A GENERALIZED APPROACH TO MODEL AND OPTIMIZE
MULTI-INPUT MULTI-OUTPUT
COMMUNICATION AND IMAGING SYSTEMS

A Dissertation in
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by
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ABSTRACT

The use of multiple elements in radio frequency (RF) wireless communication systems has been proven to be more robust to rich scattering environments, which otherwise contributes to random fading, long considered detrimental to traditional single-element systems. However, performance improvements obtained by multiple-input multiple-output (MIMO) systems can be severely limited by physical constraints such as antenna separation, which directly affects the random statistics with correlation. In this dissertation, we review various determining factors and their effects on a MIMO RF wireless system and apply the conclusions to other areas for modeling and optimization.

Until now, MIMO systems in communications have been restricted to wireless mediums. However, the basic tenets of MIMO transmission also apply to a general communication or imaging system. This is evident in copper cable Ethernet transmission, where the user has access to four parallel twisted-pair wires and the performance is limited by inter-channel-interference (ICI), not only inter-symbol-interference (ISI). These interferences are usually mitigated by employing single-input single-output (SISO) equalizers and interference cancellers. However, in a MIMO model, the cross-channel interferences carry useful information that can be salvaged by employing a MIMO equalizer/canceller, and thus contribute to signal-to-noise-ratio (SNR) increment. We investigate the effectiveness of different MIMO equalization and cancellation techniques in such semi-static channels, and provide a comparison of SNR and bit-error-rate (BER). Signal constellation design for modulation and coding is also an important factor for achieving the theoretical capacity bounds, and these are presented for hypothetical systems supporting 10Gbps, 40Gbps and 100Gbps data rates.

The mathematical analysis for MIMO systems easily extends itself to other communication systems, such as free space optical (FSO) links. These communication links suffer from log-normal fading due to scintillation prevalent in distances longer than a hundred meters. Cloud and fog also impose barriers of attenuation in these channels. A simulation model is used in conjunction with scintillation characteristics to show the benefits of using multiple transmitters and receivers for FSO links. Furthermore, MIMO
techniques are extended to indoor optical wireless communications in the infra-red wavelength range, which have the potential to provide data rates in the gigabits per second range. A MIMO modeling technique is presented here, which gives an accurate representation of the channel, and MIMO optimization techniques can be employed which use the available channel resources judiciously to improve data rate and error performances.

Active optical imaging is a parallel to communication systems, with spatial information added on to the temporal information, where the latter is the only concern in communications. The mathematics behind MIMO has the flexibility to incorporate multiple variables, in this case space and time, easily into analysis and simulation. Active optical imaging by means of laser is a well-known technology used by remote sensing and surveillance communities; however, these suffer tremendously in the presence of scattering particles and turbulence. Scintillation is the greatest obstacle in imaging cases, and it causes the ideal point spread function (PSF) to broaden and distort or blur the received image. MIMO techniques, comparable to MIMO equalization for communication systems, can be applied to blindly deconvolve the received image from the distorted PSF. These novel techniques are discussed in this thesis, along with contrast and resolution improvements they offer.

The contributions of this thesis are manifold. The concept of MIMO has been applied to several communication and imaging systems and is liberated from the RF wireless communications paradigm, and meaningful quantitative and qualitative connections have been drawn to applications other than RF wireless, namely FSO and indoor optical communications, Ethernet copper cable transmission, and deblurring for active imaging. It is expected that the value of the findings will trigger a tremendous motivation in thinking ‘jointly’ in terms of multiple variables rather than in terms of the single time or space-dependent variables which communication engineers are inclined to favor. The value of the analyses also lies in the fact that communications and imaging are shown to be similar problems, when cast into a general MIMO framework.
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My father, himself being a Ph.D. in Mechanical Engineering, has been my role model, and he has been my unofficial advisor as well. My mother, a medical doctor, has been a constant caregiver even from half a world way. My younger brothers, Mahran and Numair, have cheered me in everything I attempted. My cousin Zubair is no less of a brother and a peer who has constantly motivated me.

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Lastly, I praise the Almighty for giving me the fortune and the strength.
To R. S.,

My parents, and

To the memory of my grandparents.
Chapter 1

Introduction

1.1 Motivation

Since the formal introduction of multiple-input multiple-output (MIMO) concept in wireless communications with the seminal paper by Telatar [1], and Foschini’s analysis of multiple-antenna channels under fading [2], tremendous amount of research and development efforts have been concentrated on design and implementation of MIMO radio frequency (RF) wireless communication systems. Simpler forms of MIMO techniques, such as spatial diversity [3] and polarization diversity [4][5] have been known to improve communication performance of microwave links under random fading, long before information theoretic analyses involving multiple antennas were formulated. Extensive research in MIMO wireless systems have been proven practically effective with the inclusion of antenna arrays on both transmit and receive ends in the most recent wireless networking standard, IEEE 802.16 or WiMAX [6], as well as amendments made to IEEE 802.11 or WiFi for multiple-antenna inclusion.

The obstacles that conventional RF wireless channels encounter are multipath fading, power limitation and regulation, frequency bandwidth congestion, and inter-channel interference. These limit the permissible data rate on current wireless radio channels. Multipath fading has been substantially addressed and, furthermore, exploited to increase signal strength in Code-Division Multiple Access (CDMA) [7] and Orthogonal Frequency Division Multiplexing/Multiple Access (OFDM/OFDMA) [8]. While the former has achieved tremendous success in broadband mobile cellular data communications, the latter has found its standing in indoor low-mobility networking. Both technologies form the basis of IEEE 802.11, popularly known as WiFi [9]. Fig. 1-1 illustrates in a simple manner how a multipath-rich environment can yield better statistics
of relative power, which can either be exploited to improve SNR, and hence bit-error-rate (BER), or to enhance the data transmission capacity of the wireless channel.

Several means of mitigating the deleterious effects in wireless communications have been proposed and implemented, including space, angle, frequency, and polarization diversity techniques. Spatial diversity was used to be considered a very expensive technique of obtaining better signal-to-noise ratio (SNR) under severe Rayleigh fading, until the widespread introduction of antenna arrays with ease of manufacturing. But as antenna design has entered the miniaturization era with introduction of new wavelengths and new materials, multiple antennas have paved the way for building MIMO systems more conveniently than before.

As much as Rayleigh and Rician fading are the banes of RF wireless transmission, so is log-normal fading caused by scintillation in free-space optical (FSO) links. Use of multiple laser transmitters and receivers on such channels has also been proven conducive for obtaining higher reliability through spatial diversity [10][11][12][13]. While there is no fading present in wireline channels, such as Ethernet copper cables, they suffer from inter-channel-interference (ICI), similar to cross-polarization interference on microwave links [4]. Traditional SISO mitigation techniques can be extended to multiple transmitters/receivers in a similar manner as in [4]. Different MIMO equalization, cancellation, and coding schemes for wireline channels need to be

![Fig. 1-1: Different multiple-input-multiple-output antenna configurations and demonstration of relative power distribution enhancement by use of SIMO with 3 receive antennas.](image)
investigated thoroughly and their performances need quantification, since the attention of cable-based communication industry has not shifted much from the conventional SISO paradigm.

Interestingly enough, the mathematics in MIMO systems allows for multiple variables in time and space, as evident in the immense literature available on Space-Time Coding [14][15][16][17][18][19][20][21][22][23][24][25][26][27]. The nature of this flexibility of MIMO systems can be exploited a great deal towards analysis of recovery techniques for imaging systems. Since imaging requires two or three independent spatial coordinates, as well as the time reference, an imaging problem can be easily cast into similar linear algebraic forms as MIMO communications. This is the motivation to investigate application of MIMO equalization, in the form of deblurring distorted images.

In summary, the main advantage of using MIMO approach to system modeling and design is the availability of all spatial dimensions in mathematical analyses involving communication data or image pixels, which allow us to observe the system with a 'joint' view, rather than from a single spatial dimension perspective which is incomplete. However, a number of practical constraints have to be incorporated into MIMO analyses, before their benefits can be exploited to the advantage of communications and imaging.

1.2 Objective

The objective of this thesis is to address several theoretical and practical limitations of and mitigation approaches for MIMO system modeling and design. In a fading channel, whether it is RF wireless or free-space optical, the capacity and diversity are both limited by the amount of correlation among propagation paths [14][18][21][30] [32][33][34][35][36][37][38]. In most theoretical analyses existent in the literature, the default assumption is absence of correlation which leads to an over-estimate of MIMO performance bounds. The amount of correlation can be reduced by careful selection of propagation wavelength and antenna separation [39]; hence antenna design becomes an important factor to investigate. Space-time coding, together with proper antenna design, can exploit the most benefit out of MIMO wireless channels. Furthermore, ‘cooperative’
relays or ‘artificial’ scatterers can be placed in the propagation area to help in multi-path scattering [40]. Part of this thesis investigates the effects of correlation on MIMO propagation and also describes measures to circumvent these deleterious effects. In general in this thesis, we use the lessons learned from MIMO RF wireless systems and apply them wireline and optical communications, as well as imaging systems, to determine MIMO applicability in each of these scenarios.

In wireline transmission, many standards use multiple parallel lines to deliver data from the source to the destination, which are broadcast in a multiplexed manner, as in Digital Subscriber Line (DSL), and Ethernet. Both technologies are omnipresent today, but they suffer from multi-user and inter-channel interference, more generally termed crosstalk [21][41][42][44][45][46][47][48][49][50]. However, a multivariate approach to analyzing the received signals yields a considerably higher SNR, which leads to a better achievable data rate. This has been previously demonstrated on DSL multi-user channels, and in this thesis, we show that this is also true for Ethernet copper cables in certain circumstances. What we do not have in this scenario is the random statistics of channel coefficients that enhance the data transmission capacity of RF wireless channels. In Fig. 1-2, we can see how interference from unintended pairs on a parallel bundle of cables degrades performance, but from MIMO point of view, the interferences contain useful data.

Fig. 1-2: Interference or crosstalks existent on twisted pair cables
The random statistics are, however, available in Free-Space Optical (FSO) links, in the form of log-normal fades. These fades are caused by scintillation, which is the result of refractive index fluctuations due to temperature gradients that occur along the path of propagation of the transmitted laser \[10][11][12][13][51][52][53][54][55]. Furthermore, clouds and fog pose attenuation and scattering to incident light; laser photons are either absorbed or scattered off the particles according to Mie theory \[56][57][58][59][60][61][62]. With finite receive geometries, having multiple apertures on transmit and receive sides can capture more photons, and after proper processing, can add to the aggregate SNR. As a result, MIMO architecture also helps FSO links to increase data rate. Both the effects of scintillation and scattering are investigated in this thesis, and it is shown that MIMO techniques are superior to single-beam propagation. Fig. 1-3 shows the two atmospheric effects affecting the propagation in a FSO link.

Fig. 1-3: Atmospheric scattering and turbulence on a free-space optical link

Finally, a new signal-processing technique is presented here, which is based on the multi-variate linear algebra theory that essentially forms the basis of MIMO communications. Active optical imaging has been investigated for a long time as a means of surveillance and remote sensing. The problem of imaging through clear weather has been addressed extensively and resolved up to a satisfactory degree \[63][64][65][66]. However, accurate imaging through atmospheric phenomena such as turbulence and scatterers still remain challenging. In Fig. 1-4, we can see a distorted image that is the result of the turbulent aspect of the atmospheric imaging channel; the resolution
degradation is visually discernible from the original image. In this thesis, we present a novel approach of mitigating these deleterious effects by post-processing. This involves recasting the problem in a linear algebraic form, and then applying an iterative algorithm that ultimate outputs a reconstructed image from a distorted or blurred image. Several other deblurring techniques have been developed and these are benchmarked with the new technique to show superiority of the latter.

![Original image and distorted image obtained from an active optical imaging system operating in turbulent atmosphere](image)

Fig. 1-4: Original image and distorted image obtained from an active optical imaging system operating in turbulent atmosphere

### 1.3 Organization

The thesis is organized as follows. Chapter 2 involves an in-depth investigation of the RF wireless channel, both from SISO and MIMO perspectives. The different statistics involving channel modeling are presented; the capacity calculation approach and results are presented. The MIMO techniques reviewed in Chapter 2 serve as the basis for applying the principles to channels under considerations in following chapters. In Chapter 3, we present various ways of using crosstalk to the benefit of data transmission on wireline channels. To this end, we present the channel model, the various crosstalk parameters, SISO and MIMO equalization, pre-coding and adaptive processing techniques, and coding/modulation schemes required to approach the capacity, which we also calculated at the beginning of the chapter. Chapter 4 is dedicated to the study of FSO
links, and log-normal fading in the presence of atmospheric turbulence. The statistics of a turbulent atmosphere are presented, and simulation results are shown for multiple aperture receivers and diversity combining in presence of log-normal fading. In Chapter 5, we investigate the more interesting problem of reconstructing a blurred image, by extending the multi-variate approach used for MIMO communication systems. The blurring mechanism is simulated with the help of phase screens, and several techniques to overcome the blurring are presented with comparison. Chapter 6 deals with modeling an indoor optical wireless channel using MIMO principles, and describes an experimental system to validate the model. Furthermore, it is demonstrated that data rates in the range of Gigabits per second can be achieved on this channel, given the system is optimized carefully in terms of cell size, transmitter directivity, etc. Chapter 7 presents the conclusion of the thesis and outlines future research work.
Chapter 2

MIMO RF Wireless Channels: Correlation and Capacity

2.1 Introduction

MIMO systems have gained unprecedented attention in broadband RF wireless communications due to its promise of higher information capacity and diversity gains in probabilistic environments. Technologies that use multiple antennas have been proven to become a popular application in coming years. The limitations of regulated bandwidth and transmissible power are overcome using diversity in polarization, angle of arrival and space, in the form of MIMO. Interference among multiple multiplexed streams and multiple users are effectively utilized by MIMO systems. Wireless SISO NLOS radio channels suffer from Rayleigh fading, and MIMO systems can improve the fading statistics by various combining schemes. Joint decoding, ease of multiple access and multi-user communications, and intelligent beam-forming are other advantages of using RF MIMO techniques.

Challenges for RF MIMO systems are the effect of correlation which reduces available capacity, inaccurate channel models that assume independence of path statistics, higher antenna spacing for combating correlation, etc. This chapter investigates the determining factors of RF MIMO channel capacity, ways to improve it, and different diversity coding/decoding mechanisms to maximize the received SNR.

2.2 Wireless Channel Modeling

Wireless RF MIMO channels are subject to attenuation, multipath fading, delay spread, shadowing, Doppler Effect, co-channel and adjacent channel interferences, and
additive white (or colored) Gaussian noise. For SISO RF systems, the multipath fading is characterized by Rician and Rayleigh fading for LOS and NLOS operations, respectively.

System models for MIMO RF wireless communication channels have been classified broadly into physical models and analytical models [138]. Analytical models are more tractable for capacity and BER performance calculations, and hence are the only ones considered here. The $n \times m$ MIMO system with $n$ receiving antennas and $m$ transmitting antennas can be represented by a time-variant channel matrix of the form

$$
H(t, \tau) = \begin{bmatrix}
h_{11}(t, \tau) & h_{12}(t, \tau) & \cdots & h_{1m}(t, \tau) \\
h_{21}(t, \tau) & h_{22}(t, \tau) & \cdots & h_{2m}(t, \tau) \\
\vdots & \vdots & \ddots & \vdots \\
h_{n1}(t, \tau) & h_{n2}(t, \tau) & \cdots & h_{nm}(t, \tau)
\end{bmatrix}
$$

(2–1)

where $h_{ij}(t, \tau)$ denotes the time-variant impulse response between the $j$-th transmit antenna and the $i$-th receive antenna. The overall MIMO input-output relation between the $m \times 1$ signal vector $s(t)$ and the $n \times 1$ receive signal vector $y(t)$ is then

$$
y(t) = \int_{\tau} H(t, \tau)s(t-\tau)d\tau + n(t)
$$

(2–2)

where $n(t)$ models the noise process and interference jointly. When the channel is time-invariant, the dependence on $\tau$ vanishes and Eq. (2–2) becomes

$$
y(t) = \int_{\tau} H(\tau)s(t-\tau)d\tau + n(t)
$$

(2–3)

Furthermore, for flat (frequency-non-selective) channels, there is only one tap coefficient for the impulse responses, and Eq. (2–3) reduces to Eq. (2–4)

$$
y(t) = Hs(t) + n(t)
$$

(2–4)

After matched filtering and sampling, the discrete notation for Eq. (2–4) becomes

$$
y = Hs + n
$$

(2–5)

We shall base our simulations and analytical calculations on Eq. (2–5) which only includes the attenuation and narrowband flat fading effects. However, this model can be
easily extended to include frequency-selective fading, log-normal shadowing, and Doppler Effect by incorporating a transversal multi-tap filter and introducing correlation among random tap coefficients. In the following subsections, we introduce several models for obtaining the MIMO transfer function $H$. All these models are narrowband based on a multivariate complex Gaussian distribution of the MIMO channel coefficients. For the general Rician fading case, the channel matrix $H$ can be split into a zero-mean stochastic part $H_s$ and a purely deterministic part $H_d$ as

$$H = \sqrt{1/(1+K)}H_s + \sqrt{K/(1+K)}H_d$$  \hspace{1cm} (2–6)$$

where $K \geq 0$ denotes the Rice factor. For simplicity, we assume $K = 0$ for the following models. Therefore, the joint distribution of the components of the $H$ matrix is given by

$$f(h) = \frac{1}{(\pi^m \det[R_H])} e^{-h^\dagger R_h h}$$  \hspace{1cm} (2–7)$$

where $h = \text{vec}[H]$ and the $nm \times nm$ matrix $R_H = \mathbb{E}[hh^H]$ has the correlations between each component of $H$ as its entries, and it is known as the full correlation matrix. The operator $\text{vec}[\cdot]$ is a stacking operator, i.e. it stacks the columns of a $n \times m$ matrix on top of each other to form a $nm \times 1$ vector. The full correlation matrix $R_H$ can be obtained experimentally from the power correlation matrices $[139]$, or analytically by considering the physical setup of the transmit/receive arrays $[29]$. However, the full specification of $R_H$ involves the measurement or analytical computation of $(nm)^2$ real-valued parameters. Several different models address the issue of reducing this large number by imposing a particular structure on the MIMO correlation matrix.

### 2.2.1 Uncorrelated Rayleigh Fading Model

The most commonly used RF wireless MIMO model is the spatially i.i.d. flat-fading channel, where the elements of $H$ are i.i.d. complex Gaussian. Physically this corresponds to a spatially white MIMO channel which occurs only in rich scattering environments characterized by independent multipath components uniformly distributed in all directions. The uncorrelated fading model can be applied to cases where the
element spacing in the transmit and receive side antennas are larger than at least a quarter-wavelength, and where it can be assumed that the scatterers are distanced enough from the antennas that they reflect the transmitted signals uniformly and illuminate the receive antennas uniformly as well.

2.2.2 Kronecker Model

The Kronecker model assumes that the spatial transmit and receive correlation are separable, i.e.

\[ \mathbf{R}_H = \mathbf{R}_{Tx} \otimes \mathbf{R}_{Rx} \]  

(2–8)

where \( \otimes \) signifies the Kronecker product, and the matrices \( \mathbf{R}_{Tx}, \mathbf{R}_{Rx} \) are the transmit- and receive-side spatial correlation matrices given by

\[ \mathbf{R}_{Tx} = E\left[ \mathbf{H}^H \mathbf{H} \right], \mathbf{R}_{Rx} = E\left[ \mathbf{H} \mathbf{H}^H \right] \]  

(2–9)

The Kronecker model requires specification of the total \( n^2 + m^2 \) elements of the correlation matrices \( \mathbf{R}_{Tx} \) and \( \mathbf{R}_{Rx} \). The Kronecker model was used in [140][14] to propose a simulation model for correlated MIMO channels and in [141] to calculate their capacity. The limitation of the Kronecker model is that it assumes a separable DoD-DoA spectrum, i.e. the joint DoD-DoA spectrum is the product of the individual spectrums. This does not reproduce the exact coupling of a single DoD with a single DoA. The separability assumption implies that the scatterers are located very far away from either side so that they can reflect signals received from each transmitter equally to each receiver and illuminate them uniformly. According to [142], the spatial power correlation matrix on the transmit side is given by

\[ \left[ \mathbf{R}_{Tx} \right]_{nm, nm'} = \rho_{nm, nm'}^{Tx} = \left| \left\langle h_{n,m} h_{n', m'}^* \right\rangle \right|^2 \]

(2–10)

\[ \rho_{nm, nm'}^{Tx} = \frac{E\left[ |h_{n,m}^* h_{n', m'}|^2 \right] - E\left[ |h_{n,m}^*|^2 \right] E\left[ |h_{n', m'}|^2 \right]}{\sqrt{\left\{ E\left[ |h_{n,m}^*|^4 \right] - E\left[ |h_{n,m}^*|^2 \right]^2 \right\} \left\{ E\left[ |h_{n', m'}|^4 \right] - E\left[ |h_{n', m'}|^2 \right]^2 \right\} }} \]
where \( h_{nm} \) denote the \( H \) matrix entry for transmitter \( m_i \) to receiver \( n_j \), where \( 1 \leq m_i \leq m \) and \( 1 \leq n_j \leq n \). The spatial power correlation on the receive side is given similarly. It is assumed that the correlation values remain the same for different \( n_j \), which is indeed a huge assumption, but it enforces the separability of the full autocorrelation matrix. The matrix actually denote the correlation of received power and transmitter power on either side, and does not say anything about how the power from transmit side couples to receive side through scatterers. This is a structural deficiency in the model.

### 2.2.3 Weichselberger Model

The Weichselberger model [30] addresses the limitations of the separability of DoA-DoD spectra of the Kronecker model by incorporating the knowledge of the eigenvectors of the transmit and receive correlation matrices. The eigendecomposition of the transmit and receive correlation matrices are given by

\[
R_{tx} = U_{tx} \Lambda_{tx} U_{tx}^H, \quad R_{rx} = U_{rx} \Lambda_{rx} U_{rx}^H
\]

The channel model, then, can be synthesized from an i.i.d. zero-mean Gaussian matrix \( G \)

\[
H = U_{rx} \left( \hat{\Omega} \odot G \right) U_{tx}^T
\]

where \( \odot \) denotes the Schur-Hadamard product (element-wise multiplication), and \( \hat{\Omega} \) is the elementwise square-root of an \( n \times m \) coupling matrix \( \Omega \) whose real-valued and nonnegative elements determine the average power coupling between the transmit and receive eigenmodes. This coupling matrix allows for the joint modeling of the transmit and receive correlations. The Weichselberger model requires specification of the transmit and receive eigenmodes \( U_{tx} \) and \( U_{rx} \), and of the coupling matrix \( \Omega \). This requires computation of \( n(n-1) + m(m-1) + nm \) real parameters. However, since the capacity and diversity order of a MIMO channel are independent of the eigenmodes, the capacity calculation only needs the specification of the coupling matrix.
This seems to be a quite acceptable general model for MIMO systems, as the parameters can be easily measured experimentally by transmit-receive sides, and it takes into account not only the transmit and receive side DoD and DoA spectra, but also how the scatterers couple power from a particular transmit DoA bin to several receive DoD bins. The coupling matrix is defined as

$$\left[ \Omega \right]_{n' m'} = \omega_{n' m'} = E_H \left[ u_{R, n', m'}^H H u_{R, n', m'}^* \right]$$  \hspace{1cm} (2–13)

The coefficients of the coupling matrix specify the mean amount of energy that is coupled from the $m'$-th eigenvector of the receive side to the $n'$-th eigenvector of the transmit side. Also the eigenvalues of the transmit and receive correlation matrices are implicitly given by the elements of $\Omega$ as

$$\lambda_{R, n'} = \sum_{m'=1}^{m} \omega_{n' m'}, \quad \lambda_{T, m'} = \sum_{n'=1}^{n} \omega_{n' m'}$$  \hspace{1cm} (2–14)

2.2.4 Virtual Channel Model

In [143][144], a MIMO model called virtual channel representation is proposed as

$$H = F_m \left( \Omega \odot G \right) F_n$$  \hspace{1cm} (2–15)

Here, the DFT matrices $F_m$ and $F_n$ contain the steering vectors for $m$ virtual transmit and $n$ virtual receive scatterers, $G$ is an $n \times m$ i.i.d. zero-mean Gaussian matrix, and $\Omega$ is an $n \times m$ matrix whose elements characterize the coupling of each pair of virtual scatterers, i.e. $(\Omega \odot G)$ represents the “inner” propagation environment between virtual transmit and receive scatterers. The virtual channel model can be viewed as a special case of the Weichselberger model with the transmit and receive eigenmodes equal to the columns of the DFT matrices. In the case where $[\Omega]_{ii} = 1$, the virtual channel model reduces to the i.i.d. channel model, that is, rich scattering with full connections of (virtual) transmit and receive scatterer clusters. Due to its simplicity, the virtual channel model is mostly useful for theoretical considerations like analyzing the capacity scaling behavior of MIMO
channels. The virtual channel representation in terms of DFT steering matrices is appropriate only for uniform linear transmit and receive arrays.

2.3 Correlation Modeling

Since correlation among transmit and receive elements of a multi-element antenna array system influences analytical model, and thus the capacity of a MIMO system, a ray tracing model is presented here that facilitates its calculation.

2.3.1 One-Ring Scattering Model

In the one-ring scattering model, we deal with effective scatterers instead of actual scatterers, and apply ray tracing to obtain channel transfer matrix $\mathbf{H}$, and then the spatial correlation matrix $E[\mathbf{H}\mathbf{H}^H]$. The setup is shown in Fig. 2-1. The model is appropriate for fixed wireless communication context, where the Base Station (BS) is usually elevated and unobstructed by local scatterers, and the subscriber unit (SU) is surrounded by local scatterers.

![One-ring scattering model to calculate spatial correlation](image)

Fig. 2-1: One-ring scattering model to calculate spatial correlation
With $D$ being distance between BS and SU, $R$ the radius of the scattering ring, $\Theta$ the angle of arrival at the BS, the angles of incoming waves at a particular antenna element are confined in $[\Theta - \Delta, \Theta + \Delta]$, where $\Delta = \sin^{-1}(R/D)$ is the angle spread. It is assumed that every actual scatterer lying at an angle $\theta$ is represented by an effective scatterer $S(\theta)$ at the same angle on the scatterer ring centered at the SU, and these effective scatterers are assumed to be distributed uniformly in $\theta$. A phase $\phi(\theta)$ associated with $S(\theta)$ represents the dielectric properties and the radial displacement from the scatterer ring of the actual scatterer. Statistically, $\phi(\theta)$ is modeled as uniformly distributed in $[-\pi, +\pi]$, and i.i.d in $\theta$. The radius $R$ of the scatterer ring is determined by the RMS delay spread of the channel. Furthermore, only rays that are reflected by the effective scatterers exactly once are considered, and all rays reaching the receiving antennas are assumed to have equal power. With $K$ effective scatterers, the properly normalized path gain $H_{p,l}$ connecting transmitting antenna elements $T_{A_p}$ and receiving antenna element $R_{A_l}$ is

$$H_{p,l} = \frac{1}{\sqrt{2\pi}} \int_0^{2\pi} \frac{1}{\sqrt{K}} \sum_{k=1}^{K} \delta(\theta - \theta_k) \exp \left\{-j \frac{2\pi}{\lambda} \left(D_{T_{A_p} \rightarrow S(\theta)} + D_{S(\theta) \rightarrow R_{A_l}}\right) + j\phi(\theta)\right\} d\theta$$

(2–16)

where $D_{X \rightarrow Y}$ is the distance from $X$ to $Y$, and $\lambda$ is the wavelength. Letting $K \rightarrow \infty$ and taking expectation over all $\phi(\theta_k)$, the covariance between $H_{p,l}$ and $H_{q,m}$ is found to be [32]

$$E[H_{p,l}H_{q,m}^*] = \frac{1}{2\pi} \int_0^{2\pi} \exp \left[-\frac{2\pi j}{\lambda} \left(D_{T_{A_p} \rightarrow S(\theta)} + D_{S(\theta) \rightarrow R_{A_q}} - D_{T_{A_p} \rightarrow S(\theta)} - D_{S(\theta) \rightarrow R_{A_l}}\right)\right]$$

(2–17)

This gives the full correlation matrix of Eq. (2–7).

### 2.3.2 Two-Ring Scattering Model

The two-ring scattering model is a modification of the one-ring model to include a ring of effective scatterers around the transmitting antenna array as well. This conforms to a real environment where the transmitting array is also surrounded by local scatterers. Fig. 2-2 shows the two-ring model.
With \( K \) effective scatterers on each ring, the normalized path gain \( H_{p,l} \) connecting transmitting antenna elements \( TA_p \) and receiving antenna element \( RA_l \) is

\[
H_{p,l} = \frac{1}{2\pi} \int_0^{2\pi} \int_0^{2\pi} \frac{1}{\sqrt{K_1 K_2}} \left( \sum_{\theta_1 \in \phi_1} \delta(\theta_1 - \theta_{1,k}) \delta(\theta_1 - \theta_{2,k}) \right) \cdot \exp \left[ j \frac{2\pi f}{\lambda} \left( D_{TA_p \rightarrow S_{1,\theta_1}} + D_{S_{1,\theta_1} \rightarrow S_{1,\theta_2}} - D_{TA_p \rightarrow S_{1,\theta_2}} \right) \right] d\theta_1 d\theta_2
\]

(2–18)

After letting \( K_1 \rightarrow \infty \) and \( K_2 \rightarrow \infty \), and taking expectation over all \( \phi_2(\theta_{2,k}) \) and \( \phi_1(\theta_{1,k}) \), the covariance between \( H_{p,l} \) and \( H_{q,m} \) is found to be

\[
E[H_{p,l}H_{q,m}^*] = \frac{1}{4\pi^2} \int_0^{2\pi} \int_0^{2\pi} \left( \begin{array}{c}
\frac{2\pi f}{\lambda} \left( D_{TA_p \rightarrow S_{2,\theta_2}} + D_{S_{2,\theta_2} \rightarrow S_{2,\theta_1}} \\
+ D_{S_{1,\theta_1} \rightarrow RA_m} - D_{TA_p \rightarrow S_{1,\theta_2}} \\
- D_{S_{2,\theta_2} \rightarrow S_{1,\theta_1}} - D_{S_{1,\theta_1} \rightarrow RA_m}
\end{array} \right)
\right] d\theta_1 d\theta_2
\]

(2–19)

This, again, gives the full correlation matrix \( \mathbf{R}_H \). It is worthwhile to note that the two-ring model yields the Kronecker structure in the full correlation matrix.

### 2.4 Capacity of RF wireless MIMO channels

Since the characteristics of the channel matrix \( \mathbf{H} \) are random, the capacity for the RF wireless MIMO channel is random as well. Therefore, we must ensure a certain
Quality of Service (QoS), which is a capacity that will be available with a certain probability at a certain location. This is characterized as the outage capacity. The outage capacity $C_{out}$ is the capacity that is available with a given probability of outage $P_{out}$, i.e.

$$P_{out} = \int_{0}^{C_{out}} p(C) dC$$

(2–20)

where $p(C)$ is the PDF of the capacity random variable.

The outage capacity depends on the transmission/reception schemes, multiplexing methods, and available channel information at each end. The Shannon capacity obtained with these assumptions is the absolute limit on practically achievable capacity. Practically achievable capacity depends on modulation and error correction schemes.

We investigate MIMO Shannon capacity from two perspectives. First, an assumption is made about access to channel state information (CSI) at transmit and receive ends. The spatial power allocation scheme chosen by the transmitter can be optimized in this case. This necessitates a closed-loop system. Second, the CSI is available only at the receiver. The best power allocation scheme, in this case, is to divide the transmit power equally among all transmit antennas [2].

### 2.4.1 Capacity with CSI at Transmit and Receive Sides

We start with a single realization of $H$, which relates the input and output vectors by Eq. (2–5), where $s \in \mathbb{C}^m$, $y \in \mathbb{C}^n$, and $n \sim \mathcal{N}(0, N_y I_m)$ denote the transmitted signal, the received signal, and white (or colored) Gaussian noise. $H$ is assumed known to both transmitter and receiver. The total transmit power is $P$, and the constraint is given by

$$E[s^H s] \leq P, \text{ or } tr\left\{E[ss^H]\right\} \leq P$$

(2–21)

The capacity can be computed by decomposing the Gaussian vector channel into a set of parallel, independent scalar Gaussian scalar sub-channels, by computing the singular value decomposition (SVD) of $H$ as
where $\mathbf{U} \in \mathbb{C}^{n \times n}$ and $\mathbf{V} \in \mathbb{C}^{m \times m}$ are unitary rotation matrices, and $\mathbf{\Sigma} \in \mathbb{R}^{n \times m}$ is a rectangular matrix whose diagonal elements are nonnegative real numbers and whose off-diagonal elements are zero. The diagonal elements $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_{\min}$ are ordered singular values of $\mathbf{H}$, where $\min = \text{rank}[\mathbf{H}]$ in general. When entries of $\mathbf{H}$ are i.i.d., the columns and rows are linearly independent with $\min = \min[m,n]$. Since $\mathbf{HH}^H = \mathbf{U}\mathbf{\Sigma}\mathbf{U}^H$, the squared singular values $\sigma_i^2$ are the eigenvalues of the matrix $\mathbf{HH}^H$, and also of $\mathbf{H}^H\mathbf{H}$. The SVD can be written as

$$\mathbf{H} = \sum_{i=1}^{\min} \lambda_i \mathbf{u}_i \mathbf{v}_i^H \quad (2-23)$$

Now, if we assume that the instantaneous channel matrix $\mathbf{H}$ is known both to the receiver (through training sequences) and to the transmitter (through feedback from the receiver), i.e. we have perfect Channel State Information (CSI) on both ends, we can apply the following unitary transformation to $\mathbf{s}$ to get

$$\tilde{\mathbf{s}} = \mathbf{V}^H \mathbf{s} \quad (2-24)$$

and the following unitary transformation to $\mathbf{y}$

$$\tilde{\mathbf{y}} = \mathbf{U}^H \mathbf{y} = \mathbf{\Sigma} \tilde{\mathbf{s}} + \mathbf{U}^H \mathbf{n} \quad (2-25)$$

The transformations do not alter the statistical properties of the noise and satisfy the power constraint of the input vector $\mathbf{s}$. Therefore, the $n \times m$ MIMO channel can be broken down into $\min$ parallel Gaussian channel, with each channel input and output relationship given by

$$\tilde{\mathbf{y}}_i = \sigma_i \tilde{s}_i + \tilde{n}_i, \ i = 1, \ldots, \min \quad (2-26)$$

Therefore, the capacity of $\min$ parallel Gaussian channel is given by

$$C = \sum_{i=1}^{\min} \log \left( 1 + \frac{P^2 \sigma_i^2}{N_0} \right) \text{bits/s/Hz} \quad (2-27)$$
where $P_i^*$ are waterfilling power allocation given by

$$P_i^* = \left( \mu - \frac{N_0}{\sigma_i^2} \right)^+$$  \hspace{1cm} (2–28)

where $\mu$ is a constant chosen to satisfy the total power constraint $\sum_i P_i^* = P$, and $(x)^+$ is equal to $x$ when greater than zero, and zero else. This is the upper limit on the capacity bound given the transmitter and the receiver both have perfect knowledge of the channel, and the input symbols have i.i.d. white Gaussian distributions. The outage capacity and the ergodic mean capacity with waterfilling power allocation can be calculated by finding the PDF of $C$ from the joint PDF of squared singular values $\sigma_i^2$, i.e. the eigenvalues $\lambda_i$.

### 2.4.2 Capacity with CSI at Receive side

When CSI is not available to transmit side, the transmitting antennas are allocated equal power, which can be calculated from an alternate capacity expression [1]

$$C = \log_2 \det \left[ \mathbf{I}_m + \frac{1}{N_0} \mathbf{H} \mathbf{Q} \mathbf{H}^H \right], m \geq n$$  \hspace{1cm} (2–29)

where $\mathbf{Q}$ is the covariance matrix of the input signal $\mathbf{s}$. If power is equally distributed among the transmitters, $\mathbf{Q} = P / n_{\text{min}} \cdot \mathbf{I}_{n_{\text{min}}}$. As a result, Eq. (2–29) can be rewritten as

$$C = \log_2 \det \left[ \mathbf{I} + \frac{P}{N_0 \cdot n_{\text{min}}} \mathbf{H}^H \mathbf{H} \right]$$  \hspace{1cm} (2–30)

Using SVD of $\mathbf{H}^H \mathbf{H}$, the capacity with uniform power allocation becomes

$$C = \sum_{i=1}^{n_{\text{min}}} \log_2 \left( 1 + \frac{P}{n_{\text{min}} \cdot N_0} \cdot \frac{\sigma_i^2}{N_0} \right) \text{ bits/s/Hz}$$  \hspace{1cm} (2–31)

Again, the outage capacity and the ergodic mean capacity can be calculated from knowledge of the joint PDF of the eigenvalues of $\mathbf{H}^H \mathbf{H}$. 
2.4.3 Eigenvalue Distribution

It is imperative to analyze eigenvalue distributions of the spatial correlation matrix $\mathbf{E}[\mathbf{H}^H\mathbf{H}]$ or $\mathbf{E}[\mathbf{HH}^H]$ to analyze the capacity of a MIMO system. In this section, we review corresponding analytical expressions available for two cases: (1) when the entries of $\mathbf{H}$ are i.i.d. Gaussian, and (2) when the entries are correlated and $\mathbf{H}$ is known.

If $\mathbf{H}$ is a random matrix with zero-mean entries with unit variance, the MIMO channel autocorrelation matrix $\mathbf{HH}^H$ is a $\min(m, n) \times \min(m, n)$ random non-negative definite matrix, called the Wishart matrix, given by

$$
\mathbf{W} = \begin{cases} 
\mathbf{HH}^H, & n < m \\
\mathbf{H}^H\mathbf{H}, & n \geq m 
\end{cases}
$$

(2–32)

The distribution law of $\mathbf{W}$ is called the Wishart distribution with parameters $n_{\min} = \min(m, n)$ and $n_{\max} = \max(m, n)$, and the joint PDF of the ordered eigenvalues are given by [145]

$$
p_{\lambda,\text{ordered}}(\lambda_1, \ldots, \lambda_{n_{\max}}) = \frac{2^{-n_{\min}n_{\max}} \pi^{n_{\max}(n_{\min}-1)}}{\Gamma_{n_{\max}}(n_{\max}) \Gamma_{n_{\min}}(n_{\min})} \exp\left(-\frac{1}{2} \sum_{i=1}^{n_{\min}} \lambda_i^2\right) \prod \lambda_i^{n_{\max} - n_{\min}} \prod_{i<j} (\lambda_i - \lambda_j),
$$

(2–33)

$$
\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_{n_{\min}}
$$

were $\Gamma_m(a) = \pi^{n_{\max} (n_{\min}-1)/2} \prod_{i=1}^{n_{\min}} \Gamma(a - i + 1)$.

For a system having 2 transmit and 2 receive antennas, the CDF of the maximum and the minimum eigenvalues can be found from Eq. (2–33) as

$$
F_{\lambda_{\max}}(\lambda) = F_{\lambda_1,\lambda_2}(\lambda, \lambda),
$$

$$
F_{\lambda_{\min}}(\lambda) = F_{\lambda_1}(\lambda) + F_{\lambda_2}(\lambda) - F_{\lambda_1,\lambda_2}(\lambda, \lambda)
$$

(2–34)

The PDF of the unordered eigenvalues has been found in [1] to be

$$
p_{\lambda,\text{unordered}}(\lambda) = \frac{1}{n_{\min}} \sum_{i=1}^{n_{\min}} \varphi_i(\lambda)^2 \lambda^{n_{\max} - n_{\min}} e^{-\lambda}
$$

(2–35)
where
\[ \varphi_{k+1}(\lambda) = \left[ \frac{k!}{(k+n_{\text{max}}-n_{\text{min}})!} \right]^{1/2} L_{k}^{n_{\text{max}}-n_{\text{min}}} (\lambda), \]
and
\[ L_{k}^{n_{\text{max}}-n_{\text{min}}} (x) = \frac{1}{k!} e^{x} x^{n_{\text{min}}-n_{\text{max}}} \frac{d^{k}}{dx^{k}} (e^{x} x^{n_{\text{max}}-n_{\text{min}}+k}) \]
is the associated Laguerre polynomial of order \( k \).

An expression for capacity of doubly correlated MIMO channels can be calculated from results in [33] which gives the characteristic function of the capacity, \textit{in the case when all transmitter antennas are allocated equal power}. This may not be an optimal allocation strategy, but it is practical if open-loop systems are used where the transmitter does not have any information about the channel. Thus, the capacity of the correlated MIMO channel in nats/s/Hz is given, according to [33], as

\[ C = \ln \det \left( I_{m} + \frac{\eta}{m} \cdot H^H H \right) \text{ nats/s/Hz} \tag{2–36} \]

where \( \eta = P/\sigma_r^2 \), \( P \) being the total power transmitted, \( \sigma_r^2 \) being the noise variance per receive antenna. The characteristic function (CF) of capacity given in [33], modified for the correlated channel under consideration, is as follows

\[ \Phi_{c}(j\omega) = E\left\{ e^{j\omega C} \right\} = K_{\text{cor}}^{-1} Y_{m}(j\omega) \det \Lambda(j\omega) \tag{2–37} \]

where

\[ K_{\text{cor}} = \left( \frac{\eta}{m} \right)^{m(m-1)/2} \prod_{p < q} (\lambda_{\text{TX},q} - \lambda_{\text{TX},p}) \prod_{p < q} (\lambda_{\text{RX},q} - \lambda_{\text{RX},p}) \]

\[ Y_{m}(j\omega) = \prod_{i=1}^{m-1} (j\omega + l)^{-1} \]

and \( \Lambda(j\omega) \) is a \( n \times n \) matrix given by

\[ \left[ \Lambda(v) \right]_{i,j} = \begin{cases} \lambda_{\text{RX},j}^{i-1}, & i = 1, \ldots, n-m, j = 1, \ldots, n \\ \lambda_{\text{RX},j}^{n-m} z_{F_{0}} \left( 1, -\nu - m + 1; \frac{\eta}{m} \cdot \lambda_{\text{TX},n+m} ; \lambda_{\text{RX},j} \right), & i = n - m + 1, \ldots, n, j = 1, \ldots, n \end{cases} \tag{2–38} \]
where $\frac{2}{F_0}$ is the hypergeometric function, $\lambda_{TX,i, j}$, $\lambda_{RX,i}$ are the $i$-th eigenvalue of the transmit side and receive side spatial correlation matrices $R_{TX}$ and $R_{RX}$. The PDF is obtained from the CF in Eq. (2–37) as

$$p(C) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Phi_C^*(j\omega) e^{j\omega C} d\omega$$  \hspace{1cm} (2–39)

### 2.5 Diversity Combining Schemes

A MIMO channel can be used in two modes: (1) diversity and (2) multiplexing. In this section, we consider the receiver diversity scheme shown in Fig. 2-3. The signals that are received by different antenna branches are demodulated to baseband with a quadrature demodulator, processed with matched filter detector, and then applied to a diversity combiner. If the signal $s_m$ is transmitted, the received signal vectors on the different diversity branches are

$$r_k = g_k s_m + n_k, \quad k = 1, \ldots, L$$  \hspace{1cm} (2–40)

where $g_k = \alpha_k e^{-j\theta_k}$ is the fading gain associated with the $k$th branch, and $r_k = g_k s_m + n_k$. The AWGN processes $n_k(t)$ are independent from branch to branch. This is essentially like a SIMO system with one transmitter and $L$ receivers. To simplify analysis, diversity branches are usually assumed to be uncorrelated, which gives most optimistic results.

![Diagram](image-url)

**Fig. 2-3:** Post-detection diversity receiver
The fade distribution will affect the diversity gain. In general, the relative advantage of diversity is greater for Rayleigh fading than Rician fading, because as the Rice factor $K$ increases there is less difference between the instantaneous received signal-to-noise ratios on the various diversity branches. However, the performance will always be better with Rician fading than with Rayleigh fading, for a given average signal-to-noise ratio and diversity order. For our purpose, we consider the performance with Rayleigh fading.

With Maximal Ratio Combining (MRC), the diversity branches are weighted by their respective complex fading gains and combined, i.e. the receiver must be able to track the changes in the channel itself. $r$ gives the baseband demodulated outputs of each of the $L$ antennas as

$$r \triangleq [r_1, r_2, \ldots, r_L]$$

(2–41)

$r$ has the multivariate Gaussian distribution

$$p(r \mid g, s_m) = \frac{1}{(2\pi)^{LN}} \exp \left\{ -\frac{1}{2N_o} \sum_{k=1}^{L} \| r - g_k s_m \|^2 \right\}$$

(2–42)

where $g = [g_1, g_2, \ldots, g_L]$ is the channel vector, $N = \text{dim} \{ r_k \}$, and $L$ is number of diversity branches. From this expression, the ML receiver chooses the message vector $s_m$ that maximizes the metric

$$\mu(s_m) = -\sum_{k=1}^{L} \left\{ \| r_k \|^2 - 2 \text{Re} \langle g_k^* r_k, s_m \rangle + |g_k|^2 \| s_m \|^2 \right\}$$

(2–43)

where $\langle x, y \rangle$ denotes the inner product of vectors $x$ and $y$, i.e. $\langle x, y \rangle = xy^H = \int_0^T x(t) y^*(t) dt$.

Since $\sum_{k=1}^{L} \| r_k \|^2$ is independent of the hypothesis as to which $s_m$ was sent and $\| s_m \|^2 = 2E_m$, $E_m$ being the bandpass symbol energy, the receiver needs to maximize the metric

$$\mu_2(s_m) = \text{Re} \{ r^* s_m \} - \beta_m$$

(2–44)
where \( \mathbf{r}' = \sum_{k=1}^{L} \mathbf{g}^*_k \mathbf{r}_k \) is the combination of all \( \mathbf{r}_k \) weighted by \( g^*_k \), which shows that the ML receiver is essentially a Maximal Ratio Combiner. If signals have equal energy then the last term in Eq. (2–44) can be neglected, since it is the same for all message vectors. After weighting, co-phasing and combining, the energy of the vector \( \mathbf{r}' \) is

\[
E_r = E_o E \{ \alpha_M^2 \} + N_o E \{ \alpha_M \}
\]

(2–45)

where \( E_{av} \) is the average symbol energy, \( N_o \) is the total noise variance, and \( \alpha_M = \sum_{k=1}^{L} \alpha_k^2 \) is the envelope of the composite signal component. Hence, the instantaneous symbol energy-to-noise ratio is

\[
\gamma_s = \frac{\alpha_M^2 E_{av}}{N_o \alpha_M} = \frac{\sum_{k=1}^{L} \alpha_k^2 E_{av}}{N_o \alpha_k} = \sum_{k=1}^{L} \gamma_k
\]

(2–46)

where \( \gamma_k = \alpha_k^2 E_{av} / N_o \). Hence, \( \gamma_s \) is the sum of the symbol energy-to-noise ratios of the diversity branches. If the branches are balanced and uncorrelated, then \( \gamma_s \) has a chi-square distribution with \( 2L \) degrees of freedom

\[
p_{\gamma_s} (x) = \frac{1}{(L-1)! (\overline{\gamma}_c)} x^{L-1} e^{-x/\overline{\gamma}_c}
\]

(2–47)

where \( \overline{\gamma}_c = E \{ \gamma_k \} \), \( k = 1,\ldots,L \).

Since MRC is a coherent detection technique we must limit our attention to coherent signaling techniques, e.g., BPSK and M-QAM. For example, if BPSK is used the bit error probability is

\[
P_b = \int_{0}^{\infty} P_b (x) p_{\gamma_s} (x) dx = \left( \frac{1-\mu}{2} \right) \sum_{k=0}^{L-1} \binom{L-1+k}{k} \left( \frac{1+\mu}{2} \right)^k
\]

(2–48)

where \( \mu = \sqrt{\overline{\gamma}_c / (1+\overline{\gamma}_c)} \). Also, from Eq. (2–45), for a 2×1 diversity scheme

\[
\mathbf{r}' = \sum_{k=1}^{L} g^*_k \mathbf{r}_k = g^*_1 \mathbf{r}_1 + g^*_2 \mathbf{r}_2 = (\alpha_1^2 + \alpha_2^2) \mathbf{s}_m + g^*_1 \mathbf{n}_1 + g^*_2 \mathbf{n}_2
\]

(2–49)
2.6 Multiplexing and Multi-User Reception Techniques

This section involves investigating the different aspects of MIMO receive algorithms, namely, Zero-forcing, Minimum Mean Square Error, Maximum Likelihood and the so-called 'sphere decoding' [147]. Several aspects of spatially multiplexed MIMO systems are discussed. A spatially-multiplexed MIMO system introduces interference from one multiplexed stream or user to another. The minimization of interference is directly related to the performance of the receive algorithms. The system would behave differently for having CSI at the receiver, at both the transmitter and receiver, or at none. In order to compare the different receive algorithms, we only concentrate on a fixed channel matrix, and vary the noise. A generalized $M \times N$ system is considered with $M$ transmit antennas and $N$ receive antennas. As this is a spatially multiplexed system, there will be multiple streams or users. This can be any number between 1 and $M$.

Assumptions are as follows. The transmitted signal points are chosen from a square QAM constellation. The signal vector is denoted by $s$, and is a $M \times 1$ vector with complex entries. In order to have fair comparison of different MIMO systems and have the noise variance as the only variable, the total power launched into the $M$ streams is always kept fixed, i.e. the average power of a QAM modulation, $4E_s(L-1)/3$. Therefore, each transmitted stream has an average energy of $4E_s(L-1)/3M$. Here, $L$ is the constellation size of the QAM modulation scheme, $E_h$ is the energy of the baseband pulse shape used, and $M$ is the number of transmit antennas (and number of individual streams). Independence leads to the correlation matrix of $s$ being diagonal, i.e., $E[ss^T] = 4E_s(L-1)/3M \cdot I_M$, where $I_M$ is the $M \times M$ identity matrix. The channel matrix $H$ is $N \times M$, and has independent complex Gaussian entries, and is kept fixed over time. Each column of $H$ is normalized to have a sum of squares equal to 1, i.e. the total energy of components of a transmitted stream reaching each receive antenna sum up to the same amount of energy that was transmitted. This implies a lossless system. The channel state information (CSI) is present only at the receiver; hence linear transformation and sphere/ML decoding can be applied. The noise vector $n$ is a $N \times 1$ vector with independent
complex Gaussian entries with variance $N_o$ along each row. Therefore, the correlation matrix of $\mathbf{n}$ is given by, $\mathbf{E}[\mathbf{n}\mathbf{n}^T] = N_o \mathbf{I}_N$, where $\mathbf{I}_N$ is the the $N \times N$ identity matrix.

Complex functions are not analytical, and hence cannot be easily differentiated to obtain minimum point for some criterion; the problem may be broken into sub-problems for easier handling. The fact that the real and the imaginary axes are orthogonal, and the noise along each dimension are Gaussian, hence independent, can be used to decompose each complex vector and matrix in the following manner. Vectors are decomposed in the following manner, where $\mathbf{X}$ is a complex vector and $\hat{\mathbf{X}}$ is an equivalent decomposed vector with dimension $2M \times 1$

$$
\hat{\mathbf{X}} = \begin{bmatrix} \text{Re}(\mathbf{X}) \\ \text{Im}(\mathbf{X}) \end{bmatrix}
$$

(2–50)

Matrices are decomposed in the following manner, where $\mathbf{H}$ is a complex matrix and $\hat{\mathbf{H}}$ is an equivalent decomposed matrix with dimensions $2N \times 2M$

$$
\hat{\mathbf{H}} = \begin{bmatrix} \text{Re}(\mathbf{H}) & -\text{Im}(\mathbf{H}) \\ \text{Im}(\mathbf{H}) & \text{Re}(\mathbf{H}) \end{bmatrix}
$$

(2–51)

These transformations do not alter the characteristics of the vectors and matrices. $\text{Re}(\mathbf{s})$ and $\text{Im}(\mathbf{s})$ each has average energies of $2E_s(L-1)/3M$. The equivalent channel matrix, too, preserves the properties of $\mathbf{H}$, and is lossless.

It can be shown that, the inversion of an equivalent matrix that is invertible gives a matrix that follows the same equivalence. That is, if $\mathbf{H}^+$ is the generalized Moore-Penrose inverse of $\mathbf{H}$, then

$$
\hat{\mathbf{H}}^+ = \begin{bmatrix} \text{Re}(\mathbf{H}^+) & -\text{Im}(\mathbf{H}^+) \\ \text{Im}(\mathbf{H}^+) & \text{Re}(\mathbf{H}^+) \end{bmatrix}
$$

(2–52)
2.6.1 Zero-Forcing Linear Receiver

To obtain the zero-forcing linear receiver [147], we consider the outputs of the channel-matched filter as follows

\[ \tilde{y}' = \tilde{H}^T \tilde{r} = \tilde{H}^T \tilde{H} \tilde{s} + \tilde{H}^T \tilde{n} \]  

(2–53)

Here, \( \tilde{R}_n = \tilde{H}^T \tilde{H} \), is the \( 2M \times 2M \) auto-correlation matrix of the channel. If the antennas are placed wide apart enough to have little spatial correlation, \( \tilde{R}_n \) will be well-conditioned and will be invertible. In order to obtain the zero-forcing estimates of the received symbols, we get by multiplying both sides with \( \tilde{R}_n^{-1} \)

\[ \tilde{y} = \tilde{R}_n^{-1} \tilde{y}' = \tilde{s} + \tilde{R}_n^{-1} \tilde{H}^T \tilde{n} = \tilde{s} + \tilde{n}' \]  

(2–54)

We get back the original transmitted symbols \( \tilde{s} \), added with Gaussian noise \( \tilde{n}' \), which is no longer white. Therefore, inter-channel symbol interference is completely eliminated, but at the cost of changing the noise space. If \( \tilde{R}_n \) is ill-conditioned, this might result in the amplification of noise, and thus degrade performance.

The Signal-to-Noise Ratio (SNR) for the \( k \)-th stream can be obtained by first finding the \( k \)-th diagonal component of the auto-correlation matrix of \( \tilde{y} \), and separating the expression into signal and noise parts

\[ E\{ \tilde{y} \tilde{y}^T \} = 2E_y(L-1)3M I_{2M} + N_o \cdot \tilde{R}_n^{-1} \]  

(2–55)

Therefore, the SNR for the \( k \)-th stream is given by the total symbol energy of the in-phase and the quadrature components divided by the noise energy in those

\[ SNR_k = \frac{8E_y(L-1)/3M}{N_o \cdot \left( \left\{ \tilde{R}_n^{-1} \right\}_{diag,k} + \left\{ \tilde{R}_n^{-1} \right\}_{diag,k+M} \right)} \]  

(2–56)

To calculate the probability of symbol error (SER), given a particular symbol sequence \( \tilde{s} \) was transmitted, we can see from Eq. (2–54) that \( \tilde{y} \) is Gaussian with mean \( \tilde{s} \) and cross-covariance matrix \( \tilde{C}_y = N_o/2 \cdot \tilde{R}_n^{-1} \). Given the \( k \)-th in-phase symbol \( \tilde{s}_k \) is
\((2m-1-\sqrt{M})\sqrt{2E_h}\), from the inner \(\sqrt{M} - 2\) of the \(\sqrt{M}\) points in the \(\sqrt{M}\)-PAM constellation, it has the probability of being correct

\[ P_{s_k|\tilde{s}_k} = 1 - 2Q \left( \frac{2\sqrt{2E_h/M}}{N_o \cdot \{R^{-1}_H\}_{\text{diag},k}} \right) \]  

(2–57)

Therefore, for this case, probability of symbol error is

\[ P_{s_k|\tilde{s}_k} = 2Q \left( \frac{2\sqrt{2E_h/M}}{N_o \cdot \{R^{-1}_H\}_{\text{diag},k}} \right) \]  

(2–58)

For the outer 2 symbols, the probability of symbol error is Eq. (2–59)

\[ P_{s_k|\tilde{s}_k} = Q \left( \frac{2\sqrt{2E_h/M}}{N_o \cdot \{R^{-1}_H\}_{\text{diag},k}} \right) \]  

(2–59)

Averaging over all the possible symbols for \(\tilde{s}_k\)

\[ \overline{P}_{s_k|\tilde{s}_k} = \frac{2(\sqrt{M} - 1)}{\sqrt{M}} \cdot Q \left( \frac{2\sqrt{2E_h/M}}{N_o \cdot \{R^{-1}_H\}_{\text{diag},k}} \right) \]  

(2–60)

Similarly, given the \(k\)-th quadrature symbol \(\tilde{s}_{k+M}\) is \((2m-1-\sqrt{M})\sqrt{2E_h}\), from the inner \(\sqrt{M} - 2\) of the \(\sqrt{M}\) points in the \(\sqrt{M}\)-PAM constellation, it has the same probability of error. As a result, the average symbol error probability for the \(k\)-th complex symbol is given by

\[ P_e = 1 - \left( 1 - \overline{P}_{s_k|\tilde{s}_k} \right)^2 \]  

(2–61)

### 2.6.2 Linear MMSE Receiver

To obtain the expressions for the Linear MMSE MIMO receiver, we start from the channel-matched-filter outputs as given in Eq. (2–53). The linear transformation M
would be an LMMSE estimator of $\hat{s}$, if the expected mean square error between the estimate $My'$ and $\hat{s}$ is minimized [147]. That is,

$$M = \min_{M \in \mathbb{R}^{2M}} E \left\{ \|My' - \hat{s}\|^2 \right\}$$  \hspace{1cm} (2–62)

where $\|\cdot\|^2$ denotes the $L^2$-norm. This condition is equivalent to minimizing the trace of the matrix $(My' - \hat{s})(My' - \hat{s})^T$. That is,

$$M = \min_{M \in \mathbb{R}^{2M}} E \left[ tr \left( (My' - \hat{s})(My' - \hat{s})^T \right) \right]$$  \hspace{1cm} (2–63)

The matrix $M$ is easily found by applying the orthogonality principle, which states that the error vector $(My' - \hat{s})$ and the observation at the input of the detector $\hat{y}'$ must be orthogonal. Therefore,

$$E \left[ (My' - \hat{s})\hat{y}'^T \right] = 0 \Rightarrow M = \left[ R_h + \frac{3MN_o}{4E_h(L-1)} \cdot I_{2M} \right]^{-1}$$  \hspace{1cm} (2–64)

We can obtain the SNR for each transmitted symbol for the LMMSE receiver by decomposing the $k$-th and $k+M$-th diagonal elements of the autocorrelation matrix of the pre-quantized detected symbols into signal and noise parts

$$E \left\{ \hat{y}_k\hat{y}_k^T \right\} = \frac{2E_h(L-1)}{3M} \cdot MR_hR_h^T + \frac{N_o}{2} \cdot MR_hM^T$$  \hspace{1cm} (2–65)

Therefore the SNR of the $k$-th complex symbol can be given by

$$SNR_k = \frac{4E_h(L-1)}{3MN_o} \cdot \frac{\left\{ (MR_hR_h^T)_{\text{diag},k} + (MR_hR_h^T)_{\text{diag},k+M} \right\}}{\left\{ (MR_hM^T)_{\text{diag},k} + (MR_hM^T)_{\text{diag},k+M} \right\}}$$  \hspace{1cm} (2–66)

In reality, a more accurate measure of performance would be the SINR for the LMMSE receiver, since unlike the ZF linear receiver, LMMSE does not eliminate interference completely, but interpolates to the most likely transmitted symbol in a mean square sense. The SINR of the $k$-th complex symbol can be obtained by using the expression $\tilde{G} = M\hat{H}^T$, i.e. $G$ is the combined channel-matched filter and LMMSE receiver as
The subscripts on $g$ denote rows of $G$, the subscripts on $h$ denote columns of $H$, and the subscripts on $x$ indicate rows of $X = \tilde{G} \tilde{H}$.

The probability of symbol error, on the other hand, can be expressed as

$$P_e = 1 - \left(1 - P_e^{k(M \text{- QAM})}\right) \left(1 - P_e^{k(M \text{- PAM})}\right)$$

with

$$P_e^{k(M \text{- PAM})} = \frac{1}{\sqrt{M}} \sum_{k \in U} P_e^{k(M \text{- PAM})}$$

and

$$P_e^{k(M \text{- QAM})} = \frac{1}{\sqrt{M}} \sum_{k \in \tilde{U}} \left(\sum_{i=1}^{M} \tilde{x}_k(i) \tilde{s}(i) \right) + Q \left(\frac{\sqrt{N_a/2} \sum_{i=1}^{2M} \tilde{g}_k^2(i)}{\sqrt{\frac{2E_b}{M} \sum_{i=1}^{N_a} \tilde{g}_k^2(i)}} \right).$$

It is worthwhile to note that certain variations on both the ZF and LMMSE receivers are possible, as follows. (1) **Successive cancellation:** By decoding the first symbol and then feeding it back to cancel the interference to the next symbols, we can improve performances of the linear receivers. But this renders the receivers non-linear, and analytical calculations are thus cumbersome, and closed-form may not exist. However, the implementations are simple, and show performance improvement, although there are possibilities of error propagation. This receive algorithm is sometimes referred to as **Nulling and Cancelling.** (2) **Successive cancellation with optimal ordering:** Before doing the successive cancellation, the received symbols are ordered in terms of SNR, with the symbol corresponding to the highest SNR being decoded first. This will increase the probability of the first decoded symbol to be detected correctly, and thus following symbols can have the interference from this symbol correctly minimized, thus giving
more accurate symbol estimates successively. This reduces the possibilities of error propagation.

### 2.6.3 Maximum Likelihood Receiver

The ML detector does a search over symbol space to find the vector corresponding to the shortest distance from received vector \( \mathbf{r} \). The received vector is

\[
\mathbf{r} = \mathbf{\hat{H}} s + \mathbf{n}
\]

The output of the ML detector is given by

\[
\hat{s} = \min_{\hat{s} \in \frac{1}{\sqrt{M}} \mathbb{Z}^{2M}} \| \mathbf{r} - \mathbf{\hat{H}} \hat{s} \|^{2}
\]

where \( \mathbb{Z} \) is an integer space with \( \{\pm 1, \pm 3, \pm 5, \ldots, \pm \sqrt{M} \} \). This is equivalent to maximizing the likelihood function over the possible values of \( \hat{s} \), i.e.

\[
\hat{s} = \arg \max_{\hat{s} \in \frac{1}{\sqrt{M}} \mathbb{Z}^{2M}} p(\mathbf{r} | \hat{s}) = \arg \max_{\hat{s} \in \frac{1}{\sqrt{M}} \mathbb{Z}^{2M}} \left[ \left( 2 \langle (\mathbf{\hat{H}} \hat{s}), \mathbf{\hat{r}} \rangle - \| (\mathbf{\hat{H}} \hat{s}) \|^2 \right) \right]
\]

The ML detection algorithm searches over all possible points in the \( \sqrt{M} \)-PAM constellation for the in-phase and quadrature-phase signals, i.e. searches over all possible points in the \( M \)-QAM signal space. This is an NP-hard problem, i.e. the search complexity grows exponentially with a linear increase of the signal dimension. Since the decision process is joint, there is no interference in the ML receiver to account for in the SINR. The SNR for the \( k \)-th symbol with ML detection can be obtained from the correlation matrix of \( \mathbf{r} = \mathbf{\hat{H}} \hat{s} + \mathbf{n} \), as

\[
SNR_k = \frac{4E_k(L-1)}{3MN_o}
\]
There is no closed-form analytical expression for the probability of symbol error of ML receivers.

### 2.6.4 Sphere Decoding Receiver

The ML detection problem in Eq. (2–70) is equivalent to finding the least-squares solution to a system of linear equation, where the unknown vector is comprised of integers, but the matrix coefficients and the given vector are real-valued. The problem is equivalent to finding the closest lattice point to a given point and is known to be exponentially hard. A geometric interpretation of the problem is presented in Fig. 2-4(a).

The main idea of sphere decoding [147] is similar to ML detection in the sense that it looks for the closest lattice point, but instead of each of the possible lattice points being tested for distance, a sphere is formed around the received signal point and any lattice point lying in that sphere are possible candidates for the transmitted symbol vector, and then the distances are tested only for those points. Fig. 2-4(b) shows the main idea behind sphere decoding.

![Fig. 2-4: Geometric Interpretation of (a) the integer least-squares problem, and (b) the idea behind sphere decoding.](image)

The two important criteria for the sphere decoding algorithm are: (1) Sphere radius, (2) method of finding points within the sphere without testing all. To address (2), we can realize that although it is difficult to determine the lattice points inside a general $2M$-dimensional sphere, it is trivial to do so in the one-dimensional case. The reason is...
that a one-dimensional sphere reduces to the endpoints of an interval and so the desired lattice points will be the integer values that lie in this interval. Such an algorithm for determining the lattice points in a $2M$-dimensional sphere essentially constructs a tree where the branches in the $k$-th level of the tree correspond to the lattice points inside the sphere of radius $d$ and dimension $k$ (Fig. 2-5). The complexity of this algorithm will depend on the size of the tree, i.e. on the number of lattice points visited by the algorithm in different dimensions.

![Sample tree generated to determine lattice points in a 4-dimensional sphere.](image)

**Fig. 2-5:** Sample tree generated to determine lattice points in a 4-dimensional sphere.

We assume that $N \geq M$, i.e. there are at least as many equations as unknowns in $\tilde{r} \approx \hat{H}\tilde{s}$. The lattice point $\hat{H}\tilde{s}$ lies inside a sphere of radius $d$ centered at $\tilde{r}$ iff,

$$d^2 \geq \|\tilde{r} - \hat{H}\tilde{s}\|^2$$  \hspace{1cm} (2-73)

In order to break this problem into sub-problems, it is useful to introduce the QR factorization of the matrix $\hat{H}$, as $\hat{H} = Q\begin{bmatrix} R & 0 \\ 0 & (2N-2M)\times2M \end{bmatrix}$, where $R$ is an $2M \times 2M$ upper triangular matrix and $Q = [Q_1 \quad Q_2]$ is a $2N \times 2N$ orthogonal matrix. The inequality in Eq. (2-73) can then be written as

$$d^2 \geq \|\tilde{r} - [Q_1 \quad Q_2]\begin{bmatrix} R \\ 0 \end{bmatrix}\tilde{s}\|^2 = \|Q_1\tilde{r} - R\tilde{s}\|^2 + \|Q_2\tilde{r}\|^2$$  \hspace{1cm} (2-74)

or, $d^2 - \|Q_2\tilde{r}\|^2 \geq \|Q_1\tilde{r} - R\tilde{s}\|^2$
where \((\cdot)^*\) denotes Hermitian matrix transposition. Defining \(y = Q_1 \tilde{s}\) and \(d'^2 = d^2 - \|Q_2 \tilde{r}\|^2\) allows us to write
\[
d'^2 \geq \sum_{i=1}^{2M} \left( y_i - \sum_{j=i}^{2M} r_{ij} \tilde{s}_j \right)
\]

or, \(d'^2 \geq \left( y_{2M} - r_{2M,2M} \tilde{s}_{2M} \right)^2 + \left( y_{2M-1} - r_{2M-1,2M} \tilde{s}_{2M} - r_{2M-1,2M-1} \tilde{s}_{2M-1} \right)^2 + \ldots\) (2–75)

Here, the first term depends only on \(\tilde{s}_{2M}\), the second term on \(\{\tilde{s}_{2M}, \tilde{s}_{2M-1}\}\), and so on. Therefore, a necessary condition for \(\tilde{H}s\) to lie inside the sphere is \(d'^2 \geq \left( y_{2M} - r_{2M,2M} \tilde{s}_{2M} \right)^2\). This condition is equivalent to \(\tilde{s}_{2M}\) belonging to the interval
\[
\left[ \frac{-d' + y_{2M}}{r_{2M,2M}} \right] \leq \tilde{s}_{2M} \leq \left[ \frac{d' + y_{2M}}{r_{2M,2M}} \right]
\]
(2–76)

where \([\cdot]\) denotes rounding to the nearest larger integer element in the \(\sqrt{M}\)-PAM constellation which spans the lattice multiplied by \(\sqrt{2E_h / M}\), and \([\cdot]\) denotes rounding to the nearest smaller element.

Eq. (2–76) is by no means sufficient, for every \(\tilde{s}_m\) satisfying Eq. (2–76), defining \(d'^2_{2M-1} = d'^2 - \left( y_{2M} - r_{2M,2M} \tilde{s}_{2M} \right)^2\), and \(y_{2M-1,2M} = y_{2M-1} - r_{2M-1,2M} \tilde{s}_{2M}\), a stronger necessary condition can be found by looking at the first two terms in Eq. (2–75), which leads to \(\tilde{s}_{2M-1}\) belonging to the interval,
\[
\left[ \frac{-d'_{2M-2} + y_{2M-1,2M}}{r_{2M-1,2M-1}} \right] \leq \tilde{s}_{2M} \leq \left[ \frac{d'_{2M-1} + y_{2M-1,2M}}{r_{2M-1,2M-1}} \right]
\]
(2–77)

One can continue in this fashion for \(\tilde{s}_{2M-2}\), and so on until \(\tilde{s}_1\), thereby obtaining all lattice points belonging to Eq. (2–75). The algorithm is formalized in Table 2.1.
To determine desired sphere radius \(d\), we note that \(\frac{d^2}{\alpha^2} = \sum_{k=1}^{2M} r_k^2 \), \(y_{2M+1} = \sum_{k=1}^{2M} r_k s_k\)

is a \(\chi^2\) random variable with \(2N\) degrees of freedom. Therefore, we may choose the radius \(d\) to be a scaled variance of the noise, \(d^2 = \alpha \cdot 2N \cdot \frac{N^2}{2} = \alpha NN_o\), in such a way that with a high probability we find a lattice point inside the sphere,

\[
\int_0^{\frac{\alpha N}{2}} \frac{\lambda^{N-1} e^{-\lambda}}{\Gamma(N)} d\lambda = 1 - \varepsilon \quad (2-78)
\]

where the integrand is the probability density function of the \(\chi^2\) random variable with \(2N\) degrees of freedom, and where \(1 - \varepsilon\) is set to a value close to 1.

### 2.7 Space Time Coding

In this section, we describe the Alamouti scheme \([148]\) as an example of space time coding on a transmit diversity scheme. The scheme uses two transmit antennas and
one receiver antenna, referred to as 2×1 diversity. In any given symbol period, two data symbols are transmitted simultaneously from the two transmit antennas. We denote the symbols by \(s_{(1)}\) and \(s_{(2)}\) during the first baud period, and \(-s_{(2)}^*\) and \(s_{(1)}^*\) during next baud period. The channel gains for the two antennas are denoted by \(g_1(t)\) and \(g_2(t)\). If the channel stays constant over two consecutive baud intervals, then we can write,

\[
g_k(t) = g_k(t + T) = g_k = \alpha_k e^{i\theta_k}
\]

where \(T\) is the baud period. The received complex vectors are

\[
r_{(i)} = g_1 s_{(i)} + g_2 s_{(2)} + n_{(i)}, \quad r_{(2)} = -g_1 s_{(2)}^* + g_2 s_{(1)}^* + n_{(2)}
\]

where \(r_{(i)}\) and \(r_{(2)}\) represent the received vectors at time \(t\) and \(t+T\), respectively, and \(n_{(i)}\) and \(n_{(2)}\) are the corresponding noise vectors.

The diversity combiner for this scheme is shown in Fig. 2-6. The combiner constructs the following two signal vectors

\[
v_{(i)} = g_1^* r_{(i)} + g_2^* r_{(2)}, \quad v_{(2)} = g_1^* r_{(i)} - g_2^* r_{(2)}
\]

Afterwards, the receiver applies the vectors \(v_{(i)}\) and \(v_{(2)}\) in a sequential fashion to the metric computer in Fig. 2-6, to make decisions by maximizing the metric

\[
\mu(s_{(1),m}) = \text{Re} \langle v_{(i)}, s_{(1),m} \rangle - E_m \left( |g_1|^2 + |g_2|^2 \right)
\]

\[
\mu(s_{(2),m}) = \text{Re} \langle v_{(2)}, s_{(2),m} \rangle - E_m \left( |g_1|^2 + |g_2|^2 \right)
\]

Using Eq. (2–79) and Eq. (2–80) in Eq. (2–81) gives

\[
v_{(i)} = (\alpha_1^2 + \alpha_2^2) s_{(i)} + g_1^* n_{(i)} + g_2^* n_{(2)}, \quad v_{(2)} = (\alpha_1^2 + \alpha_2^2) s_{(2)} - g_1 n_{(i)} + g_2^* n_{(1)}
\]
Comparison of Eq. (2–83) and Eq. (2–49) shows that the combined signals in each case are the same. The only difference is the phase rotations of the noise vectors which will not change the error probability due to their circular symmetry. Therefore, the Alamouti scheme has the same performance as the MRC, i.e. ML receiver, and is optimum.

2.8 WiMAX Throughput Tests

Some tests involving WiMAX, the emerging MIMO technology for wireless communications, were carried out at Intel Corporation in Hillsboro, Oregon. The tests had the goal of characterizing system performance of fixed WiMAX (IEEE 802.16d) end-to-end network architecture. To accomplish the goals, a controlled environment was emulated through which performance can be measured against variable parameters, such as distance, noise power density, interference power, etc. over different channel models. The controlled variables included: distance variation with an attenuator; Channel Models (multipath taps and noise power density) with a Channel Emulator; co-channel
interference with a VSG. The measured variables were: RF Receiver Sensitivity, receiver
SNR, Transmit power of the SU; transmit power of the BS; and throughput and response
time of TCP data.

Fig. 2-7 shows a graphical picture of the test setup. An IxChariot server is used as
the traffic simulator. IxChariot is capable of simulating TCP and UDP traffics of different
types such as data, VoIP, Video, IPTV, etc. The IxChariot server is connected through a
switch to the WiMAX compliant mini base station, which has a BS transceiver unit for
transmission and reception. The RF signal from the transceiver is then fed by means of a
conduction path (coaxial cable) into a Channel Emulator which emulates a random RF
wireless environment according to specific model parameters. An attenuator provides
additional attenuation to simulate the variation of distance from the base station. The SU
receives the signal from the attenuator and demodulates and decodes it and delivers the
data to the client machine. The VSG simulates co-channel interference. The throughput
of the data link is measured by the IxChariot through a separate control link. The VSA
measures the power spectrum of the RF signal. The power meter measures the amount of
power transmitted from the BS transceiver unit (outdoor unit).

Fig. 2-7: WiMAX performance evaluation test setup
The different channel models for the Wireless Broadband Fixed Access links have been standardized in the IEEE 802.16 document [6]. Based on these definitions and several others, six so-called Stanford University Interim (SUI) Channel models are proposed [6]. These channel models are emulated using the RF channel emulator to obtain WiMAX performance measures.

### 2.9 Results and Discussion

This section summarizes the results of analyses, simulations, and experiments discussed in previous sections. First, we demonstrate how antenna spacing affects the correlation coefficient between individual paths on a MIMO channel. To this end, correlation coefficients are demonstrated in Fig. 2-8 for one-ring and two-ring scatterer models with 2 transmit and 2 receive antennas with broadside orientation for 836MHz propagation at a distance of 1000m, and beamwidth of 2 degrees. The radius of both transmit and receive side scattering rings are 17.4533m, and there are 200 effective scatterers uniformly spaced on each ring.

![Fig. 2-8: Correlation coefficients against antenna element separation for 2×2 uniformly spaced linear multi-element arrays.](image)

It is seen that correlation coefficients decrease with antenna spacing for both cases. The magnitude of the correlation coefficient goes below 0.4 and 0.2, respectively,
for one-ring and two-ring models, for antenna separations of greater than $35 \lambda$ and $2 \lambda$ respectively, which are equivalent to 3m and 17cm respectively. This signifies that we can rarely have a completely uncorrelated spatial MIMO channel in reality. Furthermore, it is observed that the two-ring model yields the separable Kronecker model, demonstrating its applicability for the case when both the transmit and receive arrays are surrounded by a substantial number of scatterers at a close distance.

Next, we look at simulation results for outage capacity values for the MIMO channel models described in Section 2.2, using capacity expressions given in Section 2.4. Three cases of correlation are considered: fully spatially correlated, fully uncorrelated, and partially double-correlated channels. The ratio of total transmit power to noise power density per receiver is kept fixed at 20dB for all cases. The parameters for partially correlated channels are listed in Table 2.2. The outage capacity, obtained using uniform power allocation and water-filling power allocation, for these three cases modeled with Kronecker, Weichselberger, and Virtual channel models are illustrated in Fig. 2-9 and Fig. 2-10.

Table 2.2: Parameters for simulated 2×4 partially double-correlated channel.

<table>
<thead>
<tr>
<th>Transmit side correlation coefficients</th>
<th>1</th>
<th>$\sqrt{0.3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Receive side correlation coefficients</td>
<td>$\sqrt{0.91}$</td>
<td>$\sqrt{0.73}$</td>
</tr>
<tr>
<td>Coupling matrix</td>
<td>0.002</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>0.0222</td>
<td>0.0056</td>
</tr>
<tr>
<td></td>
<td>0.4955</td>
<td>0.2124</td>
</tr>
<tr>
<td></td>
<td>1.2894</td>
<td>5.97</td>
</tr>
</tbody>
</table>

Examination of Fig. 2-9 and Fig. 2-10 reveals that the fully spatially uncorrelated MIMO channel yields the best outage capacity, which would be the upper bound of outage capacity for MIMO channel. For uncorrelated fading, the capacity at an outage probability of 1% is about 12 bps/Hz with both uniform and water-filling power allocation. When spatial correlation is introduced with spatial correlation matrices
defined in [14], this value reduces to about 8.5 bps/Hz. The worst case is experienced with full correlation, when the 1%-outage capacity diminishes to 4.6 bps/Hz, setting the lower bound.

Comparisons of water-filling and uniform power allocation also show that water-filling yields a slightly better outage capacity performance for Kronecker and Weichselberger models, whereas it performs as good as uniform power allocation for the Virtual channel representation. Therefore, there may not be much gain by employing the complex water-filling power allocation. The most revealing difference among the results, is that Weichselberger model produces a higher capacity at high outage probabilities, thus showing the possibility of having a better CDF tail.

Next, we examine the ordered and unordered eigenvalue PDFs obtained for uncorrelated 2×4 MIMO channel from the analytical expressions given in Eq. (2–33) and Eq. (2–35) and compare them with those obtained using three simulated channel models. These are illustrated in Fig. 2-11. Furthermore, the simulated PDFs are compared in Fig. 2-12 and Fig. 2-13 for fully correlated and partially correlated cases. It is evident that the eigenvalue distribution for the uncorrelated case is the best, since it shifts away the mean from zero. For partial correlation, the eigenvalues are more dispersed than in uncorrelated case, with one eigenvalue being closer to zero with more probability. For the

Fig. 2-9: Simulated Outage Probability against Capacity for uniform power allocation for uncorrelated, partially correlated, and fully correlated 2×4 MIMO channel.
fully correlated case, the eigenvalue distribution is the same as exponential distribution, which corresponds to Rayleigh fading, and is the worst case.

![Simulated Outage Probability against Capacity for uncorrelated, partially correlated, and fully correlated 2×4 MIMO channel.](image1)

**Fig. 2-10:** Simulated Outage Probability against Capacity for water-filling power allocation for uncorrelated, partially correlated, and fully correlated 2×4 MIMO channel.

![Comparison of Analytical and Simulated PDFs of (a) ordered eigenvalues and (b) unordered eigenvalues for a 2×4 uncorrelated MIMO channel.](image2)

**Fig. 2-11:** Comparison of Analytical and Simulated PDFs of (a) ordered eigenvalues and (b) unordered eigenvalues for a 2×4 uncorrelated MIMO channel.

It is also observed that the eigenvalue PDFs are similar for all three channel models, except the slight difference of Weichselberger model in the case of partial correlation, hence signifying deficiencies of the other two models.

Since analytical expression of eigenvalue PDFs are not available for partially correlated channels, we have to compare simulated results for capacity PDFs or CDFs with the expression for capacity characteristic function given in Eq. (2–37). This is only
valid for uniform power allocation, and the resulting capacity PDFs and CDFs are illustrated in Fig. 2-14. Examination of the figure shows correspondence of the analytical results with Kronecker model, whereas Weichselberger and Virtual models differ somewhat.

Fig. 2-12: Simulated PDFs of (a) ordered and (b) unordered eigenvalues of partially correlated 2×4 MIMO channel.

Fig. 2-13: Simulated PDF of the single eigenvalue of fully correlated 2×4 MIMO channel.
Next we compare multi-user reception schemes in terms of bit error rates for different reception schemes. Fig. 2-15 and Fig. 2-16(a) demonstrate the results for linear (zero-forcing and MMSE) receivers, and maximum-likelihood (ML) and ML-like Sphere Decoding receivers. These are obtained with static MIMO channels, and since no fading is introduced, the results coincide with ideal AWGN, expectedly. This demonstrates the effectiveness of the algorithms in demultiplexing multiple streams or user. The complexity of the Sphere Decoding algorithm is demonstrated in Fig. 2-16(b), from which it is evident that SD has a much lower complexity than ML at high SNR.

The symbol error rate performance of space-time coding or the Alamouti scheme, together with water-filling, applied to the three channel cases under consideration is illustrated in Fig. 2-17 for a 2×4 MIMO system. The modulation scheme used is BPSK, and the channel correlation coefficients are varied in accordance with the three discussed models. The results are also indicative of MRC performance.

Fig. 2-18, on the other hand, demonstrates the diversity order achieved using MRC or Alamouti scheme. The diversity order is calculated as the slope of the curve representing the log of inverse of the symbol error probability versus log of SNR. The calculated diversity orders are listed in Table 2.3. As expected, maximum diversity order of about 3 is obtained with uncorrelated fading; it reduces to about 2 for partial
correlation, and to the minimum of 1 when fading is completely correlated, yielding a SISO channel.

Fig. 2-19 explicitly demonstrates the comparison between MRC scheme and Alamouti scheme in terms of BER performance with BPSK modulation. For this simulation, the distribution of channel transfer matrix entries was assumed to be i.i.d. Gaussian with unit variance. As expected, a system that employs no spatial diversity performs the worst in the uncorrelated channel. Further improvement is achieved by employing higher diversity orders, demonstrated in Table 2.4. The 3dB difference between the Alamouti and the MRC schemes stems from the fact that the total transmit power in the Alamouti scheme had to be divided equally between the two antennas to keep the total transmit power fixed for both cases. If the same amount of power were
allowed to be transmitted from each antenna, the BER plot for the Alamouti scheme would overlap with that of MRC.

Fig. 2-16: (a) BER performance of ML and ML-like (Sphere Decoding) MIMO Receive Algorithms with 4QAM modulation constellation for various MIMO schemes. (b) Average number of visited nodes against noise variance for SD with 2×4 MIMO.

Fig. 2-17: Performance of Space-Time Coding on 2×4 uncorrelated, partially correlated, and fully correlated MIMO channel.
Fig. 2-18: Log of inverse symbol error probability versus log of SNR to determine diversity order through slope.

Table 2.3: Diversity orders of variously correlated channels using three models.

<table>
<thead>
<tr>
<th>Model</th>
<th>Uncorrelated</th>
<th>Partially Correlated</th>
<th>Fully Correlated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kronecker</td>
<td>3.2001</td>
<td>1.8634</td>
<td>1.0264</td>
</tr>
<tr>
<td>Weichselberger</td>
<td>3.0378</td>
<td>1.957</td>
<td>0.992</td>
</tr>
<tr>
<td>Virtual Representation</td>
<td>2.9142</td>
<td>2.0354</td>
<td>1.0256</td>
</tr>
</tbody>
</table>

Fig. 2-19: Comparison of simulated and analytical BPSK BER performances of SISO and 1×2, 2×1, 1×4, 2×2, and 2×4 MIMO systems employing Alamouti and MRC schemes.
Next, we calculate transmission capacity values as function of distance for the fixed wireless broadband models described in Section 2.8 and compare them with actual experimented throughput values. These are demonstrated in Fig. 2-20. Both experimental and simulation results show that SUI-1 and SUI-2 perform almost equally fairly, and they give better capacities with distance compared to SUI-3, which is in turn better than SUI-4, and SUI-6. The experimental results for SUI-5 are not in agreement with any conjectures, which was agreed upon to be due to an equipment failure during the execution of the tests. The theoretical capacities match almost perfectly at higher distance with the experimental capacities; however there is a big gap between the two at lower distances. This could be because of using a smaller constellation for those distances than could be used without sacrificing performance. There are only 7 adaptive modulation constellation sizes, and the largest one is required to cover as much area as possible, thus limiting the rate.

Table 2.4: Comparison of SNR values required to achieve BER of $10^{-3}$ for various diversity combining and space-time coding schemes.

<table>
<thead>
<tr>
<th>Diversity</th>
<th>Analytical</th>
<th>MRC</th>
<th>Alamouti</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single branch, no diversity</td>
<td>24dB</td>
<td>24dB</td>
<td>×</td>
</tr>
<tr>
<td>2-branch diversity</td>
<td>11dB</td>
<td>11dB</td>
<td>14dB</td>
</tr>
<tr>
<td>4-branch diversity</td>
<td>4dB</td>
<td>4dB</td>
<td>7dB</td>
</tr>
<tr>
<td>8-branch diversity</td>
<td>-0.5dB</td>
<td>×</td>
<td>2.5dB</td>
</tr>
</tbody>
</table>
Fig. 2-20: Simulated and experimental capacity and TCP throughput versus distance for SUI- (a) 1, (b) 2, (c) 3, (d) 4, (e) 5, and (f) 6 fixed wireless broadband channel models.
2.10 Conclusions

In this chapter, we investigated various channel models used to represent RF wireless MIMO setups. Correlation statistics of fading coefficients are shown influence the capacity through simulation of outage capacity values for uncorrelated and correlated channel models. Having higher correlation coefficients can greatly reduce MIMO capacity, thereby enhancing the argument of employing MIMO RF systems in rich scattering environments. Although Weichselberger channel model is found to be more accurate, Kronecker and Virtual channel models can closely approximate actual capacity values. Furthermore, it is pointed out that knowledge of eigenvalue distributions is imperative to obtain accurate measures of capacity statistics. MIMO can offer twofold advantages: *diversity* and *multiplexing*. MIMO diversity schemes are analyzed and practical methods of achieving maximum diversity is presented, as well as the diversity order being quantified. Linear and ML-like practical MIMO receivers for multiplexing schemes are presented and their performances analyzed. Zero-forcing and Linear MMSE receivers have the least complexity, whereas Sphere Decoding is found to have comparatively less complexity than ML receiver at high SNR, making it a very attractive Viterbi Decoder-like implementation. Space Time coding through Alamouti codes are investigated. Some experimental results showing comparison of achievable capacity and real TCP throughput indicate that there is more room for improvement by having a more graded adaptive modulation/coding combination for RF broadband wireless data communication. The MIMO techniques reviewed in this chapter form the bases for the modeling analysis, simulation, and optimization in the following chapters.
Chapter 3

MIMO Transmission Techniques for Category Copper Cables

3.1 Introduction

Although wireless digital systems are becoming more available and the information transfer rate is growing ever higher, there is no alternative for wired transmission systems, since they form the backbone of regional or nation-wide networks. Category copper cables are used to deliver digital data to customer premises using Digital Subscriber Lines (DSL), and also to provide Ethernet connectivity. The fastest mediums available to date for high-bandwidth long-distance data delivery are single-mode optical fibers, which are usually used in submarine cables and for very high-speed Internet backbone. Multi-mode optical fibers are more suitable for connecting to individual home and office users, since these are easier to install and are less expensive. Data centers and high performance computing centers require high-speed connectivity among content storage servers and data repositories. Traditionally, optical fibers have dominated the market for high-bandwidth data access. However, low cost and lower complexity of installation has driven the need to develop high data rate transmission systems based on copper cables. Due to its lower cost and simpler installation on one hand, and higher information capacity compared to wireless, copper cables are still a viable contender for optical fibers and digital RF microwave systems. As a result, IEEE has pursued and successfully implemented 1 Gigabit per second data transmission standard for unshielded twisted-pair copper cables in 1999 [67].

The principal difference of wired guided media from wireless is that once installed, the copper cable is mostly a static channel. The channel state information (CSI) is obtained during startup of the system using training sequences and is retained throughout long block transmissions. Therefore, from a MIMO perspective, no degree of freedom or diversity gain can be achieved for the wired media. Furthermore, in
applications like Ethernet, where four pairs of wires are used to transfer spatially multiplexed information streams, each pair already acts as a parallel path, i.e. there is no stringent requirement to further isolate the pairs. This means that if there is no significant interference from one pair to the other, and coding is done for individual pairs, there is no need to perform a MIMO analysis on wired systems.

However, this simplistic view is incomplete, because LAN and DSL cables do suffer from interferences, and some of these interferences contain useful data, which is cancelled in traditional systems. In this chapter, we present the copper cable channel model, obtain MIMO equalizers and cancellers, and design coding and modulation schemes that achieve capacity on these channels.

### 3.2 Copper Cable Channel Modeling

We are interested in high-speed Ethernet data transmission using copper cables, and Ethernet connectivity is achieved with copper cables of Category-5 and above. These cables consist of four twisted pairs covered in a plastic insulating sheath; there are two basic kinds that can support high data rate: unshielded twisted pair (UTP) and shielded twisted pair (STP) cables. In UTP type, the copper pairs inside the cable are individually covered by plastic insulation, but no metal shielding is provided, which is present in STP cables (Fig. 3-1). As a result, UTP cables suffer more from crosstalk than STP.

![Fig. 3-1: Cross-sections of UTP cable (left), e.g. Cat-5/5e/6 (without individual pair shielding), and STP cable (right), e.g. Cat-7/7+ (with individual pair shielding).](image-url)

The various interferences that the received signal contains in shielded or unshielded twisted pair cabling are listed and defined below.
1. **Insertion Loss (IL):** *Insertion loss* (IL), also commonly termed *attenuation*, is the reduction of signal strength along length of cable due to electromagnetic dissipation. Attenuation is not constant over the signaling frequency range which causes the impulse response to be distorted and spread over several symbol periods. The mitigation approach is to use equalizers that invert channel frequency response characteristics. Several methods are investigated including SISO and MIMO MMSE and adaptive equalizer and precoder.

2. **Far-End Crosstalk (FEXT):** *Far-end Crosstalk* (FEXT) is the interference that occurs at the receiving end from the transmit end of an unintended pair. With increase in length and proper isolation, FEXT can be reduced to a significantly low level having no effect on error-free detection. FEXT has MIMO information embedded in it, which leads to the expectation of more SNR and data rate when MIMO modeling is considered. Cable manufacturers limit FEXT in their cables by carefully incorporating metal sheaths on individual pairs, which increases manufacturing complexity, cost, and weight. Allowing pairs to interfere with each other may, in fact, have benefits, and this is investigated.

3. **Near-End Crosstalk (NEXT):** *Near-end Crosstalk* (NEXT) refers to the interference that originates from the transmit end of a pair and is induced on a different pair’s receiving circuitry on the same end. The information in NEXT is redundant since the transmitting end does not need to receive information it sends. As a result, this must be removed before digital symbol detection. Traditionally, this is done by canceling NEXT by means of MMSE or adaptive interference cancellers. SISO interference cancellers can be outperformed by
MIMO interference cancellers, since the interfering data on all 3 pairs acting on the intended pair is known jointly. This leads to a better design of an interference canceller, which can perform near the noise floor.

4. **Return Loss (RL):** *Return Loss (RL)*, also termed *Echo*, refers to self-interference, i.e. the signal that reflects towards receiving circuitry of a pair from the transmitter on same pair. Ideally, there should be no RL on a well-balanced cable. However, it is not possible to balance a cable perfectly even with some impedance matching circuitry present. Like NEXT, RL contains redundant information, and has to be cancelled from received signal. SISO interference cancellers based on MMSE or adaptive principles are usually employed to perform this task. When MIMO models are considered, RL can be lumped with NEXT as self-interference, and cancelled together.

5. **Alien NEXT (ANEXT):** *Alien NEXT (ANEXT)* refers to interference on a cable’s pairs that is induced by neighboring cables. ANEXT does not originate from within the same cable, i.e. it has no relevance to the data carried on the intended cable. ANEXT does not occur on STP cables, since they are doubly shielded. On UTP cables, ANEXT brings up the noise floor and cannot be eliminated appropriately in a MMSE or adaptive manner, since *a priori* statistics of the data on interfering cables cannot be estimated. UTP cables suffer from ANEXT, only when put in a bundle, as in a data center. Therefore, ANEXT has to be properly characterized with the noise floor.

Fig. 3-2 illustrates different crosstalk mechanisms in a schematic form. Both ends of the cable employ hybrid transformers to duplex incoming and outgoing data signals.
This transmission technique is termed full-duplex, and is important for increasing the net throughput of the channel. Half-duplex techniques exist, in which the data rate is one-half compared to full-duplex, but NEXT and RL do not occur. Half-duplex techniques are not considered in this thesis, since they do not contribute to higher data rates.

![Schematic diagram of various crosstalks existent on UTP cables, ANEXT is absent in STP cables](image)

Fig. 3-2: Schematic diagram of various crosstalks existent on UTP cables, ANEXT is absent in STP cables

### 3.2.1 Frequency Responses

Before we analyze the capacity of these channels, we present the measured frequency responses of various interferences in Category cables in this section. Fig. 3-3 illustrates measured frequency responses of IL, RL, NEXT, and FEXT for a Cat-7A cable, which has been made available by the cable manufacturing company Nexans Inc.

The notations used in this chapter for the frequency responses of each of these degradations are listed in Table 3.1.
Using the measured frequency responses given in Section 3.2.1, impulse responses can be obtained for the different degradations occurring in the Cat-6/7A cable, by taking inverse Fourier transforms of the quantities given in Table 3.1, i.e., 

\[ h_i(t) = \mathcal{F}^{-1}\{H_i(f)\}. \]

Proper windowing and smoothing is taken into account while working with measured data, which is prone to contain noise and interference as well as artifacts.
due to frequency-domain sampling. Some samples of the resulting impulse responses of a Cat-6A cable are illustrated in Fig. 3-4.

\[ y_i(t) = \sum_{j=1}^{4} h_{ij}(t) \ast x_{R,j}(t) + \sum_{j=1}^{4} g_{ij}(t) \ast x_{T,j}(t) + n_i(t) \]  

(3–1)

Since the transmission is digital, the received signal is sampled at a multiple of the symbol rate, and we have to transform Eq. (3–1) into a discrete form. This is obtained by using the D-transform and converting the convolutions in Eq. (3–1) to polynomial multiplications. The resulting output is,

\[ y_i(D) = \sum_{j=1}^{4} h_{ij}(D) \cdot x_{R,j}(D) + \sum_{j=1}^{4} g_{ij}(D) \cdot x_{T,j}(D) + n_i(D) \]  

(3–2)

Fig. 3-4: Samples of IR, RL, FEXT and NEXT impulse responses on CAT-6A cable.
The MIMO system model can be represented following Eq. (3–1) and Eq. (3–2), by substituting the single-dimensional variables into matrix and vector formats. Corresponding notations are given in Table 3.3, and mathematical models below,

\[
y(t) = H(t) x_r(t) + G(t) x_t(t) + n(t) \quad (3–3)
\]

\[
y(D) = H(D) x_r(D) + G(D) x_t(D) + n(D) \quad (3–4)
\]

Table 3.2: Notations in SISO representation of Category copper cable channel model

<table>
<thead>
<tr>
<th>Expression</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_{r,i}(t); i \in {1\ldots 4} )</td>
<td>Transmitted near-to-far-end signal on ( i )-th pair</td>
</tr>
<tr>
<td>( x_{r,i}(t); i \in {1\ldots 4} )</td>
<td>Transmitted far-to-near-end signal on ( i )-th pair</td>
</tr>
<tr>
<td>( y_i(t); i \in {1\ldots 4} )</td>
<td>Received near-to-far-end signal on ( i )-th pair</td>
</tr>
<tr>
<td>( h_{ij}(t); i, j \in {1\ldots 4} )</td>
<td>Impulse response of IL and FEXT from ( i )-th transmitting near-end pair to ( j )-th receiving far-end pair; this is IL where ( i = j ), and FEXT where ( i \neq j ).</td>
</tr>
<tr>
<td>( g_{ij}(t); i, j \in {1\ldots 4} )</td>
<td>Impulse response of RL and NEXT from ( i )-th transmitting near-end pair to ( j )-th receiving far-end pair; this is RL where ( i = j ), and NEXT where ( i \neq j ).</td>
</tr>
<tr>
<td>( n_i(t); i \in {1\ldots 4} )</td>
<td>Additive White Gaussian Noise (AWGN) on ( i )-th pair with variance ( \sigma^2 = N_0/2 ).</td>
</tr>
</tbody>
</table>

Table 3.3: Notations in MIMO representation of Category copper cable channel model

<table>
<thead>
<tr>
<th>Expression</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_t(t) )</td>
<td>Transmitted 4×1 near-to-far-end signal vector</td>
</tr>
<tr>
<td>( x_n(t) )</td>
<td>Transmitted 4×1 far-to-near-end signal vector</td>
</tr>
<tr>
<td>( y(t) )</td>
<td>Received 4×1 near-to-end signal vector</td>
</tr>
<tr>
<td>( H(t) )</td>
<td>4×4 matrix near-to-far-end channel impulse response; diagonal and off-diagonal elements represent IL and FEXT impulse responses.</td>
</tr>
<tr>
<td>( G(t) )</td>
<td>4×4 matrix near-to-near-end channel impulse response; diagonal and off-diagonal elements represent RL and NEXT impulse responses.</td>
</tr>
<tr>
<td>( n(t) )</td>
<td>4×1 Additive White Gaussian Noise (AWGN) vector with covariance matrix, ( \sigma^2 \cdot I = N_0/2 \cdot I ).</td>
</tr>
</tbody>
</table>
The desired signal on \(i\)-th pair, \(x_{R,i}(D)\) to be detected in a SISO environment can be isolated from Eq. (3–2) as:

\[
y_i(D) = h_{ii}(D) \cdot x_{R,i}(D) + \sum_{j \in \{1, \ldots, \tilde{q}_i\}} h_{ij}(D) \cdot x_{R,j}(D) + \sum_{j \in \{1, \ldots, 4\}} g_{ij}(D) \cdot x_{R,j}(D) + n_i(D)
\] (3–5)

In Eq. (3–5), the first term represents the desired term convolved with the corresponding impulse response. The second term represents FEXT, the third term NEXT, and the fourth term is AWGN. To excise interference, we have to design an interference canceller that will remove the second and third terms, and also an equalizer that will deconvolve \(x_{R,i}(D)\) from the first term.

In a MIMO setting, the desired signal \(x_n(D)\) is embedded in the first term of Eq. (3–4). The FEXT terms are also included in the desired signal. The only interference here is the second term which is required to be removed by interference cancellation, whereas to circumvent channel dispersion effects in the first term, we need to design a MIMO or matrix equalizer.

Note that the \(D\)-transforms in Eqs. (3–1), (3–2), (3–3), (3–4), and (3–5) use a sampling interval of \(T\), the symbol period. This is referred to as symbol-spaced model. As a result, pulse shaping has to be included. The square-root-raised-cosine (SRRC) pulse shape is optimum for channels with dispersion [68] and it is used on the transmit end.

### 3.2.2.1 Fractionally-spaced Expressions

Considering \(T/2\)-fractionally-spaced equalizers and cancellers, \(D\)-transform of the SISO impulse responses can be denoted by a \(2 \times 1\) polynomial vector as follows,

\[
h_y(D) = \left[\begin{matrix} h_{y0}^{(0)}(D) \\ h_{y0}^{(1)}(D) \end{matrix}\right]^T
\] (3–6)
This is formed by stacking split-phase pulse response samples corresponding to time $kT$ (denoted by superscript $(0)$) on top of split-phase response samples corresponding to time $kT + T/2$ (denoted by $(1)$). The received signal on $i$-th pair is,

$$y_i(D) = h_{ix}(D) \cdot x_{ri}(D) + \sum_{j=1, j\neq i}^4 h_{jx}(D) \cdot x_{rj}(D) + \sum_{j=1}^4 g_{jx}(D) \cdot x_{rj}(D) + n_i(D)$$  \hfill (3–7)

In Eq. (3–7), the noise sequences are i.i.d white, i.e. the covariance matrix is given by $R_{nn}(D) = \sigma_n^2 \cdot I_2 = 2N_0/2 \cdot I_2 = N_0 \cdot I_2$. Transmitted symbols are i.i.d. white, with average transmit energy per pair of $E_X$, i.e. $R_{xx}(D) = E_x$.

Similarly, $D$-transform of MIMO fractionally-spaced impulse response can be given by the following $8 \times 4$ polynomial matrix, with same notation as before.

$$H(D) = [H^{(0)}(D) | H^{(1)}(D)]^T$$ \hfill (3–8)

Then the output of the channel is,

$$y(D) = H(D)x(D) + G(D)x_r(D) + n(D)$$  \hfill (3–9)

which is the same as in Eq. (3–4), except the change in matrix dimensions, and hence the transmitted, received, and noise vectors have to be modified as follows,

$$y(D) = \begin{bmatrix} y^{(0)}_1(D) & y^{(1)}_1(D) & \cdots & y^{(0)}_4(D) & y^{(1)}_4(D) \end{bmatrix}^T$$ \hfill (3–10)

$$x(D) = \begin{bmatrix} x_1(D) & x_2(D) & \cdots & x_4(D) \end{bmatrix}^T$$ \hfill (3–11)

$$n(D) = \begin{bmatrix} n^{(0)}_1(D) & n^{(1)}_1(D) & \cdots & n^{(0)}_4(D) & n^{(1)}_4(D) \end{bmatrix}^T$$ \hfill (3–12)

The dimensions of these vectors are $1 \times 8$, $1 \times 4$, and $1 \times 8$, respectively. The noise sequence in Eq. (3–12) is assumed to be i.i.d. white with autocorrelation $R_{nn}(D) = \sigma_n^2 \cdot I_8 = N_0/2 \cdot I_8$. Also the autocorrelation of the input sequence in Eq. (3–11) is $R_{xx}(D) = E_x \cdot I_4$.

In the case of fractionally-spaced (FSE) structures, the matched filtering is performed discretely within the equalizing or canceling filter, and therefore need not be included separately, as in symbol-spaced structures.
3.3 Capacity Calculations

The capacity of a communication channel denotes the upper bound of practical digital transmission rate, and is related to the physical properties of the channel. A number of capacity bounds can be defined under certain conditions, and are functions of the usable bandwidth, attenuation, interference, power allocation, and noise floor. Shannon bound gives the theoretical bound which is the highest possible capacity given infinite resources for coding and decoding. Single carrier bound reflects practical constraints of finite power and processing. Waterfilling bound is another capacity bound based on the availability of channel state information at the transmitter.

3.3.1 SISO Single Carrier and Waterfilling Capacity

Shannon’s theorem states that in a noisy channel with capacity and transmission rate of $C$ and $R$ bits per second respectively, a coding technique exists that allows transmission error to arbitrarily reach zero if $R < C$. According to Shannon-Hartley Theorem [100][101], the theoretical maximum error-free data rate, $C_{sh}$ bits/second, using a signal power spectrum of $S(f)$ on a channel band limited to $W$ Hz, and with additive (or colored) Gaussian noise with noise power spectrum $N(f)$, is given by,

$$C_{sh} = \int_{0}^{W} \log_2 \left[ 1 + \frac{S(f)}{N(f)} \right] df$$

(3–13)

It is useful to obtain a discrete counterpart of Shannon capacity, which is found by dividing available bandwidth into $N$ smaller bands so that attenuation in each of them is approximately constant,

$$C_{sh} \approx \sum_{i=1}^{N} \frac{W}{N} \log_2 \left[ 1 + \frac{S(f_i)}{N(f_i)} \right]$$

(3–14)

When using multiple channels for data transmission, as in Eq. (3–14), all subchannels must use the same class of coding with a constant SNR gap from capacity [102]. The performance measure for a multitone channel such as this is geometric SNR, comparable
to Decision-Point SNR for equalized channels [102]. The single-carrier bound is obtained from the multi-channel capacity in Eq. (3–14) by limiting $N$ to infinity and including the gap to capacity $\Gamma$ in the denominator,

$$C_{\text{SC}} = \lim_{N \to \infty} \sum_{n=1}^{N} \frac{W}{N} \log_2 \left( 1 + \frac{\text{SNR}_n}{\Gamma} \right) = W \log_2 \lim_{N \to \infty} \prod_{n=1}^{N} \left( 1 + \frac{\text{SNR}_n}{\Gamma} \right)^{1/N}$$

$$= W \log_2 \exp \left[ \lim_{n \to \infty} \sum_{n=1}^{N} \ln \left( 1 + \frac{\text{SNR}_n}{\Gamma} \right)^{1/N} \right] = W \log_2 \exp \left[ \frac{1}{W} \int_0^W \ln \left( 1 + \frac{\text{SNR}(f)}{\Gamma} \right) df \right]$$

The single carrier bound is representative of the realistic throughput of a system with finite coding and processing, as opposed to the infinite paradigm of Shannon’s theorem. Real system implementations that can achieve this bound are Minimum Mean-Squared Error Decision Feedback Equalization (MMSE-DFE) and Tomlinson-Harashima Precoding (THP), discussed in Sections 3.4.1 and 3.4.3.

If the $k$-th pair of a category copper cable has a transfer function of $H_{ik}(f_n)$ sampled at every $n$-th frequency bin centered at $f_n$, the total capacity $C = \sum_{n=1}^{N} c_n$, where $c_n$ is the capacity of individual sub-channels, has to be maximized with a total power constraint of $E_s$, with individual sub-channels consuming $E_n$ power, i.e. $E_s = \sum_{n=1}^{N} E_n$.

Mathematically the optimization problem is expressed as,

$$\maximize_{E_n} W \sum_{n=1}^{N} \log_2 \left( 1 + \frac{E_n |H_{ik}(f_n)|^2}{\Gamma N(f_n)} \right)$$

subject to $\sum_{n=1}^{N} E_n \leq E_s$ \hspace{1cm} (3–16)

The solution to this optimization problem is obtained using waterfilling principle [103], which uses Lagrange multipliers to obtain the maximum of Eq. (3–16), giving the waterfilling bound, $C_{\text{WF}}$. Generally, all four pairs in the cable behave similarly; if they differ significantly, Eq. (3–16) can be modified to perform a joint optimization.

The SISO single-carrier bound needs modification by accounting for residual interference powers after equalization and crosstalk cancellation, and adding them to the noise term, giving,
where the noise plus interference term is

\[ N_i(f) = \sigma_n^2 + E_i \left( \sum_{i=1}^{4} \Gamma_{F}^f |H_{fi}(f)|^2 + \sum_{i=1}^{4} \Gamma_{RL}^u |G_{ui}(f)|^2 \right) \]

with \( \Gamma_{F}^f \) and \( \Gamma_{RL}^u \) representing the cancellation achieved by FEXT and RL/NEXT cancellers, respectively.

### 3.3.2 MIMO Capacity

Since the information about the RL/NEXT interferences are available at the transmit end, we can ignore the effect of \( G(f) \) for calculating MIMO capacity. Furthermore, the power distribution over first Nyquist set of frequencies from \(-1/2T\) to \(+1/2T\) should be uniform for optimum capacity \[104\], i.e. waterfilling and uniform power distribution are the same. The generalized MIMO capacity for parallel channels is then,

\[
C_{MIMO} = \int_{0}^{W} \log_2 \det \left[ I_N + \frac{E_i}{\sigma_n^2} H(f)H^H(f) \right] df \quad (3-18)
\]

Similarly, following Eq. (3–14), (3–15), and (3–18), the MIMO single-carrier capacity can be given by,

\[
C_{MIMO-SC} = W \log_2 \left[ \frac{1}{W} \exp \left( \frac{1}{W} \int_0^W \ln \left( 1 + \frac{E_i}{\sigma_n^2} H(f)H^H(f) \right) df \right) \right] \quad (3-19)
\]

This bound corresponds to the capacity that can be achieved by an infinite-length MIMO MMSE Decision Feedback Equalizer (DFE), discussed in section 3.4.1.

### 3.4 MIMO Equalizer and Canceller Design

In this section, we present the formulation of a MIMO equalizer. Equalizers are widely used for mitigating the long tails of inter-symbol interference (ISI) existent in
communication channels \[68\]. Several types of equalizers are available for ISI mitigation, such as the zero-forcing equalizer, minimum mean-square error (MMSE) equalizer, decision feedback equalizer (DFE) \[70\], as well as adaptive equalizers \[69\] based on the least mean square (LMS) and recursive least square (RLS) principles. Usually, the same techniques are applied to high-data-rate cables, but as SISO implementations; however, infinite-length and finite-length equalizers for multiple parallel channels with distortion and interference have been treated in literature \[71\][72]. In particular, \[44\], \[73\], and \[74\] deal with equalizer design for 1Gbps data transmission over copper. For now, we disregard NEXT and RL, and discuss their removal procedure in Section 3.5. Our choices of equalizer parameters and the justifications are as follows:

a. Decision Feedback Equalizer (DFE) has superiority over linear equalizers, since they can remove trailing ISI symbols. However, DFE suffers from error propagation at low SNR. This can be resolved by Tomlinson-Harashima precoding (THP) technique. Furthermore, the transmission link operates at significantly high SNR so that the error propagation issue rarely materializes.

b. The fractionally-spaced model as described in Section 3.2.2.1 is adopted here, since this model allows for incorporation of the channel matched filter inside the equalizer. This does not degrade performance as long as the oversampling rate is twice the symbol rate, and simplifies implementation.

c. Both infinite-length and finite-length DFE equalizers are tested. Even though the infinite-length equalizer cannot be easily implemented, it serves as a benchmark, and provides the absolute bound of ISI cancellation that any equalizer can achieve.
d. When the channel is not directly known to the receiver, training sequences must be transmitted to estimate the dispersive channel. As a result, we also investigate the performance of an adaptive LMS MIMO equalizer.

### 3.4.1 MIMO MMSE-DFE Equalizer

Throughout this section, notations from Section 3.2.2.1 and Eq. (3–9) — after neglecting RL/NEXT terms and dropping the subscript of $x_n(D)$ — are adopted. Fig. 3-5 shows a block diagram of MIMO-DFE. The causal feedforward (FF) and feedback (FB) matrix filters are denoted by $W(D)$ and $I - B(D)$, and are of sizes $4 \times 8$ and $4 \times 4$, respectively. The estimate of the transmitted vector is then,

$$\hat{x}(D) = W(D)y(D) + [I - B(D)]\hat{x}(D)$$  \hspace{1cm} (3–20)

where $\hat{x}(D)$ is symbol vector estimate after slicing. Assuming correct estimates, $\hat{x}(D) = x(D)$, which is a reasonable assumption for DFE at high SNR, we can write the vector error sequence as,

$$e(D) = B(D)x(D) - W(D)y(D)$$  \hspace{1cm} (3–21)

![Fig. 3-5: Block Diagram of an infinite-length MIMO-DFE.](image)
Applying orthogonality principle for minimum mean squared error, 
\[ E\{e(D)y^\nu(D^*)\} = 0 \], where \( D^* = 1/D^* \), we find the FF matrix filter as,

\[ W(D) = B(D)R_{\nu}(D)R_{\gamma}^{-1}(D) \] \hspace{1cm} (3–22)

where \( R_{\nu}(D) \) denotes the cross-correlation matrix of the sequences \( x(D) \) and \( y(D) \). To find the FB matrix filter for the IIR-MMSE-DFE that minimizes MSE, we form the autocorrelation matrix of the error sequence given in Eq. (3–21),

\[ R_{\omega}(D) = B(D)Q(D)B^H(D^*) \] \hspace{1cm} (3–23)

where \( Q(D) = R_{\nu}(D) - R_{\nu}(D)R_{\gamma}^{-1}(D)R_{\gamma}^{-1}(D^*) \). The crosscorrelation between input and output sequences and autocorrelation matrix of the output are,

\[ R_{\nu}(D) = E_xH^H(D^*) \] \hspace{1cm} (3–24)

\[ R_{\gamma}(D) = E_xH(D)H^H(D^*) + \sigma_x^2I \] \hspace{1cm} (3–25)

Using Eq. (3–24), Eq. (3–25), and the independent identically distributed property of \( x(D) \), and matrix inversion lemma, Eq. (3–23) can be simplified as,

\[ R_{\omega}(D) = \sigma_x^2B(D)\left[\sigma_x^2E_x + H^H(D^*)H(D)\right]^{-1}B^H(D^*) \] \hspace{1cm} (3–26)

Using techniques outlined in [75] for the factorization of spectral matrix polynomials, the argument of the inversion operation in Eq. (3–26) can be factored as \( G^H(D^*)\Lambda G(D) \), where \( G(D) \) is a causal (\( G_i = 0, i < 0 \)) and monic (\( \text{diag}\{G_o\} = 1 \)) 4×4 polynomial matrix, with \( G_o \) lower triangular, and \( \Lambda \) is a real diagonal matrix. Therefore, Eq. (3–26) becomes,

\[ R_{\omega}(D) = \sigma_x^2B(D)\left[G^H(D^*)\Lambda G(D)\right]^{-1}B^H(D^*) \] \hspace{1cm} (3–27)

Now, we can employ the fact that the optimal error sequence is always white [76], and that the error sequence becomes white with the choice \( B_{opt}(D) = G(D) \). This makes the autocorrelation matrix of the error sequence in Eq. (3–27),

\[ R_{\omega,\text{min}}(D) = \sigma_x^2\Lambda^{-1} \] \hspace{1cm} (3–28)
This detector is biased, as Eq. (3–20) can now be rewritten as,

\[ \mathbf{s}(D) = \left[ \mathbf{I} - \mathbf{M}(D) \right] \mathbf{x}(D) + \mathbf{G}(D) \mathbf{R}_{\delta}(D) \mathbf{R}_{\eta}^{-1}(D) \mathbf{N}(D) \]  

(3–29)

where \( \mathbf{M}(D) = \left( \mathbf{E}_x / \sigma^2 \cdot \mathbf{G}^H(D^*) \cdot \mathbf{A} \right)^{-1} \). \( \mathbf{M}(D) \) is causal, and,

\[ \mathbf{x}_i = (\mathbf{I} - \mathbf{M}_0) \mathbf{x}_i - \sum_{n=1}^{\infty} \mathbf{M}_n \mathbf{x}_{i-n} + \mathbf{n}_i \]  

(3–30)

Therefore, multiplying \( \mathbf{x}(D) \) by \( (\mathbf{I} - \mathbf{M}_0)^{-1} \) removes the bias, and the autocorrelation of error of the unbiased detector is,

\[ \mathbf{R}_{\text{er,\text{u,\text{min}}}}(D) = \sigma_x^2 (\mathbf{I} - \mathbf{M}_0)^{-1} \mathbf{A}^{-1} \left[ (\mathbf{I} - \mathbf{M}_0)^H \right]^{-1} \]

\[ - \mathbf{E}_x \left[ \mathbf{I} - (\mathbf{I} - \mathbf{M}_0)^{-1} \right] \left[ \mathbf{I} - (\mathbf{I} - \mathbf{M}_0)^{-1} \right]^H \]  

(3–31)

Individual decision point SNR (DP-SNR) values for each pair are obtained by forming the ratio of transmitted symbol energy to the diagonal elements of Eq. (3–31), i.e. \( \text{DP-SNR}_i = \mathbf{E}_x \left[ \text{diag} \{ \mathbf{R}_{\text{er,\text{u,\text{min}}}}(D) \} \right] \). The MMSE could be further reduced with optimal decoding order [72]. By permuting columns of \( \mathbf{H}(D) \) corresponding to descending values of \( \text{diag} \{ \mathbf{A} \} \), performance can be optimized. Since there is negligible asymmetry among pairs of the Cat-6/7 cables, this may not lead to significant performance improvement; therefore is not considered.

### 3.4.1.1 Finite-Length MIMO-MMSE-DFE

The FIR MIMO MMSE-DFE can be formulated along [72]. For a channel with \((L_c+1)\) \(T\)-spaced matrix taps, FF filter \( \mathbf{W} \) with \(2L_{FF} T/2\)-spaced matrix taps (size, \(4 \times 8L_{FF}\)), FB filter \( -B \) with \(L_{FB} T\)-spaced matrix taps (size, \(4 \times 4L_{FB}\)), we can write the \(8L_{FF} \times 4(L_{FF}+L_C)\) channel convolution matrix as,
Then, the content of the FF filter at \(kT\)-th instant is,

\[
\begin{bmatrix}
H_0^{(i)} & H_1^{(i)} & \cdots & H_{L_c}^{(i)} & 0 \cdots 0 \\
H_0^{(i)} & H_1^{(i)} & \cdots & H_{L_c}^{(i)} & 0 \cdots 0 \\
0 & H_0^{(i)} & H_1^{(i)} & \cdots & H_{L_c}^{(i)} & 0 \\
\vdots & \vdots & \ddots & \ddots & \ddots & \ddots \\
0 & 0 & 0 & \cdots & H_0^{(i)} & H_1^{(i)} \\
0 & 0 & 0 & \cdots & H_0^{(i)} & H_1^{(i)} \\
\end{bmatrix}
\]  
(3–32)

Therefore, the error vector is,

\[
\mathbf{e}_{k-\Delta} = \mathbf{x}_{k-\Delta} - \hat{\mathbf{x}}_{k-\Delta} 
\]  
(3–34)

where \(\mathbf{x}_{k-\Delta} = \mathbf{h} \mathbf{X}_k + \mathbf{N}_k\) with \(\mathbf{X}_k = [x_1^{(i)} \cdots x_{L_c-\Delta+1}^{(i)}]\), \(\mathbf{X}_k = [x_1 \cdots x_{L_c-\Delta+1}]\), and \(\mathbf{N}_k = [n_1^{(i)} \cdots n_{L_c-\Delta+1}^{(i)}]\), \(\mathbb{E}\{\mathbf{h}\mathbf{X}_k\mathbf{X}_k^T\} = \sigma_n^2 \mathbf{I}\). Suppose the decision delay is \(\Delta\), so that \((k - \Delta)\)-th symbol is decoded at \(kT\)-th instant. Then we form the observable vector \(\mathbf{Z}_k\) by augmenting \(\mathbf{Y}_k\) by previously detected symbols \(\hat{\mathbf{x}}_k = [\hat{x}_{k-\Delta} \cdots \hat{x}_{k-\Delta-L_c+1}]\), i.e., \(\mathbf{Z}_k = [\mathbf{Y}_k^T | \hat{\mathbf{x}}_{k-\Delta}^T]^T\), and the augmented filter matrix becomes \(\mathbf{V} = [\mathbf{W}^T - \mathbf{B}]\). Thus the estimate of the \((k - \Delta)\)-th symbol before slicing is,

\[
\hat{x}_{k-\Delta} = \mathbf{VZ}_k 
\]  
(3–34)

Now, assuming correct feedback, and using i.i.d. white property of \(\mathbf{x}_{k-\Delta}\), autocorrelation matrices in Eq. (3–35) are found as below.

\[
\mathbf{R}_{\mathbf{Z}_k} = \mathbb{E}\{\mathbf{x}_{k-\Delta}\mathbf{Z}_k^T\} = \mathbf{E}_X \cdot \left[ \mathbf{J}_x \mathbf{H}_c^T [\mathbf{0}\mathbf{I}_{N-\Delta N}] \right] 
\]  
(3–36)

\[
\mathbf{R}_{\mathbf{ZZ}} = \mathbb{E}\{\mathbf{Z}_k\mathbf{Z}_k^T\} = \mathbf{E}_X \cdot \left[ \mathbf{H}_c^T \mathbf{H}_c + \sigma_n^2 / \mathbf{E}_X \cdot \mathbf{J}_x \mathbf{I}_{L_c} / \mathbf{J}_x \mathbf{J}_x^T \mathbf{H}_c^T \mathbf{H}_c \right] 
\]  
(3–37)

where \(\mathbf{J}_x = [\mathbf{0}_{\Delta L_c} | \mathbf{I}_{L_c} | \mathbf{0}_{\Delta L_c}]\) and
Substituting Eq. (3–36) and Eq. (3–37) in Eq. (3–35) produces the optimal FF and FB matrix filters as,

\[
W_{opt} = J_0 H_x^H \left( H_x H_x^H + \sigma_n^2 I_x \cdot I_{4L_{FB}} \right) - H_x J_{\Delta} J_{\Delta}^H H_x^H \right)^{-1}
\]

(3–38)

\[
B_{opt} = J_0 H_c^H \left( H_c H_c^H + \sigma_n^2 I_c \cdot I_{4L_{FB}} \right) - H_c J_{\Delta} J_{\Delta}^H H_c^H \right)^{-1} H_c J_{\Delta}
\]

(3–39)

The autocorrelation of the error is then found to be,

\[
R_{ee} = E\{e_t e_t^T\} = E_{\Delta} I_4 - R_{ee} V_{opt}^H
\]

(3–40)

The optimum decision delay can be found by searching over \( \Delta \) for either minimum determinant or minimum trace of Eq. (3–40). Since both minimizations lead to the same results [72], we choose to minimize the determinant. Therefore, the corresponding noise autocorrelation matrix is,

\[
R_{ee,\min} = \min_{\Delta} \det R_{ee}
\]

(3–41)

To find the receiver bias, we note that,

\[
\tilde{x}_{t-\Delta} = \left[ W_{opt} - B_{opt} \right] \left[ H_x X_{t-\Delta} + N_t \right]
\]

(3–42)

The matrix that multiplies \( x_{t-\Delta} \) is obtained from the \((N\Delta + 1)\)-th to \(N(\Delta + 1)\)-th columns of \( W_{opt} H_c \). Calling this matrix \( M_0 \), the unbiased estimator is \( \tilde{x}_{t-\Delta, \text{opt}} = M_0^T V_{opt} Z_{\Delta} \). The error autocorrelation of the unbiased detector is,

\[
R_{ee,\min} = M_0^T R_{ee,\min} \left(M_0^T \right)^{-1} - E_{\Delta} \left(I - M_0^T \right) \left(I - M_0^T \right)^H
\]

(3–43)

Individual DP-SNR values are found similarly as in 3.4.1. A significant difference of this FIR structure from the IIR is that it does not consider successive interference cancellation by subtracting current decoded symbol on a branch from current symbol estimate for the next branch. However, by enforcing additional orthogonality conditions, such structures can be formulated [72].
3.4.2 LMS Adaptive DFE Equalizer with Steepest Descent

Sometimes, it is convenient to estimate the channel through training sequences, and obtain equalizer coefficients through an iterative procedure, during which convergence occurs under certain conditions and the resulting equalizer is used for data decoding. The simplest algorithm is the steepest descent algorithm, where one begins by arbitrarily choosing the FF and FB matrix filters $V$, say $V_0$. This initial choice of coefficients corresponds to some point in the quadratic MSE surface in the space of coefficients. The quadratic MSE, a function of equalizer coefficients, follows from Eq. (3–34) as,

$$J(V) = E\{e_{k-\Lambda}^H e_{k-\Lambda}\} = E\{(x_{k-\Lambda} - VZ_k)^H (x_{k-\Lambda} - VZ_k)\}$$  \hspace{1cm} (3–44)

In steepest descent method we use the gradient of $J(V)$, $G_m = \frac{\partial J}{\partial V_{k-V_n}}$, at a point in the coefficients space, $V_n$, corresponding to $m$-th iteration, and add an offset to $V_m$ that is proportional to $G_m$ to get $V_{m+1}$, i.e.,

$$V_{m+1} = V_m - \mu G_m$$  \hspace{1cm} (3–45)

With appropriate choice of the positive step parameter $\mu$, $J$ will converge to the optimum point, since $J(V)$ is a convex function of $V$. Hence, this algorithm is an implementation of Least Mean Square (LMS) principle. Once the minimum $J$ is reached at $M$-th iteration, the equalizer is said to reach convergence with $G_M = 0$, and no further change occurs in equalizer taps. From Eq. (3–44), $G_m$ at $m$-th iteration is found out to be,

$$G_m = \frac{1}{2} \frac{\partial J(V)}{\partial V_n} = \frac{1}{2} \frac{\partial}{\partial V_n} \left[ E\{(x_{k-\Lambda,m} - V_m Z_{k,m})^H (x_{k-\Lambda,m} - V_m Z_{k,m})\}\right]$$

$$= -E\{e_{k-\Lambda,m}^H Z_{k,m}\}$$

$$= -E\{e_{k-\Lambda,m}^H Z_{k,m}\}$$  \hspace{1cm} (3–46)

An estimate of the gradient vector can be taken without the expectation given in Eq. (3–46). As a result, the value of $V$ at $m$-th iteration is,
To better tune the convergence rate of the FF and FB matrix filters, their step coefficients can be made different as follows,

\[
V_{m+1} = V_m + \mu G_m = V_m + \mu e_{k-\Delta,m} Z_{k,m}^H
\]  

(3-47)

where \(\mu_{FF}\) and \(\mu_{FB}\) are the step parameters for the FF and FB sections respectively.

### 3.4.3 Tomlinson-Harashima Precoder

Decision Feedback Equalizers suffer from a serious drawback, since a decision error, when it occurs, propagates through the feedback filter and may induce further errors. As a result, a receiver DFE cannot use Trellis-Coded Modulation (TCM), unless parallel banks of DFEs are combined at the receiver with Viterbi decoding [77][78]. The proposed solution to encounter this problem is to place the feedback section of the DFE in the transmitters, which allows for the straightforward application of TCM, and does not suffer from DFE error propagation [79][80]. This type of precoding the data before transmission by filtering it with the feedback section of the DFE is called Tomlinson-Harashima precoding [81][82]. The block diagram of a MIMO Tomlinson-Harashima Precoder (THP) with the equalizer implementation is illustrated in Fig. 3-6. There is a minimal transmitted power increase due to removal of the feedback section to transmitter side, but no reduction in DFE SNR.

\[
V_{m+1} = V_m + e_{k-\Delta,m} \left[ \frac{\mu_{FF} Y_{k,m}}{\mu_{FB} X_{k-\Delta,m}} \right]^H
\]  

(3-48)

Fig. 3-6: MIMO MMSE Equalizer with Tomlinson-Harashima Precoding
The problem with the given structure is that the peak value of the transmitted sequence after precoding tends to increase towards infinity. This difficulty is solved by inserting an arithmetic modulo-2 operation, \( \Gamma_{M}() \), as shown in Fig. 3-7, where,

\[
\Gamma_{M}(x) = x - M\left\lfloor \frac{x + M/2}{M} \right\rfloor
\]  

(3–49)

Fig. 3-7: MIMO MMSE THP with Modulo-2 arithmetic

In Eq. (3–49), \( M \) is the number of modulation levels, \( d \) is distance between constellation points along one dimension, which is usually 2, and \( \lfloor x \rfloor \) indicates the highest integer that is less than \( x \). When optimum feedback filter is used in THP precoder section, as given by \( B_{opt}(D) = G(D) \), following from Eq. (3–27), it can be shown from Eq. (3–22) that for optimum feedforward filter \( W_{opt}(D) \), \( W_{opt}(D)H(D) = G(D) \), at reasonably high SNR values. Assuming THP precoder generates an internal signal \( x'(D) \),

\[
x'(D) = x(D) + \left[ I - G(D) \right] a(D)
\]  

(3–50)

Using sequence notation, the individual coefficients in Eq. (3–50) are,

\[
x'_k = x_k - \sum_{i=1}^{L} G_i a_{k-i}
\]  

(3–51)

Then the output of the non-linear function on the receiver side is given as,
Therefore, it is clear that the THP precoder with the non-linear function behaves exactly similarly as the DFE implementation, with a minor noise-scaling.

### 3.5 Joint NEXT and Echo Cancellation

As in the case of IL and FEXT joint equalization and cancellation, MIMO techniques can be implemented to jointly cancel NEXT and Echo interferences on the desired pair of cable. Echo cancellation has been investigated previously in the context of telephone lines, Digital Subscriber Lines (DSL), and general Discrete-Multi-Tone (DMT) transmission \[83\][84][85][86][87][88][89][90][91][92]. Several methods of echo cancellation exist, including MMSE, LMS adaptive, and pole-zero cancellation. Complexity-wise, MMSE echo cancellation performs best, however it requires channel knowledge. LMS adaptive cancellation is an effective method where training sequences are sent to estimate and subtract echo interference. Here, both MMSE and LMS adaptive Echo/NEXT cancellation techniques are presented in a MIMO setting. Fig. 3-8 illustrates a joint canceller. $\hat{G}$ is to be determined to minimize energy of the error sequence, $e_k$. 

\[
\Gamma_M [\hat{x}_k] = \Gamma_M \left[ a_k + \sum_{i=1}^{L} G_i a_{k-i} + n'_k \right] = \Gamma_M \left[ \sum_{i=1}^{L} G_i a_{k-i} + n'_k \right] = x_k + e'_k
\]  

\[(3–52)\]
3.5.1 MMSE Joint Echo and NEXT cancellation

We neglect the intended signal vector $x_R(D)$ in Eq. (3–9), and rewrite,

$$y(D) = G(D)x(D) + n(D)$$  \(3–53\)

Here, $G(D)$ contains echo impulse responses along its diagonals and NEXT impulse responses in off-diagonals, and has a similar structure for Fractionally-Spaced Implementation as in Eq. (3–8). We require a fractionally-spaced implementation, since it is assumed that the equalizer has a fractionally-spaced implementation and equalization is done after Echo & NEXT cancellation. Therefore, Eq. (3–32) and Eq. (3–33) can be followed to write,

$$Y_k = G_k X_k + N_k$$  \(3–54\)

with the $k$-th received echo/NEXT interference given by $Y_k = [y_{k}^{(0)T} \ y_{k}^{(1)T}]^T$, transmitted symbols on the near end as $X_k = [x_k^T \ \cdots \ x_{k-L_E}^T]$, and additive noise samples, $N_k = [n_{k}^{(0)T} \ n_{k}^{(1)T}]^T$, where $E\{N_iN_i^T\} = \sigma_n^2 I$, and $E\{X_iX_i^T\} = E_x I$. Each vector is of the appropriate size, with echo path length extending from 0 to $L_E$ in symbol intervals.

Since we are trying to estimate the echo/NEXT channel, i.e. $G_c$ itself and not the data $X_k$, it is more convenient to express Eq. (3–54) in a different way, as follows,
In this notation, $X_{c,k}$ and $g$ have the following definitions to conform with Eq. (3–54),

$$Y_k = X_{c,k}g + N_k$$  \hspace{1cm} (3–55)

Here, $vec(\cdot)$ denotes the vectorization operator which stacks columns of a matrix on top of each other sequentially to form an equivalent vector, and $\otimes$ denotes the Kronecker product. Furthermore, $\tilde{x}_k = \left[ x_k^T \cdots x_{k-L_c}^T \right]$. Assuming Echo/NEXT canceller length of $L_c$, with $L_c + 1 \leq L_E$, and vectorized estimated Echo/NEXT impulse response of $\tilde{g}$, we can write estimated echo/NEXT interference as,

$$\hat{Y}_k = X_{c,k-\Delta} \tilde{g}$$  \hspace{1cm} (3–58)

Here, $X_{c,k-\Delta} = I_8 \otimes \tilde{x}_{k-\Delta}$ with $\tilde{x}_{k-\Delta} = \left[ x_{k-\Delta}^T \cdots x_{k-L_c+1}^T \right]$, and the delay of the echo path, $\Delta$, has been taken into account to form the estimate. The estimation error is,

$$e_k = Y_k - \hat{Y}_k$$  \hspace{1cm} (3–59)

Therefore, the Mean Square Error is found out as follows,

$$J(\tilde{g}) = E\{e_k^T e_k\}$$

$$= E\left\{ (X_{c,k}g + N_k - X_{c,k-\Delta} \tilde{g})^T (X_{c,k}g + N_k - X_{c,k-\Delta} \tilde{g}) \right\}$$

$$= g^T E\{ X_{c,k}^H X_{c,k} \} g + E\{ N_k^H N_k \} - \tilde{g}^T E\{ X_{c,k-\Delta}^H X_{c,k-\Delta} \} \tilde{g} - g^T E\{ X_{c,k-\Delta}^H X_{c,k-\Delta} \} \tilde{g} + \tilde{g}^T E\{ X_{c,k-\Delta}^H X_{c,k-\Delta} \} \tilde{g}$$

$$= E_g g^H g + 8\sigma_n^2 - E_g \tilde{g}^H P_g \tilde{g} - E_g g^H P_g^H \tilde{g} + E_g \tilde{g}^H \tilde{g}$$

(3–60)

where $P_g = I_8 \otimes \left[ \begin{array}{cccc} 0_{4L_c+4} & 0_{4L_c+4(L_c+1)} & \cdots & 0_{4L_c+4(L_c+1)} \end{array} \right]$. To find the minimum MSE, we obtain the partial derivative of Eq. (3–60) with respect to $\tilde{g}$ and set it to zero to find,

$$\frac{\partial}{\partial \tilde{g}} \{ J(\tilde{g}) \} = -2E_g P_g \tilde{g} + 2E_g \tilde{g} = 0$$  \hspace{1cm} (3–61)

which gives the FIR MMSE Echo/NEXT canceller as,
\[ \hat{g} = P \cdot g \] (3–62)

From Eq. (3–62) it is obvious that the optimum FIR Echo/NEXT canceller has taps equal to the Echo/NEXT channel response itself, in the appropriate range as shown below,

\[
\hat{G} = \begin{bmatrix}
G_\Delta^{(0)} & \cdots & G_{\Delta+i-1}^{(0)} \\
G_\Delta^{(1)} & \cdots & G_{\Delta+i-1}^{(1)}
\end{bmatrix}
\] (3–63)

The interference isolation performance of the canceller is given by the Echo/NEXT Isolation Ratio (ENIR), which is indicative of the reduction in the error resulting from cancellation, and given for the overall Echo/NEXT canceller as,

\[
ENIR = 10 \log_{10} \left( \frac{E \left( Y_i^H Y_i \right)}{E \left( e_{\Delta}^H e_{\Delta} \right)} \right) = 10 \log_{10} \left( \frac{E_x \left( g^H g + 8\sigma_n^2 \right)}{E_x \left( \hat{g}^H \hat{g} - g^H g \right) + 8\sigma_n^2} \right)
\] (3–64)

The individual ENIR for the \( i \)-th pair can be given as follows, specifically for a \( T/2 \)-fractionally-spaced case,

\[
ENIR_i = 10 \log_{10} \left( \frac{\sum_{i=1}^{4} \text{diag} \left( E \left[ Y_i Y_i^H \right] \right)}{\sum_{i=1}^{4} \text{diag} \left( E \left[ e_{\Delta}^H e_{\Delta} \right] \right)} \right)
\] (3–65)

The numerator is found as sum of \( i \)-th and \((i + 4)\)-th diagonal entries of \( E \left[ Y_i Y_i^H \right] \), which is equal to \( E \left[ (X_{c,i}g + N_i)(X_{c,i}g + N_i)^H \right] \) or \( E \left[ (X_{c,i}gg^H X_{c,i}^H + \sigma_n^2 I) \right] \). To find the numerator of Eq. (3–65), we denote the autocorrelation of \( g \) as \( C_g = gg^H \) and it can be shown that

\[
E \left[ (X_{c,i}g + \sigma_n^2 I) \right] = E_x \sum_{i_1, i_2, j_1, j_2} \text{diag} \left( \left[ C_g \right]_{i_1, i_2, j_1, j_2} \right) + \sigma_n^2 \delta_{ij},
\]

where \( \left[ \cdot \right]_{i, j} \) indicates a submatrix formed by spanning \( i_1 \) through \( i_2 \)-th rows and \( j_1 \) through \( j_2 \)-th columns of the matrix in the argument. Here \( i_1 = 4(L_x + 1)(i-1)+1 \), \( i_2 = 4(L_x + 1)i \), \( j_1 = 4(L_x + 1)(j-1)+1 \), \( j_2 = 4(L_x + 1)j \), and \( \delta_{ij} \) is equal to 1 when \( i = j \), and 0 else. Similarly, the denominator is found as sum of \( i \)-th and \((i + 4)\)-th diagonal entries of \( E \left[ e_{\Delta}^H e_{\Delta} \right] \), which can be determined to be a matrix with \((i, j)\)-th element equal to \( E_x \sum_{i_1, i_2, j_1, j_2} \text{diag} \left( \left[ C_g \right]_{i_1, i_2, j_1, j_2} \right) + \sigma_n^2 \delta_{ij} \), with similar notations as before, and \( i_1 = 4[(L_x + 1)(i-1)+\Delta]+1 \), \( i_2 = 4[(L_x + 1)(i-1)+\Delta+L_x] \),
The delay, $\Delta$, is chosen carefully so that the ENIR is maximized for a given canceller length, $L_c$.

### 3.5.2 LMS Adaptive Joint Echo and NEXT Cancellation

In Section 3.5.1, MMSE joint Echo and NEXT canceller was found out to have identical taps to the Echo/NEXT MIMO impulse response. However, channel knowledge is required for implementing such a canceller, which may not be directly available without a channel estimator. Therefore, a Least Mean Square Adaptive Echo/NEXT channel estimator or canceller offers a more suitable alternative at the cost of some loss in ENIR. The LMS adaptive solution follows immediately from the iterative equation,

$$
\tilde{g}_{m+1} = \tilde{g}_m - \mu \frac{1}{2} \frac{\partial J(\tilde{g})}{\partial \tilde{g}} \\
= \tilde{g}_m + \mu \cdot X_{c,k-L,m}^H e_{k,m}
$$  \hspace{1cm} (3-66)

Convergence of this equation is assured when the step size $\mu$ is chosen proportional to $2/\lambda_{\text{max}}$, where $\lambda_{\text{max}}$ is the maximum eigenvalue of the received signal autocorrelation matrix, i.e., $E[XX^H]$. 

### 3.6 Block Coded LDPC Modulation

Forward Error Correction (FEC) coding is an integral part of any digital data transmission system, which helps achieve the maximum practical data rate. However, FEC coding increases overhead by adding redundant parity bits. Therefore, optimization is required to trade-off the balance between bandwidth (BW) and bit error rate (BER). In this section, we introduce Low Density Parity Check (LDPC) block coded modulation (BCM) as a means to reach achievable data rate on Category copper cables. Coded modulation was first introduced by Ungerboeck [93][94][95], which is an efficient way to trade-off BW and BER by combining coding schemes with modulation. This technique is
based on set partitioning rules discussed in section 3.6.3. This gives a high bit-rate with high coding gain by combining LDPC code [96] and PAM/QAM modulation.

3.6.1 Low Density Parity Check (LDPC) code

LDPC codes have been shown to approach the Shannon bound within 0.0045dB [97], albeit with very large block length. Furthermore, LDPC codes can be efficiently constructed from random distributions, and decoding complexity is linear with block length. Particularly, LDPC was preferred over Convolutional Turbo Codes for the ITU-T G.hn standard, as well as for 10GBase-T Ethernet. This was due to lower decoding complexity, especially when operating at data rates near and above 1Gbit/s. However, the disadvantages are achievement of capacity with large block length and the iterative nature of the decoding algorithms which is imperative for efficient capacity achievement.

LDPC code is a linear block code with a sparse parity-check matrix, H, which has only a small fraction of elements equal to 1, and the rest equal to 0. Gallager constructed LDPC codes by drawing from a random binary distribution to build a \( m \times n \) parity check matrix [96]. The conditions are for each row to have the same number, \( d_c \), of 1’s and each column \( d_r \) number of 1’s. These are called \((d_c, d_r)\)-regular LDPC codes of length \( n \). Each information bit contributes to \( d_r \) parity checks and each parity bit contributes to checking \( d_c \) information bits. The following matrix is an example 5×10 \((2, 4)\)-regular LDPC code.

\[
H = \begin{bmatrix}
1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1
\end{bmatrix}
\quad (3–67)
\]

The code can be visually represented by bipartite graphs introduced by Tanner [105], in which the nodes are partitioned into two subsets so that no edges connect nodes within a subset. The subsets are called variable nodes and check nodes. Each variable node stands for each bit in the code, and each check node represents each row of H. As a result, the LDPC code is nothing but a set of parity check equations, with each check node checking only a small number of code bits. Tanner graph for Eq. (3–67) is illustrated in Fig. 3-9.
The fraction of 1’s in \( H \) is \( \frac{d_c}{n} \), which approaches zero with larger block lengths. Similar to regular block codes, LDPC generator matrix, \( G \), can be found for \( H \) to give \( GH^T = 0 \). If we put the sparse matrix \( H \) in the form \([P^T \mid I]\) via Gaussian elimination, the generator matrix is obtained as \( G = [I \mid P] \).

3.6.2 Decoding LDPC codes

The most popular and efficient LDPC decoding algorithm is the *sum product* algorithm [98], which is based on *belief propagation* of likelihood estimates between check and variable nodes. The initial input to the LDPC decoder is the log-likelihood ratio (LLR), \( L(c_i) \), which is defined as,

\[
L(c_i) = \log \left( \frac{\Pr\{c_i = 0 \mid \text{channel output for } c_i \}}{\Pr\{c_i = 1 \mid \text{channel output for } c_i \}} \right)
\]  

(3–68)

where \( c_i \) is the \( i \)-th bit of transmitted codeword \( e \). Three log likelihood ratios in the iterative algorithm are \( L(r_{ji}) \), \( L(q_{ij}) \), and \( L(Q_i) \), where \( L(q_{ij}) \) is initialized as \( L(q_{ij}) = L(c_i) \). For each iteration, \( L(r_{ji}) \), \( L(q_{ij}) \), and \( L(Q_i) \) are updated using,

\[
L(r_{ji}) = 2 \tanh^{-1} \left[ \prod_{j \neq j'} \tanh \left( \frac{1}{2} L(q_{ij}) \right) \right]
\]  

(3–69)

\[
L(q_{ij}) = L(c_i) + \sum_{j \neq i} L(r_{ji})
\]  

(3–70)

\[
L(Q_i) = L(c_i) + \sum_{j \neq i} L(r_{ji})
\]  

(3–71)
where the index sets, \( C_i \setminus j \) and \( V_j \setminus i \) are respectively the set of edges or transitions from \( i \)-th check node to all the variable nodes except \( j \)-th variable node, and the set of edges from \( j \)-th variable node to all the check nodes except \( i \)-th check node. At the end of each iteration, \( L(Q_i) \) provides an updated estimate of the \( a \ posteriori \) log-likelihood ratio for the transmitted bit \( c_i \). The hard decision output for \( c_i \) is 1 if \( L(Q_i) < 0 \), and 0 otherwise.

### 3.6.3 Block Coded Modulation

It was first suggested by Massey [99] to combine channel coding with modulation at the transmitting end, as well as to perform joint decoding and demodulation at the receiver end. Since \( M \)-PAM or \( M \)-QAM constellation has to be used on bandwidth limited channels to obtain capacity, Ungerboeck’s suggestion [94][95] was to use an expanded signal constellation where the redundant coding bits can be used in the expanded set. The whole constellation is divided into cosets, and one of these cosets is first chosen by a coding bit. Once a coset is agreed upon, the uncoded bits are found out from this coset and the received signal. Coset construction is achieved by dividing the whole constellation in levels so that the minimum distance is maximized. This is illustrated in Fig. 3-10, where a 16-QAM signal constellation is subdivided into 3 levels using 3 code bits \( c_0, c_1 \) and \( c_2 \), respectively, and one uncoded bit to discriminate between two remaining points in the lowest level set.

The structure of block coded modulation for 10GBase-T is followed here. We wish to obtain a high-rate high-gain coding scheme from a low-rate powerful LDPC code. If \( g_c \) denotes the minimum required coding gain to achieve capacity, then we can choose a \((n, k)\) rate block code, whereby a block of \( k \) data bits is transformed into \( n \) coded bits, and the block is rearranged into another block of \( nr \) rows and \( nc \) columns. Fig. 3-11 indicates one such block where \( c_i, i = 0 \cdots p \) are the coded bits, and \( s_j, j = 0 \cdots q \) are uncoded bits, and they are stacked on top of each other to produce a point in a \( M \)-ary constellation where \( M = p + q + 2 \).
The decoding process starts with the coded bits, and these are detected by supplying the log-likelihood ratios (LLR) to the LDPC decoding algorithm iterations. These initial LLR values are found out as follows, where \( r \) is the received point, and \( \{\mu | \mu_c = 0\} \) indicates the coset of the constellation which corresponds to \( i \)-th coded bit being 0. Once these LLR estimates are transferred to the algorithm described by Eq. (3–68) through Eq. (3–71), we obtained the coded bits \( c_0, ..., c_p \), and this establishes the cosets from which the uncoded bits are decoded using any algorithm including Maximum Likelihood using LLR such as follows,

\[
LLR(c_i) = \log \left( \frac{\sum_{\mu \in \mu_{c_i} = 0} \exp\left[-(\mu_i - r)^2\right]}{\sum_{\mu \in \mu_{c_i} = 1} \exp\left[-(\mu_i - r)^2\right]} \right)
\]

(3–72)
where \( r \) again is the received point, and \( \{ \mu \mid c', s_i = 0 \} \) indicates the coset of the constellation which corresponds decoded LDPC-coded bit-sequence \( c' \) and where \( i \)-th uncoded bit is 0. Wherever the LLR is less than 0, the decoded bit is a 1, and 0 otherwise.

However, some optimization is required to find out the right block and partition sizes, so that high data rate is maintained while necessary coding gain is achieved. Supposing the block has a coded bits part with \( n_c \) columns and \( n_r \) rows, and an uncoded bits part with \( n_a \) columns and \( n_r \) rows, the rate of the block code is \( r = (k + n_r \cdot n_c) / (n + n_r \cdot n_c) \), which is greater than the original code rate, \( k / n \). The signal constellation is partitioned into a number of cosets up to \( l \)-th level so that the required coding gain \( g_c \) falls in the proper Euclidean distance difference between \( l \)-th and \( (l+1) \)-th level of partitioned sets, i.e. \( d_i^l \leq g_c \leq d_i^{l+1} \). Then, \( n_r \geq l+1 \) is required to obtain an overall coding gain of \( g_c \).

What is left to be determined is \( n_a \), the number of rows allotted to uncoded bits to achieve a high aggregate rate \( r \). For a channel with band-limitation to \( W \) Hz, and root raised cosine filter with roll-off \( \alpha \), the required bits per symbol to achieve a transmission rate of \( R \) is \( b = R(1 + \alpha) / 2W \). If two \( M \)-QAM constellations are used to send 4 real symbol streams along 4 pairs of the cable pairs, signal constellation size has to be greater than or equal to \( 2^{b/r} \), i.e. \( M^2 \geq 2^{(1+\alpha)/2W} \). The other constraint to be satisfied is \( M \geq 2^{n+\alpha} \). Lowest bound of \( n_a \) can be found out by the following algorithm:

1. **Step 1:** Initialize \( r = 0.5 \);
2. **Step 2:** Set \( M^2 = 2^{b/r} \);
3. **Step 3:** \( n_a = \lceil \log_2 M - n_r \rceil \);
4. **Step 4:** Update \( r \) as: \( r = (k + n_r \cdot n_c) / (n + n_r \cdot n_c) \);
5. **Step 5:** if \( s \leq b/2r - n_r \), stop iteration;
   - else, \( n_a = n_a - 1 \), update \( r \): \( r = (k + n_r \cdot n_c) / (n + n_r \cdot n_c) \);
6. **Step 6:** Go to Step 2.
3.7 Simulations & Results

In this section, we demonstrate results of MIMO capacity, equalization, interference cancellation, modulation and coding. Four types of cables are considered, namely standard Cat-6 and Cat-6 characterized responses from Nexans for a length of 100m, and Cat-7A cables of length 50m and 100m, respectively.

3.7.1 Capacity Results

The capacity versus transmission bandwidth curves are plotted in Fig. 3-12 and Fig. 3-13 for various samples of CAT-6 and CAT-7A cables. The parameters for capacity evaluation are listed in Table 3.4, whereas the capacity bounds in Gbps and corresponding bandwidth in MHz where these are achieved are tabulated in Table 3.5.

| Table 3.4: Parameters used for capacity evaluation of CAT-6 & CAT-7A cables |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|                | Transmit Power, | Noise Power     | Coding gain,   | System Margin, | FEXT Cancellation | Echo Cancellation |
|                | $P_t$ (dBm)    | Spectral Density, | $g_c$ (dB)    | $\gamma_m$ (dB) | Level (dB)       | Level (dB)       |
| CAT-7A 100m    | 10             | – 155           | 5              | 0               | 0               | 53             |
| CAT-7A 50m     | 10             | – 155           | 5              | 0               | 0               | 53             |
| CAT-6 Standard | 10             | – 140           | 9              | 5               | 30              | 0              |
| CAT-6 Measured | 10             | – 140           | 9              | 5               | 30              | 0              |

$P_e$
Fig. 3-12: Capacity of Cat-6 cables, (a) Standard Cat-6, (b) Cat-6A characterized at Nexans.

Fig. 3-13: Capacity of Cat-7A cables of (a) 50m length, and (b) 100 m length.
There are a number of observations to be made, which are listed below:

i. The standard Cat-6 cable demonstrates much higher capacity values with a MIMO implementation in comparison with SISO. The difference is in the order of 10 Gbps. This is due to the fact that there is considerable amount of joint information embedded in FEXT, which contributes to increase of SNR, and hence capacity. In either case, the capacity reaches well beyond the 10Gbps cap intended for these cables.

ii. The characterization data provided by Nexans for the 100m Cat-6 cable fails to show any improvement when a MIMO system is employed instead of SISO. There is no visible gain in capacity. This is an improved version of the Cat-6 cable specially designed to suppress FEXT for higher data rate delivery. As a result, when FEXT is appropriately reduced by 30dB using separate SISO FEXT cancellers, we would observe no SNR gain, nor capacity gain. However, a MIMO joint equalizer/canceller may prove to be less complex than a SISO implementation that incorporates separate equalizers and FEXT cancellers, which we shall investigate shortly in Section 3.7.3.

Table 3.5: Capacity assessment of Cat-6 and Cat-7A cables

<table>
<thead>
<tr>
<th></th>
<th>Shannon Capacity (Gbps)</th>
<th>Waterfilling Capacity (Gbps)</th>
<th>Single-Carrier Capacity (Gbps)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SISO Bandwidth (MHz)</td>
<td>SISO Bandwidth (MHz)</td>
<td>SISO Bandwidth (MHz)</td>
</tr>
<tr>
<td>Cat-6 Standard</td>
<td>20.72 @ 800</td>
<td>15.12 @ 800</td>
<td>12.51 @ 580</td>
</tr>
<tr>
<td></td>
<td>32.09 @ 800</td>
<td>28.17 @ 800</td>
<td>28.02 @ 780</td>
</tr>
<tr>
<td>Cat-6 Nexans</td>
<td>152.5 @ 4000</td>
<td>115.1 @ 3500</td>
<td>115.2 @ 3500</td>
</tr>
<tr>
<td>Cat-7A 50m</td>
<td>54.72 @ 3500</td>
<td>40.26 @ 2750</td>
<td>40 @ 1250</td>
</tr>
<tr>
<td>Cat-7A 100m</td>
<td>55.87 @ 3500</td>
<td>40.37 @ 2000</td>
<td>40.5 @ 1250</td>
</tr>
</tbody>
</table>
iii. For the Cat-7A 50m cable, there is enough realizable capacity to transmit error-free data at 100Gbps. However, MIMO and SISO implementations yield virtually the same amount in all considered capacity evaluations.

iv. For the Cat-7A 100m cable, the maximum realizable capacity can reach close to 40Gbps. Again, the difference between MIMO and SISO capacity quantities is no more than 1Gbps.

v. The coding gain required for Cat-6 cables was assumed to be 5dB, which is easily realizable using LDPC codes, without block coded modulation. For Cat-7 cables, the assumed coding gain is 9dB, which requires more complex coding schemes, and block coded modulation, the results for which are discussed in Section 3.7.4.

vi. The FEXT cancellation level for Cat-7A was assumed to be 0, since FEXT is already suppressed enough not to degrade SISO capacity, nor increase MIMO capacity. But the echo cancellation level was assumed 53dB, and this has to be achieved by joint or separate Echo cancellers. These are discussed in Section 3.7.3.

### 3.7.2 Equalization & Precoding Results

Since the Nexans Cat-6 cable and the Cat-7A cables demonstrate similar trends in terms of SISO and MIMO capacity evaluation, it is sufficient to examine and compare the equalization and precoding results for the standard Cat-6 cable and the Nexans Cat-6 cable, whose capacity values differ significantly for SISO and MIMO implementations. The parameters used for the simulation are listed in Table 3.6.

For the Nexans-provided Cat-6 cable, the resulting Decision-Point SNR quantities and total number of taps are tabulated in Table 3.7 for MIMO-IIR and MIMO-FIR joint equalizer/canceller, and SISO equalizer with separate FEXT cancellers. Fig. 3-14 shows
the comparison of DP-SNR values as the number of feedforward taps and the number of feedback taps are varied, on the same cable.

Table 3.6: Simulation parameters for equalization and interference cancellation for Cat-6 cables

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target Data Rate</td>
<td>10 Gbps</td>
</tr>
<tr>
<td>Symbol Rate</td>
<td>800 Msp</td>
</tr>
<tr>
<td>Square Root Raised Cosine</td>
<td>8%</td>
</tr>
<tr>
<td>Filter Rolloff</td>
<td></td>
</tr>
<tr>
<td>Transmitted Symbol Energy</td>
<td>10dBm</td>
</tr>
<tr>
<td>per Pair</td>
<td></td>
</tr>
<tr>
<td>Background Noise Level</td>
<td>– 140 dBm/Hz</td>
</tr>
</tbody>
</table>

Table 3.7: Comparison of MIMO and SISO equalizers & FEXT cancellers for Cat-6+.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>MIMO-IIR</th>
<th>MIMO-FIR</th>
<th>SISO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feedforward Taps</td>
<td>×</td>
<td>×</td>
<td>90</td>
</tr>
<tr>
<td>Feedback Taps</td>
<td>×</td>
<td>90</td>
<td>90</td>
</tr>
<tr>
<td>FEXT Canceller Taps</td>
<td>×</td>
<td>×</td>
<td>30</td>
</tr>
<tr>
<td>Decision Point SNR, dB (all pairs)</td>
<td>31.86</td>
<td>31.41</td>
<td>31.41</td>
</tr>
<tr>
<td></td>
<td>32.28</td>
<td>32.84</td>
<td>31.84</td>
</tr>
<tr>
<td></td>
<td>32.28</td>
<td>31.78</td>
<td>31.78</td>
</tr>
<tr>
<td></td>
<td>32.68</td>
<td>32.25</td>
<td>32.25</td>
</tr>
<tr>
<td>Total Taps</td>
<td>×</td>
<td>3360</td>
<td>2280</td>
</tr>
</tbody>
</table>

Fig. 3-14: Decision-Point SNR for different feedforward and feedback filter tap numbers for improved Cat-6 cable.
As expected earlier from capacity evaluation for the Nexans Cat-6 cable, there is no enhancement of decision-point SNR by changing the system to MIMO from SISO, which is clear from Fig. 3-14. The expectation that MIMO system implementation may lead to less complexity is not realized either, as evidenced in Table 3.7. Both MIMO-FIR equalizer/canceller and SISO equalizer/canceller reach close to the maximum DP-SNR bound that could be achieved by a joint MIMO-IIR filter, and a SISO implementation gives less number of taps than a MIMO implementation. Both of these observations result from the fact that FEXT levels are too low to contribute to any signal power increment.

However, the DP-SNR quantities for a standard Cat-6 cable are plotted in Fig. 3-15, and reveals a different picture. The DP-SNR for MIMO equalizer/canceller consistently increases with increase in feedforward filter taps, whereas it remains almost constant for SISO equalizer/canceller. With 100 feedforward taps and 30 feedback taps, the MIMO equalizer/canceller performs about 1 dB above a SISO equalizer/canceller with similar taps and 30-tap FEXT cancellers. The SISO equalizer/canceller fails to isolate FEXT levels enough in comparison with the MIMO equalizer/canceller. As a result, we can conclude that significant FEXT signal energy can be harvested by joint MIMO equalizer/cancellers when the FEXT levels are high enough.

**Fig. 3-15:** Decision-Point SNR for different feedforward and feedback filter tap numbers for standard Cat-6 cable.
Bit Error Probability curves for various SISO and MIMO equalization and precoding schemes for the Nexans Cat-6 cable are illustrated in Fig. 3-16, Fig. 3-17 and Fig. 3-18. The foremost observation is the close correspondence between the matched filter bound plots, which demonstrate the case where infinite impulse response equalization is available. This observation is consistent with the close correspondence of capacity curves in Fig. 3-12(b). There is no extra benefit to be obtained by considering MIMO implementation over a SISO system.

Furthermore, MIMO FIR DFE and THP perform very close to each other, as should be expected from the similarity of the two. The only benefit of THP is the elimination of error propagation in the DFE. Two LMS equalization algorithms, decision-directed (DD) and non-decision-directed (non-DD) were also tested. As a result of absence of channel information, the BER plots shift by a small amount, about 0.5 to 1 dB, from the MMSE cases. The SISO equalizer/FEXT canceller also performs in close agreement with MIMO DFE and THP.

---

Fig. 3-16: Bit Error Probability versus SNR (E_s/No) for SISO and MIMO Equalization and Precoding schemes for 2-level pulse amplitude modulation at 4 Gigasymbols/second.
The gap to ideal AWGN has to be filled by error correction coding schemes. The difference in SNR at the same BER between ideal AWGN and a practically realizable equalizer/FEXT canceller varies from about 6dB for 2-level PAM, about 8dB for 4-level PAM and about 9dB for 8-level PAM. This is consistent with the assumptions considered for capacity analysis in Section 3.7.1. Incidentally, 2-, 4-, and 8-level PAM signaling at 4Gsps give us data rates of 4Gbps, 8Gbps, and 12Gbps, the last scheme reaching above the standard 10Gbps over the Cat-6 cable. The number of feedforward taps and feedback taps, for all cases, is equal to 150 and 80, respectively, in terms of symbol intervals.

### 3.7.3 Joint NEXT and Echo Cancellation Results

In this section, we present results for SISO and MIMO Echo and NEXT cancellers for Cat-6/7A cables. Fig. 3-19 shows the convergence the MIMO LMS adaptive NEXT/Echo canceller for a 50m Cat-7A cable. The performance of echo cancellation is tabulated in Table 3.8 for various tap lengths of Echo/NEXT canceller. Echo cancellation performance reaches the target 53dB with a tap number of 256, and the NEXT cancellation performance reaches to about 50dB with the same tap number.
**Fig. 3-18**: Bit Error Probability versus SNR (Es/No) for SISO and MIMO Equalization and Precoding schemes for 8-level pulse amplitude modulation at 4 Gigasymbols/second.

**Fig. 3-19**: Convergence of MIMO LMS Echo/NEXT canceller, showing reduction of estimation error for Echo impulse response, with number of taps equal to (a) 200, (b) 256.
The cancellation performance is also visually demonstrated in Fig. 3-20, where the Echo and NEXT impulse responses are compared with the LMS estimated impulse responses for a 50m Cat-7A cable. The variance of error between original impulse response and estimated impulse responses are $1.3937 \times 10^{-11}$, $1.2853 \times 10^{-11}$, $1.1005 \times 10^{-11}$ and $1.3698 \times 10^{-11}$, respectively, giving an average ENIR of 45dB.

### 3.7.4 Block LDPC-coded Modulation Results

As mentioned earlier, the required bit error rate at a reasonable transmit power can only be reached by error correction coding with coding gains of 6 to 9dB. In this section, we produce results of the block LDPC-coded modulation scheme introduced in Section 3.6.3.

First, we demonstrate the performance of a generic (64800, 32400) LDPC code of rate 0.5. Both gray and binary mapping for modulation are considered. Four inner uncoded bits are encapsulated in each symbol by four outer LDPC-coded bits in a 256-QAM constellation. As a result, the aggregate coding rate is augmented to 0.75, instead of the original 0.5. The resulting bit error probability plots are illustrated in Fig. 3-21. The coding gain achieved by this low rate complex coding scheme is seen to be between 17dB for gray coded modulation and 12dB for normal binary modulation. When the transmission scheme employs block coding to increase aggregate rate, the coding gain reduces to about 9dB for both gray and binary modulation. The powerful nature of LDPC

<table>
<thead>
<tr>
<th># of taps</th>
<th>0</th>
<th>200</th>
<th>256</th>
<th>500</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_v/E_{Echo}$</td>
<td>14.4dB</td>
<td>56.1dB</td>
<td>67.9dB</td>
<td>68.6dB</td>
</tr>
<tr>
<td>Echo Isolation Ratio</td>
<td>0dB</td>
<td>41.7dB</td>
<td>53.5dB</td>
<td>54.2dB</td>
</tr>
<tr>
<td>$E_v/E_{NEXT}$</td>
<td>63.6dB</td>
<td>93.8dB</td>
<td>112.9dB</td>
<td>115dB</td>
</tr>
<tr>
<td>NEXT Isolation Ratio</td>
<td>0dB</td>
<td>30.2dB</td>
<td>49.3dB</td>
<td>51.4dB</td>
</tr>
</tbody>
</table>

Table 3.8: LMS Echo and NEXT cancellation performance

The cancellation performance is also visually demonstrated in Fig. 3-20, where the Echo and NEXT impulse responses are compared with the LMS estimated impulse responses for a 50m Cat-7A cable. The variance of error between original impulse response and estimated impulse responses are $1.3937 \times 10^{-11}$, $1.2853 \times 10^{-11}$, $1.1005 \times 10^{-11}$ and $1.3698 \times 10^{-11}$, respectively, giving an average ENIR of 45dB.

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codes is easily realized, however the coding complexity for such block lengths and latency time are the limiting factors for implementing such codes.

The algorithm described in Section 3.6.3 was used to optimize coding rate and coding gain, and the resulting bit error probability plots are shown in Fig. 3-22. The chosen modulation scheme is 256-QAM, where the imaginary and real parts of the symbols are allocated to two twisted pairs. To achieve a minimum coding gain of 6dB, a two-level set partitioning is required, and to achieve 9dB coding gain, three-level set partitioning is employed. The LDPC code used to encapsulate uncoded bits is a low-rate (3,6) code with block length of 816 and rate 0.5, obtained from [137]. To achieve 6dB (or 9dB) coding gain, the blocks are formed in 8 rows, 2 (3) with coded bits and 6 (5) with uncoded bits. The total rate can be increased by considering blocks with larger number of

Fig. 3-20: Estimated Echo and NEXT impulse responses compared with Original impulse responses: (a) Echo, \( g_{11}(k) \); (b) NEXT, \( g_{12}(k) \); (c) NEXT, \( g_{13}(k) \); (d) NEXT, \( g_{14}(k) \)
columns. For the minimum block length, the aggregate rate for these two schemes are 0.9286 and 0.9167, respectively. The results in Fig. 3-22 indicate achievement of 6dB and 9dB coding gains, respectively, as required for 10Gbps and 100Gbps over Cat-6 and over 50m of Cat-7A cables.

Fig. 3-21: Bit error probability plots for uncoded, fully coded, and block coded transmission on a Cat-6/7A cable.

Fig. 3-22: Performance of block coded modulation to achieve coding gains of 6 and 9dB.
3.8 Conclusions

In this chapter, we discussed digital transmission techniques to achieve high data rates, namely 10Gbps over 100m of Cat-6 cables and 100Gbps over 50m of Cat-7A cable. To this end, equalization techniques are described, and their performance evaluated. The important finding is that without having enough FEXT interference, MIMO implementation will not yield an improved performance over SISO designs. However, manufacturers have the option to trade-off some manufacturing complexity by allowing higher FEXT, and by using MIMO equalization and cancellation techniques. MIMO adaptive Echo/NEXT cancellation techniques are also presented and validated. Block coded modulation schemes are used to achieve the coding gain required for copper cables to reach target data rates, and prove to be efficient in terms of bandwidth.
Chapter 4

Free Space Optical Communications using Multiple Apertures

4.1 Introduction

Free Space Optical (FSO) communication is an appealing line-of-sight alternative to RF microwave links for high data rate backbone access, due to the abundant bandwidth offered in the optical wavelength range, resistance to interference and interception, and license-free operation \[106\]. FSO communications can offer data rates comparable to fiber optical communications at a fraction of deployment cost, while extremely narrow laser beam widths provide no limit to the number of FSO links that can be installed at a given location.

However, the fundamental limitation of FSO communications arises from the atmosphere through which the laser propagates. Although relatively unaffected by rain and snow, FSO systems can be severely affected by fog and atmospheric turbulence. For example, fog particles are only a few hundred microns in diameter and can modify light characteristics through a combination of absorption, scattering, and reflection, which can lead to attenuation as large as 300 dB/km \[107][108\]. Furthermore, wind and temperature gradients create pockets of air with rapidly varying refraction index which induces fading. This phenomenon is termed as turbulence or scintillation. A slower effect of turbulence is beam wander, which causes misalignment of the transmitting laser and the receiving photodetector.

In this chapter, we present modeling techniques to properly characterize the atmospheric FSO channel, by developing cloud/fog simulation tools, as well as phase screens to characterize turbulence. Furthermore, diversity achievable on a multiple laser multiple detector (MLMD) FSO link is investigated, with bit error plots showing the superiority of the same over single laser single detector (SLSD) links.
4.2 Modeling of Scattering Dispersion

Atmospheric laser communications has to be undertaken in the presence of cloud, fog, haze, rain, or snow, due to the presence of which laser photons undergo multiple scattering before reaching the receiver or completely missing it. This results in the optical pulse to be broadened in space and time, and angular, spatial, and temporal dispersions occur. First efforts to solve multiple scattering problem analytically were based on the radiative transfer equation of Chandrasekhar [109], and culminated in the outstanding works of Dell-Imagine [110] and Heggestad [111]. However, the peaked nature of the phase function in clouds makes it difficult to obtain a closed form solution for free space optical communications [112]. As a result, Monte Carlo Ray Tracing (MCRT) has emerged as a popular semi-analytical simulation approach to tracking photons in a scattering medium. In this section, MCRT simulation method is detailed in the context of angular, spatial, and temporal dispersion of laser pulse propagation through clouds.

4.2.1 Parameters

Phase function gives the directional distribution of radiation scattered by individual particles due to a single scattering event. It denotes the probability that an incident photon is scattering through a scattering angle $\theta$ into a solid angle element $d\Omega$. The phase function can be obtained using Mie theory with particle size distribution as parameter [113][114]. Fig. 4-1 demonstrates characteristic phase functions for different types of clouds. The plotting is done only against the polar scattering angle, $\theta$, since the scattering is uniform along the azimuthal scattering angle, $\phi$. The phase function is usually normalized to have an integral of unity over $4\pi$ steradians, which gives it the convenience to be used as a probability distribution function (PDF) of scattering angles. Due to the isotropy of azimuthal angle, the integration of the phase function $P(\theta, \phi)$ over $\phi$ only contributes a factor of $2\pi$. As a result, $P(\theta, \phi)$ can be simply written as $P(\theta)$, and

$$2\pi \int_0^\pi P(\theta) \sin(\theta) d\theta = \int_0^\pi P(\theta) \frac{\sin(\theta)}{2} d\theta = 1$$ (4–1)
Therefore, the PDF of $\theta$ can be obtained from this normalized phase function as,

$$f(\theta) = P(\theta) \frac{\sin \theta}{2} \quad (4\text{-}2)$$

The phase function can be approximated by the Henyey-Greenstein (HG) phase function when optical thickness is sufficiently large, given by

$$P_{\text{HG}}(\theta, g) = \frac{1}{4\pi} \frac{1 - g^2}{(1 - 2g \cos(\theta) + g^2)^{3/2}}, \quad (4\text{-}3)$$

where $g$ is the *asymmetry parameter* and is a measure of the ratio of forward to backward scattering.

![Fig. 4-1: Phase Functions of different types of clouds](image)

The *optical thickness*, $\tau$, of the medium is used as a normalized quantity representing the physical length, $L$, of the scattering medium, which are related as follows

$$\tau = \beta_{\text{ext}} L = \frac{L}{D_{\text{ave}}} \quad (4\text{-}4)$$

where $\beta_{\text{ext}}$ is the *extinction coefficient* of the scattering medium, and $D_{\text{ave}}$ is the *mean free path*, i.e. the average distance between two successive scattering sites. At each scattering there is a small probability of absorption, given by absorption coefficient, $\beta_{\text{abs}}$, as well as scattering probability, represented by scattering coefficient, $\beta_{\text{scat}}$, where $\beta_{\text{ext}} = \beta_{\text{scat}} + \beta_{\text{abs}}$. 
The distance $d$ traveled by each photon after a scattering event, until it encounters the next scattering, is an exponential random variable with mean equal to the inverse of the scattering coefficient with $D_{ave} = 1/\beta_{scat}$, i.e.,

$$P(d) = \frac{1}{D_{ave}} \exp \left( -\frac{d}{D_{ave}} \right)$$  \hspace{1cm} (4-5)

### 4.2.2 Algorithm

The schematic of the Monte Carlo Ray Tracing (MCRT) algorithm is illustrated in Fig. 4-2. In MCRT, a large number of photons are launched from the transmitter coordinates, and their paths are tracked until they either reach the receiver or they escape a cylinder containing a large volume of the scattering medium. The output of the algorithm are angular dispersion, represented by the average cosine of angle of arrival; spatial dispersion given by the average displacement of the photons from the optical axis; and temporal dispersion, represented by the maximum delay spread. By the law of large numbers, statistical fluctuations in the algorithm converge to the steady-state quantities.

Fig. 4-2: Schematic of Monte Carlo Ray Tracing for Multi-scattering Simulation
The MCRT model is, in fact, a Markov chain, where the states are described by \([x_k,y_k,z_k,\theta_k,\phi_k]\); the first three variables represent current location of the photon in a Cartesian coordinate system defined by the previous traveling direction given by \(\theta_k\) and \(\phi_k\). The relative Cartesian coordinates are transformed into global coordinates as follows,

\[
\tilde{x} = R_{\theta_{k-1}} R_{\phi_{k-1}} \tilde{x}' = B_{k-1} \tilde{x}'
\]

(4–6)

where \(\tilde{x}\) and \(\tilde{x}'\) are global and relative coordinates of the photon location, and

\[
R_{\theta_{k-1}} = \begin{bmatrix}
\cos \theta_{k-1} & 0 & \sin \theta_{k-1} \\
0 & 1 & 0 \\
-\sin \theta_{k-1} & 0 & \cos \theta_{k-1}
\end{bmatrix}
\]

and

\[
R_{\phi_{k-1}} = \begin{bmatrix}
\cos \phi_{k-1} & -\sin \phi_{k-1} & 0 \\
\sin \phi_{k-1} & \cos \phi_{k-1} & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

are rotation matrices relating the global and relative coordinates through previous traveling direction \((\theta_{k-1},\phi_{k-1})\). With these notations, the final global position of a photon after \(n\) scatterings can be given as,

\[
\tilde{x} = d_0 + \sum_{i=1}^{n-1} \left( \prod_{k=i}^{j} B_k \right) d_i
\]

(4–7)

where \(\prod_{i=1}^{j} B_i = B_1 \cdot B_2 \cdots B_j\), with

\[
B_i = \begin{bmatrix}
\cos \phi_i \cos \theta_i & -\sin \phi_i \cos \theta_i & \sin \phi_i \sin \theta_i \\
\sin \phi_i \cos \theta_i & \cos \phi_i \cos \theta_i & -\sin \phi_i \sin \theta_i \\
-\sin \theta_i & 0 & \cos \theta_i
\end{bmatrix}
\]

and \(d_i = [0 \ 0 \ d_i]^T\) is the traveled distance vector oriented along local z-axis.

The MCRT simulator keeps track of the events that the photons encounter, including escape from scattering medium, absorption, and arrival at the receiver plane. Position, angle of arrival and traveled distance are recorded for each photon.

### 4.2.3 Performance Measures

By processing MCRT results, angular, spatial, and temporal dispersions can be determined. The time delay of each photon, with respect to the line-of-sight path, i.e. no-scatter path, to travel from transmitter to receiver can be calculated as,

\[
t_d = \left( d_{\text{no}} - L_{\text{coh}} \right)/c
\]

(4–8)
where $d_{tot}$ is the total distance traveled by the photon, $L_{ch}$ is the physical length of the cloud channel, and $c$ is the speed of light. This gives a measure of temporal dispersion. The maximum delay spread can be calculated from the distribution of time delays, by truncating the long impulse responses to contain only a significant portion of the received power, typically set to 90%.

Spatial dispersion is given by the spread of the received photon from the optical axis, as the distance from the optical axis, $r = \sqrt{x^2 + y^2}$, where $x$ and $y$ are coordinates of the final location of the photon on the receiver plane. The average distance from optical axis, $E[r]$ gives a measure of the total spatial dispersion encountered by a laser beam while propagating through the scattering medium.

Angular dispersion is another important determining factor, in conjunction with the field-of-view (FOV) of the receive geometry, of the received power. This is given by the cosine of angle of arrival on the receiver plane, $\cos \theta$. $E[\cos \theta]$ gives a measure of the overall angular dispersion encountered by the photons.

### 4.3 Markov Chain Analysis of Angular Dispersion in Scattering Medium

MCRT requires long run times and very high computational resources due to the need to track millions of photons. Statistical closed-form approaches have been used in [115] to find only the first two moments of the average angular dispersion due to multiple scattering. In this section, we present an alternative analytical method to calculate only the angular dispersion by use of Markov chain analysis. State transition matrices are calculated for various optical thicknesses to find the PDFs of the final scattering angles after any number of scattering incidents. The relationship among angular, spatial, and temporal distributions can be extracted from the angular dispersion of the whole system.

The trajectory of a photon can be modeled as a two-dimensional random walk best described by a Markov chain [112]. The $k$-th state of a photon, i.e. Cartesian coordinate of its position and its traveling direction prior to $(k+1)$-th scattering event, is given by the state vector as follows,
Alternatively, directional cosines \((\mu_x, \mu_y, \mu_z)\) can be used to represent the traveling direction [116]. The state variables in Eq. (4–9) need to be quantized for finite transition matrix representation, even though the state variables, in reality, are continuous variables. If the resolution is chosen carefully so as to circumvent noticeable fluctuations, we can obtain a close approximation to a continuous-space result. A convenient assumption is that of homogeneity of the scattering medium, i.e., the phase function does not change from one scattering event to another. As a result, the associated Markov chain demonstrates a time (and space) invariant nature.

Fig. 4-3 is used to find out the relationship between the initial traveling direction \(\theta_{k-1}\), with respect to the global z-axis, and the traveling direction \(\theta_k\) after \(k\)-th scattering. The scattering angle \(\theta\) is with respect to the temporary coordinate axis \(z'\) defined by prior traveling direction, and is drawn from the PDF of \(\theta\) given by Eq. (4–2). However, for the Markov chain analysis, we have defined the state variables with respect to the global coordinate system, and thus we require the distribution of \(\theta_k\), defined with respect to the global axis. The relationship among these angles are given by,

\[
\cos \theta_k = \cos \theta_{k-1} \cos \theta - \cos \phi \sin \theta_{k-1} \sin \theta
\]

(4–10)

Then, the cumulative distribution function (CDF) of \(\theta_k\) is can be expressed as,

\[
\Pr[\theta_k < \Theta] = \Pr[\cos \theta_{k-1} \cos \theta - \cos \phi \sin \theta_{k-1} \sin \theta > \cos \Theta] = \Pr[\cos \theta_k > \cos \Theta]
\]

(4–11)

Furthermore,

\[
\Pr[\cos \phi < (-\cos \Theta + \cos \theta_{k-1} \cos \theta)/\sin \theta \sin \theta_{k-1}] = \Pr[\cos \theta_k > \cos \Theta]
\]

(4–12)

As a result the CDF of \(\theta_k\) is obtained as,

\[
\Pr[\theta_k < \Theta] = \int_0^\phi f(\theta) \int_{\cos^{-1}(-\cos \Theta + \cos \theta_{k-1} \cos \theta)/\sin \theta \sin \theta_{k-1}}^{2\pi} f(\phi) d\phi d\theta
\]

(4–13)
The probability distribution of quantized $\theta_k$ is obtained by differentiating Eq. (4–15) given each quantized value of $\theta_{k-1}$, and the state transition matrix of the Markov process can be obtained from the PDF as,

$$
\mathbf{P} = \begin{bmatrix}
\Pr(\theta_k = 0 | \theta_{k-1} = 0) & \cdots & \Pr(\theta_k = \pi | \theta_{k-1} = 0) \\
\vdots & \ddots & \vdots \\
\Pr(\theta_k = 0 | \theta_{k-1} = \pi) & \cdots & \Pr(\theta_k = \pi | \theta_{k-1} = \pi)
\end{bmatrix}
$$

(4–14)

The statistical distribution of the angle of arrival at the receiver of each photon after $k$ scattering events is found from,

$$
P_k(\theta) = (\mathbf{P}^T)^T [1 \ 0 \ \cdots \ 0]^T
$$

(4–15)

This gives $P(\theta)\sin\theta/2$; therefore the statistical probability of the arrival angle can be found out by dividing through by $\sin\theta/2$.

**4.3.1 Eigenanalysis**

In Eq. (4–14), $\mathbf{P}$ denotes a regular state transition matrix, since it has no zero entries, and all transitions are allowed between $\theta_k$ and $\theta_{k-1}$. For regular Markov chains, there is a steady state $\lim_{n \to \infty} \mathbf{P}^n = \mathbf{\Pi}$, with the columns of $\mathbf{\Pi}$ being all equal to a vector $\mathbf{v}$. This
means that the final state of the Markov chain is independent of any initial state. This implies memory loss and complete diffusion or uniform illumination by photons at the exit plane of the cloud. This uniform distribution is identical to $\sin \theta/2$. The steady state of the Markov chain satisfies the equation the condition $\Pi P = \Pi$. By Perron-Frobenius theorem, $\nu$ is the first left eigenvector of $P$, corresponding to largest eigenvalue of 1. Therefore, it is impossible to spatially confine transmitted energy for a cloud or scattering medium having a large value of optical thickness.

It is an interesting problem to find out after how many scattering events one can approximate the steady state probabilities with the memory loss. This number determines the depth up to which the laser beam can penetrate the scattering medium before becoming completely diffuse in spatial distribution. It follows from Eq. (4–10) that,

$$E[\cos(\theta_i)] = E[\cos(\theta_{i-1})]E[\cos(\theta)] = E[\cos(\theta_{i-1})] \cdot g = g^i$$

(4–16)

This leads to the conclusion that the asymmetric parameter, $g$, characterizes the convergence of the Markov chain to its steady state. On the other hand, the Second Largest Eigenvalue Modulus (SLEM) determines the mixing rate of a Markov chain [117], and spatial memory loss occurs faster with smaller SLEM. The mixing rate of a Markov chain is given by,

$$T = 1/\log(1/\lambda^*)$$

(4–17)

where $T$ is the number of steps over which the difference from the steady state decreases by a factor of $e^1$, and $\lambda^*$ is the SLEM. The SLEM of the state transition matrix $P$ indeed gives the asymmetric parameter $g$.

### 4.4 Modeling of Atmospheric Turbulence

Due to temperature fluctuations in the atmosphere, turbulent pockets are created in the air [118], and as a result, FSO communications suffer from beam wander and fading. These phenomena originate from the broadening of the point spread function
Atmospheric turbulence is well described by Kolmogorov theory and Rytov approximation, which give statistical distribution of phase fluctuations due to turbulence [118]. Atmospheric turbulence is characterized by the Hufnagel-Valley equation,

\[ C_n^2(h) = 0.00594 \left( \frac{\nu}{27} \right)^2 \left( 10^{-5} h \right)^{10} e^{-h/1000} + 2.7 \times 10^{-6} e^{-h/1500} + Ae^{-h/100} \]  

(4–18)

giving the structure parameter of refractive index fluctuations, \( C_n^2 \), as a function of \( h \), altitude in meters, \( \nu \), the RMS wind speed in meters per second, and the sea-level reference of \( A = C_n^2(0) \). The variations in refractive index directly give rise to optical phase front deformation, which under near-field conditions translates into apparent phase perturbations, and under far-field conditions becomes apparent as amplitude variations [11]. As a result, both the amplitude and intensity of the optical wave have log-normal distributions on the pupil plane.

![Hufnagel-Valley profile](image)

**Fig. 4-4**: Hufnagel-Valley profile of structure function of refractive index fluctuation, for typical reference values of \( A \)

Under weak turbulence conditions, when the scintillation index is considerably lower, the variance of log-irradiance, \( \sigma_{lnI}^2 = 4\sigma_x^2 \), where \( \sigma_x^2 \) represents the variance of log-amplitude, is approximately equal to the normalized variance of irradiance or the scintillation index, \( \sigma_I^2 = \sigma_{lnI}^2 \), which are given for plane and spherical waves as,
The log-normal distribution of the irradiance fluctuation is given by Eq. (4–20), where the mean irradiance is set to $-\sigma_i^2 / 2$, so as not to introduce amplification or attenuation.

$$f_i(I) = \left(\frac{1}{I\sqrt{2\pi\sigma_i^2}}\right)e^{-\frac{\ln(I) - \ln(I_0)}{2\sigma_i^2}} \quad (4–20)$$

### 4.4.1 Phase Screens based on Kolmogorov Theory and Fourier Expansion

Thin phase screens are used to simulate effects of phase perturbation. These are obtained from spatial power density spectrum of refractive index fluctuations, given by Kolmogorov model as,

$$\Phi_\kappa(\kappa) = 0.033C_n^2k^{-1/3} \Lambda / L_0 < \kappa < 1 / l_0 \quad (4–21)$$

where $\kappa$ is the spatial wave number, and $l_0$ and $L_0$ are inner and outer scales of turbulence, respectively. A more precise model is given by von Kàrmàn model, as follows,

$$\Phi_\kappa(\kappa) = \left[0.033C_n^2/\left(\kappa^2 + \kappa_\text{m}^2\right)^{1/6}\right]e^{-\kappa^2/\kappa_\text{m}^2} \quad (4–22)$$

where $\kappa_\text{m}^2 = 2\pi / L_0$ and $\kappa_\text{m} = 5.92 / l_0$. A more accurate model is given by the modified Hill Spectrum [118], which is a useful analytical approximation to Hill’s results,

$$\Phi_\kappa(\kappa) = \left[1 + 1.8\kappa / \kappa_l - 0.25(\kappa / \kappa_l)^{7/6}\right]e^{-\kappa^2 / \kappa_l^2} \quad (4–23)$$

where $\kappa_l = 3.3 / l_0$. These three models assume homogeneity and isotropy of the turbulent media. Fig. 4-5 illustrates the normalized Kolmogorov, von Kàrmàn, and modified Hill spectrum. From the Kolmogorov spectrum of refractive index fluctuations, the power spectrum of phase front fluctuations can be derived as,

$$\Phi_{\Phi_\delta}(k) = 0.023C_n^2k^{-5/3} |\kappa_l|^{-1/3} \Lambda / L_0 < \kappa < 1 / l_0 \quad (4–24)$$
where \( r_0 \) is the atmospheric coherence length, also called the Fried parameter, which is a measure of coherence radius of the optical field, and is approximated as,

\[
 r_0 = 0.185 \left( \frac{4 \pi^2}{k^2} \int_0^L C_s^2(h)(1 - h/L)^{3/2} \, dh \right)^{3/5}
\]  
\textbf{(4–25)}

Physical interpretation of Fried parameter is as follows: if two receivers are apart by \( r_0 \), they experience virtually independent phase fluctuation, i.e. independent fading.

One other important parameter is the phase structure function defined by,

\[
 D_p(|r|) = 6.88(|r|/r_0)^{3/5}
\]  
\textbf{(4–26)}

Using Eq. \textbf{(4–24)}, a discrete phase screen can be generated by transforming Gaussian random variables \( h(k_x, k_y) \) after filtering with the square-root of the power spectrum of phase fluctuations, and then taking IDFT, as follows,

\[
 \varphi(x, y) = \sum_{k_x} \sum_{k_y} h(k_x, k_y) \sqrt{\Phi_p(k_x, k_y)} e^{i(k_x x + k_y y)} \Delta k_x \Delta k_y
\]  
\textbf{(4–27)}

The phase screen is evaluated at discrete points separated by spatial intervals \( \Delta x \) and \( \Delta y \), and the evaluation is done over discrete spatial frequency points separated by \( \Delta k_x \) and \( \Delta k_y \). The discrete white noise process \( h(k_x, k_y) \) is obtained from the complex Gaussian
process \( g(k_x, k_y) \), which has zero mean and standard deviation of \( 1/\sqrt{2} \) along each dimension, as below,

\[
h(k_x, k_y) = g(k_x, k_y) \sqrt{\Delta k_x \Delta k_y}
\]  

(4–28)

Eq. (4–27) can be rewritten for a phase screen of spatial dimension \( (G_x, G_y) \) with \( (N_x, N_y) \) discrete points as,

\[
\varphi(m,n) = \sum_{m=-N_x/2}^{N_x/2} \sum_{n=-N_y/2}^{N_y/2} h(m',n') f(m',n') e^{j2\pi(m'n'/N_x*n'/N_y)}
\]  

(4–29)

where \( f(m',n') = 2\pi(G_x G_y)^{-1/2} \sqrt{0.00058 \rho_0^{-5/6}} \left( f_x^2 + f_y^2 \right)^{-11/12} \) from Kolmogorov spectrum, \( m' \) and \( n' \) are spatial frequency indexes, and \( m \) and \( n \) discrete point indexes on the phase screen.

This method, while simple to implement, does not account for all the spectral frequency ranges corresponding to inner and outer scales of turbulence. Accuracy can be improved by taking phase screens as large as of the order of \( L_0 \), with sampling interval of \( l_0 \). This requires a large number of samples, thereby making it complex and less efficient to obtain phase screens fast. This issue can be resolved by non-uniform sampling in spatial frequency domain [119]. Three more advanced techniques are investigated here, namely, the sub-harmonic method [119], the random mid-point displacement algorithm [120], and Zernike polynomial expansion [121].

### 4.4.2 Phase screens based on Sub-harmonic method

Since the generic phase screen generation based on Fourier Expansion and Kolmogorov theory does not accurately model the phase fluctuation at the origin, the spatial frequencies near the origin are further sampled at smaller frequency intervals, i.e. each sample point near the center is replaced by multiple number of points. The process is continued by further sub-dividing the smaller spatial frequency intervals. Several low-frequency phase screens are generated and added to the high frequency phase screen. Assuming sub-harmonic level of \( N_p \), the low frequency phase screen is obtained as,
The modified phase screen is obtained by adding the high and low frequency phase screens.

4.4.3 Phase screens using Random Mid-point Displacement Algorithm (RMDA)

In the sub-harmonics method, a limited number of sub-harmonics are added which only contributes to a few low-frequency oscillations in limited directions. This is overcome in the RMDA method by exploiting the self-similarity of phase screens between inner and outer scales of turbulence. An initial phase screen can be generated on a rectangular grid of diameter proportional to $L_0$, with fewer samples, and its central part can be interpolated to obtain a finer sampled version. The process is continued until the phase screen diameter becomes comparable to aperture diameter. The randomness is obtained by adding random perturbations to interpolated points.

The two most popular variations of the RMDA algorithm are described here. The first method is FFT-based RMDA, where a large phase screen is first generated by FFT-based methods described in Section 4.4.1, and then the process of interpolation and addition of a new FFT-based phase screen as random perturbation is continued until the phase structure function converges to the desired shape. The algorithm can be summarized as below:

1. $L_0 = 2^n D$, $n$ integer; $D$ set to aperture diameter; $r_0$ set to Fried parameter; $m = 1/2$.
2. Generate initial phase screen covering outer scale area, with spatial limits $x_{\text{max}} = L_0/2$, spatial resolution $\Delta x = L_0/N$, spatial frequency limits $f_{\text{max}} = 1/2\Delta x$, and spatial frequency resolution $\Delta f = 1/L_0$.
3. Copy central part of phase screen generated in 2. Interpolate to generate $N \times N$ grid by cubic interpolation with new parameters $x_{\text{max}} = mX_{\text{max}}$, $\Delta x = m\Delta x$, $f_{\text{min}} = f_{\text{max}}$.

The phase structure function is given by:

$$
\varphi_{x'}(m, n) = \sum_{p=-n}^{n} \sum_{m=-1}^{1} h(m', n') f(m', n') e^{i 2\pi x' \phi_{x'}(m n N, n N, m N, n N)}
$$

where $f(m', n') = 2\pi 3^{-p} \left(G_x G_y \right)^{-1/2} \sqrt{0.00058 r_0^{-3.6} (f_x^2 + f_y^2)^{-11/12}}$. The modified phase screen is obtained by adding the high and low frequency phase screens.
\[ \Delta f = \frac{1}{2} x_{\text{max}}, \text{ and } f_{\text{max}} = \frac{1}{2} \Delta x. \] Create new phase screen with nonzero spectral component in the limits \([f_{\text{min}}, f_{\text{max}}]\). Add with phase screen generated in 2.

4. If \(2x_{\text{max}} = D\), exit, otherwise, go to 2.

The second RMDA method starts with a rough approximation of the fractal surface, and then performs successive refinements in smaller localized areas. The first four initial corner samples are generated as follows,

\[
\alpha = R^c_\alpha + 0.5R^d_{\alpha\delta}; \beta = R^c_\beta + 0.5R^d_{\beta\gamma}; \gamma = R^c_\gamma - 0.5R^d_{\beta\gamma}; \delta = R^c_\delta - 0.5R^d_{\alpha\delta}
\] (4–31)

where \(R^c_\alpha, R^c_\beta, R^c_\gamma, \text{ and } R^c_\delta\) are independent zero-mean Gaussian random variables of variance \(\sigma^2_c\), and \(R^d_{\alpha\delta} \text{ and } R^d_{\beta\gamma}\) are independent zero-mean Gaussian random variables of variance \(\sigma^2_d\), where the respective variances are given by,

\[
\sigma^2_c = 0.7506(D/r_0)^{5/3}; \sigma^2_d = 10.7575(D/r_0)^{5/3}
\] (4–32)

Then, the central sample is obtained by interpolating the corner samples and adding a random perturbation, as \(m = (\alpha + \beta + \gamma + \delta)/4 + \varepsilon\). The variance of \(\varepsilon\) is determined from the phase structure function to be,

\[
\sigma^2_\varepsilon = 0.6091(D/r_0)^{5/3}
\] (4–33)

The process of interpolation is continued to find out intermediate sample values.

### 4.4.4 Phase screens by Zernike Expansion

Zernike polynomials are a set of orthonormal functions over a unit circle and are represented in the polar coordinate system as a product of radial polynomials and angular functions. Since they form a basis on the unit circle, the phase perturbation of an optical wavefront in polar coordinates, \(\psi(\rho, \phi)\), can be obtained by the following expansion,
\[
\psi(\rho, \phi) \approx \sum_{i=1}^{N} a_i Z_i(\rho, \phi)
\]  
(4–34)

where \( Z_i(\rho, \phi) \) is the \( i \)-th Zernike polynomials given by Eq. (4–35), ordered according to Noll [121]. When \( N \) tends to infinity, the equality in Eq. (4–34) becomes absolute. Here, \( \rho = r/R \), where \( R \) is the radius of the aperture, is normalized radial distance from center.

\[
Z(\rho, \theta) = \begin{cases} 
\sqrt{n+1} R_0^n(\rho) \cos(m \theta), & \text{for } i = \text{even}, m \neq 0 \\
\sqrt{n+1} R_0^n(\rho) \sin(m \theta), & \text{for } i = \text{odd}, m \neq 0 \\
R_0^n(\rho), & \text{for } m = 0 
\end{cases}
\]  
(4–35)

where \( R_m^n(\rho) = \frac{(-1)^m (n-s)!}{s! [(n+m)/2-s]! [(n-m)/2-s]!} \rho^{n-2s} \), and azimuthal and radial orders, \( m \) and \( n \), are non-negative integers so that \( m \leq n \), and \( (n - m) = \text{even} \). The multipliers \( a_i \) are Zernike coefficients which are Gaussian distributed zero mean random numbers with covariance given by,

\[
E[a_i a_j] = 0.0072 \left( \frac{D}{r_0} \right)^{5/3} (-1)^{(n_i+n_j-2n_{ij})/2} \left\lfloor (n_i+1)(n_j+1) \right\rfloor^{1/2} \pi^{5/3} \delta_{m_i m_j} \\
\times \frac{\Gamma(14/3) \Gamma((n_i+n_j-5/3)/2)}{\Gamma((n_i-n_j+17/3)/2) \Gamma((n_j-n_i+17/3)/2) \Gamma((n_i+n_j+23/3)/2)}
\]  
(4–36)

where, \( m_i \) and \( n_i \) refer to the azimuthal and radial orders associated with the \( i \)-th Zernike polynomial, and \( \delta_{ij} \) is the Kronecker delta function.

### 4.5 Channel Modeling Results

#### 4.5.1 Multi-scattering Simulation with MCRT

The simulation algorithm described in Section 4.2 is used to obtain spatial, angular and temporal dispersion of a laser beam propagating through a homogeneous scattering medium of a given optical thickness, \( \tau \). The simulation parameters are
summarized in Table 4.1 Corresponding spatial dispersion characteristics for optical thickness values of 1 and 10 are illustrated in Fig. 4-6.

Table 4.1: MCRT Simulation Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Number of Photons</td>
<td>1,000,000</td>
</tr>
<tr>
<td>Cloud Type</td>
<td>Cumulus</td>
</tr>
<tr>
<td>Scattering Coefficient</td>
<td>131.85 / km</td>
</tr>
<tr>
<td>Optical Thickness</td>
<td>1 through 50</td>
</tr>
</tbody>
</table>

Fig. 4-6: Spatial distribution of photons on the receiver plane for (a) $\tau = 1$, (b) $\tau = 10$.

The effect of optical thickness on the spatial distribution of received photons is immediately visible. For low values of $\tau$, there is hardly any multiple scattering experienced by the photons, resulting in a received impulse-like response at the receiver plane. For large values of $\tau$, a two-dimensional Gaussian shape is formed around the optical axis. The spatial broadening of the beam is also visible in the temporal domain, as
illustrated in Fig. 4-7. For low optical thickness values, again, the photon temporal distribution is impulse-like, thereby contributing to little or no ISI at high data rate. The peak of the impulse is also relatively higher than in the case of \( \tau = 10 \). With increase in optical thickness, the temporal distribution broadens, and the peak received power also reduces, contributing to attenuation as well as ISI on the communication channel.

![Graph](image)

**Fig. 4-7:** Temporal distribution of photons at receiver plane for (a) \( \tau = 1 \), and (b) \( \tau = 10 \)

The spatial and temporal dispersions are measured in terms of normalized distance from optical axis, \( \tau \), and delay spread, \( d \), respectively. These are plotted in Fig. 4-8 against optical thickness range between 1 and 50. Each graph contains two plots, one corresponding to the case where scattering coefficient \( \beta_{\text{scat}} \) is kept fixed, while physical length of the channel, \( L_{\text{ch}} = \tau / \beta_{\text{scat}} \), is varied to get the optical thickness range; whereas in the other case, the physical length of the channel is kept fixed, while the scattering coefficient is changed to obtain the range of \( \tau \)’s.
With increase in scattering coefficient for a fixed-length cloud channel, the spatial dispersion, as well as the delay spread, increases and slowly reaches saturation. For the case when spatial dispersion and delay spread are normalized to the mean free path, $D_{\text{ave}}$, and the time to travel $D_{\text{ave}}$, respectively, the graphs coincide, as expected.

The third dispersion to be quantified is angular dispersion, given by the average cosine of the arrival angle, $\overline{\cos \theta}$. This is analyzed using the Markov chain theory discussed in Section 4.3. The simulation parameters for angular dispersion

---

**Fig. 4-8**: Spatial Dispersion: absolute (a) and normalized (b); and Delay Spread: absolute (c) and normalized (d), for fixed physical length channel (red) and fixed scattering coefficient (blue).
characterization are summarized in Table 4.2. A graphical profile of the state transition matrix, $P$, is presented in Fig. 4-9, as well as $P$ raised to the 15th and 50th powers.

Table 4.2: Simulation Parameters for Markov Chain analysis of angular dispersion

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cloud Type</td>
<td>Cumulus</td>
</tr>
<tr>
<td>Propagation Wavelength</td>
<td>1.55μm</td>
</tr>
<tr>
<td>Angular resolution</td>
<td>$\pi/300$</td>
</tr>
<tr>
<td>Transition Matrix size</td>
<td>$300 \times 300$</td>
</tr>
</tbody>
</table>

It is observed from Fig. 4-9 that as optical thickness increases, there are less and less Line-of-Sight (LoS) photons, and the conditional scattering angle probability slowly becomes independent of the initial state. After 50 scattering events, the probabilities of transitioning from any initial state to another are completely identical, and as such memory loss occurs. This identical distribution is proportional to $\sin \theta/2$. This is consistent with Bucher’s observation of uniform brightness at the bottom of a cloud when sunlight shines through it [56]. Comparisons of angular dispersion obtained from MCRT, Markov chain analysis and Ciervo’s Moment method [115] are shown in Fig. 4-10.

A point to explain about Fig. 4-10 is that MCRT produces distribution of photons on the 2D receiver plane, whereas Markov chain model and moment technique generate angular distribution in 3D space. Fair comparison between Markov model and MCRT is obtained by truncating the Markov results in the range $0 < \theta < \pi/2$, and projecting them on the receiver plane after correcting by a factor of $\cos \theta$. The mean and variance of angular dispersion coincide for truncated Markov model and MCRT, and there is also a fairly close agreement between the Moment technique and the Markov model. It is clear that the variance increases with optical thickness, while the mean decreases. This implies that for small optical thickness, a receiver with a small FOV is sufficient to collect reasonable amount of optical power; however, the energy is almost uniformly distributed in a 3D space for large optical thickness values. Saturation of the variance, as illustrated in Fig. 4-10(b), suggests that the density evolution of angular distribution merges to a steady state, as predicted in Markov analysis in Section 4.3.
To demonstrate the correspondence between the SLEM obtained by eigenanalysis and the asymmetric parameter $g$, these values for various clouds are listed in Table 4.3, along with the mixing time, $T$, given by Eq. (4–17).

Fig. 4-9: (a) Profile of State Transition matrix of scattering angle, $P$; (b) $P^{15}$, (c) $P^{50}$; (d) first row of $P^{50}$.
4.5.2 Phase Screen Generation

Sample phase screens generated by the different techniques described in Section 4.4 are presented in Fig. 4-11. It is immediately visible from Fig. 4-11(a) that the FFT-based method does not produce low-frequency phase fluctuations that cause the phase-front to tilt in space, as discussed in Section 4.4.1. This problem is not prominent in the phase screen generated by the Sub-harmonics method (Fig. 4-11(b)). Further refinement is obtained by the RMDA algorithms (Fig. 4-11(c, d)) described in Sections 4.4.2 and 4.4.3. Sample phase screen generated by Zernike Expansion is illustrated in Fig. 4-11(e).
Note that Zernike Expansion produces phase screens on a polar coordinate system, as opposed to the rectangular screens obtained by the other methods.

Fig. 4-11: Phase screens generated by (a) FFT method; (b) Sub-harmonics method; (c) Random Midpoint Displacement Algorithm I; (d) Random Midpoint Displacement Algorithm 2; (e) Zernike Expansion.

One measure of accuracy of the phase screen generation techniques is the phase structure function given in Eq. (4–26). We compare it with the phase structure function obtained from the simulations. To this end, 500 sample phase screens were generated by each method, and the correlation of phase fluctuations is evaluated at points located at a distance $|r|$ from the origin. These are presented in Fig. 4-12. The simplest phase screen produced by FFT-based technique does not correspond exactly to theory; however, there is more and more correspondence between theory and sub-harmonics method as the
number of sub-harmonics is increased. Furthermore, RMDA-I produces phase structure close to theory as the number of interpolation levels is increased, whereas RMDA-II is slightly divergent from the Kolmogorov phase structure. Surprisingly enough, the Zernike Expansion method of generating phase screens fails to produce a phase structure function that completely corresponds with theory, and this is due to considering only a limited number of Zernike coefficients and ignoring the rest.

Fig. 4-12: Comparison of Theoretical Phase Structure Function with simulated phase screens: (a) Subharmonics method; (b) RMDA methods; (c) Zernike Expansion.

4.6 MIMO FSO Communication System Design

With the deployment of multiple transmitters and multiple receiving apertures on an FSO communications system, it would be possible to benefit from spatial diversity
under the fading induced by turbulence and scattering induced by clouds and fog. For example, MIMO systems have been shown to be effective in combating log-normal amplitude fading [10][11][122]. In this section, we first demonstrate the effect of clouds and other scattering medium on a MIMO FSO communications system, and then discuss the more interesting aspects of diversity and multiplexing in a turbulent atmosphere. The two phenomena are treated independently, even though scattering and turbulence are not completely independent of each other. But assumption of independence gives mathematically tractability of the problem, and it is shown that such an assumption is realistic in many scenarios.

4.6.1 Scattering Impulse Response

Since Intensity Modulation with Direct Detection (IM/DD) is preferred over phase and frequency modulation for FSO communication systems, we consider On-Off keying (OOK) due to its lower complexity in comparison with coherent modulation [11]. MCRT algorithm was used in Section 4.2 to find spatial and temporal distribution of photons at the receiver side. By considering receiver geometries, such as aperture diameter and area, and Field-of-View, the system intensity impulse responses were determined. In Section 4.5.1, it was found out that multiple scattering broadens a collimated laser beam, as well as attenuate the received optical energy. The angular dispersion is manifested in the different angles of incidence at the receiver plane. Therefore, it is important to consider receiver geometry for a particular FSO system.

The scattering impulse response will have line-of-sight component, termed the coherent component, corresponding to photon trajectories that undergo no scattering, and also a diffuse component, corresponding to multiply scattered photons. Beer-Lambert law gives the coherent component of the received intensity as [113],

$$I_{\text{LOS}} = I_0 e^{-\tau}$$  \hspace{1cm} (4–37)
where $\tau$ is the optical thickness, represented by the average number of scattering events encountered in a cloud of physical thickness $L$. This LoS component is neither broadened in space and time, and is only an attenuated version of the transmitted impulse.

The dependence of impulse response on receiver geometry is illustrated in Fig. 4-13, where impulse responses obtained by a receiver of infinite aperture diameter are compared with a receiver of 20 cm aperture diameter, for various optical thickness values. It is apparent the pulse broadening is more prominent for infinite aperture receiver than finite aperture, since most received power comes from ballistic and snake (highly forward-scattered) photons. The number of snake photons increase with optical thickness, yet LoS part of the impulse response reduces exponentially according to Eq. (4–37).

![Fig. 4-13](image.png)

Fig. 4-13: Impulse responses obtained by MCRT for various optical thickness of cumulus cloud with (a) infinite receiver aperture, and (b) receiver aperture diameter of 20 cm.

The amount of received power could be increased by using larger aperture and FoV at the receiver, which will contribute to increase of collected diffuse photons. However, this also entails temporal and spatial broadening giving rise to Inter-Symbol Interference at high bit rates. Usually for a commercial applications, photodetector diameter is chosen on the scale of $\mu$m, and as such only the coherent component becomes dominant in the impulse response. As a result, we can simplify the effect of cloud and scatterers to an exponential attenuation factor proportional to optical thickness.
4.6.2 Effects of Turbulence on Multi-Aperture Systems

The effects of turbulence on a communication system can be broadly divided into two scenarios, namely flat log-normal fading when the aperture diameter is much smaller than the coherence length, i.e. $D/r_0 << 1$, and fading due to spatial broadening, when $D$ is comparable to $r_0$. The first case has been extensively dealt with in literature [11][12][13][10], but this situation occurs only under weak turbulence (large $r_0$) or for smaller receiver aperture. As a result of being in a region of coherence, the receiver can be approximated by a point aperture, and the amplitude fading factor can be treated as log-normally distributed, as given in Eq. (4–20). The second scenario occurs when receiver aperture diameter is larger than atmospheric coherence length, when turbulence is strong (small $r_0$) or the aperture diameter is made large intentionally to exploit aperture averaging effect. As a result, intensity fluctuations in the pupil plane are reduced, whereas spatial coherence is lost due to phase aberrations and point spread function (PSF) broadening. We investigate both of these cases, and assess performance of a multi-transmitter multi-aperture FSO communication system in terms of diversity and BER.

4.6.2.1 MIMO FSO Diversity Order Assessment under log-normal fading

In order to assess the diversity order, we consider multiple laser transmitters emitting towards multiple receivers. The complete MIMO channel transfer matrix is,

$$
\mathbf{H}(\tau) = \begin{bmatrix}
    h_{11}(\tau) & \cdots & h_{1M}(\tau) \\
    \vdots & \ddots & \vdots \\
    h_{N1}(\tau) & \cdots & h_{NM}(\tau)
\end{bmatrix}
$$

(4–38)

where there are $M$ transmitters, $N$ receivers, and the channel response between the $j$-th transmitter and the $i$-th receiver is represented by $h_{ij}(\tau)$.

The effect of clouds on the laser beam is deterministic at a macroscopic level, and the only randomness of the entries of $\mathbf{H}(\tau)$ is due to turbulence-induced fading. The fading coefficients are dependent on receive aperture spacing and geometric configuration, and their variance could be reduced by having a large aperture so that
aperture averaging holds, but will not contribute to significant MIMO gain. We assume high SNR regime, where we can use Gaussian noise model. Assuming intensity-modulation/direct-detection (IM/DD) and on-off keying (OOK) transmission [123], the received signal at the \( i \)-th receive aperture is given by

\[
r_i = \eta A_r \sum_{j=1}^{M} h_{ij} s + v_i = a \sum_{j=1}^{M} h_{ij} s + v_i, \quad i = 1, \ldots, N
\]

(4–39)

where \( s \in \{0, \sqrt{P_i}\} \) is the transmitted information bit, \( \eta \) is the optical-to-electrical conversion coefficient, \( A_r \) is the receive aperture area, and \( v_i \) is additive white Gaussian noise with zero mean and variance \( \sigma_v^2 = N_0/2 \). The fading channel coefficient which models the channel from the \( j \)-th transmit aperture to \( i \)-th receive aperture can be given based on the Rytov approximation,

\[
h_{ij} = h_{0,ij} \exp \left( 2 \chi_{ij} \right)
\]

(4–40)

where \( h_{0,ij} \) is the diffraction-limited intensity without turbulence that shines from the \( j \)-th transmit aperture on the \( i \)-th receive aperture and \( \chi_{ij} \) are identically distributed, but not necessarily independent, normal random variables with mean \( \mu_{\chi} \) and variance \( \sigma_{\chi}^2 \).

Therefore, \( h_{ij} \) follows a lognormal distribution

\[
p(h_y) = \frac{1}{2h_y \sqrt{2\pi\sigma_{\chi}^2}} \exp \left( -\frac{\left( \ln(h_{ij}/h_{0,ij}) - 2\mu_{\chi} \right)^2}{8\sigma_{\chi}^2} \right)
\]

(4–41)

To ensure that the fading does not attenuate or amplify the average power, we normalize the fading coefficients such that \( E \|h_{ij}/h_{0,ij}\|^2 = 1 \). Doing so requires the choice of \( \mu_{\chi} = -\sigma_{\chi}^2 \). Assuming weak turbulence conditions, the variances of log-amplitude fluctuation of plane and spherical waves are given by [118]

\[
\begin{align*}
\sigma_{\chi}^2_{\text{pl}} &= 0.307 C_{n}^{2} k^{7/6} L^{11/6} \\
\sigma_{\chi}^2_{\text{sph}} &= 0.124 C_{n}^{2} k^{7/6} L^{11/6}
\end{align*}
\]

(4–42)
where $k = 2\pi/\lambda$ is the wavenumber, and $L$ is the link distance in meters; $C_n^2$ is given in Eq. (4–18). For FSO links near ground, $C_n^2 \approx 1.7 \times 10^{-14}$ m$^{-2/3}$ during day and $C_n^2 \approx 8.4 \times 10^{-15}$ m$^{-2/3}$ at night. In general, $C_n^2$ ranges from $10^{-13}$ m$^{-2/3}$ for strong turbulence to $10^{-17}$ m$^{-2/3}$ for weak turbulence with $10^{-15}$ m$^{-2/3}$ defined as a typical average.

Assuming $l_0 \leq \sqrt{\lambda L} \leq L_0$, where $l_0$ and $L_0$ are inner and outer scales, $r_0$, the correlation length of intensity fluctuations can be approximated by $d_0 = \sqrt{\lambda L}$. When the aperture size, $D_o \gg d_0$, the detrimental effect of turbulence-induced fading is reduced by aperture averaging. But it is not always possible to make the aperture large enough, requiring multiple photodetectors at the receiver side for scenarios with $D_0 < d_0$. The spatial correlation matrix $\mathbf{R}$ to model the correlations among receive apertures is given by

$$
\mathbf{R} = \begin{bmatrix}
1 & b(d_{12}) & \cdots & b(d_{1N}) \\
b(d_{21}) & 1 & \cdots & b(d_{2N}) \\
\vdots & \vdots & \ddots & \vdots \\
b(d_{N1}) & b(d_{N2}) & \cdots & 1
\end{bmatrix} \tag{4–43}
$$

where $d_{ij}$ is the separation between $i$-th and $j$-th receiver apertures. $b(d)$ represents the normalized log-amplitude covariance function between two points in a receiving plane perpendicular to the direction of propagation and is defined by

$$
b(d_{P_1P_2}) = \frac{E[P_1P_2] - E[P_1]E[P_2]}{\sigma_x^2} \tag{4–44}
$$

where $d_{P_1P_2}$ is the distance between $P_1$ and $P_2$. Then the spatial covariance matrix $\mathbf{\Gamma}$ is given by $\mathbf{\Gamma} = \sigma_x^2 \mathbf{R}$. Similarly a correlation matrix of size $M \times M$ and corresponding covariance matrix can be defined for modeling spatial correlations at transmitter side.

Now, we formulate SNR expressions for different diversity reception systems using multiple transmit and receive apertures. Closed-form approximations can be obtained for maximal ratio combining (MRC) and equal gain combining (EGC), using Schwartz and Yeh method [124] of approximating sum of log-normal random variables. Simulations are also conducted and compared with analytical approximations.
The received signal for a SISO system can be given by

\[ r = a h_0 e^{2\chi} s + \nu \]  \hspace{1cm} (4–45)

where \( \chi \sim N(-\sigma_\chi^2, \sigma_\chi^2) \) and \( \nu \sim N(0, N_o/2) \). The SNR is easily seen to be,

\[
\text{SNR}_{\text{SISO}} = \left( \frac{ah_0e^{2\chi}E[s^2]}{E[\nu^2]} \right) = \left( \frac{(ah_0)^2 \cdot P_t/2}{N_o/2} \right) e^{4\chi} = \left( \frac{(ah_0)^2 P_t}{N_o} \right) e^{4\chi} = \text{SNR}_0 e^{4\chi} \]  \hspace{1cm} (4–46)

Therefore the SISO SNR is a log-normal variable given by \( \hat{\chi} \sim N(-4\sigma_\chi^2, 16\sigma_\chi^2) \) and \( \text{SNR}_0 \) is the SNR when no turbulence-induced fading is present.

Next we consider a MIMO system with \( M \) transmit apertures and \( N \) receive apertures. For a fair comparison, total transmitted intensity is divided equally among \( M \) transmitters and total receiver aperture area is also kept the same by making each receive aperture have an area of \( A_r/N \). As a result, background noise for each receiver is also reduced by a factor of \( N \). The signal received at the \( i \)-th receive aperture is given by

\[ r_i = a \sum_{j=1}^{M} \frac{h_{0,ij}}{MN} e^{2\chi_0} s + \nu_i \]  \hspace{1cm} (4–47)

where \( \nu_i \sim N(0, N_o/2N) \). After maximal ratio combining (MRC), the decision metric is

\[
y = \sum_{i=1}^{N} \left( a \sum_{j=1}^{M} \frac{h_{0,ij}}{MN} e^{2\chi_0} \right)^2 s + \left( a \sum_{j=1}^{M} \frac{h_{0,ij}}{MN} e^{2\chi_0} \right) \nu_i \]  \hspace{1cm} (4–48)

The resulting SNR after MRC is

\[
\text{SNR}_{\text{MRC}|\chi|} = \frac{1}{M^2 N} \left( \sum_{i=1}^{N} \left( \sum_{j=1}^{M} e^{2\chi} \right)^2 \right)^{-1} \cdot \text{SNR}_0 \]  \hspace{1cm} (4–49)

With similar MIMO setup as for MRC, the output of an equal gain combiner (EGC) can be written as

\[
y = s \sum_{i=1}^{N} \sum_{j=1}^{M} \frac{ah_0}{MN} e^{2\chi_0} + \sum_{i=1}^{N} \nu_i \]  \hspace{1cm} (4–50)

The SNR at the output of the combiner, then, is represented as
Simulated and analytical PDFs of output SNRs of different combining schemes are illustrated in Fig. 4-14 Fig. 4-15 for a $2 \times 2$ MIMO setup for uncorrelated and correlated fading under turbulence conditions with $\sigma_x = 0.1$ (weak) and $\sigma_x = 0.3$ (strong).

In all cases MIMO configuration shifts SNR PDFs further to the right from SISO case, and reduces the variability of the SNR. This means that, a diversity-based multiple-aperture system, under the given turbulence conditions with given correlations, will perform better on the average than a SISO system. It is also observed that a large correlation coefficient performs at an intermediate point between fully uncorrelated MIMO diversity system and a SISO system. The covariance values of $\sigma_x = 0.1, 0.3$ and correlation coefficients $\rho = 0$ through 0.9, are based on the work by [12][11]. One interesting observation is that MRC does not perform better than EGC. This is counter-
intuitive to results from RF communication systems. However, here we have considered intensity-modulation and direct detection, due to which the current output of the photodetector is proportional to the power, and maximal ratio combining results in noise being enhanced, and thus SNR reduced, while EGC results in equal-gain addition and noise is not amplified. The results indicate this as a fact for IM/DD.

Spatial diversity is an effective tool to mitigate the degrading effects of fading and has been extensively studied by several authors [123][10][125][126] within the context of FSO communication. In this study, we adopt relative diversity order (RDO) [123], as a performance measure for benchmarking MIMO FSO channels against SISO channels.

Assumptions for RDO assessment study are as follows. A FSO IM-DD system employing binary pulse position modulation is considered, where the optical transmitter is “on” during half duration of the bit interval and is “off” during the other half. The receivers integrate the signal and non-signal slots and resulting signal vector is given by

![Fig. 4-15: PDF of post-detection SNR for 2×2 MIMO-FSO with SNR₀ = 1, log-normal fading variance, σ² = 0.09, correlation coefficient, ρ = {0,0.1,0.3,0.9}(a, b, c, d).](image)

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Here, $r^s$ and $r^n$ are the received electrical signals which correspond to the signal and non-signal slots of the BPPM pulse. $P_s$ and $P_b$ are the optical signal power and the background power incident on the photo-detector, $T$ is the bit duration, $\eta$ is the responsivity of the photo-detector(s), and $n^s$ and $n^n$ are the additive white Gaussian noise terms in “on” and “off” slots with zero-mean and variance $\sigma^2 = N_o/2$. The receivers are also assumed background noise-limited. Incorporating distance-dependent path loss and turbulence-induced fading, the channel gain of a link of length $d$ can be given as,

$$g = |\alpha|^2 PL(d)$$

where $|\alpha| = \exp(\chi)$ is the log-normally distributed channel fading amplitude which is commonly used to model weak turbulence conditions. The normalized path loss term $PL(d)=l(d)/l(d_{s,d})$ can be assumed to be 1. Outage probability at transmission rate $R_0$ is given by $P_{out}(R_0) = \Pr\{C(\gamma) < R_0\}$, where $C(\gamma)$ is the instantaneous capacity corresponding to the instantaneous SNR $\gamma$. Since $C(\gamma)$ is a monotonically increasing function of $\gamma$, the outage probability can also be expressed in terms of SNR as,

$$P_{out}(R_0) = \Pr\{\gamma < \gamma_{th}\}$$  \hspace{1cm} (4–53)

where $\gamma_{th} = C^{-1}(R_0)$ is the threshold SNR. If SNR exceeds $\gamma_{th}$, no outage happens and signal can be decoded with arbitrarily low error probability at the receiver.

For SISO, the optical signal power incident on the photodetector at the destination can be expressed as $P_s = P_t g_{s,d} = P_t |\alpha_{s,d}|^2$, where $g_{s,d}$ and $|\alpha_{s,d}|$ are the gain and fading amplitude of the channel linking the source and destination nodes and $P_t$ is the total average transmit power. Thus, the received electrical SNR at the destination node is

$$\gamma = \frac{\eta^2 T^2 P^2}{N_o} = \frac{\eta^2 T^2 P^2 |\alpha_{s,d}|^2}{N_o}$$  \hspace{1cm} (4–54)

The outage probability is then,

$$P_{out,SISO} = \Pr\{\gamma < \gamma_{th}\} = \Pr\left\{|\alpha_{s,d}|^2 < \frac{\gamma_{th} N_o}{\eta T^2 P^2}\right\} = \Pr\left\{|\alpha_{s,d}|^2 < 1/P_t\right\}$$  \hspace{1cm} (4–55)
where $P_M$ denotes the power margin and is defined as $P_M = P_t / P_{th}$. $P_{th}$ denotes a threshold transmit power required to guarantee that no outage happens in a direct fading-free transmission from the source to the destination. Thus, the power margin can be expressed as $P_M = \sqrt{\gamma^2 P_t T^2 / N_o \gamma_{th}}$. The outage probability can also be expressed in terms of the log-normal Cumulative Distribution Function (CDF) as follows,

$$P_{out, SISO} (P_M) = CDF_{\text{log-norm}} \left( 1 / P_M , -2\sigma_x^2 , 4\sigma_x^2 \right) \tag{4–56}$$

where $CDF_{\text{log-norm}} (x, \mu, \sigma^2) = \frac{1}{\sigma \sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{(t-\mu)^2}{2\sigma^2}} dt$.

For MIMO-MRC, the optical signal power incident on the photodetector at $i$-th receive aperture of the destination node can be expressed as $P_i(i) = (P/M) \cdot \sum_{j=1}^M s_{i,j} = (P/M) \cdot \sum_{j=1}^M |\alpha_{i,j}|^2$, where $P = P_t / M$ is the average transmitted optical power per transmit aperture. To make a fair comparison, we assume that the sum of the $N$ receive aperture areas is the same as the area of the receive aperture for a SISO link. Furthermore, the background noise at the receiving apertures is also reduced by a factor of $N$. Therefore, the received SNR for the MIMO-MRC scheme can be written as

$$\gamma = \frac{\eta^2 T^2 P_t^2}{M^2 N^2 \cdot N_o / N} \sum_{i=1}^N \left( \sum_{j=1}^M |\alpha_{i,j}|^2 \right)^2 = \frac{\eta^2 T^2 P_t^2 / M^2 N}{N_o} \sum_{i=1}^N \left( \sum_{j=1}^M |\alpha_{i,j}|^2 \right)^2 \tag{4–57}$$

The outage probability for MIMO-MRC then becomes,

$$P_{out, MRC} = \Pr \left\{ \frac{1}{M^2 N} \sum_{i=1}^N \left( \sum_{j=1}^M |\alpha_{i,j}|^2 \right)^2 < 1 / P_M^2 \right\} \tag{4–58}$$

Nothing has been mentioned so far about the correlations between the fading log-normal variables. The correlation among the log-normal fading coefficients is described by the transmit-side and receive-side correlation matrices $R_t$ and $R_r$. The $j$-th entry of these matrices, $\rho_{t,i,j}$ or $\rho_{r,i,j}$, indicate the normalized correlation coefficient between the $i$-th and $j$-th transmitters or receivers. For simplicity, it is assumed that the geometric configurations of the transmitter arrays are such that $\rho_{t,i,j} = \rho_{t,i} \neq j$ and $\rho_{t,i,i} = 1, \forall i$, and
similar for the receiver arrays. The normalized full correlation matrix describing correlation statistics among all transmitters and receivers is obtained from the Kronecker product \( R = R_t \otimes R_r \). The normalization can be removed by multiplying \( R \) by \( \sigma_y^2 \).

By employing approximation methods based on works by Schwartz and Yeh [124], the sum of log-normal variables \( \sum_{i=1}^N \left( \sum_{j=1}^M |\alpha_{i,j}|^2 \right) \) can be approximated in the following manner. We can represent the sum inside the first squared term as a single log-normal variable, \( u_{1,i} = \sum_{j=1}^M |\alpha_{i,j}|^2 \approx e^{\alpha_i} \). Variable \( u_{1,i} \) will be log-normal with mean and variance given by, \( \mu_{\alpha_i} = \log \left( \alpha_i / \sqrt{1 + e^{\beta_i^2 / \alpha_i^2}} \right) \) and \( \sigma_{\alpha_i}^2 = \log \left( 1 + e^{\beta_i^2 / \alpha_i^2} \right) \), where \( \alpha_i = \sum_{k=1}^M e^{-2\sigma_i^2 + 4\sigma_i^2} = M \) and \( \beta_i^2 = M \left( e^{4\sigma_i^2} - 1 \right) + M \left( M - 1 \right) \left( e^{4\sigma_i^2} - 1 \right) \). The outer sum \( \sum_{i=1}^N e^{2\alpha_i} \) can then be approximated as another log-normal variable \( u_2 \), i.e. \( \sum_{i=1}^N e^{2\alpha_i} = e^{\alpha_2} \). Variable \( u_2 \) is then another log-normal variable with mean and variance given by, \( \mu_{u_2} = \log \left( \alpha_2 / \sqrt{1 + e^{\beta_2^2 / \alpha_2^2}} \right) \) and \( \sigma_{u_2}^2 = \log \left( 1 + e^{\beta_2^2 / \alpha_2^2} \right) \), where \( \alpha_2 = N \cdot e^{2(\alpha_i + \sigma_i^2)} \), and \( \beta_2^2 = \left[ e^{2(\alpha_i + \sigma_i^2)} \right] \right] \left[ N \left( e^{4\sigma_i^2} - 1 \right) + N \left( N - 1 \right) \left( e^{4\sigma_i^2} - 1 \right) \right] \). Thus, the outage probability of MIMO-MRC can also be expressed in terms of the log-normal CDF as follows,

\[
P_{\text{out},\text{MRC}}(P_u) = \text{CDF}_{\text{log-norm}} \left( \frac{1}{P_M}, \mu_{u_2} - \log \left( M^2 N \right), \sigma_{u_2}^2 \right)
\]

(4–59)

For MIMO-EGC, the optical signal power incident on the photodetector at \( i \)-th receive aperture of the destination node can be expressed as \( P_i(i) = P/N \cdot \sum_{j=1}^M g_{i,j} = P/N \cdot \sum_{j=1}^M |\alpha_{i,j}|^2 \), where \( P = P_t / M \) is the average transmitted optical power per transmit aperture. To make a fair comparison, we again assume that the sum of the \( N \) receive aperture areas is the same as the area of the receive aperture for a SISO link. The received SNR for the MIMO-EGC scheme can be written as

\[
\gamma = \frac{\eta^2 T^2 P_t^2}{M^2 N^2 \cdot N_o \left( \sum_{i=1}^N \sum_{j=1}^M |\alpha_{i,j}|^2 \right)^2} = \frac{\eta^2 T^2 P_t^2 / M^2 N^2 \cdot \left( \sum_{i=1}^N \sum_{j=1}^M |\alpha_{i,j}|^2 \right)^2}{N_o}
\]

(4–60)
The outage probability then can be given by,

\[ P_{\text{out, EGC}} = \Pr \left\{ \frac{1}{MN} \sum_{i=1}^{N} \sum_{j=1}^{M} |\alpha_{ij}| < 1/P_M \right\} \] (4–61)

Again, by employing approximation methods for sum of log-normal random variables, the inner sum \( \sum_{j=1}^{M} |\alpha_{ij}| \) can be represented as a single log-normal variable \( u_{1,i} \), i.e.

\[ \sum_{j=1}^{M} |\alpha_{ij}| = \sum_{j=1}^{M} e^{z_j} \approx e^{z_{1,i}}. \]

Variable \( u_{1,i} \) will be log-normal with mean and variance given by,

\[ \mu_{u_1} = \log \left( \alpha_i \sqrt{1 + \beta_i^2 / \alpha_i^2} \right) \] and \( \sigma^2_{u_1} = \log \left( 1 + \beta_i^2 / \alpha_i^2 \right), \]

where \( \alpha_i = \sum_{k=1}^{M} e^{2z_k^2} \) and \( \beta_i = M \left( e^{z_i^2} - 1 \right) + M (M - 1) \left( e^{z_i^2} \sigma_i^2 \right) - 1 \). The outer sum \( \sum_{i=1}^{N} e^{u_{n,i}} \) can then be approximated as another log-normal variable \( u_2 \), i.e.

\[ \sum_{i=1}^{N} e^{u_{n,i}} = e^{u_2}. \]

Variable \( u_2 \) is then another log-normal variable with mean and variance given by,

\[ \mu_{u_2} = \log \left( \alpha_z \sqrt{1 + \beta_z^2 / \alpha_z^2} \right) \] and \( \sigma^2_{u_2} = \log \left( 1 + \beta_z^2 / \alpha_z^2 \right), \]

where \( \alpha_z = \sum_{i=1}^{N} e^{z_i^2} \) and \( \beta_z = M \left[ N \left( e^{z_i^2} - 1 \right) + M (M - 1) \left( e^{z_i^2} \sigma_i^2 \right) - 1 \right] \). Thus, the outage probability can also be expressed in terms of the log-normal CDF as follows,

\[ P_{\text{out, EGC}} (P_M) = \text{CDF}_{\text{log-norm}} \left( 1/P_M, \mu_{u_2} - \log (MN), \sigma^2_{u_2} \right) \] (4–62)

Diversity order is defined as the negative of the asymptotic slope of the error rate performance (e.g. bit error rate or outage probability) versus SNR. With this convention, the diversity order of a SISO transmission is given by

\[ d = - \lim_{\text{SNR} \to \infty} \frac{\partial \ln P_{\text{out}}}{\partial \ln \text{SNR}} = - \lim_{\text{SNR} \to \infty} \frac{\partial \ln P_{\text{out}}}{\partial \ln P_M} \] (4–63)

where \( P_{\text{out}} \) is the outage probability, and \( P_M \) is the power margin.

It can be shown that the conventional diversity order tends to infinity for log-normally faded FSO channels. It is due to this reason that \textit{relative} diversity order measure is necessary. The single-branch diversity system is taken as the benchmark, and then the relative diversity order (RDO) is quantified as,
Furthermore, the asymptotic relative diversity order (ARDO) can be given by

\[ \text{ARDO} = \lim_{P_M \to \infty} RDO(P_M) \]  

(4–65)

RDO and ARDO can be determined by numerically differentiating the probability of outage, \( P_{\text{out}} \), with respect to \( P_M \), the power margin.

Simulation and analytical approximation results for the RDO and ARDO of different MIMO systems are presented in Fig. 4-16 and Fig. 4-17. Overall, two conditions are considered, (i) weak turbulence, \( \sigma_\zeta^2 = 0.01 \), and (ii) strong turbulence, \( \sigma_\zeta^2 = 0.09 \). The correlation coefficients on transmit and receive sides are kept the same, and the values considered are \( \rho_{\text{Tx}} = \rho_{\text{Rx}} = \{0, 0.1, 0.3, 0.9\} \).

![Fig. 4-16: Relative Diversity Order versus Power Margin in dB for \( \sigma_\zeta^2 = 0.01, \rho_{\text{Tx}} = \rho_{\text{Rx}} = \{0, 0.1, 0.3, 0.9\} \) (a, b, c, d)]
The diversity order achieved by MIMO increases with increasing number of transmitters and receivers. The ARDO can be approximated by the value of the flattening line towards infinity, and the ARDO is seen to vary significantly with correlation values, from a maximum of $MN$ (for full correlation, $\rho_{Tx} = \rho_{Rx} = 0$) to a minimum of 1 (for full correlation, $\rho_{Tx} = \rho_{Rx} = 1$). The analytical approximations seem to fit quite well with simulated values. Simulated results and analytical approximations do not show any difference between the diversity order obtained by MRC and EGC. Oscillations caused at the tail of the simulated values should be discarded, since about $10^8$ realizations were used for the simulations, and this value was not adequate to get a smooth tail probability.

Fig. 4-17: Relative Diversity Order versus Power Margin in dB for $\sigma^2_x = 0.09, \rho_{Tx} = \rho_{Rx} = \{0, 0.1, 0.3, 0.9\}$ (a, b, c, d)
The most outstanding finding is that the analytical approximation fails to hold for $\sigma_x^2 = 0.09$. Both the EGC and MRC analytical approximations break down and diverge from the simulated values. So, caution must be exercised when employing analytical approximations for log-normal variables. The approximation would only hold well for small values of $M$ and $N$, and moderate values of $\sigma_x^2$. This was obviously overlooked in [123] for $\sigma_x = 0.3$.

The conclusions drawn from this study are threefold: (1) for turbulence conditions exceeding a certain threshold, analytical approximation is not a good tool for closed-form analyses, (2) spatial correlation must be reduced to as practically low values as possible, by employing geometric configurations, so as to achieve the maximum benefit from multiple aperture systems, both for communications and imaging, and (3) MRC and EGC perform similarly in IM-DD systems.

BER performance curves are shown in Fig. 4-18 for a SISO vertical link of 2 km length together with several comparable MIMO systems. The total transmitted power is kept fixed and the total aperture area is also kept constant, corresponding to a SISO receiver aperture diameter of 5 cm. The diversity combining is chosen to be MRC, which gives the same performance as EGC. In this case, $r_0 = 5$ cm giving $D/r_0 = 1$, and the effect of turbulence is manifested as log-normal amplitude fading only. The effectiveness of MIMO geometry in combating fading is apparent, and as the number of diversity branches is increased, MIMO BER performance slowly reaches ideal AWGN channel.

4.6.2.2 MIMO Performance in Strong Turbulence

When aperture diameter is larger than the atmospheric coherence length, due to strong turbulence or due to large aperture size to combat amplitude fading, intensity fluctuations at the pupil plane are reduced by aperture averaging effect [10][11]. The optical field decomposes into several spatial modes, and the PSF becomes distorted in place of the well-known Airy diffraction-limited PSF. This is demonstrated in Fig. 4-19, where sample PSFs for aperture diameter of 20 cm on a 5 km long link are shown for (a) no turbulence, (b) weak turbulence ($C_n^2(0) = 1.7 \times 10^{-13}$, $v = 21$ m/s), and (c) moderate
turbulence in \( C_n^2(0) = 1.7 \times 10^{-12}, \nu = 21 \text{ m/s} \). The phenomena of beam wander and PSF broadening are apparent, and they deteriorate with increasing \( D/r_0 \).

In view of the above, we now present possible MIMO FSO designs. As found in Section 4.6.2.1, diversity is maximized when the multipath propagation paths encounter uncorrelated fading. This can be achieved by placing transmit and receive apertures by at least \( r_0 \), which ranges from 20cm under good conditions to 2-4 cm under severe turbulence. A simplifying assumption is that of perfect tracking of the centroid of the

---

Fig. 4-18: BER performance of SISO and MIMO FSO link in weak turbulence, \( A=1.7 \times 10^{-13}, \nu = 21 \text{ m/s}, \sigma^2=0.09, r_0 = 5 \text{ cm}, D = 5 \text{ cm (SISO)} \)

Fig. 4-19: Point Spread Functions (PSFs) under (a) no turbulence, (b) weak turbulence, \( D/r_0 \approx 4 \), (c) moderate turbulence, \( D/r_0 \approx 15 \).

---
received point spread function, which compensates for beam wander. Removal of phase fluctuations is possible by complicated systems that incorporate adaptive optics.

BER performances for comparable SISO and MIMO systems are plotted in Fig. 4-20 for stronger turbulence conditions, i.e. $A=1.7 \times 10^{-12}$, $v = 21$ m/s, $\sigma_I^2=0.8$, $r_0 = 1.4$ cm. The aperture diameter $D$ is varied from 5 to 20 cm, to obtain different $D/r_0$ ratios: 3.57, 7.14, and 14.29, respectively. For Fig. 4-20(a), the aperture diameter is small compared to coherence length, as a result aperture averaging is not visibly effective for the SISO link, which has a much lower BER compared to the MIMO links. As the aperture diameter is increased to 10 cm, for Fig. 4-20(b), both SISO and MIMO performance curves are enhanced due to aperture averaging, but more so for MIMO than SISO. The MIMO BER quantities are comparable to log-normal fading only case, shown in Fig. 4-18. In an effort to increase aperture averaging, receiver diameter is increased in Fig. 4-20(c); but aperture averaging is not effectively present since the phase distortions over the large SISO aperture give rise to significant fluctuations. Smaller MIMO apertures, on the other hand, still exploit aperture averaging effectively, and hence show improved performance compared to the SISO link.

4.6.2.3 Adaptive Optic Corrections

Phase fluctuations induced by atmospheric turbulence can be compensated by a correction mechanism, commonly termed Adaptive Optics (AO). This is achieved by sensing the impinging phasefront by Shack-Hartmann sensor [128], and using deformable
mirrors to induce relative phase shifts, bringing the overall phase structure to conform to a uniform iso-phase plane \cite{129}. Adaptive optics can correct only for phase fluctuations; amplitude fluctuations must be compensated for with larger apertures.

Conventional adaptive optics correct only for the first few Zernike coefficients, namely tilt. Under weak turbulence, these will perform satisfactorily; but in stronger turbulence conditions more Zernike coefficients need to be removed, which may be possible by an advanced adaptive optic module capable of removing up to the first 300 Zernike coefficients. Fig. 4-21 illustrates BER performances of SISO FSO systems with and without adaptive optics under strong turbulence ($A=1.7\times10^{-12}$, $v = 21$ m/s, $\sigma_t^2=0.8$, $r_0 = 1.4$ cm). It is evident that for adaptive optic corrections improve the performance of a SISO system under turbulence. However, when the aperture diameter is increased to exploit aperture averaging, conventional adaptive optics still improves SISO BER performance, but not as significantly as for advanced adaptive optic correction module. This is due to larger contributions from higher Zernike modes on a larger aperture.

**Fig. 4-21:** BER performances of SISO FSO systems with and without Adaptive Optic Corrections, with aperture diameters (a) $D = 10$ cm, (b) $D = 20$ cm.

By comparison with Fig. 4-18 and Fig. 4-20, we see that multiple apertures on transmit and receive sides perform as good as a FSO system equipped with advanced adaptive optic correction. Since adaptive optic systems have the disadvantages of being costly, on one hand, and slow response time on the other, MIMO links can significantly improve communication performance at comparatively lower costs.
4.7 Conclusions

In this chapter, we characterized laser propagation for free-space optical wireless communications, and modeled the phenomena of multi-scattering and turbulence. Spatial, temporal and angular dispersions are introduced by scattering medium such as clouds, fog, aerosols, and haze. For a large field-of-view, these phenomena were found to be more prominent, whereas for a significantly smaller field-of-view at the receiver, these could be closely approximated by attenuation only. Furthermore, the Markov chain analysis done in this chapter is important in terms of characterizing the diffusion of photons in multiple scattering scenarios. Atmospheric turbulence was modeled using Kolmogorov theory and phase screens, and its effects are categorized into two cases: (a) log-normal amplitude fading, and (b) amplitude fading due to phase fluctuations. Several phase screen generation techniques were compared in relation to theoretical analyses.

MIMO FSO systems are shown to be robust against fading induced by turbulence, under the assumption that multiple scattering only induce attenuation. This assumption is valid given the small field-of-view which is typical of receiving apertures. MIMO systems are shown to achieve maximum possible diversity under uncorrelated fading, which can be realized by placing the receiving apertures apart by at least the coherence length, $r_0$. Strong turbulence has to be treated differently from log-normal fading encountered under weak turbulence. BER curves are presented to demonstrate the superiority of MIMO architectures in both weak and strong turbulence, as well as over complicated receiving optics comprised of adaptive optics.
Chapter 5

Active Imaging with Multiple Parallel Laser Beams

5.1 Introduction

Active Optical Imaging is another application of laser beams, primarily used for surveillance, monitoring, and reconnaissance missions. Until now, much sensing and imaging research has been conducted with Radars and RF frequency-based sensing instruments. But Laser Detection and Ranging (LADAR) or Light Detection and Ranging (LIDAR) applications have been developed [130] which use light wavelength ranges. Due to use of smaller wavelength of optical waves, LIDAR and LADAR offer higher resolution, faster area search rate, and human-friendly visual interpretation.

The primary challenges of active optical imaging in an outdoor scenario are multi-scattering, leading to back-reflections that contribute to a background noise at the imaging photodetector, thus reducing contrast; and turbulence-induced amplitude and phase-front variations, resulting in PSF broadening and resolution degradation. In this chapter, we present spatially multiplexed imaging system design using split beam transmission and pixilated imaging receivers that can effectively illuminate the target area in a uniform and efficient manner. Photons reflected from the target are detected on a focal plane array of individual photodetector pixels. We also address signal processing approaches to mitigate blurring due to PSF distortion.

5.2 Active Imaging in presence of Multiple Scattering

The dispersion mechanisms affecting communications performance through an FSO link, discussed in Chapter 4, are also applicable to an atmospheric imaging system. However there are several differences that have to be taken into account, for example:
a. For an imaging system, we have to consider two-way propagation of light, from the laser transmitter to the target, and from target back to the camera. This is different from the one-way transmission in the FSO scenario, and in fact, is worse, since in a scattering medium, photons are subjected to twice the optical thickness.

b. In case of FSO, performance metrics were mainly limited by temporal dispersion which contributes to ISI, and spatial dispersion, which mainly contributes to attenuation. These phenomena are easily countered with by using small-FoV receiving aperture. But for imaging, spatial information is of utmost importance; any spatial dispersion that occurs along the path has to be compensated for to obtain an informative image.

c. In addition to forward scattering, back scattered photons need to be considered as well. In a communications scenario, back-scattered photons are simply ‘lost’; but in an imaging system, they may pose significant interference to receiving apparatus.

As in Chapter 4, we make the simplifying assumption that scattering and turbulence are independent occurrences, and can be modeled separately. As a result, we can resort to Monte Carlo Ray Tracing (Section 4.2) for multiple scattering model, and use phase screens (Section 4.4) to represent turbulence. Both these techniques provide temporal, as well as spatial, information which are vital for imaging.

5.2.1 Monostatic Imaging

The MCRT algorithm, introduced in Section 4.2, is modified to account for two-way propagation, incorporation of a retro-reflective element as target, and illumination pattern. The setup for the simulation is illustrated in Fig. 5-1 where a mono-static imaging system is considered, which has collocated transmitter and camera receiver.
The difference of this setup from MCRT for communications is clear from Fig. 5-1. The target in our simulations is always considered to be the retro-reflector with the shape of the letter ‘H’, which is placed in free space at distance $L$ from the imaging apparatus. Following notations from Chapter 4, the optical thickness corresponding to $L$ is $\tau = \beta_{\text{ext}} L = L / \text{D}_{\text{ave}}$. The total optical thickness that transmitted photons encounter is $2\tau$. Therefore, the attenuation encountered by the transmitted photons are twice that of a communication scenario, and the line-of-sight photons are attenuated according to the following modified Beer-Lambert law,

$$I_{\text{LOS}} = I_0 e^{-2\tau}$$  \hspace{1cm} (5–1)
Spatial and temporal distributions of photons reflected from the target are illustrated in Fig. 5-2 and Fig. 5-3 for a monostatic imaging system. The information photons and the backscattered photons are labeled separately, and this shows how they can be distinguished in space and time. As optical thickness increases, the number of image information-bearing photons reduce and the number of backscattered photons increase, thus reducing SNR or contrast. By examining the temporal distribution of these photons, it is apparent that backscattered photons return to the imaging apparatus comparatively at an earlier time than photons reflected from the image. This indicates that the use of time-domain filtering may be beneficial to reduce the noise induced by backscattered photons. We can also conclude that it is mostly the photons that undergo no scattering on the two-way propagation path which carry image information. Once a photon undergoes a scattering event that drastically alters its direction of travel, it can carry no spatial information about the image, and hence contributes to clutter. As a result, field of view of the receiving aperture should be small enough to be able to reject multiply scattered photons, and allow only ballistic photons. The fact that a large FoV receiver contributes to loss of contrast and resolution has been demonstrated in [131].

Fig. 5-2: Spatial distribution of image photons (a, b) and backscattered photons (c, d) at camera plane for optical thickness of 2 (a, c) and 8 (b, d)
5.2.2 MIMO Imaging

It was pointed out earlier that locating the transmitter and the receiver separately with a slanted angle of transmission may improve contrast. Furthermore, multiple imaging apparatuses can be placed closely and be pointed towards the same target, as shown in Fig. 5-4. Here we demonstrate this fact by employing a 2×2 MIMO imaging setup, where one of the transceiver is monostatic (Tx1/Rx1 in Fig. 5-4) and the other one is bistatic (Tx2/Rx2 in Fig. 5-4). As before, the length of the imaging link is kept constant at 1km, i.e. 2km for two-way propagation, and the scattering coefficient is varied to get roundtrip optical thickness quantities of 4 and 10.

Spatial distributions of the 2×2 MIMO imaging setup are demonstrated in Fig. 5-5 and Fig. 5-6. The effect of placing transmitter and receiver apart from each other is visible from the figures, where the monostatic set has more concentration of backscattered photons than the bistatic set. The number of image photons reduces, expectedly, as optical thickness increases. Furthermore, electronically combining the image photons received on the two imaging receivers will yield a better contrast than a single imaging receiver.
Temporal distributions of the imaging setup are shown in Fig. 5-7 and Fig. 5-8. The distinction between the arrival times of image photons and backscattered photons is apparent, and implies that a time-domain filtering approach implemented by fast shutter with appropriate timing is able to excise interference from backscattered photons. However, with increase in optical thickness, the number of received image photons decreases exponentially according to Eq. (5–1), and reduces received power and contrast.

Fig. 5-4: Schematic of a MIMO active imaging apparatus

Fig. 5-5: Spatial distributions of image photons (top) and backscattered photons (bottom) on camera plane for roundtrip optical thickness 4 with a 2×2 imaging system.
Fig. 5-6: Spatial distributions of image photons (top) and backscattered photons (bottom) on camera plane for roundtrip optical thickness 10 with a 2×2 imaging system.

Fig. 5-7: Temporal distributions of image photons (top) and backscattered photons (bottom) on camera plane for roundtrip optical thickness 4 with a 2×2 imaging system.
5.3 Active Imaging in Turbulence

Following the MIMO methodology of multiple transmitted beams and pixilated imaging receiver, we present the effects of turbulence on an imaging system in this section. We assume that a $N \times N$ array of beamlets are launched towards the target, and focused by a set of concentrators on an imaging receiver having the same dimension of pixels. The effect of clouds can be treated only as an attenuation factor, given by the Beer-Lambert law, as demonstrated in Section 5.2. In Chapter 4, different phase screen generation techniques to model turbulence were discussed, which yield spatial information about the distortions imposed by turbulence. Due to phasefront distortion, point spread functions (PSF) are broadened, and this effect is illustrated in Fig. 4-19. Since PSF represents the spatial impulse response of the system, parallel reflected beams inter-mix and results in inter-pixel interference at the imaging receiver. If the reflected optical field in absence of turbulence and the PSF due to turbulence are given in
quantized space by \( f(x,y) \) and \( h(x,y) \) respectively, then the received field distribution is the spatial convolution of the two, i.e.,

\[
g(x,y) = f(x,y) * h(x,y) + n(x,y)
\]  \( \text{(5–2)} \)

where \( n(x,y) \) represents additive noise, originating from electronics and ambient light. The PSF for a coherent imaging system is found by Fourier transforming the phase front distortion, i.e.,

\[
h(x,y) = \frac{1}{\lambda} \int \int W(f \lambda df_x) e^{-j2\pi \left( x f_x + y f_y \right)} e^{j \varphi(x,y)} \lambda df_x \lambda df_y
\]  \( \text{(5–3)} \)

For an incoherent system, the average intensity observed at the imaging plane would be given by,

\[
i(x,y) = \int \int o(x',y') \left| h(x-x',y-y') \right|^2 dx' dy'
\]  \( \text{(5–4)} \)

where \( o(x',y') \) is the average intensity of the target object. Therefore, for an incoherent imaging system, the effective PSF is \( \left| h(x,y) \right|^2 \).

The resolution of an imaging system is usually measured by the inverse of the full-width half-maximum (FWHM) of the PSF. With increase in turbulence, the broadening of PSF increases FWHM, thereby reducing resolution.

The effect of turbulence on imaging is observed in Fig. 5-9, where (a) is the focal plane intensity distribution of the reflected 8×8 beamlets under no turbulence on a 5km long imaging link (10 km roundtrip), and (b) is the quantized intensity pattern formed on a 8×8 pixilated photodetector array. There is no intermixing of the beamlets and the resulting image has the highest contrast possible. Under moderate turbulence \( (C_n^2(0) = 1.7 \times 10^{-12}, r_0 = 1.4\text{cm}, D = 50\ \text{cm}) \), the PSF broaden and the beamlets intermix at the focal plane (Fig. 5-9(c)). The resulting photodetected image demonstrates the blurring due to turbulence (Fig. 5-9(d)).
5.4 Contrast and resolution enhancement techniques

In view of the dispersions caused by scattering and turbulence, we present compensation techniques in this section for a MIMO imaging system. In accordance with our findings, we treat the two problems independently of each other, and optimize the system to compensate for each of these individually. As previously mentioned, temporal filtering and combining multiple images from separately located multiple imagers may improve image quality, and we describe these techniques. The effect of turbulence is manifested in blurring and loss of resolution, and deconvolution techniques must be implemented in the signal processing stage. Adaptive optics can also correct for phase distortions and perform deblurring prior to signal processing.

Fig. 5-9: Focal plane intensity distributions (a, c) and photo-detected intensity patterns (b, d) on an 8×8 imaging receiver, under no turbulence (a, b) and moderate turbulence (c, d).
5.4.1 Temporal Filtering to combat backscattered photons

It was stipulated in Section 5.2.1 that temporal filtering can improve contrast by rejecting backscattered photons, and allowing only image photons. With the assumption that backscattered photons dominate noise, the Signal to Noise and Interference Ratio (SNIR) can be closely approximated as,

\[
\text{SNIR (dB)} = 10 \log_{10} \left( \frac{N_i}{N_b} \right)
\]

(5–5)

where \(N_i\) and \(N_b\) are the number of image photons and backscattered photons respectively. Spatial and temporal distributions obtained by MCRT are filtered using variable-width time gates, ranging between 0.1\(\mu\)s and 10.0\(\mu\)s and centered at roundtrip propagation delay, for different roundtrip optical thicknesses. The resulting SNIR quantities are tabulated in Table 5.1. One can observe the expected reduction in SNIR for all cases as the optical thickness increases. Applying time gates significantly improves the SNIR in all cases. For lower optical thicknesses, most photons are ballistic. Hence, no further improvement may is visible as gate widths are decreased. But for higher thicknesses, the improvement achieved is as high as 50 dB, compared to no time gating.

<table>
<thead>
<tr>
<th>Optical Thickness (roundtrip)</th>
<th>Infinite width</th>
<th>10(\mu)s</th>
<th>1(\mu)s</th>
<th>0.1(\mu)s</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>26</td>
<td>50</td>
<td>50</td>
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<td>8</td>
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<td>-15</td>
<td>15</td>
<td>35</td>
<td>35</td>
</tr>
<tr>
<td>12</td>
<td>-25</td>
<td>6</td>
<td>16</td>
<td>25</td>
</tr>
</tbody>
</table>

Table 5.1: SNIR (dB) for different optical thickness and gate widths after temporal filtering of received photons at camera plane.
5.4.2 Diversity Combining

Turbulence manifests itself as random perturbation of the phasefront. As a result, combining multiple received image profiles of the same object could significantly improve image quality. This is similar to diversity combining for communication systems. Under weak turbulence condition, it may be expected that the received intensity from multiple beamlets will not significantly interfere with each other, and averaging multiple images will result in a smoothing effect, thereby reducing random fading on each pixel. This scenario is similar to log-normal fading case discussed in Section 4.6.2.1. However, under moderate and strong turbulence, the PSF becomes very broad, and all the pixels interfere with each other. Therefore, contrast cannot be recovered by simply averaging over multiple images.

Fig. 5-10 demonstrates the results of combining and averaging 20 individual realizations of the image under weak and moderate turbulence, and compares with a single realization. One can observe that averaging has virtually removed all random fluctuations (Fig. 5-10(b)) compared to the single image shown in Fig. 5-10(a). However, when turbulence is more severe, mixing between neighboring pixels increases significantly. As a result, combining 20 images simply yields a ‘smoother’ image, but no resolution enhancement is observed. The blurring must be removed by either using adaptive optics in the receiver optical elements to compensate for phase distortions, or implementing a blind deconvolution algorithm in the post-processing circuitry.

5.4.3 Blind Deconvolution Techniques

From previous discussions, we realize that the effect of turbulence on an imaging system is more complicated than that of multiple scattering. Removal of blurring caused by PSF distortion can be accomplished by signal processing techniques, comparable to ISI removal by equalization in communication systems. However, the problem is more complicated by the fact that we do not have an estimate of the channel so that inverse filtering or MMSE-based techniques can be directly applied. Therefore, we have to resort
to blind deconvolution techniques. In this section, we present three techniques to perform blind deconvolution; namely, Simulated Annealing (SA), Non-negativity And Support-constrained Recursive Inverse Filtering (NAS-RIF), and Approximate Factorization of Bivariate Polynomials (AFBP).

The problem is formulated as follows. The received image is \( g(x, y) \) is a two-dimensional convolution of the blurring function or PSF, \( h(x, y) \), and the original image \( f(x, y) \) obtained in absence of blurring, i.e.,

\[
g(x, y) = h(x, y) * f(x, y) + n(x, y)
\]

(5–6)

Here, \( n(x, y) \) is the additive noise. Simultaneous blind estimation of \( h(x, y) \) and \( f(x, y) \) is possible only if the support of each of these, \( S_h \) and \( S_f \), are finite, i.e. beyond the limits described by \( S_h \) and \( S_f \), the functions can be assumed to be zero.

Fig. 5-10: Photodetected image on a 16×16 imaging receiver under weak turbulence (a, b) and moderate turbulence (c, d), showing single distorted image (a, c) and average of 20 distorted images (b, d).
(a) Simulated Annealing

Simulated annealing algorithm [132] is a Monte Carlo global minimization technique which minimizes a given cost function. The assumptions for the algorithm are as follows: \( S_h \) and \( S_f \) are finite and individually known; the image and the PSF cannot be represented as convolutions, i.e. they are irreducible; and pixel values are non-negative (enforced by incoherent imaging). The last assumption is enforced by the incoherent nature of the imaging technique. Both image and PSF are initially set to pseudorandom functions at the beginning of the iterations. The cost function is defined by the normalized squared error between the convolution of the initial estimates and the received image. Random perturbations are added to the estimates, and only the perturbations that result in lower costs are accepted. The process is continued until the squared error reaches equilibrium or within tolerance. To demonstrate the functionality of Simulated Annealing, we take the image of the letter H and convolve it with known PSFs corresponding to weak, moderate and strong turbulence. The received image is degraded by additive noise, which is 40 dB lower than the total energy contained in the received image. Simulated annealing is performed with number of cycles and scans set to 100 and 500, respectively. The results are illustrated in Fig. 5-11, Fig. 5-12 and Fig. 5-13. The successful separation of the image and the PSF is visible in all cases. However, it must be noted that the convergence of the algorithm depends on initial estimate, which affects final results.

The performance is measured by the percentage MSE, which is defined by,

\[
\text{%MSE} = 100 \left( \frac{\sum |a\hat{f}(x,y) - f(x,y)|^2}{\sum |f(x,y)|^2} \right)
\]  

(5-7)

where \( \hat{f}(x,y) \) and \( f(x,y) \) are the recovered and original images, respectively, and \( a \) is a scaling factor given by,

\[
a = \frac{\sum f(x,y)\hat{f}(x,y)}{\sum \hat{f}(x,y)^2}
\]  

(5-8)

The SNR improvement (SNRI) is another performance metric defined as,

\[
\text{SNRI} = \frac{\text{%MSE}(g)}{\text{%MSE}(\hat{f})}
\]  

(5-9)
where \( g(x, y) \) is the blurred noisy image. The SNRI quantities for simulated annealing under these three conditions are listed in Table 5.2.

<table>
<thead>
<tr>
<th>Turbulence Level</th>
<th>Weak</th>
<th>Moderate</th>
<th>Strong</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNRI (dB)</td>
<td>13.7</td>
<td>18.8</td>
<td>18.6</td>
</tr>
</tbody>
</table>

Fig. 5-11: Blind deconvolution by simulated annealing in weak turbulence: (a) distorted image, (b) estimated image, (c) convergence of MSE percentage.

Fig. 5-12: Blind deconvolution by simulated annealing in moderate turbulence: (a) distorted image, (b) estimated image, (c) convergence of MSE percentage.
Non-Negativity and Support-constrained Recursive Inverse Filtering (NAS-RIF)

The NAS-RIF algorithm \[133\] finds a reliable estimate of the true image \( f(x, y) \) from a gray-scale image \( g(x, y) \) degraded by a linear shift invariant PSF \( h(x, y) \), with partial or no information about the PSF and the true image. Non-negativity and support constraints are used as partial information to define the cost function as,

\[
E = \sum_{(x,y) \in S_f} \left[ \left( \frac{1 - \text{sgn}(\hat{f}(x,y))}{2} \right) + \frac{\hat{f}(x,y) - L_B}{\gamma} \right]^2 + \gamma \left[ \sum_{\forall (x,y) \in S_f} u(x,y) - 1 \right] \tag{5–10}
\]

Here, the image estimate \( \hat{f}(x,y) = g(x,y) * u(x,y) \) is the convolution of the received image with a variable LSI filter \( u(x, y) \), \( S_f \) is the region of support, \( L_B \) is the average background intensity, and \( \gamma \) is a variable which is nonzero only for dark background. The cost function is proven to be convex and have a global minimum \[133\]. Furthermore, a nonlinear filter is used which projects the estimate onto a convex set corresponding to the non-negativity and support constraints of the true image. The.

Fig. 5-13: Blind deconvolution by simulated annealing in strong turbulence: (a) distorted image, (b) estimated image, (c) convergence of MSE percentage
difference between the projected image and the output of the LSI filter is used as an error function to update the LSI filter coefficients.

The blind deconvolution results for the NAS-RIF algorithm are illustrated in Fig. 5-14. It is noted that under weak turbulence conditions the NAS-RIF algorithm performs satisfactorily, which is not the case under moderate turbulence. The SNRI and %MSE quantities for NAS-RIF algorithm are tabulated in Table 5.3 for cases when additive noise is absent and when the additive noise is such that received image has 70 dB SNR per pixel. A general observation is made that NAS-RIF algorithm is quite robust against additive noise, and performs similarly in both cases. However, performance degrades significantly when turbulence level becomes stronger. This is also visually apparent from Fig. 5-14.

Fig. 5-14: NAS-RIF blind deconvolution in weak turbulence (a, b, c) and moderate turbulence (d, e, f): focal plane image (a, d); 8×8 photodetected image (b, e); and deconvolved image (c, f).
Approximate Factorization of Bivariate Polynomials

It has been shown that components of a convolution in two-dimensional space can be recovered simply from the convolution itself \[134\]. The \(Z\)-transform of a composite image is the product of the original image and PSF \(Z\)-transforms, i.e,

\[
X(z_1, z_2) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} x(m, n) z_1^{-m} z_2^{-n}
\]

is the \(Z\)-transform of a 2D function \(x(m, n)\).

Therefore the \(Z\)-transform of the blurred image \(G(z_1, z_2)\) is a bivariate polynomial in \(z_1\) and \(z_2\). Hence the problem of deconvolution becomes a problem of factorizing the polynomial. In practice, the coefficients of the polynomial are not perfect and are corrupted by additive noise. An algorithm to perform approximate bivariate polynomial factorization is outlined in [135] and [136] where the \(\textit{nearest}\), in least square sense, factorizable polynomial having a total degree less than or equal to the given polynomial is determined.

Assuming \(G(z_1, z_2) \in \mathbb{Q}(i)(z_1, z_2)\) is an irreducible polynomial over the set of complex numbers \(\mathbb{C}\), where irreducibility is caused by perturbations on the coefficients of \(G\), the algorithm finds \(G^{[\text{min}]} = \prod_i F_i\), a factorizable polynomial over \(\mathbb{C}\) with

\[
\deg(G^{[\text{min}]}) \leq \deg(G),
\]

such that the normalized error \(\|G - G^{[\text{min}]}\|_2 / \|G\|_2\) is minimized.

The algorithm is based on Ruppert’s criterion of irreducibility of polynomials [135]. If \(G(z_1, z_2) \in \mathbb{C}(z_1, z_2)\) with bi-degree \((M, N)\), i.e., \(\deg_{z_1} G = M\) and \(\deg_{z_2} G = N\), then \(G\) is absolutely irreducible if and only if Eq. (5–12) has no nonzero solution \(P, Q \in \mathbb{C}(z_1, z_2)\) with \(\deg P \leq (M - 1, N)\) and \(\deg Q \leq (M, N - 2)\).
Since Eq. (5–12) is linear over C, we can construct a linear system with coefficients of \( P \) and \( Q \), whose coefficient matrix is called the Ruppert matrix \( R(G) \). \( R(G) \) is full rank if and only if \( G \) is absolutely irreducible. Furthermore, a basis of the solution space can be found from the Ruppert matrix which could be used, in conjunction with a multivariate GCD finding algorithm, to determine the approximate factors of \( G \). If the number of factors is more than 2, testing convolution combinations on subsets of \( \{ F_i \} : \forall i \) and using some a priori information about the target, the original image may be recovered.

The blind deconvolution results for the AFBP algorithm are illustrated in Fig. 5-15. It must be noted that addition of noise significantly degrades the performance of the algorithm, and are not shown. Otherwise, AFBP performs exceedingly well in comparison with the previous two techniques in both weak and moderate turbulence conditions. The SNRI and %MSE quantities for AFBP algorithm are tabulated in Table 5.4 for weak and moderate turbulence levels, with no additive noise. The general observation is the significant SNRI improvement and very low %MSE achieved by this algorithm under both turbulence levels. This is also visually apparent from Fig. 5-15.

### Table 5.4: AFBP SNRI & %MSE under various turbulence conditions

<table>
<thead>
<tr>
<th>Turbulence Level</th>
<th>Weak</th>
<th>Moderate</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNRI (dB)</td>
<td>41</td>
<td>77</td>
</tr>
<tr>
<td>%MSE</td>
<td>2.7985\times10^{-4}</td>
<td>4.8725\times10^{-7}</td>
</tr>
</tbody>
</table>

A comparison of the three blind deconvolution algorithms discussed above is presented in Table 5.5.
Fig. 5-15: AFBP blind deconvolution in weak turbulence (a, b, c) and moderate turbulence (d, e, f): focal plane image (a, d); 8×8 photodetected image (b, e); and deconvolved image (c, f).

Table 5.5: Comparison of Simulated Annealing, NAS-RIF and AFBP algorithms

<table>
<thead>
<tr>
<th></th>
<th>SA</th>
<th>NAS-RIF</th>
<th>AFBP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iterative Algorithm</td>
<td>Iterative</td>
<td>Iterative</td>
<td>Non-iterative</td>
</tr>
<tr>
<td>Computational</td>
<td>Low</td>
<td>Moderate</td>
<td>High</td>
</tr>
<tr>
<td>Complexity</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sensitivity to noise</td>
<td>Little</td>
<td>Almost no</td>
<td>Very highly sensitive to</td>
</tr>
<tr>
<td></td>
<td>sensitivity</td>
<td>sensitivity</td>
<td>additive noise</td>
</tr>
<tr>
<td>A priori</td>
<td>Known</td>
<td>Non-negativity and</td>
<td>None</td>
</tr>
<tr>
<td>information</td>
<td>support and non-</td>
<td>support</td>
<td></td>
</tr>
<tr>
<td>Overall</td>
<td>negativity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Performance</td>
<td>Upto 19dB SNR</td>
<td>Upto 37dB SNR</td>
<td>Upto 77dB SNR</td>
</tr>
<tr>
<td></td>
<td>improvement, with noise</td>
<td>improvement, with 30-40dB SNR per pixel</td>
<td>improvement, only</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>without noise</td>
</tr>
<tr>
<td>Convergence</td>
<td>Convergence dependent on initial random estimate</td>
<td>Convergence is ensured</td>
<td>Not applicable</td>
</tr>
<tr>
<td>Performance in Weak Turbulence</td>
<td>Moderate (14dB SNRI)</td>
<td>Very good (37dB SNRI)</td>
<td>Very good (41dB SNRI)</td>
</tr>
<tr>
<td>Performance in Moderate Turbulence</td>
<td>Moderate (19dB SNRI)</td>
<td>Moderate (16dB SNRI)</td>
<td>Excellent (77dB SNRI)</td>
</tr>
</tbody>
</table>
5.4.4 Adaptive Optics

As mentioned before, adaptive optics can compensate phasefront distortions by use of Shack-Hartmann sensors, and thus remove blurring from an image. In this section, we investigate the effect of adaptive optics of varying degrees of complexity on an imaging system. Zernike coefficients of the observed phase screen are first estimated by correlating the observed phase screen with known Zernike orthonormal polynomials, and these coefficients are added to obtain a corrective phase screen which is subtracted from the observed phase screen. The adaptive optics complexity varies from simple tip-tilt correction, i.e. removal of upto 3rd order Zernike coefficients, to more complex corrections by removing upto 300th order coefficients. NAS-RIF algorithm is used in conjunction with adaptive optics to investigate the performance improvements and complexity reduction achievable by such a composite system.

Sample images resulting from incorporation of adaptive optics under moderate turbulence conditions are shown in Fig. 5-16. The performance measures, i.e. SNRI and %MSE, for adaptive corrections, with and without NAS-RIF blind deconvolution, are listed in Table 5.6 for various orders of Zernike coefficients removed.

![Sample images](image_url)

Fig. 5-16: 8×8 photodetected images in moderate turbulence and adaptive optics with (a) no AO correction; and corrections upto (b) 3rd order; (c) 55th order; (d) 78th order; (e) 120th order; (f) 300th order.
It is noted from Fig. 5-16 and Table 5.6 that the highest increase in %MSE occurs by simple tip-tilt correction, which expounds the benefits of using simple adaptive optics in imaging systems. This is because tip-tilt is responsible for 86% of total phase-error in piston-removed phase [127]. As number of Zernike modes corrected for increases, the performance improves further, but at a lower rate. Of course, the complexity of incorporating an adaptive optics system capable of removing upto 300 Zernike mode is very complex. Incorporation of NAS-RIF as a post-processing technique is also seen to be beneficial and the SNRI is increased from 37dB for stand-alone NAS-RIF implementation to about 48dB after adaptive optics is included. However, the highest number of iterations are reached for higher order correction, and does not provide much complexity reduction. It is also noted that with the highest order adaptive optic correction, NAS-RIF does not provide much incremental improvement, since most of the possible compensation has already been affected by the adaptive optic stage. As a result, we can conclude that inclusion of blind deconvolution with moderately complex adaptive optics can perform as good as a highly complex adaptive optic module.

<table>
<thead>
<tr>
<th>Zernike Correction Applied</th>
<th>%MSE before NAS-RIF</th>
<th>%MSE after NAS-RIF</th>
<th>SNRI (dB)</th>
<th>NAS-RIF iterations</th>
</tr>
</thead>
<tbody>
<tr>
<td>No correction</td>
<td>30.1</td>
<td>8.1</td>
<td>5.7</td>
<td>185</td>
</tr>
<tr>
<td>3rd order correction</td>
<td>27.9</td>
<td>9.55</td>
<td>4.65</td>
<td>149</td>
</tr>
<tr>
<td>6th order correction</td>
<td>17.18</td>
<td>0.2</td>
<td>19.3</td>
<td>2000</td>
</tr>
<tr>
<td>10th order correction</td>
<td>16.17</td>
<td>0.34</td>
<td>16.68</td>
<td>2000</td>
</tr>
<tr>
<td>15th order correction</td>
<td>15.57</td>
<td>0.03</td>
<td>26.6</td>
<td>2000</td>
</tr>
<tr>
<td>55th order correction</td>
<td>6.72</td>
<td>3.73×10⁻⁴</td>
<td>42.56</td>
<td>2000</td>
</tr>
<tr>
<td>78th order correction</td>
<td>7.8</td>
<td>6.31×10⁻⁴</td>
<td>40.92</td>
<td>2000</td>
</tr>
<tr>
<td>120th order correction</td>
<td>6.38</td>
<td>1.06×10⁻⁴</td>
<td>47.8</td>
<td>2000</td>
</tr>
<tr>
<td>300th order correction</td>
<td>2.51</td>
<td>1.03×10⁻⁴</td>
<td>43.88</td>
<td>2000</td>
</tr>
</tbody>
</table>
5.5 Conclusions

In this chapter, we presented models for an imaging system operating under multiple scattering and atmospheric turbulence. It is shown that this problem is significantly different from imaging in clear weather conditions. The models presented in Chapter 4 are modified to properly accommodate the spatial degradations observed in imaging systems. A modified Monte Carlo Ray Tracing algorithm characterizes the behavior of photon propagation in clouds, fog, and other scattering particles, whereas phase screens represent effect of turbulence. These two phenomena are responsible for contrast and resolution degradation, respectively. It is shown that for very high optical thicknesses, very little spatial information is retained by photons reflected from a target, thus complicating image formation. But for low to moderate optical thicknesses, imaging can be performed if compensation techniques are employed. A spatially multiplexed system is proposed to counter backscattering and employ diversity. Temporal filtering or time-gating is one measure that can be undertaken to improve contrast by rejecting backscattered photons. Resolution enhancement is performed by means of blind deconvolution, adaptive optics, and combination of the two. Three different blind deconvolution techniques are tested and compared with respect to SNR improvement and percentage MSE. All three algorithms show satisfactory performance improvement in different turbulence conditions. Adaptive optics is also employed, in conjunction with blind deconvolution, and it is concluded that highly complex adaptive optic systems can be replaced by a composite correction mechanism comprised of moderately complex adaptive optics and a post-processor performing blind deconvolution.
Chapter 6

Indoor Optical Wireless MIMO Channels

6.1 Introduction

Indoor optical wireless systems can provide a high-bandwidth alternative to traditional RF physical layer techniques, such as WiFi. The IEEE 802.11 standard had specified diffuse infrared as a physical layer alternative operating at 2 Mbit/s, with a range of up to 20 m with no line-of-sight [9]. IrDA defines a direct point-to-point low-power short-distance communication link running up to 16 Mbit/s [149]. Whereas radio technology has enabled wireless networks to transmit data at a gross rate of slightly above 600 Mbit/s [150], these networks are prone to security breach due to interception by unauthorized users, thus requiring encryption and authentication. Furthermore, they suffer from interference from neighboring networks operating in the same frequency range, and they operate at radio frequencies as high as 2.4 to 5GHz. Indoor optical wireless links are inherently resistant to these detrimental effects, and moreover, the corresponding spectral region is unregulated worldwide.

Diffusely scattered infrared light was introduced as a medium for wireless communications as early as 1979 [151], and then continued with the development of a general computer simulation method for IR channel characterization [152][153][154][155]. At present, there are high-power laser transmitters available in the market or can be built with off-the-shelf components, which can be coupled with high-bandwidth, high-gain and high-sensitivity photo-detectors, such as the photo-multiplier tube (PMT) and the avalanche photo-diode (APD). For example, frequency characterization with narrow FOV receivers has been simulated up to 1GHz using laser diodes and PMTs in recent times [156]. This motivates the high-frequency characterization of indoor optical wireless channels under different conditions and using different optical configurations. There are two principal methods of implementing an optical wireless link, namely diffuse
and line-of-sight configurations [157]. Diffuse links suffer from low received power, while providing robustness against shadowing, but requiring more power. Line-of-sight channels, on the other hand, can offer power efficiency, but at the cost of vulnerability to shadowing. Indoor optical wireless channels usually employ intensity modulation with direct detection (IM/DD) for its simplicity and reliability in short range communications. As such, an indoor channel can be modeled as a linear system that has a real-valued impulse response.

There is also a Multi-Input-Multi-Output (MIMO) wireless optical architecture referred to as Multi-Spot Diffuse (MSD) configuration, also known as quasi-diffuse, with multi-element optical transmitters and multi-branch optical receivers [158], which can provide diversity and sufficient power to improve bandwidth and reject ambient noise. The main idea is to use multiple directed beams which are diffused at diffusing spots, and then the reflected signals are picked up by imaging receivers, aptly called fly-eye receivers, to avail of spatial diversity with better pointing features.

In this chapter, we present a simulation method, as well as an experimental setup, for the characterization of indoor optical wireless links using near-infrared wavelength. Different transmit-receive geometry and configurations are explored, along with their time-domain and frequency-domain characteristics, and means of adequately employing spatial diversity are investigated. The main idea behind the indoor propagation model is based on multiple-input multiple-output techniques, i.e. matrix manipulation.

6.2 Indoor Optical Propagation Model

In an indoor LAN scenario, the transmitter is composed of a laser diode or a LED with a given efficiency and emission pattern, and a Laser or LED driver. The receiver can be composed of a p-i-n photodiode (PIN), an avalanche photodiode (APD), or a photomultiplying tube (PMT). Photons emitted from the transmitter travel in the room and are reflected from the ceiling, floor and walls, until they impinge on the receiver.
The received signal at the photodetector depends on the light irradiance at a given point inside the room, the coordinates of which can be given by \( \mathbf{r} = (d, \theta, \varphi) \) in a spherical coordinate system with the transmitter at the origin. The transmitter can be modeled as a modified Lambertian source with mode number \( m \), with a diffuse source having \( m = 1 \). The Half-Power Angle (HPA) or viewing angle is related to the mode number by,

\[
m = -\ln \frac{2}{\ln \cos \varphi_{1/2}}
\]  

(6–1)

The diffuse source can be a performance benchmark for link budget calculations, and has a mode number of 1 and HPA of 60°. The resulting irradiance at \( \mathbf{r} \) inside the room is given by Lambert’s cosine law [151] as,

\[
I(\mathbf{r}, m, P_t) = \frac{m+1}{2\pi d^2} \cdot \cos^m \varphi \cdot P_t \quad ; \quad 0 \leq \varphi \leq \pi/2
\]

(6–2)

where the transmitted optical power from the source is \( P_t \). The irradiance normalized to \( P_t / \pi d^2 \) is illustrated in Fig. 6-1 for various HPA and mode numbers. For smaller HPA values, the stated directivity gains are with respect to a source with mode number of 1.

The power received by the PIN photodetector depends on its surface area, as well as the angle between the line connecting the detector and transmitter locations and the normal on the detector surface, \( \psi \), and the Field of View (FOV), \( \psi_c \). Furthermore, the effective area can be increased by using optical concentrators or lenses, the gain associated with which we denote as \( A_c \). The received optical power in the line of sight direction is then given by,

\[
P_r = A_c \cdot A_{det} \cdot \cos \psi \cdot I(\mathbf{r}, m) \quad ; \quad \psi < \psi_c
\]

(6–3)

Here, the optical gain of the optical concentrator is given as [159],

\[
A_c(\psi) = \frac{n^2}{\sin^2 \psi_c} \quad ; \quad 0 \leq \psi \leq \psi_c
\]

(6–4)

where \( n \) is the refractive index of the lens. Fig. 6-2 illustrates the concentrator gain for various FOV angles.
The electrical to optical conversion process in a laser diode is given as follows,

\[ P_i = R_i (i - i_{sh}) \]  \hspace{1cm} (6–5)
where $P_t$ is the output optical power for injection current $i$, $R_t = \eta_{ext} h\nu / q$ is the slope efficiency ($\eta_{ext}$ is the external quantum efficiency), and $i_{th}$ is the threshold current. As a result, lasers require a constant input current to bias it. On the other hand, LEDs have a conversion process given by,

$$P_t = R_i i$$  \hspace{1cm} (6–6)$$

where $R_i = \eta_{ext} h\nu / q$ is the LED slope efficiency. For p-i-n photodiodes, the conversion between received optical power and generated photocurrent follows the equation,

$$i_r = R_o \cdot P_r$$  \hspace{1cm} (6–7)$$

where $i_r$ is the photocurrent generated in response to received optical power $P_r$, and $R_o = \eta_{qe} q / h\nu$ is the responsivity of the photodiode, with $\eta_{qe}$ being the quantum efficiency. For APDs, the generated photocurrent is,

$$i_r = M R_o \cdot P_r$$  \hspace{1cm} (6–8)$$

where $M$ is the multiplication factor of the APD which is a function of the reverse bias voltage.

When propagation occurs inside a room, the walls, floor and ceiling act as reflectors, and these six surfaces can be divided into small reflecting elements of differential area $dA$, each having a reflectance $\rho$. The power collected by a reflecting element is then,

$$P_{surf} = I(\mathbf{\tilde{r}}, m) \cdot \cos \psi \cdot dA; \quad \psi \leq \pi / 2$$  \hspace{1cm} (6–9)$$

where $\psi$ is the angle between the normal on the reflecting surface and line connecting center of the surface to the transmitter. Each reflecting element then acts as a secondary source, and the irradiance from each of these at coordinate $\mathbf{\tilde{r}}$ can be given by,

$$I(\mathbf{\tilde{r}}, m, \rho) = \frac{\rho}{\pi d^2} \cdot \cos \varphi \cdot P_{surf}; \quad 0 \leq \varphi \leq \pi / 2$$  \hspace{1cm} (6–10)$$
With the quantities given in (6-1) to (6-10), a simulation model is developed to evaluate the impulse responses, and thereby the delay spread and bandwidth, of an indoor optical wireless link in Section 6.3.

6.3 Indoor Optical Wireless Channel Simulation Model

In this section, we present a multi-input multi-output model for characterizing the impulse response of an indoor optical wireless channel. The model is based on [152] and [160]. The room is divided into segments of reflecting elements each having a very small area. The source can be laser or LED, and hence can be collimated or diffuse. The overall impulse response from the transmitter to the receiver can be segmented into several individual responses, as below. The number of sources (or reflecting spots) is assumed to be $M$, the number of receivers or receiver branches is $N$, and the number of reflecting surfaces in the room is $N_{surf}$, which is equal to $2(N_x N_y + N_y N_z + N_z N_x)$, where $N_x$, $N_y$, $N_z$ are the number of elements along $x$, $y$, $z$ directions of the room. We normalize each source or spot to have unit power, in order to calculate the impulse response. The total power and individual power restrictions need to be considered by applying appropriate scaling, when optimization of the system is conducted.

6.3.1 Source to Receiver

The direct impulse response from the light source to the receiver is a $N \times M$ matrix $H_0(t)$, which is obtained by computing the power collected by the $n$-th receiver and the delay between $n$-th receiver and $m$-th transmitter, as,

$$P_{ji} = A_{det,j} \cos \varphi_{ji} \cos \varphi_{ji} \left( m + 1 \right) / 2 \pi d_{ji}^2 \cdot \xi \leq FOV_{det,j} \quad (6–11)$$

where the subscripts $i$ and $j$ correspond to the transmitter and receiver, respectively; $A_{det,j}$ is the effective area of the receiver branch; $\xi_{ji} = \cos^{-1} \left[ \left( \vec{r}_{ji} - \vec{r}_j \right) \cdot \vec{u}_j / d_{ji} \right]$ is the angle...
between the line connecting $i$-th transmitter and $j$-th receiver location and the $i$-th transmitter optical axis; $\zeta_{ji} = \cos^{-1}\left(\left(\tilde{r}_{s_i} - \tilde{r}_{r_j}\right) \cdot \tilde{u}_{r_j} / d_{ji}\right)$ is the angle between the connecting line and the $j$-th receiver location vectors and the $i$-th transmitter optical axis; $m$ is the transmitter mode number; and $d_{ji} = \|\tilde{r}_{r_j} - \tilde{r}_{s_i}\|$ is the propagation distance. As a result, the direct impulse response is,

$$ H_0(t) = \left[ P_{ji} \delta\left(t - d_{ji}/c\right) \right]_{1 \leq j \leq N, 1 \leq i \leq M} \quad (6-12) $$

### 6.3.2 Source to Reflecting Elements

The boundaries of the room act as reflectors with reflection coefficients less than 1, and a field of view of $\pi/2$. The impulse response is a $N_{surf} \times M$ matrix $F(t)$, obtained by calculating the power collected by each reflector, and the delay between each reflector and source, as,

$$ P_{ji} = A_{surf,j} \cos \xi_{ji}^m \cos \zeta_{ji} (m+1)/2 \pi d_{ji}^2, \zeta_{ji} \leq \pi/2 \quad (6-13) $$

where the subscripts $i$ and $j$ correspond to the transmitter and reflector, respectively; $A_{surf,j}$ is the area of $j$-th small reflector; $\xi_{ji} = \cos^{-1}\left(\left(\tilde{r}_{surf_j} - \tilde{r}_{s_i}\right) \cdot \tilde{u}_{s_i} / d_{ji}\right)$, is the angle between the line connecting $i$-th transmitter and $j$-th reflecotor location vectors and the $i$-th transmitter normal; $\zeta_{ji} = \cos^{-1}\left(\left(\tilde{r}_{s_i} - \tilde{r}_{surf_j}\right) \cdot \tilde{u}_{surf_j} / d_{ji}\right)$ is the angle between the connecting line and the $j$-th reflector normal; and $d_{ji} = \|\tilde{r}_{surf_j} - \tilde{r}_{s_i}\|$ is the propagation distance. As a result, the impulse response is,

$$ F(t) = \left[ P_{ji} \delta\left(t - d_{ji}/c\right) \right]_{1 \leq j \leq N_{surf}, 1 \leq i \leq M} \quad (6-14) $$
6.3.3 Reflecting Elements to Receivers

The reflectors can be considered as secondary sources, each reflecting back their collected power attenuated by their reflection coefficients. The impulse response $G(t)$ is a $N \times N_{surf}$ matrix. To obtain the power collected by the $j$-th receiver, first the footprint of the receiver on the corresponding room boundary is calculated as $S_{fp}$, by geometry of conic sections. If the $i$-th reflector has an area bounded by $S_{surf}$, then the overlap between the two has an effective area of $A_{eff} = A(S_{surf} \cap S_{fp})$. The receiver obtains only the fraction of power reflected from this intersection. Then, the power collected by the $j$-th receiver branch from $i$-th reflector is given by,

$$P_{ji} = \rho_i A_{det,i} \cos \xi_{ji} \cos \zeta_{ji} A_{eff} \int A_{surf} \pi d_j^2, \; \zeta \leq FOV_{det,i} \tag{6–15}$$

where $\rho_i$ is the reflection coefficient of the $i$-th reflector; $\xi_{ji} = \cos^{-1} \left[ (\mathbf{u}_{surf} - \mathbf{u}_{i}) \cdot (\mathbf{d}_j / d_j) \right]$ is the angle between the line connecting $i$-th reflector and $j$-th receiver location vectors and the $i$-th reflector normal; $\zeta_{ji} = \cos^{-1} \left[ (\mathbf{u}_{surf} - \mathbf{u}_{j}) \cdot (\mathbf{d}_j / d_j) \right]$ is the angle between the connecting line and the $j$-th receiver optical axis; $A_{surf}$ is the area of a reflector; and $d_{ji} = \| \mathbf{r}_j - \mathbf{r}_{surf} \|$ is the propagation distance. As a result, the impulse response is,

$$G(t) = \left[ P_{ji} \delta(t - d_{ji} / c) \right]_{1 \leq i \leq N; 1 \leq j \leq N_{surf}} \tag{6–16}$$

6.3.4 Reflector to Reflector

In a multiple-bounce scenario, the boundaries of the room reflect the transmitted lines multiple times before the light impinges on the receiver. Therefore, we need to calculated the elementwise impulse responses between reflector elements. This is given by a $N_{surf} \times N_{surf}$ matrix $E(t)$. The power collected by the $j$-th reflector from the $i$-th reflector is given by,
\[ P_{ji} = \rho_i A_{\text{surf}} \cos \xi_{ji} \cos \zeta_{ji} / \pi d_{ji}^2, \xi \leq \pi/2 \] (6-17)

where \( \xi_{ji} = \cos^{-1}\left( \frac{\mathbf{r}_{\text{surf}_i} - \mathbf{r}_{\text{surf}_j}}{d_{ji}} \cdot \mathbf{u}_{\text{surf}_j} / d_{ji} \right) \) is the angle between the line connecting \( i \)-th reflector and \( j \)-th reflector location and the \( i \)-th reflector normal; \( \zeta_{ji} = \cos^{-1}\left( \frac{\mathbf{r}_{\text{surf}_i} - \mathbf{r}_{\text{surf}_j}}{d_{ji}} \cdot \mathbf{u}_{\text{surf}_j} / d_{ji} \right) \) is the angle between the connecting line and the \( j \)-th reflector normal; and \( d_{ji} = \| \mathbf{r}_{\text{surf}_i} - \mathbf{r}_{\text{surf}_j} \| \) is the propagation distance. As a result, the impulse response is,

\[ E(t) = \left[ P_{ji} \delta(t - d_{ji}/c) \right]_{\xi,j \leq N_{\text{surf}}, i \leq i \leq N_{\text{surf}}} \] (6-18)

### 6.3.5 Combined Impulse Response

From the impulse responses calculated in Eq. (6-12), Eq. (6-14), Eq. (6-16), and Eq. (6-18), the combined impulse responses between all transmitters and receivers can be calculated as a \( N \times M \) matrix, by performing elementwise convolution of matrix elements, and adding the resulting responses, as follows,

\[ H(t) = H_0(t) + F(t) * G(t) + \sum_{k=1}^{N_{\text{ref}}} F(t) * \left[ E(t) \right]_k * G(t) \] (6-19)

where the first term is the direct response, the second term is the first-bounce response, and the rest of the terms represent higher number of bounces. The number of bounces to consider can be limited to \( N_{\text{ref}} \), chosen appropriately so that power from higher reflection orders are considerably attenuated. The notation \( \left[ x(t) \right]_k \) denotes a convolution with self operated \( k \) times, e.g., \( \left[ x(t) \right]_2 = x(t) * x(t) \).

Since each element of the matrices in Eq. (6-19) is a delta function with finite amplitude and delay, the convolution operation can be performed efficiently, e.g.,
The temporal resolution of the simulation is limited by the time required for light to propagate between adjacent reflectors, i.e., \( \Delta t = \frac{d}{c} \), where \( d \) is the distance between neighboring reflectors. For temporal response analysis, the time axis is divided into bins separated by \( \Delta t \), and each bin contains the total power collected by the receiver at that instant.

6.4 Noise Model

The several noise processes occurring in a photodetector, whether PIN or APD, are defined in this section. The power of shot noise induced by background illumination, such as room lighting and sunlight, can be given by \[164\],

\[
N_{0,\text{bg}} = 2qR_0P_{\text{bg}} = 2qR_0p_{\text{bg}}A_{\text{bg}}\Delta \lambda
\]

(6–21)

where \( q \) is the charge of an electron, \( R_0 \) is the photodiode responsivity, and \( P_{\text{bg}} \) is the total power of the detected background light. Furthermore, \( p_{\text{bg}} \) is the background irradiance per unit filter bandwidth, \( A_{\text{bg}} \) denotes the effective area of the optical front end as seen by the background light, and \( \Delta \lambda \) is the spectral width of the optical filter used to reject part of the ambient spectrum and to allow only signal wavelengths. If a transceiver system with bandwidth \( B \) is employed for signaling, then the additive noise variance would be \( 2N_{0,\text{bg}} \).

The noise process occurring in an APD is slightly different from a PIN photodiode, due its internal avalanche multiplication, and the power of the shot noise is given by \[164\],

\[
N_{0,\text{bg}} = M^2 F(M)qR_0P_{\text{bg}}
\]

(6–22)

where \( M \) is the multiplication factor of the APD, and \( F(M) \) is the excess noise factor, expressed as,
where \( k \) is the hole to electron ionization ratio.

The variance thermal noise process in the electronics is given by \( \frac{4kT}{R} \), with \( k \) being Boltzmann’s constant, \( T \) the temperature, and \( R \) the resistance. For high background illumination and high signal levels, the thermal noise can be neglected. The pre-amplifier stage, however, introduces FET noise, and the variance is given by,

\[
\sigma_{pr} = (\text{NEP}) \cdot \sqrt{B}
\]

where \( \text{NEP} \) is the noise-equivalent power, and is found in photodetector/amplifier package specifications. A convenient way to reduce noise is to use narrow optical filters, which allow only certain wavelengths of light that carry modulated data, and reject the rest.

### 6.5 Experimental Characterization of Indoor Wireless Optical Channels

An experimental setup comprising of a laser transmitter, APD receiver, corresponding amplifier, and a network analyzer is set up to obtain the frequency response characteristics of indoor wireless channel inside a room [165][166]. The block diagram is illustrated in Fig. 6-3, and a photograph of the setup is shown in Fig. 6-4.

The measurement system is comprised of a network analyzer, the Agilent ENA 5071C, which uses a CW swept frequency approach to obtain the amplitude and phase characteristics of the channel at frequencies between 10MHz and 1GHz. The sinusoidal signal from the network analyzer is amplified using a HP 83006A high-power amplifier with a gain of 26dB, before input to the laser transmitter. This ensures sufficient modulation depth of the transmit laser. An aspheric lens attached on the laser diode (JDS 2455-G1) produces a focused spot on the ceiling. The laser temperature is stabilized at room temperature with a Thorlabs TED350 temperature controller, and a Thorlabs LDC240C current controller. For the receiver, an avalanche photo-diode, APD210 from Menlo Systems with an integrated adjustable-gain low-noise amplifier, is chosen for its

\[
F(M) = kM + (1 - k)(2 - 1/M)
\]

(6–23)
high-sensitivity, and high gain. Due to the small APD sensitive area, it is very difficult to detect any signal at a distance without a lens assembly. Therefore, a focusing lens is attached to the photodetector to collect sufficient reflected light and focus it on the active detector area. When a 2-inch diameter lens is used, the ratio of receiving lens area to the detector area is approximately 10,000. This provides about 40 dB power enhancement to overcome path loss due to propagation. The output of the receiver is fed to a low-noise amplifier, which is connected to the measurement port of the network analyzer using a shielded coax cable. A ratio measurement is done with respect to the reference transmitted signal to find out the amplitude and phase responses over the bandwidth of interest.

Transmitted power level is set at 50mW, which is sufficiently low so as not to saturate the APD. This quantity may not be eye-safe when looked directly towards the laser source; however, the collimated light is diffused at the ceiling yielding an irradiance of 159 μW/cm² at 0.1m distance, which is much lower than the 8.37 mW/cm² hazard limit specified for 808nm light by the IEC 60825-1 Laser Safety Standard [161]. While the focusing lens assembly is useful to collect sufficient power, it renders the transmit and the receive optics sensitive to appropriate alignment. To align them properly, the receiver is mounted on a rotary assembly with gimbals, and as such can be moved to any other location inside the room for measurement.

The laser is modulated in the specified frequency range by the network analyzer using a Bias-Tee network included in the laser mount. To ensure operation in the linear regime, RF power output of the network analyzer is set to 0dBm. This generated a modulating current of about 360mA, which is well above the specified laser threshold current of 320mA. The received power at the APD was also below the maximum incident power rating of 10mW, and its gain was set properly to ensure linear operation.
Fig. 6-3: Block diagram of indoor optical wireless channel frequency response characterization.

Fig. 6-4: Experimental setup.

Fig. 6-5: Room layout showing relative positions of the optical transmitter and receiver.
Measurements are carried out at three different locations in a standard laboratory room, as shown in Fig. 6-5, with lenses of three different diameters: 1”, 2”, and 3”. The room is about 7.8m (306”) long by 4m (159”) wide. The room ceiling is about 2.5m (100”) above the communication floor. For each measurement, the transmitter is kept fixed at one location, while the receiver setup is moved from one location to another. The room is darkened during measurements to reject interference from ambient light, although the fluorescent lighting is observed to have no appreciable effect on the measured responses. The measurement parameters are summarized in Table 6.1.

This setup is effectively an indoor directed non-line-of-sight configuration (shown in Fig. 6-4), where the transmitter sends modulated collimated light towards the ceiling, which acts as a secondary Lambertian (diffuse) source, and the reflected signal is captured at the receiver through a focusing lens, which is pointed towards the ceiling spot.

### 6.5.1 Calibration of Results

To obtain the channel characteristics, the responses of the optical transmitter, receiver and amplifiers need to be subtracted from the data obtained from the measurement described above. This requires the laser transmitter and the APD to be
placed back-to-back, with the channel excluded. To protect the APD from exceeding exposure, the laser power is attenuated to a controlled level by covering with a thick black paper with a pinhole. As a result, the channel is replaced with a constant attenuation factor, which is found by measuring the unobstructed laser power and the power at the pinhole with an optical power sensor. The measured back-to-back frequency response is then given by,

\[ H_{\text{ref}}(f) = T(f) \cdot K_{\text{att}} \cdot R(f) \quad (6–25) \]

where transmitter side frequency response is \( T(f) \), the receiver side response is \( R(f) \), and \( K_{\text{att}} \) is a frequency-independent attenuation constant.

When the measurement is done with the propagation channel between the transmitter and the receiver, we obtain the non-calibrated channel response as,

\[ H(f) = T(f) \cdot H_{\text{channel}}(f) \cdot R(f) \quad (6–26) \]

To obtain the calibrated channel response, Eq. (6–26) is divided by Eq. (6–25) and multiplied by the known attenuation constant, \( K_{\text{atts}} \), which gives,

\[ H_{\text{channel}}(f) = \frac{H(f) \cdot K_{\text{atts}}}{H_{\text{ref}}(f)} \quad (6–27) \]

This quantity corresponds to the frequency response of the channel under consideration.

### 6.5.2 Obtaining Frequency Responses

At the locations shown in Fig. 6-5, frequency response measurements are conducted from 10MHz in steps of 618.75kHz to 1GHz, giving a set of 1601 uniformly spaced points in frequency for each measurement. Since the frequency response is characterized up to 1GHz, the resolution of the impulse response obtained by Fourier transform would be \( 1/(2 \times 10^9) = 0.5 \text{ ns} \). Due to the DC-blocking capacitive effects of the amplifying section of the receiver, the start frequency is chosen as high as 10MHz. The intermediate frequency (IF) bandwidth of the network analyzer was set to 10Hz to
reduce the fluctuations in the data registered on the network analyzer. Magnitude and phase responses are then calibrated according to the method described in 6.5.1.

6.5.3 Obtaining Impulse Responses

To better understand the effect of the channel on a high-speed transmission link, the electrical-domain impulse response is required. This is obtained by appropriately Fourier-transforming the frequency responses obtained by the procedures described in Section 6.5.2. Since the transfer ratios for DC and near-DC frequencies are absent, frequency measurements are extended to DC by interpolation. Savitzky-Golay filtering is applied on the frequency response to smoothen it and to reduce effects of noise. Furthermore, windowing is applied to the responses before taking the inverse Fourier transform, which helps reduce sidelobe levels in the impulse response. Several different windowing functions have been used in previous works, including Hamming [157] and Blackman. We choose the Kaiser window, since this has been established as a de facto standard for network analyzers which have impulse response transformation and display capability [162]. The Kaiser window is given by,

\[
w_n = \begin{cases} 
  I_0\left(\pi\alpha\sqrt{1-(2n/M-1)^2}\right) / I_0(\pi\alpha), & 0 \leq n \leq M \\
  0, & \text{otherwise}
\end{cases}
\] (6–28)

The reason for selecting the Kaiser window is its near-optimal maximum energy concentration near low frequencies, as required for response estimation from finite length segments of data [163]. The frequency span of the measurements, as well as the roll-off (\(\alpha\)) factor, affects the sidelobe levels and the 3-dB impulse width of the windowing function. Therefore, the desired temporal resolution of the impulse response must be taken into account before choosing \(\alpha\). The approximate impulse width of Kaiser window with \(\alpha \{0, 6, 13\}\) is \{0.6, 0.98, 1.39\} divided by the frequency span of the two-sided response [162]. We chose the \(\alpha\) parameter of our Kaiser window to be 6, giving an
impulse width of $0.98/2 \times 10^9$ or 0.49ns, which is less than the 0.5ns temporal resolution imposed by the APD rise-time limitation. The sidelobe attenuation level of the selected window is as low as -44dB with respect to the main lobe [162]. This ensures proper sidelobe level reduction, while maintaining desired temporal resolution.

6.5.4 Performance Measures

In this section, we introduce performance measures to be evaluated from the frequency-domain and time-domain response characteristics obtained through the procedures described.

*Delay spread* gives an initial estimate of the data rate performance of the channel and can be computed from the impulse responses. The average delay, $\mu_\tau$, and the rms delay spread, $\sigma_\tau$, are given by,

$$\mu_\tau = \frac{\int_0^\infty h(t)^2 dt}{\int_0^\infty h^2(t) dt} \quad (6–29)$$

$$\sigma_\tau = \sqrt{\frac{\int_0^\infty (t - \mu_\tau)^2 h^2(t) dt}{\int_0^\infty h^2(t) dt}} \quad (6–30)$$

Furthermore, the inverse of the delay spread is proportional to the *coherence bandwidth*, which is the bandwidth over which the channel can be considered “flat”, i.e. the frequencies within this band experience similar amount of attenuation, and as a result, there should be no intersymbol interference in time domain for a flat frequency response.

6.6 Simulation and Experiment Results

In this section, performance measures obtained from simulation and experiment on the indoor optical wireless channel under consideration are presented.
6.6.1 Frequency Responses

First, we present frequency-domain channel characterization results from the experimental procedure described in Section 6.5. The parameters of the experimental setup are described in Table 6.1. The measured frequency responses for the three different lens assemblies at the three locations are illustrated in Fig. 6-6, which demonstrates that the measured magnitude responses are almost flat over the entire bandwidth of interest, and furthermore, the phase responses are linear over that range. With change in location, the responses retain their shapes and attenuate by a constant amount, due to increased path loss or increased distance from the transmitter. On the other hand, the lens with the largest collecting surface experiences the least attenuation, irrespective of location. In our measurements, the receiving lens forms part of the channel, and thus is included in the channel frequency and impulse responses. The flatness of the frequency response is due to having only the line-of-sight component of the reflected light impinging on the receiving aperture. Therefore, no multiple-bounce reflection components should appear in the impulse response, and it would ideally consist of a finite impulse located at the delay corresponding to propagation distance. The magnitude also corresponds to the amount of optical power collected by the receiving optics with an ideal Lambertian source located at the ceiling spot.

Fig. 6-7 illustrates frequency response plots obtained through the simulation procedure described in Section 6.3. The simulation parameters are detailed in Table 6.2. It is observed that the magnitude responses are visibly comparable to the experimentally measured frequency response, thus validating the simulation algorithm.
Table 6.2: Simulation Parameters to obtain frequency and impulse responses.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Reflections counted</td>
<td>5</td>
</tr>
<tr>
<td>Room Size</td>
<td>(7.8m, 4.8m, 3.3m)</td>
</tr>
<tr>
<td>Number of reflectors in each dimension</td>
<td>(78, 48, 33)</td>
</tr>
<tr>
<td>Reflector Area</td>
<td>1cm²</td>
</tr>
<tr>
<td>Reflections coefficients of (Walls, Ceiling, Floor)</td>
<td>(0.8, 0.8, 0.1)</td>
</tr>
<tr>
<td>Windows</td>
<td>None</td>
</tr>
<tr>
<td>Lamps</td>
<td>None</td>
</tr>
<tr>
<td>Transmitter Location</td>
<td>(2.2m, 2m, 0.8m)</td>
</tr>
<tr>
<td>Spot Location</td>
<td>(2.2m, 2m, 3.3m)</td>
</tr>
<tr>
<td>Receiver Locations</td>
<td>(2.54m, 1.9m, 0.8m)</td>
</tr>
<tr>
<td>Receiver Orientation Angles (Elevation, Azimuth)</td>
<td>(4.14m, 1.08m, 0.8m)</td>
</tr>
<tr>
<td>Receiver Effective Areas (1, 2, 3-inch lenses)</td>
<td>(6.08m, 2.73m, 0.8m)</td>
</tr>
<tr>
<td>Time Resolution</td>
<td>0.33ns</td>
</tr>
</tbody>
</table>

Fig. 6-6: Measured and calibrated magnitude and phase responses
6.6.2 Impulse Responses

After applying the windowing and smoothing functions on these frequency responses, as detailed in Section 6.5.3, we obtain impulse responses for the three lens assemblies at the three locations shown in Fig. 6-5. The impulse responses are illustrated in Fig. 6-8. These are truncated at 50ns, which is the propagation delay corresponding to about twice the length of the room. This limits only the direct reflection and first reflections from walls to appear in the impulse response. The impulse responses are positive and their peak values occur at a delay consistent with the propagation distance.
The ideal impulse nature is not seen due to the finite resolution of the measurement system; however, as surmised previously, there are no multiple-bounce components in the impulse response. On a contrary note, having a diffuser in front of the laser transmitter and having a wider field-of-view at the receiver would have created strong multipath components, and would widen the tail of the impulse responses. This would also require more power to sustain a viable communication link.

To validate the impulse responses of the optical wireless links, simulations were carried out, following the procedure outlined in Section 6.3. The parameters for the simulation are coherent with the experiment parameters given in Table 6.1 and are listed in Table 6.2. The impulse responses are illustrated in Fig. 6-9. Fig. 6-10 further demonstrates the contributions from higher order reflections. The plots are executed in logarithmic scale to better represent these. We can conclude that the main contribution to the impulse response occurs from direct path, and inconsequential amounts of energy are concentrated in higher order reflections. As the lens aperture gets larger, it accumulates

![Fig. 6-8: Impulse responses obtained through inverse Fourier Transform of measured frequency responses.](image-url)
larger contributions from higher order reflections. Particularly, for short distance with smaller apertures, the 2nd reflection does not impinge on the receiver. As distances get longer and lens aperture gets bigger, 2nd reflections appear, thus increasing the delay spread shown in Table 6.3. But even in the worst case (lens 3 at position 3), the 2nd reflection component is ~20dB below direct path component, and therefore, does not contribute to reasonable degradation in terms of ISI.

6.6.3 Delay Spread

Delay spreads can be calculated from the simulated and experimental impulse responses using Eq. (6–29) and Eq. (6–30), and are tabulated in Table 6.3.
From Table 6.3, the root-mean-square delay spread values for all three lenses at the three locations are seen to be less than 0.5ns. This is because the measurement of the channel is limited by the devices themselves, and not by the channel. For a realistic scenario, we can approximate the delay spread to 0.5ns, taking the device limitation as the channel limitation. If the true delay spread value is taken to be on the order of the measurement resolution, the coherence bandwidth would be $1/(0.5\times10^{-9}) = 2$GHz. On a first approximation basis, this means that the wireless link under measurement can support data rates well beyond 1Gbps, without requiring complex equalization schemes as are necessary for RF wireless and most wireline communication media. Furthermore, the conclusion about the delay spread being limited by the APD rise time is corroborated by the simulation results, which consistently demonstrate lower delay spread values for all locations. Given the device limitations are shifted to give a much narrower rise time,
the channel would be able to support between 3 to 25Gbps data rates, for the three lenses at the given three locations. For receiver locations other than the experimental setup, the simulation model can be used to determine delay spread quantities, and obtain a measure of the distance over which the link quality does not degrade, in terms of being ISI-free. Fig. 6-11 illustrates delay spread profile for receiver locations spread across the room.

![Diagram of delay spread profile](image)

**Fig. 6-11:** Profile of delay spread quantities obtained from simulation corresponding to experimental setup.

What is immediately visible from Fig. 6-11 is that delay spread increases as the distance of the receiver from the transmitter increases, and as the receiver gets closer to the room corners. The lowest delay spread occurs at the closest position with respect to the transmitter, and is 0.03561ns. At the farthest corner of the room, this value increases to 0.6419ns, which is still less than the possible measurement resolution. Therefore, at least 1Gbps data transmission without ISI is still possible, given sufficient power is available for the receiver to distinguish the signal from the noise. We examine the power profile later in Section 6.7.

### 6.6.4 Peak Distortion

In addition to delay spread, peak distortion is an important indicator of ISI degradation resulting from multiple reflections. This is obtained from the impulse response $x(n) \equiv x(nT_s)$ sampled at the symbol rate $T_s$, which we take to be 1Gsymbols/s
for an OOK modulation system, by taking the ratio of the desired (maximum) sample energy over the total energy contained in the remaining samples, i.e.,

$$D = \frac{x_0}{\sum_{n=\infty}^{-\infty} |x_n|}$$

(6–31)

where $x_0$ represents the symbol-carrying sample, and would be optimally chosen at the maximum point of the impulse response. The same concept holds if matched filtering is used, but the impulse response would be replaced by the time-auto-correlation function of the combined end-to-end channel impulse response. Peak distortion represents the worst possible ISI, where transmitted OOK data has such a sequence that maximum ISI is introduced. Therefore, in reality, the introduced ISI would have a statistical distribution more favorable for data transmission than peak distortion. The profile of peak distortion at various receive locations inside the room, obtained by simulation, is illustrated in Fig. 6-11.

![Figure 6-11: Profile of peak distortion quantities obtained from simulated impulse responses.](image-url)
From Fig. 6-11, the peak distortion is observed to vary from a minimum -23.32dB to -10.2dB. The minimum values occur close the transmitter or the spot and as the receiver moves away from the transmitter, peak distortion becomes more and more significant. This is due to appearance of the second reflection component as the receiver moves further away from the transmitter and gets closer to the room walls. Therefore, to obtain reasonable performance, we should consider a cellular topology, whereby each spot or transmitter is allowed to cover a range of about 5 to 7m, beyond which a different transmitter or spot would operate with minimal interference. In any case, when ISI exceeds the tolerable limits, equalization needs to be employed to compensate for degradations.

6.6.5 Performance in Noise

Delay spread does not reflect the performance of the optical wireless system in additive noise and background radiation. To examine the effects of noise on a high-speed data transmission link using optical wireless, we calculate the variance of the background light-induced shot noise component in the output current, using Eq. (6–21), and the noise variance in the preamplifier stage of the receiver from Eq. (6–24). We assume the background light irradiance to be 6 μW/cm² [164] and Δλ to be 30nm. R is 50 A/W, and NEP is 0.4pW/√Hz according to specifications. With these quantities, the total variance of the additive noise is computed, and used to evaluate eye diagrams with impulse responses obtained experimentally and by simulation.

Simulations are run for 800Mbps and 1Gbps indoor optical wireless link with receiver located at designated three positions. The modulation technique is chosen to be On-Off Keying (OOK) modulation, since OOK is the most widely used transmission scheme for such links, and offers least complexity of implementation without unreasonable degradation in performance. The pulses transmitted from the laser are rectangular with maximum power output occurring during symbol ‘1’ and no power is output during symbol ‘0’. The average transmit power of the modulated data stream is
also changed to use the complete linear range within which the laser can be modulated. Thus, the average optical power output of the laser can be increased to 24.7dBm. This power does not exceed eye-safety limitations discussed in Section 6.7, and the chosen laser is capable of providing linear modulation in this range. The additive noise was assumed Gaussian with variance computed as before. The resulting eye diagrams are illustrated in Fig. 6-12 and Fig. 6-13 for 800Mbps and 1Gbps data rates, respectively. Pseudo-random binary sequences of 100,000 bits are used to generate these eye diagrams, which show two consecutive bit intervals in each plot.

Fig. 6-12: Eye diagrams at 800Mbps data rate.
The eye diagrams consistently show that the “open-eye” condition is satisfied for the three lenses at all three locations inside the room. The noise and background interference becomes more and more prominent in longer propagation distances, furthermore the larger lenses collect more background interference than the smaller lenses. For 800Mbps, the eye diagrams are found to be almost ideal with little overlap between consecutive symbols, whereas there is some visible overlap between symbols for 1Gbps, which results from the window function not being able to apply sufficient attenuation near the tail of the impulse response at this high rate. The eyes are still observed to be sufficiently open and binary detection can be applied with slightly degraded performance. In general, it can be concluded from the eye diagrams that transmission of at least 1Gbps data is feasible through an indoor optical wireless channel.

Fig. 6-13: Eye diagrams at 1Gbps data rate.
### 6.7 Link Budget and Eye Safety Considerations

Link budget analysis is of utmost importance to find out the required received and transmitted power limits for the indoor optical wireless system. The transmitted power, and therefore the allowable received power, is limited by eye safety limits. Specially the wavelength chosen for the presented case was 808nm, which is subject to exposure limitation for both eye and skin safety [160]. First we calculate the maximum permissible exposure (MPE) for the wavelength under consideration, with the high power levels used to simulate the eye diagrams of Section 6.6.5, a single transmitted pulse from the ceiling diffusing spot would have an irradiance (calculated using ideal Lambertian emission pattern) of 1.91mW/cm² when viewed directly from the minimum 0.1m observation distance, as specified in [160]. Hence, such a system would be theoretically eye-safe.

On the other hand, the required power for a OOK system is dependent on the SNR and BER requirements, which is given by [164],

\[
BER_{OOK,req} = Q\left(\frac{R_0P_{rx,req}}{\sqrt{N_0R_b}}\right)
\]

where \( BER_{OOK,req} \) is the required bit error ratio, \( R_0 \) is the detector responsivity, \( P_{rx,req} \) is the required received power, \( N_0 = qI_{bg} = qR_0p_{bg}\Delta\lambda A_r \) is the background-light induced noise variance per unit bandwidth, and \( R_b \) is the data rate to be considered. Therefore, the required bit-energy to noise spectral density is The received power is calculated as,

\[
P_{rx} = \frac{G_c A_r \cos\psi}{d^2} \cdot I_n(r_s, r_r)
\]

where \( G_c \) is the concentrator gain, \( A_r \cos\psi \) is the effective receiver area with \( \psi \) being the angle between transmitter-receiver connector and the receiver normal, \( d \) is the propagation distance, and \( I_n(r_s, r_r) \) is the normalized irradiance due to transmitter and receiver located at \( r_s \) and \( r_r \), respectively. The normalized irradiance is given by,

\[
I(r_s, r_r) = \frac{(m+1)\cos^m \theta}{2\pi} \cdot P_{ns}
\]
where \( m \) is the Lambertian mode number of the transmitting source, \( \theta \) is the angle between transmitter-receiver connecting line and the transmitting normal, and \( P_{tx} \) is the average transmit power. Therefore, the link budget can be written in terms of required transmit and receive optical powers, as follows,

\[
P_{rx, req[dBm]} = -94 + 5 \log_{10} \left( \frac{E_h}{N_0} \right) - 10 \log_{10} R_{\text{eff}[A/W]} + 5 \log_{10} R_{\text{bg}[\mu A]} + 5 \log_{10} I_{\text{bg}[\mu A]}
\]

Using the link budget equation and the values in Table 6.3, the required transmit and receive optical powers are calculated for a diffuse system \( (m=1) \) and a directed system \( (m=20) \) and demonstrated in Fig. 6-14. The required transmit power values are shown for 1Gbps bit rate, and are 19dBm, and 9dBm, respectively, for a completely diffuse system \( (m=1) \) and a directed system \( (m=20) \). Note that, the chosen transmit power in Section 6.6.5 was chosen to be 24.7dBm, which is significantly higher than the value
required for 1Gbps transmission from link budget analysis. The transmit power requirement can be reduced further by use of directed ‘spotlights’ which have a Lambertian order greater than 1, and are commercially available with such specifications. For example, to obtain a directivity gain of 10.2dB, we need a Lambertian source of order 20, corresponding to a half power angle of about 15°, which is available commercially.

![Graphs showing required power levels over variable distance and data rate](image)

**Fig. 6-15:** Required received and transmitted power against distance and data rate for a bit error probability of $10^{-6}$: (a) $m = 1, R_b = 1$Gbps, (b) $m = 1, d = 10$m, (c) $m = 20, R_b = 1$Gbps, (d) $m = 20, d = 10$m.

The path loss profile in the room under consideration is shown in Fig. 6-16, where we consider the direct reflection component, since this contributes to maximum power, and symbols can be decoded with higher SNR. The path loss ranges from -40.92dB optical from the nearest location to the spot to -52.92dB optical at the farthest room.
corner. From the power budget analysis, the required receive power for a reasonable BER over a 1Gbps link is -33dBm. Therefore, the required transmit power to obtain this performance across room locations would range from 8dBm to 20dBm, the latter closely corresponding to the value chosen to obtain eye diagrams in Section 6.6.5. If instead of diffuse spots, a directed light source is mounted on the ceiling with a directivity gain of 10dB, corresponding to a HPA of about 15°, the transmit power requirement would reduce to between -2dBm to 10dBm. With a directed source, the coverage would reduce, but would be compensated by mounting several light sources in a cellular configuration. Therefore, scalability holds. In conclusion, for a room of dimensions comparable to the experimental setup, the required power can be easily made available, while also being eye-safe.

Fig. 6-15: Profile of direct reflection power obtained from simulated impulse responses.
6.8 Conclusions

The simulations and experiments presented in this chapter have successfully demonstrated the capability of indoor optical wireless systems to deliver 1Gbps and possibly more data rate at a reasonable bit error probability over typical short indoor distances. Frequency and time domain characteristics have been obtained through simulation and experiments and verified. Furthermore, data rate performance is measured in terms of delay spread values and eye diagrams. The eye diagrams show that there is minimal inter-symbol interference for data links operating at 800Mbps and 1 Gbps. The immediate conclusion is that it is possible to use the optical spectrum to transmit and receive tremendous amounts of data, which has been foreseen for many years. To the author’s knowledge, these are the first set of measurements attesting bit-rates as high as 1 Gbps over directed non-line-of-sight indoor optical links. It must be noted that majority of the components used in obtaining these measurements were commercial off the shelf (COTS), and hence were not specifically optimized for communication purposes. The primary intention of using these was to demonstrate the feasibility of mentioned high data rates on indoor optical wireless links. To avail of these data rates in an efficient manner, interested users must utilize custom-built or optimized optical and opto-electronic components tailored to meet the specific requirements of the data transmission system. Application of these optical links for sensor networking can significantly overcome the bandwidth and data-rate restrictions imposed on RF sensor network communications. With advances made in photonics and electronics, it is now possible to implement optical links with inexpensive low-power laser diodes and high-sensitivity wideband photodiodes. Incorporation of these optical transmitting and receiving devices on miniaturized sensors will greatly reduce the required amount of power, and increase data rate or conversely increase number of simultaneously active links. The current analysis did not take into account the effect of interference between neighboring sensors, and optimization will be required to place and orient the sensors to reduce interference. Another problem is the loss of data due to shadowing, and an attractive and efficient way to combat shadowing and increase coverage is to use multi-spot diffuse configuration
In terms of eye safety, the hypothetical data transmission systems presented here would meet the requirements set out by Laser Safety standards. This is due to the diffuse nature of the laser propagation system. Further reduction of eye exposure can be achieved with the use of multiple diffusing spots on the ceiling generated by a beamsplitter. This would also benefit the communication system by providing spatial diversity. With the incorporation of a “fly-eye” receiver [158], required received power would also reduce by selectively rejecting background light. Holographic optical filters and concentrators promise better power efficiency, as well as noise reduction. In general, we conclude that indoor optical wireless links have communication capabilities exceeding contemporary technologies, and further research on system design and optimization of opto-electronic components for communication-specific applications are required for realizing the immense potentials of this emerging technology.

The optical system can be optimized by using concentrators, directed sources, and optical filters, and the specifications will vary depending the application for which it is required. For example, the HPA of the transmitter corresponding to $m=20$ is $15^\circ$, and gives a gain of about 10dB. White LED light sources are available commercially with HPA ranging from $8^\circ$ to $120^\circ$. Combined optical filter and concentrator can be manufactured using computer-generated holographic technology [167]. Furthermore, mechanical control mechanisms can be developed to align the transmitter and receiver appropriately to maximize received power level. The short distances covered by the system can be potentially implemented in a femtocell-like structure in a building, thereby both saving power and maximizing end-user data rate. Multi-spot diffuse configurations with fly-eye receivers are the next logical steps to optimize the system, where delay spread, available bandwidth, and transmitted power can be optimized to yield the highest bit rates possible within a small area corresponding to a typical indoor environment.
Chapter 7
Conclusions and Future Work

7.1 Conclusions

Modeling a system from a multivariate perspective provides more insight than conventional single-input-single-output systems. In RF wireless environment, multiple antennas can improve communication performance, given there is rich, and preferably uncorrelated, scattering. In this thesis, we have reviewed and examined statistical models of RF wireless channels and quantified the circumstances under which MIMO channels can improve capacity, data rate, SNR, and BER performances. Detection and coding techniques are described, that utilize the diversity of a MIMO channel to the fullest.

Many of the concepts introduced in RF MIMO can be extended to other systems, including category copper cables, where interference can limit the capacity. However, from analysis and simulation, we find out that MIMO techniques can be useful only when there is a reasonable degree of crosstalk between the pairs of the cable. Newer Cat-7 cables are shown to achieve capacity values approaching 100Gbps. Investigation of transmission techniques reveal that the complexity of equalization, interference cancellation, and error correction coding is too high by current standards, leading to lower energy efficiency. Specially, echo cancellation imposes the biggest constraint in terms of large number of filter taps and long delays. FIR implementation of echo canceller is seen to result in an aggregate of 1000 taps and above. Adaptive LMS echo cancellers are difficult to implement due to convergence problems. The other crosstalk terms can be negligible due to the double-layer shielding of the cables. For 100Gbps, the conservative estimate of distance coverage would be 50m, rather than the conventional 100m.

Achieving 40Gbps on 100m Cat-7 cables seems to be a more realizable goal and IEEE standard committees and industries are currently pursuing this. Furthermore, digital
processor chips need to be developed that operate at the ultra-high clock speeds required for data transmission in the 40-100Gbps range. Most likely, it will not be until 2 or 3 more generations of CMOS technology have passed that we would see the DSP required for an energy-efficient implementation of a 40 or 100GBase-T system.

Block LDPC coded modulation is used to achieve high coding gain, without sacrificing much needed bandwidth. The proposed hypothetical system is optimized to obtain the required coding gain for 40/100Gbps data transmission.

The general conclusion about MIMO modeling and signal processing on copper cables is that it will not perform any better or be any less complex than a SISO system for a system with excellent shielding and minimal leakage of signals. This is because the pairs of cable are already a parallel independent set of channels, and can practically be processed separately without sacrificing resources. But lower level manufacturing issues could be incorporated to find out how the cable manufacturing process could be simplified to allow some interference into the cables, which can be mitigated by a MIMO system in a more efficient manner than SISO. This requires characterizing and analyzing several cables with different twist rates and shielding materials, etc.

The multivariate analyses extended to FSO systems subjected under atmospheric turbulence or scintillation reveals a more optimistic scenario. Channel models have been developed to incorporate clouds as well. However, the effect of clouds is manifested as attenuation from scattering and for the immense number of photons involved in each pulse there is little statistical fluctuation to provide diversity. But clouds do contribute to pulse broadening in time and space, thus requiring temporal or spatial equalization for compensation. Receiver aperture and Field of view at the receiver can be optimized as well. On the other hand, the randomness required to harness MIMO capabilities come from the log-amplitude distribution of laser pulses due to scintillation. Diversity combining techniques such as EGC and MRC can be employed to improve the statistics of scintillation over multiple apertures. A conventional adaptive optics system, in addition with multiple apertures, can prove effective in combating scintillation to a reasonable degree. This is under the assumption that scattering only introduces attenuation, which is a realistic assumption for small FOV receivers. However, the
statistical variation seen with strong turbulence is considerably different from weak turbulence, and has to be treated differently than just the log-normal fading approximation.

Spatial multiplexing is a MIMO concept that is shown to offer a robust active optical imaging system. Illumination of the target in a pixelwise manner can prove less dispersive, if laser beams with smaller beam-waist are used. Noise due to back-scattered photons is effectively compensated for by time-gating, thus enhancing contrast. We show that active optical imaging beyond an optical thickness of 5 may be very impractical. This would be a thin layer of cloud or fog. However, the level of penetration depends on the power transmitted, and penetrating thicker clouds can seriously constrain available power resources. On the other hand, the problem of turbulence, as manifested in amplitude and phase variations, is tractable, but still very challenging. The loss of resolution needs to be compensated for by deconvolution or deblurring techniques. Since a priori information may not be available, blind algorithms need to be implemented. Several blind deconvolution techniques are examined, and we observed that a combination of adaptive optics with blind signal processing techniques is practical, and gives satisfactory results.

For indoor optical wireless communications, a MIMO modeling perspective is found useful. An experimental setup demonstrates the capability of these channels to support data rates in the Gbps range, which is also corroborated by a simulation using MIMO principles. Several performance measures, such as delay spread, received power, and peak distortion indicate that only short distances, typical of a room environment, can be covered. This is by no means a deterring factor, since a cellular topology would be able to provide complete coverage. Furthermore, a MIMO system can be developed with multiple diffusing spots and multiple branches of a “fly-eye” receiver. Such a system can be optimized by using MIMO diversity mechanisms to have better ambient light rejection, as well as interference excision. Field of view and effective area of the receiver and directivity gain of the transmitter are important factors to be chosen carefully. Furthermore, for a multi-spot configuration, power divided in each spot can be varied to obtain smaller delay spread and higher received power. Gains of each branch of a fly-eye
receiver can also be optimized for aggregate combined SNR. In all, a MIMO system would have more flexibility and degrees of freedom required for the system to be robust to variable environments. Power budget analysis for such a hypothetical system demonstrates that there is sufficient power in a small cell for high data rate, and it would be eye-safe. The important fact we also demonstrate is that having a line-of-sight component in receiver field-of-view is preferable over having a completely diffuse configuration, in terms of power and delay spread.

7.2 Future Work

Application of MIMO or multi-variate topology to areas other than RF wireless needs to be further investigated. There are certain scenarios where it may not be the optimum solution for system design, such as the category copper cables. But in general, a MIMO model reveals a more complete picture.

Future research in category copper cables would involve several investigations, including sparse realization of a SISO or MIMO equalizer/canceller to find out their relative complexities; development and performance quantification of a joint MIMO equalizer/canceller and coder; further reduction of effective return loss interference by employing a possible MIMO analog front-end; design of matching circuits which have joint characteristics; segmentation of the overall bandwidth and employment of asymmetric coding and modulation; and finally, an investigation of the tradeoff of cable manufacturing complexity and MIMO signal processing complexity.

Spatially-multiplexed MIMO FSO links can be optimized for better power allocation and modulation schemes. In place of adaptive optics, spatial waveforms can also be optimized with greater resistance ability in turbulence. Some of these have been recently developed and shown improvements. The spatially-multiplexed imaging system needs to be characterized in terms of realizable transmit power. The obstacles imposed by clouds cannot be completely overcome, except by transmitting pulses with higher energy. Adaptive selection of number of beamlets and power allocation seem to be a good candidate for the proposed optical imaging system to be robust under varying climate
conditions. Coherent MIMO imaging systems could be a huge improvement over the incoherent systems investigated here, since they will allow phase retrieval, and simplify 3D visualization. However, these systems would be formidably expensive and complex.

Indoor optical wireless transmission systems, similarly, can be optimized when multi-spot diffuse configuration is used along with fly-eye receivers. Power allocation, shadowing detection and compensation, and adaptive reconfiguration for mobility are promising fields. Complex modulation schemes can now be easily incorporated into small chips, and could be used in place of the simple OOK modulation assumed here. Visible light communications (VLC) is an interesting emerging area. The investigations carried out here are also applicable to VLC; however, VLC requirements are substantially different than an IR IM/DD system. Firstly, the lighting application has to be considered primary, and hence, situations such as dimming and extinction need to be addressed. Secondly, The diffuse nature of lighting application reduces available power; a tradeoff can be reached by using ‘spotlights’ with reasonable half power angles, while providing directivity gain. Fly-eye receivers would prove particularly useful for VLC applications, due to their efficient exploitation of spatial diversity. On the other hand, the ultraviolet spectrum, although not a hygienic choice, can be exploited for their excellent scattering properties for military outdoor applications, and are being investigated currently.
7.3 Publication List

The following is a list of publications which are a direct outcome of the research covered in this dissertation:


Bibliography


137 MacKay, D. J. C., Encyclopedia of Sparse Graph Codes, available online at: http://www.inference.phy.cam.ac.uk/mackay/codes/data.html#151.


162 Agilent Technologies, Agilent Time Domain Analysis using a Network Analyzer, Application Note 1287-12 [online].


VITA

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Jarir Muhammad Fadlullah, was born in 1979 in Dhaka, Bangladesh. He received B.Sc. in Electrical and Electronic Engineering from Bangladesh University of Engineering and Technology (BUET), Dhaka, Bangladesh, in 2004. He has been a lecturer in the Department of EEE at the Islamic University of Technology (IUT), Gazipur, Bangladesh, from 2004 to 2005, where he instructed undergraduate electrical and electronic engineering theory and laboratory classes.

He joined the Center of Information and Communications Technology Research at the Pennsylvania State University in August 2005 pursuing Ph.D. in Electrical Engineering. He has performed fundamental research in various fields including high speed data transmission over category copper cables (funded by Nexans Inc.), mitigation approaches for active imaging systems in presence of scattering and turbulence (funded by DARPA), and broadband optical wireless sensor networks (funded by NSF). He has presented his and his colleagues’ works at prestigious conferences including IEEE GlobeCom, IEEE MilCom, and SPIE Photonics West. He is co-author of a “Best Paper of the Year” published in the ETRI Journal in 2009. His research on indoor optical wireless communication links has been featured in several technical news media including MIT Technology Review, IEEE’s the Institute, Communications of the ACM, Penn State Live and EE Newsletter, and Photonics spectra.

He is a member of the IEEE and the SPIE, and has reviewed conference and journal papers for these societies.