HIGH PRECISION SURFACE CONTROL OF
FLEXIBLE SPACE REFLECTORS

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by
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ABSTRACT

Flexible reflectors are used for a number of applications in space, including resource monitoring, weather analysis, hazard assessment, reconnaissance, and imaging. With the rapid advances in deployable membrane and mesh antenna technologies, the feasibility of developing large, lightweight reflectors has greatly improved, though high-precision surface control is needed in order to achieve the required surface accuracy. The purpose of this research is to advance the state of the art by implementing high-precision surface control on a flexible reflector using PVDF actuators, as well as introducing methods to overcome real world problems that can develop when implementing this technology.

To facilitate the design and analysis of a reflector/actuator system, a theoretical model is derived. The reflector is modeled as a thin, shallow, spherical cap, and the system is solved using a Ritz method with Fourier-Bessel series expansion. PVDF surface actuators are modeled using the same method and are assumed to be perfectly bonded to the reflector surface. The model can be quickly modified in order to accommodate different actuator locations. Surface errors comparable to those the reflector would experience in space are modeled, including “W-error” caused by the inflation of the reflector and the error experienced from temperature changes as the reflector circles the earth. Experimental results show that while the analytical reflector model is generally correct, due to idiosyncrasies in the reflector it should not be used for online control. Therefore, a methodology is proposed for online system identification for use with an online control law.
Using the PVDF actuators, surface control is executed using a least squares control law, where photogrammetry is used to determine the out-of-plane displacement at designated points on the surface. Least squares can be used because of the quasi-static nature of the surface error. While there is a time dependence in the temperature profile, the rate of change is very low, so that at any moment in time the surface error can be assumed to be static. Using the least squares control, it is shown that on a theoretical 35 meter space reflector with the surface fully covered by PVDF actuators, the error resulting from a 40 K temperature change on the entire surface of the reflector can be controlled to within the desired tolerances.

When the surface cannot be fully covered with actuators, optimal placement of the available actuators must be considered. First, when focusing solely on the constraint on the number of possible actuators, optimal placement of the actuators is found using a genetic algorithm. In order to facilitate the convergence of the algorithm, a symmetric constraint is applied and is shown to reduce the required time to find the optimum, while having a negligible impact on the final result. Next, a constraint on how many independent power supplies are available is added. A new method to determine the optimal grouping of actuators to power supplies is derived, called the En Masse Elimination (EME) method. This method can determine the global optimal solution without having to exhaustively search every possible grouping combination. A number of improvements to the EME method are given which increase the speed of the algorithm as well as enable the EME algorithm to be used online during dynamically changing error conditions.
Finally, the EME method is experimentally validated using a single pinned-pinned beam. Using a beam rather than a reflector reduces the complexity of the problem while still showing the functionality of the EME algorithm. Experimental data show that the EME algorithm is able to quickly and accurately find the global optimal actuator grouping.
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Chapter 1

Introduction

In recent years, there has been a growing demand for improvement in space reflectors. Space reflectors are used for a number of applications including resource monitoring, weather analysis, hazard assessment, reconnaissance, and imaging. While geostationary-orbiting (GEO) satellites have been in operation for years, their science payloads have been limited to passive optical imagers for cloud-top sensing. Currently, all space-borne active and passive microwave and millimeter-wave remote sensing instruments have been operating in low-earth orbits (LEOs) in order to provide acceptable spatial resolutions and measurement sensitivity.

Active and passive microwave remote sensing from higher orbits (MEO, GEO, L1/L2, etc) are desirable due to these sensors’ unique ability to penetrate through clouds to acquire vertical profile measurements of geophysical parameters. These sensors also have orbital geometries that offer large spatial coverage and more frequent observations. To take full advantage of the benefits of these microwave instruments, the acquired measurements must have similar spatial resolution and sensitivity even when operating at orbit altitudes one to two orders of magnitude higher than those of LEO. These requirements point to the need for significantly larger antennas with very small surface errors. The large aperture increases the signal-to-noise ratio and signal resolution, while the high-precision surface will increase the sensitivity and spatial resolution.
With the rapid advances in deployable membrane and mesh antenna technologies, the feasibility of developing large, lightweight antennas has greatly improved. There are many benefits of using deployable membranes. The launch mass and stowage size are greatly reduced allowing for a larger aperture, which makes higher orbit possible. However, a major shortcoming of deployable membrane reflectors is the difficulty in maintaining a sufficiently tight surface accuracy.

The major problem is that in order to achieve high gain and low side lobes, large deployable reflectors (5-35 meter diameter) operating at 14–35 GHz must keep a very tight surface tolerance: 0.54 mm RMS (0.021 inches) at 14 GHz and 0.21 mm RMS (0.008 inches) at 35 GHz. A surface tolerance of 0.21 mm is equivalent to the thickness of three sheets of paper. Studies have shown ([Im et al., 2004]; [Im and Durden, 2005]) that without any mechanisms to correct for mechanical and thermal distortion, a deployed reflector would nominally achieve a surface tolerance of 4.3 mm RMS which is 20 times the required surface accuracy.

In order to achieve the needed surface accuracy, high-precision surface control is needed. The purpose of this research is to show the feasibility of using a large deployable reflector for high orbit sensing, as well as developing methods to overcome real world problems that can be encountered when implementing this technology.

**Background and Literature Review**

Flexible membranes have been used extensively in space for many years. Ruggiero and Inman [2006] compiled a thorough review of Gossamer structures in space.
One of their findings is that gossamer reflectors have mainly been utilized for applications where low imaging acuity is sufficient because of the difficulty in maintaining the proper shape. The main drawback to their use for high precision applications is the difficulty in controlling and maintaining their shape to the desired spherical or parabolic profile [Bao et. al., 2005].

There are two main sources of surface error that occur in inflatable reflectors. First, when a perfectly fabricated circular membrane is inflated, the structure forms a Hencky surface, which deviates from the desired spherical or parabolic profiles [Hencky, 1915], [Campbell, 1956], and [Jenkins and Marker, 1998]. This surface error is commonly known as “W-error”, for its shape.

In addition to W-error, another significant source of in-orbit figure error is due to the thermal expansion and contraction of the reflector due to solar heating and the earth’s shadowing effects throughout the orbital period. Thus, reflectors in orbit will experience both thermal gradients across the structure as well as bulk temperature shifts.

Jenkins and Faisal [2001] investigated the effect of thermal loads on precision membranes. In their model, they varied both the temperature magnitude and the spatial extent of the temperature. Preliminary experiments were conducted by heating the membrane, using infrared thermography to measure the temperature, and using a capacitance displacement sensor to measure the resulting deformation. The results were then compared to the theoretical model. Glaese et al. [2003] derived a set of equations to predict the basic structural dynamic behavior of thin-film single surface shells. Using
these equations, they were able to explain why the frequency separation between the
modes was very minimal.

There are both active and passive methods for shape control. The W-error can be
controlled passively because it does not change in orbit. Vaughn [1980] and Hart-Smith
and Crisp [1967] investigated possible ways of passively eliminating the shape error due
to the initial inflation by varying the thickness of the structure. Marker and Jenkins
[1997] demonstrated that the deviation of the inflated membrane profile may be reduced
by as much as 58% by using appropriate outward boundary displacements. However, for
thermal expansion, passive control cannot be used due to the dynamic nature of the error;
some type of active in-orbit shape control treatment is necessary.

One such active control, tried by Haftka and Adelman [1985], examined how the
application of temperature gradients to the reflector would affect changes in the surface
profile. They showed that this technique is able to reduce the RMS shape error due to a
20K on-orbit bulk temperature shift by approximately 50%.

Additionally, boundary control of thin film shells has been investigated by Linder
and Flint [2004]. In this technique, the reflector rim or outer radial locations are actuated
to adjust the overall shape. Results of experimental boundary manipulation tests were
presented and compared favorably with numerical predictions. Others [Jenkins et. al.,
1998] have also studied boundary control, and show that it is effective in correcting
higher-order disturbances, such as astigmatism and comma, associated with uneven
boundary stresses and mounting errors.
Another form of active control, similar to our current study, was attempted by Salama et al. [1994]. They performed shape control by completely covering a large (14-meter) inflatable membrane reflector with piezoelectric film. They suggested a combination of a pressure control to adjust the focal length, and piezoelectric control to correct symmetric or asymmetric local irregularities in the membrane surface arising from material imperfections or modulus variations.

Others have proposed and studied various cable actuated shape control strategies. Tabata and Natori [1995] used tip-mast cable tension adjustments on a tension-truss mesh antenna. Jenkins and Schur [2002] proposed actively controlled gore/seam cables for global control of a membrane reflector. Coleman et al. [1994] implemented control using cables excited by polyvinylidene fluoride (PVDF) end actuators. Using 104 active gore/seam cables, they were able to reduce the RMS error due to thermal loading by approximately 75%, and the inflation RMS error by approximately 95%. The cable lengths and attachment points were optimized using a genetic algorithm.

DeSmidt [2006] also utilizes gore/seam actuation, in conjunction with PVDF surface coverage to achieve greater surface control. Duvvuru et. al. [2003] utilizes the seams, but investigated the concept of PVDF actuated seams for shape refinement of Shape Memory Alloy (SMA) deployable umbrella-type reflectors.

Another area of research is focused on the optimal placement of actuators, attempted primarily because of the restrictions on the launch mass. Salama et al. [1994] used a simulated annealing algorithm to determine the optimal actuator locations and gains for an inflated parabolic antenna system. Chen et al. [1991] looked into optimal
placement of active and passive members in truss structures. In their study, they locate the active members (structural members with actuators) to dissipate the vibration energy associated with lower vibration modes. The passive members (structural members that provide energy dissipation) were located so as to reduce vibrations associated with higher order modes.

Andoh et al. [2004] investigated optimization of actuator locations by maximizing the controllability of the dominant modes of the shape error. Their optimization procedure exploited the relation among eigensolutions of the distributed parameter system, those in the modal configuration space, and those in the state space. Pattom [2006] investigated optimal placement of PVDF actuators bonded to the surface of a membrane reflector through the use of a genetic algorithm.

Each of these aforementioned studies has assumed that each actuator can be controlled individually. While ideally each actuator should be controlled by an individual power supply, at times it is not realistically feasible due to design, weight, or cost constraints. There are many instances where the only solution is to group multiple actuators together and power each group with a single power supply. For best performance, the grouping of actuators must be optimized.

Very little work in adaptive structures has attempted to optimize the actuator groupings. Rader et al. [2005] and Grewal and Tse [2000] used genetic algorithms to determine actuator groupings on a flexible fin and in an aircraft cabin, respectively. Grocott [1997] partitioned hundreds of actuators of a flexible telescope mirror using a circulant matrix theory. Jamoom et al. [1998] used a recursive algorithm based on the
Linear Quadratic Regulator (LQR) to group the actuators, and Lin [1996] used a controllability Grammian method for open loop grouping of actuators on a wing. Each of these approaches requires the user to define a stop condition, and while these approaches indeed have merit, they lack the guarantee of a global optimal solution or even the guarantee of an acceptable solution.

As has been shown in this section, a substantial amount of theoretical work has been done concerning the control of flexible materials, as well as actuator placement, with less work being done on actuator grouping. While some experimental work has been done, it has not yet been explored in-depth without making simplifications that could impact the ability to fly the technology. The overall goal of this research is to explore the feasibility of using piezoelectric actuators to realize shape error reduction in large, light-weight space reflectors given real-world constraints. A main focus will be on the optimal grouping and use of actuators given a dynamic surface error on the reflector. More specifically, we propose to advance the state of the art in the following aspects:

a) In order to accurately design the control system on a reflector, a simple yet accurate reflector model is needed. Most current finite element models are not able to interface with a control system as well as have the ability to be rapidly updated given different combinations of actuators. To assure the veracity of the conclusions, the model must compare favorably to actual experimental data.

b) Real-world constraints are added to this improved model, and methodologies are introduced to overcome these constraints. Some of
these constraints include discrepancies between the model and the actual reflector, as well as a constraint on the number of power supplies used to control the actuators.

c) Finally, special emphasis is given to the constraint on power supplies. A new method is developed in order to efficiently determine the global optimal actuator groupings, both a priori and during in-orbit operations.

**Dissertation Outline**

This dissertation consists of seven chapters, which are organized as follows.

The first chapter introduces the background information and the motivation for this research. A comprehensive review of the state of the art is presented along with the overall research objectives.

The second chapter steps through the derivation of a comprehensive membrane reflector model. The reflector as well as the actuators are modeled and compared to experimental results.

The third chapter of this dissertation focuses on the shape control applied to the reflector model. A basic least squares approach is shown, with corresponding results. A methodology is proposed for online system identification for use with the least squares approach.

The fourth chapter details work done on actuator placement. First, we look at the system when there is simply a constraint on the number of possible actuators. Second, we introduce a constraint on how many power supplies are available, and review
currently available methods. We examine a simple way to portray this constraint and show that a new method is needed in order to find the optimal grouping of actuators.

The fifth chapter introduces the En Masse Elimination (EME) algorithm, used to find the global optimal grouping of actuators. A number of improvements over the basic algorithm are given. Finally, a methodology for updating the actuator groupings online is outlined and simulation results are given.

The sixth chapter details the experimental work done to verify the EME algorithm. The experimental setup and procedure are explained, followed by the experimental results. Three cases are given to show the efficacy of the EME algorithm.

Finally, the conclusions of this research are summarized in Chapter 7, along with some possible directions for enhancing future research, including improving control authority on the reflector as well as increasing the speed of the EME algorithm.
Chapter 2

Modeling

To facilitate the design and analysis of a reflector/actuator system, a theoretical model must be established. Most current finite element models, while very accurate, are difficult to use with a control law and cannot be quickly modified for different actuator locations. It is important for the model to be able to be quickly modified for different configurations of actuator placement during the optimization of the actuator locations. The model must also compare favorably to experimental results.

Reflector Modeling

The reflector to be modeled is similar to the JPL/NIS spherical antenna reflector, as shown in Figure 2-1.
The entire reflector is composed of two circular membranes that are sealed on their edges, attached to a tensioning ring or rim structure. The reflector is internally pressurized which produces the desired curve surface. In order to simplify the model, the entire structure is not modeled, but rather only the front side of the reflector. The connection between the two faces is replaced with a simply supported boundary condition along the entire rim, while the internal inflation pressure is modeled as a pre-stress on the material.

The reflector is modeled as a thin shallow spherical cap [Soedel, 1993]. The displacements ($u$, $v$ and $w$) are defined relative to a spherical coordinate system, as shown in Figure 2-2, which coincides with the lines of principal curvature of the mid-surface.
The strain – displacement relationship at any point in the reflector is given by Eq. (2-1). The subscripts ‘1’ and ‘2’ denote the \( \hat{\epsilon}_r \) and \( \hat{\epsilon}_\theta \) directions respectively.

\[
\begin{align*}
\varepsilon_{11} &= \frac{1}{R} \frac{\partial u}{\partial \phi} + \frac{w}{R} \\
\varepsilon_{22} &= \frac{u \cot \phi}{R} + \frac{1}{R \sin \phi} \frac{\partial v}{\partial \theta} + \frac{w}{R} \\
\varepsilon_{12} &= \frac{1}{R} \frac{\partial v}{\partial \phi} + \frac{v \cot \phi}{R} + \frac{1}{R \sin \phi} \frac{\partial u}{\partial \theta}
\end{align*}
\] (2-1)

The reflector can be considered a shallow shell. The shallow shell simplifications [Mazurkiewicz and Nagorski, 1991] are reproduced in Eq. (2-2).

\[
\cos \phi = 1, \sin \phi = \frac{r}{R} \frac{1}{R} \frac{\partial}{\partial \phi} = \frac{\partial}{\partial r}
\] (2-2)
Applying these simplifications to Eq. (2-1), it is reduced to Eq. (2-3), as shown below.

\[ \varepsilon_{11} = \frac{\partial u}{\partial r} + \frac{w}{R} \]
\[ \varepsilon_{22} = \frac{u}{r} + \frac{1}{r} \frac{\partial \nu}{\partial \theta} + \frac{w}{R} \]
\[ \varepsilon_{12} = \frac{\partial \nu}{\partial r} + \frac{\nu}{r} + \frac{1}{r} \frac{\partial u}{\partial \theta} \]  

(2-3)

The constitutive equation used for the reflector is given by Eq. (2-4)

\[ \{\sigma\} = [c]\{\varepsilon\} - \{\sigma_T\} \]  

(2-4)

where

\[ \{\sigma\} = \begin{bmatrix} \sigma_{11} & \sigma_{22} & \sigma_{12} \end{bmatrix}^T \]
\[ \{\varepsilon\} = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{22} & \varepsilon_{12} \end{bmatrix}^T \]
\[ \{\sigma_T\} = \begin{bmatrix} \sigma_{1T} & \sigma_{22T} & 0 \end{bmatrix}^T \]  

(2-5)

\[ \sigma_{11T} = \sigma_{22T} = \frac{E_{Youngs}\alpha_{CTE}T}{(1 - \nu)} \]

and

\[ [c] = \frac{E_{Youngs}}{(1 - \nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & 1 - \nu \end{bmatrix} \]  

(2-6)

Here, \( T \) is the temperature distribution, assumed to be known, measured with respect to the nominal temperature. The reflector strain energy, \( U_a \), is given by Eq. (2-7). This accounts for the elastic strain energy and the energy due to the thermal expansion.

\[ U_a = \iiiint_V \frac{1}{2} \{\sigma\}^T \{\varepsilon\} - \{\varepsilon_T\} dV \]  

(2-7)
The strain growth due to the thermal expansion is represented in Eq. (2-8),

\[ \{\varepsilon_T\} = \{\varepsilon_{11T} \quad \varepsilon_{22T} \quad 0\}^T \]  \hspace{1cm} (2-8)

where

\[ \varepsilon_{11T} = \varepsilon_{22T} = \alpha_{CTE} T. \]  \hspace{1cm} (2-9)

Up to this point, we have assumed that the reflector has zero stress level at the equilibrium position. If we consider an isotropic pre-stress at this equilibrium position, which is the case when the reflector is pressurized, the additional strain energy term must be accounted for, as given by Eq. (2-10).

\[ U_b = \iiint_V \frac{1}{2} \sigma_{pre-stress} \left[ \left( \frac{\partial w}{\partial r} \right)^2 + \left( \frac{1}{r} \frac{\partial w}{\partial \theta} \right)^2 \right] dV \]  \hspace{1cm} (2-10)

Hence, the total reflector strain energy, \( U_{ref} \), is given by Eq. (2-11).

\[ U_{ref} = U_a + U_b \]

\[ = \iiint_V \frac{1}{2} \{\sigma\}^T (\{\varepsilon\} - \{\varepsilon_T\}) dV \]  \hspace{1cm} (2-11)

\[ + \iiint_V \frac{1}{2} \sigma_{pre-stress} \left[ \left( \frac{\partial w}{\partial r} \right)^2 + \left( \frac{1}{r} \frac{\partial w}{\partial \theta} \right)^2 \right] dV \]

**Actuator Modeling**

Actuator patches can be attached to the reflector. Each patch is assumed to be perfectly bonded to the surface. The strain-displacement relationship at a point in the actuator is given by Eq. (2-12).
\[
\varepsilon_{11p} = \frac{\partial u_p}{\partial r} + \frac{w_p}{R}
\]
\[
\varepsilon_{22p} = \frac{u_p}{r} + \frac{1}{r} \frac{\partial v_p}{\partial \theta} + \frac{w_p}{R}
\]
\[
\varepsilon_{12p} = \frac{\partial v_p}{\partial r} + \frac{v_p}{r} + \frac{1}{r} \frac{\partial u_p}{\partial \theta}
\]

The constitutive equation describing the ‘converse’ piezoelectric effect (i.e. using the piezo material as an actuator), is given by Eq. (2-13)
\[
\{ \sigma_p \} = [c_p] \{ \varepsilon_p \} - [e]^T \{ E \} \tag{2-13}
\]

where
\[
\{ E \} = \begin{bmatrix} 0 & 0 & E_3 \end{bmatrix}^T. \tag{2-14}
\]

The elastic constant matrix is \([c_p]\) and the piezoelectric constant matrix is given by Eq. (2-15),
\[
[e] = [d][c_p] \tag{2-15}
\]

while the electric strain constant is given by Eq. (2-16).
\[
[d] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ d_{31} & d_{32} & 0 \end{bmatrix} \tag{2-16}
\]

The total energy for the \(i^{th}\) patch is given by Eq. (2-17),
\[
U_{\text{patch}_i} = \iiint_{V_i} \frac{1}{2} \{ \sigma_p \}^T (\{ \varepsilon_p \} - \{ \varepsilon_E \}) dV \tag{2-17}
\]

where
\[
\{ \varepsilon_E \} = \begin{bmatrix} \varepsilon_{11E} & \varepsilon_{22E} & 0 \end{bmatrix}^T \tag{2-18}
\]
represents the strain ‘growth’ due to the electric field $E_3$, with

$$\varepsilon_{11E} = d_{31}E_3$$

$$\varepsilon_{22E} = d_{32}E_3$$

and $V_i$ represents the volume of each individual actuator patch.

**Love Simplification**

Because the reflector is a very thin shell, the displacements $u$ and $v$ of the reflector can be assumed to be varying linearly with the shell thickness, whereas the displacement $w$ can be assumed to not vary at all with the shell thickness. Thus, the Love simplification can be used.

$$u(r, \theta, \alpha_3) = u_0(r, \theta) + \alpha_3 \beta_1$$

$$v(r, \theta, \alpha_3) = v_0(r, \theta) + \alpha_3 \beta_2$$

$$w(r, \theta, \alpha_3) = w_0(r, \theta)$$

In Eq. (2-20), $u_0$, $v_0$ and $w_0$ represent the mid-surface displacements, $\beta_1$ and $\beta_2$ represent the rotation angles, and $\alpha_3$ is the dimension normal to the mid-surface along the thickness of the reflector. Transverse shear strains, $\varepsilon_{13}$ and $\varepsilon_{23}$ are assumed to be zero as well. Setting $\varepsilon_{13} = \varepsilon_{23} = 0$ in the strain-displacement relationship, we get expressions for $\beta_1$ and $\beta_2$.

$$\beta_1 = \frac{u_0}{R} - \frac{\partial w_0}{\partial r}$$

$$\beta_2 = \frac{v_0}{R} - \frac{1}{r} \frac{\partial w_0}{\partial \theta}$$
By substituting Eq. (2-20) and Eq. (2-21) into the strain-displacement relationship, we can express the net strains in terms of a membrane strain \( \{ \varepsilon_o \} \) and a bending strain \( \{ \kappa \} \), as shown in Eq. (2-22),

\[
\{ \varepsilon \} = \{ \varepsilon_o \} + \alpha_3 \{ \kappa \}
\]

where

\[
\{ \varepsilon_o \} = \begin{bmatrix} \varepsilon_{11o} & \varepsilon_{22o} & \varepsilon_{12o} \end{bmatrix}^T
\]

and

\[
\{ \kappa \} = \begin{bmatrix} \kappa_{11} & \kappa_{22} & \kappa_{12} \end{bmatrix}^T.
\]

Also,

\[
\varepsilon_{11o} = \frac{\partial u_o}{\partial r} + \frac{w_o}{R}
\]

\[
\varepsilon_{22o} = \frac{u_o}{r} + \frac{1}{r} \frac{\partial v_o}{\partial \theta} + \frac{w_o}{R}
\]

\[
\varepsilon_{12o} = \frac{\partial v_o}{\partial r} + \frac{v_o}{r} + \frac{1}{r} \frac{\partial u_o}{\partial \theta}
\]

and

\[
\kappa_{11} = \frac{\partial \beta_1}{\partial r}
\]

\[
\kappa_{22} = \frac{1}{r} \frac{\partial \beta_2}{\partial \theta} + \frac{\beta_1}{r}
\]

\[
\kappa_{12} = \frac{\partial \beta_2}{\partial r} - \frac{\beta_2}{r} + \frac{1}{r} \frac{\partial \beta_1}{\partial \theta}.
\]

Similarly, for the patches we have
\[ u_p(r, \theta, \alpha_3) = u_{op}(r, \theta) + \alpha_3 \beta_1 \]
\[ v_p(r, \theta, \alpha_3) = v_{op}(r, \theta) + \alpha_3 \beta_2 \]  
\[ w_p(r, \theta, \alpha_3) = w_{op}(r, \theta) \]  

and

\[ \beta_1 = \frac{u_{op}}{R} - \frac{\partial w_{op}}{\partial r} \]  
\[ \beta_2 = \frac{v_{op}}{R} - \frac{1}{r} \frac{\partial w_{op}}{\partial \theta} \]  

If it is assumed that there is perfect bonding at the reflector/patch interface, then the mid-surface displacements of the reflector are related to the mid-surface displacements of the patch as shown in Eq. (2-29).

\[ u \left( r, \theta, \frac{h_{ref}}{2} \right) = u_p \left( r, \theta, -\frac{h_p}{2} \right) \]
\[ v \left( r, \theta, \frac{h_{ref}}{2} \right) = v_p \left( r, \theta, -\frac{h_p}{2} \right) \]  
\[ w(r, \theta) = w_p(r, \theta) \]

Here, \( h_{ref} \) is the thickness of the reflector while \( h_p \) is the thickness of the actuator patch.

By using Eq. (2-20), Eq. (2-21), Eq. (2-27), Eq. (2-28), and Eq. (2-29), we can relate the mid-surface displacements as shown in Eq. (2-30).

\[ u_{op} = u_o - \frac{\partial w_o}{\partial r} \frac{(h_{ref} + h_p)}{2} \]
\[ v_{op} = v_o - \frac{1}{r} \frac{\partial w_o}{\partial \theta} \frac{(h_{ref} + h_p)}{2} \]  
\[ w_{op} = w_o \]
### Solution using Ritz Method

The total energy of the reflector and patch system is given by

\[ U_{total} = U_{ref} + \sum_{i} U_{patch_i}. \tag{2-31} \]

The system is solved using the Ritz method. The boundary conditions to be satisfied at the rim \((r = a)\) and the center \((r = 0)\) of the reflector by the shape functions are given by

\[ u_o(a, \theta) = v_o(a, \theta) = w_o(a, \theta) = u_o(0, \theta) = v_o(0, \theta) = 0. \tag{2-32} \]

The boundary condition at the center is required to prevent the strains \(\varepsilon_{11}\) and \(\varepsilon_{22}\) from becoming singular. A Fourier-Bessel series modal expansion is given by Eq. (2-33).

\[
\begin{align*}
    u_o(r, \theta) &= \sum_{n=0}^{N_n} \sum_{m=0}^{N_m} J_{n+1}(\lambda_{n+1,m} r) \left[ U_{cnm} \cos n\theta + U_{snm} \sin n\theta \right] \\
    v_o(r, \theta) &= \sum_{n=0}^{N_n} \sum_{m=0}^{N_m} J_{n+1}(\lambda_{n+1,m} r) \left[ V_{cnm} \cos n\theta + V_{snm} \sin n\theta \right] \\
    w_o(r, \theta) &= \sum_{n=0}^{N_n} \sum_{m=0}^{N_m} J_n(\lambda_{n,m} r) \left[ W_{cnm} \cos n\theta + W_{snm} \sin n\theta \right]
\end{align*}
\]  

(2-33)

Here, \(J_n(\lambda_{n,m} r)\) is a Bessel function of the first kind and \(\lambda_{n,m}\) are the characteristic roots satisfying \(J_n(\lambda_{n,m} a) = 0\). These expansions satisfy the boundary conditions specified in Eq. (2-32). Furthermore, the Fourier-Bessel series expansion order utilized in this analysis is \(N_n = 6\) and \(N_m = 6\). These numbers were chosen as a trade-off between accuracy and computational time. A variety of cases were run varying the expansion order, and it was found that no improvement in model accuracy was.
noticed after $N_n = N_m = 6$. The unknown coefficients can be grouped together into a single vector, $\{X\}$.

$$\{X\} = \{U_{cmn}, U_{smn}, V_{cmn}, V_{smn}, W_{cmn}, W_{smn} \} \quad (2-34)$$

Minimizing the total energy with respect to the unknown coefficients, we derive a linear system of equations which can be solved for the unknown coefficients.

$$\frac{\partial U_{total}}{\partial X_i} = 0 \Rightarrow \left[ K_{ref} + \sum_i K_{patch_i} \right] \{X\} = \{F_T\} + \{F_E\} \quad (2-35)$$

Here, $\{F_T\}$ and $\{F_E\}$ are the generalized force vectors representing the thermal load and actuator force, respectively.

**Surface Errors**

As discussed in DeSmidt [2006] and Pattom [2006], there are two main types of shape distortions that the reflector would experience in space. When the reflector is initially inflated, it does not perfectly form the desired spherical shape. The difference between the desired shape at the actual inflated shape is called the ‘W-error’. The second kind of distortion is due to thermal loading, which is a result of solar heating and earth shadowing. These two distortions, ‘W-error’ and thermal loading, are discussed in the following sections.
W-error

Before being inflated, the reflector is assumed to have a spherical shape with a non-zero central rise height of $z_0 < z_r$, where $z_r$ is the central rise height of the desired spherical shape. Eq. (2-36) represents the reflector profile before inflation, where $a$ is the planform diameter. The planform is simply the plane that intersects the outside boundary of the reflector.

$$z_{\text{initial}}(r) = \sqrt{R_0^2 - r^2} - \sqrt{R_0^2 - a^2}$$  \hspace{1cm} (2-36)

$$R_0 = \frac{z_0^2 + a^2}{2z_0}$$

The desired spherical profile is given by Eq. (2-37).

$$z_{\text{desired}}(r) = \sqrt{R^2 - r^2} - \sqrt{R^2 - a^2}$$  \hspace{1cm} (2-37)

$$R = \frac{z_r^2 + a^2}{2z_r}$$

The equation for large deflections due to inflation of an initially flat circular membrane to a final rise height was derived by Hencky [1915]. This theory is used to approximately calculate the inflated reflector profile, which is given by Eq. (2-38).

$$z_{\text{Hencky}} = (z_r - z_0) \left[ 1 + a_2 \left( \frac{r}{a} \right)^2 + a_4 \left( \frac{r}{a} \right)^4 + a_6 \left( \frac{r}{a} \right)^6 + \cdots \right], \text{with}$$

$$b_0 = 1.724, \quad a_0 = 0.653, \quad a_2 = \frac{-1}{b_0}, \quad a_4 = \frac{-1}{2b_0^4}, \quad a_6 = \frac{-5}{9b_0^7}$$
The approximate profile for a structure that is initially a spherical cap is calculated by adding the Hencky profile to the initial (un-inflated) reflector spherical profile. This profile is given by Eq. (2-39).

\[ z_{\text{inflated}} = z_{\text{initial}} + z_{\text{Hencky}} \]  \hspace{1cm} (2-39)

The W-error is given by Eq. (2-40)

\[ W_{\text{error}} = z_{\text{inflated}} - z_{\text{desired}} \]  \hspace{1cm} (2-40)

The initial profile, desired spherical profile, the inflated profile, and the resulting W-error are illustrated in Figure 2-3.

![Initial, inflated, and desired reflector profiles.](image)

Furthermore, the inflation pressure, \( P_0 \), required to achieve the desired central rise height, \( z_r \), is given by Eq. (2-41).

\[ P_0 = \left[ \frac{z_r - z_0}{a_0 a} \right]^3 E_{\text{Youngs}} h_{\text{ref}} \frac{a}{a} \]  \hspace{1cm} (2-41)
Thermal Loading

The reflector moves in a geosynchronous orbit around the earth, so its position relative to the sun changes. This is illustrated in Figure 2-4.

![Figure 2-4](image)

Figure 2-4  Different positions of reflector relative to the sun.

![Figure 2-5](image)

Figure 2-5  Temperature distribution of reflector at different positions relative to the sun.

Figure 2-5 shows the temperature distributions over the reflector at different locations relative to the sun. This distribution can be approximated by a bulk temperature shift of
the entire reflector, combined with a linear temperature gradient. The temperature is modeled according to Eq. (2-42), where \( T_0 \) is the bulk temperature shift at the center and \( \Delta T \) is the linear temperature gradient across the reflector with respect to the temperature at the center. This distribution is relative to the nominal temperature in space, \( T_{\text{nom}} \). This temperature profile approximately capture the change in temperature that will be seen on the reflector at different points through its’ orbit.

\[
T(r, x) = T_0 + \Delta T \frac{x}{L}
\]

In polar coordinates, this becomes

\[
T(r, \theta) = T_0 + \Delta T \frac{r}{2a} \cos \theta.
\]

A graphical representation of this equation is shown in Figure 2-6, which shows the parameters used as well as how the temperature change is applied to the reflector.

![Figure 2-6 Illustration of parameters used in modeling temperature distribution](image-url)
As the reflector is moved from earth nominal temperature, $T_{nom-earth}$, to orbit nominal temperature, $T_{nom-space}$, an additional inflation pressure, $P_{nom}$, is required to overcome the thermal contraction of the reflector. This additional pressure is given by Eq. (2-44).

$$P_{nom} = 2\alpha_{CTE}(T_{nom-earth} - T_{nom-space})\frac{E_{Yours}h_{ref}}{R(1 - v)}$$  \hspace{1cm} (2-44)

Thus, the total required inflation pressure is $P = P_0 + P_{nom}$. The resultant pre-stress required is given by Eq. (2-45).

$$\sigma_{pre-stress} = \frac{PR}{2h_{ref}}$$  \hspace{1cm} (2-45)

**Actuator Approximations**

The previous sections have presented the reflector and actuator models, as well as the surface error that will be controlled. As mentioned, both the reflector and actuator models have been derived using polar coordinates. By so doing, the most natural way to model each actuator is in the shape of a wedge, as seen in Figure 2-7.
Unfortunately, as can also be seen from this illustration, when using wedge-shaped actuators, there will be many different sizes of actuators, which would increase the difficulty of manufacturing the actuators. In order to facilitate the experimental portion of this study, rectangular actuators are used in the model when there will be a comparison between the analytical and experimental work.

While rectangular actuators can be modeled in polar coordinates, this would drastically increase the computational time needed to build the model. One simple solution is to approximate each rectangular actuator with one or more wedges. To measure the accuracy of the approximation, a measure of overlap is used, as given by Eq. (2-46).

\[
\text{Measure of Overlap} = \frac{\text{Area(Strip } \cap \text{ Wedge)}}{\text{Area(Strip } \cup \text{ Wedge)}}
\]  

(2-46)

The numerator is the area of the intersection of the rectangular actuator and the approximating wedge(s), which will always be less than or equal to one, while the denominator is the area of the union of the rectangular actuator and the approximating
wedge(s), which will always be greater than or equal to one. As the overlap percentage approaches 100%, the wedge approximation becomes exact. In this study, two reference points are used, as shown in Figure 2-8.

![Diagram](image)

**Figure 2-8** Wedge approximations for rectangular actuator

These two points are the centre of the reflector (pt. O) and the mid-point of a side edge of the rectangular strip (pt. M). The line OM cuts the bottom edge at L and is extended to cut the extension of the top edge at N. Line LMN forms one side of the radial wedge. The same is done for the other side edge. Arcs are drawn with radii OL and ON to form the radial wedge. The graph to the right is the measure of overlap for two approximations: using a single wedge per actuator strip, and using three wedges per actuator strip. The radial patch number in the figure refers to how far the actuator is from the center. The greater the patch number the further the actuator is from the center of the reflector. As an actuator is moved further from the center, the approximation accuracy
increases. As can be seen, using three wedges yields very similar coverage to using a rectangular strip. Using the results of this study, we have determined to approximate rectangular strips using three wedges per actuator. This is done in order to reduce the computational time for the model.

**Reflector Test-beds**

Throughout this study, two distinct reflector test-beds are used. The first, a 2.4 meter diameter reflector is developed so that model comparisons can be made with an actual reflector. This reflector was tested at the Jet Propulsion Lab (JPL) and is used to validate the aforementioned model. The second test-bed used in this study is a 35 meter diameter model which conforms to desired attributes of the NASA NEXRAD reflector. This test-bed is used to show the efficacy of the algorithms and recommendations of this study. For both cases, the actuator used is a Polyvinylidene Fluoride (PVDF) flexible actuators, aligned with the primary actuation direction (polling direction) in the reflector radial direction.

The relevant geometric properties for both of these test-beds are listed in Table 1 while the material properties, which are the same for both, are listed in Table 2.
### Table 1 Geometric Properties of 2.4 meter test-bed and 35 meter test-bed

<table>
<thead>
<tr>
<th></th>
<th>2.4 Meter Test-bed</th>
<th>35 Meter Test-bed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius of Curvature</td>
<td>$R = 1.45$ m</td>
<td>$R = 56$ m</td>
</tr>
<tr>
<td>Planform Radius</td>
<td>$a = 1.2$ m</td>
<td>$a = 17.5$ m</td>
</tr>
<tr>
<td>Thickness</td>
<td>$h_{ref} = 50$ $\mu$m</td>
<td>$h_{ref} = 50$ $\mu$m</td>
</tr>
</tbody>
</table>

### Table 2 Material properties used for both test-beds

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflector Density</td>
<td>$\rho = 1420$ kg/m$^3$</td>
</tr>
<tr>
<td>Reflector Elastic Modulus</td>
<td>$E_{Young's} = 2.5$ GPa</td>
</tr>
<tr>
<td>Reflector Poisson’s Ratio</td>
<td>$\nu = 0.34$</td>
</tr>
<tr>
<td>Coefficient of Thermal Expansion</td>
<td>$\alpha_{CTE} = 0.4 \times 10^{-6}$ K$^{-1}$</td>
</tr>
<tr>
<td>Actuator Density</td>
<td>$\rho = 1780$ kg/m$^3$</td>
</tr>
<tr>
<td>Actuator Elastic Modulus</td>
<td>$E_{Young's} = 2.27$ GPa</td>
</tr>
<tr>
<td>Actuator Poisson’s Ratio</td>
<td>$\nu = 0.225$</td>
</tr>
<tr>
<td>Actuator Thickness</td>
<td>$h_{ref} = 65$ $\mu$m</td>
</tr>
<tr>
<td>Piezoelectric Constants</td>
<td>$d_{31} = 15 \times 10^{-12}$ m/Volt</td>
</tr>
<tr>
<td></td>
<td>$d_{32} = 6 \times 10^{-12}$ m/Volt</td>
</tr>
<tr>
<td>Maximum Allowed Voltage</td>
<td>$V_{max} = 2000$ V</td>
</tr>
</tbody>
</table>

**Validation on 2.4 Meter Reflector**

The 2.4 meter reflector was built by ManTech NeXolve Technologies while the actuators were attached to the surface at JPL. For this reflector, 168 rectangular actuators
were used. These actuators were 2 cm wide by 27 cm long, and were arranged on the reflector surface in three rings, as shown in Figure 2-9. The actuators on the right are ‘double’ actuators; one placed directly on top of the other, which give twice the output of a single actuator. In this figure, the long red lines that extend outside the reflector are flexible circuits designed to take power to each actuator.

Figure 2-9  Actuator placement on 2.4 meter reflector

While each actuator on the experimental model is independently wired, when testing was performed, only six power supplies were available. The different colors in the figure signify which power supply is attached to each actuator. Figure 2-10 shows the actual reflector. The white dots seen on the reflector surface are for the photogrammetry system, while the long, slender orange tabs that run from the center to the edge are flexible circuits which connect to each actuator.
The following figures, (Figure 2-11 to Figure 2-13) show the analytical and experimental displacement results for varying powering scenarios. The analytical results are on the left, while the experimental results, obtained from the JPL reflector, are on the right. The out-of-plane displacement is measured in millimeters. Figure 2-11 shows the analytical and experimental displacement results for the case when all single actuators are powered with 1000 volts. Similarly, Figure 2-12 shows the displacement when 1000 volts is applied to only the double actuators. Finally, Figure 2-13 shows the out-of-plane displacement when all actuators are powered to 1000 volts.
Figure 2-11  Comparison between the analytical and experimental model when 1000 V is applied to all SINGLE actuators. Displacement is in mm.

Figure 2-12  Comparison between the analytical and experimental model when 1000 V is applied to all DOUBLE actuators. Displacement is in mm.
While there is some discrepancy between the analytical and experimental results in these cases, the general magnitude is correct, as well as the overall movement trend. It does appear that some of the actuators create a larger displacement on the surface than do others. One possible explanation is that the actuators themselves are not perfectly bonded to the reflector. Many photogrammetry points are located directly on the actuators. Therefore, the photogrammetry system will read the movement of the actuator it’s attached to and not the movement of the reflector. If the actuator is not fully attached to the surface, then the surface will not move as much as the actuator, causing the photogrammetry measurement at that point to be incorrect. However, even with these errors there is good correlation between the general magnitude of the model and experimental results, as well as the general movement trend.

Figure 2-13 Comparison between the analytical and experimental results when 1000 V is applied to all actuators. Measurement is in mm.
Conclusions

To precisely control the surface of a space reflector, as well as to determine optimal actuator grouping and control, an accurate model of the reflector/actuator system is necessary. In this chapter, a reflector model was developed which allows for accurate numerical analysis while concurrently being able to be quickly modified for different actuator placement. The observations of this investigation are summarized as follows:

1) The reflector/actuator system was modeled in MATLAB, allowing easy modifications for actuator placement and design changes in the reflector and actuators.

2) Common surface errors prevalent in space systems have been integrated into the reflector model, allowing studies to be performed to determine actuator placement.

3) A method was determined to approximate rectangular actuators on the reflector surface.

4) A 2.4 meter reflector was built in order to experimentally verify the analytical model. Results are given which show that, while the analytical model is not accurate enough for use in a feed forward control system, it is correlated well enough with the experimental results that it can be used to determine nominal placement of actuators and efficacy of control systems.
Chapter 3
Shape Control

In this investigation, the shape control law is designed based on a least squares (LS) approach, where photogrammetry is used to determine the out-of-plane displacement at certain points on the surface. Least squares can be used because of the quasi-static nature of the surface error. While there is a time dependence in the error, the rate of change is very low, so that at any moment in time the surface error can be assumed to be static. The formulation for least squares is described in the following sections, along with results.

Least Squares Control

The general form of the reflector/actuator system is given by Eq. (3-1)

\[
[K_{sys}]{X} = \{F_T\} + \{F_E\}
\]  

(3-1)

where \([K_{sys}] = [K_{ref} + \sum_i K_{patch_i}]\) is the system stiffness matrix, and \(\{F_T\}\) and \(\{F_E\}\) are generalized force vectors due to thermal loading and control inputs, respectively. Furthermore, \(\{F_E\} = [B_u]{U}\), where \(\{U\}\) is a vector of control input electric fields for the PVDF patch actuators and \(B_u\) is the corresponding control input distribution matrix. Recall that \(\{X\}\) is a vector consisting of all the coefficients of the Bessel functions, as
stated in Eq. (2-34). For control purposes, we are only interested in the transverse surface displacements, so a vector of surface transverse displacements \( \{Y\} = [C]\{X\} \) is used. The matrix \([C]\) transfers the transverse Bessel coefficients to a rectangular grid of sensors, which are measured as \( \{Y\} \). For this study, a grid totaling 1184 sensors was used, where the spacing between each grid point is \( 1/40 \) of the planform diameter of the reflector. This vector can be considered the sum of two parts, as written in Eq. (3-2).

\[
\{Y\} = \{Y_o\} + \{Y_E\}
\]  

(3-2)

In this equation, \( \{Y_o\} \) is a vector of sensor measurements representing the shape error at the sensor locations (which can be due to thermal loading or ‘W-error’) and \( \{Y_E\} \) is the vector of sensor measurements as a result of only the patch actuator inputs. For a thermal loading case,

\[
\{Y_o\} = [C][K_{sys}]^{-1}\{F_T\}
\]

(3-3)

and

\[
\{Y_E\} = [C][K_{sys}]^{-1}[B_u]\{U\}.
\]

(3-4)

The algorithm used to determine the optimal electric fields for a given patch configuration and loading/shape error is the reflective Newton method [Coleman, 1994]. The PVDF material can retain its piezoelectric properties only up to a certain value of the electric field. Beyond this value, it gets depoled (loses its piezoelectric properties). This threshold value determines the bounds on the electric field in the algorithm. The
objective function, \( J = \{Y\}^T\{Y\} \) is minimized in the least-squares sense, keeping the electric fields within the selected bounds.

**Least Square Control Results**

This section illustrates how particular cases of error are corrected by the application of the least squares control law. The results shown are for the case of a 35 meter reflector, assuming 128 actuators divided into 8 rings and 16 rays, entirely covering the surface of the reflector. This would be the best case scenario, before taking into account possible hardware and model limitations inherent in real world systems. The surfaces errors applied are consistent with what a reflector would undergo while in orbit.

The first case given in Figure 3-1 shows the reflector with a uniform temperature shift, \( T_0 \), of 40 K applied to the entire reflector. On the left of the figure is the surface error of the reflector without any control, while the middle plot shows the surface error of the reflector after the least squares control is applied. The plot on the right shows the voltage applied to each actuator patch, where the patch number starts with 1 at the center and circles around to the outside edge. As shown, the RMS error is reduced from 0.88 mm to 0.09 μm, a reduction of 99.9%.
Figure 3-1 Graphical result for 35 meter reflector, 100% coverage, given a uniform temperature shift, $T_0 = 40$ K. All displacements are in mm.

The second case, Figure 3-2, illustrates the application of the control law to correct the shape error caused by a gradient temperature shift $\Delta T$ of 40 K, while $T_0$ is kept at 0 K. The RMS error is reduced from 0.28 mm to 0.8 $\mu$m, a reduction of 99.9%.

Figure 3-2 Graphical result for 35 meter reflector, 100% coverage, given a gradient temperature shift, $\Delta T = 40$ K. All displacements are in mm.
The third case is a combination of the first two cases. A uniform temperature shift, $T_0$, of 40 K is applied with a gradient temperature shift, $\Delta T$, of 40 K, as seen in Figure 3-3. In this case, the maximum applied temperature is 80 K where the two temperatures are added together. The uncontrolled RMS error is 0.93 mm, while after the control, the RMS error is reduced to 0.2 μm, or a reduction of 99.9%.

![Graphical result for 35 meter reflector, 100% coverage, given a gradient temperature shift, $\Delta T = 40$ K and a uniform temperature shift, $T_0 = 40$ K. All displacements are in mm.](image)

The final case presented here illustrates the control ability given an inflation error ("W-error") on the surface, as shown in Figure 3-4. The uncontrolled RMS error is 1.89 mm, while after control, the RMS error is reduced to 0.01 mm.
These results show that given full surface coverage and an exact model, accurate control of the surface is possible using PVDF actuators. In actuality, we cannot assume that our model is perfect. The remainder of this dissertation will be investigating real world deficiencies in the modeling and control and possible ways to overcome each one. The first constraint to be examined here is the possible discrepancies between the developed analytical model and the actual reflector. Each reflector built will have its own idiosyncrasies. These may be caused by manufacturing error, differences across the material, or any number of other sources. Because of these discrepancies, an on-line identification method must be developed for use with the control algorithm on the reflector. The Influence Coefficient Matrix (ICM) method which we have developed performs such an online model identification and can be used in place of the analytical model.

Figure 3-4 Graphical result for 35 meter reflector, 100% coverage, with a W-error of 3.0 mm. All displacements are in mm.
Influence Coefficient Matrix

The ICM can be used when the analytical model might not properly model the surface deformations. The ICM is found by assuming there are no external forcing on the reflector, so in Eq. (3-2), \( Y_0 = 0 \). Eq. (3-4) then becomes

\[
Y = [M][U]
\]  
(3-5)

where

\[
[M] = [C][K_{sys}]^{-1}[B_u]
\]  
(3-6)

is an \( n \) by \( m \) matrix, with \( n = \) number of sensor points, and \( m = \) number of voltage supplies. In the case where each actuator is independently controlled, the number of voltage supplies is equal to the number of actuators. To determine \( M \) analytically, it can be decomposed into the following

\[
M = \begin{bmatrix}
\begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} & \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix} & \cdots & \begin{pmatrix} m_1 \\ m_2 \\ \vdots \\ m_n \end{pmatrix}
\end{bmatrix}
\]  
(3-7)

where each vector \( \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \) is called the Influence Coefficient Vector, where \( x = a, b, \ldots, m \). This vector corresponds to a single independently controlled power supply. By keeping all voltages at zero, and varying one power supply at a time, each influence coefficient vector can be determined. This vector is found by applying a known voltage from one power supply and recording the change in displacement at each sensor point. As long as the
displacement is linear, then the superposition principle holds and each vector will be linearly independent.

**Experimental Validation**

The Influence Coefficient Method has been tested on the 2.4 meter reflector to determine if the linearity constraint is appropriate. Instead of powering each actuator individually to test the ICM theory, the actuators were connected to only two power supplies. This was done so that a large area of the surface would be activated, insuring the surface deflection would be large enough to stand out of the noise in the measurements. First, 1000 volts was applied only to the left half of the actuators, and photogrammetry was used to determine the surface deflection. Second, 1000 volts was applied to the right half of the actuators, and the deflection determined. Finally, the voltage was applied to all actuators simultaneously, and the deflection again was measured. If superposition holds, the summation of the displacement from the first two runs should equal the movement from the third run. Figure 3-5 shows the individual deflection from the left actuators, as well as the right.
Figure 3-5 Experimental results from applying 1000 volts to left and then right actuators. All measurements are in mm.

Figure 3-6 shows the comparison between the calculated and measured surface deflection. On the left is the calculated surface deflection, a summation of the two plots in Figure 3-5. On the right is the measured surface deflection, where 1000 volts is applied to all actuators simultaneously.

Figure 3-6 Calculated (left) and measured (right) displacement assuming 1000 V actuation. Displacement is measured in mm.
Finally, Figure 3-7 shows the error between the measured and the calculated deflection at 1000 volts. The RMS value of the difference is less than 0.05 mm, which shows that the superposition principle does hold, and that the ICM method can be used on the reflector to experimentally determine the model for use in the control system.

![Image of Figure 3-7](image.png)

Figure 3-7 Difference between measured and calculated deflection.

Conclusions

In this chapter, a least squares shape control is presented and applied to a 35 meter reflector model. The least squares control law requires a grid of photogrammetry points to determine the RMS error on the surface of the reflector. Results are presented as well
as a method to determine an online model of the reflector/actuator system. The major observations of this investigation are summarized as follows:

1) Given an accurate measurement of the surface error, the least squares control is a simple yet effective means to accurately control the surface error. Using least squares control, it has been shown that on a fully covered reflector, the RMS error for common surface errors can be greatly reduced.

2) A new online identification method (ICM) has been developed which allows a model to be generated using the actual reflector. Experimental results on the 2.4 meter reflector show that this is an appropriate method for determining the reflector model and can be used in a control system.
Chapter 4

Actuator Location

For the results shown thus far, the actuator location has been arbitrarily set, either fully covered for the 35 meter reflector, or evenly distributed around the reflector for the 2.4 meter reflector. For better control authority, the location of each actuator can be optimized. This chapter presents a methodology for determining actuator placement so that the surface error will be minimized. An additional constraint on the number of power supplies is then added.

Actuator Placement with Genetic Algorithm

On a large reflector covered with actuators, it is possible when determining actuator placement that a very large number of decision variables will exist. For example, a 2.4 meter reflector, with 1 cm by 18 cm rectangular actuators, would require more than 360 actuators to cover only 15% of the surface, with many possible locations for each actuator. Due to the large number of decision variables and a highly nonlinear design space, a Genetic Algorithm (GA) is used to determine the placement of each actuator.

The GA is set up by first determining a set of possible actuator locations. These locations are determined arbitrarily, given the constraints of the system. These
constraints might be the space needed for wiring each actuator, the size, shape, and number of available actuators, or any other constraints imposed by the design team. While any location or orientation could be used for each actuator, in this study the possible locations were simplified by placing all actuators into rings around the reflector and evenly spacing the actuators in each ring. For example, in Figure 4-1, approximately 650 possible locations are shown, assuming 1cm by 18 cm actuators on a 2.4 meter reflector surface. Each location is numbered, starting from the innermost ring and moving outward.

Figure 4-1  Possible locations for 1 cm by 18 cm actuators on a 2.4 meter reflector. These locations are the input to the genetic algorithm which determines actuator location.
The genetic algorithm used was developed specifically for this study. An initial population of 100 trial solutions is randomly generated and subjected to a fitness function, in this case the RMS error of the surface. The best solutions are kept while the others are discarded. Each individual solution is subjected to the same fitness function to determine if it will be kept or discarded, allowing the population to converge over many generations to find an optimum solution.

In this work, each individual design is comprised of a single chromosome containing a single gene. This gene is represented by an \( n \)-size string of non-repeating integers from 1 to \( m \), where \( n \) is the number of actuators, and \( m \) is the total number of possible locations. The resulting GA is a hybrid, incorporating features of both binary and continuous algorithms. Although, like binary, only a limited number of integer values are able to represent the genes, the genes are not limited to 1s or 0s but instead to non-repeating numbers representing the actuator positions.

The GA utilizes reproduction, asexual crossover and mutation. For reproduction, a tournament is utilized to determine each parent. Two designs are randomly chosen and compared to each other. The winner is selected as a parent while the other is discarded. The children are then determined through asexual crossover, meaning that a crossover point is randomly selected in the chromosome of both parents and all data after the crossover point is swapped between the two parents. Finally, mutation is used to introduce diversity into the GA. The likelihood of mutation for each chromosome was set to 30\%, where if a chromosome is mutated, 1/8 of the data is changed.
The most basic fitness function used for the optimal placement control is the RMS surface error value after the optimal voltages are applied to the actuators. For each individual in the GA, the RMS surface error is minimized through the application of the least squares control.

A case study was run using the same possible actuator locations as shown in Figure 4-1, using a total of 360 actuators. Multiple runs were made using different initial errors, and a sample of the results is given in Figure 4-2. As can be seen, the only noticeable trend is that the three inside rings are filled, while the outside ring has no actuators assigned. For all but the outside ring, no discernable reason for the actuator placement can be established.

Figure 4-2  Sample result of actuator placement using a genetic algorithm
When a majority of the possible actuator locations are filled, like in the previous example, there does not appear to be a problem with this solution. However, when we increase the possible number of actuator positions, and reduce the number of actuators, we get a much different solution, an example of which is shown in Figure 4-3. We would expect to see a symmetrical solution because we have a symmetrical surface error; however, we see a very unsymmetrical result. This plot shows the difficulty for even a genetic algorithm in converging to the global optimum.

![Actuator placement for large number of possible locations using genetic algorithm](image)

Figure 4-3 Actuator placement for large number of possible locations using genetic algorithm

Due to the symmetric nature of the reflector and its surface errors, a good way to simplify the model so that an optimum can be more easily reached is to artificially create symmetry in the model. To create symmetry, the reflector is divided into slices, where each slice contains a number of actuator locations. The optimal placement found for this slice is replicated onto all the other slices. As the number of divisions is increased, there
are fewer variables that need to be determined by the GA, helping it to converge more quickly. An example of this method is shown in Figure 4-4. In this example, there will be 4 slices for symmetry. The non-filled rectangles represent an actuator location without an actuator present, while the solid blue rectangles represent a placed actuator. The upper right slice shows the 11 possible actuator locations, and the 7 locations chosen. This pattern is then copied to the other 3 quadrants. Thus, in the GA the size of the chromosome will drop from 28 to 7, while the possible integers that each gene can take drops from 44 to 11. This same method can be applied for any number of divisions and any number of actuators.

![Figure 4-4 Schematic of a 4-division symmetry on actuator](image)

At this point, the only symmetry enforced is for actuator placement and not for the voltage applied to each actuator. The voltage is still an independent variable which is chosen using the LS control during each function evaluation. Figure 4-5 shows the RMS value for a number of different symmetry conditions. The plot shows the best individual
at each function evaluation of the GA. As more symmetry divisions are used, the number of variables decrease, thus making the GA converge faster. At the same time, the converged result yields a higher RMS error when more divisions are used. However, this difference is very small, making the symmetric placement of actuators a very useful tool when determining actuator location when the time needed for convergence is limited.

Figure 4-5 Results for different symmetry constraints using Genetic Algorithm

Power Supply Constraint

A major constraint for real world application is the total number of available power supplies for the reflector. Up to this point, it has been assumed that each actuator can be powered independently. If there is a constraint on the amount of independent voltages, there are two options: have the same number of actuators as power supplies, or connect actuators together so that the applied voltages are not independent. If this second
approach is taken, the constraint must be applied to the system model before any optimization can take place.

**Binary Matrix Representation**

The simplest way to enforce this constraint is to add a binary matrix \([G]\) to Eq. (3-4), as shown in Eq. (4-1).

\[
\{Y_e\} = [C][K_{sys}]^{-1}[B_u][G]\{U\} \tag{4-1}
\]

This binary matrix \([G]\) is \(p\) by \(q\), where \(p\) = number of actuators and \(q\) = number of power supplies. Each row will be all 0s except for a single 1, while there are no constraints enforced on the columns. A 1 at location \((p,q)\) would denote that actuator \(p\) is connected to power supply \(q\). This architecture can be used with the influence coefficient matrix approach, along with any of the control techniques discussed above.

By adding the constraint of a limited number of power supplies, the number of variables that must be decided becomes very large. Even assuming that the locations of the actuators are fixed, the total possible combinations are staggering: for 168 actuators, and only 6 power supplies, an exhaustive search would require more than \(1e^{130}\) function evaluations. Even a fairly simple system of 16 actuators and 5 power supplies would require more than \(1.5e^{11}\) function evaluations to try every combination.

As mentioned previously, there have been attempts at optimizing the actuator groupings, although no method to date is able to supply the global optimal solution without doing an exhaustive search (ie, testing every possible grouping combination to
determine the global optimal solution). The required number of function evaluations for this method increases exponentially, according to \( m^n \), where \( m \) is the number of groups (power supplies) and \( n \) is the number of actuators to be grouped. As the number of actuators increases, the computational time of using an exhaustive search method makes it prohibitively time consuming. An alternative to the exhaustive search that is commonly employed is to use a genetic algorithm, but due to the incredibly large number of possible combinations and the inability of a genetic algorithm to guarantee a result sufficiently close to the global optimum, an effective and efficient method needs to be developed that can arrive at the global optimal solution without the need to test every possible combination of actuator grouping.

### Conclusions

For better control authority, it is helpful to optimize the actuator locations. A methodology for actuator placement optimization has been described using a genetic algorithm. Due to the high complexity of determining the actuator locations, a genetic algorithm was chosen for the optimization algorithm. It has been shown that the GA works well when the surface is mostly covered, but fails to consistently find adequate solutions when the surface is sparsely covered. To overcome this deficiency, a symmetry constraint is added to the reflector. Investigation reveals that increasing the symmetry on the reflector does not drastically change the final result found, and can decrease the number of function evaluations necessary to converge to an acceptable solution.
Chapter 5

En Masse Elimination Method

As mentioned in the previous chapter, when there is a limit on the number of power supplies available for control, a method is needed to determine how to group the actuators to the available power supplies.

We have developed a new method to determine the actuator grouping and achieve optimal control performance with a limited number of power supplies – the En Masse Elimination (EME) technique. This technique was developed in order to reduce the computational time needed to find the global optimal grouping of actuators. The EME method searches the entire design space (all of the possible combinations) but is able to eliminate areas without testing each combination. In this chapter, the basic EME method is explained and illustrated using a simple beam example. Improvements on the basic method are given, including the modification of the EME method for online use. Finally, computational results of the new online method are given.

Basic EME Method

The basic premise of the EME method is that by temporarily relaxing the power supply constraint, the resultant objective function measurement (RMS error) is a lower bound for multiple possible actuator combinations, which can be used to eliminate large
areas of the design space. While the EME algorithm is still an exponential algorithm similar to the exhaustive search, large numbers of function evaluations can be eliminated without testing each possibility. A detailed step by step process of the EME method is given below, along with a flowchart in Figure 5-1.

![Flowchart for basic EME algorithm](image-url)
To start the EME algorithm, a baseline value of the minimized RMS error must be found where all power supply constraints are satisfied. An initial grouping can be determined, either by simply choosing a case at random or utilizing a quick algorithm, the RMS error is calculated. This design is called the Current Best Case (CBC). The results from the EME algorithm are always compared to the results of the CBC design to determine which grouping of actuators can be eliminated. More in-depth analysis on choosing the initial CBC design is found further on in this chapter.

During each iteration of the EME algorithm, the actuators are divided into two distinct groups: first, some actuators are “set”, meaning that they are grouped to the given power supplies. Second, we allow some actuators to be “free”, meaning they are not grouped together, but are connected to additional individual power supplies. These extra power supplies are not counted towards the power supply constraint. Let us take, for example, a system with three power supplies and fifteen actuators. We can have ten “set” actuators, connected to the three power supplies. Then we allow the remaining five actuators to be “free” by connecting them to their own individual power supplies, relaxing the power supply constraint from three to eight power supplies. This allows the “free” actuators to take on any voltage value instead of being forced to take on only the voltage value of a group. For each iteration of the EME algorithm, the “set” actuators are placed into a given grouping, where the number of groupings is constrained by the number of power supplies available. The “free” actuators are each connected to individual power supplies. Using this combination of “set” and “free” actuators, the RMS error is then calculated. It is at this point in the algorithm that large areas of the
design space can be removed. The minimized RMS error is the lower bound for any actuator groupings that have the same combination of “set” actuators as in this specific case. If the RMS error calculated is greater than the RMS error of the CBC design, then regardless of which group each “free” actuator could be placed in, the minimized RMS error will always continue to be greater than this lower bound. Therefore, every grouping that has this combination of “set” actuators can be removed from consideration.

To illustrate the concept, a simple example is given in Figure 5-2. Assume that during an iteration of the EME algorithm, actuators 1-3 are “set.” In this case, actuators 1 and 3 are set to power supply #1 and actuator 2 is set to power supply #2, while actuators 4 and 5 are allowed to be “free,” as illustrated in Figure 5-2(a). If the RMS error of this configuration is higher than that of the CBC design, we know that the optimal solution cannot contain this configuration, where actuators 1 and 3 are connected to one power supply while actuator 2 is connected to another. Thus, we can eliminate each design with this combination. These designs are given in Figure 5-2(b)-(e).

![Set Free Matrix](image)

**Figure 5-2** Matrix (a) contains three set actuators and two free actuators. The error from this combination will provide a lower bound for the error of matrices (b) – (e).
In this situation, with only two free actuators, we can remove a total of four possible designs. As the number of free actuators increases, the number of combinations that can be removed increase according to $2^n$, where $n$ is the number of free actuators.

The basic EME algorithm runs through each possible combination iteratively, attempting to remove the largest amount of the design space possible. It starts by setting the majority of the actuators to be “free”. If the minimized RMS error is larger than the RMS error of the CBC design, multiple design possibilities can be removed, the algorithm updates to a new combination of “set” variables, and another iteration is run. If, on the other hand, the minimized RMS error is less than that of the CBC design, no new knowledge is gained and none of the designs can be removed. Under this condition, if all the actuators are “set”, meaning the design is now feasible, a new CBC has been found. If not all of the actuators are “set”, then one “free” actuator is placed into a group and becomes “set”. The RMS error is again minimized using the new grouping configuration and is again compared to the RMS error of the CBC design. As more actuators become “set”, the probability that we can remove some actuator groupings is increased, though the number of actuator groupings that can be removed during each iteration becomes smaller.

The EME algorithm is stopped once every possible grouping combination has either been tested or removed from the design space. In this way, the global optimal solution is guaranteed to be found, without the need for an exhaustive search of each possible combination.
Numerical Results of En Masse Elimination Method

To show the results of the EME method, a simplified system consisting of a single pinned-pinned beam with up to 10 actuators and 5 power supplies is used as an example test-bed, shown in Figure 5-3. This system allows us to visualize the effect of the EME as well as to see how the system responds to different control inputs. The structure of the mathematical construct of the system is exactly the same as with the reflector, so any findings are directly applicable to the reflector case.

![Figure 5-3 Beam test-bed consisting of a pinned-pinned beam and multiple linear actuators.](image)

**Beam Test-bed**

Assuming a linear system and superposition, the deflection of the beam due to multiple point forces can be considered as the sum of the beam deflection under each individual point force.
Figure 5-4 Dimensions for a single point force on a pinned-pinned beam

Given Figure 5-4, the deflection of the beam due to a static load at \( x = a \) is

\[
\begin{align*}
w_n(x) &= P_n \begin{cases} 
\frac{bx}{6Ell} \left( l^2 - b^2 - x^2 \right); & 0 \leq x \leq a \\
\frac{a(l-x)}{6Ell} \left( a^2 + x^2 - 2lx \right); & a \leq x \leq l
\end{cases}
\end{align*}
\] (5-1)

where \( E \) is the modulus of elasticity, \( I \) is the moment of inertia, \( w \) is the out-of-plane displacement, while \( P_n \) is the force applied by a single actuator. The total deflection due to the combination of all actuators is given in Eq. (5-2), where \( m \) is the total number of actuators.

\[
w(x) = \sum_{n=1}^{m} w_n(x)
\] (5-2)

The displacement measured at discrete points, evenly distributed along the beam, can be given by the vector \( \{y(x')\} \). This displacement can be put in matrix form, as shown in Eq. (5-3),

\[
\{y\} = [B]\{P\}
\] (5-3)
where $B$ is a matrix determined from Eqs. (5-1) and (5-2) while $P$ is a vector of all actuation forces. The error applied to the beam is also in vector form, $\{e\}$, where the error is measured at the same distributed points. The error is controlled using a least squares approach, minimizing the squared error at each point, as seen in Eq. (5-4),

$$RMS = \sqrt{\frac{\sum_{n=1}^{d}(y_n - e_n)^2}{d}}$$  \hspace{1cm} (5-4)

where $d$ is the total number of displacement measurement points. For this case study, the least squares procedure is the objective function used to determine the optimal actuator forces. When there are more available actuators than power supplies, the actuators are grouped together using a binary grouping matrix, $[G]$. This matrix is the same binary grouping matrix used in the reflector modeling. The vector $P$ is replaced with the grouping matrix and a new vector of forces $\{F\}$, corresponding to the force of each group, as shown in Eq. (5-5).

$$\{y\} = [B \overline{G}]\{F\}$$  \hspace{1cm} (5-5)

This equation is now in the identical form as Eq. (4-1), allowing the EME algorithm to proceed exactly as it would if being applied to the reflector rather than a beam. By substituting Eq. (5-5) into Eq. (5-4), we now have the objective function that will be minimized in the EME algorithm to determine the optimal actuator groupings.
EME Results on Beam

The ability of the EME algorithm to find the global optimal grouping is illustrated using the beam example. The geometric and material properties of the beam and actuators are shown in Table 1. A total of 5 power supplies are used, connected to 10 actuators.

Table 3 Geometric and material properties used for beam example

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l$</td>
<td>1.0 m</td>
</tr>
<tr>
<td>$E$</td>
<td>2.3 GPa</td>
</tr>
<tr>
<td>$I$</td>
<td>$2.08 \times 10^{11}$ m$^4$</td>
</tr>
<tr>
<td>$a$</td>
<td>actuator every 0.1 m</td>
</tr>
<tr>
<td>Number of actuators</td>
<td>10</td>
</tr>
<tr>
<td>Number of power supplies</td>
<td>2,3,4,5</td>
</tr>
<tr>
<td>Number of measurement points on beam</td>
<td>100</td>
</tr>
</tbody>
</table>

The original displacement error $\{e\}$, is shown in Figure 5-5. This error profile is used for all cases to allow for comparisons.
The great benefit of the EME algorithm is that it will find the global optimum without having to search every possible combination. In this case with 10 actuators and 5 power supplies, there are over 10 million possible combinations that would have to be checked if an exhaustive search method was used to determine the optimal grouping. To show the benefits of the EME algorithm, we will compare it to a Genetic Algorithm (GA), which is currently the most commonly used method for determining groupings. In order to have a fair and objective comparison between the EME and the GA, we define one iteration to be a single function call to the least squares procedure. The computational time needed for the least squares algorithm, where the voltage for each actuator is determined as well as the overall RMS error, is at least an order of magnitude
greater than the other calculations used in either algorithm. Both algorithms (as well as the exhaustive search algorithm) call the same least squares procedure, so using that to determine the number of iterations is an appropriate means of comparison. It is noted that the term iteration possibly has a different connotation for genetic algorithms, where one iteration could refer to running an entire family of function evaluations. For ease of use here, one iteration is one function evaluation of the least squares procedure.

The GA was run for 200,000 iterations for the case of 10 actuators and 5 power supplies, with the results given in Figure 5-6. The plot shows the best RMS error result found by the GA at each function evaluation. As can be seen, after about 110,000 iterations, the GA found its best case, with an RMS error of $1.48 \times 10^{-8}$ mm. The results do not change from that point until the algorithm is stopped at 200,000 iterations.
The EME algorithm is then tested using the identical initial error displacement on the same beam. The initial Current Best Case design was found by randomly grouping the actuators, and the results are shown in Figure 5-7.
Figure 5-7 Results of random start EME Algorithm optimization on beam, combining 10 actuators into 5 groups

The RMS error of the initial CBC was found to be $6.15 \times 10^{-2}$ mm, substantially worse than the Genetic Algorithm results. After only 11,000 iterations, the CBC had become $4.98 \times 10^{-8}$ mm, better than the result ($6.14 \times 10^{-8}$ mm) found by the Genetic Algorithm in the same number of iterations. After 40,000 iterations the global optimal result was found, $3.70 \times 10^{-10}$ mm. After 218,000 iterations, all grouping possibilities had been exhausted. This is a reduction of 98% of required iterations to guarantee an optimal solution as compared to the exhaustive search method.

The EME algorithm can also be used to determine the tradeoff between using fewer or more power supplies. Although more power supplies increase controllability,
they also add to the cost and weight of the system. Using the same error profile and randomly choosing the initial CBC design, results were obtained for systems having between 2 and 5 power supplies. These results are shown in Figure 5-8 through Figure 5-10. As expected, larger numbers of power supplies allows better possible control result (lower RMS error), but require substantially more iterations to search the entire design space. Table 4 shows the total number of iterations needed for each case as well as the final RMS error.

![Graph showing RMS error vs. number of iterations for EME Algorithm optimization on beam, combining 10 actuators into 4 groups.](image)

Figure 5-8  Results of random start for EME Algorithm optimization on beam, combining 10 actuators into 4 groups.
Figure 5-9 Results of random start for EME Algorithm optimization on beam, combining 10 actuators into 3 groups.

Figure 5-10 Results of random start for EME Algorithm optimization on beam, combining 10 actuators into 2 groups.
Table 4  Results for beam case study test using between 2 and 5 power supplies

<table>
<thead>
<tr>
<th>Number of power supplies</th>
<th>Total number of function evaluations</th>
<th>Final RMS error</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>206</td>
<td>4.46 x 10^{-5} mm</td>
</tr>
<tr>
<td>3</td>
<td>1885</td>
<td>6.85 x 10^{-6} mm</td>
</tr>
<tr>
<td>4</td>
<td>17556</td>
<td>3.66 x 10^{-7} mm</td>
</tr>
<tr>
<td>5</td>
<td>218081</td>
<td>3.72 x 10^{-10} mm</td>
</tr>
</tbody>
</table>

EME Results on Reflector

This section illustrates how particular cases of error are corrected by the application of the EME algorithm on the reflector. The results shown are for the case of a 35 meter reflector, assuming 32 actuators divided into 4 rings, and 8 rays, which entirely cover the surface of the reflector. A constraint of 4 power supplies is enforced on the reflector.

The first case given in Figure 5-11 shows the reflector with a uniform temperature shift, $T_0$, of 40 K. On the left is the surface error of the reflector without any control, while the middle plot shows the surface error of the reflector after the EME algorithm is used to determine the actuator grouping and the least squares control is applied. The plot on the right shows the voltage applied to each actuator. As seen, the RMS error is reduced from 0.99 mm to 0.36 mm, a reduction of 64%.
Figure 5-11 Graphical result for 35 meter reflector, 100% coverage, and 4 power supplies, given a uniform temperature shift, $T_0 = 40$ K. All displacements are in mm.

The second case, Figure 5-12, illustrates the application of the EME algorithm and control law to correct the shape error caused by a gradient temperature shift $\Delta T$ of 40 K, while $T_0$ is kept at 0 K. Again, the EME algorithm determines the optimal grouping assuming 4 power supplies, and then the least squares control is applied to the groups. The RMS error is reduced from 0.24 mm to 0.099 mm, a reduction of 58.8%.

Figure 5-12 Graphical result for 35 meter reflector, 100% coverage, and 4 power supplies, given a gradient temperature shift, $\Delta T = 40$ K. All displacements are in mm.
The third case is a combination of the first two cases. A uniform temperature shift, $T_0$, of 40 K is applied with a gradient temperature shift $\Delta T$ of 40 K, as seen in Figure 5-13. In this case, the maximum applied temperature is 80 K, where the two temperatures are added together. The EME algorithm is used to determine the optimal grouping, and the least squares control is used to determine the optimal voltages. The uncontrolled RMS error is 1.00 mm, while after control the RMS error is reduced to 0.37 mm, or a reduction of 63%.

![Graphical result for 35 meter reflector, 100% coverage, given a gradient temperature shift, $\Delta T = 40$ K and a uniform temperature shift, $T_0 = 40$ K. All displacements are in mm.]

**EME Improvements**

In this section, a number of improvements made to the basic EME algorithm will be explained. First, a case study was performed to determine the optimal method for
establishing the initial Current Best Case design. Second, a new method that has been
developed to generate the RMS error is explained. This method drastically increases the
speed of the EME algorithm, allowing more function evaluations to be done in a shorter
amount of time. Finally, a methodology for utilizing the EME algorithm for time-
dependent error signals is explained and some results are given.

**Initial CBC Optimization**

While there are many different ways that the initial CBC can be chosen, this
section will focus on only three: a set (arbitrarily chosen) CBC design, a genetic
algorithm, and a greedy algorithm tailored for this situation.

**Set Algorithm**

First, the most basic method for determining the initial CBC is to randomly
choose a design. Although one could attempt to intuitively pick a good design by
matching the error profile with actuator combinations, in this study we simply assign
each actuator iteratively to a group. We place actuator 1 into group 1, actuator 2 into
group 2, etc. Once each group has one actuator, we start over again with group 1. This
continues until all actuators are placed into groups. One benefit of the set algorithm is
that no time or effort needs to be spent on determining the initial CBC design.
**Genetic Algorithm**

In this study, two identical genetic algorithms are used, where the only difference is the length that each is run. The short genetic algorithm (SGA) is run for 10,000 function evaluations, while the long genetic algorithm (LGA) is run for 2,000,000 function evaluations. A GA works well in this situation because of its ability to quickly find a good solution, even if it is not the global optimum. There is a trade-off when determining how long the GA should be run in order to find the initial CBC. A longer run GA may find a better CBC design, which could reduce the number of EME function evaluations needed to find the optimal solution. However, continuing to run the GA for a long time will add function evaluations without necessarily finding a better CBC design, thus increasing the total number of function evaluations needed.

**Greedy Algorithm**

Greedy algorithms are a type of heuristic algorithm that attempts to find a localized optimal solution without focusing on the global optimum. For this application, we have developed a greedy algorithm that requires fewer function evaluations than the total number of actuators, making this a very fast algorithm. The greedy algorithm starts by assuming there are no limits on the number of power supplies, so each actuator is placed in its own group. The RMS error is minimized, and the desired voltage for each group is recorded. These voltages are compared, and the two groups with the closest desired voltage are combined, eliminating one group. The RMS error is again
minimized, and another two groups are combined. This continues until the constraint on power supplies is met. This method is very fast, and has the possibility of giving a better CBC design than the set algorithm while not requiring the larger number of function evaluations of a GA.

**Initial CBC Case Study on Reflector**

We wish to compare these three cases not only against themselves, but also against the best possible scenario. The best CBC design (BCBC) would be where we somehow use the global optimal solution as the CBC design. This would be the lower limit on the number of function evaluations needed for the EME algorithm. We will compare these five cases (Set, SGA, LGA, Greedy, and BCBC) given three different temperature profiles ($T_0 = 40K / \Delta T = 0K$, $T_0 = 0K / \Delta T = 40K$, and $T_0 = 40K / \Delta T = 40K$). Table 5 shows the results of these three temperature profiles. While the results from these scenarios are not all inclusive, some general trends can be seen.

<table>
<thead>
<tr>
<th>Temperature Profile</th>
<th>Set</th>
<th>SGA</th>
<th>LGA</th>
<th>Greedy</th>
<th>BCBC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_0 = 40K / \Delta T = 0K$</td>
<td>887,039</td>
<td>869,638</td>
<td>2,044,322</td>
<td>42,631</td>
<td>42,602</td>
</tr>
<tr>
<td>$T_0 = 0K / \Delta T = 40K$</td>
<td>17,505,332</td>
<td>15,599,918</td>
<td>10,694,078</td>
<td>8,349,135</td>
<td>5,813,856</td>
</tr>
<tr>
<td>$T_0 = 40K / \Delta T = 40K$</td>
<td>196,730,884</td>
<td>196,624,932</td>
<td>110,849,719</td>
<td>103,806,568</td>
<td>412,130</td>
</tr>
</tbody>
</table>

Table 5  Total number of function evaluations for EME convergence for different algorithms used to determine initial Current Best Case design.
For the first case, where $T_0 = 40K / \Delta T = 0K$, the greedy algorithm is very similar to the BCBC, which shows that before the EME algorithm was started, the greedy algorithm was able to get very close to the actual optimal solution. The long GA was able to also find a near optimal solution, but the 2 million function evaluations cause the total number of function evaluations to be large. The short GA and the Set case are similar, and require many more function evaluations than the BCBC.

For the second case, where $T_0 = 0K / \Delta T = 40K$, many more function evaluations are needed for the BCBC. Again, the greedy algorithm is the best of the four, while the long GA is now better than the short GA. Again, the Set design is the worst.

For the final case, where $T_0 = 40K / \Delta T = 40K$, none of the initial algorithms are able to get close to the optimal solution, and require many more function evaluations than the BCBC. Again, the greedy algorithm is the best of the four, with the long GA considerably better than the short GA. The Set design is considerably worse than any other.

Regardless of the way the CBC is found, there is a tradeoff between the benefit of finding a better CBC, and how many function evaluations it takes to find the CBC. One difficulty in our ability to choose a single algorithm for choosing the initial CBC is the fact that the minimum number of function evaluations needed can change by orders of magnitude from one error profile to another. In spite of this difficulty, at least for these three cases, the greedy algorithm is able to outperform not only the Set design, but also both genetic algorithms.
RMS Error Calculation

When using the EME algorithm for surface control of the reflector, the required computational time is highly dependent on the call to the RMS error calculation. Previously in this study, the RMS error is calculated by calling the MATLAB function \textit{lsqlin}, which uses a reflective Newton Method to optimize the voltages in order to minimize the RMS error. This generic function allows a least squares optimization to be performed while considering constraints on the independent variables, which is needed in our case because of the saturation limitations of the actuators. While this function is very effective at determining the optimal solution, it requires a considerable amount of computation for each iteration. For each iteration of the EME algorithm, 99\% of the computational time is spent in the \textit{lsqlin} function. A new least squares calculation approach allows us to drastically reduce the computational time spent on this step of the EME algorithm.

New Least Squares Derivation

To solve this least squares problem, it is helpful to utilize geometric principles. A least squares approach, given \( m \) points and \( n \) inputs, is identical to finding the minimum distance between an \( n \)-dimensional hyperplane and a single point in an \( m \)-dimensional space. This is easily visualized in 3D space with a 2D hyperplane, as seen in Figure 5-14, but it is equally valid for any \( m \)-dimensional space with an \( n \)-dimensional hyperplane, given \( n < m \). In our situation, \( m = \) number of photogrammetry points used, while \( n = \)
number of groups of actuators. It should be noted that the voltage vectors \((V_1, V_2, \text{ etc})\) do not need to be orthogonal.

Figure 5-14 Geometric representation of Least Squares method. The gray area is the hyperplane proscribed by two input voltages, while \(E_1\), \(E_2\), and \(E_3\) are the errors we are attempting to minimize.

Let point \(O\) in Figure 5-14 be the target point \((\text{RMS} = 0)\) while point \(A\) is the unconstrained optimal point on the hyperplane; i.e., the optimal applied voltages needed to minimize the RMS error, or the closest point of the hyperplane to the target point. This point is found by determining the location where the slope of the hyperplane is
orthogonal to the line segment $\overrightarrow{OA}$. This unconstrained location is easily found when no constraints are applied to the system.

However, when a constraint is applied to a single input, it divides the plane into a feasible region and an infeasible region, and if point $A$ is not in the feasible region, our constrained optimal solution must be moved to a feasible region. We will prove that, given a constraint on one input, the constrained optimal location must be located on the constraint line.

First, let us take the counterexample where the constrained optimal solution (point $C$) is not on the constraint line. For any $C$, we are able to draw a straight line ($\overrightarrow{OA}$) on the hyperplane which crosses the constraint line at $B$. Recall that $\overrightarrow{OA}$ is, by definition, orthogonal to the hyperplane, and due to the fact that line segment $\overrightarrow{AC}$ is on the hyperplane, $\overrightarrow{AC}$ and $\overrightarrow{OA}$ are orthogonal. Due to this orthogonality, triangles $\overrightarrow{OAB}$ and $\overrightarrow{OAC}$ are right triangles with a common side, and by using Pythagorean’s theorem, we can see that the length of $\overrightarrow{OB}$ is always less than the length of $\overrightarrow{OC}$, thus contradicting the assumption that $C$ is the constrained optimal solution. Therefore, the constrained optimal solution must lie on the constraint line, meaning the saturated voltage must be set to the saturation limit, and then the least squares method can again be applied to the remaining voltages, which will minimize the length of $\overrightarrow{OB}$. It should be noted that minimizing the length of $\overrightarrow{OB}$ also minimizes the length of $\overrightarrow{AB}$, and vice versa.

An interesting dilemma occurs when there is more than one voltage value over the saturation limit. Figure 5-15 shows a close-up of the hyperplane when two constraints
are placed on the system. Again, the dotted lines represent the constraints on \( V_1 \) and \( V_2 \), the darker gray shading is the feasible space, and point \( A \) is the unconstrained optimum.

![Figure 5-15](image) Close-up of geometric representation of Least Squares method with two constraints.

One might assume that, given an unconstrained solution that violates both constraints, the proper technique would be to set both voltages to their constrained values, point \( D \). This would be incorrect. Recall that the constrained minimum is the closest point on the feasible area of the hyperplane to the unconstrained optimum. In this case, \( \overline{AB} \) is shorter than \( \overline{AD} \). The proper technique is to determine which constraint is most violated, which is the binding constraint. Once the individual voltage is set to this constraint, the least squares method can again be applied to the remaining voltages to
determine if other voltages lay outside of the feasible space. If not, the length of $\overline{AB}$, will be minimized, finding the optimal constrained point $B$.

**New Least Squares vs. lsqlin**

Utilizing this new least squares method for determining the optimal voltages and minimized RMS error rather than using *lsqlin* immediately results in a much lower computation time. The average time required to run 100,000 function evaluations of the EME algorithm using the *lsqlin* function was 615 seconds, while running the same 100,000 function evaluations using the new least squares method reduced the required time to 57 seconds, an improvement of more than ten times.

**Online EME Algorithm**

The EME algorithm is able to determine the global optimal solution without having to check every possible combination, but at times it still requires a large number of function evaluations to find the optimum. As shown previously, runs that require up to 100 million function evaluations are possible. This is still a drastic reduction in the required number of function evaluations compared to exhaustive search, but makes using the EME algorithm in real time a challenge. This challenge has been overcome with the new online EME algorithm (OHEME). The OHEME algorithm is based on the fact that the number of required function evaluations greatly depends on the number of actuators that must be grouped. As the temperature changes on the reflector, there will be some
actuators that will not change groups, so they can be removed from the optimization routine.

**Online EME Methodology**

The OEME algorithm works as follows: first, a grid of data points must be determined offline by using the normal EME algorithm. These data points will be used for the online implementation. When a temperature profile is given, the OEME algorithm first determines a set of data points that will bracket the temperature profile. For example, if the online temperature profile is $T_0 = 30K$, we might use the data points of $T_0 = 20K$ and $40K$, $ΔT = 70K$ and $90K$. The OEME algorithm compares all four data points and determines which actuators do not change groups over the temperature range. Setting these actuators as fixed, the EME algorithm is run on the remaining actuators, allowing them to be placed in the optimal grouping. If enough actuators are fixed, the EME algorithm can find the optimal solution in almost real time, allowing it to be used for online grouping optimization.

Determining the actuators that do not change groups is not an insurmountable problem; however, neither is it trivial. The numbering of the groups is arbitrary, so comparing the group number from different runs does not suffice. One simple way to determine those actuators that do not move groups is to determine the optimal group numbering where the most actuators are located in the same group in each of the four runs. For example, if actuators 1 and 2 are grouped together in group 1 for the first run, in group 2 for the second and third, and group 3 for the fourth run, the optimal group
numbering would be for each of these groups to be labeled with the same group number. To find the optimal group numbering, an exhaustive search can be used because of the small number of function evaluations needed. The groups are renumbered, and the total number of actuators that are in the same group for all four runs is determined. After iteratively comparing all group numbering possibilities, the case with the most actuators in the same groups is chosen for the group numbers. This allows us to remove the most number of actuators from the OEME algorithm, increasing the speed of the algorithm.

*Online EME Results*

The OEME algorithm has been tested for a number of different temperature profiles. The first is when the online temperature profile is $T_0 = 30K / \Delta T = 80K$, and using the data points of $T_0 = 20K / \Delta T = 70K$, $T_0 = 20K / \Delta T = 90K$, $T_0 = 40K / \Delta T = 70K$, and $T_0 = 40K / \Delta T = 90K$. These data points were run offline, with an average of just over 52 million function evaluations needed to determine each optimal grouping. Using the OEME algorithm, it was determined that 20 of the 32 actuators did not change groups for the four locations chosen. Letting these actuators be fixed in their respective groups, the OEME algorithm was able to search the entire design space in only 974 function evaluations, which can easily be run in real time. For comparison, the normal EME algorithm was also run for the case of $T_0 = 30K / \Delta T = 80K$, which required 43.6 million function evaluations to search the design space, generating the same optimal grouping, shown in Figure 5-16, as the OEME algorithm.
Figure 5-16  Optimal actuator grouping for $T_0 = 30K / \Delta T = 80K$, found using the OEME algorithm. The color represents the optimal applied voltage to each group.

The four temperature profiles utilized for the OEME algorithm do not have to perfectly box the desired temperature profile, simply bracket the desired profile. Using the temperature profiles of $T_0 = 0K / \Delta T = 40K$, $T_0 = 40K / \Delta T = 0K$, $T_0 = 40K / \Delta T = 40K$, and $T_0 = 10K / \Delta T = 10K$, the OEME algorithm is used to calculate the optimal grouping at $T_0 = 25K / \Delta T = 25K$. The OEME algorithm was only able to find 10 actuators that did not change groups, and a total of 306,948 function evaluations were needed to search the entire design space. This is still a great improvement over the 196 million function evaluations needed for the basic EME algorithm, and required less than 3 minutes to converge. Due to the slow change in temperature on the reflector, this is sufficiently fast. The optimal grouping found is shown in Figure 5-17.
Conclusions

In this chapter a new method, the En Masse Elimination technique, has been developed to derive the optimal actuator-power supply combinations for the case when there are more actuators than power supplies. This method is presented and a number of improvements to the basic method have been proposed. The observations and conclusions are summarized as follows:

1) An effective and efficient method (EME) has been developed to group actuators to power supplies that can determine the global optimal solution (not achievable with commonly used GA approaches) without the need to test every possible combination of actuator groupings as an exhaustive search.

Figure 5-17 Optimal actuator grouping for $T_0 = 25\,\text{K} / \Delta T = 25\,\text{K}$, found using the OEME algorithm. The color represents the optimal applied voltage to each group.
would require. Through numerical simulations on a beam as well as on a reflector test-bed, the EME algorithm has been shown to find the global optimal grouping while also reducing the total number of needed function evaluations by 99.9%.

2) Through the use of a case study, it has been shown that the greedy algorithm outperforms both the randomly chosen initial CBC and those chosen by a short and long genetic algorithm.

3) A new method for minimizing the RMS error is introduced. This method decreases the total required computational time from 615 seconds per 100,000 function evaluations to 57 seconds per 100,000, an improvement of more than 10 times.

4) The EME method has been expanded to be used for online grouping. By determining which actuators do not change groups over time, the number of required function evaluations is drastically reduced. While this method is not instantaneous, as long as the surface error is not changing rapidly the online EME is an effective means for online grouping.
Chapter 6

Experimental Results for EME

In order to validate the EME algorithm, an experimental system has been developed. In this chapter, the experimental setup will be explained, followed by a step-by-step experimental procedure. Finally, the experimental results are shown, demonstrating the validity of the EME method.

Experimental Test Setup

The EME algorithm was validated using an experimental system comprised of a pinned-pinned beam and 8 linear force actuators. The dimensions given in Table 6 correspond to the symbols in Figure 6-1.

Table 6  Physical dimensions of experimental beam

<table>
<thead>
<tr>
<th></th>
<th>Symbol</th>
<th>Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of beam</td>
<td>l</td>
<td>600 mm</td>
</tr>
<tr>
<td>Height of beam</td>
<td>h</td>
<td>6.35mm</td>
</tr>
<tr>
<td>Width of beam</td>
<td>w</td>
<td>25.4 mm</td>
</tr>
<tr>
<td>Distance between actuators</td>
<td>a</td>
<td>50 mm</td>
</tr>
</tbody>
</table>
The actuators are 24 V linear solenoid actuators, each with a maximum force of 22 N. Each actuator is powered using a controllable DC power supply (TENMA, 72-6610) and controlled using dSpace software and hardware on a PC. The surface displacement is measured using a Polytec OFV-303 Laser Vibrometer mounted on a linear stage with a maximum travel of 350 mm. The experimental setup can be seen in Figure 6-2 and Figure 6-3. An overhead view is shown in Figure 6-4, where the actuator numbering is shown. This numbering will be used throughout the experiment.

Figure 6-1 Dimensions for beam/actuator system used for experimental validation.
Figure 6-2 Schematic of experimental setup.

Figure 6-3 Experimental setup for beam/actuator system. The beam, actuators, and laser vibrometer are shown, while the PC and power supplies are not.
There are a number of physical constraints on the system that must be accounted for prior to running the experiment. First, due to the limitations on the travel of the linear stage, the entire 600 mm beam could not be measured; only the middle 350 mm. Second, the solenoid actuators are “push” actuators, meaning they can only supply a positive force to the structure. This requires that the applied voltage must be positive, with a maximum applied voltage of 24 V.
Experimental Procedure

Measuring Beam Displacement

To measure the beam displacement, a number of steps must be followed. First, as mentioned previously, the laser vibrometer is mounted to a linear stage. The beam is scanned by utilizing an S-curve jog in the linear stage, with a pause at each end of the beam. The S-curve was used to reduce vibrations caused by starting and stopping the linear stage. The pause allows us to co-locate the beam location with the time series data from the laser vibrometer. A sample run is shown in Figure 6-5.

Figure 6-5 Time series laser Vibrometer data for a sample run of beam measurement.
The laser vibrometer output is divided into three distinct sections. The first steady state is recorded while the linear stage is not moving. The linear stage then begins to move, causing the laser to scan the surface of the beam. Finally, the linear stage finishes the scan, and pauses for another steady state measurement. The stage then returns to its initial position to repeat the measurements. As can be seen here, the beam is not perfectly parallel to the path of the linear stage. This was intentionally done so that the start and stop of the linear stage could be easily deduced from the measurement data. By knowing the S-curve used by the linear stage, as well as locating the time stamp of the start and stop of the linear stage, we can extract the beam location signal from the laser vibrometer signal. To find the beam displacement signal, we must compare the measurement of the beam after actuation with the measurement of the beam before actuation. We can see how this has been accomplished in Figure 6-6, where multiple beam measurements are shown.
Figure 6-6  Time series laser vibrometer data for a sample run of multiple measurements of the beam. The beam is measured once without actuation, followed by another measurement with action, and then repeated.

First, the beam is measured without actuation. As the linear stage is moved back to its original location, the actuation voltage is applied to the actuators, and the beam is then measured again. After this measurement, the actuation voltage is removed, and the beam is again measured without any applied voltage. As can be seen in the figure, there is some drift that occurs over time. It is believed that this is caused by the laser vibrometer itself. To determine displacement, the vibrometer integrates a velocity function. Any small error in the laser reading can cause the signal to slowly drift over time. To overcome this possible problem, the data collected for each actuated run is
compared to the non-actuated data taken immediately before and immediately after. In this way, any linear drift will be removed from the measurement.

Even after utilizing this method to remove drift, there is still some uncertainty in the measurement. It has been determined that by taking the average of at least 5 measurements, repeatable results can be found. Figure 6-7 shows the beam displacement when only actuator 4 is powered. The beam was measured 8 different times, and these runs along with their average are shown below.

Figure 6-7 Data for 8 measurements and their average when actuator 4 is powered.
Hysteresis

One other problem must be overcome before the EME algorithm can be tested on the physical experiment is that the linear actuators used exhibit some hysteresis. It is thought that the actuators stick slightly as voltage is applied, and thus show some hysteretic effect, as seen in Figure 6-8.

![Graph showing hysteresis](image)

Figure 6-8  Out of plane deformation at center point of beam given a sinusoidally applied voltage on actuator 5.

A sinusoidal voltage is applied to actuator 5, and the displacement is measured at only the center point of the beam. This voltage was applied for two different conditions; first, a sinusoidal voltage varying from 0 to 15 volts is shown in blue, while a sinusoidal
voltage that varies from 0 to 22.5 volts is shown in green. The hysteretic effect is clearly shown. The system remains stationary from 0 to 11.5 volts, and then begins to move. Once the initial stickiness is overcome, the system is fairly linear. In order to get repeatable results, for each test, all voltages are turned off, the system is allowed to reach steady state, and then the voltages are slowly increased to their desired magnitude. In this way, we will always follow the same hysteresis curve up, giving consistent results.

**ICM on Experimental Beam**

To determine a model for the beam/actuator system, the ICM method mentioned in Chapter 3 was used. The influence of each individual actuator was determined by measuring the beam displacement while the actuator was powered with 16 volts, and subtracting the beam displacement measured while the actuator was powered with 15 volts. This gives us the influence vector for an incremental voltage of 1 volt on each actuator. The influence vector was found for each actuator and the results are shown in Figure 6-9. The actuators numbers are the same as in Figure 6-4.
Figure 6-9 Influence of an increment of 1 V applied to individual actuators on beam surface displacement.

Using the influence vector from each actuator and assuming linearity, the total displacement due to the influence of all actuators can be determined for any given voltage values.

**Error Profiles**

The EME algorithm was used to attempt to control the beam deflection to match three desired surface profiles, as shown in Figure 6-10.
The first profile was determined by combining the actuators into three groups (1,3,4), (2,5,7,8) and (6) and applying a voltage of 22 V, 10 V, and 17 V to the groups, respectively. These voltages were applied experimentally to the actuators on the beam, and the profile of the beam was measured. The EME algorithm is then used to determine the optimal grouping and voltage given this beam profile, without knowledge of the grouping or voltages used to generate the profile. Utilizing this profile allows us to know \textit{a priori} the optimal grouping and voltage, allowing us to check our work. All that the EME algorithm sees, however, is the desired profile. The second profile was also
determined by grouping the actuators into three groups, (8), (1,2,3,4,5), and (6,7) and applying voltage of 15 V, 0 V, and 24 V to the groups, respectively. The third profile is a half-sine wave, with a magnitude similar to the first two profiles.

**Experimental Results**

Utilizing the desired profile as well as the actuator influence vectors, the EME algorithm is used to determine the optimal grouping and voltage for each profile. The final RMS error between the profile determined by the EME algorithm and the desired profile is then calculated. Figure 6-11 shows the first desired profile along with the EME calculated profile.
Figure 6-11 Comparison of beam displacement for first desired profile and EME calculated profile.

As expected, the profile determined by the EME algorithm is a close match to the desired profile. After only 42 function evaluations, the optimal grouping is found with a measured RMS error of 0.24 μm. Table 7 shows the grouping and voltage determined through the EME algorithm, which compares very well with the originally applied grouping and voltage which was used to obtain the desired shape.
Table 7  EME iteration results for profile 1 compared to expected values.

<table>
<thead>
<tr>
<th>EME grouping</th>
<th>Expected grouping</th>
<th>EME voltages</th>
<th>Expected voltages</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1,3,4)</td>
<td>(1,3,4)</td>
<td>20.8 V</td>
<td>22.0 V</td>
</tr>
<tr>
<td>(2,5,7,8)</td>
<td>(2,5,7,8)</td>
<td>10.0 V</td>
<td>10.0 V</td>
</tr>
<tr>
<td>(6)</td>
<td>(6)</td>
<td>16.8 V</td>
<td>17.0 V</td>
</tr>
</tbody>
</table>

To verify that the optimal grouping and voltage has in fact been found, every possible grouping combination was numerically tested. In this small example, only 6561 function evaluations are needed to check each combination. This is not feasible for even a slightly more complex system; as stated previously, having 10 actuators and 5 power supplies would require over 10 million function evaluations to check each combination. For this case, the optimal voltages for each possible combination were found and the RMS error was calculated. Figure 6-12 shows a histogram of the RMS error for each grouping. The RMS error ranges from a low of 0.24 μm to a high of 1.53 μm with a mean of 0.60 μm.
For the second profile, 92 function evaluations were required to determine the optimal grouping, which produced a measured RMS error of 0.13 μm. The desired and EME calculated profiles are given in Figure 6-13. The histogram of the RMS error for each grouping is shown in Figure 6-14. The RMS error ranges from a low of 0.13 μm to a high of 2.4 μm, with a mean of 1.35 μm. Table 8 gives the applied grouping and voltage, as well as the grouping and voltage determined through the EME algorithm; it is again clear that the two sets compare well.
Figure 6-13 Comparison of beam displacement for second desired profile and EME calculated profile.

Figure 6-14 Histogram of RMS error for each grouping for second profile. Each bar represents a range of approximately 0.05 $\mu$m while the height of the bar represents the number of groupings in the range.
Table 8  EME iteration results for profile 2 compared to expected values.

<table>
<thead>
<tr>
<th>EME grouping</th>
<th>Expected grouping</th>
<th>EME voltages</th>
<th>Expected voltages</th>
</tr>
</thead>
<tbody>
<tr>
<td>(8)</td>
<td>(8)</td>
<td>13.2 V</td>
<td>13.0 V</td>
</tr>
<tr>
<td>(1,2,3,4,5)</td>
<td>(1,2,3,4,5)</td>
<td>0.0 V</td>
<td>0.0 V</td>
</tr>
<tr>
<td>(6,7)</td>
<td>(6,7)</td>
<td>24.0 V</td>
<td>24.0 V</td>
</tr>
</tbody>
</table>

Finally, the half-sine profile required 58 function evaluations to find the optimal grouping, which yielded an RMS error of 0.70 μm. The desired and EME calculated profiles are given in Figure 6-15, while Figure 6-16 shows the histogram of the RMS error for each grouping. The RMS error ranges from a low of 0.7 μm to a high of 1.40 μm with a mean of 1.00 μm. Table 9 shows the optimal grouping and voltages found through the EME algorithm.
Figure 6-15  Comparison of beam displacement for third desired profile and EME calculated profile.

Figure 6-16  Histogram of RMS error for each grouping for third profile. Each bar represents a range of approximately 0.05 μm while the height of the bar represents the number of groupings in the range.
Table 9  EME iteration results for profile 3.

<table>
<thead>
<tr>
<th>EME grouping</th>
<th>EME voltages</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3,4,5,6)</td>
<td>0.0 V</td>
</tr>
<tr>
<td>(1)</td>
<td>16.7 V</td>
</tr>
<tr>
<td>(2,7,8)</td>
<td>23.7 V</td>
</tr>
</tbody>
</table>

Table 10 summarizes the iteration results from the experimental validation effort. Even for a system with tens of actuators, rather than hundreds or thousands, the EME algorithm was able to outperform the exhaustive search method by reducing the number of function evaluations needed to find the optimal grouping for each profile.

Table 10  EME iteration results for experimental validation.

<table>
<thead>
<tr>
<th>Profile</th>
<th>EME Iterations</th>
<th>Possible Combinations</th>
<th>% Reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>42</td>
<td>6561</td>
<td>99.4</td>
</tr>
<tr>
<td>2</td>
<td>92</td>
<td>6561</td>
<td>98.6</td>
</tr>
<tr>
<td>3</td>
<td>58</td>
<td>6561</td>
<td>99.1</td>
</tr>
</tbody>
</table>

Conclusions

In this chapter, the EME algorithm has been shown experimentally to be effective in finding the optimal solution while significantly reducing the total number of function evaluations needed to guarantee the global optimal grouping. The observations of this experimental investigation are summarized as follows:
1) In order to effectively use the EME algorithm, care must be taken when acquiring surface displacement data. The EME algorithm performs well, but requires an accurate surface error in order to function properly.

2) The ICM method performed admirably for the experimental validation. By using the ICM method, the analytical model was not needed, thus greatly reducing the complexity of the experiment.

3) While the EME algorithm was envisioned to be used as a tool with large systems of actuators, it still can reduce the number of function evaluations needed even on a relatively small system like that shown here.
Chapter 7

Conclusions and Recommendations

The purpose of this chapter is to summarize the research efforts of this dissertation and showing the conclusions that can be drawn for the previous chapters. The contributions of this work are discussed as well as recommendations for future work.

Summary

The purpose of this research was to advance the state of the art of surface control by implementing high-precision control on a flexible reflector using PVDF actuators. Along with this control, methodologies have been determined to overcome real world problems that can develop when implementing this technology.

To facilitate the design and analysis of a flexible reflector/actuator system, a theoretical model was derived. The reflector was modeled as a thin, shallow, spherical cap, and the system was solved using a Ritz method with Fourier-Bessel series expansion. PVDF surface actuators were modeled using the same method and were assumed to be perfectly bonded to the reflector surface. A contribution of this modeling technique is that it can be quickly modified in order to accommodate different actuator locations. Surface errors comparable to those experienced by a reflector in space were modeled. Specifically, “W-error”, which is caused by the inflation of the reflector, and the error experienced from temperature changes as the reflector circles the earth.
Experimental results have shown that while the analytical reflector model is generally correct, due to idiosyncrasies in the reflector it should not be used for online control. Therefore, the Influence Coefficient Method (ICM) was proposed for online system identification for use with an online control law.

Using the PVDF actuators, surface control was executed using a least squares control law, where photogrammetry was used to determine the out-of-plane displacement at designated points on the surface. Least squares can be used because of the quasi-static nature of the surface error. While there is time dependence in the temperature profile, the rate of change is very low, so that at any moment in time the surface error can be assumed to be static. It has been shown that on a fully covered theoretical 35 meter space reflector, the error resulting from a 40 K temperature change on the surface of the reflector can be controlled to within the desired tolerances.

When the surface cannot be fully covered with actuators, the optimal placement of actuators must be considered. In this dissertation, acceptable results have been found using a genetic algorithm. When the surface is sparsely covered, a symmetric constraint is applied to the genetic algorithm in order to facilitate the convergence of the algorithm. This has been shown to reduce the required time to find the optimal solution while having a negligible impact on the final result.

Additionally, a focus of this work has been implementing the constraint of having more actuators than available power supplies. This real world constraint has been incorporated into the model through the use of a binary matrix. One method to address this issue of less power supplies than actuators is to group multiple actuators together and
power each group with a single power supply. A new method to determine the optimal grouping of actuators given this constraint has been developed, called the En Masse Elimination (EME) method. This method determines the global optimal solution without having to exhaustively search every possible grouping combination. A number of improvements to the EME method have been given which increase the speed of the algorithm, and expand the possible uses of the algorithm. The EME method was then experimentally validated using a single pinned-pinned beam. Using a beam rather than a reflector reduced the complexity of the problem while still showing the functionality of the EME algorithm.

Conclusions

Much work has been done in the modeling and experimental validation of different concepts relating to the flexible reflector. The conclusions drawn in this study can be divided into the following two subsections.

**Reflector**

1) Through experimental validation on a 2.4 meter reflector, it has been shown that the reflector/actuator model developed here is adequate for analytical simulations for case studies such as actuator placement, but due to idiosyncrasies inherent in each reflector, is not sufficiently accurate for use in a feed-forward control system.
2) When rectangular actuators are present, a method has been determined to approximate these actuators in the model. Utilizing no more than 3 wedges for each actuator provides an excellent approximation for a rectangular actuator when modeled in polar coordinates.

3) Given an accurate measurement of the surface error, the least squares control is a simple yet effective means to accurately control the surface error of a reflector. Using least squares control, it has been shown that on a fully covered reflector, the RMS error for commonly seen surface errors can be reduced below the required threshold limit.

4) A new online identification method (ICM) has been developed which allows a model to be generated using the actual reflector. Experimental results from the 2.4 meter reflector show that this is an appropriate method for determining the reflector model and can be used in a control system.

5) A methodology utilizing symmetry and a genetic algorithm has been developed for actuator placement optimization. It has been shown that by requiring symmetry of actuator locations reduces the number of function evaluations necessary to converge to an acceptable solution without drastically changing the final result found.

**En Masse Elimination**

1) The EME algorithm has been developed to group actuators to power supplies, guaranteeing the global optimal solution (not achievable with commonly used
GA approaches) without the need to test every possible combination of actuator groupings as an exhaustive search would require. Through numerical simulations on a beam as well as on a reflector test-bed, the EME algorithm has been shown to find the global optimal grouping while also reducing the total number of needed function evaluations by 99.9%

2) Through the use of a case study, it has been shown that the speed of the basic EME algorithm is dependent on the initial Current Best Case (CBC) design. A simple yet effective greedy algorithm has been developed which allows us to determine a good choice for the initial CBC. This algorithm outperforms both a short run genetic algorithm (10,000 function evaluations) and a long run genetic algorithm (2,000,000 function evaluations).

3) A new method for minimizing the RMS error has been introduced. This method decreases the total required computational time from on average 615 seconds per 100,000 function evaluations to 57 seconds per 100,000, an improvement of more than 10 times.

4) The EME method has been expanded to be used for online grouping. By determining which actuators do not change groups over time, the number of needed function evaluations is drastically reduced. While this method still requires multiple function evaluations, as long as the surface error is not changing rapidly, it is an effective means for online grouping.

4) The EME algorithm has been effectively validated using an experimental beam test-bed. It has been shown that accurate surface error is needed for the
EME algorithm to perform properly. The ICM method was used for online model identification and performed admirably. By using the ICM method, the analytical model was not needed, thus greatly reducing the complexity of the experiment.

**Recommendations for Future Work**

This section contains recommendations for furthering the research performed in this dissertation. While some of these recommendations are for incremental improvements of the current methods, other ideas are postulated to increase the scope and applicability of this technology.

**Reflector Control**

Current work has focused entirely on utilizing PVDF actuators for surface control of a flexible surface. PVDF was chosen because it is flexible enough to be attached directly to the surface of the reflector, thus allowing for localized control. While this is a definite advantage of PVDF, this material does not produce the force necessary for large-scale deformation. There are a number of possibilities that could be considered in future work, such as using a hybrid approach with multiple types of actuators. PVDF could be used for localized deformation, while a cable actuated boundary control could be used for larger, global deformations. Another possibility is to use the inflation pressure for global control, concurrently with the PVDF. This would add another layer of difficulty to the
modeling, due to the interaction between the stiffness of the membrane and the tension in the membrane.

Another area of focus is in dealing with the actual measurement of the surface. An accurate surface profile is integral to the control algorithms performance. For this study, it was assumed that photogrammetry would be used, but this can be challenging to implement, especially in space. Two avenues of research exist here: how can we more effectively measure the surface deformation, and is there a way to tailor the control algorithm so that the full surface profile is not needed. Alternative methods can be explored and compared to the current state of the art.

**EME Improvements**

There are two main avenues of future research for the EME algorithm. The first is improvement on the speed of convergence of the algorithm. The current method is able to determine the global optimum in reasonable time for systems with 10s and 100s of actuators. In order to use this method on larger systems, the speed of convergence of the algorithm must be improved. The choice at each iteration for which actuators to group together for testing can increase or decrease the speed of the algorithm greatly. Currently, an iterative approach is used to determine which actuators are grouped together, for example, actuators 1 and 2 are grouped together, and then 1 and 3, 1 and 4, etc. It is possible that by more intelligently choosing the order of the grouping, the total number of function evaluations needed could be greatly reduced. Additional future work could be done to determine how to make the algorithm smarter.
Second, the current applicability of the EME algorithm is very small. It is only when actuator grouping is required that the EME algorithm valid, and not for other optimization problems. Thought should be taken to determine if there are any other applications with which the EME algorithm can be employed. In large structures that require large number of actuators for control, there is great need for an efficient yet effective algorithm to determine the optimal placement of actuators. For example, a cable bridge could have a small number of actuated cables, but a large number of possible locations for the actuators. This can be a difficult problem to solve. Currently, the EME algorithm is not capable of determining the actuator locations for this example, because it does not require actuator groupings. Expanding the methodology behind the EME algorithm into other types of optimization is a promising application of this research.


VITA

Jeffrey R. Hill was born in Tacoma, WA in 1979. He received his B.S. degree in Mechanical Engineering from Brigham Young University in Provo, UT in 2004. In 2005, he received his M.S. degree in Mechanical Engineering, also from Brigham Young University. In August 2005, he enrolled in the graduate program in Mechanical Engineering at The Pennsylvania State University and began to pursue his Ph.D. degree. His research interests include vibrations and control, non-linear dynamics, and large scale optimization.