A FIELD-FREQUENCY LOCK IMPLEMENTED WITH A SAMPLED-DATA FEEDBACK CONTROL ALGORITHM DERIVED FROM A SMALL-SIGNAL NMR MODEL

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by
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Abstract

High field resistive magnets offer many advantages for nuclear magnetic resonance (NMR) experiments. The spectral resolution of a NMR measurement increases linearly with the magnetic field $B_0$, the signal-to-noise ratio (SNR) increases with $B_0^{7/4}$, and the data acquisition time decreases as $1/B_0^{3/4}$. However, resistive magnets require a power supply and cooling system, both of which introduce temporal fluctuations in the magnetic field. Earlier studies that used active methods for suppressing fluctuations can be categorized by the techniques used to sense the fluctuations. In one category, a pickup coil is used to sense changes in the magnetic field through Faraday induction; the voltage across the coil terminals provides a measure of the field fluctuations. Due to the low signal-to-noise ratio of the inductive measurement system at low frequencies, another method is also used to sense field fluctuations. In this second category, the field fluctuations are measured in terms of their effect on an NMR experiment and the resulting controller is known as a field-frequency lock (FFL). In the past, FFL controllers have been used to reduce long-term drift in permanent and superconducting magnets, where field fluctuations are low in both amplitude and frequency. Control gains were traditionally chosen by trial and error and not derived using a dynamic model of the plant. This thesis describes a FFL controller which is designed using a small-signal model of the nonlinear NMR plant. The FFL is tested on a 7 Tesla superconducting magnet in the presence of artificial magnetic field disturbances injected with a solenoidal coil. In order to establish the region of operation where the linear controller is
applicable, its performance is quantified for a range of disturbance amplitudes and frequencies. An understanding of the limitations of the linear controller is essential to the long-term goal of implementing a field-frequency lock in resistive magnets, where field fluctuations have much larger amplitudes and frequencies.
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Flux Regulation

1.1 Field Fluctuations in Resistive Magnets

A nuclear magnetic resonance (NMR) experiment requires a static magnetic field to polarize the magnetic moment of the sample nuclei. In most NMR experiments, the static field is generated by a superconducting magnet. These magnets are favored because they produce spatially uniform fields with amplitudes that vary slowly in comparison to experiment duration. The magnetic field in a superconductor is nearly constant in time by design. When a superconducting magnet is commissioned, an external power supply establishes a current in the superconducting coil. After the field reaches its desired value, a superconducting shunt is placed across the winding and the external power supply is removed. As a result, as long as the magnetic windings are cooled, the current in the coil is constant.

Increasing the static magnetic field improves a NMR measurement in several significant ways. The spectral resolution of the measurement increases linearly with the magnetic field $B_0$ \cite{4}, the signal-to-noise ratio (SNR) increases with $B_0^{7/4}$ \cite{7}, and the data acquisition time decreases as
Unfortunately, superconducting materials are characterized by a critical current value, which limits the maximum field available in a superconducting magnet. For magnets made of niobium-titanium, the critical current limits the maximum field to 10 T. Magnets composed of niobium-tin are more expensive, but can generate fields up to 30 T. Unfortunately, this compound is extremely brittle and therefore difficult to wind in a superconducting coil. For this reason, composites of niobium-tin and niobium-titanium are often used in superconducting magnets.

Figure 1.1 shows the increase in field strength in superconducting magnets over the last 50 years [19]. Currently, the maximum achievable field has plateaued at approximately 21 T [23]. To achieve fields above this value, several research centers, including the National High Magnetic Field Laboratory (NHMFL), have been developing resistive magnets using non-superconducting metals that provide field strengths up to 33 T. These magnets require external power supplies, and their field windings must be water cooled to prevent thermal damage from the large currents required to achieve high field strengths.

**Figure 1.1.** Evolution of magnetic field strength in spectrometer design [19].

Although resistive magnets overcome the field strength limitations imposed by existing superconductor technology, they present their own engineering challenges. The first arises from their need for external power supplies. Despite efforts to eradicate it, power supply ripple cannot be completely eliminated. The low-amplitude residual ripple results in a magnetic field fluctuation
that adversely affects NMR experiments [24].

The Keck magnet is a resistive magnet currently in use at the NHMFL. It achieves field strengths up to 25 T using three windings connected in series. The magnet is 920 mm in diameter and 1070 mm tall and has a 52 mm diameter bore. Full field operation requires a current of 40 kA that results in a voltage drop of 500 V across the coil [24].

Figure 1.2 shows a block diagram of the DC power supply. Substation power at 12.38 kV is stepped down to 460 V RMS, and then rectified. A passive filter eliminates a portion of the DC ripple. The remaining ripple is partially removed by a feedback loop consisting of an active filter, a Holec device that measures variation in current, and a Model 36 computer control system. Despite these two stages of filters to eliminate ripple, a ripple current of 15 ppm is measured by the Holec sensor. The spectrum of the ripple includes harmonics of 60 Hz, as well as 1440 and 50 Hz components from the rectifier and Holec device, respectively [24].

\[
\text{Figure 1.2. Block diagram of power supply [24].}
\]

The effect of a 15 ppm ripple in the power supply current on the NMR experiment is now considered. The nuclear resonance transition frequency, \( f_{NMR} \), is related to the amplitude of the static magnetic field \( B_0 \) as
\[ f_{NMR} = \gamma B_0 \]  

(1.1)

where the gyromagnetic ratio \( \gamma \) for hydrogen nuclei is 42.58 MHz/T. The magnetic field strength is directly proportional to current. As a result, a 15 ppm variation in the supply current results in a 15 ppm variation in the resonant frequency. At full field, where \( B_0 = 25 \) T, the supply current is \( I_s = 40 \) kA. The variation in current is

\[
\frac{15 \text{ ppm} \times I_s}{10^6 \text{ ppm}} = 0.6 \text{ A},
\]  

(1.2)

while the variation in resonant frequency is

\[
\frac{15 \text{ ppm} \times B_0 \times \gamma}{10^6 \text{ ppm}} = 16 \text{ kHz}
\]  

(1.3)

Although the 0.6 A variation in current is small in comparison with the peak current of 40 kA, the 16 kHz shift in resonant frequency is large compared to the 100 Hz linewidth of typical NMR transitions.

The second basic engineering challenge unique to resistive magnets arises from the considerable power dissipated as heat in the self-resistance of their windings. The temperature of the windings is regulated by a water cooling system, and variations in water flow rate and temperature result in magnetic field fluctuations.

At full field, the Keck magnet dissipates 20 MW (40 kA \( \times \) 500 V) in the coil windings, which are contained in a volume roughly equivalent to that of a 55 gallon drum. High field resistive magnets such as the Keck are designed to be water cooled in order to mitigate the power dissipation in the windings. The windings of these magnets are composed of copper plates, known as Bitter plates, separated by insulating layers as shown in Figure 1.3. A series of holes in each plate allows water to flow through and cool the resistive windings. Approximately
107 liters of cooling water per second flow through the magnet. The water enters the magnet at 9 °C and is heated to 35 °C by the time it leaves [24]. Variations in water flow rate and temperature result in magnetic field fluctuations.

The relationship between the cooling system and its effects on magnetic field fluctuations is complex. Since resistance varies proportionally with respect to temperature, the winding resistance changes with fluctuations in water temperature, and the current and field strength change accordingly. However, the temperature variations also change the area of the Bitter plates, thereby affecting the current distribution and resulting magnetic field [21]. Furthermore, changes in the flow rate of the water vary the position of the plates and introduce another set of magnetic field fluctuations.

![Diagram of cooling system](image)

**Figure 1.3.** Diagram of cooling system [20].

Due to the intricate relationship between the cooling system and the magnetic field, the change in field strength cannot be easily predicted for a given change in water temperature or
flow rate as it was for a given value of power supply ripple. However, experimental NMR data has confirmed the link between the variations in coolant temperature and magnetic field. This is shown in Figure 1.4, in which the bold line represents water temperature and the thin line depicts field strength. A 17 ppm change in magnetic field per degree Celsius corresponds to the thermal expansion coefficient of copper at room temperature [26]. The 2 minute period of the magnetic field fluctuation in Figure 1.4 directly corresponds to the period of water temperature fluctuation [21], [23].

![Figure 1.4](image)

**Figure 1.4.** Relationship between variations in coolant temperature and magnetic field. The bold line represents water temperature and the thin line depicts field strength [24].

Figures 1.5 and 1.6 show the effect of temporal magnetic field fluctuations on a NMR experiment. The NMR signals in the figures were generated by repeated use of the spin echo sequence, an RF pulse sequence commonly used in NMR experiments and described in section 2.2.1. The spin echo experiment was repeated 100 times in the Keck resistive magnet operating at 7 Tesla. To provide a basis for comparison, the same experiment was repeated in 7 Tesla superconducting magnet. The resulting time-domain NMR responses are shown in Figure 1.5 for the superconducting magnet (a) and Keck resistive magnet (b). In the superconducting magnet, the NMR response is nearly identical among trials. However, in the Keck magnet, the power supply and cooling system introduce magnetic field fluctuations which vary the phase of the NMR response from trial to trial. Because the NMR response is not repeatable, signal averaging cannot be used.
to improve the SNR of NMR signals obtained with the Keck magnet. Figure 1.6 compares the magnitude spectra of the NMR responses in the superconducting and resistive magnets for one of the trials. The most notable difference between subplots (a) and (b) is the linewidth of the spectrum. In the superconducting magnet, the spectrum’s full width at half of its maximum value (FWHM) is approximately 0.47 Hz. In the Keck magnet, the FWHM is nearly 46 times larger at 21.7 Hz. The resistive magnet produces a response with a broader spectrum due to its temporally fluctuating and inhomogeneous magnetic field.

Figure 1.5. NMR signal in a superconducting (a) and resistive (b) magnet.

Figure 1.6. NMR spectrum in a superconducting (a) and resistive (b) magnet.
Several techniques have been proposed for reducing magnetic field fluctuations. These include passive systems, such as inductive shields \cite{12}, \cite{22}, and active systems. The active systems can be categorized as inductive and NMR based. Section 1.2 discusses flux regulation using inductive measurements. Section 1.3 discusses flux regulation using a field-frequency lock based on NMR measurements.

1.2 Flux Regulation Using Inductive Measurements

In the category of active systems that use a Faraday detector to sense magnetic field fluctuations, a pickup coil is placed in the magnet and used to sense temporal fluctuations. From Faraday’s law of induction, the open-circuit voltage of the coil is proportional to the product of the field amplitude and frequency. The integrating preamplifier removes the frequency dependence from this voltage, and the compensator determines the magnetic field necessary to counteract the fluctuations and generates a corresponding current. Driven by the compensator current, the correction coil produces a magnetic field to compensate for the disturbances measured by the pickup coil. The compensation signal is determined by either an analog or digital feedback system, which can employ various control methods. A block diagram of the inductive control scheme is shown in Figure 1.7.

Primas and Gunthard were the first to implement a controller based on inductive measurements of magnetic field fluctuations \cite{17}, \cite{5}. In the late 1950s, they demonstrated its successful operation in a superconducting magnet. The technique was extended to resistive magnets in 1972, when Gottlieb et. al. used an inductive control system to reject the higher frequency temporal field fluctuations present in a resistive magnet \cite{3}. 
1.2.1 Recent Results

In a 2000 publication, Soghomonian describes an inductive feedback system that was implemented on the Keck magnet at the NHMFL. The compensation signal was generated by an analog proportional controller, and the performance of the controller was quantified by its effect on an NMR signal. The gain of the proportional controller was determined experimentally and not designed based on a model of the system. Feedback control reduced the peak-to-peak variation in the NMR signal from 16 to 12 ppm [24]. These results, shown in Figure 1.8, display a subtle reduction in high frequency components.

In a successful attempt to further attenuate the temporal magnetic field fluctuations due to the power supply, Penn State graduate student Mingzhou Li designed several control schemes based on an analysis of the open-loop transfer function of the system. In order to quickly switch between compensator designs, Li implemented the controllers digitally. The simplest control design, a proportional controller, attenuated the 60 Hz component of the power supply ripple by over 20 dB for the maximum value of proportional gain. Increasing the control gain beyond this level caused the closed-loop system to become unstable. In order to increase the maximum
loop gain that resulted in a stable closed-loop system, Li turned to phase-lead-lag control. The phase-lag component of the controller maintained the closed-loop stability while allowing an increased low-frequency loop gain. The phase-lead component decreased the loop gain near DC to a value less than the maximum allowed for closed-loop stability; the maximum value was determined by the offset of the integrating preamplifier. Implementation of the phase-lead-lag controller reduced the 60 Hz disturbance component by an additional 30 dB as compared with the proportional controller [11].

In an attempt to even further reduce the field-fluctuations at 60, 120, 180, and 720 Hz, Li augmented the phase-lead-lag controller by applying the internal model principle (IMP) and placing poles at these frequencies. In order to ensure the controller stability while maintaining the shape of the loop transfer function, the poles were placed slightly to the left of the $j\omega$ axis, and corresponding zeros were placed to the left of the poles at the same frequencies. Inclusion of the internal model principle reduced the 60 Hz component by more than 10 dB compared with
the phase-lead-lag controller alone.

Li quantified the effects of his controllers using two techniques. He measured field fluctuations directly with a dynamic signal analyzer (DSA) connected to the output of the integrating preamplifier, and he observed the effect of the temporal fluctuations on the nuclear transition frequency by measuring the standard deviation of the spin echo phase. Since variations in spin echo phase correspond to fluctuations in magnetic field, measuring the standard deviation of this parameter provides a means of determining the magnitude of these fluctuations [11]. Figure 1.9 shows the impact of the proportional and phase-lead-lag IMP controllers on the magnitude spectra of the field within the Keck resistive magnet at 7 Tesla. The effect of the controllers on the standard deviation of spin echo phase is shown in figure 1.10.

![Figure 1.9](image_url). Magnitude spectra of the temporal magnetic field fluctuations in the Keck resistive magnet operating at 7 T, with and without feedback compensation [11].
1.2.2 Limitations on Disturbance Rejection

At low frequencies, the SNR of the inductive measurement system is limited by several factors. The open-circuit pickup coil voltage is proportional to the frequency of the sinusoidal disturbance. In the absence of noise, there is no limit on the smallest voltage that can be observed. However, the finite resistance of the pickup coil gives rise to a thermal noise voltage that is frequency independent. Furthermore, at low frequencies, 1/f noise dominates the pickup coil voltage. Finally, the inductive sensor is unable to distinguish between voltages induced by time-varying magnetic fields within the magnet and those generated by induced currents in the cable between the pickup coil and integrating preamplifier. The combination of these noise sources results in a cutoff frequency below which the noise exceeds the amplitude of the disturbance. Below this frequency, inductive techniques can no longer be used to sense field fluctuations. Section 2.1.1 discusses the sources which limit low-frequency disturbance rejection, while section 2.1.2 experimentally determines the cutoff frequency below which inductive methods fail to give an accurate measure of field fluctuations.
1.3 Flux Regulation Using a Field-Frequency Lock

Although inductive methods are adequate to detect and correct for high frequency field fluctuations produced by the power supply, they are limited by low SNR at low frequencies. Therefore, another method must be used to measure and mitigate the low frequency disturbances due to the cooling water and other sources. Nuclear resonance measurements provide the basis for a feedback system known as the field frequency lock (FFL), which works well at low frequencies. NMR systems measure magnetic field fluctuations by sensing the effect of the time-varying field on the resonant frequency of the nuclei, which is typically above 100 MHz. As a result, they are therefore unaffected by the noise sources that limit the low-frequency SNR of the flux regulator.

1.3.1 Nuclear Resonance Measurements

In order to understand how measurements of the magnetization may be used to regulate magnetic field fluctuations, consider the following explanation of the nuclear magnetization vector, shown in figure 1.11. In thermal equilibrium, the magnetization vector $M_0$ lies along the $z$ axis, parallel to the static magnetic field. If a rotating magnetic field is introduced in the $xy$ plane, the magnetization tips an angle $\theta$ away from the $z$ axis. It precesses about the $z$ axis at a frequency $\gamma(B_0 + \Delta B(t))$, where $\gamma$ is the gyromagnetic ratio in radians per Tesla, $B_0$ is the static magnetic field, and $\Delta B(t)$ represents the undesirable field fluctuations. We define the $uv$ plane, which rotates at angular frequency $\gamma B_0$, and the transverse magnetization $M_{uv}$, which is the projection of $M_0$ into the $uv$ plane. In the absence of temporal field fluctuations $\Delta B(t)$, $M_{uv}$ lies along the $u$ axis and is stationary in the $uv$ plane. In this case, the in-phase transverse magnetization component $M_u$ is maximum and the quadrature component $M_v$ is zero. If magnetic field fluctuations are present, $M_{uv}$ rotates in the $uv$ plane with frequency $\gamma \Delta B(t)$. The angle between $M_{uv}$ and the $u$ axis is given by

$$\phi(t) = \int_0^t \gamma \Delta B(p) dp.$$ (1.4)
In the case where $\Delta B(t) = 0$, $M_v$ and $\phi$ are also zero. Therefore, we may eliminate the magnetic field fluctuations by measuring either $M_v$ or $\phi$ and driving it to zero. In the next subsection, we will see that previous field-frequency lock designs are classified according to which of the two quantities they measure.

Figure 1.11. Diagram of magnetization vector

1.3.2 Field-Frequency Lock Realizations

Although all field-frequency locks use measurements of a NMR signal to reduce magnetic field fluctuations, they can be categorized by their pulse sequence parameters. In his patent, Howard Hill characterizes the previous FFLs according to whether they used high-power pulses at a low repetition rate or low-power pulses at a high repetition rate [6]. We refer to these implementations by the abbreviations LRR and HRR. The HRR class of field-frequency locks excites the nuclei with a series of low-power pulses ($\theta << \pi/2$) at a high repetition rate on the order of several kHz. An illustration of the pulse sequence and response of this type of lock is shown in figure 1.12. Because the pulses are spaced much closer than $T_1$ and $T_2^*$, the NMR response is approximately constant between pulses and is sampled only once in each inter-pulse interval. The spectrometer uses a phase-
sensitive detector to resolve the measured magnetization signal into quadrature components. If the spectrometer is exciting and receiving at the nuclear transition frequency, then the in-phase component (absorption signal) consists of an exponentially decaying signal while the quadrature component (dispersion signal) is zero. If the spectrometer is not receiving on resonance, both channels will have a nonzero component. As the field fluctuates, the resonant frequency changes, and the quadrature channel produces a nonzero response which serves as the error signal for the compensator.

![Figure 1.12. HRR pulse sequence.](image)

In the LRR field-frequency lock, illustrated in figure 1.13, the NMR response is generated by a train of high-power pulses \( \theta = \pi/2 \) at a low repetition rate of around 10 Hz. The resulting NMR signal is known as a free induction decay (FID). It consists of an exponentially decaying sinusoid with a frequency equivalent to the resonant frequency of the nuclei. The FFL samples the FID and uses a phase-locked loop (PLL) to drive the frequency of the FID to that of the transmitter. A phase-sensitive detector outputs the phase difference between the FID and the signal from the transmitter. The compensator attempts to drive this phase difference to zero by changing the magnetic field until the resonant frequency of the nuclei matches the transmitter frequency. Once the FID has decayed past a certain level, it can no longer be used to provide a measure of the resonant frequency. At this time, another high-power pulse excites the nuclei and produces another free-induction decay, and the process repeats. An integrator is used to maintain the level of the correction field during pulses.

Table 1.1 compares the HRR and LRR field-frequency lock implementations with respect to pulse spacing \( T \) and tip angle \( \theta \) and lists the advantages and limitations of each method. Because
the pulse spacing is much smaller for the HRR lock, this implementation has the advantage of a higher bandwidth than the LRR lock. The major advantage of the LRR implementation is that dynamics of the system are set by the instrumentation and not the NMR plant. In the HRR lock, the nonlinear plant dynamics present a significant challenge for control design.

**Table 1.1. Comparison of lock techniques.**

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<th>Low Repetition Rate (LRR)</th>
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<tr>
<td><strong>Pulse Sequence</strong></td>
<td>$T \ll T_2^*, T \ll T_1$, $\theta \ll \pi/2$</td>
<td>$T \gg T_2^*, T &gt; T_1$, $\theta = \pi/2$</td>
</tr>
<tr>
<td><strong>Advantages</strong></td>
<td>Higher bandwidth</td>
<td>Dynamics set by instrumentation</td>
</tr>
<tr>
<td><strong>Disadvantages</strong></td>
<td>Nonlinear NMR dynamics</td>
<td>Lower Bandwidth</td>
</tr>
</tbody>
</table>

Field-frequency lock systems may also be classified according to other aspects of their implementation. Several categories are of particular interest to this thesis, including the type of magnet used (resistive, superconducting, or permanent), the controller instrumentation, the control algorithm, and the type of spectrometer used (continuous-wave (CW) or pulsed). The literature review classifies previous work according to these categories as well as the HRR and LRR methods of design. For the research efforts that used CW spectrometers, the controller implementation is equivalent to the method that uses low-power pulses at a high repetition rate.

### 1.3.3 Literature Survey

Over the past six decades, researchers have implemented various systems that used NMR to regulate magnetic field fluctuations. The first field-frequency lock was published by Packard in 1948 [13]. In 1957, research team Baker and Burd [2] described a FFL which preserved the resonance condition by varying the spectrometer frequency to follow changes in the resonant
frequency due to field fluctuations. In 1960, Pound and Freeman [16] use a similar design to lock an oscillator to the NMR frequency for a different purpose. In their 1970 paper, Packer and Strike [14] presented the first description of a field-frequency lock which used high power pulses at a low repetition rate. This method was further developed in two 1978 publications by research teams Hoult, Richards, and Styles [8] and Kan, Gonord, Fan, and Sauzade [9]. Hoult’s team was the first to consider a model of the NMR plant in the design of a FFL [8]. In 1979, Hill published a patent [6] describing a dual mode field-frequency lock. His design combined the HRR and LRR approaches by including a provision to switch from one to the other depending on the proximity to resonance. In 2000, Soghomonian’s research team [24] implemented a field-frequency lock using a HRR pulse sequence. Her controller was used in conjunction with a compensator based on inductive measurements and was implemented in the Keck high field resistive magnet.

Packard’s FFL [13] was intended for use with magnets used in nuclear spectroscopy, mass spectrographs, and cyclotrons. The FFL was tested with a CW spectrometer in a resistive magnet system, where it augmented a current regulator already present in the system. In Packard’s design, the error signal is determined by a magnetic field discriminator with a time constant of two seconds. The discriminator outputs the quadrature component of the magnetization for a given magnetic field, and the correction signal is generated using a proportional analog control scheme. Packard notes that increasing the proportional control gain eventually causes the regulator to oscillate and suggests that derivative control could be added to allow a higher proportional gain; however, he does not consider a model for determining the control gains.

Most field-frequency locks were designed to regulate the NMR transition frequency, $\gamma B_0$, to a fixed spectrometer frequency. However, the 1957 publication by Baker and Burd describes a FFL which preserves the resonance condition by varying the spectrometer frequency to follow changes in $\gamma B_0$ resulting from field fluctuations. According to the authors, this was done in order to avoid the use of the correction coil, the induction of which would introduce time delays in the response of the controller. Baker’s system included an electromagnet, a CW spectrometer,
and an analog proportional controller. In his paper, he mentions the inclusion of a low pass filter, which would add an integral or phase-lead component to the controller. However, he does not show this component in the block diagram. In their 1960 paper [16], Pound and Freeman describe a system that operates on the same principles as Baker’s field-frequency lock. They use the method to build a super-regenerative oscillator that generates a signal at the NMR frequency.

The research team of Packer and Strike [14] was the first to implement the FFL on a pulsed spectrometer using a sequence of high-power pulses at a low repetition rate. Although they didn’t explicitly state it in their paper, the phase-locked loop is the principle behind their method of locking the spectrometer frequency to that of the free induction decay. In their system, the error signal is determined by the phase difference between the free induction decay (FID) following a 90° RF pulse and a reference signal obtained from the spectrometer oscillator. The correction signal is given by an analog proportional-integral (PI) compensator and injected into the power supply of the electromagnet. The correction signal is updated once after each pulse, and the pulses are spaced far enough apart to prevent the resulting FIDs from interfering. As each FID decays to zero after a time equivalent to approximately ten times the relaxation constant $T_1$, the controller updates every $10T_1$.

Two 1978 publications also describe field-frequency lock systems which use phase-locked loops to regulate field fluctuations. Research teams Hoult, Richards, and Styles [8] and Kan, Gonord, Fan, and Sauzade [9] implemented PLL-based field-frequency locks in superconducting magnets. Both designs used the FIDs from a train of 90° RF pulse as the voltage-controlled oscillator (VCO). The error signal, given by the phase difference between the VCO and the spectrometer oscillator, was fed into an analog PI controller and the resulting correction signal was used to shift the magnetic field toward resonance. Although Packer’s system predates these systems by eight years, there are several significant differences between the earlier and later designs. While Packer’s system samples only one point between pulses, the systems designed by Hoult and Kan use the entire length of the FID to lock the NMR frequency to the spectrometer frequency.
Therefore, these systems have a higher bandwidth than their predecessor.

Another major difference among the publications describing the phase-locked-loop is that Hoult et. al. consider a model of the NMR plant in the analysis of their controller. A high-level block diagram for this controller appears in figure 1.14. The sinusoidal voltage \( v_{\text{ref}}(t) \) is derived from the spectrometer transmitter and serves as a reference frequency \( \omega_r \). The NMR plant results in a free induction decay represented by \( v_{\text{FID}}(t) \), which oscillates at the NMR frequency \( \omega_0 \). The error \( \xi \) is given by the output of the phase sensitive detector, which corresponds to the sine of the phase difference between \( v_{\text{ref}}(t) \) and \( v_{\text{FID}}(t) \). Because the FID decays as \( 1/T_2^* \), the error is given by

\[
\xi = A \sin \theta e^{-t/T_2^*}, \tag{1.5}
\]

where \( A \) is the amplitude of the FID at \( t = 0 \). The phase difference between \( v_{\text{ref}}(t) \) and \( v_{\text{FID}}(t) \) is

\[
\theta = \int_0^t (\omega_0 - \omega_r) dt + \theta_0, \tag{1.6}
\]

where \( \theta_0 \) is the receiver phase. After applying the small angle approximation \( \sin \theta \approx \theta \), the error becomes

\[
\xi \approx A \left[ \int_0^t (\omega_0 - \omega_r) dt + \theta_0 \right] e^{-t/T_2^*}. \tag{1.7}
\]

![Figure 1.14. Block diagram of controller.](image)

To understand why changing the value of the magnetic field will drive \( \omega_0 \) to \( \omega_r \), we assume
that the spectrometer operates on resonance if the total magnetic field sensed by the nuclei is $B_0$. As the spectrometer frequency is fixed at $\omega_r$, this implies that $\omega_r = \gamma B_0$, where $\gamma$ is the gyromagnetic ratio of the nuclei. Define the magnetic field fluctuations as $B_d(t)$. By applying a correction field $B_c(t)$ that exactly cancels $B_d(t)$, the proportional controller drives $B_\Sigma(t)$ to $B_0$. As the NMR frequency is given by $\omega_0 = \gamma B_\Sigma(t)$, $B_\Sigma(t) = B_0$ corresponds to the case where the NMR frequency is equal to the spectrometer frequency, $\omega_0 = \omega_r$. If this is the case, $\xi$ is a constant which can be regulated to zero with the application of integral control at time $t_1$.

Besides regulating the error to zero, the integrator serves another purpose. The voltage in the integrator is held constant when the next RF pulse is applied, allowing the controller to lock over several pulses [8]. Figure 1.15 plots the error $\xi$ as a function of time; the sharp spikes represent the 90° RF pulses. The PI controller drives the error to zero over several periods of the pulse sequence, as expected.

![Figure 1.15. Reduction of temporal instabilities with field frequency lock [8].](image)

In 1979, Hill published a patent [6] describing a dual mode field-frequency lock implemented with a pulsed spectrometer. In "search mode", which is active far from resonance, the system uses the phase-locked-loop approach to drive the NMR frequency to resonance. As soon as the frequency passes through resonance, the system switches into "lock mode," which is characterized
by low amplitude pulses at a high repetition rate. In this mode, the dispersion voltage provides the error signal which drives the analog proportional controller. In his patent, Hill claims that his controller switches to the "lock mode" close to resonance in order to eliminate low-frequency instabilities.

In their 2000 paper, Soghomonian, Sabo, Powell, Murphy, Rosanske, T. A. Cross, and Schneider-Muntau describe a field-frequency lock developed for the Keck high-field resistive magnet. Although the system uses a pulsed spectrometer, the pulse sequence consists of low-power pulses at a high repetition rate of about 5 kHz. The dispersion signal is generated by sampling the response due to the pulses, which are transmitted 300 Hz off resonance. The error is obtained by comparing the dispersion signal to a reference voltage; as the pulses are not transmitted on resonance, the reference signal is nonzero. The error signal drives an analog proportional controller, and the resulting correction signal sums with that of the flux regulator based on inductive measurements. Together they shift the magnetic field toward resonance.

Figure 1.16 demonstrates the ability of this FFL to minimize low-frequency field perturbations in the Keck resistive magnet. The vertical scale shows the deviation from resonance, and hence the temporal change in magnetic field. The thick and thin lines represent the locked and unlocked signals, respectively. When used in conjunction with the flux regulator, the lock reduced variations from 8.5 ppm to 0.9 ppm [24].

1.3.4 Shortcomings of Previous Implementations

The limitations of the existing field-frequency locks are due to two major factors: the methods used to design the controllers, and the system instrumentation. Most researchers failed to consider the underlying NMR model in designing their controllers. Instead, they implemented a proportional or PI controller and tuned its gains until the most desirable response was achieved. The only exception is the effort by Hoult’s team [8], who included a model and analysis describing their phase-locked-loop approach. Without a model for the NMR plant, researchers failed to
consider certain important factors as they implemented their controllers. For instance, none of the researchers who implemented a HRR field-frequency lock considered whether the amplitude of the magnetic field fluctuations would push their linear controllers out of the linear region of operation of the plant. This oversight is especially significant in Soghomonian’s work [24] because of the high amplitude of magnetic field fluctuations in the Keck resistive magnet. Furthermore, researchers Packard [13] and Packer and Strike [14] implemented systems that updated very slowly compared to the NMR time constants. As most NMR experiments have very short durations, their systems would not acquire measurements quickly enough to adequately attenuate magnetic field fluctuations. However, neither publication included an analysis of the impact of their choice of controller parameters on the response of the system.

Another shortcoming of the existing field-frequency locks stems from the instrumentation used to implement the controllers. Most of the systems used analog instrumentation to generate the correction signal, and Hill’s patent [6] described the use of discrete digital components such as flip-flops. However, none of the research teams used a dedicated digital signal processor (DSP) to implement their controllers. A DSP has several advantages over analog control implementation. First, it offers the user greater flexibility in implementing control designs. In an analog system, implementing a change in the control design requires time-consuming changes in the hardware.
In a DSP, control designs are updated with simple software modifications. The use of a DSP also allows complex systems to be implemented quickly and relatively easily, while increasing the complexity of an analog system requires the addition of hardware components. Finally, the digital circuits in a DSP are more robust to noise and electromagnetic radiation than analog circuits.

The shortcomings in control design and instrumentation are explained by the fact that most FFL designs were attempted by researchers with extensive backgrounds in the physics and instrumentation of NMR but little experience in control theory or implementation. As a result, except for the paper by Hoult, there have no designs based on a model of the NMR plant that take advantage of advances in control theory and instrumentation.

Each of the two methods of implementing a field-frequency lock has its own benefits and disadvantages. Because of the long interval between pulses, the phase-locked-loop approach is ineffective at attenuating high frequency fluctuations. If the magnetic field is fluctuating slowly compared to the pulse repetition rate, the integrator will store the value of the correction field from pulse to pulse so that the controller can achieve lock over several periods of the sequence. However, if the magnetic field fluctuations are fast compared to the pulse repetition rate, the signal stored in the integrator will not provide an accurate measure of the correction field necessary to attenuate the field fluctuations. Pulsing faster would allow the controller to better keep up with the time-varying magnetic field; however, if the pulses are closer together, the controller has less time to lock in each interval. Because it samples faster than the LRR field-frequency lock, the HRR controller is better equipped to deal with higher frequency magnetic field fluctuations. However, this system is approximately linear only in a small region around the resonant condition. A linear controller using the HRR approach is unable to handle disturbance amplitudes larger than the width of this linear region. The phase-locked-loop approach is not affected by this limitation; it results in an approximately linear system regardless of the amplitude of the magnetic field disturbance.
The field-frequency lock described in this thesis uses a sequence of low power pulses at a high repetition rate. This method was chosen over the phase-locked-loop method because of limitations imposed by the hardware of the system. Although the FFL shares the same basic structure as many of the previous implementations, the controller is unique because it is designed based on a dynamic model of the plant. Due to the inherent nonlinearities of this model, the FFL suffers the same limitations as others in the HRR category. In this case, however, knowledge of the plant model allows us to determine a priori what range of disturbance amplitudes is acceptable for the linear controller.

1.4 Thesis Contributions and Organization

Low-frequency magnetic field fluctuations present a major obstacle to the goal of performing NMR experiments in resistive magnets [24]. Successful integration of a controller that reduces these fluctuations is essential to preparing resistive magnets for NMR. In existing field-frequency locks, control gains are set by trial and error. Although these systems are adequate to remove magnetic field drift in superconducting magnets, they are unable to reject the high-amplitude, high-frequency disturbances in resistive magnets. Furthermore, their ad hoc implementation prohibits analysis of their limitations.

This thesis addresses the need for a model-based linear controller which is adequate to reject disturbances below a certain amplitude and frequency. Analysis of its limitations is essential to the eventual goal of adapting the controller for use in resistive magnets. The contributions of this thesis are the following.

- **Nonlinear plant model:** The thesis considers a nonlinear dynamic model for the relationship between the magnetization and magnetic field.

- **Small-signal plant model and linear control design:** Design of the sampled-data controller is based on a linearized model of the nonlinear plant.
• **Predicted limitations of controller:** The nonlinear plant model is used to predict the system response to magnetic field disturbances of various amplitudes and frequencies.

• **Measured limitations of controller:** The predicted response of the nonlinear system is verified experimentally.

The thesis is organized as follows. In order to motivate the need for a control technique that will attenuate low frequency fluctuations, Chapter 2 quantifies the limitations of the inductive sensor and examines the sensitivity of the standard deviation of spin echo phase to field fluctuations of various frequencies. The next sections discuss the derivation and simulation of several models for the NMR system, including the small signal model later used for control design. The chapter concludes by presenting experimental measurements that verify the models presented in the previous section.

Chapter 3 discusses the sampled-data field frequency lock feedback controller. It begins by describing the process used to design the controller and then presents simulation results in which the feedback system is integrated with a model of the NMR plant. Finally, the chapter describes implementation of the field frequency lock using a dSPACE digital signal processor and shows experimental results of the operation of the controller on the actual plant.

Chapter 4 summarizes the performance of the controller for both constant and sinusoidally-varying disturbance fields. It also discusses the major limitations inherent to the design of the controller. The chapter concludes with a discussion on overcoming these limitations in order to adapt the system for use in resistive magnets.
This chapter examines the indirect method of measuring magnetic field fluctuations by observing their effect on an NMR signal. Section 2.1 motivates the use of this method by exploring the limitations of the inductive sensor at low frequencies. By examining the impact of low-frequency fluctuations on the standard deviation of the phase at the center of a repeated spin echo experiment, section 2.2 motivates the need for a system that will target this class of fluctuations. In section 2.3.1, a nonlinear dynamic model is derived for the NMR response to the field frequency lock pulse sequence. The following subsections simplify this model to yield steady-state and linear dynamic models. In section 2.4, the three models are simulated using measured parameters from the actual system. Section 2.5 experimentally verifies the simulation results for the three models.
2.1 Limitations of Inductive Measurements

This section discusses mechanisms that reduce the signal-to-noise ratio of inductive measurements at low frequencies, and therefore motivates the advantage of NMR as a method for sensing low-frequency fluctuations. Noise in inductive measurements can arise from many sources. In this section, we consider several sources that determine the smallest magnetic field fluctuations that can be observed. We carefully distinguish between magnetic field noise, which is the signal that we wish to measure, and the noise sources which obscure these measurements. Magnetic field noise is not included in these noise sources. The three sources considered are the thermal noise of the pickup coil, the 1/f noise in the pickup coil and integrating preamplifier, and the noise due to the cabling between the pickup coil and preamplifier.

Figure 2.1 depicts the inductive measurement system consisting of the pickup coil and integrating preamplifier. In the presence of a time-varying magnetic field, the pickup coil generates an open-circuit voltage \( v_B(t) \) that is proportional to the time derivative of the field according to Faraday’s law of induction. Assuming the magnetic field fluctuations consist of sinusoidal components at frequencies \( \omega_p \) with corresponding amplitudes \( B_p \), \( v_B(t) \) has the form

\[
v_B(t) = \sum_p \omega_p NAB_p \cos(\omega_p t + \phi_p),
\]

where \( N \) and \( A \) are the number of turns and cross-sectional surface area of the pickup coil. The output of the pickup coil is fed into the integrating preamplifier, which amplifies the signal and removes the frequency dependence. The output of the integrating preamplifier is measured with a dynamic signal analyzer (DSA), which yields the spectra of the magnetic field fluctuations.

In practice, this method of measuring field fluctuations is subject to noise resulting from the pickup coil, preamplifier, and cabling between the two. First, we will consider thermal noise due to the self-resistance of the pickup coil. Figure 2.1 shows that the pickup coil is modeled as a series combination of a self-resistance \( R \) and a self-inductance \( L \). The self-resistance \( R \) gives rise
Figure 2.1. Inductive measurement system, including electrical model of the pickup coil.

to a thermal noise with power spectral density

$$S_t = 2k_BT \frac{W}{Hz} \quad (2.2)$$

where $k_B = 1.3810^{-23} m^2 \cdot kg \cdot s^{-2} \cdot K^{-1}$ is the Boltzmann constant and $T$ is the coil temperature in Kelvins [25]. Therefore, we expect the noise at the input of the preamp to have a voltage spectrum defined by

$$V_t = \sqrt{2k_BT R} \frac{V}{\sqrt{Hz}} \quad (2.3)$$

For the pickup coil used in this study, $R = 114 \, \Omega$.

Figure 2.1 defines the frequency response of the integrating preamplifier as

$$H_{IP}(j\omega) = \frac{K}{j\omega\tau + 1} \quad (2.4)$$

where $K = 4700 \, V/V$ and $\tau = 0.44$ seconds. From the figure, it is clear that all measurements are taken at the output of the preamplifier; however, in the following analysis, we sometimes refer to the equivalent magnitude spectra at the input of the preamplifier. These spectra are obtained by scaling the output of the integrating preamplifier by the inverse of the magnitude response of the preamplifier $|H_{IP}(j\omega)|^{-1}$. Other magnitude spectra are plotted in terms of the equivalent magnetic field within the pickup coil. These plots are obtained by scaling the
preamplifier output data by the inverse of the magnitude responses of the pickup coil \((\omega NA)^{-1}\) and preamplifier \(|H_{1p}(j\omega)|^{-1}\).

As the DSA is only used to display the spectra at the output of the integrating preamplifier, it is not considered a part of the inductive measurement system. However, in order to accurately interpret the DSA measurements, we must account for its resolution bandwidth \(B_R\). When a spectrum analyzer is used to measure an incoherent signal such as thermal noise, the measured value of the linear spectrum of the noise scales proportionally with \(\sqrt{B_R}\). Although the DSA is a FFT analyzer, we can approximate it conceptually as a bank of \(M\) parallel bandpass filters [27]. Then the resolution bandwidth is given by \(f_{\text{max}}/M\), where \(f_{\text{max}}\) is the highest frequency measured [27]. Unless otherwise specified, the DSA measurements in this section were taken using a resolution bandwidth of \(B_R = 4\) Hz.

We now show several calculations and measurements used to determine the SNR limitations of the inductive measurement system. First, we calculate the predicted thermal noise at the input of the integrating preamplifier due to the self-resistance of the pickup coil. Next, we measure the equivalent noise at the input of the preamplifier when it is terminated by a 110 \(\Omega\) resistor. Using a curve-fitting algorithm, we obtain an equation for the baseline of this spectra and compare this equation with the predicted thermal noise. Finally, we measure the noise sensed by the pickup coil in the Keck magnet at 0 Telsa and suggest several explanations for the features of this spectrum. Figure 2.2 presents the four curves and is followed by a discussion of the results.

The noise floor for the inductive measurement system is limited by thermal noise arising from the pickup coil self-resistance. We predict the voltage spectrum of this noise at the input to the integrating preamplifier, as measured by the DSA with a bandwidth of \(B_R = 4\) Hz. As the preamplifier has an infinite input impedance, this spectrum is given by [25]

\[
V_{t,\text{meas}} = \sqrt{\frac{1}{2\pi} \int_{-2\pi B_R}^{2\pi B_R} 2k_BT R d\omega} = \sqrt{4k_BT R B_R} \text{ V rms. (2.5)}
\]
We evaluate equation 2.6 using $T = 295$ K and $B_R = 4$ Hz. In a later experiment, we model the pickup coil with a fixed 110 Ω resistor. Accordingly, in equation 2.6 we use 110 Ω as the value of $R$ even though 114 Ω is the measured self-resistance of the pickup coil. From equation 2.6, we find that $V_{t,meas} = 2.68$ nV rms, or -171 dBV rms. This value represents the theoretical noise floor for the inductive measurement system when the noise due to the preamplifier and cabling is neglected. In figure 2.2, it is represented by the dash-dotted curve.

In order to measure the noise due to the self-resistance of the pickup coil, we perform the experiment shown in figure 2.3. In this experiment, the input of the integrating preamplifier is terminated by a 110Ω resistor, which approximates the self-resistance of the pickup coil. The output of the preamplifier is measured by the DSA, and the measured magnitude spectrum is scaled by $|H_{IP}(j\omega)|^{-1}$ to provide the equivalent spectrum at the input of the preamplifier. The resulting spectrum is shown as the solid curve in figure 2.2.

In equation 2.6, we predict a flat spectrum for the noise at the input to the integrating preamplifier. The shape of the measured spectrum requires some explanation, as it differs greatly from this prediction. In describing the measured spectrum, it is useful to define two regions...
Figure 2.3. Experimental setup for measuring the noise due to the pickup coil self-resistance and integrating preamplifier.

corresponding to frequencies below and above 130 Hz.

In the lower frequency range, the baseline of the experimental data may be approximated by the dashed curve in figure 2.2. This curve is defined by

\[ y = \frac{3.2 \times 10^{-8}}{f^{0.68}} + 12.2 \times 10^{-9}, \]  \hspace{1cm} (2.7)

where \( f \) is frequency in Hz. Equation 2.7 is the sum of two terms, a constant and a "1/f" term that is inversely proportional to \( f^{0.68} \). The constant term represents the thermal noise due to the self-resistance of the pickup coil and internal noise of the integrating preamplifier. Note that the measured value of this term, 12.2 nV, is about 13.2 dB larger than the predicted value of 2.68 nV. This increase is defined as the noise figure of the integrating preamplifier. The 1/f term is due to 1/f noise. Unlike thermal noise, 1/f noise is poorly understood. As its name suggests, it is a function of inverse frequency, but no well-defined formula exists to relate the two. Instead, 1/f noise \( V_{1/f} \) has the general form

\[ V_{1/f} = \frac{b}{f^a}, \]  \hspace{1cm} (2.8)

where \( b \) and \( 0 < a < 2 \) must be experimentally determined. Comparing equations 2.7 and 2.8, we find that \( b = 3.2 \times 10^{-8} \) and \( a = 0.68 \) for the spectrum in figure 2.2.

We now consider the equivalent input spectrum at frequencies greater than 130 Hz. In this regime, the measured data no longer fits the curve defined by 2.7. Instead, the noise voltage appears to be increasing with frequency. This trend is erroneous, and results from the method used to generate the equivalent spectra at the input of the preamplifier. Recall that these spectra
were obtained by multiplying the preamplifier output by the inverse magnitude response of the preamplifier, given by

\[ |H_{IP}(j\omega)|^{-1} = \frac{\sqrt{\omega^2 \tau^2 + 1}}{K}. \]  

(2.9)

Scaling by this response is equivalent to differentiating the output, which reverses the integrating action of the preamplifier. However, if thermal noise is introduced in the preamplifier after the integrator stage, it will not be integrated and its output spectrum will appear flat. Differentiation will produce a spectrum that increases proportionally with frequency. As this linear trend is observed in the measured spectra above 130 Hz, we assume that this data is due to noise introduced after the preamplifier stage and ignore this portion of the plot.

By terminating the input of the preamplifier by the self-resistance of the pickup coil and measuring the resulting spectra, we obtain a more accurate estimate of the noise floor of the inductive measurement system than the thermal noise estimate given by equation 2.6. In order to obtain an even better estimate, we consider measuring the spectra at the output of the integrating preamplifier when the input is connected to the pickup coil and the pickup coil is placed inside the Keck magnet. Figure 2.4 shows a plot of this measurement when the Keck is operating at 7 Tesla.

For this measurement, the resolution bandwidth of the DSA was 2 Hz. The vertical axis shows the magnitude of the magnetic field sensed by the pickup coil in decibels relative to 1 Gauss rms. The magnetic field plot was obtained by scaling the preamplifier output data by the inverse transfer function of the pickup coil and preamplifier. Because the plot includes only frequencies above 1 Hz, the integrating preamplifier may be approximated as a pure integrator with magnitude response \((K/\tau)/\omega\). The largest component of the magnetic field fluctuations occurs at 60 Hz and has an amplitude of about 0.18 Gauss rms. Because this peak results from a coherent signal (a 60 Hz sinusoid), the measurement does not scale with the resolution bandwidth of the DSA.
Figure 2.4. Magnetic field fluctuations within the Keck resistive magnet operating at 7 Tesla.

The spectrum in figure 2.4 represents magnetic field noise. Recall that this is actually the signal that the inductive sensor measures and not the noise that obscures the measurements. Therefore, it cannot be used to determine the noise floor of the inductive measurement system. However, if we measure the output of the integrating preamplifier when the input is connected to the pickup coil, the pickup coil is placed inside the Keck magnet, and the magnet is turned off, the resulting spectrum corresponds to background noise and not magnetic field noise. This spectrum can be used to determine the noise floor of the inductive sensor.

In figure 2.2, the dotted curve shows the equivalent preamplifier input due to the background noise within the Keck magnet. Two salient observations arise from this data. First, we note that the baseline of the background noise in the Keck is over 30 dB higher than the baseline of the solid curve, which was obtained by terminating the preamplifier with a 110 Ω resistor. The origin of this extra noise is unknown; however, one explanation for the increase could be the noise in the cable between the pickup coil and preamplifier.

Second, we observe that the dotted curve exhibits the same linear dependence on frequency as the solid curve corresponding to the spectrum obtained by terminating the preamplifier with
a 110 Ω resistor. Once again, this trend might be caused by thermal noise introduced in the preamplifier after the integrator stage. However, there is another possible cause. In this case, the voltage at the input of the preamplifier corresponds to the voltage at the output of the pickup coil, which represents the time derivative of the magnetic field within the coil. Therefore, the linear dependence on frequency could be a result of time-varying magnetic fields produced by induced currents in the cable between the pickup coil and the preamplifier.

Whatever its origin, it is clear that the noisy environment in the Keck will define the noise floor of the inductive measurement system. Currently, we have no low-frequency measurements of this environment. Therefore, we assume the SNR of the inductive measurement system is limited by the 1/f noise and thermal noise expressed in equation 2.7. This equation represents only noise due to the preamplifier and self-resistance of the pickup coil; the true noise floor is much higher due to the presence of other noise sources in the Keck system. Therefore, this assumption will provide a lower bound on the lowest disturbance amplitude that may be sensed with the inductive system at a given disturbance frequency.

Equation 2.7 defines the noise floor for the inductive measurement system as measured at the input of the integrating preamplifier. In order to determine the equivalent noise floor in Gauss rms, we scale by the inverse of the magnitude response of the pickup coil \((\omega NA)^{-1}\). In figure 2.5, the solid line shows the noise floor in decibels relative to 1 Gauss rms. The dashed line corresponds to the amplitude of the 60 Hz component of the magnetic field fluctuations in the Keck magnet operating at 7 T. It intersects the extrapolated noise floor at about 0.015 Hz. This implies that a magnetic field disturbance with a frequency of 0.015 Hz and an amplitude of 0.18 Gauss rms will have a SNR of approximately 0 dB. Magnetic field fluctuations with lower frequencies or lower amplitudes will fall below the noise floor of the inductive measurement system.
2.2 Standard Deviation of Spin Echo Phase

An NMR pulse sequence consisting of two closely spaced RF pulses produces a waveform known as a spin echo. Because the spin echo response is an NMR signal that is unaffected by the ringdown of the RF probe, it is used to measure temporal fluctuations in the main magnetic field. The pulse sequence parameters set the time at which the center of the echo occurs. In the absence of temporal magnetic field fluctuations, the phase at the center of the spin echo remains constant over multiple trials of the experiment. The presence of a temporally fluctuating magnetic field introduces significant phase variations among trials, as explained in the paper by Sigmund, Calder, Thomas, Mitrovic, Bachman, Halperin, Kuhns, and Reyes [21]. The standard deviation of spin echo phase is a metric used to gauge the effect of temporal magnetic field fluctuations on an NMR signal. A high value of the standard deviation results from phase incoherence among trials and indicates the presence of significant temporal field fluctuations.
2.2.1 Definition of Spin Echo Phase

The spin echo sequence is an NMR pulse sequence that consists of two radio frequency (RF) pulses and an acquisition window. This particular sequence produces a waveform called a spin echo, the center of which occurs at the time to echo (TE). Before the first pulse is applied, the magnetic moments of the nuclei are aligned along the axis of the static magnetic field \( B_0 \), generally denoted as the \( z \) axis. The \( \pi/2 \) pulse rotates the nuclei into the \( xy \) plane, where the precessing magnetic moments induce a signal in the NMR probe. This voltage decays exponentially, in part because the nuclei have slightly different precession frequencies due to a spatial variation of magnetic field over the sample. This exponential decay is called a free induction decay (FID). The destructive interference can be reversed by applying a \( \pi \) pulse \( TE/2 \) seconds after the \( \pi/2 \) pulse. The \( \pi \) pulse causes the magnetization of the nuclei to rephase and add constructively, producing the spin echo.

![Spin echo sequence and response.](image)

Figure 2.6 shows an example of the in-phase magnetization component \( M_u \) of the spin echo signal. Equation 2.10 defines \( M_u \) as a function of time \( t \) as

\[
M_u(t) = e^{-TE/T_2} \cdot e^{-[(t-T_E)/T_2^*]} \cdot \cos(\omega_r (t - TE) + \phi(TE)),
\]

(2.10)
where \( \omega_r \) is the difference between the resonant frequency of the nuclei and the spectrometer frequency, \( T_2 \) and \( T_2^\ast \) are time constants defined by the NMR sample and magnet system, and \( \phi(TE) \) is the phase \( \phi \) of the NMR signal at time \( t = TE \), which corresponds to the center of the spin echo.

If the NMR spectrometer transmits and receives on resonance, \( M_u \) has a maximum at time \( TE \). If the spectrometer is operating off resonance, the maximum value of \( M_u \) will occur at a time different from \( TE \). The latter case is represented in figure 2.6. The phase \( \phi(TE) \) at the spin echo center is proportional to the time difference between \( TE \) and the time corresponding to the maximum value of \( M_u \). Therefore, if the spectrometer is operating on resonance, \( \phi(TE) \) is zero. In the presence of a time-varying magnetic field \( B(t) \), the resonant frequency of the nuclei will vary as \( \gamma B(t) \), forcing the spectrometer to operate off resonance. In this case, \( \phi(TE) \) is the sum of a constant component introduced by the quadrature phase detector and a time-varying component due to the temporal magnetic field fluctuations. In terms of the magnetic field, the time-varying portion is evaluated at the center of the spin echo as

\[
\phi(TE) = \gamma \int_0^{TE/2} B(p)dp - \gamma \int_{TE/2}^{TE} B(p)dp \tag{2.11}
\]

where the magnetic field \( B(t) \) is equal to the sum of the static magnetic field \( B_0 \) and the temporal fluctuations \( B_f(t) \) [21], [10].

The standard deviation of the phase at the spin echo center describes the consistency of the echo waveform from one sequence to the next. As such, it also characterizes the temporal fluctuations of the applied magnetic field. The standard deviation of \( N \) values of \( \phi \) is

\[
\sigma_{SE} = \sqrt{E \left\{ [\phi_k(TE) - E\{\phi_k(TE)\}]^2 \right\}}. \tag{2.12}
\]

If there are no field fluctuations, \( B(t) \) and \( \phi(TE) \) are constant, yielding a standard deviation of
zero. This case was shown in figure 1.5(a), which depicted a series of spin echo signals obtained using a superconducting magnet with negligible field fluctuations. The phase at the spin echo centers are nearly identical; correspondingly, the standard deviation of the phase is approximately zero. Conversely, a high standard deviation implies a large phase difference among echo trials, which suggests the existence of substantial field fluctuations. An example appears in figure 1.5(b), which shows a series of spin echo signals obtained using the Keck resistive magnet. In this case, the phase at the spin echo center varies from trial to trial due to the significant time-varying component of the magnetic field.

2.2.2 Experimental Measurements

Section 1.2.1 describes Li’s implementation of a flux regulator based on inductive measurements to attenuate field fluctuations at 60 Hz harmonics. In order to show the improvements afforded by the flux regulator, the section presents magnitude spectra of the temporal magnetic field fluctuations in the Keck resistive magnet with and without feedback compensation. As an indirect metric for quantifying the performance of his controller, Li compares $\sigma_{SE}$ for series of spin echo experiments run with and without feedback compensation.

In order to predict $\sigma_{SE}$, background noise is neglected and the field fluctuations are approximated by the first twenty-five harmonics of the power supply ripple. Figure 2.7 shows the measured and predicted reduction in $\sigma_{SE}$ afforded by the proportional and PLL-IMP control schemes. The predicted values of $\sigma_{SE}$ were generated using the definition for standard deviation shown in equation 2.12, where the expectation is approximated by

$$E\{x\} = \frac{1}{N} \sum_{k=1}^{N} x.$$  (2.13)
The phase at the $k^{th}$ echo is given by

$$
\phi_k(TE) = \gamma \int_{(k-1)T}^{TE/2+(k-1)T} B_f(p)dp - \gamma \int_{UE/2+(k-1)T}^{TE+(k-1)T} B_f(p)dp
$$

$$
= \gamma \sum_{n=1}^{25} \frac{B_n}{n\omega_0} \left\{ \sin\left(n\omega_0 \left[(k-1)T + TE/2\right]\right) - \sin\left(n\omega_0(k-1)T\right) - \sin\left(n\omega_0 \left[(k-1)T + TE\right]\right) \right\}
$$

(2.14)

In equation 2.14, background noise is neglected, and the field fluctuations $B_f(t)$ are approximated by the sum of the first 25 harmonics of the power supply ripple

$$
B_f(t) = \sum_{n=1}^{25} B_n \cos(n\omega_0 t).
$$

(2.15)

When predicting $\sigma_{SE}$ for the proportional and phase-lead-lag IMP closed-loop systems, $B_f(t)$ is approximated by the sum of the first 25 harmonics of the diminished power supply ripple that results when each controller is applied.

Substituting equation 2.14 into equation 2.12 yields

$$
\sigma_{SE}(TE) = \frac{\gamma}{N} \sqrt{\sum_{k=1}^{N} \left( X_k(TE) - \frac{1}{N} \sum_{m=1}^{N} X_m(TE) \right)^2}
$$

(2.16)

where

$$
X_k(TE) = \sum_{n=1}^{25} \frac{B_n}{n\omega_0} \alpha_{n,k}(TE)
$$

(2.17)

and

$$
\alpha_{n,k}(TE) = 2 \sin\left(n\omega_0 \left[(k-1)T + TE/2\right]\right) - \sin\left(n\omega_0(k-1)T\right)
$$
\[ -\sin(n\omega_0 [(k-1)T + TE]). \] (2.18)

Figure 2.7 shows that the values predicted for the standard deviation of spin echo phase are consistently lower than those measured. This discrepancy results from the assumption made in equation 2.15, which approximates \( B_f(t) \) as the sum of the first twenty-five harmonics of the power supply ripple. In reality, \( B_f(t) \) also contains components at frequencies lower than 60 Hz and higher than 1.5 kHz.

2.2.3 Sensitivity to Low-Frequency Field Fluctuations

In order to determine whether low-frequency or high-frequency fluctuations have a greater impact on \( \sigma_{SE} \), we calculate the sensitivity of \( \sigma_{SE} \) to \( B_n \), the amplitude of the \( n^{th} \) harmonic of the power supply ripple. From the sensitivity function, we can learn how changes in \( B_n \) affect \( \sigma_{SE} \) for various values of \( TE \). The magnitude of the sensitivity function represents the size of the change in \( \sigma_{SE} \) due to a change in \( B_n \). The sign of the sensitivity function indicates whether the
changes in $B_n$ and $\sigma_{SE}$ have the same or opposite signs.

Equation 2.19 defines the formula for the sensitivity of $\sigma_{SE}$ to $B_n$ as

$$S_{B_n}^{\sigma_{SE}}(TE) = \frac{\partial \sigma_{SE}}{\partial B_n} \frac{B_n}{\sigma_{SE}(TE)}. \quad (2.19)$$

Using equations 2.16 through 2.18,

$$\frac{\partial \sigma_{SE}}{\partial B_n} = \frac{\gamma N}{2} \left( \sum_{k=1}^{N} \left( X_k(TE) - \frac{1}{N} \sum_{m=1}^{N} X_m(TE) \right) \right)^{-1/2} \times \sum_{k=1}^{N} \left\{ \left( X_k(TE) - \frac{1}{N} \sum_{m=1}^{N} X_m(TE) \right) \right\} \times \frac{1}{n\omega_0} \left( \alpha_{n,k} - \frac{1}{N} \sum_{m=1}^{N} \alpha_{n,m} \right). \quad (2.20)$$

Equation 2.19 is numerically evaluated using equations 2.16 and 2.20.

Figure 2.8 shows the sensitivity of $\sigma_{SE}$ to $B_n$ for TE values of 0.5 ms, 2 ms, and 3.5 ms. As TE increases, $\sigma_{SE}$ becomes more sensitive to amplitude changes in low-frequency harmonic components. Since low-frequency fluctuations have a larger impact on $\sigma_{SE}$, they are targeted for attenuation.

### 2.3 FFL Pulse Sequence and Response Dynamics

As discussed in chapter 1, there are three categories of techniques that use measurements of the NMR signal to attenuate field fluctuations: point by point within a FID, points across different FIDs, and a combination of both approaches [6]. In the existing experimental systems for both the 7 Tesla superconducting and Keck resistive magnets, the hardware is set up for the second method. For this reason, the thesis focuses on this approach. However, unlike all earlier attempts at this method, this thesis

1. obtains a nonlinear model of the process
2. by considering a steady-state model, finds limits on the maximum amplitude and frequency that can be tolerated by a linear control scheme, and

3. designs a linear control system based on the small-signal model.

Section 2.3.1 describes the nonlinear model. The steady-state and small-signal models are derived from the nonlinear model in sections 2.3.2 and 2.3.3, respectively.

### 2.3.1 First Principle Discrete Time Model

Consider the FFL sequence of RF pulses of width $t_w$ and spacing $T$, as shown in figure 2.9, applied to a sample in the presence of a static magnetic field $B = B_0 + \Delta B$ directed along the $z$ axis. This subsection presents a discrete-time model of the magnetization $M$ as a function of $\Delta B$, as derived by Schiano et. al. [18]. The model is derived by mapping $M = [M_u, M_v, M_z]^T$ first from time $t_k$ to time $t_k + t_w$ and then from time $t_k + t_w$ to time $t_{k+1}$.

At $t_k$, the magnetization has some value denoted by $M(t_k)$. During the time interval from $t_k$ to $t_k + t_w$, the applied RF pulse produces a magnetic field $B_1$ along the $u$ axis, causing the
magnetization to rotate in the $uz$ plane. The angle of rotation is given by

$$\theta = \gamma B_1 t_w,$$

(2.21)

where $\theta$ is measured in radians and $\gamma$ is the gyromagnetic ratio measured in radians per second per Gauss. Assuming $t_w$ is very small compared to the relaxation and decay constants of the NMR sample, the rotation matrix

$$V(\theta) = \begin{bmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{bmatrix}$$

maps $M(t_k)$ to $M(t_k + t_w)$ as

$$M(t_k + t_w) = V(\theta)M(t_k).$$

(2.22)

If we define the operating frequency of the spectrometer as $\gamma B_0$, on-resonance excitation of the nuclei occurs if the magnetic field is $B_0$. In this case, the magnetization $M$ is stationary in the rotating reference frame for $t$ between $t_k + t_w$ and $t_{k+1}$. As the total magnetic field is actually given by $B(t) = B_0 + \Delta B(t)$, the magnetization vector precesses about the $z$ axis at a frequency equal to $\Delta \omega(t) = \gamma \Delta B(t)$. Between times $t_k + t_w$ and $t_{k+1}$, the magnetization sweeps out an angle

$$\phi = \int_{t_k + t_w}^{t_{k+1}} \gamma \Delta B(\tau) d\tau.$$
We make two assumptions to simplify the relationship between $\phi$ and $\Delta B(t)$. First, we assume that $\Delta B(t)$ remains at a constant value over the interval from $t_k + t_w$ to $t_{k+1}$. In this case, equation 2.23 reduces to

$$\phi = \gamma \Delta B [t_{k+1} - (t_k + t_w)]. \quad (2.24)$$

Second, we assume that $T$ is much greater than $t_w$, in which case $T - t_w$ can be approximated as $T$ and the angle $\phi$ is given by

$$\phi = \gamma T \Delta B. \quad (2.25)$$

Between $t_k + t_w$ and $t_{k+1}$, the $M_u$ and $M_v$ components decay as $e^{-T/T_2}$ and the $M_z$ component relaxes back to the thermal equilibrium magnetization $M_0$ as $e^{-T/T_1}$. Due to the combination of precession, relaxation, and decay, the magnetization maps from $t_k + t_w$ to $t_{k+1}$ as

$$M(t_{k+1}) = A(T_1, T_2) W(\phi) M(t_k + t_w) + B(T_1), \quad (2.26)$$

where

$$A(T_1, T_2) = \begin{bmatrix} e^{-T/T_2} & 0 & 0 \\ 0 & e^{-T/T_2} & 0 \\ 0 & 0 & e^{-T/T_1} \end{bmatrix},$$

$$B(T_1) = \begin{bmatrix} 0 \\ 0 \\ M_0 (1 - e^{-T/T_1}) \end{bmatrix},$$

and

$$W(\phi) = \begin{bmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Although equation 2.26 is derived under the assumption that the magnetization is sampled imme-
diately before the RF pulse at time $t_{k+1}$, the equation is valid for any sampling instant between $t_k + t_w$ and $t_{k+1}$ as long as $T << T_1, T_2$. In this case, $e^{-T/T_1}$ and $e^{-T/T_2}$ are approximately unity, and the longitudinal and transverse magnetization components change only negligibly in the interval between the pulses.

To obtain the map from $M(t_k)$ to $M(t_{k+1})$, we substitute equation 2.22 into equation 2.26. We assume that $\theta$ is fixed in time and calculate the following difference equation for $M$ as a function of $\phi$ by denoting $M(t_k)$ as $M(k)$ and $M(t_{k+1})$ as $M(k+1)$

$$M(k+1) = AW(\phi(k))VM(k) + B.$$ (2.27)

Equation 2.27 fully describes the response of the magnetization vector $M(k)$ to changes in the input $\Delta B(k) = \phi(k)/(\gamma T)$. Because the matrix $W$ contains trigonometric functions of the input variable $\Delta B$, the difference equation is nonlinear. As this property makes the system difficult to analyze, we derive two simplified models by placing restrictions on the parameters $T_1/T$, $T_2/T$, $\theta$, and $\phi$.

### 2.3.2 Steady-State Response to Constant Disturbance Field

If the disturbance field $\Delta B(k)$ remains at a constant value $\Delta B_0$ over many sample instants, the magnetization $M(k)$ will eventually reach a steady-state value. In this case, we can approximate equation 2.27 by a nonlinear but instantaneous map from $\Delta B_0$ to $M$.

In steady-state, $M(k) = M(k+1) = M^{ss}$ for $k$ sufficiently large, and equation 2.27 is rewritten as

$$M^{ss} = \bar{A}\bar{W}(\phi)\bar{V}M^{ss} + \bar{B}.$$ (2.28)

To derive the steady-state model, we assume $T << T_1, T_2$ and $\phi, \theta \approx 0$. This allows us to use
Taylor expansions to simplify $A$, $B$, $W$, and $V$ to

\[
\bar{A} = \begin{bmatrix}
1 - \frac{T}{T_2} & 0 & 0 \\
0 & 1 - \frac{T}{T_2} & 0 \\
0 & 0 & 1 - \frac{T}{T_1}
\end{bmatrix},
\]

\[
\bar{B} = \begin{bmatrix}
0 \\
0 \\
M_0 \left(\frac{T}{T_1}\right)
\end{bmatrix},
\]

\[
\bar{V} = \begin{bmatrix}
1 & 0 & -\theta \\
0 & 1 & 0 \\
\theta & 0 & 1
\end{bmatrix},
\] and

\[
\bar{W}(\phi) = \begin{bmatrix}
1 & -\phi & 0 \\
\phi & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

Solving equation 2.28 for $M^{ss}$ yields

\[
M^{ss} = (I - \bar{A}\bar{W}\bar{V})^{-1}\bar{B},
\] (2.29)

where $(I - \bar{A}\bar{W}\bar{V})$ is evaluated as

\[
(I - \bar{A}\bar{W}\bar{V}) = \begin{bmatrix}
\frac{T}{T_2} & \phi \left(1 - \frac{T}{T_2}\right) & \theta \left(1 - \frac{T}{T_2}\right) \\
-\phi \left(1 - \frac{T}{T_2}\right) & \frac{T}{T_2} & \phi \theta \left(1 - \frac{T}{T_2}\right) \\
-\theta \left(1 - \frac{T}{T_1}\right) & 0 & \frac{T}{T_1}
\end{bmatrix}.
\]
By approximating $1 - T/T_1$ and $1 - T/T_2$ as unity and $\phi \theta$ as zero, $(I - \bar{A}\bar{W}\bar{V})$ is simplified to

\[
(I - \bar{A}\bar{W}\bar{V}) = \begin{bmatrix}
-\frac{T}{T_2} & \phi & \theta \\
-\phi & \frac{T}{T_2} & 0 \\
-\theta & 0 & \frac{T}{T_1}
\end{bmatrix},
\]

Substituting this simplified expression into equation 2.29 yields equations for all three components of the steady-state magnetization as a function of known parameters $M_0$, $T_1$, $T_2$, $T$, $\theta$, and $\Delta B_0 = \phi/(\gamma T)$, as shown in equations 2.30 through 2.32. These equations result from Abragam’s analysis in *Principles of Nuclear Magnetism* [1].

\[
M_{u}^{ss} = \frac{-\theta T_2 M_0}{1 + (\frac{\theta}{T})^2 T_1 T_2 + (\gamma \Delta B_0 T_2)^2} \tag{2.30}
\]

\[
M_{v}^{ss} = \frac{-\gamma \Delta B_0 \theta T_2^2 M_0}{1 + (\frac{\theta}{T})^2 T_1 T_2 + (\gamma \Delta B_0 T_2)^2} \tag{2.31}
\]

\[
M_{z}^{ss} = \frac{1 + (\gamma \Delta B_0 T_2)^2}{1 + (\frac{\theta}{T})^2 T_1 T_2 + (\gamma \Delta B_0 T_2)^2} M_0 \tag{2.32}
\]

Figure 2.10 shows the steady-state value of $M_v$ as a function of $\Delta B_0$. In a region near $\Delta B_0 = 0$, $\Delta B_0$ and $M_{u}^{ss}$ have an approximately linear relationship. The following analysis derives the slope and width of this linear region, as defined in figure 2.10.

The slope of the linear region is the slope of the tangent line at $\Delta B_0 = 0$. It is calculated as

\[
\frac{\partial M_{u}^{ss}}{\partial \Delta B_0} \bigg|_{\Delta B_0 = 0} = \frac{-\gamma \theta T_2 M_0}{1 + \theta^2 \frac{T}{T_2}}. \tag{2.33}
\]

The width of the linear region is determined by locating the values of $\Delta B_0$ that yield the minimum
and maximum values of $M_v^{ss}$. At these extrema, $\Delta B_0$ satisfies

$$\frac{\partial M_v^{ss}}{\partial \Delta B_0} = 0$$ \hfill (2.34)$$

Equation 2.34 has the two solutions

$$\Delta B_0^{max} = \frac{1}{\gamma T_2} \sqrt{1 + \theta^2 \frac{T_1 T_2}{T^2}}, \quad \text{and}$$

$$\Delta B_0^{min} = \frac{1}{\gamma T_2} \sqrt{1 + \theta^2 \frac{T_1 T_2}{T^2}}$$ \hfill (2.36)$$

Between $\Delta B_0^{max}$ and $\Delta B_0^{min}$,

$$M_v^{ss} \approx \Delta B_0 \cdot \frac{\partial M_v^{ss}}{\partial \Delta B_0} \bigg|_{\Delta B_0=0}$$ \hfill (2.37)$$

Subtracting equation 2.35 from equation 2.36 gives the full width of the linear region (FWLR) as

$$FWLR = \frac{2}{\gamma T_2} \sqrt{1 + \theta^2 \frac{T_1 T_2}{T^2}}$$ \hfill (2.38)$$
The slope of the linear region represents the DC gain of the dynamic system near $\Delta B = 0$. Close to this point, $M_v(k)$ may be approximated as the product of the slope and $\Delta B(k)$. The width of the linear region determines the range of disturbance amplitudes $\Delta B_0$ that a linear controller can compensate, as the parameters of the linear controller are not calibrated to reject magnetic field disturbances with amplitudes outside the linear region. Because the slope of the curve changes sign immediately outside the linear region, the linear controller would be unstable outside this region. For the system used in this research, the FWLR is measured in tens of mG.

### 2.3.3 Small-Signal Model

In the previous subsection, we developed a static, nonlinear approximation for the full model of the NMR plant. In this subsection, we will linearize the plant around an equilibrium point, producing a dynamic, linear approximation. As a stepping stone, we will first derive the linearization of a generalized nonlinear discrete-time system with one input and one state.

Assume the generalized nonlinear system has the form

$$x(k+1) = f(x(k), u(k)), \quad (2.39)$$

where $x$ is the state, $u$ is the input, and $f$ represents a nonlinear function. A discrete-time system is in equilibrium if the state $x$ remains at the same value for all time. The equilibrium state $x^e$ satisfies $x(k) = x(k+1) = x^e$ for $k$ sufficiently large and so

$$x^e = f(x^e, u^e), \quad (2.40)$$

where $u^e$ denotes the value of the input that will allow the equilibrium condition to persist. For small perturbations around $x^e$ and $u^e$, the nonlinear system may be approximated by a linear system that is derived according to the following analysis.
We denote small perturbations in the state and input as $\delta x$ and $\delta u$, respectively. To obtain the linearization about $x^e$ and $u^e$, we use the first-order Taylor expansion shown in equation 2.42.

$$\delta x(k + 1) + x^e = f(\delta x(k) + x^e, \delta u(k) + u^e)$$
$$\approx f(x^e, u^e) + \frac{\partial f}{\partial x} |_{x^e, u^e} \delta x(k) + \frac{\partial f}{\partial u} |_{x^e, u^e} \delta u(k)$$

By substituting equation 2.40 into equation 2.42, we can eliminate the $x^e$ term from the left hand side of the equation and rewrite it as

$$\delta x(k + 1) = \frac{\partial f}{\partial x} |_{x^e, u^e} \delta x(k) + \frac{\partial f}{\partial u} |_{x^e, u^e} \delta u(k)$$

The linearization of the NMR model derived in section 2.3.1 is analogous to the linearization of the single-state system. The full model for the nonlinear system, given in equation 2.27, can be expressed as

$$\begin{bmatrix} M_u(k + 1) \\ M_v(k + 1) \\ M_z(k + 1) \end{bmatrix} = A W(\phi(k)) V \begin{bmatrix} M_u(k) \\ M_v(k) \\ M_z(k) \end{bmatrix} + B = \begin{bmatrix} f_1(M_u(k), M_v(k), M_z(k), \phi(k)) \\ f_2(M_u(k), M_v(k), M_z(k), \phi(k)) \\ f_3(M_u(k), M_v(k), M_z(k), \phi(k)) \end{bmatrix}.$$

At equilibrium, equation 2.27 can be written as

$$M^e = A W(\phi^e) V M^e + B,$$

where $M^e = [M_u^e, M_v^e, M_z^e]^T$, and $M_u^e, M_v^e$, and $M_z^e$ represent the equilibrium values of $M_u, M_v,$ and $M_z$, respectively. In equilibrium, $\phi$ has a value of $\phi^e = 0$, corresponding to input

$$\Delta B^e = 0.$$
Solving equation 2.44 yields the equilibrium magnetization components

\[
M^e_u = \frac{e^{\frac{x}{T_2}} \sin \theta M_0 \left(1 - e^{\frac{x}{T_1}}\right)}{1 - \cos \theta \left(e^{\frac{x}{T_1}} + e^{\frac{x}{T_2}}\right) + e^{\frac{x}{T_1}} e^{\frac{x}{T_2}}} \quad (2.46)
\]

\[
M^e_v = 0, \quad \text{and} \quad (2.47)
\]

\[
M^e_z = \frac{\left(1 - e^{\frac{x}{T_2}} \cos \theta\right) M_0 \left(1 - e^{\frac{x}{T_1}}\right)}{1 - \cos \theta \left(e^{\frac{x}{T_1}} + e^{\frac{x}{T_2}}\right) + e^{\frac{x}{T_1}} e^{\frac{x}{T_2}}} \quad (2.48)
\]

Equation 2.47 shows that the equilibrium value of \(M^e_v\) is zero. This result agrees with the steady-state analysis in the previous subsection.

The linearization about \(M^e_u, M^e_v, M^e_z\), and \(\Delta B^e\) is given by

\[
M(k + 1) \approx A_{lin} M(k) + B_{lin} \Delta B(k) \quad (2.49)
\]

for small changes in \(M\) and \(\Delta B\). We can express \(A_{lin}\) and \(B_{lin}\) as

\[
A_{lin} = \begin{bmatrix}
\frac{\partial f_1}{\partial M_u} & \frac{\partial f_1}{\partial M_v} & \frac{\partial f_1}{\partial M_z} \\
\frac{\partial f_2}{\partial M_u} & \frac{\partial f_2}{\partial M_v} & \frac{\partial f_2}{\partial M_z} \\
\frac{\partial f_3}{\partial M_u} & \frac{\partial f_3}{\partial M_v} & \frac{\partial f_3}{\partial M_z}
\end{bmatrix}
\]

and

\[
B_{lin} = \begin{bmatrix}
\frac{\partial f_1}{\partial \Delta B} \\
\frac{\partial f_2}{\partial \Delta B} \\
\frac{\partial f_3}{\partial \Delta B}
\end{bmatrix}
\]

For \(f_1, f_2,\) and \(f_3\) are linear functions of \(M_u, M_v,\) and \(M_z,\)

\[
A_{lin} = AW(\Delta B^e)V. \quad (2.50)
\]
\( B_{\text{lin}} \) is evaluated as

\[
B_{\text{lin}} = \begin{bmatrix} 0 \\ b_2 \\ 0 \end{bmatrix},
\]

where

\[
b_2 = \gamma T \frac{e^{-\frac{T}{\tau_2}} \sin \theta M_0 \left( 1 - e^{-\frac{T}{\tau_1}} \right)}{1 - \cos \theta \left( e^{-\frac{T}{\tau_1}} + e^{-\frac{T}{\tau_2}} \right) + e^{-\frac{T}{\tau_1}} e^{-\frac{T}{\tau_2}}} \tag{2.51}
\]

The transfer function from \( \Delta B \) to \( M_\nu \) is given by

\[
\frac{M_\nu(z)}{\Delta B(z)} = [0 1 0] (zI - A_{\text{lin}})^{-1} B_{\text{lin}} \tag{2.52}
\]

\[
= \frac{b_2}{z - e^{-\frac{T}{\tau_2}}} \tag{2.53}
\]

\[
= \frac{g \left( 1 - e^{-\frac{T}{\tau_2}} \right)}{z - e^{-\frac{T}{\tau_2}}} \tag{2.54}
\]

where \( g \) is the DC gain and can be approximated by the slope of the linear region in the steady-state model.

The small-signal model is a good approximation for the nonlinear system around the equilibrium point. Because the model is linear and time-invariant, we can use it as the basis for designing a linear controller and predicting how the controller will behave for small perturbations in the input.

### 2.4 Simulated Responses to FFL Pulse Sequence

The previous section derived three models for the response of NMR system to changes in magnetic field. In this section, we examine each of these models in simulation. First, we use the nonlinear, time-varying model to determine the transient and steady-state response of \( M_\nu \) to magnetic field step changes of various amplitudes \( \Delta B_0 \). In order to test the accuracy of the steady-state model,
we compare the steady-state values of $M_v$ generated using the time-varying model with those obtained from the instantaneous steady-state approximation. Finally, we test the accuracy of the small-signal model by comparing its step response to that of the nonlinear model. In order to numerically evaluate the models, parameters $T$, $T_1$, $T_2$, and $\theta$ are set to their measured values of $T = 500 \mu s$, $T_1 = 300 ms$, $T_2 = 32 ms$, and $\theta = 0.5^\circ$, where details of the measurement of these parameters is described in section 2.5.2. The input to the system is

$$\Delta B(t) = \frac{\phi(t)}{\gamma T}.$$  \hspace{1cm} (2.55)

For the deuterium sample used, the gyromagnetic ratio is known to be $\gamma = 6.5 \text{ MHz/T} = 4.08 \times 10^3 \text{ rad/s/G}$.

The input waveform consists of a step from zero to $\Delta B_0$ at time zero. To ensure that the magnetization is in thermal equilibrium before the step is applied, $\Delta B(t) = 0$ for at least $10T_1$ (approximately 3 seconds) prior to $t = 0$. In order to observe the transient and steady-state response of the magnetization to $\Delta B(t)$, the step is applied for 3 seconds. A plot of the input $\Delta B(t)$ as a function of time is shown in figure 2.11.

Figure 2.12 shows the transient and steady-state responses of the quadrature component of the transverse magnetization ($M_v$) for several values of $\Delta B_0$. The traces in the figure were generated by numerically solving the nonlinear difference equation 2.27, which describes the full model of the NMR plant. As shown in the figure, $\Delta B(t) = 0$ yields $M_v(t) = 0$ for all time. This result is predicted in the derivation of the small-signal model in section 2.3.3. The plots are symmetric about the time axis; for each $\Delta B_0$ resulting in $M_v(t)$, $-\Delta B_0$ yields $M_v(t)$. Step amplitudes $\Delta B_0 = -15 \text{ mG}$ and $\Delta B_0 = 15 \text{ mG}$ correspond to the maximum and minimum steady-state values of $M_v$, which are normalized to $\pm 1$ to eliminate the dependence of $M_v$ on $M_0$. The normalization factor is $0.141M_0$. The nonlinearities in the system are apparent in the transient responses of $M_v$. In a linear system, the transient characteristics of the output are
the same regardless of the value of the input. In figure 2.12, the dashed curves corresponding to $\Delta B_0 = \pm 40 \text{ mG}$ exhibit a greater number of oscillations than the solid curves corresponding to $\Delta B_0 = \pm 15 \text{ mG}$. The further $\Delta B_0$ is from 0 G, the more pronounced the nonlinear effects become.

Figure 2.13 shows the accuracy of the steady-state approximation derived in section 2.3.2. The curve of open circles was generated by simulating the response of the full model to the input shown in figure 2.11. Each circle represents the final value of $M_v$ for some value of $\Delta B_0$ between -250 mG to 250 mG. The thin solid curve was generated using the steady-state model in equation 2.31. The values of $\Delta B_0$ span approximately $\pm 250 \text{ mG}$, corresponding to values of $\phi$ from $-30^\circ$ to $30^\circ$.

The steady-state model was derived by assuming that $T << T_1, T_2$, and $\theta, \phi \approx 0$. The measured values of $T$, $T_1$, $T_2$, and $\theta$ satisfy the approximations made. However, we must determine whether it is valid to use small angle approximations for $\sin \phi$ and $\cos \phi$ for all values of $\phi$ between $-30^\circ$ and $30^\circ$. As $\phi = 30^\circ$ represents the worst case scenario, we will use this value to calculate
Figure 2.12. Simulated output $M_v(t)$ for step perturbation field with several different values of $\Delta B_0$.

the error in the small angle approximations.

Figure 2.13. $M_v^{ss}$ vs. $\Delta B_0$ curve generated from steady-state model.
Given \( \phi = 30^\circ = \pi/6 \) radians, we know that

\[
\sin \phi = 0.5 \quad \text{and} \quad (2.56)
\]
\[
\cos \phi = \frac{\sqrt{3}}{2} \approx 0.866 \quad (2.57)
\]

In equation 2.28, these trigonometric functions were approximated as

\[
\sin \phi \approx \phi = \pi/6 \approx 0.524 \quad (2.58)
\]
\[
\cos \phi \approx 1 \quad (2.59)
\]

For \( \phi = 30^\circ \), the errors in the sine and cosine approximations are 4.7% and 15.5%, respectively. These relatively low percentages show that the small angle approximations are valid at \( \phi \) values less than \( 30^\circ \).

Because it was generated using the full NMR model, the curve of open circles in figure 2.13 is used as a baseline for determining the accuracy of the steady-state model derived in section 2.3.2. It coincides almost exactly with the thin solid curve, which represents the steady-state value of \( M_n \) as calculated using the steady-state model in equation 2.31. The overlap of the two curves demonstrates the accuracy of the steady-state model.

Figure 2.13 also includes a thick solid line to represent the tangent line at \( \Delta B_0 = 0 \) and a thick dashed line which spans the width of the linear region. Substituting the measured values for \( T, T_1, T_2, \) and \( \theta \) into equations 2.33 and 2.38 yield values of \( -18.6M_0 \) V/G and 30.3 mG for the slope and width, respectively, of the linear region. After dividing the value of the slope by the 0.141\( M_0 \) normalization factor, we find that the slope of the normalized response is -132 V/G.

The final goal of the simulations was to verify the accuracy of the small-signal approximation to the nonlinear model. Using the measured values of \( T, T_1, T_2, \) and \( \theta \), and setting \( M_0 = 1 \),
equation 2.53 is evaluated as

\[
\frac{M_v(z)}{\Delta B(z)} = \frac{-0.2883}{z - 0.9845} = g \frac{1 - 0.9845}{z - 0.9845}.
\]  

(2.60)

Evaluating equation 2.33, we find the DC gain

\[
\left. \frac{\partial M_v^{ss}}{\partial \Delta B_0} \right|_{\Delta B_0=0} = g = -18.6 \text{ V/G}
\]  

(2.61)

and substitute it into equation 2.60 to find

\[
\frac{M_v(z)}{\Delta B(z)} = -18.6 \frac{(1 - 0.9845)}{z - 0.9845}.
\]  

(2.62)

In order to verify that the linearized model is a valid approximation to the nonlinear system for small perturbations around the equilibrium input \( \Delta B^e = 0 \), we simulate both models with the input shown in figure 2.11. To show that the small-signal model is valid only near the equilibrium value of \( \Delta B(k) \), we choose two values of \( \Delta B_0 \): one near the center of the linear region and the other corresponding to the maximum of the \( M_v^{ss} \) vs. \( \Delta B_0 \) curve. When evaluated for the measured parameter values, equation 2.38 shows that the \( M_v^{ss} \) vs. \( \Delta B_0 \) curve has a linear region of width 30.3 mG. An input of \( \Delta B_0 = -1 \) mG falls well within the linear region, while \( \Delta B_0 = -15 \) mG corresponds to the edge of the nonlinear region.

Figure 2.14 (a) shows the simulated output \( M_v \) for the nonlinear and small-signal models in the presence of a perturbation field with \( \Delta B_0 = -1 \) mG. The dashed and solid lines represent the linear and nonlinear models, respectively. For this small value of \( \Delta B_0 \), the small signal model is a very accurate approximation of the full nonlinear plant. Figure 2.14 (b) shows the simulated response of \( M_v \) for both models to the perturbation field with \( \Delta B_0 = -15 \) mG. The dashed and solid lines represent the linear and nonlinear models, respectively. This far from equilibrium, the
small-signal model is no longer adequate to describe the nonlinear plant. The small-signal plant fails to model the oscillations in the transient response of $M_v$. Furthermore, the linearized model predicts a steady-state value of $M_v$ that is twice the value obtained using the full plant.

![Graphs showing response of linear and nonlinear models](images/graphs.png)

**Figure 2.14.** Comparison of the response of the linear and nonlinear models for various perturbation fields.

### 2.5 Experimental Verification of Dispersion Signal Models

#### 2.5.1 Instrumentation

All experiments are performed using the 7 Tesla, 52 mm bore superconducting magnet shown in Figure 2.15. At this field strength, the nuclear magnetic frequencies for hydrogen and deuterium are approximately 300 MHz and 46 MHz, respectively. Most experiments also make use of a drive
coil assembly consisting of a correction and disturbance coil, a current driver, a NMR probe, and a spectrometer console, as shown in figure 2.16.

![Image of a superconducting magnet](image)

**Figure 2.15.** 7 Tesla superconducting magnet.

The drive coil assembly, shown in figure 2.17, consists of two coils wound one over another on a 44.28 mm diameter fiberglass tube. The assembly mounts from above the magnet. Both coils were wound using adhesive copper tape 0.125 inches wide and 0.003 inches thick and have identical characteristics. In order to distinguish between the two, we denote one as the disturbance coil and the other as the correction coil. Table 2.1 lists other specifications of the disturbance and correction coils.

The NMR probe is made by Bruker and is used to deliver the RF pulse to the deuterium oxide sample used in these experiments. The probe mounts from below the magnet and is adjusted so that its center coincides with that of the drive coils. The current driver, a wide-bandwidth pulsed gradient amplifier, is model BFA-310A from Resonance Research, Inc. (RRI). Each of
Figure 2.16. Block diagram of instrumentation for NMR measurements.

Figure 2.17. Drive coil assembly, including disturbance and correction coils.
Correction Coil (Disturbance Coil)

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<th>Value</th>
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<td>Copper Tape</td>
</tr>
<tr>
<td>Turn Spacing (mm)</td>
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</tr>
</tbody>
</table>

**Table 2.1.** Specifications of the correction and disturbance coils.

its three channels can source up to 10 amperes pulsed current or up to 3.5 amperes continuous current. The Tecmag spectrometer has three transmit and two receive channels, as well as a field-frequency lock with its own transmitter and receiver.

Figure 2.18 shows the instrumentation setup for experimental verification of the NMR models presented in section 2.2. The dSPACE DS1104 controller board is a real-time DSP processor that is programmed using the Simulink block diagram environment. dSPACE provides additional software to generate C code based on the block diagram and transfer it to the DSP processor on the controller board.

![Instrumentation block diagram for NMR model verification.](image)

**Figure 2.18.** Instrumentation block diagram for NMR model verification.

In Simulink, a signal generator is used to output a voltage \( V_{DAC} \) through the dSPACE digital-to-analog converter (DAC). \( V_{DAC} \) is transmitted through an attenuator to the current driver, which injects a current into the windings of the disturbance coil. The disturbance coil induces
a magnetic field $B_p$, which sums with the field of the superconducting magnet. The purpose of the attenuator is to best use the dynamic range of the dSPACE DAC.

The actuator gain $G_{act}$ from $B_p$ to $V_{DAC}$ is determined as follows. A simple $\pi/2$-acquire deuterium NMR experiment is repeated on the 7 Tesla superconducting magnet while a 0.01 Hz, 1 V peak-to-peak sinusoid is injected into the disturbance coil. The disturbance coil generates a time-varying magnetic field, which shifts the NMR signal from resonance. For the 1 V peak-to-peak injected disturbance, the measured peak-to-peak frequency shift is 660 Hz. The corresponding peak-to-peak magnetic field is found by dividing the frequency shift by the proton gyromagnetic ratio $\gamma = 650 \text{ Hz/G}$. Using this relationship, the gain $G_{act}$ is calculated as

$$G_{act} = \frac{B_p}{V_{DAC}} = \frac{660 \text{ Hz}}{1 \text{ V}} \times \frac{1 \text{ G}}{650 \text{ Hz}} = 1.02 \frac{\text{G}}{\text{V}}.$$  

(2.63)

The Tecmag spectrometer includes a HRR field-frequency lock that uses a proportional-integral-derivative (PID) controller. The pulse spacing and tip angle are fixed, while the user can set the control gains and receiver filter bandwidth. Unfortunately, there is no public domain information regarding the details of the receiver filter. The Tecmag FFL applies the pulse sequence, and the output of the PID controller drives the input to a current amplifier, which powers the correction coil. In order to obtain a signal directly proportional to the measured quadrature component, the integral and derivative gains are set to zero, while P is fixed at a constant value. In this way, the output voltage to the correction coil is proportional to $M_v$. The receiver filter is set to fast to reduce the effect of the Tecmag dynamics on our closed-loop system, and the proportional gain is set to 10,000 to maximize the dynamic range used by the ADC.

The lock pulse sequence shown in figure 2.19 consists of a train of RF pulses of length $t_w$ transmitted on-resonance and spaced $T$ apart. Using an oscilloscope, the pulse sequence parameters were measured as $t_w = 70 \mu s$ and $T = 500 \mu s$. In the time between two pulses, the digital receiver samples the NMR response once. The resulting signal is phased so that its magnitude is
zero for on-resonance excitation ($\Delta B = 0$). For this reason, it is labeled as the dispersion signal, $M_v$. After the sampled-data dispersion signal is converted back to analog, it is sampled once again by the dSPACE ADC. The dSPACE operates at 2 kHz, the original sampling frequency of the spectrometer receiver. MATLAB is used to store and manipulate the acquired data.

![Diagram of field-frequency lock pulse sequence.](image)

**Figure 2.19.** Tecmag field-frequency lock pulse sequence.

The pulse width $t_w$ of the FFL pulse sequence is proportional to the rotation angle $\theta$. Typically, the value of $\theta$ corresponding to $t_w = 70 \ \mu s$ would be found by applying a $\pi/2 - \text{acquire}$ pulse sequence, determining the pulse width $t_{w,180}$ corresponding to $\theta = 180^\circ$, and calculating $\theta$ as $180^\circ \times 70 \ \mu s/t_{w,180}$. Unfortunately, the FFL pulse width is not variable, and so another method was needed to determine $\theta$. Using one of the main transmitter channels on the spectrometer, we determined that $180^\circ$ corresponds to a pulse width of about $58 \ \mu s$. We used an oscilloscope to measure the rms power output by this transmitter and the lock transmitter, and found that the main transmitter output 104 W rms, while the lock transmitter output 0.625 mW rms. The main transmitter output 166,400 times the power of the lock transmitter. Therefore, a $180^\circ$ pulse from the lock transmitter must be $\sqrt{166,400}$ times as long as a $180^\circ$ pulse from the main transmitter, or about 23 ms. For the 70 $\mu s$ pulses in the lock sequence, the corresponding rotation angle is $180^\circ \times 70 \ \mu s/23 \ \text{ms}$, or about $0.5^\circ$.

### 2.5.2 Sample Characteristics

In order to simulate the dispersion models in section 2.2, we measured the relaxation and decay constants $T_1$ and $T_2$ for the NMR sample. The sample of pure deuterium oxide has gyromagnetic
ratio $\gamma = 6.5 \, MHz/T = 4.08 \times 10^3 \, rad/s/G$. The relaxation constant was measured to be $T_1 = 300 \, ms$ using an inversion recovery pulse sequence. The decay constant would typically be measured with a series of spin echo sequences with varying TE. However, the field-frequency lock pulse spacing is 500 $\mu s$, much smaller than typical TE values, which are usually measured in milliseconds. A different sequence is needed to measure $T_2^{eff}$, the effective value of the decay constant.

The sequence used is shown below in figure 2.20. It consists of a $\pi/2$ pulse followed by a train of $\pi$ pulses. The spacing between the first two pulses is 250 $\mu s$ and the following pulses are spaced 500 $\mu s$ apart. A spin echo is acquired between every two $\pi$ pulses at 500 $\mu s$, 1 $ms$, 1.5 $ms$, etc. The peak of the Fourier transform of each spin echo is related to the time $t$ at which the center of the echo occurs by $e^{-t/T_2^{eff}}$.

![Figure 2.20. Repeated π sequence to measure $T_2^{eff}$.](image)

Figure 2.21 shows a plot of the base 10 logarithm of the spin echo Fourier transform peaks as a function of time. The plot is not linear as expected but characterized by peaks and troughs, representing constructive and destructive interference among the spin echoes. The interference occurs because each echo decays as $e^{-t/T_2^*}$, while the time between subsequent echos is much less than $T_2^*$. Therefore, the signal at an echo peak represents the sum of signals from more than one echo. Due to the overlap, this method does not produce an exact value for $T_2^{eff}$. Taking the negative reciprocal of the slope of the line through the peaks yields an approximate $T_2^{eff}$ value of 32 $ms$. 
2.5.3 Measurement of Steady-State Response

In this section, we present experimental results to verify the simulations of the full nonlinear and steady-state models. The step input shown in figure 2.11 was applied to the actual NMR plant using the actuator in figure 2.18 for a range of $\Delta B_0$. However, as the spectrometer transmits and receives slightly off-resonance when $\Delta B_0 = 0$, the range of offset fields was not centered about zero for the actual plant.

In simulation, we assumed on-resonance excitation and acquisition corresponding to a frequency of $\gamma B_0$. Under these circumstances, $M_v$ is zero for a total field of $B_0$, or equivalently, an offset field $\Delta B(t) = 0$. In practice, the spectrometer transmits and receives slightly off-resonance at $\gamma (B_0 + B_{offset})$. In this case, on-resonance operation corresponding to $M_v = 0$ occurs when the total field is $B_0 + B_{offset}$ (when $\Delta B(t) = \Delta B_0 = B_{offset}$). To facilitate comparison between the experimental results and the results of the simulations, this section specifies the amplitude of the step disturbance field in terms of $\Delta B_0 - B_{offset}$. For the simulated systems, $B_{offset} = 0G$; the experimental value of $B_{offset}$ is approximately -252 mG.

Figure 2.22 shows the transient and steady-state responses of $M_v$ for six values of $\Delta B_0$ —
The experimental $M_v(t)$ curves are normalized so that the largest steady-state value of $M_v$ has a mean of one; the mean is based on the last second of data. The experimental results differ from the simulation results in figure 2.12 in several ways. The experimental $M_v(t)$ vs. $t$ curves include high frequency noise, which is not modeled in simulation. Experimentally, the minimum and maximum steady-state values of $M_v$ occur for $\Delta B_0 - B_{offset} = \pm 12$ mG, as opposed to $\Delta B_0 = \pm 15$ mG. However, the most notable difference between the two sets of results is the marked asymmetry in the experimental responses. In simulation, opposite values of $\Delta B_0$ produce $M_v$ responses that mirror each other. The experimental data does not display the same characteristic. For $\Delta B_0 - B_{offset}$ values of 12 mG and -12 mG, the corresponding $M_v(t)$ responses have opposite steady-state values but completely different transient responses. To further illustrate this point, opposite steady-state $M_v$ values of about ±0.65 V are produced by asymmetric $\Delta B_0 - B_{offset}$ values of -38 mG and 62 mG. The asymmetries apparent in the experimental data are likely caused by the spatial inhomogeneity of both the superconducting magnet generating $B_0$ and the disturbance coil generating $\Delta B$, which was not accounted for in the derivation of the nonlinear plant model.

Figure 2.23 shows the similarity between the normalized steady-state responses of the experimental and simulated systems. The curve of open circles was calculated using the steady-state model in equation 2.31, while the curve of open squares represents experimental data. In order to reduce distortion due to the high frequency noise, the experimental steady-state curve was obtained from the average value of $M_v^{ss}$ instead of the last sample of $M_v(t)$. Each square represents the mean value of the last second the $M_v$ response for some value of $\Delta B - B_{offset}$ between -248 mG to 252 mG.

From figure 2.23, we can see that the experimental and simulated steady-state responses have linear regions with approximately the same width. As both curves are normalized so that their maximum values are unity, there is little information in the observation that their linear regions have approximately the same slope. The major difference between the two plots is the asymmetry
present in the experimental results. The minimum experimental value of $M_{ss}^*$ does not occur exactly at -1, although the maximum value is normalized to +1. In general, the experimental steady-state curve is not symmetric about the origin, as the simulation results predict. Once again, this discrepancy is most likely due to the unmodeled spatial inhomogeneities present within the superconducting magnet and disturbance coil.

Equations 2.33 and 2.38 show that the slope has a linear dependence on the tip angle $\theta$, while the peak separation has no dependence on $\theta$. In order to experimentally determine the effect

Figure 2.22. A comparison of the simulated and measured output $M_v(t)$ for a step perturbation field with five different values of $\Delta B_0$. 

Equations 2.33 and 2.38 show that the slope has a linear dependence on the tip angle $\theta$, while the peak separation has no dependence on $\theta$. In order to experimentally determine the effect
of the tip angle on the steady-state parameters, we varied $\theta$ while injecting the input in figure 2.11 for various $\Delta B_0$. As $\theta$ is set by the width of the RF pulse, which is fixed at 70 $\mu$s, and the power level of the spectrometer transmitter, we varied $\theta$ by changing the transmitter power. The steady-state $M_v$ responses were measured for seven different values of transmitter power, as shown in figure 2.24. All tip angles are given as fractions of the largest angle $\theta$.

The slope and peak separation were numerically calculated for each tip angle, and are plotted in figure 2.25. The experimental results bear out the theory for the steady-state equations. Inspection of the figure confirms that the slope has a linear dependence on $\theta$, while the peak separation is independent of $\theta$.

2.5.4 Parametric Identification of Small-Signal Model Parameters

In order to check the accuracy of the predicted small-signal model, we identified the transfer function of the experimental model using batch least-squares estimation (LSE). Before presenting the results of this effort, we explain the derivation of the LSE algorithm [15]. We consider a
Figure 2.24. Effect of tip angle $\theta$ on slope and width of linear region.

Figure 2.25. Slope and FWLR for various tip angles $\theta$.

general first-order system with transfer function

$$\frac{Y(z)}{U(z)} = \frac{b_0}{z - a_0}.$$  (2.64)
In the time domain, this transfer function corresponds to the difference equation

\[ y(k + 1) = a_0 y(k) + b_0 u(k), \tag{2.65} \]

where \( u(k) \) and \( y(k) \) denote the input and output at the \( k^{th} \) time step. Suppose we obtain measurements of \( u(k) \) and \( y(k) \) for values of \( k \) from 0 to \( N - 1 \). As each set of measurements must satisfy equation 2.65 with some error, we have a series of \( N \) equations

\[
\begin{align*}
y(1) &= a_0 y(0) + b_0 u(0) + e(1) \\
&\vdots \\
y(N) &= a_0 y(N - 1) + b_0 u(N - 1) + e(N)
\end{align*}
\tag{2.66}
\tag{2.67}
\]

which we can use to solve for \( a_0 \) and \( b_0 \). Each error term \( e(k) \) represents the error between the measured value of \( y(k) \) and the predicted value given by equation 2.65.

We can express equations 2.66 through 2.67 by the matrix equation

\[ Y = F \Theta + E, \tag{2.68} \]

where

\[
Y = \begin{bmatrix}
y(1) \\
\vdots \\
y(N)
\end{bmatrix},
\quad F = \begin{bmatrix}
y(0) & u(0) \\
\vdots & \vdots \\
y(N - 1) & u(N - 1)
\end{bmatrix}.
\]
\[ \Theta = \begin{bmatrix} a_0 \\ b_0 \end{bmatrix}, \text{ and} \]
\[ E = \begin{bmatrix} e(1) \\ \vdots \\ e(N) \end{bmatrix}. \]

By minimizing the sum of the squared error, we can determine the "least-squares" estimate for the parameter vector \( \Theta \). We first define the cost function \( J(\Theta) \), which is equal to the sum of the squared error terms, as

\[ J(\Theta) = \sum_{i=1}^{N} e^2(i) \]
\[ = E^T E \]
\[ = [Y - F\Theta]^T [Y - F\Theta]. \]

Next, we minimize \( J \) with respect to \( \Theta \) by solving

\[ \frac{\partial J(\Theta)}{\partial \Theta} = 0 \]

for \( \Theta = \Theta_{\text{min}} \). The derivative of \( J \) with respect to \( \Theta \) is calculated as

\[ \frac{\partial J(\Theta)}{\partial \Theta} = -F^T [Y - F\Theta] - [Y - F\Theta]^T F \]
\[ = -F^T Y + F^T F\Theta - Y^T F + \Theta^T F^T F \]
\[ = -2F^T Y + 2F^T F\Theta. \]

Solving 2.72 yields

\[ \Theta_{\text{min}} = [F^T F]^{-1} F^T Y. \]
Equation 2.76 describes the estimates of $a_0$ and $b_0$ that result in minimal error between the measured and predicted values of $y(k)$. However, the least-squares estimation algorithm has several limitations. First, implementation of LSE requires prior knowledge of the order of the numerator and denominator of the transfer function. Therefore, the algorithm will produce the best parameter estimates for a transfer function of that particular order; however, a transfer function of another order might produce an even lower minimum value for the cost function $J$ [15]. Another shortcoming of the LSE algorithm is its sensitivity to outliers in the measured output data. If one sample of $y(k)$ is very different from the others, the error produced by this sample will weigh heavily in the calculation of the minimum cost function. The estimated parameters may not accurately describe the "good" data [15].

From equation 2.53, we know that the small-signal NMR model is represented by a first-order transfer function with one pole and no finite zeros. We estimate the numerator and pole of the transfer function by using the least-squares algorithm. The input $\Delta B(t)$ is a square wave with amplitude 0.01 G and frequency 0.1 Hz, and the output $M_v(t)$ is measured over 10 cycles. To compensate for the slightly off-resonance operation of the Tecmag field-frequency lock, the input is offset by $B_{offset}$, the value of $\Delta B_0$ for which $M_{v\ast\ast}$ is zero. Before applying the LSE algorithm, the offset is removed from both $\Delta B(t)$ and $M_v(t)$. Figure 2.26 shows a plot of the first four cycles of the detrended input and output, as well as the output estimated from the transfer function identified using LSE.

From the LSE algorithm, the transfer function is found to be

\[
\frac{M_v(z)}{\Delta B(z)} = \frac{-0.294}{z - 0.9574} \quad (2.77)
\]

\[
= -6.9 \frac{(1 - 0.9574)}{z - 0.9574} \quad (2.78)
\]

The pole in equation 2.78 is close to that predicted in equation 2.62, with a measurement error of only 2.8 %. However, the DC gain is smaller by a factor of approximately 2.7. As the theoretical
DC gain was determined by equation 2.33 with $M_0$ set to unity; the measured DC gain suggests that at the receiver output, $M_0$ maps closer to $-6.9/ -18.6 = 0.371$ V.
Field-Frequency Lock Design

Using the small-signal linear model derived in chapter 2, this chapter presents the design of a feedback control system that reduces magnetic field fluctuations using NMR measurements. Section 3.1 lists the objectives of the controller and outlines its design. In section 3.2, the controller is simulated for constant and sinusoidal disturbances. Section 3.3 describes the implementation of the controller using Simulink and the dSPACE development platform. Section 3.4 presents experimental results which show the response of the controller to various constant and sinusoidal disturbances. These are compared with the simulation results from section 3.2.

3.1 Dead Beat Control Design

The field-frequency lock is designed with two objectives in mind. The controller must attenuate magnetic field fluctuations under about 10 Hz, and it must result in a stable closed loop system. To fulfill both requirements, we made use of a sampled-data linear control scheme known as a deadbeat controller. The major advantages of the deadbeat controller are its fast response time and its ability to drive a constant magnetic field disturbance to zero. There are also several limitations associated with the controller. The major shortcoming of the deadbeat control design
is that it amplifies high-frequency noise. Another limitation is due to the hardware: as the sample rate of the controller is fixed by the Tecmag field-frequency lock pulse sequence to 500 \( \mu s \), this places a bound on the highest frequency of magnetic field disturbance that can be rejected. Finally, because the controller is based on a small-signal linear model, variations in \( M_v \) must be kept within a certain region for the model to be valid.

The block diagram for the closed-loop system is shown in figure 3.1. The input \( N(k) \) represents the noise observed on \( M_v \) in the experimental system. Because more noise is present on the lock channel than on the main spectrometer channels, we attribute it to measurement noise that originates at the output of the plant, and not process noise, which is a part of the plant itself. Some of the simulations in the following sections account for measurement noise on \( M_v \). In these simulations, the noise \( N(k) \) is introduced as shown in figure 3.1.

The desired transfer function from the disturbance input \( B_d(k) \) to the error in the dispersion component of the magnetization \( M_{v,rr} \) has a zero at DC (\( z = 1 \)) and two poles at the origin. For a linear plant, this implies that the controller cancels a constant disturbance two sample instants after it occurs. The pole is repeated in order to ensure the causality of the controller.

We choose the controller \( G_c(z) \) to yield the closed-loop transfer function

\[
\frac{M_{v,rr}(z)}{B_d(z)} = \frac{K(z - 1)}{z^2},
\]

(3.1)
where $K$ is a parameter determined in the control design. The controller is given by

$$G_c(z) = \frac{s_1 z + s_0}{z - 1}. \quad (3.2)$$

The NMR plant is approximated by the linearized model originally given in equation 2.54 and repeated in equation 3.3

$$G_p(z) = \frac{M_e(z)}{\Delta B(z)} = g \frac{1 - e^{-\frac{T}{T_2}}}{z - e^{-\frac{T}{T_2}}}. \quad (3.3)$$

In equation 3.3, the DC gain $g$ is measured in V/G.

In terms of $G_c$ and $G_p$, the closed-loop transfer function is

$$\frac{M_{corr}(z)}{B_d(z)} = \frac{-G_p(z)}{1 - G_p(z)G_c(z)} \quad (3.4)$$

$$= \frac{-g \left(1 - e^{-\frac{T}{T_2}}\right)}{z - e^{-\frac{T}{T_2}}} \frac{z - e^{-\frac{T}{T_2}}}{1 - g \left(1 - e^{-\frac{T}{T_2}}\right) z^{-1}} \quad (3.5)$$

$$= \frac{-g \left(1 - e^{-\frac{T}{T_2}}\right) (z - 1)}{\left(z - e^{-\frac{T}{T_2}}\right) (z - 1) - g \left(1 - e^{-\frac{T}{T_2}}\right) (s_1 z + s_0)} \quad (3.6)$$

Equating the denominators of equations 3.1 and 3.6 yields the controller coefficients

$$s_0 = \frac{e^{-\frac{T}{T_2}}}{g \left(1 - e^{-\frac{T}{T_2}}\right)}, \text{ and} \quad (3.7)$$

$$s_1 = -\frac{\left(1 + e^{-\frac{T}{T_2}}\right)}{g \left(1 - e^{-\frac{T}{T_2}}\right)}. \quad (3.8)$$

Equating the numerators yields the closed-loop parameter

$$K = -g \left(1 - e^{-\frac{T}{T_2}}\right). \quad (3.9)$$
Substituting equations 3.7 and 3.8 into equation 3.2 yields the final controller transfer function

\[
G_c(z) = \left[ -\frac{1}{g \left( 1 - e^{-\frac{T}{T_2}} \right)} \right] \frac{\left( 1 + e^{-\frac{T}{T_2}} \right) z - e^{-\frac{T}{T_2}}}{(z - 1)}
\]  

(3.10)

The controller \( G_c(z) \) has a pole at \( z = 1 \), corresponding to a discrete-time integrator. The integrator allows the controller to regulate constant disturbances to zero, as expected. The high-frequency gain of the controller is determined by setting \( z = 0 \) in the transfer function. Because \( T << T_2 \), \( e^{-T/T_2} \) is close to unity, and the high-frequency gain is very large. This result is expected in light of the fact that the controller amplifies the high-frequency noise.

To study the effect of the closed-loop system on noise present in \( M_{err}^e \), we consider the transfer function from \( N(z) \) to \( M_{err}^e(k) \)

\[
\frac{M_{err}^e(z)}{N(z)} = \frac{-1}{1 - G_p(z)G_c(z)}.
\]  

(3.11)

Substituting equations 3.3 and 3.10 into equation 3.11 yields

\[
\frac{M_{err}^e(z)}{N(z)} = \frac{-(z - 1) \left( z - e^{-\frac{T}{T_2}} \right)}{z^2}.
\]  

(3.12)

Equation 3.12 shows that \( M_{err}^e(z)/N(z) \) has two poles at \( z = 0 \), corresponding to high frequencies. As expected, the closed-loop transfer function shows that the controller amplifies high-frequency components of the measurement noise \( N(k) \).

In preparation for implementing the controller on noisy systems in simulation and experimentally, control gain \( K_c \) is included in \( G_c(z) \) order to reduce the high-frequency gain of \( M_{err}^e / B_d \). The resulting controller transfer function is given by

\[
G_c(z) = K_c \left[ -\frac{1}{g \left( 1 - e^{-\frac{T}{T_2}} \right)} \right] \frac{\left( 1 + e^{-\frac{T}{T_2}} \right) z - e^{-\frac{T}{T_2}}}{(z - 1)}
\]  

(3.13)
If the controller input is a noisy measurement of \( M_{\text{err}}^v \) and \( K_c \) is not included, the large high-frequency gain of the controller may result in a correction field that drives \( M_v \) out of the linear region of operation. By moving the poles of \( M_{\text{err}}^v(z)/B_d(z) \) and \( M_{\text{err}}^v(z)/N(z) \) away from \( z = 0 \) and closer to \( z = 1 \), the factor \( K_c \) prevents the closed-loop system from becoming unstable at the expense of slowing down its response and reducing low-frequency disturbance attenuation.

The closed-loop transfer functions from \( B_d \) and \( N \) to \( M_{\text{err}}^v \) for general \( K_c \) are given by

\[
\frac{M_{\text{err}}^v(z)}{B_d(z)} = \frac{-g \left(1 - e^{-\frac{T}{T_2(z)}}\right) (z - 1)}{z^2 + z (K_c - 1) \left(1 + e^{-\frac{T}{T_2(z)}}\right) - (K_c - 1) e^{\frac{T}{T_2(z)}}},
\]

(3.14)

and

\[
\frac{M_{\text{err}}^v(z)}{N(z)} = \frac{-(z - 1) \left(z - e^{\frac{T}{T_2(z)}}\right)}{z^2 + z (K_c - 1) \left(1 + e^{\frac{T}{T_2(z)}}\right) - (K_c - 1) e^{\frac{T}{T_2(z)}}}.
\]

(3.15)

The system in figure 3.1 was implemented in simulation with parameter values \( T = 500 \) \( \mu \)s, \( T_2 = 32 \) ms, and \( g = -18.6 \) G/V. Using these values, equations 3.14 and 3.15 are evaluated as

\[
\frac{M_{\text{err}}^v(z)}{B_d(z)} = \frac{0.288(z - 1)}{z^2 + 1.985 (K_c - 1) z - 0.985 (K_c - 1)},
\]

(3.16)

and

\[
\frac{M_{\text{err}}^v(z)}{N(z)} = \frac{-(z - 1) (z - 0.985)}{z^2 + 1.985 (K_c - 1) z - 0.985 (K_c - 1)}.
\]

(3.17)

Figures 3.2 and 3.3 show the magnitude responses from \( M_{\text{err}}^v \) to \( B_d \) and \( M_{\text{err}}^v \) to \( N \) for \( K_c = 1 \) (solid curves) and \( K_c = 0.001 \) (dashed curves). The responses have similar characteristics because \( M_{\text{err}}^v(z)/B_d(z) \) and \( M_{\text{err}}^v(z)/N(z) \) have the same poles. For the system with \( K_c = 1 \), the magnitude increases monotonically due to the fact that both closed-loop transfer functions have a double pole at the origin and a zero at \( z = 1 \), implying low gain at low frequencies and high gain at high frequencies. In the other system, control gain \( K_c \) was set to 0.001 in order to limit the magnitude of \( M_{\text{err}}^v(z)/N(z) \) at high frequencies. Figure 3.3 shows that it is successful
in doing so. However, by moving the closed-loop poles away from \( z = 0 \), the gain \( K_c = 0.001 \) also decreases the controller’s ability to attenuate low frequency disturbances. Below about 10 Hz, using \( K_c = 0.001 \) instead of \( K_c = 1 \) results in 60 dB less attenuation in both magnetic field disturbances \( B_d \) and measurement noise \( N \). Between 10 Hz and 300 Hz, setting \( K_c = 1 \) still affords larger reductions in \( B_d \) and \( N \) than setting \( K_c = 0.001 \), but the improvement is less than 60 dB. Above 300 Hz, greater attenuation in \( B_d \) and \( N \) is achieved for \( K_c = 0.001 \). As expected, reducing \( K_c \) limits the controller’s amplification of high frequency components, at the expense of limiting the attenuation of low frequency components.

![Bode magnitude plot](image)

**Figure 3.2.** Bode magnitude plot for transfer function from \( B_d \) to \( M_{\text{err}}^v \).

### 3.2 Simulation

All simulations ran at 2 kHz, the sampling frequency of the Tecmag spectrometer, to ensure that the simulation and experimental results would be comparable. Parameters \( T, T_2 \), and \( g \) were set to the values defined in section 3.1, resulting in controller transfer function

\[
G_c(z) = K_c \frac{6.89z - 3.42}{z - 1}.
\] (3.18)
The control gain was set to $K_c = 1$ for noiseless simulations ($N(k) = 0$) and $K_c = 0.001$ for noisy simulations ($N(k) \neq 0$).

The first set of simulations was designed to compare the controller responses for the linear and nonlinear plant models with $N(k) = 0$. In order to measure the effectiveness of the controller at attenuating a constant disturbance, a square wave with amplitude 8 mG and frequency 0.1 Hz was used as the disturbance input $B_d$. The disturbance amplitude was chosen to place $M_v$ well within the linear region of operation of the system. The period was chosen to allow $M_v$ to reach steady-state before the next step change was applied. Figures 3.4 shows the disturbance field $B_d$, correction field $B_c$, and magnetization $M_v$ for the linearized plant model. Figure 3.5 details $B_d$, $B_c$, and $M_v$ at individual sample instants around a step in the disturbance field.

As predicted by the double pole in the closed-loop transfer function, the deadbeat controller takes two sample instants to cancel the disturbance field. As the sample time is only 500 µs, this corresponds to a 1 ms response for the linearized plant. In order to better predict how the controller will respond experimentally, the simulation was repeated on the full nonlinear plant model detailed in section 2.2.1. The results are shown in figures 3.6 and 3.7.

A comparison of figures 3.4 and 3.6 shows that the controller responds similarly to the lin-
earized and nonlinear plants, with one notable difference. When the linearized plant is simulated, an overshoot of about 200% characterizes the response of $B_c$ to a step change in $B_d$. In contrast, the overshoot is less than 25% for the simulations using the nonlinear model.

Contrasting figures 3.5 and 3.7 brings to light another fundamental difference. When the
nonlinear plant is simulated, the controller takes approximately six time steps to cancel a step change in the disturbance field, as opposed to the two sample instants observed in the simulations using the linearized plant, and expected for a deadbeat controller with two poles at the origin. These discrepancies arise from the unmodeled nonlinearities of the plant. Therefore, they become
more significant the further the system is perturbed from equilibrium.

Two sets of simulations were performed to test the performance of the field-frequency lock on a range of constant disturbances. The controller, with $K_c = 1$, was first tested in simulation using the nonlinear plant model given in equation 2.27. In the interest of more accurately approximating the NMR plant, the output $M_v$ was augmented with zero-mean noise $N(k)$ acquired open-loop from the actual plant. In the second experiment, the controller was tested in simulation using the nonlinear model and the noisy magnetization signal. In this simulation, $K_c$ was set to 0.001 in order to reduce the controller’s amplification of $N(k)$ and preserve the stability of the closed-loop system. Each experiment had a ten second duration.

The linear regions of the simulated $M_v$ vs. $\Delta B$ curve spans approximately $\pm 15.15$ mG. As the controller is only effective where the small-signal model of the plant is valid, the disturbance field amplitudes were chosen within this region. The field-frequency lock was tested on each plant for eight values of $B_d$ ranging from 1 to 8 mG in 1 mG steps. As a basis for comparison, both plants were also run open-loop for ten seconds. The results of the two experiments are presented in figure 3.8. Disturbance amplitude $B_d$ is shown on the horizontal scale, while the vertical axis displays the corresponding steady-state value of $M_v$. For the system simulated without noise, $M_v$ is taken as the last sample of $M_v(k)$. In the system where noise is present, the values of $M_v$ plotted in figure 3.8 are obtained by averaging the steady-state data after acquiring it. In order to sufficiently reduce the effects of the high frequency noise while ensuring that $M_v$ is in steady-state, each data point on the graph is represented by the mean value of the last five seconds of $M_v(k)$. Although the data points depict average values, $M_v(k)$ was not averaged before being input to the controller.

After the effect of the noise is averaged out of the second simulation, it produces very similar results to the first. The simulated open-loop magnetization is approximately linear with respect to the disturbance amplitude; their relationship is given by the slope of the steady-state $M_v$ vs. $\Delta B$ curve. The closed-loop simulation magnetization is zero for all values of disturbance
Controller effectiveness for various disturbances with constant amplitudes, simulated plants.

As the deadbeat controller is designed specifically for attenuating DC magnetic field fluctuations, it is expected that it will perform best at low frequencies. To determine the range of frequencies for which the FFL provides adequate disturbance rejection, we designed a series of experiments similar to those just explained. Once again, the lock was tested in simulation on both the noiseless and noisy nonlinear NMR models. The two experiments again implemented the controller in equation 3.13, with $K_c = 1$ and $K_c = 0.001$, respectively, and both plants were once again run open-loop to provide a comparison for measuring the disturbance rejection afforded by the controller.

In this series of experiments, the injected disturbance fields were sinusoids with frequencies from 0.06 Hz to 120 Hz and amplitudes within the linear region of the $M_v$ vs. $\Delta B$ curve. In order to determine the effectiveness of the controller, the discrete time Fourier transform (DTFT) was calculated for the open and closed-loop response to each disturbance frequency. The disturbance attenuation due to the controller was measured as the difference between the open and closed-loop DTFT peaks at each disturbance frequency. In order to produce an accurate numerical
approximation to the DTFT at each disturbance frequency, the duration of each experiment was equivalent to the time for 30 cycles of the disturbance sinusoid. Figures 3.9 and 3.10 show the magnitude spectra of $M_v$ plotted in decibels relative to 1 V for each disturbance frequency. Figure 3.11 shows the attenuation due to the controller at each disturbance frequency for the noiseless and noisy systems.

![Figure 3.9. Sinusoidal disturbance rejection using simulated plant without noise.](image)

As expected, the attenuation in $M_v$ decreases with increasing disturbance frequency. For the simulated plant without noise, the dB attenuation in $M_v$ is an approximately linear function of the disturbance frequency, and ranges from 110 dB at 0.06 Hz to only 23 dB at 120 Hz. The controller has a much less significant effect on sinusoidal disturbances in the simulated noisy system; it attenuates the 0.06 Hz fluctuation by less than 50 dB and actually increases the 120 Hz fluctuation by 0.1 dB. This effect is explained by figure 3.2, which compares the magnitude of $M_v^{err}(z)/B_d(z)$ for $K_c = 1$ and $K_c = 0.001$. Below about 10 Hz, the controller with $K_c = 1$ reduces disturbance components by 60 dB more than the controller with $K_c = 0.001$. Above 10 Hz, there is less disparity in the disturbance attenuation offered by the two controllers.
3.3 Implementation

The controller is implemented on the real NMR plant using the instrumentation described in section 2.3.1. Figure 3.12 shows a block diagram of the closed-loop system. The disturbance field
$B_d$ is injected through the disturbance coil on the drive coil assembly, while the correction field $B_c$ is generated by the correction coil. Since one coil is wound directly on top of the other, they have the same characteristics, as shown in table 2.1.

![Block diagram of controller with injected disturbance.](image)

**Figure 3.12.** Block diagram of controller with injected disturbance.

However, there is a slight difference in the generation of the disturbance and correction fields. The output of DAC 1, which is used to produce $B_c$, is reduced by approximately 5x by an external analog attenuator before it reaches the current driver. As discussed in section 2.3.1, the attenuator is in place to allow the use of a greater portion of the dynamic range of the DAC, thereby increasing the resolution of the correction field. The requirements on the resolution of the disturbance field are more relaxed, so there is no need for an analog attenuator at the output of DAC 2.

As found in section 2.3.1, the gain from $B_c$ to the output of DAC 1 is approximately 1 G/V. To ensure that the gain from $B_d$ to the signal generator output is also 1 G/V, the signal generator output is multiplied by one-fifth before it exits the dSPACE board through DAC 2. Matching
these gains is necessary for proper interpretation of the signals acquired within the dSPACE.

As in the system identification experiments, the Tecmag lock receiver filter is set to ‘fast’ to minimize its impact on the observed signal $M_v$, and the proportional gain of the Tecmag controller is set to 10,000 to best use the dynamic range of the ADC. The integral and derivative gains are set to zero so that the receiver outputs a scaled version of the open-loop magnetization $M_v$. The dSPACE samples at 2 kHz, the sampling frequency of the Tecmag field-frequency lock.

When simulating the controller operation, we assumed on-resonance excitation and acquisition corresponding to a frequency of $\gamma B_0$. Under these circumstances, $M_v$ is zero for a total field of $B_0$, or equivalently, an offset field of zero. In practice, the spectrometer transmits and receives slightly off-resonance at $\gamma (B_0 + B_{\text{offset}})$. In this case, on-resonance operation corresponding to $M_v = 0$ occurs when the total field is $B_0 + B_{\text{offset}}$ (when the offset field is $B_{\text{offset}}$).

In simulation, the input to the plant was simply the quantity $\Delta B = B_d - B_c$, as shown in figure 3.1. The error signal was driven to zero for $\Delta B = 0$, or $B_c = B_d$. In the true system, the error signal is driven to zero for $\Delta B = B_{\text{offset}}$, which corresponds to $B_c = B_d$ if $\Delta B$ is redefined as $B_d - B_c + B_{\text{offset}}$. This change is shown in the revised block diagram in figure 3.13.

![Figure 3.13. Closed-loop block diagram for controller operation on physical plant.](image)

As knowledge of $B_{\text{offset}}$ is necessary for successful operation of the controller, a preliminary routine is used to find its value before the controller becomes operational. The Simulink block diagram for the overall system is shown in figure 3.14. The three subsystems 'Find Min, Max', 'Find Boffset', and 'Controller' represent the first, second, and third modes of operation of the system. When enable signal 1, 2, or 3 is greater than zero, the corresponding mode is operational,
and the output of that subsystem is sent to the DAC. 'Find Min, Max' locates the linear region of
the steady-state $M_v$ curve, and 'Find Boffset' searches within the linear region for the magnetic
field value which corresponds to $M_v = 0$. Once $B_{offset}$ is determined, the 'Controller' mode
implements the closed-loop system in figure 3.13.

The 'Find Min, Max' and 'Find Boffset' subsystems operate open-loop. These subsystems
take two inputs: a 100 second bidirectional sweep of the offset field between -0.5 G and 0.5 G,
and the corresponding average value of 2000 samples of $M_v$ from the ADC. Figure 3.15 shows a
plot of the offset field $\Delta B$, $M_v$, and $M_v^{avg}$ as a function of time. From the plot, it is clear that
$M_v$ is very noisy; for this reason, its average value is used in determining $B_{offset}$.

The 'Find Min, Max' routine stores the minimum and maximum values of $M_v^{avg}$ and the
corresponding values of $\Delta B$ for both the increasing and decreasing field sweeps. The 'Find
Boffset' routine uses a zero-crossing detector to determine the values of $\Delta B$ that yield $M_v^{avg} = 0$
on the sweep up and down; $B_{offset}$ is calculated as the average of these two values. Any zero-
crossings that occur outside the region between the two peaks are discarded.

Operating in the 'Controller' mode implements the system shown in figure 3.13. The controller
transfer function in equation 3.10 is evaluated using the linearized system parameters identified
in section 2.3.4. From equation 2.78, we find that $g = -6.9$ G/V and $e^{-T}$ T = 0.9574. In order
to maintain the stability of the closed-loop system, the compensator also includes control gain
$K_c = 0.001$. Altogether, the controller implemented is given by

$$G_c(z) = K_c \frac{6.66z - 3.26}{z - 1}.$$ (3.19)

The 'Controller' subsystem includes a method for switching between open-loop and closed-
loop operation and provides the input to the disturbance coil through the second DAC. 'Con-
troller' takes input $M_v$ and outputs $B_{offset} - B_c$, the opposite of the correction field offset by
$B_{offset}$. This quantity is output to the correction coil through the first DAC. In the magnet, the
Figure 3.14. Simulink block diagram of control scheme and preliminary routines.
total sum of fields is $B_0 + B_{offset} + B_d - B_c$. If $B_d$ and $B_c$ are equivalent, the field inside the magnet is $B_0 + B_{offset}$, yielding on-resonance operation ($M_v = 0$).

### 3.4 Experimental Verification

The controller was implemented experimentally using the system described in the previous section, where the controller transfer function is given by 3.19 with $K_c = 0.001$. Figure 3.16 shows the controller response to a square 0.04 G, 0.1 Hz disturbance field. Disturbance field $B_d$, correction field $B_c$, and magnetization $M_v$ are plotted as functions of time. Comparing the results for the actual plant and simulated nonlinear plant in figure 3.4, we see that the greatest difference is the presence of high-frequency noise in the experimental results. Experimental operation of the controller yields an average value of $M_v$ that is approximately zero. However, the controller also amplifies the high-frequency noise already present in the magnetization. The increase in noise is quantified in the next chapter.

In order to characterize its response to disturbances of various amplitudes, the field-frequency

**Figure 3.15.** Initial sweep to determine $B_{offset}$. 
lock was tested experimentally in the presence of constant disturbances lasting ten seconds and ranging from 1 mG to 8 mG in 1 mG steps. As a basis for comparison, the system was also run open-loop for ten seconds in the presence of each disturbance. The range of disturbances falls well within the linear region of the experimental $M_v$ vs. $\Delta B$ curve, which spans about $\pm12$ mG. Therefore, the experimental small-signal plant model given by equation 2.78 is valid for this experiment, and the linear controller is viable. The results of the experiment are presented in figure 3.17. Disturbance amplitude $B_d$ is shown on the horizontal scale, while the vertical axis displays the corresponding steady-state value of $M_v$. Because of the high-frequency noise present in the system, $M_v$ is once again given by the mean value of the last five seconds of $M_v(k)$. Although the data points depict average values, $M_v(k)$ was not averaged before being input to the controller.

The results from this experiment were compared with the corresponding simulation results shown in figure 3.8. The experimental data exhibits the same trends as the simulation data, although the slope of the open-loop data is less steep in this case. This observation is explained by section 2.5.4, which shows that the DC gain of the simulated small-signal model is approxi-
inately 2.7 times greater than that of the model identified using least squares estimation. As the closed-loop experimental data is approximately zero for all disturbance amplitudes, the controller operates as expected on the true plant.

In order to determine the effectiveness of the field-frequency lock in the presence of time-varying magnetic field fluctuations, we injected sinusoidal disturbance fields with frequencies ranging from 0.06 Hz to 120 Hz and measured the magnetization output $M_v$ of the experimental plant with and without the controller running. In order to obtain an accurate approximation to the Fourier transform of the magnetization for each disturbance frequency, the duration of each experiment was equivalent to the time for 30 cycles of the disturbance sinusoid. The Fourier transforms of the open and closed-loop magnetization were taken for each disturbance frequency, and the results are shown in figure 3.18.

In section 3.2, we examined the effectiveness of the field-frequency lock at attenuating time-varying magnetic field fluctuations in simulation. We now compare the results of the simulations with the results obtained using the experimental plant. Figure 3.19 summarizes the results for
the three systems by showing the attenuation due to the controller at each disturbance frequency.

Figure 3.19. Comparison of sinusoidal disturbance rejection among experimental and simulated plants.

In all three systems, the controller is less effective when faced with higher disturbance frequencies. This trend is predicted by the magnitude response of $M_v^\text{err}$ to $B_d$, as shown in figure
3.2. We can also see that the presence of noise in the system severely limits the effectiveness of the controller, because it is necessary to include control gain $K_c < 1$ to ensure the stability of the noisy systems. Figure 3.3 shows that setting $K_c = 0.001$ limits the amplification of high frequency noise due to the controller, while figure 3.2 shows that it also limits the effectiveness of the controller at attenuating high frequency magnetic field disturbances. As mentioned in section 3.2, the controller reduces the 0.06 Hz disturbance by 110 dB in the simulated noiseless case and by 49 dB in the simulated noisy case. In the experimental case, the controller attenuates this component by only 43 dB.
Chapter 4

Discussion and Future Work

This chapter summarizes the performance of the deadbeat controller and makes several recommendations for future revisions. The first section discusses the effectiveness of the controller at attenuating constant and sinusoidal disturbances, including limitations due to measurement noise and disturbance amplitude and bandwidth. The second section recommends several avenues for further investigation, including characterization of inductive measurement noise, research into the HRR and LRR lock implementations, and improvements to the control design.

4.1 Controller Performance and Limitations

This section discusses the performance of the field-frequency lock on the simulated and experimental systems. Section 4.1.1 describes the effect of the deadbeat controller on measurement noise. Section 4.1.2 examines the response of the controller to constant disturbances of various amplitudes. Section 4.1.3 quantifies the attenuation due to the controller for sinusoidal disturbances of various frequencies.
4.1.1 Effect on Noise

In the previous chapter, we noted that the magnetization measurements included a significant amount of high frequency noise. Assuming that the noise $N(k)$ entered the system at the output of the NMR plant, we calculated the closed-loop transfer function from $N$ to $M_{v}^{err}$ in equation 3.15. Figure 4.1 shows the squared magnitude of $M_{v}^{err}(z)/N(z)$ evaluated using $K_c = 0.001$ and the experimental plant parameters ($g = -6.9 \text{ G/V}, e^{-\tau/2} = 0.9574$) estimated in section 2.5.4. From the magnitude response of the closed-loop system, we expect the controller to attenuate frequency components below about 7.5 Hz, amplify components between 7.5 Hz and about 60 Hz, and leave components greater than 70 Hz unchanged. The largest gain of about 2 dB occurs at approximately 13 Hz, and total attenuation occurs at DC.

In order to experimentally assess the noise increase due to the controller, we use the dSPACE to acquire $M_{v}^{err}$ for 100 seconds ($2 \times 10^5$ samples) with the controller disabled ($B_c = 0$) and no injected disturbance field ($B_d = 0$). Under these conditions, $M_{v}^{err} = -N$, so the data acquired represents both the error signal and the opposite of the measurement noise. We also acquire
100 seconds of $M_{v,err}^t$ while the controller is running and the injected disturbance field is zero. After detrending both sets of data to remove a constant offset, we calculate the rms value of the magnetization error with and without the controller running as

$$M_{v, \text{err, rms}} = \sqrt{\frac{1}{N} \sum_{k=1}^{N} [M_{v, \text{err}}^t(k)]^2},$$

(4.1)

where $N = 2 \times 10^5$. For the open-loop system, $M_{v, \text{err, rms}} = 0.0239$ V, while for the closed-loop system, $M_{v, \text{err, rms}} = 0.0815$ V; the controller more than triples the rms value of $M_{v, \text{err}}^t$. The difference between the highest and lowest peaks of the open-loop noise is about 0.7 V; the controller increases the peak-to-peak value to 0.95 V for the closed loop system.

If the measurement noise $N(k)$ is a wide-sense stationary (WSS) process, its power spectral density (PSD) $S_N(z)$ is related to the PSD of the closed-loop magnetization error $S_{M_{v, \text{err}}^t}(z)$ by

$$S_{M_{v, \text{err}}^t}(z) = \frac{|M_{v, \text{err}}^t(z)|^2 S_N(z)}{N(z)},$$

(4.2)

In order to use this result, we assume that $N$ is WSS and predict the PSD of $M_{v, \text{err}}^t$ using equation 4.2. We use the MATLAB function `pwelch` to numerically approximate the PSD of $N$ and $M_{v, \text{err}}^t$ using Welch’s method.

Figure 4.2 shows the PSD of $N(k)$ and the predicted and measured PSDs of $M_{v, \text{err}}^t$. The predicted PSD of $M_{v, \text{err}}^t$ was obtained by numerically evaluating equation 4.2. There is clearly a large discrepancy between the measured and predicted spectral densities at frequencies below 60 Hz. Although the two share the same general shape, the closed-loop system amplifies the noise much more than predicted. The largest amplification of nearly 30 dB occurs at approximately 6.5 Hz, compared with the predicted maximum of 2 dB at 13 Hz. Above 60 Hz, both the measured and predicted PSDs of the magnetization error coincide with the PSD of the noise.

The large discrepancy between the measured and predicted spectral densities of the closed-
loop magnetization error likely results from at least two sources. First, we assumed that all of the noise on $M_v$ may be represented as measurement noise at the spectrometer output, as shown in figure 3.1. In reality, some of this noise is actually process noise that is embedded within the NMR plant. As we did not account for process noise when modeling the plant, the closed-loop transfer function $M_{v \text{err}}(z)/N(z)$ is not accurate. Second, we assumed that the noise $N$ was a wide-sense stationary process in order to predict the spectral density of $M_{v \text{err}}$ using equation 4.2. The inaccuracy of the predicted model shows that this assumption is probably incorrect.

### 4.1.2 Response to a Constant Disturbance

As shown in the simulated and experimental results in sections 3.2 and 3.4, the controller is very effective at attenuating constant disturbances that correspond to the linear region of the NMR plant. If noise-free measurements of $M_{v \text{err}}$ form the input of the controller, the closed-loop error due to a constant disturbance remains at zero. If the measurements of $M_{v \text{err}}$ include high-frequency noise, the controller drives the average error to zero, but the instantaneous error at any sample instant is nonzero in general. These results are expected due to the frequency response.
of the closed-loop system, which has a zero gain at DC and high gain at high frequencies.

Because the controller is designed from a small-signal model of the nonlinear NMR system, its operating range is limited to the linear region of the plant. Outside the linear region, the slope changes sign, and the control gains based on the old slope render the system unstable. Figure 4.3 shows the effect on the correction field $B_c$ of a large step change in the disturbance field $B_d$. The instant that the change in $B_d$ exceeds the width of the linear region, its relationship to $M_{\text{err}}^v$ is no longer given by the small signal model derived in section 2.2.3. Because the controller is linear, it cannot account for this change in the plant, and produces correction field $B_c$ as if the linearized model were still accurate. However, instead of regulating $M_{\text{err}}^v$ to zero, $B_c$ has the effect of increasing the error signal. The positive feedback loop in the closed-loop system eventually drives $B_c$ to the maximum value allowed by the instrumentation.

![Figure 4.3. Effect of large amplitude change in $B_d$ on controller stability.](image)

4.1.3 Response to a Sinusoidal Disturbance

When the disturbance input is sinusoidal, the attenuation in $M_{\text{err}}^v$ decreases with increasing disturbance frequency. From the simulation results in section 3.2, we saw that for the noiseless
system, the controller attenuates the 0.06 Hz component by 110 dB and the 120 Hz component by 23 dB. When noise is included in the simulation and when the system is implemented experimentally, the controller transfer function includes a gain $K_c = 0.001$ in order to reduce the amplification of the noise and maintain closed-loop stability. The introduction of $K_c$ moves the closed-loop poles away from $z = 0$, resulting in lower disturbance attenuation at frequencies below 300 Hz. For $K_c = 0.001$, the attenuation at 0.06 Hz is 49 dB for the simulated system with noise and 43 dB for the experimental system.

The ability of the digital compensator to attenuate high frequency disturbances is limited by the 2 kHz sampling frequency of the controller. According to the Nyquist sampling theorem, the controller will detect frequency components below half of its sampling frequency, or up to 1 kHz. Practically, the FFL requires at least ten points per cycle to significantly attenuate a periodic disturbance. This restricts the maximum frequency of a controllable magnetic field disturbance to about 200 Hz. However, figures 3.9 through 3.19 show that the controller is inadequate to provide significant rejection even for 100 Hz disturbances. Clearly, the performance of the present controller is severely limited by the frequency of the field fluctuations it attempts to correct. The deadbeat controller produces an infinite loop gain at DC, therefore, this disturbance is effectively eliminated. As disturbance frequency increases, the gain of the deadbeat controller rolls off and disturbances are no longer attenuated.

### 4.2 Recommendations for Future Research

This thesis describes a field-frequency lock which may be successfully used to attenuate magnetic field disturbances with restricted amplitudes and frequencies, such as those artificially injected into the field of a superconducting magnet. The long-term goal of the project is to remove the constraints on the field fluctuations so that the system may be implemented in the Keck resistive magnet. In order to meet this goal, it is suggested that future research be directed along several
avenues in parallel.

A significant research effort is needed to quantify the limits of the inductive measurement system and characterize the low frequency components of the magnetic field fluctuations within the Keck. Knowledge of both the noise floor of the inductive sensor and the Keck disturbance field at low frequencies is necessary for defining performance expectations for the field-frequency lock.

Another important research step is characterization of the noise present on the magnetization signal from the spectrometer. Knowledge of the frequency components of the spectrometer noise would allow us to reject these components when measuring the magnetization. Information on the statistics of the noise is necessary for producing meaningful simulations of the system.

Another area for future research involves a detailed examination of the HRR and LRR implementations as they apply to resistive magnets. Studying the advantages and disadvantages of these two implementations will allow the researcher to make an informed decision as to which is more suited for use in the Keck magnet. As the LRR implementation requires access to both magnetization components, the instrumentation must be altered to provide them if this model is chosen. Even if the lock is designed based on the HRR model, the controller would benefit from the addition of the feedback variable $M_u$. With measurements of both $M_u$ and $M_v$, it may be possible to estimate and exactly cancel magnetic field fluctuations at a range of frequencies.

Finally, significant control design changes are needed if the field-frequency lock is to be adapted for use in resistive magnets. Instead of a linear controller based on a small-signal model of the plant, the future FFL might employ a nonlinear compensation scheme based on the full model of the NMR plant in order to eliminate the restriction on disturbance amplitudes. In order to obtain a cleaner feedback variable, and hence a cleaner compensation field, the next-generation lock should also employ an algorithm to estimate the true value of the magnetization from its noisy measurement.


