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THE EFFECTS OF GEOMETRY AND ADJACENT REGENERATORS ON SHELL-AND-TUBE HEAT EXCHANGERS IN OSCILLATING FLOWS

A Dissertation in

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by

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ABSTRACT

An experimental study into the effects of geometry and the presence of adjacent screens on the acoustic impedances and heat transfer performance of shell-and-tube heat exchangers in oscillating flow was conducted. Measurements of linear and nonlinear acoustic impedances were conducted simultaneously with heat transfer measurements.

The results showed that rounded tube-ends produce less nonlinear resistance than flat tube-ends. A stack of screens placed adjacent to an exchanger results in nonlinear resistances that are within 5% of those that result when no adjacent screens are used. The screens also act to reduce the drop in the inertance of the exchanger at higher displacements.

The length of the exchanger was found to influence the amount of nonlinear acoustic resistance. Correlations for this effect were found, but the cause is unknown.

Heat transfer measurements showed that the aspect ratio of the exchanger tubes (the ratio of length to diameter) is an important parameter in predicting heat transfer. The presence of adjacent screens increases this effect. Correlations including these effects were found.

It was found that when screens were placed adjacent to an exchanger, the heat transfer effectiveness dropped by as much as 20%. Likewise, when the ends of the exchanger tubes were rounded (instead of flat) effectiveness dropped by as much as 25% again.
Sudden increases in effectiveness were observed at higher frequencies and displacements. It was found that these increases correspond to the onset of turbulent bursts during velocity peaks.

Application of the Chilton and Colburn-J Factor analogy to oscillating flows was also investigated. It was found that at higher friction factors the analogy did not hold. Some agreement may exist at lower friction factors; however, there is insufficient data within this range to derive reliable correlations.

Comparisons between measurements and the heat exchanger model in the TX segment of the DeltaEC software package were made. Comparisons to models using steady-flow heat transfer correlations (using the TASFE approximation) were also made. It was found that none of these models accurately predicted heat transfer performance.

Data and correlations are presented in dimensionless form, and an explanation as to the application of the results to other exchanger sizes, different gases, and different conditions is provided.
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Chapter 1

Introduction

There are many applications in which heat exchangers are used in oscillating flows, ranging from thermoacoustic and Stirling devices to water de-salinification processes. However, very little is understood about their performance in oscillating flow. There are two main considerations: maximizing heat transfer, and minimizing acoustic losses. Both of these effects depend on flow conditions. This suggests that the geometry—or shape—of the exchanger impacts its performance. The aim of this study is to experimentally quantify that impact using tube-and-shell type exchangers in oscillating flow.

1.1 Heat Exchangers in Oscillating Flow: Heat Transfer

Calculating heat exchanger performance in steady-flow is generally straightforward. If the fluid inlet and outlet temperatures and flow rates are known, the Log-Mean Temperature Difference (LMTD) method allows the designer to select a heat exchanger type and then calculate the surface area (or exchanger size) required. If on the other hand the heat exchanger type and size are known, the Effectiveness-Number of Transfer Units (Effectiveness-NTU) method enables the designer to predict the fluid outlet temperatures for given flow rates and input temperatures. If an application requires
a compact exchanger, the engineer can find a suitable model by studying Colburn-J factor/Reynolds number relationships\(^1\) provided by the manufacturer.

Heat transfer in oscillating flow is not nearly so well defined, however. For example, “parallel flow” and “counter flow” have little meaning when one of the flows is oscillatory. Likewise, in steady flow Colburn-J factors and Reynolds numbers are proportional to velocity. In oscillatory flow though, a given velocity can be produced from infinite combinations of frequency and displacement.

Besides definitions, the physics of oscillating flow is different. Steady internal flow profiles (both thermal and viscous) become “fully developed”, whereas oscillating flow profiles are much more complex functions of space and time. In steady internal flow, the heat transfer rate at a particular position is a constant. In oscillating flow it becomes a function of time, as the velocity is constantly changing in both magnitude and direction.

### 1.2 Heat Exchangers in Oscillating Flow: Acoustic Power Losses

In addition to heat transfer, an exchanger presents an acoustic impedance (the ratio of acoustic pressure drop to volumetric velocity) to the oscillating fluid inside the thermoacoustic device. This impedance is complex, consisting of both real and imaginary parts. Acoustic power loss occurs when the real part of this impedance is non-zero. Linear acoustic theory describes this loss well when fluid velocities are small. However, additional nonlinear losses caused by the exchanger’s geometry become
significant as the velocity increases. At present, our understanding of these nonlinearities is limited. A similar effect occurs in steady flow, however.

In steady flow, pressure losses occur at transitions between channels or changes in direction (i.e. tees, expansions, elbows, etc.). These losses are relatively small contributors to the overall pressure drops in long piping systems, and are therefore called “minor losses.” They are described by the following equation:

\[ \Delta P = \frac{1}{2} \kappa \rho v^2. \]  

Here \( \Delta P \) is the pressure loss, \( \rho \) is the fluid density, \( v \) is the cross-sectionally averaged particle velocity, and \( \kappa \) is the “minor loss coefficient”. This coefficient is a function of both the geometry of the transition and the fluid flow profile through it. The value of the coefficient is usually experimentally derived (especially for turbulent flows), as analytic expressions are possible for only a very few cases.

The mechanism of these losses is intuitively straightforward. Consider a steady flow through an ideal sudden expansion. Bernoulli’s law states that some of the fluid’s kinetic energy will be converted into additional pressure in the wider cross-section, and that this occurs without loss.

In an actual expansion, however, this idealization is too simple. As the flow passes from narrow to wide cross-sections some of the kinetic energy goes into eddies and turbulence. This portion of the kinetic energy is eventually dissipated as heat, resulting in a loss of stagnation pressure.

This occurs in oscillating flow as well; but while these losses are sometimes negligible in piping systems, they are often very significant in thermoacoustic engines
and refrigerators. Accurate prediction of acoustic minor losses is therefore a necessity for accurate thermoacoustic system design. However, analytic descriptions of these losses require some rather drastic simplifying assumptions.

The first and most important of these is codified in what has become known as the “Iguchi hypothesis.” This hypothesis claims that at high enough velocities the flow is “quasi-steady”—that is, that at every instant in time the behavior of the oscillating flow is the same as that of steady flow at the same instantaneous velocity. Based on this assumption, equation (1.1) is then adapted for time-harmonic flow, and division of both sides by volumetric velocity (the product of cross-sectionally averaged particle velocity and the cross-sectional area—νA) yields an expression describing a velocity-dependent acoustic resistance:

$$R_m = \frac{4}{3\pi A^2} \kappa \rho U.$$  (1.2)

Here $U$ is the peak volumetric velocity, and $\rho$ is the mean fluid density.

The next assumptions that are made are: first, that the volumetric velocity is sinusoidal; and second, that the minor loss coefficient $\kappa$ is not itself a function of velocity. A third assumption—that minor loss coefficients derived for steady flow also hold for oscillating flow—is forced by a lack of experimental data.

There are problems with this quasi-steady analysis. First, it is largely unproven—experimental investigations have been conducted for only a few transition geometries. Second, the Iguchi hypothesis is rigorous only when the oscillating flow is completely turbulent—in other words, when the oscillating velocity is very high. These conditions are not likely to ever occur in thermoacoustic heat exchangers, however. Third, equation
(1.2) is far too simple if the flow has any additional frequency components or if a steady flow is also present. Fourth, acoustic minor losses are nonlinear and generate harmonic content that is not described by these equations. Fifth, this analysis does not take into account nor predict whatever nonlinear inertial effects might be caused by the transition.

1.3 Other Implications for Heat Exchangers

There are other implications specific to the design of thermoacoustic heat exchangers that have not yet been addressed. First, minor losses are caused by flow separation and turbulence dissipation at transitions in channels. Does this alteration of the flow impact heat transfer? For example, if minor losses are large is heat transfer increased?

Second, it is easy to envision minor losses occurring at the outer end of a heat exchanger—the end facing a thermal buffer tube, for example—but what about at the inner end—the end that faces a regenerator? Does the close proximity of the regenerator change or reduce the minor loss at that end? If so, is there a corresponding impact on the heat transfer performance of the exchanger?

1.4 Scope of the Present Work

There are two main goals of the present work: first, to experimentally quantify the effects of the shape of the ends of the exchanger tubes on heat transfer (if any) and acoustic power loss; and second, to identify the effects of an adjacent regenerator on both
the heat transfer performance and the acoustic impedance of these exchangers. Performance correlations are then developed from this data. This will allow designers to not only predict the performance of various exchanger geometries, but also to weigh the heat transfer effectiveness versus the acoustic power loss of those geometries and thereby optimize the design of the exchanger for their particular application.

The effects of two tube-end shapes have been examined: flat and rounded ends. Only one radius of curvature was tested however (1/8 in. radius). These cases are likely to be sufficient for designers’ purposes.

1.5 Dimensionless Numbers in this Study

Several dimensionless numbers are used in this study. For the benefit of the reader, a brief review of each of those numbers is included here.

The Reynolds number \( \text{Re} \) is the ratio of the inertial to viscous forces within a fluid flow, and is often used as an indicator of flow conditions (laminar vs. turbulent flow, for example). For flows through a pipe this ratio is given by:

\[
\text{Re} = \frac{D \rho v}{\mu},
\]

where \( D \) is the inner diameter of the pipe; \( \rho \) is the mean density of the fluid; \( v \) is the cross-sectionally averaged particle velocity of the fluid; and \( \mu \) is the viscosity of the fluid.
The Prandtl number of the fluid Pr is an indicator of the thickness of the viscous boundary layer in relation to that of the thermal boundary layer of the fluid. It is given by:

\[ \text{Pr} = \frac{\mu c_p}{k}. \]  

(1.4)

In this expression, \( c_p \) is the constant-pressure specific heat of the fluid, and \( k \) is the thermal conductivity of the fluid.

The Nusselt number Nu is a dimensionless form of the heat transfer coefficient \( h \). They are related by the following expression:

\[ \text{Nu} = h \frac{L}{k}, \]  

(1.5)

where \( h \) is the convection coefficient; \( L \) is the length of the heat exchanger; and \( k \) is the thermal conductivity of the fluid moving through the exchanger. The Nusselt number is related to heat transfer produced by an exchanger by the following:

\[ \dot{Q} = \text{Nu} \frac{k}{L} A_s \Delta T, \]  

(1.6)

where \( A_s \) is the surface area of the exchanger in thermal contact with the fluid, and \( \Delta T \) is the difference between the surface temperature of the exchanger and the spatially averaged temperature of the fluid within the exchanger.

Heat exchanger effectiveness \( E \) is defined as the ratio of the heat transfer produced by an exchanger to the maximum possible heat transfer. In a perfect exchanger, the fluid would enter at one temperature and come completely to the surface temperature of the exchanger before exiting. Thus, effectiveness can be expressed as:
where \( \dot{m} \) is the mass flow-rate of the fluid through the exchanger; \( c_p \) is the constant-pressure specific heat of the fluid; \( T_{\text{fluid in}} \) is the cross-sectionally averaged temperature of the fluid as it enters the exchanger; and \( T_{\text{HX surface}} \) is the surface temperature of the heat exchanger. This ratio can also be re-written as:

\[
E = \frac{\Delta T_{\text{fluid}}}{T_{\text{fluid in}} - T_{\text{HX surface}}},
\]  

(1.8)

This expression states that the heat transfer effectiveness is the ratio of the change in the fluid temperature due to heat transfer to the maximum possible temperature change the fluid could experience.

The Colburn-J factor is very often used in steady-flow applications. Its usefulness comes from the Chilton and Colburn-J Factor analogy. This analogy states that heat, mass, and momentum transfer in thermally and viscously fully-developed steady flows are all related to each other by simple constant conversion factors. The Colburn-J factor can be defined as:

\[
J_H = \frac{\text{Nu}}{\text{Re} \cdot \text{Pr}^{1/3}},
\]  

(1.9)

In this expression \( \text{Re} \) is the Reynolds number, and \( \text{Pr} \) is the Prandtl number of the fluid.
1.6 Limitations of the Present Study

There are a few significant limitations to this study. First, this study considers the thermoacoustic side of the exchanger only—the performance and power requirements of the internal side of the exchanger—the side through which liquid flows—are not addressed. Second, the flow during all measurements was approximately incompressible. In thermoacoustic devices however, acoustic pressure oscillations are often ten to fifteen percent of the mean pressure. The effect of these pressure oscillations on heat transfer was not examined. Third, the only gas used in this study was air. The results have been analyzed with dimensionless parameters and in theory can be applied to other gases in similar exchangers; but this has not been tested. Fourth, this experiment operates on the assumption that the gas temperature and flow will be uniform throughout the cross-section of an actual shell-and-tube exchanger. It is assumed that under these conditions, the performance of four adjacent exchanger tubes can be extended to predict the performance of the hundreds or thousands of such tubes that would be found in an actual exchanger.

A review of the related literature follows this introduction. The development of the experimental apparatus is outlined next, followed by an explanation of the measurements, data analysis, and experimental error. A discussion of the results and a summary of conclusions complete this study.
1.7 Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>cross-sectional area</td>
</tr>
<tr>
<td>$A_s$</td>
<td>surface area</td>
</tr>
<tr>
<td>$A_{HX}$</td>
<td>cross-sectional area of exchanger tubes</td>
</tr>
<tr>
<td>$c_p$</td>
<td>constant-pressure specific heat</td>
</tr>
<tr>
<td>$C_d$</td>
<td>discharge coefficient</td>
</tr>
<tr>
<td>$c$</td>
<td>sound speed</td>
</tr>
<tr>
<td>$D$</td>
<td>tube inner diameter</td>
</tr>
<tr>
<td>$E$</td>
<td>heat transfer effectiveness</td>
</tr>
<tr>
<td>$E_{in}$</td>
<td>heat exchanger effectiveness during the instroke</td>
</tr>
<tr>
<td>$E_{raw}$</td>
<td>uncorrected heat exchanger effectiveness</td>
</tr>
<tr>
<td>$f$</td>
<td>frequency of oscillation in Hz.</td>
</tr>
<tr>
<td>$J_H$</td>
<td>Colburn-J factor</td>
</tr>
<tr>
<td>$L$</td>
<td>heat exchanger length</td>
</tr>
<tr>
<td>$\dot{m}$</td>
<td>mass flow-rate</td>
</tr>
<tr>
<td>$Nu$</td>
<td>Nusselt number</td>
</tr>
<tr>
<td>$Pr$</td>
<td>Prandtl number</td>
</tr>
<tr>
<td>$P_m$</td>
<td>mean pressure</td>
</tr>
<tr>
<td>$P$</td>
<td>pressure</td>
</tr>
<tr>
<td>$\dot{Q}$</td>
<td>heat transfer in watts</td>
</tr>
<tr>
<td>$R$</td>
<td>acoustic resistance</td>
</tr>
</tbody>
</table>
\( r \)  inner radius of a tube
\( r_{\text{end}} \)  radius of curvature of tube-ends
\( r_h \)  hydraulic radius
\( R_{\text{ml}} \)  minor loss resistance
\( T \)  temperature
\( T_{\text{air in}} \)  cross-sectionally averaged air temperature as it enters a heat exchanger
\( T_{\text{fluid in}} \)  cross-sectionally averaged fluid temperature as it enters a heat exchanger
\( T_{\text{HX surface}} \)  surface temperature of a heat exchanger
\( T_{\text{in}} \)  water temperature at an inlet
\( T_L \)  cross-sectionally averaged temperature of a Lagrangian segment of gas
\( T_{\text{LM}} \)  log-mean temperature
\( T_m \)  mean temperature
\( T_{\text{out}} \)  water temperature at an outlet
\( U \)  volumetric velocity
\( v \)  cross-sectionally averaged particle velocity
\( V \)  volume
\( x \)  peak displacement within an exchanger
\( x_{\text{gap}} \)  thickness of a gap between hot and cold exchangers
\( x_m \)  mean gas position
\( x_{\text{pistons}} \)  peak piston displacement
\( Z \)  acoustic impedance
\( Z_{\text{norm}} \)  normalized acoustic impedance
Greek Symbols

\( \delta_c \) thermal penetration depth

\( \delta_v \) viscous penetration depth

\( \Delta P \) pressure component due to minor loss

\( \Delta T_{\text{air in}} \) change in air temperature within an exchanger during an instroke

\( \Delta T_{\text{air out}} \) change in air temperature within an exchanger during an outstroke

\( \Delta T_{\text{duct air}} \) change in air temperature within the lower duct

\( \Delta T_{\text{fluid}} \) change in fluid temperature due to heat transfer

\( \Delta T_{\text{L,iso}} \) temperature change of a Lagrangian gas segment in isothermal contact with a wall

\( \Delta T_{\text{LM}} \) log-mean temperature difference

\( \Delta V_L \) change in volume of a Lagrangian gas segment due to heat transfer

\( \phi \) phase angle between acoustic pressure and volumetric velocity through an exchanger

\( \gamma \) ratio of specific heats of an ideal gas

\( \kappa \) minor loss coefficient

\( \mu \) viscosity

\( \nu \) kinematic viscosity

\( \rho \) mean gas density

\( \sigma \) porosity

\( \omega \) angular frequency of oscillation
Chapter 2

Literature Review

Although a good deal of work exists on the topics of minor losses and thermoacoustic heat exchanger performance, there is none that this author is aware of that addresses the questions investigated in the present study. The following literature review comprises two sections. The first summarizes the current understanding of the nonlinear impedance of cross-sectional changes, and is arranged in a conceptual progression. The second summarizes the present understanding of the thermal performance of heat exchangers in oscillating flow. In contrast to the first section, the second is grouped into three main bodies of work, each arranged in chronological order.

2.1 Nonlinear Impedances at Cross-Sectional Changes

2.1.1 Nonlinear Resistance of Orifices

Sivian\(^4\) conducted the first study of nonlinear orifice impedances. He observed that a sinusoidal fluid velocity through an orifice produces an oscillating pressure with two components: one that is complex and proportional to the fluid velocity, and another that is mainly real and proportional to the square of the velocity. Sivian correlated the first component with linear acoustic theory; but the second required new insight.

Sivian’s experimental data showed that this second pressure component occurred at Reynolds numbers as low as 300, effectively ruling out turbulence as the cause. He argued that the effect was therefore velocity-dependent, and could therefore be described
using a quasi-steady flow approximation. From this, he derived the following expression for the nonlinear resistance of an orifice \((R)\) is the nonlinear resistance of the orifice; \(\rho\) is the fluid density; and \(v\) is the particle velocity through the orifice):

\[
R = \frac{\Delta P}{v} = \frac{1}{2} \rho v.
\]  

Sivian’s experimental data agreed with this expression to within 17\%, and these results were later repeatedly verified.\textsuperscript{5-12}

Ingard and Labate\textsuperscript{6} were the first to directly confirm Sivian’s hypothesis that turbulence was not the source of the nonlinearity. In this study, smoke was injected into an impedance tube capped with an orifice and photographed the fluid at the exit. They observed that the resulting flow was not simply oscillatory but that it included other distinct patterns as well, ranging from slow circular eddies to vortices and jets. These patterns were repeatable, and were found to be functions of the fluid velocity through the orifice.

Ingard and Labate then compared the energy of the high-velocity jets to the energy of the nonlinear pressures generated simultaneously at the orifice. They found a direct correlation between the two, and concluded that at all fluid velocities the nonlinear component of the generated pressure provided the additional energy required to drive the additional flows they had observed. These conclusions were verified by Salikkuddin and Ahuja,\textsuperscript{13} and later Jin and Sun.\textsuperscript{14}
2.1.2 Nonlinear Effects on Inertance

In addition to a nonlinear resistance, Bolt and Labate,\textsuperscript{5} Thurston,\textsuperscript{15} and Ingard\textsuperscript{16} observed an accompanying velocity-dependent decrease in the imaginary part of the orifice impedance as well. Ingard\textsuperscript{16} argued that the vortices and jets he had observed must transport some of the usually-coupled fluid mass away from the orifice, reducing the inertial component of the orifice impedance. Thurston, Hargrove, \& Cook\textsuperscript{11} later described this effect as a reduction in the linear orifice end correction (Rayleigh’s end-correction), and Panton and Goldman\textsuperscript{17} later used this as a basis for normalizing the imaginary part of the acoustic impedance measured in their experiment.

Ingard and Ising\textsuperscript{18} noted that the acoustic inertance appeared to approach one-half of the value predicted by linear theory as the fluid velocity increased. Thurston et. al.\textsuperscript{11} and Jin and Sun\textsuperscript{14} observed an apparent asymptotic limit of seven-tenths of the linear value instead. In a separate study, Westervelt\textsuperscript{19} estimated that the inertance should approach five-eighths of the linear value—about midway between the limits given by Ingard, et. al.\textsuperscript{18} and Thurston, et. al.\textsuperscript{11} None of these studies provided a general correlation for this effect, although Melling\textsuperscript{12} did derive an expression giving the velocity that corresponds to the inertance minimum.

2.1.3 Geometric Effects on Nonlinearities

Ingard and Labate stated in their 1950 paper\textsuperscript{6} that because the nonlinear resistance of an orifice is a velocity-dependent effect rather than a turbulence-induced effect, the Reynolds number is a bad correlation; and if such a non-dimensional number were to be
found, it must be a function of the geometry of the orifice. Ingard’s subsequent study\textsuperscript{16} explored the beginnings of this hypothesis and showed that nonlinear orifice resistance is not a function of the thickness of the orifice. Thurston, et. al.\textsuperscript{11} found that orifice nonlinearities are independent of diameter and thickness, and argued that they are a function of fluid velocity only. Panton and Goldman\textsuperscript{17} concluded that when the length of an orifice is at least three times the diameter, nonlinear effects at each end can be considered separately and independently of each other.

Later studies focused on the shape of the orifice edges or tube ends. Atig, Dalmont, and Gilbert\textsuperscript{20} studied the nonlinear impedances of square and round-edged tubes. They found that nonlinear resistance decreases as the radius of curvature of the end increases, and also observed that the imaginary part of the impedance of a tube does not decrease from its linear value (at least not more than their experimental error of 0.03 radii of the tube) if the radius of curvature at the end is greater than 0.5 diameters.

Using the same experiment as his previous study, Thurston\textsuperscript{10} measured the nonlinear impedances of orifices with square, rounded, symmetrically beveled, and asymmetrically beveled edges. He found that both the nonlinear resistance and decrease in inertance are functions of the orifice edge geometry. He suggested that a geometry-dependent and experimentally determined coefficient ($\sigma$) should be added to Sivian’s original expression for the nonlinear resistance:

$$R = \frac{1}{2} \sigma \rho v.$$  \hfill (2.2)

Panton and Goldman\textsuperscript{17} proposed a slightly different version of Thurston’s coefficient. They proposed that the discharge coefficient $C_d$ (the ratio of the actual mass
flow-rate through a nozzle or transition to that of the ideal, lossless mass flow-rate) could be used in the same equation instead, giving:

\[ R = \frac{\rho v}{2C_d^2}. \]  

(2.3)

This coefficient was widely used in steady-flow applications at the time, and was generally provided by the manufacturer for a given part. Panton and Goldman argued that at high enough velocities the quasi-steady approximation should be valid and the value of the discharge coefficient should not depend on velocity. Based on these assumptions, they derived the following expression for the nonlinear resistance:

\[ R = \frac{4 \rho v}{3 \pi C_d^2}. \]  

(2.4)

This equation is widely used today, except that the discharge coefficient has been replaced by the “minor loss” coefficient \( \kappa \):

\[ R = \frac{4 \kappa \rho v}{3 \pi}. \]  

(2.5)

The minor loss coefficient is commonly taken from steady-flow data such as that provided by Idelchik\(^{35} \) unless experimental oscillating flow data is available. No such correlation exists for the velocity dependence of the acoustic inertance, however.

### 2.1.4 Other Geometric & Flow Effects

The results of Ingard and Labate’s first study\(^6 \) differed significantly from Sivian’s, but Ingard and Ising\(^18 \) later correctly attributed this to the influence of the walls of the impedance tube near the orifice. Bies and Wilson\(^9 \) noted that when connecting a
Helmholtz resonator to an impedance tube, the nonlinearity at the end of the neck depends on whether it is connected to the end of the tube or as a T-branch. They conducted no further studies of this effect, however.

Melling\textsuperscript{12} found that the nonlinear resistance of an orifice or perforated plate depends only on the particle velocity through the holes and the shape of the edges. In other words, for a given porosity, a given fluid velocity, a given thickness, and a given edge shape, the nonlinear resistance of a single orifice would be the same as that of a perforate with holes of arbitrary shape and distribution. Salikkuddin & Ahuja\textsuperscript{13} and Salikkuddin & Brown\textsuperscript{21} continued these studies, and confirmed Melling’s conclusions as well.

In addition to geometric effects, nonlinearities are affected by flow conditions as well. Wakeland and Keolian\textsuperscript{22} demonstrated analytically that the nonlinear resistance of a sudden, square-edged expansion can be significantly different for fully developed laminar flow (Poiseuille flow) as compared to fully turbulent flow (approximately uniform flow profile). The authors used well-established laminar oscillating boundary layer theory and derived an expression for a time-averaged “effective” minor loss coefficient based on frequency, velocity, and porosity of the expansion.

\subsection*{2.1.5 Generation of Harmonics & DC Pressure}

Sivian\textsuperscript{4} hypothesized that the nonlinear impedance of an orifice should generate odd-order pressure harmonics in addition to increasing the pressure at the oscillation frequency, but such harmonics were not visible in his data. Later, Thurston, Hargrove, &
Cook\textsuperscript{11} successfully measured odd-order harmonic content produced by square-ended orifices. Ingard and Ising\textsuperscript{18} later confirmed these results.

Thurston\textsuperscript{10} showed that in addition to odd-order harmonics, asymmetric orifice edges generate a dc pressure and even-order harmonics. Petculescu & Wilen\textsuperscript{23} investigated the nonlinear impedances of jet pumps—a long conical taper with a rounded narrow exit—and confirmed the presence of a dc pressure and even-order harmonics.

Steady-flow data suggests that minor loss coefficients could depend on flow direction as well—that is, that the loss coefficient could have different values during each half of an acoustic cycle for certain geometries. Swift and Smith\textsuperscript{24} measured the minor loss coefficients of rounded exits into free space. By assuming that the minor loss would be negligible during the “sucking” half of the acoustic cycle (an assumption based on steady-flow data), they were able to characterize the minor loss coefficient of the “blowing” half by measuring the time-averaged (or dc) pressure in the exit.

### 2.1.6 The Quasi-Steady Approximation and The Iguchi Hypothesis

Sivian\textsuperscript{4} argued that because nonlinear orifice resistance appeared to be a velocity-dependent effect rather than turbulence-induced, there was some justification for the use of the quasi-steady approximation in his analysis. Thurston, et. al.\textsuperscript{11} implied a quasi-steady approximation in the derivation of nonlinear resistance. Panton & Goldman’s data\textsuperscript{17} showed that the assumption was appropriate when the ratio of particle velocity to the square-root of the product of kinematic viscosity and angular frequency was greater than 100. Ibrahim & Hashim,\textsuperscript{25} on the other hand, argued that during the acoustic cycle...
the instantaneous minor loss coefficient can be as much as a factor of two higher or lower than the steady flow coefficient at the same velocity; and therefore steady flow coefficients could not be applied to oscillating flow. Ultimately, any analytic analysis of nonlinear resistance must of necessity involve a quasi-steady assumption; the question is whether or not steady-flow minor loss coefficients hold for oscillating flow as well.

Iguchi and Ohmi,\textsuperscript{26} and later Iguchi, Ohmi, and Maegawa\textsuperscript{27} studied the transition between laminar and turbulent oscillating flows in a U-tube. They found that once the flow had become turbulent the velocity profile followed the turbulent steady-flow $1/7$ power law $v = v_{\text{max}} \left(1 - \frac{y}{r}\right)^{(1/7)}$, where $v$ is the particle velocity at a distance $y$ from the center; $v_{\text{max}}$ is the particle velocity at the center of the tube; and $r$ is the radius of the tube), and that the instantaneous friction factor followed Blassius’ steady-flow law as well. They also found that the critical Reynolds number that indicated a fully turbulent oscillating flow could be given by the expression:

$$\text{Re} = 800r \sqrt{\frac{\omega}{v}}.$$  \hfill (2.6)

In this expression $r$ is the tube radius, $\omega$ is the angular frequency, and $v$ is the kinematic viscosity. Further studies by Iguchi et. al.\textsuperscript{28,29} investigated this transition in greater detail.

Iguchi and Ohmi then conducted the same U-tube experiment with an orifice placed in the bend.\textsuperscript{30} They found that even in the laminar flow regime, steady-flow minor loss coefficients used with the quasi-steady approximation achieved good agreement with experimental results. Because of these studies, the argument that steady-flow minor loss coefficients can be used in oscillating flow with the quasi-steady approximation has come to be known in some circles as the “Iguchi hypothesis.”
Since the work of Iguchi et. al., several other investigations have been made to confirm his hypothesis. Swift and Olson\textsuperscript{31} provided evidence for Iguchi’s hypothesis by repeating his U-tube experiment (without the orifice) and extending it to a broader range of velocities, pipe radii, and radii of curvature, including tubes with multiple coils. Petculescu & Wilen’s study on jet pumps\textsuperscript{23} also appear to confirm Iguchi’s results. Wakeland and Keolian\textsuperscript{32} conducted measurements of the acoustic resistance of parallel-plate heat exchangers and also found good agreement with Iguchi.

At the present time, the state-of-the-art treatment of nonlinearities at changes in cross-section consist of using the formula given by Panton & Goldman for nonlinear resistances with the minor loss coefficient taken either from steady-flow data or experimentally obtained correlations. No correlation exists for the corresponding decrease in inertance, and this effect is usually neglected. Harmonics generated by nonlinear impedances are usually neglected as well.

### 2.2 Heat Transfer in Oscillating Flow

#### 2.2.1 Heat Transfer in Oscillating Pipe Flow

Stirling engines operate via the same thermodynamic cycle as recent thermoacoustic engines and share many of the same challenges. Like thermoacoustic devices, Stirling engines rely on a pair of heat exchangers to enable the device to function. The performance of these exchangers is critical, and researchers within the
Stirling community have worked to understand their proper design. A chronological review of the related literature follows.

Organ\textsuperscript{33} derived an analytic expression for the heat transfer between a duct and an internally oscillating fluid of constant pressure and laminar flow. The duct consisted of a hot segment of constant wall temperature with cool segments to either side. The fluid temperature on either side of the hot segment was forced to be constantly low (the heat exchange in the cool segments was assumed to be perfect, in other words). The model showed that under these conditions, exchanger efficiency reaches a maximum when the length of the exchanger equals the peak-to-peak gas displacement inside it, and that further increasing the length provides no additional benefit.

Although the model assumed incompressible flow, Organ realized that if pressure oscillations were considered as well, increasing the exchanger length beyond the acoustic displacement would actually cause work to be lost. The thermoacoustic cycle would be reversed in the additional length—that is, the gas in that segment of the exchanger would lose heat during compression and gain heat during expansion.

Wakeland and Keolian\textsuperscript{54} found similar results through a numeric study of their own. Their results showed that heat transfer is increased by pressure-driven temperature oscillations in standing-wave refrigerators, and that it is decreased by the same mechanism in standing-wave engines.

Hwang and Dybbs\textsuperscript{34} experimentally investigated the efficiency of the coolers in Stirling engines. An oscillating piston drove gas through a pre-heated length of tubing and into a liquid-cooled test segment. This test segment connected to an additional length of cooled tubing. This then exited into a large cooled cavity. The cavity allowed
the experiment to operate in approximately constant pressure conditions. The walls of
the various tube segments and cavity were held at approximately constant temperatures,
facilitating heat transfer between the hot and test sections. By measuring the air
temperatures to either side of the test segment, the mass flow-rate of the water in the test
segment, and the corresponding change in water temperature during operation, Hwang
and his colleagues were able to measure the heat transferred into the test section. Their
data indicated that for a fixed displacement amplitude, increasing the operating frequency
increases heat transfer. For a fixed frequency however, the experiment was not sensitive
enough to find a correlation based on varying displacement.

Zhao and Cheng\textsuperscript{36} conducted a similar numeric study. Geometry and boundary
conditions were like the models of Organ\textsuperscript{33}. The flow was assumed to be laminar,
incompressible, and fully developed with a developing thermodynamic layer, and was
calculated in two dimensions.

Their results showed that heat transfer between the gas and tube depends on four
parameters: the Prandtl number, the ratio of gas displacement to tube diameter, the ratio
of tube length to diameter, and the oscillating Reynolds number (the product of the
square of the diameter and the angular frequency, divided by the gas viscosity). For a
fixed ratio of length to diameter, they found that heat transfer increases with either
increasing displacement or Reynolds number (in other words, frequency). They also
found that the ratio of length to diameter significantly impacts heat transfer.

Walther, Kuhl, and Shulz\textsuperscript{37} numerically studied heat transfer in fully turbulent
oscillating pipe flows with a similar model. They argued that the quasi-steady approach
using steady-flow heat transfer coefficients is inadequate, because such methods under-
predict the amount of heat transfer and completely ignore the temporal phase difference between heat transfer and wall/gas temperature difference or gas velocity. The authors suggested entirely new correlations for both the magnitude and phase of a complex Nusselt number based on their results. It should be noted, however, that their numerical data was calculated at high Reynolds numbers (20,000 to 50,000), based on the oscillating peak particle velocity.

Bouvier, Stouffs, and Bardon\textsuperscript{38} conducted an additional experimental study. A 5.7 m stainless steel tube was heated at its midpoint with a 0.76 m heater band. Water-cooled heat exchangers were placed at either end of the tube, and instantaneous temperatures within the tube wall were taken at the midpoint of the heated segment via a series of imbedded thermocouples. Bouvier and colleagues also measured the instantaneous cross-sectional temperature distribution of the gas at the same point.

They presented their observations mainly in graphical form, and showed that the temperature profiles in the gas and tube wall were accurately described by existing theory. They found that the steel tube wall acts like a low-pass filter in temperature relative to the temperature changes in the gas, and also argued that unlike in steady flow, a Nusselt-like dimensionless number may not exist that can correlate local heat flux to overall heat flux in oscillating flow.

Several important results emerge from the previously cited literature. First, that at least in the conditions described in these studies, quasi-steady approximations under-predict heat transfer. Second, that there is a phase delay between heat transfer and gas/wall temperature and/or gas velocity. Third, the concept of a critical exchanger length—that is, that there is an optimal exchanger length, and that any other length can
decrease efficiency. Fourth, that heat transfer is proportional to frequency for a given gas displacement, and that it is proportional to displacement for a given frequency.

2.2.2 Thermoacoustic Studies of Heat Exchangers

Researchers in thermoacoustics have conducted similar studies, although based on a different set of assumptions and conditions. Linear thermoacoustic theory is well-established, so a review is not included here.

Brewster, Raspet, and Bass\textsuperscript{39} investigated temperature discontinuities between heat exchangers and stacks in thermoacoustic devices. They derived an analytic expression for the heat transfer coefficient between an exchanger and the end of a stack based on linear acoustic theory, the assumption of perfect heat transfer within the exchanger, an assumption of laminar flow, and an identical parallel-plate geometry in both the exchangers and the stack. They then checked their expression against experimentally obtained data.

When the experiment was run at low acoustic amplitudes, their analytic expression overestimated the stack/exchanger heat transfer coefficient by about 20%. At higher amplitudes however, the predicted results were too low. The authors gave no explanation for this effect; but it is important to note that gaps existed between the heat exchangers and stack in the experimental device, and that the potential impact of these gaps was not taken into account by their model.

Mozrukewich\textsuperscript{40} developed his own analytic model based on parallel-plate geometry of both stack and exchanger, the assumption of laminar flow, and an additional
assumption that variations in gas temperature occurred only along the length of the
exchanger or stack.

The model provided several interesting results. The gas temperature within the
exchanger can be quite non-uniform, rather than being anchored to the wall temperature.
In addition, a temperature difference between the exchanger and the adjacent stack end
increases net heat transfer between the gas and exchanger. The optimal exchanger length
given by the model was about equal to the peak-to-peak acoustic displacement, and
optimal plate spacing was shown to be equivalent to a few thermal penetration depths in
the gas.

Dong, Lucentini, and Nazo\textsuperscript{41} used a simple analytic model to predict parallel-plate
exchanger performance in oscillating flows. Their model assumed laminar flow with
simple parallel-plate geometries, and consisted of expressions for heat transfer in two
simplified cases: perfect heat transfer within the exchanger, and boundary layer flow
between widely spaced plates (their analysis assumed that steady-flow boundary layer
theory and Nusselt number correlations were sufficient to describe the oscillating thermal
boundary layer, however). In the first case, heat transfer was limited by the mass flow-
rate of the gas between plates (a function of plate spacing). In the second case, the model
was based on a steady-flow expression for the Nusselt number. In both cases, exchanger
length was set to be equal to the peak-to-peak gas displacement. The authors plotted the
results of these two expressions as a function of plate spacing, and suggested that optimal
spacing was indicated by the intersection of the two curves.

Mozurkewich\textsuperscript{42} experimentally investigated the accuracy of the assumption that
heat transfer in oscillating flow could be approximated by averaging the steady-flow
correlations over a sinusoidal distribution of gas velocities (known as the TASFE approximation, or Time-Averaged Steady Flow Equivalent). His experiment consisted of an array of parallel tubes arranged transversely to the oscillating gas and placed next to a stack. He found that the quasi-steady assumption based on correlations for a single tube in cross flow predicted heat transfer well when the acoustic Reynolds number was less than 1000 (Reynolds number based on the peak acoustic velocity between tubes), and noted that this was likely due to the relatively wide spacing of the tubes. At Reynolds numbers above 1000, the correlation deteriorated. Mozurkewich hypothesized that this result would likely hold for other geometries as well, as long as the flow could be approximated as external rather than duct flow.

Wakeland and Keolian argued that heat exchanger effectiveness should be used to characterize exchanger performance rather than a heat transfer coefficient. Effectiveness is defined as the ratio of the actual amount of heat transfer to the amount of heat transfer that would have occurred had the fluid and exchanger been in perfect thermal contact. The authors showed that for parallel-plate type exchangers, the effectiveness method predicted heat transfer at narrow plate spacings much better than the boundary-layer model used in the DeltaE software (the de facto standard software used to predict thermoacoustic device performance). In addition, the experimental data showed that for a given displacement effectiveness falls as frequency rises. For a given frequency, effectiveness falls as the acoustic displacement becomes larger than the exchanger length.

Paek, Braun, and Mongeau also investigated the accuracy of the quasi-steady assumption via an experimental test of an “off the shelf” micro-channel heat exchanger.
They measured the time-averaged heat transfer in both steady and oscillating flows as functions of Reynolds numbers and compared the results to both the TASFE approximation previously investigated by Mozurkewich, as well as a boundary-layer approximation proposed by Zukauskas for tubes in cross-flow.

Their results showed that the TASFE approximation over-predicted heat transfer by as much as 36% at Reynolds numbers less than 1000 (the Reynolds number based on acoustic particle velocity), and that it significantly under-predicted heat transfer at higher Reynolds numbers. By comparison, the boundary-layer approximation was off by as much as 114% at the same lower Reynolds numbers, and also under-predicted heat transfer at higher Reynolds numbers. The authors found however, that good agreement could be achieved between the TASFE approximation and their experimental data if they multiplied the acoustic Reynolds number by a factor of 0.353. This factor accounted for the diminished heat transfer during the second half of the acoustic cycle—an effect not accounted for in the original TASFE, boundary layer, or other Reynolds number-based models.

Herman and Chen studied the performance of parallel-plate heat exchangers and stacks via numerical modeling. Their model assumed that the exchanger and stack plates were of the same thickness, width, and spacing, with either no gap or a small gap between. Flow was assumed to be laminar, and relied on heat transfer coefficients derived in previous studies. Their results indicated that the thermal performance of one exchanger had a negligible impact on the performance of the exchanger at the other end of the stack, suggesting that the performance of the two exchangers could be considered independently during the design process. They also found that the temperature of the
stack plates was one-dimensional and linear along the stack, except at the edges near the
exchangers where the temperature distribution became two-dimensional and very
nonlinear. High heat fluxes occurred near the edges as well, with zero net heat flux
elsewhere along the stack. Although not stated by the authors, these results suggest that
the geometry of the ends of a heat exchanger or stack could have a considerable impact
on heat transfer between an exchanger and stack.

Piccolo and Pistone\textsuperscript{48} numerically examined parallel-plate heat exchanger
performance as well. Their model was based on assumptions of simple geometry and
laminar flow, and integrated over the local cross-section using linear thermoacoustic
theory. Their results showed that the optimal length of the exchanger is a function of
plate spacing, and concluded that the length of the exchanger should match the acoustic
peak-to-peak displacement when the plates are spaced by one or two gas thermal
penetration depths.

Their results correlated well with the TASFE approximation at Reynolds numbers
below 700 (based on acoustic particle velocity and exchanger length). At a Reynolds
number of 2000, however, the TASFE approximation overestimated heat transfer by
about 40\%. The authors pointed out that the TASFE and other quasi-steady
approximations should not work well at plate spacings less than a couple of thermal
penetration depths, because steady flow correlations are based on external flow rather
than duct-like flow.

Several points are worth mentioning. First, there is some agreement between the
thermoacoustic studies just cited and the previously reviewed Stirling engine literature on
the point of quasi-steady approximations. Both bodies of literature show that quasi-
steady approximations fail at higher oscillating velocities. Additionally, the thermoacoustic literature shows that quasi-steady approximations fail when applied to parallel-plate exchangers with narrow plate spacing because steady-flow heat transfer correlations for such geometries are generally based on external boundary-layer flows. At lower velocities, however, quasi-steady approximations appear to work quite well. Since flow conditions in actual applications are likely to never be fully turbulent, the TASFE, modified TASFE, or other quasi-steady models are probably reasonable.

Good agreement exists on the “critical length” concept as well—that is, that there is an optimal exchanger length for a given plate spacing. The studies of Mozurkewich,\textsuperscript{40} Dong et. al.,\textsuperscript{41} and Piccolo and Pistone\textsuperscript{48} in particular confirm this conclusion.

Curiously, no mention is made within the thermoacoustic literature reviewed here of the phase shift between heat transfer and gas/wall temperature or acoustic velocity. It appears that in all of the analytic studies reviewed here (Mozurkewich,\textsuperscript{40} Dong, Lucentini, and Naso,\textsuperscript{41} Herman and Chen,\textsuperscript{47} and Piccolo and Pistone\textsuperscript{48}), time-related effects are either intentionally ignored in favor of time-averaging (Mozurkewich\textsuperscript{40}), or strictly real-valued local Nusselt numbers are used in heat transfer derivations (Herman and Chen\textsuperscript{47}). The latter case is the equivalent of assuming that heat transfer occurs instantly whenever a difference of temperature exists, rather than as a function of time. Obviously, TASFE and other quasi-steady models based on steady-flow correlations address only the magnitude of heat transfer, and completely ignore the issue of phase as well.
2.2.3 The Effect of a Sudden Expansion on Heat Transfer

One major drawback in all of the studies previously reviewed is that they are all based on simple geometries with no changes in cross-section. Heat transfer and temperature differences appear to be concentrated along the ends of the exchanger plates (Brewster et al.\textsuperscript{39} and Herman & Chen\textsuperscript{47}), and it would seem reasonable that any turbulence or disturbance generated by those ends could impact heat transfer. To my knowledge, there are very few studies of the influence of cross-sectional changes on heat transfer. The following section is a review of what is available.

El-Mehlawi and Mankbadi\textsuperscript{49} studied the impact of a sudden expansion on heat transfer in a non-zero mean, pulsatile flow via numeric modeling. They found that the instantaneous, local heat transfer is greater than that of strictly steady flow at the same velocities, and that the effect is frequency-dependent. The authors determined that an optimum frequency exists at which a small oscillation amplitude results in substantial increases in heat transfer (a super-imposed oscillation amplitude of 3% of the steady-flow velocity caused an increase in heat transfer of 14%). Their results also showed that this increase is due to turbulence generated at the sudden expansion.

Devalba and Rispoli\textsuperscript{50} conducted their own numeric investigation of heat transfer in sudden expansions. Their model involved strictly oscillatory flow (zero mean); however, they chose to use a pressure source to drive the model rather than a velocity source (they chose to specify an oscillating pressure at the model inlet rather than an oscillating velocity). They recognized that because of this, the results would not apply to Stirling engines in general and were of limited value. Their model did show an increase
in instantaneous heat transfer relative to steady flow, lending support to the conclusions cited in the previous study.

Ibrahim\textsuperscript{51} conducted a similar computational investigation as well. His numeric model assumed strictly laminar, incompressible flow, and showed that the instantaneous heat transfer coefficient near the expansion could be an order of magnitude higher than that of steady flow of the same conditions. He also found that instantaneous heat transfer increased even more when the thermal expansion of the gas was taken into account. He reasoned that this was due to an increase in turbulence at the expansion caused by the additional gas velocity.

Oseid and Patankar\textsuperscript{52} developed a numeric model of fully turbulent, incompressible flow through a pipe between two expansions. Their model fixed all walls at a constant hot temperature and forced the gas on either side to be cold. As in the previously mentioned studies, they also found that the presence of an expansion increased the instantaneous local heat transfer. In addition though, their model showed that a complex phase relationship existed between the increased heat transfer and the gas/wall temperature difference. They found no simple way to correlate the Nusselt number as a function of axial space and time, however.

2.2.4 The Present Study

All of the previously reviewed studies consider flow through simple geometries or square-edged, sudden expansions. In an actual Stirling-type thermoacoustic device, however, heat exchangers are placed in close proximity to a regenerator. It would appear
reasonable that the proximity of the regenerator may alter the nonlinear impedances of
the exchanger by reducing jetting and vortex shedding. Likewise, if the flow is altered in
such a way as to reduce these effects, there may be a corresponding effect on heat
transfer. This argument holds for the ends of the exchanger tubes as well—that is, that
turbulence produced by the shape of the ends of the tubes may impact heat transfer as
well as generate minor losses. This study is the first to consider such questions.
Chapter 3

Development of the Experiment

Figure 3.1 is a picture of an experiment that was designed and built by Ray Wakeland\(^5\) to study the effectiveness of parallel-plate heat exchangers. The arm of an electrodynamic shaker at the base of the apparatus was coupled to two rubber bellows—one just above the shaker, and another at the top of the apparatus. The bellows were in turn mounted to two large acrylic ducts. Model exchangers constructed of parallel, hollow, flat tubes (used to approximate parallel plates) were placed between the ducts, and were held at hot or cold temperatures by pumping heated or cooled water through the tubes. As the shaker drove the bellows, air oscillated through the ducts and model exchangers, causing a time-averaged transfer of heat. Heat transfer was calculated by taking the product of measurements of the mass flow-rate and temperature change of the water and the water’s heat capacity. The volumetric velocity of the air through the model exchangers was inferred from the shaker displacement.

Originally, my intent was to use this same experiment to conduct further testing of shell-and-tube exchangers in support of Dr. Robert Keolian’s thermoacoustic generator project. However as I began to consider the problem, it became clear that testing scale models of entire exchangers would be impractical. In addition, more fundamental questions emerged as to the optimal exchanger design.

After some consideration, it was decided that a more fundamental study was necessary. Rather than testing several specific exchanger designs, I decided to examine the effects of the shape of an individual tube on heat transfer and acoustic power losses.
Since a complete exchanger is simply a bundle of identical tubes, the lessons learned from testing a single tube should apply to the entire exchanger if the flow through each of the exchanger’s tubes and the temperatures of the walls could be considered to be identical.

The electro-dynamic shaker the apparatus was based on was incapable of operating at the same frequencies as are typical in thermoacoustic devices. As a result, the experiment operated over a much lower range of frequencies (0.22 Hz to 8.5 Hz). I wanted to examine the performance of shell-and-tube exchangers with tubes that had inner radii that were on the order of the thermal penetration depth of the gas. This, combined with the available range of frequencies of the shaker, forced the inner radius of the tubes that I used to be 8.13 mm.

### 3.1 First Attempt

My first attempt consisted of mounting two tubes end-to-end between the two large acrylic ducts in Dr. Wakeland’s experiment, as in figure 3.2. The walls of the tubes were of hollow construction. Hot water was pumped through the walls of the upper tube, while cold water was pumped through the walls of the lower tube. In this way, air oscillating in the middle of the tubes would transfer heat from the hot to cold tubes.

However, a serious problem emerged. When built in this fashion, the entire experiment became a large double-Helmholtz resonator at a frequency of about 5 Hz. This was a major problem, because the velocity of the air through the tubes would be very different in both magnitude and phase from the velocity of the pistons below. In
Figure 3.1: Photograph and cross-sectional diagram of Ray Wakeland’s original experiment.
Figure 3.2: Cross-sectional diagram of the modified apparatus. Notice the double-Helmholtz resonator shape. The resonance of this design occurred at about 5 Hz, making it very difficult to know the velocity through the exchangers.
addition, this effect would be difficult to correct analytically because the Q of the resonance depended on the acoustic impedance of the tubes—one of the very things I wanted to measure.

The ducts in the experiment served several purposes: they provided an approximately adiabatic region on either side of the exchangers, as well as conditions similar to those in an actual thermoacoustic device (where wide thermal buffer tubes or cavities exist to one side of an exchanger). Because of this, they could not be eliminated. However, their dimensions could be altered.

### 3.2 Second Attempt

Figure 3.3 is a cross-sectional diagram of the modified apparatus. The large acrylic ducts were replaced with narrow PVC ducts, and the rubber bellows on the top and bottom were replaced with four dashpots (made by Airpot, Inc.). The inner diameter of the PVC ducts was chosen so that the porosity of the duct/tube interface would match that of the exchangers that were planned to be used in a future project.

This new design raised the Helmholtz resonance to a frequency of 50 Hz, with a Q of 5. Since data was to be taken over a range from 0.22 Hz to 8.5 Hz, any resonance effects on the velocity of the air in the tubes would be negligible. The rubber bellows of the previous design had been prone to bursting, and introduced errors in the gas velocity due to flexing. The new dashpots eliminated these issues completely.

Pressure sensors were mounted in the PVC ducts; and I had anticipated that the signals at each of these locations would be out of phase with each other because the air in
Figure 3.3: Cross-sectional diagram of the second re-design. Heat transfer in the constant-volume condition of this design caused large oscillations in the mean pressure, masking the much smaller acoustic pressures.
one duct would compress while that in the other duct would simultaneously expand due to the oscillating pistons. However, I observed exactly the opposite during operation. The signals at both locations were nearly exactly in phase at all frequencies, and their magnitudes were much larger than I had anticipated.

The pistons at both the top and bottom of the apparatus created a constant-volume condition. As the air oscillated between hot and cold sections of the apparatus, the change in the volumetrically-averaged temperature caused a corresponding change in the average pressure within the experiment. This was the cause of the in-phase signals measured by the pressure sensors. Additionally, the magnitude of this common-mode pressure signal was much greater than that of the differential-mode signal that I wanted to measure, making simultaneous measurement of heat transfer and acoustic impedance of the tubes difficult.

The quality of the heat transfer measurement in this arrangement was poor as well. Figure 3.4 is a plot of the normalized effectiveness as measured in this version of the experiment. Two key features emerge: first, the poor signal-to-noise ratio evident at smaller displacements (the left side of the graph); and second, the sudden increase in effectiveness as the gas moves beyond the tubes and into the neighboring duct ($x/L > 1$). This last feature seemed to suggest that perhaps the narrow PVC ducts weren’t acting as an ideal adiabatic space, as had been previously assumed. This caused the heat transfer measured in the tubes to be lower than it would have been had the ducts been adiabatic. To address these issues, further changes were needed.
Figure 3.4: Effectiveness data for medium and long model exchangers ($L/D = 7.813$ in black, and 15.63 in green) with flat ends. This data was taken with the second re-designed experiment, and shows the signal-to-noise problems in the heat transfer measurement at lower displacements.
3.3 Third Attempt

The experiment was modified to operate in a constant-pressure condition instead by removing the pistons at the top of the apparatus (see figure 3.5). An off-the-shelf transmission oil cooling radiator was positioned above the upper duct to act as a flow straightener and thermal guard. A U-shaped hood was placed above the oil cooler to act as a warm air trap, providing another layer of thermal protection from the cooler room air. In addition, the previously thin layer of foam was replaced by wide layers of extruded polystyrene insulation (as seen in figure 3.5). To further improve the signal-to-noise ratio of the heat transfer measurement, two bundles of four tubes were used instead of two individual tubes. The ducts were rebuilt by soldering hollow walls together using brass plates and spacers. By pumping water through the duct walls, their surface temperatures could be well-controlled. This also enabled direct measurement of the heat transfer within the ducts as well.

The physical symmetry of the apparatus was very important. Because the air temperature was never directly measured, the effectiveness of the tubes could only be calculated if the effectiveness of the hot and cold sides were identical (this was a key requirement of Ray Wakeland’s original experiment as well). This same limitation, however, made it very difficult to isolate the effectiveness of the tubes from that of the ducts.

To correct for this, two complete sets of measurements were required for each tube geometry—one with the top duct and tubes kept hot while the bottom tubes and duct were kept cold; and another with only the top duct kept hot while both sets of tubes and
Figure 3.5: Cross-sectional diagram of the third re-designed apparatus and photograph with insulation. Notice the black plastic bags near the top of the insulation. These bags created a pressure-release condition so that no oscillation in the mean pressure due to heat transfer would occur.
the bottom duct were kept cold. By looking at the differences in the heat transfer measured in the tubes and ducts under these two temperature conditions, the effectiveness of the tube-to-tube heat transfer could be isolated from the tube-duct heat transfer.

Although functional, this proved to be extremely time consuming—a complete set of data for one tube geometry required sixteen days to acquire. In addition, the analysis was somewhat complex. Persistent water leaks in the solder joints of the duct walls were an issue as well.

3.4 Fourth Attempt

After some consideration, I decided on an entirely different approach (see figure 3.6). The pistons were moved to the top of the upper duct, while the lower tubes and duct were replaced with a foam channel. The duct and tubes were kept hot, while a fan blew chilled air through the channel below the open end of the remaining tubes. In this design, the time-averaged temperature of the air at the open end of the tubes was measured directly using a thermocouple. This allowed a direct calculation of the air inlet temperature (the temperature of the air as it entered the tubes) from the measured total heat transfer in the tubes and duct.

Because of the direct measurement of the average air temperature at the open end of the tubes, the requirement for physical symmetry was eliminated. The purpose of the channel and cool breeze was simply to ensure that completely cooled air entered the tubes with each acoustic cycle, maximizing the heat transfer signal.
Figure 3.6: Cross-sectional diagram and photograph of the fourth re-designed asymmetric apparatus. The lower exchanger and duct were replaced by a cold wind duct.
Under these conditions, an analytic expression was found that corrected the measured effectiveness of the tubes based on the heat transfer that was simultaneously measured in the duct. This eliminated the need to conduct multiple measurements with different temperature conditions, greatly reducing the amount of time needed to collect the data.

Several problems emerged upon consideration of the data, however. While the heat transfer measurements seemed believable, the acoustic impedance measurements did not. In addition, the soldered-wall duct assembly continued to leak. This forced a final change in the design.
Chapter 4

The Working Experiment

4.1 Description of the Experiment

After several iterations, a successful experiment was designed, constructed, and carried out. This chapter discusses the experiment and results in four sections. The first describes the physical design and operation of the experiment. The second section describes the measurements and collection of raw data. The third explains several quantities that are deduced from the raw data gathered with each model exchanger: volumetric air velocity through the exchangers; acoustic impedances and associated minor loss coefficients; acoustic power losses; and heat transfer performance expressed as effectiveness, Nusselt numbers, and Colburn-J factors. The final section in this chapter discusses the scaling of these results to exchangers of other sizes and to gasses other than air of arbitrary mean pressure and temperature.

4.2 Apparatus and Operation

4.2.1 Experimental Apparatus

The final apparatus is shown as a cross-sectional diagram and image in figure 4.1. The arm of an electro-dynamic shaker (APS Dynamics, Inc.) at the base of the experiment was connected to the rods of seven pistons (piston and cylinder set 2k444 by Airpot Corp.), as seen in figure 4.2. These pistons were mounted to the base of a round
Figure 4.1: Cross-sectional diagram and photograph of the working apparatus.
Figure 4.2: Electro-dynamic shaker and pistons.
aluminum duct with a custom-made assembly (throughout the rest of this document the pistons and assembly are referred to together as the “piston assembly”). Model heat exchangers were mounted at the top of the duct, with symmetric exchangers mounted above those via a PVC spacer plate. A second round duct was then mounted above these symmetric exchangers (the purpose of the symmetric exchangers and upper duct will be explained in the next section).

The upper and lower ducts were both 305 mm in length, and were constructed with two concentric aluminum tubes to create a hollow wall. The inner diameters of the ducts were both 70.0 mm, and the walls of the inner aluminum tubes were 9.53 mm thick.

The model exchangers were built by mating four aluminum tubes between two aluminum end-plates and surrounding these with an aluminum outer shell (see figures 4.3 and 4.4). The walls of these tubes were 2.98 mm thick, and had inner diameters of 16.3 mm. The symmetric exchangers were built in exactly the same manner.

Two separate water loops were plumbed into the apparatus, and are shown in figure 4.5. Room-temperature water (22°C) was pumped through the first from a PolyScience chiller unit into a small plate-and-fin radiator (the purpose of this radiator—a Swiftech MCR-80—was to ensure that the air entering the duct from the pistons below was at the same temperature as the surface of the duct). The radiator outlet was connected to an inlet at the base of the lower duct. The water filled and moved through the hollow duct wall, and exited through an outlet at the top. This outlet connected directly to the inlet of the adjacent model exchanger, so that water flowed into and through the shell of the exchanger. The water then exited through an outlet at the top of the exchanger and returned to the chiller unit.
Figure 4.3: The exchangers were constructed with four aluminum tubes mounted between two end-caps.

Figure 4.4: The assembly was surrounded with an outer aluminum shell. Notice the water inlet and outlet at the top and bottom of the shell.
Figure 4.5: Diagram of the flows through the hot (in red) and room-temperature (in blue) water loops.
Likewise, heated water (65°C) was pumped from a second PolyScience heater/chiller unit into an inlet at the base of the symmetric heated exchanger. Water flowed up through the shell of the exchanger and through an outlet at the top. This was immediately connected to the inlet of the duct above. The water then flowed up through the walls of the duct and passed through an outlet at the top. This outlet was then connected directly to the inlet of another small radiator (also a Swiftech MCR-80). The outlet of this radiator was then connected to the inlet of a larger radiator above (Black Ice GTS140-Stealth). The outlet of this radiator then returned to the PolyScience heater/chiller. The purpose of these radiators was to ensure that all air entering the top of the apparatus was at the same temperature as the walls of the duct.

Distilled water was used in both loops. Sodium nitrate was added at a concentration of about 2g/liter to prevent corrosion of the aluminum parts. Additionally, a few drops of biocide (PolyScience Lab Algicide) were added in both loops to prevent algae growth.

The flow-rate of the room-temperature loop was kept between 0.3 and 0.6 kg/min, and was maintained by a user-adjustable auxiliary pump. The flow-rate of the heated water loop was fixed by the internal pump of the associated heater/chiller unit (this pump was set to its maximum pumping capacity). The flow-rate of the heated water loop was not measured.
4.2.2 Operation

During operation, the two water loops created heated and cooled regions within the apparatus. As the shaker moved the pistons the air within the apparatus oscillated between these regions, thereby generating a non-zero time-averaged transfer of heat into the cooled model exchanger and duct below.

At the same time, oscillating air pressures were produced as the air moved through the apparatus. By measuring these pressures in various locations the volumetric air velocity through the model exchangers, their associated acoustic impedances, and resulting acoustic power losses (referred to simply as “acoustic losses” hereafter) could be deduced.

A LABVIEW program stepped the shaker through a series of frequencies and displacement amplitudes, while collecting data at each step. Eight frequencies were used: 0.22000 Hz, 0.37000 Hz, 0.62500 Hz, 1.05000 Hz, 1.77500 Hz, 3.00000 Hz, 5.00000 Hz, and 8.50000 Hz (the zeros are significant, but will be neglected hereafter). Data was taken at fifteen displacements at every frequency. The only exception was at 8.5 Hz where fewer displacements were used because of the limitations of the shaker. Combinations of frequencies and amplitudes were ordered so that measurements were made in groups of approximately constant rates of heat transfer, rather than stepping through sequential amplitudes or frequencies. The same program recorded all raw data. Analysis was conducted later using MATLAB, IGOR PRO, and EXCEL software.

To test the effects of geometry on heat transfer and acoustic losses, two groups of model exchangers were made. The first group was built so that the ends of the exchanger
tubes were flat—in other words, that there was no measureable radius on the ends. The second group was built so that the ends of the tubes were rounded. This was accomplished by machining a 1/8 in. radius around the holes in the outer face of the end-plates. In both groups, three lengths of exchangers were used: 76.2 mm, 127.0 mm, and 254.0 mm.

In addition to the effects of geometry, the effects of an adjacent stack of screens were also investigated. Three layers of 10 x 10 mesh (10 wires per inch) stainless steel screens (813 μm wire diameter) were spaced by placing 1/16 in. thick balsa wood shims around the edges. A balsa wood manifold was made on both sides of the stack to mate the stack to the model exchanger on one side and the heated symmetric exchanger on the other (see figure 4.6). This manifold formed an air-tight seal around the exchangers while leaving a 1/16 in. air gap between the stack and the exchanger end-plates. Epoxy was used to glue the assembly together. Measurements with adjacent screens were made with all three lengths of model exchangers with both flat and rounded tube-ends (both ends of the tubes were radiused when measurements were made with rounded tube-ends).

When the screen assembly was not used, a PVC spacer plate was used instead (see figure 4.7). This plate had holes through it that were carefully machined to match the model exchanger and symmetric heated exchanger tubes. In this way, the path through the model exchanger tubes, spacer plate, and symmetric heated exchanger appeared to the oscillating air to be long, continuous surfaces with an abrupt change in temperature. When model exchangers with rounded tube-ends were used with the spacer plate, however, the ends adjacent to the spacer plate were not radiused—only the
Figure 4.6: Screen assembly with thermocouples and pressure probe.

Figure 4.7: PVC separator plate with thermocouple. The holes in this plate were carefully machined to match the diameters and positions of the exchanger tubes.
opposite ends were rounded. The thickness of the plate between the two exchangers was 7.62 mm.

By comparing measurements made with different variations in geometry and with or without adjacent screens, the related effects on both heat transfer and acoustic losses could be simultaneously determined.

4.2.3 Operating Conditions

The experiment operated in an approximately constant-pressure condition. This was accomplished by placing a large hot air trap made from several layers of extruded polystyrene insulation over the guard radiator at the top of the apparatus. Vents at the bottom of the trap created a pressure-release condition. The trap served both as an additional layer of protection against the cooler room-temperature air and as a mechanical guard to keep dust from entering the interior of the apparatus.

It should be noted that in all cases, only time-averaged temperatures were measured. As a result, only the time-averaged heat transfer was calculated. It is true that there would have been an oscillating component to the heat transfer as well; however, in a thermoacoustic application this oscillating component does nothing useful for the device. Because of this, this oscillating component was neglected in this study.
4.3 Measurements and Raw Data

The measurements and initial conversions that were performed by the LabVIEW program are described in this section.

4.3.1 Cold Water-Loop Mass Flow-Rate

The mass flow-rate of the cooled water loop is measured using a technique described by Swift\(^3\) (section 9.2.2) and used by Wakeland.\(^53\) A power resistor was submerged in the water within an insulated tube that was plumbed between the outlet of the model exchangers and the return to the chiller. A known electrical power was dissipated through the resistor, while water temperatures upstream and downstream from the resistor were measured.

These measurements, combined with a knowledge of the heat capacity of water, enabled calculation of the mass flow-rate of the water through the following equation:

$$\dot{m}_{\text{water}} \cdot c_{p\text{water}} \cdot \Delta T_{\text{water}}.$$  \(4.1\)

In this equation, \(m_{\text{water}}\) is the mass flow-rate; \(c_{p\text{water}}\) is the heat capacity of the water; and \(\Delta T_{\text{water}}\) is the difference in water temperature as measured upstream and downstream from the submerged resistor.

\(\dot{Q}_{\text{heater}}\), however, is not given by the electrical power dissipated in the resistor alone. It actually consists of two parts:

$$\dot{Q}_{\text{heater}} = \dot{Q}_{\text{resistor}} + \dot{Q}_{\text{leak}},$$  \(4.2\)
where \( \dot{Q}_{\text{resistor}} \) is the electrical power dissipated in the resistor, and \( \dot{Q}_{\text{leak}} \) is the heat leak to or from the exterior environment.

To deal with this, the difference between the water temperatures upstream and downstream of the resistor were measured while running the water at the chosen flow-rate with the electrical power to the resistor shut off. This temperature difference was then subtracted from the difference that was measured while the electrical power was turned on.

It was assumed that the heat leak in the insulated tube was constant while the experiment was in operation. This was in fact only approximately true, as the temperature of the room varied by about ±2.5°C during any given 24-hour period. Subsequent measurements showed, however, that the resulting variation in heat leak was only ±50 mW, or 0.224% of the 22.3 W dissipated in the resistor.

### 4.3.2 Heat Transfer in the Lower Duct and Model Exchangers

As warmed air moved down into the cold model exchanger, heat transfer between the air and the exchanger caused the temperature of the water within to increase. By measuring the difference between the temperatures of the water at the inlet and outlet of the exchanger, the heat transfer within the exchanger could be calculated by using the previously measured water mass flow-rate and the following equation:

\[
\dot{Q}_{\text{HX total}} = m_{\text{water}} \cdot c_p \cdot \text{water} \cdot \Delta T_{\text{HX total}}.
\]

(4.3)
Likewise, if the air displacement was larger than the model exchangers, heat transfer occurred within the duct as well. This heat transfer was given by:

\[
\dot{Q}_{D\,\text{total}} = m_{\text{water}} \cdot c_{p\,\text{water}} \cdot \Delta T_{D\,\text{total}}
\]

As in the water mass flow-rate measurement, however, \( \dot{Q}_{\text{HX\,total}} \) and \( \dot{Q}_{D\,\text{total}} \) also include heat leaks. The temperature of the water in the lower duct and model exchangers was kept at or near room temperature to minimize heat leaks to or from the room. In addition, a great deal of extruded polystyrene insulation surrounded the apparatus (see figure 4.8). Heat leaks still occurred between the hot and cold sections of the apparatus, however.

These leaks were between one and three watts, whereas the heat transfer signals in the duct and model exchangers were as small as a few tens of milliwatts. To deal with this, the LabVIEW program would stop the shaker, wait for temperatures within the experiment to stabilize, and measure the heat leak directly with the shaker off. The shaker would then be set to the next combination of frequency and amplitude, and heat transfer measurements would resume. While heat transfer signals were small, the program measured the heat leak in this manner between every heat transfer measurement. As the signals increased, heat leaks were measured less frequently. The measured heat leaks were later subtracted from the data during analysis.

In addition, every measurement—whether the shaker was on or off—was actually the calculated average of several consecutive measurements. The number of averages
Figure 4.8: Insulated apparatus.
was dynamically adjusted depending on the magnitude of the heat transfer signal, and ranged from a minimum of 16 averages to a maximum of 64. When conducted in this manner, consecutive measurements at a constant shaker amplitude and frequency showed a variation in the heat transfer measured within the duct and model exchangers of ±20 mW.

### 4.3.3 Water Temperature Measurements

All water temperatures were measured with thermistors manufactured by RDP Corporation. Each of these thermistors contained a 2252 Ω YSI series 55000 glass-encapsulated thermistor, sheathed in 6 in. long stainless steel tubes with 1/8 in. diameters. During operation of the experiment, the resistance of each of the thermistors was measured by a Hewlett Packard 34970A data acquisition box. The LabVIEW control program collected these values via a GPIB bus. Figure 4.9 shows the position of the thermistors relative to the apparatus.

The thermistors were calibrated over the range of temperatures experienced during operation of the experiment (about 2°C) by bundling them together in a copper cylinder. The cylinder was then filled with water and sealed (care was taken to ensure that the thermistors did not touch the copper walls or end-cap), and was then submerged in a heater/chiller bath. The bath temperature was stepped through the previously mentioned range of temperatures at 0.1°C intervals. The resistances of the thermistors were measured at each temperature using the hardware and software previously
Figure 4.9: Thermistors were used to measure the water temperatures at the inlet at the base of the lower duct, the outlet at the top of the duct, the inlet at the base of the lower exchanger, and the outlet at the top of the lower exchanger. A thermocouple was used to measure the time-averaged air temperature between the lower exchanger and the screens or spacer plate.
described. Correlations from this data were derived, yielding each of the thermistors’ resistances as a function of temperature.

In all cases, heat transfer measurements depended on differential temperature measurements rather than absolute measurements. The individual thermistor responses were correlated to the average of the responses over the range of calibration temperatures. The resulting differential temperature measurements were accurate to within ±0.5 mK.

4.3.4 Time-Averaged Air Temperature

In addition to changes in water temperatures, the time-averaged temperature of the air was measured immediately between the model exchanger and adjacent screen assembly or separator plate. This was accomplished with thermocouples that had been calibrated with the previously described thermistors. In figure 4.6 three thermocouples are visible in the screen assembly. It was found that the differences between the three were negligible. Consequently only the output of one was used during analysis, and only one thermocouple was used with the separator plate (as seen in figure 4.7).

The outputs of these thermocouples were measured by the same Hewlett Packard 34970A that measured the resistances of the thermistors, and were averaged in the same manner by the LabVIEW program. The resulting measurement was accurate to ±8 mK.
4.3.5 Acoustic Pressures

Acoustic pressures were measured within the assembly between the pistons and the lower duct, within the duct just below the model exchangers, and either just above the model exchangers (between the model exchangers and adjacent screens) or just above the symmetric heated exchangers in the upper duct if the separator plate was used instead. Figure 4.10 shows these locations. These pressures were measured using piezo-resistive pressure sensors (Endevco 8510-B-1 and 8510-B-2 sensors, biased with custom in-house electronics). The outputs of these sensors were amplified using Stanford Research SR560 preamps.

The analog outputs of the preamps were digitized by a National Instruments PCI-MIO-16E1 data acquisition card installed in a PC computer. The LabVIEW control program then calculated FFT’s of each signal. The sampling rate was held at 6 kHz for all operating frequencies, and the number of samples recorded for each measurement was set so that an integer number of oscillations were digitized. In addition, the number of samples was set so that FFT’s for all measurements were approximately the same length—about 300 seconds. In this way, frequency resolution was approximately the same for every measurement while providing both magnitude and phase information without the use of windowing functions.

The three pressure sensors with all associated electronics were calibrated using a manometer technique and the previously described LabVIEW code. Figures 4.11 through 4.13 are the results. Actual measurements are given by red dots, while least-squares fits are given by solid black lines. The text boxes in each of these figures report
Figure 4.10: Piezo-resistive pressure sensors were positioned in the dashpot assembly just below the lower duct and in the lower duct just below the lower exchanger. If screens were used, a pressure probe was located immediately between the screens and lower exchanger. If the spacer plate was used, the pressure was measured in the upper duct just above the upper exchanger.
Figure 4.11: Calibration of the first pressure sensor. The coefficients $a$ and $b$ are the intercept and slope of the least-squares fit.

Figure 4.12: Calibration of the second pressure sensor.
Figure 4.13: Calibration of the third pressure sensor.

Coefﬁcient values ± one standard deviation

\[ a = 0 \pm 0 \]
\[ b = -1.599 \times 10^{-5} \pm 3.05 \times 10^{-8} \]

Endevco Output = b \times \text{Manometer Pressure} + a
values for two coefficients: a and b. These are the calculated intercepts and slopes of the least-squares fits, with standard deviations. The calibrated sensitivities of each of the pressure sensors and all associated electronics and software are indicated by these slopes in units of V/Pa.

The relatively low sensitivity of the piezoresistive sensors (from $1.899 \cdot 10^{-5}$ V/Pa to $2.924 \cdot 10^{-5}$ V/Pa) combined with the wide dynamic range of the individual pressure signals (dynamic range was on the order of five orders of magnitude, with peak signals as high as about 1000 Pa) created a challenging measurement. The digital converters in the National Instruments data acquisition card were twelve-bit converters. Even if they had been ideal, they would have only had four orders of magnitude of dynamic range. As a result, a great deal of preamp gain was required at low frequencies and low displacements, while much less gain was required as frequencies and displacements increased. To accomplish this, a routine was written into the master LABVIEW program that dynamically adjusted the preamp gains (via serial bus control) according to pre-determined levels.

The differential pressures across the model exchangers were calculated by taking the difference between the FFT’s of the pressure signals (after converting to pascals) measured at the duct just below the model exchangers and at the adjacent screens (or if the PVC separator plate was used instead, at the upper duct just above the symmetric heated exchanger). In the case of measurements made with the separator plate, the resulting differential pressure FFT was divided by a factor of two to account for the additional length and end-effects of the symmetric heated exchangers. The result was a
differential pressure measurement that was accurate to ±35 mPa (this was approximately 0.02% of the measured peak differential pressure).

4.3.6 Piston Displacement

The displacement of the pistons was measured with a Schaevitz Linear Variable Differential Transformer (hereafter referred to as an LVDT) that had been mounted to the shaker arm. The LVDT was biased and demodulated by a Schaevitz ATA2001 demodulator. The demodulator output was then digitized by the National Instruments data acquisition card. The LabVIEW control program calculated the FFT of the signal using the same routine that was used to calculate the FFT’s of the pressure signals.

Calibration of the LVDT and ATA2001 demodulator was accomplished in two stages. First, the demodulator was calibrated by externally multiplying the 5 kHz bias signal produced by the ATA2001 with a signal output of a function generator using an external analog circuit. The result was then sent back to be demodulated by the ATA2001. The output was then compared with the original function generator signal using the LabVIEW FFT routine previously described. In this way, both the magnitude and phase error of the ATA2001 were directly measured (see figures 4.14 and 4.15). During operation, the LabVIEW program used these functions to correct the measured output of the ATA2001.

The LVDT was then calibrated as biased and demodulated by the ATA2001, while the shaker arm displacement was measured by hand. Figure 4.16 is a graph of the LVDT output as a function of shaker arm displacement. Red dots indicate
Figure 4.14: Magnitude of the ATA2001 output error.

Figure 4.15: Phase error of the ATA2001 output.
Figure 4.16: LVDT calibration.

Coefficient values ± one standard deviation

\[ a = 0 \pm 0 \]
\[ b = 58.00 \pm 3.510 \times 10^{-2} \]

LVDT Output = \[ b \times \text{Shaker Displacement} + a \]
measurements, while the solid black line indicates the least-squares fit of the data. The coefficients \(a\) and \(b\) reported in the text box of figure 4.16 are the intercept and slope of the least-squares fit, respectively. The sensitivity of the LVDT is given by the slope of the least-squares fit and is accurate to about 0.06% of the peak signal.

### 4.3.7 Physical Properties of Air

Atmospheric pressure was read from a mercury barometer at the beginning of each set of measurements. During analysis, the density of the air within the experiment was calculated using the measured atmospheric pressure and the time-averaged air temperature given by the thermocouples between the model exchanger and screen assembly or spacer plate. The ideal gas law was used:

\[
P_m = \rho r T_m \tag{4.5}
\]

where \(P_m\) is the atmospheric pressure; \(r\) is the specific gas constant of air; and \(T_m\) is the time-averaged air temperature.

All other physical properties of the air—specific heat, thermal conductivity, and viscosity—were assumed to be constant over the range of air temperatures within the experiment during operation.

### 4.3.8 Surface Temperatures of the Duct and Model Exchangers

The surfaces of the model exchangers and duct that were in contact with the oscillating air were assumed to be held at constant temperatures. Analysis showed that
the greatest change in water temperature in either duct or exchanger was only 0.935°C, so this approximation is reasonable. During analysis, the logarithmic means of the water inlet and outlet temperatures of the duct and model exchangers were used as the surface temperatures of each [the logarithmic mean is given by $T_{LM} = T_{out} - T_{in} / \ln \frac{T_{out}}{T_{in}}$].

It was assumed that the temperature drops through the aluminum walls of the model exchanger tubes and the inner aluminum wall of the duct were negligible. The largest measured heat transfer within the model exchangers was 77.2 W, while that measured within the lower duct was 35.3 W. The surface area of the model exchanger tubes was 5.19×10^{-2} m^2, while that of the duct was 6.69×10^{-2} m^2. The thickness of the model exchanger tube walls was 3 mm, while that of the duct wall was 10 mm.

Given the thermal conductivity of aluminum it was calculated that the largest temperature drop across the walls of the model exchanger tubes was 22 mK, while the drop across the duct wall was 25 mK. By comparison, the largest difference between the temperature of the air as it entered the model exchangers and the surface temperature of the model exchangers was 29.0° K. Therefore, the small temperature drops through the walls of the model exchangers and duct were negligible.

### 4.3 Deduced Quantities

Several quantities are deduced from the raw measurements and calculations during analysis. These include the volumetric air velocity through the model exchangers, the acoustic impedances of the exchangers, the associated minor loss coefficients, acoustic losses, and the heat transfer performance of the model exchangers expressed as
effectiveness, Colburn-J factors, and Nusselt numbers. Discussions of each of these follow.

4.4.1 Volumetric Air Velocity

The peak volumetric air velocity produced at the pistons is given by:

\[ U_{\text{pistons}} = 2\pi f x_{\text{pistons}} A_{\text{pistons}} , \]

(4.6)

where \( f \) is the frequency of operation, \( x_{\text{pistons}} \) is the measured peak piston displacement, and \( A_{\text{pistons}} \) is the cross-sectional area of the pistons. This quantity is known to 0.06% of the peak volumetric velocity produced.

The volumetric air velocity through the model exchangers is, however, not the same as that produced at the pistons. Compliance of the air within the piston assembly and duct alter both the magnitude and phase of the velocity through the exchangers.

To account for this, I measured the compliance of the air within the piston assembly directly by blocking the base of the duct and measuring the pressure within the piston assembly at each frequency of operation. Then, I unblocked the bottom of the duct and measured the total compliance of the piston assembly and duct together by blocking the top of the duct and measuring the pressure in the same manner. These compliances were then known to the same amount of precision and accuracy as the initial volumetric air velocity calculation. The same measurements showed that DC air leaks past the pistons were negligible.
I then took the imaginary part of the measured acoustic impedances in the pistons and duct at each frequency (the real parts were negligibly small in comparison) and treated the system as if the piston assembly and duct were analogous to ideal parallel capacitors in an electrical circuit (see figure 4.17). The value of the first capacitor (that is, the compliance of the air within the piston assembly) was known from direct measurement. The value of the second capacitor (the isolated compliance of the duct) could then be calculated from simple algebra. The result was a measurement of the frequency-dependent compliance of both sections of the apparatus.

In the analogous circuit of figure 4.17, the impedance of the guard exchanger between the piston assembly and duct is represented by the impedance $Z_{\text{guard}}$. Likewise, the equivalent load presented by the top half of the apparatus is represented by the unknown impedance $Z_{\text{load}}$. The analogous voltages at $C_1$ and $C_2$ were measured directly in the experiment with pressure sensors in the piston assembly and lower duct. The complex volumetric velocities lost to compliance in the piston assembly and duct were then calculated directly.

The volumetric velocity into the duct was taken to be the difference between the volumetric velocity at the pistons and that lost to compliance in the piston assembly. Likewise, the volumetric velocity through the model exchangers was given by the difference between the volumetric velocity into the duct and that lost to compliance within the duct.

In addition to the compliance of the air within the piston assembly and duct, acoustic Helmholtz resonances produced within the apparatus also altered the volumetric velocity through the model exchangers. At a certain frequency, the air within the model
Figure 4.17: Analagous electrical circuit showing the compliances in the pistons and duct. $Z_{\text{guard}}$ is the impedance of the small radiator at the base of the lower duct. $Z_{\text{load}}$ is the impedance of the apparatus above the lower duct.
exchanger acts like a simple mass while the air within the duct and pistons below it acts like a simple spring. This condition is called a Helmholtz resonance. If such a resonance were to occur at or near the frequencies of operation in the experiment, it would cause significant changes in the magnitude and phase of the air velocity through the model exchangers.

Elimination of this resonance was impossible in this apparatus; but its effects were minimized by ensuring that it occurred at a high enough frequency and Q. To accomplish this, the dimensions of the ducts and model exchangers were carefully chosen. The result was a Helmholtz resonance frequency of 50 Hz with a Q of 5, successfully making any resonance effects negligible at the frequencies of operation (any residual effects were taken into account by calculating the effects of compliance in the pistons and duct, as described previously).

The last effect that had to be taken into account was the change in air velocity within the model exchanger and duct due to thermal expansion and contraction of the air caused by heat transfer. The magnitude of this effect was easily accounted for because of the simultaneous heat transfer measurements and constant-pressure condition of the experiment, and was about a 12% effect at its peak (under constant pressure conditions, the isobaric ideal gas law states that $\Delta V/V = \Delta T/T$. The result was an estimate of the magnitude of the additional volumetric velocity due to thermal effects that was known to 0.2% of the peak effect.

The phase shift due to this effect could not be measured directly, however, and had to be accounted for by using linear thermoacoustic theory (a summary of which is
given in Swift\textsuperscript{3}). It is entirely possible that this application of the linear theory is not very appropriate, however, given the conditions of the experiment. The theory was derived based on assumptions of perfectly laminar flow and a linear temperature gradient in space. Both of these were violated within the experimental conditions. The Reynolds number of the air within the model exchangers ranged from 32 to 22,000, and the change in temperature between hot and cold sections of the experiment was more abrupt than linear. However, the greatest possible phase error (according to linear theory) would be within 7° (the inverse tangent of 12%).

With these three effects quantified, the magnitude and phase of the volumetric velocity of the air through the model exchangers was reasonably well-known. The calculated magnitude was known to about 0.1% of the peak velocity, and the maximum phase error was 7°.

### 4.4.2 Acoustic Impedances

The acoustic impedances of the various model exchangers were calculated by dividing the measured differential pressure across the exchangers by the calculated volumetric air velocity through them. Both of these values were complex (having both real and imaginary parts).

The inertial part of the impedance was given by the imaginary part of this ratio. Likewise, the acoustic resistance was given by the real part.
4.4.3 Minor Loss Coefficients

The acoustic resistances of the model exchangers consisted mainly of two parts: a linear component, and the nonlinear minor loss. At low velocities, the linear component of the resistance (and the imaginary part of the impedance, for that matter) is described by well-established linear theory (an excellent summary of which can be found in Swift\textsuperscript{3}).

At higher velocities, however, the nonlinear minor losses become significant. The pressures generated by these losses are given by:

\[ \Delta P = \frac{1}{2} \kappa \rho n^2, \]

where \( \rho \) is the average air density, \( v \) is the cross-sectionally averaged particle velocity, and \( \kappa \) is the minor loss coefficient.

Dividing this expression by the magnitude of the volumetric air velocity \( |U| \) results in the following velocity-dependent resistance:

\[ R_{\text{nl}} = \frac{\kappa \rho |U|}{2A_{\text{HX}}} \]

In this expression, \( A_{\text{HX}} \) is the total open cross-sectional area of all four of the model exchanger tubes. The value of the minor loss coefficient depends only on the shape of the ends of the exchanger tubes. Determination of the value of this coefficient is one of the primary reasons for the acoustic impedance measurements.

4.4.4 Acoustic Losses

Acoustic losses are given by:
\[ \Pi = \frac{1}{2} |P||U| \cos \phi, \]  

where \( \phi \) is the phase angle between the acoustic differential pressure and the volumetric air velocity through the exchangers.

### 4.4.5 Heat Exchanger Effectiveness

The goal of the heat transfer measurements is to calculate the heat transfer effectiveness of the lower exchanger in isolation from any other part of the apparatus. In other words, had the lower duct been adiabatic. Heat transfer within the duct causes the heat transfer within the model exchanger to be less than it would have been had the duct been in that case, however.

To illustrate this, consider the following process, shown in figure 4.18: as warm air passed down into the model exchanger during one part of the acoustic cycle, it would cool. Then, if the duct had been ideally adiabatic, no additional cooling would take place between the air and the duct walls. The air would then have returned to the model exchanger at the same temperature it was when it entered the duct. Additional cooling would then take place as the air passed back up through the model exchanger. In these conditions, the effectiveness of the model exchanger could have been calculated directly from the heat transfer measured within the exchanger.

In reality, the warm air passed through the model exchanger and then entered the duct where it continued to cool. It then returned through the model exchanger, but at a cooler temperature than it was when it first entered the duct. Under these conditions, the
Adiabatic Duct

Actual Duct

Figure 4.18: No additional cooling would take place if the duct walls were adiabatic. Instead, additional cooling within the duct reduces the amount of heat transfer within the exchanger.
measured heat transfer in the model exchanger was less than it would have been had the duct walls been adiabatic. This lead to estimates of the exchanger effectiveness that were too low. This effect, however, was taken into account as shown here.

An expression was derived to correct the measured exchanger effectiveness, based on the assumption that the effectiveness of the exchanger does not change with the direction of the oscillating air. In other words, the effectiveness of the exchanger as the air entered it (hereafter called $E_{in}$) was assumed to be the same as the effectiveness as the air reversed direction and exited the exchanger (called $E_{out}$).

As the air moves into and through the model exchanger from the hot side of the apparatus, it will experience temperature change $\Delta T_{air\ in}$. This change in temperature is given by:

$$\Delta T_{air\ in} = E(T_{air\ in} - T_{HX\ surface}).$$

Here, $T_{air\ in}$ is the warm temperature of the air as it enters the model exchanger and $T_{HX\ surface}$ is the cold surface temperature of the exchanger.

While the air is in the duct, it will experience a further temperature change $\Delta T_{duct\ air}$. Then, as the air passes back through the model exchanger it will experience a further temperature change $\Delta T_{air\ out}$. This is given by:

$$\Delta T_{air\ out} = E_{in}(T_{air\ in} - T_{HX\ surface} - \Delta T_{air\ in} - \Delta T_{duct\ air}).$$

Or,

$$\Delta T_{air\ out} = E_{in}(T_{air\ in} - T_{HX\ surface} - \Delta T_{duct\ air} - E_{in}(T_{air\ in} - T_{HX\ surface})).$$

The measured heat transfer in the model exchanger enables calculation of $\Delta T_{air\ in} + \Delta T_{air\ out}$. This quantity is given by:
\[
\Delta T_{\text{air in}} + \Delta T_{\text{air out}} = E_{\text{in}}(T_{\text{air in}} - T_{\text{HX surface}}) + E_{\text{in}}[T_{\text{air in}} - T_{\text{HX surface}} - \Delta T_{\text{duct air}} - E_{\text{in}}(T_{\text{air in}} - T_{\text{HX surface}})] .
\] (4.13)

Dividing \( \Delta T_{\text{air in}} + \Delta T_{\text{air out}} \) by \( T_{\text{air in}} - T_{\text{HX surface}} \) gives the uncorrected, raw effectiveness of the model exchanger:

\[
E_{\text{raw}} = \frac{\Delta T_{\text{air in}} + \Delta T_{\text{air out}}}{T_{\text{air in}} - T_{\text{HX surface}}} = 2E_{\text{in}} - \frac{E_{\text{in}}\Delta T_{\text{duct air}}}{T_{\text{air in}} - T_{\text{HX surface}}} - E_{\text{in}}^2 .
\] (4.14)

If the duct had been adiabatic, \( \Delta T_{\text{duct air}} \) would be zero. Then, \( E_{\text{raw}} \) would be the true model exchanger effectiveness (or “corrected” effectiveness, as it is called through the rest of this derivation), or \( E \). Then,

\[
E = E_{\text{in}}(2 - E_{\text{in}})
\] (4.15)

and

\[
E_{\text{in}} = 1 - (1 - E)^{1/2}.
\] (4.16)

Then by substitution,

\[
E_{\text{raw}} = 2 - 2(1 - E)^{1/2} - \frac{\Delta T_{\text{duct air}}}{T_{\text{air in}} - T_{\text{HX surface}}}[1 - (1 - E)^{1/2}] - [1 - (1 - E)^{1/2}]^2 .
\] (4.17)

Through manipulation, this reduces to:

\[
E_{\text{raw}} = E + \frac{\Delta T_{\text{duct air}}}{T_{\text{air in}} - T_{\text{HX surface}}}[1 - (1 - E)^{1/2} - 1] .
\] (4.18)

At this point, let \( x = (1 - E)^{1/2} \) and \( E = 1 - x^2 \). Also, let:

\[
E_{\text{duct}} = \frac{\Delta T_{\text{duct air}}}{T_{\text{air in}} - T_{\text{HX surface}}} .
\] (4.19)

Then by substitution, use of the quadratic equation to solve for \( x \), and with some manipulation, it can be shown that:
\[ E = 1 - \frac{1}{4} \left( E_{\text{duct}} + \sqrt{E_{\text{duct}}^2 - 4(E_{\text{duct}} + E_{\text{raw}} - 1)} \right)^2. \] (4.20)

In this equation, \( E \) is the corrected effectiveness of the model exchanger. Note that I have neglected one of the solutions obtained from the quadratic equation. This is because that solution gives a negative result for \( E \)—a result that is physically impossible. In all cases in this study, this equation is used to calculate exchanger effectiveness.

In practice, the assumption that the effectiveness of the model exchangers is the same in both directions may not be correct. A sensitivity analysis showed that if the effectiveness in one direction were only 50% of the effectiveness in the other direction (it is not important which is \( E_{\text{in}} \) and which is \( E_{\text{out}} \)), the results given by equation (4.20) would differ only by 5%.

### 4.4.6 Heat Exchanger Nusselt Number

The Nusselt number can be thought of as a dimensionless heat transfer convection coefficient, and is related to the convection coefficient \( h \) through the following relationship:

\[ \text{Nu} = \frac{hL}{k}. \] (4.21)

In this expression \( k \) is the thermal conductivity of the gas; \( L \) is the length of the heat exchanger; and \( h \) is the convection coefficient.

The Nusselt number can be calculated from the model exchanger effectiveness. We begin with the following expression:
The effectiveness $E$ is given by equation (4.20). The heat transfer $\dot{Q}$ then is the heat transfer that would have taken place within the exchanger had the duct been adiabatic.

The heat transfer caused by the exchanger [$\dot{Q}$ in equation (4.22)] can also be described using the spatially-averaged Nusselt number $\text{Nu}$ as follows:

$$\dot{Q} = \frac{kA_s \text{Nu}(T_{\text{air in}} - T_{\text{HX surface}})}{L \Delta T_{LM}}. \quad (4.23)$$

In this expression $k$ is the thermal conductivity of the air; $L$ is the length of the model exchanger; $A_s$ is the surface area of the model exchanger that is in contact with the air; and $\Delta T_{LM}$ is the log-mean temperature difference given by:

$$\Delta T_{LM} = \frac{T_{\text{air in}} - T_{\text{HX surface}} - T_{\text{air out}} - T_{\text{HX surface}}}{\ln \left( \frac{T_{\text{air in}} - T_{\text{HX surface}}}{T_{\text{air out}} - T_{\text{HX surface}}} \right)}. \quad (4.24)$$

The use of this log-mean temperature difference comes from the convention used in internal steady flow (see pages 403-404 of Incropera and DeWitt$^1$).

The exchangers used in this study all have four tubes, so equation (4.23) reduces to:

$$\dot{Q} = 8\pi r k \text{Nu} \Delta T_{LM}, \quad (4.25)$$

where $r$ is the inner radius of the exchanger tubes (the slight change in $A_s$ for rounded tube-ends is ignored).
Substituting equation (4.25) into equation (4.22) and rearranging results in the following expression:

\[ \text{Nu} = \frac{E m_{air} c_{p air} T_{air_{in}} - T_{HX\text{ surface}}}{2\pi r k \Delta T_{LM}}. \]  

(4.26)

It should be noted that while the Nusselt numbers for any of the exchangers tested in this study vary by as much as five orders of magnitude (because of the wide variation in \( \dot{m} \) with frequency and amplitude), the effectiveness always ranges between 0 and 1. Therefore, graphs of effectiveness are often better able than graphs of \( \text{Nu} \) to highlight interesting variations of heat transfer with variables other than \( \dot{m}_{air} \).

4.4.7 Colburn-J Factors

The Colburn-J factor is another dimensionless number that is widely used in steady-flow applications. Its utility comes from the Chilton Colburn-J Factor analogy—that is, that under fully-developed conditions in steady flow, heat, momentum, and mass-transfer are analogous and differ only by constant scaling factors. This analogy was derived from experimental data gathered in steady-flow conditions, rather than from analytic derivations. As far as I know, there have been no studies to determine if this analogy holds in oscillating flows as well.

The Colburn-J factor is defined as the following:

\[ J_H = \frac{\text{Nu}}{\text{Re Pr}^{1/3}}. \]  

(4.27)
The two dimensionless numbers in this expression are the Reynolds number and the Prandtl number of the gas. The Reynolds number is defined as:

\[ Re = \frac{2r \rho \nu}{\mu}, \] (4.28)

where \( r \) is the inner radius of the exchanger tubes; \( \rho \) is the density of the air (calculated at atmospheric pressure and the mean air temperature measured with the thermocouple); and \( \nu \) is the particle velocity of the air through the exchanger. The Prandtl number \( Pr \) is defined as:

\[ Pr = \frac{\mu c_p}{k}, \] (4.29)

where \( \mu \) is the viscosity of the air, \( c_p \) is the constant-pressure specific heat of the air, and \( k \) is the thermal conductivity of the air.

Equation (4.27) can also be expressed as:

\[ J_H = \frac{Pr^{\frac{1}{2}} E(T_{\text{air in}} - T_{\text{HX surface}})}{4\Delta T_{LM}}, \] (4.30)

This is the equation used to calculate Colburn-J Factors in this study.
Chapter 5

Scaling to Other Exchangers and Gases

For the results of this study to be of any practical use they must be applicable to shell-and-tube exchangers of other sizes than the ones used in this study, to gases other than air, and to conditions other than atmospheric pressure and room temperature. This section discusses the scaling of the acoustic impedance, minor loss, acoustic loss, and heat transfer data.

5.1 Acoustic Impedance

The linear acoustic impedance of simple shell-and-tube heat exchangers (such as those modeled in this study) is easily calculated from existing acoustic theory for any ideal gas (a summary of which can be found in Swift\(^3\)). This study will show that the impedance of a shell-and-tube exchanger can be very reasonably described by a superposition of linear impedances and nonlinear minor losses. Because of this, dimensional analysis of the linear impedance data in this study is not necessary.

However, in keeping with the dimensionless analysis of all other data in this study, the imaginary (or inertial) part of acoustic impedances will be normalized as follows:

\[
\text{Im}[Z_{\text{norm}}] = \frac{4\pi r^2 c \text{Im}[Z]}{P_m}.
\] (5.1)
The choice of mean pressure $P_m$, sound speed $c$, and cross-sectional area of the exchanger tubes to normalize impedances is not random. Rather, they are consistent with Swift’s non-dimensionalization of ideal gases for thermoacoustic applications. The speed of sound was calculated at atmospheric pressure and at the time-averaged temperature of the air as measured at the interface between the model exchangers and screen assembly or spacer plate.

Acoustic resistances, on the other hand (the real parts of acoustic impedances), are not presented in dimensionless form. The minor loss coefficients are already dimensionless, and are the primary purpose for investigating these resistances.

### 5.2 Minor Losses

Previous literature has shown that the minor loss coefficient $\kappa$ is a function of the shape of the ends of the tubes only—it does not depend on the physical properties of the gas. Because of this, the values of the coefficients found in this study are expected to be immediately applicable to similar geometries. As long as this is the case, the only dimensionless parameter that is necessary for similarity is the dimensionless radius of curvature at the ends of the tubes, given by:

$$r_{ml} = \frac{r_{end}}{r_h},$$  \hspace{1cm} (5.2)

where $r_{end}$ is the radius of curvature at the ends of the exchanger tubes, and $r_h$ is the hydraulic radius of the tubes.
Only two radii of curvature were tested in this study: \( r_{ml} = 0 \) (flat tube-ends), and \( r_{ml} = 0.781 \) (1/8 in. radius of curvature).

### 5.3 Acoustic Losses

Although the acoustic losses generated by the model exchangers used in this study have been calculated, their usefulness is somewhat limited. This study was conducted under approximately constant-pressure conditions—in other words, the acoustic pressure oscillations were negligible compared to atmospheric pressure. The pressure oscillations in actual applications, on the other hand, are often very significant. For example, the acoustic pressure oscillations within thermoacoustic devices are routinely 5%-10% of the mean pressure.

Besides increasing the magnitude of acoustic losses, these pressure oscillations cause the temperature of the gas to oscillate significantly. These temperature oscillations will affect heat transfer not only because of their magnitude, but also because of their phase relative to the velocity of the gas.

Wakeland and Keolian\(^5^4\) assumed that the effects of these pressure oscillations could be considered separately from those of an oscillating velocity. This study operates on the same assumption.
5.4 Heat Transfer

Heat transfer data has been normalized as effectiveness and Nusselt numbers in this study. The following is a discussion of the dimensionless parameters that are necessary for the effectiveness and Nusselt number correlations to apply in such conditions.

Zhao and Cheng\(^\text{36}\) conducted a numerical study of heat transfer from the walls of a heated tube to an oscillating internal gas, with cold reservoirs located at each end. The model was based on assumptions of strictly laminar and incompressible flow. Through dimensional analysis of the coupled continuity, momentum, and energy equations they found that the model depended on four dimensionless groups: first, the Reynolds number based on the thermal penetration depth of the gas (non-dimensional frequency); second, the ratio of the peak gas displacement to the tube length; third, the ratio of the tube length to inner diameter; and fourth, the Prandtl number of the gas. Through numeric simulation, they arrived at a correlation for three of these four parameters (they did not examine the effects of the Prandtl number, as all of their calculations were done with air as the gas).

The assumption of strictly laminar and incompressible flow fails in this study, however. The flows through the model exchangers can be much more complex than those in Zhao and Cheng’s study, and likely affect heat transfer. For this reason, this study proposes a slightly different set of dimensionless parameters. These are the dimensionless radius of curvature of the tube-ends given in equation (5.2); the ratio of the peak gas displacement within the exchanger to the length of the exchanger, or
the ratio of the hydraulic radius to the thermal penetration depth of the gas, or

\[ \frac{r_h}{\delta_k}, \]  

and the Reynolds number as given by

\[ Re = \frac{D \rho v}{\mu}, \]  

where \( D \) is the diameter of the exchanger tubes, \( \rho \) is the mean density of the gas, \( v \) is the cross-sectionally averaged particle velocity of the gas through the exchanger, and \( \mu \) is the viscosity of the gas.

The choice of the parameters \( r_{na} \) and \( x_{norm} \) is obvious. The remaining parameters—\( r_h/\delta_k \), \( Re \), and \( Pr \)—and the omission of the aspect ratio of the exchanger (Zhao and Cheng defined this as \( L/D \)) require further discussion.

### 5.4.1 Normalized Frequency

The operating frequency \( f \) has been normalized by taking the ratio of the hydraulic radius of the model exchanger tubes to the thermal penetration depth of the air, as in equation (5.4). The thermal penetration depth is given by:

\[ \delta_k = \sqrt{\frac{2k_{air}}{\rho \rho_{air} c_{p,air}}}. \]  

In this equation, $k_{\text{air}}$ is the thermal conductivity of the air; $\omega$ is the angular frequency of operation; and $\rho_{\text{air}} c_{\text{p,air}}$ is the product of the density and heat capacity of the air at constant pressure.

This equation gives the thickness of the thermal boundary layer in oscillating flow, and has units of meters. Notice that thickness of the boundary layer is inversely proportional to the square-root of frequency—in other words, as the frequency rises the thickness of the thermal boundary layer decreases.

Now consider again the ratio of the hydraulic radius of the model exchanger tubes to the thermal penetration depth:

\[
\frac{r_h}{\delta_c} = \frac{r}{\sqrt{\frac{2k_{\text{air}}}{\omega \rho_{\text{air}} c_{\text{p,air}}}}}.
\]  

Notice that as the angular frequency decreases the ratio does also, indicating that the thermal boundary layer occupies more of the space within the exchanger tube. In other words, the gas within the tube would be in better thermal contact with the walls of the tube. The other advantage to this ratio is that it normalizes the thermal conductivity, heat capacity, and density of the gas.

Most previous papers (in the thermoacoustic and Stirling communities, that is) have used an “oscillating Reynolds number” to indicate a normalized frequency. To my mind, this is confusing as the usual Reynolds number is often simultaneously used to normalize the oscillating velocity. Also, the ratio presented in equation (5.4) is more
physically intuitive, as it directly relates the thickness of the thermal boundary layer to
the relevant dimension of the exchanger.

5.4.2 Normalized Velocity

The Reynolds number $Re$ is almost always used as a normalization of velocity. It
is the ratio of inertial to viscous forces in the flow, and is very often used to characterize
different flow regimes (such as laminar and turbulent flows) in both steady and
oscillating flows. Matching this parameter will—in absence of other effects—match flow
conditions. It is defined as:

$$Re = \frac{D \rho v}{\mu}, \quad (5.8)$$

where $D$ is the diameter of the exchanger tubes; $\rho$ is the density of the gas; $v$ is the
cross-sectionally averaged particle velocity of the gas; and $\mu$ is the viscosity of the gas.

5.4.3 Prandtl Number

Zhao and Cheng found in their dimensional analysis that the Prandtl number of
the gas was a required parameter; however, their study did not investigate its relative
importance. At this point, I am not aware of any correlations of heat transfer in
oscillating flow that characterize the effect of Prandtl number. However, with some
theoretical analysis it is possible to derive one.
The following is a derivation of an expression for the heat transfer effectiveness of an idealized tube under constant-pressure conditions. With this expression, we can examine the influence of the Prandtl number on heat transfer effectiveness.

Suppose that an ideal gas oscillates through the interior of an infinitely long, round tube of radius $r$ (the mean pressure and density are not important for now) with peak volumetric velocity $U_1$. We will assume that the flow is laminar and the walls of the tube are smooth.

We will also impose a linear mean-temperature gradient along the length of the tube (along the $x$-direction, given by $dT_m/dx$, and will also assume that the time-averaged temperature of the gas along the $x$-direction (and over the cross-section) matches this temperature gradient.

We will impose the condition that heat transfer through the gas along the $x$-direction is not allowed. Heat transfer between the walls of the tube and the gas (in the $y$-direction, that is) is allowed, however. We will also assume that the heat capacity of the walls is infinite, so that the temperature of the walls remains constant even though the temperature of the gas may be different. We will include the effects of viscosity.

As the gas oscillates within the tube, the cross-sectionally averaged gas temperature at any given location along the $x$-direction will oscillate about the fixed temperature of the wall at that location (given by $T_m$). This causes an oscillating heat transfer to occur between the walls of the tube and the gas, as well as thermal expansion and contraction of the gas. This then causes a change in the oscillating volumetric
velocity of the gas. This change is described by Rott’s thermoacoustic version of the continuity equation (equation 4.70, page 90 of Swift):}

\[
dU_1 = \frac{-i\omega A dx}{\gamma P_m} \left[1 + (\gamma - 1) f_k \right] p_1 + \frac{f_k - f_v}{(1 - f_k)(1 - Pr)} \frac{dT_m}{T_m} U_1, \tag{5.9}
\]

In this expression, \( p_1 \) is the oscillating pressure; \( U_1 \) is the oscillating volumetric velocity of the gas averaged across a control volume; \( P_m \) is the mean pressure of the gas; \( T_m \) is the fixed temperature of the tube wall at any given location along the \( x \)-direction, and is also the time-averaged temperature of the gas throughout the cross-section at that location; \( \gamma \) is the ratio of specific heats of the gas; \( \omega \) is the angular frequency of oscillation; \( A \) is the cross-sectional area of the tube; \( Pr \) is the Prandtl number of the gas; and \( f_k \) and \( f_v \) are the thermo-viscous functions as described by Swift\(^3\) (in the case of round tubes, equation 4.60 on page 88 of Swift).

We will neglect the first term on the right of equation (5.9), because we will assume that the oscillating pressure \( p_1 \) is 0. This is approximately the case in this experimental study, where the peak differential pressure oscillation was 245 Pa—or about 0.245% of the mean pressure. From the ideal gas law, \( \Delta P / P + \Delta V / V = \Delta T / T \).

Therefore, the change in the temperature of the air due to pressure oscillations was at its peak 0.245% of the mean temperature. In comparison, the peak temperature change of the air measured in the experiment was about 4.71% of the mean temperature (a change of 14.1°C), and is nearly 20 times higher than that due to pressure oscillations.

Multiplying by \( T_m / i\omega A \), equation (5.9) then becomes:

\[
\frac{dx_1}{dx} T_m = \frac{f_k - f_v}{(1 - f_k)(1 - Pr)} \frac{dT_m}{dx} x_1, \tag{5.10}
\]
where \( x_i \) is the Eulerian cross-sectionally averaged peak displacement.

Equation (5.10) is written in the control-volume Eulerian perspective. The \( x_i dT_m/dx \) term on the right of equation (5.10), however, can also be identified as the temperature swing \( \Delta T_{Liso} \) of a Lagrangian segment of gas moving a distance \( x_i \) from a mean position \( x_m \) while in perfect isothermal contact with the wall. Therefore, we may write equation (5.10) as:

\[
T_m \frac{dx_i}{dx} = \frac{f_k - f_v}{(1-f_e)(1-Pr)} \Delta T_{Liso},
\]

the subscript \( L \) indicating a Lagrangian quantity.

We will focus our attention now on the Lagrangian segment of gas that happens to be in the Eulerian control volume between \( x_m \) and \( x_m + dx \) at \( t = 0 \), where \( t = 0 \) is defined as the moment when \( Re \ x_i e^{\text{isot}} = 0 \). As the gas oscillates back and forth the Lagrangian segment distorts in shape, fluctuates first-order in volume because of thermal expansion and contraction, and has a time-averaged volume given by \( V_L = Adx \).

To first order, \( dx_i \) in equation (5.11) is constant over the cross section of the Lagrangian segment as it oscillates from side-to-side. This is shown by taking the first two terms of the Taylor series of \( x_i \) at \( x = x_m + x_i \):

\[
x_i \bigg|_{x=x_m+x_i} = x_i \bigg|_{x=x_m} + \left( \frac{dx_i}{dx} \right)_{x=x_m} x_i;
\]

and then taking the derivative with respect to \( x \):

\[
\left( \frac{dx_i}{dx} \right)_{x=x_m+x_i} = \left( \frac{dx_i}{dx} \right)_{x=x_m} + \frac{d}{dx} \left[ \left( \frac{dx_i}{dx} \right)_{x=x_m} x_i \right].
\]
Looking at equation (5.10), $dx_i$ is seen to be proportional to $x_i$. Therefore, the second-order term on the right of equation (5.13) is negligible as $x_i$ approaches 0. Hence, $dx_i$ is constant to first-order for all the elements of the Lagrangian segment across the cross-section of the tube and for all changes of profile of the segment, as long as the elements of the segment stay within a distance of order $x_i$ from $x_m$.

Given this result, we may set the change in volume $\Delta V_L$ of the Lagrangian segment equal to the change of volume $A dx_i$ calculated at the fixed Eulerian control volume at $x = x_m$:

$$Adx_i = \Delta V_L.$$  

Equation (5.11) can then be re-written as:

$$T_m \frac{\Delta V_L}{V_L} = \frac{f_k - f_v}{(1 - f_v)(1 - Pr)} \Delta T_{L \text{iso}}.$$  

Then, the isobaric gas law states that:

$$\frac{\Delta V_L}{V_L} = \frac{\Delta T_L}{T_L}.$$  

Then, to first order, $T_L = T_m$ when the Lagrangian slice of gas is located at $x_m$.

Similarly, to first-order, the mean temperature $T_L$ of the Lagrangian segment of gas is equal to the Eulerian mean temperature $T_m$ of the gas in the control volume at $x = x_m$. Substituting these results into equation (5.15) and manipulating gives:

$$\frac{\Delta T_L}{\Delta T_{L \text{iso}}} = \frac{f_k - f_v}{(1 - f_v)(1 - Pr)} = E_T,$$

where we have identified $\Delta T_L / \Delta T_{L \text{iso}}$ as a heat transfer effectiveness $E_T$. 
The system described in this derivation is very much like that studied by Wakeland and Keolian.\textsuperscript{43} In their study, two identical parallel plate heat exchangers—one hot and one cold—were placed adjacent to each other in an oscillating air flow. In our case, the portion of the tube to the right of \(x_m\) can be considered to be one exchanger while the portion to the left can be considered to be another. The effectiveness \(E_t\) in equation (5.17) then, is the total effectiveness of the two-exchanger system rather than the effectiveness \(E\) of the individual exchangers.

Wakeland and Keolian showed that in a system such as this the total effectiveness of the dual exchanger system is related to the effectiveness of the individual exchangers \(E\) by:

\[
E_t = \frac{E}{2 - E}. \tag{5.18}
\]

Solving for the magnitude of the tube effectiveness \(E\) using equations (5.17) and (5.18) gives:

\[
E = \frac{2}{\left(1 - f_c\right)(1 - \text{Pr})} + 1. \tag{5.19}
\]

Figure 5.1 is a graph of the output of this equation as a function of dimensionless frequency \((\nu/\delta_e)\) and the Prandtl number of air at room temperature (0.707). As the frequency decreases, the output of equation (5.19) approaches a value of one. Likewise, as the frequency increases the output approaches a value of zero.

By solving equation (5.19) numerically, the relative importance of the Prandtl number can be observed. This was accomplished by solving equation (5.19) using the Prandtl numbers of
Figure 5.1: Output of equation 5.20 with $Pr = 0.707$.

Figure 5.2: Ratio of the output of equation 5.20 as calculated for air and helium. Notice that the variation is within 1%.
air (0.707) and helium (0.680) while holding the ratio $r_h/\delta_k$ the same for both. The thermoviscous functions $f_\kappa$ and $f_\nu$ are functions of the ratios $2r_h/\delta_k$ and $2r_h/\delta_\nu$, respectively; and the Prandtl number of a gas is given by the ratio:

\[
Pr = \frac{\delta_\nu}{\sqrt{\delta_k}}. \tag{5.20}
\]

Figure 5.2 is a graph of the ratio of the calculated effectiveness of the helium-filled tube to that of the air-filled tube. Notice that although the Prandtl number of helium is about 4.0% less than that of air, figure 5.2 shows that the effectiveness of the helium-filled tube is about 1% greater (at most) than that of the air-filled tube. This ratio may be used as an approximate Prandtl number correction factor to convert the effectiveness measurements made in this thesis in air for use in helium. It should be reliable for laminar flow with small $x/L$—conditions where the Rott theory is expected to hold—and may not be too far off for higher Reynolds number flows.

### 5.4.4 Aspect Ratio

Perfect similarity between two exchangers and gases would require that $r_h/\delta_k$, $Re$, $x/L$, $Pr$, and $r/L$ would all be the same. However, if the Prandtl numbers of the two gases are different, we can compensate with an inequality in any one of these parameters. To illustrate this, the following is a derivation that shows the change required in $r/L$ if the Prandtl number changes while the other parameters are held constant.
Suppose that we have two different shell-and-tube exchangers. The inner radius of the tubes in the first is given by \( r_1 \) while that of the second is given by \( r_2 \). The lengths of the exchangers are given by \( L_1 \) and \( L_2 \), respectively. The ratios of the radii of curvature of the ends of the tubes to the inner radii of the tubes for both exchangers are equal as well.

First, we will require the ratios of the inner radii of the exchanger tubes to the thermal penetration depths in the gasses to be equal. This begins as:

\[
\frac{r_1}{2k_1} = \frac{r_2}{2k_2} \sqrt{\frac{\omega_1 \rho_1 c_{p1}}{\omega_2 \rho_2 c_{p2}}}.
\]  

With some manipulation, the following equality is achieved:

\[
\frac{\omega_1}{\omega_2} = \left( \frac{r_1}{r_2} \right)^2 \frac{\rho_2 Pr_2 \mu_1}{\rho_1 Pr_1 \mu_2}.
\]  

Next, we will require that the Reynolds numbers of both gasses be the same:

\[
\frac{2r_1 \rho_1 \omega_1 x_1}{\mu_1} = \frac{2r_2 \rho_2 \omega_2 x_2}{\mu_2}.
\]  

By substituting equation (5.22) into (5.23) and rearranging, we arrive at:

\[
\frac{x_1}{x_2} = \frac{r_1 Pr_1}{r_2 Pr_2}.
\]  

Remembering that the ratios of gas displacement to exchanger length must also be the same, equation (5.24) can then be re-written as:

\[
\frac{L_1}{r_1} \frac{r_1}{Pr_1} = \frac{L_2}{r_2} \frac{Pr_2}{Pr_2}.
\]
So, requiring that $r_h/\delta_k$, Re, and $x/L$ of the two systems be the same forces the aspect ratios of the two exchangers to be different if the Prandtl numbers of the gases in the two systems are different.
Chapter 6

Acoustic Impedance Data

In addition to heat transfer performance, the acoustic impedance of shell-and-tube heat exchangers is important to the designer. Acoustic losses rob thermoacoustic devices of efficiency and are directly related to the real part of the acoustic impedance.

The real part of the impedance is comprised of at least two parts: a linear resistance, and the nonlinear resistance caused by minor losses. Linear acoustic theory is well-established, and impedances can be easily calculated for any ideal gas and most geometries (a summary of this theory is found in Swift\textsuperscript{3}). The nonlinear resistance is described by the equation:

\begin{equation}
R_{ml} = \frac{\kappa \rho v}{2A},
\end{equation}

where $\rho$ is the density of the gas; $v$ is the cross-sectionally averaged particle velocity of the gas; $A$ is the total open cross-sectional area of the exchanger; and $\kappa$ is the minor loss coefficient.

The minor loss coefficient is a function only of the geometry of the transition in a sudden expansion or constriction and not of any of the properties of the gas. Because of this, minor loss coefficient values obtained through this study would apply directly to exchangers of similar geometries (but different sizes) and to different gases. Finding these coefficients is the primary objective of this chapter.

Wakeland and Keolian\textsuperscript{32} conducted a similar study of the minor losses of parallel-plate heat exchangers. In their study, minor loss coefficients were found by
subtracting the acoustic resistance predicted by the established linear theory from that measured in their experiment. The result was plotted as a function of volumetric velocity. The slope of this difference was then used to calculate the minor loss coefficient as follows:

$$\kappa = \frac{2A \cdot dR}{\rho \cdot dU}.$$  \hfill (6.2)

In this expression, $A$ is the open cross-sectional area of the exchanger; $\rho$ is the mean density of the gas; and $dR/dU$ is the slope of the plotted difference. This method will be used to calculate all minor loss coefficients in this study.

The imaginary part (the inertance) of the impedance of each exchanger will also be given. There are two reasons for this: first, in most cases comparison with linear theory will validate the data; and second, to illustrate the effects of heat transfer and nonlinear impedances on the inertance.

The imaginary part of the impedance will always be presented in both dimensional and dimensionless forms. The resistance will never be presented in dimensionless form, as the primary objective is the value of the minor loss coefficient.

In all cases, the ratio $x/L$ indicates the ratio of the peak gas displacement within the exchangers to the exchanger length. The ratio of hydraulic radius to viscous penetration depth (or $r_h/\delta_v$) has been used to indicate normalized angular frequency.
6.1 Validating the Impedance Measurements

To test the validity of the measurements, I first measured the impedance of the short exchanger \((L/D = 4.688)\) with flat tube-ends, no adjacent screens, and with the entire apparatus held at room temperature. In this way, any effects due to heat transfer were eliminated so that an accurate comparison to the impedances predicted by linear acoustic theory could be made. If accurate, the measured impedance should agree well with linear theory at the lowest gas displacements (where minor losses and any other nonlinear effects would be minimal).

Figure 6.1 is a graph of the imaginary part of the measured impedance of this exchanger as a function of normalized displacement. The impedance predicted by linear theory is plotted as solid black squares. Agreement between measurement and theory is excellent at low displacements. In fact, the measured impedance is within 5% of theory at low displacements.

Figure 6.2 is a graph of the phase of the measured impedance vs. normalized displacement. The phase predicted by linear theory is given as solid black squares. Agreement is again excellent, with a maximum difference of about 5° at the lowest frequency.
6.2 Exchangers with Flat Tube-Ends, No Screens, and Heat Transfer

6.2.1 Imaginary Impedance

Figure 6.3 through 6.5 show the imaginary part of the impedance measured with the short ($L/D = 4.688$), medium ($L/D = 7.813$), and long exchangers ($L/D = 15.63$), respectively. The ends of the exchanger tubes were flat, and the spacer plate was used instead of screens. In addition, these measurements were made simultaneously with heat transfer measurements. In every case, the impedance predicted by linear theory is indicated with solid black squares.

In all three cases, agreement with theory is excellent at higher frequencies. At lower frequencies, however, a new pattern emerges. At low displacements, the impedance is significantly lower than theory predicts. As displacement rises, however, the impedance rises to about that predicted. The only difference between the measurements shown in figure 6.3 and figure 6.1 is that heat transfer had been enabled in figure 6.3.

As displacement increases, the inertance decreases slightly. This has been observed in several previous studies as well\textsuperscript{8,10,11,13,14,16–19} and was explained as a reduction due to turbulence and vortex shedding of the mass end-correction (given by $8/3\pi D$ as shown on page 274 of Kinsler, Frey, Coppens, and Sanders\textsuperscript{56}).
Figure 6.1: Imaginary part of the impedance of the short \((L/D = 4.688)\) exchanger with flat tube-ends, no adjacent screens, and no heat transfer. Data is given in colored lines and dots, while values predicted by linear theory are given in black squares. Notice that the data agrees well with acoustic theory at the lowest displacements, where nonlinear effects are negligible.

Figure 6.2: Measured phase of the impedance of the short \((L/D = 4.688)\) exchanger with flat tube-ends, no adjacent screens, and no heat transfer. Data is given in colored lines and dots, while values predicted by linear theory are given in black squares. Notice that the largest difference between data and theory at low displacements (at \(r_i/\delta_k = 0.584\)) is about \(5^\circ\).
Figure 6.3: Imaginary part of the impedance of the short \((L/D = 4.688)\) exchanger with flat tube-ends, no adjacent screens, and with heat transfer. Notice the significant drop at low \(x/L\) and low \(r_h/\delta_c\).

Figure 6.4: Imaginary part of the impedance of the medium \((L/D = 7.813)\) exchanger with flat tube-ends, no adjacent screens, and with heat transfer. The same drop at low \(x/L\) and low \(r_h/\delta_c\) is present.
Figure 6.5: Imaginary part of the impedance of the long \((L/D = 15.63)\) exchanger with flat tube-ends, no adjacent screens, and with heat transfer. The trends shown in the other two exchangers \((L/D = 4.688 \text{ and } 7.813)\) are evident here.
6.2.2 Minor Losses

Figure 6.6 shows the measured resistance (as a function of normalized displacement) of the short \((L/D = 4.688)\) exchanger with flat tube-ends, no adjacent screens, and heat transfer enabled. Figure 6.7 shows the calculated difference between the data in figure 6.6 and the resistance predicted by linear theory as a function of volumetric velocity.

A least-squares fit of the result placed too much emphasis on the data points at very low displacements. Instead, the slope and intercept of a line that approximated the difference at all displacements was found by inspection. The solid black line in figure 6.6 is that result. This line is given by:

\[
\Delta R = (5.68 \times 10^5 \text{ kg/m}^7) U. \tag{6.3}
\]

Using equation (6.2), this correlation results in a minor loss coefficient \(\kappa\) of 0.693.

Figure 6.8 shows the measured resistance of the medium \((L/D = 7.813)\) exchanger (as a function of normalized displacement), while figure 6.9 shows the calculated difference between the data and linear theory (as a function of volumetric velocity). The result is approximated (using the method previously described) by:

\[
\Delta R = (5.90 \times 10^5 \text{ kg/m}^7) U. \tag{6.4}
\]

This corresponds to a minor loss coefficient \(\kappa\) of 0.720.
Figure 6.6: Real part of the impedance of the short \((L/D = 4.688)\) exchanger with flat tube-ends, no adjacent screens, and with heat transfer. Notice the strong dependence on \(x/L\), indicating that nonlinear resistances are dominating.

Figure 6.7: Correlation of the difference between the measured resistance of the short exchanger \((L/D = 4.688)\) with flat tube-ends, no screens, and heat transfer, and that predicted by linear theory.
Figure 6.10 is the measured resistance of the long \((L/D = 15.63)\) exchanger, while figure 6.11 is the calculated difference between the data and linear theory. This figure shows greater complexity than previous graphs, but can still be approximated by:

\[
\Delta R = (6.40 \times 10^5 \text{ kg/m}^2) \cdot U.
\] (6.5)

This gives a minor loss coefficient \(\kappa\) of 0.781.

Notice that as the exchanger length increases, the differences shown in figures 6.7, 6.9, and 6.11 become more and more complex. Clearly, more is going on than is indicated by the simple nonlinear resistance equation [equation (6.1)]. Similar results were observed by Ingard\(^6\) while measuring the impedances of orifices using an impedance tube technique, and were later attributed to the influence of the nearby walls of the impedance tube.\(^{16}\) Thurston et al.\(^{11}\) found that the minor loss coefficient \(\kappa\) is not a constant as is usually assumed. He also pointed out that no obvious correlation existed for the variation in \(\kappa\), and argued that the use of a constant value was of “considerable utility”. Wakeland and Keolian\(^{22}\) showed analytically that the value of \(\kappa\) depends on the flow profile as well, ranging from 0.6 to 1.1 for uniform and Poisseuille flow, respectively. Given that the Reynolds number in these measurements ranged from 32 to 22,000, it is reasonable to expect that some variation in the flow profile may have contributed to the fluctuating value of \(\kappa\) seen in these results as well.
Figure 6.8: Real part of the impedance of the medium ($L/D = 7.813$) exchanger with flat tube-ends, no adjacent screens, and with heat transfer. The cause of the peak at $x/L = 0.3$ and $r_{h}/\delta_{k} = 0.756$ is unknown.

Figure 6.9: Correlation of the difference between the measured resistance shown in the previous figure and that predicted by linear theory.
Figure 6.10: Real part of the impedance of the long \( (L/D = 15.63) \) exchanger with flat tube-ends, no adjacent screens, and with heat transfer.

Figure 6.11: Correlation of the difference between the measured resistance shown in the previous figure and that predicted by linear theory.
The minor loss coefficients of these three exchangers can be correlated as a function of exchanger length, as shown in figure 6.12. This results in the expression:

\[ \kappa = 8.03 \times 10^{-3} \left( \frac{L}{D} \right) + 0.656. \]  

(6.6)

Extrapolating back to an exchanger of zero length (or in other words, an orifice) gives a minor loss coefficient of 0.656.

It would be tempting to conclude that this correlation says that the minor loss coefficient of the nonlinear resistance of an orifice (a tube of zero length) is 0.656. However, this is not the case. It is important to recall how these measurements were taken. Two pressure sensors were used—one in the lower duct just before the model exchangers, and another in the upper duct just above the symmetric heated exchanger.

We are only interested in the impedance of the model exchanger—not the model exchanger and symmetric exchanger together. To account for this, the difference was taken between the two pressure signals and was then divided in half. The resulting pressure was then divided by the volumetric velocity through the exchanger to calculate the impedance. As a result, calculated minor losses were due only to one end of the model exchanger—the end away from the spacer plate.
Figure 6.12: Least-squares fit of the minor loss coefficients measured with the three different lengths of exchangers with flat tube-ends and no adjacent screens. Notice the excellent correlation is a function of $L/D$. 

Coefficient values ± one standard deviation

\[
\begin{align*}
a &= 0.656 \pm 0 \\
b &= 8.03 \times 10^{-3} \pm 5.83 \times 10^{-5}
\end{align*}
\]

$K = b(L/D) + a$
When extrapolating this data down to an exchanger of zero length, the result should be compared to one-half the nonlinear resistance of an orifice. Previous literature\textsuperscript{4-12} has shown that the nonlinear resistance of an orifice has a minor loss coefficient of 1. The result given by extrapolating equation (6.6) down to zero length was 0.656. This is about 31\% higher than 0.5 (half of the nonlinear resistance of an orifice).

6.3 Exchangers with Flat Tube-Ends and Adjacent Screens

6.3.1 Imaginary Impedances

Figures 6.13 through 6.15 are graphs of the imaginary part of the measured impedances of the short, medium, and long exchangers ($L/D = 4.688, 7.813,\text{ and } 15.63$) with flat tube-ends, but with an adjacent stack of screens. Two details stand out here: first, agreement with linear theory is significantly improved in the lower frequencies over that observed with the same exchangers but without adjacent screens; and second, there is less of an apparent drop at higher displacements.

6.3.2 Minor Losses

Figure 6.16 shows the real part of the measured impedance (the resistance) of the short exchanger ($L/D = 4.688$) with flat tube-ends and adjacent screens. Figure 6.17
Figure 6.13: Imaginary part of the impedance of the short \((L/D = 4.688)\) exchanger with flat tube-ends and adjacent screens. Notice that the drop at low \(x/L\) and low \(r_h/\delta_k\) is not as apparent with the adjacent screens. The cause of the peak at \(x/L = 0.2\) and \(r_h/\delta_k = 1.66\) is unknown.

Figure 6.14: Imaginary part of the impedance of the medium \((L/D = 7.813)\) exchanger with flat tube-ends and adjacent screens.
Figure 6.15: Imaginary part of the impedance of the long \((L/D = 15.63)\) exchanger with flat tube-ends and adjacent screens.
Figure 6.16: Real part of the impedance of the short $(L/D = 4.688)$ exchanger with flat tube-ends and adjacent screens. The cause of the two peaks at $x/L = 0.2$ and $0.6$ is unknown.

Figure 6.17: Correlation of the difference between the measured resistance and that predicted by linear theory.
Figure 6.18: Real part of the impedance of the medium ($L/D = 7.813$) exchanger with flat tube-ends and adjacent screens. The cause of the peaks at $x/L = 0.2$ is unknown.

Figure 6.19: Correlation of the difference between the measured resistance and that predicted by linear theory.
shows the difference between this measured resistance and that predicted by linear theory. As before, a correlation of the result is given by:

$$\Delta R = (6.40 \times 10^5 \text{ kg/m}^2) U - 150 \text{ kg/m}^2.$$

(6.7)

Using this result with equation (6.2) gives a minor loss coefficient of 0.769.

Figure 6.18 shows the measured resistance of the medium exchanger ($L/D = 7.813$) with flat tube-ends and adjacent screens. Figure 6.19 shows the difference between this data and the values predicted by linear theory. The result is the correlation:

$$\Delta R = (6.70 \times 10^5 \text{ kg/m}^2) U - 200 \text{ kg/m}^2.$$

(6.8)

The resulting minor loss coefficient has a value of 0.817.

Figure 6.20 shows the resistance measured with the long exchanger ($L/D = 15.63$) with flat tube-ends and adjacent screens. Figure 6.21 is the calculated difference between this resistance and that predicted by linear theory. The resulting correlation is given by:

$$\Delta R = (7.80 \times 10^5 \text{ kg/m}^2) U - 300 \text{ kg/m}^2.$$

(6.9)

The corresponding minor loss coefficient has a value of 0.951.

The minor loss coefficients calculated in this section are correlated as before in figure 6.22. The resulting expression is:

$$\kappa = 1.67 \times 10^{-2} \left( \frac{L}{D} \right) + 0.689.$$

(6.10)

Extrapolating back to an exchanger of zero length results in a value for $\kappa$ of 0.689.

Notice that this value is not very different from the one obtained from equation (6.6). That equation is the correlation obtained from the minor loss coefficients found with the same exchangers but without adjacent screens. The value in that case
Figure 6.20: Real part of the impedance of the long \((L/D = 15.63)\) exchanger with flat tube-ends and adjacent screens.

Figure 6.21: Correlation of the difference between the measured resistance and that predicted by linear theory.
Figure 6.22: Least-squares fit of the minor loss coefficients measured with the three different lengths of exchangers with flat tube-ends and adjacent screens.

Coefficient values ± one standard deviation
\[a = 0.689 \pm 0\]
\[b = -1.67 \times 10^{-2} \pm 1.24 \times 10^{-4}\]

\[K = b(L/D) + a\]
was 0.656—only 4.8% less than 0.689.

Figure 6.23 illustrates why this result is important. In the case of the exchangers with flat tube-ends and no adjacent screens, only one end of the exchanger tubes created nonlinearities in the measurement. In the case of the exchangers with flat tube-ends and adjacent screens, effects caused by both ends of the exchanger tubes were measured. The fact that the intercepts of equations (6.6) and (6.10) are within 4.8% of each other seems to indicate that the adjacent screens act to greatly reduce the minor losses at that end of the exchanger.

6.4 Exchangers with Rounded Tube-Ends and Adjacent Screens

6.4.1 Imaginary Impedance

Figures 6.24 through 6.26 show the imaginary part of the measured impedance of the three lengths of exchangers with rounded tube-ends and adjacent screens. The results are very similar to those of the exchangers with flat tube-ends and adjacent screens. No obvious differences are apparent.

6.4.2 Minor Losses

Figure 6.27 shows the acoustic resistance measured with the short exchanger ($L/D = 4.688$) with rounded tube-ends and adjacent screens, while figure 6.28 shows the
Figure 6.23: The differential pressure measurement of the exchangers with flat tube-ends and without adjacent screens included nonlinearities caused by only one end of the exchanger tubes. The differential pressure measurement of the same exchangers with adjacent screens, however, included nonlinearities caused by both ends of the exchanger tubes.
Figure 6.24: Imaginary part of the impedance of the short \((L/D = 4.688)\) exchanger with rounded tube-ends and adjacent screens.

Figure 6.25: Imaginary part of the impedance of the medium \((L/D = 7.813)\) exchanger with rounded tube-ends and adjacent screens. The cause of the small peak at \(x/L = 0.4\) is unknown.
Figure 6.26: Imaginary part of the impedance of the long ($L/D = 15.63$) exchanger with rounded tube-ends and adjacent screens.
Figure 6.27: Real part of the impedance of the short \((L/D = 4.688)\) exchanger with rounded tube-ends and adjacent screens.

Figure 6.28: Correlation of the difference between the measured resistance and that predicted by linear theory.
Figure 6.29: Real part of the impedance of the medium \((L/D = 7.813)\) exchanger with rounded tube-ends and adjacent screens. The cause of the peak at \(x/L = 0.4\) is unknown.

Figure 6.30: Correlation of the difference between the measured resistance and that predicted by linear theory.
calculated difference between those measurements and linear theory. The resulting correlation is given by:

$$\Delta R = (4.50 \times 10^5 \text{ kg/m}^7) U - 150 \text{ kg/m}^7. \quad (6.11)$$

The corresponding value of the minor loss coefficient is 0.549.

Figure 6.29 shows the resistance measured with the medium exchanger ($L/D = 7.813$). The difference between these results and linear theory is shown in figure 6.30. The resulting correlation is given by:

$$\Delta R = (5.10 \times 10^5 \text{ kg/m}^7) U - 100 \text{ kg/m}^7. \quad (6.12)$$

The corresponding minor loss coefficient has a value of 0.622.

The measured resistance of the long exchanger ($L/D = 15.63$) is shown in figure 6.31. The difference between this data and linear theory is shown in figure 6.32. The resulting correlation is given by:

$$\Delta R = (6.10 \times 10^5 \text{ kg/m}^7) U - 300 \text{ kg/m}^7. \quad (6.13)$$

The value of the minor loss coefficient is 0.744.

As before, a least-squares fit of the minor loss coefficients of these exchangers is shown in figure 6.33. The resulting expression is:

$$\kappa = 1.74 \times 10^{-2} \left( \frac{L}{D} \right) + 0.475. \quad (6.14)$$

This result is 31% lower than that given by the exchangers with flat tube-ends and adjacent screens, indicating that the rounded ends cause less of a nonlinearity for a given gas velocity than the flat ends do.
Figure 6.31: Real part of the impedance of the long \((L/D = 15.63)\) exchanger with rounded tube-ends and adjacent screens.

Figure 6.32: Correlation of the difference between the measured resistance and that predicted by linear theory.
Figure 6.33: Least-squares fit of the minor loss coefficients measured with the three different lengths of exchangers with rounded tube-ends and adjacent screens.
6.5 Summary

The inertance data taken with the short exchanger ($L/D = 4.688$) with flat tube-ends, no adjacent screens, and no heat transfer shows excellent agreement with linear theory. In addition, the imaginary part of the impedances measured with all of the exchangers (with only the exception of the data at the lower frequencies taken with the exchangers with flat tube-ends and adjacent screens) agree very well with linear theory at low displacements. This gives confidence that the estimate of the volumetric air velocity through the exchangers is accurate (this gives confidence to the heat transfer measurements presented in the next chapter as well, as they were conducted simultaneously with the impedance measurements).

In all cases, the minor loss coefficients were found to be functions of exchanger length as well as the shape of the ends of the tubes. This was noticed by Petculescu and Wilen\textsuperscript{23} as well, and indicates that there are more sources of nonlinearity than just the ends of the tubes in these measurements. Correlations were derived, however, that allowed extrapolation of the minor loss coefficients to exchangers of zero length. Ideally, this would isolate minor losses from other length-dependent effects.

The value of the zero-length minor loss coefficient for the flat tube-ends without adjacent screens was 0.656. By contrast, the value of the zero-length coefficient for the flat tube-ends with adjacent screens was 0.689. This is only a 4.8\% increase, even though effects due to both ends of the exchanger tubes were part of the measurements with adjacent screens. From this, we can conclude that the adjacent screens act to greatly reduce the minor loss effects at that end of the exchanger.
In comparison, the value of the zero-length minor loss coefficient for the rounded tube-ends with adjacent screens was 0.475. This result is 31% less than that obtained with the flat tube-ends and adjacent screens. This indicates that minor losses can be reduced by rounding the ends of the exchanger tubes.

The presence of adjacent screens causes some additional effects as well. Consider the slopes of the minor loss correlations just discussed [as seen in equations (6.6), (6.10), and (6.14)]. In the case of the exchangers with flat tube-ends and no adjacent screens, the slope is $8.03 \times 10^{-3}$ (unitless). In the case of the same exchangers but with adjacent screens the slope becomes $1.67 \times 10^{-2}$ (unitless). This is about a factor of two greater than the slope previously mentioned. For the exchangers with rounded tube-ends and adjacent screens the slope is $1.74 \times 10^{-2}$ (unitless).

Apparently, the adjacent screens cause a dramatic increase in the length-dependent nonlinearities observed in this data. It may be then that the source of the additional nonlinearity is turbulence generated inside the tubes, either by the increased length of the tube walls or by the screens themselves.

Comparing the imaginary part of the impedances of the exchangers with screens to those without screens, it appears that the screens act to produce two additional effects. The first is that the discrepancy with linear theory at low frequencies is reduced. The second is that the drop in the impedance at higher displacements is also reduced, so that the impedance at each frequency is more constant.
Chapter 7

Heat Transfer Data

This chapter discusses the heat transfer effectiveness of several different model shell-and-tube heat exchangers. The effectiveness of an exchanger is defined as the ratio of the actual amount of heat transfer generated by the exchanger to that that would have occurred had the fluid been in perfect thermal contact with the exchanger, and is given generally by:

$$ E = \frac{\dot{Q}}{m c_p (T_{\text{fluid in}} - T_{\text{HX surface}})}. $$

(7.1)

In this expression, $m$ is the mass flow-rate of the fluid; $c_p$ is the constant-pressure specific heat of the fluid; $T_{\text{fluid in}}$ is the temperature of the fluid as it enters the exchanger; and $T_{\text{HX surface}}$ is the surface temperature of the exchanger. For a perfect exchanger, this ratio has a value of one. For a completely imperfect exchanger, the value of this ratio is zero.

In this chapter, a comparison is made between the results obtained by Wakeland and Keolian and the heat transfer effectiveness measured with the three lengths of exchangers with flat tube-ends and no adjacent screens—the measurements most closely resembling those of Wakeland and Keolian. Then, measurements taken with adjacent screens and then rounded tube-ends will be discussed.

In all cases, normalized displacement and frequency are calculated using Wakeland and Keolian’s convention. That is: normalized displacement is given by the ratio of peak gas displacement within the exchanger tubes $x$ to the exchanger length $L$. 
The length was measured from the face of one end-plate to the face of the other. No correction was made when rounded tube-ends were used, as the radius of curvature of the ends was very small compared to the overall length.

7.1 Exchangers with Flat Tube-Ends and No Adjacent Screens

7.1.1 Wakeland and Keolian’s Experiment

Wakeland and Keolian’s study was an experimental investigation of the heat transfer performance of parallel-plate heat exchangers in oscillating flows, and resulted in correlations for the effectiveness of the exchangers based on the frequency of oscillation and gas displacement.

In their experiment, two identical parallel-plate exchangers (see figure 7.1) were placed adjacent to each other, with a gap between. The exchangers were laid horizontally within the apparatus, and positioned relative to each other so that the exchanger plates were aligned vertically (see figure 7.2). The edges of the plates were rounded.

Water was pumped through the manifolds and plates of each exchanger. The water pumped through the upper exchanger was hot, while that pumped through the lower exchanger was cold. As the air within the experiment oscillated between and through the two exchangers, a non-zero time-averaged transfer of heat occurred. This heat transfer was calculated by measuring the mass flow-rates of the water through the exchangers and the temperature differences between water inlets and outlets.
Figure 7.1: One of the parallel-plate heat exchangers used by Wakeland and Keolian.

Figure 7.2: Cross-sectional diagram showing the positioning of the parallel-plate exchangers in Wakeland and Keolian’s experiment. Identical exchangers were placed one above the other with a gap in between with the plates aligned vertically.
7.1.2 Wakeland and Keolian’s Air Mass Flow-Rate

A gap existed between the hot and cold exchangers in Wakeland and Keolian’s study. Because of this, the mass flow-rate of the air through the exchangers was calculated as follows (in my notation, rather than Wakeland and Keolian’s):

\[
m^*_\text{air} = \rho A f \frac{2(x - x_{\text{gap}})}{\sigma}.
\]  

(7.2)

In this expression, \( \rho \) is the density of the air at the time-averaged air temperature; \( A \) is the total cross-sectional area of the exchangers; \( f \) is the operating frequency in Hz; \( x \) is the displacement of the air outside of the exchangers; \( x_{\text{gap}} \) is one-half of the gap between the hot and cold exchangers; and \( \sigma \) is the porosity—the ratio of the open cross-sectional area to the total cross-sectional area—of the exchangers. Dividing \( x - x_{\text{gap}} \) by \( \sigma \) converts the gas displacement outside the exchangers to that within the exchangers. This is the convention used elsewhere in this dissertation.

Notice that gas displacements that were smaller than \( x_{\text{gap}} \) caused an oscillation between the exchangers; but air that began at the end of one exchanger did not pass into the other exchanger. Thus, no heat transfer between exchangers would occur at these displacements (ideally). Because of this, \( x_{\text{gap}} \) was subtracted from \( x \) to calculate the amount of mass per acoustic cycle that actually moved from one exchanger into the other.

In comparison, when the experiment of the present study was set up with model exchangers with flat tube-ends and no adjacent screens, a separator plate was used to keep the hot symmetric exchanger from conducting heat directly into the model exchanger under test. This plate had holes through it that were carefully machined to
match the placement and diameter of the four tubes through the model exchanger and the symmetric heated exchanger above. The segment of the plate between the two exchangers was 7.6 mm thick.

The temperature of the spacer plate was anchored to that of the cool model exchanger on one side and to that of the heated exchanger on the other, creating a temperature gradient through the thickness of the plate. Because of this, it is not clear if the thickness of the spacer plate should be subtracted from the gas displacement [as in $x - x_{\text{gap}}$ in equation (7.2)] in Wakeland and Keolian’s study.

To investigate whether or not this spacing should be taken into account, I used equation (7.2) and calculated the effectiveness of all three lengths of model exchangers with $x_{\text{gap}}$ set to zero and to 3.8 mm.

Figures 7.3 through 7.5 are graphs of the effectiveness of the short, medium, and long exchangers ($L/D = 4.688, 7.813, \text{ and } 15.63$) with $x_{\text{gap}} = 0$. The feature to notice is the slope of the lines at $x/L < 0.5$.

Wakeland and Keolian argued that the effectiveness at any given frequency in this region should be constant, rather than changing as is seen here. Under strictly laminar conditions and for any $x/L < 1$ (and neglecting any effects caused by the ends of the exchanger), the mass of gas oscillating into the exchanger would spend exactly the same amount of time in thermal contact with the exchanger (this is because the period of the acoustic oscillation is independent of displacement amplitude), regardless of displacement. Given this, the heat transfer per unit mass would be the same. This is also the equivalent of saying that the spatially and temporally averaged change in gas
Figure 7.3: Effectiveness of the short exchanger ($L/D = 4.688$) with flat tube-ends and no adjacent screens. The value of $x_{gap}$ was zero. Notice the slight slope in the data at $x/L < 0.5$, and also the slight increase in effectiveness at $1 < x/L < 2$ and $r_h/\delta_k = 4.43$.

Figure 7.4: Effectiveness of the medium exchanger ($L/D = 7.813$) with flat tube-ends and no adjacent screens. The value of $x_{gap}$ was zero. Notice the slope in the data at $x/L < 0.5$, and also the increase in effectiveness at $1 < x/L < 2$ and $r_h/\delta_k = 4.43$ and 3.39.
Figure 7.5: Effectiveness of the long exchanger \( (L/D = 15.63) \) with flat tube-ends and no adjacent screens. The value of \( x_{\text{gap}} \) was zero. Notice again the slight slope in the data at \( x/L < 0.5 \). The increase in effectiveness at \( 1 < x/L < 2 \) is much more apparent as well.

Figure 7.6: Effectiveness of the short exchanger \( (L/D = 4.688) \) with flat tube-ends and no adjacent screens. The value of \( x_{\text{gap}} \) in this case was 3.8 mm. In general, this value of \( x_{\text{gap}} \) appears to flatten the slope in the data at \( x/L < 0.5 \).
Figure 7.7: Effectiveness of the medium exchanger \((L/D = 7.813)\) with flat tube-ends and no adjacent screens. The value of \(x_{\text{gap}}\) was 3.8 mm. The slope in the data at \(x/L < 0.5\) is more even than that in figure 7.4, and the increase in effectiveness at \(x/L = 1\) is more apparent.

Figure 7.8: Effectiveness of the long exchanger \((L/D = 15.63)\) with flat tube-ends and no adjacent screens. The value of \(x_{\text{gap}}\) was 3.8 mm. There is still some slope in the data at \(x/L < 0.5\), and the increase in effectiveness at \(x/L = 1\) is very apparent.
temperature (while the gas is in thermal contact with the exchanger) is the same. This, in turn, implies that the effectiveness of the exchanger is independent of displacement amplitude for $x/L < 1$.

Given this argument, it would seem plausible that the slight slope in the effectiveness at $x/L < 0.5$ in figures 7.3 through 7.5 might be related to the value of $x_{\text{gap}}$ in equation (7.2). To test this, I calculated the effectiveness of the three exchangers again, but with a value for $x_{\text{gap}}$ of 3.8 mm (one-half of the thickness of the spacer plate between the exchangers).

Figures 7.6 through 7.8 are the effectiveness of the same three exchangers, but this time with $x_{\text{gap}}$ equal to 3.8 mm. It can be seen that this adjustment does cause the effectiveness curves below $x/L = 0.5$ to be more-or-less flat, compared to figures 7.3 through 7.5. However, a slight slope within this region persists as $L/D$ increases.

From these results it is not clear if the sloping effectiveness at lower displacements is due to an adjustment in $x_{\text{gap}}$ or if it is actual physics. For the rest of the analysis in section 7.1, however, I will use $x_{\text{gap}}$ with a value of 3.8 mm in all calculations of effectiveness to be consistent with Wakeland and Keolian’s analysis.

### 7.1.3 Effectiveness at $r_h/\delta_k > 1$ and $(x-x_{\text{gap}})/L < 0.5$

Wakeland and Keolian found that at $r_h/\delta_k > 1$ and $(x-x_{\text{gap}})/L < 1$ the effectiveness of the parallel-plate exchangers used in their study was given by:
To compare this correlation to the performance of the three model exchangers with flat tube-ends and no adjacent screens, I took the effectiveness of each at \((x-x_{\text{gap}})/L = 0.5\) (this value was chosen to avoid features in the effectiveness that will be discussed later) and \(r_h/\delta_k > 1\). I then generated least-squares fits of these points as a function of \(r_h/\delta_k\) assuming the form

\[ E = C \left( \frac{r_h}{\delta_k} \right)^M, \tag{7.4} \]

where \(C\) and \(M\) are constants.

The resulting correlation for the short exchanger \((L/D = 4.688)\) was:

\[ E_{L/D=4.688} = 0.886(r_h/\delta_k)^{-0.743}. \tag{7.5} \]

The correlation for the medium exchanger \((L/D = 7.813)\) was:

\[ E_{L/D=7.813} = 0.855(r_h/\delta_k)^{-0.737}. \tag{7.6} \]

The correlation for the long exchanger \((L/D = 15.63)\) was:

\[ E_{L/D=15.63} = 0.939(r_h/\delta_k)^{-0.807}. \tag{7.7} \]

The corresponding values of the correlation coefficients were 0.997, 0.999, and 0.939, respectively.

None of these correlations match equation (7.3), found by Wakeland and Keolian.

Notice, however, that the values of the coefficients and powers depend on exchanger length.
To make direct comparison possible between the correlations obtained here and that of Wakeland and Keolian, the aspect ratio \( L/D \) is converted instead into the ratio of exchanger length to hydraulic radius \( L/r_h \). Figure 7.9 shows a linear least-squares fit of the coefficients in these three equations as a function of this ratio, while figure 7.10 shows a similar least-squares fit of the powers. The correlation for the coefficients was:

\[
C = 1.51 \times 10^{-3} (L/r_h) + 0.837, \tag{7.8}
\]

and that for the powers was:

\[
M = -1.61 \times 10^{-3} (L/r_h) - 0.702. \tag{7.9}
\]

The ratio \( L/r_h \) of the parallel-plate exchangers in Wakeland and Keolian’s study was 6.93. Inserting this value into equations (7.8) and (7.9) yields the following equation:

\[
E = \frac{0.847}{r_h/\delta_c^{0.713}}. \tag{7.10}
\]

The coefficient 0.847 is 21% greater than that in Wakeland and Keolian’s correlation, while the power of 0.713 is 28.7% less. It is important to remember, however, that the exchangers in Wakeland and Keolian’s study were parallel-plate exchangers, and that the ends of the plates were rounded. By contrast, the exchangers used in this comparison were shell-and-tube exchangers with flat tube-ends. While some difference is to be expected, equation (7.10) is not substantially different from equation (7.3), lending some plausibility to the results.

I conducted this analysis a second time following exactly the same procedure, but with \( x_{\text{gap}} \) set to a value of zero. The resulting correlation was:
Figure 7.9: Least-squares fit of the coefficients from equations (7.5) through (7.7).

Figure 7.10: Least-squares fit of the powers from equations (7.5) through (7.7).
\[
E = \frac{0.828}{\left( \frac{r_h}{\delta_k} \right)^{0.757}}.
\]

(7.11)

The coefficient is 2.2% less than that in equation (7.10), while the power is 6.2% greater. Again, from these results it is not clear whether any particular value for \(x_{\text{gap}}\) should be used.

7.1.4 Effectiveness Increase Near \((x-x_{\text{gap}})/L = 1\)

In figures 7.3 through 7.8, sudden increases occur in the effectiveness near \((x-x_{\text{gap}})/L = 1\) at higher frequencies (this is especially evident in figures 7.5 and 7.8). This feature might be correlated with the critical Reynolds number.

Iguchi and Ohmi\(^{26}\) found that the Reynolds number corresponding to the onset of fully-turbulent bursts in oscillating flows within tubes (called the critical Reynolds number) is given by:

\[
\text{Re}_c = 800 \sqrt{\frac{r^2 \omega}{\nu}}
\]

(7.12)

In this expression, \(r\) is the radius of the tube; \(\omega\) is the angular frequency of oscillation; and \(\nu\) is the kinematic viscosity of the gas.

The sudden increase in effectiveness in figure 7.8 at \((x-x_{\text{gap}})/L = 1\) and \(r_h/\delta_k = 4.43\) occurs at a Reynolds number (\(\text{Re} = D \rho \nu \mu\)) of about 14,800. For this frequency, the value of the critical Reynolds number is 8,820. The peak at the same displacement but at \(r_h/\delta_k = 3.39\) occurs at \(\text{Re} = 8,560\). For this frequency, \(\text{Re}_c = 9,140\).
At the next lowest frequency \((r_0/\delta_k = 2.64)\) a similar increase in effectiveness occurs near \((x-x_{\text{gap}})/L = 1\); but in this case it is a smaller effect than at the higher frequencies. Re at this point \([(x-x_{\text{gap}})/L = 1]\) is 4810, while \(Re_c\) at this frequency is 7170.

Notice the pattern: in figures 7.3 through 7.8, as the frequency decreases the Reynolds number corresponding to the sudden effectiveness increase at \((x-x_{\text{gap}})/L = 1\) gets further away from the critical Reynolds number of that frequency. At the same time, the magnitude of this effect decreases. At the lowest frequencies, there is no apparent effect at all.

It appears that the Reynolds numbers that correspond to the onset of these sudden effectiveness increases are correlated to the critical Reynolds numbers of those frequencies. This would suggest then that bursts of turbulent flows increase heat transfer within the exchanger.

By contrast, the Reynolds numbers corresponding to the data that Wakeland and Keolian used to generate equation (7.3) range from 38.8 to 383—values that would indicate basically laminar flows. It is likely that this is why similar effects never occur in their data.

It is possible that the ends of the tubes could cause some of this effect as well. At \((x-x_{\text{gap}})/L > 1\) the gas passes through and beyond the exchanger. If this was the only or major cause of this effect though, I would expect to see the same increase in effectiveness regardless of frequency. This does not happen, however.
7.1.5 Effectiveness at \((x-x_{\text{gap}})/L > 1\)

As the gas displacement rises above \((x - x_{\text{gap}})/L = 1\) the effectiveness falls. This is because the gas spends less time in thermal contact with the exchanger during an acoustic cycle. As a result, it does not experience as great a change in temperature, resulting in decreased effectiveness.

Wakeland and Keolian found a correlation that fit their data at \((x-x_{\text{gap}})/L > 1\). The turbulence effects that are evident in the data of this study, however, do not fit their correlation. At this point, there is insufficient data to find a good correlation that describes the behavior of the effectiveness above \((x-x_{\text{gap}})/L = 1\) for these three exchangers.

7.1.6 Effectiveness and Nusselt Number Correlations for \((x-x_{\text{gap}})/L < 1\) and \(r_h/\delta > 1\)

Figures 7.11 and 7.12 show correlations for the exchangers with \(x_{\text{gap}} = 3.8\) mm and \(x_{\text{gap}} = 0\), respectively, for \((x-x_{\text{gap}})/L < 0.5\) and \(r_h/\delta > 1\). These correlations were obtained by inspection, rather than by analytic means. The resulting correlation for the effectiveness of all three lengths of exchangers with \(x_{\text{gap}}\) of 3.8 mm is:

\[
E = 0.852 \left( \frac{r_h}{\delta} \right)^{-0.750} \left( \frac{x - x_{\text{gap}}}{L} \right)^{-2.00 \times 10^{-3}}.
\]  

(7.13)

The correlation with \(x_{\text{gap}} = 0\) is:

\[
E = 0.921 \left( \frac{r_h}{\delta} \right)^{-0.750} \left( \frac{x}{L} \right)^{0.104}.
\]  

(7.14)
Figure 7.11: Least-squares fit (for \((x-x_{\text{gap}})/L < 0.5\)) of the effectiveness of the short, medium, and long exchangers \( (L/D = 4.688, 7.813, \text{ and } 15.63) \) with flat tube-ends, no adjacent screens, and \( x_{\text{gap}} = 3.8 \) mm. Notice that this value of \( x_{\text{gap}} \) causes the correlation to be nearly independent of \( x/L \).

\[
E = 0.852 \left( \frac{r_n}{\delta_x} \right)^{0.75} \left( \frac{(x-x_{\text{gap}})/L}{2.01 \times 10^{-3}} \right)
\]

Figure 7.12: Least-squares fit (for \( x/L < 0.5 \)) of the effectiveness of the short, medium, and long exchangers \( (L/D = 4.688, 7.813, \text{ and } 15.63) \) with flat tube-ends, no adjacent screens, and \( x_{\text{gap}} = 0 \).

\[
E = 0.921 \left( \frac{r_n}{\delta_x} \right)^{0.750} (x/L)^{0.104}
\]
Figure 7.13: Normalized Nusselt numbers for the short, medium, and long exchangers \((L/D = 4.688, 7.813, \text{ and } 15.63)\) with flat tube-ends, no adjacent screens, and \(x_{\text{gap}} = 3.8 \text{ mm}\). The correlation is valid only for \(x/L < 0.5\).

\[
Nu = 5.0 \left(\frac{L}{D}\right) \left(\frac{r_h}{\delta_k}\right)^{0.75} \left(\frac{x-x_{\text{gap}}}{L}\right)^{1.05}
\]

Figure 7.14: Normalized Nusselt numbers for the short, medium, and long exchangers \((L/D = 4.688, 7.813, \text{ and } 15.63)\) with flat tube-ends, no adjacent screens, and \(x_{\text{gap}} = 0\). Again, the correlation is valid only for \(x/L < 0.5\).

\[
Nu = \left(\frac{r_h}{\delta_k}\right)^{0.60} \left(\frac{L}{D}\right)^{1.15} 10^{4.30 \ln(4+\log(x/L)) - 6.25}
\]
Notice that these correlations are functions only of \( \frac{r_h}{\delta_k} \) and \( x/L \), and not of \( L/D \). This would seem to contradict the findings presented earlier, that showed that equations (7.8) and (7.9) were functions of \( L/D \).

In reality, both correlations are approximately correct. In this case the variation due to exchanger length is small, and is not adequately discernable by inspection (in data to be presented later, however, this will not be the case).

The Nusselt numbers for these exchangers can be correlated in a similar manner. Figure 7.13 is a graph of the correlation for the exchangers with \( x_{\text{gap}} = 3.8 \text{ mm} \), while figure 7.14 is graph of the correlation produced when \( x_{\text{gap}} = 0 \). The resulting correlations are given by:

\[
\text{Nu} = 5.50 \left( \frac{r_h}{\delta_k} \right)^{0.75} \left( \frac{L}{D} \right) \left( \frac{x - x_{\text{gap}}}{L} \right)^{1.05},
\]

and

\[
\text{Nu} = \left( \frac{r_h}{\delta_k} \right)^{0.60} \left( \frac{L}{D} \right)^{1.15} 10^{4.30 \ln \left( \frac{x}{L} \right) - 4.25},
\]

respectively. Notice that \( \ln \) indicates the base-e logarithm, while \( \log \) indicates the base-ten logarithm.
7.2 Exchangers with Flat Tube-Ends and Adjacent Screens

This section discusses the effectiveness of the same three exchangers, but with the addition of an adjacent stack of screens. By comparing the effectiveness measured in these conditions with that of the previous section, the effects of the screens can be directly ascertained.

Correlations in this section (and for the remainder of this chapter) are given in the general form:

\[ E = B \left( \frac{L}{D} \right)^G \left( \frac{X}{L} \right)^H. \]  

(7.17)

The values of B, F, G, and H were found largely by inspection. This method yielded three digits of precision. Note that equation (7.17) is a more general form of equation (7.4).

7.2.1 Air Mass Flow-Rate

In the previous section a spacer plate was placed between the model exchangers and the symmetric heated exchangers above. This created a well-defined region that was much like the gap between the parallel-plate exchangers in Wakeland and Keolian’s study. In this case, however, that plate was replaced by the screen assembly described in Chapter Four. Under these conditions, it is not clear at all whether the \( x_{gap} \) term should be used in calculating the mass flow-rate of the air through the exchangers.

Figure 7.15 is a graph of the effectiveness of the short exchanger \((L/D = 4.688)\) with a value for \( x_{gap} \) of zero. Figure 7.16 is a graph of the effectiveness as calculated
with a value for $x_{\text{gap}}$ of 3.8 mm. It is immediately apparent that the results are not nearly as good as in figure 7.6. Using $x_{\text{gap}}$ has caused the effectiveness at $r_h/\delta_k = 4.43$ to flatten out somewhat at lower $(x-x_{\text{gap}})/L$, but not at all frequencies. In fact, there is not much apparent change in the slope of the curves at lower frequencies.

Increasing the value of $x_{\text{gap}}$ does not yield any better results—in fact they get worse, as shown in figure 7.17. Rather than flattening out, the effectiveness curves simply develop a low dip and then rise drastically at very low $(x-x_{\text{gap}})/L$. For these reasons, I have decided to leave the value of $x_{\text{gap}}$ at zero for this analysis.

### 7.2.2 Effectiveness at $x/L < 0.5$ and $r_h/\delta_k > 1$

Figures 7.18 through 7.19 are graphs of the effectiveness of the medium ($L/D = 7.813$) and long ($L/D = 15.63$) exchangers as functions of normalized displacement. In all cases, the effectiveness below $x/L = 0.5$ at each frequency are not independent of displacement, but rather appear to be more proportional to displacement.

### 7.2.3 Effectiveness at $x/L \geq 1$

The same sudden increases in effectiveness that were observed without the adjacent screens are present in this data as well. They are somewhat obscured by the slope of the effectiveness curves at lower displacements, however. They are most visible at higher frequencies in figure 7.15.
Figure 7.15: Effectiveness of the short exchanger \((L/D = 4.688)\) with flat tube-ends and adjacent screens. The value of \(x_{\text{gap}}\) was zero. Notice the obvious slope in the data at \(x/L < 0.5\).

Figure 7.16: Effectiveness of the short exchanger \((L/D = 4.688)\) with flat tube-ends and adjacent screens. The value of \(x_{\text{gap}}\) was 3.8 mm. Notice that in this case, the use of a non-zero value for \(x_{\text{gap}}\) does not appear to even out the slope as well as it did previously.
Figure 7.17: Effectiveness of the short exchanger \((L/D = 4.688)\) with flat tube-ends and adjacent screens. The value of \(\chi_{\text{gap}}\) was 7.64 mm. This value appears to increase the slope at \(x/L < 0.5\) far too much.

Figure 7.18: Effectiveness of the medium exchanger \((L/D = 7.813)\) with flat tube-ends and adjacent screens. The value of \(\chi_{\text{gap}}\) was zero.
Figure 7.19: Effectiveness of the long exchanger ($L/D = 15.63$) with flat tube-ends and adjacent screens. The value of $x_{gap}$ was zero.
Effectiveness falls as displacement increases beyond $x/L > 1$ for the same reasons as in
the previous data set (the same exchangers, but without adjacent screens).

### 7.2.4 Effectiveness and Nusselt Number Correlation for $x/L < 0.5$ and $r_h/\delta_\kappa > 1$

The data for all three exchangers can again be correlated to a single function at
$x/L < 0.5$. Figure 7.20 shows the least-squares fit for $x/L < 0.5$ and $r_h/\delta_\kappa > 1$. The
resulting correlation is given by:

$$E = 0.696 \left( \frac{r_h}{\delta_\kappa} \right)^{-0.750} \left( \frac{x}{L} \right)^{0.213} \left( \frac{L}{D} \right)^{2.69 \times 10^{-2} \cdot (r_h/\delta_\kappa + 0.130)}. \quad (7.18)$$

The Nusselt numbers for these three exchangers can be correlated as well. This is
shown in figure 7.21, and is given by:

$$Nu = \left( \frac{r_h}{\delta_\kappa} \right)^{1.15} \left( \frac{L}{D} \right)^{1.30} 10^{4.00\ln[\log(x/L+1)-5.00]}. \quad (7.19)$$

### 7.2.5 Impact of Adjacent Screens on Effectiveness

The effect of adding the adjacent screens on the performance of the three
exchangers with flat tube-ends can now be characterized for $x/L < 0.5$ and $r_h/\delta_\kappa > 1$.
This is done by dividing the least-squares correlation found for the exchangers with
adjacent screens by that found for the exchangers without.
Figure 7.20: Least-squares fit (for $x/L < 0.5$) of the effectiveness of the short, medium, and long exchangers ($L/D = 4.688, 7.813,$ and $15.63$) with flat tube-ends, adjacent screens, and $x_{gap} = 0$.

Figure 7.21: Normalized Nusselt numbers for the short, medium, and long exchangers ($L/D = 4.688, 7.813,$ and $15.63$) with flat tube-ends, adjacent screens, and $x_{gap}=0$. The correlation is valid only for $x/L < 0.5$. 
There are two cases for the exchangers without screens, however: the effectiveness with $x_{\text{gap}} = 3.8 \text{ mm}$ and with $x_{\text{gap}} = 0$. The result for the first case is the following:

$$\frac{E_{\text{screens}}}{E_{\text{without}}} = 0.756 \left( \frac{x}{L} \right)^{0.107} \left( \frac{L}{D} \right)^{0.269 (\frac{x}{L})+0.130}, \quad (7.20)$$

while the result for the second case is:

$$\frac{E_{\text{screens}}}{E_{\text{without}}} = 0.817 \left( \frac{x}{L} \right)^{0.216} \left( \frac{L}{D} \right)^{2.69 \times 10^{-2} (\frac{x}{L})+0.130}. \quad (7.21)$$

The important information in these two results is the slightly different dependence on $x/L$, and the dependence on $L/D$. The power of the $x/L$ factor indicates in both cases that the screens cause the effectiveness to be more dependent on displacement. The $L/D$ factor, on the other hand, was introduced by the adjacent screens (there was no $L/D$ factor in either of the two cases without screens). So, the screens cause the effectiveness of the exchangers to be a function of the length of the exchangers as well.

In addition to these effects, there is some effect on magnitude as well. Figure 7.22 through 7.23 are graphs of the ratio of the effectiveness measured with adjacent screens to that without, with $x_{\text{gap}}$ set to a value of zero. Each graph corresponds to one of the three exchanger lengths.

In all three cases, the screens act to reduce the effectiveness of the exchanger except for at the highest frequencies tested. This is as much as a 20% effect, neglecting large peaks or dips at low displacements that are most likely caused by noise.
Figure 7.22: Ratio of the effectiveness of the short exchangers ($L/D = 4.688$) with flat tube-ends and adjacent screens to that without screens. Notice that at low $r_h/\delta_k$ the effectiveness is decreased by up to 25%. At $x/L > 1$ minor losses become significant and dominate as $r_h/\delta_k$ rises, which may explain the rising effectiveness in this region.

Figure 7.23: Ratio of the effectiveness of the medium exchangers ($L/D = 7.813$) with flat tube-ends and adjacent screens to that without screens. The same patterns are evident here as in the previous figure.
Figure 7.24: Ratio of the effectiveness of the long exchangers ($L/D = 15.63$) with flat tube-ends and adjacent screens to that without screens. The same general patterns are evident as in the previous two figures.
No comparisons are given for the ratio of the effectiveness when $x_{\text{gap}}$ is 3.8 mm for the exchangers without screens. The effectiveness for this case is higher than when $x_{\text{gap}} = 0$. This in turn causes the ratio of the screens to no-screens effectiveness to be even lower than that presented in figures 7.22 through 7.24.

7.3 Exchangers with Rounded Tube-Ends and Adjacent Screens

This section discusses the effectiveness of the three lengths of exchangers with adjacent screens, but with rounded instead of flat tube-ends. By comparing the effectiveness measured in these conditions with that of the previous section, the impact on effectiveness caused by rounding the ends of the tubes can be directly ascertained.

7.3.1 Air Mass Flow-Rate

The data for the exchangers with rounded tube-ends shows the same problems as in the previous section when adjusting $x_{\text{gap}}$. Because of this, $x_{\text{gap}}$ will be set to zero for this analysis as well.

7.3.2 Effectiveness at $x/L < 0.5$ and $\rho_{\text{h}}/\delta_c > 1$

Figures 7.25 through 7.27 are graphs of the effectiveness of the short ($L/D = 4.688$), medium ($L/D = 7.813$), and long ($L/D = 15.63$) exchangers with rounded tube-ends as functions of normalized displacement. As with the flat tube-ends and adjacent
screens, the effectiveness below $x/L = 0.5$ at each frequency are not independent of displacement.

### 7.3.3 Effectiveness at $x/L \geq 1$

The sudden increases in effectiveness are still present, but they are not very apparent. They are most visible at high frequencies in figure 7.25. The effectiveness then falls as displacement increases, as in the previous two data sets.

### 7.3.4 Effectiveness and Nusselt Number Correlations for $x/L < 0.5$ and $\frac{r_h}{\delta_k} > 1$

The data for all three exchangers can again be correlated to a single function at lower displacements. Figure 7.28 shows the least-squares fit for $x/L < 0.5$ and $\frac{r_h}{\delta_k} > 1$. The resulting correlation is given by:

$$E = 0.744 \left( \frac{r_h}{\delta_k} \right)^{-0.850} \left( \frac{x}{L} \right)^{0.280} \left( \frac{L}{D} \right)^{4.04 \times 10^{-1} (\frac{r_h}{\delta_k}) + 0.120} .$$ (7.22)

The correlation for the Nusselt numbers of these exchangers is shown in figure 7.27, and is given by:

$$\text{Nu} = \left( \frac{r_h}{\delta_k} \right)^{1.05} \left( \frac{L}{D} \right)^{1.35} 10^{4.00 \ln(\log(x/L) + 4) - 5.35} .$$ (7.23)
Figure 7.25: Effectiveness of the short exchanger \((L/D = 4.688)\) with rounded tube-ends and adjacent screens. The value of \(x_{gap}\) was zero. Notice the slope in the data at \(x/L < 0.5\), and the slight rise at \(x/L = 1\) and \(r_h/\delta_k = 1\).

Figure 7.26: Effectiveness of the medium exchanger \((L/D = 7.813)\) with rounded tube-ends and adjacent screens. The value of \(x_{gap}\) was zero.
Figure 7.27: Effectiveness of the long exchanger ($L/D = 15.63$) with flat tube-ends and adjacent screens. The value of $x_{gap}$ was zero.

Figure 7.28: Least-squares fit (for $x/L < 0.5$) of the effectiveness of the short, medium, and long exchangers ($L/D = 4.688$, $7.813$, and $15.63$) with rounded tube-ends, adjacent screens, and $x_{gap} = 0$. 

$$E = 0.744 \left( \frac{r_f}{\delta_k} \right)^{0.050} \left( \frac{x}{L} \right)^{0.200} \left( \frac{U}{D} \right)^{4.04 \times 10^2 \left( \frac{r_f}{\delta_k} \right) + 0.120}$$
Figure 7.29: Normalized Nusselt numbers for the short, medium, and long exchangers ($L/D = 4.688$, $7.813$, and $15.63$) with rounded tube-ends, adjacent screens, and $x_{\text{gap}}=0$. The correlation is valid only for $x/L < 0.5$. 

The correlation is valid only for $x/L < 0.5$. 

$$Nu = (r_i/\lambda)_{1.05} (L/D)^{1.35} 10^{4.01(4+\log_2(x/L)) - 5.35}$$
7.3.5 Effects of Rounded vs. Flat Tube-Ends

The effect of the rounded tube-ends on the performance of the three exchangers with can now be characterized as before for $\chi/L < 0.5$ and $\eta_h/\delta_k > 1$. The ratio of the effectiveness correlations for the flat tube-ends vs. the rounded tube-ends gives:

$$\frac{E_{\text{flat}}}{E_{\text{rounded}}} = \left(\frac{\eta_h}{\delta_k}\right)^{0.100} \left(\frac{\chi}{L}\right)^{-0.670} \left(\frac{L}{D}\right)^{-1.35 \times 10^{-2}(\eta_h/\delta_k) + 0.100}.$$  \hspace{1cm} (7.24)

From this result, it appears that the rounded ends slightly alter the dependence on frequency, displacement, and exchanger length.

The rounded ends act to alter the magnitude of the effectiveness of the exchanger as well. Figures 7.30 through 7.32 show the ratios of the effectiveness of the exchangers with flat tube-ends to those of the exchangers with rounded tube-ends. In all cases, the flat tube-ends produce greater effectiveness. This is as much as a 20% to 25% effect at higher displacements, and about a 10% to 15% effect at $\chi/L = 1$ (neglecting peaks at low displacements that are caused by noise).

It was shown in Chapter Six that the minor loss coefficient of exchangers with flat tube-ends and adjacent screens [a value of 0.689, indicated by the intercept in equation (6.10)] is about 31% greater than that of exchangers with rounded tube-ends and adjacent screens [a value of 0.475, indicated by the intercept in equation (6.14)]. This indicates that for a given velocity, flat tube-ends cause more turbulence and jetting than the rounded ends. This may be the cause of the increased effectiveness at higher displacements in figures 7.30 through 7.32.
Figure 7.30: Ratio of the effectiveness of the short exchangers ($L/D = 4.688$) with flat tube-ends and adjacent screens to that with rounded tube-ends. Notice that at $x/L > 1$ the effectiveness of the flat tube-ends is greater than that of the rounded ends, and that this appears to be a function of velocity (this ratio increases with frequency at a given displacement). This may be related to the increased minor losses of the flat tube-ends relative to those of the rounded tube-ends.

Figure 7.31: Ratio of the effectiveness of the medium exchangers ($L/D = 7.813$) with flat tube-ends and adjacent screens to that with rounded ends. Patterns are similar to those in the previous figure.
Figure 7.32: Ratio of the effectiveness of the long exchangers \((L/D = 15.63)\) with flat tube-ends and adjacent screens to that with rounded ends. Again, the patterns evident in the last two figures are evident here.
7.4 Further Discussion

Thus far, effectiveness and Nusselt numbers have been discussed only in the context of gas displacement. A brief discussion of these as functions of Reynolds number is warranted as well. In addition, new features in the data deserve further discussion. These are the sloping effectiveness curves at $x/L < 0.5$, and the apparent dependence on exchanger length evident in all of the data.

7.4.1 Effectiveness and Nusselt Numbers vs. Reynolds Number

Figure 7.33 is a graph of the effectiveness vs. Reynolds number ($Re = D\rho v/\mu$) of the three exchangers with flat tube-ends, no adjacent screens, and $x_{gap} = 3.8$ mm, while figure 7.34 is the same data but with $x_{gap} = 0$. The patterns are a little difficult to see in these graphs; but there appears to be two regimes present: in the first, at lower $Re$ the effectiveness is approximately independent of exchanger length and is a function of frequency only. In the second, the effectiveness becomes a function of exchanger length and is independent of frequency.

Figure 7.35 shows these two regimes more clearly by presenting the effectiveness of the three exchangers at only one frequency. Notice that at lower $Re$ the effectiveness of the three exchangers converges roughly onto a single horizontal line. At higher $Re$, however, the effectiveness diverge into lines that can easily be correlated with the different exchanger lengths.
Figure 7.33: Effectiveness vs. Reynolds number for the three exchangers with flat tube-ends, no adjacent screens, and $x_{gap} = 3.8$ mm. It appears that there are two regimes: one in which effectiveness is dependent on frequency (at lower Re); and another where effectiveness becomes dependent on exchanger length instead (at higher Re).

Figure 7.34: Effectiveness vs. Reynolds number for the three exchangers with flat tube-ends, no adjacent screens, and $x_{gap} = 0$. The patterns evident in the previous figure are evident here as well.
Figure 7.35: Effectiveness vs. Reynolds number at $r_{ih}/\delta_k = 2.02$ for the three exchangers with flat tube-ends, no adjacent screens, and $x_{gap} = 3.8$ mm. This graph shows more clearly the transition of effectiveness as a function of frequency to a function of exchanger length.

Figure 7.36: Effectiveness vs. Reynolds number for the three exchangers with flat tube-ends, adjacent screens, and no $x_{gap}$. In addition to the frequency-dependent and exchanger length-dependent regimes, effectiveness has become a stronger function of Re at lower values.
Figure 7.37: Effectiveness vs. Reynolds number for the three exchangers with rounded tube-ends, adjacent screens, and no $\chi_{gap}$. The patterns evident in the previous figure are evident here as well.
Figure 7.36 shows the effectiveness of all three exchangers with flat tube-ends, adjacent screens, and no $x_{gap}$, while figure 7.37 shows the same information but for exchangers with rounded tube-ends, adjacent screens, and no $x_{gap}$. Similar trends are seen as those in figure 7.33 and 7.34.

Figure 7.38 is a graph of the Nusselt number as a function of Reynolds number of the three exchangers with flat tube-ends, no adjacent screens, and $x_{gap} = 3.8$ mm. Figure 7.39 is the corresponding graph for the exchangers with flat ends, adjacent screens, and no $x_{gap}$; and figure 7.40 is the same information but for the exchangers with rounded ends. The same two regimes are visible in these graphs as well.

Correlations can be found for both effectiveness and Nusselt number as functions of Reynolds number, but only for a certain range. In every case, the effectiveness or Nusselt number transitions from one regime into the other at a Reynolds number that corresponds to an $x/L$ of about 0.5. For Reynolds numbers that correspond to $x/L < 0.5$, correlations could be found. However, since the Reynolds number can easily be calculated from gas displacement, frequency of oscillation, gas density, and gas viscosity, it seems redundant to provide these correlations here. For Reynolds numbers corresponding to $x/L > 0.5$, there is insufficient data at all exchanger lengths to generate correlations.
Figure 7.38: Nusselt number vs. Reynolds number for the three exchangers with flat tube-ends, no adjacent screens, and $x_{gap} = 3.8$ mm. Again, two regimes are suggested in this figure.

Figure 7.39: Nusselt number vs. Reynolds number for the three exchangers with flat tube-ends, adjacent screens, and $x_{gap} = 0$. Two different regimes are clear in this figure.
Figure 7.40: Nusselt number vs. Reynolds number for the three exchangers with rounded tube-ends, adjacent screens, and \( x_{\text{gap}} = 0 \). Two regimes are evident in this figure.
7.4.2 Sloping Effectiveness at $x/L < 0.5$

The fact that the effectiveness of the exchangers with adjacent screens is dependent on gas displacement at $x/L < 0.5$ is a mystery. I thought that perhaps this might be a velocity-dependent effect; but considering figures 7.33, 7.35, and 7.36, it appears that this is not the case.

An argument for constant effectiveness at $x/L < 0.5$ [or $(x-x_{gap})/L < 0.5$] was presented in section 7.1.2. However, it may be that this argument is too simplistic. For example, it may be possible that a transitional region may exist between laminar and turbulent bursting flows. Increasing the velocity (and therefore the Reynolds number) in this region may lead to an increase in effectiveness, even if the gas displacement is less than the length of the exchanger. Further research could shed additional light on this question.

7.4.3 Impact of Exchanger Length on Effectiveness

When the effectiveness of the exchangers with flat tube-ends and no adjacent screens was examined using the approach given by Wakeland and Keolian, three correlations of the form $E = C(r_h/\delta_x)^M$ were found—one for each length of exchanger. Linear correlations were then found for $C$ and $M$ as functions of $L/r_h$.

Although these correlations work for the range of frequencies and exchanger lengths used in this study (and those of Wakeland and Keolian’s as well), they are obviously erroneous when taken to extremes. For example, as $L/r_h$ becomes infinite (as
in an infinitely long tube) the resulting value of $C$ becomes infinite as well. Likewise, the value of $M$ remains negative and becomes infinite as $L/r_h$ increases, indicating no dependence on frequency at all. Both of these results are physically impossible of course, as the maximum possible effectiveness under any condition is 1.

Likewise, the correlations found for the exchangers with adjacent screens [equations (7.18) and (7.22)] indicate that effectiveness increases with exchanger length, and that this effect increases with frequency. At this point I do not know why this would be, nor what the asymptotic limits of these functions should be. These effects are plainly visible in the data, however.

**The Numeric Study of Zhao and Chang**

This is not the only study to have found an effect due to exchanger length, however—Zhao and Cheng\(^\text{36}\) also found a dependence on exchanger length. They numerically investigated heat transfer caused by oscillating air in a tube connected at both ends to larger reservoirs. The walls of the tube were assumed to remain at a fixed hot temperature, while the temperature of the air within the reservoirs was assumed to remain at a fixed cold temperature. As air oscillated from the reservoirs to the tube and back, a non-zero time-averaged flow of heat occurred. Their numeric model relied on the assumptions of incompressible and strictly laminar flow.

The dimensionless parameters in their model consisted of a dimensionless air displacement (the ratio of peak air displacement in the tube to the tube diameter; or $x/D$); the kinetic Reynolds number (defined as $Re_\omega = \omega D^2 / \nu$, where $\nu$ is the kinematic
viscosity of the air); the ratio of the tube length to its diameter (or $L/D$); and the Prandtl number. The Prandtl number was fixed at that of air (0.707), while the other three parameters were varied extensively.

From the results, Zhao and Cheng derived the following correlation for the temporally and spatially-averaged Nusselt number of the tube:

$$\text{Nu} = 0.00495 \frac{x}{D}^{0.9} \text{Re}^{0.656} [43.74 \frac{D}{L}^{1.18} + 0.06].$$

(7.25)

Notice that this equation states that two tubes of the same diameter but different lengths will produce different amounts of heat transfer—even for the same absolute gas displacement and frequency of oscillation. No explanation was given for this effect, however.

*The Study of Walther, Kuhl, and Schulz*

In addition to Zhao and Cheng, a separate numeric study was conducted by Walther, Kuhl, and Schulz. Their study assumed fully turbulent oscillating flow in a pipe, with Reynolds numbers ranging between 20,000 and 50,000. The Prandtl number used was 0.707. An important detail is that they assumed that the energy equation was not coupled with the momentum and continuity equations. This was accomplished by assuming that all gas properties were constant, even in the presence of heat transfer.

Their study resulted in the following expression for the instantaneous, spatially-averaged Nusselt number during turbulent flows:

$$\text{Nu} = 1.846 \times 10^{-6} \text{Re}^{1.236} \left[ 1 + 104.7 \left( \frac{L}{d_h} \right)^{0.1} \right].$$

(7.26)
The only new variable in this expression is the hydraulic diameter \( d_h \). Notice that this expression says that during fully turbulent flows, the Nusselt number (and therefore the effectiveness) is a function of the length of the exchanger.

While this correlation says that heat transfer increases with increasing length, Zhao and Cheng’s correlation indicated that the Nusselt number decreased with increasing length. These studies examine two different flow regimes, however (strictly laminar flow in Zhao and Cheng’s study, with strictly turbulent flow in Walther, Kuhl, and Schulz’s study).

On a tangential note, there is one other important detail to point out about the study done by Walther, Kuhl, and Schulz. They found that at the point in an acoustic cycle when turbulence occurred, the instantaneous heat transfer increased dramatically over that during laminar portions of the cycle. This was not stated quantitatively (there was no statement of how much the heat transfer increased relative to laminar flows; only that a “rapid increase in heat transfer can be observed”); but it does seem to give support to the idea presented earlier in this chapter that the sudden increases in effectiveness observed in the data correspond to the critical Reynolds numbers, and are therefore caused by turbulent bursts during the oscillating flows.

### 7.5 Summary

The effectiveness data for the three exchangers with flat tube-ends and no adjacent screens results in correlations that are similar to that found by Wakeland and
Keolian. This lends plausibility to the effectiveness and Nusselt number calculations of this study.

In the case of the exchangers with flat tube-ends and no adjacent screens, it is not clear if some value of \( x_{gap} \) should be used in calculating the mass flow-rate of the air through the exchangers. It appears, however, that in all of the measurements made with adjacent screens subtracting \( x_{gap} \) is probably not correct.

Sudden increases in the effectiveness of every exchanger tested appear to correspond with the critical Reynolds number, indicating that bursts of turbulent flows within the exchangers increase heat transfer. Effectiveness falls as \( x/L \) increases beyond a value of one, because the air spends less time during an acoustic cycle in thermal contact with the exchanger.

It appears that an adjacent stack of screens has no effect on the frequency dependence of the effectiveness, but it does introduce a dependence on exchanger length. The screens act to decrease the effectiveness by as much as 20\%, except at higher frequencies. They also cause the effectiveness to be dependent on gas displacement at \( x/L < 0.5 \) instead of constant.

Rounded tube-ends slightly alter the dependence of the effectiveness on frequency, exchanger length, and gas displacement. The biggest effect is that the rounded ends produce as much as 25\% lower effectiveness, compared to the flat tube-ends. In no case did the rounded ends produce greater effectiveness. This may be caused by increased turbulence generated by the flat tube-ends relative to that produced by the rounded tube-ends at a given velocity.
Correlations were also given for the Nusselt numbers of the exchangers in each case. Similar effects caused by adjacent screens and different tube-end conditions are visible in these correlations as well.
Chapter 8

Chilton and Colburn-J Factor Analogy

The most widely used analogy between heat, momentum, and mass transfer in steady flow is the Chilton and Colburn-J Factor Analogy. Unlike the Rayleigh or Prandtl-Taylor analogies, the Chilton and Colburn-J Factor analogy was derived from experimental data. It is applicable to gases with Prandtl numbers in the range of 0.7 to 160, and is applicable to flows in tubes with aspect ratios \((L/D)\) greater than 60.

This analogy states that heat and momentum transfer in fully developed thermal and flow conditions are related locally by the following expression:

\[
J_H = \frac{1}{2} F, \tag{8.1}
\]

where \(J_H\) is the Colburn-J factor, and \(F\) is the friction factor used in fluid dynamics.

The friction factor \(F\) is defined as:

\[
F = \Delta P \frac{2D}{\rho L v^2}, \tag{8.2}
\]

where \(\Delta P\) is the pressure loss through the pipe or channel, \(D\) is the diameter of the pipe, \(L\) is the length of the pipe, \(\rho\) is the mean density of the gas, and \(v\) is the cross-sectionally averaged particle velocity through the pipe.

The Colburn-J factor is defined as:

\[
J_H = \text{St} \text{Pr}^{2/3}, \tag{8.3}
\]

and the Stanton number \(\text{St}\) is defined as:

\[
\text{St} = \frac{h}{\rho vc_p} = \frac{\text{Nu}}{\text{Re Pr}}, \tag{8.4}
\]
where \( h \) is the local convection coefficient, \( \nu \) is the cross-sectionally averaged particle velocity, \( \rho \) is the density of the gas, and \( c_p \) is the specific heat of the gas.

Notice that the friction factor \( F \) in equation (8.2) and the Nusselt number in equation (8.4) are both local quantities, rather than spatially-averaged. For a tube with no changes in cross-section or surface roughness, the friction factor does not change along the length of the tube. Likewise, under fully-developed thermal conditions the local Nusselt number does not change along the tube length. For tubes that are much longer than the thermal entry length (given by \( x/D = 0.05 \text{RePr} \) on page 393 of Incropera and DeWitt\(^1\)), the local and spatially-averaged Nusselt numbers and friction factors are approximately the same.

The Colburn-J factor can also be expressed in terms of the experimental variables:

\[
J_T = \frac{E \text{Pr}^{2/3} (T_{\text{air in}} - T_{\text{HX surface}})}{4 \Delta T_{\text{LM}}},
\]

(8.5)

where \( \Delta T_{\text{LM}} \) is the log-mean gas temperature [given by equation (4.24) and discussed on pages 77-78], \( T_{\text{air in}} \) is the cross-sectionally averaged temperature of the air as it entered the exchanger, and \( T_{\text{HX surface}} \) is the surface temperature of the exchanger.

At the present, it is not known whether the Chilton and Colburn-J Factor analogy is applicable in oscillating flows. In this chapter, friction factors have been calculated using equation (8.2) and the real part of the peak acoustic pressure as discussed in Chapter Six. Colburn-J factors were calculated using the effectiveness data from Chapter Seven with equation (8.5).
8.1 Exchangers with Flat Tube-Ends and No Adjacent Screens

Figure 8.1 is a graph of the Colburn-J factors measured with the short, medium, and long exchangers ($L/D = 4.688, 7.813, \text{ and } 15.63$) with flat tube-ends, no adjacent screens, and an $x_{gap}$ of 3.8 mm. The Colburn-J factors predicted by the Chilton and Colburn-J Factor analogy are indicated by the thick, solid red line.

It is immediately apparent that the oscillating frequency is not taken into account by the Chilton and Colburn-J Factor analogy, as it is possible to generate the same friction factor with an infinite number of frequencies (this is because any given velocity can be produced by any combination of frequency and displacement).

Notice that this graph suggests two different regimes. The first is characterized by the horizontal colored lines at higher friction factors. In this regime, the Colburn-J factors for each exchanger length converge into approximately single lines at each $r_h/\delta_k$. In the second regime, the Colburn-J factors at all $r_h/\delta_k$ become functions of the friction factor and converge into three sloped lines, corresponding to each of the three exchanger lengths.
Figure 8.1: Colburn-J factors of the short, medium, and long exchangers ($L/D = 4.688$, 7.813, and 15.63) with flat tube-ends, no adjacent screens, and $x_{gap} = 3.8$ mm. Notice that there appear to be two regimes in this figure: one in which the Colburn-J factors are relatively independent of friction factor but are a strong function of frequency; and another in which the Colburn-J factors become a strong function of friction factor and exchanger length instead.

Figure 8.2: Colburn-J factors of the short, medium, and long exchangers ($L/D = 4.688$, 7.813, and 15.63) with flat tube-ends, no adjacent screens, and $x_{gap} = 0$. 
Figure 8.2 is a graph of the Colburn-J factors of the same three exchangers, but with $x_{gap} = 0$. The same two regimes are evident, with the only difference being the slight slope to the lines at higher friction factors.

### 8.2 Exchangers with Flat Tube-Ends and Adjacent Screens

Figure 8.3 is a graph of the Colburn-J factors as measured with the same three exchangers ($L/D = 4.688$, 7.813, and 15.63 and flat tube-ends), but with the addition of a stack of adjacent screens and $x_{gap} = 0$. The same patterns are evident in this graph as well.

### 8.3 Exchangers with Rounded Tube-Ends and Adjacent Screens

Figure 8.4 is a graph of the Colburn-J factors as measured with the short, medium, and long exchangers ($L/D = 4.688$, 7.813, and 15.63) with rounded tube-ends, adjacent screens, and $x_{gap} = 0$. The same two regimes are evident in this graph as well.

### 8.4 Discussion

At this point, it is unclear what the cause of these two regimes might be. They are consistently present, however, under all experimental conditions. The Chilton and Colburn-J Factor analogy does not agree at all with the data at higher friction factors.
Figure 8.3: Colburn-J factors of the short, medium, and long exchangers \((L/D = 4.688, 7.813, \text{ and } 15.63)\) with flat tube-ends, adjacent screens, and \(x_{\text{gap}} = 0\). The data is a bit more noisy, but the same to regimes seen in previous figures are apparent here as well. Also, notice that the Colburn-J factors are now stronger functions of friction factor at higher values of the friction factor.

Figure 8.4: Colburn-J factors of the short, medium, and long exchangers \((L/D = 4.688, 7.813, \text{ and } 15.63)\) with rounded tube-ends, adjacent screens, and \(x_{\text{gap}} = 0\). The patterns evident in the previous figure are evident here as well.
There may be some agreement at lower friction factors, but insufficient data is present to derive reliable correlations.
Chapter 9

Comparisons to DeltaEC and the TASFE Approximation

In this chapter, the Nusselt numbers and effectiveness measured with the short exchanger \((L/D = 4.688)\) with flat tube-ends, no adjacent screens, and \(x_{gap} = 0\) will be compared to those predicted by the TX segment of the DeltaEC software package (of Los Alamos National Laboratory\(^4^4\)) and the Time-Averaged Steady Flow approximation (TASFE approximation) discussed in previous literature.

9.1 Comparison to the DeltaEC Model

DeltaEC is a software package designed specifically for thermoacoustic device design, and includes a basic model to predict heat exchanger performance.\(^4^4\) The DeltaEC TX segment assumes that the convection coefficient of the heat exchanger is given by:

\[
h = \frac{k}{y_0},
\]

where \(k\) is the thermal conductivity of the gas and \(y_0\) is either the thermal penetration depth of the gas or the radius of the tube, whichever is smaller.

By substitution, it can be shown that the DeltaEC expression results in the following expression for the Nusselt number of the exchanger:

\[
Nu = \frac{L}{y_0}.
\]
In this expression, \( L \) is either the length of the exchanger or the peak-to-peak displacement of the gas—whichever is smaller.

Figure 9.1 is a graph of the Nusselt number as predicted by the DeltaEC TX segment (solid black line) and as measured with the flat-ended exchangers without adjacent screens (colored lines with markers). It is immediately apparent that at \( x/L < 0.5 \) the DeltaEC model drastically overestimates the Nusselt number, except for at the highest frequencies. At \( x/L > 0.5 \), DeltaEC begins to underestimate the Nusselt number.

These results are then converted into effectiveness using the following expression:

\[
E = \frac{2\pi r k \text{Nu}(T_{\text{air in}} - T_{\text{HX surface}})}{m c_p \Delta T_{\text{LM}}}.
\]

Figure 9.2 is a graph of these results. The effectiveness as predicted by DeltaEC is shown in solid colored lines, while that measured with the short exchanger with flat tube-ends and no adjacent screens is shown in colored lines with markers.

At the highest frequency and low displacements, the DeltaEC model overestimates the effectiveness by a factor of about two. At the lowest frequency and low displacements, the DeltaEC model underestimates the effectiveness by a factor of about five. At high displacements the DeltaEC model underpredicts effectiveness at all frequencies.

These problems were first noted by Ray Wakeland.\(^{43,53}\) He correctly pointed out that the model in the DeltaEC HX and TX segments does not take into account the limited heat capacity of the gas oscillating through the exchanger, resulting in impossible values of effectiveness at low frequencies (a new segment has been added to DeltaEC that does attempt to take these effects into account, although no comparison is offered.
Figure 9.1: Comparison of the Nusselt numbers predicted by the DeltaEC TX segment and those measured with the short model exchanger with flat tube-ends and no adjacent screens. At $x/L < 0.5$ there is some agreement; but not at higher displacements.

Figure 9.2: Effectiveness of the short exchanger ($L/D = 4.866$, lines with markers) with flat tube-ends and no adjacent screens, and that predicted by DeltaEC (solid lines).
here). He also pointed out the problems with using a convection coefficient in the standard steady-flow manner—that is, that the convection coefficient is assumed to be a constant, regardless of velocity, while temperature differences and surface area cause differences in heat transfer. The model in the DeltaEC TX segment uses this approach, as shown by equation (9.1).

As of January 2009, a new segment has been added to the DeltaEC software package that addresses these issues. This new segment takes into account the changing time-averaged temperature of the gas along the length of the exchanger. Comparisons of the output of this new segment to the data presented in this chapter will be provided in a future publication.

### 9.2 Comparison to TASFE models

The TASFE approximation is the heat transfer equivalent of the Iguchi hypothesis—that is, that steady-flow correlations can be used with the instantaneous oscillating velocity to predict the time-averaged Nusselt number in oscillating flows. To my knowledge, no previous study has been conducted to verify the accuracy of this assumption with internal flows in tubes, as exists in shell-and-tube exchangers.

The choice of the steady-flow correlation used in these calculations is very important. For example, figure 9.3 is a graph comparing the Nusselt number as measured using the short \((L/D = 4.688)\), flat-ended exchanger without adjacent screens, and that calculated using the TASFE approximation and the correlation for thermal and viscous developing laminar flows with constant surface temperature by Sieder and Tate.\(^1\) Data is
Figure 9.3: Comparison of actual Nusselt number data and a laminar TASFE approximation for the short model exchanger with flat tube-ends and no screens. Agreement is generally poor.

Figure 9.4: Comparison of actual Nusselt number data and a fully-turbulent TASFE approximation for the short model exchanger with flat tube-ends and no screens. The TASFE model achieves close to the same slope as the data at higher $x/L$, but still underpredicts the Nusselt number considerably.
indicated by colored lines with markers, while the TASFE calculations are indicated by solid colored lines. By contrast, figure 9.4 is a graph comparing the same data to another TASFE calculation using the Petukhov correlation for fully-developed turbulent flow.¹ Obviously, the choice of steady-flow correlation has a large impact on the results.

Of the steady-flow correlations I am aware of for internal flow in tubes, these correlations most closely approximated the actual data. However, as is obvious in figures 9.3 and 9.4, they do not agree well with the oscillating flow data. Based on these results, it appears that the TASFE approximation (at least when used with the steady-flow correlations cited here) is probably not of value for shell-and-tube exchanger designs.
Chapter 10

Summary and Conclusions

This study has experimentally investigated the effects of geometry and the presence of adjacent screens on the acoustic losses and heat transfer performance of shell-and-tube heat exchangers in oscillating flow under constant-pressure conditions. The results were validated by comparing acoustic impedance measurements to linear acoustic theory, and by showing that effectiveness correlations derived from the data agreed somewhat with those of a prior study by Wakeland and Keolian.\(^{43}\)

Correlations were derived in Chapter Six for the minor loss coefficient as a function of exchanger length for three cases: exchangers with flat tube-ends and no adjacent screens; exchangers with flat tube-ends and adjacent screens; and exchangers with rounded tube-ends and adjacent screens. Those correlations are as follows:

\[ \kappa_{\text{flat, no screens}} = 8.03 \times 10^{-3} \left( \frac{L}{D} \right) + 0.656, \quad (9.4) \]

\[ \kappa_{\text{flat with screens}} = 1.67 \times 10^{-2} \left( \frac{L}{D} \right) + 0.689, \quad (9.5) \]

and

\[ \kappa_{\text{rounded with screens}} = 1.74 \times 10^{-2} \left( \frac{L}{D} \right) + 0.475. \quad (9.6) \]

These correlations also show that regardless of the shape of the ends of the tubes (flat or rounded), the screens act to increase the rate of change of the minor loss coefficients (as functions of exchanger length) by a factor of two. This may be caused by the development of turbulence within the exchangers induced by the screens.
The intercepts in equations (9.4) and (9.5)—the minor loss coefficient correlations for exchangers with flat tube-ends, with and without adjacent screens—differ by about 5%. This seems to indicate that the screens minimize the development of an additional minor loss at that end of the exchangers. The results of this chapter also showed that the screens act to reduce the drop in the imaginary part of the impedance observed at high \(x/L\), as well as the drop at low \(x/L\) and \(r_i/\delta_c\).

The results of Chapter Seven were surprising. It was shown that the aspect ratio of the exchanger (the ratio \(L/D\), or exchanger length to tube diameter) is an important parameter in predicting the effectiveness and Nusselt number. The presence of adjacent screens appears to make this even more so.

It was not clear if a small gap should be subtracted from the air displacement through the exchangers when calculating the mass flow-rate of the air (as in Wakeland and Keolian’s study\(^{43}\)). In the case of the exchangers with flat tube-ends and no adjacent screens a small gap of 3.8 mm was justifiable because of the thickness of the spacer plate, and resulted in reasonably constant effectiveness vs. \(x/L\) [or \((x-x_{\text{gap}})/L\)] for a given frequency and \(x/L < 0.5\).

This did not appear to work nearly as well in the case of exchangers with adjacent screens. The effectiveness of these exchangers at \(x/L < 0.5\) is not independent of \(x/L\), however. The presence of the screens seems to be a key factor, but the exact cause is unknown.

In all cases, sudden increases in the effectiveness near \(x/L = 1\) were visible to varying degrees. These increases appeared to be correlated to the onset of fully turbulent
bursts during the velocity peaks. This was determined by comparing the Reynolds numbers at the effectiveness increases to the critical Reynolds number defined by Iguchi and Ohmi. This was also corroborated by observations in a study by Walther, Kuhl, and Schulz.

Measurements showed that screens adjacent to exchangers with flat tube-ends caused a decrease in effectiveness of up to 20%. In addition, exchangers with rounded tube-ends and adjacent screens produced up to 25% lower effectiveness than exchangers with flat tube-ends and adjacent screens. It may be that the rounded tube-ends generate less turbulence than the flat tube-ends, thereby reducing the heat transfer effectiveness.

Chapter Eight showed that for $\chi/L < 0.5$ (corresponding to larger friction factors) and $r_b/\delta_k > 1$ the Chilton and Colburn-J Factor analogy does not hold. Colburn-J factors are either not functions of friction factors, or are very weak functions within this range. At smaller friction factors, however, this was not the case. There is insufficient data in this range to derive reliable correlations, however.

Chapter Nine showed that the heat exchanger model in the TX segment of the DeltaEC software does not accurately predict the Nusselt number or effectiveness of shell-and-tube heat exchangers. In addition, comparisons showed that models based on the application of steady-flow heat transfer correlations (the TASFE approximation) are not accurate.
Further Study

There are several observations in this study that invite further examination. First, the dependence of exchanger effectiveness on $x/L$ at values less than 0.5 is a mystery. It appears that this may be related to the presence of adjacent screens; but the cause is unknown.

The importance of the aspect ratio of the exchanger tubes ($L/D$) also merits further investigation. Only two other studies that I know of—that of Zhao and Cheng\textsuperscript{36} and that of Walther, Kuhl, and Schulz\textsuperscript{37}—have found that the aspect ratio of an exchanger was a factor in predicting heat transfer of an oscillating flow in a tube.

This study also showed that heat transfer depends on the shape of the ends of the exchanger tubes. With further experimentation, it may be possible to derive a correlation for the impact on heat transfer as a function of the minor loss coefficient.

The importance of the Prandtl number on heat transfer deserves greater attention. Dimensional analysis by Zhao and Cheng\textsuperscript{36} showed that the Prandtl number is important, but they did not investigate this. Analytic theory derived in Chapter Five shows that the Prandtl number is probably negligible; but all measurements in this study were conducted in air.

When considering effectiveness as a function of Reynolds number, two distinct regimes are suggested by the data. The first (at lower Re) can be described by correlations; however, there is insufficient data to derive correlations for the regime at higher Re. The criteria for transition between the two regimes, as well as this second regime itself, require further investigation.
It is not clear from this study if the Chilton and Colburn-J Factor analogy holds in oscillating flows through heat exchangers. Further investigation is definitely warranted.
References


VITA

John Feurman Brady

John Feurman Brady was born on July 2, 1972 while his father worked to complete a doctoral degree in Philosophy at the University of Texas. In 1990 he graduated from Patrick Henry High School in San Diego, California, and began his undergraduate studies at Brigham Young University in Utah later that year. In August 1991 he left school to serve a full-time religious mission in Chile from 1991-1993.

John returned to his undergraduate studies in the fall of 1993 and graduated with a Bachelor of Arts degree in Music and a Professional Certificate in Sound Recording Technology. After graduation, he launched his own small business providing digital recording and encoding services to local web site developers. This business ran successfully for several years and grew to include five employees.

After several years in business, John decided to pursue an advanced degree in Acoustics at the Pennsylvania State University. He returned to Brigham Young University as a non-degree seeking student and earned an additional twenty-five credits in advanced math, physics, and electronics as prerequisites for admission to the program at Penn State. He began his studies at Penn State in the fall of 2004.

John has been married to his lovely wife Jami for fifteen years, and is the father of three daughters.