The Pennsylvania State University
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QUANTUM-CORRECTED BLACK HOLES: CONSTRUCTING AND INVESTIGATING MODIFIED BLACK HOLE MODELS WITH QUANTUM CORRECTIONS AND EXPLORING AVENUES FOR TESTABLE PREDICTIONS

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Abstract

The extreme spacetime environments of modified black holes are an ideal context in which to study possible quantum corrections. This is critical for reconciling general relativity and quantum mechanics and creating a theory of quantum gravity. In this dissertation, I use canonical gravity methods to construct, reinterpret, and probe the properties of quantum-corrected black holes, with the goal of refining modified gravity models, in the pursuit of a theory of quantum gravity. First, I construct a quasi-classical static black hole model with an additional scalar field introduced in the Hamiltonian constraint, and I derive the form of the resulting quantum effects surrounding the horizon and asymptotically. Then, I demonstrate that this model can be similarly constructed as a superposition of classical black holes of varying mass by deriving a quantum modification to the Newtonian potential in the asymptotic limit. Finally, I calculate the effect of a related quantum correction on established volume calculations for the interior of the event horizon. Together, this work provides key insights into the possible structures and behaviors of quantum black holes, opening avenues to probe the information paradox, black hole "deaths," mass uncertainty, and other mysteries of black hole physics. These advances lay the groundwork for potential future predictions such as quantum switch behaviors around quantum black holes and gravitational wave quasinormal mode observables from mergers of modified black holes.

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Dedication

I dedicate this dissertation to the underrepresented scientists who helped clear this path ahead of me, to the allies and loved ones who supported me, and to the future scientists who, I hope, will be inspired and supported by my research and advocacy.

Chapter 1 Introduction

The core of my dissertation consists of three papers, each exploring different aspects of constructing, interpreting, and investigating quantum-corrected black holes in canonical gravity. In this introduction, I provide big-picture background and motivation, in an effort to make this work accessible to a wider audience and a useful resource for future researchers. I also provide a brief overview of the core papers. In chapters 2-4, I summarize the key procedures and results of each of these papers, followed by the full text for further detail. In Chapter 5, I highlight my novel results and lay out a variety of follow-up projects, including plans for observable predictions. In Chapter 6, I document additional contributions to the department and the physics community in general.

1.1 Background and Motivation

How can we reconcile general relativity with quantum mechanics, and what are the necessary attributes of a superseding theory of Quantum Gravity? What is the nature of spacetime, quantum information, and black holes? We can use canonical quantization to probe these questions via modified black hole spacetimes. Introducing quantum corrections into black hole physics has the potential to address open questions such as the information paradox, singularity divergences, and the mysterious late stages of a black hole's life. What observable consequences would distinguish such modified gravity models? Ultimately, I am working to make potentially observable predictions for quantum gravity effects in quantum switch experiments and the quasinormal modes of gravitational wave signals.

General relativity and quantum mechanics are the foundation of modern physics and are essential to modern life, from GPS and space travel, to nuclear power and quantum computing. General relativity has been tested on planetary and intergalactic scales, and quantum mechanics has been tested on sub-atomic scales, but they come into conflict when their regimes of validity overlap and the mathematics of general relativity affects the behavior of particles on a quantum scale. How to reconcile these two physical models, combining them into a cohesive theory of quantum gravity, is one of the largest open questions in physics.

Black holes are an excellent testing ground for theories of quantum gravity because the extreme curvature of spacetime makes general relativity relevant on small quantum mechanical scales. This is why I research Modified Gravity, specifically quantum-corrected black holes. I construct black hole spacetimes with quantum corrections and calculate their behavior, exploring their dynamics, volumes, and distinctive properties. There are a number of features of black holes which these modified models can potentially explain or predict, such as information capacity, singularity replacement, and black hole death. These predictions would allow us to constrain what models are likely candidates for a theory of quantum gravity.

This dissertation revolves around black holes, using the mathematics of canonical gravity to explore quantum effects. Specifically, this dissertation consists of three projects involving two modified black hole spacetimes, two representations of a single modification, and novel investigations of relevant quantum effects. These investigations include: interior volume calculations, near-horizon effects, and modified potentials in the asymptotic regime. A successful theory for quantum gravity explaining quantum behaviors involving black holes would have the potential to solve many open questions in physics, astronomy, cosmology, and beyond.

1.1.1 Black Holes

A black hole is any amount of mass compressed into effectively zero volume, creating a singularity. The diverging density of the singularity corresponds to an infinitely increasing gravitational force as one approaches the center. Thus, at some radius around this central mass, the velocity necessary to escape the gravitational pull would exceed the speed of light. This boundary, from which not even light can escape, is known as an event horizon and is why these astrophysical objects appear black.

The only well-established mechanism for forming black holes is stellar core collapse during specific supernova events, first proposed in 1939 in [1]. However, most galaxies have supermassive black holes at the center [2], with masses hundreds of thousands or billions of times the mass of our sun, and current models cannot account for how large these black holes have grown in the age of the universe. There is a limit to the size that stars can form, so any black hole born from a supernova would be limited in size. To grow this large, black holes must consume dust, gas, planets, and hundreds of thousands of solar systems or other black holes. Even making generous estimates of how early stars formed, how large they were, how many went supernova, how many of those formed black holes, and how frequently they were able to merge, the sizes of supermassive black holes we see today defy explanation. Thus, we know that there is much more left to discover about black holes and the evolution of our universe. Thus, we need to understand black holes, and any quantum behaviors related to them, at a more fundamental level.

Another problem with current models is that the infinite density of a singularity should be physically impossible, and yet we observe black holes and their extreme gravitational effects. Astronomers have long observed the jets of active galactic nuclei [3], the gravitational slingshot of stars orbiting the galactic center [4], gravitational waves emitted from merging black holes [5], and direct light from the accretion disc and Einstein ring around an (astrophysically speaking) nearby black hole [6]. All of these measurements confirm the existence of gravitational forces so concentrated that light cannot escape. We know this extreme warping of spacetime occurs, but not physically what happens behind the curtain of the horizon. There is some room to modify the mathematical models of black hole spacetimes and preserve the dominant external curvature effects, while sidestepping the problems of a central singularity. Avoiding these mathematical and physical problems is another motivation for various modified black hole models, such as those in Chapter 4.

Hawking radiation is an important mechanism by which black holes can reduce their mass, in a process known as evaporation [7]. A simplified visualization of this process is an entangled particle-antiparticle pair emerging from vacuum energy, as allowed by quantum mechanics, at the edge of an event horizon. One particle falls into the black hole and annihilates a particle inside, identical to its original partner, while the other escapes to be detected. The mass of the black hole will be reduced by the mass of the particle annihilated on the interior, and the black hole will appear to have emitted the identical escaped particle. This is the mechanism of black hole evaporation, though the detected particle of Hawking radiation did not originate inside the black hole, and therefore never had to exceed the speed of light to reach an observer outside. A more accurate way to conceptualize Hawking Radiation is simply as positive energy flux from near the event horizon towards positive infinity outside the black hole and negative energy flux from the horizon towards the singularity inside the black hole. Already being a quantum process, Hawking radiation could be significantly affected by quantum corrections in black hole models.

The information paradox, also known as the problem of information loss, is another mystery which could benefit from a quantum description of black holes [8]. Any matter/information which falls into the black hole becomes inaccessible beyond the horizon. Is this information effectively destroyed, which would violate the law of conservation of information? Understanding the geometry, volume, and longevity of the space inside the event horizon could provide mechanisms to store and/or recover the information consumed by a black hole.

Black hole death, or lack thereof, is a hotly debated topic. What happens as Hawking radiation depletes the last of the mass in a singularity? Are there violent explosions? Black hole to white hole transitions? Remnants? Any quantum properties of black holes will become most relevant at this stage, and could therefore hold the answers to what happens to a black hole at this point in its life. This would determine whether information is destroyed, recovered, or stored away.

1.1.2 Modified Gravity

The term "modified gravity" usually refers to cases where the line element is modified directly. This can also refer to a method of modifying the action, such as modifying the higher curvature term in scalar-tensor theories. While this is worthwhile, it tends to have challenges with stability. Adding higher order terms with second derivatives in time produces equations higher than second order. This requires more initial values, but most possible initial values are unstable [9]. High curvature effective actions often have problems with the speed of gravitational waves differing from the speed of light, which is a departure from observations.

In my derivation of modified gravity theories from canonical quantum gravity, I avoid these instabilities, first by maintaining the second-order nature of equations of motion, such that there are no new independent solutions that would otherwise result from additional initial values. Second, I respect the observational constraint that the speed of gravitational waves is close to the speed of light by formulating a coupled system for gravity and light based on one spacetime geometry [10–13]. This is a relative physical criteria that these two speeds must be the same, therefore their agreement is not affected by my modification magnitudes. This advantage is inherent in modifying the spacetime itself.

I choose to work with canonical gravity because it centers around preserving covariance, so that physical results have the critical property of being coordinate independent.

The condition for covariance is that the Poisson brackets of the Hamiltonian and diffeomorphism constraints with themselves, and each other, must remain closed. The Hamiltonian constraint is a statement of conservation of energy, which also functions as the generator of time translations. The corresponding statement of conservation of momentum, which generates spatial translations, is known as the diffeomorphism constraint. The relationship between these constraints, determined from hypersurface deformations, enforces covariance. This means that the output of any Poisson bracket between them must depend on one or more of the constraints themselves, as part of a closed space. This condition is utilized in the construction of quantum-corrected black holes in canonical gravity. Our work is distinct, in that we modify the Hamiltonian constraint and propagate that through to the line element by imposing covariance.

1.2 Overview

This research is presented in three parts, projects I conducted in collaboration with my advisor, Martin Bojowald, and each of my undergraduate research mentees: Gianni Sims (now an astrophysics grad student at Florida Atlantic University), Aurora Colter (continuing to build on our work for her senior thesis), and Allison Colarelli (starting grad school at University of Arizona in general relativity). We have published, submitted, and drafted papers respectively, which comprise each of the next three chapters.

1.2.1 Quantum-Corrected Black Hole

In Chapter 2, I will elaborate on my work in, *Quasiclassical solutions for static quantum black holes*, written in collaboration with Gianni Sims, Manuel Díaz, and Martin Bojowald. Our goal was to calculate quasiclassical space-time dynamics with non-local quantum corrections, using canonical methods of non-adiabatic quantum dynamics, and ultimately extend this to an effective quantum field theory. We start from the simplest case, a spherically symmetric black hole spacetime, introduce a quantum correction into the Hamiltonian constraint, impose our covariance condition on the constraints, calculate the equations of motion, introduce perturbations, impose a static simplification, and use these results to constrain what form the modification can take. The success of this approach takes us a step closer to a quantum gravity theory.

1.2.2 Quantum Black Hole Superposition

Collaborating with Aurora Colter, Manuel Díaz, and Martin Bojowald, I have submitted *Space-time superpositions as fluctuating geometries* for publication in Physical Review D. In this paper, we reinterpret my previous black hole model as a quantum superposition of classical black hole spacetimes of varying mass. We set the modified Hamiltonian constraint to zero, recalculate the equation of motion using only the first order correction to the Hamiltonian, take the weak field limit, and solve for power series solutions for the scalar fields and lapse function. We then used these solutions to construct the quantum-corrected Newton potential, complete with bounds on the constants. This broadens the applicability, both of our previously established quantum-corrected black hole, and of related black hole superposition studies. We were able to derive quantum corrections to Newton's potential in the weak field limit, and the corresponding corrections to metric components.

1.2.3 Quantum Black Hole Volume

Collaborating with Allison Colarelli and Martin Bojowald, I have written *Volumes of Quantum Corrected Black Holes*, in which we calculate the volume on the horizon interior for a black hole spacetime with a related modification. Our goal was to use established definitions for the volume inside the event horizon as constrained slices of the spacetime (Maximally Slicing a Black Hole, Estabrook and Wahlquist) and recreate this calculation for a spacetime model with a similar modification to ours (An effective model for the quantum Schwarzschild black hole, Alonso-Bardaji, Brizuela, and Vera). We expected this modification to shift the radius at which these slices were found to converge inside the horizon. Such a result could offer insights into both the information paradox and the late-stages of black hole evaporation. We are in the final stages of this calculation, now analyzing second order effects.

Chapter 2 Constructing a quantum-corrected black hole

In my recent paper, *Quasiclassical solutions for static quantum black holes*, I constructed a spacetime with quantum modifications, using Canonical Gravity methods adjacent to Loop Quantum Gravity. My goal was to calculate quasiclassical space-time dynamics with non-local quantum corrections, using canonical methods of non-adiabatic quantum dynamics. I modeled coordinate-independent black hole quantum corrections and established the presence of nonlocal quantum effects surrounding this modified black hole. I already have projects in motion to make testable predictions, and ultimately, I hope to extend this to an effective quantum field theory.

2.1 Summary

Starting from a spherically symmetric, static black hole model, we introduce corrections to the Hamiltonian constraint, impose a covariance condition, calculate the equations of motion, simplify to the static case, and use these calculations to restrict the form of the scalar field introduced in the modification.

2.1.1 Construction

Most non-local effects in black hole models assume a specific non-local action. We used a new systematic quasiclassical formulation with non-local corrections derived in a canonical quantization to model a coordinate-independent black hole with quantum corrections, deriving lon-local effects. We introduce an additional field in the Hamiltonian constraint, ϕ_3, and treat it as a quantum correction on the existing metric field, ϕ_2, making a static gauge choice for the other metric field, ϕ_1 equal to x^2.

Starting with the classical Hamiltonian for a static, spherically symmetric black hole,

$$H[N] = -\int \mathrm{d}x N(x) \left(\frac{\phi_2 p_2^2}{2\sqrt{\phi_1}} + 2\sqrt{\phi_1} p_1 p_2 + \left(1 - \left(\frac{\phi_1'}{\phi_2}\right)^2\right) \frac{\phi_2}{2\sqrt{\phi_1}} - 2\left(\frac{\phi_1'}{\phi_2}\right)' \sqrt{\phi_1} \right) \tag{2.1}$$

we introduce our first correction, dependent on a new scalar field, ϕ_3:

$$\begin{aligned} H_2[N] &= \int \mathrm{d}x N(x) \left(\frac{1}{2}\frac{\partial^2 H}{\partial p_2^2}\left(p_3^2 + \frac{U}{\phi_3^2}\right) + \frac{\partial^2 H}{\partial\phi_2 \partial p_2}\phi_3 p_3 + \frac{1}{2}\frac{\partial^2 H}{\partial \phi_2^2}\phi_3^2 + \frac{\partial^2 H}{\partial\phi_2\partial\phi_2'}\phi_3\phi_3' \right) \\ &= -\int \mathrm{d}x N(x) \left(\frac{\phi_2 p_3^2}{2\sqrt{\phi_1}} + \frac{\phi_3 p_2 p_3}{\sqrt{\phi_1}} + \left(6\frac{\sqrt{\phi_1}\phi_1'\phi_2'}{\phi_2^4} - \frac{1}{2}\frac{(\phi_1')^2}{\sqrt{\phi_1}\phi_2^3} - 2\frac{\phi_1''\sqrt{\phi_1}}{\phi_2^3}\right)\phi_3^2 \right. \\ &\quad \left. -4\frac{\sqrt{\phi_1}\phi_1'\phi_3\phi_3'}{\phi_2^3} + \frac{U(x)\phi_2}{2\sqrt{\phi_1}\phi_3^2} \right) . \end{aligned} \tag{2.2}$$

This correction introduces an additional scalar field in the Hamiltonian constraint. We then interpret the new scalar field ϕ_3 as a quantum correction on the existing metric field ϕ_2. Note that x is the spatial, radial coordinate measuring distance from the center of the black hole, C is a constant, and M is the mass of the black hole. The ϕ's and p's are degrees of freedom in the metric. They are functions of the tetrad variables, which relate to physical properties such as extrinsic curvature.

Note that the diffeomorphism constraint is affected, modified with E_x/E_ϕ^2, but this is not relevant for the static analysis where momenta are set to zero. The diffeomorphism constraint modification simplifies conveniently in the static regime we chose. Next, we preserve covariance by requiring the quantum-corrected constraints to maintain the property that the Poisson brackets are closed.

$$\{D[M_1], D[M_2]\} = D[M_1 M_2' - M_2 M_1'] \tag{2.3}$$

$$\{H[N], D[M]\} = -H[MN'] \tag{2.4}$$

$$\{H[N_1], H[N_2]\} = -D[E^x (E^\varphi)^{-2}(N_1 N_2' - N_2 N_1')]$$

In the process of enforcing this condition, we discovered the need for a higher order constraint, specifically, it required the introduction of a higher order correction to the Hamiltonian constraint:

$$H_{\phi_2}[L] = \int \mathrm{d}x L(x) \left(\frac{\partial H}{\partial \phi_2}\phi_3^2 + \frac{\partial H}{\partial \phi_2'}\phi_3\phi_3' + \frac{\partial H}{\partial p_2}\phi_3 p_3 \right)$$

$$
\begin{aligned}
= & -\int \mathrm{d}x L(x)\left(\left(\frac{p_2^2}{2\sqrt{\phi_1}}+\frac{1}{2\sqrt{\phi_1}}+\frac{(\phi_1')^2+4\phi_1\phi_1''}{2\sqrt{\phi_1}\phi_2^2}-4\frac{\sqrt{\phi_1}\phi_1'\phi_2'}{\phi_2^3}\right)\phi_3^2\right. \\
& \left.+\frac{2\sqrt{\phi_1}\phi_1'}{\phi_2^2}\phi_3\phi_3'+\left(\frac{\phi_2 p_2}{\sqrt{\phi_1}}+2\sqrt{\phi_1}p_1\right)\phi_3 p_3\right). \qquad (2.5)
\end{aligned}
$$

2.1.2 Investigation

With both quantum corrections to the Hamiltonian constraint, we calculated the equations of motion to see how our modified spacetime behaves, specifically how the fields and their momenta evolve in time. These are calculated by taking the Poisson bracket of the unfixed fields and their momenta with the sum of modified Hamiltonian constraints (again the Diffeomorphism constraint does not have a relevant effect):

$$
\begin{aligned}
\dot{\phi}_2 &= \{\phi_2, \bar{H}[N]+H_{\phi_2}[L]\} = \frac{\delta \bar{H}[N]}{\delta p_2}+\frac{\delta H_{\phi_2}[L]}{\delta p_2} \qquad (2.6) \\
&= \left(2xp_1+\frac{\phi_2 p_2}{x}+\frac{\phi_3 p_3}{x}\right)N+\frac{\phi_3}{x}\left(\phi_3 p_2+\phi_2 p_3\right)L
\end{aligned}
$$

$$
\begin{aligned}
\dot{\phi}_3 &= \{\phi_3, \bar{H}[N]+H_{\phi_2}[L]\} = \frac{\delta \bar{H}[N]}{\delta p_3}+\frac{\delta H_{\phi_2}[L]}{\delta p_3} \qquad (2.7) \\
&= \frac{1}{x}(\phi_2 p_3+\phi_3 p_2)N+\left(\frac{\phi_2 p_2}{\sqrt{\phi_1}}+2\sqrt{\phi_1}p_1\right)\phi_3 L\,,
\end{aligned}
$$

$$
0=\dot{p}_2=\{p_2, \bar{H}[N]+H_{\phi_2}[L]\} = -\frac{\delta \bar{H}[N]}{\delta \phi_2}-\frac{\delta H_{\phi_2}[L]}{\delta \phi_2} \qquad (2.8)
$$

$$
0=\dot{p}_3=\{p_3, \bar{H}[N]+H_{\phi_2}[L]\} = -\frac{\delta \bar{H}[N]}{\delta \phi_3}-\frac{\delta H_{\phi_2}[L]}{\delta \phi_3} \qquad (2.9)
$$

We then use perturbation theory, introducing small fluctuations around the classical lapse function N (which governs the passage of time) and around the classical metric field ϕ_2.

$$
\begin{aligned}
\phi_2 &= \phi_2^{(0)}+\delta\phi_2 = \frac{2x}{\sqrt{1-\mu/x}}+\delta\phi_2 \qquad (2.10) \\
N &= N^{(0)}+\delta N = \sqrt{1-\mu/x}+\delta N
\end{aligned}
$$

Implementing the static condition that momenta are zero, we solve the system of equations to restrict what properties our quantum correction can have. This produces a constrained form of the scalar field, ϕ_3, which we initially introduced into the Hamiltonian constraint.

$$\phi_3(x) = \frac{C}{(1-\mu/x)^{3/2}} \tag{2.11}$$

We solve these equations of motion and the established constraints for these perturbations, introducing an integration constant for each. These new degrees of freedom imply that quantum extended theories like this are more complex than classical ones. We then analyze asymptotic behavior around the horizon and infinity to see the effects of our quantum corrections.

$$\delta\phi_2 = \sqrt{\frac{E\sqrt{x}}{(1-\mu/x)^{5/2}} - \frac{3C^2}{(1-\mu/x)^3} + \frac{2\sqrt{x}}{C^2(1-\mu/x)^{5/2}} \int U(x)\left(1-\frac{\mu}{x}\right)^{7/2}\sqrt{x}\mathrm{d}x}\,. \tag{2.12}$$

$$\delta N \sim -\frac{2}{3}\frac{\sqrt{E}}{x^{3/4}(1-\mu/x)^{1/4}}\,, \tag{2.13}$$

2.1.3 Results

We successfully restricted the form the modification ϕ_3 can take. I also conducted a numerical analysis of the behavior of the necessary Casimer function, $U(x)$, showing asymptotically constant behavior. The final modified metric takes the form:

$$\begin{aligned} \mathrm{d}s^2 &= -(N^{(0)}+\delta N)^2\mathrm{d}t^2 + \frac{(\phi_2^{(0)}+\delta\phi_2)^2}{4x^2}\mathrm{d}x^2 + x^2\mathrm{d}\Omega^2 \\ &\sim -(N^{(0)2}+2N^{(0)}\delta N)\mathrm{d}t^2 + \frac{\phi_2^{(0)2}+2\phi_2^{(0)}\delta\phi_2}{4x^2}\mathrm{d}x^2 + x^2\mathrm{d}\Omega^2 \end{aligned} \tag{2.14}$$

valid for describing the exterior spacetime, outside the horizon.

By analyzing the asymptotic behavior around the event horizon and at infinity, we see that the quantum corrections have a ripple effect that extends beyond their local area. We succeeded in identifying quantum effects from canonical quantization of this spherically symmetric constrained system and obtained solutions in almost-closed form. The Planck-scale corrections have more effect than one might expect. We found potential new effects near the horizon and asymptotically, demonstrating that our model is sensitive to new, possibly non-local, corrections while maintaining general covariance. These corrections may be crucial for understanding the behavior around the event horizon of quantum black holes, and our methods are promising for future work.

2.2 Paper: Quasiclassical solutions for static quantum black holes

Abstract:
A new form of quasiclassical space-time dynamics for constrained systems reveals how quantum effects can be derived systematically from canonical quantization of gravitational systems. These quasiclassical methods lead to additional fields, representing quantum fluctuations and higher moments, that are coupled to the classical metric components. The new fields describe non-adiabatic quantum dynamics and can be interpreted as implicit formulations of non-local quantum corrections in a field theory. This field-theory aspect is studied here for the first time, applied to a gravitational system for which a tractable model is constructed. Static solutions for the relevant fields can be obtained in almost closed form. They reveal new properties of potential near-horizon and asymptotic effects in canonical quantum gravity and demonstrate the overall consistency of the formalism.

2.3 Introduction

For some time now, black holes have presented a popular testing ground for possible implications of quantum gravity. Examples include quantum corrections to Newton's law, modified horizon dynamics, implications for Hawking radiation, tools to address the information loss problem, potential resolutions of the central singularity, or speculations about the post-singular life of a black hole. A large variety of methods have been applied, ranging from effective field theory [14–16] to proposed non-perturbative ingredients of approaches such as string theory or loop quantum gravity.

Here, we present new results using a formulation situated on the middle ground between standard effective field theory on one hand and non-perturbative effects on the other: We extend effective field theory by applying non-adiabatic quantum dynamics, foregoing the derivative expansion of quantum corrections that is implicitly assumed when they are expressed in higher-curvature form. Our formulation will therefore be sensitive to new (and possibly non-local) corrections, while maintaining crucial consistency conditions for constraint equations and an application to space-time physics. The importance of such consistency, related to the question of whether general covariance can be maintained by quantum corrections, has recently been highlighted by the finding that most black-hole models or other space-time descriptions proposed in the field of loop quantum gravity violate covariance [17–19]. (A more careful approach that aims to maintain covariance as much as possible has been studied in [20–29], using a variety of models.) One of these no-go theorems that ruled out covariance for certain modifications encountered in models of loop quantum gravity, derived in [18], relies on the local nature of current models. The no-go theorem could therefore be evaded by constructing suitable non-local quantum corrections, possibly leading to consistent implementations of modifications in covariant models. The present paper can be considered a first step in this direction, studying non-local quantum corrections in spherically symmetric canonical quantum gravity. We will formalize our consistency conditions in more detail when we introduce relevant ingredients of space-time physics in Section 3.4.

Our formulation is based on canonical methods of non-adiabatic quantum dynamics, used for some time in various fields such as quantum chaos or quantum chemistry [30–33] mainly for systems with finitely many classical degrees of freedom. Related methods [34–36] have been applied recently to spherically symmetric models of collapsing shells [37]. Our task will be to extend these methods to quantum field theories, and to incorporate access to space-time structures in order to implement consistency conditions required for general covariance. In particular, we will consider a generalization of quasiclassical methods to constrained systems, applied to the Hamiltonian and diffeomorphism constraints of canonical general relativity. By requiring that quantum-corrected constraints obey suitable Poisson brackets, known from hypersurface deformations [38–41], we will show that such constraints can be imposed consistently and solved for modified metric components and their quantum fluctuations.

In order to reduce the complexity of these tasks, we will work with spherically symmetric models and analyze, for now, only static solutions. In a field theory, even static solutions are sensitive to non-adiabatic methods because they may vary significantly in a spatial direction. An implementation of non-adiabatic quantum dynamics with methods from other fields is therefore of interest. In this way, we will be able to explore new quantum effects in a tractable manner.

We will review canonical effective methods, which provide the mathematical basis for non-adiabatic, quasiclassical dynamics in Section 2.4. We will first summarize the well-developed version of these methods applied to the quantum mechanics of a single degree of freedom, as well as an extension to constrained systems. Section 3.4 is the central part of our paper, in which we generalize quasiclassical methods to the field theory given by spherically symmetric gravity. We will describe the form of effective constraints encountered in this system, and derive the equations to be solved for static solutions with leading quasiclassical corrections. Although multiple integrations will be required, interesting information about these solutions can be obtained in closed form, in particular regarding the near-horizon and the asymptotic behaviors. We will discuss the self-consistency of our solutions from the perspective of an intuitively expected behavior of quantum fluctuations, demonstrating that they are smaller in the asymptotic regime.

2.4 Canonical effective theories

Canonical, non-adiabatic methods of quantum dynamics provide a quasiclassical formulation in which the classical phase space, say (q, p), is extended by a certain number of quantum degrees of freedom, depending on the order in an expansion by $\hbar$. To leading order, the classical variables q and p are combined with a second canonical pair, (s, p_s) where $s = \Delta q$, such that a classical potential $V(q)$ is turned into a specific effective potential $V_{\rm eff}(q, s)$. The derivation of this effective potential (and the physical meaning of the momentum p_s) requires making use of methods of Poisson geometry.

2.4.1 Effective Hamiltonians

First, if the classical system is described by a Hamiltonian

$$H = \frac{p^2}{2m} + V(q)\,, \tag{2.15}$$

one can define an effective Hamiltonian as the expectation value $H_{\rm eff} = \langle \hat{H} \rangle$ of the corresponding Hamilton operator, taken in an arbitrary state. The effective Hamiltonian is therefore a function on the state space of the system. A systematic semiclassical description parameterizes suitable states by their expectation values of basic operators, $q = \langle \hat{q} \rangle$ and $p = \langle \hat{p} \rangle$, as well as a series of moments such as $\Delta(q^n) = \langle (\hat{q} - \langle \hat{q} \rangle)^n \rangle$. Taking into account ordering choices, we follow [42, 43] and define a specific set of moments of a state by

$$\Delta(q^n p^m) = \langle (\hat{q} - \langle \hat{q} \rangle)^n (\hat{p} - \langle \hat{p} \rangle)^m \rangle_{\rm symm} \tag{2.16}$$

in completely symmetric (or Weyl) ordering. The moment order, $n + m$, corresponds to the order in a semiclassical expansion, given by $\hbar^{(n+m)/2}$.

Moments, together with the basic expectation values, form a phase space equipped with a Poisson bracket that is obtained by extending the definition

$$\{\langle\hat{A}\rangle,\langle\hat{B}\rangle\}=\frac{\langle[\hat{A},\hat{B}]\rangle}{i\hbar} \tag{2.17}$$

using linearity and the Leibniz rule. With this bracket, the effective Hamiltonian $H_{\text{eff}}=\langle\hat{H}\rangle$ indeed generates the correct Hamiltonian dynamics: The equation

$$\{\langle\hat{A}\rangle,H_{\text{eff}}\}=\frac{\langle[\hat{A},\hat{H}]\rangle}{i\hbar}=\frac{\mathrm{d}\langle\hat{A}\rangle}{\mathrm{d}t} \tag{2.18}$$

is equivalent to quantum evolution of generic expectation values implied by the Schrödinger equation. At fixed order in $\hbar$, the resulting Poisson manifold is, in general, not symplectic. That is, it is described by a family of symplectic leaves, on which certain Casimir functions take constant values. A Casimir function has vanishing Poisson brackets with any other function on the same Poisson manifold. It therefore implies a degeneracy of the Poisson tensor which cannot be inverted to obtain a symplectic form. The dynamics are nevertheless determined uniquely because Hamilton's equations, used in what follows for evolution as well as gauge transformations, only require a Poisson bracket.

The effective Hamiltonian $H_{\text{eff}}=\langle\hat{H}\rangle$ used in (3.53) can be interpreted as a function of the moments obtained from the state that appears in the expectation value. It can be computed explicitly to order $N/2$ in $\hbar$, for any integer N, by applying a Taylor expansion to $\langle\hat{H}\rangle$ around any fixed pair of basic expectation values:

$$\begin{aligned} H_{\text{eff}} &= \langle H(\hat{q},\hat{p})\rangle=\langle H(q+(\hat{q}-q),p+(\hat{p}-p)\rangle \\ &= H(q,p)+\sum_{n+m=2}^{N}\frac{1}{n!m!}\frac{\partial^{n+m}H(q,p)}{\partial q^n\partial p^m}\Delta(q^np^m)\,. \end{aligned} \tag{2.19}$$

(Here, we assume that the Hamilton operator is Weyl ordered. For a Hamiltonian polynomial in q and p, the series always truncates at a finite order. It merely rewrites bare moments $\langle\hat{q}^n\hat{p}^m\rangle$ in terms of central moments $\Delta(q^np^m)$. These are centered around basic expectation values, according to (3.49). For non-polynomial Hamiltonians, the series is in general asymptotic.)

Written as a phase-space function, the Hamiltonian (3.54) generates equations of motion. This is accomplished by coupling basic expectation values and moments, such as $\langle\hat{q}\rangle$ and $\langle\hat{p}\rangle$, by applying Hamilton's equations with the Poisson bracket (3.53). However, while it can easily be seen that $\{\langle\hat{q}\rangle,\langle\hat{p}\rangle\}=1$ is of canonical form, the moments are not canonical variables. For instance, $\{\Delta(q^2),\Delta(p^2)\} = 4\Delta(qp)$. More generally, second-order moments of M classical degrees of freedom have brackets equivalent to the Lie algebra $\mathrm{sp}(2M,\mathbb{R})$ [44,45], while higher-order moments have brackets quadratic in moments [42,46].

It is therefore convenient to apply a transformation from moments to canonical coordinates. Such a transformation always exists locally, according to the Darboux theorem [47] or its extension to Poisson manifolds [48]. To second order for a single classical degree of freedom, canonical coordinates for the moments $\Delta(q^2)$, $\Delta(qp)$ and $\Delta(p^2)$ are given by (s,p_s) such that [30,31,33]

$$\Delta(q^2) = s^2 \quad , \quad \Delta(qp) = sp_s \quad , \quad \Delta(p^2) = p_s^2 + \frac{U}{s^2} \tag{2.20}$$

with a Casimir function U, restricted by Heisenberg's uncertainty relation to obey the inequality $U \geq \hbar^2/4$. As a Casimir function, U has vanishing Poisson brackets with any other phase-space function that depends only on basic expectation values and second-order moments. In particular, its Poisson bracket with the Hamiltonian vanishes in a second-order truncation, which means that U is conserved to this order. In quantum mechanics, the phase-space function U is reduced to a constant on any given solution which determines how close the evolving state is to saturating the uncertainty relation. One of the more technical aims of the present paper will be to explore the role of U in a field theory, where it may be a function of spatial coordinates.

Inserting the canonical form (3.55) of moments in the expansion (3.54) for $N = 2$, assuming a classical-mechanics Hamiltonian with generic potential $V(q)$, we obtain

$$H_{\mathrm{eff}} = \frac{p^2}{2m} + \frac{p_s^2}{2m} + \frac{U}{2ms^2} + V(q) + \frac{1}{2}V''(q)s^2\,. \tag{2.21}$$

The last three terms together form the effective potential

$$V_{\mathrm{eff}}(q,s) = \frac{U}{2ms^2} + V(q) + \frac{1}{2}V''(q)s^2\,. \tag{2.22}$$

The independent quantum degree of freedom s describes quantum corrections by two terms in the effective potential: The first term, $U/(2ms^2)$, originates in the kinetic energy or momentum fluctuations. In the effective picture, its U/s^2-form (where U is strictly positive) prevents position fluctuations s from reaching zero. The other term, $\frac{1}{2}V''(q)s^2$, may be positive or negative depending on the classical potential. It is positive around local minima, where it raises the ground-state energy by a term analogous to zero-point fluctuations. The term is negative around local maxima, which would be relevant in quasiclassical descriptions of tunneling phenomena.

The description is non-adiabatic because no assumption has been made about the rate of change of s compared with q. If, by contrast, one assumes that s changes slowly and merely tracks its q-dependent minimum

$$s_{\min}(q) = \sqrt[4]{\frac{U(x)}{mV''(q)}} \tag{2.23}$$

of a ground state in the potential (2.22), one obtains a q-dependent effective potential

$$V_{\mathrm{low-energy}}(q) = V(q) + \sqrt{\frac{U(x)V''(q)}{m}}\,. \tag{2.24}$$

This quasiclassical result equals the standard low-energy effective potential for the minimum value $U = \hbar^2/4$ [42,49]. For the harmonic oscillator, for instance, $V''(q) = m\omega^2$ implies the correct zero-point energy $\frac{1}{2}\hbar\omega$. A higher-order adiabatic approximation implies higher-derivative corrections to the classical equations of motion [50]. An adiabatic approximation to all orders would imply a non-local theory with time derivatives of arbitrarily high orders. Such a non-local theory, which is often complicated because it cannot be analyzed by solving local partial differential equations, can more easily be studied by keeping s as an independent field in a non-adiabatic quasiclassical formulation. (From the point of view of the non-local theory, s would be considered an auxiliary field that makes it possible to write non-local equations in local form. Here, however, s has physical meaning; s represents quantum fluctuations in one of the classical degrees of freedom. The local formulation is therefore more physical than an alternative non-local theory obtained by eliminating s by partially solving equations for it in an adiabatic expansion.)

Along similar lines, a canonical moment description of field theories has been performed in [51], where the analog of (2.24) is the Coleman–Weinberg potential [52]. Here, we apply canonical moment methods to a field theory motivated by spherically symmetric gravitational systems. This formluation retains independent quantum degrees of freedom, such as a field version of s. We therefore derive non-adiabatic or non-local effects of quantum gravity.

2.4.2 Effective constraints

Relativistic systems are subject to constraints, instead of Hamiltonian evolution with respect to an absolute time. The formalism of effective and quasiclassical methods therefore has to be generalized to constrained systems, as done in [53–55]. The main observation is that the presence of new quantum degrees of freedom, such as $\Delta(q^2)$, implies additional constraints compared with the classical theory.

An effective constraint,

$$C_{\text{eff}} = \langle \hat{C} \rangle \tag{2.25}$$

for a constraint operator $\hat{C}$, is defined just like an effective Hamiltonian. An effective constraint is a function on the phase space of basic expectation values and moments, which can be computed by Taylor expansion as in (3.54). The Hamilton's equations generated by C_{eff} correspond to gauge transformations rather than strict evolution. According to Dirac's quantization procedure for constrained systems, effective constraints must vanish on physical solutions, $C_{\text{eff}} = 0$. This is because the constraint operator $\hat{C}$ annihilates any admissible state upon which it acts. (As a general phase-space function, C_{eff} is obtained for states in the so-called kinematical Hilbert space of states not necessarily annihilated by $\hat{C}$. In addition, solving the equation C_{eff} implicitly restricts solutions to the physical Hilbert space of states annihilated by $\hat{C}$.)

Classical constraints, where $C(q,p)$ and $f(q,p)C(q,p)$ as phase-space functions imply the same gauge flow on the constraint surface, and have the same solution space as long as $f \neq 0$. In contrast, expressions such as $\langle \hat{C} \rangle$, and $\langle f(\hat{q},\hat{p})\hat{C} \rangle$ in general, imply independent functions when expressed in terms of basic expectation values and moments. (For instance, in the simple case of $\hat{C} = \hat{p}$ and $f(\hat{q},\hat{p})$, the constraint $\langle \hat{C} \rangle = \langle \hat{p} \rangle = 0$ restricts the expectation value $\langle \hat{p} \rangle$, while $\langle f(\hat{q},\hat{p})\hat{C} \rangle = \langle \hat{p}^2 \rangle = 0$ then requires zero variance as well. In a kinematical state, the expectation value $\langle \hat{p}$ and the variance $\Delta(p^2)$ can be chosen independently, for instance in a standard Gaussian wave function.) Effective descriptions of singly-constrained classical systems are therefore subject to multiple constraints. These constraints are of a number that depends on the order of moments considered. Based on [53, 54], it is convenient to organize higher-order constraints by powers of the same basic operators used in the moments that describe a given system. In addition to $C_{\text{eff}} = \langle \hat{C} \rangle$, we have independent constraints

$$C_{q^n p^n} = \left\langle ((\hat{q} - \langle \hat{q} \rangle)^n (\hat{p} - \langle \hat{p} \rangle)^m)_{\text{Weyl}} \hat{C} \right\rangle \tag{2.26}$$

for integer n and m such that $n + m \geq 1$.

While we symmetrize products of non-commuting $\hat{q}$ and $\hat{p}$, we have to keep $\hat{C}$ to the right, to make sure that it always acts on the state used in the expectation value. In general, higher-order effective constraints therefore take complex values. Solving them for moments then results in complex values. This indicates that the inner product used on the kinematical Hilbert space which defines effective constraint functions is adjusted when a physical Hilbert space is introduced for the solution space. In an effective constrained system, the transition from a kinematical to a physical Hilbert space, which can be very complicated in generic quantum systems and is in general uncontrolled, is implicitly performed by simply imposing reality conditions for combinations of moments that solve the constraints. The consistency of this approach has been demonstrated in several examples [53, 54, 56–61].

Because $\langle \hat{O} - \langle \hat{O} \rangle \rangle = 0$ for any operator $\hat{O}$, all terms in higher-order constraints (2.26) contain at least one moment factor. Therefore, they can be considered as constraints on the moments, supplementing the effective constraint (2.25) which restricts basic expectation values, subject to quantum corrections depending on moments. Since moments up to a given order in general form a Poisson manifold that is not symplectic, applying the usual constraint formalism requires a generalization to Poisson manifolds as given in [62]. In particular, it is possible for a number N of first-class constraints (that is, C_i with $i = 1, \ldots, N$ such that all Poisson brackets $\{C_i, C_j\} \approx 0$ vanish on the solution space of the constraints C_i) to generate gauge flows that span a hypersurface of dimension less than N.

The formalism of effective constraints has a straightforward generalization to systems with more than one classical constraint. If the classical constraints are first class, the corresponding effective and higher-order constraints are then guaranteed to be first class as well. A new feature arises in constrained systems with structure functions, as in general relativity. If there is a first-class quantization with constraint operators $\hat{C}_i$ such that $[\hat{C}_i, \hat{C}_j] = i\hbar \sum_k \hat{f}_{ij}^k \hat{C}_k$ with operator-valued coefficients $\hat{f}_{ij}^k$, effective constraints have the Poisson-bracket relations [55]

$$\{C_{i,\text{eff}}, C_{j,\text{eff}}\} = \sum_k \langle \hat{f}_{ij}^k \hat{C}_k \rangle = \sum_k f_{ij\,\text{eff}}^k C_{k,\text{eff}} + \cdots \tag{2.27}$$

where $f_{ij\,\text{eff}}^k$ are effective structure functions obtained from $\langle \hat{f}_{ij}^k \rangle$, and the dots indicate neglected higher-order constraints. For systems with structure functions, the basic effective constraints (2.25) and higher-order constraints (2.26) are therefore coupled in the constraint algebra, forming an enlarged system of underlying gauge symmetries.

It is an interesting question whether such an enlarged system in models of gravity can be interpreted as an extended space-time structure. Here, we will not address this question in complete generality because we will restrict our attention to static solutions. However, our constraints will have higher-order corrections, allowing us a glimpse on what moment-based extended space-time structures might entail. In our technical analysis, we will combine the formalism of effective constraints with a field-theory version of the canonical variables (3.55) for moments, restricted to spherical symmetry. The metric components that determine the fields of spherically symmetric gravity will be complemented by an additional canonical field, ϕ_3, representing quantum fluctuations of ϕ_2.

2.5 Space-time in quasiclassical form

In a classical canonical formulation of general relativity, the line element of spherically symmetric space-times is defined by

$$\begin{aligned} \mathrm{d}s^2 &= -N(t,x)^2\mathrm{d}t^2 + q_{xx}(t,x)\left(\mathrm{d}x + M(t,x)\mathrm{d}t\right)^2 \\ &\quad + q_{\varphi\varphi}(t,x)\mathrm{d}\Omega^2 \end{aligned} \tag{2.28}$$

with the lapse function N, the radial component M of the shift vector, and two independent spatial metric components, q_{xx} and $q_{\varphi\varphi}$.

The definition of a line element entails that it implies coordinate invariant geometrical statements such as distances, areas or volumes as well as physically important concepts such as geodesics or horizons. A geometry described by a line element can therefore be evaluated with any choice of coordinates, or any conditions slicing space-time into spatial hypersurfaces. However, individual metric components such as N or q_{xx} are not invariant and must transform in a specific way under coordinate changes for the line element to be invariant. Classically, this consistency condition is described by the tensor-transformation law for the space-time metric. But it is not clear that quantization (even in a quasiclassical form, which avoids operators but amends terms—such as N and q_{xx}—by quantum corrections δN and δq_{xx}) can maintain this condition.

We will use a canonical approach, in which the space-time metric is replaced by time-dependent families of fields (for $q_{xx}(t)$ and $q_{\varphi\varphi}(t)$, as well as their momenta). We will do this such that fixing the value of t is classically equivalent to fixing a constant-t

hypersurface in space-time. The lapse function N and shift vector M then appear as coefficients in evolution equations for these fields. These evolution equations are obtained as Hamilton's equations generated by a phase-space function, which can be written in the form $H[N]+D[M]$ with the Hamiltonian constraint H and the diffeomorphism constraint D. Since changes of hypersurfaces are gauge transformations, their generators H and D are constrained to vanish. Several consistency conditions then immediately arise, because the constraints $H=0$ and $D=0$ must hold at all times. Therefore, they must be preserved by Hamiltonian evolution generated by $H[N]+D[M]$, and the combination of two slicing changes must be another slicing change. In technical terms, the constraints must therefore be first-class. They must also have Poisson brackets suitable for the geometrical form of hypersurface deformations in space-time. Since it is difficult to evaluate these conditions for quantum-corrected constraints, we will do so here only for a specific class of gauge transformations that preserve the static nature of solutions. We will therefore check that the Poisson brackets of constraints have the correct form, only in the case of vanishing momenta. What we will now refer to as consistency conditions has the following ingredients:

- There is a quasiclassical set of constraints, of the form $H+\delta H$ and $D+\delta D$, where H and D are the classical expressions. Additionally, δH and δD depend on quantum fluctuations, in a specific way derived from the classical constraints following (3.54).

- The constraint brackets remain first class, and of hypersurface-deformation form, when restricted to the phase-space submanifold of vanishing momenta. This value, $\{(H+\delta H)[N_1],(H+\delta H)[N_2]\}$, is proportional to the diffeomorphism constraint, and therefore vanishes when restricted to the submanifold of vanishing momenta. We set the momenta equal to zero, only after evaluating the Poisson bracket, which therefore is not trivially zero.

- We will be able to go slightly beyond the preceding condition by comparing momentum-dependent terms in the Poisson bracket $\{(H+\delta H)[N_1],(H+\delta H)[N_2]\}$ with terms expected from the classical structure function resulting from this bracket. Some terms are as expected, but others are not. This observation highlights the necessity of vanishing momenta at the current stage of developments for quasiclassical constraints.

- In practical terms, we will explicitly demonstrate that all the constraint and evolution equations of the quasiclassical system have mutually consistent static solutions with the desired classical limit.

We will now recall detailed definitions of the phase-space variables and properties of the constraints.

2.5.1 Variables and constraints

At any x, the phase space of metric components has a boundary given by the inequality $\det q = q_{xx}q_{\varphi\varphi}^2 > 0$. A canonical quantization of these variables therefore requires some care [63, 64]. Here, we avoid this issue by using a triad formulation, with two components E^x and E^φ of a (densitized) triad at each x, related to the metric components by the canonical transformation

$$q_{xx} = \frac{(E^\varphi)^2}{|E^x|} \quad , \quad q_{\varphi\varphi} = |E^x|\,. \tag{2.29}$$

(These components of a spherically symmetric metric are derived from the general relationship $q^{ab} = E^a_i E^{bi}/|\det(E^c_j)|$ between the inverse spatial metric and a densitized triad E^a_i.) In our explicit calculations, we will assume $E^x > 0$, corresponding to a right-handed triad. But in general, E^x, unlike the metric components, may take negative values for a left-handed triad thanks to absolute values in (2.29), and the sign of E^φ does not matter thanks to the quadratic appearance in (2.29). In a triad formulation, it is therefore possible to apply standard canonical quantization of a phase space without boundaries. According to the appearance of E^x and E^φ in the spatial metric, the former (times 4π) represents the areas of 2-spheres at a constant radial coordinate x, while the latter determines the radial distance.

Momenta of the triad fields are classically given by the components of extrinsic curvature, such that we have basic Poisson brackets [65–67]

$$\begin{aligned} \{K_x(x), E^x(y)\} &= 2G\delta(x,y) \quad \text{and} \\ \{K_\varphi(x), E^\varphi(y)\} &= G\delta(x,y) \end{aligned} \tag{2.30}$$

with Newton's constant G. (There is no factor of two in the second equation because the angular direction represents two degrees of freedom on a 2-sphere that are strictly related by spherical symmetry.) The relationship between K_x and K_φ and derivatives of the triad components follows from equations of motion of the classical theory. Classically, K_φ is proportional to the change in time of E^x or of 2-sphere areas, while K_x determines the change in time of the radial distance. These relationships, in general, may be modified by quantum effects introduced in the canonical dynamics.

Specific equations of motion are generated by a combination of constraints, the Hamiltonian constraint $H[N]$, corresponding to conservation of energy, and the diffeomorphism constraint $D[M]$, corresponding to conservation of momentum, such that $\dot{f} = \{f, H[N]+D[M]\}$ for any phase-space function f. The dot refers to a time derivative in the direction of an evolution vector field $t^a = Nn^a + Me^a$ determined by lapse and shift [40], where n^a is the future-pointing unit normal to a space-like foliation and e^a a unit vector tangential to the foliation. In classical spherically symmetric gravity, the constraints as phase-space functions take the form

$$\begin{aligned} H[N] &= -\frac{1}{G}\int \mathrm{d}x\, N\left(\frac{E^\phi}{2\sqrt{E^x}}K_\varphi^2 + \sqrt{E^x}K_\varphi K_x \right. \\ &\quad + \frac{E^\phi}{2\sqrt{E^x}} - \frac{((E^x)')^2}{8\sqrt{E^x}E^\phi} \\ &\quad \left. + \frac{\sqrt{E^x}(E^x)'(E^\phi)'}{2(E^\phi)^2} - \frac{\sqrt{E^x}(E^x)''}{2E^\phi}\right) \end{aligned} \tag{2.31}$$

and

$$D[M] = \frac{1}{2G}\int \mathrm{d}x\, M\left(2K_\varphi' E^\varphi - K_x(E^x)'\right) . \tag{2.32}$$

They form a first-class system with brackets

$$\{D[M_1], D[M_2]\} = D[M_1M_2' - M_2M_1'] \tag{2.33}$$

$$\{H[N], D[M]\} = -H[MN'] \tag{2.34}$$

and

$$\begin{aligned} &\{H[N_1], H[N_2]\} \\ &= -D[E^x(E^\varphi)^{-2}(N_1N_2' - N_2N_1')] \end{aligned} \tag{2.35}$$

corresponding to deformations of spacelike hypersurfaces in classical space-times with spherical symmetry. The structure function $E^x/(E^\varphi)^2$ in the last equation is the only component of the inverse spatial metric that contributes if spherical symmetry is imposed.

Any $(1+1)$-dimensional triad theory subject to brackets (3.19), (3.20) and (3.21) is generally covariant [68], in the sense that solutions of the theory are subject to gauge transformations equivalent to space-time coordinate transformations. The general form of the brackets should therefore be maintained by quantum corrections. More generally, it may be possible that quantum corrections preserve the first-class nature of the two constraints, $H[N]$ and $D[M]$, but with modified brackets. In particular, as in (2.27) the phase-space function $E^x(E^\varphi)^{-2}$ in (3.21) may be quantum corrected if E^x and E^φ are quantized. Such a theory would still be consistent, but it may not describe space-time with Riemannian geometry. It would rather describe a quantum version of space-time with a structure that depends on the detailed modification of the coefficient $E^x(E^\varphi)^{-2}$, or on higher-order versions (2.26) of the gravitational constraints.

Quantum corrections considered in the present paper will not present a clear modification of the structure function, but information about this possibility is limited by the restriction to static configurations that we will make for tractable equations. Further analysis of non-static solutions will be necessary before statements about the quasiclassical structure of space-time can be made. Nevertheless, within the setting to be developed here, it is possible to study implications of quantum effects on specific static solutions. To this end, we initiate and apply here a canonical description of quasiclassical quantum field theory. The canonical nature makes it possible to extend the Poisson brackets used in (3.19) and (3.21) to constraints amended by quantum corrections. Consistent orderings of constraint operators are known in spherically symmetric quantum gravity [69, 70], which guarantees that closed effective constraint brackets of the form (2.27) exist. The required equations have been derived explicitly in [71], where consistency was confirmed independently. The quasiclassical nature means that we will be able to include key features such as quantum fluctuations or uncertainty relations, in our analysis. (In addition, factor ordering choices matter in quantum constraints, which in our context imply certain imaginary contributions to effective constraints that we will not consider in detail here.)

It will also turn out to be important that the methods we use, which are generalized versions of what has been known for some time in quantum chemistry [33], are non-adiabatic. In our context, this non-adiabaticity means that we will not be required to express quantum corrections in the form of a derivative expansion, as implicitly done by common methods of quantum field theory such as low-energy potentials or Feynman expansions. Quantum corrections are rather expressed in terms of independent degrees of freedom that physically correspond to fluctuations or higher moments of a state.

2.5.2 Canonical fields

Before we implement fluctuation variables, we transform our current fields to strictly canonical form, removing a factor of two in (2.30). (From now on, we choose units such that $2G = 1$.) Also renaming the fields, this transformation is accomplished by introducing

$$\phi_1 = E^x \quad , \quad p_1 = -K_x \quad , \quad \phi_2 = 2E^\varphi \quad , \quad p_2 = -K_\varphi \,. \tag{2.36}$$

In this notation, ϕ_1 and ϕ_2 therefore represent the metric components with momenta p_1 and p_2. Classically, the momentum fields have the usual interpretation as extrinsic curvature, but this relationship will be modified by quantum corrections. In these variables, the Hamiltonian constraint takes the form

$$H[N] = -\int \mathrm{d}x N(x) \left(\frac{\phi_2 p_2^2}{2\sqrt{\phi_1}} + 2\sqrt{\phi_1} p_1 p_2 + \left(1 - \left(\frac{\phi_1'}{\phi_2} \right)^2 \right) \frac{\phi_2}{2\sqrt{\phi_1}} - 2 \left(\frac{\phi_1'}{\phi_2} \right)' \sqrt{\phi_1} \right) \tag{2.37}$$

while

$$D[M] = \int \mathrm{d}x M(x) \left(-\phi_1' p_1 + p_2' \phi_2 \right) \,. \tag{2.38}$$

The number of independent fields can be reduced by making a gauge choice for E^x or ϕ_1 such that x is the usual area radius: $\phi_1 = x^2$. The gauge-fixing condition, $g(x) = \phi_1(x) - x^2$ for all x, then forms a second-class pair of constraints, together with the diffeomorphism constraint. This is because $\{g(x), D[M]\} = -2M(x)\phi_1(x)\phi_1'(x) \approx -4M(x)x^3 \neq 0$, unless $x = 0$ or $M(x) = 0$. Here, $\approx$ indicates that we have used $g(x) = 0$ in this step. For second-class constraints, we have to solve both conditions, $D[M] = 0$ for all M and $g(x) = 0$ for all x. We do this while removing the diffeomorphism constraint and fixing its gauge freedom, by using a specific radial coordinate x, such that $\phi_1(x) = x^2$. In the static case, $D[M]$ is automatically zero. However, its gauge flow, restricted to the submanifold of zero momenta in phase space, does not identically vanish. This is because it may still change ϕ_1 and ϕ_2. This freedom is fixed by imposing the condition $g(x) = 0$.

The remaining flow generated by the Hamiltonian constraint will be time evolution for a given lapse function N. The only fluctuating field will then be ϕ_2, for which we introduce an independent quantum degree of freedom ϕ_3 as a field version of $s = \Delta q$ as recalled for quantum mechanics in Section 2.4.1, together with a momentum field p_3. Therefore,

$$\Delta(\phi_2^2) = \phi_3^2 \quad , \quad \Delta(\phi_2 p_2) = \phi_3 p_3 \quad , \quad \Delta(p_2^2) = p_3^2 + \frac{U(x)}{\phi_3^2} \,. \tag{2.39}$$

As our notation indicates, the Casimir function U, which was a function on phase space but constant along solutions in quasiclassical quantum mechanics, may now be a function of the spatial coordinate x, just like the other canonical fields. There are no equations of motion for U because it does not have a momentum field. One of the aims of this paper is to look for additional consistency conditions that may be used to determine U based on U-dependent equations of motion for the other fields.

In order to determine how the new fields appear in an effective Hamiltonian, we need to perform a Taylor expansion of $H[N]$ by ϕ_2 and p_2, which is rather lengthy. The result is that the effective Hamiltonian constraint is of the form

$$\bar{H}[N] = H[N] + H_2[N] \tag{2.40}$$

with the classical $H[N]$ from (2.37) and a correction

$$\begin{aligned} H_2[N] &= \int \mathrm{d}x N(x) \left(\frac{1}{2} \frac{\partial^2 H}{\partial p_2^2} \left(p_3^2 + \frac{U}{\phi_3^2} \right) + \frac{\partial^2 H}{\partial \phi_2 \partial p_2} \phi_3 p_3 + \frac{1}{2} \frac{\partial^2 H}{\partial \phi_2^2} \phi_3^2 + \frac{\partial^2 H}{\partial \phi_2 \partial \phi_2'} \phi_3 \phi_3' \right) \\ &= -\int \mathrm{d}x N(x) \left(\frac{\phi_2 p_3^2}{2\sqrt{\phi_1}} + \frac{\phi_3 p_2 p_3}{\sqrt{\phi_1}} + \left(6 \frac{\sqrt{\phi_1} \phi_1' \phi_2'}{\phi_2^4} - \frac{1}{2} \frac{(\phi_1')^2}{\sqrt{\phi_1} \phi_2^3} - 2 \frac{\phi_1'' \sqrt{\phi_1}}{\phi_2^3} \right) \phi_3^2 \right. \\ &\quad \left. -4 \frac{\sqrt{\phi_1} \phi_1' \phi_3 \phi_3'}{\phi_2^3} + \frac{U(x) \phi_2}{2\sqrt{\phi_1} \phi_3^2} \right) . \end{aligned} \tag{2.41}$$

As in (3.54), the effective Hamiltonian follows from a Taylor expansion, here in terms of $\phi_2(x)$ at any x. The leading corrections are expressed in terms of second-order partial derivatives in the first line of the preceding equation, which are evaluated in the next two lines.

The last term in (3.68), $\frac{1}{2} U(x) \phi_2 \phi_1^{-1/2} \phi_3^{-2}$, is implied by (3.61). Its analog in quantum mechanics has a contribution from zero-point fluctuations [51] that would be subtracted out in a quantum field theory, or be subject to renormalization. (See also the simple example we gave after (2.24).) For this reason, and because uncertainty relations for operator-valued fields are less clear than those of quantum mechanics, we will not impose a non-zero lower bound on $U(x)$ such as $\hbar^2/4$. We will, however, require that $U(x)$ be positive for all x, motivated by its interpretation as a remnant of zero-point fluctuations. The value of $U(x)$ at a given position can then be used as an indication of the strength of quantum effects.

From the perspective of hypersurface deformation generators, the U-term in (3.68) does not contribute to the Poisson bracket of two Hamiltonian constraints because it does not contain any spatial derivatives or momenta. Therefore, it does not have an effect on the main consistency test performed in this paper, given by closure of the quasiclassical constraints in the static limit. The term will, however, affect our static solutions to be derived below.

2.5.3 Spatial diffeomorphisms

It is noteworthy that the function $U(x)$, according to its first appearance in (3.61), should have spatial density weight two so as to be consistent with a density weight one of ϕ_3 (inherited from ϕ_2) and density weight zero of p_3. This property provides further motivation for allowing $U(x)$ to be a function of x, rather than a constant which would be possible for a density only in a specific spatial coordinate choice. Moreover, any lower bound such as $\hbar^2/4$, imposed on a density, would not be respected by transformations of the spatial coordinate, while positivity $U(x) \geq 0$ is compatible with a density weight.

The density weight of $U(x)$ also implies that the U-term in (3.68) has the correct density weight one, as expected for any contribution to a spatial integrand. If the density weight were ignored, the term would have density weight minus one because ϕ_2 and ϕ_3 have the same transformation property according to (3.61), and ϕ_2 has density weight one. This unconventional transformation behavior, if it were used, would be analogous to a property studied in the minisuperspace context [72–74], where it originated in a contribution to the dynamics from infrared modes included in a symmetric model. Spatially homogeneous minisuperspace models do not provide control over the density weight because the spatial dependence of all functions is ignored. The present paper is the first one that studies this phenomenon in a field-theory setting in which density weights can be determined unambiguously. We will see below that the density weight of $U(x)$ may be ignored consistently if only spatial transformations are considered that are generated by a quantum corrected diffeomorphism constraint equal to the Poisson bracket of two Hamiltonian constraints (including the structure function). This generator is sufficient for formal consistency of the quasiclassical constraints. However, if one tries to analyze full covariance under all spatial coordinate transformations, which lies outside the scope of the present paper, there may be further subtleties related to spatial transformations in spatially inhomogeneous quantum midisuperspace models.

The effective diffeomorphism constraint does not follow directly from the quantum-mechanics model because its structure is rather different from a Hamiltonian. However, we may expect that the effective diffeomorphism constraint should be of the form

$$\bar{D}[M] = \int \mathrm{d}x M(x)(-\phi_1' p_1 + p_2'\phi_2 + p_3'\phi_3) \,. \tag{2.42}$$

Unlike ϕ_1, which transforms as a standard scalar field in the symmetry reduced model, the field ϕ_2 transforms with density weight one, a property that is inherited by the original appearance of ϕ_2 in the metric components. The new field ϕ_3, which represents quantum fluctuations of ϕ_2, is assigned the same density weight. These considerations explain the different signs and positions of spatial derivatives in the three terms of (2.42).

The new term, compared with the classical constraint, can be derived from a quantized $p_2'\phi_2$ after applying a point-splitting procedure: In order to evaluate the expectation value of a product of field operators, defining the effective diffeomorphism constraint, we follow the quantum mechanics example of (3.54). We first introduce two slightly different positions for $\hat{p}_2'(x)$ and $\hat{\phi}_2(y)$, such that the prime uniquely refers to a derivative only of $\hat{p}_2$. This holds, even in a product of these two operators or in the quantum covariance $\langle\hat{p}_2'(x)\hat{\phi}_2(y)\rangle_{\rm symm} = {\rm d}\langle\hat{p}_2(x)\hat{\phi}_2(y)\rangle_{\rm symm}/{\rm d}x$. Taking the limit $x\to y$ after moving the derivative out of the expectation value, we obtain $\langle\hat{p}_2'(x)\hat{\phi}_2(x)\rangle_{\rm symm} = \lim_{y\to x}{\rm d}\langle\hat{p}_2(x)\hat{\phi}_2(y)\rangle_{\rm symm}/{\rm d}x$, without any ambiguity as to which operator the derivative is acting on. Continuing with this equation, we have

$$\begin{aligned}\langle\hat{p}_2'(x)\hat{\phi}_2(x)\rangle_{\rm symm} &= \lim_{y\to x}\frac{\rm d}{{\rm d}x}\langle\hat{p}_2(x)\hat{\phi}_2(y)\rangle_{\rm symm} = \lim_{y\to x}\frac{\rm d}{{\rm d}x}\Big(\langle\hat{p}_2(x)\rangle\langle\hat{\phi}_2(y)\rangle + \Delta(p_2(x)\phi_2(y))\Big)\\ &= \lim_{y\to x}\frac{\rm d}{{\rm d}x}\Big(\langle\hat{p}_2(x)\rangle\langle\hat{\phi}_2(y)\rangle + p_3(x)\phi_3(y)\Big) = p_2'\phi_2 + p_3'\phi_3\,. \end{aligned} \tag{2.43}$$

(We may assume the symmetric ordering of $\hat{p}_2'$ and $\hat{\phi}_2$ because reordering terms of the quadratic expression would merely be introduce constants.) This form of the diffeomorphism constraint is also consistent with the transformation behavior of ϕ_3 which, like ϕ_2, should be a scalar density, as already observed in (2.42).

A schematic operator version of the diffeomorphism constraint can also be used to determine which higher-order constraints should contribute to the effective constraint brackets, as in (2.27). The classical bracket (3.21), after fixing $\phi_1 = E^x = x^2$ to be non-dynamical, shows that the two expectation values $-4\langle\hat{\phi}_2^{-2}\phi_1\phi_1'p_1\rangle$ and $4\langle\phi_1\hat{\phi}_2^{-1}\hat{p}_2'\rangle$ will be relevant, which we should expand by moments of ϕ_2 and p_2. Ignoring ordering questions for now, we therefore expect the replacements

$$\begin{aligned} -4\frac{\phi_1\phi_1'p_1}{\phi_2^2} &\to -4\left\langle\frac{\phi_1\phi_1'p_1}{(\phi_2+\widehat{\Delta\phi}_2)^2}\right\rangle\\ &\sim -4\frac{\phi_1\phi_1'p_1}{\phi_2^2} - 12\frac{\phi_1\phi_1'\phi_3^2p_1}{\phi_2^2} \end{aligned} \tag{2.44}$$

and

$$
\begin{aligned}
4\frac{\phi_1 p_2'}{\phi_2} &\to 4\left\langle \frac{\phi_1(p_2+\widehat{\Delta p_2})'}{\phi_2+\widehat{\Delta\phi_2}} \right\rangle \\
&\sim 4\frac{\phi_1 p_2'}{\phi_2} + \frac{\phi_1\phi_3^2 p_2'}{\phi_2^3} - 4\frac{\phi_1\phi_3 p_3'}{\phi_2^2}
\end{aligned}
\tag{2.45}
$$

where $\widehat{\Delta\phi_2} = \hat{\phi}_2 - \phi_2$ and $\widehat{\Delta p_2} = \hat{p}_2 - p_2$. Interestingly, the last term in the preceding equation cancels out completely with the last term in (2.42) once the latter equation is evaluated with the structure function according to (3.21). We therefore do not expect a term proportional to p_3' in the bracket of two Hamiltonian constraints, even though it appears in (2.42).

With this result we can return to the U-term in (3.68), proportional to $U\phi_2/(\sqrt{\phi_1}\phi_3^2)$. Even if the density weight of $U(x)$ is ignored, this term is consistent with gauge transformations generated by a quantum-corrected diffeomorphism constraint that includes the structure function expected for the bracket of two Hamiltonian constraints: Due to the fact that the p_3-term is expected to cancel out in this expression, these gauge transformations do not act on the ϕ_3-dependence of the U-term. If this dependence is ignored for the purpose of counting density weights relevant for a Poisson bracket with the diffeomorphism constraint, the remaining dependence on ϕ_2 provides the expected density weight of one, suitable for an integrand. (If the density weight of ϕ_3 is included in the count, one has to assign a density weight two to $U(x)$. As already mentioned, this definition is likely necessary if one attempts to extend diffeomorphism to arbitrary shift vectors. If the structure function is not included in the diffeomorphism constraint, the latter depends on p_3 and is sensitive to the density weight of $|\phi_3\rangle$.)

As a further test of mutual consistency of the quasiclassical constraints, we now evaluate the bracket of two Hamiltonian constraints in more detail. The derivation of $\{\bar{H}[N], \bar{H}[M]\}$ can be split up into smaller calculations using

$$
\begin{aligned}
&\{\bar{H}[N], \bar{H}[M]\} \\
&\quad = \{H[N], H[M]\} + \{H[N], H_2[M]\} + \{H_2[N], H[M]\} + \{H_2[N], H_2[M]\} \\
&\quad = \{H[N], H[M]\} + \{H[N], H_2[M]\} - \{H[M], H_2[N]\} + \{H_2[N], H_2[M]\}
\end{aligned}
\tag{2.46}
$$

based on the antisymmetry of the Poisson bracket. We already have the first term in (2.46), so we only need to derive the second term, $\{H[N], H_2[M]\}$, and the last term,

$\{H_2[N], H_2[M]\}$. The third term can then be obtained from the second term by flipping N and M. The last bracket, $\{H_2[N], H_2[M]\}$, and the combination $\{H[N], H_2[M]\} - \{H[M], H_2[N]\}$ are antisymmetric in N and M. It is therefore sufficient to consider only terms in which a spatial derivative of ϕ_2 or ϕ_3 appears, which after integration by parts then leads to the non-zero antisymmetric combination $NM' - N'M$. We are interested here in the Poisson brackets of gauge generators, relevant for our consistency conditions, as well as asymptotically flat solutions close to the classical case for large x. Therefore, the lapse function is required to drop off to zero at infinity, while the ϕ-dependent terms in the constraint remain finite. (The fields ϕ_1 and ϕ_2 asymptotically grow like x^2 and x, respectively, but the momentum-independent terms in the Hamiltonian constraint contain only ratios or derivatives of these fields with finite limits.) We can then ignore boundary terms when integrating by parts for gauge generators. Lapse functions with non-zero limits at infinity correspond to symmetry generators in the asymptotically flat region, which we do not consider here.

A lengthy calculation produces the result

$$
\begin{aligned}
\{\bar{H}[N], \bar{H}[N]\} = & \int \mathrm{d}x (NM' - N'M) \left(-4\frac{\phi_1\phi_1'}{\phi_2^2} p_1 + 4\frac{\phi_1}{\phi_2} p_2' \right) + \\
& \left(12\frac{\phi_1\phi_1'\phi_3^2}{\phi_2^4} p_1 - 4\frac{\phi_1\phi_3^2}{\phi_2^3} p_2' - 2\frac{\phi_1'\phi_3}{\phi_2^2} p_3 \right) \\
= & \int \mathrm{d}x (NM' - N'M) \frac{4\phi_1}{\phi_2^2} (-\phi_1' p_1 + \phi_2 p_2' + 3\frac{\phi_3^2}{\phi_2^2} \phi_1' p_1 \\
& - \frac{\phi_3^2}{\phi_2} p_2' - \frac{1}{2} \phi_1' \phi_3 p_3)
\end{aligned}
\tag{2.47}
$$

for the Poisson bracket of two Hamiltonian constraints. The first two terms are the classical diffeomorphism constraint with the correct structure function, while the next two terms are quantum corrections as expected from the expansions (2.44) and (2.45). The last term does not correspond to a contribution in the diffeomorphism constraint. It can be seen as a consequence of our reduction, which includes quantum corrections only of ϕ_2 but not of ϕ_1. In particular, a complete effective constraint would include moments such as $\Delta(\phi_2\phi_1')$ as well as $\Delta(p_1p_2)$ with a bracket that can contribute to the ϕ_3p_3-term we obtained here. As shown by the consistency check in [71], all such contributions indeed cancel out in the complete effective system, while they do not completely cancel out in our reduction. The left-over contribution here re-introduces a p_3-dependence that generates non-trivial transformations on the U-term in (3.68). Our system is therefore not fully consistent if generic spherically symmetric configurations are considered, but it may be used for static solutions for which the last term in $\{\bar{H}[N], \bar{H}[N]\}$ vanishes. The solutions we obtain are also reliable as consistent configurations in the complete system in which all cross-correlations between ϕ_1 and ϕ_2 vanish.

2.5.4 Higher-order constraint

In addition to $\bar{H}[N]$ and $D[M]$, there is one higher-order constraint of the form (2.26) that is relevant for static solutions at second order in moments:

$$H_{\phi_2}[L] = \langle(\hat{\phi}_2 - \langle\hat{\phi}_2\rangle)\hat{H}[L]\rangle\,. \tag{2.48}$$

This constraint does not directly contribute to the brackets of hypersurface deformations, but it provides additional restrictions on the fields that are implied by imposing the quantum constraint. For a derivation of $H_{\phi_2}[L]$ in terms of moments, we need a Taylor expansion of $\hat{H}[L]$ to first order in $\hat{\phi}_2 - \langle\hat{\phi}_2\rangle$ and $\hat{p}_2 - \langle\hat{p}_2\rangle$. These terms, together with the factor of $\langle\hat{\phi}_2 - \langle\hat{\phi}_2\rangle\rangle$ included in the definition of $H_{\phi_2}[L]$, then produce second-order moments. Considering the fact that $\hat{H}$ locally depends ϕ_2 as well as on $\hat{\phi}_2'$, we write

$$\begin{aligned} H_{\phi_2}[L] &= \int \mathrm{d}x L(x)\left(\frac{\partial H}{\partial\phi_2}\phi_3^2 + \frac{\partial H}{\partial\phi_2'}\phi_3\phi_3' + \frac{\partial H}{\partial p_2}\phi_3 p_3\right) \\ &= -\int \mathrm{d}x L(x)\Bigg(\left(\frac{p_2^2}{2\sqrt{\phi_1}} + \frac{1}{2\sqrt{\phi_1}} + \frac{(\phi_1')^2 + 4\phi_1\phi_1''}{2\sqrt{\phi_1}\phi_2^2} - 4\frac{\sqrt{\phi_1}\phi_1'\phi_2'}{\phi_2^3}\right)\phi_3^2 \\ &\quad + \frac{2\sqrt{\phi_1}\phi_1'}{\phi_2^2}\phi_3\phi_3' + \left(\frac{\phi_2 p_2}{\sqrt{\phi_1}} + 2\sqrt{\phi_1}p_1\right)\phi_3 p_3\Bigg)\,. \end{aligned} \tag{2.49}$$

In the first line, the common factor of ϕ_2 in all three terms is implied by the explicit factor of $\langle\hat{\phi}_2 - \langle\hat{\phi}_2\rangle\rangle$ in the definition of $H_{\phi_2}[L]$. The remaining factors of ϕ_3, ϕ_3' and p_3, respectively, are correspond to first-order terms in a Taylor expansion of H.

The presence of higher-order constraints implies that evolution is not uniquely determined by the classical pair of two functions, lapse and shift, but also requires the specification of additional functions such as L. The latter determine the direction of a time evolution vector field in moment or state space. The general form of evolution equations with moment terms is therefore given by

$$\dot{f} = \{f, \bar{H}[N] + D[M] + H_{\phi_2}[L] + \cdots\} \tag{2.50}$$

for any phase-space function f, where the dots indicate further higher-order constraints that would involve higher moments or higher-order versions of the diffeomorphism constraint. The former do not appear to second order as considered here, while the latter, just like the classical $D[M]$, is not included for static solutions. Our evolution equations will therefore be given by $\dot{f} = \{f, \bar{H}[N] + H_{\phi_2}[L]\}$. In the static case, N is classically determined by the consistency condition that evolution equations be compatible with static behavior. As we will see, the same is true for L if quantum fluctuations are required to be static too.

2.5.5 Solutions

We will derive properties of static solutions in radial gauge, choosing $\phi_1 = x^2$ such that x is the area radius. Since our extended system is first class according to (2.27), we are allowed to fix the gauge in order to determine solutions. All momenta vanish for static solutions, and we are left with four free functions, ϕ_2, ϕ_3, N and L.

2.5.5.1 Equations

The diffeomorphism constraint (along with its higher-order versions) is identically satisfied for static solutions, and we have fixed its flow. Two equations of motion,

$$\begin{aligned}\dot{\phi}_2 &= \{\phi_2, \bar{H}[N] + H_{\phi_2}[L]\} = \frac{\delta \bar{H}[N]}{\delta p_2} + \frac{\delta H_{\phi_2}[L]}{\delta p_2} \\ &= \left(2xp_1 + \frac{\phi_2 p_2}{x} + \frac{\phi_3 p_3}{x}\right) N + \frac{\phi_3}{x}\left(\phi_3 p_2 + \phi_2 p_3\right) L\end{aligned} \tag{2.51}$$

and

$$\begin{aligned}\dot{\phi}_3 &= \{\phi_3, \bar{H}[N] + H_{\phi_2}[L]\} = \frac{\delta \bar{H}[N]}{\delta p_3} + \frac{\delta H_{\phi_2}[L]}{\delta p_3} \\ &= \frac{1}{x}(\phi_2 p_3 + \phi_3 p_2) N + \left(\frac{\phi_2 p_2}{\sqrt{\phi_1}} + 2\sqrt{\phi_1} p_1\right) \phi_3 L\,,\end{aligned} \tag{2.52}$$

are identically satisfied in the static case.

The remaining equations are therefore given by two constraints, $\bar{H}[N] = 0$ and $H_{\phi_2}[L] = 0$, and two equations of motion,

$$0 = \dot{p}_2 = \{p_2, \bar{H}[N] + H_{\phi_2}[L]\} = -\frac{\delta \bar{H}[N]}{\delta \phi_2} - \frac{\delta H_{\phi_2}[L]}{\delta \phi_2} \tag{2.53}$$

and

$$0 = \dot{p}_3 = \{p_3, \bar{H}[N] + H_{\phi_2}[L]\} = -\frac{\delta \bar{H}[N]}{\delta \phi_3} - \frac{\delta H_{\phi_2}[L]}{\delta \phi_3} \tag{2.54}$$

in static form. These implement the correct flow generated by the Hamiltonian constraint as required for static solutions. The full equations are rather lengthy, and will be shown in a more specific form when we start solving them below. With these conditions, we obtain the Hamiltonian constraint

$$\bar{H}[N] = -\int \mathrm{d}x N(x) \left(\frac{\phi_2}{2x} - \frac{2x}{\phi_2} - 4x\left(\frac{x}{\phi_2}\right)' + \left(12\frac{x^2\phi_2'}{\phi_2^4} - \frac{6x}{\phi_2^3}\right)\phi_3^2 - 8\frac{x^2\phi_3\phi_3'}{\phi_2^3} + \frac{U\phi_2}{2x\phi_3^2}\right), \tag{2.55}$$

the higher-order constraint

$$H_{\phi_2}[L] = -\int \mathrm{d}x L(x) \left(\left(\frac{1}{2x} + \frac{6x}{\phi_2^2} - 8\frac{x^2\phi_2'}{\phi_2^3}\right)\phi_3^2 + \frac{4x^2}{\phi_2^2}\phi_3\phi_3'\right) \tag{2.56}$$

and the two equations of motion.

These four equations are coupled differential equations for the four free functions. In order to simplify the solution procedure, we proceed perturbatively and assume that ϕ_2 and N are given by their classical solutions (according to the Schwarzschild line element) plus small corrections of the order of ϕ_3^2:

$$\begin{aligned} \phi_2 &= \phi_2^{(0)} + \delta\phi_2 = \frac{2x}{\sqrt{1-\mu/x}} + \delta\phi_2 \\ N &= N^{(0)} + \delta N = \sqrt{1-\mu/x} + \delta N \end{aligned} \tag{2.57}$$

where $\phi_2^{(0)}$ and $N^{(0)}$ are obtained from the Schwarzschild line element. The constant μ is equal to the mass in our units, having set $2G$ equal to one in order to simplify several numerical factors in the constraints and Poisson brackets. Transforming to the more standard choice where G equals one can easily be achieved by equating μ to twice the mass.

The higher-order constraint equation $H_{\phi_2}[L] = 0$ then takes the form

$$\begin{aligned} 0 &= H_{\phi_2}[L] \\ &= -\int \mathrm{d}x L(x)\phi_3 \left(\frac{3\mu}{2x^2}\phi_3 + \left(1-\frac{\mu}{x}\right)\phi_3'\right) + O(\phi_3^2\delta\phi_2) \end{aligned} \tag{2.58}$$

and can be interpreted as a first-order differential equation for ϕ_3. Its general solution is

$$\phi_3(x) = \frac{C}{(1-\mu/x)^{3/2}} \tag{2.59}$$

with an integration constant C. This solution diverges at the horizon, which is not surprising because this is where our background solutions (3.96) break down in the Schwarzschild coordinate system. At spatial infinity, ϕ_3 approaches a constant while ϕ_2 diverges. Fluctuations are therefore small at low curvature.

2.5.5.2 Metric correction

Using our solution for ϕ_3, the constraint (3.97) implies a differential equation for $\delta\phi_2$, coupled to δN. Only the classical part of the constraint contributes to the dependence on $\delta\phi_2$ and δN in our perturbative treatment because $H_2[N]$ is quadratic in the small ϕ_3, such that any contribution from $\delta\phi_2$ or δN would be of higher order. For this contribution, we have

$$\begin{aligned} H[N] = H[N]|_{\phi_2^{(0)}} &+ \int \mathrm{d}x (N^{(0)} + \delta N)\left(\frac{\partial H}{\partial \phi_2}\delta\phi_2 + \frac{\partial H}{\partial \phi_2'}\delta\phi_2'\right) \\ &+ \int \mathrm{d}x N^{(0)}\left(\frac{1}{2}\frac{\partial^2 H}{\partial \phi_2^2}(\delta\phi_2)^2 + \frac{\partial^2 H}{\partial\phi_2\partial\phi_2'}\delta\phi_2\delta\phi_2'\right) \end{aligned} \tag{2.60}$$

where all coefficients are evaluated at the classical solution. Therefore, $H[N]|_{\phi_2^{(0)}}$ is set to zero by definition of $\phi_2^{(0)}$ There is no second-order term in $\delta\phi_2'$ because the dependence of $H[N]$ on ϕ_2' is linear. In the first line, we can integrate by parts in the last term. Several resulting contributions then equal the integral of $\delta\phi_2$ times the classical

$$\begin{aligned} -\dot{p}_2(y)|_{N^{(0)}} &= -\{p_2(y), H[N^{(0)}]\} = \frac{\delta H[N^{(0)}]}{\delta\phi_2(y)} \\ = \int \mathrm{d}x N^{(0)}(x)&\left(\frac{\partial H(x)}{\partial\phi_2(y)}\delta(x-y) + \frac{\partial H(x)}{\partial\phi_2'(y)}\frac{\partial\delta(x-y)}{\partial x}\right) = N^{(0)}\frac{\partial H}{\partial\phi_2} - \left(N^{(0)}\frac{\partial H}{\partial\phi_2'}\right)' \end{aligned} \tag{2.61}$$

which vanishes for static background solutions. For the δN-terms, we can also integrate by parts,

$$\begin{aligned} \int \mathrm{d}x\delta N\left(\frac{\partial H}{\partial\phi_2}\delta\phi_2 + \frac{\partial H}{\partial\phi_2'}\delta\phi_2'\right) &= \int \mathrm{d}x\frac{\delta N}{N^{(0)}}\left(N^{(0)}\frac{\partial H}{\partial\phi_2}\delta\phi_2 + N^{(0)}\frac{\partial H}{\partial\phi_2'}\delta\phi_2'\right) \\ = \int \mathrm{d}x\delta\phi_2&\left(\left(N^{(0)}\frac{\partial H}{\partial\phi_2} - \left(N^{(0)}\frac{\partial H}{\partial\phi_2'}\right)'\right)\delta N - N^{(0)}\frac{\partial H}{\partial\phi_2'}\left(\frac{\delta N}{N^{(0)}}\right)'\right) \end{aligned} \tag{2.62}$$

The first δN-term in this expression vanishes, again by virtue of (3.26), but one term now remains, containing $(\delta N/N^{(0)})'$. Including this term in the expanded Hamiltonian constraint, we are left with

$$H[N] = \int \mathrm{d}x N^{(0)} \left(-\frac{\partial H}{\partial \phi_2'} \delta\phi_2 \left(\frac{\delta N}{N^{(0)}} \right)' + \frac{1}{2} \frac{\partial^2 H}{\partial \phi_2^2} (\delta\phi_2)^2 + \frac{\partial^2 H}{\partial \phi_2 \partial \phi_2'} \delta\phi_2 \delta\phi_2' \right) . \quad (2.63)$$

The expansion of $\bar{H}[N]$ contributes additional terms depending on ϕ_3, which by construction of $H_2[N]$ from a Taylor expansion have the same coefficients as the last two $\delta\phi_2$-terms in (2.63). All these terms can be combined to

$$\begin{aligned} \bar{H}[N] &= \int \mathrm{d}x N^{(0)} \left(-\frac{\partial H}{\partial \phi_2'} \delta\phi_2 \left(\frac{\delta N}{N^{(0)}} \right)' \right. \quad (2.64) \\ &\left. + \frac{1}{2} \frac{\partial^2 H}{\partial \phi_2^2} \left((\delta\phi_2)^2 + \phi_3^2 \right) + \frac{1}{2} \frac{\partial^2 H}{\partial \phi_2 \partial \phi_2'} \left((\delta\phi_2)^2 + \phi_3^2 \right)' + \frac{U(x)\phi_2}{2\sqrt{\phi_1}\phi_3^2} \right) . \end{aligned}$$

(The U-term should be considered second-order because it is derived from $\Delta(p_2^2)$ in (3.61). The function $U(x)$ is therefore of fourth order in the quasiclassical expansion. If moments of a semiclassical or Gaussian state are used, one order in the quasiclassical expansion corresponds to a factor of $\sqrt{\hbar}$.) Inserting classical solutions, from the background upon which we evaluate the perturbations, in the coefficients, we obtain

$$\begin{aligned} \bar{H}[N] &= \int \mathrm{d}x \left(-\left(1 - \frac{\mu}{x}\right) \delta\phi_2 \delta N' + \frac{\mu}{2x^2} \delta\phi_2 \delta N + \frac{U(x)}{\phi_3^2} \right. \quad (2.65) \\ &\left. + \frac{3}{4x^2} \left(1 - \frac{\mu}{x}\right) \left(1 - \frac{2\mu}{x}\right) \left((\delta\phi_2)^2 + \phi_3^2 \right) - \frac{1}{2x} \left(1 - \frac{\mu}{x}\right)^2 \left((\delta\phi_2)^2 + \phi_3^2 \right)' \right) . \end{aligned}$$

In order to simplify this expression, we can combine it with the a non-vanishing contribution to the full $\dot{p}_2$ to linear order in $\delta\phi_2$ and δN, which will allow us to eliminate δN from (2.64). Using (3.26) for the expanded solution, this linear contribution to $\dot{\phi}_2$ is given by

$$\begin{aligned} \dot{p}_2|_{\mathrm{linear}} &= -\frac{\partial H_{\mathrm{linear}}}{\partial \phi_2} N^{(0)} - \frac{\partial H^{(0)}}{\partial \phi_2} \delta N + \left(\frac{\partial H_{\mathrm{linear}}}{\partial \phi_2'} N^{(0)} + \frac{\partial H^{(0)}}{\partial \phi_2'} \delta N \right)' \\ &= -\left(\frac{\partial^2 H}{\partial \phi_2^2} \delta\phi_2 + \frac{\partial^2 H}{\partial \phi_2' \partial \phi_2} \delta\phi_2' - \left(\frac{\partial^2 H}{\partial \phi_2' \partial \phi_2} \delta\phi_2 \right)' \right) N^{(0)} \\ &\quad + \frac{\partial^2 H}{\partial \phi_2' \partial \phi_2} \delta\phi_2 N^{(0)\prime} - \left(\frac{\partial H}{\partial \phi_2} - \left(\frac{\partial H}{\partial \phi_2'} \right)' \right) \delta N + \frac{\partial H}{\partial \phi_2'} \delta N' \end{aligned}$$

$$= -\left(\frac{\partial^2 H}{\partial \phi_2^2} - \left(\frac{\partial^2 H}{\partial \phi_2' \partial \phi_2}\right)'\right) \delta\phi_2 N^{(0)}$$

$$+\frac{\partial^2 H}{\partial \phi_2' \partial \phi_2} \delta\phi_2 N^{(0)\prime} - \left(\frac{\partial H}{\partial \phi_2} - \left(\frac{\partial H}{\partial \phi_2'}\right)'\right) \delta N + \frac{\partial H}{\partial \phi_2'} \delta N' \,. \tag{2.66}$$

Upon inserting background solutions in the coefficients, the δN-terms in

$$\dot{p}_2|_{\rm linear} = -\frac{1}{2x^2}\left(1-\frac{\mu}{x}\right)\delta\phi_2 - \frac{\mu}{2x^2}\delta N + \left(1-\frac{\mu}{x}\right)\delta N' = 0 \tag{2.67}$$

are of the same form as those of (2.64) and can therefore be eliminated from this equation. The simplified second-order constraint,

$$\bar{H}[N] = \int {\rm d}x \left(-\frac{1}{2x^2}\left(1-\frac{\mu}{x}\right)(\delta\phi_2)^2 + \frac{U(x)}{\phi_3^2}\right. \tag{2.68}$$

$$\left. + \frac{3}{4x^2}\left(1-\frac{\mu}{x}\right)\left(1-\frac{2\mu}{x}\right)\left((\delta\phi_2)^2+\phi_3^2\right) - \frac{1}{2x}\left(1-\frac{\mu}{x}\right)^2\left((\delta\phi_2)^2+\phi_3^2\right)'\right) \,,$$

provides a differential equation for $\delta\phi_2$ if we use the known solution (2.59) for ϕ_3. Keeping some of the ϕ_3^2-terms for now, we write this differential equation as an inhomogeneous one for $(\delta\phi_2)^2+\phi_3^2$:

$$\frac{1}{4x^2}\left(1-\frac{\mu}{x}\right)\left(1-\frac{6\mu}{x}\right)\left((\delta\phi_2)^2+\phi_3^2\right) - \frac{1}{2x}\left(1-\frac{\mu}{x}\right)^2\left((\delta\phi_2)^2+\phi_3^2\right)'$$

$$= -\frac{C^2}{2x^2(1-\mu/x)^2} - \frac{U(x)}{C^2}\left(1-\frac{\mu}{x}\right)^3 \,. \tag{2.69}$$

The corresponding homogeneous equation can easily be solved for

$$(\delta\phi_2)^2+\phi_3^2 = \frac{D\sqrt{x}}{(1-\mu/x)^{5/2}} \,, \tag{2.70}$$

which then implies the differential equation

$$D' = \frac{C^2}{(x-\mu)^{3/2}} + \frac{2U(x)}{C^2}\frac{(x-\mu)^{7/2}}{x^3} \tag{2.71}$$

for solution of the inhomogeneous equation of the form (2.70) with x-dependent D.

Solving this equation, we obtain

$$(\delta\phi_2)^2+\phi_3^2=\frac{E\sqrt{x}}{(1-\mu/x)^{5/2}}-\frac{2C^2}{(1-\mu/x)^3}+\frac{2\sqrt{x}}{C^2(1-\mu/x)^{5/2}}\int U(x)\left(1-\frac{\mu}{x}\right)^{7/2}\sqrt{x}\mathrm{d}x \tag{2.72}$$

with a new integration constant E, or

$$\delta\phi_2=\sqrt{\frac{E\sqrt{x}}{(1-\mu/x)^{5/2}}-\frac{3C^2}{(1-\mu/x)^3}+\frac{2\sqrt{x}}{C^2(1-\mu/x)^{5/2}}\int U(x)\left(1-\frac{\mu}{x}\right)^{7/2}\sqrt{x}\mathrm{d}x}\,. \tag{2.73}$$

(For constant U, there is a closed-form logarithmic expression for $\int(1-\mu/x)^{7/2}\sqrt{x}\mathrm{d}x$, but it is lengthy.)

Notice that the second term dominates near the horizon, where it is negative. The perturbative solution therefore breaks down before the horizon is reached, where ϕ_3 is large but still finite. For $x \gg \mu$, the dominant behavior of $\phi_2(x)$ is determined by the last term in (2.73), which, for an asymptotically constant $U(x)$, behaves like Ux^2 (the integral can then be approximated as $\int x^{1/2}\mathrm{d}x=\frac{2}{3}x^{3/2}$). In this case, therefore, $\delta\phi_2\sim\sqrt{U}x$ grows with x, but so does the classical solution $\phi_2(0)$. Since $\phi_2^{(0)}\sim x$ for $x\gg\mu$, the ratio $(\delta\phi_2)/\phi_2^{(0)}\sim\sqrt{U}$ implies a nearly constant correction of the order of $\hbar$ for semiclassical states, where $U\approx\hbar^2/4$ remains asymptotically constant. The first term in (2.73) may also be relevant in intermediate regimes, where it would imply a $\delta\phi_2$ that behaves like $x^{1/4}$. The correction to ϕ_2 then increases asymptotically, unlike ϕ_3, but less slowly than $\phi_2^{(0)}$: we have $(\delta\phi_2)/\phi_2^{(0)}\sim x^{-3/4}$ from the first term in (2.73).

2.5.5.3 Lapse correction

Given this solution for $\delta\phi_2$, we can go back to (2.67) as a differential equation for δN. So far, we have not fully solved this equation and only used it to eliminate δN from (2.64). Our solution for $\delta\phi_2$ obtained in this way now makes it possible to solve (2.67) for δN, although the lengthy form of (2.73) makes it hard to find a complete analytical solution. Nevertheless, the form of the solution in certain limits will turn out to be instructive.

We first rewrite equation (2.67) as

$$0=\left(1-\frac{\mu}{x}\right)^{3/2}\left(-\frac{1}{2x^2\sqrt{1-\mu/x}}\,\delta\phi_2+\left(\frac{\delta N}{\sqrt{1-\mu/x}}\right)'\right) \tag{2.74}$$

such that

$$\delta N = -\frac{1}{2}\sqrt{1-\frac{\mu}{x}}\int \frac{\delta\phi_2}{x^2\sqrt{1-\mu/x}}\mathrm{d}x \tag{2.75}$$

where (2.73) should be inserted in the integral. A simple integration is obtained in regimes in which both C^2-terms in (2.73) can be ignored, in which case

$$\delta N \sim -\frac{2}{3}\frac{\sqrt{E}}{x^{3/4}(1-\mu/x)^{1/4}} + F\sqrt{1-\frac{\mu}{x}} \tag{2.76}$$

with a new integration constant F. The F-term just changes the background lapse function by a constant factor $1+F$, which can be absorbed in the time coordinate. The remaining correction to the lapse function,

$$\delta N \sim -\frac{2}{3}\frac{\sqrt{E}}{x^{3/4}(1-\mu/x)^{1/4}}\,, \tag{2.77}$$

shows an interesting asymptotic behavior of the correction which falls off more slowly than the classical curvature correction $-\mu/x$ of the lapse function. Using this term as a correction of Newton's potential in a weak-field line element shows that non-local effects could imply larger corrections than effective field theory in a derivative expansion, where the leading correction would be of the order $1/x^3$ [15]. However, our simplified solution (2.77), based on the E-term in (2.73), does not apply in the completely asymptotic regime where the U-term in (2.73) would be dominant. Since this term, for asymptotically constant $U(x)$, implies an asymptotic behavior of $\delta\phi_2 \sim \sqrt{U}x$, the corresponding δN according to (2.75) is $\delta N \sim \sqrt{U}\log(\mu/x)$. Interpreted as a correction to Newton's potential, this term suggests a relationship with infrared contributions, consistent with the interpretation of fluctuation terms in quantum cosmological models that have the same origin as U here [72, 73].

We are left with the equation $\dot{p}_3 = 0$, a differential equation for L. It is straightforward to solve

$$\begin{aligned}
\frac{\dot{p}_3}{\phi_3} &= -\frac{\partial^2 H}{\partial\phi_2^2}N^{(0)} + \left(\frac{\partial^2 H}{\partial\phi_2\partial\phi_2'}N^{(0)}\right)' + \frac{U(x)\phi_2^{(0)}N^{(0)}}{\sqrt{\phi_1}\phi_3^4} - 2\frac{\partial H}{\partial\phi_2}L + \left(\frac{\partial H}{\partial\phi_2'}L\right)' \\
&= -\frac{1-\mu/x}{2x^2} + \frac{2U(x)}{C^4}\left(1-\frac{\mu}{x}\right)^6 - \frac{2\mu}{x^2}L + \left(1-\frac{\mu}{x}\right)L' \\
&= \left(1-\frac{\mu}{x}\right)^3\left(-\frac{1}{2(x-\mu)^2} + \frac{2U(x)}{C^4}\left(1-\frac{\mu}{x}\right)^3 + \left(\frac{L}{(1-\mu/x)^2}\right)'\right) = 0
\end{aligned} \tag{2.78}$$

for L, where we have used background solutions in all coefficients. The result is

$$L = -\frac{1-\mu/x}{2x} + G\left(1-\frac{\mu}{x}\right)^2 \tag{2.79}$$
$$-\frac{2U(x)x}{C^4}\left(1-\frac{\mu}{x}\right)^2\left(\left(1-\frac{\mu}{x}\right)^3 + \frac{3\mu}{2x}\left(1-\frac{\mu}{x}\right)^2 + \frac{3\mu^2}{x^2}\left(1-\frac{\mu}{x}\right) + \frac{3\mu}{x}\log\left(\frac{\mu}{x}\right)\right)$$

with a new integration constant G. Since L vanishes at $x=\mu$, the evolution of fluctuations freezes at the horizon, just as the evolution of the classical metric.

2.5.5.4 Quantum effects

We have obtained complete solutions, up to two remaining integrations. These are not only lengthy in analytical form but also require additional information about the function $U(x)$, which quantifies the strength of quantum effects. So far, we have mainly discussed U-dependent modifications in asymptotic low-curvature regimes, in which we assumed that $U(x)$ is nearly constant. The results were encouraging, in that they showed that a nearly constant U also implies a nearly constant relative metric fluctuation, given by $\delta\phi_2/\phi_2^{(0)}$. Nevertheless, it is of interest to obtain independent information about the possible form of $U(x)$.

Since the field $U(x)$ does not have a momentum, in the truncation to second-order moments used here, it is not subject directly to an evolution equation. (At higher moment orders, the uncertainty product $\Delta(\phi_2^2)\Delta(p_2^2)-\Delta(\phi_2 p_2)$, which equals U to second order, is not conserved. The momentum of U can therefore be thought of as a combination of higher-order moments that are eliminated in our truncation.) However, it turns out that we can use another equation of motion in order to derive a consistency condition for $U(x)$: We have implemented the leading non-zero terms in the equation $\dot{p}_2=0$, which were of linear order in $\delta\phi_2$ and δN. Since we used second-order constraints, there is also a second-order contribution to $\dot{p}_2$. Setting this contribution equal to zero for static solutions allows us to test the self-consistency of the formalism. A long calculation (performed using Mathematica) implies an equation for $U(x)$ of the form

$$\begin{aligned} 0 &= f_1(x)+f_2(x)U(x)+f_3(x)U(x)^2+f_4(x)U'(x) \\ &\quad +f_5(x)I[U]+f_6(x)U(x)I[U]+f_7(x)U'(x)I[U] \\ &\quad +f_8(x)I[U]^2 \end{aligned} \tag{2.80}$$

where

$$I[U] = \int \sqrt{x}(1-\mu/x)^{7/2} U(x)\,\mathrm{d}x \tag{2.81}$$

and the U-independent coefficient functions are

$$\begin{aligned}
f_1(x) &= 36C^8\sqrt{1-\frac{\mu}{x}}\left(\sqrt{1-\frac{\mu}{x}}\left(1+\frac{3\mu}{2x}-\frac{3\mu^2}{2x^2}\right)+1\right) \qquad (2.82)\\
&\quad -3C^6Ex^{1/2}\left(1-\frac{\mu}{x}\right)\left(\sqrt{1-\frac{\mu}{x}}\left(5+\frac{7\mu}{x}-\frac{6\mu^2}{x^2}\right)+9\right)+5C^4E^2x\left(1-\frac{\mu}{x}\right)^{3/2}\\
f_2(x) &= 12C^4x^2\left(1-\frac{\mu}{x}\right)^{11/2}\left(\sqrt{1-\frac{\mu}{x}}\left(3+\frac{13\mu}{x}\right)-1\right)\\
&\quad -4C^2Ex^{5/2}\left(1-\frac{\mu}{x}\right)^6\left(\sqrt{1-\frac{\mu}{x}}\left(3+\frac{14\mu}{x}\right)-1\right) \qquad (2.83)\\
f_3(x) &= 16x^4\left(1-\frac{\mu}{x}\right)^{11} \qquad (2.84)\\
f_4(x) &= 16C^2x^3\left(1-\frac{\mu}{x}\right)^7\left(3C^2-E\sqrt{x}\sqrt{1-\frac{\mu}{x}}\right) \qquad (2.85)\\
f_5(x) &= -6C^4x^{1/2}\left(1-\frac{\mu}{x}\right)\left(\sqrt{1-\frac{\mu}{x}}\left(5+\frac{7\mu}{x}-\frac{6\mu^2}{x^2}\right)+9\right)+20C^2Ex\left(1-\frac{\mu}{x}\right)^{3/2} \qquad (2.86)\\
f_6(x) &= -8x^{5/2}\left(\sqrt{1-\frac{\mu}{x}}\left(3+\frac{14\mu}{x}\right)-1\right)\left(1-\frac{\mu}{x}\right)^6 \qquad (2.87)\\
f_7(x) &= -32x^{7/2}\left(1-\frac{\mu}{x}\right)^{15/2} \qquad (2.88)\\
f_8(x) &= 20x\left(1-\frac{\mu}{x}\right)^{3/2}. \qquad (2.89)
\end{aligned}$$

This long equation can be analyzed in the asymptotic regime if we assume that $U(x)$ is of power-law form there. In the derivative terms, $xU'(x)$ is then of the same order as $U(x)$, and asymptotically for $x \gg \mu$ with nearly constant U the integral behaves like $x^{3/2}$. In (2.80), the contributions with coefficient functions $f_3(x)$, $f_6(x)$, $f_7(x)$, and $\mathrm{f}_8(x)$ are then dominant, such that the equation simplifies to

$$\begin{aligned}
&aU(x)^2 + bU(x)I[U]/x^{3/2} \qquad (2.90)\\
&+cxU'(x)I[U]/x^{3/2} + dI[U]^2/x^3 = 0
\end{aligned}$$

with x-independent coefficients a, b, c and d. For nearly constant U at $x \gg \mu$, we have $I[U] \sim \frac{2}{3}x^{3/2}U$, and therefore our equation takes the form

$$\tilde{a}U(x)^2 + cxU(x)U'(x) = 0 \tag{2.91}$$

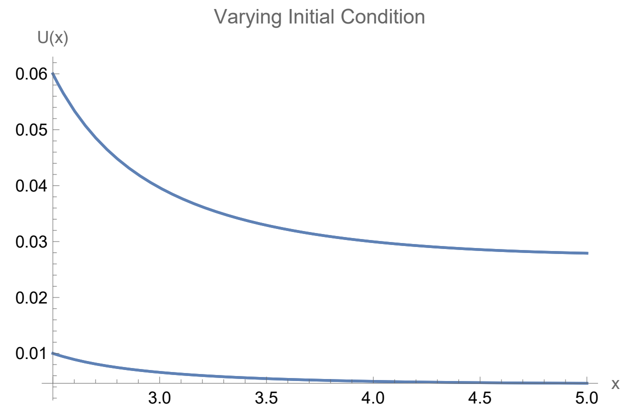

Figure 2.1. An example of two numerical results for $U(x)$ following from equation (2.80). Two choices of initial values were set for $U(x)$ at $x = 2.5$, given by $U_{\rm in} = 0.01$ in the lower curve and $U_{\rm in} = 0.06$ for the upper curve, respectively. Constants are set as follows: $C = 0.01, E = 0.0001, \mu = 1$.

with a new constant $\tilde{a}$. The simplified equation therefore has solutions $U(x) = 0$ or a power law for $U(x)$. Asymptotically, these solutions are consistent with our condition that $U(x)$ not be negative. Numerical solutions at smaller x, shown in Fig. 2.1 confirm this behavior.

This result is encouraging because the non-negativity condition is motivated by the quantum-mechanics origin of our modifications, which is independent of the consistency conditions we checked for the constraint brackets. The observation that solutions respect the quantum condition indicates that the equations are self-consistent, not only as a model of modified gravity, but also from the perspective of quantum physics.

2.6 Conclusions

Any quantum theory, and in particular quantum gravity, is expected to imply non-local behavior. Non-local action principles and their equations of motion are usually hard to solve, but if one assumes a specific non-local action, it can often be analyzed by mapping the theory to a local one in which classical degrees of freedom are coupled to auxiliary fields. We have introduced here a new, systematic quasiclassical formulation of spherically symmetric models in quantum gravity with non-local corrections derived in a canonical quantization. By implementing quantum fluctuations and correlations as physical versions of what would usually be called auxiliary fields in a non-local theory, a multi-field local theory is obtained in which coupling terms are completely determined by the rules of canonical quantization.

The presence of new degrees of freedom implies that such quantum extended theories are more complex than the classical model. Working with vacuum spherically symmetric models, we constructed a tractable constrained system in which one of the metric components, ϕ_1, is fixed by using the area radius (a partial gauge fixing of the theory). Doubling the classical field content by introducing second-order quantum moments, we therefore obtained a theory for two independent fields that represent a single classical metric component (the radial distance measure ϕ_2) and its quantum fluctuation (ϕ_3). While the reduced system ignores cross-correlations between the radial distance $\phi_2/(2\sqrt{\phi_1})$ and the area radius $\sqrt{\phi_1}$, it is formally consistent for static solutions and allows explicit solutions in almost complete closed form.

The fluctuation field ϕ_3 couples dynamically to expectation value ϕ_2, representing one of the metric components. The former field cannot vanish owing to uncertainty relations, and through the coupling terms it implies changes $\delta\phi_2$ of the metric field compared with its classical behavior. Through canonical equations of motion, the staticity condition determines the lapse function N for a given ϕ_2, such that δN inherits certain changes from $\delta\phi_2$. Using the appearance of these fields in a classical-type line element, we obtain a quantum-corrected space-time geometry from

$$\begin{aligned} \mathrm{d}s^2 &= -(N^{(0)}+\delta N)^2\mathrm{d}t^2 + \frac{(\phi_2^{(0)}+\delta\phi_2)^2}{4x^2}\mathrm{d}x^2 + x^2\mathrm{d}\Omega^2 \\ &\sim -(N^{(0)2}+2N^{(0)}\delta N)\mathrm{d}t^2 + \frac{\phi_2^{(0)2}+2\phi_2^{(0)}\delta\phi_2}{4x^2}\mathrm{d}x^2 + x^2\mathrm{d}\Omega^2 \end{aligned} \tag{2.92}$$

to first order in δN and $\delta\phi_2$. The latter values are given by the rather lengthy expressions (2.75) and (2.73), respectively. However, a word of caution is in order when we organize our solutions in this form: So far, we have checked the consistency of our quasiclassical constraints only for static configurations, and therefore we can use a line element of the form (2.92) only for static slicings. It might be tempting to apply a more general coordinate transformation once solutions have been put into the form of a line element, but by doing so we would leave the range of validity of our derivations here. The cosmological analysis [75] extended our static constraints to non-static ones, observing that consistency then requires an inclusion also of fluctuations of ϕ_1. An application to black-hole models remains to be completed.

We have observed several interesting features of our solutions. In particular, the quasiclassical approximation breaks down before the horizon is reached, which suggests that non-local effects may be crucial for horizon dynamics of quantum black holes. A confirmation of this expectation would, however, have to await a solution of higher-order quasiclassical approximations, as well as an extension to non-static configurations that would allow us to use different space-time slicings.

The asymptotic behavior is more reliable within the restrictions of our model. We analyzed it by studying solutions for one of the new quantum fields that corresponds to the uncertainty of a state in quantum mechanics. For this field, we found an asymptotic fall-off behavior consistent with a positivity condition. Our quasiclassical solutions are therefore consistent with the existence of an underlying quantum state of static, spherically symmetric space-times. In a full quantum field theory, important properties such as positivity would be implied by unitary evolution. The fact that we observed a positivity property without explicitly deriving unitary evolution from the quasiclassical constraints indicates that our treatment is self-consistent and does reveal features of an underlying quantum theory of gravity. Our analysis therefore shows that quasiclassical methods are promising in applications to inhomogeneous models of quantum gravity. They allow explicit derivations of quantum corrections without requiring additional assumptions beyond what is provided by canonical quantization.

Chapter 3 | Reinterpretation as a black hole superposition

My recent preprint, *Space-time superpositions as fluctuating geometries*, is building on my recent paper. In the previous chapter, I described *Quasiclassical solutions for static quantum black holes* and how I established the presence of quantum effects in a black hole spacetime, modified with novel quantum-corrections. This new approach involves recreating these quantum effects, while modeling the quantum-corrected black hole as, instead, a quantum superposition of classical black holes. Rather than a spacetime with quantum fluctuations, we now have a quantum superposition of classical spacetimes. This reinterpretation supports my earlier numerical results and produces quantum corrections in the Newtonian potential in the asymptotic limit. Shifting the framework in which these quantum corrections are interpreted broadens the applicability of my results.

3.1 Summary

It is valuable to examine phenomena in a variety of formalisms. Here, we start with a spherically symmetric, static black hole model with quantum corrections shown to produce nonlocal effects in a canonical gravity formalism. We relate this to a quantum superposition of classical black hole spacetimes of varying mass, investigating the generation of analogous nonlocal effects. Having examined the asymptotic limit in isotropic coordinates, we have reinforced the previous numerical results for fall-off of $U(x)$, derived quantum corrections to the Newtonian potential, and articulated a metric with quantum uncertainty present in both the temporal and spatial components.

We set the modified Hamiltonian constraint to zero, recalculate the equation of motion using only the first order correction to the Hamiltonian, take the weak field limit, and solve for power series solutions for the scalar fields and lapse function. We then used these solutions to construct the quantum-corrected Newton potential, complete with bounds on the constants.

Our work is distinct in that we utilized the general formalism laid out for models of canonical quantum gravity in [76]. The previous Schwarzschild gauge treatment was relevant for horizon properties, and here, we replace it with a derivation in isotropic coordinates. This choice is more appropriate for taking the complete weak-field limit. This gauge choice is enabled by our construction of a covariant space-time formulation. Thus we have a universal theory for spherically symmetric black holes with quantum fluctuations. This enables derivation of a large set of physical properties.

3.1.1 Calculation

Transforming to isotropic coordinates, we start with the modified Hamiltonian constraint from the previous chapter, with only the first order correction:

$$0 = H[N] + H2[N] \tag{3.1}$$

And we re-derive the equation of motion for $\dot{p}_2$ using only the classical Hamiltonian, since this will already be a higher-order equation than the Hamiltonian constraint.

$$0 = \dot{p}_2 \tag{3.2}$$

We introduce power series for the quantum correction, ϕ_3, the metric field ϕ_2, and the lapse function N:

$$\phi_3(x) = C\left(1 + \frac{3b}{x} + \frac{3b^2}{x^2} + \cdots\right). \tag{3.3}$$

$$\phi_2 = \phi_2^{(0)} + c_1 + \frac{c_2}{x} + \cdots \tag{3.4}$$

$$N = N^{(0)} + \frac{d_1}{x} + \frac{d_2}{x^2} + \cdots \tag{3.5}$$

We impose the isotropy condition

$$\phi_2 = 2\frac{\phi_1}{x} \tag{3.6}$$

and take the asymptotic limit for large radius. By collecting the constraint equations by powers of radius and equating each coefficient individually to zero, we get a system of equations solvable for the values defined in our power series expansions. Note that some coefficients in the constraint equations were trivially zero, so we went to higher powers to have enough constraint equations for our system. The solution to said system of equations being:

$$
\begin{aligned}
c_1 &= \epsilon\frac{8\sqrt{bU_0}}{C}\sqrt{1-C^4/(64bU_0)} && (3.7)\\
c_2 &= \epsilon b\frac{3\sqrt{bU_0}}{8C}\frac{70-9C^4/(8bU_0)}{\sqrt{1-C^4/(64bU_0)}} && (3.8)\\
d_2 &= \epsilon b\frac{\sqrt{bU_0}}{96C}\frac{394-47C^4/(8bU_0)}{\sqrt{1-C^4/(64bU_0)}} && (3.9)\\
c_3 &= \epsilon b^2\frac{\sqrt{bU_0}}{80C}\frac{2445-79C^4/(bU_0)+163C^8/(256b^2U_0^2)}{(1-C^4/(64bU_0))^{3/2}} && (3.10)\\
d_3 &= -\epsilon b^2\frac{\sqrt{bU_0}}{640C}\frac{9735-589C^4/(2bU_0)+569C^8/(256b^2U_0^2)}{(1-C^4/(64bU_0))^{3/2}} && (3.11)
\end{aligned}
$$

With these solutions, we can derive Newton's potential with quantum corrections.

$$V(x)=\frac{c^2}{2}(N(x)^2-1)\approx-\frac{GM}{x}+\frac{G^2M^2}{x^2c^2}+\frac{d_2}{2x^2}+O(x^{-3}) \tag{3.12}$$

And the corresponding modification to the metric:

$$
\begin{aligned}
\mathrm{d}s^2 &= -\left(N^{(0)}+\frac{d_1}{x}+\frac{d_2}{x^2}+O(x^{-3})\right)^2 c^2\mathrm{d}t^2\\
&\quad+\left(\frac{\phi_2^{(0)}}{2x}+\frac{c_1}{2x}+\frac{c_2}{2x^2}+O(x^{-3})\right)\left(\mathrm{d}x^2+x^2(\mathrm{d}\vartheta^2+\sin^2\vartheta\mathrm{d}\varphi^2)\right) && (3.13)
\end{aligned}
$$

Note that the metric contains uncertainty in both the temporal and spatial components, via this relationship:

$$\phi_3\sim C\leq 2\sqrt{2}(bU_0)^{1/4} \tag{3.14}$$

We derived characteristic effects in the line element. Specifically, the time and space components of the space-time metric are affected differently by quantum corrections, in contrast to previous assumptions. Our results imply a more precise space-time geometry than past constructions, combining interrelated gravitational and quantum effects.

3.1.2 Results

This project broadens the applicability of both our quantum-corrected black hole model and the existing literature on black hole mass superpositions. This work has demonstrated that our past numerical result for the asymptotically constant behavior of $U(x)$ in the large radius limit is robust, as outlined in Chapter 2. We have calculated corrections to the Newtonian potential in the asymptotic limit, showing the impacts of our quantum corrections even at large radii. We went on to derive the corresponding metric, with quantum uncertainties in both spatial and temporal components. This result takes us another step closer to a theory of Quantum Gravity and has clearly outlined paths for further study, as will be discussed in Chapter 5.

3.2 Preprint: Space-time superpositions as fluctuating geometries

Abstract:
Superpositions of black holes can be described geometrically using a combined canonical formulation for space-time and quantum states. A previously introduced black-hole model that includes quantum fluctuations of metric components is shown here to give full access to the corresponding space-time geometry of weak-field gravity in terms of suitable line elements with quantum corrections. These results can be interpreted as providing covariant formulations of the gravitational force implied by a distribution of black holes in superposition. They can also be understood as a distribution of quantum matter constituents in superposition for a single black hole. A detailed analysis in the weak-field limit reveals quantum corrections to Newton's potential in generic semiclassical states, as well as new bounds on quantum fluctuations, implied by the covariance condition, rather than the usual uncertainty principle. These results provide additional control on quantum effects in Newton's potential that can be used in a broad range of predictions to be compared with observations.

3.3 Introduction

A quantum state formed by a superposition of different massive objects is expected to generate a gravitational force dependent on quantum effects. Detailed derivations and studies of the resulting implications on test objects, which may themselves be in non-trivial quantum states, require ingredients from gravitational as well as quantum physics. Given the current lack of a complete and consistent quantum theory of gravity, only partial answers can be given to the question of how quantum test objects behave in a superposition state of different masses. Nevertheless, this setting is promising because it does not necessarily require high curvature, where a detailed theory of quantum gravity is believed to be essential, and can likely be addressed by semiclassical or other approximations.

3.3.1 Current approaches

Recent developments in quantum information methods applied to gravitating states and quantum reference frames have led to strong interest in the possible implications of superpositions of states that experience or generate gravitational fields. For instance, various proposals in [77–80] analyze potential aspects of quantum superpositions of test-particle states at locations that experience different values of proper time, due to their location in the gravitational potential well of a larger mass. In these and other cases, a classical background gravitational field or a classical space-time geometry are assumed, usually at weak fields, which is experienced in different ways by observers or quantum test particles depending on their states. For example, particles moving along the two arms of an interferometer experience different time dilations, if they travel at slightly displaced altitudes. Since the combined superposition state simultaneously experiences two different values of time dilation, there may be characteristic signatures in the interference pattern that differ from the pattern in the absence of gravity. An analysis of the resulting state suggests the possibility to study gravitational effects in a quantum context, potentially using new experiments.

Models based on a classical background space-time are feasible because they do not directly involve quantum gravity or quantum space-time, even if quantum states are used for test objects. A related question closer to quantum aspects of the gravitational field itself is how test objects might experience the gravitational force generated by two (or more) masses in a quantum superposition. Recent examples of such studies include [81,82] for superpositions of spherically symmetric masses or shells. In a first approximation, the question may again be analyzed in a weak-field context, using superpositions of Newtonian potentials. However, even though a Newtonian description is not necessarily relativistic, geometrical properties of space-time are used in the definition of proper time that determines how the internal degrees of freedom of a quantum test particle evolve at its position. This is particularly relevant to one of our results, as it involves deriving quantum corrections to the Newtonian potential.

The concept of proper time requires an understanding of space-time geometry because it is derived from a line element. In [83–85], proper time experienced by an object in the gravitational field generated by a superposition of two masses was defined using the individual space-time geometries of each mass, and applying transformations between quantum reference frames to combine the individual classical-type results into a time duration experienced in a superposition of the two geometries. However, there was no space-time geometry that could describe the combined implication of both masses. From a fundamental perspective, such a description is incomplete because it is unable to describe important gravitational phenomena such as gravitational waves: If the two masses in superposition are orbiting around each other, they emit gravitational waves. Since these waves are produced by the combined system of both masses, they cannot be formulated as excitations on each of these individual space-times for a single mass; the system must be considered in a combined space-time description. Such studies may be sufficient to derive a specific kind of new, and potentially observable, effect related to the behavior of test objects. However, they do not show whether (or how) space-time geometry around a superposition of masses can be used for a complete description of gravitational phenomena. Field-theoretical aspects of gravity and general covariance require a consistent geometrical formulation of space-time, on which excitations such as gravitational waves can propagate. An open question is whether such a formulation can retain its classical form of Riemannian geometry if it is generated by superposition states. It is possible that geometrical concepts eventually have to be generalized in a suitable way to create such a formalism.

3.3.2 Geometrical space-time structures

A key issue in these endeavors is therefore whether a consistent geometrical structure can be derived from a superposition of two gravitational fields, and how to do so, if it is possible. The existence of basic geometrical concepts such as line elements or suitable generalizations compatible with a superposition state is also required. This is necessary for a complete description that makes it possible to extend the analysis to strong fields. In such cases, gravity is understood through general relativity as an implication of space-time curvature. The concept of space-time curvature, in turn, requires a meaningful definition of space-time geometry, tensor calculus, and general covariance. However, it is currently unclear what the superposition of two geometries should be like in a geometrical setting. For instance, is the superposition of two Riemannian geometries (described by space-time line elements) still Riemannian, or do we need a more general concept of geometry? If the superposition is still Riemannian, how do we derive a valid line element which faithfully describes geometrical properties of the superposition? Compared to the motion of quantum test masses on a given background space-time, these questions are much closer to fundamental issues in quantum gravity. Answering them could potentially suggest new classes of physical phenomena, such as, unique properties of gravitational waves produced by a black hole (in a superposition of the massive constituents of a progenitor star), or new behaviors of Hawking radiation.

Even in weak fields, a geometrical picture of superpositions of gravitational states is expected to have important distinctions. Superpositions of masses with a Newtonian gravitational potential are restricted by standard conditions of quantum mechanics, such as the uncertainty relation. Well-defined space-time geometries, and their potential superpositions, are also subject to the important condition of general covariance. This symmetry, expressed as 4-dimensional coordinate invariance or the freedom to choose arbitrary spacelike slicings of the space-time manifold, imposes restrictions on possible theories. One standard application is the classification of possible modifications of general relativity. For instance, the properties of curvature tensors and their invariants determine the form of possible higher-curvature effective actions. Similarly, general covariance is expected to restrict possible superpositions of gravitational fields, even if they are applied only in a weak-field limit. It would be impossible to notice such restrictions from general covariance, if one does not attempt to construct a geometrical space-time picture of superposed Newtonian potentials. In this context, the question posed here is whether there is an action principle or some other fundamental description that has superpositions of gravitational fields among its solutions. This important field-theoretical question cannot be addressed by current formulations of quantum reference frames, which focus on the properties of test objects.

General covariance is usually difficult to control if one starts with a non-covariant formulation, such as a spatially dependent potential. As is well known from decades of research on quantum gravity, the problem becomes even more challenging if quantum effects are included. For instance, a path-integral formulation may formally work with space-time tensors, but could still violate general covariance if there are anomalies in the measure used to integrate over the space of metrics. Superpositions of masses lead to non-Gaussian states on a field space, which may imply additional challenges for common evaluations of path integrals based on saddle-point approximations. It is therefore important to develop a direct and tractable approach to analyze covariance conditions and their potential implications. A possible covariant approach has been developed in [86,87] and used for an analysis of space-time superpositions in [81]. However, in this case, the space-time geometry follows indirectly from the evolution of matter fields, based on field correlators. Here, we require that a fundamental description of gravitational superposition states be independent of matter properties and also work in vacuum situations. It should therefore be directly applied to the geometry of space-time, described by the metric or an alternative mathematical object.

It might seem that a full-fledged theory of quantum gravity is required to address these questions about the quantum properties of the gravitational field. However, we will show here that this is not the case. The situation is similar to derivations of quantum corrections to Newton's potential [15], based on perturbative quantum gravity as an effective field theory [14, 16]. These results are obtained from Feynman expansions. Alternatively, they can be calculated using path integrals. This effectively traces out quantum degrees of freedom such as fluctuations, quantum correlations, or higher moments of an underlying state. Quantum corrections obtained in this way depend on dynamical relationships between the moments of a state and expectation values of basic field operators, but they do not use explicit expressions for these moments. In a quantum mechanical context, these methods, described for this setting in [49], can be extended to a more general treatment [42, 43] in which moments and expectation values are initially independent and subject to equations of motion. An adiabatic approximation can then be used to solve for the moments in terms of expectation values. Inserting these solutions back into the equations of motion for expectation values is equivalent to the path-integral effective action from [49]. The generalization lies in the fact that it is not necessary to perform the adiabatic approximation. Non-adiabatic properties may indeed be important in a gravitational context, even for static backgrounds. This is due to the fact that a space-time covariant treatment implies non-adiabatic behavior if there are relevant variations of the fields, not only in time, but also in space. Moreover, the general methods of [42, 43] provide direct information about properties of moments which may lead to additional physics insights. For instance, we will derive new bounds on quantum fluctuations of static black holes in addition to standard uncertainty relations.

3.3.3 Combined geometry of space-time and quantum physics

Importantly, these generalizations also allow us to consider non-Gaussian and possibly mixed states. In the present context, we are looking for superposition states of masses. In a Newtonian picture, such states could be understood as wave functions in an energy representation of a mass superposition, or in a position representation describing the location of the superposed masses. However, masses or positions are not necessarily the fundamental degrees of freedom of a quantum theory of gravity in which a geometrical superposition state may be constructed. The precise nature of fundamental degrees of freedom depends on the approach to quantum gravity, but it is to be expected that a rather simple state for the superposition of two masses or two positions can only be obtained after a complicated procedure of tracing out infinitely many local degrees of freedom of a fundamental theory of quantum gravity. It is therefore important that physical results do not depend on assumptions of Gaussianity, purity, or other properties of states that are being superposed. The only reliable assumption is semiclassicality in a general sense, such as the existence of an expansion in $\hbar$, in order to preserve the classical limit in a Newtonian regime. A general parameterization of states by moments up to a certain order gives us enough freedom to obey this condition. As we will see, specific solutions for moments can be derived from geometrical constraints and staticity conditions.

We use these general methods in order to propose a new approach to the question of superposition geometries based on a canonical formulation of general relativity. This procedure has two main advantages in the present context. First, while the usual canonical formulation of general relativity does not work with space-time tensors and is not manifestly covariant, it implies strict algebraic conditions on possible equations of motion, which are compatible with general covariance. These conditions can be described by the requirement that the Poisson brackets between the local generators of space and time translations remain closed. These generators are formally given by the diffeomorphism and Hamiltonian constraints of canonical gravity, respectively. In theories known to be generally covariant, such as general relativity, evaluating canonical equations is usually considered more tedious than evaluating equations for space-time tensors, such as Einstein's equation. However, canonical equations have the great advantage of allowing us to analyze the covariance of proposed modifications of general relativity without having to know beforehand that the modifications are covariant, or whether they may be related to non-Riemannian geometries. Canonical gravity is therefore a useful tool for analyzing possible geometrical descriptions of superpositions of gravitational fields.

Secondly, canonical methods are useful because they employ phase-space formulations of the gravitational field, which can be combined with suitable phase-space formulations of quantum mechanics, such as the canonical effective methods introduced in [42, 43].

General covariance is implemented in this picture by having a suitable form of gauge generators, as phase-space functions that obey certain algebraic relations based on the Poisson bracket. One of the gauge generators is closely related to the gravitational Hamiltonian because the Hamiltonian generates time translations, which are one example of covariant transformationa. The quantum Hamiltonian, written in a phase-space formulation of quantum mechanics, then provides a candidate for possible quantum modifications of the gauge generators. For instance, one can add fluctuation terms derived from the classical form of the Hamiltonian. A phase-space formulation of quantum mechanics also extends the Poisson bracket to quantum degrees of freedom, which in our case will be fluctuations of metric components. It is therefore possible to evaluate algebraic relations of the gauge generators, even if they include quantum terms. This procedure results in strict covariance conditions for fluctuating or superposed geometries. The quantum-corrected gauge generators can be viewed as semiclassical approximations of transformations between quantum reference frames in a full field-theory context of space-time.

In this paper, we apply canonical methods of gravity and quantum mechanics in the weak-field regime of static spherically symmetric space-times, reviewed in Sections 3.4 and 3.5, respectively. By including phase-space degrees of freedom for quantum fluctuations, we will be able to analyze the geometrical structure and physical properties of the gravitational field implied by two or more masses in superposition at the same central point. Our methods are based on [76], where the general formalism has been laid out for models of canonical quantum gravity. The previous treatment in a Schwarzschild gauge, relevant for horizon properties, is replaced here by a derivation in isotropic coordinates, which is more suitable for a complete weak-field limit. This choice of various gauges is possible only because we are dealing with a covariant space-time formulation. There is therefore a universal theory of spherically symmetric black holes and their quantum fluctuations that can be used to derive a large set of physical properties. Our new weak-field results, such as novel terms in quantum Newton's potentials, as well as bounds on fluctuations allowed in a covariant context, are contained in Section 3.6.

3.4 Canonical description of space-time structure

The canonical formulation of generally covariant systems describes space-time as an evolving geometry on spacelike hypersurfaces. From the space-time point of view, the spatial geometry on a given hypersurface is determined by the induced spatial metric q_{ab}, and its velocity or momentum p^{ab} is related tothe extrinsic curvature of the hypersurface in space-time [39, 40, 88]. These relationships define a phase-space structure on the space of geometries. The canonical formulation replaces the derivation of an induced metric and extrinsic curvature from a 4-dimensional line element. It is replaced with coupled evolution equations for two phase-space fields, q_{ab} and p^{ab}, together with gauge transformations. These gauge transformations ensure that a 4-dimensional geometry exists, from which these fields may be induced. Geometrically, the gauge transformations correspond to infinitesimal deformations of the spatial hypersurfaces in the normal and tangential directions [41]. These deformations are parameterized mathematically by the lapse function and shift vector, as described in this section.

Explicit expressions simplify if spatial geometries are restricted to be spherically symmetric, which we will assume from now on. The general spatial line element is then given by

$$\mathrm{d}s^2 = q_{xx}\mathrm{d}x^2 + q_{\vartheta\vartheta}(\mathrm{d}\vartheta^2 + \sin^2\vartheta\mathrm{d}\varphi^2) \tag{3.15}$$

with two functions, q_{xx} and $q_{\vartheta\vartheta}$, that depend on the radial coordinate x. If a family of hypersurfaces is considered, labeled by time coordinate t, then the metric components also depend on t.

The definition of the time derivative $\dot{q}_{xx} = \mathcal{L}_t q_{xx}$ as a Lie derivative requires a time direction that relates points on two nearby hypersurfaces with the same spatial coordinates. Such a time direction is not unique and implies additional free functions. It can be parameterized by the time-evolution vector field

$$t^a = c(Nn^a + M^a) \tag{3.16}$$

with the speed of light c and the unit normal n^a to a hypersurface. The free components of this space-time vector field are then separated into a normal component, the lapse function N, and the three components of a spatial vector field, the shift vector M^a, tangential to the hypersurface.

In a covariant theory, the spatial metric, together with the time-evolution vector field, can be used to reconstruct a space-time metric. As inverse metric tensors, the standard relationship is given by $g^{ab} = q^{ab} - n^a n^b$, because it implies two properties: (i) $q^{ab}n_a = 0$ with the timelike unit normal n_a, $n^a n_a = -1$, such that the spatial metric is induced on a hypersurface normal to n^a. And (ii) $q^{ab}s_a = g^{ab}s_a$ for any vector s_a tangential to the hypersurface, $n^a s_a = 0$, such that the spatial metric agrees with the space-time metric in this case. Solving (3.16) for n^a and inserting the result in g^{ab}, we obtain

$$g^{ab} = q^{ab} - \frac{1}{N^2}(t^a/c - M^a)(t^b/c - M^b)\,. \tag{3.17}$$

Inversion of this metric tensor implies the reconstructed space-time line element:

$$\mathrm{d}s^2 = -N^2c^2\mathrm{d}t^2 + q_{ab}(\mathrm{d}x^a + M^a c\mathrm{d}t)(\mathrm{d}x^b + M^b c\mathrm{d}t)\,. \tag{3.18}$$

For (3.18) to provide a generally covariant description of the original canonical theory, we must have slicing independence. Every family of spacelike hypersurfaces (or every time coordinate t that defines spacelike hypersurfaces $t = \mathrm{const}$) implies a time-dependent family of induced spatial metrics and extrinsic curvatures, determined by (3.18). Each such family must evolve in a manner consistent with the canonical equations of motion. Since evolution is canonically determined as Hamilton's equations generated by a Hamilton function on phase space, this function itself must be transformed in a consistent way if the family of hypersurfaces changes. There must, therefore, be a set of Hamiltonian phase-space functions that obey specific algebraic relations through Poisson brackets. The classical theory of general relativity in canonical form can be used to derive these relations, which, in spherical symmetry, turn out to be:

$$\{D[M_1], D[M_2]\} = D[M_1M_2' - M_2M_1'] \tag{3.19}$$
$$\{H[N], D[M]\} = -H[MN'] \tag{3.20}$$
$$\{H[N_1], H[N_2]\} = -D[q_{xx}^{-1}(N_1N_2' - N_2N_1')] \tag{3.21}$$

Here, $H[N]$ is the Hamilton function for a given time component N in (3.16). $D[M]$ is the generator of a spatial shift tangential to a hypersurface, using the radial component of a spherically symmetric version of (3.16).

These relations describe the symmetry of infinitesimal deformations of spatial hypersurfaces. In a physical theory presented in canonical form, abstract generators $D[M]$ and $H[N]$, fulfilling the relations (3.19)–(3.21), are realized by specific phase-space functions. These functions depend on the geometrical canonical variables q_{ab} and p^{ab}, such that their Poisson brackets model (3.19)–(3.21). Since the underlying symmetries are gauge transformations, the generators must vanish for physical phase-space solutions: $D[M] = 0$ for all M and $H[N] = 0$ for all N. These constraint equations depend on M and N, as explicitly shown, but also on q_{ab} and p^{ab} through their phase-space realizations. The constraints do not directly depend on position coordinates x because they are globally defined expressions given by spatial integrals. We use the conventional expression $H[N]$ to denote the Hamiltonian constraint, and similar notation for its quantum-corrected versions. In addition, the notation H by itself will represent the Hamiltonian constraint's integrand excluding the lapse function. The lapse function always appears as a single factor in the full integrand of $H[N]$ in order to be compatible with relations (3.19)–(3.21) linear in D and H.

Explicit expressions of the gravitational constraints in spherical symmetry are given by

$$H[N] = -\frac{c^3}{2G}\int \mathrm{d}x N(x)\left(\frac{\phi_2 p_2^2}{2\sqrt{\phi_1}} + 2\sqrt{\phi_1}p_1p_2 + \left(1 - \left(\frac{\phi_1'}{\phi_2}\right)^2\right)\frac{\phi_2}{2\sqrt{\phi_1}} - 2\left(\frac{\phi_1'}{\phi_2}\right)'\sqrt{\phi_1}\right) \tag{3.22}$$

and

$$D[M] = \frac{c^3}{2G}\int \mathrm{d}x M(x)\left(-\phi_1' p_1 + p_2'\phi_2\right) . \tag{3.23}$$

We parameterize the metric components as scalar fields

$$\phi_1 = q_{\vartheta\vartheta} \quad \text{and} \quad \phi_2 = 2\sqrt{q_{xx}q_{\vartheta\vartheta}} \tag{3.24}$$

with momenta p_1 of ϕ_1 and p_2 of ϕ_2, such that $\{\phi_1(x), p_1(y)\} = 2Gc^{-3}\delta(x-y)$ and $\{\phi_2(x), p_2(y)\} = 2Gc^{-3}\delta(x-y)$. Here, G is Newton's constant and we use primes to indicate partial derivatives by x. We assign units of length to the coordinate x and use unitless angles ϑ and φ. Since the line element has units of length squared, q_{xx} is unitless and $q_{\vartheta\vartheta}$ has units of length squared. Therefore, ϕ_1 has units of length squared as well, while ϕ_2 has units of length. The momenta's units are then determined by the form of our Poisson brackets: A Poisson bracket of a 1-dimensional field theory has units of $1/\hbar$ (from the product of partial derivatives of position and momentum components) times inverse length from $\delta(x-y)$. Using the expression $\ell_{\mathrm{P}} = \sqrt{G\hbar/c^3}$ for the Planck length, the factors of G/c^3 in our Poisson brackets therefore provide units of length squared divided by $\hbar$. This is compatible with the units of ϕ_1 and ϕ_2 if p_1 has units of inverse length and p_2 is unitless. Our Hamiltonian constraint then has units of energy divided by c. Thus, a Poisson bracket with this expression generates a spatial derivative, rather than a time derivative, as appropriate for a geometrical object.

The relationships between the momenta and time derivatives of the metric components, related to extrinsic curvature, need not be imposed independently. They follow from Hamilton's equations applied to ϕ_1 and ϕ_2: If our time evolution vector field is given by

(3.16) with spherically symmetric components N and M, then the time derivatives are

$$\frac{\dot{\phi}_1}{c} = \{\phi_1, H[N] + D[M]\} = -2N\sqrt{\phi_1}p_2 - M\phi_1' \tag{3.25}$$

and

$$\frac{\dot{\phi}_2}{c} = -N\frac{\phi_2}{\sqrt{\phi_1}}p_2 - 2N\sqrt{\phi_1}p_1 - (M\phi_2)'\,. \tag{3.26}$$

These equations can be solved for p_1 and p_2 in terms of $\dot{\phi}_1$, $\dot{\phi}_2$, N and M, in agreement with expressions for extrinsic curvature. Then the time derivatives of p_1 and p_2, derived in the same way, determine the evolution equations.

For the weak-field limit, we need the Schwarzschild solution in isotropic coordinates, which can be derived from the canonical equations following the standard Schwarzschild example in [89, 90]. These coordinates, by definition, imply a line element

$$\begin{aligned} \mathrm{d}s^2 \;\; &= \;\; -\left(1 + \frac{2\Phi(x)}{c^2} + O(\Phi(x)/c^2)\right) c^2\mathrm{d}t^2 \\ &\quad + \left(1 - \frac{2\Phi(x)}{c^2} + O(\Phi(x)/c^2)\right)\left(\mathrm{d}X^2 + \mathrm{d}Y^2 + \mathrm{d}Z^2\right) \end{aligned} \tag{3.27}$$

in Cartesian spatial coordinates (X, Y, Z) with $x^2 = X^2 + Y^2 + Z^2$. In this form, the line element directly implies a proper-time interval

$$\mathrm{d}\tau = \sqrt{-\mathrm{d}s^2/c^2} = \sqrt{1 + 2\frac{\Phi(x)}{c^2} - \left(1 - 2\frac{\Phi(x)}{c^2}\right)\frac{|\vec{V}|^2}{c^2}}\;\mathrm{d}t \tag{3.28}$$

to leading order in Newton's potential, $\Phi(x)/c^2$, and with the velocity vector $\vec{V} = \mathrm{d}\vec{X}/\mathrm{d}t$ in these coordinates. For small potential and non-relativistic speeds, a further expansion implies

$$\mathrm{d}\tau = \left(1 + \frac{\Phi(x)}{c^2} - \frac{1}{2}\frac{|\vec{V}|^2}{c^2}\right)\mathrm{d}t \tag{3.29}$$

with a combination of gravitational and special-relativity redshift effects. The metric components in isotropic coordinates therefore directly determine the gravitational potential $\Phi(x)$. (The leading-order potential may be obtained also in standard Schwarzschild coordinates. However, any corrections such as those studied below may require the specific form of isotropic coordinates, in which the gravitational potential and the speed are clearly separated in the non-relativistic limit. In this context, note that Newton's potential is not covariant because it was originally defined assuming an absolute time.

In general relativity, this non-covariant concept can be defined only in a specific slicing, which is the one given by isotropic coordinates.)

The isotropic line element can be expressed in the general spherically symmetric form by transforming the original (x, ϑ, φ) in (3.18) to Cartesian coordinates by

$$X = x\sin\vartheta\cos\varphi \quad , \quad Y = x\sin\vartheta\sin\varphi \quad , \quad Z = x\cos\vartheta\,. \tag{3.30}$$

The line element then takes the form

$$\begin{aligned} \mathrm{d}s^2 &= -\left(1+\frac{2\Phi(x)}{c^2}+O(\Phi(x)/c^2)\right)c^2\mathrm{d}t^2 \\ &\quad +\left(1-\frac{2\Phi(x)}{c^2}+O(\Phi(x)/c^2)\right)\left(\mathrm{d}x^2+x^2(\mathrm{d}\vartheta^2+\sin^2\vartheta\mathrm{d}\varphi^2)\right)\,. \end{aligned} \tag{3.31}$$

By comparing with (3.18), we read off the condition

$$q_{xx} = \frac{q_{\vartheta\vartheta}}{x^2} \tag{3.32}$$

which can be used to define isotropic coordinates in the canonical formulation, or

$$\phi_2 = 2\frac{\phi_1}{x} \tag{3.33}$$

for the fields used here. In addition, since the line element in isotropic coordinates is static, we require that all time derivatives, as well as the $\mathrm{d}x\mathrm{d}t$-component M of the line element, vanish. These conditions, taken together, uniquely determine the isotropic slicing into hypersurfaces. In other words, they determine the gauge of the canonical theory.

It immediately follows that the constraint $D[M] = 0$, as well as (3.25) and (3.26), are identically satisfied. The remaining constraint simplifies to

$$H[N]_{\mathrm{static}} = -\frac{c^3}{2G}\int \mathrm{d}x N(x)\left(\frac{\sqrt{\phi_1}}{x}+\frac{3x(\phi_1')^2}{4\phi_1^{3/2}}-\frac{x\phi_1''}{\sqrt{\phi_1}}-\frac{\phi_1'}{\sqrt{\phi_1}}\right)\,. \tag{3.34}$$

This expression vanishes for all $N(x)$ if ϕ_1 satisfies the differential equation

$$\phi_1'' - \frac{3}{4}\frac{(\phi_1')^2}{\phi_1} - \frac{\phi_1}{x^2} + \frac{\phi_1'}{x} = 0\,. \tag{3.35}$$

The substitution $f(x) = \sqrt{x}\phi_1(x)^{1/4}$ us employed for the convenience of simplifying this equation to $f'' = 0$, such that

$$\phi_1(x) = \left(a\sqrt{x} + \frac{b}{\sqrt{x}}\right)^4 \tag{3.36}$$

with two constants, a and b. Therefore,

$$q_{xx} = \frac{\phi_1}{x^2} = \left(a + \frac{b}{x}\right)^4 . \tag{3.37}$$

We can choose $a = 1$ without loss of generality because a different value (other than zero) could always be absorbed into the Cartesian coordinates. The weak-field limit then relates

$$b = \frac{GM}{2c^2} \tag{3.38}$$

to the mass M, with Newton's constant G. The equation

$$\phi_1(x) = x^2\left(1 + \frac{b}{x}\right)^4 = x^2_{\text{Schwarzschild}} \tag{3.39}$$

provides the coordinate transformation between isotropic and Schwarzschild coordinates.

The remaining equations to solve are Hamilton's equations for p_1 and p_2, given by

$$\begin{aligned}\frac{\dot{p}_1}{c} &= -N''\frac{2\sqrt{\phi_1}}{\phi_2} + N'\frac{2\phi_1\phi_2' - \phi_2\phi_1'}{\sqrt{\phi_1}\phi_2^2} \\ &\quad + N\frac{-\left(p_2^2+1\right)\phi_2^2 + p_1p_2\phi_1\phi_2 + \phi_1\phi_1'\phi_2'/\phi_2 + \frac{1}{4}(\phi_1')^2 - \phi_1\phi_1''}{\phi_1^{3/2}\phi_2}\end{aligned} \tag{3.40}$$

and

$$\frac{\dot{p}_2}{c} = \frac{2N'\sqrt{\phi_1}\phi_1'}{\phi_2^2} + N\frac{\left(p_2^2+1\right)\phi_2^2 - (\phi_1')^2}{2\sqrt{\phi_1}\phi_2^2} . \tag{3.41}$$

With staticity, $p_1 = 0 = \dot{p}_2$, $p_2 = 0 = \dot{p}_2$, and the isotropy condition $\phi_2 = 2\phi_1/x$, these equations simplify to

$$\begin{aligned}0 &= -2x\phi_1N'' + (x\phi_1' - 2\phi_1)N' \\ &\quad - \frac{-5x^2(\phi_1')^2 + 4x\phi_1\left(\phi_1' + x\phi_1''\right) + 4\phi_1^2}{4x\phi_1}N\end{aligned} \tag{3.42}$$

and

$$0 \quad = \quad \frac{\phi_1^2 - \frac{1}{4}x^2(\phi_1')^2}{\phi_1} N - x^2\phi_1' N' . \tag{3.43}$$

Using (3.35) and (3.43), equation (3.42) is turned into

$$N'' = N\left(\frac{(\phi_1')^2}{8\phi_1^2} + \frac{\phi_1'}{4x\phi_1} - \frac{\phi_1}{x^3\phi_1'} - \frac{1}{2x^2}\right) = \frac{-4b}{(x+b)^2(x-b)} N \tag{3.44}$$

with our solution for $\phi_1(x)$, while (3.43) takes the form

$$\frac{N'}{N} = \frac{\phi_1}{x^2\phi_1'} - \frac{\phi_1'}{4\phi_1} = \frac{1}{x-b} - \frac{1}{x+b} \,. \tag{3.45}$$

The functions

$$N(x) = N_0 \frac{x-b}{x+b} \tag{3.46}$$

solve both equations, with an integration constant N_0 that can be absorbed into the time coordinate. Applying the coordinate transformation (3.39) confirms that this is the correct expression, which replaces the well-known lapse function in the Schwarzschild line element with a corresponding expression in isotropic coordinates upon implementing the coordinate transformation (3.39). Note that the alternative derivation of $N(x)$ is by using the coordinate transformation (3.39) in the original Schwarzschild lapse function.

3.5 Canonical description of quantum mechanics

Given a quantum system with canonically conjugate basic operators $\hat{q}$ and $\hat{p}$ with $[\hat{q}, \hat{p}] = i\hbar$, it is possible to equip states and operators with geometrical meaning on a quantum phase space. We first interpret operators $\hat{O}$ as functions $f_{\hat{O}}$ on the space of states ψ, given by the evaluation $f_{\hat{O}}(\psi) = \langle \psi, \hat{O}\psi \rangle$. There is a unique complex-valued function with a dense domain of ψ for every operator $\hat{O}$ acting on the Hilbert space of the system. In general, this function is not defined for all ψ because expectation values of unbounded operators may be infinite in some states, but it is defined for a dense subset in the topological sense. Useful examples for the following developments are the functions

$$q \quad = \quad f_{\hat{q}} = \langle \hat{q} \rangle \tag{3.47}$$

$$p \quad = \quad f_{\hat{p}} = \langle \hat{p} \rangle \tag{3.48}$$

as well as functions for products of multiple $\hat{q}$'s and $\hat{p}$'s, from which we can construct the central moments

$$\Delta(q^n p^m) = \langle(\hat{q} - \langle\hat{q}\rangle)^n(\hat{p} - \langle\hat{p}\rangle)^m\rangle_{\rm symm} \tag{3.49}$$

with operator products in completely symmetric (or Weyl) ordering.

3.5.1 Dynamics

On this function space, we define a Poisson bracket by

$$\{f_{\hat{A}}, f_{\hat{B}}\} = \frac{f_{[\hat{A},\hat{B}]}}{i\hbar} \tag{3.50}$$

or, equivalently,

$$\{\langle\hat{A}\rangle, \langle\hat{B}\rangle\} = \frac{\langle[\hat{A},\hat{B}]\rangle}{i\hbar} \tag{3.51}$$

which is extended to products of functions by using the Leibniz rule. This Poisson bracket turns Ehrenfest's theorem into a statement about Hamiltonian dynamics: The evolution of expectation-value functions is given by Hamilton's equations generated by the expectation value

$$H_{\rm eff} = f_{\hat{H}} = \langle\hat{H}\rangle \tag{3.52}$$

because

$$\{\langle\hat{A}\rangle, H_{\rm eff}\} = \frac{\langle[\hat{A},\hat{H}]\rangle}{i\hbar} = \frac{{\rm d}\langle\hat{A}\rangle}{{\rm d}t}\,. \tag{3.53}$$

If we use the basic expectation values q and p, together with central moments, as coordinates on the quantum phase space, we should write the effective Hamiltonian $H_{\rm eff}$ as a function of these variables. We can do so by using a formal Taylor expansion in

$$\begin{aligned} H_{\rm eff} &= \langle H(\hat{q},\hat{p})\rangle = \langle H(q + (\hat{q} - q), p + (\hat{p} - p)\rangle \\ &= H(q,p) + \sum_{n+m=2}^{N} \frac{1}{n!m!} \frac{\partial^{n+m} H(q,p)}{\partial q^n \partial p^m} \Delta(q^n p^m) \end{aligned} \tag{3.54}$$

assuming that the Hamilton operator is Weyl ordered. (If this is not the case, there will be additional reordering terms that explicitly depend on $\hbar$.)

The Poisson bracket of two moments is not constant, as can be seen by a direct calculation [42,46]. Therefore, the moments are not canonical coordinates on the quantum phase space. The Darboux theorem and its generalization to Poisson manifolds [47,48] guarantees the existence of local canonical coordinates with the standard values zero and one of basic Poisson brackets. However, suitable coordinates of this form may be difficult to derive in general. For low orders, several examples exist. We are particularly interested in the second-order canonical parameterization [30, 31, 33, 34]

$$\Delta(q^2) = s^2 \quad , \quad \Delta(qp) = sp_s \quad , \quad \Delta(p^2) = p_s^2 + \frac{U}{s^2} \tag{3.55}$$

where s and p_s are canonically conjugate, $\{s, p_s\} = 1$, and U has vanishing Poisson brackets with both s and p_s. The uncertainty principle restricts the values of U by $U \geq \hbar^2/4$. Further examples have been derived in [44, 45] for moments of up to fourth order or for two independent degrees of freedom. With these canonical variables, the effective Hamiltonian of a 1-dimensional mechanical system takes the form

$$H_{\rm eff} = \frac{p^2}{2m} + \frac{p_s^2}{2m} + \frac{U}{2ms^2} + V(q) + \frac{1}{2}V''(q)s^2\,, \tag{3.56}$$

which can directly be used to compute Hamiltonian equations of motion, with coupling terms (or quantum back-reaction) between (q, p) and (s, p_s).

The example of the harmonic oscillator, where $V(q) = \frac{1}{2}m\omega^2q^2$, shows that U determines the value of zero-point energies: In this case, the Hamiltonian contains the s-dependent potential

$$V_s(s) = \frac{U}{2ms^2} + \frac{1}{2}m\omega^2 s^2 \tag{3.57}$$

which is minimized by

$$s^2 = \frac{\sqrt{U}}{m\omega} = \frac{\hbar}{2m\omega}\,, \tag{3.58}$$

using the minimal value for U in the second step. At this value of s, the effective potential evaluates to a constant contribution $V_s(s) = \omega\sqrt{U} = \frac{1}{2}\hbar\omega$. We will consider this role of U in our extension to a 1-dimensional field theory in the radial direction of our spherically symmetric models. Quantization then require a suitable regularization procedure that subtracts zero-point energies or vacuum expectation values from the Hamiltonian, just as in standard quantum field theory.

For relativistic systems and the physics of space-time, we need two extensions of this formalism: a canonical formulation of constraint operators such as $\hat{H}$ and $\hat{D}$, and a suitable description for quantum field theories. The first extension [53–55] is technically involved because it requires general properties of constrained systems, but it follows directly from a basic definition of an effective constraint

$$C_{\text{eff}} = f_{\hat{C}} = \langle \hat{C} \rangle \tag{3.59}$$

for every constraint operator $\hat{C}$ in the system. Physical states then define a submanifold of the quantum phase space defined by $C_{\text{eff}} = 0$ for each constraint.

However, it turns out that this restriction is not sufficient because the quantum constraint equation $\hat{C}|\psi\rangle = 0$ implies that not only does $\langle \psi, \hat{C}\psi \rangle$ vanish, but also $\langle \psi, \hat{O}\hat{C}\psi \rangle$ for any operator $\hat{O}$. A single quantum constraint therefore implies infinitely many constrained expectation values. If we parameterize these constraints by

$$\langle (\hat{q} - q)^n (\hat{p} - p)^m \hat{C} \rangle = 0 \tag{3.60}$$

with positive integers n and m, there is a finite number of such constraints for any set of central moments up to some finite order. Such a system of constraints can be treated by the general methods for constrained systems as given in [**?**]. It constitutes a semiclassical approximation of infinitesimal transformations between quantum reference frames of space-time.

In spherically symmetric gravitational systems, we have two constraints, $H[N]$ and $D[M]$. For static solutions, D and its higher-order versions (3.60) vanish identically, and higher-order versions of H need only be considered for the ϕ-fields, but not for their momenta, such that $m = 0$ in (3.60). For constraints on second-order moments, including quantum fluctuations, it is sufficient to consider only $n = 0$ and $n = 1$ in (3.60). Moreover, the isotropy condition strictly relates ϕ_1 and ϕ_2, and therefore their fluctuations.

3.5.2 Ingredients from quantum field theory

In order to derive the relevant constraints and higher-order versions, we need to determine how quantum parameters such as s and p_s can be implemented for fields. In general, quantum fields may be in a state with non-zero correlations between their values of different positions, which would require infinitely many correlation fields. Here, for a first analysis that aims mainly to include quantum fluctuations, it will be sufficient to ignore such non-local correlations and only implement a version of (s, p_s) at each point on the radial line. Therefore, we will introduce an additional fluctuation field, ϕ_3 with a canonical momentum p_3, such that

$$\Delta(\phi_2^2) = \phi_3^2 \quad , \quad \Delta(\phi_2 p_2) = \phi_3 p_3 \quad , \quad \Delta(p_2^2) = p_3^2 + \frac{U}{\phi_3^2} \tag{3.61}$$

independently at each radial position x. The treatment of U now requires some care because, as shown by our discussion of the effective potential (3.57) of the harmonic oscillator, this parameter is related to zero-point energies. For a quantum field theory on Minkowski space-time, we would simply remove the U-term from any Hamiltonian as a subtraction of zero-point energies. In our background-independent treatment, however, we can only assume that we will be near Minkowski space-time in a suitable asymptotic region of small curvature, close to the regime where we will apply our main weak-field analysis. In regions of curved space-time that are not strictly Minkowskian, which are relevant for any radial dependence of quantities such an effective gravitational potential, the vacuum state and corresponding zero-point energies are not uniquely defined. As a result, we cannot simply remove the entire U-term from our Hamiltonian but rather subtract only the Minkowski limit, as we will see in more detail when we discuss the specific Hamiltonian.

For now, we conclude that some U-dependent contributions are likely to remain in the Hamiltonian and are fully subtracted only in the asymptotic Minkowski region at $x \to \infty$. The presence of a subtraction implies that we should no longer subject U to the inequality $U \geq \hbar^2/4$ from quantum mechanics. We will therefore impose only positivity of U without using a general non-zero lower limit. And since the subtraction is complete only at $x \to \infty$, we expect that remaining effects described by U may be given by a function $U(x)$ of x that should be determined by consistency conditions within the quantum theory but cannot be known a priori.

Another basic implication of a treatment of our model as a quantum field theory is that quantizations of $\phi_2(x)$ and $p_2(x)$ are operator-valued distributions. Their commutator equals $i\hbar G/c^3$ times a 1-dimensional delta function, where the factor of G/c^3 comes from our Poisson bracket. Combining all factors, including an inverse length scale for units of the delta function, the commutator has units determined by $G\hbar/(c^3 L) = \ell_{\rm P}^2/L$ with the Planck length $\ell_{\rm P}$ and a relevant macroscopic, $\hbar$-independent length L such as the Schwarzschild radius. The uncertainty relation, derived from this field-theory commutator, then implies that $U = \Delta(\phi_2^2)(\Delta(p_2^2) - p_3^2)$ has units of $\ell_{\rm P}^4/L^2$. This result differs from the units of $\hbar^2/4$ obtained for U in quantum mechanics. Therefore, just based on dimensional reasoning, it is now impossible to impose a lower bound of $\hbar^2/4$ on

U in a quantum field theory. This observation provides further motivation to require only positivity of $U(x)$ but not a stronger non-zero lower bound.

Following our discussion of units for the classical fields, the individual fields used here have units of length for ϕ_2 and ϕ_3, inherited from the metric, and no units for p_2 and p_3. The quantum fields ϕ_3 and p_3, defined as square roots of second-order moments of a state, are expected to be proportional to $\sqrt{\hbar}$ for semiclassical solutions. These conditions are consistent with one another if $\phi_3 \sim \ell_{\rm P}$ and $p_3 \sim \ell_{\rm P}/L$. For U, we then obtain $U \sim \ell_{\rm P}^4/L^2$, in agreement with the result suggested by field commutators.

3.5.3 Gauge choice and constraints

Finally, we must implement the isotropy condition (3.33). Since momentum fluctuations may contribute non-zero terms such as U/ϕ_3^2, even in static situations, we first find the accompanying equations to $2\phi_1 - x\phi_2 = 0$, a corresponding condition on momenta. For the isotropy condition to be preserved over time and in the zero M limit, the equations of motion (3.25) and (3.26) imply that the momenta must be related by

$$xp_1 - p_2 = 0\,. \tag{3.62}$$

This pair of conditions is second class: The smeared Poisson bracket

$$\{2\phi_1(x) - x\phi_2(x), \int \lambda(y)(yp_1(y) - p_2(y))\mathrm{d}y\} = 6\frac{G}{c^3}x\lambda(x) \tag{3.63}$$

is non-zero for $x \neq 0$. We should therefore impose the two conditions contained in the second-class pair before we quantize, for instance by eliminating (ϕ_1, p_1) in favor of (ϕ_2, p_2). The remaining field will then be supplied by quantum variables, as in (3.61). (After imposing the second-class constraints, the Dirac bracket of ϕ_2 and p_2 is 2/3 times the Poisson bracket. The additional factor provides a constant rescaling of all time derivatives, which we may ignore since we are interested in static configurations.)

The reduced classical Hamiltonian constraint after imposing the second-class isotropy conditions equals

$$H[N] = -\frac{c^3}{2G}\int \mathrm{d}x N(x)\sqrt{\frac{\phi_2}{2x}}\left(3p_2^2 + \frac{3}{4} - \frac{3}{2}\frac{x\phi_2'}{\phi_2} + \frac{3}{4}\frac{x^2(\phi_2')^2}{\phi_2^2} - \frac{x^2\phi_2''}{\phi_2}\right) \tag{3.64}$$

A Taylor expansion as in (3.54) then leads to

$$\bar{H}[N] = H[N] + H_2[N] \tag{3.65}$$

with the classical $H[N] = \int \mathrm{d}x NH$ and and a quantum correction

$$\begin{aligned} H_2[N] &= \int \mathrm{d}x N(x) \left(\frac{1}{2}\frac{\partial^2 H}{\partial p_2^2}\left(p_3^2 + \frac{U}{\phi_3^2}\right) - H_0 + \frac{\partial^2 H}{\partial\phi_2 \partial p_2}\phi_3 p_3 \right. \\ &\quad \left. + \frac{1}{2}\frac{\partial^2 H}{\partial\phi_2^2}\phi_3^2 + \frac{\partial^2 H}{\partial\phi_2\partial\phi_2'}\phi_3\phi_3' + \frac{1}{2}\frac{\partial^2 H}{(\partial\phi_2')^2}(\phi_3')^2 + \frac{\partial^2 H}{\partial\phi_2\partial\phi_2''}\phi_3\phi_3'' \right) \end{aligned} \tag{3.66}$$

Partial derivatives of H follow the form of the expanded effective Hamiltonian in (3.54), and H_0 is the vacuum expectation value to be subtracted from the Hamiltonian. We can write this terms as

$$H_0 = \frac{1}{2}\lim_{b\to 0}\frac{U}{\phi_3^2}\frac{\partial^2 H}{\partial p_2^2} \tag{3.67}$$

if we use b, related to the black-hole mass in specific solutions, as a parameter that determines deviations from Minkowski space-time. Partial derivatives of the classical Hamiltonian then implies the following coefficients:

$$\begin{aligned} H_2[N] &= -\frac{c^3}{2G}\int \mathrm{d}x N(x) \left(3\frac{\sqrt{\phi_2}p_3^2}{\sqrt{2x}} + 3\frac{\phi_3 p_2 p_3}{\sqrt{2x\phi_2}} - \frac{3}{32}\frac{\phi_3^2}{\sqrt{2x}\phi_2^{3/2}} - \frac{9}{16}\frac{\sqrt{x}}{\sqrt{2}\phi_2^{5/2}}\phi_2'\phi_3^2 \right. \\ &\quad + \frac{45}{32}\frac{x^{3/2}}{\sqrt{2}\phi_2^{7/2}}(\phi_2')^2\phi_3^2 - \frac{3}{8}\frac{x^{3/2}}{\sqrt{2}\phi_2^{5/2}}\phi_2''\phi_3^2 + \frac{3}{4}\frac{\sqrt{x}}{\sqrt{2}\phi_2^{3/2}}\phi_3\phi_3' \\ &\quad - \frac{9}{4}\frac{x^{3/2}}{\sqrt{2}\phi_2^{5/2}}\phi_2'\phi_3\phi_3' + + \frac{3}{4}\sqrt{\frac{\phi_2}{2x}}\frac{x^2}{\phi_2^2}(\phi_3')^2 + \frac{1}{2}\frac{x^{3/2}}{\sqrt{2}\phi_2^{3/2}}\phi_3\phi_3'' \\ &\quad \left. + 3\left(\sqrt{\frac{\phi_2}{2x}}\frac{U}{\phi_3^2} - \lim_{b\to 0}\frac{U}{\phi_3^2}\right)\right) . \end{aligned} \tag{3.68}$$

In the vacuum subtraction, we used the Minkowski limit $\phi_2 \to 2x$ of ϕ_2. At this point, the b-dependence of ϕ_3 and possibly U remains to be determined from field equation.

The only higher-order constraint we need to consider after the reduction is given by

$$H_{\phi_2}[L] = \langle(\hat{\phi}_2 - \langle\hat{\phi}_2\rangle)\hat{H}[L]\rangle . \tag{3.69}$$

Also here, a Taylor expansion provides the constraint as a function of moments:

$$H_{\phi_2}[L] = \int \mathrm{d}x L(x) \left(\frac{\partial H}{\partial\phi_2}\phi_3^2 + \frac{\partial H}{\partial\phi_2'}\phi_3\phi_3' + \frac{\partial H}{\partial\phi_2''}\phi_3\phi_3'' + \frac{\partial H}{\partial p_2}\phi_3 p_3 \right)$$

$$
\begin{aligned}
&= -\frac{c^3}{2G}\int \mathrm{d}x L(x) \\
&\left(\left(\frac{3}{2}\frac{p_2^2}{\sqrt{2x\phi_2}} + \frac{39}{8}\frac{1}{\sqrt{2x\phi_2}} + \frac{3}{4}\frac{\sqrt{x}}{\sqrt{2}\phi_2^{3/2}}\phi_2' - \frac{9}{8}\frac{x^{3/2}}{\sqrt{2}\phi_2^{5/2}}(\phi_2')^2 + \frac{x^{3/2}}{2\sqrt{2}\phi_2^{3/2}}\phi_2''\right)\phi_3^2\right. \\
&\left. + \left(-\frac{6}{4}\sqrt{\frac{x}{2\phi_2}} + \frac{6}{4}\frac{x^{3/2}}{\sqrt{2}\phi_2^{3/2}}\phi_2'\right)\phi_3\phi_3' - \frac{x^{3/2}}{\sqrt{2\phi_2}}\phi_3\phi_3'' + 5\sqrt{\frac{2\phi_2}{x}}p_2\phi_3 p_3\right) . \qquad (3.70)
\end{aligned}
$$

3.5.4 Solutions

Inserting the background solution

$$
\sqrt{\frac{\phi_2}{2x}} = \left(1 + \frac{b}{x}\right)^2 \qquad (3.71)
$$

based on (3.36) with $a = 1$ and (3.33), as well as $p_2 = 0$, we have

$$
\begin{aligned}
&H_{\phi_2}[L] \qquad (3.72) \\
= \quad & -\frac{c^3}{2G}\int \mathrm{d}x \frac{L\phi_3}{x(1+b/x)^4}\left(3\frac{b}{x}(1-b/x)\phi_3 - 3\frac{b}{x}(1+b/x)x\phi_3' - \frac{1}{2}(1+b/x)^2 x^2\phi_3''\right) .
\end{aligned}
$$

The constraint $H_{\phi_2}[L] = 0$ is fulfilled for all $L(x)$ if ϕ_3 obeys the second-order differential equation

$$
3\frac{b}{x}(1-b/x)\phi_3 - 3\frac{b}{x}(1+b/x)x\phi_3' - \frac{1}{2}(1+b/x)^2 x^2\phi_3'' = 0 \qquad (3.73)
$$

which does not seem to have simple closed-form solutions. However, thanks to the factor of b/x in each of the two lower-order derivative terms, there is a unique solution (up to a constant multiplicative factor) which permits an asymptotic expansion $\phi_3(x) \propto 1 + a_1/x + a_2/x^2 + \cdots$, compatible with the weak-field limit. The coefficients a_1, a_2, $\ldots$ can be computed iteratively from

$$
(3b - a_1)x^{-1} + ((3ba_1 - 3b^2) + 3ba_1 - (2a_1 b + 3a_2))x^{-2} + O(x^{-3}) = 0 \qquad (3.74)
$$

and collecting terms with the same factor of x^{-n}. The three individual terms in the second parenthesis result from the three independent derivative expressions in (3.73). From the first two terms of the expansion, we obtain the solution

$$
\phi_3(x) = C\left(1 + \frac{3b}{x} + \frac{3b^2}{x^2} + \cdots\right) . \qquad (3.75)
$$

To first order in b/x, this result is consistent with the corresponding solution derived for Schwarzschild coordinates in [76]. Since ϕ_3 should have units of length, like ϕ_2, and be proportional to $\sqrt{\hbar}$ for semiclassical solutions, we have $C \propto \ell_{\rm P}$.

We recall that the new field ϕ_3 describes quantum fluctuations of the metric component ϕ_2, or of the full spatial metric, since ϕ_1 is strictly related to ϕ_2 by the isotropy condition (3.33). In order to derive implications of quantum fluctuations on Newton's potential, we have to solve the remaining constraint and evolution equations for a correction δN of the classical lapse function, which happens to depend on the correction $\delta\phi_2$ of ϕ_2 implied by quantum effects. The required equations are considerably longer than those used for our derivation of ϕ_3 and are therefore collected in Appendix 3.8. For the weak-field behavior, it is sufficient to derive the asymptotic form of solutions for ϕ_2 and N for large x. Given the leading orders $\phi_2 \sim x$ and $N \sim 1$, our equations can be solved for the coefficients c_i and d_j in

$$\phi_2 = \phi_2^{(0)} + c_1 + \frac{c_2}{x} + \cdots \quad , \quad N = N^{(0)} + \frac{d_1}{x} + \frac{d_2}{x^2} + \cdots \tag{3.76}$$

where $\phi_2^{(0)}$ and $N^{(0)}$ are the classical solutions in isotropic coordinates. Corrections to Newton's potential can be read off directly from $N(x)^2 = 1 + 2\Phi(x)/c^2$, in which $\Phi(x)$ is the quantum-corrected gravitational potential.

A general implication of the dynamical equations, in particular using $\dot{p}_2 = 0$ for static solutions and to leading order in $1/x$, is that $d_1 = 0$; see equation (3.107) in the appendix. The leading term in Newton's potential is therefore unmodified, but there are higher-order terms in $1/x$. The remaining coefficients are determined by terms in $\dot{p}_2 = 0$ of higher order in $1/x$, coupled to the Hamiltonian constraint. With the quantum extension, the latter depends on the function U, for which we should assume a power-law form $U(x) \propto x^{-\kappa}$, with some $\kappa \geq 0$, in order for a weak-field expansion to exist. It turns out that there are also rather strong reality conditions implied by the explicit terms of the Hamiltonian constraint, which is quadratic in ϕ_3 and the perturbation $\phi_2 - \phi_2^{(0)}$ and should be compatible with real c_i. A direct evaluation shows that these conditions can be realized only if $U(x) = U_0/x$ with a constant U_0, but not for constant U or $U \propto 1/x^2$. Relevant conditions can be seen, for instance, in (3.105), derived for $U(x) = U_0/x$: The second-order term in this expansion of $H[N]$ in $1/x$ has mixed signs, such that there is a range of quantum parameters for which c_1 is real. For $U(x) = U$ or $U(x) = U_0/x^\kappa$ with $\kappa \geq 2$, by contrast, all signs in this term would be the same, implying imaginary solutions for c_1. These properties are discussed in more detail in App. 3.8.3.

We evaluate the expansions (3.105) and (3.107) for the coefficients c_i and d_i up to

$i = 3$, or third order in $1/x$, using the power-law $U(x) = U_0/x$ that allows real solutions. The new constant U_0 introduced by this parameterization should, according to the discussion following equation (3.61), have units of $\hbar^2/L$ with a classical length scale L. A natural choice for the latter is the Schwarzschild radius or b in the present context. The same classical parameter also appears in the equations for c_i and d_i because they have been obtained after perturbing ϕ_2 and N around their b-dependent classical values. The coupled set of conditions for d_2 and d_3 in N as well as c_1, c_2 and c_3 in ϕ_2, contains linear and quadratic equations which are solved by

$$c_1 = \epsilon \frac{8\sqrt{bU_0}}{C}\sqrt{1 - C^4/(64bU_0)} \tag{3.77}$$

$$c_2 = \epsilon b \frac{3\sqrt{bU_0}}{8C} \frac{70 - 9C^4/(8bU_0)}{\sqrt{1 - C^4/(64bU_0)}} \tag{3.78}$$

$$d_2 = \epsilon b \frac{\sqrt{bU_0}}{96C} \frac{394 - 47C^4/(8bU_0)}{\sqrt{1 - C^4/(64bU_0)}} \tag{3.79}$$

$$c_3 = \epsilon b^2 \frac{\sqrt{bU_0}}{80C} \frac{2445 - 79C^4/(bU_0) + 163C^8/(256b^2U_0^2)}{(1 - C^4/(64bU_0))^{3/2}} \tag{3.80}$$

$$d_3 = -\epsilon b^2 \frac{\sqrt{bU_0}}{640C} \frac{9735 - 589C^4/(2bU_0) + 569C^8/(256b^2U_0^2)}{(1 - C^4/(64bU_0))^{3/2}} . \tag{3.81}$$

There is a single sign choice $\epsilon = \pm 1$ in c_1, which is not fixed by the constraint but determines the sign choices in c_2, c_3, d_2, and d_3.

3.6 Implications

The values (3.77)–(3.79) have several unexpected implications. The solution for d_2 immediately gives the leading quantum correction to Newton's potential and can therefore be compared (although not directly, as we will see) with calculations in perturbative quantum gravity. The same coefficient, together with c_1 and c_2, implies characteristic features of a line element that may be used for a covariant description of superpositions of central masses.

3.6.1 Effective potentials

First, using $d_1 = 0$, the quantum-corrected Newton potential is given by

$$V(x) = \frac{c^2}{2}(N(x)^2 - 1) \approx -\frac{GM}{x} + \frac{G^2M^2}{x^2c^2} + \frac{d_2}{2x^2} + O(x^{-3}) \tag{3.82}$$

where we used $b = GM/(2c^2)$. We also expanded the lapse function $N^{(0)}(x)$ in isotropic coordinates to second order in x^{-1}. Since $d_1 = 0$, Newton's constant is not renormalized. The leading quantum correction d_2 from (3.79) is of the order $GM\ell_{\rm P}/c^2$ because C, for a semiclassical state, is of the order $\sqrt{\hbar} \propto \ell_{\rm P}$ and $\sqrt{bU_0}$ is of the order C^2. The classical term in (3.82) quadratic in x^{-1} agrees with the perturbative result from [15], and it has the same origin. However, the leading quantum correction in this case is of the order $M\ell_{\rm P}^2/x^3$, which is smaller than our $M\ell_{\rm P}/x^2$ for generic quantum corrections. If we extend $\delta\phi_2$ and δN to higher orders in x^{-1}, we obtain additional terms of the order $b^n\ell_{\rm P}$ with integers $n > 1$. Their dependence on $\ell_{\rm P}$ or $\hbar$ remains of first order, which is fixed in our approximation by using second-order moments of order $\hbar$ in the constraint. As a quadratic expression in $\delta\phi_2$ and δN, the constraint then implies corrections in δN of the order $\sqrt{\hbar}$. Higher orders in $\sqrt{\hbar}$ would require higher moments.

We could try to impose $d_2 = 0$, eliminating our larger term compared with perturbative quantum field theory, by choosing suitable values for C and U_0. This is possible only if

$$C^4 = \frac{3152bU_0}{47} \approx 67.1bU_0\,, \tag{3.83}$$

but this value is not compatible with the upper bound on $C^4 < 64bU_0$ implied by the square root in the denominator of d_2 being real. Therefore, our quantum corrections are always greater than what is expected from perturbative quantum gravity as derived (with varying results) for instance in [?, 14, 15, 91–95]. This discrepancy can be explained by the fact that we are considering different physical settings, such that the underlying states, which determine quantum corrections, need not be the same. Here, we use a generic semiclassical state parameterized by its moments. The moments, derived from the gravitational constraints, describe an entire black hole (or a superposition of black holes) that has been formed by some collapse process and eventually settled down to a static configuration. In contrast, the result from perturbative quantum gravity implicitly refers to 2-particle states close to the interacting vacuum. This vacuum is itself close to the Gaussian vacuum state of a free field theory in perturbative situations. Moreover, perturbative calculations are usually done for two masses on Minkowski space-time, while our results are for the effective Newton potential experienced by a light test mass in the curved background of a heavy mass. The physical settings are therefore different, which means that differing results are not problematic. Coefficients, and even orders of quantum corrections, may well be different in our case.

We already referred to an upper bound on C, implied by reality conditions on the expansion coefficients (3.77)–(3.79). This upper bound is of interest and is explicitly given by

$$\phi_3 \sim C \leq 2\sqrt{2}(bU_0)^{1/4} \,. \tag{3.84}$$

If the classical length scale L in U_0 is equated with b, we have $(bU_0)^{1/4} \sim \sqrt{\hbar}$, which is the expected order of C for semiclassical solutions. With the required length units for ϕ_2 and ϕ_3, we can parameterize bU_0 as $u\ell_{\rm P}^4$ with a unitless number u. Given the inequality in (3.84), a space-time geometry is compatible with such quantum corrections only if C, or the quantum fluctuations ϕ_3 of ϕ_2, are subject to the upper bound

$$\Delta\phi_2 \leq 2\sqrt{2}u^{1/4}\ell_{\rm P} \,. \tag{3.85}$$

Since (3.61) implies

$$(\Delta p_2)^2(\Delta\phi_2)^2 = U + p_3^2\phi_3^2 \geq U = \frac{u\ell_{\rm P}^4}{b^2} \,, \tag{3.86}$$

momentum fluctuations are bounded from below by

$$\Delta p_2 \geq \frac{\sqrt{u}\ell_{\rm P}^2}{b\Delta\phi_2} \geq \frac{u^{1/4}\ell_{\rm P}}{2\sqrt{2}b} \,. \tag{3.87}$$

Static solutions as derived here, which by assumption have a vanishing expectation value of p_2, can therefore exist only with non-zero momentum fluctuations. The geometrical meaning of the momentum as extrinsic curvature of spatial slices suggests that quantum black holes that are static on average are, in fact, superpositions of collapsing and expanding geometries, corresponding to wave functions that have support on positive and negative values of p_2. An open question is whether such quantum oscillations would imply the emission of gravitational waves if non-spherical perturbations are included in our models. Since $u \propto b^{1/4}$, according to its definition, the lower bound on Δp_2 decreases with the mass and is negligible for Schwarzschild radii much greater than the Planck length. This superposition effect in the momentum and possible quantum oscillations would therefore be relevant only for microscopic or primordial black holes.

Compared with the previous [76], on which the present paper is based, we have performed calculations in isotropic rather than Schwarzschild coordinates. This change of coordinates implies that a clear analysis of the weak-field limit could be performed with an unambiguous definition of Newton's potential. Crucially, our space-time description made it possible to consider different coordinate systems within the same model, which is a requirement for a simultaneous interpretation of physical effects in the weak-field regime (derived here) as well as close to the horizon of a black hole (considered in [76]). Our novel weak-field results qualitatively confirm the fall-off behavior of the function $U(x)$ which previously was found only numerically. They also led us to re-interpret this function by introducing a suitable length scale, L, implied by the density-behavior of fields compared with point particles in quantum mechanics, as discussed in detail in the passage following equation (3.61). The function $U(x)$ here therefore has units of $\hbar^2/L$, rather than $\hbar^2$, which is important for estimating orders of magnitude for quantum corrections.

3.6.2 Black-hole superpositions

We have obtained specific expressions (3.77)–(3.79), together with $d_1 = 0$, for the expansion coefficients c_i of $\delta\phi_2$ and d_i of δN. In canonical gravity, ϕ_2 is a phase-space degree of freedom that would be quantized in canonical quantum gravity and therefore have quantum fluctuations. Here, these fluctuations are described by a new independent field ϕ_3 and determined in our solutions by the parameter C. Solving the Hamiltonian constraint also implies a dependence of c_i and d_i on the spatial function $U(x)$ that can be interpreted as the uncertainty product, imposing a lower bound $(\Delta\phi_2)^2(\Delta p_2)^2 - \phi_3^2 p_3^2$ as an expression of the uncertainty relation.

In contrast to the c_i, the coefficients d_i in an expansion of δN do not appear in a phase-space degree of freedom because N does not play such a role in the formulation of canonical gravity used here. (An extended phase-space formulation could include N as a phase-space degree of freedom, but only in a limited role because its momentum would be constrained to vanish.) Canonically, the coefficients c_i and d_i, respectively, therefore play different roles in how they may be related to wave functions or quantum fluctuations. The c_i in $\delta\phi_2$ have a more direct relationship with quantum fluctuations than the d_i in δN. In the derivation of Newton's potential, however, the d_i are more relevant because the gravitational potential in the weak-field limit appears in N to leading order.

An effective Newton's potential therefore depends on quantum fluctuations in a rather indirect way. One such dependence is given by the relationships between c_i and d_i implied by the constraints and staticity conditions, which we solved in order to obtain our solutions. Moreover, if we use the full space-time metric, all the coefficients c_1, c_2, d_1 and d_2 introduced in our solution procedure appear in the corrected metric components through δN and $\delta\phi_2$ in isotropic coordinates. The corresponding line element is of the form

$$\begin{aligned} \mathrm{d}s^2 &= -\left(N^{(0)} + \frac{d_1}{x} + \frac{d_2}{x^2} + O(x^{-3})\right)^2 c^2\mathrm{d}t^2 \\ &\quad + \left(\frac{\phi_2^{(0)}}{2x} + \frac{c_1}{2x} + \frac{c_2}{2x^2} + O(x^{-3})\right)\left(\mathrm{d}x^2 + x^2(\mathrm{d}\vartheta^2 + \sin^2\vartheta\mathrm{d}\varphi^2)\right) . \end{aligned} \tag{3.88}$$

One implication is that the coefficients c_i in $\delta\phi_2$ change the radial length according to the underlying geometry. If a corrected Newton's potential is expressed as a function of the geometrical radius $r = \int(\phi_2^{(0)} + \delta\phi_2)^{1/2}\,\mathrm{d}x$, rather than the coordinate x, all coefficients contribute to the potential in higher-order corrections from

$$\frac{d_2}{x^2} = \frac{d_2}{r^2}\left(x^{-1}\int(\phi_2^{(0)} + \delta\phi_2)^{1/2}\,\mathrm{d}x\right)^2 . \tag{3.89}$$

The second factor depends on the c_i, but overall corrections that include the first factor of d_2 are at least of second order.

Our weak-field results therefore depend on the behavior of quantum fluctuations, indirectly to leading order in relationships between d_2 and the c_i implied by the constraints, and directly at higher orders when the geometrical radial distance is used in the effective Newton potential. The equations we solve for these expansion coefficients are implied by covariance conditions, implemented here in a model of curved space-time. They could not have been derived from quantum mechanics alone, which would not suggest any bounds such as those found for C and Δp_2, in addition to the uncertainty principle. General covariance, or the existence of a curved space-time geometry for the gravitational force of a quantum state of masses, therefore implies non-trivial conditions.

There are also characteristic effects in the line element. In particular, since $d_1 = 0$ while $c_1 \neq 0$ generically, the time and space components of the space-time metric are affected in different ways by quantum corrections. This result is in contrast to previous assumptions, for instance in [82], where Schwarzschild-like patches of space-time, with closely related time and space components, were glued to each other in order to construct a geometry suitable for superposition states. Our results imply a more precise space-time geometry that combines interrelated gravitational and quantum effects.

In this way, our solutions can be used for consistent descriptions of quantum superpositions, defined by suitable values of the moments or of the parameters C and U_0. The setting of spherical symmetry implies that a superposition state can only be formulated for masses at the same position, defining the center of symmetry. Quantum fluctuations are therefore not given by position fluctuations, but rather indirectly by mass fluctuations: the primary quantum operator, as always in canonical gravity, is given by the spatial metric, or ϕ_2 in the present formulation. Quantum momenta of ϕ_2, parameterized by C and U_0, therefore determine the fundamental fluctuations. The mass enters only indirectly via the weak-field limit of the lapse function N, which is coupled to ϕ_2 and its moments by the constraints and evolution equations (or staticity conditions in the present case). These indirect implications on the mass or its superpositions are the reason why superposition geometries cannot simply be constructed from wave functions, but rather have to be derived through various consistency conditions implied by the gravitational constraints. The final result, expressed in the form of a line element, can then be analyzed by standard means, for instance by computing geodesics and proper-time intervals.

Our space-time geometry can be used not only for standard relativity analysis, such as geodesics, but also for additional quantum-information studies. For instance, there is a promising line of inquiry calculating quantum switch predictions in the context of our (and related) black hole mass superpositions. If the quantum switch experiment in [82] were conducted outside one such black hole superposition, how would the uncertainty within our quantum-corrected metric components impact the quantum switch experimental output? We would go about defining the measurement events in terms of proper time, which will be subject to fluctuation effects inherited from the metric components. One implication is that measurements would have to be made far enough apart in space and time for the quantum switch order to be distinct despite the uncertainties in radial and temporal distances. Increasing these distances should improve the result of the quantum switch measurement, as long as the necessary mass entanglement is preserved, throughout the necessary time and over the necessary distances. Such a quantum switch experiment would need to be conducted repeatedly to generate sufficient statistics on the results so as to associate any variation in output with the mass (and therefore metric) uncertainty from the space-time quantum corrections.[1]

[1]We thank Natália Móller for discussions about these questions.

3.7 Conclusions

We have derived the first fundamental description of fluctuating space-time geometries that may be interpreted as superpositions of classical black holes, or as superpositions of matter constituents in a single black hole. Our analysis here was made with the assumption of spherical symmetry. Intuitively, all superposed ingredients are therefore located at the same central point. Quantum fluctuations and superposition effects first appear in metric components of the resulting geometry. By standard relativistic analysis, such as taking the weak-field limit, they then imply indirect effects on mass superpositions in the central object or on corrections to Newton's potential.

The canonical methods used here, for both gravitational and quantum physics, made it possible to derive state properties. These include results such as new conditions on quantum fluctuations from general principles, one key example being general covariance. Further conditions follow from our assumption that our solutions are static, describing a non-rotating superposition of a black hole that has settled down after a collapse process. These conditions made it possible to derive state properties, without having to impose strong assumptions on the nature of the state, such as Gaussianity or purity. Our results, derived from leading corrections by second-order moments in the gravitational Hamiltonian, could therefore belong to either a pure or a mixed state.

The generality of our formalism is crucial for bridging fundamental questions in quantum gravity with potentially observable implications. In particular, identifying what are to be considered fundamental gravitational degrees of freedom depends on one's approach to quantum gravity. On a basic level, wave functions or superpositions of states would be formulated for these degrees of freedom. Most of these would have to be traced out to obtain a state relevant for a given observational situation. The actual tracing process is expected to be challenging, if not impossible, to perform explicitly in any complete quantum theory of gravity that includes all possible degrees of freedom, but the outcome determines the relevant final state. We eliminate assumptions on the final state, other than that it be semiclassical in the weak-field regime but it may well be mixed. In this way, we obtained results which may be considered a universal implication of quantized space-time geometries.

The condition of general covariance not only gave us a restrictive set of equations to solve, it also made it possible to derive gravitational effects in different coordinate systems. In particular, we used isotropic coordinates in the present paper, which are relevant for the weak-field limit. Our methods were originally developed in [76] and evaluated in

Schwarzschild-type coordinates. We have seen several results that demonstrate general agreement, in particular the behavior of quantum fluctuations as a function of the radial distance. The previous paper focused on horizon properties, while the new physical result of the present paper is an effective Newton's potential, with corrections from quantum fluctuations. We found new terms in this potential that are expected to be larger than those previously derived in perturbative quantum gravity. The general nature of our quantum states explains this difference because our effects pertain to a test mass in the curved, fluctuating space-time of a large central mass. Standard results of perturbative quantum gravity instead produce an effective potential between two test masses on a flat background.

Our conditions on state parameters imply a novel lower bound on momentum fluctuations (geometrically related to the extrinsic curvature of spacelike slices). Unlike the standard uncertainty relation of quantum mechanics, this lower bound (3.85) is independent of fluctuations of the configuration variable (a metric component) conjugate to this momentum. The lower bound does, however, depend inversely on the mass of the central object. Heuristically, this bound means that momentum fluctuations cannot be arbitrarily small. A quantum black hole that is static on average, with vanishing momentum expectation values as assumed here, can therefore be viewed in a new way. We interpret the black hole as an oscillating system in a superposition of expanding and collapsing classical geometries. Implications for the stability of black holes would require an extension of our model to non-static and non-spherical geometries. In such an extension, the quantum oscillation could involve higher multipoles that could source gravitational waves.

The present results for static and spherical configurations can be used in multiple ways for further analysis, mainly as a background space-time. Several questions of current interest in relativistic quantum information theory make use of the concept of proper time, which so far has mainly been defined for a classical space-time. Our effective line elements extend this important notion to quantum backgrounds with fluctuation terms. On these backgrounds, additional systems of interest for quantum information can then be set up. One example is the analysis of quantum switch experiments described in [82]. It is also possible to study the quantum effects of matter fields on our backgrounds. One could, for instance, study possible effects on their entanglement or other properties, as analyzed in [96,97]. Another promising avenue for exploration is to look at related effects in analog gravity [98].

3.8 Detailed equations for metric corrections calculation

This appendix collects details of our solution procedure of quantum-corrected constraints.

3.8.1 Equations of motion

Calculating the equations of motion from Poisson brackets of our fields and momenta with the Hamiltonian, we obtain:

$$\frac{\dot{\phi}_2}{c} = \{\phi_2, \bar{H}[N] + H_{\phi_2}[L]\} = \frac{\delta \bar{H}[N]}{\delta p_2} + \frac{\delta H_{\phi_2}[L]}{\delta p_2} \tag{3.90}$$

$$= -\frac{3\phi_3\left(2p_3\phi_2 + p_2\phi_3\right)}{\sqrt{2x\phi_2}}L - \frac{3x\left(4p_3\phi_2\phi_3 + p_2\left(8{\phi_2}^2 - {\phi_3}^2\right)\right)}{4\sqrt{2}\left(x\phi_2\right)^{3/2}}N \tag{3.91}$$

and

$$\frac{\dot{\phi}_3}{c} = \{\phi_3, \bar{H}[N] + H_{\phi_2}[L]\} = \frac{\delta \bar{H}[N]}{\delta p_3} + \frac{\delta H_{\phi_2}[L]}{\delta p_3} \tag{3.92}$$

$$= -\frac{3\sqrt{2}p_2\sqrt{x\phi_2}\phi_3}{x}L - \frac{3\left(2p_3\phi_2 + p_2\phi_3\right)}{\sqrt{2x\phi_2}}N\,. \tag{3.93}$$

These equations are identically satisfied in the static case.

In addition, we have non-trivial equations of motion

$$0 = \frac{\dot{p}_2}{c} = \{p_2, \bar{H}[N] + H_{\phi_2}[L]\} = -\frac{\delta \bar{H}[N]}{\delta \phi_2} - \frac{\delta H_{\phi_2}[L]}{\delta \phi_2} \tag{3.94}$$

$$0 = \frac{\dot{p}_3}{c} = \{p_3, \bar{H}[N] + H_{\phi_2}[L]\} = -\frac{\delta \bar{H}[N]}{\delta \phi_3} - \frac{\delta H_{\phi_2}[L]}{\delta \phi_3} \tag{3.95}$$

for static behavior. We evaluate the first equation after imposing the Hamiltonian constraint. The second equation, as seen in [76] can be used to derive L, which is not required for our purpose of finding corrections to Newton's potential. (Unlike N, the multiplier L does not appear in metric coefficients and instead determines how fluctuations contribute to the evolution generator. This function does contribute to the equation (3.94) that we will solve below, but only in terms of second or higher order in the quantum parameter ϕ_3. As we will see, for our purposes it will be sufficient to solve (3.102) to first order, in which L does not appear.)

3.8.2 Perturbations around classical background

Fluctuation terms in the quantum Hamiltonian constraint modify the classical solutions for ϕ_2 and N. As a first approximation, we apply a perturbative treatment to both ϕ_2 and N around the classical solutions, $\phi_2^{(0)}$ and $N^{(0)}$:

$$\begin{aligned} \phi_2 &= \phi_2^{(0)} + \delta\phi_2 = 2x\left(1+\frac{b}{x}\right)^4 + \delta\phi_2 \\ N &= N^{(0)} + \delta N = \frac{x-b}{x+b} + \delta N \end{aligned} \tag{3.96}$$

As discussed earlier, we absorb the original integration constant N_0 in $N^{(0)}$ into the definition of the time coordinate, effectively setting $N_0 = 1$ here. Applying these perturbations to the Hamiltonian constraint, we can arrange the terms according to whether they contain the quantum field ϕ_3 and its momentum p_3 or only the classical fields, taking into account the perturbed portions for ϕ_2 in the latter case. We write

$$\bar{H}[N] = H[N] + H_2[N] \tag{3.97}$$

with

$$\begin{aligned} H_2[N] &= \int \mathrm{d}x N(x)\left(\frac{1}{2}\frac{\partial^2 H}{\partial p_2^2}\left(p_3^2 + \frac{U}{\phi_3^2}\right) - H_0 + \frac{\partial^2 H}{\partial\phi_2\partial p_2}\phi_3 p_3\right. \\ &\quad \left.+\frac{1}{2}\frac{\partial^2 H}{\partial\phi_2^2}\phi_3^2 + \frac{\partial^2 H}{\partial\phi_2\partial\phi_2'}\phi_3\phi_3' + \frac{1}{2}\frac{\partial^2 H}{(\partial\phi_2')^2}(\phi_3')^2 + \frac{\partial^2 H}{\partial\phi_2\partial\phi_2''}\phi_3\phi_3''\right) \end{aligned} \tag{3.98}$$

and

$$\begin{aligned} H[N] &= H[N]|_{\phi_2^{(0)}} + \int \mathrm{d}x(N^{(0)} + \delta N)\left(\frac{\partial H}{\partial\phi_2}\delta\phi_2 + \frac{\partial H}{\partial\phi_2'}\delta\phi_2' + \frac{\partial H}{\partial\phi_2''}\delta\phi_2''\right) \\ &\quad + \int \mathrm{d}x N^{(0)}\left(\frac{1}{2}\frac{\partial^2 H}{\partial\phi_2^2}(\delta\phi_2)^2 + \frac{\partial^2 H}{\partial\phi_2\partial\phi_2'}\delta\phi_2\delta\phi_2' + \frac{1}{2}\frac{\partial^2 H}{(\partial\phi_2')^2}(\delta\phi_2')^2 + \frac{\partial^2 H}{\partial\phi_2\partial\phi_2''}\delta\phi_2\delta\phi_2''\right) \end{aligned} \tag{3.99}$$

where $H[N]|_{\phi_2^{(0)}} = 0$ for classical background solutions. We consider the quantum field ϕ_3 and the perturbations $\delta\phi_2$ and δN induced by it to be of the same order. Therefore, $H_2[N]$ is already of second order, such that we can simply insert the classical solutions in the coefficients defined by partial derivatives of H. Similarly, all non-zero correction terms in (3.99) are of second order because the only linear terms (in the first line of (3.99) proportional to $N^{(0)}$) are, upon integration by parts, proportional to the classical equation of motion for p_2, which vanishes for static solutions.

Having applied the perturbations of N and ϕ_2 to both $H[N]$ and $H_2[N]$, we now have their sum, the perturbed form of our corrected Hamiltonian $\bar{H}[N]$. We then apply the classical background solutions into the equation. At this stage, we also insert the solution for ϕ_3, resulting in the equation

$$\begin{aligned}\bar{H}[N]|_{N^{(0)},\phi_2^{(0)}} \quad = \quad & -\frac{3x^2(b+x)^2U}{C^2\left(3b^2+3bx+x^2\right)^2}+\frac{3U}{C^2}-\frac{3x^4(x-b)\left(13b^2-10bx+x^2\right)\delta\phi_2{}^2}{16(b+x)^9} \\ & +\frac{3C^2\left(-24b^6+16b^3(b+x)^3-(b+x)^6\right)}{16(b+x)^8}+\frac{3x^6(b-x)\delta\phi_2'{}^2}{16(b+x)^7} \\ & +\frac{x^3\delta N\delta\phi_2''}{2(b+x)^2}+\frac{3bx^2\delta N\delta\phi_2'}{(b+x)^3}+\delta\phi_2\left(\frac{x^6(b-x)\delta\phi_2''}{8(b+x)^7}\right) \\ & +\delta\phi_2\left(\frac{3x^5(x-5b)(x-b)\delta\phi_2'}{8(b+x)^8}+\frac{3bx(b-x)\delta N}{(b+x)^4}\right). \end{aligned} \tag{3.100}$$

In the second term, given by the vacuum subtraction $-H_0 = 3U/C^2$, we have assumed that U does not depend on b. This assumption will be sufficient for our purposes but could easily be relaxed if needed.

We need a second constraint equation, since the Hamiltonian constraint provides a single equation coupling $\delta\phi_2$ and δN. Such a condition can be obtained from the equation of motion for p_2 which, unlike the background equation (3.102), includes perturbations from $\delta\phi_2$. Since we only need one additional equation, it is sufficient to consider the equation of motion expanded to linear order in $\delta\phi_2$. Since ϕ_3 is of the same order as $\delta\phi_2$ and $\bar{H}$ is quadratic in ϕ_3, the relevant equation of motion is generated by the expanded $H[N]$ without the contribution from $H_2[N]$. With

$$H_{\text{linear}} = \frac{\partial H}{\partial\phi_2}\delta\phi_2 + \frac{\partial H}{\partial\phi_2'}\delta\phi_2' + \frac{\partial H}{\partial\phi_2''}\delta\phi_2'' \tag{3.101}$$

and

$$-\frac{\dot{p}_2|_{N^{(0)},\phi_2^{(0)}}}{c} = N^{(0)}\frac{\partial H}{\partial\phi_2} - \left(N^{(0)}\frac{\partial H}{\partial\phi_2'}\right)' + \left(N^{(0)}\frac{\partial H}{\partial\phi_2''}\right)'' = 0. \tag{3.102}$$

we have separated the ϕ_2, ϕ_2' and ϕ_2'' derivatives that appeared in the original functional derivative by ϕ_2, allowing us to treat them as separate variables for the purposes of these partial derivatives. We then have

$$\frac{\dot{p}_2|_{\text{linear}}}{c} = -\frac{\partial H_{\text{linear}}}{\partial\phi_2}N^{(0)} - \frac{\partial H^{(0)}}{\partial\phi_2}\delta N + \left(\frac{\partial H_{\text{linear}}}{\partial\phi_2'}N^{(0)} + \frac{\partial H^{(0)}}{\partial\phi_2'}\delta N\right)'$$

$$
\begin{aligned}
& -\left(\frac{\partial H_{\text{linear}}}{\partial \phi_2''} N^{(0)} + \frac{\partial H^{(0)}}{\partial \phi_2''}\delta N\right)'' \\
= & -\Bigg(\frac{\partial^2 H}{\partial \phi_2^2}\delta\phi_2 + \frac{\partial^2 H}{\partial \phi_2' \partial \phi_2}\delta\phi_2' + \frac{\partial^2 H}{\partial \phi_2'' \partial \phi_2}\delta\phi_2'' \\
& -\left(\frac{\partial^2 H}{\partial \phi_2' \partial \phi_2}\delta\phi_2 + \frac{\partial^2 H}{(\partial \phi_2')^2}\delta\phi_2'\right)' - \left(\frac{\partial^2 H}{\partial \phi_2'' \partial \phi_2}\delta\phi_2)\right)''\Bigg) N^{(0)} \\
& +\left(\frac{\partial^2 H}{\partial \phi_2' \partial \phi_2}\delta\phi_2 + \frac{\partial^2 H}{(\partial \phi_2')^2}\delta\phi_2' - 2\left(\frac{\partial^2 H}{\partial \phi_2'' \partial \phi_2}\delta\phi_2\right)'\right) N^{(0)\prime} - \frac{\partial^2 H}{\partial \phi_2'' \partial \phi_2}\delta\phi_2 N^{(0)\prime\prime} \\
& -\left(\frac{\partial H}{\partial \phi_2} - \left(\frac{\partial H}{\partial \phi_2'}\right)' + \left(\frac{\partial H}{\partial \phi_2''}\right)''\right)\delta N + \left(\frac{\partial H}{\partial \phi_2'} - 2\left(\frac{\partial H}{\partial \phi_2''}\right)''\right)\delta N' - \frac{\partial H}{\partial \phi_2''}\delta N'' \\
= & -\Bigg(\left(\frac{\partial^2 H}{\partial \phi_2^2} - \left(\frac{\partial^2 H}{\partial \phi_2' \partial \phi_2}\right)' - \left(\frac{\partial^2 H}{\partial \phi_2'' \partial \phi_2}\right)''\right)\delta\phi_2 - \left(\left(\frac{\partial^2 H}{(\partial \phi_2')^2}\right)' + 2\left(\frac{\partial^2 H}{\partial \phi_2'' \partial \phi_2}\right)'\right)\delta\phi_2' \\
& -\frac{\partial^2 H}{(\partial \phi_2')^2}\delta\phi_2''\Bigg) N^{(0)} \\
& +\left(\left(\frac{\partial^2 H}{\partial \phi_2' \partial \phi_2} - 2\left(\frac{\partial^2 H}{\partial \phi_2'' \partial \phi_2}\right)'\right)\delta\phi_2 + \left(\frac{\partial^2 H}{(\partial \phi_2')^2} - 2\frac{\partial^2 H}{\partial \phi_2'' \partial \phi_2}\right)\delta\phi_2'\right) N^{(0)\prime} \\
& -\frac{\partial^2 H}{\partial \phi_2'' \partial \phi_2}\delta\phi_2 N^{(0)\prime\prime} \\
& -\left(\frac{\partial H}{\partial \phi_2} - \left(\frac{\partial H}{\partial \phi_2'}\right)' + \left(\frac{\partial H}{\partial \phi_2''}\right)''\right)\delta N + \left(\frac{\partial H}{\partial \phi_2'} - 2\left(\frac{\partial H}{\partial \phi_2''}\right)''\right)\delta N' - \frac{\partial H}{\partial \phi_2''}\delta N'' \qquad (3.103)
\end{aligned}
$$

Applying the classical background solutions and the solution for ϕ_3, we obtain:

$$
\begin{aligned}
\frac{\dot{p}_2|_{N^{(0)},\phi_2^{(0)}}}{c} = & \frac{bx^4\left(33b^2 - 51bx + 16x^2\right)\delta\phi_2}{4(b+x)^9} + \frac{3x^6(b-x)\delta\phi_2''}{8(b+x)^7} \qquad (3.104) \\
& + \frac{bx^5(15b-16x)\delta\phi_2'}{4(b+x)^8} - \frac{x^3\delta N''}{2(b+x)^2} + \frac{3bx\left(b(x-2)+x^2\right)\delta N'}{(b+x)^4}.
\end{aligned}
$$

3.8.3 Asymptotic expansion

In order to solve this system, we first expand δN and $\delta\phi_2$ as power series, similar to our earlier handling of ϕ_3: $\delta N = d_1/x + d_2/x^2 + d_3/x^3$ and $\delta\phi_2 = c_1 + c_2/x + c_3/x^2$. The classical solution for N is asymptotically constant, while the classical ϕ_2 grows like x. Our perturbations include only terms higher than the classical order in $1/x$ in order to preserve the Minkowski limit.

The remaining undetermined function is $U(x)$. It is not subject to a constraint or equation of motion, but its dependence on x turns out to be restricted by solvability conditions on the expansion coefficients c_i and d_i. For $U(x)$ to be compatible with the weak-field limit, it must have an expansion in $1/x$. For simplicity, we only consider power-law forms $U(x) = U_0 x^{-\kappa}$, in which the exponent is an integer $\kappa \geq 0$. We will first provide detailed expressions of the constraint and equation of motion, both expanded in $y = b/x$ around zero, for the case of $\kappa = 1$ and then show why the remaining power laws do not result in non-trivial real solutions.

The expanded Hamiltonian constraint is given by

$$\begin{aligned}
\bar{H}[N] \;=\; & y^2\left(-\frac{3c_1^2}{16b^2}-\frac{3C^2}{16b^2}+\frac{12U_0}{bC^2}\right) \\
& +y^3\left(\frac{c_2 d_1}{b^3}-\frac{c_1 c_2}{b^3}-\frac{3c_1 d_1}{b^2}+\frac{15c_1^2}{4b^2}+\frac{3C^2}{8b^2}-\frac{30U_0}{bC^2}\right) \\
& +y^4\left(\frac{3c_3 d_1}{b^4}+\frac{c_2 d_2}{b^4}-\frac{c_2^2}{b^4}-\frac{15c_1 c_3}{8b^4}-\frac{8c_2 d_1}{b^3}-\frac{3c_1 d_2}{b^3}+\frac{59c_1 c_2}{4b^3}+\frac{15c_1 d_1}{b^2}\right) \\
& +y^4\left(-\frac{501c_1^2}{16b^2}-\frac{9C^2}{16b^2}+\frac{54U_0}{bC^2}\right)+O(y^5) \qquad (3.105)
\end{aligned}$$

This equation can be simplified because the constraint has to vanish for any lapse function. Instead of using the expanded lapse with coefficients d_i, a simpler expression is obtained for $N = N^{(0)}$. The resulting constraint can simply be obtained by setting $d_1 = 0$ and $d_2 = 0$ in the preceding expression:

$$\begin{aligned}
\bar{H}[N^{(0)}] \;=\; & y^2\left(-\frac{3c_1^2}{16b^2}-\frac{3C^2}{16b^2}+\frac{12U_0}{bC^2}\right)+y^3\left(-\frac{c_1 c_2}{b^3}+\frac{15c_1^2}{4b^2}+\frac{3C^2}{8b^2}-\frac{30U_0}{bC^2}\right) \\
& +y^4\left(-\frac{c_2^2}{b^4}-\frac{15c_1 c_3}{8b^4}+\frac{59c_1 c_2}{4b^3}-\frac{501c_1^2}{16b^2}-\frac{9C^2}{16b^2}+\frac{54U_0}{bC^2}\right)+O(y^5) \qquad (3.106)
\end{aligned}$$

and we have the equation of motion

$$\begin{aligned}
\frac{\dot{p}_2}{c} \;=\; & -\frac{d_1 y^2}{b^2}+y^3\left(-\frac{3c_2}{4b^3}-\frac{3d_2}{b^3}+\frac{4c_1}{b^2}-\frac{d_1}{b^2}\right)+y^4\left(-\frac{9c_3}{4b^4}-\frac{6d_3}{b^4}+\frac{14c_2}{b^3}-\frac{195c_1}{4b^2}+\frac{6d_1}{b^2}\right) \\
& +O(y^5), \qquad (3.107)
\end{aligned}$$

after gathering terms by powers of $y = b/x$. We then equate the second, third, and fourth order coefficients to zero for both $\bar{H}[N] = 0$ and $\dot{p}_2 = 0$. This produces a system of six equations, which we can solve for the coefficients in each power series. For instance, the quadratic term in y of the Hamiltonian constraint has real solutions from a quadratic equation for c_1 since $U_0 > 0$ by our positivity condition. These solutions then successively determine c_2, d_2, c_3 and d_3 from the remaining terms, while $d_1 = 0$ from the quadratic term in $\dot{p}_2/c = 0$. These results are used and discussed in the main part of our paper.

Direct evaluation of the resulting Hamiltonian constraint for constant U or $U \propto 1/x^2$ show that non-trivial real solutions cannot be obtained in these cases. For constant U, we have

$$\begin{aligned}
\bar{H}[N] &= \frac{12Uy}{C^2} + y^2\left(-\frac{3c_1^2}{16b^2} - \frac{3C^2}{16b^2} - \frac{30U}{C^2}\right) \\
&\quad + y^3\left(\frac{c_2 d_1}{b^3} - \frac{c_1 c_2}{b^3} - \frac{3c_1 d_1}{b^2} + \frac{15c_1^2}{4b^2} + \frac{3C^2}{8b^2} + \frac{54U}{C^2}\right) \\
&\quad + y^4\left(\frac{3c_3 d_1}{b^4} + \frac{c_2 d_2}{b^4} - \frac{c_2^2}{b^4} - \frac{15c_1 c_3}{8b^4} - \frac{8c_2 d_1}{b^3} - \frac{3c_1 d_2}{b^3} + \frac{59c_1 c_2}{4b^3} + \frac{15c_1 d_1}{b^2} - \frac{501c_1^2}{16b^2}\right) \\
&\quad + y^4\left(-\frac{9C^2}{16b^2} - \frac{63U}{C^2}\right) + O(y^5) \qquad (3.108)
\end{aligned}$$

where a constant contribution $-3U/C^2$ has been removed by our general vacuum subtraction. However, a linear term in y now remains, which as a part of the Hamiltonian constraint requires $U = 0$. The second-order term in y then implies imaginary solutions for c_1 if $C \neq 0$, making the remaining coefficients complex as well. (If we choose $C = 0$, we have a real $c_1 = 0$ but also remove all quantum corrections.)

For $U \propto 1/x^2$, we obtain

$$\begin{aligned}
\bar{H}[N] &= y^2\left(-\frac{3c_1^2}{16b^2} - \frac{3C^2}{16b^2}\right) + y^3\left(\frac{c_2 d_1}{b^3} - \frac{c_1 c_2}{b^3} - \frac{3c_1 d_1}{b^2} + \frac{15c_1^2}{4b^2} + \frac{12U_0}{b^2C^2} + \frac{3C^2}{8b^2}\right) \\
&\quad + y^4\left(\frac{3c_3 d_1}{b^4} + \frac{c_2 d_2}{b^4} - \frac{c_2^2}{b^4} - \frac{15c_1 c_3}{8b^4} - \frac{8c_2 d_1}{b^3} - \frac{3c_1 d_2}{b^3} + \frac{59c_1 c_2}{4b^3} + \frac{15c_1 d_1}{b^2} - \frac{501c_1^2}{16b^2}\right) \\
&\quad + y^4\left(-\frac{30U_0}{b^2C^2} - \frac{9C^2}{16b^2}\right) + O(y^5) \qquad (3.109)
\end{aligned}$$

after removing a second-order term in y as per our vacuum subtraction. The remaining quadratic term implies imaginary c_1, and complex values for the remaining coefficients. This behavior remains the same if we use larger exponents $\kappa > 2$ because the first non-zero U-term then appears in higher-order contributions in y and do not change the imaginary nature of the resulting c_1. As a power law, the form $U(x) = U_0/x$ is therefore uniquely determined by solvability conditions.

Chapter 4
Current work: Quantum corrected black hole volume

Studying the volume inside the event horizon can have interesting implications for information theory, black hole evolution, and the importance of quantum effects. For this reason, I have recreated an established volume calculation from [99] and applied it to a black hole spacetime with a covariant quantum correction λ [100]. This spacetime is derived using similar covariant methods to those in the previous chapter, and has the benefit of an interior solution with the potential to affect these volume calculations. This volume definition has an interesting feature, convergence at a radius of $r = 3M/2$, subject to the influence of said quantum corrections.

4.1 Summary

Our goal is to use established definitions for the volume inside the event horizon as maximal spatial hypersurfaces [99] and recreate that volume calculation for the spacetime model in [100]. Broadening the scope of my work, this current paper draft utilizes a distinct, though related, modified spacetime [100]. This spacetime is valid for the non-static case, and therefore has a full interior solution, with a quantum parameter introduced using covariant methods similar to those utilized in Chapter 2.

4.1.1 Calculations

Defining volume as maximal spatial hypersurfaces, we start with the definition of this slicing, contracting extrinsic curvature (K_{ij}) with the spatial metric (γ^{ij}) and setting it to zero.

$$\gamma^{ij}K_{ij} = 0 \tag{4.1}$$

We then utilize the Hamiltonian constraint, diffeomorphism constraint, and an equation of motion for the quantum-corrected spacetime. With these equations, together with the trace condition, we are able to solve for the radial component of the spatial metric (γ) in terms of the modification (λ) and a function of time (T).

$$\gamma^{-1} = \left(1 - 2r + \frac{r^4T^2}{M^4}\right) + \lambda^2\left(-3r + 4r^2 - \frac{r^4T^2}{M^4} - \frac{2r^5T^2}{M^4}\right) \tag{4.2}$$

From the structure of expressions for lapse and shift, we can argue that the roots of the inverse of the radial component of the spatial metric coincide at the new convergence of spatial leaves. The following is a preliminary result for the modified radius of convergence of spatial volume slices (x).

$$x = \frac{3}{2}M\left(1 \mp \frac{\lambda}{\sqrt{2}}\right) \tag{4.3}$$

Note that when the quantum modification λ goes to zero, we recover the original result $x = 3M/2$.

4.1.2 Results

The quantum modification, λ, shifts the radius at which these spatial leaves converge, which could offer insights into both the information paradox and the late-stages of black hole evaporation. We are in the final stages of this calculation, with preliminary results of first-order corrections in λ.

4.2 Paper Draft: Volume of a Quantum Corrected Black Hole

Abstract:
We seek to refine which types of black hole models are successful by exploring the information capacity/entropy/computability of a quantum-corrected black hole interior [100, 101]. Confirming established definitions for the volume inside the event horizon, we calculate the maximal spatial hypersurfaces used in [99]. This foliation used to define volume has interesting features inside the horizon, notably a convergence at $r = 3M/2$ in the classical case. We recreate this calculation with the quantum-corrected black hole

spacetime. We derive the quantum correction's effect on the slicing convergence result. The quantum modification has unique interactions with the convergence of these spatial leaves, in certain limits, which could offer insights into both the information paradox and the late-stages of black hole evaporation.

4.3 Introduction

We can test possible theories of quantum gravity by studying black hole spacetimes, because settings with dramatic spacetime curvature make the mathematics of general relativity relevant on the small quantum-mechanical scales. Many areas of study will benefit from a better understanding of these environments, including the phenomena of Hawking radiation, information loss, removing singularities, and speculation about black hole "deaths." Many approaches to these problems, such as non-perturbative approaches to string theory, loop quantum gravity, and effective field theories [14–16] have been investigated. We seek to refine which models work by exploring the information capacity/entropy/computability of a particular black hole interior.

Building on the canonical quantum black hole models in [100, 101], we recreate the volume calculations conducted in [102], using the definitions outlined in [99]. By defining the volume inside the event horizon using maximal spatial hypersurfaces, we can explore the shift of the $3M/2$ boundary (to which these slices converge)due to the quantum-correction.

We are particularly interested in the late-time limit approaching the final stages of evaporation, when these boundaries inside the horizon approach each other. This is explored in a limited capacity, given the static assumption for some of these spacetime models. There is ample opportunity for further studies of this type.

4.4 Background and Motivation

General relativity and quantum mechanics are essential to modern life, from GPS and space travel, to nuclear power and quantum computing. GR works well on large scales and quantum mechanics works well on small scales, but they are at odds. It is one of the largest open questions on the frontiers of physics to find a way to reconcile the two, taking the parts that work and combining them into a cohesive theory of Quantum Gravity. Black holes are an excellent testing ground for such theories because the extreme curvature of spacetime makes the mathematics of general relativity relevant on the small quantum mechanical scales.

There is a great deal of motivation for studying black hole volumes, as they determine the potential for information storage, remnants post-evaporation, and whether the divergences inherent in singularities can be removed. We hope to address big questions about information paradox, singularity removal, and black hole death.

For this investigation, we chose to analyze the covariant spacetime in [101]. Covariance is necessary for results to be coordinate independent, as discussed in [103]. We are interested in the interpretations/applications of the λ modification in [100]. This spacetime model has the necessary properties from a canonical derivation method, similar to [103]. In addition, this model has the advantage of not being restricted to the static case. The absence of this restriction means that the spacetime is valid on the interior of the horizon, where the temporal and radial properties switch. This complete solution allows us to analyze the interior region, including implementing this volume calculation.

Introducing modifications directly into the Hamiltonian and/or Diffeomorphism constraints allows one to impose the covariance condition by requiring that the Poisson brackets between these constraints are closed. Covariance is necessary for any results to be coordinate independent. One can consider the physical interpretations of these constraints as conservation laws for energy and momentum respectively. They can also be considered the generators of temporal and spatial translations respectively. For example, taking the Poisson bracket of canonical variables with the Hamiltonian constraint results in the time derivative of that variable.

Note that in these calculations, we will be using smeared constraints and tetrad coordinates.

4.5 Verifying the foundation

We recreated the calculations in [99] to verify the relationships. We first attempt to rederive the classical boundary using the method and definitions from [99]. We begin with the trace condition:

$$\gamma^{ij} K_{ij} = 0 \tag{4.4}$$

The extrinsic curvature is given by:

$$K_{ij} = \frac{1}{2}\ \alpha^{-1}(-\gamma_{ij,0} + \mathcal{L}_{\beta}\gamma_{ij}) \tag{4.5}$$

where α is the lapse function, γ_{ij} is the spatial metric, commas denote partial derivatives, and $\mathcal{L}_\beta$ denotes the Lie derivative with respect to the shift vectors β_i. We expand the Lie derivative so it simplifies to:

$$\mathcal{L}_\beta \gamma_{ij} = \beta^k \frac{\partial \gamma_{ij}}{\partial x^k} + \gamma_{kj} \frac{\partial \beta^k}{\partial x^i} + \gamma_{ik} \frac{\partial \beta^k}{\partial x^j} \tag{4.6}$$

We apply these conditions to the new line element in [101]. Note the notation differences: N represents the lapse function, N^x represents the shift vector, and E^x and E^ϕ represent two independent components of a densitized triad, with K_x and K_ϕ as their respective conjugate momenta.

$$ds^2 = -N^2 dt^2 + \left(1 - \frac{r_0}{\sqrt{E^x}}\right)^{-1} \frac{E^{\phi 2}}{E^x} (dx + N^x dt)^2 + E^x d\Omega^2 \tag{4.7}$$

The length scale r_0, in terms of the quantum correction λ and mass M, is given by:

$$r_0 = \frac{2M\lambda^2}{1+\lambda^2} \tag{4.8}$$

We also make use of the Hamiltonian constraint, diffeomorphism constraint, and equations of motion [101]. To rederive the classical case. We set $\lambda = 0$, giving the following constraints:

$$H = -\frac{E^\phi (1 + K_\phi^2)}{2\sqrt{E^x}} - 2K_x K_\phi \sqrt{E^x} + \frac{1}{2}\left(\frac{(E^x)'}{2E^\phi}(\sqrt{E^x})' + \sqrt{E^x}\left(\frac{(E^x)'}{E^\phi}\right)'\right) \tag{4.9}$$

$$D = -K_x (E^x)' + E^\phi K_\phi' \tag{4.10}$$

In this case, the equations of motion reduce to:

$$\dot{E^x} = (E^x)' N^x + 2\sqrt{E^x} K_\phi N \tag{4.11}$$

$$\dot{E^\phi} = (N^x E^\phi)' + 2\sqrt{E^x} K_x N + K_\phi N \frac{2E^\phi}{2\sqrt{E^x}} \tag{4.12}$$

$$\begin{aligned} \dot{K_x} = (N^x K_x)' + \frac{\sqrt{E^x} N''}{2E^\phi} + \frac{\sqrt{E^x} N'}{2E^{\phi 2}} \left(\frac{(E^x)' E^\phi}{2E^x} - (E^\phi)'\right) \\ + N\left(\frac{E^\phi \left(1 + K_\phi^2\right)}{4(E^x)^{\frac{3}{2}}} + \frac{1}{4\sqrt{E^x} E^\phi}\left(-\frac{(E^x)'(E^\phi)'}{E^\phi} - \frac{E^{x2}}{4E^x} + (E^x)''\right) - \frac{K_x K_\phi}{\sqrt{E^x}}\right) \end{aligned} \tag{4.13}$$

$$\dot{K}_\phi = N^x K_\phi' + N' \frac{\sqrt{E^x}(E^x)'}{2E^{\phi 2}} - N\frac{K_\phi^2}{2\sqrt{E^x}} + N\frac{((E^x)')^2}{8\sqrt{E^x}E^{\phi 2}} \tag{4.14}$$

Using time as our 0$\hat{\text{th}}$ coordinate, we then expand the maximality constraint (the trace condition) in terms of E^x, E^ϕ, K_x, and K_ϕ . The maximality constraint can then be solved for K_x:

$$K_x = \frac{E^\phi K_\phi}{E^x} \tag{4.15}$$

Setting $E^x = x^2$ and making use of the fact that the time derivative of this expression is zero allows us to solve for the shift vector, N^x. Note that the sign of the shift vector was changed to be consistent with the conventions in [99].

$$N^x = K_\phi N \tag{4.16}$$

The diffeomorphism constraint is left as a differential equation for K_ϕ. Solving gives $K_\phi = \frac{T}{x^2}$, where the constant of integration T is dependent only on time. This expression allows us to eliminate K_ϕ from the expression for N^x, which in turn allows us to transform the Hamiltonian constraint into a differential equation for E^ϕ. We set the second constant of integration to be $-2M$ and time-independent (add more about this). We then define our radial component of the spatial metric as $\gamma = \frac{(E^\phi)^2}{E^x}$, so that γ can be explicitly written as:

$$\gamma = \frac{x^4}{-2Mx^3 + T^2 + x^4} \tag{4.17}$$

We are now equipped to replicate the calculations used to determine the boundary [99]. Using our expressions for [variables], we then reduce the maximality constraint to be in terms of T, M, N, and x:

$$0 = \frac{T\left(-2Mx^3 + T^2 + x^4\right)\left(x^2 N'\left(-2Mx^3 + T^2 + x^4\right) + Nx^4(3M - 2x) + \dot{T}x^4\right)}{Nx^4\left(x^4 - 2Mx^3 + T^2\right)^2} + \frac{2T}{x^3} \tag{4.18}$$

This is a differential equation for N. Solving for N', the expression reduces to:

$$N' = (-N)\left(\frac{x^2(3M - 2x)}{-2Mx^3 + T^2 + x^4} + \frac{2}{x}\right) - \frac{\dot{T}x^2}{-2Mx^3 + T^2 + x^4} \tag{4.19}$$

We know that $\frac{\partial}{\partial x}(\alpha\gamma^{\frac{1}{2}}) = \alpha'\gamma^{\frac{1}{2}} + \frac{1}{2}\alpha\gamma^{-\frac{1}{2}}\frac{\partial\gamma}{\partial x}$. We also have an explicit expression for γ and can compute its partial derivative with respect to x. Through careful manipulation of equation (alpha' eq), we arrive at

$$\frac{\partial}{\partial x}(N\gamma^{\frac{1}{2}}) = -\gamma^{\frac{3}{2}}\frac{\dot{T}}{x^2} \tag{4.20}$$

After integrating out the partial derivative in this equation and making the substitution $x = \frac{M}{r}$, we get:

$$N = (-2Mx^3 + T^2 + x^4)^{\frac{1}{2}}(1 + \frac{\dot{T}}{M}\int_0^{Mx^{-1}} (1 - 2r + T^2M^{-4}r^4)^{-\frac{3}{2}}\, dr) \tag{4.21}$$

The roots of the polynomial inside the integrand lead to the desired result: we confirm that the two real roots of the quartic coincide at $T = \frac{3\sqrt{3}M^2}{4}$, which, when plugged back in, yields a root at $r = \frac{2}{3}$. Therefore, the boundary is confirmed to occur at $x = \frac{3}{2}M$.

We solve for alpha and note that the $3M/2$ condition arises from roots of the polynomial in the square root within the integral.

4.6 Applying volume calculation to modified spacetime

We now recreate these calculations using the modified black hole in [101]. This spacetime is interesting because of the unique boundary inside the horizon, characterized by the parameter λ.

Conceptually, we know that the mass of the black hole should not be changing with time, so time derivatives of M can be removed from our equation.

Hamiltonian (non-expanded):

$$\begin{aligned} H = &\frac{\cos^2(\lambda \mathrm{K}_\phi)\left(\sqrt{E^x}\left(\frac{(E^x)'}{E^\phi}\right)' + \frac{E^{x\prime}\sqrt{E^x}'}{2E^\phi}\right)}{2\sqrt{\lambda^2+1}} \\ &- \frac{\sqrt{E^x}K_x \sin(2\lambda \mathrm{K}_\phi)\left(\left(\frac{\lambda \mathrm{E}^{x\prime}}{2E^\phi}\right)^2 + 1\right)}{\lambda\sqrt{\lambda^2+1}} + \frac{\left(-E^\phi\right)\left(\frac{\sin^2\left(\lambda \mathrm{K}_\phi\right)}{\lambda^2} + 1\right)}{2\sqrt{\lambda^2+1}\sqrt{E^x}} \end{aligned} \tag{4.22}$$

Motion equations:

$$\dot{E^x} = E^{x\prime}N^x + \frac{N\sqrt{E^x}\sin(2\lambda \mathrm{K}_\phi)}{\lambda\sqrt{\lambda^2+1}}\left(\left(\frac{\lambda \mathrm{E}^{x\prime}}{2E^\phi}\right)^2 + 1\right) \tag{4.23}$$

The following equation is needed to solve for α'.

$$\dot{E^\phi} = \left(E^\phi N^x\right)' + \frac{2N\sqrt{E^x}K_x \cos\left(2\lambda \mathrm{K}_\phi\right)}{\sqrt{\lambda^2+1}}\left(\left(\frac{\lambda E^{\mathrm{x}\prime}}{2E^\phi}\right)^2+1\right)+ \frac{\left(N\sin\left(2\lambda \mathrm{K}_\phi\right)\right)}{\lambda\sqrt{\lambda^2+1}}\left(\frac{1}{2}\lambda^2\left(\sqrt{E^x}\left(\frac{E^{\mathrm{x}\prime}}{E^\phi}\right)'+\frac{E^{\mathrm{x}\prime}\sqrt{E^x}'}{2E^\phi}\right)+\frac{E^\phi}{2\sqrt{E^x}}\right) \tag{4.24}$$

The trace condition, $\gamma_{ij}K^{ij}=0$, is calculated using the same constraint in the previous section: (need trace condition definition included or cited here first

$$0 = \frac{2N^x}{Nx} + \frac{x^2\left(1-\frac{r_0}{x}\right)}{2NE^{\phi 2}}\Bigg(-\frac{2E^\phi}{x^2\left(1-\frac{r_0}{x}\right)}\Bigg(N^xE^{\phi\prime} + \frac{2NxK_x\left(\frac{\lambda^2x^2}{\mathrm{E}\phi^2}+1\right)\left(1-2\lambda^2 s(t,x)^2\right)}{\sqrt{\lambda^2+1}} + \frac{2Ns(t,x)\sqrt{1-\lambda^2 s(t,x)^2}\left(\frac{1}{2}\lambda^2\left(x\left(\frac{2}{E^\phi}-\frac{2xE^{\phi\prime}}{E^{\phi 2}}\right)+\frac{x}{E^\phi}\right)+\frac{E^\phi}{2x}\right)}{\sqrt{\lambda^2+1}} + E^\phi N^{\mathrm{x}\prime}\Bigg) + \frac{2E^{\phi 2}N^{\mathrm{x}\prime}}{x^2\left(1-\frac{r_0}{x}\right)} + N^x\left(-\frac{r_0E^{\phi 2}}{x^4\left(1-\frac{r_0}{x}\right)^2} - \frac{2E^{\phi 2}}{x^3\left(1-\frac{r_0}{x}\right)} + \frac{2E^\phi E^{\phi\prime}}{x^2\left(1-\frac{r_0}{x}\right)}\right)\Bigg) \tag{4.25}$$

Solving Trace Condition $=0$ for K_x:

$$K_x = -\frac{\sqrt{1+\lambda^2}E^\phi}{2x\left(\frac{\lambda^2x^2}{E^{\phi 2}}+1\right)\left(1-2\lambda^2 s(t,x)^2\right)}\left(\frac{2s(t,x)\sqrt{1-\lambda^2 s(t,x)^2}\left(\frac{1}{2}\lambda^2\left(x\left(\frac{2}{E^\phi}-\frac{2xE^{\phi\prime}}{E^{\phi 2}}\right)+\frac{x}{E^\phi}\right)+\frac{E^\phi}{2x}\right)}{\sqrt{\lambda^2+1}E^\phi} - \frac{x^2N^x\left(1-\frac{r_0}{x}\right)\left(-\frac{r_0E^{\phi 2}}{x^4\left(1-\frac{r_0}{x}\right)^2}-\frac{2E^{\phi 2}}{x^3\left(1-\frac{r_0}{x}\right)}+\frac{2E^\phi E^{\phi\prime}}{x^2\left(1-\frac{r_0}{x}\right)}\right)}{2NE^{\phi 2}} + \frac{E^{\phi\prime}N^x}{NE^\phi} - \frac{2N^x}{Nx}\right) \tag{4.26}$$

Solving $\dot{E^x}=0$ for N^x :

$$N^x = -\frac{N\left(\lambda^2x^2+E^{\phi 2}\right)s(t,x)\sqrt{1-\lambda^2 s(t,x)^2}}{\sqrt{1+\lambda^2}E^{\phi 2}} \tag{4.27}$$

Plugging N^x into K_x, then plugging in $K_x$to solve the diffeomorphism constraint for s, which represents K_ϕ:

$$s(t,x) = \frac{1}{\sqrt{2}}\sqrt{\frac{1}{\lambda^2} - \frac{\sqrt{\frac{4r_0\lambda^2E^{\phi 4}T^2-4\lambda^2xE^{\phi 4}T^2+\lambda^4x^9+2\lambda^2x^7E^{\phi 2}+x^5E^{x4}}{x^5\left(\lambda^2x^2+E^{\phi 2}\right)^2}}}{\lambda^2}} \tag{4.28}$$

The next step is then to solve the Hamiltonian for E^ϕ. However, this differential equation is not solvable. We instead use a second-degree Taylor expansion about $\lambda = 0$ of the Hamiltonian and Taylor expanded all results.

Here, we have the Hamiltonian, Taylor expanded around $\lambda = 0$:

$$H = \frac{3E^{\phi 3}T^2 - 2x^7 E^{\phi\prime} + 3x^6 E^\phi - x^4 E^{\phi 3}}{2x^5 E^{\phi 2}} + \frac{\lambda^2}{4x^9 E^{\phi 2}}\big(-16Mx^3E^{\phi 3}T^2 - 4x^7E^{\phi\prime}T^2 - 2x^6E^\phi T^2 - 3x^4E^{\phi 3}T^2 + 14E^{\phi 3}T^4 + 2x^{11}E^{\phi\prime} - 3x^{10}E^\phi + x^8E^{\phi 3}\big) \tag{4.29}$$

We Taylor expand the key equations around $\lambda = 0$:

$$E^\phi = x^3\sqrt{\frac{1}{-2Mx^3 + T^2 + x^4}} + \frac{1}{2}\lambda^2 x^3\left(Mx^3 + T^2\right)\left(\frac{1}{-2Mx^3 + T^2 + x^4}\right)^{3/2} \tag{4.30}$$

N^x (beta, in [99]):

$$N^x = -\frac{T\alpha}{x^2} + \frac{\lambda^2\alpha\left(2MT^2 + xT^2\right)}{2x^3T} \tag{4.31}$$

$s(t,x)$, representing K_ϕ:

$$s(t,x) = \frac{T}{x^2} - \frac{\lambda^2\left(-2Mx^3T^2 + 2x^4T^2 + T^4\right)}{2x^6T} \tag{4.32}$$

$$K_x = -\frac{T}{x\sqrt{-2Mx^3 + T^2 + x^4}} + \lambda^2\sqrt{\frac{1}{-2Mx^3 + T^2 + x^4}}\frac{\left(6M^2Tx^6 - 7MT^3x^3 - 8MTx^7 + 2T^5 + 3T^3x^4 + 2Tx^8\right)}{2x^5\left(-2Mx^3 + T^2 + x^4\right)} \tag{4.33}$$

$$E^\phi = x^3\sqrt{\frac{1}{-2Mx^3 + T^2 + x^4}} + \frac{1}{2}\lambda^2 x^3\left(Mx^3 + T^2\right)\left(\frac{1}{-2Mx^3 + T^2 + x^4}\right)^{3/2} \tag{4.34}$$

We now have an explicit expression for γ:

$$\gamma = \frac{\left(x^3\sqrt{\frac{1}{-2Mx^3+T^2+x^4}} + \frac{1}{2}\lambda^2x^3\left(Mx^3 + T^2\right)\left(\frac{1}{-2Mx^3+T^2+x^4}\right)^{3/2}\right)^2}{x^2\left(1 - \frac{2\lambda^2 M}{(1+\lambda^2)x}\right)} \tag{4.35}$$

Making the substitution $x = \frac{M}{r}$, taking the inverse of the above equation, and Taylor expanding, we have:

$$\gamma^{-1} = \left(1 - 2r + \frac{r^4T^2}{M^4}\right) + \lambda^2\left(-3r + 4r^2 - \frac{r^4T^2}{M^4} - \frac{2r^5T^2}{M^4}\right) \tag{4.36}$$

By setting the discriminant of this polynomial in r equal to zero, solving for T, and once again Taylor expanding the result, we find that the roots of the above equation coincide at $T = \frac{3\sqrt{3}M^2}{4} + \frac{9\sqrt{3}M^2}{8}\lambda^2$.

γ^{-1} is of the form $p(r) + \lambda^2 q(r)$. Evaluated at $r = r_c + \lambda\delta r$, close to the classical root $r_c = \frac{2}{3}$, it then takes the form:

$$\gamma^{-1} = \frac{1}{2}p''(r_c)\lambda^2\delta r^2 + \lambda^2 q(r_c) \tag{4.37}$$

To first order in λ, setting this to zero gives:

$$\delta r = \sqrt{-\frac{2q(r_c)}{p''(r_c)}} \tag{4.38}$$

From the above expression for γ^{-1}, we have:

$$p''(r) = \frac{12r^2}{M^4}\left(\frac{3\sqrt{3}M^2}{4} + \frac{9\sqrt{3}M^2}{8}\lambda^2\right)^2 \tag{4.39}$$

$$\begin{aligned} q(r) = &-3r + 4r^2 - \frac{r^4}{M^4}\left(\frac{9\sqrt{3}M^2}{8}\lambda^2 + \frac{3\sqrt{3}M^2}{4}\right)^2 \\ &- \frac{2r^5}{M^4}\left(\frac{9\sqrt{3}M^2}{8}\lambda^2 + \frac{3\sqrt{3}M^2}{4}\right)^2 \end{aligned} \tag{4.40}$$

It has already been established that at the classical root $r_c = \frac{2}{3}$, $p(r_c) = 0$. Since local extrema can occur where two roots coincide, the additional condition $p'(r_c) = 0$ is implied. To first order, $q(r_c)$ and $p''(r_c)$ reduce to:

$$p''(r_c) = 9 \tag{4.41}$$

$$q(r_c) = -1 \tag{4.42}$$

We then have:

$$\delta r = \pm\frac{\sqrt{2}}{3} \tag{4.43}$$

This gives us a final root at:

$$r = \frac{2}{3} \pm \frac{\sqrt{2}}{3}\lambda \tag{4.44}$$

Since $x = \frac{M}{r}$:

$$x = \frac{3M}{2 \pm \lambda\sqrt{2}} \tag{4.45}$$

Taylor expanding this result for x gives:

$$x = \frac{3}{2}M\left(1 \mp \frac{\lambda}{\sqrt{2}}\right) \tag{4.46}$$

4.7 Preliminary Conclusions

This is a preliminary result, subject to change as we review our calculations and finalize this paper for submission to Physical Review D. However, these calculations are promising, and we are confident that we have found first order, and perhaps also second order, corrections to the volume convergence boundary in terms of the quantum correction λ. This is an exciting outcome, indicating that the volume inside the event horizon of black holes maybe subject to quantum corrections. This has implications for the information capacity of black holes, potentially providing a resolution to the information paradox. Further study is warranted and particularly intriguing for the late-stages of black hole evaporation.

Chapter 5
Results and Future Projects

In this dissertation, I have used canonical gravity methods to: (1) construct a modified black hole spacetime with nonlocal quantum corrections, (2) reinterpret this spacetime as a quantum superposition of masses which produce modifications in the Newtonian potential in the asymptotic limit, and (3) calculate the implications for black hole volume slices when another quantum correction modifies a black hole interior. Each of these papers has produced novel results and opened new avenues for further exploration. These projects provide many avenues for further research: extending our modified spacetime solution, applying superposition and volume analyses to additional black hole models, and calculating a variety of potentially observable quantum effects.

5.1 Results

5.1.1 Chapter 2 results

In Chapter 2, we obtained a theory for two independent fields, representing a single classical metric component and its quantum fluctuation, using canonical methods in the spherically symmetric, static case. We analyzed the asymptotic behavior around the event horizon and at infinity, surrounding our canonical quantum-corrected black hole, and found that our quantum corrections have a ripple effect that extends beyond their local area. Our work is unique because we modify the spacetime via the Hamiltonian constraint, we do not violate energy conditions, we avoid common instabilities, and we preserve covariance by maintaining that the brackets of canonical constraints are closed. The quasiclassical approximation breaks down before one reaches the horizon, meaning that non-local effects may be crucial for horizon dynamics of quantum black holes. We need higher-order quasiclassical approximations to confirm this, but we have demonstrated that quasiclassical methods are promising for inhomogeneous models of quantum gravity, allowing explicit derivations of quantum corrections with only canonical quantization requirements. Our quasiclassical methods are promising for future work on inhomogeneous models of quantum gravity. There are many more future avenues for study discussed at the end of this chapter.

5.1.2 Chapter3 results

In Chapter 3, our goal was to interpret our quantum-corrected spacetime as providing covariant formulations of the gravitational force implied by a distribution of black holes in superposition, or by quantum matter constituents in superposition around a single black hole. Our detailed analysis in the weak-field limit reveals quantum corrections to Newton's potential, enables us to construct a spacetime with quantum uncertainty in the temporal and spatial components, and paves the way for a broad range of potential predictions. Our results also corroborate the fall-off behavior of the Casimir function, $U(x)$, from the numerical analysis in our previous paper described in Chapter 2. Without constructing a geometrical space-time picture of these superposed Newtonian potentials, one could not derive the relevant restrictions imposed by general covariance. Preexisting formulations for quantum reference frames, which emphasize test object properties, cannot address the question of whether the space-time has an action principle or other fundamental description with solutions involving gravitational field superpositions. This is an important field-theoretical question we address with our novel approach.

Our equations are implied by the covariance conditions, implemented in a model of curved space-time, which could not have been derived from quantum mechanics alone. Without our work, said equations would not suggest any bounds for C and δp_2, on top of the uncertainty principle. General covariance, specifically the existence of a curved space-time geometry for gravitational force of a quantum superposition of masses, implies non-trivial conditions. This work forms a bridge, connecting spacetime models with quantum corrections to superpositions of mass with uncertainties in the metric and potential. This connection broadens the applicability of results in both areas and sets the stage for a new connection to quantum switch experiments.

In the black hole superposition I derived in Chapter 3, the time and space components of the metric are affected differently by our quantum corrections. Our results imply a more precise geometry combining interrelated gravitational and quantum effects. This can be used for consistent descriptions of quantum superpositions, defined by suitable values of the moments or parameters C and U_0. Spherical symmetry implies a superposition state can only be formulated for masses at the same position, defining the center of symmetry. The quantum fluctuations given indirectly via mass fluctuations, rather than via position fluctuations. The final line element can be analyzed by computing geodesics and proper-time intervals.

5.1.3 Chapter 4 results

In chapter 4, using a related canonical quantum-corrected black hole model with a full interior extension, we calculated the first-order quantum effects on the volume inside the event horizon. This volume is defined by maximal spatial hypersurfaces, which converge at a specific radius. We show that this radius shifts depending on the quantum correction λ. We also found a second-order quantum correction to the redefined time function. This is the start of many such investigations of other quantum-corrected spacetimes. These quantum effects on black hole volume will have implications for information storage, black hole evaporation, and post-evaporation remnants.

5.2 Future Projects

Future projects fall into three general categories. The first is generalizing quantum-corrected spacetime models beyond static, spherically symmetric, and/or asymptotic regimes. The second is applying our interior volume and mass superposition calculations to more of these quantum-corrected models. The third is connecting these modified black holes to other work relevant for making testable predictions, such as gravitational wave quasinormal modes and quantum switch experiment designs.

5.2.1 Extending our quantum black hole model

We plan to extend our quantum-corrected spacetime to a full solution, adding complexity one step at a time. The extendability of this approach to the non-static case has already been established, with the addition of another scalar field in the Hamiltonian constraint, modifying the metric field ϕ_1. The foundation for this extension is provided in the 2019 Masters Thesis by Manuel Díaz, *Semiclassical consistent constraints with moments in spherically symmetric quantum gravity.* This provides proof of concept that this modified black hole model can be extended beyond the static case, with additional corrections.

There are plenty of open questions to pursue from here. It is not clear whether the extra factor between the Dirac and Poisson brackets, which rescales all time derivatives, will appear in the non-static case because it depends on which gauge conditions we choose. If generalised by superposition states, can the geometric formalism retain its classical form of Riemannian geometry? Do these geometric concepts eventually have to be generalized in a suitable way? Once we can talk about time evolution, we can explore formation and evaporation scenarios. And once we can extend beyond the spherically symmetric case, we can then consider quantum effects on rotating black holes.

It will also be interesting to explore whether our quantum corrections can be adapted to the context of a cosmological singularity. Preliminary steps have been taken in *Quasi-classical model of inhomogeneous cosmology* by Martin Bojowald and Freddy Hancock. There is more to investigate about how the quantum effects manifest in a cosmological context, and there are many open questions about the formation and evolution of the universe which could be affected by quantum fluctuations around the cosmological singularity. These include primordial fluctuations, cosmological black holes, gravitational radiation from the early universe, and the formation of supermassive black holes.

5.2.2 Expanding upon black hole superposition model

I propose a geodesic follow-up at smaller radii for our black hole superposition. It is a natural next step to analyze the non-asymptotic approximation near the horizon. Then we could expand to the non spherically symmetric case, incorporating rotation, as in a Kerr spacetime. I am also interested in whether the spacetime used in Chapter 4 would be well-suited to a similar reinterpretation as a quantum superposition of masses.

5.2.3 Applications to other quantum black hole models

Increasing complexity by removing statisticity and spherical symmetry is the natural progression for expanding upon other modified spacetimes as well. Generalizations of astrophysical black hole models to cosmological singularitiy models is another natural line of enquiry for extending the applicability for other models, such as for the spacetime considered in Chapter 4. Once our spacetime, outlined in Chapter 2, is fully extended to the non-static case, we expect to be able to recreate this volume calculation to calculate the effect of the quantum fluctuations in the metric field on the interior volume slices. This is yet another way to probe the validity and impact of various black hole quantum corrections.

5.2.4 Delocalized particles near quantum-corrected black holes

Our work is also promising in combination with that of Joseph Balsells' work considering the varied gravitational effect of black hole curvature near the horizon on particles viewed as probability distributions [104]. This work involves the interplay of geodesic moment terms, test mass moments, central mass moments, and correlations between two quantum objects [104]. I am interested in analyzing these particle probability distributions in the presence of various quantum-corrected black holes, including my own. This could be repeated with the other canonical quantum-corrected black hole spacetimes mentioned, which include modifications to the lapse function and the addition of scalar field corrections.

5.2.5 Gravitational Waves

Karim Noui at Université Paris-Saclay has been building on our, and related, modified black hole spacetimes [105]. Hugo Roussille, in his related doctoral dissertation, presents novel quasinormal mode computations in modified gravity. This is the start of a new calculation method, enabling quasinormal mode computations for a large class of modified gravity, potentially including loop quantum gravity. We have designed a collaborative project around this shared topic of modified gravity, while introducing their quasinormal mode work. In a merger of two black holes, such as those constructed in Chapter 2, there could be modifications in the properties of quasinormal modes. For example, their frequency spectrum could be affected, which is not highly constrained by current observations. I have begun reconciling our formalisms to calculate quasinormal modes of the gravitational wave signals from black hole merger ring-down, using the same modified spacetime derived in Chapter 2.

We know that the static spherically symmetric constraints are closed, with a non-rotating background. Quasinormal modes are not static. Nor are they exactly symmetric; the plane of coalescence breaks symmetry, but we are using spherical symmetry with perturbations. Starting from a static, spherically symmetric background, we will introduce a small perturbation in all canonical fields. Then, we will expand the constraints to get new equations. We will solve these linear partial differential equations, not the full nonlinear Einstein equations. Even with the spherically symmetric inhomogeneity, we will ensure that covariance is maintained. It should be possible to derive quasinormal modes with our modified spacetime directly from the modified Hamiltonian constraint by determining the correct gauge condition. Deriving this procedure is the first stage of our work. I will apply this procedure to my calculations of quasinormal modes.

My goal is to adapt this novel derivation method for quasinormal mode equations from Hugo Roussille's Dissertation, making it applicable to a wide variety of modified spacetimes. I will then calculate gravitational wave quasinormal mode signals detectable by next generation detectors, as predicted by each of the spacetimes discussed in this section. We expect that these predicted gravitational wave signals will be visible to the next generation of detectors, such as LISA, and therefore testable on that timescale. These measurements could either rule out or support certain modifications, limiting possible theories of quantum gravity.

5.2.6 Quantum Switch

There is a promising line of enquiry calculating Quantum switch predictions in the context of our (and related) black hole mass superpositions. If the quantum switch experiment in [81, 82] were conducted outside one such black hole superposition, how would the uncertainty within our quantum-corrected metric components impact the quantum switch experimental output? We would go about defining the measurement events in terms of proper time, which will have some uncertainty inherited from the metric components. Measurements would have to be made far enough apart in space and time for the quantum switch order to be distinct, despite the uncertainties in radial position and time. Increasing these distances should, in principal, improve the result of the quantum switch measurement, as long as the necessary mass entanglement is preserved over the course of the event separation, throughout the necessary time and over the necessary distances. Said quantum switch experiment would need to be conducted repeatedly to generate statistics on the results and associate any variation in output with the mass (and therefore metric) uncertainty from the space-time quantum corrections. [81, 82]

Chapter 6
Contributions to Teaching, Service, and Advocacy

The work in this section is neither required, nor universally valued, but I believe it is just as important as my quantum gravity research. I hope that, one day soon, sections like this will be standard components of dissertations. I am passionate about generating new knowledge through research, and I share my passion through teaching, mentorship, and outreach. Throughout this work, I smooth the way for those who come after me by advocating for equitable systemic change and furthering an inclusive cultural shift. This chapter contains materials and resources I have written for students, instructors, and mentors. These documents cover advice and activities for teaching, research, outreach, mentorship, and allyship. If you make use of them, I would love to hear about it.

6.1 Teaching and Education Research

I have actively developed my teaching skills, including taking a Physics Pedagogy course, attending physics education research talks, and serving as a TA for six years. I have implemented these skills by contributing to the curricula, course designs, and overall culture of the department. I have even conducted a physics education research study.

6.1.1 Teaching Philosophy

My teaching philosophy centers on promoting growth mindset, scientific identity, and belonging. I teach physics as a skill learned through effort, rather than as an innate ability. I foster adaptable problem solving skills and expert-type thought patterns, emphasizing curiosity, creativity, and collaboration. I am always learning and adapting, in order to utilize the latest evidence-based pedagogy. You can read my full statement of teaching philosophy in Appendix A.

6.1.2 Building Inclusive Communities: Presentation and Physics Education Research Study

I created a *Building Inclusive Communities* presentation to uplift all students, particularly underrepresented groups, by providing tools to build inclusive communities. I set an open and inclusive tone by addressing the existence of systemic bias, its manifestations as microaggressions, and healthy ways to respond. I have presented for my students over the last five years, with unanimously positive feedback on my anonymous surveys. I am in the process of formalizing this effect by measuring the impact of this intervention on students' sense of belonging, scientific identity, and class performance. Several colleagues at various universities have already adopted and adapted this presentation, and I am working to increase its use and impact to uplift more students. I am currently collaborating with Eric Hudson to study the impact on students' sense of belonging, STEM identity, and overall course performance. You can access this presentation and slides introducing the study in Appendix B.

6.1.3 Adaptable Activities/Worksheets

I designed several adaptable class activities. The first teaches students how to write their own practice questions, which has been shown to be one of the most effective study methods. The second is a worksheet designed to train students in an expert-type problem-solving process. Both of these activities are highly adaptable and available in Appendix C.

6.1.4 "Hidden Curriculum" Course Design

I created a syllabus draft for an extended grad student orientation/survival guide. It is crucial that institutions provide instruction on the "hidden curriculum" of navigating academic bureaucracies, connecting with support systems, and generally setting themselves up for success. This was intended as a redesign of the first-year grad "seminar course," known as PHYS590 at Penn State, but it can easily be implemented as an orientation course at any institution, even adapted for other departments and fields of study. The curriculum I designed is available in Appendix C.

6.2 Outreach and Science Communication

I believe outreach is both a serious responsibility and a great joy. I have organized, designed, and led countless workshops and demos for events such as AstroFest, Envision, Haunted University, Young Women in STEM, ArtsFest Kids' Day, local schools, etc. Promoting science literacy is crucial for building public trust, promoting science funding, and improving science policy. I have engaged in continuous science communication work, presenting for organizations and events such as Astronomy on Tap, the Osher Lifelong Learning Institute, the Central Pennsylvania Observers, Astro Night, Nerd Nite, etc. I even visit my high school, Chico Senior High, on occasion to present for the Science Club which I founded there.

I am including a list of opportunities I found for presenting outreach. I hope this list makes it easier for future students and educators to engage in outreach surrounding Penn State and inspire others to connect with their communities elsewhere. I really enjoyed sharing my work and/or fun science topics at Nerd Nite, Astronomy on Tap, the Osher Lifelong Learning Institute, the Central Pennsylvania Observers, and high school science clubs. Good places to propose outreach are local schools, museums, libraries, community centers, and relevant organizations for the topic you want to share. I encourage everyone to contribute to, or create, science demos for students at various levels, particularly underrepresented students and underserved communities. Some examples from my outreach at Penn State are Young Women in STEM, ENVISION, the Special Olympics, and PAW Pals. There are also often annual community events to get involved in. Some examples from what I have done are AstroFest, Astro Night, Haunted University, Science U, and ArtsFest Kids' Day.

I have made various worksheets, slides, opportunities, and advice documents available in Appendix D, for others' future use and adaptation. I encourage everyone to keep intersectional representation in mind when creating or adapting such educational materials. It is also important to make sure our outreach efforts reach underserved communities.

6.3 Allyship

As a queer, neurodiverse woman in physics, I have faced bias, harassment, and lack of support at every stage of my education, and I am committed to improving accessibility and inclusion for those who come after me. To enact change, I engage with the system at all levels, joining professional development programs, serving on committees, founding organizations, and supporting my peers in various capacities. Throughout this work, I place an emphasis on community-building and adaptable support structures.

I am always working to improve my allyship with other underrepresented groups through professional development and service, such as joining the Rainbow Science Network and serving as a Gender Equity Center Ambassador. I have posted countless informational brochures and fliers around the department, especially the grad/undergrad lounges and study rooms. These contain crucial information about university resources, community support systems, professional development, and opportunities to get more involved. I created and improved community spaces like the grad lounge to make them more welcoming and inclusive, strengthening community and connection, particularly among students.

Resources I collected for underrepresented groups and allies can be found in Appendix F. My primary advice for allyship is to learn about the experiences of underrepresented groups you are unfamiliar with by seeking out resources created **by** members of those groups. Use any privileges and platforms you have to make space for marginalized voices, from large-scale public forums to small-scale group conversations. Empower them and follow their lead.

6.4 Leadership and Service

I work on various committees and initiatives at the departmental, university, national, and international levels, emphasizing equity and inclusion in my service and advocacy.

6.4.1 Committee Work

I have been a member of the Eberly College of Science Climate Committee Grad Student Subcommittee, the Physics Climate, Inclusion, and Diversity Committee, the American Institute of Physics TEAM-UP program, and the American Physical Society IDEA Team. Together, we have replaced an installation of Nobel prize posters, representing a biased and exclusionary history of physics, with colorful art and informational posters celebrating women and underrepresented groups in science history. We organized a successful APS site visit to the physics department, which produced a wealth of actionable items to improve our department and strengthen our community. We enabled town halls, created weekly Community Lunches, and are continuing to find ways to improve communication within the department, such as committee reports at these lunches. We created a Community tab on the department website and filled it with resources for underrepresented students, including an optionally anonymous online dropbox for sharing feedback and concerns. I also coauthored guidelines to make colloquiums throughout the university more inclusive. These guidelines can be found in Appendix E.

6.4.2 Student Organization Leadership

I emphasize community building and mutual support through my work as President of PSU Physics and Astronomy for Women+, my membership in Graduate Women in Science, and my participation in a variety of professional organizations.

In my five years as PSU Physics and Astronomy for Women+ President, we have more than doubled the size of the leadership team and increased undergraduate participation. I implemented evidence-based shared leadership practices to empower each member with more agency to benefit the community. I earned an APS Women in Physics Group Grant and used it to launch an ongoing poster series, which celebrates underrepresented scientists throughout history. These posters, and the means to make more, are available on the PAW+ website.

Our leadership team continues to thrive, despite the pandemic. We hosted the 2023 APS Conference for Undergraduate Women+ in Physics, with such success that we were asked to share our efforts with hosts for the following year. We re-launched a peer mentorship program, expanding it to all physics grad students. We are also supporting the Society of Physics Students in structuring their own mentorship program for undergraduates. We collaborated with the Physics Grad Student Association to establish regular Town Halls and elect representatives to share our suggestions with the department, greatly improving communication, student involvement, and the potential for improvements to the graduate program. We also expanded our outreach efforts, creating PAW Pals to regularly bring volunteers to present science demos for local elementary school students, and we are creating a database of demos to reach a wider and less-privileged audience online.

I have researched, written, and edited many PAW+ posters celebrating women and minorities in science, now displayed around the department and other universities. I have designed many inclusive and uplifting physics-themed stickers for PAW+, which will continue to be printed and distributed for years to come, including at outreach and recruiting events, such as conferences. I also sourced, printed, and posted countless mini-posters of underrepresented physicists and astronomers around the physics and astronomy department bulletin boards. These collections are available through PAW+.

6.4.3 Founding an International Organization

I am Co-Founder of Women+ International in Theoretical Physics, an online community for networking and mutual support among women and gender minorities in theoretical physics. Please share and join either the group or our upcoming networking list of allies! https://sites.google.com/view/withphys/home

6.4.4 Proposals

I have written and coauthored numerous proposals to improve our program, most notably, a proposal for an Equity, Diversity, and Inclusion Assistantship. This would be a funded TA position for at least two grad students to implement initiatives to improve the department climate, serve as a liaison between grad/undergrad students and the department, and be a friendly point of contact to connect students with vital resources. My proposals for these EDI Assistantships, a grad student retreat, and improvements to the grad lounge are included in Appendix E.

6.5 Mentorship

I believe each of us have a responsibility, at whatever level we are able, to give those who come after us the benefit of our experience. I approach this through personal mentorship, sharing resources, creating resources, and advocating for positive change. I consider myself a mentor and advocate in all my professional roles, supporting my students, undergraduate Learning Assistants, friends, collaborators, and formal mentees alike. I take pride in facilitating others' mentorship work as well.

My mentorship extends to all my students, providing resources and advice on study strategies, allyship, research, and grad applications, earning me TA of the Year. I have drafted many documents of advice and resources for mentors, mentees, students, and educators, shared on the PAW+ resources website, enabling me to reach more people and provide lasting support. I mentor students, helping them connect with university resources, empowering them to seek accommodations, and encouraging them to seize professional development opportunities.

I mentor undergraduate researchers, helping my students connect with research opportunities or publishing papers with them myself. I mentor student leaders, recruiting more student engagement and building a lasting structure within groups like PAW+, to continue our work after I leave. I mentor friends and peers, helping them navigate the hidden curriculum of academic bureaucracies and persevere through personal and professional challenges. This is exhausting work, but very rewarding, and the more people step up, the more distributed this emotional and intellectual labor will be.

I donate my time and share my experiences through mentorship, creating lasting resources, and by presenting and serving on panels, both at conferences and within the department. I organized and presented a physics colloquium and panel discussion about the experiences of underrepresented students, sharing my presentation about building inclusive communities. I also served on countless panels for advising undergrads, incoming grad students, and young women in STEM.

My advice and resources for mentors, mentees, and students are available in Appendix F. You can also find the structure and resources for the PAW+ peer mentorship program on our website.

6.6 Feedback

We each have the power to shape our communities, so it is important to use our voices, share our insights, and collectively envision the future we want to create. I have been providing continuous feedback to the department and university throughout my PhD, advocating for policy changes to uplift our community and set a positive example for other institutions. My current list of feedback is available in Appendix G, for use by Penn State, future advocates, and other institutions.

Appendix A
Statement of Teaching Philosophy

I love sharing my passion through teaching. As a mentor and role model, I set a respectful and inclusive tone, designing the classroom and curriculum to encourage active engagement and collaboration. My goal as an educator is to foster critical thinking skills and growth mindset. I teach physics, not as a collection of facts, but as a process and a set of tools with which to explore. I use well-tested, innovative strategies, such as Active Learning, which been shown to benefit all students, with additional benefits for underrepresented groups (Freeman 2014). My teaching strategy is adaptive, as I continuously incorporate new advances in the field, and respond to feedback from students and colleagues. I am particularly eager to implement SCALE-UP practices and an Integrated Peer Leadership Program, as presented at APS April Meeting.

For course planning, I use Backward Design as described in "Understanding by Design" by Wiggins and McTighe: Starting from students' background, I create learning objectives for concepts and skills I want them to gain. I then plan how to assess that progress and design learning activities around each assessment. I gauge the starting context, while simultaneously demonstrating to the students that they are valued and respected, by creating a survey of background material as well as students' goals and interests, asking them to articulate what they want to get out of the class. These responses are taken into account when constructing learning objectives for both subject knowledge and general skills.

I value clear goals and expectations, publishing learning objectives, course timeline, code of conduct, and resources like accommodations in the syllabus. I emphasize mutual respect and inclusion by modeling allyship strategies such as Amplification, and I find that being vulnerable about my own struggles creates space for students to ask for the support they need. I want my classes to be empowering experiences each student carries forward to uplift their future communities. I promote Growth Mindset (recognizing that physics is a skill acquired through perseverance, as opposed to an innate ability with a limit) by embracing mistakes as part of the learning process. Through positive reinforcement focused on effort, creativity, and follow through, I change how students define success. This allows me to train them in expert-type thinking by empowering them to interrogate their own understanding, identify misconceptions, and follow up with questions.

Assessing student progress towards objectives is essential for assessing the success of my teaching. For formative assessments, students collaborate on i-clicker questions, worksheets, homework, and lab activities. For summative assessments, students solve short-answer and multiple choice questions in small Objective Checks throughout the semester. These assessments are presented as learning opportunities for students to gauge their mastery of a topic, and they are offered continuously throughout the semester. Rather than moving on from material that has not been mastered after a traditional exam, students can do additional tutorials on a topic to access another objective check until they have understood the material. It is wonderful to see things "click" for different students at various points in the semester and watch them reach similar mastery by the end.

Readings are assigned to prepare students for group worksheets and labs, accompanied by small pre-class assignments, which the students share with each other at the beginning of class and submit a group copy for feedback. When readings are assigned digitally, participation can be tracked automatically. Homework assignments with immediate feedback systems, such as Mastering Physics, are useful for practice, self-assessment, and monitoring participation outside of class. Participation is also tracked by routinely spot-checking notebooks and grading how well the work is communicated. Pre-class assignments are graded primarily on completeness, to incentivize making an attempt, while in-class assignments are graded primarily on correctness, to incentivize students supporting each other and doing their best work. In class, students complete hands-on, collaborative activities, consulting with the teaching team. Labs have scaffolding questions, but are largely open-ended and exploratory. Worksheets are derivations and calculations, broken into questions designed to prompt metacognition. For example, students are encouraged to look for multiple routes to a solution and to ask, “What if there were X change in the setup?”

I have developed an activity in which students choose a learning objective and follow a flow-chart to construct their own practice assessment. Creating questions in this way engages students in all levels of Bloom’s Taxonomy, and self-testing is the most effective study strategy (What Works, What Doesn’t by Dunlosky). Ideally students will incorporate this into their study habits. They each write a question before class, then engage in small group discussion before building a question together. After implementing feedback from TA’s and myself, they solve and provide feedback on another group’s question. After revisions, all questions are posted for practice, and at least one question will be included as an assessment. I have received remarkably positive feedback on these activities in anonymous student surveys, and they are now being implemented by other instructors as well.

The physical space can be used to optimize learning and encourage collaboration. Students sit in discussion groups, with boardspaper to solve problems together. I avoid having a focal pointfront of the room, since students seated in the back dramatically underperform (Perkins & Wieman 2005). It is also beneficial to broaden the physical boundaries of the classroom through office hours, scheduled study spaces for homework collaboration, and extra review sessions. I engage students in research, teaching, and community service. I connect students with labs for tours and research, encourage and provide opportunities for volunteering/outreach, and prompt students to synthesize their learning by teaching others. Students are encouraged to attend colloquia and can earn extra credit for researching topics relevant to course material.

I look forward to learning from, and collaborating with, colleagues who share these values, improving the classroom experience for all by focusing on the needs of underrepresented students. WPI has unique initiatives aligned with my work, such as Project Based Learning, and I am eager to contribute to these programs.

Appendix B
Physics Education Research Study: Building Inclusive Communities

Abstract:
Collaborating with Eric Hudson and Jackson Henry, and I conducted an IRB-approved physics education research study to measure the impact of my equity, diversity, and inclusion presentation, administered at the beginning of the semester in an intro physics course, on students' sense of belonging, STEM identity, and overall performance.

B.1 Introduction

As members of a society, we have all been conditioned to have a variety of conscious and unconscious biases, which manifest as systemic obstacles, harassment, and microaggressions. In order to build an inclusive and supportive community, which facilitates the best learning environment for all students, we must acknowledge the presence of these biases, learn how to identify their manifestations, and provide our students with constructive ways to engage with these issues. Having created a presentation on these topics for my courses over the last several years, which has been consistently well-received. We have now conducted a physics education research study to quantify the impact such an intervention can have on students' sense of belonging, scientific identity, and class performance.

B.2 Motivation

As a neurodiverse woman in physics, facing microaggressions, imposter syndrome, stereotype threat, and harassment throughout my educational career, I wanted to create safer environments for my students and provide context for their experiences. I sought to establish the institutional acknowledgement, baseline awareness, and healthy dialogue about systemic bias, and access to wellness resources, that I lacked as a student.

My presentation covers the existence of systemic bias in society, implicit bias in ourselves, and common manifestations as microaggressions. Next, we discuss how we can respond, support victims, and care for ourselves. Then, we go over how to be allies, be called out, and educate ourselves.

I created this presentation for my classes, to facilitate building inclusive communities. I workshopped it over the last five years in the intro Mechanics and E&M courses for physics majors. Now 50% of current undergrad physics majors have experienced it, and I believe this has contributed to a meaningful cultural shift. I designed it to be easily adaptable to any class, field, or workplace, so feel free to adopt or adapt. After overwhelmingly positive feedback, I conducted this Physics Education Research Study to measure the impact on students' sense of belonging, identity, and course performance.

B.3 Experimental Design and Implementation

Our goal is to measure the impact of this intervention on students' sense of belonging, STEM identity, and grades, via anonymized surveys. We collected data for the Newtonian Mechanics course for non physics majors, for larger sample. In the first lab/recitation sections, half received this intervention and half had a placebo activity (a peer discussion of Chegg and AI ethics, with no participation from the TA). These discussions were administered by volunteer TA's from other courses (two for each type of presentation). Surveys administered before, after, and at the end of the semester.

We chose the PHYS 211 intro physics course on classical mechanics because we wanted an intro course to shape students' first experience of the department. We also wanted a large course ($\tilde{1}$,000 students) to have statistically significant results, particularly being able to do a demographics analysis without identifiable data.

Using before/after surveys, we have students self-report their sense of identity and belonging. Survey responses were associated with student ID's and matched with final grades, for those who consented to grade collection. A third party combined the data and removed the ID's and any other identifying information before sharing the statistical results included here.

B.4 Results and Analysis

We are currently processing the data. We documented a strong positive responses in-person: Students thanking the TA's and expressing appreciation for the presentation topics, particularly underrepresented students. Students followed up for community, advice, and accommodations.

We ran a two sample independent T tests used to determine statistical significance in changes before and after the presentation. Many survey questions were inconclusive but very significant effect for some measures. Making plots of changes in question responses, the effect was varied, but the most statistically significant results are in support of the intervention's effectiveness. Data still being analyzed, but we have included a special preview of some of the results.

End of semester effect analysis is limited by lower participation in the final survey, but we have plans for addressing this in further studies. Analysis of changes from before to after the initial intervention have trends of varying statistical significance. The strongest change noted so far is included below as an example.

This is a bar graph showing the responses to whether students identify as a Physics Person, and how the number of positive and negative responses changed between the first and second survey. P value: $0.017 << 0.05$ Mean test difference: 0.166 Mean control difference: 0.0348 While not all survey questions had statistically significant changes, this P value is well below the 0.05 threshold for statistical significance. The figure shows a clear positive trend in the test case receiving my presentation.

B.5 Conclusions

This intervention has received unanimously positive feedback during development, and this study begins to quantify the impact on students identity and experience of the course.

Many colleagues at Penn State, and at other universities, have used or adapted this intervention in their own classes. The content is designed to be generally applicable, so that only the local resources for support and community building need be updated for use elsewhere. We hope that the results of this study will encourage others to adopt this, or similar interventions, and help shape academic culture and communities in a more inclusive and uplifting direction.

For future studies, we would like to repeat this experiment with other classes including

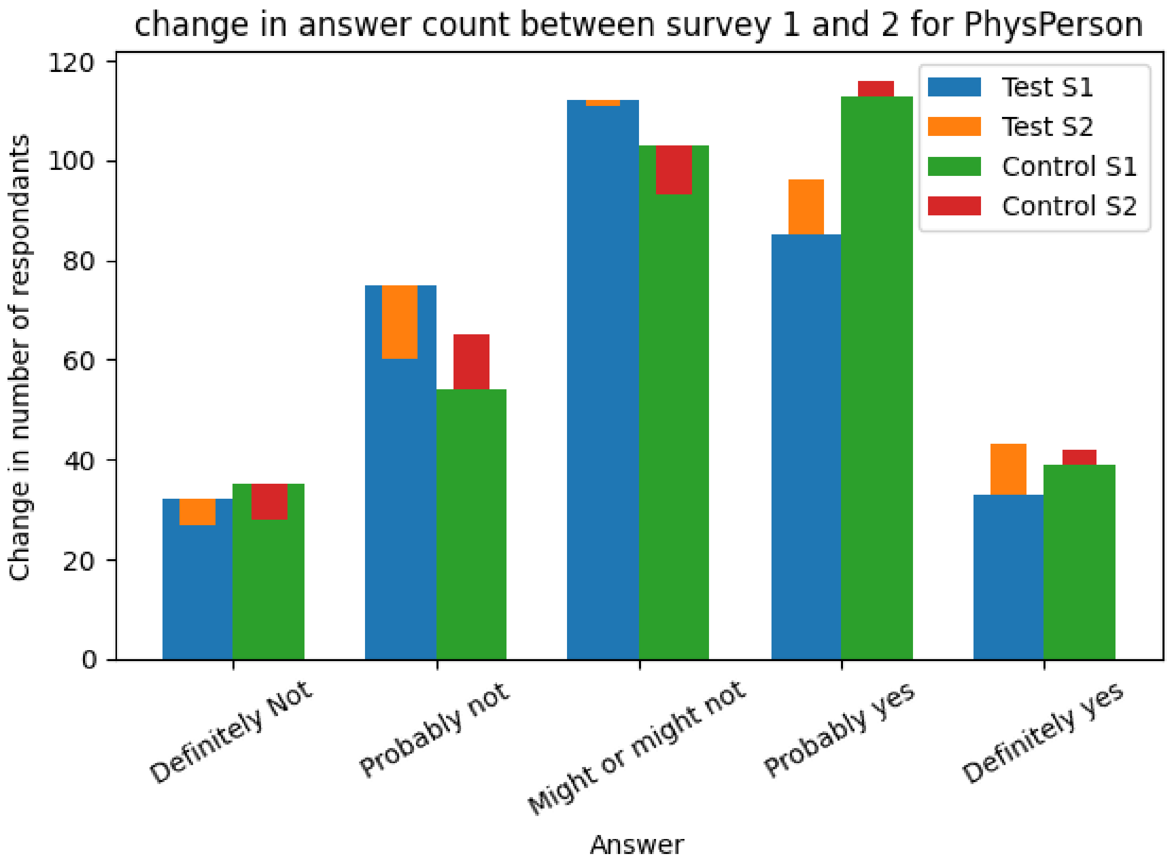

Figure B.1. Bar graph showing responses to whether students identify as a Physics Person, and how the number of positive and negative responses changed between the first and second survey, before and after the interventions. The blue columns to the left of each pair represent the responses of the test group before the presentation, with the smaller orange bar showing the change in number of responses after the presentation. The green columns to the right of each pair represent the responses of the control group, and the smaller red bars show the change in responses after the placebo discussion.

the intro courses for physics majors (hypothesizing stronger results in a course with a pedagogical structure aligned with the values of the presentation), other departments (to demonstrate a transferable effect), grad students (to see how past experience in a variety of physics department cultures changes the effectiveness), and other institutions (any statistically stronger or weaker affect could point us in the direction of policies or practices to adopt or avoid, to best support the values of the presentation in our community). We are also interested in creating a related intervention focused entirely on mental health and accommodations, as that portion prompted the most students to reach out for support. Finally, we would like to do a longer study to measure long-term impacts over four years or beyond.

Our takeaway is that making space in science classes for dialogue about systemic bias, and providing relevant resources, has a positive impact on students' sense of physics identity— potentially belonging and course performance as well. This effect may be stronger for underrepresented groups! This type of intervention is worth implementing widely and studying further.

B.6 Acknowledgements

Thank you to Dan Costantino for allowing us to conduct this study on his class. Thank you to Emma Steinebronn, Julian Mintz, and Sanika Khadkikar for assisting with presenting the intervention and placebo discussions. Thank you to the students who agreed to participate in this study. And a special thank you to all the students in my 211M/212M courses over the past few years who engaged with this presentation, as I developed it, and offered such thoughtful and supportive comments.

Appendix C
Course Design and Adaptable Activities

This Appendix contains some of my most useful and adaptable course and activity designs. First, an outline for an extended orientation, teaching students about the "hidden curriculum." Second, an activity designed to teach students a useful study strategy: writing their own practice questions. Third, an example of a worksheet structured to teach matery-level problem solving procedures. Fourth, pre-class and in-class activities to accompany my *Building Inclusive Communities* presentation.

C.1 "Hidden Curriculum" Extended Orientation Course Design

C.1.0.1 Redesigning grad first year seminar

This originated as my Department Feedback on PSU PHYS 590, but I now see it as a general guide for an intro/orientation course for any program, particularly graduate programs, though it could easily be adapted for undergrads. This course is a valuable way to get important information to the first-years all together, and the scope should include an extended orientation: "Student Survival Skills" or "The Hidden Curriculum."

- Start with a presentation/discussion about creating an inclusive community, like the one I created in APPENDIX. This will set the tone and expectations for conduct and community in the department.
- Make sure students understand the flexibility in course scheduling: taking undergrad courses, testing out of classes, signing up for research credits while focusing on

coursework, medical leave, etc. Assign creating a tentative schedule for their course credits over a few years, in consultation with their faculty mentors. This will also help establish who is getting a successful start in their mentorship relationship.

- Highlight the importance of community-building and have a day where you invite representatives from a variety of student groups, especially on a departmental level and university groups supporting underrepresented students, to talk about their events, both to support students and to offer edification for allies. GPSA, Coalition of Graduate Employees, and Coalition for a Just University should be included in this as well.
- Incorporate lessons on growth mindset and study strategies, similar to the freshman FYS. (This could be covered in the Pedagogy course instead, as long as it's towards the beginning of the program, to shape the formation of their study habits.)
- Have guests from CAPS and the Accommodations Office present about mental health and accessibility. Perhaps have students volunteer to share their experiences to have a peer reinforce these messages.
- Have someone from the Title IX office and related campus organizations talk about best-practices for community growth as well as reporting.
- Have students do small activities/discussions in a variety of groups during class to get to know their cohort, facilitate the formation of study groups through schedule surveys, the grad student discord channel, and other platforms, and encourage students to share other interests with the class to build community (groups interested in Dungeons and Dragons, hiking, cooking, etc). Emphasize the importance of work-life balance.
- Outline the bureaucracy of the department/university so students understand how to navigate it: funding, reporting, feedback, and participation. Highlight the optionally anonymous department and college drop-boxes.
- Assign students to investigate/try a few various campus resources like libraries, the writing center, etc. and share what they find with the class, to make sure everyone is aware of what is available for support.
- Have the Ombuds present about healthy relationships, warning signs, and conflict resolution strategies, as well as explain their role as confidants and mediators.

(Note, the Ombuds program needs to be improved; students are generally not satisfied with the results when they ask for help, and are sometimes traumatized by breaches of trust.) Highlight that ombuds in other departments are also available, and students are encouraged to talk with whomever they choose.

- Have a day where representatives from all committees open to grad students share a brief description of their purpose and work, recruiting present or future grad student participation. Have students sit-in on a committee meeting of their choice and write a paragraph proposing an initiative for the committee.

- Have student and PI representatives from research groups taking new students present, not just about their work, but about the group (community, expectations, opportunities, and general experience). Highlight resources for connecting with research groups, like the newly organized department website and pages like the Rainbow Science initiative.

- Discuss how to approach potential advisors, assign practice emails, offer mock-interviews, etc. to build confidence and guide students through the process of finding an advisor. (Some of this could be optional, since some people will already have these skills.)

- Assign a short report in which students identify research groups they are interested in joining. Offer to make introductions, if students are hesitant to reach out themselves. You could even invite some PI's to class, put the students in groups, and run little "speed-dating" interviews so the class can meet the potential advisors.

- Offer strategies for productive research meetings (take notes, prepare questions beforehand, ask at least one question and push forward with follow-up questions, assert your ignorance rather than trying to seem like you understand something that you do not, send a recap email summarizing the discussion and action items for both parties, etc.). Outline the expectations for treatment, like not being routinely stood-up for meetings and having support for professional development like grant applications and conferences. Spell-out that it is normal and ok to change advisors and that they should never feel stuck on a research path; consider bringing in a student to share their experience changing projects, to make the message tangible.

- Discuss the ideals and expectations of advising and mentoring experiences. Have students share some advice they found valuable from a mentorship experience, to learn from each other. Check-in that mentors are in productive contact with the students.

- Encouraging colloquium attendance is good, especially now that the quality and diversity of invited speakers has gone up, but it does not need to be weekly and could be broadened to include seminars: Attend X colloquiums/seminars of your choice and write about them, perhaps for a variety of audiences. You could specify a few that everyone should attend, like the climate colloquia.
- Practicing presentation skills is also important: Give one presentation at the end of the semester (perhaps one in the middle too, so there is time and opportunity for implementing feedback) on a physics topic of your choice, like a colloquium or prospective research project.
- These assignments can also be used to practice strategies for reading/skimming and interpreting physics publications (reference a paper and at least one of the papers it cites, summarize the main arguments, recreate a calculation from a paper, etc.)
- Require them to submit some anonymous feedback on their first-year experience at the end of the class, for participation points. Remind students about the drop-boxes.
- Foster physics identity throughout. Maybe have them write about some future plans in consultation with their mentor?
- Consider requiring students to attend at least one discussion event hosted by PAW+ or a related group and report on it in class discussion.
- Have supplemental workshops focused on the needs of different groups, like international students, students with disability, neurodiverse students, first-generation students, etc. There could be professional development workshops too, like a journal club, critical reading, scientific writing, outreach, etc. Require students to attend at least X number of these workshops, whichever are most relevant to them.

C.2 Adaptable Activities/Worksheets

I will be using versions of these activities regularly in all my courses, and they can be easily adapted for others' use as well. I designed them to train students in evidence-based study strategies and expert-type problem solving processes.

C.2.1 Writing Practice Questions

Studies show that writing and solving practice problems is one of the most effective ways to study physics. I made a flowchart to assist students in this process. Then I built a class activity around it to introduce the students to the process. I usually incentivise this activity by including at least one of the student problems in an upcoming assessment. I usually introduce this activity before the first assessment of the semester and repeat it before each, to keep build students' skills, keep them engaged, and keep this resource in the forefront of their minds as they study.

C.2.1.1 Pre-Class Question Writing Activity

Physicist (Your) Name:
Group Number:
Question Writing PLA

1. Read and Review the Learning Objectives for the recent class material.
2. Choose a learning objective from the recent class material relevant for the next exam and use the flow-chart to construct a question for the next exam. You will use this question for group discussion/feedback and to create a new question as a group, which will be reviewed in class.
3. When you come to class, work with your groups of three. Rotate questions among your group, and write feedback for at least one other group member. Staple the feedback that you receive to your question, with any adjustments that you make to your question.
4. You can earn extra credit by deconstructing a question from a past assessment (re-create the flow-chart that was used to generate the question) and handing it in at the beginning of class.

Choose a learning objective from the recent class material and use the flow-chart to construct a practice question assessing that learning objective.
Chosen Objective:

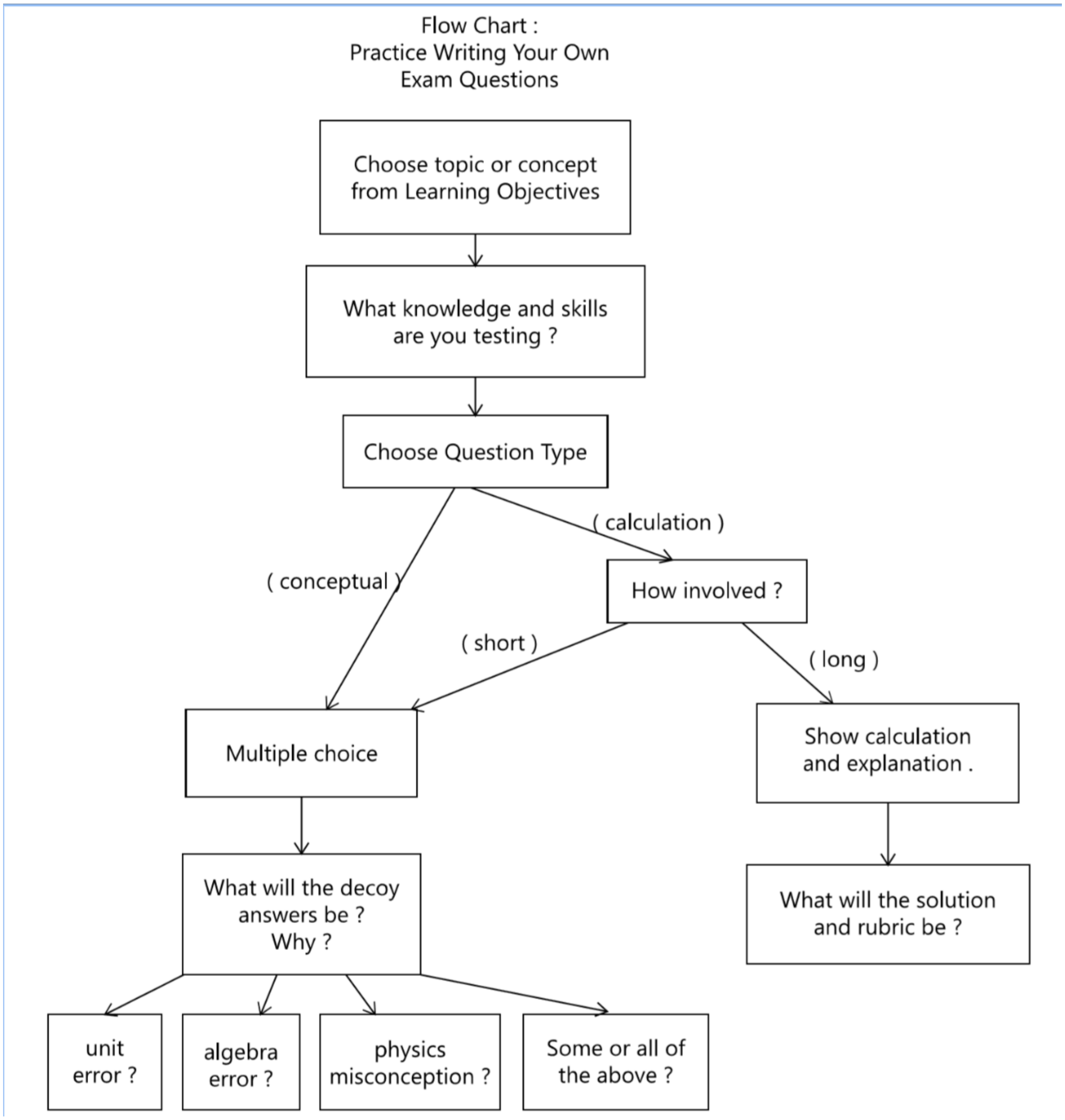

Figure C.1. Flowchart for constructing practice questions

C.2.1.2 In-Class Question Writing Activity

Physicist (Your) Names:

Group Number:

Question Writing LA

1) Write Question:

Working from the PLA questions and the i-clicker questions from recent lectures, create a finalized question that you want to propose for the next exam. Use the flow-chart as an outline, then write your finalized question on a separate page. Write your solution/rubric on another separate page.

2) Self-Check:

Make sure you have a clearly drawn and labeled diagram illustrating your question, with a defined coordinate system. If any values are needed for the problem, define them (as variables and/or numbers). Could your setup be confused with anything else? Eliminate ambiguities.

3) TA Check:

Have two TA/LA's review your question and solution/rubric in detail.

LA initials 1:

LA initials 2:

4) Trade:

Trade questions with another group.

5) Test:

Solve their question individually and record how long it took you. (You are not graded on the time, we just want to be sure it is a reasonable length for the exam. If we don't catch it now, exam questions could take too long.) Average these times and record them in your feedback.

6) Feedback:

Discuss the question with your group and write feedback. Reconstruct their flow chart and add any constructive comments that you have. Return their question with your feedback attached, including the average time it took the members of your group to solve the problem.

C.2.2 Problem Solving Procedure Training Activity

I introduce background information on slides, with lots of class discussion questions throughout, to build-up to a multi-stage problem. Then, I distribute this worksheet for the students to start in groups of three. Whatever doesn't get done in class becomes homework, still to be done in groups. Most of the questions are general prompts for approaching a physics problem, with a few topic-specific questions sprinkled in.

C.2.2.1 In-Class Worksheet

Exoplanets are winking at us!

Physicist (your) name:
Names of collaborators (inside and outside of class):

Useful concepts from class:
What is the gravitational force between masses M and m, separated by r?
What is Newton's 2nd Law?
What net force and acceleration cause circular motion?
How is centripetal acceleration related to tangential velocity?
How is period related to tangential velocity?

Group Problem Solving: For a planet of mass m, in a circular orbit around a star of mass M, you measure an orbital period T. What is the radius of the orbit?

What are you trying to find? What information is given? Draw/label a diagram.

What physics is relevant here? (Laws, geometry, types of motion, etc.) What physical relationships exist between the given values and the value you want to find?

How do these relationships connect? Use them to derive the relationship between the orbital radius and the given variables. Write out your steps and reasoning. (This is good practice for communicating your thoughts, will help you catch mistakes, and will make it easier to correct errors).

Reality check your result and explain: (ex: Are the units and limits reasonable?)

What do you notice about the relationship between the period T and the planet's mass m, in this calculation?

Discuss your work with classmates/instructors. What thoughts/comments stood out to you? What have you learned?

Useful concepts from class, for reference and study:

What is the gravitational force between masses M and m, separated by r?
$F = GMm/r^2$
What is Newton's 2nd Law?
$F = ma$
What net force and acceleration cause circular motion?
Centripetal force and centripetal acceleration (radially inward)
How is centripetal acceleration related to tangential velocity?
$a = V^2/r$
How is period related to tangential velocity?
$V = 2 * pi * r/T$

C.3 Building Inclusive Communities Activity

I created a presentation on Building an Inclusive Community for the class that I TA, and I'd love to see similar presentations done in intro courses of every university and every subject. It's a great way to set the tone and give students tools for constructive dialogue and allyship as they join the university community. Informally, I have collected very positive (anonymous) feedback from students, and I am planning a pedagogy study to officially measure and publish the impact of this intervention.

Presentation for my class: https://docs.google.com/presentation/d/1LOCLXmSfwBe_PvuFeqzMT

Pre-class activity: Choose an underrepresented group you are not a part of. Find at least two sources by people in that community discussing their experiences with bias/prejudice (articles, blogs, videos, podcasts, etc.), and reflect on any privileges you may have (financial stability, parents experienced with higher education, cicgender, etc.)

This can also be assigned as a follow-up to the presentation with a brief discussion the next day to share what people learned, perhaps with a shared document where students can access the sources that their classmates found.

When in-person, the discussion slide can also include an activity in which each participant writes anonymously about a time they witnessed bias, on a provided note card, which is then shuffled, and redistributed. This way everyone starts with an anonymous experience of a classmate to share with their group and start discussion. (Once the ice is broken with the anonymous experiences, students are encouraged to share their own.)

I also adapted this presentation for a PSU Physics Colloquium, preceding a panel discussion on the experiences of underrepresented students: https://science.psu.edu/physics/equity-and-inclusion/dialogues-and-media

Appendix D
Outreach Activities, Slides, and Advice

Here is my advice, resources, and examples for the outreach activities I have curated over the years. I focus on intersectional representation when talking about scientists, and I design activities to build kids' growth mindset and scientific identity. Feel free to adapt these activities yourselves and reach out for slides I could not include in this format.

D.1 Advice for Designing Outreach

Curriculum design:
1-3 General physics concepts (color mixing, magnetism, center of mass/balance, etc.)
Hands-on demos of these concepts (the physics department has class demos you can check out) Craft activity that generates something they can take home (like a constellation tube or paper orbit demo, etc.)
Real astrophysics research that the students can do (identify exoplanets from light curves, or citizen science projects like classifying galaxies, etc.) Feature a female or nonbinary physicist, preferably from an additionally underrepresented group, who is leading research in that area (include a photo and use empowering language, like referring to said researcher as a "leader" and using their titles, such as "Doctor" or "Professor")

Presentations:
Whenever using photos in a presentation, make sure that the people featured represent a lot of different demographics.
Ask lots of simple questions for the students to answer to keep them engaged. Dedicate a lot of time to their exploration and discussion of demos.

Have all presenters/volunteers introduce themselves at the beginning as physicists/astronomers and say a little about their research. Invite questions about their work at the end of the presentation.

Demo stations:
Modify the content of the presentation to fit a short activity
Focus on hands-on demos and craft activities and have something they create that they can take home and use to teach others
Print a few visual aids

General tips:
Always refer to the participants as scientists, encourage them to make predictions and test their hypothesis, then say things like "Now you are a physicist!"
Compliment students on their questions and problem-solving processes (don't just call them smart, be specific and complement their effort and thought process).
Emphasize that engaging in the science is a success, regardless of whether they made the correct prediction, because we all learned something in the end.

D.2 Edible Science Activities

Food always gets people interested and is a great way to engage young kids in science. Here are a few examples of activities I have refined over the years.

D.2.1 Cake Core Sampling

This can be accompanied by a presentation, or simply a few visual aids, about geology and the many contexts in which cores samples are used (the moon/asteroids/mars for astronomy, coral reefs for marine biology, salt marshes for ecology, glaciers for climate science, etc.). Try to use photos of diverse scientists engaged in taking cores samples.

Bake cupcakes (with opaque wrappers) which have uneven layers of different types of cake, bits, and fillings. Have participants use straws to take core samples and sketch what they think the interior looks like, before testing their hypothesis by slicing and eating it.

Discuss the different strategies: What if you take all your samples along one line? Is it more useful to take a lot of samples in one area, or spread them out? How could you get more information with fewer samples?

You can use a frosted confetti sheet cake for a group collaboration, having each participant contribute a data point to a larger effort to map the interior.

D.2.2 Cookie Phases of the Moon

Using "oreo" cookies, have participants open them up and scrape the frosting into a phase of the moon: waxing/waning, crescent/gibbous, full/new, first/third quarter.

Have a movable model solar system handy, to illustrate how the position of the sun and moon create the phases. You can discuss the offset of the moon's orbit/Earth's rotational axis from the plane of the solar system/ecliptic to explain the "tilt" of the moon.

Ask questions to lead people to the realization that the right hand side is illuminated when the moon is waxing, and the left hand side is illuminated when the moon is waning. Challenge them to notice and name the phase of the moon when they are out at night.

This is also a good lead into discussion of equinoxes and eclipses!

D.2.3 Frosting the Sun

Frosting cookies to look like suns is a great open-ended opportunity to discuss solar features. Mixing colors, crating textures, and adding various sprinkles can be associated with sunspots, flares, prominences, granulation, etc. Color can be used to discuss the type, age, and temperature of stars. More advanced discussion can include the coiling of the sun's magnetic field from differential roation rates, and how that creates the eleven year sunspot cycle.

D.3 Science Crafting Activities

I try to include craft activities in my outreach, particularly for young audiences, because it is engaging, gives them a sense of accomplishment, and continues the educational experience beyond the lesson. When children create their own demonstration apparatus or artwork and take it home, they are more likely to share it with siblings, parents, friends, and teachers. This reinforces and spreads the learning experience.

D.3.1 Glittering Galaxies Activity

These are talking points for a science craft activity, in which attendees use glitter, glue, and stickers to draw their own galaxy on black construction paper. The different colors and sizes of stars, structure of spiral arms, density of objects, etc. create many opportunities to interpret their creations with a scientific lens. For example: Are there more red stars? Then it's an old galaxy without a lot of new, blue stars!

Glittering Galaxies:
Galaxies are collections of billions of stars orbiting around a central, super-massive black hole. Our galaxy, the Milky Way, is a spiral galaxy.

Spiral galaxy: Spiral galaxies are flat and can have different numbers of spiral arms, which can be wound tightly or loosely. Some spiral galaxies have a round bulge of stars in the center, and some may also have a bright bar of stars across the center. There tend to be older stars in the central bulge and newer stars in the spiral arms.

How many spiral arms does your galaxy have? And how tightly wound is your spiral?
Is there a central bulge?
Is there a bar across the middle? Is the bar large or small?

Elliptical galaxy: Elliptical galaxies are not flat. Instead, they are large balls of stars, which can be round like a soccer ball, or longer like a football. Elliptical galaxies tend to be large and have more old, red stars and fewer young, blue stars.

Is your galaxy round or long?
Is it large or small?

Irregular galaxy: Irregular galaxies are strange shapes, often resulting from two galaxies passing near or through each other, and sometimes merging.

Was your galaxy a spiral or elliptical that warped into a new shape?
Is your galaxy actually two galaxies that have run into each other and are combining into one galaxy?
Are the two black holes near each other or far apart?
Does it have a lot of hot, blue stars or colder, red stars?

D.3.2 Planet Planning Activity

These are talking points for a science craft activity in which attendees use watered-down glue to layer colorful tissue paper over a small paper plate, adding geographic features and pipecleaner rings. This is an open-ended opportunity to discuss the types of planets (terrestrial or gaseous?), extreme environments (acid rain, frozen nitrogen, etc.), and other engaging features of exoplanets and planets in our own solar system.

Planet Planning

Size: Is your planet large like Jupiter, small like Mercury, or in-between?

Temperature: Is your planet close to the sun and boiling hot, like Venus? Is it far from the sun and freezing cold, like Mars? Or is it in the habitable zone, where liquid water can exist, like Earth?

Type: Is your planet a gas giant, like Jupiter, Saturn, Uranus, and Neptune? Or is it a small, terrestrial planet with a rocky surface, like Mercury, Venus, Earth, and Mars?

Atmosphere: Is there air for an atmosphere, and what is it like? Is there oxygen to breathe, like on Earth? Or is there poisonous gas, like on Venus?

Water: Is there water on the surface? If so, is it frozen solid, flowing liquid, or clouds of gas?

Life: Is there life on your planet? If so, what is it like? You can draw pictures!

D.4 Outreach Proposals

Here are a couple of example proposals from successful outreach activities and workshops I have led.

D.4.1 Exoplanet Workshop

Girl Scout Workshop Proposal Spring 2019

On Behalf of the Physics and Astronomy for Women at Penn State
By: Källan Berglund, PAW Outreach Chair

Finding Planets Dancing with the Stars

This activity focuses on physics that is not visible to the human eye, not because it is small, but because it is so far away from us. Exoplanets are so small in the sky that they can barely be seen with the most powerful telescopes, so we need to be clever in how we detect them. We will be teaching the students about how planets are detected around other stars, when they are too small, dark, and far away to be observed directly. This lesson incorporates the topic of center of mass through tangible, hands-on demos, includes building a take-home demo, and culminates in applying the detection method to simulated data to engage in the research themselves.

We will introduce ourselves as grad students and mention the topics of our own research, as examples of the things they could do in the future. We will introduce the concept of center of mass, with a moment of dance demonstration and a few hands-on activities to get them engaged. We will encourage them to reflect on the demos with questions about why and how things are balanced.

We will have a powerpoint for visual aids throughout the discussion, but the time will be spent alternating between hands-on activities and discussion/reflection on the activities. First we will have them hold hands and lean back, with their feet together, to see how taller people can't lean back as far relative to their shorter partners, because the center of mass has to remain over their base of support between their feet. We will then have them assemble miniature exoplanet orbit demos from paper to take home, placing the point of rotation where the center of mass should be.

We will then discuss Doppler shift of light, using audio examples of passing carr sirens. Next we will introduce the application of all this to how exoplanets are detected from Doppler shift of the sun's light as it orbits the center of mass of the sun-planet system. We will talk about Debra Fischer, as an example of a scientist conducting this research, and as a role model for the students.

The lesson culminates in the students being given example data from doppler shift detections to identify which samples contain exoplanet signals, engaging in Dr. Fischer's research themselves. They will also be directed to a Citizen Science project where they can help identify real exoplanets that have never been seen before, using a website accessible from home. Time will be saved for additional questions and discussion of the activity and the research conducted by volunteers.

D.4.2 Stellar Spectra Workshop

Envision STEM Workshop Proposal 2018
By Kallan Berglund and Parul Maheshwari
PAW Outreach Coordinators

Workshop Type: Place an 'X' by the type of workshop that you would like to develop 60-minute hands-on STEM workshops for MIDDLE SCHOOL audience (grades 6-8)

Workshop Scheduling: We are available for presentations as needed.

Workshop Title: Decoding the Secrets of Stars from Starlight

Workshop Leads (names and emails): Källan Berglund (kallan_berglund@alumni.brow.edu) and Parul Maheshwari (parul.dkm@gmail.com)

Other Volunteers (names and emails): TBD

Objective #1: Participants will learn about light and optics through hands-on demonstrations.

Objective #2: Participants will apply this understanding and creative problem solving in the context of determining stellar composition and evolution.

Objective #3: Participants will learn about the historical narrative of how this knowledge was originally discovered by pioneering female astronomers Cecilia Payne and Annie Jump Cannon who overcame many societal obstacles to have their discoveries recognized by the scientific community.

Content Area/Discipline: Physics; specifically optics, astronomy, and history.

INTRODUCTION (5-10 min): We will introduce ourselves by name, educational history, area of study, and a few personal research topics (which can be revisited in general questions at the end). We will present the goal of understanding what stars are made of and how they evolve, along with the intermediate goal of understanding light in order to decode information about the stars. We will present some demonstrations of optical phenomena, comparing them to "magic tricks," but explaining that today we are all physicists who *will* reveal the secrets to magicians' tricks.

MAIN ACTIVITY (30-40 min):

Demos:
Prism and flashlight separating white light into a color spectrum.
CD reflecting light of different colors at different angles.
Dipping pencil in water to see it appear to bend at the surface.
Small spheres with the same refractive index as water disappear when submerged.
Cardboard tubes with constellations poked through tape at one end to simulate stargazing and help students relate to specific stars of a given type.
Example spectra of red giant, blue dwarf, and main-sequence stars to study and interpret.

We will have students seated in small groups within view of a demo station. Students will come up in small groups to conduct demonstrations of refraction and scattering of light. (We are working to gather enough materials, so that each demo can be done at each table simultaneously, instead of at one station.) The students will have a few minutes to discuss each demo with their groups and fill out a worksheet designed to prompt connections between the observed phenomena. Volunteers will be leading class discussions to introduce the concept of different types/colors of light and how they interact differently with surfaces and boundaries to create rainbows and (prisms and CD's) optical illusions (bent pencil and disappearing sphere).

A powerpoint presentation will be incorporated into the workshop for visual aids when introducing the applications of optics and spectroscopy to stellar astronomy and astrophysics, as well as the historical figures associated with the material. The interactive constellation demo will be introduced to connect the new ideas to specific stars. The students will be asked to use their new knowledge, and some example spectra, to identify different stellar types/compositions from the patterns of light that were measured, putting themselves in the shoes of those pioneering astronomers, Cecilia Payne and Annie Jump Cannon, who first observed these patterns.

CONCLUSION (5-10 min): After recreating the discoveries of influential women in astronomy, the students will record some of their own questions to be discussed as potential research areas. Volunteers will reiterate their interests/experience and the students will have the opportunity to ask questions about their career path, research, and experiences. If time permits, the students will have a chance to revisit the earlier demos for further inspection and questions.

Materials: We have most of the necessary materials already, though we may need a few more glass prisms and flashlights in order to have materials for each demo at each table. Some sharing may be possible without significant loss of time. We will also need to print a worksheet for each student and a few sheets of example spectra, but this will not be expensive.

Facilities needed/requested: We would prefer a classroom with group seating, like round tables, but we do not need any special equipment besides a projector.

Appendix E
Proposals for Improvements

E.1 Introduction

This Appendix contains proposals for initiatives intended to improve equity, diversity, and inclusion (EDI). The first is a general proposal for an EDI Assistantship, funding grad students to be liaisons between students and the department, participate in departmental service like committees, and connect students with essential resources. I hope that similar roles are implemented widely at Penn State and at other institutions. The latter two proposals are specific funding proposals for improvements to the grad lounge and for a grad student retreat.

E.2 Equity, Diversity, and Inclusion TA Roles

Proposal for Inclusivity Assistantships

E.2.1 Grad-Focused EDI TA

Building on the Grad Pedagogy TA role
Beyond but including being a TA for 590
Full-time appointment
Facilitating grad peer mentorship program
Weekly hybrid office hours
Updating grad handbook
Friendly face to help new grad students feel welcome
Ease transition into grad school
Connect students with mental health, community building, and professional development

resources
Call out the fact that everyone has imposter syndrome and their needs are valid
Facilitate self-advocacy to help students express and stand up for their needs
Open up conversations about mental health and accommodations
Synthesize and offer feedback to professors and the department on the workload and student struggles, keeping an eye on the big picture of the grad courses collectively
Maintaining bulletin board postings of EDI info and professional opportunities in the first year office, grad lounge, and department website
Nurture sense of belonging and physics identity
Facilitate connections like organizing a Q&A with underrepresented postdocs
Relay student needs to the department

E.2.2 Undergrad-Focused EDI TA

Full-time appointment
Present for intro courses about building an inclusive community
Facilitate presentations each year about how to find summer research, applying to grad school, and other relevant topics for undergrad physics majors
Friendly face
Ease transition into college
Connect students with mental health, community building, and professional development resources
Call out the fact that everyone has imposter syndrome and their needs are valid
Facilitate self-advocacy
Open up conversations about mental health and accommodations
Hold weekly hybrid office hours for students to seek support, resources, and advice
Maintain postings in plastic sleeves in bathroom stalls (rotating info on Autism Acceptance Month, Pride Month, etc.) and by the sinks (resources like CAPS and accommodations)
Propose other such EDI initiatives, as needs become clear
Nurture sense of belonging and physics identity
Facilitate connections like a Q&A with underrepresented grad students
Relay student needs to the department

E.2.3 General Notes

Advertise these people/roles/resources in posters and on each syllabus, with a QR code and link directing people to a page on the department website (updating bio and contact info as new people take on the role)
These roles will help fill the Robinett void (lighten the load for program heads)
This will help with retention of underrepresented students and support all students' success in the program
Serve on or keep in contact with various departmental committees and student organizations
Each managing a page on the department website with resources or simply making recommendations to committees like EDI on what content is needed online
Ideally there would be two of each role, to support each other, provide an additional point of contact if someone is uncomfortable approaching one for any reason, and cover for each other when ill.
Starting with the Graduate Inclusivity Assistantship focused on supporting the grad student population, with a focus on first-years, would be a good start to trial this program.

E.2.4 Addressing Potential Objections

Money: pay as TA's
TA supply: Pay more LA's to reduce demand for TA's (separate, critical EDI issue that LA's should be paid, at least after an initial semester of LAing for credit and training, supplemented by a book scholarship to offset opportunity cost of not working a paying job)
Stepping on Ombud's toes: These fill a different role and would help connect more students with Ombuds. Students are already working to meet this need but are getting burned out because this labor is not accounted for in their responsibilities.
Not enough labor to justify payment: I can list more ideas, but this should be plenty.
Liability for…? Bad recommendations, mishandling information. Solution: Proper training. And there's less liability than untrained TA's already being approached about sensitive topics.

E.2.5 Final appeal

As previously stated, students are currently doing this work because the need is there and we cannot stand by while others struggle and suffer, but it is hurting us, and we are getting worn out. Please pay us for this essential labor and account for the time and energy required by making this an official assistantship and recognizing it as part of our workload.

E.3 Funding Proposals for Programs and Improvements

These proposed initiatives are intended to build community and gather feedback for the department. These could also be adapted for other applications.

E.3.1 Upgrading the Grad Lounge

Dear Alumni Society Board,

We, the leadership of Physics and Astronomy for Women+ and the Physics Graduate Student Association, request $5,000 to facilitate making our small grad student lounge a more welcoming, inclusive, and effective space.

The lounge, 120 Osmond Lab, was formerly a storage closet and is currently furnished with discarded furniture and broken chairs. We would like to create a sensory corner for neurodiverse students and a more welcoming gathering space that students can be proud of. This is part of our plan to build back a healthy community post-pandemic.

This space is heavily used for student meals, Physics and Astronomy for Women+ Tuesday Tea, and even hosting discussions with guest speakers. Thus, these upgrades will have a wide-reaching impact on the lives of physics grad students, recruitment, and the university's public-facing image.

We will source basics like tables, shelves, and chairs from Lion Surplus to keep costs down, and we have already taken steps to save costs by soliciting donations of smaller luxuries, like fidget toys/pillows from professors and graduating students. Salvage will remove the non-functional items we currently have to make room for new items to be brought in. Our organizations will host a free food event to recruit students for assembling/moving new pieces into place.

Most of the budget will be used for key items to make better use of the small space and to accommodate various sensory needs among our current and future student populations. We have the support of the Climate, Inclusion, and Diversity Committee.

Kitchenette:$770
Furniture:$1510
Lighting:$263
Sensory Needs:$135
Recreation/Community Building:$1365
Health/Safety:$390
Decor and Contingency:$555
Total:$4,988

Sincerely,
The Physics Graduate Student Association
& Physics and Astronomy for Women+

E.3.2 Grad Student Retreat

The Physics Climate, Inclusion, and Diversity Committee
February 23, 2024

Dear Alumni Society Board,

On February 14th the Physics Climate, Inclusion, and Diversity (CID) Committee released a proposal for overhauling the graduate student qualifying process and solicited feedback from students. We, the CID, request $3,000 to fund a weekend retreat, with transportation, food, and lodging provided, for as many members of the graduate program as possible to attend. Your funding will reduce barriers to attendance and encourage participation in community building and feedback collection from the physics graduate students as we finalize major policy changes in our department. It is crucial to collect feedback from graduate students on these issues that will affect their community deeply moving forward.

The physics graduate students have previously demonstrated excellent community organizing and self-advocacy in self-organized town halls. These culminated in electing representatives to share prioritized concerns and suggestions with our new Department Head. We would like to recognize and further this effort. Because mental health is a known issue among graduate populations and getting physical distance from campus in a peaceful, natural setting is known to have positive effects, we believe that a weekend retreat is a natural and useful tool for our goals.

Through the retreat, our graduate students will connect with one another over meals, hikes, and games, while spending an allocated amount of time reading the proposal draft, discussing ideas, making comments, and thinking ahead to propose other initiatives. An active and engaged grad student community is essential to the success of our educational mission and fostering this will help significantly with creating a healthy climate in our department and the university.

We are already taking steps to make this trip affordable, such as using buses, volunteers to pick up food, cooking simple meals ourselves, and reserving university lodging. A summary budget for the full event includes:

Transportation:$400 Lodging:$1000 Food:$1136 Total:$2536

Sincerely,
The Physics Climate, Inclusion, and Diversity Committee

Appendix F
Advising and Mentorship Resources

Students looking for support and advice, or instructors looking for resources to share with their students, here are resources I compiled to share with my students. Some of the links have university-specific information, but most universities will have similar programs and policies that you can look up. Fellow mentors, feel free to take inspiration from any of this and adapt your own versions to meet your needs. If you choose to use any of these materials, please cite me as appropriate, and I'd love to hear how it goes. I welcome any feedback you would like to share. These resources will continue to evolve throughout my career.

F.1 Facilitating Advisor/Advisee Communication

Seek resources to educate yourself about the experiences of underrepresented groups by looking up sources *by* the group you want to learn about, such as how to properly support neurodivergent students. There are lots of existing trainings related to these topics, such as the Rainbow Science Network. The department should provide time and incentives for professions to engage in such professional development. To get started, look up resources by, for, and about some underrepresented groups. There are many groups to consider in this way, such as students of color, first generation students, indigenous students, international students, students with disabilities, etc.

There are often small changes you can make to the environment or your word choice, which can have a big impact on a student's experience. Discuss potential stressors with the accommodations office, such as buzzing/bright lights, making sure students are not cornered away from the door, etc. Remember that positive feedback (particularly focused on effort and growth) goes a long way towards building students' physics identity, sense of belonging, and growth mindset. To get things started, here are a few suggestions to facilitate advisor/advisee communication during research meetings:

Questions to facilitate a successful advising meeting:

1. How are you doing in general? Is there anything about your life or work that you would like to share with me?
2. How are you feeling about your work?
3. What questions do you have? (not "Do you have any questions?")
4. Are there any resources I can provide that would be helpful?
5. What challenges have you faced recently? I'd like to offer support.
6. Are these meetings helpful? Are there any adjustments we can make for the better?
7. Do you feel that we have good communication?
8. How would you summarize our objectives?
9. What is your understanding of the next steps?

Agree on each person's action-items for the next meeting, ideally over a follow-up email.

F.2 Advice and Resources for Students

F.2.0.1 Approaching Physics Problems

Here is a general guide for approaching physics problems and writing up your work well:

1. What do I want?
2. What do I know? (Draw and label a diagram, or several for different stages)
3. Make sure you are using uniform and compatible units.
4. What tools do I have? (Look for equations that contain only things you know and the thing you want to know; sometimes you'll need several. This can include physics relationships and mathematical relationships, like geometry.)
5. Break the problem into stages of motion or stages of conserved quantities.

6. Break the problem into single dimensions (like separating motion in the x and y direction)
7. Take advantage of symmetries, conservation laws, and transition states.
8. Write out your steps and reasoning (This is good practice for communicating your thoughts, will help you catch mistakes, and will make it easier to correct errors).
9. Reality check your result and explain (Are the units correct? Is this a reasonable order of magnitude?...)
10. Discuss your work with a classmate or instructor.

F.2.1 Tips for lab work

1. Safety first. Read instructions, treat equipment with respect, and ask if you are unsure.
2. Keep in mind that someone should be able to understand and recreate your work, so be clear and thorough.
3. Make sure you draw and label diagrams for all stages of your experiment. This is the most useful for communicating your experimental design.
4. Clearly define all variables in the equations you use and be consistent.
5. Include units on all your numbers and be consistent.
6. Include all your graphs (with title, axis labels with variables and units, scaled appropriately to display your data).
7. Interpret all features of your graphs/data; label notable features in your graphs and describe what is physically happening at that point (including but not limited to: maxima, minima, inflection points, plateaus, spikes, x intercepts, asymptotes, slopes, noisy data)
8. Consider and explain possible sources of error in your data.
9. Suggest alterations to improve future experiments or ideas for follow-up experiments.
10. Ask for help if you are stuck.

F.2.2 Tips for research progress

These are strategies I have found helpful and recommend trying out to establish healthy habits. But the most important thing is to pay attention to your own needs and build from there, from your ideal work environment to your preferred communication styles.

Keep notes on useful terminology and tools, ideally collaborating with mentors and collaborators. Some examples I created from my group are tips for Mathematica and a living dictionary of useful terminology.

Start the document you'll be using to write your paper/dissertation and write a few sentences of paragraphs each week to track your progress and document your insights. Overleaf is a useful platform to collaborate on LaTeX files. To collaborate on code, setup a GitHub repository.

Meetings:

1. Have a consistent place for research work (notebook, tablet folder, etc.)
2. Prepare question(s) ahead of time.
3. Take quick notes during meetings (have your advisor/collaborators draw/write, photograph the board, etc.)
4. Consolidate your notes into "insights," "questions," and "action items."
5. Send a follow-up email to communicate and document your understanding of the action items for each person, or at least yourself.

Communication:

1. Be brave and vulnerable enough to express your confusion and assert your ignorance until it is adequately explained/resolved.
2. Revisit questions if past explanations are insufficient.
3. Establish clear communication channels and expectations with coworkers.
4. Consider collaborative workflows, like Github and Overleaf.
5. Having a written record of action items is very helpful, and perhaps shared documents for notes and/or paper writing.

Independent work:

1. Learn what environments support your focus. For example, I work best with other people co-working in the space with me. This is known as "body doubling."
2. Prioritize your action items and break them into small steps.
3. Work through steps until you get stuck.
4. Write out why you are stuck, any questions you have, and ideas about how to move forward. Articulating this usually gives you an idea of what to try next.
5. Repeat.
6. Pretend you are explaining the problem to a lay person and/or your advisor to fully articulate your confusion.
7. Try multiple approaches to see what works. If you are missing a piece of information; you could assume one way, then the other, and see how that goes.
8. Look up terms, ask for help/clarification, and/or take a break and look at it again with a fresh perspective.
9. Work on other action items to make progress in as many areas as you can between meetings.
10. Write up your questions/sticking points for each, to discuss at the next meeting, or send them in an email to your advisor/team.
11. Have different types of work that you can switch between for variety: reading, writing, calculations, coding, etc.
12. Have readings you can go through, take notes on, and generate questions from to switch to when stuck on calculations/coding/etc.

Setting yourself up for success:

1. Be kind to yourself.
2. Prioritize your physical and mental health. Make sure you have good care providers.
3. Make sure you are getting good food, sleep, hydration, and exercise.

4. Build community, drawing support from multiple sources (friends, family, clubs, etc.)

5. Pay attention to your needs (Sensory stimuli, fidgets, quiet/music, comfortable workspace, etc.)

6. Cultivate and maintain a Growth Mindset in yourself and others.

7. Don't hesitate to ask for help from a variety of sources.

8. Use the resources available for support and community building.

9. Practice acknowledging progress in all of these areas and celebrating those successes.

F.2.3 Guide to reading scientific papers

This document is a tool for learning, not judgment. Use it as much or as little as is helpful. My students are encouraged to share their reports for me/their advisor to learn from and answer questions.

Strategy for reading papers, with different levels of depth:

1. Title

2. Abstract

3. Figures/labels

4. If you have background/context questions, read the Introduction

5. If you have follow-up questions/clarifications, read the conclusion

6. If you want to know more, read the whole paper

7. If you want more context, explore the sources,* especially those cited in the introduction, conclusion, and/or written by the same authors.

*Citations are made for various reasons. If a source is cited as being wrong/disproven, skip it. If it's cited as the foundation/previous work being built upon, or as being related work, check it out.

My students: As you find unfamiliar or key terms, look them up in the living dictionary. Other researchers: Create a living dictionary for your own research area/collaborators.

Be courageous about acknowledging confusion and asking questions, to help mentors and collaborators communicate better with you and improve everyone's learning.

Comprehension Reflections:
Essentials:

1. Subject: What is the topic?
2. Objective: What is the goal?
3. Motivation: Why does it matter?
4. Uniqueness: What makes this work stand out?
5. Result: What do they accomplish?
6. Deeper reading:
7. Context: What is the starting point?
8. Toolkit: What tools are used? (formalism, properties, constraints, assumptions, etc.)
9. Obstacles: What challenges were faced?
10. Adaptations: How were the challenges resolved?
11. Observables: How can this theory be proven/disproven?
12. Future: What can/will be investigated next?

As you read, note what works and what does not in the writing, for when you write your own papers.

F.2.3.1 Tips for finding research

Everyone approaches this differently, with varying amounts of time/energy to spare, so prioritize your physical and mental health and find what works for you. *Some of these resources are specific to Penn State. **Unfortunately, some are restricted to US citizens/permanent residents.

Congratulations on pursuing research experience! You're on your way.

General good practices for networking and building your CV/resume throughout undergrad **(but don't put too much pressure on yourself, just take one step at a time and you'll get where you want to go)**:

1. Seek research experience whenever you are interested; it is very valuable, particularly if you want to go to grad school
2. Reach out for mentors among peers and faculty for support and advice
3. Get to know advisors/professors, to learn and to get letters of recommendation
4. Go to office hours and seminars, challenge yourself to ask questions and figure out your interests
5. Seek professional development opportunities like conferences
6. Participate and take on leadership roles or found student groups like the Society of Physics Students and PAW+
7. Build your math and/or computer skills (Python is valuable, particularly in Astrophysics. Mathematica is useful for classes and research. LaTeX will be useful for all CV's and publications...)
8. Work on communication skills with community outreach and presentations (opportunities available through PAW+ and other groups https://science.psu.edu/outreach)
9. Be proactive about your mental health, and seek accommodations if needed! Education is a lifelong endeavor, so pace yourself.

Applications:

1. Request letters of recommendation in advance (two week minimum in general, a month or two is better), and send reminders as the deadlines approach. Sharing a spreadsheet of application materials and deadlines is useful. Providing your transcript, CV, and/or application essay(s) can be useful resources for letter-writers.
2. Always have at least one person edit your statement/application materials (family, friend, or university writing support services)
3. Use active verbs that keep the focus on your initiative, highlighting your perseverance

4. Mention a few specifics about the program you are applying for to demonstrate your interest and highlight how you are a good fit for this opportunity.

5. Connect with potential advisors in advance if you can.

Places to look for research:

1. Talk to older students, grad students, and mentors about their past research, and ask for email introductions to their contacts.

2. Look up universities and talk to professors you're interested in working with and search the corresponding university website for funding opportunities.

3. Read professors' web pages and abstracts of their papers to see what you're interested in working on and reach out to them and/or their grad students to talk about their work. (Don't be intimidated if you don't understand the papers! Just ask for explanations of a few key terms to get the conversation going.) Ask if they have funding or can arrange course credit for an undergrad researcher.

The remainder of these lists are particular programs, funding sources, or searches.

1. Perimeter Institute

2. **Research Experience for Undergraduates, REU's. These are available at other universities!

3. **NASA Space Grant, PA. These are available in every state! (Pro-tip: smaller states tend to have smaller application pools for the same funding.)

4. **One Stop Shopping Initiative, OSSI

5. JPL/NASA

6. *Site to help with your search: https://urfm.psu.edu/

7. *PSU Eberly College of Science Undergraduate Research Program

8. *More sources for opportunities in the PSU College of Science: science.psu.edu and scienceengagement.psu.edu

9. PSU Summer Research Opportunities Program, SROP (focused on underrepresented students)

10. The PAW+ website resources page will be updated with additional suggestions like this soon, so keep an eye out for that.
11. ECoS SURE
12. The American Physical Society
13. The American Institute for Physics

This is not an exhaustive list. If you find opportunities that should be added to this list, particularly for international students, please contact the PAW+ president and web chair.

F.2.3.2 Advice for applying to grad school

Everyone approaches this differently, with varying amounts of time/energy to spare, so prioritize your physical and mental health and find what works for you.

Thinking ahead as an undergrad:

1. Get as much research experience as you can (REU, NASA Space Grant, OSSI, etc.)
2. Reach out for mentors among peers and faculty
3. Get to know advisors/professors to learn and to get letters of recommendation
4. Go to office hours and seminars, challenge yourself to ask questions
5. Seek professional development opportunities like conferences
6. Participate and take on leadership roles in student groups
7. Build you math and/or computer skills
8. Work on communication skills with community outreach and presentations (like PAW
9. Pals through Physics and Astronomy for Women+ —All are welcome!)
10. Be proactive about your mental health, and seek accommodations if needed

Things to look for/ask about:

Advisors:

Make sure there are at least 2-3 people you would like working with at each school you apply to. Reach out to discuss what type of work you would do with them and connect with their grad students to learn about the environment and professional development opportunities. Make sure the professors are taking new students and not retiring soon. Open a dialogue with potential advisors, and mention that in your applications. If a professor tells the admissions committee they'd work with you, that can help a lot. In person discussions are most effective, then video chatting, then phone calls, then email. Try to schedule a visit or video call with prospective advisors to talk about their work. People love to talk about their work. Responsiveness to your emails can also be a gauge of how welcoming/helpful professors are, but don't take it personally if they are too overwhelmed to respond. Send a follow-up email or two at 1-2 week intervals. How well funded are an advisor's students? Have to TA a lot? Summer support? What are a potential advisor's past students doing now?

Climate:

1. Is there an inclusivity webpage? What kind of info/resources does it have?
2. Is the GRE required? (good sign if not)
3. Are there underrepresented student groups/clubs?
4. Are there department/university climate committees?
5. Is there a grad student union? How does the university respond to student organizing?
6. Is there a faculty or peer mentoring program?
7. What percentage of incoming grad students actually complete the PhD? (Some schools intentionally admit more than they intend to graduate to use them for TA labor and force them out with a masters degree.) What is the average time to graduate?
8. How did the university/department respond to covid? Did they prioritize money or safety?
9. What recent changes have been made in response to activism?

10. What is the representation like among professors, postdocs, grads, undergrads, etc?

Opportunity:

1. What professional development opportunities does the department support?
2. Conference funding?
3. Networking events?
4. Giving presentations?
5. If you might change research areas, is the program strong in many areas?

Money
Fees:
Many places accept unofficial transcripts/scores, at least until you are admitted. You can order an official transcript/test scores to yourself and scan it instead of sending one to every school. Many places accept this. If you have been on financial aid or can demonstrate financial need, you can apply for fee waivers for sending test scores like the GRE. You can also often get fee waivers for ordering transcripts. You need to arrange these in advance. Many schools offer application fee waivers upon request. Even if a program doesn't list application fee waivers available, you can often get one by reaching out to the people in charge of applications. If that fails, you could ask them to review your application unofficially and then submit the application & fee only if they accept you.

Funding:
Apply for as many fellowships and scholarships as you can. Sometimes they aren't well advertised, but most schools have internal fellowships/scholarships you can apply for. You can ask the program directors about opportunities if the school doesn't have a website for it. If you get a fellowship like NSF GRFP that isn't attached to a school, it makes you a more appealing applicant because you come with funding. Even if you haven't gotten results from fellowship apps, mention what you applied for in your grad apps.

Applications:

1. Request letters of recommendation in advance (two week minimum in general, a month or two is better), and send reminders as the deadlines approach. Sharing

a spreadsheet of application materials and deadlines is useful. Providing your transcript, CV, and/or application essay(s) can be a useful resource for letter-writers.

2. Always have at least one person edit your statement/application materials (family, friend, or university writing support services)

3. Use active verbs that keep the focus on your initiative, highlighting your perseverance

4. Mention a few specifics about the school you are applying for (advisors, research, university programs, etc.)

5. Connect with potential advisors in advance.

F.2.3.3 Tips for professional development

This is a checklist to go over yourself or with advisors/mentees to encourage seeking professional development opportunities. You can ask about these opportunities or research them yourself to see what interests you.

Skills to learn and develop, according to your interests:
Computer skills, particularly programming, especially Python
Math (different fields depending on your research/career interests)
Mathematica
LaTeX

Categories of opportunities to consider/research:

1. Conferences

2. Colloquia

3. Seminars

4. Workshops

5. Panel discussions

6. Meetings/meals with speakers

7. Professional Organization Memberships (like the American Physical Society)
8. University Groups/Clubs (like Physics and Astronomy for Women+)
9. Community Organizations
10. Awards (Put yourself out there! You're awesome. You won't get what you don't ask for.)
11. Funding Sources
12. Employment/jobs
13. Other/related research
14. Readings (journals, papers, newsletters, etc.)
15. Publications
16. Initiatives (departmental and beyond)
17. Committees (departmental and beyond)

Forms of engagement:

1. Attending (remotely or in person)
2. Asking questions
3. Networking
4. Submitting posters
5. Presenting slides
6. Nominating and/or inviting speakers
7. Leadership roles
8. Teaching
9. Outreach
10. Advocacy

11. Collaboration

Logistics: Time
Subject/content of presentations
Transportation
Funding
Relevance/usefulness

Opportunities:
Some are specific to field/institution, relevant for my students, or examples for others.

Professional Organizations:

1. American Physical Society (APS) (Free membership for Student Ambassadors)
2. American Astronomical Society (AAS)
3. Women+ International in Theoretical Physics (WIThPhys)
4. Basic Research Community for Physics (BRCP)
5. Women in Quantum

Student Groups:

1. Physics and Astronomy for Women+ (PAW+)
2. Society of Physics Students (SPS)
3. Women+ in Astronomy (W+iA)
4. Towards a More Inclusive Astronomy (TaMIA)
5. More: orgcentral
6. Graduate Women in Science (GWiS)

Conferences and Symposiums:

1. Graduate Women in Science (GWiS) Empower (Open to undergrads and other genders)

2. APS Conference for Undergraduate Women+ in Physics (CUWiP) (Also open to other genders, and lots of ways to get involved/volunteer too!)
3. APS General
4. AAS Meeting
5. LOOPS conference on loop quantum gravity and related work (every two years)
6. Departmental, College, or University Research Symposiums (present posters/talks)

Workshops/summer schools (in person and/or virtual):

1. Summer School on Quantum Gravity (every two years before LOOPS, varying hosts)
2. Indian Association for GR Gravitation School on Gravitation and Cosmology
3. Informational Architecture of Spacetime
4. Sejny Summer Institute on the Foundations of Physics
5. Lindau Nobel Laureate Meeting
6. Perimeter Institute

Colloquia/seminars (live and/or recorded, in person and/or virtual):

1. Department Colloquia
2. GAPP Seminars
3. PUG Seminars
4. Astro Colloquia/Seminars
5. Quantum Information Structure of Spacetime (QISS)
6. Quantum Gravity Across Approaches (QGAA)

Outreach opportunities:

1. PAW Pals
2. MRSEC

3. Envision
4. Haunted University
5. ArtsFest Kids' Day
6. Reach out to science communicators you admire online
7. PAW+ posters
8. Science comics (put up online or submit to journals/campus literary outlets)

Mentorship (sign up or step up to be a mentor and/or mentee):
Research group
Student groups (SPS should launch undergrad mentorship program soon)
Organizations like APS

Presentation venues:

1. Outreach events like AstroFest and AstroNight
2. Local schools (often science clubs looking for presenters; reach out!)
3. Clubs on campus or at other Universities
4. Astronomy on Tap
5. Nerd Nite
6. Central Pennsylvania Observers

Awards/funding:

1. Student org leadership awards
2. PAW+ Travel Award
3. NASA Space Grant
4. Philanthropic Education Organization (PEO) International
5. Chateaubriand Fellowship
6. Penn State listings for Scholarships/Fellowships:

7. APS Women in Physics Group Grant

Readings/journals:

1. Notice which papers are cited the most in the ones you are working on and read those.
2. Look for papers by the same authors as ones you find interesting.
3. Read more papers by your advisor or potential advisors, starting with the most recent.
4. Set ArXiv and google alerts for key words you're interested in to be notified when new articles are posted
5. Universities and organizations like APS typically provide access to many publications. Talk to your mentors, advisors, professors, and librarians about the options. (I recommend Physical Review D, but it's good to keep tabs on general publications like Nature too.)

Trainings (google/ask librarians):
Workshops on coding, public speaking, scientific writing, etc.
Workshops on teaching, outreach, science communication, etc.
Workshops on mentorship, allyship, negotiating, collaboration, etc.

Initiatives:
APS IDEA Team/Program
Quantum Information Structure of Spacetime (QISS)

Committees:
Departmental Climate, Community and Diversity Committee
Eberly College of Science Climate Committee, Sub Committees
Reimagine Committee
Admissions Committee
And more! Talk to your mentors, advisors, professors, and/or department head.

Thinking ahead to other research:
Consider what topics you're most interested in. (It's ok if it's not what we work on.)
What do you like/dislike about your current work? Coding? Calculations? Predictions?...

What kind of people/groups do you want to work with? 1-on-1? Big collaborations?... Where do you want to work/live? Institution size, climate, weather, country... Network with your mentors, advisors, professors, and peers with these goals in mind.

F.2.3.4 Advice for writing a CV

Career Services at has workshops and resources on this, and the Writing Center is available for proofreading/editing. *ask for an example CV and Resume *ask for an example poster

Content:
A resume is typically a page and is geared towards industry jobs. A CV is a longer academic resume with more emphasis on research, publications, presentations, etc. Very similar when you are first starting, but CV's will grow long. You can rearrange/reformat the content depending on your audience. Ask more experienced peers/mentors for examples and have your advisor and/or Career Services representatives proofread and edit your draft.

As an early-career scientist, your CV will be sparse, so include everything from high school and this year, phrasing it all to emphasize your skills and experience relevant to the jobs you want. Include things that are current, like your status as a Penn State physics major (or other major).

Don't overlook or dismiss any of your achievements or experience. Voted "Best Team Player" by a team/club? List it. Wrote a blog? You can present it as "writing exercises." Play Dungeons and Dragons? → "Co-founded community group where I met regularly with peers for team-building and problem-solving exercises." You can list languages you are learning; you can specify your level of proficiency if you like.

Think about how you want to be perceived. Your strengths that you want to showcase. Think about how you have spent your time, and what you gained from it. What are the transferable skills? Leadership? Creative thinking? Working independently or with a team?...

Choose a format/template that includes the date and location of any experiences you list. Keep this up-to-date, so you don't have to track down random calendar events or emails to figure out when you did things. Make sure your name and contact info are at the top.

When describing activities, be concise; they don't have to be complete sentences. Start with an action word emphasizing your involvement/contribution and include info about what skills you used (computer programs, lab equipment, etc.), and who you worked with (this is the place to name-drop).

Formatting/presentation:
You can use Word or Google Docs if you like, but I recommend getting familiar with the markup language LaTeX, which is used for most scientific publications (and is a skill you can list!). LaTeX is a markup language, so it looks a bit like code, but don't be intimidated. There's just a little syntax to learn and you can google anything you want to know how to do. I like using the online LaTeX editor Overleaf. They have lots of templates you can start from:
https://www.overleaf.com/gallery/tagged/cv

Put your name and contact info at the top.
You can also create a "tag line" with a few words that describe you.
Put the most important/relevant sections first (usually education).
Make hyperlinks to any relevant websites/publications.

If you don't have much in a category, combine it with something else, so you have longer lists. For example, outreach and volunteer work can be combined, etc. Format to make the space look full. You can include a photo if you want to.

List things from most recent to least recent (with some wiggle room to put the most relevant/impressive information at the top)

Category ideas for CV sections (need at least some form of education, experience, and skills):

1. Education (Official degrees and diplomas, but can include additional summer , trainings, and workshops, like the "Professional Development Workshop" I did over zoom covering this CV advice for you)

2. Research Experience

3. Employment/Work Experience (Any official role or something you got paid for: tutoring, babysitting, selling jewelry, being a Learning Assistant for this class etc.)
4. Professional Organization Memberships (like the Society of Physics Students, American Physical Society, American Astronomical Society, Physics and Astronomy for Women+... Most memberships aren't citizenship restricted, so don't let that stop you from joining.)
5. Publications (School paper? Science blog? Grant applications? You can include stuff line that until you have scientific papers to include.)
6. Awards/Honors (any time you were singled-out for recognition; make up a name for it if there wasn't an official one)
7. Presentations/Invited Talks (Conference posters, invited to present for a club, outreach talks, etc. Later this will be only talks at conferences or colloquia.)
8. Outreach and/or Volunteer Work (PAW Pals, MRSEC, judging local science fairs, any community service. . .)
9. Computer Skills (operating systems, coding languages, useful programs, electronics. . .)
10. Certifications (like metal shop, wood shop, first aid, etc.)
11. Additional Skill and Interests (everything else that shows who you are, your values, your skills, your work ethic, etc. Ex: athletics, art, outdoors work, crafting, readings. . . you can even include general skills and/or personality traits. You never know what will catch someone's attention and make your application stand out.)

F.3 Resources for Mental Health and Community Building

F.3.1 Connecting with community

(some info specific to Penn State) General healthcare: studentaffairs.psu.edu/health/myuhs

Connecting with peers:
Org Central is a database of student groups always happy to have new members and working hard to stay connected remotely.Physics and Astronomy for Women+ (which

welcomes all genders) has many community-building events (including Tuesday Teas in Osmond 120 at 3pm) and a growing collection of resources, particularly for underrepresented groups. The Physics Department has a similar list or resources. Stay connected, use the resources available, and reach out to the teaching team, department, and university for support.

F.3.2 Mental health resources

1. Crisis line: studentaffairs.psu.edu/counseling/crisis-intervention
2. PSU has Case Managers to help you navigate the process of getting care.
3. CAPS has resources for students in crisis, group counseling events, free therapy appointments, and more, though they can take a while to schedule for non-crisis case.
4. The Herr Clinic may be faster to access care, offered by doctoral and master's level students in the Counselor Education program: https://ed.psu.edu/epcse/cedar-clinic/cedar-clinic
5. Penn State Psychological Clinic has a longer intake process, but it is guaranteed to connect you with a long-term care provider: https://psych.la.psu.edu/psychological-clinic/services-1
6. The Office of Educational Equity has groups, such as Student Disability Resources, with specialists to help you determine what policies would help you succeed. A lot of people don't expect to qualify for accommodations, but actually do, so it's always good to ask. They also have an online calendar of events.

F.3.3 Resources for women in physics

Groups/Organizations:

1. PSU Physics and Astronomy for Women+
2. Women+ International in Theoretical Physics
3. Penn State Specific Resources
4. Center for Women Students

5. Penn State Commission for Women

6. Committee on the Status of Women in Astronomy

Blogs/Websites:
APS Women in Physics website
Women in Astronomy blog
Ohio State Women in Physics blog

ArXiv Papers:
arxiv.org/abs/0804.2026 - A Case Study of Gender Bias at the Postdoctoral Level in Physics, and its Resulting Impact on the Academic Career Advancement of Females
arxiv.org/abs/1206.4112 - Gender and Sexual Diversity Issues in Physics: The Audience Speaks
arxiv.org/abs/1403.3091 - Studying Gender in Conference Talks - data from the 223rd meeting of the American Astronomical Society

F.3.4 Resources for underrepresented physicists

Resources for underrepresented groups in physics/STEM (some are USA specific)
Organizations:

1. Women+ International in Theoretical Physics

2. National Society of Black Physicists

3. National Society of Hispanic Physicists

4. Society of Indegenous Physicists

5. Out in Physics

6. Nonbinary in STEM

7. Neurodivergent in STEM

8. Disabled in STEM

Examples of resources to educate yourself about the experiences of underrepresented groups: Look for any sources *by* members of the group whose experiences you want to learn about.

1. Black Lives Matter: https://blacklivesmatter.com/resources/
2. National Indigenous Women's Resource Center: https://www.niwrc.org/
3. LGBT+ community: https://lgbtphysicists.org/media.html
4. Neurodiversity: https://www.facebook.com/EdWileyAutismAcceptance
5. Unmasking Autism by Devon Price, PhD

Appendix G
Feedback

This is my cumulative feedback and recommendations for Penn State and the physics department, much of which would be useful to other institutions. Many of the challenges these suggestions address originate from the inherent biases in our society, the imbalanced power structures we have inherited, and the perverse incentive structures present in for-profit education. Thus, these are common topics that other institutions are trying to address as well.

G.0.1 University level suggestions

- Rebuild the community's trust in the university as an institution. Earn it by implementing shared leadership, decentralizing power away from the overpaid people who are not involved in, or even aligned with, the university's educational mission.

- Start with transparency (in hiring, committee membership, promotions, awards, budgeting, funding decisions, handling of feedback and misconduct, etc.) (There has been some progress, such as Dean Langkilde's involvement and increased community involvement in the hiring of the IGC Director and Physics Department Head.)

- Follow through with accountability for misconduct and failures of the system (such as addressing instances of bias) (Recommended reading: Complaint! By Sarah Ahmed)

- Increase funding and staffing for CAPS, Title IX, and the accommodations office

- Create and fully support a title VI office

- Increase scholarships and fellowships for underrepresented students. (The relatively new fellowship in the physics department to support incoming grad employees working on equity, diversity, and inclusion is fantastic for the students, community, and recruiting.)
- Provide funding to continue the very successful trial program in the physics department in which first-year TA's take a pedagogy class instead of teaching their first semester. This helps them adjust to grad school, become better TA's, and become better students.
- Strengthen rules surrounding hate speech in codes of conduct and regulations on student organizations by limiting what university funds can be used for. Separate the concept of free speech from the distinct concept of to whom we as a community, and as an institution, provide a platform for their speech.
- Make sure all identities and write-in options are represented when soliciting demographic information on university forms.
- Gender inclusive bathrooms should be as plentiful as any others. All should have well-stocked menstrual products available for free
- Changing tables in all bathrooms. Safe, comfortable, accessible spaces for nursing.
- Make Penn DOT invest in pedestrian-first infrastructure and traffic calming, particularly on East Beaver Ave, where the university's negligence has been responsible for many instances of injury and death from traffic disasters. We need raised crossings, narrower lanes, protected bike lanes, barriers like large planters between traffic and pedestrians. See https://sthv.org/
- Advocate for local governments to invest in public transportation/bike infrastructure, improving mobility/accessibility, particularly for underserved, low-income populations.
- Advertise thoroughly and consistently to all grad employees that our stipends are exempted income and citizens qualify for SNAP benefits (food stamps). Pay grad employees equitably; imagine the research/community building possible if we were not rent-burdened.

G.0.2 Department level suggestions

- Implement shared leadership by decentralizing power, enabling more democratic selection of leaders and committee members, and including more grad employees, staff, undergrads, and postdocs on committees.

- Be transparent about the selection processes and open about the membership of committees, keeping an up-to-date list of members, with one or two designated contact people per committee. Be explicit about the overarching responsibilities of each committee and their short and long term goals.

- Make information accessible to grad employees that: (1) grad employees can join Physics Department committees. (2) How to do that. (Email grad employees each year asking for committee participation. Add a lecture to the first year colloquium course about committee participation, perhaps with members from the committees to talk about what they do. Add a statement to the committee websites and the grad employee canvas page welcoming grad employee participation and explaining the process.)

- Find funding to continue the experimental first-year pedagogy course instead of immediate teaching duties. This is a wonderful step and will help a lot of people!

- Alter the grad first year seminar to be an extended orientation, teaching students how to navigate academia and connect with advisors.

- Modify/remove the qualifying exam. Let students choose a few from a selection of problems in each subject. Allow flexibility such as letting students opt into a verbal assessment instead, or a presentation project. There are many options to work around the grad school imposed requirements. The UNC Chapel Hill physics department made the final exams of core grad courses count as the qualifying exam, so there were no extra tests. The astro department has the qual split over two tests, a year apart, one on the material of the core courses the students have taken in grad school, and one as an exercise in reading comprehension and deeper investigation into the sources and methods of a published paper (both of these exams are untimed, take-home over the course of three days, open-book, open-internet, closed-people).

- Value mentorship more formally in the hiring and promotion process for faculty. Have guidelines for mentors and mentees to meet at least once per semester, more often in the first year. A good portion of the meetings should be one-on-one, but meeting in groups with other mentees or professors for discussion and professional development is good too. Good progress is being made on strengthening the mentorship program; keep it up. We need oversight, transparency, follow-through, and accountability.

- Keep going with the new hiring practices (community nominations for search/hiring committees, anonymous feedback opportunities from students after potential hire presentations, etc.) Make sure all members of the community are participating in the nomination and feedback processes (grad employees, staff, etc.). Screen all potential hires to make sure they have a growth mindset.

- Train all advisors and mentors on strategies for communicating with students of different backgrounds, whether it is navigating a language/cultural barrier, or ways to make autistic students more comfortable in the conversation. (example resource: https://cimerproject.org/training/) Provide questions for professors and students to prompt discussion and open communication during meetings. I have written example questions myself, but this should be supplemented with professional training and advice.

- Assign a trained, volunteer peer mentor to each incoming grad employee and each undergrad (including satellite campuses!) when they declare their major, in addition to a faculty mentor (opt-out not opt-in). Good steps are being taken on this! Include undergrads at satellite campuses to help recruit them into the major.

- Open lines of communication between student groups in the department and undergrads at satellite campuses. Normalize professors advertising student groups like PAW+ at the beginning of each semester.

- Provide more flexibility in course scheduling, especially for first year grad employees (reducing credit minimum or offering more research credits to focus on classes; There could be ways around the credit minimum that are within the department's power, like creating a credit-filler course to help students concentrate on their other classes, allowing students to sign up for research credits without much expectation of research, or perhaps increasing the number of credits for the colloquium course?) Reducing the required courses was an excellent step!

- In consultation with a psychologist, create a mandatory in-person workshop series for faculty, staff, grad employees, and undergraduates to discuss A: that systemic bias exists, B: the common manifestations such as microaggressions, and C: how to have a constructive conversation about these issues when they arise. Introduce people to the different kinds of bias: sexism, racism, ableism, heteronormativity, xenophobia, islamophobia, etc. Bring in professionals to explain the concepts and importance of neurodiversity, the gender spectrum, etc. Example presentation I created for my class: Inclusivity Presentation I would like to see something similar implemented in all classes, especially intro courses and first-year-seminars for grad and undergrad employees.

- Create paid positions for grad employees and undergrads to function as equity, diversity, and inclusion liaisons who can help people report misconduct and access support resources within the university, these individuals can also propose their own initiatives, like posting educational information and support resources in bathroom stalls, or providing empowering posters for common spaces, etc. EDI TA Proposal

- Hire more professors for their teaching skills, and train all professors to use good teaching practices like active learning, flipped classrooms, and growth mindset. Provide more classrooms designed for this group work, like 207 Osmond.

- Train faculty, staff, and students to default to gender-neutral pronouns (they/them) when referring to students, especially if a student has not stated their gender identity. Use a mix of pronouns in examples and homework problems, so the assumption is not always that students and scientists are male. Make sure nonbinary options are available on university forms as well.

- Make sure that gender neutral bathrooms are as equally accessible as each of the gendered bathrooms. Ex: make one bathroom on each floor gender-inclusive, and alternate m/f for the other one on each floor.

- Making pads/tampons available in all bathrooms is an excellent step in the right direction! Please persist in replacing the supply when vandalized and take appropriate action to address the hate crime of some people putting the basket of products from the men's rooms in the trash.

- Have the department engage the community more on occasions like Autism Acceptance Week, Pride, and other such occasions that provide an opportunity for dialogue about inclusivity. (the EDI TA roles can help with this)

- Create additional orientation, geared towards first-generation and international grad employees that outlines the terminology and bureaucracy of the department. This benefits everyone, especially individuals with additional challenges like social anxiety.

- Create a matching system like online dating or an in-person speed-dating forum to help students find advisers with matching research interests. Keep in mind that some students have social anxiety or other concerns which make reaching out to professors on their own much more of an obstacle.

- Require students to sit-in on a few meetings with different research groups in the department to find a good fit. —this could be incorporated into the colloquium course. See MIT for a good example. (Note, advisors should receive departmental guidance to build community within their groups and have effective group meetings.)

- Host free food events to boost attendance and invite CAPS to come present about mental health best practices and the resources available inside and outside the university to support students. Mental health is a huge problem in academia, especially among grad employees, and many people go undiagnosed. Relationship violence and abuse is also a significant factor in cases where underrepresented students left the department or delayed their degrees. Teaching about red flags, green flags, and how to look out for your friends and peers could go a long way.

- Train instructors to appropriately discuss the syllabus section on accommodations, explain what accommodations are and how to get them, without implying any stigma. Normalize discussion of mental health and accommodations.

- Listen to and act on the concerns expressed by the Coalition for a Just University and the Coalition of Graduate Employees about topics ranging from Covid safety and to social justice.

- Hire LA's to assist in recitations and labs for courses like 211 and 212 to reduce demand on grad TA's there.

- The Wednesday Community Meetings are a great forum for general community building and discussing and addressing challenges in the department! Incentivse more participation from all stakeholder groups.

- Reserve a few community meetings annually for each of the committees to present who they are, what they do, what their goals are, and solicit ideas and feedback from the department.

- Schedule a community meeting each year with departmental student groups to share their work and recruit participation.

- Use a community meeting each year for a town hall for each of the department groups: staff, faculty, postdocs, grad employees, undergrads, underrepresented groups, etc.

- Schedule meetings to accommodate families (no committee meetings/colloquiums after elementary school days end, etc.)

- Maintain hybrid event structures to accommodate health challenges, families, etc.

- Provide more detailed information on the "hidden curriculum," university resources, and community beyond the university to help students adjust upon arrival and be less isolated.

G.0.3 Additional Feedback

G.0.3.1 Useful precedents from UMass Lowell

- ACTIVE GRADUATE ADVISORY COMMITTEES supporting students and advocating to the department

- Supportive departmental faculty and leadership present and involved

- More people means a more distributed workload

- GRAD UNION provides organizational structure for mutual aid and advocacy

- DEI grad representative on committees

- Peer mentorship (currently opt-in) well advertized and run jointly by faculty, and grad employees

- Grad representatives are PAID FOR THEIR LABOR

- People serve on the committee 1-2yrs (self-nominate in January)

- Grad reps involved in admissions
- Grad reps have EQUAL VOTING POWER
- Department head established these committees
- Permanent lecturer position serving as a DEDICATED STUDENT ADVOCATE and grad program coordinator in charge of the research credit course

G.0.4 Recruitment

We want to expand our existing efforts in outreach and recruitment.

Children: Conducting outreach and providing role models for children of all ages to engage in physics and see themselves as physicists: PAW+, ENVISION, Young Women in STEM, AstroNight, AstroFest, Haunted University, Education U, etc.

Grad and undergrad applicants: Eric's conference (specific name?) recruiting as an example of success (do we have data on increased applications from students of color?). List of other opportunities to build on that, reaching out to organizations like Women in Science and Engineering, Historically Black Colleges, Women's Colleges, etc.

G.0.5 Climate

We want to continue making institutional changes to improve climate, and we are advocating for improvements on multiple fronts: physical, environmental, bureaucratic, pedagogical, and cultural.

Changes to infrastructure:

- new building
- more gender inclusive bathrooms
- more classrooms configured for active learning and group work
- greater accessibility like ramps
- automatic doors

- elevators
- wheelchair accessible tables for labs
- improving grad and undergrad lounges
- small & large posters of underrepresented scientists in all three buildings

Bureaucratic changes:

- removing GRE requirements
- modifying/removing the qualifying exam (There are some options to work around the grad school imposed requirements. The UNC Chapel Hill physics department made the final exams of core grad courses count as the qualifying exam, so there were no extra tests. That idea sounds great to me, but there are other options.)
- adding more accountability to mentoring (I would like mentorship to be valued more formally in the hiring and promotion process for faculty. I also think it would be useful to have guidelines for mentors and mentees to meet at least once per semester, perhaps more often in the first year. A good portion of the meetings should be one-on-one, but meeting in groups with other mentees or professors for discussion and professional development is good too.
- I would also support a training for all advisors and mentors on strategies for communicating with students of different backgrounds, whether it is navigating a language/cultural barrier, or ways to make autistic students more comfortable in the conversation)
- adding a peer mentor in addition to a faculty mentor
- providing more flexibility in course scheduling especially for first year grad employees (reducing credit minimum or offering more research credits to focus on classes; There could be ways around the credit minimum that are within the department's power, like creating a credit-filler course to help students concentrate on their other classes, allowing students to sign up for research credits without much expectation of research, or perhaps increasing the number of credits for the colloquium course?
- improving the TA experience (scheduling, workload, class preferences. . .)

Pedagogy changes:

- active learning
- Flipped classrooms
- learning assistants
- group work
- training teachers and students "growth mindset."

Cultural changes:

- education and communication (awareness and allyship)
- Generating dialogue about systemic bias (meal discussions and guest speakers)
- allyship workshops on implicit bias for all members of the department community
- class presentations on subconscious bias for students

Retention: Having recruited applicants and worked to create a welcoming and inclusive climate, there are additional steps we can take to actively support underrepresented students and improve attrition rates.

- Departmental and student organization mentoring matching students with peers who share their demographic experiences.
- Check-ins with mentors, advisors, and advisees to make sure students are on track and teams have good communication about goals and professional development opportunities.
- Funding professional development opportunities like students attending conferences and workshops about their research as well as about supporting underrepresented groups.

Mentoring: Mentoring is a key part of improving climate and retention of vulnerable students, and we have plans to build on our existing programs.

- Check-ins and accountability for regular, clear communication
- Responsibility to recommend professional development opportunities
- Peer mentors as well as faculty mentos (opt-out not opt-in)

- Group and individual mentoring meetings
- Providing mentors with as much as possible in common with a student's experience

Advising:
I think we should have the grad-student activity report and advisor evaluation required, with someone in charge of looking them over and conducting an annual check-in, like an Ombudsperson, to make sure there is good communication about goals, performance, and professional development. It might help to update and/or have an in-person review of the responsibilities of mentors and mentees.

For Mentors, Advisors, and Instructors:
Landmark College has a wonderful teacher training program for these topics that has been utilized successfully by other universities. Send a few faculty and have them present what they have learned in faculty meetings, colloquiums, and TA trainings. Here are some useful links recommended to me by advocates in other universities: https://www.landmark.edu/news/landmark-college-professors-present-world-of-learners-wheel-poster https://www.landmark.edu/research-training/blog/supporting-college-students-with-adhd-an-introduction https://anautismobserver.wordpress.com/ https://www.youtube.com/watch?v=GZp459zssr0

Community: Creating space for underrepresented groups and building the department community as a whole.
Student groups have expanded inclusivity efforts, and we would like to support more groups creating community, camaraderie, and mutual support for traditionally underrepresented groups. We also want to continue building community between faculty, staff, students, and postdocs by improving communication paths and continuing to organize social events like the holiday potluck.

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Education

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  Penn State University, State College, PA
  2017 - 2024
  Collaboration Abroad: University of Paris-Saclay, 2022
- **Loop Quantum Gravity (LQG) Summer School**
  Online
  Summer 2021
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- **Summer School on Quantum Gravity**
  Bard College, Annandale-On-Hudson, NY
  Summer 2019
- **Astrophysics Honors ScB**
  Brown University, Providence, RI
  2012 - 2016
  Honors: Smiley Award for Excellent Contribution to the Astronomy Program
  Study Abroad: University of Edinburgh, Spring 2015

Employment

- **Assistant Teaching Professor of Physics**
  Worcester Polytechnic Institute, Worcester, MA
  Starting Fall 2024
  Teaching physics, astronomy, and astrophysics courses
- **Physics Teaching and Research Assistant**
  Penn State University, State College, PA
  2017 - Present
  Led labs, recitations, and lectures for physics courses, including designing new activities
- **Ladd Observatory Assistant**
  Brown University, Providence, RI
  2014 - 2016
  Helped setup telescopes, gave constellation tours, and discussed astronomy with the public

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