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ONE-GROUP INTERFACIAL AREA TRANSPORT MODELING OF HORIZONTAL BUBBLY FLOW WITH RESTRICTIONS: 90-DEGREE OR 45-DEGREE ELBOW

A Thesis in

Nuclear Engineering

by

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ABSTRACT

The one-group interfacial area transport equation applicable to air-water horizontal bubbly flows with a 90-degree or 45-degree horizontal elbow is developed. The local database provided by the U.S. Nuclear Regulatory Commission is used to benchmark the present models. In total, 105 area-averaged data points obtained in 15 different flow conditions for both the 90-degree and 45-degree experiments at four and three different axial locations, respectively, are employed for model evaluation. A predictive model for the characteristic pressure loss due to the flow restriction is developed and used as a constitutive relation for the pressure gradient term in the model. The non-uniform distribution of two-phase flow parameters due to both the flow orientation and the flow restrictions is taken into consideration in the present model by a distribution parameter analogous to covariance. In general, it is found that the Velocity Gradient (VG), Random Collision (RC), and Pressure Drop (PD) play the major roles for interfacial area transport in low void fraction flow conditions, whereas the Turbulence Impact (TI), Random Collision (RC), and Pressure Drop (PD) mechanisms become dominant as the void fraction increases. Overall, the present model predicts the data well with the average percent difference of ±20%.
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Chapter 1

Literature Survey

1.1 Theoretical Background

In general, physical problems of two-phase flow are represented by macroscopic field equations and constitutive relations using a continuous formulation. The drift-flux model (Zuber and Findlay, 1965), an advanced mixture model, has been extensively used in two-phase flow calculations. However, a more detailed treatment of two-phase motion is possible through the two-fluid model (Ishii, 2007). The two-fluid model is formulated by considering each phase separately in terms of two sets of conservation equations governing the mass, momentum and energy of each phase. In the two-fluid formulation, the interaction terms, which couple the transport of mass, momentum and energy of each phase across the interface, appear in the field equations. These interfacial transfer terms are directly related to the interfacial area concentration and to the local transfer mechanisms. Therefore, an accurate model for the interfacial area is essential in view of the two-fluid model formulation. The lack of proper mechanistic models for the interfacial structure and interfacial transfer processes lead to inaccurate predictions of these phenomena, and thus become a major concern in current two-phase flow modeling practice.

In the current nuclear reactor system analysis codes, the interfacial area concentrations are calculated by flow regime dependent correlations. The use of steady-state based flow regime transition criteria imposes significant shortcomings in modeling the changes of interfacial structures because they do not model the evolution of interfacial structures dynamically. Mortensen (1995) and Kelly (1997) identified several shortcomings related to the flow regime based static approach, and they can be summarized as follows:
• Since the flow regime transition criteria are algebraic relations for steady-state, fully developed flows, they reflect neither the true dynamic nature of changes in the interfacial structure, nor the gradual regime transition.

• The compound errors due to the two-step flow regime based method can be significant.

• The existing flow regime dependent correlations and criteria are valid in limited parameter ranges for certain operational conditions. Often the geometrical scale effects are not taken into account correctly. Hence, these models may cause significant discrepancies, artificial discontinuities and numerical instabilities.

Therefore, a mathematical model that can take into account the dynamic change of the interfacial structure should be employed in the two-fluid model to accurately model the effects of interfacial structure and regime transition.

The interfacial area transport equation can dynamically predict the changes of interfacial area concentration by the use of mechanistically modeled source and sink terms. Therefore, the artificial bifurcations related to the flow regime transitions in the conventional models can be eliminated. Furthermore, since it dynamically models the two-phase flow evolution across flow regime transition boundaries, it also prevents artificial discontinuities. A dynamic approach is particularly important to two-phase flow in channels that are interconnected via various junctions, because two-phase flow in these conditions undergo a significant change in interfacial structures. Therefore, use of the interfacial area transport equation in analyzing the interfacial transfer and regime transitions is not only rational, but can also make a significant improvement in thermal-hydraulic reactor system analysis.

The foundation of the interfacial area transport equation was first established by Kocamustafaogullari and Ishii (1995). It was followed by Wu et al. (1998), where they established the source and sink terms in the interfacial area transport equation by mechanistically
modeling the major bubble interaction phenomena in the bubbly flow regime. Later, with improved source and sink terms, Kim (1999) established the interfacial area transport equation applicable to a vertical confined bubbly flow. A similar approach on the treatment of interfacial area concentration can be also found in the studies by Millies et al. (1996), Morel et al. (1999), and Hibiki and Ishii (1999). In the model evaluation studies by Ishii et al. (2002) and Kim et al. (2002), the model showed good agreement with an extensive database acquired in various sizes of vertical round pipes and a confined test duct. Furthermore, a preliminary study of implementing the interfacial area transport equation in the thermal-hydraulic system analysis code TRAC clearly demonstrated that it made a significant improvement in the calculated results (Ishii et. al, 2000). The interfacial area transport equation was also developed for vertical co-current downward bubbly flow (Ishii et. al, 2004) and the model showed promising results. Sun (2001) and Fu (2001) developed the interfacial area transport equation for a wide range of two-phase flow regimes spanning from vertical bubbly to churn-turbulent two-phase flow for rectangular and round flow channels, respectively. In all of the evaluation studies, the model predictions compare well with the experimental data with a relative error of less than ±20% in general. The comprehensive mathematical formulations of both the one-group and two-group interfacial area transport equations can be found in the work by Ishii and Kim (2004).

1.2 Interfacial Area Transport Equation

Since the interfacial area of fluid particles is closely related to the particle number, the interfacial area transport equation can be formulated based on the Boltzmann transport equation. By defining \( f(V, x, v, t) \), the particle number density distribution function per unit mixture and bubble volume, which is assumed to be continuous and specifies the probable number density of
fluid particles moving with particle velocity \( v \), at a given time \( t \), in a spatial range \( dx \) with its center-of-volume located at \( x \) with particle volumes between \( V \) and \( V+dV \), we can obtain:

\[
\frac{\partial f}{\partial t} + \nabla \cdot (fv) + \frac{\partial}{\partial V} \left( f \frac{dV}{dt} \right) = \sum_j S_j + S_{ph}
\]  

(1-1)

which is analogous to the Boltzmann transport equation of particles with the distribution function \( f(V,x,t) \) (where \( d/dt \) denotes the substantial derivative). Then, the particle number, void fraction and the interfacial area concentration can be specified by:

\[
n(x,t) = \int_{V_{min}}^{V_{max}} f(V,x,t) dV
\]

(1-2)

\[
a(x,t) = \int_{V_{min}}^{V_{max}} f(V,x,t)V dV
\]

(1-3)

\[
a_i(x,t) = \int_{V_{min}}^{V_{max}} f(V,x,t)A_i(V) dV
\]

(1-4)

where \( V \) and \( A_i \) represent the volume and surface area of a fluid particle, respectively. Since the transport equation given by Eq. (1-1) is too detailed to be employed in practice, a more practical form of the transport equation can be obtained by averaging Eq. (1-1) over all particle sizes. Hence, the transport equations for particle number, void fraction and interfacial area concentration can be obtained, respectively as:

\[
\frac{\partial n}{\partial t} + \nabla \cdot (n v_{pm}) = \sum_j R_j + R_{ph}
\]

(1-5)

\[
\frac{\partial \alpha}{\partial t} + \nabla \cdot (\alpha v_g) = - \frac{\alpha}{\rho_g} \left[ \frac{\Gamma_g - \eta_{ph} \rho_g}{\alpha} - \frac{d \rho_g}{dt} \right] = \int_{V_{min}}^{V_{max}} \left[ \sum_j S_j V + S_{ph} V \right] dV
\]

(1-6)
\[
\frac{\partial a_i}{\partial t} + \nabla \cdot (a_i v_i) = \frac{2}{3} \left( \frac{a_i}{\alpha} \right) \left( \frac{\partial \alpha}{\partial t} + \nabla \cdot \alpha v_g \right) + \eta_{ph} \left( \frac{\alpha}{a_i} \right)^2 \sum_j R_j + \pi D_{ph}^2 R_{ph} \quad (1-7)
\]

where the particle velocity \((v_{pm})\), bubble velocity \((v_g)\) and the interfacial velocity \((v_i)\) are defined, respectively as:

\[
v_{pm}(x,t) = \frac{\int_{V_{\text{mix}}} ^{V_{\text{mix}}} f(V,x,t) v(V,x,t) dV}{\int_{V_{\text{mix}}} ^{V_{\text{mix}}} f(V,x,t) dV} \quad (1-8)
\]

\[
v_g(x,t) = \frac{\int_{V_{\text{mix}}} ^{V_{\text{mix}}} f(V,x,t) v(V,x,t) dV}{\int_{V_{\text{mix}}} ^{V_{\text{mix}}} f(V,x,t) dV} \quad (1-9)
\]

\[
v_i(x,t) = \frac{\int_{V_{\text{mix}}} ^{V_{\text{mix}}} f(V,x,t) A_i(V) v(V,x,t) dV}{\int_{V_{\text{mix}}} ^{V_{\text{mix}}} f(V,x,t) A_i(V) dV} \quad (1-10)
\]

In Eq. (1-6), \(S_j\) and \(S_{ph}\) represent the particle source/sink rates per unit mixture volume due to \(j\)th particle interactions (such as disintegration or coalescence) and that due to phase change, respectively. Hence, the number source/sink rate is defined by:

\[
R(x,t) = \int_{V_{\text{mix}}} ^{V_{\text{mix}}} S(V,x,t) dV \quad (1-11)
\]

and similarly, the nucleation source rate per unit mixture volume is defined by:

\[
\eta_{ph} = \int_{V_{\text{mix}}} ^{V_{\text{mix}}} S_{ph} V dV \quad (1-12)
\]

Furthermore, \(\Psi\) in Eq. (1-7) originates from:

\[
n = \Psi \frac{A_i}{\alpha^2} \quad (1-13)
\]
and it is defined by:

\[
\psi = \frac{1}{36\pi} \left( \frac{D_{mc}}{D_s} \right)^3
\]

(1-14)

First, the interfacial area transport equation for dispersed bubbles can be formulated by mechanistically modeling the major bubble interaction phenomena that are responsible for creating and destroying the bubble interfaces. In formulating the equation for dispersed bubbles, the bubbles are assumed to be spherical in shape and considered as one group in view of drag. Hence, the transport equation for dispersed bubbles is termed as the “one-group” transport equation.

The mechanisms that constitute the source and sink terms in the one-group equation include:

a. Break-up due to the impact of turbulent eddies \( (TI) \)

b. Coalescence through random collision driven by turbulent eddies \( (RC) \)

c. Coalescence due to the acceleration of the following bubble in the wake of the preceding bubble \( (WE) \)

As a result, the one-group interfacial area transport equation for vertical air-water bubbly flow can be given by (Wu et al., 1998; Kim, 1999):

\[
\frac{\partial a_i}{\partial t} + \nabla \cdot \left( a_i \mathbf{v}_i \right) = \frac{2}{3} \left( \frac{a_i}{\alpha} \right) \left( \frac{\partial \alpha}{\partial t} + \nabla \cdot \alpha \mathbf{v}_g \right) + \frac{1}{3\psi} \left( \frac{\alpha}{a_i} \right)^2 \left[ R_{HT} - R_{RC} - R_{WE} \right]
\]

(1-15)

where:

\[
R_{HT} = C_{HT} \left( \frac{n_{ni}}{D_b} \right) \exp \left( - \frac{We_{cr}}{We} \right) \sqrt{1 - \frac{We_{cr}}{We}} \quad \text{where } We > We_{cr},
\]

(1-16)
\[ R_{RC} = C_{RC} \left[ \frac{n^2 u_t D_b^2}{\alpha_{max}^{1/3} \left( \alpha_{max}^{1/3} - \alpha^{1/3} \right)} \right] \left[ 1 - \exp \left( -C_t \frac{\alpha_{max}^{1/3}}{\alpha_{max}^{1/3} - \alpha^{1/3}} \right) \right] \]  

(1-17)

\[ R_{WE} = C_{WE} C_D n^2 D_b^2 u_r \]  

(1-18)

where \( C_{Ti} \), \( C_{RC} \) and \( C_{WE} \) are the coefficients determined based on experimental data, and \( D_b \), \( We \), \( \alpha_{max} \), \( u_t \) and \( u_r \) denote the bubble diameter, Weber number, maximum packing limit, turbulent velocity and relative velocity, respectively. Here, the source term due to phase change \( (R_{ph}) \) has been omitted accounting for an adiabatic condition.

To account for the interfacial area transport in various two-phase flow regimes, the two-group approach is employed. This is based on the fact that the differences in size or shape of the bubbles causes substantial differences in their interfacial transport and interaction phenomena. In mixed two-phase flow, the bubbles can be categorized in two characteristic groups based on their relative motion and drag, namely; smaller spherical bubbles and larger cap-shaped bubbles. Therefore, the interfacial area transport for small bubbles should be considered separately from that of larger cap, slug or churn-turbulent bubbles. Hence, two transport equations for the two characteristic groups of bubbles are sought. The group 1 equation describes the transport of small dispersed and distorted bubbles, while the group 2 equation describes the transport phenomena of cap/slug/churn-turbulent bubbles.

As a result, the following two-group interfacial area transport equation is established (Ishii and Kim, 2004):

\[ \frac{\partial a_i}{\partial t} + \nabla \cdot (a_i \mathbf{v}_i) = \left( \frac{2}{3} - \chi D_i \right) \frac{a_i}{\alpha_i} \left[ \frac{\partial \alpha_i}{\partial t} + \nabla \cdot (\alpha_i \mathbf{v}_i) - \eta_{ph1} \right] + \sum_j c_j a_j + \phi_{ph1} \quad \text{for Group 1} \]  

(1-19)
\[
\frac{\partial a_{i,2}}{\partial t} + \nabla \cdot (a_{i,2} \mathbf{v}_{i,2}) = 2 \frac{a_{i,2}}{3 \alpha_{i}^{2}} \left[ \frac{\partial \alpha_{i}}{\partial t} + \nabla \cdot (\alpha_{i} \mathbf{v}_{i,2}) - \eta_{ph,2} \right]
\]

\[
+ \chi D_{c_{i,1}}^{*} \frac{a_{i,1}}{a_{i}} \left[ \frac{\partial \alpha_{i}}{\partial t} + \nabla \cdot (\alpha_{i} \mathbf{v}_{i,2}) - \eta_{ph,1} \right] + \sum_{j} \phi_{i,j} + \phi_{ph,2}
\]

for Group 2 \hspace{1cm} (1-20)

where the subscripts 1 and 2 denote group 1 and group 2, respectively. In Eqs. (1-19) and (1-20), the non-dimensional parameter \( D_{c_{i,1}}^{*} \) is a measure of the average size of group 1 bubbles and defined as:

\[
D_{c_{i,1}}^{*} = \frac{D_{c_{i}}}{D_{sm_{i}}}
\]

(1-21)

while \( D_{c_{i}} \) and \( D_{sm_{i}} \) are the volume-equivalent diameter of a bubble with critical volume \( V_{c} \) and the Sauter mean diameter of group 1 bubbles, respectively. In addition, the coefficient \( c_{i} \) accounts for the inter-group transfer at the group boundary between the two groups due to expansion, compression, and/or phase change. It should be noted that in general applications where no rapid condensation occurs, the phase change terms for group 2 bubbles (i.e., \( h_{ph,2} \) and \( f_{ph,2} \)) can be neglected in Eq. (1-20).

In the development of the two-group interfacial area transport equation, additional bubble interaction mechanisms need to be added to account for the large bubble transport. These are (1) shearing off of small bubbles at the base rim of large cap bubbles (SO) and (2) break-up of large cap bubbles due to the surface instability at the interface (SI). In addition to these, the mechanisms applicable to both group 1 and group 2 bubbles are carefully considered. The mechanistic modeling studies on various two-group bubble interaction mechanisms are established for both the round pipe and rectangular flow geometries by Fu (2001) and Sun (2001), respectively.
Chapter 2

Experimental Studies

2.1 Experimental Facility and Test Conditions

A comprehensive description of the 90-degree and 45-degree experimental studies can be found in form of University of Missouri-Rolla (UMR) (now Missouri Institute of Science and Technology) Internal Reports (Report Nos: UMR/NE-TFTL-05-01 and UMR/NE-TFTL-05-02) in March and July of 2005, respectively. In the present study the work of Callender (2007) is extended, and essential information related to the interfacial area transport in the test facility is summarized.

The data presented in this report was acquired in air-water bubbly two-phase flow through a horizontal tube of 50.3 mm inner diameter (ID) in two separate experiments performed at the University of Wisconsin, Milwaukee (UWM). The first set of data was collected with a 90-degree elbow installed and then a 45-degree elbow was added to the existing facility to acquire the second set of data. A simplified top view schematic diagram of the test facility is shown in Fig. 2.1. It is composed of round Pyrex pipes of 50.3 mm inner diameter with a 90-degree elbow installed at L/D=206.6 from the inlet, and a 45-degree elbow installed at L/D=353.5. The 90-degree elbow has a radius of curvature of 89 mm with a \((L/D)_{elbow}\) of approximately 6, while the 45-degree elbow has a \((L/D)_{elbow}\) of approximately 4. The inset of Fig. 2.1 shows the detailed dimensions of the 90-degree and 45-degree elbows.

In total, eight pressure taps are installed in the instrumentation ports along the test section, including one directly after the two-phase mixing chamber (P_{ref}), and are denoted in the figure as
P1 through P7. The port P1 is located at L/D=197 from Pref, with P2, P3, and P4 located at L/D=225, 250, and 329 from Pref, respectively. The port P5 is located at L/D=342 from Pref, with P6, and P7 located at L/D=363 and L/D=419 from Pref, respectively.

The first experiment, with only the 90-degree elbow present, was performed using 5 measurement ports (i.e. Pref, P1, P2, P3, and P4). For the second experiment, after the addition of the 45-degree elbow, 4 measurement ports (i.e. P1, P5, P6, and P7) are used. At each measurement location, local two-phase flow parameters were acquired by the double-sensor conductivity probe. The local parameters acquired by the probe included bubble frequency ($f_b$), bubble velocity ($v_g$), void fraction ($a$) and interfacial area concentration ($a_i$).

The local measurements are made across the entire vertical tube diameter because of the asymmetric distribution of bubbles in horizontal two-phase flow. In the 90-degree experiment, at ports P1 and P4, a local probe is traversed only in the vertical direction along the tube cross-section diameter ($r/R_v$), assuming that the flow is symmetric along the vertical axis. At ports P2 and P3, on the other hand, the 90-degree elbow may significantly affect the interfacial structures. Hence, the probe is traversed along both vertical and horizontal ($r/R_h$) directions of the tube cross-section diameters at ports P2 and P3. For the 45-degree experiment, at ports P5 and P7, the local probe is traversed only in the vertical direction, while at port P6 the probe is traversed along both the vertical and horizontal tube diameters due to the effects of the flow restriction. To identify the inner and outer section of the elbow, $r/R=0$ is chosen to be at the center of tube cross-section. Then, a positive $r/R$ denotes the inner or upper half of the horizontal and vertical tube diameters, respectively. Similarly, a negative $r/R$ indicates the outer or lower half of the horizontal and vertical tube diameters, respectively. The measurement coordinate system described here is shown in the inset of Fig. 2.1.
For each experiment 15 different \( j_g \) & \( j_f \) combinations are investigated, all in bubbly flow conditions. The test conditions of each experiment are labeled as Runs 1 through 15 and are summarized in Tables 2.1 and 2.2 for both the 90-degree and 45-degree experiments, respectively. Since the local gas flow rate is a function of local pressure, the gas flow rates shown in Tables 2.1 and 2.2 are based on the standard atmospheric pressure condition.
Table 2-1: 90-degree experiment test conditions.

<table>
<thead>
<tr>
<th></th>
<th>Run 1</th>
<th>Run 2</th>
<th>Run 3</th>
<th>Run 4</th>
<th>Run 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$j_{g,\text{atm}}$ [m/s] $^*$</td>
<td>0.116</td>
<td>0.124</td>
<td>0.127</td>
<td>0.312</td>
<td>0.320</td>
</tr>
<tr>
<td>$j_f$ [m/s]</td>
<td>3.762</td>
<td>4.051</td>
<td>4.335</td>
<td>3.765</td>
<td>4.047</td>
</tr>
<tr>
<td></td>
<td>Run 6</td>
<td>Run 7</td>
<td>Run 8</td>
<td>Run 9</td>
<td>Run 10</td>
</tr>
<tr>
<td>$j_{g,\text{atm}}$ [m/s] $^*$</td>
<td>0.329</td>
<td>0.644</td>
<td>0.659</td>
<td>0.673</td>
<td>0.985</td>
</tr>
<tr>
<td>$j_f$ [m/s]</td>
<td>4.338</td>
<td>3.772</td>
<td>4.048</td>
<td>4.338</td>
<td>3.764</td>
</tr>
<tr>
<td></td>
<td>Run 11</td>
<td>Run 12</td>
<td>Run 13</td>
<td>Run 14</td>
<td>Run 15</td>
</tr>
<tr>
<td>$j_{g,\text{atm}}$ [m/s] $^*$</td>
<td>1.004</td>
<td>1.031</td>
<td>1.336</td>
<td>1.372</td>
<td>1.406</td>
</tr>
<tr>
<td>$j_f$ [m/s]</td>
<td>4.049</td>
<td>4.313</td>
<td>3.760</td>
<td>4.051</td>
<td>4.332</td>
</tr>
</tbody>
</table>

$^*j_{g,\text{atm}}$ is the superficial gas velocity equivalent to the standard atmospheric pressure condition.

Table 2-2: 45-degree experiment test conditions.

<table>
<thead>
<tr>
<th></th>
<th>Run 1</th>
<th>Run 2</th>
<th>Run 3</th>
<th>Run 4</th>
<th>Run 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$j_{g,\text{atm}}$ [m/s] $^*$</td>
<td>0.135</td>
<td>0.140</td>
<td>0.145</td>
<td>0.348</td>
<td>0.355</td>
</tr>
<tr>
<td></td>
<td>Run 6</td>
<td>Run 7</td>
<td>Run 8</td>
<td>Run 9</td>
<td>Run 10</td>
</tr>
<tr>
<td>$j_{g,\text{atm}}$ [m/s] $^*$</td>
<td>0.368</td>
<td>0.707</td>
<td>0.736</td>
<td>0.760</td>
<td>1.090</td>
</tr>
<tr>
<td>$j_f$ [m/s]</td>
<td>4.342</td>
<td>3.760</td>
<td>4.050</td>
<td>4.341</td>
<td>3.767</td>
</tr>
<tr>
<td></td>
<td>Run 11</td>
<td>Run 12</td>
<td>Run 13</td>
<td>Run 14</td>
<td>Run 15</td>
</tr>
<tr>
<td>$j_{g,\text{atm}}$ [m/s] $^*$</td>
<td>1.134</td>
<td>1.176</td>
<td>1.504</td>
<td>1.577</td>
<td>1.630</td>
</tr>
<tr>
<td>$j_f$ [m/s]</td>
<td>4.042</td>
<td>4.342</td>
<td>3.769</td>
<td>4.042</td>
<td>4.340</td>
</tr>
</tbody>
</table>

$^*j_{g,\text{atm}}$ is the superficial gas velocity equivalent to the standard atmospheric pressure condition.
In order to obtain the area-averaged two-phase flow parameters, the local two-phase flow parameters were averaged by:

$$\langle \psi \rangle = \frac{1}{A} \int \psi dA \sum_{i=1}^{N} \frac{1}{A_i} \psi_i \Delta A_i$$

(2-1)

where $\psi$, $A$ and $\Delta A_i$ are any two-phase flow parameter of interest, total flow area and the $i^{th}$ partial area segment, respectively. Accounting for the asymmetry of the flow in horizontal pipe and through the elbow junction, two different methods were employed in defining the $\Delta A_i$ based on the measurement orientation as shown in Fig. 2.2. Hence, for the data acquired along $R_V$, the $\Delta A_i$ is defined by:

$$\Delta A_i = \frac{R_i^2}{2} \left( \theta_i - \theta_{i-1} \right) - R \left( h_i \cdot \sin \frac{\theta_i}{2} - h_{i-1} \cdot \sin \frac{\theta_{i-1}}{2} \right)$$

(2-2)

On the other hand, for the data acquired along both the $R_V$ and $R_H$, the $\Delta A_i$ is defined by:

$$\Delta A_i = \frac{\pi}{4} \left( h_i^2 - h_{i-1}^2 \right)$$

(2-3)

Here, $R_V$ and $R_H$ are the vertical and horizontal radii with respect to the top and bottom of the elbow cross-section, respectively. Therefore, Eq. (2-2) was employed to calculate the area-averaged parameters from the data acquired in ports P1 and P4 for the 90-degree experiment and at ports P5 and P7 for the 45-degree experiment. Similarly, Eq. (2-3) was employed for the data acquired at ports P2 and P3 for the 90-degree experiment and at port P6 for the 45-degree experiment.
After the area-averaged values of the two-phase flow parameters were obtained, other averaged parameters were calculated as follows:

\[ \langle \langle u_g \rangle \rangle = \frac{\langle \alpha u_g \rangle}{\langle \alpha \rangle} : \text{void-weighted bubble velocity} \quad (2-4) \]

In calculating the average Sauter mean diameter, it was weighted by the bubble frequency because the number of bubbles may differ significantly depending on the location in the pipe cross-section. Hence, it is given by:

\[ \langle \langle D_{sm} \rangle \rangle_f = \frac{\langle f_b D_{sm} \rangle}{\langle f_b \rangle} : \text{frequency-weighted bubble Sauter mean diameter} \quad (2-5) \]

In view of benchmarking the reliability of the acquired local data, the local superficial gas velocity, \( <j_{g, loc}> \), at each measurement port is compared with that calculated based on the \( \alpha \) and \( v_g \) acquired with the conductivity probe by:
\[ f_{g,loc} = \left[ \langle \alpha \rangle \langle u_g \rangle \right]_{loc} \]  

(2-6)

These values are found to agree well for both experiments within a ±10% difference.

2.2 Characteristic Results in Transport of Two-Phase Flow Parameters

The effects of the 90-degree and 45-degree elbows are characterized in part through a comprehensive analysis on the local interfacial structures of the flow. This section presents the effect of gas and liquid flow rates as well as the location of the measured experimental data in relation to the flow restrictions on the local void fraction and interfacial area concentration profiles. An investigation on the effect of the elbows in relation to the one-dimensional transport of two-phase flow parameters will be presented in Section 2.3.

2.2.1 Local Profiles of Void Fraction and Interfacial Area Concentration

In Figs. 2.3 (a) and (b), characteristic profiles of local \( \alpha \) and \( a_i \) obtained in Run 5 \((j_{g,atm}=0.32 \text{ m/s and } j_f=4.05 \text{ m/s})\) of the 90-degree experiment are shown as an example. In the figures, profiles along the vertical and horizontal radii are shown for the parameters acquired at four different measurement ports. As noted earlier, the measurements are made only along the vertical radius, \( R_V \), at ports P1 and P4 (or L/D=197 and 329) for the 90-degree experiment, assuming that the flow is symmetric along the vertical axis of the tube cross-section. At ports P2 and P3, on the other hand, profiles along the horizontal radius, \( R_H \), are also acquired for the 90-degree experiment.
As shown in the figures, the $\alpha$ and $a_i$ profiles show a similar trend. It is interesting, however, that both $\alpha$ and $a_i$ along the horizontal direction are distributed such that bubbles reside in the outer-half of the tube cross-section ($R_{\text{H}} < 0$) right after the elbow (L/D=225), and then migrate toward the inner-half ($R_{\text{H}} > 0$) further downstream (L/D=250). This suggests that the

Figure 2-3: Profiles of 90-degree experiment local two-phase flow parameters in Run 5 at different L/D’s along the vertical and horizontal radii.
elbow induces significant flow oscillation in the horizontal direction of the tube cross-section. In Run 5, the flow oscillation phenomenon can be found in both the vertical and horizontal directions across the tube cross-section. In fact, these oscillations are found in all of the 90-degree flow conditions examined in the present study and that such an oscillation is a characteristic elbow effect. The degree of oscillation varies depending on the flow conditions as will be shown later.

In Figs. 2.4 (a) and (b), characteristic profiles of local $\alpha$ and $a_i$ obtained in Run 5 ($j_{g,atm}=0.36$ m/s and $j_f=4.05$ m/s) of the 45-degree experiment are shown as an example. In the figures, profiles along the vertical and horizontal radii are shown for the parameters acquired at three different measurement ports. As noted earlier, the measurements are made only along the vertical radius, $R_V$, at ports P5 and P6 (or L/D=342 and 419) for the 45-degree experiment, assuming that the flow is symmetric along the vertical axis of the tube cross-section. At port P6, on the other hand, profiles along the horizontal radius, $R_H$, are also acquired.

As shown in the figures, the $\alpha$ and $a_i$ profiles of the 45-degree experimental generally show a similar trend, but it can be seen that now the void peaking in the lower half of the tube just downstream of the elbow is much greater. As in the 90-degree experiment, the 45-degree data is found to also exhibit vertical oscillations in all investigated flow conditions and is found to be a characteristic elbow effect. In several high void fraction runs, this high void peaking induces the bubbles to coalesce and drive the interfacial area concentration down. It is also found that the void peaking for the 45-degree case typically occurs lower in the tube at $r/R_V \approx -0.6$ to -0.7, compared to $r/R_V \approx -0.5$ in the 90-degree experiment. As in the 90-degree case, the vertical oscillation in the 45-degree profiles dampens downstream of the elbow, as the profiles are recovering to their original configuration. While data is only acquired along the horizontal axis at
port P6 in the 45-degree experiment, it is believed that a similar horizontal oscillation, as shown in Figure 2.3 for the 90-degree experiment, also occurs in the 45-degree experiment.

Figure 2-4: Profiles of 45-degree experiment local two-phase flow parameters in Run 5 at different L/D’s along the vertical and horizontal radii.
As shown in the figures, the $\alpha$ and $a_i$ profiles of the 45-degree experimental generally show a similar trend, but it can be seen that now the void peaking in the lower half of the tube just downstream of the elbow is much greater. As in the 90-degree experiment, the 45-degree data is found to also exhibit vertical oscillations in all investigated flow conditions and is found to be a characteristic elbow effect. In several high void fraction runs, this high void peaking induces the bubbles to coalesce and drive the interfacial area concentration down. It is also found that the void peaking for the 45-degree case typically occurs lower in the tube at $r/R_v \approx -0.6$ to -0.7, compared to $r/R_v \approx -0.5$ in the 90-degree experiment. As in the 90-degree case, the vertical oscillation in the 45-degree profiles dampens downstream of the elbow, as the profiles are recovering to their original configuration. While data is only acquired along the horizontal axis at port P6 in the 45-degree experiment, it is believed that a similar horizontal oscillation, as shown in Figure 2.3 for the 90-degree experiment, also occurs in the 45-degree experiment.

### 2.2.2 Transport of Local Two-Phase Flow Parameters in the 90-degree Experiment

In view of examining the effects of gas and liquid flow rates on the development of local two-phase flow parameters in the 90-degree experiment, profiles of $\alpha$ and $a_i$ along the vertical radius of the tube cross-section, $R_v$, are plotted for different flow conditions in Figs. 2.5 and 2.6. In Fig 2.5, the superficial gas flow rates, $j_{g,atm}$ are varied to 0.13, 0.67 and 1.41 m/s while the superficial liquid flow rate, $j_l$, is fixed at 4.3 m/s (Runs 3, 9 and 15 of the 90-degree experiment). In Fig. 2.6, on the other hand, $j_l$'s are varied to 3.8, 4.0 and 4.3 m/s while $j_{g,atm}$ is fixed at 0.3 m/s (Runs 4, 5 and 6 of the 90-degree experiment).

In general, it is found that the 90-degree local profiles of $\alpha$ resemble those of $a_i$, which is characteristic of the bubbly flow regime. Several flow conditions, such as Runs 12 through 15 of the 90-degree experiment, however, do not always share the same trend between $\alpha$ and $a_i$. In
these conditions, the flow may transition from the bubbly flow regime to the elongated bubbly flow regime and group two bubbles may be present. The locations of the local peak and the degree of peaking depend on both gas and liquid flow rates. In all of the 90-degree test conditions, both $\alpha$ and $a_i$ tend to peak near the top of the tube, $0.75 < r/R_V < 1$, upstream of the elbow. However, the distribution pattern changes right after the 90-degree elbow ($L/D=225$), which indicates that the elbow has significant effects on the flow structure.

After examining Figs. 2.5 and 2.6, the following characteristics can be found in the flow structures along the vertical tube radius, $R_V$:

At $L/D=197$ (9.6 diameters before the 90-degree elbow): In all flow conditions, bubbles reside mostly in upper half of the tube, resulting in local peaks of $\alpha$ and $a_i$ near the top of the tube cross-section ($0.75 < r/R_V < 1$). Bubble distribution tends to spread more along $R_V$ with increasing gas flow rates.

At $L/D=225$ (18.4 diameters downstream of the 90-degree elbow): The distribution of bubbles and location of local peaks of $\alpha$ and $a_i$ depend primarily on the gas flow rates. In lower gas flow rates up to $j_{G,atm}=0.33 \text{ m/s}$ (Runs 1 through 6 of the 90-degree experiment), bubbles migrate toward the lower half of the tube radius regardless of the liquid flow rates. Hence, both $\alpha$ and $a_i$ peak near the center of the lower half of tube cross-section (i.e. $r/R_V \approx -0.5$), and the peaking phenomenon is more pronounced with increasing gas and liquid flow rates. When the gas flow rate is increased to $j_{G,atm}=0.67 \text{ m/s}$ (Runs 7 through 9 of the 90-degree experiment), bubbles migrate toward the center of tube cross-section, and $\alpha$ and $a_i$ peak near $r/R_V=0$. As the gas flow rates increases further to $j_{G,atm}=1.41 \text{ m/s}$ (Runs 10 through 15 of the 90-degree experiment), bubbles migrate further into the upper half of the cross-section, and both $\alpha$ and $a_i$ peak near the center of upper half of the tube cross-section (i.e. $r/R_V \approx +0.5$). It was found in all 90-degree flow
conditions that the degree of peaking depends primarily on the gas flow rates, such that local peaks of $\alpha$ and $a_i$ become more pronounced with increasing gas flow rates.

At L/D=250 (or 43.4 diameters downstream of the 90-degree elbow): In this region, bubbles redistribute themselves to recover their distribution pattern before the 90-degree elbow. As will be shown later from the local profiles along the horizontal radius, $R_{H}$, however, significant elbow effects still remain and are characterized by flow oscillations in the horizontal direction. Both $\alpha$ and $a_i$ display a single peak in the upper half of the tube cross-section ($r/R_{V}>+0.5$). The degree of peaking depends on both liquid and gas flow rates, such that the local peaking phenomenon becomes more pronounced with increasing liquid flow rates, whereas it damps out with increasing gas flow rates. In conditions with lower gas flow rates (Runs 1 through 3 of the 90-degree experiment), the local profiles of $\alpha$ and $a_i$ are almost fully recovered to those before the 90-degree elbow, whereas the flow structures are yet to be recovered and bubbles are more distributed when $j_{g,atm} > 0.3$ m/s.

At L/D=329 (or 122.4 diameters downstream of the 90-degree elbow): In this region, the flow structures are fully recovered in Runs 1 through 5 and 7 of the 90-degree experiment ($j_{g,atm} < 0.3$ m/s, except for Run 7). In Runs 6 and 8 through 15 of the 90-degree experiment ($j_{g,atm} > 0.6$ m/s, except for Run 6), on the other hand, the flow structures are yet to be recovered. The characteristic features in the local two-phase flow parameters in this region are similar to those at L/D=250.

The effects of gas and liquid flow rates on the flow structures along the horizontal tube radius, $R_{H}$, of the 90-degree experiment are examined in Figs. 2.7 (a) and (b), respectively. As mentioned earlier, the local measurements along $R_{H}$ are performed only at L/D=225 and 250, where the 90-degree elbow effects can be most significant. The following characteristics can be found in the flow structures along the horizontal tube radius, $R_{H}$:
At L/D=225 (or 18.4 diameters downstream of the 90-degree elbow): As shown in Fig. 2.7 (a), bubbles reside toward the inner radius of the tube cross-section (i.e., \( R_H > 0 \)) when \( j_{g,atm} \approx 0.1 \) m/s (Runs 1 through 3 of the 90-degree experiment), but migrate toward the outer radius (\( R_H < 0 \)) as the gas flow rate increases. Hence, when \( j_{g,atm} \) is increased to 0.3 m/s (Runs 4 through 6 of the 90-degree experiment), both \( \alpha \) and \( \alpha_i \) are distributed more uniformly across \( R_H \). Then, the profiles peak near the outer side of the tube cross-section, at \( r/R_H \approx -0.5 \), as the liquid flow rates increase further to \( j_{g,atm} > 0.6 \) m/s (Runs 7 through 15 of the 90-degree experiment). In this region, the liquid flow rates show a similar effect as the gas flow rates, such that the peak locations change from the inner radius (\( R_H > 0 \)) to the outer radius (\( R_H < 0 \)) as the liquid flow rates increase. This is shown in Fig. 2.7 (b).

At L/D=250 (or 43.4 diameters downstream of the 90-degree elbow): In this region, bubbles migrate toward the inner radius of the tube cross-section regardless of the gas or liquid flow rates. Hence, both \( \alpha \) and \( \alpha_i \) peak near \( r/R_H \approx +0.75 \). The peaking phenomenon becomes more pronounced for both \( \alpha \) and \( \alpha_i \) with increasing gas and liquid flow rates as shown in Figs. 2.7 (a) and (b), respectively.

### 2.2.3 Transport of Local Two-Phase Flow Parameters in the 45-degree Experiment

In view of examining the effects of gas and liquid flow rates in the development of local two-phase flow parameters in the 45-degree experiment, profiles of \( \alpha \) and \( \alpha_i \) along the vertical radius of the tube cross-section, \( R_V \), are plotted for different flow conditions in Figs. 2.8 through 2.11. In the 45-degree experiment there exists a two-fold effect of the gas and liquid flow rates on the profiles of \( \alpha \) and \( \alpha_i \). Figs. 2.8 and 2.9 show the two effects of the gas flow rate. In Fig. 2.8 the superficial gas flow rates, \( j_{g,atm} \), are varied to 0.14, 0.35, and 0.71 m/s while the superficial liquid
flow rate, \( j_n \), is fixed at 3.8 m/s. In Fig 2.9, however, the superficial gas flow rates, \( j_{g, atms} \), are varied to 0.71, 1.09, and 1.50 m/s while the superficial liquid velocity is again fixed at 3.8 m/s
Figure 2-5: 90-degree experiment: Effect of gas flow rates on the flow structure alone the vertical radius, $R_V$, at different L/D’s. $j_f = 4.3$ m/s and $j_{g, atm}$=varied (Runs 3, 9 and 15). Arrows denote the direction of increasing gas flow rates.
Figure 2-6: 90-degree experiment: Effect of liquid flow rates on the flow structure along the vertical radius, $R_Y$, at different L/D’s. $j_{g,atm}=0.3$ m/s and $j_f$ varied (Runs 4, 5 and 6). Arrows denote the direction of increasing liquid flow rates.
Figure 2-7: 90-degree experiment: Effect of gas and liquid flow rates on the flow structures along the horizontal radius, $R_{H}$, at different L/D’s. Arrows denote the direction of increasing flow rates.
As shown in Fig. 2.8, for the lower superficial gas velocities \( (j_{g,atm} < 0.82 \text{ m/s}) \), the effect of an increasing gas flow rate causes the peaking phenomena in both the upper and lower halves of the tube to become more pronounced. The second effect, on the other hand, shown in Fig. 2.9 for the higher superficial gas velocities \( (j_{g,atm} > 0.82 \text{ m/s}) \), is that increasing the gas flow rate uniformly increases the void profile in the upper half of the tube while making the void peaking in the lower half of the tube less pronounced. This implies that the two phases become more mixed. The interfacial area concentration follows the same trend of increasing in the upper half of the pipe and either increasing or decreasing in the lower half of the pipe depending on the gas superficial velocity. Figures 2.10 and 2.11 show the two effects of the liquid flow rate. In Fig. 2.10 the superficial liquid flow rates, \( j_f \), are varied to 3.78, 4.04 and 4.34 m/s while the superficial gas flow rate, \( j_{g,atm} \), is fixed at 0.3 m/s (Runs 4, 5 and 6 of the 45-degree experiment). In Fig. 2.11, on the other hand, \( j_f \)'s are varied to 3.78, 4.04 and 4.34 m/s while \( j_{g,atm} \) is fixed at 1.57 m/s (Runs 13, 14 and 15 of the 45-degree experiment). As shown in Fig. 2.10, for a lower superficial gas velocity \( (j_{g,atm} = 0.3 \text{ m/s}) \) condition, the effect on the void and interfacial area concentration profiles of an increasing superficial liquid velocity is minimal. For a higher superficial gas velocity \( (j_{g,atm} = 1.57 \text{ m/s}) \) condition, on the other hand, the effect of an increasing superficial liquid velocity is to decrease the local peaking phenomena in the upper half of the tube while increasing the peaking in the lower half. The effect on the interfacial area concentration by \( j_f \) tends to be small in comparison to the \( j_{g,atm} \) effect and has no significant effect at the measurement ports just before the 45-degree elbow or downstream of the elbow (i.e. L/D=342 and 419, respectively).

In general, it is found that the profiles of \( \alpha \) resemble to those of \( a_i \), which is characteristic of the bubbly flow regime. Several conditions, however, such as Runs 8, 9 and 12 of the 45-degree experiment undergo high local void peaking, between 60-75%, where the interfacial area concentration profile is not similar to the void profile. As mentioned previously, this high degree of void peaking can induce the bubbles to coalesce and drive the interfacial area concentration down. In
these conditions, the flow may transition from the bubbly flow regime to the elongated bubbly flow regime and group two bubbles may be present. As in the 90-degree experiment, the locations of local peaking and the degree of peaking depend on both gas and liquid flow rates. In all of the 45-degree test conditions, both $\alpha$ and $a_i$ tend to peak near the upper half of the tube before the elbow. However, the distribution pattern changes right after the 45-degree elbow (L/D=363), which indicates that the 45-degree elbow also has significant effects on the flow structure. After examining Figs. 2.8 through 2.11, the following characteristics can be found in the flow structures along the vertical tube radius, $R_V$:

**At L/D=342 (11.5 diameters before the 45-degree elbow):** In all flow conditions, bubbles reside mostly upper half of the tube, resulting in local peaks of $\alpha$ and $a_i$ near the top of the tube cross-section ($0.75 < r/R_V < 1$). The bubble distribution tends to spread more along $R_V$ with increasing gas flow rates.

**At L/D=363 (9.5 diameters downstream of the 45-degree elbow):** The distribution of bubbles and location of local peaks of $\alpha$ and $a_i$ depend primarily on the gas flow rates. In flow conditions Runs 4 through 9 of the 45-degree experiment, where the gas flow rate is $j_{g,atm} < 0.76$ m/s, bubbles migrate toward the lower half of the tube radius regardless of the liquid flow rates. Hence, both $\alpha$ and $a_i$ peak near the center of the lower half of tube cross-section (i.e. $r/R_V \approx -0.6$ to -0.7), and the peaking phenomenon is more pronounced with increasing gas and liquid flow rates. When the gas flow rate is increased to or above $j_{g,atm}=1.09$ m/s (Runs 10 through 15, except Run 12 of the 45-degree experiment), the bubbles tend to distribute more uniformly along the vertical tube radius, $R_v$, as shown in Fig. 2.9. It is found in all 45-degree flow conditions that the degree of peaking depends primarily on the gas flow rates, such that local peaks of $\alpha$ and $a_i$ become more pronounced with increasing gas flow rates, as long as $j_{g,atm} < 1.09$ m/s.

**At L/D=419 (or 65.5 diameters downstream of the 45-degree elbow):** In this region, the flow structures are fully recovered in Runs 1 through 3 of the 45-degree experiment ($j_{g,atm} < 0.15$ m/s). In
Runs 4 through 15, except Run 9 of the 45-degree experiment the void profile is recovered, while the longer lasting effect of the elbow is seen in the interfacial area concentration. In fact, the interfacial area concentration profiles of Runs 4 through 15, except Run 9 are never recovered by port P7.

The effects of gas and liquid flow rates on the flow structures along the horizontal tube radius, \( R_H \), for the 45-degree experiment are examined in Figs. 2.12 (a) and (b), respectively. As mentioned earlier, the local measurements along \( R_H \) are performed only at L/D=363, where the 45-degree elbow effects can be most significant. The following characteristics can be found in the flow structures along the horizontal tube radius, \( R_H \):

At L/D=363 (or 9.5 diameters downstream of the 45-degree elbow): As shown in Fig. 2.12 (a) and (b), bubbles reside toward the inner radius of the tube cross-section (i.e, \( R_H > 0 \)) when \( j_{g,atm} < 0.4 \) m/s (Runs 1 through 6 of the 45-degree experiment), but distribute more uniformly across the horizontal radius as the gas flow rates increase. Hence, when \( j_{g,atm} \) is increased to 0.7 m/s (Runs 7 through 9 of the 45-degree experiment), both \( \alpha \) and \( a_i \) are distributed more uniformly across \( R_H \). As the gas flow rates increase further to \( j_{g,atm}>1.09 \) m/s (Runs 10 through 15 of the 45-degree experiment) the profiles grow uniformly, but in general still keep a dominant peak toward the inner radius of the tube cross-section. This uniform growth implies that the increased void fraction serves to mix the two phases more. In this region, the liquid flow rates show a similar effect as the liquid flow rates on the vertical distribution, such that the \( j_{g,atm} \) effect on the interfacial area concentration is much greater that the \( j_f \) effect as shown in Fig. 2.12 (b). Also, in general, the profiles of \( \alpha \) resemble to those of \( a_v \) which is characteristic of the bubbly flow regime.
Figure 2-8: 45-degree experiment: Effect of lower gas flow rates on the flow structure along the vertical radius, $R_v$, at different L/D’s. $j_f=3.8$ m/s fixed and $j_{g,atm}$ varied (Runs 1, 4, and 7). Arrows denote the direction of increasing gas flow rates.
Figure 2-9: 45-degree experiment: Effect of higher gas flow rates on the flow structure along the vertical radius, $R_V$, at different L/D’s. $j_f=3.8$ m/s fixed and $j_{g,atm}$ varied (Runs 7, 10, and 13). Arrows denote the direction of increasing gas flow rates.
Figure 2-10: 45-degree experiment: Effect of liquid flows rates on the flow structure along the vertical radius, $R_V$, for lower $j_{g,atm}$ conditions at different L/D’s. $j_{g,atm}=0.3$ m/s fixed & $j_f$=varied (Runs 4, 5, and 6). Arrows denote the direction of increasing liquid flow rates.
Figure 2-11: 45-degree experiment: Effect of liquid flow rates on the flow structure along the vertical radius, $R_V$, for higher $j_{g,atm}$ conditions at different L/D’s. $j_f=3.8$ m/s fixed and $j_{g,atm}$ varied (Runs 13, 14, and 15). Arrows denote the direction of increasing gas flow rates.
Figure 2-12: 45-degree experiment: Effect of gas and liquid flow rates on the flow structure along the horizontal radius, \( R_{hf} \), at L/D=363. Arrows denote the direction of increasing flow rates.
2.3 One-dimensional Transport of Two-Phase Flow Parameters

In view of one-dimensional transport of $\alpha$ and $a_i$, results obtained in four different characteristic flow conditions, Runs 3, 4, 8 and 14 of the 90-degree experiment are plotted in Figs 2.13 (a) through (d), respectively. Here, the one-dimensional void fraction and interfacial area concentration are calculated by taking the average over the tube cross-section via equations (2-1) through (2-3).

It can be seen in Runs 3 and 4 of the 90-degree experiment, Fig. 2.13 (a) and (b) respectively, where $j_{g,atm} < 0.312$ m/s, that overall $<\alpha>$ increases while $<a_i>$ decreases as the flow passes across the elbow (i.e. $197 < L/D < 250$). Further downstream of the 90-degree elbow ($250 < L/D < 329$), however, $<a_i>$ starts to increase while there is a small, or almost no, change in void fraction. This is more pronounced in Run 4, such that $<a_i>$ increases notably ($\sim$30%) with only a small increase ($\sim$7%) in $<\alpha>$. This implies that in both Runs 3 and 4, the elbow promotes coalescence mechanisms, then the disintegration mechanisms dominate as flow develops further downstream of the elbow.

In Runs 8 and 14 of the 90-degree experiment, Fig. 2.13 (c) and (d) respectively, where $j_{g,atm}$ is relatively high with: $0.65$ m/s $< j_{g,atm} < 1.4$ m/s, on the other hand, overall $<\alpha>$ increases while $<a_i>$ decreases along the axial direction of the flow regardless of the elbow. This implies that the higher gas flow rates in Runs 8 and 14 make the coalescence mechanisms more dominant than the disintegration mechanisms.
Figure 2-13: 90-degree experiment: Axial development of one-dimensional void fraction and interfacial area concentration in four different flow conditions.
For the 45-degree experiment, in all flow conditions except for Runs 5 and 6, the void fraction can increase up to 30% after the elbow, compared to the value before the elbow. While the effect of the elbow on the $<\alpha>$ can be significant, the effect on the $<\alpha_i>$ is generally greater. As shown in Runs 9 and 15 of the 45-degree experiment, Figs. 2.14 (a) and (b), respectively, the elbow can cause either an increase or decrease in the $<\alpha_i>$. For the lower void fraction conditions, Fig. 2.14 (a), it is found that the elbow promotes coalescence of the bubbles. In flow conditions where coalescence is promoted, Runs 2 through 6 and Run 9 of the 45-degree experiment, $<\alpha_i>$ decreases by up to 45%. For the higher void fraction conditions, Fig. 2.14 (b), however, it is found that the elbow promotes disintegration mechanisms. In such cases, Runs 1, 7, 8 and Run 10 through 15 of the 45-degree experiment, $<\alpha_i>$ can increase by as much as 70%. For all 45-degree flow conditions, with the exception of Run 6, coalescence mechanisms are found to be dominant farther downstream of the elbow as the flow recovers.

(a) Run 9: $j_{g,\text{atm}}=0.76$ m/s & $j_f=4.34$ m/s     (b) Run 15: $j_{g,\text{atm}}=1.63$ m/s & $j_f=4.34$ m/s

Figure 2-14: 45-degree experiment: Axial development of one-dimensional void fraction and interfacial area concentration in two different flow conditions.
The comprehensive results of axial development of \( \langle \alpha \rangle \) and \( \langle a_i \rangle \) in 15 different flow conditions are shown in Figs. 2.15 and 2.16 for the 90-degree and 45-degree experiments, respectively. They are summarized based on three different liquid flow rates. As can be clearly seen in the figures, both \( \langle a_i \rangle \) and \( \langle \alpha \rangle \) increase with increasing gas flow rates for a given liquid flow rate. In general, the effect of the elbows, located between \( L/D=197 \) and \( 225 \) and \( L/D=342 \) and \( 363 \), respectively for the 90-degree and 45-degree experiment, is more evident in the change of \( \langle a_i \rangle \). It is highlighted by a sudden increase or decrease of \( \langle a_i \rangle \) in that region. This clearly indicates that bubble interaction mechanisms are promoted by the elbow. However, it is also interesting to note in many 90-degree flow conditions that drastic changes in both \( \langle a_i \rangle \) and \( \langle \alpha \rangle \) do not appear across the elbow (between \( L/D=197 \) and \( 225 \)), but further downstream of the elbow between \( L/D=225 \) and \( 250 \). This indicates that the effects of 90-degree elbow propagates further downstream and affects the development of interfacial structures by promoting either coalescence or disintegration depending on the given flow conditions.
Figure 2-15: 90-degree experiment: One-dimensional transport of void fraction and interfacial area concentration along the axial direction of the flow for different conditions (a) $j_f=3.8$ m/s and $j_g,atm$ varied, (b) $j_f=4.0$ m/s and $j_g,atm$ varied and (c) $j_f=4.3$ m/s and $j_g,atm$ varied. Arrows denote the direction of increasing gas flow rates.
Figure 2-16: 45-degree experiment: One-dimensional transport of void fraction and interfacial area concentration along the axial direction of the flow for different conditions (a) $j_f=3.8$ m/s and $j_{g,atm}$ varied, (b) $j_f=4.0$ m/s and $j_{g,atm}$ varied and (c) $j_f=4.3$ m/s and $j_{g,atm}$ varied. Arrows denote the direction of increasing gas flow rates.
2.4 Other Hydrodynamic Phenomena

2.4.1 Pressure Drop

In Fig. 2.17 (a) and (b), the change in pressure per unit length, \( dp/dz \), over the test section is plotted with respect to the various superficial gas velocities at three different superficial liquid velocities for both the 90-degree and 45-degree experiments, respectively. It is evident from the figures that the pressure loss increases with increasing gas and liquid flow rates.

![Figure 2-17: Pressure drop per unit length across the test section for different superficial gas and liquid velocities. (a) 90-degree experiment between L/D=0 and L/D=329 (b) 45-degree experiment between L/D=197 and L/D=419. Arrow denotes the direction of increasing liquid flow rates.](image)

In Fig. 2.18, the local static pressure acquired at five different axial positions along the entire test section in the 90-degree experiment is plotted for all flow conditions. Each figure represents pressure change at a fixed liquid flow rate with varying gas flow rates. The pressure is measured at L/D=0, 197, 225, 250 and 329, along which a 90-degree elbow is located at L/D=206.6. Characteristic geometric effects of the elbow on pressure loss is clearly demonstrated in all flow conditions. It is
interesting to note, however, that there is little effect in the immediate downstream of the elbow (L/D=225). The effect of the elbow becomes more pronounced in the region further downstream of the elbow, between L/D=225 and 250, and is characterized by a drastic loss in pressure in that region. As the flow develops after L/D=250 into further downstream (L/D=329), the effect of elbow diminishes, and the pressure drop slope almost recovers to its initial slope before the elbow.

In Fig. 2.19, the local static pressure acquired at four different axial positions along the entire test section in the 45-degree experiment is plotted for all flow conditions. Each figure represents the pressure change at a fixed liquid flow rate with varying gas flow rates.

The pressure is measured at L/D=197, 342, 363 and 419, along which a 90-degree elbow is located at 206.6 and a 45-degree elbow is located at L/D=353.5. Characteristic geometric effects of the elbow on pressure loss are clearly demonstrated in all flow conditions. The effect of the 45-degree elbow is characterized by a drastic loss in pressure in that region. As the flow develops further downstream of L/D=363 (L/D=419), the effect of elbow diminishes, but the pressure drop slope is less negative than its initial value between L/D=197 and L/D=342. This is due to the 90-degree elbow, installed at L/D=206.6, which also has an effect on the pressure in that region and does not represent a fully developed flow.
Figure 2-18: 90-degree experiment: Change in local gage pressure measured along the axial direction of the flow. The vertical line in the figures represents the location of the 90-degree elbow. Arrows denote the direction of increasing gas flow rates.
Figure 2-19: 45-degree experiment: Change in local gage pressure measured along the axial direction of the flow. The vertical line in the figures represents the location of the 45-degree elbow. Arrows denote the direction of increasing gas flow rates.
2.4.2 Bubble Velocity

Another characteristic geometric effect induced by the elbows is a change in bubble velocity along the flow direction. The void-weighted gas velocity of two characteristic flow conditions for the 90-degree experiment are plotted in Fig. 2.20 for Runs 1 and 13, where gas flow rates are varied and liquid flow rate is fixed at 3.8 m/s.

As the flow passes through the elbow (197 < L/D < 225), it is clear that bubbles accelerate for the lower gas flow condition (Run 1), but decelerate for the higher gas flow condition (Run 13). The oscillations in bubble velocity after the elbow (225 < L/D < 329) for both conditions are presumed to be due to the “recovery” process such that the bubbles try to recover to their original velocity before the elbow. This is shown by the amplitude of fluctuation, which damps out gradually as the flow develops further into the downstream. Such phenomenon is observed in all 90-degree test conditions.

Similarly, two characteristic flow conditions for the 45-degree experiment are plotted in Fig. 2.21 for Runs 2 and 11. As in the 90-degree case, for the lower gas flow condition (Run 2) the bubbles accelerate as they pass through the 45-degree elbow (342 < L/D < 363), but decelerate for the higher gas flow condition (Run 11). It can be seen, however, that the degree of acceleration or deceleration in the 45-degree case is smaller than in the 90-degree test conditions. Again it can be seen that the bubble velocity oscillates downstream of the elbow (363 < L/D < 419) and is characteristic of all 45-degree test conditions.
Figure 2-20: 90-degree experiment: Axial development of one-dimensional void-weighted gas velocity in two characteristic flow conditions.

Figure 2-21: 45-degree experiment: Axial development of one-dimensional void-weighted gas velocity in two characteristic flow conditions.
2.4.3 Bubble Distribution

In order to better represent the characteristic vertical and horizontal oscillations induced by the elbows, qualitative figures of the interfacial structures are generated. For the 90-degree experiment, these figures are generated from the data collected along the vertical and horizontal axes at ports P1 through P4 (L/D=197, 225, 250, and 329, respectively). In the 45-degree experiment, these figures are generated from the data collected at ports P5 through P7 (L/D=342, 363, and 419, respectively). Changes in the distribution of the gas phase with increasing superficial gas velocity for a fixed superficial liquid velocity are shown in Figs. 2.22 and 2.23 for the 90-degree and 45-degree experiments, respectively. In these figures, a void fraction threshold of 5-10% is used for the borders.

In Fig. 2.22 it is seen that before the elbow the gas phase resides mainly in the upper half of the pipe, whereas directly after the elbow the gas phase is pushed toward the inner half of the tube. As the superficial gas velocity increases it is observed that the gas resides mainly along the vertical radius of the tube. For the higher void fraction conditions, \(j_{g,atm}=0.985\) m/s and \(j_{g,atm}=1.336\) m/s, a gas core now occupies the upper central portion of the tube. It is also seen that to some degree the distribution oscillates horizontally between ports P2 and P3 for all values of \(j_{g,atm}\). As the two-phase mixture moves farther downstream of the elbow the oscillations in the flow dampen out and the flow structure returns to its original configuration before the elbow. It can be seen however that in the higher void fraction conditions that the amount of gas present as port P4 is greater than the amount at port P1. This increase in the area-averaged void fraction is caused by the deceleration of the bubble velocity through the elbow in these conditions.

It can be seen in Fig. 2.23 that before the elbow the gas phase again resides mainly in the upper half of the tube whereas directly after the elbow the gas phase is pushed toward the inner half of the tube. As the superficial gas velocity is increased it is observed that the gas rotates farther around the tube wall with a majority of the gas being present in the bottom half of the tube. For the
higher void fraction conditions, $j_{g, atm} = 1.090$ m/s and $j_{g, atm} = 1.504$ m/s, a gas core now occupies the center of the tube. It is speculated that this coring of the gas phase is caused by large scale eddies induced by the 45-degree elbow. As the two-phase mixture moves farther downstream of elbow the oscillations in the flow dampen out and the flow structure regains the same configuration as before the elbow.
Figure 2-22: Qualitative time-averaged void profiles through the 90-degree elbow for Runs 1, 4, 7, 10 and 13.
Figure 2-23: Qualitative time-averaged void profiles through the 45-degree elbow for Runs 1, 4, 7, 10 and 13.
Chapter 3

Development of One-Dimensional Interfacial Area Transport Equation for Horizontal Bubbly Flow

To develop the one-dimensional interfacial area transport equation for the given flow conditions, characteristic differences between vertical and horizontal bubbly flow need to be taken into consideration. In addition to the geometric effect due to the orientation, other effects induced by the existence of the 90-degree and 45-degree elbows need to also be carefully examined. In the preceding chapter, the experimental data is analyzed in detail. Three characteristic features of horizontal bubbly flow with elbow flow restrictions different from vertical bubbly flow are identified as follows:

- Additional pressure drop across the elbow restrictions;
- Acceleration / deceleration of bubbles across the elbow restrictions. Hence non-uniform bubble velocity along the flow direction;
- Non-uniform bubble distribution across the tube cross section.

3.1 Bubble Interaction Mechanisms in Horizontal Bubbly Flow

The source and sink terms of the interfacial area transport equation can summarized into three categories:

- Bubble break-up or coalescence via various interaction mechanisms,
- Bubble volume expansion or contraction, and
- Evaporation / Condensation / Nucleation.

Among them, the break-up or coalescence mechanisms are attributed to different types of bubble interactions. The major bubble interaction mechanisms of one-group transport in vertical flow include break-up due to the impact of turbulent eddies (TI), coalescence through random collision driven by
turbulent eddies (RC), and coalescence due to the acceleration of the following bubble in the wake of a preceding bubble (WE).

The mathematical models previously developed for these mechanisms are also applicable to horizontal bubbly flow with elbow flow restrictions, because they are mechanistically modeled and do not depend on the flow orientation. In the present study, however, the WE interaction mechanism is neglected. This is because the wake entrainment mechanism is directly related to the relative velocity between the leading bubble and the following bubble within the wake region in vertical systems, and may not be applicable to a horizontal system.

It should be noted though that the relative velocity between the bubbles may not be negligible at the elbows, and the contribution from the wake entrainment mechanism in these regions may therefore also not be negligible. However, modeling of the relative velocity between the bubbles at the elbows is beyond the scope of present study.

3.2 Covariance of the Interfacial Area Transport Equation Terms

3.2.1 Transport of Local Two-Phase Flow Parameters in the 45-degree Experiment

The background and the theoretical formulation of the interfacial area transport equation are given in Sections 1.1 and 1.2. For bubbly flow, the two-group interfacial area transport equation reduces to the one-group interfacial area transport equation given by:
\[
\frac{\partial a_i}{\partial t} + \mathbf{v}_i \cdot \nabla a_i = -a_i \nabla \cdot \mathbf{v}_i + \frac{2}{3} \left( \frac{a_i}{\alpha} \right) \left( \frac{\partial \alpha}{\partial t} + \nabla \cdot (\alpha \mathbf{v}_g) \right) \\
+ C_{TI} \frac{1}{18} \left( \frac{a_i^2}{\alpha} u_i \right) \sqrt{1 - \frac{We_{cr}}{We}} \left( \exp \left( -\frac{We_{cr}}{We} \right) \right)
\]
\[
- C_{RC} \frac{1}{3\pi} \frac{a_i^2 u_i}{\alpha_{\text{max}}^{1/3} (\alpha_{\text{max}}^{1/3} - \alpha^{1/3})} \left( 1 - \exp \left( -C \frac{\alpha_{\text{max}}^{1/3} \alpha^{1/3}}{\alpha_{\text{max}}^{1/3} - \alpha^{1/3}} \right) \right)
\]
\[
+ C_{WE} C_{D}^{1/3} \frac{1}{3\pi} a_i^2 u_r \\
+ \pi D_{r}^2 R_{ph}
\]

where the left-hand-side (LHS) of the equation represents the total rate of change in interfacial area concentration and the RHS of the equation represents various sources and sinks that contribute to the change. The source and sink terms in the RHS of the equation include those due to bubble contraction or expansion via bubble velocity gradient (VG), source due to bubble expansion via pressure drop (PD), source due to break-up via turbulence impact (TI), sink due to coalescence via random collision (RC), sink due to coalescence via wake entrainment (WE), and source or sink due to phase change such as evaporation and condensation (Ph), respectively.

Accounting for the adiabatic test condition, the steady-state one-group interfacial area transport equation applicable to the present flow configuration reduces to:

\[
\nabla \cdot (a_i \mathbf{v}_i) = \left( \frac{2a_i}{3\alpha} \right) \left( \nabla \cdot \alpha \mathbf{v}_g \right) \\
+ C_{TI} \frac{1}{18} \left( \frac{a_i^2}{\alpha} u_i \right) \sqrt{1 - \frac{We_{cr}}{We}} \left( \exp \left( -\frac{We_{cr}}{We} \right) \right)
\]
\[
- C_{RC} \frac{1}{3\pi} \frac{a_i^2 u_i}{\alpha_{\text{max}}^{1/3} (\alpha_{\text{max}}^{1/3} - \alpha^{1/3})} \left( 1 - \exp \left( -C \frac{\alpha_{\text{max}}^{1/3} \alpha^{1/3}}{\alpha_{\text{max}}^{1/3} - \alpha^{1/3}} \right) \right)
\]

(3-2)

As mentioned previously, the WE mechanism contribution has been omitted since the current model for WE may not be applicable in horizontal systems. For practical applications, the one-
dimensional form of the interfacial area transport equation is area-averaged over the cross-sectional area, which yields:

\[
\left\langle \frac{\partial}{\partial z} (a_i v_i) \right\rangle = \left\langle \frac{2a_i}{3\alpha} \left( \frac{\partial \alpha v_g}{\partial z} \right) \right\rangle + \left\langle C_{\alpha} \frac{1}{18} \left( \frac{a_i^2}{\alpha} u_i \right) \sqrt{1 - \frac{W_{er}}{W}} \left( \exp\left( -\frac{W_{er}}{W} \right) \right) \right\rangle \\
- \left\langle C_{\alpha} \frac{1}{3\pi} \frac{a_i^2 u_i}{\alpha_{\text{max}}^{1/3} (\alpha_{\text{max}}^{1/3} - \alpha^{1/3})} \left( 1 - \exp\left( -C \frac{\alpha_{\text{max}}^{1/3} \alpha^{1/3}}{\alpha_{\text{max}}^{1/3} - \alpha^{1/3}} \right) \right) \right\rangle
\]

(3-3)

Here, considering that one-group transport assumes bubbles are spherical in shape, the void fraction and interfacial area are linearly related by:

\[ D_{sm} = \frac{6\alpha}{a_i} \] : Bubble Sauter mean diameter

(3-4)

Hence, the \( a_i \)-weighted bubble interfacial velocity can be approximated by the \( \alpha \)-weighted bubble velocity as:

\[
\left\langle \left\langle v_i \right\rangle \right\rangle \equiv \frac{\left\langle a_i v_i \right\rangle}{\left\langle a_i \right\rangle} \approx \frac{\left\langle \alpha v_g \right\rangle}{\left\langle \alpha \right\rangle} = \left\langle \left\langle v_g \right\rangle \right\rangle
\]

(3-5)

Then, the LHS of Eq. (3-3) can be written in terms of \( \left\langle a_i \right\rangle \) and \( \left\langle \left\langle v_g \right\rangle \right\rangle \) as:

\[
\left\langle \frac{\partial}{\partial z} (a_i v_i) \right\rangle = \frac{\partial}{\partial z} \left( \left\langle a_i \right\rangle \left\langle \left\langle v_g \right\rangle \right\rangle \right) = \left\langle a_i \right\rangle \frac{\partial}{\partial z} \left\langle \left\langle v_g \right\rangle \right\rangle + \left\langle a_i \right\rangle \frac{\partial}{\partial z} \left\langle \left\langle v_i \right\rangle \right\rangle
\]

(3-6)

Furthermore, by employing the steady-state gas-phase continuity equation and the ideal gas law, first term in the RHS of Eq. (3-3) can be expressed in terms of pressure as:

\[
\left\langle \frac{2a_i}{3\alpha} \left( \frac{\partial \alpha v_g}{\partial z} \right) \right\rangle = \left\langle \frac{2\left\langle a_i \right\rangle \left\langle \left\langle v_g \right\rangle \right\rangle}{3\left\langle p \right\rangle} \right\rangle - \frac{\partial}{\partial z} \left\langle \left\langle p \right\rangle \right\rangle
\]

(3-7)
3.2.2 Distribution Parameter: Covariance

In vertical two-phase flow, the profiles of local two-phase flow parameters can be assumed uniform, unless there is a strong local phenomenon such as wall-peaking. Nevertheless, the covariance in vertical two-phase flow is nearly unity and can be neglected for simplicity. In horizontal two-phase flow, however, it is clear from the data that the profiles of local two-phase flow parameters across the tube cross-section are highly non-uniform, and the covariance in the area-averaging process can be significant. Therefore, a mathematical parameter analogous to the covariance is defined as:

\[ COV(AB) = \frac{\langle AB \rangle}{\langle A \rangle \langle B \rangle} \]  \hspace{1cm} (3-8)

Hence, the TI and RC in Eq. (3-3) can be written as:

\[ \left\langle C_{\eta} \frac{1}{18} \left( \frac{a_i^2}{\alpha} \right) u_t \right\rangle \sqrt{1 - \frac{We_{cr}}{We} \left( \exp \left( -\frac{We_{cr}}{We} \right) \right)} = \]  \hspace{1cm} (3-9)

and:

\[ \left\langle C_{\pi} \frac{1}{18} \left( \frac{a_i^2}{\alpha} \langle u_t \rangle \right) \sqrt{1 - \frac{We_{cr}}{We} \left( \exp \left( -\frac{We_{cr}}{We} \right) \right)} \right\rangle = \]  \hspace{1cm} (3-10)

where the covariance of TI and RC are given by:
\[
\text{COV}_{\text{tr}} = \frac{\left(\frac{a_i^2}{\alpha} u_i \right) \sqrt{1 - \frac{We_{cr}}{We} \left(\exp\left(-\frac{We_{cr}}{We}\right)\right)}}{\left(\frac{a_i}{\alpha}\right)^2 \left(\frac{u_i}{\langle u_i \rangle}\right) \sqrt{1 - \frac{We_{cr}}{\langle We \rangle} \left(\exp\left(-\frac{We_{cr}}{\langle We \rangle}\right)\right)}}
\]

(3-11)

and:

\[
\text{COV}_{\text{bc}} = \frac{\left(\frac{a_i^2}{\alpha^{1/3}} u_i \right) \left(\frac{\alpha_{max}^{1/3}}{\alpha^{1/3}}\right) \left(\frac{\alpha_{max}^{1/3}}{\alpha^{1/3}}\right) \left(\frac{\alpha_{max}^{1/3}}{\alpha^{1/3}}\right)}{\left(\frac{a_i}{\alpha}\right)^2 \left(\frac{u_i}{\langle u_i \rangle}\right) \left(\frac{\alpha_{max}^{1/3}}{\alpha^{1/3}}\right) \left(\frac{\alpha_{max}^{1/3}}{\alpha^{1/3}}\right) \left(\frac{\alpha_{max}^{1/3}}{\alpha^{1/3}}\right)}
\]

(3-12)

Here, \(We_{cr}\), \(\alpha_{max}\) and \(C\) are constants that need to be specified before calculating the covariance.

The values employed in vertical upward flow in round pipes, vertical co-current downward flow in round pipes, and vertical upward flow in a rectangular duct, as well as for the present model are shown in Table 3.1.

<table>
<thead>
<tr>
<th>Flow geometry/orientation</th>
<th>Model constants</th>
<th>(We_{cr})</th>
<th>(\alpha_{max})</th>
<th>(C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal w/ 90° or 45° elbow: round pipe</td>
<td></td>
<td>8</td>
<td>0.75</td>
<td>3</td>
</tr>
<tr>
<td>Vertical up: round pipe</td>
<td></td>
<td>6</td>
<td>0.75</td>
<td>3</td>
</tr>
<tr>
<td>Vertical co-current down: round pipe</td>
<td></td>
<td>6</td>
<td>0.75</td>
<td>3</td>
</tr>
<tr>
<td>Vertical up: rectangular duct</td>
<td></td>
<td>8</td>
<td>0.75</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 3-1: Constants in the model for various flow geometry and orientation.
In order to calculate the covariance of the TI and RC terms, the local values of $a_i$, $\alpha$ and $u_t$, are required. Since only $a_i$ and $\alpha$ data are available at each measurement port, the value for $u_t$ and $We$ need to be expressed as functions of $a_i$, $\alpha$ and fluid properties.

Assuming isotropic homogenous turbulence and spherical bubbles, the local turbulent velocity, $u_t$, can be estimated through the relation given by Betchelor (1958) as:

$$u_t = 1.4\varepsilon^{1/3} \left( \frac{6\alpha}{a_i} \right)^{1/3}$$  \hspace{1cm} (3-13)\]

where:

$$\varepsilon = \frac{f_{TP}v_m^3}{2D_h} : \text{turbulent dissipation rate}$$ \hspace{1cm} (3-14)\]

with:

$$f_{TP} = 0.316 \left( \frac{\mu_m}{\text{Re}_m \mu_f} \right)^{1/4} : \text{two-phase friction factor}$$ \hspace{1cm} (3-15)\]

$$\text{Re}_m = \frac{\rho_f v_m D_h}{\mu_m} : \text{mixture Reynolds number}$$ \hspace{1cm} (3-16)\]

$$\mu_m = \frac{\mu_f}{1 - \alpha} : \text{mixture viscosity}$$ \hspace{1cm} (3-17)\]

$$v_m = \frac{\rho_f j_f + \rho_g j_g}{\alpha \rho_g + (1 - \alpha) \rho_f} : \text{mixture velocity}$$ \hspace{1cm} (3-18)\]

where the subscripts $f$, $g$, and $m$ are the phase indices for liquid, gas, and two-phase mixture, respectively.

Assuming that the flow is isothermal, $v_m$ can be written in terms of pressure as:
Here, considering the significant difference in densities of air and water, and assuming the pressure along the flow channel does not vary significantly, we may assume:

\[
\frac{\rho_{\text{go}} P}{P_o} \ll \rho_f
\]

(3-20)

and:

\[
v_m \approx \frac{\rho_f j_f + \rho_{\text{go}} j_{\text{go}}}{\rho_f (1 - \alpha)}
\]

(3-21)

Combining Eqs. (3-17) and (3-21) with Eq. (3-16), the mixture Reynolds number can be approximated as a constant value for a given flow condition:

\[
\text{Re}_m \approx \frac{D_h}{\mu_f} \left( \rho_f j_f + \rho_{\text{go}} j_{\text{go}} \right) = \text{constant}
\]

(3-22)

Hence, the turbulent dissipation rate can be written in terms of void fraction as:

\[
\varepsilon \approx \frac{0.316}{2} \left( \frac{\rho_f j_f + \rho_{\text{go}} j_{\text{go}}}{\rho_f D_h \text{Re}_m^{1/4}} \right)^3 \left( \frac{1}{(1 - \alpha)^{3/4}} \right)
\]

(3-23)

The turbulent velocity, \( u_t \), can be obtained in terms of the \( a_i \) and \( \alpha \) by substituting Eq. (3-23) into Eq. (3-13):

\[
u_t \approx \frac{1.3753 \left( \rho_f j_f + \rho_{\text{go}} j_{\text{go}} \right)}{\rho_f D_h^{1/3} \text{Re}_m^{1/12}} \left( \frac{1}{(1 - \alpha)^{3/4}} \right) \left( \frac{\alpha}{a_i} \right)^{1/3}
\]

(3-24)

Here, defining:
\[ B = \frac{1.3753 \left( \rho_f j_f + \rho_{g0} j_{go} \right)}{\rho_f D_f^{1/3} Re_n^{1/12}} \approx \text{constant at any given flow condition} \quad (3-25) \]

and approximating \( \frac{13}{12} \approx 1.0 \), Eq. (3-25) reduces to:

\[ u_i \approx B \left( \frac{1}{(1 - \alpha)} \right) \left( \frac{\alpha}{a_i} \right)^{1/3} \quad (3-26) \]

Similarly, by employing Eq. (3-26) the Weber number defined as:

\[ We = \frac{\rho_f u_i^2 D_f}{\sigma} \quad (3-27) \]

can be reduced to:

\[ We = \frac{6 \rho_f B^2}{\sigma (1 - \alpha)^2} \left( \frac{\alpha}{a_i} \right)^{5/3} \quad (3-28) \]

Thus, both the TI and RC covariance terms can be expressed as functions of \( \alpha \) and \( a_i \) as:

\[
\text{COV}_{TI} = \left\langle \left( \frac{a_i}{\alpha} \right)^{5/3} \left( \frac{\alpha}{1 - \alpha} \right) \sqrt{1 - \frac{We_{cr}}{\langle We \rangle}} \exp \left( -\frac{We_{cr}}{\langle We \rangle} \right) \right\rangle \quad \text{for} \quad \langle We \rangle \geq We_{cr} \quad (3-29)
\]

and:

\[
\text{COV}_{RC} = \left\langle \left( \frac{a_i}{\alpha} \right)^{5/3} \left( \frac{\alpha^2}{1 - \alpha} \right) \left( \frac{1}{\alpha_{max}^{1/3} - \alpha^{1/3}} \right) \left( 1 - \exp \left( -C \frac{\alpha_{max}^{1/3} - \alpha^{1/3}}{\alpha_{max}^{1/3} - \alpha^{1/3}} \right) \right) \right\rangle \quad (3-30)
\]

where \( \langle We \rangle \) is given by:
\[
\langle \text{We} \rangle = \frac{6 \rho_f B^2}{\sigma (1 - \langle \alpha \rangle)^2} \left( \langle \alpha \rangle \right)^{5/3} \]  

(3-31)

In performing numerical calculations, the Weber number is continuously calculated per the numerical mesh along the flow direction. Since the TI mechanism is valid only when \(\text{We}>\text{We}_{cr}\), and the square-root terms in Eq. (3-29) becomes non-physical when \(\text{We}<\text{We}_{cr}\), some numerical treatment of \(\text{COV}_{TI}\) is needed. Hence, the following equation is used for the case when \(\text{We}<\text{We}_{cr}\):

\[
\text{COV}_{TI} = \frac{\left( \frac{a_i}{\alpha} \right)^{5/3} \left( \frac{\alpha}{1 - \langle \alpha \rangle} \right) \left( \exp \left( - \frac{\text{We}_{cr}}{\text{We}} \right) \right)}{\left( \frac{\langle a_i \rangle}{\langle \alpha \rangle} \right)^{5/3} \left( \frac{\langle \alpha \rangle}{1 - \langle \alpha \rangle} \right) \left( \exp \left( - \frac{\text{We}_{cr}}{\langle \text{We} \rangle} \right) \right)} \quad \text{for } \langle \text{We} \rangle < \text{We}_{cr} \]  

(3-32)

The covariance of the TI and RC terms calculated at each port for all 90-degree and 45-degree test conditions are summarized in Tables 3.2 and 3.3, respectively. It should be noted that at one local point in the 45-degree data set, Port 6 of Run 9, the local void fraction exceeded \(\alpha_{max}=0.75\). Physically this gives that the probability of a random collision is 1, but numerically this causes \(\text{COV}_{RC}\) to approach infinity as seen in Eq. 3-30. To treat this problem numerically, the local void fraction value at \(r/R=-0.5149\) is modified from 0.752 to 0.740, which still generates a relatively large \(\text{COV}_{RC}\) of 87.64, compared to the typical range of 1 to 18 for this measurement port.
Table 3-2: Calculated values for TI and RC covariance in the 90-degree experiment at each measurement port.

<table>
<thead>
<tr>
<th>Run Number</th>
<th>Port 1</th>
<th>Port 2</th>
<th>Port 3</th>
<th>Port 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>COVTI</td>
<td>COVRC</td>
<td>COVTI</td>
<td>COVRC</td>
</tr>
<tr>
<td>1</td>
<td>1.69</td>
<td>21.28</td>
<td>1.03</td>
<td>2.05</td>
</tr>
<tr>
<td>2</td>
<td>1.54</td>
<td>18.02</td>
<td>1.03</td>
<td>2.14</td>
</tr>
<tr>
<td>3</td>
<td>1.32</td>
<td>11.71</td>
<td>1.01</td>
<td>2.25</td>
</tr>
<tr>
<td>4</td>
<td>2.61</td>
<td>28.52</td>
<td>1.06</td>
<td>1.97</td>
</tr>
<tr>
<td>5</td>
<td>2.66</td>
<td>20.89</td>
<td>1.05</td>
<td>2.11</td>
</tr>
<tr>
<td>6</td>
<td>2.51</td>
<td>27.32</td>
<td>1.03</td>
<td>2.43</td>
</tr>
<tr>
<td>7</td>
<td>2.45</td>
<td>28.08</td>
<td>1.08</td>
<td>2.81</td>
</tr>
<tr>
<td>8</td>
<td>2.41</td>
<td>29.79</td>
<td>1.10</td>
<td>3.54</td>
</tr>
<tr>
<td>9</td>
<td>1.99</td>
<td>21.63</td>
<td>1.11</td>
<td>4.00</td>
</tr>
<tr>
<td>10</td>
<td>2.08</td>
<td>19.47</td>
<td>1.08</td>
<td>3.53</td>
</tr>
<tr>
<td>11</td>
<td>1.98</td>
<td>21.96</td>
<td>1.09</td>
<td>3.70</td>
</tr>
<tr>
<td>12</td>
<td>1.84</td>
<td>18.94</td>
<td>1.09</td>
<td>4.77</td>
</tr>
<tr>
<td>13</td>
<td>1.81</td>
<td>13.80</td>
<td>1.09</td>
<td>2.99</td>
</tr>
<tr>
<td>14</td>
<td>1.75</td>
<td>14.30</td>
<td>1.07</td>
<td>2.28</td>
</tr>
<tr>
<td>15</td>
<td>1.51</td>
<td>8.54</td>
<td>1.13</td>
<td>4.99</td>
</tr>
<tr>
<td>Average</td>
<td>2.01</td>
<td>20.28</td>
<td>1.07</td>
<td>3.04</td>
</tr>
</tbody>
</table>
Table 3-3: Calculated values for TI and RC covariance in the 45-degree experiment at each measurement port.

<table>
<thead>
<tr>
<th>Run Number</th>
<th>Port 1</th>
<th>Port 2</th>
<th>Port 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>COVTI</td>
<td>COVR_C</td>
<td>COVTI</td>
</tr>
<tr>
<td>1</td>
<td>1.83</td>
<td>24.06</td>
<td>1.82</td>
</tr>
<tr>
<td>2</td>
<td>7.53</td>
<td>19.45</td>
<td>1.60</td>
</tr>
<tr>
<td>3</td>
<td>3.90</td>
<td>14.43</td>
<td>1.73</td>
</tr>
<tr>
<td>4</td>
<td>2.53</td>
<td>29.63</td>
<td>0.94</td>
</tr>
<tr>
<td>5</td>
<td>2.30</td>
<td>25.87</td>
<td>0.92</td>
</tr>
<tr>
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<td>2.24</td>
<td>18.39</td>
<td>0.93</td>
</tr>
<tr>
<td>7</td>
<td>1.75</td>
<td>12.37</td>
<td>1.10</td>
</tr>
<tr>
<td>8</td>
<td>1.64</td>
<td>11.08</td>
<td>1.11</td>
</tr>
<tr>
<td>9</td>
<td>1.77</td>
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<td>1.36</td>
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</tr>
<tr>
<td>11</td>
<td>1.26</td>
<td>4.16</td>
<td>1.04</td>
</tr>
<tr>
<td>12</td>
<td>1.27</td>
<td>4.14</td>
<td>1.08</td>
</tr>
<tr>
<td>13</td>
<td>1.23</td>
<td>3.73</td>
<td>1.03</td>
</tr>
<tr>
<td>14</td>
<td>1.17</td>
<td>3.09</td>
<td>1.02</td>
</tr>
<tr>
<td>15</td>
<td>1.14</td>
<td>2.68</td>
<td>1.04</td>
</tr>
<tr>
<td>Average:</td>
<td>2.19</td>
<td>12.75</td>
<td>1.17</td>
</tr>
</tbody>
</table>

3.3 Pressure Loss Correlation

In general, the two-phase frictional pressure drop is calculated by the Lockhart and Martinelli (1949) correlation given as:

\[
\phi^2_f = 1 + \frac{C}{X} + \frac{1}{X^2}
\]  

(3-33)

These are defined by:
\[ \phi_f^2 = \frac{\left( \frac{dp}{dz} \right)_F^{2\phi}}{\left( \frac{dp}{dz} \right)_F^f} \quad \text{and} \quad X^2 = \frac{\left( \frac{dp}{dz} \right)_F^g}{\left( \frac{dp}{dz} \right)_F^g} \]

(3-34)

In Eq. (3-34), superscripts \( f \), \( g \) and \( 2\phi \) are the phase indices for liquid, gas and two-phase mixture, respectively, and subscript \( F \) indicates the frictional loss. Hence, \( \left( \frac{dp}{dz} \right)_F^f \), \( \left( \frac{dp}{dz} \right)_F^g \) and \( \left( \frac{dp}{dz} \right)_F^{2\phi} \) in Eqs. (3-34) denote the frictional pressure losses due to the single-phase liquid, single-phase gas and the two-phase mixture, respectively.

Here, the pressure drop due to the \( k \)th phase can be correlated via the friction factor, \( f \), by:

\[ \left( \frac{dp}{dz} \right)_F^k = 2f \frac{\rho_k j_k^2}{D} \quad \text{where the subscript } k = f \text{ or } g. \]  

(3-35)

where the friction factor can be obtained from the Blasius formulation by:

\[ f = m \frac{Re}{n} \quad \text{with } Re = \frac{\rho_k j_k D}{\mu_k} \]

(3-36)

where the subscript \( k=f \) or \( g \), and the coefficients \( m \) and \( n \) for round pipe flow are given by:

\( m = 64 \) & \( n = 1 \) \quad : \text{for laminar flow} \\
\( m = 0.079 \) & \( n = 0.25 \) \quad : \text{for turbulent flow}  

(3-37)

Therefore, by finding appropriate values for the parameter \( C \) in Eq. (3-33), one can estimate the two-phase frictional pressure loss. The parameter \( C \) for the gas-liquid two-phase flow in straight horizontal pipe without any flow obstructions is given by Chisholm (1967) and is summarized in Table 3.4.
In two-phase flow through a channel with a flow restriction, however, additional pressure loss stemming from the restriction needs to be considered and Eq. (3-33) is not applicable for the elbow region. Therefore, the present study develops a simple correlation for the two-phase minor loss due to the 90-degree or 45-degree elbow by employing an approach analogous to that of Lockhart and Martinelli.

First, it is noted that Eq. (3-33) originates from the hypothesis that the two-phase frictional pressure drop can be expressed as the pressure drop of each phase and its combination by:

\[
\left( \frac{dp}{dz} \right)_{F}^{2} = \left( \frac{dp}{dz} \right)_{F}^{f} + \left( \frac{dp}{dz} \right)_{F}^{g} + C \left[ \left( \frac{dp}{dz} \right)_{F}^{f} \left( \frac{dp}{dz} \right)_{F}^{g} \right]^{1/2}
\]  

(3-38)

Table 3-4: Suggested values for parameter C in Eq. 3-33.

<table>
<thead>
<tr>
<th>Liquid – Gas</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turbulent – Turbulent (tt)</td>
<td>20</td>
</tr>
<tr>
<td>Turbulent – Laminar (tl)</td>
<td>12</td>
</tr>
<tr>
<td>Laminar – Turbulent (lt)</td>
<td>10</td>
</tr>
<tr>
<td>Laminar – Laminar (ll)</td>
<td>5</td>
</tr>
</tbody>
</table>

Since Eq. (3-38) is written for the flow through pipes without any minor loss, it does not account for the effect of flow restrictions in the two-phase pressure drop. When there is a flow restriction in a two-phase flow system, the total frictional pressure drop should account for losses due to both the wall friction and the restrictions (or minor losses). Hence, the liquid-only frictional pressure drop is given by:
\[
\left( \frac{dp}{dz} \right)_F^f = \left( \frac{4f}{D} + \frac{k}{L} \right) \rho f f^2 \frac{f}{2} \text{ with a flow restriction (3-39)}
\]

where the second term in the parenthesis in the RHS of the equation is due to the minor loss, and \( k \) and \( L \) are the minor loss factor and characteristic length scale of the restriction specific to the restriction geometry, respectively. Hence, Eq. (3-39) can be broken into two terms such that:

\[
\left( \frac{dp}{dz} \right)_F^f = \left( \frac{dp}{dz} \right)_{Ff}^f + \left( \frac{dp}{dz} \right)_{FM}^f \tag{3-40}
\]

where \( \left( \frac{dp}{dz} \right)_{Ff}^f \) and \( \left( \frac{dp}{dz} \right)_{FM}^f \) denote the contributions from frictional loss by the liquid phase and that due to the flow restriction, respectively.

Assuming that the change in gas-phase frictional pressure drop due to the restriction is negligibly small compared to the effect of the restriction on the liquid-phase, the Martinelli parameter given by Eq. (3-34) is redefined in two forms as:

\[
X^2 = \frac{\left( \frac{dp}{dz} \right)_{Ff}^f}{\left( \frac{dp}{dz} \right)_F^g}
\]

\[
X^2_M = \frac{\left( \frac{dp}{dz} \right)_{Ff}^f}{\left( \frac{dp}{dz} \right)_{FM}^f} \tag{3-42}
\]

Eq. (3-41) is essentially same as Eq. (3-33) and represents the contribution to the frictional pressure loss by the liquid-only flow with respect to the gas-only flow without including the minor
loss. A new parameter $X_{M}^2$, given by Eq. (3-42), on the other hand, reflects the contribution from the flow restriction.

Now, combining Eq. (3-38) with Eqs. (3-39) through (3-42), a new correlation accounting for both the wall friction and minor losses is obtained as:

$$
\phi_{fM}^2 = \left[ 1 + \frac{1}{X_{M}^2} \right] + \left[ 1 + \frac{1}{X_{M}^2} \right]^{1/2} \frac{C}{X} + \frac{1}{X^2}
$$

(3-43)

where $\phi_{fM}$ is the two-phase friction multiplier defined similar to Eq. (3-33), but accounts for contributions from both the wall friction and minor losses by $X$ and $X_{M}$. Therefore, by finding the value of parameter $C$ that best fits experimental data, the two-phase frictional pressure drop accounting for both wall friction and minor losses can be estimated. The minor loss factor $k$ can be obtained for a given flow restriction geometry by noting that:

$$
X_{M}^2 = 4 \left( \frac{f}{k} \right) \left( \frac{L}{D} \right)_{Restriction}
$$

(3-44)

where $\left( \frac{L}{D} \right)_{Restriction}$ is specified by the restriction geometry, and $f$ is given by Eq. (3-36).

In the present study, Eqs. (3-43) and (3-44) are used to determine coefficients, $C$ and $k$ by plotting the logarithmic graph of $\phi_{fM}$ versus $X$. It is noted that the parameter $C$ essentially determines the slope of the asymptotic profile for $\ln(\phi_{fM})$, and the $k$-factor determines the level of the asymptotic value. Optimal values for both $C$ and $k$ are found by varying them to find the best fit with the data.

In calculating the Martinelli parameter, $X$, in the present analysis, a correlation for turbulent flow is employed for the friction factor, $f$, regardless of the gas flow rate whether it is laminar or turbulent. Namely, Eq. (3-36) with $m=0.079$ and $n=0.25$ is employed for $f$. This is because, in bubbly flow, the
transport of dispersed gas bubbles is determined essentially by the liquid phase. Since all of the liquid flow rates in the present test conditions indicate that the flow is turbulent, the turbulent flow correlation is employed to calculate the friction factors for both the gas and liquid phases.

Furthermore, the present model is correlated based on the pressure drop between the ports P1 and P3 (instead of ports P1 and P2) in the 90-degree experiment and between the ports P5 and P6 in the 45-degree case. This is essentially because the most significant pressure drop occurs in these regions for all of the test conditions as shown earlier in Figs. 2.18 and 2.19. Therefore, the present analysis is based on \((L/D)_{\text{elbow}}=53.5\) for the 90-degree elbow, and \((L/D)_{\text{elbow}}=21.1\) for the 45-degree elbow.

The results are shown in Figs. 3.1 and 3.2 for the 90-degree and 45-degree experiments, respectively. In Figs. 3.1(a) and Fig. 3.2(a), the pressure drop across the test section for the 90-degree experiment \((L/D=0\) to 329) and 45-degree experiment \((L/D=342\) to 419), respectively, is calculated from the conventional Lockhart-Martinelli correlation given by Eq. (3-32). In Figs. 3.1(b) and 3.2(b), pressure drop across the 90-elbow \((L/D=197\) to 250) and the pressure drop across the 45-degree elbow \((L/D=342\) to 363) is correlated by the newly developed Eq. (3-43).

Figure 3.1(a) shows that the pressure drop across the entire test section for the 90-degree experiment (i.e., between \(L/D = 0\) and 329) can be predicted well by the conventional Lockhart-Martinelli correlation with \(C=25\), which is slightly higher than the recommended value of \(C=20\) developed for turbulent-turbulent gas-liquid two-phase flow through a straight flow channel without flow restriction. The increase in \(C\) value is presumed to be attributed to the existence of the 90-degree elbow in the test section at \(L/D=206.6\).

In Fig. 3.1(b), the newly developed correlation with modified parameter, \(C=65\) and \(k=0.58\) is plotted against the 90-degree experimental data. The present model predicts the data well with an average percent difference of only 2%. The two-phase minor loss factor, \(k\), acquired via Eq. (3-43), is
found to be \( k=0.58 \) and is approximately 3 times higher than the \( k=0.20 \) recommended for single-phase flow through a 90-degree elbow (White, F. M, 2006).

Fig. 3.2(a) shows that the pressure drop across the test section for the 45-degree experiment (i.e., between \( L/D = 342 \) and 419) can be predicted well by the conventional Lockhart-Martinelli correlation with \( C=40 \), which is again higher than the recommended value of \( C=20 \) developed for turbulent-turbulent gas-liquid two-phase flow through a straight flow channel without flow restriction. This increase in \( C \) value is also again presumed to be attributed to the existence of the 45-degree elbow in the test section at \( L/D=353.5 \). In addition, this value may be higher than the \( C=25 \) for the 90-degree elbow case, since to the pressure effect of the 90-degree elbow may still be present in the region upstream of the 45-degree elbow.

In Fig. 3.2(b), the newly developed correlation with modified parameter, \( C=65 \) and \( k=0.35 \) is plotted against the 45-degree experimental data. The present model predicts the data well with an average percent difference of less than 2%. The two-phase minor loss factor, \( k \), acquired via Eq. (3-43), is found to be \( k=0.35 \) and is also approximately 3 times higher than the \( k=0.13 \) recommended for single-phase flow through a 45-degree elbow (White, F. M, 2006).

The comprehensive results of the pressure prediction are shown in Appendices A.1 and A.2 for the 90-degree and 45-degree experiments, respectively.
Figure 3-1: $\phi_M$ vs. $X$. (a) Prediction made by the conventional Lockhard-Martinelli correlation for pressure drop across the 90-degree test section. (b) Prediction made by the new correlation for pressure drop across the 90-degree elbow. Error bars shown: $\pm 5\%$. 
Figure 3-2: $\phi_M$ vs. $X$. (a) Prediction made by the conventional Lockhart-Martinelli correlation for pressure drop across the 45-degree test section. (b) Prediction made by the new correlation for pressure drop across the 45-degree elbow. Error bars shown: $\pm 5\%$. 
In view of expressing the pressure loss as a function of void fraction, the parameters \( X \) and \( X_M \) can be rearranged in terms of void fraction as:

\[
\frac{1}{X^2} = S^2 \left( \frac{\rho_g}{\rho_f} \right) \left( \frac{\alpha}{1-\alpha} \right)^2 \left( \frac{\text{Re}_f}{\text{Re}_g} \right)^{1/4} = \left[ S \left( \frac{\alpha}{1-\alpha} \right) \right]^{7/4} \left[ \frac{\rho_g}{\rho_f} \right]^{3/4} \left( \frac{\mu_g}{\mu_f} \right)^{1/4}
\]  

\( (3-45) \)

\[
\frac{1}{X_M^2} = \left( \frac{D}{4L} \right) \left( \frac{k}{f} \right) = 3.165 \left( \frac{k}{L/D} \right) \text{Re}_f^{0.25}
\]  

\( (3-46) \)

where:

\[
S = \text{slip ratio} = \frac{u_g}{u_f}
\]  

\( (3-47) \)

By defining the following non-dimensional numbers:

\[
L^* = (L/D)_{\text{restriction}} \text{ : non-dimensional restriction length}
\]  

\( (3-48) \)

\[
\rho^* = \frac{\rho_g}{\rho_f} \text{ : non-dimensional density}
\]  

\( (3-49) \)

\[
\mu^* = \frac{\mu_g}{\mu_f} \text{ : non-dimensional viscosity}
\]  

\( (3-50) \)

Equation (3-45) reduces to:

\[
\frac{1}{X^2} = S^2 \rho^* \left( \frac{\alpha}{1-\alpha} \right)^2 \left( \frac{\text{Re}_f}{\text{Re}_g} \right)^{1/4} = \left[ \rho^* \mu^* S^7 \left( \frac{\alpha}{1-\alpha} \right) \right]^{1/4}
\]  

\( (3-51) \)

and when the slip ratio is close to unity, as in general for horizontal bubbly flow, Eq. (3-51) further reduces to:

\[
\frac{1}{X^2} \approx \rho^* \left( \frac{\alpha}{1-\alpha} \right)^2 \left( \frac{\text{Re}_f}{\text{Re}_g} \right)^{1/4} = \left[ \rho^* \mu^* \left( \frac{\alpha}{1-\alpha} \right) \right]^{7/4} \text{ when } S \approx 1
\]  

\( (3-52) \)
Thus, when the slip ratio can be approximated as unity, Equation (3-43) can be written in terms of void fraction and other non-dimensional numbers as:

\[
\phi_{jM}^2 = 1 + C \left[ \rho^* \mu^* \left( \frac{\alpha}{1-\alpha} \right)^{7/4} \left[ 1 + \frac{3.165k}{L} \Re_{f}^{0.25} \right]^{1/2} \right] + \left[ \rho^* \mu^* \left( \frac{\alpha}{1-\alpha} \right)^{7/4} \right] + \frac{3.165k}{L} \Re_{f}^{0.25} \text{ when } S \equiv l
\]  

(3-53)

In summary, the following conclusions can be made based on the present results:

(1) The geometric effect of the 90-degree and 45-degree horizontal elbows is clearly shown in the experimental data. Additional pressure loss due to the minor loss across the elbows is clearly characterized by the steeper slope in the plot of pressure with respect to development length. It is also shown that the pressure drop increases with increasing gas and liquid flow rates.

(2) In the present test facility, the effect of the 90-degree elbow on pressure drop is found to be more pronounced further downstream of the elbow than immediately downstream of the elbow.

(3) The pressure drop across the entire test section matches well with the existing Lockhart-Martinelli correlation with \( C=25 \) and \( C=40 \) for the 90-degree and 45-degree data, respectively. The higher values of \( C \) compared to the conventional value of \( C=20 \) is presumed to be due to the remaining effect of the elbows. It is evident from Figs. 2.18 and 2.19 in the previous chapter that the slope in pressure drop (i.e., \( dp/dz \)) in all flow conditions is not fully recovered to their initial slope before their respective elbows. This implies that the flow is yet to be fully recovered from the elbow effect even after 122.4 or 65.5 diameters downstream of the elbows, for the 90-degree and 45-degree experiments respectively.
(4) The newly developed correlation, given by Eq. (3-43) matches well with the 90-degree experimental data when the minor loss factor \(k=0.58\) and \(C=65\) are employed. The correlation also matches well with the 45-degree data when the minor loss factor \(k=0.35\) and \(C=65\) are employed. The significantly higher value of \(C=65\) compared to the conventional value of \(C=20\) indicates the geometric effect of the elbows on two-phase frictional pressure loss. The minor loss factors, \(k=0.58\) and \(k=0.35\), for the 90-degree and 45-degree elbows respectively, are approximately 3 times higher than the conventional \(k\)-factors, \(k=0.20\) and \(k=0.13\) recommended for single-phase flow. These values signify the additional pressure loss due to the two-phase flow around the elbow. The present correlation matches very well with the data with an average percent difference of \(\pm 2\%\).

(5) The current model needs to be verified by additional experimental data. The present model coefficients for the 90-degree data are developed based on the pressure measurements across ports P1 and P3, instead of ports P1 and P2 because the most significant pressure loss occurs across ports P1 and P3. Port P3 is located far downstream of the elbow and may not reflect the actual pressure drop at the elbow. The model coefficients for the 45-degree data are also developed based on the pressure measurements across ports P5 and P6, where the most significant pressure loss occurs.

### 3.4 Drift Flux Model for Horizontal Bubbly Flow

In order to solve the one-dimensional interfacial area transport equation, constitutive relations are needed for \(\langle< v_g \rangle \rangle\) and \(\langle< \alpha \rangle \rangle\). The present study employs a simple approach by using the conventional drift flux model. It should be noted, however, that the use of the drift flux model is not conceptually accurate for flow with acceleration. In fact, a predictive model for \(\langle< v_g \rangle \rangle\) in the presence of a flow restriction (hence flow acceleration/deceleration) would be important to complete
the model for the present test condition. Nevertheless, the conventional drift flux model does a good
job in estimating the bubble velocity change when employed for each section of the flow pipe.

The drift flux model is given by (Zuber and Findley, 1965):

\[
\frac{j_g}{\alpha} = \langle v_g \rangle = C_o \langle j \rangle + \langle v_g \rangle
\]  

(3-54)

where \( C_o \) is the distribution parameter defined by:

\[
C_o \equiv \frac{\langle \alpha j \rangle}{\langle \alpha \rangle \langle j \rangle}
\]  

(3-55)

and \( \langle v_g \rangle \) is the void-weighted drift velocity defined by:

\[
\langle v_g \rangle \equiv \frac{\langle \alpha (v_g - j) \rangle}{\langle \alpha \rangle}
\]  

(3-56)

The values of \( C_o \) and \( \langle v_g \rangle \) can be determined by plotting the experimental \( \langle v_g \rangle \) versus \( \langle j \rangle \) for all flow conditions, and by determining the best-fit line through the experimental data points. This approach assumes that \( C_o \) and \( \langle v_g \rangle \) are relatively constant for all flow conditions, which is generally acceptable in vertical flow. However, if the bubble distribution is highly non-uniform as in the present study, the use of the drift-flux model can be erroneous, and non-physical \( \langle v_g \rangle \) values can be obtained. Mathematically, this can be better understood by noting that if \( C_o \) and \( \langle v_g \rangle \) increase constantly with increasing \( \langle j \rangle \), then a higher \( C_o \) will be acquired from the best-fit line and a much smaller value can be acquired for \( \langle v_g \rangle \).

If the change in acceleration is negligible, single values for \( C_o \) and \( \langle v_g \rangle \) can be used to predict \( \langle v_g \rangle \) along the z-direction. However, due to the significant change in velocity at the 90-degree and 45-degree elbows, the void-weighted gas velocities at ports P2 through P7 are estimated by finding different \( C_o \) and \( \langle v_g \rangle \) values for each port such that:
\[
\langle \langle v_g \rangle \rangle_i = (C_o)_i \langle j \rangle_i + \langle \langle V_{g} \rangle \rangle_i 
\]

(3-57)

where \(i\) is varied for integer values 2 through 7.

The \(C_o\) and \(\langle \langle V_{g} \rangle \rangle\) values used to estimate the \(\langle v_g \rangle\) at ports P2 through P7 are obtained by plotting \(\langle v_g \rangle\) versus \(j\) for each port. The \(\langle v_g \rangle\) versus \(j\) plots for each port are shown in Figs. 3.3 and 3.4 for the 90-degree and 45-degree experiments, respectively. The values for \(C_o\) and \(\langle \langle V_{g} \rangle \rangle\) at each port are summarized in Table 3.5. As the void-weighted gas velocity at each port is estimated by Eq. (3-57), the \(\langle v_g \rangle\) along the test section is linearly interpolated by:

\[
\langle \langle v_g \rangle \rangle = \langle \langle v_g \rangle \rangle_i + \left( \frac{\langle \langle v_g \rangle \rangle_{i+1} - \langle \langle v_g \rangle \rangle_i}{z_{i+1} - z_i} \right) (z - z_i) 
\]

(3-58)

where \(i\) is varied for integer values 2 through 7, and \(\langle v_g \rangle\) is applicable in the region between Port \(i\) and \(i+1\).

Some characteristic results of \(\langle v_g \rangle\) prediction made by the drift flux model described above are shown in Figs. 3.5 and 3.6 for the 90-degree and 45-degree experiments, respectively. For both the 90-degree and 45-degree experiments, the average error of the \(\langle v_g \rangle\) prediction made by the drift flux model is ±4%. The comprehensive results of \(\langle v_g \rangle\) prediction are shown in Appendices A.3 and A.4 for the 90-degree and 45-degree experiments, respectively.
Table 3-4: Summary of distribution parameters, $C_0$, and void-weighted drift velocities, $\langle\langle V_{gj} \rangle\rangle$, for each measurement port.

<table>
<thead>
<tr>
<th>Port</th>
<th>$C_0$</th>
<th>$\langle\langle V_{gj} \rangle\rangle$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Port 1</td>
<td>1.99</td>
<td>-4.38</td>
</tr>
<tr>
<td>Port 2</td>
<td>1.23</td>
<td>-0.54</td>
</tr>
<tr>
<td>Port 3</td>
<td>1.62</td>
<td>-2.1</td>
</tr>
<tr>
<td>Port 4</td>
<td>1.91</td>
<td>-3.88</td>
</tr>
<tr>
<td>Port 5</td>
<td>1.97</td>
<td>-4.36</td>
</tr>
<tr>
<td>Port 6</td>
<td>1.19</td>
<td>-0.79</td>
</tr>
<tr>
<td>Port 7</td>
<td>1.67</td>
<td>-2.66</td>
</tr>
</tbody>
</table>
Figure 3-3: Determination of $C_0$ and $\langle \langle V_g \rangle \rangle$ for each 90-degree measurement port. Error bars shown: ±5%.
Figure 3-4: Determination of $C_0$ and $<\langle V_{gj} \rangle>$ for each 45-degree measurement port. Error bars shown: ±5%.
Figure 3-5: $\langle v_g \rangle$ predicted by the drift-flux model versus the 90-degree experimental data for Runs 5 ($j_{g,atm}=0.32$ m/s and $j_f=4.05$ m/s), Run 8 ($j_{g,atm}=0.66$ m/s and $j_f=4.05$ m/s), Run 11 ($j_{g,atm}=1.00$ m/s and $j_f=4.05$ m/s) and Run 14 ($j_{g,atm}=1.37$ m/s and $j_f=4.05$ m/s). Error bars shown: ±5%.
Figure 3-6: $\langle v_g \rangle$ predicted by the drift-flux model versus the 45-degree experimental data for Runs 5 ($j_{g,atm}$=0.36 m/s and $j_f$=4.05 m/s), Run 8 ($j_{g,atm}$=0.74 m/s and $j_f$=4.05 m/s), Run 11 ($j_{g,atm}$=1.13 m/s and $j_f$=4.04 m/s) and Run 14 ($j_{g,atm}$=1.58 m/s and $j_f$=4.04 m/s). Error bars shown: ±5%.
Chapter 4

Model Evaluation

The steady-state one-group interfacial area transport equation is evaluated with experimental data acquired from the adiabatic horizontal tube with a 90-degree and 45-degree elbow installed. Two models are created, one for the 90-degree experiment, and the other for the 45-degree experiment. The models are evaluated in one-dimensional form by taking the area-average of the local data acquired at different axial locations.

4.1 Steady-State One-Dimensional One-Group Interfacial Area Transport Equation

The one-dimensional interfacial area transport equation applicable to the present flow configuration is obtained by area-averaging the three-dimensional equation over the cross-sectional area. The area-averaging of each term in the equation is described in Section 3.2. Thus, the steady-state one-dimensional one-group interfacial area transport equation applicable to the present test conditions is given by:

\[
\frac{d}{dz} \langle \langle v_s \rangle \rangle = -\langle a_i \rangle \frac{d}{dz} \langle \langle v_s \rangle \rangle + \left[ \frac{2}{3} \frac{\langle a_i \rangle \langle \langle v_s \rangle \rangle^2}{\langle p \rangle} \right] \left( -\frac{d}{dz} \langle p \rangle \right) + C_{TT} COV_{TT} \frac{1}{18} \left( \frac{\langle a_i \rangle}{\langle \alpha \rangle} \right)^2 \langle u_i \rangle \sqrt{1 - \frac{We_{cr}}{\langle We \rangle}} \exp \left( -\frac{We_{cr}}{\langle We \rangle} \right) - C_{RC} COV_{RC} \frac{1}{3\pi} \frac{\langle a_i \rangle^2}{\langle \alpha \rangle^{1/3}} \langle u_i \rangle \left[ 1 - \exp \left( -C \frac{\langle \alpha \rangle^{1/3}}{\langle \alpha \rangle_{max}^{1/3} - \langle \alpha \rangle^{1/3}} \right) \right]
\] (4-1)
The source and sink terms in the RHS of equation are sink/source due to the bubble velocity gradient (VG) along the flow direction, source/sink due to pressure drop (PD), source due to bubble break-up by turbulence impact (TI), and sink due to bubble coalescence via random collision (RC).

Given that the model is evaluated against experimental data, the local parameters measured by the local conductivity probe are averaged over the cross-sectional area of the tube. The area-averaging schemes and measurement meshes used to obtain the experimental area-averaged two-phase flow parameters are described in Section 2.1.

### 4.2 Model Evaluation Scheme

In solving the interfacial area transport equation given by Eq. (4-1), the terms below are calculated. In each case, the first experimental value is used as an initial condition and all downstream locations are determined based on the following:

- **Pressure Estimation:** The 90-degree data shows that the effect of the elbow becomes more pronounced in the region further downstream of the elbow, between ports P2 and P3. After port P3, the effect of the elbow diminishes, and the pressure drop can be assumed to be mainly due to wall friction. Therefore, two correlations are used to predict the pressure drop along the tube. First, the correlation developed in Section 3.3 for the two-phase frictional loss with an elbow, given by Eq. (3-42), is employed for pressure estimation between ports P1 through P3. The second correlation is the conventional Lockhart-Martinelli two-phase frictional pressure drop given by Eq. (3-32). This correlation is employed for the pressure estimation between ports P3 and P4. A similar approach is used in estimating the pressure for the 45-degree experiment. The correlation given by Eq. (3-42) is employed between ports P5 and P6 while Eq. (3-32) is employed between ports P6 and P7. The full set of pressure predictions are given in Appendices.
A.1 and A.2 for the 90-degree and 45-degree experiments, respectively with an average error of ±2%.

- **Void-weighted Bubble Velocity Estimation:** Another parameter to be estimated is the void-weighted gas velocity, \(<<v_g>>\). As discussed in detail in Section 3.4, \(<<v_g>>\) is predicted by the drift flux model. The full set of void-weighted gas velocity predictions are given in Appendices A.3 and A.4 for the 90-degree and 45-degree experiments, respectively with an average error of ±4%.

- **Void Fraction Estimation:** Estimation of void fraction is relatively simple and is based on the definition of void-weighted bubble velocity given by:

  \[
  \langle \alpha \rangle = \frac{\langle j_g \rangle}{\langle v_g \rangle}
  \]  

  (4-2)

  Hence, after the local gas superficial velocity and the void-weighted gas velocity are estimated via local pressure and the drift flux model, respectively, the area-averaged void fraction along the z-direction is calculated by Eq. (4-2). The full set of void fraction predictions are given in Appendices A.5 and A.6 for the 90-degree and 45-degree experiments, respectively with an average error of ±7%.

  It should be noted that since the experimental void fraction value at port P1 does not always exactly match the value calculated by Eq. 4-2, there exists a jump at the first calculated position downstream of port P1. This jump can be due to error in the pressure prediction, error in the drift flux model prediction for void-weighted gas velocity, or measurement error of the probe.

- **Interfacial Area Concentration Estimation:** A finite forward difference method is employed in the numerical calculation of Eq. (4-1), such that \(a_i\) is calculated by:
\[
\langle a_i \rangle_{i+1} = \langle a_i \rangle_i + \frac{z_{\text{step}}}{\langle \phi_{\text{VG}} \rangle_i + \langle \phi_{\text{PD}} \rangle_i + \langle \phi_{\text{TI}} \rangle_i + \langle \phi_{\text{RC}} \rangle_i}
\]

(4-3)

where the \( i \) subscript represents the value of the parameter at the current step, \( z_{\text{step}} \) represents the step size along \( z \), and \( \langle \phi \rangle \) represent various area-averaged source and sink terms. A schematic diagram of the numerical solution procedure is illustrated in Figure 4.1. Parameters such as the tube diameter, tube length, and fluid properties are input as experimental conditions. Other parameters such as pressure, superficial gas velocity, and superficial liquid velocity measured at port P1 are used as initial values for the constitutive relations. The void-weighted gas velocity, area-averaged void fraction and area-averaged interfacial area concentration measured by the local conductivity probe at port P1 are also used as initial values for the drift flux model and the interfacial area transport equation, respectively. The covariance of the turbulent impact (TI) and random collision (RC) terms at ports P1 through P4 for the 90-degree model and at ports P5 through P7 for the 45-degree model are input as the covariance coefficients. The covariance coefficients are then linearly interpolated along the entire test section. This is because in all the flow conditions of the present study the bubble distribution pattern changes significantly along the flow direction due to flow restrictions, which results in changes in the covariance values. In the present study, MATLAB is employed to perform the numerical calculations. The complete MATLAB programs used for model evaluation are given in Appendices A.7 and A.8 for the 90-degree and 45-degree cases, respectively. The full set of interfacial area concentration predictions, which will be discussed in Section 4.3, are given in Appendices A.9 and A.10 for the 90-degree and 45-degree experiments, respectively with an average error of ±20%.
Figure 4-1: Schematic diagram of the numerical solution procedure.
4.3 Evaluation Results and Discussion

In evaluating the model, the coefficients specific to the TI and RC mechanisms need to be determined, which include: $C_{TI}$, $We_{cr}$, $C_{RC}$, $\alpha_{max}$, and $C$. Here, $\alpha_{max}$ and $C$ denote the maximum packing limit and the “distance” within which turbulent eddies are effective for bubble collision, respectively. Since these coefficients are not sensitive to the geometry or orientation, the values used in vertical bubbly flow are employed, such that: $\alpha_{max}=0.75$ and $C=3$.

In previous studies (Ishii et. al., 2002, Kim et. al., 2002, and Ishii et. al., 2004) the values for $We_{cr}$ were found to be 6 and 8 for round pipe and rectangular channels, respectively. Hence, in the present study the $We_{cr}$ value is varied between 6 and 8 in flow conditions with small RC effect to find the best fit to the data. As a result, it was found that a $We_{cr}$ of 8 yields the best prediction results. Therefore, the present model uses a critical Weber number of $We_{cr}=8$ throughout the calculations.

In determining the remaining coefficients (i.e.; $C_{TI}$ and $C_{RC}$), the dominant bubble interaction mechanism is identified based on the flow conditions. For example, in Runs 1 through 3 of the 90-degree experiment, where the $We$ is smaller (or only slightly larger) than the $We_{cr}$, there is no (or minimal) contribution from the TI mechanism in the $<a_i>$ change. Therefore, the random collision coefficient, $C_{RC}$, is adjusted to provide the best fit for these conditions. Then the TI coefficient, $C_{TI}$, is adjusted until the value that yields the best agreement between the experimental data and the model is obtained. Fine adjustments of these coefficients are made based on all the available experimental data. In the present study the determination of the coefficients $C_{TI}$ and $C_{RC}$ are based primarily on the 90-degree data. This is because the differences in $\alpha$ and $a_i$ distributions after the elbow are accounted for by the covariance terms, and the velocity information of the 45-degree experiment is thought to be less reliable due to the larger local peaking phenomena. A summary of the coefficients for the turbulent impact and random collision terms is shown in Table 4.1.
In comparing the coefficients of the present study with those determined in vertical round pipe flows, it is found that $C_{TI}$ is 17 times smaller and $C_{RC}$ is 2 times smaller than those determined in vertical round pipe flows. However, considering the covariance, this does not necessarily mean that the TI and RC mechanisms are less active in the present study. In fact, the covariance for RC mechanism typically ranges from 2 to 30, and the product of $C_{RC}$ and $COV_{RC}$ can be as high as 15 times that of $C_{RC}$ in vertical flow. Similarly for the TI mechanism, the covariance typically varies between 1 and 3, and the product of $C_{TI}$ and $COV_{TI}$ is 6 to 17 times less than that of $C_{TI}$ in vertical flow. Hence, it can be concluded that in the present test conditions, the RC mechanism is promoted while TI mechanism is reduced, which is believed to be due to the flow configuration and turbulence structure. It is speculated that in the present test conditions, the agglomeration of bubbles toward the tube wall due to the channel orientation and flow restrictions promotes the random collision interaction mechanism. On the other hand, the TI mechanism is presumed to be reduced by large eddies created by the elbows that results in a flow oscillation along the flow direction. Hence, it is speculated that bubbles are swept by the large eddies instead of being disintegrated. A similar phenomenon was also observed in the vertical co-current downward flow, where large eddies in the core resulted in reduction in the TI break-up mechanism (Ishii et. al., 2004).

The comprehensive evaluation results, categorized by the superficial liquid velocity, are shown in Figs. 4.2 and 4.3 for the 90-degree and 45-degree experiments, respectively. As shown in the figures, agreement between the model prediction and the experimental data is generally good within ±20% difference. For the significant deviation in Run 6 of the 90-degree experiment prediction, however, it is believed that the experimental data is erroneous and as such this run is ignored in the model evaluation. In the higher void fraction conditions, the 45-degree model over predicts the interfacial area concentration, which is consistent with the presence of group two bubbles implied by the local data. The error in the predicted area-averaged interfacial area concentration at each port for the 90-degree and 45-degree experiments is shown in Tables 4.2 and 4.3, respectively.
In general, the 90-degree model over predicts the data acquired at the last measurement port (P4) under higher gas flow rates (i.e.: Runs 12, 13, 14 and 15). This is believed to be due to the appearance of two-group bubbles in such flow conditions. Since the current model accounts only for one-group bubble transport, and the data is acquired by the double-sensor conductivity probe, the model will over predict the data when group two bubbles exist. In fact, examining the local data in such flow conditions, it is evident that group two bubbles are present.

Table 4-1: Summary of Model Coefficients.

<table>
<thead>
<tr>
<th></th>
<th>Horizontal with 90° or 45° elbow</th>
<th>Vertical Upward in Round Pipes</th>
<th>Vertical Co-current Downward in Round Pipes</th>
<th>Vertical Upward in Rectangular Channel</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{RC}$</td>
<td>0.002</td>
<td>0.004</td>
<td>0.004</td>
<td>0.003</td>
</tr>
<tr>
<td>$C_{TI}$</td>
<td>0.005</td>
<td>0.085</td>
<td>0.034</td>
<td>0.026</td>
</tr>
<tr>
<td>$We_{cr}$</td>
<td>8</td>
<td>6</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>$C$</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>$\alpha_{max}$</td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
</tr>
</tbody>
</table>
Table 4-2: 90-degree model absolute percent errors in predicted $\langle a_i \rangle$

<table>
<thead>
<tr>
<th>Run Number</th>
<th>Port P1</th>
<th>Port P2</th>
<th>Port P3</th>
<th>Port P4</th>
<th>Run Avg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.76</td>
<td>0.40</td>
<td>10.51</td>
<td>5.89</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.99</td>
<td>11.85</td>
<td>5.01</td>
<td>5.95</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2.94</td>
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<td>0.83</td>
<td>3.49</td>
<td></td>
</tr>
<tr>
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<td>12.49</td>
<td>11.86</td>
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<tr>
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<td>5.67</td>
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<td></td>
</tr>
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<tr>
<td>7</td>
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<td>18.97</td>
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<tr>
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<td>11.86</td>
<td>2.46</td>
<td>7.53</td>
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<td>13.17</td>
<td>7.69</td>
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<tr>
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<td>25.11</td>
<td>1.90</td>
<td>37.28</td>
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</tr>
<tr>
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<td>18.59</td>
<td>3.39</td>
<td>7.50</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>5.08</td>
<td>4.35</td>
<td>10.68</td>
<td>6.70</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>30.32</td>
<td>7.92</td>
<td>54.48</td>
<td>30.91</td>
<td></td>
</tr>
<tr>
<td>Port Avg.</td>
<td></td>
<td>8.18</td>
<td>6.84</td>
<td>14.67</td>
<td></td>
</tr>
<tr>
<td>Total Avg.</td>
<td></td>
<td>9.90</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4-3: 45-degree model absolute percent errors in predicted $\langle a_i \rangle$

<table>
<thead>
<tr>
<th>Run Number</th>
<th>Port P5</th>
<th>Port P6</th>
<th>Port P7</th>
<th>Run Avg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>15.15</td>
<td>38.25</td>
<td>26.70</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>6.43</td>
<td>7.45</td>
<td>6.94</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>10.24</td>
<td>10.28</td>
<td>10.26</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>9.59</td>
<td>70.45</td>
<td>40.02</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>36.90</td>
<td>81.14</td>
<td>59.02</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>58.35</td>
<td>41.94</td>
<td>50.15</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>8.42</td>
<td>61.13</td>
<td>34.77</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>0.84</td>
<td>100.94</td>
<td>50.89</td>
</tr>
<tr>
<td>9</td>
<td></td>
<td>7.34</td>
<td>18.03</td>
<td>12.68</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>30.73</td>
<td>29.66</td>
<td>30.20</td>
</tr>
<tr>
<td>11</td>
<td></td>
<td>30.73</td>
<td>29.66</td>
<td>30.20</td>
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<tr>
<td>12</td>
<td></td>
<td>9.35</td>
<td>63.33</td>
<td>36.34</td>
</tr>
<tr>
<td>13</td>
<td></td>
<td>15.86</td>
<td>39.56</td>
<td>27.71</td>
</tr>
<tr>
<td>14</td>
<td></td>
<td>19.25</td>
<td>15.94</td>
<td>17.59</td>
</tr>
<tr>
<td>15</td>
<td></td>
<td>30.64</td>
<td>16.48</td>
<td>23.56</td>
</tr>
<tr>
<td>Port Avg.</td>
<td></td>
<td>19.32</td>
<td>41.62</td>
<td></td>
</tr>
<tr>
<td>Total Avg.</td>
<td></td>
<td>30.47</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
In view of studying the contributions from individual mechanisms, the contribution from each source and sink of \(<a_i>\) is also examined. The characteristic results of the 90-degree experiment for lower void fraction conditions (Runs 1 and 3) are plotted in Fig. 4.4, while results for higher void fraction conditions (Runs 8 and 14) are plotted in Fig. 4.5. Similarly the characteristic results of the 45-degree experiment for lower void fraction conditions (Runs 2 and 3) are plotted in Fig. 4.6, while the results for higher void fraction conditions (Runs 9 and 13) and shown in Fig. 4.7.

As can be seen in Fig. 4.4, the contribution from the bubble velocity gradient (VG) is a dominant sink of \(<a_i>\). Furthermore, in low void fraction conditions, the TI mechanism plays no role until downstream near port P4. This is because the turbulent velocity is not high enough to reach the critical Weber number in that flow condition. For low void fraction flows, it can also be seen that the contribution from the RC mechanism is generally comparable to or larger than the TI contribution.

The contributions from each source and sink of \(<a_i>\) for the intermediate and high void fraction flow conditions of the 90-degree experiment (Run 8 and Run 14) are also shown in Figure 4.4. Different from the low void fraction conditions, the RC and TI mechanisms become the dominant sink and source mechanisms in \(<a_i>\) transport for these flow conditions.

In general, for the 90-degree experiment, the VG and RC are major sink mechanism for lower void fraction conditions, while PD is the major source mechanism. As the void fraction increases, the RC becomes the major sink mechanism while TI and PD are relatively equal as the dominant source mechanisms. The comprehensive evaluation results and the contributions from each mechanism for all the 90-degree flow conditions are shown in Appendix A.9.

As shown in Fig. 4.6, the contribution from the bubble velocity gradient (VG) is a dominant sink of \(<a_i>\). Furthermore, in low void fraction conditions of the 45-degree experiment, the TI mechanism and PD mechanism are nearly identical source terms. The contributions from each source and sink of \(<a_i>\) for the intermediate and high void fraction flow conditions of the 45-degree
experiment (Run 9 and Run 13) are also shown in Figure 4.7. Different from the low void fraction conditions, the RC mechanism becomes the dominant sink in $<a_i>$ while PD becomes the major source mechanism in higher void fraction flow conditions.

In general, for the 45-degree experiment, VG and RC are the major sink mechanisms for lower void fraction conditions, while PD and TI are the major source mechanisms. As the void fraction increases, the PD and RC mechanisms become the major source and sink. The comprehensive evaluation results and the contributions from each mechanism for all the 45-degree flow conditions are shown in Appendix A.10.
Figure 4-2: 90-degree model evaluation results. Error bars shown: ±20%.
Figure 4-3: 45-degree model evaluation results. Error bars shown: ±20%.
Figure 4-4: 90-degree experiment: Contribution from each interaction mechanism for low void fraction conditions (a) $j_{g,atm} = 0.12 \text{ m/s}$ and $j_f = 3.76 \text{ m/s}$ (Run 1) (b) $j_{g,atm} = 0.13 \text{ m/s}$ and $j_f = 4.34 \text{ m/s}$ and (Run 3). Error bars shown: ±20%.

Figure 4-5: 90-degree experiment: Contribution from each interaction mechanism for high void fraction conditions (a) $j_{g,atm} = 0.66 \text{ m/s}$ and $j_f = 4.05 \text{ m/s}$ (Run 8) (b) $j_{g,atm} = 1.37 \text{ m/s}$ and $j_f = 4.05 \text{ m/s}$ and (Run 14). Error bars shown: ±20%.
Figure 4-6: 45-degree experiment: Contribution from each interaction mechanism for low void fraction conditions (a) $j_{g,atm} = 0.14 \text{ m/s}$ and $j_f = 4.04 \text{ m/s}$ (Run 2) (b) $j_{g,atm} = 0.15 \text{ m/s}$ and $j_f = 4.34 \text{ m/s}$ and (Run 3). Error bars shown: ±20%.

Figure 4-7: 45-degree experiment: Contribution from each interaction mechanism for high void fraction conditions (a) $j_{g,atm} = 0.76 \text{ m/s}$ and $j_f = 4.34 \text{ m/s}$ (Run 9) (b) $j_{g,atm} = 1.58 \text{ m/s}$ and $j_f = 4.04 \text{ m/s}$ and (Run 14). Error bars shown: ±20%.
Chapter 5

Summary and Recommendations

The one-dimensional interfacial area transport equation applicable to steady-state horizontal bubbly flow with a 90-degree or 45-degree horizontal elbow is developed. In the course of developing the theoretical model, the following characteristics are analyzed:

- Development of local two-phase flow structures and their transport characteristics
- Transport of one-dimensional two-phase flow parameters
- Pressure drop characteristics due to the existence of a 90-degree or 45-degree elbow
- Bubble velocity gradient along the flow direction
- Covariance in area-averaging due to the non-uniform bubble distribution

The geometric effects of a 90-degree or 45-degree elbow on the development of the flow structure is clearly demonstrated in the profiles of the two-phase flow parameters. Additional pressure loss resulting from the minor loss across the elbow is characterized by a steeper slope in the graph of pressure loss versus development length. It is interesting to note that the effect of the 90-degree elbow is more pronounced further downstream of the elbow than immediately downstream. It is also shown that the pressure drop increases with increasing gas and liquid flow rates. A predictive model for two-phase frictional loss that accounts for the both the wall friction and minor loss across the elbows is developed. The pressure predicted by the model agrees well with the data with an average error of ±2%.

In general, the bubble velocity increases across the elbow for lower $j_{g,atm}$ ($j_{g,atm} < 1.0 \text{ m/s}$) while it decreases for higher $j_{g,atm}$ ($j_{g,atm} = 1.0 \text{ m/s}, 1.4 \text{ m/s}$) conditions. Such a gradient in the bubble velocity
along the flow direction results in large oscillations downstream of the elbow and provides an
additional source/sink for the interfacial area concentration. A simple approach employing the
conventional drift flux model is employed in predicting the bubble velocity along the flow field.
While it is noted that the use of the drift flux model for accelerating flow may not be conceptually
consistent, the predictions made by this approach are acceptable with an average error of ±4%. To
develop a practical one-dimensional equation, an area-averaging method is employed. In the process,
covariance arising due to the non-uniform distribution of the two-phase flow parameters across the
flow cross-section is carefully evaluated. As a result, it is found that the covariance for RC typically
varies in the range between 2 and 30, while that for TI typically varies between 1 and 3. Inclusion of
the covariance in the one-dimensional source and sink mechanism accounts for the effect of non-
uniform distribution in the bubble interaction mechanisms.

The model coefficients are determined based primarily on the 90-degree flow conditions as well
as the coefficients developed earlier for different flow configuration. In view of interfacial area
transport, it is found that the elbows promote bubble coalescence while they diminish the
disintegration mechanisms. The increase in bubble coalescence is believed to be due to the
agglomeration of bubbles attributed to both the channel orientation and the elbows. The reduction in
bubble disintegrations, on the other hand, is believed to be due to the large scale eddies generated by
the elbows, resulting in the sweeping of bubbles instead of breaking them. Such phenomenon is also
observed in the vertical co-current downward two-phase flow configuration.

The one-dimensional interfacial area transport equation developed for the adiabatic air-water
horizontal bubbly flow with a 90-degree or 45-degree horizontal elbow restriction is evaluated against
the data acquired by the double-sensor conductivity probe. In total, 60 area-averaged data points
obtained in 15 different flow conditions for each of the two experiments are used to evaluate the
model. In the 90-degree experiment the local data is acquired at four different axial locations while in
the 45-degree experiment three different axial locations are used. Contributions from the individual
source and sink mechanisms to the total change in interfacial area concentration are also studied. In general, it is found that the VG, RC and PD are the dominant mechanisms in low void fraction flow conditions, whereas the TI, PD and RC become dominant as the void fraction increases. Overall, the model predicts the data well with an average error of ±20%. Accounting for the limitations in employing a one-dimensional approach for the present flow configuration, the present model predicts the data relatively well.

Based on the current study, the following areas are identified for future improvement:

- The current model is developed for horizontal bubbly flow with a 90-degree or 45-degree flow restriction. However, there is no model available for normal horizontal bubbly flow without a restriction. To establish a more reliable model, it is essential to develop a model for a normal horizontal two-phase flow configuration. Such a model will serve as a reference for interfacial area transport in horizontal two-phase flow and highlight the effect of the flow restrictions. Therefore, it is recommended that a more extensive database in a normal horizontal two-phase flow configuration, as well as additional databases in horizontal two-phase flow with various flow restrictions, be established in the future.

- It is evident that group two bubbles start to appear in some of the present test conditions. Therefore, expanding the database of data acquired in purely bubbly flow conditions will be needed to enhance the reliability of the model.

- It is evident from the data that in nearly all test conditions the flow after an elbow does not fully recover to its original condition. The local data clearly shows flow oscillations in both the vertical and horizontal directions across the tube cross-section. The degree of flow oscillation caused by the elbows may play a major role in distributing bubbles across the tube cross-section and hence be important in bubble interaction mechanisms. Additional
experiments with a longer development length after the flow restrictions can provide a database to successfully quantify both the degree of oscillation and the bubble distribution.

- Even though the present method employing the drift flux model predicts the change in bubble velocity relatively well, the drift flux model is developed for a steady-state fully developed two-phase flow. A predictive model other than the drift flux model to estimate the change in bubble velocity across a flow restriction is needed for a more reliable model.

- In view of a one-dimensional approach, it is important to establish a methodology to estimate the covariance. The bubble distribution in horizontal pipe is inherently non-uniform and the covariance stemming from the area-averaging process plays a major role in determining contributions from various source and sink mechanisms. A predictive model that qualitatively estimates the bubble distribution across the tube cross-section, and hence the covariance, is essential in the one-dimensional interfacial area transport equation specific to the horizontal configuration.

- The present interfacial area transport equation relies on the source and sink mechanisms identified for vertical two-phase flow systems. While the existing source and sink phenomena are mechanistically modeled, there may exist additional mechanisms that play a major role in the evolution of the interfacial area concentration for horizontal systems. A detailed flow visualization study is recommended to evaluate whether there exists other bubble interaction mechanisms to consider specific to the present flow configuration.
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Fu, X. Y., 2001, Interfacial area measurement and transport modeling in air-water two-phase flow, *Ph. D. Thesis*, School of Nuclear Engineering, Purdue University.

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Kim, S., 1999, Interfacial area transport equation and measurement of local interfacial characteristics, PhD. Thesis, Purdue University, West Lafayette, IN, USA.


Sun, X., 2001, Two-group interfacial area transport equation for a confined test section, PhD. Thesis, Purdue University, West Lafayette, IN, USA.


Appendix A1

Pressure Prediction along the Axial Direction (90-deg)
Figure A1-1: Pressure Prediction along the Axial Direction for Run 1. Error bars shown: ±2%.

Figure A1-2: Pressure Prediction along the Axial Direction for Run 2. Error bars shown: ±2%.
Figure A1-3: Pressure Prediction along the Axial Direction for Run 3. Error bars shown: ±2%.

Figure A1-4: Pressure Prediction along the Axial Direction for Run 4. Error bars shown: ±2%.
Figure A1-5: Pressure Prediction along the Axial Direction for Run 5. Error bars shown: ±2%.

Figure A1-6: Pressure Prediction along the Axial Direction for Run 6. Error bars shown: ±2%.
Figure A1-7: Pressure Prediction along the Axial Direction for Run 7. Error bars shown: ±2%.

Figure A1-8: Pressure Prediction along the Axial Direction for Run 8. Error bars shown: ±2%.
Figure A1-9: Pressure Prediction along the Axial Direction for Run 9. Error bars shown: ±2%.

Figure A1-10: Pressure Prediction along the Axial Direction for Run 10. Error bars shown: ±2%.
Figure A1-11: Pressure Prediction along the Axial Direction for Run 11. Error bars shown: ±2%.

Figure A1-12: Pressure Prediction along the Axial Direction for Run 12. Error bars shown: ±2%.
Figure A1-13: Pressure Prediction along the Axial Direction for Run 13. Error bars shown: ±2%.

Figure A1-14: Pressure Prediction along the Axial Direction for Run 14. Error bars shown: ±2%. 
Figure A1-15: Pressure Prediction along the Axial Direction for Run 15. Error bars shown: ±2%.
Appendix A2

Pressure Prediction along the Axial Direction (45-deg)
Figure A2-1: Pressure Prediction along the Axial Direction for Run 1. Error bars shown: ±2%.

Figure A2-2: Pressure Prediction along the Axial Direction for Run 2. Error bars shown: ±2%.
Figure A2-3: Pressure Prediction along the Axial Direction for Run 3. Error bars shown: ±2%.

Figure A2-4: Pressure Prediction along the Axial Direction for Run 4. Error bars shown: ±2%.
Figure A2-5: Pressure Prediction along the Axial Direction for Run 5. Error bars shown: ±2%.

Figure A2-6: Pressure Prediction along the Axial Direction for Run 6. Error bars shown: ±2%.
Figure A2-7: Pressure Prediction along the Axial Direction for Run 7. Error bars shown: ±2%.

Figure A2-8: Pressure Prediction along the Axial Direction for Run 8. Error bars shown: ±2%.
Figure A2-9: Pressure Prediction along the Axial Direction for Run 9. Error bars shown: ±2%.

Figure A2-10: Pressure Prediction along the Axial Direction for Run 10. Error bars shown: ±2%.
Figure A2-11: Pressure Prediction along the Axial Direction for Run 11. Error bars shown: ±2%.

Figure A2-12: Pressure Prediction along the Axial Direction for Run 12. Error bars shown: ±2%.
Figure A2-13: Pressure Prediction along the Axial Direction for Run 13. Error bars shown: ±2%.

Figure A2-14: Pressure Prediction along the Axial Direction for Run 14. Error bars shown: ±2%.
Figure A2-15: Pressure Prediction along the Axial Direction for Run 15. Error bars shown: ±2%. 
Appendix A3

Drift Flux Model Prediction of Void-weighted Bubble Velocity (90-deg)
Figure A3-1: Drift Flux Model prediction for void-weighted bubble velocity along the axial direction for Run 1. Error bars shown: ±5%.

Figure A3-2: Drift Flux Model prediction for void-weighted bubble velocity along the axial direction for Run 2. Error bars shown: ±5%.
Figure A3-3: Drift Flux Model prediction for void-weighted bubble velocity along the axial direction for Run 3. Error bars shown: ±5%.

Figure A3-4: Drift Flux Model prediction for void-weighted bubble velocity along the axial direction for Run 4. Error bars shown: ±5%.
Figure A3-5: Drift Flux Model prediction for void-weighted bubble velocity along the axial direction for Run 5. Error bars shown: ±5%.

Figure A3-6: Drift Flux Model prediction for void-weighted bubble velocity along the axial direction for Run 6. Error bars shown: ±5%.
Figure A3-7: Drift Flux Model prediction for void-weighted bubble velocity along the axial direction for Run 7. Error bars shown: ±5%.

Figure A3-8: Drift Flux Model prediction for void-weighted bubble velocity along the axial direction for Run 8. Error bars shown: ±5%.
Figure A3-9: Drift Flux Model prediction for void-weighted bubble velocity along the axial direction for Run 9. Error bars shown: ±5%.

Figure A3-10: Drift Flux Model prediction for void-weighted bubble velocity along the axial direction for Run 10. Error bars shown: ±5%.
Figure A3-11: Drift Flux Model prediction for void-weighted bubble velocity along the axial direction for Run 11. Error bars shown: ±5%.

Figure A3-12: Drift Flux Model prediction for void-weighted bubble velocity along the axial direction for Run 12. Error bars shown: ±5%.
Figure A3-13: Drift Flux Model prediction for void-weighted bubble velocity along the axial direction for Run 13. Error bars shown: ±5%.

Figure A3-14: Drift Flux Model prediction for void-weighted bubble velocity along the axial direction for Run 14. Error bars shown: ±5%.
Figure A3-15: Drift Flux Model prediction for void-weighted bubble velocity along the axial direction for Run 15. Error bars shown: ±5%.
Appendix A4

Drift Flux Model Prediction of Void-weighted Bubble Velocity (45-deg)
Figure A4-1: Drift Flux Model prediction for void-weighted bubble velocity along the axial direction for Run 1. Error bars shown: ±5%.

Figure A4-2: Drift Flux Model prediction for void-weighted bubble velocity along the axial direction for Run 2. Error bars shown: ±5%.
Figure A4-3: Drift Flux Model prediction for void-weighted bubble velocity along the axial direction for Run 3. Error bars shown: ±5%.

Figure A4-4: Drift Flux Model prediction for void-weighted bubble velocity along the axial direction for Run 4. Error bars shown: ±5%.
Figure A4-5: Drift Flux Model prediction for void-weighted bubble velocity along the axial direction for Run 5. Error bars shown: ±5%.

Figure A4-6: Drift Flux Model prediction for void-weighted bubble velocity along the axial direction for Run 6. Error bars shown: ±5%.
Figure A4-7: Drift Flux Model prediction for void-weighted bubble velocity along the axial direction for Run 7. Error bars shown: ±5%.

Figure A4-8: Drift Flux Model prediction for void-weighted bubble velocity along the axial direction for Run 8. Error bars shown: ±5%.
Figure A4-9: Drift Flux Model prediction for void-weighted bubble velocity along the axial direction for Run 9. Error bars shown: ±5%.

Figure A4-10: Drift Flux Model prediction for void-weighted bubble velocity along the axial direction for Run 10. Error bars shown: ±5%.
Figure A4-11: Drift Flux Model prediction for void-weighted bubble velocity along the axial direction for Run 11. Error bars shown: ±5%.

Figure A4-12: Drift Flux Model prediction for void-weighted bubble velocity along the axial direction for Run 12. Error bars shown: ±5%.
Figure A4-13: Drift Flux Model prediction for void-weighted bubble velocity along the axial direction for Run 13. Error bars shown: ±5%.

Figure A4-14: Drift Flux Model prediction for void-weighted bubble velocity along the axial direction for Run 14. Error bars shown: ±5%.
Figure A4-15: Drift Flux Model prediction for void-weighted bubble velocity along the axial direction for Run 15. Error bars shown: ±5%.
Appendix A5

Drift Flux Model Prediction of Void Fraction (90-deg)
Figure A5-1: Drift Flux Model prediction for void fraction along the axial direction for Run 1. Error bars shown: ±10%.

Figure A5-2: Drift Flux Model prediction for void fraction along the axial direction for Run 2. Error bars shown: ±10%.
Figure A5-3: Drift Flux Model prediction for void fraction along the axial direction for Run 3. Error bars shown: ±10%.

Figure A5-4: Drift Flux Model prediction for void fraction along the axial direction for Run 4. Error bars shown: ±10%.
Figure A5-5: Drift Flux Model prediction for void fraction along the axial direction for Run 5. Error bars shown: ±10%.

Figure A5-6: Drift Flux Model prediction for void fraction along the axial direction for Run 6. Error bars shown: ±10%.
Figure A5-7: Drift Flux Model prediction for void fraction along the axial direction for Run 7. Error bars shown: ±10%.

Figure A5-8: Drift Flux Model prediction for void fraction along the axial direction for Run 8. Error bars shown: ±10%.
Figure A5-9: Drift Flux Model prediction for void fraction along the axial direction for Run 9. Error bars shown: ±10%.

Figure A5-10: Drift Flux Model prediction for void fraction along the axial direction for Run 10. Error bars shown: ±10%.
Figure A5-11: Drift Flux Model prediction for void fraction along the axial direction for Run 11. Error bars shown: ±10%.

Figure A5-12: Drift Flux Model prediction for void fraction along the axial direction for Run 12. Error bars shown: ±10%.
Figure A5-13: Drift Flux Model prediction for void fraction along the axial direction for Run 13. Error bars shown: ±10%.

Figure A5-14: Drift Flux Model prediction for void fraction along the axial direction for Run 14. Error bars shown: ±10%.
Figure A5-15: Drift Flux Model prediction for void fraction along the axial direction for Run 15. Error bars shown: ±10%.
Appendix A6

Drift Flux Model Prediction of Void Fraction (45-deg)
Figure A6-1: Drift Flux Model prediction for void fraction along the axial direction for Run 1. Error bars shown: ±10%.

Figure A6-2: Drift Flux Model prediction for void fraction along the axial direction for Run 2. Error bars shown: ±10%.
Figure A6-3: Drift Flux Model prediction for void fraction along the axial direction for Run 3. Error bars shown: ±10%.

Figure A6-4: Drift Flux Model prediction for void fraction along the axial direction for Run 4. Error bars shown: ±10%.
Figure A6-5: Drift Flux Model prediction for void fraction along the axial direction for Run 5. Error bars shown: ±10%.

Figure A6-6: Drift Flux Model prediction for void fraction along the axial direction for Run 6. Error bars shown: ±10%.
Figure A6-7: Drift Flux Model prediction for void fraction along the axial direction for Run 7. Error bars shown: ±10%.

Figure A6-8: Drift Flux Model prediction for void fraction along the axial direction for Run 8. Error bars shown: ±10%.
Figure A6-9: Drift Flux Model prediction for void fraction along the axial direction for Run 9. Error bars shown: ±10%.

Figure A6-10: Drift Flux Model prediction for void fraction along the axial direction for Run 10. Error bars shown: ±10%.
Figure A6-11: Drift Flux Model prediction for void fraction along the axial direction for Run 11. Error bars shown: ±10%.

Figure A6-12: Drift Flux Model prediction for void fraction along the axial direction for Run 12. Error bars shown: ±10%.
Figure A6-13: Drift Flux Model prediction for void fraction along the axial direction for Run 13. Error bars shown: ±10%.

Figure A6-14: Drift Flux Model prediction for void fraction along the axial direction for Run 14. Error bars shown: ±10%.
Figure A6-15: Drift Flux Model prediction for void fraction along the axial direction for Run 15. Error bars shown: ±10%.
Appendix A7

MATLAB Program (90-deg)

Main_90.m

clear all
Experimental_Conditions_90
Initial_Conditions_90
Coefficients_90
Weer8_Covariance_90
for k = 1 : length(exp_ai)
    z = zo + zstep * (0 : No_zstep - 1);
    a(1) = exp_a(k,1);
    ai(1) = exp_ai(k,1);
    vgz(1) = exp_vgz(k,1);
    jg(1) = exp_jg(k,1);
   jf = exp_jf(k);
PDM_90
Model_90
ai_comp(k,:) = ai;
VG_comp(k,:) = VG * (-1);
PD_comp(k,:) = PD;
TI_comp(k,:) = TI;
RC_comp(k,:) = RC * (-1);
a_comp(k,:) = a;
vgz_comp(k,:) = vgz;
ppz(k,:)=pz;
delta_ai(k,:)= ai_comp(k,:) - exp_ai(k,1);
end
Model_Output_90

Experimental_Conditions_90.m

zo = 9.91;
axial_length = 16.57 - zo;
Dh = 0.0503;
rhof = 998;
rhog = 1.23;
muf = 0.001;
sigma = 0.07278;

Initial_Conditions_90.m

zstep = 0.01;
No_zstep = round(axial_length / zstep);
exp_z = [9.91, 11.30, 12.60, 16.56];
exp_ai = [
56.509
57.299
55.410
93.267
108.992
167.681
142.281
154.440
138.088
164.694
156.270
166.579
168.547
182.491
173.208
];
exp_a = [
0.026
0.027
0.025
0.063
0.063
0.107
0.112
0.110
0.101
0.150
0.140
0.138
0.178
0.179
0.166
];

exp_po = [
124767
126835
128903
126835
130281
133038
130281
133039
136485
133728
135796
139932
135796
]
exp_jf = [
3.759
4.058
4.34
3.759
4.048
4.334
3.778
4.042
4.334
3.772
4.045
4.331
3.759
4.035
4.337
];

exp_jg = [
0.090 0.102 0.108 0.114
0.097 0.103 0.112 0.121
0.101 0.103 0.111 0.120
0.247 0.256 0.279 0.307
0.250 0.256 0.283 0.309
0.250 0.258 0.285 0.317
0.501 0.516 0.566 0.624
0.499 0.513 0.573 0.639
0.501 0.515 0.580 0.648
0.755 0.766 0.857 0.947
0.750 0.767 0.864 0.972
0.749 0.774 0.875 0.991
0.995 1.033 1.151 1.293
0.999 1.034 1.172 1.321
0.999 1.037 1.184 1.356
];

exp_vgz = [
3.303
3.546
3.835
3.879
3.960
4.110
4.458
4.552
4.946
5.080
5.371
];
```
5.499
5.757
5.777
6.195
];

Coefficients_90.m

amax = 0.75;
C = 3;
Wecr = 8;
CTI = 0.005
CRC = 0.002

Wecr8_Covariance_90.m

CovTI = [
1.694 1.029 1.551 1.630
1.541 1.033 1.305 2.673
1.319 1.014 1.128 1.920
2.607 1.064 1.277 2.271
2.660 1.052 1.205 2.235
2.505 1.033 1.118 1.932
2.449 1.082 1.213 1.936
2.410 1.104 1.120 1.726
1.990 1.110 1.132 1.502
2.075 1.080 1.107 1.342
1.984 1.086 1.059 1.229
1.838 1.090 1.062 1.161
1.807 1.086 1.051 1.189
1.750 1.073 1.077 1.170
1.508 1.130 1.082 1.072
];

CovRC = [
21.277 2.050 12.393 18.744
18.018 2.141 5.773 19.752
11.712 2.246 3.417 15.741
28.523 1.965 11.765 28.876
20.894 2.109 4.495 22.244
27.320 2.430 2.858 18.411
28.082 2.811 5.907 15.897
29.792 3.543 3.392 12.000
21.626 3.998 3.393 8.991
19.466 3.526 4.483 5.238
21.959 3.698 2.974 4.020
18.939 4.768 2.585 3.167
13.795 2.992 3.991 3.689
14.302 2.282 2.846 3.181
8.538 4.994 2.759 2.293
];
```
CM = 65;
CFL = 25;
kM = 0.58;
L_D = (0.48+0.91+1.3)*1000/50.3;
mug = 1.73 * 10^(-5);
m = 0.079;
n = 0.25;
f = m * (rhof * jf * Dh / muf)^(-n);
fdpdz = -1 * (2 * f * rhof * jf^2) / Dh;
X2=((rhog*exp_jg(k,3)*muf)/(rhof*jf*mug))^n*(rhof*jf*jf)/(rhog*exp_jg(k,3)* exp_jg(k,3));
XM2 = 4 * (f / kM) * (L_D);
mu2 = X2 / XM2;
phiM = 1 + CM / (X2^0.5) * (1 + 1 / XM2)^0.5 + (1 + mu2) / X2;
Mdpdz = phiM * fdpdz;
X2FL=((rhog*exp_jg(k,4)*muf)/(rhog*jf*mug))^n*(rhog*jf*jf)/(rhog*exp_jg(k,4)* exp_jg(k,4));
phiFL = 1 + CFL / (X2FL^0.5) + 1 / X2FL;
FLdpdz = phiFL * fdpdz;
for i = 1 : No_zstep
    if (z(i) >= exp_z(1) && z(i) <= exp_z(3));
        po = exp_po(k);
        dpdz = Mdpdz;
        omegaz(i) = 1 - (z(i) - exp_z(1)) * abs(dpdz / po);
    elseif (z(i) > exp_z(3));
        po = pz(270);
        dpdz = FLdpdz;
        omegaz(i) = 1 - (z(i) - exp_z(3)) * abs(dpdz / po);
    end
    pz(i) = po * omegaz(i);
    dpdz_pz(i) = dpdz / pz(i);
end

Model_90.m

linearCovTI(k,:) = interp1(exp_z,CovTI(k,:),z,'linear');
linearCovRC(k,:) = interp1(exp_z,CovRC(k,:),z,'linear');
rhogz = rhog * pz / exp_po(k);
for i = 1 : No_zstep
    jg(i) = jg(1) * exp_po(k) / pz(i);
end
for i = 1 : No_zstep - 1
    if (z(i) >= exp_z(1) && z(i) <= exp_z(2));
        vgz(i+1)=vgz(i)+(1.23*(jg(1)+ jf)-0.54-vgz(1))/(exp_z(2)-exp_z(1))*zstep;
        a(i + 1) = jg(i + 1) / vgz(i + 1);
    elseif (z(i) > exp_z(2) && z(i) <= exp_z(3));
        vgz(i+1)=vgz(i)+(1.23*(jg(1)+ jf)-0.54-vgz(1))/(exp_z(2)-exp_z(1))*zstep;
        a(i + 1) = jg(i + 1) / vgz(i + 1);
    end
end
vgz(i+1)=vgz(i)+(1.62*(jg(270)+jf)-2.10-vgz(140))/(exp_z(3) - exp_z(2))*zstep;
a(i + 1) = jg(i + 1) / vgz(i + 1);
else;
vgz(i+1)=vgz(i)+(1.91*(jg(666)+jf)-3.88-vgz(270))/(exp_z(4)-exp_z(3)) *zstep;
a(i + 1) = jg(i + 1) / vgz(i + 1);
end
end
Db(1) = 6 * a(1) / ai(1);
for i = 1 : No_zstep - 1
SVG(i) = ai(i) * (vgz(i+1) - vgz(i)) / zstep;
Model_90.m (cont.'d)
SPD(i) = -2 / 3 * ai(i) * vgz(i) * dpdz_pz(i);
vfz(i) = jf / (1 - a(i));
mum = muf / (1 - a(i));
vm = (rhof*(1-a(i))*vfz(i)+rhogz*a(i)*vgz(i))/((1-a(i))*rhof+a(i)*rhogz);
Rem = rhof * vm * Dh / mum;
fTW = 0.316 * (mum / muf) ^ 0.25 / Rem ^ 0.25;
e(i) = fTW * (vm ^ 3) / 2 / Dh;
ult(i) = 1.4 * e(i) ^ (1 / 3) * Db(i) ^ (1 / 3);
term_1 = ult(i)*ai(i)^2/amax^(1/3)/(amax^(1/3)-a(i)^(1/3));
term_2 = 1-exp(C*amax^(1/3)*a(i)^(1/3)/(amax^(1/3)-a(i)^(1/3)));
SRC(i) = linearCovRC(k,i)*CRC * term_1 * term_2 / 3 / pi;
term_1 = ult(i) * ai(i) ^ 2 / a(i);
We = rhof * ult(i) ^ 2 * Db(i) / sigma;
if We > Wecr;
term_2 = (1 - Wecr / We) ^ 0.5 * exp(-Wecr / We);
else
term_2 = 0;
end
STI(i) = linearCovTI(k,i) * CTI * term_1 * term_2 / 18;
VG(1) = 0;
PD(1) = 0;
TI(1) = 0;
RC(1) = 0;
ai(i+1)=ai(i)+zstep*(STI(i)-SRC(i)+SPD(i)-SVG(i))/vgz(i);
Model_90.m (cont.'d)
VG(i + 1) = VG(i) + zstep * SVG(i) / vgz(i);
PD(i + 1) = PD(i) + zstep * SPD(i) / vgz(i);
TI(i + 1) = TI(i) + zstep * STI(i) / vgz(i);
RC(i + 1) = RC(i) + zstep * SRC(i) / vgz(i);
Db(i + 1) = 6 * a(i + 1) / ai(i + 1);
fid=fopen('Data01.txt', 'w');
M=[z; ppz(1,:); ai_comp(1,:); delta_ai(1,:); RC_comp(1,:);
   TI_comp(1,:); PD_comp(1,:);
   VG_comp(1,:); a_comp(1,:); vgz_comp(1,:)];
fprintf(fid,'%7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %9.7f
   %7.3f\n', M);
fclose(fid);

fid=fopen('Data02.txt', 'w');
M=[z; ppz(2,:); ai_comp(2,:); delta_ai(2,:); RC_comp(2,:);
   TI_comp(2,:); PD_comp(2,:);
   VG_comp(2,:); a_comp(2,:); vgz_comp(2,:)];
fprintf(fid,'%7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %9.7f
   %7.3f\n', M);
fclose(fid);

fid=fopen('Data03.txt', 'w');
M=[z; ppz(3,:); ai_comp(3,:); delta_ai(3,:); RC_comp(3,:);
   TI_comp(3,:); PD_comp(3,:);
   VG_comp(3,:); a_comp(3,:); vgz_comp(3,:)];
fprintf(fid,'%7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %9.7f
   %7.3f\n', M);
fclose(fid);

fid=fopen('Data04.txt', 'w');
M=[z; ppz(4,:); ai_comp(4,:); delta_ai(4,:); RC_comp(4,:);
   TI_comp(4,:); PD_comp(4,:);
   VG_comp(4,:); a_comp(4,:); vgz_comp(4,:)];
fprintf(fid,'%7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %9.7f
   %7.3f\n', M);
fclose(fid);

fid=fopen('Data05.txt', 'w');
M=[z; ppz(5,:); ai_comp(5,:); delta_ai(5,:); RC_comp(5,:);
   TI_comp(5,:); PD_comp(5,:);
   VG_comp(5,:); a_comp(5,:); vgz_comp(5,:)];
fprintf(fid,'%7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %9.7f
   %7.3f\n', M);
fclose(fid);

fid=fopen('Data06.txt', 'w');
M=[z; ppz(6,:); ai_comp(6,:); delta_ai(6,:); RC_comp(6,:);
   TI_comp(6,:); PD_comp(6,:);
   VG_comp(6,:); a_comp(6,:); vgz_comp(6,:)];
fprintf(fid,'%7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %9.7f
   %7.3f\n', M);
fclose(fid);
fid=fopen('Data07.txt', 'w');
M=[z; ppz(7,:); ai_comp(7,:); delta_ai(7,:); RC_comp(7,:);
TI_comp(7,:); PD_comp(7,:);
VG_comp(7,:); a_comp(7,:); vgz_comp(7,:)];
fprintf(fid,'%7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %9.7f
%7.3f\n', M);
close(fid);

Model_Data_90.m (cont.'d)
 fid=fopen('Data08.txt', 'w');
M=[z; ppz(8,:); ai_comp(8,:); delta_ai(8,:); RC_comp(8,:);
TI_comp(8,:); PD_comp(8,:);
VG_comp(8,:); a_comp(8,:); vgz_comp(8,:)];
fprintf(fid,'%7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %9.7f
%7.3f\n', M);
close(fid);

fid=fopen('Data09.txt', 'w');
M=[z; ppz(9,:); ai_comp(9,:); delta_ai(9,:); RC_comp(9,:);
TI_comp(9,:); PD_comp(9,:);
VG_comp(9,:); a_comp(9,:); vgz_comp(9,:)];
fprintf(fid,'%7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %9.7f
%7.3f\n', M);
close(fid);

fid=fopen('Data10.txt', 'w');
M=[z; ppz(10,:); ai_comp(10,:); delta_ai(10,:); RC_comp(10,:);
TI_comp(10,:); PD_comp(10,:);
VG_comp(10,:); a_comp(10,:); vgz_comp(10,:)];
fprintf(fid,'%7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %9.7f
%7.3f\n', M);
close(fid);

fid=fopen('Data11.txt', 'w');
M=[z; ppz(11,:); ai_comp(11,:); delta_ai(11,:); RC_comp(11,:);
TI_comp(11,:); PD_comp(11,:);
VG_comp(11,:); a_comp(11,:); vgz_comp(11,:)];
fprintf(fid,'%7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %9.7f
%7.3f\n', M);
close(fid);

fid=fopen('Data12.txt', 'w');
M=[z; ppz(12,:); ai_comp(12,:); delta_ai(12,:); RC_comp(12,:);
TI_comp(12,:); PD_comp(12,:);
VG_comp(12,:); a_comp(12,:); vgz_comp(12,:)];
fprintf(fid,'%7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %9.7f
%7.3f\n', M);
close(fid);
fid=fopen('Data13.txt', 'w');
M=[z; ppz(13,:); ai_comp(13,:); delta_ai(13,:); RC_comp(13,:);
TI_comp(13,:); PD_comp(13,:);
VG_comp(13,:); a_comp(13,:); vgz_comp(13,:)];
fprintf(fid,'%7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %9.7f %7.3f
', M);
fclose(fid);

fid=fopen('Data14.txt', 'w');
M=[z; ppz(14,:); ai_comp(14,:); delta_ai(14,:); RC_comp(14,:);
TI_comp(14,:); PD_comp(14,:);
VG_comp(14,:); a_comp(14,:); vgz_comp(14,:)];
fprintf(fid,'%7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %9.7f %7.3f
', M);
fclose(fid);

Model_Data_90.m (cont.'d)
fid=fopen('Data15.txt', 'w');
M=[z; ppz(15,:); ai_comp(15,:); delta_ai(15,:); RC_comp(15,:);
TI_comp(15,:); PD_comp(15,:);
VG_comp(15,:); a_comp(15,:); vgz_comp(15,:)];
fprintf(fid,'%7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %9.7f %7.3f
', M);
fclose(fid);
Appendix A8

MATLAB Program (45-deg)

Main_45.m

clear all
Experimental_Conditions_45
Initial_Conditions_45
Coefficients_45
Wecr8_Covariance_45
for k = 1 : length(exp_ai)
    z = zo + zstep * (0 : No_zstep - 1);
    a(1) = exp_a(k,1);
    ai(1) = exp_ai(k,1);
    vgz(1) = exp_vgz(k,1);
    jg(1) = exp_jg(k,1);
    jf = exp_jf(k);
end
Model_45
ai_comp(k,:) = ai;
VG_comp(k,:) = VG * (-1);
PD_comp(k,:) = PD;
TI_comp(k,:) = TI;
RC_comp(k,:) = RC * (-1);
a_comp(k,:) = a;
vgz_comp(k,:) = vgz;
ppz(k,:)=pz;
delta_ai(k,:)= ai_comp(k,:) - exp_ai(k,1);
end
Model_Output_45

Experimental_Conditions_45.m

zo = 17.22;
axial_length = 21.09 - zo;
Dh = 0.0503;
rhof = 998;
rhog = 1.23;
muf = 0.001;
sigma = 0.07278;

Initial_Conditions_45.m

zstep = 0.01;
No_zstep = round(axial_length / zstep);
exp_z = [17.22, 18.28, 21.08];
exp_ai = [66.320]
exp_a = [0.035, 0.033, 0.031, 0.077, 0.073, 0.072, 0.132, 0.124, 0.121, 0.187, 0.172, 0.167, 0.239, 0.220, 0.204];
exp_po = [115108, 117184, 119942, 116495, 118564, 121321, 118562, 121320, 122698, 120286, 123387, 125456, 123389, 126835, 128214];
exp_jf = [169]
3.756
4.042
4.340
3.778
4.042
4.343
3.762
4.048
4.343
3.775
4.042
4.343
3.772
4.042
4.340
];
exp_jg = [ 
0.119 0.125 0.131
0.122 0.127 0.134
0.122 0.128 0.139
0.304 0.315 0.332
0.304 0.320 0.338
0.309 0.326 0.351
0.605 0.639 0.673
0.617 0.652 0.698
0.629 0.670 0.723
0.915 0.985 1.037
0.931 1.003 1.069
0.952 1.023 1.103
1.240 1.325 1.402
1.267 1.356 1.459
1.291 1.391 1.503
];
exp_vgz = [ 
3.349
3.588
3.856
3.884
4.166
4.244
4.660
5.056
5.233
5.186
5.698
6.081
5.565
6.089
6.717
];
Coefficients_45.m

amax = 0.75;
C = 3;
Wecr = 8;
CTI = 0.005
CRC = 0.002

Wecr8_Covariance_45.m

CovTI=
[ 1.830 1.817 1.685 
 7.529 1.601 1.881 
 3.898 1.730 1.812 
 2.527 0.942 1.567 
 2.298 0.918 1.456 
 2.236 0.933 1.467 
 1.750 1.096 1.409 
 1.638 1.105 1.186 
 1.766 1.129 1.093 
 1.360 1.027 1.229 
 1.262 1.036 1.211 
 1.265 1.084 1.202 
 1.228 1.029 1.121 
 1.165 1.021 1.113 
 1.143 1.044 1.114 ];
CovRC=[
24.061 14.397 23.284 
19.450 13.977 19.955 
14.434 17.881 13.055 
29.631 3.104 17.769 
25.868 3.605 15.028 
18.393 3.688 12.495 
12.370 3.462 8.045 
11.084 5.936 4.771 
13.039 87.642 3.196 
5.100 1.396 3.827 
4.163 1.757 3.610 
4.139 3.125 3.743 
3.732 1.428 2.428 
3.094 1.365 2.501 
2.684 1.794 2.222 ];

PDM_45.m

CM=65;
CFL=40;
kM=0.35;
L_D=(0.56+0.5)*1000/50.3;
mug=1.73*10^(-5);
m=0.079;
n=0.25;
f = m * (rhof * jf * Dh / muf)^(-n);
fdpdz = -1 * (2 * f * rhof * jf^2) / Dh;
X2=((rhog*exp_jg(k,2)*muf)/(rhof*jf*mug))^n*(rhof*jf*jf)/(rhog*exp_jg(k,2)*exp_jg(k,2));
XM2 = 4 * (f / kM) * (L_D);
mu2 = X2 / XM2;
phiM = 1 + CM / (X2^0.5) * (1 + 1 / XM2)^0.5 + (1 + mu2) / X2;
Mdpdz = phiM * fdpdz;
X2FL=((rhog*exp_jg(k,3)*muf)/(rhof*jf*mug))^n*(rhof*jf*jf)/(rhog*exp_jg(k,3)*exp_jg(k,3));
phiFL = 1 + CFL / (X2FL^0.5) + 1 / X2FL;
FLdpdz = phiFL * fdpdz;
for i = 1 : No_zstep
    if (z(i) >= exp_z(1) && z(i) <= exp_z(2));
        po = exp_po(k);
        dpdz = Mdpdz;
        omegaz(i) = 1 - (z(i) - exp_z(1)) * abs(dpdz / po);
    elseif (z(i) > exp_z(2));
        po = pz(107);
        dpdz = FLdpdz;
        omegaz(i) = 1 - (z(i) - exp_z(2)) * abs(dpdz / po);
    end
    pz(i) = po * omegaz(i);
    dpdz_pz(i) = dpdz / pz(i);
end

Model_45.m

linearCovTI(k,:)=interp1(exp_z,CovTI(k,:),z,'linear');
linearCovRC(k,:)=interp1(exp_z,CovRC(k,:),z,'linear');
rhogz = rhog * pz / exp_po(k);
for i = 1 : No_zstep
    jg(i) = jg(1) * exp_po(k) / pz(i);
end
for i = 1 : No_zstep - 1
    if (z(i) >=exp_z(1) && z(i) < exp_z(2));
        vgz(i+1)= vgz(i)+(1.19*(jg(107)+jf)-0.79-
vgz(1))/(exp_z(2)-exp_z(1))*zstep;
        a(i+1) = jg(i+1) / vgz(i+1);
    else;
        vgz(i+1)=vgz(i)+(1.67*(jg(387)+jf)-2.66-vgz(107))/(exp_z(3)-exp_z(2)) * zstep;
        a(i+1) = jg(i+1) / vgz(i+1);
    end
end
Db(1) = 6 * a(1) / ai(1);
for i = 1 : No_zstep - 1
SVG(i) = ai(i) * (vgz(i+1) - vgz(i)) / zstep;
SPD(i) = -2 / 3 * ai(i) * vgz(i) * dpdz_pz(i);
vfz(i) = jf / (1 - a(i));
mum = muf / (1 - a(i));
vm = (rhof*(1-a(i))*vfz(i)+rhogz*a(i)*vgz(i))/((1-a(i))*rhof+a(i)*rhogz);
Rem = rhof * vm / Dh / mum;
Model_45.m (cont.'d)
fTW = 0.316 * (mum / muf) ^ 0.25 / Rem ^ 0.25;
e(i) = fTW * (vm ^ 3) / 2 / Dh;
ut(i) = 1.4 * e(i) ^ (1 / 3) * Db(i) ^ (1 / 3);
term_1 = ut(i)*ai(i)^2/amax^(1/3)/(amax^(1/3)-a(i)^(1/3));
term_2 = 1-exp(-C*amax^(1/3)*a(i)^(1/3)/(amax^(1/3)-a(i)^(1/3)));
SRC(i) = linearCovRC(k,i)*CRC * term_1 * term_2 / 3 / pi;
term_1 = ut(i) * ai(i) ^ 2 / a(i);
We = rhof * ut(i) ^ 2 * Db(i) / sigma;
if We > Wecr;
term_2 = (1 - Wecr / We) ^ 0.5 * exp(-Wecr / We);
else
    term_2 = 0;
end
STI(i) = linearCovTI(k,i) * CTI * term_1 * term_2 / 18;
VG(1) = 0;
PD(1) = 0;
TI(1) = 0;
RC(1) = 0;
ai(i+1)=ai(i)+zstep*(STI(i)-SRC(i)+SPD(i)-SVG(i))/vgz(i);
VG(i+1) = VG(i) + zstep * SVG(i) / vgz(i);
PD(i+1) = PD(i) + zstep * SPD(i) / vgz(i);
TI(i+1) = TI(i) + zstep * STI(i) / vgz(i);
RC(i+1) = RC(i) + zstep * SRC(i) / vgz(i);
Db(i+1) = 6 * a(i + 1) / ai(i + 1);
end

Model_Output_45.m

fid=fopen('Data01.txt', 'w');
M=[z; ppz(1,:); ai_comp(1,:); delta_ai(1,:); RC_comp(1,:);
    TI_comp(1,:); PD_comp(1,:);
    VG_comp(1,:); a_comp(1,:); vgz_comp(1,:)];
fprintf(fid,'%7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f
', M);
fclose(fid);

fid=fopen('Data02.txt', 'w');
M=[z; ppz(2,:); ai_comp(2,:); delta_ai(2,:); RC_comp(2,:);
    TI_comp(2,:); PD_comp(2,:);
    VG_comp(2,:); a_comp(2,:); vgz_comp(2,:)];
fprintf(fid,'%7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %9.7f
', M);
fclose(fid);

fid=fopen('Data03.txt', 'w');
M=[z; ppz(3,:); ai_comp(3,:); delta_ai(3,:); RC_comp(3,:);
  TI_comp(3,:); PD_comp(3,:);
  VG_comp(3,:); a_comp(3,:); vgz_comp(3,:)];
fprintf(fid,'%7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f
', M);
fclose(fid);

fid=fopen('Data04.txt', 'w');
M=[z; ppz(4,:); ai_comp(4,:); delta_ai(4,:); RC_comp(4,:);
  TI_comp(4,:); PD_comp(4,:);
  VG_comp(4,:); a_comp(4,:); vgz_comp(4,:)];
fprintf(fid,'%7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f
', M);
fclose(fid);

fid=fopen('Data05.txt', 'w');
M=[z; ppz(5,:); ai_comp(5,:); delta_ai(5,:); RC_comp(5,:);
  TI_comp(5,:); PD_comp(5,:);
  VG_comp(5,:); a_comp(5,:); vgz_comp(5,:)];
fprintf(fid,'%7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f
', M);
fclose(fid);

fid=fopen('Data06.txt', 'w');
M=[z; ppz(6,:); ai_comp(6,:); delta_ai(6,:); RC_comp(6,:);
  TI_comp(6,:); PD_comp(6,:);
  VG_comp(6,:); a_comp(6,:); vgz_comp(6,:)];
fprintf(fid,'%7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f
', M);
fclose(fid);

fid=fopen('Data07.txt', 'w');
M=[z; ppz(7,:); ai_comp(7,:); delta_ai(7,:); RC_comp(7,:);
  TI_comp(7,:); PD_comp(7,:);
  VG_comp(7,:); a_comp(7,:); vgz_comp(7,:)];
fprintf(fid,'%7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f
', M);
fclose(fid);

Model_Data_45.m (cont.'d)
fid=fopen('Data08.txt', 'w');
M=[z; ppz(8,:); ai_comp(8,:); delta_ai(8,:); RC_comp(8,:);
  TI_comp(8,:); PD_comp(8,:);
  VG_comp(8,:); a_comp(8,:); vgz_comp(8,:)];
fprintf(fid,'%7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %9.7f %7.3f\n', M);
fclose(fid);

fid=fopen('Data09.txt', 'w');
M=[z; ppz(9,:); ai_comp(9,:); delta_ai(9,:); RC_comp(9,:);
TI_comp(9,:); PD_comp(9,:);
VG_comp(9,:); a_comp(9,:); vgz_comp(9,:)];
fprintf(fid,'%7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %9.7f %7.3f\n', M);
fclose(fid);

fid=fopen('Data10.txt', 'w');
M=[z; ppz(10,:); ai_comp(10,:); delta_ai(10,:); RC_comp(10,:);
TI_comp(10,:); PD_comp(10,:);
VG_comp(10,:); a_comp(10,:); vgz_comp(10,:)];
fprintf(fid,'%7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %9.7f %7.3f\n', M);
fclose(fid);

fid=fopen('Data11.txt', 'w');
M=[z; ppz(11,:); ai_comp(11,:); delta_ai(11,:); RC_comp(11,:);
TI_comp(11,:); PD_comp(11,:);
VG_comp(11,:); a_comp(11,:); vgz_comp(11,:)];
fprintf(fid,'%7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %9.7f %7.3f\n', M);
fclose(fid);

fid=fopen('Data12.txt', 'w');
M=[z; ppz(12,:); ai_comp(12,:); delta_ai(12,:); RC_comp(12,:);
TI_comp(12,:); PD_comp(12,:);
VG_comp(12,:); a_comp(12,:); vgz_comp(12,:)];
fprintf(fid,'%7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %9.7f %7.3f\n', M);
fclose(fid);

fid=fopen('Data13.txt', 'w');
M=[z; ppz(13,:); ai_comp(13,:); delta_ai(13,:); RC_comp(13,:);
TI_comp(13,:); PD_comp(13,:);
VG_comp(13,:); a_comp(13,:); vgz_comp(13,:)];
fprintf(fid,'%7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %9.7f %7.3f\n', M);
fclose(fid);

fid=fopen('Data14.txt', 'w');
M=[z; ppz(14,:); ai_comp(14,:); delta_ai(14,:); RC_comp(14,:);
TI_comp(14,:); PD_comp(14,:);
VG_comp(14,:); a_comp(14,:); vgz_comp(14,:)];
fprintf(fid,'%7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %9.7f %7.3f\n', M);
fclose(fid);
fclose(fid);

Model_Data_45.m (cont.'d)
fid=fopen('Data15.txt', 'w');
M=[z; ppz(15,:); ai_comp(15,:); delta_ai(15,:); RC_comp(15,:);
  TI_comp(15,:); PD_comp(15,:);
  VG_comp(15,:); a_comp(15,:); vgz_comp(15,:)];
fprintf(fid,'%7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %7.3f %9.7f
  %7.3f
', M);
fclose(fid);
Appendix A9

Model Evaluation Results (90-deg)
Figure A9-1: Interfacial area concentration prediction along the axial direction for Run 1. Error bars shown: ±20%.

Figure A9-2: Contributions to the interfacial area concentration along the axial direction for Run 1. Error bars shown: ±20%.
Figure A9-3: Interfacial area concentration prediction along the axial direction for Run 2. Error bars shown: ±20%.

Figure A9-4: Contributions to the interfacial area concentration along the axial direction for Run 2. Error bars shown: ±20%.
Figure A9-5: Interfacial area concentration prediction along the axial direction for Run 3. Error bars shown: ±20%.

Figure A9-6: Contributions to the interfacial area concentration along the axial direction for Run 3. Error bars shown: ±20%.
Figure A9-7: Interfacial area concentration prediction along the axial direction for Run 4. Error bars shown: ±20%.

Run 4: $j_g, at=0.35 \text{ m/s}; j_f=3.78 \text{ m/s}$

Figure A9-8: Contributions to the interfacial area concentration along the axial direction for Run 4. Error bars shown: ±20%.
Figure A9-9: Interfacial area concentration prediction along the axial direction for Run 5. Error bars shown: ±20%.

Figure A9-10: Contributions to the interfacial area concentration along the axial direction for Run 5. Error bars shown: ±20%.
Figure A9-11: Interfacial area concentration prediction along the axial direction for Run 6. Error bars shown: ±20%.

Figure A9-12: Contributions to the interfacial area concentration along the axial direction for Run 6. Error bars shown: ±20%.
Figure A9-13: Interfacial area concentration prediction along the axial direction for Run 7. Error bars shown: ±20%.

Figure A9-14: Contributions to the interfacial area concentration along the axial direction for Run 7. Error bars shown: ±20%.
Figure A9-15: Interfacial area concentration prediction along the axial direction for Run 8. Error bars shown: ±20%.

Figure A9-16: Contributions to the interfacial area concentration along the axial direction for Run 8. Error bars shown: ±20%.
Figure A9-17: Interfacial area concentration prediction along the axial direction for Run 9. Error bars shown: ±20%.

Figure A9-18: Contributions to the interfacial area concentration along the axial direction for Run 9. Error bars shown: ±20%.
**Figure A9-19:** Interfacial area concentration prediction along the axial direction for Run 10. Error bars shown: ±20%.

**Run 10: jg,atm=1.09 m/s; jf=3.78 m/s**

- Predicted
- Experimental

**Figure A9-20:** Contributions to the interfacial area concentration along the axial direction for Run 10. Error bars shown: ±20%.
Figure A9-21: Interfacial area concentration prediction along the axial direction for Run 11. Error bars shown: ±20%.

Figure A9-22: Contributions to the interfacial area concentration along the axial direction for Run 11. Error bars shown: ±20%.
Figure A9-23: Interfacial area concentration prediction along the axial direction for Run 12. Error bars shown: ±20%.

Figure A9-24: Contributions to the interfacial area concentration along the axial direction for Run 12. Error bars shown: ±20%.
Figure A9-25: Interfacial area concentration prediction along the axial direction for Run 13. Error bars shown: ±20%.

Figure A9-26: Contributions to the interfacial area concentration along the axial direction for Run 13. Error bars shown: ±20%.
Figure A9-27: Interfacial area concentration prediction along the axial direction for Run 14. Error bars shown: ±20%.

Figure A9-28: Contributions to the interfacial area concentration along the axial direction for Run 14. Error bars shown: ±20%.
Figure A9-29: Interfacial area concentration prediction along the axial direction for Run 15. Error bars shown: ±20%.

Figure A9-30: Contributions to the interfacial area concentration along the axial direction for Run 15. Error bars shown: ±20%.
Appendix A10

Model Evaluation Results (45-deg)
Figure A10-1: Interfacial area concentration prediction along the axial direction for Run 1. Error bars shown: ±20%.

Figure A10-2: Contributions to the interfacial area concentration along the axial direction for Run 1. Error bars shown: ±20%.
Figure A10-3: Interfacial area concentration prediction along the axial direction for Run 2. Error bars shown: ±20%.

Figure A10-4: Contributions to the interfacial area concentration along the axial direction for Run 2. Error bars shown: ±20%.
Figure A10-5: Interfacial area concentration prediction along the axial direction for Run 3. Error bars shown: ±20%.

Figure A10-6: Contributions to the interfacial area concentration along the axial direction for Run 3. Error bars shown: ±20%.
Figure A10-7: Interfacial area concentration prediction along the axial direction for Run 4. Error bars shown: ±20%.

Run 4: \(j_g, \text{atm}=0.35 \text{ m/s}; j_f=3.78 \text{ m/s}\)

Predicted
Experimental

Figure A10-8: Contributions to the interfacial area concentration along the axial direction for Run 4. Error bars shown: ±20%.
Figure A10-9: Interfacial area concentration prediction along the axial direction for Run 5. Error bars shown: ±20%.

Figure A10-10: Contributions to the interfacial area concentration along the axial direction for Run 5. Error bars shown: ±20%.
Figure A10-11: Interfacial area concentration prediction along the axial direction for Run 6. Error bars shown: ±20%.

Figure A10-12: Contributions to the interfacial area concentration along the axial direction for Run 6. Error bars shown: ±20%.
Figure A10-13: Interfacial area concentration prediction along the axial direction for Run 7. Error bars shown: ±20%.

Figure A10-14: Contributions to the interfacial area concentration along the axial direction for Run 7. Error bars shown: ±20%.
Figure A10-15: Interfacial area concentration prediction along the axial direction for Run 8. Error bars shown: ±20%.

Figure A10-16: Contributions to the interfacial area concentration along the axial direction for Run 8. Error bars shown: ±20%.
Figure A10-17: Interfacial area concentration prediction along the axial direction for Run 9. Error bars shown: ±20%.

Figure A10-18: Contributions to the interfacial area concentration along the axial direction for Run 9. Error bars shown: ±20%.
Figure A10-19: Interfacial area concentration prediction along the axial direction for Run 10. Error bars shown: ±20%.

Figure A10-20: Contributions to the interfacial area concentration along the axial direction for Run 10. Error bars shown: ±20%. 
Figure A10-21: Interfacial area concentration prediction along the axial direction for Run 11. Error bars shown: ±20%.

Figure A10-22: Contributions to the interfacial area concentration along the axial direction for Run 11. Error bars shown: ±20%.
Figure A10-23: Interfacial area concentration prediction along the axial direction for Run 12. Error bars shown: ±20%.

Figure A10-24: Contributions to the interfacial area concentration along the axial direction for Run 12. Error bars shown: ±20%.
Figure A10-25: Interfacial area concentration prediction along the axial direction for Run 13. Error bars shown: ±20%.

Figure A10-26: Contributions to the interfacial area concentration along the axial direction for Run 13. Error bars shown: ±20%.
Figure A10-27: Interfacial area concentration prediction along the axial direction for Run 14. Error bars shown: ±20%.

Figure A10-28: Contributions to the interfacial area concentration along the axial direction for Run 14. Error bars shown: ±20%.
Figure A10-29: Interfacial area concentration prediction along the axial direction for Run 15. Error bars shown: ±20%.

Figure A10-30: Contributions to the interfacial area concentration along the axial direction for Run 15. Error bars shown: ±20%.