The Pennsylvania State University
The Graduate School
The Smeal College of Business

SERVICE-LEVEL AGREEMENTS
AND SUPPLY CHAIN COORDINATION

A Dissertation in
Business Administration
by
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Abstract

In this dissertation I investigate contracting and coordination in a two-tier supply chain. First, in Chapter 2 I experimentally investigate how altering the inventory risk affects both the total supply chain profits and the distribution of profits between the two parties. I evaluate three contracts, each differing in which party incurs the risk associated with unsold inventory; a Push contract where the retailer incurs the risk, a Pull contract where the supplier incurs the risk, and an Advance-Purchase Discount (APD) contract where both parties share the risk. I find that, contrary to standard theory, the APD contract maintains or exceeds the Push contract in terms of both retailer and supplier profits, suggesting that a Push contract should never be considered when an APD contract is a possibility.

In Chapter 3 I narrow the focus of the dissertation and consider only Pull contracts. I investigate three Pull contracts in a controlled laboratory environment: a wholesale price contract and two coordinating contracts. The primary result of this section is that the theoretical benefit of the two coordinating contracts over the wholesale price contract does not perfectly translate into practice. Instead, subjects set wholesale prices too high and the second contract parameter too low in the coordinating contracts. I explore alternative models to explain this behavior and find that a model of anticipated regret fits the data well.

In Chapter 4 I again narrow the dissertation's focus and study service-level agreements (SLAs), a type of contract where a retailer pays a bonus to a supplier when the supplier satisfies a certain fraction of the retailer’s demand (fill rate). Specifically, I investigate a unique class of SLAs which I call “Grace” SLAs. Grace SLAs consider the fill rate separately for each stocking decision, as opposed to aggregating the fill rate, and awards the bonus if the fill rate is achieved a certain number of times out of all stocking decisions. I find that Grace SLAs increase both retailer and supplier profitability beyond that of a wholesale price contract in a number of scenarios.
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To my family
Chapter 1

Introduction

In this dissertation I investigate contracting and coordination in a two-tier supply chain. In Chapter 2 I take a behavioral approach to answering a simple, yet understudied, research question “Does shifting the risk associated with unsold inventory between the supplier and retailer impact total supply chain profits?” Specifically, I test a number of theoretical results in a controlled laboratory setting, and find that both parties, the retailer and supplier, are the same or better off when the parties share the risk of unsold inventory rather than a setting where the retailer incurs all of it.

In Chapter 3 I consider a group of contracts, called Pull contracts, used in supply chains where the supplier always incurs the cost of unsold inventory. In practice, Pull contracts have increased in popularity with the emergence of e-commerce industries. Similar to Chapter 2, I take a behavioral approach, and attempt to answer the following research question “Which Pull contract structure generates the most profits for the retailer?” In theory, many complex contracts should outperform a simple wholesale price contract. However, I find that human subjects, when setting parameters in the more complex contracts, act regretful, driving down their overall profits to a level in line with a simple wholesale price contract.

In Chapter 4 I take a theoretical approach to studying a class of contracts called service-level agreements (SLAs). SLAs state that a bonus be paid from the retailer to the supplier in situations where the supplier carries enough inventory to satisfy a predetermined portion of the retailer’s demand, called the target fill
rate. I consider a setting with a multi-period time horizon, and investigate how the profits of both parties and the supply chain are affected when the supplier must achieve the target fill rate a certain number of stocking decisions over the entire time horizon. Specifically, I evaluate how the presence of “grace” periods, where the supplier is forgiven for potentially missing a target fill rate for one or more stocking decisions over the time horizon, impacts both parties. I find that for longer time horizons, incorporating an extra grace period can improve both the retailer’s and supplier’s profits in a number of numerical simulations.

Lastly, in Chapter 5, I summarize the results of the dissertation and discuss the implications for practitioners.
Chapter 2

The Effect of Inventory Risk on Supply Chain Performance

2.1 Introduction

A primary aim of supply chain management is to reduce costs and improve channel profits. One driver for accomplishing this goal is effective inventory management. In many theoretical models where the supply chain consists of a single retailer and single supplier with uncertain demand, the inventory, and its associated risks of unsold product, lie with the retailer. However, recent advances in technology like the internet and collaborative planning, forecasting, and replenishment, allow supply chains to consider alternative arrangements for the inventory risk. For example, the transaction cost for a retailer to satisfy end demand directly with inventory located at the supplier has been dramatically reduced through the introduction of the internet.

Supply chains operating under wholesale-price contracts generally have three alternative inventory risk arrangements. In the first contract, called a “Push” contract, the supplier offers a single wholesale price to a retailer, and the retailer orders a single stocking quantity in advance of the selling season. In this setting, the supplier produces an amount equal to the retailer’s order, and therefore pushes all of the inventory risk onto the retailer. Push contracts are used in practice for those retailers that purchase and carry products in-store for end consumers.

In a second contract setting, called a “Pull” contract, the retailer offers a single
wholesale price to a supplier\(^1\), and the supplier sets its own production quantity in advance of the selling season. In the Pull contract, the inventory risk lies with the supplier, because individual units of the inventory are pulled by the retailer as the selling season takes place only when demand exists for the product. E-commerce, such as Amazon.com, or supply chains carrying specialty products, often utilize Pull contracts (Netessine & Rudi 2006).

Lastly, a third contract setting is the Advance-Purchase Discount, “APD”, contract. With an APD contract, a supplier offers a discount wholesale price, \(\text{and}\) regular wholesale price to a retailer. A retailer then makes a prebook inventory decision at the discount wholesale price in advance of the selling season, and a supplier, expecting the retailer to potentially run out of stock once demand is realized, sets its production above the prebook order, hoping to sell additional units at the regular wholesale price to the retailer as demand occurs. In this setting, the retailer incurs the risk of unsold product associated with the prebook amount, and the supplier incurs the risk of unsold product for the difference between the production amount and prebook order. One example of a company utilizing the APD contract is O’Neill Inc., an apparel company for water sports (Cachon 2004).

The location of the inventory risk can have serious consequences on total supply chain profits and the split of profits between the two parties in the supply chain. Regarding total supply chain profits, Cachon (2004) shows that taking the inventory risk and shifting it from the retailer to the supplier (moving from the Push contract to the Pull contract) improves total profits, and that an arrangement where both parties can share the risk (moving from the Pull contact to the APD contract) improves total profits even more. Additionally, Cachon (2004) illustrates in the same paper that in a supply chain where one party (the proposer) has substantial bargaining power, and can make a one-shot take-it-or-leave-it offer to the other party (the responder), the division of profits largely favors the proposer. Our objective in this study is to evaluate the effect of inventory risk on the supply chain from an experimental standpoint. Past literature has shown that in simple supply chain settings decision makers are susceptible to a number of behavioral factors, such as bounded rationality (Su 2008), contract complexity (Kalkanci, Chen &

\(^1\)This is equivalent to a retailer offering a commission rate to a supplier for each unit sold through the retailer's store or location.
Erhun 2010), and regret (Davis 2011). In our study, which we believe is the first to directly test the effect of inventory risk on supply chain performance in a laboratory setting, we identify if retailers and suppliers conform with the normative prescriptions of the standard game theoretic models, or if subjects behave in a way that ultimately generates different results than the theoretical recommendations.

Our primary results suggest that the standard theory is correct in that simply switching the inventory risk from the retailer to the supplier increases supply chain profits. However, we find two results in the lab that contradict the standard normative predictions. The first is that the Pull contract generates total supply chain profits equal to those of the APD contract, where the APD contract should exceed the Pull contract. The second difference between our results and the standard theory, is that of all three contracts evaluated, the APD contract results in the most equitable distribution of profits between the two parties, and dominates the Push contract in terms of both retailer and supplier profit, contrary to theory.

Our results translate into one key managerial recommendation: when a supply chain has the flexibility to pick among the three contracts without restrictions\(^2\), the Push contract should not be considered. This stems from two supporting managerial insights, (1) retailers with substantial bargaining power should offer the Pull contract to suppliers (which agrees with theory), as it generates the most retailer profits, and (2) for all other supply chains, the APD contract is best, as it generates the highest profits for the supplier, generates the same or more retailer and supplier profit than the Push contract, and also has the most equitable distribution of payoffs.

In the next section we detail our experimental design and provide a brief overview of the theory regarding the Push, Pull, and APD contracts. Following this, in Section 2.3, we provide a top-down approach of our results, starting with summary statistics and eventually analyzing individual behavior. We then perform a supply chain efficiency analysis to determine which party, the retailer or supplier, impacts total supply chain profits most in Section 2.4. In Section 2.5, we conclude our study with an outline of future research and a summary of our findings.

\(^2\)A practical example of a restriction here would be a supply chain that supplies perishable food items. In this case a Push contract is really the only feasible option.
2.2 Experimental Design

We evaluated three supply chain contracts, each in its own experimental treatment for a 3 x 1 between-subjects design. We selected contract settings that differed in both who proposed the contract terms and who incurred the inventory risk; the retailer, the supplier, or both. In the first setting we evaluated, the Push contract, the supplier proposed contract terms and the retailer incurred the inventory risk. In the second contract setting, the Pull contract, the retailer proposed the contract terms and the supplier incurred the inventory risk. In the third setting, the APD contract, the supplier proposed the contract terms but both parties potentially shared the inventory risk. These three treatments allow us to directly compare if varying the inventory risk between the parties causes the retailer, supplier, or supply chain profits to change.

In the Push and Pull treatments, each round was comprised of three steps. First, the party making contract offers (supplier in Push, retailer in Pull, referred to as the “proposer”) proposed a wholesale price to the other party (referred to as the “responder”). Second, the responder then either rejected the proposer’s offer or set a stocking quantity. Third, following both players’ decisions, demand and profits were realized.

As mentioned previously, the APD contract differs from the Push and Pull contracts in that it requires two wholesale prices, a wholesale and discount wholesale price, and both parties may share the inventory risk. Specifically, the retailer incurs the inventory risk for a prebook quantity ordered in advance of realized demand, and the supplier incurs the inventory risk on the difference between its own production amount and the retailer’s prebook quantity. Therefore, in the APD treatment, each round was comprised of four steps rather than three. First, the supplier proposed both a wholesale price and discount wholesale price to its retailer. Second, the retailer rejected the proposal or set a prebook quantity. Third, the supplier, after receiving the prebook amount (if the offer was accepted), set a production amount. Following these three steps, demand and profits were then realized for both players. Figure 2.1 depicts the decision sequence for all three treatments.

If a responder decided to reject a proposal, in all three treatments, then both
parties earned a profit of 60. The outside option profit of 60 is just below the lowest theoretical prediction of profit for either player under any contract (75 for the supplier under the Pull contract), ensuring that we could use the same outside option for all treatments and that responders should never reject a proposer’s offer that was in line with theoretical predictions.³

Our 3 x 1 between-subjects design totalled 120 subjects. Subjects were randomly assigned their role at the beginning of each treatment, retailer or supplier, which remained fixed for the duration of the session. Subjects made decisions in 30 rounds. Retailers and suppliers were placed into a cohort of 6 to 8 participants, and a single retailer was randomly matched with a single supplier within the cohort in each round, replicating a one-shot game. Each experimental treatment had six cohorts, where the cohort is the main statistical unit of analysis in this study.

We used the same per unit revenue and cost parameters in all three treatments (revenue $r = 15$ and cost $c = 3$), which were known by both parties. We set demand to be a random integer uniformly distributed between 0 and 100 that was independent each round. Table 2.2 summarizes our final design of experiment and number of participating subjects.

We conducted the experiment at a large northeast U.S. university in 2010. Participants in all treatments were students, mostly undergraduates, from a variety of majors. Before each session subjects were allowed a few minutes to read over the instructions themselves. Following this, we read the instructions aloud and

³The experimental profit predictions will be illustrated in the next section.
Table 2.1: Experimental design and number of participating subjects.

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Push</th>
<th>Pull</th>
<th>APD</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subjects</td>
<td>40</td>
<td>40</td>
<td>40</td>
<td>120</td>
</tr>
</tbody>
</table>

answered any questions. Each individual participated in a single session only and was recruited through an online system. Cash was the only incentive offered, where subjects were paid a $5 show-up fee plus an additional amount that was based on their personal performance. Average compensation for the participants, including the show-up fee, was $25. Each session lasted approximately 1 to 1.5 hours and all software was programmed using the zTree system (Fischbacher 2007).

2.2.1 Theoretical Background

Here we provide a short summary of the theory behind all three contracts along with predictions based on our experimental parameters. We refer the interested reader to Cachon (2004) for more details on the theory.

In all three contracts, a retailer $R$ receives revenue $r$, for each unit sold, incurs no fixed ordering costs, and loses sales if demand exceeds inventory. A supplier $S$ produces inventory at a fixed per unit cost of $c$. Let $D$ represent a random variable, with an increasing generalized failure rate (IGFR), for demand with realization $x$, with cumulative distribution $F$. There is full information of all cost parameters, where retailers and suppliers are risk-neutral expected-profit maximizers. Let $S(q) = q - \int_0^q F(x)dx$ represent the expected sales for a stocking quantity $q$. Lastly, let efficiency be defined as the percent ratio between the decentralized supply chain profit and the centralized supply chain profit.

2.2.1.1 Push

For the Push contract, a supplier offers a per unit wholesale price $w$ to a retailer. The retailer sets a stocking quantity $q$, for a given $w$ that maximizes its expected profit, $\mathbb{E}[\pi_R(q)] = rS(q) - wq$. Let $q^*$ be the quantity that maximizes the retailer’s expected profit. It is well known that the optimal stocking quantity for the retailer,
$q^*(w)$, must satisfy
\[ F(q^*(w)) = \frac{r - w}{r}. \] (2.1)

The supplier’s decision under a Push contract is $w$, where $w^*$ maximizes the supplier’s profit, and the supplier’s profit is $\pi_S(w) = (w - c)q^*(w)$. Because there is a one-to-one relationship between $w$ and $q$, we can rearrange (2.1) to solve for $w^*$ in terms of $q$,
\[ w^*(q) = r(1 - F(q)). \] (2.2)

Assuming the retailer sets the stocking quantity according to (2.1), then the supplier should set $w = w^*(q^*)$ from (2.2).

2.2.1.2 Pull

Under the Pull contract, the decisions of the retailer and supplier are reversed compared to the Push contract. A retailer establishes a per unit wholesale price $w$. The supplier then sets a stocking quantity $q$, for a given $w$ that maximizes its expected profit, $E[\pi_S(q)] = wS(q) - cq$. Let $q^*$ be the optimal stocking quantity that maximizes the supplier’s expected profit, $q^*(w)$ must satisfy
\[ F(q^*(w)) = \frac{w - c}{w}. \] (2.3)

The retailer’s decision under the Pull contract is $w$, let $w^* = \arg\max E[\pi_R(w)]$, where $E[\pi_R(w)] = (r - w)S(q)$. As with the Push contract, because there is a one-to-one relationship between $w$ and $q$, we can rearrange (2.3) to solve for $w^*$ in terms of $q$
\[ w^*(q) = \frac{c}{1 - F(q)}. \] (2.4)

Assuming the supplier sets the stocking quantity according to (2.3), then the retailer should set $w = w^*(q^*)$ as in (2.4).

2.2.1.3 APD

Under the APD contract, a supplier proposes two wholesale prices; a regular wholesale price $w$, and a discount wholesale price $w_d$, where $w \geq w_d$. A retailer then sets a prebook stocking quantity $q_R$, where the retailer incurs a cost of $w_d$ for each
unit of the prebook quantity. Following this, a supplier sets a production quantity $q_S$, where $q_S \geq q_R$. The expected profit for the supplier is

$$
\mathbb{E}[\pi_S(w, w_d, q_S)] = w_d q_R + w(S(q_S) - S(q_R)) - c q_S.
$$

The first term represents immediate revenue from the retailer’s prebook quantity, the second term represents the marginal revenue from the retailer running out of its prebook amount and ordering more from the supplier at the regular wholesale price, and the third term is the supplier’s total production cost.

Let $q^*_S$ be the supplier’s optimal production quantity that maximizes the supplier’s expected profit, $q^*_S(w)$ must satisfy

$$
F(q^*_S(w)) = \frac{w - c}{w}.
$$

(2.5)

Where the supplier’s production quantity is independent of $w_d$.

The retailer’s decision is the prebook quantity, $q_R$. Let $q^*_R = \arg \max \mathbb{E}[\pi_R(q_R)]$, where the expected profit for the retailer is

$$
\mathbb{E}[\pi_R(q_R)] = r S(q_R) + (r - w)(S(q_S) - S(q_R)) - w_d q_R.
$$

The first term represents the total revenue from the prebook quantity, the second term the marginal profits when the retailer must purchase extra units from the supplier at the higher wholesale price, and the third term is the total prebook cost. $q^*_R(w, w_d)$ must satisfy

$$
F(q^*_R(w, w_d)) = \frac{w - w_d}{w}.
$$

(2.6)

Under the APD contract, the supplier can achieve 100% efficiency in the supply chain when it sets $w = r$ (and the corresponding $q^*_S(w)$ as in (2.5)). Assuming the retailer plays the best response according to (2.6), then $w_d$ determines the division of supply chain profits between the two parties. For example, if both parties set $q_R$ and $q_S$ optimally, according to (2.6) and (2.5), then, if $w_d = w$ the supplier achieves 100% of the supply chain profits, on the other hand, if $w_d = c$, the retailer receives 100% of the supply chain profits.
2.2.1.4 Experimental Predictions

Table 2.2 illustrates the theoretical predictions for retailer and supplier profit, and supply chain efficiency given our experimental parameters ($r = 15, c = 3$, demand uniform on $[0, 100]$ and the outside option of 60).

Table 2.2: Predicted retailer profit, supplier profit, and supply chain efficiency.

<table>
<thead>
<tr>
<th></th>
<th>Push</th>
<th>Pull</th>
<th>APD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retailer Profit</td>
<td>120.00</td>
<td>337.50</td>
<td>60.16</td>
</tr>
<tr>
<td>Supplier Profit</td>
<td>240.00</td>
<td>75.00</td>
<td>419.84</td>
</tr>
<tr>
<td>Efficiency</td>
<td>75.00%</td>
<td>85.94%</td>
<td>100.00%</td>
</tr>
</tbody>
</table>

Notice that with these predictions the proposer always makes more than the responder (i.e. in the Push contract, the supplier makes more than the retailer). Also in the APD contract the supplier should, in theory, extract 99.99% of the supply chain profits, but because the outside option is 60 he must propose a set of contract terms that will leave the retailer with a profit of at least 60.

Table 2.3 depicts the decisions that correspond to the profit predictions in Table 2.2. Note that in the experiment subjects could not enter more than two decimal places for wholesale prices, and only integers were allowed for stocking quantities.

Table 2.3: Predicted parameters.

<table>
<thead>
<tr>
<th></th>
<th>Push</th>
<th>Pull</th>
<th>APD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wholesale Price</td>
<td>9.00</td>
<td>6.00</td>
<td>15.00</td>
</tr>
<tr>
<td>Discount Wholesale</td>
<td>-</td>
<td>-</td>
<td>10.76</td>
</tr>
<tr>
<td>Quantity/Prebook</td>
<td>40.00</td>
<td>50.00</td>
<td>28.33</td>
</tr>
<tr>
<td>Production</td>
<td>-</td>
<td>-</td>
<td>80.00</td>
</tr>
</tbody>
</table>

2.3 Results

We present our results in a top-down approach. We begin with summary statistics for the supply chain, retailer, and supplier. We then illustrate the average decisions by all subjects along with an analysis of subjects' behavior over time. Lastly, we
look at individual decisions and determine what proportion of subjects achieved profits equal to theoretical predictions.

We calculate the expected profit for each subject’s decision and report it as the “observed profit” to replicate a managerial setting where each decision affects several periods. Also, in an attempt to help interpret the results, for all tables and figures, we will present the proposers’ statistics and decision variables in a boldface font, whereas the responders’ statistics and decisions will be in a regular typeface.

### 2.3.1 Summary Statistics

Figure 2.2 illustrates the predicted and observed supply chain profits along with the corresponding supply chain efficiency (located at the top of each column). We see that for the Push and Pull contracts there is no significant difference between observed and predicted supply chain profits. However, in the APD contract, there is a very large deviation between the observed supply chain profits and the predicted supply chain profits, where this difference is significant at the 5% level (given by a Wilcoxon signed-rank test).

![Figure 2.2: Predicted and observed supply chain profits.](image)

In Figure 2.2 we also see that the observed supply chain profits increase as the supply chain switches from the Push contract to the Pull contract, and from the
Push contract to the APD contract, but that the supply chain profits for the Pull contract and the APD contract are nearly identical. The differences between the Push contract and the other two contracts are statistically significant at the 5% level given by a Mann-Whitney U-test. These results suggest that the normative prediction of shifting the inventory risk from the retailer to the supplier (changing from the Push contract to the Pull contract) to improve supply chain efficiency is correct. However, this increase in efficiency does not exist when moving from the Pull contract to the APD contract. Instead, a Pull contract achieves the same efficiency as an APD contract.

When looking at the observed profits for retailers in Figure 2.3a, we observe that when retailers act as proposers, in the Pull contract, they earn significantly less than what theory predicts (significant at the 5% level with a Wilcoxon signed-rank test). However, when acting as responders, retailers earn the same as theory predicts in the Push contract, and in the APD contract, they achieve profits higher than what theory predicts (significant at the 5% level with a Wilcoxon signed-rank test).

When selecting among the three contracts, a retailer makes the most in the Pull contract, as theory suggests. The retailer profits from the Pull contract are significantly higher than both the Push and APD contracts at the 1% level with a Mann-Whitney U-test. Also note that the Pull contract is the only setting where the retailer gets to propose contract terms.

Observed supplier profits, depicted in Figure 2.3b, are slightly below theory in the Push contract (weakly significant at the 10% level with a Wilcoxon signed-rank test). For the Pull and APD contracts, when looking at the observed supplier profit with respect to the theoretical predictions, the results are essentially opposite to those of retailers; suppliers make more than the theoretical prediction in the Pull contract, when acting as the responder, and less than the theoretical prediction in the APD contract, when acting as the proposer. Both deviations to theory for the Pull contract and the APD contract are significant at the 5% level with a Wilcoxon signed-rank test.

Comparing the supplier profits across the three contracts, we see that the Push and APD contracts are essentially the same, but both are statistically higher than the Pull contract at the 1% level given by a Mann-Whitney U-test. Also, note that
Figure 2.3: Predicted and observed retailer and supplier profits. Bold numbers represent proposers’ results.

in both the Push and APD contracts, the supplier acts as a proposer.

The results thus far indicate that it is beneficial to be the proposer in all three contracts (proposers always make more than responders), which agrees with the theoretical model. However, we find that proposers always make less than what the theory predicts, and responders more than what the theory predicts. This suggests that for all three contracts, the observed split of profits between the two parties is more equitable than the normative prescriptions. In the APD contract, where the distribution of profits should be the most one-sided (12.5% by the retailer and 87.5% by the supplier), we actually find the most equitable split among the three contracts; retailers earn 46.4% of the supply chain profits and suppliers earn 53.6%.

As mentioned in the Introduction, of this analysis, many supply chains currently operate under the Push contract. If we compare the Push contract profits for both the retailer and supplier to the APD results, we find that both parties are the same or better off with an APD contract (the APD contract is Pareto-improving over the Push contract). This suggests that the Push contract should never be considered when a supply chain has the option of using the APD contract.

Our results thus far indicate that when retailers have substantial bargaining power, and act as proposers, they should prefer the Pull contract, as theory suggests. However, for all other scenarios, such as a powerful supplier proposing contract terms or a situation where both parties have equal bargaining power, the
APD contract is preferred as it achieves the highest profit for the supplier, has the added benefit of an equitable split in total profits, and is Pareto-improving over the Push contract.

### 2.3.2 Decisions and Behavior

Table 2.4 illustrates the average contract offers made by proposers. For all three contracts, proposers set parameters that are significantly different than the theoretical predictions. Additionally, for all three contracts, proposers made offers that were more generous, towards the responder, than theory predicts. In the Push contract and APD contract, where proposers set a wholesale price(s) that is paid by the responder, the wholesale price(s) are below the standard prediction(s). In the Pull contract, where proposers set a wholesale price that is paid by the proposer, we observe that participants set the wholesale price higher than the theoretical prediction.

<table>
<thead>
<tr>
<th></th>
<th>Push</th>
<th>Pull</th>
<th>APD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Predicted</td>
<td>Observed</td>
<td>Predicted</td>
</tr>
<tr>
<td>Wholesale Price</td>
<td>9.00</td>
<td>8.38**</td>
<td>6.00</td>
</tr>
<tr>
<td>Discount Wholesale Price</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 2.4: Average contract decisions made by proposers.

Note: Standard errors are reported in square brackets. Significance of Wilcoxon signed-rank test given by ***, ** p−value < 0.01 and ** p−value < 0.05. Hotelling T-square given by ††† p−value < 0.01. Bold numbers represent proposers’ decisions.

In order to interpret the responder’s stocking quantities correctly, we calculate the optimal stocking quantities, conditioned on the proposers’ contract terms, and then average them for the conditional predicted values. Table 2.5 shows these results.

In Table 2.5 we observe significant differences in the Pull quantity and the APD production amount compared to the conditional predictions. In the Push contract we fail to reject that the observed quantity is the same as the conditional
prediction, and in APD the observed prebook amount is almost exactly the same as the conditional prediction.

Table 2.5: Average stocking decisions conditioned on proposers’ offers.

<table>
<thead>
<tr>
<th></th>
<th>Push</th>
<th></th>
<th>Pull</th>
<th></th>
<th>APD</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Predicted</td>
<td>Observed</td>
<td>Predicted</td>
<td>Observed</td>
<td>Predicted</td>
<td>Observed</td>
</tr>
<tr>
<td>Quantity/Prebook</td>
<td>44.11</td>
<td>39.67</td>
<td>60.46</td>
<td>53.29***</td>
<td>33.73</td>
<td>34.93</td>
</tr>
<tr>
<td></td>
<td>[3.37]</td>
<td></td>
<td>[1.79]</td>
<td></td>
<td>[2.00]</td>
<td></td>
</tr>
<tr>
<td>Production</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>73.59</td>
<td>55.99***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[2.93]</td>
<td></td>
</tr>
</tbody>
</table>

Note: Standard errors are reported in square brackets. Significance of Wilcoxon signed-rank test given by *** \( p-value < 0.01 \), ** \( p-value < 0.05 \), and * \( p-value < 0.10 \). Bold numbers represent proposers’ decisions.

Assume for a moment that the APD contract is a combination of both a Push contract and a Pull contract, where the prebook amount is ordered through a Push system, and the production quantity through a Pull system. If we interpret the APD contract this way, then we observe a stocking result that is consistent across all three contracts; in both the Push contract and the Push aspect of APD, participants set stocking quantities in line with theory, whereas in both the Pull contract and the Pull aspect of APD, participants understock.

Recall that responders, when receiving a set of contract terms from the proposer, had the option to reject the contract and earn an outside option profit of 60. In our data we find that the rejections in the Pull contract are lower than the Push contract and the APD contract, contributing to its favorable supply chain efficiency presented earlier in this analysis. Specifically, the average rejection rate across all decisions was 8.5% for the Push contract, 3.3% for the Pull contract, and 7.0% for the APD contract.

In an effort to determine if responders were acting rationally when making rejections, we can calculate the average responder profits, conditioned on both the proposers’ offers and the responders’ optimal stocking quantities, and compare them to the outside option. If we find that this profit is above the outside option of 60, then responders behaved suboptimally according to standard theory. If we find that the profit is below the outside option of 60, then the responders behaved
optimally according to standard theory. After conducting this analysis for each rejection, we find that for the Push contract, responders could have made, on average, 101 in profit, which is far higher than the outside option of 60. In the Pull contract, responders could have made, on average, 61 in profit, almost equal to the outside option of 60. Lastly, in the APD contract, responders could have made, on average, 173 in profit for the rejected offers.

Past literature has shown that subjects, when participating in a study that repeats for multiple decisions, may learn or adapt based on experience (Bostian, Holt & Smith 2008). Therefore we ran regressions with random effects of the decision period on the observed profit for all three contracts and both parties.\footnote{A Hausman test was performed for all of the regressions, confirming that random effects results in consistent coefficients.} We centered the decision period so that the intercept coincided with the average observed profits reported earlier in this study. Also, for each contract and for each party (retailer and supplier), we performed two regressions. The first was done using the observed profits as its independent variable, the second used conditional observed profit as its independent variable. For each of the regressions in the second set, this conditional profit represents the participants’ profits if the responder set the stocking quantity optimally (for responders it also considers the proposers’ actual offers). Table 2.6 reflects the results of these 12 regressions.

Starting with the Push contract in Table 2.6, we find evidence of experience effects only for the retailer’s (responder) conditional profits, coefficient of 0.53, suggesting that retailers could have made more profits over time if they had stocked optimally. However, there are no experience effects when looking at the regular observed profits. In the Pull contract, once again we find evidence of positive experience effects for the retailers (proposers), when conditioned on the supplier stocking optimally, but no evidence of experience effects for regular profits. Also, we see that the coefficient on \textit{Centered Period} for the conditional profit is the same between the retailer and supplier, but opposite in sign (0.47 versus -0.47), suggesting that the retailer learned to extract more profit at the supplier’s expense.

In the APD contract we find substantial experience effects. The supplier, who acts as the proposer, has a positive and highly significant coefficient for both observed profits and profits conditioned on the responder (retailer) setting the
Table 2.6: Experience effects on observed profit and conditional observed profit.

<table>
<thead>
<tr>
<th></th>
<th>Push Profit</th>
<th>Conditional Profit</th>
<th>Pull Profit</th>
<th>Conditional Profit</th>
<th>APD Profit</th>
<th>Conditional Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Retailer</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>124.17</td>
<td>149.67</td>
<td>256.40</td>
<td>292.27</td>
<td>184.00</td>
<td>226.65</td>
</tr>
<tr>
<td></td>
<td>[5.95]</td>
<td>[3.89]</td>
<td>[8.79]</td>
<td>[8.68]</td>
<td>[6.62]</td>
<td>[7.18]</td>
</tr>
<tr>
<td>Centered</td>
<td>0.37</td>
<td>0.53</td>
<td>-0.20</td>
<td>0.47</td>
<td>-2.01</td>
<td>-2.10</td>
</tr>
<tr>
<td>Period</td>
<td>[0.28]</td>
<td>[0.22]</td>
<td>[0.37]</td>
<td>[0.17]</td>
<td>[0.42]</td>
<td>[0.39]</td>
</tr>
<tr>
<td><strong>Supplier</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>204.59</td>
<td>229.88</td>
<td>140.94</td>
<td>154.32</td>
<td>212.92</td>
<td>200.13</td>
</tr>
<tr>
<td></td>
<td>[9.71]</td>
<td>[2.37]</td>
<td>[6.27]</td>
<td>[6.74]</td>
<td>[12.55]</td>
<td>[13.41]</td>
</tr>
<tr>
<td>Centered</td>
<td>-0.12</td>
<td>0.05</td>
<td>-0.48</td>
<td>-0.47</td>
<td>1.97</td>
<td>2.02</td>
</tr>
<tr>
<td>Period</td>
<td>[0.45]</td>
<td>[0.06]</td>
<td>[0.33]</td>
<td>[0.31]</td>
<td>[0.39]</td>
<td>[0.29]</td>
</tr>
</tbody>
</table>

Note: Standard errors are reported in square brackets. Significance given by *** $p$-value $< 0.01$, ** $p$-value $< 0.05$, and * $p$-value $< 0.10$. Bold numbers represent proposers’ results.

prebook optimally. For suppliers, the experience effects for both observed profits and conditional profits are negative and highly significant. In fact, once again we witness that the coefficients between the retailer and supplier are almost equal, but opposite in sign (-2.10 versus 1.97 and -2.10 versus 2.02), indicating that as the supplier learned to extract more profits over time, much of this increase in profits came directly at the retailers’ expense.

2.3.3 Individual Heterogeneity

The analysis to this point has only looked at summary statistics and average decisions by participants. Next we focus on individual behavior and determine what percentage of participants performed in line with the standard theoretical predictions.

Rather than evaluate each subject’s individual decisions (which in the APD contract could be multiple decisions), we consider each subject’s observed profits and compare them to a theoretical benchmark using regression. To account for any heteroscedasticity due to possible experience effects within each subject, we ran all regressions with robust standard errors (White 1980).

For proposers we ran a regression of the conditional profit on the decision
period, where we shifted the decision period by 30. This shift in the decision period was done to account for any potential learning or experience effects by the participant, and ultimately test if subjects were behaving in line with the standard theory by the end of the experiment. We implemented the following regression for each proposer:

$$E[\pi_{\text{prop}}|\theta] = \alpha + \beta(\text{Period} - 30)$$

(2.7)

where “prop” stands for the proposer, and $\theta$ represents the offered contract parameters by the proposer ($w$ in the Push and Pull contracts, $(w, w_d)$ in the APD contract). We compared the 95% confidence interval for the intercept, $\alpha$, to the standard theoretical profit prediction for the proposers. If the standard theoretical profit was in the 95% confidence interval for the intercept, then we classified this proposer as acting optimally.

We performed a similar procedure for responders, the only difference being that we compared the observed profit to conditional profit to see how subjects performed. For each responder, we ran a regression of observed profit on the shifted decision period, where each regression equation had the following form:

$$E[\pi_{\text{resp}}] = \alpha + \beta(\text{Period} - 30)$$

(2.8)

where “resp” stands for the responder.\(^5\) For each responder, we compared the 95% confidence interval for the intercept, $\alpha$, to the intercept, $\gamma$, from the following regression:

$$E[\pi_{\text{resp}}|\theta] = \gamma + \delta(\text{Period} - 30)$$

(2.9)

In Equation 2.9, $\gamma$ represents the average conditional profit at the end of the experiment, which accounts for the proposers offers to a responder and whether those offers change over time\(^6\). If the value for $\gamma$ fell inside the 95% confidence interval for the observed profits by a responder, $\alpha$, then that responder was classified as

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\(^5\)Note that the observed profit here for the APD contract uses the optimal production amount by proposers, since responders, when making decisions, should assume that the proposer will set production optimally.

\(^6\)We cannot use the average conditional profit for responders here as a benchmark, because we are incorporating experience effects. For example, if we took this approach, we may incorrectly conclude that a responder behaved optimally, because that responder may have received more generous offers over time up to period 30.
behaving optimally.

Table 2.7 depicts the percentage of subjects who were classified as acting optimally. As with the previous analysis, numbers in bold are proposers and regular typeface represents responders.

Table 2.7: Percentage of subjects who made decisions that resulted in profits being in line with theory. Bold numbers represent proposers’ results.

<table>
<thead>
<tr>
<th></th>
<th>Push</th>
<th>Pull</th>
<th>APD</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retailer</td>
<td>75.00%</td>
<td><strong>20.00%</strong></td>
<td>95.00%</td>
<td>63.33%</td>
</tr>
<tr>
<td>Supplier</td>
<td><strong>55.00%</strong></td>
<td>100.00%</td>
<td><strong>0.00%</strong></td>
<td>51.67%</td>
</tr>
</tbody>
</table>

As one can see in Table 2.7, responders generally did quite well when accounting for heterogeneity. The percentage of subjects who earned profits in line with their conditional prediction was only as low as 75% in the Push contract, and as high as 100% in the Pull contract. This result with the Pull contract may seem counterintuitive; recall earlier we saw that responders in the Pull contract (suppliers) set stocking quantities, on average, significantly different from theory (conditional prediction of 60.46 versus observed of 53.29). The difference here is that we are focusing on profits, and the Pull contract appears to be quite robust for deviations in the stocking quantity. For example, if the optimal stocking quantity in the Pull contract is 60.46, then the expected profits for the responder are around 139. If instead, the responder sets a stocking quantity of 53.29, then the expected profits are around 137. In other words, a difference of 12% in the stocking quantity results in a difference of 1.4% in expected profit.

Proposers, in Table 2.7, do not fare as well as responders. In the Push contract, 55% of proposers (suppliers) set the wholesale price in a way that should achieve a profit equal to the theoretical prediction, and in the Pull contract 20% of proposers (retailers) set the wholesale price in a way that should result in theoretical profit predictions. Lastly, no subjects acting as proposers in the APD contract found the set of contract parameters that would generate the profits predicted by standard theory.
2.4 Supply Chain Efficiency Analysis

Thus far we have seen that participants, particularly proposers, make errors in their decisions with respect to their own profits, but how do these errors affect total supply chain efficiency? In order to investigate this we partitioned the deviation between the observed supply chain efficiency and the predicted supply chain efficiency based on the two parties’ decisions.

The simplest way to convey our analysis here is by example. Let $\Delta_S$ represent the retailer’s contribution to the deviation in supply chain efficiency in the Pull contract, and $\Delta_S$ represent the supplier’s contribution to the deviation in supply chain efficiency in the Pull Contract. The portion of the deviation in total supply chain profits attributed to the supplier (responder) and retailer (proposer) in the Pull contract is given by:

$$\Delta_S = (\mathbb{E}[\pi_R(w, q(w))] + \mathbb{E}[\pi_S(w, q(w))]) - \left(\mathbb{E}[\pi_R(w, q^*(w))] + \mathbb{E}[\pi_S(w, q^*(w))]\right)$$

(2.10)

$$\Delta_R = (\mathbb{E}[\pi_R(w, q^*(w))] + \mathbb{E}[\pi_S(w, q^*(w))]) - \left(\mathbb{E}[\pi_R(w^*, q^*(w^*))] + \mathbb{E}[\pi_S(w^*, q^*(w^*))]\right).$$

(2.11)

In the Pull contract, the first equation, (2.10), represents the total observed supply chain profits minus the total supply chain profits if the supplier had stocked optimally given the retailer’s proposed $w$. This equation essentially looks at the difference in total supply chain profits for a suboptimal stocking decision by the supplier. The second equation, (2.11), represents the total supply chain profits assuming the supplier stocks optimally for a retailer’s proposed $w$, minus the total supply chain profits if the retailer had set $w$ according to the standard theoretical prediction $w = w^*$. This equation considers the difference in total supply chain profits for a suboptimal wholesale price by the retailer.

We applied this approach to total supply chain profits in each round of all three contracts and identified which party’s behavior contributed most to the deviation.
in total profits. The results of this analysis are presented for all three contracts in Table 2.8.

Table 2.8: The retailer and supplier’s impact on the deviation in supply chain efficiency. Bold numbers represent proposer’s results.

<table>
<thead>
<tr>
<th></th>
<th>Push</th>
<th>Pull</th>
<th>APD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Predicted Efficiency</td>
<td>75.00%</td>
<td>85.94%</td>
<td>100.00%</td>
</tr>
<tr>
<td>Retailer Impact</td>
<td>-10.59%</td>
<td>7.10%</td>
<td>0.00%</td>
</tr>
<tr>
<td>Supplier Impact</td>
<td>4.08%</td>
<td>-10.26%</td>
<td>-17.32%</td>
</tr>
<tr>
<td>Observed Efficiency</td>
<td>68.49%</td>
<td>82.78%</td>
<td>82.68%</td>
</tr>
</tbody>
</table>

Recall that earlier we said that proposers made decisions which hurt their prospective profits. For the Push and Pull contracts, when looking at total supply chain profits, proposers made decisions that actually increased the overall potential supply chain profits. This is because the negative impact on the proposers’ profits, in the Push and Pull contracts, were offset by a potentially more favorable impact on the responders’ profits. However, responders, as we saw in previous results, generally understocked (or set quantities optimally), rather than overstocked relative to conditional predictions, hence hurting the total supply chain profits.

In the APD contract, recall that the proposer (supplier) has control of all three parameters that impact supply chain efficiency; wholesale price, discount wholesale price, and production. Therefore, the reduction in supply chain efficiency is directly attributed to the proposer. We observe that of all three decisions by the proposer, the deviation in total supply chain profits stems primarily from poor production amounts. Recall that proposers, on average, given the wholesale prices they set, should have also set a corresponding production amount of approximately 74 units. If they had set stocking quantities in line with this, the supply chain efficiency could have been 99.6%. However, instead they set the production amounts too low, resulting in an efficiency closer to 83%.
2.5 Conclusion

In this study we evaluate three wholesale-price contracts that differ in how the inventory risk is placed in the supply chain. We study these contracts in a controlled laboratory setting and determine how the results differ from the normative prescriptions of standard theory. From our experimental results we identify which inventory setup results in the highest total supply chain efficiency, and also which setup generates the most profits for the retailer and supplier.

We find that the theoretical prediction of increasing supply chain profits by switching the inventory risk from the retailer to the supplier is correct, however, we also find that the total supply chain profits do not improve when moving the inventory risk from the supplier to both parties (the Pull contract to the APD contract). Similarly, our results suggest that the Pull contract does indeed generate the most profits for the retailer, but for suppliers, and supply chains where the retailer and supplier have equal bargaining power, the APD contract is best. In fact, the APD contract performs the same or better than the Push contract in terms of retailer and supplier profit.

Another result we observe is that proposers always earn more in profits than responders. However, compared to the standard theory, proposers earn less than they should, and responders earn more than they should. One possible explanation for this result is fairness. Given that in our study, subjects did not know the earnings of their partner, the presence of fairness or social preferences is not salient. Similarly, if fairness were driving the results of this study, then proposers in the APD contract would not set the regular wholesale price different from 14.99. It is possible that a fairness model, in conjunction with an error model such as the quantal response equilibria (McKelvey & Palfrey 1995, McKelvey & Palfrey 1998), could be driving this behavior. However, given that these alternative theoretical models nest the standard theory within them, they will always fit our data the same or better than the standard theory, even when they may not be the true cause of subject behavior. In short, the goal of this study is not to investigate the cause of human behavior, but its consequences on the supply chain.

Regardless of the cause of participants’ behavior, the managerial results of our analysis remain the same; when possible, powerful retailers should favor
the Pull contract, and suppliers and supply chains with equal partners, should adopt the APD contract. We admit that these three inventory risk arrangements may not always be possible for certain supply chain products, such as perishable food products where the Push contract is the only option. However, when those restrictions are not present, a supply chain may find that it can increase profits by simply altering the inventory risk setup. For example, if a supply chain is operating under the Push contract, it appears that both parties, the retailer and supplier, can maintain or increase profits by switching to the APD contract. In conclusion, businesses should carefully evaluate the location of their inventory in the supply chain, as it can have serious consequences on both the total supply chain profits and their own.
Chapter 3

An Experimental Investigation of Pull Contracts

3.1 Introduction

CommerceHub provides software solutions for retailers operating under a setting where retailers satisfy demand directly with their supplier’s inventory. Specifically, CommerceHub’s software allows retailers to integrate with any one of 6,000 suppliers, all prepared to ship product when a retailer has realized demand. CommerceHub’s customers include, among others, BestBuy, Sears, Staples, Costco, Dell, Toys R Us, and Kohls (CommerceHub 2010). In each of these cases, these powerful retailers must establish contracts with the available suppliers. Contracts used in this setting, known as pull contracts, can take on a variety of structures. In this paper we attempt to investigate, using controlled laboratory experiments, what types of pull contracts perform best for retailers and identify what reasons may drive the results.

CommerceHub is but one example of pull contracts in practice. Specialty items in stores are frequently ordered using pull contracts, and e-commerce retailers utilize pull contracts extensively. Take Amazon.com for example; they claim that an order under a pull setting (also called drop shipping) saves 50% of the fulfillment costs compared to a traditional distribution center (Patsuris 2001). Despite the benefits of pull contracts over push contracts\(^1\), they have yet to be understood.

\(^1\)Under a push contract, a retailer orders from a supplier and incurs the entire inventory cost
from an empirical standpoint. Our paper is the first attempt to investigate this problem.

Pull contracts, from a theoretical perspective, have yielded interesting results, even when focusing solely on wholesale price contracts. Cachon (2004) illustrates that wholesale-price pull contracts are more efficient than wholesale-price push contracts. The reason for this higher efficiency is that, under the pull contract, the supplier and the retailer both share the risk associated with uncertain demand. This creates an incentive for the supplier (the party setting the stocking quantity under the pull contract) to order more in equilibrium than the retailer orders in equilibrium under the push contract.

Our objective in this study is to analyze pull contracts in the laboratory, in order to test how they perform and understand the underlying reasons. We evaluate three pull contracts; a wholesale price contract, and two coordinating contracts—an overstock-allowance contract and a service-level agreement (SLA). A wholesale price contract states that a wholesale price be paid from the retailer to the supplier for each unit sold. Both an overstock-allowance contract and an SLA, build on the wholesale price contract by adding an extra contract parameter that helps coordinate the supply chain. In the overstock-allowance contract, the extra term is an overstock-allowance amount that is paid from the retailer to the supplier for each unit that the supplier produces but is not sold. The overstock-allowance contract presents a number of attractive theoretical characteristics, and is therefore included in this study to understand how it performs in practice. In an SLA, a relatively understudied contract but one that is prevalent in industry (a recent study showed that 91% or organizations utilize SLAs to manage suppliers and external customers (Oblicore 2007)), the extra parameter is a bonus, that is paid from the retailer to the supplier when the supplier satisfies a predetermined fraction of the retailer’s demand, known as the target fill rate. We compare and contrast these three contracts, both theoretically and experimentally, in a setting where a retailer makes a one-shot, take-it-or-leave-it contract proposal to a single supplier.

Experiments have been utilized with a variety of operations management models; including the bullwhip effect (Croson & Donohue 2006), revenue management (Bearden, Murphy & Rapoport 2008), the bandit problem (Gans, Knox & before demand is realized.
Croson 2007), and forecast sharing (Özer, Zheng & Chen 2011). Much of past behavioral operations management literature on contracts focuses on push contracts in supply chains, and specifically how retailers set stocking quantities (see Schweitzer & Cachon (2000), Bolton & Katok (2008), Rudi & Drake (2008), and Ho, Lim & Cui (2010) and references therein). In our work we focus on pull contracts and place a particular emphasis on understanding how the party establishing the contract terms, the retailer, behaves.

There have only been a few select works that experimentally study how contract terms are set. Lim & Ho (2007) study how two and three-part tariffs, under a push setting, impact channel efficiency between two human participants. Ho & Zhang (2008) extend this work by examining how the framing of the fixed fee affects results. In both papers the authors find that a quantal response equilibrium framework fits the data particularly well. Katok & Wu (2009) compare wholesale-price push contracts to two coordinating push contracts where each participant is partnered with a computerized player. Kalkanci, Chen & Erhun (2010) evaluate the complexity of contracts and find that an experience-weighted attraction learning model explains the data well. Our study differs from these works in two ways: first, we evaluate pull contracts, which have different theoretical properties compared to push contracts, and second, our experimental design has two treatments; one where a retailer is partnered with a computerized supplier who stocks optimally, and a second where a retailer is partnered with a human supplier. By including treatments with computerized suppliers as well as human suppliers, we can directly observe and measure the effect of social preferences (Cui, Raju & Zhang 2007, Loch & Wu 2008).

Our results are twofold. First, in terms of retailer profits, our data suggest that the theoretical benefit of the overstock-allowance contract and the SLA over the wholesale price contract does not perfectly translate into practice. Instead, retailers set the wholesale price too high, and the second parameter too low, in both coordinating contracts, thus foregoing substantial profits. Second, we find that a model of anticipated regret explains retailer behavior quite well. Specifically, we show that it explains the data better than the standard theory or risk aversion. We also run a number of experimental treatments that provide subjects with additional feedback and decision support to eliminate the possibility that,
instead of being regretful, subjects simply struggled with the complexity of the co-
ordinating contracts. For each of the additional treatments, we observe essentially the same retailer behavior as in our baseline treatments, thus supporting a theory of regretful retailers in the coordinating contracts.

The primary implication of our work is that, when one considers both the administrative simplicity of a wholesale price contract and its performance being only slightly below that of the two coordinating contracts, it may be the preferred alternative for retailers in the pull setting. Even in a controlled laboratory setting with many different forms of decision support, in the two coordinating contracts, retailers exhibit a natural bias to set the wholesale price too high, and the second parameter too low. In more complicated settings it is likely that retailers will be even more susceptible to behavioral biases, driving down profits even further. As such, the theoretical benefit of coordinating contracts may not be realized in practice.

We begin in Section 3.2 by following the work that Cachon (2004) and Netessine & Rudi (2006) have completed on wholesale-price pull contracts by deriving the equilibria of the overstock-allowance contract and the SLA. We describe the design of our experiment in Section 3.3, and present our results in Section 3.4. We follow this with Section 3.5, where we propose two alternative models and fit our data to them using parameter estimation techniques. After this, in Section 3.6 we present three additional experimental treatments in which we provide subjects with extra decision support to determine whether subjects struggled with the task because of complexity, or because they were maximizing some alternative utility function such as regret. Lastly, in Section 3.7, we conclude with a final summary.

3.2 Pull Model Overview

Our model is closely related to the classic newsvendor model between a single supplier and a single retailer. However, in our model the supplier $S$, as opposed to the retailer $R$, sets a stocking quantity $q$ for a single product, for a single period. The supplier sets this stocking quantity based on its production cost per unit $c$, and wholesale price per unit $w$ (and potentially other contract parameters outlined below). The retailer purchases products instantaneously from the supplier at the
wholesale price per unit once consumer demand is realized, receives revenue \( r \), for each unit sold, incurs no holding cost, and loses sales if demand is greater than the supplier stocking quantity \( D > q \). Let \( D \) represent a random variable for demand with cumulative distribution \( F \), complement \( \bar{F} \), and density \( f \). We assume no fixed ordering costs and full information of all cost parameters, where retailers and suppliers are risk-neutral expected-profit maximizers. This setup is often termed vendor-managed-inventory with consignment or drop-shipping.

Each period begins with the retailer offering contract terms to its supplier. The supplier, upon receipt of the retailer’s proposed terms, can either reject those terms or set a stocking quantity. After both players’ decisions are made, demand is realized. The supplier’s goal is to maximize its expected profit, \( E[\pi_S] \), with respect to the stocking quantity, and the retailer’s goal is to maximize its expected profit, \( E[\pi_R] \), with respect to the contract terms.\(^2\) Lastly, let efficiency be defined as the ratio between the decentralized supply chain profit and the centralized supply chain profit.

### 3.2.1 Wholesale Price Model

Under a wholesale price (\( WP \)) contract, in a pull setting, a retailer establishes a wholesale price \( w \). The supplier then sets a stocking quantity \( q \), for a given \( w \) that maximizes its expected profit, \( E[\pi_S(q,D)] \), where

\[
E[\pi_S(q,D)] = \int_0^q (wx - cq)dF(x) + \bar{F}(q)(w - c)q.
\]

Let \( q^* \) be the quantity that maximizes the supplier’s expected profit,
\[
q^* = \arg \max \ E[\pi_S(q,D)],
\]
where \( q^* \) is implicitly defined by \( w \), so that \( q^*(w) \). It is well known that \( q^*(w) \) must satisfy the result commonly referred to as the critical fractile, which is

\[
F(q^*(w)) = \frac{w - c}{w}.
\]  

\(^{(3.1)}\)

The retailer’s decision under a \( WP \) contract is \( w \), let \( w^* = \arg \max \ E[\pi_R(w,D)] \),

\(^2\)This is essentially a Stackelberg game where the retailer is the leader and the supplier is the follower. A combination of the supplier’s optimal stocking strategy, which is his best response to the retailer, and the retailer’s optimal decision, given this behavior by the supplier, constitutes the subgame perfect Nash equilibrium of the game.
where $E[\pi_R(w, D)]$ is

$$E[\pi_R(w, D)] = \int_0^{q^*(w)} ((r - w)x) dF(x) + \bar{F}(q^*(w))(r - w)q^*(w).$$

It is straightforward to show that $w^*$ exists and is unique when demand has the increasing failure rate property (see Appendix 6.1 for details). For more details regarding the $WP$ contract in a pull setting, see Cachon (2004) and Netessine & Rudi (2006).

### 3.2.2 Overstock-Allowance Model

An overstock-allowance ($OA$) contract builds on a $WP$ pull contract in that the retailer now proposes, in addition to a wholesale price per unit, an overstock-allowance amount per unit $\alpha$, that is paid from the retailer to the supplier for each unit that the supplier overstocks. Under an $OA$ contract, a supplier then sets a stocking quantity for a given $w$ and $\alpha$, that maximizes its expected profit, $E[\pi_S(q, D)]$, where $E[\pi_S(q, D)]$ under an $OA$ contract is

$$E[\pi_S(q, D)] = \int_0^q (wx - cq + \alpha(q - x)) dF(x) + \bar{F}(q)(w - c)q,$$

where $\alpha(q - x)$ in the first term is what differentiates an $OA$ contract from a $WP$ contract.

The supplier’s optimal stocking quantity, $q^* = \arg \max E[\pi_S(q, D)]$, which is implicitly defined by $w$ and $\alpha$, so that $q^*(w, \alpha)$, under an $OA$ contract must satisfy

$$F(q^*(w, \alpha)) = \frac{w - c}{w - \alpha}.$$

Let $(w^*, \alpha^*) = \arg \max E[\pi_R(w, \alpha, D)]$, where the retailer’s expected profit, $E[\pi_R(w, \alpha, D)]$ is

$$E[\pi_R(w, \alpha, D)] = \int_0^{q^*(w, \alpha)} ((r - w)x - \alpha(q^*(w, \alpha) - x)) dF(x) + \bar{F}(q^*(w, \alpha))(r - w)q^*(w, \alpha).$$

$^3$The increasing failure rate property (IFR) is common in many well known distributions, such as Normal and Uniform, and utilized frequently in newsvendor models (Lariviere & Porteus 2001).
A *push* version of the overstock-allowance contract is part of a larger class of coordinating push supply chain contracts (see Cachon (2003) for a review) and identical to a buyback contract, also known as “markdown money” (Tsay 2001). For many of the risk-sharing push contracts, it can be shown that they can perfectly coordinate the supply chain with an arbitrary split of profits between the retailer and supplier. As one might expect, in a pull setting, a similar result exists for an overstock-allowance contract.

**Proposition 1** If $0 < \lambda < 1$ represents the supplier’s share of the total profit, then for each $\lambda$, a wholesale price $w$, and overstock-allowance $\alpha$, can be found that perfectly coordinate the supply chain (100% efficiency) under an overstock-allowance contract, when the following equations are satisfied:

\[
\alpha = (1 - \lambda)c \\
w = \alpha + \lambda r
\]  

(3.2)

Please see Appendix 6.1 for the proof.

### 3.2.3 SLA Model

Under a service-level agreement (*SLA*) a retailer sets not only a wholesale price, but also a bonus $\beta$, which is paid to the supplier when the realized fill rate $\psi(q, D)$, is equal to or greater than an exogenously set target fill rate $\tau$. The realized fill rate $\psi(q, D)$, is defined as the fraction of satisfied demand for the retailer with the supplier stocking quantity,

\[
\psi(q, D) = \frac{\min(q, D)}{D},
\]

and the probability of the supplier achieving the target fill rate and earning the bonus is

\[
\Pr[\psi(q, D) \geq \tau] = F\left(\frac{q}{\tau}\right).
\]

The supplier’s expected profit, $E[\pi_S(q, D)]$, under an *SLA* is

\[
E[\pi_S(q, D)] = \int_0^q (wx - cq)dF(x) + \bar{F}(q)(w - c)q + \beta F\left(\frac{q}{\tau}\right),
\]
where the last term, differentiating an SLA from a WP contract, represents the expected value of the bonus. The supplier’s optimal stocking quantity, \( q^* = \arg \max \mathbb{E}[\pi_S(q, D)] \), is now implicitly determined by \( w \) and \( \beta \) so that \( q^*(w, \beta) \).

Under an SLA the supplier’s optimal stocking quantity must satisfy

\[
F(q^*(w, \beta)) = \frac{w - c}{w} + \frac{\beta}{w\tau} f \left( \frac{q^*(w, \beta)}{\tau} \right).
\] (3.3)

Notice that (3.3) is simply the WP contract solution with an additional term relating to the bonus and the target fill rate. In fact, for a fixed \( \tau \), any optimal solution \( q^*(w, \beta) \) is non-decreasing in \( \beta \).

Let \( (w^*, \beta^*) = \arg \max \mathbb{E}[\pi_R(w, \beta, D)] \), where the retailer’s expected profit, \( \mathbb{E}[\pi_R(w, \beta, D)] \) is

\[
\mathbb{E}[\pi_R(w, \beta, D)] = \int_0^{q^*(w, \beta)} ((r - w)x)dF(x) + F(q^*(w, \beta))(r - w)q^*(w, \beta) - \beta F \left( \frac{q^*(w, \beta)}{\tau} \right).
\]

As with the supplier’s profit, the retailer’s profit under an SLA is the same as the profit from a WP contract except for an extra term representing the expected value of the bonus.

An SLA is also a member of the class of coordinating pull contracts, and can perfectly coordinate the supply chain when demand is uniformly distributed from 0 to \( Z \) (which will be used in the experiment of this study).

**Proposition 2** If \( 0 < \lambda < 1 \) represents the supplier’s share of the total profit in the supply chain, where the target fill rate \( \tau \) is exogenously set, and demand is uniformly distributed from a lower bound of 0 to an upper bound of \( Z \), then for each \( \lambda \), a wholesale price \( w \), and bonus \( \beta \), can be found that perfectly coordinate the supply chain when the following equations are satisfied:

\[
\beta = c\tau(1 - \lambda)Z
\]
\[
w = \lambda r
\] (3.4)

Please see Appendix 6.1 for the proof.
3.3 Experimental Design

Our experiment involves a game between a retailer and a supplier. In each round a retailer proposes contract terms to its supplier. The supplier, upon receiving the retailer’s proposed terms, then decides whether to reject those terms or what stocking quantity to produce (a rejection resulted in a stocking quantity of zero). Following both of these decisions, demand is realized.

We evaluated a WP contract, an OA contract, and an SLA, each in separate treatments (a between subjects design) to avoid order effects. In the WP treatments, the retailer proposed a wholesale price per unit \( w \), for each unit sold. In the OA treatments, the retailer proposed \( w \), and an overstock-allowance amount \( \alpha \), for each unit of the supplier’s stocking quantity that exceeded demand. Lastly, in the SLA treatments, the retailer proposed a \( w \) and a bonus \( \beta \), where \( \beta \) was paid to the supplier whenever the supplier’s stocking quantity satisfied 100% of demand\(^4\). We selected these three contract types because, as a group, they provided one that is simple (WP), another that has ideal theoretical characteristics (OA), and a third because it is prevalent in industry (SLA).

In our experiment a retailer acts as a proposer, which replicates our motivating example of powerful retailers dealing with smaller suppliers. As such, because retailers are proposers, they can greatly influence the outcome for both parties. For example, retailers may set terms that extract the most possible profit for themselves, or decide to offer a more equitable split of profits with their supplier. Therefore, in all of our treatments, a human subject played the role of the retailer, but we used two different treatments for the supplier; a computerized supplier or a human subject supplier. In the treatments with a computerized supplier, the supplier always played its best response and set a stocking quantity that would maximize its expected profit. In the treatments with a human subject supplier, the supplier had the opportunity to reject the retailer’s proposed contract terms. If the supplier did not reject the retailer’s offer, he/she set a stocking quantity given the retailer’s proposal.

\(^4\)Note that a retailer setting a wholesale price here is equivalent to a setting where a supplier approaches a retailer to sell inventory at the retailer’s location, and the retailer establishes the commission rate for each unit sold. We felt setting \( w \) was easier for subjects to comprehend and the most accurate representation of the theoretical model.
This design allows us to understand retailer behavior under a controlled setting, where retailers should extract essentially the entire channel profit, and another where we allow social preferences to come into play, and study how they affect retailer decisions.

Our 3 x 2 between-subjects design totalled six treatments. Three represents the contract types (WP, OA, and SLA) and two represents the different roles for the supplier (computerized versus human). We used the same revenue and cost parameters in all six treatments, $r = 20$ and $c = 4$, which were known by both parties. We set demand to be a uniform integer between 0 and 100 that was independent each period of the game. To keep the game in the domain of gains rather than losses, we provided each party with an endowment of 400 laboratory dollars for each round of all six treatments. Lastly, subjects made 60 decisions in the “Computer” treatments, and 30 decisions in the “Human” treatments, where retailers and suppliers were randomly matched each period.\(^5\) Table 3.1 summarizes our design of experiment and sample sizes.

<table>
<thead>
<tr>
<th>Treatment</th>
<th>WP</th>
<th>OA</th>
<th>SLA</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Computer</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>60</td>
</tr>
<tr>
<td>Human</td>
<td>40</td>
<td>40</td>
<td>38</td>
<td>118</td>
</tr>
<tr>
<td>Total</td>
<td>60</td>
<td>60</td>
<td>58</td>
<td>178</td>
</tr>
</tbody>
</table>

The standard theory we outlined in Section 3.2 assumes that all decision makers have the ability to make optimal choices, even when they involve complex calculations. Therefore, to give the standard theory its best opportunity of being confirmed, we provided subjects with a decision support tool where they could move scroll bar(s) that corresponded to their decision(s) and test different values. Specifically, for a retailer, the screen showed the supplier’s optimal stocking quantity for a given set of test values, along with a graph illustrating their profit for every value of demand assuming this supplier stocking quantity. The supplier (in the Human treatments), was then shown a graph illustrating their profit for every value of demand, given their test stocking quantity and the retailer’s proposal.

\(^5\)In the OA contract Human treatment, due to network problems, 6/40 subjects completed 22 of 30 rounds while 6/40 others completed 27 of 30 rounds.
Using the results outlined in Section 3.2, and assuming the retailer will extract all but the minimum positive amount of the channel profits, we identified the predicted decisions and corresponding profits for each party given our experimental parameters. Those values are provided in Table 3.2. In our experiment, stock-

Table 3.2: Predicted values for each contract.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>WP</th>
<th>OA</th>
<th>SLA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retailer profit ($\pi^*_R$)</td>
<td>450</td>
<td>638</td>
<td>638</td>
</tr>
<tr>
<td>Supplier profit ($\pi^*_S$)</td>
<td>100</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Efficiency</td>
<td>85.9%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Quantity ($q^*$)</td>
<td>50.0</td>
<td>80.0</td>
<td>80.0</td>
</tr>
<tr>
<td>Wholesale price ($w^*$)</td>
<td>8.0</td>
<td>4.04</td>
<td>0.05</td>
</tr>
<tr>
<td>Overstock-allowance ($\alpha^*$)</td>
<td>-</td>
<td>3.99</td>
<td>-</td>
</tr>
<tr>
<td>Bonus ($\beta^*$)</td>
<td>-</td>
<td>-</td>
<td>399</td>
</tr>
</tbody>
</table>

ing quantities and bonuses were restricted to integers, and wholesale prices and overstock-allowances were restricted to two decimals, hence our predictions satisfy these rounding requirements. Also note that in Table 3.2 we have removed the endowment for presentation purposes and will continue with this format throughout the rest of our analysis.

We conducted all sessions at a large northeast U.S. university through the Fall of 2009 to the Summer of 2010. Participants in all six treatments were students, mostly undergraduates, from a variety of majors. Before each session subjects were allowed a few minutes to read over the instructions themselves. Following this, we read the instructions aloud and answered any questions. Each individual participated in a single session only and was recruited through an online system. Cash was the only incentive offered, where subjects were paid a $5 show-up fee plus an additional amount that was based on their personal performance. Average compensation for the participants, including the show-up fee, was $22. Each session lasted approximately 45 minutes to 1.5 hours and all software was programmed using the zTree system (Fischbacher 2007).
3.4 Results

In this section we first focus on our results from the Computer treatment; comparing and contrasting the contracts to predicted values and each other. Following this, we compare the Computer treatment directly to the Human treatment and discuss any significant differences between the two. For all results, we calculate expected profit of subjects’ decisions and report it as “observed” profit. This allows us to understand how subjects’ decisions translate into a managerial setting where they experience many demand realizations for a single contract decision.

3.4.1 Computer Treatment Results

Figure 3.1 depicts the predicted and observed retailer profit for the Computer treatment. As one can see, observed retailer profits are slightly below predicted values for the WP contract, and far below predicted levels for the OA contract and SLA (all differences with a two-sided t-test have \( p \)-values < 0.01).

![Figure 3.1: Predicted and observed retailer profit for the Computer treatment.](image)

If we directly compare the observed profits of each contract to each other, we find that there is a significant difference in retailer profit between the WP contract and the two coordinating contracts; the OA contract and SLA (\( p = 0.032 \) and...
However the predicted benefit of the coordinating contracts over the WP contract is far below what it should be in theory.\textsuperscript{6}

To understand why the retailer profits are different from predicted values we turn to the retailers’ proposals. Table 3.3 delineates the retailer proposals for the Computer treatment. As seen in the left-most columns of this table, retailers

<table>
<thead>
<tr>
<th>Table 3.3: Retailer proposals for the Computer treatment.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>WP</strong></td>
</tr>
<tr>
<td>Predicted</td>
</tr>
<tr>
<td>Wholesale price</td>
</tr>
<tr>
<td>[0.11]</td>
</tr>
<tr>
<td>Overstock-allowance</td>
</tr>
<tr>
<td>[0.22]</td>
</tr>
<tr>
<td>Bonus</td>
</tr>
<tr>
<td>[19.54]</td>
</tr>
</tbody>
</table>

Note: Standard errors are reported in square brackets. Significance of two-tailed t-tests given by \textsuperscript{**} \( p \text{-value} < 0.01 \) and \textsuperscript{**} \( p \text{-value} < 0.05 \). Hotelling T-square given by \textsuperscript{†††} \( p \text{-value} < 0.01 \).

in the WP treatment set \( w \) slightly higher than optimal. While the difference between \( w \) and \( w^* \) in the WP treatment is statistically significant, it appears as though retailers set \( w \) fairly well. In the OA and SLA treatments, retailers made decisions poorly relative to predicted values. In both of these contracts, it appears that retailers set \( w \) too high and the other parameter, \( \alpha \) or \( \beta \), too low (we may refer to \( \alpha \) or \( \beta \) as the “coordinating” parameter at times). We will investigate potential explanations for this retailer behavior further in Section 3.5.

Table 3.3 also indicates that in theory, \( w^* \) under a WP contract should be higher than \( w^* \) under a OA contract, which should be higher than \( w^* \) under an SLA (\( w^*_{WP} > w^*_{OA} > w^*_{SLA} \)). Despite subjects setting \( w \) too high within each contract, we do observe this comparative static across all three contracts. This suggests that participants generally responded correctly to each contract when setting \( w \), but not as strongly as they should (the differences are statistically

\textsuperscript{6}There is no difference between any of the three contracts for suppliers (not depicted), and therefore total supply chain profits have the same results as the retailer profit; WP is significantly lower than the two coordinating contracts, but the expected gain of the coordinating contracts is smaller than theory predicts.
significant for all comparisons of \( w \) except between the \( WP \) and \( OA \) contracts).

Past work has suggested that over time, subjects may learn with experience for their given task (Bostian, Holt & Smith 2008). Figure 3.2 illustrates the observed retailer profit over time for the Computer treatment. As one can see in Figure 3.2, in all three contracts, subjects increased profits over the first half of decisions (30), and then leveled off for the remainder of the experiment. To check directly for experience effects, we ran piecewise regressions, with random effects, of retailer profit on the decision periods with a knot at period 30.\(^7\)

We find that subjects did indeed learn to increase retailer profits over time up until period 30, with the strongest learning taking place in the \( SLA \). However, there does not appear to be learning after period 30. Because of the presence of learning in early periods, we can compare the retailer profit of all three contracts to each other only considering the second half of periods. After doing this we find the same conclusion as in the previous section; that the \( OA \) contract and \( SLA \) are statistically significantly better than the \( WP \) contract, that the \( OA \) contract and \( SLA \) are not statistically different from each other, and the expected benefit of the coordinating contracts over the \( WP \) is much smaller than predicted.

\(^7\)The Hausman test fails to reject that random effects are consistent for all regressions in this study.
3.4.2 Computer Treatment versus Human Treatment

We are primarily interested in one question when comparing the Computer treatment to the Human treatment: do retailers make different decisions when partnered with a human supplier?

Figure 3.3 partially addresses this question, and compares the two treatments for retailer profit, conditional on the supplier playing his best response and stocking optimally (since this is the case in the Computer treatment). As seen in Figure 3.3, retailer profit is essentially the same between the Computer and Human treatments, where there is only a significant difference in the WP treatment, but this difference is rather small.

Table 3.4 provides the retailer proposals for the Computer treatment and Human treatment. In the WP and OA treatments, we find that retailers make nearly identical proposals to suppliers, computerized or human. In the SLA treatment, retailers offer slightly higher wholesale prices and lower bonuses to human suppliers, but these differences are not statistically significant. Also note that the primary result seen in the Computer treatment for the OA contract and the SLA, that retailers set the wholesale price too high and the coordinating term too low, failing to extract all of the supply chain profits, continues to exist with a human supplier.
In sum, retailers do not appear to make different offers to human suppliers.

Table 3.4: Retailer proposals for the Computer and Human treatments.

<table>
<thead>
<tr>
<th></th>
<th>WP Human</th>
<th>WP Computer</th>
<th>OA Human</th>
<th>OA Computer</th>
<th>SLA Human</th>
<th>SLA Computer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wholesale price</td>
<td>8.92 [0.31]</td>
<td>8.57 [0.11]</td>
<td>8.39 [0.28]</td>
<td>8.03 [0.32]</td>
<td>6.57 [0.50]</td>
<td>4.98 [0.53]</td>
</tr>
<tr>
<td>Overstock-allowance</td>
<td>-</td>
<td>-</td>
<td>1.60 [0.16]</td>
<td>1.47 [0.22]</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Bonus</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>158.5 [13.01]</td>
<td>214.2 [19.54]</td>
</tr>
</tbody>
</table>

Note: Standard errors are reported in square brackets. No significant differences within each contract.

Table 3.4 has already provided us with evidence regarding whether retailers make more generous offers to human or computerized suppliers. However, fairness concerns may also exist over what proportion of total profits are achieved by one party (Bolton & Ockenfels 2000). Therefore, we can perform a regression where the dependant variable is the fraction of total profits that would go to the retailer, conditioned on the retailer’s proposal. In other words, this model assumes that the supplier plays his best response and stocks optimally, given the retailer’s offer, then considers the portion that would go to the retailer. We ran the regression with data from both the Computer and Human treatments with three independent variables; centered period, an indicator for the Human treatment, and an interaction term between the centered period and Human indicator. If the coefficient on the indicator for the Human treatment is negative and significant, then retailers offered a more equitable split of total profits to the supplier. We include an interaction term between the decision period and the indicator for the Human treatment to see if any sort of experience and fairness effects are stronger for the Computer or Human treatment. Table 3.5 provides these results.

In Table 3.5 we observe that retailers offer human suppliers a slightly larger portion of total profits in the SLA treatment, as shown by the weakly significant coefficient on Human for the SLA. In the WP and OA treatments, the coefficient on Human is not significant at any level (although there is significance of the in-
Table 3.5: Regression with the fraction of total profits going to the retailer, conditioned on the retailer’s proposal and assuming the supplier plays his best response and stocks optimally, as the dependant variable for the Computer and Human treatments.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>WP</th>
<th>OA</th>
<th>SLA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>Approx. fraction of profit for retailer in Computer</td>
<td>0.783***</td>
<td>0.787***</td>
<td>0.825***</td>
</tr>
<tr>
<td></td>
<td>(Per − Per) avg. number of periods</td>
<td>0.0003***</td>
<td>0.0009***</td>
<td>0.0016***</td>
</tr>
<tr>
<td>Human</td>
<td>Indicator for the Human treatment</td>
<td>-0.023</td>
<td>-0.026</td>
<td>-0.047*</td>
</tr>
<tr>
<td>(Per − Per) × Human</td>
<td>Interaction between period and Human</td>
<td>-0.0021***</td>
<td>0.0009*</td>
<td>0.0011**</td>
</tr>
</tbody>
</table>

Note: ***p-value < 0.01, **p-value < 0.05, *p-value < 0.10.

...interaction term for the WP treatment, implying that, over time, retailers may have made slightly more equitable offers in the Human WP treatment. Additionally, it appears as though retailers, in all three treatments, learned to extract a greater share of the available profits over time, evidenced by the positive and significant coefficient on (Per − Per), and, this same effect appears to be even stronger in the Human treatment for the two coordinating contracts (evidenced by the coefficients on the interaction terms).

In sum, in our data, it does not appear as though retailers are more fair when dealing with a human supplier, both in terms of total potential supplier profit or the fraction of total profit going to the supplier.

### 3.4.2.1 Rejections

Recall that in the Human treatment, suppliers are given the opportunity to reject the retailer’s offer, which results in a stocking quantity of 0. According to the standard theory, a supplier should never reject an offer that results in a positive expected profit (which occurred more than 99% of the time in our data). However, we observe that the rejection rates across all three treatments, when looking only
at offers that would result in a positive expected profit for the supplier, are positive. Specifically, suppliers rejected retailers’ offers 10% of the time in the WP treatment, 11% of the time in OA treatment, and 8% of the time in the SLA treatment.

Figure 3.4: Potential profit for the retailer and supplier, conditioned on retailer proposals and assuming the supplier stocks optimally, for rejections and acceptances for the Human treatment. The percentages on top represent the potential supply chain efficiency.

To gain insight into suppliers rejections, we can compare the expected profit, conditioned on the retailer proposals, for both accepted and rejected offers (the expected profit assuming the supplier plays his best response and stocks optimally). After doing this, we first find that the supplier’s potential profit is lower for rejections. However, interestingly, we also observe that the retailer’s potential profit for accepted offers is equal to or higher than that of rejected offers. In other words, when proposals were rejected, there was almost certainly a contract proposal that was Pareto improving (both parties could have been better off). Figure 3.4 illustrates the potential expected profit for both parties when offers are accepted and rejected.\footnote{Also, we do not find that retailers make more equitable offers over time (Pavlov & Katok}
In short, retailers could have avoided rejections, without hurting their profit, by offering a set of contract terms that resulted in a higher efficiency for the supply chain. This provides evidence that retailers have an incentive to increase supply chain coordination for their own gain.

### 3.5 Alternative Models

Now that we have observed how retailers make decisions under various pull contracts, we need to better understand the underlying cause. Two plausible reasons for the retailer decisions we observe are:

1. Subjects were systematically deviating from the standard predictions because they were maximizing a different utility function than what the standard theory assumes.

2. Subjects simply could not recognize the optimal contract proposal because of feedback variability or complexity.

In this section we attempt to address the first reason, that subjects were maximizing some alternative utility function, and identify which model fits the data best through parameter estimation techniques (we will investigate the second reason for retailer behavior in the next section).

There have been many alternative models that have been applied to newsvendor type experiments. Despite designing our experiment in an effort to mitigate these as possible explanations, and considering that we can generally rule out learning as a possible model, there are still undoubtedly many models that may explain our results. Therefore, we consider two alternatives that seem to fit the characteristics of our retailer data; risk aversion and anticipated regret.

To determine the predicted behavior of the retailer, we assume that they are proposing terms to a risk-neutral expected-profit maximizing supplier. This is exactly the scenario in the Computer treatment and, as such, we will focus exclusively on fitting the alternative models to the Computer data.
3.5.1 Risk Aversion

Given that retailers in the OA and SLA treatments set \( w \) too high, and the second coordinating parameter too low, it seems obvious that a first attempt to explain this behavior would be risk aversion. Let \( u(x) \) represent the decision maker’s Bernoulli utility function. We assume a form of risk aversion that is common in newsvendor settings; constant absolute risk aversion (CARA) where the utility function is of the form \( u(x) = -e^{-\phi x} \), and \( \phi \) is the degree of risk aversion, \( \phi > 0 \) (Eeckhoudt, Gollier & Schlesinger 1995). As \( \phi \) increases, a retailer’s risk aversion increases.

If we maximize this utility function for each contract using our experimental parameters, \( r = 20, c = 4 \), and demand uniformly distributed between 0 and 100, subject to the supplier’s profit being non-negative, and assume the supplier plays his best response by setting the optimal supplier stocking quantity from Section 3.2, we can find the optimal set of contract parameters for different degrees of risk aversion. In short, as a retailer becomes more risk averse, they set \( w \) lower in the WP contract. In the two coordinating contracts, a more risk averse retailer increases \( w \) and decreases the coordinating contract parameter (\( \alpha \) or \( \beta \)) until, for higher levels of risk aversion, the coordinating parameter is driven to 0 and \( w = 8 \) (the risk-neutral prediction for the WP contract).

3.5.2 Anticipated Regret

Another model that seems reasonable to consider for retailer behavior is that of anticipated regret (Bell 1982, Bleichrodt, Cillo & Diecidue 2010). In a regret model, a decision maker feels worse when their decisions do not result in capturing all of the potential profit after demand is realized. For example, in the SLA treatment, a retailer may experience regret from setting the bonus too high and paying out a larger amount than was necessary to induce the supplier to stock a quantity that would satisfy all of demand. Similarly, a retailer in the SLA treatment may feel regret over setting too low a bonus, and missing out on potential sales. We will call the first type of regret winner’s regret and the second type, loser’s regret.

It is intuitive that there be more regret in the OA contract and SLA compared to the WP contract, due to the coordinating parameter that may be paid to
the supplier depending on the supplier stocking quantity and demand realization. Therefore, we consider a model of regret that is only applied to the two coordinating contracts; OA and SLA. Specifically, we posit a model of regret that only considers the retailer being regretful over the overstock-allowance amount or the bonus. In the case of the WP contract, we do not allow any sort of regret to exist, and hence the standard theory is the regret model for the WP contract (which we have already seen predicts retailer behavior quite well).

In the case of the OA contract and SLA, we model winner’s regret as proportional to the total amount paid back to the supplier if all of the demand is satisfied (for the OA contract this is the total overstock-allowance amount, \( \alpha(q-D)^+ \), and for the SLA this amount is the bonus, \( \beta \)). We then compound this by how far the stocking quantity is from realized demand, \((q-D)^+\), so that a retailer has more winner’s regret when the supplier sets \( q \) further from realized demand (note that this makes the OA contract’s winner’s regret term quadratic in \((q-D)^+\) because the total overstock-allowance amount is \( \alpha(q-D)^+ \)). This approach is also similar to incorporating reference dependance in newsvendor stocking decisions (Ho, Lim & Cui 2010).

In the case of loser’s regret, we use a similar formulation as that of winner’s regret, except we invert the overstock-allowance and bonus terms so that more regret is experienced the lower a retailer sets \( \alpha \) or \( \beta \). Analogous to the winner’s regret function, we then multiply these amounts by how far the supplier’s stocking quantity is from realized demand, \((D-q)^+\).

The winner’s and loser’s regret terms are given in Table 3.6, where \( \delta_\omega \) is the amount of winner’s regret, and \( \delta_\ell \) is the amount of loser’s regret, \( \delta_\omega \geq 0, \delta_\ell \geq 0 \). The numerators in the loser’s regret terms, \( \kappa \) and \( 100\kappa \), are simply scaling constants that we introduce to make the estimated parameters easier to interpret.\(^9\)

Given these two regret formulations, the expected utility of the retailer for the OA contract and SLA under a regret model is

\[
OA: \quad E[u(\pi_R(w, \alpha, D))] = E[\pi_R(w, \alpha, D)] - \int_0^{q^*(w, \alpha)} \delta_\omega \alpha(q^*(w, \alpha) - x)^2 dF(x)
\]

\(^9\)The range of potential overstock-allowances per unit in the experiment was 0 to 4 and bonuses in the experiment was 0 to 400, hence a 1 to 100 ratio. In the upcoming parameter estimation section, we set \( \kappa = 250 \).
Table 3.6: Anticipated regret terms for the OA contract and the SLA.

<table>
<thead>
<tr>
<th>Regret Type</th>
<th>OA</th>
<th>SLA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Winner’s regret</td>
<td>$\delta_\omega \alpha ((q(w, \alpha) - D)^+)^2$</td>
<td>$\delta_\omega \beta (q(w, \beta) - D)^+$</td>
</tr>
<tr>
<td>Loser’s regret</td>
<td>$\delta_\ell \left( \frac{\alpha}{\alpha} \right) (D - q(w, \alpha))^+$</td>
<td>$\delta_\ell \left( \frac{100\kappa}{\beta} \right) (D - q(w, \beta))^+$</td>
</tr>
</tbody>
</table>

$$- \int_{q^*(w,\alpha)}^{\infty} \delta_\ell \left( \frac{\kappa}{\alpha} \right) (x - q^*(w,\alpha))dF(x)$$

$$SLA_{\tau=1}: \quad \mathbb{E}[u(\pi_R(w, \beta, D))] = \mathbb{E}[\pi_R(w, \beta, D)] - \int_0^{q^*(w,\beta)} \delta_\omega \beta (q^*(w, \beta) - x)dF(x)$$
$$- \int_{q^*(w,\beta)}^{\infty} \delta_\ell \left( \frac{100\kappa}{\beta} \right) (x - q^*(w,\beta))dF(x),$$

where the WP profit for the retailer under a regret model is the same as in Section 3.2.

If we maximize these functions using our experimental parameters subject to the supplier profit being non-negative, and assume the supplier plays his best response by setting the optimal stocking quantity from Section 3.2, we find that a retailer more concerned about winner’s regret than loser’s regret will tend to set a lower $\alpha$ or $\beta$, and increase $w$, much like that of a risk averse retailer.

### 3.5.3 Parameter Estimation

We use the maximum-likelihood estimation (MLE) method to fit our data to the two alternative models and the standard theory. We perform this technique first for all decisions by retailers for each separate contract, and then allow for retailers to be fit separately to determine which model fits the highest proportion of subjects.

We can use the alternative theories outlined above to predict the optimal retailer decisions as a function of the parameters of each model. For simplicity, we will outline the parameter estimation process we used for the OA contract and regret model as an example.\(^{10}\) Let $i$ denote the index for a retailer’s deci-\(^{10}\)The SLA uses the same procedure as the OA contract, and the risk aversion model fits one
sion, \( i = 1, \ldots, T \) where \( T \) is the total number of decisions for the OA contract. We assume the retailer’s decision errors are distributed normally with mean 0 and variance \( \sigma_w^2 \) and \( \sigma_\alpha^2 \), with correlation \( \rho_{wa} \), where the individual decisions \( w_i \) and \( \alpha_i \) follow the bivariate normal distribution for the OA contract (normal distribution for the WP contract):

\[
\begin{pmatrix}
  w_i \\
  \alpha_i
\end{pmatrix}
\sim
N
\left(
\begin{pmatrix}
  w^*(\delta_\omega, \delta_\ell) \\
  \alpha^*(\delta_\omega, \delta_\ell)
\end{pmatrix},
\begin{pmatrix}
  \sigma_w^2 & \rho_{wa}\sigma_w\sigma_\alpha \\
  \rho_{wa}\sigma_w\sigma_\alpha & \sigma_\alpha^2
\end{pmatrix}
\right).
\]

Let \( w^*(\delta_\omega, \delta_\ell) \) and \( \alpha^*(\delta_\omega, \delta_\ell) \) be the optimal wholesale price and overstock-allowance amount under the regret model for a particular selection of \( \delta_\omega \) and \( \delta_\ell \).

The log-likelihood (LL) function, in the case of the OA contract for the regret model, which is maximized over five parameters, is

\[
LL(\delta_\omega, \delta_\ell, \sigma_w, \sigma_\alpha, \rho_{wa}) = \\
\sum_{i=1}^{T} \left( -\ln(2\pi) - \frac{1}{2} \ln |\Omega| - \frac{1}{2} \left( w_i - w^*(\delta_\omega, \delta_\ell) \right)' \Omega^{-1} \left( w_i - w^*(\delta_\omega, \delta_\ell) \right) \right)
\]

where \( \Omega = \begin{bmatrix}
  \sigma_w^2 & \rho_{wa}\sigma_w\sigma_\alpha \\
  \rho_{wa}\sigma_w\sigma_\alpha & \sigma_\alpha^2
\end{bmatrix} \).

Table 3.7 delineates the LL values and parameter estimates for retailers in the Computer treatment.\(^{11}\)

Before comparing the models to each other, we interpret the parameter estimates for the risk aversion and regret models. As one can see from Table 3.7, the value of \( \phi \) for the WP contract is almost zero and generates roughly the same \( LL \) as the standard theory, which is not surprising, as subjects set \( w \) only slightly higher than the value of 8 that the standard theory predicts. Values of \( \phi \) for the OA contract and SLA, for the risk aversion model, are relatively small. For example, \( \phi = 0.0027 \) implies that a retailer would be indifferent between a 50-50 chance

\(^{11}\) We omit reporting the estimation results for the variances and correlations for presentation purposes. Additionally, we generated standard errors for each parameter using 1,000 bootstraps of the data. Each parameter estimate is significant at 99%. 

less parameter.
Table 3.7: Maximum likelihood estimates for standard theory, risk aversion, and anticipated regret for the Computer treatment.

<table>
<thead>
<tr>
<th>Model</th>
<th>WP</th>
<th></th>
<th>OA</th>
<th></th>
<th>SLA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LL</td>
<td>MLE</td>
<td>LL</td>
<td>MLE</td>
<td>LL</td>
</tr>
<tr>
<td>Standard theory</td>
<td>-2,073</td>
<td>-</td>
<td>-5,516</td>
<td>-</td>
<td>-10,838</td>
</tr>
<tr>
<td>Risk aversion</td>
<td>-2,078</td>
<td>φ = 0.0001</td>
<td>-4,689</td>
<td>φ = 0.0027</td>
<td>-10,366</td>
</tr>
<tr>
<td>Anticipated regret</td>
<td>-</td>
<td>-</td>
<td>-4,645</td>
<td>δω = 0.0710, δℓ = 0.0026</td>
<td>-9,984</td>
</tr>
</tbody>
</table>

of $0 or $100, or a guaranteed amount of $46.64. In short, our subjects did not exhibit high levels of risk aversion.

To get a better insight as to what the levels of regret represent, we can plug in the average values of \( w, \alpha, \) and \( \beta \) for the Computer treatment, and calculate the expected value of the regret functions given in Table 3.6 with the parameter estimates. When we do this, we find that for both the OA contract and SLA, that the amount of winner’s regret outweighs the amount of loser’s regret.\(^{12}\)

Now we turn to which model fits which contract best. For the WP contract, as mentioned previously, subjects are best fit by the standard theory. For the OA contract and SLA, a likelihood ratio test shows the risk aversion and regret models are favored over the standard theory. To compare the two non-nested models directly to each other, we performed a Vuong test (Vuong 1989). It shows that the regret model provides a statistically significantly higher LL than the risk aversion model, for both the OA contract and SLA, thus favoring a model of anticipated regret overall.

The previous results can be slightly misleading if one does not consider two additional factors. First, the regret model has a clear advantage over the standard theory and risk aversion models by having two extra parameters (\( δ_ω, δ_ℓ \)). Therefore, to more formally rank these models, we employ the Bayesian information criterion.

\(^{12}\)Recall that the regret functions incorporate the stocking quantity’s deviation from realized demand, \( (q - D)^+ \) and \( (D - q)^+ \). For the OA contract and SLA average quantities were roughly 62. Therefore, the winner’s regret parameter, by itself, does not have to be very large for the entire winner’s regret term to become greater than the total loser’s regret term.
(BIC), which introduces a penalty for models that increase in the number of parameters.\footnote{The BIC generally provides a larger penalty for extra parameters compared to other penalty functions.} The second caveat with the results in Table 3.7 is that, while a regret model may fit the data best on aggregate, it may also be the case that retailers, when allowed to have their parameters estimated individually, may be better fit by one of the other two models. Therefore, we repeated the same estimation process for each retailer for each model. These results, along with the BIC values for the aggregate data, are presented in Table 3.8, where a lower BIC value is preferred.

Table 3.8: BIC values and percentage of subjects best fit for standard theory, risk aversion, and anticipated regret for the Computer treatment.

<table>
<thead>
<tr>
<th>Model</th>
<th>BIC</th>
<th>% of Subjects Best Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>WP  OA  SLA</td>
<td>WP  OA  SLA</td>
</tr>
<tr>
<td>Standard theory</td>
<td><strong>4,154</strong> 11,053 21,697</td>
<td><strong>90%</strong> 0% 0%</td>
</tr>
<tr>
<td>Risk aversion</td>
<td>4,169 9,403 20,757</td>
<td>10% 45% 0%</td>
</tr>
<tr>
<td>Anticipated regret</td>
<td>- <strong>9,325</strong> 20,003</td>
<td>- <strong>55%</strong> 100%</td>
</tr>
</tbody>
</table>

Table 3.8 provides a more accurate depiction of which model fits our data best on both an aggregate and individual level. As one can see, the standard theory best describes the WP contract on aggregate and when allowing for individual heterogeneity. For the OA contract and SLA, the lower values of the BIC generated by the regret model indicate that the regret model, despite having an extra free parameter, is still favored over a model of risk aversion. Similarly, one can see that when we allow for heterogeneity and estimate each subject separately, that a vast proportion of subjects are best fit by the regret model. In the SLA, 100% of subjects were best fit by this model. However, in the OA treatment the advantage is not as large, as 45% of subjects were actually best fit with a risk aversion model.
3.5.3.1 Overall Fit

While the regret model may be preferred over the other models, it does not provide us with evidence as to how well the regret model can explain our data overall. To get a better idea as to how well the regret model fits our data, we performed a Hotelling T-square test (Hotelling 1931) comparing the best predicted values from the regret parameters, in Table 3.7, of the retailer’s decision variables to our actual data, and found that we cannot reject the null hypothesis that the two vectors of $w$ and $\beta$ are equal in the SLA treatment (predicted values of $w = 4.92$ and $\beta = 217.1$ versus observed values of $w = 4.98$ and $214.2$). However, in the OA treatment, the difference between our best predictions and the actual data remains statistically significant (predicted values of $w = 6.46$ and $\alpha = 1.44$ versus observed values of $w = 8.03$ and $\alpha = 1.47$). This implies that the regret model fits our data very well for the SLA, but there may still be an alternative model that would fit the OA data more completely (which is not surprising as an all-or-nothing bonus term is a more salient “regretful” parameter, compared to a per unit overstock-allowance amount).

3.6 Support Treatments

Thus far we have seen that a simple model of anticipated regret can account for the behavior of retailers in the coordinating contracts. However, just because a model fits the data best does not prove that there wasn’t some other cause that generated this behavior. Therefore, in this section we provide the results from three additional experimental treatments (referred to as “Support” treatments), each providing retailers with various support mechanisms to assist with their decision making task. Ultimately through these treatments we can identify if retailers are behaving deliberately and maximizing some other utility function, or if they simply require additional support or information to find optimal solutions predicted by the standard theory.

In all three Support treatments suppliers were computerized. In the first Support treatment, “10P (10 Periods),” we manipulated the experiment so that retailers’ decisions would be held constant for 10 periods of demand (rather than 1
previously). This eliminates much of the demand variability and provides subject with additional feedback about how their decisions fare on average rather than for a single demand realization.

In the second Support treatment, “ExpSup (Expected Profit Support),” we provided subjects with an additional piece of decision support; expected profit. This allowed retailers to test different values and see, in addition to their previous decision support, their expected profit. If the same results exist as in the original baseline Computer treatments, then it is reasonable that confusion is not the cause of the results, and subjects are instead maximizing some alternative utility function.

In the third Support treatment, “SingDec (Single Decision),” we altered the coordinating contracts so that the decision space was now a single variable; the coordinating parameter. For example, in the SLA treatment, subjects set a bonus, and for each bonus they set, the corresponding optimal wholesale price, conditioned on that bonus, was automatically used. This made it so that all three contracts, WP, OA, and SLA, had the same number of decision variables (one) for subjects. This provided a direct test to determine if the level of contract complexity in the two coordinating contracts is pushing their profits close to the WP level. The experimental design and sample sizes for the three Support treatments are given in Table 3.9.

Table 3.9: Experimental design and sample sizes for the Support treatments.

<table>
<thead>
<tr>
<th>Treatment</th>
<th>OA</th>
<th>SLA</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>10P</td>
<td>10</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>ExpSup</td>
<td>10</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>SingDec</td>
<td>10</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>30</td>
<td>30</td>
<td>60</td>
</tr>
</tbody>
</table>

Figure 3.5 depicts the observed retailer profit for the original baseline Computer treatment and the three Support treatments. As one can see there is only a marginal improvement in retailer profit for each Support treatment over the original Computer treatment. The only statistically significant difference between the Support treatments and the Computer treatment, for both contracts, was between the OA Computer baseline and OA SingDec treatment, where the OA retailer
profit of 504 is still far from the theoretical prediction of 638 (not shown). Also, it is important to note that in the SingDec treatments the retailer profit for any decision can be no worse than the optimal retailer profit for the $WP$ contract, 450, because a bonus or overstock-allowance of zero would result in an optimal wholesale price of 8, and the retailer’s profit is strictly increasing in the coordinating parameter (since the corresponding optimal wholesale price will be utilized).

![Figure 3.5: Retailer profit for the Computer treatment and three Support treatments.](image)

Table 3.10 shows the actual contract proposals by retailers in the baseline Computer treatment and three Support treatments. The only significant difference comparing the Computer baseline with the Support treatments, is the SingDec treatments for both contracts. While reducing the complexity to a single decision variable slightly improves the actual decisions, the values are still far from theoretical predictions. Also, we performed the same MLE parameter estimation process in Section 3.5 on the Support treatments and find the results are largely unchanged.

As with the original Computer treatments, there is slight learning in the early stages by retailers in the 10P and ExpSup treatments, and none in the SingDec treatments. This illustrates that subjects once again had a sufficient number of decisions to determine what they believed was the optimal solution for their given...
Table 3.10: Retailer proposals for the **Computer** treatment and three **Support** treatments.

<table>
<thead>
<tr>
<th></th>
<th>OA</th>
<th></th>
<th></th>
<th>SLA</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Computer</td>
<td>10P</td>
<td>ExpSup</td>
<td>SingDec</td>
<td>Computer</td>
<td>10P</td>
<td>ExpSup</td>
<td>SingDec</td>
</tr>
<tr>
<td>Wholesale price</td>
<td>8.03</td>
<td>7.43</td>
<td>6.77</td>
<td>6.86††</td>
<td>4.98</td>
<td>5.36</td>
<td>5.50</td>
<td>5.55+++</td>
</tr>
<tr>
<td></td>
<td>[0.32]</td>
<td>[0.55]</td>
<td>[0.50]</td>
<td>[0.14]</td>
<td>[0.53]</td>
<td>[0.74]</td>
<td>[0.94]</td>
<td>[0.29]</td>
</tr>
<tr>
<td>Overstock allowance</td>
<td>1.47</td>
<td>2.06</td>
<td>2.08</td>
<td>1.66+++</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>[0.22]</td>
<td>[0.37]</td>
<td>[0.38]</td>
<td>[0.18]</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Bonus</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>214.2</td>
<td>188.7</td>
<td>194.2</td>
<td>144.9+++</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>[19.5]</td>
<td>[32.4]</td>
<td>[45.6]</td>
<td>[16.4]</td>
</tr>
</tbody>
</table>

Note: Standard errors are reported in square brackets. Significance of Hotelling T-square test, comparing the Computer baseline to each Support treatment, given by † $p-value < 0.10$, †† $p-value < 0.05$, and ††† $p-value < 0.01$.

preferences.

In sum, it appears that subjects, despite considerable information feedback, decision support, and complexity reduction, fail to set contract proposals that match standard theoretical predictions. We believe that these additional experimental treatments suggest that it seems reasonable subjects did not set suboptimal contracts because of confusion or complexity in the original treatments, but because they were maximizing some alternative utility function.

### 3.7 Conclusion

In this study we evaluate the performance of three pull contracts; a wholesale price contract, an overstock-allowance contract, and a service-level agreement. We compare the results of each contract to theoretical predictions and each other under both a controlled treatment for supplier behavior and a second treatment where the supplier is a human subject.

We find that the theoretical advantage of the two coordinating contracts over the wholesale price contract does not perfectly translate into practice. Instead, the overstock-allowance contract and service-level agreement seem to induce regretful behavior by retailers with respect to the coordinating parameter, thus driving their profits down to levels only marginally higher than a simple wholesale price
contract.

To test whether retailers are truly acting in a way that resembles regret, or simply struggling with the complexity of the task, we ran additional treatments where we provided subjects with additional feedback, decision support, and reduced the decision variables to one. We find that the same phenomena observed in the original treatments continues to persist. As such, we believe that our results are not due to complexity of the coordinating contracts, but instead retailers are maximizing an alternative utility function that resembles regret.

As mentioned previously, as technology advances so does the adoption and utilization of pull contracts. Improved integration and communication between a retailer and supplier has allowed more and more products to be shipped directly from suppliers to retailers’ customers. Our work suggests that retailers operating, or considering operating, under a pull setting should tread carefully when deciding how to structure their contracts. We have shown that retailers, in a simple controlled laboratory environment, can generally set contract parameters close to optimal in a wholesale price contract. However, when it comes to coordinating contracts, retailers do not perform well. This is primarily due to a natural bias for humans to regret setting the coordinating contract high, and instead offsetting this by increasing the wholesale price, thus foregoing substantial profits. In settings more complicated than our laboratory experiment, it is reasonable that retailers are even more susceptible to these natural biases, and that the benefits of coordinating contracts posed in theory are even less likely to be realized in reality.
Chapter 4

Evaluating the Profitability of Grace Service-Level Agreements on Supply Chains

4.1 Introduction

In 2001 Amazon.com, the popular internet provider of everything from books to home furniture, claimed to have saved 50% of its fulfillment costs by shipping its product directly to customers from its suppliers’ warehouses (Patsuris 2001). This method of satisfying demand, commonly called drop shipping, has increased in popularity since its widespread adoption at Amazon.com and other online businesses. However, for a retailer similar to Amazon.com, when it receives an order and its supplier is out of stock, then the retailer misses a potential sale and incurs a penalty cost for not having the product readily available, contributing to a poor reputation for the retailer, but not necessarily the supplier. To avoid such situations, a retailer could offer a bonus to its supplier, where the bonus would be paid to the supplier if it satisfied a specific portion of the retailer’s demand, known as a target fill rate. In short, the retailer could propose a service-level agreement (SLA) to its supplier and help avoid situations that hurt profitability and reputation.

The standard SLA states that a bonus is paid to a supplier when the fill rate, aggregated over a number of stocking decisions (review horizon), is above some target fill rate. However, there is another class of SLAs, which we call Grace SLAs,
that state the fill rate be calculated separately for each stocking decision over a review horizon. Under Grace SLAs, the supplier may have to satisfy the target fill rate every stocking decision over a review horizon to be awarded the bonus, or, the supplier may be allowed to miss the target fill rate for a specific number of stocking decisions over the review horizon (the number of grace periods), and still be awarded the bonus. A recent study showed that 91% or organizations utilize some type of SLA to manage suppliers and external customers (Oblicore 2007), in this investigation we address the following research questions pertaining to Grace SLAs:

(1) What are the optimal bonuses and target fill rates from the retailer’s perspective?

(2) Do Grace SLAs increase profits for both parties over wholesale price contracts?

(3) Can a retailer increase its profit without dictating all of the contract terms to its supplier?

We derive the optimal conditions of Grace SLAs from the retailer’s point-of-view and evaluate their performance over a variety of scenarios. We demonstrate that the profitability of Grace SLAs in equilibrium is higher than that of a wholesale price contract for both retailers and suppliers, thus generating more profits for the supply chain. This increase in profits also persists if the supplier makes errors randomly or systematically from its optimal stocking quantity.

We then investigate how the length of the review horizon impacts supply chain performance for various SLAs. Our results suggest that the length of the review horizon can have dramatic consequences on the benefits SLAs pose. Particularly, as the length of the review horizon increases, it is in both the retailer’s and supplier’s interest to increase the number of grace periods. Intuitively, one might think that adding grace periods would be costly for the retailer, as it gives the supplier a better chance at achieving the target fill rate and paying out a bonus, but this cost is offset by a bigger improvement in overall fill rates and sales.

Our results also indicate that retailers can concede some terms to suppliers when proposing a Grace SLA and still achieve considerable profits compared to
a wholesale price contract. For example, when the target fill rate is set by the supplier, the retailer can increase profits by adjusting the time horizon and/or the number of grace periods.

The results from our study can help retailers to better understand how they can improve profitability through offering a Grace SLA to their suppliers, and how terms within the Grace SLA should be set in order to maximize profits.

In the next section we provide a brief summary of related research. In §4.3 we introduce the basic model for Grace SLAs and relate certain fill rate requirements to different varieties of Grace SLAs. In §4.4, we provide our results relating to Grace SLAs when the target fill rate is exogenously or endogenously set, along with the performance of SLAs when the supplier does not act optimally. Lastly, we conclude our investigation and discuss future work in §4.5.

### 4.2 Related Literature

There are three research areas that pertain to our study; the item-level fill rate, contracting under drop shipping scenarios, and contracting with service-constraints.

The item-level fill rate has been well studied. Chen, Lin & Thomas (2003) prove that the expected fill rate over a finite review horizon for a fixed stocking quantity is greater than the expected fill rate over an infinite review horizon. Sobel (2004) illustrates how short supply chains have higher expected fill rates than longer supply chains. Banerjee & Paul (2005) generalize Chen et al. (2003)'s work by proving the expected fill rate is decreasing in the number of stocking decisions. Thomas (2005) examines how the expected fill rate and probability of achieving that fill rate behaves over different demand distributions and review horizons.

A moderate amount of research has been completed on contracting in supply chains under drop shipping. Netessine & Rudi (2004) consider drop shipping between a single supplier and single retailer with a multiple period model where the demand facing the retailer depends on a level of effort. Cachon (2004) studies push (traditional supply chain) and pull (drop shipping) contracts and illustrates, among other results, how a drop shipping scenario always generates higher supply chain profits over the push contract. Netessine & Rudi (2006) study multiple retailers and investigate how a structure that incorporates both a traditional supply
chain setup and drop shipping setup can result in both parties stocking less product. Mukhopadhyay, Zhu & Yue (2009) examine a setting where a supplier sells directly to customers through internet sales, as if the direct channel were a certain type of drop shipping.

Service constraints in supply chains can take on a variety of forms. Swaminathan & Srinivasan (1999) provide an algorithm for satisfying individual service-constraints with a single source of inventory for multiple customers for traditional supply chains. Choi, Dai & Song (2004) look at a traditional vendor managed inventory scenario and examine the effects of supplier service levels and stockout rates. Ferguson, Guide & Souza (2006) apply service levels to reverse supply chains for false failure returns. Bernstein & Federgruen (2007) assume retailers face random demand that depend on prices and the availability of products (service-levels) and compare those mechanisms to contracts when demand only depends on prices. Gan, Sethi & Zhou (2009) consider drop shipping with penalty contracts and asymmetric information. Additionally, service-levels have been thoroughly studied for call centers Koole (2008).

Lastly, we would be remiss to ignore general contracting in traditional supply chains. Lariviere (1999) and Cachon (2003) provide full reviews of supply chain contracting under a variety of settings.

4.3 Model

We assume the following model; a supplier, $S$, sets a base-stock, order-up-to level of a single item, $Q$, which is held constant for each period, $t$, over a review horizon $T$ where $t = \{1, ..., T\}$ (each period in the review horizon can be considered a single stocking decision). The supplier chooses its stocking level based on its cost per unit, $c$, and wholesale price per unit $w$. The retailer, $R$, purchases units from the supplier at the wholesale price per unit, receives revenue, $r$, for each unit sold, incurs no holding cost, and loses sales if demand is greater than the supplier stocking quantity ($X_t > Q$). Let $X_t$ be a random variable that represents realized demand for period $t$, which is independently and identically distributed with cumulative distribution $F$ and density $f$.

We assume lead time is zero with no ordering costs. Let $SC$ represent the
supply chain, and $\pi_j$ the corresponding expected profit where $j \in \{R, S, SC\}$. There is full information of all cost parameters, and both parties are risk-neutral expected-profit maximizers.

Up to this point our model represents a traditional wholesale price contract between a retailer and supplier. Now we introduce the two parameters that differentiate our Grace SLA from a wholesale price contract; a per period bonus, $\beta$, and a target fill rate, $\alpha$ (from here on, unless otherwise stated, SLA will refer to the class of Grace SLAs). The bonus, $\beta$, is paid from the retailer to the supplier when the realized fill rate each period, $t$, denoted by $\psi_t$, is equal to or greater than the target fill rate, $\alpha$, a specific number of times over the review horizon. The realized fill rate is defined as the fraction of satisfied demand for the retailer with the supplier stocking quantity (more details to follow).

The retailer plays the Stackelberg leader, offering bonus and target fill rate terms that the supplier can accept or reject. Upon receipt of the SLA terms, the supplier sets a stocking quantity, and demand is then realized. Note that for our analysis we assume a wholesale price contract is already in place and the retailer decides to offer specific SLA terms to its supplier on top of the current agreement.

The retailer’s goal is to maximize its expected profit, with respect to the bonus and target fill rate, subject to the supplier’s profit being no worse than under a traditional wholesale price contract. When this condition is satisfied, the supplier always accepts. Lastly, let efficiency be defined as the ratio between the decentralized supply chain profit and the centralized supply chain profit.

The expected profit functions, $\pi_j$, per period for the relevant parties are given by:

$$
\pi_S(Q) = wE[\min(Q, X)] - cQ + \beta \times \Pr(\beta)
$$

$$
\pi_R(\beta, \alpha) = (r - w)E[\min(Q, X)] - \beta \times \Pr(\beta)
$$

$$
\pi_{SC}(Q) = rE[\min(Q, X)] - cQ
$$

(4.1)

where $\Pr(\beta)$ represents the probability of the bonus being paid out.

Unless otherwise stated, we will suppress the arguments on the left-hand side of these profit equations. Note that these are the traditional wholesale price profit equations with an additional term relating to the expected value of the bonus.
Next we turn to how different fill rate targets can create different varieties of SLAs.

4.3.1 Fill Rate Targets and SLA Variations

Understanding how the probability of achieving the bonus behaves is instrumental in investigating SLAs. The fill rate for a single period review horizon, $\psi_t(Q)$ is:

$$\psi_t(Q) = \min(Q, X_t)/X_t.$$  \hfill (4.2)

The probability of achieving a bonus in a single period is:

$$\Pr[\psi_t(Q) \geq \alpha] = F\left(\frac{Q}{Q/\alpha}\right).$$ \hfill (4.3)

We use this definition of achieving the fill rate in a single period to partition the Grace SLA class into two types of SLAs.

**All Periods SLA**: Under the All Periods (AP) SLA, the supplier is awarded the bonus, $\beta$, when fulfilling a target fill rate, $\alpha$, each and every period over a review horizon. The AP – SLA can be viewed as a general case of the single period time horizon SLA where the probability of achieving the bonus is:

$$\Pr(\beta) = F\left(\frac{Q}{\alpha}\right)^T.$$ \hfill (4.4)

**Grace($k$) SLA**: Under a Grace($k$) (Gr($k$)) SLA, there are two target fill rates; one that the supplier can miss for a fixed number of periods over a review horizon, and a second that the supply can never miss to achieve the bonus. Let $\alpha$ represent the target fill rate that must be achieved $(T - k)$ periods and $\alpha_L$ the minimum target that must be satisfied for the remaining $k$ periods ($k$ can be considered the number of grace periods over a review horizon). In practice, $\alpha_L$ represents a breach of contract minimum service level, we assume in our model that $\alpha_L$ is exogenous.

The probability of attaining the bonus under the Gr($k$) – SLA is given by:

$$\Pr(\beta) = \sum_{j=T-k}^{T} \binom{T}{j} F\left(\frac{Q}{\alpha}\right)^j \left( F\left(\frac{Q}{\alpha_L}\right) - F\left(\frac{Q}{\alpha}\right) \right)^{(T-j)}.$$ \hfill (4.5)
Note that in this model we assume the supplier can never miss both fill rate targets. If that were the case, the previous equation, (4.5), would simply be described by a multinomial distribution.

4.4 Results

For both the All Periods SLA \((AP - SLA)\), and Grace\((k)\) SLAs \((Gr(k) - SLA)\), we evaluate the effects of review horizon length on a variety of metrics. These include retailer profit, supplier profit, supply chain efficiency, optimal supplier stocking quantities, optimal bonus levels, the probability of achieving the bonus, and optimal target fill rates.

First we will review our results when the target fill rate is set at 100%, later we will investigate how alternative target fill rates affect the results. After we present both of those sections, we will then perform a number of robustness checks on the performance of SLAs when a supplier stocks suboptimally.

4.4.1 Fixed Service Level: 100% Target Fill Rate

We begin with a setting where the target fill rate is fixed and equal to 100%. Under the \(AP - SLA\), the supplier’s expected profit over a review horizon of \(T\) periods is (please see Appendix 7.1 for all derivations):

\[
\pi_s = wE[\min(Q, D)] - cQ + \beta F(Q)^T
\]

(4.6)

We wish to find the optimal bonus, \(\beta^*\) for the retailer. First, we must derive the supplier’s optimal stocking quantity, \(Q^*(\beta)\), where we suppress the arguments for notational convenience.

The supplier’s optimal stocking quantity must satisfy:

\[
(w - c) = wF(Q^*) - T\beta F(Q^*)^{T-1}f(Q^*)
\]

(4.7)

Because (4.6) does not necessarily have a unique maximum (particularly for large \(T\), which will be shown momentarily, Figure 4.1a), the second order condition must
also be satisfied for $Q^*$ to be a local maximum:

$$ wf(Q^*) > T\beta((T - 1)F(Q^*)^{T-2}f(Q^*)^2 + F(Q^*)^{T-1}f'(Q^*)) \quad (4.8) $$

Let a stocking quantity that satisfies the local maximum conditions be denoted by $\hat{Q}$. Then, the optimal bonus for the retailer must satisfy:

$$ F(\hat{Q}) = \frac{Tf(\hat{Q})(r(1 - F(\hat{Q})) - c)}{wf(\hat{Q}) - T\beta^*((T - 1)F(\hat{Q})^{T-2}f(\hat{Q})^2 + F(\hat{Q})^{T-1}f'(\hat{Q}))} \quad (4.9) $$

In sum, conditions (4.7), (4.8), and (4.9) must be satisfied for $Q^*$ and $\beta^*$ to be locally optimal. However, $\pi_R(\beta)$ may not be differentiable at $\beta^*$. As an illustration, consider the $AP - SLA$ for $T = \{3, 4, 5\}$. We begin by looking at the supplier’s profit function with respect to $Q$ given an optimal bonus (in other words, the retailer has already solved the problem for $\beta^*$ but wants to investigate how the supplier reacts to it). The supplier’s profit function with respect to $Q$, given $\beta^*$, is illustrated in Figure 4.1a. Note how the supplier’s profit function is unimodal for $T = 3, 4$, but bimodal for $T = 5$. While difficult to see, the second mode is actually slightly higher than the first for $T = 5$, making the supplier “jump” to a different stocking quantity. This is because, as $T$ increases, the bonus also becomes larger, and when we combine the traditional wholesale price supplier profit function with a bonus that can be achieved for larger quantities, it creates a bimodal function.

Figure 4.1: Supplier profit with respect to $Q$, and optimal stocking quantities with respect to $\beta$ with $D \sim N(100, 20)$ with $r = 10$, $w = 5$, $c = 3$, and $\alpha = 100\%$. 
Turning to Figure 4.1b, we plot the supplier’s optimal stocking quantity with respect to different bonuses. Once again, when $T = 5$ we see a situation where, when the bonus exceeds some threshold ($\beta^*$), the supplier “jumps” to a higher stocking quantity, which corresponds to the second mode in Figure 4.1a.

![Graph showing retailer profit with respect to $\beta$](image)

**Figure 4.2:** Retailer profit with respect to $\beta$ with $D \sim N(100, 20)$ with $r = 10$, $w = 5$, $c = 3$, and $\alpha = 100\%$.

Given this behavior of the supplier, how does it impact the retailer? In Figure 4.2, there is a discontinuity in the retailer’s profit function for $T = 5$ at the same bonus that caused the supplier’s optimal stocking quantity to jump in Figure 4.1a. This is due to the benefit, for the retailer, of marginally increasing the bonus and receiving a dramatic increase in the stocking quantity (note that it is not always profit maximizing for the retailer to set a bonus where this jump takes place, we are simply illustrating that this situation does exist). Thus, $\pi_R(\beta)$, for large $T$, may have a discontinuity that requires numerical computations for finding the optimal bonus.

We conduct a similar process to find the optimal conditions for the $Gr(k) - SLA$, when $k = 1$. Recall that under the $Gr(k) - SLA$, with $k = 1$, the retailer will pay the bonus even if the supplier misses the higher target fill rate one period (one grace period) and still manages to hit the lower target fill rate during that
specific period. Let:

\[ \lambda(Q) = \Pr(\beta) = \left( \frac{T}{T-1} \right) F(Q)^{T-1} \left( F\left( \frac{Q}{\alpha_L} \right) - F(Q) \right) + F(Q)^T \]  

(4.10)

Which we denote as \( \lambda(Q) \). The supplier’s optimal stocking quantity must satisfy:

\[ (w - c) = wF(Q^*) - \beta \frac{\partial \lambda}{\partial Q} \]  

(4.11)

Once again, the supplier’s profit is not necessarily concave, and the second order condition must be satisfied for \( Q^* \) to be a local maximum.

\[ wf(Q^*) > \beta \frac{\partial^2 \lambda}{\partial Q^2} \]  

(4.12)

Let \( \hat{Q} \) represent any \( Q \) in the set that satisfies the local maximum conditions.

The final condition for \( \beta^* \) for the retailer is:

\[ \lambda(Q) = \left( (r - w)(1 - F(\hat{Q})) - \beta^* \frac{\partial \lambda}{\partial Q} \right) \frac{\partial \hat{Q}}{\partial \beta} \]  

(4.13)

In sum, conditions (4.11), (4.12), and (4.13) must be satisfied for \( Q^* \) and \( \beta^* \) to be locally optimal. However, once again, \( \pi_R(\beta) \) may not always be differentiable at \( \beta^* \) for large \( T \). For the \( Gr(k) - SLA \) with a single grace period, the same behavior as the \( AP - SLA \) exists, only it occurs for larger \( T \).

Recall in our earlier example that when \( T = 5 \), there is a jump in \( Q \) (Figure 4.2), and the retailer profit is lower than when \( T = 4 \) and the supplier has a unique maximum. If the review horizon is fixed at \( T = 5 \), then adding one grace period makes the supplier’s profit function unimodal again and increases the retailer’s profit. The supplier’s profit and retailer’s profit, for the \( AP - SLA \) and \( Gr(k) - SLA \) with a single grace period and \( T = 5 \) are plotted in Figures 4.3a and 4.3b.

**Performance over Review Horizons**

Figure 4.4 illustrates the percent per period profit improvement for the retailer and supplier over a \( WP \) contract, the resulting supply chain efficiency, and the optimal supplier stocking quantity for various review horizons. All results are based on the retailer maximizing its profit, given that the supplier will stock optimally.
Figure 4.3: Retailer and supplier profit with respect to $\beta$ and $Q$ with $D \sim N(100, 20)$ with $r = 10$, $w = 5$, $c = 3$, and $\alpha = 100\%$.

For all four figures, $r = 10$, $w = 5$, $c = 3$, and $\alpha_L = 70\%$. Demand is assumed to be normally distributed with mean 100 and standard deviation 20. Four contracts are plotted; All Periods ($AP$), Grace($k$) with a single grace period per review horizon ($Gr(1)$), Grace($k$) with two grace periods per review horizon ($Gr(2)$), and a wholesale price contract ($WP$).
(a) Retailer profit $\pi_R$

(b) Supplier profit $\pi_S$

(c) Optimal supplier stocking quantity $Q^*$

(d) Efficiency

Figure 4.4: Per period metrics for $D \sim N(100, 20)$ with $r = 10$, $w = 5$, $c = 3$, $\alpha = 100\%$, and $\alpha_L = 70\%$, for various $T$.

From Figure 4.4a, we can see that as $T$ increases, a retailer should initially favor an $AP - SLA$ and then switch to a $Gr(k) - SLA$ with an increasing number of grace periods. This is because when the retailer finds the supplier stocking at the second mode of its profit function, the retailer must pay out a large bonus very frequently, and instead finds it optimal to incorporate an extra grace period with a smaller bonus where the “jump” in modes for the supplier does not take place. In fact, we find that whenever a specific SLA induces the supplier to set a stocking quantity that corresponds to the second mode of its profit function, the retailer always finds it profit maximizing to add one grace period, and create a supplier’s profit function that has a unique maximum.

Looking at the supplier’s profitability improvement to a $WP$ contract, in Figure 4.4b, we see that the exact values of $T$ where each SLA are preferred slightly vary
from that of the retailer. Unlike the retailer, the supplier prefers to switch SLAs at lower values of $T$. It is also worth noting that for very large time horizons (i.e. $T > 15$), all of the contracts evaluated here converge to the $WP$ profit, but SLAs can address this by incorporating additional grace periods.

Figures 4.4a and 4.4b suggest that the maximums of each SLA profit function, when connected, are monotonically increasing and concave for the retailer, and monotonically decreasing and convex for the supplier. For a number of scenarios we consider, it seems as though SLAs pose an even greater benefit to retailers for large $T$ when $\alpha$ is restricted to 100% and there are more grace periods.

Note that $Q^*$, in Figure 4.4c, actually exceeds the optimal supply chain quantity, $\approx 110$, for certain scenarios. When this occurs, the retailer ends up paying out the bonus more frequently, and finds that an alternative SLA, often one with an additional grace period, is much more attractive in terms of its own profitability.

It follows from the profit results that the efficiency of SLAs is sensitive to the review horizon. As seen in Figure 4.4d, these SLAs frequently result in nearly 100% efficiency. For example, for $T = 4$, the $AP - SLA$ results in nearly 100% efficiency, and then the $AP - SLA$ efficiency, for $T > 4$, drops rapidly to the $WP$ contract efficiency. This pattern, repeats for the $Gr$ SLAs as $T$ grows. Lastly, note that the $WP$ contract efficiency is rather high. This is to be expected, as supply chain efficiency, when the supplier holds the inventory, has been proven to be higher when compared to the more traditional newsvendor scenario, where the retailer holds the inventory (Cachon 2004).

Another observation for changing review horizons pertains to $\beta^*$. Although not depicted, $\beta^*$ for all SLAs is monotonically increasing in $T$. Additionally, $\beta^*$ for the $AP - SLA$ is strictly greater than $\beta^*$ for both $Gr$ SLAs (where $Gr(1)$ has an optimal bonus strictly greater than $Gr(2)$). There is also a clear relationship between $Q^*$ and the probability of achieving $\beta^*$. Specifically when there is a jump in $Q^*$, there is also a jump in the probability of achieving the bonus, despite the increase in $T$ (which naturally reduces the probability of the bonus for a constant $Q$ and fixed SLA).

One theme that stands out from Figure 4.4 is that of recurrence. When looking at all four plots in Figure 4.4, each SLA plot appears to be one function, roughly copied and pasted further and further along the x-axis. For example, the
retailer and supplier profit improvement plots show that as one moves along $T$, the $AP-SLA$ from $T = \{1, ..., 6\}$ is almost identical to the $Gr(1) - SLA$ from $T = \{3, ..., 10\}$. The same takes place in regards to supply chain efficiency. Furthermore, the plot of $Q^*$ for the $AP-SLA$ from $T = \{1, ..., 7\}$ is very similar to the $Gr(1) - SLA$ for $T = \{4, ..., 10\}$.

We also investigated how all of these supply chain metrics performed when $\alpha$ was fixed at values less than 100% (i.e. 95% and 90%). We find that the same sort of trends exist as when $\alpha = 100\%$ except for one difference. As $\alpha$ decreases, the benefit of each specific SLA over a WP contract exists for a greater range of review horizons. For example, when $\alpha = 100\%$ the retailer benefit of the $AP-SLA$ takes place roughly for $T = \{1, ..., 9\}$, however, when $\alpha = 90\%$, the retailer benefits from an $AP-SLA$ roughly for $T = \{1, ..., 22\}$. This trend exists for the other SLAs as well.

These results indicate that even when an exogenous target fill rate is fixed for an SLA, the length of the review horizon can have dramatic consequences on both the retailer’s and supplier’s profits. Specifically as $T$ increases, the retailer, and supplier, find it profit maximizing to propose additional grace periods.

### 4.4.2 Flexible Target Fill Rate

Now assume that $\alpha$ is no longer restricted to 100%, and is a free parameter that can be optimized. To find the optimal supplier stocking quantities, bonuses, and target fill rates, we can repeat those steps outlined in the previous section. For the $AP-SLA$, the supplier’s optimal stocking quantity must satisfy:

$$ (w - c) = wF(Q^*) - \frac{1}{\alpha} T \beta F \left( \frac{Q^*}{\alpha} \right)^{T-1} f \left( \frac{Q^*}{\alpha} \right) $$

Which is not a maximum unless the second order condition is satisfied as well. Let a member of the set of stocking quantities that satisfy the first and second order conditions be denoted by $\hat{Q}$.

Differentiating the retailer’s profit with respect to the bonus and substituting
in accordingly, results in the following condition for \( \beta^* \) to be optimal:

\[
F \left( \frac{\hat{Q}}{\alpha} \right) = \frac{\frac{1}{\alpha} T f(\hat{Q}) (r(1 - F(\hat{Q})) - c)}{w f(\hat{Q}) - \frac{1}{\alpha} T \beta^* ((T - 1) F(\hat{Q}) T^{-2} f(\hat{Q})^2 + \frac{1}{\alpha} F(\hat{Q}) T^{-1} f'(\hat{Q}))} (4.15)
\]

To find the optimal target fill rate, one must find \( \frac{\partial \beta^*}{\partial \alpha} \) and replace it in the following equation to solve for \( \alpha^* \):

\[
\frac{\partial \pi_R}{\partial \alpha} = (r - w)(1 - F(\hat{Q})) \frac{\partial \hat{Q}}{\partial \beta} \frac{\partial \beta^*}{\partial \alpha} - T \beta^* F \left( \frac{\hat{Q}}{\alpha^*} \right)^{T-1} f \left( \frac{\hat{Q}}{\alpha^*} \right) \left( \frac{1}{\alpha^*} \frac{\partial \hat{Q}}{\partial \beta} \frac{\partial \beta^*}{\partial \alpha} - \frac{\hat{Q}}{\alpha^*} \right)
- F \left( \frac{\hat{Q}}{\alpha^*} \right)^T \frac{\partial \beta^*}{\partial \alpha} (4.16)
\]

For the Grace\((k)\) SLA with one grace period per review horizon, the same steps can be performed. And, as mentioned previously, one must be careful because \( \pi_R \), for all SLAs, may not always be differentiable at \( \beta^* \) for large \( T \).

**Performance over Review Horizons**

As before, demand is normally distributed with mean 100 and standard deviation 20. Also, we continue with \( r = 10, w = 5, \) and \( c = 3 \) with \( \alpha_L = 70\% \). We evaluate target fill rates that are integer valued. Figure 4.5 illustrates all four of the contracts evaluated for the percent improvement in retailer and supplier profit over a WP contract, supply chain efficiency, optimal stocking quantities, optimal bonuses, and optimal target fill rates.
Figure 4.5: Per period metrics for $D \sim N(100, 20)$ with $r = 10$, $w = 5$, $c = 3$, and $\alpha_L = 70\%$, for various $T$. 
As expected all of the SLAs outperform the WP contract. However, compared to when \( \alpha \) was restricted to 100\%, the retailer profit functions no longer have a concave shape. Instead, they are monotonically increasing in \( T \), which implies that it is in the retailer’s interest to set a review horizon that is as long as possible when \( \alpha \) is a free parameter.

Another difference between these results and those when \( \alpha \) was restricted to 100\%, relates to the expected value of \( \beta^* \). In the \( \alpha = 100\% \) case, the expected value of \( \beta^* \) was increasing in \( T \) and varied between 0 and at least 30 for all three of the SLAs evaluated. However, when \( \alpha \) is unrestricted, we find that the expected value of \( \beta^* \) fluctuates between just a few units. For example, for the \( AP-SLA \), \( \mathbb{E}[\beta^*] \approx 13 \) for almost every \( T > 1 \).

The results depicted in Figure 4.5 depend on \( \alpha \) being a free parameter. However, as seen in Figure 4.5e, for shorter time horizons, \( \alpha^* \) is actually 100\% in many cases. In fact, for \( Gr(2) \), \( T \) must exceed 10 before \( \alpha^* \) drops below 100\% (not graphed). Therefore, if a retailer cannot get its supplier to agree on a longer review horizon, then the retailer should state that in concession for a short review horizon the target fill rate must be 100\%.

Figure 4.6: Relationship between \( \beta^* \) and \( \alpha^* \) for \( D \sim N(100, 20) \) with \( r = 10 \), \( w = 5 \), \( c = 3 \), and \( \alpha_L = 70\% \), for various \( T \) under the \( AP-SLA \).

Figure 4.6 plots both \( \beta^* \) and \( \alpha^* \) for the \( AP-SLA \) on the same chart. As one can see, \( \beta^* \) and \( \alpha^* \) are highly correlated. Particularly, when there is a “kink” in either of the lines plotted, the other follows in the same direction. For example,
when $\alpha$ is restricted to 100%, $\beta^*$ is monotonically increasing in $T$. However, if $\alpha^*$ drops, then this effect offsets any natural increase in $\beta^*$ from an increase in $T$.

Summarizing these results for the profitability of both parties, it appears allowing $\alpha$ to be a free parameter primarily benefits the retailer, as the profit function for the retailer is monotonically increasing in $T$, while the supplier is only slightly better depending on the specific scenario.

The main conclusion to draw from all of these results ($\alpha$ restricted and unrestricted) is that, even for a fixed review horizon, two key levers can improve a retailer’s profitability; the target fill rate and the number of grace periods. If a retailer does not have a target fill rate that it desires, then any SLA will benefit the retailer. However, if a retailer is fixated on a certain fill rate, say 95%, and the supplier is firm on a review horizon length, then the retailer can be just as well off as when the target fill rate was a free parameter, by increasing the number of grace periods as $T$ increases.

4.4.3 SLA Robustness

Past literature has shown that suppliers may not always stock optimally when facing a WP contract or SLA (Schweitzer & Cachon (2000) and Katok, Thomas & Davis (2008)). Therefore, we now consider how SLAs perform when the suppliers deviate from $Q^*$.

We continue with $D \sim N(100, 20)$, $r = 10$, $w = 5$, $c = 3$, and fix $T = 8$. Now, let $Q$ represent a uniform random variable centered on $Q^*$, with its support given by a percent deviation in $Q^*$. Modeling $Q$ this way replicates a simple scenario where a supplier sets $Q$ optimally on average, but tends to make systematic errors in both directions (overstocks and understocks).

Using this setup along with $\beta^*$ and $\alpha^*$, we can calculate the corresponding per period profits for the retailer and supplier, along with the supply chain efficiency, and probability of achieving the bonus. Figures 4.7a and 4.7b illustrate the percent reduction in per period profit for the retailer and supplier for $T = 8$, given different supports for $Q$. Note that the supplier’s profit function can be bimodal function in certain conditions with respect to $Q$. We assume that for this robustness section, the supplier initially finds the correct mode, and then proceeds to deviate from this
value within some margin of error. Figures 4.7c and 4.7d also depict the resulting absolute per period profits for the retailer and supplier.

Figure 4.7: Robustness of SLAs and WP contracts to deviations in $Q$ for $D \sim N(100, 20)$ with $r = 10$, $w = 5$, $c = 3$, $\alpha_L = 70\%$, and $T = 8$, where $Q$ is a uniform random variable with mean $Q^*$. 
According to Figures 4.7a and 4.7b, the sensitivity of SLAs to deviations in $Q$, for the retailer, is greater than that of a WP contract. However, it appears as though SLAs are rather robust to changes in $Q$ for supplier profitability. The robustness to changes in $Q$ for the supplier implies that when a retailer proposes an SLA to a supplier, the supplier may be more inclined to accept.

Figures 4.7c and 4.7d illustrate that, despite the sensitivity of the retailer’s profit for SLAs to $Q$, the absolute profit for the retailer still favors all three SLAs (and more so for the supplier).

The preference of the SLAs to specific parties depends heavily on the review horizon. For example, the same robustness check was also completed with $T = 4$ and $T = 10$. In all cases the sensitivity of the SLAs to changes in $Q$ was rather high compared to the WP contract for the retailer (not for the supplier). But the corresponding absolute profits for both parties always favored the three SLAs over the WP contract, where the order of preference between the SLAs varied.

The supply chain efficiency when $Q$ is modeled as a uniform random variable around $Q^*$ for SLAs significantly outperforms the WP contract. For the given parameters, supply chain efficiency never drops below 98% for any of the three SLAs evaluated for every support of $Q$. On the other hand, the efficiency for the WP contract at best, reaches 96.2%. This results also holds for $T = 4$ and $T = 10$. In sum, these results suggest that when $Q$ is not always stocked optimally, but is set optimally on average, the supply chain efficiency of SLAs generally exceeds that of the traditional WP contract.

Past literature has also shown that suppliers may set $Q$ in a systematic way depending on the profit margin of the product for sale ((Schweitzer & Cachon 2000) and (Bostian et al. 2008)). For high margin products (critical fractile $> 50\%$), suppliers tend to understock $Q$, and for low margin products (critical fractile $< 50\%$), suppliers tend to overstock $Q$. There have been numerous explanations for this behavior such as the pull-to-center effect and bounded rationality (Su 2008). Regardless of the reason for the result, this behavior clearly exists in practice. As such, we now consider how SLAs perform when the supplier sets $Q$ above or below $Q^*$ consistently. Figure 4.8 illustrates the relative performance of profits with $Q^*$ to those with consistent deviations in $Q$ (Figures 4.8a and 4.8b) and the corresponding absolute profits (Figures 4.8c and 4.8d).
(a) Reduction in retailer profit $\pi_R$

(b) Reduction in supplier profit $\pi_S$

(c) Retailer profit $\pi_R$

(d) Supplier profit $\pi_S$

Figure 4.8: Robustness of SLAs and WP contracts to consistent deviations in $Q$ for $D \sim N(100, 20)$ with $r = 10$, $w = 5$, $c = 3$, $\alpha_L = 70\%$, $T = 8$.

In Figure 4.8a the retailer’s profit actually has a negative reduction in profit (an increase in profit), in certain cases. This is to be expected for the WP contract because the retailer incurs no holding cost under the drop shipping scenario. However, this same phenomenon occurs for the $Gr(2) - SLA$. This is because, when the supplier overstocks and there are two grace periods, the supplier increases its probability of achieving a small bonus, and the retailer finds that the increase in profits from the higher $Q$ offsets this cost of the bonus (although not shown, this trend also exists for the $Gr(1) - SLA$ for $T = 4$). When the supplier understocks, on the other hand, all three SLAs are more robust than the WP contract for the retailer.

For the supplier, illustrated in Figure 4.8b, it appears as though all four of the
contracts evaluated perform rather similarly to consistent changes in $Q$. The one exception relates to when $Q$ is understocked by 10% for the $WP$ contract. In this case, the supplier incurs a large reduction in profit.

Once again, we plot the absolute profits for both the retailer and supplier for these changes in $Q$ in the bottom two plots of Figure 4.8. As when $Q$ was modeled as a uniform random variable, we find that for any consistent deviations in $Q$, except $+10\%$, all three SLAs outperform the $WP$ contract. This same result holds for $T = 4$ and $T = 10$.

We observe that SLAs should generally be favored by retailers and suppliers even when the supplier may not always stock optimally. While the relative performance of SLAs is not always as resilient to changes in $Q$ as a $WP$ contract, the corresponding absolute profits favor SLAs. For retailers, the only time this is not true is when a supplier consistently overstocks by a large amount. For suppliers, it is true in every scenario we evaluate. These results imply that not only do SLAs benefit the retailer, but the supplier has a higher likelihood of accepting the SLA because it benefits the supplier even if they stock suboptimally, either randomly around $Q^\ast$ or consistently in one direction.

4.5 Conclusion

This work is motivated by witnessing different variations of SLAs, Grace SLAs, in practice and observing the benefits they pose over wholesale price contracts. Our work suggests that retailers, operating under a drop shipping environment, have a number of options if they offer a Grace SLA to their suppliers on top of an existing wholesale price contract. Those options, when the target fill rate is exogenously set, are the length of the review horizon and the number of grace periods. If the parties agree on a longer review horizon, then both parties benefit by incorporating more grace periods. Additionally, if the target fill rate is not exogenously fixed, and the length of the review horizon is long, then adjusting the target fill rate for any number of grace periods will result in higher profit for the retailer. In summary, retailers proposing Grace SLAs to suppliers generally have the ability to adjust any one of three levers to maximize profits; the length of the review horizon, the number of grace periods, and the target fill rate.
Given our model, there are some extensions which need addressing. One natural extension is the case where there are multiple products between many retailers and suppliers (although the presence of an SLA would likely benefit a retailer even more were the supplier pooling inventories). Another extension is to consider a dynamic inventory policy.

Businesses are constantly looking for ways to improve profitability. One way companies can accomplish this is by considering a Grace SLA as its contract for operations. Not only are Grace SLAs cost-effective to administer (the fill rate is calculated by companies even operating under a wholesale price contract), but they provide a retailer with the flexibility to propose and negotiate a Grace SLA to its supplier, that not only augment the retailer’s profitability, but its supplier’s profitability as well.
Chapter 5

Appendix for Chapter 1

5.1 Sample Instructions

You are about to participate in a decision making experiment. If you follow these instructions carefully and make good decisions you could earn a considerable amount of money that will be paid to you in cash at the end of the session. Your earnings will depend on your decisions and the decisions of other participants. If you have a question at any time, please raise your hand and I will answer it. Please do not talk with one another, or use any sort of electronic devices (i.e. cellphones, texting devices, iPods, etc) for the duration of the experiment.

How to make money

You will have one of two roles for the duration of the session, a retailer or a supplier. You will play 30 periods, each representing one selling season. The retailer sells units to customers at 15 per unit. The supplier produces the units at a cost of 3 per unit.

The supplier sets a wholesale price, which is the amount per unit the retailer pays for units he pre-orders at the beginning of the season. The retailer, after seeing the supplier’s offered wholesale price, sets a pre-order amount, or rejects the contract, in which case both players earn the outside option of 60.

Customer demand is randomly determined each period, from 0 to 100, each integer in that range equally likely. You will be randomly matched with a different person each period.

Profit calculations

Profits depend on the number of units sold, called sales. Sales are determined as follows:
• Sales = Demand if (Demand < Pre-Order)
• Sales = Pre-Order if (Demand ≥ Pre-Order)

The profit calculations, if the retailer accepts the supplier’s wholesale price are:

• Retailer Profit = (15 x Sales) - (Wholesale Price x Pre-Order)
• Supplier Profit = (Wholesale Price - 3) x Pre-Order

The profit calculations, if the retailer rejects the supplier’s wholesale price are the outside option:

• Retailer Profit = 60
• Supplier Profit = 60

Flow of Each Period

At the beginning of each period the supplier will first set the wholesale price. To help suppliers make their decision they will see a scroll bar that allows them to test different wholesale prices and calculates the retailer’s optimal pre-order for the test value they choose, please note that this does not mean that the retailer will set this pre-order. Below is an example of the screen retailers will see:

Suppliers can try as many wholesale prices as they want, by using the scroll bar, before submitting their decision. When ready, suppliers should enter the wholesale
price into the text box on the screen and click the submit button. The wholesale price has to be between 3 and 15.

Next, the retailer will see the wholesale price and will decide whether to reject the supplier’s proposal or set a pre-order. To help retailers make their decision, they will see a graph that allows them to enter a test pre-order and calculates their profit for every possible demand realization. Below is an example of a screen the retailers will see.

![Retailer Screen Example](image)

Retailers can try as many pre-orders as they want, by using the scroll bar, before submitting their decision. When ready, retailers should make their decision by either setting a pre-order, which must be between 0 and 100, or rejecting the supplier’s proposal.

**Information you will see after each round**

After each round, you will see the following information (where applicable).

- The period
- The wholesale price the supplier offered
- Whether the proposal was accepted by the retailer
- The pre-order submitted by the retailer
- Realized demand
- Sales
• Your profit for the round.

You will also see this information for all past rounds on the bottom of each screen, and this information will be for different partners (since you will be randomly matched each period).

**Example**

Suppose the supplier enters a wholesale price of 7.00.

**Retailer accepts**

Imagine that the retailer accepts the supplier’s proposal then sets a pre-order of 50. Suppose demand for that period turns out to be 60, sales are then 50. The retailer earns the selling price for sales (15 x 50), minus the wholesale cost to purchase the pre-order (7.00 x 50).

• Retailer Profit = $(15 \times 50) - (7.00 \times 50) = 400$

The supplier earns the wholesale price per unit minus its cost per unit for the retailer pre-order.

• Supplier Profit = $(7.00 - 3) \times 50 = 200$

Now suppose the demand turns out to be 30. In this case sales are 30 because the demand of 30 is smaller than the pre-order of 50.

• Retailer Profit = $(15 \times 30) - (7.00 \times 50) = 100$
• Supplier Profit = $(7.00 - 3) \times 50 = 200$

**Retailer rejects**

Imagine the retailer rejects the supplier’s proposal. Then the profits are equal to the value of the outside option for each party:

• Retailer Profit = 60
• Supplier Profit = 60

**How you will be paid**

At the end of the session the actual earnings from the game will be converted to US dollars at the rate of 300 laboratory dollars for $1 US dollar. These profits will be added to your $5 show-up fee, displayed on your screen, and paid to you in cash at the end of the session.
Chapter 6

Appendix for Chapter 2

6.1 Proofs

Proof that \( w^* \) exists and is unique for the retailer under a pull wholesale price contract when demand has the IFR property. Considering that the retailer knows the supplier’s best response for a given a wholesale price, and by implicitly differentiating (3.1) with respect to \( w \) so that \( f(q^*) \frac{\partial q^*}{\partial w} = \frac{c}{w^2} \). We can plug this into the retailer’s first order condition with respect to \( w \) to obtain:

\[
\frac{\partial \pi_R}{\partial w} = c(r - w^*) \frac{\bar{F}(q^*)}{(w^*)^2} f(q^*) - \min(q^*, D).
\]

This first order condition is decreasing, from positive to negative, in \( w \) when the inverse failure rate function, \( \frac{\bar{F}(x)}{f(x)} \), is decreasing (or when the failure rate function is increasing), which is sufficient to guarantee uniqueness of \( w^* \).

Proof of Proposition 1: If \( 0 < \lambda < 1 \) represents the supplier’s share of the total profit, then for each \( \lambda \), a wholesale price \( w \), and an overstock-allowance \( \alpha \), can be found that perfectly coordinate the supply chain (100% efficiency) under an overstock-allowance contract, when the following equations are satisfied:

\[
\alpha = (1 - \lambda)c \\
w = \alpha + \lambda r
\]

(6.1)

Let \( 0 < \lambda < 1 \) represent the supplier’s share of the total profit in the supply chain.
Using the conditions from (6.1), then it is simple to verify that the profit for the supplier from an overstock-allowance contract can be expressed as a portion $\lambda$, of the total supply chain profit $\Pi$, and that the profit for the retailer from an overstock-allowance contract can be expressed as the remaining portion $(1 - \lambda)$, of the total supply chain profit $\Pi$.

\[
\mathbb{E}[\pi_S(q, D)] = (w - c)E[D] - (w - c)E[D - q]^+ - (c - \alpha)E[q - D]^+ \\
= (\alpha + \lambda r - c)E[D] - (\alpha + \lambda r - c)E[D - q]^+ - (c - (1 - \lambda)c)E[q - D]^+ \\
= ((1 - \lambda)c + \lambda r - c)E[D] - ((1 - \lambda)c + \lambda r - c)E[D - q]^+ - \lambda cE[q - D]^+ \\
= \lambda(r - c)E[D] - \lambda(r - c)E[D - q]^+ - \lambda cE[q - D]^+ \\
= \lambda \Pi
\]

\[
\mathbb{E}[\pi_R(w, \alpha, D)] = (r - w)E[D] - (r - w)E[D - q]^+ - \alpha E[q - D]^+ \\
= (r - \alpha - \lambda \alpha)E[D] - (r - \alpha - \lambda \alpha)E[D - q]^+ - (1 - \lambda)cE[q - D]^+ \\
= (r - (1 - \lambda)c - \lambda r)E[D] - (r - (1 - \lambda)c - \lambda r)E[D - q]^+ - (1 - \lambda)cE[q - D]^+ \\
= (1 - \lambda)(r - c)E[D] - (1 - \lambda)(r - c)E[D - q]^+ - (1 - \lambda)cE[q - D]^+ \\
= (1 - \lambda)\Pi
\]

**Proof of Proposition 2:** If $0 < \lambda < 1$ represents the supplier’s share of the total profit in the supply chain, where the target fill rate $\tau$ is exogenously set, and demand is uniformly distributed from a lower bound of 0 to an upper bound of $Z$, then for each $\lambda$, a wholesale price $w$, and bonus $\beta$, can be found that perfectly coordinate the supply chain when the following equations are satisfied:

\[
\beta = c\tau(1 - \lambda)Z \\
\alpha = \lambda \alpha \\
w = \lambda r
\]

(6.2)

Let $0 < \lambda < 1$ represent the supplier’s share of the total profit in the supply chain. Using the conditions from (6.2), then it is simple to verify that the profit for the supplier from an SLA can be expressed as a portion $\lambda$, of the total supply chain profit $\Pi$, and that the profit for the retailer from an SLA can be expressed as the remaining portion $(1 - \lambda)$, of the total supply chain profit, $\Pi$.

\[
\mathbb{E}[\pi_S(q, D)] = (w - c)E[D] - (w - c)E[D - q]^+ - cE[q - D]^+ + \beta \left( \frac{q}{Z} \right)
\]
\[
E[\pi_R(w, \beta, D)] = (r - w)E[D] - (r - w)E[D - q]^{+} - c(1 - \lambda)q \\
= (r - \lambda r)E[D] - (r - \lambda r)E[D - q]^{+} - c(1 - \lambda)q \\
= (1 - \lambda)rE[D] - (1 - \lambda)rE[D - q]^{+} \\
- (1 - \lambda)cE[D] - E[D - q]^{+} - E[q - D]^{+} \\
= (1 - \lambda)(r - c)E[D] - (1 - \lambda)(r - c)E[D - q]^{+} - (1 - \lambda)cE[q - D]^{+} \\
= (1 - \lambda)\Pi
\]

### 6.2 Sample Instructions

You are about to participate in a decision making experiment. If you follow these instructions carefully and make good decisions you could earn a considerable amount of money that will be paid to you in cash at the end of the session. Your earnings will depend on your decisions and the decisions of other participants. If you have a question at any time, please raise your hand and I will answer it. Please do not talk with one another for the duration of the experiment. On your desk you should have a check-out form, a copy of the instructions, a pen and a copy of the consent form.

**How to make money**

You will play one of two roles, a **retailer** or a **supplier**. Your role will not change for the duration of the experiment. The retailer purchases product from the supplier at a **Wholesale Price** per unit and sells it to customers at a rate of 20 laboratory dollars per unit. The supplier produces each unit at a rate of 4 laboratory dollars and sells those products at the Wholesale Price per unit to the retailer’s customers.

The retailer satisfies demand by shipping product **directly** from the supplier to the end customers. The number of units the supplier produces, called the supplier
Stocking Quantity, will depend on the Wholesale Price.

Each period starts with the retailer making their decision, the Wholesale Price per unit, in a way that would maximize profit. The supplier then receives the retailer’s proposed Wholesale Price per unit, and then makes a decision either to Reject the Wholesale Price or how many units to produce for the Stocking Quantity. If the supplier Rejects the retailer’s Wholesale Price per unit, the supplier’s Stocking Quantity will be 0.

You will each make a decision in 30 consecutive rounds, where in each round you will be randomly matched with a different person. In each round you will start with a revenue of 400 laboratory dollars.

Profit calculations

Each period, the number of units you sell (for both the retailer and supplier) is determined as follows:

- \[ \text{Units Sold} = \text{Demand} \quad \text{if} \quad (\text{Demand} < \text{Stocking Quantity}) \]
- \[ \text{Units Sold} = \text{Stocking Quantity} \quad \text{if} \quad (\text{Demand} \geq \text{Stocking Quantity}) \]

Demand is randomly determined each period, from 0 to 100, each integer in that range equally likely.

The profit calculations, if the supplier accepts the retailer’s Wholesale Price are:

- \[ \text{Retailer Profit} = 400 + (20 - \text{Wholesale Price}) \times (\text{Units Sold}) \]
- \[ \text{Supplier Profit} = 400 + (\text{Wholesale Price}) \times (\text{Units Sold}) - 4 \times (\text{Stocking Quantity}) \]

The profit calculations, if the supplier Rejects the retailer’s Wholesale Price are:

- \[ \text{Retailer Profit} = 400 \]
- \[ \text{Supplier Profit} = 400 \]

Information to help retailers

At the beginning of each period the retailer will set a Wholesale Price that will be paid to the supplier for each unit sold. To help retailers make their decision they will see a graph that allows them to enter a test Wholesale Price and calculates their profit for every possible demand realization. It will also show the supplier’s optimal Stocking Quantity for the test Wholesale Price they choose, please note that this does not mean that the supplier will set this Stocking Quantity. Below is an example of a screen the retailers will see.
At the beginning of each period the retailer will set a Wholesale Price that will be paid to the supplier for each unit sold. To help retailers make their decision they will see a graph that allows them to enter a test Wholesale Price and calculates their profit for every possible demand realization. It will also show the supplier’s optimal Stocking Quantity for the test Wholesale Price they choose, please note that this does not mean that the supplier will set this Stocking Quantity.

Retailers can try as many Wholesale Prices as they want, by using the scroll bar, before submitting their decision. When ready, retailers should enter the Wholesale Price into the text box on the screen and click the Submit button. The Wholesale price has to be between 4 and 20.

Information to help suppliers

After the retailer has submitted a Wholesale Price, the supplier will then be shown this information, and must decide whether to accept the retailer’s proposed Wholesale Price or set a Stocking Quantity. To help suppliers make their decision, they will see a graph that allows them to enter a test Stocking Quantity and calculates their profit for every possible demand realization. Below is an example of a screen the suppliers will see.

Suppliers can try as many Stocking Quantities as they want, by using the scroll bar, before submitting their decision. When ready, suppliers should make their decision by either setting a Stocking Quantity, which must be between 0 and 100, or Rejecting the retailer’s proposal.

Information you will see after each round

After each round, you will see the following information.

- The Round Number
- The Wholesale Price submitted by the retailer
Suppliers can try as many Stocking Quantities as they want, by using the scroll bar, before submitting their decision. When ready, suppliers should make their decision by either setting a Stocking Quantity, which must be between 0 and 100, or Rejecting the retailer's proposal.

Information you will see after each round:

- The Round Number
- The Wholesale Price submitted by the retailer
- Whether the proposed Wholesale Price was accepted by the supplier
- The Stocking Quantity submitted by the supplier, if applicable
- Realized Demand
- Your Profit and a Profit Graph for the decision you made.

You will also see this information for all past rounds on the bottom of each screen, and this information will be for different partners (since you will be randomly matched each period).

**Example**

Suppose the retailer enters a Wholesale Price of 6.00 corresponding to the screenshot above.

**Supplier accepts**

Imagine that the supplier accepts the retailer’s proposal then sets a Stocking Quantity of 40. Suppose demand for that round turns out to be 60, the unit sold are then 40. The retailer earns the starting revenue, plus the selling price (20.00) minus the Wholesale Price (6.00) for each unit sold.

- Retailer Profit = \(400 + (20.00 - 6.00) \times 40 = 960\)
The supplier earns the starting revenue, plus the Wholesale Price per unit for each unit sold, minus the cost to produce the Stocking Quantity of 40.

- Supplier Profit = $400 + (6.00 \times 40) - (4 \times 40) = 480$

Now suppose the demand turns out to be 20. In this case units sold is 20 because the demand of 20 is smaller than the stocking quantity of 40.

- Retailer Profit = $400 + (20.00 - 6.00) \times 20 = 680$
- Supplier Profit = $400 + (6.00 \times 20) - (4 \times 40) = 360$

**Supplier rejects**

Imagine the supplier Rejects the retailer’s proposal. Then the profits are the starting revenue for each party:

- Retailer Profit = 400
- Supplier Profit = 400

**How you will be paid**

At the end of the session the actual earnings from the game will be converted to US dollars at the rate of 1200 laboratory dollars for $1 US dollar. These profits will be added to your $5 show-up fee, displayed on your screen, and paid to you in cash at the end of the session.
Chapter 7

Appendix for Chapter 3

7.1 Derivations

Derivations for the AP−SLA with 100% target fill rate in Section 4.4:

Under the AP−SLA, the supplier’s expected profit over a review horizon of $T$ periods is:

$$
\pi_S = w\mathbb{E}[\min(Q, D)] - cQ + \beta F(Q)^T
$$

(7.1)

Let $Q(\beta)$ where we wish to find the optimal bonus, $\beta^*$, for the retailer. The supplier’s optimal stocking quantity must satisfy:

$$
(w - c) = wF(Q^*) - T\beta F(Q^*)^{T-1}f(Q^*)
$$

(7.2)

Because (7.1) does not necessarily have a unique maximum, the following second order condition must be satisfied for $Q^*$ to be a local maximum:

$$
w f(Q^*) > T\beta((T - 1) F(Q^*)^{T-2} f(Q^*)^2 + F(Q^*)^{T-1} f'(Q^*))
$$

(7.3)

Because we may have multiple local optima for $Q$, let a stocking quantity that satisfies the local maximum conditions be denoted by $\hat{Q}$. We can now differentiate the retailer’s profit function with respect to the bonus:

$$
\frac{\partial \pi_R}{\partial \beta} = \left[ (r - w)(1 - F(\hat{Q})) \frac{\partial \hat{Q}}{\partial \beta} \right] - T\beta^* F(\hat{Q})^{T-1}f(\hat{Q}) \frac{\partial \hat{Q}}{\partial \beta} - F(\hat{Q})^T
$$

(7.4)
Using \( [T \beta F(\hat{Q})^{T-1} f(\hat{Q}) = w F(\hat{Q}) - (w - c)] \) from (7.2), and plugging it into (7.4) results in:

\[
\frac{\partial \pi_R}{\partial \beta} = \frac{\partial \hat{Q}}{\partial \beta} \left[ r(1 - F(\hat{Q})) - c \right] - F(\hat{Q})^T \tag{7.5}
\]

Where differentiating the supplier’s first order condition, (7.2), with respect to \( \beta \) yields:

\[
\frac{\partial \hat{Q}}{\partial \beta} = \frac{TF(\hat{Q})^{T-1} f(\hat{Q})}{wf(\hat{Q}) - T\beta((T - 1)F(\hat{Q})^{T-2}f(\hat{Q})^2 + F(\hat{Q})^{T-1}f'(\hat{Q}))} \tag{7.6}
\]

Plugging (7.6) into (7.5) provides the following condition for \( \beta^* \) for the retailer:

\[
F(\hat{Q}) = \frac{TF(\hat{Q})^{T-1} f(\hat{Q})}{wf(\hat{Q}) - T\beta^{*(T - 1)F(\hat{Q})^{T-2}f(\hat{Q})^2 + F(\hat{Q})^{T-1}f'(\hat{Q}))} \tag{7.7}
\]

In sum, conditions (7.2), (7.3), and (7.7) must be satisfied for \( Q^* \) and \( \beta^* \) to be locally optimal. However, \( \pi_R(\beta) \) may not be differentiable at \( \beta^* \).

**Derivations for the Gr(1) – SLA with 100% target fill rate in Section 4.4:**

A similar process can be conducted to find the optimal conditions for the \( Gr(k) – SLA \), when \( k = 1 \). Recall that under the \( Gr(k) – SLA \), with \( k = 1 \), the retailer will pay the bonus if the supplier misses the higher target fill rate one period (one grace period) and still manages to hit the lower target fill rate during that specific period. Let:

\[
\lambda(Q) = \Pr(\text{Bonus}) = \left( \frac{T}{T - 1} \right) F(Q)^{T-1} \left( F \left( \frac{Q}{\alpha_L} \right) - F(Q) \right) + F(Q)^T \tag{7.8}
\]

Which we denote as \( \lambda(Q) \). The supplier’s optimal stocking quantity must satisfy:

\[
(w - c) = w F(Q^*) - \beta \frac{\partial \lambda}{\partial Q} \tag{7.9}
\]

Once again, the supplier’s profit is not necessarily concave, and the second order condition must be satisfied for \( Q^* \) to be a local maximum.

\[
w f(Q^*) > \beta \frac{\partial^2 \lambda}{\partial Q^2} \tag{7.10}
\]
Let \( \hat{Q} \) represent any \( Q \) in the set that satisfies the local maximum conditions.

The first and second order conditions of \( \lambda(Q) \) are given by:

\[
\frac{\partial \lambda}{\partial Q} = \left( \frac{T!}{(T-1)!} \right) \left( (T-1)F(\hat{Q})^{T-2}f(\hat{Q}) \left( F \left( \frac{\hat{Q}}{\alpha_L} \right) - F(\hat{Q}) \right) \right.
\]

\[
+ F(\hat{Q})^{T-1} \left( f \left( \frac{\hat{Q}}{\alpha_L} \right) \frac{1}{\alpha_L} - f(\hat{Q}) \right) \right) + TF(\hat{Q})^{T-1}f(\hat{Q}) \tag{7.11}
\]

and

\[
\frac{\partial^2 \lambda}{\partial Q^2} = \left( \frac{T!}{(T-1)!} \right) \left( (T-1)(T-2)F(\hat{Q})^{T-3}f(\hat{Q})^2 \left( F \left( \frac{\hat{Q}}{\alpha_L} \right) - F(\hat{Q}) \right) \right.
\]

\[
+ (T-1)F(\hat{Q})^{T-2}f'(\hat{Q}) \left( F \left( \frac{\hat{Q}}{\alpha_L} \right) - F(\hat{Q}) \right) + 2(T-1)F(\hat{Q})^{T-2}f(\hat{Q}) \left( f \left( \frac{\hat{Q}}{\alpha_L} \right) \frac{1}{\alpha_L} - f(\hat{Q}) \right)
\]

\[
+ F(\hat{Q})^{T-1} \left( f' \left( \frac{\hat{Q}}{\alpha_L} \right) \frac{1}{\alpha_L} - f'(\hat{Q}) \right) \right) + T(T-1)F(\hat{Q})^{T-2}f(\hat{Q})^2 + TF(\hat{Q})^{T-1}f'(\hat{Q}) \tag{7.12}
\]

To find the optimal bonus for the retailer, we differentiate the retailer’s profit function with respect to the bonus:

\[
\frac{\partial \pi_R}{\partial \beta} = (r - w)(1 - F(\hat{Q})) \frac{\partial \hat{Q}}{\partial \beta} - \left( \lambda(Q) + \beta \frac{\partial \lambda}{\partial Q} \frac{\partial \hat{Q}}{\partial \beta} \right) \tag{7.13}
\]

Where:

\[
\frac{\partial \hat{Q}}{\partial \beta} = \frac{\partial \lambda}{\partial Q} \frac{\partial Q}{\partial \beta} \tag{7.14}
\]

And the final condition for \( \beta^* \) simplifies to:

\[
\lambda(Q) = \left( (r - w)(1 - F(\hat{Q})) \right) \frac{\partial \lambda}{\partial \beta} \frac{\partial \hat{Q}}{\partial \beta} \tag{7.15}
\]

In sum, conditions (7.9), (7.10), and (7.15) must be satisfied for \( Q^* \) and \( \beta^* \) to be locally optimal. However, once again, \( \pi_R(\beta) \) may not always be differentiable at \( \beta^* \).
Bibliography


 Management Science (Forthcoming).


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