dialogue that ensued between collaborating participants, they noticed that they engaged in co-constructing difficult areas of the language related task to arrive at a correct response. Each participant offered alternatives for a problem they were experiencing in a language related episode which led to increased opportunities to reflect on their own language use and jointly construct a correct response. Swain (2000) maintains that the joint construction of the participants’ performance served as a transitional mechanism to move the collective cognitive activity from the social plane to intramental psychological functioning (pp. 111). The collective dialogue of the participants served to mediate their second language development.

**Questions.** The mediating effects of questions used in classroom interactions with ELLs have been documented in the literature. Research reports by two scholars are relevant here. A study conducted by Kim (2010) that focused on the instructional strategies of two successful ELL teachers revealed their use of questions to scaffold ELLs learning and language development. Through an observation study, Kim (2010) identified three different types of
questions that facilitated changes in students’ participation and use of English language in oral activities. The three types of questions, coaching, facilitating, and collaborating were used by teachers throughout the course of the year to enhance student learning. Coaching questions required a high level of teacher authority and were used to help students monitor their own utterances and thinking, add more information, give commands, remind them of their goals, summarize, focus, and model a class activity. Facilitating questions were most often used by teachers to help students articulate and elaborate on utterances, encourage interaction, voice their opinions, seek opinions, and validate a response. The third type of questions, collaborating were those with that required the least amount of teacher responsibility and invited students to share about personal experiences that occurred throughout the year (Kim, 2010). Collaborative questions were open ended and enabled students to collaborate with the teacher in order to continue a conversation, develop oral language skills, and exercise their ownership of learning by asking questions to peers or teachers.

The study concluded with an examination of student performance on the Qualitative Reading Inventory (QRI) that revealed significant progress in the advancement of the students’ reading levels. Kim (2010) maintains that the scaffolding effect of teacher questions in a strength–based classroom, that is a classroom that focuses on student strengths as opposed to their weaknesses, enabled students to reach these goals.

The importance of teacher questions in ELL language development was also evident in an Australian school. Gibbons (2003) conducted a study to examine how teacher/student talk in content based classrooms contributes to ELLs language development. Drawing on the constructs of mediation from sociocultural theory and mode continuum from systemic functional linguistics, Gibbons (2003) identified how teacher questions based on student response can
enable students to work within their ZPD. In a study conducted in an economically deprived Australian school, Gibbons (2003) focused on 8 and 9 year old students (90% were ELLs), in a mainstream science classroom. She identified that teachers who scaffold the language of their questions based on student response, that is to gradually increase the sophistication and specialized vocabulary of the content area within their questions, increased the students' ability to internalize these forms of language for later use. Gibbons (2003) found that too close a match between the vocabulary used in teacher questions and student response did not allow learning to occur, however, when teachers elevated the level of language used in their questions that recast their simpler language into more sophisticated forms, the students were provided an opportunity to elaborate and use more technical vocabulary.

**Wait time.** Wait time is considered the length of a pause or moment of silence between teacher questions/student response and student response/teacher reaction. Mary Budd Rowe (1974, 1986) conducted pioneering work in the area of wait time. Cazden (2001) reports that Rowe conducted research on wait time in classes that spanned from “elementary schools to college, from special education teachers to museum guides” and found that most teachers generally wait 1 second or less between questions/answer episodes, that is between the time the teacher finishes the question and the student begins to respond (pp. 94). Budd Rowe (1986) further elaborated this in an article in which she discussed the results of wait time on an analysis of over 300 tape recordings. She (Budd Rowe) identified two types of pauses that occur during classroom instruction: the pause after teachers pose questions and students respond, and the pause that takes place when students hesitate momentarily in their replies. She analyzed these two types of pauses to determine mean wait time associated with them. For the first type of pause, Budd Rowe revealed between a teacher’s question and the beginning of a student’s
response averaged 1 second. For the second type of pause, she revealed that the average mean wait time between a student’s response and teacher’s reaction averaged .09 seconds. When the teacher initiated wait time between pauses that averaged 3-5 seconds and revealed a marked increase in the length of student’s response, number of appropriate response items, and frequency of student questions.

Stahl (1994) investigated thinking time, which is similar to Budd-Rowe’s studies of wait time. He argues that students must have undisturbed periods of time to process information, reflect on what has been said or observed, and arrange their thoughts to form a response. Stahl revealed eight categories of “think time” where periods of silence occur. The eight categories where think time occurs includes: (1) post-teacher question wait-time: the time after a teacher poses a question until they either continue talking or allow a student to respond (typical teacher waits between 0.7-1.4 seconds); (2) within-student's response pause-time: the time between when a student begins to answer and hesitates (typical teacher waits .5 seconds); (3) post-student's response wait-time: after a student completes a response and others are preparing to comment, question, or answer; (4) student pause-time: students hesitate during self-initiated question, comment, or statement; (5) teacher pause-time: occurs throughout discourse; (6) within-teacher presentation pause-time: teacher pauses during lecture, presentations, or extended explanations; (7) student task-completion work-time: time allotted for students to compete a task; and (8) impact pause-time: focus attention. Providing students 3 seconds or more of silence within each of the eight categories positively affects students’ performance as they are offered an opportunity to process information and create richer, more elaborate, and correct responses and interact more with their peers.
Providing a sufficient amount of wait time is critical when instructing ELLs. Already at a disadvantage due to their limited English language proficiency, ELLs must use extra cognitive resources to process questions asked in English, think of an answer, and translate the answer back into English (Abedi, Courtney, & Leon, 2003; Lacina and Newman, 2005). Allowing between 5-20 seconds of wait time for an ELL to respond conveys the message that the teacher has confidence in the student’s ability to answer the question (Slavin, 2006) which greatly increases the chances the ELL will participate more often in class.

The Sheltered Instruction Observation Protocol (SIOP), a model for sheltered instruction for ELLs from (Echevarria, Vogt, & Short, 2004) includes wait time as a key component in their framework. Sheltered instruction is a method of teaching ELLs content within the ESL classroom in a means to promote increased comprehension of the content area. This framework addresses the importance of wait time in conjunction with an ELLs’ cultural background maintaining that some cultures allow more sufficient amounts of silence between teacher/student question/response episodes (Echevarria, Vogt, & Short, 2004).

**Interaction and Mathematic Development**

Theoretical similarities between second language learning and mathematics learning are evident in the sociocultural approaches researchers employ when studying interactional discourse and math. Sociocultural theories provide a resource from which researchers and educators can better understand the interconnectivity of discourse, second language acquisition and mathematics learning. Rooted in the work of L.S. Vygotsky, “the fundamental tenet of sociocultural theory holds that sociocultural and mental activity are bound together in a dependent, symbolically mediated relationship” (Frawley & Lantolf, 1985 pp. 108). From this
perspective, sociocultural theory provides a means to identify how language in the form of discourse, as a psychological tool, can mediate ELLs learning of math.

The competing perspectives towards educational research were not confined to SLA and scholars studying the learning of math from a sociocultural perspective as opposed to the dominant constructivist view, began to emerge. Although both sociocultural and constructivist theorists highlight the important role activity plays in mathematics learning and development (Cobb, 1994), constructivists view activity as an individual process that occurs in developmental stages whereas sociocultural theorists “link activity to participation in socially culturally organized practices” (Cobb, 1994, pp. 14). Similar to the sociocultural theorists of SLA, those that subscribed to the same point of view in the field of mathematics education looked to Vygotsky and his colleagues as a means to justify their views. The constructivists, whose views dominated mathematics education research, primarily followed the work of Piaget.

Several scholars conducted pioneering work from a sociocultural perspective in mathematics education. Scholars including Scribner (1984), Lave (1988), and Saxe (1991) conducted empirical analyses of mathematics learning from a situated sociocultural perspective to reveal mathematics learning as a culturally, socially, situated practice. These scholars among others taking a sociocultural stance argued that knowledge is a “product of activity, context, and culture in which it is developed and used,” a process of enculturation within a community of practice (Brown, Collins, & Duguid, 1989 pp.32; Minick, 1989). In a move to examine mathematics learning from a socially situated perspective, many researchers studied mathematics practices that emerged from every-day practices, primarily the workplace.
Differing Instructional Perspectives within the Sociocultural Theoretical Domain

Many studies oriented towards examining mathematics learning from a sociocultural perspective focused on mathematics as cultural tool in authentic situations, primarily the workplace. This was brought about as a means to determine if the mathematical learning that takes place in school, meaning the rigid whole group teacher demonstration, drill, and practice method benefit students once they left school or if they adapted their calculating habits to suit their immediate needs. Could students that failed mathematics in school, function mathematically in the real world? Scribner (1984), for example, focused her study on the socially situated cultural tools used by dairy workers to compute math. The "case price technique" developed and used by milk drivers allowed them a simple and uncomplicated process to compute delivery charges. This revealed an example of mathematical knowledge developed through culturally situated practices and tools derived from the workplace (Scribner, 1984).

Another example comes from a study conducted by Saxe (1991) on the body part counting practices of the Oksapmin people of Papua New Guinea. He identified how the 27 body-part counting system, a trajectory from the thumb on one hand up and around the head to the little finger on the opposite hand, was the primary method of counting used within a community of practice. However, as Western influences became more prevalent in Papua New Guinea, trade stores and schools were established that began to change the methods used for counting. Saxe, interested in how the Oksapmin’s would adjust their culturally situated counting system to perform complex mathematical tasks new to them such as change making, studied the language used by trade shop owners as a means to identify shifting methods of quantification that occurred over generations.
Saxe (1996) also conducted a study in which he observed unschooled Brazilian children selling candy. The children were able to conduct simplified computations working with round numbers as they bought, repackaged, and sold candy. Although their strategies did not mimic those taught in school, the children were still able to calculate in a way that enabled them to earn money.

Lave (1988) conducted similar experiments in which she studied apprentices of Liberian tailors, children selling coconuts in Brazil, and dieters and shoppers in California, in a means to determine if one could acquire general mathematic skills in a classroom and then use them in authentic context dependent situations. She identified two kinds of mathematics instruction; school and every day which are further labeled as formal (school) and informal (every day). She determined in her studies that the algorithms learned in school are most often not used to compute in the authentic situations and vice versa. Her study, as well as Scribner’s and Saxe’s are of particular interest because they ignited a conflict as to which type of math should be taught in the classroom, thus creating an inevitable rift among the teaching approaches adopted by mathematic educators.

The studies presented in this section helped define my research focus through results that reveal mathematics learning as culturally, socially, situated practice. This evidence dispels the misconception that mathematics classrooms must be conducted in teacher fronted lock step fashion and supports the use of interaction and teaching strategies from a sociocultural perspective.

**Tensions in teaching.** The similarities between SLA and mathematics education do not end with their theoretical orientations to research. Methods or approaches to teaching in SLA underwent significant changes with the introduction of *Communicative Competence* (Canale &
Swain, 1980; Hymes, 1971; Savignon, 1991). Teachers who had previously followed behaviorist approaches that included the audio-lingual or translation methods were now being instructed to focus on a collaborative approach, to focus on language *use* as opposed to *usage* (Savignon, 1991). Discourse and communication in the classroom became the locus of instruction as teachers began to focus on the four elements of communicative language teaching; discourse competence, strategic competence, grammatical competence, and sociolinguistic competence.

Differing teaching approaches were also finding their way into the mathematics classroom. Calls for mathematics education reform became more intense encouraging math teachers to promote instructional activities that facilitate meaningful learning and inquiry (Cobb, Wood, Yackel, & McNeal, 1992). In a move to accomplish this goal, NCTM outlined a set of instructional goals in their Professional Standards for Teaching Mathematics (1989, 1991, 2000), which required teachers to promote discourse and communication in the mathematics classroom. Although educators began to adopt a more language focused approach to teaching math, they were further divided as to what type of math to teach.

Scholars oriented towards a sociocultural theoretical perspective of mathematics education jointly agree that learning occurs through participation in socially situated and culturally organized practices (Cobb, 1994), however, they are divided as to their beliefs in what type of math should be instructed in the school and offer two recommendation’s. The first recommendation emphasizes learning authentic mathematical problem solving skills by mimicking authentic situations in the classroom similar to what Lave refers to as *every day* or *informal* math (Moschkovich, 2002). The second recommendation mimics NCTM reform oriented teaching practices, and is similar to Lave’s *school* or *formal* math instruction, in that it
requires students to form and test hypotheses, generalizations, conjectures, and initiate mathematic discussions (Moschkovich, 2002).

Using Lave’s term *everyday* math for the informal type of mathematics instruction and *academic* to refer to instruction that is *formal*, Moschkovich conducted a comparison study between the two instructional recommendations. Her findings revealed that teachers who adopted an everyday math approach to teaching mathematics focused on authentic work-place type of activities that include buying and selling. Moschovich also revealed that classrooms using this type of approach were more teacher-fronted in that the teacher managed the discourse and activities used in the lesson, however, due to the familiarity of the activities (e.g. buying and selling), and simplicity of the language used in the activities allowed the teacher to reach a broad range of individuals. Academic math, on the other hand, places more emphasis on student- centered instruction and includes activities that require discussion, debate, verbal interaction, and use of sophisticated mathematic terms. Although this style of math was not as easy to comprehend, it did help to better prepare the students for problems they would be required to compute on large-scale assessments. At the end of the comparison analysis, Moschkovich (2002) concluded that a synthesis of both types of mathematics instruction would be most beneficial. Incorporating academic math would help students reach NCTM content standards while everyday math would help prepare them for authentic experiences outside the classroom. By incorporating discursive interaction that promotes authentic problem solving skills through communication in the form of defending, justifying, and explaining mathematical processes, students can navigate in authentic situations as well as successfully access and solve problems on large-scale tests of math.
Challenges English Language Learners Face in Reform-Oriented Mathematics Classrooms

Language

Research indicates a high correlation between English language proficiency and mathematics achievement citing linguistic complexities of mathematics vocabulary as one of the primary causes of academic failure of ELLs (Abedi & Herman, 2010; Abedi & Lord, 2001). Ensuring “that all students have the opportunity to develop their mathematical potential, regardless of a lack of proficiency in the language of instruction” (NCTM, 1989, p. 142) is a difficult task to execute provided the linguistic diversity of the ELL population attending our schools. ELLs enter U.S. schools with varied levels of native language proficiency, literacy, education, and English language proficiency. Most secondary ELLs have attended schools in their home country however; many arrive with little to no formal education. Some ELLs have some background experience in English from English as a foreign language classes while living in their native country, however most arrive as non-English speakers. Research conducted on second language acquisition maintains that it takes approximately five to ten years to acquire the cognitive academic language proficiency (CALP) needed for academic success (Cummins, 1979; Thomas & Collier, 1995). This means the majority of secondary school aged ELLs will not have sufficient time to develop this level of English proficiency to actively participate and successfully achieve in the mathematics classroom.

Studies focusing on the effects of English language proficiency on mathematics performance of ELLs reveal language used in mathematics instruction and assessment requires an advanced sophistication of English that most ELL students, particularly at the beginner and intermediate levels have yet to acquire (Abedi, Lord, and Plummer, 1997; Cummins, 1979).
Abedi and his colleagues studied the language used in mathematics assessments and identified indexes of vocabulary that are ambiguous, abstract, uncommon, cognitively taxing, and require extra time to decode and interpret (Abedi, 2006; 2004; Abedi, Lord, & Plummer, 1997; Mid Atlantic Comprehensive Center, 2009; Rivera & Collum, 2006).

Studies conducted to determine the effects of language, primarily through reading, on performance levels of ELLs in assessments of math and science reveal that ELL students tend to score lower than non-ELL students on math assessments at all grade levels, including the mathematic subsections of the Graduate Record Examination (GRE) and Scholastic Aptitude Test (SAT) (Cocking & Chipman, 1988; Abedi, Hofstetter, Baker, & Lord, 2001). An analysis of standardized assessments in mathematics from Grades 3 and 11 reveal that approximately two thirds of test items include uncommon words and vocabulary used in an atypical manner (Abedi, Courtney, & Leon, 2003). The remaining one third of the assessment used syntactic structures that were unusual and complex (Abedi, Courtney, & Leon, 2003). Both studies indicate the difficulties ELLs experience accessing content due to the linguistic complexities of math items, the primary reason ELLs do not perform well on large-scale content assessments (Abedi & Lord, 2001). Although these studies were conducted to determine language as an inhibiting feature of large scale assessments the results reveal important implications regarding language used in mathematics instruction. Due to the high stakes associated with large scale assessment scores, most teachers teach “to the test” which means the language used in the classroom mimics the language found in the test. This indicates that if ELLs are exhibiting difficulty accessing and comprehending mathematic content on large-scale assessments, chances are high they are experiencing this in the classroom as well.
ELLs in the Reform-Oriented Mathematics Classroom

In reviewing the prior studies for patterns in discourse, one can identify certain norms that thread throughout each of the studies: the power of questions, mathematics vocabulary, and wait time. These constructs are highlighted in the following research studies conducted to incorporate discourse and communication, two NCTM goals, in the mathematics classroom. Although the following studies primarily focus on the elementary level classroom, they offer insight as to the power discursive interaction, primarily questions and wait time, play in developing ELL ownership of the mathematics problem solving activities.

The importance of discourse and interaction in achieving the reform oriented teaching goals set by NCTM, has prompted researchers to identify characteristics of effective teaching and general principles of instruction that help children make sense of mathematics and organize them into a pedagogical frameworks. One such framework, Advancing Children’s Thinking (ACT) (Fraivillig, Murphy, & Fuson, 1999), contains three components that use discourse as a means to advance students thinking in mathematics. Each of the three component s of the ACT framework contains instructional practices that support the teacher’s role in promoting mathematical thinking and learning by (1) *Eliciting* children’s solution methods, (2) *Supporting* children’s conceptual understandings, (3) *Extending* mathematical thinking (Fraivillig, Murphy, & Fuson, 1999). Describing the main goal of the ACT framework as helping students create their own meaningful understanding of mathematical concepts, Fraivillig expands on each of the three components:

The eliciting component reminds teachers to consider how they might get children’s thinking out in the open for discussion and build instruction to support that thinking. The supporting component describes instructional strategies for assisting children at their current levels of understanding. The extending component
promotes teachers to challenge children’s thinking regardless of the students’ initial efforts (Fraivillig, 2001, pp. 459)

Another theoretical framework developed to promote discourse in the mathematics classroom is the Math-Talk Learning Community (MTLC) by Hufferd-Ackles, Fuson, and Sherin (2004). The primary goal of MTLC is to support and extend the mathematical thinking of all students by developing a discourse community within the NCTM reform oriented mathematics classroom (Hufferd-Ackles, Fuson, and Sherin, 2004). Four distinct, but related components of the MTLC framework capture the growth of the math-talk learning community by focusing on specific teacher and/or student actions (Hufferd-Ackles, Fuson, and Sherin, 2004, pp. 87). These developmental trajectories (1) questioning, (2) explaining mathematical thinking, (3) source of mathematical ideas, (4) responsibility for learning, “build on one another to measure changes in students’ and teacher’s actions over time” (Hufferd-Ackles, Fuson, and Sherin, 2004, pp. 87). Each trajectory consists of 4 levels, from 0-4, that provide information regarding the progressive development of the math-talk learning community as the teacher becomes less in control of the discourse and the students gradually take on more responsibility for their mathematical learning. A gradual process, MTLC advocates the teacher’s use of wait time, identified as a patient increase in the amount of time between questions and answer episodes, to allow the student an opportunity to process the question and produce a response. The framework also supports a rich use of mathematics vocabulary throughout discourse to increase the appropriation and use of these specialized terms.

Both frameworks, ACT and MTCL, were developed as part of research studies conducted on elementary students learning math. Instructional strategies of effective teachers in both
studies were utilized as a means to develop both frameworks and both studies were conducted throughout the course of a year. ACT was developed as a pedagogical framework in light of a case study conducted in a mainstream first grade classroom while MTCL, was developed through a case study of a bilingual third grade teacher whose students’ were primarily ELLs (Fraivillig, Murphy, & Fuson, 1999; Hufferd-Ackles, Fuson, and Sherin, 2004).

Moschkovich (1999) conducted research in an elementary bilingual classroom in a move to reveal methods teachers used to integrate language with mathematics instruction. Moschkovich (1999) compared three perspectives, the vocabulary (correct terms), constructing multiple meanings (fluency across language registers), and sociocultural perspectives (discourse and communication strategies), that impact the instructional practices of teachers with bilingual students.

In a bilingual classroom, a group of students were engaged in small group discussion in which they were trying to identify a geometric shape. The students, Spanish speakers, attempted to produce the word in Spanish; however, they could not produce the correct term for rectangle. Following the small-group discussion, their teacher asked several questions from the front of the class. In response, Alicia, one of the students in this small group, attempted to describe a relationship between the length of the sides of a rectangle and its perimeter, but could not use the term rectangle correctly. The teacher asked Alicia questions to elicit a response and eventually, Alicia was able to explain the problem using gestures in conjunction with language to get her point across. If the teacher had assessed Alicia’s mathematical knowledge from either a vocabulary perspective on her failed attempt to produce the correct term or multiple meanings perspective, she would not have uncovered Alicia’s competence in math. The sociocultural perspective on the other hand enabled the teacher to view the discourse as a whole and identify
how the student generalized, described patterns or abstract features of math in response to her questions. This perspective enables bilinguals to articulate the process of solving the mathematical problems therefore demonstrating competency of the problem solving process. Applying a sociocultural perspective enables the teacher to engage students in active participation and provide them a forum to practice their mathematics skills and vocabulary.

Khisty & Chval (2002) analyzed the discursive interactions of a successful mathematics teacher of ELL students in a move to identify if teacher talk contributed or detained the mathematic achievement of their students. The first teacher, Mrs. Martinez, teaches a fifth grade class of ELL and special education students. Most of her students enter her class one to two grade levels below in mathematics and leave one to two grades levels above the norm (Khisty & Chval, 2002). By focusing on the processes of interaction and the characteristics of talk in the classroom, the researchers, Khisty & Chval, were able to identified patterns of discourse that were conducive to learning. First, Mrs. Martinez actively engages all of her students in the class through discussion. Whether in small groups or whole class discussion, Mrs. Martinez orchestrates the activities in her room so that all students are actively engaged in the problem solving process and collaboration through verbal and/or written modes of communication (Khisty & Chval, 2002). Mrs. Martinez also actively participates in the classroom discussion. Rather than constrain discourse by focusing on one student at a time or lecturing, she guided students with her questions towards explaining, justifying, and defending their independent and collaborative problem solving process. Another salient feature of Mrs. Martinez’s teaching methods is that she gave sufficient wait time between questions and answers for students to process information which transferred to the students when working in dyads or small groups. In one instance, in a dyadic pair, one of the participants is struggling to get his answer out. Mrs.
Martinez, understanding his dilemma, reminds his partner to allow him sufficient time before expecting a response which resulted in a more complete sentence. Mrs. Martinez utilized wait time often in conjunction with her questions. Rather than constantly depending on known answer questions, Mrs. Martinez utilized open ended ones and then gave students an appropriate amount of time to respond. Khisty and Chval (2002) also revealed the richness of Mrs. Martinez’s talk. Mrs. Martinez’s talk, either in the form of questions, statements, recasts, or explanations, was rich with arithmetic vocabulary. She began her lessons with discourse that was heavily populated with mathematical terms and continued to build on the discourse throughout the year. Mrs. Martinez relied on the response of students as a means to scaffold instruction, thus allowing them to work within their ZPD.

Cobb, Yackel, & Wood (1993) and Moschkovich (2007) identify a set of social norms that evolve in mathematics discourse classrooms that impact instruction and learning. Referred to as sociomathematical norms (Cobb & Yackel, 1996), these norms consist of regular, noticeable patterns that occur in discursive interactions. Traditionally, these patterns refer to discursive interaction between teacher and students and general classroom behaviors that regulate collaborative problem solving, explanation, justification, and argumentation of mathematical processes (Yackel & Cobb, 1996).

The discursive characteristics of the ACT and MTCL frameworks and studies from Moschovich (1999) and Khisty & Chval (2002) indicate that purposive questioning serves to engage students in mathematical thinking and increase their participation in the mathematics activities with the most salient features of effective discourse occurring between Mrs. Martinez and her students. Her questions allowed the students to become active participants of the problem solving process and demonstrate this process through acquisition of mathematical terms.
One can infer that the patterns of discourse in her classroom reflect positive sociomathematical norms that enhance mathematics learning.

Differences between explicit (closed answer) and implicit (open answer) questions within a mathematics classroom were explored through a study in a socially and ethnically diverse third grade classroom (Parks, 2010). The move for teachers to adopt implicit questioning in their instruction has been advocated by educators for the past two decades, initiated by a shared belief among educators that implicit questions that are probing, neutral, and explorative offer students more opportunities to learn mathematics than explicit questions which are narrow, direct, and authoritative. Parks (2010) conducted a study to identify the effects of implicit and explicit questions used in mathematics with regard to equity in a third grade mathematics class. Analyzed from two teaching perspectives; reform oriented “teaching that includes process skills as well as content” and traditional “teaching that is more narrowly focused on content.” (pp. 1872), Parks (2010) conducted her study through a yearlong investigation that focused on five students in a third grade classroom.

Parks (2010) expected to find two broad categories of questions in her data, either implicit or explicit (i.e. reform oriented or traditional). However, as mathematics currently falls under reform style teaching, there is a process or solution for every mathematical problem, students need to acquire strategies in order to solve a problem and produce an answer, her data revealed that the teacher used both types of questions in her reform oriented methods of teaching. This resulted in a reclassification of the question types as reform/implicit, reform/explicit, traditional/implicit and traditional/explicit.

Reform oriented implicit questions resembled those that align most closely to NCTM goals in that they required students to choose from a variety of means; explain, juxtapose,
summarize to explain the problem solving processes. Reform oriented explicit questions are open
ended requiring a student to explain a process; however, they are used in context and had a
tendency to direct the student’s attention more narrowly. Results from Parks’ study revealed that
the students did not fare well on the implicit/reform oriented questions, usually responding to
these questions in silence. As she analyzed their reactions to these questions throughout the year,
Parks maintained that there was no difference in the response of students to these questions by
the year’s end as compared to that in the beginning. On the other hand, explicit reform oriented
questions elicited more opportunities to engage in discourse and results indicate that answered
explicit reform questions most

Additional studies on teacher questioning strategies in the mathematics classroom come
from Lampert (2001). Lampert advocated beginning lessons with questions as a means to invite
student participation and explore the broad range of answers that could be generated as opposed
to explicit, single answer questions. Similar to Parks (2010) study, Lampert’s (2001) study was
conducted in a mainstream classroom and although this style of questioning may benefit English
speakers, it is highly probable that beginning a mathematics instruction lesson with implicit
questions would benefit ELLs.

The importance of small group or pair work. In light of 21st century teaching reform
movements for discourse oriented classrooms and a move to include more collaborative, small
group or pair work, it is paramount that teachers be aware of the challenges facing both students
and instructors. In studying classroom discourse, Markee (2004) identified two areas of
interaction that may cause problems for teachers and ELLs; counter questioning techniques and
tactical fronting talk. Markee (2004) maintained that “when teachers and learners make the
transition from one speech exchange system to another, it is quite common for problems of
various kinds to occur as members adjust to the turn-taking and repair practices of the new speech exchange system “(p.584). What is meant primarily is that when students work in small groups together and then the instruction changes to a teacher fronted class where students are expected to engage in discourse with the teacher, they encounter difficulties making this speech exchange transition. Markee (2004) revealed that when students participate in a peer-peer interaction, there is more equal power in the speech exchange system, however, when this switches to teacher-fronted discourse and questioning, there is unequal power in the speech exchange system which may lead to misunderstandings, reduced risk taking on the part of the student, and frustration.

In a move to foster collaborative learning, Yackel, Cobb, & Wood (1991) conducted an experiment from a second grade mathematics classroom that replaced traditional classroom instruction with small group activities for one year. Children were grouped in pairs and spent half of each math class working in dyads together and the second half of the class in whole group, teacher fronted discussions reflecting their paired activity.

Yackel, Cobb, & Wood (1991) also paid homage to the effects of homogeneous and heterogeneous groupings of the children. Often times, when children of differing ability levels were paired in the same group, the teacher was forced to intervene in a move to provide a resolution to a problem that could not reach a consensus or to keep the activity moving. Often times, this resulted in a dominance over the activity on the part of the more advanced level participant which implies that students of differing proficiency levels should not be paired together as the partnership should prove mutually beneficial for both children.

An additional study conducted by Neomy Storch (2004) applied activity theory as a means to explain differences in dyadic interactions of a university level ESL class. Similar to
Yackel, Cobb, & Wood (1991), Storch identified that the patterns of interaction within dyad participants often differ; however, Storch did not look at proficiency levels or personality differences, but identified changes in patterns of interaction by focusing on the participants’ orientation i.e. attitudes, to the activity. Storch (2004) further clarifies this distinction by examining the dyad participants’ attitude in regard to the tasks assigned, placement in dyads or small groups, and their motives and goals in completing the tasks. By examining the assumptions dyad participants had about appropriate goals, roles, motives, and means to achieve the goals, Storch was able to identify some variability in the groups’ behavior. Citing a study conducted by Donato (1988) in which he applied activity theory to identify underlying goals of dyad participants in a means to “uncover the mutual effects of learners on each other’s interlanguage system” (Donato in Lantolf & Appel, 1998 pp. 42), Storch’s study unfolds in much the same way.

Similar to Donato’s (1988) study, Storch used activity theory as a means to identify to identify patterns of dyadic interaction. As Donato’s study produced results that explained variability in the behavior of the dyad participants ranging from “loosely knit” to “collective” (Storch, pp 459), Storch’s study examined dyads participants to determine equality of contribution and mutuality. Storch defines equality as control “over the direction of the task” and mutuality as “the participants level of engagement with each another’s contributions” (pp. 461).

Storch (2004) revealed patterns labeled as collaborative, dominant/dominant, dominant/passive, and expert/novice. Collaborative pairs indicated a high level of equality and mutuality; dominant/dominant exhibited a high level of equality, however, mutuality was low; dominant/passive included interaction where both equality and mutuality were low, and in
expert/novice pairs, one participant took the role of the expert and one as participant which led to a naturally low level of equality, however, a relatively high level of mutuality.

Summary

Although there is a paucity of research that specifically focuses on discourse and interaction of secondary ELLs in the mainstream mathematics classroom (Brenner, 1994, 1998; Moschkovich 1999, 2002), those that are available in SLA and mathematics that focus on elementary ELLs and English speaking students offer invaluable insight into the nature of interaction and how it should be arranged for optimal participation and demonstration of mathematics problem solving and vocabulary, two areas identified by NCTM that students need to develop in order to be successful in math. Discursive interaction that is heavily populated with mathematical terms and engages all class participants has proven to be successful, particularly in the case of Mrs. Martinez. Focusing on discourse as a whole as opposed to a narrow focus of correct vocabulary use or fluency inform teachers of successful practices that they could incorporate into their own classroom as revealed by Moschkovich (1999). Both the ACT and MTCL frameworks have research based evidence that indicate usefulness for promoting discourse and children’s conceptual development of learning math. However, there are pitfalls to each. ACT does not address the language skills necessary for ELLs to reach the reform oriented goals of mathematics learning and MTCL is a gradual process that requires several months to achieve. Khisty & Chval’s (2002) study on Mrs. Martinez and the discourse practices from Moschkovich’s (1999) study provide invaluable insight as to what is needed for a successful mathematics classroom for ELLs. However, neither addresses mainstream secondary mathematics teachers or a mix of mainstream secondary English speakers and ELLs in the same
class. For these teachers, interaction must engage all students and be challenging enough for everyone, ELLs and English speakers, to work within their ZPD.

Research reviewed from studies that focus on questions from Kim (2010), Parks (2010), and Gibbons (2003) provide invaluable tools to help teachers identify questioning strategies that benefit students, however, they either focus exclusively on ELLs or mathematics lessons of mainstream students. They do not include secondary students as their focus. Finally, the research on small group work and dyads, (Yackel, Cobb, & Wood (1991), Markee (2004), and particularly Storch (2004) provided insight as to how activity theory can inform changes in interaction of small group activities.

Although all of the literature reviewed provided interesting insight, the overarching question that begs to be answered is how can teachers interact with ELLs to develop ownership of mathematical problem solving activities in reform oriented classrooms where students are both ELLs and English speakers and time is of the essence? It is through my study that I attempt to answer this question.
CHAPTER THREE

METHODOLOGY

Participants: Recruitment and Selection

Research participants were initially recruited on a voluntary basis: I announced during my ESL class one week prior to my departure that I was looking for ELLs to participate in mathematics tutoring sessions. Initial information meeting took place at a local library where they provided me with their most recent English language proficiency scores and mathematics GPA. I selected a total of 8 English language learners between the ages of 16 and 20 who had been enrolled in the same school district for at least 3 years and members of the same ESL class so that there was consistency in the tests used to measure English language proficiency. Among them, five ELLs were to serve as tutors and three ELLs would serve as tutees. Tutors were required to have an English language proficiency level of at least a 4.0 as indicated by their most recent WIDA ACCESS exams (a score of 5.0 is necessary to exit the ESL program) and grades of C+ or better in math. The tutees were required to have had an English language proficiency score of 3.9 or lower according to their most recent WIDA ACCESS or W-APT exam scores and grades of C- and lower in math (for additional information regarding the English language proficiency levels (see Appendix A). A certified high school math teacher was also recruited to tutor the same three tutees as the student tutors.

I considered student interest in selecting tutors. For example, the students who fulfilled initial criteria necessary to participate in the study were then asked to state reasons for their interest in volunteering their time. This was done in a means to identify tutors who were genuinely interested in providing free tutoring sessions and tutees who wanted the extra help. This portion of the selection process also helped me to determine what participants would most likely remain in the study from start to finish as it would require at least five hours of their time.
From the total of eight tutors who initially applied, the five selected demonstrated the strongest interest in serving as tutors and assured they would remain throughout the time needed to complete the data collection for the study. From the initial nine who applied for the tutee role, the three that were most responsive to receiving extra help and wanted to use this study as a means to improve their math skills were selected. The mathematics teacher, Mrs. T was selected because she was one of the co-teachers in the class I taught and was genuinely interested in learning different techniques that might help the ELLs in her class. In the following section I introduce the tutors and tutees in more detail. All names of participants and schools are pseudonyms.

**Tutors.** The student tutors consisted of four females and one male from three different native language backgrounds and mathematical skills. Two spoke Spanish, one Tagalog, and two German. Erin and Cindy were sophomores, Jean, a junior, and Shiela and Tina were seniors from the same school at the time this study was conducted.

Erin, a sophomore, transferred to ABC school district from another Pennsylvania school in the eighth grade. Although, she had attended her previous school for 4 years before transferring to ABC, she had not been tested or placed in an ESL class there. Upon arrival to ABC, I checked her home language survey which indicated she spoke a language other than English at home and according to Pennsylvania law, this information made her eligible to be tested for ESL. I tested Erin and she scored at the *bridging* level of English language proficiency and therefore qualified for one hour of ESL per day (see Appendix A for description of the English language proficiency levels). Since her arrival at ABC school, Erin had failed pre-algebra once and was then placed in remedial math for one year. She is currently enrolled in a general education algebra I class, performing at grade level, and helps tutor some of the ELLs in
my class during her study hall at my request.

Cindy is in the same sophomore class as Erin and enrolled in ABC middle school during the sixth grade. Cindy did not transfer from a local school district, but arrived at ABC with no prior records from a school abroad. Although one of her parents is from England, Cindy’s fluency in English was limited and she tested into the ESL program at the beginning level which qualified her for two hours of ESL per day. Since her arrival, Cindy, as well as most newcomer ELL students, was placed in grade level mainstream core content classes and although she demonstrated some trouble in science and social studies, she excelled in math. Placed in a pilot sheltering program I developed for one year (the ESL teacher teaches core content classes to ELLs in the ESL classroom), she caught up in science and social studies and has made the honor roll every quarter since the seventh grade. She is currently at the bridging level of English language proficiency and excelling in an advanced algebra II class. Cindy is highly recommended by teachers as a tutor for ELL and mainstream math students at the secondary level and has been tutoring students struggling in math for the past 2 years at ABC High during her study halls.

Jean, a junior at the secondary level, has been enrolled at ABC school district and in the ESL class since the sixth grade. He transferred from a school district outside of Pennsylvania where he had been enrolled in ESL classes for two years prior to arriving at ABC Middle School. Jean was re-tested for ESL services using the Pennsylvania mandated placement test, the WAPT, and scored at the beginning level which recommends two hours of ESL per day. Since his arrival at ABC, Jean struggled in mathematics and was placed in remedial math during the seventh and eighth grade school years. Currently at the bridging level of English language proficiency, Jean receives ESL one hour per day and is currently at grade level in his Algebra II class. Jean is the brother of one of the tutees named Jerry who I will introduce in the tutee section.
Shiela is a high school senior who was adopted from an orphanage abroad by parents who speak both English and her native language. She arrived in the U.S. at the age of 12 and was enrolled in sixth grade at ABC middle school. Tested for ESL services, Shiela scored at the lower end of the beginning level which qualified her for two hours of ESL per day. Placed in mainstream content classes of math, science, and social studies, Shiela excelled in all of her classes, particularly math and science. Currently at the bridging level of English language proficiency, Shiela receives 1 hour of ESL per day, completed all of her mathematics requirements (Algebra 1 & 2) in her junior year, and recently passed the PSSA math exam. For extra credit, Shiela is enrolled in AP mathematics and advanced physics. Highly recommended by math and physics teachers as a tutor, Shiela spends most of her time tutoring mainstream students both subjects during her study halls.

Tina, another senior in Shiela’s class, arrived at ABC school district in the seventh grade from a school in a country abroad. Similar to Cindy, one of her parents is a native English speaker and Tina grew up bilingual. Although her social language was strong, her academic English proficiency and reading skills were weak and she tested for ESL at the developing level. Tina struggled in math and science and was placed in remedial classes during the seventh grade and received after school tutoring in math during her eighth grade year. Since then has excelled in science and is working at grade level in her Algebra II class. Currently at the bridging level of English language proficiency, Tina receives one hour of ESL per day. Similar to Shiela, Tina also passed the PSSA math assessment and upon completion of this math class will have fulfilled all the requirements for graduation.

The mathematics teacher in the study, Mrs. T, is an employee of the same school district the students attend and co-teaches in a mathematics class for mainstream, special education, and
newly arrived ELL students. She possesses sophisticated mathematical skills as she is a certified mathematics and special education teacher and has spent most of her eight years teaching in secondary life skills and learning support classes. Mrs. T was assigned as a second math teacher in the co-taught high school Algebra I and II mathematics classes for the past two years, and she was responsible for helping the special education and the ELL students keep pace and comprehend the math lessons taught to the mainstream students by her colleague. Mrs. T was one of the teachers that I taught with in the co-taught classroom. I identify her as the other co-teacher of the ELLs because I taught in this classroom three days per week and the other two days, Mrs. T took over my place with the ELLs in addition to her responsibilities with the special education students. Although she has never taught any of the ELLs in this study, upon my departure, Mrs. T completely took over helping the ELL students in this class and she currently works with recently arrived ELLs in the school. It is interesting to note that Mrs. T has had note prior experience or training in teaching ELLs and was assigned to this position through school administration. Placing ELLs at the secondary and middle school levels in math and science classrooms co-taught my mainstream and special education teachers with no prior training or experience in teaching ELLs is a common practice in most Pennsylvania schools. This information I have gathered through experience and communication with multiple teachers in various school districts across the state and remains a common complaint discussed among ELL practitioners at most professional events.

**Tutees.** All three tutees were male from two different language backgrounds. Two of them spoke Spanish and the third spoke Russian. All three had been placed in remedial math or were seeing a math tutor daily and had repeated pre-algebra or Algebra 1 at least once during their high school careers. Peter and Jerry are high school seniors and Arnie is a sophomore.
Arnie came to ABC middle school from an orphanage overseas. Placed in sixth grade, he was tested for ESL and qualified at the *entering* level. In addition to the 2 hours of ESL he received per day, Arnie was also placed in my pilot sheltering class for math, science, and social studies. The following year, Arnie was placed in mainstream classes and although he did not fare well, he passed them with the exception of math. He was then placed in remedial math during his ninth grade year and is currently repeating algebra I in a mainstream class. Arnie must pass this class or he will be at risk of not meeting the education requirements to qualify for the district’s vocational technical (vo-tech) program. He will not be permitted access to the vo-tech program if he fails any of his core content areas. Also, Arnie will take the PSSA test next year which he needs to pass in order to fulfill graduation requirements for his senior year. During his study hall, his math teacher has arranged for tutoring from a peer tutor, however, this does not seem to be helping and Arnie remains after school for extra help from the teacher. It is interesting to note that unlike Sheila who was adopted by an American and native speaker of her language, Arnie was adopted into an American family that speaks only English. Arnie’s writing system is unlike the American writing system, and unlike Sheila who has a strong accent, Arnie exhibits native like oral language proficiency. His most recent English language proficiency test scores place his speaking and listening skills at the *bridging* level of English language proficiency, but his reading and writing skills just slightly above *beginning*. Through my twenty plus year experience teaching ELLs, this discrepancy in reading/writing versus speaking/listening is a usual occurrence for many foreign born children adopted by American families, however, although this poses no problem in an ESL class, Arnie’s mainstream teachers concur with one another that Arnie’s poor math scores are due to laziness. This results in a multitude of problems for Arnie in the mainstream mathematics classroom.
Currently enrolled in the twelfth grade, Peter and his family moved from a country overseas to the ABC school district during his ninth grade year. Issued the WAPT placement test, Peter’s English language proficiency was at the entering level and he received two hours of ESL per day. Placed in pre-algebra during his first year of entry in ABC high school, Peter failed and took the class again during summer school. In the tenth grade he took algebra I and failed, however, this time he retook the class during the eleventh grade in a co-taught algebra I class. Unfortunately, he failed it again and had to retake the class during summer school. Currently taking algebra II in his senior year, the stakes are high for Peter as he must pass this class in order to fulfill graduation requirements. Also, he is required to retake the PSSA math exam and if he fails that, he must take a school based assessment to fulfill the remainder of his math requirements. Currently at the lower spectrum of the beginning level, Peter has made most of his progress this past year and although he receives ESL for two hours per day, I had arranged for him to remain an extra hour so that he can receive tutoring in math from one of the available peer tutors during that time.

Jerry is also a senior at ABC high and is the brother of Jean, one of the tutors in the study. One year older than Jean, Jerry entered the school district and ESL program during his seventh grade year and scored at the beginning English language proficiency level when tested for ESL. Similar to Jean, Jerry received two hours of ESL per day and struggled in math and science, failing both in the ninth grade and retaking them during summer school. Jerry passed pre-algebra in the tenth grade, algebra I in the eleventh grade, and is taking algebra II this year in a move to fulfill graduation requirements. Similar to Peter, the stakes are high for Jerry as he is required to pass algebra II in order to fulfill graduation requirements. In addition, he must also re-take the PSSA due to a failing score (below basic) last year and if he doesn’t pass, must take and pass a
school based math exam. Currently at the expanding level of English language proficiency, Jerry receives ESL one hour per day and remains an additional hour for tutoring in math from any of the available peer tutors.

Demographics of tutors and tutees are presented in Tables 1 & 2 Description of the WIDA ELP proficiency levels are presented in Appendix A and PSSA mathematics scoring guidelines are included in Appendix B.

<table>
<thead>
<tr>
<th>Table 1</th>
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<tbody>
<tr>
<td><strong>Tutor Demographics</strong></td>
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<tr>
<td><strong>Teacher Tutor</strong></td>
</tr>
<tr>
<td>Name</td>
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<td>---</td>
</tr>
<tr>
<td>Mrs. T</td>
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</table>

<table>
<thead>
<tr>
<th>Student Tutors</th>
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</thead>
<tbody>
<tr>
<td><strong>Name</strong></td>
</tr>
<tr>
<td>Shiela</td>
</tr>
<tr>
<td>Cindy</td>
</tr>
<tr>
<td>Jean</td>
</tr>
<tr>
<td>Tina</td>
</tr>
<tr>
<td>Erin</td>
</tr>
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Table 2

**Tutee Demographics**

<table>
<thead>
<tr>
<th>Name</th>
<th>ELP</th>
<th>Grade</th>
<th>Current Grade in math</th>
<th>Native Language</th>
<th>Remedial Math</th>
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</thead>
<tbody>
<tr>
<td>Jerry</td>
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<td>12</td>
<td>Failing</td>
<td>Spanish</td>
<td>No</td>
</tr>
<tr>
<td>Arnie</td>
<td>3.8</td>
<td>10</td>
<td>Failing</td>
<td>Russian</td>
<td>Yes</td>
</tr>
<tr>
<td>Peter</td>
<td>2.5</td>
<td>12</td>
<td>Failing</td>
<td>Spanish</td>
<td>No</td>
</tr>
</tbody>
</table>

**Research Focus and Questions**

As I mentioned in the introduction, the focus of this study is to understand the nature of interaction used by tutors contribute to tutees’ ownership in the mathematics problem solving process. By “ownership,” I refer to the tutee’s successful acquisition and demonstration of the vocabulary and problem solving processes involved in mathematic activities. As previously mentioned and presented in Appendix B, the PSSA mathematics tests are scored on the student’s ability to not only arrive at a correct answer, but to explain the process using the specialized vocabulary of mathematics. Failure to provide a full explanation in mathematical terms results in points deducted from the overall score of a problem set.

My ultimate goal is to observe the connection between tutor questions and mathematical learning of the tutee. An overarching research question was: *How do tutors interact with tutees to develop ownership of mathematical problem solving activity? How do effective tutors interact differently from ineffective tutors?* I then developed two sub questions:

a). What types of tutor questions support active participation, independent
problem solving, and accurate completion of problem sets as tutees engage in small group mathematic activities?

b). In what way does tutors’ use of wait time contribute to tutees’ use of arithmetic vocabulary?

**Procedure and Data Collection**

Upon initial participant selection is completed, I divided participants into dyads. The students/students and teacher/students worked in dyads (total of 18 dyads) to solve approximately three to five math problems chosen from the Grade 11 Pennsylvania System of School Assessment (PSSA) 2009 test item sampler. I recruited tutors and tutees on a voluntary basis. After the initial recruitment phase the tutors, tutees, and PI met for dinner to discuss the logistics of the study. The tutees were given a variety of mathematical problems and instructed to circle five they did not understand or could not compute. The tutors were given copies of the tutee problem sets after the initial meeting and asked to rate them 1-5 as to their level of comfort in teaching e.g. 1-very comfortable, 5-extremely uncomfortable. From this sample I divided the problem sets among the tutors ensuring that they each had at least one problem that required computational skills and one that required an understanding of the specialized vocabulary of mathematics. During the tutoring sessions, the tutors were allotted no more than 30 minutes with each tutee to solve a sample set of 2-5 problems (although it was rare that the interactions during the study exceeded more than 10 minutes per episode). My role in the study was to observe and record data through tape recordings, video recordings, and field notes. In addition, as an observer and non-participant in the activity, I gave explicit directions in what they were to accomplish during the activity. My instructions did not include suggestions for how tutoring sessions should be done considering that the focus of this research is to identify discourse
strategies, primarily questions and wait time, used by tutors to help tutees become active participants and independent problem solvers of the mathematics activity.

**Data Analysis**

Interaction between each dyad was recorded via video and/or tape recorder for later transcription. During each dyadic interaction, field notes were generated that focused specifically on verbal questions used by tutors to promote active participation of tutees and independent problem solving. I also generated field notes as to the places during dialogue where there was an episode of silence between tutor question and tutee response and tutee response and tutor reaction and indicated reasons why the episode of silence occurred. I labeled these episodes as possible wait time. At the close of each tutoring session, which primarily consisted of three mathematic problem solving activities, copies of problem sets used by tutors were collected by the PI to use in better understand the context of data that was recorded.

Data from the recordings were transcribed via long hand for initial analysis in conjunction with the field notes. Areas where possible wait time occurred were highlighted and the length of silence within these episodes calculated by using the second counter on both the video and audio recorders as well as a secondary stop watch. This possible wait time was then placed in the areas of the transcription where it occurred within discourse between dyad participants. After the initial longhand transcription was completed and the possible wait time included, I compared this to the video recordings to ensure I had not missed any type of dialogue, behavior, or possible wait time that needed to be included in the transcription. Once assured I my transcription matched the record, I entered the data into a word processor and using an open-coding, line by line process (Charmaz, 2002), made detailed changes to the font and generated notes to identify questions, mathematics vocabulary, and wait time. The approach
was inductive as I compared and contrasted the data from each problem solving episode within each small group activity. Patterns and themes within each dyadic activity began to emerge which I color-coded and put into a variety of charts.

I analyzed data in three stages. During the first stage, I focused on teasing out patterns that existed in the questions used between and within each dyadic interaction. I noticed that all tutors used questions in conjunction with an explanation of the process and often visual representation of the problem statement to solve problems. They also used questions during the interaction as they solved the problems. In a move to determine the function each question served, I first compared them to the types of response they generated from tutees and placed them in two broad categories, implicit (open ended questions) and explicit (known answer questions). I color coded explicit and implicit questions and further analyzed the corresponding tutee responses to determine consistency within and across dyads. Implicit questions generated similar responses from tutees across dyads; however, differing patterns of responses emerged within the explicit type questions. Further analysis revealed that certain types of explicit questions generated a complete answer from the tutee and others only generated a yes or no response. The differences in tutee response prompted further categorization of elicit questions which I coded as yes/no or known answer questions.

Once the questions had been properly coded into three categories, I then focused on the degree each category of questions served in encouraging tutees to participate in co-constructing the problem solving process. Remaining consistent with Glaser & Strauss’ (1967) approach, I coded these questions according to the degree of participation they elicited from the tutee assigning PP for a question that resulted in the passive participation of the tutee, AP for those that generated active participation, and ID for those that generated independent demonstration of
the problem solving process. PP was assigned to questions that did not meaningfully engage the tutee in co-constructing the problem solving process. By meaningful, I refer to questions that do not ask tutee to solve a step within the process or carry out a mathematical operation. AP was assigned to questions that encouraged the tutee to actively co-construct the problem. Although these types of questions could be subject to guessing on the part of the tutee or when broken in steps, solving for only the immediate step, he was still co-constructing the problem solving process which was something he could not have previously done alone according to his initial selection of problem sets at the first meeting. The final coding scheme resulted in ID as a means to identify those questions that provided tutees an opportunity to independently demonstrate the problem solving process with minimal to no support from the tutor. The successful response to questions such as this provided evidence that the tutee had comprehended the problem solving process and could demonstrate it independently or with minimum support.

Once the tutor questions had been categorized in conjunction with the tutee response and further coded with regard to the degree of tutee participation in the activity, I developed an additional coding scheme for level of mediation or support questions provided tutees to accurately co-construct the mathematics problem solving processes. Similar to the degree of participation tutor questions encouraged, the level of support they offered also changed dependent on tutee response. I applied a number scheme to identify the level of support provided by each question and created a scale from 1-3, 1 indicating little to no support, 2 indicating mid-level of support, and 3 as highly supportive questions. Questions coded a 1 typically revealed a minimal amount or absence of support in that they did not help the tutee accurately co-construct mathematics problems within his ZPD or independently. For example: a tutor who explains a process and then asks the tutee "do you understand what I said?" before moving does not
provide support if the tutee answers yes/no because he is not demonstrating accuracy of understanding the question, actually he is not demonstrating anything except that he heard the question therefore, for this type of question, I assigned a 1. Although most yes/no questions received a 1, there were some that did provide a high level of support, for example a response to a problem revealed that the tutee did not arrive at the correct number of exponents, the tutor asked “did you add all the X’s?” This type of yes/no question is provided as mediation to prompt the tutee that something is wrong with his answer and based on the tutee’s response of the correct or expanded answer as opposed yes/no, provides evidence that the tutee did work within the ZPD and would therefore receive a score of 3 for providing support.

The known answer questions were generally used by tutors to guide tutees towards the appropriate solution of a problem. Often used to model a step by step process, for example in this Excerpt between Erin and Jerry, Erin (T5) makes use of known answer questions to guide Jerry (A) to a correct answer for the problem.

(1) T5: Alright, the probability a battery is defective is 8%. In a shipment of 500 batteries, how many can be expected to be defective? (Points to the words with her pen as she reads the problem statement) alright? How many hundreds are in 500?
(2) A: 5
(3) T5: 5, right. What’s 5 times 8?
(4) A: 40
(5) T5: 40, that’s how many we can expect to be defective. So the answer is 40, right?
(6) A: yeah.

Known answer questions similar to those posed by Erin that support the students in arriving at the correct solution to a problem were rated as level 3 indicating a high level of support.

Finally, the implicit questions received a level 2 indicating a mid-level of support. This level was assigned to implicit questions that required tutees to demonstrate problem solving through verbal or physical explanation and although this type of question provided a prompt for
the tutee to solve the problem, they only resulted in a mid-level of also provided him a midlevel range of support as he was usually able to complete the problem somewhat independently.

Three types of questions emerged; *procedural* which were yes/no answered questions that resulted in passive participation (PP) and level 1 support, *guiding* which were explicit, provided active participation (AP), and level 3 support, and *reflection* which were implicit, led to independent demonstration of problem solving process (ID), and received a level 2 for support.

The second stage of my analysis was conducted in 2 parts to answer the second research question: *In what way does tutors’ use of wait time contribute to tutees’ use of arithmetic vocabulary?* I first analyzed the frequency and use of vocabulary used by tutors and tutees that is more specific to math than that used in other academic content areas or in social situations. Consistent with constant comparative methodology (Glaser & Strauss, 1967), I began the second stage of my data analysis with another reading of the transcription and highlighted each word I inferred to be mathematics vocabulary used in tutor questions, explanations, and tutee response. I created a list and separated the vocabulary into categories labeled mathematical operations, symbols, measurements, and shapes. I then compared the vocabulary within each category against the vocabulary found in the glossary of a middle-school mathematics text book. As the sample participants of my study are high school level ELLs and have been enrolled in math classes within the U.S. school system for at least 2 years or more, simple vocabulary that included numeric forms, fractions, over, equal, plus, and minus were eliminated. Vocabulary out of the ordinary that appeared in the mathematics text book remained as vocabulary considered specific to mathematics and highlighted throughout each transcription and placed in a table (see Table 3).
Next, I focused on identifying what I would consider as wait time within each episode operationalizing wait time as any pause in discourse exceeding 3 seconds or more that occurred between tutor questions, question and answer items, tutee response and tutor reaction, and problem solving processes. I made inferences regarding the amount of time necessary to allow the tutee to process a question and demonstrate a problem solving process and referenced the SIOP model developed by Echevarria, Vogt, & Short (2007) as a means to establish boundaries as to what I would consider an appropriate amount of wait time. I analyzed the transcriptions and used the counter on both the voice and video camera as a means to determine the amount of seconds for wait time to be established. Any length of silence between tutor question/tutee response or tutee response/tutor reaction under 3 seconds was discarded and considered a pause. I correlated the amount of wait time indicated by the clock on the recording device with that of a computer generated stop watch to ensure accuracy. Wait time was then calculated in whole or half second intervals on any amount of wait time that averaged 3 or more seconds. I reanalyzed my data and excluded wait time that occurred when the tutee was solving an extended mathematical problem. By extended, I refer to math problems that require two steps or newly learned computation skills to solve. For example: in the algebraic equation -4x + =15, a two-step process is required to get the variable 4x by itself: 7 must be reversed and applied to both sides. In a situation such as this or that required additional steps, wait time was not calculated. Instead, I focused on wait time that occurred primarily between tutor question/tutee answer sequences and tutee response/tutor reaction sequences. I initially considered this 3 second amount of silence as an appropriate amount of wait time based on my experiences teaching ELLs and English speakers learning German as a foreign language. However, upon completion of my literature review, I was able to confirm my initial inference with data revealed from extensive studies.
conducted by Mary Budd Rowe that considered 3 seconds an appropriate amount of wait time for beneficial changes in student learning to occur.

After I established what I considered as appropriate wait time between question and answer sequences, I retraced my steps and focused on all the question and answer segments in which wait time was used. Patterns began to emerge in that most of the wait time in question/answer segments occurred when the tutee provided an incorrect or incomplete answer. In a move to cue the tutee to expand on his answer, the tutor often utilized wait time in addition to repeating the tutee’s incomplete answer. This usually prompted the tutee to expand his answer and include arithmetic vocabulary. I analyzed these question/answer episodes, to determine if the answers provided by the tutee after wait time was established led to an increased use of mathematics language within his utterance and identified this as “richness” of language. I coded wait time according to the context in which it occurred, calculated the richness of utterance by highlighting the occurrence of mathematics vocabulary, and the length of wait time in which this richness of vocabulary was observed. Through this stage of the data analysis, I was able to identify if wait time was an enabling factor in allowing tutees to demonstrate their appropriation of mathematics vocabulary as it was used in the process of the activity (see Table 3).

Table 3

<table>
<thead>
<tr>
<th>Arithmetic Vocabulary</th>
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<tbody>
<tr>
<td><strong>Vocabulary found in textbook</strong></td>
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<tr>
<td>Negative/positive</td>
</tr>
<tr>
<td>Integer</td>
</tr>
<tr>
<td>Variable</td>
</tr>
<tr>
<td>Divide</td>
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<td>Slope</td>
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</tbody>
</table>
In the third and final stage of the analysis I holistically analyzed each case episode to identify patterns that occurred in the types of questions used within dyadic interactions that may account for the success or failure of the tutee’s ability to demonstrate “ownership” of the mathematic problem solving activity. By ownership I refer to the tutees ability to demonstrate the vocabulary and problem solving processes involved in the mathematics activity. By mathematics activity, I refer to the collaborative problem solving process from beginning to end of each tutoring episode. As is, each tutor had three tutoring activities, and I present one from each dyad as a case study to answer the overarching research questions; How do tutors interact with tutees to develop ownership of mathematical problem solving activity? How do effective tutors interact differently from ineffective tutors? More specifically, I examine the activity holistically to determine how tutor questions worked in tandem with wait time and tutee response to enable the tutee to actively participate and independently demonstrate arithmetic vocabulary and problem solving processes. To accomplish this goal, I carefully looked at the collective activity of the most successful tutors who were able to use questions as tools to encourage tutees to become active participants of co-constructing the mathematics activities and then work beyond that by independently demonstrating the problem solving process. I then compared effective tutors’ use of questions and wait time within dyadic interactions to those of tutors deemed less effective in achieving the collective goal of active participation and independent problem solving. It is through the collective results of the case studies that I base my recommendations for mathematics teachers of ELLs in the discussion section.
CHAPTER FOUR
RESEARCH RESULTS

This study aims to analyze how tutors interact with tutees to develop ownership of mathematical problem solving activity and the differences in the nature of interaction between effective tutors and ineffective tutors. I specifically look at the types of questions used by effective tutors in conjunction with wait time and tutee response that promote active participation, independent problem solving, and use of mathematics vocabulary. In this section I discuss the analysis of tutor questions based on tutee responses and reveal how each category of questions function differently in supporting tutee participation in the problem solving process. I then examined the questions to determine the level of support they provided the tutee in arriving at a correct answer to the mathematic activity. Upon completion of the questions analysis, I turned my attention to the use of wait time by tutors between tutor question/tutee response and tutee response/tutor reaction to determine the effects of wait time on the use of arithmetic vocabulary. Finally, I present case studies of tutoring dyads from the most effective tutor and least effective tutor to identify differences within the nature of interaction that accounted for the success in developing tutee ownership of the mathematical problem solving activity.

Because I was interested in understanding specific types of support during interaction that contributed to tutee learning, I focused on the types of questions tutors used that resulted in tutee’s active participation and independent problem solving. This is consistent with the socio-cultural theory that places emphasis on both mediation and active participation in interaction as a means to promote independent mastery over a task. It seemed reasonable for me to hypothesize that a certain type of questions tutors use support (mediate) tutees’ ability to “stretch beyond
independent performance” (Lantolf & Poehner, 2010 p. 14) and become active participants in accurately co-constructing the problem solving process. I also hypothesized that tutees’ demonstration of independent problem solving would occur if tutees’ comprehension of the problem solving process has occurred.

Research findings are organized into three sections. The first section contains the results of my analysis on the types of questions used by tutors and their effect on tutee participation and support. The second section focuses on wait time as a means to increase tutee’s use of arithmetic vocabulary. The third section presents case studies from the most effective and least effective tutors and provides a rationale as to why some tutors were successful in developing tutee ownership of the mathematics problem solving activity.

Before I explain my research findings, I will briefly summarize data analytic methods, which involved two steps. In the first step, I identified patterns in tutors’ questions based on the responses of tutees. I coded the questions as explicit (closed/known answer) and implicit (open ended/inquiry) questions. Implicit questions within and between dyads elicited similar types of responses in that tutees were required to explain or demonstrate the problem solving process. For example, a tutor question “Now why would you circle above negative one?” required more elaboration in the form of an explanation or justification from the tutee than explicit questions and consisted primarily of the words how and why. Explicit questions, on the other surface, seemed relatively simple to code, however, upon further examination of tutee responses, I revealed two differing categories of explicit questions. Explicit questions such as “Is it safe to say that this answer forms a pattern?” that prompted tutees to answer with a yes or no I coded as yes/no items and others that required the tutee to think deeper or calculate a problem, for example “what times 5 is 65?” were coded as known answer questions. This analysis resulted
in three categories of questions, *implicit, yes/no, and known answer.*

The second step of the question analysis focused on the relationship between tutor questions and degree of tutee participation in co-constructing the mathematic activity. I coded these questions according to the level of participation they elicited from the tutee assigning PP for a question that resulted in passive participation, AP for those that encouraged active participation, and ID for those that generated independent demonstration of the problem solving process. For example, an implicit question posed by the tutor “*So what is the answer to this one, how would you do that?*” would require a tutee to demonstrate the problem solving process the tutor had previously explained or he had collaboratively solved. This type of question requires independent problem solving on the part of the tutee and received an ID for degree of participation. Known answer questions required a full response from the tutee and were primarily used to encourage students to actively participate in the problem solving process. For example, Erin’s use of known answer questions in a mathematics activity with Peter enabled him to sequence the steps of the problem solving process and actively participate in co-constructing both the process and solution to the problem.

**Excerpt A (T5) Erin and (C) Peter**

*What is the greatest common factor (GCF) of these monomials? 18 x^3 y    30 x^2 y^2*  

(1) T5: O.K….they’re asking what the GFC….greatest common factor, is for these two monomials (pointing to the problem statement)  

   *O.K, so, what number is divisible by 18 and 30?*  

(2) C:  

   (5 seconds pass)  

   6  

(3) T5: Yeah, 6..and what is the lowest X…the number…is it X squared or what?  

(4) C:  

   Umm, yeah..its X squared.  

(5) T5: *OK, and the lowest Y?*  

(6) C:  

   Y  

(7) T5: OK, then the answer is 6 X squared Y  

(8) C:  

   OK
As indicated in the dialogue, Erin posed known answer questions to elicit a response from Peter and collaboratively guide him through the steps needed to solve the problem. These known answer questions are necessary and integral steps of the problem solving process Peter must learn in order to complete the problem and arrive at a correct solution. Peter’s correct response to the answers Erin posed enabled him to actively participate in the collaborative problem solving process and accurately complete the mathematic activity. Questions from this category were coded AP for active participation.

In contrast to known answer questions that required active participation from the tutee to co-construct the mathematics problem solving process, explicit questions that resulted in a yes/no response from the tutee most often elicited a passive level of participation. Unless a request was further used to expand upon the response, most yes/no questions were primarily used to check comprehension, remain focused, and acknowledge the introduction or restatement of the problem or procedure. For example, in the dialogue between the tutor Shiela and tutee Jerry, Shiela shows Jerry a picture of a protractor, begins to tell him it’s a protractor “this is a protractor”, then rather than end the statement there includes the word right accompanied by a slight intonation turning the prior statement into a question that is asking Jerry to acknowledge or agree that the picture they are looking at is a protractor (see Excerpt B).

Excerpt B (T2) Shiela and (A) Jerry

(1) T2: O.K. *This is a protractor right?*
(2) A: Yeah.
(3) T2: O.K. it’s asking what is (pointing at the problem statement with her pen) X, Y and Z. *O.K?*
(4) A: *O.K.

As is visible in the interaction, Jerry agrees in line 2 that the picture is indeed a protractor which leads Shiela to provide a brief explanation of what the problem is asking him to solve for and follows this again with a question that requires his response to be in affirmation or denial.
Neither the questions posed in lines 1 or 3 have provided Jerry an opportunity to participate in the problem solving process and were coded PP for passive participation as Jerry remained a passive participant of this activity.

Once the tutor questions had been categorized in conjunction with the tutee response and further coded with regard to the degree of tutee participation in the activity, I developed a numeric coding system based on the level of support tutor questions provided tutees in arriving at a correct response during the problem solving process. I applied a scale of 1-3 (1) low level of support, (2) mid-range level of support, and (3) high level of support, based on the support offered through the tutor’s questions in helping the tutee arrive at a correct response. Similar to the level of participation tutor questions encouraged, the degree of support they offered also changed dependent on tutee response. Questions coded as (1) typically revealed a minimal amount of support in that they did not meaningfully engage the tutee in the problem solving process, help him accurately co-construct the solution, or demonstrate independent problem solving. For example: a tutor who asks “This isn’t an easy problem is it?” is not providing any level of support if the tutee answers yes/no, therefore this type of question was coded (1) for low level of support. Although most yes/no questions were coded as providing low levels of support, further analysis of the questions in context revealed extreme differences in tutee response. For example, on the surface, a tutor question of; “8x?” or “you know how to do this one?” seemed to be a simple yes/no question. Upon reconsideration the questions in conjunction with the context they were delivered and the tutee response revealed both questions to be supportive in that they prompted the tutee for additional information or that his answer was wrong leading him to reanalyze his initial response and produce a more expanded or correct version. Both questions are presented in the following dialogues between Jean and Arnie in Excerpts C and D.

Excerpt C (T1) Jean and (B) Arnie
In Excerpt C, Arnie’s response of $8x$ was not a correct calculation of the variable exponents and he did not arrive at the correct answer. Although Jean’s question “$8x?$” could have been met with a yes/no reply, this recast of Arnie’s response in the form of a question prompted Arnie to reevaluate his first answer and arrive at the correct response. In Excerpt D, Jean’s question “you know how to do this one?” could also have been met with a yes/no reply; however, because of the context in which it was presented, it elicited a different response. Prior to this question, Jean and Arnie had collaboratively solved a problem and Jean produced a new sample problem mimicking the first, therefore when he posed the question “you know how to do this one?” to Arnie, Arnie infers that Jean wants him to attempt to solve the problem on his own and complies.

Excerpt D (T1) Jean and (B) Arnie

(1) T1: You know how to do this one? (indicating a sample problem he made similar to the one they just completed together)
(2) TS2: um, you put this over 1 (writing the equation on the paper) then you go to the graph and start from here (pointing to the graph) and go one, two, three (mumbling to himself)
(5 seconds pass)
And here.
(3) T1: Good. (smiles) Done. Let’s go to the next one.

Excerpts C and D revealed questions that initially could have been answered in a yes/no reply, but based on the context in which they were posed or the dialogue that occurred prior to them, they were initiated and received as correction to a mistake and/or independent demonstration of a problem solving process. These questions were coded as (3) and (2) respectively for level of support as the first yes/no question provided a high level of support for Arnie to arrive at the correct solution to the step of the activity and the second provided a mid-range level of support because the tutor prompted the tutee to solve the problem independently and was available to
help if necessary. In addition, if the tutee arrived at the incorrect answer, the tutor provided correction and/or mediation.

In summary, the analysis of the questions in relation to tutee response first revealed two categories of questions, implicit and explicit. Upon additional analysis of tutors’ questions in regard to level of tutee participation and further coding within and across dyads revealed that although implicit questions generated similar responses that required independent demonstration of the problem solving process, the explicit questions were further divided into two categories, yes/no items and known answer questions. These two types of questions were coded as PP for yes/no and AP for known answer questions. During the final stage of the analysis, I examined tutor questions to determine the level of support or mediation they provided tutees in arriving at a correct answer to the mathematic activity. In this instance, the yes/no questions were divided depending on the context they were used, prior discussion, and tutee response. This further indicated that some yes/no questions did invite the tutee to participate and provided a high level of support (level 3) for the tutee to arrive at a correct accurate answer or independently demonstrate the problem solving process. This further coding and categorization evolved into three types of tutor questions; *Procedural* questions are primarily yes/no questions posed by tutors that elicit passive participation (PP) and provide a low (1) level of support for the tutee, *Guiding* questions are explicit in nature and include some yes/no questions that encourage active participation (AP) and provide the tutee a high (3) level of support, and *Reflection* questions which are primarily implicit questions that enable the tutee to demonstrate independent problem solving (ID) and provide a mid-range (2) level of support.

It is important to note that all of the questions are reflexive, meaning, their level of support and degree of tutee participation is dependent on the tutee response and tutee response is ultimately
based on the question. For example, a tutor who has worked through a problem set with the tutee and poses a procedural question as a comprehension check i.e. “Did you get it?” would normally be met with yes/no. However, there are times when the tutee may answer yes, followed with an explanation of the problem solving process without additional prompts. This would then classify the procedural question as a reflection question because it supported or prompted the tutee to demonstrate independent problem solving. This is also the case for reflection questions that ask “Can you show me how to do this one?” and the tutee responds with “no.” In this sense the reflection question is now serving the same role as most procedural questions in that it serves as a comprehension check and is met with a yes/no answer.

The following section provides some additional examples of the three student questions followed by a description, the sample excerpt with explanations as to how the context, question, and tutee response all work together to determine participation and support.

**Research Finding 1: Three Types of Tutor Questions**

**Procedural questions.** Procedural questions provided the least amount of instructional support or opportunities for the tutee to contribute to the activity or demonstrate independent problem solving. Consisting of primarily yes/no questions, procedural questions were used by tutors as comprehension checks, prompts for acknowledgement or agreement, and to direct attention to the problem solving process. Posed after the introduction of an activity, an explanation of the activity procedures, or as a means to elicit the degree of tutee confidence in reproducing the activity, procedural questions were associated with moving the activity forward. As a result, tutees were not offered an opportunity to participate, demonstrate, or expand their understanding of the mathematical problem solving process. For example, procedural questions that emerged in the problem solving activity between Shiela and Peter (Excerpt E) do not encourage active
participation or provide a high level of support for the tutee in arriving at the correct solution to the activity. The procedural question Shiela uses after her detailed explanation to Peter regarding the closed and open signs, two very important facts necessary to solve the problem, serve two purposes. The first is to ensure Peter is focusing on her explanation and the second is to provide him an opportunity to affirm he understands the concept of the closed and open circle or to request assistance. This procedural question is used to call the tutee’s attention to an important component of the problem and to serve as a comprehension check.

Excerpt E (T2) Shiela and (C) Peter

1) T2: O.K. Now for this one, it says, which **graph represents** the **solution** of **negative** 1 is **greater than times**.. is **greater than** 2. So what you do is like, if it is like this (draws an open circle 0 next to > on the paper) like that (uses her fingers to make the shape of an open circle) it’s a **open, like open?** (makes the shape of the circle with her fingers and holds it closer to her partner, shaking her hand to emphasize the shape she is making with her fingers) and then if it is with a line in it, (using her pen to draw an example ≤ •) anything like that, or like this (pointing to both symbols •≤), it is **closed. O.K.?**

2) C: O.K.

3) T2: O.K.?

4) C: So if, if, it is a line down there on the bottom it is **closed..if it is**

5) T2: (interrupts her partner) **less than**, like that (pointing to her drawing) **less than**, if there’s no line in it, it’s **open.**

6) C: O.K.

7) T2: Like, the **circle is open** (using her fingers and hand to make an open sign) an **open shape**

8) C: Oh! uh, huh.

9) T2: It’s like that (continues to hold the sign for open circle with her hand in and shake it in front of her partner) There’s like, no **shade** in it, like, it’s like that (pointing to the open circle drawing with her pen) anything like that, its **open**, but if its have a line in it (tracing the ≤ drawing)

As Peter responds to Shiela’s question of “OK?” in affirmation that he does understand, Shiela follows with another procedural “OK?” to draw Peter’s attention to the fact that understanding of open and closed circles is an important component of the problem solving process and attention must be paid to this fact. Although not explicitly stated, this is inferred by her following statements which include further explanation of open and closed circles. Although this use of procedural questions in Shiela’s dialogue with Peter does not involve active participation or support in collaboratively arriving at a correct answer, they are important as they
draw attention to the problem solving process and provide the tutee an opportunity to ask for clarification or request help. Unfortunately, most of the tutees rarely acknowledged difficulty in following along or comprehending a tutor’s explanation of the problem process and it was not until they were offered the chance to participate in the activity that their lack of comprehension was revealed. This occurred in the next interaction between Shiela and Peter.

Procedural questions that emerged in this problem solving activity between Shiela and Peter (Excerpt F) did not encourage active participation or provide a high level of support for the tutee in arriving at the correct solution to the activity, however, they did serve as comprehension checks that Shiela acted upon to determine if Peter was following her explanation and understanding the problem solving process required to solve the activity. Shiela was teaching Peter how to identify the angles on a protractor and offered Peter an explanation of the process in conjunction with a visual demonstration of her drawing lines on the protractor they were to measure. In the opening line (line 1) Shiela asks “This is a protractor, right?” as she shows Peter the picture of a protractor. Similar to the previous exchange between Shiela and Peter, this is also done for two reasons; to introduce the activity and to assess background knowledge. If Peter had replied no to her opening question, Shiela would have revealed that this instrument, the protractor, was not a part of Peter’s background knowledge and would have to explain its purpose in more detail to familiarize Peter with the physical properties of the instrument before moving onto the explanation of the problem solving process. Peter replied to Shiela’s opening answer that he was familiar with the tool and this prompted her to continue with an explanation of its properties and how to find the correct measure.

Shiela gave a detailed explanation (line 3) indicating the importance of using the middle line on the protractor to identify what number, bottom or top, you would use as the measure. She further
reinforced this explanation by highlighting the midline on the picture of the protractor and explained through gestures that any lines that fall to the left of the mid-line were identified by the bottom numbers on the protractor and any that landed to the right were identified by the top numbers. (See Appendix C and Excerpt F)

Excerpt F (T2) Shiela and Peter (C)

(1) T2: Alright Peter. *This is ah, a protractor right?*
(2) C: Yeah.
(3) T2: O.K...So, its asking us, like, what this (pointing at the problem statement with her pen), like X, Y, Z, is pointing at (pointing to X-Y-Z and the picture of the protractor as she explains), like what *degree*.
(4) C: O.K.
(5) T2: *Ya know?*
(6) C: yeah.
(7) T2: Uh, so, how you figure it out is like, half of this (using her pen to draw a line through the middle of the protractor) whenever its passing over here (waving her hand over the right side of the protractor) you look at the bottom. Like, if it is passing here (waving her hand over the right side of the protractor)
(8) C: (nods while looking at the paper)
(9) T2: *So, like, what, what is this?* (making a line on the right side of the protractor at the 63/120 mark) *what does it seem? ...what?*
(10) C: (Pointing to the line drawn on the protractor) 63? No, 62.
(11) T2: That’s the thing. If this line is pointing over down there (waving her hand over the left side of the protractor) it would be 63. But it’s over here (waving her hand over the right side of the protractor) so we’re
(12) C: (Interrupting the tutor) 122. er 3..123!
(13) T2: *Yeah...So do you get it?*
(14) C: Yeah.
(15) T2: So like, if this line is going over to the right ..(pointing with her finger in the air to the right)
(16) C: (interrupts the tutor) it is *plus*
(17) T2: (simultaneously with the tutee's interruption) it is the bottom.. *the bottom*
(18) C: Oh
(19) T2: (using her pen to point) the bottom. OK?
(20) C: aaah, O.K.
(21) T2: *But if its going over there (pointing to the left with her thumb), like down there its like.. the top,. O.K?*
(22) C: (nods) O.K.
(23) T2: So let’s say, this line..lets give you an example (marking a new line on the protractor) Is over, down here. *What is that?*
(24) C: *Um..50?*
(25) T2: (shakes her head no)
(26) C: *No?*

Throughout most of the dialogue (lines 1-21), Shiela uses detailed explanations and procedural questions to ensure Peter is paying attention and comprehending the problem. Peter responds
positively that he understands through verbal agreement *OK* (line 6) and gestures (nodding in line 8) and Shiela, believing Peter does understand the process, provides him an example measurement that she draws on the protractor for him to identify (line 9). Peter has two choices depending on where Shiela drew the line; it can be either the bottom or the top number. Peter ‘s response is incorrect which leads Shiela to use gestures to accompany her detailed explanation to help Peter better comprehend the number he needs to use to choose for the measurement based on which side it falls in relation to the mid line (line 11). Peter interrupts Shiela with the correct answer (line 12) and Shiela seems to think Peter had understood rather than that he may have simply restated the other number since his first response was incorrect. Shiela poses a procedural question in acknowledgement that yes; this is the correct answer and a comprehension check “*Yeah...So do you get it?*” to give Peter an opportunity to pose a question or request clarification identify if he does not understand (line 13) and again Peter acknowledges comprehension of the problem solving process. Shiela reiterates (line 15) her prior explanation in response to Peter’s previous acknowledgement to her comprehension and again explains where Peter needs to look on the protractor to find the right number for the measurement depending on which side of the midline the line passes. As she explains what side of the mid line to look at left or right, Peter interrupts with “*plus*” (line 16) which cues Shiela that again, Peter has not understood her explanation of the activity thus far since the vocabulary plus, minus, negative, or positive was never mentioned. This is an obvious breakdown in communication that needs to be repaired. Shiela seems to recognize this and attempts repair by reiterating the importance of using the bottom number when the line falls to the right of the mid-line. She repeats and enunciates the word “*bottom*” (lines 17 and 19) so that Peter not only understands the importance of looking at the bottom number to identify the correct measure of the line, but to call attention to the correct
terminology.

It now seems that Peter might be understanding Shiela’s explanation and she further expands on this by providing a final explanation which number to look at depending on the side of the midline it falls on and uses gestures to accompany this before following with a procedural comprehension check (line 21) “But if it’s going over there (pointing to the left with her thumb), like down there it’s like.. the top.., OK?” Peter responds that he understands Shiela.

Most tutors or teacher by now would believe that Peter has understood since he was required to comprehend a seemingly simple task that required a choice of two possible numbers, bottom or top, to use in identifying the measurement of the angle depending on what side of the midline, left or right, the angle line passed through. However, even with Shiela’s detailed explanation, Peter did not understand the process and even used the term “plus” out of place. There has been a miscommunication throughout the process so far, although Peter continually acknowledges he has understood Shiela’s explanation of the process. This is quite believable as the explanations provided by Shiela thus far were very detailed and included visual and gestural support. Many tutors would have stopped the activity here and took Peter’s response in conjunction with the detailed explanation as indication that he did understand the activity and end the activity at that point. Thankfully, Shiela does not end the activity there, but tests Peter’s ability to identify the correct number before she moves on. She draws a line on the protractor (line 23) and asks Peter to identify what number it falls on. Peter (line 24) responds with the wrong answer. This not only frustrates Shiela, but requires her to reexplain the process which she does (included in Appendix C).

This episode is important in that Shiela uses procedural questions after she provides detailed explanation using gestures and visuals for support. Although the procedural questions do not
actively engage the tutee or provide support in producing a correct solution, they do serve as comprehension checks to ensure Peter understands the parts of the problem solving process she had explained to that point. Peter’s affirmation is believable due to the simplicity of the problem and Shiela’s detailed explanations. However, when Shiela asks Peter to solve the problem, his response is incorrect leading to another explanation. Shiela continues this process a second time and is again disappointed with an incorrect response. Had Shiela relied on procedural questions only, she would not have revealed that Peter truly did not understand the activity and her goal of teaching Peter how to measure angles on a protractor would not have been accomplished.

In the next example, Cindy, a recommended school wide peer tutor, use of procedural questions do not encourage active participation or provide support for the tutee in arriving at the correct solution to the activity. The procedural questions Cindy uses serve as comprehension checks to an explanation she offers after an explanation regarding how to solve for the square root of 360. She begins the tutoring episode by offering Peter a visual representation of the problem statement they will complete together by pointing to the problem on the worksheet and explains the procedures they will follow to solve the problem (see Excerpt G).

**Excerpt G: Cindy (T3) and Peter (C)**

*Simplify completely \( \sqrt{360} \)

(1) T3: OK, for this (pointing to the problem statement) you need the square root of 360. So, (writing the equation) you have to **divide** that by 2 (indicating the 360) until it doesn't go anymore…so that would equal to 6 and a negative 10. **OK?**
(2) C: Yes
(3) T3: *Do ya get it?*...
(4) C: *(interrupts) by 2?*
(5) T3: Yeah, you just keep goin until it doesn’t go anymore…**OK?**
(6) C: OK…..alright! *(smiles)*

Cindy explains that they must find the square root in order to solve the problem (line 1) and continues explaining the method they will use to accomplish this goal "**divide that by 2** *(pointing to
the 360) until it doesn't go anymore...so that would equal to 6 and a negative 10” and follows this with a procedural question “O.K.? ” as a comprehension check to determine that Peter has understood how she arrived at the solution of 6 and negative 10.

Her explanation up to this point has been very brief and vague, however, Peter replies “yes” to Cindy’s comprehension check that he does understand the process. Possibly hesitant because of the lack of clarity and briefness of the explanation or her prior knowledge of Peter’s limited English background (Peter’s English language proficiency level is much lower than the other tutees), Cindy recasts the question again as a comprehension check “Do ya get it?” (line 3). This time, however, rather than give a yes/no response, Peter asks “by 2?” which Cindy acknowledges as Peter’s correct interpretation of the problem solving process. She responds to him “Yeah, you just keep goin until it doesn’t go anymore...OK?” (line 4) in which Peter responds “OK”. In this instance Cindy interprets Peter’s OK as affirmation that he understands the problem solving process and closes the activity.

The interactional dialogue of question and response that occurred during this episode revealed that Cindy’s use of procedural questions did not promote active participation in co-constructing the problem solving process to take place. These questions resulted in passive participation PP, as Peter remained a passive participant of the activity and there was no evidence of support in arriving at a correct response. Unfortunately the one line (4) where Cindy can expand on the process or offer Peter a chance to demonstrate comprehension of her prior explanation is not acted upon through Peter’s question to her comprehension check and instead, Cindy follows with a confirmation to Peter’s questions that you do divide by 2 following with another procedural question “OK?” in a prompt for Peter to either affirm or deny that comprehension of the activity had been achieved. Peter responds that he did agree, however, due to the absence of participation in this activity or level of support, it is doubtful that comprehension was achieved.
Guiding questions. Guiding questions provided a high level of instructional support and opportunities for the tutee to contribute to the problem solving activity. Consisting of closed, known/answer, or display questions, guiding questions were often used by the tutors during the task to effectively model a problem solving process, assess background knowledge, elicit a response, elaborate on a response, or focus tutee attention on the activity. Guiding questions used throughout the activity provided tutees an opportunity to actively participate (AP) in the problem solving process and work within their ZPD. The provided tutees with a high level of support (3) in that they encouraged remediation or the tutor used them to demonstrated proper techniques or steps necessary to collaboratively arrive at a correct solution to the mathematics activity. In the following excerpt (Appendix D and Excerpt H.1) between Jean and Jerry, Jerry provides Jean with guiding questions as a means to collaboratively engage in the problem solving process and arrive at a correct solution to the activity.

Excerpt H.1: Jean (T1) and Jerry A)

(1) T1: To do this problem (shows his partner the problem statement with his pen) you have to **multiply negative** 2 X (uses his pen to circle negative 2X in the problem statement) **times** all the numbers that are here (uses his pen to draw an arrow from negative 2 x to all the other numbers in the equation) So its **negative 2X times negative** 3 X **squared**, **negative 2x times** 4X, and **negative 2X times** 5. (Points to each number and draws an arrow to the others as he speaks)

(3 seconds pass)

and what’s **negative 2X times negative** 3X squared?

(2) A: 6 …..square

(3 seconds pass)

(3) T1: **Did you add** the X's? (points to the X's within the equation with his pen)

(4) A: Six …..cubed……..six cubed……..Is cubed 3?

(5) T1: Just say X to the **third power** (the tutor writes the answer next to the equation) So its 6X to the **third**.

**negative 2 x times 4 x which is?** (writes out the next equation (step) as he speaks)

(3 seconds pass)

which is?

(6 seconds pass)

(6) A: **negative 8**

(3 seconds pass)

squared...X squared.

(7) T1: (writes the answer next to the equation) yeah, and it’s a **negative** so it’s minus.....minus 8…So it’s **negative 8 squared** (writes the equation)
And then what’s negative 2X times 5?

(8) A: negative 10...X?

(9) T1: (writes the answer) and then what’s negative 8 times negative 3Xsquared?

(10) A: its 24

(3 seconds pass)

X squared

(11) T1: (writes his partners response next to the equation)

And what’s negative 8 times 4X?

(12) A: 32X, negative 32?

(13) T1: (writes partners response next to the equation)

What’s negative 8 times 5?

(14) A: negative 40

(15) T1: (writes the response next to the equation)

Then you got to add the like terms, like this one (points to a particular number), it doesn't have any...so you just rewrite it (rewrites the number)

This one has X squared and X squared (pointing his pen to 2 numbers)

(writes the equation)

What’s negative 8 minus 24?

(16) A: 16 squared (mumbling)

(17) T1: (writes the response of his partner)

16, but is it positive or negative?

(18) A: pos...negative?

(19) T1: its positive (writing the positive sign next to the answer)

what’s negative 8 and negative 24

(20) A: negative 8 plus negative 24?

(21) T1: Yeah, what’s negative 8 plus negative 24?

(22) A: Negative 32?

In line 1, Jerry opens with an explanation of the problem solving process and offers Jerry a preview into the steps they will recreate in order to solve the problem as he circles bits of the problem to further reinforce the importance of solving the problem in steps “you have to multiply negative 2 X (uses his pen to circle negative 2X in the problem statement) times all the numbers that are here (uses his pen to draw an arrow from negative 2 x to all the other numbers in the equation). So its negative 2X times negative 3 X squared, negative 2x times 4X, and negative 2X times 5.” Jean then directly begins to engage Jerry in the problem solving process by offering moment of silence (3 seconds) for Jerry to process the information then begins with a guiding question “and what’s negative 2X times negative 3X squared?” When Jerry answers using the wrong exponent for the variable x, Jean allows a moment of time for Jerry to reconsider his
answer, then asks him “Did you add the X's?” which prompts Jerry to reconsider his reply and come up with the correct answer in arithmetic terms (line 4) although he questions the use of this term, Jerry provides him an alternative, “to the third power” he knows as being correct. Jerry continues this line of questioning using guiding questions throughout the dialogue which actively engages Jerry in the problem solving process, provides remediation for an incorrect response, and supports Jerry’s arrival at a correct answer through effective modeling of the steps involved in solving the problem.

Although it seems as though Jerry comprehends the problem solving process in its entirety, care must be taken not to mistakenly associate success in solving individual chunks the ability to independently solve the problem. In order to accomplish that goal, reflection questions must follow, which do in the next segment of the problem solving activity presented in the next section in Except H.2 and Appendix D.

**Reflection questions.** Reflection questions provided a mid-range level of instructional support and a high level of opportunities for the tutee to contribute to and participate in the problem solving activity. Consisting of questions that are open, implicit, or vague, reflection questions were often used by the tutor at the close of the task to determine internalization or comprehension of the problem solving activity. Offered by the tutor as a means for the tutee to demonstrate the problem solving process or vocabulary, reflection questions provided the tutor a concrete assessment of what the tutee had learned. This true assessment enabled the tutor to relinquish ownership of the activity or take back ownership when the tutee failed to provide a correct response. Reflection questions did not offer the tutee as much support as the guiding questions, however, by this time there should have been no need for a high level of support, only guidance should the tutee make a mistake. Far removed from simple questions, reflection
questions offered tutees the chance to reflect on a previous answer or process, deepen their thinking, ask questions, and demonstrate independent problem solving under the guidance of a more capable other. Reflection questions not only allowed the tutee to work within his ZPD, but to independently practice the problem solving skills and vocabulary he had internalized from the tutoring episode. Reflection questions are best utilized when posed after guiding questions or an elaborate explanation has taken place.

For example, in the prior excerpt between Jean and Jerry, we saw Jean’s use of guiding questions as a means to actively engage Jerry in the problem solving process and support him towards an accurate solution to the problem. In the remainder of the excerpt (Excerpt H.2) Jean releases ownership of the mathematic activity by encouraging Jerry to demonstrate independent problem solving on a similar problem set he created. Jean begins with a direct statement that offers Jerry an opportunity to complete the problem set they had previously worked on solving together (Excerpt H.1) and demonstrate the remainder of the problem solving process (line 23). Jerry obliges, however, when Jean begins to a correct Jerry’s paper, he notices a mistake and calls Jerry’s attention to it by posing a reflection question “Yeah, O.K., 16 x squared and then what?” to prompt Jerry that something is wrong with this portion of his answer. In this instance, Jean provides Jerry support through prompting him that the process is incorrect and a reexamination necessary in order to continue (line 24). Although not as explicit as a guiding question, it still offers Jerry support in the process of arriving at a correct answer (line 24). In a move to provide additional support in directing Jerry to the correct problem solution, Jean rephrases the reflection question as a guiding question (line 24). Jerry replies to Jean’s question “There are 2 left, what’s negative 10 minus 40?” with a counter question “40?” (line 25) in which he recognizes and points out, using Jean’s previous style of remediation, that the 40 in Jean’s question is incorrect
This moment of correction on the part of the tutee is interesting in that it offers a unique example of how Jerry has appropriated some of Jean’s prior questioning strategies that cued Jerry that an answer was incorrect or needed to be expanded upon. Jean acknowledged Jerry’s remediation and replies with the correct response of negative 40.

Excerpt H .2 Jean (T1) and Jerry (B)

(23) T1: Ok, now finish adding this (points to the equations he has written and hands the pen and paper over to the tutee).
(15 seconds pass and the tutee writes his answers and slides the paper and pen over to the tutor)
(24) T1: Yeah, O.K., 16 x squared and then what?
   There are 2 left, what’s negative 10 minus 40?
   (5 seconds pass)
(25) A: 40?
(26) T1: I mean negative 40.
(27) A: negative 50
(28) T1: OK (shows the equation to the tutee)
   Now you just multiply all of this with that (pointing to the numbers)
   (3 seconds pass)
   OK...now can you show me how it’s done? (Writes similar equations and has the tutee complete it on his own)
   (twenty seconds pass as the tutee works on the problem)
(29) T1: Then add like terms (talking through the steps as he observes the tutee working)
   (40 seconds pass)
(30) A: This is... (points to a problem he is having difficulty with)
(31) T1: This is x to the 3rd that’s x squared (pointing at the number sets). They can't go together. If there’s only one you just rewrite it again.
(32) A: So 8 plus?
(33) T1: What’s that? (pointing to a number)
(34) A: 2
(35) T1: So its 8 minus 2
   (20 seconds pass)
   (the tutor takes the tutee's paper and checks his work)
   What’s negative 8 x and negative 1x? (gives the paper back)
(36) A: Negative 9x (writing the answer)
(37) T1: So we just minus 1 (taking the pen from his partner and correcting the sign made on the paper) you don't need to plus(waving his hand over the paper) and that’s it!

The exchange between reflection questions and direct statements or commands support Jerry through the independent problem solving continues throughout the remainder of the dialogue. They work in tandem to support and encourage Jerry to independently solve the problem. Jean also uses reflection questions to redirect Jerry or help him explain or think about why he used a
particular rule or process (line 33). Jean’s question “What’s that” (line 33) prompts Jerry that something is wrong and that he needs to internally reexamine the process he used in order to justify it to Jean. Jean’s guiding questions, commands, and reflection questions successfully bring Jean from peripheral, novice participant, to full, independent problem solver of the mathematics activity.

**Summary**

Table 4.1 is offers a summary of an excerpt from each of the three question types including the degree of participation, level of support, and the context in which the questions are used.

**Table 4.1 : Categorization and Characteristics of Questions used in Small Group Activity**

<table>
<thead>
<tr>
<th>Category</th>
<th>Participatory influence</th>
<th>Independent demonstration of the problem solving process</th>
<th>Level of support</th>
<th>Characteristic and Context</th>
<th>Example</th>
<th>Line</th>
</tr>
</thead>
<tbody>
<tr>
<td>Procedural</td>
<td>PP</td>
<td>N0</td>
<td>1</td>
<td>Yes/no questions</td>
<td>This is a protractor right?</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Yeah...So do you get it?</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>The bottom. OK?</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>But if it’s going over there like down there it’s like.. the top...OK?</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>21</td>
</tr>
</tbody>
</table>
The following examples are taken from Excerpt H.1 between Jean and Jerry

<table>
<thead>
<tr>
<th>Category</th>
<th>Participatory influence</th>
<th>Independent demonstration of the problem solving process</th>
<th>Level of support</th>
<th>Characteristic and Context</th>
<th>Example</th>
<th>Line</th>
</tr>
</thead>
<tbody>
<tr>
<td>Guiding</td>
<td>AP</td>
<td>N0</td>
<td>3</td>
<td>Explicit/Known answer questions</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Model problem solving process</td>
<td>And what’s negative 2X times negative 3X squared?</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Assess background knowledge</td>
<td>Did you add the X’s?</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Focus attention on problem solving process</td>
<td>Which is?</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Elicit a response</td>
<td>But is it positive or negative?</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Elaborate on a response</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The following examples are taken from Excerpt H.2 between Jean and Jerry

<table>
<thead>
<tr>
<th>Category</th>
<th>Participatory influence</th>
<th>Independent demonstration of the problem solving process</th>
<th>Level of support</th>
<th>Characteristic and Context</th>
<th>Example</th>
<th>Line</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflection</td>
<td>ID</td>
<td>YES</td>
<td>2</td>
<td>Implicit/Vague questions</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Prompts to reflect on previous answer, that additional information is needed</td>
<td>Yeah, O.K., 16 x squared and then what?</td>
<td>24</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Deepen thinking, to demonstrate remainder of the process.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Demonstrate problem solving skills and vocabulary</td>
<td>OK...now can you show me how it’s done?</td>
<td>28</td>
</tr>
</tbody>
</table>
Research Finding 2: Wait Time and Tutee’s use of Arithmetic Vocabulary

During the study, time between question and response sequences that totaled three seconds or more were considered wait time. Responses before and after wait time were analyzed to determine if this time sequence contributed to tutees’ increased use of arithmetic vocabulary. During the dyad interactions, it was determined that four tutors, Jean, Tina, Cindy, and Erin, utilized wait time as a means to elicit a response from their partner, however only 2 tutors, Tina and Jean used wait time as a means to elicit both a response and mathematics vocabulary. For example, in the exchange between Tina and Arnie, Tina used the terms *multiply* and *times* interchangeably throughout her explanation of the activity as well as her guiding questions. Tina then asked Arnie reflection questions to prompt a verbal explanation of the problem solving process (see Table 4.2). In one exchange, she asks Arnie the question “Why?” regarding why a number from the table they are working with would not fit the pattern needed to arrive at a different equation. This question prompts Arnie to respond with “Ummm ..there’s no 5” (line 36) which is somewhat correct, however, Tina needs additional information. She does not respond to Arnie’s answer, but instead allots him time to think and reflect on his initial response in which 4 seconds later, Arnie responds with “there’s no 5…you need 5 to multiply” (line 36). Only when Arnie had formed a complete response and used mathematical language (multiply) did Tina acknowledge his answer with a response of “good” (line 37).
Table 4.2: Wait Time

<table>
<thead>
<tr>
<th>Line</th>
<th>Question</th>
<th>Initial Tutee Response</th>
<th>Wait Time</th>
<th>Response</th>
<th>Feedback</th>
</tr>
</thead>
<tbody>
<tr>
<td>31-32</td>
<td>What about this one? Why wouldn’t this work?</td>
<td>It works for the first one</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>33-35</td>
<td>Yeah, but what about the rest?</td>
<td>No.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>35-37</td>
<td>Why?</td>
<td>Ummm ..There’s no 5.</td>
<td>4</td>
<td>There’s no 5….you need 5 to <strong>multiply</strong>.</td>
<td></td>
</tr>
</tbody>
</table>

In another exchange with Jerry, Tina is showing Jerry how to determine which graph represents the solution set of \(-1 < X \leq 2\). She explains the process and meaning of the closed and open circles used in the graph (see Table 4.3 for more information) then poses a reflection question to enable Jerry to demonstrate his comprehension of her explanation:

Excerpt I (T4) Tina and Jerry (A)

(3) Tina (T4): Now what if we did this **problem** (pointing to a new problem set) **what would you do?**
(4 ) Jerry (A): **Circle it?**
(5) T4: **Circle what?**
(6) A: Start circling it.
(7 ) T4: No, just a circle because it’s **less than** X
**And then what?...would you do a straight line to 3 and a darker circle? Or what?**
(7 seconds)
(8) J: **A dark line?**
(4 seconds) **and a dark circle..to 3?**
(9) T4: **Yep, and a dark circle to 3..why?**
(10) J: Because its **less than**

Tina used a guiding question backed up with a reflection question to elicit a response (line 9).

When Jerry answers Tina’s question “**Yep, and a dark circle to 3..why?**” in this exchange with “**because it’s...**” (line 10) Tina does not offer an answer but instead waits 3 seconds for Jerry to reflect on his initial response and reply which he finally does using “**less than**” which is the
arithmetic term Tina was waiting for (Table 4.3 lines 9-11). Although Jerry did reply with an incorrect answer, his use of “less than” revealed his understanding of the arithmetic vocabulary necessary to solve the problem.

**Table 4.3 Wait Time**

<table>
<thead>
<tr>
<th>Line</th>
<th>Question</th>
<th>Initial Tutee Response</th>
<th>Wait Time</th>
<th>Response</th>
<th>Feedback</th>
</tr>
</thead>
<tbody>
<tr>
<td>7-8</td>
<td>No, just a circle because it’s less than X and then what?...would you do a straight line to 3 and a darker circle? Or what?</td>
<td>A dark line?</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>And a dark circle..to 3?</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9-11</td>
<td>Yep, and a dark circle to 3..why?</td>
<td>Because it’s</td>
<td>3</td>
<td>less than</td>
<td>Cuz it’s greater than</td>
</tr>
<tr>
<td>12-13</td>
<td>Oh greater than.</td>
<td></td>
<td></td>
<td></td>
<td>Yeah....Now you could try that one (indicating a new equation)</td>
</tr>
</tbody>
</table>

Although in the previous examples Tina did not use feedback as a means to elicit a response, in her exchange with Peter, Tina posed reflection questions to initiate the problem solving sequence, however, when she wanted Peter to elaborate rather than initiate another question or wait for a reply, Tina responded with “yep” as a means to keep the activity moving and a cue for Peter that more information was needed (Excerpt J and Table 4.4). This use of a simple verbal cue in the form of “yep” along with wait time prompted Peter to expand on his answer and increase his use of mathematics vocabulary.

**Excerpt J** Tina (T4) and Peter (C)

*Which graph represents the solution set of -1< X ≤ 2?*

13 T4: **Minus** 1 is less than, is less than X, but **positive** 2 is greater or equal to X (pointing to the equation)
So what you do is, mark **negative** one, , because its less than X you just do, just do (making a circle on the graph) a circle.
You don't fully cover it.
14 C: OK
15 T4: And, but, because 2 is greater than or equal to X…you go up (marking the graph) the number line
16 C: OK
17 T4: And you darken all of that in (demonstrating). so you go from negative 1 to positive 2 (retracing the arrow)
18 C: Oh, OK!
19 T4: So you know what I mean?
20 C: Yes
21 T4: OK
22 C: So and minus 1 here is this (pointing to the open circle) is quick over to this part (drawing darkened line to positive 2 and coloring in the circle there)
23 T4: Yep
24 C: Because this one (pointing to less than equal to)…the symbol is..is.. (3 seconds pass)
Greater…so we go up because it is 2 positive (indicating the direction of the line from negative 1)
26 T4: Yep
27 C: OK

In line 23, Tina’s reply of yep prompts Peter to continue with his explanation (line 24) “Because this one (pointing to less than equal to)…the symbol is..is. “and even though Peter is stumbling, Tina recognizes that Peter is probably searching for the right word and allows him enough wait time for him to complete his thought “Greater…so we go up because it is 2 positive.” Tina’s use of wait time, possibly because she is also an ELL and can relate to Peter’s dilemma, enabled Peter to find the right word in English and complete his thought. Often times, teachers with no prior experience in ESL or who are not familiar with having to use a second language do not realize the extra time needed to translate or find the correct word within one’s interlanguage to complete a thought.

Table 4.4 Wait Time

<table>
<thead>
<tr>
<th>Line</th>
<th>Question</th>
<th>Initial Tutee Response</th>
<th>Wait Time</th>
<th>Response</th>
<th>Feedback</th>
</tr>
</thead>
<tbody>
<tr>
<td>19-20</td>
<td>So you know what I mean?</td>
<td>Yes</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>21-23</td>
<td>O.K., what?</td>
<td>So and minus 1 here is this …is quick over to this part</td>
<td></td>
<td></td>
<td>Yep</td>
</tr>
</tbody>
</table>
Because this one ..the symbol...is...is...

3 Greater...so we go up because it is 2 positive

During an exchange between Jean and his dyad partner Jerry, wait time without any type of additional questioning, prompting, or feedback was used. In the exchanges between Jean and Jerry (Table 4.5), Jean’s use of wait time prompted Jerry to expand his response using arithmetic vocabulary (Excerpt K).

Excerpt K Jean (T1) and Jerry (A)

5 T1: Just say X to the third power (the tutor writes the answer next to the equation) So its 6X to the third.

negative 2 x times 4 x which is? (writes out the next equation (step) as he speaks)
(3 seconds pass)
which is?
(6 seconds pass)
6 A: negative 8
(3 seconds pass)
squared...X squared.

Table 4.5 Wait Time

<table>
<thead>
<tr>
<th>Line</th>
<th>Question</th>
<th>Initial Tutee Response</th>
<th>Wait Time</th>
<th>Response</th>
<th>Feedback</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-4</td>
<td>Did you add the X’s?</td>
<td>Six …cubed six cubed Is cubed 3?</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5-6</td>
<td>negative 2 x times 4 x which is?</td>
<td>Negative 8</td>
<td>3</td>
<td>squared...X squared</td>
<td></td>
</tr>
<tr>
<td>9-10</td>
<td>and then what’s negative 8 times negative 3X squared?</td>
<td>It’s 24</td>
<td>3</td>
<td>X squared</td>
<td></td>
</tr>
<tr>
<td>11-12</td>
<td>And what’s negative 8 times 4X?</td>
<td>32X, negative 32?</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Although Jean used this method to elicit tutee response most frequently, he also recast the tutee’s response in the form of a question followed by wait time to prompt the tutee to expand upon his initial response (Table 4.6 and Excerpt L). This use of wait time is evident in the exchange
between Jean and Arnie (lines 9-12 and 23-26) when Arnie provides an incorrect response to Jean’s question. In the first section (Excerpt L.1), Jean asks Arnie “what’s negative 2x times” and Arnie interrupts with the remainder of the equation “4x?”. “That would be 8x.” This answer is not quite correct as Arnie did not arrive at the correct exponent for x. Jean allows six seconds wait time in which Arnie realizes something must be wrong, reevaluates his answer and then produces the correct response “squared” in arithmetic terms. Six seconds, although not a lengthy amount of time enables Arnie to correctly solve this step of the problem using an appropriate arithmetic term. Further into the During the same problem activity, Jean asks Arnie to solve for “negative 8 times 5” (Excerpt L.2) and pauses in silence for 10 seconds allowing sufficient wait time for Arnie to reflect and possibly change his answer. Arnie’s second response of “40” results in another 3 second pause followed by Jean’s recast of Arnie’s answer in the form of a question as a cue that 40 was not enough and an expansion on his response was necessary. Arnie picked up on Jean’s cue and responded with the correct answer of “negative 40.”

Excerpt L.1 Jean (T1) and Arnie (B)

9 T1: what’s negative 2x times (B interrupts)
10 B: (interrupting) 4x? That would be 8x
6 seconds pass
11 T1: 8 X?
4 seconds pass
12 B: squared.

Excerpt L.2
23 T1: what’s negative 8 times 5?
24 B: 40
10 seconds lapse
25 B: 40
3 seconds lapse
26 T1: 40?
27 B: negative 40.

Excerpt L.2 provides an example how wait time in conjunction with recast can prompt the tutee that his original answer is not correct and a reevaluation is in order to arrive at the correct
response. Tina and Jean used wait time to ensure tutees expanded their answers to include more appropriate vocabulary which is a necessity for large scale assessments and to become an active, meaningful participant of the mathematics classroom. A slight pause that lasts 3 seconds or more is all that is needed to expand, use, and acquire mathematics vocabulary.

**Table 4.6 Wait Time**

<table>
<thead>
<tr>
<th>Jean and Arnie</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Line</strong></td>
</tr>
<tr>
<td>3-4</td>
</tr>
<tr>
<td>5-6</td>
</tr>
<tr>
<td>7-8</td>
</tr>
</tbody>
</table>

**Case Studies**

This section presents findings from case studies of Jean, the most effective tutor, and Cindy the least effective tutor. The purpose is to show differences in interactions between them in developing tutee’s ownership of mathematical problem solving activity. My explanation focuses on how the effective tutors utilized varying types of questions to encourage active participation, independent problem solving, support to reach a correct answer, and incorporated wait time to elicit arithmetic vocabulary and compare them with the less effective tutors in a move to explain these differences that can ultimately impact instructional strategies used with ELLs in the mathematics classroom.

Each case study contains a tutor and a tutee. As I stated in the methods section, the tutor, considered an advanced student of English and math, was instructed to teach the tutee, a novice student of math and English, a set of two to three mathematical problems from the PSSA test item sampler. I gave the dyad tutors one instruction; help the tutees solve the problem sets so that
they (the tutees) could accomplish it independently should it appear on a test. The tutees also received only one directive; solve the problem sets with help from the tutors so it could be successfully executed if encountered on a large-scale assessment. What occurred through the interactions were patterns in which various types of questions emerged followed by different uses of wait time which either enabled or disabled the pair from reaching their collective goal. For some tutors, the actions driven by their choice and placement of questions jelled into the collective fulfillment of the problem solving goal. For others, there were contradictions within the actions that could not be negotiated resulting in the breakdown of activity or abandoned efforts to accomplish the collective problem solving goal. In the individual cases, I present the interaction that ensued between each tutor and tutee dyad during one problem solving activity. I summarize this with a chart indicating the differences across dyadic interactions.

Dyad One: Jean and Jerry. In the beginning of the activity, Jean establishes his position as tutor as he sits next to Jerry at the work table and places the worksheet between them so it is mutually visible. Both students are aware that the collective goal is to transform Jerry from a novice to independent problem solver of the three mathematic activities they are to solve. As Jerry begins the tutoring session, he establishes the objective of the activity through his goals, which are to understand and accurately solve the mathematical problem sets. As a high school senior, Jerry needs to fulfill graduation requirements which include achieving a score of basic on his PSSA math exam and passing his algebra II class. Jerry’s goals are to comprehend the problem solving process and arithmetic vocabulary so he could successfully execute them independently and fulfill his graduation requirements. His goals can only be achieved if he follows the tutor’s directions, models his procedures, answers questions, and pays attention to explanations. With
stakes this high, Jerry is prepared to listen and do whatever the tutor asks so he could learn to solve the problem sets and graduate high school.

The object of Jean’s activity is to help Jerry arrive at the correct solutions to the mathematic problem sets independently. To accomplish this goal, Jean needs to actively engage Jerry in co-constructing the problem solving process, support his arrival at an accurate solution, and encourage independent problem solving and use of arithmetic vocabulary. Jean begins with an oral introduction of the problem set in which he breaks down the steps of the problem solving process they will need to compute in order to successfully solve the problem. He populates his explanation with arithmetic vocabulary and provides wait time after his explanation and opening question allowing Jerry a chance to process what they will be doing. Jean immediately follows this wait time with a guiding question that encourages Jerry to actively participate in the problem solving process.

Excerpt M.1 Jean (T1) and Jerry (A)

Problem set: Multiply: 

\((-3x^2 +4x +5)(-2x -8)\)

1 Jean (T1): To do this problem (shows his partner the problem statement with his pen) you have to multiply negative 2 X (uses his pen to circle negative 2X in the problem statement) times all the numbers that are here (uses his pen to draw an arrow from negative 2 x to all the other numbers in the equation) So its negative 2X times negative 3 X squared, negative 2x times 4X, and negative 2X times 5. (Points to each number and draws an arrow to the others as he speaks)

(3 seconds pass)
and what’s negative 2X times negative 3X squared?

2 Jerry (A): 6 ………square

(3 seconds pass)

3 T1: Did you add the X's? (points to the X's within the equation with his pen)

4 A: Six …cubed
six cubed
is cubed 3?

5 T1: Just say X to the third power (the tutor writes the answer next to the equation)
So its 6X to the third.

negative 2 x times 4 x which is? (writes out the next equation (step) as he speaks)

(3 seconds pass)
which is?

(6 seconds pass)

6 A: negative 8

(3 seconds pass)
squared...X squared.

7 T1: (writes the answer next to the equation) yeah, and it’s a negative so it’s minus.....minus 8…So it’s negative 8 squared (writes the equation) And then what’s negative 2X times 5?

Providing guiding questions this early in the activity enables Jerry to establish himself as active participant in the problem solving process and focus his attention on collaboratively solving the mathematics activity. In line 2, Jerry produces a wrong answer to Jean’s initial question “and what’s negative 2X times negative 3X squared?” as he fails to respond with the correct exponent. Jean calls this to Jerry’s attention by asking him if he added the x’s which prompts Jerry to reevaluate his answer and arrive at the correct one. Jerry even goes as far as to use mathematic vocabulary and questions Jean if the use of cubed in this instance is correct. Jean, possibly unsure of this term, produces another for Jerry to use (line 5) “Just say X to the third power” And reinforces that this is the term they will use for the remainder of the activity “So its 6X to the third.”

Jean continues with another guiding question (line 5) “negative 2 x times 4 x which is?” and as he writes this equation on the shared paper and waits 3 seconds for Jean to respond. When the wait time fails to elicit a response, Jean prompts Jerry again that he is requesting an answer to the question by posing “which is?” followed by another episode of wait time in which Jean responds with “negative 8.” This answer is again wrong due to the variable exponent, however, rather than prompting Jerry with a question, Jean utilizes wait time to see if Jerry recognizes his mistake, which he does and adds the correct exponent value, “squared” to his answer. Jean continues using guiding questions to actively engage Jerry in the problem solving process and uses wait time again in line 10 to prompt Jerry into a correct response:

9 Jean (T1): and then what’s negative 8 times negative 3X squared?
10 Jerry (A): Its 24 (3 seconds pass)
Jean’s use of wait time in conjunction with his guiding questions have encouraged Jerry to elaborate his responses resulting in the correct answer to the problem and use of arithmetic vocabulary (lines 6 and 10). Jean continues the activity with another explanation of the next step needed to arrive at the correct solution and continues using guiding questions to actively engage Jerry in the process. Jean relinquishes control of the problem activity when he asks Jerry to complete the problem (line 23). Jerry obliges by demonstrating the problem solving process under the guidance of Jean as he (Jean) notices that Jerry has not fully completed the task. In a move to support Jerry in arriving at a correct solution, Jean prompts him to think about what he has completed up until now and that there are two numbers left he needs to compute (line 24). As the discourse progressed, Jerry’s confidence was obvious as he points out a mistake Jean made.

As Jean asks Jerry (line 24) “what’s negative 10 minus 40?” Jerry replies “40?” (line 25) which is a similar method Jean used prior to this moment in the exchange to alert Jerry to a possible mistake. This is important in that it reveals Jerry is appropriating Jean’s questioning strategies and understands their function as prompts to a mistake. Jean realizes his mistake and recasts his question as “I mean negative 40” (line 26).

15 Jean (T1): (writes the response next to the equation) Then you got to add the like terms, like this one (points to a particular number), it doesn't have any...so you just rewrite it (rewrites the number) This one has X squared and X squared (pointing his pen to 2 numbers) (writes the equation) What’s negative 8 minus 24?
16 Jerry (A): 16 squared (mumbling)
17 T1: (writes the response of his partner) 16, but is it positive or negative?
18 A: pos...negative?
19 T1: its positive (writing the positive sign next to the answer)

what’s negative 8 and negative 24

20 A: negative 8 plus negative 24?

21 T1: Yeah, what’s negative 8 plus negative 24?

22 A: Negative 32?

23 T1: Ok, now finish adding this (points to the equations he has written and hands the pen and paper over to the tutee).

(15 seconds pass and the tutee writes his answers and slides the paper and pen over to the tutor)

24 T1: Yeah, O.K., 16 x squared and then what?

There are 2 left, what’s negative 10 minus 40?

(5 seconds pass)

25 A: 40?

26 T1: I mean negative 40.

27 A: negative 50

The dialogue continues (28 -37) with Jean and Jerry solving the problem together through a mixture of guiding and reflection questions. Jean never uses procedural questions as he engages and supports Jerry from the opening question until the end. Jean offers Jerry a sample problem he made similar to the one they completed (line 39) and under the guidance of Jean, he demonstrates independent problem solving and arrives at the correct solution to the second activity (see Appendix D for transcription).

28 T1: OK (shows the equation to the tutee)

Now you just multiply all of this with that (pointing to the numbers)

(3 seconds pass)

OK...now you can show me how it’s done. (Writes similar equations and has the tutee complete it on his own)

(20 seconds pass as the tutee works on the problem)

29 T1: Then add like terms (talking through the steps as he observes the tutee working)

(40 seconds pass)

33 T1: What’s that? (pointing to a number)

34 A: 2

35 T1: So its 8 minus 2

(20 seconds pass)

(the tutor takes the tutee's paper and checks his work)

What’s negative 8 x and negative 1x? (gives the paper back)

36 A: Negative 9x (writing the answer)

37 T1: So we just minus 1 (taking the pen from his partner and correcting the sign made on the paper) you don't need to plus (waving his hand over the paper) and that’s it!

As noted in Table 5.1, Jean’s use of questions and incorporating wait time contributed Jerry to
actively participate and take ownership in the mathematic problem solving activity as he demonstrated acquisition of the vocabulary and problem solving processes. Jean and Jerry both reached the collective goal of transforming Jerry from a novice to an independent problem solver of the mathematics activities.

**Table 5.1**

<table>
<thead>
<tr>
<th>Line</th>
<th>Type of Question</th>
<th>Characteristics of tutee discourse</th>
<th>Characteristics of tutee response</th>
<th>Level of tutee participation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-14</td>
<td>Guiding</td>
<td>(Opens activity) Provides an explanation of the problem solving process. Poses guiding questions punctuated by episodes of wait time Writes tutee response next to the equations he has written to solve the problem.</td>
<td>Use of arithmetic vocabulary, pays attention, actively participates in problem solving process, asks a questions regarding an exponent in line 4</td>
<td>Active participation</td>
</tr>
<tr>
<td>15-22</td>
<td>Guiding/Reflective</td>
<td>Provides an explanation of the next step involved in the problem solving Poses guiding questions punctuated by episodes of wait time Writes tutee response next to the equations he has written to solve the problem.</td>
<td>Use of arithmetic vocabulary, pays attention, actively participates in problem solving process, asks a questions regarding an exponent in line 4 and 10</td>
<td>Active participation</td>
</tr>
<tr>
<td>23-52</td>
<td>Guiding/Reflective</td>
<td>(Closes the Activity) Directs tutee as to how to finish the problem providing only minimal guidance</td>
<td>Demonstrates understanding of problem solving process, poses questions, asks for help, asks for additional clarification of the problem solving process</td>
<td>Independent problem solving</td>
</tr>
</tbody>
</table>

Dyad 2: Jean and Arnie. Similar to Dyad 1, both Jean and Arnie established their positions within the problem solving activity through Jean’s oral introduction of the problem set and opening question. As Arnie begins the tutoring session, he establishes the objective of the activity through his motives which are to understand the mathematical problem sets so that he could successfully solve them independently on classroom tests. Although he will take the PSSA
exams the following year, Arnie’s initial motives do not involve large scale assessment scores, but a more immediate objective which is to pass his Algebra I class so that he can remain in the vocational-technical program (tech) to which he was recently been accepted. Although Arnie must establish passing scores in all his content classes, he struggles most in math. Similar to Jerry, the stakes are also high for Arnie as he would like to remain in the tech program. Arnie’s motives are to comprehend the problem solving process and arithmetic vocabulary so he could successfully pass his classroom exams and remain in the tech program prompt him to follow the tutor’s directions, model his procedures, answer questions, and pay attention to explanations. The objective of Jean’s activity is to help Arnie independently arrive at the correct solutions to the problem sets in the mathematic activity. To accomplish this goal, Jean needs to make the problem solving processes and vocabulary of the problem sets comprehensible so that Arnie could solve them independently when he encounters them on a test.

Similar to the previous interaction between Jean and Jerry, the activity opens with Jean explaining the problem solving process in a step by step manner as he explains to Arnie how to choose which coordinate plane shows the graph of $y \geq 2x + 1$. Arnie, sits next to Jean and watches as he demonstrates how to graph this equation on a sample he has made. Arnie initially interrupts Jean as he begins the tutoring session, however, Jean remains undeterred and continues with his explanation of the problem solving process using and demonstrating the letter variables they will use in place of the numbers.

Excerpt M.2: (T1) Jean and (B) Arnie

Problem Set: Which coordinate plane shows the graph of $y \geq 2x + 1$?

1 T1: To do this, you ... this is 2 (talking to himself as he looks at the problem set)
2 B: (interrupts) plus 1.
3 T1: No, This is $Y$ equals $M$ X plus B. (writes $MX + B$ under the equation)
   Its B you start out with... (making a graph)..one, put a dot, then up 2 over 1..this way (moving
his pen on the graph to show his partner) up 2 and over 1.. you have to go up 2 and over 1 (makes a dot on the graph) and draw a line (draws a straight line between the dots. Now look for the one that has this line. (pointing to the various choices of graphs on the worksheet)

4 B:  What?
5 T1:  Which one has this?  (pointing to the sample graph )
6 B:  Which one has that?  (pointing to the sample graph)
8 B:  This one..B (pointing to the correct choice of graphs)
9 T1:  How do you know that?
10 B:  Huh?
11 T1:  Why do you think it's the right one?
12 B:  Cuz it’s up 2 over 1 from B and this one, (pointing to graph A) is up one over 2 and those ones (pointing to C and D), the lines are too close to the middle.

Jean explains the problem solving process in a step by step manner as he models the steps for Arnie. He then instructs Arnie to choose a graph from a variety of choices he has on a worksheet. Arnie complies and chooses the correct one. Jean follows this with a reflection question (line 9-11) “How do you know that?” to prompt Arnie to explain his choice of graph and when Arnie’s response “Huh?” indicates a communication breakdown had occurred, Jean rephrases his question to make it more comprehensible “Why do you think it’s the right one?”

Arnie complies and correctly explains why he chose that particular graph (line 12). Jean continues the activity by producing a sample equation similar to the one they had just completed which is partially solved and asks Arnie to complete it and choose a graph that best represents it from a choice of graphs on the worksheet (lines 13-15). Arnie complies by correctly modeling the step needed to complete the equation and chooses the correct graph that best represents it. Jean then produces an equation and has Arnie complete it in its entirety (lines 16-22).

13 T1: You know how to do this one?  (indicating a sample problem he has partially completed)
14 B:  Umm, you put this over 1 (writing the equation on the paper), then you go to the graph and start from here (pointing to the graph) and go one, two, three (mumbling to himself) (5 seconds pass)
And here.
15 T1:  Good. (smiles) Done. Let’s go to the next one.
(22 seconds go by as the tutor prepares a sample graph and equation)

16 T1: Start off with that..*do you think you can do it?* (indicates a sample problem he wrote)
17 B:  (Mumbling the steps) it would be up 2 over 1 (plotting a bank graph)
18 T1:  Yeah, cuz ya start from 1 you go up one (showing his partner on the graph) then up 2, don't want to go up 2
19 B:  Yeah it would be..*that would be then, one?* (marking a point on the graph)
20 T1: No, because you start from 1 (showing his partner on the graph)
21 B:  Oh, yeah, yeah, yeah.. so it would be up 2, should be 3 in it..over 1, would be positive 1,
22 T1:  (nods)

Arnie mumbles the steps he needs to follow in order to begin the problem solving process. This is an indication that he has internalized the steps Jean had previously modelled and he (Arnie) had followed to complete the prior problem. As Arnie begins and mumbles the steps, Jean acknowledges Arnie’s utterance and reinforces Arnie’s decision by providing him a brief explanation why his choice thus far is correct. Jean not only has actively engaged Arnie in the problem solving process and provided him an opportunity to demonstrate independent problem solving; he also supports Arnie in choosing the correct answer as he works. Arnie acknowledges this and asks Jean a question to ensure he is marking the graph correctly (lines 19-20) “*that would be then, one?*” (indicating a point he is about to make on the graph) and Jean replies by guiding Arnie to arrive at the correct solution “*No, because you start from 1*” (showing his partner on the graph).

**Table 5.2**

<table>
<thead>
<tr>
<th>Line</th>
<th>Type of Question</th>
<th>Characteristics of tutor interaction</th>
<th>Characteristics of tutee response</th>
<th>Level of tutee participation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-8</td>
<td>Guiding</td>
<td>(Opens Activity)</td>
<td>Use of arithmetic vocabulary,</td>
<td>AP</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Provides an explanation of the</td>
<td>pays attention, actively</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>problem solving process.</td>
<td>participates in problem solving</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Poses guiding question for Arnie to</td>
<td>process</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>choose correct graph</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9-24</td>
<td>Reflection</td>
<td>Offers tutee a chance to demonstrate</td>
<td>Arnie displays use of inner</td>
<td>ID</td>
</tr>
<tr>
<td></td>
<td></td>
<td>problem solving process by</td>
<td>speech in his mumbling as he</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>independently attempts to solve</td>
<td></td>
</tr>
</tbody>
</table>
The close of the mathematic activity has resulted in the successful completion of both participants’ goals, Arnie’s ownership of the mathematic activity. Jean’s use of questions serve to both support and encourage Arnie to actively participate and demonstrate independent problem solving of the mathematic activity.

Episode Three: Jean and Peter. Similar to the previous episodes, both Jean and Peter established their positions within the problem solving activity through Jean’s oral introduction of the problem set and opening question. Peter’s motives for successfully completing the mathematic activities are high as similar to Jerry, he must also fulfill graduation requirements and as a senior, the stakes are high. His objective is to pay attention, follow Jean’s instructions, and ultimately comprehend the problem solving process of each activity so he can successfully complete it on his own.

The objective of Jean’s activity is to help Peter solve the various problem sets given to him so he can repeat it independently on a test or in class. During the interaction between Jean and Peter, Jean once again used questions and wait time in conjunction with his explanations as a means to engage Peter in the problem solving activity. Although all three tutees’ English language proficiency levels were lower than the tutor’s, Peter’s was significantly lower than Arnie and Jerry’s.

Jean begins the tutoring episode by telling Peter to “*listen up.*” Although he does this in a joking manner as both boys laugh, this is a word problem and Peter’s level of English language
proficiency can significantly hinder his ability to complete this problem. Jean begins by repeating a portion of the problem question (line 1) and follows this with a guiding question to test Peter’s knowledge of the meaning of increase. He gives Peter choices “So, when it increases, do you go up, down, sideways...?” to support him in arriving at the correct answer and also to ensure Peter understands his question. Peter complies to Jean’s expectations and Jean offers Peter an opportunity to choose the graph that indicates an increase.

Excerpt M.3 (T1) Jean and (C) Peter

Which scatter plot best supports the conclusion that as the number of years of education increases, yearly earnings increase?

1 T1: OK Pete, listen up. (both boys laugh) Which scatter plot best supports the conclusion that the number of years of education increases? So, when it increases, do you go up, down, sideways...?

2 C: O.K.... aaah up.

3 T1: Yeah, so which one of these shows that the scatter plots going up. (points to the selection of various scatter plots)

4 C: This one..right here ....A (pointing to scatter plot example A)

Peter makes the correct selection from Jean’s answer choices and continues the interaction with a reflection question that functions as a test of Peter’s ability to produce the arithmetic vocabulary needed to understand the problem as it would be worded on a test. Jean asks Peter (line 5) “and when its goin this way,(points to scatter plot B) what’s it doin?” and offers Peter sufficient wait time, 8 seconds and then an additional 3 after he begins an utterance, to provide a response. Peter chooses a term “left” that would not be used on an exam or in class to identify the direction of a scatter plot, therefore, Jean provides Peter with the correct term increasing and the reason for the term (lines 5-7). He then provides Peter another question in an attempt to prompt Peter to use more mathematically appropriate terms. Peter complies and responds to Jean’s question “This
one’s?” as he points to a point that indicates a decrease, with the correct term of decreasing (line 8).

Jean  5  T1: (Circles A) and when its goin this way.(points to scatter plot B) what’s it doin? (8 seconds pass)
Peter  6  (C) Ummm its going..ah..going…
(3 seconds pass)
To..to the left?
7 T1:  (Uses his pen to follow the line of the previous scatter plot example A)
   This one’s increasing because it’s going up.
   This one’s?  (points to the scatter plot (B) and follows the line with his pen)
(3 seconds pass)
C:  decreasing?
8 T1:  Yeah, because its going down from here .see? (traces the line with his pen)
9 C:  O.K.
10 T1:  And this one?  (points to scatter plot example C)
11 C:  Its going to the right. Its uh...equal.
12 T1:  It’s goin no where. It’s staying the same, equal. And this one? (pointing to scatter plot example (D))
13 C:  Its going aaaaall the ways (laughs)
14 T1:  So, do you understand it?
15 C:  Yes.
16 T1:  Now show me, which one shows that as the number of years of education increases, yearly earnings increase?
18 C:  aahhh….this one! (points to the correct scatter plot).
19 T1:  Why?
20 C:  It’s increasing

Jean follows Peter’s response of decreasing with a justification of why the point on the scatter plot indicates it is decreasing and uses the basic vocabulary term down to draw a connection between decrease and down as he did previously with increase and up (lines 1-3).

This seems to be a pattern that Jan continues through the remainder of the interaction. When Peter replies in line 12 with the term equal, Jean reinforces this mathematical term with basic vocabulary “its goin nowhere.” Jean asks Peter a procedural question in line 15 to determine if Peter comprehends the direction of the scatter plots thus far and then provides a reflection question that enables Peter to demonstrate his ability to solve the problem independently and use arithmetic vocabulary. Peter complies by choosing the correct answer and the activity ends. The close of the mathematic activity has
resulted in the successful completion of both participants’ goals, Peter’s ownership of the
mathematic activity. Jean’s use of questions and wait time served to both support and encourage
Peter to actively participate and demonstrate independent problem solving of the mathematic
activity and acquisition of the vocabulary.

Table 5.3

<table>
<thead>
<tr>
<th>Line</th>
<th>Type of Question</th>
<th>Characteristics of tutor discourse</th>
<th>Characteristics of tutee response</th>
<th>Level of tutee participation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-14</td>
<td>Guiding Reflection</td>
<td>(Opens the activity) Provides an explanation of the problem question Uses guiding and reflection questions to assess Peter’s knowledge of mathematic vocabulary and comprehension of directionality of the scatter plots</td>
<td>Initially choses correct term from those offered by Jean to indicate direction on the scatter plot, however, when Jerry removes the answer choices uses a term that is not as appropriate Demonstrates eventual Use of arithmetic vocabulary, pays attention, actively participates in problem solving process</td>
<td>AP</td>
</tr>
<tr>
<td>15-18</td>
<td>Reflection</td>
<td>Closes activity with a question using arithmetic vocabulary that requires Peter to choose the correct scatter plot</td>
<td>Comprehends vocabulary and chooses the correct response. Uses mathematic vocabulary</td>
<td>ID</td>
</tr>
</tbody>
</table>

Summary of Jean’s performance

Although the motive for the activity was different for all participants, the overall goal was the
same: tutee ownership of the mathematic activity which was accomplished in all of the problem
sets Jean completed with the tutees. It is inferred that Jean’s use of questions and wait time
enabled the tutees to comprehend and use arithmetic vocabulary as well as accurately and
independently demonstrate the problem solving activity.

Dyad 4: Cindy and Jerry. Both Cindy and Jerry established their positions within the problem
solving activity as they sat next to one another at the table. Similar to the dyad episodes with
Jean, Cindy also arrived with a worksheet with the problems she and her partner were going to
solve, but unlike Jean, she did not make them mutually visible. Jerry’s motives have not
changed which are to understand the mathematical problem sets so that he could successfully
solve them independently on classroom tests.

The objective of Cindy’s activity is to help Jerry independently arrive at the correct solutions to
the problem sets in the mathematic activity. To accomplish this goal, Cindy must attempt to
make the problem solving processes and vocabulary of the problem sets comprehensible so that
Jerry could solve them independently when he encounters them on a test.

Unlike the interaction between Jerry and the tutees, Cindy does not open with an explanation of
the problem set or show the tutee the equation which is, but begins directly with announcing
“This is the factor” and posing a guiding question “What times what equals 32?”

N.1 Cindy (T3) and Jerry (A)

Factor: 15x^2 + 74x - 32
1 Cindy T3: This is the factor, What times what equals 32? (6 seconds pass)
2 Jerry A: 6?
(2 seconds pass)
No, I don’t know...(laughs)

Although Cindy allows Jerry sufficient wait time, Jerry replies with the wrong answer. Cindy
tries to find out of Jerry does not the answer, however, Jerry’s next answer suggests he may
as he replies with “4 times 8.” Cindy acknowledges his attempt and prompts him to go higher.

She allows a more than sufficient amount of time to pass (14 seconds) before she provides Jerry
the correct answer “16 times 2” (lines 3-6)

3 T3: You don’t know? (smiles)
4 TS1: No, na, is it um,
(4 seconds pass)
4 times 8?
5 T3: Yeah, but you could go higher.
(14 seconds pass)
16 times 2
The interaction between the dyad participants thus far has resulted in minimal participation on the part of the tutee and little support from the tutor. This continues through the remainder of the dyad activity with the participants focusing on the problem 16 times 2. Cindy asks Jerry again to solve the equation 16 times 2 and when there is no response after 3 seconds, Cindy begins to break down the problem into steps by asking Jerry to solve 2 times 6 and then 1 times 2, and finally add the 1 (lines 14-19). As Jerry arrives at the correct answer if 32, Cindy asks him “where the plus goes to the 16 or the 2 to get 74?” Jerry provides the correct response; however, it is doubtful that he knew what he was solving for.

Although Cindy did eventually engage Jerry in the activity, it was a menial one at best in that she had him solve 16 times 2 by breaking it down. As a high school senior in an algebra II class, I would think by now that would not be necessary for Jerry and perhaps a pen and paper would have provided him an opportunity to answer the first time. Cindy also does not use basic arithmetic vocabulary as she replaces addition or add with the word plus. I infer that Cindy watered down the content to make it more comprehensible, something that ESL practitioners are warned against. Unfortunately, the support Cindy provided and the participatory role Jerry played were not conducive to his developing ownership in the mathematic problems solving.
activity and I am left with the distinct feeling that Jerry never knew what exactly they were solving for.

**Table 5.4**

<table>
<thead>
<tr>
<th>Line</th>
<th>Type of Question</th>
<th>Characteristics of tutee discourse</th>
<th>Characteristics of tutee response</th>
<th>Level of tutee participation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-6</td>
<td>Guiding</td>
<td>Does not provide an introduction to the problem solving process. Poses guiding questions punctuated by episodes of wait time</td>
<td>Claims not to know the answer to the problem Answers questions to the problem steps involved</td>
<td>Active/passive participation</td>
</tr>
<tr>
<td>6-22</td>
<td>Guiding</td>
<td>Communication breaks down when Jerry cannot solve 16 times 2. Cindy breaks this down into steps to help him solve the problem Asks him what number the “plus” goes on from a choice of 2. Evidence of watering down the content</td>
<td>Simple tasks, 16 times 2 Correctly chooses the right answer choice.</td>
<td>Active/passive participation</td>
</tr>
</tbody>
</table>

Dyad 5: Cindy and Arnie. Similar to the prior dyadic interactions, Cindy and Arnie established their roles as tutor and tutee by taking seats next to one another at the table they would be working at. Arnie’s goals have not changed and he is working to understand the mathematical problem sets so that he could successfully solve them independently on classroom tests. Cindy’s goal also remains the same in that she is attempting to help Arnie independently arrive at the correct solutions to the problem sets in the mathematic activity by making the problem solving processes and vocabulary of the problem sets comprehensible.

Although Cindy and Arnie take their seats next to one another as Cindy prepares to begin the session, unlike the beginning interaction she previously had with Jerry, she places the worksheet
on the table so it is mutually visible to both of them. She also gives Arnie direction as to what they will need to do to solve the problem, poses a procedural question as a comprehension check, and then continues with an explanation of the problem solving process. She then follows her explanation with a guiding question after she writes the equation (line 1).

N.2 Cindy (T3) and Arnie (B)

What is the slope of the line that passes through the points (5, -1) and (-3, 3)?

1 (Cindy)T3: **Now for this one you need the slope formula, OK?** And you go from Y to minus Y one over X to minus Y..no, over X one, so that there is 3 minus 1 over negative 3 minus 5 (writing the equation)

What does that equal to, 3 minus 1?

Arnie complies with the correct answer (line 2) and she continues with a guiding question “and? “ that more information is needed. Once again Arnie complies as Cindy asks him to reduce the answer. However, when Arnie gives the appropriate answer, Cindy mistakes it as wrong and when as she attempts to correct Arnie, He explains the reason why his answer is correct.

In this episode, Cindy did actively engage Arnie in the problem solving process, however, she did not provide support for arriving at a correct solution as she tried to correct Arnie’s accurate response. Cindy also never provided an opportunity for Arnie to demonstrate independent problem solving. Although Arnie was able to justify his final response, of “8 divided by 2 is 4”, this is a basic math concept all students by this grade level should know and it also did not provide evidence that he understood the problem solving process as a means to solve the original problem statement. There is no evidence that Cindy’s use of questions and wait time supported Arnie’s developing ownership of the mathematic activity:

2 B: 2
3 T3: (writes the number) and?
4 B: **Negative 8**
5 T3: (writes the number) and that reduces to?
6 B: Four, oh, one over four
7. T3: *No, 8 divided by 2?*
8 B: Yeah, *8 divided* by 2 is 4.
9 T3: Oh (laughs) yeah!

**Table 5.5**

<table>
<thead>
<tr>
<th>Line</th>
<th>Type of Question</th>
<th>Characteristics of tutee discourse</th>
<th>Characteristics of tutee response</th>
<th>Level of tutee participation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Procedural</td>
<td>(Opens activity) Provides an explanation of the problem solving process. Poses a procedural question to check comprehension</td>
<td>No response</td>
<td>Passive participation</td>
</tr>
<tr>
<td>2-5</td>
<td>Guiding</td>
<td>Poses guiding questions to answer portions of the problem</td>
<td>Responds to the guiding questions</td>
<td>Active participation</td>
</tr>
</tbody>
</table>

Dyad Six: Cindy and Peter. This episode began with Cindy explaining the purpose of the activity and populating her words with arithmetic terms. She explains the process briefly by explaining that Peter is to divide by 2 throughout the process (line 1) and follows this with a procedural question to assess comprehension. When Peter provides an affirmative response to her question, Cindy hesitates in moving forward with the activity and poses yet another simple question as a means to reassure herself. Peter responds with the question “*by 2?*” thus indicating uncertainty that if Cindy had explored would have possibly revealed some confusion by Peter. Instead, Cindy accepts his question as an affirmation of comprehension of her explanation and closes the activity. Peter is not afforded an opportunity to co-construct the activity or demonstrate independent problem solving. This interaction did not lead to Peter’s developing ownership in the mathematic activity.

N.3 : (T3) Cindy and (C) Peter

*Simplify completely* $\sqrt{360}$
OK, for this (pointing to the problem statement) you need the *square root* of 360. So, (writing the equation) you have to *divide* that by 2 (indicating the 360) until it doesn't go anymore…so that would *equal* to 6 and a *negative* 10. OK?

Yes

Do ya get it?...

(interrupts) by 2?

Yeah, you just keep goin' until it doesn't go anymore…OK?

OK…..alright! (smiles)

Table 5.6

<table>
<thead>
<tr>
<th>Line</th>
<th>Type of Question</th>
<th>Characteristics of tutee discourse</th>
<th>Characteristics of tutee response</th>
<th>Level of tutee participation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-6</td>
<td>Procedural</td>
<td>Begins with an explanation of the problem solving process</td>
<td>Responds that he understands</td>
<td>Passive Participation</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Poses a simple question to assess comprehension</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Poses another simple question to ensure comprehension has been achieved</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Summary of Cindy’s performance

To summarize Cindy’s performance as a tutor with some inferences I made as I analyzed the material, she came highly recommended, yet provided tutees the least amount of opportunities to actively participate and take ownership of the activity. This may have occurred for many reasons; however, I identify some patterns in her background that may shed light on this problem.

Cindy was a high achiever in mathematics before she acquired proficiency in English which may mean that she is are naturally predisposed to learning math. This would create difficulties in her ability to match strategies that had been helpful to her with strategies needed for a student
struggling in math. This might explain the low incidence of guiding questions and absence of reflection questions within her discursive interactions with the tutees.

Cindy indicates a degree of impatience, particularly with Peter. As a bilingual ELL (one parent is a native English speaker) and advanced speaker of English, Cindy may not have experienced learning math in a completely foreign language. She may not be aware of the vocabulary and terms that populate this content area and the effect this has on Peter’s ability to learn math. Rather than explain the terms to all dyad partners, she skims over them and simply executes the process of the activity. Once she reaches the product (which she often completes on her own) the activity ends.

**Differences in the nature of interaction between Jean and Cindy**

Jean uses a mixture of procedural, guiding, and reflection questions to encourage active participation and independent problem solving of the tutees he worked with. Jean introduces the lesson and gives the tutees a preview of what they will do in order to solve the problem, then he fractures the lesson into manageable multiple pieces. Jean then uses questions, particularly guiding questions, to encourage tutees to co-construct the pieces into a single unified problem set again. He then offers the tutees the opportunity to independently demonstrate the problem solving process through physical demonstration and supports them in arriving at a correct solution to the problem. In some cases, after the demonstration, Jean occasionally asks the tutee to explain or justify the process they chose to solve the problem. Jerry displayed a fine example of how a tutor could develop tutee ownership in the mathematics activity in a small amount of time (5-10 minutes).

Cindy does not always offer a demonstration, explanation, or preview to the problem she will solve with the tutee. Although her occasional use of guiding questions did encourage active participation, it was unclear what the tutees were solving for, particularly in Jerry’s case. Often
times Cindy also fractured the problem, but often watered down the content and vocabulary in the process. Due to the absence of reflection questions, it is very difficult to determine if comprehension of the problem solving process had been achieved, however, it was clear that she did not develop tutee ownership in the mathematic activity.
CHAPTER FIVE

DISCUSSION

The purpose of this study is to identify interactive discourse strategies secondary mathematic teachers can use in the classroom to develop ELL ownership of the mathematic problem solving activities. More specifically, the study investigated differences in the use of questions and wait time between effective tutors and those deemed less effective in promoting the acquisition and demonstration of vocabulary and mathematic problem solving processes. Following the fundamental tenets of sociocultural theory, mathematic development was enhanced through social interactions and mediation made possible by questions and wait time. My analysis of tutor questions that supported tutee’s active participation and independent mathematical problem solving revealed that three types of questions (i.e., procedural, guiding, and reflective) were used most often by tutors. Similar to Mary Budd Rowe (1974), this study also revealed that wait time exceeding more than three seconds between tutor question/tutee answer or tutee response/tutor reflection provided tutees an opportunity to increase their use of arithmetic language. The addition of wait time in combination with a guiding question was particularly useful if an elaborated or correct version of a prior answer was needed.

The study revealed how effective tutors, such as Jean and Tina, used questions and wait time as catalysts in moving ELLs forward from peripheral participants of the mathematics activity to full active participants capable of independent problem solving and use of arithmetic vocabulary. Most all of the tutors used explanation followed by procedural questions to check comprehension and draw attention to important features or vocabulary necessary to solve the problem. Effective tutors, such as Jean and Tina, fractured their problem sets into pieces or steps, which in and of themselves, were similar to mini lessons. They used a variety of questioning
types, procedural, guiding, and reflection to help tutees focus on each individual piece of the problem to recombine as a whole. When the tutees encountered a problem, the tutors used guiding and reflection questions to help the tutee examine the incorrect response and reconfigure to arrive at a proper fit to the entire solution. Fracturing of the problem solving process helped students to process it in manageable pieces. As the tutees worked with these manageable pieces, shaping them and analyzing their fit and correcting problems with the help of the tutor’s guiding questions, they began to internalize the process resulting in a shift from role of novice to independent problem solver in a relatively short amount of time, approximately 5-10 minutes per problem set. Effective tutors used reflection questions to prompt tutees to explain, justify, and physically demonstrate the problem solving process which provided evidence that this shift in cognitive development from the intermental to the intramental plane had actually occurred. Using questions to help the tutees solve the problem sets supported active engagement and eventual ownership of the mathematic activity.

Together, these three questions types, wait time, and interaction, contributed to developing tutee ownership of the mathematic activity. In the opening section of the study, I provided my personal reasons for conducting this study. Recall that one of them was to dispel the misconception that mathematics classrooms must be conducted in lock step fashion through demonstration, mimicry and practice in silence. Another misconception I hoped to dispel was to learning through social interaction leads to bad habits such as cheating or copying from one another. Inspired by the beliefs of my colleague, I conducted this study to prove that interaction in which knowledge is co-constructed through the strategic use of questions and wait time, in either small group or whole group settings, can benefit all students regardless of English language proficiency.
The results of this study indicate that interaction, regardless of English language proficiency level or native language background, has a positive effect on mathematics learning and vocabulary use. As pair work is an integral instructional strategy utilized in many classrooms, the results of this study like previous studies conducted by Storch (2004), Markee (2004), and Yackel, Cobb, & Wood (1991), indicate that student personality, mathematics experiences, and attitudes to group work greatly affect the outcomes of peer tutoring sessions and that care must be taken when pairing students for collaborative work.

Adding to the previous research, this study also maintained that even among ELLs, those predisposed to learning math are best not paired with novice students. The dyads in this study that were least effective in supporting tutees’ in active participation, independent problem solving, and use of arithmetic vocabulary were the dyads that contained the ELL tutors who were most recommended by math teachers and used as tutors throughout the school. In contrast, the tutors who were most effective in attaining the goal of supporting tutees’ active participation, independent problem solving, and use of mathematics vocabulary were the tutors who had been placed in remedial math at one time during their school career due to poor math grades. These findings are beneficial for mathematics teachers in that they do not have to place the most advanced math learners with ELLs as peer tutors, but can place a mediocre problem solver from their class to work with the ELL if the peer tutor he or she demonstrates effective questioning strategies similar to the results of this study.

This instance of developing tutee ownership of the mathematics activity is of key importance for teachers of secondary ELLs in mainstream mathematics classroom where everyone is held accountable for successful scores on high stakes tests and there is little time available to prepare the students. In a heterogeneous classroom where time is a constraining
factor, teachers must work extra hard to ensure that their non-English speaking students successfully achieve mathematically alongside their English speaking peers. However, with the increase in ELLs and minimal number of secondary mathematics teachers with ESL training, this student sub group is at risk for failure unless teachers adapt their teaching techniques to include ELLs.

The results of this study offer secondary mathematics teachers of ELLs practical solutions for a complex situation. Teachers must not entirely abandon their current methods of teaching; simply add more communicative and interactive components to them. By investigating the backgrounds of their students, they could choose those who would best work well with the ELLs, offer them a brief training session on the questioning strategies and use of wait time that work best for ELLs, and employ the peer tutors during each class. This ensures all students are active participants of the mathematics community, either as tutors or tutees, and actively involved in the problem solving process. The focus is on building on what the ELL already knows and supporting his gaining of new information in the least intimidating manner.

For example, the teacher presents the problem to be solved on the board or through a projector, explaining the basic procedures and calling attention to arithmetic vocabulary. He asks a procedural question as a comprehension check or to call attention to the importance of a particular aspect of the problem before moving on. The teacher then fractures the problem into manageable pieces and poses guiding questions to students as they collaboratively work through the process together in a whole group activity. Although instructional strategies that include fracturing or chunking information and questioning find support in studies that focus on reading comprehension of junior high ELLs with learning disabilities additional support is found in the use of graphic organizers to categorize information (Barrera, Liu, Thurlow, & Chamberlain,
This collaborative effort supports student’s confidence in that their answer is collaborative and not individual which places responsibility on the group rather than one student. In other words, the students can actively participate in whole group discussion through side-bar collaboration after a question is posed without fear of being singled out or humiliated due to their limited English language proficiency. In addition, this collaborative interaction within the whole group work offers students an opportunity to reflect and build on prior answers, understand why a particular answer might not be a good fit, and to come to a complete understanding of the problem solving process as a whole. This whole group collaboration encourages students to form a community of learners (Savignon, 2007).

Once the whole group problem solving process is completed, the teacher assigns students problem sets to work on together in dyads or small groups. In this instance, the teacher must be sure that the peer tutors placed with the ELLs understand their role in helping the ELLs become active participants of the problem solving process. This allows students to work in more intimate settings and reduces the insecurities that affect ELLs from participating in whole group discussion. This also gives the teacher time to circulate the room as a facilitator, providing guidance and deport to individual groups when needed.

Upon completion of the tutoring episode, the teacher can revisit the sample problem sets she had previously assigned and with the class as a group, fracture them again and through questions and wait time, actively engage the entire class in reassembling the problem sets and solving them as a whole. Finally, in keeping with the traditional method of mathematics teaching, the teacher can assign a few problem sets for independent practice and provide additional help to students that may need it.
There are many benefits in using teaching strategies that actively engage students through offering students a chance to co-construct and demonstrate problem solving skills in small groups, as a class, and independently. As the teacher begins the lesson, provides an example, and begins to pose questions to the entire class, students have an opportunity to interact and pose counter questions that other students may be wanting to ask, but are intimidated. It also gives the teacher an opportunity to identify where some of the problems may lie, particularly when none of the students provide an answer to a question. By utilizing wait time, the teacher can allow students time to process and collaborate on a question. All students can ask for help as the teacher circulates the room without being cast in the spotlight and all students develop ownership and become active participants in the mathematic community.

The results of this study have significant implications for teacher professional development and pre-service teacher education. As most mathematic teachers have little to no training in ESL, this offers them an opportunity to increase the success of their ELL students while ensuring mainstream students are not left behind. By promoting a class that encourages active participation and interaction, every student will benefit. Providing teachers with useful strategies that they can utilize daily in the classroom without cumbersome planning or requiring an abundance of time, teachers and pre-service teachers can be prepared for ELLs in their classroom. Although this study focused specifically on math, it reveals that questioning strategies in conjunction with wait time and interaction are practical strategies that can be used in any content classroom and are practical for teachers from all content and grade levels.

Teachers, students, and school districts all face challenges in light of an education system that is not quite prepared for the influx of English Language Learners entering our schools. Unfortunately as the number of ELLs entering school districts increase, the budgetary allowance
set aside for teacher in-service and professional development training for ESL decreases. This results in more secondary teachers relying on effective and efficient teaching strategies to use with ELLs in their mainstream classrooms. Unfortunately, the number of research studies that focus specifically on teaching strategies for secondary ELLs in the mainstream mathematics classroom are scarce.

With the rising numbers of secondary ELLs entering our schools, the high stakes large scale assessment carry, the paucity of research on secondary ELLs in mainstream mathematics classes, and the time constraint teachers are forced to work with to ensure the success of their students, instructional strategies that incorporate interaction, questions, and wait time, in small groups, dyads, or large group instruction can provide important benefits for the mathematic performance of all students, particularly ELLs.
References


Rowe, M.B. (1986). Wait time: Slowing down may be a way of speeding up! *Journal of Teacher Education* 37, pp. 43-50


## APPENDIX A

### WIDA ELP Proficiency Levels

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<table>
<thead>
<tr>
<th>Level</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>6- Reaching</td>
<td>Specialized or technical language reflective of the content areas at grade level. A variety of sentence lengths of varying linguistic complexity in extended or written discourse as required by the specified grade level. Oral or written communication in English comparable to English speaking peers.</td>
</tr>
<tr>
<td>5-Bridging</td>
<td>Specialized or technical language of the content areas. A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse, including stories, essays or reports. Oral or written language approaching comparability to that of English language proficient peers when presented at grade level.</td>
</tr>
<tr>
<td>4-Expanding</td>
<td>Specific and some technical language of the content area. A variety of sentence lengths of varying linguistic complexity in oral discourse or multiple, related sentences or paragraphs. Oral or written language with minimal phonological, syntactic or semantic errors that do not impede the overall meaning of the communication when presented with oral or written connected discourse with sensory, graphic or interactive support.</td>
</tr>
<tr>
<td>3-Developing</td>
<td>General and some specific language of the content area. Expanded sentences in oral interaction or written paragraphs. Oral or written language with phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written, narrative or expository descriptions with sensory, graphic or interactive support.</td>
</tr>
<tr>
<td>2-Beginning</td>
<td>General language related to the content areas. Phrases or short sentences. Oral or written language with phonological, syntactic, or semantic errors that often impede the meaning of the communication when presented with one- to multiple-step commands, directions, questions, or a series of statements with sensory, graphic or interactive support.</td>
</tr>
<tr>
<td>1-Entering</td>
<td>Pictorial or graphic representation of the language of the content areas. Words, phrases or chunks of language when presented with one-step commands, directions, WH-, choice or yes/no questions, or statements with sensory, graphic or interactive support. Oral language with phonological, syntactic, or semantic errors that often impede meaning when presented with basic oral commands, direct questions, or simple statements with sensory, graphic or interactive support.</td>
</tr>
</tbody>
</table>
APPENDIX B

PSSA Mathematics Scoring Guidelines

GENERAL DESCRIPTION OF MATHEMATICS SCORING GUIDELINES

4 – The response demonstrates a thorough understanding of the mathematical concepts and procedures as required by the task.

The response provides correct answer(s) with clear and complete mathematical procedures shown and a correct explanation, as required by the task. Response may contain a minor “blemish” or omission in work or explanation that does not detract from demonstrating a thorough understanding.

3 – The response demonstrates a general understanding of the mathematical concepts and procedures as required by the task.

The response and explanation, as required by the task, are mostly complete and correct. The response may have minor errors or omissions that do not detract from demonstrating a general understanding.

2 – The response demonstrates a partial understanding of the mathematical concepts and procedures as required by the task.

The response is partially correct with partial understanding of the required mathematical concepts and/or procedures demonstrated and/or explained. The response may contain some work that is incomplete or unclear.

1 – The response demonstrates a minimal understanding of the mathematical concepts and procedures as required by the task.

0 – The response has no correct answer and insufficient evidence to demonstrate any understanding of the mathematical concepts and procedures as required by the task for that grade level.

Response may show only information copied from the question.

Produced from Grade 11 Mathematics Item Sampler 2006–2007
APPENDIX C

The following transcription occurs during tutoring session between two ELL students as they work towards solving mathematical problems.

Shiela, is an ELL student acting as tutor.
Arnie is an ELL student in need of tutoring.

*Questions* are italicized and underlined.

*Wait time* is bolded.

Wait time is identified as wait time between questions and an answer, between questions, during a problem solving process, and after the explanation of a concept.

Language particular to math that is used in the response of the tutee and/or feedback from the tutor is both bolded and highlighted.

The correctness of the tutee's ability to solve the mathematical problem is indicated at the close of the transcription from each problem set.

**Excerpt 1.4: Shiela (T2) and Arnie (B)**

A protractor is labeled below

![Protractor Diagram]

What is the measure of angle XYZ?

1 T2:  *O.K., this is a protractor, right?* (points to the protractor on the paper)

2 B:  Yeah

3 T2:  That’s a picture of it. So, they’re asking (points to the words of the problem statement as she says them) *what is the measure of X, Y, and Z? So, which is X (pointing to each point on the protractor as she says the coordinating letter) Y, Z?*

4 B:  So it would be (points to the paper)
5 T2: No, no, (interrupts) just let me, let me, I'll tell you. Yeah, that…So anything (going that’s like, *this is half right?*) (draws a line through the middle of the protractor) that’s *half* anything from here (waves her pen from the middle of the protractor to the right) going here, you look at the top, I mean, the bottom number.

(2 seconds pass)

O.K.?

6 B: Yeah, *wha*...? (T2 interrupts)

7 T2: And anything from here (using her pen to draw an arrow from the middle of the protractor pointing to the left) you look at the top bottom, I mean top.... the number on the top (pointing to the top number on the protractor). *So if let’s say, like, so this is like, right here right?* (placing a line on the protractor) *So what do you? What is the answer?*

8 B: (looks at the paper) thirttyyyyy

(7 seconds pass)

*ummm...looks like 62, sixty?*

9 T2: No, no...look....(using her pen to trace the arrow pointing to the right side of the protractor) I said anything from here you look at the bottom number (tapping on the bottom number of the line she had drawn on the protractor)

10 B: I looked at…..(pointing to the number where the line falls) (T2 interrupts)

11 T2: (interrupts) Yeah, yeah, you were looking at (B interrupts)

12 B: Oh! Oh! You didn’t say..(T2 interrupts)

(Both students speak over one another)

13 T2&B: The top

14 T2: There’s 2 numbers (pointing to the numbers)
15 B: Yeah (smiles points to the numbers) Yeah, you didn't say which it...OK (smiles) then it would be *120...122-3?*

16 T2: 123.

17 B: 123

18 T2: 123 (counting the lines from the number 120 where the line was drawn on the protractor)

O.K.
APPENDIX D

The following transcription occurs during tutoring session between two ELL students as they work towards solving mathematical problems.

Jean, is an ELL student acting as tutor.

Jerry, is an ELL student in need of tutoring.

Questions are italicized and underlined.

Wait time is bolded

Wait time is identified as wait time between questions and an answer, between questions, during a problem solving process, and after the explanation of a concept.

Language particular to math that is used in the response of the tutee and/or feedback from the tutor is both bolded and highlighted.

The correctness of the tutee's ability to solve the mathematical problem is indicated at the close of the transcription from each problem set.

Multiply:
\((–3x^2 +4x +5) (–2x –8)\)

1 T1: To do this problem (shows his partner the problem statement with his pen) you have to **multiply negative** 2 X (uses his pen to circle negative 2X in the problem statement) **times** all the numbers that are here (uses his pen to draw an arrow from negative 2 x to all the other numbers in the equation)

So its **negative 2X times negative** 3 X **squared**, **negative 2x times** 4X, and **negative 2X times** 5. (Points to each number and draws an arrow to the others as he speaks)

**(3 seconds pass)**

*and what's negative 2X times negative 3X squared?*

2 A: 6

**square**

**(3 seconds pass)**

3 T1: **Did you add** the X's? (points to the X's within the equation with his pen)

4 A: Six …**cubed**

**six cubed**
Is cubed 3?

5 T1: Just say X to the third power (the tutor writes the answer next to the equation) So its 6X to the third. negative 2x times 4x which is? (writes out the next equation (step) as he speaks)

(3 seconds pass)

which is?

(6 seconds pass)

6 A: negative 8

(3 seconds pass)

squared...X squared.

7 T1: (writes the answer next to the equation) yeah, and it’s a negative so it’s minus.....minus 8...So it’s negative 8 squared (writes the equation) And then what’s negative 2x times 5?

8 A: negative 10...X?

9 T1: (writes the answer next to the equation) and then what’s negative 8 times negative 3X squared?

10 A: its 24

(3 seconds pass)

X squared

11 T1: (writes his partners response next to the equation) And what’s negative 8 times 4X?

12 A: 32X, negative 32?

13 T1: (writes partners response next to the equation) What’s negative 8 times 5?

14 A: negative 40

15 T1: (writes the response next to the equation) Then you got to add the like terms, like this one (points to a particular number), it doesn’t have any...so you just rewrite it (rewrites the number)
This one has \textit{X squared} and \textit{X squared} (pointing his pen to 2 numbers) (writes the equation) \textit{What’s negative 8 minus 24?}

16 A: 16 \textit{squared} (mumbling)

17 T1: (writes the response of his partner) \textit{16, but is it positive or negative?}

18 A: pos...\textit{negative}?

19 T1: its \textit{positive} (writing the positive sign next to the answer) \textit{what’s negative 8 and negative 24}

20 A: \textit{negative 8 plus negative 24}?

21 T1: \textit{Yeah, what’s negative 8 plus negative 24}?

22 A: \textit{Negative 32}?

23 T1: Ok, now finish \textit{adding} this (points to the equations he has written and hands the pen and paper over to the tutee).

(\textbf{15 seconds pass} and the tutee writes his answers and slides the paper and pen over to the tutor)

24 T1: Yeah, O.K., \textit{16 x squared and then what}?

(1 second)

There are 2 left, \textit{what’s negative 10 minus 40}?

(\textbf{5 seconds pass})

25 A: \textit{40}?

26 T1: I mean \textit{negative} 40.

27 A: \textit{negative} 50

28 T1: OK (shows the equation to the tutee) Now you just \textit{multiply} all of this with that (pointing to the numbers)

(\textbf{3 seconds pass})

OK...now you can show me how it’s done. (Writes similar equations and has the
tutee complete it on his own)

(twenty seconds pass as the tutee works on the problem)

29 T1: Then add like terms (talking through the steps as he observes the tutee working)

(40 seconds pass)

30 A: This is…(points to a problem he is having difficulty with)

31 T1: This is x to the 3rd that’s x squared (pointing at the number sets). They can’t go together. If there’s only one you just rewrite it again.

32 A: So 8 plus?

33 T1: What’s that? (pointing to a number)

34 A: 2

35 T1: So its 8 minus 2

(20 seconds pass)

(the tutor takes the tutee’s paper and checks his work)

What’s negative 8 x and negative 1x? (gives the paper back)

36 A: Negative 9x (writing the answer)

37 T1: So we just minus 1 (taking the pen from his partner and correcting the sign made on the paper) you don’t need to plus (waving his hand over the paper) and that’s it!

PROBLEM 2 part B

(–3x^2 +4x) (–2x –8)

39 T1: O.K., this is like the same as the other one only with different numbers. We’ll do this one because we already did this one (pointing to one of the problem sets) You have to multiply everything (hands the paper and pen over to the tutee).

(57 seconds pass as the tutee works on the problem set)

40 T1: (looks at the tutees paper) O.K. …you have to multiply this and then multiply this by that (pointing to the various numbers on the problem set).
A: (points to his paper) yeah, yeah I got that.

T1: Oh yeah, you got that (looks at additional numbers in the problem set as the tutee continues to work)

(25 seconds pass while the tutee works)

T1: (points to what the tutee has written) if it’s negative you only have to write negative, you don’t have to write the plus.

A: (Puts down the pen indicating he is finished)

T1: (Takes the paper to check if the problem is correct)

A: Was it right?

T1: I’m checking......

(5 seconds pass)

You messed up on the plus for the negative...just this (points to the incorrect sign) just the sign...otherwise it is all right.

A: Cool

T1: (showing the tutee the incorrect problem) When you have this (circles the number) you just write negative you don’t have to write the plus.

A: (pointing to the problem set) But what about this?

T1: You don’t have to write the negative (circling the negative sign)

A: Oh, O.K.
VITA

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Experience
Part-time Faculty (full time temporary 8/2010)
Bloomsburg University  Bloomsburg, PA  Jan '06 - Present
Graduate Program Development Team
Misericordia University  Dallas, PA  May '09 - Present
Instructional Design Team/Adjunct faculty (on-line and traditional)
Misericordia University  Dallas, PA  Jun '02 - Present
ESL/German/Instructional Support Teacher
Tamaqua Area School District  Tamaqua, PA  Nov '02 - Nov '10
Adjunct Faculty
Pennsylvania State University  Frackville, PA  Jan '04 - Aug '05

Education
PhD (Curriculum and Instruction)
Pennsylvania State University  University Park, PA  Dec '11
Passed thesis defense June 24, 2011
Master of Science (Instructional Technology)
Wilkes University  Wilkes Barre, PA  May '04
Master of Science (Education)
Wilkes University  Wilkes Barre, PA  Jun '02
Bachelor of Science (Elementary Education)
Clarion University  Clarion, PA  Jun '83
Certificates Elementary Education, Instructional Technology, English as a Second Language, German, English Language Arts: Pennsylvania Department of Education, PA

Presentations
July, 2011: Jones Center Summer Institute, Bloomsburg, PA: Effective Instruction and Assessment Strategies for English Language Learners in the Mainstream Classroom.
April, 2010: Tamaqua Area School District: Educating English Language Learners in the Content Area Classroom.
March, 2008: Harvard Graduate School of Education: "What's at Stake? Using Traditional Standardized Assessment to Measure Academic Progress of English Language Learners."
October, 2007: The Northeastern Speech-Language-Hearing Association of Pennsylvania Fall Seminar; Title: Communication Disorders in ELL's :Helping Educators and Speech Language Pathologists identify Communication Disorders in Culturally Linguistic and Diverse Learners.
October, 2007 NESHAP student chapter monthly meeting; Cultural Awareness and Diversity