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Abstract

This dissertation consists of two chapters, both of which address the geographic distribution of economic activity.

In the United States, four million African Americans migrated from the South to the North between 1940 and 1970. Chapter 1 studies the effects of this great Black migration on aggregate US output and the welfare of African Americans and others. For this purpose, I develop and quantify a dynamic general equilibrium model of the spatial economy in which cohorts of African Americans and others migrate across locations. I compare the baseline equilibrium matched with US data from 1940 to 2010 with counterfactual equilibria in which African Americans or others cannot relocate across the North and the South between 1940 and 1970. The mobility of African Americans and others increased aggregate output by 0.7 and 0.3 percent, respectively. Although African Americans accounted for about 10 percent of the US population, their relocation impacted the aggregate economy more than the relocation of the other 90 percent did. The mobility of African Americans induced a large increase in the welfare of African Americans in the South, a small decrease in the welfare of African Americans in the North, and little change in the welfare of others.

Chapter 2 studies the effect of a productivity change in a foreign country on unemployment across US states. I develop a model of involuntary unemployment in multiple geographic locations. The model merges a quantitative general equilibrium model of international trade and spatial economy and the efficiency-wage model (Shapiro and Stiglitz, 1984). I quantify it for 27 countries and 50 US states and compute the counterfactual of the 5% increase in China's productivity. The model predicts that real wages increase in all the US states, but unemployment increases in 44 states, and the overall US welfare increases. The counterfactual result highlights heterogeneous effects of foreign shocks on unemployment and real wages across the US states.

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Acknowledgments

At the commencement ceremony in 2017, the then-dean of the faculty of economics at the University of Tokyo, Kiyohiko G. Nishimura, said finding a good mentor/advisor by oneself is key in one's life. I came to appreciate his remark through my PhD at Penn State.

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Last but not least, I would like to dedicate this dissertation to African Americans who made a journey from the South to the North or chose to stay in the South.

Chapter 1
The Aggregate Effects of the Great Black Migration

1.1 Introduction

Slavery was a place-based policy in the history of the United States. Before Emancipation, about 70 to 80 percent of the Black population in the US resided in the South, and the vast majority of them were enslaved. The largest change in the spatial distribution of the Black population occurred from 1940 to 1970. In this period, 4 million African Americans migrated from the South to the North, and the fraction of the Black population in the South dropped from 69 percent to 45 percent. This is called the great Black migration.

How did the great Black migration impact aggregate US output and the welfare of cohorts of African Americans and others? To answer this question, I develop and quantify a dynamic general equilibrium model in which cohorts of African Americans and others migrate across states.

In the period of the great Black migration, African Americans in the South were more likely to move to the North than others in the South were. 44 percent of African Americans born in the South in the 1930s migrated to the North, whereas only 17 percent of others born in the South in the same decade did so. Accordingly, my model delivers different migration patterns across races.

In the great migration period, African Americans who moved from the South to the North earned much higher wages than African Americans who stayed put in the South. The degree of this wage gap between movers and stayers was higher for African Americans from the South than for any other group of people. These facts suggest that there was pecuniary incentive for the great Black migration. I primarily attribute wages to productivity parameters flexibly varying at race, age, time, and location levels to capture heterogeneous pecuniary incentive for migration.

Higher housing rent in the North partly offset higher wages in the North. But, in the great migration period, only about one fourth of the wage gap between movers and stayers for African Americans from the South was absorbed by higher rent in the North. In the model, I assume that individuals spend fixed expenditure shares on freely tradeable goods and locally supplied housing following the Cobb-Douglas period utility function. Therefore, in the model, real wages are nominal wages divided by the power function of housing rent. The supply of housing is imperfectly elastic, so it serves as a congestion force in the local economy.

The great Black migration did not make everyone better off. The migration of African Americans to the North shifted up the labor supply of the Black labor force and put stronger downward pressure on African Americans' wages than on others' wages in the North. Therefore the great Black migration made worse off African Americans who had already lived in the North (Boustan, 2009). Allowing for imperfect substitutability across African Americans and others, my general equilibrium model generates such ramifications of the great Black migration in the North.

Since people of different cohorts and ages migrate differently in data, the model includes overlapping generations, so that the model has distinct notions of cohorts and ages. African Americans have lower survival probabilities and life expectancies than others in US data. Accordingly, African Americans and others of the same cohort face different survival probabilities and life expectancies in the model.

I parameterize the model in two steps. In the first step, I estimate elasticities: parameters mapping a percent change in one endogenous variable to another. Elasticities include migration elasticity mapping percent changes in real wages and non-pecuniary amenities to percent changes in migration flows, elasticity of substitution across ages and races in the production function, and rent elasticity mapping a percent change in aggregate local income to a percent change in housing rent. In so doing, I follow standard methods in trade, labor, and urban economics literature (Artuc and McLaren, 2015; Borjas, 2003; Card, 2009; Glaeser et al., 2005; Saks, 2008). My estimate for the elasticity of substitution between African Americans and others is about 9.0, which falls in the range of the estimates by Boustan (2009) from 8.3 to 11.1.

In the second step, I back out the other parameters such as amenities, productivity, and migration costs for different age and racial groups across states over time. Rent shifters, which govern levels of housing rent given aggregate local income, are also recovered. The model delivers explicit formulae to pin down these parameters given elasticities and relevant data, as the models in Allen and Arkolakis (2014) and Ahlfeldt et al. (2015) do. The migration costs thus backed out are higher for African Americans than for others, but the racial gap in the migration costs shrank over time.

Armed with the parameter values, I compare the baseline equilibrium that resembles the factual path of the US economy to two counterfactual equilibria. In the first counterfactual equilibrium, African Americans could not migrate across the North and the South between 1940 and 1970 (the no great Black migration scenario). In the second counterfactual equilibrium, others could not migrate across the North and the South for the same period (the no others' migration scenario). The first counterfactual helps me understand the role of the great Black migration in the US economy. Comparing the first and second counterfactuals contrast the role of African Americans' migration to the role of others' migration.

In the no great Black migration scenario, aggregate US output in 1970 would have been lower by 0.74 percent than in the baseline equilibrium. In the no others' migration scenario, aggregate output in 1970 would have been lower by 0.28 percent. Therefore, although African Americans accounted for about 10 percent of the US population, their migration had a larger impact on the aggregate economy than the migration of the other 90 percent of the population did.

If the great Black migration did not occur, fewer African Americans would have worked in the North. Because of the imperfect substitutability across races, this would have put upward pressure on the wage of African Americans in the North. Indeed, in the no great Black migration scenario, the average wage of African Americans in the North would have been higher by 5.3 percent than in the baseline equilibrium. This number is a little smaller than, but comparable to, the predictions made by Boustan (2009) from 7.2 to 9.6.[1]

Welfare is measured by the expected value at the beginning of life. In the no great Black migration scenario, the welfare for African Americans born in Mississippi in the 1930s would have been 3.5 percent lower than in the baseline equilibrium. This is because they lost opportunities of migrating to the productive, high-wage North. In the no great Black migration scenario, the welfare for African Americans born in Illinois in the 1930s would have been 0.2 percent higher. This is because the wage of African Americans in the North would have been higher in the no great Black migration scenario than in the baseline equilibrium. The welfare of others in the no great Black migration scenario is not substantially different from the welfare of others in the baseline equilibrium.

[1] See her table 6.

In the no others' migration scenario, the welfare of others born in Mississippi in the 1930s would have been 1.4 percent lower than in the baseline equilibrium. This number is smaller than 3.5 percent, the welfare loss of African Americans born in Mississippi from the baseline equilibrium to the no great Black migration scenario. These two counterfactual experiments highlight African Americans' strong incentive for the outmigration from the South. In the no others' migration scenario, the welfare of others born in Illinois in the 1930s would have been 0.37 percent lower than in the baseline equilibrium. Others in Illinois were already in the productive location, but they forwent varieties in location choices in the no others' migration scenario, leading to the welfare loss.

My quantitative model delivers implications for racial inequality. In the no great Black migration scenario, the nationwide average nominal wage ratio between African Americans and others in 1970 would have been 10.2 percent lower than in the baseline equilibrium. This is in line with the prediction by Smith and Welch (1989), who adopt a reduced-form decomposition technique to measure the impact of the great Black migration on nominal wage gaps between African Americans and white people. In the no great Black migration scenario, the nationwide average real wage ratio between African Americans and others in 1970 would have been 8.8 percent lower than in the baseline equilibrium.[2] Therefore, my quantitative model suggests that the great Black migration substantially reduced the racial gap in nominal and real wages.

This paper relates to two strands of literature. First, this paper contributes to the literature on the great Black migration and economic geography of African Americans in the US, including Smith and Welch (1989), Gregory (2006), Boustan (2009, 2010, 2017), Black et al. (2015), Chay and Munshi (2015), Derenoncourt (2022), Calderon et al. (2022), Althoff and Reichardt (2022). To the best of my knowledge, this paper is the first to quantify the aggregate, general equilibrium effects of the great Black migration.

Among the papers I have listed, the most related is Boustan (2009). She estimates a constant elasticity of substitution (CES) production function and finds imperfect substitutability between African Americans and white people. She extrapolates the estimated production function to predict what African Americans' wages would have been in the North if the great Black migration did not occur. This paper integrates her idea of imperfect substitutability across races into a quantitative general equilibrium setting. Moreover, due to its overlapping generations structure, the model speaks to the

[2]Smith and Welch (1989) do not discuss the impact of the great Black migration on the racial gap in real wages.

effects of the great migration on not only migrants and their contemporaries, but also later generations. In this sense, this paper relates to Derenoncourt (2022) who studies the intergenerational effects of the great Black migration.

Second, I take advantage of the recent development of quantitative general equilibrium models of the dynamic spatial economy, including Desmet and Rossi-Hansberg (2014), Caliendo et al. (2019), Monras (2020), Ahlfeldt et al. (2021),Kleinman et al. (2022), Allen and Donaldson (2022), Eckert and Peters (2022), Nagy (2022), Pellegrina and Sotelo (2022), and Takeda (2022). Following Allen and Donaldson (2022), Eckert and Peters (2022), Pellegrina and Sotelo (2022), and Takeda (2022), the model has an overlapping generations structure in the spatial economy. My model differs from theirs in that individuals work for more than one period because decennial US census data enable me to keep track of wages, residential places, and migration of cohorts for multiple decades. In comparison to the existing literature, my model allows for different productivity, amenities, migration costs, and survival probabilities across ages and races, delivering heterogeneous migration patterns across ages and races.[3]

The remainder of the paper is organized as follows. Section 1.2 describes motivating facts including the data mentioned in this introduction. Section 1.3 lays out the model. In Section 1.4, I estimate the elasticities and back out the other parameters. Section 1.5 discusses the fit of the model with the data. Section 1.6 compares the baseline equilibrium with counterfactual equilibria. Section 1.7 concludes.

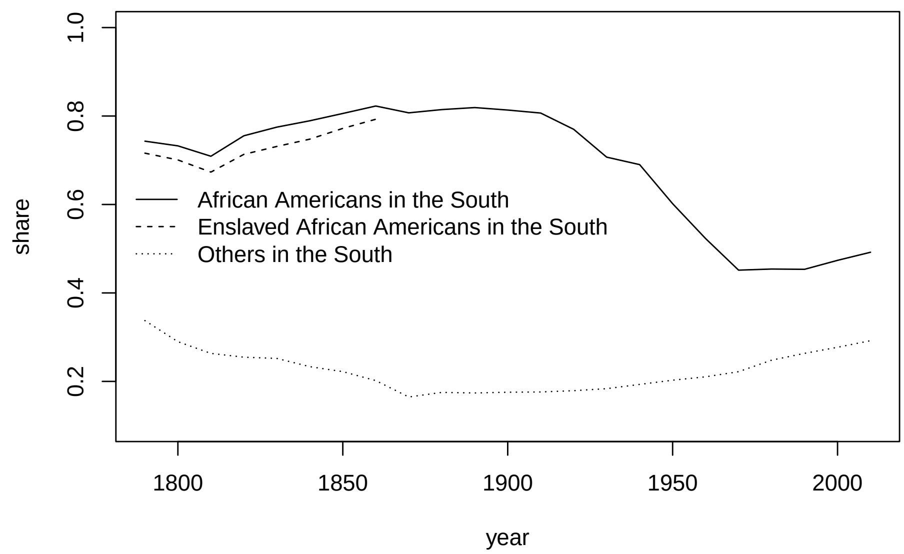
1.2 Motivating Facts

How have African Americans been spatially distributed in the US? Before Emancipation, about 70 to 80 percent of African Americans in the US lived in the South,[4] as the solid line in Figure 1.1 shows. The dashed line shows the fraction of enslaved African Americans in the South out of African Americans in the US. The vast majority of African Americans in the South were enslaved. In the meantime, only about 20 to 30 percent of the people other than African Americans (henceforth, others) in the US resided in the South, as in the dotted line. The fraction of African Americans in the South stayed high around 80 percent even after Emancipation.

[3]Suzuki (2021) integrates heterogeneous migration costs and survival probabilities across ages into the model of Caliendo et al. (2019).

[4]The US refers to the area of the current US states except Alaska and Hawaii (the contiguous US). The South refers to all confederate states. The North refers to the area of the contiguous US except the South.

Figure 1.1. Fractions of Populations in the South for African Americans and Others



Notes: The solid line is the ratio of the number of African Americans in the South to the number of African Americans in the US. The dashed line is the ratio of the number of enslaved African Americans in the South to the number of African Americans in the US. The dotted line is the ratio of the number of the people other than African Americans (others) in the South to the number of others in the US. Source: US census 1940-2000, American Community Survey 2010.

African Americans started leaving the South to the North circa 1910, and the fraction of African Americans in the South dropped from 81 percent to 71 percent between 1910 and 1930. This amounts to the migration of 1.5 million African Americans from the South to the North and is called the first great Black migration. Following the Great Depression, Black migration paused for a decade. The largest migration occurred after the pause. The fraction of African Americans in the South declined from 69 percent to 45 percent between 1940 and 1970. 4 million African Americans left the South to the North in this period. This is called the second great Black migration on which this paper focuses.[5]

To see migration behavior by demographic group, I define movers and stayers for

[5]The second great Black migration is often just referred to as the great Black migration. See footnote 1 of Derenoncourt (2022).

races (African Americans or others), birthplaces (the North or the South), and cohorts.[6] I consider 10-year windows as cohort bins and call those who were born in 1930-1939 as cohort 1930, and so on.[7] For each cohort c, I collect the individuals who lived in either the North or the South as of year $c + 50$. Then for each race, birthplace, and cohort c,

- movers are the individuals who lived in the other place than the birthplace as of year $c + 50$,
- stayers are the individuals who lived in the birthplace as of year $c + 50$.

For each race r, birthplace p, and cohort c, I compute

$$\frac{\text{movers}_{r,p,c}}{\text{movers}_{r,p,c} + \text{stayers}_{r,p,c}}, \tag{1.1}$$

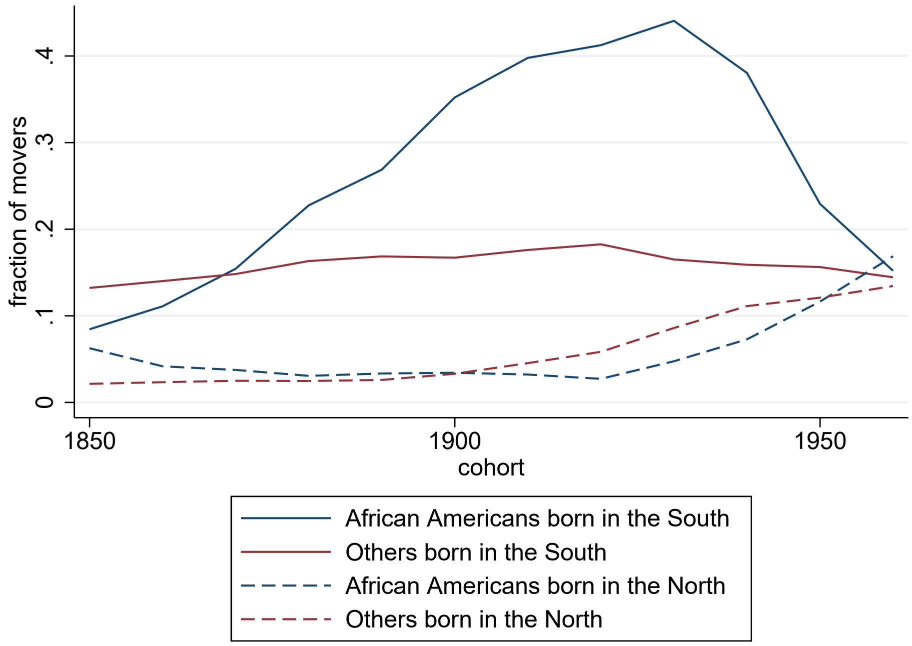
where $\text{movers}_{r,p,c}$ and $\text{stayers}_{r,p,c}$ denote the numbers of movers and stayers for race r, birthplace p, and cohort c, respectively. I call the ratio (1.1) as the fraction of movers.

The migration patterns of African Americans from the South were different from the migration patterns of other groups of people. Figure 1.2 provides the fractions of movers for African Americans and others born in the North or the South. The fraction of movers for African Americans born in the South exhibits remarkable changes over time. It steadily increased from cohort 1850 and peaked at 0.44 in cohort 1930. That is, over 40 percent of African Americans born in the South in the 1930s moved to the North by 1980. After that, the fraction of movers for African Americans born in the South sharply declined to 0.15. The trajectory of the fraction of movers for African Americans born in the South highlights different migration behavior across cohorts within the race-birthplace bin. This motivates the model with a notion of cohorts in Section 1.3. The trajectory of the fraction of movers for African Americans born in the South is clearly different from the trajectory of others born in the South, which was always around 0.15. This suggests that African Americans and others had different economic incentive for the migration from the South to the North. The fraction of movers for African Americans and others born in the North exhibit similar patterns. The fraction of movers for them was stable and less than 0.05 from cohort 1860 to cohort 1910. After that, the fraction of movers increased and reached 0.15 and 0.13 for African Americans and others in cohort 1960, respectively. The recent net migration from the North to

[6]See Figure 1 of Black et al. (2015) and Chapter 1 of Boustan (2017) for earlier tabulation of the great Black migration by cohort.

[7]Formally, I refer to the individuals who were born in the period from year c to year $c + 9$ as cohort c for calendar year c whose 4th digit is zero.

Figure 1.2. Fractions of Movers for Races, Cohorts, and Birthplaces



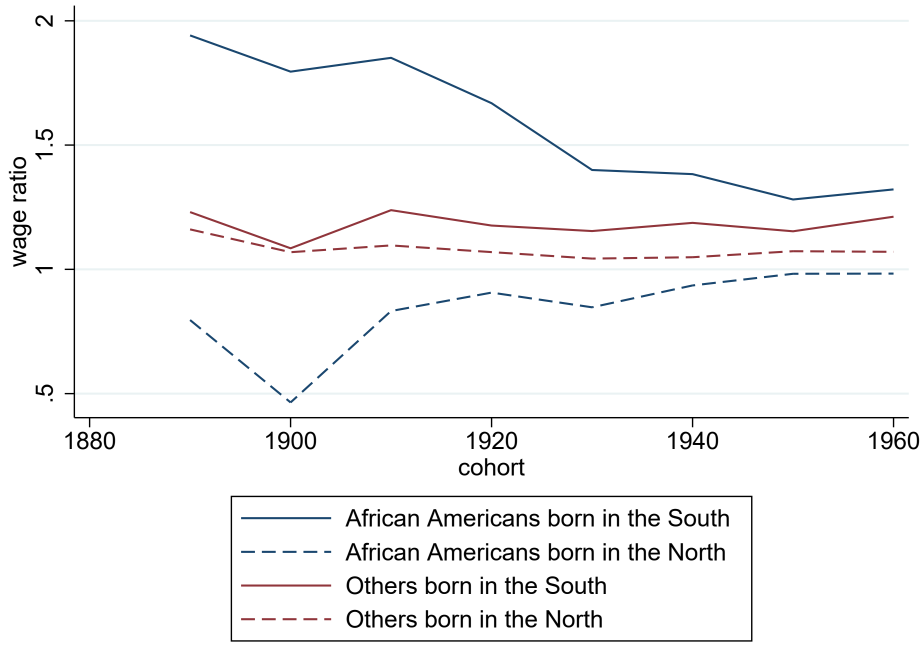
Notes: Cohort 1850 refers to those who were born in 1850-1859, and so on. For each cohort (say c), race, and birthplace (the North or the South) tuple, the fraction of movers is the ratio of the number of movers to the sum of the numbers of movers and stayers. Source: US census 1900-2000, American Community Survey 2010.

the South is called the reverse great migration. But its magnitude is smaller than the magnitude of the original great migration.

In the great migration period, African Americans who moved from the South to the North earned much higher nominal wages than African Americans who stayed put in the South. To see this, I compute a measure I call the mover-stayer wage ratio. For each cohort c, race, and birthplace, I compute the ratio of the average wage of movers to the average wage of stayers as of year $c + 50$.[8] Figure 1.3 provides mover-stayer wage ratios for race-birthplace tuples. As in the blue solid line, African Americans who moved from the South to the North earned 79 to 94 percent higher wages than African Americans who stayed in the South from cohort 1890 to cohort 1910. This wage differential is extremely high compared with the other race-cohort-birthplace tuples. Since cohort

[8]The average wage means the average of the wages of all individuals who earn positive wages for each race, birthplace, and cohort tuple. In Appendix A.1, Figure A.1 reports the mover-stayer ratios of per capita payrolls and yields a similar result.

Figure 1.3. Mover-Stayer Wage Ratios for Cohorts, Races, and Birthplaces



Notes: For each cohort (say c), race, and birthplace (the North or the South), this graph provides the ratio of the average wage of movers to the average wage of stayers as of year $x + 50$. Source: US census 1940-2000, American Community Survey 2010.

1910, the mover-stayer wage ratio for African Americans born in the South declined and reached 1.39 in cohort 1930. That is, on average, African Americans who moved from the South to the North earned 39 percent higher wages than African Americans who stayed in the South at the peak of the great migration. Since then, the mover-stayer wage ratio for African Americans born in the South moderately declined. As the blue dashed line shows, African Americans who moved from the North to the South earned 17 to 54 percent lower wages than African Americans who stayed in the North from cohort 1890 to cohort 1910. The mover-stayer ratio for African Americans born in the North increased after cohort 1910 and is 0.98 in cohort 1960. As in the red solid and dashed lines, the mover-stayer wage ratios for others were relatively stable over time: 1.08 to 1.24 for others born in the South and 1.04 to 1.16 for others born in the North.

Housing rent in the North was higher than housing rent in the South, but the rent gap absorbed only a small part of the mover-stayer wage gap for African Americans from the South. For each cohort c, the first row of Table 1.1 shows

$$\text{movers' wage} - \text{stayers' wage}.$$

for African Americans' born in the South. Similarly, the second row of Table 1.1 shows

$$\text{movers' rent} - \text{stayers' rent}.$$

Both of wages and rent are deflated by the consumer price index and measured in 2010 US dollars. For cohort 1890, the magnitude of the rent gap was comparable to the magnitude of the wage gap. But for cohorts 1920-1940, the rent gaps were only about one-fourth of the wage gaps. Therefore, at the peak of the great Black migration, the rent gap between the North and the South was unlikely to absorb the mover-stayer wage gaps for African Americans born in the South.[9]

Table 1.1. Wage and Rent Gaps between Movers and Stayers for African Americans Born in the South

cohort	1890	1900	1910	1920	1930	1940	1950	1960
wage gaps	5,479	7,232	10,222	12,839	9,826	11,543	9,714	10,789
rent gaps	2,978	2,365	2,644	2,875	2,303	2,987	2,183	2,765

Notes: For cohort c, the first row refers to the average wage of movers minus the average wage of stayers as of year $c + 50$ for African Americans born in the South. Analogously, for cohort c. the second row refers to the average rent of movers minus the average rent of stayers as of year $c + 50$ for African Americans born in the South. Wages and rent are deflated by the consumer price index and measured in 2010 US dollars.

I summarize the empirical facts I have described so far to four points.

1. The migration rate of African Americans from the South in the great migration period was higher than the migration rate of people of the other race-birthplace-cohort tuples.

2. African Americans who moved from the South to the North in the great migration period earned much higher wages than African Americans who stayed put in the South.

[9] Wages are defined for individuals, but rent is defined for households. So I match housing rent with the household head's race, cohort, birthplace, and current place bins. If a household has multiple wage-earners, the mover-stayer rent gap relative to the mover-stayer wage gap can be even smaller at household levels.

3. The mover-stayer wage gap for African Americans from the South in the great migration period was higher than the mover-stayer wage gap for people of the other race-birthplace-cohort tuples.

4. The mover-stayer rent gap accounted for only about one fourth of the mover-stayer wage gap for African Americans from the South in the great migration period.

1.3 Model

I develop a dynamic general equilibrium model that delivers different migration patterns across races and cohorts over time. Individuals of different races and cohorts migrate, taking into account the future flows of real wages and non-pecuniary amenities in potential destinations, and migration costs across locations.

1.3.1 Environment

The economy consists of a finite set of locations $\mathcal{N}$. Let $N = |\mathcal{N}|$, that is, N is the number of locations. Time is discrete and denoted by $t = 0, 1, \cdots$. Goods are perishable in each period. Individuals cannot save their income.

Individuals are characterized by race r, age a, and location i in period t. The set of races is $\{b, o\}$, where b denotes African Americans, and o denotes others. The set of ages is $\{0, 1, \cdots, \bar{a}\}$, where $\bar{a} > 0$ denotes the age of the oldest group in each period. Individuals can live through at most age $\bar{a}$, but they may die before age $\bar{a}$ due to exogenous survival probabilities. Specifically, individuals of race r and age a in period t can survive to period $t+1$ with probability $s_{r,a,t}$. Note that the maximum periods of life is $\bar{a}+1$.

I can trace trajectories of individuals' behavior by cohort. Individuals of cohort c are born in period c. If all relevant survival probabilities are strictly greater than 0,[10] some of them survive up to period $c+\bar{a}$. Individuals of cohort c are age 0 in period c, age 1 in period $c+1$, $\cdots$, age $\bar{a}$ in period $c+\bar{a}$. Thus tracing behavior of individuals of these age-period pairs pins down the life course of cohort c.

Individuals' only source of income is their wages. They supply a fixed length of work hours in each period and earn the market wage (no intensive margin of the labor supply). Individuals of age 0 do not work. Individuals of ages $1, \cdots, \bar{a}$ work.

[10] Specifically, $s_{r,a,c+a} > 0$ for any $a = 0, 1, \cdots, \bar{a}-1$.

1.3.2 Period Utility

The period utility of individuals of race r and age a in period t and location i, $u^i_{r,a,t}$, is

$$u^i_{r,a,t} = \begin{cases} 0 & \text{for } a = 0, \\ \log C^i_{r,a,t} + \log B^i_{r,a,t} & \text{for } a = 1, \cdots, \bar{a}, \end{cases} \tag{1.2}$$

where $C^i_{r,a,t}$ is the consumption of individuals of race r, age a in period t and location i (henceforth individuals of (r, a, t, i)), and $B^i_{r,a,t}$ is the exogenous parameter of the amenites for individuals of (r, a, t, i).

For age $a = 1, \cdots, \bar{a}$, workers of (r, a, t, i) consume the Cobb-Douglas composite of homogeneous goods and housing

$$C^i_{r,a,t} = \left(\frac{G^i_{r,a,t}}{1-\gamma}\right)^{1-\gamma} \left(\frac{H^i_{r,a,t}}{\gamma}\right)^{\gamma}, \tag{1.3}$$

where $G^i_{r,a,t}$ and $H^i_{r,a,t}$ are the consumption of homogeneous goods and housing by the individuals of (r, a, t, i), and γ is the exogenous parameter for the expenditure share on housing. Homogeneous goods are freely tradeable across locations. Housing is not tradeable across locations. Homogeneous goods are the numeraire in each period. Let r^i_t be the unit rent in location i and period t. Then for age $a = 1, \cdots, \bar{a}$, individuals of (r, a, t, i) are subject to the following budget constraint

$$G^i_{r,a,t} + r^i_t H^i_{r,a,t} \leq w^i_{r,a,t}, \tag{1.4}$$

where $w^i_{r,a,t}$ is the nominal wage of the individuals of (r, a, t, i). Since the amenities $B^i_{r,a,t}$ are exogenous, the maximization of the period utility (1.2) (or, equivalently, (1.3)) subject to the budget constraint (1.4) yields the demand functions for homogeneous goods and housing service $G^i_{r,a,t} = (1-\gamma)w^i_{r,a,t}$ and $H^i_{r,a,t} = \gamma w^i_{r,a,t}/r^i_t$. Substituting these demands into the composite (1.3), the consumption level is equalized to the real wage

$$C^i_{r,a,t} = \frac{w_{r,a,t}}{(r^i_t)^\gamma}.$$

Substituting this into the period utility yields the indirect period utility

$$\bar{u}^i_{r,a,t} = \begin{cases} 0 & \text{for } a = 0, \\ \log\left(\frac{w_{r,a,t}}{(r^i_t)^\gamma}\right) + \log B^i_{r,a,t} & \text{for } a = 1, \cdots, \bar{a}. \end{cases}$$

1.3.3 Values

Individuals of age 0 to $\bar{a}-1$ make migration decisions, and arrive in destinations next period. Individuals of age $\bar{a}$ do not make migration decisions because they are not alive next period. The value of individuals of (r,a,t,i), $v^i_{r,a,t}$, is

$$v^i_{r,a,t} = \begin{cases} \bar{u}^i_{r,a,t} + \max_{j\in\mathcal{N}} \left\{ s_{r,a,t} E[v^j_{r,a+1,t+1}] - \tau^{j,i}_{r,a,t} + \nu \epsilon^j_{r,a,t} \right\} & \text{for } a = 0,\cdots,\bar{a}-1, \\ \bar{u}^i_{r,a,t} & \text{for } a = \bar{a}. \end{cases}$$

where $\tau^{j,i}_{r,a,t}$ is the migration cost for individuals of race r and age a in period t from location i to location j, $\epsilon^j_{r,a,t}$ is the idiosyncratic preference shock, and ν adjusts the variance of the idiosyncratic preference shock. The expectation is taken over the next period's idiosyncratic preference shocks $\epsilon^k_{r,a+1,t+1}$ for $k \in \mathcal{N}$.

Assume that the idiosyncratic preference shock $\epsilon^j_{r,a,t}$ is independently and identically distributed Type-I extreme value distribution $F(x) = \exp(-\exp(x))$ across all infinitesimal individuals. Then the expected value of workers of (r,a,t,i), $V^i_{r,a,t} = E[v^i_{r,a,t}]$, is

$$V^i_{r,a,t} = \begin{cases} \bar{u}^i_{r,a,t} + \nu \log \left(\sum_{j\in\mathcal{N}} \exp(s_{r,a,t} V^j_{r,a+1,t+1} - \tau^{j,i}_{r,a,t})^{1/\nu} \right) & \text{for } a = 0,\ldots,\bar{a}-1, \\ \bar{u}^i_{r,a,t} & \text{for } a = \bar{a}. \end{cases} \tag{1.5}$$

1.3.4 Migration

For age $a = 0,\cdots,\bar{a}-1$, the fraction of individuals of (r,a,t,i) who migrate to location j, $\mu^{j,i}_{r,a,t}$, is

$$\mu^{j,i}_{r,a,t} = \frac{\exp(s_{r,a,t} V^j_{r,a+1,t+1} - \tau^{j,i}_{r,a,t})^{1/\nu}}{\sum_{k\in\mathcal{N}} \exp\left(s_{r,a,t} V^k_{r,a+1,t+1} - \tau^{k,i}_{r,a,t}\right)^{1/\nu}}. \tag{1.6}$$

I call $\mu^{j,i}_{r,a,t}$ the migration share.

1.3.5 Populations

Then for $a = 1,\cdots,\bar{a}$, the population of (r,a,t,i) is

$$L^i_{r,a,t} = \sum_{j\in\mathcal{N}} \mu^{j,i}_{r,a-1,t-1} s_{r,a-1,t-1} L^j_{r,a-1,t} + I^i_{r,a,t}, \tag{1.7}$$

where $I^i_{r,a,t}$ denotes the number of immigrants of race r and age a who arrive from abroad in location i in period t. Individuals of age 0 are born according to

$$L^i_{r,0,t} = \sum_{a=1,\cdots,\bar{a}} \alpha_{r,a,t} L^i_{r,a,t}, \tag{1.8}$$

where, as before, the second subscript of $L^i_{r,0,t}$ denotes age (0), and $\alpha_{r,a,t}$ denotes the exogenous parameter of how many individuals of age 0 are born per person of race r and age a in period t.

1.3.6 Firms and Wages

A representative firm exists in each location. The firm sells homogeneous goods in the competitive goods market and hires individuals of various races and ages from its location. The production function of the firm in location i is

$$Y^i_t = A^i_t L^i_t, \tag{1.9}$$

where A^i_t is the parameter of the productivity in location i and period t, and L^i_t is the labor input in location i and period t. L^i_t has a nested CES structure. At the outer nest, L^i_t aggregates labor of different age groups within period t and location i

$$L^i_t = \left(\sum_{a=1}^{\bar{a}} (\kappa^i_{a,t})^{\frac{1}{\sigma_a}} (L^i_{a,t})^{\frac{\sigma_a - 1}{\sigma_a}} \right)^{\frac{\sigma_a}{\sigma_a - 1}}, \tag{1.10}$$

where $\kappa^i_{a,t}$ is the parameter of the productivity of individuals of age a in period t and location i, and σ_a is the parameter of the elasticity of substitution across age groups within location-period bins. Then $L^i_{a,t}$, in turn, aggregates labor of different racial groups within age a, period t and location i

$$L^i_{a,t} = \left(\sum_{r \in \{b,o\}} (\kappa^i_{r,a,t})^{\frac{1}{\sigma_r}} (L^i_{r,a,t})^{\frac{\sigma_r - 1}{\sigma_r}} \right)^{\frac{\sigma_r}{\sigma_r - 1}}, \tag{1.11}$$

where $\kappa^i_{r,a,t}$ is the parameter of the productivity of individuals of race r and age a in period t and location i, and σ_r is the parameter of the elasticity of substitution across races within age-period-location bins. This production function is similar to, but different from Boustan (2009). She controls for education, but I do not. She considers one representative producer in the entire North (which is her only geographic location), whereas I consider different producers in different geographic locations.

The firm in location i solves the following profit maximization problem

$$\max_{\{L^i_{r,a,t}\}_{r,a}} A^i_t L^i_t - \sum_a \sum_r w^i_{r,a,t} L^i_{r,a,t}.$$

The first-order conditions imply that wages are priced at the marginal product of labor

$$\begin{aligned} w^i_{r,a,t} &= A^i_t \frac{\partial L^i_t}{\partial L^i_{a,t}} \frac{\partial L^i_{a,t}}{\partial L^i_{r,a,t}} \\ &= A^i_t (L^i_t)^{\frac{1}{\sigma_a}} (\kappa^i_{a,t})^{\frac{1}{\sigma_a}} (L^i_{a,t})^{-\frac{1}{\sigma_a}+\frac{1}{\sigma_r}} (\kappa^i_{r,a,t})^{\frac{1}{\sigma_r}} (L^i_{r,a,t})^{-\frac{1}{\sigma_r}}. \end{aligned} \tag{1.12}$$

Note that migration decisions are made one period ahead, so L^i_t, $L^i_{a,t}$, $L^i_{r,a,t}$ are all predetermined from the viewpoint in period t.

1.3.7 Rent

Let H^i_t be the quantity of housing in location i and period t. Then the housing market clearing condition is

$$r^i_t H^i_t = \gamma \sum_{r\in\{b,o\}} \sum_{a=1}^{\bar{a}} L^i_{r,a,t} w^i_{r,a,t}, \tag{1.13}$$

where the right-hand side is the Cobb-Douglas expenditure share on housing service γ multiplied by the total income in location i and period t. Therefore, the right-hand side is the total housing expenditure in location i and period t. I assume that the quantity of housing service H^i_t is determined by the housing supply function

$$H^i_t = \frac{1}{\bar{r}^i} \left(\gamma \sum_{r\in\{b,o\}} \sum_{a=1}^{\bar{a}} L^i_{r,a,t} w^i_{r,a,t} \right)^{1-\eta},$$

where the inverse of $\bar{r}^i$ is the exogenous time-invariant and location-specific housing supply shifter, and η is the exogenous parameter governing the elasticity of housing service with respect to local housing expenditure. Substituting this into (1.13) yields

$$r^i_t = \bar{r}^i \left(\gamma \sum_{r\in\{b,o\}} \sum_{a=1}^{\bar{a}} L^i_{r,a,t} w^i_{r,a,t} \right)^{\eta}. \tag{1.14}$$

Rent r^i_t is decomposed into $\bar{r}^i$ and the power function of the local housing expenditure. I call $\bar{r}^i$ the location-specific rent shifter. Because $\eta = d\log r^i_t / d\log \left(\sum_{r\in\{b,o\}} \sum_{a=1}^{\bar{a}} L^i_{r,a,t} w^i_{r,a,t} \right)$

holds, I call η as the rent elasticity with respect to local income, or simply the rent elasticity.

1.3.8 Equilibrium and Steady State

Now I am equipped with all equilibrium conditions.

Equilibrium. Given populations in period 0 $\{L^i_{r,a,0}\}^i_{r,a}$, an equilibrium is a tuple of expected values $\{V^i_{r,a,t}\}^i_{r,a,t=0,1,\cdots}$, wages $\{w^i_{r,a,t}\}^i_{r,a,t=0,1,\cdots}$, populations $\{L^i_{r,a,t}\}^i_{r,a,t=1,2,\cdots}$, migration shares $\{\mu^{j,i}_{r,a,t}\}^{j,i}_{r,a,t=0,1,\cdots}$, housing rent $\{r^i_t\}^i_{t=0,1,\cdots}$ that satisfies (1.5), (1.6), (1.7), (1.8), (1.12), and (1.14).

I compute transition paths to steady states, given the initial populations of all demographic groups in all locations. For this purpose, I characterize steady states.

Steady state. A steady state is a tuple of time-invariant variables: expected values $\{V^i_{r,a}\}^i_{r,a}$, wages $\{w^i_{r,a}\}^i_{r,a}$, populations $\{L^i_{r,a}\}^i_{r,a}$, migration shares $\{\mu^{j,i}_{r,a}\}^{j,i}_{r,a}$, housing rent $\{r^i\}^i$ satisfying (1.5), (1.6), (1.7), (1.8), (1.12), and (1.14), dropping time subscripts in all equations.

1.4 Quantification

I load the parameter values from 1940 to 2010 into the model. The geographic units are 36 states including all confederate and border states, the District of Columbia, and the constructed rest of the North. In total, there are 38 locations in the sample. The rest of the North aggregates states with less than 5,000 Black population as of 1940.[11] The rest of the North accounts for 0.1 and 1 percent of the Black population in the US as of 1940 and 2010, respectively. One period is ten years.

I estimate a set of parameters that I call elasticities: migration elasticity $1/\nu$, elasticities of substitution σ_a and σ_r, and rent elasticity η. The model delivers explicit formulae mapping elasticities and relevant data to amenities, migration costs, productivity, and location-specific rent shifters. Specifically, with relevant data, migration elasticity $1/\nu$ pins down amenities $B^i_{r,a,t}$ and migration costs $\tau^{j,i}_{r,a,t}$. Elasticities of substitution σ_a and σ_r pin down productivity A^i_t, $\kappa^i_{a,t}$, $\kappa^i_{r,a,t}$. Rent elasticity η pins down location-specific rent

[11]The rest of the North consists of Idaho, Maine, Montana, Nevada, New Hampshire, New Mexico, North Dakota, Oregon, South Dakota, Utah, Vermont, and Wyoming.

shifters $\bar{r}^i$. Subsection 1.4.4 touches on survival probabilities, fertility, and immigrants from abroad. As in Ahlfeldt et al. (2015), I set the Cobb-Douglas share on housing $\gamma = 0.25$ following Davis and Ortalo-Magné (2011). Recall that the mover-stayer rent gap was about one-fourth of the mover-stayer wage gap in Section 1.2.

The main data source is the US censuses from 1940 to 2000 and the American Community Survey (ACS) from 2001 to 2019, both of which are tabulated in IPUMS (Ruggles et al., 2022; Manson et al., 2022). Migration shares, populations, wages, and fertility (babies per person) for race and age bins in states and time periods are from these data. Median rent across states from 1940 to 2010 is published by the US Census Bureau or IPUMS. All prices (wages and rent) are deflated by the consumer price index and measured in the 2010 US dollars. I use payrolls per capita as wages and head counts as populations for race, age, location, and time tuples. See Appendix A.2 for further details on the data sources.

Table 1.2. Age Bins in the Model and in the Data

model	0	1	$\cdots$	$\bar{a} = 6$
data	1-10	11-20	$\cdots$	61-70

Notes: The first row lists age bins used in the model. The second row lists the corresponding age bins in the data.

Since US censuses are decennial, one period in the model corresponds to ten years in the data. Accordingly, age bins in the model correspond to 10-year windows as in Table 1.2. In the quantification of the model, ages run from 0 to 6, so the maximum length of life is 7 periods. Age 0 in the model corresponds to ages 1 to 10 in the data, $\cdots$, age $\bar{a} = 6$ in the model corresponds to ages 61 to 70 in the data.

1.4.1 Migration Elasticity, Migration Costs, and Amenities

I estimate migration elasticity $1/\nu$, following the two-step estimation developed by Artuc and McLaren (2015). Artuc and McLaren (2015) used this method to study sectoral and occupational choices by workers, and Caliendo et al. (2021) applied it to the context of migration.

Define the option value of race r, age a, period t (hereafter (r, a, t)), in location j, $\Omega^j_{r,a,t}$, by

$$\Omega^j_{r,a,t} = \nu \log \left(\sum_{k \in \mathcal{N}} \exp(s_{r,a,t} V^k_{r,a+1,t+1} - \tau^{k,j}_{r,a,t})^{1/\nu} \right).$$

Then the expected value of (r, a, t) in location j (1.5) is rewritten as

$$V^j_{r,a,t} = \bar{u}^j_{r,a,t} + \Omega^j_{r,a,t}. \tag{1.15}$$

The migration share of (r, a, t) from location i to j (1.6) is also rewritten as

$$\mu^{j,i}_{r,a,t} = \exp\left\{\frac{1}{\nu}(s_{r,a,t}V^j_{r,a+1,t+1} - \tau^{j,i}_{r,a,t}) - \frac{1}{\nu}\Omega^i_{r,a,t}\right\} \tag{1.16}$$

Multiplying both sides by the population of (r, a, t) in origin location i, $L^i_{r,a,t}$, I have the number of migrants

$$L^i_{r,a,t}\mu^{j,i}_{r,a,t} = \exp\left\{\frac{1}{\nu}(s_{r,a,t}V^j_{r,a+1,t+1} - \tau^{j,i}_{r,a,t}) - \frac{1}{\nu}\Omega^i_{r,a,t} + \log(L^i_{r,a,t})\right\}$$

I decompose the number of migrants into destination fixed effects $\upsilon^j_{r,a,t}$, origin fixed effects $\omega^i_{r,a,t}$, and the remaining variation $\tilde{\tau}^{j,i}_{r,a,t}$ by race r, age a, and period t

$$L^i_{r,a,t}\mu^{j,i}_{r,a,t} = \exp\{\upsilon^j_{r,a,t} + \omega^i_{r,a,t} + \tilde{\tau}^{j,i}_{r,a,t}\}. \tag{1.17}$$

Comparing equations (1.16) and (1.17) yields

$$\upsilon^j_{r,a,t} = \frac{1}{\nu}s_{r,a,t}V^j_{r,a+1,t+1}, \tag{1.18}$$

$$\omega^i_{r,a,t} = -\frac{1}{\nu}\Omega^i_{r,a,t} + \log(L^i_{r,a,t}), \tag{1.19}$$

$$\tilde{\tau}^{j,i}_{r,a,t} = -\frac{1}{\nu}\tau^{j,i}_{r,a,t}. \tag{1.20}$$

Note that $\upsilon^j_{r,a,t}$, $\omega^i_{r,a,t}$, and $\tilde{\tau}^{j,i}_{r,a,t}$ capture expected values, option value, and migration costs, respectively. Equations (1.15), (1.18), and (1.19) imply

$$\begin{aligned}\frac{\upsilon^j_{r,a,t}}{s_{r,a,t}} + \omega^j_{r,a+1,t+1} - \log(L^j_{r,a+1,t+1}) &= \frac{1}{\nu}\bar{u}^j_{r,a,t} \\ &= \frac{1}{\nu}\left\{\log\left(\frac{w^j_{r,a+1,t+1}}{(r^j_{t+1})^\gamma}\right) + \log(B^j_{r,a+1,t+1})\right\}.\end{aligned} \tag{1.21}$$

That is, regressing migration shares on origin and destination fixed effects recovers period utilities for each (r, a, t) in location j.

Guided by the derivations so far, I implement two step estimation. Following equation

(1.17), in the first step, I run the following regression

$$L^i_{r,a,t}\mu^{j,i}_{r,a,t} = \exp\left\{v^j_{r,a,t} + \omega^i_{r,a,t} + \tilde{\tau}^{j\neq i}_t + \tilde{\tau}^{\{i,j\}}_{r,G(t)} + \tilde{\tau}^{\{i,j\}}_{a,G(t)}\right\} + \epsilon^{j,i}_{r,a,t}. \tag{1.22}$$

For race r, age a, and period t, $v^j_{r,a,t}$ is the destination fixed effect, and $\omega^i_{r,a,t}$ is the origin fixed effect. The terms $\tilde{\tau}$ with various subscripts and superscripts capture migration costs. $\tilde{\tau}^{j\neq i}_t$ denotes the fixed effect for year t and moving, that is, destination j is a different location from origin i. The sample years are decennial: $1930, 1940, \cdots, 2000, 2010$.[12] In the subscripts of $\tilde{\tau}^{\{i,j\}}_{r,G(t)}$ and $\tilde{\tau}^{\{i,j\}}_{a,G(t)}$, function $G(\cdot)$ groups years as in Table 1.3. I call partitions of years defined by G as year groups. In the superscripts of $\tilde{\tau}^{\{i,j\}}_{r,G(t)}$ and $\tilde{\tau}^{\{i,j\}}_{a,G(t)}$, $\{i,j\}$ represents the unordered pair of locations i and j.[13] Thus $\tilde{\tau}^{\{i,j\}}_{r,G(t)}$ is the race $\times$ year group $\times$ location pair fixed effect, and $\tilde{\tau}^{\{i,j\}}_{a,G(t)}$ is the age $\times$ year group $\times$ location pair fixed effect. Finally, $\epsilon^{j,i}_{r,a,t}$ is the error term. Notice that I assume symmetric migration costs.

Since one period is 10 years, migration shares in the regression (1.22) must be of 10-year windows. Migration is, however, reported in 1- or 5-year windows in the US censuses and ACS. Appendix A.3 details how I map 1- or 5-year migration in the data to 10-year migration in the quantification of the model.

Table 1.3. Grouping Sample Years

year	1930	1940	1950	1960	1970	1980	1990	2000	2010
group	1		2		3		4		

Notes: This table defines function G used in equation (1.22).

Suppose that I obtained estimates of the destination fixed effects $\hat{v}^j_{r,a,t}$ and the origin fixed effects $\hat{\omega}^i_{r,a,t}$ by race r, age a, ant time t from the first step. In the second step, guided by equation (1.21), I run the regression (whose result is in column (3) in Table 1.4)

$$\frac{\hat{v}^j_{r,a,t}}{s_{r,a,t}} + \hat{\omega}^j_{r,a+1,t+1} - \log(L^j_{r,a+1,t+1}) = \frac{1}{\nu}\log(w^j_{r,a+1,t+1}) + \tilde{B}^j_{r,a+1} + \tilde{B}^j_{r,t+1} + \epsilon^j_{r,a,t}, \tag{1.23}$$

where $\log(w^j_{r,a+1,t+1})$ is the log of the nominal wage of individuals of $(r, a+1, t+1)$ in

[12] The migration shares in 1930 are necessary to compute amenities in 1940. See time subscripts in equation (1.21).

[13] More precisely, for locations $i \neq j$, I assume $\tilde{\tau}^{\{i,i\}}_{r,G(t)} = \tilde{\tau}^{\{j,j\}}_{r,G(t)}$ and $\tilde{\tau}^{\{i,i\}}_{a,G(t)} = \tilde{\tau}^{\{j,j\}}_{a,G(t)}$. Thus I have the fixed effects for staying and all unordered pairs of different locations.

location j, $\tilde{B}^j_{r,a+1}$ is the race × age × location fixed effect, $\tilde{B}^j_{r,t+1}$ is the race × year × location fixed effect, and $\epsilon^j_{r,a,t}$ is the error term. $\tilde{B}^j_{r,a+1}$ and $\tilde{B}^j_{r,t+1}$ are to control for the amenities $B^j_{r,a+1,t+1}$. Since rent r^j_{t+1} in equation (1.21) varies at location-time levels, it is absorbed by the race × year × location fixed effect $\tilde{B}^j_{r,t+1}$.

Table 1.4. Migration Elasticity

Dependent variable:	period utility × migration elasticity		
	(1)	(2)	(3)
log(real wage)	0.4976***	0.6129***	0.7676***
	(0.1323)	(0.1665)	(0.1952)
fixed effects:			
race-location	✓	✓	✓
age-location	✓	✓	✓
year-location	✓	✓	✓
age-race	✓	✓	✓
year-race	✓	✓	✓
age-race-location		✓	✓
year-race-location			✓
Observations	2,660	2,660	2,660

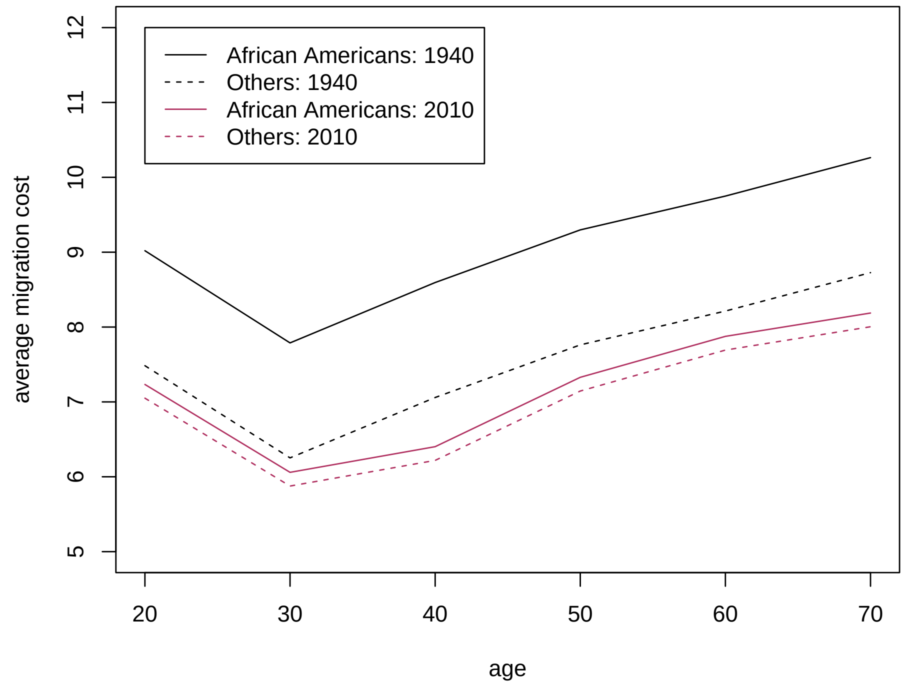
Notes: The second step of the migration elasticity estimation. Dependent variables are constructed of the estimates from the first step and represent period utilities multiplied by the migration elasticity. Units of observations are year-age-race-location tuples. Robust standard errors clustered at locations are in parentheses. Significance codes: ***: 0.01, **: 0.05, *: 0.1.

Nominal wages are directly from the data. Following Artuc and McLaren (2015), I instrument the log of the nominal wage for individuals of $(r, a+1, t+1)$ in location j $\log(w^j_{r,a+1,t+1})$ by the log of the nominal wage for individuals of $(r, a+1, t)$ in location j $\log(w^j_{r,a+1,t})$ (the lagged instrumental variable). Since one period is ten years, I instrument the nominal wage of each race-age-year-location quadruple by the nominal wage of the same race-age-location triple but ten years before.

Table 1.4 reports the results of the second step. Column (3) reports the result of the specification (1.23), and the other columns report the results of the specifications with fewer fixed effects. The estimates for the migration elasticity range from 0.50 to 0.77, which are between 0.5 estimated by Caliendo et al. (2021) for EU countries and 2.0 estimated by Suzuki (2021) for Japanese prefectures. I use 0.77 as the value of the migration elasticity.

Given $\hat{\nu}$, I can back out migration costs. Recall that in the first step (1.22), I have

Figure 1.4. Average Migration Costs for Races and Ages: 1940 and 2010



Notes: For races, ages, and years, I compute the averages of the induced migration costs across location pairs. The migration costs are induced by the estimate of the migration elasticity and the fixed effects in equation (1.22).

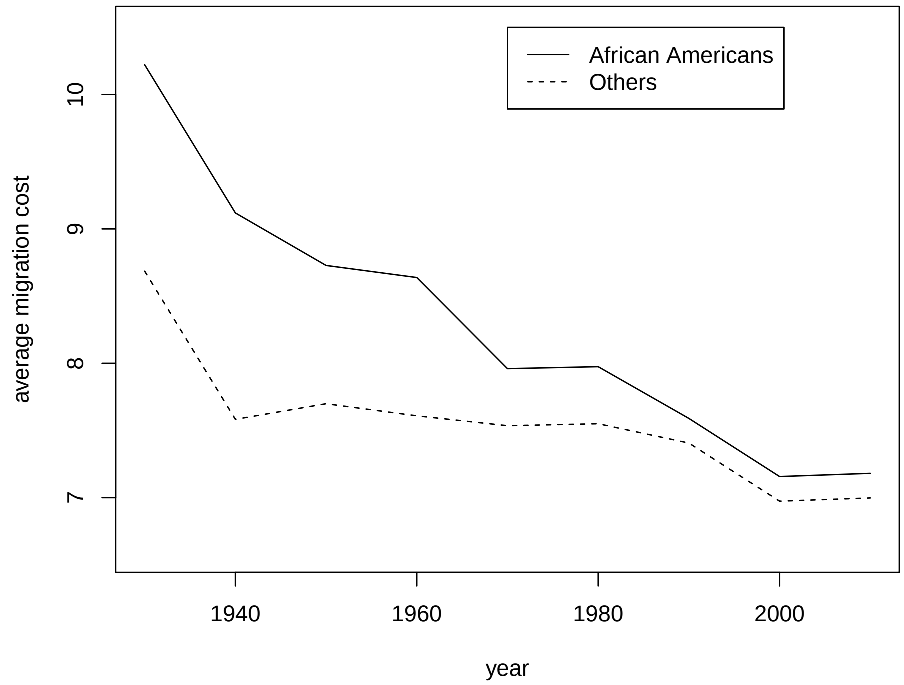
estimated the fixed effects $\tilde{\tau}_t^{j\neq i}$, $\tilde{\tau}_{r,G(t)}^{\{i,j\}}$, and $\tilde{\tau}_{a,G(t)}^{\{i,j\}}$. Let $\hat{\tilde{\tau}}_t^{j\neq i}$, $\hat{\tilde{\tau}}_{r,G(t)}^{\{i,j\}}$, and $\hat{\tilde{\tau}}_{a,G(t)}^{\{i,j\}}$ be the estimates of these fixed effects. By equation (1.20), I obtain migration costs induced by these estimates of the fixed effects,

$$\hat{\tau}_{r,a,t}^{j,i} = -\hat{\nu}\left(\hat{\tilde{\tau}}_t^{j\neq i} + \hat{\tilde{\tau}}_{r,G(t)}^{\{i,j\}} + \hat{\tilde{\tau}}_{a,G(t)}^{\{i,j\}}\right).$$

Figure 1.4 shows the averages of the induced migration costs for races and ages. 20 in the horizontal axis refers to age bin 11-20, and so on.[14] The migration costs are the lowest for people of the ages of 21-30. Migration costs increase after the ages 21-30. The migration costs in 2010 increase with ages less steeply than the migration costs in 1940 do. This is perhaps because seniors are more physically mobile or infrastructure is better

[14]Age $x \in \{20, \cdots, 70\}$ on the horizontal axis refers to ages from $x-9$ to x.

Figure 1.5. Average Migration Costs for Races and Years



Notes: For races and years, I compute the averages of the induced migration costs across (ordered) location pairs and ages. The migration costs are induced by the estimate of the migration elasticity and the fixed effects in equation (1.22).

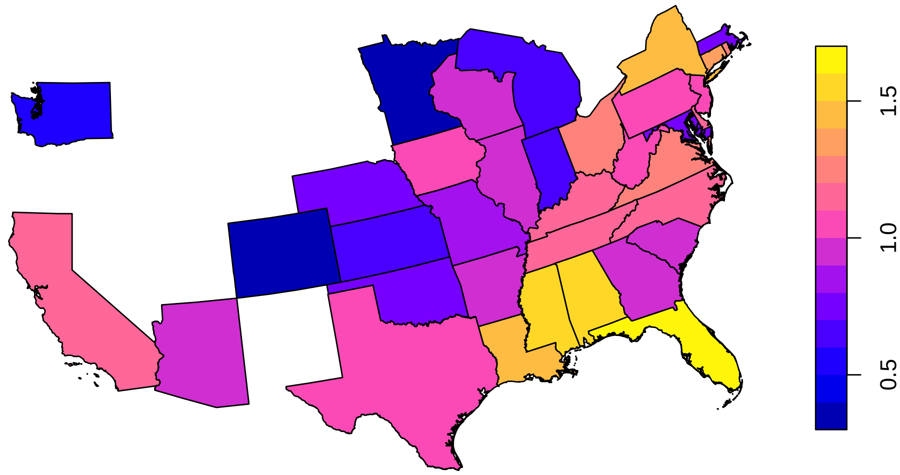
in 2010. African Americans faced higher migration costs than others in 1940, but the racial gap in migration costs shrank by 2010.

Figure 1.5 illustrates the averages of the induced migration costs for races and years. The migration costs of African Americans were always higher than those of others. The migration costs, however, have steadily declined over the sample periods, particularly for African Americans. The racial gap in migration costs declined over time.

Using $\hat{\nu}$ and the fixed effects estimated in the second step (1.23), I back out amenities. Let $\hat{\tilde{B}}^j_{r,a+1}$ and $\hat{\tilde{B}}^j_{r,t+1}$ be the estimates of the fixed effects in the second step (1.23). Then by comparing equations (1.21) and (1.23), the induced amenities, $\hat{B}_{r,a,t}$, are

$$\hat{B}^j_{r,a,t} = \exp\left\{\hat{\nu}\left(\hat{\tilde{B}}^j_{r,a} + \hat{\tilde{B}}^j_{r,t}\right) + \gamma \log(r^j_t)\right\}, \tag{1.24}$$

Figure 1.6. Amenities for African Americans in 1960



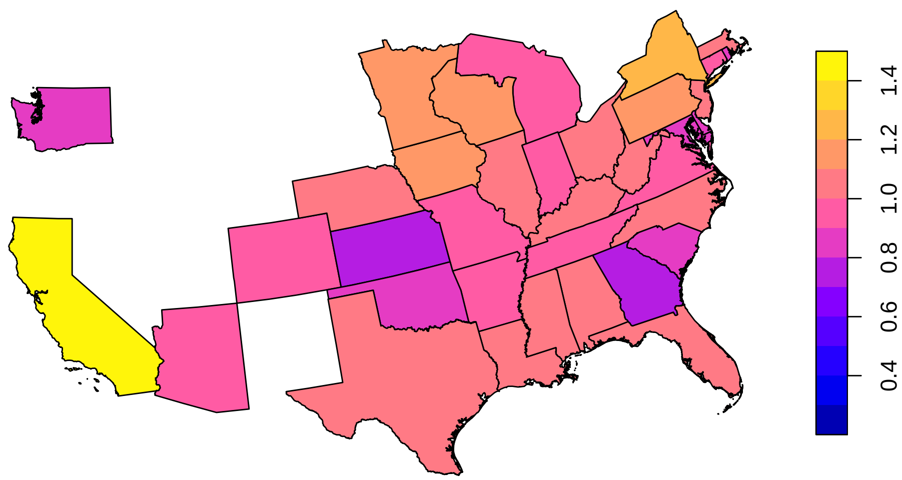
Notes: The amenities for African Americans in 1960 averaged across ages. The amenities are induced by equation (1.24). The rest of the North is excluded from the map.

where rent r_t^j is directly from the data, and I set $\gamma = 0.25$. I normalize amenities $\{\hat{B}^j_{r,a,t}\}^j$ so that the mean of $\{\hat{B}^j_{r,a,t}\}^j$ is 1 for each (r, a, t). In the model, migration decisions are made in period t foreseeing real wages and amenities in period $t+1$. I have the data on wages and rent from 1940 to 2019. So I can compute the induced amenities for the years 1950 to 2010 in this way. The reason that I cannot obtain the amenities in 1940 is that I do not have the data on wages as of 1930, so I do not have the lagged instrumental variable for wages. For 1940, I directly back out the amenities using equation (1.21)

$$\hat{B}^j_{r,a,1940} = \exp\left\{\nu\left(\frac{\hat{\nu}^j_{r,a-1,1930}}{s_{r,a-1,1930}} + \hat{\omega}^j_{r,a,1940} - \log(L_{r,a,1940})\right)\right\} \Big/ \left\{\frac{w^j_{r,a,1940}}{(r^j_{1940})^\gamma}\right\}.$$

Recall that in Table 1.2, the age 6 in the model is the highest age and corresponds to the ages 61 to 70 in the data. I do not include the age 7 (71-80 year olds) in the sample because including the age 7 in the estimation makes estimates of the migration elasticity unstable. Since the origin fixed effect for the age 7 $\hat{\omega}^j_{r,7,t+1}$ is needed to induce the amenities for the age 6 (see (1.23)), I cannot obtain the amenities for the age 6

Figure 1.7. Amenities for Others in 1960



Notes: The amenities for others in 1960 averaged across ages. The amenities are induced by equation (1.24). The rest of the North is excluded from the map.

by using (1.23) and (1.24). I assume that within race r, location i, and period t, the amenities for the age 6 (61-70 year olds) are the same as the amenities for the age 5 (51-60 year olds). I use the estimates for the amenities for the age 5 for the amenities for the age 6.

Figures 1.6 and 1.7 show the induced amenities for African Americans and others for states in 1960 averaged across the age bins from 1 (11-20 year olds) to 5 (51-60 year olds). The peak of the great migration was in the 1950s, and in the model, individuals make migration decisions in 1950 foreseeing real wages and amenities from 1960 onward. This is why I pick up the year 1960. The amenities of the rest of the North are not in the figures, although they are assigned in the quantification of the model. As in Figure 1.6, the induced amenities for African Americans were high in states in the South such as Florida, Alabama, Mississippi, and Louisiana in 1960. In contrast, states in the North such as Michigan, Minnesota, Kansas, Colorado, and Washington had low amenities for African Americans then. Figure 1.7 shows a different geographic pattern of amenities for others. California provided high amenities for others, but there is not a clear North-South pattern in the amenities for others.

1.4.2 Elasticity of Substitution and Productivity

1.4.2.1 Races

I turn to the estimation of the elasticities of substitution. I start with the elasticity of substitution across races within age, location, and time bins σ_r. Equation (1.12) implies

$$\frac{w^n_{b,a,t}}{w^n_{o,a,t}} = \frac{(\kappa^n_{b,a,t})^{\frac{1}{\sigma_r}} (L^n_{b,a,t})^{-\frac{1}{\sigma_r}}}{(\kappa^n_{o,a,t})^{\frac{1}{\sigma_r}} (L^n_{o,a,t})^{-\frac{1}{\sigma_r}}}, \tag{1.25}$$

where I recall that the first subscripts b and o denote African Americans and others respectively. Taking logs of both sides,

$$\log\left(\frac{w^n_{b,a,t}}{w^n_{o,a,t}}\right) = -\frac{1}{\sigma_r}\log\left(\frac{L^n_{b,a,t}}{L^n_{o,a,t}}\right) + \frac{1}{\sigma_r}\log\left(\frac{\kappa^n_{b,a,t}}{\kappa^n_{o,a,t}}\right). \tag{1.26}$$

Since productivity ratio between races $\kappa^n_{b,a,t}/\kappa^n_{o,a,t}$ is not observable in data, my main econometric specification is

$$\log\left(\frac{w^n_{b,a,t}}{w^n_{o,a,t}}\right) = -\frac{1}{\sigma_r}\log\left(\frac{L^n_{b,a,t}}{L^n_{o,a,t}}\right) + f_a + f_t + f_{a,t} + \epsilon^n_{a,t}, \tag{1.27}$$

where f_a denotes the age fixed effect, f_t denotes the time fixed effect, $f_{a,t}$ denotes the age $\times$ time fixed effect, and $\epsilon^n_{a,t}$ is the error term. Notice that age-time fixed effects $f_{a,t}$ capture cohorts. For earlier cohorts, the education gap between African Americans and others was larger. As the education gap is a reason for the productivity gap between races, controlling for cohorts is important in the regression (1.27).

In his seminal work, Borjas (2003) considers the nationwide labor market. But, here I consider different locations in the US as different labor markets. Suppose that African Americans migrate to a location where their productivity is high relative to others within age bins. Then productivity ratio $\kappa^n_{b,a,t}/\kappa^n_{o,a,t}$ is positively correlated with population ratio $L^n_{b,a,t}/L^n_{o,a,t}$, which causes an upward bias for the estimator of $-1/\sigma_r$ in ordinary least squares (OLS). Note that the concern here is that productivity ratio $\kappa^n_{b,a,t}/\kappa^n_{o,a,t}$ may work as a pull factor of migration.

To deal with this potential bias, I follow Card (2009). In Appendix A.4, I pursue a different approach following the first difference estimation of Monras (2020). Here I consider two instrumental variables. The first one is the ratio of shift-share predicted populations. The shift-share predicted population of race r and age a in period t and location n is

$$\hat{L}^n_{r,a,t} = \sum_{j\in\mathcal{N}} \mu^{n,j}_{r,a-1,t-1-X} \cdot s_{r,a-1,t-1} L^j_{r,a-1,t-1}. \tag{1.28}$$

If $X = 1$, this equation would be the same as equation (1.7), omitting immigrants. But I use the value of $X > 1$. That is, I interact the current (actually one period before) survival probability and populations with the old-time migration shares to make shift-share predicted populations. As in Goldsmith-Pinkham et al. (2020), the assumption for identification is that the error term $\epsilon^n_{a,t}$ is mean-independent from the old-time migration shares $\{\mu^{n,j}_{r,a-1,t-1-X}\}_{j\in\mathcal{N}}$

$$E[\epsilon^n_{a,t} \mid \{\mu^{n,j}_{r,a-1,t-1-X}\}_{j\in\mathcal{N}}] = 0.$$

This is satisfied if the old-time migration shares $\mu^{n,j}_{r,a-1,t-1-X}$ do not react to shocks to the current productivity ratio between races $\kappa^n_{b,a,t}/\kappa^n_{o,a,t}$. Here I am teasing out the push factor of migration because I am extracting variation in $L^n_{b,a,t}/L^n_{o,a,t}$ that is orthogonal to the pull factor of migration $\kappa^n_{b,a,t}/\kappa^n_{o,a,t}$. The relevance (correlation with the actual population ratio) of this IV hinges on the so-called network effect of migration; migrants tend to go to a destination to which their precursors went.

Table 1.5. Elasticity of Substitution across Races: Level Estimation

Dependent variable:	$\log(w^n_{b,a,t}/w^n_{o,a,t})$		
Model:	OLS	IV 1	IV 2
$\log(L^n_{b,a,t}/L^n_{o,a,t})$	-0.1154***	-0.1108***	-0.1120***
	(0.0120)	(0.0127)	(0.0139)
fixed effects:			
year	✓	✓	✓
age	✓	✓	✓
year-age	✓	✓	✓
Observations	1,368	1,368	1,328
First-stage F-statistic		3,112.5	2,107.5

Notes: The results of the level estimation of the elasticity of substitution across races. Block bootstrap standard errors are in parentheses. See Appendix A.5 for the computation of standard errors. Significance codes: ***: 0.01.

I also consider a leave-one-out version of (1.28) as the second IV, removing $\mu^{n,n}_{r,a-1,t-1-X}$.

$s_{r,a-1,t-1}L^n_{r,a-1,t-1}$ from its right-hand side. Then I obtain

$$\hat{L}^{n,-n}_{r,a,t} = \sum_{j \neq n} \mu^{n,j}_{r,a-1,t-1-X} \cdot s_{r,a-1,t-1} L^j_{r,a-1,t-1}. \tag{1.29}$$

The economic interpretation for this is shift-share predicted gross inflows because the right-hand side collects inflows of people from all locations but n itself.

For either IV, I set $X = 2$. Since 1 period is 10 years, I use the migration shares 20 years before period $t-1$.

Table 1.5 provides the estimation results.[15] The first column shows the result of OLS. The second and third columns show the results of two-step least squares using the first and second IVs (1.28) and (1.29), respectively. The OLS and the two IV estimations produce similar estimates around -0.11. From columns 1, 2, and 3, let the OLS estimate, the first IV estimate, and the second IV estimate for σ_r be $\hat{\sigma}^{OLS}_r$, $\hat{\sigma}^{IV1}_r$, $\hat{\sigma}^{IV2}_r$, respectively. Then

$$\begin{aligned} \hat{\sigma}^{OLS}_r &= 1/0.1154 = 8.67, \\ \hat{\sigma}^{IV1}_r &= 1/0.1108 = 9.02, \\ \hat{\sigma}^{IV2}_r &= 1/0.1120 = 8.93. \end{aligned}$$

In the quantification of the model, I use $\hat{\sigma}^{IV1}_r$ as the value of the elasticity of substitution across races.

How do my estimates for the elasticity of substitution across races compare with estimates in the literature? The estimates of the elasticity of substitution across races range from 8.7 to 9.0. In Appendix A.4, the first difference estimation produces the estimate of 4.9. Boustan (2009) estimates the elasticity of substitution across races within education-experience bins in the entire US North. Her preferred values range from 8.3 to 11.1 and coincide with my estimates here. Her paper also includes an estimate of 5.4, which is somewhat similar to my estimate from the first difference estimation.

Given the estimate of the elasticity of substitution across races $\hat{\sigma}_r$, I can back out race-specific productivity $\kappa^n_{r,a,t}$. Rearranging equation (1.25), I obtain

$$\frac{\hat{\kappa}^n_{b,a,t}}{\hat{\kappa}^n_{o,a,t}} = \left(\frac{w^n_{b,a,t}}{w^n_{o,a,t}}\right)^{\hat{\sigma}_r} \cdot \left(\frac{L^n_{b,a,t}}{L^n_{o,a,t}}\right). \tag{1.30}$$

Since wages $w^n_{r,a,t}$ and populations $L^n_{r,a,t}$ are directly observable for $r = b, o$, I can back

[15] Appendix A.5 details the computation of standard errors.

out productivity ratio between African Americans and others $\hat{\kappa}^n_{b,a,t}/\hat{\kappa}^n_{o,a,t}$. Comparing equations (1.10) and (1.11), multiplying all $\kappa^n_{r,a,t}$ ($r = b, o$) by scalar $x > 0$ is equivalent to multiplying $\kappa^n_{a,t}$ by $x^{(\sigma_a-1)/(\sigma_r-1)}$ in the production function. Thus I normalize $\hat{\kappa}^n_{r,a,t}$ for $r = b, o$, so that $\sum_{r=b,o} \hat{\kappa}^n_{r,a,t} = 1$. With equation (1.30), this normalization pins down $\hat{\kappa}^n_{r,a,t}$ for $r = b, o$.

1.4.2.2 Ages

Dual to age-level labor (1.11) in the production function, age-level wages within location-time bins are

$$w^i_{a,t} = \left(\sum_{r'} \kappa^i_{r',a,t} (w^i_{r',a,t})^{1-\sigma_r} \right)^{\frac{1}{1-\sigma_r}}. \tag{1.31}$$

Then I obtain

$$\frac{w^n_{a,t}}{w^n_{a',t}} = \frac{(\kappa^n_{a,t})^{\frac{1}{\sigma_a}} (L^n_{a,t})^{-\frac{1}{\sigma_a}}}{(\kappa^n_{a,t})^{\frac{1}{\sigma_a}} (L^n_{a',t})^{-\frac{1}{\sigma_a}}}. \tag{1.32}$$

Taking logs of both sides of (1.32), I have

$$\log\left(\frac{w^n_{a,t}}{w^n_{a',t}}\right) = -\frac{1}{\sigma_a} \log\left(\frac{L^n_{a,t}}{L^n_{a',t}}\right) + \frac{1}{\sigma_a} \log\left(\frac{\kappa^n_{a,t}}{\kappa^n_{a',t}}\right).$$

Fix an age bin a'. For any age $a \neq a'$, the econometric specification is

$$\log\left(\frac{w^n_{a,t}}{w^n_{a',t}}\right) = -\frac{1}{\sigma_a} \log\left(\frac{L^n_{a,t}}{L^n_{a',t}}\right) + f_a + f_t + f_{a,t} + \epsilon^n_{a,t}, \tag{1.33}$$

where f_a is the age fixed effect, f_t is the time fixed effect, and $f_{a,t}$ is the age × time fixed effect. Note that $w^n_{a,t}$ and $L^n_{a,t}$ are computed using $\hat{\sigma}_r$ and $\hat{\kappa}^n_{r,a,t}$ for $r = b, o$. A concern is that people of an age group migrate to a location where relative productivity of the age group is high, causing positive correlation between population ratios across ages $L^n_{a,t}/L^n_{a',t}$ and productivity ratios across ages $\kappa^n_{a,t}/\kappa^n_{a',t}$.

To deal with this endogeneity concern, I make use of the shift-share predicted populations and gross inflows at race-age-location-time levels in equations (1.28) and (1.29). The first IV is constructed of the aggregates of the shift-share predicted populations

$$\hat{L}^n_{a,t} = \left[\sum_{r' \in \{b,o\}} (\hat{\kappa}^n_{r',a,t})^{\frac{1}{\hat{\sigma}_r}} (\hat{L}^n_{r',a,t})^{\frac{\hat{\sigma}_r - 1}{\hat{\sigma}_r}} \right]^{\frac{\hat{\sigma}_r}{\hat{\sigma}_r - 1}}, \tag{1.34}$$

where $\hat{L}^n_{r',a,t}$ is the shift-share predicted populations defined in equation (1.28). The second IV is constructed of the aggregates of the shift-share predicted gross inflows

$$\hat{L}_{a,t}^{n,-n} = \left[\sum_{r' \in \{b,o\}} (\hat{\kappa}_{r',a,t}^{n})^{\frac{1}{\hat{\sigma}_r}} (\hat{L}_{r',a,t}^{n,-n})^{\frac{\hat{\sigma}_r - 1}{\hat{\sigma}_r}} \right]^{\frac{\hat{\sigma}_r}{\hat{\sigma}_r - 1}}, \tag{1.35}$$

where $\hat{L}_{r',a,t}^{n,-n}$ is the shift-share predicted gross inflows defined in equation (1.29). In either case, I instrument the population ratios across ages $L_{a,t}^{n}/L_{a',t}^{n}$ by the ratios of the aggregates of shift-share predicted populations $\hat{L}_{a,t}^{n}/\hat{L}_{a',t}^{n}$ or gross inflows $\hat{L}_{a,t}^{n,-n}/\hat{L}_{a',t}^{n,-n}$.

Age-level wages $w_{a,t}^{i}$, age-level labor $L_{a,t}^{i}$, and instruments (1.34) and (1.35) are all constructed with the estimate of σ_r and the race-specific productivity. Therefore, to compute standard errors for estimates of σ_a, I need to take into account variability in the estimate of σ_r and the race-specific productivity. I compute block bootstrap standard errors to address this issue. See Appendix A.5 for details.

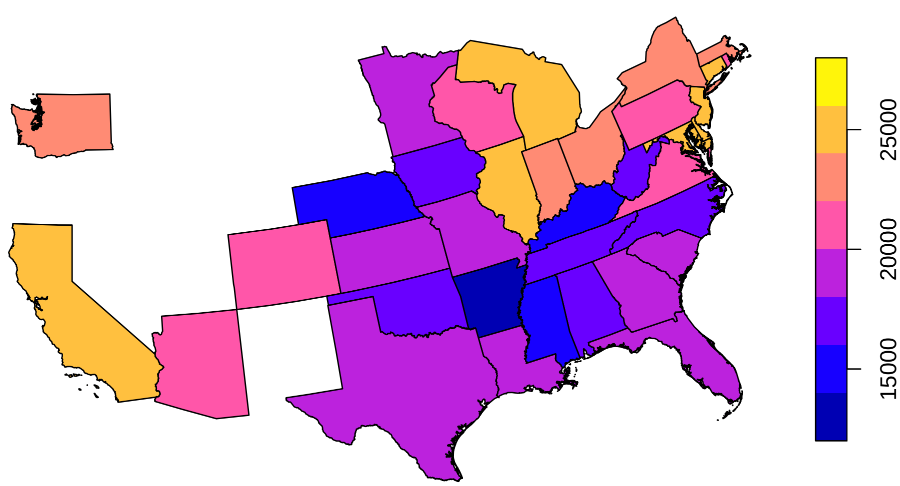
Table 1.6 reports three estimation results for the elasticity of substitution across ages. In the column 1, I use $\hat{\sigma}_r^{OLS}$ and its associated race-specific productivity to compute age-level wages $w_{a,t}^{n}$ and populations $L_{a,t}^{n}$, respectively. Here I use actual race-level populations $L_{r,a,t}^{n}$ to construct $L_{a,t}^{n}$ using equation (1.11). Likewise, in columns 2 and 3, I use $\hat{\sigma}_r^{IV1}$ and $\hat{\sigma}_r^{IV2}$ and the race-specific productivity induced by these two estimates to construct age-level wages $w_{a,t}^{n}$ and populations $L_{a,t}^{n}$. Columns 2 and 3 use the ratios of the aggregates of shift-share predicted populations (1.34) and the ratios of the aggregates of shift-share predicted gross inflows (1.35) as IVs, respectively. The two IVs seem to correct a positive bias in the OLS (column 1). In the quantification of the model, I use $1/0.3401 = 2.94$ as the value of the elasticity of substitution across ages. The induced elasticities of substitution across ages range from 1.8 to 3.5. They are lower than the estimates in the prior literature in labor economics. Card and Lemieux (2001) and Ottaviano and Peri (2012) report estimates of 3.8-4.9 and 3.3-6.3 for the US, respectively; Manacorda et al. (2012) provide estimates of 5.1-5.2 for the UK. There are three differences between these papers and mine. First, they consider the nationwide labor market whereas I consider the state-level labor markets. Second, they control education levels, but I do not. Third, their age bins are five-year windows, whereas mine is of ten years because the wage and population data are from the decennial censuses. My age bins are twice as large as the bins in the literature, and it is possible that substitution across larger age bins exhibits a larger degree of imperfection.

Table 1.6. Elasticity of Substitution across Ages

Dependent variable:	$\log(w^n_{a,t}/w^n_{a',t})$		
Model:	OLS	IV 1	IV 2
$\log(L^n_{a,t}/L^n_{a',t})$	-0.2849***	-0.3401*	-0.5429***
	(0.0672)	(0.1922)	(0.1579)
fixed effects:			
year	✓	✓	✓
age	✓	✓	✓
year-age	✓	✓	✓
Observations	1,520	1,140	1,140
1st-stage F-statistic		1,247.3	186.4

Notes: The estimates of the elasticity of substitution across ages. Block bootstrap standard errors are in parentheses. See Appendix A.5 for the computation of standard errors. Significance codes: ***: 0.01, **: 0.05, *: 0.1.

Figure 1.8. Productivity in 1960



Notes: The induced location-level productivity $\hat{A}^i_t$ in 1960. The rest of the North is excluded from the map.

1.4.2.3 Locations

Dual to equation (1.10), location-time-level wages w_t^i are the aggregate of age-location-time level wages $w_{a,t}^i$

$$w_t^i = \left(\sum_{a'=1}^{\bar{a}} \kappa_{a',t}^i (w_{a',t}^i)^{1-\sigma_a} \right)^{\frac{1}{1-\sigma_a}}.$$

In equilibrium, the representative firm in location i makes zero profit. Thus the revenue equates the cost

$$A_t^i L_t^i = \sum_{a=1}^{\bar{a}} \sum_{r \in \{b,o\}} w_{r,a,t}^i L_{r,a,t}^i. \tag{1.36}$$

By a property of the CES function, I have

$$\sum_{a=1}^{\bar{a}} \sum_{r \in \{b,o\}} w_{r,a,t}^i L_{r,a,t}^i = w_t^i L_t^i. \tag{1.37}$$

Equations (1.36) and (1.37) imply

$$A_t^i = w_t^i.$$

Thus I can back out location-level productivity A_t^i by computing location-level wages w_t^i.

I compute the induced location-level productivity $\hat{A}_t^i$ using $\hat{\sigma}_a^{IV1} = 1/0.3401 = 2.94$ from the column 2 of Table 1.6 and the age-level productivity $\hat{\kappa}_{a,t}^{i,IV1}$ induced by $\hat{\sigma}_a^{IV1}$. Figure 1.8 shows the induced location-level productivity across locations except for the rest of the North in 1960. Along with California, Northern manufacturing states such as Illinois, Michigan, and Ohio had higher productivity. These places were destinations of the great Black migration. In contrast, states in the South such as Arkansas and Mississippi had lower productivity. The two states were typical origins of the great Black migration.

I have backed out migration costs, amenities, and productivity using the formulae implied by the model. A possible story from the induced parameters is as follows. As in Figure 1.5, migration costs were high for African Americans in the great migration period. Figure 1.6 showed states in the North provided lower amenities for African Americans in the great migration period. Despite these impediments or disincentive, African Americans made the journey from the South to the North for higher wages or workplaces of higher productivity as in Figure 1.8.

1.4.3 Rent Elasticity

I estimate the rent elasticity η, which governs how much local rent increases if aggregate local income increases by one percent. Taking logs of both sides in equation (1.14), I have

$$\log r_t^i = \log \bar{r}^i + \eta \log \left(\gamma \sum_{r \in \{b,o\}} \sum_{a=1}^{\bar{a}} L_{r,a,t}^i w_{r,a,t}^i \right). \tag{1.38}$$

Note that $\sum_r \sum_a L_{r,a,t}^i w_{r,a,t}^i$ is the total income in location i in my model. Taking time differences of equation (1.38), I obtain

$$\Delta \log r^i = \eta \Delta \log(\text{income}^i),$$

where income^i is the total income in location i. The location-specific rent shifter $\bar{r}^i$ and the expenditure share on housing γ are washed out by taking time differences.

Table 1.7. Rent Elasticity

Dependent variable:	$\Delta \log r^i$	
Model:	OLS	IV
$\Delta \log(\text{income}^i)$	0.3948***	0.4092***
	(0.0254)	(0.0264)
Observations	38	38
First-stage F-statistic		162.4

Notes: The estimation of the rent elasticity. The regressions are weighted by populations as of 1970. Robust standard errors are in parentheses. Significance code: ***: 0.01.

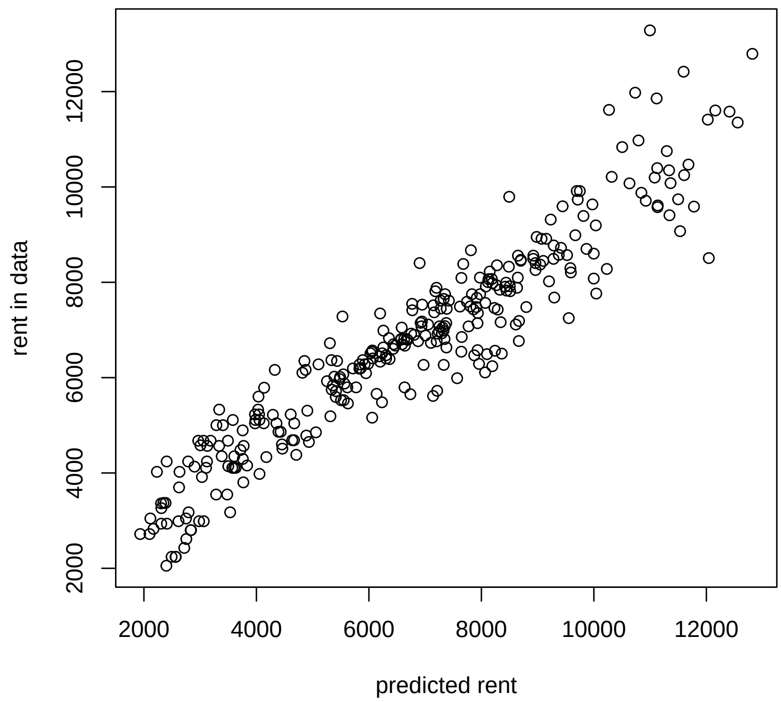
The econometric specification is

$$\Delta \log r^i = \eta \Delta \log(\text{income}^i) + \epsilon_i, \tag{1.39}$$

where ϵ_i is the error term. Time differences are taken between 1970 and 2010. I use median rent for rent in each location.[16] Threat to identification is that an increase in rent may increase local income, causing a positive correlation between the growth rate in local income $\Delta \log(\text{income}^i)$ and the error term ϵ_i. To deal with this threat, I instrument the growth rate in local income $\Delta \log(\text{income}^i)$ by the manufacturing share in employment

[16] For the rest of the North, I take the mean of the median rents across the states within the rest of the North. The reason that I use median rent for the measure of local rent is that it is only the available rent measure for 1940.

Figure 1.9. Predicted and Actual Rent



Notes: Actual rent in the US state data from 1940 to 2010 on the vertical axis against the predicted rent on the horizontal axis. Units of observations are state-time.

and the share of college graduates in population as of 1950.[17] In this IV estimation, I pick up only the variation in the income growth predicted by sectoral and educational composition in the old time. The regression is weighted by populations as of 1970. Table 1.7 reports the result. The OLS and IV estimations produce similar estimates around 0.4.

I back out location-specific rent shifters $\bar{r}^i$ given the estimate of rent elasticity $\hat{\eta} = 0.41$, the expenditure share on housing service $\gamma = 0.25$, and the income data. Rearranging equation (1.14) yields

$$\bar{r}^i = \frac{r_t^i}{\left(\gamma \sum_r \sum_a L_{r,a,t}^i w_{r,a,t}^i\right)^\eta}. \tag{1.40}$$

[17]Glaeser et al. (2005) and Saks (2008) estimate the housing supply elasticity with an IV constructed of old-time sectoral shares.

But if I replace the model objects with data counterparts, the left-hand side and the right-hand side of equation (1.40) cannot perfectly equate because the left-hand side depends on only location i, but the right-hand side depends on both location i and time t. By taking the averages of the numerator and the denominator on the right-hand side over time, I compute the sample counterpart to the location-specific rent shifter $\hat{\bar{r}}^i$

$$\hat{\bar{r}}^i = \frac{\frac{1}{8}\sum_{t=1940}^{2010} r_t^i}{\frac{1}{8}\sum_{t=1940}^{2010}\left(\gamma \cdot \text{income}_t^i\right)^{\hat{\eta}}},$$

where $t = 1940, \cdots, 2010$ runs the sample periods, and income_t^i denotes the total income in location i and period t.

Using the estimates $\hat{\eta}$ and $\{\hat{\bar{r}}^i\}^i$, I can predict rent by

$$\hat{r}_t^i = \hat{\bar{r}}^i \left(\gamma \cdot \text{income}_t^i\right)^{\hat{\eta}}. \tag{1.41}$$

Figure 1.9 plots actual rent in the data against rent predicted by equation (1.41). Predicted rent has a tight and linear relationship with actual rent. The correlation between actual and predicted rent is 0.94.

1.4.4 Fertility, Survival Probabilities, and Immigrants

Fertility. Recall that age 0 in the model corresponds to ages 1 to 10 in the data. I attribute each person of ages 1 to 10 to the parents (in the household including a married couple) or the single parent (in the single parent household) of race-age bins in each period. Averaging the number of children in each race-age bin in each period yields the data counterpart to the number of babies per person $\alpha_{r,a,t}$ for (r, a, t). This procedure is detailed in Appendix A.6.

Survival probabilities. The Centers for Disease Control and Prevention (CDC) publish life tables documented by several different government agencies. I use life tables for 1940, 1950, $\cdots$, 2010. These tables provide the annual survival probabilities for white Americans and African Americans at each age for these sample years. Since periods and age bins are of 10-year windows in the quantification of the model, I map annual survival probabilities for 1-year age bins in the life tables to 10-year survival probabilities for 10-year age bins. This procedure is detailed in Appendix A.7.

Immigrants from abroad. In the model, population dynamics (1.7) take into account immigrants from abroad. Recall that locations in my quantification cover all US states and DC except Alaska and Hawaii. Thus all migrants from outside of the contiguous US are regarded as immigrants from abroad. I tabulate the numbers of immigrants for each race r and age a in period t and location i using the census and ACS data. See Appendix A.8 for details.

1.4.5 Computation of Steady States and Transition Paths

I consider two types of equilibria in Section 1.6. First, I keep all the parameters except fertility as in 1940 to see whether the US economy in 1940 was close to the steady state. I compute another equilibrium replacing parameter values for the first 5 periods from such forever 1940 scenario.[18] I set fertility such that individuals of the age 2 (21 to 30 year olds) have children such that the population size of African Americans and others will be sustained, that is, $\alpha_{r,2,t} = 1/(s_{r,0,1940} \cdot s_{r,1,1940})$. Loading the populations in 1940 as the initial populations, $L^i_{r,a,0} = L^i_{r,a,1940}$, I compute the equilibrium forward. I assume that the economy converges to the steady state in period 105.[19]

Second, I compute the baseline equilibrium that resembles the US economy from 1940 to 2010. I compute counterfactual equilibria, too, replacing a subset of parameters in the baseline equilibrium with hypothetical ones. For the baseline equilibrium, I load all the parameter values from 1940 to 2000. From 2010 onward, I assume that all parameters are as of 2010 with one exception. The exception is, again, fertility. If I use the actual fertility in the 2010 data for fertility from 2010 onward, the US population would shrink forever. To avoid this, I assume that individuals of the age 2 (21 to 30 year olds) have children such that the populations of African Americans and others will be sustained, that is, $\alpha_{r,2,t} = 1/(s_{r,0,2010} \cdot s_{r,1,2010})$. I assume that the economy converges to the steady state in period 107. Period 0 is the year 1940, ..., period 7 is the year 2010. So in the remaining 100 periods, the economy converges to the steady state with time-invariant parameters. Replacing a subset of parameters, I similarly compute counterfactual equilibria.

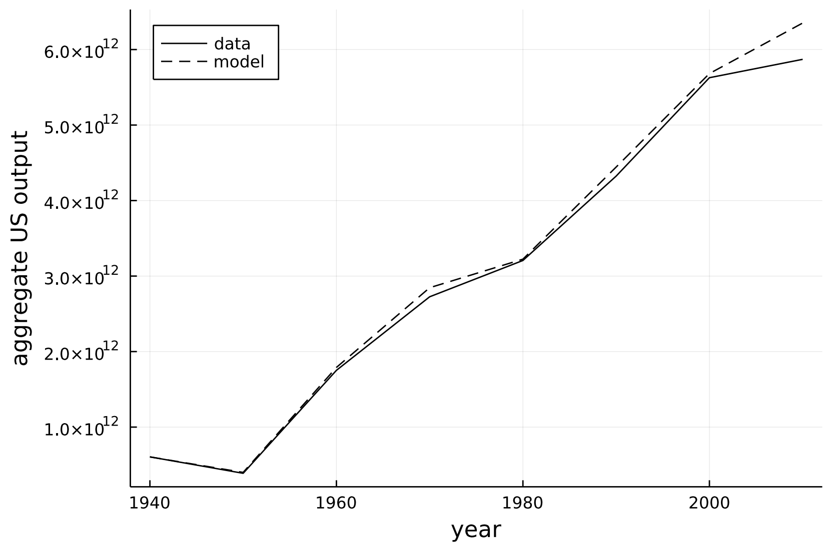
For either type of equilibria, I first compute steady states toward which the economy converges. Then I compute transition paths. Appendix A.9 details the algorithm to compute steady states. Appendix A.10 explains how to compute transition paths toward given steady states.

[18] See Section 1.6 for the purposes of such exercises.
[19] The initial period is period 0, so I have 106 periods.

1.5 Model Fit

I compare variables in the baseline equilibrium of the model with their data counterparts.

Figure 1.10. Aggregate US Output: Model vs Data



Notes: Following (1.42), I plot aggregate US output generated by the baseline equilibrium of the model and its data counterpart.

First, I compare aggregate US output between the baseline equilibrium and the data. Aggregate output in period t, Y_t, is

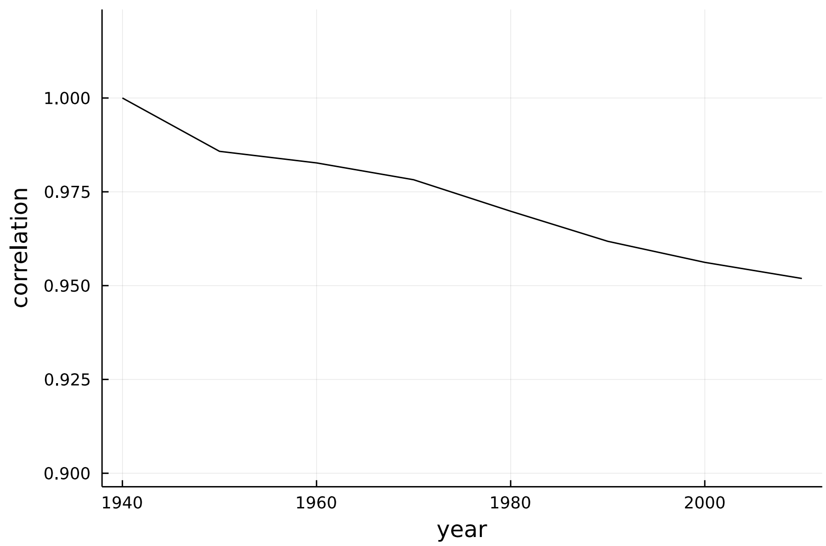
$$Y_t = \sum_{i \in \mathcal{N}} Y_t^i = \sum_{i \in \mathcal{N}} A_t^i L_t^i = \sum_{i \in \mathcal{N}} \sum_{r \in \{b,o\}} \sum_{a=1}^{\bar{a}} w_{r,a,t}^i L_{r,a,t}^i, \tag{1.42}$$

where the second and third equalities follow from equations (1.9) and (1.36), respectively.[20] The right-most object has the data counterpart because it depends on only wages and populations. Figure 1.10 plots aggregate US output (or labor income) in the baseline equilibrium of the model and in the data over time. From 1940 to 2000, the baseline equilibrium closely resembles the data in aggregate output. In 2010, the baseline equilibrium overstates aggregate output by 8 percent.

Second, I compare populations in the baseline equilibrium with those in the data. Pick up any sample year t from $\{1940, \cdots, 2010\}$. For t, let $(L_{r,a,t}^{i,\text{baseline}})_{r,a}^i$ and $(L_{r,a,t}^{i,\text{data}})_{r,a}^i$

[20]Hsieh and Moretti (2019) discuss the impact of spatial misallocation on aggregate output defined similarly. See their equation (7).

Figure 1.11. Correlations between the Populations in the Baseline Equilibrium and in the Data



Notes: For each year, this graph shows the correlation between the population vector in the baseline equilibrium $(L_{r,a,t}^{i,\text{baseline}})_{r,a}^{i}$ and the population vector in the data $(L_{r,a,t}^{i,\text{data}})_{r,a}^{i}$. Since I load the actual population vector in 1940 to the model as the initial population vector, the correlation is one in 1940 by construction.

be the vectors of the populations in the baseline equilibrium and in the data, respectively. Then for each sample year t, I compute the correlation coefficient between these two population vectors $Cor((L_{r,a,t}^{i,\text{baseline}})_{r,a}^{i}, (L_{r,a,t}^{i,\text{data}})_{r,a}^{i})$. Figure 1.11 plots such correlations over time. In 1940, the correlation is one because I load the actual populations in 1940 as the initial population. The correlation between the model and data population vectors declined over time. But, even in 2010, the correlation between the model and data population vectors is over 0.95. Throughout the sample years, the baseline equilibrium captures the spatial distribution of populations fairly well.

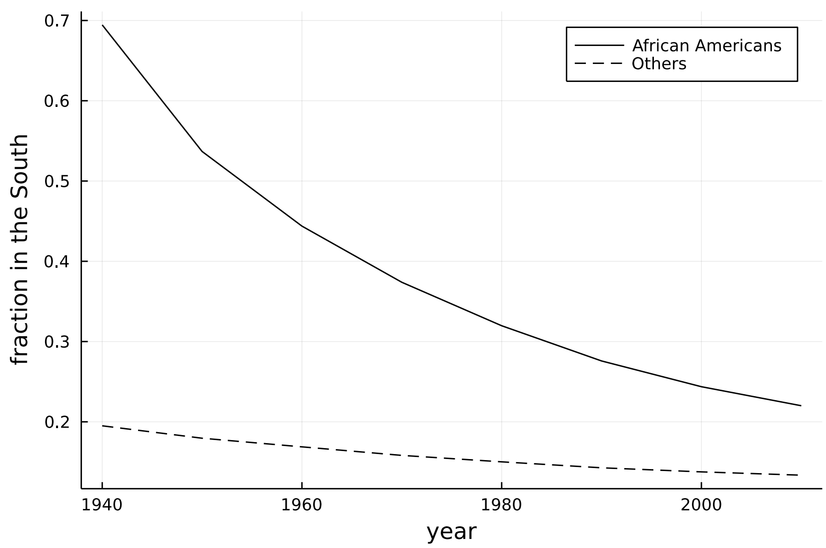
1.6 Counterfactual Results

1.6.1 Forever 1940

First, I consider the counterfactual equilibrium in which all the parameter values except fertility are as of 1940 every period. Fertility is such that individuals of the age 2 (21 to 30 year olds) have children such that the population size will not change for each race

r, $\alpha_{r,2,t} = 1/(s_{r,0,t} \cdot s_{r,1,t})$. I call this equilibrium the forever 1940 equilibrium. Figure 1.12 plots the fractions of African Americans and others in the South in the forever 1940 equilibrium. As the solid line shows, 69 percent of African Americans lived in the South in 1940, but the fraction of African Americans in the South drops to 22 percent by 2010. The fraction of others in the South declines from 20 percent to 13 percent from 1940 to 2010 as in the dashed line. Although the parameter values such as productivity, amenities, and migration costs are constant over time, the spatial distribution of populations change drastically. This suggests that the US economy in 1940 was far from the steady state induced by the parameter values in 1940.

Figure 1.12. Fractions of African Americans and Others in the South: Forever 1940



Notes: The fractions of the populations of African Americans and others in the South over time in the equilibrium in which the parameter values are as of 1940 forever.

In addition to the forever 1940 equilibrium, I consider the equilibrium in which African Americans cannot migrate across the North and the South for 5 periods since 1940. Let $\mathcal{N}_N$ be the set of locations in the North, and $\mathcal{N}_S$ be the set of locations in the South. Then I set $\tau^{j,i}_{b,a,t} = \infty$ for any pair of locations j, i such that $(j, i) \in \mathcal{N}_N \times \mathcal{N}_S$ or $(j, i) \in \mathcal{N}_S \times \mathcal{N}_N$ and $t = 1940, \cdots, 1980$. If individuals make migration decisions in 1980, they arrive in destinations in 1990. Therefore, this shuts down the relocation of individuals until 1990. All the other parameters are as in the forever 1940 equilibrium. For the forever 1940 equilibrium and the equilibrium without the North-South migration of African Americans, Figure 1.13 shows (nationwide) per capita output Y_t/L_t, where

$L_t = \sum_{i \in \mathcal{N}} \sum_{r \in \{b,o\}} \sum_{a=0}^{\bar{a}} L_{r,a,t}^i$. Per capita output is normalized by the initial level in 1940. The solid line shows that even without any change in productivity, amenities, and migration costs, per capita output increases by 11.3 percent by 1990. The increase in per capita output is caused by migration because only dynamic change in the forever 1940 equilibrium is the relocation of individuals. This suggests that there was an opportunity to increase output by relocating the work force in 1940. If African Americans cannot migrate across the North and the South, per capita output increases by 9.7 percent by 1990. Thus the remaining $11.3 - 9.7 = 1.6$ percentage point increase is explained by the migration of African Americans across the North and the South. Putting differently, in the 11.3 percent increase in per capita output, the migration of African Americans across the North and the South accounts for 14 percent of it because

$$\frac{0.113 - 0.097}{0.113} = 0.14.$$

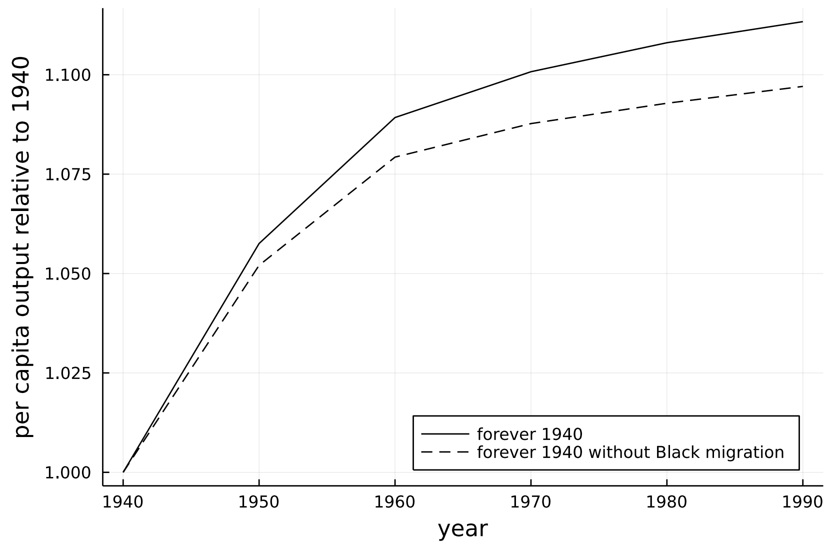
The remaining 86 percent is explained by the migration of others within or across the North and the South and the migration of African Americans within the North and the South. The relocation of African Americans accounts for a substantial part of room for improving aggregate output.

So far I have considered the role of migration, especially of African Americans, in equilibria in which parameters are time-invariant. In the actual US economy, however, all of productivity, amenities, and migration costs changed over time across locations. I turn to equilibria with time-varying parameters.

1.6.2 The Effects of the Great Black (and Others') Migration

I compare the baseline equilibrium that resembles the US economy from 1940 to 2010 with two counterfactual equilibria. In the first counterfactual equilibrium, African Americans cannot migrate across the North and the South from 1940 to 1960. That is, $\tau_{b,a,t}^{j,i} = \infty$ for any pair of locations j, i such that $(j,i) \in \mathcal{N}_N \times \mathcal{N}_S$ or $(j,i) \in \mathcal{N}_S \times \mathcal{N}_N$, any age a, and $t = 1940, \cdots, 1960$. Here I bilaterally shut down the migration of African Americans from the North to the South and from the South to the North. Since migration decisions are made one period ahead of arrival, this shuts down African Americans' relocation until 1970, the end of the great Black migration. I call this equilibrium the equilibrium of African Americans' immobility. In the second counterfactual equilibrium, others cannot migrate across the North and the South from 1940 to 1960. That is, $\tau_{o,a,t}^{j,i} = \infty$ for any pair of locations j, i such that $(j,i) \in \mathcal{N}_N \times \mathcal{N}_S$ or $(j,i) \in \mathcal{N}_S \times \mathcal{N}_N$, any age a, and

Figure 1.13. Per Capita Output in the Forever 1940 Equilibrium and the Equilibrium without Black Migration



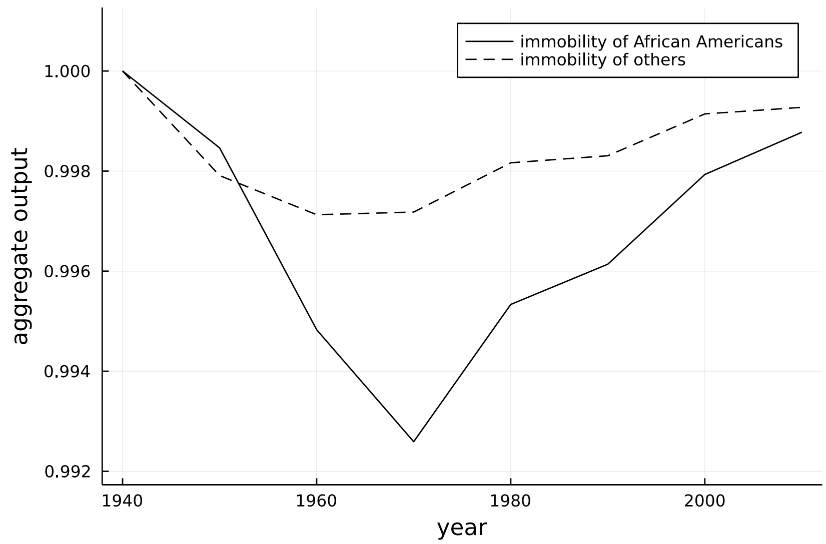
Notes: Per capita output relative to the 1940 level for the forever 1940 equilibrium and the equilibrium in which African Americans cannot migrate across the North and the South from 1940 to 1980.

$t = 1940, \cdots, 1960$. I call this equilibrium the equilibrium of others' immobility.

Figure 1.14 plots aggregate output in the two counterfactual equilibria relative to the baseline equilibrium. In 1970, aggregate output in the equilibrium of African Americans' immobility is 0.74 percent lower than aggregate output in the baseline equilibrium, as in the solid line. The dashed line shows that in the same year, aggregate output in the equilibrium of others' immobility is 0.28 percent lower than aggregate output in the baseline equilibrium. These two results imply that African Americans' relocation across the North and the South increased aggregate output more than others' relocation did, although African Americans accounted for only about 10 percent of the US population. The back-of-the-envelope calculation in Appendix A.11 predicts that if African Americans were spatially distributed as in 1940, aggregate labor income in 1970 would have been lower than actual aggregate labor income in the data by 0.86 percent. Therefore the quantitative model and the back-of-the-envelope calculation yield similar predictions for the aggregate impact of the great Black migration. Figure 1.15 plots aggregate real wages

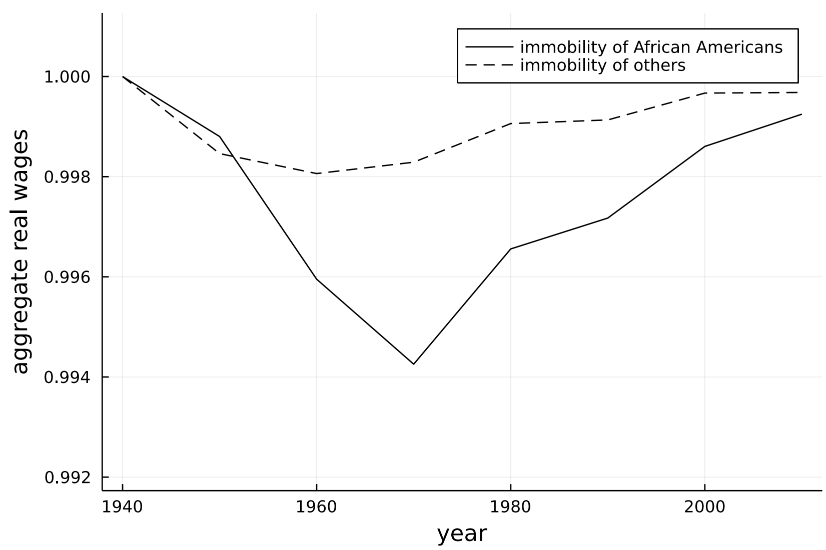
$$\sum_{i \in \mathcal{N}} \sum_{r \in \{b,o\}} \sum_{a=1}^{\bar{a}} L^i_{r,a,t} \frac{w^i_{r,a,t}}{(r^i_t)^\gamma}$$

Figure 1.14. Aggregate Output



Notes: Aggregate output in the equilibria of African Americans' or others' immobility relative to the baseline equilibrium.

Figure 1.15. Aggregate Real Wages



Notes: Aggregate real wages in the equilibria of African Americans' or others' immobility relative to the baseline equilibrium.

in the two counterfactual equilibria relative to the baseline equilibrium. Shutting down the North-South migration of African Americans and others decreases aggregate real wages by 0.63 percent and 0.17 percent, respectively. Shutting down the North-South relocation decreases real wages less than output because higher nominal wages are partly offset by higher housing rent. But the difference between the decrease in aggregate output and the decrease in aggregate real wages is not large.

Table 1.8. Nominal Wage Changes due to the Great Black Migration

		this paper	Boustan (2009)	
			OLS	IV
North	African Americans	0.053	0.096	0.072
	Others	-0.002	-0.005	-0.004
South	African Americans	-0.038	-	-
	Others	0.005	-	-

Notes: Percent changes in nominal wages from the baseline equilibrium to the equilibrium of African Americans' immobility (or the no great Black migration scenario). The result of Boustan (2009) is from her table 6.

What wages would African Americans and others have earned if the great Black migration did not occur? Let region g index the North N or the South S, $g \in \{N, S\}$. Then for $g \in \{N, S\}$, define the average nominal wage of race $r \in \{b, o\}$ in region g and period t by

$$\text{average nominal wage}_{r,t}^{g} = \frac{\sum_{i \in \mathcal{N}_g} \sum_{a=1}^{\bar{a}} L_{r,a,t}^{i} w_{r,a,t}^{i}}{\sum_{i \in \mathcal{N}_g} \sum_{a=1}^{\bar{a}} L_{r,a,t}^{i}}.$$

Let average nominal $\text{wage}_{r,t}^{g}$ and average nominal $\text{wage}_{r,t}^{g,\text{no mig}}$ be such average nominal wages of race r in region g and period t in the baseline equilibrium and in the equilibrium of African Americans' immobility, respectively. Then the percent change in the average nominal wage of race r in region g and period t from the baseline equilibrium to the equilibrium of African Americans' immobility is

$$\frac{\text{average nominal wage}_{r,t}^{g,\text{no mig}}}{\text{average nominal wage}_{r,t}^{g}} - 1.$$

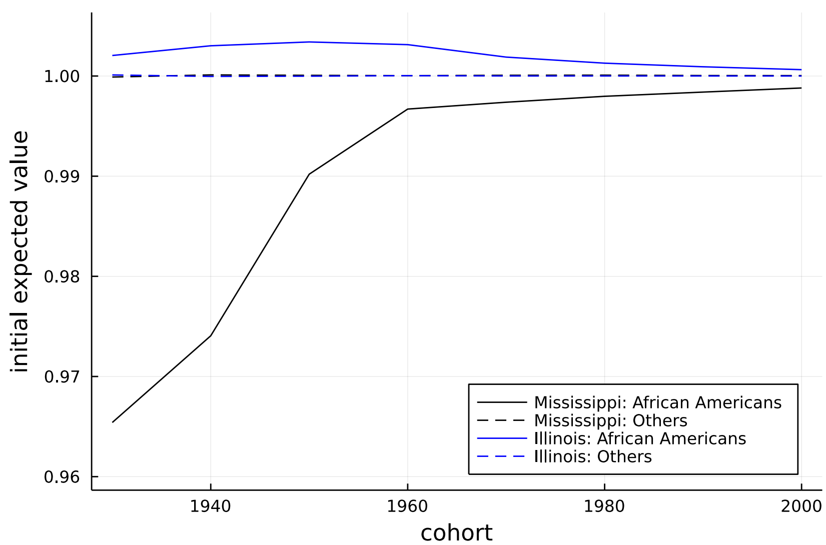
Table 1.8 shows the percent changes in the average nominal wages in its first column. The year is 1970. According to my quantitative model, if the great Black migration did not occur, the average wage of African Americans in the North would have been higher by 5.3 percent, and the average wage of others in the North would have been lower by 0.2 percent. As is common in Boustan (2009) and this paper, African Americans and others are imperfectly substitutable. In the baseline equilibrium, the inflow of African

Americans from the South to the North decreased the wages of African Americans in the North. But in the equilibrium of African Americans' immobility, African Americans in the North would have had fewer competitors in their local labor market than in the baseline equilibrium and would have received higher wages. The second and third columns show the predictions by Boustan (2009). My result for the change in African Americans' wages in the North, 5.3 percent, is a little smaller than her predictions ranging from 7.2 percent to 9.6 percent. My result for the change in others' wages in the North, -0.2 percent, is, again, a little smaller than her predictions, -0.4 or -0.5 percent, in the absolute values. I differ from Boustan (2009) in that I make wage predictions not only in the North but also in the South. If African Americans could not migrate to the North, more African Americans would have remained in the South. The average nominal wage of African Americans in the South would have been lower by 3.8 percent in the no great Black migration scenario than in the baseline. Others' wages in the South would have been higher by 0.5 percent.

I turn to the welfare effects of the migration of African Americans and others across the North and the South. Figure 1.16 plots the initial expected values for each cohort of African Americans and others born in Mississippi and Illinois in the equilibrium of African Americans' immobility relative to the baseline equilibrium. As they lost opportunities of migrating to the high-wage North, in the equilibrium of African Americans' immobility, the expected value of African Americans born in Mississippi in the 1930s would have been 3.5 percent lower than in the baseline equilibrium. Although African Americans born in Illinois lost opportunities of migrating to the South till 1970, in the equilibrium of African Americans' immobility, their welfare for the cohort 1950 would have been 0.34 percent higher than in the baseline equilibrium. The expected values of others in the equilibrium of African Americans' immobility are very similar to the expected values of others in the baseline equilibrium. As in the blue solid line, for African Americans in Illinois, the expected values for the cohorts 1950 and 1960 are higher than the expected values for the cohorts 1930 and 1940 in the equilibrium of African Americans' immobility relative to the baseline equilibrium. This is because the earlier generations cannot move to the South in their youth (till 1970), but the later generations can move to the South in their youth and still benefit from fewer competitors in the labor market. As in the blue solid line, if the great migration did not occur, African Americans in the North would have been better off for generations. This is reminiscent of the result of Derenoncourt (2022), who argues intergenerational negative impacts of the great migration on locations in the North which received relatively large Black migrants from the South.

In Appendix A.12, Figure A.2 shows the expected values of African Americans born in the 1930s in the equilibrium of African Americans' immobility relative to those in the baseline equilibrium across states. In the equilibrium of African Americans' immobility, the expected values of African Americans born in the South would have been lower than in the baseline equilibrium by 1.6 to 3.5 percent, depending on the states. The welfare loss from the baseline equilibrium to the equilibrium of African Americans' immobility is particularly large in South Carolina, Mississippi, and Arkansas. In the equilibrium of African Americans' immobility, the expected values of African Americans born in most states in the North would have been higher than in the baseline equilibrium by less than 1 percent.[21] As in Figure A.3, moving from the baseline equilibrium to the equilibrium of African Americans' immobility does not substantially change the expected values of others born in the 1930s across states.

Figure 1.16. Initial Expected Values: African Americans' Immobility

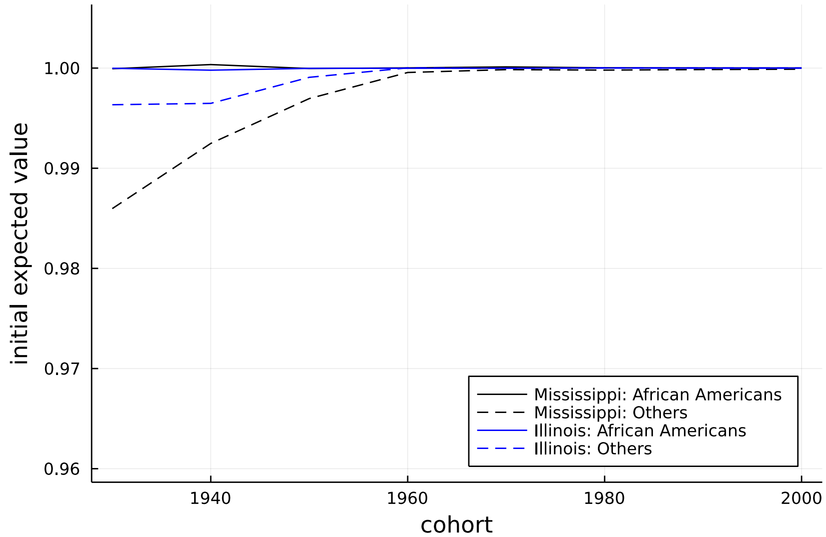


Notes: Initial expected values of African Americans and others born in Mississippi and Illinois in the equilibrium of African Americans' immobility relative to those in the baseline equilibrium.

Figure 1.17 plots the expected values for each cohort of African Americans and others born in Mississippi and Illinois in the equilibrium of others' immobility relative to the

[21] In the equilibrium of African Americans' immobility, the expected values of African Americans born in 5 locations in the North would have been lower than in the baseline equilibrium by less than 1 percent. These 5 locations are DC, Colorado, the Rest of the North, Oklahoma, and Arizona in the order of the magnitude of the welfare loss from the baseline equilibrium to the equilibrium of African Americans' immobility.

Figure 1.17. Initial Expected Values: Others' Immobility



Notes: Initial expected values of African Americans and others born in Mississippi and Illinois in the equilibrium of others' immobility relative to those in the baseline equilibrium.

baseline equilibrium. The expected value of others born in Mississippi in the 1930s in the equilibrium of others' immobility would have been 1.4 percent lower than that in the baseline equilibrium. Recall that in Figure 1.16, for this cohort, the welfare of African Americans born in Mississippi would have been 3.5 percent lower in the equilibrium of African Americans' immobility than in the baseline equilibrium. Thus these two figures jointly highlight African Americans' strong incentive for outmigration from the South. Others in Illinois would also have been worse off by closing the North-South border for others because they lost varieties in location choices. The effects of others' immobility on African Americans' welfare are small.

In Appendix A.12, Figure A.5 shows the expected values of others born in the 1930s in the equilibrium of others' immobility relative to those in the baseline equilibrium across states. In the equilibrium of others' immobility, the expected values of others in the North and the South would have been lower than in the baseline equilibrium by 0.2 to 1.3 percent and 1.2 to 2.8 percent, respectively, depending on the states. As in Figure A.4, moving from the baseline equilibrium to the equilibrium of others' immobility does not substantially change the expected values of African Americans across states.

In the great migration period, the gaps in wages and living standards between African Americans and others shrank. How did the relocation of African Americans across the

Table 1.9. Real Wage Ratios between African Americans and Others

	1940	1950	1960	1970	1980	1990	2000	2010
the baseline equilibrium	0.448	0.576	0.563	0.634	0.677	0.696	0.731	0.717
African Americans' immobility	0.448	0.565	0.522	0.578	0.644	0.669	0.718	0.710

Notes: Nationwide average real wage ratios between African Americans and others in the baseline equilibrium and in the equilibrium of African Americans' immobility.

North and the South contribute to reducing the racial gaps? For period t and race $r \in \{b, o\}$, I define the (nationwide) average real wage by

$$\text{average real wage}_{r,t} = \frac{\sum_{i \in \mathcal{N}} \sum_{a=1}^{\bar{a}} L^i_{r,a,t} \left(\frac{w^i_{r,a,t}}{(r^i_t)^\gamma} \right)}{\sum_{i \in \mathcal{N}} \sum_{a=1}^{\bar{a}} L^i_{r,a,t}}.$$

Then I compute the ratio of average real wages between African American and others

$$\frac{\text{average real wage}_{b,t}}{\text{average real wage}_{o,t}}.$$

Table 1.9 reports the Black-other average real wage ratios in the baseline equilibrium and in the equilibrium of African Americans' immobility. If African Americans could not migrate across the North and the South, the Black-other average real wage ratio is smaller. In 1970, the Black-other average real wage ratio was 0.634 in the baseline equilibrium. In the same year, the Black-other average real wage ratio was 0.578 in the equilibrium of African Americans' immobility. Therefore, the relocation of African Americans across the North and the South decreased the racial gap in average real wages by 8.8 percent, because

$$\frac{0.634 - 0.578}{0.634} = 0.088.$$

Since the prior literature primarily addresses nominal wages, I similarly define the average nominal wage for race $r \in \{b, o\}$ and time t by

$$\text{average nominal wage}_{r,t} = \frac{\sum_{i \in \mathcal{N}} \sum_{a=1}^{\bar{a}} L^i_{r,a,t} w^i_{r,a,t}}{\sum_{i \in \mathcal{N}} \sum_{a=1}^{\bar{a}} L^i_{r,a,t}}.$$

I take the ratio of average nominal wages between African Americans and others

$$\frac{\text{average nominal wage}_{b,t}}{\text{average nominal wage}_{o,t}}.$$

Table 1.10. Nominal Wage Ratios between African Americans and Others

	1940	1950	1960	1970	1980	1990	2000	2010
the baseline equilibrium	0.431	0.562	0.555	0.629	0.673	0.694	0.728	0.716
African Americans' immobility	0.431	0.549	0.508	0.565	0.634	0.662	0.712	0.707

Notes: Nationwide average nominal wage ratios between African Americans and others in the baseline equilibrium and in the equilibrium of African Americans' immobility

Table 1.10 reports the Black-other ratios of average nominal wages in the baseline equilibrium and in the equilibrium of African Americans' immobility. In 1970, the Black-other average nominal wage ratios were 0.629 and 0.565 in the baseline equilibrium and in the equilibrium of African Americans' immobility, respectively. Therefore, the great Black migration reduced the racial gap in nominal wages by $(0.629 - 0.565)/0.629 = 0.102$, 10.2 percent. This is a similar number to Smith and Welch (1989) who use a reduced-form decomposition technique to compute the contribution of the great migration to reducing the racial gap in nominal wages.

1.7 Conclusion

4 million African Americans migrated from the South to the North between 1940 and 1970, which is called the great Black migration. This paper has quantified the aggregate and distributional effects of the great Black migration. A dynamic general equilibrium model of the spatial economy has served this purpose. I have estimated the elasticities in the model and backed out the other parameters. My quantitative model and the existing reduced-form studies have produced comparable predictions about nominal wages and racial inequality in the no great migration scenario. The quantitative model revealed that between 1940 and 1970, the mobility of African Americans across the North and the South increased aggregate US output more than the mobility of others did. I view this paper as the first step in understanding the connection between the geography of African Americans and the aggregate performance of the US economy.

Chapter 2 | Trade and the Spatial Distribution of Unemployment

2.1 Introduction

How does productivity improvement in China affect unemployment in the US states? Autor et al. (2013) find that unemployment increased more in commuting zones that were hit by Chinese import competition more severely. This is about effects on a commuting zone relative to others. The bottom-line effect is out of the scope of their paper, and requires quantification of a general equilibrium model.

This paper develops a static general equilibrium model of unemployment in multiple geographic locations. Specifically the model merges a standard quantitative trade and spatial model and the efficiency-wage model of Shapiro and Stiglitz (1984). The model is built on monopolistic competition with a fixed number of firms. But wages are not determined in a Walrasian labor market. Workers can shirk rather than contribute to production. Firms can punish shirkers by firing them, but imperfectly monitor whether an employee shirks or not. To prevent shirking, firms set a higher wage than the market clearing wage, so that higher foregone income upon being fired disciplines employees to contribute to production.

Given wages, monopolistic firms set optimal prices. Given prices and income, the constant elasticity of substitution (CES) demand system determines demand for varieties in each destination. To meet demand for all destinations, firms set production levels, which in turn determines labor demand. The total labor demand in each location is less than the labor force there, which causes unemployment. Unlike a Walrasian economy, wages do not adjust flexibly to clear labor markets. This is because wages are set to prevent shirking, whose condition does not coincide with the labor market clearing condition.

I quantify the model for 27 countries and the 50 US states. Following Dekle et al. (2007), I express a system of equations for changes from the factual equilibrium to a counterfactual. I compute the counterfactual of the 5% increase in China's productivity. The model predicts that real wages increase in all the US states, but unemployment rates increase in 44 US states. The overall US welfare increases.

The counterfactual result highlights heterogeneous effects of China's productivity improvement on the US states. If productivity in China increases by 5%, real wages increase more in states in the west coast than those in the upper midwest. Unemployment decreases only in states in the west coast and a few others, while unemployment increases in the other 44 states. Labor forces are reallocated from the heartland to the west coast and a handful of other states.

Signs of welfare changes are mixed. By the 5% increase in China's productivity, 14 countries out of 28 countries (including the US) have welfare gains, while the other 14 have welfare losses. This result is at adds with Caliendo et al. (2019), for they argue that any country in their calibration had welfare gains in response to the China shock from 2000 to 2007.

This paper contributes to general equilibrium models of international trade (Eaton and Kortum, 2002; Anderson and van Wincoop, 2003) and spatial economy (Allen and Arkolakis, 2014; Redding, 2016). In this literature, Caliendo et al. (2019) study the effect of the China shock on labor markets across the US states with a dynamic Ricardian model with migration. In their paper, individuals choose a sector to work in and a US state to live in, and sector 0 is labeled as non-employment. That is, individuals voluntarily choose non-employment. In contrast, individuals are involuntarily unemployed in this paper.

This paper is not the first attempt to apply the efficiency-wage model to international trade. Davis and Harrigan (2011) and Wang and Zhao (2015) combine the efficiency-wage model and the Melitz (2003) model. As far as I know, however, the efficiency-wage model has not been applied to a many-country quantitative trade model.

This paper belongs to literature of many-region models that comprise involuntary unemployment. Bilal (2019) develops a dynamic spatial model with job search. Goods are freely traded in his model, while trade costs are incurred between different locations in this paper. Eaton et al. (2013) assume fixed wages and endogenize employment in the Eaton-Kortum model. Rodriguez-Clare et al. (2019) introduce downward nominal wage rigidity into the multi-sector Eaton-Kortum model of Caliendo and Parro (2015). These two papers exogenously assume wage rigidity, while I rely on efficiency wages to model unemployment.

The remainder of this paper is organized as follows. Section 2.2 reports empirical facts on unemployment in the US states. Section 2.3 develops the model. Section 2.4 describes data and parameterization. Section 2.5 shows the counterfactual result. Section 2.6 concludes.

2.2 Facts on Unemployment in the US States

This sections lays out facts about unemployment across the US states. Unemployment rates are different across the US states. Figure 2.1 is a map of average unemployment rates from 2011 to 2019 across the US states. States with darker blue have higher unemployment rates. Nevada has the highest of 7.5%, and California has the second highest of the 7.0%. North Dakota has the lowest of 2.8%, and Nebraska has the second lowest of 3.4%. A cluster of states in the southwest including California and Nevada have high unemployment, while a cluster of states in the upper midwest have low unemployment.

Difference in unemployment across the US states is persistent. Figure 2.2 plots average unemployment rates from 2011 to 2019 against those from 2001 to 2010 for the US states. Observations are close to the 45 degree line, implying strong persistence of unemployment rates in the US states. The correlation coefficient is 0.86. Bilal (2019) also reports strong persistence of unemployment rates in French commuting zones.

Persistent difference in unemployment across the US states may be attributed to skill levels and sectoral composition. Guided by hypothesis, I regress unemployment rates on skill levels and sectoral shares as in the following specification

$$u_j^t = \alpha_0 + \alpha_1 \text{skilled labor share}_j^t + \sum_{k=1}^{13} \alpha_{1+k} \text{sectoral share}_{k,j}^t + \epsilon_j^t,$$

where u_j^t is the unemployment rate of state j in year t, skilled labor share$_j^t$ is the number of college graduates who are 25 years old or older divided by the population who is 25 years old or older, sectoral share$_{k,j}^t$ is the GDP of sector k in state j as of year t divided by the GDP in state j as of year t, α_l $(l = 1, \cdots, 14)$ are parameters, and ϵ_j^t is the disturbance. I run the regression (2.2) separately for two years $t = 2012, 2017$. The data for sectoral GDPs in the US states is from SAGDP2N Gross domestic product (GDP) by state of the US Bureau of Economic Analysis. The data for population who is 25 years old or older and college graduates in it is from the American Community Survey of the US Census Bureau. The data for unemployment rates for the US states is from Expanded State Employment Status Demographic Data of the US Bureau of Labor Statistics.

Table 2.1 reports the result for the regression (2.2).[1] For 2012, only constant, the share of college graduates in population, and the share of agriculture in GDP are statistically significant. The share of college graduates in population and the share of agriculture in GDP are negatively related to unemployment rates. For 2017, in addition to the shares of agriculture in GDP, the share of manufacturing in GDP and the share of arts, entertainment, recreation, accommodation and food services in GDP are also statistically significant at the 5% level, and are negatively related to unemployment rates.

However, persistent difference in unemployment across the US states is not fully attributed to skill levels and sectoral composition. The left panel of Figure 2.3 plots unemployment rates in 2017 against those in 2012. I observe positive relationship between them. The correlation coefficient is 0.56. Observations are below the 45 degree line, because the macroeconomic condition in 2012 was worse than that in 2017. The right panel of Figure 2.3 plots the residuals from the regression (2.2) for the year 2017 against those for the year 2012. The residuals from the two different regressions have a positive correlation, 0.52, whose magnitude is not very smaller than the correlation coefficient between the unemployment rates in 2012 and those in 2017. This suggests that variation in unemployment that is unexplained by sectoral composition and skill levels is still persistent across the US states. A possible reason is geography. The model provided in the following section pursues this possibility.

2.3 Model

Let N_{US} be the set of the 50 US states. Let N_{NUS} be the set of countries but the US, where the subscript NUS stands for "not the US." The economy consists of $N = N_{US} \cup N_{NUS}$. A location $j \in N$ is either a US state or a non-US country.

An individual in the US endogenously chooses a US state to live in. The mass of the labor force L_j in $j \in N_{US}$ is endogenously determined in equilibrium. Let L_{US} be the total labor force in the US. Then $\sum_{j \in N_{US}} L_j = L_{US}$. An individual in non-US country j cannot emigrate from her country. Thus the mass of the labor force L_j in $j \in N_{NUS}$ is exogenously given.

[1] Table 2.2 gives a North American Industry Classification System (NAICS) 2-digit code for each industry that appears in Table 2.1.

A timing assumption follows. An individual in the US chooses her state to live in. Then, she may or may not be employed in her destination. She cannot emigrate from her state to another, even if she is unemployed.

This section proceeds as follows. Subsection 2.3.1 describes consumers' utility maximization and firms' profit maximization given the labor forces in the US states. Subsection 2.3.2 states location choices of individuals in the US, which pins down the distribution of the labor forces over the US states. Subsection 2.3.4 defines an equilibrium. Subsection 2.3.5 characterizes a counterfactual equilibrium in terms of changes from the factual.

2.3.1 Consumer and Firm Behavior

Utility maximization

If individual i lives in location $j \in N$, her utility is

$$U_{i,j} = \frac{1}{\tilde{\eta}_i} C_{i,j} A_j \nu_{i,j},$$

where $C_{i,j}$ is the composite good consumed by individual i who lives in location j, $\tilde{\eta}_i$ captures the disutility from making an effort, A_j is the amenity of location j that is common to anyone, and $\nu_{i,j}$ is individual i's idiosyncratic amenity shock for location j.[2] A unit continuum of firms produce differentiated varieties in any location $k \in N$. The composite good $C_{i,j}$ for individual i in location j is defined by

$$C_{i,j} = \left(\sum_{k \in N} \int_0^1 C_{i,k,j}(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}},$$

where $C_{i,k,j}(\omega)$ is individual i's consumption of variety ω shipped from location k to her location j, and σ is the parameter of CES. The associated price index P_j is

$$P_j = \left[\sum_{k \in N} \int_0^1 (p_k(\omega) t_{k,j})^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}},$$

where $p_k(\omega)$ is the f.o.b. price of variety ω produced in location k, and $t_{k,j}$ is the iceberg trade costs of any variety shipped from location k to location j. Note that the price of

[2] The form of the utility function (2.3.1) follows Davis and Harrigan (2011), who assume that the disutility from making an effort is multiplicative. It is slightly different from the specification in Shapiro and Stiglitz (1984), who assume that the disutilty from making an effort is subtractive.

variety ω from location k that consumers in location j face is $p_k(\omega)t_{k,j}$.

Each individual is either employed or unemployed. If individual i is employed, she chooses to make an effort or to shirk. If she makes an effort, her utility is divided by $\eta > 1$, but if she shirks, she does not incur this disutility. That is, using the notation $\tilde{\eta}_i$ in equation (2.3.1),

$$\tilde{\eta}_i = \begin{cases} \eta > 1 & \text{if } i \text{ makes an effort,} \\ 1 & \text{if } i \text{ shirks.} \end{cases}$$

If individual i is unemployed, she does not incur the disutility from making an effort, $\tilde{\eta}_i = 1$.

If an individual in location j is employed (and is not caught shirking), she receives the nominal wage w_j. If she is unemployed, she receives the nominal home production $b_j P_j$, where b_j is the real home production in location j.

Besides the wage or home production, no matter whether she is employed or not, an individual receives a share of profits. Let π_j be total profits of firms in location j. If she lives in non-US country $j \in N_{NUS}$, she receives the share of profits $\frac{\pi_j}{L_j}$. If she lives in US state $j \in N_{US}$, she receives the share of profits $\frac{\pi_{US}}{L_{US}}$, where $\pi_{US} = \sum_{k \in N_{US}} \pi_k$ is total profits in the US. In other words, anyone in non-US country $j \in N_{NUS}$ owns the same share of ownership of firms in her country j. Anyone in the US owns the same share of ownership of firms in the US, no matter which state she lives in.

The nominal income for individual i in location j, $I_{i,j}$, is

$$I_{i,j} = \begin{cases} w_j + \frac{\pi_j}{L_j} & \text{if } i \text{ is employed in } j \in N_{NUS}, \\ b_j P_j + \frac{\pi_j}{L_j} & \text{if } i \text{ is unemployed in } j \in N_{NUS}, \\ w_j + \frac{\pi_{US}}{L_{US}} & \text{if } i \text{ is employed in } j \in N_{US}, \\ b_j P_j + \frac{\pi_{US}}{L_{US}} & \text{if } i \text{ is unemployed in } j \in N_{US}. \end{cases}$$

Then, the budget constraint for individual i in location $j \in N$ is

$$\sum_{k \in N} \int_0^1 p_k(\omega) t_{k,j} C_{i,k,j}(\omega) d\omega \leq I_{i,j}.$$

To solve utility maximization, first I consider consumers' choice of consumption bundle subject to the budget constraint. Then I turn to consumers' choice on whether to make an effort or not.

Individual i's demand for variety ω shipped from location k to her location j, $C_{i,k,j}(\omega)$, is

$$C_{i,k,j}(\omega) = \left(\frac{p_k(\omega)t_{k,j}}{P_j}\right)^{-\sigma} \left(\frac{I_{i,j}}{P_j}\right).$$

Since the budget constraint (2.3.1) is binding, the CES demand aggregator for individual i in location j, $C_{i,j}$, satisfies

$$P_j C_{i,j} = I_{i,j}.$$

Any firm in location j can catch shirking with probability $q_j \in (0,1)$. If a shirker is caught, she is fired and ends up unemployed. Her shirking is not caught with probability $1 - q_j$, then the shirker receives the same wage w_j as those who make an effort. The parameter q_j represents imperfect contract in the labor market in location j.

Substituting $C_{i,j} = \frac{I_{i,j}}{P_j}$ into utility (2.3.1), I obtain the following expressions for indirect utilities. If individual i in location j is unemployed, her indirect utility is

$$\left(b_j + \frac{\pi_j}{L_j P_j}\right) A_j \nu_{i,j} \quad \text{for } j \in N_{NUS},$$
$$\left(b_j + \frac{\pi_{US}}{L_{US} P_j}\right) A_j \nu_{i,j} \quad \text{for } j \in N_{US}.$$

If individual i in location j is employed and makes an effort, her indirect utility is

$$\frac{1}{\eta}\left(\frac{w_j}{P_j} + \frac{\pi_j}{L_j P_j}\right) A_j \nu_{i,j} \quad \text{for } j \in N_{NUS},$$
$$\frac{1}{\eta}\left(\frac{w_j}{P_j} + \frac{\pi_{US}}{L_{US} P_j}\right) A_j \nu_{i,j} \quad \text{for } j \in N_{US}.$$

If individual i in location j is employed and shirks, her expected indirect utility is

$$(1 - q_j) \cdot \left(\frac{w_j}{P_j} + \frac{\pi_j}{L_j P_j}\right) A_j \nu_{i,j} + q_j \cdot \left(b_j + \frac{\pi_j}{L_j P_j}\right) A_j \nu_{i,j} \quad \text{for } j \in N_{NUS},$$
$$(1 - q_j) \cdot \left(\frac{w_j}{P_j} + \frac{\pi_{US}}{L_{US} P_j}\right) A_j \nu_{i,j} + q_j \cdot \left(b_j + \frac{\pi_{US}}{L_{US} P_j}\right) A_j \nu_{i,j} \quad \text{for } j \in N_{US}.$$

Individual i makes an effort if the indirect utility of making an effort (2.3.1) is greater than the expected indirect utility of shirking (2.3.1). She shirks if (2.3.1) is less than (2.3.1). She is indifferent between making an effort and shirking if (2.3.1) is equal to (2.3.1).

Production function

Suppose that firm $\omega \in [0,1]$ in location j hires the measure $l_j(\omega)$ of workers. If the measure $l'_j(\omega) \in [0, l_j(\omega)]$ of employees make an effort, the production of firm ω in location j, $y_j(\omega)$, is

$$y_j(\omega) = z_j \left(\frac{l'_j(\omega)}{1-\beta}\right)^{\beta} \left(\frac{m_j(\omega)}{\beta}\right)^{1-\beta}.$$

where z_j is the productivity that is common to all firms in location j, and $m_j(\omega)$ is the input of intermediate goods and β is the parameter that represents the labor share in total costs. The input bundle of intermediate goods is the same as consumers' composite good. Shirkers do not contribute to production.

No shirking condition

Assume that any individual receives a wage offer with probability e_j, once she chooses her location j.[3] The probabilities $\{e_j\}_{j \in N}$ are endogenously determined in general equilibrium.[4] A wage offer arrives from at most one firm to an individual. Firms have the full bargaining power, and a wage offer is a take-it-or-leave-it offer. If an individual receives a wage offer and accepts it, she will be hired by a firm. If an individual receives a wage offer and rejects it, she will be unemployed. If an individual does not receive a wage offer, she will be unemployed.

Suppose that the indirect utility of making an effort (2.3.1) and the expected indirect utility of shirking (2.3.1) are equal, that is,

$$\frac{1}{\eta}\left(\frac{w_j}{P_j} + \frac{\pi_j}{L_j P_j}\right) A_j \nu_{i,j} = (1-q_j)\cdot\left(\frac{w_j}{P_j} + \frac{\pi_j}{L_j P_j}\right) A_j \nu_{i,j} + q_j \cdot \left(b_j + \frac{\pi_j}{L_j P_j}\right) A_j \nu_{i,j} \quad \text{for } j \in N_{NUS},$$
$$\frac{1}{\eta}\left(\frac{w_j}{P_j} + \frac{\pi_{US}}{L_{US} P_j}\right) A_j \nu_{i,j} = (1-q_j)\cdot\left(\frac{w_j}{P_j} + \frac{\pi_{US}}{L_{US} P_j}\right) A_j \nu_{i,j} + q_j \cdot \left(b_j + \frac{\pi_{US}}{L_{US} P_j}\right) A_j \nu_{i,j} \quad \text{for } j \in N_{US}.$$

Solving this for w_j, I obtain

$$w_j = \begin{cases} \frac{1}{1-\eta(1-q_j)}\left(\eta q_j b_j P_j + (\eta-1)\frac{\pi_j}{L_j}\right) & \text{for } j \in N_{NUS}, \\ \frac{1}{1-\eta(1-q_j)}\left(\eta q_j b_j P_j + (\eta-1)\frac{\pi_{US}}{L_{US}}\right) & \text{for } j \in N_{US}. \end{cases}$$

[3] Location choices do not take place in non-US countries $j \in N_{NUS}$. Thus the timing assumption about location choices and wage offers does not apply in $j \in N_{NUS}$.

[4] I will show that e_j is the employment rate in location j, because no one rejects a wage offer in equilibrium.

The idiosyncratic amenity shock $\nu_{i,j}$ does not appear in the wage (2.3.1). That is, the nominal wage (2.3.1) equalizes the indirect utility of making en effort and the expected indirect utility of shirking not only for individual i. but also for anyone in location j. In any firm $\omega \in [0,1]$ in location j, all employees make an effort if its wage $w_j(\omega)$ is strictly higher than (2.3.1). All employees of ω shirk if its wage $w_j(\omega)$ is strictly lower than (2.3.1).

In equilibrium, indeed, the nominal wage in location j satisfies (2.3.1). That is, any firm $\omega \in [0,1]$ offers the wage (2.3.1). I show this by the way of contradiction. Let w_j be the wage defined by (2.3.1), and $w_j(\omega)$ generically denote the wage that firm ω in location j offers. Suppose, to the contrary, that there exists firm ω such that $w_j(\omega) \neq w_j$. On the one hand, suppose that $w_j(\omega) > w_j$. Then firm ω would have an incentive to decrease the wage to, say, $w'_j(\omega) \in [w_j, w_j(\omega)) \neq \emptyset$. This is because, if the firm reduces the wage to $w'_j(\omega)$, any employee would make an effort so that the firm would sustain the production level as of $w_j(\omega)$, and the firm would reduce the labor cost. On the other hand, suppose that $w_j(\omega) < w_j$. Then no employee makes an effort, and by the production function (2.3.1), the production level is zero. The profits are non-positive. Therefore the firm has an incentive to increase the wage to $w'_j(\omega) \in [w_j, \infty)$, so that the firm can produce a positive amount of the product. Later I will see that any firm makes positive profits as long as it produces a positive amount, because of monopolistic competition with a fixed number of firms.

Since $\eta > 1$ and $0 < q_j < 1$ for any $j \in N$, the no-shirking wage (2.3.1) is strictly greater than the nominal home production,

$$w_j > b_j P_j.$$ [5]

Therefore anyone accepts a wage offer, if she receives it. As a result, e_j represents the employment rate in location j as well as the probability that an individual in location j receives a wage offer.

The measure of employees who shirk is zero, for the wage (2.3.1). Suppose, to the contrary, that a positive measure of employees of firm $\omega \in [0,1]$ shirk for the wage (2.3.1). Then the firm would increase the wage slightly, so that any employee makes an effort, then the production level and the profits would discontinuously increase. Therefore the case where a positive measure of employees shirk for the wage (2.3.1) is not sustained in equilibrium. In other words, in equilibrium, if the $l_j(\omega)$ measure of workers are hired by

[5] A sufficient condition for this is $1 - 2\eta + \eta q_j < 0$. $\eta > 1$ and $0 < q_j < 1$ satisfy this.

firm $\omega \in [0,1]$ in location j, the same $l_j(\omega)$ measure of employees make an effort, for the nominal wage (2.3.1). I refer to equation (2.3.1) as the no shirking condition, and the equilibrium nominal wage (2.3.1) as the no shirking wage.

Given the no-shirking wage (2.3.1) and the zero measure of shirkers, the unit cost for any firm in location j is

$$\frac{w_j^{\beta} P_j^{1-\beta}}{z_j}.$$

Suppose temporarily that the price index P_j and profits π_j are held fixed in the no-shirking wage (2.3.1) for $j \in N_{NUS}$, although they are actually general equilibrium objects. Then, two properties hold. First, the no-shirking wage w_j is increasing in η, the disutility from making an effort. If the disutility from making an effort is larger, firms have to compensate employees with a higher wage. Second, the no-shirking wage w_j is decreasing in q_j, the probability that firms catch shirking. If shirkers are more likely to be caught, workers voluntarily make an effort with a lower wage. Then firms do no longer have to offer a high wage. Now I return to the general equilibrium model where $\{P_j\}_{j\in N}$ and $\{\pi_j\}_{j\in N}$ are endogenous.

Aggregate nominal income and expenditure

The aggregate nominal income in location j, I_j, is given by

$$I_j = \begin{cases} e_j L_j \left(w_j + \frac{\pi_j}{L_j}\right) + (1-e_j)L_j \left(b_j P_j + \frac{\pi_j}{L_j}\right) & \text{for } j \in N_{NUS}, \\ e_j L_j \left(w_j + \frac{\pi_{US}}{L_{US}}\right) + (1-e_j)L_j \left(b_j P_j + \frac{\pi_{US}}{L_{US}}\right) & \text{for } j \in N_{US}. \end{cases}$$

This is the sum of the aggregate nominal incomes of the employed (the first term) and the unemployed (the second term).

The aggregate expenditure in location $j \in N$ is

$$X_j = I_j + \frac{1-\beta}{\beta} w_j e_j L_j,$$

where the first term on the right-hand side is the final absorption and the second term on the right-hand side is the purchase of intermediate goods.

Profit maximization - Constant markup

Let $C_{j,k}(\omega)$ be the aggregate demand for variety ω shipped from location j to location k. Since preferences are homothetic, by replacing an individual's nominal income $I_{i,j}$

in equation (2.3.1) with the aggregate nominal expenditure X_k, I obtain the aggregate demand for variety ω shipped from location j to location k, $C_{j,k}(\omega)$, by

$$C_{j,k}(\omega) = \left(\frac{p_j(\omega)t_{j,k}}{P_k}\right)^{-\sigma} \left(\frac{X_k}{P_k}\right).$$

From the viewpoint of monopolistic firm ω in location j, equation (2.3.1) means that how much the demand in location k would be if firm ω sets the f.o.b. price $p_j(\omega)$.

Note that firm ω in location k needs to ship $t_{k,j}C_{k,j}(\omega)$ to meet the demand $C_{k,j}(\omega)$ in location j, because of the iceberg trade costs $t_{k,j}$. Thus to meet the demands from all destinations, firm ω in location j must produce the amount

$$y_j(\omega) = \sum_{k \in N} t_{j,k}C_{j,k}(\omega).$$

Given the no-shirking wage (2.3.1), firm $\omega \in [0,1]$ in location j maximizes its profits $\pi_j(\omega)$ given by

$$\begin{aligned}
\pi_j(\omega) &= p_j(\omega)y_j(\omega) - \frac{w_j^\beta P_j^{1-\beta}}{z_j} y_j(\omega) \\
&= \left(p_j(\omega) - \frac{w_j^\beta P_j^{1-\beta}}{z_j}\right)\left(\sum_{k \in N} t_{j,k}C_{j,k}(\omega)\right) \\
&= \left(p_j(\omega) - \frac{w_j^\beta P_j^{1-\beta}}{z_j}\right)\left[\sum_{k \in N} t_{j,k}\left(\frac{p_j(\omega)t_{j,k}}{P_k}\right)^{-\sigma} \frac{X_k}{P_k}\right],
\end{aligned}$$

where the first line means revenue minus cost, the second line follows from the goods market clearing (2.3.1), and the third line follows from the CES aggregate demand (2.3.1). Taking the first order condition with respect to $p_j(\omega)$, the optimal price for any monopolistic firm ω in location j is

$$p_j(\omega) = \frac{\sigma}{\sigma - 1}\frac{w_j^\beta P_j^{1-\beta}}{z_j},$$

which is the constant markup $\frac{\sigma}{\sigma-1}$ multiplied by the unit cost. Substituting the optimal price (2.3.1) into the price index (2.3.1) (with modifying subscripts), the price index in location j is

$$P_j = \left[\sum_{k \in N}\left(\frac{\sigma}{\sigma - 1}\frac{w_k^\beta P_k^{1-\beta}}{z_k} t_{k,j}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}.$$

Since the measure one of firms exist in each location, substituting the optimal price (2.3.1) into the profits (2.3.1), I have the aggregate profits in location j as

$$\pi_j = \frac{1}{\sigma} \sum_{k \in N} \left(\frac{\sigma}{\sigma - 1} \frac{w_j^{\beta} P_j^{1-\beta} t_{j,k}}{z_j P_k} \right)^{1-\sigma} X_k$$

for any $j \in N$.

The aggregate labor cost is equalized to the aggregate labor income

$$\beta \frac{\sigma - 1}{\sigma} \sum_{k \in N} \left(\frac{\sigma}{\sigma - 1} \frac{w_j^{\beta} P_j^{1-\beta} t_{j,k}}{z_j P_k} \right)^{1-\sigma} X_k = w_j e_j L_j$$

for any $j \in N$. Usual trade models have labor market clearing conditions in value terms. They are similar to (2.3.1), but have $w_j L_j$ on the right-hand side, because usual trade models preclude unemployment, that is, $e_j = 1$. Since e_j can be less than one, I refer to (2.3.1) as the labor market unclearing condition.

2.3.2 Location Choices

I have stated all equilibrium conditions for non-US countries $j \in N_{NUS}$. I turn to location choices of individuals in the US.

Individual i in the US chooses a US state to live in after she draws her idiosyncratic amenity shock $\nu_{i,j}$ for $j \in N_{US}$. Individual i chooses her location to maximize her expected indirect utility. That is, individual i solves

$$\max\{V_{i,j} : j \in N_{US}\},$$

where $V_{i,j}$ is her expected indirect utility of living in US state j,

$$\begin{aligned} V_{i,j} &= e_j \cdot \frac{1}{\eta} \left(\frac{w_j}{P_j} + \frac{\pi_{US}}{L_{US} P_j} \right) A_j \nu_{i,j} + (1 - e_j) \cdot \left(b_j + \frac{\pi_{US}}{L_{US} P_j} \right) A_j \nu_{i,j}, \\ &= \Phi_j \nu_{i,j}, \end{aligned}$$

and Φ_j is the baseline expected indirect utility of living in US state j which is common to anyone in the US

$$\Phi_j = \left[e_j \frac{1}{\eta} \left(\frac{w_j}{P_j} + \frac{\pi_{US}}{L_{US} P_j} \right) + (1 - e_j) \left(b_j + \frac{\pi_{US}}{L_{US} P_j} \right) \right] A_j \text{ for } j \in N_{US}.$$

Equation (2.3.2) means that the expected indirect utility of living in US state j is the weighted sum of the indirect utilities of being employed/unemployed in US state j, with the weights being the probabilities of being employed/unemployed. Note that an individual foresees that the no-shirking condition will hold and she will make an effort upon being employed.

The amenity shock $\nu_{i,j}$ follows the Fréchet distribution whose cumulative distribution function is $F(\nu) = e^{-\nu^{-\theta}}$, independently and identically across individuals i's and the US states $j \in N_{US}$. The labor force in location j is

$$L_j = \frac{\Phi_j^\theta}{\sum_{k \in N_{US}} \Phi_k^\theta} L_{US} \text{ for } j \in N_{US}.$$

I assume that individual i in any non-US country $j \in N_{NUS}$ also draws the amenity shock $\nu_{i,j}$ from the Fréchet distribution $F(\nu) = e^{-\nu^{-\theta}}$ independently across individuals in country j. This affects none of equilibrium outcomes, because individuals in a non-US country cannot emigrate from their country.

2.3.3 Welfare

Welfare in the US is given by the ex-ante expected indirect utility before individuals in the US draw idiosyncratic amenity shocks. Let W_{US} be welfare in the US, then

$$W_{US} = E\left[\max_{j \in N_{US}} V_{i,j}\right] = E\left[\max_{j \in N_{US}} \Phi_j \nu_{i,j}\right] = \Gamma\left(1 - \frac{1}{\theta}\right)\left(\sum_{j \in N_{US}} \Phi_j^\theta\right)^{\frac{1}{\theta}},$$

where $\Gamma(\cdot)$ is the gamma function. Welfare in non-US country $j \in N_{NUS}$, W_j, is given by

$$W_j = E[V_{i,j}] = E[\Phi_j \nu_{i,j}] = \Gamma\left(1 - \frac{1}{\theta}\right)\Phi_j,$$

where Φ_j is the baseline expected indirect utility of living in non-US country j which is common to anyone

$$\Phi_j = \left[e_j \frac{1}{\eta}\left(\frac{w_j}{P_j} + \frac{\pi_j}{L_j P_j}\right) + (1 - e_j)\left(b_j + \frac{\pi_j}{L_j P_j}\right)\right] A_j \text{ for } j \in N_{NUS}.$$

2.3.4 Equilibrium

An equilibrium is defined to be a tuple of price indices $\{P_j\}_{j\in N}$, nominal wages $\{w_j\}_{j\in N}$, employment rates $\{e_j\}_{j\in N}$, aggregate profits $\{\pi_j\}_{j\in N}$, aggregate nominal expenditures $\{X_j\}_{j\in N}$, labor forces in the US states $\{L_j\}_{j\in N_{US}}$ that satisfies equations (2.3.1), (2.3.1) (with (2.3.1)), (2.3.1), (2.3.1), (2.3.1), (2.3.2) (with (2.3.2)). Let $n = |N|$ and $n_{US} = |N_{US}|$, where $|\cdot|$ denotes the cardinality of a set. Then this is a system of $5n + n_{US}$ equations with $5n + n_{US}$ unknowns.

2.3.5 Hat Algebra

Following Dekle et al. (2007), I characterize a counterfactual equilibrium as a solution to a system of equations for changes in endogenous variables from the factual equilibrium to a counterfactual equilibrium. For a generic variable x, let $\hat{x} = \frac{x'}{x}$ be the change in the variable x from the factual equilibrium to a counterfactual equilibrium, where x' and x are the counterfactual and factual value of the variable, respectively. In the following, I consider exogenous changes in productivity and trade costs from the factual to a counterfactual, and assume that any other parameter does not change.

Taking the ratio of (2.3.1) between a counterfactual and the factual, I obtain the changes in no-shirking wages

$$\hat{w}_j = \frac{\eta q_j b_j P_j \hat{P}_j + (\eta - 1)\frac{\pi_j \hat{\pi}_j}{L_j}}{\eta q_j b_j P_j + (\eta - 1)\frac{\pi_j}{L_j}} \quad \text{for } j \in N_{NUS},$$

$$\hat{w}_j = \frac{\eta q_j b_j P_j \hat{P}_j + (\eta - 1)\frac{\pi_{US} \hat{\pi}_{US}}{L_{US}}}{\eta q_j b_j P_j + (\eta - 1)\frac{\pi_{US}}{L_{US}}} \quad \text{for } j \in N_{US},$$

where $\hat{\pi}_{US}$ is the change in the US total profits, that is,

$$\hat{\pi}_{US} = \frac{\pi'_{US}}{\pi_{US}} = \frac{\sum_{j\in N_{US}} \pi_j \hat{\pi}_j}{\sum_{j\in N_{US}} \pi_j}.$$

Taking the ratio of (2.3.1) between a counterfactual and the factual, the changes in aggregate nominal incomes are expressed as

$$\hat{I}_j = \frac{e_j\hat{e}_j\left(w_j\hat{w}_j + \frac{\pi_j\hat{\pi}_j}{L_j}\right) + (1 - e_j\hat{e}_j)\left(b_jP_j\hat{P}_j + \frac{\pi_j\hat{\pi}_j}{L_j}\right)}{e_j\left(w_j + \frac{\pi_j}{L_j}\right) + (1 - e_j)\left(b_jP_j + \frac{\pi_j}{L_j}\right)} \quad \text{for } j \in N_{NUS},$$

$$\hat{I}_j = \frac{e_j\hat{e}_j\hat{L}_j\left(w_j\hat{w}_j + \frac{\pi_{US}\hat{\pi}_{US}}{L_{US}}\right) + (1 - e_j\hat{e}_j)\hat{L}_j\left(b_jP_j\hat{P}_j + \frac{\pi_{US}\hat{\pi}_{US}}{L_{US}}\right)}{e_j\left(w_j + \frac{\pi_{US}}{L_{US}}\right) + (1 - e_j)\left(b_jP_j + \frac{\pi_{US}}{L_{US}}\right)} \quad \text{for } j \in N_{US}.$$

Then, taking the ratio of (2.3.1) between a counterfactual and the factual, the changes in aggregate expenditures are

$$\hat{X}_j = \frac{I_j\hat{I}_j + \frac{1-\beta}{\beta}w_j\hat{w}_je_j\hat{e}_jL_j}{I_j + \frac{1-\beta}{\beta}w_je_jL_j} \text{ for } j \in N_{NUS},$$

$$\hat{X}_j = \frac{I_j\hat{I}_j + \frac{1-\beta}{\beta}w_j\hat{w}_je_j\hat{e}_jL_j\hat{L}_j}{I_j + \frac{1-\beta}{\beta}w_je_jL_j} \text{ for } j \in N_{US}.$$

For any pair of locations $(k, j) \in N \times N$, define location k's share in location j's imports, $\gamma_{k,j}$, by

$$\gamma_{k,j} = \frac{X_{k,j}}{\sum_{n \in N} X_{n,j}},$$

where $X_{n,j}$ denotes the aggregate trade value from location n to location j. Taking the ratio of (2.3.1) between a counterfactual and the factual, I have changes in price indices

$$\hat{P}_j = \left[\sum_{k \in N} \gamma_{k,j}\hat{t}_{k,j}^{1-\sigma}\hat{w}_k^{\beta(1-\sigma)}\hat{P}_k^{(1-\beta)(1-\sigma)}\hat{z}_k^{\sigma-1}\right]^{\frac{1}{1-\sigma}} \quad \text{for } j \in N.$$

For any pair of locations $(k, j) \in N \times N$, define location k's share in location j's exports, $\alpha_{j,k}$, by

$$\alpha_{j,k} = \frac{X_{j,k}}{\sum_{n \in N} X_{j,n}}.$$

Taking the ratio of (2.3.1) between a counterfactual and the factual, I have changes in employment

$$\hat{e}_j = \hat{w}_j^{\beta(1-\sigma)-1}\hat{P}_j^{(1-\beta)(1-\sigma)}\hat{z}_j^{\sigma-1}\sum_{k \in N}\alpha_{j,k}\hat{t}_{j,k}^{1-\sigma}\hat{P}_k^{\sigma-1}\hat{X}_k \text{ for } j \in N_{NUS},$$

$$\hat{e}_j = \hat{w}_j^{\beta(1-\sigma)-1}\hat{P}_j^{(1-\beta)(1-\sigma)}\hat{z}_j^{\sigma-1}\hat{L}_j^{-1}\sum_{k \in N}\alpha_{j,k}\hat{t}_{j,k}^{1-\sigma}\hat{P}_k^{\sigma-1}\hat{X}_k \text{ for } j \in N_{US}$$

Taking the ratio of (2.3.1) between a counterfactual and the factual, I obtain changes in aggregate profits

$$\hat{\pi}_j = \hat{w}_j^{\beta(1-\sigma)} \hat{P}_j^{(1-\beta)(1-\sigma)} \hat{z}_j^{\sigma-1} \sum_{k \in N} \alpha_{j,k} \hat{t}_{j,k}^{1-\sigma} \hat{P}_k^{\sigma-1} \hat{X}_k \text{ for } j \in N.$$

Let $\mu_j = \frac{L_j}{L_{US}}$ for any $j \in N_{US}$, that is, the share of state j in the total labor force in the US. Then taking the ratio of (2.3.2) between a counterfactual and the factual, I have

$$\hat{L}_j = \hat{\mu}_j = \frac{\hat{\Phi}_j^\theta}{\sum_{k \in N_{US}} \mu_k \hat{\Phi}_k^\theta} \text{ for } j \in N_{US},$$

where

$$\hat{\Phi}_j = \frac{e_j \hat{e}_j \frac{1}{\eta} \left(\frac{w_j \hat{w}_j}{\hat{P}_j} + \frac{\pi_{US} \hat{\pi}_{US}}{L_{US} P_j} \right) + (1 - e_j \hat{e}_j) \left(b_j P_j + \frac{\pi_{US} \hat{\pi}_{US}}{L_{US} P_j} \right)}{e_j \frac{1}{\eta} \left(w_j + \frac{\pi_{US}}{L_{US}} \right) + (1 - e_j) \left(b_j P_j + \frac{\pi_{US}}{L_{US}} \right)} \text{ for } j \in N_{US}.$$

Taking the ratio of (2.3.3), the change in the US welfare from the factual to a counterfactual, $\hat{W}_{US}$, is

$$\hat{W}_{US} = \left(\sum_{k \in N_{US}} \mu_k \hat{\Phi}_k^\theta \right)^{\frac{1}{\theta}}.$$

Taking the ratio of (2.3.3), the change in welfare in non-US country j from the factual to a counterfactual, $\hat{W}_j$, is

$$\hat{W}_j = \hat{\Phi}_j,$$

where, by taking the ratio of (2.3.3) between a counterfactual and the factual, $\hat{\Phi}_j$ is

$$\hat{\Phi}_j = \frac{e_j \hat{e}_j \frac{1}{\eta} \left(\frac{w_j \hat{w}_j}{\hat{P}_j} + \frac{\pi_j \hat{\pi}_j}{L_j \hat{P}_j} \right) + (1 - e_j \hat{e}_j) \left(b_j P_j + \frac{\pi_j \hat{\pi}_j}{L_j \hat{P}_j} \right)}{e_j \frac{1}{\eta} \left(w_j + \frac{\pi_j}{L_j} \right) + (1 - e_j) \left(b_j P_j + \frac{\pi_j}{L_j} \right)}$$

for $j \in N_{NUS}$.

An equilibrium in changes is defined to be a tuple of changes in price indices $\{\hat{P}_j\}_{j \in N}$, nominal wages $\{\hat{w}_j\}_{j \in N}$, employment rates $\{\hat{e}_j\}_{j \in N}$, aggregate profits $\{\hat{\pi}_j\}_{j \in N}$, aggregate nominal expenditures $\{\hat{X}_j\}_{j \in N}$ and labor forces in the US states $\{\hat{L}_j\}_{j \in N_{US}}$ that satisfies equations (2.3.5), (2.3.5) (with (2.3.5)), (2.3.5), (2.3.5), (2.3.5), (2.3.5) (with (2.3.5)). This is a system of $5n + n_{US}$ equations with $5n + n_{US}$ unknowns. I refer to this system of equations as hat algebra, following Costinot and Rodriguez-Clare (2014).

2.4 Taking the Model to Data

This section details data source and how parameters and factual values in the hat algebra are assigned from the data. I consider the 5% increase in China's productivity. I do not have to assign productivity, amenity and trade costs to compute this counterfactual because the hat algebra cancels them out. However, some parameter values and the factual equilibrium values remain to be assigned.

The elasticity of substitution σ is in (2.3.5), (2.3.5) and (2.3.5). Following Broda and Weinstein (2006), I set the elasticity of substitution $\sigma = 4$.[6]

Trade values are needed in (2.3.5) and (2.3.5). I collect trade values among 27 non-US countries and the 50 US states as of 2012. The trade values among the 80 locations constitute the 77×77 matrix whose (j, k) element is the trade value from location j to location k, $X_{j,k}$. Trade values between non-US countries come from the United Nations comtrade database. Trade values between the US states and the non-US countries come from the US Census Bureau U.S. Import and Export Merchandise trade statistics on USA trade online. Trade values between the US states are from the commodity flow survey that is uploaded on the US Census Bureau American Fact Finder. A problem is that 288 values out of $50 \times 50 = 2500$ are missing in the US inter-state trade data. Suppose that the trade value from US state k to US state j as of 2012, $X_{k,j}^{2012}$, is missing. Then if I have the trade value from k to j as of 2007, say $X_{k,j}^{2007}$, I fill the missing value $X_{k,j}^{2012}$ with

$$X_{k,j}^{2007} \times g_{US}^{2007,2012},$$

where $g_{US}^{2007,2012} = 1.12$ is the growth rate of the US nominal GDP from 2007 to 2012. This procedure fills 194 missing values out of 288. I set zeros for the remaining 94 missing trade values among the US states, which are 3.8% of all the 2500 inter-state trade values. Some of international trade values and trade values between non-US countries and US states are zeros or missing in the data sources. I set zeros for missing values in them. After all, I have 129 zero values in the 77×77 whole trade value matrix. That is, 2% of the trade values are zeros in my data.

Given the parameter value $\sigma = 4$ and trade values, I back out aggregate profits in each location using

$$\pi_j = \frac{1}{\sigma} \sum_{k \in N} X_{j,k}$$

[6]The mean of the point estimates for the elasticities of substitution for US imports across SITC-3 industries is 4, as in pp. 568, Table IV of Broda and Weinstein (2006). The elasticity of substitution varies across SITC-3 industries from 1.2 of thermionic cold cathode to 22.1 of crude oil.

for any $j \in N$. This equation is implied by (2.3.1).

Factual levels of nominal wages $\{w_j\}_{j \in N}$ are needed in (2.3.5), (2.3.5), (2.3.5). The nominal wages of all the non-US countries but China come from OECD's data of average annual wages as of 2012. The average nominal wages of the OECD countries are measured in national currency units such as euros for EU and yens for Japan, and I translate them in terms of the US dollars with the nominal exchange rates in 2012. The nominal wage in China as of 2012 is taken from China Labour Statistical Yearbook 2016. Again I translate it in terms of the US dollars with the nominal exchange rate. For the nominal wages for the US states, I use the data of average annual pays from Bureau of Labor Statistics Quarterly Census of Employment and Wages.

Factual levels of labor forces $\{L_j\}_{j \in N}$ are needed in (2.3.5), (2.3.5), (2.3.5) and (2.3.5). Labor forces in the non-US countries come from the World Bank. Labor forces in the US states are taken from US Bureau of Labor Statistics Local Area Unemployment Statistics.

The factual levels of employment rates $\{e_j\}_{j \in N}$ are needed in (2.3.5), (2.3.5) and (2.3.5). For any location j, the employment rate e_j satisfies $e_j = 1 - u_j$, where u_j denotes the unemployment rate. Therefore it is sufficient to have unemployment rates to assign the factual employment rates to the hat algebra. I obtain the unemployment rates in the non-US countries except China at the World Bank Open Data, where the data, in turn, is from the ILOSTAT database of International Labour Organization (ILO). The unemployment rate in China as of 2012 is taken from China Labour Statistical Yearbook 2016.[7] The unemployment rates of the US states come from the US Bureau of Labor Statistics Expanded State Employment Status Demographic Data, as I referred to in Section 2.2.

The labor share in total costs β is in (2.3.5), (2.3.5), (2.3.5) and (2.3.5). The labor market unclearing condition (2.3.1) implies

$$\beta = \frac{\sigma}{\sigma - 1} \frac{w_j e_j L_j}{\sum_{k \in N} X_{j,k}}$$

for any $j \in N$. Since all values on the right-hand side are already given, I could compute the value of the right-hand side. However, the value of the right-hand side varies for different locations j's. I rather take an average of the right-hand side across locations.

[7]In the data of the World Bank-ILO, the unemployment rate of China in 2012 is 4.6%. In the China Labour Statistical Yearbook 2016, it is 4.1%. I use the value of 4.1%.

That is, I compute β by

$$\beta = \frac{1}{n} \sum_{j \in N} \frac{\sigma}{\sigma - 1} \frac{w_j e_j L_j}{\sum_{k \in N} X_{j,k}} = 0.51.$$

The value 0.51 is close to Alvarez and Lucas (2007)'s preferred value of 0.5 for the labor share.

I assign the data of nominal unemployment benefits to nominal home production $\{b_j P_j\}_{j \in N}$ in (2.3.5), (2.3.5) and (2.3.5). The data of unemployment benefits in 2012 come from three sources. First, the data of the unemployment benefits in the non-US countries except China come from OECD.Stat Net Replacement Rates in unemployment. The website provides the percentages that an unemployed person can receive from unemployment insurance relative to her previous wage that she received before unemployment. This data is provided for a variety of countries, wage levels and spells of unemployment. For example, I can obtain how much an unemployed single person receives from unemployment insurance if she has been unemployed for 1 year and had received the national average wage before unemployment. I use the value of insurance payment for this profile (single, unemployed for 1 year, previous in-work earnings of the national average wage) to assign the values of unemployment benefits for non-US countries except China. Second, I assume that unemployed people receive 20% of the wage in China, based on the description in Qian (2014), because I cannot find the data of unemployment benefits of China in sources from the government or public organizations. He says "Benefits, which could be valid for as long as 104 weeks, can account up to about 20% of average wage." Thus assuming that anyone unemployed receives 20% of the average wage admittedly overstates the unemployment benefits in China. Third, the data of the unemployment benefits in the US states come from UI Replacement Rates Report by the US Department of Labor, Employment and Training Administration. The webpage presents the replacement rate which is defined by the weighted average of

$$\frac{\text{the weekly benefit amounts (WBA)}}{\text{the claimants' normal hourly wage times 40 hours}}.$$[8]

The replacement rate is the weighted average rather than the simple average across unemployed people because each unemployed person has a different spell of unemployment. The weights are lengths of unemployment spells. I multiply the average nominal wage of a US state by the replacement rate to compute the level of the nominal unemployment

[8]This replacement rate is defined to be the "replacement ratio 1" at the webpage.

benefit in the US state.

The values of the disutility from making an effort η and the probabilities that firms detect shirking $\{q_j\}_{j\in N}$ are needed in (2.3.5), (2.3.5) and (2.3.5). I set $\eta = 1.05$, which is admittedly arbitrary. This means that making an effort reduces utility by 5% relative to shirking, if consumption and amenity are held fixed. Later I will compare the result of $\eta = 1.05$ with those of $\eta = 1.01, 1.1$. Rewriting (2.3.1), $\eta = 1.05$ and the factual values that I have assigned together determine the values of $\{q_j\}_{j\in N}$ by

$$q_j = \frac{(\eta - 1)\left(\frac{\pi_j}{L_j} + w_j\right)}{\eta(w_j - b_j P_j)} \quad \text{for } j \in N_{NUS},$$
$$q_j = \frac{(\eta - 1)\left(\frac{\pi_{US}}{L_{US}} + w_j\right)}{\eta(w_j - b_j P_j)} \quad \text{for } j \in N_{US}.$$

Table 2.3 reports the detection probabilities in non-US countries $\{q_j\}_{j\in N_{NUS}}$ associated with $\eta = 1.05$. Switzerland has the highest of 0.27, whereas China and Italy have the lowest of 0.08. Figure 2.4 presents the detection probabilities in the US states $\{q_j\}_{j\in N_{US}}$ associated with $\eta = 1.05$. The US states have smaller variation than the non-US countries, ranging from 0.16 in Kansas and Iowa to 0.11 in New York and Illinois.

Now the hat algebra (2.3.5), (2.3.5) (with (2.3.5)), (2.3.5), (2.3.5), (2.3.5) and (2.3.5) (with (2.3.5)) is equipped with all the parameters and the factual values to compute counterfactuals.

2.5 Counterfactual Result

Based on the model in Section 2.3 and the data and the parameter values in Section 2.4, I compute the counterfactual of the 5% increase in China's productivity.

Figure 2.5 represents the percent changes in real wages across the US states $\left\{\frac{\hat{w}_j}{\hat{P}_j}\right\}_{j\in N_{US}}$. Real wages increase in all the states. States with dark blue have large increases in real wages, while states with light blue have small increases. California has the largest increase in the real wage of 0.13%, and Tennessee has the second largest increase of 0.10%. On the other hand, Alaska has the smallest increase in the real wage of 0.0487%, and Louisiana has the second smallest increase of 0.0493%. I observe gradation from the west coast with dark blue to the upper midwest with light blue.

Figure 2.6 represents the percent changes in real profits across the US states $\left\{\frac{\hat{\pi}_j}{\hat{P}_j}\right\}_{j\in N_{US}}$. Real profits increase in only 10 states including states in the west coast. Real profits decrease in the other 40 states. States with the darkest blue have increases in real profits, and states with the other colors have decreases. Alaska has the largest increase in the real profits of 0.91%, and California has the second largest increase of 0.58%. On the other hand, Hawaii has the largest decrease in the real profits of 0.40%, and South Dakota has the second largest decrease of 0.35%. I observe gradation from the west coast with dark blue to the heartland with light blue.

Figure 2.7 represents the percentage point changes in unemployment across the US states. Unemployment decreases only in 6 states including states in the west coast. Unemployment increases in the other 44 states. States with the lightest blue have decreases in unemployment, while states with the other colors have increases. Hawaii has the largest increase in unemployment of 0.32 percentage points, and Wyoming has the second largest of 0.25 percentage points. On the other hand, Alaska has the largest decrease in unemployment of 0.52 percentage points, and Washington has the second largest decrease of 0.26 percentage points. Again I observe gradation from the west coast with dark blue to the heartland with light blue.

My model predicts that California has a decrease in unemployment in response to the productivity improvement in China, while Caliendo et al. (2019) argue that California has the largest increase in non-employment among the US states in response to the China shock from 2000 to 2007. A possible reason for the difference in prediction is sectors. The model in this paper has a single sector, while their model has multiple sectors. Computers and electronics have a large sectoral share in both California and China, and demands for computers and electronics shift from California to China, as China's productivity rises. This reduces labor demands in California in their model. In contrast, since my model does not have multiple sectors, the competition in the computers and electronics sector does not happen. My quantification just picks up geographic proximity between California and China, thus unemployment in California decreases as China's productivity rises.

Figure 2.8 represents the percent changes in labor forces across the US states. Labor forces increase in only 9 states including states in the west coast. Labor forces decrease in the other 41 states. States with the darkest blue have increases in labor forces, while states with the other colors have decreases. Alaska has the largest increase in the labor force of 0.31%, and California has the second largest increase of 0.27%. On the other hand, South Dakota has the largest decrease in the labor force of 0.16%, and Wyoming has the second largest decrease of 0.14%. Once again, I observe gradation from the west coast with dark blue to the heartland with light blue.

Table 2.4 shows changes in equilibrium outcomes across countries. The first column reports the percent changes in real wages. Real wages increase in all countries. The second column reports the percent changes in real profits. Real profits increase in 10 out of 27 non-US countries. The third column reports the percentage point changes in unemployment. Unemployment decreases only in China and South Korea, and increases in the other 25 non-US countries. A problem is that unemployment decreases by 5.6 percentage points in China. This is impossible because the unemployment rate in China is 4.1% in the data for 2012. The model can predict an impossible counterfactual value of an unemployment rate because (2.3.1), the equation that pins down employment rates, does not discipline employment rates to be in $[0, 1]$. The fourth column reports the percent changes in welfare. Welfare increases in 14 countries out of 28 countries (including the US), while welfare decreases in the other 14 countries. This result is at odds with Caliendo et al. (2019), for Caliendo et al. (2019) argue that the China shock increases welfare in all countries in their calibration.

Table 2.5 shows the percent changes in the US welfare in response to the 5% increase in China's productivity for $\eta = 1.01, 1.05, 1.1$. As the disutility from making an effort, η, increases from 1.01 to 1.1, the percent change in the US welfare increases. The productivity improvement in China has a favorable and an unfavorable effect on the US. The favorable effect is that the US can import cheaper goods. The unfavorable effect is that unemployment increases in most states. η does not affect the price index, thus does not change the magnitude of the former (favorable) effect. But a high η makes working less attractive, thus mitigates the latter (unfavorable) effect. In total, a high η enhances the US welfare gains from China's productivity improvement.

2.6 Conclusion

This paper develops a model of involuntary unemployment in multiple geographic locations. The model merges a general equilibrium model of international trade and spatial economy and the efficiency-wage model of Shapiro and Stiglitz (1984). I quantify the model for 27 countries and the 50 US states. The counterfactual simulation of the 5% increase in China's productivity highlights its heterogeneous effects on real wages and unemployment across the US states.

Figure 2.1. Average Unemployment Rates from 2011 to 2019 across the US States

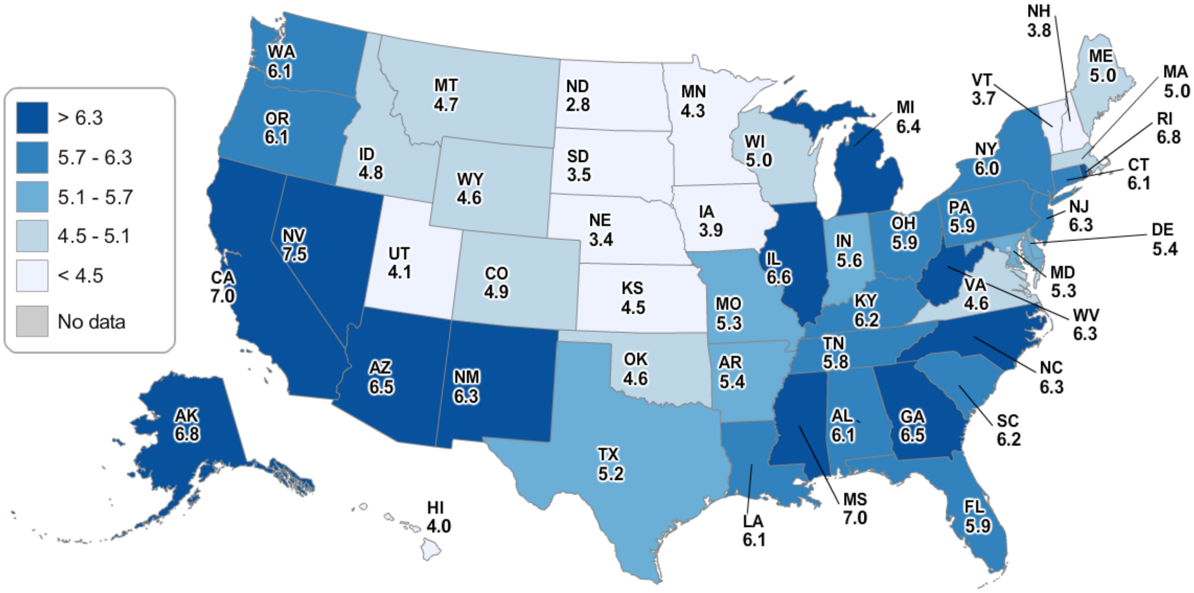


Figure 2.2. Average Unemployment Rates from 2011 to 2019 against Those from 2001 to 2010 across the US States

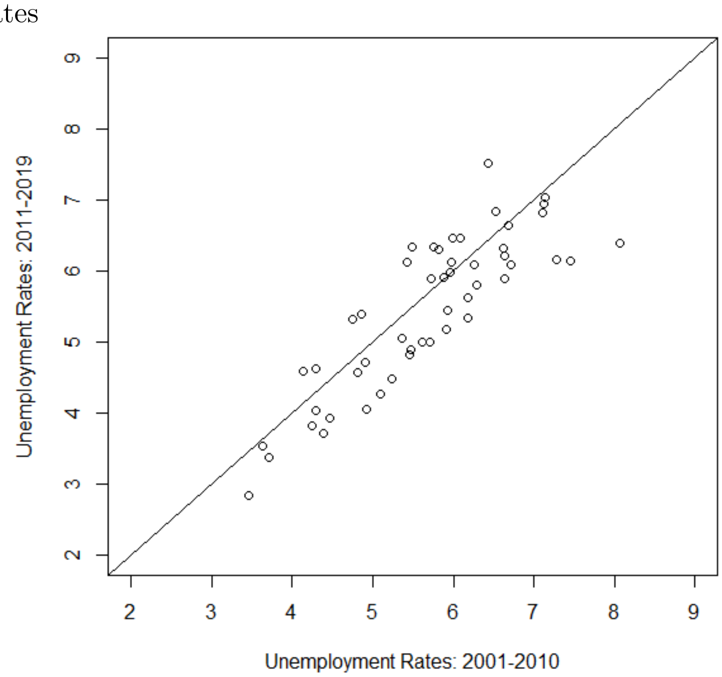


Figure 2.3. Unemployment Rates and Residuals in 2012 and 2017

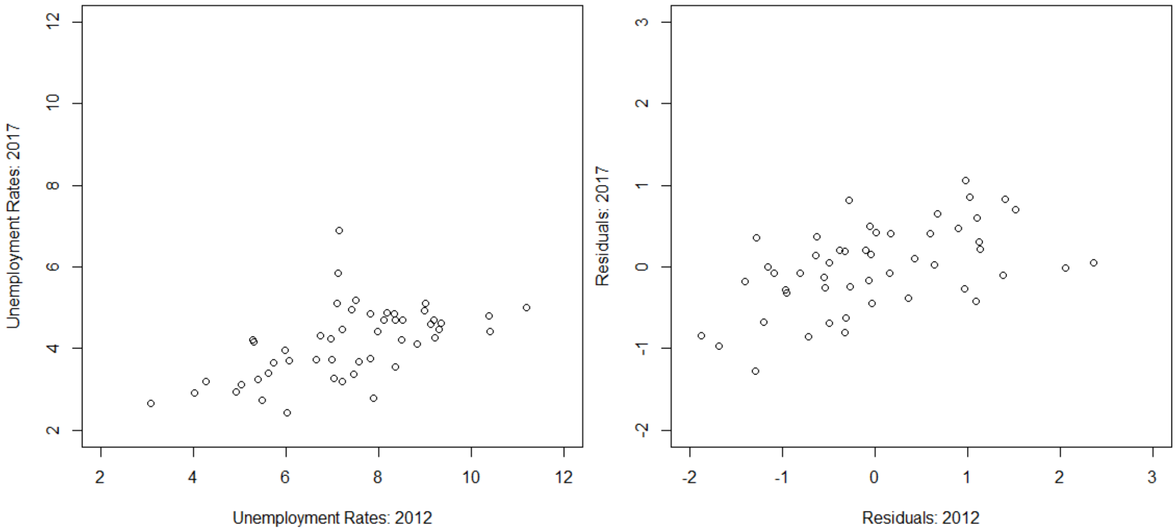


Figure 2.4. Detection Probabilities for the US States $\{q_j\}_{j \in N_{US}}$ Associated with $\eta = 1.05$

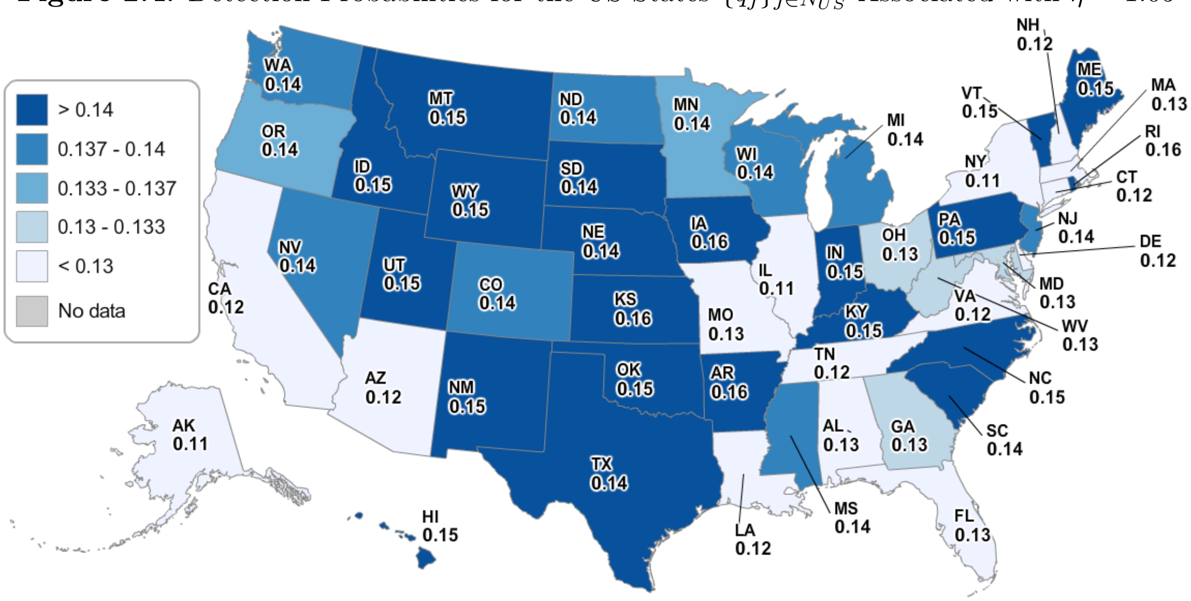


Figure 2.5. % Changes in Real Wages in the US States in Response to 5% Increase in China's Productivity

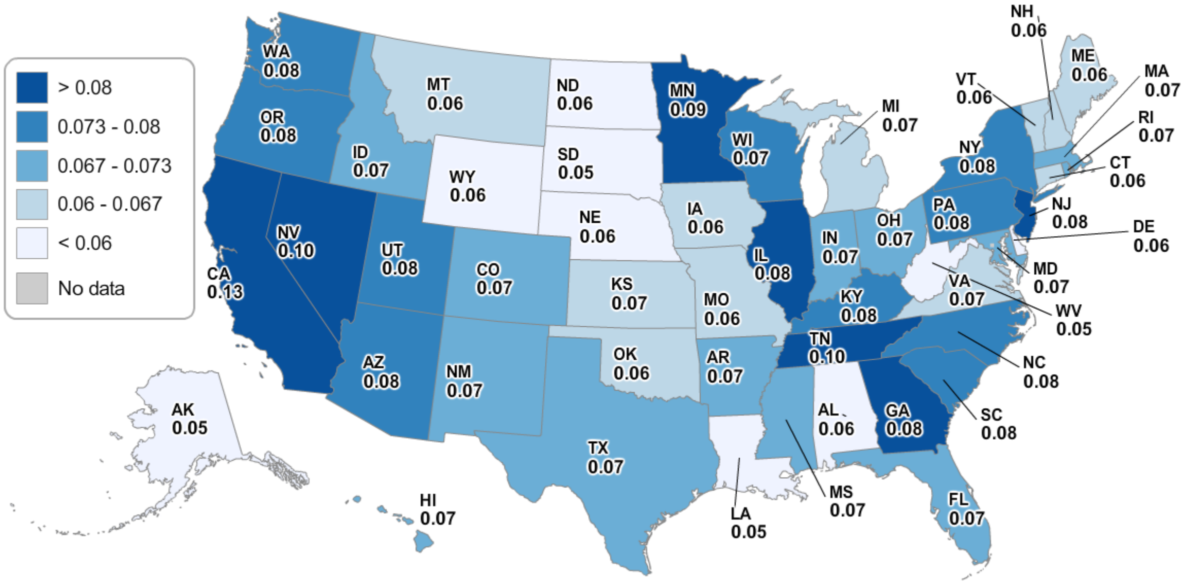


Figure 2.6. % Changes in Real Profits in the US States in Response to 5% Increase in China's Productivity

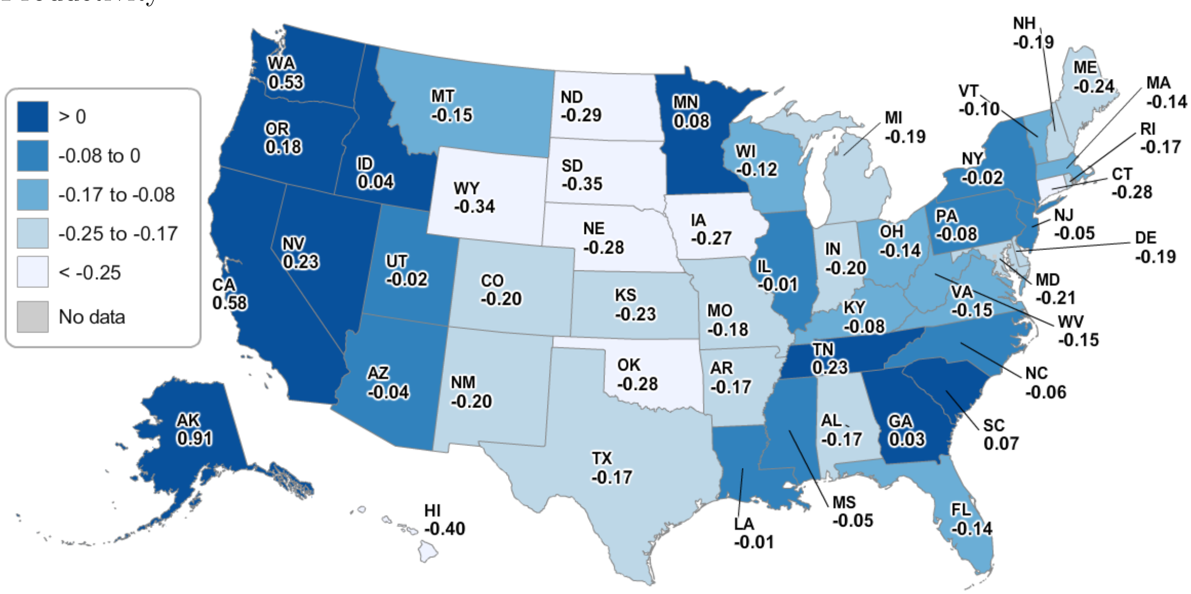


Figure 2.7. Percentage Point Changes in Unemployment in the US States in Response to 5% Increase in China's Productivity

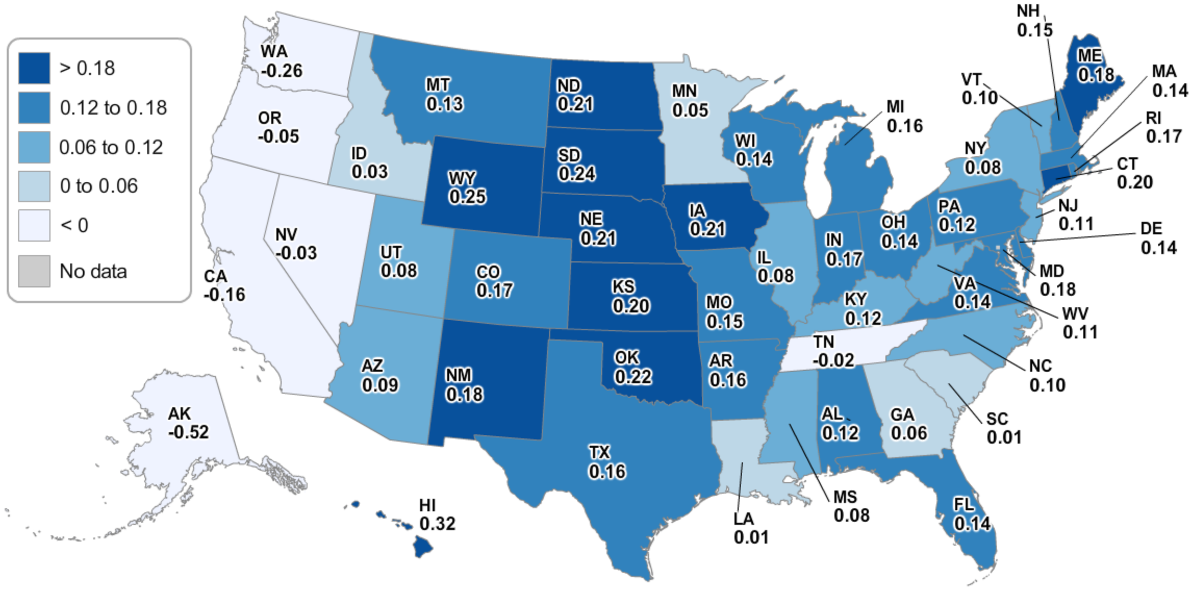


Figure 2.8. % Changes in Labor Forces in the US States in Response to 5% Increase in China's Productivity

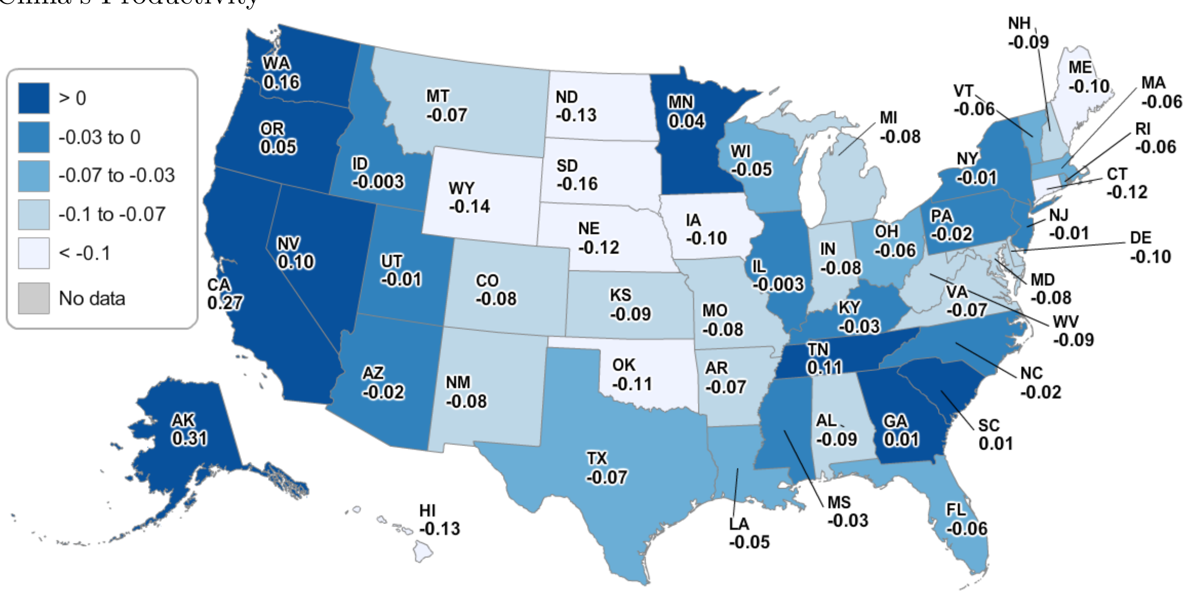


Table 2.1. Regression of Unemployment Rates on Skilled Labor Shares and Sectoral Shares

	Unemployment Rate	
	2012	2017
College graduates	-15.519***	-12.902***
	(5.624)	(2.874)
Agriculture, forestry, fishing and hunting	-49.552***	-35.060***
	(14.973)	(10.917)
Mining, quarrying, and oil and gas extraction	-17.091	-8.460
	(10.742)	(6.811)
Utilities	-27.260	-31.946
	(47.705)	(24.085)
Construction	-56.423	-22.966*
	(36.402)	(12.874)
Manufacturing	-10.610	-10.548**
	(8.627)	(4.874)
Wholesale trade	24.949	-3.075
	(16.682)	(8.939)
Retail trade	-28.910	-18.062
	(29.549)	(15.247)
Transportation and warehousing	-24.575	-4.773
	(20.256)	(8.580)
Information	2.382	2.808
	(16.110)	(7.450)
Finance, insurance, real estate, rental, and leasing	-13.277	-8.449*
	(9.073)	(4.911)
Professional and business services	-6.000	-4.257
	(15.608)	(8.246)
Educational services, health care, and social assistance	-13.854	-5.649
	(13.511)	(7.149)
Arts, entertainment, recreation, accommodation,	12.349	-14.727**
and food services	(11.808)	(6.769)
Other services (except government)	-174.037	-111.677*
	(106.417)	(57.651)
Constant	25.739***	18.608***
	(8.514)	(4.717)
Observations	50	50
R^2	0.673	0.661
Adjusted R^2	0.529	0.511

Note: *p<0.1; **p<0.05; ***p<0.01

2012 NAICS 2 digit codes corresponding to industry descriptions are given in Table 2.2.

Table 2.2. Industry Description and 2012 NAICS 2-digit Codes

Description	NAICS 2-digit
Agriculture, forestry, fishing and hunting	11
Mining, quarrying, and oil and gas extraction	21
Utilities	22
Construction	23
Manufacturing	31-33
Wholesale trade	42
Retail trade	44-45
Transportation and warehousing	48-49
Information	51
Finance, insurance, real estate, rental, and leasing	52, 53
Professional and business services	54, 55, 56
Educational services, health care, and social assistance	61, 62
Arts, entertainment, recreation, accommodation, and food services	71, 72
Other services (except government)	81

Table 2.3. Detection Probabilities for non-US Countries $\{q_j\}_{j \in N_{NUS}}$ Associated with $\eta = 1.05$

Country	q_j	Country	q_j
Australia	0.10	Italy	0.08
Belgium	0.21	Japan	0.12
Canada	0.09	Korea, South	0.09
China	0.08	Netherlands	0.26
Czech Republic	0.14	New Zealand	0.10
Denmark	0.19	Norway	0.22
Estonia	0.15	Poland	0.14
Finland	0.18	Slovakia	0.11
France	0.22	Slovenia	0.11
Germany	0.18	Spain	0.17
Greece	0.10	Sweden	0.14
Hungary	0.18	Switzerland	0.27
Ireland	0.12	United Kingdom	0.11
Israel	0.09		

Table 2.4. Changes in Equilibrium Outcomes of Countries in Response to 5% Increase in China's Productivity

Country	Real Wage %	Real Profits %	Unemployment p.p.	Walfare %
Australia	0.13	0.10	0.03	0.10
Belgium	0.08	-0.03	0.10	0.01
Canada	0.07	-0.02	0.09	-0.01
China	7.42	13.65	-5.56	12.63
Czech Republic	0.13	0.10	0.02	0.11
Denmark	0.06	-0.13	0.18	-0.04
Estonia	0.09	0.03	0.06	0.04
Finland	0.07	-0.07	0.12	-0.01
France	0.06	-0.14	0.18	-0.05
Germany	0.09	0.03	0.06	0.05
Greece	0.04	-0.08	0.09	-0.06
Hungary	0.04	-0.05	0.08	-0.04
Ireland	0.08	-0.02	0.08	0.01
Israel	0.07	-0.03	0.09	-0.01
Italy	0.05	-0.03	0.07	-0.03
Japan	0.10	0.06	0.04	0.07
Korea, South	0.25	0.26	-0.01	0.26
Netherlands	0.09	0.09	0.01	0.09
New Zealand	0.11	0.08	0.03	0.08
Norway	0.06	-0.15	0.20	-0.06
Poland	0.07	-0.03	0.09	0.00
Slovakia	0.10	0.03	0.06	0.04
Slovenia	0.07	-0.04	0.10	-0.02
Spain	0.05	-0.16	0.16	-0.07
Sweden	0.05	-0.11	0.14	-0.06
Switzerland	0.08	-0.05	0.13	0.01
United Kingdom	0.06	-0.08	0.12	-0.04
United States				0.05

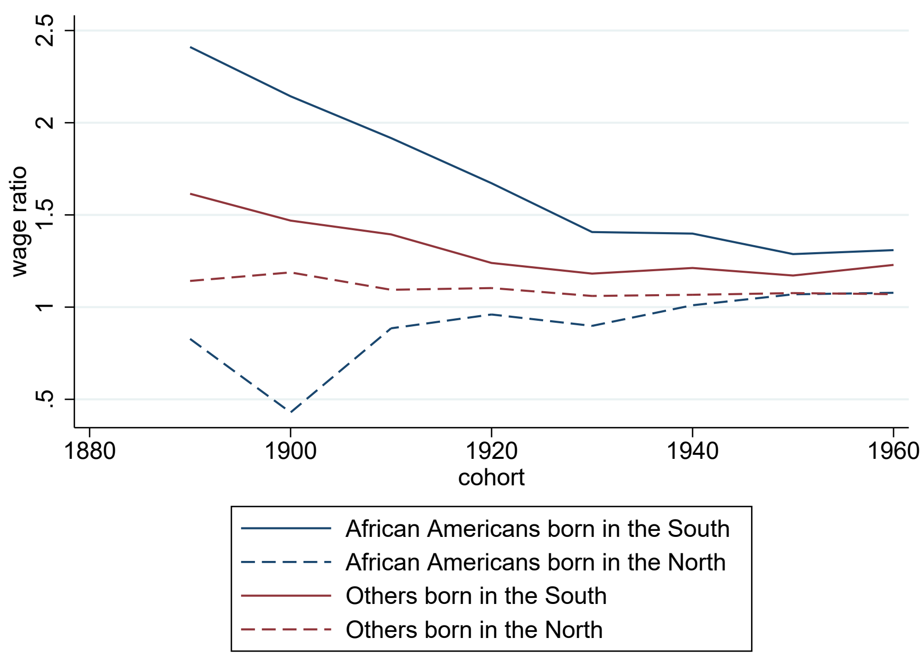
Table 2.5. The US Welfare Changes from the 5% Increase in China's Productivity for Three Values of η

η	1.01	1.05	1.1
% Change in the US Welfare	0.047	0.052	0.058

Appendix A

A.1 An Additional Figure on Motivating Facts

Figure A.1. Mover-Stayer Ratios of Payrolls per Capita for Cohorts, Races, and Birthplaces



Notes: For each cohort (say c), race, and birthplace (the North or the South), this graph provides the ratio of the payroll per capita of movers to the payroll per capita of stayers as of year $c + 50$. Source: US census 1940-2000, American Community Survey 2010.

A.2 Details on Data Sources

Wages, populations, migration shares, and fertility. The data on wages, populations, migration shares, and fertility (babies per person) are from the US censuses from 1940 to 1990, and the ACS from 2000 to 2019. I use the full count data for the US census 1940, 5 percent samples for the US censuses 1960, 1980, and 1990, and 1 percent samples for the US censuses 1950 and 1970, and the ACS of all the sample years. All of them are tabulated in IPUMS USA (Ruggles et al., 2022). Figure 1.1 requires the data on the populations of African Americans, enslaved African Americans, and others in the North and the South since 1790. These data are from the US censuses tabulated in IPUMS NHGIS (Manson et al., 2022).

Rent. IPUMS does not provide the data on rent in 1950. The website of the US Census Bureau provides median rent in states from 1940 to 2000.[1] I obtain rent in 2010 and 2019 from IPUMS NHGIS (Manson et al., 2022).

Survival probabilities. Survival probabilities are from life tables published on the CDC's website.[2]

Aggregate income, college graduates, manufacturing shares. Aggregate incomes in states are used in the regression (1.39). In this regression, I use the manufacturing shares in employment and the shares of college graduates in the population in 1950 as IVs. All of these variables are from IPUMS NHGIS (Manson et al., 2022).

Consumer price index. Wages and rent are deflated by the consumer price index and measured in the 2010 US dollars. The Bureau of Labor Statistics publishes consumer price indices on its website.[3]

A.3 Tabulation of Migration Shares

In the quantification of the model, time periods are 10 years. The US censuses and ACS report individuals' locations 1 or 5 years ago, depending on sample years. Table A.1

[1] `https://www.census.gov/data/tables/time-series/dec/coh-grossrents.html` (accessed on 10/31/2022)

[2] `https://www.cdc.gov/nchs/products/life_tables.htm` (accessed on 10/31/2022)

[3] `https://data.bls.gov/cgi-bin/surveymost?bls` (accessed on 11/01/2022)

reports which sample year includes 1- or 5-year migration shares. I need to map 1- or 5-year migration shares in the data to 10-year migration shares in the model. In the model, individuals make migration decisions in period t and arrive in destinations in period $t+1$. Thus, the census data of 1940 inform me of migration decisions as of 1930, the census data of 1950 inform me of migration decisions as of 1940, and so on.

Let $\mu_{r,a,t}^{i,j,10}$ be the model-consistent 10-year migration share for race r and age a in year t from location j to location i. $\mathbf{M}_{r,a,t}^{10}$ is the matrix whose (i,j) element is $\mu_{r,a,t}^{i,j,10}$.

The censuses in the years 1940, 1960, 1970, 1980, and 1990 yield 5-year migration shares. Let $\mu_{r,a,t}^{i,j,5,\text{data}}$ be such 5-year migration share of race r and age a in (the sample) year t from location j to location i. $\mu_{r,a,t}^{i,j,5,\text{data}}$ is directly computed from the census data of the aforementioned sample years. Let $\mathbf{M}_{r,a,t}^{5,\text{data}}$ be the matrix whose (i,j) element is $\mu_{r,a,t}^{i,j,5,\text{data}}$. I assume that for such census year $t+10$, 5-year migration shares are constant between years t and $t+10$. Then for $t = 1930, 1950, 1960, 1970, 1980$, the model-consistent migration matrix $\mathbf{M}_{r,a,t}^{10}$ is computed by

$$\mathbf{M}_{r,a,t}^{10} = \left(\mathbf{M}_{r,a,t+10}^{5,\text{data}}\right)^2.$$

The census in 1950 yields 1-year migration shares. Let $\mu_{r,a,t}^{i,j,1,\text{data}}$ be the 1-year migration share for race r and age a in the census or ACS year t from location j to location i. Let $\mathbf{M}_{r,a,t}^{1,\text{data}}$ be the matrix whose (i,j) element is $\mu_{r,a,t}^{i,j,1,\text{data}}$. I assume that from 1940 to 1950, 1-year migration shares are constant. Then the model-consistent migration matrix $\mathbf{M}_{r,a,1940}^{10}$ is computed by

$$\mathbf{M}_{r,a,1940}^{10} = \left(\mathbf{M}_{r,a,1950}^{1,\text{data}}\right)^{10}.$$

Since 2000, ACS reports current locations and locations 1 year ago for individuals every year. Then $\mathbf{M}_{r,a,2000}^{10}$ is computed by

$$\mathbf{M}_{r,a,2000}^{10} = \mathbf{M}_{r,a,2001}^{1,\text{data}}\mathbf{M}_{r,a,2002}^{1,\text{data}}\cdots\mathbf{M}_{r,a,2010}^{1,\text{data}}.$$

Since I would like to avoid picking up irregularity caused by the COVID-19 pandemic in 2020, $\mathbf{M}_{r,a,2010}^{10}$ is computed by

$$\mathbf{M}_{r,a,2010}^{10} = \mathbf{M}_{r,a,2011}^{1,\text{data}}\cdots\mathbf{M}_{r,a,2018}^{1,\text{data}}\mathbf{M}_{r,a,2019}^{1,\text{data}}\mathbf{M}_{r,a,2019}^{1,\text{data}},$$

where the 1-year migration shares in the 2019 data are double-counted and the 1-year migration shares in the 2020 data are excluded.

Table A.1. 1- or 5-Year Migration Shares

year	source	location X years ago
1940	census	5
1950	census	1
1960	census	5
1970	census	5
1980	census	5
1990	census	5
2000-2019	ACS	1

Notes: The US censuses and American Community Survey report individuals' locations 1 or 5 years ago depending on sample years.

A.4 First Difference Estimation of the Elasticity of Substitution across Races

I estimate the elasticity of substitution across races within age, time, and location bins σ_r by the first difference estimation, following Monras (2020). Taking the time difference in equation (1.26), I have

$$\Delta \log \left(\frac{w^n_{b,a}}{w^n_{o,a}} \right) = -\frac{1}{\sigma_1} \Delta \log \left(\frac{L^n_{b,a}}{L_{o,a}} \right) + \frac{1}{\sigma_1} \Delta \log \left(\frac{\kappa^n_{b,a}}{\kappa^n_{o,a}} \right).$$

Since the growth rate of the productivity ratio $\Delta \log \left(\frac{\kappa^n_{b,a}}{\kappa^n_{o,a}} \right)$ is unobservable, the econometric specification is

$$\Delta \log \left(\frac{w^n_{b,a}}{w^n_{o,a}} \right) = -\frac{1}{\sigma_1} \Delta \log \left(\frac{L^n_{b,a}}{L_{o,a}} \right) + f_a + \epsilon^n_a, \tag{A.1}$$

where f_a is the age fixed effect, and ϵ^n_a is the error term. The time differences are taken between 1940 and 2010. If the population of African Americans relative to the population of others increase in a location with a high growth rate of productivity ratio between African Americans and others $\Delta \log \left(\frac{\kappa^n_{b,a}}{\kappa^n_{o,a}} \right)$, OLS estimation of equation (A.1) produces an upward bias for $-1/\sigma_1$. Following Monras (2020), I instrument $\Delta \log \left(\frac{L^n_{b,a}}{L_{o,a}} \right)$ by the level of population ratio between African Americans and others in the old time, 1930, $\log \left(\frac{L^n_{b,a,1930}}{L^n_{o,a,1930}} \right)$. I make two assumptions. One is that the population ratio between African Americans and others as of 1930 is not correlated with the growth rate of the productivity ratio between African Americans and others between 1940 and 2010. The other assumption is that the growth rate of the population ratio between African Americans and others between 1940 and 2010 is correlated with the level of the population ratio between the two racial groups as of 1930. An example of the latter assumption is that from 1940 to 2010, African Americans migrated to Illinois or New York where a moderate number of African Americans resided in 1930, but very few African Americans migrated to Wyoming or Montana where very few African Americans resided in 1930.

Table A.2. Elasticity of Substitution across Races: First Difference Estimation

Dependent variable:	$\Delta \log(w^n_{b,a}/w^n_{o,a})$	
Model:	OLS	IV
$\Delta \log(L^n_{b,a}/L^n_{o,a})$	-0.1510***	-0.2048***
	(0.0235)	(0.0276)
fixed effects:		
age	✓	✓
Observations	228	228
First-stage F-statistic		621.3

Notes: The first difference estimation of the elasticity of substitution across races. Robust standard errors are in parentheses. Significance codes: ***: 0.01.

Table A.2 reports the results. The first column reports the result of OLS, and the second column reports the result of the IV estimation. In line with my conjecture, the IV estimation seems to correct a positive bias in OLS.

A.5 Standard Errors of the Elasticities of Substitution

The estimation of the elasticity of substitution across ages σ_a involves an estimate of σ_r, as equations (1.11), (1.31), and (1.33) imply. Let $\hat{\sigma}_r$ and $\hat{\sigma}_a$ be estimates for σ_r and σ_a, respectively. Then the standard error of $\hat{\sigma}_a$ need to take into account variability of $\hat{\sigma}_r$.

For this purpose, I compute block bootstrap standard errors. This is a similar approach to Glitz and Wissmann (2021). I have the data of wages and populations from 1940 to 2010 and the data of migration shares from 1930. To construct shift-share predicted populations (1.28) and gross inflows (1.29), I need migration shares 30 years before the year for wages and populations. The earliest migration data are of 1930, so my sample years are 1960 to 2010. I split the sample years to two groups. Group 1 consists of the years 1960, 1970, and 1980. Group 2 consists of the years 1990, 2000, and 2010.

The reason that I resample block of years is that bootstrap samples would understate serial correlation over time if each year is resampled separately.[4]

The procedure of block bootstrap is the following. Set the number of bootstrap samples $B = 10,000$. Recall that the number of locations, N, is 38. Then for $b = 1, \cdots, B$,

1. Randomly choose one of N locations $2N$ times, allowing for replacement.[5] I get x_b, a $2N$-dimensional vector of locations . Treat them as $2N$ distinct locations.

2. Draw a Bernoulli random number $2N$ times with the success probability 1/2. I get y_b, a $2N$-dimensional vector whose element is either 0 or 1.

3. Let $x_{b,i}$ and $y_{b,i}$ be the i-th elements of x_b and y_b, respectively. If $y_{b,i} = 0$, location $x_{b,i}$ has the sample years in group 1. If $y_{b,i} = 1$, location $x_{b,i}$ has the sample years in group 2. Then $y_{b,i} + 1$ is the group number of sample years.

4. Collect all observations in the pairs of $(x_{b,i}, y_{b,i}+1)_{i=1}^{2N}$ from the original sample. Note that all age bins are collected within location-sample year group pairs $(x_{b,i}, y_{b,i} + 1)$. Call such bootstrap sample S_b.

5. For bootstrap sample S_b, compute the OLS, IV1, and IV2 estimates for σ_r, following Subsubsection 1.4.2.1. Denote such OLS, IV1, and IV2 estimates by $\hat{\sigma}_{r,b}^{OLS}$, $\hat{\sigma}_{r,b}^{IV1}$, and $\hat{\sigma}_{r,b}^{IV2}$, respectively.

6. Compute the race-specific productivity induced by each of $\hat{\sigma}_{r,b}^{OLS}$, $\hat{\sigma}_{r,b}^{IV1}$, and $\hat{\sigma}_{r,b}^{IV2}$.

7. Using $\hat{\sigma}_{r,b}^{OLS}$, $\hat{\sigma}_{r,b}^{IV1}$, and $\hat{\sigma}_{r,b}^{IV2}$ and the race-specific productivity induced by each of the three estimates, compute the OLS, IV1, and IV2 estimates for σ_a following Subsubsection 1.4.2.2. Denote such OLS, IV1, and IV2 estimates by $\hat{\sigma}_{a,b}^{OLS}$, $\hat{\sigma}_{a,b}^{IV1}$, and $\hat{\sigma}_{a,b}^{IV2}$.

Now I have six vectors: $\vec{\hat{\sigma}}_r^{OLS} = (\hat{\sigma}_{r,b}^{OLS})_{b=1}^B$, $\vec{\hat{\sigma}}_r^{IV1} = (\hat{\sigma}_{r,b}^{IV1})_{b=1}^B$, $\vec{\hat{\sigma}}_r^{IV2} = (\hat{\sigma}_{r,b}^{IV2})_{b=1}^B$, $\vec{\hat{\sigma}}_a^{OLS} = (\hat{\sigma}_{a,b}^{OLS})_{b=1}^B$, $\vec{\hat{\sigma}}_a^{IV1} = (\hat{\sigma}_{a,b}^{IV1})_{b=1}^B$, and $\vec{\hat{\sigma}}_a^{IV2} = (\hat{\sigma}_{a,b}^{IV2})_{b=1}^B$. The standard deviations of $\vec{\hat{\sigma}}_r^{OLS}$, $\vec{\hat{\sigma}}_r^{IV1}$, and $\vec{\hat{\sigma}}_r^{IV2}$ are the standard errors in Table 1.5. The standard deviations of $\vec{\hat{\sigma}}_a^{OLS}$, $\vec{\hat{\sigma}}_a^{IV1}$, and $\vec{\hat{\sigma}}_a^{IV2}$ are the standard errors in Table 1.6.

[4] See Glitz and Wissmann (2021) for details.

[5] I collect $2N$ locations because the number of sample years in each group (1 or 2) is one half of the number of sample years in the original sample.

A.6 Tabulation of Fertility

I compute the data counterparts to babies per person of race r, age a and period t $\alpha_{r,a,t}$ in equation (1.8) in the following way. Note that the households are sampled in the census data, and information of all members in the sampled households are presumably recorded.

1. Fix census year t.

2. For each household i, count the number of 1-10 year-old children of the household head. Denote such number by b_i.

3. (a) If household i has both the household head and his or her spouse, apportion $x = 0.5b_i$ to the household head's race-age bin, and apportion $x = 0.5b_i$ to his or her spouse's race-age bin.

   (b) If household i has only the household head, and not his or her spouse, apportion $x = b_i$ to the household head's race-age bin.

4. Now I have a list of parents with various values of x. Sum x across all parents within each race-age bin. Let $b_{r,a,t}$ denote such summation of x for race-age bin (r, a).

5. Compute shares of babies of age bin a within race-time tuple (r, t), $\xi_{r,a,t}$,

$$\xi_{r,a,t} = \frac{b_{r,a,t}}{\sum_{a'} b_{r,a',t}}.$$

6. Let $L_{r,0,t}$ be the number of 1-10 year-old people of race r in period t. The babies per person of race-age tuple (r, a), $\alpha_{r,a,t}$ are

$$\alpha_{r,a,t} = L_{r,0,t} \cdot \xi_{r,a,t}.$$

Step 2 captures only 1-10 year-old children whose biological parent is the household head in their household. This may understate the number of children because they may live without biological parents or their parent may not be a household head. Step 5 and 6 correct this understatement of the number of children. First I compute the relative importance of age a in reproduction $\xi_{r,a,t}$ within race-time tuple (r, t). Then I attribute all children $L_{r,0,t}$ of race-time tuple (r, t) to various ages within (r, t) using $\xi_{r,a,t}$. This yields babies per person $\alpha_{r,a,t}$ for race-age-time bin (r, a, t).

A.7 Tabulation of Survival Probabilities

I assume that survival probabilities are common across locations within race-age-time bin (r, a, t). The source of survival probabilities is life tables in the website of CDC.[6] Age 0 in the model corresponds to age 1 to 10 in the data, so I do not consider infant mortality that is the probability of death before one becomes 1 year old. The life table provides the annual survival probability for each race-age bin (r, a), where ages are counted as $0, 1, \cdots$. But in my quantification of the model, one period is 10 years, and age bins are of 10-year windows.

I map survival probabilities in life tables to those in the setting of my model in the following way. Take any census year t and race r. My model assumes that people of age $\bar{a} = 6$ cannot survive to the next period, so I need to compute survival probabilities from age 0 to age $\bar{a} - 1 = 5$. Pick up any age bin a from the 6 age bins that can survive to the next period. Notice that age bin a in the model includes people of the ages from $10a - 9$ to $10a$ in the data. For example, age bin 3 is the set of people who are 21 to 30 years old. According to the life table of year t, the oldest within 10-year-window age bin a survive to the next census year $t + 10$ with probability

$$s_{r,10a,t} \times s_{r,10a+1,t} \times \cdots \times s_{r,10a+9,t}, \tag{A.2}$$

where $s_{r,a',t}$ is the annual probability that people of race r, age a' (of 1-year windows) can survive to the next year in the life table of year t. The youngest within 10-year window age bin a survive to the next census year $t + 10$ with probability

$$s_{r,10a-9,t} \times s_{r,10a-8,t} \times \cdots \times s_{r,10a,t}. \tag{A.3}$$

I take the average of probabilities (A.2) and (A.3), and obtain the 10-year-window survival probability of people of race r, 10-year age bin a, and time t.

A.8 Immigrants from Abroad

The US census 1950 and the ACS 2010 report residential places 1 year ago. For each race r, age a, period $t = 1950, 2010$, and location i, I count the number of individuals who came from abroad (including Alaska and Hawaii) to location i in the last one

[6] https://www.cdc.gov/nchs/products/life_tables.htm (accessd on 09/10/2022)

year. Assuming that the number of immigrants is constant every year within 10-year windows, I multiply the number of immigrants in the last one year by 10 to obtain the number of immigrants in 10-year windows. The US censuses from 1960 to 1990 and the ACS in 2000 report residential places 5 years ago. Similarly, for each race r, age a, period $t = 1960, \cdots, 2000$, and location i, I count the number of individuals who came from abroad to location i in the last five years. Assuming that the number of immigrants is constant every 5-year window within 10-year windows, I multiply the number of immigrants in the last 5 years by 2 to obtain the number of immigrants in 10-year windows.

A.9 Computation of Steady States

Given parameter values, I compute steady states by iterating populations $\{L^i_{r,a}\}^i_{r,a}$. To achieve a steady state, fertility $\alpha_{r,a}$ and survival probabilities $s_{r,a}$ are such that populations will not explode or shrink.

1. Guess populations $\{L^i_{r,a}\}^i_{r,a}$.

2. Given populations $\{L^i_{r,a,t}\}^i_{r,a}$, compute wages $\{w^i_{r,a}\}^i_{r,a}$, rent $\{r^i\}^i$, and eventually period indirect utilities $\{\bar{u}^i_{r,a}\}^i_{r,a}$, using (1.12), (1.14), and (1.2) respectively.

3. In steady state, expected values $\{V^i_{r,a}\}^i_{r,a}$ are fully characterized by period indirect utilities $\{\bar{u}^i_{r,a}\}^i_{r,a}$ by (1.5). Thus I get expected values $\{V^i_{r,a}\}^i_{r,a}$.

4. Given expected values $\{V^i_{r,a}\}^i_{r,a}$, compute migration shares $\{\mu^{j,i}_{r,a}\}^{j,i}_{r,a}$ using (1.6).

5. Given populations $\{L^i_{r,a}\}^i_{r,a}$ and migration shares $\{\mu^{j,i}_{r,a}\}^{j,i}_{r,a}$, update populations $\{\tilde{L}^i_{r,a}\}^i_{r,a}$ using (1.7) and (1.8).

6. Let $\epsilon > 0$ be a prespecified small number.

   (a) Go back to Step 1 with updated guesses $\{\tilde{L}^i_{r,a}\}^i_{r,a}$ if

$$\max_{r,a,i} \left| \frac{\tilde{L}^i_{r,a} - L^i_{r,a}}{L^i_{r,a}} \right| > \epsilon.$$

   (b) End the process with the converged populations $\{\tilde{L}^i_{r,a}\}^i_{r,a}$ otherwise.

A.10 Computation of Transition Paths

I compute transition paths by value function iteration in the following way. Assume that the economy converges to a steady state in period T.

1. Compute the steady state expected values $\{V^i_{r,a,\infty}\}^i_{r,a}$ and populations $\{L_{r,a,\infty}\}^i_{r,a}$ as in Appendix A.9.

2. Load the steady state expected values and populations into those in period T. That is, for any race r, age a, location i,

$$V^i_{r,a,T} = V^i_{r,a,\infty},$$

$$L^i_{r,a,T} = L^i_{r,a,\infty}.$$

3. Load the populations from the 1940 data to those in the first period 0. That is, for any race r, age a, location i,

$$L^i_{r,a,0} = L^i_{r,a,1940}.$$

4. Guess expected values $\{V^i_{r,a,t}\}^i_{r,a}$ for $t = 0, \cdots, T-1$.

5. Given expected values $\{V^i_{r,a,t}\}^i_{r,a}$ for $t = 0, \cdots, T$, compute populations $\{L^i_{r,a,t}\}^i_{r,a}$ for $t = 1, \cdots, T-1$ forward from period 1 to period $T-1$, using (1.6), (1.7), (1.8).

6. Given populations $\{L^i_{r,a,t}\}^i_{r,a}$ for $t = 0, \cdots, T-1$, compute wages $\{w^i_{r,a,t}\}^i_{r,a}$, rent $\{r^i_t\}^i$, and eventually period indirect utilities $\{\bar{u}^i_{r,a,t}\}^i_{r,a}$ for $t = 0, \cdots, T-1$, using (1.12), (1.14), and (1.2) respectively.

7. Given period indirect utilities $\{\bar{u}^i_{r,a,t}\}^i_{r,a}$ for $t = 0, \cdots, T-1$ and expected values in the last period $\{V^i_{r,a,118}\}^i_{r,a}$, compute new expected values $\{\tilde{V}^i_{r,a,t}\}^i_{r,a}$ for $t = 0, \cdots, T-1$.

8. Let $\epsilon > 0$ be a prespecified small number.

   (a) Go back to Step 4 with updated guesses $\{\tilde{V}^i_{r,a,t}\}^i_{r,a}$ for $t = 0, \cdots, T-1$ if

$$\max_{r,a,t,i} \left| \frac{\tilde{V}^i_{r,a,t} - V^i_{r,a,t}}{V^i_{r,a,t}} \right| > \epsilon.$$

(b) End the process with the converged expected values $\{\tilde{V}^i_{r,a,t}\}^i_{r,a}$ for $t = 0, \cdots, T-1$ otherwise.

A.11 Back-of-the-Envelope Calculation

I attempt to calculate the effects of the great Black migration on aggregate output (or labor income) (1.42) without a structural model. Individuals are classified to two races $r \in \{b, o\}$ and two regions $i \in \{N, S\}$, where b and o denote African Americans and others, and N and S denote the North and the South, respectively. Let $L^i_{r,t}$ and $w^i_{r,t}$ be the population and the wage of race r in region i and year t. As in Section 1.4, I use data of head counts and per capita payrolls as populations and wages, respectively. Then the actual aggregate labor income in 1970, labor income$_{1970}$, is

$$\text{labor income}_{1970} = L^N_{b,1970} \cdot w^N_{b,1970} + L^S_{b,1970} \cdot w^S_{b,1970} + L^N_{o,1970} \cdot w^N_{o,1970} + L^S_{o,1970} \cdot w^S_{o,1970}.$$

I seek the counterfactual aggregate labor income as of 1970 in the situation where the Black population is apportioned to the North and the South as in 1940. Let $L_{b,t} = L^N_{b,t} + L^S_{b,t}$ be the total Black population in year t. Then define $s^i_{b,1940}$ by

$$s^i_{b,1940} = \frac{L^i_{b,1940}}{L_{b,1940}}$$

for $i = N, S$. That is, $s^i_{b,1940}$ denotes the fraction of the Black population in region i in 1940. Then the counterfactual aggregate labor income in 1970, labor income$^{cf}_{1970}$, is

$$\begin{aligned}\text{labor income}^{cf}_{1970} =& L_{b,1970} \cdot s^N_{b,1940} \cdot w^N_{b,1970} + L_{b,1970} \cdot s^S_{b,1940} \cdot w^S_{b,1970} \\ &+ L^N_{o,1970} \cdot w^N_{o,1970} + L^S_{o,1970} \cdot w^S_{o,1970}.\end{aligned}$$

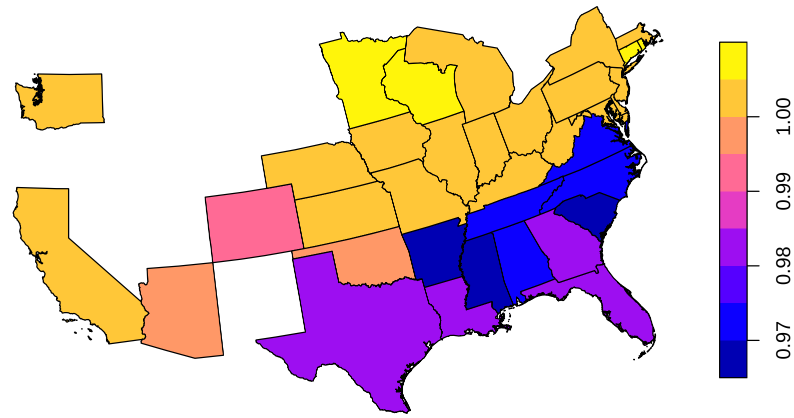
From the census data, I obtain

$$\frac{\text{labor income}^{cf}_{1970}}{\text{labor income}_{1970}} = 0.9914.$$

Therefore, if the Black population was distributed across the North and the South as in 1940, aggregate labor income in 1970 would have been 0.86 percent lower. In Section 1.6, Figure 1.14 shows the quantitative model predicts that aggregate labor income would have been 0.74 percent lower without the North-South migration of African Americans between 1940 and 1970. These two numbers are in the same ballpark.

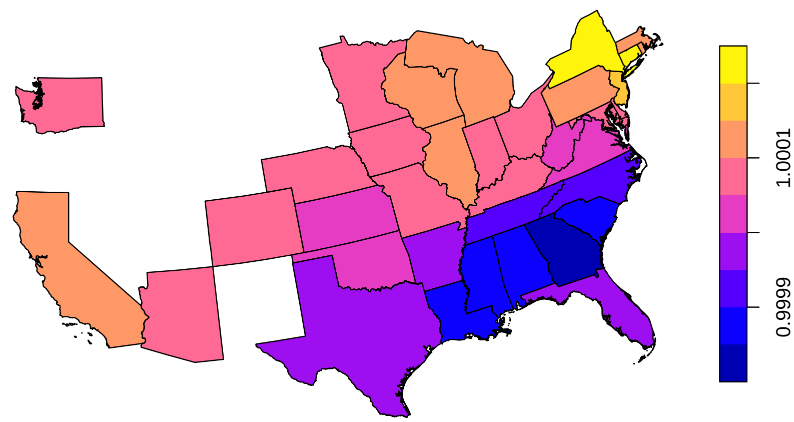
A.12 Additional Figures on the Counterfactual Results

Figure A.2. The Expected Values of African Americans in the Equilibrium of African Americans' Immobility



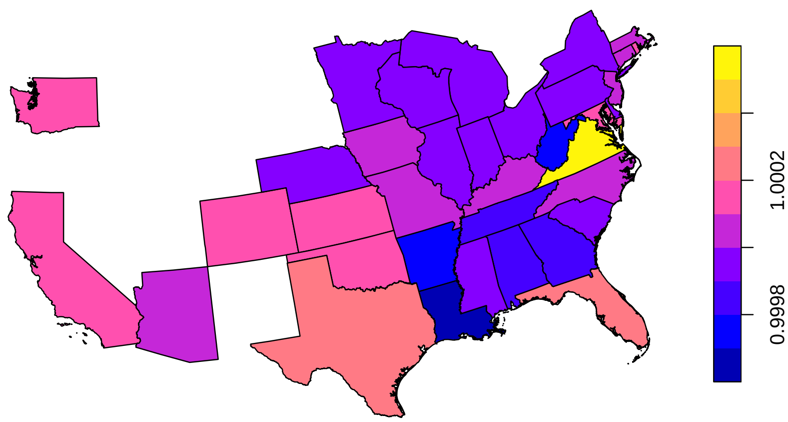
Notes: The expected values of African Americans born in the 1930s in the equilibrium of African Americans' immobility relative to those in the baseline equilibrium. The rest of the North is excluded from the map.

Figure A.3. The Expected Values of Others in the Equilibrium of African Americans' Immobility



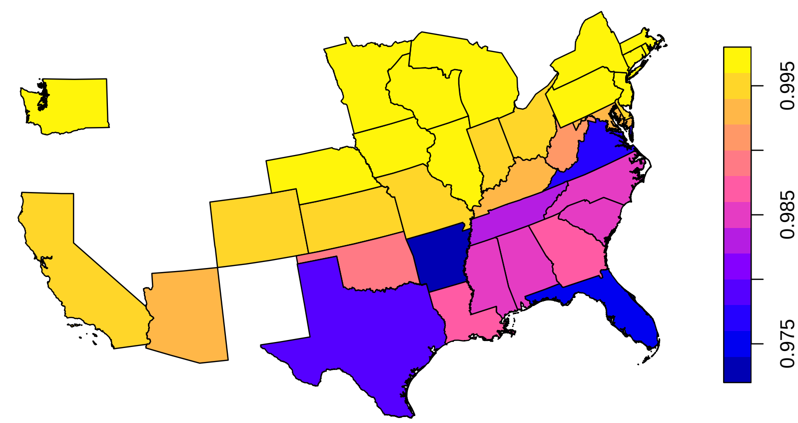
Notes: The expected values of others born in the 1930s in the equilibrium of African Americans' immobility relative to those in the baseline equilibrium. The rest of the North is excluded from the map.

Figure A.4. The Expected Values of African Americans in the Equilibrium of Others' Immobility



Notes: The expected values of African Americans born in the 1930s in the equilibrium of others' immobility relative to those in the baseline equilibrium. The rest of the North is excluded from the map.

Figure A.5. The Expected Values of Others in the Equilibrium of Others' Immobility



Notes: The expected values of others born in the 1930s in the equilibrium of others' immobility relative to those in the baseline equilibrium. The rest of the North is excluded from the map.

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Vita

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