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PROGRESSIVE COLLAPSE ANALYSES
OF STEEL FRAMED MOMENT RESISTING STRUCTURES

A Thesis in
Civil Engineering

by
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ABSTRACT

Progressive collapse has been an important issue in building failures since the collapse of the Ronan Point apartment building in 1968. Progressive collapse is a failure sequence that relates local damage to large scale collapse in a structure. If any load exceeds the load-carrying capacity of any member, it will cause additional local failures. Such sequential failures can propagate through the structure. Therefore, a local member failure analysis is the basic element for the progressive collapse analysis.

Three different failure criteria have been considered in this study. They are material failure, buckling failure, and connection failure. Material and buckling failures were analyzed by using a second-order inelastic method. Connection failures were analyzed by using a moment-curvature relationship calculated by a power model using three parameters.

The finite element code ABAQUS/Explicit has been used for the analyses. Single column failure results from the ABAQUS/Explicit simulations and from the NFA developed for this study based on the second-order inelastic method were compared for a verification purpose.

Various numbers of spans and stories with rigid, semi-rigid, and reinforced semi-rigid frames were studied for 2D frame analyses. As the number of spans increased, the collapse mode tended to change from total collapse to partial collapse. As the number of stories increased, the collapse mode tended to change from partial collapse to total collapse. However, the analyses of the semi-rigid frames showed different trends. All semi-rigid
frames collapsed partially by joint failures. The 2D frame analyses showed that a vertical failure was caused by connection failures, a horizontal failure was caused by column buckling.

The 3D frame analyses showed a different tendency. Four different span frames with rigid and semi-rigid connection with a single column size, were used for six initial failure cases. The assumed rigid and semi-rigid connection did not make a big difference in the 3D cases. Therefore, semi-rigid connection would not be a critical factor if the initial damage is localized. Inner columns were more critical than outer columns.

However, the single column initial failure caused total collapse for the most of the frame with a more realistic design. The 3D frame with realistic designs showed three facts. First, a more realistic design, using two different column sizes, is more vulnerable to progressive collapse than a single size column design. Second, a semi-rigid connection could trigger total collapse of a structure, while a rigid frame just caused internal damage. Third, the lengths of columns did not affect collapse modes. However, it could be a factor when the frame is overdesigned to resist progressive collapse.
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1. Introduction

1.1. Problem Statement

Progressive collapse has been one of the issues in building failures since the collapse of the Ronan Point apartment building in 1968 (Griffiths, et al. 1968). It was a 22-story precast concrete panel construction building in England. A gas explosion in the kitchen of an apartment on the 18th floor of the building blew out an exterior wall panel. That caused a chain reaction of structural collapse from the roof down to the ground, as debris from above fell on successive floors below (Figure 1-1). The particular type of joint detail used in the Ronan Point apartment building relied heavily on joint friction between elements. This resulted in a structure that has been termed a “house of cards”, indicating that buildings with similar joint characteristics were particularly susceptible to progressive collapse (Breen 1980).

![Figure 1-1. Damage at the Ronan Point Apartment Building](image)

(a) Immediate local damage  
(b) Progressive Collapse

Figure 1-1. Damage at the Ronan Point Apartment Building
Progressive collapse is a failure sequence that relates local damage to large scale collapse in a structure. The local failure can be defined as a loss of the load-carrying capacity of one or more structural components that are part of the whole structural system. Preferably, once any structural component fails, the structure should enable an alternative load-carrying path. After the load is redistributed through a structure, each structural component will support different loads. If any load exceeds the load-carrying capacity of any member, it will cause another local failure. Such sequential failures can propagate through the structure. If a structure loses too many members, it may lead to partial or total collapse. This type of collapse behavior may occur in framed structures, such as buildings (Griffiths, et al. 1968, Burnett, et al. 1973, Ger, et al. 1993, Sucuoglu, et al. 1994, Ellis and Currie 1998, Bazant and Zhou 2002), trusses (Murtha-Smith 1988, Blandford 1997), and bridges (Ghali and Tadros 1997, Abeysinghe 2002).

Progressive collapse is an important issue because local damage may cause massive destruction of a structural system. Small accidents, such as a local gas explosion or vehicle collision, can cause total collapse of a structure. It is an obvious undesired risk since small accidents cannot be prevented completely. Therefore, investigating the nature of progressive collapse is very important.

1.2. Objective

The main objective of this study is to enable the development of a rational numerical analysis of progressive collapse by studying simple frame systems with
simple material models focusing on the role of material models and buckling. Several factors may contribute to progressive collapse. It usually occurs when a structure is subjected to an abnormal loading event. (i.e., such loading conditions are not normally considered in the design of structures (Breen and Siess 1979)). Abnormal loads could generate local failure, which may lead to progressive collapse. Geometry of structure, material properties of members could be other factors to influence progressive collapse. However, it is unrealistic to include all the factors in a single study. Therefore, only a few key factors in progressive collapse will be included to simplify the analysis.

Three key issues need to be considered. Those are structural stability, semi-rigid joint connection behavior, and dynamic frame analysis. Each issue requires a separate analysis step. However, they will be combined in this study because they could occur simultaneously and influence each other in progressive collapse. Therefore, this study will be focused on developing a simplified numerical analysis method and setting up the numerical procedure that combines these issues.

1.3. Scope

This research will be limited to understanding the characteristics of progressive collapse in simple, moment resisting steel frame buildings. The initial structural responses to abnormal loadings (e.g., an explosion) will not be considered. Damage for the conditions cited above will be assumed to be the removal of one or more columns at critical locations.
Structural details such as walls and partitions can affect the response of the structure. Including these secondary elements in the analysis would increase the complexity of the responses. Therefore, the effect of secondary elements will not be considered because the responses of the major load carrying members should be clarified first. The analysis including the effects of the secondary elements could be a future study.

A buckling analysis procedure will be set up at the first stage of the study. Buckling failure in steel structure includes flexural buckling, torsional buckling, lateral-torsional buckling, and local buckling. Only flexural member buckling will be considered in this study. A single structural member will be analyzed to verify a buckling analysis procedure to be included in the overall analysis of a structure.

Simple material properties such as elastic, elasto-plastic will be used for the collapse analysis.

Influence of connection rigidities will be studied next. Since real connection behavior is neither rigid nor pinned, semi-rigid behavior will be considered. Steel connection simulations could be used to obtain the moment-curvature relationship that will be included in the structural models.

Once each analysis step has been completed, it will be combined into the dynamic analysis of steel frame structures. Several configurations of structures will be used from a simple structure such as a 2X2X2 bay and story configuration to larger structures such as up to 5X5X5 bay and story configurations. Critical damage cases will be assumed. Columns at the corner, the center, and other locations will be removed
to simulate the initial failures. Each configuration will use the simplified analysis procedures obtained in the previous steps.

Results from these analyses will be used to analyze the general behavior of progressive collapse. Behavior differences between failure criteria, geometries, and connections rigidities will be compared and discussed. Conclusions and recommendations will be provided as the last stage of the study.
2. Background

2.1. Introduction

Definition

Progressive collapse is characterized by the loss of the load-carrying capacity of a relatively small portion of a structure due to an abnormal load which, in turn, triggers a cascade of failure affecting a major portion of the structure (Breen and Siess 1979, Fintel and Schultz 1979, Gross and McGuire 1983). Analyzing progressive failure of a structure requires one to assess the response of a structure due to the failure or damage of one or more members of the structure. The loss or failure of a member or members causes force redistribution to the remaining structural members and may lead to collapse (Blandford 1997). All these definitions explain the relationship between a relatively small local failure and a major portion failure of a structure.

Cause of Progressive Collapse – Abnormal Loading

It is estimated that at least 15-20% of the total number of building failures are due to progressive collapse (Leyendecker and Burnett 1976). A notable example of such a failure is the Ronan Point Collapse (Griffiths, et al. 1968). In the years following the Ronan Point incident, hundreds of engineering articles and reports on these subjects have been published (Breen 1980). Since an explosion caused progressive collapse at Ronan Point, a number of studies were devoted to the relationships between abnormal loadings and progressive collapse (Astbury 1969, Astbury, et al. 1970, Burnett, et al. 1973, Mainstone 1973, Burnett 1974, Leyendecker, et al. 1975, Leyendecker and
An abnormal load is any loading condition a designer does not include in the normal and established practice course design (Gross and McGuire 1983). Abnormal loadings include explosion, sonic boom, wind-induced localized overpressure, vehicle collision, missile impact, service system malfunctions, and debris resulting from some incident (Burnett 1974). Leyendecker and Burnett (1976) discussed plausible sources of abnormal loading events and estimated the risk of the loadings in residential building design with regard to progressive collapse. They concluded that gas explosions occurred with an annual frequency of 1.6 events per million dwelling units, bomb explosions occurred with an annual frequency of 0.22 events per million dwelling units, vehicular collisions with buildings that cause severe damage occurred with an annual frequency of 6.8 events per million dwelling units.

For certain abnormal loading events, the probability of an event occurring in a building increases with building size. In particular, high-rise buildings tend to be at a higher risk for gas and bomb explosions. In contrast, vehicular collisions affect primarily ground story areas. It was found that the hazard due to a gas explosion exceeds that for vehicular collisions in buildings greater than about 5 stories in height (Leyendecker and Burnett 1976). However, the structural designer has very little control over the probability of an abnormal event. Therefore, structural engineers should consider other approaches to prevent progressive collapse.
Propagation of Failure

Mainstone (1973) pointed out that there are two ways in which local damage might spread progressively to other parts of the structure:

“1. It might spread upwards and/or sideways through the removal of supports and the impairment of the stability of the immediately superincumbent and/or adjacent structure.

2. It might spread downwards, primarily as a result of the impact loads of falling debris, as happened in the Ronan Point collapse. “

These two types of damage propagation can occur simultaneously in a structure.

2.2. Prevention of Progressive Collapse

The probability of structural failure in the event of an abnormal load, may be stated as (Leyendecker and Burnett 1976):

$$P(F) = P\left(\frac{F}{A}\right) \times P(A)$$

where, $P(F)$ = the probability of failure

$P\left(\frac{F}{A}\right)$ = the probability of failure given that an abnormal load occurs

$P(A)$ = the probability of occurrence of an abnormal load

To reduce the probability of failure, the structural designer has two alternatives. One is to reduce the probability of an abnormal load. The other is to design a structure
so that the probability of failure, given the occurrence of an abnormal load, is acceptably small.

There are three approaches for reducing the risk of progressive collapse failures, as follows (Ellingwood 1981, Gross and McGuire 1983):

(1) Event Control

(2) Indirect Design Approach

(3) Direct Design Approach.

Event Control

Event control refers to avoiding or protecting against an incident that might lead to progressive failure. In other words, it is to reduce the probability of an abnormal load $P(A)$ in Equation (1). Examples might include regulating the distribution and use of gas, or installing barriers to protect against vehicular collisions. This approach may be effective for certain types of loads. However, it does not increase the inherent resistance of a structure to progressive collapse, and does not protect against other sources of accidents. Moreover, it causes structural integrity to depend on factors that are outside of the control of the designer (Ellingwood 1981).

Indirect Design Approach

An indirect design approach develops resistance to progressive collapse by specifying minimum levels of strength, continuity and ductility in members and joints, often in terms of minimum transverse, vertical, longitudinal and peripheral tie forces.
The specification of minimum tie forces for continuity is relatively easy to implement in a design specification. The ACI building code (ACI 2002) uses a structural integrity concept to implement this approach. Section 7.13 introduces the requirements for structural integrity. It states that the intent of this section of the code is to improve the redundancy and ductility in structures so that in the event of damage to a major supporting element or an abnormal loading event, the resulting damage may be confined to a relatively small area and the structure will have a better chance to maintain overall stability. The ACI building code (ACI 2002) also includes design recommendations for structural integrity of precast concrete construction (section 16.5), because precast construction tends to be more susceptible to progressive collapse than does conventional cast-in-place construction (Breen and Siess 1979). Provisions for structural integrity focus on tension ties for precast concrete construction and continuous reinforcements for cast-in-place concrete construction, so that each structural member can develop a minimum resistance to survive damage induced by an abnormal loading event. The ASCE code (ASCE 2000) also mentions that all parts of the structure between separation joints shall be interconnected to form a continuous path for the seismic force-resisting system. The connections shall be capable of transmitting the seismic force induced by the parts being connected. On the other hand, the AISC Building Code (AISC 1999) does not consider a structural integrity concept explicitly. It states that due to strain hardening, a ductile steel bar loaded in axial tension can resist, without fracture, a force greater than the product of its gross area and
its coupon yield stress. Therefore, it does not require additional attention like for structural concrete continuity and ductility.

**Direct Design Approach**

The direct design approach enables one to explicitly consider resistance to progressive collapse and the ability of a structure to absorb localized damage. Two basic means of direct design are the specific local resistance method and the alternate load path method.

The intent of the specific local resistance method is to provide sufficient strength to resist an abnormal load. That means that the load-bearing structural elements must be able to function under an extreme load (Gross and McGuire 1983). However, it is unrealistic to expect that individual structural elements can resist all abnormal loading cases. Therefore, the specific local resistance method is appropriate for special cases, where abnormal loads may be estimated with reasonable accuracy.

The alternate load path method, in contrast, permits local damage to occur but provides alternate paths around the damaged area so that the structure is able to absorb the abnormal load without collapse. In the event of the loss of the load-carrying capacity of a structural member, its forces are redistributed to adjacent members and the loss in load capacity is accommodated by bridging over the damaged area.

These alternatives are not mutually exclusive. The resistance to progressive collapse may be evaluated by analyzing the structure to determine whether alternate load paths around the damaged area can be developed. On the other hand, alternate
path studies may be used as guides for developing rules for the minimum levels of
strength, ductility, and continuity required to assure general structural integrity (Fintel
and Schultz 1979, Gross and McGuire 1983). For some buildings, it may not be
possible to develop alternate load paths in the event of failure of certain members.
When this is the case, it may be desirable to redesign the critical elements for specific
local resistances. In this manner, the various approaches may be combined to provide a
progressive collapse resistant design (Gross and McGuire 1983).

2.3. Analyses of Progressive Collapse

Progressive collapse may occur in any type of framed structure. This includes
steel or concrete framed buildings, space trusses, and bridges. Progressive collapse is
not necessarily related to any one type of structural material. However, some types of
materials may be more prone to progressive collapse in that they may have less
ductility or weaker connection details than others (Breen and Siess 1979). Although
moment resisting steel frames will be focused on, several other structural systems will
be discussed.

Precast Concrete Structure

The risk of progressive collapse in precast concrete large panel and bearing wall
structures is greater than the risk in traditional cast-in-place structures (Breen 1980).
Since precast concrete structures are vulnerable to progressive collapse, the problem
was studied to address structural integrity and design (Speyer 1976, Burnett 1979,
Hendry 1979, Pekau 1982). The collapse of the Ronan Point building (Griffiths, et al. 1968) is an example of this type of failure. The report revealed several deficiencies in the British codes and standards of that time, particularly as they applied to multistory construction. The report focused on the lack of redundancy or “alternate load paths” in the structure. As a consequence of that investigation, the British building regulations were changed to require that multistory structures be designed either to provide an “alternate path”, in case of loss of a critical member, or to have sufficient local resistance so as to withstand the effects of a gas-type explosion (Breen 1980). More recent British code (Steel Construction Institute 1990) states that a building should be checked to see whether at each story in turn any single column, or beam carrying a column, could be removed without causing collapse of more than a limited portion of the building local to the member concerned. The ACI building code (ACI 2002) provides a separate chapter for precast concrete structures. It requires that adequate horizontal, vertical, and peripheral ties be provided between all structural elements to develop tensile continuity and ductility of the elements. This combination of system continuity and ductility should enable the structure to either absorb the abnormal loads with minimal damage, or bridge localized damage as a result of the abnormal load. The provision of general structural integrity will bring the safety of precast large panel structures closer to that of the traditional cast-in-place reinforced concrete buildings (Fintel and Schultz 1979).
Concrete Slab Structures

Cast-in-place concrete construction has a better performance against progressive collapse than precast concrete construction (Fintel and Schultz 1979). However, there were progressive collapses of reinforced concrete slab structures (Feld 1964, Leyendecker and Fattal 1973, Litle 1975, Lew, et al. 1982, Hueste and Wight 1999). The mechanism most likely to trigger such a collapse is a punching shear failure at an interior column (Hawkins and Mitchell 1979). The key to preventing such progressive collapse may be to design and detail slabs such that are able to develop secondary load carrying mechanisms after initial failures have occurred. If the continuous reinforcement is properly anchored, the slab can develop tensile membrane action after initial failure. If the final collapse load is higher than the initial failure load, then a means of preventing progressive collapse has been provided (Mitchell and Cook 1984). The ACI building code (ACI 2002) states that even though top reinforcement is continuous over the support, it will not guarantee integrity without stirrup confinement or continuous bottom reinforcement. This is because top reinforcement tends to pull out of the concrete when a support is damaged. Hawkins and Mitchell (1979) concluded that only bottom reinforcement continuous through a column or properly anchored in a wall or beam should be considered effective as tensile membrane reinforcement.
Truss Structures

Space trusses are highly redundant structures. This statement means that space truss structures are expected to survive even after loss of several members. However, the failure of the Hartford Connecticut Coliseum space roof truss in 1978 (Ross 1984) showed that this assumption was not always correct.

Progressive collapse could occur following the loss of one of several potentially critical members when the structures are subject to full service loading (Murtha-Smith 1988). Force redistributions may cause members to exhibit nonlinear behavior and yield in the case of a tension member, or buckle in the case of a compression member (Schmidt and Hanoar 1979). However, because of strain hardening, a yielded tension member can typically absorb additional force, whereas a compression member will carry lower loads after reaching its buckling load. Thus, a compression member cannot resist additional force but has to shed force, and cause additional force redistributions into other members. These other members might also buckle and cause further force redistributions, and, thus, failure can progress through a structure to cause collapse. In addition, because the snap-through phenomenon is rapid, dynamic effects can increase the force redistribution intensity further (Murtha-Smith 1988). Morris (1993) pointed out that neglect of the dynamic response due to member snap-through leads to a significant over-estimate of the structure. Malla and Nalluri (1995) presented variations of natural frequencies of the truss structures due to the member failure.
Steel Frame Buildings

Steel moment resisting frame structures have similar structural characteristics as truss structures, except trusses do not resist moments in the members. Each steel member can have different failure modes because it will be subject to different load, cross sectional shape, and material property combinations. Failure modes can be categorized as follows (Salmon and Johnson 1996):

“1. Material Failure
   a. Ductile failure
      i. Plastic Deformation Failure
      ii. Failure induced by high temperature
   b. Brittle failure
      i. Fracture failure
      ii. Fatigue failure

2. Buckling
   a. Flexural column buckling, (i.e., global buckling)
   b. Torsional buckling
   c. Lateral-torsional buckling
   d. Local buckling ”

Material failures are expected mostly in tension. Steel member shows ductile behavior before failure in an ideal condition such as a uniaxial tension test. However, steel members can fail in a brittle manner in a practical case due to an initial flaw or notch. Steel members can lose stiffness in a high temperature induced by fire. Repeated
loading and unloading may eventually result in a fatigue failure even if the yield stress is never exceeded. Material failure in compression does not occur easily because steel has a redundant capacity due to strain hardening. Therefore, steel tends to buckle before it reaches the material failure strain in compression.

Steel members will buckle if the load exceeds the critical load. If a steel member has relatively low torsional stiffness, such as angles, tees, zees, and channels, it may buckle torsionally while the longitudinal axis remains straight. Steel beams can also buckle under bending without a proper lateral restraint, because the flange can be considered as a column when it is subject to compression by bending. Components such as flanges, webs, angles, and cover plates, which are combined to form a column section may themselves buckle locally prior to the entire section achieving its maximum capacity.

The reasons for progressive collapse may be several, as follows (Christiansson 1982).

“1. Material failure due to high stresses
2. Failure as a result of the inability of the structure to sustain the large deformations.
3. Stability failure of the entire structure.
4. Local stability failure in the form of buckling, etc.”

Christiansson (1982) said that there is no sharp distinction between Cases 1 and 2. Case 1 applies mainly for a brittle material, while Case 2 may occur when the deformation capacity of the material has been exhausted. In the same way, there is no
sharp distinction between Cases 3 and 4. Therefore, we can reconsider only two categories, material and buckling failures. There is another important failure mechanism, which is the joint connection failure. It will be explained in the following chapters.

2.4. Concluding Remarks

Progressive collapse is a sequential failure induced by a local failure. It can occur at any type of framed structure. However, some types of construction may be more prone in that they may have less ductility or have weaker connection details than others (Breen and Siess 1979).

Each construction type has a weakness. A precast concrete structure has a weak point in joint continuity and tensile resistance. A cast-in-place slab structure is vulnerable to punching shear failures at supports. A steel frame structure tends to buckle. Therefore, one should use a different approach to analyze each type of structure.

The focus of this study will be on progressive collapse of moment resisting steel frame structures. The alternative load path method can be used. The alternate load path method permits local damage to occur but provides alternate paths around the damaged area, so that the structure is able to absorb the abnormal load without collapse. In the event of the loss of load-carrying capacity of a structural member, the loss in load capacity is accommodated by bridging over the damaged area and the forces in the adjacent members are modified accordingly. The resistance to progressive collapse may be evaluated by analyzing the structure to determine whether alternate load paths
around the damaged area can be developed. Several studies were done using this method (Pekau 1982, Gross and McGuire 1983, Murtha-Smith 1988, Sucuoglu, et al. 1994). However, most of those analyses used static analysis methods and simplified assumptions, such as elastic or piece-wise nonlinear material properties, static loadings and responses, no buckling member or constant critical load. Since abnormal loadings and propagation of failures are dynamic events, it is reasonable to apply a dynamic analysis approach for progressive collapse.

Most of the previous studies focused on specific events, such as punching shear, buckling, yielding failures. They were based on incomplete or simplified assumptions, mainly because there was no sufficient computing power available to perform dynamic and highly nonlinear analysis at that time. Many numerical approaches that were not possible in the past are now available.

It seems that there were no studies of steel structure that includes all of followings:

1. Intensive dynamic analysis
2. Complete nonlinear material behavior
3. Dynamic local member buckling
4. Evaluation of the status of the whole structure while a local failure is advancing
5. Joint connection rigidity

This research will consider these factors to achieve a more comprehensive analysis of progressive collapse.
3. Analysis Theory of Steel Frame Structure

3.1. Introduction

There are two types of failures for the steel frame structure. These are material and buckling failures. Each failure has subcategories such as ductile, brittle material failure, and flexural, local, lateral-torsional buckling failure, as explained before. Material failure involves analysis of member strength. It includes nonlinearity and inelasticity of material such as proportional limit, yield stress (or surface), yield plateau, strain hardening, necking, and eventually rupture. Buckling failure requires analysis of structure and structural member stability. It includes second-order effect, such as $P-\Delta$, or $P-\delta$ effect ($P-\Delta$ effect is induced by the deformation of the whole structure and $P-\delta$ effect is induced by the deformation within the member).

The main cause of progressive collapse in steel frame structures may be propagation of member buckling. Therefore, it is important to calculate member buckling, as accurately as possible. However, it is very hard to estimate the member buckling load accurately, because each single member connects and interacts with each other.

3.1.1. The Effective Length Factor K Method

To evaluate a member buckling, one can estimate the effective length of a member. The effective length approach for column stability has been a very famous method to predict the buckling load of each column. In current engineering practice,
the interaction between the structural system and its members is represented by the effective length factor (Chen 2000).

Figure 3-1 shows the subassemblage model for the evaluation of the effective length of a member. We can obtain the effective length of column c2 by calculating the $G_A$ and $G_B$ which represent the stiffness ratio between columns and beams at joint A and B.

(a) Side Sway Prohibited       (b) Side Sway Permitted

Figure 3-1. Subassemblage Models

However, it is known that isolated columns analyzed by the conventional effective length approach involves inaccurate assessment of critical loads and the corresponding structural response predictions (Hellesland and Bjorhovde 1996). There have been many efforts to evaluate the correct effective lengths of columns (Ermopoulos 1991, Hellesland and Bjorhovde 1996, Aristizabal-Ochoa 1997, Kishi, et
al. 1997, Essa 1998). Chen (2000) also pointed out that the approach has major limitations, despite its popular use in the past and present as a basis for design, because it does not consider the interaction of strength and stability between the member and structural system in a direct manner, so that it will not give an accurate indication of the factor against failure.

The effective length is calculated based on following assumptions (Chen, Lui 1987).

1. All members are prismatic and behave elastically.
2. The axial forces in the beams are negligible.
3. All columns in a story buckle simultaneously.
4. At a joint, the restraining moment provided by the beams is distributed among the columns in proportion to their stiffnesses.
5. At buckling, the rotations at the near and far ends of the girders are equal and opposite (i.e., the girders are bent in single curvature – for braced frame)
6. At buckling, the rotations at the near and far ends of the girders are equal in magnitude and direction (i.e., the girders are bent in double curvature – for unbraced frame)

These assumptions set many limitations for analyzing the extreme behavior of structures. First, the inelastic behavior cannot be calculated because the effective length is based on elastic assumption. Second, it excludes the possibility of beam buckling by neglecting the axial force in the beam. Third, the assumption of simultaneous buckling
of columns in a story is not always true. Since each column may have different size, boundary condition, and applied load, it probably fails at different time step. Progressive collapse analysis should be able to track the failure sequence. Therefore, this assumption does not provide an adequate background for the analysis. The fourth assumption indicates the rigid frame. Therefore, the effective length K factor method has to be modified to model the semi-rigid frame (Kishi, et al. 1997, 1998). The fifth and sixth assumptions describe the buckling mode of the member for braced and unbraced frame. However, failure mode can change while the failure sequence progresses. It is obvious that these assumptions are not valid for progressive collapse analysis. Therefore, it seems that a different approach is required to analyze the progressive collapse.

3.1.2. Advanced Analysis of Steel Frame Structure

Advanced analysis is referred to any method of analysis that sufficiently represents the strength and stability behavior such that separate specification member capacity checks are not required (Chen, Toma 1994). The main distinction between advanced analysis and other simplified analysis methods is that advanced analysis combines, for the first time, the theories of plasticity and stability in the limit states design of structural steel frameworks. Other analysis and design methods treat stability and plasticity separately – usually through the use of beam-column interaction equations and member effective length factors (Liew, et al. 1991).

The strength of a column may be expressed by
\[ P_{cr} = \frac{\pi^2 E_t}{(KL/r)^2} A_g = \frac{P_{cr}}{A_g} \]  \hspace{1cm} (3-1)

where \( E_t \) = tangent modulus of elasticity at stress \( \frac{P_{cr}}{A_g} \)

\( A_g \) = gross cross-sectional area of member

\( KL/r \) = effective (or equivalent pinned-end) slenderness ratio

\( K \) = effective length factor

\( L \) = length of member

\( r \) = radius of gyration = \( \sqrt{I/A_g} \)

\( I \) = moment of inertia

This is a modified Euler critical load equation. The tangent modulus of elasticity was used instead of the modulus of elasticity. Failure modes of column depend on the load, boundary condition, shape of cross section and especially slenderness ratio. If a column is long enough (high slenderness ratio), it will buckle. If a column is short enough (low slenderness ratio), it will crush (material failure). Figure 3-2 shows a typical range of column strength vs slenderness ratio.
Figure 3-2 shows that a column fails at the Euler buckling load for a high slenderness ratio, while it fails at the material failure load for a low slenderness ratio. Since the slenderness ratio is the only factor that changes a failure mode, it is the most important factor to calculate a column strength. As this figure implies, the accurate strength of column can be obtained by considering strength and stability of a member simultaneously. Load and Resistance Factor Design (AISC 1994) also provides design procedures to estimate critical column stress.
Figure 3-3 depicts critical column stress $F_{cr}$ vs $KL/r$ according to Load and Resistance Factor Design, for various yield stresses (Salmon and Johnson 1996).

Figure 3-3 depicts critical column stress vs slenderness ratio for various yield stresses. It shows that a column fails at yield stress if it has a low slenderness ratio, and fails at much lower stress if it has high slenderness ratio. These two regions are connected with smooth curve. It implies that the transition area is affected by both strength and stability of a member.

There is another issue called inelastic bucking (Salmon and Johnson 1996). Euler’s theory pertains only to situations where compressive stress below the elastic limit acts uniformly over the cross-section when buckling failure occurs. However, in
many cases, some of fibers in the cross section usually yield when the member buckles. This is called inelastic buckling. If all fibers in the cross section yield before the member reaches its critical load, it is called material failure. The inelastic buckling is affected by strength and stability simultaneously. Therefore, inelastic buckling is in the transition area in Figures 3-2 and 3-3. It connects material and buckling failure with smooth curve. Since many of structural members have slenderness ratios in the vicinity of transition area, advanced analysis is essential to obtain accurate results.

When a change in the geometry of a structure or structural component under compression will result in the loss of its ability to resist loadings, this condition is called instability. Equilibrium equations must be written based on the geometry of structure that becomes deformed under load. This is known as a second-order analysis (Chen and Lui 1987). The stiffness of a structure changes as the geometry changes. If the load and deformation keeps changing or increasing, the stiffness of a structure may reach a point of stiffness vanishing. It is called buckling. In ordinary structural analysis, the original geometry does not change even if the load goes to an extreme value. Therefore, ordinary structural analysis cannot capture the buckling phenomenon.

A second-order analysis is essential for stability analysis, as explained above. One also needs a plastic analysis approach for strength consideration. Therefore, a second-order inelastic analysis is required.

It is proposed to adopt an advanced analysis, based on Chen (2000) as outlined below:

1. Elastic-plastic hinge
A. Zero length plastic hinges
B. No spread of yielding through the cross-section, or along the length
C. No consideration of residual stresses
D. Second-order geometric effects can be considered.

2. Plastic zone
   A. Discretized finite elements along the length and through the cross-section.
   B. Captures the incremental load-versus-deflection response considering the second-order geometric distortion.
   C. A constant residual stress pattern can be assumed.
   D. The spread of plasticity is traced.

3. Quasi-plastic zone
   A. A compromise between plastic zone and elastic plastic hinge methods
   B. The spread of plasticity is considered by flexibility coefficients.
   C. A simplified residual stress pattern can be used.
   D. The fully plastic cross-section is calibrated to the plastic zone solution.
   E. There is no potential to upgrade this from its current two-dimensional restriction.

4. Refined plastic hinge method
   A. A step up from the elastic plastic model for two dimensions
   B. Distributed plasticity-smooth stiffness degradation of a hinge
C. Inelasticity is considered indirectly by forces rather than strains. The tangent \( E_t \) modulus is used to describe the effect of residual stresses.

D. Stiffness degradation function is used for gradual yielding.

E. Connection flexibility can be modeled using rotational springs.

5. Practical refined plastic hinge method

A. The refined model (4, above) is made practical by calibration to the LRFD empirical code equations.

B. A separate modification of tangent modulus \( E_t \) is imposed to consider geometric imperfections.

C. The CRC tangent modulus model is used allowing residual stresses to be considered separately.

Methods 3, 4, and 5 are modifications of methods 1 and 2. Method 1 (Elastic-plastic hinge) considers plasticity by using zero length plastic hinges. Spreading yielding does not appear in this method. Part of a member between plastic hinges behaves elastically. Therefore, the status of structure will suffer a sudden change from elastic to plastic range, when yield occurs. However, method 2 (Plastic zone) considers gradual spread of yielding through the cross section and along the length of the member. Therefore, it shows a smooth transition from elastic to plastic range, and gives more accurate responses, as expected. Figure 3-4 shows moment-curvature relationship for a concentrated plastic hinge and a spread of plasticity model.
3.1.3. Semi-Rigid Connections

It is very common practice to use rigid or pinned connection between steel members for analysis purpose. However, experiments have shown that a real steel connection is neither rigid nor pinned (Kameshki 2003). Furthermore, experiments have also shown that when a moment is applied to a ductile connection, the relationship between the moment and the beam column rotation is nonlinear (Kameshki and Saka 2003). There have been studies to evaluate the effective length with semi-rigid connection (Ermoupolos 1991, Kishi, et al. 1997, Aristozabal-Ochoa 1997, Kishi, et al. 1998, Kameshki and Saka 2003, Liew, et al. 2000).
Figure 3-5 shows moment-rotation curves for various steel connections. A connection should be considered as an individual member of the structure when its behavior is semi-rigid. Axial springs and rotational springs can be adopted to simulate the behavior of semi-rigid nonlinear connection. Figure 3-6 shows the conceptual shape of connection model.

Figure 3-5. Connection Moment-Rotation Curve (Kameshki and Saka 2003)
Only the rotational nonlinear springs will be used to simulate the semi-rigid connection, assuming that the moment-curvature relationship of a semi-rigid connection dominates its behavior.

3.2. Formulation of 2D Finite Element Method for Plastic Zone Analysis

3.2.1. General

As far as geometric nonlinearity is concerned, one may employ one of three approaches to deriving the stiffness matrices of a beam element, namely the total Lagrangian, the updated Lagrangian, and the corotational formulations. The updated Lagrangian formulation is known to be more complicated than the other two alternatives because the volume integrations in an updated Lagrangian formulation
should strictly be carried out over the deformed shape of the element at the last computed configuration. In contrast, the integration procedures in a total Lagrangian or corotational formulation are relatively straight-forward because they are performed over the undeformed beam element (Teh and Clarke 1999).

The solution for the geometric nonlinearity formulation cannot be obtained by simply inverting a stiffness matrix. The essential difficulty of a geometrically nonlinear analysis is that equilibrium equations must be written with respect to the deformed geometry, which is not known in advance (Cook, et al. 2001). Since the response is nonlinear, the solution must be obtained iteratively until the solution converges. Solution convergence can be judged by user defined tolerance criteria. If inappropriate criteria are used, the iteration may terminate before the necessary solution accuracy is reached, or continue after the required accuracy has been reached. If the solution does not converge, the time step or load step should be decreased to try another iteration. The most famous iterative method is the Newton-Rhapson iteration, which will be introduced later.

Clarke developed a plastic-zone using the total Lagrangian formulation and Newton-Rhapson method (Chen and Toma 1994). In Clarke’s finite element nonlinear formulation, a curved line element is used. The element stiffness matrices and nodal residual force vectors are integrated numerically along the element length using three-point Gaussian quadrature.

The assumptions employed in the analysis are generally those of a conventional beam-column and shallow arch theory, as summarized next:
1. The beam thickness is small compared to its length and radius of curvature.

2. The normal strain perpendicular to the beam axis is negligible, thus the cross-section preserves its shape during deformation.

3. Normals to the beam axis remain plane and normal during the deformation, so that the strain at any point in the beam can be expressed in terms of the membrane strain and curvature (Bernoulli-Euler hypothesis)

4. The strains are small.

In the Lagrangian formulations of the continuum mechanics equations of motion, all the static and kinematic variables are referred to a reference configuration. The following description of the plastic-zone analysis is based on the total Lagrangian formulation, which means that the reference configuration corresponds to the original, undeformed configuration (Bathe 1996).

The stress and strain measures utilized in Lagrangian geometric nonlinearity are the second Piola-Kirchhoff stress tensor and the Green-Lagrange strain tensor (Bathe 1996). For small strains, the components of the second Piola-Kirchhoff stress tensor and the Green-Lagrange strain tensor have a physical significance close to that of the conventional engineering stress and strain measure.

The finite element formulation process involves two basic steps:

1. Derivation of a set of governing equations that describe the nonlinear problem. This step is not in fact unique to the finite element process, but is common to all problems of structural mechanics.

   A. Selection of an appropriate theory to relate strains to displacements.
B. Adoption of an appropriate constitutive model to deduce stresses from strains.

C. Application of the principle of virtual displacement to derive the governing equilibrium equations.

2. Solution of these nonlinear equations by the finite element process involving discretization of the structure and adoption of nodal displacements as the fundamental unknowns. Solution of the governing equations by the finite element method involves the following procedures:

A. Selection of an appropriate finite element and set of nodal displacements.

B. Definition of the displacement functions expressing general element displacements in terms of nodal displacements.

C. Expression of the governing equilibrium equations in terms of nodal displacements.

D. Solution of the resulting nonlinear equations for the nodal displacements.

3.2.2. Strain-Displacement Relationships

For in-plane beam-column problems, the only strain component of relevance is the longitudinal strain, denoted by $\varepsilon_s$, which may be expressed as follows:

$$
\varepsilon_s = (\bar{\varepsilon}_s - \bar{\varepsilon}_s^0) + y(\kappa_s - \kappa_s^0)
$$

(3-2)

or, in the following matrix and vector notation,
\[
\varepsilon_s = \left[ 1 \ y \right] \begin{bmatrix} \varepsilon_s \\ \kappa_s \end{bmatrix} = \left[ 1 \ y \right] \begin{bmatrix} \varepsilon_s \\ \kappa_s \end{bmatrix} - \begin{bmatrix} \varepsilon_{s0} \\ \kappa_{s0} \end{bmatrix} \tag{3-3}
\]

\[
= \left[ 1 \ y \right] \left( \begin{bmatrix} \varepsilon_0 \\ \kappa_0 \end{bmatrix} \right) \tag{3-4}
\]

where, \( y \) = the distance above the reference line in the cross section.

\( \varepsilon_s, \kappa_s \) = the membrane strain and curvature, respectively, of the reference line that are caused by deformation.

\( \varepsilon_{s0}, \kappa_{s0} \) = the initial strains.

The Green-Lagrange strain measure of the reference line is defined by

\[
\bar{\varepsilon}_s = \frac{\left( \frac{d\pi}{ds} + \bar{v} \frac{d\phi}{ds} \right)}{1} + \frac{1}{2} \left( \frac{d\pi}{ds} + \bar{v} \frac{d\phi}{ds} \right)^2 + \frac{1}{2} \left( \frac{d\pi}{ds} - \bar{u} \frac{d\phi}{ds} \right)^2 \tag{3-5}
\]

and the curvature of the reference line is given for moderate rotations by

\[
\kappa_s = -\frac{d}{ds} \left( \frac{d\pi}{ds} - \bar{u} \frac{d\phi}{ds} \right) \tag{3-6}
\]

\[
= -\frac{d^2\pi}{ds^2} + \frac{d\pi}{ds} \bar{u} \frac{d\phi}{ds} + \bar{u} \frac{d^2\phi}{ds^2} \tag{3-7}
\]

where, \( \bar{u} \) = membrane displacement of a curved element

\( \bar{v} \) = normal direction displacement to the axis of a curved element

\( \phi \) = angle to the measured point of strain \( \left( \frac{d\phi}{ds} = \kappa \right) \)

\( s \) = coordinate along the axis of a curved element
To express the strain-displacement relations in matrix and vector notation, the generalized strain vector \( \{\varepsilon\} \) can be expressed as the sum of linear and nonlinear components as

\[
\{\varepsilon\} = \{e\} + \{\eta\} \quad (3-8)
\]

in which,

\[
\{\varepsilon\} = \begin{bmatrix} \varepsilon_x \\ \kappa_x \end{bmatrix} \quad (3-9)
\]

\[
\{e\} = \begin{bmatrix} \theta_{xx} \\ -\frac{d\theta_{ss}}{ds} \end{bmatrix} \quad (3-10)
\]

\[
\{\eta\} = \frac{1}{2} \begin{bmatrix} \theta_{xx} & \theta_{ss} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_{xx} \\ \theta_{ss} \end{bmatrix} = \frac{1}{2} [A] \{\theta\} \quad (3-11)
\]

and the displacement gradients \( \theta_{xx} \) and \( \theta_{ss} \) are defined by

\[
\theta_{xx} = \frac{d\bar{u}}{ds} + \bar{v} \frac{d\phi}{ds} \quad (3-13)
\]

\[
\theta_{ss} = \frac{d\bar{v}}{ds} - \bar{u} \frac{d\phi}{ds} \quad (3-14)
\]

Taking a variation of Eq. (3-8) with respect to displacements results in

\[
\delta\{\varepsilon\} = \delta\{e\} + [A] \delta\{\theta\} \quad (3-15)
\]

since it can be shown readily by expanding out terms that

\[
\delta [A] \{\theta\} = [A] \delta \{\theta\} \quad (3-16)
\]
3.2.3. Stress-Strain Relationships

In the present work, only longitudinal strains ($\varepsilon_s$) are considered and so at any point in the cross-section, yielding is assumed to occur as a result of longitudinal stress ($\sigma_s$) only. In addition, the material stress-strain curve is assumed to be identical in tension and compression, and yield surface is defined by isotropic hardening rule. An incremental theory of plasticity (Mendelson, 1983) is adopted for this formulation.

An increment in longitudinal strain ($d\varepsilon_s$) can be considered to be composed of an elastic component ($d\varepsilon_{se}$) and a plastic component ($d\varepsilon_{sp}$).

$$d\varepsilon_s = d\varepsilon_{se} + d\varepsilon_{sp} \hspace{1cm} (3-17)$$

The nonnegative equivalent plastic strain increment is defined by

$$d\varepsilon_{p} = \sqrt{d\varepsilon_{sp}^2} \hspace{1cm} (3-18)$$

and the nonnegative effective stress is defined by

$$\bar{\sigma} = \sqrt{\sigma_s^2} \hspace{1cm} (3-19)$$

The initial and subsequent yield surfaces can be expressed in the form

$$F(\sigma_s, k) = \bar{\sigma} - \sigma_y \hspace{1cm} (3-20)$$

in which $k$ is a ‘hardening parameter’, and $\sigma_y$ is the instantaneous yield stress. Specific yield surfaces are not required because this is a uniaxial case. For isotropic hardening, $k$ can be interpreted as the total equivalent plastic strain, $\bar{\varepsilon}_p = \int d\varepsilon_p$. The instantaneous yield stress $\sigma_y$ is given for isotropic hardening by
\[ \sigma_y = \sigma_{y0} + \int d\sigma_y = \sigma_{y0} + \int_0^\epsilon H' d\bar{\varepsilon}_p \] (3-21)

in which \( \sigma_{y0} \) is the initial yield stress and \( H' \) is the slope of the effective stress-equivalent plastic strain curve, as shown in Figure 3-7. The parameter \( H' \) can be computed from

\[ H' = \frac{EE_T}{E - E_T} \] (3-22)

where, \( E = \) the elastic modulus of material

\( E_T = \) the tangent modulus of material

(a) Uniaxial Stress-Strain Relation

Figure 3-7. Stress-Strain Relationships (cont.)
(b) Effective Stress-Equivalent Plastic Strain Relation

Figure 3-7. Stress-Strain Relationships

The instantaneous yield stresses are evaluated at every time step. Each monitoring point stores calculated yield stress, so that it can be used for the next instantaneous yield stress calculation.

\[ H_p = \frac{d\bar{\sigma}}{d\bar{\varepsilon}_p} \]

\[ \bar{\varepsilon} = \int d\bar{\varepsilon}_p \]

3.2.4. Stress Resultants

For the in-plane analysis of beam-columns, a stress resultants vector \( \{\sigma\} \) can be defined as

\[
\{\sigma\} = \begin{bmatrix} N \\ M \end{bmatrix} = \int_A [y]^T \sigma_x dA \tag{3-23}
\]

where, \([y] = [1 \ y]\)

A denotes cross-sectional area. The stress resultant of axial force \( (N) \) and bending moment \( (M) \) are therefore defined by

\[
N = \int_A \sigma_x dA \tag{3-24}
\]
\[ M = \int_A y \sigma_y dA \quad (3-25) \]

Taking a variation of Eq. (3-23) gives

\[
\delta \{ \sigma \} = \int_A [Y]^T E_T [Y] J dA \delta \left( \{ \varepsilon \} - \{ \varepsilon_o \} \right) = [D_T] \delta \left( \{ \varepsilon \} - \{ \varepsilon_o \} \right) \quad (3-26)
\]

in which \([D_T]\) is the tangent modulus matrix, given by

\[
[D_T] = \begin{bmatrix}
\int_A E_T dA & \int_A E_T y dA \\
\int_A E_T y dA & \int_A E_T y^2 dA
\end{bmatrix}
\quad (3-28)
\]

where, \(y\) = the coordinate in the cross-section above the element reference line

If the material behavior is elastic and the reference line is chosen as the centroidal axis, the tangent modulus matrix simplifies to the elastic constitutive matrix

\[
[D] = \begin{bmatrix}
EA & 0 \\
0 & EI
\end{bmatrix}
\quad (3-29)
\]

where, \(A\) = the cross-sectional area

\(I\) = the second moment of inertia about the centroidal axis

If the material behavior is inelastic and \(y\) is measured from the original elastic centroid, the off-diagonal terms in Eq. (3-28) become nonzero.

### 3.2.5. The Principle of Virtual Displacements

In the total Lagrangian formulation, equilibrium is expressed using the principle of virtual displacements (Bathe 1996) as
\[ \int_V \sigma \cdot \delta \varepsilon \cdot dV = \int_V \rho q_i \delta u_i \cdot dV + \int_S p_i \delta u_i \cdot dS \tag{3-30} \]

where, \( \sigma \cdot \varepsilon \) = the longitudinal stress

\( \varepsilon \) = the longitudinal strain

\( \rho \) = the mass density

\( q_i \) = body forces per unit mass

\( p_i \) = surface traction

\( \delta \varepsilon_i \) = virtual strain

\( \delta u_i \) = virtual displacement field

\( V \) = the original volume of the element in the undeformed configuration (left-hand side)

\( V \) = volume of the body subject to body force \( q_i \) (right-hand side)

\( S \) = surface area subject to surface traction \( p_i \)

The left-hand side of Eq. (3-30) constitutes the internal virtual work performed when the element is subject to a virtual displacement field \( \delta u_i \). The right-hand side of Eq. (3-30) corresponds to the external virtual work performed in moving the external loading system, consisting of body forces per unit mass \( q_i \) and surface traction \( p_i \).

Using Eq. (3-4) and Eq. (3-23), the virtual work equation [Eq. (3-30)], can be expressed in terms of generalized strains and stresses as

\[ \int_s \delta \{ \varepsilon \}^T \{ \sigma \} ds = \int_s \delta \{ u \}^T \{ p \} ds \tag{3-31} \]
where, \( \{u\} \) = the displacement vector of any point on the reference axis

\( \{p\} \) = distributed forces vector at any point on the reference axis

\( s \) = arc-length along the reference axis

The next step in the finite element formulation is to discretize Eq. (3-31) so that the resulting equation is expressed in terms of nodal displacements.

3.2.6. Discretization of the Virtual Work Equation

Element Geometric Description

The element described here is isoparametric (Bathe 1996) which means that the following two conditions are satisfied:

1. nodal coordinates and nodal degrees of freedom are in one-to-one correspondence.

2. the same shape functions are used to interpolate the coordinates of a point within the element from the nodal coordinates, and the displacements of a point within the element from the nodal displacements.

The curved eight-degree of freedom element used for the plastic-zone analysis is illustrated in Figure 3-8. The element geometry is defined in terms of a reference line. The element nodal coordinates are defined in a global Cartesian axis system (X, Y) coordinates.
Figure 3-8. Geometry and Local and Global Displacement Fields of the Element

The slope of the element is described by the angle $\phi$ and the rates of change of the X and Y coordinates with respect to arc-length $s$, given by $\frac{dX}{ds}$ and $\frac{dY}{ds}$, respectively. The gradients $\frac{dX}{ds}$ and $\frac{dY}{ds}$ are related by

$$\frac{dX}{ds} = \cos \phi$$  \hspace{1cm} (3-32)

$$\frac{dY}{ds} = -\sin \phi$$  \hspace{1cm} (3-33)

The geometric curvature of the element and its rate of change with respect to arc-length $s$ in the reference configuration are described by $\frac{d\phi}{ds}$ and $\frac{d^2\phi}{ds^2}$. 
respectively. The sign convention for $\phi$ results in positive values of curvature $\frac{d\phi}{ds}$ for the element oriented as shown in Figure 3-8.

The geometry within the element is interpolated from the nodal parameters using cubic Hermitian polynomial interpolating functions as

$$X(\xi) = \sum_{i=1}^{2} N_{0i}(\xi) X_i + N_{1i}(\xi) \left( \frac{dX}{ds} \right)_i$$  \hspace{1cm} (3-34)

$$Y(\xi) = \sum_{i=1}^{2} N_{0i}(\xi) Y_i + N_{1i}(\xi) \left( \frac{dY}{ds} \right)_i$$  \hspace{1cm} (3-35)

$$\frac{d\phi}{ds}(\xi) = \sum_{i=1}^{2} N_{0i}(\xi) \left( \frac{d\phi}{ds} \right)_i + N_{1i}(\xi) \left( \frac{d^2\phi}{ds^2} \right)_i$$  \hspace{1cm} (3-36)

in which $N_{0i}, N_{1i}, N_{02}, N_{12}$ are the element shape functions, illustrated in Figure 3-9 and defined by

$$N_{0i} = \frac{1}{4} (\xi^3 - 3\xi^2 + 2)$$  \hspace{1cm} (3-37)

$$N_{1i} = \frac{l}{4} (\xi^3 - \xi^2 - \xi + 1)$$  \hspace{1cm} (3-38)

$$N_{02} = \frac{1}{4} (-\xi^3 + 3\xi^2 + 2)$$  \hspace{1cm} (3-39)

$$N_{12} = \frac{l}{4} (\xi^3 + \xi^2 - \xi - 1)$$  \hspace{1cm} (3-40)
In Eqs. (3-37) to (3-40), $l$ is the half-length of the element in the initial configuration, and $\xi$ is a dimensionless curvilinear coordinate ranging from -1 at node 1 of the element ($s = -l$) to +1 at node 2 of the element ($s = +l$), so that

\[ s = l \xi \quad (3-41) \]
\[ ds = ld\xi \quad (3-42) \]

The element reference line local displacements in the curvilinear coordinate system consist of the local tangential displacement $\bar{u}$ and the local transverse displacement $\bar{v}$. For a curved element, simple polynomial displacement fields defined in terms of the local displacements may produce spurious straining when the element undergoes rigid body translation, although convergence towards strain-free rigid body
motion occurs as the angle subtended by each element decreases to zero. To overcome this problem of spurious straining, the element nodal displacements and strain-displacement matrix are expressed in global coordinates.

The vector of element nodal displacements in global coordinates is defined as

\[
\{a\} = \{a_1, a_2\}^T
\]  

(3-43)

with

\[
\{a\}_i = \left\{U_i, \left(\frac{dU}{ds}\right)_i, V_i, \left(\frac{dV}{ds}\right)_i\right\}^T
\]  

(3-44)

The element displacement field is written in terms of the shape functions defined in Eqs. (3-37) to (3-40) as

\[
U(\xi) = \sum_{i=1}^{2} \left[ N_{0i}(\xi) U_i + N_{1i}(\xi) \left(\frac{dU}{ds}\right)_i \right]
\]  

(3-45)

\[
V(\xi) = \sum_{i=1}^{2} \left[ N_{0i}(\xi) V_i + N_{1i}(\xi) \left(\frac{dV}{ds}\right)_i \right]
\]  

(3-46)

which can be expressed in matrix notations as

\[
\{u\} = [N]\{a\}
\]  

(3-47)

where, \(\{u\} = [U \ V]^T\)

\[
[N] = \begin{bmatrix} [N]_1 & [N]_2 \end{bmatrix}
\]

with

\[
[N]_i = \begin{bmatrix} N_{0i} & N_{1i} & 0 & 0 \\ 0 & 0 & N_{0i} & N_{1i} \end{bmatrix}
\]  

(3-48)

The transformation between the local and global displacement components is
\[
\begin{bmatrix}
\bar{u} \\
\bar{v}
\end{bmatrix} =
\begin{bmatrix}
\cos \phi & -\sin \phi \\
\sin \phi & \cos \phi
\end{bmatrix}
\begin{bmatrix}
U \\
V
\end{bmatrix}
\]  
(3-49)

\[
\{\bar{u}\} = [T]\{u\}
\]  
(3-50)

**Strain-Displacement Matrices**

The vector of generalized strains could be expressed as the sum of linear and nonlinear components

\[
\{\varepsilon\} = \{\varepsilon\} + \{\eta\} = \left[B_0\right] + \frac{1}{2} \left[B_{\ell}\right]\{\alpha\}
\]  
(3-51)

where, \left[B_0\right] = \left[B_0\right]_1 \left[B_0\right]_2

\[
[B_0]_i = \begin{bmatrix}
\frac{N'_{0i}}{l} \cos \phi & \frac{N'_{li}}{l} \cos \phi & \frac{-N'_{0i}}{l} \sin \phi & \frac{-N'_{li}}{l} \sin \phi \\
\frac{-N'_{0i}}{l} \frac{d\phi}{ds} \cos \phi & \frac{-N'_{li}}{l} \frac{d\phi}{ds} \cos \phi & \frac{N'_{0i}}{l} \frac{d\phi}{ds} \sin \phi & \frac{N'_{li}}{l} \frac{d\phi}{ds} \sin \phi \\
\frac{-N''_{0i}}{l^2} \sin \phi & \frac{-N''_{li}}{l^2} \sin \phi & \frac{N''_{0i}}{l^2} \cos \phi & \frac{N''_{li}}{l^2} \cos \phi
\end{bmatrix}
\]  
(3-52)

\[
[B_{\ell}] = [A][G]
\]  
(3-53)

with,

\[
[G] = \left[G\right]_1 \left[G\right]_2
\]  
(3-54)

\[
\left[G\right]_1 = \begin{bmatrix}
\frac{N'_{0i}}{l} \cos \phi & \frac{N'_{li}}{l} \cos \phi & \frac{-N'_{0i}}{l} \sin \phi & \frac{-N'_{li}}{l} \sin \phi \\
\frac{N''_{0i}}{l} \sin \phi & \frac{N''_{li}}{l} \sin \phi & \frac{-N''_{0i}}{l} \cos \phi & \frac{-N''_{li}}{l} \cos \phi
\end{bmatrix}
\]  
(3-55)

\[
\left[G\right]_2 = \begin{bmatrix}
\frac{N'_{0i}}{l} \cos \phi & \frac{N'_{li}}{l} \cos \phi & \frac{-N'_{0i}}{l} \sin \phi & \frac{-N'_{li}}{l} \sin \phi \\
\frac{N''_{0i}}{l} \sin \phi & \frac{N''_{li}}{l} \sin \phi & \frac{-N''_{0i}}{l} \cos \phi & \frac{-N''_{li}}{l} \cos \phi
\end{bmatrix}
\]  
(3-56)

\[
\left[G\right]_2 = \begin{bmatrix}
\frac{N'_{0i}}{l} \cos \phi & \frac{N'_{li}}{l} \cos \phi & \frac{-N'_{0i}}{l} \sin \phi & \frac{-N'_{li}}{l} \sin \phi \\
\frac{N''_{0i}}{l} \sin \phi & \frac{N''_{li}}{l} \sin \phi & \frac{-N''_{0i}}{l} \cos \phi & \frac{-N''_{li}}{l} \cos \phi
\end{bmatrix}
\]  
(3-57)
\[ [A] = \begin{bmatrix} \theta_{ux} & \theta_{uy} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \theta' \\ 0 & 0 \end{bmatrix} \] (3-58)

\[ \theta = [G][a] \] (3-59)

The matrix \([B_0]\) is the infinitesimal strain-displacement which relate the linear strain component \(\{\epsilon\}\) to the element nodal displacements \(\{a\}\). In the above equations, \((\cdot)\)' denotes differentiation with respect to the dimensionless variable \(\xi\). The nonlinear strain components \(\{\eta\}\) is expressed in terms of nodal displacements gradients \(\{\theta\}\).

The variation of Eq. (3-52) may be determined as

\[ \delta\{\epsilon\} = [B_0] + [B_\xi] \delta\{a\} \] (3-60)

\[ = [B] \delta\{a\} \] (3-61)

**Equilibrium Equations**

The variation of the element displacement field defined by Eq. (3-47) can be written

\[ \delta\{u\} = [N] \delta\{a\} \] (3-62)

in which \([N]\) is the matrix of shape functions defined by Eqs. (3-37) to (3-40).

Eq. (3-31) leads to the following equation by substituting Eq. (3-61) and (3-62).

\[ \delta\{a\}^T \left( \int [B]^T \{\sigma\} ds - \{R\} \right) = \delta\{a\}^T \{\psi\} = 0 \] (3-63)

where, \(\{R\} = [N]^T \{p\}\), \(\{p\}\) is a vector of distributed loads.
\( \{ \psi \} \) is the nodal residual force vector for the element. The element equilibrium is achieved by the condition \( \{ \psi \} = 0 \).

3.2.7. Solving the Nonlinear Equilibrium Equations

The Newton-Phapson solution algorithm has been used for solving nonlinear equations as a basic iterative method. The fundamental aim is to obtain the set of nodal displacement \( \{ a \} \) for which \( \{ \psi (\{ a \}) \} = \{ 0 \} \). The tangent stiffness matrix should be calculated at every iterative step to achieve this.

Suppose an initial estimate \( \{ a \}^n \) of the nodal displacements is known for which the structure is not in equilibrium. For an increment in nodal displacements of \( \{ \Delta a \}^{n+1} \), the residual function \( \{ \psi \} \) can be expanded by the Taylor’s series about \( \{ a \}^n \), ignoring third and succeeding terms as

\[
\{ \psi ([\{ a \}^n + \{\Delta a\}^{n+1}]\} = \{ \psi ([\{ a \}^n])\} + \frac{\partial \{ \psi \}}{\partial \{ a \}} \bigg| \{ a \} = [\{ a \}^n] \{ \Delta a \}^{n+1} + \ldots \quad (3-64)
\]

For the requirement \( \{ \psi ([\{ a \}^n + \{\Delta a\}^{n+1}]\} = 0 \), the linearized approximation to the equilibrium equations is

\[
\{ \psi ([\{ a \}^n])\} + [K_T]\{ \Delta a \}^{n+1} = \{ 0 \} \quad (3-65)
\]

where, \( [K_T] = \frac{\partial \{ \psi \}}{\partial \{ a \}} \bigg| \{ a \} = [\{ a \}^n] \)

The \( [K_T] \) is the tangent stiffness matrix. The displacement increment \( \{ \Delta a \}^{n+1} \) can be calculated as
\[ \{\Delta a\}^{n+1} = -[K_T]^{-1}\{\psi\}^n \quad (3-66) \]

Then, the total displacement is

\[ \{a\}^{n+1} = \{a\}^n + \{\Delta a\}^{n+1} \quad (3-67) \]

To evaluate \([K_T]\), a variation of the residual with respect to the nodal displacements \(\{a\}\) is used as,

\[ \delta \{\psi\} = \int \delta \{\sigma\} ds + \int \delta [B]^T \{\sigma\} ds - \delta \{R\} \quad (3-68) \]

If the loading is assumed to be conservative (i.e., not dependent on the deformation), \(\delta \{R\} = \{0\}\).

Using Eqs. (3-27) and (3-61), the first term of \(\delta \{\psi\}\) leads to

\[ \int [B]^T \delta \{\sigma\} ds = \delta \{a\} \int [B]^T [D]B] ds \quad (3-69) \]

The second term is more complicated as

\[ \int \delta [B]^T \{\sigma\} ds = \int \delta \{G\}^T \{A\} \{\sigma\} ds \quad (3-70) \]

\[ = \int \{G\}^T \delta \{A\} \{\sigma\} ds \quad (3-71) \]

\[ = \delta \{a\} \int \{G\}^T \{\bar{S}\}G ds \quad (3-72) \]

where, \([\bar{S}] = \begin{bmatrix} N & 0 \\ 0 & N \end{bmatrix}\)

Therefore, the stiffness matrix can be calculated as

\[ \frac{\delta \{\psi\}}{\delta \{a\}} = \frac{\partial \{\psi\}}{\partial \{a\}^\prime} \bigg|_{\{a\}^\prime = \{a\}^n} = [K_T] = \int [B]^T [D]B ds + \int \{G\}^T \{\bar{S}\}G ds \quad (3-73) \]
The first term of stiffness matrix is the displacement matrix. It considers small, large, and initial displacement. The second term of stiffness matrix is the geometric stiffness matrix and accounts for the effects of internal stresses on the instantaneous stiffness of the structure.

3.2.8. Transformation, Condensation, and Recovery of Nodal Variables

The strain-displacement matrices $[B]$ and $[G]$ used in the formation of the element tangent stiffness matrix $[K_T]$ from local to global coordinates is therefore not required before assembly at the structure level. However, an alternative transformation and condensation procedure is required in order to enforce the appropriate connectivity of nodal variables at junctions of element.

Transformation of Nodal Variables

The four nodal variables used in the derivation of the element tangent stiffness matrix and the nodal residual vector were $U_i, (dU/ds)_i, V_i$ and $(dU/ds)_i$. These four variables are satisfactory for structures consisting of a single uniform straight or smoothly curving member, such as beams and arches. However, excessive continuity at element junctions occurs if these four nodal variables are retained for use in problems where a discontinuity in element slope or cross-section occurs at a node. Figure 3-10 shows the discontinuity of frame members.
\[
\frac{dU}{ds} = \text{Axial Force} \\
\frac{dV}{ds} = \text{Rotation}
\]

\[
\frac{dU}{ds} = \text{Rotation} \\
\frac{dV}{ds} = \text{Axial Force}
\]

Figure 3-10. Compatibility at a Discontinuity of Element Slope

The only variables which should be connected between all members framing into a common node are the two translations \(U_i\) and \(V_i\) and the in-plane rotation \(\theta_i\).

First, the global displacement derivatives should be transformed as

\[
\{\varepsilon_i\} = \begin{bmatrix} \cos \phi_i & -\sin \phi_i \\ \sin \phi_i & \cos \phi_i \end{bmatrix} \begin{bmatrix} (dU / ds)_i \\ (dV / ds)_i \end{bmatrix}
\]

(3-74)

The incremental equilibrium equations are transformed as

\[
[\hat{K}_T] \{\Delta \hat{a}\} = -\{\hat{\psi}\}
\]

(3-75)

where, \([\hat{K}_T] = [\hat{f}]^T [K_T] \hat{f}\)

\[
\{\hat{\psi}\} = [\hat{f}]^T \{\psi\}
\]

\[
\{\hat{a}\} = \{\hat{a}_i\} \{\hat{a}_i\}^T
\]

\[
\{\hat{a}_i\} = \{U_i, \varepsilon_i, V_i, \theta_i\}^T
\]
The transformed variable \( \varepsilon_i \) should not be connected to other elements and so must be condensed out of the element stiffness matrix before assembly of the global stiffness matrix. Suppose the transformed degree of freedom vector \( \{ \hat{a} \} = \{ \{ a_r \}, \{ a_e \} \}^T \), in which \( \{ a_r \} \) are the retained degree of freedom \( U_i, V_i, \theta_i \) and \( \{ a_e \} \) are the eliminated degrees of freedom, \( \varepsilon_i \). The element stiffness matrix can be written in partitioned form as

\[
\begin{bmatrix}
[K_{rr}] & [K_{re}] \\
[K_{re}]^T & [K_{ee}]
\end{bmatrix}
\begin{bmatrix}
\{ \Delta a_r \} \\
\{ \Delta a_e \}
\end{bmatrix}
= 
\begin{bmatrix}
-\{ \psi_r \} \\
-\{ \psi_e \}
\end{bmatrix}
\] (3-76)

The lower partition can be solved for \( \{ \Delta a_e \} \) as

\[
\{ \Delta a_e \} = [K_{ee}]^{-1} \left( -\{ \psi_e \} - [K_{re}]^T \{ \Delta a_r \} \right)
\] (3-77)

Substituting Eq. (3-77) into the Eq. (3-76) yields

\[
[K_{re}] \{ \Delta a_r \} = -\{ \psi_e \}
\] (3-78)

where, \( [K_{re}] = [K_{rr}] - [K_{re}] [K_{ee}]^{-1} [K_{re}]^T \)
\[
\{\psi_e\} = \{\psi_r\} - [K_{re}]^{-1}\{\psi_e\}
\]

The eliminated degrees of freedom \(\{a_e\}\) are allowed to float in the analysis and so small discontinuities may occur at the nodes of a single uniform straight or smoothly curved member.

**Recovery of Nodal Variable**

Evaluation of the element stiffness matrix and total equilibrium equations depends on the nonlinear strain matrix \([B_L]\), which requires the displacements at any point in the element to be known. These internal displacements are expressed in terms of the nodal displacements using Eqs. (3-45) and (3-46). Recovery of the complete set of nodal displacements \(\{a\}\) for each element is therefore necessary for evaluation of \([B_L]\).

### 3.2.9. Cross-Sectional Analysis

**General**

The plastic-zone formulation is characterized by the subdivision of the member cross-sections into elemental areas, or into grids of ‘monitoring points’ or ‘fibers’. As a consequence of the path-dependent nature of plasticity, the current stress, current yield stress, and the current level of equivalent plastic strain must be stored at each monitoring point and updated incrementally throughout the analysis. In this way, the spread of yielding throughout the cross-section can be captured.
The grid of monitoring points is obtained by dividing a cross-section into straight or uniformly curved strips. The layout of monitoring points for a typical strip is illustrated in Figure 3-11.

**Numerical Integration Procedure**

It is required two kinds of numerical integration procedures for the plastic-zone method implementation of the beam element formulation. First, a numerical integration for the longitudinal axial is needed to build the stiffness matrix. A Gaussian integration technique is usually adopted for this procedure. Stress and strain resultants are traced at the integration points within the element. Second, each integration point has cross-sectional properties, such as cross-sectional area and second moment of inertia. If the formulation follows linear elastic behavior, these cross-sectional properties may be given as constants. However, a numerical integration procedure should be used to
obtain cross sectional properties for the plastic-zone analysis, because each monitoring point can have an inelastic value, which affects stress and strain resultants. Therefore, a numerical integration procedure for cross-section is introduced here.

The integral of a general two-dimensional function \( f(x, y) \) over an area can be approximated using Simpson’s rule as

\[
\int_A f(x, y)dA \approx \sum_{i=1}^{m} \sum_{j=1}^{n} f(x_{ij}, y_{ij})w_{ij}
\]

where,

- \( m = \) the number of through-thickness monitoring points \( (m \geq 3, m \text{ odd}) \)
- \( n = \) the number of longitudinal monitoring points \( (n \geq 3, n \text{ odd}) \)
- \( (x_{ij}, y_{ij}) = \) the coordinates of monitoring point \( (i, j) \)
- \( w_{ij} = \) the \( (i, j) \)th element of the 2D Simpson’s rule weight array \([W]\)

The 2D Simpson’s rule weight array \([W]\) is defined by

\[
[W] = [H_s][W]H_t]
\]

where,

\[
[H_s] = \begin{bmatrix}
\Delta s_1 \\
\vdots \\
\Delta s_m 
\end{bmatrix}
\]

\[
[H_t] = \begin{bmatrix}
\Delta t_1 \\
\vdots \\
\Delta t_n 
\end{bmatrix}
\]
\[ [\mathbf{W}] = \frac{1}{9} \begin{bmatrix}
1 & 4 & 2 & 4 & 2 & \cdots & 4 & 1 \\
4 & 16 & 8 & 16 & 8 & \cdots & 16 & 4 \\
2 & 8 & 4 & 8 & 4 & \cdots & 8 & 2 \\
4 & 16 & 8 & 16 & 8 & \cdots & 16 & 4 \\
2 & 8 & 4 & 8 & 4 & \cdots & 8 & 2 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
4 & 16 & 8 & 16 & 8 & \cdots & 16 & 4 \\
1 & 4 & 2 & 4 & 2 & \cdots & 4 & 1 
\end{bmatrix} \]

\( \Delta s_i \) and \( \Delta t_j \) are the spacings of the longitudinal monitoring points in row \( i \) and the through-thickness monitoring points in column \( j \), respectively.

**Numerical Integration of Stress Resultants**

The axial force and bending moment acting on a single strip can be expressed as

\[
N_{\text{strip}} = \sum_{i=1}^{m} \sum_{j=1}^{n} \sigma_{ij} w_{ij} \quad (3-81)
\]

\[
M_{\text{strip}} = \sum_{i=1}^{m} \sum_{j=1}^{n} \sigma_{ij} y_{ij} w_{ij} \quad (3-82)
\]

in which \( m, n \) and \( w_{ij} \) are described above, \( \sigma_{ij} \) is the longitudinal stress acting at monitoring point \((i, j)\) above the reference axis of the cross-section. The stress resultants acting on the entire cross-section are simply obtained by summing the contributions of all strips,

\[
N = \sum_{\text{strips}} N_{\text{strip}} \quad (3-83)
\]
\[ M = \sum_{\text{strips}} M_{\text{strip}} \quad (3-84) \]

Numerical Integration of the Tangent Modulus Matrix

The elastic-plastic cross-section properties in the tangent modulus matrix (Eq. 3-28) are also computed using Simpson’s rule. The individual terms are computed as

\[ \int_A E_T dA = \sum_{\text{strips}=1}^{m} \sum_{j=1}^{n} E_{Tij} w_{ij} \quad (3-85) \]

\[ \int_A E_T y dA = \sum_{\text{strips}=1}^{m} \sum_{j=1}^{n} E_{Tij} y_{ij} w_{ij} \quad (3-86) \]

\[ \int_A E_T y^2 dA = \sum_{\text{strips}=1}^{m} \sum_{j=1}^{n} E_{Tij} y_{ij}^2 w_{ij} \quad (3-87) \]

So far, the plastic-zone finite element method has been formulated. Since the second order inelastic analysis is essential for analyzing the stability of structures, this method can be a very good solution. Therefore, this formulation will be used to verify the performance of the ABAQUS (Hibbitt, et al. 2002). The finite element code, the NFA (Nonlinear Frame Analysis) was written by the author. Flow-chart, Matlab source code, and implementation details will be shown in Appendix A. This code is based on the theoretical procedures presented previously in this section.
3.3. Semi-Rigid Connections

3.3.1. General

For simplicity, it is very common practice in structural analysis to assume rigid or pinned connection between members. A rigid connection implies that the moment in the beam is completely transferred to the column. There is no relative rotation between the column and the beam. A pinned connection implies that no moment in the beam is transferred to the column. Only shear and axial forces are transferred. However, experiments have shown that a real steel connection is neither rigid nor pinned (Kameshki 2003), and one may use the term semi-rigid connections. Each type of semi-rigid connections shows different responses, as shown in Figure 3-5. The LRFD design code (AISC 1994) designated the strength of members and the types and strength of connections, as follows.

Type FR (fully restrained), commonly designated as “rigid-frame” (continuous frame), assumes that connections have sufficient rigidity to maintain the angles between intersecting members.

Type PR (partially restrained) assumes that connections have insufficient rigidity to maintain the angles between intersecting members.

There are several types of semi-rigid connections. Each connection type has different capacity of load transfer.

A semi-rigid connection transfers part of the moment in the beam to the column. Moment-curvature relationships in the semi-rigid connection are usually nonlinear, as shown in Figure 3-5. The nonlinear characteristics of beam-to-column connections play
a very important role in frame stability, because semi-rigid connections enable larger story drift than rigid connections. This excessive story drift influences the $P - \Delta$ effect, which can cause column buckling.

### 3.3.2. Modeling of Semi-Rigid Moment Connections

It is necessary to know the moment-curvature behavior of semi-rigid connections to analyze the frame as accurately as possible. At present, the most commonly used approach to describe the moment-curvature relationship is to curve-fit experimental data with simple expressions (Chen and Toma 1994).

Frye and Morris (1975) have reported a polynomial model to evaluate the behavior of several types of connections. In that model, the moment-curvature behavior is represented by an odd power polynomial.

Lui and Chen (1986) used an exponential function to curve-fit an experimental moment-curvature curve. Kishi and Chen (Chen and Toma 1994) refined the Lui-Chen exponential model to accommodate any sharp changes in slope in the moment-curvature curve.

Richard and Abbot (1975) proposed a power model using three parameters: initial connection stiffness, ultimate moment capacity, and shape parameters.

The generalized form of this model is

$$
M = \frac{R_u \theta_r}{\left\{ 1 + \left( \frac{\theta_r}{\theta_0} \right)^n \right\}^{1/n}}
$$

(3-88)
where, \( R_{ki} \) = initial connection stiffness

\[ \theta_0 = \text{a reference plastic rotation} \left( \frac{M_u}{R_{ki}} \right) \]

\( M_u \) = ultimate moment capacity

\( n \) = shape parameter

Figure 3-12 shows the moment-curvature curves for various \( n \) values. From this figure, it is recognized that the larger the power index \( n \), the steeper the curve.

![Figure 3-12. Three-parameter Power Model](image-url)

The power model is an effective tool for designers to execute a second-order nonlinear structural analysis, because the tangent connector stiffness \( R_k \) and relative rotation \( \theta_r \) can be determined directly from Eq. (3-88) without iteration, as follows:
\[ R_k = \frac{dM}{d\theta_r} = \frac{R_{ki}}{1 + \left( \frac{\theta_r}{\theta_0} \right)^n} \]  \hspace{1cm} (3-89)

and the rotation \( \theta_r \) is:

\[ \theta_r = \frac{M}{R_{ki} \left[ 1 - \left( \frac{M}{M_u} \right)^n \right]^{1/n}} \]  \hspace{1cm} (3-90)

Kishi and Chen (1990) proposed a method to determine these parameters using a simple mechanical procedure with an assumed failure mechanism. Kishi, et al. (1993) presented a practical design procedure which uses angles for the connection based on their previous work. In that paper, the initial connection stiffness \( R_{ki} \), the ultimate moment capacity \( M_u \), and the shape parameter \( n \) of the three-parameter power model can be determined easily.

The top and seat angle with double web angle connection has been chosen to simulate semi-rigid connection behavior in this dissertation. Because it can be considered as an ordinary beam connection. The design procedures are as follows:

**Assumptions and Notations**

- \( t \) = angle thickness
- \( k \) = gauge distance from heel to the top of fillet
- \( l \) = angle length
- \( g \) = distance between heel to the center of fastener closest to web or flange
of beam

\[ W = \text{nut width} \]

\[ I_0 = \frac{t^3}{12} = \text{geometrical moment of inertia} \]

\[ M_0 = \frac{\sigma_y t^2}{4} = \text{pure plastic bending moment} \]

where \( \sigma_y \) is the yielding stress of steel, and \( I_0 \) and \( M_0 \) are the values per unit length of plate element of angle. The top and seat angle are assumed to have the same dimensions.

The following parameters are also used:

\[ \beta = \frac{g}{l}, \gamma = \frac{l}{t}, \delta = \frac{d}{t}, \kappa = \frac{k}{t}, \omega = \frac{W}{t}, \rho = \frac{tw}{t} \]

where \( d \) is the height of beam and subscripts \( t \) and \( w \) denote top angle and web angle respectively.

**Top and Seat Angle without Double Web-Angle Connection**

Assuming that the center of rotation is located at the angle leg adjacent to the compression beam flange and the top angle acts as a cantilever beam to resist surcharged moment, the initial connection stiffness \( R_{k_{its}} \) is obtained as follows (Kishi and Chen 1990).

\[ \frac{R_{k_{its}}}{EI_{0r}} = (1 + \delta_r)^2 D_{is} \]  

(3-91)
where,
\[
D_{ts} = \frac{3}{\beta_i'(\gamma_i^2 \beta_i'^2 + 0.78)}
\]
(3-92)

\[
\beta_i' = \beta_i - \frac{1 + \omega_i}{2 \gamma_i}
\]
(3-93)

The ultimate capacity \( M_{uts} \) is obtained by assuming a simple failure mechanism.

The equation for \( M_{uts} \) is given by

\[
\frac{M_{uts}}{M_{\omega_0 t}} = \gamma_i \left[ 1 + \xi_i \left[ 1 + \beta_i^* + 2(\kappa_i + \delta_i) \right] \right]
\]
(3-94)

where the variable \( \xi_i \) is a non-dimensional ultimate shearing force acting at the plastic hinge. As in the case of single web-angle connection, it is obtained by solving following equation:

\[
\xi_i^4 + \beta_i^* \xi_i - 1 = 0
\]
(3-95)

where,

\[
\beta_i^* = \beta_i \gamma_i - \kappa_i
\]
(3-96)

Top and Seat Angle with Double Web-Angle Connection

The initial connection stiffness \( R_{ki} \) and the ultimate moment capacity \( M_u \) can be evaluated by separating the top and seat angle part and the double web angle part as follows:

\[
\frac{R_{ki}}{EI_{0t}} = \frac{R_{kts}}{EI_{0t}} + \frac{R_{kiw}}{EI_{0t}}
\]
(3-97)
\[
\frac{M_u}{M_{0t}t_e} = \frac{M_{uts}}{M_{0t}t_e} + \frac{M_{uw}}{M_{0t}t_e}
\]  

(3-98)

As for the web angle, it acts as a cantilever beam similar to the behavior of the top angle, the initial connection stiffness \( R_{kiw} \) is related to the double web angle connection part as follows (Kishi and Chen 1990),

\[
\frac{R_{kiw}}{EI_{0t}} = (1 + \delta_c)^2 \rho D_w
\]

(3-99)

where,

\[
D_w = \frac{3}{2\beta'_w (\gamma_w^2 \beta'_w^2 + 0.78)}
\]

(3-100)

\( \beta'_w \) is defined the same as \( \beta'_t \) in Eq. (3-93).

In the limit state, choosing a simple failure mechanism of web angle, and taking moment about the center of rotation at the angle leg adjacent to the compression beam flange, the ultimate moment capacity \( M_{uw} \) is

\[
\frac{M_{uw}}{M_{0t}t_e} = \gamma_w (1 + \xi_w) \left\{ \frac{\xi_w - 1/3(\xi_w + 1)}{\gamma_w + \delta_w + 1/\rho} \right\} \rho^3
\]

(3-101)

where \( \xi_w \) can be obtained by solving following equation:

\[
\xi_w^4 + (\beta_w \gamma_w - \kappa_w)\xi_w - 1 = 0
\]

(3-102)
Determination of Parameters in the Power Model

The three parameter power model is given in Eq. (3-88). The only parameter which is not determined is the shape parameter \( n \). The equation to determine the shape parameter \( n \) is given by (Kishi, et al. 1993).

1. if \( \log_{10} \theta_0 > -2.880 \), then \( n = 2.003 \log_{10} \theta_0 + 6.070 \) (3-103)
2. if \( \log_{10} \theta_0 \leq -2.880 \), then \( n = 0.302 \) (3-104)

Rotational Capacity of Joint Connection

A capacity of joint connection is measured by rotation. Figure 3-13 shows the rotational capacity of joint connection.

![Figure 3-13. Rotational Capacity of Joint Connection](image)

\( M_u \) is the ultimate moment capacity of the joint, \( R_{ki} \) is the initial connection stiffness, and \( \theta_u \) is the theoretical elastic rotation magnitude. \( \theta_u \) can be obtained as,
\[ \theta_u = \frac{M_u}{R_{ki}} \quad (3-105) \]

The rotational capacity of joint can be obtained by a multiple of \( \theta_u \), such that the total rotation becomes \( k\theta_u \). The actual magnitude of \( k \) may vary, depending on the design code requirement. In this study, a \( k \) value of six was chosen, on the premise that this would be satisfactory for all but the most severe seismic requirement (Bjorhovde, et al. 1990).
4. Analysis

4.1. Numerical Implementation

4.1.1. Code Selection

Two time integration techniques are available when nonlinear dynamic analysis is performed with ABAQUS. Those are the implicit and explicit methods (Bathe 1996). ABAQUS/Standard (implicit solver) is designed to analyze the overall dynamic response of a structure, in contrast to a wave propagation solution (ABAQUS/Explicit) associated with relatively local response in continua (Hibbitt, Karlsson & Sorensen, 2002).

The equilibrium of the system at time \( t + \Delta t \) can be obtained in nonlinear analysis by iterative methods, such as the Newton-Rhapson iteration that was introduced in section 3.2.7. Using the modified Newton-Rhapson iteration, the governing equilibrium equations are (neglecting the effect of damping matrix):

\[ M^{t+\Delta t} \ddot{u}^{(k)} + t^{t+\Delta t}K \Delta u^{(k)} = -^{t+\Delta t}R -^{t+\Delta t}F^{(k-1)} \]  

\[ t^{t+\Delta t}u^{(k)} = t^{t+\Delta t}u^{(k-1)} + \Delta u^{(k)} \]  

where, \( M \) = time-independent mass matrix

\( t^{t}K \) = linear strain incremental stiffness matrix at time \( t \)

\( t^{t+\Delta t}R \) = vector of externally applied nodal point loads at time \( t + \Delta t \)

\( t^{t+\Delta t}F^{(k-1)} \) = vector of nodal point forces equivalent to the element stresses at time \( t + \Delta t \) with iteration \( k - 1 \)

\( t^{t+\Delta t}u^{(k)} \) = vector of nodal point accelerations at time \( t + \Delta t \) with
iteration \( k \)

\[ \Delta u^{(k)} = \text{vector of nodal point displacement increment with iteration } k \]

Using the trapezoidal rule of time integration, the following assumptions are employed:

\[
^{t+\Delta t} u = u' + \frac{\Delta t}{2} (\dot{u}' + \dot{u}'') \tag{4-3}
\]

\[
^{t+\Delta t} \ddot{u} = \ddot{u}' + \frac{\Delta t}{2} (\dddot{u}' + \dddot{u}'') \tag{4-4}
\]

Using the relations in Eq. (4-2) to Eq. (4-4),

\[
^{t+\Delta t} \dddot{u}^{(k)} = \frac{4}{\Delta t^2} (^{t+\Delta t} u^{(k-1)} - u' + \Delta u^{(k)}) - \frac{4}{\Delta t} \dddot{u}' - \dddot{u}' \tag{4-5}
\]

substituting into Eq. (4-1),

\[
^{t} \hat{K}\Delta u^{(k)} = ^{t+\Delta t} R - ^{t+\Delta t} F^{(k-1)} - M \left( \frac{4}{\Delta t^2} (^{t+\Delta t} u^{(k-1)} - u') - \frac{4}{\Delta t} \dddot{u}' - \dddot{u}' \right) \tag{4-6}
\]

where, \( ^{t} \hat{K} =^{t} K + \frac{4}{\Delta t^2} M \)

The iterative equations in dynamic nonlinear analysis using implicit time integration have the same form as the equations used for static nonlinear analysis, except that both the coefficient matrix and the nodal point force vector contain contributions from the inertia of the system (Bathe 1996). If the system is nonlinear, an iterative method, such as the Newton-Rhapson method, should be used to obtain the displacement increment vector at each time increment. This procedure can be performed with ordinary structural dynamics problems. However, the situation is different in progressive collapse analysis. The stiffness matrix is approaching a singular
state if members of the system buckle, so that the convergence is hard to achieve. It means that the implicit method can not be guaranteed to obtain solutions when buckling problems are considered. Therefore, another method is required to perform the progressive collapse analysis.

There is another time integration method called the explicit method. The most common explicit time integration operator used in nonlinear dynamic analysis is probably the central difference operator (Bathe 1996). Neglecting the effect of a damping matrix, each discrete time step solution of the equilibrium equations is given, as follows:

\[ M^\prime \ddot{u} = ^\prime R - ^\prime F \]  \hspace{1cm} (4-7)

The equations of motion for the body are integrated using the explicit central difference integration rule (Hibbitt, Karlsson & Sorensen 2002):

\[ ^{r+\frac{1}{2}} \ddot{u} = \frac{^{r+\frac{1}{2}} \ddot{u} + ^{r+1} \Delta t \Delta t \dddot{u}}{2} \]  \hspace{1cm} (4-8)

\[ ^{r+1} u = ^{r} u + ^{r+1} \Delta t \left( ^{r+\frac{1}{2}} \ddot{u} \right) \]  \hspace{1cm} (4-9)

The notation is the same as for the equations of the implicit method, except for \( \Delta t \) and \( \left( t + \frac{1}{2} \right) \) and \( \left( t - \frac{1}{2} \right) \). \( \Delta t \) is the stable time increment and \( \left( t + \frac{1}{2} \right) \) and \( \left( t - \frac{1}{2} \right) \) refer to mid-increment values. The central difference integration operator is explicit in that the kinematic state can be advanced using known values of \( ^{r+\frac{1}{2}} \ddot{u} \) and \( ^{r+1} \dddot{u} \) from the previous increment. The explicit integration rule is quite simple but it does not provide
the computational efficiency associated with the explicit dynamics procedure. The key to the computational efficiency of the explicit procedure is the use of diagonal element mass matrices that enable the efficient inversion of the mass matrix that is used in the computation for the accelerations at the beginning of the increment:

\[ \ddot{u} = M^{-1}(R - \dot{F}) \]  \hspace{1cm} (4-10)

The explicit procedure requires no iterations and no tangent stiffness matrix (Hibbitt, Karlsson & Sorensen 2002). Once \( \ddot{u} \) is obtained in Eq. (4-10), \( u^{+1} \) is calculated by using Eqs. (4-8) and (4-9). \( u^{+1} \) is calculated based on the equilibrium condition at time \( t \).

Special treatments of the mean velocities \( u^{-\frac{1}{2}} \), \( u^{\frac{1}{2}} \) etc. are required for initial conditions, certain constraints, and presentation of results. For presentation of results, the state velocities are stored as a linear interpolation of the mean velocities:

\[ u^{+1} = \frac{1}{2} \ddot{u} + \frac{1}{2} \Delta t \dddot{u} \]  \hspace{1cm} (4-11)

The central difference operator is not self-starting because the value of the mean velocity \( u^{\frac{1}{2}} \) needs to be defined. The initial values at time \( t = 0 \) of velocity and acceleration are set to zero or given values. The following condition can be used:

\[ \frac{1}{2} \ddot{u} = \ddot{u} + \frac{1}{2} \Delta t \dddot{u} \]  \hspace{1cm} (4-12)
Substituting this into the update expression for \( u_{t+\frac{1}{2}} \) yields the following definition of \( \frac{1}{2} \dot{u} \):

\[
\frac{1}{2} \dot{u} = \dot{u}_0 - \frac{\Delta t}{2} \ddot{u}_0
\]  

(4-13)

The explicit method integrates through time by using many small time increments. The central difference operator is conditionally stable, and the stability limit for the operator (with no damping) is given in terms of the highest eigenvalue in the system as

\[
\Delta t \leq \frac{2}{\omega_{\text{max}}}
\]  

(4-14)

which is extremely small.

This method is fast, reliable, and well suited for problems with many degrees of freedom with extreme loads. Most of all, the explicit method does not require the inversion of the stiffness matrix which can cause matrix singularity problems associated with buckling. Therefore, ABAQUS/Explicit (explicit solver) has been selected for this study.

4.1.2. Mesh Sensitivity

Structural components, such as columns and girders, were modeled with several beam elements. Four elements were used for each column and beam in this study. However, eight elements were also used in this section to test the solution for mesh
sensitivity. The model was one of the 3D rigid frame models, which will be introduced in the section 4.4. It is shown in Figure 4-1.

The Failure Case 1 was adopted to simulate an initial column failure. The corner column in the center of the Figure 4-1 was assumed to be an initial failure. The vertical displacement at the point A, the axial forces at the column C and E, and the moment at the beam B and D are provided for comparisons, as shown in the Figure 4-2. Figure 4-2 shows the results of displacements, axial forces, and moments at the several locations. Both results from the case with four elements and eight elements formulation were almost identical. Therefore, four elements will be used for the 2D and 3D frame cases.
(a) The Vertical Displacements at the Point A

(b) The Axial Forces at the Column C

Figure 4-2. Results of 4 and 8 Elements for Each Column Formulation (Cont.)
(c) The Axial Forces at the Column E

(d) The Moments at the Beam B

Figure 4-2. Results of 4 and 8 Elements for Each Column Formulation (Cont.)
(e) The Moments at the Beam D

Figure 4-2. Results of 4 and 8 Elements for Each Column Formulation
4.2. Single Column Analysis

4.2.1. General

Figure 4-3 shows a perfectly straight column and its deformed shape at the buckling load. If \( P \) is small, the column will remain in a straight position and it undergoes only axial deformation. The column at this state is said to be in a stable equilibrium state since any lateral displacement produced by a slight disturbing lateral force will disappear when the lateral force is removed (Chen and Lui 1987). As \( P \) is increased, a condition is reached in which equilibrium in a straight position of the column ceases to be stable. Under this condition, a very small lateral force will produce a very large lateral deflection that does not disappear when the lateral force is removed. The axial load that makes the column transform to the unstable equilibrium is called the critical load, or Euler buckling load (\( P_c \)). The critical load is given as

\[
P_c = \frac{\pi^2 EI}{(Kl)^2}
\]

(4-15)

where, 
\( E \) = modulus of elasticity 
\( I \) = second moment of inertia
\( K \) = the effective length factor
\( l \) = length of the member

The effective length factor depends on the boundary conditions of the member, as shown in Figure 4-4.
Figure 4-3. The Critical Load (Euler Load)

Figure 4-4. The Effective Length Factors
It is possible to calculate the critical load of a member with Eq. 4-1. However, this equation is only valid if the member is slender enough. Fibers in the cross-section would yield before the load reaches the critical load if the member is relatively short, as explained in Chapter 3. Therefore, there are several strength curves available that compensate for the slenderness of columns (Salmon and Johnson 1996). The strength curves that were provided by AISC (AISC 1994) will be used for comparison purposes, because these curves were derived based on many theoretical and experimental works.

The nominal strength $P_n$ of rolled shape compression members is given by

$$P_n = A_g F_{cr}$$  \hspace{1cm} (4-16)

where, $A_g$ = area of the cross section

$F_{cr}$ = critical stress

The slenderness parameter $\lambda_c$ is defined by

$$\lambda_c^2 = \frac{F_y}{F_{cr (Euler)}} = \frac{F_y}{\pi^2 E \left( \frac{KL}{r} \right)^2}$$  \hspace{1cm} (4-17)

$$\lambda_c = \frac{KL}{r \pi} \sqrt{\frac{F_y}{E}}$$  \hspace{1cm} (4-18)

If $\lambda_c \leq 1.5$, the critical stress is given by

$$F_{cr} = (0.658 \lambda^2) F_y$$  \hspace{1cm} (4-19)

If $\lambda_c > 1.5$, the critical stress is given by
\[ F_{cr} = \left( \frac{0.877}{\lambda_c^2} \right) F_y \]  

(4-20)

LRFD strength curves for various yield stress are shown in Figure 4-5.

Figure 4-5. Critical Column Stress vs Slenderness Ratio according to LRFD for Various Yield Stress

These curves show that critical stress is much lower than yield stress at high slenderness ratio. It is required to use an inelastic method to compute an accurate critical stress of relatively short column.

4.2.2. Specifications of the Models

Three computer codes have been used in this study to analyze and compare the buckling behavior of a single column. They are ABAQUS/Standard (Hibbitt, et al. 2002), ABAQUS/Explicit (Hibbitt, et al. 2002), and NFA (Nonlinear Frame Analysis),
which was introduced in Chapter 3. ABAQUS/Standard is a commercial finite element software package that uses an implicit integration method while ABAQUS/Explicit uses an explicit integration method.

The cross section W12X106 was chosen for the calculations. The geometry of W12X106 and load configuration are shown in Figure 4-6.

![Figure 4-6. Geometry and Load Configuration of W12X106 Column](image)

The column is fixed at the bottom and free at the top. Therefore, the effective length factor $K$ is 2.0. The column length $l$ is adjusted to slenderness ratios 20, 60, 100, 160, and 200, so that the second-order effect and material failure can be clarified. A horizontal force $P/n$ is applied at the top of the column to provide the eccentric load. The variable $n$ in the denominator of the eccentric load are given as 1000, 500, 200, 100, and 50. The modulus of elasticity and yield stress of the column are assumed to be
\(2.9 \times 10^7\) psi, and \(3.6 \times 10^4\) psi, respectively. The plastic behavior of the model is elastic-perfectly plastic.

This column has three elements. ABAQUS/Standard and NFA used two-node cubic interpolation elements. ABAQUS/Explicit used two-node linear interpolation elements. In a 2D plane, ABAQUS has 5 monitoring points through the cross section of the element. NFA has 45 monitoring points through the cross section.

The results will be shown in the next section based on five slenderness ratios, such as 20, 60, 100, 160 and 200. It covers from very a short column to a very long (slender) column. The results of each slenderness ratio consist of results from ABAQUS/Explicit, ABAQUS/Standard (Implicit), and NFA with various horizontal forces. The results from each code will be compared and discussed.
4.2.3. Results

**Slenderness Ratio: 20 (Column Length: 54.7 in)**

A column that has a slenderness ratio 20 is very short. It has a much higher Euler buckling load than the LRFD critical load. It is expected that this column will crush (i.e. material failure) when the load approaches critical state. Figure 4-7 (a), (b), and (c) show similar behavior. With smallest eccentric load ($P/1000$), all cases show that the column failed at the LRFD critical load. As the eccentric load increases, each column failed at a lower load. The responses of the columns are virtually linear. All failures occurred suddenly when the cross section reached the yield stress. For these short columns, the elasto-plastic behavior dominates the responses. Therefore, the second-order effect of a short column is not significant, as expected.

Figure 4-7 (d) shows comparison of the results for different codes. They are virtually the same for the smallest eccentric load. However, ABAQUS/Explicit showed a different slope for the biggest eccentric load ($P/50$). ABAQUS/Standard and NFA showed very close results. It seems that differences in the interpolation functions are responsible for these results. However, these cases have almost the same critical load, regardless of their slope differences.

Figure 4-7 shows the yielding occurred at the almost same level between the codes. However, the formation of plastic hinges were different between the codes. ABAQUS/Standard showed a rather slow plastic hinge formation compared to ABAQUS/Explicit. However, it did not make big differences for overall behaviors of the columns.
The results from the NFA code showed earlier failure than the results from ABAQUS (Explicit and Standard). It is because the solution did not converge when the second layer of monitoring points in the cross section began to yield in the NFA. It seems that equilibrium was hard to be achieved while the external load was increasing, because the stress resultants in the cross section were limited due to the elasto-plastic behavior.

![Graph](image)

Figure 4-7. Single Column Behavior at Slenderness Ratio 20 (Cont.)
Figure 4-7. Single Column Behavior at Slenderness Ratio 20 (Cont.)
A column which has a slenderness ratio 60 is still rather short. The behavior of this column is basically linear like the previous case. The Euler buckling load is still much higher than the LRFD critical value. However, it shows a little nonlinear behavior. The moderate eccentric load \((P/200)\) caused the failure load to be equal to the same LRFD critical load. All failure loads are much lower than the Euler buckling load, as expected for short columns. Figure 4-8 (d) shows that the behaviors obtained by the three codes matched well.
Figure 4-8. Single Column Behavior at Slenderness Ratio 60 (Cont.)
Figure 4-8. Single Column Behavior at Slenderness Ratio 60
Slenderness Ratio : 100 (Column Length : 273.5 in)

Figure 4-9 shows the behavior for a slenderness ratio of 100. This column is no longer short. The Euler buckling load approaches the LRFD critical load. Since the LRFD critical load considered the initial crookedness of the column and eccentric load effects, the LRFD critical load still lower than the Euler buckling load.

The nonlinear behavior of the columns became clear in this region, because the second order effect is activated. The displacement increased rapidly as the load approaches the critical value. The moderate eccentric load ($P/200$) caused the failure load to be close to the LRFD critical load. Figure 4-9 (d) shows that results from these codes are matched well.

Figure 4-9. Single Column Behavior at Slenderness Ratio 100 (Cont.)
Figure 4-9. Single Column Behavior at Slenderness Ratio 100 (Cont.)
Slenderness Ratio : 160 (Column Length : 437.6 in)

Figure 4-10 shows the behavior for a slenderness ratio of 160. This column is rather slender. The Euler buckling load and the LRFD critical load are closer than in the previous cases. The nonlinear behavior of the column is now clearer. The smallest eccentric load ($P/1000$) produced the same failure load as the Euler buckling load. The moderate eccentric load ($P/200$) caused the failure load to be close to the LRFD critical value. The results of three codes matched well as shown in Figure 4-10 (d).
Figure 4-10. Single Column Behavior at Slenderness Ratio 160 (Cont.)
Figure 4-10. Single Column Behavior at Slenderness Ratio 160
Slenderness Ratio : 200 (Column Length : 547.0 in)

The results for the slenderness ratio 200 column is shown in Figure 4-11. This column is very slender. Therefore, the critical load with small eccentric load approached the Euler buckling load (i.e. elastic buckling load). The critical loads with the smallest eccentric load \((P/1000)\) of ABAQUS/Explicit and ABAQUS/Standard exceeded the Euler buckling load by 7.2 and 4.7 percent, respectively. The eccentric load \(P/500\) made the critical load the same as the Euler buckling load. The eccentric load \(P/100\) made the critical load the same as the LRFD critical load. However, the critical load obtained with NFA and with the smallest eccentric load \((P/1000)\) seemed to approach the Euler buckling load. Figure 4-11 (d) shows these differences clearly. Therefore, it seems that the difference of monitoring points in the cross section induced different results, because the number of monitoring points in ABAQUS(Explicit and Standard) is only 5 while NFA has 45 monitoring points. However, the results from ABAQUS(Explicit and Standard) are still reasonable. Besides, the horizontal displacement of ABAQUS/Explicit reached almost 25 inches when the load was at the Euler buckling load.
Figure 4-11. Single Column Behavior at Slenderness Ratio 200 (Cont.)
Figure 4-11. Single Column Behavior at Slenderness Ratio 200
4.2.4. Concluding Remarks

Results from three codes, column slendernesses, and eccentric loads combinations have been shown. The number of elements in a column was limited to three. Results of each code matched each other well and provided reasonable solutions compared with the Euler buckling load and the LRFD critical load. Since ABAQUS (Explicit and Standard) has shown that it has an ability to handle the second-order effect and material nonlinearity, these codes could be adopted to analyze more complicated frame structures (e.g., 2D portal frame and 3D building frame).
4.3. 2D Frame Analysis

4.3.1. General

Analyses of 2D frames have been performed for several combinations of numbers of spans and stories. Frames are from 2x2 to 5x5 spans. Frame columns were based on a simple LRFD design procedure manual (AISC 1994).

Each analysis includes second order effects and material nonlinearity with different connection stiffnesses to explore possible failure and progressive collapse. First order inelastic and second order elastic with stress failure criterion analyses were also implemented for comparison purpose. The initial failure (i.e. column removal) was assumed in the outside column of the first floor. The external loads were self weight and typical office load. The gravity acceleration was given as 386 in/sec$^2$, a typical office load was given as 50lb/ft$^2$. Analyses were performed up to 7 seconds after the initial failure.

4.3.2. Specifications of the Models

The 16 frames have been used for the 2D simulations. The dimensions of the frames are shown in Figure 4-12.
Figure 4-12. Dimensions of the 2D Frames (cont.)
Figure 4-12. Dimensions of the 2D Frames (cont.)
Each beam has a cross section of W12x26. Column sizes depend on the number of stories at each frame. The width of each span is 24 feet; the height of each story is 12 feet. The Young's modulus of structural steel is $7 \times 10^9$ psi and yield stress is $4 \times 10^6$ psi. The moment curvature relationship of the connection was calculated based on the procedure introduced in section 3.3.2. It is shown in Figure 4-13.
The failure rotation was 0.0192 radian, the initial stiffness was \(2.311 \times 10^8\) lb-in/rad. The external loads were only the office load (50 lb/ft\(^2\)) and self weight. The gravity acceleration was applied gradually to the mass density of the members to avoid dynamic effect due to the gravity load.

Figure 4-14 shows the column naming convention. Column lines are called column A, column B, to column F in 5x5 span from the right side of the frame. All initial column removals were Column A in the first floor.
Figure 4-14. Naming Convention of the Frame
4.3.3. Results

4.3.3.1. Rigid Frame

A rigid frame transfers all moments in the beams and the columns to the neighboring members.

2x2 Span Frame

Most of the 2x2 frame collapsed after column A was removed, and the first span fell down to the ground, as shown in Figure 4-15. Plastic hinges were formed in the beams above the initial failure. Column B buckled after the first span came down, and only column C survived after all.

Figure 4-15. 2x2 Frame – Column A Failure (cont.)
(e) Mises Stress at the Beam in the First Floor above the Initial Failure

(f) Mises Stress at the Beam in the Second Floor above the Initial Failure

Figure 4-15. 2x2 Frame – Column A Failure

Figure 4-15 (e) and (f) shows that Mises stress variations at the beams above the initial column failure. The stress level increased due to the initial failure, then reached the yield stress. Plastic deformation was observed while the span above the initial failure was falling. After the span fell to the ground, stress levels fluctuated while rest of the frame was collapsing.
2x3 Span Frame

Figure 4-16 shows the total collapse of the 2x3 span frame. The failure sequence is vertical span failure, horizontal column failure, and total collapse. It is a typical total failure sequence as will be shown in the following cases.

![Figure 4-16. 2x3 Frame – Column A Failure](image)

2x4 Span Frame

The 2x4 span frame collapsed similarly to the previous case, as shown in Figure 4-17. The frame collapsed faster than the 2x3 frame.
The 2x5 span frame collapsed similarly to the previous cases, as shown in Figure 4-18.

2x5 Span Frame

Figure 4-18. 2x5 Frame – Column A Failure (cont.)
The 3x2 span frame showed a partial collapse, as shown in Figure 4-19. Only the first span above the initial column failure fell to the ground.

The 3x3 span frame showed a total collapse, as shown in Figure 4-20. However, the columns in the second floor buckled instead of the columns in the first floor while the horizontal failure was progressing.
Figure 4-20. 3x3 Frame – Column A Failure

(a) Before Initial Failure  
(b) Vertical Failure (1.15 sec)  
(c) Horizontal Failure (3.4 sec)  
(d) Total Failure (5.95 sec)

3x4 Span Frame

The 3x4 span frame collapsed similarly to the 3x3 span case, as shown in Figure 4-21. The only difference is that the column B in the first floor buckled instead of the second floor while the horizontal failure was progressing.

Figure 4-21. 3x4 Frame – Column A Failure (cont.)

(a) Before Initial Failure  
(b) Vertical Failure (1.15 sec)
3x5 Span Frame

The 3x5 span frame collapsed similarly to the previous 3x4 span frame, as shown in Figure 4-22.
4x2 Span Frame

The 4x2 span frame showed a partial collapse, as shown in Figure 4-23. Only the first span above the initial column failure fell to the ground.

![4x2 Frame - Column A Failure](image)

(a) Before Initial Failure  (b) Final Failure (2.20 sec)

Figure 4-23. 4x2 Frame – Column A Failure

4x3 Span Frame

The 4x3 shows the same failure type as the previous 4x2 frame case, as shown in Figure 4-24. Only the first span above the initial column failure fell to the ground.

![4x3 Frame - Column A Failure](image)

(a) Before Initial Failure  (b) Final Failure (2.20 sec)

Figure 4-24. 4x3 Frame – Column A Failure
4x4 Span Frame

The 4x4 span frame shows a typical total collapse mode, as shown in Figure 4-25.

4x5 Span Frame

The 4x5 span frame collapsed similarly to the 4x4 span case, as shown in Figure 4-26. The only difference is that column B in the first floor buckled instead of the second floor while the horizontal failure was progressing.
5x2 Span Frame

The 5x2 span frame shows a typical 2 story frame collapse mode, as shown in Figure 4-27. Only the first span above the initial column failure fell to the ground.

Figure 4-27. 5x2 Frame – Column A Failure
5x3 Span Frame

The 5x3 span frame failed similarly to the previous 5x2 frame case, as shown in Figure 4-28.

![Figure 4-28. 5x3 Frame – Column A Failure](image)

(a) Before Initial Failure                  (b) Final Failure (2.20 sec)

5x4 Span Frame

The 5x4 span frame shows a typical total collapse mode, as shown in Figure 4-29. However, it took a longer time for the failure process, as compared to the other cases.

![Figure 4-29. 5x4 Frame – Column A Failure (cont.)](image)

(a) Before Initial Failure                  (b) Vertical Failure (1.15 sec)

Figure 4-29. 5x4 Frame – Column A Failure (cont.)
5x5 Span Frame

The 5x5 span frame shows an interesting result. The failure of this frame was localized to the first span above the initial column failure. It seems that the additional span provided the resistance to the horizontal displacement, which usually increased the $P-\Delta$ effect. However, the previous 5x4 frame showed a total collapse. Therefore, it may imply that the column design in the 5x5 frame is more robust than the 5x4 frame, so that the horizontal failure propagation will not be initiated.

Figure 4-30. 5x5 Frame – Column A Failure

Table 4-1 shows the collapse summary of the rigid frames. As the number of spans increases, the collapse mode tends to change from the total collapse to partial
collapse. As the number of stories increases, the collapse mode tends to change from the partial collapse to the total collapse except in the 5x5 span frame. Clearly, a frame structure has a higher survival probability if it has more spans and fewer stories.

Table 4-1. Collapse Modes of Rigid Frames

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<tr>
<th>Story</th>
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</table>

4.3.3.2. Semi-Rigid Frame

There are no rigid frames in the real world, and it is only assumed to simplify the analysis. This assumption does not induce wrong solutions when the structure is subject to the ordinary design loads. However, a joint connection will be subject to much higher loads in a progressive collapse situation. A semi-rigid connection will show the plastic, nonlinear behavior, and eventually it will fail. A rigid assumption is no longer valid for this situation.

Therefore, it is very important to include the behavior of semi-rigid connections in progressive collapse situation.
Ordinary Semi-Rigid Frame

The semi-rigid connection was designed according to the procedure in section 3.3. All design assumptions were made on the ordinary design loads. Therefore, it is expected that semi-rigid connection will show large deformation or even failure.

All the cases from the 2x2 span frame to the 5x5 span frame failed at connections after column A was removed. Figure 4-31 shows results of all semi-rigid frames.

Figure 4-31. Failures of the Semi-Rigid Frames (cont.)
(d) 2x5 Span – Column A Failure - Final Failure (2.20 sec)

(e) 3x2 Span – Column A Failure - Final Failure (1.45 sec)

(f) 3x3 Span – Column A Failure - Final Failure (1.75 sec)

(g) 3x4 Span – Column A Failure - Final Failure (1.90 sec)

Figure 4-31. Failures of the Semi-Rigid Frames (cont.)
(h) 3x5 Span – Column A Failure - Final Failure (2.20 sec)

(i) 4x2 Span – Column A Failure - Final Failure (1.45 sec)

(j) 4x3 Span – Column A Failure - Final Failure (1.75 sec)

(k) 4x4 Span – Column A Failure - Final Failure (1.90 sec)

Figure 4-31. Failures of the Semi-Rigid Frames (cont.)
(l) 4x5 Span – Column A Failure - Final Failure (2.20 sec)

(m) 5x2 Span – Column A Failure - Final Failure (1.45 sec)

(n) 5x3 Span – Column A Failure - Final Failure (1.75 sec)

(o) 5x4 Span – Column A Failure - Final Failure (1.90 sec)

Figure 4-31. Failures of the Semi-Rigid Frames (cont.)
All semi-rigid frame collapsed partially because of joint connection failure. However, only the first span above the initial column failure collapsed. The collapse was limited to the local area. This failure mode is different from the local failure mode of the rigid frame cases. The first span of the semi-rigid frames was demolished completely, while the first span of the rigid frames reached the ground with severe damage. Therefore, connection failures caused a whole bay destruction.

Since a semi-rigid connection limits moment transfer, and enables localized rotations, there was no horizontal failure propagation which usually induces a total frame collapse. Therefore, it is possible to prevent the whole frame collapse with ordinary joint design at the cost of a whole bay destruction above the failed span.

The collapse modes of rigid frames and semi-rigid frames are very different. There may exist some transition between them as connections are reinforced.
4.3.3.3. Reinforced Semi-Rigid Frames

To study the possible effect of connection reinforcement, various semi-rigid frames with reinforced joints were analyzed. The moment capacities of each joint were increased between 2 and 5 times, as shown in Figure 4-32. However, to simplify the analysis, the same rotational capacity was used.

![Graph showing joint moment-curvature relationships of various frames.](image)

Figure 4-32. Joint Moment-Curvature Relationships of the Frames

The results are tabulated in the Tables 4-2 and 4-3. Table 4-2 shows that the failure mode is getting close to the rigid case as the joint resistance is increased. The responses of the 4-times reinforced semi-rigid frame were almost the same as the responses of the rigid frame. It shows that if a rigid frame is required, joint connection should be reinforced 4-times the ordinary joint design. Table 4-3 shows the trends of collapse modes.
Table 4-2. Collapse Modes with Various Joint Rigidities

<table>
<thead>
<tr>
<th>Story span</th>
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<td>P,C</td>
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<td>(b) 2-Times Reinforced Semi-Rigid Frame</td>
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<td>(c) 3-Times Reinforced Semi-Rigid Frame</td>
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<td>(d) 4-Times Reinforced Semi-Rigid Frame</td>
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<td>(e) 5-Times Reinforced Semi-Rigid Frame</td>
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T: Total Failure, P: Partial Failure, C: Connection Failure
*1: Almost Total Collapse
*2: Total Collapse after Connection Failure
Table 4-3. Collapse Modes by Various Semi-Rigid Connections (cont.)

<table>
<thead>
<tr>
<th>Story</th>
<th>Span</th>
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(a) Partial Collapse by Connection Failure

(b) Total Collapse

: No Collapse

: Collapse
Table 4-3. Collapse Modes by Various Semi-Rigid Connections

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<tr>
<th>Story</th>
<th>Span</th>
<th>Ordinary Frame</th>
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(c) Partial Collapse with Damage

Table 4-3 (a) shows the trend of partial collapse by connection failure. This collapse mode is dominated by the rigidities of the connections. All of the ordinary and 2-times reinforced semi-rigid frame collapse in this mode. Three quarters of the 3-times and only 2 cases of the 4-times reinforced frame collapsed in this mode. If the connection is reinforced more than 4 times, this failure mode disappears.

However, the total collapse mode appears as the joint is more reinforced, as shown in Table 4-3 (b). This collapse mode dominates for strong connection with high story frames and strong connection with a few stories and spans frames. Since this
failure mode is the worst, it is not always a good idea to reinforce the connection as much as possible.

Table 4-3 (c) shows partial collapse with damage. It is the best collapse case. This failure mode dominated when frames had a few stories and many spans (i.e. low frames). It seems that this failure mode disappears for more than 4 story frames except the 5x5 span frame.

4.3.3.4. Member Buckling

Horizontal member failure is propagated by buckling. Figure 4-33 shows the axial force time history result in rigid frames of column line B. The axial forces had a sudden jump at 8 seconds, when the initial column failure occurred. The responses of the 5x2, 5x3, and 5x5 span frames stabilized over time, because they had only partial collapses, while the 5x4 frame collapsing totally. Therefore, the axial force of the 5x4 frame vanished.

Figure 4-34 shows the horizontal displacement of the top of the column line B. The displacement of the partial collapse cases (5x2, 5x3, and 5x5 span frame) stabilized between 5 to 10 inches. The displacement of the 5x4 span frame reached -20 inches, because it had a total collapse. It rebounded to -5 inches, because it was dragged by other collapsing parts on the ground.

The axial force and displacement relationship can be obtained by combining Figure 4-33 and 4-34. Figure 4-35 represents this relationship. When the axial forces in each frame reached a certain level, the horizontal displacement increased suddenly. It
shows the buckling behavior. It implies that large plastic deformations occurred in the neighboring columns due to the second order effect (i.e. buckling) even if they did not fail.

Figure 4-33. Axial Forces of Column B in Rigid Frames

(a) Axial Forces in 5x2 and 5x3 Span Frames

(b) Axial Forces in 5x4 and 5x5 Span Frames
Figure 4-34. Horizontal Displacement of the Top of the Column B

Figure 4-35. Force-Displacement Relationship of the Column B
Figure 4-36 shows the Mises stress of the top of the column B in the first floor. Yielding occurred in all of the cases. However, only the 5x4 span frame collapsed totally. The 5x2, 5x3, and 5x5 span frames did not totally collapsed and had large plastic deformations. It indicates that certain levels of damage occurred in structural members that survived the collapse. Therefore, the evaluation of member damage is required if the frames are considered for rehabilitation.

Figure 4-36. Mises Stresses in Column B of Rigid Frame (cont.)
4.3.3.5. First Order Inelastic and Second Order Elastic with Stress Failure Criterion

Analyses of Rigid Frames

So far, all analyses were performed based on the second order inelastic method (i.e. advanced analysis). The first order inelastic and the second order elastic analysis of rigid frames were done to compare with the results of the second order inelastic analysis. This chapter will reveal the limitation of the conventional analysis methods, such as the first order inelastic and second order elastic with stress failure criterion analyses.
The First Order Inelastic Analysis

The first order analysis is performed based on the initial geometry of the structure. The deformed geometry does not contribute to the response. Therefore, the buckling phenomenon can not be computed. The first order inelastic analysis considers only the material inelasticity, which forms plastic hinges in a member.

Table 4-4 shows the collapse mode of rigid frames.

Table 4-4. Collapse Mode of the First Order Inelastic Analysis

<table>
<thead>
<tr>
<th>Story</th>
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</tbody>
</table>

P : Partial Collapse

All frames collapsed partially due to plastic hinges in the beams of the first span next to the removed column. Total collapse did not occur because the horizontal buckling failure propagation was not initiated, as expected.

The Second Order Elastic with Stress Failure Criterion Analysis

The second order elastic with stress failure criterion analysis is basically an Euler buckling with stress failure problem. It can capture the buckling phenomenon. However, it fails when the stress reaches the yield stress. Table 4-5 shows the results.
Table 4-5. Collapse Mode of Rigid Frames

<table>
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<th>Story</th>
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</tbody>
</table>

T : Total Collapse

All frames collapsed totally. This table and the conclusion show that this approach does not provide plastic capacity to accommodate large deformations. Therefore, this analytical procedure cannot be used to analyze the progressive collapse due to its lack of ductility.

4.3.4. Concluding Remarks

Collapse modes of the 2D frame structures depend on various factors, such as the geometries, material properties, loadings, and especially rigidities of connections. Rigid frames have been assumed in many studies due to its simplicity for analyses. It provides very good solutions within the ordinary working conditions. However, there is no guarantee that the rigid joint assumption can provide realistic solutions for extreme cases, such as blast, earthquake, and progressive collapse. For example, many buildings were damaged due to the Northridge earthquake. It turned out that connections were
damaged in more than 200 buildings (Whittaker, et. al 1998, Joh and Chen 1999). However, this would not be realized if rigid joints were assumed in the analysis. Therefore, various joint rigidities have been used for this study.

Combinations of various numbers of stories and spans in rigid connection showed that collapse modes depended on the number of stories and spans. As the number of spans increased, the collapse mode tended to change from total collapse to partial collapse. As the number of stories increased, the collapse mode tended to change from partial collapse to total collapse. Therefore, rigid frame structures can survive better with more bays and fewer stories.

However, the analyses of the semi-rigid frames showed totally different results. All semi-rigid frames collapsed partially by joint failure. Only the first bay above the initial column failure collapsed to the ground. The collapse was limited to the local area. Since the joints failed before all moments and forces were transferred to the adjacent members, the horizontal failure propagation did not occur. Therefore, it is possible to prevent the whole frame collapse with ordinary semi-rigid joint design, and to sacrifice only the bay with the failed span.

These two extreme cases imply that there would be transitional collapse modes if the rigidities of the joint approached the rigid case. Table 4-3 shows that the collapse trends were divided into partial collapse by connection failure, total collapse, and partial collapse with damage. It shows that ordinary and 2-times reinforced semi-rigid frames collapsed due to joint connection failure. As the joints became stronger, this collapse mode disappeared. Total collapse mode appeared with more than 4-times
reinforced joint connection. The results of 4-times reinforced connection are similar to those of the rigid frames. Partial collapse with damage also appeared in this range. This collapse mode occurs when frames have a few stories and many spans. It seems that this collapse mode disappears for more than 4-story frame.

The first order inelastic and the second order elastic with stress criterion analyses were tested in this study. These methods are not suitable for progressive collapse analysis, because they used simplified assumptions, such as small deflection (the first order), or failure at yield stress which are not appropriate for extreme cases.

The best result is of course no damage after the initial failure. However, it is practically not possible, because the nature of the abnormal loading. Therefore, it seems that damage propagation control is the best strategy we can choose.

The second best damaged case is the partial collapse with damage. It happened in frames of a few stories or many spans with strong or rigid joints. However, strong or rigid joints can cause total collapse, if the frame has too many stories or too few bays. Ordinary semi-rigid connections can be used for that situation. Such connections disconnect members when subjected to abnormal loadings, stop transferring moments and forces, and prevent failure propagation. However, it can also cause total destruction of the failed span. Therefore, the use of mixed connection types should be considered carefully. Besides, specific responses can change if the geometry, material properties, and external loads are different.
4.4. 3D Frame Analysis

4.4.1. General

Analyses of the 2D frames have been done based for several combinations of numbers of spans, stories, and rigidities of connections. Column A in the first floor was the only initial failure. However, it was a severe damage mode for the 3D frame. A single column failure in 2D frame means that all columns in one row fail in a 3D frame. This could easily cause a total frame collapse. Therefore, the combinations of spans, stories, and rigidities of connections have been investigated instead of failure cases.

In the 3D frame analyses, only four different frames were used. These were 2x2x2, 3x3x3, 4x4x4, and 5x5x5 span frames. However, six initial failures with rigid and semi-rigid connections were used to analyze for progressive collapse. Frame columns were based on a simple LRFD design procedure manual (AISC 1994). The external loads were self weight and typical office load. The gravity acceleration was given as 386 in/sec\(^2\), a typical office load was given as 50 lb/ft\(^2\). Analyses were performed up to 7 seconds after the initial failure.

4.4.2. Specifications of the Models

Four frames have been used for 3D simulations. The dimensions of the frames are shown in Figure 4-37.
Each beam has a cross section of W12x26. Column sizes depend on the number of stories at each frame. Each floor has a concrete slab. Finite membrane strain and large rotation shell elements were used to model a concrete slab. The Young’s modulus of concrete is $3.5 \times 10^6$ psi, and yield stress is 10500 psi. The thickness of the slab is assumed to be 3.25 inches. The concrete slab and the beam were fully connected, so that the deflections were continuous between the slabs and the beams. Steel decks and
reinforcements were not considered for simplicity of the analyses. The width of each span is 24 feet; the height of each story is 12 feet. The Young’s modulus of structural steel is $2.9 \times 10^7$ psi and yield stress is $3.6 \times 10^4$ psi. The same moment curvature relationship of the connection as 3D cases was used for 3D cases, as shown in Figure 4-13.

### 4.4.3. Initial Column Failure Cases

Six different initial failure cases were used for 3D frame analyses. Failed columns at each case are shown in Figure 4-38.

![Diagram of Initial Column Failure Cases](image)

**Figure 4-38. Initial Column Failure Cases for 3D frame**

All initial column failure cases were assumed at the first floor. Case 1 was assumed to have a single corner column failure. Case 2 assumed a single column failure in an inside span. Case 3 and 4 assumed a single corner and an adjacent column failure. These two failure cases were not symmetric because of the direction of the
column cross sections, as shown in Figure 4-38. Case 5 assumed a corner column and two adjacent column failures. Case 6 assumed two inner column failures.

4.4.4. Results

4.4.4.1. 2x2x2 Span Frame

2x2x2 span frame is the smallest structure between the models. Six initial failure cases with rigid and semi-rigid connections were performed.

Rigid Frame

The only frame collapsed totally was cases 5. Figure 4-39 shows the results.
Figure 4-39. 2x2x2 Span Rigid Frame Results

Collapse did not occur in Cases 1 to 4 and 6. These cases involved a single or two column failures. However, collapse occurred in Cases 5, which had three outer column failures. Therefore, it seems that if an initial failure includes more than two outer columns, collapse will occur in 2x2x2 span rigid frames.
Semi-Rigid Frame

Analyses of 2x2x2 span frames with semi-rigid connection were performed with the same failure cases. Figure 4-40 shows the results.

Figure 4-40. 2x2x2 Span Semi-Rigid Frame Results (cont.)
The collapse conditions of these frames were not different from the 2x2x2 rigid frames. It seems that small number of initial column failures produced the same results for the 2x2x2 rigid and semi-rigid frames.

4.4.4.2. 3x3x3 Span Frame

Rigid Frame

Same initial failure cases were applied to these frames, as shown in Figure 4-41.
Figure 4-41. 3x3x3 Span Rigid Frame Results (cont.)
Figure 4-41. 3x3x3 Span Rigid Frame Results

Figure 4-41 showed a few interesting points. Frames with failure Cases 1 to 4, and 6 survived. However, the frame with failure Case 2 and 6 had inner column bucklings. Figures 4-41 (j) and (k) show the first floor frame with failure Case 6. Two adjacent columns to the initial column failure buckled. Only failure Case 5 showed a typical collapse sequence.
Semi-Rigid Frame

In 3x3x3 span frame, semi-rigid connection did not make any differences from rigid frames. Figure 4-42 shows the results.

Figure 4-42. 3x3x3 Span Semi-Rigid Frame Results (cont.)
The frame with failure Case 1 to 4 and 6 survived. However, the frame with failure Case 2 and 6 had adjacent column bucklings like in the rigid frame cases.

### 4.4.4.3. 4x4x4 Span Frame

**Rigid Frame**

The same six failure cases were applied to these frames, as shown in Figure 4-43.
Figure 4-43. 4x4x4 Span Rigid Frame Results (cont.)
Figure 4-43. 4x4x4 Span Rigid Frame Results

Figure 4-43 showed that failure Cases 1 to 4 did not cause collapse. The frame with failure Case 5 and 6 collapsed totally while the 2x2x2 and 3x3x3 frames with failure Case 6 survived. It implies that a bigger frame is weaker for two internal column failures (failure Case 6) than smaller frames. It took little more time for the frame with Case 6 to collapse than with Case 5.

Semi-Rigid Frame

Figures 4-44 shows the 4x4x4 semi-rigid frame results.
Figure 4-44. 4x4x4 Span Semi-Rigid Frame Results (cont.)
Figure 4-44 shows that semi-rigid frames did not make a difference from the corresponding rigid frames.

4.4.4.4. 5x5x5 Span Frame

Rigid Frame

Figure 4-45 shows the 5x5x5 span rigid frame results.
Figure 4-45 5x5x5 Span Rigid Frame Results (cont.)
Frames with failure Cases 1 to 4 did not collapse. However, frame with failure Case 5 and 6 collapsed totally. These results are similar to the results from the 4x4x4 span frame.

Semi-Rigid Frame

Figure 4-46 shows the 5x5x5 span semi-rigid frame results.
Figure 4-46. 5x5x5 Span Semi-Rigid Frame Results (cont.)
Figure 4-46. 5x5x5 Span Semi-Rigid Frame Results

Figure 4-46 shows similar results to those of the rigid frame results. The overall collapse modes of rigid and semi-rigid frames were virtually the same.

4.4.4.5. 5x5x5 Span Frame with a More Realistic Design

Two Different Size Columns

Only a single size column for each frame has been used in the previous cases due to the simplification of the frames. However, columns of different sizes are used for a realistic design of a frame. It may affect the responses of a frame.

Two different sizes of columns were used for the 5x5x5 span frames in this section. Figure 4-47 shows the specification.
W8x24 columns were used for the exterior columns of the frame, and W8x40 columns were used for the interior columns of the frame. Other structural components were the same as the previous section, as were the initial damage cases.

Figure 4-48 shows the results of the 5x5x5 span rigid frames, and Figure 4-49 shows the results of the 5x5x5 span semi-rigid frames.
Figure 4-48. 5x5x5 Span Rigid Frames with Two Column Sizes (cont.)
Figure 4-48. 5x5x5 Span Rigid Frames with Two Column Sizes (cont.)
Figure 4-48. 5x5x5 Span Rigid Frames with Two Column Sizes

(a) Case 1                                          (b) Case 2 (0.00 sec)

(c) Case 2 (2.72 sec)                                  (d) Case 2 (3.56 sec)

(e) Case 2 (5.00 sec)                                  (f) Case 3 (0.00 sec)

Figure 4-49. 5x5x5 Span Semi-Rigid Frames with Two Column Sizes (cont.)
Figure 4-49. 5x5x5 Span Semi-Rigid Frames with Two Column Sizes (cont.)
Figure 4-49. 5x5x5 Span Semi-Rigid Frames with Two Column Sizes
Both results were almost identical except the frame with Failure Case 2. The rigid frame with Failure Case 2 showed four column bucklings, while the semi-rigid frame with the same failure case collapsed totally. Only the frame with Failure Case 1, single corner column initial failure, survived.

Columns of Two Different Sizes and Lengths

Longer columns of two different sizes in the first floor were used in this section. The length of the column in the first floor was 22 feet, while other columns were all 12 feet, as shown in Figure 4-50.

![Figure 4-50. Dimension of the 5x5x5 Span Frame with Longer Columns](image)
The new column sizes were given in Figure 4-51.

![Figure 4-51. Two Column Sizes of the 5x5x5 Span Frame with Longer Columns in a Plan View](image)

Columns were designed with bigger sizes, due to the length. Figure 4-52 and 4-53 show the results.

![Figure 4-52. 5x5x5 Span Rigid Frames with Longer Columns with Two Sizes](image)
Figure 4-52. 5x5x5 Span Rigid Frames with Longer Columns with Two Sizes (cont.)
Figure 4-52. 5x5x5 Span Rigid Frames with Longer Columns with Two Sizes (cont.)
Figure 4-52. 5x5x5 Span Rigid Frames with Longer Columns with Two Sizes

Figure 4-53. 5x5x5 Span Semi-Rigid Frames with Longer Columns with Two Sizes (cont.)
Figure 4-53. 5x5x5 Span Semi-Rigid Frames with Longer Columns with Two Sizes (cont.)
Figure 4-53. 5x5x5 Span Semi-Rigid Frames with Longer Columns with Two Sizes (cont.)
The results of rigid and semi-rigid are virtually identical. Only Failure Case 1 survived. The 12 feet columns and the 22 feet columns in the first floor did not make a big difference, as compared to the previous frame, because the design procedure already considered the column length.

The results show three facts. First, a more realistic design, using two different column sizes, is more vulnerable to progressive collapse than a single size column design. Second, a semi-rigid connection could trigger total collapse of a structure, while a rigid frame just caused internal damage. Third, the lengths of columns do not affect considerably the collapse modes. However, it could be a factor when the frame is overdesigned to resist progressive collapse.

The GSA (2003) and the DoD (2003) required that a structural failure due to a single initial member failure should be confined to the damaged local area. Since the single column size cases with a single initial member failure did not cause total collapse, it could be a possible solution to meet the requirement of the GSA (2003) and the DoD (2003). However, this is an overdesign condition.
4.4.5. Concluding Remarks

The results of the 3D rigid and semi-rigid frames with single size columns have been shown. The overall results are tabulated in Table 4-6.

Table 4-6. The Results of the 3D Frame Cases

<table>
<thead>
<tr>
<th>Frame Failure Case</th>
<th>2x2x2 Span Frame</th>
<th>3x3x3 Span Frame</th>
<th>4x4x4 Span Frame</th>
<th>5x5x5 Span Frame</th>
<th>5x5x5 Span Frame</th>
<th>5x5x5 Span Frame</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td></td>
<td></td>
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<td>Case 2</td>
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<td>Case 3</td>
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<td>Case 4</td>
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<td>Case 5</td>
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<tr>
<td>Case 6</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

: No Collapse  
: Damaged (column Buckling)  
: Total Collapse

* : Single Size Columns with 12 feet  
** : Two Sizes Columns with 12 feet  
*** : Two Sizes Columns with 22 feet in the 1st Floor

For the single column size cases, it is obvious that frames of a single column size with Failure Cases 1 to 4 were much safer than with Failure Case 5 and 6. Failure Cases 1 to 4 included a corner column failure, an inner column failure, and two outer column failures. Therefore, it can be considered that two outer column failures or a single inner column failure are allowed.

Three outer column failures caused total collapse for all the frames. An inner column failure caused internal column damages except for the 2x2x2 span frame. Two inner column failures caused inner column bucklings or total collapse except for the
2x2x2 frame case. It implies that a bigger frame is weaker for internal column failures (failure Case 6) than smaller frames. Inner column bucklings occurred with the inner column initial failure cases (i.e. Failure Case 2 and 6). Since serious damages were caused by two inner column failures or three outer column failures, it seems that inner columns are more critical than outer columns.

However, the responses changed when a more realistic design was adopted. Most of these frames collapsed totally. It shows that the GSA (2003) and the DoD (2003) requirements are valid. The results show three facts. First, a more realistic design, using two different column sizes, is more vulnerable to progressive collapse than a single size column design. Second, a semi-rigid connection could trigger total collapse of a structure, while a rigid frame just caused internal damage. Third, the lengths of columns did not affect the collapse modes. However, it could be a factor when the frame is overdesigned to resist progressive collapse. Since the single column size cases with a single initial member failure did not cause total collapse, it could be a possible solution to meet the requirement of the GSA (2003) and the DoD (2003). However, this is an overdesign condition.

3D frame analyses with rigid and semi-rigid connection did not show big differences while 2D frame analyses showed such differences. Therefore, it seems that the connection type is not an important factor when a structure is subjected to relatively small initial damage. However, it can be a critical factor when a structure suffers heavy internal damages, as shown in Section 4.4.4.5, or serious initial damage, as shown for 2D frame analyses.
5. Conclusions

5.1. Conclusions

Progressive collapse is an important issue because limited local damage may cause destruction of a structural system. The main objective of this study was to enable the development of a rational numerical analysis of physical phenomena associated with progressive collapse by studying simple frame systems with simple material models, focusing on the role of material models, geometry and buckling.

Many factors may affect progressive collapse. These include material properties of members, external loadings, connection resistances, and structural geometries. Since progressive collapse analyses have to handle extreme responses of a structure, analysis methods must be able to handle material and geometric nonlinearities. These nonlinearities are very important, because all member failure phenomena involve material yielding and/or buckling. Therefore, simplified assumptions, such as elastic and first order (i.e. small) deformation, are no longer valid, as shown in Section 4.2.4.5. Linear analysis methods can not be used because they can not represent failure states. Even linear analysis methods with failure criteria can not be used, because they can not track plastic deformations which absorb much energy while a failure occurs. Advanced methods that can perform second order inelastic analyses should be adopted. ABAQUS/Explicit was selected for this study. It was compared with results from ABAQUS/Standard and the NFA code written based on advanced analysis method derived in Section 3.2 for verification purposes.
There have been many studies for progressive collapse. Most of studies used simplified analysis methods, such as linear elastic, bi-linear elasto-plastic, the effective length factor $K$, and even static. However, this study combined, for the first time, material, buckling, and connection failure models in a dynamic 3D analysis method for progressive collapse of steel frame structures. Therefore, these simulations would produce most reliable results.

2D and 3D frame analyses have been performed to explore the nature of progressive collapse. Influences from several factors, such as numbers of spans and numbers of stories, connection rigidities, and initial column failure cases were studied. Since a single column failure in a 2D frame could be severe in a view of a 3D frame, only a single outer column failure in the first floor was assumed. The 2D frame analyses provided the following results.

As the number of spans increased, the collapse mode tended to change from total collapse to partial collapse. As the number of stories increased, the collapse mode tended to change from partial collapse to total collapse. However, the analyses of the semi-rigid frames showed different trends. All semi-rigid frames collapsed partially by joint failure. The 2D frame analyses showed that a vertical failure was caused by connection failure, a horizontal failure was caused by column buckling.

When the semi-rigid connections were four times stronger than originally analyzed, the semi-rigid frames collapsed similarly to the rigid frames. For the rigid frames, if the low-rise frames (2 or 3 stories) had more number of spans than number of stories, they would not collapse. This might be expressed, as follows:
\[
\frac{n_{\text{span}}}{n_{\text{story}}} \leq 1 \text{ for total collapse of low-rise 2D frames} \tag{5-1}
\]

\[
\frac{n_{\text{span}}}{n_{\text{story}}} > 1 \text{ for partial collapse with damage of low-rise 2D frames} \tag{5-2}
\]

\(n_{\text{span}}\) is the number of spans and \(n_{\text{story}}\) is the number of stories. If the inequality (5-1) is valid, low-rise 2D frames may collapse totally. If the inequality (5-2) is valid, low-rise 2D frames may collapse partially with damage.

3D frame analyses showed a different tendency. Four different span frames with rigid and semi-rigid connection were used for six assumed initial failure cases. The assumed rigid and semi-rigid connection did not make a big difference in the 3D cases. Therefore, semi-rigid connections are not a critical factor if the initial damage is localized unless a structure suffers heavy damages, as shown in Section 4.4.4.5.

However, the single column initial failure caused total collapse for the most of the frame with a more realistic design. It shows that the GSA (2003) and the DoD (2003) requirements are valid. Since single column size frames with single initial column failure did not cause total collapse, it could be a possible solution to meet the requirement of the GSA (2003) and the DoD (2003). However, this is an overdesign condition.

The 3D frame with realistic designs showed three facts. First, a more realistic design, using two different column sizes, is more vulnerable to progressive collapse than a single size column design. Second, a semi-rigid connection could trigger total collapse of a structure, while a rigid frame just caused internal damage. Third, the
lengths of columns did not seriously affect collapse modes. However, it could be a factor when the frame is overdesigned to resist progressive collapse.

Progressive collapse analyses have shown that once failure propagation (i.e. horizontal column buckling) was initiated, it would not stop until it caused total collapse (or almost total collapse). If a structure has to be protected from progressive collapse, horizontal column buckling propagation is the most critical factor to be controlled. Additional column bracing can be a solution. Connection rigidities would be a critical factor only if serious initial damage is expected. It is also recommended that a structure be built with a large numbers of spans and low height. Therefore, it seems that a frame that has a large numbers of spans, low height, and relatively strong connections with less than three outer columns damaged is the safest structure.

5.2. Recommendations

The limitations of this study are clear. Only simplified frame structures were analyzed. There are other structural components, such as the elevator core and shear walls, that could influence the responses of the structure. Therefore, secondary structural components should be included when existing buildings are analyzed.

Only simplified geometries, such as symmetric low-rise frames, were adopted to analyze progressive collapse. Therefore, asymmetric high-rise building should be analyzed in the future study.

The initial damage states were also idealized. The specific columns were removed, as the initial damage in this study. However, the anticipated initial damages
caused by vehicle impact, or explosives would be significantly different. Therefore, actual damage states should be included for the analysis of real buildings.

Another issue is boundary conditions of the supports. Boundary conditions were assumed to be fixed in this study. This is a reasonable assumption for a preliminary study. The fixed boundary condition provides the biggest buckling capacity. If the boundary condition of a real building is not completely fixed, the buckling capacity of columns will be reduced. Therefore, boundary conditions (i.e. fixing rigidities) should be checked for real building analyses.

Field experiments can be the next step to verify the numerical analyses. Those may include a dynamic test for a collapse of a steel frame and/or a rupture of a steel connection. Abnormal loads, such as a vehicle impact or a blast, can be used as an initial damage loading.
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APPENDIX A. Flow Charts of the NFA

The formulation of the NFA (Nonlinear Frame Analysis) was shown in chapter 3.2. NFA is for a second order inelastic analysis code that was written in MATLAB programming language.

NFA consists of several subroutines. The main body of the program is the file called “nfa.m”. It controls the whole process and subroutines. The flow chart and the source code of “nfa.m” are shown in Figure A-1 and APPENDIX A-1, respectively.

“nBeam2D.m” is one of the subroutines. It calculates a local stiffness matrix, a nodal force vector, and updates cross sectional properties. The flow chart and the source code of “nBeam2D.m” are shown in Figure A-2 and APPENDIX A-2, respectively. Since NFA has 45 monitoring points and 3 Gaussian integration points in an element, it needs an organized data structure that stores cross sectional properties. It is shown in Figure A-3 (a). The monitoring points of an I-shape cross section are shown in Figure A-3 (b).

Other subroutines are given in APPENDIX A-3. “check_convergence.m” checks a convergence of the solution. “inv_K” inverts the global stiffness matrix and returns the global displacement vector. “read_input.m” reads the input file. “Partition_Matrix.m” and “Partition_Vector.m” rearrange a matrix and a vector to perform a condensation.
START
Input nodes, element definitions
Initialize variables
See "read_input.m"

N1 = 1 to nload (Load increment iteration)

Inc = 1 to incMax (Iteration limit)

I = 1 to n_elem (Iteration for the number of elements)

Calculate local stiffness matrix (Eq. 3-73), nodal force vector (Eq. 3-63), cosine and sine of element direction (Eq. 3-32, 33), and update cross-sectional properties (Sec. 3.2.9)

Transformation (Eq. 3-74, 75) and condensation (Eq. 3-76, 77, 78) of local stiffness matrix and nodal force vector

Assemble global stiffness matrix and global residual vector

Converged?
Yes
No

Calculate displacement increment (Eq. 3-66)

I = 1 to n_elem (Iteration for the number of elements)

Calculate local stiffness matrix (Eq. 3-73), nodal force vector (Eq. 3-63), cosine and sine of element direction (Eq. 3-32, 33), and update cross-sectional properties (Sec. 3.2.9)

Transformation (Eq. 3-74, 75) and condensation (Eq. 3-76, 77, 78) of local stiffness matrix and nodal force vector

Calculate the eliminated axial strains

Update global displacement vector (Eq. 3-67)

END
nBeam2D.m

Set up the Gaussian integration parameters:
\[ w(1) = -0.77460; \quad \alpha(1) = 0.55556 \]
\[ w(2) = 0; \quad \alpha(2) = 0.88889 \]
\[ w(3) = 0.77460; \quad \alpha(3) = 0.55556 \]

For \( I = 1 \) to 3; Iteration for Gaussian Integration

Calculate variables for stiffness and cross sectional properties:
\[ B01(w(i)), B02(w(i)), B0, G1, G2, G, \]
\[ \text{theta, } A, \text{ BL, } \overline{B} \]

Update current stress, current yield stress, and level of equivalent plastic strain at each monitoring point.

Calculate cross sectional properties:
\[ D, S \]

Calculate stiffness matrix:
\[ K = K + (B^T * D * B + G * S * G) * \alpha(i)^2 \]

Calculate nodal force vector:
\[ nForce = nForce + B^T * (B0 + \frac{1}{2} * BL) * u^\alpha(i) \]

Return \( nForce, K \)
(a) Data Structure of Cross Sectional Properties

(b) Strips and Monitoring Points

Figure A-3. Data Structure of Cross Sectional Properties

Each parameter is a matrix which has the same number of cells with monitoring points in a strip.
APPENDIX A-1. The Main Routine of the NFA

- “nfa.m”

%% Program to solve second-order inelastic buckling analysis
%% Written by Joonhong Lim (Sep-2003)

clear all;
close all;

% These global variables are all temporary. You can erase it freely.
global N M d_es inc_gU stresses

(elem, node, bc, cross_section) = read_input('input_beam.dat');

n_node = size(node, 1);
n_elem = size(elem, 1);

% result vectors
x_dis = [];
y_dis = [];
load_inc = [];

incMax = 20; % Newton-Rhapson iteration limit

% gU is global displacement vector which contains global displacements based on each element.
% This vector uses same configuration with the external load vector which explained above.
gU = zeros(n_elem*8, 1);
prev_gU = gU;

% External load increment vector
ld = [0:100000:130000 131000:1000:150000 150100:100:212000]';
nload = size(ld, 1);

for n1=1:nload
    no_load = zeros(1, 8);
    Load = [no_load no_load 0 0 0 0 0 ld(n1)/50 0 -ld(n1) 0]; % Load vector

% The external loads should be given on nodes of each element unit. For example,
% ‘Load’ vector will have [8 DOFs of element #1 8 DOFs of element #2 8 DOFs of element #3...
% and so on]. It indicates that the nodes belonging to more than 2 element should appear more than
% 2 times in the Load vector. If we have a beam which has 2 elements, each element has 2 node,
% each node has 4 DOFs (x, dx/ds, y, dy/ds). If an external load 100 is applied at the center of the
% beam, we will have a Load vector like following:
% Load = [0 0 0 0 0 100 0 0 0 0 0 0 0 0 0 0 0 0] or Load = [0 0 0 0 0 100 0 0 0 0 0 0 0 0 0 0 0 0]
% Each Load produces same results.
    gUtc = transpose(gU);

% gUtc is the transformed and condensed vector of gU. Refer to
% 'Advanced Analysis of Steel Frames, P. 280'

\( gU_{tc} = \text{zeros}(n_{\text{elem}} \times 6, 1); \)

\% This vector will be filled with eliminated axial strains.

\( ae = \text{zeros}(n_{\text{elem}} \times 2, 1); \)

for inc=1:incMax

\( gResidual = \text{zeros}(n_{\text{node}} \times 3, 1); \)

\( gK = \text{zeros}(n_{\text{node}} \times 3, n_{\text{node}} \times 3); \)

\% Stiffness Matrix will be smaller than expected. Because
\% each element stiffness matrices will be transformed
\% and condensed out.

\( \text{prev\_cross\_section} = \text{cross\_section}; \)

for i=1:n_elem

\( i1 = \text{elem}(i, 1) \times 4 - 3; \) \( i2 = \text{elem}(i, 1) \times 4 - 2; \) \( i3 = \text{elem}(i, 1) \times 4 - 1; \) \( i4 = \text{elem}(i, 1) \times 4; \)

\( i5 = \text{elem}(i, 2) \times 4 - 3; \) \( i6 = \text{elem}(i, 2) \times 4 - 2; \) \( i7 = \text{elem}(i, 2) \times 4 - 1; \) \( i8 = \text{elem}(i, 2) \times 4; \)

\( j1 = \text{elem}(i, 1) \times 3 - 2; \) \( j2 = \text{elem}(i, 1) \times 3 - 1; \) \( j3 = \text{elem}(i, 1) \times 3; \)

\( j4 = \text{elem}(i, 2) \times 3 - 2; \) \( j5 = \text{elem}(i, 2) \times 3 - 1; \) \( j6 = \text{elem}(i, 2) \times 3; \)

\( k1 = i^8 - 7; \) \( k2 = i^8 - 6; \) \( k3 = i^8 - 5; \) \( k4 = i^8 - 4; \)

\( k5 = i^8 - 3; \) \( k6 = i^8 - 2; \) \( k7 = i^8 - 1; \) \( k8 = i^8; \)

\( \text{current\_U} = [gU(k1) \ gU(k2) \ gU(k3) \ gU(k4) \ gU(k5) \ gU(k6) \ gU(k7) \ gU(k8)]; \)

\( \text{prev\_U} = [\text{prev\_gU}(k1) \ \text{prev\_gU}(k2) \ \text{prev\_gU}(k3) \ \text{prev\_gU}(k4) \ ... \ \text{prev\_gU}(k5) \ \text{prev\_gU}(k6) \ \text{prev\_gU}(k7) \ \text{prev\_gU}(k8)]; \)

\( \text{current\_node} = [\text{node}(\text{elem}(i, 1), :); \ \text{node}(\text{elem}(i, 2), :)]; \)

\( [K, \ n\text{Force}, \ cc, \ ss, \ \text{cross\_section}] = \text{nBeam2D(prev\_U, current\_U ...} \)
\( \text{, current\_node, cross\_section, i);}; \)

% From now on, this matrices and vectors should be transformed and condensed
% to maintain continuity. Refer to 'Advanced Analysis of Steel Frames, P. 280'

% Transformation

\( T1 = \begin{bmatrix} 1 & 0 & 0 & 0; & 0 & cc & 0 & ss; & 0 & 0 & 1 & 0; & 0 & -ss & 0 & cc \end{bmatrix}; \)
\( T2 = \begin{bmatrix} 1 & 0 & 0 & 0; & 0 & cc & 0 & ss; & 0 & 0 & 1 & 0; & 0 & -ss & 0 & cc \end{bmatrix}; \) \% T1 and T2 are same because
\% the element is assumed to be straight.

\( T = [T1 \ \text{zeros}(4:4); \ \text{zeros}(4:4) \ T2]; \)

\( K = T^*K*T; \)

\( n\text{Residual} = n\text{Force} - [\text{Load}(k1) \ \text{Load}(k2) \ \text{Load}(k3) \ \text{Load}(k4) \ \text{Load}(k5) \ \text{Load}(k6) \ ... \ \text{Load}(k7) \ \text{Load}(k8)]; \)

\( n\text{Residual} = T^*n\text{Residual}; \)

% Condensation
K = Partition_Matrix(K);
nResidual = Partition_Vector(nResidual);

Krr = K(1:6, 1:6); Kre = K(1:6, 7:8); Kee = K(7:8, 7:8); inv_Kee = inv(Kee);

Ktc = Krr - Kre*inv_Kee*Kre';  % Ktc is transformed and condensed K
nResidualtc = nResidual(1:6) - Kre*inv_Kee*nResidual(7:8);

% nResidualtc is transformed and condensed nResidual

% gResidual(j1) = gResidual(j1) + nResidualtc(1);
gResidual(j2) = gResidual(j2) + nResidualtc(2);
gResidual(j3) = gResidual(j3) + nResidualtc(3);
gResidual(j4) = gResidual(j4) + nResidualtc(4);
gResidual(j5) = gResidual(j5) + nResidualtc(5);
gResidual(j6) = gResidual(j6) + nResidualtc(6);

% gK(j1, j1) = gK(j1, j1) + Ktc(1, 1);
gK(j1, j2) = gK(j1, j2) + Ktc(1, 2);
gK(j1, j3) = gK(j1, j3) + Ktc(1, 3);
gK(j1, j4) = gK(j1, j4) + Ktc(1, 4);
gK(j1, j5) = gK(j1, j5) + Ktc(1, 5);
gK(j1, j6) = gK(j1, j6) + Ktc(1, 6);
gK(j2, j1) = gK(j2, j1) + Ktc(2, 1);
gK(j2, j2) = gK(j2, j2) + Ktc(2, 2);
gK(j2, j3) = gK(j2, j3) + Ktc(2, 3);
gK(j2, j4) = gK(j2, j4) + Ktc(2, 4);
gK(j2, j5) = gK(j2, j5) + Ktc(2, 5);
gK(j2, j6) = gK(j2, j6) + Ktc(2, 6);
gK(j3, j1) = gK(j3, j1) + Ktc(3, 1);
gK(j3, j2) = gK(j3, j2) + Ktc(3, 2);
gK(j3, j3) = gK(j3, j3) + Ktc(3, 3);
gK(j3, j4) = gK(j3, j4) + Ktc(3, 4);
gK(j3, j5) = gK(j3, j5) + Ktc(3, 5);
gK(j3, j6) = gK(j3, j6) + Ktc(3, 6);
gK(j4, j1) = gK(j4, j1) + Ktc(4, 1);
gK(j4, j2) = gK(j4, j2) + Ktc(4, 2);
gK(j4, j3) = gK(j4, j3) + Ktc(4, 3);
gK(j4, j4) = gK(j4, j4) + Ktc(4, 4);
gK(j4, j5) = gK(j4, j5) + Ktc(4, 5);
gK(j4, j6) = gK(j4, j6) + Ktc(4, 6);
gK(j5, j1) = gK(j5, j1) + Ktc(5, 1);
gK(j5, j2) = gK(j5, j2) + Ktc(5, 2);
gK(j5, j3) = gK(j5, j3) + Ktc(5, 3);
gK(j5, j4) = gK(j5, j4) + Ktc(5, 4);
gK(j5, j5) = gK(j5, j5) + Ktc(5, 5);
gK(j5, j6) = gK(j5, j6) + Ktc(5, 6);
gK(j6, j1) = gK(j6, j1) + Ktc(6, 1);
gK(j6, j2) = gK(j6, j2) + Ktc(6, 2);
gK(j6, j3) = gK(j6, j3) + Ktc(6, 3);
gK(j6, j4) = gK(j6, j4) + Ktc(6, 4);
gK(j6, j5) = gK(j6, j5) + Ktc(6, 5);
gK(j6, j6) = gK(j6, j6) + Ktc(6, 6);
end

con = check_convergence(gResidual, bc, .01);
if (con == 1)
    disp(['Iteration number : ' int2str(n1)])
    disp(['   Number of increment in this step : ' int2str(inc)]);
    disp(['   Load : ' num2str(Load(21)) ' ' num2str(Load(23))]);
    disp(['   X displacement : ' num2str(gU(21))]);
    stresses
    x_dis = [x_dis(:);gU(21)]; y_dis = [y_dis(:);-gU(23)]; load_inc=[load_inc(:);-Load(23)];
disp(' ');
    break;
end

gUtc_inc = zeros(n_node*3, 1);
gUtc_inc = -1 * inv_K(gK, gResidual, bc);

%% Calculate the axial strains of each element using the results obtained above.
%% This procedure is inefficient, because it has to calculate, transform, and condense
%% each element stiffness matrix and residual again.

gU_inc = zeros(n_node*4, 1);
for i=1:n_elem
    i1 = elem(i, 1)*4-3; i2 = elem(i, 1)*4-2; i3 = elem(i, 1)*4-1; i4 = elem(i, 1)*4;
    i5 = elem(i, 2)*4-3; i6 = elem(i, 2)*4-2; i7 = elem(i, 2)*4-1; i8 = elem(i, 2)*4;
    j1 = elem(i, 1)*3-2; j2 = elem(i, 1)*3-1; j3 = elem(i, 1)*3;
    j4 = elem(i, 2)*3-2; j5 = elem(i, 2)*3-1; j6 = elem(i, 2)*3;
    k1 = i8-7; k2 = i8-6; k3 = i8-5; k4 = i8-4;
    k5 = i8-3; k6 = i8-2; k7 = i8-1; k8 = i8;

    prev_U = [prev_gU(k1) prev_gU(k2) prev_gU(k3) prev_gU(k4) ...
        prev_gU(k5) prev_gU(k6) prev_gU(k7) prev_gU(k8)];
    current_U = [gU(k1) gU(k2) gU(k3) gU(k4) gU(k5) gU(k6) gU(k7) gU(k8)];
    current_node = [node(elem(i, 1),); node(elem(i, 2),)];

    [K, nForce, cc, ss, prev_cross_section] = nBeam2D(prev_U, current_U ...
        , current_node, prev_cross_section, i);

    % From now on, this matrices and vectors should be transformed and condensed
    % to maintain continuity. Refer to 'Advanced Analysis of Steel Frames, P. 280'

    % Transformation
    T1 = [1 0 0 0; 0 cc 0 ss; 0 0 1 0; 0 -ss 0 cc];
    T2 = [1 0 0 0; 0 cc 0 ss; 0 0 1 0; 0 -ss 0 cc]; % T1 and T2 are same because
    % the element is assumed to be straight.
    T = [T1 zeros(4,4); zeros(4,4) T2];
\[ K = T^*K^*T; \]
\[ n\text{Residual} = n\text{Force} - \begin{bmatrix} \text{Load(k1)} & \text{Load(k2)} & \text{Load(k3)} & \text{Load(k4)} & \text{Load(k5)} & \text{Load(k6)} & \ldots & \text{Load(k7)} & \text{Load(k8)} \end{bmatrix}; \]
\[ n\text{Residual} = T^*n\text{Residual}; \]

% Condensation

\[ K = \text{Partition\_Matrix}(K); \]
\[ n\text{Residual} = \text{Partition\_Vector}(n\text{Residual}); \]
\[ K_{rr} = K(1:6, 1:6); \quad K_{re} = K(1:6, 7:8); \quad K_{ee} = K(7:8, 7:8); \quad \text{inv\_Kee} = \text{inv}(K_{ee}); \]

\[ \begin{bmatrix} g\text{Utc\_inc}(j1); \ g\text{Utc\_inc}(j2); \ g\text{Utc\_inc}(j3); \ldots \ g\text{Utc\_inc}(j4); \ g\text{Utc\_inc}(j5); \ g\text{Utc\_inc}(j6) \end{bmatrix}; \]
\[ \text{ae\_inc} = \text{inv\_Kee}^*(-n\text{Residual}(7:8) - K_{re}^* \text{ar\_inc}); \]
\[ \text{tU} = [\text{ar\_inc}; \text{ae\_inc}]; \]
\[ n\text{U\_inc\_transformed} = [\text{tU}(1); \text{tU}(7); \text{tU}(2); \text{tU}(3); \text{tU}(4); \text{tU}(8); \text{tU}(5); \text{tU}(6)]; \]
\[ n\text{U\_inc} = \text{inv}(T)^*n\text{U\_inc\_transformed}; \]
\[ g\text{U\_inc}(k1) = n\text{U\_inc}(1); \quad g\text{U\_inc}(k2) = n\text{U\_inc}(2); \]
\[ g\text{U\_inc}(k3) = n\text{U\_inc}(3); \quad g\text{U\_inc}(k4) = n\text{U\_inc}(4); \]
\[ g\text{U\_inc}(k5) = n\text{U\_inc}(5); \quad g\text{U\_inc}(k6) = n\text{U\_inc}(6); \]
\[ g\text{U\_inc}(k7) = n\text{U\_inc}(7); \quad g\text{U\_inc}(k8) = n\text{U\_inc}(8); \]

end
\[ \text{prev\_gU} = g\text{U}; \]
\[ g\text{U} = g\text{U} + g\text{U\_inc}; \]
end
if(inc >= incMax)
    disp("Solution does not converge!!");
    break;
end
end
APPENDIX A-2. A Subroutine for the Stiffness Matrix of the NFA

- “nBeam2D.m”

% % Subroutine for the local stiffness matrix and nodal force of % non-linear 2D curved beam element. % Written by Joonhong Lim. Sep-2003 %

function [K, nForce, cc, ss, cross_section] = nBeam2D(prev_U, gU, node, cross_section, n_el);
   % gU is the parameters that are % part of real global displacement % vector. node has global coordinates of the nodes of the % element which will be calculated in this function. % therefore, they are only used in this function locally.

global N M d_es inc_gU sigma stresses

%% calculate 'l' which is the half length of the element

vx = node(2, 1) - node(1, 1);
vy = node(2, 2) - node(1, 2);
l = norm([vx vy]) / 2;

%% calculate sine and cosine value
%% cc = cos(phi), ss = sin(phi)
%% the element is assumed to be straight.

cc = vx / l / 2;
ss = -vy / l / 2;

%% set up the Gaussian integration parameters

w(1) = -0.77460; alpha(1) = 0.55556;
w(2) = 0; alpha(2) = 0.88889;
w(3) = 0.77460; alpha(3) = 0.55556;

%% initialize the local stiffness matrix and nForce

Kbar = zeros(8, 8);
Ksig = zeros(8, 8);
K = zeros(8, 8);
nForce = zeros(8, 1);

%% calculate parameters for cross sectional integration

element = cross_section.element(n_el);

for i = 1:3  % iteration for Gaussian integration
dpds = 0; \%\% dpds is (\phi)/d(s). It is zero because this element is straight.
dN01 = dN01(w(i)); dN11 = dN11(w(i)); dN02 = dN02(w(i)); dN12 = dN12(w(i));
ddN01 = ddN01(w(i)); ddN11 = ddN11(w(i)); ddN02 = ddN02(w(i)); ddN12 = ddN12(w(i));

B01 = [dN01*cc/l dN11*cc/l -dN01*ss/l -dN11*ss/l; ...
     -dN01/l*cc*dpds-ddN01/l^2*ss -dN11/l*cc*dpds-ddN11/l^2*ss ...
     dN01/l*ss*dpds -ddN01/l^2*cc  dN11/l*ss*dpds -ddN11/l^2*cc];

B02 = [dN02*cc/l dN12*cc/l -dN02*ss/l -dN12*ss/l; ...
     -dN02/l*cc*dpds-ddN02/l^2*ss -dN12/l*cc*dpds-ddN12/l^2*ss ...
     dN02/l*ss*dpds -ddN02/l^2*cc  dN12/l*ss*dpds -ddN12/l^2*cc];

B0 = [B01 B02];

G1 = [dN01*cc/l dN11*cc/l -dN01*ss/l -dN11*ss/l; ...
     dN01*ss/l dN11*ss/l dN01*cc/l dN11*cc/l];

G2 = [dN02*cc/l dN12*cc/l -dN02*ss/l -dN12*ss/l; ...
     dN02*ss/l dN12*ss/l dN02*cc/l dN12*cc/l];

G = [G1 G2];

theta = G*gU;

A = [theta'; 0 0];

BL = A*G;

B = B0 + BL;

\% D = [E*area 0; 0 E*1]; \% Linear elastic material
inc_gU = gU - prev_U;
inc_membrain_strain = ( B0 + 0.5*BL ) * inc_gU;
axial_strain = inc_membrain_strain(1); curvature = inc_membrain_strain(2);
if (abs(curvature) < 10^-15)
    curvature = 0;
end
if (abs(axial_strain) < 10^-15)
    axial_strain = 0;
end

\% Update current stress, current yield stress, and level of equivalent plastic strain
\% at each monitoring point.

for k2=1:element.n_strip \% iterate for the number of strips
    y = element.Gauss_Int_pt(i).strip(k2).y;
    sigma = element.Gauss_Int_pt(i).strip(k2).sigma;
    sigma_y = element.Gauss_Int_pt(i).strip(k2).sigma_y;
    ep_p = element.Gauss_Int_pt(i).strip(k2).ep_p;
    Et = element.Gauss_Int_pt(i).strip(k2).Et;
    orig_sigma_y = element.Gauss_Int_pt(i).strip(k2).orig_sigma_y;
    orig_E = element.Gauss_Int_pt(i).strip(k2).orig_E;
    ...
for k3=1:3
   for k4=1:5
      d_es = axial_strain + y(k3, k4) * curvature;

      current_sigma = sigma(k3,k4)+orig_E(k3,k4)*d_es;
      % strain of the current position in the cross section
      if ( abs(current_sigma) > sigma_y(k3,k4) ) % will it yield at this increment?
         if (current_sigma > 0)
            strn_inc = (current_sigma - sigma_y(k3,k4)) / orig_E(k3,k4);
            Et(k3,k4) = 1 / ( 1/orig_E(k3,k4) ... 
                           + 3/7/(orig_E(k3,k4))^50*(current_sigma/orig_sigma_y(k3,k4))^49 );
            % tangent stiffness
            sigma(k3,k4) = sigma_y(k3,k4) + Et(k3,k4)*strn_inc;
         end
         iter=0;
         while( abs(current_sigma - sigma(k3,k4)) > 100 )
            % iterate until current_sigma equals to sigma
            current_sigma = sigma(k3,k4);
            Et(k3,k4) = 1 / ( 1/orig_E(k3,k4) ... 
                           + 3/7/(orig_E(k3,k4))^50*(current_sigma/orig_sigma_y(k3,k4))^49 );
            % tangent stiffness
            sigma(k3,k4) = sigma_y(k3,k4) + Et(k3,k4)*strn_inc;
            iter = iter + 1;
            if iter > 5
               disp('Et does not converge!');
               break;
            end
         end
         H_prime = orig_E(k3,k4) * Et(k3,k4) / (orig_E(k3,k4) - Et(k3,k4));
         sigma_y(k3,k4) = sigma_y(k3,k4) + H_prime * strn_inc;
         ep_p(k3,k4) = ep_p(k3,k4) + strn_inc;
      else
         strn_inc = (current_sigma + sigma_y(k3,k4)) / orig_E(k3,k4);
         Et(k3,k4) = 1 / ( 1/orig_E(k3,k4) ... 
                           + 3/7/(orig_E(k3,k4))^50*(abs(current_sigma)/orig_sigma_y(k3,k4))^49 );
         % tangent stiffness
         sigma(k3,k4) = -sigma_y(k3,k4) + Et(k3,k4)*strn_inc;
         iter=0;
         while( abs(current_sigma - sigma(k3,k4)) > 100 )
            % iterate until current_sigma equals to sigma
            current_sigma = sigma(k3,k4);
            Et(k3,k4) = 1 / ( 1/orig_E(k3,k4) ... 
                           + 3/7/(orig_E(k3,k4))^50*(abs(current_sigma)/orig_sigma_y(k3,k4))^49 );
            % tangent stiffness
            sigma(k3,k4) = -sigma_y(k3,k4) + Et(k3,k4)*strn_inc;
            iter = iter + 1;
            if iter > 5
               disp('Et does not converge!');
               break;
         end
\[
H_\text{prime} = \text{orig}_E(k3,k4) \div (\text{orig}_E(k3,k4) - \text{Et}(k3,k4));
\]
\[
\sigma_{y}(k3,k4) = \sigma_{y}(k3,k4) - H_\text{prime} \times \text{strn}_\text{inc};
\]
\[
\varepsilon_{p}(k3,k4) = \varepsilon_{p}(k3,k4) + \text{strn}_\text{inc};
\]
\[
\text{else } \% \text{ not yield}
\]
\[
\sigma(k3,k4) = \text{current}_\sigma;
\]
\[
\text{end}
\]
\[
\text{end}
\]
\[
\text{end}
\]
\[
\text{element.Gauss}_\text{Int_pt}(i).\text{strip}(k2).\sigma = \sigma; \% \text{current stress}
\]
\[
\text{element.Gauss}_\text{Int_pt}(i).\text{strip}(k2).\sigma_{y} = \sigma_{y}; \% \text{current yield stress}
\]
\[
\text{element.Gauss}_\text{Int_pt}(i).\text{strip}(k2).\varepsilon_{p} = \varepsilon_{p}; \% \text{current level of equivalent plastic strain}
\]
\[
\text{element.Gauss}_\text{Int_pt}(i).\text{strip}(k2).\text{Et} = \text{Et}; \% \text{current tangent stiffness}
\]
\[
\text{if } n_{\_}\text{el}==1
\]
\[
\text{if } i==1
\]
\[
\text{if } k2==3
\]
\[
\text{stresses} = \sigma;
\]
\[
\text{end}
\]
\[
\text{end}
\]
\[
\text{end}
\]
\[
\text{end}
\]
\[
\text{cross}_\text{section.element}(n_{\_}\text{el}) = \text{element};
\]
\[
\% \text{ Calculate } D \text{ and stress resultants}
\]
\[
D1 = 0; \ D2 = 0; \ D3 = 0;
\]
\[
N = 0; \ M = 0; \ % N \text{ is axial force, } M \text{ is moment.}
\]
\[
\text{for } k2=1: \text{element.n}_\text{strip} \% \text{iterate for the number of strips}
\]
\[
\text{for } k3=1:3
\]
\[
D1 = D1 + \text{element.Gauss}_\text{Int_pt}(i).\text{strip}(k2).\text{Et}(k3,k4) \times \text{element.Gauss}_\text{Int_pt}(i).\text{strip}(k2).w(k3,k4);
\]
\[
N = N + \text{element.Gauss}_\text{Int_pt}(i).\text{strip}(k2).\sigma(k3,k4) \times \text{element.Gauss}_\text{Int_pt}(i).\text{strip}(k2).w(k3,k4);
\]
\[
D2 = D2 + \text{element.Gauss}_\text{Int_pt}(i).\text{strip}(k2).\text{Et}(k3,k4) \times \text{element.Gauss}_\text{Int_pt}(i).\text{strip}(k2).y(k3,k4) \times \text{element.Gauss}_\text{Int_pt}(i).\text{strip}(k2).w(k3,k4);
\]
\[
M = M + \text{element.Gauss}_\text{Int_pt}(i).\text{strip}(k2).\sigma(k3,k4) \times \text{element.Gauss}_\text{Int_pt}(i).\text{strip}(k2).y(k3,k4) \times \text{element.Gauss}_\text{Int_pt}(i).\text{strip}(k2).w(k3,k4);
\]
\[
D3 = D3 + \text{element.Gauss}_\text{Int_pt}(i).\text{strip}(k2).\text{Et}(k3,k4) \times \text{element.Gauss}_\text{Int_pt}(i).\text{strip}(k2).y(k3,k4)^2 \times \text{element.Gauss}_\text{Int_pt}(i).\text{strip}(k2).w(k3,k4);
\]
\[
\text{end}
\]
\[
\text{end}
\]
\[
D = [D1 \ D2; \ D2 \ D3];
\]
\[
\% \text{stress_resultant} = D \times (B0 + 0.5 \times BL) \times gU
stress_resultant = [N; M];

S = [stress_resultant(1) 0; 0 stress_resultant(1)];

Kbar = Kbar + alpha(i)*B'*D*B';

Ksig = Ksig + alpha(i)*G'*S*G';

nForce = nForce + alpha(i)*B'*stress_resultant';

end;

K = Kbar + Ksig;

%% Subroutine for the shape function N01
function [r01] = N01(x)
    r01 = 1/4 * (x^3 - 3*x + 2);

%% Subroutine for the shape function N02
function [r02] = N02(x)
    r02 = 1/4 * (-x^3 + 3*x + 2);

%% Subroutine for the shape function N11
function [r11] = N11(x, length)
    r11 = length/4 * (x^3 - x^2 - x + 1);

%% Subroutine for the shape function N12
function [r12] = N12(x, length)
    r12 = length/4 * (x^3 + x^2 - x - 1);

%% Subroutine for the shape function dN01
function [dr01] = dN01(x)
    dr01 = 1/4 * (3*x^2 - 3);

%% Subroutine for the shape function dN02
function [dr02] = dN02(x)
    dr02 = 1/4 * (-3*x^2 + 3);

%% Subroutine for the shape function dN11
function [dr11] = dN11(x, length)
    dr11 = length/4 * (3*x^2 - 2*x - 1);

%% Subroutine for the shape function dN12
function [dr12] = dN12(x, length)
\( dr_{12} = \frac{\text{length}}{4} \times (3x^2 + 2x - 1); \)

\%\% Subroutine for the shape function \( \text{ddN}01 \)
function \( [\text{ddr01}] = \text{ddN01}(x) \)
\[
\text{ddr01} = \frac{1}{4} \times (6x);
\]

\%\% Subroutine for the shape function \( \text{ddN}02 \)
function \( [\text{ddr02}] = \text{ddN02}(x) \)
\[
\text{ddr02} = \frac{1}{4} \times (-6x);
\]

\%\% Subroutine for the shape function \( \text{ddN}11 \)
function \( [\text{ddr11}] = \text{ddN11}(x, \text{length}) \)
\[
\text{ddr11} = \frac{\text{length}}{4} \times (6x - 2);
\]

\%\% Subroutine for the shape function \( \text{ddN}12 \)
function \( [\text{ddr12}] = \text{ddN12}(x, \text{length}) \)
\[
\text{ddr12} = \frac{\text{length}}{4} \times (6x + 2);
\]
APPENDIX A-3. Other Subroutines of the NFA

- “check_convergence.m”

% This function checks the convergence of the solution.
% If convergence is achieved, it will return 1. Otherwise,
% it will return 0.
[

function convergence = check_convergence(R, bc, criteria)

% These global variables are all temporary. You can erase it freely.
global N M d_es inc_gU stresses

n_bc = size(bc, 1);
n_R = size(R, 1);
order = zeros(n_R, 1); % This vector contains the order should be
 % used for rearranging the stiffness matrix
 % and the residual vector

for i=1:n_R
    order(i) = i;
end

% Push order numbers which corresponds to the zero displacement
% boundary condition to the upper part of the order vector
for i=1:n_bc
    for j=i:n_R
        if (bc(i) == order(j))
            temp1 = order(i);
            order(i) = order(j);
            order(j) = temp1;
            break;
        end
    end
end

% Arrange the order vector so that the rest of the vector in
% a descending order.
for i=n_bc+1:n_R-1
    for j=i:n_R
        if (order(i) > order(j))
            temp1 = order(i);
            order(i) = order(j);
            order(j) = temp1;
            break;
        end
    end
end
% Arrange the residual vector along the order vector

    temp1 = [];
    for i=1:n_R
        temp1 = [temp1; R(order(i))];
    end
    R = temp1;

    if (norm(R(n_bc+1:n_R)) < criteria)
        convergence = 1;
    else
        %     norm(R(n_bc+1:n_R))
        %     stresses
        convergence = 0;
    end

- "inv_K.m"

% This function calculates the global displacement vector (which is
% transformed and condensed already). It considers the boundary condition
% vector to exclude rows and columns that are zeros in the global stiffness
% matrix and the global residual

function gU = inv_K(K, R, bc)

n_bc = size(bc, 1);
    n_R = size(R, 1);
    order = zeros(n_R, 1); % This vector contains the order should be
    % used for rearranging the stiffness matrix
    % and the residual vector
    back_order = zeros(n_R, 1); % This vector contains the back order
    % will be used for the recover of the global
    % displacement vector

    for i=1:n_R
        order(i) = i;
    end
    back_order = order;

    % Push order numbers which corresponds to the zero displacement
    % boundary condition to the upper part of the order vector
    for i=1:n_bc
        for j=i:n_R
            if (bc(i) == order(j))
                temp1 = order(j);
                order(j) = order(i);
                order(i) = temp1;
                break;
            end
        end
    end
% Arrange the order vector so that the rest of the vector in
% a descending order.
for i=n_bc+1:n_R-1
    for j=i:n_R
        if (order(i) > order(j))
            temp1 = order(i);
            order(i) = order(j);
            order(j) = temp1;
        end
    end
end

% Arrange the residual vector along the order vector

% Arrange the stiffness matrix along the order vector

% Calculate the global displacement vector

gU = zeros(n_R, 1);
gU(n_bc+1:n_R) = inv(K(n_bc+1:n_R, n_bc+1:n_R)) * R(n_bc+1:n_R);

% Calculate the back order. This calculation assumes that
% the original node number is an ascending order.
for i=1:n_R-1
    for j=i+1:n_R
        if (order(j) < order(i))
            temp1 = back_order(i);
            back_order(i) = back_order(j);
            back_order(j) = temp1;
            temp1 = order(i);
            order(i) = order(j);
        end
    end
end
order(j) = temp1;
end
end

% Rearrange the global displacement vector to the original order

temp1 = [];
for i=1:n_R
    temp1 = [temp1; gU(back_order(i))];
end
gU = temp1;

- “read_input.m”

%%% This function reads the input file

function [elem, node, bc, cross_section] = read_input(input_file_name)

fid = fopen(input_file_name);
node = []; elem = []; bc = [];

cross_section = [];

while ~feof(fid)
    tline = fgetl(fid);
    switch deblank(tline)
    case 'node'
        cont = 1; % temporary value
        while (cont==1)
            tline = fgetl(fid);
            tline = deblank(tline);
            try
                [t1, t2, t3] = strread(tline, '%d%f%f', 'delimiter',',');
                node = [node; t2 t3];
            catch
                cont = check_comment(tline, fid);
            end
        end
    case 'element'
        cont = 1; % temporary value
        while (cont==1)
            tline = fgetl(fid);
            tline = deblank(tline);
            try
                [t1, t2, t3] = strread(tline, '%d%d%d', 'delimiter',',');
                elem = [elem; t2 t3];
            catch
                cont = check_comment(tline, fid);
            end
        end
    end
end
catch
    cont = check_comment(tline, fid);
end
end

case 'property'
    cont = 1; % temporary value
    while (cont==1)
        tline = fgetl(fid);
        tline = deblank(tline);
        if strncmp(sscanf(tline, '%c', 1), 'I')
            \[t1, t2, t3, t4, t5, t6, t7, t8, t9, t10\] = ...
                strread(tline, '%c%d%f%f%f%f%f%f%f%f', 'delimiter','
            cross_section = ...
                fill_cross_section(cross_section, t1, t2, t3, t4, t5, t6, t7, t8, t9, t10);
        elseif strncmp(sscanf(tline, '%c', 1), 'R')
            %%%
            %%% code for the rectangular section
            %%%
        else
            cont = check_comment(tline, fid);
        end
    end
end

case 'boundary'
    cont = 1; % temporary value
    while (cont==1)
        tline = fgetl(fid);
        tline = deblank(tline);
        try
            \[t1, t2\] = strread(tline, '%d%s', 'delimiter','
            if strcmp(t2, 'f')
                t3 = \[t1*3-2 t1*3-1 t1*3\]';
            elseif strcmp(t2, 'h')
                t3 = \[t1*3 -2 t1*3-1\]';
            else
                disp('Boundary condition input error!');
                break;
            end
            bc = [bc; t3];
            catch
                cont = check_comment(tline, fid);
            end
        end
    case 'end'
        break
    otherwise
        if (sscanf(tline, '%c', 1) ~= '%')
            disp('Input Error!!');
            break;
        end
    end
end
fclose(fid);

% This function checks the line is a just comment. Otherwise, % it moves the file position back to a line.
%
function cont = check_comment(tline, fid)
if ~strcmp(sscanf(tline, '%$c', 1), '%$')
    cont = 0;
    fseek(fid, -(size(tline, 2)+2), 'cof'); % rewind the file % position for the 'tline'
else
    cont = 1;
end

% This function fills the cross_section data structure which contains material and % cross sectional properties for various cross sections.
%
function cross_section = fill_cross_section(cross_section, t1, t2, t3, t4, t5, t6, t7, t8, t9, t10)
if strcmp(t1, 'I')
    cross_section.element(t2).n_strip=3;
    for i=1:3
        % for strip 1
        m=3; n=5; delta_s=t3/4; delta_t=t4/2;
        Hs = zeros(m, m);
        for j=1:m
            Hs(j, j) = delta_s;
        end
        Ht = zeros(n, n);
        for j=1:n
            Ht(j, j) = delta_t;
        end
        W_bar = 1/9*[1 4 2 4 1; 4 16 8 16 4; 1 4 2 4 1];
        cross_section.element(t2).Gauss_Int_pt(i).strip(1).w = Hs*W_bar*Ht;

        sigma=zeros(m, n); sigma_y=zeros(m, n); ep_p=zeros(m, n);
        Et=zeros(m, n); y=zeros(m, n);
        for k1=1:m
            for k2=1:n
                sigma(k1, k2) = 0.; sigma_y(k1, k2) = t10; ep_p(k1, k2) = 0;
                Et(k1, k2) = t9; y(k1, k2) = t5/2 + t4 - (t4/2 * (k1-1));
            end
        end
        cross_section.element(t2).Gauss_Int_pt(i).strip(1).sigma = sigma;
        cross_section.element(t2).Gauss_Int_pt(i).strip(1).sigma_y = sigma_y;
    end
end
% for strip 2
m=3; n=5; delta_s=t5/4; delta_t=t6/2;
Hs = zeros(m, m);
for j=1:m
    Hs(j, j) = delta_s;
end
Ht = zeros(n, n);
for j=1:n
    Ht(j, j) = delta_t;
end
W_bar = 1/9*[1 4 2 4 1; 4 16 8 16 4; 1 4 2 4 1];
cross_section.element(t2).Gauss_Int_pt(i).strip(2).w = Hs*W_bar*Ht;

sigma=zeros(m, n); sigma_y=zeros(m, n); ep_p=zeros(m, n);
Et=zeros(m, n); y=zeros(m, n);
for k1=1:m
    for k2=1:n
        sigma(k1, k2) = 0.; sigma_y(k1, k2) = t10; ep_p(k1, k2) = 0;
        Et(k1, k2) = t9;  y(k1, k2) = t5/4 * (3 - k2);
    end
end
cross_section.element(t2).Gauss_Int_pt(i).strip(2).sigma = sigma;
cross_section.element(t2).Gauss_Int_pt(i).strip(2).sigma_y = sigma_y;
cross_section.element(t2).Gauss_Int_pt(i).strip(2).ep_p = ep_p;
cross_section.element(t2).Gauss_Int_pt(i).strip(2).Et = Et;
cross_section.element(t2).Gauss_Int_pt(i).strip(2).y = y;
cross_section.element(t2).Gauss_Int_pt(i).strip(2).orig_sigma_y = sigma_y;
cross_section.element(t2).Gauss_Int_pt(i).strip(2).orig_E = Et;

% for strip 3
m=3; n=5; delta_s=t7/4; delta_t=t8/2;
Hs = zeros(m, m);
for j=1:m
    Hs(j, j) = delta_s;
end
Ht = zeros(n, n);
for j=1:n
    Ht(j, j) = delta_t;
end
W_bar = 1/9*[1 4 2 4 1; 4 16 8 16 4; 1 4 2 4 1];
cross_section.element(t2).Gauss_Int_pt(i).strip(2).w = Hs*W_bar*Ht;

sigma=zeros(m, n); sigma_y=zeros(m, n); ep_p=zeros(m, n);
Et=zeros(m, n); y=zeros(m, n);
for k1=1:m
    for k2=1:n

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sigma(k1, k2) = 0.; sigma_y(k1, k2) = t10; ep_p(k1, k2) = 0;
Et(k1, k2) = t9; y(k1, k2) = - t5/2 - t8 + (t8/2 * (k1-1));
end
end
cross_section.element(t2).Gauss_Int_pt(i).strip(3).sigma = sigma;
cross_section.element(t2).Gauss_Int_pt(i).strip(3).sigma_y = sigma_y;
cross_section.element(t2).Gauss_Int_pt(i).strip(3).ep_p = ep_p;
cross_section.element(t2).Gauss_Int_pt(i).strip(3).Et = Et;
cross_section.element(t2).Gauss_Int_pt(i).strip(3).y = y;
cross_section.element(t2).Gauss_Int_pt(i).strip(3).orig_sigma_y = sigma_y;
cross_section.element(t2).Gauss_Int_pt(i).strip(3).orig_E = Et;
end
end

- “input_beam.dat” – An example of the input file

% Input file deck
% '%' is reserved for comments
% Empty line is not allowed
%node
1, 0., 0.
2, 0., 182.33
3, 0., 364.67
4, 0., 547.
%element
1, 1, 2
2, 2, 3
3, 3, 4
%property
% R(Rectangular cross section), element number, width, height, Young's Modulus, Yield Stress
% R, 1, 1., 1., 29000000., 36000.
% R, 2, 1., 1., 29000000., 36000.
% R, 3, 1., 1., 29000000., 36000.
% I(I cross section), element number, upper flange width, upper flange thickness, web height
% , web thickness, lower flange width, lower flange thickness, Young's Modulus, Yield Stress
% I, 1, 12.22, 0.99, 10.91, .61, 12.22, 0.99, 29000000., 36000.
% I, 2, 12.22, 0.99, 10.91, ,61, 12.22, 0.99, 29000000., 36000.
% I, 3, 12.22, 0.99, 10.91, ,61, 12.22, 0.99, 29000000., 36000.
%boundary
% boundary condition supports only 'f' (fixed) or 'h' (hinge)
1, f
end

- “Partition_Matrix.m”

%%
% This function rearrange the matrix.
% It will exchange the 2nd row and column to the 7th,
function R = Partition_Matrix(K)

traffic = [K(1,:); K(3,:); K(4,:); K(5,:); K(7,:); K(8,:); K(2,:); K(6,:)];
R = [traffic(:,1) traffic(:,3) traffic(:,4) traffic(:,5) traffic(:,7) traffic(:,8) traffic(:,2) traffic(:,6)];

function R = Partition_Vector(V)

R = [V(1,:); V(3,:); V(4,:); V(5,:); V(7,:); V(8,:); V(2,:); V(6,:)];
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