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MULTI-VARIATE PRODUCTION OPTIMIZATION OF A NATURAL GAS FIELD

A Thesis in

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by

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ABSTRACT

Any production well is drilled and completed for the extraction of oil or gas from its original location in the reservoir to the stock tank or the sales line. During their transportation from the reservoir to the surface, these fluids require energy to overcome friction losses and to lift products to the surface. The production system in use in an oil or gas field consists of several components where pressure losses may occur, thus affecting the well performance in terms of production rate. In order to optimize production performance and determine the exact effect of each component on the production rate, it is important to analyze the entire production system from the reservoir to the surface network. A model is presented here that will analyze the effect of selected parameters on the final performance of the production system of a natural gas field. A sensitivity analysis will be conducted for these parameters, thus displaying their individual and combined impact on the system performance in terms of production rate. This will be done by presenting several production scenarios and by analyzing the technical and economic influence of the selected parameters on the field, the objective being to determine the combination of parameters that will best allow the attainment of a given field production target.

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NOMENCLATURE

A= Drainage area (ft^2)

C_w= Total drilling and completion cost per well ($US)

D= Non-Darcy coefficient (D/MSCF)

d= Pipe diameter (in)

d_{choke}= Choke size (in)

d_{tubing}= Tubing diameter (in)

$d_{tubingnominal}$= Nominal tubing diameter (in)

$d_{pipesurf}$= Surface Flow-line diameter (in)

f= Fanning friction factor (Dimensionless)

G_p= Gas cumulative production (SCF)

h= Reservoir thickness (ft)

i= Discount rate (%)

k= Permeability (md)

L_{surf}= Length of the surface flow-line (ft)

L_{vert}= Well depth (ft)

NPV= Net Present Value ($US)

N_{Re}= Reynolds number (Dimensionless)

N_w= Total number of wells to be drilled (Dimensionless)

OGIP =Original Gas In Place (SCF)

P_{ab}= Abandonment pressure (psia)

P_{downs}= Choke downstream pressure (psia)

$P_{downscalc}$= Calculated choke downstream pressure (psia)

$P_{downscritical}$= Critical choke downstream pressure (psia)

P_e = Pressure at the edge of the reservoir (psia)

P_g= Gathering tank pressure (psia)

P_{pc}= Pseudo-critical pressure (psia)

P_{ups}= Choke upstream pressure (psia)

P_w = Wellbore pressure (psia)

P_{wh}= Wellhead pressure (psia)

P_{wf} = Flowing Bottom hole pressure (psia)

P_{wfprod} = Flowing Bottom hole pressure at well deliverability conditions (psia)

q= Flow-rate (MSCFD)

q_{prod}= Flow-rate at well deliverability conditions (MSCFD)

q_{well}= Flow-rate at well deliverability conditions (MSFCD)

Q_{field}= Total gas field production rate per year (MSFCD)

r = Radius (ft)

r = Pressure ratio (Dimensionless)

r_c= Critical pressure ratio (Dimensionless)

r_e= Drainage radius (ft)

r_w= Wellbore radius (ft)

$RF_{contract}$= Recovery factor (%)

s= Skin factor (Dimensionless)

t = Time (years)

t_{max}= Project lifetime (years)

t_p= Project lifetime (years)

T_{downs}= Choke downstream temperature (Rankine)

T_g= Gathering tank temperature (Rankine)

T_{pc}= Pseudo-critical temperature (Rankine)

T_{res}= Reservoir temperature (Rankine)

T_{wh}= Wellhead temperature (Rankine)

T_{wf}= Flowing Bottom hole temperature (Rankine)

v = Fluid velocity (ft/s)

Z= Z-factor (Dimensionless)

Z_{res}= Z-factor at the current reservoir pressure (Dimensionless)

$Z_{resinitial}$= Z-factor at the initial reservoir pressure (Dimensionless)

GREEK

α= Inflow performance relationship constant ($psia^2$/MSCFD)

β= Inflow performance relationship constant ($psia^2/MSCFD^2$)

γ= Specific heat ratio (Dimensionless)

γ_g = Gas specific gravity (Dimensionless)

ε = Pipe roughness (in)

μ = Gas viscosity (cp)

θ = Inclination angle (°)

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Chapter 1

Introduction

In everyday life, we come across various kinds of decision-making problems ranging from personal decisions related to investment, travel, and career development to business decisions related to procuring equipment, hiring staff, product design and modifications to existing design and manufacturing procedures. In the oil industry as well, engineers have to make decisions that will involve the investment of billions of dollars and can jeopardize the future of a company, if they lead to the loss of the investment instead of generating profit. Therefore, they need tools and techniques that will allow them to make the best choice possible in order to solve a particular problem.

Optimization consists in making a design, a process, or a system as fully perfect, functional or effective as possible. This method can be modeled mathematically by the definition of an objective criterion which will be described by an objective function, decision variables and by the definition of constraints. Then, optimizing becomes the process of finding the value of the decision variables that will maximize or minimize the objective function subject to restrictions.

In the oil and gas industry, the topic of production optimization has been covered for several years. Much work has been done in this area and various mathematical techniques, numerical tools and software have been developed, in order to solve some of the optimization problems that occur in the industry.

Any production well is drilled and completed for the transportation of hydrocarbon fluids from the underground reservoir to the surface facilities. The production system in use in oil and gas fields can be relatively simple or involve many components in which pressure drops may occur. Nonetheless, three main units influence petroleum production: the reservoir, the well tubing and the surface flow-line.

The reservoir fluids require energy to overcome friction losses in the system and for their transportation from the reservoir to the surface. The success of this complicated process depends not only on the type of reservoir you are dealing with, but also on how well you design the production system.

The flow in the surface pipeline is not autonomous from what is happening in the tubing system, which is in constant communication with the reservoir. It is imperative to select the best values for parameters such as the tubing size, the wellhead pressure, the choke size and the surface flow-line diameter. Too often, production engineers face problems due to ill-sized wells or ill-sized chokes when the selection of the values of those parameters that will best fit their field properties and production demand, is incorrect.

Since, numerous factors such as the ones previously mentioned are interrelated and can have a significant impact on the production rate; production system design cannot be separated into reservoir performance and piping system performance and handled independently. It is important to analyze the flow as a whole from the reservoir to the surface facilities and to understand the interplay between those factors.

In this study, a numerical tool has been developed that describes the flow from the reservoir to the surface for a single well in a natural gas field. The model is able to give the set of decision variables among a panel of alternatives that will allow best to reach a targeted field production rate; taking into account reservoir depletion over the lifetime of the project. In order to reach optimum results, reservoir properties, tubing system properties and surface flow-line properties need to be provided, as well as the expected recovery factor that is to be achieved and the lifetime of the project, which will enable the determination of the yearly constant field production rate.

The numerical model presented in this work, has been designed for a dry gas reservoir but its use can be extended to a wet gas reservoir, a condensate reservoir or an oil reservoir. It will just be required to make some adjustments in the equations used.

Chapter 2

Literature review

Performance optimization is a common problem that is met by production engineers worldwide. Several methods have been developed since the late 1960's and can be found in the petroleum engineering literature. In this chapter, a summary of these studies and their field application is given.

2.1. Gas well optimization using nodal analysis

Ueda, Samizo and Shirakawa (1991) presented in their paper, the application of production system analysis to an offshore oil field. Total production system analysis was applied to optimize the production of the Khafji field, an oil field situated offshore Saudi Arabia that has been producing from more than 100 wells since 1961. Nodal analysis was used to investigate the flow through the wellhead choke and through the flow-line network. A well choke model based on Ashford formula and flow-line networks models were developed. Bottlenecks were identified and solutions such as rerouting and resizing of certain flow-lines were advised, based on the results of this study. It was estimated that the implementation of these solutions would allow reaching an increase of production rate of 30 MBPD.

Leong and Tenzer (1994) presented the successful use of nodal analysis as well as production enhancing techniques such as acidizing, water entry exclusion and soap injection in order to increase the production of a mature gas field at minimal cost.

Bitsindou and Kelkar (1999) presented a computer tool, which can be utilized for the production optimization of a gas well using dynamic nodal analysis. The concept of dynamic nodal analysis combines the advantages offered by well-known techniques such as nodal analysis, material balance and decline curve analysis. Used individually, each one of the previously mentioned techniques can allow the attainment of a good but an incomplete result.

Nodal analysis enables the appropriate selection of the individual components involved in the production system. However, it only provides the user with a snapshot picture of the well production. It is therefore impossible to assess how production will change with time.

Decline curve analysis is the most commonly used technique in order to include the effect of time on the well production. It involves matching the prior production data using one of the decline curve types (exponential, hyperbolic or harmonic) and using the estimated decline parameters, thus predicting future performance under existing conditions. However, it may not reflect how the well will behave under altered conditions.

The material balance technique was proven useful in understanding how much gas can be produced from the well. The drawback of this technique is that it cannot predict production as a function of time; it can only describe production as a function of reservoir pressure. The impact of alterations cannot be assessed.

The algorithm that has been designed, allows the determination of a sensitivity analysis of future performance for gas wells once a satisfactory match of the past production performance is obtained. The method provided excellent results for both synthetic and field data, and is an improvement to conventional nodal analysis.

2.2. Numerical optimization techniques for oil and gas fields

Van Dam (1968) presented in his paper a method for the selection of an optimum production pattern for a gas field. His method offers the advantage of including economic considerations. Indeed, unlike oil production, gas production is highly influenced by market conditions, since the demand for natural gas is seasonal. There is usually a peak in the demand for natural gas during the winter season and a decrease of the demand during the summer season.

Van Dam started by emphasizing the importance of typical characteristics of the producing field such as total natural gas reserves, well performance, reservoir deliverability, gas field performance, as well as market conditions, in the conception of an optimum development plan for a gas field. Then, based on a dimensionless system of units used to represent the future life of any particular gas field, he was able to determine the production pattern that would allow obtaining the optimum profit in terms of present value. Finally, following his method, it is possible to know the optimum way for the build up phase, constant production phase and decline phase to take place and at what production rate.

Murray and Edgar (1978) presented a method to optimize well location and the sequence of flow rates and compression needs at each well, according to a pre-specified demand schedule. Their work is an extension of previous research on reservoir optimization *((Coats 1969), (Cooksey, Henderson and Dempsey 1969), (Dempsey, et al. 1971), (Henderson, Dempsey and Tyler 1968), (Sharp, et al. 1970))*. Two separate optimization techniques were derived from the concept of zero-one (no-yes) integer programming, the integer variable representing the decision to drill or not to drill a well, at a particular location. A BEMIP (binomial enumerative mixed integer program) was used to solve the scheduling problem and was found to be as successful as the traditional BBIMP (branch-and-bound mixed integer program), since it led to the same results with a gain in computing time of 75-80%. A non-linear optimization algorithm for continuous variables (the gradient projection method) was used to determine the scheduling of production that would meet demand at minimal costs. A non-linear term served to approximate the zero-one decision to drill. This algorithm appeared to be an improvement over existing methods for selecting well locations and scheduling flow rates from a multiwell reservoir.

Eme (2005) presented in his paper a simple technique to evaluate gas deliverability. Its method offers the advantage to require less data than other conventional methods, in order to be able to assess gas well performance. Three components were needed for performance prediction: initial rate estimate, prediction of rate decline as reservoir pressure depletes, prediction of reservoir pressure decline due to production.

It was shown in his study that the gas rate decline for various tubing sizes on a dimensionless or normalized scale was similar. The resulting relationship combined with the material balance equation for depletion drive reservoirs allowed the prediction of gas well performance for most reservoir types (dry gas and condensate). The method was validated by comparison with the results obtained from simulation software.

2.3. Multivariate optimization

Carroll and Horne (1990) introduced the use of multivariate optimization technique to reach an optimum criterion for a petroleum field, which in their case was the maximization of the revenue stream generated from the production of a petroleum field. Traditional analysis of production systems has treated individual nodes one at a time, calculating feasible but not necessarily optimal solution. Multivariate optimization involves finding the extreme values of a function of multiple variables, thus giving the combination of these variables that will allow best the attainment of a predefined extreme value.

Carroll and Horne's study's objective was to demonstrate the effectiveness of multivariate optimization when applied to the performance of hydrocarbons wells. They first developed a well model that was able to give the production profile for a given time step and a given project life, for any combination of variables. That well model contained 4 components:

- The reservoir model, which was based on a model developed by Borthne. It was a black oil model that could perform a generalized material balance calculation in concert with an inflow performance relationship.
- The tubing component, which described multiphase flow through the vertical flow-string, was based on 3 multiphase flow correlations *((Hagedorn and Brown 1965)*, (*Orkiszewski 1967*) and *(Aziz, Govier and Fogarasi 1972)*).
- The choke component, which described flow through the surface choke in both critical and subcritical regimes, was based on the Sachdeva et al model *(Sachdeva and al 1986)*.
- The separator component, which modeled surface facilities, was also included.

Since the initial idea was to test the multivariate optimization technique, they chose the present value of the revenue stream generated over the life of the project, as the objective function.

The model was tested and plots were obtained for the present value of the revenue stream as a function of the separator pressure and the tubing diameter. Several non-linear solvers were used such as the unmodified Newton's method, a modified Newton's method based on Cholesky factorization and the polytope algorithm. Convergence was obtained with the Newton's method only when initial estimates were within certain well-defined regions. Significant improvement was achieved in convergence to the maximum value, when using the modified Newton's method. However, it was difficult to obtain meaningful derivatives with both methods. The polytope method seemed to be more effective for noisy functions.

To conclude, *Caroll and Horne (1990)* succeeded in displaying the effectiveness of multivariate optimization as compared to univariate sequential optimization, in the area of petroleum production. Many advantages such as the number of decision variables that can be simultaneously chosen and the convergence speed that can be attained, were emphasized.

Tavakkolian et al (2004) presented in their paper the use of another kind of multivariate optimization technique for production optimization. Their study dealt with the investigation of the effectiveness of the genetic algorithm method in optimizing the performance of hydrocarbon producing wells. It is well-known that the performance of a producing well depends on several variables such as tubing size, choke size, separator pressure, etc. In other words, it is a function of these variables. Therefore, changing any of them will alter the well performance. The method presented in this paper is a new stochastic method that enables the analysis of a system of mathematical equations with a number of decision variables and to determine the optimum values of these variables that should give the most economic result. Genetic algorithms offer the advantage of coping with all categories of optimization problems, optimizing with continuous or discrete parameters or a combination of both types, searching simultaneously through a large number of decision variables, providing a list of optimum parameters. A code was developed with MATLAB environment based on genetic algorithm and it was possible to determine the most optimum values for the tubing string size (single size or dual sized tubing), the depth at which the tubing size should vary, the choke size, the number of separators and the separator pressure. The method was applied to a real gas condensate production system. The obtained results were compared to those of PROSPER simulator and a good agreement was found between the two software packages, which demonstrated the effectiveness of the genetic algorithm method.

Kappos and Economides (2005) developed a multi-well network optimization scheme, which combines the three systems at stake in petroleum production: the reservoir, the well tubing and the surface network. They analyzed the flow from the reservoir to the gathering tank. Then, they solved a system of equations describing the flow as a whole, from the reservoir to the surface, with the help of a mathematical solver. Thus, they were able to find the production system optimum rate, the appropriate choke size as reservoir pressure declines and the adequate surface pipeline diameter. The model was applied to a six gas well system, for which they tried to determine the optimal surface pipe diameter subject to maximal production rate, as the reservoir pressure declines. Their study showed that production optimization is ensured when the operating flowing bottom hole pressure and the flow rate are dictated by the intersection of the inflow performance relationship (IPR) and the vertical lift performance (VLP) curves. It was recommended that the flow at the choke be close-critical as it allows obtaining maximal rate for a given downstream pressure. It was also shown that keeping a well in production is not only a function of reservoir pressure, but the choice of parameters such as the choke size and the surface pipe diameter is crucial; the latter being even more important, considering the fact that it plays a determining role in field economics.

2.4. Summary

From the literature review, we can notice that extensive work has been performed in the area of production optimization of hydrocarbon fluids. Several techniques such as nodal analysis and dynamic nodal analysis have been introduced and the importance of optimal design has been emphasized. It is also noticeable that most recent studies have focused on multivariate production optimization. This technique combined with the use of genetic algorithm has been proven to be effective.

The individual impact of production units such as the reservoir, the well tubing and the surface network has been mentioned. However, although some work has been done in order to demonstrate the importance of optimizing the production system by looking at all the components simultaneously instead of sequentially, there are still some questions which remain unanswered:

- What is the combined influence of the previously mentioned production units on well production rate?
- How can we optimize the combined effect of these parameters during the design phase of the production system in order to reach predefined optimum production conditions for a gas field?

Chapter 3

Problem statement

Optimizing the production from a single well, in terms of mass rate is an exercise that encompasses all three units that influence petroleum production: the reservoir, the tubing system and the surface network. It is important to fully understand their interplay and discover the determining factors or parameters, which will materialize the effect of each unit on the well production rate. There are four key parameters, which play an important role in the optimal functioning of these units:

- The tubing size
- The wellhead pressure
- The choke size
- The separator or gathering tank pressure

A good combination of these parameters will lead to an optimum production of the well at a specific reservoir pressure. It is therefore essential to mathematically describe the flow as a whole from the reservoir to the surface and analyze the interdependency of these parameters, as well as their effect on well production rate.

Several methods have been introduced in the petroleum engineering literature, which aim at attaining an optimum functioning of a production well. However, very few of the previous studies truly defined the individual impact of the decision parameters that constitute the wellhead pressure, the tubing size, the choke size and the separator or gathering tank pressure. In the same way, very few of them included these four parameters in a multivariate optimization procedure. In terms of multivariate optimization, it is essential to define an objective function to be maximized or minimized. This objective function depends on several variables.

Every gas production project is closely linked to market conditions. Usually, a gas production contract for a given field is conceived as follows: given the gas field properties (reservoir properties, fluid properties, total natural gas reserves), a company will agree to produce a constant specified amount of gas Q_{field} per year, over the life of the project, t_p. In other words, the company will agree to reach a certain gas cumulative production or recovery factor from the field, at the end of t_p years. In order to meet the requirements, a production pattern needs to be chosen, based upon the individual well performance, which is related to the appropriate selection of parameters such as the wellhead pressure, the tubing size, the choke size and the gathering tank pressure.

The evaluation of the effectiveness of the chosen operating conditions can be accomplished by selecting net present value as the objective function and by estimating the net present value of the project for several production scenarios determined by the different combinations of decision variables that are available. The optimal combination of decision variables will maximize the net present value of the project.

Therefore, the goals of this study are:

- To demonstrate the impact of each decision variable on the well performance and display how they are interrelated, by conducting a sensitivity analysis.
- To obtain the optimal combination of decision variables that will maximize the net present value of the project.

Chapter 4

Methodology

In this chapter, the methods that are used to solve the problem and the mathematical background behind these methods are explained. First, the general workflow to be followed to solve the problem is presented. Then, the four main equations that are incorporated in the gas well model and their adaptation to the problem are described. Finally, the modified gas well model is also explained.

4.1. Workflow of the study

First, a clear description of the problem in terms of mathematical objectives needs to be accomplished. Then, those objectives need to be translated numerically with the use of a numerical tool that will encompass all the aspects of our problem. In this study, this step is accomplished by using MATLAB to design a gas well model. A sensitivity analysis for each decision variable is conducted using this model in combination with an in-house case study.

The results from this analysis are included in the design of the optimization procedure. A modified gas well model is conceived. This model is an improvement of the previous one that takes into account the objective function and generates the optimum set of decision variables for the objective function.

Thus, final results can be obtained, which will describe the parameters to be chosen in order to best meet the requirements of a gas project contract. Figure 4-1 displays a brief workflow of the procedure to be followed in order to achieve the goals.

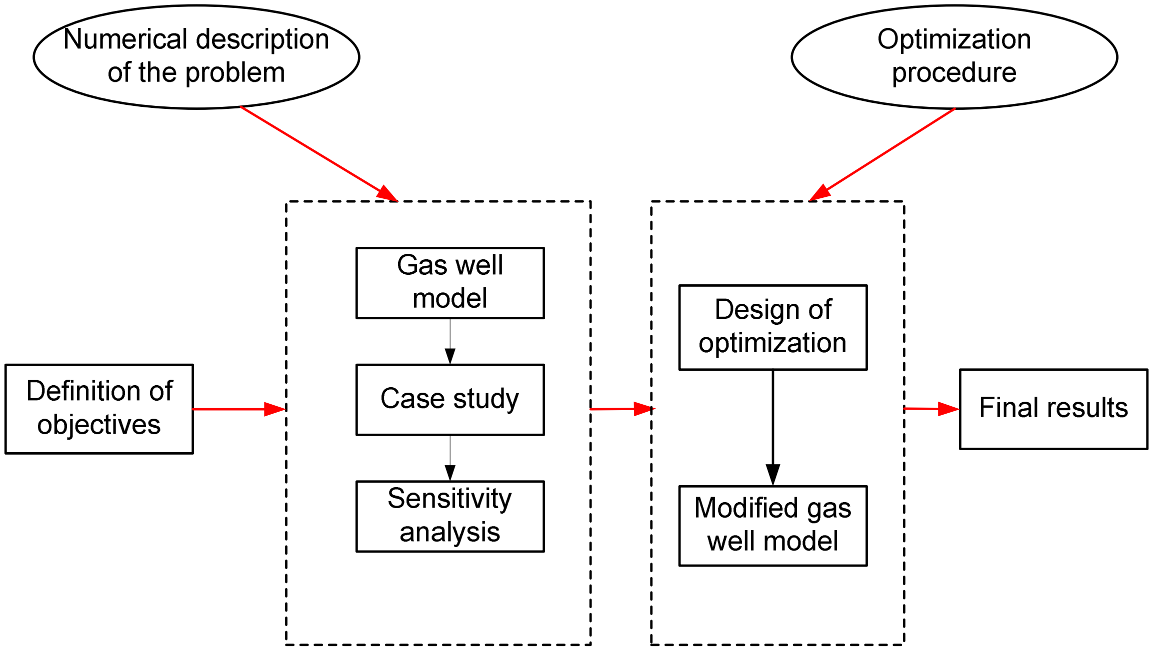

Figure 4-1: Workflow of this study

4.2. Equations

The gas well model has been built to describe the flow from the underground reservoir to the surface facility. It takes into account the pressure drops that occur in the production system when gas flows through the following units: the reservoir, the tubing system and the surface network.

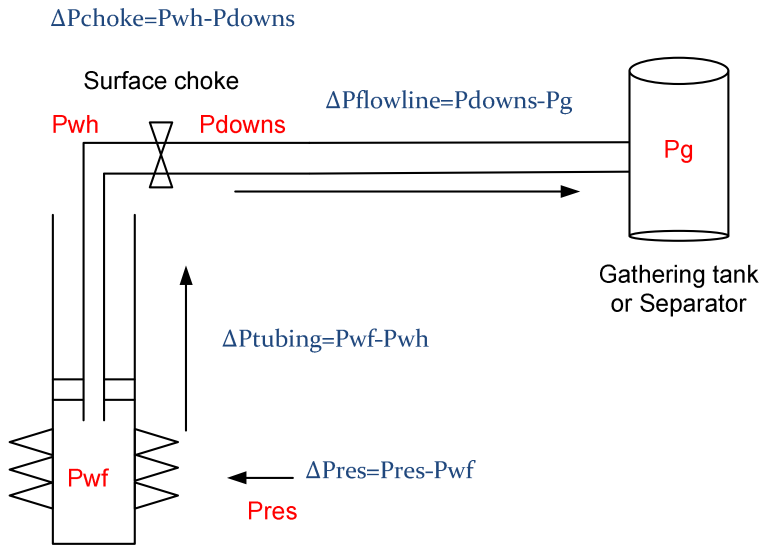

Figure 4-2: Pressure drops in the production system

Figure 4-2 shows the pressure drops occurring in the production system that will be of interest for this study. Equations have been derived that explains the flow in these units.

4.2.1. Reservoir performance equation

To calculate the pressure drop occurring in a reservoir, an equation that expresses the energy or pressure losses due to viscous shear or friction losses as a function of velocity or flow rate is required.

The following assumptions must be considered:

- Radial flow:

We are considering the existence of radial flow through the reservoir. The fluid is converging in a radial way into a relatively smaller hole. Unlike linear flow configuration, the cross-sectional area open to flow is not constant for radial flow. At any radius r, this area can be calculated using the following equation where h is the reservoir thickness:

$$A = 2 * \pi * r * h \tag{4-1}$$

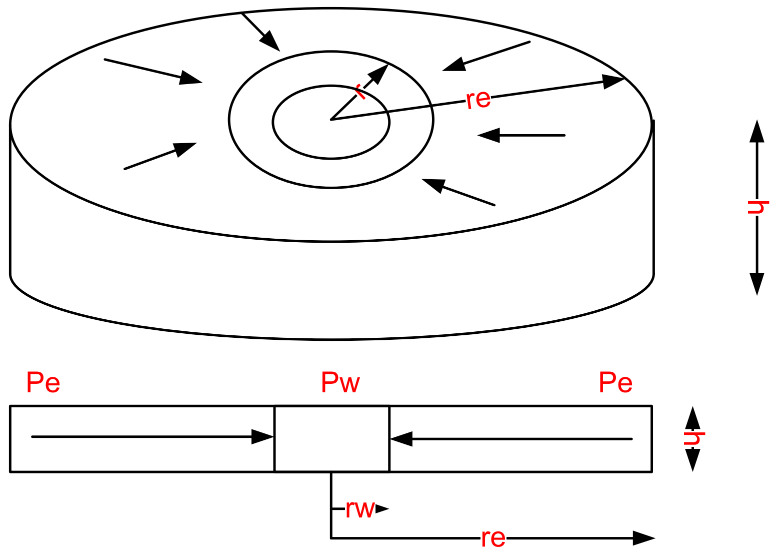

Figure 4-3: Radial flow system

Figure 4-3 portrays the configuration of a one-dimensional radial flow system.

- Single-phase fluid:

We are dealing with single phase fluid. The only fluid that is flowing through our producing system is gas. We do not account for any phase transition.

- Compressible fluid:

Gas is the flowing fluid in this study and it is a compressible fluid, which means that its density is highly dependent on pressure.

- Presence of skin:

The well is rarely drilled under ideal conditions. In most cases, the formation permeability is altered near the wellbore, during the drilling and completion of the well. This alteration typically results in reducing the formation permeability near the wellbore. On the other hand, clean-up stimulation, or fracturing treatments can cause an increase of the permeability around the wellbore. These phenomena are taken into account using the skin factor.

- Non-Darcy effect :

In the majority of reservoirs, Darcy's law can be used to represent fluid flow. Darcy's law assumes that pressure drop varies linearly with velocity. However, for high fluid velocities, experimental observations show that the pressure drop increases more rapidly with velocity than what a simple linear relationship would suggest. As stated before in equation 4-1, the cross-sectional area open to flow A, is proportional to the reservoir thickness h and the radius r. For a given reservoir, as the radius becomes smaller, the area decreases. Fluid velocity can be calculated from the following relationship:

$$v = \frac{q}{A} \tag{4-2}$$

As the area decreases, the velocity increases, which explains why the non-Darcy effect is more likely to be noticeable around the wellbore, where we have the smallest radii.

Another reason for increasing velocity is the expansion of the reservoir fluid. From Figure 4-3, we can notice that for the fluid to be able to flow from the reservoir to the wellbore, the well flowing pressure Pw needs to be smaller than the pressure at the edge of the reservoir Pe. Thus, as reservoir fluid approaches the wellbore, its pressure decreases, which results in fluid expansion. This expansion causes an increase in the volumetric flow rate. In the meantime, the cross-sectional area A, decreases. The result is an increase in the velocity (see Equation 4-2).

Based on the previous observations, the non-Darcy effect will have more impact on the flow rate of a compressible fluid like gas than it will have impact on the flow rate of a slightly-compressible fluid like single phase oil, because the fluid expansion that occurs near the wellbore due to the pressure drop is more significant in the case of a compressible fluid than in the case of a slightly-compressible one. Since we are dealing with a compressible fluid, this is definitely an element that should be taken into account.

- Pseudo-steady state conditions:

We are going to experience three types of flow regimes during the producing life of a reservoir: transient flow, pseudo-steady state flow and steady state flow. Initially, in a virgin reservoir, the pressure at any fixed depth is constant. As production begins the pressure near the wellbore drops significantly, since near wellbore fluids expand to satisfy the imposed production condition. However, far away from the wellbore, no measurable pressure drop can be observed at early times. The reservoir is said to be “infinite-acting” since at locations far away from the reservoir, no pressure drop is noticeable, despite the fact that the reservoir is producing. The flow is said to be transient. After a long time, pressure drops can be measured at all reservoir locations, and the entire reservoir is contributing to the well production. At this time, the pressure changes at the same rate at every reservoir location. In other words, dp(x,t)/dt=constant. The reservoir is in the pseudo-steady state flow regime. Finally, steady state flow occurs in the reservoir when you replace reservoir fluids at the same rate that you remove them. This occurs in secondary recovery or enhanced oil recovery operations. During that time, nothing is changing in the reservoir, i.e. dp(x,t)/dt=0. Figure 4-4 shows the pressure profile in a reservoir and illustrates these three flow regimes.

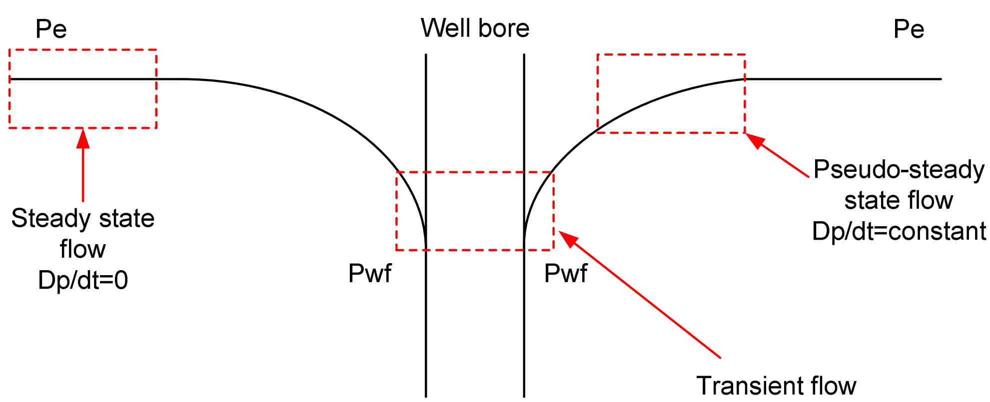

Figure 4-4: Pressure profile in the reservoir

Based on the previous assumptions and starting with the Forchheimer equation rather than Darcy's law, it is possible to come up with the following equation, which describes the flow from the reservoir to the wellbore, for a gas reservoir *((Economides, Hill and Ehlig-Economides 1994), (Kelkar 2008))*:

$$P_{res}^{\ 2} - P_{wf}^{\ 2} = \alpha * q + \beta * q^2$$

4-3

With:

$$\alpha = \frac{1424 * \mu * Z * T_{res}}{kh} * \left(ln\left(0.472 * \frac{r_e}{r_w}\right) + s \right)$$

4-4

$$\beta = \frac{1424 * \mu * Z * T_{res}}{kh} * D$$

4-5

P_{res} is the average reservoir pressure in psia; P_{wf} is the wellbore flowing bottom hole pressure in psia; q is the gas flow rate in MSCF/D;µ is the gas average viscosity obtained at average reservoir pressure in cp; Z is the average gas compressibility factor obtained at average reservoir pressure; T_{res} is the reservoir temperature in °R; k is the reservoir permeability in md; h is the reservoir thickness in ft; r_e is the drainage radius in ft; r_w is the wellbore radius in ft; s is the skin factor; D is the non-Darcy coefficient .

From equations 4-3, 4-4, and 4-5, it is possible to generate an inflow performance relationship curve (IPR). This curve gives the wellbore flowing bottom hole pressure as a function of the flow rate.

4.2.2. Tubing system performance

In order to relate the pressure drop that occurs in the tubing system to the volumetric flow rate, an equation describing the flow from the bottom hole to the wellhead is required. In fact, the generalized gas flow equation through an inclined pipe for a single-phase fluid is used. The equation has been derived from the mechanical energy balance relationship, considering the following assumptions:

- Single-phase flow:

There are only one fluid and one phase that are flowing through the well tubing. No phase transition is accounted for in our study.

- Compressible fluid:

Gas is the flowing fluid and it is compressible. Gas density is highly non-linear with regards to pressure. This has an impact on properties such as viscosity, compressibility factor and every pressure related property because you need to come up with an expression that will account for the high non-linearity of density with regards to pressure.

- Newtonian fluid:

The fluid follows Newton's law. The fluid's stress varies linearly with the rate of strain. Fluid viscosity is the constant of proportionality. For a Newtonian fluid, fluid viscosity depends only on pressure and temperature. In an isothermal reservoir, we are only considering the dependence of fluid viscosity with pressure.

- Inclined pipe:

This assumption allows accounting for any elevation difference in the general flow equation through a pipeline. In the case of a horizontal pipe, the effect of gravity can be neglected. But, this becomes incorrect when dealing with an inclined pipe. For this study, the equation is developed for a vertical tubing string. Therefore, the value of the angle is 90°, which will simplify the calculations later on.

Figure 4-5 describes the configuration of an inclined pipe and the meaning of all the parameters involved and which appear in the derivation of the equation.

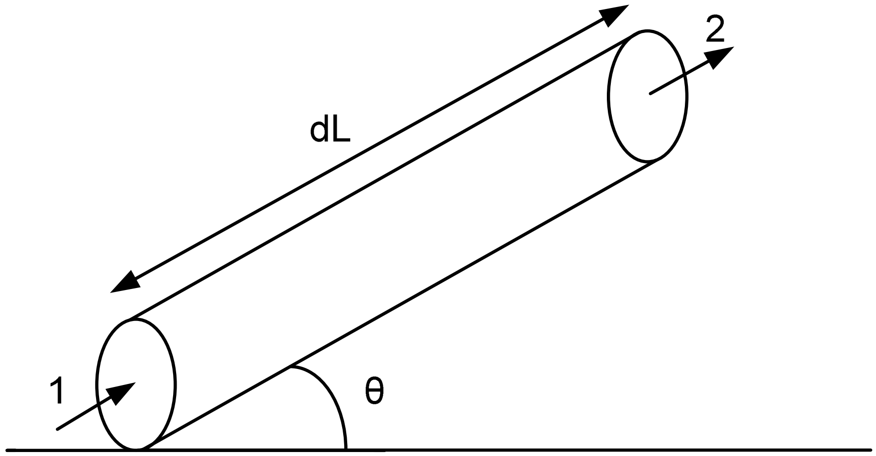

Figure 4-5: Flow through an inclined pipe

- Steady state flow:

We assume that there is steady state flow in the pipe. There is no accumulation or creation of matter inside the pipe. The amount of fluid that comes in is the amount of fluid that comes out of the pipe. Thus, the mass flow rate is constant.

As a result, it is possible to obtain the following expression for the pressure drop in the well tubing. This relationship is also called the vertical lift performance relationship (VLP) *((Economides, Hill and Ehlig-Economides 1994), (Kelkar 2008))*:

$$P_{wf}{}^2 - P_{wh}{}^2 * exp(-S) = 2.685 * 10^{-3} * \frac{f * \left(Z_{avg} * T_{avg}\right)^2 * q^2 * (exp(-S) - 1)}{sin\theta * d_{tubing}{}^5} \quad \textbf{4-6}$$

With:

$$S = \frac{-0.0375 * \gamma_g * sin\theta * L_{vert}}{Z_{avg} * T_{avg}} \quad \textbf{4-7}$$

P_{wf} is the upstream pressure. It is the wellbore flowing bottom hole pressure in psia; P_{wh} is the downstream pressure. It is the wellhead pressure in psia; Z_{avg} and T_{avg} are the average compressibility factor and temperature, accounting for the entire length of the well tubing; q is the volumetric gas flow rate in MSCF/D; d_{tubing} is the tubing diameter in inches; S is a coefficient accounting for the elevation aspect; γ_g is the gas specific gravity; L_{vert} is the length of the tubing string or the well depth in ft.

4.2.3. Gas flow through the choke

A surface choke is a restriction in a production system. There are two types of flow that can take place in a surface choke: critical flow and subcritical flow. The critical flow occurs when the fluid velocity at the smallest cross-section in the restriction is equal to the velocity of sound in that medium. When the velocity is less than the velocity of sound in the medium, we experience what we call subcritical flow. When the fluid velocity is greater than the velocity of sound in the medium, we have supercritical flow. This type of flow is rarely encountered in petroleum production systems. However, both critical and subcritical flows are very common. The basic difference between those two flow regimes reside in the way the fluid flow rate is affected.

When the flow through the restriction is critical, the flow rate is maximal and insensitive to pressure changes downstream of the given restriction. The rate is only related to the pressure upstream of the choke. Therefore, reduction in downstream pressure does not influence the flow rate, since the reduction is never transmitted to the choke.

During subcritical flow however, reduction of the downstream pressure will result in an increase of the flow rate, for a fixed upstream pressure. Further reduction of the pressure downstream of the choke can be obtained by picking a larger choke size. It also leads to further increase in the flow rate.

Therefore, if the purpose of installing a choke in the production system is to isolate downstream fluctuations from upstream conditions, it is better to operate the restriction under critical conditions. For example, a choke can be installed at the wellhead and operated under critical conditions so that the downstream fluctuations in the pipeline or the separator conditions will not affect the wellhead pressure, thus assuring a smooth production. On the other hand, if the purpose is to control the flow rate or to minimize gas hydrate blockages, the surface choke should be operated under subcritical conditions *(Kelkar 2008)*.

This is the assumption that we are making in our study, in order to be able to assess the influence of the surface choke on production. We have to make sure that this assumption is always verified while building up the gas well model.

The following assumptions are considered:

- Single-phase :

We keep the same assumption as in the previous units of the production system. The flowing fluid is still single-phase. No phase transition is taking place in the restriction.

- Compressible :

Gas is the flowing fluid and it is compressible, which means that the density is highly non-linear with pressure.

- Frictionless and adiabatic flow:

The flow through the restriction is considered to be adiabatic. There is no heat transfer with the environment. We are also assuming that they are no friction losses when using the energy balance equation. However, a discharge coefficient is added later on in the derivation to account for any friction losses that might occur.

- Subcritical flow :

As previously explained, the flow through the choke is assumed to be subcritical. The flow rate is affected by downstream pressure changes.

Based on the previous assumptions, an equation for gas flow through a choke is developed, starting from the energy balance. The following equation is obtained, with P_{wh} as the upstream pressure:

$$q = 798.796 * d_{choke}^2 * P_{wh} * \left(\frac{\gamma}{\gamma_g * T_{wh} * (\gamma - 1)} * \left(\left(\frac{P_{downs}}{P_{wh}}\right)^{\frac{2}{\gamma}} - \left(\frac{P_{downs}}{P_{wh}}\right)^{\frac{\gamma+1}{\gamma}}\right)\right)^{0.5} \tag{4-8}$$

q is the gas flow rate in MSCF/D; d_{choke} is the choke diameter or choke size in inches; P_{wh} is the wellhead pressure in psia; P_{downs} is the pressure downstream of the choke; γ is the specific heat ratio; γ_g is the gas specific gravity; T_{wh} is the wellhead temperature in ° R.

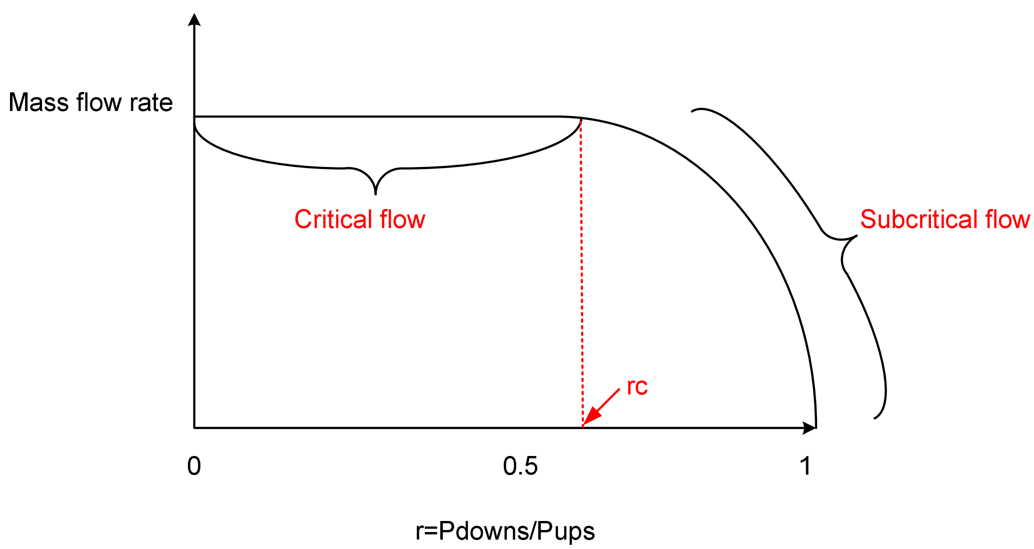

Figure 4-6: Mass flow rate versus pressure ratio for a surface choke

Figure 4-6 shows what we obtain when we plot the mass flow rate through a restriction (here a surface choke) versus the ratio of the pressure downstream of the restriction to the pressure upstream of the restriction. This ratio is noted r in most petroleum production engineering books.

As we can see, during subcritical flow, the flow rate increases as r decreases, for a given upstream pressure. This is true up to a certain point r_c, which is the critical pressure ratio. Once that point is reached, we enter into the critical flow region and the flow rate stays constant despite the decrease in the pressure ratio r.

Thus, under critical flow conditions, the flow rate is no more affected by changes in the pressure ratio r. This statement can be translated by the following equation:

$$\frac{dq}{dr} = 0 \tag{4-9}$$

By taking the derivative of equation 4-8 and by applying $r=P_{downs}/P_{wh}$, the following expression can be obtained for the critical pressure ratio r_c:

$$r_c = \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma-1}} \tag{4-10}$$

The following table describes critical and subcritical flow conditions in terms of equations:

Table 4-1: Choke flow conditions for a given upstream pressure

TYPE OF FLOW	CONDITIONS
CRITICAL FLOW	$r \leq r_c \Leftrightarrow P_{downs} \leq P_{downscritical}$
SUBCRITICAL FLOW	$r > r_c \Leftrightarrow P_{downs} > P_{downscritical}$

This relationship can be used to calculate $P_{downscritical}$ for a given upstream choke pressure P_{ups}:

$$P_{downscritical} = r_c * P_{ups} = \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma - 1}} * P_{ups}$$

4-11

4.2.4. Surface flow-line equation

Here, the generalized flow equation for gas flow through a horizontal pipe is used. The following assumptions are considered as explained in section 4.2.2: single-phase, compressible, Newtonian fluid and steady state flow. As stated before, the equation is derived from the generalized energy balance equation *(Economides, Hill and Ehlig-Economides 1994)*. When applied to the surface flow-line, we obtain:

$$P_{downs}{}^2 - P_g{}^2 = 1.007 * 10^{-4} * \frac{\gamma_g * f * Z_{avg} * T_{avg} * q^2 * L_{surf}}{d_{pipesurf}{}^5}$$

4-12

P_{downs} is the choke downstream pressure in psia; P_g is the gathering tank pressure or separator pressure; γ_g is the gas specific gravity; f is the Fanning friction factor; Z_{avg} and T_{avg} are respectively the average compressibility factor and the average temperature in °R for the entire length of the pipe; q is the gas flow rate in MSCF/D; L_{surf} is the length of the surface flow line in ft and $d_{pipesurf}$ is its diameter in inches.

In both section 4.2.2 and section 4.2.4, the following equation is used to determine the Fanning friction factor:

$$\frac{1}{\sqrt{f}} = -4 * log * \left(\frac{\varepsilon}{3.7065} - \frac{5.0452}{N_{Re}} * \log\left(\frac{\varepsilon^{1.1098}}{2.8257} + \left(\frac{7.149}{N_{Re}}\right)^{0.8981}\right)\right)$$

4-13

ε is the pipe roughness in inches; N_{Re} is the Reynolds number.

This correlation is called the Chen equation. It has been derived from the commonly used Colebrook-White equation. The advantage that comes from the choice of this equation is that it is explicit in f. Therefore, there is no need to use an iterative procedure like the Newton-Raphson method to be able to generate values for f.

The Reynolds number can be determined from this relationship:

$$N_{Re} = 20.09 * \frac{\gamma_g * q}{d * \mu} \tag{4-14}$$

4.3. Gas well model

The gas well model is a computer program that allows linking the reservoir unit to the equipment at the surface. It has been developed in the MATLAB environment. It mathematically analyzes the flow as a whole from the reservoir to the surface using the equations described in section 4.2.

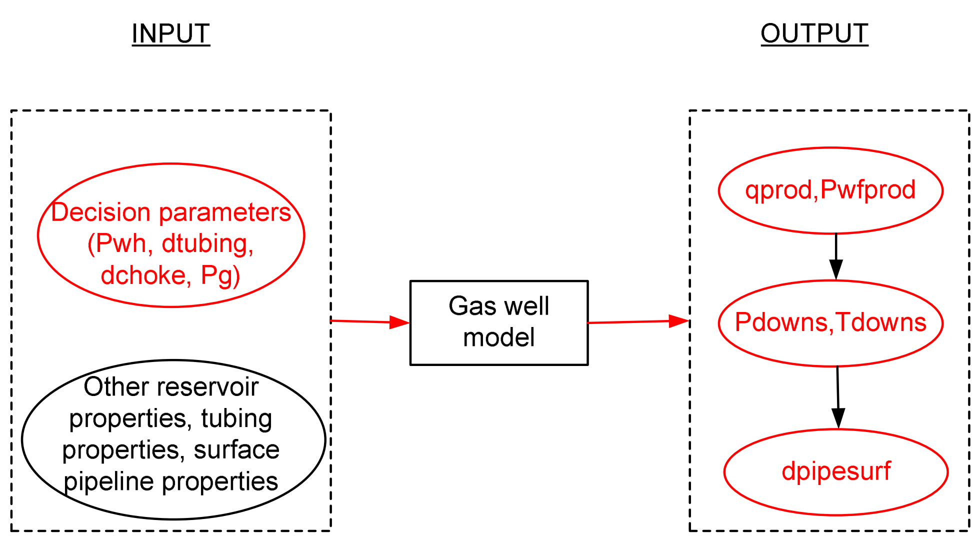

Figure 4-7: Gas well model input/ output flowchart

Figure 4-7 displays the input and output of the gas well model. You need to provide a combination of decision variables (P_{wh}, d_{tubing}, d_{choke}, P_g). These combinations can vary whenever you are running the program. You also need to provide other properties defining the three units of production (reservoir, tubing and surface flow-line) and the flowing fluid:

- Reservoir properties: reservoir pressure, reservoir temperature, formation permeability, drainage radius, wellbore radius, skin factor, non-Darcy coefficient, reservoir thickness, water saturation, porosity, recovery factor, project lifetime
- Fluid properties: gas specific gravity, Molecular weight
- Tubing system properties: pipe roughness, tubing string length, bottom hole temperature, wellhead temperature
- Surface flow-line properties: pipe roughness, length, gathering tank or separator temperature

Once these parameters have been provided to the program, it will first solve a system of equations constituted of the IPR relationship and the VLP relationship. This action will allow to obtain the well deliverability, described by the production rate q_{prod} and the corresponding flowing bottom hole pressure P_{wfprod}. In fact, the well operating point is dictated by the intersection of the following curves:

- The IPR curve: it is a plot of the flowing bottom hole pressure for various gas flow rates q. It can be obtained using equation 4-3. It defines the flow coming from the reservoir to the wellbore (inflow).
- The VLP curve: it is a plot of the flowing bottom hole pressure for various gas flow rates q. It can be obtained using equation 4-6. It describes the flow from the bottom hole of the well to the surface (outflow).

A well is optimized when it is operating at the intersection point of both curves. Those curves are closely linked to parameters such as the wellhead pressure and the tubing size. Changing one of them will have an impact on the well deliverability.

Once the well deliverability (q_{prod}, P_{wfprod}) has been found for the corresponding input parameters, the production rate values serve to determine the pressure and the temperature downstream of the choke, for a given choke size, selected at the beginning of the program. This process is executed using equation 4-8.

Finally, the downstream choke pressure and temperature are used to determine the diameter of the flow-line, which is necessary to the flow of gas at the surface. This can be done with the help of equation 4-12 as the length of the surface pipeline is given. The size of the surface pipeline is extremely important since it affects pipeline economics. Figure 4-8 represents the flow chart of the gas well model MATLAB subroutine and summarizes all the elements described above.

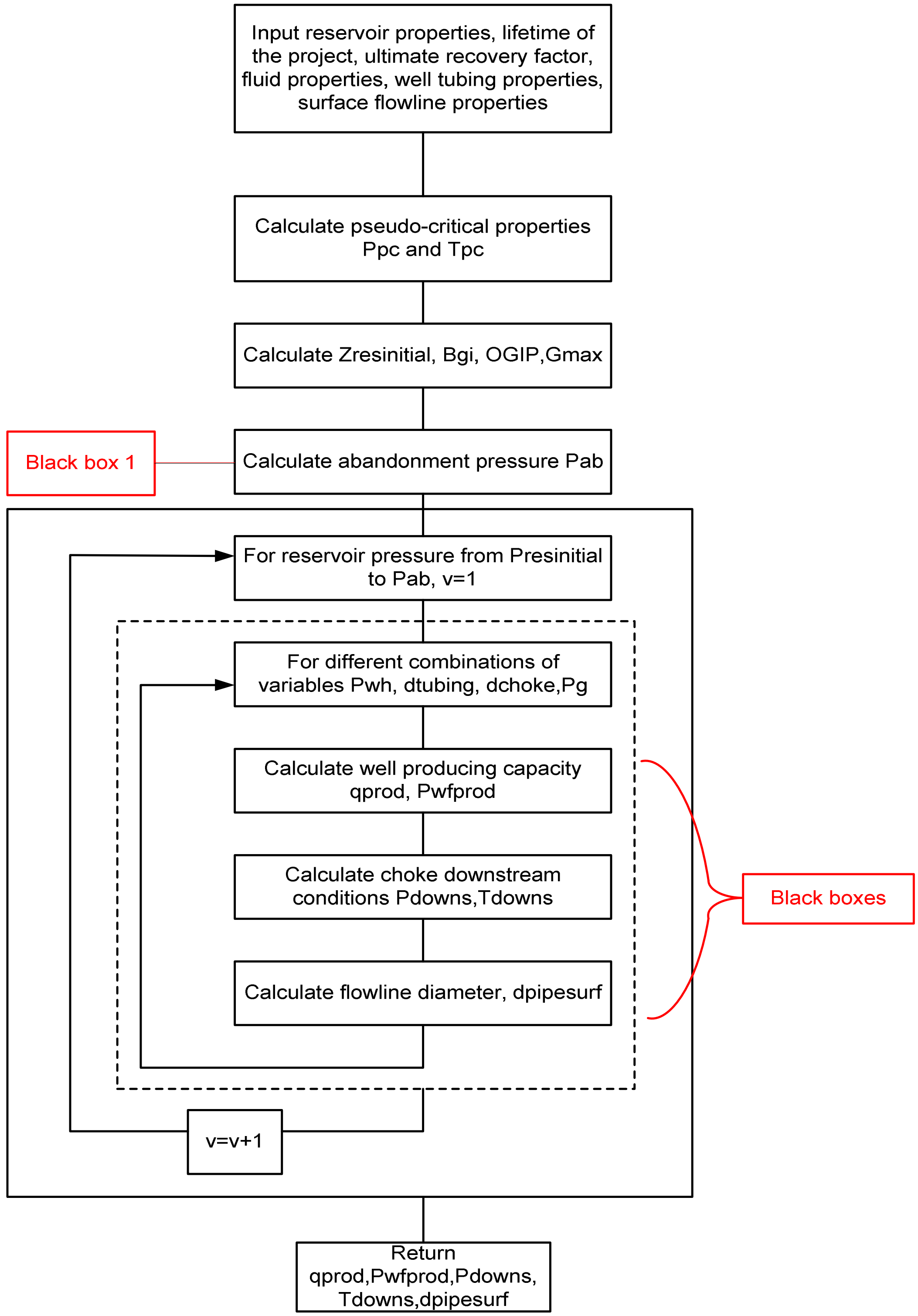

Figure 4-8: Flow chart for the gas well model MATLAB subroutine

In the following sections, we will describe in more detail, the different black boxes contained in the flow chart presented on Figure 4-8 and explain how the equations of section 4.2 are adapted for the problem.

4.3.1. Calculation of the abandonment pressure

Any gas production contract includes an ultimate recovery factor, which combined with the original gas in place and the lifetime of the project, permits the determination of the reservoir pressure at the end of the project, i.e. the abandonment pressure. This can be accomplished using the gas material balance equation:

$$\frac{P_{res}}{Z_{res}} = \frac{P_{resinitial}}{Z_{resinitial}} * \left(1 - \frac{G_p}{OGIP}\right)$$

4-15

P_{res} is the current reservoir pressure in psia; Z_{res} is the current Z-factor; $P_{resinitial}$ is the initial reservoir pressure in psia; $Z_{resinitial}$ is the Z-factor at initial reservoir pressure; G_p is the cumulative gas production of the field at the end of the project lifetime in SCF; OGIP is the initial gas in place in the reservoir in SCF.

Equation 4-15 has been derived for a dry gas reservoir, which is assumed to be volumetric: no water is produced from the reservoir during the entire lifetime of the project and there is no water influx; the initial water saturation stays constant.

The following objective function is used:

$$Func = \frac{P_{res}}{Z_{res}} - \frac{P_{resinitial}}{Z_{resinitial}} * \left(1 - \frac{G_p}{OGIP}\right)$$

4-16

Func will be equal to 0 only when you reach the reservoir pressure that allows you to have the ultimate cumulative production for the gas field $G_{p.}$ Therefore, with the use of equation 4-16, it is possible to get the interval containing the root of Func.

Then, the bisection method *(Harris and Stocker 2006)* is applied in order to obtain the value of the abandonment pressure. Figure 4-9 provides a complete description of the procedure.

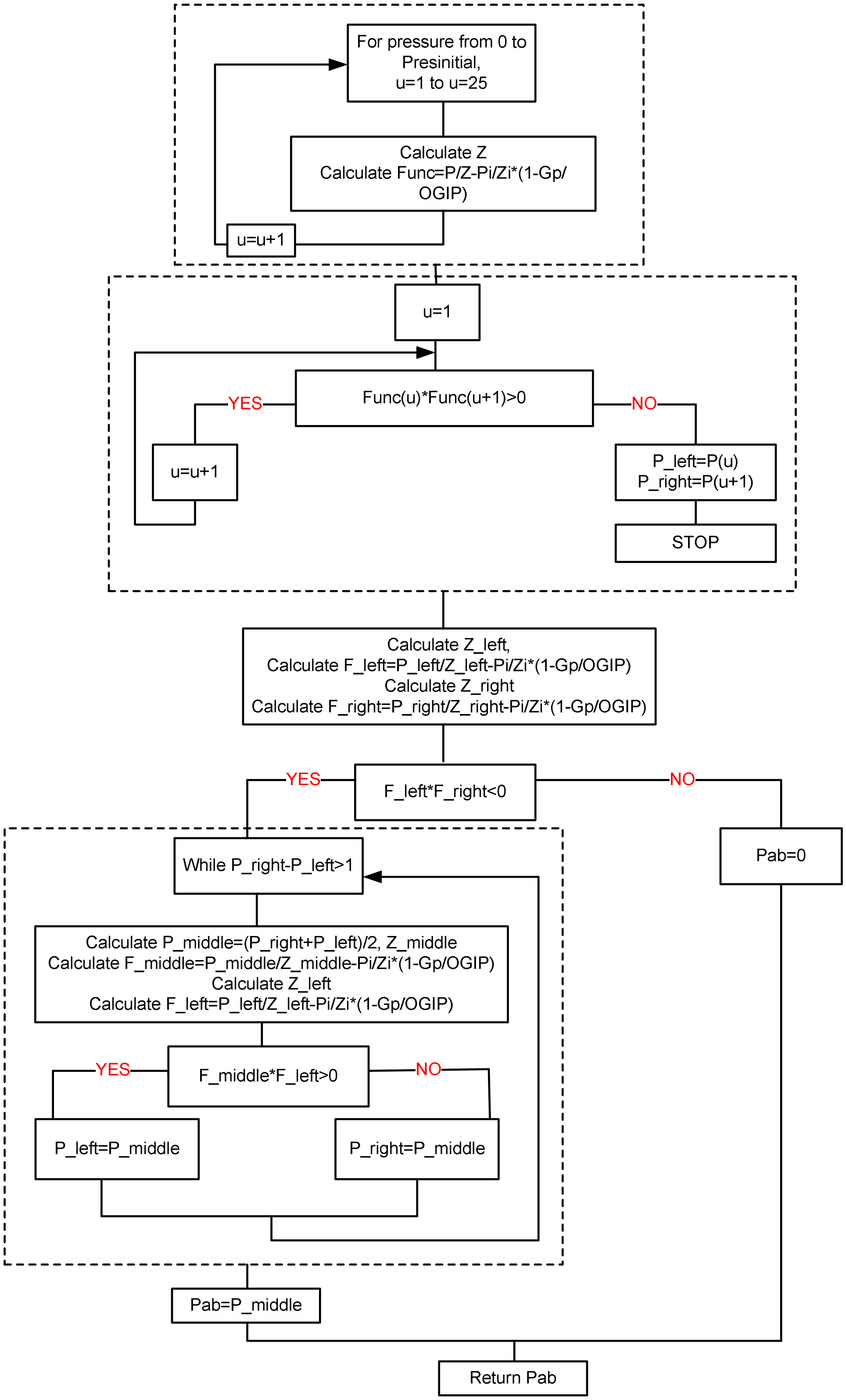

Figure 4-9: Flow chart for abandonment pressure MATLAB subroutine

4.3.2. Calculation of the well producing capacity

The bisection method is used in order to determine the well producing capacity. In other words, the bisection method allows the determination of the well producing flow rate q_{prod} and the corresponding flowing bottom hole pressure P_{wfprod}, for each reservoir pressure and each set of values for the decision variables P_{wh}, d_{tubing}, d_{choke} and P_g. Equations 4-3, 4-4, 4-5, 4-6, 4-7, 4-13 and 4-14 are involved in the determination of the flowing bottom hole pressure for the inflow from the reservoir to the wellbore and for the outflow from the bottom hole to the wellhead, using different flow rate values.

The following expression enables to get an idea of the well producing flow rate:

$$diffP = P_{wf1}^2 - P_{wf2}^2$$

4-17

P_{wf1} is the flowing bottom hole pressure in psia that is calculated from the IPR relationship (Equations 4-3, 4-4, and 4-5) for different values of flow rate q; P_{wf2} is the flowing bottom hole pressure in psia that is calculated from the VLP relationship (Equations 4-6, 4-7, 4-13 and 4-14). diffP will be equal to 0 only when you reach the producing capacity q_{prod} of the well.

Then, the bisection method is applied to the following objective function F_q:

$$F_q = {P_{res}}^2 - (\alpha * q + \beta * q^2) - \left(P_{wh}^2 * exp(-S) + 2.685 * 10^{-3} * \frac{f * (Z_{avg} * T_{avg})^2 * q^2 * (exp(-S) - 1)}{sin\theta * {d_{tubing}}^5}\right)$$

4-18

Figure 4-10 summarizes all the procedures used in order to obtain the producing capacity of the well.

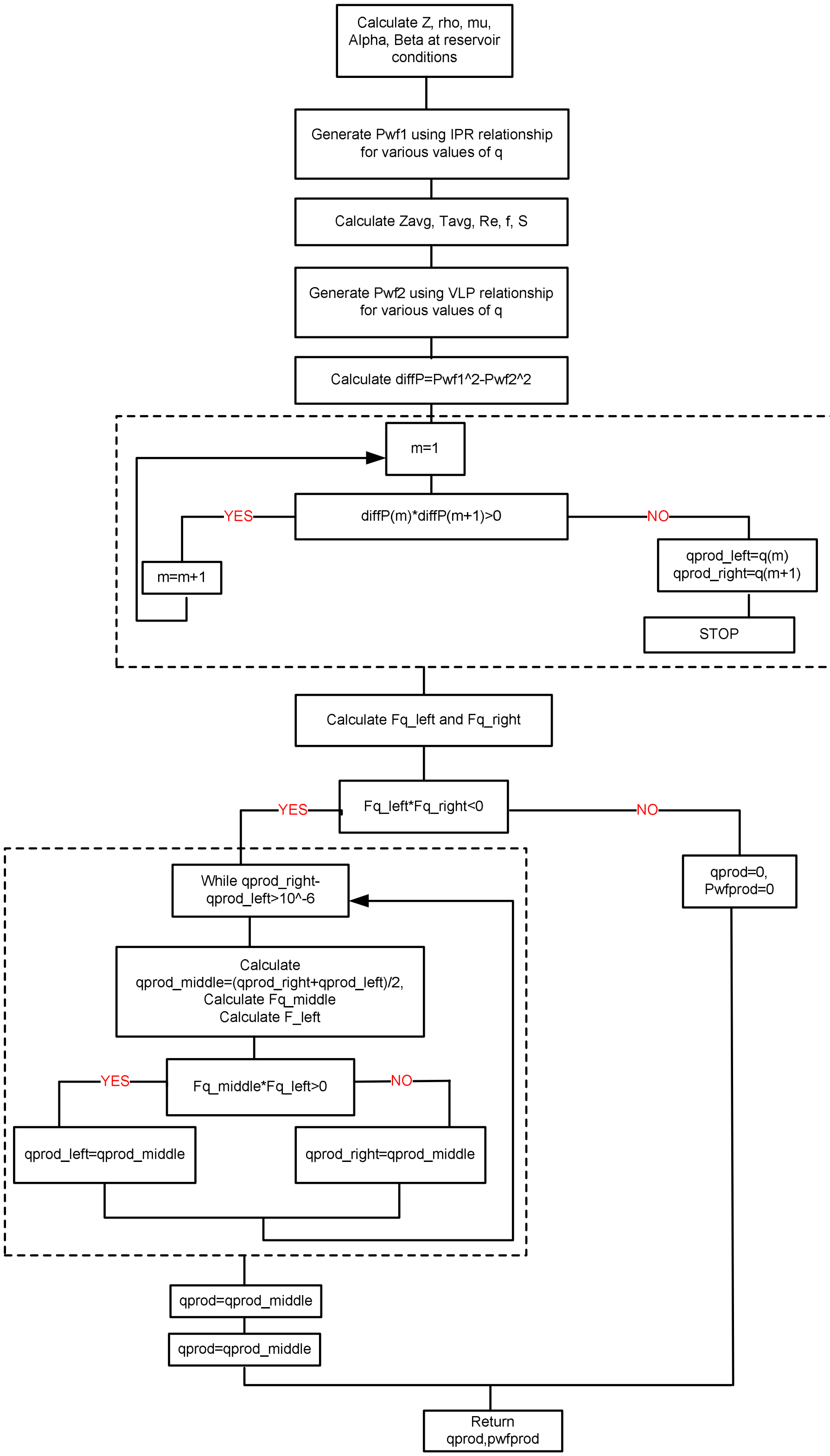

Figure 4-10: Flow chart for the well producing capacity MATLAB subroutine

4.3.3. Calculation of the choke downstream conditions

The next step in the program is to find the choke downstream pressure and temperature, which correspond to the calculated well producing capacity and to the given choke size. This is accomplished with the help of equation 4-8.

As explained in section 4.1.3, we have to make sure that the choke operates in the subcritical flow regime. Therefore, we have to calculate first the choke downstream critical pressure using equation 4-11. The following function helps to determine the value of the choke downstream pressure that corresponds to the calculated well producing capacity:

$$Q = -798.796 * \mathrm{d}_{\mathrm{choke}}^{2} * P_{wh} * (\frac{\gamma}{\gamma_g * T_{wh} * (\gamma - 1)} * \left(\left(\frac{P_{downs}}{P_{wh}}\right)^{\frac{2}{\gamma}} - \left(\frac{P_{downs}}{P_{wh}}\right)^{\frac{\gamma+1}{\gamma}}\right)^{0.5} + \mathrm{q}_{\mathrm{prod}} \tag{4-19}$$

d_{choke} is the choke size in inches; P_{wh} is the wellhead pressure in psia; γ is the specific heat ratio; γ_g is the gas specific gravity; T_{wh} is the wellhead temperature in ˚R; q_{prod} is the calculated well producing capacity in MSCFD.

Q is estimated for several values of P_{downs} and Q is also estimated at the critical choke downstream pressure $P_{downscritical.}$ This value of the objective function Q is called $Q_{critical}$. The objective function Q will be equal to 0 for a specific value of the choke downstream pressure and only for some combinations of the decision variables P_{wh}, d_{tubing}, d_{choke} and P_g. In fact, in certain cases, the choke size will be too small to support the well producing capacity and the objective function Q will not be equal to 0. Figure 4-11 displays the relationship between Q and P_{downs}.

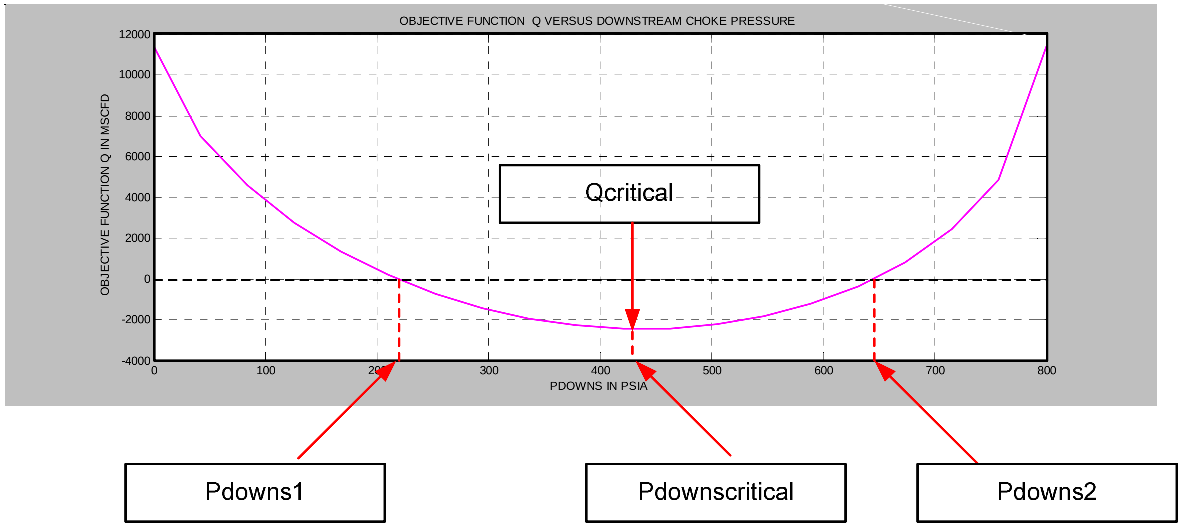

Figure 4-11: Objective function Q versus Pdowns

As it is explained in section 4.2.3 and as it is showed in the previous figure, we have to select the interval containing the choke downstream pressure P_{downs2} as it is the pressure value that will ensure that the choke operates in the subcritical flow regime. Finally, the bisection method is applied to function Y=-Q. The procedures used in the computer program are summarized on Figure 4-12.

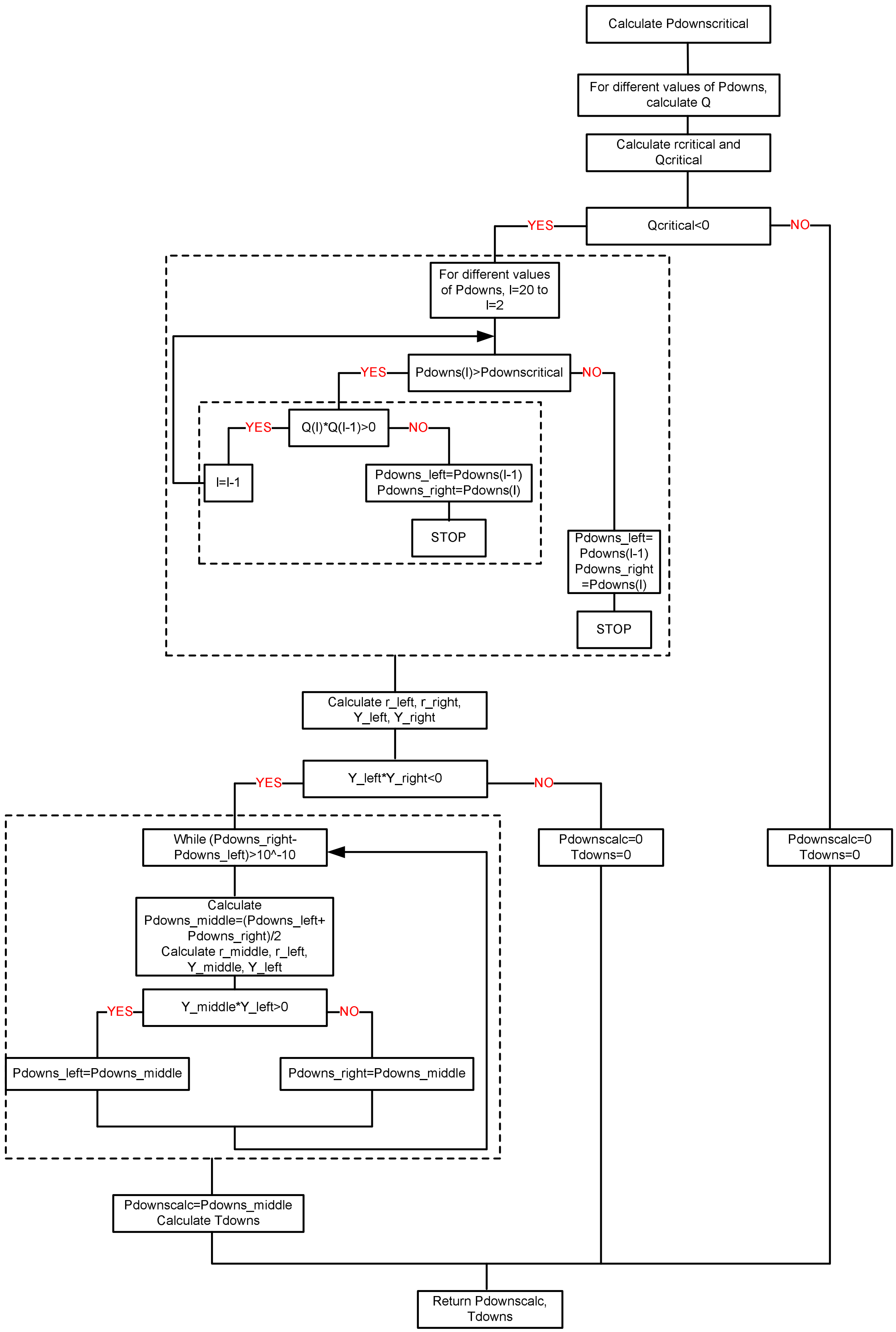
Calculate Pdownscritical
For different values of Pdowns, calculate Q
Calculate rcritical and Qcritical
Qcritical<0
YES
NO
For different values of Pdowns, I=20 to I=2
Pdowns(I)>Pdownscritical
YES
NO
Q(I)*Q(I-1)>0
YES
NO
I=I-1
Pdowns_left=Pdowns(I-1) Pdowns_right=Pdowns(I)
STOP
Pdowns_left= Pdowns(I-1) Pdowns_right =Pdowns(I)
STOP
Calculate r_left, r_right, Y_left, Y_right
Y_left*Y_right<0
YES
NO
While (Pdowns_right- Pdowns_left)>10^-10
Calculate Pdowns_middle=(Pdowns_left+ Pdowns_right)/2 Calculate r_middle, r_left, Y_middle, Y_left
Y_middle*Y_left>0
YES
NO
Pdowns_left=Pdowns_middle
Pdowns_right=Pdowns_middle
Pdownscalc=0 Tdowns=0
Pdownscalc=0 Tdowns=0
Pdownscalc=Pdowns_middle Calculate Tdowns
Return Pdownscalc, Tdowns

Figure 4-12: Flow chart for the choke downstream condition MATLAB subroutine

4.3.4. Calculation of the flow-line diameter

The final step of the program enables to determine the diameter of the surface pipeline which connects the surface choke to the gathering tank. The following objective function is used:

$$K = d - \left(\frac{\left(1.007 * 10^{-4} * \gamma_g * f * T_{avg} * Z_{avg} * q_{prod}^2 * L_{surf}\right)}{P_{downscalc}^2 - P_g^2}\right)^{\frac{1}{5}} \quad \text{4-20}$$

d is the surface flow-line diameter in inches; f is the Fanning friction factor; T_{avg} and Z_{avg} are the average temperature and the average compressibility factor for the entire length of the pipeline; q_{prod} is the calculated well producing capacity in MSCFD; L_{surf} is the length of the surface flow-line in ft; $P_{downscalc}$ is the calculated choke downstream pressure in psia; P_g is the gathering tank pressure in psia.

Function K is estimated for different values of the flow-line diameter d. These results are then used to determine the interval containing the root of the objective function. K will be equal to 0 for the only value of the flow-line diameter that corresponds to the calculated well producing capacity and choke downstream pressure, which are the results to the selected combination of decision variables P_{wh}, d_{tubing}, d_{choke}, and P_g. Finally, knowing that interval, the bisection method can be applied to the function K in order to find the flow-line diameter associated to the chosen operating conditions. The procedures used in the MATLAB subroutine are summarized on Figure 4-13.

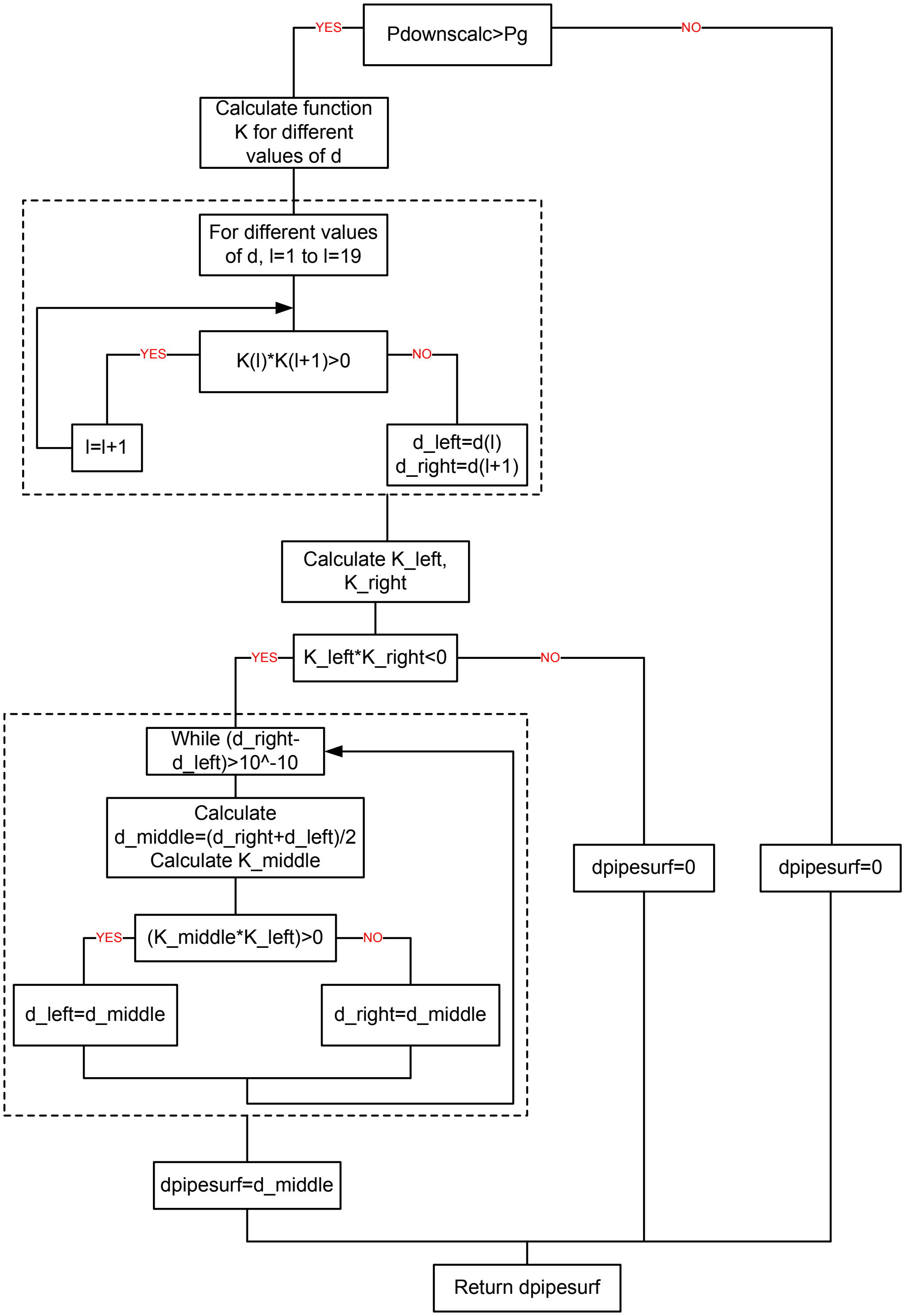

Figure 4-13: Flow chart for the flow line diameter MATLAB subroutine

4.4. Modified gas well model

The modified gas well model takes into account the optimization aspect of the problem. Given different combinations of decision variables (P_{wh}, d_{tubing}), the gas project requirements (recovery factor, project lifetime), other reservoir properties, fluid properties, and given well tubing properties, the modified gas well model will determine the well producing capacity at abandonment conditions. Indeed, if the chosen operating conditions can sustain the production at abandonment pressure, they will be capable of sustaining the production during the entire life of the project.

A gas production contract for a given field is defined as follows: given reservoir properties, fluid properties, an estimation of the total natural gas reserves (OGIP, Original Gas In Place at initial reservoir pressure), a company will agree to produce a constant specified quantity of gas Q_{field} per year, over the life of the project t_p. This automatically fixes the ultimate recovery factor of that field, which is defined by this relationship:

$$RF_{contract} = 365000 * \frac{Q_{field} * t_p}{OGIP} = \frac{G_p}{OGIP} \tag{4-21}$$

G_p is the cumulative gas production at the end of t_p years in SCF; t_p is the lifetime of the project; OGIP is the Original Gas In Place in SCF; Q_{field} is the total gas field production rate per year in MSCFD

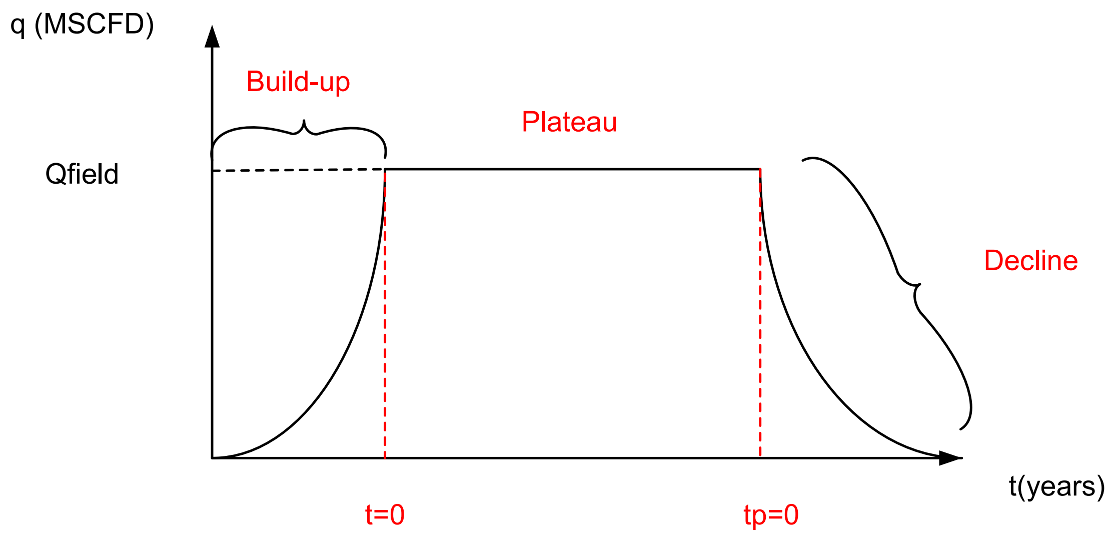

Figure 4-14: Typical production pattern of a natural gas field

Figure 4-14 describes the production pattern for most gas production fields. The build-up phase coincides with the drilling phase. As you drill more wells, the total field production rate increases until you reach the number of wells N_w that will allow you to attain the pre-specified total field production rate target.

N_w is defined as follows:

$$Q_{field} = q_{well} * N_w \quad \textbf{4-22}$$

Q_{field} is the total gas field production rate per year in MSCFD; q_{well} is the well producing capacity in MSCFD; N_w is the number of wells.

q_{well} is defined by nodal analysis. A well is called optimized when it is producing at its well capacity, which is the solution to the inflow performance relationship and the outflow performance relationship. Figure 4-15 presents the inflow performance relationship curve and the outflow curve.

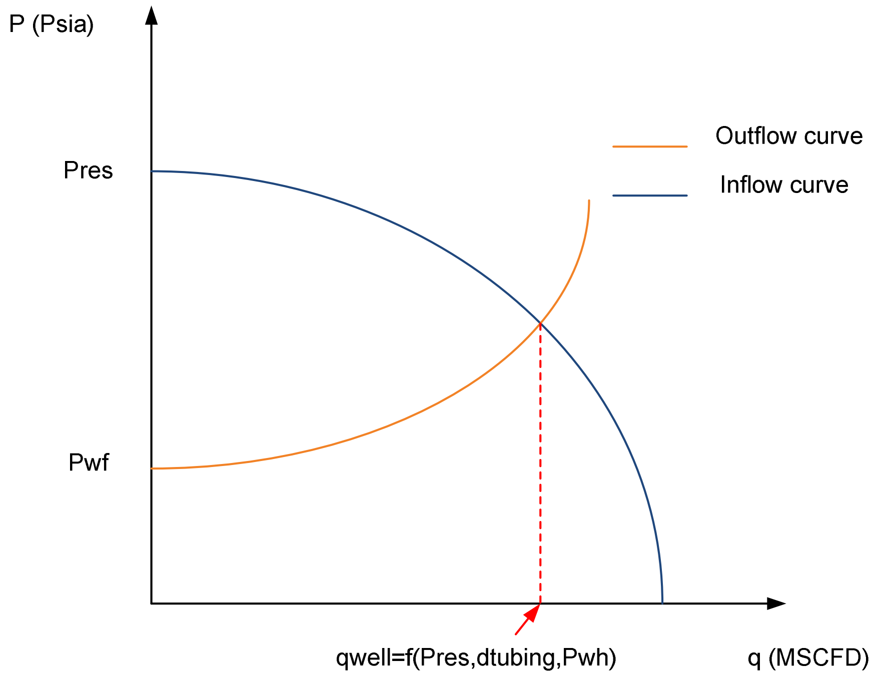

Figure 4-15: Well producing capacity

At a given reservoir pressure, the well producing capacity is affected by the combination of design parameters. As reservoir pressure declines, you will have different sets of design parameters that will enable the attainment of the field production target.

We are going to use net present value as objective function and we will try to find the values of the chosen design parameters that maximize it. Net present value can be defined as follows:

$$NPV = PV(Annual\ cashflow) - Initial\ investment \tag{4-23}$$

With :

$$PV(Annual\ cashflow) = (Yearly\ revenue - Yearly\ cost) * \frac{(1+i)^{t_{max}} - 1}{i * (1+i)^{t_{max}}} \tag{4-24}$$

PV(Annual cashflow) is the present value of the annual cash flow generated by the cumulative gas production; Yearly revenue is the revenue generated per year by selling the gas production; Yearly cost is the total operating cost per year for all the wells drilled; t_{max} is the project lifetime; i is the discount rate. Figure 4-16 illustrates how the right selection of the decision variables will influence the net present value of the project.

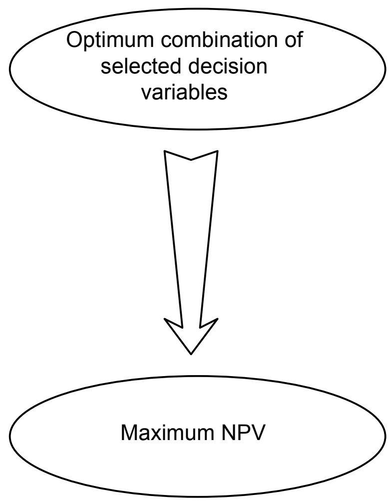

Figure 4-16: NPV maximization flow chart

It is essential to evaluate the initial investment of the project which is related to the number of wells to be drilled, but also to the type of wells to drill. The following function can be used to estimate the initial investment and it is inspired from the power law and sizing model which is often used for equipment cost estimating in the oil and gas industry (*M. Mian 2002*):

$$Initialcost = C_w * \left(\frac{d_{tubing}}{d_{tubing\ nominal}}\right)^5 * N_w \quad \text{4-25}$$

C_w is the total cost of drilling and completion per well in US dollars for a well with the same depth as the wells drilled in our study and with a tubing diameter of $d_{tubing\ nominal}$; d_{tubing} is the tubing size of interest in inches; N_w is the total number of wells drilled in the gas field.

For the gas well model, C_w can be expressed as follows:

$$C_w = \frac{60.17\$}{ft} * L_{vert} \tag{4-26}$$

L_{vert} is the well depth in ft; 60.17$/ft is the average cost of drilling and completion per ft and per well. This value has been inspired by the results of the Joint Association Survey on drilling costs for onshore gas wells (*JAS 1992*).

A diameter of 8 inches has been chosen for the nominal tubing diameter. This value of the nominal tubing diameter and the value of the exponent in equation 4-25, have been chosen via trial and error and gave a good agreement with the concept that we were trying to model; which is that the drilling cost for a large tubing size (7 in-12 in) should be much higher than the drilling cost for a small tubing size (0.5 in-3 in). The proposed cost function is an attempt to capture the influence of tubing size on the total cost of the project. Figure 4-17 presents the flow chart of the MATLAB subroutine for the modified gas well model.

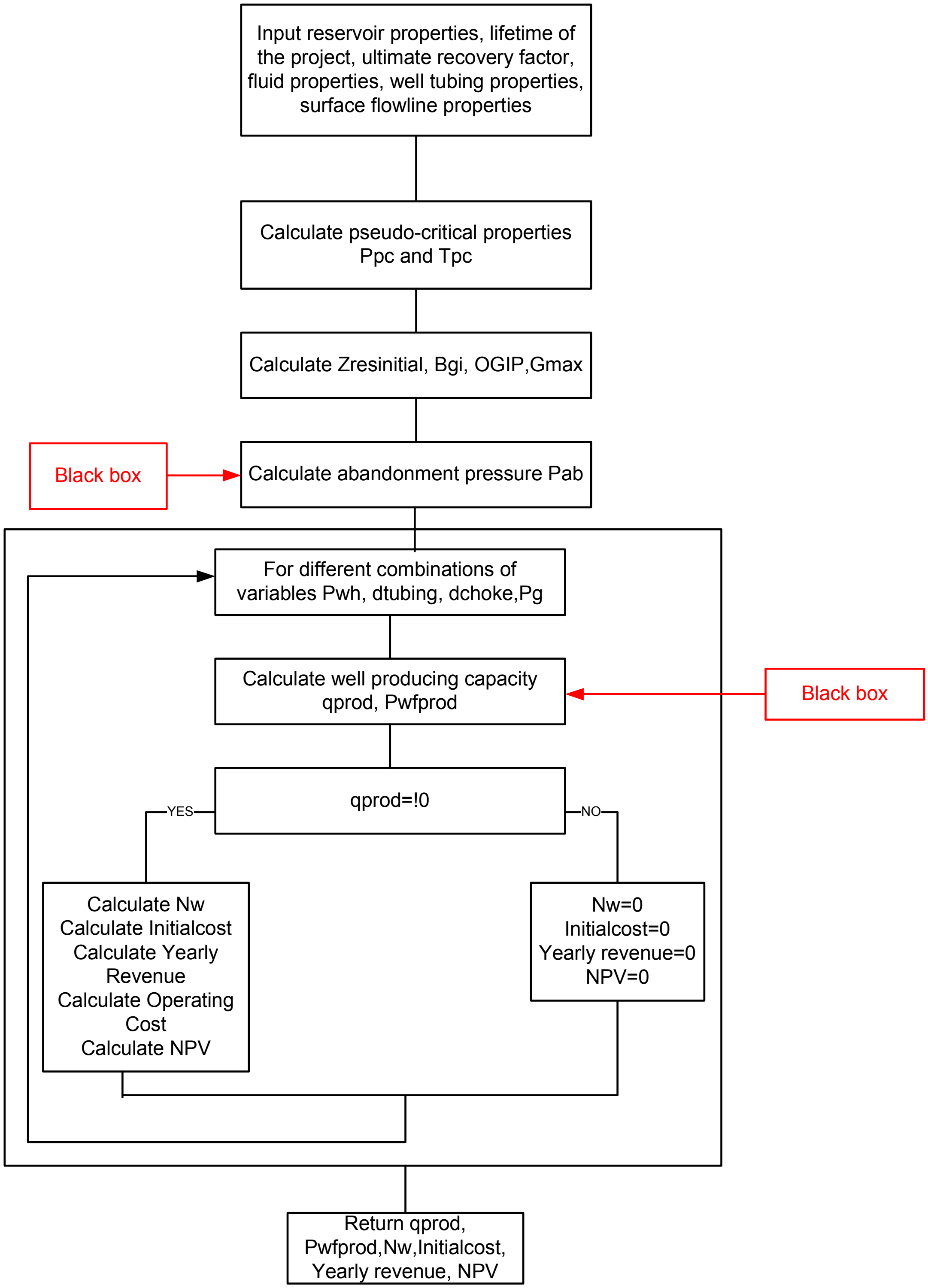

Figure 4-17: Flow chart of the MATLAB subroutine for the modified gas well model

Chapter 5

Sensitivity analysis

In this chapter, we will present the results obtained from the gas well model by applying it to different production scenarios. We will test several ranges of values for the selected decision variables P_{wh}, d_{tubing}, d_{choke}, and P_g and analyze the results. We will be looking at sensitivity analysis in pairs in order to show the interconnectivity between those variables and we will examine the effect of the decision variables on production.

These are the scenarios that are going to be tested:

- Scenario 1: effect of wellhead pressure and tubing size
- Scenario 2: effect of wellhead pressure and choke size
- Scenario 3: effect of wellhead pressure and gathering tank pressure
- Scenario 4: effect of tubing size and choke size
- Scenario 5: effect of choke size and gathering tank pressure

5.1. Presentation of the case study

In the following sections, we are describing the parameters that need to be provided to the gas well model and which describe every unit of interest in the production system. The values associated to the parameters used for this case study are inspired by field data used by Kappos and Economides for their work on gas production optimization *(Kappos and Economides 2005)*. These values are fixed when applying the gas model for different scenarios as we are only trying to determine the influence of the wellhead pressure P_{wh}, the tubing size d_{tubing}, the choke size d_{choke} and the gathering tank pressure $P_{g.}$.

5.1.1. Reservoir properties

We are studying a dry gas reservoir with the following properties:

Table 5-1: Reservoir properties for the case study

PARAMETERS	VALUES
Initial reservoir pressure, psia	12000
Reservoir temperature, °R	640
Permeability, md	650
Porosity	19%
Water saturation	0.15
Reservoir thickness, ft	100
Drainage radius, ft	6000
Wellbore radius, ft	0.328
Skin factor	2
Non-Darcy coefficient	0.049

Here are the requirements for production from this gas field:

- The lifetime of the project is set to be 30 years
- The ultimate recovery factor is 80%

Changing one of these requirements will modify the total field production rate per year, which will then have an impact on the cash-flow generated from the project and therefore change the net present value.

We are going to take into account reservoir depletion in the choice of the design parameters. Given, the project requirements, the program is able to evaluate the abandonment pressure that is the pressure at which the field will reach the ultimate recovery factor of 80%. In our case, the abandonment pressure is equal to 1418.9 psia.

Table 5-2 contains the reservoir pressure values that are going to be tested in the program from the initial reservoir pressure to the abandonment pressure.

Table 5-2: Reservoir pressure values

P_{res} in psia
12000
11443.1
10886.2
10329.3
9772.4
9215.5
8658.6
8101.7
7544.8
6987.9
6431
5874.1
5317.2
4760.3
4203.4
3646.5
3089.6
2532.7
1975.8
1418.9

5.1.2.Fluid properties

We are considering single phase gas as the flowing fluid. No phase transition takes place in the production system during the entire life of the project. Table 5-3 contains the gas specific gravity and the molecular weight.

Table 5-3: Flowing fluid properties

PARAMETERS	**VALUES**
Gas gravity	0.55
Molecular weight, lbm/lbmol	16

5.1.3.Tubing system

Here are the parameters that need to be provided to the gas well model concerning the tubing system:

Table 5-4: Tubing system properties

PARAMETERS	VALUES
Pipe roughness ε, in	0.006
Tubing string length, ft	10000
Bottom hole temperature, °R	560
Wellhead temperature, °R	550

5.1.4.Surface flow-line properties

The following parameters are used to describe the pipeline connecting the surface choke to the gathering tank.

Table 5-5: Surface flow-line properties

PARAMETERS	VALUES
Pipe roughness ε, in	0.0025
Length, ft	13055
Gathering tank temperature, °R	540

5.2. Scenario 1: Effect of wellhead pressure and tubing size

In this scenario, we are going to run the gas well model using the input parameters described in section 5.1.We have chosen a broad range for the wellhead pressure and the tubing diameter. The decision variables, which are the wellhead pressure P_{wh}, the tubing size d_{tubing}, the choke size d_{choke}, and the gathering tank pressure P_g, will take the following magnitudes:

- The gathering tank pressure value is set at 1500 psia
- The choke size is set at 36/64 inches
- The wellhead pressure will vary from 100 psia to 11500 psia with a constant increment
- The tubing size will vary from 0.1 inches to 12 inches with a constant increment

The values are contained in Table 5-6:

Table 5-6: Wellhead pressure and tubing sizes for scenario 1

P_{wh} in psia	d_{tubing} in psia
100	0.1
700	0.72632
1300	1.3526
1900	1.9789
2500	2.6053
3100	3.2316
3700	3.8579
4300	4.4842
4900	5.1105
5500	5.7368
6100	6.3632
6700	6.9895
7300	7.6158
7900	8.2421
8500	8.8684
9100	9.4947
9700	10.121
10300	10.747
10900	11.374
11500	12

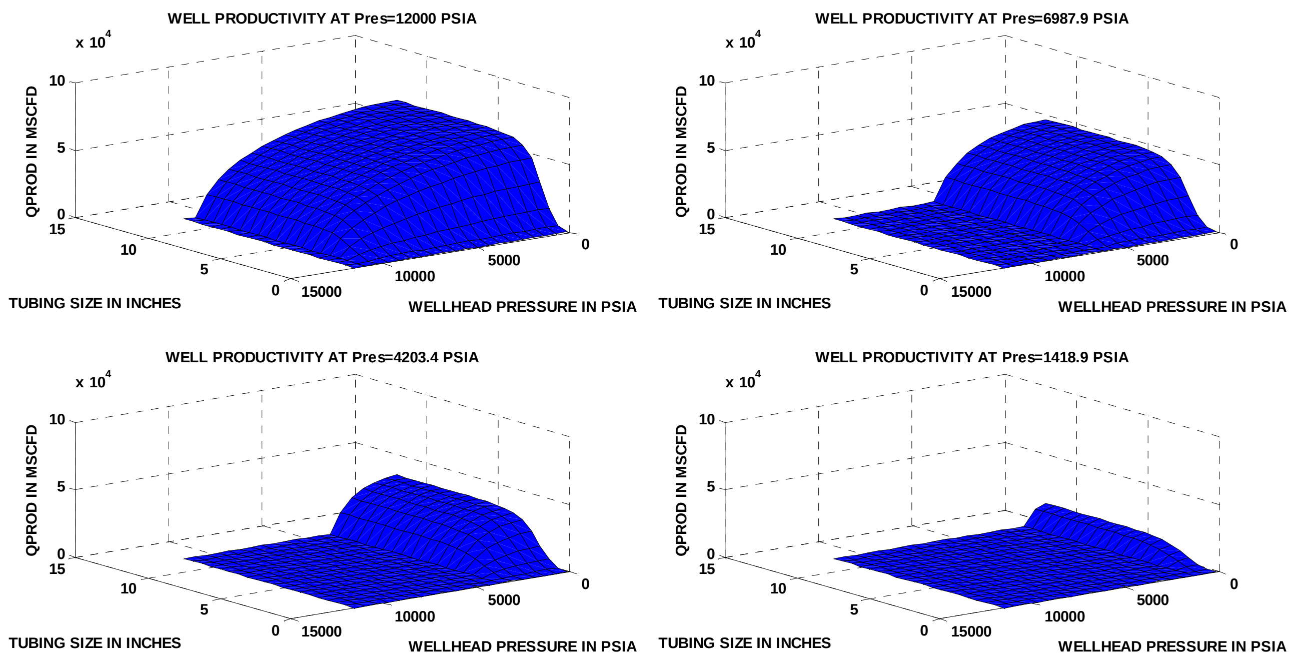

Figure 5-1: Well producing capacity versus tubing size and wellhead pressure (scenario 1) for Pres=12000 psia, Pres=6987.9 psia, Pres=4203.4 psia and Pres=1418.9 psia

Figure 5-1 displays how the calculated well producing capacity varies with changes in wellhead pressure and tubing size as the reservoir depletes in time. As we can see on Figure 5-1, when the reservoir pressure depletes due to gas production from the reservoir, there is a decrease in the producing capacity from the well. This can be explained by the fact that the IPR curve is shifted downward as reservoir pressure depletes in time, and therefore at fixed operating conditions, the well producing capacities become smaller and smaller. In our case, the well producing capacity goes from 61628 MSCFD (highest point on the “WELL PRODUCTIVITY AT P_{res}=12000 psia” surface) to 14276 MSCFD (highest point on the “WELL PRODUCTIVITY AT P_{res}=1418.9 psia” surface). Those two points correspond to a tubing size of d_{tubing}=12 in and to a wellhead pressure of P_{wh}=100 psia . For all the reservoir pressures that are presented on Figure 5-1, the highest well producing capacity is obtained for the same tubing size and wellhead pressure.

It is noticeable that some areas on the previous plots presented on Figure 5-1 are flat. Those areas describe the combinations of wellhead pressure and tubing size that are not doable; the program returns q_{prod}=0 for these combinations of wellhead pressure and tubing size. For all reservoir pressures, we can also observe an increase in the well producing capacity by increasing the tubing size of the well and by decreasing the wellhead pressure.

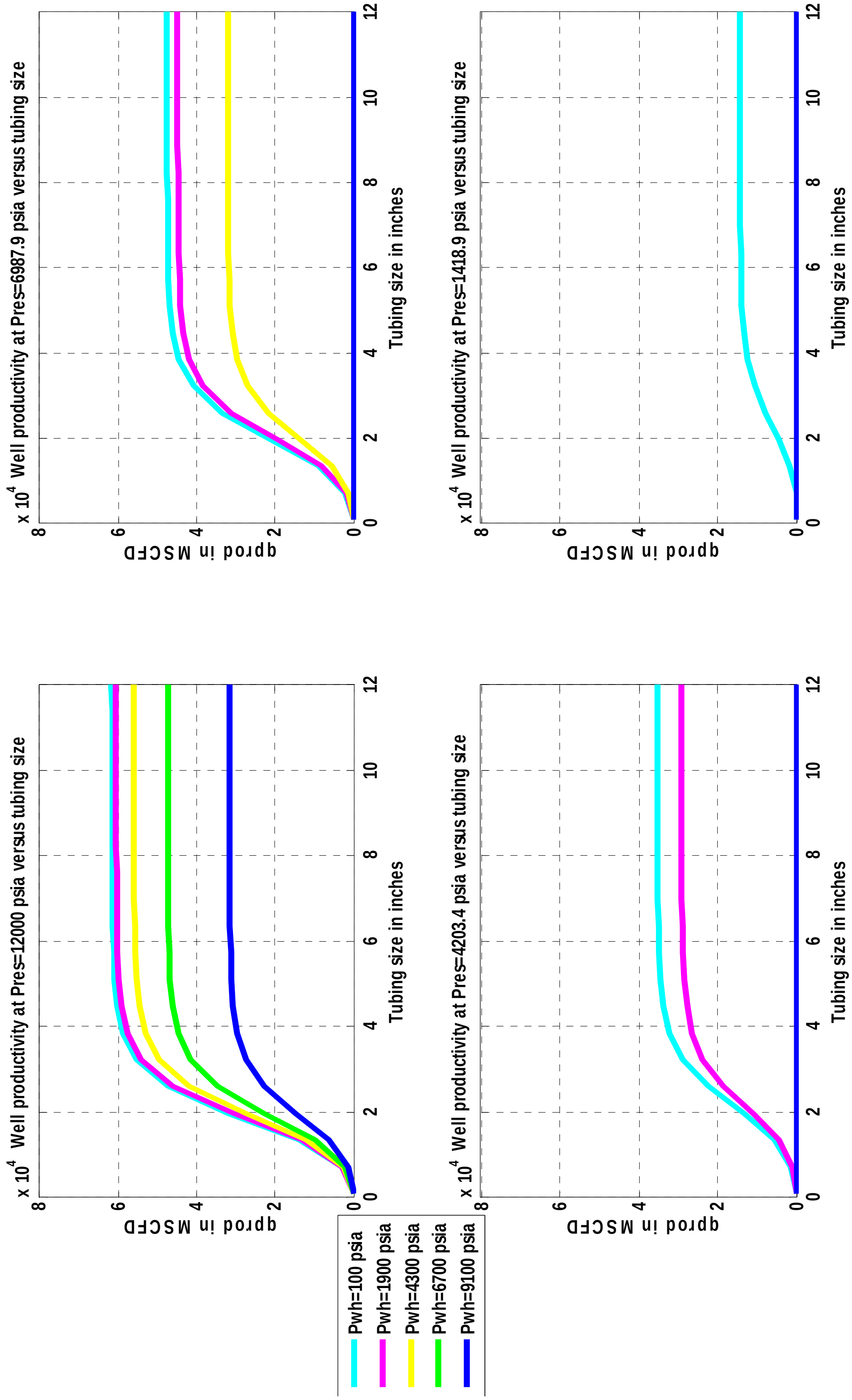

Figure 5-2: Well producing capacity versus tubing size for Pres=12000 psia, Pres=6987.9 psia, Pres=4203.4 psia and Pres=1418.9 psia

Figure 5-2 displays the well producing capacity versus the tubing size, for different wellhead pressures and at different reservoir pressures. Once again, we can see that as we decrease the reservoir pressure, we decrease the well producing capacity, despite the chosen operating conditions.

At all reservoir pressures, for a fixed wellhead pressure, we realize that by increasing the tubing size of the well, we are increasing its producing capacity. This is due to the fact that for a fixed pressure drop in the well tubing, an increase in the tubing size will cause an increase in the gas flow rate (see Vertical Lift Performance relationship in section 4.2). More importantly, we notice that this is true up to a tubing size value which is between 5 in and 6 in. Above this value, a further increase in the size of the tubing string, will create no significant improvement in the production rate; the well productivity curves become flat. This can be explained by the fact that choosing a tubing size, which is too large, can cause the well to load up with liquids and die. It will also generate an increase in the project initial investment. This will be explained in more details in chapter 6, where we will be discussing the optimization procedure.

As the reservoir depletes due to production, it is noticeable that certain wellhead pressures cannot be used to produce the well. In fact, in this scenario, wellhead pressures vary between 100 psia and 11500 psia. The wellhead pressures that can be used at initial reservoir pressure range between 100 psia and 10300 psia; however, as reservoir pressure decreases, the highest wellhead pressure values do not enable to continue to produce the well. There is no common solution to the Inflow Performance Relationship and to the Vertical Lift Performance Relationship. Therefore, the well producing capacity becomes equal to 0. Thus, at the abandonment pressure (Pres=1418.9 psia), the only curve displayed gives the well producing capacity at Pwh=100 psia, for different tubing sizes. This result is in accordance with what we observe on the well productivity surface at the abandonment pressure, when looking at Figure 5-2. The well produces only for wellhead pressures ranging from 1300 psia to 100 psia. The following tubing size range (0.72 in-12 in) can be used at all reservoir pressures. There is no gas production at d_{tubing}=0.1 in because this tubing diameter is too small to be able to sustain production from this reservoir, at any point in time.

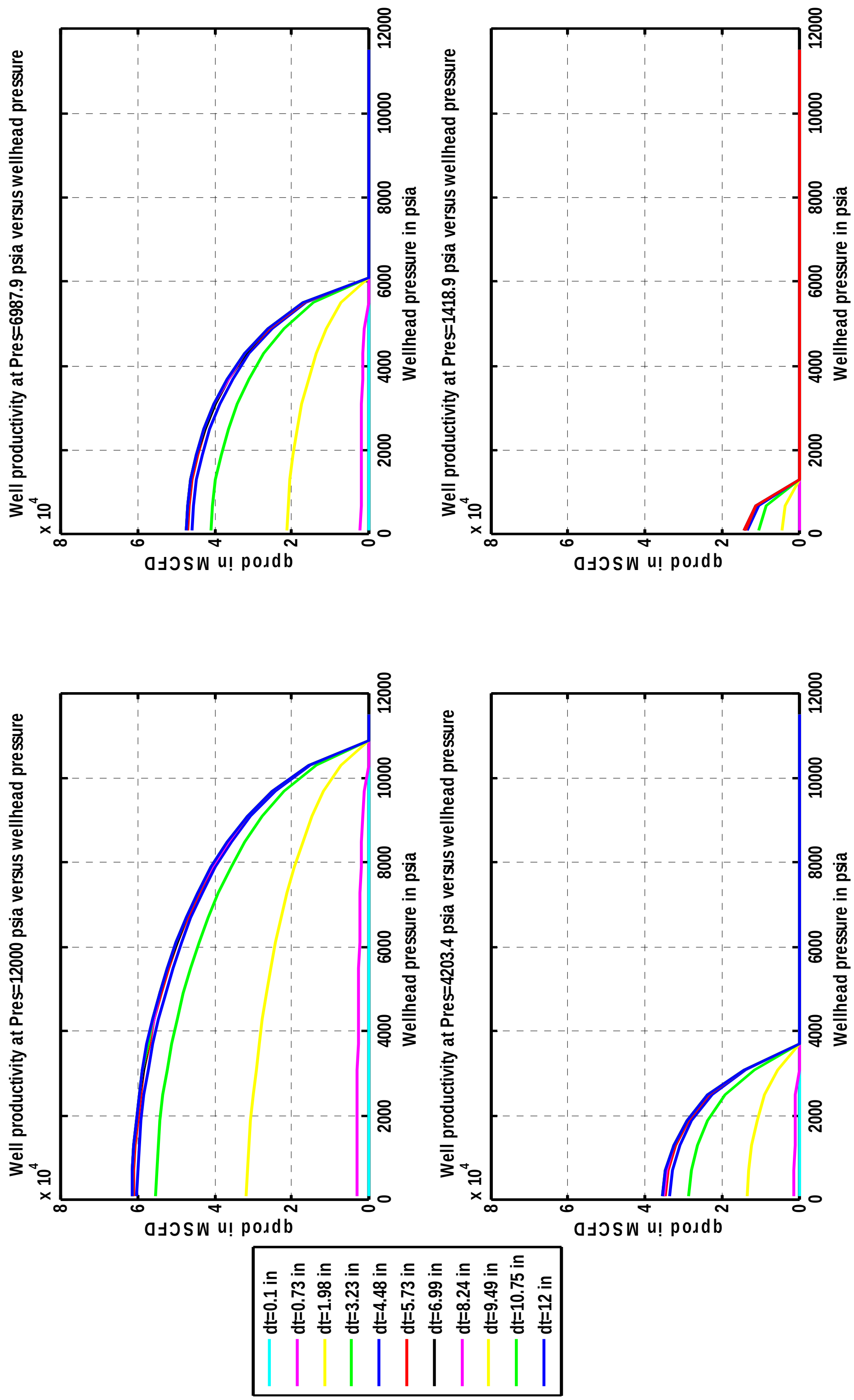

Figure 5-3: Well producing capacity versus wellhead pressure for Pres=12000 psia, Pres=6987.9 psia, Pres=4203.4 psia and Pres=1418.9 psia

Figure 5-3 is showing the influence of wellhead pressure on the well producing capacity, at a given tubing size and a given reservoir pressure. The gas flow rate has been plotted for the following tubing sizes: 0.1 in, 0.73 in , 1.98 in , 3.23 in , 4.48 in , 5.73 in, 6.99 in, 8.24 in , 9.49 in, 10.47 in and 12 in. The following reservoir pressures are considered: 12000 psia, 6987.9 psia, 4203.4 psia and 1418.9 psia.

By decreasing the wellhead pressure, we can generate an increase in the well production rate. This can be explained by the fact that for a constant well tubing size and a constant reservoir pressure, decreasing the wellhead pressure implies increasing the pressure drop in the well tubing, which results in an increase of the production rate (see Vertical Lift Performance Relationship in section4.2). For P_{res} =12000 psia, P_{res}=6987.9 psia and P_{res} =4203.4 psia, we notice that above a wellhead pressure of 2000 psia, there is no more significant increase in the gas production rate from the well. This is true for every tubing size represented on the plots. It is also noticeable that for each reservoir pressure, there is a limit to the highest allowable wellhead pressure value.

At Pres=12000 psia, the maximum allowable wellhead pressure is 10300 psia. This value decreases as the reservoir pressure depletes and at $P_{abandonment}$=1418.9 psia, the maximum allowable wellhead pressure is 1300 psia.

Furthermore, we can observe that there is an increase in the gas production rate as you increase the tubing size from 0.72 in to 4.48 in. For d_{tubing}= 0.1 in, the gas production rate equals 0 at all reservoir pressures and for all wellhead pressures. This is due to the fact that this tubing size is too small to allow production from the well and the well is choked. For tubing sizes that are higher than 4.48 in, it is remarkable that the gas production rate curves are superimposed. This is another way of displaying the fact that passed a certain tubing size, there is no significant improvement in the production rate.

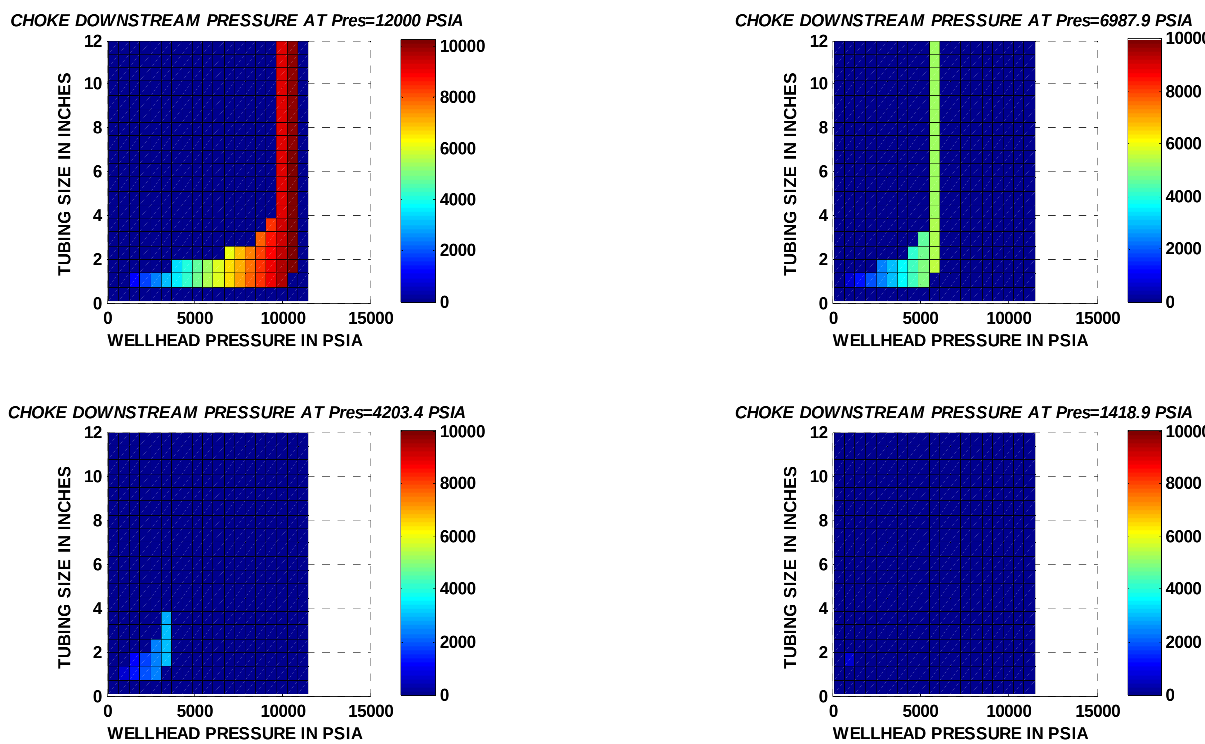

Figure 5-4: Choke downstream pressure versus tubing size and wellhead pressure (scenario 1) for Pres=12000 psia, Pres=6987.9 psia, Pres=4203.4 psia and Pres=1418.9 psia

Figure 5-4 is a top view representing the calculated choke downstream pressure for each available couple formed by the design parameters of interest in this scenario: tubing size and wellhead pressure. Since we are looking at 20 wellhead pressures and 20 tubing sizes, there are 400 possible combinations (P_{wh}, d_{tubing}). From the previous analysis, we can infer that for different reservoir pressures, only some combinations among the 400 available will allow to have gas production from the well. Figure 5-4 displays that among the doable combinations of tubing size and wellhead pressure, only a small portion will enable to have flow through the choke. This result is linked to the chosen choke size. In scenario 1, the choke size is set to be equal to 36/64 in.

The colorbar, which is located next to each subplot, gives the choke downstream pressure range for each color displayed on the plot. Thus all the areas colored in blue represent the non-doable combinations of wellhead pressure and tubing size at a given reservoir pressure. It is noticeable that the non-doable combination area gets larger as reservoir pressure decreases, due to the fact that you need to lower the wellhead pressure in order to continue production from the well until you reach abandonment conditions. At P_{res}=1418.9 psia, the doable tubing sizes range between 1.35 in and 1.98 in whereas the doable wellhead pressures range between 700 psia and 1300 psia. This implies that there is a link between choke size, wellhead pressure and tubing size, which we will try to display in the next scenario.

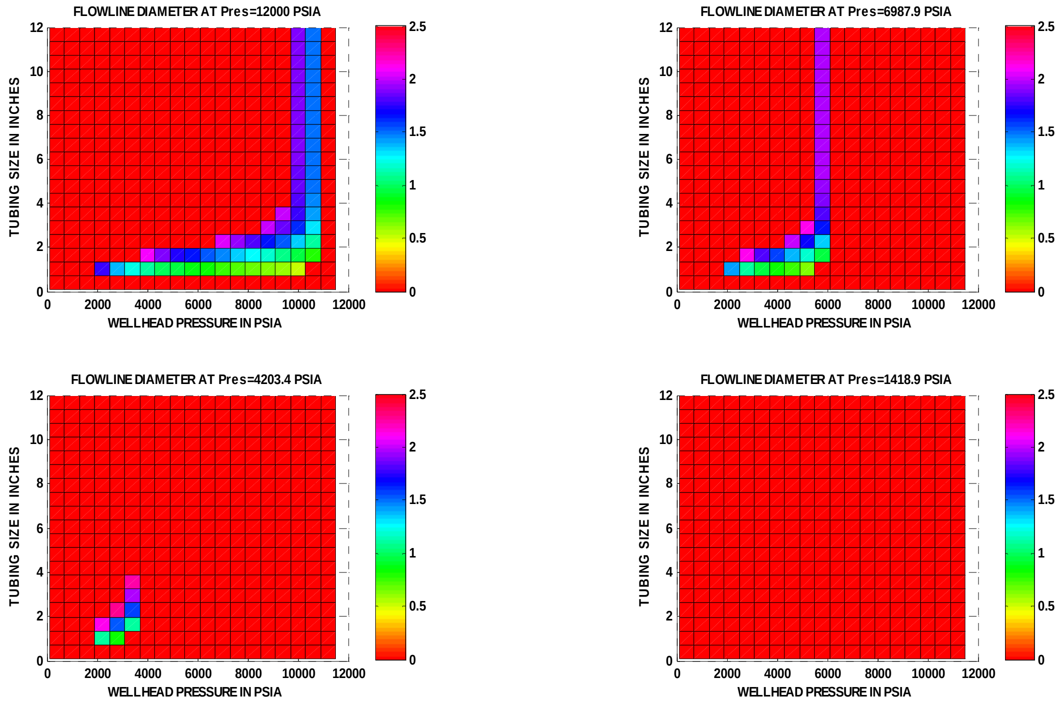

Figure 5-5: Flow-line diameter versus tubing size and wellhead pressure (scenario 1) for Pres=12000 psia, Pres=6987.9 psia, Pres=4203.4 psia and Pres=1418.9 psia

Figure 5-5 displays the calculated flow-line diameter for different combinations of tubing size and wellhead pressure, at different reservoir pressures. It shows the results of the final step in the gas well model which is the calculation of the surface flow-line diameter corresponding to the given conditions. It ends the series of calculations that are conducted from the reservoir to the surface and links the possible production from the well to the surface conditions. The results shown on Figure 5-5 have been obtained using the well producing capacity calculated in the gas well model and the choke downstream conditions that have been obtained from the chosen operating conditions (P_{wh}, d_{tubing}, d_{choke}, P_g). The choke size and the gathering tank pressure have been fixed.

Once again, we can see, whether the doable combinations of wellhead pressure and tubing size resulting from the calculation of the choke downstream conditions, can enable gas flow from the choke to the gathering tank. This is primarily linked to the chosen gathering tank pressure. Indeed, if for a given combination of wellhead pressure and tubing size among the doable solutions according to Figure 5-4, the calculated choke downstream pressure is lower than the gathering tank pressure, there is no solution to the gas flow equation describing the flow through the surface flow-line. In that case, the program returns 0 for the flow-line diameter value.

The colorbar, which is located next to each subplot gives the value of the flow-line diameter that corresponds to the colors displayed on the plot. When looking at Figure 5-5, we notice that the area corresponding to the non-doable combinations of wellhead pressure and tubing size is colored in red. This area enlarges as reservoir pressure depletes from 12000 psia to 4203.4 psia. At Pres=1418.9 psia, the area is entirely red. This translates the fact that none of the (P_{wh}, d_{tubing}) combinations will allow gas flow at the surface, even if the well is technically able to produce. From Figure 5-1 and Figure 5-4, we have seen that if we choose a tubing size within the range 1.35 in-1.98 in and a wellhead pressure within the range 700 psia-1300 psia, there will be gas flow from the reservoir to the surface choke at P_{res}=1418.9 psia. However, the choke downstream pressure obtained for these operating conditions is 686.84 psia, which is lower than the chosen gathering tank pressure. In order to allow flow to occur, you will need to decrease the gathering tank pressure, to increase the wellhead pressure or to increase the choke size.

To conclude, the wellhead pressure and the tubing size directly affect the well productivity. In terms of gas flow from the underground reservoir to the surface, we can observe that all combinations of wellhead pressure and tubing size are not doable when associated to the chosen gathering tank pressure and choke size. From the results obtained at the abandonment pressure for the well productivity, the choke downstream pressure and the flow-line diameter, we can see that you will be able to have gas flow from the reservoir to the choke throughout the life of the project by selecting a tubing size within the range 1.35 in-1.98 in. The corresponding wellhead pressures range from 1900 psia to 10300 psia at the initial reservoir pressure. As the reservoir pressure declines in time, the range of doable wellhead pressures shrinks. At abandonment conditions, the doable wellhead pressures vary from 700 psia to 1300 psia. However, no flow is possible from the surface choke to the gathering tank.

5.3. Scenario 2: Effect of wellhead pressure and choke size

In this case, the objective is to display the impact of the choke size and the wellhead pressure on the production system performance. We have the following input for the decision variables:

- The tubing size is set to be equal to 3.5 inches
- The gathering tank pressure is set to be equal to 1500 psia
- The wellhead pressure will vary between 100 psia and 11500 psia, with a constant increment allowing to have 20 different wellhead pressures

- The choke size will take subsequent values between 8/64 inches and 84/64 inches.

The values used in the program are contained in Table 5-7:

Table 5-7: Wellhead pressure and choke size values for scenario 2

P_{wh} in psia	d_{choke} in inches
100	0.125
700	0.1875
1300	0.25
1900	0.3125
2500	0.375
3100	0.4375
3700	0.5
4300	0.5625
4900	0.625
5500	0.6875
6100	0.75
6700	0.8125
7300	0.875
7900	0.9375
8500	1
9100	1.0625
9700	1.125
10300	1.1875
10900	1.25
11500	1.3125

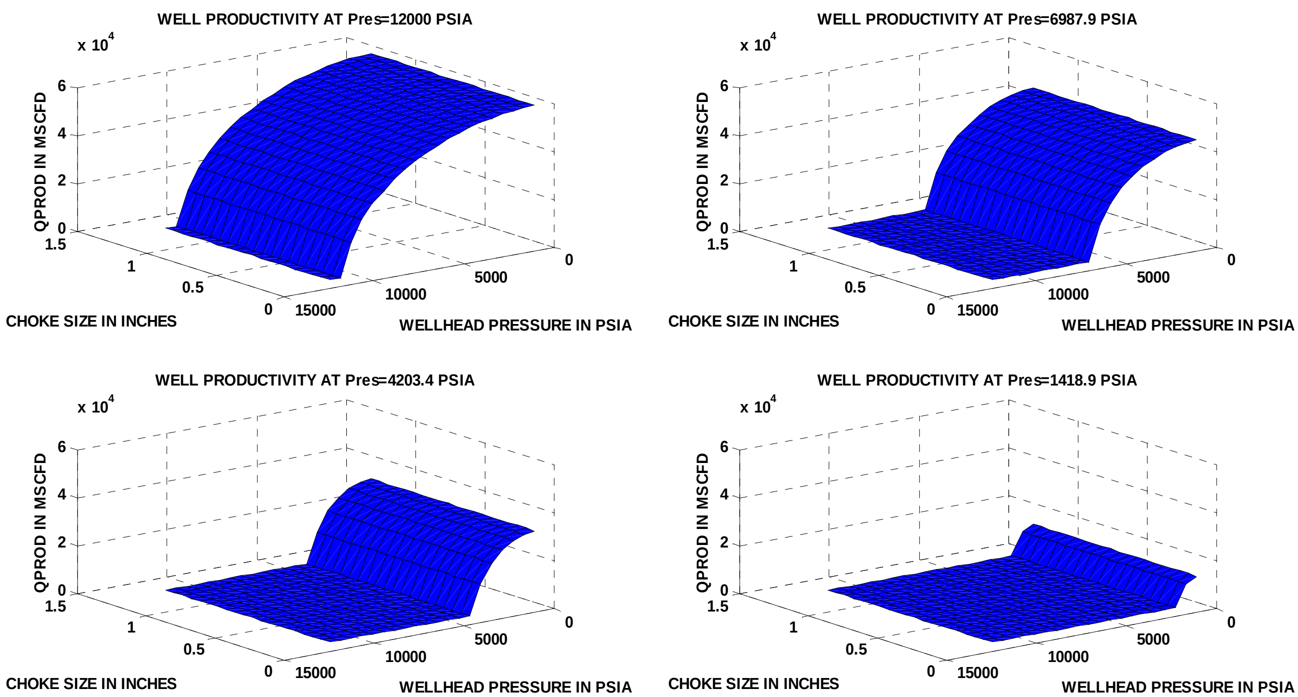

Figure 5-6: Well producing capacity versus choke size and wellhead pressure (scenario 2) for Pres=12000 psia, Pres=6987.9 psia, Pres=4203.4 psia and Pres=1418.9 psia

Figure 5-6 represents the results obtained for the well producing capacity when running the gas well model at different reservoir pressures. As we can see on this plot, the well producing capacity increases as you decrease the wellhead pressure for the four cases of reservoir pressure that are presented. On the well productivity surface at the initial reservoir pressure, P_{res}=12000 psia, gas production rates vary from q_{prod}= 57277 MSFCD at P_{wh}=100 psia to q_{prod}= 14129 MSFCD at Pwh=10300 psia. On the well productivity surface at the abandonment pressure P_{res}=1418.9 psia, gas production rates vary from q_{prod}= 11485 MSFCD at P_{wh}=100 psia to

q_{prod}= 9177.3 MSFCD at P_{wh}=700 psia. However, we can notice that for a given wellhead pressure, the calculated production rate q_{prod} stays constant despite the selected choke size. This allows observing the fact that the choke size has no influence on the upstream gas flow rate, which is the gas production rate, coming from the wellhead.

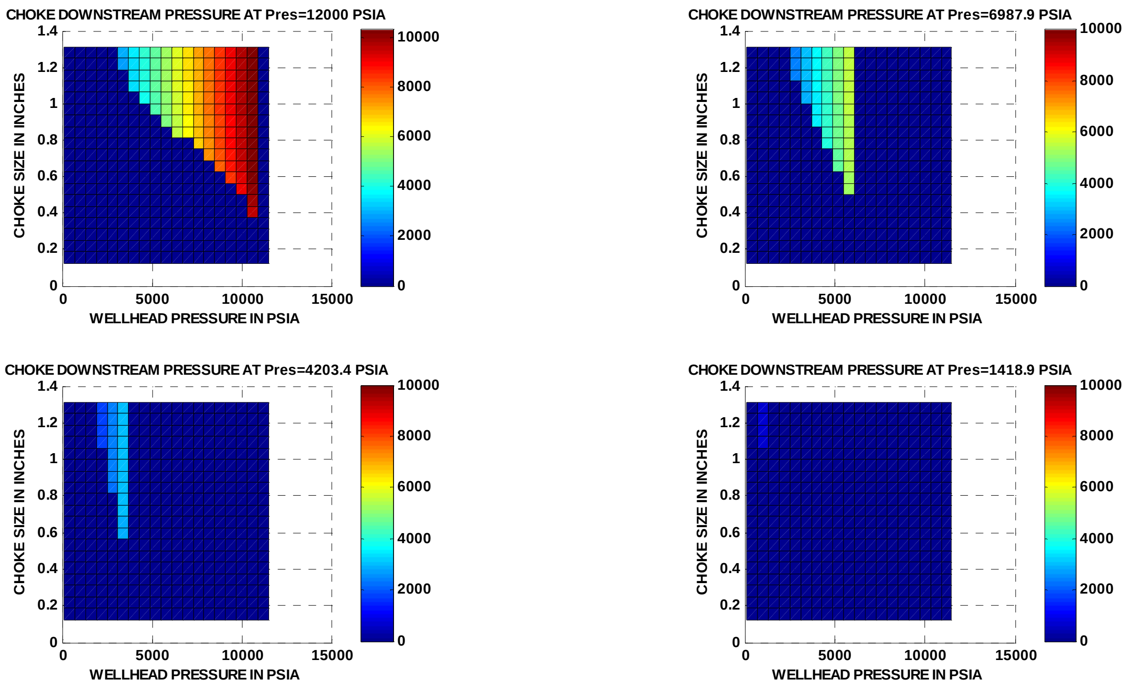

Figure 5-7: Choke downstream pressure versus choke size and wellhead pressure (scenario 2) for Pres=12000 psia, Pres=6987.9 psia, Pres=4203.4 psia and Pres=1418.9 psia

Figure 5-7 is a top view of the values obtained for the choke downstream pressure after running the gas well model at different reservoir pressures. The colorbar, which is placed next to each plot gives the range of values taken by the choke downstream pressure for the different cases presented on Figure 5-7. Notice that, for scenario 2, the tubing size is kept constant and equal to 3.5 in. However, we still have 400 cases corresponding to the different possible combinations of wellhead pressure and choke size. According to the colorbars on Figure 5-7, areas colored in blue on the plots represent the combinations of choke size and wellhead pressure for which P_{downs}=0, which translates the fact that there is no solution to the choke flow equation for the selected combination and the corresponding well producing capacity. This said, we can realize that as you decrease the reservoir pressure, the blue area gets larger, which translates that as you decrease the reservoir pressure less combinations of choke size and wellhead pressure are doable.

At P_{res}=12000 psia, the doable choke sizes range between 0.375 in and 1.3125 in. The corresponding wellhead pressures vary between 2500 psia and 10900 psia. It is important to observe that the highest choke size value (1.3125 in) enables to work with the widest range of wellhead pressures. As you decrease the choke size, the wellhead pressure range gets smaller and smaller. The smallest choke size of that range is only doable for wellhead pressures between 10300 psia and 10900 psia. This is in accordance with the fact that for a given tubing size, the highest wellhead pressure gives the smallest flow rate.

At the abandonment pressure (P_{res} =1418.9 psia), we have the smallest range of doable combinations of choke size and wellhead pressure: the choke sizes range between 1.0625 in and 1.3125 in and the doable wellhead pressures range from 700 psia to 1300 psia. This information is extremely important for the design phase. Indeed, we know that as we produce gas from the reservoir, the reservoir pressure decreases and therefore the amount of gas produced becomes smaller and smaller. In order to keep a constant production rate, you have to lower the wellhead pressure as reservoir pressure depletes. From scenario 1, we notice that the maximum wellhead pressure that can be chosen for the well to continue to produce is between 700 psia and 1300 psia. The complement information given by the analysis of Figure 5-7 tells us that for a tubing size of 3.5 in, only wellhead pressures between 700 psia and 1300 psia and choke sizes between 1.0625 in and 1.3125 in will allow gas to flow from the reservoir to the wellhead and then to flow through the choke. Therefore, if you assign a choke size within that range to the well in addition to the other parameters, you can have the insurance that gas will flow from the reservoir to the wellhead and then through the choke, during the entire lifetime of the project.

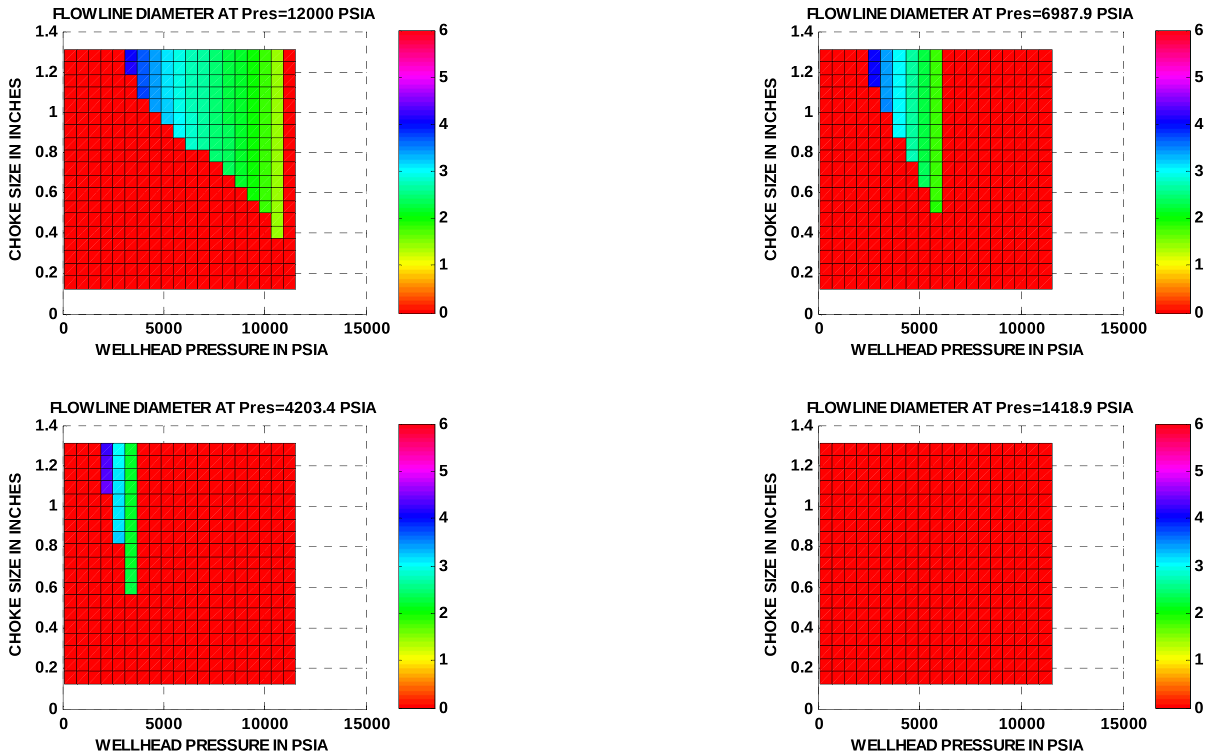

Figure 5-8: Flow-line diameter versus choke size and wellhead pressure (scenario 2) for Pres=12000 psia, Pres=6987.9 psia, Pres=4203.4 psia and Pres=1418.9 psia

Figure 5-8 displays a top view of the calculated surface flow-line diameter for different combinations of choke sizes and wellhead pressures, at different reservoir pressures. The surface flow-line is the pipeline that connects the surface choke to the gathering tank. This pipeline is described by its length and its diameter. In this case study, the length has been fixed. The calculation of the diameter is the last operation conducted by the gas well model and it tells us if gas will flow from the choke to the gathering tank for the selected conditions.

The gas model uses the well producing capacity q_{prod} and the choke downstream pressure obtained from the previous steps in the program, to calculate the surface flow-line diameter that will allow the amount of gas q_{prod} to be transported from the choke to the gathering tank.

The colorbar, which is placed next to each plot, gives the value of the surface flow-line diameter corresponding to each color encountered on the plot. As we can see on Figure 5-8, for P_{res} from 12000 psia to 4203.4 psia, areas colored in red represent the combinations of choke size and wellhead pressure for which $d_{pipesurf}$=0 i.e. no flow is possible at the surface. In the program, the calculation of $d_{pipesurf}$ is conducted only when the calculated choke downstream pressure P_{downs} for the selected conditions is higher than the chosen gathering tank pressure $P_{g.}$. When this condition is not verified, it is physically impossible to have gas flowing from the surface choke to the gathering tank. Therefore, the program returning 0 as the result for $d_{pipesurf}$ translates the fact that the choke downstream pressure obtained from the previous step in the program is not high enough to allow fluid flow from the surface choke to the gathering tank. Analyzing the results presented on Figure 5-8, we can notice that, from P_{res}=12000 psia to P_{res}=4203.4 psia, only certain combinations of choke size and wellhead pressure will enable to have gas flow at the surface.

At P_{res}=12000 psia, the doable choke sizes range from 0.375 in to 1.3125 in. The doable wellhead pressures range from 2500 psia to 10900 psia. As the reservoir pressure decreases, we realize that the area corresponding to the doable combinations of choke size and wellhead pressure, becomes smaller and smaller. In fact, as you decrease the reservoir pressure, the allowable wellhead pressures are also decreasing. Therefore, you will get lower values for the calculated choke downstream pressure and fewer combinations of choke size and wellhead pressures will enable gas flow at the surface.

At P_{res}=1418.9 psia, the area is entirely red. For every combination of choke size and wellhead pressure, the program returns $d_{pipesurf}$=0. This can be explained by the fact that at this reservoir pressure, we have the lowest allowable wellhead pressures, which range from 700 psia to 1300 psia. The calculated choke downstream pressure for the given choke sizes, is too low to allow gas flow at the surface. In the design phase, you want to select the parameters that will allow best the production system to reach the field production target during the entire life of the project. The result from this analysis tells us that even if the following ranges of choke size (1.0625 in -1.3125 in) and wellhead pressure (700 psia-1300 psia), enable flow through the choke for the conditions of scenario 2, gas cannot flow from the choke to the gathering tank since these combinations of choke sizes and wellhead pressures result in very low choke downstream pressures as compared to the selected gathering tank pressure for scenario 2.

To sum up, if you select a choke size within the range 1.0625 in -1.3125 in, you will be able to have gas flow from the reservoir to the surface, when associated with a tubing diameter of 3.5 in and a gathering tank pressure of 1500 psia. However, you also have to associate this to the adequate wellhead pressure. At initial reservoir pressure, the allowable wellhead pressures vary from 2500 psia to 10900 psia. At abandonment conditions, a wellhead pressure within the range 700 psia-1300 psia will enable gas flow from the reservoir to the surface choke, but no flow will be possible between the surface choke and the gathering tank for the selected choke size range.

5.4. Scenario 3: Effect of wellhead pressure and gathering tank pressure

In this case, we will focus on the interplay between the wellhead pressure and the gathering tank pressure. We have the following input for the decision variables:

- The tubing size is set to be equal to 3.5 in
- The choke size is set to be equal to 1.0625 in
- The wellhead pressure will vary from 100 psia to 11500 psia with a constant increment allowing to have 20 subsequent values

- The gathering tank pressure will vary from 500 psia to 5000 psia with a constant increment allowing to have 20 subsequent values.

Table 5-8 contains the wellhead pressure values and gathering tank pressure values which, are tested in the gas well model.

Table 5-8: Wellhead pressure and Gathering tank pressure values for scenario 3

P_{wh} in psia	P_g in psia
100	500.0
700	736.8
1300	973.7
1900	1210.5
2500	1447.4
3100	1684.2
3700	1921.1
4300	2157.9
4900	2394.7
5500	2631.6
6100	2868.4
6700	3105.3
7300	3342.1
7900	3578.9
8500	3815.8
9100	4052.6
9700	4289.5
10300	4526.3
10900	4763.2
11500	5000.0

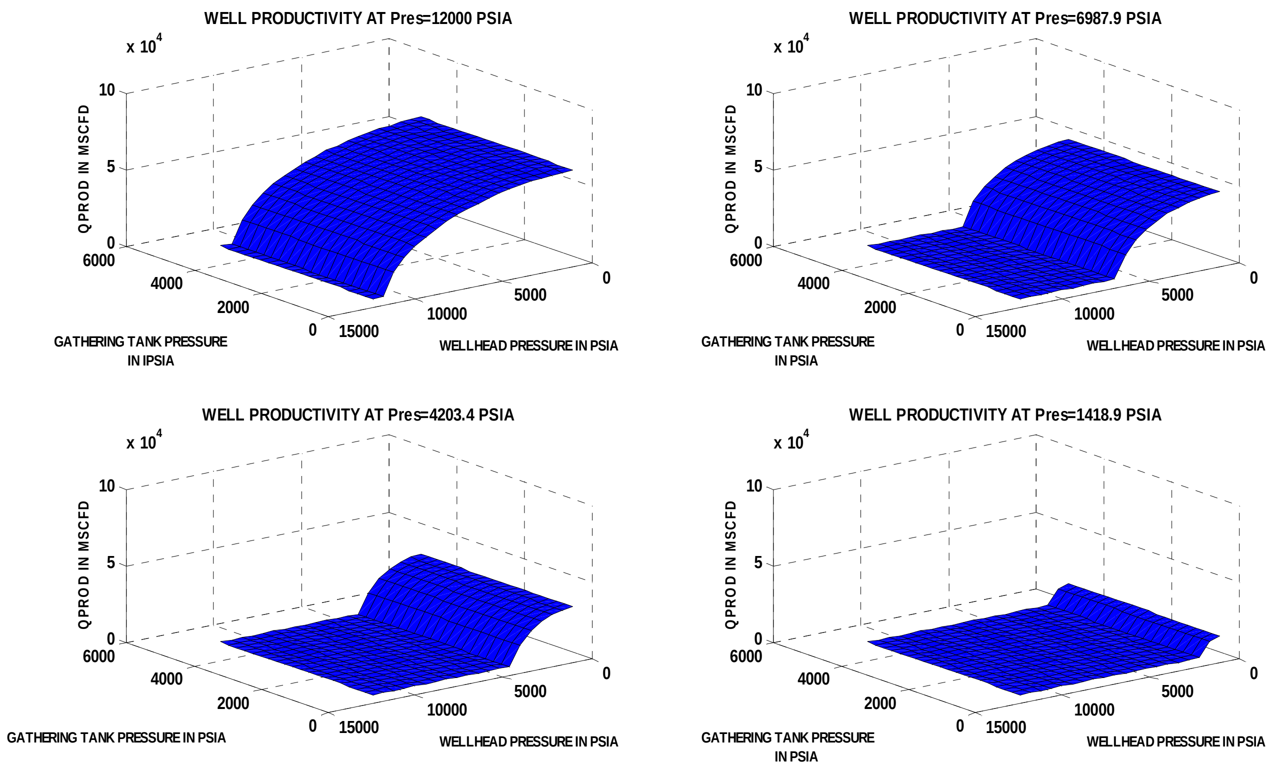

Figure 5-9: Well producing capacity versus wellhead pressure and gathering tank pressure (scenario 3) for Pres=12000 psia, Pres=6987.9 psia, Pres=4203.4 psia and Pres=1418.9 psia

Figure 5-9 shows how the well producing capacity obtained by running the gas well model with the conditions of scenario 3, changes with wellhead pressure and gathering tank pressure at different reservoir pressures. As we can see on Figure 5-9, the well producing capacity is not influenced by the gathering tank pressure. The only parameter having an impact on the value of q_{prod} is the wellhead pressure. At a given gathering tank pressure, q_{prod} increases as you decrease the wellhead pressure. However, for a fixed wellhead pressure, q_{prod} stays constant as you change the gathering tank pressure value. At P_{res}=12000 psia, q_{prod} varies from 57277

MSCFD at P_{wh}=100 psia to 14129 MSCFD at P_{wh}=10300 psia. At P_{res}=1418.9 psia, q_{prod} varies from 11485 MSCFD at P_{wh}=100 psia to 9177.3 MSCFD at P_{wh}=700 psia.

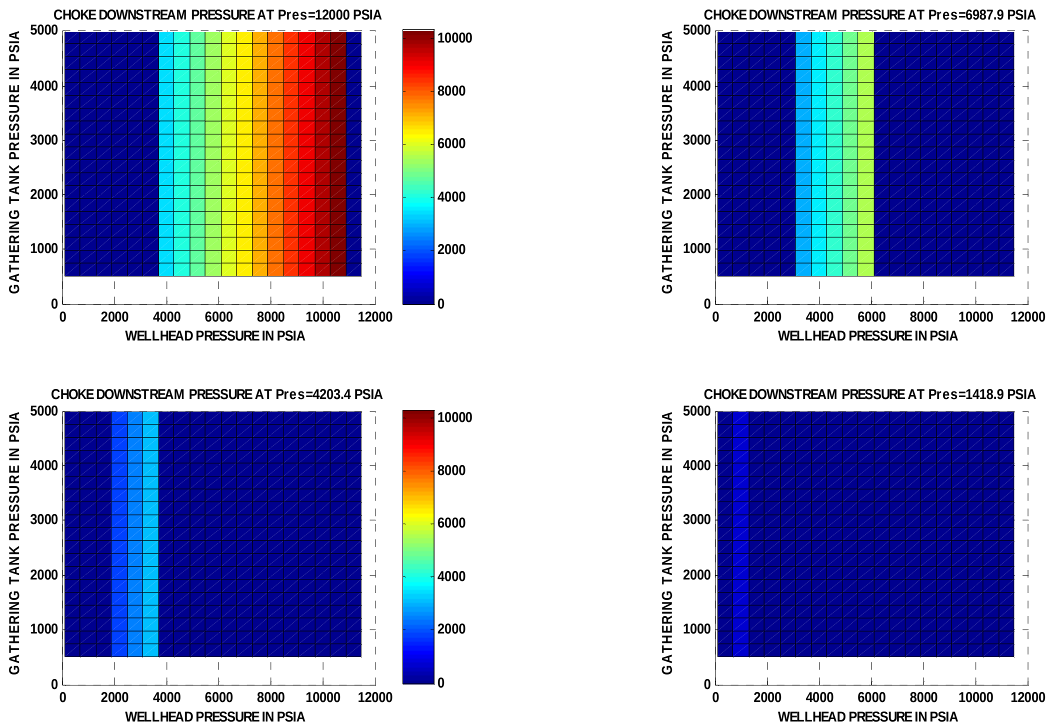

Figure 5-10: Choke downstream pressure versus wellhead pressure (scenario 3) and gathering tank pressure for Pres=12000 psia, Pres=6987.9 psia, Pres=4203.4 psia and Pres=1418.9

Figure 5-10 is a top view of the calculated choke downstream pressure obtained when running the gas well model for the conditions of scenario 3. The colorbar next to each plot gives the range of values corresponding to each color represented on the plot. Here again, we can notice that the gathering tank pressure has no impact on the choke performance. At a given wellhead pressure, P_{downs} is constant for the entire range of gathering tank pressure (500 psia-5000 psia).

The choke downstream pressure is only influenced by the wellhead pressure. At each reservoir pressure, there is a range of wellhead pressure values among the doable wellhead pressure values that can be obtained from Figure 5-9, that allow you to have flow through the choke. At P_{res}=12000 psia, the doable combinations of wellhead pressure range between 3700 psia and 10900 psia. As you decrease the reservoir pressure, this range of doable wellhead pressures shrinks, since you have to use lower wellhead pressures to be able to keep on producing from the well. At P_{res}=1418.9 psia, there is gas flow through the choke only for P_{wh} between 700 psia and 1300 psia.

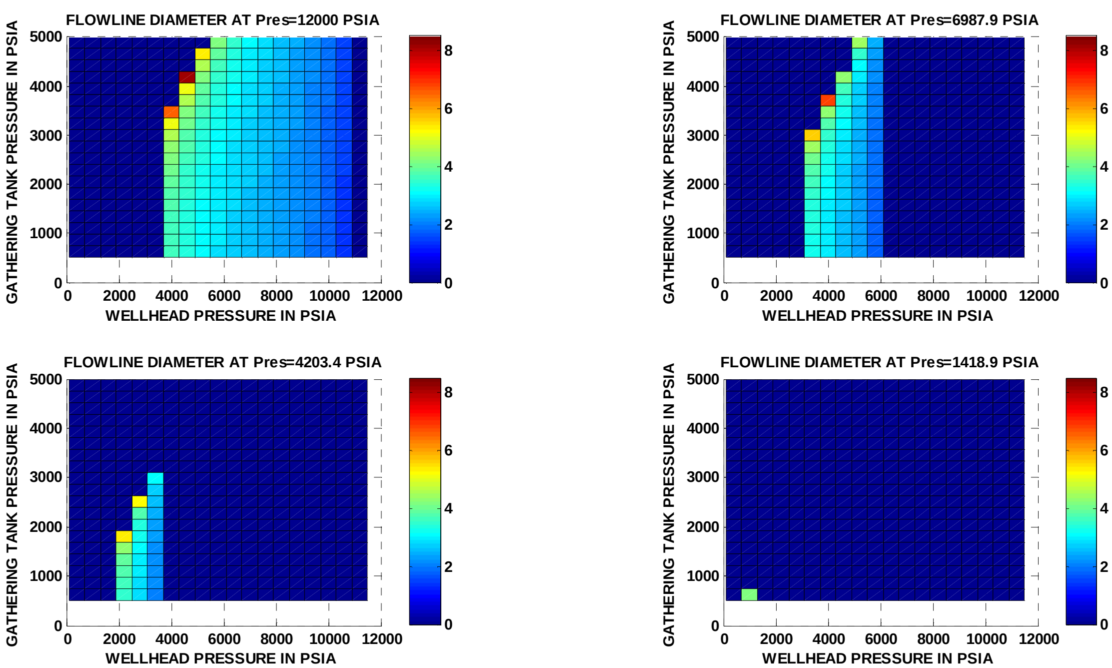

Figure 5-11: Flow-line diameter versus gathering tank pressure and wellhead pressure (scenario 3) at Pres=12000 psia, Pres=6987.9 psia, Pres=4203.4 psia and Pres=1418.9 psia

Figure 5-11 is a top view of the surface flow-line diameter obtained when running the gas well model for scenario 3.The colorbar that is located next to each plot gives the flow-line diameter value corresponding to the colors encountered on the plot. As we can see on Figure 5-11, there is a range of doable combinations of wellhead pressures and gathering tank pressures, at each reservoir pressure. On each plot, the blue area designates combinations of wellhead pressures and gathering tank pressures for which the program returns $d_{pipesurf}$=0. For these combinations, there is no gas flowing in the surface flow-line connecting the surface choke to the gathering tank.

At P_{res}=12000 psia, the doable wellhead pressures are between 3700 psia and 10900 psia. At a given wellhead pressure, the doable gathering tank pressures correspond to the pressure values that are lower than the wellhead pressures in consideration. Thus at P_{wh}=3700 psia, the maximum allowable gathering tank pressure is P_g= 3578.9 psia. As reservoir depletion takes place, the area corresponding to the doable combinations shrinks, since you need to lower the wellhead pressure to keep on producing from the well. At P_{res}=1418.9 psia, the only doable gathering tank pressure is P_g= 500 psia for wellhead pressures between P_{wh}=700 psia and P_{wh}=1300 psia.

Finally, it is noticeable that at a fixed wellhead pressure, as you increase the gathering tank pressure, you increase the diameter of the surface flow-line. This can be explained using the gas flow equation through the surface pipeline (see section 4.2). For a fixed wellhead pressure, a fixed tubing size and choke size, you will get a fixed choke downstream pressure and a fixed well producing capacity. The square of the pressure differential between the surface choke and the gathering tank will decrease as you increase the gathering tank pressure. The square of the pressure differential between the surface choke and the gathering tank, multiplied by the surface flow-line diameter raised to the fifth power is equal to the square of the well producing capacity, multiplied by a constant. Therefore, in order to keep the second term of the equation (with the well producing capacity) constant, you have to increase the surface flow-line diameter as you decrease the pressure differential between the surface choke and the gathering tank. In other words, when you increase the gathering tank pressure, the surface flow-line diameter also increases. For instance, at P_{res}=12000 psia, for P_{wh}=3700 psia, the flow-line diameter varies from 3.62 in to 6.56 in with gathering tank pressures from 500 psia to 3578.9 psia. This is important in terms of field economics because having a very large diameter for the surface flow-line will be expensive. However, choosing a very low gathering tank pressure will also turn out to be expensive since it means that you will have to install some compressors in order to be able to take the gas produced to the sales lines. But more importantly, the selected combination of wellhead pressure and gathering tank pressure shall allow gas production during the entire life of the project. In this scenario, at P_{res}=1418.9 psia, the only doable combination of wellhead pressure and gathering tank pressure is P_{wh}=700 psia and P_g=500 psia; the obtained flow-line diameter is 4.24 in. Generally, the gathering tank pressure is fixed throughout the life of the reservoir. Thus, we can infer from the previous analysis that a gathering tank pressure of 500 psia should be chosen for production from the reservoir. We will start with a wellhead pressure between 3700 psia and 10900 psia and reduce its value as reservoir depletes with time to reach a wellhead pressure of 700 psia at the abandonment pressure.

5.5. Scenario 4: Effect of tubing size and choke size

In this scenario, we will examine how the tubing size and the choke size are interrelated and how they influence the performance of the production system. We have the following input for the decision variables:

- The wellhead pressure is set to be equal to 1000 psia
- The gathering tank pressure is set to be equal to 500 psia

- The tubing size varies between 0.1 in and 12 in with a constant increment
- The choke size varies between 8/64 in and 84/64 in with a constant increment.

The tubing size values and choke size values are contained in Table 5-9:

Table 5-9: Tubing size values and choke size values for scenario 4

d_{tubing} in inches	d_{choke} in inches
0.1	0.125
0.72632	0.1875
1.3526	0.25
1.9789	0.3125
2.6053	0.375
3.2316	0.4375
3.8579	0.5
4.4842	0.5625
5.1105	0.625
5.7368	0.6875
6.3632	0.75
6.9895	0.8125
7.6158	0.875
8.2421	0.9375
8.8684	1
9.4947	1.0625
10.121	1.125
10.747	1.1875
11.374	1.25
12	1.3125

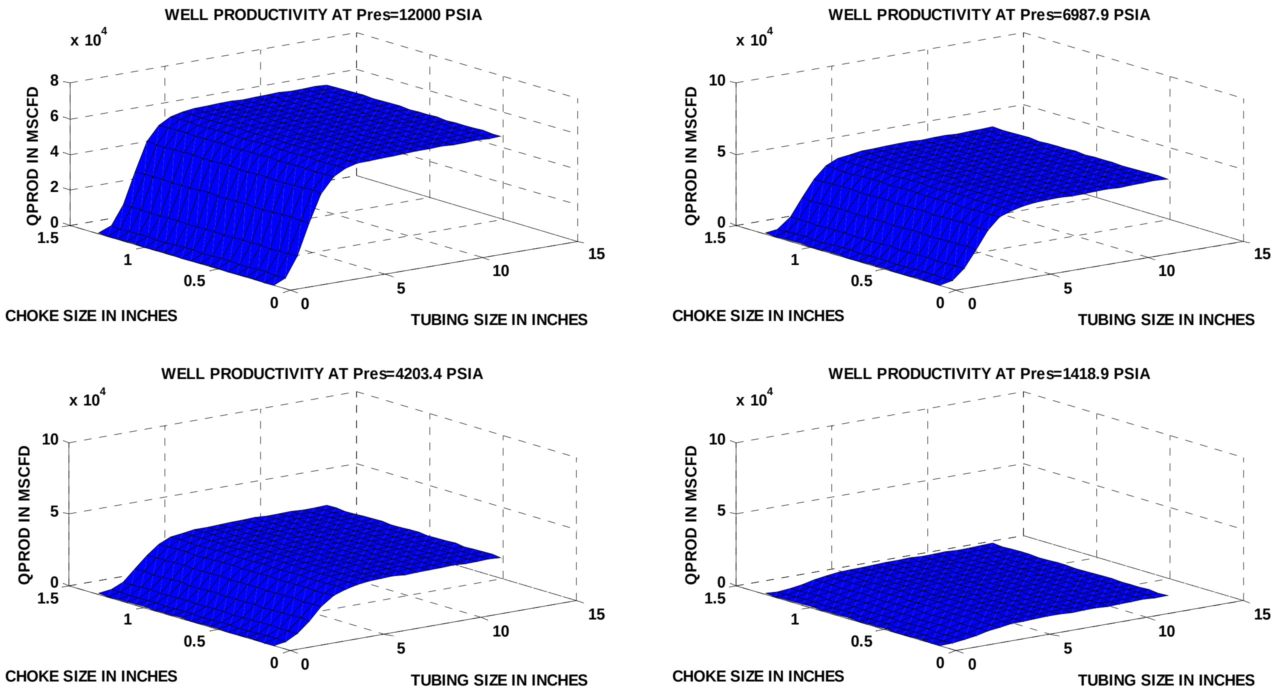

Figure 5-12: Well producing capacity versus tubing size and choke size (scenario 4) at Pres=12000 psia, Pres=6987.9 psia, Pres=4203.4 psia and Pres=1418.9 psia

Figure 5-12 presents how the well producing capacity obtained when running the gas well model for scenario 4, evolves with the tubing size and choke size, at different reservoir pressures. As we can see on Figure 5-12, the choke size has no direct impact on the well producing capacity. For a given tubing size, the gas flow rate is constant for any choke size that is tested by the program. This is related to the fact that we have a fixed value for the wellhead pressure. At a given choke size, the gas flow rate increases with the tubing size until it reaches an optimum tubing diameter, which is between 5 in and 6 in. At P_{res}=12000 psia, the gas flow rate varies from 2976.1 MSCFD for a tubing size of 0.73 in to 61330 MSCFD for a tubing size of 12 in. At P_{res}=1418.9 psia, the gas flow rate varies from 892.61 MSCFD for a tubing size of 1.35 in to 6987.3 MSCFD for a tubing size of 12 in. We can still notice that the well producing capacity decreases as the reservoir pressure depletes with time.

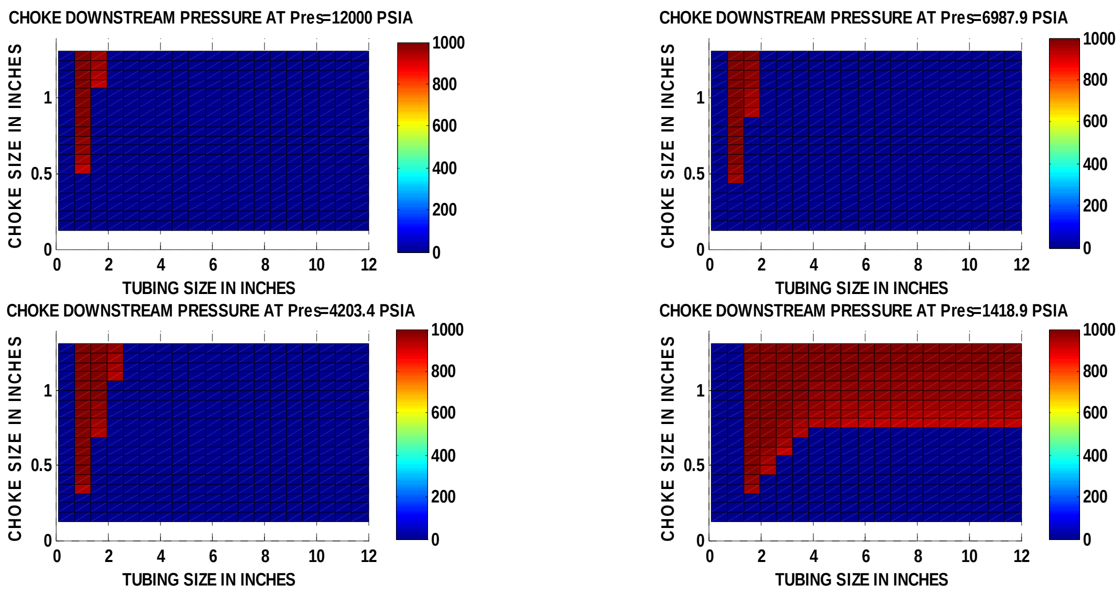

Figure 5-13: Choke downstream pressure versus tubing size and choke size (scenario 4) at Pres=12000 psia, Pres=6987.9 psia, Pres=4203.4 psia and Pres=1418.9 psia

Figure 5-13 is a top view of the choke downstream pressure obtained for different combinations of tubing sizes and choke sizes, at different reservoir pressures. The colorbar that is located next to each plot, enables to give the choke downstream pressure associated to each color represented on the plot. The fact that the gas well model returns a value for P_{downs} which is different from 0, tells us that there is gas flow through the choke.

As we can notice on Figure 5-13, the blue area on each plot designates the combinations of tubing sizes and choke sizes that cannot be used to produce the well, given the chosen wellhead pressure and the chosen gathering tank pressure. As reservoir pressure depletes in time, the blue area becomes smaller and smaller.

At Pres=12000 psia, the following results are obtained:

- For tubing sizes between 0.73 in and 1.35 in, the doable choke sizes are between 0.5 in and 1.3125 in. For a choke size below 0.5 in, P_{downs}=0; there is no gas flow through the choke. In fact, the choke sizes below 0.5 in are too small to sustain the well producing capacity associated with the selected tubing size and the wellhead pressure.
- For tubing sizes between 1.35 in and 1.98 in, the doable choke sizes range between 1.0625 in and 1.3125 in. There is no flow for a choke size below that range; which is due to the fact that by increasing the tubing size at a fixed wellhead pressure, you increase the well producing capacity and therefore you impose an additional restriction to the choke sizes that can be used for production.

At Pres=1418.9 psia, we get the following results:

- The doable tubing sizes range from 1.35 in to 12 in
- For tubing sizes from 3.86 in to 12 in, the doable choke sizes are between 0.75 in and 1.3125 in.
- For tubing sizes between 3.23 in and 3.86 in, the doable choke sizes range from 0.6875 in to 1.3125 in
- For tubing sizes between 2.6 in and 3.23 in, the doable choke sizes range from 0.5625 in to 1.3125 in
- For tubing sizes between 1.98 in and 2.6 in, the doable choke sizes range from 0.4375 in to 1.3125 in

- For tubing sizes between 1.35 in and 1.98 in, the doable choke sizes range from 0.3125 in to 1.3125 in

When you decrease the tubing size, you enlarge the doable choke size range. This can be explained by the fact that as you decrease the tubing size for a fixed wellhead pressure, you decrease the gas production rate. Therefore, you can use smaller choke sizes at the surface.

It is also noticeable that we have the largest number of doable combinations of choke size and tubing size at the abandonment pressure for this case study. It is important to see how the operating conditions influence the well performance throughout the life of the reservoir so that you can choose values for the decision parameters that will sustain production from the well from the initial reservoir pressure to the abandonment pressure.

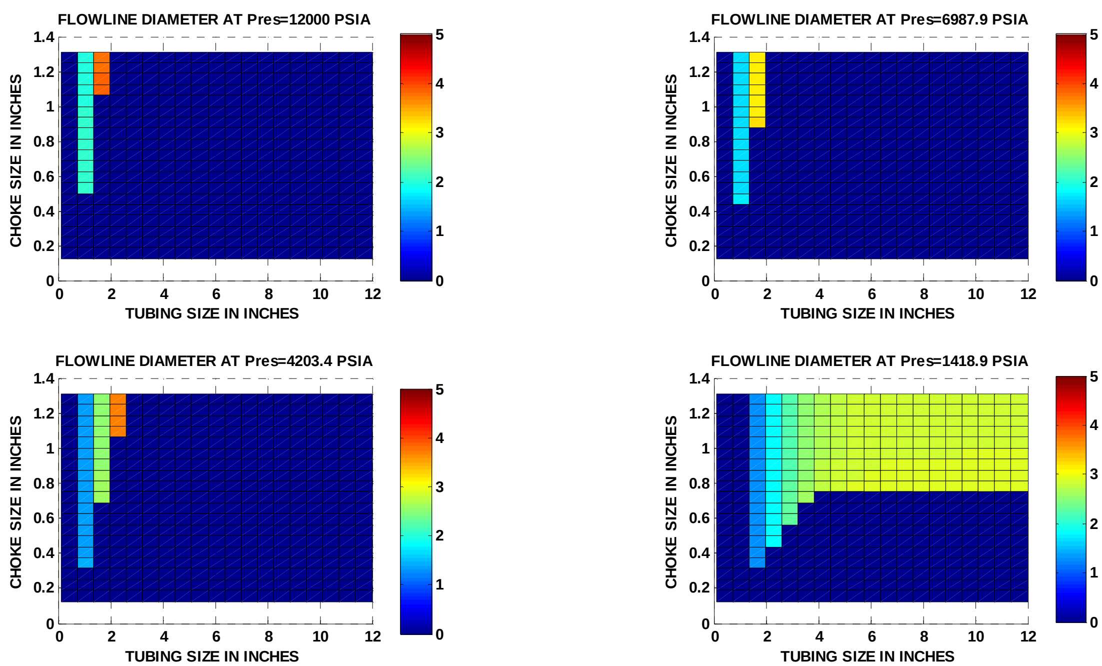

Figure 5-14: Flow-line diameter versus tubing size and choke size (scenario 4) at Pres=12000 psia, Pres=6987.9 psia, Pres=4203.4 psia and Pres=1418.9 psia

Figure 5-14 is a top view of the surface flow-line diameter obtained when running the gas well model for scenario 4. It displays how the calculated surface flow-line diameter changes with tubing size and choke size. The colorbar, which is located next to each plot gives the surface flow-line diameter value corresponding to the combinations of tubing size and choke size. The calculation of the surface flow-line diameter is the last step in the gas well model and tells us if there is gas flow from the choke to the gathering tank.

As we can see on Figure 5-14, we obtain the same ranges for the doable combinations of tubing size and choke size, as for the choke downstream pressure. On any of the plot, at a given tubing size, the flow-line diameter is constant for every choke size. We can infer from this observation that the choke size has no influence on the surface flow-line diameter. For a fixed choke size, the flow-line diameter increases with the tubing size. This can be explained by the fact that as you increase the tubing size for a fixed wellhead pressure, you produce more and more gas out of the well; therefore, you will need a bigger surface flow-line to accommodate the gas flow at the surface.

5.6. Scenario 5: Effect of choke size and gathering tank pressure

In this case, we will try to display the interplay between the choke size and the gathering tank pressure and their impact on the production system. We have the following input for the gas well model:

- The wellhead pressure is set to be equal to 1000 psia
- The tubing size is set to be equal to 1.35 in
- The choke size varies between 8/64 and 84/64 with a constant increment
- The gathering tank pressure varies between 400 psia and 1000 psia with a constant increment.

The values used for the gathering tank pressure and the choke size in this scenario, are contained in Table 5-10:

Table 5-10: Choke size and gathering tank pressure values for scenario 5

d_{choke} in inches	P_g in psia
0.125	400.0
0.1875	431.6
0.25	463.2
0.3125	494.7
0.375	526.3
0.4375	557.9
0.5	589.5
0.5625	621.1
0.625	652.6
0.6875	684.2
0.75	715.8
0.8125	747.4
0.875	778.9
0.9375	810.5
1	842.1
1.0625	873.7
1.125	905.3
1.1875	936.8
1.25	968.4
1.3125	1000.0

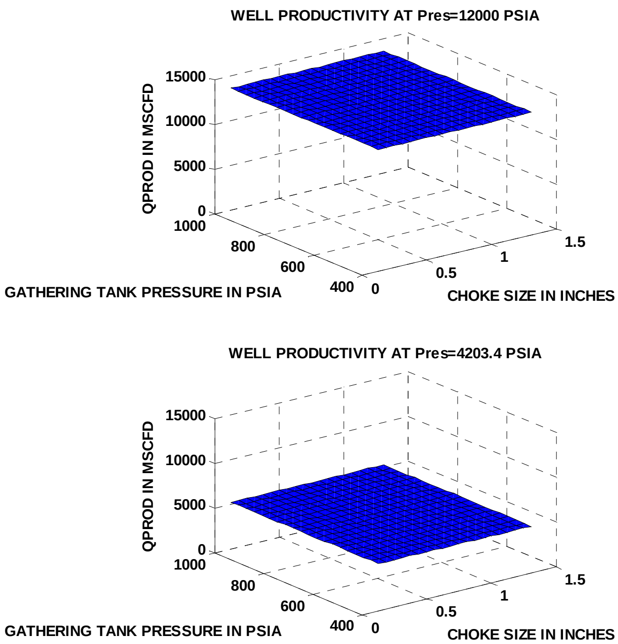

Figure 5-15 : Well producing capacity versus choke size and gathering tank pressure (scenario 5) at Pres=12000 psia, Pres=6987.9 psia, Pres=4203.4 psia and Pres=1418.9 psia

Figure 5-15 shows the well producing capacity that is obtained when running the gas well model for scenario 5, for various combinations of choke size and gathering tank pressure and at different reservoir pressures. As we can see on Figure 5-15, once the wellhead pressure and the tubing size have been fixed, the gathering tank pressure and the choke size have no influence on the gas flow rate. The well producing capacity is constant for all the combinations of choke size and gathering tank pressures that are tested in this scenario. It goes from 13684 MSCFD at P_{res} =12000 psia to 888.35 MSCFD at P_{res} =1418.9 psia.

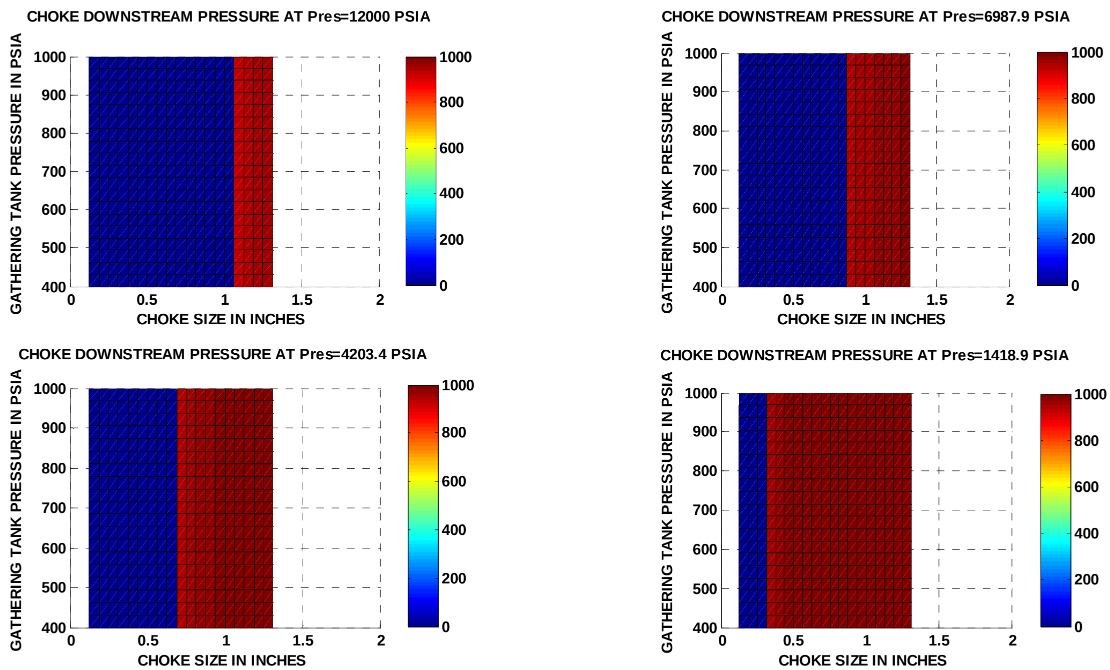

Figure 5-16: Choke downstream pressure versus choke size and gathering tank pressure (scenario 5) for Pres=12000 psia, Pres=6987.9 psia, Pres=4203.4 psia and Pres=1418.9 psia

Figure 5-16 is a top view of the choke downstream pressure obtained when running the gas well model for scenario 5. It displays how this pressure is influenced by the choke size and the gathering tank pressure. The colorbar, which is located next to each plot helps obtain the pressure value corresponding to each color displayed on the plot. The doable combinations of choke size and gathering tank pressure are the combinations for which P_{downs} is different from 0.

As we can see on Figure 5-16, at a given tubing size, the calculated choke downstream pressure is constant for any gathering tank pressure. The range of doable choke sizes increases as the reservoir pressure depletes with time. When you decrease the reservoir pressure, it becomes possible to use lower and lower choke sizes at the surface. At Pres=12000 psia, the doable choke sizes range between 1.0625 in and 1.3125 in. At Pres=1418.9 psia, the doable choke sizes range between 0.3125 in and 1.3125 in.

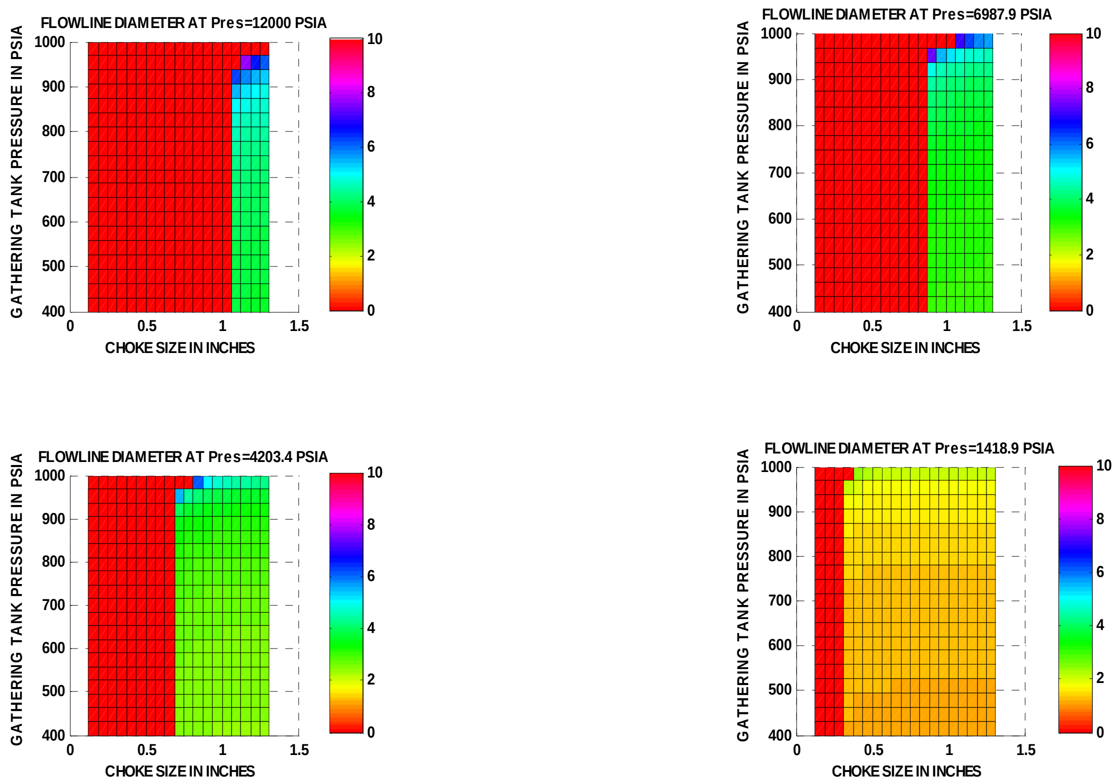

Figure 5-17: Flow-line diameter versus choke size and gathering tank pressure (scenario 5) at Pres=12000 psia, Pres=6987.9 psia, Pres=4203.4 psia and Pres=1418.9 psia

Figure 5-17 is a top view of the calculated surface flow-line diameter versus the choke size and the gathering tank pressure, for various reservoir pressures. It displays the influence of these two decision variables on the surface flow-line. The colorbar that is located next to each plot enables to see the value of the flow-line diameter associated with each color represented on the plot.

There is a discontinuity on the surface obtained when plotting the surface flow-line diameter versus the choke size and the gathering tank pressure. This is due to the fact that for any combination of choke size and gathering tank pressure associated to the tubing size and wellhead pressure, for which there is no solution to the choke equation, the gas well model returns P_{downs}=0. The doable ranges of choke size are the same as the ones observed on Figure 5-16. The only difference resides in the gathering tank pressure. At any reservoir pressure, there is a limitation to the gathering tank pressure that is linked to the value of the choke downstream pressure. When calculating the surface flow-line diameter, the program verifies that the choke downstream pressure is higher than the gathering tank pressure that is considered. If not, the program returns $d_{pipesruf}$=0; in other words, there is no gas flow in the surface flow-line. The red area on each plot corresponds to the combinations of choke sizes and gathering tank pressure for which there is no gas flow in the surface flow-line. As reservoir pressure depletes, the red area gets smaller and smaller. This is due to the fact that when you lower the reservoir pressure, the well producing capacity also decreases and therefore, it becomes possible to use smaller choke sizes in the production system.

To cap it all, at P_{res} =12000 psia, by selecting any pressure in the range 400psia-936.8 psia, there is gas flow for any choke size between 1.0625 in and 1.3125 in. At P_{res}=1418.9 psia, there is gas flow in the surface flow-line for any gathering tank pressure in the range 400 psia-968.4 psia and for any choke size between 0.3125 in and 1.3125 in. Since the objective is to keep the well producing throughout the entire life of the reservoir (from initial conditions to abandonment conditions), in this case, you should select a choke size between 1.0625 in and 1.3125 in , and a gathering tank pressure within the range 400 psia and 936.8 psia.

5.7. Summary of the results

The chosen decision variables which are the wellhead pressure P_{wh}, the tubing size d_{tubing}, the choke size d_{choke}, and the gathering tank pressure $P_{g,}$ are interconnected. In this sensitivity analysis, we were able to display the interplay between the decision variables. There are layers of interconnectivity between these variables. The values obtained for the doable combinations of the decision variables depend on the ranges chosen in the different scenarios.

However, the selected ranges for this sensitivity analysis are very wide and do not necessarily reflect the ranges that are available in the industry for the wellhead pressure, the tubing size, the choke size and the gathering tank pressure. For example, viable tubing sizes that are used in the industry vary between 1.3 in and 7 in *(Brown 1977-1984).* Extreme values were taken into account in order to obtain results which would be as general as possible. The results from this sensitivity analysis are summarized in Table 5-11:

Table 5-11: Summary of results for the sensitivity analysis

Scenario	Doable combinations	Additional comments
Scenario 1 d_{choke}=0.5625 in P_g=1500 psia	At P_{res}=12000 psia: P_{wh}: 1900 psia-10300 psia d_{tubing}: 0.72 in-12 in At P_{res}=1418.9 psia and P_{wh}: 700 psia-1300 psia d_{tubing}: 1.35 in-1.98 in	Select the doable range of tubing size at abandonment pressure for the entire life of the reservoir and associate the adequate wellhead pressure. No flow is possible from the surface choke to the gathering tank at abandonment conditions
Scenario 2 d_{tubing}=3.5 in P_g=1500 psia	At P_{res}=12000 psia: P_{wh}: 2500 psia-10900 psia d_{choke}: 0.375 in-1.3125 in At P_{res}=1418.9 psia P_{wh}: 700 psia-1300 psia d_{choke}: 1.0625 in-1.3125 in	Select the doable choke size range at abandonment conditions for the entire life of the reservoir and match with the adequate wellhead pressure. No flow is possible from the surface choke to the gathering tank at abandonment conditions
Scenario 3 d_{choke}=1.0625 in d_{tubing}=3.5 in	At P_{res}=12000 psia: P_{wh}: 3700 psia-12000 psia P_g: 500 psia-5000 psia At P_{res}=1418.9 psia P_{wh}: 700 psia P_g: 500	Select the doable gathering tank pressure at abandonment conditions for the entire life of the reservoir and match with the adequate wellhead pressure.
Scenario 4 P_{wh}=1000 psia P_g= 500 psia	At P_{res}=12000 psia: d_{tubing}: 0.72 in-1.98 in d_{choke}: 0.5 in-1.3125 in At P_{res}=1418.9 psia d_{tubing}: 1.35 in-12 in d_{choke}: 0.3125 in-1.3125 in	Select the choke size between 0.5 in and 1.3125 in and associate with a tubing size between 1.35 in and 12 in for the entire life of the project.
Scenario 5 d_{tubing}=1.35 in P_{wh}=1000 psia	At P_{res}=12000 psia: P_g: 400 psia-936.8 psia d_{choke}: 1.0625 in-1.3125 in At P_{res}=1418.9 psia P_g: 400 psia-968.4 psia d_{choke}: 0.3125 in-1.3125 in	Keep the doable combinations of choke size and gathering tank pressure at initial reservoir conditions, for the entire life of the reservoir.

Chapter 6

Optimization

The objective of this section is to provide the combination of decision variables that will optimize the production from a natural gas field in terms of net present value. We are only going to focus on the two following units in the production system from a well:

- The reservoir
- The well tubing

Therefore, we will examine the following decision variables: the wellhead pressure P_{wh} and the tubing size d_{tubing}.We will use the modified gas well model to generate the combinations of these two variables which will maximize the net present value of the project.

6.1. Presentation of the case study

We are using the same reservoir as in Chapter 5 with the same properties.

6.1.1. Reservoir properties

The reservoir of interest is a dry gas reservoir with the following properties:

Table 6-1: Reservoir properties for the optimization procedure

PARAMETERS	VALUES
Initial reservoir pressure, psia	12000
Reservoir temperature, °R	640
Permeability, md	650
Porosity	19%
Water saturation	0.15
Reservoir thickness, ft	100
Drainage radius, ft	6000
Wellbore radius, ft	0.328
Skin factor	2
Non-Darcy coefficient	0.049

The following requirements are provided concerning the gas production contract:

- The lifetime of the project is 30 years
- The recovery factor is 80%

Given the conditions, we obtain the following results for the abandonment pressure, the total natural gas reserves, the cumulative gas production at the end of t_p years and the total gas field daily production rate per year:

- P_{res}=1418.9 psia.
- OGIP=7.745*10^11 SCF
- G_p= 6.196*10^11 SCF
- Q_{field}=56585 MSCFD

6.1.2. Fluid properties

The reservoir fluid is single-phase dry gas with the following properties:

Table 6-2: Fluid properties for the optimization procedure

PARAMETERS	VALUES
Gas gravity	0.55
Molecular weight, lbm/lbmol	16

6.1.3. Tubing system

The following values are provided for the tubing properties:

Table 6-3: Tubing system properties for the optimization procedure

PARAMETERS	VALUES
Pipe roughness ε, in	0.006
Tubing string length, ft	10000
Bottom hole temperature, °R	560
Wellhead temperature, °R	550

6.1.4. Economic parameters

In order to calculate the net present value, certain parameters need to be defined:

- The discount rate: it allows the discount of future cash-flows generated over the life of the project in order to evaluate them in the present. For most oil and gas companies, the general trend is to use a discount rate between 10% and 15%. Mian *(M. Mian 2002)* recommends the use of the weighted average cost of capital approach in order to evaluate the discount rate. This technique involves estimating the current costs of each source of funds (common equity, preferred stock, debt and any other elements in the capital structure) and finding the weighted average of the costs.

For this case study, we assume the discount rate to be equal to 10%.

- The total annual operating costs per well are assumed to be equal to $80000/well *(Abdel-Aal, Bakr and Al-Sahlawi 1992)*.
- The Initial Investment or Initial Cost per well is evaluated using the power law and sizing model also suggested by Mian *(M. Mian 2002)*; the cost equation can be found in section 4
- The gas price is set at $5/MSCF *(EIA 2009)*. This assumption is inspired by results from surveys on gas prices conducted by the Energy Information Administration and by values found for gas prices in the petroleum engineering literature.

6.1.5. Decision variables

We have the following input for the decision variables which are the wellhead pressure and the tubing diameter:

- The wellhead pressure will vary from 700 psia to 1050 psia
- The tubing size will vary from 1.35 in to 12 in.

The wellhead pressure and tubing size ranges are inspired from the results obtained in section 5. We are using such low wellhead pressures as compared to the initial reservoir pressure, because we are going to calculate the well producing capacity at P_{res}=1418.9 psia. The magnitudes for the wellhead pressure and tubing size are contained in Table 6-4.

Table 6-4: Wellhead pressure and tubing diameter values for the optimization procedure

P_{wh} in psia	d_{tubing} in inches
700.00	1.3500
718.42	1.9105
736.84	2.4711
755.26	3.0316
773.68	3.5921
792.11	4.1526
810.53	4.7132
828.95	5.2737
847.37	5.8342
865.79	6.3947
884.21	6.9553
902.63	7.5158
921.05	8.0763
939.47	8.6368
957.89	9.1974
976.32	9.7579
994.74	10.3184
1013.16	10.8789
1031.58	11.4395
1050.00	12.0000

6.2. Results

The typical production pattern of a gas field goes through three different phases: the build-up phase, the constant production phase and the decline phase. During the build-up phase, the cumulative gas production for the field Q_{field} increases as you drill more wells, until you reach the number of wells N_w which will allow you to meet the production requirement for the given field. Once you reach that condition, you enter into the constant production phase and you have to maintain that field production during the number of years specified in the gas project contract or the lifetime of the project. However, as you produce from the wells, the reservoir pressure depletes with time until it reaches the abandonment pressure value at the end of the project lifetime. Our objective in this optimization procedure is to establish the type of wells to drill in order to maximize the net present value of the project. The types of wells that are going to be drilled are determined by the operating conditions selected during the design phase. In our case, we are focusing on the tubing diameter d_{tubing} and the wellhead pressure P_{wh}. There is a unique well producing capacity that can be obtained from any combination of tubing size and wellhead pressure. Therefore, you will get a corresponding total number of wells in order to reach Q_{field}, a corresponding initial cost and a corresponding net present value. You also have to make sure that the chosen conditions will be able to sustain production in time, as the reservoir pressure declines.

The tubing size does not change throughout the life of the reservoir. This can be explained by the fact that once a tubing size is selected, it fixes the different casing diameters of the well and therefore the diameter of the hole. The wellhead pressure on the other hand, will change as reservoir depletion takes place. At higher reservoir pressures, the well has the ability to produce much more than at the abandonment pressure. Therefore, it is essential to lower the wellhead pressure for keeping a constant well productivity from the initial reservoir pressure to the abandonment pressure. This is illustrated on Figure 6-1.

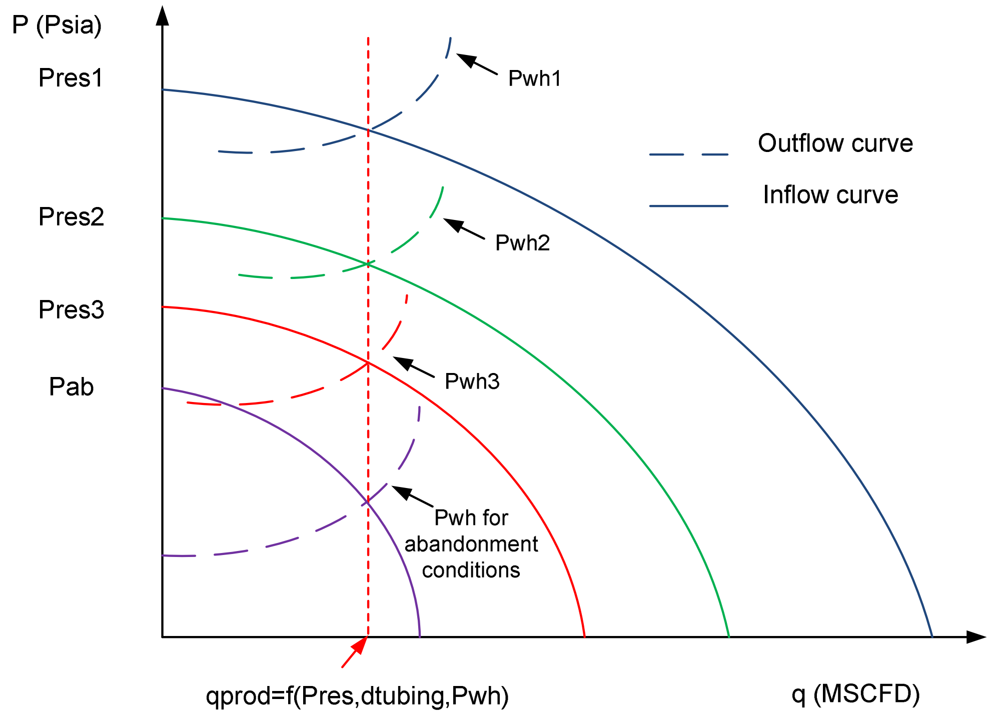

Figure 6-1: The effect of reservoir depletion on well producing capacity

Thus the well production capacity obtained at abandonment pressure will determine the lowest allowable wellhead pressure, the tubing size that should be selected and therefore the number of wells to be drilled and the well productivity to be maintained throughout the life of the reservoir. The following results have been obtained when running the modified gas well model for the case study at P_{res}=1418.9 psia.

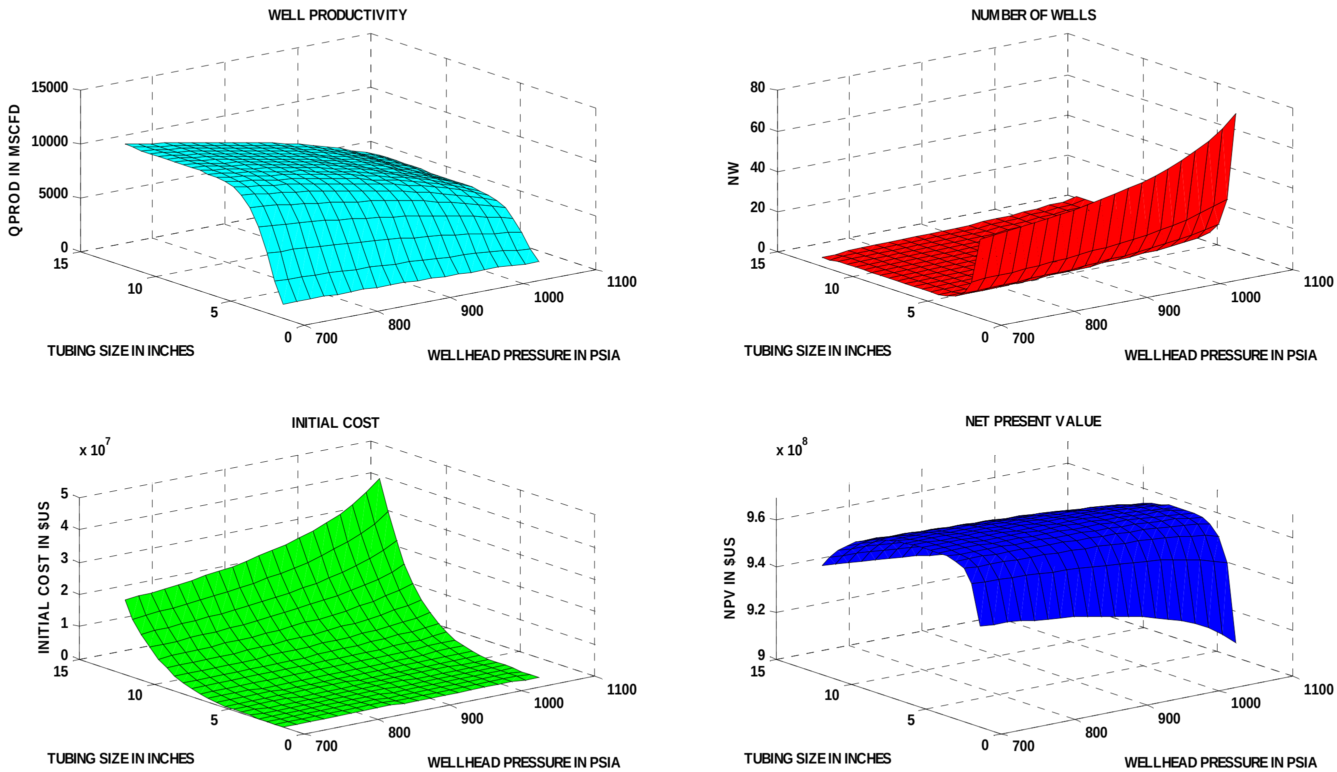

Figure 6-2: Well productivity, Initial Cost, Number of wells and Net Present Value versus tubing size and wellhead pressure.

Figure 6-2 presents the results obtained when running the modified gas well model for the different combinations of wellhead pressure and tubing size. It shows how the well producing capacity, the number of wells, the initial cost and the net present value of the project change with tubing size and wellhead pressure.

As we can see on Figure 6-2, at a given wellhead pressure, the well producing capacity increases with the tubing size. At a given tubing size, the well producing capacity increases as you decrease the wellhead pressure. The well producing capacity goes from q_{prod}=729.75 MSFCD obtained for d_{tubing}=1.35 in and P_{wh}=1050 psia to q_{prod}=11355 MSFCD obtained for d_{tubing}=12 in and P_{wh}=700 psia.

At a given wellhead pressure, the number of wells decreases when the tubing size increases. This can be accounted for by the fact that as you produce more from a well with a larger tubing diameter, you will need to drill less wells to be able to meet the total production of the field. At a given tubing size, the number of wells increases with the wellhead pressure. This is related to the fact that you produce less from the reservoir with high wellhead pressures, therefore you need to drill more wells in order to reach the total field production. The number of wells goes from N_w= 77 for d_{tubing}=1.35 in and P_{wh}=1050 psia to N_w= 5 for d_{tubing}=12 in and P_{wh}=700 psia.

The initial cost takes into account the total drilling and completion cost for a given type of well. The type of well drilled is linked to the selected tubing size. The initial cost has been estimated for various combinations of tubing size and wellhead pressure. It increases with the tubing size and the wellhead pressure. This is due to the fact that it becomes more and more expensive to drill larger wells, even if you only drill a small number of wells. As the wellhead pressure increases, the number of wells to be drilled also increases, which makes the project cost more. The initial cost varies from $3258.1 for d_{tubing}=1.35 in and P_{wh}=700 psia to $4.52*10^7 for d_{tubing}=12 in and P_{wh}=1050 psia.

The net present value is positive for all the combinations of wellhead pressure and tubing size that have been tested. Therefore the project is feasible for all the tested combinations of wellhead pressure and tubing size. At a given wellhead pressure, we can see that the net present value increases up to certain value of the tubing size and then starts decreasing. For a given tubing size, there is an increase of the net present value as you decrease the wellhead pressure. Therefore, the net present value surface suggests that there is an optimum value that can be selected for the tubing size in order to meet the contract requirements.

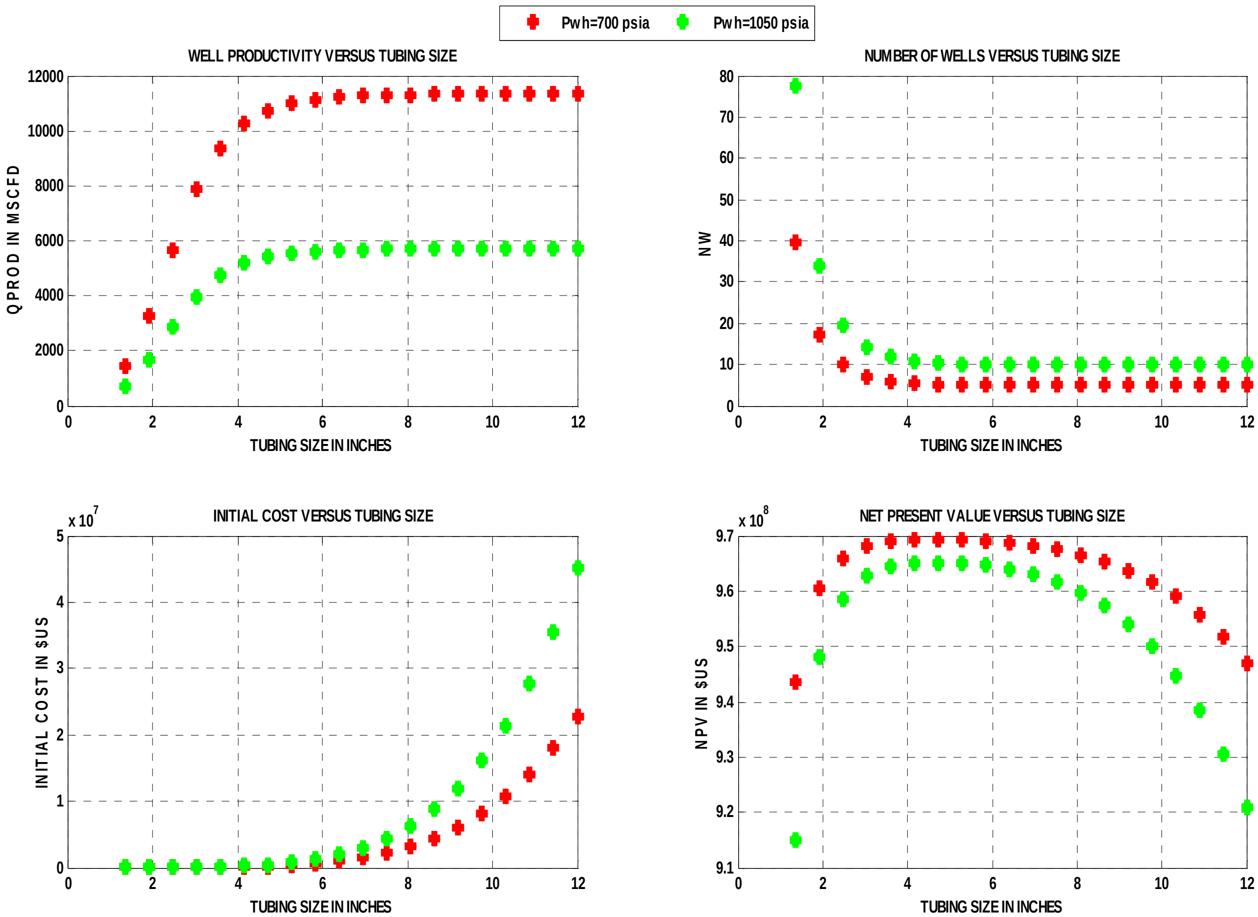

Figure 6-3: Initial Cost, Number of wells, Net Present Value and Well productivity versus tubing size

Figure 6-3 presents the initial cost, the number of wells, the net present value and the well productivity versus the tubing size for two different wellhead pressures.

When looking at the net present value plot, we can see that for a certain tubing diameter, the net present value reaches its maximum. Passed this tubing diameter, the net present value starts decreasing. We can estimate that the optimum tubing size is between 4.71 in and 5.27 in. The location of the optimum tubing size does not change with the wellhead pressure; only the value of the net present value changes. We have higher net present values for small wellhead pressures.

When looking at the well productivity plot, we can notice that the well producing capacity increases up to a tubing diameter which is between 5 in and 6 in. Passed this value, there is no significant improvement of the well producing capacity. The number of wells to be drilled does not change significantly for tubing sizes above 6 in and it is even at its lowest value. However, the initial cost plot and net present value plot tell us that drilling wells with a tubing diameter larger than 6 in will cost more and generate less and less profit. The suggested tubing size range which has been obtained from the calculation of the net present value is in accordance with the conclusions from scenario 1 of the sensitivity analysis.

We can infer from the previous analysis that the best option is to select a tubing size within the range 4.71 in and 5.27 in. The wellhead pressure chosen to operate at the abandonment conditions should be equal to 700 psia.

The economic evaluation is the last task to be fulfilled during the design phase of a gas project. An economic analysis is essential because it allows the production engineer to evaluate if the selected decision variables will enable to reach a production rate, which will ensure that the project generates profit. The sensitivity analysis conducted in chapter 5 not only displayed the effect of the different decision variables on the production system performance, but it also generated a set of doable combinations for these decision variables. However, the conclusions obtained from the sensitivity analysis only described the technical and physical aspects of the project. Thus, the application of these technical requirements to the production system might be too expensive for the company to carry out the project. Including an economic analysis in the design procedure, is essential because it allows you to translate the conclusions from the technical analysis in terms of generation of revenues which is the first goal of any oil and gas company and therefore to close the gap between the technical aspect of the project and the finite product which is the gas that is to be sold.

To conclude, for the case analyzed in this study, selecting a tubing size between 4.71 in and 5.27 in and matching it with the adequate wellhead pressure as reservoir depletion takes place, will not only help meet the technical requirements for the well to produce, but it will also generate the maximum amount of profit among all the available alternatives.

Chapter 7

Concluding remarks

The objectives of this study were to examine the influence of the tubing diameter, the wellhead pressure, the choke size and the gathering tank pressure on a gas production system and to determine the optimal combination of decision variables that will maximize the net present value of a gas production contract. A gas well model and a modified gas well model have been designed in the MATLAB environment in order to help attain the goals of this study. The following conclusions have been reached:

- The combination of tubing diameter, wellhead pressure, choke size and gathering tank pressure has an influence on the performance of the gas production system. Any association of these four decision variables will not always cause the gas to flow from the reservoir through the well tubing, through the surface choke and then through the surface flow-line. In the design phase, it is important for the production engineer to test all the available alternatives by analyzing the flow from the reservoir to the surface in a similar method as the one presented in this work, in order to ensure that none of its design parameters will cause the well not to produce.
- The well producing capacity is only influenced by the wellhead pressure and the tubing size, at a given reservoir pressure. In order to produce more from the well, you need to associate large diameters to low wellhead pressures. However, there is a limitation to how large the selected tubing size can be. You can determine this by calculating the well producing capacity for the different tubing diameters that are available.

- The choke size and the gathering tank pressure have an impact on the performance of the surface flow-line. Not all choke sizes can be associated with a given wellhead pressure or tubing size. Not all gathering tank pressures can be used with certain wellhead pressures and tubing sizes.
- The economic analysis in this study was based on the wellhead pressure and the tubing size. However, it displayed the influence of the tubing size on the project economics. It was proven that there is an economic tubing size optimum that can be chosen among several alternatives. In our case, the economic optimum happens to be in a range that superimposes itself with the range given for the physical optimum that was found in the sensitivity analysis.

However, there are some restrictions that need to be taken into account:

- The quantitative results of this study apply only to the reservoir properties, fluid properties, tubing system properties, surface flow-line properties and to the assumed gas contract requirements. Changing the input parameters, will therefore change the values obtained in the sensitivity analysis as well as the result of the suggested optimal combination of wellhead pressure and tubing size in order to maximize the net present value of the project. The qualitative results on the other hand, can be applied to any dry gas reservoir with the same assumptions that have been taken in this case.
- Only discrete values were tested in this study. Therefore, it is possible to get more precise results for the range of values of the tubing size, the wellhead pressure, the choke size and the gathering tank pressure, by using smaller increments.
- Wellhead pressure and tubing size were the only decision variables tested in the optimization procedure. An optimum was found for the tubing size. We could only notice that we have higher profits for lower wellhead pressures. However, we know that having

low wellhead pressures can become costly later on in the project life because you will have to compress the gas to be able to reach the conditions at the sales line. This aspect was not included in the initial cost function, since you might need to install compressors at any time during the project lifetime and it is therefore difficult to generalize. In addition, wellhead pressures as high as 11500 psia, imply the use of heavy equipment, which is extremely expensive and increases the initial cost of the project. The influence of the choke size and the gathering tank pressure on the profit from the project was also not evaluated.

- The chosen ranges for the decision variables in this study, are very wide and do not necessarily reflect the ranges that are available in the industry.
- It was noticed that the allure of the net present value plot is somewhat related to the initial cost function. The initial cost equation used in that study is based on the economics of scale. After several trial and errors, the selected cost function is the one that gave the best allure in terms of net present value. In fact, no clear relationship has yet been established between the drilling cost of a well and the tubing size. It has been proven that the drilling cost varies exponentially with the well depth. It is also recognized that drilling costs increase when you drill bigger wells but there is no explicit equation relating drilling costs to the tubing diameter, as there is for the well depth. Therefore, the equation given in this study is an attempt to quantify the relationship between the drilling costs and the tubing diameter. A more precise expression can be established using statistical techniques and sufficient field data. However, this was not the objective of this research.

For future work, it will be interesting to adapt the well model and the optimization procedure to any kind of hydrocarbon reservoir and any kind of flowing fluid. Multi-phase flow can be analyzed from the reservoir to the surface and the influence of all decisions variables presented in this work can also be examined. Finally, a more exhaustive cost function accounting for all decision variables can be established and incorporated to the optimization procedure.

APPENDIX: SOURCE CODE FOR THE MODIFIED GAS WELL MODEL

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% 03/01/2009
%% ANNICK NAGO
%% MODIFIED GAS WELL MODEL
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function modifiedgaswellmodel()
clear all;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%DATA INPUT
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
R=10.73;
Psc=14.7;
Tsc=520;
Tres=640;
Swi=0.15;
Phi=0.19;
Presinitial=input('input reservoir initial pressure:');
tmax=input('input the lifetime of the project in years:');% Lifetime of
the gas contract in years
REC=input('input the expected recovery factor:');% recovery factor
Pwh=linspace(700,1050,20);% wellhead pressure values
dpipe=linspace(1.35,12,20);% tubing diameter values
fprintf('Wellhead pressure:');
fprintf('\n');
fprintf('%8.3f\n',Pwh);
fprintf('\n');
fprintf('Tubing size:');
fprintf('\n');
fprintf('%8.3f\n',dpipe);
fprintf('\n');
Pres=Presinitial;
q=linspace(0,65000,100);% flow rate values
k=650;
h=100;
re=6000;
rw=0.328;
Area=pi*(re^2-rw^2);% drainage area in ft^2
Areac=Area/43560;% drainage area in acres
s=2;
D=0.049;
gammag=0.55; % specific gravity
MW=gammag*28.97;
gamma=(2.738-log10(gammag))/2.328;% specific heat ratio
epsilonw=0.006;
epsilons=0.0025;
Lvert=10000;
Tg=540;
Twf=560;
Twh=550;
Lsurf=13055;
Cw=60.17*Lvert;% Nominal cost of drilling a well in dollars
```

```
dnominal=8;% nominal tubing size
Opw=80000;% Annual operating cost per well
rate=0.1;% discount rate
Price=5;% Gas price in dollars per MSCF
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% CALCULATING PSEUDO-CRITICAL PROPERTIES
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%
Ppc=677+15*gammag-37.5*gammag^2;
Tpc=168+325*gammag-12.5*gammag^2;
%% PARAMETERS RELATED TO RESERVOIR PROPERTIES
[Z,Rho,mu]=calcfluidproperty1(Tres,Pres,MW,R,Ppc,Tpc);
Zresinitial=Z;
Bgi=0.0283*Z*Tres/Pres;%Gas Formation Volume Factor in RCF/SCF
OGIP=Area*h*Phi*(1-Swi)/Bgi;% original gas in place in SCF
Gmax=REC*OGIP;% total cumulative production at the end of tmax years
Qfield=Gmax/(1000*tmax*365);  %  total  field  production  per  year  in
MSCF/D,
%constant for all the years of production
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% FINDING Reservoir pressure at the end of tmax
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
P=linspace(0,Presinitial,25);
for u=1:25
    [Z,Rho,mu]=calcfluidproperty1(Tres,P(u),MW,R,Ppc,Tpc);
    Func(u)=P(u)/Z-Presinitial/Zresinitial*(1-REC);
end
%% BISECTION METHOD TO FIND THE RESERVOIR PRESSURE AFTER tmax YEARS OF
%%PRODUCTION
for u=1:24
    if (Func(u)*Func(u+1))>0
        continue
    end
    P_left=P(u);
    P_right=P(u+1);
    break
end
[Z,Rho,mu]=calcfluidproperty1(Tres,P_left,MW,R,Ppc,Tpc);
Z_left=Z;
F_left=P_left/Z_left-Presinitial/Zresinitial*(1-REC);
[Z,Rho,mu]=calcfluidproperty1(Tres,P_right,MW,R,Ppc,Tpc);
Z_right=Z;
F_right=P_right/Z_right-Presinitial/Zresinitial*(1-REC);
if (F_left*F_right)<0
    while (P_right-P_left)>1
        P_middle=(P_right+P_left)/2;
        [Z,Rho,mu]=calcfluidproperty1(Tres,P_middle,MW,R,Ppc,Tpc);
        Z_middle=Z;
        F_middle=P_middle/Z_middle-Presinitial/Zresinitial*(1-REC);
        [Z,Rho,mu]=calcfluidproperty1(Tres,P_left,MW,R,Ppc,Tpc);
        Z_left=Z;
        F_left=P_left/Z_left-Presinitial/Zresinitial*(1-REC);
        if (F_middle*F_left)>0
            P_left=P_middle;
        else
            P_right=P_middle;
```

```
        end
    end
    Pres=P_middle;
else
    disp('no solution:');
    Pres=0;
end
[Z,Rho,mu]=calcfluidproperty1(Tres,Pres,MW,R,Ppc,Tpc);
Zmax=Z;
Alpha=1424*mu*Z*Tres*(log(0.472*re/rw)+s)/(k*h);
Beta=1424*mu*Z*Tres*D/(k*h);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% CALCULATING THE BOTTOMHOLE FLOWING PRESSURE FOR VARIOUS FLOW RATES
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
for m=1:100
    Exp1(m)=Pres^2-(Alpha*q(m)+Beta*q(m)^2);
    Pwf1(m)=Exp1(m)^(1/2);
end
%% FOR EACH SET OF DESIGN PARAMETERS
for n=1:20
    disp('outer loop n :');
    disp(n);
    for j=1:20


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Generating flowing bottom-hole pressure with the outflow relationship
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    Tavg=(Twf+Twh)/2;
    Pavg=(Pres+Pwh(n))/2;
    [Z,Rho,mu]=calcfluidproperty1(Tavg,Pavg,MW,R,Ppc,Tpc);
    for m=1:100

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
        %   CALCULATING   Pwf   for   various   sizes   of   the   tubing
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
        Zfactor=Z;
        density=Rho;
        Visc=mu;
        Re(m)=20.09*gammag*q(m)/(dpipe(j)*Visc);
        ff(m)=(1/(-4*log10(epsilonw/3.7065-
5.0452/Re(m)*log10(epsilonw^1.1098/2.8257+(7.149/Re(m))^0.8981))))^2;
        S(m)=-0.0375*gammag*Lvert/(Zfactor*Tavg);
        Exp2(m)=Pwh(n)^2*exp(-S(m))+2.685*10^(-
3)*ff(m)*(Zfactor*Tavg)^2*q(m)^2*(exp(-S(m))-1)/dpipe(j)^5;
            if Exp2(m)>=0
                Pwf2(m)=(Pwh(n)^2*exp(-S(m))+2.685*10^(-
3)*ff(m)*(Zfactor*Tavg)^2*q(m)^2*(exp(-S(m))-1)/dpipe(j)^5)^(1/2);
            else
        diff2(j,i)=1;
            Pwf2_new(m)=1000;
            while min(min(diff2))>0.5
                Pwf2(m)=Pwf2_new(m);
                F_Pwf2(m)=Pwf2(m)^2-Exp2(m);
                dF_Pwf2(m)=2*Pwf2(m);
```

```
                Pwf2_new(m)=Pwf2(m)-F_Pwf2(m)/dF_Pwf2(m);
                diff2(m)=abs(Pwf2_new(m)-Pwf2(m));
                disp(diff2(m));
            end
            Pwf2(m)=Pwf2_new(m);
            end
    end

    for m=1:100
        diffP(m)=Pwf1(m)^2-Pwf2(m)^2;

    end
    for m=2:100
        y(m)=log(q(m));
        x(m)=log(Pres^2-Pwf1(m)^2);
    end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% CALCULATING THE WELL PRODUCING CAPACITY FOR EACH COMBINATION OF PWH
%% AND Dpipe AND FINDING THE NET PRESENT VALUE
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%% beginning of bisection method

    disp('step:');
    disp(j);
    S(j)=-0.0375*gammag*Lvert/(Zfactor*Tavg);
    for m=1:99
        if(diffP(m)*diffP(m+1))>0
            continue
        end
        qprod_left(j)=q(m);
        qprod_right(j)=q(m+1);
        break
    end
    K1(j)=epsilonw/3.7065;
    K2_qleft(j)=5.0452*Visc*dpipe(j)/(20.09*gammag*qprod_left(j));
    K3(j)=epsilonw^1.1098/2.8257;

K4_qleft(j)=(7.149*Visc*dpipe(j)/(20.09*gammag*qprod_left(j)))^0.8981;
    c_qleft(j)=log10(K3(j)+K4_qleft(j));
    b_qleft(j)=K1(j)-K2_qleft(j)*c_qleft(j);
    a_qleft(j)=log10(b_qleft(j));
    ffleft(j)=1/(16*(a_qleft(j))^2);
    F_qleft(j)=Pres^2-(Alpha*qprod_left(j)+Beta*qprod_left(j)^2)-
(Pwh(n)^2*exp(-S(j))+2.685*10^(-
3)*ffleft(j)*(Zfactor*Tavg)^2*qprod_left(j)^2*(exp(-S(j))-
1)/dpipe(j)^5);
    K2_qright(j)=5.0452*Visc*dpipe(j)/(20.09*gammag*qprod_right(j));

K4_qright(j)=(7.149*Visc*dpipe(j)/(20.09*gammag*qprod_right(j)))^0.8981
;
    c_qright(j)=log10(K3(j)+K4_qright(j));
    b_qright(j)=K1(j)-K2_qright(j)*c_qright(j);
    a_qright(j)=log10(b_qright(j));
    ffright(j)=1/(16*(a_qright(j))^2);
```

```
    F_qright(j)=Pres^2-(Alpha*qprod_right(j)+Beta*qprod_right(j)^2)-
(Pwh(n)^2*exp(-S(j))+2.685*10^(-
3)*ffright(j)*(Zfactor*Tavg)^2*qprod_right(j)^2*(exp(-S(j))-
1)/dpipe(j)^5);
    disp('F_qleft(j)');
    disp(F_qleft(j));
    disp('F_qright(j)');
    disp(F_qright(j));

    if (F_qright(j)*F_qleft(j))<0
        while (qprod_right(j)-qprod_left(j))>10^-3
        qprod_middle(j)=(qprod_right(j)+qprod_left(j))/2;
        K1(j)=epsilonw/3.7065;
        K2_q(j)=5.0452*Visc*dpipe(j)/(20.09*gammag*qprod_middle(j));
        K3(j)=epsilonw^1.1098/2.8257;

K4_q(j)=(7.149*Visc*dpipe(j)/(20.09*gammag*qprod_middle(j)))^0.8981;
        c_q(j)=log10(K3(j)+K4_q(j));
        b_q(j)=K1(j)-K2_q(j)*c_q(j);
        a_q(j)=log10(b_q(j));
        ff(j)=1/(16*(a_q(j))^2);
        S(j)=-0.0375*gammag*Lvert/(Zfactor*Tavg);
        F_q(j)=Pres^2-(Alpha*qprod_middle(j)+Beta*qprod_middle(j)^2)-
(Pwh(n)^2*exp(-S(j))+2.685*10^(-
3)*ff(j)*(Zfactor*Tavg)^2*qprod_middle(j)^2*(exp(-S(j))-1)/dpipe(j)^5);
        K2_qleft(j)=5.0452*Visc*dpipe(j)/(20.09*gammag*qprod_left(j));

K4_qleft(j)=(7.149*Visc*dpipe(j)/(20.09*gammag*qprod_left(j)))^0.8981;
        c_qleft(j)=log10(K3(j)+K4_qleft(j));
        b_qleft(j)=K1(j)-K2_qleft(j)*c_qleft(j);
        a_qleft(j)=log10(b_qleft(j));
        ffleft(j)=1/(16*(a_qleft(j))^2);
        F_qleft(j)=Pres^2-(Alpha*qprod_left(j)+Beta*qprod_left(j)^2)-
(Pwh(n)^2*exp(-S(j))+2.685*10^(-
3)*ffleft(j)*(Zfactor*Tavg)^2*qprod_left(j)^2*(exp(-S(j))-
1)/dpipe(j)^5);
        disp('F_q(j)');
        disp(F_q(j));
        disp('F_qleft(j)');
        disp(F_qleft(j));

        if (F_q(j)*F_qleft(j))>0
            qprod_left(j)=qprod_middle(j);
        else
            qprod_right(j)=qprod_middle(j);
        end
    end
    qprod(j)=qprod_middle(j);
    Pwfprod(j)=(Pres^2-(Alpha*qprod(j)+Beta*qprod(j)^2))^0.5;
    disp(qprod(j));
    disp(Pwfprod(j));
    else
        disp('no solution');
        qprod(j)=0;% no solution
        Pwfprod(j)=0;% no solution
```

```
    end
if qprod(j)~=0
    Nw(j)=Qfield/qprod(j);
    InitialCost(j)=Cw*(dpipe(j)/dnominal)^5*Nw(j);
    Yearlyrevenue(j)=qprod(j)*365*Nw(j)*Price;
    OperatingCost(j)=Opw*Nw(j);
    Npv(j)=(Yearlyrevenue(j)-OperatingCost(j))*((1+rate)^tmax-
1)/(rate*(1+rate)^tmax)-InitialCost(j);
else
    Nw(j)=0;
    InitialCost(j)=0;
    Yearlyrevenue(j)=0;
    Npv(j)=0;
end

    end
%% SAVING THE RESULTS%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

    QPROD(n,:)=qprod;
    PWFCALC(n,:)=Pwfprod;
    NW(n,:)=Nw;
    INITIALCOST(n,:)=InitialCost;
    YEARLYREVENUE(n,:)=Yearlyrevenue;
    NPV(n,:)=Npv;
end
%%PRINT RESERVE ESTIMATES, QFIELD, AND RESERVOIR PROPERTIES AT
%%ABANDONMENT
%%CONDITIONS%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

disp('Presinitial:');
disp(Presinitial);
disp('Zresinitial:');
disp(Zresinitial);
disp('Pres in psia at tmax:');
disp(Pres);
disp('Zmax:');
disp(Zmax);
disp('OGIP in SCF:');
disp(OGIP);
disp('Gmax in SCF:');
disp(Gmax);
disp('Qfield in MSCF:');
disp(Qfield);
disp('Field area in acres:');
disp(Areac);
dlmwrite('wellproductivitynpv.txt',QPROD,'delimiter','\t');
dlmwrite('numberofwells.txt',NW,'delimiter','\t');
dlmwrite('initialcost.txt',INITIALCOST,'delimiter','\t');
dlmwrite('yearlyrevenue.txt',YEARLYREVENUE,'delimiter','\t');
dlmwrite('netpresentvalue.txt',NPV,'delimiter','\t');
%%FIGURES%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
figure;
subplot(2,2,1);
surf(Pwh,dpipe,QPROD', 'FaceColor', 'cyan','EdgeColor', 'black' );
```

```
title('WELL PRODUCTIVITY')
subplot(2,2,2);
surf(Pwh,dpipe,NW','FaceColor', 'red','EdgeColor','black');
title('NUMBER OF WELLS')
subplot(2,2,3);
surf(Pwh,dpipe,INITIALCOST','FaceColor','green','EdgeColor','black');
title('INITIAL COST')
subplot(2,2,4);
surf(Pwh,dpipe,NPV','FaceColor','blue','EdgeColor', 'black');
title('NET PRESENT VALUE')
hold off;

figure;
subplot(2,2,1);
plot(dpipe,INITIALCOST(1,:),'r-+');
hold on;
plot(dpipe,INITIALCOST(20,:),'g-*');
title('INITIAL COST VERSUS TUBING SIZE')
subplot(2,2,2);
plot(dpipe,NW(1,:),'r-+');
hold on;
plot(dpipe,NW(20,:),'g-*');
title('NUMBER OF WELLS VERSUS TUBING SIZE')
subplot(2,2,3);
plot(dpipe,NPV(1,:),'r-+');
hold on;
plot(dpipe,NPV(20,:),'g-*');
title('NET PRESENT VALUE VERSUS TUBING SIZE')
subplot(2,2,4);
plot(dpipe,QPROD(1,:),'r-+');
hold on;
plot(dpipe,QPROD(20,:),'g-*');
title('WELL PRODUCTIVITY VERSUS TUBING SIZE')
hold off;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% CALCULATING FLUID PROPERTIES WITH THE DRANCHUK AND ABU-KASSEM
%% CORRELATION FOR THE Z-FACTOR AND THE LEE ET AL CORRELATION FOR THE
%% GAS VISCOSITY
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [Z,Rho,mu]=calcfluidproperty1(T,P,MW,R,Ppc,Tpc)
% Calculating Z factor
A1=0.3265;
A2=-1.0700;
A3=-0.5339;
A4=0.01569;
A5=-0.05165;
A6=0.5475;
A7=-0.7361;
A8=0.1844;
A9=0.1056;
A10=0.6134;
A11=0.7210;
Tpr=T/Tpc;
Ppr=P/Ppc;
```

```
%Newton Raphson Method
diff=1;
    Z_new=0.1;
    while diff>10^-10
        Z=Z_new;
        F=Z-
((A1+A2/Tpr+A3/Tpr^3+A4/Tpr^4+A5/Tpr^5)*0.27*Ppr/(Z*Tpr)+(A6+A7/Tpr+A8/
Tpr^2)*(0.27*Ppr/(Z*Tpr))^2-
A9*(A7/Tpr+A8/Tpr^2)*(0.27*Ppr/(Z*Tpr))^5+A10*(1+A11*(0.27*Ppr/(Z*Tpr))
^2)*(0.27*Ppr/(Z*Tpr))^2/Tpr^3*exp(-A11*(0.27*Ppr/(Z*Tpr))^2)+1);

dF=(1/Tpr)*(1+(A1+A2/Tpr+A3/Tpr^3+A4/Tpr^4+A5/Tpr^5)*0.27*Ppr/(Z*Tpr)*1
/Z+2*(A6+A7/Tpr+A8/Tpr^2)*(0.27*Ppr/(Z*Tpr))^2*1/Z-
5*A9*(A7/Tpr+A8/Tpr^2)*(0.27*Ppr/(Z*Tpr))^5*1/Z+2*A10*(0.27*Ppr/(Z*Tpr)
)^2/(Tpr^3*Z)*(1+A11*(0.27*Ppr/(Z*Tpr))^2-
(A11*(0.27*Ppr/(Z*Tpr))^2)^2)*exp(-A11*(0.27*Ppr/(Z*Tpr))^2));
        Z_new = Z-(F/dF);
        diff= abs(Z_new-Z);
    end

     Z=Z_new;
%Calculating Rho
Rho=P*MW/(Z*R*T);
% Calculating Mu
X=3.5+986/T+0.01*MW;
Y=2.4-0.2*X;
M=X*(Rho/62.4)^Y;
K=(9.4+0.02*MW)*T^1.5/(209+19*MW+T);
mu=10^(-4)*K*exp(M);
end



end
```

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