ALL-OPTICAL SWITCHING WITH DYE-DOPED LIQUID CRYSTALS

A Dissertation in
Electrical Engineering
by
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Submitted in Partial Fulfillment
of the Requirements
for the Degree of

Doctor of Philosophy

May 2011
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Abstract

A passive all-optical limiter to clamp high-power incident lasers or intense lights to protect human eyes or optical sensors from being damaged is being investigated. Here we present a dye-doped liquid crystal limiter based on a twist-nematic alignment configuration that switches off a high-power incident beam but allows a low-power beam to get through.

Detailed theoretical analysis and experimental investigation of twist-nematic liquid crystal (TNLC) cell switching behaviors under various conditions are presented. Two main mechanisms that contribute to liquid crystal switching include: (1) laser-induced birefringence change and (2) photo-chemical molecular confirmation change. Both will disorder the liquid crystal alignment and destroy the light polarization guidance in the sample. A 90-degrees twist alignment nematic liquid crystal doped with suitable dye could impart the required photonic absorption and order parameter modulation. The Jones matrix and Landau-de Gennes theory are introduced and used for the simulation of light propagation within the liquid crystal samples.

Experimentally, we have demonstrated ultrafast all-optical liquid crystal switching operation for lasers spanning the visible to near-infrared spectral region (488nm; 532nm; 750nm; 1064nm; 1550nm). With increasing intensity, the switching time decreases from microseconds to the nanoseconds regime, in such a manner that the transmitted light energy/intensity is clamped to below the eye- or sensor-safe levels.

We also present the simulation models for steady-state and transient TNLC switching behaviors. The simulation starts from solving temperature distribution and then order parameter profiles. Refractive index anisotropy is then evaluated and the light transmission response is calculated and discussed. The theory, simulation, and experiment results are in good accordance.
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Acknowledgment

I would like to express my gratitude to my thesis advisor, Professor I. C. Khoo, for his support and guidance over the past few years of my graduate studies at Penn State University. He always gave me full support when I was facing forthcoming challenges. I really appreciate all the opportunities he had provided to me in establishing my vision and advance toward success.

I would also like to thank my thesis committee members, Dr. Douglas Werner, Dr. Stuart Yin, and Dr. Tom Mallouk for their comments and suggestions. Also, my thanks go to Dr. Zhiwen Liu and Dr. Ruyan Guo for their encouragement and the models they have set.

Special appreciation to all my previous and present colleagues: Dr. Andres Diaz, Dr. Kan Chen, Dr. Yana Williams, Dr. Jae-Hong Park, Dr. Shoichi Kubo, Dr. Mike Stinger, Junbin Huang, Yi Ma, KuanLun Hong, Shuo Zhao, and all of friends who had helped me of any kind in the past.

Last but not least, full gratitude goes to my parents and family for their unconditional love and support.
Chapter 1  Liquid Crystal – A Brief Introduction

1.1 Brief History of Liquid Crystals

It is believed liquid crystal was first observed in the 19th century by Reinitzer [1], but research on the physics of liquid crystal by Georges Friedel did not begin until the early 1920s [2]. After almost a half century of inactivity, in the 1970s, under the influence of Pierre-Gilles de Gennes [3], liquid crystals physics finally entered a highly studied era and has significantly changed our lives. Pierre-Gilles de Gennes is the 1991 Nobel Laureate in Physics for his studies involving liquid crystals and polymers [4].

Friedel is believed to be the first one to use the term “mesomorphic states.” By using this meso- (means ‘intermediate’ in Greek) prefix, he meant to distinguish these phases of the genuine states of matter that possess phase properties that are intermediary between those of crystals and ordinary liquids. At the time, he used the plural term “states” and proposed the idea of polymorphism for these mesomorphic states, just like polymorphism for crystals. Even though he only knew two mesomorphic states, nematics and cholesterics, at the time, his idea had extended to a complete study today and numerous mesophases have been classified based on different symmetries. These mesomorphic states, or mesophases, are the so-called liquid crystals in general today.

Some of the most common mesophases include nematic, cholesteric, smectic, columnar, and blue phases. We will briefly introduce these different phases but will focus on the nematic liquid crystal phase throughout this dissertation.

1.2 Mesogenic Anatomy [5]

Liquid crystals (LC) phases (or mesophases) are found in a variety of materials. They can be distinguished by their molecular alignment symmetries and macroscopic behaviors. Some of the common mesogenic types include:

1. Lyotropic LCs – aqueous solutions of amphiphiles
2. Polymetric LCs
3. Colloidal LCs – suspensions of colloidal particles
4. Thermotropic LCs – low molecular mass species

Each type of the above liquid crystals is worthy of a book-length discussion. Nematic liquid crystals, the one we will be using in the later chapters, belongs to the thermotropic type and will be detailed in the next section, and the physical properties of the other mesogenic types, except thermotropic, are introduced briefly in this section.
1.2.1. Lyotropic Liquid Crystals

Lyotropic liquid crystalline mesophases were actually the first mesophases discovered from the mixtures of myelin and water in 1857. Basically, a liquid crystalline material is called lyotropic if phases of long-ranged orientational order are induced by the addition of a solvent. The molecules of this type usually have amphiphilic properties. In other words, these molecules possess two parts of different chemical affinities. One end of the molecules is hydrophobic while the other end is hydrophilic. In this respect they are very similar to the common soap molecules and work like surfactants. Examples include the widely used sodium dodecyl sulphate (SDS, \( \text{C}_{12}\text{H}_{25}\text{SO}_4\text{Na} \)) contained in our daily cleaning products, cf. Fig. 1.1. The dodecyl end is the linear alkyl chain and is hydrophobic (insoluble in water), while the \( \text{SO}_4\text{Na}^+ \) end is the ionic group and is very hydrophilic. Another example of amphiphilic molecules could be the non-ionic surfactant \( \text{C}_{12}\text{EO}_6 \) (hexa-ethylene glycol mono-n-dodecyl ether), which comprise a linear alkyl chain, just like SDS, and a six-unit oligomer of ethylene oxide, which is non-ionic but soluble in water due to the hydrogen bonds it can form with water.

![Sodium dodecyl sulphate (SDS)](image)

When this type of surfactant is dissolved in water, its molecules will form a variety of different alignments under different concentrations. When the concentration is low, it will form a thin film. When the concentration is higher than a certain value, these surfactant molecules saturate the film at the water surface and form “micelle” in the bulk water, as shown in Fig 1.2. These micelle can be spherical, cylindrical, or lamella shape. Their hydrophilic ends are pointed outward while the hydrophobic ends are enclosed inside from contacting water molecules. The driving force for this alignment and aggregation is the reduction of the surface tension for the surfactant molecules and in this way, reduces their potential energy to a minimum. Under different concentration and temperature, different types of alignment symmetry (different phases) can be observed and plotted as a phase diagram for further analysis (similar to what we did for alloys).
1.2.2. Polymeric Liquid Crystals

Also called polymer liquid crystals (PLC), this type of material has both the properties of polymers and liquid crystals. Rod-like or disk-like liquid crystal mesogens are linked into chains and show flexible macroscopic characteristics. By its molecular structure, it can be subdivided into main-chain polymer liquid crystals (MC-PLCs) - for those the mesogens are part of the main chain, and side-chain polymer liquid crystals (SC-PLCs) - for those the mesogens are connected as side chains of the polymer (cf. Fig 1.4).
1.2.3. Colloidal Liquid Crystals

The aqueous suspension of certain materials is found to have a nematic phase when they are at a sufficiently high concentration and the right pH values. Other metastable states are found by mixing two or more thermodynamically incompatible materials and blending in a particular suspension matter. These mesophases are generally categorized as colloidal liquid crystals. Some examples include the tobacco mosaic virus in aqueous suspension at pH 7–8. Under these conditions, the virus has the appearance of stiff monodisperse cylinders aligned in the nematic phase.[7] Another example would be a soft solid made with the mixture of well-known colloidal materials PMMA (polymethyl methacrylate) and the thermotropic nematic liquid crystal 5CB. This suspension mixture shows a birefringent waxy property and possesses a strong mechanical modulus.[8]

1.3 Thermotropic Liquid Crystals

1.3.1 Chemical potential perspective

Thermotropic liquid crystals are generally referred to synthesized substances that possess liquid crystals mesogenic phases of their own, without the need of being an aqueous solution or suspension in another matrix within a particular concentration range. Thermotropic liquid crystals are also the most widely used and extensively studied liquid crystals. Usually they are of low molecular weight and not polymers.

As indicated by the term “thermotropic,” the mesogenic phase of a thermotropic liquid crystal is determined thermodynamically by its “chemical potential”[9]. Before we go through chemical potential $\mu$, we have to define Gibbs energy $G$.

$$G = H - TS$$
$$= U + pV - TS$$

(1.1)
where $H$ is the enthalpy, $T$ is the absolute temperature, $U$ is the internal energy, and $S$ is the entropy. The chemical potential $\mu$ is defined as

$$\mu = \left( \frac{\partial G}{\partial n} \right)_{T,p}$$

where $n$ is the number of mole of the substance. The chemical potential indicates the change of the Gibbs energy system as this substance is added to it under constant temperature and pressure. In other words, the chemical potential is the “molar Gibbs energy.” At each phase of a substance, the chemical potential is a function of temperature and pressure. The phase that exists is the phase with the lowest chemical potential at the specified temperature and pressure because it is the most stable one, as shown in Fig 1.5. Generally, when the substance is not in the lowest chemical potential phase, the phase transition will happen spontaneously from higher potential phase to lowest potential phase in order to reach thermodynamic stability.

![Fig 1.5](image)

Fig 1.5 The schematic temperature dependence of the chemical potential of (a) common substance, & (b) liquid crystal

There are a few exceptions such as diamond, which is not in the lowest chemical potential phase, but the rate for diamond to change carbon atoms arrangement to become graphite, the one of lowest potential phase, is immeasurably slow at room temperature. This phase is the metastable phase and is beyond the scope of our discussion of liquid crystals.

### 1.3.2 Molecular frustration

To observe the phase transition of thermotropic liquid crystals, we would like to take the most common liquid crystal, 5CB, as an example. Fig 1.6 shows the phase sequence of three substances. The first two belong to very dissimilar and nonmesogenic molecules: (a) pentane, and (b) 4-cyanobiphenyl. The third one is that of (a) & (b) chemically bonded together as one molecule, 4-cyano-4’-pentylbiphenyl, also known as 5CB.

For pentane and 4-cyanobiphenyl, the solid crystalline phase S is in competition with the liquid phase L. The solid-liquid phase transition will take place at the temperature at which the
curves $\mu_L$ and $\mu_S$ cross (in Fig 1.5). At the crossing point, the chemical potential of the system remains unchanged, $\Delta G=0$ (or $\Delta \mu=0$ per mole), but all the other energy and the phase are changing. In the three-dimensional crystal structure (solid), molecules are placed on the sites of crystalline lattice and possess the lowest internal potential energy $U$ when $T=0$ K. At constant pressure $p$, when the system keeps absorbing energy and the temperature increases to the melting point $T_{SL}$, the entropy gained $\Delta S=S_L-S_S$, as the molecules jump away from their fixed positions, would balances the internal potential energy increase $\Delta U=U_L-U_S$ plus the work done by the system $p\Delta V=p(V_L-V_S)$, or

$$\Delta G_{SL} = \Delta U + p\Delta V - T\Delta S = 0.$$  (1.3)

It is known that the attraction between two organic molecules can be due to:

- Direct interaction between permanent electric dipoles
- Permanent dipole-induced dipole interaction – depends on the molecular polarizability
- Induced dipole-induced dipole interaction (van der Walls force) – also depends on the molecular polarizability

In alkanes such as pentane, the first two interactions listed above do not exist due to non-existence of permanent dipole, thus at ambient pressure, pentane melts at a low temperature $T_{SL}=$
-129.8°C. On the other hand, 4-cyanobiphenyl has a cyano group \(-\text{C}=\text{N}\) that has a strong dipolar moment and could cause high polarizability of the aromatic rings. This way its melting point is around \(T_{\text{SL}} = 88^\circ\text{C}\).

When the two completely different molecules are chemically associated by the covalent bond, the system must find a different solution to the conflicting properties of these two molecules. Forming liquid crystal mesophases is the solution it reaches. For 5CB, the mesophase that exists is the nematic phase, with \(T_{\text{SN}} = 18^\circ\text{C}\) and \(T_{\text{NL}} = 35^\circ\text{C}\). In fact, for the compounds in the nCB family with different aliphatic chain (\(\text{C}_n\text{H}_{2n+1}\)), the nematic phase appears between the crystalline phase and the liquid isotropic phase when \(n \geq 4\). For \(n \geq 8\), such as 8CB and 9CB, an additional “smectic A” phase appears between the crystal and the nematic phase.

### 1.3.3 Eutectic mixtures

Another important property for thermotropic liquid crystals is eutectic behavior. It is very similar to the phase diagrams of binary metallic alloys, which can be found in many engineering material handbooks. When two similar but different materials are mixed together in certain proportions, the transition from the crystal to nematic phase is sharp, without a coexistence range of nematic and crystalline phases. Moreover, this transition takes place at a temperature even lower than the melting points of either of the two pure substances, and hence the nematic phase temperature range is stretched, as seen in Fig 1.7 for the phase diagram of the mixture of 5CB and 7CB. This way the mixtures would have a wider dynamic working temperature range under the nematic phase and is desirable for application purposes, such as the liquid crystal display industry.

![Phase diagram of the binary mixture of 5CB and 7CB](image-url)
1.4 Classifications and Structures

Different mesophases comprise different molecule arrangement symmetries. For liquid crystals, the symmetry and collective order is in between 3D crystals and the liquid isotropic phase. Here we summarize a diagram of mesophase classification in Fig. 1.8.

In electrodynamics, we learned dipole, quadrupole, octupole…etc, but mostly in pure mathematical forms. In fact, physicists found that molecules of a variety of liquid crystalline mesophases are aligned with quadrupolar orientational order. For quadrupolar arrangement with positional spatially homogeneous, the mesophase presents only orientational long-range order and is called the nematic phase. For quadrupolar arrangement with spatially modulated order, depends on different types of spatial order, it is either the chiral cholesteric phase or the blue phase I, II, or III. For the molecules with even higher spatial order, if in lamellar arrangement, they are named smectic phases. If in they are in a columnar arrangement, they are called columnar phases. However, so far, mesophases of dipolar or octupolar orientational order have not been experimentally observed.

![Mesophase classification diagram](image)

1.4.1 Nematic phase

As we mentioned earlier, nematic phase presents only orientational long-range order and its molecules are randomly positioned. The directional correlated molecules are aligned in a rather uniform direction, even with some small variations between individual molecules, and this direction is defined as the “director axis” and designated by a unit vector $\mathbf{n}$. For nematics,
because no dipole exists and the bulk behaves centrosymmetric, these molecules orientated in \textbf{n} and -\textbf{n} with equal probability. In the diagrams, we often treat these elongated-shaped molecules as individual rods and don’t differentiate their \textbf{n} and -\textbf{n} directions.

1.4.2 Cholesteric phase

The cholesteric phase, also called chiral nematic, is very similar to the nematic phase except that the molecules tend to align in a helical manner in space along the axis called the helicoidal axis. This alignment can be naturally formed, such as cholesterol esters, or sometimes it can be human-made by adding chiral molecule to a nematic liquid crystal.
1.4.3 Smectic phases

The aforementioned cholesteric phase, though it possesses more structural helical orientational order, still does not possess a positional order. For the phases starting from smectics, they are distinguished by different molecular positioning arrangements and can be subdivided into various types. They can be either parallel or tilted with respect to the layer’s normal axis in a uniform or repetitive periodic structure. Due to the number of smectic types, we cannot go through all of them here but would like to show the most fundamental ones, Smectic A, Smectic C, and Smectic O, as in Fig 1.11.

![Smectic A, Smectic C, Smectic O](image)

Fig 1.11 Some varieties of smectic phase liquid crystal molecule alignment

The following figure summarizes all the major smectic phase types.

![Smectic liquid crystal phase types](image)

Fig 1.12 Types of smectic liquid crystal phase
Nematic and smectic liquid crystals are mostly rod-shaped and are generally formed with similar chemistry structures. The most stable series usually consist of a rigid body, formed by at least two aromatic groups, and are attached by one or more aliphatic chains. There are numerous possible combinations and we have no way to list all of them. Here are just a few of the common combinations.

\[
\begin{array}{ccc}
R & A & R' \\
C_nH_{2n+1} & -\text{CH=N-} & R \\
C_nH_{2n+1}-O & -\text{COO-} & -\text{C\equiv N} \\
C_nH_{2n+1}-\text{COO} & -\text{C\equiv C-} & -\text{Cl} \\
C_nH_{2n+1}-\text{OCOO} & & -\text{Br}
\end{array}
\]

Fig 1.13 Table of some common liquid crystal structural combinations

1.4.4 Columnar phases

For columnar phases, discoidal (disk-shaped) or cone-like molecules stack up in a infinitely long columnar shape. They are free to slide with respect to each other and have no positional correlations between each column. Similar to smectic phases, the molecules of columnar phases can be either normal to the column axis or have a tilt angle with respect to the axis.

(a) (b) (c)

Fig 1.14 Some examples of columnar mesophases: (a) hexagonal $D_h$ alignment, (b) molecules orthogonal to columns, (c) molecules oblique to columns
Here is one example of discoidal molecules: benzene-hexa-n-alkanoates (BHn), where n is the number of carbon atoms on the aliphatic chain. It is found that only n=7 to 9 gives rise to the mesomorphic phases.

![Diagram of BHn molecule]

**Fig 1.15** The structure of a series of discoidal molecules BHn, and its phase sequence

### 1.5 Order parameter for nematic liquid crystals

As we mentioned earlier, liquid crystals molecules, under certain temperature and pressure conditions, tend to align themselves naturally to a certain direction and form mesophases (liquid crystal phases) so as to reach the minimum chemical potential (or Gibbs Free Energy) of the system. However, it is not at the solid crystalline phase yet and each individual molecule may still deviate from the collective (average) orientation randomly due to thermo fluctuation, geometric constrain, defects, etc. To describe how uniform these molecules align themselves, “order parameter” is used, generally for ordinary uniaxial nematic liquid crystals.

Assume the overall average direction of a nematic liquid crystal sample is in z-axis direction (i.e. \( \mathbf{n} \)). And \( \mathbf{a} \) is the unit vector parallel to each of the rod-shaped molecules. The order parameter in tensor form is defined by

\[
S_{ij} = \langle a_i a_j \rangle - \frac{1}{3} \delta_{ij}
\]  

(1.5)

This tensor is symmetric, diagonalizable, and traceless. For uniaxial nematics, two of its eigenvalues must be equal and can be expressed in this generic form.
\[
\begin{bmatrix}
S_y
\end{bmatrix} = S \cdot \begin{bmatrix}
-\frac{1}{3} & 0 & 0 \\
0 & -\frac{1}{3} & 0 \\
0 & 0 & \frac{2}{3}
\end{bmatrix}
\]  

(1.6)

with the amplitude

\[
S = \frac{1}{2} \left\langle 3 \cos^2 \theta - 1 \right\rangle 
\]

(1.7)

\[
= \int_{\pi/2}^{\pi} \frac{1}{2} \left( 3 \cos^2 \theta - 1 \right) \cdot f(\theta) \cdot 2\pi \sin \theta d\theta
\]

where \( f(\theta) \) is the probability function for the molecules tilted at angle \( \theta \) deviated from director \( \mathbf{n} \). This scalar amplitude \( S \) is a measure of the degree of molecular alignment along the director \( \mathbf{n} \) with the maximum value 1 when all molecules are perfectly parallel to director \( \mathbf{n} \), i.e., \( f(\theta)=1 \) when \( \theta=0 \) and \( f=0 \) for else angle. Alternatively, the minimum value of \( S \) is -1/2 when probability distribution function \( f(\theta) \) is only peaked at \( \theta=\pi/2 \) but in random in-plane directions, which is almost impossible to happen due to Gibbs free energy minimization purpose. When the mesogens turn into the isotropic liquid phase, \( S \) turns to 0 because molecules align in random directions and \( f(\theta) \) is evenly distributed and is a flat line for every angle. For most cases when in ordinary nematic phase, \( S \) is always a positive value between 0 and 1.

The above definition is only for rigid rod-shaped molecules. With a more rigorous and general definition applicable to any molecular types, the concept of multipolar moments in electrostatics has to be introduced.

electric charge:  
\[
e = \sum_{\alpha} e_{\alpha}
\]

(1.8)

dipolar moment  
\[
p_i = \sum_{\alpha} e_{\alpha} r_{\alpha i}
\]

(1.9)

quadrupolar moment  
\[
q_{ij} = \frac{1}{2} \sum_{\alpha} e_{\alpha} \left( 3r_{\alpha i}r_{\alpha j} - r_{\alpha i}^2 \delta_{ij} \right)
\]

(1.10)

In electrostatics, each of these quantities listed above describes the space orientation of the charge distribution \( \{e_{\alpha}\} \) in the system. For liquid crystal order parameter definition purposes, the first one (electric charge) is zero as the molecule is neutral. The second one is dipolar moment and is not zero for a single molecule but the average value \( \langle p \rangle \) is zero for the collective molecules.

Finally, the first non-zero moment is the quadrupole \( Q_{ij} = \langle q_{ij} \rangle \). Similarly, for uniaxial nematics, these quadrupolar moment can be expressed as
\[
Q_{ij} = Q \cdot \begin{bmatrix}
-\frac{1}{3} & 0 & 0 \\
0 & -\frac{1}{3} & 0 \\
0 & 0 & \frac{1}{3}
\end{bmatrix}
\] (1.11)

For any arbitrary reference coordinate \((e_1, e_2, e_3)\), this quadrupole components become

\[
Q_{ij} = Q(T) \left( n_i n_j - \frac{1}{3} \delta_{ij} \right)
\] (1.12)

where the amplitude \(Q\) is a function of temperature and \(n_i\) is the component of director \(n\) in \((e_1, e_2, e_3)\) coordinate. The fundamental property of order parameter \(Q\) (or \(S\)) and its determination associated with the temperature will be detailed in the next chapter.

### 1.6 Frank-Oseen free energy

For uniaxial nematics, when in the absence of external constrains, the molecule director orientation is almost everywhere parallel to a fixed uniform direction \(n\), and the system is in the most relaxed (lowest energy) state. When the system is exposed to external constrains, it will induce elastic distortion and, as a result, the elastic potential energy will be elevated. This elastic deformation energy is called “Frank-Oseen free energy,” or just “free energy” for short, and is different from Gibbs free energy, which we discussed in earlier sections.

The elastic continuum theory concluded that the most general form of this elastic free energy \(f\) can be expressed as:

\[
f = \frac{1}{2} K_1 (\nabla \cdot n)^2 + \frac{1}{2} K_2 (n \cdot \nabla \times n)^2 + \frac{1}{2} K_3 \left| (n \times \nabla \times n) \right|^2
\] (1.13)

where each term stands for a particular type of deformation: splay, twist, and bend respectively and \(K_1, K_2, K_3\) are the respective Frank elastic constants. Most situations can be analyzed with these three types of deformation. To illustrate this, we can assume a local reference coordinate \((x_1, x_2, x_3)\) such that at a point \(p\) the axis \(x_3\) is collinear with director \(n\) direction. For the molecules close to point \(p\) in space, their director vector \(n=(n_1, n_2, n_3)\) can be expressed as

\[
n \approx \left( n_1, n_2, 1 - \frac{1}{2} n_1^2 - \frac{1}{2} n_2^2 \right)
\] (1.14)

since \(n\) is a unit vector. It can be shown that \(n_{3,i}=0\) where the tensor notation \(n_{i,j}\) is defined as

\[
n_{i,j} = \left. \frac{\partial n_i}{\partial x_j} \right|_{p}
\] (1.15)

such that

\[
n_{3,j} = \left. \frac{\partial}{\partial x_j} \left( 1 - \frac{1}{2} n_1^2 - \frac{1}{2} n_2^2 \right) \right|_{p} = n_1 \frac{\partial n_1}{\partial x_j} + n_2 \frac{\partial n_2}{\partial x_j} = 0
\] (1.16)
since \( n_1 = n_2 = 0 \) near point \( p \). In addition, it is obvious that
\[
\nabla \cdot \mathbf{n} = n_{1,1} + n_{2,2}
\]  
(1.17)
and
\[
\mathbf{n} \cdot \nabla \times \mathbf{n} = -n_1n_{2,3} + n_2n_{1,3} + n_3n_{2,1} - n_3n_{1,2} = n_{2,1} - n_{1,2}
\]  
(1.18)
and similarly
\[
\mathbf{n} \times \nabla \times \mathbf{n} = -(n_{1,3} + n_{2,3})
\]  
(1.19)
We can conclude that the free elastic energy is
\[
f = \frac{1}{2} K_1 \left( n_{1,1} + n_{2,2} \right)^2 + \frac{1}{2} K_2 \left( n_{2,1} - n_{1,2} \right)^2 + \frac{1}{2} K_3 \left( n_{1,3} + n_{2,3} \right)^2
\]  
(1.20)
Now it has become easier to visualize these terms

Fig 1.16  Schematics of three different types of molecule alignment for continuum studies and their mathematic expression

1.7 Free energy minimization

When the liquid crystal system suffers from boundary constrains, it will naturally deform (align) to the state such that the elastic free energy is minimized. To demonstrate how the liquid crystal sample realigns itself, we would take twist-nematics as an example, since it is also the one we will be dealing with in this dissertation. Twist-nematics is also one of the most widely used configurations of which the liquid crystal thin film sandwiched between a pair of orthogonal upper and lower boundary alignment constrains, as shown in Fig 1.17.
Fig 1.17 The director orientation of a twist-nematic liquid crystal thin film sandwiched between two glass slides

At the bottom boundary \( z=0 \), the director is set to be parallel to the \( x \)-axis, \( \phi(z=0) = 0 \), and at the upper boundary it is set to be a specific angle \( \phi(z=d) = \Phi \). We can reasonably assume that the liquid crystal molecules will remain in the horizontal plane and twist in a certain way along with \( z \) axis, such that the twist angle is a function of \( z \), \( \phi(z) \) and the director \( \mathbf{n} = (n_1, n_2, n_3) = (\cos \phi, \sin \phi, 0) \). That is, only the twist term exists in the free energy equation Eq.(1.13), and it becomes:

\[
f = \frac{1}{2} K_2 (\mathbf{n} \cdot \nabla \times \mathbf{n})^2 \tag{1.21}
\]

where

\[
\mathbf{n} \cdot \nabla \times \mathbf{n} = (n_1, n_2, 0) \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ n_1 & n_2 & 0 \end{vmatrix} = -n_1 \frac{\partial}{\partial z} n_2 + n_2 \frac{\partial}{\partial z} n_1
\]

\[
= -\cos^2 \phi \frac{\partial \phi}{\partial z} - \sin^2 \phi \frac{\partial \phi}{\partial z}
\]

\[
= -\frac{\partial \phi}{\partial z}
\]
and

\[ f = \frac{1}{2} K_2 \left( \frac{\partial \phi}{\partial z} \right)^2 \]  

(1.23)

To minimize \( f \), which is a function of \( \frac{\partial \phi}{\partial z} \) and \( \phi(z) \) is to be evaluated, we would have to apply the Euler-Lagrange equation, of calculus of variations[10]. In this case, the Euler-Lagrange equation for \( f(\phi, \phi', z) \) can be expressed as

\[ \frac{\partial f}{\partial \phi} - \frac{d}{dz} \left( \frac{\partial f}{\partial \phi'} \right) = 0 \]  

(1.24)

The \( f \) minimization can be reached when the above equation is satisfied, and we can write:

\[ 0 - \frac{d}{dz} \left( K_2 \frac{\partial \phi}{\partial z} \right) = 0 \]  

(1.25)

and thus

\[ K_2 \frac{d^2 \phi}{dz^2} = 0 \]  

(1.26)

With the above boundary conditions, we can solve this second-order ordinary differential equation easily and obtain:

\[ \phi(z) = \frac{\Phi + N\pi}{d} z \]  

(1.27)

where \( N \) is an integer stand for different local minima \( f \), and the global minimum energy is given by \( N=0 \) when \( 0 < \Phi < \pi/2 \). Therefore,

\[ \phi(z) = \frac{\Phi}{d} z \]  

(1.28)

and now we have proved that the twisted nematic liquid crystal molecules would align themselves “linearly” when the sample is subject to twist boundary conditions. Even though the results might look straightforward, the reason it works this way is not trivial.

1.8 Dielectric anisotropy

Due to the nature of rod-shaped liquid crystal molecules and its collective orientational alignment, for uniaxial nematic liquid crystals, almost all the physical properties differ in values along the director axis \( \mathbf{n} \) and perpendicular to \( \mathbf{n} \). These anisotropic physical properties include dielectric constants, refractive indices, electric conductivities, thermo conductivities, etc.

As we already know, the dielectric constant \( \epsilon \) is a tensor of rank 2 and is defined as
\[ \mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} = \left(1 + \varepsilon_0 \chi \right) \varepsilon_0 \mathbf{E} = \varepsilon_0 \mathbf{E} \]  

(1.29)

and thus

\[ \varepsilon_0 \left(1 + \varepsilon_0 \chi \right) \mathbf{E} \]  

(1.30)

where \( \mathbf{D} \) is the electric displacement (electric flux density), \( \mathbf{E} \) is the electric field, \( \mathbf{P} \) is the electric polarization (dipole moment density), \( \varepsilon_0 \) is the permittivity of free space, \( \chi \) is the electric susceptibility tensor and is linearly related to dielectric constant \( \varepsilon \) tensor. On a local material orthogonal coordinate basis \((x_1, x_2, x_3)\) such that the director \( \mathbf{n} \) collinear with \( x_3 \) axis (ie. \( \mathbf{n} = (0,0,1) \)), the \( \varepsilon \) tensor is diagonal for a uniaxial nematic liquid crystal:

\[
\varepsilon = \begin{pmatrix}
0 & 0 & 0 \\
0 & \varepsilon_\perp & 0 \\
0 & 0 & \varepsilon_p
\end{pmatrix}
\]

in the local material basis \((x_1, x_2, x_3)\).

(1.31)

When using tensor notation, \((x_1, x_2, x_3)\) can be treated as individual vectors (rank 1 tensor) \((\frac{x_1}{\partial x_1}, \frac{x_2}{\partial x_2}, \frac{x_3}{\partial x_3})\) in lab coordinate \((\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)\), and \( \mathbf{n} = \frac{x_3}{\partial x_3} \). Thus \( \varepsilon \) can be expressed in terms of dyads \( \frac{x_\perp}{\partial x_\perp} \) as

\[
\varepsilon = \varepsilon_\perp \left( \frac{x_\perp x_\perp}{\partial x_\perp \partial x_\perp} + \varepsilon_p \frac{x_\perp x_\perp}{\partial x_\perp \partial x_\perp} \right)
\]

(1.32)

For any orthogonal system, the second-order identity tensor is just the identity matrix

\[
I = \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

(1.33)

or in index notation

\[
I = \delta_{\mu\nu}\frac{x_\mu x_\nu}{\partial x_\mu \partial x_\nu}
\]

(1.34)

thus

\[
\varepsilon = \varepsilon_\perp \left( I - \frac{x_\perp x_\perp}{\partial x_\perp \partial x_\perp} + \varepsilon_p \frac{x_\perp x_\perp}{\partial x_\perp \partial x_\perp} \right)
\]

(1.35)

Rearranging this equation gives the most general form of anisotropic second-order tensor:
\[ \mathbf{\varepsilon} = \mathbf{I} + \Delta \varepsilon \mathbf{nn} \]  

(1.37)

or in index notation

\[ \varepsilon_{ij} = \delta_{ij} + \Delta \varepsilon n_i n_j \]  

(1.38)

The advantage of Eq.(1.37) over the matrix representation Eq.(1.33) is that Eq.(1.37) can be calculated in lab coordinate \( \{i\} \) without the need of coordinate transformation. In other words, if director \( \mathbf{n} \) in the lab coordinate \( \{i\} \) is of the value \((n_1, n_2, n_3)\), then the dielectric permittivity tensor \( \varepsilon \) in matrix form is

\[ \mathbf{\varepsilon} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \Delta \varepsilon \begin{bmatrix} n_1 n_1 & n_1 n_2 & n_1 n_3 \\ n_2 n_1 & n_2 n_2 & n_2 n_3 \\ n_3 n_1 & n_3 n_2 & n_3 n_3 \end{bmatrix} \]  

(1.39)

and then the electric displacement \( \mathbf{D} \) in tensor index form would be

\[ D_i = \varepsilon_{ij} E_j = \delta_{ij} E_j + \Delta \varepsilon n_i n_j E_j \]  

(1.40)

It is found that typical values of \( \varepsilon_\perp \) and \( \varepsilon_\parallel \) are on the order of \( \approx \varepsilon_0 \), and for most nematic liquid crystals (such as 5CB, 7CB, etc), they are positive anisotropic, \( \varepsilon_\parallel > \varepsilon_\perp \), while there are some exceptions of negative anisotropic, \( \varepsilon_\perp > \varepsilon_\parallel \), such as MBBA.

\( \varepsilon_\perp \) and \( \varepsilon_\parallel \) also suffer from dispersion. Some liquid crystals possess negative dielectric anisotropy \( \Delta \varepsilon = \varepsilon_\parallel - \varepsilon_\perp < 0 \) when under certain frequency but become positive dielectric anisotropy \( \Delta \varepsilon = \varepsilon_\parallel - \varepsilon_\perp > 0 \) when the frequency goes higher, or the other way around. This type of liquid crystals that change the anisotropic sign could be utilized for some special applications such as fast-switching display by modulating the applied electric field frequency. They are commonly called “dual-frequency liquid crystal” or DFLC. Some examples include MLC-2048,
which has $\Delta \varepsilon = 3.2$ at $f = 51$ kHz and $\Delta \varepsilon = -3.1$ at 50 kHz [11]. In fact, 5CB also change the dielectric anisotropic sign twice when the frequency is very high (~40 MHz and ~110 MHz) [12], but the degree of anisotropy gets smaller and thus not utilizable in general.

![Dielectric anisotropy of 5CB as a function of applied field frequency](Fig 1.18)

Dielectric constants are also a function of temperature and closely-related to order parameter as well. $\Delta \varepsilon$ gets slightly smaller when the temperature goes up and then becomes zero, where $\varepsilon_\perp$ and $\varepsilon_p$ merge to a single value $\varepsilon_{iso}$, when the temperature reaches the clearing point $T_c$ and the liquid crystal phase turns into isotropic liquid phase. The statistic value of $\varepsilon_{iso}$ is

$$\varepsilon_{iso} = \frac{\varepsilon_p + 2\varepsilon_\perp}{3}$$ (1.41)

![Dielectric constants as a function of temperature at 1kHz for 7CB and MBBA](Fig 1.19)

Similarly, other anisotropic physical parameters can be expressed and analyzed in a similar manner, such as magnetic susceptibility. More details can be referred to [13]

$$\chi_{ij}^m = \chi_\perp^m \delta_{ij} + \Delta \chi_\perp^n n_i n_j$$ (1.42)
References:
Chapter 2  Optical properties of liquid crystals

2.1 Field-induced reorientation[1]

As discussed in the previous chapter, dielectric anisotropy $\Delta \varepsilon$ (also written as $\varepsilon_a$) plays an important role for various applications such as liquid crystal switching behaviors because the anisotropy gives rise to the torque on the molecules when the material is exerted by an external electric field. From electrodynamics, we learned that the torque is generated when an electric field is applied on a dipole and it has the relationship:

$$\Gamma_{elec} = P \times E$$  \hspace{1cm} (2.1)

For simplicity, we would like to use

$$\Gamma_{elec} = D \times E$$  \hspace{1cm} (2.2)

since $D = \varepsilon_0 E + P$ and the first term will be zero when cross product with $E$. This way we will only have to work on $\varepsilon$ rather than $\chi$ ($\chi$ expression can be easily obtained at the very end as they are linear related).

To evaluate the torque $\Gamma_{elec} = D \times E$, we have to start from the $D = \varepsilon_0 E + P$. For the first time, we analyze the problem in both vector notation and tensor notation due to the fact that most books do not detail this aspect. In vector notation, we decompose $D$ into the components parallel and perpendicular to the director $n$

$$D = D_p + D_\perp$$

$$= \varepsilon_p E_p n_p + \varepsilon_\perp E_\perp n_\perp$$  \hspace{1cm} (2.3)

by assuming we can rotate the basis $(x_1, x_2, x_3)$ about $n$ such that $E = (E_\perp, 0, E_p)$. The term $E_\perp n_\perp$ can be expressed in terms of $n$ (ie. $n_p$) as

$$E_\perp n_\perp = E - (n \cdot E)n$$  \hspace{1cm} (2.4)

and the term $E_p n_p$ also

$$E_p n_p = (n \cdot E)n$$  \hspace{1cm} (2.5)

Thus we have
\[
D = \varepsilon_p((n \cdot E)n) + \varepsilon_\perp (E - (n \cdot E)n) \\
= \varepsilon_\perp E + (\varepsilon_p - \varepsilon_\perp)(n \cdot E)n
\]

(2.6)

therefore

\[
\Gamma_{\text{elec}} = D \times E \\
= \Delta \varepsilon (n \cdot E)(n \times E)
\]

(2.7)

For tensor notation, again:

\[
D_i = \varepsilon_{ij} E_j = \varepsilon_\perp \delta_{ij} E_j + \Delta \varepsilon n_j E_j
\]

(2.8)

And the torque \( \Gamma_{\text{elec}} = D \times E \) become

\[
\Gamma_i = D_m E_n \varepsilon_{mnl}
\]

(2.9)

where \( \varepsilon_{mnl} \) is the 3D permutation symbol in tensor analysis and has nothing to do with dielectric constant. Plug Eq.(2.8) into Eq.(2.9), we get

\[
\Gamma_i = D_m E_n \varepsilon_{mnl} \\
= (\varepsilon_\perp \delta_{m} E_j + \Delta \varepsilon n_m n_j E_j) E_n \varepsilon_{mnl} \\
= \varepsilon_\perp \delta_{m} E_j E_n \varepsilon_{mnl} + \Delta \varepsilon n_m n_j E_j E_n \varepsilon_{mnl}
\]

(2.10)

rearrange it and we obtain

\[
\Gamma_i = \varepsilon_\perp E_m E_n \varepsilon_{mnl} + \Delta \varepsilon n_j E_j n_m E_n \varepsilon_{mnl} \\
= \Delta \varepsilon (n_j E_j)n_m E_n \varepsilon_{mnl}
\]

(2.11)

Here we drop the first term since \( E_m E_n \varepsilon_{mnl} \) is exactly \( E \times E = 0 \). We find Eq.(2.7) is exactly the same as our tensor index notation Eq.(2.11) and thus confirm our derivation.

This torque generated from the applied electric field \( E \) is exerting on the liquid crystal molecules and is positive if positive anisotropy \( \Delta \varepsilon > 0 \). Thus molecules have the tendency to align their director \( n \) with the applied electric field \( E \). For the material with negative anisotropy \( \Delta \varepsilon < 0 \), the molecules tend to align the director normal to the field. It disturbs the original equilibrium state (the one defined by free elastic energy minimization and boundary conditions) and will reach a new equilibrium when the molecules reach the new energy minimum. This electric field can be a DC field, an AC field, or an optical field.

The additional orientational potential energy of the dipolar molecules caused by this applied electric field is

\[
U = \int dU = \int -P \cdot dE \\
= -\int (D - \varepsilon_0 E) \cdot dE
\]

(2.12)
Plug (2.6) into it and then simplify the equation to obtain

\[ U = -\int D \cdot dE + \int \varepsilon_0 E \cdot dE \]

\[ = -\int (\varepsilon_1 E + \Delta \varepsilon (n \cdot E) n) \cdot dE + \int \varepsilon_0 E \cdot dE \]

\[ = -\int (\varepsilon_1 - \varepsilon_0) E \cdot dE - \int \Delta \varepsilon (n \cdot E) n \cdot dE \]

\[ = -\frac{\varepsilon_1 - \varepsilon_0}{2} E^2 - \frac{1}{2} \Delta \varepsilon E^2 \cos^2 \theta \]

(2.13)

where \( \theta \) is the angle between \( n \) and \( E \). The first term at the right-hand side is independent of orientation angle \( \theta \) and is the constant electric energy stored in the space and is generally not considered in discussion. Therefore, the energy associated with the application of electric field can be written as

\[ U = -\int_0^E \Delta \varepsilon (n \cdot E) n \cdot dE \]

\[ = - \frac{1}{2} \Delta \varepsilon (n \cdot E)^2 \]

\[ = - \frac{1}{2} \Delta \varepsilon E^2 \cos^2 \theta \]

(2.14)

If we set the potential \( U=0 \) at \( \theta=0 \), then, at \( \theta=\theta \)

\[ U (\theta) = - \frac{1}{2} \Delta \varepsilon E^2 \cos^2 \theta \]

\[ = \frac{1}{2} \Delta \varepsilon E^2 \sin^2 \theta \]

(2.15)

This relation is exactly the same as the work required to counteract the torque and rotate the molecule from \( \theta=0 \) to \( \theta=\theta \), as indicated in Eq.(2.7):

\[ W = \int_0^0 \Gamma_{elec} \cdot d\theta \]

\[ = \Delta \varepsilon \int_0^\theta (n \cdot E)(n \times E) \cdot d\theta \]

\[ = \Delta \varepsilon \int_0^\theta E^2 \cos \theta \sin \theta d\theta \]

\[ = \frac{1}{2} \Delta \varepsilon E^2 \sin^2 \theta \]

(2.16)

The new equilibrium will be reached when the sum of elastic free energy and induced electric potential reaches its minimum. This external field induced electric potential plays an important part in liquid crystal molecules’ reorientation and will be discussed in further detail in the next chapter.
2.2 Field-induced director axis reorientation and Freedericksz transition [2]

2.2.1 Freedericksz transition

As discussed in the previous chapter, liquid crystal molecules possess an additional potential energy when suffering external electric and/or magnetic fields. These potential energy and elastic free energy tend to reorient the liquid crystal molecule director to another direction to reach overall energy minimum.

However, the applied field has to reach a minimum value to initiate the reorientation. This threshold value plays an important role on all active liquid crystal switching devices, such as liquid crystal display (LCD) and liquid crystal modulator. For simplicity, we start with a twist-only example.

In Fig. 2.1, a planar sample is set to align parallel to the x-direction from top and bottom of the sample and is suffered an external electric field E in y direction.

![Fig 2.1 The geometry of the Freedericksz transition under an applied electric field](image)

The total free energy of the system consists of the elastic energy Eq. (1.21) or (1.23) and the field-induced dipolar potential Eq.(2.15). (Note: \( \theta = \frac{\pi}{2} - \phi \))

\[
U_i = \frac{1}{2} K_2 \left( \frac{d\phi}{dz} \right)^2 + \frac{1}{2} \Delta \varepsilon E^2 \cos^2 \phi
\]  

(2.17)

Again, to minimize total free energy of the system by applying the Euler-Lagrange equation

\[
\frac{\partial U_i}{\partial \phi} - d \frac{d}{dz} \left( \frac{\partial U_i}{\partial \phi} \right) = 0
\]  

(2.18)

we have

\[
K_2 \frac{d^2 \phi}{dz^2} + \Delta \varepsilon E^2 \sin \phi \cos \phi = 0
\]  

(2.19)
Our goal is to find $\phi(z)$. However, this equation has no direct simple analytical solution and requires a special *Elliptic Integral* treatment. Let us first normalize the length unit. Set

$$\zeta = \frac{z}{d}$$  \hspace{1cm} (2.20)

such that $\zeta$ varies from 0 to 1 as $z$ ranges from 0 to $d$. This way we have $dz = d \cdot d\zeta$ and Eq.(2.19) becomes

$$\frac{K_2}{d^2} \frac{d^2\phi}{d\zeta^2} + \Delta \varepsilon E^2 \sin\phi \cos\phi = 0$$  \hspace{1cm} (2.21)

Also, set

$$p^2 = \frac{K_2}{\Delta \varepsilon E^2 d^2}$$  \hspace{1cm} (2.22)

Eq.(2.21) can be expressed as

$$p^2 \frac{d^2\phi}{d\zeta^2} \sin\phi \cos\phi = 0$$  \hspace{1cm} (2.23)

or

$$\frac{d^2\phi}{d\zeta^2} = -\frac{1}{p^2} \sin\phi \cos\phi$$  \hspace{1cm} (2.24)

To transform this equation to standard Elliptic Integral form, multiply $d\phi$ and take integral at both sides.

$$\int d\left(\frac{d\phi}{d\zeta}\right) \frac{d\phi}{d\zeta} = -\int \frac{1}{p^2} \sin\phi \cos\phi \cdot d\phi$$  \hspace{1cm} (2.25)

such that

$$\frac{1}{2} \left(\frac{d\phi}{d\zeta}\right)^2 = -\frac{1}{2p^2} \left(\sin^2\phi + C\right)$$  \hspace{1cm} (2.26)

where $C$ is the integration constant to be determined. Now, we know this system is symmetric to the middle plan $z = \frac{d}{2}$, i.e. $\zeta = \frac{1}{2}$ and the reorientation angle $\phi$ reaches its maximum $\phi_m$ such that $\frac{d\phi}{d\zeta} = 0$ at $z = \frac{d}{2}$. Plug this condition into the above equation and obtain the constant $C$.

$$C = -\sin^2\phi_m$$  \hspace{1cm} (2.27)

and
\[
\left( \frac{d\phi}{d\zeta} \right)^2 = \frac{1}{p^2} \left( \sin^2 \phi_m - \sin^2 \phi \right)
\]  

(2.28)

Take the square root

\[
\frac{d\phi}{d\zeta} = \pm \frac{1}{p} \sqrt{\sin^2 \phi_m - \sin^2 \phi}
\]  

(2.29)

The system can reorient either in positive \( \phi \) direction or negative \( \phi \) direction. Here we take positive direction as the example, such that

\[
d\zeta = \frac{p \cdot d\phi}{\sqrt{\sin^2 \phi_m - \sin^2 \phi}}
\]  

(2.30)

The **incomplete elliptic integral of the first kind** can be expressed in the form of

\[
F(k,\phi) = \int_0^\phi \frac{d\phi}{\sqrt{1 - k \sin^2 \phi}}
\]  

(2.31)

for \( k < 1 \), or in the form as

\[
\bar{F}(k,x) = \int_0^x \frac{dz}{\sqrt{1 - z^2 \sqrt{1 - k z^2}}}
\]  

(2.32)

when \( z = \sin \phi \). To have Eq.(2.30) in the standard form, we can set \( m = \sin^2 \phi_m \), and \( t = \sin \phi / \sin \phi_m \). This way

\[
d\phi = \frac{\sin \phi_m}{\sqrt{1 - \sin^2 \phi}} \ dt = \frac{1}{\sqrt{m - t^2}} \ dt
\]  

(2.33)

and Eq.(2.31) becomes

\[
\frac{d\zeta}{p} = \frac{d\phi}{p \sin \phi_m \sqrt{1 - \sin^2 \phi / \sin^2 \phi_m}} = \frac{d\phi}{p \sqrt{m \sqrt{1 - t^2}}}
\]  

(2.34)

Plug Eq.(2.33) into Eq.(2.34), we obtain

\[
\int_{\zeta=0}^\zeta \frac{d\zeta}{p} = \int_{t=0}^t \frac{dt}{\sqrt{1-t^2 \sqrt{1-m t^2}}}
\]  

(2.35)

where \( m < 1 \) and it is in the standard form of the **incomplete elliptic integral of the first kind** \( F(m,t) \), as Eq.(2.32), and \( F(m,t) \) can be evaluated from the integral table or numerical simulation. Thus

\[
\frac{1}{p} \zeta = F(m,t)
\]  

(2.36)
For \( z \) location range between \( \zeta : (0 \to \frac{1}{2}) \), \( \phi : (0 \to \phi_m) \) and thus \( t : (0 \to 1) \), now Eq.(2.35) becomes the **complete elliptic integral of the first kind** \( K(m) \)

\[
\frac{1}{2p} = \frac{1}{\sqrt{1-t^2}} \sqrt{1-mt^2} = F(m, t = 1) = K(m)
\]  

(2.37)

Before we go further, we should keep their relations in mind:

\[
E \leftrightarrow p \leftrightarrow K(m) \leftrightarrow m \leftrightarrow \phi_m
\]

or

\[
\frac{Ed}{2} \sqrt{\frac{\Delta e}{K_2}} = \frac{1}{2p} = K(m) \leftrightarrow m \leftrightarrow \sin^2 \phi_m
\]  

(2.38)

For each given electric field \( E \), we can obtain its corresponding maximum twist angle \( \phi_m \) through these relations. Our goal is to find out the threshold electric field \( E_t \), which would happen when \( m = \sin^2 \phi_m = 0 \). Fig 2.2 shows \( K(m) \) in terms of \( m \)

![Graph of K(m)](image)

Fig 2.2  The function plot of the complete elliptic integral of the first kind \( K(m) \)

\( K(m) \) starts from \( \pi/2 \) when \( m \) is zero and goes to infinity when \( m \) approaches 1. Physically, this means \( \phi_m = 0 \) as long as \( K(m) \leq \pi/2 \), or

\[
E \leq \frac{\pi}{d} \sqrt{\frac{K_2}{\Delta e}}
\]  

(2.39)

and finally we have this Freedericksz transition threshold value:
\[ E_i = \frac{\pi}{d} \sqrt{\frac{K_2}{\Delta\varepsilon}} \]  

(2.40)

It is worthy to note that this threshold switching voltage is independent of the sample thickness and is only the function of liquid crystal intrinsic properties – elastic constant and dielectric anisotropy.

\[ V_i = E_i d = \pi \sqrt{\frac{K_2}{\Delta\varepsilon}} \]  

(2.41)

Now, for \( E > E_i \), \( \phi_m > 0 \) (\( m>0 \)),

\[ \frac{Ed}{2} \sqrt{\frac{\Delta\varepsilon}{K_2}} = \frac{1}{2p} = K(m) > \frac{\pi}{2} \]  

(2.42)

and

\[ \frac{E_i d}{2} \sqrt{\frac{\Delta\varepsilon}{K_2}} = K(m = 0) = \frac{\pi}{2} \]  

(2.43)

Divide Eq.(2.42) by Eq.(2.43). We have the ratio

\[ \frac{E}{E_i} = 2 \pi K(m) = 2 \pi K(\sin^2 \phi_m) \]  

(2.44)

\[ \phi_m \]

\[ \pi/2 \]

**Fig 2.3** \( \phi_m \) plot in terms of the applied field ratio

<table>
<thead>
<tr>
<th>( \phi_m )</th>
<th>( E/E_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi/4 )</td>
<td>1.18034</td>
</tr>
<tr>
<td>0.3686( \pi )</td>
<td>1.500</td>
</tr>
<tr>
<td>0.4438( \pi )</td>
<td>2.000</td>
</tr>
<tr>
<td>0.48855( \pi )</td>
<td>3.000</td>
</tr>
<tr>
<td>( \pi/2 )</td>
<td>( \infty )</td>
</tr>
</tbody>
</table>
The threshold behavior is evident in this figure at $E = E_t$, and the maximum twist angle will approach 90 degree when the applied electric filed is several times of the threshold value.

We would also like to know how the twist angle $\phi(z)$ distributes along the $z$ axis under each specific applied field. Again, back to Eq.(2.36)

$$\frac{1}{p} \zeta = F(m,t) = F\left( m = \sin^2 \phi_m, t = \frac{\sin\phi}{\sin\phi_m} \right) = F(z,\phi)$$

and from Eq.(2.38) and Eq.(2.44),

$$\frac{1}{p} \zeta = Ed \sqrt{\frac{\Delta e}{K_2}} \zeta = 2K(m) \cdot \zeta = \frac{E}{E_t} \pi \cdot \zeta$$

Plug Eq.(2.46) into Eq.(2.45), we obtain

$$\zeta = \frac{F(z,\phi)}{\frac{E}{E_t} \pi}$$

For each given specific $E/E_t$ value, the maximum twist angle $\phi_m$ can be determined (or vice versa), and here the twist angle $\phi(z)$ in terms of normalized position $\zeta$ for some specific $E/E_t$ ratio can be plotted as:

Finally, we obtained the molecule orientation distribution when the sample is suffering external electric field in the twist geometry. For the electric field that could induce splay or bend geometries, their deduced solutions are found to be very similar to the result we just derived but with different elastic constants involved.
2.2.2 Optical field-induced director axis reorientation

Similar to the applied DC/low frequency field case in the previous section, when liquid crystal sample is subject to an applied high frequency AC or even optical frequency field, the light-induced molecular reorientation will occur. The optical Freedericksz transition threshold also exists due to the same nature as the DC electric field case.

For a more complex example, we have a homeotropically aligned nematic liquid crystal cell as its profile depicted in Fig 2.5. When the sample is shined with a linearly polarized light, the director axis of the molecules will rotate to an angle toward the light-polarization orientation in order to reach the free energy $F = F_d + F_{op}$ minimization. In this case, there is no twist but splay and bend existing in the system. Therefore, the elastic free energy $F_d$ is

$$F_d = \frac{1}{2} K_1 (\nabla \cdot \hat{n})^2 + \frac{1}{2} K_3 |\hat{n} \times \nabla \times \hat{n}|^2$$

(2.48)

Fig 2.5 Schematics of a homeotropic aligned liquid crystal cell subjected to an oblique polarized light and the optical field-induced-molecular dipole potential $F_{op}$ is similar to what we had in Eq.(2.12) and (2.13):

$$F_{op} = \int dU = -\int \mathbf{P} \cdot d\mathbf{E} = -\int (\mathbf{D} - \varepsilon_0 \mathbf{E}) \cdot d\mathbf{E}$$

(2.49)

and thus
\[ F_{op} = -\int (\varepsilon_\perp - \varepsilon_0) \mathbf{E} \cdot d\mathbf{E} - \frac{1}{2} \Delta \mathbf{E} \cdot (\mathbf{n} \cdot \mathbf{E}) \cdot d\mathbf{E} \]
\[ = -\frac{\varepsilon_\perp - \varepsilon_0}{2} E^2 - \frac{1}{2} \Delta \mathbf{E} \cdot (\mathbf{n} \cdot \mathbf{E})^2 \] (2.50)

From the above figure, we have \( \mathbf{n} = \sin \theta \hat{x} + \cos \theta \hat{z} \) and \( \mathbf{E}_{op} = E_x \hat{x} + E_z \hat{z} \), and we assume \( \phi \) is only the function of \( z \), thus \( \frac{\partial \phi}{\partial x} = \frac{\partial \phi}{\partial y} = 0 \). Also, we define \( \phi' \triangleq \frac{\partial \phi}{\partial z} \). It can be shown that,
\[
(\nabla \cdot \mathbf{n})^2 = \sin^2 \phi \left( \frac{\partial \phi}{\partial z} \right)^2
\] (2.51)
\[
|\mathbf{n} \times \nabla \times \mathbf{n}|^2 = \cos^2 \phi \left( \frac{\partial \phi}{\partial z} \right)^2
\] (2.52)
and
\[
(\mathbf{n} \cdot \mathbf{E})^2 = E_x^2 \sin^2 \phi + E_z^2 \cos^2 \phi + 2 E_x E_z \sin \phi \cos \phi
\] (2.53)

Plug them into total \( F = F_d + F_{op} \) and we have
\[
F = F_x + F_{op}
\]
\[
= \frac{1}{2} K_\perp (\nabla \cdot \mathbf{n})^2 + \frac{1}{2} K_\perp |\mathbf{n} \times \nabla \times \mathbf{n}|^2 - \frac{\varepsilon_\perp - \varepsilon_0}{2} E^2 - \frac{1}{2} \Delta \mathbf{E} \cdot (\mathbf{n} \cdot \mathbf{E})^2
\] (2.54)
\[
= \frac{1}{2} \left( \frac{\partial \phi}{\partial z} \right)^2 \left( K_\perp \sin^2 \phi + K_\perp \cos^2 \phi \right) - \frac{\varepsilon_\perp - \varepsilon_0}{2} \left( E_x^2 - E_z^2 \right) - \frac{\Delta \mathbf{E}}{4} \left( E_x^2 \sin^2 \phi + E_z^2 \cos^2 \phi + 2 E_x E_z \sin \phi \cos \phi \right)
\]

where \( \left( E_z^2 \right) = |E|^2/2 \). Now we know \( F \) is a function of \( (\phi, \phi', z) \), and \( \phi \) is a function of \( z \).

To find the minimum of \( F \), again, the Euler-Lagrange Equation is applied:
\[
\frac{\partial F}{\partial \phi} - \frac{\partial}{\partial z} \frac{\partial F}{\partial \phi'} = 0
\] (2.55)

First term of Eq.(2.55) is
\[
\frac{\partial F}{\partial \phi} = \frac{1}{2} \left( \frac{\partial \phi}{\partial z} \right)^2 \left( 2 K_\perp \sin \phi \cos \phi - 2 K_\perp \sin \phi \cos \phi \right) - \frac{\Delta \mathbf{E}}{4} \left( 2 E_x^2 \sin \phi \cos \phi - 2 E_z^2 \sin \phi \cos \phi + 2 E_x E_z \cos 2 \phi \right)
\] (2.56)
and
\[
\frac{\partial F}{\partial \phi'} = \frac{\partial \phi}{\partial z} \left( K_\perp \sin^2 \phi + K_\perp \cos^2 \phi \right)
\] (2.57)

take derivative on (2.57) with respect to \( z \)
\[
\frac{\partial}{\partial z} \frac{\partial F}{\partial \phi} = \frac{\partial^2 \phi}{\partial z^2} \left( K_1 \sin^2 \phi + K_3 \cos^2 \phi \right) + \left( \frac{\partial \phi}{\partial z} \right)^2 \left( 2K_1 \sin \phi \cos \phi - 2K_3 \sin \phi \cos \phi \right). \tag{2.58}
\]

Thus the minimum condition Eq.(2.55) could be rewritten as
\[
-\frac{1}{2} \left( \frac{\partial \phi}{\partial z} \right)^2 \left( K_1 - K_3 \right) \sin \phi \cos \phi + \frac{\Delta e}{4} \left( 2E_z \sin \phi \cos \phi - 2E_z \sin \phi \cos \phi + 2E_z E_z \cos 2\phi \right) - \frac{\partial^2 \phi}{\partial z^2} \left( K_1 \sin^2 \phi + K_3 \cos^2 \phi \right) = 0,
\]
which is the equilibrium condition for the reorientation angle. It can be rearranged as
\[
-\left( \frac{\partial \phi}{\partial z} \right)^2 \left( K_1 - K_3 \right) \sin \phi \cos \phi + \frac{\Delta e}{4} \left( 2\sin 2\phi \left( E_z^2 - E_z^2 \right) + 2E_z E_z \cos 2\phi \right) + \frac{\partial^2 \phi}{\partial z^2} \left( K_1 \sin^2 \phi + K_3 \cos^2 \phi \right) = 0 \tag{2.59}
\]

When \( E \) is complex, the above (2.60) can be further expressed as
\[
-\left( \frac{\partial \phi}{\partial z} \right)^2 \left( K_1 - K_3 \right) \sin \phi \cos \phi + \frac{\Delta e}{4} \left( 2\sin 2\phi \left( |E_z|^2 - |E_z|^2 \right) + \cos 2\Theta \left( E_z E_z^* + E_z E_z^* \right) \right) + \frac{\partial^2 \phi}{\partial z^2} \left( K_1 \sin^2 \phi + K_3 \cos^2 \phi \right) = 0 \tag{2.60}
\]

Now, for a special case that the director axis reorientation is very small \( \phi \to 0 \) and hence \( \sin \phi \to \phi \), \( \cos \phi \to 1 \). Also, the square of them are infinitesimal can be neglected, ie. \( \sin^2 \phi \to 0 \) and \( \left( \frac{\partial \phi}{\partial z} \right)^2 \to 0 \). The preceding equilibrium equation reduce to
\[
K_3 \frac{\partial^2 \phi}{\partial z^2} + \frac{\Delta e}{4} \left( 2\phi \left( |E_z|^2 - |E_z|^2 \right) + (E_z E_z^* + E_z E_z^*) \right) = 0. \tag{2.62}
\]

We can see only \( K_3 \) exists under this small \( \phi \) approximation, which mean only “bend” dominates and it makes sense to what we learned from the illustration in the previous chapter. Even the applied light field \( E \) is given, \( E_x \) and \( E_z \) are not constants but a function of \( z \). \( E \) field will change its direction as the medium (liquid crystal molecules) is changing the direction of anisotropy as well. This means Eq.(2.62) is not that easy to be solved yet.

How \( E_x \) and \( E_z \) change depends on the corresponding dielectric anisotropy at each location. Since the system is charge neutral, the Maxwell equation \( \nabla \cdot \mathbf{D} = 0 \) applies. In addition, we have seen \( \mathbf{D} = \varepsilon_0 \mathbf{E} \) or \( D_i = \varepsilon_{ij} E_j \) in the previous chapter. Since this system is all within \( x-z \) plane, we analyze it in two-dimensional mathematics. Along the direction of director axis, we know
\[
\begin{bmatrix} D_p \\ D_\perp \end{bmatrix} = \begin{bmatrix} \varepsilon_p \\ \varepsilon_\perp \end{bmatrix} \begin{bmatrix} E_p \\ E_\perp \end{bmatrix} \tag{2.63}
\]

To express this relation to the \( x-z \) coordinate system, we need to multiply the rotation matrix such that
\[
\begin{bmatrix}
D_z \\
D_x
\end{bmatrix} = \begin{bmatrix}
\cos \phi & -\sin \phi \\
\sin \phi & \cos \phi
\end{bmatrix}
\begin{bmatrix}
\varepsilon_p \\
\varepsilon_x
\end{bmatrix}
\begin{bmatrix}
\cos \phi & \sin \phi \\
-\sin \phi & \cos \phi
\end{bmatrix}
\begin{bmatrix}
E_z \\
E_x
\end{bmatrix}
\]
\[= \left[ E_z \left( \varepsilon_p \cos^2 \phi + \varepsilon_x \sin^2 \phi \right) + E_x \left( \varepsilon_p - \varepsilon_x \right) \sin \phi \cos \phi \right]
\]
\[+ \left[ E_z \left( \varepsilon_p - \varepsilon_x \right) \sin \phi \cos \phi + E_x \left( \varepsilon_p \sin^2 \phi + \varepsilon_x \cos^2 \phi \right) \right]
\]  
(2.64)

Assume the beam size is very wide when compared with the thickness \( d \), such that \( \frac{\partial}{\partial x} = 0 \) for every property. This way \( D_x, D_z, E_x, E_z \), and \( \phi \) are only the functions of \( z \). Therefore, we have \( \frac{\partial}{\partial z} D_z = 0 \), since \( \nabla \cdot D = \frac{\partial}{\partial x} D_x + \frac{\partial}{\partial z} D_z = 0 \).

so
\[
\frac{\partial}{\partial z} D_z = 0 = \frac{\partial}{\partial z} \left[ E_z \left( \varepsilon_p \cos^2 \phi + \varepsilon_x \sin^2 \phi \right) + E_x \left( \varepsilon_p - \varepsilon_x \right) \sin \phi \cos \phi \right]
\]  
(2.65)

Again, apply the small angle assumption \( \phi \rightarrow 0 \), \( \sin \phi \rightarrow \phi \), \( \cos \phi \rightarrow 1 \), and \( \sin^2 \phi \rightarrow \phi^2 \rightarrow 0 \), we have
\[
\frac{\partial}{\partial z} \left[ E_z \left( \varepsilon_p + \varepsilon_x \right) + E_x \left( \varepsilon_p - \varepsilon_x \right) \phi \right] = 0
\]  
(2.66)

We need to keep in mind that \( \varepsilon_p \) and \( \varepsilon_x \) are constants, and \( \phi \left( z \right), E_x \left( z \right), \text{ and } E_z \left( z \right) \) are all functions of \( z \), and they are all coupled with each other. This equation indicates
\[
\varepsilon_p E_z \left( z \right) + \left( \varepsilon_p - \varepsilon_x \right) E_x \left( z \right) \phi \left( z \right) = \text{const.}
\]  
(2.67)

Under our hard boundary condition \( \theta \left( z = 0 \right) = \theta \left( z = d \right) = 0 \) and we assume \( E_z \left( z = 0 \right) = E_{z0} \).

Plug them into the above equation to obtain their relation
\[
E_z \left( z \right) = E_{z0} - \frac{\Delta \varepsilon}{\varepsilon_p} E_x \left( z \right) \phi \left( z \right)
\]  
(2.68)

thus
\[
E_x E_z = E_{z0} E_x - \frac{\Delta \varepsilon}{\varepsilon_p} \phi E_x^2
\]  
(2.69)

and
\[
\left| E_z \right|^2 - \left| E_x \right|^2 = E_z^2 - E_{z0}^2 + 2 \frac{\Delta \varepsilon}{\varepsilon_p} E_{z0} E_x \phi
\]  
(2.70)

Eq.(2.62) becomes
\[ K_3 \frac{\partial^2 \phi}{\partial z^2} + \frac{\Delta \varepsilon}{4} \left( 2\phi \left( E_x^2 - E_{z0}^2 \right) + 2 \left( E_{z0} E_x - \frac{\Delta \varepsilon}{\varepsilon_p} \phi E_x^2 \right) \right) = 0 \]  

(2.71)

and further simplified as

\[ K_3 \frac{\partial^2 \phi}{\partial z^2} + \phi \left( \frac{\Delta \varepsilon}{2} \left( 1 - \frac{\Delta \varepsilon}{\varepsilon_p} \right) E_x^2 - E_{z0}^2 \right) + \frac{\Delta \varepsilon}{2} E_{z0} E_x = 0 \]  

(2.72)

Since \( E_x \) is also a function of \( z \), this second-order differential equation is non-linear and can only be solved numerically. For a special case when the light is at normal incidence, i.e. \( E_{z0}=0 \), we can reach a much more simplified form

\[ K_3 \frac{\partial^2 \phi}{\partial z^2} + \frac{\Delta \varepsilon}{2} \left( \frac{\varepsilon_\perp}{\varepsilon_p} \right) E_x^2 \phi = 0 \]  

(2.73)

The solution will be in the form of

\[ \theta \left( z \right) = A \sin \left( |E_x| \sqrt{\frac{\Delta \varepsilon}{2K_3} \frac{\varepsilon_\perp}{\varepsilon_p} - z} \right) + B \cos \left( |E_x| \sqrt{\frac{\Delta \varepsilon}{2K_3} \frac{\varepsilon_\perp}{\varepsilon_p} - z} \right) \]  

(2.74)

Again, applying hard B.C. \( \theta \left( z = 0 \right) = 0 \), we get \( B=0 \), then

\[ \theta \left( z \right) = A \sin \left( |E_x| \sqrt{\frac{\Delta \varepsilon}{2K_3} \frac{\varepsilon_\perp}{\varepsilon_p} - z} \right) \]  

(2.75)

For the other end \( \theta \left( z = d \right) \) to reach 0, the argument of sine has to meet the value of \( \pi \). Therefore,

\[ |E_x| \sqrt{\frac{\Delta \varepsilon}{2K_3} \frac{\varepsilon_\perp}{\varepsilon_p}} \geq \pi \]  

(2.76)

Finally, we have reached the optical Freedericksz transition threshold field

\[ |E_x| \geq \sqrt{\frac{2K_3 \varepsilon_p \pi}{\Delta \varepsilon \varepsilon_\perp d}} \]  

(2.77)

which is in the similar form as we had in the “twist” sample as Eq.(2.40).

2.3 Self-phase modulation, self-focusing, and self-defocusing

In the preceding section, we assumed the beam width is very large when compared with the
sample thickness. However, in a more practical case, the beam size can be comparable to sample thickness and then it can no longer be treated as a laterally uniform beam. A Gaussian profile could be a typical example for a TEM<sub>00</sub> mode laser beam. Due to this spatial non-uniform intensity profile, the liquid crystal orientation change caused by the applied light field higher than the threshold value discussed in the preceding section would modify the local refraction index and form a spatial refractive index distribution. This non-uniform spatial refractive index distribution would further alter the light propagation. For homeotropic-aligned geometry, the central part of the Gaussian beam with higher intensity would cause a larger tilt angle of the molecules and hence the higher corresponding refractive index seen by the light. This higher index at the beam center and lower index around the beam peripheral distribution works exactly like the positive (convex) lens and would focus the beam passing through.

Experimentally, for a Gaussian beam with diameter \( a \), light intensity across the beam can be expressed as

\[
I(r) = I_0 \exp\left(-\frac{2r^2}{a^2}\right) \quad (2.78)
\]

And the refractive index is to be written in the form of

\[
n = n_0 + n_2 (I - I_{FR}) \exp\left(-\frac{2r^2}{a^2}\right) \quad (2.79)
\]

It can be shown the focal length has the relation to \( n_2 \) as

\[
\frac{1}{f} = 4 \frac{n_2}{a^2} (I - I_{AF}) d \quad (2.80)
\]

where \( I_{AF} = I_{FR} + \frac{1}{4n_2dk} \), \( k \) is the wave vector, and \( I_{FR} \) is the Freedericksz threshold intensity.

Fig 2.7  Focal length reciprocal as a function of the pump beam intensity for 6CB homeotropic samples of 100\( \mu \)m (open circles and 700 \( \mu \)m (solid circles) (from [3]))
Theoretically, to evaluate the effective Kerr constant $n_2$, we would have to plug the intensity profile Eq.(2.78) into Eq.(2.73) or (2.75) to obtain $\phi(r,z)$, and then put it in the index ellipsoid relation to obtain the refractive index

$$n(\phi) = n(r,z) = \frac{n_n n_r}{\sqrt{n_o^2 \sin^2 \phi + n_z^2 \cos^2 \phi}}$$

(2.81)

And then the phase delay would become

$$\Delta \phi(r) = \frac{2\pi}{\lambda} \int_0^d (n(r,z) - n_o)\,dz$$

(2.82)

This way, the $\Delta \phi$ will look nasty and is not in the form of a paraxial spherical lens. This means this system is suffering an aberration (not a perfect lens) and, in fact, this is the real situation.

For thick media with light passing through, solving the beam profile would be very complex as the modulated beam pattern would then change the refractive index, which would cause the beam pattern to be changed again. In addition, it is not necessary for the system to reach a steady state. The “coupled wave equations” generally have to be solved numerically and are an important topic in nonlinear optics.

If the media is thin, such as our liquid crystal film samples, the beam propagation problem is greatly simplified. The sample with index variation can be seen as an intensity-dependent phase plate and the transmitted light far-field pattern is the diffraction pattern of the incident field and the phase plate.

Typically, when “self-phase modulation” (SPM) is discussed in nonlinear optics textbooks, it usually means temporal SPM, such as pulse dispersion, red / blue shift, and temporal soliton. Here, we are discussing the spatial SPM, which is induced by a laser beam with Gaussian profile, in general. Again, the CW laser beam profile can be expressed as in Eq.(2.78). The phase delay for the light passing through this sample is

$$\phi = \phi_0 + \phi_{NL} = \frac{2\pi}{\lambda} \cdot \frac{d}{n}$$

(2.83)

where $d$ is the sample thickness and

$$n = n_o + n_z I$$

$$= n_o + n_z I_o \exp \left(-\frac{2r^2}{a^2}\right)$$

(2.84)

Thus the phase shift is given by
\[ \varphi_{NL} = k \cdot d \cdot n_2 I_0 \exp\left( -\frac{2r^2}{a^2} \right) \]  \hspace{1cm} (2.85)

where \( k = 2\pi/\lambda \). From Fourier optics, we know the far field diffraction is actually a Fourier transform of the field at the exit plane of the sample. In cylindrical coordinate, the Fourier transform of the function \( g(r) \) would be in the form of Bessel function \( J_0 \). It is also called Fourier-Bessel transform or Hankel transform of zeroth order.

\[ G(\rho) = B\{g(r)\} = 2\pi e^{j\frac{2\pi z}{\lambda}} \int_0^\infty g(r) \cdot r \cdot J_0(2\pi r \rho) dr \]  \hspace{1cm} (2.86)

where \( \rho = r'/\lambda z \). Here our light field \( g(r) \) is the field at the exit plane of the sample:

\[ g(r) = \sqrt{I_0} \exp\left( -\frac{r^2}{a^2} \right) \cdot \exp(-j\varphi_{NL}) \]  \hspace{1cm} (2.87)

Plug it into Eq.(2.86) and then take absolute square to obtain the intensity pattern:

\[ I(r', z) = \left( \frac{2\pi}{\lambda z} \right)^2 I_0 \left| \int_0^\infty \exp\left( -\frac{r^2}{a^2} \right) \cdot \exp(-j\varphi_{NL}) \cdot r \cdot J_0 \left( 2\pi r \frac{r'}{\lambda z} \right) dr \right|^2 \]  \hspace{1cm} (2.88)

Moreover, If the incident laser beam is not a plane wave but a converging wave with wave front converging radius \( R \), the \( g(r) \) would possess another phase term \( \exp(-j\varphi_D) \), where

\[ \varphi_D = \frac{\pi}{\lambda R} r^2 = k \frac{r^2}{2R} \]. That is:

\[ g(r) = \sqrt{I_0} \exp\left( -\frac{r^2}{a^2} \right) \cdot \exp\left[ -j\left( \varphi_{NL} + \varphi_D \right) \right] \]  \hspace{1cm} (2.89)

This way, at distance \( z \) far away from the sample, the diffracted field could be written as

\[ I(r', z) = \left( \frac{2\pi}{\lambda z} \right)^2 I_0 \left| \int_0^\infty \exp\left( -\frac{r^2}{a^2} \right) \cdot \exp\left[ -j\left( \varphi_{NL} + \varphi_D \right) \right] \cdot r \cdot J_0 \left( 2\pi r \frac{r'}{\lambda z} \right) dr \right|^2 \]  \hspace{1cm} (2.90)

This diffracted field could be either focusing or defocusing depending on different parameters such as nonlinearity \( n_2 \), input beam converging radius \( R \), distance \( z \), and incident intensity \( I_0 \). Numerical simulation based on this model can be found in [4], which clearly shows all the possible diffraction patterns along with different intensity \( I_0 \) and \( z \). Experimental verification can also be found in [5].
2.4 Orientation photorefractive effect in LC

2.4.1 Introduction to photorefractive effect

The term “photorefractive effect” is sometimes used to describe any light-induced refractive index change phenomenon in a material. However, here we mean the refractive index change specifically results from the optically induced redistribution of electrons and holes in the material, which is the definition applied by most nonlinear optics literatures. We shall see that the photorefractive effect is different from other typical nonlinear optical effects in the fact that it cannot be described simply by the nonlinearity of susceptibility $\chi^{(n)}$ or permittivity $\varepsilon^{(n)}$. The change of refractive index is not the result of light intensity, but the result of intensity spatial modulation. In other words, if there is only one single spatially uniform light shined into the photorefractive material, there will be no refractive index change, even if the light is very strong. Therefore, a special model must be employed to describe the photorefractive effect.

The photorefractive effect was first discovered in 1966 by Ashkin et al. [6] in Bell Labs when they found the wavefront being disturbed by the non-uniformly distributed refractive index in the EO crystal. At the time, they called it “optical damage” as it was not the phenomenon they wanted to observe. Two years later, Chen et al. [7] successfully recorded the holographic interferogram in a poled lithium niobate (LiNbO$_3$) crystal, and it had been given this rather positive term “photorefractive effect” and started the glorious era of the research on photorefractive effect for more than 30 years. Most of the confirmed photorefractive materials are inorganic and they are called photorefractive crystal (PRC). In addition to lithium niobate, some of the other common ones include KNbO$_3$, BaTiO$_3$ (BTO), LiTaO$_3$, Bi$_{12}$GiO$_{20}$ (BGO), Bi$_{12}$SiO$_{20}$ (BSO), GaAs, InP, CeTe,… etc.

Several models had been proposed to describe this effect, and the Band Transport Model concluded by Kukhtarev [8, 9] is the most accepted and widely used one. It has also been called SPM (Standard Photorefractive Model). Although this model is mostly for solid photorefractive crystal, its basic concept also applies to our liquid crystal orientational photorefractive effect and is still worth an introduction.

As shown in Fig 2.8, within the PRC, some impurities and imperfections play the role of donors or acceptors between the valence band and conduction band. Depending on the type of materials, the free-charge carriers, which we assume are electrons here, for example, will be generated from the donors through photoionization at the rate proportional to the local value of the optical intensity in appropriate wavelength. The excited charge careers (electrons) can move freely in the conduction band and will be redistributed through the crystal based on three mechanisms: (1) diffusion, (2) drift, and (3) photovoltaic effect. Generally, when the crystal is under non-uniformly distributed illumination, the excited free careers will then migrate to vicinity
and could be trapped at dark region until being re-excited. A corresponding steady state will be reached when all three mechanisms balance each other locally.

![Fig 2.8 Schematics of the photorefractive “band transport model”](image)

When the photorefractive crystal is illuminated by two coherent beams with an angle intersecting each other within the crystal, the spatially sinusoidal interference pattern $I(x)$ will be produced. After the excited charge careers have enough time to diffuse and reach the steady state, charge density distribution would become $\rho(x)$ as in shown Fig. 2.9. This spatially varying charge distribution would give rise to a spatially varying electric field distribution $E(x)$. Note that this space-charged field $E(x)$ is shifted by $\pi/2$ with respect to $\rho(x)$ due to the fact that $\mathbf{\nabla} \cdot \mathbf{D} = \rho$ and it implies that $dE/dx = \rho/\varepsilon$ for one-dimensional case. The spatial derivative is the reason that it leads to a $\pi/2$ phase shift.

![Fig 2.9 Photorefractive space-charge induced index variation and spatial phase shift](image)

If the material exhibits linear electro-optic effect (Pockel’s Effect) and high dielectric
anisotropy of the photorefractive crystal, the local refractive index is then being modulated based on the space charge field it suffers. It can be shown that the index modulation is associated to the magnitude of the space-charge field $E_{sc}$ by

$$\Delta n = -\frac{1}{2} \gamma_e n^2 E_{sc}$$  \hspace{1cm} (2.91)

where $\gamma_e$ is the effective electro-optic coefficient of the light polarization and incident angle between the light beam and the PR crystal. More detailed derivation can be found in [10].

The non-local ($\pi/2$ phase-shifted) index grating is one of the important features among the photorefractive effect and it has led to enormous research and optical applications. As the photorefractive effect in solid crystals is a huge research topic in nonlinear optics and is beyond the scope of this dissertation, a more complete introduction and derivation on photorefractive governing equations and its applications can be found in some reference such as [11-14].

### 2.4.2 Raman-Nath & Bragg scattering

When talking about the refraction (or scattering) caused by an index grating, it is important to distinguish the case from two different regimes. One is the Bragg regime and the other is the Raman-Nath regime. We would like to briefly describe their features and conditions for applications. More detail and derivations can be referred to [15] and [16].

**Bragg scattering**

![Fig 2.10 Schematics of a thick grating and beam diffraction for Bragg scattering](image)

When the grating depth (interaction length) $L$, as shown in Fig 2.10, is sufficiently long enough for light to interact with the grating, it is in Bragg regime. This condition has to be satisfied:

$$L \tan \theta \geq \Lambda$$  \hspace{1cm} (2.92)

The Bragg condition must be satisfied as well:

$$2\Lambda \sin \theta = \lambda$$  \hspace{1cm} (2.93)
With these conditions, Bragg refraction leads to one single diffracted beam. When the incident angle is much smaller than unity, (2.92) and (2.93) become

\[ \frac{\lambda L}{\Lambda^2} \approx 1 \]  

(2.94)

The diffraction efficiency can be derived by solving the wave equation

\[ \nabla^2 E - \frac{n^2 + \Delta\epsilon^*}{c^2} \frac{\partial^2 E}{\partial t^2} = 0 \]

(2.95)

where the light field amplitude \( E = A_1 + A_2 \) and \( \Delta\epsilon^* \) is the dielectric constant function

\[ \Delta\epsilon^*(x) = \Delta\epsilon \cdot \exp(iq \cdot r) + c.c. \]

(2.96)

where \( q \) is the grating constant obviously. If the phase-matching condition is met (not shown in the figure)

\[ k_z = k_1 + q \]

(2.97)

The coupled equations of the transmitted and diffracted beam can be solved and their amplitudes are in the form of

\[
\begin{cases}
A_1(z) = A_1(0) \cos(|\kappa|z) \\
A_2(z) = \frac{i\kappa^*}{|\kappa|} A_1(0) \sin(|\kappa|z)
\end{cases}
\]

(2.98)

where \( \kappa = \frac{\omega^2 \Delta\epsilon^*}{2k_\epsilon c^2} = \frac{2\pi^2 \Delta\epsilon^*}{k_\epsilon \lambda^2} \). The diffraction efficiency is given by

\[ \eta = \left| \frac{A_2(L)}{A_1(0)} \right|^2 = \sin^2(|\kappa|L) = \sin^2\left(\frac{\pi \Delta\epsilon^* L}{\lambda}\right) \]

(2.99)

**Raman-Nath scattering**

Contrary to the Bragg regime, at the other limit when the grating depth is rather thin

\[ \frac{\lambda L}{\Lambda^2} < 1 \]

(2.100)

Raman-Nath scattering applies. Of this regime, several diffracted light beams can be present at the same time and the phase-matching condition is not important.
If the index grating is of the form
\[ n = n_0 + \frac{1}{2} \Delta n \cdot \left( e^{i q x} + c.c. \right) \]
(2.101)
and the electric field of the incident beam is
\[ E = A e^{i(kz - \omega t)} + c.c. \]
(2.102)
The phase shift this light beam experienced after passing through this grating would be
\[ \phi = \Delta n \frac{\omega}{c} L \cos(qx) \]
(2.103)
We can use sine function instead of cosine for simplicity. The transmitted field becomes
\[ E = A \exp \left[ i \left( k z - \omega t + \frac{\Delta n \omega L}{c} \sin(qx) \right) \right] + c.c. \]
(2.104)
By applying the Jacobi–Anger expansion
\[ e^{i \alpha \cos \beta} = \sum_{m=-\infty}^{\infty} i^m J_m(\alpha) e^{im\beta} \]
(2.105)
or to use
\[ e^{i \alpha \sin \beta} = \sum_{m=-\infty}^{\infty} J_m(\alpha) e^{im\beta} \]
(2.106)
The transmitted field can be expressed as
\[ E = A \sum_{m=-\infty}^{\infty} J_m \left( \frac{\Delta n \omega L}{c} \right) \exp \left[ i \left( kz + mqx - \omega t \right) \right] + c.c. \]
(2.107)
The transmitted field is in the form of superposition of multiple plane waves with different wave vector \( k + mq \) where \( m \) is an integer.
Thus the $m^{th}$ order diffracted wave is emitted at the angle

$$\theta_m = \tan^{-1}\left(\frac{mq}{k}\right) = \frac{mq}{k} = \frac{m\lambda}{\Lambda}$$

(2.108)

And the diffraction efficiency of the $m^{th}$ order beam is

$$\eta_m = J_m^2 \left(\frac{\Delta n d L}{c}\right)$$

(2.109)

It is worthy to note that the $\Delta n$ presented here is the amplitude of the sinusoidal diffractive index variation (as defined in Eq.(2.101)), not the refractive index anisotropy.

Eq.(2.109) is also often written as

$$\eta_m = J_m^2 \left(\frac{2\pi \Delta n L}{\lambda}\right)$$

(2.110)

as $\omega/c$ is substituted by $2\pi/\lambda$. A special case when $\frac{2\pi \Delta n d}{\lambda} = 1$ for the first-order diffraction ($m=1$), Eq.(2.110) can be approximated as

$$\eta \approx \left(\frac{\pi \Delta n L}{\lambda}\right)^2$$

(2.111)

[proof] Since Bessel Function can be expended in a polynomial series

$$J_p(x) = \sum_{m=0}^{\infty} (-1)^m \frac{x^{2m+p}}{m!(p+m+2)!}$$

When $p=1$ and $x<<1$, all the terms of $m>0$ can be neglected, thus

$$J_1(x) \approx \frac{x}{2}$$

and

$$J_1^2(x) \approx \frac{x^2}{4}$$

Therefore, here

$$\eta_1 = J_1^2 \left(\frac{2\pi \Delta n L}{\lambda}\right) \approx \frac{1}{4} \left(\frac{2\pi \Delta n L}{\lambda}\right)^2 = \left(\frac{\pi \Delta n L}{\lambda}\right)^2$$
Eq.(2.99) for Bragg diffraction and Eq.(2.110) for Raman-Nath diffraction are very similar to each other except the former one is in the form of sine square and dielectric anisotropy $\Delta \varepsilon$ while the latter one is of Bessel square and refractive index anisotropy $\Delta n$. Besides, it can be shown that $\Delta n = \Delta \varepsilon / 2n_0$. Experimentally, it is very common to have the need to evaluate the grating modulation depth, generally $\Delta n$. For this purpose, we can measure the power ratio of the diffraction beam to the incident beam and plug it into either of these two equations based on the sample conditions and to solve the equation.

### 2.4.3 Orientation photorefractive effect in LC

Nematic liquid crystals appear to be a very complex photorefractive material. The nature of molecule reorientation and elastic torque balancing when subject to applied field distinguishes liquid crystalline from other inorganic photorefractive crystals. For the case similar to previous discussion, when a nematic liquid crystal thin film is crossed with two coherent laser beams, the sinusoidal intensity grating would form inside the liquid crystal layer. Molecules situated at the brighter intensity region will undergo reorientation while those in the darker region will stay relatively unperturbed. In addition, the space charge field will also occur and strengthen the modulation. This nonuniformly distributed liquid crystal molecule reorientation will incur the index grating and induce the beam diffraction as well, in most cases, within Raman-Nath diffraction regime.

The very early discovery and study of orientational photorefractivity in nematic liquid crystal was initiated by Rudenko and Sukhov [17], and later on Khoo [18] developed a more complete theory in 1994. After that, numerous studies on various aspects of orientational photorefractivity in liquid crystalline systems had been published, such as [19], [20] and [21].

In 2006, Khoo and Chen [22] further concluded the theory of orientational photorefractive effect in nematic liquid crystal, with the analysis from both the bulk effect and surface-field effect. The bulk theory describes the general features of the self-diffraction and threshold DC field. This bulk effect is mainly due to the photo-induced space-charge field, which can occur from charge separation and dielectric anisotropies (Helfrich-Carr effect).
Fig 2.12 Schematics of a homeotropically aligned liquid crystal film subjected to orientational photorefractive effect and its interaction fields and molecule orientation

Fig 2.12 shows the typical two-wave mixing NLC photorefractive configuration where two equal-power mutually coherent beams with polarization lying in x-z plane are overlapped on a homeotropically aligned nematic liquid crystal cell with a small angle $\alpha$. The optical intensity grating is sinusoidally distributed in $q$ direction and the wave vector $q$ is tilted away from x direction by angle $\beta$. The director axis reorientation angle is $\theta$ such that $\mathbf{n}=(\sin \theta, 0, \cos \theta)$. To evaluate the director axis reorientation distribution $\theta(r)$, again we would have to come back to solve the free energy of the system

$$F = \frac{K}{2} \left[ \left| \mathbf{n} \cdot \mathbf{n}(r) \right|^2 + \left| \nabla \times \mathbf{n}(r) \right|^2 \right] - \frac{\Delta \varepsilon}{2} \left| \mathbf{E} \cdot \mathbf{n}(r) \right|^2 - \frac{\Delta \varepsilon_{\text{op}}}{2} \left| \mathbf{E}_{\text{op}}(r) \cdot \mathbf{n}(r) \right|^2$$

(2.112)

Here, single elastic constant approximation $K_1=K_2=K_3=K$ is assumed. After applying the Euler-Lagrange equation to evaluate the director axis orientation when the torque reaches balance (minimum free energy), it can be shown the angle $\theta$ will obey this equation

$$K \frac{d^2 \theta}{dz^2} + K \frac{d^2 \theta}{d\xi^2} + \Delta \varepsilon \left[ E_\Delta E_z \cos \beta + E_z E_{\text{ph}} \cos \beta \right] + \frac{\Delta \varepsilon_{\text{op}}}{2} \left[ \sin 2 \beta + 2 \cos 2 \beta \right] = 0$$

(2.113)

where $E_{ph} = E_{ph}^0 \cos (q\xi)$, $E_\Delta = \left( \frac{\Delta \varepsilon}{\sigma_z} + \frac{\Delta \varepsilon}{\xi_z} \right) E_{dc}$ and the solution is in the form of
\[
\theta = \theta_0 \sin \left( \frac{\pi z}{d} \right) \cos(q\xi) \tag{2.114}
\]

where
\[
\theta_0 = \frac{1}{2} \frac{\Delta e_w}{\Delta e} \frac{E^2_{op}}{E_{dc}^2} \frac{\sin 2\beta + E_{ph}^0 E_{dc} \cos \beta}{E_{dc} \cos \beta + \frac{\Delta e_w}{\Delta e} E_{op}^2 \cos 2\beta} \left[ 1 + \left( \frac{q_{PH}}{\pi} \right)^2 \right]^{-1} \tag{2.115}
\]

and \( E_F = \frac{\pi}{d} \sqrt{\frac{K}{\Delta \varepsilon}} \) is the Freedericksz threshold field.

For a 20µm thick 5CB homeotropic sample, for the grating constant \(2\pi/q \sim 20\mu\text{m}, \beta \sim 22.5^\circ\), at a low optical intensity \( \sim 1.4 \text{ mW/cm}^2 \), the 1st order Raman-Nath diffraction grating can be obtained once the DC voltage exceeds the threshold value 3.4 V.

It has also been shown that the photo-induced surface charge from the alignment layer (such as HTAB, hexadecyl-trimethyl ammonium bromide) could also lower the DC threshold value. A complete discussion and experimental demonstrations can be found in [23].
References:


Chapter 3  Optical Limiting with Liquid Crystals

3.1 Introduction to passive all-optical limiting/switching

Optical power limiting / switching is always of considerable interest to researchers. One of the main interests is to prevent damage to human eyes and optical devices from high-intensity light sources, such as lasers, sun glares, or welding flashes. However, it is quite challenging to achieve the protection with broadband coverage both in the temporal and spectral domain of the light sources. A process that is efficient in one time scale often becomes rather inefficient in another time scale. And material that reacts well in one spectrum region is often inactive in other spectrum regions.

Take human eyes, for example. To protect the eye, laser radiation reaching the retina must be less than 0.1 J/cm$^2$, which corresponds to a laser energy of 1 μJ (considering the eye as an optical system with focused-intensity gain of $10^5$). The device must also respond quickly enough within the duration of the pulse, which could be from picoseconds to nanoseconds. All-optical-driven techniques are required since other electronic/feedback-driven techniques may not meet the high-speed response requirement.

In this chapter, we begin with this protection objective and discuss the research that has been conducted in the past, then introduce our current development.

3.2 Literature Review

An ideal optical limiter for protection purpose should exhibit a linear transmission below a certain intensity value (threshold) and clamp the output to a constant when the incident power is above the threshold, i.e. performing high transmittance for weak-incident light but low transmittance for strong-incident light. Many optical-limiting techniques have been developed and Tutt et al. conducted a comprehensive review [1]. Most of the limiting techniques proposed rely on at least one material that exhibits certain nonlinear optical mechanisms. Those mechanisms include nonlinear absorption, nonlinear refraction, induced scattering, self-focusing, self-defocusing, and thermal-phase transitions. Each mechanism can be further subcategorized into a few different origins, and some materials may perform multiple simultaneous nonlinearities in a single limiting device. They have been intensively studied for materials and devices capable of such performance (for example: [2-4]):

Among these mechanisms, nonlinear absorption demonstrates a compelling nonlinear effect, particularly on materials that possess reverse saturable absorption (RSA), two-photon absorption (TPA), and excited-state absorption (ESA) [5, 6]. Saturable absorber dye was initially investigated for the application to Q-switch the pulse laser. In 1967, Giuliano et al. [7] first
recognized the “reverse” saturable absorption effect that some dyes not only do not bleach at low power but instead darken at high intensity. Generally, RSA materials have a low switching threshold (<mJ) but are absorptive and can be easily bleached at very high intensity (i.e. the dynamic range is rather short). TPA materials are more desirable for their faster energy absorption processes since their intermediate energy transition states are virtual and no actual energy needs to be converted to the intermediate state. Several TPA or even multi-photon absorption materials have been studied and synthesized in the form of dye solutions [8], dye-doped liquid crystals [9], and some organic compounds [8, 10]. For a material that possesses ESA properties its ESA cross-section is larger than that of the ground state, and thus it causes the absorption to increase with the input fluence [7]. These theoretical models on optical limiting can be found in [5, 11]. Recently, Khoo et al. [5, 9] proposed an unique liquid-cored fiber-array structure implemented with highly nonlinear absorption liquid L34 and liquid crystals that have shown limiting efficacy with clamp threshold below 1 μJ for picosecond- to nanosecond-pulse lasers [12].

On the other hand, anisotropic material, such as liquid crystal with large birefringence, also stands out as a highly promising optical-limiting material. As we have discussed in previous chapters, the birefringence of liquid crystals originates from the molecule shape and its self-alignment tendency, which shows different responsiveness to different electric field orientation (polarization). Several unique properties of liquid crystal that contribute to the optical switching behavior include laser-induced director reorientation, laser-induced order parameter modification [13], photochemical trans-cis phase transitions [14], and laser-induced temperature and phase changes [15].

Twist-Nematic (TN) liquid crystal cells, widely utilized for displays or modulators, are now proposed to be a passive all-optical self-limiter [16]. With the aid of various organic dyes, the doped nematic LCs demonstrate significant enhancement on their nonlinear optical responsiveness. In the past two decades, the nonlinear optical properties of dye-doped LCs in the nematic phase have been intensively studied. Some of these properties such as giant optical nonlinearity (GON) [17], supranonlinear effect [18], and surface-induced nonlinear effect (SINE) [19] are still the active subjects in this field. For limiting purpose, the basic idea is to produce a light-induced LC bulk orientation or phase change or anisotropy changes that then affect the light propagation with a resulting onset of a nonlinear optical response, and thus inducing optical switching / limiting behavior.

Due to the fact that limiting studies mentioned above are rather broad, in this dissertation, we would have to narrow down to the research on twist-nematic liquid crystal cell limiter, its mechanisms, theoretical model, and experiments. For the counterpart of this study including the research on nonlinear liquid switching and its mechanism such as TPA, ESA-based L34 nonlinear
fluid, and their fiber-cored array implementations for optical limiting, readers can refer to Stinger [20].

3.3 Optical Limiting with Liquid Crystals

Light modulation by liquid-crystal devices has become one of the most dominant liquid crystal applications today. From amplitude modulator, phase modulator, to liquid crystal displays (LCD), the importance of these tasks needs no further emphasis, and they have been widely investigated and commercialized in the past decades. However, its development is still far from complete. Among most of these electro-optical light modulators / switching devices, active control with electronic circuits is required. If they are used for limiting purpose, the response time is rather slow (up to ms scale, not including the feedback circuit time). For our optical-limiting purposes, we propose a passive all-optical twist-nematic liquid crystal cell that will not only effectively clamp down the strong incident light but also respond in the microsecond scale and work in broad wavelength region to meet our protection purpose [16].

3.3.1 Wave propagation in twist-nematic liquid crystal

Before we proceed to the liquid crystal switching mechanism, we like to visualize how the light propagates in a twist-nematic liquid crystal (TNLC) cell. Intuitively, we can roughly state that the polarization of a linearly polarized light will follow the liquid crystal director orientation while propagating through the liquid crystal cell. If the twist is 90°, then the polarization will rotate 90°; if it is a 180° or 270° super-twist (STN) cell, the polarization will rotate 180° or 270°, respectively. The rotation only happens during the propagation within the TNLC cell and the light is still linearly polarized when it exits the cell. It does not exit the TNLC cell in a circularly nor elliptically polarized configuration, as long as Mauguin condition, which will be discussed shortly, is satisfied.

![Fig 3.1 Schematic of the exploded TNLC sample diagram and polarizers](image-url)
Fig. 3.1 shows a 90° twist-nematic liquid crystal (TNLC) cell structure accompanied by a pair of crossed polarizers.

Assume this TNLC sample with thickness $d$ can be divided into $M$ layers ($M$ can be any large numbers), and the liquid crystal molecules of each layer have their own director orientation, which can be expressed by twist angle $\phi$ and tilt (out of x-y plane) angle $\theta$. These angles change continuously and gradually layer by layer because liquid crystal phase is a continuum. The thickness of each layer is $h$, i.e. $d = hM$. We can therefore treat each layer as a single phase retardation plate and the phase delay between extraordinary and ordinary axes of each layer is given by

$$\gamma = \frac{2\pi h \Delta n_m}{\lambda}$$  \hspace{1cm} (3.1)

where

$$\Delta n_m = \frac{n_e n_o}{\sqrt{n_e^2 \sin^2 \theta_m + n_o^2 \cos^2 \theta_m}} - n_o$$  \hspace{1cm} (3.2)

where $n_e, n_o$ are extraordinary and ordinary refractive index of the liquid crystal respectively and $\theta_m$ is the tilt angle of the $m^{th}$ layer as shown in Fig. 3.1.

To calculate the total effect of numerous layers of phase retarders, Jones matrix has to be applied, and we can represent one phase retarder as matrix

$$G = \begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & e^{\frac{j \gamma}{2}} \end{pmatrix}$$  \hspace{1cm} (3.3)

While this phase retarder is rotated $\phi$ with respect to lab coordinate x-axis, the total effect of this layer of liquid crystal (phase retarder) can be expressed as

$$J_m = S^{-1}(\phi_m) \cdot G \cdot S(\phi_m)$$  \hspace{1cm} (3.4)

where the axis rotation matrix $S(\phi_m)$ is

$$S(\phi_m) = \begin{pmatrix} \cos\phi_m & \sin\phi_m \\ -\sin\phi_m & \cos\phi_m \end{pmatrix}$$  \hspace{1cm} (3.5)

The total effect of the multiple layers of these phase retarders is the product of all phase retardation of each $J_m$

$$R = J_M \cdot J_{M-1} \cdots J_1 = \prod_{n=1}^{M} J_{M-(n-1)}$$  \hspace{1cm} (3.6)
Now, consider a polarizer of the orientation angle $\phi$ with respect to x-axis. This transfer matrix of this polarizer can be expressed as

$$P(\phi) = S^{-1}(\phi) \cdot \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \cdot S(\phi)$$

$$= \begin{pmatrix} \cos^2 \phi & \cos \phi \sin \phi \\ \sin \phi \cos \phi & \sin^2 \phi \end{pmatrix}$$  \hspace{1cm} (3.7)

Therefore, for the system with a polarizer, TNLC sample, and an analyzer in sequence, the transfer function of the whole system should be expressed as

$$A = P(\phi_{\text{Entr}}) \cdot R \cdot P(\phi_{\text{Exit}})$$  \hspace{1cm} (3.8)

where $\phi_{\text{Entr}}$ and $\phi_{\text{Exit}}$ are the orientation angle of the polarizer and analyzer, respectively.

When a linear polarized light $\Psi(z = 0) = \begin{pmatrix} E_x \\ E_y \end{pmatrix}$ enters the system, the output $\Psi(z = d)$ should be

$$\Psi(z = d) = \begin{pmatrix} E'_x \\ E'_y \end{pmatrix} = A \cdot \Psi(z = 0) = A \cdot \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$  \hspace{1cm} (3.9)

Thus the (irradiance) transmittance can be defined as

$$T = \frac{|E'_x|^2 + |E'_y|^2}{|E_x|^2 + |E_y|^2}$$  \hspace{1cm} (3.10)

With this relation, we can now verify if the polarization does follow the liquid crystal director axes.

### 3.3.2 Examples of wave propagation in twist-nematic liquid crystal

[Case 1] Parallel polarizers

Here we take two most encountered situations as examples. The first one is that both polarizer and analyzer are parallel to each other and aligned to x-axis, i.e. $\phi_{\text{Entr}} = \phi_{\text{Exit}} = 0$. Also we assume the sample is a pure twist, so only twist angles $\phi_m$ exist and there is no tilt (out-of-plane) orientation, i.e. $\theta_m = 0$. As mentioned in Section 1.7, the twist angle distribution under equilibrium will be linear with respect to the position $z$, and, therefore, for a 90° degree TN sample, its twist and tilt angle, as illustrated in Fig 3.2, is a straight line. Thus,

$$\phi_m = \frac{\pi / 2}{M} \cdot m$$  \hspace{1cm} (3.11)

and

$$\Delta n = n_e - n_o$$  \hspace{1cm} (3.12)
such that

$$\gamma = \Delta n \cdot \frac{2\pi}{\lambda} \frac{d}{M}$$

(3.13)

Fig 3.2  Liquid crystal tilt and twist angle distribution in z direction

Here we take $M=150$ for the calculation but it can be any fairly large integer, and we find that the transmission is a function of thickness $d$ and is plotted below in Fig 3.3. This curve is found to fit the curve of the Gooch & Tarry Equation [21].

Fig 3.3  Transmission curve of a TNLC sample as a function of thickness and anisotropy

[Case 2]  Crossed polarizers

For our TNLC limiting configurations, the analyzer is set perpendicularly to the entrance polarizer, i.e. $\phi_{\text{Exit}} = 0$ and $\phi_{\text{Entr}} = 90^\circ$. Assume no absorption, the transmission diagram of this case will be the value 1 minus the result of the first case. In other words, the diagram will be the same as Fig. 3.3 but flipped upside down.

For both cases, one thing worth our attention is that the transmission will approach to either zero (all blocked, a.k.a. Normally Black Mode in display terminology) for parallel case, or 1 (all
passed, a.k.a. *Normally White Mode*) for crossed case when the sample thickness $d$ is large enough, i.e. $d \Delta n / \lambda \gg 1$. This is the condition for the polarization of light to be able to “catch up and follow” the liquid crystal director orientation. This condition is the so-called Mauguin condition. In other words, for a TNLC cell to rotate the light polarization effectively, the multiplication of sample thickness and index anisotropy $d \Delta n$ must be much larger than the wavelength $\lambda$.

### 3.4 TNLC switching/limiting effect from laser-induced birefringence change

#### 3.4.1 TNLC switching mechanism -- A simplified model

As discussed in Section 1.8, liquid crystal dielectric constant and index of refraction are related to each other and both are the function of temperature. The degree of anisotropy will become smaller and smaller and finally merge to a single value at the clearing temperature $T_c$ (the temperature liquid crystal to change from nematic phase to isotropic liquid phase). As seen in Eq.(3.13), if the index anisotropy $\Delta n$ is zero, that means there will be no phase retardation and no fast and slow axes, and thus no polarization switching. The incident polarized light without being $90^\circ$ rotated will be blocked by the crossed analyzer and this is one of our approaches to limit (clamp down) the output for strong input.

To demonstrate the idea of the TNLC switching behavior without losing generosity, we propose a simple linear model, which we assume the birefringence $\Delta n$ decreases linearly with the increase of incident beam intensity $I$ until $n_e$ and $n_o$ reach the same value at $n_{iso}$, the index at isotropic phase, at the intensity $I=I_{iso}$, the intensity needed to turn the liquid crystal into the isotropic phase. Again, taking the liquid crystal 5CB as our example, the extraordinary and ordinary index of refraction at zero optical intensity are $n_e (I = 0) = 1.681$ and $n_o (I = 0) = 1.511$ at room temperature. Isotropic index will be the spatial statistical average of $n_e$ and $n_o$, $n_{iso} = \frac{1}{3} n_e + \frac{2}{3} n_o = 1.568$. Or, we can express the indices as a linear function of intensity $I$ in terms of $I_{iso}$ before they reach $I_{iso}$ as:

\begin{align*}
    n_o (I) &= 1.511 + \left( \frac{0.057}{I_{iso}} \right) I \\
    n_e (I) &= 1.681 - \left( \frac{0.113}{I_{iso}} \right) I
\end{align*}

(3.14) (3.15)
and thus \[ \Delta n(I) = n_e(I) - n_o(I) = 0.170 \left( 1 - \frac{I}{I_{iso}} \right) \] (3.16)

These relations can be easily illustrated in Fig 3.4 as shown below.

![Diagram showing index anisotropy as a function to intensity](image)

Fig 3.4 Dye-doped LC index nonlinearity as a function to intensity

The index anisotropy can be further expressed as \( \Delta n(I) = 0.170 - n_2 I \) where \( n_2 = 0.170/I_{iso} \), which can be easily realized from the diagram in Fig 3.4.

From Section 3.3, we know that as long as the Mauguin criterion \((d\Delta n \gg \lambda)\) is satisfied, the transmission is ideally 1 (100%). Since the index anisotropy is decreasing, the Mauguin condition will not be satisfied gradually and thus the transmission drops. After following the modified Jones matrix calculation as discussed in the previous section, we obtain the transmission curves as shown in Fig. 3.5 (b). It is clear that when the index anisotropy diminishes with the increasing input intensity, the output intensity is clamped down, creating the desired limiting effect.

![Fig 3.5 Transmitted output energy under (a) linear \( n_2 = 0 \), and (b) nonlinear \( n_2 \neq 0 \) cases for three different TNLC sample thickness](image)
3.4.2 Preliminary verification

5CB doped with a near IR absorbing dye Epolite® III 125 [Appendix A] is used for our liquid crystal cell. This dye absorbs broadly in the spectral range from 800 ~ 1600 nm. The TNLC cell is 20 μm in thickness, and the dye concentration used here is such that the transmission of the cell is ~60 percent at 1550nm wavelength. The Er-doped laser beam is not focused and has the diameter of 2mm. The results of this experiment on limiting are plotted in Fig 3.6.

![Graph](image1)

Fig 3.6 Measured dye-doped TNLC optical limiting curve

And these results can be compared with our previous developed theoretical simulation curve shown in Fig. 3.7.

![Graph](image2)

Fig. 3.7 Normalized limiting experiment data vs. simulation curve

As we can see, the experiment data agree with our model prediction curve very well except at the tail. The threshold is ~100mW. This result significantly supports our model but still has
some deviations indicating that our theory must be modified and improved. We also repeated the experiment with a lightly focused beam and obtained similar experiment data (Fig 3.8). Its threshold reduces to ~1mW.

![Fig 3.8 Dye-doped TNLC optical limiting curve under lightly focused laser beam](image)

In addition, we repeated the same series of experiments with methyl-red (MR) [Appendix B] dye-doped TNLC in the visible wavelength regime ($\text{Ar}^+ \text{ laser at 488nm}$) and observed similar results (Fig 3.9).

![Fig 3.9 Limiting curve of MR-doped 5CB at 488nm wavelength](image)

### 3.5 Landau-de Gennes Theory and Simulation

The preceding simplified model is just to demonstrate a rough idea of the liquid crystal birefringence changes due to strong incident beam power. To be more precise, this high-power laser-induced refractive index (birefringence) change in the TNLC sample is through a variety of mechanisms, including temperature-induced order parameter modification, field-induced
molecular realignment, and the order parameter disturbance due to dye absorption, etc. Several possible reasons can all contribute to the untwisting in the TNLC sample, and the switching results we have seen in the previous section are the overall behavior of all these possible factors. We are now introducing the Landau-deGennes theory to further depict the association between the birefringence change and the temperature change. This gives us a better understanding of the switching behaviors and the direction to meet a better performance.

### 3.5.1 Temperature distribution in a liquid crystal cell – a steady-state case

Landau-deGennes theory describes the coupling of order parameter and temperature. Before we reach the core of Landau-de Gennes theory, we would have to analyze the temperature distribution for each given incident light power.

For our experiment setup in which the transverse dimension (~ few mm) of the incident laser beam is much larger than the LC layer thickness (10~50 μm), we can employ a one-dimensional model to describe the laser absorption and heat-diffusion process. The entire LC cell can be represented by three layers (glass–liquid crystal–glass) as schematically depicted in Fig 3.10, where the LC layer (tens of micrometers) is much thinner than the enclosing glass slides (thickness ~1 mm). We also assume that: (1) conduction dominates the heat transfer process, (2) there is no mass flow inside the LC cell thus convection plays a negligible role, and (3) the temperature at the outer surfaces of the glass plate stays at room temperature $T_0$ (300 K).

![Fig 3.10](image)

Since there is no heat source in the glass slides, Fourier Law of Conduction applies.

$$ q = -k_s \frac{dT}{dz} $$

(3.17)
where \( q \) is the rate of heat transferring, \( k_g (=1 \text{ W/mK}) \) is the thermal conductivity of the glass, and \( T \) is the temperature. Assuming \( k_g \) is a constant, it is obvious that the temperature distribution is a linear function of the position \( z \) (as shown in Fig 3.10). The temperature distribution at the left and right side of the glass are respectively given by

\[
T_L(z) = Cz + D \\
T_R(z) = Ez + F
\] (3.18)

where \( C, D, E, \) and \( F \) are the constants that can be determined from the boundary conditions.

For the LC layers, the dye-doped LC absorbs part of the incident light and transforms the energy into heat. For the material with internal heat generation, the Fourier Law of Conduction becomes

\[
\frac{d}{dz}\left(k_{LC} \frac{dT}{dz}\right) + \Phi = 0
\] (3.19)

where \( \Phi \) is the heat generation rate per unit volume and \( k_{LC} (=0.15 \text{ W/mK}) \) is the LC thermal conductivity. We can assume the light absorption follows the regular exponential form, i.e. \( I = I_0 \exp(-\alpha z) \), where \( \alpha (=2 \times 10^4 \text{ m}^{-1}) \) is the absorption coefficient of the doped LC, and the rate of light absorbed at each particular \( z \) position is equal to the heat generated at that position. Therefore,

\[
\Phi = -\frac{dI}{dz} = -\frac{d}{dz}(I_0 e^{-\alpha z}) = I_0 \alpha e^{-\alpha z}
\] (3.20)

Eq. (3.19) results in the following solution

\[
T(z) = Az + B - \frac{I_0}{k_{LC} \alpha} e^{-\alpha z}
\] (3.21)

where \( A \) and \( B \) are constants and can be determined from the boundary conditions.

As for the boundary conditions, the temperature at two outer surfaces is assumed to be air temperature \( T_0 \) and temperature at the inner boundaries are continuous. In addition, heat transfer rate \( q \) also continues at each boundary. The six coefficients \( A \sim F \) can then be solved analytically and expressed in terms of the thickness and their thermal parameters of each of the corresponding layers:

\[
A = -\frac{e^{-\alpha d} I_0}{\alpha k_{LC}} \left( k_g \frac{e^{\alpha d}}{d} k_g + \alpha k_{LC} + e^{\alpha d} \alpha k_{LC} \right)
\] (3.22)

\[
B = I_0 \frac{1}{\alpha k_{LC}} \left( k_g \frac{e^{\alpha d}}{d} k_g + \alpha k_{LC} + e^{\alpha d} \alpha k_{LC} \right) + \frac{T_0}{\alpha k_{LC}} \left( d k_g + 2 \alpha k_{LC} \right)
\] (3.23)
Once we have obtained the temperature distribution of the TNLC sample, we can now proceed to the calculation of the order parameter distribution.

### 3.5.2 Landau-de Gennes Theory

A phenomenological model proposed by Landau-de Gennes [22, 23] states the free energy density $f$ in space can be expressed in terms of temperature distribution $T(z)$ and the power series of order parameter $S(z)$:

$$
C = \frac{e^{-\alpha} I_0 \left( k_x - e^{a t} k_x + d e^{a t} \alpha k_x - \alpha k_{LC} + e^{a t} \alpha k_{LC} \right)}{\alpha k_s \left( dk_s + 2tk_{LC} \right)}
$$

(3.24)

$$
D = \frac{e^{-\alpha} I_0 \left( k_x - e^{a t} k_x + d e^{a t} \alpha k_x - \alpha k_{LC} + e^{a t} \alpha k_{LC} \right)}{\alpha k_s \left( dk_s + 2tk_{LC} \right)} + T_0
$$

(3.25)

$$
E = \frac{e^{-\alpha} I_0 \left( -k_x + e^{a t} k_x - d \alpha k_x - \alpha k_{LC} + e^{a t} \alpha k_{LC} \right)}{\alpha k_s \left( dk_s + 2tk_{LC} \right)}
$$

(3.26)

$$
F = \frac{e^{-\alpha} \left( -(d + t) I_0 \left( \left(1 - e^{a t} + d \alpha \right) k_x - (1 + e^{a t}) \alpha k_{LC} \right) + e^{a t} \alpha k_s \left( dk_s + 2tk_{LC} \right) T_0 \right)}{\alpha k_s \left( dk_s + 2tk_{LC} \right)}
$$

(3.27)

Where $a$, $T^*$, $b$, $c$ and $L$ are material parameters. $g_1$ and $g_2$ are the surface potentials per unit volume and $S_1$ and $S_2$ are the surface order parameters. We assume that the low concentration of Epolite-dye does not change much of the overall thermodynamic properties of LC (5CB). Hence, the parameters for pure 5CB are used [24]: $(a = 6.5 \times 10^3 \text{ J/m}^3 \text{K} \text{ }, \ b = -5.3 \times 10^5 \text{ J/m}^3 \text{ }, \ c = 9.8 \times 10^5 \text{ J/m}^3 \text{ }, \ T^* = 307.14 \text{ K} \text{ and } \ L = 4.5 \times 10^{-12} \text{ J/m} \text{ )}$. For simplicity, we also assume $g_1 = g_2 = 0$, which means the transition of the surface order parameter follows that of the bulk order parameter. When the temperature distribution $T(z)$ is obtained, the equilibrium state of the order parameter $S(z)$ can be determined by minimization of Eq. (3.28) via the solution of the Euler-Lagrange equation as given by

$$
\frac{\partial f}{\partial S} - \frac{d}{dz} \left( \frac{\partial f}{\partial \left( \partial S / \partial z \right)} \right) = 0
$$

(3.29)

with boundary conditions
From Eq. (3.29) and (3.30), we obtained the following equations

\[ 2a(T(z) - T^*)S(z) + 3bS(z)^2 + 4cS(z)^3 - 2L \left( \frac{d^2S(z)}{dz^2} \right) = 0 \]

\[ -2L \left. \frac{dS}{dz} \right|_{z=0} - g_1 = 0, \quad 2L \left. \frac{dS}{dz} \right|_{z=d} - g_2 = 0 \]

With (3.31) and (3.32), we can further evaluate the position-dependent order parameter distribution \( S(z) \) for each given temperature profile \( T(z) \) by using numerical method (relaxation method). From this result, the optical phase retardation of each LC thin layer as discussed in Section 3.4.1 can be determined by using this index anisotropy relationship:

\[ \Delta n(z) = S(z) \cdot \Delta n_{\text{max}} \]

Finally, we can calculate the transmittance as a function of input power by using the Jones matrix method as described previously in Section 3.4.1.

### 3.5.3 Steady-state (CW) light-induced TNLC switching simulation and analysis based on Landau-de Gennes Theory

From the derivation described in the preceding sections, the temperature and order parameter distribution profile depending on the input power of light can be plotted out as shown in Fig 3.11 (a) and (b), respectively, for a TNLC (5CB) sample with 50 \( \mu \)m thickness near the threshold condition. In Fig 3.11(a), asymmetric distributions were obtained in the z direction because the beam intensity decreases (get absorbed) exponentially while it propagates in the doped LC film. As the power of the incident laser beam increases, the highest temperature near the center region also increases. In addition, Eq.(3.28) mathematically indicates that the theoretical nematic-isotropic transition temperature (\( T_{NI} \)) in the bulk LC is determined by \( T_{NI} = T^* + b^2 / 4ac = 308.24 \) K, which is plotted as the dotted horizontal line in Fig 3.11(a). Correspondingly shown in Fig 3.11(b), as the temperature becomes close to \( T_{NI} \), the order parameter tends to decrease dramatically. Ultimately, when the temperature becomes larger than \( T_{NI} \), the order parameter reaches zero, and this isotropic region is getting wider as the input power increases. This order-parameter-dependent optical phase retardation due to the change of index anisotropy is used to calculate the transmitted power based on the modified Jones matrix method as we detailed earlier. Fig 3.11(c) shows the calculated transmitted power as a function of the input power. As expected, the transmitted light begins to diminish as the input optical power is raised above the threshold value, and when at very high optical power, the NLC becomes isotropic.
and the output power is down to zero.

Fig 3.11  (a) The temperature distribution and (b) the corresponding order parameter distribution, for a 50 µm thick sample under various incident power. \( T_{NI} (= 308.24 \text{ K} ) \) represents the nematic-isotropic phase transition temperature. (c) The transmitted power as a function of the input power for this 50 µm thick sample

Fig 3.12(a) shows the calculated temperature distribution for various film thickness when the input power of 0.055 W is incident on the sample. Apparently, the temperature of the thicker sample is higher than the thinner ones, and thus the corresponding order parameter of the thicker sample is much smaller than the thinner ones as shown in Fig 3.12(b). Again, same as the case above, for the one with temperature higher than 308.24 K, the order parameter reaches zero completely.

Fig 3.12  (a) The temperature distribution and (b) the corresponding order parameter distribution for various film thicknesses at the fixed incident power 0.055 W
Fig 3.13 shows the calculated transmitted power under a wide range of the input power for various film thicknesses. The transmitted power is almost linear with respect to the input power until it reaches the threshold and then the output drops down (switches off) quickly due to reduced index anisotropy and lack of ability to rotate the light polarization. As expected, the threshold becomes lower as the film thickness increases.
So far, we have limited our discussion on thickness dependence. However, from the same theory and derivation given above, we may also predict the threshold-changing trend for other different sample parameters. Fig 3.14 demonstrates the trends of each different parameter. Fig 3.14(a) shows the threshold-reducing effect while the room temperature increases. Fig 3.14(b) shows the threshold also gets lower when the absorption coefficient (dye concentration) is higher. In addition, the thermal conductivity of cell glass also plays a role (Fig 3.14(c)). If the conductivity of the glass plate is larger, the LC switching threshold also becomes larger. This make perfect sense since the higher glass thermal conductivity means faster heat dissipating rate, and that lowers the central temperature in the liquid crystal layer. Similarly, higher thermal conductivity of liquid crystal also helps to increase the threshold, as shown in Fig 3.14(d). As for glass thickness, thicker glass slides induce lower threshold (Fig 3.14(e)).

3.6 Experiment on steady-state TNLC switching

For an experimental verification, we prepared a slightly wedged sample (0.12° off from parallel between two slides) with its alignment layers rubbed orthogonal to each other. This way, several different thicknesses can be obtained from one single sample rather than requiring several different samples and that avoids some system errors or deviations between samples. The inside of the cell is filled with Epolite III 125-dye doped liquid crystal (5CB), which forms a 90-degree twisted-nematic film. The optical properties are measured using a linearly polarized CW near-infrared Er⁺ fiber laser (wavelength 1550 nm) under the crossed-polarizers as illustrated in Fig 3.1. Fig 3.15 shows the experimental result of the transmitted power as a function of the input power for various film thicknesses. The threshold characteristics are in very good agreement with the theoretical expectations we have, although the curves behave slightly differently for the region that power above the threshold. We believe a more rigorous model is necessary to fully describe the behaviors. However, it is clear that the input threshold depending on the film thickness, is well-described by our model.
In Fig 3.16, the theoretical thresholds are compared with the experimental ones. We can see from both plots that threshold is inversely proportional to the film thickness and are in pretty good accordance with each other and within acceptable range of the deviation.

3.7 Azo-Photochemical Enhanced Liquid Crystal Nonlinearity

In 1991, Jánossy et al. [25] reported the dye-doped nematic liquid crystals under optical fields possess some anomalous behavior. The threshold of the optical Freedericksz transition for the homeotropic samples of guest-host mixture (dye concentration 1~2 percent) is two orders of
magnitude lower than the pure nematics. Later, Khoo [18] also showed the nonlinear coefficient $n_2$ of dye-doped nematic liquid crystal samples can reach a value as high as 6 cm$^2$/W. Thereafter, several investigations of liquid crystal optical nonlinearities, including optical Freedericksz transition, four wave mixing [26] and Z-scan [27], have shown that the thermodynamic coupling due to heating of the sample cannot explain the observations because different dyes having the same absorption coefficient gave rise to different nonlinear responses. Besides, the low concentrations of dyes in the liquid crystal mixtures only slightly modify the material properties such as dielectric anisotropy $\Delta \varepsilon$ or elastic constants $K$, and this cannot account for the huge change in optical nonlinearity.

Some theories trying to explain these behaviors have been proposed. Some are macroscopic stating that additional liquid crystal elastic torque could be induced by the dyes [26-28]. Further microscopic model was proposed by Marrucci and Paparo [29] on the angular momentum transfer between rotating molecules and its neighbor host molecules.

Among the dyes that could enhance liquid crystal nonlinearities, the photoisomerization featured in azo-dye is still the most effective and widely studied [14]. Azo-dyes are the derivatives of azobenzene, which is a compound with two phenyl rings linked by a N=N double bond. These azobenzene derivatives possess two geometric isomers, the $\text{trans}$ and the $\text{cis}$, as shown in Fig 3.17

![Fig 3.17 Trans-Cis isomerization of azobenzene molecules](image)

The $\text{trans}$ state is more stable and the overall molecular structure is more elongated. When exposed to a specific wavelength of light (UV ~green), the compound will be excited and undergo a trans-to-cis conversion to reach the less-stable $\text{cis}$ state, which has a geometrically distorted configuration and is harder to align with order. The $\text{cis}$-state will thermally relax back to the $\text{trans}$ state via cis-to-trans isomerization. The $\text{trans}$ isomer is more stable by approximately 50 kcal/mol, and the barrier to photo-isomerization is approximately 200 kJ/mol.

The mechanism of the trans-cis photoisomerization pathway has been a longstanding controversy among chemists. The possible conversion channels include the C-N=N bending
(inversion), C-N=N-C torsion (rotation), as shown in Fig 3.18, and the concerted C-N=N bending (concerted inversion) (not shown). The energy barrier and quantum yield of each excitation and relaxation (between states of $S_0 \leftrightarrow S_1$ and $S_0 \leftrightarrow S_2$) pathway was precisely calculated by Diau in 2004 [30], and he proposed a new mechanism which amazingly ruled out the widely accepted inversion channel. More detail of this complexity could be referred to [30].

Putting aside the *trans-cis* transition mechanism that we may leave to the chemists, this photoisomerization in nematic liquid crystal does effectively reorient the neighbor liquid crystal molecules, disrupt the overall alignment, and thus lower the order parameter and enhance the optical nonlinearity.

In the next chapter, we will show that the azo-dye in nematic liquid crystal not only enhances the nonlinearity but also boosts up the switching speed when compared with non-azo-dye doped samples, which have been tuned to possess the same absorption coefficient and thickness.
References:
27 I. Jánossy and T. Kósa, "Influence of anthraquinone dyes on optical reorientation of


Chapter 4 Multi Time Scale TNLC Switching Behaviors – Simulation and Experiment

4.1 Dynamic TNLC switching response under various conditions

Previously we demonstrated the steady-state switching threshold lowering upon increasing the incident cw laser power to the TNLC dye-doped sample. Here, we will investigate how fast the TNLC sample can switch off (clamp down) the output power.

§ Epolite® III 125-doped 5CB TNLC cell at 1550nm

Our very preliminary experiment [1] indicates the switching gets faster when the input laser power is higher. Fig. 4.1 shows a switching experiment of an Epolite® III 125-doped 5CB TNLC sample being switched by a lightly focused near-IR 1550 μm Er+ fiber laser. Epolite® III-125 is a broad-band IR dye with the maximum absorption at 950nm. Its characteristics and absorption spectrum are introduced in Appendix A. The beam diameter at the sample is ~0.2 mm and the sample thickness is 20μm. For input power above the switching threshold, the onset time (defined as the intensity drops to $e^{-2}$ of the initial value) decreases dramatically with the increase of laser power. The response time varies from ~3ms at moderate input power (~100mW) down to ~300 μs at high power (~766 mW). The latter value is actually limited by the mechanical shutter opening time, which is about 250~300 μs. A much shorter response time on the order of microseconds or even sub-microseconds can be achieved with the use of nanosecond laser pulses [2-4] or with the aid of other techniques.

![Switching off curves of a 20μm thick Epolite® III 125-doped 5CB TNLC sample under 1550 μm laser of ~0.2 mm spot size with different powers of step input](image)

**Fig. 4.1** Switching off curves of a 20μm thick Epolite® III 125-doped 5CB TNLC sample under 1550 μm laser of ~0.2 mm spot size with different powers of step input
As mentioned, in the rudimentary experiments, the mechanical shutter was used to generate a step inputs from a cw laser of various powers. However, the time required for the shutter to switch on is about 250~300 μs (varies from different units) and it is the bottleneck for this experiment. Later, we employed an E-O amplitude modulator together with a high-voltage amplifier and a function generator, which gave us the capability to switch on the system up to the speed of ~100ns (Bandwidth: DC~200MHz) and was able to meet our need for the experiment. (Fig. 4.2)

![Switching Speed Comparison](image)

Fig 4.2 A switching speed comparison of an EO modulator and a mechanical shutter

The EO amplitude modulator we selected is New Focus® 4102, which uses a MgO:LiNbO₃ as the EO crystal with the capable working wavelength 500~900 nm for DC~200MHz driving frequency. It was paired with a high-voltage amplifier, for which a PZT-based New Focus® 3121 was selected for its gain up to +/- 40, dynamic range from -200~+200 V, and up to 0.5MHz bandwidth. A special four-axis tilt aligner was used to mount the EO modulator in order to meet the alignment requirement for most experiments.

Following, we present some of our best TNLC switching results from our experiments.

§ Methyl Red doped 5CB TNLC cell at 532 nm

Methyl Red is one of the most widely used azo-dyes and is a common selection for liquid crystal absorption of green to blue wavelength. It was originally used as a pH indicator and it is red in pH under 4.4, yellow in pH over 6.2, and orange in between. The molecule structure of Methyl Red is shown in Fig 4.3 and we can easily recognize its azo trans-cis isomerization nature from the structure. The absorption spectrum of Methyl Red is attached in Appendix B.

![Methyl Red Structure](image)

Fig 4.3 Schematics of Methyl Red molecule structure

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Here we have a Methyl Red doped 5CB 0.5%wt 30 μm thickness sample, located at the focal point of a 81 mW 532nm laser beam, focal beam size ~0.4 mm. We can see the switch-on time is as fast as the speed of the EO modulator, and the switch-off time by the sample itself is ~200 μs. (Fig 4.4)

![Zoom in](image)

**Fig 4.4** A switching curve for the experiment when EO modulator is used

It is critical to employ a faster switch-on tool because a shorter switch-on time indicates it is faster to reach the maximum energy throughput, and thus shorter time is required to reach the switching threshold energy. Fig 4.5 shows a comparison of two experiments under the exact same condition (MR-5CB 0.5%wt TNLC, 30 μm thickness, 532nm Laser beam 122 mW, at the proximity of the focal point with beam diameter ~0.4 mm) but with different switch-on tools. The faster performance of the EO modulator setup confirmed not only the switch-on time but also the switch-off time are greatly reduced.

![Input](image) ![Output](image)

**EO Modulator**

![Input](image) ![Output](image)

**Mechanical Shutter**

*Fig 4.5* These two shots were taken under the same conditions, except the incident beam switch-on speed. The first one was switched on by EO modulator and the second one was switched on by a mechanical shutter. It is apparent that the response time gets longer when the input is switched on slower, even when they are under the same laser power.
Previous results were captured at the first laser shot for a clean/new spot of the sample; i.e. that spot of the sample has not been exposed to any laser beam before the ongoing trial. This way, we are sure all the results we obtained here are independent of the others and any kind of the memory effect from previous trial can be avoided. Here, on the contrary, we show that the switching speed is getting faster and faster when the same spot of the sample are exposed to a series of laser pulse train, as shown in Fig 4.6. Since the TNLC switching is mostly based on thermal heating and order parameter disturbing, the heat takes some time to diffuse and the liquid crystal molecules need some time to relax back to their original state. If multiple pulse trains were shined to the same spot of the sample without enough time for that spot to relax, the switching time we observed will certainly “look” smaller and smaller. Nevertheless, we have to know those results are not based on the same initial condition and are not the real improvement.

![Multiple pulse train and signal response](image)

Fig 4.6 t1–t5 stand for the time sequence after initially turning on the repetitive pulse trains. Repetition rate 5Hz, Δt~2 sec for each ON-state duration. We can see the limiting effect switches faster after more pulses exposure.

Indeed, the initial sample temperature does affect the switching speed and we will discuss the temperature-controlled experiment momentarily.
Fig 4.7 TNLC dynamic switching behavior under the condition of MR-5CB 0.5% wt TNLC cell, 30 μm thickness, 532nm laser of 105 mW, beam diameter ~0.1 mm. (a) and (b) are of the same trial data while (a) is in photo detector voltage signal and (b) is in calculated real output power. (c) and (d) are of the same experiment condition as (a) but just of different trials.

Here, we show some more switching curves with even shorter switching times obtained under a more optimized condition and a more focused beam size: MR-5CB 0.5% wt TNLC, 30 μm thickness, 532nm laser beam of 105 mW, beam diameter ~0.1 mm, as in Fig 4.7. The switch-off time is down to the order of 10 μs and the instantaneous peak output before being completely switched-off is ~1.9mW, or 1.8% in transmittance.

Similar to the aforementioned multiple experiment data we obtained, we have to realize that even under the same condition, the switching curves can still behave slightly differently in different trials. In order to compare the results, we quantitate the switching curves in terms of their switching time. When taking a closer look at these curves, it is worth noting that the switching curves seem to have two components: a fast decaying part and a slow decaying part. This phenomenon deserves a theory to explain it, and we shall discuss this matter later. To quantitate how fast the switching is, the decay time of both fast and slow switching components can be evaluated from curve-fitting data points of each trial, as shown in the example of Fig 4.8. Least-mean-square regression was used to evaluate the coefficients, and thus the decay time constants.
\[ f = y_0 + a \exp(-bt) + c \exp(-dt) \]

**Calculated curve-fitted value:**
\[ y_0 = 0.0025 \]
\[ a = 0.0220 \]
\[ b = 30.9985 \]
\[ c = 0.0053 \]
\[ d = 2979.7661 \]

Time constant 1 = 1/b = 33.2 \( \mu \text{s} \)
Time constant 2 = 1/d = 336 \( \mu \text{s} \)

Fig 4.8 Switch-off curve fitting

§ Thickness Dependence

A wedged 0.5%wt Methyl-Red doped 5CB TNLC sample was made for thickness dependence evaluation under the same experiment setup discussed previously (532 nm laser at 105 mW power, focal beam size ~0.1 mm). Several trials were conducted for each thickness and the best results of each condition are selected and shown in Table 4.1 and plotted in Fig 4.9 and Fig 4.10. From now on, not every captured switching response curve will be shown individually, but will be summarized in numbers and plotted or tabulated instead. The table “Transmittance at low power” indicates the “linear” transmission values measured under the incident light as weak as few mW and no any reaction can take place. “Peak transmittance” stands for the highest transmission value of the output spike generated when the high-power (~105mW) laser beam is turned on.

<table>
<thead>
<tr>
<th>Thickness (( \mu \text{m} ))</th>
<th>Time const 1 (( \mu \text{s} ))</th>
<th>Time const 2 (( \mu \text{s} ))</th>
<th>2 Transmittance at low power (%)</th>
<th>Peak output (mV)</th>
<th>Peak transmitted value (mW)</th>
<th>Peak transmittance (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.74</td>
<td>14.1</td>
<td>76.3</td>
<td>710</td>
<td>53.25</td>
<td>48.85</td>
</tr>
<tr>
<td>20</td>
<td>1.65</td>
<td>13.7</td>
<td>64.6</td>
<td>490</td>
<td>36.75</td>
<td>33.72</td>
</tr>
<tr>
<td>30</td>
<td>2.14</td>
<td>15.9</td>
<td>50.0</td>
<td>265</td>
<td>19.875</td>
<td>18.23</td>
</tr>
<tr>
<td>40</td>
<td>2.20</td>
<td>26.3</td>
<td>33.3</td>
<td>38</td>
<td>2.850</td>
<td>2.61</td>
</tr>
<tr>
<td>50</td>
<td>1.85</td>
<td>51.8</td>
<td>30.6</td>
<td>33</td>
<td>2.475</td>
<td>2.27</td>
</tr>
</tbody>
</table>

All the samples switch off the high-power input eventually without doubt. However, the peak transmittance of high-power input is only somewhat lower than its corresponding low-power transmittance (Fig 4.9). It is worth noting that the fast (1\text{st}) decay time constants are about the
same among all thickness, but the slower ($2^{nd}$) decay time constants are getting larger with thickness (Fig 4.10).

Fig 4.9  Instant peak transmittance upon laser incidence

Fig 4.10  Switching time for the samples of different thickness
§ Thickness and dye concentration dependence

A series of similar experiments were conducted under the conditions of different dye concentration and thickness, and the results are calculated and plotted in the following figures. For each condition, at least 20 to 30 measurements were conducted and about 5 to 10 best results among them were selected for further numerical evaluation. Each of the data point in the figures comprises these 5 to 10 selected ones and the error bar indicates their variation. All of them were measured under 532 nm cw laser at ~105 mW power, and focal point beam size ~0.1 mm.

► For 0.2% Sample
Fig 4.12 0.2% Methyl-Red doped 5CB TNLC switch-off speed of different sample thickness (a) 1\textsuperscript{st} decay time constant, (b) 2\textsuperscript{nd} decay time constant, (c) the combination of (a) and (b)

► For 0.5% sample

Fig 4.13 0.5% Methyl-Red doped 5CB TNLC switching time. The shortest 1\textsuperscript{st} switching time is 0.528 μs for the 50 μm sample

► For 1% sample
Fig 4.14 1.0% Methyl-Red doped 5CB TNLC switching time

For 1.5% sample

Fig 4.15 1.5% Methyl-Red doped 5CB TNLC switching time. (a)1st switching time, (b)2nd switching time, (c) the combination of (a) and (b). The shortest 1st switching time is 0.66 μs (avg) for the 20 μm sample

For 5μm Sample
Fig 4.16  Methyl-Red doped 5CB TNLC switching time for 5 μm sample but different dye concentration

All of them together in one single plot is shown in Fig 4.17 (a) and (b).
Unfortunately, from the results shown above, the lines are somewhat crooked, and not each tendency is consistent with the others. One possible reason for this inconsistency may come from the imperfection or non-uniformity of the samples. Even though every sample was made carefully under the same standard procedure, chances are non-uniformly distributed anchoring energy may come from non-uniformed manual rubbing, dust may cause localized liquid crystal alignment dislocation and uneven stress/strain plus sample thickness deviation, and some inconsistent cell glass heat treatment condition may affect the surface anchoring capability as well. Besides the factor from the samples, human errors from measurement data selection is also highly possible since we humans tend to select those curves that appear to have the best visual performance, but not necessarily the best numbers behind them.

Nevertheless, there are still few things worthy of remark. (1) Under this order of laser power exposure (532 nm cw laser, at 105 mW power, focal beam size ~0.1 mm), the first (fast) switching decay constant is in the order of few microseconds. (2) Also, in general, the higher dye concentration, the shorter the first switching time constant. When the absorption is rather high (dye concentration 1.5% sample), this first switching time can be down to sub-microsecond scale. (3) When the sample gets thicker, the switching speed gets slightly faster, but not too obvious.

The second (slower) decay time constants, they are all distributed in the 10~50 microseconds range. Similarly, the higher the concentration, the faster the second switching time constant, in general. However, the correlation between thickness and this slower part of switching speed is
not obvious.

Further, if we rearrange all of their first decay constants and put the dye concentration as the abscissa, we have the plot Fig 4.18.

![Graph](image)

**Fig 4.18** First switching time constant of each sample thickness and dye concentration

Even though Fig 4.17(a) and Fig 4.18 are of the same data, we may notice a rather uniform trend in Fig 4.18. That is, for the same thickness, the higher the concentration, the faster switching speed, generally.

The low-power (linear) transmission and the peak transmission upon a step high-power incidence of all these samples with different thickness and dye concentration are summarized in the following figures for reference purposes.
MR-5CB TNLC cell transmission under low power 532 nm wavelength with different dye concentration and thickness

![Graph showing transmittance vs sample thickness for different dye concentrations.](image)

Fig 4.19 Low-power transmittance for all different Methyl-Red doped 5CB TNCL samples

MR-5CB TNLC cell dynamic switching peak transmission of different dye concentration and thickness samples

![Graph showing instantaneous peak transmittance vs sample thickness for different dye concentrations.](image)

Fig 4.20 Instantaneous peak transmittance for each sample upon laser incidence

§ **IR-780 Iodide doped 5CB TNLC switching under 750 nm cw laser irradiance**

Similar experiments were conducted for 750 nm wavelength to validate the feasibility of the
switching behavior under the same TNLC configuration. Another dye has to be used as methyl-red is not absorptive in the near IR region. We selected “IR-780 Iodide” (CAS # 207399-07-3), which is a laser dye, from Sigma-Aldrich for its appropriate absorption spectrum, availability, and price. Its molecular structure, optical absorption spectrum, and fluorescence spectrum are included in Appendix C for reference.

For the 50 μm thick IR-780 Iodide 0.3%wt doped 5CB TNLC sample, its low-power transmittance is ~25% at 750 nm wavelength (measured by the spectrometer). A similar optical setup was used to detect its switching curves under high-power incidence. The laser beam was focused down to 40 μm by a microscope objective, and the input / output curves were recorded by the oscilloscope under different levels of input power. These switching curves are shown in Fig 4.21 (a), (b), & (c). Since the EO modulator for this wavelength was not available at the time, we were only able to show the slower switch-on version of this experiment conducted with the mechanical shutter. With the same mechanism and the similar switching behavior done with the mechanical shutter, a similar sub-microsecond switching speed can be foreseen when the input is controlled by an appropriate EO modulator.

<table>
<thead>
<tr>
<th>Input</th>
<th>Peak Output</th>
<th>1st time const</th>
<th>2nd time const</th>
</tr>
</thead>
<tbody>
<tr>
<td>41mW</td>
<td>13.97 mW</td>
<td>0.322 ms</td>
<td>3.12 ms</td>
</tr>
<tr>
<td>84mW</td>
<td>18.275 mW</td>
<td>0.278 ms</td>
<td>4.25 ms</td>
</tr>
</tbody>
</table>

![Fig 4.21 (a)](image1)

![Fig 4.21 (b)](image2)
Fig 4.21  Some switching curves of the IR-780 Iodide doped 5CB TNLC sample under 750 nm cw laser of different power (a) the smallest power and (c) the highest power of the three.

Similar to previous cases, the output will finally be clamped down to zero. There was an instantaneous onset peak output power increase along with the increasing incident power, but not to the same proportion. In other words, the onset peak transmittance goes down as the input power gets higher. The switching speed gets faster, too, which again meet our purpose of TNLC switching / limiting. The aforementioned data were summarized in Fig 4.22.

Fig 4.22  Switching decay time and input/output relations of a 50 μm thick IR-780 Iodide 0.3% wt doped 5CB TNLC sample

§ IR-780 Iodide doped 5CB TNLC at 532 nm cw laser irradiance

Even though IR-780 Iodide is specifically for near IR 700~850 nm range, it also absorbs 532 nm wavelength somewhat. Therefore, we would also like to know if it can help on switching just like methyl-red did at 532 nm wavelength. A series of trials have confirmed our idea and it shows
thickness dependence indicated in Fig 4.23

![Graph of 1st Time Constant vs Sample Thickness](a)

![Graph of 2nd Time Constant vs Sample Thickness](b)

Fig 4.23 Switching decay time of various IR-780 Iodide doped 5CB TNLC at 532 nm cw laser radiation

For reference, the low-power transmittance measured by the spectrometer at 532 nm for each sample with different thickness is shown in Fig 4.24.

![Graph of Transmittance vs Sample Thickness](IR-780 1% doped 5CB TNLC at 532 nm)

Fig 4.24 Low-power transmittance for the IR-780 Iodide doped 5CB TNLC samples

§ IR-780 Iodide doped 5CB TNLC switching under 750 nm pulse laser incidences

To further extend our investigation to the nanosecond time scale, this IR dye doped 5CB TNLC sample was again tested under the setup with a 750 nm pulse laser. This pulse alexandrite laser, model number 101-PAL (Pulse Alexandrite Laser), was manufactured by Light Age, Inc. of New Jersey. The emitted laser wavelength is tunable between 720 to 795 nm and was set to 750.8 nm for our experiment. This laser unit can operate in either Q-switched or free-running (longer pulse duration) modes, with the pulse duration 325 ns and an average of 3 μs, respectively. The
repetition rate is tunable between 5 to 10 Hz.

The optics setup can be illustrated as in Fig 4.25. Experiment wise, because this laser unit does not have the single-shot capability, an additional mechanical shutter was installed so that the shutter opening time was set to the hundred millisecond range, so only one laser pulse among a series of pulse train would pass through the system upon our manual triggering. A low-power visible cw laser (He-Ne 543 nm) was installed collinearly for alignment purposes since our human eyes are not sensitive to 750 nm wavelength but very sensitive to green wavelength regime. Also, it is generally better to use a lower-cost cw laser for alignment before the actual experiment is conducted in order to save the lifetime of the rather expensive pulse laser.

![Diagram of TNLC switching at 750 nm pulse laser experiment configuration](image)

**Fig 4.25** The schematic of TNLC switching at 750 pulse laser experiment configuration

A similar 50 μm thick 5CB TNLC sample doped with IR-780 Iodide dye 0.3% was made for this experiment. When the sample was placed a few millimeters away from the focal plane, an almost linear relationship was obtained, as shown in Fig 4.26(a). This plot is just to verify the sample does possess the linear property under low intensity (per unit area) even the exposed radiation is a nanoseconds-scale laser pulse rather than a cw laser beam. When the sample was moved toward the focal point, a nonlinear limiting effect was exhibited as the output was muted down gradually, as indicated in Fig 4.26 (b). The best result of this series of trials is pointed out in the figure and shown in Fig 4.26 (c) on an enlarged scale.
Fig 4.26  IR-780 Iodide-doped 5CB sample being shot with a 750 nm laser pulse when the sample is (a) away from the focal plane, (b) moving toward the focal plane, (c) the zoom-in plot for the best shot indicated in (b)

One thing worth mentioning again is that each trial (one data point) in the plots was done with the same sample but at a different ‘fresh’ spot. This way we can be sure each data point is independent and can be compared to each other without the concern of some accumulated effects such as thermal or chemical reactions. For each data trial, its input and output pulse shapes were also recorded. As indicated in Fig 4.27, the muted (limited) output pulse not only retains much less energy than the input pulse, but it also switches off earlier than the input pulse. Our theory and simulation also confirmed with this experimental result and will be presented later.
Fig 4.27 Input and output pulse signals of the best shot indicated in Fig 4.26 (b) captured by the oscilloscope. The output pulse is obviously being muted.

Fig 4.28 A series of more trials of the same 50µm 0.3% doped sample located at the 750 nm laser pulse focal plane (the focal location of the best shot indicated in Fig 4.26 (b)). The best clamped transmittance is as low as 0.038%.

Fig 4.29 A series of more switching experiments of the samples of different dye concentration and thickness under 750 nm laser pulse. The letter “d” indicates the sample start to damage at that input energy.
For thickness dependence, a series of trials on samples with different thickness and dye concentration were conducted. The main idea is to keep the sample about 50% transmitted under low radiation while tuning the dye concentration and thickness, and to observe their switching / limiting behaviors under high fluence laser pulses accordingly. Some more results are shown in Fig 4.28 and Fig 4.29.

Few even thicker samples of similar low-power transmittance were evaluated under the same setup and focal distance as in the previous trials (Fig 4.30).

The switching behavior under free-running mode was also conducted and the results are shown in Fig 4.31 and Fig 4.32.
Fig 4.32 Some pulse shapes captured by the oscilloscope of the data points indicated in Fig 4.31

Unfortunately, the “free-running” mode of this Alexandrite laser does not generate the radiation in a uniform “long-pulse” shape but instead in the form of a series of random spikes within one pulse package. However, after taking a closer look at its transmission distribution and comparing it with its counterpart of short-pulse version (Fig 4.29), we found that the result from the free-running mode actually provided a more clean-cut curve, almost like a right angle, which indicates a lower switching / limiting threshold.

This better performance is understandable because the sample has more time to absorb the laser energy and more time to react and switch, which of course provides better efficiency.

For reference, a series of transmission values of IR-780 Iodide doped TNLC 5CB sample with different dye concentration and thickness are tabulated in Appendix D.

§ Epolite® III 125 doped 5CB TNLC Switching under 1064 nm cw laser incidences

A 1064 nm Nd:YAG DPSS cw laser was used for further exploration on TNLC switching behavior. A similar optical setup was adopted and a series of Epolite® III 125 doped 5CB TNLC 50 μm cells were made with different dye concentration to be evaluated. The laser beam was focused by a microscope objective and the focal beam size was 60 μm in diameter. The samples were placed at the same focal plane for all the trials. Their steady-state switching behaviors of increasing incident laser powers are plotted in the following figures.
Fig 4.33 1% 50 µm Epolite® III 125-5CB at 1064 switching

Fig 4.34 0.5% 50 µm Epolite® III 125-5CB switching under 1064 nm cw laser

Fig 4.35 0.25% 50 µm Epolite® III 125-5CB switching under 1064 nm cw laser
Fig 4.36  0.125% 50 µm Epolite® III 125-5CB switching under 1064 nm cw laser

Fig 4.37  0.1% 50 µm Epolite III 125-5CB switching under 1064 nm cw laser

Or to plot all them together:

Fig 4.38  50 µm Epolite® III 125-5CB switching under 1064 nm cw laser with all different dye concentrations
§ Epolite® III 125 doped 5CB TNLC dynamic switching under 1064 nm laser incidences

To once again demonstrate the dynamic switching capability for dye doped 5CB TNLC cells, a mechanical shutter was added into the system and the output power was recorded upon the onset of laser incidence. Due to the similarity of their switching patterns, the values of their switching speed were not calculated individually for the fact that we already know the speed under current setup is limited by the mechanical shutter and it could have been a lot faster if we have the EO modulator for this wavelength. It is less meaningful to evaluate the values of speed limited by something other than the by TNLC cells themselves physically.

Instead, we present a series of switching curves for the same sample located at a different focal distance (Z-scan), as shown in Fig 4.39. The input laser power is 8.35 mW fixed at 1064 nm, and the Epolite® III 125 doped TNLC sample thickness is 50 μm and 0.5% wt concentrated. It can be seen clearly that when the sample is farther from the focal point, the irradiated area gets larger and the intensity gets lower and thus switches slower.

Fig 4.39 “Z-scan” perspective of the 0.5% 50 μm Epolite® III 125-5CB switching under 1064 nm cw laser
The diagram (Fig 4.40) shown below presents how the switching speed changes along with the change of incident laser power while keeping the sample at the same focal position. The smallest beam waist is ~60 μm and all the other sample parameters are the same as the ones used previously (0.5% wt Epolite® III 125 doped 5CB TNLC 50 μm at 1064nm wavelength).

![Switching curves for different input power](image)

Fig 4.40  Switching curves for different input power, from small (back) to large (front), of this 0.5% 50 μm Epolite® III 125-5CB sample under 1064 nm cw laser

### 4.2 Azo dye isomerization enhanced TNLC switching verification

As discussed in Section 3.7, not all the dyes work in the same manner. Azo-dye stands out from other dyes for TNLC switching purpose because its structure is capable of changing from *trans* to *cis* confirmation when subjected to strong illumination at the right wavelength. This spatial confirmation change thus perturbs the nearby alignment of liquid crystal molecules and reduces the order parameter abruptly. For non-azo dye doped samples, it is mostly just a thermal effect. However, for azo dye doped samples, there is the additional isomerization effect on top of the thermal effect. From our experiment, we noticed that even the samples are tuned to possess the same low-power absorbance of the same wavelength, azo dye doped TNLC samples still
switch faster than other non-azo dye doped samples in any condition. This indicates there must be some additional factor that helps boost the switching process. Hence, it supports our theory on azo-dye-assisted switching mechanism.

The TNLC samples we made are 1% IR-780 Iodide doped 5CB in five different thicknesses and 0.02% Methyl-Red doped 5CB in the thicknesses similar to IR-780 ones. Their measured low-power transmittance at 532 nm is plotted in Fig 4.41.

![Fig 4.41](image1)

Fig 4.41 The low-power transmittance for the samples we made for IR-780 and Methyl Red comparison purpose

Although we did our best to fabricate each pair of samples carefully and uniformly, a little deviation between samples might still exist and it is hard to make them of the exact same transmittance values. Nevertheless, their switching performances are in unison and showing methyl-red doped ones do switch faster in every case, as shown in Fig 4.42. These trials were done under the same setup as in Section 4.1 of 532 nm cw DPSS laser with EO modulator installed. The input laser power is 112 mW and the focal beam size is ~100 μm.

![Fig 4.42](image2)

Fig 4.42 Switching speed comparison for IR-780 Iodide and Methyl Red doped sample of same transmittance
4.3 Temperature effect on TNLC switching

In section 3.5, we discussed the association between liquid crystal order parameter and the switching behavior. One of the factors that determines the values of order parameter is temperature. Without loss of generality, 300 K was assumed to be the initial sample temperature for most of the simulations we performed. In addition, we also acknowledge that the higher the temperature, the lower the order parameter, as long as the sample is still in liquid crystal phase. If we can raise the initial sample temperature, less beam energy is required for the liquid crystal to reach the clearing temperature, and thus the complete switching. To verify this idea, a temperature-controlled experiment was conducted. (Fig 4.43 and Fig 4.44)

![Fig 4.43 Temperature effect on 5CB TNLC switching](image)

![Fig 4.44 Temperature effect on E7 TNLC switching](image)

The digital scanning calorimeter (DSC) measurement data of 5CB and E7 are attached in Appendix E where the clearing temperature, specific heat, and latent heat information can be determined. For 5CB, the clearing temperature is ~35°C. We did observe some enhancement on switching speed, but the difference was not too much since it is only a few degrees from room
temperature to 35°C and there is no too much room to improve. For E7, the clearing temperature is ~58°C and that gives us about 40 degrees dynamic range for temperature tuning, and we observed a prominent speed improvement. This improvement again confirms the trend we predicted from the theory.

4.4 Simulation on dynamic TNLC switching with the aid of COMSOL Multiphysics and Matlab

4.4.1 Transient switching model

Similar to the previous steady-state case in Section 3.6, our goal here is to simulate the input / output relations for the TNLC system under laser irradiance through the sequence illustrated below:

\[ \text{Input}(t) \rightarrow T(z,t) \rightarrow S(z,t) \rightarrow \Delta n(z,t) \rightarrow \text{trans}(t) \]

For the first arrow in this sequence, temperature distribution in the TNLC sample of each instance is to be evaluated. Previously, we assumed the conduction dominates the heat transfer process and neglected convection and radiation. We shall keep this assumption for dynamic response evaluation and for the reasons that: (1) there is no time for molecules to flow around and incur mass transfer, and (2) the boundary layer effect plus already-aligned liquid crystal phase with high viscosity in such a thin layer make it difficult to flow within the sample cell. However, the dynamic heat diffusion equation is not as simple as the steady-state one. For any conductive system, the general form of heat diffusion equation with heat source is in the form of:

\[
\rho_0 C_p \frac{\partial T(y,t)}{\partial t} - \nabla \cdot (k \nabla T(y,t)) = \phi(y,t)
\]  

(0.1)

where \( \rho_0 \) is the density of the medium, \( C_p \) is the specific heat of constant pressure for this medium, \( k \) is thermal conductivity, and \( \phi \) is the heat generation rate per unit space. Again, if all the absorbed light energy is transformed into heat, for the one-dimensional case,

\[
\phi = -\frac{dI}{dz} = -\frac{d}{dz}(I_0 e^{-az}) = I_0 \alpha e^{-az}
\]  

(0.2)

Here, though similar to Eq.(3.26), we have to add another assumption that the absorption coefficient is a constant within the system and is not a function of either location \( z \) or time \( t \).

For the previous steady-state case, the first-time derivative term is zero and can be dropped such that the remaining terms become the so-called “Fourier’s Law of Conduction” as we discussed in Section 3.6. Here, however, there is no reason to neglect the time derivative term for dynamic cases.

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A more complete and rather complex model for this transient (dynamic) switching behavior should be considered, with appropriate boundary conditions and initial condition.

As seen in this figure, our TNLC sample can be shown as a simple three-layer system exposed in the air of room temperature. Note that this figure is not to scale and the glass slide thickness $d$ (1 mm) is $\gg$ liquid crystal layer thickness $t$ ($\sim 50 \, \mu\text{m}$). We know the heat transfer between the ambient air and the glass slides is convective. However, since $d \gg t$, glass slides are like heat reservoirs to the thin liquid crystal layer and the heat does not have enough time to dissipate through the other end of the glass slide under the transient time regime we are observing ($\ll \text{ms}$). This way, we can ignore the convection with air at the outer surface and put the glass thickness as infinity for calculation.
Therefore, we are now able to specify the heat diffusion equation for each region of this model:

**Liquid crystal region** \(0 \leq z \leq d\)

\[
\rho_{LC} C_{p,LC} \frac{\partial T(z,t)}{\partial t} - k_{LC} \frac{\partial^2 T(z,t)}{\partial z^2} = \delta(z) = \alpha I_0 e^{-\alpha z}
\]  
(0.3)

with the initial condition:

\[T(z,t=0) = T_0\]  
(0.4)

and boundary conditions:

at \(z=0\), temperature continues and heat transfer rate continues,

\[q = k_{LC} \left. \frac{dT}{dz} \right|_{z=0^+} = k_g \left. \frac{dT}{dz} \right|_{z=0^-}\]  
(0.5)

and

\[T_g \left( z=0^-, t \right) = T_{LC} \left( z=0^+, t \right)\]  
(0.6)

similarly, at \(z=d\)

\[q = k_{LC} \left. \frac{dT}{dz} \right|_{z=d^-} = k_g \left. \frac{dT}{dz} \right|_{z=d^+}\]  
(0.7)

and

\[T_{LC} \left( z=d^-, t \right) = T_g \left( z=d^+, t \right)\]  
(0.8)

**Left side glass region** \(z < 0\)

\[
\rho_g C_{p,g} \frac{\partial T(z,t)}{\partial t} - k_g \frac{\partial^2 T(z,t)}{\partial z^2} = 0
\]  
(0.9)

with the same initial condition:

\[T(z,t=0) = T_0\]  
(0.10)

and the boundary condition:

at \(z=0\), heat flux and temperature continues, same as Eq.(0.5) and (0.6),

at \(z \to -\infty\)

\[T(z \to -\infty, t) = T_0\]  
(0.11)

**Right side glass region** \(z > d\), it is similar to the life side glass region

\[
\rho_g C_{p,g} \frac{\partial T(z,t)}{\partial t} - k_g \frac{\partial^2 T(z,t)}{\partial z^2} = 0
\]  
(0.12)

initial condition:
and boundary condition:
at \( z=d \), same as Eq.(0.7) and (0.8).
at \( z \to \infty \)

\[
T(z \to \infty, t) = T_0
\]  

Now, we have reached the point to solve these equations. We actually only need the solution \( T(z, t) \) of the liquid crystal region, and, if possible, we don’t have to solve the glass regions completely. However, even though this model looks quite simple and straightforward, solving this problem analytically is not. Taking a simple semi-infinite object as an example, when it is exposed to a constant surface temperature or a constant surface heat flux without a heat source, its solution would involve the Gaussian Error function. For a finite-size object with a heat source inside, when it has homogeneous boundary conditions, method of eignefunction expansion needs to be used. With non-homogeneous boundary conditions, Green’s function would have to be used. In our rather complex case, it has one finite-sized object with non-uniform heat generation heat source \( (\mathcal{Q}(z) \neq \text{const.}) \) and the non-homogeneous boundary conditions, plus two semi-infinite objects with non-constant (non-homogeneous) boundary conditions. After seeking the solution from various heat transfer literature thoroughly, we believe there is no well-developed analytical solution for our case so far. We could, perhaps, develop this analytical mathematics by ourselves. However, in considering the research progress, it would probably not provide the best interest for us. Furthermore, even if the analytical solution for \( T(z, t) \) can be reached, a numerical method for solving the order parameter \( S(z,t) \) is still required since we are sure there is no analytical solution for \( S(z,t) \); thus it is not very meaningful to spend extra effort just trying to obtain the analytical form of the temperature profile, if it does exist. We could just simulate this model numerically and still get a quite accurate result.

We could, of course, write up our own programming code to evaluate the instantaneous temperature profile by solving this heat transport governing equation, and then carry the temperature result into the same Matlab program we had in Chapter 3 to evaluate the order parameter transient profile \( S(z,t) \). Yet we do not really have to handle all of these jobs from the very bottom by ourselves since there are already several well-developed simulation packages commercially available with the features that can fit our needs.
4.4.2 COMSOL-aided simulation

COMSOL Multiphysics® (abbreviated COMSOL) was chosen for our simulation for its powerful “multiphysics” feature, user interface, accuracy, and availability. COMSOL is actually a famous finite-element-method-based analysis (FEA) and simulation package. It was formerly called FEMLAB for years and the name was changed to COMSOL a few years ago. Its full features and application-specific modules are quite broad and beyond the scope of our discussion. Interested readers should refer to the website and user’s manual for more information.

Part 1 One-dimensional examples

We can start with one-dimensional cases as follows:

Example 1

<table>
<thead>
<tr>
<th>Absorption coefficient $\alpha$</th>
<th>$2 \times 10^4$ [m$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intensity $I_0$</td>
<td>$7.96 \times 10^7$ [W/m$^3$]</td>
</tr>
<tr>
<td></td>
<td>(Input power 100mW, beam diameter 40µm)</td>
</tr>
<tr>
<td>Sample thickness</td>
<td>50 [µm]</td>
</tr>
<tr>
<td>Initial temp</td>
<td>300K</td>
</tr>
</tbody>
</table>

At $t=80$ µs, the sample temperature distribution is shown in this color map. Here, only the very center region of the sample is shown and most of the glass regions at both sides are not displayed. The laser irradiance comes from the left side and that causes the left side of liquid crystal layer to heat up faster. The temperature profile of each instance after the sample being exposed to the laser can also be expressed in the form of curves as shown at the right-hand side figure.

Fig 4.47 (left, Z-Y coordinate, in [m] and [m]) Temperature distribution of a TLNC sample after being exposed to a laser radiance from the left side for 80 µs. The two “dark blue sea” at the sides are glass slides and play the role as thermal reservoir. The unit for the legend is Kelvin. (right, Z-T coordinate, in [µm] and [K]) Temperature profile of
each instance after the sample has exposed to the laser beam. The unit in the legend is $\mu s$.

One special feature of COMSOL is that it can provide a way to solve more than one physical property at the same time. Then, the Landau-de Gennes Theory Eq.(3.31) was applied to the model with all appropriate boundary and initial conditions. This way we can obtain the order parameter $S(z,t)$ all together with temperature profiles, as shown in Fig 4.48. Upon obtaining these numerical order parameter values for each instance, we can calculate the transmittance by using the Jones matrix algorithm described in Chapter 3 by using the Matlab code we developed previously. The transient switching curve is shown in Fig 4.49.

![Fig 4.48 Calculated order parameter for the temperature profile shown in Fig 4.47](image)

![Fig 4.49 Calculated switching curve for example 1](image)

we can note that at trans=50% $t \approx 19\ \mu s$, at trans=10% $t \approx 26\ \mu s$
Example 2

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Absorption coefficient $\alpha$</td>
<td>$2 \times 10^4 \text{ [m}^{-1}]$</td>
</tr>
<tr>
<td>Intensity $I_0$</td>
<td>$1.99 \times 10^7 \text{ [W/m}^3$]</td>
</tr>
</tbody>
</table>
  (Input power 100mW, beam diameter 80µm) |
| Sample thickness              | 50 [µm]                |
| Initial temp                  | 300K                   |

Fig 4.50 Calculated temperature and order parameter profile at different time incidence under laser irradiance

Fig 4.51 Calculated switching curve for Example 2
We can notice that when trans=50% $t \sim 80\mu s$, trans=13% $t \sim 104\mu s$
Example 3

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Absorption coefficient $\alpha$</td>
<td>$4 \times 10^4$ [m$^{-1}$]</td>
</tr>
<tr>
<td>Intensity $I_0$</td>
<td>$7.96 \times 10^7$ [W/m$^3$]</td>
</tr>
<tr>
<td></td>
<td>(Input power 100mW, beam diameter 40µm)</td>
</tr>
<tr>
<td>Sample thickness</td>
<td>50 [µm]</td>
</tr>
<tr>
<td>Initial temp</td>
<td>300K</td>
</tr>
</tbody>
</table>

![Image](image.png)

Fig 4.52  Calculated temperature plot and profile for example 3

Fig 4.53  Calculated temperature and its corresponding order parameter distribution for some selected instance.
Fig 4.54  Calculated switching curve for example 3
(\text{trans}=50\% \quad t\sim 15.5\mu s, \text{trans}=13\% \quad t\sim 28\mu s)

Example 4

<table>
<thead>
<tr>
<th>Absorption coefficient ( \alpha )</th>
<th>( 8 \times 10^4 ) ( [\text{m}^{-1}] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intensity ( I_0 )</td>
<td>( 7.96 \times 10^7 ) ( [\text{W/m}^3] )</td>
</tr>
<tr>
<td>(Input power 100mW,</td>
<td>beam diameter 40\mu m)</td>
</tr>
<tr>
<td>Sample thickness</td>
<td>50 ( [\mu \text{m}] )</td>
</tr>
<tr>
<td>Initial temp</td>
<td>300K</td>
</tr>
</tbody>
</table>
Fig 4.56 Calculated switching curve for example 4
Example 5

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Absorption coefficient α</td>
<td>$2 \times 10^4$ [m$^{-1}$]</td>
</tr>
<tr>
<td>Intensity $I_0$</td>
<td>$7.96 \times 10^7$ [W/m$^3$]</td>
</tr>
<tr>
<td></td>
<td>(Input power 100mW, beam diameter 40µm)</td>
</tr>
<tr>
<td>Sample thickness</td>
<td>20 [µm]</td>
</tr>
<tr>
<td>Initial temp</td>
<td>300K</td>
</tr>
</tbody>
</table>

Fig 4.57 Calculated temperature plot and profile for example 5

Fig 4.58 Calculated switching curve for example 5
(trans=50% $t\sim 14.5\mu$s, trans=13% $t\sim 16.5\mu$s)

Part 2 Three-dimensional examples

For a spatial Gaussian beam shining on the TNLC sample, we could also evaluate the three-dimensional temperature and order parameter spatial distribution by using COMSOL. In fact, we may build up the simulation on cylindrical coordinate system with just $r$ and $z$ axes but no
ϕ coordinate because Gaussian beam is axial symmetric and we assume all the quantities are invariant with respect to ϕ coordinate. Dropping one variable could help save a lot of computation time and resources.

For a Gaussian beam with beam diameter 80 μm and power 100 mW, shining on a TNLC sample of thickness 50 μm and absorption coefficient $2 \times 10^4$ [m$^{-1}$], the temperature and order parameter distribution at 100ms after exposed to the laser can be illustrated as

![Fig 4.59 Calculated temperature and order parameter distribution based on a two-dimensional cylindrical symmetry](image)

**At 100 ms after laser exposure**
- Absorption $\alpha$: $2 \times 10^4$
- Input Power: 100 mW
- Sample thickness: 50 μm
- Spot diameter: 80 μm
- Initial temp: 297.13K (20C)

**5CB parameters used in this model**
- Heat capacity $C_p$: 1.5 J/gK
- Density: 1.019 g/cm$^3$
- Thermal conductivity $k$: 0.15 W/mK

Local order parameter profile can also be illustrated:
A cross-cut order parameter profile

Or 3D pie revolving plot

A 3-D pie-cut demonstration on LC temperature distribution upon laser irradiance

A numerical-based three-dimensional polarization switching and beam diffraction due to refractive index anisotropy change is rather complex and may require FDTD computation capability for further analysis. For research progress consideration, this would be a good project for future continuing research.
References:


Chapter 5 Conclusion and Future

5.1 Summary and Conclusion

In summary, we have discussed the nature of twisted-nematic liquid crystal (TNLC) cells, and the mechanisms of TNLC cells working as all-optical passive light limiters. The untwisting of TNLC cells mostly comes from laser-induced liquid crystal birefringence change, which is rooted from heat-induced order parameter modulation and photo-chemical dye molecule conformation change. We also utilized the Jones matrix method to calculate the TNLC transmission under various conditions. The Landau-de Gennes theory was presented and used for order parameter evaluation. We also built the heat transfer models to evaluate the temperature distribution when TNLC samples are irradiated by laser beams, both for steady-state cases (analytically) and for transient cases (numerically). The effects of various LC sample parameters to the switching threshold are also discussed.

Experimentally, we confirmed this all-optical switching capability and demonstrated the switching behaviors of various dye-doped TNLC under different conditions. Broad spectral operating capability is validated from VIS to NIR spectral regime, including 532 nm, 750 nm, 1064 nm, and 1550 nm wavelengths. In addition, broad temporal switching response ranging from cw to hundred ns scale has been reached. The switching speeds under different thickness, temperature, dye concentration, and dye type were examined and compared. Last, two-dimensional and three-dimensional dynamic temperature and order parameter models were built and simulated with the aid of COMSOL Multiphysics. Reasonable simulation results were obtained and further confirmed our theory and experiments.

5.2 Future Potential Research

There are certainly still many physics about liquid crystal switching underlying all the subtle phenomena and mechanisms that remain unclear and are waiting for us to discover. To push the switching/limiting process to an even faster temporal response and a lower switching threshold, deeper understanding and investigation of every aspect of liquid crystal physics is desired. Here we propose some potential research directions based on what we encountered or observed.

5.2.1 Photo-chemical dopant investigation

We have known that dyes with azobenzene structures, such as methyl red, can provide two different geometric conformations, which could disturb the liquid crystal alignment upon incidence of light. Methyl Red has been found to be the most efficient one of this type to work
with nematic liquid crystals. We could certainly believe the existence of some other compounds that may also provide an even better photo-chemical reaction when mixed with liquid crystal. It may work either in trans-cis isomer conformation change, or even in some other structure conformation modulation. A strong chemistry background would be required to perform this investigation.

5.2.2 TNLC switching oscillation

During our TNLC dynamic switching experiment, we noticed that among approximately half of all the trials, the switching curves actually oscillate more or less before they reach the steady downward trend. To be more specific, we built a setup as in Fig 4.1, where an additional detector (#2 in the figure) was installed for the other polarization to be detected.

![Fig 5.1 Potential experiment setup](image)

L : Focusing Lens  
QWP: quarter wave plate

With this configuration, we can obtain some extra information for the part of the light whose polarization has not been twisted to the other orthogonal direction (i.e. the one supposed to be blocked originally). Some of our observations are shown below (Fig. 4.2).
From these trials, we notice that the transmitted light does not always decay in a smooth fashion. It actually oscillates back and forth between both P and S polarizations and finally reaches the steady decaying tendency. What is the reason it oscillates? What makes the initial decay curve come back to a higher value and then finally decay in the end? Why are they not always switching in a clear and smooth fashion? A theory to explain this effect has to be proposed, and the appropriate experiment has to be designed and conducted to confirm the proposed model.
Appendix A -- Epolite III 125 dye

Epolite III 125 dye was the old name of a proprietary formula owned and manufactured by Epolin, Inc. of 358-364 Adams Street, Newark, NJ 07105. Its name had been changed to Epilight™1125. It is a broad band IR dye with maximum absorption at 950 nm and can absorb IR good to 1650 nm. It was designed for coatings or injection molding purpose originally.

The absorption spectrum of this dye is provided by Epolin. More product information can be referred to Epolin [http://www.epolin.com/](http://www.epolin.com/)

![Normalized absorption spectrum of Epilight™ 1125 in acetone](image)

Fig A.1 Epolite III 125 absorption spectrum
Appendix B – Methyl Red

CAS number: 493-52-7
Molecular formula: C_{15}H_{15}N_{3}O_{2}

Fig B.1 Molecular structure of methyl red

Fig B.2 Absorption spectrum of methyl red doped in 5CB with two different light polarization with respect to liquid crystal alignment
Appendix C – IR-780 Iodide

![IR-780 Iodide structure](image)

Fig C.1 IR-780 Iodide structure

Transmission Measurement for
IR-780 in 5CB 0.3%, 150µm TN sample

Absorption Measurement for
IR-780 in 5CB 0.3%, 150µm TN sample

750nm, t=15%

750nm, OD=0.82

Fig C.2 Absorption and Transmission spectrum of IR-780 Iodide in 5CB

IR-780 Iodide Dye Fluorescence

![Fluorescence spectrum of IR-780 Iodide in methane](image)

Fig C.3 Fluorescence spectrum of IR-780 Iodide in methane
Appendix D – Some measured concentration, thickness, and absorption dependence for the IR-780 Iodide doped TNLC 5CB sample

<table>
<thead>
<tr>
<th>Sample thickness (μm)</th>
<th>IR-780 concentration (wt%)</th>
<th>Transmission at 750 nm (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>0.3</td>
<td>40</td>
</tr>
<tr>
<td>25</td>
<td>0.28</td>
<td>42</td>
</tr>
<tr>
<td>25</td>
<td>~0.2</td>
<td>60</td>
</tr>
<tr>
<td>40</td>
<td>0.10</td>
<td>72</td>
</tr>
<tr>
<td>50</td>
<td>0.13</td>
<td>61 (07-07-2009)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>81 (07-16-2009)</td>
</tr>
<tr>
<td>50</td>
<td>0.15</td>
<td>54 (07-08-2009)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>51 (07-16-2009)</td>
</tr>
<tr>
<td>65</td>
<td>0.10</td>
<td>68</td>
</tr>
<tr>
<td>100</td>
<td>0.09</td>
<td>63 (07-07-2009)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>73</td>
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<tr>
<td>100</td>
<td>0.13</td>
<td>64 (07-16-2009)</td>
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<td>42</td>
</tr>
<tr>
<td>150</td>
<td>0.3</td>
<td>15</td>
</tr>
</tbody>
</table>
Appendix E – Digital scanning calorimeter (DSC) measurement on liquid crystals

5CB DSC Chart

E7 DSC Chart
EDUCATION:

Ph.D.
Dept. of Electrical Engineering, 2011
Penn State University, University Park, PA
Dissertation: “All-optical Switching with Dye-doped Liquid Crystals”
Advisor: Dr. I.C. Khoo, Distinguished Professor, Fellow – IEEE, OSA, & IOP

Master of Science
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Thesis paper: “Liquid Crystal Nonlinear Switching and Its NIM Applications”
Advisor: Dr. I.C. Khoo

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Institute of Applied Mechanics, 2002
National Taiwan University, Taipei, Taiwan
Thesis: “Photorefractive-Crystal-Based Holographic Interferometry System for Full-field Wave Propagation Metrology”
Advisor: Dr. Chih-Kung Lee, Lifetime Distinguished Professor, Fellow – IOP & ASME

Bachelor of Science
Mechanical Engineering, 2000
National Taiwan University, Taipei, Taiwan

WORKING EXPERIENCE:

Research Assistant
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Dept of Electrical Engineering, Penn State University
Fall 2005~Fall 2010

Teaching Assistant
E Mch 210 (Engineering Mechanics)
Dept of Engineering Science and Mechanics, Penn State University
Fall 2004~Spring 2005

Research Assistant
MEMS & NEMS Group
Institute of Applied Mechanics, National Taiwan University
Fall 2000~Spring 2003

RESEARCH EXPERIENCE:

- Liquid crystal and nonlinear optics
- Sculptured thin film
- Laser-assisted transfer of quantum dot emitters
- Optical metrology system & holographic interferometry

PUBLICATIONS AND PRESENTATIONS (Selected):


As of May 2011, 13 co-authored journal papers, 13 conference proceeding papers, 6 conference poster presentations, 2 magazine/book articles, and 1 patent have been published.

Complete citations and curriculum vitae available upon request.

HONORS & AWARDS (Selected):

- SPIE / Newport & Spectra-Physics Research Excellence Travel Award, 2007 & 2008
- Fellowship - Melvin P. Bloom Memorial Outstanding Doctoral Research in Electrical Engineering, April 2009

LEADERSHIP & SERVICES:

- President, SPIE Penn State University Student Chapter, 2007~2008
- Secretary, SPIE Penn State University Student Chapter, 2006~2007

LICENSES / CERTIFICATIONS:

- FE (Fundamentals of Engineering) / EIT of NCEES (National Council of Examiners for Engineering and Surveying)
- CVP (Certified Vision Professional)-I of AIA (Automated Imaging Association)
- CSWP (Certified SolidWorks Professional) of Dassault Systèmes SolidWorks Corp.
- CSWA (Certified SolidWorks Associate) of Dassault Systèmes SolidWorks Corp.