CHARACTERIZATION OF POROUS MATERIAL USED FOR THERMOACOUSTIC DEVICES

A Thesis in
Acoustics
by
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Abstract

Three new compact experimental apparatuses were developed and calibrated to quickly and accurately characterize heat transfer and fluid drag in bulk porous media that may be useful candidates for standing-wave (stack-based) and traveling-wave (regenerator-based) thermoacoustic machines. In the first apparatus, the thermal properties of the testing sample were determined by exploiting the transition of heat transfer from the isothermal to the adiabatic limits for gas compressions and expansions within the porous media, using the thermoviscous $f$-function. For the first time, the complex Nusselt number is related to the thermoviscous $f$-function allowing the heat transfer coefficient for oscillating flow to be determined without measuring temperatures and without requiring the use of heat exchangers. In the limit appropriate to heat exchangers, the new technique produced results that agreed with several other theoretical and experimental determinations of the heat transfer coefficient for oscillatory flow through heat exchangers. In the second apparatus, the pressure gradient for the oscillating flow through several samples was measured by a phase sensitive technique to resolve the pressure drop into two parts: the viscous drag and the oscillating inertia (Kelvin drag). Unlike the measurements of previous investigators, who did not separate the viscous and inertial effects, these measurements of viscous drag agreed with the published drag coefficients made by steady flow techniques in the appropriate limit. An additional dimensionless variable was created that allowed pressure drop measurements to be correlated through the transition from the low-frequency Poiseuille flow (parabolic) to high-frequency plug flow (boundary layer) regimes. The third apparatus was designed specifically to separate thermal and viscous losses in porous ceramic materials (automotive catalytic converter substrates) used in standing-wave thermoacoustic machines. By sealing the walls of the ceramic using an $\textit{in situ}$ polymerization technique, it was shown that wall porosity had no effect on thermal-relaxation dissipation at low frequencies.
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<th>Symbol</th>
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<tr>
<td>$A$</td>
<td>area, $m^2$</td>
</tr>
<tr>
<td>$a$</td>
<td>acceleration, $m/s^2$</td>
</tr>
<tr>
<td>$Bl$</td>
<td>transduction coefficient, $N/Amp = volt/(m/s)$</td>
</tr>
<tr>
<td>$c_p$</td>
<td>isobaric heat capacity per unit mass, $J/kg\cdot K$</td>
</tr>
<tr>
<td>$D$</td>
<td>diameter, $m$</td>
</tr>
<tr>
<td>$e$</td>
<td>$2.71818...$</td>
</tr>
<tr>
<td>$f$</td>
<td>frequency, $Hz$</td>
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<td>$f$</td>
<td>spatially averaged thermoviscous function</td>
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<tr>
<td>$F$</td>
<td>force, $N$</td>
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<tr>
<td>$h$</td>
<td>thermoviscous function</td>
</tr>
<tr>
<td>$h$</td>
<td>heat transfer coefficient, $W/m^2\cdot K$</td>
</tr>
<tr>
<td>$i$</td>
<td>$\sqrt{-1}$</td>
</tr>
<tr>
<td>$i$</td>
<td>electrical current, A</td>
</tr>
<tr>
<td>$k$</td>
<td>spring constant, $N/m$</td>
</tr>
<tr>
<td>$k$</td>
<td>thermal conductivity, $W/m\cdot K$</td>
</tr>
<tr>
<td>$K$</td>
<td>wavenumber, $m^{-1}$</td>
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</table>
\[ L \] electric inductance, henry

\[ L \] length, m

\[ Lc_\kappa \] Lautrec number, \( r_h/\delta_\kappa \)

\[ Lc_\nu \] viscous Lautrec number, \( r_h/\delta_\nu \)

\[ m \] mass, kg

\[ n \] mode number

\[ Nu \] Nusselt number

\[ \dot{m} \] mass flow, kg/s

\[ p \] pressure, Pa

\[ q'' \] heat flux, W/m\(^2\)

\[ Q \] quality factor

\[ R \] electric resistance, ohm

\[ R \] mechanical resistance, kg/s

\[ Re \] Reynolds number

\[ r \] radius, m

\[ S \] surface area, m\(^2\)

\[ T \] temperature, K

\[ T \] period, s

\[ t \] time, s

\[ U \] volume flow rate, m\(^3\)/s

\[ u \] x component of velocity, m/s

\[ V \] volume, m\(^3\)

\[ X_L \] inductive reactance, ohm

\[ V \] voltage, volt
v  vector velocity, m/s

Z  electrical impedance, ohm

**Greek letters**

\( \gamma \)  ratio of isobaric to isochoric specific heats
\( \delta \)  penetration depth, m
\( \eta \)  efficiency
\( \varepsilon_s \)  ratio of available heat capacity of the gas to that of the solid
\( \theta \)  phase angle
\( \kappa \)  thermal diffusivity, m\(^2\)/s
\( \lambda \)  wavelength, m
\( \mu \)  dynamic viscosity, Pa·s
\( \nu \)  kinetic viscosity, m\(^2\)/s
\( \xi \)  displacement of gas particle, m
\( \Pi \)  perimeter, m
\( \Pi \)  power dissipated by voice coil and mechanical resistance, W
\( \pi \)  3.14159...
\( \rho \)  density, kg/m\(^3\)
\( \sigma \)  Prandtl number
\( \phi \)  volumetric porosity
\( \phi \)  phase angle in radians
\( \omega = 2\pi f \), angular frequency, s\(^{-1}\)

**Subscripts**

\( ac \)  alternating current

\( back \)  back volume
\( dc \) direct current
\( e \) electrical
\( el \) electrical
\( eff \) effective
\( front \) front volume
\( h \) hydraulic
\( iner \) inertance component
\( m \) mean
\( mech \) mechanical
\( \nu \) viscous
\( rms \) root-mean-squared value
\( s \) solid
\( s \) surface
\( vis \) viscous component
\( 0 \) “environment” or “ambient”
\( \kappa \) thermal

**Special symbols**

\( \text{Im}[] \) Imaginary part
\( \text{Re}[] \) Real part

\( \langle \rangle \) spatial average perpendicular to \( x \)

overdot time derivative, time rate
Acknowledgments

I would like to gratefully acknowledge my advisor Dr. Steven Garrett for his guidance, support and patience throughout these years. I am also very grateful to Dr. Thomas Gabrielson, Dr. Victor Sparrow, and Dr. James Brasseur for serving on my committee. Thanks to Dr. Matthew Poese and Bob Smith for all their help in the lab.

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Chapter 1

Introduction

1.1 Motivation

Porous media play a critical role in thermoacoustic machines (engines and refrigerators) since they can permit these devices to span far greater temperature differences than would be found under simple adiabatic conditions. The penalty for using porous media to enhance the temperature span is that viscous energy losses are increased. Choosing the right porous material for efficient heat transfer and minimum possible viscous loss is essential for designing efficient machines.

Among acousticians, the thermoviscous $f$-function (Rott, 1969) has long been used for characterizing porous media. The thermoviscous $f$-function characterizes both the heat transfer and dynamics of oscillating flow in porous media. The details of the $f$-function will be discussed in Section 2.1. In the laminar flow regime, the heat transfer and dynamics of oscillating flow inside porous media are fully characterized by the thermoviscous $f$-function. Closed form solutions for the $f$-function are calculable in simple geometries such as arrays of circular pores, square pores, pin arrays, and parallel plates (Swift, 2002). But the regenerators of many traveling wave machines comprise screen beds, randomly stacked spheres, and other tortuous porous geometries for which the fundamental data are scarce. One of the motivations of this study is to develop and calibrate new experimental apparatuses for the efficient and accurate determination of flow and thermal transfer properties of small porous samples to enable the search for new potential porous media for
Researchers who study Stirling machines are also interested in characterizing porous media used as regenerators. Their approach is different from acousticians: friction (drag) factor and Nusselt numbers are used to describe the dynamic and heat transfer features of the porous media, whereas acousticians use the thermoviscous $f$-functions.

Establishing the connection between these two approaches was another goal of this study. It will be shown that introducing complex Nusselt number is the appropriate approach for characterizing heat transfer behavior for oscillating flow in porous media. For the first time, the complex Nusselt number is expressed in terms of the thermoviscous $f$-function. The friction (drag) factor for oscillating flow was split into two parts by using phase-sensitive measurements: the viscous part and the inertial (oscillating) part. When frequency approaches zero, the inertial part goes to zero and the viscous part approaches the well-known steady (DC) results. This work resolves an anomalous discrepancy caused by earlier measurements of the total pressure drop, which includes both the viscous effects and oscillating inertial effects, that did not agree with the steady (DC) results.

To determine the accuracy of the two apparatuses and their associated experimental techniques, this thesis focuses mostly on materials with calculable behavior. Following work has already begun at PSU to use these techniques to study novel potential regenerator materials including glass capillary arrays, metal felts, isotropic, and anisotropic reticulated vitreous carbon.

### 1.2 Thermoacoustics

The word “thermoacoustics” as introduced by Rott (1980) includes all effects in acoustics in which heat conduction and entropy variations of the medium play a role. Theoretical thermoacoustics began in 1868 with Kirchhoff’s calculation of acoustic attenuation in a duct of circular cross-section due to oscillatory heat transfer and viscous losses between the solid duct wall and the oscillating gas (Kirchhoff, 1868). Rott (1969) and coworkers published a series of papers which established the foundation of today’s linear thermoacoustic theory. Swift (1988)
has reviewed much of this early work.

During the last 20 years, thermoacoustics has been developed to provide a new paradigm for production of environmentally friendly and energy-efficient alternatives to internal combustion engines and vapor-compression refrigerators. The thermoacoustics approach attempts to use the pressure oscillations and gas motions associated with sound waves to execute engine and refrigeration cycles at high efficiency with a minimum of mechanical moving parts.

Since the invention of the thermoacoustic machine by Wheatley, Swift, and Migliori (1983) there has been a continuous effort to produce thermoacoustic engines (Migliori and Swift, 1988), (Swift, 1992), and refrigerators (Hofler, 1986), (Garrett et al., 1993), (Garrett, 1991), (Garrett, 1997), that have the simplicity and robustness that comes with the elimination of most mechanical parts, while achieving efficiencies that were comparable to or better than internal combustion engines and vapor-compression refrigerators.

There are two categories of thermoacoustic machines classified by the oscillating wave type: standing wave and traveling wave. Most of the thermoacoustic machines that have been built so far are standing wave engines or refrigerators. In 1999, Scott Backhaus and Greg Swift published the results of an experiment that used the thermoacoustic paradigm to produce a Stirling cycle engine that had a thermal efficiency of 30% (Backhaus and Swift, 2000). Their experimental device combined an acoustic phasing network and acoustic resonator to produce a one-horsepower Stirling cycle engine that was as efficient as a gas-powered automotive internal combustion engine but required no moving parts. As Ceperley (1979) first pointed out, a traveling acoustic wave propagating through a regenerative heat exchanger (regenerator) undergoes a thermodynamic cycle similar to the Stirling cycle, which is why traveling wave machines are also called Stirling cycle machines here.
1.3 Porous media used for thermoacoustics devices

Both stack-based machines and regenerator-based machines require oscillating acoustic pressure and velocity. Both use a structure made of a porous solid substrate spanning the temperature difference. When an acoustic wave interacts with a stationary porous substrate the acoustic wave is modified by its presence, resulting in two important effects: (i) there is a time-averaged heat flux near the surface of the plate, along the direction of acoustic vibration, and (ii) acoustic power is absorbed (for a refrigerator) or generated (for an engine) near the surface.

In standing wave devices, the porous substrate is called a “stack”. The hydraulic radius, \( r_h = \frac{A}{\Pi} \), that characterizes the size of the stack’s pore is about the same order as the penetration depth. \( A \) is the cross-sectional area of the pore and \( \Pi \) is the perimeter of the pore. The penetration depths are defined as

\[
\delta_\nu = \sqrt{\frac{2\mu}{\omega \rho}} = \sqrt{\frac{2\nu}{\omega}} \tag{1.1}
\]
\[
\delta_\kappa = \sqrt{\frac{2k}{\omega c_p \rho}} = \sqrt{\frac{2\kappa}{\omega}} \tag{1.2}
\]

where \( k \) and \( \kappa \) are the thermal conductivity and diffusivity of the gas, \( \mu \) and \( \nu \) are its dynamic and kinematic viscosities, and \( c_p \) is its specific heat per unit mass at constant pressure. The penetration depths tell how far heat (\( \delta_\kappa \)) or momentum (\( \delta_\nu \)) can diffuse laterally during a time interval on the order of the period of the oscillation divided by \( \pi \). The temperature difference or velocity gradient created at the wall diffuses into the flow and is attenuated at the same time.

1.3.1 Stacks in standing waves

Figure 1.1, (a) illustrates one cycle of the plane standing wave. Following a small gas parcel confined between planes A and B, for a standing wave the motion of planes A and B are in-phase, but differ in amplitude. This leads the gas parcel to its maximum pressure amplitude when it reaches its maximum displacement (zero velocity). For stack materials, the pore size is on the same order as the
Figure 1.1. (a) Standing wave pressure oscillation. (b) Standing wave refrigerator (c) Standing wave prime mover. (d) Traveling wave pressure oscillation. (e) Traveling wave refrigerator. (f) Traveling wave prime mover. Colors are used to indicate temperature. Red: hot, blue: cold, yellow: ambient temperature.
penetration depth, so the thermal contact of the fluid and the stack is neither perfect nor negligible. A temperature difference is created between the gas and the wall and diffuses into the flow. The imperfect thermal contact provides the required phasing between heat transfer and gas motion to complete a thermal dynamic cycle with no moving parts.

Heat flowing from the plate to the gas at one part of the cycle and back from the gas to the plate at another part of the cycle is shown in Fig. 1.1 (b) and (c). In Fig. 1.1 (b), the gas parcel absorbs heat at the cold end and exhausts heat at the hot end to function as a heat pump or refrigerator. In Fig. 1.1 (c), the gas parcel absorbs heat at the hot end and exhausts heat at the cold end to function as a heat engine thereby converting thermal energy to acoustic energy. For stack-based devices, heat transfer happens over a finite temperature difference so its efficiency is lower than the Second Law (Carnot) limit.

1.3.2 Regenerators in traveling waves

Regenerators are used for traveling wave thermoacoustic devices. Both regenerators and stacks provide temporary heat storage but function differently in a sound wave field. The hydraulic radius, $r_h$, of regenerators is much smaller than the thermal penetration depth. A regenerator absorbs heat from the working fluid during half of the cycle and reheats the fluid during the other half of the cycle. Due to its small pores ($r_h \ll \delta$) and much larger thermal capacity, the heat transfer between the fluid and the regenerator is almost isothermal here. This reversible heat transfer allows the thermal efficiency to approach the Carnot (Second Law) efficiency if one ignores viscous losses.

As shown in Fig. 1.1 (d), (e) and (f), the oscillating pressure is in-phase with the particle velocity. When this oscillation happens isothermally inside the regenerator, the gas will absorb heat from the regenerator during the first half of the cycle and exhaust heat back to the regenerator during the second half of the cycle. Heat is moved in the reverse direction from the power flow of the traveling wave. Ideally, at steady state, the net heat transfer per cycle between the working gas and the regenerator matrix is zero. The regenerator is merely a temporary thermal storage medium.
1.3.3 Regenerator optimization

The hydrodynamics and thermodynamics of oscillatory flow through solid matrices are of great interest to researchers studying Stirling machines because the highest efficiencies are achieved only with the most carefully engineered regenerators (Backhaus and Swift, 2001). A small pore size is required to approximate isothermal heat exchange but viscous dissipation will increase substantially when the pore size is reduced. A carefully engineered regenerator should minimize the viscous loss while providing required heat transfer efficiency.

The first mathematical theories to describe regenerator operation were published in the late 1920s, more than 100 years after it’s invention by Robert Stirling (1816). Although oscillatory flow is theoretically well understood in simple geometries, many Stirling regenerators are comprised of stacked screens and other complex and tortuous geometries for which the analysis relies on empirical correlation functions only. Organ (1992) points out that the oscillating flow within regenerators is commonly treated by the traditional method for steady, incompressible viscous flow, whereas a friction factor is correlated with geometry and Reynolds number. When analysis and computer simulation based on such correlation yield pressure distributions which do not agree with measurement in running machines, it is common practice to ‘improve’ matters by arbitrarily adjusting the correlations. The technique is part of a process which has become known as ‘validation’. Exercises in validation have been reported which called for friction factors at a given Reynolds number to be multiplied by factors between 4 and 7.

1.4 Thesis outline

Dynamic and thermodynamic behavior of gas oscillations in porous media were studied in this thesis. Three experiments were designed and conducted separately. Porous media samples with calculable geometries were used first for verifying the experimental techniques and theoretical extensions.

In Chapter 2, the thermoviscous $f$-function, which characterizes both the heat transfer and dynamics of the oscillating flow in porous medium, were measured. Placing the testing sample against the wall of the sample holder, the velocity of
the fluid inside the sample was minimized and the pressure and density oscillation were maximized. Due to the oscillating density, the temperature of the fluid would oscillate such that heat would flow between the fluid and the porous media. The irreversible heat transfer causes a phase shift between the averaged oscillating temperature and the pressure which is characterized by the thermal $\kappa$-function. Instead of measuring the temperature directly to characterize this irreversible thermal loss, the complex ratio of displacement to pressure was measured and the thermal $f_\kappa$-function were deduced from these measurements.

Thermoviscous $f$-functions are used mostly by acousticians. Researchers studying oscillating flow Stirling engines traditionally use the Nusselt number to characterize the heat transfer in regenerators. Nusselt number is the dimensionless temperature gradient at the gas-wall interface. It is also the ratio of the heat transfer coefficient to the heat conductivity times a scale length. Empirical correlation functions of Nusselt number from steady-state measurements were commonly used to characterize oscillating flow heat transfer. In Chapter 2, the use of a complex Nusselt number for oscillating flow is derived and expressed in terms of the thermoviscous $f$-function. This provides an acoustic method to experimentally characterize the heat transfer coefficient of porous media subject to oscillating flow without requiring a heat exchanger and without need for temperature measurements.

Due to the similar feature of the momentum equation and the general heat transfer equation, the thermoviscous $f$-function fully characterizes both the dynamic and thermal behavior of oscillating fluid within the porous media in the laminar flow range. But in many applications, the velocity of the flow through the porous media is high enough that the assumption of small perturbation based linear theory is not valid anymore. It is important to characterize when the transition occurs and provide the empirical correlations, since thermoacoustic machines are less efficient when the flow in the regenerator becomes turbulent.

In Chapter 3, a new test apparatus was designed based on the “bellows bounce” (Poese et al., 2004) pressure vessel to study the pressure drop due to oscillating flow through the porous media up to high Reynolds numbers. The key to these measurements was the use of phase-sensitive detection using two lock-in amplifiers.
to permit simultaneous independent determination of both the inertial and viscous components of the pressure gradient. A previous study (Zhao and Cheng, 1996) found that the value of the cycle-averaged pressure drop due to the oscillatory flow in a packed column of stacked-screen sample was four to six times higher than that of a steady flow at the same Reynolds number based on the cross-sectional mean velocity. The cycle-averaged pressure drop for oscillatory flow includes the combined effects of viscosity and oscillating inertia. Comparing the steady (DC) flow pressure drop without incorporating phase information was bound to produce anomalous discrepancies between the friction factor for AC and DC flows.

It is also shown that for oscillatory flow, the Reynolds number is not sufficient to characterize the flow and another dimensionless group, the viscous Lautrec number is introduced. The viscous Lautrec number is the ratio of the hydraulic radius to the viscous penetration depth, $Lc_v = r_h/\delta_v$. In these experiments, the testing sample was placed in the flow path between two volumes to maximize the velocity and minimize the density oscillation when the fluid passes through the porous testing sample. The apparatus provides a combination of operating pressures, gases, amplitudes, and frequencies that can cover a large range of Reynolds numbers, and viscous Lautrec numbers.

Results obtained with this apparatus using two ceramic Celcor™ samples (400 and 600 cells/in$^2$), over four orders-of-magnitude in Reynolds number will be reported in that chapter. The hydraulic radius of the square pores in the 400 Celcor™ is 0.28 mm and it is 0.23 mm for the 600 Celcor™. The experimental results were in good agreement with the linear acoustic approximation at low Reynolds numbers, but deviate at higher Reynolds numbers. The frequency dependence of the pressure drop due to viscosity was observed when the viscous Lautrec number was greater than unity. It was also found that the characteristic lengths in the lateral direction for shear and velocity gradients are also dependent upon the viscous Lautrec number.

The wall pores of the ceramic material are a potential concern when using them as stack or regenerator material. In Chapter 4, an independent measurement of excess attenuation due to wall porosity of Celcor™ was conducted to study the attenuation due to viscous and thermal effects separately for a thermoacoustic re-
frigeration application. A 38 mm long sample was placed at the center of a 700 mm long electrodynamically-driven plane wave resonator. The central location of the Celcor\textsuperscript{TM} within the standing wave would select different contributions from viscosity and thermal conductivity depending on whether an odd or even mode of the resonator was excited. Following the technique described by Moldover (Moldover et al., pg. 395-399), the resonator quality factors $Q$ were measured between 230 Hz and 1.6 kHz in air at atmospheric pressure on a pristine sample. Wall pores were then blocked by an \textit{in situ} polymerization sealing process. There was no observed decrease in attenuation of even modes (dominated by thermal loss) for the coated sample below 1.0 kHz. The volume exclusion due to pore sealing was inferred by the increase in even-mode resonance frequencies and is in good agreement with other determinations of wall porosity. Thermoviscous losses were computed with DeltaE and compared to measured losses.

The linear motor used in the bellow-bounce test apparatus for flow measurement was characterized and the results were reported in Appendix A. The transduction coefficient, $Bl$, was observed to be a function of position of the magnet. But as shown in this appendix, the variation in $Bl$ with position has a reduced effect on the driver’s output power because $Bl$ is largest around the equilibrium position, where the piston velocity is also largest. By mechanical co-linear joining of the armature of two such motors, an electrodynamic load (dynamometer) was created to measure the efficiency as a function of energy dissipated in the dynamometer. The measured efficiencies are shown to be in good agreement with the predictions if a position-averaged effective transduction coefficient is introduced. Based on these results, this linear motor is judged to be an attractive power source in small electrically driven thermoacoustic refrigerator applications.
Chapter 2

Measurements of the thermoviscous $f_\kappa$-function and the complex Nusselt number

2.1 Theory

The exact solutions for sound propagation in a single circular tube which takes both viscous loss and thermal conductivity loss into account was first given by Kirchhoff (1868). Zwikker and Kosten (1949) introduced a simpler, approximate treatment. The effects of viscosity and thermal conductivity are treated separately and expressed in terms of complex density and compressibility functions subject to the condition that the transverse fluid velocities are much smaller than the longitudinal fluid velocity. The numerical calculations developed by Stinson (1991) confirmed that these approximate expressions and the complete Kirchhoff theory give nearly identical results for frequencies and tube radii in the range that the tube radius is greater than $r = 10^{-3}$ cm and sound frequencies $f_r$ such that $rf_r^{3/2} < 10^6$ cm·s$^{-3/2}$.

In this section, the solution for spatial averaged oscillating velocity and temperature of oscillating flow inside arbitrary shaped pores will be discussed. It will be shown that they both depend on the thermoviscous $f$-function.

The coordinate system used in this section is shown in Fig. 2.1. Following
Swift’s (2002) notation and using Rott’s acoustic approximations (the time dependent variables are small, the penetrations depths are smaller than wavelengths, and only ideal gases are considered), the steady-state sinusoidal variables such as pressure, temperature, and density can be expanded in complex form as:

\[
p(x, y, z, t) = p_m(x) + \text{Re}[p_1(x, y, z)e^{i\omega t}],
\]

\[
v(x, y, z) = \text{Re}[v_1(x, y, z)e^{i\omega t}],
\]

\[
\rho(x, y, z, t) = \rho_m(x) + \text{Re}[\rho_1(x, y, z)e^{i\omega t}],
\]

\[
T(x, y, z, t) = T_m(x) + \text{Re}[T_1(x, y, z)e^{i\omega t}] .
\]

Where subscript \( m \) and 1 imply the mean value and the oscillating component.

Assuming the accessible solid heat capacity per unit volume is much larger than the fluid heat capacity per unit volume inside the pore, then for sinusoidal
oscillations momentum equation in the $x$ direction becomes,

$$
i\omega \rho_m u_1 = -\frac{dp_1}{dx} + \mu \left[ \frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2} \right], \quad (2.5)$$

where $\mu$ is the shear viscosity, $u(x, y, z, t) = \text{Re}[u_1(x, y, z)e^{i\omega t}]$ is the $x$ component of the velocity. The derivative in the $x$ direction was ignored because the wavelength is much larger than the penetration depth.

The acoustic approximation for the general equation for heat transfer is

$$
\rho_m c_p \left( i\omega T_1 + u_1 \frac{dT_m}{dx} \right) - i\omega p_1 = k \left[ \frac{\partial^2 T_1}{\partial y^2} + \frac{\partial^2 T_1}{\partial z^2} \right]. \quad (2.6)
$$

After the spatial average, $\frac{1}{A} \int \text{[ ]} dA$, the averaged solutions for $\langle u_1 \rangle$ and $\langle T_1 \rangle$ are (Swift, 2002):

$$
\langle u_1 \rangle = \frac{1}{i\omega \rho_m} \left( 1 - f_\nu \right) \frac{dp_1}{dx}, \quad (2.7)
$$

$$
\langle T_1 \rangle = \frac{1}{\rho_m c_p} \left( 1 - f_\kappa \right) p_1 - \frac{1}{i\omega A} \frac{dT_m}{dx} \frac{(1 - f_\kappa) - \sigma (1 - f_\nu)}{(1 - f_\nu)(1 - \sigma)} U_1. \quad (2.8)
$$

where $U_1$ is volumetric flow rate:

$$
U_1 = \langle u_1 \rangle A. \quad (2.9)
$$

Without mean temperature gradient, $\frac{dT_m}{dx}$, $\langle T_1 \rangle$ becomes,

$$
\langle T_1 \rangle = \frac{1}{\rho_m c_p} \left( 1 - f_\kappa \right) p_1. \quad (2.10)
$$

It will be necessary to distinguish between the value of a variable at a point in space and its spatially averaged value over a small region. In the case of velocity, such an average value will be in the $x$ direction; we denote it as $\langle u_1 \rangle$. Mean variables do not have microscopic spatial dependence, nor does the pressure amplitude, but for velocity, temperature, and density amplitudes we will distinguish between their microscopic values $u_1$, $T_1$, $\rho_1$ and their local spatial averages $\langle u_1 \rangle$, $\langle T_1 \rangle$, and $\langle \rho_1 \rangle$. The microscopic values have the spatial dependence but the spatial averaged value depend on the $x$ direction.
Figure 2.2. Thermoviscous \( f \)-function for different geometries (Swift, 2002, pp. 88, 89).

\( f_\kappa \) and \( f_\nu \) are the thermoviscous “\( f \)-functions” (also known as the Rott functions and thermoacoustic \( f \)-functions (Wilen, 1998)). They are functions which depend on the ratio of the hydraulic radius of pores (\( r_h \)) to the penetration depths (\( \delta_\nu \), \( \delta_\kappa \)). \( f_\kappa \) depends on \( r_h/\delta_\kappa \) and \( f_\nu \) depends on \( r_h/\delta_\nu \). Physically, \( f_\kappa \) represents the transition from isothermal to adiabatic and \( f_\nu \) represents the transition from viscous flow to inviscid flow for oscillating flow inside pores. The thermoviscous \( f \)-function has been calculated for many geometries such as parallel plates, rectangular pores, circular pores. (Swift, 2002, pp. 88, 89)

The thermoviscous \( f \)-function for parallel plates, rectangular pores, and circular pores are plotted in Fig. 2.2. For small tight pores when \( r_h/\delta \) approaches zero, the real part of the \( f \)-function approaches one and the imaginary part of the \( f \)-function approaches zero. In this limit, from Eq. 2.7 and Eq. 2.10, both the spatial averaged particle velocity \( \langle u_1 \rangle \) and the spatial averaged oscillating temperature \( \langle T_1 \rangle \) approach zero. The pores are so tight that the particle does not move and
the fluid has perfect thermal contact with the solid to expand and compress with no temperature oscillating.

In the other limit of very large pores, when $r_h \gg \delta$, both the real and imaginary parts of the $f$-function approach zero. Eq. 2.7 is reduces to $\langle u_1 \rangle = \frac{1}{i \omega \rho_m} \frac{dp}{dx}$, which is the momentum equation for oscillating inviscid flow, and Eq. 2.10 reduces to $\langle T_1 \rangle = \frac{p_1}{\rho_m c_p}$, which means the fluid expands and compresses adiabatically inside the pore. In the limit of large pores, the fluid inside the pores does not feel the influence of the boundary and behaves as a lossless bulk flow.

For stacked screens and other more complex geometries, the $f$-functions are only approximate (Swift and Ward, 1996) or must be determined empirically.

### 2.2 Measurements of the thermal $f_\kappa$-function

Using the technique described by Wilen (1998), the complex ratio of oscillating displacement, $\xi_1$, to oscillating pressure, $p_1$, is measured to find the thermal $f_\kappa$-function for bulk porous media.

Substituting Eq. 2.10 and the spatially-averaged first order equation of state,

$$\langle \rho \rangle = -\frac{\rho_m}{T_m} \langle T \rangle + \frac{\rho_m}{\rho_m} p_1,$$

into the continuity equation,

$$i \omega \langle \rho \rangle + \frac{d}{dx} \left( \rho_m \langle u \rangle \right) = 0,$$

yields

$$dU_1 = \frac{i \omega A dx}{\gamma \rho_m} \left[ 1 + (\gamma - 1) f_\kappa \right] p_1.$$  (2.13)

Therefore, the ratio of the displacement to the oscillating pressure, $\xi_1/p_1$, equals,

$$\frac{\xi_1}{p_1} = -\frac{L_{\text{eff}}}{\gamma \rho_m} \left[ 1 + (\gamma - 1) f_\kappa \right].$$  (2.14)

The effective length, $L_{\text{eff}}$, was found by extrapolating the experimental results for $\xi_1/p_1$ to zero frequency, $\omega = 0$. At zero frequency, the oscillation of the fluids
inside the pore is isothermal so that,

\[
\frac{\xi_1 (\omega = 0)}{p_1 (\omega = 0)} = -\frac{L_{\text{eff}}}{p_m}.
\]  

(2.15)
2.2.1 Experimental apparatus

![Diagram of experimental apparatus]

**Figure 2.3.** Cross-sectional diagram of experimental apparatus modified for the thermal $f_0$-function measurement. The mean bellows diameter is 1.97" (50.0 mm). The diameter of the testing sample is 1.7" (43.2 mm) and it is 1.8" (45.7 mm) long.

The schematic drawing of the experimental apparatus is shown in Fig. 2.3. This apparatus was original the driver for a thermoacoustic standing-wave refrigerator (called the Shipboard ThermoAcoustic Electronics Chiller(Garrett, 1997)). The electro-magnetic force drives the voice coil, which is attached to the piston moving
up and down sinusoidally. The pressure sensor is a ±10 psid Endevco model 8514-10 piezoresistive differential pressure sensor. The reference port of the pressure sensor is vented to a small chamber and capillary leak that creates a low-pass filter network to permit a 10 psid sensor to function at mean pressures up to 2 MPa. The accelerometer is a Single-Axis ±50 g Analog Devices AD22280 accelerometer. Both sensors were calibrated by comparison to NIST-traceable sensors in the lab.

EG&G Princeton Applied Research Model 5210 lock-in amplifiers were used to measure the amplitude and phase of the oscillating pressure and oscillating acceleration of the piston motion. For sinusoid motion, the displacement is $\xi_1 = -a/\omega^2$, where $a$ is the acceleration of the piston. The two lock-in amplifiers were calibrated to determine their relative phase error ($\pm0.1^\circ$) by measuring the same signal before the measurements. This apparatus could be evacuated and pressurized up to 200 psia (1.4 MPa) and could use different gases to provide a wide range of $r_h/\delta_k$.

The AD22280 accelerometer has a built-in 400 Hz, 2-pole Bessel filter. It was calibrated by comparing to a lab standard accelerometer [Bruel & Kjaer Type 4302, S/N: 1753085] from 30 Hz to 310 Hz. A complex correction factor was applied to correct both the amplitude and the phase shift by the 2-pole Bessel filter. The amplitude correction factor with respect to the B&K is plotted in Fig. 2.4, and the phase correction is plotted in Fig. 2.5
Figure 2.4. The amplitude correction factor for the AD22280 accelerometer with respect to lab standard Bruel & Kjaer Type 4382, S/N: 1753085.

Figure 2.5. The phase correction factor of the AD22280 accelerometer with respect to lab standard Bruel & Kjaer Type 4382, S/N: 1753085. The linear rate of change of phase with respect to frequency is characteristic of a Bessel filter.
2.2.2 Measurements of the 400 cells/in² Celcor™

As shown in Fig. 2.3, the testing sample was placed against the sample holder wall. No fluid could flow through the sample, therefore losses inside the testing sample were dominated by thermal relaxation loss and the viscous losses was ignored since the length of the sample is much smaller than the wavelength. As discussed in Chapter 4, flow losses are dramatically reduced relative to thermal effects because the acoustic impedance $Z_{ac} = p_1/u_1$ is nearly infinite. Equation 2.14 was used to calculate the thermal $f_κ$-function based on the measured ratio of $ξ_1/p_1$. At very low frequency, when the thermal penetration depth is much larger then the pore size, the fluid expands and compresses inside the pore isothermally, and $f_κ ≈ 1$. At very high frequency, the fluid expands and compresses inside the pore adiabatically, and $f_κ ≈ 0$. In the intermediate frequency range, the oscillating temperature, $T_1$, is no longer in-phase with the oscillating pressure, $p_1$, due to the irreversible heat transfer between the fluid and the testing sample; Thus, the thermal $f_κ$-function is a complex number.

From the experimental setup, it is obvious the measured ratio, $ξ_1/p_1$, depends on both the sample and the volume in front of the sample, which includes the inside of the bellows. In all the measurements the wavelength was much larger than the length of the testing sample so that it was reasonable to assume that the pressure was uniform inside the bellows and sample. Based on this assumption, the ratio $ξ_1/p_1$ could be split to two parts:

$$\frac{ξ_{total}}{p_1} = \frac{ξ_{bellows}}{p_1} + \frac{ξ_{sample}}{p_1}$$  \hspace{1cm} (2.16)

Therefore,

$$\frac{ξ_{total}}{p_1} - \frac{ξ_{bellows}}{p_1} = \frac{ξ_{sample}}{p_1} = -\frac{L_{eff}^{sample}}{γp_m} \left[ 1 + (γ - 1) f_κ^{sample} \right].$$  \hspace{1cm} (2.17)

The ratio of $ξ_{bellows}^{sample}/p_1$ was measured by replacing the sample with a flat plate. The subtraction could only be done when both measurements occurred at the same mean pressure, at the same frequency, and using the same fluid. All the measurements were done at the lowest possible amplitude to eliminate amplitude-dependent effects.
A 400 cells/in² Celcor™ sample was tested using the method described above. The square pore size is 1.12 mm, the diameter of the bulk sample is 43.2 mm and the length is 46 mm. The sample, therefore, contained about 900 parallel pores.

\[ y = -6 \times 10^{-9} x^2 - 9 \times 10^{-6} x + 0.0307 \]
\[ R^2 = 0.9982 \]

0.027
0.0275
0.028
0.0285
0.029
0.0295
0.03
0.0305
0 100 200 300 400
frequency (Hz)

Figure 2.6. Curve fitting of \( \xi_{1}^{sample} / p_1 \) at low Lautrec range, \( 0.31 < Lc_\kappa < 0.65 \), to extrapolate to find the equivalent length, \( L_{eff} \), at zero frequency. The test results were taken with helium at 15 psia (103 kPa) from 70 to 310 Hz.

Figure 2.6 shows the curve fit at low Lautrec number \( (Lc_\kappa = r_h/\delta_\kappa) \) range to extrapolate to find the equivalent length of the sample, \( L_{eff} \), at zero frequency. The tests were done using helium at 15 psia (103 kPa) from 70 to 310 Hz. The Lautrec number range was from 0.31 to 0.65. The zero intercept of the curve gave \( L_{eff} = 0.0307(\pm1.3\%) \). Using \( L_{eff} \) and Eq. 2.17, the thermal \( f_\kappa \)-function of 400 cells/in² Celcor™ was calculated and plotted in Fig. 2.7. To cover the Lautrec number from 0.31 to 2.4, the sample was tested at three different mean pressures using helium only. The circles are the test results at 15 psia (103 kPa), the crosses are test results at 50 psia (345 kPa), and the stars are test results at 200 psia (1.38 MPa). The continuous lines were the calculated results also shown in Fig. 2.2. The comparison of the experimental results and the theory showed fairly good
Figure 2.7. Thermal $f_\kappa$-function for 400 cells/in$^2$ Celcor$^{TM}$. The circles are test results at 15 Psia (103 kPa) helium, the crosses are test results at 50 Psia (345 kPa) helium, and the stars are test results at 200 Psia (1.38 MPa) helium. The continuous lines are the theory (Swift, 2002, pp. 88, 89).

agreement for this test. The experimental technique was based on the assumption that the length of the testing volume is much smaller than the wavelength to have uniform pressure inside the testing volume for the subtraction. Helium was chosen as the working gas for this reason. The total empty volume inside the porous testing sample is the volume remaining after subtraction to determine the $f$-function for the testing sample. The total empty volume inside the porous sample should be much larger than the volume occupied by the bellows and piston to minimized the relative error caused by the subtraction.


2.3 Acoustic measurement of porosity

Equation 2.17, which is the complex ratio of displacement to oscillating pressure for the testing sample, is multiplied by the effective cross-sectional area, \( A_{\text{eff}} \), to give \( \frac{dU_{1}^{\text{sample}}}{p_{1}} \), which is the complex ratio of oscillating volume change to oscillating pressure:

\[
\frac{dU_{1}^{\text{sample}}}{p_{1}} = -\frac{V_{0}^{\text{sample}}}{\gamma p_{m}} \left[ 1 + (\gamma - 1) f_{\kappa}^{\text{sample}} \right].
\] (2.18)

At zero frequency, \( V_{0}^{\text{sample}} = L_{\text{eff}} A_{\text{eff}} \) is the total gas volume inside the sample. The porosity of the sample is \( \phi = V_{0}/V_{\text{total}} \). \( V_{\text{total}} \) is the total volume of the testing sample.

The equivalent cross-sectional area of the piston was measured using the same apparatus without the testing sample. The sample holder cap was threaded on the inside wall. A 1/4 inch thick plate was threaded into the the sample holder cap and moved to different places to change the volume of the sample holder \( (V_{0} + V_{i}) \). Since the diameter and length of the sample holder is much larger than the thermal penetration depth, gas compressions and expansions were treated as adiabatic. Therefore,

\[
\frac{\xi_{1} A_{\text{eff}}}{p_{1}} = -\frac{(V_{0} + V_{i})}{\gamma p_{m}}.
\] (2.19)

Plotting \( \xi_{1} \gamma p_{m}/p_{1} \) vs. added volume, \( V_{i} \), the inverse of the slope gave the effective cross section area, \( A_{\text{eff}} \). The test was done using air at 450 Hz, 500 Hz, 550 Hz and 600 Hz. The effective cross section area is \( A_{\text{eff}} = 2.06 \times 10^{-3} \) (±1.6%) m\(^2\). The test results at 550 Hz are shown in Fig. 2.8. The porosity of 400 cells/in\(^2\) Celcor\textsuperscript{TM} measured using this method is \( \phi = 0.90 \) (±5%).
2.4 Complex Nusselt number

For oscillating flow, the heat transfer between the fluid and the pores causes a phase shift between \( \langle T_1 \rangle \) and \( p_1 \), which is characterized by the thermal \( f_\kappa \)-function as shown in Eq. 2.10. It is not an easy task to measure the averaged oscillating temperature, \( \langle T_1 \rangle \), inside the pore. Instead of measuring \( p_1 \) and \( \langle T_1 \rangle \) directly, the complex ratio of the displacement to pressure, \( \xi_1/p_1 \), was measured to determine the \( f_\kappa \)-function for different geometries. The technique was discussed in section 2.2.2.

In this section, the relationship between the Nusselt number and the thermoviscous \( f \)-function is discussed. This provides an acoustic method to characterize the heat transfer behavior of porous media without measuring the temperature and requires no heat exchangers. This is unlike the technique of Gedeon and Wood (1996) or Kornhauser and Smith (1989), thereby making the apparatus much smaller and less complex. The penalty for not measuring \( T_1 \) and heat flow through the heat exchanger is that the measurement method described here is only capable of measuring the heat transfer in the low flow-rate limit. Fortunately for thermoacoustic
applications, that is the important limit. As will be shown later in this chapter, the effects of flow can be related directly to the thermoviscous \( f \)-functions.

Newton’s Law of Cooling states that convective heat flux, \( q'' \), is proportional to the difference between the surface and mean fluid temperatures (Incropera and DeWitt, 1996),

\[
q'' = h(T_s - T_{\text{mean}}),
\]

where it is customary to define a convective heat transfer coefficient, \( h \). The mean velocity-weighted average temperature, \( T_{\text{mean}} \) is defined as

\[
T_{\text{mean}} = \frac{\int_{A_c} \rho u c_v T dA_c}{m c_v},
\]

and \( T_s \) is the temperature of surface. The exact expression of Fourier’s Law of Conduction states that heat transfer is proportional to the temperature gradient at the wall.

\[
q'' = -k \frac{\partial T}{\partial y} \bigg|_{y=0}.
\]

The Nusselt number, \( Nu_r \), can be interpreted as a characteristic dimensionless gas temperature gradient at the wall.

\[
Nu_r = \frac{h r_h}{k} = r_h \frac{\partial \left( \frac{T_{\text{gas}} - T_{\text{reg}}}{(T_{\text{gas}} - T_{\text{reg}})} \right)}{\partial y} \bigg|_{y=0},
\]

where \( T_{\text{reg}} \) is the temperature of the regenerator and \( \langle \rangle \) means the spatial average.

The convective heat transport coefficient has been converted to a non-dimensional Nusselt number by introducing a length scale which allows comparison to the thermal conductivity of the stationary gas, \( k \). The length scale is usually chosen to reflect the ratio of the cross-sectional area of a single pore, \( A_{\text{pore}} \), to the wetted perimeter of the pore, \( \Pi \), that is the hydraulic radius, \( r_h = A_{\text{pore}}/\Pi \). The most common choice for this length scale in the literature on heat exchange is the hydraulic diameter, \( D_h = 4r_h \), therefore \( Nu_D = 4Nu_r \). In this study we chose to use the hydraulic radius to be consistent with the thermoviscous \( f \)-function which is defined as a function of the ratio of the hydraulic radius to the penetration depth.

In steady (DC) flow, the gas-wall temperature gradient, \( \frac{\partial T}{\partial y} \bigg|_{y=0} \), is proportional
and in-phase with the bulk gas-wall temperature difference, $T_{\text{mean}} - T_s$. Therefore, the heat transfer coefficient and Nusselt number are real numbers and Nusselt number is a real constant for fully developed internal flow. But for oscillating flow, the bulk gas-wall temperature difference is not always in-phase with the wall temperature gradient.

As shown in Fig. 2.9, the instantaneous temperature profile of $T_1$ for oscillating flow inside circular pore was plotted using the solution given by Swift (2002, Eq. 4.59). In this plot, the hydraulic radius is five times the thermal penetration depth. Two important features appear in this plot: First, the center of the fluid exhibits “plug flow”; the bulk temperature-wall gradient occurs mostly in a thin layer of fluid close to the wall. Therefore, the thinner the thermal penetration
depth, the greater the temperature gradient. Second, all of the fluid inside the pore is not oscillating in-phase; the thin layer of the fluid next to the wall is leading the phase of the plug flow at the center.

Because of these features of oscillating flow, it is not appropriate to use the heat transfer coefficient calculated or measured from steady-state flow. A complex heat transfer coefficient and thus a complex Nusselt number is required to account for the phase-shifts in the flow. The literature on complex Nusselt numbers is sparse. Kornhauser and Smith (1989) experimentally studied the complex Nusselt number for oscillating pressure and oscillating flow by measuring the surface heat flux and gas wall temperature differences. Later Kornhauser and Smith (1994) did a similar study of complex Nusselt number from pressure-volume measurements only: instantaneous surface-averaged heat transfer and mixed mean gas temperature were calculated from the pressure, volume, and time data. Gedeon (1986) calculated the exact solution for complex Nusselt number in incompressible flow between parallel plates based on a mean-parameter model.

The calculated complex Nusselt number based on the thermoviscous $f$-function will be calculated for the first time in next section. The agreement with Gedeon’s theoretical expression was excellent. Gedeon also pointed out the advected energy calculated from the mean-parameter model deviates from the exact solution, a correction factor that needs to be added when using the mean-parameter model.
2.4.1 Complex Nusselt number for heat transfer due to compression and expansion of oscillating flow

Consider the general equation of heat transfer for an inviscid ideal gas (Laudau and Lifshitz, 1959),

\[ \rho c_p \frac{\partial T}{\partial t} - \frac{\partial p}{\partial t} + \rho c_p \mathbf{v} \cdot \nabla T - \mathbf{v} \cdot \nabla p = \nabla \cdot (k \nabla T) \]  

This is the exact equation from the microscopic view in the fluid-filled space. The \( \mathbf{v} \cdot \nabla p \) term which is proportional to \( uKp \) is small compared to \( \frac{\partial p}{\partial t} \) term which is proportional to \( \omega p \) because \( \omega / K = c \gg v \), where \( c \) is the sound speed, \( u \) is the particle velocity, and \( K \) is the wavenumber. In the absence of a static temperature gradient, Eq. 2.24 can be reduced to

\[ \rho c_p \frac{\partial T}{\partial t} - \frac{\partial p}{\partial t} = \nabla \cdot (k \nabla T). \]  

Next, we perform a local spatial average of Eq. 2.25 over the gas volume inside the pore of length \( dx \) and cross-sectional area \( A \) (Swift and Ward, 1996). On the right hand side, Divergence theorem is used to express the volume integral as the divergence of the heat flux density \( k \nabla T \) as a surface integral over the gas-solid wetted area \( S \) within the volume of integration. The result is

\[ Adx \left( \left\langle \frac{\rho c_p}{\partial t} \right\rangle - \frac{\partial p}{\partial t} \right) = \int dS k \nabla T. \]  

where \( \langle \rangle \) is the spatial average.

Using the definition of heat transfer coefficient in the presence of an oscillating pressure, \( h_p \), and the definition of hydraulic radius \( r_h \), which is the ratio of the gas volume to gas-wetted surface area, we obtain

\[ \left\langle \frac{\rho c_p}{\partial t} \right\rangle - \frac{\partial p}{\partial t} = -\frac{h_p}{r_h} (\langle T \rangle - T_m). \]  

Spatial averaged temperature \( \langle T \rangle = \frac{1}{A} \int T dA \) is used here instead of the velocity-weighted section average temperature in Eq. 2.21. Using Rott’s approximation, Eq. 2.4, \( T(x, y, z, t) = T_m(x) + \text{Re}[T_1(x, y, z)e^{i\omega t}] \), and substituting into Eq. 2.27,
\[
\left\langle \rho c_p \frac{\partial T_1}{\partial t} \right\rangle - \frac{\partial p_1}{\partial t} = -\frac{h_p}{r_h} \langle T_1 \rangle. \quad (2.28)
\]

Solving for \( T_1 \) and substituting into equation Eq. 2.10, the complex heat transfer coefficient can be written in term of thermal \( f_\kappa \)-function,

\[
h_p = i \omega \rho_m c_p r_h \left( \frac{f_\kappa}{1 - f_\kappa} \right). \quad (2.29)
\]

Using the definition of the penetration depth,

\[
\delta_\kappa^2 = \frac{2k}{\rho c_p \omega}, \quad (2.30)
\]

\( h_p \) can be expressed in terms of the penetration depth as

\[
h_p = \frac{2ikr_h}{\delta_\kappa^2} \left( \frac{f_\kappa}{1 - f_\kappa} \right). \quad (2.31)
\]

When the penetration depth is much smaller than the hydraulic radius of the pores, the boundary layer limits for the thermoviscous \( f \)-function are (Swift, 2002),

\[
f_\kappa \approx (1 - i) \frac{\delta_\kappa}{2r_h} \quad (2.32)
\]

\[
f_\nu \approx (1 - i) \frac{\delta_\nu}{2r_h}. \quad (2.33)
\]

Substituting Eq. 2.32 into Eq. 2.31 and taking the limit that

\[
\lim_{\omega \to \infty} \frac{\delta_\kappa}{r_h} = 0, \quad (2.34)
\]

the boundary layer limit of \( h_p \) becomes

\[
\lim_{\omega \to \infty} h_p = \frac{k}{\delta_\kappa} (1 + i). \quad (2.35)
\]

The complex Nusselt number for oscillating flow, subject to oscillating pressure, is \( Nu_p \)

\[
Nu_p = \frac{h_p r_h}{k} = 2i(Lc_\kappa)^2 \left( \frac{f_\kappa}{1 - f_\kappa} \right). \quad (2.36)
\]
Figure 2.10. Complex Nusselt number for circular pores, square pores, and parallel plates subject to oscillating pressure. The circles are measured results of 400 cells/in$^2$ Celcor$^{\text{TM}}$. The black diamond symbols are the boundary layer limit from Eq. 2.37.

where the Lautrec number is $Lc_\kappa = r_h/\delta_\kappa$. The boundary layer limit for $Nu_p$ is

$$\lim_{\omega \to \infty} Nu_p = \frac{r_h}{\delta_\kappa}(1 + i).$$  \hfill (2.37)

The complex Nusselt numbers for circular pores, square pores, and parallel plates, subject to oscillating pressure, are plotted in Fig. 2.10. The circles are from the measured $f_\kappa$-function for 400 cells/in$^2$ Celcor$^{\text{TM}}$. The boundary layer limit based on Eq. 2.37 are plotted in black diamond symbols. The real part of the Nusselt number is much larger than the imaginary part at low Lautrec numbers. That means the spatial averaged temperature is in-phase with the gas-wall temperature gradient when the penetration depth is much larger than the pore size as it would be in a regenerator.
2.4.2 Complex Nusselt number for oscillating flow subject to temperature gradient

The dimensionless form of Eq. 2.24 is

\[ \frac{L}{u/i \omega} T^* + u^* \frac{\partial T^*}{\partial x^*} - \frac{i \omega u_m L}{c_p T_m} p^* - \frac{u_m^2}{c_p T_m} u^* \frac{\partial p^*}{\partial x^*} = \frac{k/r_h}{\rho c_p u_m} \frac{L}{r_h} \frac{\partial^2 T^*}{\partial y^2} \]  

(2.38)

\[ u^* = \frac{u}{u_0}, T^* = \frac{T}{T_m}, p^* = \frac{p}{\rho u_0^2}. \]

\( L \) is the scale length in the longitudinal direction, \( r_h \) is the scale length in the lateral direction, and \( u_0 \) is the reference velocity amplitude of the oscillating velocity. In regenerators and stacks, \( c_p T_m \gg \omega u_m L \), therefore the coefficient of the third and fourth terms are both much smaller than unity since \( u_m \sim \omega L \). Ignoring those two terms and assuming the temperature gradient is dominated by mean temperature gradient in the \( x \)-direction only, the dimensional heat transfer equation of oscillating flow becomes

\[ \rho c_p \left( \frac{\partial T}{\partial t} + u \frac{d T_m}{d x} \right) = \nabla \cdot (k \nabla T). \]  

(2.39)

After the spatial average and defining another heat transfer coefficient, \( h_T \), the heat transfer coefficient of oscillating flow subject to a temperature gradient, we obtain

\[ \langle \rho c_p \frac{\partial T_1}{\partial t} \rangle + \rho c_p u \frac{d T_m}{d x} = - \frac{h_T}{r_h} \langle T_1 \rangle. \]  

(2.40)

Solving for \( \langle T_1 \rangle \), then equating \( \langle T_1 \rangle \) with solution given by Eq. 2.8 by setting \( p_1 = 0 \), the heat transfer coefficient \( h_T \) can be expressed in terms of thermoviscous \( f \)-functions,

\[ h_T = -i \omega \rho_m c_p r_h \left[ 1 - \frac{(1 - f_\nu)(1 - \sigma)}{(1 - f_\kappa) - \sigma(1 - f_\nu)} \right]. \]  

(2.41)

\( \sigma \) is the Prandtl number. The complex Nusselt number for oscillating flow subject to temperature gradient, \( Nu_T \), is
Figure 2.11. Comparison between Gedeon’s $Nu_T$ and the results derived in this chapter. Only one line is seen for the real part of $Nu_T$ because both results are in exact agreement.

$$Nu_T = \frac{h_T r_h}{k} = 2i(Lc_k)^2 \left[ \frac{(1-f_\nu)(1-\sigma)}{(1-f_\kappa)-\sigma(1-f_\nu)} - 1 \right]. \quad (2.42)$$

$Nu_p$ depends on thermal boundary layer only but $Nu_T$ depends on both viscous and thermal boundary layer. That is because temperature profile is related to the velocity profile when a velocity is also imposed. Gedeon (1986) solved the mean-parameter model of $Nu_T$ for oscillating flow between parallel plates. Gedeon’s results and $Nu_T$ from Eq. 2.42 are plotted in Fig. 2.11. The agreement is excellent for the real part and for the imaginary part at high Lautrec numbers. In Gedeon’s paper, the imaginary part of $Nu_T$ smaller than 0.1 was not plotted.

From Gedeon’s paper (Gedeon, 1986), using hydraulic diameter, $D_h = 4r_h$, as
the characteristic length,

$$
\lim_{\omega \to 0} Nu_T^{D_h} = 10 = Nu_0.
$$

(2.43)

$Nu_0$ is the Nusselt number for steady laminar flow between parallel plates under condition of uniform heat flux and when the film coefficient is in terms of section-average temperature rather than bulk (velocity-weighted section averaged) temperature (Jacob, 1949). Using $r_h$ in this work, it converges to 2.5 as shown in Fig. 2.11. Therefore, when penetration depth is much larger than the pore size, the steady state flow results are applicable to oscillating flow.

The complex Nusselt number for oscillating flow inside circular pores, square
pores and parallel plates subject to a temperature gradient and choosing Prandtl number to be 0.68 are plotted in Fig. 2.12.

Substituting Eq. 2.32 and Eq. 2.33 into Eq. 2.41, using the fact that, $\delta_\nu = \sqrt{\sigma} \delta_\kappa$, and taking the limit $\delta_\kappa/r_h \to 0$, the boundary layer limit of $h_T$ becomes

$$\lim_{\omega \to \infty} h_T = \frac{k}{\delta_\kappa} (1 + i) \frac{1 - \sqrt{\sigma}}{1 - \sigma}.$$  (2.44)

The coefficient, $C_T = \frac{1 - \sqrt{\sigma}}{1 - \sigma}$ is Prandtl number dependent. It is plotted in Fig. 2.13. For $\sigma = 0.68$, Eq. 2.44 becomes

$$\lim_{\omega \to \infty} h_T = 0.55 \frac{k}{\delta_\kappa} (1 + i).$$  (2.45)

When the thermal penetration depth is smaller than the hydraulic radius, the
heat transfer coefficient is proportional to the thermal conductivity of the gas divided by the thermal penetration depth. Mozurkewich (1998) approximated the coefficient to be 0.61 for \( \sigma = 0.68 \) from the eigen-solutions of a boundary-value problem he formulated to study the time-averaged temperature distribution in a thermoacoustic stack. The gas temperature he used was that at the center of the pore. He also pointed out that this coefficient is Prandtl number dependent.

From Eq. 2.44, the boundary layer limit of \( Nu_T \) becomes

\[
\lim_{\omega \to \infty} Nu_T = \frac{r_h}{\delta_\kappa} (1 + i) \frac{1 - \sqrt{\sigma}}{1 - \sigma}.
\]

(2.46)

It is plotted in Fig. 2.12 in black diamond symbols.

The boundary value limit for \( h_T \) is important for the design of heat exchangers used in thermoacoustic machines. In addition to the value of \( C_T \) calculated here (0.55) and calculated by Mozurkewich (0.61), Garrett, Perkins, and Gopinath (1994) assumed \( C_T = 1 \) but then reduced its value to 0.76 to account for the sinusoidal velocity variation. Wakeland and Keolian (2004) gave a value of 0.45 at small gas displacements although a value of unity is approached when gas displacements are comparable to the thickness of the heat exchanger. The specific values of the boundary layer limit depend on which assumptions and what kind of averaged temperature were chosen. In this study the spatially averaged temperature was used whereas Mozurkewich used the temperature at the center of the pore, and Wakeland averaged the hot and cold heat-exchanger temperatures.

Application of similitude at \( Lc_\kappa > 1 \) guarantees

\[
\lim_{\omega \to \infty} h \propto \frac{k}{\delta_\kappa}.
\]

(2.47)

Because the velocity gradient and temperature gradient both depend on the penetration depth when \( Lc_\kappa > 1 \), the heat transfer coefficient and the Nusselt number are geometry independent. However, when \( Lc_\kappa < 1 \), both the velocity gradient and temperature gradient are characterized by the pore size so that the Nusselt number and heat transfer coefficient are geometry dependent. These features are shown in Fig. 2.10 and Fig. 2.12.
2.5 Summary and conclusions

A test apparatus was designed for measuring the thermal $f_\kappa$-function. Test results for a calculable geometry, the ceramic Celcor\textsuperscript{TM} with parallel square pores, matched favorably with the linear theory prediction.

It is shown that oscillatory flow requires a complex Nusselt number and the complex Nusselt number can be related to the thermoviscous $f$-functions. The complex Nusselt number for heat transfer due to compression and expansion of oscillating flow, $Nu_p$, and the complex Nusselt number for oscillating flow subject to temperature gradient, $Nu_T$, were calculated for the first time in terms of the thermoviscous $f$-function. The agreement with Gedeon’s (1986) results for $Nu_T$ is excellent.

By relating the complex Nusselt number to the thermoviscous $f$-function, the heat transfer behavior of oscillating flow in porous media can be characterized more quickly and accurately without measuring the temperature and requires no heat exchangers. It has also been shown that the $f$-function interpretation of the Nusselt number for oscillating flow produces results that are consistent with several previous theoretical and experimental investigations of heat exchangers in oscillating flow in the appropriate limit ($Lc_\kappa \geq 1$).
Chapter 3

Dynamic similitude and nonlinearity of oscillating flow through porous media

3.1 Introduction

In Chapter 2 a method for measuring the thermal $f_\kappa$-function was presented. Due to the similarities between the momentum equation and the general heat transfer equation, the thermoviscous $f$-function fully characterizes both the dynamic and thermal behavior of oscillating flow in porous media in the laminar flow range. In many applications the velocity of the flow through the porous media can be high enough that the assumption of small perturbation-based linear theory is not valid.

Experimental results for oscillating flow through porous media are less common than experimental results for steady flow. Zhao and Cheng (1996) measured the pressure drop due to oscillating flow through stacked screens. They pointed out that the oscillatory pressure drop (friction) factor, which is the dimensionless pressure gradient, increases with the kinetic Reynolds number, $\omega D_h^2/\nu$, and with the dimensionless fluid displacement. $D_h$ is the hydraulic diameter which is four times the hydraulic radius, $D_h = 4r_h$. A correlation equation for pressure drop using these two parameters, the kinetic Reynolds number and the dimensionless displacement was given. They found that the value of the cycle-averaged pressure
The cycle-averaged total pressure drop for oscillatory flow includes the combined effects of viscosity and (oscillating) inertia. Comparing the cycle-averaged total pressure drop to the steady (DC) flow pressure drop without incorporating phase information was bound to produce discrepancies between the friction factor for AC and DC flows.

Tanaka, Yamashita, and Chisaka (1990) studied various regenerator materials subject to oscillating flow with fixed fluid displacement amplitude. Their correlation function for the friction factor used the Reynolds number as the only parameter and the data collapsed when using the hydraulic radius as the characteristic length for the Reynolds number. No phase information between pressure drop and velocity was given. For most of their test results, the viscous penetration depth was larger than the hydraulic radius of the pores, so the measured pressure drop was dominated by viscous effects. For that case, the test data correlate to Reynolds number only.

Gedeon and Wood (1996) measured the pumping dissipation inferred from the work done by the piston to move fluid through the samples to determine the pressure drop. Their correlation function of the friction drag factor was comparable to the steady flow results (Kays and London, 1998). The correlation function inferred from piston work was due to viscous dissipation only, so that it is comparable to the steady flow results. No frequency dependency was observed in their test results. This is probably because the hydraulic radius is smaller than the penetration depth for most of their samples. Since their investigations were directed toward improvements in Stirling engine design, the focus on hydraulic radii smaller than penetration depth was justified.

In this chapter, we develop a measurement technique that exploits phase-sensitive detection of the pressure drop and flow using lock-in amplifiers to separate the effects of viscosity and inertia in bulk samples of porous media. A similar approach was employed by Professor Wilen and his students at Ohio University (Wilen, 1998) to measure thermoviscous behavior within individual pores.
As a further extension of Wilen’s approach, the apparatus described in this chapter has allowed exploration of the friction factor in Reynolds numbers regimes that show significant deviations from the “laminar flow” results that are shown to be characterized completely by the Rott thermoviscous $f$-functions.

### 3.2 Dimensional analysis

Similitude is used to reduce the number of independent variables in order to correlate results of experiments done under different conditions and reduce experimentation time to acquire data in regions of parameter space that have not been explored (Strutt, 1915). Similitude is complete as long as all the necessary variables have been included in the problem. The functional form will scale correctly even for turbulent flow when an analytical solution is not available. Olsen and Swift (1994) have shown this approach to be very useful when applied to thermoacoustic engines. Similitude fails if the dimensionless parameters do not form a complete representation of the physical effects.

The following dimensionless variable groups are obtained by non-dimensionizing the one-dimensional Navier-Stokes equation for flow oscillating at angular frequency $\omega = 2\pi f$, as shown in Eq. 3.1.

$$
\frac{i\omega r_h u^*}{u_0} + u^*_x \frac{\partial u^*}{\partial x^*} = -\frac{\partial p^*}{\partial x^*} + \frac{1}{Re} \frac{\partial^2 u^*}{\partial y^*^2} \tag{3.1}
$$

$$
u^* = \frac{u}{u_0}, y^* = \frac{y}{r_h}, x^* = \frac{x}{r_h}, \Delta p^* = \frac{\Delta p}{\rho u_0^2}, Re = \frac{u_0 r_h}{\mu}
$$

$f$ is the frequency, $\mu$ is the shear viscosity of the fluid, $\rho$ is the density of the fluid, $u_0$ is the reference velocity amplitude of the oscillating velocity, and $L$ is the sample length along the direction of oscillatory flow. The gradient of the velocity in $x$ direction is characterized by the wavelength, $\lambda$. The gradient of the velocity in $y$ direction is characterized by the viscous penetration depth, $\delta_\nu$. The derivatives in the $x$ direction are neglected here since $\lambda \gg \delta_\nu$.

For uniform capillary arrays with square cross-section, the advection term $u^*_x \frac{\partial u^*}{\partial x^*}$ is absent because there is no velocity change due to the change of the cross-sectional
area. In addition, if the flow is inviscid, the dimensionless pressure gradient is only a function of the dimensionless displacement.

\[
\frac{i \omega r h}{u_0} u^* = -\frac{\partial p^*}{\partial x^*}.
\]  

(3.2)

For the steady (DC) viscous flow, the dimensionless pressure gradient is only a function of the Reynolds number,

\[
\frac{\partial p^*}{\partial x^*} = \frac{1}{Re} \frac{\partial^2 u^*}{\partial y^2}.
\]  

(3.3)

Comparing Eq. 3.3 for steady (DC) flow to Eq. 3.1 for oscillatory flow, the oscillatory flow has one more dimensionless variable, \( \frac{u}{\omega r h} \), the dimensionless displacement, due to the dependence on frequency.

As shown in Eq. 3.3, Reynolds number is the coefficient of the dimensionless equation where the pressure was nondimensionalized by the momentum effect, \( \rho u^2 \), so that Reynolds number is the ratio of momentum effects to viscous effects. Similarly, dimensionless displacement is the ratio of momentum effects to oscillating inertial effects. In steady (DC) flow, the pressure gradient depends only on the momentum and viscous effects, therefore systems having the same Reynolds number would satisfy dynamic similitude. For oscillating flow, the pressure gradient depends on a momentum effects, a viscous effects, and an oscillating inertia effects; to satisfy dynamic similitude we need one more dimensionless parameter in addition to the Reynolds number. For oscillating flow to obey dynamic similitude, both the Reynolds number and dimensionless displacement \( \frac{u}{\omega r h} \) must be the same.

The ratio of oscillating inertial effects to viscous effects is \( \frac{r^2}{\delta^2} \). The viscous penetration depth tells how far the momentum can diffuse laterally during a time interval on the order of the period of the oscillation divided by \( \pi \). In accordance with an earlier publication (Garrett, 2004) the dimensionless parameter \( Lc_\nu = \frac{r_\nu}{\delta_\nu} \) is introduced here as viscous Lautrec number. The Reynolds number, viscous Lautrec number, and dimensionless displacement are dependent upon each other in such a way that, \( \frac{u}{\omega r h} = \frac{1}{2} \frac{Re}{Lc_\nu^2} \). Dynamic similitude will be satisfied for oscillatory flow if any two of those three dimensionless groups are the same in two different systems. In the following analysis, we choose to correlate the dimensionless pressure gradient...
with Reynolds number and viscous Lautrec number.

### 3.3 Experimental apparatus

A cross-sectional diagram of the apparatus is shown in Fig. 3.1. The porous sample is contained within a plastic sample holder that has the same dimensions as the holder used by Gedeon and Wood (1996). This choice was made so that the same samples used in the Gedeon & Wood apparatus could be re-examined using our apparatus described in this section. (The results of the re-measurement of the Gedeon/Wood samples will be included in future work).

Inside the gas filled metal bellows, a “volume excluder” is fitted in the empty space to increase the amplitude of the oscillating pressure for a given piston stroke. The bellows and piston are attached to a high-power, moving-magnet linear motor. Details on the parameters which characterized the motor and the measurement of these parameters are presented in Appendix A. The harmonic linear motion of the piston drives the fluid back and forth between the front volume (bellows) and the back volume through the porous sample.

The pressure oscillations within the front volume, \( p_{\text{front}} \), and back volume, \( p_{\text{back}} \), together with the phase difference between \( p_{\text{front}} \) and \( p_{\text{back}} \), would show the dynamics of the fluid flowing through the porous material and allow separation of viscous and inertial effects. Since the characteristic lengths, \( V_3^{\frac{1}{3}} \), of either volume are small compared to the wavelength of sound in the gas they contain \( \left( V_3^{\frac{1}{3}} < \frac{\lambda}{20} \right) \), both the front volume and the back volume can be treated as “lumped elements” (Morse, 1981).

The driver, bellows, and sample, were all enclosed within the pressure vessel. The pressure vessel could be evacuated or pressurized up to 200 psia (1.4 MPa) with different gases that provided a wide range of testing parameters. A cylindrical spring (Garrett, 2003) was used to increase the resonance frequency and make it less dependent on mean pressure. A detailed DELTAE output file of this bellows-bounce apparatus is included in Appendix B.

The pressure sensors shown in Fig. 3.1 are both Endevco model 8530B-200 piezoresistive sensors. EG&G Princeton Applied Research Model 5210 lock-in
amplifiers were used to measure the amplitude and phase of the oscillating pressure in the front volume and the back volume. The two lock-in amplifiers were calibrated to determine their relative phase error (± 0.1°) by measuring the same electrical signal before making the pressure measurements. The phase-sensitive measurement allows separation of viscous and inertial effects (Petculescu and Wilen, 2003).

Knowing the back volume and the oscillating pressure amplitude, the volume
velocity, $U_1$, passing through the sample could be calculated (Swift, 2002):

$$U_1 = \frac{-i\omega p_{\text{back}} V_{\text{back}}}{\gamma p_m}.$$  \hfill (3.4)

$p_m$ is the mean pressure, $p_{\text{back}}$ is the amplitude of the oscillating pressure, $V_{\text{back}}$ is the back volume, and $\gamma$ is the ratio of specific heats (polytropic coefficient) for the gas. The portion of the volume velocity due to thermal relaxation is ignored here. The thermal relaxation effect is only 2.9% of the adiabatic compression effect of the gas (Eq. 3.4) at the lowest frequency and lowest mean pressure operating point where the effect of the isothermal boundary layer would be greatest.

$V_{\text{back}}$ was measured acoustically. The testing sample was replaced by an open duct to form a Helmholtz resonator with the back volume. The resonance frequency was measured with an HP 3562A Dynamic Signal Analyzer using the non-linear pole-zero curve fit technique (The details of this technique are presented in Section 4.2.3). Varying the back volume by adding standard copper weights, the changes of the resonance frequency were measured. The inverse of the frequency squared was plotted and curve-fit to the change of the back volume as described previously in Section 2.3 and Fig. 2.8. The slope and the intercept were used to extract the back volume based on the relation,

$$\omega = c \sqrt{\frac{s}{L' V_{\text{back}}}}$$  \hfill (Kinsler et al., 2000, Eq. 10.8.8),

where $c$ is the sound speed, $S$ is the cross-section area of the open tube, and $L'$ is the effective length of the duct. The measured result is $V_{\text{back}} = 2.8 \times 10^{-4}$ m$^3$ (±3.8%).

Fig. 3.2 is a schematic representation of the phasor diagram of the pressure drop of the fluid through the porous sample. $\theta$ is the phase difference between the front volume and back volume oscillating pressure measured by the lock-in amplifiers. Assuming the oscillating pressure in the back volume is lossless, the volume velocity, $U_1$, is 90° out-of-phase with the oscillating pressure in the back volume $p_{\text{back}}$. The pressure drop across the sample, $\Delta p$, could be split into two parts with respect to $U_1$: The dissipative pressure drop due to viscosity, $\Delta p_{\text{vis}}$, is 180° out-of-phase with the volume velocity, and the pressure drop due to oscillation inertia, $\Delta p_{\text{iner}}$, lags 90° behind the volume velocity. $p_{\text{front}}$, $p_{\text{back}}$, and $\theta$ were measured quantities; $\Delta p_{\text{vis}}$ and $\Delta p_{\text{iner}}$ are calculated as shown in Fig. 3.2.
Figure 3.2. Phasor diagram showing measured front and back volume oscillating pressure and their relative phase, $\theta$, the volumetric velocity through the porous samples, and the viscous and inertial components of the pressure drop, $\Delta p$, across the sample.

$$\Delta p_{vis} = p_{front} \sin \theta$$  \hspace{1cm} (3.5)

$$\Delta p_{iner} = p_{back} - p_{front} \cos \theta.$$  \hspace{1cm} (3.6)

3.4 Results for samples of calculable geometry

Two Celcor™ samples were tested using the apparatus described above. Both samples had a diameter of 19 mm. One sample is a 0.8 inch (2.03 cm) long 600 cells/in$^2$ Celcor™; the other one is a 1.6 inch (4.06 cm) long 400 cells/in$^2$ Celcor™. The hydraulic radius of the 600 cells/in$^2$ Celcor™ is 0.23 mm and the hydraulic radius of the 400 cells/in$^2$ Celcor™ is 0.28 mm. Three different
gases: helium (atomic weight 4.0023 amu), argon (atomic weight 39.944 amu) and sulfur hexafluoride (SF$_6$, atomic weight 146.07 amu), were used for the test and the operating ambient pressure was varied from 0.5 atm to 12 atm. The operating frequency was tuned to the mechanical resonance frequency determined by the combined stiffness of the pressurized gas, bellows and cylindrical spring, and the moving mass of the piston and motor at each operating condition to allow the maximum stroke using the linear motor. This combination of gases, pressures, and piston stroke provided a Reynolds number range from 6 to over 2000 and the viscous Lautrec number range from 0.27 to 7.7.

### 3.4.1 Pressure gradient due to viscosity

Figure 3.3 shows the test results for viscous drag in the 400 Celcor™ sample using helium as the testing fluid. The viscous Lautrec number ranges from 0.24 to 0.76. Four groups of data with different viscous penetration depths, all larger than the hydraulic radius, collapsed together over 2.5 orders-of-magnitude in Reynolds number by plotting the dimensionless pressure gradient vs. Reynolds number, using $r_h$ as the characteristic length. For internal flow, the viscous pressure gradient in the axial direction depends on the shear force gradient in the lateral direction; shear force in the axial direction depends on the velocity gradient in the lateral direction so that two characteristic lengths are used in the lateral direction in the dimensionless correlation function. For fully developed internal steady (DC) flow, both the velocity gradient and the shear gradient in the lateral direction were characterized by the physical geometry of the pore, so the dimensionless pressure gradient correlates to Reynolds number only. Hydraulic radius is the only relevant characteristic length. For oscillating flow, at $Lc_\nu < 1$, the velocity profile has a parabolic shape that is characteristic of Poiseuille flow (Laudau and Lifshitz, 1959); The velocity gradient and shear gradient also depend only on pore geometry so that the dimensionless pressure gradient due to the fact that viscosity correlates to Reynolds number only.

Figure 3.4 shows the correlation of the dimensionless viscous pressure gradient vs. the Reynolds number for the 0.8 inch long 600 Celcor™ sample using argon as the test fluid. The viscous Lautrec number ranges from 1.36 to 3.7. This implies
that the viscous penetration depth is smaller than the hydraulic radius for all of these five measurements. The straight lines are the prediction of linear theory (Swift, 2002, Eq. 4.54 and 4.62). The experimental results match the linear theory at only low Reynolds numbers and begin to deviate at Reynolds number (based on hydraulic radius) above 50. As shown in Fig. 3.4, use of the Reynolds number only was not adequate to collapse all the data. The pressure gradients depend on both the Reynolds number and the viscous Lautrec number.

For oscillating flow, when the viscous penetration depth is smaller than the hydraulic radius ($Lc_\nu > 1$), the velocity gradient in the lateral direction is characterized by the viscous penetration depth instead of the pore’s geometry. This
Figure 3.4. Dimensionless pressure gradient due to viscosity vs. Reynolds number for 600 cells/in$^2$ Celcor™ sample testing in argon. The straight lines are the linear theory predictions which match the measured data only at the lowest values of Reynolds number.

explains the viscous Lautrec number dependency observed in Fig. 3.4. The thinner the viscous penetration depth is, the bigger the velocity gradient would be, thus the bigger the pressure gradient due to viscosity.

When $Lc_v \gg 1$, in the boundary layer regime, all of the viscous effects will be restricted to a very thin layer close to the pore wall. In this thin layer, the fluid will oscillate in-phase with the pressure gradient, unlike the fluid in the center of the pore which oscillates with a “plug flow” profile and is 90 degree out-of-phase with the pressure gradient. Both velocity and shear gradients only occur in this thin layer and depend on the viscous penetration depth only. In other words, for a given oscillating pressure gradient, no matter how big the pore is the amplitude
of the instantaneous velocity will depend on the penetration depth only.

The velocity we measured here is the spatial-averaged velocity, \( \langle u \rangle = \frac{1}{A} \int u(r) dA \). Although the instantaneous velocity is the same for different pores with the same penetration depth, the viscous part of the spatial-averaged velocity is smaller for big pores since the ratio of the cross-sectional area occupied by the viscous boundary layer (\( \approx 2\pi r\delta_v \)) to the total cross-sectional area (\( \pi r^2 \)) is smaller for bigger pores (for pores of circular cross-section, \( r = 2r_h \)). Therefore, correlating to the Reynolds number using spatial averaged velocity in the region where the penetration depth is smaller than pore size, both penetration depth and hydraulic radius should be included as characteristic lengths.

Taking this into consideration, using both the viscous penetration depth and hydraulic radius as the characteristic lengths in the lateral direction, the dimensionless pressure gradient was correlated to the acoustic Reynolds number, \( \text{Re}_v = \frac{u \rho \delta_v}{\mu} = \frac{\text{Re}}{L_c} \), and the correlation function becomes \( \frac{\mu u \rho \delta_v}{\rho u \delta_v} \left( \frac{r_h}{L_c} \right) = f \left( \frac{\mu u \rho \delta_v}{\rho u \delta_v} \right) \). Figure 3.5 shows that by using this the same five groups of data from Fig. 3.4 collapse to form a single line.

To summarize, when \( L_c > 1 \) the dimensionless pressure gradient normalized by the ratio of hydraulic radius to sample length correlates to acoustic Reynolds number, where the scale length for acoustic Reynolds number is the viscous penetration depth not the pore hydraulic radius. The correlation function used both viscous penetration depth and hydraulic radius as the characteristic lengths in the lateral direction. When \( L_c < 1 \), the dimensionless pressure gradient correlates to Reynolds number only, while the correlation function used hydraulic radius only as the characteristic length. The transition from using the hydraulic radius only to using both the Reynolds number and the viscous penetration depth happens smoothly when the viscous penetration depth and the hydraulic radius are equal. To combine these rules together, both the helium and argon results of dimensionless viscous pressure gradient were plotted vs. \( \text{Re}/L_c \) modified by setting \( L_c = 1 \) when \( L_c < 1 \). The data from two different fluid and two different pore size collapsed nicely to a single line spanning three orders-of-magnitude in this modified Reynolds number as shown in Fig. 3.6.

To extend the viscous Lautre number for the viscous penetration depth much
smaller then the pores, sulfur hexafluoride was chosen for its high density to test with the 600 Celcor\textsuperscript{TM} sample. This extended the range of the Lautrec number from 4 to 7.7. The dimensionless pressure gradients due to viscosity are plotted in Fig. 3.7. The straight lines are the linear theory prediction for the SF\textsubscript{6}. The linear theory prediction matched the experimental results poorly at both high Reynolds number and high viscous Lautrec number. This deviation will be discussed more in Section 3.6.2.

The pressure drop due to viscosity vs. velocity is plotted in Fig. 3.8. The symbols are the experimental data, and the two lines are the linear theory predictions. The red line corresponds to the red cross symbols for SF\textsubscript{6} data at $Lc_{\nu} = 7.7$. The

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.5.png}
\caption{Dimensionless pressure gradient due to viscosity vs. acoustic Reynolds number for the same data that was presented in Fig. 3.4 but incorporating both $r_h$ and $\delta_{\nu}$ together as the lateral characteristic lengths, thus collapsing the data from Fig. 3.4 on to a single line.}
\end{figure}
blue line corresponds to the blue circles for helium data at \( Lc_\nu = 1.37 \). It is shown in this plot that the viscous pressure drop departs from the linear theory prediction quickly and depends on the velocity quadratically at larger values of velocity. The deviation occurs more quickly at smaller viscous penetration depths.
Figure 3.7. Dimensionless pressure gradient due to viscosity vs. Reynolds number for 600 cells/in$^2$ Celcor$^\text{TM}$ SF$_6$ test data. The straight lines are the linear theory prediction.

### 3.4.2 Pressure gradient due to inertance at $L_c > 1$

As shown in Fig. 3.2 (the phasor diagram) the pressure drop due to inertance lags by 90° with respect to the mean velocity. Figure 3.9 shows the correlation of dimensionless pressure gradient due to the inertance vs. Reynolds number for the 600 cells/in$^2$ Celcor$^\text{TM}$ argon test results. The straight lines are the linear theory predictions. The test results are in good agreement with the theory except at high Reynolds numbers, where the test results slightly bend down from the straight line. This will be discussed further in Section 3.6.1 together with the stacked-screen results.

In Fig. 3.10, the dimensionless pressure gradient due to inertance is correlated to the dimensionless displacement, $\frac{u}{\omega r h} = \frac{Re}{2L_c}$. The same five groups of data from
Figure 3.8. Comparison of test results with the linear theory. The two lines are the linear theory prediction. The red line is the prediction for the red cross symbols ($Lc_\nu = 7.7$) and the blue line is the prediction for the blue circles ($Lc_\nu = 1.37$).

Fig. 3.9 collapsed on to a single line.

When $Lc_\nu > 1$, the velocity profile of the flow inside the pore starts to become like a “plug flow” and the total pressure drop starts to change from viscous dominant to oscillating inertance dominant. The “plug flow” at the center of the pore oscillates like the inviscid flow, therefore at $Lc_\nu > 1$, the dimensionless pressure gradient due to inertance correlates to the dimensionless displacement only.
Figure 3.9. Dimensionless pressure gradient due to inertance vs. Reynolds number for 600 cells/in$^2$ Celcor$^\text{TM}$ sample testing in argon. The straight lines are the linear theory prediction.

3.5 Results for a stacked-screen sample

The test results of one stacked-screen sample is discussed in this section. The wire diameter of the screen is 0.0022 in (0.056 mm) and it has 145 wires per inch (mesh). The porosity and the hydraulic radius are calculated using the expression from Organ (1992),

$$\phi = 1 - \frac{\pi D_{\text{wire}}}{4},$$  \hspace{1cm} (3.7)

$$r_h = \frac{D_{\text{wire}}}{4} \frac{\phi}{(1 - \phi)}.$$  \hspace{1cm} (3.8)

The results are $r_h = 41.9 \mu\text{m}$ and $\phi = 75\%$. 
Figure 3.10. Dimensionless pressure gradient due to inertance vs. $\frac{Re}{Lc^2_\nu}$ for 600 cells/in$^2$ Celcor™ sample testing in argon. The data of Fig. 3.9 collapsed on to a single line.

The test result of dimensionless pressure gradient due to viscosity for the stacked-screen sample is plotted vs. Reynolds number in Fig. 3.11. The viscous Lautrec number varies from 0.007 to 1.58. For $Lc_\nu < 1$, all the test results collapse nicely. When $Lc_\nu$ approaches and then exceeds unity, the effect of the viscous penetration depth starts to show up and the results lie on different lines. Compared to Fig. 3.4, the test result of 600 cells/in$^2$ Celcor™, the effects of the penetration depth for stacked-screen showed up on both low and high Reynolds numbers.

As discussed before, when the viscous penetration depth is comparable or smaller than the hydraulic radius, both hydraulic radius and penetration depth should be used as the characteristic length. Since the effect of penetration depth showed up on both high and low Reynolds numbers for the stack-screen, instead of
dividing the Reynolds number by the Lautrec number, the dimensionless pressure gradient due to viscosity was divided by Lautrec number and plotted vs. Reynolds numbers, $\frac{dp_{vis}^*}{\rho u^2 \delta^*} = f(\frac{urh}{\nu})$. This is shown in Fig. 3.12 and for comparison one 600 cells/in² Celcor™ data set is also plotted in pink circles. The deviation from the laminar flow behavior happens at much lower Reynolds numbers for stack-screen compared to the Celcor™ sample which has uniform parallel square pores. This means that for similar flow conditions, the viscous losses for the stacked-screen are much higher than for uniform parallel porous material. Backhaus and Swift (2001) doubled the efficiency, at the highest acoustic power output, in a thermoacoustic-Stirling heat engine by switching from a screen-based regenerator to a parallel-plate regenerator.
Figure 3.12. Dimensionless pressure gradient due to viscosity vs. the Reynolds number for stacked-screens using both the hydraulic radius, $r_h$, and viscous penetration depth, $\delta_\nu$, as the characteristic length. The pink circles are test results from Celcor™ for comparison.

3.6 Discussions

3.6.1 Pressure drop due to inertance

In Fig. 3.13, the dimensionless pressure gradient due to inertance is correlated to the dimensionless displacement, $\frac{u}{\omega r_h} = \frac{Re}{2Lc_\nu}$, for the 600 cells/in$^2$ Celcor™ sample. All the results for helium, argon, and sulfur hexafluoride are plotted. For Lautrec number larger than one, $\frac{dp_{\text{iner}}}{dx}$ correlates to the dimensionless displacement $\frac{u}{\omega r_h}$ and forms a straight line. For $Lc_\nu < 1$, the plotted results are slightly above this line due to the viscosity. At extremely low frequencies, when all the fluid is within the viscous boundary layer, the drag from the wall of the pores makes
the fluid density “appears” larger compared to inviscid flow. Also, in this regime, the pressure drop are dominated by the viscous part. Shown in Fig. 3.14, the ratio of the pressure gradient due to viscosity to the pressure gradient due to the oscillating inertance was plotted for different geometries. When $L_{c\nu} \ll 1$, the pressure gradient is dominated by the viscous effect and when $L_{c\nu} \gg 1$, the pressure gradient is dominated by the oscillating inertial effect.

The oscillating inertial part of the dimensionless pressure gradient vs. dimensionless displacement for stacked-screen is plotted in Fig. 3.15. For $L_{c\nu} > 1$, the results collapse into a single line, and for $L_{c\nu} < 1$ the results are above this line. Also shown in this plot, for high Reynolds numbers, this correlation doesn’t hold any more. When the Reynolds number is high enough and the flow is no longer
Figure 3.14. The ratio of the pressure drop due to viscosity to the pressure drop due to inertance as the function of viscous Lautrec number, \( \frac{r_h}{\delta_v} \), for different geometries.

In laminar flow, the inertial part of the pressure gradient drops quickly.

The pressure drop due to viscosity and inertance for the stacked-screen sample was plotted vs. velocity separately in Fig. 3.16. In the region when the viscous pressure drop depends on the velocity quadratically, the pressure drop due to inertance decreases dramatically. The pressure gradient is dominated by viscous effect when the flow is no longer laminar flow even at high Lautrec numbers.

Based on the data shown in both Fig. 3.15 and Fig. 3.16, it appears that behavior of both viscous and inertial pressure gradients is qualitatively similar to the behavior observed in straight pores.

We hypothesize that the behavior of the dramatic drop of the inertial part pressure drop in Fig. 3.16 might result from transition to turbulence. As evident in Fig. 3.16, the viscous pressure drop grows quadratically at \( L_{c_v} > 1 \), rather than
Figure 3.15. Dimensionless pressure gradient due to inertance vs. the dimensionless displacement \( \left( \frac{u}{\omega r_h} \right) \) for the stacked-screen sample.

linearly, as it does for \( Lc_\nu < 1 \). The same behavior that we ascribe to turbulence is reinforced by the behavior of the inertial component of the pressure drop which reaches a maximum at the same value of velocity where the slope of the viscous pressure drop transitions from a linear to a quadratic dependence on flow velocity. Above this transition the inertial pressure drop is smoothly overwhelmed by the rapidly (but also smoothly) increasing dissipative (turbulent) pressure drop.

We have attributed this to the transition from laminar flow to disorder turbulent flow. More detailed study could be conducted, including a broad-band frequency analysis, to compare the power density spectrum before and after the transition occurs. This type of measurement might provide another tool that could give further insight into the transition from laminar to turbulent flow.
Figure 3.16. Pressure drop across the stacked screen sample vs. velocity. Top: pressure drop due to friction; Bottom: pressure drop due to inertia.

The Celcor™ testing samples with uniform parallel pores exhibit similar but less pronounced behavior as shown in Fig. 3.9. At large Reynolds numbers, the test results for the pressure due to inertance are slightly bent down from the straight line.

3.6.2 Deviation of pressure drop due to viscosity at high Lautrec numbers

In this chapter, we reported the measurements of samples with calculable geometry, the Celcor™ sample with uniform square pores, and the stacked-screen sample. Nonlinearity was observed at high Reynolds number and the characteristic length scales for the dimensionless pressure gradient was observed to be frequency depen-
dent as summarized in Table 3.1.

Deviation from the theory was observed at high viscous Lautrec numbers, \( \left( Lc_\nu > 5 \right) \), as shown in Fig. 3.7. The measurement matched poorly with the theory even at low Reynolds numbers.

<table>
<thead>
<tr>
<th>( Lc_\nu &gt; 1 )</th>
<th>( Lc_\nu &lt; 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{dp_{visc}}{\rho u^2} \frac{r_h}{L} = f \left( \frac{\mu}{u \rho \delta_\nu} \right) )</td>
<td>( \frac{dp_{visc}}{\rho u^2} \frac{r_h}{L} = f \left( \frac{\mu}{u \rho \delta_\nu} \right) )</td>
</tr>
</tbody>
</table>

Table 3.1. The characteristic length of Poiseuille flow is geometry dependent only. For oscillatory boundary layer flow, it depends on both geometry and viscous penetration depth.

More measurement were taken by keeping the Reynolds number small enough to eliminate the nonlinear effect so as to compare with the linear theory at different Lautrect numbers. Instead of operating the apparatus at its resonance frequency to achieve the highest possible Reynolds numbers, for these measurements the driving amplitude was kept low and frequency was swept to access viscous Lautrec numbers approaching 10.

The 400 cells/in\(^2\) Celcor\textsuperscript{TM} sample was tested, and \( \frac{dp_{visc}}{dx} \frac{1}{\omega \rho a} \) was plotted vs. \( \frac{r_h}{\delta_\nu} \) in Fig. 3.17. The straight line is the prediction from the linear theory. The experimental results deviate from the theory when \( Lc_\nu > 4 \). For all the measurements, the sample holder and threaded holes were carefully sealed to ensure the testing sample is the only flow path between the front volume and the back volume. until we can find an explanation for this deviation, results for experiments with \( Lc_\nu > 5 \) should be considered suspect. The test results of the cross symbols around Reynolds number 0.9 are also below the straight line, this is because the poor signal to noise ratio for this set of the test results.

The behavior of porous media in thermoacoustic machines that employ traveling-wave phasing for acoustic execution of the Stirling cycle use porous media operated in the limit of small Lautrect number \( \left( Lc_\nu \leq 0.1 \right) \), so the anomalies observed in this
Figure 3.17. In the laminar flow region, the pressure gradient due to viscosity was observed to deviate from the linear theory at high Lautrec numbers but low Reynolds number.

apparatus at $Lc_{\nu} > 5$ are not important. On the other hand, for standing-wave thermoacoustic machines that employ a “stack” ($Lc_{\nu} > 2$) rather than a regenerator, another apparatus was used to investigate sound speed (dispersion) and attenuation that will be described in Chapter 4.
Chapter 4

Separation of thermoviscous losses in Celcor\textsuperscript{TM} ceramic

4.1 Introduction

The use of Celcor\textsuperscript{TM} as a thermoacoustic stack material was first suggested by scientists at the National Center for Physical Acoustics (NCPA). Due to a long-standing interest at NCPA in the acoustics of porous media, particularly soils, several measurements of the effects of the porosity of the ceramic cell walls were undertaken. In thermoacoustic applications, Celcor\textsuperscript{TM} has been an attractive stack material due to its regular pore geometry, ease of shaping, low cost, and availability. The Celcor\textsuperscript{TM} ceramic material has been used as the stack material for several standing-wave thermoacoustic devices (Garrett and Chen, 2000) (Johnson et al., 2000). Due to the wall porosity, as sketched in Fig. 4.1, the possibility of excess attenuation due to the wall porosity was a cause for concern.

Theory and experiments were reported for sound propagation in porous media consisting of straight capillary tubes having square cross sections. The theory (Roh et al., 1991) assumed a non-porous wall. A single microphone measurement method was used to determine the propagation constants in the square pore media. A tortuosity factor of 1.1 was chosen to improve the agreement among theory and experiment. Tortuosity is a structure form factor depend only on the geometry of the frame. It is the ratio of the square of the microscopic velocity to the square
Figure 4.1. Schematic representation of the cell structure of Celcor™ ceramic with an expanded view of a portion of cell wall illustrating the porous nature of the ceramic wall surface. Photo micrographs of actual cell walls are shown in Fig. 4.2.

of the macroscopic velocity of the fluid (Allard, 1993). The measured absorption coefficient exhibited good agreement with the theory at low frequencies.

Arnott, Sabatier, and Raspet (1991), enhanced this theory by taking the wall pores into consideration. The continuity equation was modified to include the transverse velocity gradient due to the compressibility of the gas in the wall pores. Density changes of the gas in the wall pores are assumed to be isothermal. Using
this model, the agreement with the measurement were improved both in attenuation and phase velocity and no tortuosity factor was required. But at high frequency, the predicted attenuation was still observed to be lower than the measurement. Bernard, Velea, and Sabatier (1996) reported a method to seal the wall pores by coating them with a very thin layer of polyurethane varnish diluted with equal parts of mineral spirits and verified the application of the theory of sound propagation in the square tube porous sample at frequencies above 1500 Hz.

The absorption coefficient is a combined effect of both thermal and viscous losses. In a standing-wave thermoacoustic refrigerator (Garrett, 2004), the position of the stack in the standing wave field determines how much of the losses will be due to viscosity and how much is due to thermal relaxation effects between the gas and the stack. In typical electrically-driven standing wave thermoacoustic refrigerators (Garrett et al., 1993) (Garrett, 1997), the stack is located much closer to a pressure anti-node than to a velocity anti-node, so the effects of thermal relaxation dominate the losses due to viscosity.

Because thermal relaxation loss dominates viscous loss in a practical standing-wave thermoacoustic machine, interest in “excess” attenuation due to Celcor™ wall porosity is not merely academic. The apparatus and measurements described in this chapter were undertaken to determine whether Celcor™ would be a suitable stack material for a 10 kW thermoacoustic chiller (TRITON) that was under development.

The measurements reported here separate the thermal and viscous effects by placing the Celcor™ sample at the center of a plane-wave resonator and examining the quality factor $Q$ of the even and odd longitudinal modes separately. At low longitudinal mode numbers, the effects of thermal relaxation dominate the even modes, when the sample is centered at a velocity node. The odd modes place the sample near a pressure node thereby enhancing the viscous loss mechanism and suppressing the thermal relaxation effects. This separation of thermo-viscous effects is only applicable when the length of the testing sample is much smaller than the wavelength. In our measurements, at the first mode, the length of the sample is 2.5% of the wavelength. Based on the DELTAE model, the viscous loss is 98.8% of the total loss in this mode. To explore the minor loss (end effect), the
driving voltage was increased from 100 mV to 3V and the change of $Q$ is within 0.7%. So, the minor loss (Swift, 2002, pg. 164) is not a concern at the amplitudes used to conduct this experiment. The cutoff frequency above which nonplanar modes can propagate in the tube is 5,900 Hz.

In this study, we also investigate several pore sealing techniques, including different kinds of polymer solutions. We discover that even with the use of vacuum impregnation, the sealing of the ceramic pores is difficult to accomplish with any acceptable degree of uniformity. For that reason, an in situ polymerization technique described herein was used.

### 4.2 Experimental technique

#### 4.2.1 Sealing

Sealing the wall pores uniformly is important in comparing the measured data of coated sample with the uncoated sample. Shellac and several polymers: polystyrene, polymethyl methacrylate, polyethylene oxide, polybutyl norbornene, and branch polyethylene were tried. Those pore sealants were applied using the following method: The Celcor\textsuperscript{TM} samples is soaked in the polymer solution for hours then drained so to keep the channels clear before drying it. But the wall pores could not be sealed uniformly inside the channels, even if the outside surface was coated with a thin layer of the polymer. Vacuum impregnation and ultrasonic agitation were added to that procedure before draining the sample, but no obvious improvement was observed.

An in situ polymerization was found to work well. The Celcor\textsuperscript{TM} sample was soaked in the solution of hydroxyethyl methacrylate and azobisisobutronitrile (AIBN). After the sample was drained, it was heated to 70\(^\circ\)C. At this temperature, the monomers linked together to form polymers with the help of the initiator AIBN. Under microscopic inspection, the Celcor\textsuperscript{TM} sample was coated uniformly both inside and outside. Fig. 4.2 photomicrographs of the Celcor\textsuperscript{TM} wall in the uncoated (left) and coated (right) conditions.
4.2.2 Quality factor of resonator

The quality factor, $Q$, of the resonant cavity is a dimensionless measure of the sharpness of the peak of the power curve of a resonance when amplitude is plotted vs. frequency. There are many equivalent ways of expressing the $Q$. This multiplicity allows us to connect the most convenient experimental method to the parameter of interest. If a resonance is excited and then allowed to freely decay, $Q$ is the ratio of $2\pi$ times energy stored to energy dissipated per cycle. For a cylindrical resonator, it is proportional to $R/\delta$. $R$ is the radius of the resonator and $\delta$ is the penetration depth. The penetration depth is inversely proportional to $\sqrt{n}$ for the plane wave resonance where $n$ is the mode number. For all the plots in this chapter, $Q/\sqrt{n}$ will be plotted versus mode numbers.
4.2.3 Instrumentation and modal analysis

The experimental setup is shown schematically in the Fig. 4.3. A 0.7 m long cylindrical resonator with a 0.033 m inner diameter was used (Burmaster, 1985). One end of the resonator was driven by a loudspeaker, modified by Fitzpatrick (1988). The response was measured by an electret microphone that was charged
by liquid contact (Chudleigh, 1976) located at the other end of the resonator. The microphone had the same cross-sectional area as the tube. An HP 3562A Dynamic Signal Analyzer was used to measure the transfer function between speaker input voltage and microphone output of the resonator. A non-linear pole-zero curve fit (HP, p. 15-9) was used to determine two complex poles that are complex conjugates, $a \pm ib$. In terms of these poles, the resonance frequency can be expressed as $f = \sqrt{a^2 + b^2}$ and the quality factor as $Q = \frac{1}{2a} \sqrt{a^2 + b^2}$.

Placing a Celcor™ sample at the center of the cylindrical resonator, per the technique described by Moldover (Moldover et al., pp. 395-399), the thermoviscous losses due to the Celcor™ sample could be evaluated by comparing the $Q$ of the resonator with the Celcor™ sample to the $Q$ of the empty resonator. The theoretical model for resonance frequency and quality factor was developed using the DELTAE software (Ward and Swift, 1994).

The pressure and velocity profile of the first 2 modes of the resonator are shown in Fig. 4.4 as well as an approximation by a lumped-element model (Morse, 1981). For all of the odd modes, the Celcor™ sample is located at the velocity antinode but pressure node. This corresponds to the maximum viscous loss but minimum thermal relaxation loss. For all the even modes, the Celcor™ sample is located at the velocity node but pressure antinode to isolate the loss mechanism of interest in this study: the change in thermal relaxation (Kirchoff) loss in porous or impenetrable walls.
4.3 **DELTAE model**

The Design Environment for Low-Amplitude ThermoAcoustic Engines (DELTAE) is a computer program that can predict how a given thermoacoustic apparatus will perform or can allow the user to design an apparatus to achieve desired performance (Ward and Swift, 1994). DELTAE solves the one-dimensional wave equation in a gas or liquid based on the usual low-amplitude, ‘acoustic’ approximation. The wave equation is solved in a geometry specified by the user as a sequence of “segments”. Segments such as ducts, compliances, transducers, heat exchangers, and thermoacoustic stacks or regenerators are available to the user. A solution to the appropriate 1-d wave equation is found for each segment, with the complex
pressures, and volume flow rates, and real temperatures matched at the junctions between segments.

DELTAE does allow the user to include some nonlinear effects that arise at high amplitudes. For example, turbulent losses can be included for oscillatory flow in “ducts” and “cones” using a Moody friction factor that is applied at each instant of time during the flow. It is also possible to use various “math segments” to compute nonlinear effects such as “minor loss.” For this work, care was taken to keep amplitudes low enough that nonlinear effects could be neglected.

A 5-segment DELTAE model was developed for the empty resonator and a 7-segment model for the resonator with a Celcor™ sample at the center. The Celcor™ sample was modeled as a STKREC segment using the pore geometry information provided by the manufacturer. A custom thermophysical parameter file was created based on the ceramic’s heat capacity at 25°C ($c_p = 616.2 \text{ J/K-kg}$) and thermal conductivity ($k_s = 2.6 \text{ W/K-m}$) as provided by the manufacturer. A solution to the appropriate 1-d wave equation is found for each segment, with complex pressures and volume flow rates matched at the junctions between segments. One of the complete DELTAE output files is included in the Appendix C.

The quality factor, $Q$, was calculated from the results of the DELTAE model. The phase difference between the pressure and the volume velocity was calculated at the neighborhood of the resonance frequency. The rate-of-change of the the phase with respect to frequency was determined by curve fitting the calculated result. The quality factor was then calculated using the rate-of-change of phase with respect to frequency at the resonance frequency,

$$Q = \frac{f_n}{2} \left| \frac{d\phi}{df} \right|_{f_n}^1$$

(4.1)

$f_n$ is the resonance frequency of the $n^{th}$ mode and $\phi$ is the phase in radians between pressure and the velocity at the driving point.

$^1$Starting from Eq. 3.1-8 (Thomson, 1981), $\phi = \arctan \left[ \frac{\omega/\omega_0}{Q(1-(\omega/\omega_0)^2)} \right]$. Differentiating this expression and evaluating at $\omega/\omega_0 = 1$, it follows that $\frac{d\phi}{df} |_{f_0} = \frac{2Q}{f_0}$. 
4.4 Results

Figure 4.5. Quality factors divided by the square-root of mode number (n) for the empty resonator as measured and as calculated by DELTAE.

Figure 4.5 compares the DELTAE prediction with the measured quality factor for the seven lowest frequency plane wave modes. As expected, the $Q$ predicted by DELTAE exceeds the measured values. We attribute the majority of this discrepancy to the existence of various small gaps between the two 1/2 inch microphone ports, a 1/4 inch gas fill port, and the plugs used to seal those ports. Those gaps provide additional surface area for both thermal and viscous surface losses. We are convinced that the gaps between the resonator end caps, and the possibility of flow into small volumes created by spaces between the o-ring groove and o-ring were not responsible. Those seals were opened and closed several times in the course of these experiments to change samples and no change in the empty resonator $Q$ was ever observed. Due to this reproducibility, we felt it was appropriate to introduce an excess loss that would account for non-ideal resonator behavior.

The plugging of the microphone ports and the gas fill port were not disturbed
during all the measurements. The resonant frequency was accurately predicted by
the DeltaE model within 0.03% for all the seven modes. Therefore the stored
energy is accurately predicted by the DeltaE model based on the Rayleigh’s
“method” (Strutt and Rayleigh, 1945, Sec. 88, pp. 109, 112)(i.e., the frequency
is a measure of the ratio of potential to kinetic energies). The dissipated power
due to the small gaps at the gas filling port and microphone ports were underesti-
mated because of the difficulty to include the accurate details of these small gaps
in the model. The discrepancy between DeltaE predication and measurement was
designated as $Q_{Excess}$ at each resonant frequency as shown in Eq. 4.2. $Q_{Excess}$
is used to correct the DeltaE predication in cases that included the Celcor™
samples.

$$\frac{1}{Q_{Measured}} = \frac{1}{Q_{DeltaE}} + \frac{1}{Q_{Excess}}.$$  \hspace{1cm} (4.2)

### 4.4.1 Resonator frequency shifts with Celcor™ sample

As shown in Fig. 4.4, for the odd modes, the center of the sample was located at
a pressure node and velocity antinode. At all the even modes, the center of the
sample was located at the pressure antinode but velocity node. When the Celcor™
sample was placed at the center of the resonator, both the resonant frequency and
the quality factor of the resonator were changed due to the volume exclusion and
the excess losses introduced by the Celcor™ sample.

The change of the resonance frequency compared with the empty resonator is
plotted in the Fig. 4.6. After the Celcor™ sample was inserted, the resonance
frequencies for all odd modes were decreased and the resonant frequencies for all
even modes were increased. Because the Celcor™ samples located at the velocity
antinode, the decrease of the frequency at the odd modes was produced by the
smaller cross section of the channel. This increases the “lumped inertance” based
on the model of Fig. 4.4. The increase of the resonant frequency at all even modes
was due to the volume exclusion produced by the sample at the pressure antinode,
making the “lumped compliance” of the gas smaller in that region. Although we
use this lumped parameter analysis to motivate our interpretation of these reso-
nance frequency shift effects introduced by the presence of the Celcor™ sample,
the DeltaE model produces a complete one dimensional standing-wave solution
Figure 4.6. The change in resonance frequency relative to the resonance frequency of the empty resonator, $\Delta f/f$, after the Celcor™ sample was inserted.

that matches the boundary condition at both ends of the resonator as well as preserving continuity of complex pressure and volume velocity within both the empty resonator and resonator with the Celcor™ inserted at the center.

Figure 4.7 showed the discrepancy between the measured resonance frequencies and the DELTAE predictions for the lowest seven modes of the resonator. It shows that the discrepancies at the even modes, when the sample is located at the pressure antinode, are larger than the discrepancies at the odd modes. This frequency depression for the even modes is expected, since the STKRECT segment in the DELTAE model assumes that the walls are rigid and impervious. The STKRECT segment defines the geometry and physical properties of rectangular pores. The consistent 0.8% depression of the even-mode frequencies suggests that wall porosity can be inferred from this acoustic measurement. Modifying the total porosity used in the DELTAE model to match the measured resonant frequencies at even modes, the total porosity inferred was $\phi = 0.89 \pm 0.02$. This agrees within the experimental error with the value $\phi = 0.87 \pm 0.05$ determined by Champoux, Stion, and Daigle
Figure 4.7. The frequency difference relative to the DELTAE prediction for the resonator with uncoated Celcor™ sample.

(1991), and φ = 0.82 determined by Arnott, Sabatier, and Raspet (1991). After the wall pores were sealed off, the discrepancy in the even mode frequencies was entirely eliminated as shown in Fig. 4.10.

After the Celcor™ sample was inserted in the resonator, the quality factors at each resonant frequency were expected to decrease. The quality factor $Q$ of the empty resonator (Swift, 1988) is

$$\frac{1}{Q} = \frac{\delta_\nu}{R} + \frac{\delta_\kappa (\gamma - 1)}{R(1 + \varepsilon_s)} + \frac{2\delta_\kappa (\gamma - 1)}{L(1 + \varepsilon_s)},$$

(4.3)

The first term corresponds the ratio of the viscous energy loss on the resonator wall to the stored energy. The second and third terms correspond to the ratio of the thermal losses on the wall and two ends of the resonator to the stored energy. $L$ is the length of the resonator, $\delta_\nu$ is the viscous penetration depth, $\delta_\kappa$ is the thermal penetration depth, and $\gamma$ is the ratio of specific heats. The ratio of the thermally-active heat capacity of the gas to that of the solid is given by $\varepsilon_s$ and
indicates how well the solid can keep the gas at the interface isothermal (Swift, 1988).

\[
\varepsilon_s = \frac{\rho_m c_p \delta}{\rho_s c_s \delta_s} \frac{\tanh [(1 + i) r_h / \delta]}{\tanh [(1 + i) l / \delta_s]},
\]

(4.4)

where the subscript \(s\) stands for the properties of the solid and \(l\) is the thickness of the solid. This effect is included in the DeltaE model but, in this case, is negligible: \(\varepsilon_s \approx 0.003\).

For the empty resonator, since \(Q\) is inversely proportional to the penetration depth, it is proportional to \(\sqrt{f}\). The frequency shift due to the insertion of the Celcor\textsuperscript{TM} sample is shown in Fig. 4.6. Taking the small frequency changes into consideration, the quality factors of the empty resonator were modified by, \(\frac{dQ}{Q} = \frac{1}{2} \frac{df}{f}\), before compared with the quality factors of the resonator after the sample was inserted. This modification of \(Q\) due to the slight shift in frequency is also applied in Fig. 4.9, the comparison between uncoated sample and DeltaE prediction.

![Figure 4.8](image)

**Figure 4.8.** Relative change, \(\Delta Q/Q\), in measured quality factor \((Q)\) compared to the empty resonator after the uncoated Celcor\textsuperscript{TM} sample was inserted in the resonator.
Fig. 4.8 shows the relative decrease of the measured quality factors, $\Delta Q/Q$, after the sample was inserted compared to the empty resonator. The reduction at the odd modes was about twice as much as at all the even modes. This implies that the viscous losses (Stokes) produced by the Celcor™ is more significant than the thermal relaxation (Kirchhoff) loss. There is also a trend apparent in Fig. 4.8 and other plots that show the difference in behavior of the even modes and the odd modes decreasing at higher frequencies. This is due to the length of the Celcor™ sample, which is becoming comparable to the wavelength at higher frequencies. For example, at the first mode, the length of the sample is about 2.5% of the wavelength. At the seventh mode, the length of the sample is about 18% of the wavelength. The Lautrec and viscous Lautrec number for the testing sample at each mode are listed in Table 4.1.

The measured quality factor exhibits fairly good agreement with the DELTAE prediction after correction for excess loss in the empty resonator. This is shown in the Fig. 4.9.
Table 4.1. The Lautrec and viscous Lautrec numbers for the testing sample at each mode.

<table>
<thead>
<tr>
<th>mode number (n)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Lc_\kappa = r_h/\delta_\kappa$</td>
<td>1.6</td>
<td>2.31</td>
<td>2.79</td>
<td>3.27</td>
<td>3.62</td>
<td>4.01</td>
<td>4.29</td>
</tr>
<tr>
<td>$Lc_\nu = r_h/\delta_\nu$</td>
<td>1.9</td>
<td>2.75</td>
<td>3.32</td>
<td>3.89</td>
<td>4.30</td>
<td>4.77</td>
<td>5.10</td>
</tr>
</tbody>
</table>
4.4.2 The coated Celcor™ sample

For an uncoated Celcor™ sample, the favorable agreement of $Q$ between the measured data and DeltaE model for the odd modes, along with the larger discrepancy of the resonance frequency at all the even modes, encourages a further exploration of the wall pores effects using this technique.

After coating the same Celcor™ sample with a very thin polymer layer to block only the wall pores, the quality factor and the resonance frequencies were measured again in the same resonator. The results are shown in Fig. 4.10 and Fig. 4.11.

![Figure 4.10. Comparison of the relative frequency shift of coated and uncoated sample to the DeltaE prediction.](image)

Figure 4.10 shows the frequency changes compared with DeltaE prediction for both coated and uncoated samples. The discrepancies at all the even modes decreased to less than 0.1% after the sample was coated. Blocking the wall pores makes the sample “stiffer” thus increasing the resonance frequency to the value expected for a sample with “rigid, impenetrable” walls.
Figure 4.11. Quality factor for the resonator with the coated and uncoated Celcor™ sample.

Figure 4.11 compares the measured $Q$ for the uncoated sample to coated Celcor™ samples. At the lowest frequency, which is 230 Hz, there is only a 0.49% change for thermal relaxation losses. This is the measurement we sought in our effort to determine whether the Celcor™ stack that was to be used in the TRITON thermoacoustic air conditioner should be coated to reduce loss. These results assure us that coating of the Celcor™ ceramic was unnecessary to reduce loss in a standing-wave, thermoacoustic refrigerator application.
4.5 Summary

The goal of this investigation was to determine if wall pores would introduce extra losses for the Celcor\textsuperscript{TM} stack that was to be used in a thermoacoustic air conditioner. To do this, the thermal loss needed to be separated from the viscous loss, unlike the previous investigation of Celcor\textsuperscript{TM}. (Arnott et al., 1991)

The use of \textsc{DeltaE} allowed subtraction of dissipative effects in the empty resonator that were constant but hard to quantify. Our conclusion regarding the absence of additional wall losses at low frequencies is independent of the \textsc{DeltaE} model and only relied on the measured difference in quality factor ($Q$) between losses in coated and uncoated samples. The total sample porosity, including the wall pores, was determined by adjusting the porosity used in the \textsc{DeltaE} model to match the measured resonant frequency at the even modes when the sample was located at the pressure antinodes. We were pleased to see that our technique produced a wall porosity that agreed with other determinations made by more traditional techniques (Sec. 4.4.1).
Chapter 5

Conclusions and recommendations for future work

5.1 Conclusions

Porous media play a critical role in thermoacoustic machines. In this work, new experimental apparatuses and techniques were developed to quickly and accurately characterize porous media for thermoacoustic applications using small samples. To determine the accuracy of these techniques this thesis focused mostly on materials with calculable behavior.

For oscillating flow inside porous media, the transition of heat transfer, which accompanies the expansion and compression of the oscillating flow, from isothermal to adiabatic, is fully characterized by the thermoviscous $f$-function. The thermoviscous $f$-function was determined from measurements of the complex ratio of displacement to the oscillating pressure amplitude. The tested results for calculable geometry consisting of an array of uniform square parallel pores, matched favorably the theoretical prediction.

Analogous to AC circuits using complex impedance, the complex Nusselt number is one correct way to describe the heat transfer behavior for oscillating flow inside porous media. For the first time, the complex Nusselt numbers were expressed in terms of the thermoviscous $f$-function allowing the heat transfer coefficient for oscillating flow in porous media to be determined quickly and accurately without
measuring the temperature and requiring no heat exchangers. The asymptomatic results of the real part of the complex Nusselt number at zero frequency converged to the steady (DC) result and at high frequency it matched qualitatively the limit derived by other researchers interested in the behavior of heat exchangers used in oscillating flows (Mozurkewich, 1998),(Garrett et al., 1994),(Wakeland and Keolian, 2004).

The pressure drop for the oscillating flow across the sample was measured by a phase sensitive technique to resolve the pressure drop to two parts: the viscous part and the oscillating inertial part. The viscous part of pressure drop is comparable to the steady (DC) results at low frequencies when the viscous penetration depths are larger than the pore size. Frequency dependence of the pressure drop due to viscosity was observed when the viscous penetration depth is smaller than the pore hydraulic radius.

The possibility of excess attenuation due to wall porosity of Celcor™ was studied per the technique described by Moldover (Moldover et al., pg. 395-399). The resonator quality factors $Q$ were measured between 230 Hz and 1.6 kHz in air at atmospheric pressure on a uncoated sample and a coated sample. There was no observed decrease in attenuation of even modes (dominated by thermal loss) for the coated sample below 1.0 kHz. The volume exclusion due to pore sealing was inferred by the increase in even mode resonance frequencies in good agreement with other determinations of wall porosity.

It is hoped that the research described in this thesis has established the relationship between earlier measurements made by the Stirling engine community and the thermoacoustic understanding of heat transfer and pressure drop expressed in terms of thermoviscous $f$-functions. In addition, an accurate and efficient way to investigate other materials, which might provide performance improvement for future thermoacoustic and Stirling machines, has been demonstrated.

### 5.2 Recommendations for future work

Experimental apparatus has been developed for characterizing porous material subject to oscillating flow and calibrated by testing in samples with geometries that
allow measurements to be compared to established theory. The complex ratio of the
displacement to the oscillating pressure was measured to determine the thermal \( f_\kappa \)-
function. The experimental technique was based on the assumption that the length
of the testing volume is much smaller than the wavelength to have uniform pressure
inside the testing volume for the subtraction. The total empty volume inside the
porous testing sample is the volume remaining after subtraction to determine the
\( f \)-function for the testing sample. The total empty volume inside the porous
sample should be much larger than the volume occupied by the bellows and piston
to minimize the relative error caused by the subtraction. Also, in this study, the
acceleration of the piston was measured to determine the displacement, since
\[ a_1 = -\omega^2 \xi_1 \]
for oscillatory flow. The signal-to-noise ratio was poor at low frequencies due
to the limitation of the stroke of the piston, and this causes difficulties for testing
large loose pores at low frequencies to extrapolate the test results to zero frequency.
The experimental apparatus could be optimized by minimizing the bellows and
piston volumes and improving the displacement measurement technique for the
future study.

The test apparatus for the flow measurement was designed and calibrated by
testing samples with uniform parallel square pores. Good agreement between
the calculation and measurements was achieved for values of the Lautrec number
smaller than 4. This deviation observed at high Lautrec number, as shown in
Fig. 3.17, needs further exploration to determine its source. Despite that limitation
(\( Lc_\nu > 4 \)), this test apparatus is capable of testing various porous media for which
the flow and thermal properties could only be determined experimentally. This will
provide useful experimental data for thermoacoustic machine design and Stirling
engine design.

The phase-sensitive technique was used to study the dynamics of oscillating
flow in porous media. The scale lengths used for Reynolds number and the di-

dimensionless pressure gradient were observed to change from depending on the
hydraulic radius only to depending on both the hydraulic radius and the viscous
penetration depth. This transition happens smoothly when the hydraulic radius
is equal to the viscous penetration depth. For an irregular geometry, when the hy-
draulic radius is not calculable, this provides an acoustic method to determine the
hydraulic radius. The $r_h$ is an essential parameter in computer models to design thermoacoustic machines.

The phase-sensitive technique could also be used to measure the viscous $f$-function, $f_\nu$, in the laminar flow range by keeping the Reynolds number very low. The thermal $f$-function, $f_\kappa$, and the viscous $f$-function, $f_\nu$, are the same for uniform simple pores as shown in Fig. 2.2. But for inhomogeneous and irregular geometries, the $f_\kappa$ and $f_\nu$ functions could exhibit different features since for the flow measurement, the fluid will choose to flow through larger pores. The differences between $f_\kappa$ and $f_\nu$ could reveal some of the details of these materials with irregular geometries.

Shown in Fig. 3.16, the viscous part of the dimensionless pressure gradient transits from linear dependency of the velocity to quadratic dependency of the velocity when the velocity increases. This transition was accompanied by an apparent dramatic decrease of the inertial part of the pressure drop. We have attributed this to the transition from laminar flow to disorder turbulent flow. More detailed study could be conducted, including a broad-band frequency analysis, to compare the power density spectrum before and after the transition occurs. This type of measurement might provide another tool that could give further insight into the transition from laminar to turbulent flow.
Appendix A

Characterization of a small moving-magnet electrodynamic linear motor

A.1 Introduction

The small moving-magnet linear motor (Model LM-1) used in the bellow-bounce test apparatus for flow measurement was developed by the EnduraTEC® Systems Group at Bose Corporation (Froeschle and Carreras, 1993). The LM-1 motor is currently used in medical instrumentation for the testing of biomedical materials. The small size and weight of the LM-1, its 200 watt power handling capacity, 12 mm stroke, and demonstrated reliability make it attractive for smaller scale thermo-acoustic refrigeration applications. This appendix reports detailed measurements that were made on the LM-1 to determine its suitability.

The transduction coefficient, $B_l$, was observed to be a function of position. But the variation in $B_l$ with position has a reduced effect on the driver’s output power because $B_l$ is largest around the equilibrium position, where the piston velocity is also largest. By mechanical co-linear joining of the armature of two such motors, an electrodynamic load (dynamometer) was created to measure the efficiency as a function of energy dissipated in the dynamometer. The measured efficiencies are shown to be in good agreement with the predictions if a position-
averaged effective transduction coefficient is introduced. These linear motors were originally characterized for Ben & Jerry ice cream cooler pre-prototype design, based on these results, this linear motor is judged to be an attractive power source in small electrically driven thermoacoustic refrigerator applications.

A.2 Theoretical framework

Wakeland (2000) has shown that the maximum electroacoustic efficiency of a linear electrodynamic motor (driver or loudspeaker) depends only upon the transduction coefficient \((Bl\)-product), the real (dissipative) part of the electrical impedance of the voice coil, \(R_e = \text{Re}[Z_{el}]\), where \(Z_{el}\) is the blocked complex electrical impedance of the voice coil, and the driver’s open-circuit mechanical resistance, \(R_m\). From these driver properties, Wakeland defines two dimensionless parameters: \(\beta = (Bl)^2/R_eR_m\) and \(\sigma = (\beta + 1)^{1/2}\). If the purely dissipative load, \(R_a\), that is presented to the driver is optimized by making \(R_a = \sigma R_m\), and the driver suspension is adjusted so that the driver is mechanically resonant at the intended frequency of operation, \(f = \omega/2\pi = (k/m)^{1/2}\), where \(k\) is the driver’s suspension stiffness and \(m\) is the driver’s moving mass, then the maximum electroacoustic transduction efficiency, \(\eta_{\text{max}}\), is

\[
\eta_{\text{max}} = \frac{\sigma - 1}{\sigma + 1}.
\]

The maximum electroacoustic transduction efficiency can also be derived for drivers with large \(\beta(\geq 5)\) by adjusting the load so that the average power dissipated by Joule heating of the voice coil \((\Pi_{el} = R_ei^2/2)\) is equal to the average power dissipated in the mechanical resistance of the driver \((\Pi_{\text{mech}} = R_mv^2/2)\).

Wakeland’s analysis assumes that the \(Bl\)-product is a constant that is independent of the driver’s amplitude of motion, \(x\). Although that is not true for the LM-1, nor for other moving-magnet electrodynamic motors that have been studied at the Penn State Applied Research Laboratory (Smith, 2001), (Smith et al., 1997), (Heake, 2001), it is shown in a subsequent section of this chapter that the variation in \((Bl)\) with piston position has a small effect on the driver’s output power. This is because \((Bl)\) is largest around the equilibrium position, where the
piston velocity is also largest.

A.3 Driver mechanical parameter measurements

The suspension stiffness and dynamic mass of the driver can be determined non-destructively by measuring the mechanical resonance frequency as a function of the mass, \( m_i \), added to the armature. The resonance frequencies, \( f_i \), corresponding to \( m_i \), are approximated by tracking the zero-crossing of the phase of the input electrical impedance (using an HP4192A Impedance Analyzer) of the voice coil. Since the quality factor of the mechanical resonance is typically greater than ten, the neglect of the motor winding inductance \( (L \approx 21mH) \) that contributes a gradual linear variation in impedance through its inductive reactance \( (X_L = \omega L) \) is not significant for determination of dynamic mass and suspension stiffness.

The armature of the LM-1 has a rectangular mounting plate that is approximately 38 mm long, 15.5 mm wide, and 1.8 mm thick. The plate has five tapped (4-40) mounting holes; one near each corner of the plate and one at the center. We attached a fixture with a 1/4-20 threaded rod to that mounting plate to accommodate rigid attachment of standard slotted laboratory weights to the threaded rod. The mass of that fixture is 0.0241 kg, including the 1/4-20 nut and washer used to constrain the added masses.

The resonance frequency, \( f_i \), with a mass, \( m_i \), added to the piston, can be plotted to produce a straight line.

\[
\frac{1}{4\pi^2 f_i^2} = \frac{m}{k} + \frac{m_i}{k} \tag{A.2}
\]

The inverse of the slope of the line in Eq. A.2 where \( m_i \) is the independent variable determines the suspension’s stiffness and the ratio of the intercept to the slope determines the dynamic mass. These measurements were made on both LM-1 drivers (S/N: 100070 and S/N: 100181) using ten different weights with a maximum added mass of 0.30 kg. The correlation coefficients for the curve fits to Eq. A.2 gave uncertainties in the slope (Higbie, 1991) that were under 1%. The measurements were repeated twice for each driver. The results reported in Table A.1 include an uncertainty that is half of the difference of the results from the two measurements.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Units</th>
<th>Result</th>
<th>Error</th>
<th>Result</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Suspension Stiffness</td>
<td>$k$</td>
<td>N/m</td>
<td>7760</td>
<td>±0.1%</td>
<td>6470</td>
<td>±0.5%</td>
</tr>
<tr>
<td>Dynamic Mass</td>
<td>$m$</td>
<td>Kg</td>
<td>0.1265</td>
<td>±0.8%</td>
<td>0.1071</td>
<td>±0.8%</td>
</tr>
<tr>
<td>Natural Frequency</td>
<td>$f$</td>
<td>Hz</td>
<td>39.4</td>
<td>±0.4%</td>
<td>39.1</td>
<td>±0.4%</td>
</tr>
<tr>
<td>Mechanical Resistance</td>
<td>$R_m$</td>
<td>Kg/sec</td>
<td>2.34</td>
<td>±1.7%</td>
<td>3.75</td>
<td>±1.5%</td>
</tr>
<tr>
<td>DC Electrical Resistance</td>
<td>$R_{dc}$</td>
<td>Ω</td>
<td>1.36</td>
<td>±2.5%</td>
<td>1.39</td>
<td>±2.2%</td>
</tr>
<tr>
<td>Inductance</td>
<td>$L$</td>
<td>mH</td>
<td>23.0</td>
<td>±0.3%</td>
<td>21.7</td>
<td>±1.4%</td>
</tr>
<tr>
<td>$Bl$-product at $x = 0$</td>
<td>$c$</td>
<td>N/A</td>
<td>18.36</td>
<td>±0.1%</td>
<td>18.12</td>
<td>±0.2%</td>
</tr>
<tr>
<td>$Bl$-product(linear term)</td>
<td>$b$</td>
<td>N/A-m</td>
<td>266.33</td>
<td>±50%</td>
<td>36.9</td>
<td>±14.3%</td>
</tr>
<tr>
<td>$Bl$-product(quadratic term)</td>
<td>$a$</td>
<td>N/A-m$^2$</td>
<td>-178000</td>
<td>±2.3%</td>
<td>-131000</td>
<td>±4.26%</td>
</tr>
<tr>
<td>Maximum efficiency</td>
<td>$\eta_{\text{max}}$</td>
<td></td>
<td>81.8%</td>
<td>±2.8%</td>
<td>77.4%</td>
<td>±2.7%</td>
</tr>
</tbody>
</table>

Table A.1. Summary of driver parameters measured for the two linear motors. The estimated uncertainties are based on the differences in the results obtained with the driver in two opposite vertical orientations. These uncertainties were consistent with the statistical uncertainties in the slopes of the least-squares fits to the measurements. The value for maximum efficiency, $\eta_{\text{max}}$, is calculated (Wakeland, 2000) from the measured parameters in the table.

made with different orientations (up and down) of the driver.

The mechanical resistance was measured with the same added mass fixture by recording the open-circuit voltage generated by the voice coil (recorded on a Nicolet 310 Digital Oscilloscope). A current applied at a frequency near the mechanical resonance excited oscillations of the driver. The current was abruptly removed and the digitized waveform was captured and curve-fit with a four parameter chi-squared algorithm to the form $V(t) = V_0 e^{-t/\tau} \sin(\omega t + \phi)$, to extract values for the exponential decay time, $\tau$. The mechanical resistance can be expressed in terms of the dynamic mass and the decay time, $R_m = 2m/\tau$. The decay time was plotted against the dynamic mass for eight different added masses up to 0.15 kg. The mechanical resistance, $R_m$, is twice the reciprocal of the slope. Uncertainties in the individual slopes are typically 2%, so the mechanical damping seems to be independent of frequency and armature orientation in the neighborhood of the
A.4 Driver \( Bl \)-product vs. piston position

The \( Bl \)-product was measured in a static experiment using the apparatus shown in Fig. A.1. A calibrated 100 lbf. stainless steel load cell (Omega LCCB-100, S/N: K736020N), energized by a precision 10.000 V\( _{dc} \) bias supply, was used to measure the force, \( F(i_{dc}) \) produced by a dc-current, \( i_{dc} \), flowing through the voice coil as measured by a Yokogawa WT-110 Power Meter. The load cell was attached to one end of a turnbuckle that was connected by a short section of chain to the added mass fixture attached to the rectangular mounting plate of the armature. A machinist’s dial indicator was placed in contact with the armature to determine its displacement from equilibrium caused by rotation of the turnbuckle. The \( Bl \)-product was measured from equilibrium to 0.25” (6.35 mm) away from equilibrium in steps of 0.05” (1.27 mm). As shown in Fig. A.2, the load cell output voltage was plotted against the current for twenty different currents ranging up to 5.0 A\( _{dc} \) at each position. The relative statistical uncertainties in the slopes are typically ±0.1%. This measurement was repeated on each driver for both directions of pull.

The \( Bl \)-product as a function of position for S/N: 100181 is plotted in Fig. A.3. The points are fit (least-squares) with a second-degree polynomial of the form 

\[
Bl(x) = ax^2 + bx + c.
\]

It can be seen that \( (Bl) \) decreases by about 30% when the armature is pulled as far away from its equilibrium position as possible (6.25 mm). Although this decrease is substantial, its effect on the time-averaged acoustic power that the driver can deliver, \( \langle \Pi_{ac} \rangle_t \), is

\[
\langle \Pi_{ac} \rangle_t = \frac{1}{T} \int_0^T F \cdot \vec{v} dt = \frac{1}{T} \int_0^T Bl(x)i(t)v(t)dt.
\]  

(A.3)

The power reduction is significantly smaller since the armature undergoes simple harmonic motion with the armature velocity being largest at the equilibrium position.

If the armature stroke is taken to be \( 2x_0 \), substitution for, \( x(t) = x_0 \sin \omega t \), and \( i(t) = i_0 \cos \omega t \), can create an effective \( Bl \)-product, \( (Bl)_{eff} \), based on the definition of average power in Eq. A.3. The velocity and current (hence force) are assumed
Figure A.1. Static test fixture for measurement of $Bl$-product as a function of the armature’s displacement from equilibrium. The S-shaped load cell at the top of the test stand is connected to the turnbuckle that is attached to the added mass fixture by a chain to avoid application of any torques to either the driver or the load cell. The dial indicator used to measure the piston position is visible behind the chain.
Figure A.2. Output of the load cell as a function of coil current for the driver’s armature displaced by 3.81mm (0.15 inches) from equilibrium. Using the calibrated sensitivity of the load cell, this plot corresponds to $Bl = 17.21 \pm 0.008$ N/A.

to be in-phase since the linear motors are typically operated at their mechanical resonance frequency. Operation at resonance is frequently ensured by use of a feed-back control system (Garrett et al., 1993) (Shearer et al., 2004). The resulting integral can be simplified if the time average is taken over only one quarter-cycle ($\pi/2$ radians) as shown in Eq. A.4 below:

$$\frac{(Bl)_{eff}}{Bl(x=0)} = \frac{\int_0^{T/4} (ax^2 + bx + c) \cos^2 \omega t dt}{\int_0^{T/4} c \cos^2 \omega t dt}$$  \hspace{1cm} (A.4)

Substitution of $x(t)$ into Eq. A.4 leads to simple integrals over products of trigonometric functions which yield the stroke-dependent result for the normalized effective $Bl$-product.
Figure A.3. The triangle-shaped points show the measured $Bl$-product as a function of the displacement of the piston from its equilibrium position for LM-1, S/N: 1000181. The line connecting the triangles is a second-order polynomial fit ($R^2 = 0.9995$). The effective $Bl$ product, $(Bl)_{eff}$, (squares connected by solid line) is based on those fit coefficients $[a = -176.232 \text{ N/A-m}^2; \ b = 266.3 \text{ N/A-m}; \ c = 18.36 \text{ N/A}]$.

\[
\frac{(Bl)_{eff}}{Bl(x = 0)} = \frac{a}{4c} x_0^2 + \frac{4b}{3\pi c} x_0 + 1
\]  

(A.5)

The value of $(Bl)_{eff}$ as a function of the armature displacement is plotted along with the static values of $Bl(x)$ in Fig. A.3. It can be seen clearly that although $Bl(x = 6.35\text{mm})$ is only 70% of its equilibrium value, $(Bl)_{eff}$ is still 94.3% of $Bl(x = 0)$ when averaged over a 12.7 mm stroke.

It should be noted that the variation of the $Bl$-product with armature position also has the effect of introducing motion of the armature at frequencies that are harmonic multiples of the frequency of the electrical current. Because these linear motors are driven simple harmonic oscillators that have quality factors that are
typically of order ten, the response of the armature at the harmonics of the drive are about a factor of $Q(f_n/f_0)$ smaller than the response at resonance.

### A.5 Voice coil electrical impedance

The average power dissipated by the voice coil is caused by three effects. For the LM-1 at the frequencies of interest ($<100$ Hz), Joule heating due to the current passing the electrical resistance of the coil, $R_{dc}$, is the largest contributor. Additional losses are created by the dissipation due to eddy currents generated in the laminations around which the coil is wound, as well as to magnetic hysteresis in the lamination material and the magnet carrier. The eddy current and hysteretic losses are frequency dependent (Smith, 2001).

$R_{dc}$ was determined by a four-wire technique which measured the current and voltage over the current range of interest (typically 0.5 - 5.0 A $\text{dc}$). A straight line was fit to the data and the slope and uncertainty in the slope provided the dc-resistance and its uncertainty reported in Table A.1. The complex input electrical impedance of the coil was measured with an impedance analyzer (HP 4192A). Care was taken to immobilize the driver’s armature using a clamping fixture during these measurements. This minimizes the contribution of motional impedance during the measurement.

All of these three loss mechanisms can be represented by the frequency dependent dissipative (real) component, $R_e(f) = \text{Re}[Z_{el}]$, of the voice coil’s blocked electrical impedance. The frequency dependent blocked electrical impedance of S/N:100070 is plotted in Fig. A.4. The frequency dependent inductive (imaginary) component of the blocked input electrical impedance of the coil, $L$, increased linearly with frequency corresponding to a voice coil inductance of $21.7 \pm 0.3$ mH.

### A.6 Driver dynamometer power measurements

A direct determination of the efficiency of one driver (S/N: 100181) was made by rigidly connecting its armature to the armature of the other driver (S/N: 100070) together with a spring (Garrett et al., 2001) to increase the resonant frequency to
Figure A.4. Dissipative portion of the voice coil’s blocked input electrical impedance of driver SN 100070, $R_e(f) = \text{Re}[Z_{el}]$, as a function of frequency. Extrapolation of the curve to zero frequency give an intercept of $R_{dc} = 1.41 \, \Omega$. That result is within experimental error for the value of $R_{dc} = 1.39 \pm 0.03 \, \Omega$ in Table A.1.

104 Hz. A photograph of the high-frequency dynamometer is shown in Fig. A.5. Alternating current was applied to the driver being tested, while different electrical load resistances, $R_{load}$, and a load-matching 80 $\mu$F capacitor were connected in series with the terminals of the driver (used as an alternator) that was acting as the load for the driver under test (Auyer and Miller, 1976).

Electromechanical conversion efficiency was measured at fixed stroke. The stroke was monitored by an accelerometer attached to the armature (PCB Model 353B65, S/N: 33055). One Yokogawa WT-110 electrical power meter measured the electrical power delivered to the driver under test and a second Yokogawa WT-110 electrical power meter measured the power dissipated in the load resistor, $R_{load}$. The total power dissipated by the load is the sum of the measured electrical power dissipated in $R_{load}$, the electrical power dissipated in $R_{dc}$ by the coil, and the
Figure A.5. Bottom view of the 104 Hz LM-1 dynamometer test apparatus. The higher frequency was achieved under atmospheric conditions by a sixteen-legged flexure spring [US Pat. No. 6,307,287 (Oct. 23 2001)], developed for the CFIC C-2 motor alternator. Under the spring, two LM-1’s are visible. Their armatures are joined by an impedance head. The top LM-1 was operated as an alternator to provide an adjustable load on the lower LM-1 that was operated as a motor. The mass of the steel and aluminum structure held together by four 1\(\frac{1}{4}\) inch diameter bolts is approximately 140 kg. Since the total moving mass of the two armatures and spring is approximately 1 kg, motion of the structure was neglected.
mechanical power dissipated by the measured mechanical resistance of the load, $R_{mks}$. The mechanical resistance of the load side alternator (SN10070) with the spring was measured via the technique described in Section A.3. The mechanical resistance of the alternator, together with the spring, is $R_{mks} = 6.6 \pm 1.34$ Kg/sec. The dissipation in the driver under test (SN 100181) is one contribution to its “inefficiency” that is being measured. Since both stroke and frequency were held constant as the electrical load resistance was varied, the mechanical power dissipated in the load was also constant. For the data shown in Fig. A.6, the stroke was $2x_0 = 10.7$ mm and the frequency was 104 Hz, corresponding to a root-mean-squared piston velocity of $v_{rms} = 2.47$ m/sec, and an average mechanical power dissipation, $\Pi_{mech} = R_{mks} v_{rms}^2$. The maximum total dissipated power is 170 watts.
at $R_{\text{load}} = 4.7 \, \Omega$.

\section*{A.7 Conclusion}

The electrodynamic parameters of two Bose LM-1 moving magnet linear motors were measured. The theoretical efficiencies for both motors were calculated. The metal structures that restrain the NdFeB magnet within the armature (i.e., magnet carriers) of the motors were different. S/N: 100181 has a titanium carrier and S/N: 100070 has a magnesium carrier. It appears that the titanium carrier experienced less mechanical damping and was therefore more efficient. The electrical resistivity of titanium (43 $\mu\Omega$-cm at 22°C) is ten times higher than the resistivity of magnesium (4.3 $\mu\Omega$-cm at 22°C), so it is likely that the difference in performance was due to eddy current losses in the magnet carrier (Campbell, 1994).

The efficiency of S/N: 100181 was measured as a function of load by connecting its armature to that of S/N: 100070 and varying the electrical resistance shunting the electrical terminals of S/N: 100070 while maintaining the stroke of both motors. The maximum measured efficiency as shown in Fig. A.6 (approximately 81.8±2.8%) of the driven motor was found to agree with the Wakeland (2000) formalism ($\eta_{\text{max}} = 82 \pm 2\%$) within experimental error.
# Appendix B

## DELTAE output file for the bellows bounce flow test apparatus using Celcor™ as the testing sample

```
!->celpdrop.out !Created@20:56:58 29-Mar-05 with DeltaE Vers. 5.3b5
for the IBM/PC-Compatible
--------------------------------- 0---------------------------------
BEGIN the setup
1.2400E+06 a Mean P Pa 367.03 A |p| G( 0d) P
71.000 b Freq. Hz 2071.2 B Ph(p) G( 0e) P
300.00 c T-beg K 8.8832E-05 C |U| G( 0f) P
367.03 d |p| Pa G
2071.2 e Ph(p) deg G
8.8832E-05 f |U| m^-3/s G
0.0000 g Ph(U) deg
0.000 hear Gas type
stainless Solid type
!---------------------------------1---------------------------------```

## THERMO properties of working fluid

```
1.6667 A gamma
```
322.58  B  a  m/s
19.861  C  rho  kg/m^3
520.27  D  c_p  J/kg/K
1.8372E-02  E  K0  W/m/K
2.3395E-05  F  mu  kg/s/m
3.3333E-03  G  beta  1/K
8.9281E-05  H  deltaK  m
7.2669E-05  I  deltaN  m
1.0000E+08  J  rho_s  kg/m^3

sameas  0  Gas type
ideal  Solid type

!------------------------------------------------------------2--------------------------------------------------

HARDEnd  End of the helmholtz resonator
0.0000  a  R(1/z)  (t)  367.03  A  |p|  Pa
0.0000  b  I(1/z)  (t)  -88.844  B  Ph(p)  deg
 8.8832E-05  C  |U|  m^3/s
 0.0000  D  Ph(U)  deg
 3.2901E-04  E  Hdot  W
 3.2901E-04  F  Edot  W
 2.7941E-03  G  R(1/z)

sameas  0  Gas type
ideal  Solid type

!------------------------------------------------------------3--------------------------------------------------

DUCT  front volume
7.0000E-03  a  Area  m^2  374.94  A  |p|  Pa
 2.1700  b  Perim  m  -88.888  B  Ph(p)  deg
 0.1040  c  Length  m  2.9899E-05  C  |U|  m^3/s
 0.0000  d  Srough  -1.1988  D  Ph(U)  deg
 2.2598E-04  E  Hdot  W
 2.2598E-04  F  Edot  W
 1.0302E-04  G  HeatIn  W

sameas  0  Gas type
ideal  Solid type

!------------------------------------------------------------4--------------------------------------------------

RPNTARGET  p1 and u1
0.0000  a  Target  (t)  (2.9893E-05, -6.2552E-07)  A
(7.2753, -374.87)  B
STKRECT the sample

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DUCT back volume

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</tr>
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sameas 0 Gas type 1.3637E-17 H I(1/z)
ideal Solid type 300.07 I T K

RPNTARGET the Lautruc number
0.0000 a Target (t) 3.8531 A RPNval
5d 2 / 1I /

RPNTAR pressure drop
0.0000 a Target (t) (16.362, 103.74) A
4B 6B -

RPNTAR dp/U
0.0000 a Target (t) (4.8570E+05, 3.5349E+06) A
10A 6A /

RPNTAR dpvis/(rho*u*w*L)
281.00 a Target (t) 0.1563 A RPNval
11A real f / 2 / pi / 5a * 5b * rho / 5c /

RPNTAR dposc/(rho*u*w*L)
0.0000 a Target (t) 1.1373 A RPNval
11A imag f / 2 / pi / 5a * 5b * rho / 5c /

RPNTAR Reynolds number
0.0000 a Target (t) 61.347 A RPNval
5C 5a / 5b / 5d 2 / * rho * mu /

RPNTAR dp*U power dissipated
0.0000 a Target (t) (2.1040E-04, -1.5313E-03) A
10A conj 6A * 0.5 *

RPNTAR Pin*Vin-Pout*Uout
0.0000 a Target (t) (2.1964E-04, 1.4445E-03) A
4A conj 4B * 6A conj 6B * - 0.5 *
RPNTAR match the back volume pressure

479.00 a Target = 17A? 479.00 A RPNval

7A
Appendix C

DELTAE output file for the Quality factor measurement with the Celcor\textsuperscript{TM} sample located at the center of the resonator

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230.57 b Freq. Hz G 22.604 B |p| G( 0d) P
296.50 c T-beg K
22.604 d |p| Pa G
0.0000 e Ph(p) deg
5.0000E-06 f |U| m^{-3}/s
0.0000 g Ph(U) deg
0.000000w Gas type ideal
0.000000w Solid type

ENDCAP  First end
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105

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sameas 1a a Area m^-2 3.6300 A |p| | Pa |
0.1080 b Perim m -33.331 B Ph(p) deg |
0.3400 c Length m 5.0410E-05 C |U| | m^3/s |
-89.177 D Ph(U) deg |
5.1367E-05 E Hdot W |
air Gas type 5.1367E-05 F Edot W |
ideal Solid type -5.0604E-06 G HeatIn W |

STKRECT Celcor Stack
sameas 1a a Area m^-2 3.0430 A |p| | Pa |
0.7400 b GasA/A -175.75 B Ph(p) deg |
3.8100E-02 c Length m 5.0376E-05 C |U| | m^3/s |
5.4500E-04 d a m -89.557 D Ph(U) deg |
9.0000E-05 e Lplate m 5.1367E-05 E Hdot W |
5.4500E-04 f b m 5.0860E-06 F Edot W |
296.50 G T-beg K |
air Gas type 296.50 H T-end K |
celcor Solid type -4.6281E-05 I StkEdt W |

DUCT  Duct
sameas 2a a Area m^-2 22.571 A |p| | Pa |
sameas 2b b Perim m -179.62 B Ph(p) deg |
0.3400 c Length m 7.4029E-09 C |U| | m^3/s |
-179.62 D Ph(U) deg |
8.3546E-08 E Hdot W |
air Gas type 8.3546E-08 F Edot W |
copper Solid type -5.1283E-05 G HeatIn W

ENDCAP Second End
sameas a Area m^-2 22.571 A |p| Pa -179.62 B Ph(p) deg 2.0000E-20 C |U| m^3/s -91.491 D Ph(U) deg 7.3848E-21 E Hdot W

copper Solid type -8.3546E-08 G HeatIn W

HARDEND Final
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0.0000 b I(1/z) = 6H? -179.62 B Ph(p) deg

air Gas type 7.3848E-21 F Edot W

ideal Solid type 296.50 I T K
Bibliography


Vita

Jin Liu

Jin Liu was born and grew up in southern China along the Yangze river. In 1993, Jin Liu received her Bachelor of Science degree in Mechanical Engineering, from Chengdu University of Science and Technology (renamed as Sichuan University in 1999) majoring in Precision Instrumentation. Teaching and doing research, she worked in the Manufacturing School at Nanjing University of Science and Technology from 1993 to 1998. In fall 1998, she entered the Graduate School of Tennessee Technological University and received the Master of Science degree in Mechanical Engineering in August 2000. The title of her master’s thesis was “Condenser Microphone Calibration by the Reciprocity Method”. In the summer of 2000, she moved to Penn State and joined the thermoacoustics group in 2001. She worked on characterization of porous material subject to oscillatory flow to characterize stack and regenerator materials used in thermoacoustic refrigerators and engines.