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ESSAYS ON ECONOMIC THEORY

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Abstract

This thesis consists of three chapters. The first chapter is a joint work with Kenjiro Asami. We study a long-term principal-agent problem involving experimentation with hidden actions. The principal delegates a costly experiment on the quality of the project to the agent. Since the agent is financially constrained in exerting effort, they require monetary transfers before exerting effort each time. The agent can privately divert these funds for personal use. When news arrives, the agent has the option to conceal the news' arrival to continue receiving monetary transfers without exerting effort. The principal designs a time-dependent bonus to incentivize the agent to report the news. We show that a contract encouraging immediate reporting is more profitable than one that allows for delayed reporting.

In the second chapter, I consider a Bayesian persuasion problem with hidden action. When the principal can decide on both information structure and monetary transfer, which relies on the stochastic output, then full separation or full pooling contract is optimal.

In the third chapter, I study a sender-receiver game in which the sender has two kinds of private information. One is the payoff-relevant state, and the other is evidence. The sender can communicate by revealing evidence. The sender can manipulate evidence downwards not upwards. I show that all equilibria have a cutoff structure and characterize these cutoffs. These cutoffs need not be aligned, that is, two types with the same state might send different messages even though both messages are available to both types. My main result is that the cutoffs are aligned if the equilibrium is monotone, that is, higher messages lead to higher actions. Depending on the bias, there might be three types of equilibria: pooling, full disclosure, and those in which low-state senders conceal evidence.

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Acknowledgments

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Chapter 1
Reporting Incentives in Delegated Experimentation

1.1 Introduction

We investigate a delegated experimentation problem with financial constraints as follows: There is a risky R&D project whose quality is unknown. Cutting-edge research is typically developed by a startup (referred to as the 'agent'). Since the research is costly and the agent is financially constrained, the agent requires funds to initiate the R&D ex ante. The agent is funded by an investor (referred to as the 'principal'). As the R&D process is complex, the investor cannot monitor the actions taken by the startup directly. Consequently, the startup may privately divert their investment for personal use. While the startup exerts effort, the agent privately observes news about the project with some probability. This news is critical, hard information indicating whether the R&D project's quality is good or bad. Once the news is reported by the agent, the principal decides whether to implement the project.

There are two main issues in this scenario:

The first is moral hazard: The agent must be incentivized to exert effort through a long-term contract. Since the actions are chosen privately by the agent, they are motivated to work by receiving a time-dependent bonus for reporting the news.

The second is hiding news: Once the agent privately observes the news, they no longer need to exert effort. Hence, the agent may have an incentive to hide the news to continue receiving benefits without further work. Since the news is observed privately by the agent, they are encouraged to report it by a time-dependent bonus for doing so.

Both issues are addressed by implementing a time-dependent bonus contract. Suppose there is a contract that induces a delayed report. We can then construct a new contract, under which the expected payment remains the same as in the original contract but induces immediate reporting. Consequently, a contract that causes delayed reporting is not optimal. We derive the optimal contract to induce full effort until the deadline as a differential equation.

The rest of the paper is organized as follows: The next subsection discusses related literature. In section 2, we explore the good news model. In section 3, we delve into the bad news model, where the implementation of the project is feasible at any time. In section 4, we conclude the paper.

1.1.1 Related Literature

Our paper contributes to the extensive literature on exponential bandit experimentation and dynamic contract theory. We build upon the seminal work in the exponential bandit field by Keller, Rady and Cripps (2005). Our study extends the exponential bandit model to include a long-term contract, thereby incorporating delegated experimentation. The literature on delegated experimentation was initiated by Bergemann and Hege (2005), who explored relational funding to financially constrained agents in a discrete time framework. In our model, unlike previous ones, we assume that the principal possesses full commitment power to the contract, and we introduce the possibility of the agent concealing news. Hörner and Samuelson (2013) analyze relational funding to financially constrained agents in a continuous time framework. They characterize both Markov and non-Markov equilibrium payoff sets.

The work most closely related to our study is by Halac, Kartik and Liu (2016), who consider hidden information about the agent's abilities and the agent's hidden actions. They characterize the optimal contract under a discrete-time setting. Our model distinguishes itself by being set in continuous time and involving a financially constrained agent who has an incentive to conceal news.

Green and Taylor (2016) delve into delegated experimentation with a moral hazard, where success requires two instances of good news, but the state is known to be good. Guo (2016) examines delegated experimentation and adverse selection within a continuous time framework, demonstrating that the cut-off policy is both optimal and time-consistent.

Lastly, Halac and Kremer (2020) address the issues of concealing good or bad news in the absence of agency problems. They argue that, under certain conditions, an agent will continue experimentation indefinitely if a significant period has elapsed before the arrival of news, due to potential reputational gains.

1.2 Good News Model

Time $t \in [0, \infty)$ is continuous. There are a principal (she) and an agent (he). The principal delegates the agent to conduct an experiment. There is one project whose state is unknown. The state ω of the project can either be good (G) or bad (B), thus, $\omega \in \{G, B\}$. Neither the principal nor the agent knows the state. The common prior of the good state is $\pi_0 \in (0, 1)$.

At each time t, the agent can conduct research $e(t) \in \{0, 1\}$, incurring a cost of $c \cdot e(t)$, where $c > 0$. When the state is good ($\omega = G$), good news arrives by time t with probability $G(t) = 1 - \exp\left(-\lambda \int_0^t e(s)\, ds\right)$, where $\lambda \in (0, 1)$, and its density is given by $g(t) = \lambda e(t) \exp\left(-\lambda \int_0^t e(s) ds\right)$. Hence, if the state is good, the probability of the good news arriving within an interval of length dt is $\lambda e(t) dt$. When the state is bad ($\omega = B$), the good news never arrives.

The agent privately observes any good news. After observing the good news, he can either report or hide it from the principal. The agent is credit-constrained; thus, he needs c at hand to choose $e(t) = 1$. The principal provides an ex-ante base wage of $A(t)$ to the agent, which can be used to finance costly research or be consumed immediately. The agent cannot save the payment. The principal cannot observe the agent's actions but pays an ex-post payment of $B(t)$ if good news is reported at time t.

The good news is verifiable hard evidence, meaning the agent cannot falsely report the arrival of good news without actual realization. Once the good news is observed, the agent no longer needs to exert effort since he can submit the evidence. We interpret good news as discovering specific steps the principal can take to implement the project. Therefore, the principal benefits by $V^* > 0$ if the good news is reported. The good news itself holds no intrinsic value to the agent. Both the principal and the agent are risk-neutral and have quasi-linear utilities. The common discount rate is $r > 0$. We assume $\lambda > r$, indicating that the players are patient enough and/or the rate of news arrival is sufficiently high.

Let $T \geq 0$ denote the timing of news arrival. When the news never arrives, set $T = \infty$. The agent chooses the effort level $e(t)$ at each time point t, as well as the reporting time $\hat{T} \geq T$. We assume that the agent reports the news if indifferent.

The principal offers the contract with full commitment power before the agent starts experimentation. The contract is denoted by $\boldsymbol{C} = \langle A(t), B(t) \rangle$, where $A(t) \in \{0, c\}$ is the base wage enabling the agent to exert $e(t) = 1$ at time t, and $B(t) \in \mathbb{R}_+$ is the bonus paid conditional on the good news being reported at time t. The contract must specify a termination date $t^* \in [0, \infty)$, after which both $A(t)$ and $B(t)$ are zero. Furthermore, we focus on contracts where $e(t) = 1$ for $t \leq \min\{t^*, T\}$, with T being the time of news arrival. This assumption implies that the principal prefers to avoid allowing the agent to have inactive periods during the contract's duration.

The timing of the game is as follows.

1. The principal offers the contract $\boldsymbol{C} = \langle A(t), B(t) \rangle$ to the agent.
2. The nature determines the state.
3. The game ends if he rejects the offer. The agent starts experimentation if he accepts the offer.
4. The agent chooses $e(t)$ and the principal pays $A(t), B(t)$ until the game ends.
5. If the good news is reported, or the time reaches the termination date t^*, the game ends.

1.2.1 Analysis

Firstly, as an initial benchmark, we analyze the socially optimal scenario where the principal can conduct the experimentation independently. Secondly, we consider two additional benchmarks: the first concerns the optimal contract when the agent's effort is observable, and the second relates to scenarios where the timing of news arrival is observable. Thirdly, we demonstrate that in situations where neither the effort nor the timing of news arrival is observable, the optimal contract incentivizes immediate reporting. We then explicitly derive the form of this optimal contract.

As long as no news is received, the belief is updated according to the following differential equation:

$$\dot{\pi}(t) = -\pi(t)(1 - \pi(t))\lambda e(t),$$

where for any function $X : \mathbb{R}_+ \to \mathbb{R}$, $\dot{X}$ denotes the time derivative of X. Thus, conditional on the absence of news, the belief decreases over time t.

1.2.2 Benchmarks

1.2.2.1 First Best

Suppose that the principal can do experimentation by herself. Let $V(\pi)$ denote the principal's value as a function of the initial belief π.

$$
\begin{aligned}
V(\pi) &= \max_{e(t),t\geq 0} \mathbb{E}_0[-\int_0^T e^{-rt}c\cdot e(t)dt + e^{-rT}V^*] \\
&= \max_{e(t),t\geq 0} \pi \int_0^\infty \lambda e(T)\exp\left(-\lambda\int_0^T e(s)ds\right)\left(-\int_0^T e^{-rt}ce(t)dt + e^{-rT}V^*\right)dT \\
&- \pi\int_0^\infty e^{-rt}ce(t)dt\cdot\exp\left(-\lambda\int_0^\infty e(s)ds\right) - (1-\pi)\int_0^\infty e^{-rt}ce(t)dt \\
&= \max_{e(t),t\geq 0} \pi \int_0^\infty \lambda e(T)\exp\left(-\lambda\int_0^T e(s)ds - rT\right)V^*dT \\
&- \pi\int_0^\infty\int_t^\infty \lambda e(T)\exp\left(-\lambda\int_0^T e(s)ds\right)dT\exp(-rt)ce(t)dt \\
&- \pi\int_0^\infty \exp(-rt)ce(t)dt\cdot\exp\left(-\lambda\int_0^\infty e(s)ds\right) \\
&- (1-\pi)\int_0^\infty \exp(-rt)ce(t)dt \\
&= \max_{e(t),t\geq 0} \pi\int_0^\infty(\lambda V^*-c)\,e(t)\exp\left(-\lambda\int_0^t e(s)ds - rt\right)dt \\
&- (1-\pi)\int_0^\infty ce(t)\exp(-rt)dt
\end{aligned}
$$

In the third equality, we use the change of the order of integration. Now we derive the Hamilton Jacobi Bellman (HJB) equation in a heuristic way. Taking any small Δ,

$$
\begin{aligned}
&V(\pi) = \max_{e_0} \pi\,(\lambda V^* - c)\,e_0\Delta - (1-\pi)ce_0\Delta \\
&+ \max_{e(t),t\geq\Delta} \pi\int_\Delta^\infty(\lambda V^*-c)\,e(t)\exp\left(-\lambda\int_0^t e(s)ds - rt\right)dt \\
&- (1-\pi)\int_\Delta^\infty ce(t)\exp(-rt)dt \\
&= \max_{e_0} \pi\,(\lambda V^* - c)\,e_0\Delta - (1-\pi)ce_0\Delta \\
&+ \max_{e(t),t\geq\Delta} \pi\exp\left(-\left(\lambda e_0 + r\right)\Delta\right)\int_\Delta^\infty(\lambda V^*-c)\,e(t)\exp\left(-\lambda\int_\Delta^t e(s)ds - r(t-\Delta)\right)dt \\
&- (1-\pi)\exp(-r\Delta)\int_\Delta^\infty ce(t)\exp(-r(t-\Delta))dt \\
&= \max_{e_0} \pi\,(pV^* - c)\,e_0\Delta - (1-\pi)ce_0\Delta
\end{aligned}
$$

$$
\begin{aligned}
&+ \max_{e(t), t \geq \Delta} \pi \exp\left(-\left(\lambda e_0 + r\right)\Delta\right) \int_0^\infty \left(\lambda V^* - c\right) e(t+\Delta) \exp\left(-\lambda \int_0^{t+\Delta} e(s)ds - rt\right) dt \\
&- (1-\pi)\exp(-r\Delta) \int_0^\infty ce(t+\Delta)\exp(-rt)dt \\
&= \max_{e_0} \left(\pi\lambda V^* - c\right) e_0 \Delta \\
&+ \exp(-r\Delta)\exp(-\pi\lambda e_0\Delta) \max_{e(t), t\geq\Delta} \frac{\pi\left(1-\lambda e_0\Delta\right)}{1-\pi\lambda e_0\Delta} \int_0^\infty \left(\lambda V^* - c\right) e(t+\Delta)\exp\left(-\lambda\int_0^{t+\Delta} e(s)ds - rt\right) dt \\
&- \frac{1-\pi}{1-\pi\lambda e_0\Delta} \int_0^\infty ce(t+\Delta)\exp(-rt)dt \\
&= \max_{e_0} \left(\pi\lambda V^* - c\right) e_0\Delta + \left(1 - r\Delta - \pi\lambda e_0\Delta\right) V\left(\pi'\right),
\end{aligned}
$$

where $\pi' = \frac{\pi(1-\lambda e_0\Delta)}{1-\pi\lambda e_0\Delta}$. In the fourth equality, we use the Taylor expansion. Dividing both sides by Δ and taking the limit as $\Delta \to 0$, we get the HJB equation as follows.

$$
rV(\pi) = \max_{e_0} -ce_0 + \pi\lambda e_0 \left(V^* - V(\pi) - (1-\pi)V'(\pi)\right)
$$

Solving differential equation, we get

$$
V(\pi) = \begin{cases} \frac{\pi\lambda V^* - c}{r+\lambda} - (1-\pi)\frac{c\lambda}{r(r+\lambda)}\left(\frac{c}{\lambda V^* - c}\right)^{\frac{r}{\lambda}} \frac{c\lambda}{r(r+\lambda)} \frac{(1-\pi)^{\frac{r+\lambda}{\lambda}}}{\pi^{\frac{r}{\lambda}}} & \text{if } \pi \geq \frac{c}{\lambda V^*} \\ 0 \quad \text{if } \pi \leq \frac{c}{\lambda V^*} \end{cases}
$$

We used the value matching condition $V(\frac{c}{\lambda V^*}) = 0$ and the smooth pasting condition $V'(\frac{c}{\lambda V^*}) = 0$.

To implement this socially optimal value function, consider the policy that $e(t) = 1$ if and only if $t \leq t^*$. Denote $V(\pi, t^*)$ as the principal's value function of the belief and the stopping time t^*.

$$
V\left(\pi, t^*\right) = \pi \int_0^{t^*} \left(\lambda V^* - c\right)\exp(-(\lambda + r)t)dt - (1-\pi)\int_0^{t^*} c\exp(-rt)dt
$$

Since this function is concave (the second-order condition is negative), the first-order condition determines the optimal stopping time t^*:

$$
\frac{\partial V}{\partial t^*} = \pi\left(\lambda V^* - c\right)\exp\left(-(\lambda + r)t^*\right) - (1-\pi)c\exp\left(-rt^*\right) = 0
$$

Then we get $\frac{\lambda V^{*}-c}{c}=\frac{1-\pi}{\pi}\exp\left(rt^{*}\right)$. By defining $l=\frac{1-\pi}{\pi}$,

$$t_{1}^{*}=\frac{1}{\lambda}\log\left(\frac{\lambda V^{*}-c}{cl}\right)$$

Plugging this into $V\left(\pi,t^{*}\right)$,

$$\begin{aligned}V\left(\pi,t_{1}^{*}\right)&=\frac{\pi\left(\lambda v^{*}-c\right)}{r+\lambda}\left(1-\exp\left(-(r+\lambda)t_{1}^{*}\right)\right)-\frac{(1-\pi)c}{r}\left(1-\exp\left(-rt_{1}^{*}\right)\right)\\&=\frac{\pi\left(\lambda V^{*}-c\right)}{r+\lambda}\left(1-\left(\frac{\lambda V^{*}-c}{cl}\right)^{-\frac{r+\lambda}{\lambda}}\right)-\frac{(1-\pi)c}{r}\left(1-\left(\frac{\lambda V^{*}-c}{cl}\right)^{-\frac{r}{\lambda}}\right)\\&=V(\pi).\end{aligned}$$

Thus, the social optimum is achievable by the deadline policy.

Proposition 1. *When the principal can conduct the experimentation, she stops experimentation at* $t_{1}^{*}=\frac{1}{\lambda}\log\left(\frac{\lambda V^{*}-c}{cl_{0}}\right)$ *where* $l_{0}=\frac{1-\pi_{0}}{\pi_{0}}$ *if* $\pi_{0}\geq\frac{c}{\lambda V^{*}}$. *She does not conduct any experimentation otherwise.*

1.2.2.2 Observable Effort

The second benchmark is the case of observable effort where effort is contractible. Consider a contract such that $A(t)=c$ for $t\leq t_{1}^{*}$, $A(t)=0$ for $t>t_{1}^{*}$ and $B(t)=0$ for all t. Then, there is no incentive for the agent to delay reporting.

1.2.2.3 Observable News Arrival

The third benchmark is the case where the news arrival is observable by the principal. Consider a contract such that $A(t)=c$ for $t\leq t^{*}$, $A(t)=0$ for $t>t^{*}$, and $B(t)=0$ for $t>t^{*}$. Given the above contract, the value function $U(\pi,t)$ of the agent is

$$\begin{aligned}U(\pi,t)&=\max_{e(s),s\geq t}\mathbb{E}_{t}\left[\int_{t}^{\min\{T,t^{*}\}}e^{-r(s-t)}c(1-e(s))ds+e^{-r(T-t)}B(T)\right]\\&=\max_{e(s),s\geq t}\pi\int_{t}^{\infty}\lambda e(T)\exp\left(-\lambda\int_{t}^{T}e(s)ds\right)\left[\int_{t}^{\min\{T,t^{*}\}}\exp(-r(s-t))c(1-e(s))ds+e^{-r(T-t)}B(T)\right]dT\\&+(1-\pi)\int_{t}^{t^{*}}\exp(-r(s-t))c(1-e(s))ds\end{aligned}$$

$$
\begin{aligned}
&= \max_{e(s), s \geq t} \pi \left\{ \int_t^{t^*} \lambda e(T) \exp\left(-\lambda \int_t^T e(s) ds\right) \left[\int_t^T \exp(-r(s-t)) c(1-e(s)) ds + e^{-r(T-t)} B(T)\right] dT \right. \\
&\left. + \exp\left(-\lambda \int_t^{t^*} e(s) ds\right) \int_t^{t^*} \exp(-r(s-t)) c(1-e(s)) ds \right\} \\
&+ (1-\pi) \int_t^{t^*} \exp(-r(s-t)) c(1-e(s)) ds \\
&= \max_{e(s), s \geq t} \pi \left\{ \int_t^{t^*} \int_s^{t^*} \lambda e(T) \exp\left(-\lambda \int_t^T e(s) ds\right) dT \exp(-r(s-t)) c(1-e(s)) ds \right. \\
&+ \int_t^{t^*} \lambda e(T) B(T) \exp\left(-r(T-t) - \lambda \int_t^T e(s) ds\right) dT \\
&\left. + \exp\left(-\lambda \int_t^{t^*} e(s) ds\right) \int_t^{t^*} \exp(-r(s-t)) c(1-e(s)) ds \right\} \\
&+ (1-\pi) \int_t^{t^*} \exp(-r(s-t)) c(1-e(s)) ds \\
&= \max_{e(s), s \geq t} \pi \int_t^{t^*} \exp\left(-r(s-t) - \lambda \int_t^s e(s) ds\right) (c(1-e(s)) + \lambda e(s) B(s)) ds \\
&+ (1-\pi) \int_t^{t^*} \exp(-r(s-t)) c(1-e(s)) ds.
\end{aligned}
$$

In the third equality, we use the change of the order of integration. Now we derive the Hamilton Jacobi Bellman (HJB) equation in a heuristic way. Taking any small Δ,

$$
\begin{aligned}
U(\pi, t) &= \max_{e_t} \{\pi(c(1-e_t) + \lambda e_t B(t)) + (1-\pi) c(1-e_t)\} \Delta \\
&+ \max_{e(s), s \geq t+\Delta} \pi \int_{t+\Delta}^{t^*} \exp\left(-r(s-t) - \lambda \int_{t+\Delta}^s e(s) ds - \lambda e_t \Delta\right) (c(1-e(s)) + \lambda e(s) B(s)) ds \\
&+ (1-\pi) \int_{t+\Delta}^{t^*} \exp(-r(s-t)) c(1-e(s)) ds \\
&= \max_{e_t} \{\pi(c(1-e_t) + \lambda e_t B(t)) + (1-\pi) c(1-e_t)\} \Delta \\
&+ \max_{e(s), s \geq t+\Delta} \pi \exp(-r\Delta - \lambda e_t \Delta) \int_{t+\Delta}^{t^*} \exp\left(-r(s-t-\Delta) - \lambda \int_{t+\Delta}^s e(s) ds\right) (c(1-e(s)) + \lambda e(s) B(s)) d \\
&+ (1-\pi) \exp(-r\Delta) \int_{t+\Delta}^{t^*} \exp(-r(s-t-\Delta)) c(1-e(s)) ds \\
&= \max_{e_t} \{\pi(c(1-e_t) + \lambda e_t B(t)) + (1-\pi) c(1-e_t)\} \Delta + (1 - r\Delta - \pi \lambda e_t \Delta) U(\pi', t+\Delta),
\end{aligned}
$$

where $\pi' = \frac{\pi(1-\lambda e_t \Delta)}{1-\pi\lambda e_t \Delta}$. In the last equality, we use the Taylor expansion. Dividing by Δ and taking Δ to 0, we get HJB equation as follows.

$$rU(\pi,t) = \max_{e_t} c(1-e_t) + \pi\lambda e_t B(t) - \pi\lambda e_t U(\pi,t) + \frac{\partial U}{\partial t} - \frac{\partial U}{\partial \pi}\pi(1-\pi)\lambda e_t.$$

If $e(s) = 1$ for $s \in [t, t^*]$ optimal,

$$U(\pi,t) = \pi\lambda \int_t^{t^*} \exp(-(r+\lambda)(s-t))B(s)ds$$

To check the incentive-compatibility conditions that the agent will not deviate at time t, i.e., from $e(t) = 1$ to $e(t) = 0$,

$$\pi\lambda \int_t^{t^*} \exp(-(r+\lambda)(s-t))B(s)ds \geq \Delta c + e^{-r\Delta}\pi\lambda \int_{t+\Delta}^{t^*} \exp(-(r+\lambda)(s-(t+\Delta)))B(s)ds.$$

Dividing by Δ and taking Δ to 0, we get

$$\pi\lambda B(t) \geq c + \lambda U(\pi,t).$$

Then the incentive-compatibility condition is

$$B(t) \geq \frac{c}{\pi\lambda} + \lambda \int_t^{t^*} \exp(-(r+\lambda)(s-t))B(s)ds$$

Let $\pi(t)$ denote the on-path belief, that is, $\pi(t) = \frac{1}{1+l_0 e^{\lambda t}}$ for $t \leq t^*$. Off-path belief is higher than on-path belief. Therefore, the incentive conditions for on-path belief are sufficient for off-path. Since this incentive-compatibility condition is binding under the cost-minimizing bonus,

$$B(t) = \frac{c}{\lambda\pi(t)} + \lambda \int_t^{t^*} \exp(-(r+\lambda)(s-t))B(s)ds.$$

Solving the above equation, we get

$$B^*(t) = \alpha + \beta e^{\lambda t} + \gamma e^{rt}$$
$$\alpha = \frac{c(\lambda+r)}{r\lambda} \quad \beta = \frac{-rcl_0}{\lambda(\lambda-r)}$$
$$\gamma = \frac{cl_0}{\lambda-r}\exp\left((\lambda-r)t^*\right) - \frac{c}{r}\exp\left(-rt^*\right).$$

The optimal stopping time t_2^* can be determined as follows.

$$\max_{t^* \geq 0} V(\pi_0, t^*) - \pi_0 \int_0^{t^*} \lambda \exp(-\lambda t) \exp(-rt) B^*(t) dt$$

The first-order condition implies that if it is positive,

$$\lambda V^* = c\left(1 + e^{\lambda t_2^*}\right)\left(1 + l_0 e^{\lambda t_2^*}\right).$$

Note that the solution to this equation is nonnegative if and only if $\pi_0 \geq \frac{2c}{\lambda V^*}$. That is, $t_2^* = 0$ if $\pi_0 \leq \frac{2c}{\lambda V^*}$. It can be shown that this stopping time is smaller than the one for the first best. The figure 1.1 illustrates the shape of the bonus $B^*(t)$. The bonus is influenced by two temporal effects. The first effect is the positive impact of belief on the bonus: as time progresses, the agent becomes more pessimistic. To motivate the agent to continue working, the bonus must increase. The second effect is the negative impact of the approaching deadline on the bonus: as time advances, the deadline draws nearer, diminishing the agent's incentive to delay effort in anticipation of a higher future bonus.

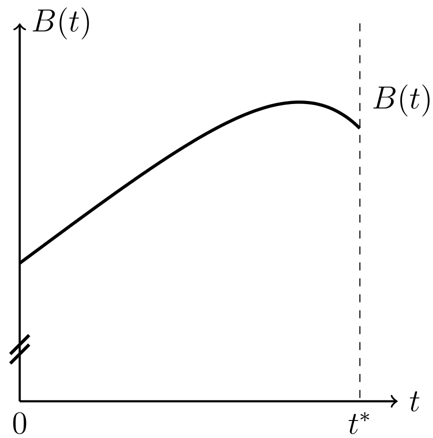

Figure 1.1. Shape of Bonus

Proposition 2. *When the principal can observe the timing of news arrival, the optimal contract is of the following form:* $A(t) = c$ *for* $t \leq t_2^*$, $A(t) = 0$ *for* $t > t_2^*$, $B(t) = B^*(t)$ *for* $t \leq t_2^*$, *and* $B(t) = 0$ *for* $t > t_2^*$, *where* t_2^* *and* B^* *are defined above.*

Proposition 3. *Under the optimal contract under the observable news arrival model, the contract terminates earlier than the first best timing of stopping experimentation.*

1.2.3 Second Best

Suppose both news arrival and effort are not observable. Let $W(t)$ denote the value of the agent when the good news has already arrived. Choosing the optimal report time $\hat{T}$,

$$W(t) = \max_{\hat{T}\in[t,t^*]} \int_t^{\hat{T}} e^{-r(s-t)}A(s)ds + e^{-r(\hat{T}-t)}B(\hat{T})$$

$$\hat{T}(t) \equiv \min \operatorname*{argmax}_{\hat{T}\in[t,t^*]} \int_t^{\hat{T}} \exp(-r(s-t))A(s)ds + \exp(-r(\hat{T}-t))B(\hat{T})$$

We show that for the optimal contract, we can restrict our attention to the ones that induce immediate reporting, i.e., $\hat{T}(t) = t$. We start by the following lemma.

Lemma 1. *For any contract $\langle A, B\rangle$, there is a non-decreasing sequence $\{t_0, t_1, \cdots\}$ such that, for each (t_n, t_{n+1}), the agent either reports immediately or delays report until t_{n+1}.*[1]

Proof. Suppose $t < \hat{T}(t)$. Then for $t' \in (t, \hat{T}(t))$,

$$\begin{aligned}
\hat{T}(t') &= \min \operatorname*{argmax}_{\hat{T}\in[t',t^*]} \int_{t'}^{\hat{T}} e^{-r(s-t')}A(s)ds + e^{-r(\hat{T}-t')}B(\hat{T}) \\
&= \min \operatorname*{argmax}_{\hat{T}\in[t',t^*]} \exp(r(t'-t)) \left[\int_t^{\hat{T}} e^{-r(s-t)}A(s)ds + e^{-r(\hat{T}-t)}B(\hat{T})\right] - \int_t^{t'} e^{-r(s-t')}A(s)ds \\
&= \min \operatorname*{argmax}_{\hat{T}\in[t',t^*]} \int_t^{\hat{T}} e^{-r(s-t)}A(s)ds + e^{-r(\hat{T}-t)}B(\hat{T}) \\
&= \hat{T}(t).
\end{aligned}$$

Thus, the optimal reporting time $\hat{T}(t')$ is the same as $\hat{T}(t)$. This suggests that we have an interval of the form $(\tilde{T}(t), \hat{T}(t))$ or $[\tilde{T}(t), \hat{T}(t))$ such that the agent delays report until $\hat{T}(t)$. The complement of such intervals is the set of t's at which the agent report immediately, which is a collection of intervals. □

Theorem 1. *For any contract $\langle A, B\rangle$ under which there is a collection of open intervals $\{(t_n, t'_n)\}$ where the agent delays his report, there exists $\langle A, B'\rangle$ that the principal prefers to $\langle A, B\rangle$ and induces immediate reporting.*

Proof.

$$W(t) = \max_{\hat{T}\in[t,t^*]} \int_t^{\hat{T}} \exp(-r(s-t))A(s)ds + \exp(-r(\hat{T}-t))B(\hat{T})$$

[1] One interpretation is that delay does not exhibit constant delay after the realization of good news.

$$\hat{T}(t) \equiv \min \operatorname*{argmax}_{\hat{T}\in[t,t^*]} \int_t^{\hat{T}} \exp(-r(s-t))A(s)ds + \exp(-r(\hat{T}-t))B(\hat{T})$$

Suppose there is a collection of intervals $\{(t_n, t'_n)\}$ when the agent delays his report under the contract $\langle A, B\rangle$. Consider a new contract $\langle A, W\rangle$. We can show that it induces immediate reporting as follows. Define

$$\tilde{W}(t) = \max_{\hat{T}\in[t,t^*]} \int_t^{\hat{T}} \exp(-r(s-t))A(s)ds + \exp(-r(\hat{T}-t))W(\hat{T})$$
$$\tilde{\hat{T}}(t) \equiv \min \operatorname*{argmax}_{\hat{T}\in[t,t^*]} \int_t^{\hat{T}} \exp(-r(s-t))A(s)ds + \exp(-r(\hat{T}-t))W(\hat{T}).$$

Then,

$$\begin{aligned}\tilde{W}(t) = &\max_{\hat{T}\in[t,t^*]} \int_t^{\hat{T}} \exp(-r(s-t))A(s)ds \\ &+ \exp(-r(\hat{T}-t)) \max_{\hat{\hat{T}}\in[\hat{T},t^*]} \int_{\hat{T}}^{\hat{\hat{T}}} \exp(-r(s-\hat{T}))A(s)ds + \exp(-r(\hat{\hat{T}}-\hat{T}))B(\hat{T}) \\ &= \max_{\hat{T}\in[t,t^*]} \int_t^{\hat{T}} \exp(-r(s-t))A(s)ds + \max_{\hat{\hat{T}}\in[\hat{T},t^*]} \int_{\hat{T}}^{\hat{\hat{T}}} \exp(-r(s-t))A(s)ds + \exp(-r(\hat{\hat{T}}-t))B(\hat{T}) \\ &= \max_{\hat{T}\in[t,t^*]} \max_{\hat{\hat{T}}\in[\hat{T},t^*]} \int_t^{\hat{\hat{T}}} \exp(-r(s-t))A(s)ds + \exp(-r(\hat{\hat{T}}-t))B(\hat{T})\end{aligned}$$

Therefore, $\tilde{\hat{T}}(t) = t$, i.e., $\langle A, W\rangle$ induces immediate reporting. Moreover, $\tilde{W} = W$, which ensures that the incentive conditions for effort exertion are unchanged.

We now show that the principal's expected payment is unchanged. The distribution of the timing T of the news arrival is unchanged, denoted by G. Expected payment under $\langle A, W\rangle$ given $T \in (t_n, t'_n)$ is

$$\begin{aligned}&\int_{t_n}^{t'_n} e^{-rt}W(t)g(t)dt + \int_{t_n}^{t'_n}\int_{t_n}^{T} e^{-rt}A(t)dtg(T)dT \\ &= \int_{t_n}^{t'_n} e^{-rt}\left\{\int_t^{t'_n} \exp(-r(s-t))A(s)ds + \exp(-r(t'_n-t))B(t'_n)\right\} g(t)dt + \int_{t_n}^{t'_n}\int_{t_n}^{T} e^{-rt}A(t)dtg(T)dT \\ &= \int_{t_n}^{t'_n} \left\{\int_t^{t'_n} e^{-rs}A(s)ds + \exp(-rt'_n)B(t'_n)\right\} g(t)dt + \int_{t_n}^{t'_n}\int_{t_n}^{T} e^{-rt}A(t)dtg(T)dT \\ &= \left\{\int_{t_n}^{t'_n} e^{-rs}A(s)ds + \exp(-rt'_n)B(t'_n)\right\} \int_{t_n}^{t'_n} g(t)dt\end{aligned}$$

which is the expression for the expected payment under $\langle A, B\rangle$.

Since the news is reported earlier while the expected payment is unchanged, the principal prefers $\langle A, W\rangle$.

$\square$

Consider a contract such that $A(t) = c$ for $t \leq t^*$, $B(t) = B^*(t)$ for $t \leq t^*$, $A(t) = 0$ for $t > t^*$, and $B(t) = 0$ for $t > t^*$ to induce the effort $e(t) = 1$ for $t \leq t^*$.

Given the above contract, the value function $U(\pi, t)$ of the agent is

$$
\begin{aligned}
U(\pi, t) &= \max_{e(s), s\geq t} \mathbb{E}_t \left[\int_t^{\min\{T,t^*\}} e^{-r(s-t)} c(1-e(s))ds + e^{-r(T-t)}W(T)\right] \\
&= \max_{e(s), s\geq t} \pi \int_t^{t^*} \exp\left(-r(s-t) - \lambda \int_t^s e(u)du\right)(c(1-e(s)) + \lambda e(s)W(s))ds \\
&+ (1-\pi)\int_t^{t^*} \exp(-r(s-t))c(1-e(s))ds
\end{aligned}
$$

HJB is

$$
rU(\pi, t) = \max_{e_t} c(1-e_t) + \pi\lambda e_t W(t) - \pi\lambda e_t U(\pi, t) + \frac{\partial U}{\partial t} - \frac{\partial U}{\partial \pi}\pi(1-\pi)\lambda e_t.
$$

If $e(s) = 1$ for $s \in [t, t^*]$ optimal,

$$
U(\pi, t) = \pi\lambda \int_t^{t^*} \exp(-(r+\lambda)(s-t))W(s)ds
$$

Then the incentive-compatibility condition for effort exertion is

$$
W(t) \geq \frac{c}{\lambda\pi} + \lambda \int_t^{t^*} \exp(-(r+\lambda)(s-t))W(s)ds
$$

By Theorem 1, we know that under the optimal contract $W = B$. Therefore, the optimal contract B^* under observable news arrival (but hidden action) is an obvious candidate. We show that it in fact satisfies the immediate reporting condition.

The first order condition for the immediate reporting to be optimal is that the derivative of the maximand with respect to $\hat{T}$ evaluated at t is nonpositive. That is,

$$
A(t) - rB(t) + \dot{B}(t) \leq 0.
$$

Then,

$$
\begin{aligned}
&A(t) - rB^*(t) + \dot{B}^*(t) \\
&= c - rB^*(t) + \dot{B}^*(t) \\
&= c - r\alpha + \beta e^{\lambda t}(\lambda - r) \\
&= -\frac{r}{\lambda}c - \frac{r}{\lambda}cl_0 e^{\lambda t} < 0
\end{aligned}
$$

Recall that $\lambda > r$. This implies $\beta < 0$. The sufficiency of the immediate reporting is the second-order condition of $\int_t^T e^{-r(s-t)}A(s)ds + e^{-\lambda(T-t)}B^{**}(T)$ with respect to T, that is,

$$
\exp(-r(T-t))\left(-rc + e^{\lambda t}\beta(\lambda - r)^2\right) < 0.
$$

Thus, $B^*(t)$ induces the immediate reporting.

Proposition 4. *When the principal can observe neither the agent's effort nor the timing of news arrival, the optimal contract is of the following form: $A(t) = c$ for $t \leq t_2^*$, $A(t) = 0$ for $t > t_2^*$, $B(t) = B^*(t)$ for $t \leq t_2^*$, and $B(t) = 0$ for $t > t_2^*$, where t_2^* and B^* are defined above.*

1.3 Bad News Model

We now turn our attention to a slightly modified model, which we refer to as the "bad news model." In contrast to the original model, when the state is bad ($\omega = B$), bad news arrives by time t with probability $G(t) = 1 - \exp\left(-\lambda \int_0^t e(s)ds\right)$, where $\lambda \in (0, 1)$, and its density is given by $g(t) = \lambda \cdot e(t) \exp\left(-\lambda \int_0^t e(s)ds\right)$. When the state is good ($\omega = G$), bad news never arrives. The bad news is privately observed by the agent.

At any time, the principal can implement the project at a cost of $c_I > 0$, which also reveals the state and results in a gain of $V^* > 0$ if the state is good.

Let $T \geq 0$ denote the timing of news arrival. When the news never arrives, set $T = \infty$. The agent chooses the effort $e(t)$ at each time point t, as well as the reporting time $\hat{T} \geq T$. Let $\bar{T} \geq 0$ denote the time when the principal implements the project. If she never implements it, set $\bar{T} = \infty$. The principal offers the contract with full commitment power before the agent starts experimentation. The contract is denoted by $\boldsymbol{C} = \langle A(t), B(t) \rangle$, where $A(t) \in \{0, c\}$ is the base wage enabling the agent to exert $e(t) = 1$ at time t, and $B(t) \in \mathbb{R}_+$ is the bonus paid conditional on bad news being reported at time t. The contract must specify a termination date $t^* \in [0, \infty)$, after which both $A(t)$ and $B(t)$ are zero. We further narrow our focus to contracts where $e(t) = 1$ for $t \leq t^*$, interpreting this assumption as the principal's preference to avoid allowing the agent to have inactive periods during the contract's duration.

The timing of the game is as follows.

1. The principal offers the contract $\boldsymbol{C} = \langle A(t), B(t) \rangle$ to the agent.
2. The nature determines the state.
3. The game ends if he rejects the offer. The agent starts experimentation if he accepts the offer.
4. The agent chooses $e(t)$ and the principal pays $A(t), B(t)$ until the game ends.
5. If the bad news is reported, the time reaches the termination date t^*, or the principal implements the project, then the game ends.

1.3.1 Analysis

Firstly, as an initial benchmark, we analyze the socially optimal scenario where the principal can conduct the experimentation independently. Secondly, we assess two additional benchmarks: the first evaluates the optimal contract when the agent's effort is observable, and the second addresses scenarios where the timing of news arrival is observable. Thirdly, we show that in cases where neither the effort nor the timing of news arrival is observable, the optimal contract promotes immediate reporting. We subsequently explicitly derive the form of this optimal contract.

The belief is updated according to the following differential equation:

$$\dot{\pi}(t) = \pi(t)(1 - \pi(t))\lambda e(t).$$

Therefore, conditional on the absence of news, the belief increases over time t.

1.3.2 Benchmarks

1.3.2.1 First Best

Suppose that the principal can do experimentation by herself. Let T denote the time the bad news arrives. Let $V(\pi)$ denote the principal's value as a function of the belief π. The absence of dynamic inconsistency indicates that the timing of the principal implementing the project can be determined at $t = 0$. Let τ denote the anticipated timing of implementation when no news is received. That is, if no news is received until τ, she implements it at that time. Then, $\bar{T} = \tau$ if $T > \tau$ and $\bar{T} = \infty$ otherwise.

$$
\begin{aligned}
V(\pi) &= \max_{e,\tau} \mathbb{E}_0 \left[\int_0^{\min\{T,\tau\}} -e^{-rt} ce(t)dt + \mathbb{I}\{\tau < T\} e^{-r\tau} \left(\mathbb{I}\{\omega = G\} V^* - c_I \right) \right] \\
&= \max_{e,\tau} \pi \left\{ \int_0^{\tau} -e^{-rt} ce(t)dt + e^{-r\tau} \left(V^* - c_I \right) \right\} \\
&+ (1-\pi) \left\{ \int_0^{\infty} \int_0^{\min\{T,\tau\}} -e^{-rt} ce(t)dt \lambda e(T) \exp\left(-\lambda \int_0^T e(s)ds \right) dT \right. \\
&\left. + \Pr(\tau < T) e^{-r\tau} \left(-c_I \right) \right\} \\
&= \max_{e,\tau} \pi \left\{ \int_0^{\tau} -e^{-rt} ce(t)dt + e^{-r\tau} \left(V^* - c_I \right) \right\} \\
&+ (1-\pi) \left\{ \int_0^{\tau} \int_0^{T} -e^{-rt} ce(t)dt \lambda e(T) \exp\left(-\lambda \int_0^T e(s)ds \right) dT \right. \\
&+ \exp\left(-\int_0^{\tau} \lambda e(s)ds \right) \int_0^{\tau} -e^{-rt} ce(t)dt \\
&\left. + \exp\left(-\int_0^{\tau} \lambda e(s)ds \right) e^{-r\tau} \left(-c_I \right) \right\} \\
&= \max_{e,\tau} \pi \left\{ \int_0^{\tau} -e^{-rt} ce(t)dt + e^{-r\tau} \left(V^* - c_I \right) \right\} \\
&+ (1-\pi) \left\{ \int_0^{\tau} \int_t^{\tau} \lambda e(T) \exp\left(-\lambda \int_0^T e(s)ds \right) dT (-e^{-rt} ce(t))dt \right. \\
&\left. - \exp\left(-\int_0^{\tau} \lambda e(s)ds \right) \left(e^{-r\tau} c_I + \int_0^{\tau} e^{-rt} ce(t)dt \right) \right\} \\
&= \max_{e,\tau} \pi \left\{ \int_0^{\tau} -e^{-rt} ce(t)dt + e^{-r\tau} \left(V^* - c_I \right) \right\} \\
&+ (1-\pi) \left\{ \int_0^{\tau} \left[-\exp\left(-\lambda \int_0^T e(s)ds \right) \right]_t^{\tau} \left(-e^{-rt} ce(t) \right) dt \right. \\
&\left. - \exp\left(-\int_0^{\tau} \lambda e(s)ds \right) \left(e^{-r\tau} c_I + \int_0^{\tau} e^{-rt} ce(t)dt \right) \right\} \\
&= \max_{e,\tau} \pi \left\{ \int_0^{\tau} -e^{-rt} ce(t)dt + e^{-r\tau} \left(V^* - c_I \right) \right\} \\
&+ (1-\pi) \left\{ \int_0^{\tau} \exp\left(-\lambda \int_0^t e(s)ds \right) \left(-e^{-rt} ce(t) \right) dt \right. \\
&\left. - \exp\left(-\lambda \int_0^{\tau} e(s)ds \right) e^{-r\tau} c_I \right\} \\
&= \max_{e,\tau} \pi \left\{ \int_0^{\tau} -e^{-rt} ce(t)dt + e^{-r\tau} \left(V^* - c_I \right) \right\} \\
&- (1-\pi) \left\{ \int_0^{\tau} \exp\left(-\int_0^t \lambda e(s)ds - rt \right) ce(t)dt \right.
\end{aligned}
$$

$$+\exp\left(-\int_0^\tau \lambda e(s)ds - r\tau\right)c_I\bigg\}.$$

Now we derive the Hamilton Jacobi Bellman (HJB) equation in a heuristic way. Taking any small Δ,

$$\begin{aligned}
&V(\pi) = \max\{\pi V^* - c_I,\\
&\max_{e_0} -ce_0\Delta + \max_{e,\tau}\pi\left\{\int_\Delta^\tau -e^{-rt}ce(t)dt + e^{-r\tau}(V^* - c_I)\right\}\\
&+(1-\pi)\left\{\int_\Delta^\tau -\exp\left(-rt - \lambda\int_\Delta^t e(s)ds - \lambda e_0\Delta\right)ce(t)dt\right.\\
&\left.-\exp\left(-r\tau - \lambda\int_\Delta^\tau e(s)ds - \lambda e_0\Delta\right)c_I\right\}\quad\bigg\}\\
&= \max\{\pi V^* - c_I,\\
&\max_{e_0} -ce_0\Delta + e^{-r\Delta}\max_{e,\tau}\pi\left\{\int_\Delta^\tau -e^{-r(t-\Delta)}ce(t)dt + e^{-r(\tau-\Delta)}(V^* - c_I)\right\}\\
&+(1-\pi)\exp(-\lambda e_0\Delta)\left\{\int_\Delta^\tau -\exp\left(-r(t-\Delta) - \int_\Delta^t \lambda e(s)ds\right)ce(t)dt\right.\\
&\left.-\exp\left(-r(\tau-\Delta) - \int_\Delta^\tau \lambda e(s)ds\right)c_I\right\}\bigg\}\\
&= \max\{\pi V^* - c_I,\\
&\max_{e_0} -ce_0\Delta + e^{-r\Delta}\max_{e,\tau}\pi\left\{\int_0^{\tau-\Delta} -e^{-rt'}ce(t'+\Delta)dt' + e^{-r(\tau-\Delta)}(V^* - c_I)\right\}\\
&+(1-\pi)\exp(-\lambda e_0\Delta)\left\{\int_0^{\tau-\Delta} -\exp\left(-rt' - \lambda\int_0^{t'} e(s)ds\right)ce(t'+\Delta)dt'\right.\\
&\left.-\exp\left(-r(\tau-\Delta) - \lambda\int_0^{\tau-\Delta} e(s+\Delta)ds\right)c_I\right\}\bigg\}\\
&= \max\left\{\pi V^* - c_I, \max_{e_0} -ce_0\Delta + (1 - r\Delta - (1-\pi)\lambda e_0\Delta)V(\pi')\right\},
\end{aligned}$$

where $\pi' = \frac{\pi}{1-(1-\pi)\lambda e_0\Delta}$. Suppose $\tau > 0$ and $e_0 = 1$, then

$$V(\pi) = -c\Delta + (1 - r\Delta - (1-\pi)\lambda\Delta)V(\pi')$$

Dividing by Δ and taking Δ to 0, we get HJB equation as follows:

$$0 = V'(\pi)\pi(1-\pi)\lambda - c - (\lambda + (1-\pi)\lambda)V(\pi)$$

Solving the above differential equation,

$$V(\pi) = -\frac{c\lambda\pi}{r(\lambda+r)} - \frac{c}{\lambda+r} + k_1 \frac{\pi^{\frac{\lambda+r}{\lambda}}}{(1-\pi)^{\frac{r}{\lambda}}}$$
$$V'(\pi) = -\frac{c\lambda}{r(\lambda+r)} + k_1 \left\{ \frac{\lambda+r}{\lambda}\left(\frac{\pi}{1-\pi}\right)^{\frac{r}{\lambda}} + \frac{r}{\lambda}\left(\frac{\pi}{1-\pi}\right)^{\frac{\lambda+r}{\lambda}} \right\}$$

Let π^* denote the belief at which the principal implement the project. Then the value matching condition is

$$V(\pi^*) = \pi^* V^* - c_I$$

The smooth pasting condition is

$$V'(\pi^*) = V^*$$

Using the above,

$$\pi^* = \frac{(\lambda+r)c_I - c}{rV^* + \lambda c_I}$$
$$k_1 = \frac{c_I - \frac{c}{\lambda+r}}{\frac{r}{\lambda}\pi^*\left\{\left(\frac{\pi^*}{1-\pi^*}\right)^{\frac{r}{\lambda}} + \left(\frac{\pi^*}{1-\pi^*}\right)^{\frac{\lambda+r}{\lambda}}\right\}} = \frac{\lambda}{r(\lambda+r)} \frac{(r(V^*-c_I)+c)^{\frac{\lambda+r}{\lambda}}}{((\lambda+r)c_I - c)^{\frac{r}{\lambda}}}$$

So far we have ignored the case in which the principal neither experiment nor implement the project. Let π^{**} be the solution to

$$0 = -\frac{c\lambda\pi^{**}}{r(\lambda+r)} - \frac{c}{\lambda+r} + k_1 \frac{(\pi^{**})^{\frac{\lambda+r}{\lambda}}}{(1-\pi^{**})^{\frac{r}{\lambda}}}.$$

When $\pi \leq \pi^{**}$, the principal neither experiment nor implement the project.

Proposition 5. *When the principal can conduct the experimentation, she experiments while $\pi(t) \in [\pi^{**}, \pi^*)$, implement the project if $\pi(t) \geq \pi^*$, and neither experiment nor implement the project otherwise.*

1.3.2.2 Observable Effort

The second benchmark is the case of observable action. If effort is observable and therefore contractible, consider $A(t) = c$ for $\pi(t) \in [\pi^{**}, \pi^*]$, $A(t) = 0$ otherwise and

$B(t) = 0$ for all t. Then, there is no incentive for the agent to delay reporting.

1.3.2.3 Observable News Arrival

The third benchmark is the case where the news arrival is observable by the principal. Consider a contract $A(t), B(t)$ that induces the effort $e(t) = 1$ for $t \in [0, t^*]$ until the bad news arrives. The agent's expected utility is

$$U(\pi, t) = \max_{e(s), s \geq t} \mathbb{E}_t \left[\int_t^{\min\{T, t^*\}} e^{-r(s-t)} c(1 - e(s)) ds + e^{-r(T-t)} B(T) \right]$$

$$= \max_{e(s), s \geq t} \pi \int_0^{t^*} e^{-r(s-t)} c(1 - e(s)) ds$$

$$+ (1 - \pi) \left\{ \int_t^{t^*} \lambda e(T) \exp\left(- \int_t^T \lambda e(s) ds \right) \left[\int_t^T e^{-r(s-t)} c(1 - e(s)) ds + e^{-r(T-t)} B(T) \right] dT \right.$$

$$\left. + \exp\left(- \int_t^{t^*} \lambda e(s) ds \right) \int_t^{t^*} \exp(-r(s-t)) c(1 - e(s)) ds \right\}$$

$$= \max_{e(s), s \geq t} \pi \int_0^{t^*} e^{-r(s-t)} c(1 - e(s)) ds$$

$$+ (1 - \pi) \left\{ \int_t^{t^*} \int_s^{t^*} \lambda e(T) \exp\left(- \int_t^T \lambda e(s) ds \right) dT e^{-r(s-t)} c(1 - e(s)) ds \right.$$

$$+ \int_t^{t^*} \lambda e(T) \exp\left(-r(T-t) - \int_t^T \lambda e(s) ds \right) B(T) dT$$

$$\left. + \exp\left(- \int_t^{t^*} \lambda e(s) ds \right) \int_t^{t^*} \exp(-r(s-t)) c(1 - \mathrm{e}(s)) ds \right\}$$

$$= \max_{e(s), s \geq t} \pi \int_t^{t^*} e^{-r(s-t)} c(1 - e(s)) ds$$

$$+ (1 - \pi) \int_t^{t^*} \exp\left(-r(s-t) - \int_t^s \lambda e(s) ds \right) \{ c(1 - e(s)) + \lambda e(s) B(s) \} ds.$$

Now we derive the Hamilton Jacobi Bellman (HJB) equation in a heuristic way. Taking any small Δ,

$$U(\pi, t) = \max_e \{ c(1 - e) + (1 - \pi) \lambda e B(t) \} \Delta$$

$$+ \max_{e(s), s \geq t + \Delta} \pi \int_{t+\Delta}^{t^*} e^{-r(s-t)} c(1 - e(s)) ds$$

$$+ (1 - \pi) \int_{t+\Delta}^{t^*} \exp\left(-r(s-t) - \int_t^s \lambda e(s) ds \right) \{ c(1 - e(s)) + \lambda \, \mathrm{e}(s) B(s) \} ds$$

$$= \max_e \{ c(1 - e) + (1 - \pi) \lambda e B(t) \} \Delta$$

$$+ \max_{e(s), s \geq t+\Delta} e^{-r\Delta} \pi \int_{t+\Delta}^{t^*} e^{-r(s-t-\Delta)} c(1-e(s)) ds$$

$$+ \exp(-r\Delta - \lambda e \Delta)(1-\pi) \int_{t+\Delta}^{t^*} \exp\left(-r(s-t-\Delta) - \int_{t+\Delta}^{s} \lambda e(s) ds\right) \{c(1-e(s)) + \lambda e(s) B(s)\} ds$$

$$= \max_{e} \{c(1-e) + (1-\pi)\lambda e B(t)\}\Delta + (1 - r\Delta - (1-\pi)\lambda e \Delta) U(\pi', t+\Delta),$$

where $\pi' = \frac{\pi}{1-(1-\pi)\lambda e_0 \Delta}$. Then the HJB is

$$rU(\pi, t) = \max_{e} \left\{ c(1-e) + (1-\pi)\lambda e B(t) - (1-\pi)\lambda e U(\pi, t) + \frac{\partial U}{\partial \pi} \pi (1-\pi) \lambda e + \frac{\partial U}{\partial t} \right\}$$

Suppose the agent chooses $e(s) = 1$ for $s \in [t, t^*]$.

$$U(\pi, t) = (1-\pi)\lambda \int_{t}^{t^*} \exp(-(r+\lambda)(s-t)) B(s) ds$$

To check the incentive-compatibility conditions that the agent will not deviate to $e(t) = 0$ from $e(t) = 1$,

$$\begin{aligned} &(1-\pi)\lambda \int_{t}^{t^*} \exp(-(r+\lambda)(s-t)) B(s) ds \\ &\geq \Delta c + e^{-\lambda\Delta}(1-\pi)\lambda \int_{t+\Delta}^{t^*} \exp(-(r+\lambda)(s-(t+\Delta))) B(s) ds \end{aligned}$$

Dividing by Δ and taking Δ to 0, we get

$$B(t) \geq \frac{c}{\lambda(1-\pi)} + \lambda \int_{t}^{t^*} \exp(-(r+\lambda)(s-t)) B(s) ds$$

Note that off-path belief is lower than on-path belief $\pi(t)$. Therefore, the incentive conditions for on-path are sufficient for off-path. Since this incentive-compatibility condition is binding under the cost-minimizing bonus,

$$B(t) = \frac{c}{\lambda(1-\pi(t))} + \lambda \int_{t}^{t^*} \exp(-(r+\lambda)(s-t)) B(s) ds$$

Solving the above equation, we get,

$$B^{**}(t) = \alpha + \beta e^{\lambda t} + \gamma e^{rt}$$

$$\alpha = \frac{c(\lambda + r)}{r\lambda} \qquad \beta = \frac{-rcl_1}{\lambda(\lambda - r)}$$
$$\gamma = \frac{cl_1}{\lambda - r} \exp\left((\lambda - r)t^*\right) - \frac{c}{r} \exp\left(-rt^*\right),$$

where $l_1 = \frac{\pi_0}{1-\pi_0}$.

The optimal stopping time t_3^* can be determined as follows.

$$\max_{t^* \geq 0} \pi_0 \left\{ - \int_0^{t^*} e^{-rt} cdt + e^{-rt^*} \left(V^* - c_I\right) \right\}$$
$$+ (1 - \pi_0) \left\{ \int_0^{t^*} \lambda \exp(-\lambda T) \left\{ \int_0^T -e^{-\lambda t} cdt - e^{-rT} B^{**}(T) \right\} dT \right.$$
$$\left. - \exp\left(-\lambda t^*\right) \int_0^{t^*} e^{-rt} cdt - \exp\left(-\lambda t^*\right) \exp\left(-rt^*\right) c_I \right\}$$

Suppose the above equation is concave. Then the first-order condition implies

$$-\pi_0 r e^{-rt^*} \left(V^* - c_I + \frac{c}{r}\right) - (1 - \pi_0) \left(c - (\lambda + r)c_I + B^{**}\left(t^*\right)\right) \exp\left(-(\lambda + r)t^*\right) = 0$$

Then the optimal stopping time is

$$t_3^* = \frac{1}{\lambda} \log \left(\frac{(\lambda + r)c_I - \frac{1+\lambda}{\lambda} c}{l_1 \left(rV^* - rc_I + c\frac{1+\lambda}{\lambda}\right)} \right)$$

Note that the solution to this equation is nonnegative if and only if $\pi \leq \hat{\pi}$, where

$$\hat{\pi} = \frac{(\lambda + r)c_I - \frac{1+\lambda}{\lambda} c}{rV^* + \lambda c_I}.$$

That is, $t_3^* = 0$ if $\pi_0 \geq \hat{\pi}$. Compared to the socially optimal π^*, the stopping belief $\hat{\pi}$ is smaller. Therefore, the stopping time is earlier than the socially optimal one.

Proposition 6. *When the principal can observe the timing of news arrival, the optimal contract is of the following form:* $A(t) = c$ *for* $t \leq t_3^*$, $A(t) = 0$ *for* $t > t_3^*$, $B(t) = B^{**}(t)$ *for* $t \leq t_3^*$, *and* $B(t) = 0$ *for* $t > t_3^*$, *where* t_3^* *and* B^{**} *are defined above.*

Proposition 7. *Under the optimal contract under the observable news arrival model, the contract terminates earlier than the first best timing of stopping experimentation.*

1.3.3 Second Best

Suppose both news arrival and effort are not observable. Denote the value of the agent when the bad news has already arrived as $W(t)$. Choosing the optimal report time $\hat{T}$,

$$W(t) = \max_{\hat{T} \in [t, t^*]} \int_t^{\hat{T}} e^{-r(s-t)} A(s) ds + e^{-r(\hat{T}-t)} B(\hat{T})$$

$$\hat{T}(t) \equiv \min \operatorname*{argmax}_{\hat{T} \in [t, t^*]} \int_t^{\hat{T}} \exp(-r(s-t)) A(s) ds + \exp(-r(\hat{T}-t)) B(\hat{T})$$

By exactly the same argument as Theorem 1 for the good news model, we can show that for the optimal contract we can restrict our attention to the ones that induce immediate reporting.

Corollary 1. *For any contract $\langle A, B \rangle$ under which there is a collection of open intervals $\{(t_n, t'_n)\}$ where the agent delays his report, there exists $\langle A, B' \rangle$ that the principal prefers to $\langle A, B \rangle$ and induces immediate reporting.*

Consider a contract such that $A(t) = c$ for $t \leq t^*$, $A(t) = 0$ for $t > t^*$, and $B(t) = 0$ for $t > t^*$ to induce the effort $e(t) = 1$ for $t \geq t^*$.

Given the above contract, the value function $U(\pi, t)$ of the agent satisfies the following HJB.

$$rU(\pi, t) = \max_e \left\{ c(1-e) + (1-\pi)\lambda e W(t) - (1-\pi)\lambda e U(\pi, t) + \frac{\partial U}{\partial \pi} \pi (1-\pi) \lambda e + \frac{\partial U}{\partial t} \right\}$$

Then the incentive-compatibility condition for effort exertion is

$$W(t) \geq \frac{c}{\lambda(1-\pi)} + \lambda \int_t^{t^*} \exp(-(r+\lambda)(s-t)) W(s) ds$$

By Corollary 1, we know that under the optimal contract $W = B$. Therefore, the optimal contract B^{**} under observable news arrival (but hidden action) is an obvious candidate. We show that it in fact satisfies the immediate reporting condition.

The first order condition for the immediate reporting to be optimal is that the derivative of the maximand with respect to $\hat{T}$ evaluated at t is nonpositive. That is,

$$A(t) - rB(t) + \dot{B}(t) \leq 0.$$

Then,

$$\dot{B}^{**}(t) - rB^{**}(t) + c = -\frac{r}{\lambda}c - \frac{r}{\lambda}cl_1 e^{\lambda t} < 0,$$

which is a necessary condition for the immediate reporting. Recall that $\lambda > r$. This implies $\beta < 0$. The sufficiency of the immediate reporting is the second-order condition of $\int_t^T e^{-r(s-t)}A(s)ds + e^{-\lambda(T-t)}B^{**}(T)$ with respect to T, that is,

$$\exp(-r(T-t))\left(-rc + e^{\lambda t}\beta(\lambda - r)^2\right) < 0.$$

Then B^{**} induces the immediate reporting.

Proposition 8. *When the principal can observe neither the agent's effort nor the timing of news arrival, the optimal contract is of the following form:* $A(t) = c$ *for* $t \leq t_3^*$*,* $A(t) = 0$ *for* $t > t_3^*$*,* $B(t) = B^{**}(t)$ *for* $t \leq t_3^*$*, and* $B(t) = 0$ *for* $t > t_3^*$*, where* t_3^* *and* B^{**} *are defined above.*

1.4 Conclusion

We study delegated experimentation with a financially constrained agent. The agent could conceal their discoveries. However, the optimal bonus contract induces immediate reporting. We find that the incentive-compatible condition for reporting is subsumed by the incentive-compatible condition for exerting effort.

There are several potential future extensions. The first is collaborative or competitive experimentation by agents. If the probability of news arrival is the sum of each agent's effort level, the free-riding problem arises. In a competitive environment, such as a winner-take-all market, the bonus could be reduced since agents are more likely to exert effort.

The second extension concerns cumulative technology. Our model assumes a constant arrival rate. If the arrival rate of good news is a function of the cumulative effort level, then agents will become more optimistic over time. The concavity or convexity of the functional form could potentially affect equilibrium outcomes.

The third extension addresses the zombie venture capital problem. While our focus is on the relationship between investors and entrepreneurs, in reality, investments are managed by venture capital firms. These firms collect money from investors and invest in entrepreneurs. If entrepreneurs are not successful, they cannot generate capital gains through an IPO. Even if the venture capital should exit, there is an incentive to delay due to management fees. Investigating intermediaries like venture capital is another potential research avenue.

Chapter 2
Designing Contracts and Information Jointly

2.1 Introduction

We examine a principal-agent problem where the principal jointly designs outcome-contingent monetary rewards as well as the information that is available to the agent. A project's success depends both on the agent's effort and the quality of the project. The principal chooses an information structure that informs the agent of the quality of the project. In addition, the principal offers the agent a menu of contracts. Each contract specifies monetary rewards based on the outcome, and the agent might choose a contract based on his information. As a result, our setting combines hidden action with endogenously specified private information.

There are several market settings where a principal jointly controls monetary rewards and information. An example is an upstream firm (principal) that delegates the design of a new product to a downstream firm (agent). The upstream firm possesses information about the market's demand for this new product. By disclosing the project details (e.g., market research) and formulating a contract, the upstream firm incentivizes the downstream firm to exert effort. Another scenario arises in laboratory settings, where a principal researcher (principal) assigns a risky research project to a graduate student (agent) in her laboratory. To motivate the student, the researcher may highlight the project's potential and promise the student first authorship if successful.

Our main result is that a full separation or full pooling contract is optimal. Full separation means the agent can perfectly know the true quality after observing the signal. Full pooling means the posterior is the same as the prior. Semi-separation is never optimal. To show this result, we first observe that without loss of generality, we can focus on binary signals and a single contract. We then show that in a separating contract, the principal's payoff is convex in the information structure's noise. As a result, full information or no information is optimal.

The synergy of Bayesian persuasion and hidden action, coupled with monetary transfer, remains unexplored in prior literature. The optimal form of the information structure has been investigated by the past literature. Lewis and Sappington (1991) show that full disclosure or full pooling is optimal under adverse selection when the principal can choose the probability that the agent knows the realization of the random cost of production. Our model is more general because we allow for arbitrary signal structures whereas their signal structures are parametric, but our model is less general because we allow for only binary efforts. Based on the binary action example of Kamenica and Gentzkow (2011), Li (2017) adds the monetary transfer to a simple Bayesian persuasion model. That paper shows limiting the amount of monetary transfer increases the principal's incentive to provide information. Our paper also adds monetary transfer to a Bayesian persuasion game under moral hazard. We show one of two extreme information structures, full pooling or full separation is optimal.

2.2 Model

There are one agent and one principal. There is a payoff relevant information, which we call quality, $\theta_i \in \Theta = \{\theta_G, \theta_B\}$, where $1 \geq \theta_G > \theta_B \geq 0$. Neither the principal nor the agent know θ. The agent will choose effort level e_H or e_L whose cost is $C(e_j)$. We assume $1 \geq e_H > e_L \geq 0$ and $C(e_H) = c_H > C(e_L) = c_L = 0$[1]. The outcome of the project is binary X or 0. X means the project is successful. The probability of the success is $\theta_i e_j$. The principal will design a signal structure and a menu of contracts.

A signal π consists of a finite realization space S and a family of distributions $\{\pi(\cdot \mid \theta_i)\}_{\theta_i \in \Theta}$ over S. A menu of contracts $\{(a_s, b_s)\}_{s \in S}$ is as follows. For every signal realization s, a_s is the payment from the principal to the agent when the project is successful and b_s is the payment from the principal to the agent when the project is not successful. We assume the limited liability, $a_s \geq b_s \geq 0$ for all s.

If the agent chooses contract (a, b), The utility of the principal is

$$\begin{cases} X - a & \text{When the project is successful} \\ -b & \text{When the project is not successful} \end{cases}$$

[1] $c_L = 0$, IC, and LL imply that the individual rationality holds under the equilibrium.

The utility of the agent when he chooses contract (a, b) and effort $e_j, j \in \{L, H\}$, is

$$\begin{cases} a - c_j & \text{When the project is successful} \\ b - c_j & \text{When the project is not successful} \end{cases}$$

The timing of the game is as follows.

1. The principal commits the signal strategy $\{\pi(\cdot|\theta_i)\}_{\theta_i \in \Theta}$ and contract $\{(a_s, b_s)\}_{s \in S}$.
2. Nature draws the quality whose prior is p to good type θ_G and $1 - p$ to bad type θ_B.
3. The agent observes the signal strategy, the contract, and the signal realization.
4. The agent chooses a contract from the menu and exerts effort.
5. The principal observes output and pays wages based on output and the agent's contract.

Our solution concept is a Sender-preferred subgame perfect equilibrium. We denote $\Delta e = e_H - e_L$, $\Delta\theta = \theta_G - \theta_B$.

Without loss of generality, we can focus on a single contract. The first step is to use a result from Gottlieb and Moreira (2022). They study a principal-agent model with a fixed signal and show the optimal mechanism offers a single contract to all types if there are two outputs.

Theorem 2. *(from Corollary 1 of Gottlieb and Moreira (2022)) Fix a signal π. There exists an optimal mechanism where $a_s = a$ and $b_s = 0$ for all s.*

The second step is to show we can focus on binary signal realizations. We can do this by pooling all the signal realizations that lead to low effort and also pooling all the signals that lead to high effort.

As a result, for the rest of the paper, we assume $S = \{s_\ell, s_h\}$, where we think of s_ℓ as a recommendation to choose e_L and s_h as the recommendation to choose e_H. Because without loss of generality, we can focus on obedient contracts, if signal s_h is realized, then the contract induces high effort, and if s_ℓ is realized, then the contract induces low effort. Let $\epsilon_H = Pr(s_\ell|\theta_G)$ be the probability of low action recommendation in the good state and $\epsilon_L = Pr(s_h|\theta_B)$ the probability of high action recommendation in the bad state, as shown in Figure 2.1. Let q_i be the posterior that the true state is θ_G after observing s_i. Let θ_{q_i} be the expected value of the θ given the posterior q_i. Let θ_p be the expected value of the θ given the prior p. The principal further chooses a single a which is the payment for a successful project, and pays 0 if the project is not successful.

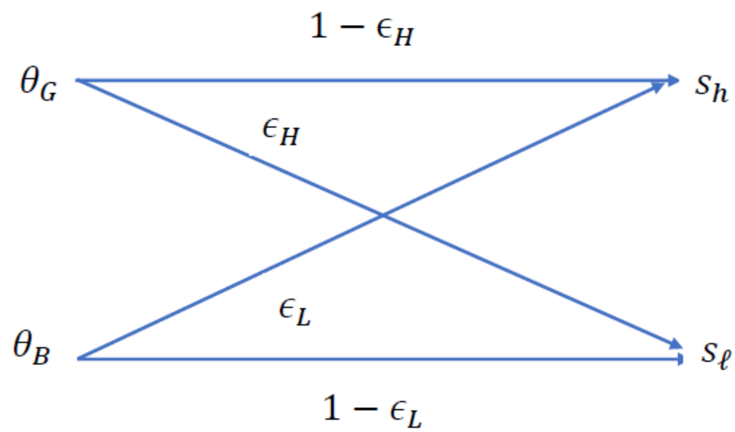

Figure 2.1. Signal Structure

2.3 Analysis

Before presenting our main result, we discuss a special class of contracts, which we call pooling contracts. A "high effort pooling" contract is one where $Pr(s_h) = 1$, and a "low effort pooling" contract is one where $Pr(s_\ell) = 1$. Contracts in this class require a separate analysis because they have a single obedience constraint and a different objective function.

The high effort pooling contract is as follows. Since the agent always takes high effort, the objective function of the principal is

$$\theta_p e_H (X - a).$$

Since the posterior is the same as the prior p, the incentive condition for the agent to take e_H is

$$\begin{aligned} &p\theta_G e_H a + (1-p)\theta_B e_H a - c_H \\ &\geq p\theta_G e_L a + (1-p)\theta_B e_L a. \end{aligned}$$

Then the incentive condition implies,

$$a \geq \frac{c_H}{\Delta e \theta_p}.$$

The maximum payoff of the principal under the high effort pooling contract is $\theta_p e_H(X - \frac{c_H}{\Delta e \theta_p})$.

The low effort pooling contract is as follows. Since there is no need to motivate high effort, $a = 0$.

2.3.1 General Analysis

The probability of the high signal is

$$Pr(s_h) \equiv p(1 - \epsilon_H) + (1 - p)\epsilon_L.$$

The probability of the low signal is

$$Pr(s_\ell) \equiv p\epsilon_H + (1 - p)(1 - \epsilon_L).$$

The general analysis is as follows. The objective function of the principal is

$$\begin{aligned} & e_H[p(1-\epsilon_H)\theta_G + (1-p)\epsilon_L\theta_B](X-a) + e_L[p\epsilon_H\theta_G + (1-p)(1-\epsilon_L)\theta_B](X-a) \\ & = Pr(s_h)e_H\theta_{q_h}[X-a] + Pr(s_\ell)e_L\theta_{q_\ell}[X-a]. \end{aligned}$$

The incentive conditions for the agent to take e_H is, given q_h,

$$q_h\theta_G e_H a + (1 - q_h)\theta_B e_H a - c_H \geq q_h\theta_G e_L a + (1 - q_h)\theta_B e_L a.$$

The incentive conditions for the agent to take e_L is, given q_ℓ,

$$q_\ell\theta_G e_L a + (1 - q_\ell)\theta_B e_L a \geq q_\ell\theta_G e_H a + (1 - q_\ell)\theta_B e_H a - c_H.$$

The combining these incentive conditions implies

$$\begin{aligned} & \frac{c_H}{\Delta e \theta_{q_h}} \leq a \leq \frac{c_H}{\Delta e \theta_{q_\ell}} \\ & \theta_{q_h} \geq \theta_{q_\ell}. \end{aligned}$$

The objective function of the principal is

$$e_H[p(1-\epsilon_H)\theta_G + (1-p)\epsilon_L\theta_B](X-a) + e_L[p\epsilon_H\theta_G + (1-p)(1-\epsilon_L)\theta_B](X-a).$$

Fix a signal structure. Since the objective function is decreasing with respect to a, the incentive constraint is binding. Therefore,

$$a = \frac{c_H}{\Delta e \theta_{q_h}}.$$

Given this optimal payment, we will derive the optimal signal structure. When we take the derivative of the objective function with respect to ϵ_H, the objective function of the seller is negative. Since $\frac{d\theta_{q_h}}{\epsilon_H} < 0$, the first derivative of the seller's objective function with respect to ϵ_H is

$$\begin{aligned}
&-\Delta e p\theta_G\left(X - \frac{c_H}{\Delta e \theta_{q_h}}\right)\\
&+ e_H[p(1-\epsilon_H)\theta_G + (1-p)\epsilon_L\theta_B]\left(X - \frac{c_H}{\Delta e}\right)\frac{c_H}{\Delta e}\frac{\frac{d\theta_{q_h}}{\epsilon_H}}{(\theta_{q_h})^2}\\
&+ e_L[p\epsilon_H\theta_G + (1-p)(1-\epsilon_L)\theta_B]\left(X - \frac{c_H}{\Delta e}\right)\frac{c_H}{\Delta e}\frac{\frac{d\theta_{q_h}}{\epsilon_H}}{(\theta_{q_h})^2} < 0.
\end{aligned}$$

Therefore, we set $\epsilon_H = 0$. Then the posterior after observing s_ℓ is $q_\ell = 0$.

Note that $q_h = \frac{p}{p+(1-p)\epsilon_L}$. When we denote $y \equiv \frac{1-p}{p}\epsilon_L$, $q_h = \frac{1}{1+y}$.
The objective function of the principal who will choose ϵ_L through y is

$$\begin{aligned}
&Pr(s_h)e_H\theta_{q_h}[X-a] + Pr(s_\ell)e_L\theta_{q_\ell}[X-a]\\
&= \left[e_H(p\theta_G + py\theta_B) + e_L(1-p)(1-\frac{py}{1-p})\theta_B\right]\left[X - \frac{c_H}{\Delta e}\frac{1+y}{\theta_G + y\theta_B}\right].
\end{aligned}$$

Since $\epsilon_L \in [0,1]$, $y \in [0, \frac{1-p}{p}]$. When $y=0$, it is full separation, that is, $\epsilon_H = \epsilon_L = 0$. When $y = \frac{1-p}{p}$, it coincides with high effort pooling contract.

Theorem 3. $y = 0$ *(Full separation) or* $y = \frac{1-p}{p}$ *(Full pooling) is optimal under separation contract.*

Proof. We want to show that the principal's objective function is convex in y. Mathematically, $\left[e_H(p\theta_G + py\theta_B) + e_L(1-p)(1-\frac{py}{1-p})\theta_B\right]\left[X - \frac{c_H}{\Delta e}\frac{1+y}{\theta_G+y\theta_B}\right]$ is convex for $y \in$

$[0, \frac{1-p}{p}]$.

$$V(y) \equiv \left[e_H(p\theta_G + py\theta_B) + e_L(1-p)(1 - \frac{py}{1-p})\theta_B\right]\left[X - \frac{c_H}{\Delta e}\frac{1+y}{\theta_G + y\theta_B}\right].$$

The second derivative of $V(y)$ is

$$V''(y) = \frac{2c_H\Delta\theta p\theta_B}{\Delta e(\theta_G + y\theta_B)^2}(e_H - \Delta e) + \{e_L(1-p-py)\theta_B\}\frac{c_H\Delta\theta}{\Delta e}\frac{2\theta_B}{(\theta_G + y\theta_B)^3} > 0.$$

The inequality is from $e_H - \Delta e = e_L > 0$ and $y \in [0, \frac{1-p}{p}]$. □

This theorem asserts that this function is convex in y and ϵ_L. If ϵ_L becomes higher, then θ_{q_h} decreases, which leads to a higher probability of taking high effort and higher payment. This is the trade-off. Even though a higher probability of taking a high effort increases the expected success probability, the higher expected payment to the agent decreases the profit.

Chapter 3 Hiding Successes in Communication

3.1 Introduction

We investigate a new sender-receiver game in which the sender possesses two types of private information: the payoff-relevant state and evidence. In our model, the sender communicates by revealing evidence and has the ability to manipulate evidence downwards but not upwards.

A novel feature of our model is that the set of messages the sender can send depends on the sender's evidence. The payoff structure follows the standard quadratic loss function of Crawford-Sobel, where the state represents the bliss point of the receiver and the state plus a known bias represents the bliss point of the sender.

Consider, for example, the scenario of investment recommendations, where the sender is a financial analyst aiming to sell a stock to an individual investor. The sender seeks to sell as many units of stocks as possible while being reputationally concerned. The state denotes the stock volume maximizing the receiver's payoff, while the parameter "bias" determines the extent to which the sender's optimal volume exceeds this. The sender possesses two types of information: the payoff-relevant state (the ideal volume for the receiver) and some evidence (privately observed outcomes of binary signals positively correlated with the true state).

In our model, the sender can choose how many positive signals to reveal to the receiver. Notably, the sender cannot directly convey the true payoff-relevant state and can only communicate through evidence, while the receiver cannot directly access the positive signals.

A key contrast from the literature is that the sender is privately informed about the set of messages they can send, unlike in Crawford and Sobel's model where senders can send any message regardless of their private information. In the literature on cheap talk, where senders can choose any message, the unique equilibrium under high bias is a pooling equilibrium, where the receiver's posterior is the same as the prior. There is no credible way to increase the posterior without message restriction. In our model, with message restriction and under high bias, there exists an equilibrium where the sender reveals every piece of evidence.

The first step of our analysis is to show that all equilibria have a cutoff structure. The cutoffs are the thresholds of the states where the sender chooses different messages. This is a natural and standard property because the sender in our game has only a finite number of evidence. Our Proposition 1 pins down these cutoffs as a function of the receiver's strategy. It says that each type chooses a message among all available messages that leads to the closest action to the state plus the bias.

We show with an example that these cutoffs need not be aligned. That is, there are equilibria in which the cutoffs are not the same given the realized evidence. We then ask if there exist equilibria in which the cutoffs are aligned, and if so, what types of equilibria lead to aligned cutoffs. An appealing feature of equilibrium with aligned cutoffs is that beyond the fact that the available evidence in our model restricts messages, the messages sent depend only on the relevant payoff state. That is, two types with the same state send the same message unless that message is unavailable to one of them.

One of our main results is that cutoffs are aligned if the equilibrium is monotone. A monotone equilibrium is one where higher messages lead to higher actions. Alignment of the cutoffs in a monotone equilibrium follows because the indifference conditions at the cutoffs between different actions are the same among the different realized evidence.

Qualitatively, there are three cases depending on whether the bias is small, medium, or large:

1. If the bias is small, only a pooling equilibrium is available.

2. If the bias is medium, a cutoff equilibrium exists where the low-state sender conceals evidence.

3. If the bias is high, the sender discloses the realized evidence.

Therefore, the role of bias differs from the Crawford-Sobel model, which is our second main result. This is because the utility of the sender is monotone under high bias. Since we assume that the posterior of the payoff-relevant state increases with the realized number of evidence, the sender with a high bias has the incentive to reveal the realized evidence to induce a higher action. If the bias is not big enough, there is an equilibrium where the sender hides some piece of evidence. Since the evidence implies a higher state, then we can interpret this as "hiding successes".

The rest of the paper is organized as follows: The next subsection discusses related literature. In section 2, we present the model. In section 3, we analyze cutoff equilibria and monotone equilibria. In section 4, we show an example where the number of available evidence is one. In section 5, we conclude this paper. The appendix explains other message space restrictions and the case where the number of available evidence is two.

3.1.1 Related literature

We rely on Crawford and Sobel (1982) in which an informed sender strategically transmits information to an uninformed receiver who decides without any restriction of message space. They show any equilibrium has a cutoff structure. Crawford and Sobel (1982) also shows that full information revelation is impossible unless the player's interests align perfectly. We analyze how an informed sender strategically transmits information to an uninformed receiver with a restriction of message space. Message space is limited to the evidence realization.

In our paper, the message space is constrained by the realized evidence. Thus, our paper is also related to the literature on evidence disclosure games. Seminal papers of the verifiable disclosure game by Milgrom (1981) and Grossman (1981) show that if the message space is showing the type or hiding the type, the sender uses the fully revealing strategy. In our paper, the sender cannot directly use the payoff-relevant state. Instead, the sender communicates only through the realized evidence. Bias plays an essential role in the existence of the full disclosure equilibrium. Our paper can also be interpreted as a combination of cheap talk and disclosure game by introducing a multidimensional type model whose type is consist of payoff-relevant state and evidence.

In the literature concerning constrained sender information, Argenziano, Severinov and Squintani (2016), Pei (2015) and Ivanov (2010) are related to this paper. In Argenziano et al. (2016), an uninformed sender strategically transmits information to an uninformed receiver after the sender knows the result of the costly experiment. Pei (2015) analyzes the model under which an uninformed sender costly chooses information partition and strategically sends a message to an uninformed receiver after the sender knows the realization. Ivanov (2010) analyzes the model under which an uninformed receiver limits the information of a sender before the sender knows the realization. After realization, the sender strategically transmits information to the receiver. Our paper restricts the message space of the sender while the past literature investigates the case where the sender's information is restricted.

3.2 Model

There are two agents, a sender (he) and a receiver (she). The sender privately observes a payoff-relevant state $s \in [0,1]$ and evidence $e \in \{0, \ldots, n\}$, where n indicates the number of possible evidence. We interpret e as the number of positive signals. We refer to (s, e) as the type of the sender. The sender's type is drawn from a distribution with cumulative density F, where $F(s', e')$ denotes the probability that the state s is at most s' and the evidence e is at most e'. We assume that the distribution has a probability density f such that $f(s', e') > 0$ on the support. Let P denote the cumulative density of the prior distribution of the state, so $P(s) = F(s, n)$.

After privately observing his type, the sender chooses a message $m \in \{0, \ldots, n\}$. The receiver then chooses an action $a \in [0,1]$. The utility of the receiver depends only on the action and the state,

$$U^R(a, s) := -(a - s)^2.$$

The utility of the sender is specified given a fixed bias parameter b as follows,

$$U^S(a, s, m, e) := \begin{cases} U^S(a, s) := -(a - s - b)^2 & \text{if } m \leq e, \\ -L & \text{if } m > e, \end{cases}$$

where L is a large constant representing the loss of the sender from sending a message that is infeasible given the sender's available evidence. We assume that L is large enough (more than the largest value $|U^S(a, s)|$ might take which is $(1 + b)^2$) so that the sender never chooses a message that is larger than his evidence in any equilibrium. This captures the idea that the agent may under-report his evidence but cannot over-report his evidence. Given this constraint, the sender's payoff also only depends on the receiver's action and the state and is captured by $U^S(a, s)$. The fixed parameter b represents the bias of the sender. A positive bias means that the sender prefers higher actions than the receiver, and a negative bias has the opposite effect. The bias therefore represents the degree of misalignment between the agents' interests.

The timing of the game is as follows.

1. Nature draws the sender's type (s, e) from the distribution F.

2. The sender privately observes his type (s, e).

3. The sender chooses message m.

4. The receiver observes the message m and chooses an action a.

We adopt the standard notion of pure-strategy perfect Bayesian equilibrium as our equilibrium concept. The sender's strategy $\hat{m} : [0, 1] \times \{0, \ldots, n\} \to \{0, \ldots, n\}$ is a mapping from the sender's type to a message. The receiver's strategy $\hat{a} : \{0, \ldots, n\} \to [0, 1]$ is a mapping from all possible messages to an action.

The strategy profile $(\hat{a}, \hat{m})$ is a pure strategy perfect Bayesian equilibrium if

1. The receiver maximizes her payoff given the strategy of the sender. Formally, for each message m that is sent with positive probability given the strategy profile $\hat{m}$,

$$\hat{a}(m) = \underset{a}{\operatorname{argmax}} \, \mathbb{E}[U^R(a, s) \mid \hat{m}(s, e) = m].$$

2. The sender has no profitable deviation given the strategy of the receiver, which is reflected in two constraints. First, the message must be at most the evidence, $\hat{m}(s, e) \leq e$. Second, for any type of the sender (s, e), and any message $m' \leq e$, we have

$$U^S(\hat{a}(\hat{m}(s, e)), s) \geq U^S(\hat{a}(m'), \theta).$$

Let us now describe a special case of our model that we study throughout the paper. We refer to this special case as the *independent successes* case. In this special case,

$$f(s, e) = \frac{n!}{e!(n-e)!} s^e (1-s)^{n-e}$$

Our interpretation is that the agent has access to n experiments. Each experiment has binary outcomes, “success" and “failure". The outcome of each experiment is drawn independently from all other experiments conditioned on s, where the probability of each success is s. The marginal distribution of state s is uniform. The marginal distribution of e has its density of $\frac{1}{n+1}$. The evidence e denotes the number of successes. The conditional probability of e successes given s is

$$f(e|s) = \frac{(n+1)!}{e!(n-e)!} s^e (1-s)^{n-e},$$

which gives the joint distribution over (e, s) specified in Equation 3.2.

3.3 Cutoff equilibria

In this section, we characterize equilibria and show that all equilibria must have a cutoff structure. The cutoffs are the points on state space $[0, 1]$ where the actions or the messages change.

We start by simplifying the equilibrium constraints given the quadratic utility functions. Fix the sender's strategy $\hat{m}$. Given the quadratic form of the receiver's payoff, upon observing any message m, she chooses an action that matches her posterior mean of the state. Therefore, the receiver's optimality condition becomes

$$\hat{a}(m) = \mathbb{E}[s \mid \hat{m}(s, e) = m].$$

for all messages m that are sent with positive probability. Given the quadratic form of the receiver's payoff, the sender will choose a message that induces an action that is as close as possible to $s + b$. Therefore, the sender prefers message $\hat{m}(s, e)$ to m' if

$$|s + b - \hat{a}(\hat{m}(s, e))| \leq |s + b - \hat{a}(m')|, \tag{3.1}$$

which is equivalent to

$$-(\hat{a}(\hat{m}(s, e)) - \hat{a}(m'))(\hat{a}(\hat{m}(s, e)) + \hat{a}(m') - 2s - 2b) \geq 0.$$

To summarize, an equilibrium can be described by a pair $(\hat{m}, \hat{a})$ where $\hat{a}(m)$ is the posterior mean of the state given the message m for all messages m that have positive probability under $\hat{m}$, and $\hat{m}$ satisfies the incentive constraint (3.1).

We next show that all equilibria have a cutoff structure. Intuitively, the cutoff strategy is a strategy where the number of partitions on the set of $[0, 1]$ is at most the realized number of evidence e, and the types belonging to each partition send the same message. If this strategy forms perfect Bayesian equilibrium, then we call it cutoff equilibrium.

The definition concerns the set of all actions that are taken in equilibrium, which we define next. The set of action induced by strategy $\hat{m}$ is $A_{\hat{m}} \equiv \{\hat{a}(0), \cdots, \hat{a}(n)\}$. To relabel, we define $\tilde{a}_i \in A_{\hat{m}}$ subject to $\tilde{a}_i \leq \tilde{a}_{i+1}$, so $\tilde{a}_i$ is the $i+1$'th lowest element of $A_{\hat{m}}$. Let $\hat{A}_e = \{\hat{a}(m) \in A_{\hat{m}} \mid m \leq e\}$ be the set of actions that are available given evidence e. Again, we sort the elements of $\hat{A}_e$ and write the set as $\hat{A}_e = \{\tilde{a}^e_1, \tilde{a}^e_2, \ldots, \tilde{a}^e_\kappa\}$, where $\tilde{a}^e_i < \tilde{a}^e_{i+1}$ and $\kappa \leq e$ is the cardinality of $\hat{A}_e$.

Each type (s, e) in equilibrium chooses an available action from $\hat{A}_e$ that maximizes payoff. Given constraint (3.1), the chosen action is the one that is the closest to $s + b$. As a result, for each evidence e we can partition $[0, 1]$ into a set of intervals, each associated with an action in $\hat{A}_e$, so that states s that are within an interval all choose the same action associated with that interval. This is illustrated in Figure 3.1 and is formalized in our first result below.

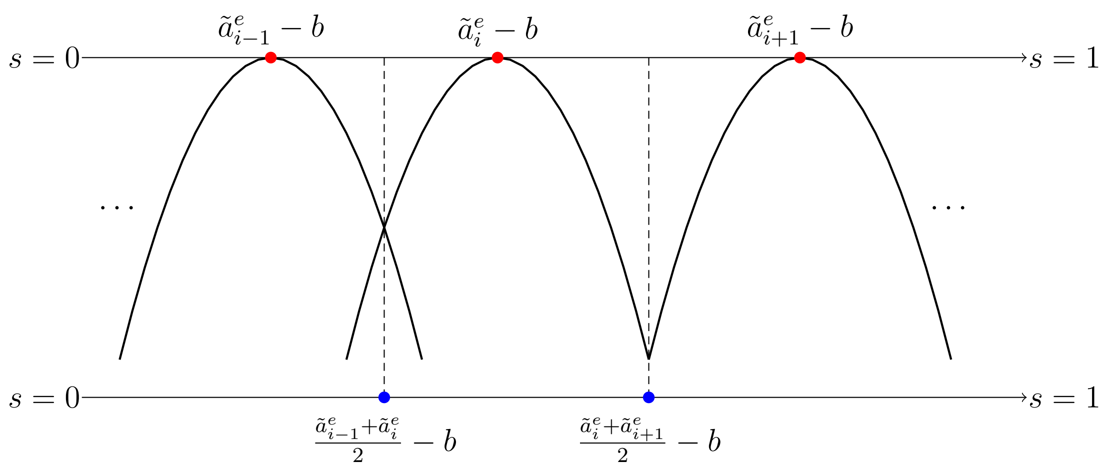

Figure 3.1. $\tilde{a}^e_i$ will be taken if $s \in [\max\{\min\{\frac{\tilde{a}^e_{i-1}+\tilde{a}^e_i}{2} - b, 1\}, 0\}, \min\{\max\{\frac{\tilde{a}^e_i+\tilde{a}^e_{i+1}}{2} - b, 0\}, 1\}]$.

Proposition 9. *For any equilibrium, the equilibrium action is as follows. Consider any realized evidence $e \geq 1$[1]. Suppose $\tilde{a}^e_1, \tilde{a}^e_1, \ldots, \tilde{a}^e_\kappa$ are on-path equilibrium actions.*

- $\tilde{a}^e_0$ *is chosen if* $s \in [0, \min\{\frac{\tilde{a}^e_0+\tilde{a}^e_1}{2} - b, 1\}]$ *and* $\frac{\tilde{a}^e_0+\tilde{a}^e_1}{2} - b \geq 0$.
- $\tilde{a}^e_i$ *is chosen if* $s \in [\max\{\frac{\tilde{a}^e_{i-1}+\tilde{a}^e_i}{2} - b, 0\}, \min\{\frac{\tilde{a}^e_i+\tilde{a}^e_{i+1}}{2} - b, 1\}]$, $\frac{\tilde{a}^e_{i-1}+\tilde{a}^e_i}{2} - b \leq 1$, $\frac{\tilde{a}^e_i+\tilde{a}^e_{i+1}}{2} - b \geq 0$ *and* $0 < i < \kappa$ *for* $i \in \{0, \cdots, e\}$.
- $\tilde{a}^e_\kappa$ *is chosen if* $s \in [\max\{\frac{\tilde{a}^e_{\kappa-1}+\tilde{a}_\kappa}{2} - b, 0\}, 1]$ *and* $\frac{\tilde{a}^e_{i-1}+\tilde{a}^e_i}{2} - b \leq 1$.

Proof. Consider $\tilde{a}^e_i$ where $0 < i < \kappa$. The sender prefers the action $\tilde{a}^e_i$ to $a' \neq \tilde{a}^e_i \in \hat{A}_e$ if

$$-(\tilde{a}^e_i - a')(\tilde{a}^e_i + a' - 2s - 2b) \geq 0.$$

[1]When $e = 0$, only $\tilde{a}_0$ is available.

First suppose $\tilde{a}_i^e - a' > 0$. For the incentive constraint to hold, the second term

$$\tilde{a}_i^e + a' - 2s - 2b$$

must be negative. We can write this as

$$s \geq \frac{\tilde{a}_i^e + a' - 2b}{2}.$$

So type (s,e) prefers message $\tilde{a}_i^e$ to a' if and only if s is at least $\frac{\hat{a}(\hat{m}(s,e))+\hat{a}(m')-2b}{2}$. This means that type (s,e) prefers the action $\tilde{a}_i^e$ to *every* $a' \in \hat{A}_e$ where $\tilde{a}_i^e - a' \geq 0$ if and only if

$$s \geq \max_{a':\tilde{a}_i^e - a' \geq 0} \frac{\tilde{a}_i^e + a' - 2b}{2} = \frac{\tilde{a}_i^e + \tilde{a}_{i-1}^e - 2b}{2}.$$

If $\frac{\tilde{a}_i^e+\tilde{a}_{i-1}^e-2b}{2} > 1$, then this is not equilibrium.

Second suppose $\tilde{a}_i^e - a' < 0$. For the incentive constraint to hold, the second term

$$\tilde{a}_i^e + a' - 2s - 2b$$

must be positive. We can write this as

$$s \leq \frac{\tilde{a}_i^e + a' - 2b}{2}.$$

So type (s,e) prefers message $\tilde{a}_i^e$ to a' if and only if s is at most $\frac{\hat{a}(\hat{m}(s,e))+\hat{a}(m')-2b}{2}$. This means that type (s,e) prefers the action $\tilde{a}_i^e$ to *every* $a' \in \hat{A}_e$ where $\tilde{a}_i^e - a' \leq 0$ if and only if

$$s \leq \min_{a':\tilde{a}_i^e - a' \leq 0} \frac{\tilde{a}_i^e + a' - 2b}{2} = \frac{\tilde{a}_i^e + \tilde{a}_{i+1}^e - 2b}{2}.$$

If $\frac{\tilde{a}_i^e+\tilde{a}_{i+1}^e-2b}{2} < 0$, then this is not equilibrium.

Then $\tilde{a}_i^e$ is chosen if $s \in [\max\{\frac{\tilde{a}_{i-1}^e+\tilde{a}_i^e}{2} - b, 0\}, \min\{\frac{\tilde{a}_i^e+\tilde{a}_{i+1}^e}{2} - b, 1\}]$, $\frac{\tilde{a}_{i-1}^e+\tilde{a}_i^e}{2} - b \leq 1$, $\frac{\tilde{a}_i^e+\tilde{a}_{i+1}^e}{2} - b \geq 0$ and $0 < i < \kappa$ for $i \in \{0, \cdots, e\}$.

Next, consider $\tilde{a}_0^e$. The sender prefers the action $\tilde{a}_i^e$ to $a' \neq \tilde{a}_0^e \in \hat{A}_e$ if

$$-(\tilde{a}_0^e - a')(\tilde{a}_i^e + a' - 2s - 2b) \geq 0.$$

Since $\tilde{a}_0^e - a' < 0$. For the incentive constraint to hold, the second term

$$\tilde{a}_0^e + a' - 2s - 2b$$

must be positive. We can write this as

$$s \leq \frac{\tilde{a}_0^e + a' - 2b}{2}.$$

So type (s, e) prefers message $\tilde{a}_0^e$ to a' if and only if s is at most $\frac{\hat{a}(\hat{m}(s,e))+\hat{a}(m')-2b}{2}$. This means that type (s, e) prefers the action $\tilde{a}_0^e$ to *every* $a' \in \hat{A}_e$ where $\tilde{a}_0^e - a' \leq 0$ if and only if

$$s \leq \min_{a':\tilde{a}_0^e - a' \leq 0} \frac{\tilde{a}_0^e + a' - 2b}{2} = \frac{\tilde{a}_0^e + \tilde{a}_1^e - 2b}{2}.$$

If $\frac{\tilde{a}_0^e+\tilde{a}_1^e-2b}{2} < 0$, then this is not equilibrium. Then $\tilde{a}_0^e$ is chosen if $s \in [0, \min\{\frac{\tilde{a}_i^e+\tilde{a}_{i+1}^e}{2} - b, 1\}]$ and $\frac{\tilde{a}_i^e+\tilde{a}_{i+1}^e}{2} - b \geq 0$.

Finally,consider $\tilde{a}_\kappa^e$. The sender prefers the action $\tilde{a}_\kappa^e$ to $a' \neq \tilde{a}_\kappa^e \in \hat{A}_e$ if

$$-(\tilde{a}_\kappa^e - a')(\tilde{a}_\kappa^e + a' - 2s - 2b) \geq 0.$$

Since $\tilde{a}_\kappa^e - a' > 0$. For the incentive constraint to hold, the second term

$$\tilde{a}_\kappa^e + a' - 2s - 2b$$

must be negative. We can write this as

$$s \geq \frac{\tilde{a}_\kappa^e + a' - 2b}{2}.$$

So type (s, e) prefers message $\tilde{a}_\kappa^e$ to a' if and only if s is at least $\frac{\hat{a}(\hat{m}(s,e))+\hat{a}(m')-2b}{2}$. This means that type (s, e) prefers the action $\tilde{a}_\kappa^e$ to *every* $a' \in \hat{A}_e$ where $\tilde{a}_\kappa^e - a' \geq 0$ if and only if

$$s \geq \max_{a':\tilde{a}_\kappa^e - a' \geq 0} \frac{\tilde{a}_\kappa^e + a' - 2b}{2} = \frac{\tilde{a}_\kappa^e + \tilde{a}_{\kappa-1}^e - 2b}{2}.$$

If $\frac{\tilde{a}_\kappa^e+\tilde{a}_{\kappa-1}^e-2b}{2} > 1$, then this is not equilibrium. Then $\tilde{a}_\kappa^e$ is chosen if $s \in [\max\{\frac{\tilde{a}_{\kappa-1}^e+\tilde{a}_\kappa}{2} - b, 0\}, 1]$ and $\frac{\tilde{a}_{i-1}^e+\tilde{a}_i^e}{2} - b \leq 1$. $\square$

The above proposition claims that $\frac{\tilde{a}^e_{i-1}+\tilde{a}^e_i}{2} - b$ are the cutoffs of state space for the different actions. We can call it a cutoff structure. What does this structure look like? Figure 3.1 shows that the point $\frac{\tilde{a}^e_{i-1}+\tilde{a}^e_i}{2} - b$ are the middle point between two equilibrium actions net of bias. Since the message space is finite, the number of equilibrium actions is finite. Then the cutoffs for the different actions are finite. How about the cutoffs of state space for different messages? Given any equilibria, there is always an outcome-equivalent equilibrium whose state space cutoffs for messages are finite. We call such an equilibrium a simple equilibrium.

Definition 1. *An equilibrium $(\hat{m}, \hat{a})$ is simple equilibrium if $\hat{a}(m) = \hat{a}(m')$ implies $m = m'$.*

A simple equilibrium has finite cutoffs for different messages. Note that in any equilibrium, $\hat{a}(\hat{m}(s,e)) \le \hat{a}(\hat{m}(s',e))$ for all $s < s'$. We prove this by contradiction. Suppose $\hat{a}(\hat{m}(s,e)) > \hat{a}(\hat{m}(s',e))$. We define the middle point $M \equiv \frac{\hat{a}(\hat{m}(s,e))+\hat{a}(\hat{m}(s',e))}{2}$. There are 2 cases. If $s \le M$, the type s deviates to report $\hat{m}(s',e)$ since the action $\hat{a}(\hat{m}(s',e)$ is closer for the type s. If $s > M$, the type s' deviates to report $\hat{m}(s,e)$ since the action $\hat{a}(\hat{m}(s,e))$ is closer for the type s'. Contradiction.

The above discussion implies the following.

Corollary 2. *Suppose that $\hat{a}(\hat{m}(s_1,e)) = \hat{a}(\hat{m}(s_2,e))$ and $s_1 < s_2$. Then $\hat{a}(\hat{m}(s,e)) = \hat{a}(\hat{m}(s_1,e))$ for $s_1 \le s \le s_2$.*

This corollary implies that the state s who sends the same messages are connected. Since the number of different equilibrium actions is finite, the simple equilibrium cutoffs for different messages are finite.

Definition 2. *Two equilibria $\hat{m}, \hat{a}$ and $\hat{m}'$, $\hat{a}'$ are outcome-equivalent if $\hat{a}'(\hat{m}'(s,e)) = \hat{a}(\hat{m}(s,e))$ for any s, e.*

We will focus on simple equilibria without loss of generality from the following proposition.

Proposition 10. *Every equilibrium is outcome-equivalent to simple equilibrium. Babbling equilibrium always exists and is a simple equilibrium.*

Proof. We prove the outcome equivalent statement by "downward relabelling". Consider any equilibrium with $\hat{m}, \hat{a}$. From this $\hat{m}, \hat{a}$, we can construct new outcome equivalent equilibria $\hat{m}', \hat{a}'$.

Suppose $\hat{m}(s,e)$ is an equilibrium such that for some (s_1,e_1) and (s_2,e_2) such that $\hat{a}(\hat{m}(s_1,e_1)) = \hat{a}(\hat{m}(s_2,e_2))$, $\hat{m}(s_1,e_1) \neq \hat{m}(s_2,e_2)$, and $e_1 \leq e_2$. Let's define the message $l_1 \equiv \hat{m}(s_1,e_1)$, the message $l_2 \equiv \hat{m}(s_2,e_2)$, and the type of set who sends the same message $S_m = \{(s,e)|\hat{m}(s,e) = m\}$.

We want to construct another equilibrium $\hat{m}', \hat{a}'$ by changing messages. For any $(s,e) \in S_{l_2}$, $\hat{m}'(s,e) = l_1$. This means the higher message is relabelled to the lower one if these two messages induce the same action. For any $(s,e) \notin S_{l_2}$, $\hat{m}'(s,e) = \hat{m}(s,e)$ for keeping the other messages the same as the original equilibrium. The above procedure is that the higher message which induces the same action is relabelled to the lower one. Let's call this procedure "downward relabelling".

We will check, for any s, e, $\hat{a}'(\hat{m}'(\cdot,\cdot)) = \hat{a}(\hat{m}(\cdot,\cdot))$ since we want to prove $\hat{m}', \hat{a}'$ is an equilibrium.

To claim this is an equilibrium, we have to check the receiver's expectation is equal to the action.

For any $(s,e) \in S_{l_2}$,

$$\begin{aligned}
&\hat{a}'(\hat{m}'(s,e)) = \mathbb{E}[s|(s,e) \in S_{l_2} \cup S_{l_1}] \\
&= Pr((s,e) \in S_{l_2}|(s,e) \in S_{l_2} \cup S_{l_1})\hat{a}(l_2) + Pr((s,e) \in S_{l_1}|(s,e) \in S_{l_2} \cup S_{l_1})\hat{a}(l_1) \\
&= \hat{a}(\hat{m}(s,e)).
\end{aligned}$$

We use $\hat{a}(l_1) = \hat{a}(l_2)$ in the last equality. For any $(s,e) \notin S_{l_2}$,$\hat{a}'(\hat{m}(s,e)) = \hat{a}(\hat{m}(s,e))$ since the set of types who send each message is invariant. Then this is a best response for the receiver.

Since the induced action is invariant in terms of any s, e, this message is a best response for the sender. Repeating the above downward relabelling gets the result.

Babbling equilibrium where the sender always sends message $m = 0$ is an equilibrium since this is a mutually best response that the receiver takes the action which is equal to the ex-ante expectation of state s with the belief under which the sender always sends $m = 0$. Babbling equilibrium is simple since the sender always sends 0. □

The above proposition shows that we can focus on simple equilibria without loss of generality. Even though we have shown that for each evidence e, the sender's equilibrium action is increasing in the state s, the sender's *messages* in equilibrium need not be increasing in s. The following example elaborates.

Example 1. *Consider the* independent successes *case described in Section 3.2 with* $n = 1$ *and a uniform distribution over* s*,* $P(s) = s$*. With* $n = 1$*, the prior is* $f(s,e) = s^e(1-s)^{1-e}$,

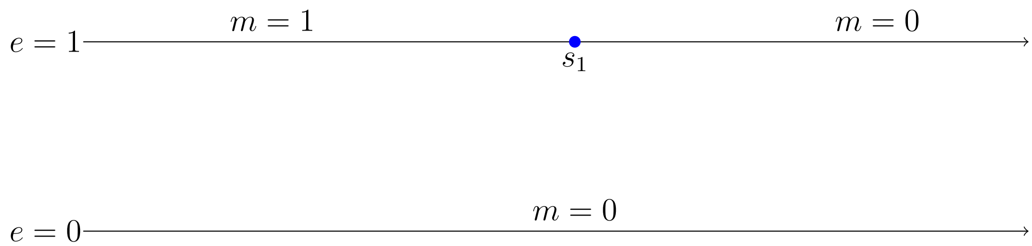

Figure 3.2. Higher message implies lower action (Example 1).

that is $f(s, e) = s$ if $e = 1$ and $f(s, e) = 1 - s$ if $e = 0$. We want to characterize the cutoff strategy as follows. When evidence $e = 1$ and state $s \geq \hat{s}$, then message m is equal to 0. When evidence $e = 1$ and $s \leq \hat{s}$, then $m = 1$. Let the action (the receiver's expectation of the state s) $a_m(\hat{s})$ denote the expected value of state s for the receiver given this strategy. The smaller message induces the higher action in this example (Figure 3.2). Since the incentive-compatible condition (1) *is equivalent to*

$$-(\hat{a}(\hat{m}(s, e)) - \hat{a}(m'))(\hat{a}(\hat{m}(s, e)) + \hat{a}(m') - 2s - 2b) \geq 0,$$

we have to verify the ranking of action $a_0(\hat{s}) \geq a_1(\hat{s})$ in this equilibrium. This pattern could be an equilibrium under some b.

$$a_1(\hat{s}) = E[s|m = 1] = \int_0^{\hat{s}} s\frac{2s}{\hat{s}^2}ds = \frac{2}{3}\hat{s}.$$
$$a_0(\hat{s}) = \mathbb{E}[s|m = 0] = Pr(e = 1|m = 0)\int_{\hat{s}}^1 s\frac{2s}{1 - \hat{s}^2}ds + Pr(e = 0|m = 0)\int_0^1 2s(1 - s)ds$$
$$= \frac{1 - \hat{s}^2}{2 - \hat{s}^2}\int_{\hat{s}}^1 s\frac{2s}{1 - \hat{s}^2}ds + \frac{1}{2 - \hat{s}^2}\int_0^1 2s(1 - s)ds$$
$$= \frac{1}{2 - \hat{s}^2}\frac{2}{3}(1 - \hat{s}^3) + \frac{1}{2 - \hat{s}^2}\frac{1}{3}.$$

To be an equilibrium, we have to check the ranking of the actions $a_0 \geq a_1$ and the indifference condition $a_0 - \hat{s} - b = \hat{s} + b - a_1$. For example, when $b = -\frac{1}{14}$, then $\hat{s} = \frac{1}{2}, a_1(\frac{1}{2}) = \frac{1}{3}, a_0(\frac{1}{2}) = \frac{11}{21}$ is an equilibrium.
When $b = \frac{4}{51}$, then $\hat{s} = \frac{1}{3}, a_1(\frac{1}{3}) = \frac{2}{9}, a_0(\frac{1}{3}) = \frac{25}{51}$ is an equilibrium.
$\hat{s} = \frac{3}{4}$ cannot be this class of equilibrium since $a_0(\frac{3}{4}) < a_1(\frac{3}{4})$.

Since the message is just a label, we also wonder if there is an equilibrium under which the action is not monotonic with respect to message. The following example shows that the cut-offs of the state are not necessarily aligned.

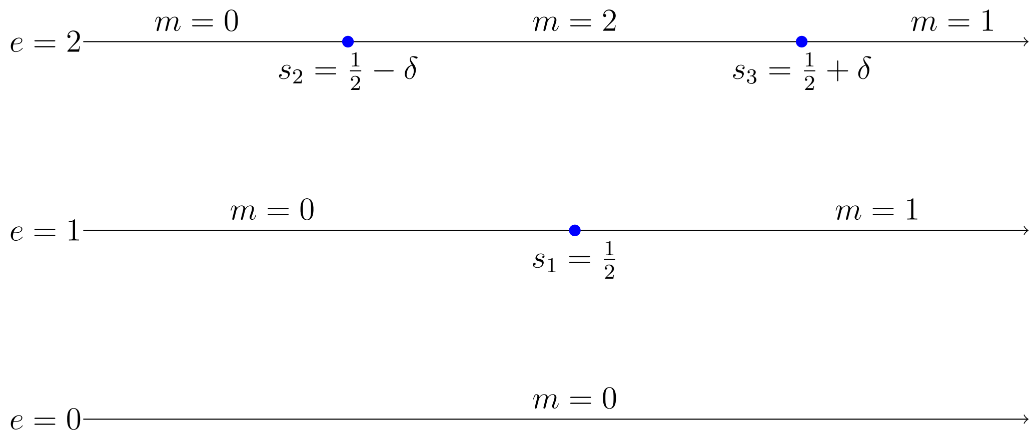

Figure 3.3. A non-monotonic example (Example 2)

Example 2. *Suppose* $b = 0$, $n = 2$, $f(s, e) = 1/2$ *for* $e = 1, 2$, *and* $f(s.e) = 0$ *for* $e = 0$. *Then* $f(s|e) = 1$ *for* $e = 1, 2$. *We want to characterize the following strategy. Suppose* $s_2 \equiv \frac{1}{2} - \delta < s_1 \equiv \frac{1}{2} < s_3 \equiv \frac{1}{2} + \delta$ *When* $e = 2$ *and* $s \leq s_2$, *then* $m = 0$. *When* $e = 2$ *and* $s_2 \leq s \leq s_3$, *then* $m = 2$. *When* $e = 2$ *and* $s \geq s_3$, *then* $m = 1$. *When* $e = 1$ *and* $s \leq s_1$, *then* $m = 0$. *When* $e = 1$ *and* $s \geq s_1$, *then* $m = 0$. *By definition,* $\hat{a}(0) < \hat{a}(2) < \hat{a}(1)$*(Figure 3.3). There exists* δ *such that the above strategy forms an equilibrium. The system of equations of indifferent conditions is as follows.*

$$0 = \hat{a}_(1) + \hat{a}(0) - 2s_1.$$
$$0 = \hat{a}(2) + \hat{a}(0) - 2s_2.$$
$$0 = \hat{a}(1) + \hat{a}(2) - 2s_3.$$

By symmetry, $\hat{a}(1) = 1 - \hat{a}(0)$. *Then the first equation above* $0 = \hat{a}(1) + \hat{a}(0) - 2s_1$*holds. Also,*$0 = \hat{a}(2) + \hat{a}(0) - 2s_2$ *is identical to* $0 = \hat{a}(1) + \hat{a}(2) - 2s_3$. *Let's define* $\hat{a}(0) = \hat{a}_0(\delta)$ *since it depends on* δ.

$$\hat{a}_1(\delta) = \frac{1}{2}(\frac{1}{2} - \delta)\frac{\frac{1}{2} - \delta}{\frac{1}{2} - \delta + \frac{1}{2}} + \frac{1}{4}\frac{\frac{1}{2}}{\frac{1}{2} - \delta + \frac{1}{2}}.$$

$0 = \hat{a}(2) + \hat{a}(0) - 2s_2$ *is the indifference condition we want to check. Then when* $\delta = 0$

$$0 > \frac{1}{2} + \frac{1}{4} - 1 = \hat{a}(2) + \hat{a}_0(\delta) - 2(\frac{1}{2} - \delta).$$

Then when $\delta = \frac{1}{2}$

$$0 < \frac{1}{2} + \frac{1}{4} - 0 = \hat{a}(2) + \hat{a}_0(\delta) - 2(\frac{1}{2} - \delta).$$

Using the intermediate theorem, there exists $\delta \simeq 0.13962$ such that $b = \hat{a}(2) + \hat{a}_0(\delta) - 2(\frac{1}{2} - \delta)$. In this example, a higher message does not lead to a higher action.

The above example shows that the cut-offs of the state are not necessarily aligned. We call an equilibrium monotone if a higher message induces a higher action.

Definition 3. *An equilibrium $(\hat{m}, \hat{a})$ is monotone if $\hat{a}(m)$ is increasing in m and strictly increasing until $\hat{a}(m) = 1$.*

We want to claim the cut-offs are aligned under monotone equilibria. The following proposition asserts that if the neighbor message m, m' are on-path under different pieces of evidence e, e', then the cut-offs are the same against different evidence e, e'.

Proposition 11. *Suppose, under monotone equilibrium, the different actions $\tilde{a}_i$ and $\tilde{a}_{i-1}$ are on path actions under different evidences $e.e'$. Then the cutoffs of state s are the same among e, e'. In addition, the cutoffs decrease in bias b.*

Proof. Define the state $s(i, e)$ as the cutoff where the type $(s(i, e), e)$ sender is indifferent choosing between the outcome $\tilde{a}_i$ and $\tilde{a}_{i-1}$. Therefore, by indifference condition,

$$\tilde{a}_i - s(i, e) - b = s(i, e) + b - \tilde{a}_{i-1}.$$
$$\tilde{a}_i - s(i, e') - b = s(i, e') + b - \tilde{a}_{i-1}.$$

Then the cutoff can be written as $s(i, e) = s(i, e') = \frac{\tilde{a}_i + \tilde{a}_{i-1} - 2b}{2}$.

The cutoffs are the functions of actions and bias. The actions are the functions of cutoffs. $s(i, e)$ is a solution of the system of equations defined as indifferent conditions. Then $s(i, e)$ is a function of b. We denote it as $s_i(b)$.

Next, we claim that the cutoff $\frac{ds_i(b)}{db} > 0$. Since it is indifferent for the type $s(i, e)$ sender to report $m = i$ and $m = i - 1$, then

$$-(\tilde{a}_i - s_i(b) - b)^2 = -(\tilde{a}_{i-1} - a_i(b) - b)^2.$$

Since $\tilde{a}_i > \tilde{a}_{i-1}$, the indifference condition implies

$$\tilde{a}_i - s_i(b) - b = s_i(b) + b - \tilde{a}_{i-1}.$$

There exists $\epsilon > 0$ such that by another bias $b' = b + \epsilon$,

$$-(\tilde{a}_i - s_i(b) - b')^2 > -(\tilde{a}_{i-1} - s_i(b) - b')^2.$$

So, the type $s_i(b)$ prefers to send $m = i$ under another bias b' when i is feasible. Note that

$$\tilde{a}_i - s_i(b) - b' < s_i(b) + b' - \tilde{a}_{i-1}.$$

We prove that the cutoff $s_i(b)$ is decreasing in b by contradiction. Suppose $s_i(b') > s_i(b)$. Since the sender takes the cutoff strategy in which the type smaller than the cutoff prefers to report $m = i - 1$, the type $s_i(b)$ should prefer to report $m = i - 1$ under another bias b'. Contradiction. □

The intuition of this proposition is that whether the sender underreports or not depends on the payoff-relevant state s. The evidence e itself is irrelevant to the payoff. Therefore, the cut-offs are the same among the different states s. In addition, higher bias implies that the lower state sender prefers higher action. Then the cut-offs decrease to include the lower-state sender.

Since the monotonicity of action also holds under off-equilibrium message, then the on-path message should be consecutive from 0.

Proposition 12. *There is no monotone equilibrium such that $m > m' > m''$ and m, m'' are on-path messages and m' is off-path messages.*

Proof. Without loss of generality, we assume $\hat{a}(m'')$ is the largest on-path action which is less than $\hat{a}(m)$. Define $s_{m',m}$ as the threshold between sending m, m' when $m, m' \leq e$ and $m > m'$. Then the cutoff $s_{m'',m}$ meets

$$\hat{a}(m) - s_{m'',m} - b = s_{m'',m} + b - \hat{a}(m'').$$

Then the type $(e = m', s_{m'',m})$ sender profitably deviates to report m'. Contradiction. □

3.3.1 Full disclosure

Since there are no closed-form solutions for the cutoffs which we will discuss later in uniform prior examples, we focus on the full disclosure equilibrium where the sender reveals the realized evidence e.

Definition 4. *An equilibrium $(\hat{m}, \hat{a})$ is full disclosure equilibrium if $\hat{m} = e$.*

Under full disclosure equilibrium, the equilibrium action is $\mathbb{E}[s|m = e]$, which is easy to calculate under uniform prior examples. The full disclosure equilibrium does not use the information from the state s. However, the evidence e has some information about the state through the conditional expectation. Then the full disclosure equilibrium is better than the pooling equilibrium. The only pooling equilibrium is available in the Crawford-Sobel cheap talk game under high bias. In our model, as we discussed later, the full disclosure equilibrium is available under high bias.

In this subsection, we claim the full disclosure equilibrium is available under high bias.

Proposition 13. *Full disclosure is an equilibrium if and only if $b \geq \hat{b}$, where*

$$\hat{b} = \frac{E[s|e = n] + E[s|e = n-1]}{2}.$$

As a result, $\hat{b}$ can be bounded as follows, $E[s|e = n-1] \leq \hat{b} \leq E[s|e = n]$.

Proof. The sufficient condition for full disclosure is for all n'

$$E[s|e = n'] - s - b \leq s + b - E[s|e = n' - 1].$$

Therefore, the full disclosure equilibrium exists if and only if $b \geq \hat{b}$ (the tightest condition is $n' = n, s = 0$). □

This result says that if the bias b is large enough, in particular if b is at least $E[s|e = n]$, then full disclosure is an equilibrium. If b is small enough, in particular, if b is at most $E[s|e = n-1]$, then full disclosure is *not* an equilibrium. In the Crawford-Sobel model, the only pooling equilibrium exists under high bias. In our model, the full disclosure equilibrium exists under high bias. Note that the full disclosure equilibrium is better than the pooling in terms of welfare. Then under high bias, the welfare in our model is better than in the Crawford-Sobel model.

3.4 One piece of evidence

In this section, we will focus on $n = 1$ case of monotone equilibrium. We are interested in the existence of non-babbling monotone equilibrium. To rank the actions for monotone equilibrium, we assume an additional condition for the distributions of state s given evidence e.

Definition 5. *The definition of hazard rate dominance is as follows.* $X \leq_{\text{hr}} Y$ *when* X *and* Y *have absolutely continuous distributions* F *and* G *with densities* f *and* g*, respectively, is the following:* $\frac{f(x)}{\bar{F}(y)} \geq \frac{g(x)}{\bar{G}(y)}$ *for all* $x \leq y$*. We call* Y *hazard rate dominates* X*.*

Definition 6. *The definition of first-order stochastic dominance is as follows.* $P\{X > x\} \leq P\{Y > x\}$ *for all* $x \in (-\infty, \infty)$*. We call* X *first order dominates* Y*.*

We assume the conditional random variable state s given e hazard rate dominates the conditional random variable state s given evidence e' for $e' \leq e$. In addition, those have different distributions. Note that X hazard rate dominates Y if and only if $[X \mid X > t]$ first order dominates $[Y \mid Y > t]$ for all $t \in \mathbb{R}$. Therefore, the higher evidence e implies the higher expected value of state s if the true state s is not known. This hazard rate dominance will be used to rank the equilibrium actions. The intuition of hazard rate dominance is that the higher evidence e implies the higher state s. Therefore, if the evidence e is high and the state s is low, then the sender has an incentive to hide the realized evidence.

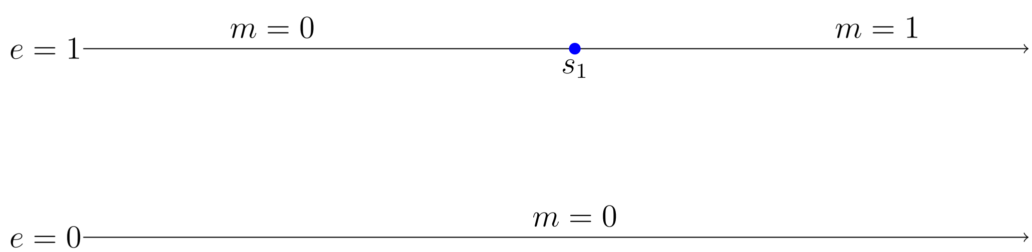

Figure 3.4. n=1

We want to characterize the cutoff strategy as follows. See figure 3.4. Suppose s_1 is a cutoff where the action is changed. When $e = 1$ and $s \leq s_1$, then $m = 0$. When $e = 1$ and $s \geq s_1$, then $m = 1$.

Let $a_m(s_1)$ denote the expected value of state s for the receiver after the message m is observed given this cutoff s_1. When $s_1 = 1$, $a_1(s_1)$ cannot be defined because the denominator is 0. So, we assume $s_1(1) = 1$ since $\lim_{s_1 \to 1} a_1(s_1) = 1$ by L'Hopital's rule. Then we can claim continuity of a. This means if $m = 1$ is off-equilibrium path, $\hat{a}(1) = 1$.

Lemma 2. $a_1(s_1) > a_0(s_1)$ *for any* $s_1 \in [0, 1]$*.*

Proof. $a_0(s_1)$ is a weighted sum of $E[s|e = 1, s \leq s_1]$, $E[s|e = 0, s \leq s_1]$ and $E[s|e = 0, s \geq s_1]$.

$a_1(s_1) = E[s|e = 1, s \geq s_1]$.

Since $E[s|e = 1, s \geq s_1] > E[s|e = 1, s \leq s_1]$ by definition , $E[s|e = 1, s \geq s_1] > E[s|e = 0, s \leq s_1]$ by definition, and $E[s|e = 1, s \geq s_1] > E[s|e = 0, s \geq s_1]$ by hazard rate dominance, then $a_1(s_1) > a_0(s_1)$. □

From the continuous density of s, $a_m(s_1)$ is continuous. Also $a_1(s_1)$ is increasing.

Theorem 4.

The equilibrium where the sender always sends $m = 0$(babbling) always exists.

The equilibrium where the sender always sends $m = 1$ when $e = 1$ exists if and only if $b \geq \frac{a_1(0)+a_0(0)}{2}$.

If $\frac{1+E(s)}{2} - 1 \leq b \leq \frac{a_1(0)+a_0(0)}{2}$, then there exists $s_1(b) \in [0, 1]$ such that the sender discloses his success($m = 1$) if $s \geq s_1(b)$ and hides his evidence ($m = 0$) if $s \leq s_1(b)$.

$s_1(b)$ is weakly decreasing in b and strictly decreasing when $0 < s_1 < 1$.

Proof. The pooling equilibrium always exists since sending $m = 0$ for any s, e and taking the unconditional expectation of s as action are mutually best replying.

$s_1 = 0$ is full disclosure. (If part) Suppose $b \geq \frac{a_1(0)+a_0(0)}{2}$. We can construct the full disclosure equilibrium. The incentive condition for full disclosure is, for any s,

$$a_1(0) - s - b \leq s + b - a_0(0).$$

The tightest condition is when $s = 0$. Therefore, the incentive condition holds if $b \geq \frac{a_1(0)+a_0(0)}{2}$

(Only if part) Suppose the full separation is equilibrium. The incentive condition for full separation is, for any s,

$$a_1(0) - s - b \leq s + b - a_0(0).$$

The tightest condition is when $s = 0$. This incentive condition implies $b \geq \frac{a_1(0)+a_0(0)}{2}$.

Next, we claim the existence of the cutoff $s_1(b) \in (0, 1)$ such that

$$s_1(b) + b - a_0(s_1(b)) = a_1(s_1(b)) - s_1(b) - b.$$

The continuous function $2s + a_1(s) + a_(s)$ takes from $a_0(1) + a_1(1) - 2 = E(s) + 1 - 2 = E(s) - 1$ to $a_0(0) + a_1(0)$.

Therefore, we can apply the intermediate value theorem to claim the existence of $s_1(b) \in (0, 1)$ for $\frac{1+E(s)}{2} - 1 \leq b \leq \frac{a_1(0)+a_0(0)}{2}$.

Finally, we claim that the cutoff $s_1(b)$ is weakly decreasing in b and strictly decreasing when $0 < s_1 < 1$ Since it is indifferent for the type s_1 sender to report $m = 1$ and $m = 0$, then

$$-(a_1(s_1(b)) - s_1(b) - b)^2 = -(a_0(s_1(b)) - s_1(b) - b)^2.$$

Since $a_1(s_1) > a_0(s_1)$,

$$a_1(s_1(b)) - s_1(b) - b = s_1(b) + b - a_0(s_1(b)).$$

There exists ϵ such that by defining $b' = b + \epsilon$,

$$-(a_1(s_1(b)) - s_1(b) - b')^2 > -(a_0(s_1(b)) - s_1(b) - b')^2.$$

So, the type $s_1(b)$ prefers to send $m = 1$ when the number of evidence is $e = 1$.

$$0 \leq a_1(s_1(b)) - s_1(b) - b' < s_1(b) + b' - a_0(s_1(b)).$$

We prove that the cutoff $s_1(b)$ is decreasing in b by contradiction. Suppose $s_1(b') > s_1(b)$. Since the sender takes the cutoff strategy in which the type smaller than the cutoff prefers to report $m = 0$, the type $s_1(b)$ should prefer to report $m = 0$. Contradiction.

□

Let us revisit the *independent successes* case with $n = 1$ and the uniform prior $P(s) = s$, where $f(s, e) = s$ if $e = 1$ and $f(s, e) = 1 - s$ if $e = 0$. Note that the density $f(s \mid m = e)$ is as follows given the evidence e and n[2].

$$f(s \mid 1) = 2s.$$

$$f(s \mid 0) = 2(1 - s).$$

[2]The probability of e given n, s is $f(s, e) = \frac{n!}{e!(n-e)!} s^k (1-s)^{n-e}$. The probability of evidence e is $\Pr(e \mid n) = \int_0^1 \frac{n!}{e!(n-e)!} s^e (1-s)^{n-e} ds = \frac{1}{n+1}$. The probability of s given e, n is $f(s \mid e, n) = \frac{(n+1)!}{e!(n-e)!} s^e (1-s)^{n-e}$ for $0 \leq s \leq 1$. The expectation of the state s given the evidence e is $\frac{e+1}{n+2}$.

We can rank the action $a_1 > a_0$ since the conditional distribution hazard rate dominance holds between $f(s \mid 1)$ and $f(s \mid 0)$. At $s = s_1$, taking $m = 1$ and $m = 0$ are indifferent. Therefore,

$$-(a_1 - s_1 - b)^2 = -(a_0 - s_1 - b)^2$$

, which implies

$$a_1 - s_1 - b = s_1 + b - a_0.$$

Then,

$$\frac{2(s_1^2 + s_1 + 1)}{3(1 + s_1)} - s_1 - b = s_1 + b - \frac{2s_1^3 + 1}{s_1^2 + 1}\frac{1}{3}.$$

The cutoff s_1 is a solution of this equation. The existence of this cutoff under some b is guaranteed by the intermediate value theorem.

$$Q(s_1, b) \equiv a_1(s_1) - 2s_1 - 2b + a_0(s_1).$$

Note that $Q(0, b) = 1 - 2b$ and $Q(1, b) = -\frac{1}{2} - 2b$. Therefore, for any $b \in [-\frac{1}{4}, \frac{1}{2}]$, $Q(0, b) \geq 0$ and $Q(1, b) \leq 0$. Because Q is continuous in s, by the intermediate value theorem for any $b \in [-\frac{1}{4}, \frac{1}{2}]$, there exist $s_1 \in [0, 1]$ such that $Q(s_1, b) = 0$.

3.5 Conclusion

We analyze a model where the sender can choose how many positive signals to reveal to the receiver. We characterize the equilibrium cutoff structure. We show with examples that there exist non-monotone equilibria where a type with a higher state sends a lower message and that the cutoffs in these equilibria need not be aligned. A conceptual contribution of this paper is to introduce monotone equilibria and to show that they exist and have a simple structure. In particular, if an equilibrium is monotone, then the cutoffs are aligned and the cutoffs decrease as the bias goes up. We also show that the full disclosure equilibrium exists under high bias. Therefore the bias plays an opposite role to the original Crawford-Sobel model. Proposing a model where the bias plays an opposite role is another contribution to the literature on strategic information transmission.

3.6 Appendix

3.6.1 Appendix for other evidence disclosure games

The standard disclosure game is a situation where the sender reveals his type or hides his type. In this section, we will discuss the other message restrictions. The first one is the situation where both underreporting and overreporting are allowed. The second one is the situation where the available message is only the null message or the realized evidence. The second one is a kind of disclosure game.

3.6.2 Full disclosure never happens if overreporting is possible

Suppose that both overreporting and underreporting are possible. Sufficient conditions for full disclosure are for all n' and all s,

$$E[s|e = n'] - s - b \leq s + b - E[s|e = n' - 1].$$

and

$$s + b - E[s|e = n'] \leq E[s|e = n' - 1] - s - b.$$

The first one is downward IC. The second one is upward IC. Therefore, the full disclosure equilibrium exists if and only if $b \geq \frac{E[s|e=n]+E[s|e=n-1]}{2}$ (the tightest condition is $n' = n, s = 0$ for the first one) and $b \leq -\frac{E[s|e=n]+E[s|e=n-1]}{2}$ (the tightest condition is $n' = 0, s = 1$ for the second one). Then full disclosure is not possible. This result is not consistent with the verifiable information disclosure literature. As we discussed, the monotone preference of the sender and the continuous action of the sender. The upward IC constraints require the negative monotonicity of the sender's utility for the action since the bias should be very low. The downward IC constraints require the positive monotonicity of the sender's utility for the action since the bias should be very high. Since the upward IC constraints contradict the downward IC constraints, there is no full disclosure of evidence e.

3.6.3 A comparison to the disclosure game

The setting where message space is $\{\phi, e\}$ is a kind of verifiable disclosure model. Null report or revealing the evidence is the available message. We want to claim there is no pooling and full disclosure monotone equilibrium plus some reasonable assumption. The reason we need more assumptions is that off-equilibrium action issue. In this verifiable disclosure game, we call an equilibrium where the type (s, e) sender always sends $m = \phi$ as the pooling equilibrium. We call an equilibrium where the type (s, e) sender always sends the realized evidence $m = e$ as the full disclosure equilibrium. Therefore $m = e$ is off equilibrium message under pooling. $m = \phi$ is off equilibrium message under full disclosure. The pooling equilibrium trivially exists by setting the off equilibrium action $\hat{a}(m) = E[s]$. But this off equilibrium action is not intuitive. Therefore, we need to define monotone equilibrium under this disclosure game.

Definition 7. *An equilibrium under the disclosure game* $(\hat{m}, \hat{a})$ *is monotone if* $\hat{a}(m)$ *is increasing in* m*, strictly increasing until* $\hat{a}(m) = 1$*,* $\hat{a}(0) < \hat{a}(\phi) < \hat{a}(n)$ *and* $\hat{a}(\phi) \neq \hat{a}(n')$ *for any* $n' \in \{0, \cdots n\}$*.*

Intuitively, the receiver's posterior given the null message should not be extreme. From the above definition, we can rank every actions including off-equilibrium actions. Then we can claim the following result.

Proposition 14. *There is no monotone pooling equilibrium under the disclosure game. In addition, there is no monotone full disclosure equilibrium under the disclosure game.*

Proof. Without loss of generality, we can assume $b \geq 0$. First, we claim there is no monotone pooling equilibrium. We prove it by contradiction. Suppose that the pooling is monotone equilibrium. Then the necessary condition for equilibrium is for any state $s \in [0, 1]$,

$$\hat{a}(n) + \hat{a}(\phi) - 2s \geq 2b$$

Since the left hand side is negative under $s = 1$, then the above equation contradicts $b \geq 0$.

Second, we claim there is no monotone full disclosure equilibrium. There is n' where $\hat{a}(n') < \hat{a}(\phi) < \hat{a}(n' + 1)$. Then the necessary condition for equilibrium is for any state $s \in [0, 1]$,

$$\hat{a}(n') + \hat{a}(\phi) - 2s \geq 2b$$

Since the left-hand side is negative under $s = 1$, then the above equation contradicts $b \geq 0$. □

With off-equilibrium path refinement, pooling, and full disclosure equilibria are eliminated. These equilibria disregard information from the true state. However, since the sender possesses two-dimensional information $(s.e)$, such information ignorance never occurs.

3.6.4 Appendix for two pieces of evidence.

In this subsection, we focus on the monotone equilibrium under $n = 2$.

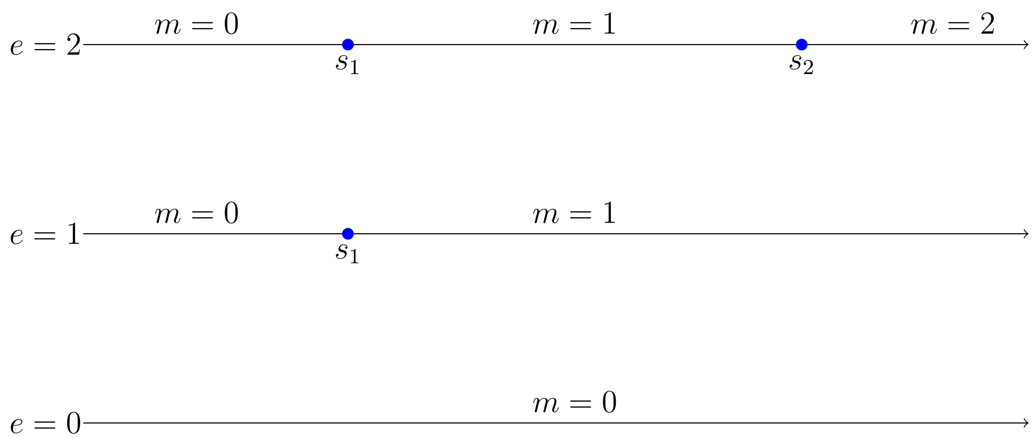

Figure 3.5. n=2

We focus on the following monotone strategy. See Figure 3.5. We use two cutoffs s_1, s_2.
When $e = 2$, then the sender sends $m = 2$ when $s \geq s_2$.When $e = 2$, then the sender sends $m = 1$ when $s_2 \geq s \geq s_1$. When $k = 2$, then the sender sends $m = 0$ when $s_1 \geq s \geq 0$.
When $e = 1$, then the sender sends $m = 1$ when $s \geq s_1$. When $k = 1$, then the sender sends $m = 0$ when $s_1 \geq s \geq 0$.
When $e = 0$, then the sender sends always sends $m = 0$.
We define $a_m(s_1, s_2)$ as the expected value of s for the receiver after the message m is observed.

We want to check the existence of the non-full disclosure and the non-babbling equilibrium. $a_2(s_1, s_2) = a_2(s_2)$ and $a_0(s_1, s_2) = a_0(s_1)$ because of the pooling pattern. To avoid the problem that the denominator is zero, we directly define $y_2(1) = 1$, which can be justified by L'Hopital's rule. We also directly define $a_1(1, 1) = 1$ for the same reason. $a_m(s_1, s_2)$ is continuous from the full support continuous density and the definition of $a_2(1)$ and $a_1(1, 1)$.

To make the cutoff equilibrium exist, we need the following assumptions. We use them to rank the actions and to apply the intermediate value theorem. These assumptions will be used to claim the monotonicity of $G(s_1, s_2)$, which will be defined later.

Assumption 1. $a_0(s_1) - s_1$ *is decreasing in* s_1.

Assumption 2. $a_1(0, s_2) \geq \mathbb{E}[s|e = 0]$ *for all* s_2.

Assumption 3. $a_2(s_2) - 2s_2$ *is decreasing in* s_2.

Lemma 3. *For any cutoffs such as* $0 \leq s_1 \leq s_2 \leq 1$, $a_2(s) \geq a_1(s) \geq a_0(s)$.

Proof. By hazard rate dominance, $a_2(s_2) \geq a_1(s_1, s_2)$.

We want to claim $a_1(s) \geq a_0(s)$. We use the assumption $a_1(0, s_2) \geq \mathbb{E}[s|e = 0]$ for all s_2. By definition, $a_0(s) \leq \max\{s_1, \mathbb{E}[s|k = 0]\}$. We also use $a_1(s_1, s_2)$ is increasing in s_1 since the minimum value of $a_1(s_1, s_2)$ is s_1. If $s_1 \leq \mathbb{E}[s|k = 0]$, then $a_1(s_1, s_2) \geq a_1(0, s_2) \geq \mathbb{E}[s|k = 0] \geq a_0(s)$. If $s_1 \geq \mathbb{E}[s|k = 0]$, then $a_1(s_1, s_2) \geq s_1 \geq a_0(s)$. Then $a_2(s) \geq a_1(s) \geq a_0(s)$.

□

The cutoffs are the solution to the indifference condition. However, if the s_0 or s_1 takes 0 or 1, then the indifference condition does not hold. To incorporate such a slackness, we need to define the following two functions. Let's define

$$f_1(s_1, s_2, b) \equiv a_2(s) - 2s_2 - 2b + a_1(s).$$
$$f_2(s_1, s_2, b) \equiv a_1(s) - 2s_1 - 2b + a_0(s_1).$$

$f_1(s_1, s_2, b) > 0$ implies $m = 1$ is preferred at when $s = s_2$. $f_2(s_1, s_2, b) > 0$ implies $m = 0$ is preferred at when $s = s_1$.

Since the utility function is quadratic, the equilibrium is determined by the following indifference conditions between reporting different messages at the cutoff.

$$a_2(s) - s_2 - b = s_2 + b - a_1(s).$$
$$a_1(s) - s_1 - b = s_1 + b - a_0(s).$$

To allow the slackness of the system of equations, we define

$$G(s_1, s_2) \equiv a_2(s_2) - 2s_2 - (a_0(s_1) - 2s_1).$$

Note that $f_1(s_1, s_2, b) = 0$ and $f_2(s_1, s_2, b) \leq 0$ if and only if $f_1(s_1, s_2, b) = 0$ and $G(s_1, s_2) \geq 0$ We want to use the single variable intermediate value theorem to verify the existence of a cutoff strategy. So, we now construct $s_2(s_1)$ using f_1 and G.

Proposition 15. *There is a continuous function $s_1(s_2)$ such that*

- $f_1(s_1(s_2), s_2, b) = 0.$
- $G(s_1(s_2), s_2) = 0$ *and* $0 \leq s_1(s_2) \leq s_2$ *or,* $G(s_1(s_2), s_2) > 0$ *and* $s_1(s_2) = 0.$
- $s_1(s_2)$ *is weakly increasing.*

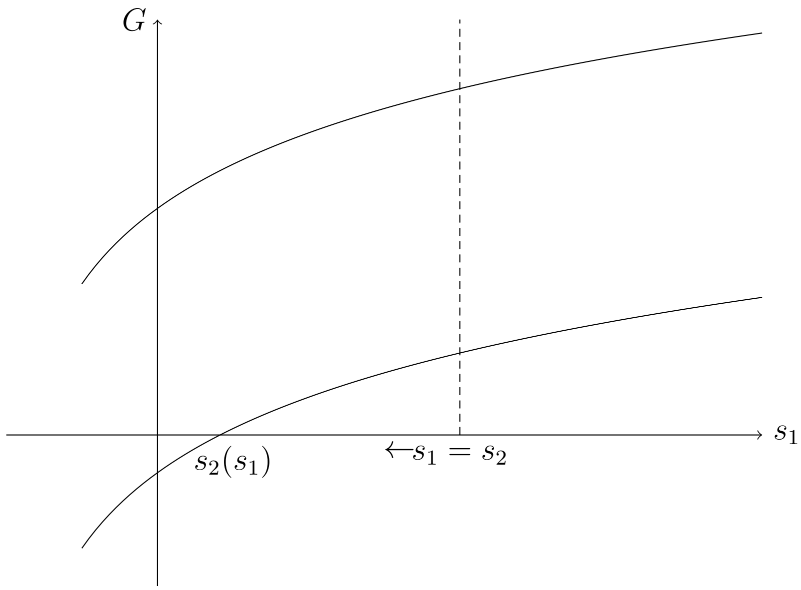

Figure 3.6. Construction of $s_2(s_1)$.

Proof. We can construct cutoff $s_2(s_1)$ as follows. See Figure 3.6.
Starting $s_1 = s_2 = s$, $G(s_1, s_2) > 0$. Making s_2 fixed as s and reducing s_1 to 0. Since $G(s_1, s_2)$ is increasing in s_1, $G(0, s_2) < 0$ or $G(0, s_2) \geq 0$.
When $G(0, s_2) < 0$, then there exist an unique $s_1(s_2)$ such that $G(s_1(s_2), s_2) = 0$ and $0 < s_1(s_2) < s_2$ by intermediate value theorem. Uniqueness comes from the monotone property of G with respect to s_1.
When $G(0, s_2) > 0$, then we set $s_1(s_2) = 0$.

Now we claim the continuity as follows. Fix $s_2^0 \in [0, 1]$. $s_1^0 \equiv s_1(s_2^0)$. We want to argue for any $\epsilon > 0$, there exists δ such that $|s_2^0 - s_2| < \delta$ implies $|s_1^0 - s_1(s_2)| < \epsilon$. There

are two cases ($s_1^0 > 0$ or $s_1^0 = 0$).
Suppose $s_1^0 > 0$. For any arbitrary small ϵ,

$$G(s_1^0 - \epsilon, s_2^0) < G(s_1^0, s_2^0) = 0 < G(s_1^0 + \epsilon, s_2^0).$$

By continuity of G, there exist δ such that $|s_2^0 - s_2| < \delta$ and

$$G(s_1^0 - \epsilon, s_2) < 0 < G(s_1^0 + \epsilon, s_2).$$

By continuity, there exist $\overline{s_1} \in [s_1^0 - \epsilon, s_1^0 + \epsilon]$ such that $G(\overline{s_1}, s_2) = 0$. Then this $\overline{s_1}$ locates within ϵ distance from s_1^0.
Suppose $s_1^0 = 0$.
In this case $G(0.s_2^0) > 0$. By monotonicity, there exists ϵ such that $G(\epsilon, s_2^0) > 0$. If $G(\epsilon, s_2) > 0$ for any s_2, then we can set $s_1(s_2) = 0$, which locates within ϵ distance from s_1^0. If $G(\epsilon, s_2) = 0$ for some s_2, then we can set $s_1(s_2) = \epsilon$, which locates within ϵ distance from s_1^0.

By the implicit function theorem and the monotonicity assumptions about $a_0(s_1) - 2s_1$ and $a_2(s_2) - 2s_2$, we get the result that $s_1(s_2)$ is weakly increasing.

□

We want to check the range of b where $f_1(s_1, s_2, b) = 0$ and $G(s_1, s_2) \geq 0$.
$G(0,0) > 0$, $f_1(0,0,b) = a_2(0) - 0 + a_1(0,0) - 2b = 0$. So, b can take $\frac{1}{2}(a_2(0) + a_1(0,0))$. $G(1,1) > 0$ and $G(0,1) < 0$, then there exists $l_1 \in [0,1]$ such that $G(l_1, 1) = 0$. Since $f_1(l_1, 1, b) = 1 - 2 + s_1(l_1, 1) - 2b$, $f_1(l_1, 1, b) = 0$ implies $b < 0$.

Therefore, b can take $[0, \frac{1}{2}(a_2(0) + a_1(0,0)]$. Note that this pattern is that at least s_2's condition is indifferent. s_1's indifference condition can be slack as sending $m = 1$ is preferred for every s when $k = 1$.

Also note that there is $l_2 \in (0,1)$ such that $G(0, l_2) = 0$ because $G(0,0) > 0, G(0,1) < 0$ and $G(a_1, 1) = 0$ and the construction of $s_1(s_2)$. So, we can claim that the existence of the equilibrium where both s_1 and s_2 are strictly bounded above 0 and strictly bounded below 1 since $G(s_1(a_2 + \epsilon), a_2 + \epsilon) = 0$ and choose b based on f_1.

Next, we want to allow the slackness of $f_1 \geq 0$ and check the value of b. Note that

$f_1(s_1, s_2, b) \geq 0$ and $f_2(s_1, s_2, b) = 0$ if and only if $f_2(s_1, s_2, b) = 0$ and $G(s_1, s_2) \geq 0$. We want to use the single variable intermediate value theorem to verify the existence of a cutoff strategy. So, we now construct $s_2(s_1)$ using f_2 and G.

Proposition 16. *There is a continuous function $s_2(s_1)$ such that*

- $f_2(s_1, s_2(s_1), b) = 0$.
- $G(s_1, s_2(s_1)) = 0$ *and* $0 \leq s_1 \geq s_2(s_1)$ *or,* $G(s_1, s_2(s_1)) > 0$ *and* $s_2(s_1) = 1$.
- $s_2(s_1)$ *is weakly increasing.*

Proof. Note that $G(s_1, s_2) > 0$ if $s_1 = s_2$. We can construct $s_1(s_2)$ as follows.
Starting $s_1 = s_2 = s$, $G(s_1, s_2) > 0$. Making s_1 fixed as s and increasing s_2 to 1. Since $G(s_1, s_2)$ is decreasing in s_2, $G(s_1, 1) < 0$ or $G(s_1, 1) \geq 0$.
When $G(s_1, 1) < 0$, then there exists an unique $s_2(s_1)$ such that $G(s_1, s_2(s_1)) = 0$ and $0 < s_1 < s_2(s_1) < 1$ by intermediate value theorem. Uniqueness comes from the monotone property of G with respect to s_2.
When $G(s_1, 1) > 0$, then we set $s_2(s_1) = 1$.
The proof of continuity and monotonicity of $s_2(s_1)$ is almost identical to Proposition 7. So, we omit. □

We want to check the range of b. $G(1, 1) > 0$, $f_2(1, 1, b) = a_2(1) + a_1(1, 1) - 2 - 2b$. So, b can take $\frac{a_2(1)+a_1(1,1)-2}{2}$. Since $G(0, 1) < 0$ and $G(0, 0) > 0$, there exist $l_3 \in [0, 1]$ such that $G(0, l_3) = 0$ by intermediate value theorem.Since $f_2(0, l_3, b) = a_0(0) + a_1(0, l_3) - 2b$, $f_2(0, l_3, b) = 0$ implies $b > 0$.

Therefore b can take $[\frac{a_2(1)+a_1(1,1)-2}{2}, 0]$. Note that this pattern is that at least s_1's condition is indifferent. s_2's condition can be slack.

Also note that there is $l_4 \in (0, 1)$ such that $G(l_4, 1) = 0$ because $G(1, 1) > 0, G(0, 1) < 0$ and $G(0, l_3) = 0$ and the construction of $s_2(s_1)$. So, we can claim that the existence of the equilibrium where both s_1 and s_2 are strictly bounded above 0 and strictly bounded below 1 since $G(l_4 - \epsilon, s_2(l_4 - \epsilon)) = 0$ and choose b based on f_2.

By summing up the above propositions,

Theorem 5. *Full pooling equilibrium ($s_1 = 1, s_2 = 1$) always exists.*
Full disclosure equilibrium exists if and only if $b \geq \frac{1}{2}(a_2(0) + a_1(0, 0))$.
If $\frac{1}{2}(a_2(1) + a_1(1, 1) - 2) < b < \frac{1}{2}(a_2(0) + a_1(0, 0))$, there exist cutoff s_1 and s_2 such that

- $0 \leq s_1 \leq s_2 \leq 1$
- *one of* s_1 *and* s_2 *is strictly smaller than* 1 *and strictly larger than* 0.
- *If* $0 \leq s \leq s_1$, *he sends* $m = 0$ *when* $e \geq 1$.
- *If* $1 \geq s \geq s_1$, *he sends* $m = 1$ *when* $e = 1$.
- *If* $s_1 \leq s \leq s_2$, *he sends* $m = 1$ *when* $e = 2$.
- *If* $1 \geq s \geq s_2$, *he sends* $m = 2$ *when* $e = 2$.

Proof. The remaining part of the proof is for full disclosure. Suppose the full disclosure exists. Then the incentive compatible conditions are, for any $s \in [0, 1]$,

$$a_2(0) - s - b \leq s + b - a_1(0, 0).$$
$$a_1(0, 0) - s - b \leq s + b - a_0(0).$$

Because $\frac{1}{2}(a_2(0) + a_1(0, 0)) > \frac{1}{2}(a_0(0) + a_1(0, 0))$, the incentive conditions imply $b \geq \frac{1}{2}(a_2(0) + a_1(0, 0))$.
Suppose $b \geq \frac{1}{2}(a_2(0) + a_1(0, 0))$. Then the above incentive conditions hold. □

An example of the above is uniform prior case. Suppose s follows uniform distribution on $[0, 1]$.

When $s_1 = 0$ and $s_2 = 0$, then the full disclosure happens. The sufficient condition for full disclosure is

$$\frac{3}{4} - s_2 - b \leq s_2 + b - \frac{1}{2}.$$
$$\frac{1}{2} - s_1 - b \leq s_1 + b - \frac{1}{4}.$$

Therefore, if $b \geq \frac{5}{8}$, then the full disclosure occurs.

Then we want to check the existence of the non-full disclosure and the non-full pooling equilibrium. Let $a_m(s)$ be the expected value of s after observing the message m given the cutoff is the vector $s = (s_1, s_2)$.

$$a_2(s_2) = \int_{s_2}^{1} \frac{1}{1 - s_2^3} s 3s^2 ds = \frac{3(1 - s_2^4)}{4(1 - s_2^3)}.$$

$$a_1(s_1,s_2) = Pr(e=2|m=1)\int_{s_1}^{s_2} s\frac{3s^2}{\int_{s_1}^{s_2} 3s^2 ds}ds + Pr(e=1|m=1)\int_{s_1}^{1} s\frac{6s(1-s)}{\int_{s_1}^{1} 6s(1-s)ds}ds$$
$$= \frac{1}{(s_2^3-s_1^3)+6(\frac{1-s_1^2}{2}-\frac{1-s_1^3}{3})}\frac{3}{4}(s_2^4-s_1^4) + \frac{1}{(s_2^3-s_1^3)+6(\frac{1-s_1^2}{2}-\frac{1-s_1^3}{3})}6(\frac{1-s_1^3}{3}-\frac{1-s_1^4}{4}).$$

$$y_0(s_1) = Pr(e=2|m=0)\int_0^{s_1} s\frac{3s^2}{\int_0^{s_1} 3s^2 ds}ds + Pr(e=1|m=0)\int_0^{s_1} s\frac{6s(1-s)}{\int_0^{s_1} 6s(1-s)ds}ds$$
$$+ Pr(e=0|m=0)\int_0^1 s3(1-s)^2 ds$$
$$= \frac{1}{s_1^3+6(\frac{s_1^2}{2}-\frac{s_1^3}{3})+1}\frac{3}{4}s_1^4 + \frac{1}{s_1^3+6(\frac{s_1^2}{2}-\frac{s_1^3}{3})+1}(2s_1^3-\frac{3}{2}s_1^4) + \frac{1}{s_1^3+6(\frac{s_1^2}{2}-\frac{s_1^3}{3})+1}\frac{1}{4}.$$

The assumptions that we use in general cases hold under this setting.

To claim, $a_2 \geq a_1 \geq a_0$. we want to check the hazard rate dominance. Hazard rates for each realization of e are as follows. The hazard rate of s after observing $e=2$ when true s is not known for $x \in [0,1]$ is

$$\frac{3x^2}{\int_x^1 3s^2 ds} = \frac{x^2}{\frac{1}{3}(1-x^3)}.$$

The hazard rate of s after observing $e=1$ when true s is not known for $x \in [0,1]$ is

$$\frac{6x(1-x)}{\int_x^1 6s(1-sds)} = \frac{6x(1-x)}{1-3x^2+2x^3}.$$

The hazard rate of s after observing $e=0$ when true s is not known for $x \in [0,1]$ is

$$\frac{3(1-x^2)}{\int_x^1 3(1-s^2)} = \frac{x^2-2x+1}{\frac{1}{3}(1-x^3)-(1-x^2)+(1-x)}.$$

We can check hazard rate dominance as follows.

$$\frac{x^2}{\frac{1}{3}(1-x^3)} - \frac{6x(1-x)}{1-3x^2+2x^3} = \frac{3x(x+2)}{(x-1)(2x+1)(x^2+x+1)} \leq 0.$$

$$\frac{6x(1-x)}{1-3x^2+2x^3} - \frac{x^2-2x+1}{\frac{1}{3}(1-x^3)-(1-x^2)+(1-x)} = \frac{3}{(x-1)(2x+1)} \leq 0.$$

Then, $a_2 \geq a_1 \geq a_0$.

Lemma 4. *$a_0(s_1) - s_1$ is decreasing under uniform prior.*

Proof.

$$a_0(s_1) - s_1 = \frac{1 + 8s_1^3 - 3s_1^4}{4 + 12s_1^2 - 4s_1^3} - s_1.$$

By taking the derivative with respect to s_1,

$$\frac{-4 - 6s_1 + 3s_1^2 - 4s_1^3 - 12s_1^4 + 6x^5 - x^6}{4(1 + 3x^2 - x^3)^2} < 0.$$

I used $0 \leq s_1 \leq 1$. □

Lemma 5. *$a_1(0, s_2) \geq \mathbb{E}[s|e = 0]$ for all s_2.*

Proof. $a_1(0, s_2) = \frac{2+3s_2^3}{4(1+s_2^3)} \geq \frac{2}{4(1+s_2^3)} \geq \frac{2}{4(1+1)} = \frac{1}{4} = \mathbb{E}[s|e = 0]$. □

Lemma 6. *$a_2(s_2) - 2s_2$ is decreasing under uniform prior.*

Proof.

$$a_2(s_2) - 2s_2 = \frac{3(1 - s_2^4)}{4(1 - s_2^3)} - 2s_2.$$

Taking the derivative for s_2,

$$\frac{-(5s_2^4 + 10s_2^3 + 15s_2^2 + 16s_2 + 8)}{4(s_2^2 + s_2 + 1)^2} < 0.$$

□

Using proposition 1, we can detect some b for the existence of cutoff strategy. We want to check the range of b where $f_1(s_1, s_2, b) = 0$ and $G(s_1, s_2) \geq 0$.
Since $G(0, 0) = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$, $f_1(0, 0, b) = a_2(0) - 0 + a_1(0, 0) - 2b = \frac{3}{4} + \frac{1}{2} - 2b = \frac{5}{4} - 2b$.
So, b can take $\frac{5}{8}$.
Since $G(1, 1) = \frac{1}{2}$ and $G(0, 1) = 1 - 2 - \frac{1}{4} < 0$, then there exists $l_5 \in [0, 1]$ such that $G(l_5, 1) = 0$. Since $f_1(l_5, 1, b) = -1 + a_1(l_5, 1) - 2b$, $f_1(l_5, 1, b) = 0$ implies $b < 0$.
Therefore, b can take $[0, \frac{5}{8}]$.

Using theorem 2, we can detect some b for the existence of cutoff strategy.

We want to check the range of b. Since $G(1,1) = a_2(1) - a_0(1) = \frac{1}{2}$, $f_2(1,1,b) = -\frac{1}{2} - 2b$. So, b can take $-\frac{1}{4}$. Since $G(0,1) = 1 - 2 - \frac{1}{4} < 0$ and $G(0,0) = \frac{1}{3}$, there exists $l_6 \in [0,1]$ such that $G(0,a) = 0$ by intermediate value theorem. Since $f_2(0,,b) = a_0(0) + a_1(0,l_6) - 2b$, $f_2(0,l_6,b) = 0$ implies $b > 0$.
Therefore b can take $[-\frac{1}{4}, 0]$.

By summing up the above,

Theorem 6. *Full disclosure equilibrium exist if and only if* $b \geq \frac{5}{8}$.
If $-\frac{1}{4} < b < \frac{5}{8}$, *there exist cutoff* s_1 *and* s_2 *such that*

- $0 \leq s_1 \leq s_2 \leq 1$.
- *one of* s_1 *and* s_2 *is strictly smaller than* 1 *and strictly larger than* 0.
- *If* $0 \leq s \leq s_1$, *he sends* $m = 0$ *when* $e \geq 1$.
- *If* $1 \geq s \geq s_1$, *he sends* $m = 1$ *when* $e = 1$.
- *If* $s_1 \leq s \leq s_2$, *he sends* $m = 1$ *when* $e = 2$.
- *If* $1 \geq s \geq s_2$, *he sends* $m = 2$ *when* $e = 2$.

Bibliography

Argenziano, Rossella, Sergei Severinov, and Francesco Squintani (2016) "Strategic information acquisition and transmission," *American Economic Journal: Microeconomics*, 8 (3), 119–155.

Bergemann, Dirk and Ulrich Hege (2005) "The financing of innovation: Learning and stopping," *RAND Journal of Economics*, 719–752.

Crawford, Vincent P and Joel Sobel (1982) "Strategic information transmission," *Econometrica: Journal of the Econometric Society*, 50 (6), 1431–1451.

Gottlieb, Daniel and Humberto Moreira (2022) "Simple contracts with adverse selection and moral hazard," *Theoretical Economics*, 17 (3), 1357–1401.

Green, Brett and Curtis R Taylor (2016) "Breakthroughs, deadlines, and self-reported progress: Contracting for multistage projects," *American Economic Review*, 106 (12), 3660–3699.

Grossman, Sanford J (1981) "The informational role of warranties and private disclosure about product quality," *The Journal of Law and Economics*, 24 (3), 461–483.

Guo, Yingni (2016) "Dynamic delegation of experimentation," *American Economic Review*, 106 (8), 1969–2008.

Halac, Marina, Navin Kartik, and Qingmin Liu (2016) "Optimal contracts for experimentation," *The Review of Economic Studies*, 83 (3), 1040–1091.

Halac, Marina and Ilan Kremer (2020) "Experimenting with career concerns," *American Economic Journal: Microeconomics*, 12 (1), 260–288.

Hörner, Johannes and Larry Samuelson (2013) "Incentives for experimenting agents," *The RAND Journal of Economics*, 44 (4), 632–663.

Ivanov, Maxim (2010) "Informational control and organizational design," *Journal of Economic Theory*, 145 (2), 721–751.

Kamenica, Emir and Matthew Gentzkow (2011) "Bayesian persuasion," *American Economic Review*, 101 (6), 2590–2615.

Keller, Godfrey, Sven Rady, and Martin Cripps (2005) “Strategic experimentation with exponential bandits,” *Econometrica*, 73 (1), 39–68.

Lewis, Tracy R and David EM Sappington (1991) “All-or-nothing information control,” *Economics Letters*, 37 (2), 111–113.

Li, Cheng (2017) “A model of Bayesian persuasion with transfers,” *Economics Letters*, 161, 93–95.

Milgrom, Paul R (1981) “Good news and bad news: Representation theorems and applications,” *The Bell Journal of Economics*, 380–391.

Pei, Harry (2015) “Communication with endogenous information acquisition,” *Journal of Economic Theory*, 160, 132–149.

Vita

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