ANALYSIS AND CONTROL OF THE TRANSIENT AEROELASTIC RESPONSE OF ROTORS DURING SHIPBOARD ENGAGEMENT AND DISENGAGEMENT OPERATIONS

A Thesis in
Aerospace Engineering

by

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ABSTRACT

An analysis has been developed to predict the transient aeroelastic response of a helicopter rotor system during shipboard engagement and disengagement operations. The coupled flap-lag-torsion equations of motion were developed using Hamilton’s Principle and discretized spatially using the finite element method. Aerodynamics were simulated using nonlinear quasi-steady or time domain nonlinear unsteady models. The ship airwake environment was simulated with simple deterministic airwake distributions, results from experimental measurements or numerical predictions. The transient aeroelastic response of the rotor blades was then time-integrated along a specified rotor speed profile.

The control of the rotor response for an analytic model of the H-46 Sea Knight rotor system was investigated with three different passive control techniques. Collective pitch scheduling was only successful in reducing the blade flapping response in a few isolated cases. In the majority of cases, the blade transient response was increased. The use of a discrete flap damper in the very low rotor speed region was also investigated. Only by raising the flap stop setting and using a flap damper four times the strength of the lag damper could the downward flap deflections be reduced. However, because the flap stop setting was raised the upward flap deflections were often increased. The use of extendable/retractable, gated leading-edge spoilers in the low rotor speed region was also investigated. Spoilers covering the outer 15% of the rotor blade were shown to
significantly reduce both the upward and downward flap response without increasing rotor torque.

Previous aeroelastic analyses developed at the University of Southampton and at Penn State University were completed with flap-torsion degrees of freedom only. The addition of the lag degree of freedom was shown to significantly influence the blade response. A comparison of the two aerodynamic models showed that the nonlinear quasi-steady aerodynamic model consistently yielded a larger blade response than the time domain nonlinear unsteady model. The transient blade response was also compared using the simple deterministic and numerically predicted ship airwakes for a frigate-like ship shape. The blade response was often much larger for the numerically predicted ship airwakes and was also much more dependent on deck location, wind speed and wind direction. Fluctuating flow components in the range of frequencies measured in previous ship airwake tests were shown to increase the blade response.

A separate analysis was developed to investigate the feedback control of gimballed rotor systems using swashplate actuation. The equations of motion for a rigid, three-bladed gimballed rotor system were derived and aerodynamic forces were simulated with a simple, linear attached flow model. A time domain Linear Quadratic Regulator optimal control technique was applied to the equations of motion to minimize the transient rotor response. The maximum transient gimbal tilt angle was reduced by as much as half within the current physical limits of the control system; further reduction was achieved by increasing the limits.
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**LIST OF SYMBOLS**

- **$a$**: 2-D lift curve slope
- **$c$**: Chord length
- **$c_{el}$**: Elemental damping matrix
- **$C$**: Rotor damping matrix
- **$C_c$**: Chord force coefficient
- **$C_{c}$**: Constrained blade damping matrix
- **$C_d$**: Drag force coefficient
- **$C_l$**: Lift force coefficient
- **$C_m$**: Moment coefficient about the $\frac{1}{4}$ chord
- **$C_n$**: Normal force coefficient
- **$C_u$**: Transformed blade damping matrix
- **$C_u$**: Unconstrained blade damping matrix
- **$C_v$**: Lag damper strength for hingeless rotors
- **$C_w$**: Flap damper strength for hingeless rotors
- **$C_\beta$**: Flap damper strength for articulated rotors
- **$C_\zeta$**: Lag damper strength for articulated rotors
- **$D$**: Blade section aerodynamic drag
- **$e$**: Modal amplitudes
\( e_c \) Blade section \( \frac{1}{4} \) chord offset from elastic axis

\( e_g \) Blade section center of gravity offset from elastic axis

\( e_\beta \) Flap hinge offset

\( e_\theta \) Pitch bearing offset

\( e_\zeta \) Lag hinge offset

\( EB_1, EB_2 \) Blade pre-twist stiffnesses

\( EC_1, EC_2 \) Warping stiffnesses

\( El_{yy}, El_{zz} \) Normal and chordwise stiffnesses

\( f_{el} \) Elemental force vector

\( F \) Rotor force vector

\( F_c \) Constrained blade force vector

\( F_r \) Transformed blade force vector

\( F_u \) Unconstrained blade force vector

\( g \) Gravitational acceleration

\( GJ \) Torsional stiffness

\( H \) Shape functions

\( \hat{i}, \hat{j}, \hat{k} \) Unit vectors in blade undeformed coordinate system

\( \hat{i}_\xi, \hat{j}_\eta, \hat{k}_\zeta \) Unit vectors in blade deformed coordinate system

\( I \) Identity Matrix

\( \hat{i}, \hat{j}, \hat{k} \) Unit vectors in hub-fixed rotating coordinate system

\( \hat{i}_F, \hat{j}_F, \hat{k}_F \) Unit vectors in vehicle-fixed coordinate system
\( \hat{\mathbf{i}}_H, \hat{\mathbf{j}}_H, \hat{\mathbf{k}}_H \) Unit vectors in hub-fixed nonrotating coordinate system

\( \hat{\mathbf{i}}_{\text{SHIP}}, \hat{\mathbf{j}}_{\text{SHIP}}, \hat{\mathbf{k}}_{\text{SHIP}} \) Unit vectors in ship-fixed coordinate system

\( \hat{\mathbf{i}}^l_{\text{SHIP}}, \hat{\mathbf{j}}^l_{\text{SHIP}}, \hat{\mathbf{k}}^l_{\text{SHIP}} \) Unit vectors in ship-fixed inertial coordinate system

\( \mathbf{k}_{el} \) Elemental stiffness matrix

\( \mathbf{K} \) Rotor stiffness matrix

\( \mathbf{K}_c \) Constrained blade stiffness matrix

\( \mathbf{K}_r \) Transformed blade stiffness matrix

\( \mathbf{K}_u \) Unconstrained blade stiffness matrix

\( \mathbf{K}_{\text{stop}_{\text{wup}}} \) Rigid flap/lag stop spring stiffness for hingeless rotors

\( \mathbf{K}_\beta \) Flap hinge spring stiffness

\( \mathbf{K}_{\beta_{\text{stop}}} \) Rigid flap/lag stop spring stiffness for articulated rotors

\( \mathbf{K}_\phi \) Control system spring stiffness

\( \mathbf{K}_\zeta \) Lag hinge spring stiffness

\( \mathbf{K}_{\zeta_{\text{stop}}} \) Rigid flap/lag stop spring stiffness for articulated rotors

\( l \) Element length

\( L \) Blade section aerodynamic lift

\( m \) Blade section mass per unit length

\( m_0 \) Blade uniform mass per unit length

\( \mathbf{m}_{el} \) Elemental mass matrix

\( mk^2_m, mk^2_{m_1}, mk^2_{m_2} \) Polar, normal and chordwise mass moments of inertia

\( M \) Mach number
M  Rotor mass matrix
M_c  Constrained blade mass matrix
M_{c/4}  Aerodynamic moment about the ¼ chord
M_r  Transformed blade mass matrix
M_u  Unconstrained blade mass matrix
N_b  Number of blades
N_{dof}  Number of degrees of freedom
N_{el}  Number of finite elements
N_m  Number of modes
q  Rotor degrees of freedom
q_{el}  Elemental degrees of freedom
q_c  Constrained blade degrees of freedom
q_r  Transformed blade degrees of freedom
q_u  Unconstrained blade degrees of freedom
R  Blade radius
s  Distance within element
t  Time
t_f  Final time
T  Kinetic energy
T_{ship}  Ship roll period
u  Axial deflection
U  Strain energy
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$U_R, U_T, U_p$</td>
<td>Blade section velocities</td>
</tr>
<tr>
<td>$v$</td>
<td>Lag deflection</td>
</tr>
<tr>
<td>$v$</td>
<td>Lag node displacements</td>
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<td>$v_{stop}$</td>
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</tr>
<tr>
<td>$v_{tip}$</td>
<td>Lag tip deflection</td>
</tr>
<tr>
<td>$\vec{V}$</td>
<td>Blade section total velocity vector</td>
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<tr>
<td>$\vec{V}_x, \vec{V}_y, \vec{V}_z$</td>
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</tr>
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<td>$V'_x, V'_y, V'_z$</td>
<td>Ship airwake fluctuating velocities</td>
</tr>
<tr>
<td>$V_{WOD}$</td>
<td>Wind over deck speed</td>
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<td>$w$</td>
<td>Flap deflection</td>
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<td>$w$</td>
<td>Flap node displacements</td>
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<td>Flap stop deflection</td>
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<tr>
<td>$w_{tip}$</td>
<td>Flap tip deflection</td>
</tr>
<tr>
<td>$W$</td>
<td>External work</td>
</tr>
<tr>
<td>$x$</td>
<td>Undeformed blade length coordinate</td>
</tr>
<tr>
<td>$x, y, z$</td>
<td>Blade undeformed coordinate system</td>
</tr>
<tr>
<td>$x_1, y_1, z_1$</td>
<td>Distance from rotor hub to blade section</td>
</tr>
<tr>
<td>$x_{nw}, y_{nw}, z_{nw}$</td>
<td>Distance from ship cg to helicopter nosewheel</td>
</tr>
<tr>
<td>$x_{rh}, y_{rh}, z_{rh}$</td>
<td>Distance from helicopter nosewheel to rotor hub</td>
</tr>
<tr>
<td>$x_\xi, y_\eta, z_\zeta$</td>
<td>Blade undeformed coordinate system</td>
</tr>
<tr>
<td>$X,Y,Z$</td>
<td>Hub-fixed rotating coordinate system</td>
</tr>
<tr>
<td>$X_H,Y_H,Z_H$</td>
<td>Hub-fixed nonrotating coordinate system</td>
</tr>
</tbody>
</table>
$X_F, Y_F, Z_F$  Vehicle fixed coordinate system

$X_{SHIP}, Y_{SHIP}, Z_{SHIP}$  Ship-fixed coordinate system

$X_{SHIP}^\prime, Y_{SHIP}^\prime, Z_{SHIP}^\prime$  Ship-fixed inertial coordinate system

$\alpha$  Blade section angle of attack

$\alpha$  Mode shape

$\alpha_s$  Longitudinal shaft angle

$\beta_{1c}$  Fixed frame rotor longitudinal tilt

$\beta_{1s}$  Fixed frame rotor lateral tilt

$\beta_{hinge}$  Flap hinge angle

$\beta_{\max}$  Maximum flap angle

$\beta_p$  Precone angle

$\beta_{stop}$  Flap/droop stop angle

$\delta$  Variational operator

$\varepsilon$  Ordering scheme coefficient

$\varepsilon_{xx}, \varepsilon_{xy}, \varepsilon_{xz}$  Blade engineering strains

$\phi$  Elastic twist in undeformed blade coordinate system

$\hat{\phi}$  Elastic twist in deformed blade coordinate system

$\hat{\phi}$  Elastic twist node displacements

$\hat{\phi}_{hinge}$  Pitch bearing angle

$\phi_{ship}$  Ship roll angle
<table>
<thead>
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<th>Symbol</th>
<th>Description</th>
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<tr>
<td>$\phi_{\text{ship_max}}$</td>
<td>Maximum ship roll angle</td>
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<tr>
<td>$\Phi$</td>
<td>Modal matrix</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Lock number</td>
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<tr>
<td>$\eta$</td>
<td>Blade section chordwise coordinate</td>
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<tr>
<td>$\eta_r$</td>
<td>Location of $\frac{3}{4}$ chord</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Gust factor</td>
</tr>
<tr>
<td>$\lambda_i$</td>
<td>Induced inflow</td>
</tr>
<tr>
<td>$\lambda_T$</td>
<td>Blade section warping constant</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>Blade section skew angle</td>
</tr>
<tr>
<td>$\nu_\beta$</td>
<td>Nondimensional flap frequency</td>
</tr>
<tr>
<td>$\nu_\theta$</td>
<td>Nondimensional torsion frequency</td>
</tr>
<tr>
<td>$\nu_\zeta$</td>
<td>Nondimensional lag frequency</td>
</tr>
<tr>
<td>$\Pi$</td>
<td>Total energy</td>
</tr>
<tr>
<td>$\theta_0$</td>
<td>Geometric pitch angle</td>
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<tr>
<td>$\theta_1$</td>
<td>Total pitch angle</td>
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<tr>
<td>$\theta_{1c}$</td>
<td>Lateral cyclic pitch angle</td>
</tr>
<tr>
<td>$\theta_{1s}$</td>
<td>Longitudinal cyclic pitch angle</td>
</tr>
<tr>
<td>$\theta_{75}$</td>
<td>Collective pitch angle at $75%R$</td>
</tr>
<tr>
<td>$\theta_{tw}$</td>
<td>Blade built-in twist</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Air density</td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>Blade mass density</td>
</tr>
</tbody>
</table>
\( \sigma_{xx}, \sigma_{yy}, \sigma_{zz} \)  Stress

\( \sigma_x, \sigma_y, \sigma_z \)  Ship airwake turbulence intensities

\( \Omega \)  Rotor speed

\( \Omega_0 \)  Full rotor speed

\( \psi \)  Master blade azimuth angle

\( \tilde{\psi} \)  Master blade azimuth angle at full rotor speed

\( \psi_0 \)  Initial master blade azimuth angle

\( \psi_{\text{max}} \)  Azimuth of maximum flap angle

\( \psi_{WOD} \)  Wind over deck direction

\( \zeta \)  Blade section normal coordinate

\( \zeta_{\text{hinge}} \)  Lag hinge angle

\( \zeta_{\text{stop}} \)  Lead/lag stop angle

(\( ' \))  \( = \frac{\partial}{\partial \tilde{\psi}} \)

(\( ' \))  \( = \frac{\partial}{\partial x} \)

(\( _{AF} \))  Contribution from aerodynamic forces

(\( _b \))  Contribution from blade motion

(\( _c \))  Contribution from circulatory effects

(\( _{CS} \))  Contribution from control system stiffness

(\( _f \))  Contribution from fuselage motion
\( C_{FD} \) Contribution from flap damper

\( C_{FS} \) Contribution from flap stop

\( g \) Contribution from gravity

\( i \) Contribution from induced inflow

\( I \) Contribution from inertial forces

\( C_{LD} \) Contribution from lag damper

\( L \) Contribution from lag stop

\( C_{NC} \) Contribution from noncirculatory effects

\( C_{ship} \) Contribution from ship motion

\( w \) Contribution from ship airwake
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Philadelphia, Mr. David Popelka of Bell Helicopter, and Mr. Robert Hansford of Westland Helicopters made themselves available for technical discussions.
1.1 Background and Motivation

Helicopters have become a critical part of successful civilian and military operations in maritime environments in the last half-century. For example, modern navies use helicopters to fulfill attack, search and rescue, cargo and troop carrying, antisubmarine, and mine sweeping missions; while oil companies use helicopters to efficiently ferry crews to and from off shore oil rigs. Naval helicopter crews regularly face hazardous flight conditions that land-based helicopter crews do not. High winds, low visibility, sea-spray, a moving landing platform, and unusual airflow around ships and oil rigs can increase the difficulty of naval helicopter operations, especially during the takeoff and landing phases of the mission.

A less well-known problem can occur even before the helicopter takes off from or even after it lands on a sea-based platform. At the beginning of every helicopter mission the rotor must be accelerated from rest to full rotor speed and vice versa at the end of every mission. This process is called the engagement and disengagement of the rotor system and normally is a very mundane procedure. In calm winds, both the aerodynamic lift and centrifugal stiffening forces increase proportionately with rotor speed. However in a maritime environment, atmospheric wind speeds are often much higher than on land.
Furthermore, when most ships were designed their mission dependence on helicopters was not foreseen. Little attention was paid to the aerodynamics of the superstructure, resulting in an unusual airwake environment around the ship. Because of the high atmospheric wind speeds and unusual flow patterns around the ship superstructure, the rotor blades can occasionally generate appreciable aerodynamic forces. Combined with low rotor speed and low centrifugal stiffening, excessive amounts of aeroelastic flapping can occur during engagement or disengagement operations. This potentially dangerous phenomenon has been termed "blade sailing" [1].

The consequences of blade sailing can range from large bending moments in the blades to physical contact between the rotor blades and the helicopter fuselage. Excessive flapping of the rotor to the point that it physically contacts the fuselage is known as a "tailboom strike" for traditional rotor configurations and a "tunnel strike" for tandem rotor configurations, shown in Figure 1.1.

(U.S. Navy Photo)

*Figure 1.1:* Illustration of an H-46 Tunnel Strike
Blade sailing most frequently occurs in severe weather conditions. North Sea oil companies and the UK Royal Navy have documented examples of blade sailing. For example, a commercially operated Boeing H-47 Chinook was reported having blade sailing problems when operating from an oil rig [2]. Another Chinook operated by the UK during the Falkland Islands conflict of 1982 could not disengage its rotors because of continuous 70-knot winds. Until the winds dropped to an acceptable level, the rotor system was kept turning with crew changes performed "on the fly". Some rescue helicopters operating off the southwest coast of England have had to spin up their rotors while remaining inside the ship's hangar because of extremely high wind speeds. Contact between the blade tips and the tailboom, and even the ground in severe cases, have been reported for the H-3 Sea King. In the most severe cases, fatalities have occurred in connection with blade sailing events [1].

The phenomenon of blade sailing is familiar not just to North Sea oil companies and the UK's military but also to the US Navy and Marines. The H-46 Sea Knight is a tandem rotor helicopter that has been operated by the US Navy and Marines since 1964. Since its introduction, there have been 114 documented H-46 tunnel strike incidents [3-5]. These tunnel strikes, which typically occur at rotor speeds around 20% of the nominal rotor speed, or 20%NR, pose a significant danger to both aircrew and ground personnel. Physical damage can range from minor to the complete loss of the helicopter. The dollar cost can range from just man-hour costs when inspections are required for minor tunnel strikes, to upwards of $500,000 for a tunnel strike which involves the sudden stoppage of the drive train system [6]. If the entire airframe is damaged beyond
repair, the true cost cannot be calculated because the H-46 is no longer in production. Hidden costs of a tunnel strike can be even more expensive; for example, an interruption in a ship's resupply schedule can lead to changes in the entire fleet schedule [7].

Because of the unusual aerodynamic qualities of many ships, engagement and disengagement problems have occurred at wind speeds as low as 15 knots [8]. Most H-46 tunnel strikes have occurred on resupply-style AE, AFS, AOE and AOR ships; or on amphibious assault-style LPH, LHA and LHD ships. Examples of each ship type are shown in Figure 1.2. Note the difference in the shapes of each ship type. The resupply-style ships have a single landing position on a small flight deck located behind a relatively large hangar; while the amphibious assault-style ships have many landing positions on a large, rectangular flight deck relatively free of obstructions.

![a) AE Class (U.S. Navy Photo)](image1)

![b) LHD Class (U.S. Navy Photo)](image2)

*Figure 1.2: Examples of USN Ship Types*
Blade sailing is not a concern just for naval helicopter operators. Tunnel strikes have occurred during land-based engagements of the Boeing model 107, the civil version of the H-46 [3]. In the future, the US Army intends to operate its helicopters from Navy ships in the Joint Shipboard Helicopter Integration Process (JSIP) program. The Army primarily operates articulated rotor systems like the AH-64 Apache, H-47 Chinook, and H-60 Blackhawk. Historically, blade sailing has been a problem for articulated rotors more often than hingeless rotors.

Since blade sailing is a phenomenon that occurs at low rotor speeds, it is relevant to discuss an important feature of rotor systems operating in this region. The majority of current naval helicopters use articulated rotor blades, such as the H-3 Sea King and the H-46 Sea Knight mentioned previously. All articulated rotor systems must use some type of mechanical stop to limit excessive flap and lag motions of the blades when the rotor is at rest and at low rotor speeds. The limiters that restrain the excessive downward and upward flap motion are typically called the droop and flap stops, while the limiters that restrain the excessive forward and backward lag motion are typically called the lead and lag stops. These stops are usually only used during the low rotor speed region of the engagement or disengagement. Beyond a pre-set rotor speed, the stops typically retract under centrifugal force and become inoperative. Hingeless, bearingless, teetering and gimballed rotor systems may or may not utilize such stops depending on the specific vehicle.
1.2 Determination of Engagement and Disengagement Limits

Due to the hazardous nature of maritime helicopter operations, limits must be established on the atmospheric wind conditions and ship motions in which shipboard helicopter operations can be safely performed. The shipboard helicopter operations, including vehicle stowage, engagement/disengagement, launch/recovery, vertical replenishment, and helicopter in-flight refueling operations, are usually referred to as Dynamic Interface (DI) operations. The safe limits are called the Ship/Helicopter Operational Limits (SHOLs). Naturally, larger SHOLs increase the helicopter's ability to perform its mission in adverse weather conditions, which in turn increases the ship's tactical flexibility.

The most important factor in determining the SHOLs for engagement and disengagement operations is the relative speed and direction of the wind, or Wind-Over-Deck (WOD), as it approaches the ship. In general, the faster the WOD speed, the more likely blade sailing will occur. The "generic" SHOLs showing all the safe combinations of WOD speed and direction conditions for engagement/disengagement operations for both the H-2 Sea Sprite and the H-46 Sea Knight are shown in Figure 1.3. In each figure, increasing relative WOD speeds are measured outward from the center of the circle and the relative WOD directions are measured in the clockwise direction. The combinations of WOD speeds and directions inside the grids are the safe conditions, while those outside are the unsafe conditions for engagement and disengagement operations.
After a series of H-46 tunnel strikes in the late 1960's, the Navy began to determine the SHOLs for engagement and disengagement operations in full-scale at-sea experimental tests. A specially outfitted helicopter, shown in Figure 1.4, was used. A box-like wooden structure was strapped to the fuselage/synchronization shaft cover area to protect the fuselage from blade strikes. Because its top surface was covered with grease to help deflect the blade, it was aptly named the “greasy board”. A series of Styrofoam pegs were placed over the synchronization shaft in the path of the rear rotor blade. Test pilots then performed a series of engagements and disengagements in this helicopter. After each test, the number of Styrofoam pegs remaining on the fuselage indicated the minimum blade clearance. For the H-46, an engagement or disengagement of the rotor was deemed “safe” if there were eight or more inches of blade clearance. The
tests were then repeated, first in very calm winds and then in progressively more severe winds for multiple wind directions, until a safe operational limit was reached. These tests were conducted for *every* helicopter/ship/landing spot combination. Typical DI test reports are available in Refs. 10-18.

Unfortunately, this process of experimentally determining the SHOLs presented several problems. First, it was very labor intensive and time consuming. The at-sea portion of DI testing typically required four to five days and a crew of four pilots, two air crewmen, four test engineers, and five maintenance personnel [19]. In 1985, a backlog of 11 helicopter types and 20 ship types required testing for DI characteristics. At that time, this testing was to be completed in 15 years, primarily due to a lack of ship availability [20]. Furthermore, DI testing was very expensive - a typical test cost approximately $75k to $150k per ship/helicopter combination. One set of DI tests cost approximately $45k and did not even yield useful data because the weather was calm during the entire test.
period [21]. Finally, experience has shown that DI testing did not even consistently produce safe results because of the variability of sea-state and wind conditions during the test period. For example, a significant number of tunnel strikes occurred aboard AOR type ships while operating inside of the "safe" envelope [7]. For these reasons, the Navy ceased the portion of DI testing devoted to engagement and disengagement SHOL determination in 1990.

The analytical determination of the SHOLs has been proposed as a viable alternative to testing because it offers substantial benefits in terms of reducing cost, improving safety, and developing larger operating envelopes [22]. The Technical Co-operation Programme (TTCP) group, an international organization whose members conduct DI testing, has led interest in analytic DI research. It is composed of the Defence and Evaluation and Research Agency (DERA) in the United Kingdom, the National Research Council - Institute for Aerospace Research (NRC - IAR) in Canada, the Naval Air Warfare Center (NAWC) in the United States, and the Defence Science and Technology Organisation (DSTO) in Australia.

1.3 Related Research

The following sections outlines the previous research related to the analytical and experimental simulation of engagement and disengagement operations. The previous research has been categorized into “early” research occurring from approximately the mid 1960’s to the mid 1980’s and “recent” research conducted from the mid 1980’s to the present.
1.3.1 Early Engagement and Disengagement Modeling

The earliest attempts at modeling the blade sailing phenomenon began in the early 1960's. Since this is before the advent of high-speed computing power, very simple analytic solutions were sought. They are characterized by the assumptions that the rotor speed is constant at some low value, typically 5% to 20%NR, and that the flow is fully attached. Blade deformations were typically represented with single mode approximations.

1.3.1.1 Research at Westland Helicopters

The British first initiated blade sailing research for the Westland Whirlwind and Wasp [23]. Both helicopters featured articulated rotor systems. The blade was assumed to remain in contact with the droop stop at all times; therefore, cantilevered blade modes were used to approximate the blade shape. The reverse flow region was modeled by assuming the entire retreating side of the rotor disk generated no lift. Three different ship airwake distributions, called the horizontal, constant and step were considered, and are shown in Figure 1.5. The horizontal airwake distribution modeled winds purely in the
plane of the rotor. Blade tip deflections in a 30-knot horizontal wind at a rotor speed of 2
RPM were $8\%R-11\%R$ for the Whirlwind and $6\%R-12\%R$ for the Wasp. Considering
that the normal static tip deflection is approximately $5\%R$, these deflections were rather
small. The constant airwake distribution was intended to simulate the effect of a
statically rolled ship deck. The sole effect of the ship roll was to rotate a component of
incident wind perpendicular to the entire rotor disk. Greater blade motions were
predicted for both rotors. The step airwake distribution was intended to simulate the
"cliff edge" effect of the ship hull in abeam winds. It simulated the air as it is forced
upward and over the ship deck on the windward side and then downward and off the ship
deck on the leeward side. This effect was modeled by using a constant vertical upflow
component on the windward side of the rotor disk and a constant downflow component
on the leeward side of the rotor disk. The Whirlwind had tip deflections of $15\%R-19\%R$
while the Wasp had deflections of only $6\%R$. The primary result of this study was to
demonstrate the susceptibility of articulated rotors to blade sailing, particularly in the
presence of vertical winds.

A later analysis for the Sea King was intended to examine the performance of the
then new composite main rotor, which had approximately $\frac{1}{2}$ of the stiffness of the
original metal blades [24]. Blade modes were used to approximate the response and the
resulting equations of motion were time integrated. It was discovered that the second
blade mode was very important to the blade response during contact with the droop or
flap stops.
A later analysis was intended as a "back of the envelope" calculation to predict and compare the blade sailing behavior for the articulated Sea King and the hingeless Lynx rotors [25]. A very simple expression for the flap equation of motion was derived for a single, rigid blade. Blade element theory was used to solve for the steady state response of the rotor. It was estimated that in a 60-knot horizontal wind with a 15-knot vertical component, the articulated Sea King blade would have maximum flapping excursions of $29\%R$, while the Lynx rotor system only $14\%R$. Once again, the susceptibility of articulated rotors to blade sailing was displayed.

### 1.3.1.2 Research at Boeing Vertol

Boeing Vertol researched blade sailing of articulated rotors as early as 1964 [26]. Initially, this research was motivated by concerns of excessive flap bending moments at the root of an articulated rotor blade during a droop stop impact. In this analysis, the flap bending moment distribution along an articulated rotor blade during such an impact was derived. Only the fundamental cantilevered mode was considered in the formulation. The rotor speed was considered constant so a steady-state solution was obtained. Using a full-scale aircraft operating at full rotor speed, an excessive cyclic pitch input was used to cause a droop stop impact. Experimental measurements of the blade flapping motion and root bending moment were recorded with an oscilloscope. Good agreement existed between the predicted and the measured root bending moment.

A specific tunnel strike incident during an engagement of an H-46 helicopter operating from the flight deck of the USS Kalamazoo, an AOR class ship, was also
modeled [27]. The strike incident was of the most severe type - all three aft rotor blades struck the synchronization shaft and fuselage. Two of the blades were completely broken off the rotor hub while the third shattered outboard of the blade attachment; and a hole 14 inches deep was torn in the fuselage. It was estimated that the tunnel strike occurred at approximately 10%NR in 42-knot winds from 25° starboard of the ship's centerline. In the analytic investigation, steady-state solutions for the flapping response at RPM values of 2%NR to 20%NR were calculated. The maximum blade deflections were found to be 34%R upwards and 23%R downwards, occurring at a rotor speed of approximately 10%NR. The blade deflection required to contact the tunnel is 18%R for the H-46.

1.3.2 Recent Engagement and Disengagement Modeling

With the advent of faster computers, serious attention was paid to blade sailing starting in the early 1980's. The blade equations of motion could be time integrated to calculate the transient blade motion. Simple attached flow aerodynamic models yielding analytic expressions were abandoned; instead, important nonlinear aerodynamic effects such the reverse flow region and blade stall were included. Blade structural models were also greatly improved. The use of a single mode approximation was no longer necessary; instead, models with multiple elastic mode shapes were used.
1.3.2.1 Research at the Naval Postgraduate School

The Naval Postgraduate School attempted to model the tunnel strike problem for the H-46 [28]. The aeroelastic rotor code, called the DYnamic System COupler program, was an elastic flap-lag-torsion code. A converged solution using DYS-Co was never obtained so further study was abandoned.

1.3.2.2 Research at NASA

At NASA, research into disengagement behavior was motivated by a wind tunnel test. It is common procedure to stop the rotor system as quickly as possible if a problem is encountered. Normally, this would not pose any safety considerations. However, in one set of tests the rotor shaft was tilted up to 30° to reduce ground effects. It was feared that an emergency power shutdown of the rotor would induce a large blade lag response leading to failure of structural components.

A rigid flap-lag blade analysis was developed to simulate a rotor in a wind tunnel undergoing an emergency power shutdown [29]. Hub motion degrees of freedom in longitudinal translation and pitch rotation were also considered. A linear, quasi-steady aerodynamic model and a dynamic inflow model were used to represent the airloads. The rotor speed profile was assumed to decay exponentially over a period of 40 seconds. In the case of an emergency power shutdown, it was recommended that the collective pitch and rotor shaft angle be quickly reduced to suppress the lag response.
1.3.2.3 Research at McDonnell Douglas

Engagement and disengagement research at McDonnell Douglas was motivated not by aeroelastic flapping, but by concerns of excessive in-plane loads in the elastomeric lead-lag dampers of the AH-64 Apache during brake-on startup operations [30]. In this type of operation, the rotor brake is kept locked which prevents the rotor from turning while the engine torque is increased. After the release of the rotor brake, the rotor is essentially "jump started" and builds up speed very quickly. Applying a large impulsive torque to the rotor tends to induce a large lag response. Accurate prediction of the lag response is essential because it is a large contributor to the loads in the blade retention system and because contact between the blade and the lag stops must be prevented.

A full-scale experiment and an analytic simulation of an Apache brake-on rotor engagement operation were conducted [30]. In the experiment, the time histories of the rotor torque, rotor speed, lag response and damper loads were measured. In the simulation, a nonlinear, rigid-blade, multi-body model of the Apache rotor was constructed. A constant profile drag coefficient was used to model aerodynamic drag. Linear and nonlinear models of a Voigt-Kelvin solid were used to simulate the elastomeric lag dampers. Using the measured torque input, the rotor speed profile, lag response and damper loads were predicted with both damper models. Results of the analysis are shown in Figure 1.6. The predicted rotor speed profiles with both damper models correlated very well with the experimentally measured profile. However, both the predicted blade lag angles and loads were significantly different from their measured counterparts, especially for rotor speeds less than 50% NR.
1.3.2.4 Research at the University of Southampton

Beginning at Westland Helicopters in the late 1970's, Dr. Simon J. Newman began to construct an analytical model and to conduct model-scale tests of the blade sailing phenomenon [1]. Blade sailing was already a well-known problem for the Sea King and the Westland EH101 Merlin helicopter, intended as a replacement for the Sea King, was in the design phase.

In 1985, Newman moved to the University of Southampton and continued his research. It was suspected that the ship airwake around the flight deck was an important factor in blade sailing, so initial efforts were focused in this direction [31-32].
experimental investigation of the airflow about a model and full-scale RFA Rover Class ship was performed without the presence of a helicopter. Mean, maximum and minimum velocities, standard deviations and power spectral densities were measured for both model-scale and full-scale flowfields. Two orthogonal velocity components, one along the direction of the free stream air and one in the vertical direction, were taken over the full-scale ship with golf ball and propeller anemometers. Data was taken for several wind directions and speeds ranging from 15 to 30 knots. A 1/120th-scale ship was used for the wind tunnel tests. No atmospheric boundary layer was simulated and the turbulence intensity of the free stream air was 0.4%. A three-axis hot wire anemometer was used to obtain the flow velocities over the same points and wind conditions on the model flight deck as the full-scale tests. Comparisons of the mean flow components and power spectral densities for both measurements showed satisfactory correlation. Significant upwash and downwash velocities at the windward and leeward deck edges were measured.

In addition to the airwake studies, a transient blade sailing model for hingeless rotors was described [31-32]. The flapping equations of motion for a single blade were derived and simulated with a modal superposition method. The effect of gravity and the variation in centrifugal force with changing rotor speed were included. The ship airwake was modeled with a step airwake distribution, which was based on the ship airwake tests. A quasi-steady aerodynamic model, including trailing edge separation effects via a Kirchhoff method, was used to calculate the aerodynamic loads. The induced velocity was assumed zero, since the nominal condition during rotor engagements and disengagements
is zero rotor thrust. The Westland Lynx was used as the helicopter for the study. The aerodynamic properties of a NACA 0012 airfoil section were assumed instead of the actual Lynx NPL 9615 airfoil section. Simple analytical expressions for the rotor speed profile during the engagement and disengagement operations were used. Ship roll was also simulated. No interaction between the ship motion and the rotor dynamics was modeled since the rotor speed was typically much higher than the ship roll frequency. Therefore, the sole effect of the ship motion was to rotate the plane of the rotor relative to the ship airwake and to induce a lateral airflow component due to rotational motion about the ship’s center of gravity. A fourth order Runge-Kutta integrator was used to time integrate the resulting equations of motion. The analysis showed the loads, calculated from a curvature method, exceeded the blade fatigue limits in 50-knot winds with a 10-knot vertical component and a 7.5° ship roll amplitude. Blade tailboom strike limits were exceeded in 60-knot winds with a 15-knot vertical component and a ship roll amplitude of 15°.

To obtain a greater understanding of the model-scale ship airwake, additional measurements were taken with a laser Doppler anemometer [33]. Three orthogonal velocity components were measured at 12 locations equally spaced around the 75%R blade position and one at the rotor hub. Wind directions of 90° (from starboard), 135°, and 180° (from astern) were examined. After examining the flow patterns, it was determined that the ship airwake could be specified with simple variations. The two airwake types that most resembled the wind tunnel data were termed the step airwake and the linear airwake and are shown in Figure 1.7. Turbulence was modeled by multiplying
the measured flow standard deviation by a Gaussian distributed random number updated at a Strouhal scaled frequency. The blade structural model was improved with flap-torsion coupling. The largest rotor deflections occurred in 50-knot abeam winds. The linear airwake model results consistently matched the original wind tunnel results much better than the step airwake model results. Torsion was shown to increase tip deflections by 30%. Extended engagement run-up times were shown to increase tip deflections by 10%. Ship roll effects were shown to increase blade deflections by 12%.

Newman later extended his theoretical model to include articulated rotor systems [34]. The flap stops were simulated with conditional, high-rate, linear springs located just outboard of the flap hinge. The helicopter chosen for the analysis was the Sea King. A modal convergence study was completed, with flap tip deflection as the convergence criteria, and full convergence was displayed for only four flap modes. Several engagements and disengagements in a 50-knot wind were simulated for the Sea King and the hingeless Lynx rotor systems. The results for the Sea King and the Lynx are shown in Figure 1.8. Blade flap deflections were much greater for the articulated Sea King, on the order of $22\%R$, than for the hingeless Lynx. It should be noted that a deflection of $22\%R$ is required to contact the tail boom on the Sea King. In addition, a full-scale
experimental run-up and run-down test with an SA330 Puma helicopter in a 10-knot headwind was conducted and correlation with analytical results was performed. Blade flap hinge angle deflections were measured and converted to tip deflections by assuming a rigid blade. Since there was difficulty in accurately measuring the rotor speed, collective pitch, and cyclic pitch at low rotor speed, a detailed investigation was not warranted.

![Graphs of rotor engagements and disengagements for Sea King and Lynx](image)

- a) Sea King Rotor Engagement (from Ref. 34)
- b) Sea King Rotor Disengagement (from Ref. 34)
- c) Lynx Rotor Engagement (from Ref. 34)
- d) Lynx Rotor Disengagement (from Ref. 34)

*Figure 1.8: Sea King and Lynx Rotor Engagements and Disengagements*
After the extension of the theoretical model to include articulated rotors, actual experimental validation tests of model-scale radio-controlled helicopter engagements and disengagements were undertaken [35-36]. The tests were intended to demonstrate blade sailing and establish the effect of the ship airwake on the rotor behavior. In these tests, a radio-controlled model helicopter was mounted atop a model ship deck. A Kalt Cyclone kit was used because it had approximately the same rotor height/rotor diameter ratio as the Westland Lynx. Only the main fuselage and rotor were used in the tests. The main rotor system as originally provided in the kit was a two-bladed teetering rotor configuration. The supplied gas engine was discarded and replaced with a ½ HP electric motor. The main helicopter body was bolted to a sliding base plate on the ship’s deck and an electric motor was fitted below the deck. The ship deck itself consisted of nothing more than a large rectangular box - all small details were omitted. The ship hull size was scaled to approximate a Westland Lynx/RFA Rover Class ship combination.

The helicopter/ship combination was too large to be placed in the normal 5x7 foot tunnel test section, so instead it was placed in the settling chamber. The ship deck was placed in abeam wind conditions since previous work showed the greatest upwash and downwash velocities were encountered in this situation. Engagements and disengagements of the rotor system were then completed at a total of 5 locations on the ship deck, from fully windward (A) to fully leeward (E), as shown in Figure 1.9. The tunnel speed was set at 5 m/s, which represented a 47-knot full-scale wind. Tests were performed in a teetering rotor configuration and in an articulated rotor configuration. For each test, the rotor speed variation was a linear run-up followed by a constant period,
followed by a linear run-down. Linear motion potentiometers were used to measure the blade flapping and pitch angles while rotary encoders were used to measure the blade azimuth and rotor speed. The blades were restrained from flapping above $+23.7^\circ$ and $-11.7^\circ$ by mechanical stops. The collective pitch was set to give nominally zero thrust. Because the model rotor blade was quite rigid, the theoretical model was modified to include a rigid blade assumption. To further simplify the analysis, the assumption of a constant lift curve slope with no stall provision was made.

![Diagram of Model Helicopter/Ship Deck Configuration](image)

*Figure 1.9: Model Helicopter/Ship Deck Configuration*

The experimental tests showed several interesting trends. First, the articulated rotor consistently displayed more flapping than the teetering rotor. Second, two different types of blade behavior were seen depending on the location of the rotor on the flight deck. Very violent blade flapping behavior was seen at positions A, B, and C while much milder blade flapping was seen at positions D and E. A brief survey of the airflow over the ship deck without the helicopter was made using a wand and tuft. It was found that deck location B coincided with the boundary between relatively smooth and separated flow regions over the deck. This meant that when the rotor was at position B, the rotor blades would experience both smooth and separated flows in one revolution.
A much more detailed investigation of the airflow was deemed necessary to accurately model the ship airwake and was completed with a traversing LDA [37-40]. Flow measurements of the mean and standard deviation of the horizontal and vertical wind components were taken at 51 points along the ship's deck at the height of the rotor. As with the wand and tuft experiments, results showed that position B was right on the dividing line between relatively smooth and highly separated flow regions, while positions D and E were entirely in the separated flow region. The measured flow velocity vectors are shown in Figure 1.10.

![Figure 1.10: Ship Airwake Survey Results](from Ref. 1)

After completion of the detailed flow survey, the resulting velocity components were used as input to the theoretical model. In addition, the assumption of a constant lift curve slope with no stall provision was also deemed inappropriate. Instead, a quasi-steady aerodynamic model with a provision for a Kirchoff trailing edge separation model was used. The assumption of zero rotor downwash was retained. As in previous work, turbulence was modeled by multiplying the measured turbulence intensity by a Gaussianly distributed random number updated at a Strouhal scaled frequency.
Comparison with the experimental results showed the blade behavior was well modeled at all positions except for a small discrepancy in the mean coning angle. Correlation was further improved when the theoretical predictions were recalculated with -1.4° of longitudinal cyclic pitch. The results for the articulated blade tests with cyclic pitch are shown in Figure 1.12 through Figure 1.15. The discrepancy between the predictions and measurements was attributed to the variation in the flow velocities at different heights over the flight deck. The two-bladed teetering rotor tests show very similar trends [1].

![Figure 1.11](image1.png)

**Figure 1.11: Articulated Blade Sailing Results for Deck Position A**

![Figure 1.12](image2.png)

**Figure 1.12: Articulated Blade Sailing Results for Deck Position B**
Figure 1.13: Articulated Blade Sailing Results for Deck Position C

Figure 1.14: Articulated Blade Sailing Results for Deck Position D

Figure 1.15: Articulated Blade Sailing Results for Deck Position E
Newman's research was the first to successfully marry a detailed ship airwake model with a transient aeroelastic rotor analysis. It demonstrated that knowledge of the ship airwake is vital to the accurate prediction of blade sailing. However, lag motion and unsteady aerodynamic effects were ignored. Moreover, only the prediction of blade sailing was addressed. The control of blade sailing behavior was not attempted.

1.3.2.5 Research at Penn State University

Research into the blade sailing phenomenon has been conducted at Penn State University since the mid 1990's. The first-generation analysis was designed to predict the transient motion of a single articulated rotor blade [41-42]. Following a similar approach to the University of Maryland Advanced Rotorcraft Code (UMARC) [43], the blade flap-torsion equations of motion were derived using Hamilton's principle and discretized using the finite element method. Like Newman's work, the effects of gravity were included and ship roll motion was simulated with a sinusoidal motion at a specified frequency. The interaction of the ship roll motion and the rotor dynamics was also neglected; the only effect of ship motion was the rotation of airwake components. Furthermore, the influence of rotor acceleration was neglected. Unlike Newman's work, the droop stop was modeled with high-rate, conditional rotational springs instead of linear springs. The flap stop was initially neglected. Modeling of the airloads was accomplished with either nonlinear quasi-steady aerodynamics, including noncirculatory airloads [44], or time domain unsteady aerodynamics, including nonlinear separation and dynamic stall effects [45]. Following Newman's work, the ship airwake was simplified to horizontal, constant,
step or linear airwakes. Blade motions were discretized with cubic Hermitian shape functions in flap and quadratic Lagrangian shape functions in torsion resulting in seven degrees of freedom per element. Modal decomposition was used to reduce the size of the problem. A fourth order Runge-Kutta method was then used to time integrate the resulting equations of motion along a specified rotor speed profile.

Upon the completion of the analytic model, validation studies with the H-46 blade fan diagram and static deflection data were completed. Experimentally measured engage/disengage rotor speed data were used in the simulations. A comparison of the assumed rotor speed profile used by Newman and the measured H-46 rotor speed profile is shown in Figure 1.16. Note the assumed rotor speed profile accelerates much faster than the H-46. For example, the assumed rotor speed profile reaches 50%NR in just over 5 seconds but the H-46 requires twice as long to reach that speed. For the case considered, blade deflections increased by 12% using the experimentally measured, slower rotor speed profile. The unsteady aerodynamic model was shown to increase tip deflections by as much as 13% for wind speeds greater than 40 knots. Both collective

![Figure 1.16: Comparison of Rotor Speed Profiles](image-url)

a) Assumed (from Ref. 1)  
b) Measured H-46 (from Ref. 41)
and cyclic pitch inputs were shown to have a moderate effect on the maximum tip deflections.

Several parametric studies focusing on the effects of pilot procedures during engagements and disengagements were performed [46-47]. During a rotor engagement, the only procedure the pilot can vary to speed the acceleration of the rotor is to increase the advancement rate of the engine throttles. It was found that faster throttle advancement only moderately reduced the downward tip deflections during an engagement, typically by about 3%\( R \). During rotor disengagement, the only procedure the pilot can vary to speed the deceleration of the rotor is to change the point at which the rotor brake is applied. For the H-46, the point of rotor brake application is restricted between rotor speeds from 65% to 45%\( NR \). Thus, the deceleration of the rotor during the critical period from 25%\( NR \) to rest is not affected. It was shown that the tip deflections were insensitive to the point at which the rotor brake was applied. Another parametric study investigated the effect of small variations in the collective and cyclic pitch settings. A baseline engagement envelope for the standard settings, an envelope with an additional 2° collective and an envelope with an additional 2° lateral cyclic are shown in Figure 1.17. In the baseline case the H-46 is limited to wind speeds of 30 knots for port WOD conditions and is limited to 35 knots for most starboard WOD conditions. For the +2° collective case, the safe engagement region was increased by 5 knots for WOD directions of 150° to 240° and 285° to 345. For the 2° lateral cyclic case, the safe engagement region was reduced by 10 to 15 knots for WOD directions of 120° to 150° and 225° to
330°. The results of this study demonstrated that collective and cyclic changes could moderately impact rotor deflections.

Results of theoretical analyses showed that impacts between an articulated rotor blade and droop or flap stops always coincided with periods of excessive flapping. The correct modeling of a transient impact event between an elastic blade and essentially rigid stop was considered highly important, so further analytical and experimental research was focused in this area [48-50]. For the analytical investigation, a flap-only finite element model was used to simulate the rotor blade. During an impact between an articulated rotor blade and droop stop, the boundary conditions of the blade change from a hinged to a cantilevered condition. This change in the boundary conditions leads to the possibility of three different methods of time integrating the equations of motion. The first was a direct integration in the full finite element space; the second was a modal space integration using only hinged modes; and the third was a modal space integration using either hinged or cantilevered modes depending on blade-droop stop contact.
In the experimental study, a 1/8 th Froude-scaled articulated model rotor blade was constructed [48-50]. Given a range of initial flap hinge angles, drop tests of the model blade at zero rotational speed were conducted and the transient tip deflection, flap hinge angle, and strain were measured. The root end of the blade was held in a vice and an electromagnet was used to hold the blade up at an initial flap hinge angle of up to 10°. The test commenced by disengaging the electromagnet, allowing the blade flap downward. At a flap hinge angle of 0°, the blade struck a rigid droop stop and continued to bend downward elastically. After bending downward to its maximum point, the blade would rebound upwards. A drop test measured the blade's motion through one cycle of downward bending and upward rebounding.

Good correlation was found between all three analytical methods and the experimental results. The results of one specific drop test are shown in Figure 1.18. It is interesting to note that for a tip deflection of -17%R, both the measured and predicted strain at the 20%R radial location are five times the static strain. It was also shown that Bernoulli-Euler beam bending theory was sufficient to predict tip deflections of up to 25%R, which is near the maximum encountered in the blade sailing phenomenon.

![Figure 1.18: Drop Test Time Histories](image-url)
After a thorough examination of the blade and droop stop impact event, further research focused on expanding the capabilities of the full aeroelastic engagement and disengagement simulation to model not just articulated and hingeless, but also teetering and gimballed rotors. Modern tiltrotor configurations feature the use of a gimballed rotor system. A gimballed rotor has three or more blades rigidly attached to the hub. The hub is attached to the rotor shaft via a universal joint, or gimbal. Elastomeric rubber springs are used to provide the gimbal stiffness in the flapping direction; however, rigid mechanical restraints are used to prevent excessive hub flapping motion. A schematic of the hub for a gimballed rotor is shown in Figure 1.19.

**Figure 1.19: Gimballed Rotor Schematic**

Compared to articulated or hingeless rotor systems, the gimballed rotor has some unique structural and aerodynamic characteristics related to shipboard engagement and disengagement operations. Articulated and hingeless rotors feature relatively long, flexible blades so potential contact between the blades and the fuselage is the primary concern of blade sailing. Gimballed tiltrotors feature relatively short, stiff blades so contact between the blades and the fuselage is quite unlikely; however, impacts between the hub and the restraint mechanism are still possible. In this case, excessive bending moments are the primary concern. Furthermore, for articulated and hingeless rotors the
hub is rigidly attached to the rotor shaft so the motion of each individual blade is uncoupled from the others. For gimballed and teetering rotors, the hub is free to pivot on the rotor shaft so the blade motions are coupled. Lastly, most articulated and hingeless rotors have relatively small amounts of twist, on the order of 6° to 10°. The V-22 Osprey tiltrotor has 45° of twist, which introduces significant structural flap-lag couplings.

An initial analysis was developed to model multi-bladed gimballed tiltrotors during engagement and disengagement sequences [51-52]. A two-degree of freedom model calculating the transient response of the gimbal pitch and roll resulted. Satisfactory correlation with the two-bladed teetering rotor experimental wind tunnel tests in Ref. 1 was displayed. The possibility of gimbal impacts for a gimballed rotor similar to the V-22 was shown. A simple linear run-up and run-down rotor speed profile was assumed requiring 100 seconds to reach full speed. Multiple impacts with the gimbal restraint were seen for wind speeds as low as 30 knots.

The full aeroelastic simulation was then extended to model all three elastic blades of a gimballed tiltrotor. Since the large amounts of blade twist result in significant flap-lag coupling, lag degrees of freedom were added to the flap-torsion degrees of freedom already included in the simulation. The capability to predict blade bending moments using a curvature method was also added. The elastic code was first validated against experimentally measured flap and lag bending moments for an aeroelastically scaled model tiltrotor in hover and satisfactory correlation was observed. Engagements and disengagements for the full scale V-22 in 35-knot winds were simulated. Several gimbal restraint impacts were shown to occur.
1.3.2.6 Multi-body Modeling Techniques

Recently, a finite element based multi-body dynamics analysis was developed to examine engagement and disengagement operations [53]. The multibody modeling technique offers the advantage of easily modeling different rotor hub types with an extensive library of pre-validated elements. In the technique, no modal reduction is performed. Special implicit time integration techniques for nonlinear systems were used containing high frequency numerical dissipation. The time integration scheme also utilized a variable time step to maximize computational efficiency. The multi-body analysis was first validated with the experimental rotor blade-droop stop impact tests performed in Ref. 50 and good correlation was displayed. Engagements and disengagements of an H-46 rotor system were then simulated in a constant airwake distribution. Time histories of the flap tip deflection and pitch link loads were examined.

1.4 Ship Airwake Modeling

As seen in earlier blade sailing research, especially in the experimental wind tunnel tests at the University of Southampton, the ship airwake environment plays a large role in the aeroelastic response of the rotor. As such, it is relevant to review past efforts in the measurement and prediction of the ship airwake environment. This review is by no means comprehensive, but intended to introduce some of the most relevant research. In general, the ship airwake is not considered during the design of ships. The boxy
configuration of the superstructure acts much like a bluff body and leads to large separated wakes and recirculation zones that vary intermittently in size and shape.

Three avenues exist for the investigation of ship airwakes: full-scale testing, model-scale testing, and numerical simulation. Full-scale testing is attractive because all the relevant aerodynamic qualities of the atmosphere and geometric qualities of the ship are modeled; however, problems with data collection exist. The conditions during any full-scale test are far from controlled; wind speed, direction, and the ship motion can all change. Furthermore, the collection of the airwake data at a significant number of locations is difficult. Significant costs are usually incurred in any full-scale testing. Model-scale tests are much cheaper and offer a controlled environment; but accurate simulation of the atmosphere and geometric characteristics of the ship is not an easy task. Numerical simulation offers the greatest amount of detail, but requires a significant amount of computing resources. In addition, the airwake around a ship tends to be unsteady and separated in nature making accurate numerical simulation difficult.

The simplest model of the airwake behind a ship's hangar is approximated by the flow around a cube [54]. A schematic drawing of the flow around a cube is shown in Figure 1.20. Note the presence of an inverted U-shape vortex whose ends remain in contact with the ground on the leeward side of the cube. Numerous horseshoe vortices are wrapped around the upstream base of the body. These basic flow structures will tend to repeat themselves, even for more complicated ship shapes. Another source of information on ship airwakes comes from research into the aerodynamic characteristics of buildings [55-58].
1.4.1 Research at the Naval Postgraduate School

In the mid 1980's, extensive research into the accurate measurement of model-scale ship airwakes was conducted at the Naval Postgraduate School. First, a review of previous attempts at measuring and modeling ship airwakes was conducted [20]. A full-scale ship operates in the Earth's Atmospheric Boundary Layer (ABL). It was concluded that almost all of the previous attempts at simulating and understanding the ship airwake environment in wind tunnels were faulty because they did not correctly model the nonuniform velocity profile and turbulence in the ABL. Instead, they assumed that the freestream flow had low turbulence levels and had a uniform velocity profile, which simulates winds that are due only to the ship speed. In reality, the relative airflow over a ship deck is a combination of the velocity of the ship and of the velocity of the winds in the ABL. To correctly model the ship airwake environment in the presence of the ABL, several quantities must be correctly simulated. The mean wind speed velocity profile as a function of height must be modeled correctly. The standard deviation of the freestream
wind speed about the mean must also match the ABL. Dividing the standard deviation of the freestream wind speed by the mean wind speed yields the turbulence intensity. Typical turbulence intensities in the ABL usually range from 13% to 17%. The ratio of the longitudinal length scale of the freestream turbulence to the ship's beam must also match the ABL. The longitudinal, or integral, length scale is a measure of the mean size of the most energetic eddies in the turbulence. Lastly, the spectrum function of the turbulence, which indicates how the energy is distributed amongst the frequencies present in the turbulence, must be matched.

A series of wind tunnel tests measuring the airwake around several different US Navy ship types was then completed. The primary variable in each of the flow studies was the ship yaw angle. At first, the flow studies described the correct modeling of the ABL in the NPS wind tunnel and made only qualitative measurements of the ship airwake environment.

The first ship examined was a 1/205th scale US Navy LHA class amphibious assault ship [59]. Later, a 1/140th scale US Navy DD class destroyer was examined [60-61]. Schematic drawings of the airflow over the aft landing pad are shown in Figure 1.21. Only yaw angles of 0°, 30° and 330° were investigated. At 0° yaw, the flow along the centerline of the ship flows over the hangar and splits. At that point, the flow then curls upward almost to the top of the hangar face. There, it splits into a starboard and port component, both of which then flow downstream. Note that much of the flight deck is exposed to the hangar wake. The flow patterns are somewhat similar for 30° yaw; however, more of the flight deck is exposed to the freestream airflow.
A more detailed investigation was later conducted for a 1/171st scale AOR class ship [21, 62]. This study provided not only basic flow pattern but also velocity information and was specifically intended to aid in solving the H-46 tunnel strike problem. Six different ship yaw angles of 0°, 30°, 50°, 70°, 90°, and 110° were investigated. Three-dimensional hot-wire measurements of the flow at four points around the locus of the helicopter blade tip were made. Since the flight deck on an AOR ship is at a relatively high elevation from the waterline, significant upflow was seen on the windward side of the flight deck for nonzero ship yaw angles. Qualitative results for the flow patterns for ship yaw angles of 0° and 110° are shown in Figure 1.22. Note the resemblance to the flow behind a cube, especially in the upside down U-shaped vortex right behind the hangar. The measured turbulence intensities over the flight deck for the range of ship yaw angles tested are shown in Figure 1.23. The turbulence intensities range from 2% to as high as 19%.
In a later study, hot-wire measurements showed extreme velocity gradients over the flight deck of a generic 1/141st scale model ship [8]. The measured turbulence intensities over the flight deck ranged from 6% to 10%. The measured velocity spectra over the flight deck are shown in Figure 1.24. The most energetic frequencies at model-scale were from 6 to 30 Hz. Translated to full-scale, these frequencies range from 0.3 to 1.5 Hz.
1.4.2 Research by The Technical Cooperation Programme

More recently, research by the TTCP group has focused not only on wind tunnel tests of the ship airwake but also on predictions through modern CFD methods. The aim of each of the studies was to examine the ship airwake characteristics around a typical helicopter approach path to the ship. Thus, the majority of the ship airwake research is more closely related to the launch/recovery problem rather than to the engage/disengage problem.

The US Navy calculated the airwake about a simplified DD class ship shape with CFL3D, a NASA developed thin-layer Navier-Stokes code [63-64]. The major flow features and mean flow velocities observed in earlier wind tunnel experiments at the Naval Postgraduate School were seen. The airwake around more complex LPD [65] and LHD [66] class ship shapes was also calculated with the same thin-layer Navier-Stokes method. In addition, the airwake around a very complicated model of a DDG class ship was simulated with FAST3D, an unsteady, inviscid low-order method [67].

Figure 1.24: Velocity Spectra Behind a Generic Ship
At DERA, an in-house code capable of predicting flows around ships was developed [68]. It was proposed that the generation and early spatial development of vortices in the flow could be predicted with the steady-state Euler equations, while the downstream development of the vortices required viscous effects. A simulated helicopter body was included above the flight deck. The ship airwake affects the rotor downwash and vice versa; therefore, an iterative process was used to determine the final solution. Results were generated for an AOR ship and a Westland Merlin helicopter. The rotor downwash was shown to significantly affect the airwake close to the hangar face.

Analytical and experimental studies were also completed at the Canadian NRC on several frigate types. Wind tunnel flow studies were first completed on a 1/300th scale "generic" frigate [69]. Both mean wind velocities, time averaged turbulence intensities, and velocity spectra were calculated around the ship. The measured turbulence intensities are shown in Figure 1.25. The along-wind turbulence intensities were as high as 20% while the vertical turbulence intensities were only 5%. The measured velocity spectra are shown in Figure 1.26. When scaled to the prototype, they showed the most energy frequencies were between 0.1 and 2 Hz.

![Turbulence Intensities Behind a Generic Frigate](image1.png)

*Figure 1.25: Turbulence Intensities Behind a Generic Frigate*
Additional wind tunnel tests were performed on a 1/50th scale Canadian Patrol Frigate [70]. Flow asymmetry, due to a gun turret mounted on top of the hangar, was investigated. A large recirculation zone, including the upside down U-shaped vortex behind the hangar, was again observed. The wind tunnel results were also compared to full-scale measurements at select points on the flight deck and favorable agreement was seen. CFD predictions were also made for the 1/50th scale Canadian Patrol Frigate with a finite-volume Navier-Stokes solver. The upside down U-shaped vortex was predicted behind the hangar and the recirculation zone was shown to extend over half of the flight deck. A detailed investigation of the airwake around the 1/60th scale Simple Frigate Shape (SFS) was also completed [71]. This frigate shape, somewhat resembling an FFG Class frigate as seen in Figure 1.27, has been set forth by The Technical Cooperation Program as a baseline case for validation of different CFD codes. Flow visualization information from oil-flows and smoke-flows along with surface pressure coefficients at

![Figure 1.26: Velocity Spectra Behind a Generic Frigate](image)

a) Along-wind (from Ref. 69)

b) Vertical (from Ref. 69)
various combinations of wind speeds and directions were obtained. No atmospheric boundary layer was simulated. Flow visualization work was completed at ship yaw angles of 0° to 135° in 15° increments while pressure coefficient data was taken at a range of ship yaw angles from 0° to 120°.

At the DSTO, the airwake around the SFS has also been numerically predicted with FLUENT [72]. The analysis was performed on a structured grid with a $k-\varepsilon$ turbulence method. Ship yaw angles of 0°, 45° and 90° were examined. The accuracy of the solution was checked by comparing with the experiments in Ref. 71. Circulation zones and shed vortices were shown to dominate the flow field in the region of the flight deck. The general flow field features from the numerical predictions compared reasonably well with the experimental data; however, small differences were found in the location of the flow reattachment points.
1.4.3 Research at Penn State University

Ship airwake prediction through modern CFD methods has been the subject of much research at Penn State University by Long et al. Two different solution methods were tested. Initially, the method chosen to simulate the ship airwake used the Nonlinear Disturbance Equations (NLDE) [73]. In this solution technique, the general Navier-Stokes equations are written and the viscous terms are neglected. The resulting flowfield variables (mass density, momentum, and energy) are then split into mean and fluctuating parts. The mean flow terms can then be solved using an "off-the-shelf" CFD code such as CFL3D or PUMA. These mean flow terms, in turn, can be used as a source term for the unsteady part of the Navier Stokes equations, or the NLDE. The NLDE are then solved numerically using a fourth order accurate Dispersion Relation Preserving scheme and a fourth order accurate Runge-Kutta scheme. This method has several advantages - it allows usage of the most effective algorithm and computational grid for the steady and unsteady parts and it minimizes round-off error. Results were generated for the SFS for a 0° yaw angle [74-75]. The mean flow components were calculated with CFL3D, a structured finite-difference flow solver. Indications of the upside-down U-shaped vortex directly behind the hangar were observed. Comparison with oil-flow visualization experiments showed that the attachment point of the U-shaped vortex was farther outboard than measured, probably due to the inviscid assumption. Using the NLDE, perturbation velocities were also predicted. Large magnitude longitudinal and vertical turbulence intensities were measured aft of the bow and hangar.
Recently, a modified version of the Parallel Unstructured Maritime Aerodynamics (PUMA) code was used to predict the flowfield around complex ship shapes [76-77]. PUMA, originally developed at Virginia Tech by Bruner, was designed to predict flows over arbitrarily complex 3D geometries [78-79]. Based on the finite volume method (FVM), it allows the use of unstructured grids and easily facilitates the mapping of complex ship shapes.

After gaining confidence in PUMA through the validation studies, the Simple Frigate Shape was again studied. Qualitative comparisons using the inviscid NLDE and the viscous PUMA approaches were made with the wind tunnel oil-flow visualizations. The results of these comparison studies are shown in Figure 1.28 through Figure 1.30. Three locations on the SFS ship are shown: the bow, the exhaust stack, and the flight deck. Figure 1.28 shows the results for the oil flow visualizations, Figure 1.29 shows the results for the NLDE analysis and Figure 1.30 shows the results for the PUMA analysis. The viscous PUMA solution was better at predicting the separated flow aft of the bow and hangar.
Figure 1.28: Oil Flow Visualization of the Airwake Over the SFS

Figure 1.29: NLDE Airwake Velocity Contours Over the SFS

Figure 1.30: PUMA Airwake Velocity Contours Over the SFS
After validation with experimental flow results for the SFS, PUMA was used to generate ship airwakes for the SFS for ship yaw angles from 0 to 180° in 30° increments and at flow speeds of 40 and 50 knots. The longitudinal (along the ship axis direction) velocity component on the surface and in the plane of an H-46 rotor at a ship yaw angle of 30° is shown in Figure 1.31. In addition, flow solutions were also generated for CVN and LHA class ships.

![Velocity Components for the SFS at 330° Yaw](image)

**Figure 1.31**: Velocity Components for the SFS at 330° Yaw

### 1.5 Summary of Related Research

When rotorcraft are flown from ships they often operate in a unique and hazardous environment. A troublesome problem known as “blade sailing” can occur during the engagement and disengagement of the rotor system. Appreciable aerodynamic forces combined with low centrifugal stiffening at low rotor speeds can result in
excessive aeroelastic flapping of the rotor. For articulated rotors, the blade tips may contact the fuselage and endanger the airframe, flight crew, and anyone standing near the helicopter. For teetering or gimballed rotors, excessive gimbal tilt angles may cause the rotor hub to strike the gimbal stops resulting in potentially high bending moments in the blade flexures.

Due to the potential of blade sailing, conservative limits are placed on the conditions in which rotor systems can be engaged and disengaged on board ships, which tends to severely limit the helicopter's operational capability. These limits are termed the Ship/Helicopter Operational Limits (SHOLs) and in the past were determined in full-scale, at-sea experimental tests. The tests had to be conducted for every helicopter/ship/landing spot combination because of the distinct performance characteristics of helicopter types, the variability in size and shape of ship types, and the various characteristics of landing spots on the ship's deck. Unfortunately, this process of experimentally determining the SHOLs was very labor intensive, time consuming, expensive, and somewhat unreliable. Because of these problems, the analytical determination of SHOLs has been proposed as a viable alternative to experimental testing. The use of modeling and simulation offers substantial benefits in terms of reducing cost, improving safety, and developing larger operating envelopes.

Dr. Simon Newman at the University of Southampton was the first researcher to conduct extensive blade sailing research. Model-scale, rigid blade, experimental tests were conducted and successfully demonstrated blade sailing. Later, an aeroelastic flap-torsion simulation code capable of predicting blade sailing for both hingeless and
articulated rotors was developed. The effect of the ship airwake on the rotor system behavior was demonstrated.

Motivated by the H-46 Sea Knight tunnel strike problem, Geyer et al at Penn State University have conducted research into blade sailing phenomenon. Initially, an aeroelastic flap-torsion simulation code was developed and capable of simulating a single hingeless or articulated rotor blade. The model was later modified to include lag degrees of freedom and the capability of modeling multiple rotor blades for gimballed and teetering rotors.

Substantial improvements in the modeling of ship airwakes have been made in the past decade. Recently, Long et al at Penn State have modified PUMA, a CFD program using unstructured grids and run on an inexpensive PC cluster, to predict the ship airwake environment. Recent comparisons with experimental results for a generic frigate with both viscid and inviscid models have increased confidence in the airwake predictions.

1.6 Objectives of the Current Research

Since 1995, substantial research has been completed and has been focused on constructing and validating a finite-element based blade sailing model at Penn State University to predict the transient aeroelastic response of rotorcraft during engagement and disengagement operations. The subjects covered in the current research are organized into the validation, the prediction, or the control of engagement and disengagement behavior.
The first objective is to validate the analysis with the model-scale helicopter and wind tunnel experiments conducted at the University of Southampton. The second objective is to address recently added prediction capabilities including:

1. A comparison of the effects of the nonlinear quasi-steady and unsteady aerodynamic models.

2. The effect of lag motion on the rotor response during engagement and disengagement operations. Previous engagement and disengagement research at the University of Southampton and Penn State University was completed with flap-torsion only blade motion and its effect has yet to be assessed.

3. The effectiveness of recently added time integration techniques. The only time integration technique used in previous engagement and disengagement simulations was a fourth order Runge-Kutta method.

4. The effect of more realistic ship airwakes. The distribution of the steady airwake components has been shown to substantially influence blade sailing behavior. Recent work has made sophisticated ship airwake models more readily available for use in engagement and disengagement research.

The third objective of this dissertation is to examine methods to control blade sailing behavior. Little research to date has seriously addressed the subject of reducing excessive flapping. Each of the following control methods will be examined:

1. The simplest method to reduce flapping motion would be to introduce a discrete flap damper. Such a flap damper could dissipate excessive flap motion as a lag
damper dissipates excessive lag motion. Naturally, such a flap damper would have to become inactive in normal operating conditions.

2. Most articulated rotor blades are relatively elastic in the normal direction, but are much stiffer in the chordwise direction. At high values of collective pitch, the effective flapwise stiffness of the blade could be increased through the rotation of the blade cross section. The effectiveness of a large change in collective pitch setting during a short period in the engagement will be examined.

3. Blade sailing is caused by appreciable aerodynamic lift generated at low rotor speeds. The simplest method of reducing blade flapping would be to reduce the excessive lift. The effectiveness of reducing the excessive lift through spoilers will be investigated. Naturally, the spoilers would only be used in the critical rotor speed region and be inactive during normal operating conditions.

4. Future naval rotorcraft may have on-board control systems that allow feedback from the blade motion to the control system inputs. The effectiveness of using a control system to reduce blade flapping during rotor engagements will be examined.
In this chapter, the analytic model and transient analysis of rotors during shipboard engagement and disengagement operations is presented. The ship is assumed to be rolling about its centerline and the helicopter is assumed to be stationary on the ship deck. The distribution of the ship airwake velocities over the rotor disk is modeled with simple analytic expressions or with arbitrary spatially varying results from experimental tests or numerical predictions. Only a single main rotor of the helicopter is modeled. The rotor is, by definition, accelerating or decelerating in engagement and disengagement operations. Each rotor blade is modeled as an isotropic elastic beam undergoing moderate out-of plane (flap), in plane (lag), and twisting (torsion) motions. The nonlinear governing equations of motion are derived using Hamilton’s Principle and are spatially discretized using the finite element method. The formulation is applicable to rotor hubs with articulated, hingeless, teetering and gimballed rotor blades.

2.1 Basic Considerations

In this section the coordinate systems, nondimensionalization and ordering schemes used in the analysis are explained. They are based on the system used in Ref. 43, but have modifications specific to the simulation of engagement and disengagement operations.
2.1.1 Coordinate Systems

The coordinate systems used in the present analysis are a combination of those defined in Ref. 43, plus one other specific to the modeling of engagement and disengagement operations. The top level coordinate system is the ship-fixed inertial coordinate system, \( (X_{\text{ship}}^I, Y_{\text{ship}}^I, Z_{\text{ship}}^I) \), with corresponding unit vectors \( \hat{X}_{\text{ship}}^I, \hat{Y}_{\text{ship}}^I, \hat{Z}_{\text{ship}}^I \) and is shown in Figure 2.1. This coordinate system is attached to the ship center of gravity and does not move with the ship motion. Also attached to the ship center of gravity, but rotating with the ship roll motion, is the ship-fixed coordinate system, \( (X_{\text{ship}}, Y_{\text{ship}}, Z_{\text{ship}}) \), with corresponding unit vectors \( \hat{X}_{\text{ship}}, \hat{Y}_{\text{ship}}, \hat{Z}_{\text{ship}} \). The \( X_{\text{ship}} \) axis points to stern, the \( Y_{\text{ship}} \) axis points to starboard and the \( Z_{\text{ship}} \) axis points upwards. The ship roll motion is a prescribed time-varying function. Assuming that the ship roll motion is small, the transformation matrix between these two coordinate systems is given by

\[ \begin{vmatrix} X_{\text{ship}}^I & Y_{\text{ship}}^I & Z_{\text{ship}}^I \\ \hat{X}_{\text{ship}} & \hat{Y}_{\text{ship}} & \hat{Z}_{\text{ship}} \\ \end{vmatrix} \]

Figure 2.1: Ship and Vehicle Coordinate Systems
\[
\begin{bmatrix}
\hat{I}_{\text{SHIP}} \\
\hat{J}_{\text{SHIP}} \\
\hat{K}_{\text{SHIP}}
\end{bmatrix} = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & -\phi_{\text{ship}} \\
0 & \phi_{\text{ship}} & 1
\end{bmatrix} \begin{bmatrix}
\hat{I}'_{\text{SHIP}} \\
\hat{J}'_{\text{SHIP}} \\
\hat{K}'_{\text{SHIP}}
\end{bmatrix}
\]

(2.1)

Where \( \phi_{\text{ship}} \) is the ship roll angle and is positive rolled to starboard. The position of the helicopter on the ship deck in actual fleet operations is determined by the location of the helicopter nosewheel. Each different position on the ship deck is called a landing spot.

In this analysis, it is assumed that the position of the nosewheel from the ship center of gravity, \( x_{nw}, y_{nw}, z_{nw} \), is specified. The rotor hub is then located a distance \( x_{rh}, y_{rh}, z_{rh} \) relative to the nosewheel. Centered at the rotor hub and parallel to the ship-fixed coordinate system is the vehicle-fixed coordinate system, \( (X_F, Y_F, Z_F) \), with corresponding unit vectors \( \hat{I}_F, \hat{J}_F, \hat{K}_F \). Unlike Ref. 43, there is no need for a coordinate system centered at the vehicle center of gravity because the helicopter is stationary on the ship deck.

The rest of the coordinate systems used in this analysis are identical to those in Ref. 43 and for reference are detailed in Appendix A.1. Centered at the rotor hub and rotated through the longitudinal shaft angle is the hub-fixed nonrotating coordinate system. Only longitudinal shaft tilt, \( \alpha_s \), is considered; lateral shaft tilt is assumed zero. Also centered at the rotor hub is the hub-fixed rotating coordinate system, which rotates with the blades in the counterclockwise direction at an angular velocity of \( \Omega \) relative to the hub-fixed nonrotating coordinate system. Note that in this analysis the rotor speed is not a constant; therefore, the blade azimuth angle is equal to the time integral of the rotor speed.
\[ \psi = \int_{0}^{t} \Omega(t) \, dt + \psi_0 \]  

(2.2)

Where \( \psi_0 \) is the azimuth angle of the blade at the start of the engagement or disengagement. The \textit{undeformed blade coordinate system} is inclined through the blade precone angle, \( \beta_p \), from the hub-fixed rotating coordinate system. The final coordinate system is the \textit{deformed blade coordinate system}. The deformed blade coordinate system accounts for the axial displacement \( u \), in plane displacement \( v \), and out-of-plane displacement \( w \) of the blade. It is also rotated about the deformed elastic axis through the total blade pitch angle. The total blade pitch angle, \( \theta_i \), is defined by

\[ \theta_i = \theta_0 + \hat{\phi} \]  

(2.3)

Where \( \theta_0 \) is the rigid pitch angle due to the control system settings and blade pretwist and \( \hat{\phi} \) is the elastic twist angle. Both angles are measured in the deformed blade coordinate system. The rigid pitch angle is given by

\[ \theta_0 = \theta_{75} + \theta_{lc} \cos \psi + \theta_{ls} \sin \psi + \theta_{tw} (x) + \Delta \theta \]  

(2.4)

Where \( \theta_{75} \) is the collective pitch at the 75% span location, \( \theta_{lc} \) and \( \theta_{ls} \) are the lateral and longitudinal and cyclic pitch controls and \( \theta_{tw} \) is the blade pretwist angle referenced to the 75% span location. In general, the blade pretwist angle is a nonlinear function. The change in the pitch angle, \( \Delta \theta \), is due to pitch-flap or pitch-lag coupling from sources such as skewed hinges, location of the pitch link, relative position of the hinges or composite couplings.
2.1.2 Nondimensionalization

The entire analysis is completed with nondimensional quantities. The primary advantage of using non-dimensional numbers is that it allows more direct comparison between helicopters of different sizes. All basic physical quantities are nondimensionalized using the reference parameters in \textit{Table 2.1}.

\begin{table}[h]
\centering
\begin{tabular}{|l|l|}
\hline
Physical Quantity & Reference Parameter \\
\hline
Length & $R$ \\
Time & $1/\Omega_0$ \\
Mass/Length & $m_0$ \\
Velocity & $\Omega_0 R$ \\
Acceleration & $\Omega_0^2 R$ \\
Force & $m_0 \Omega_0^2 R^2$ \\
Moment, Energy or Work & $m_0 \Omega_0^2 R^3$ \\
\hline
\end{tabular}
\caption{Nondimensionalization Parameters}
\end{table}

In this analysis the rotor speed is not a constant; therefore, all quantities are nondimensionalized by the full rotor speed $\Omega_0$. It is also convenient to nondimensionalize the time derivatives. Because the rotor speed is not constant, time derivatives are transferred to azimuthal derivatives taken at full rotor speed

$$\frac{\partial}{\partial t} = \frac{\partial \psi}{\partial t} \frac{\partial}{\partial \psi} = \Omega_0 \frac{\partial}{\partial \psi} \quad (2.5)$$
2.1.3 Ordering Scheme

Inclusion of every term in the analysis would become very cumbersome. To reduce the total number of terms in the formulation, an ordering scheme is applied. The ordering scheme provides a systematic method for neglecting terms based on their relative magnitudes. The nondimensional quantity $\varepsilon$ is introduced and defined such that $\varepsilon \ll 1$. Each term is then associated with a relative magnitude, expressed in powers of $\varepsilon$. The order of the terms in the analysis is given in Table 2.2. In the majority of the analysis, only terms of order $\varepsilon^2$ or lower are retained; however, in the torsion equations some terms of order $\varepsilon^3$ are retained. Furthermore, special notice is taken to the torsion terms $k_{m_1}^2$ and $k_{m_2}^2$. These terms normally would normally be order $\varepsilon^2$, but they are treated as order $\varepsilon$, or are “promoted” in this analysis. Otherwise, their effects in the torsion equations would be ordered out because of larger flap and lag terms.
Table 2.2: Ordering Scheme

<table>
<thead>
<tr>
<th>EA</th>
<th>$O\left(\varepsilon^{-2}\right)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\partial}{\partial \Omega}, \Omega, \psi$</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>$x, x_{nw}, y_{nw}, z_{nw}, x_{rh}, y_{rh}, z_{rh}$</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>$V_{x}, V_{y}, V_{z}, \cos \psi, \sin \psi$</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>$\theta_{0}, \theta_{75}, \theta_{1c}, \theta_{1s}, \theta_{nw}$</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>$m, EI_{yy}, EI_{zz}, GJ$</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>$v, w, \phi, \dot{\phi}$</td>
<td>$O(\varepsilon)$</td>
</tr>
<tr>
<td>$k_{A}, k_{m_{1}}, k_{m_{2}}, k_{m_{1}^{2}}, k_{m_{2}^{2}}$</td>
<td>$O(\varepsilon)$</td>
</tr>
<tr>
<td>$\phi_{ship}, \alpha_{s}, \beta_{p}$</td>
<td>$O(\varepsilon)$</td>
</tr>
<tr>
<td>$\lambda_{s}, \eta_{r}$</td>
<td>$O(\varepsilon)$</td>
</tr>
<tr>
<td>$EB_{2}, EC_{2}$</td>
<td>$O(\varepsilon)$</td>
</tr>
<tr>
<td>$e_{e}, e_{g}, e_{A}$</td>
<td>$O\left(\varepsilon^{3/2}\right)$</td>
</tr>
<tr>
<td>$u$</td>
<td>$O(\varepsilon^{2})$</td>
</tr>
<tr>
<td>$\lambda_{T}, EB_{1}, EC_{1}$</td>
<td>$O(\varepsilon^{2})$</td>
</tr>
</tbody>
</table>
2.2 Formulation Using Hamilton’s Principle

In the following sections, the generalized Hamilton’s principle is used to derive the partial differential equations governing the blade motion. The generalized Hamilton’s principle, applied to a nonconservative system, is expressed as

\[
\delta \Pi = \int_0^t \left[ \sum_{i=1}^{N_i} (\delta U_i - \delta T_i - \delta W_i) \right] dt = 0 \tag{2.6}
\]

The variations in the kinetic energy, strain energy and virtual work are composed of contributions from the blade \((b)\), flap \((FS)\) and lag \((LS)\) spring stiffnesses, control system stiffness \((CS)\), gravitational forces \((g)\), flap \((FD)\) and lag damper forces \((LD)\), and aerodynamic forces \((AF)\)

\[
\delta U_i = \delta U_b + \delta U_{FS} + \delta U_{LS} + \delta U_{CS} \\
\delta T_i = \delta T_b \\
\delta W_i = \delta W_g + \delta W_{FD} + \delta W_{LD} + \delta W_{AF} \tag{2.7}
\]

Note that several specialized terms are included in this analysis that are not normally included in other comprehensive analyses. The rotor speed is by definition not a constant; therefore, several terms due to the rotor acceleration are included. Furthermore, since the rotor speed is very low gravitational effects cannot be ignored. The ship airwake environment contributes appreciable in plane and out-of-plane airflow velocities. These extra ship airwake velocities result in additional aerodynamic forces, which must be included. Because the rotor speed is very low and the ship airwake environment is very unusual, the blades can experience any incidence angle from ranging 0º to 360º. In addition, large regions of the rotor disk may be immersed in reverse flow.
2.2.1 Strain Energy

In this section, expressions for the total strain energy of the system will be derived. Contributions from the elastic deformation of the blade, the flap and lag spring stiffnesses and the control system stiffness are also considered.

2.2.1.1 Blade Strain Energy

The expressions for the blade strain energy are derived in a manner similar to Ref. 43. The only difference is that the blade is assumed inelastic in the axial direction. The variation in strain energy of each rotor blade is given by

\[
\delta U_b = \int_0^1 \int_A \left( E \varepsilon_{xx} \delta \varepsilon_{xx} + G \varepsilon_{yy} \delta \varepsilon_{yy} + G \varepsilon_{zz} \delta \varepsilon_{zz} \right) d\eta d\zeta dx
\]  

(2.8)

The strains are expressed as

\[
\varepsilon_{xx} = u' + \frac{1}{2} v'' + \frac{1}{2} w'' - \lambda \phi'' + \left( \eta^2 + \zeta^2 \right) \left( \theta_0 \phi' + \frac{1}{2} \phi'' \right)
\]

\[-v'' \left( \eta \cos \theta_1 - \zeta \sin \theta_1 \right) - w'' \left( \eta \sin \theta_1 + \zeta \cos \theta_1 \right)
\]

\[
\varepsilon_{yy} = -\dot{\phi}'
\]

\[
\varepsilon_{zz} = \dot{\eta} \phi'
\]

(2.9)

Taking the variation of each yields

\[
\delta \varepsilon_{xx} = \delta u' + v' \delta v' + w' \delta w' - \lambda \delta \phi'' + \left( \eta^2 + \zeta^2 \right) \left( \theta_0 + \phi' \right) \delta \phi'
\]

\[-\left( \eta \cos \theta_1 - \zeta \sin \theta_1 \right) \left( \delta v'' + w'' \delta \theta_1 \right) - \left( \eta \sin \theta_1 + \zeta \cos \theta_1 \right) \left( \delta w'' - v'' \delta \theta_1 \right)
\]

\[
\delta \varepsilon_{yy} = -\dot{\zeta} \delta \phi'
\]

\[
\delta \varepsilon_{zz} = \dot{\eta} \delta \phi'
\]

(2.10)

Expressions for the axial deflection, \( u \); the elastic twist in the undeformed blade coordinate system, \( \phi \); and the total blade pitch angle, \( \theta_1 \), are then substituted.
Appropriate structural quantities can then be easily defined. For reference, this process is fully outlined in Appendix A.2.

2.2.1.2 Flap Spring Strain Energy

Depending on the type of rotor system, different means are employed to restrict blade motions at rest and at low rotor speeds. The following sections describe the formulas necessary to model the mechanical stops for articulated, hingeless, teetering, and gimballed rotor types.

2.2.1.2.1 Articulated Rotors

Articulated rotors can have discrete rotational springs at the flap hinge. Such a spring may be used to model friction in the hinge itself or the stiffness properties of a flap damper. In the case of a continuous spring at the flap hinge, the strain energy is

\[ U_{FS} = \frac{1}{2} K_\beta \beta_{hinge}^2 \]  

(2.11)

Taking the variation yields

\[ \delta U_{FS} = K_\beta \beta_{hinge} \delta \beta_{hinge} \]  

(2.12)

The flap hinge angle, \( \beta_{hinge} \), is defined as the difference in the flap slope, \( w' \), evaluated on either side of the flap hinge

\[ \beta_{hinge} = \tan^{-1}\left(\Delta w'_{flap hinge}\right) \]  

(2.13)

If the flap slope is small, the flap hinge angle simplifies to
Articulated rotors also require rigid mechanical limiters to support the blade weight when the rotor is at rest and to limit the blade flap angle at low rotor speeds. These mechanical limiters are called droop and flap stops. In normal flight conditions, the stops are not needed to restrain the blade motion. The stops are designed to retract and become inactive above a certain rotor speed, usually about 50%NR. The stops are also designed such that they may not retract if the blade is in contact with the stop. Conversely, the stops are also designed such that they may not extend if the blade is beyond the stop angle. An additional discrete rotational spring is used to simulate the droop and flap stops; however, the spring stiffness is conditional on the blade rotation at the flap hinge. If the blade is in between either stop, the spring stiffness is zero. If the stops are extended and the flap hinge angle is sufficient to contact either stop, the spring stiffness is equal to the stop stiffness. This stop stiffness is set arbitrarily large, so that the blade rotation at the flap hinge is limited to less than 0.01° of rotation at the flap hinge upon blade-stop contact. The strain energy for the flap stop spring is

\[ U_{FS} = \frac{1}{2} K_\beta (\beta_{hinge} - \beta_{stop})^2 \]

Where \( \beta_{stop} \) is the flap hinge angle at which contact between the blade and the stop is made. Taking the variation yields

\[ \delta U_{FS} = K_\beta (\beta_{hinge} - \beta_{stop}) \delta \beta_{hinge} \]

\[ K_\beta = \begin{cases} 0 & \beta_{hinge} \leq \beta_{stop} \\ K_{\beta_{stop}} & \beta_{hinge} > \beta_{stop} \end{cases} \]
2.2.1.2.2 Hingeless Rotors

Hingeless rotors do not necessarily require rigid stops to support the blade weight when the rotor is at rest; however, some models use them to limit the blade deflection at low rotor speed. Like an articulated rotor, a discrete conditional spring is used to simulate the droop and flap stops. However, the contact is based not upon the flap hinge angle but upon the flap deflection at a point just beyond the flexbeam. The strain energy for a hingeless rotor flap stop spring is

\[ U_{FS} = \frac{1}{2} K_w \left( w_{\text{hinge}} - w_{\text{stop}} \right)^2 \]

\[ K_w = \begin{cases} 0 & w_{\text{hinge}} \leq w_{\text{stop}} \\ K_{w_{\text{stop}}} & w_{\text{hinge}} > w_{\text{stop}} \end{cases} \quad (2.17) \]

Where \( w_{\text{hinge}} \) is the flap deflection at the flap stop location, and \( w_{\text{stop}} \) is the flap deflection at which contact between the blade and the stop is made. Taking the variation yields

\[ \delta U_{FS} = K_w \left( w_{\text{hinge}} - w_{\text{stop}} \right) \delta w_{\text{hinge}} \quad K_w = \begin{cases} 0 & w_{\text{hinge}} \leq w_{\text{stop}} \\ K_{w_{\text{stop}}} & w_{\text{hinge}} > w_{\text{stop}} \end{cases} \quad (2.18) \]

Like articulated rotors, in normal flight conditions, the stops are no longer needed to restrain the blade motion.

2.2.1.2.3 Teetering Rotors

Teetering rotors have a rigid mechanical restraint that restricts the maximum flapping angle of the rotor. Unlike articulated and hingeless rotors, this mechanical restraint is usually always extended. The restraint is modeled with a conditional rotational spring at the root of the rotor. The strain energy for such a spring is given by
\[ U_{FS} = \frac{1}{2} K_\beta \left( \beta_{max} - \beta_{stop} \right)^2 \]

\[ K_\beta = \begin{cases} 0 & \beta_{max} \leq \beta_{stop} \\ \frac{1}{\beta_{stop}} & \beta_{max} > \beta_{stop} \end{cases} \quad (2.19) \]

Taking the variation yields

\[ \delta U_{FS} = K_\beta \left( \beta_{max} - \beta_{stop} \right) \delta \beta_{max} \]

\[ K_\beta = \begin{cases} 0 & \beta_{max} \leq \beta_{stop} \\ \frac{1}{\beta_{stop}} & \beta_{max} > \beta_{stop} \end{cases} \quad (2.20) \]

Where \( \beta_{max} \) is the maximum flap angle of the rotor. Because the blades are rigidly attached to each other for a teetering rotor, the flapping of the entire rotor disk can be referenced to any blade. In this analysis, the flapping of the rotor disk is referenced to blade 1, also called the master blade. In terms of the motion of the master blade, the fixed frame flap angles of a teetering rotor are given by

\[ \beta_{lc} = \tan^{-1} \left( w'_1 \right) \cos \psi_1 \]

\[ \beta_{ls} = \tan^{-1} \left( w'_1 \right) \sin \psi_1 \quad (2.21) \]

Where \( w'_1 \) is the master blade root flap slope. Assuming that the master blade root flap slope is small, Eqn. 2.21 reduces to

\[ \beta_{lc} = w'_1 \cos \psi_1 \]

\[ \beta_{ls} = w'_1 \sin \psi_1 \quad (2.22) \]

The azimuthal location of the maximum flapping angle, \( \psi_{max} \), is given by

\[ \psi_{max} = \tan^{-1} \left( \frac{\beta_{ls}}{\beta_{lc}} \right) \quad (2.23) \]

The maximum flap angle of the rotor is then given by

\[ \beta_{max} = \beta_{lc} \cos \psi_{max} + \beta_{ls} \sin \psi_{max} \quad (2.24) \]

Substituting Eqn. 2.22 into Eqn. 2.24 yields
\[ \beta_{\text{max}} = w'_1 \cos \psi_i \cos \psi_{\text{max}} + w'_1 \sin \psi_i \sin \psi_{\text{max}} \]  

(2.25)

Taking the variation yields

\[ \delta \beta_{\text{max}} = \delta w'_1 \cos \psi_i \cos \psi_{\text{max}} + \delta w'_1 \sin \psi_i \sin \psi_{\text{max}} \]  

(2.26)

### 2.2.1.2.4 Gimballed Rotors

Gimballed rotors also have a rigid restraint that restricts the tilt angle of the rotor. Like the teetering rotor, this restraint is modeled with a conditional rotational spring at the gimbal. The strain energy expression for such a flap spring is identical to Eqn. 2.19. Unlike a teetering rotor, the maximum rotor flapping angle is a function of the master blade root flap slope and twist. In terms of the motion of the master blade, the fixed frame flap angles of the rotor disk for a gimballed rotor are given by

\[ \beta_{tc} = \tan^{-1} \left( \frac{w'_1}{\hat{\phi}_i} \right) \cos \psi_i - \hat{\phi}_i \sin \psi_i \]  

\[ \beta_{ts} = \tan^{-1} \left( \frac{w'_1}{\hat{\phi}_i} \right) \sin \psi_i + \hat{\phi}_i \cos \psi_i \]  

(2.27)

Assuming that the master blade root flap slope is small, Eqn. 2.27 reduces to

\[ \beta_{tc} = w'_1 \cos \psi_i - \hat{\phi}_i \sin \psi_i \]  

\[ \beta_{ts} = w'_1 \sin \psi_i + \hat{\phi}_i \cos \psi_i \]  

(2.28)

The position and tilt of the rotor disk in the rotating frame are still given by Eqns. 2.23 and 2.24. Substituting the new expressions for the fixed frame rotor motion yields

\[ \beta_{\text{max}} = \left( w'_1 \cos \psi_i - \hat{\phi}_i \sin \psi_i \right) \cos \psi_{\text{max}} + \left( w'_1 \sin \psi_i + \hat{\phi}_i \cos \psi_i \right) \sin \psi_{\text{max}} \]  

(2.29)

Taking the variation yields

\[ \delta \beta_{\text{max}} = \left( \delta w'_1 \cos \psi_i - \delta \hat{\phi}_i \sin \psi_i \right) \cos \psi_{\text{max}} + \left( \delta w'_1 \sin \psi_i + \delta \hat{\phi}_i \cos \psi_i \right) \sin \psi_{\text{max}} \]  

(2.30)
2.2.1.3 Lag Spring Strain Energy

Articulated and even some hingeless rotor designs must also employ mechanical limiters to limit the blade lag motion at low rotor speeds. The following sections describe the formulas necessary to model the mechanical stops for each rotor type.

2.2.1.3.1 Articulated Rotors

Articulated rotors can have discrete rotational springs at the flap hinge. Such a spring may be used to model friction in the hinge itself or stiffness properties of a lag damper. In the case of a continuous spring, the strain energy is

\[
U_{LS} = \frac{1}{2} K_\zeta \zeta_{hinge}^2
\]  

(2.31)

Taking the variation yields

\[
\delta U_{LS} = K_\zeta \zeta_{hinge} \delta \zeta_{hinge}
\]  

(2.32)

The lag hinge angle, \( \zeta_{hinge} \), is defined as the difference in the lag slope, \( \nu' \), evaluated on either side of the lag hinge

\[
\zeta_{hinge} = \tan^{-1} \left( \Delta \nu \right)_{lag hinge}
\]  

(2.33)

If the lag slope is small, the lag hinge angle simplifies to

\[
\zeta_{hinge} = \Delta \nu \bigg|_{flap hinge}
\]  

(2.34)

Articulated blades also employ mechanical limiters to restrict the blade lag motion at low rotor speeds. These limiters are called lag stops. Unlike the flap stops, the
lag stops typically never retract. For an articulated blade lag stop spring the strain energy is given by

\[ U_{LS} = \frac{1}{2} K \zeta \left( \zeta_{hinge} - \zeta_{stop} \right)^2 \]

\[ K = \begin{cases} 
0 & \zeta_{hinge} \leq \zeta_{stop} \\
K_{\zeta_{stop}} & \zeta_{hinge} > \zeta_{stop} 
\end{cases} \]

(2.35)

Where \( \zeta_{stop} \) is the lag hinge angle at which contact between the blade and the stop is made. Taking the variation yields

\[ \delta U_{LS} = K \zeta \left( \zeta_{hinge} - \zeta_{stop} \right) \delta \zeta_{hinge} \]

\[ K = \begin{cases} 
0 & \zeta_{hinge} \leq \zeta_{stop} \\
K_{\zeta_{stop}} & \zeta_{hinge} > \zeta_{stop} 
\end{cases} \]

(2.36)

Again, the stop stiffness is set arbitrarily large so that the lag hinge rotation is restricted to less than 0.01° upon blade-stop contact.

2.2.1.3.2 Hingeless Rotors

Likewise, mechanical limiters can be used to limit the blade lag motion for a hingeless rotor. A discrete conditional spring is used to simulate the lag stops; however, the contact is based not upon the lag hinge angle but upon the lag deflection at a point just beyond the flexbeam. The strain energy for a conditional linear lag stop spring is

\[ U_{LS} = K_v \left( v_{hinge} - v_{stop} \right)^2 \]

\[ K_v = \begin{cases} 
0 & v_{hinge} \leq v_{stop} \\
K_{v_{stop}} & v_{hinge} > v_{stop} 
\end{cases} \]

(2.37)

Where \( v_{hinge} \) is the lag deflection at the lag stop location, and \( v_{stop} \) is the lag deflection at which contact between the blade and the stop is made. Taking the variation yields
\[ \delta U_{LS} = K_v (v_{hinge} - v_{stop}) \delta v_{hinge} \]

\[ K_w = \begin{cases} 0 & v_{hinge} \leq v_{stop} \\ K_{i_{top}} & v_{hinge} > v_{stop} \end{cases} \]

(2.38)

### 2.2.1.4 Control System Strain Energy

Articulated, hingeless, teetering and gimballed rotors may all exhibit stiffness in the control system. This stiffness can represent friction in the pitch bearing itself or elastic stiffness in the pitch system linkages. A continuous rotational spring acting at the pitch bearing is used to simulate the control system stiffness. The strain energy for such a spring is

\[ U_{CS} = \frac{1}{2} K_{\phi_{hinge}} \hat{\phi}_{hinge}^2 \]

(2.39a)

Taking the variation yields

\[ \delta U_{CS} = K_{\phi_{hinge}} \hat{\phi}_{hinge} \delta \hat{\phi}_{hinge} \]

(2.40)

Where the pitch bearing angle, \( \hat{\phi}_{hinge} \), is defined as the difference in the elastic twist angle in the deformed blade coordinate system, \( \hat{\phi} \), when evaluated on either side of the pitch bearing

\[ \hat{\phi}_{hinge} = \Delta \hat{\phi}_{pitch bearing} \]

(2.41)

### 2.2.2 Kinetic Energy

In this section, expressions for the total kinetic energy of the system are derived. The blade motion is derived in terms of the flap, lag, torsion, and axial deformation of the
blade, although axial elastic dynamics are not included in the analysis. Axial foreshortening resulting from flap and lag deflections is included. Much of the derivation follows that in Ref. 43; however, in this analysis extra terms are included because of the variable rotor speed.

2.2.2.1 Blade Kinetic Energy

The kinetic energy of the blade is dependent upon the total blade velocity and is given by

\[ T_b = \frac{1}{2} \int_0^1 \int_A \rho_s \tilde{V} \cdot \tilde{V} \, d\eta \, d\zeta \, dx \]  

(2.42)

Strictly speaking, the total velocity of the blade is due to the motion of the blade relative to the hub and the motion of the hub itself \( \tilde{V} = \tilde{V}_b + \tilde{V}_r \). In this analysis, it is assumed that the contribution to the kinetic energy due to the motion of the hub is much smaller than the contribution due to the motion of the blade relative to the hub. Therefore, the kinetic energy simplifies to

\[ T_b = \frac{1}{2} \int_0^1 \int_A \rho_s \tilde{V}_b \cdot \tilde{V}_b \, d\eta \, d\zeta \, dx \]  

(2.43)

Taking the variation yields

\[ \delta T_b = \int_0^1 \int_A \rho_s \tilde{V}_b \cdot \delta \tilde{V}_b \, d\eta \, d\zeta \, dx \]  

(2.44)

The blade velocity can be written as

\[ V_b = (\hat{\tau}_b)^{rel} + \hat{\omega}_b \times \hat{\nu}_b \]  

(2.45)
Where $\tilde{r}_p$ is the position vector from the hub to a given point on the blade and $\tilde{\omega}_b$ is the angular velocity. Using notation from Ref. 43, a point $P$ on the undeformed elastic axis moves to a point $P'$ on the deformed elastic axis. The vector from the hub to $P'$ is

$$\tilde{r}_p = x_i \hat{i} + y_j \hat{j} + z_k \hat{k} \quad (2.46)$$

The angular velocity of the coordinate system is simply due to the rotor speed

$$\tilde{\omega}_b = \Omega \hat{K}_H \quad (2.47)$$

Substitution of Eqn. 2.46 and 2.47 into Eqn. 2.45 yields

$$\tilde{V}_b = V_{b_i} \hat{i} + V_{b_j} \hat{j} + V_{b_k} \hat{k} \quad (2.48)$$

Where

$$V_{b_i} = \dot{x}_i - \Omega y_i$$
$$V_{b_j} = \dot{y}_j + \Omega x_i - \Omega \beta_p z_i$$
$$V_{b_k} = \dot{z}_k + \Omega \beta_p y_i \quad (2.49)$$

Taking the variation yields

$$\delta \tilde{V}_b = \delta V_{b_i} \hat{i} + \delta V_{b_j} \hat{j} + \delta V_{b_k} \hat{k} \quad (2.50)$$

Where

$$\delta V_{b_i} = \delta \dot{x}_i - \Omega \delta y_i$$
$$\delta V_{b_j} = \delta \dot{y}_j + \Omega \delta x_i - \Omega \beta_p \delta z_i$$
$$\delta V_{b_k} = \delta \dot{z}_k + \Omega \beta_p \delta y_i \quad (2.51)$$

Substitution into the variation of kinetic energy, Eqn. 2.44, and integration by parts yields

$$\delta T_b = \int_0^1 \iint_A \left( T_{x_i} \delta x_i + T_{y_j} \delta y_j + T_{z_k} \delta z_k \right) d\eta d\zeta dx \quad (2.52)$$

Where
Note that the double underlined terms are due to the rotor acceleration and are unique to this analysis. The deflections in the undeformed frame are given by

\[
\begin{align*}
    x_i &= x + u - \lambda_i \phi' - \nu'(\eta \cos \theta_i - \zeta \sin \theta_i) - w'(\eta \sin \theta_i + \zeta \cos \theta_i) \\
    y_i &= v + (\eta \cos \theta_i - \zeta \sin \theta_i) \\
    z_i &= w + (\eta \sin \theta_i + \zeta \cos \theta_i)
\end{align*}
\]

Taking the variation yields

\[
\begin{align*}
    \delta x_i &= \delta u - \lambda_i \delta \phi' - \nu'(\eta \cos \theta_i - \zeta \sin \theta_i) - \delta w'(\eta \sin \theta_i + \zeta \cos \theta_i) \\
    &\quad + \delta \theta_i \left[ \nu'(\eta \sin \theta_i + \zeta \cos \theta_i) - w'(\eta \cos \theta_i - \zeta \sin \theta_i) \right] \\
    \delta y_i &= \delta v - \delta \theta_i (\eta \sin \theta_i + \zeta \cos \theta_i) \\
    \delta z_i &= \delta w + \delta \theta_i (\eta \cos \theta_i - \zeta \sin \theta_i)
\end{align*}
\]

The expressions for the axial deformation, \( u \), elastic twist in the undeformed blade coordinate system, \( \phi \), and total blade pitch angle, \( \theta \), are then substituted. Retaining terms up to second order yields

\[
\delta T_b = \int_0^1 \left( T_v \delta v + T_v \delta v + T_w \delta w + T_w \delta \phi + T_F \hat{\delta} \phi + T_F \right) dx
\]

Where

\[
T_v = m \left[ e_g \left( \Omega^2 \cos \theta_0 + \dot{\theta}_0 \sin \theta_0 \right) + \Omega^2 v - \Omega^2 e_g \hat{\phi} \sin \theta_0 + 2 \omega \beta_p \dot{w} + \Omega \beta_p w \\
+ 2 \omega \dot{v}' e_g \cos \theta_0 + \dot{\Omega} \dot{v}' e_g \cos \theta_0 + 2 \omega \dot{w}' e_g \sin \theta_0 + \Omega \dot{w}' e_g \sin \theta_0 \right]
\]

\[
= \Omega^2 e_g \cos \theta_0 + \dot{\Omega} \dot{v}' e_g \cos \theta_0 + 2 \omega \dot{w}' e_g \sin \theta_0 + \Omega \dot{w}' e_g \sin \theta_0
\]
\[ T_v = m \left[ -e_g \left( \Omega^2 x \cos \theta_0 - \Omega^2 x \dot{\phi} \sin \theta_0 + 2\Omega \dot{v} \cos \theta_0 + \hat{\Omega} v \cos \theta_0 \right) \right] \quad (2.57b) \]

\[ T_w = m \left[ -\Omega^2 \beta \rho x - \dot{\theta}_e \epsilon_g \cos \theta_0 - 2\Omega \beta \rho \dot{v} - \hat{\Omega} \beta \rho \dot{v} - \ddot{w} \left( \dot{\phi} + \hat{\Omega} \beta \rho \right) e_g \cos \theta_0 \right] \quad (2.57c) \]

\[ T_w' = m \left[ -e_g \left( \Omega^2 x \sin \theta_0 + \Omega^2 \dot{\phi} x \cos \theta_0 + 2\Omega \dot{v} \sin \theta_0 + \hat{\Omega} v \sin \theta_0 \right) \right] \quad (2.57d) \]

\[ T_{\dot{\phi}} = m \left[ -k_m^2 \ddot{\phi} - \Omega^2 \left( k_m^2 - k_m^2 \right) \cos \theta_0 \sin \theta_0 - \Omega^2 x \beta \rho e_g \cos \theta_0 - \Omega^2 \dot{v} e_g \sin \theta_0 + \Omega^2 \dot{v} e_g \sin \theta_0 - \Omega^2 \ddot{\phi} \left( k_m^2 - k_m^2 \right) \cos 2\theta_0 \right. \]
\[ + \Omega^2 \dot{v} x e_g \sin \theta_0 - \Omega^2 \dot{w} x e_g \cos \theta_0 + \dot{w} x e_g \sin \theta_0 - \Omega^2 \dot{\phi} \left( k_m^2 - k_m^2 \right) \cos 2\theta_0 \]
\[ - \dot{w} e_g \cos \theta_0 - k_m^2 \dot{\theta}_e + \hat{\Omega} x e_g \sin \theta_0 + \hat{\Omega} \dot{x} e_g \cos \theta_0 \right] \]

\[ T_f = m \left[ -\left( \Omega^2 x + 2\Omega \dot{v} + \hat{\Omega} v \right) \int_0^1 \left( v' \dot{v}' + w' \dot{w}' \right) dx \right] \quad (2.57f) \]

The cross-sectional integrals are defined as

\[ m = \int_A \rho \, d\eta \, d\zeta \quad \text{and} \quad m e_g = \int_A \rho \, \eta \, d\eta \, d\zeta \]

\[ mk_m^2 = \int_A \rho \, \zeta^2 \, d\eta \, d\zeta \quad mk_m^2 = \int_A \rho \, \eta^2 \, d\eta \, d\zeta \quad (2.58) \]

\[ mk_m^2 = \int_A \rho \left( \eta^2 + \zeta^2 \right) d\eta \, d\zeta \]

Note that in the derivation of the blade kinetic energy, the blade cross-section has been assumed symmetric about the \( \xi-\eta \) plane. Therefore, all integrals of the form
\[ \int_A \zeta ( \cdots ) d\eta d\zeta \] are equal to zero.
2.2.3 Virtual Work

In this section, expressions for the virtual work due to both conservative and nonconservative external forces are derived. The virtual work includes contributions from gravity, flap and lag hinge dampers, and aerodynamic forces.

2.2.3.1 Virtual Work due to Gravitational Forces

Because the rotor speed can range from zero to full rotor speed in this analysis, the centrifugal stiffening forces acting on the blade are not always large. Therefore, the effects of gravity cannot necessarily be neglected as in most comprehensive rotor codes. The virtual work due to gravitational forces is

\[ \delta W_g = \int_A \int_0^1 \rho_s \ddot{g} \cdot \delta \vec{r}_b \, d\eta \, d\zeta \, dx \]  \hspace{1cm} (2.59)

Where the position vector, \( \vec{r}_b \), is given in Eqns. 2.46 and 2.54 and the gravitational acceleration vector is

\[ \ddot{g} = -g \hat{k}_{SHIP} \]  \hspace{1cm} (2.60)

Substituting the above expressions and retaining terms up to second order yields

\[ \delta W_g = \int_0^1 \left[ -mg \delta w + e_g \cos \theta_\delta \delta \phi \right] \, dx \]  \hspace{1cm} (2.61)

The second term in the preceding equation represents the offset of the mass center from the shear center and statically twists the blade.
2.2.3.2 Virtual Work due to a Flap Hinge Damper

A discrete flap damper can be modeled in two manners, either with a linear or rotational hydraulic damper, depending on the rotor system. For articulated blades, the damper is assumed to provide a damping moment that resists the blade rotational velocity at the flap hinge. The virtual work done by a discrete rotational flap damper is

\[ \delta W_{FD} = -C_\beta \dot{\beta}_{hinge} \delta \beta_{hinge} \]  

(2.62)

Where \( C_\beta \) is the rotational flap damper strength.

For hingeless blades, the damper is assumed to provide a damping force that resists the blade linear velocity at a specified point outboard of the flexbeam. The virtual work done by a discrete linear flap damper is

\[ \delta W_{FD} = -C_w \omega_{hinge} \delta \omega_{hinge} \]  

(2.63)

Where \( C_w \) is the linear flap damper strength.

2.2.3.3 Virtual Work due to a Lag Hinge Damper

For an articulated blade, the virtual work done by a rotational lag damper is

\[ \delta W_{LD} = -C_\zeta \dot{\zeta}_{hinge} \delta \zeta_{hinge} \]  

(2.64)

Where \( C_\zeta \) is the rotational flap damper strength. For a hingeless blade, the virtual work done by a linear lag damper is

\[ \delta W_{LD} = -C_v \dot{v}_{hinge} \delta v_{hinge} \]  

(2.65)

Where \( C_v \) is the linear lag damper strength.
2.2.3.4 Virtual Work due to Aerodynamic Forces

The virtual work done by the aerodynamic forces can be expressed as

\[
\delta W_{AF} = \int_{0}^{1} \left\{ L_{AF} \delta V + L_{AF_{u}} \delta W + M_{AF_{b}} \delta \theta \right\} dx
\]

(2.66)

Naturally, the aerodynamic forces are proportional to the resultant velocity over the blade section. The individual contributions of the blade section velocities will be derived first, followed by a discussion of the modeling of the aerodynamic forces using both nonlinear quasi-steady and unsteady aerodynamic models.

2.2.3.4.1 Blade Section Velocities

The velocity of the air flowing over the blade section includes contributions from several sources. Airflow is due to the motion of the blade relative to the hub, the motion of the hub (or ship), induced inflow and the airwake around the ship

\[
\vec{V} = \vec{V}_{b} + \vec{V}_{\text{ship}} - \vec{V}_{i} - \vec{V}_{w}
\]

(2.67)

Each component of the resultant velocity will be discussed in the following sections.

Motion of the Blade Relative to the Hub

The motion of the blade relative to the hub was defined earlier in Eqns. 2.48 and 2.49. For convenience, it is restated here

\[
\vec{V}_{b} = V_{b_{x}} \hat{i} + V_{b_{y}} \hat{j} + V_{b_{z}} \hat{k}
\]

(2.68)

\[\text{where}\]
For use in the aerodynamic model, the deflections in the undeformed reference frame stated in Eqn. 2.54 are substituted into Eqn. 2.69. The only velocity of interest is the velocity at the \( \frac{3}{4} \) chord location, where \( \eta = \eta_i \) and \( \zeta = 0 \). Making this substitution yields

\[
V_{b_v} = \dot{x}_i - \Omega y_i \\
V_{b_y} = \dot{y}_i + \Omega x_i - \Omega \beta_p \ z_i \\
V_{b_z} = \dot{z}_i + \Omega \beta_p \ y_i
\]

\[ (2.69) \]

Ship Motion

Ship motion effects are accounted for by writing a vector from the ship center of gravity to a given point on the blade section. Since the motion of the blade relative to the hub was accounted for in the previous section and the helicopter fuselage is stationary relative to the ship deck, the velocity due to ship motion can be expressed as

\[
\vec{V}_{\text{ship}} = \vec{\omega}_{\text{ship}} \times \vec{r}_{\text{ship}}
\]

\[ (2.71) \]

Where \( \vec{r}_{\text{ship}} \) is the position vector from the ship center of gravity to a point \( P' \) on the deformed elastic axis and \( \vec{\omega}_{\text{ship}} \) is the angular velocity of the ship. The position vector is composed of contributions from the ship center of gravity to the rotor hub, defined in \textit{Figure 2.1}, and then from the rotor hub to the point \( P' \)

\[
\vec{r}_{\text{ship}} = \vec{r}_{nw} + \vec{r}_{rh} + \vec{r}_{p}
\]

\[ (2.72) \]
The position vectors $\tilde{r}_{nw}$ and $\tilde{r}_{rh}$ are
\[ \tilde{r}_{nw} + \tilde{r}_{rh} = (x_{nw} + x_{rh}) \hat{i}_{SHIP} + (y_{nw} + y_{rh}) \hat{j}_{SHIP} + (z_{nw} + z_{rh}) \hat{k}_{SHIP} \] (2.73)

The ship is assumed to roll only, so the ship angular velocity is simply
\[ \hat{\omega}_{ship} = -\phi_{ship} \hat{i}_{SHIP} \] (2.74)

Substitution of Equations 2.72 and 2.74 into Equation 2.71, and transferring to the blade undeformed coordinate system yields
\[ \tilde{V}_{ship} = V_{ship,x} \hat{i}_{ship} + V_{ship,y} \hat{j}_{ship} + V_{ship,z} \hat{k}_{ship} \] (2.75a)

Where
\[ V_{ship,x} = \dot{\phi}_{ship} (z_{nw} + z_{rh} + w) \sin \psi \]
\[ V_{ship,y} = \dot{\phi}_{ship} (z_{nw} + z_{rh} + w) \cos \psi \] (2.75b)
\[ V_{ship,z} = -\dot{\phi}_{ship} [y_{nw} + y_{rh} + v \cos \psi + x \sin \psi] \]

The ship roll angle is also assumed to have a sinusoidal variation
\[ \phi_{ship} = \phi_{ship, max} \sin \left( \frac{2\pi t}{T_{ship}} \right) \] (2.76)

Both the maximum ship roll angle, $\phi_{ship, max}$, and the ship roll period, $T_{ship}$, are dependent upon the type of ship and the sea and wind states. Approximate values are 3° to 5° for the maximum ship roll angle and 10 seconds for the ship roll period.
Induced Inflow

A complex calculation of the time-varying inflow distribution over a blade moving at low rotor speed in the unusual ship airwake environment would be difficult at best. Luckily, pilots are typically required to use a low collective pitch setting during engagements and disengagements to ensure that the helicopter does not slip on a wet ship deck. Even when the rotor finally reaches full speed the helicopter is not generating large amounts of thrust, relative to hover or forward flight conditions, since the helicopter has not lifted off the deck. The induced inflow, even in the high rotor speed portion of the engagement or disengagement, is much lower than in hover or forward flight conditions. Therefore, the induced inflow in this analysis is assumed zero.

However, it is occasionally necessary to model the induced inflow in hover or forward flight conditions for validation purposes. In this case, the induced inflow is modeled as constant across the rotor disk

\[ \tilde{V}_i = -\lambda_i \hat{K}_H \]  

(2.77)

Transforming to the blade undeformed coordinate system

\[ \tilde{V}_i = V_{i,\hat{i}} \hat{i} + V_{i,\hat{j}} \hat{j} + V_{i,\hat{k}} \hat{k} \]  

(2.78)

Where

\[ V_{i,\hat{i}} = -\beta_p \lambda_i \]
\[ V_{i,\hat{j}} = 0 \]
\[ V_{i,\hat{k}} = -\lambda_i \]  

(2.79)
Ship Airwake Components

The total ship airwake components are expressed in the inertial coordinate system and are given by

\[ \tilde{V}_w = V_x \hat{J}_w^{\text{SHIP}} + V_y \hat{J}_v^{\text{SHIP}} + V_z \hat{K}_w^{\text{SHIP}} \]  \hspace{1cm} (2.80)

Transferring to the blade undeformed coordinate system yields

\[ \tilde{V}_w = V_{w_x} \hat{i} + V_{w_y} \hat{j} + V_{w_z} \hat{k} \]  \hspace{1cm} (2.81)

Where

\[ V_{w_x} = (V_x + \alpha_z V_z) \cos \psi + (V_y - \phi_{\text{ship}} V_z) \sin \psi + \beta_p V_z \]
\[ V_{w_y} = -(V_x + \alpha_z V_z) \sin \psi + (V_y - \phi_{\text{ship}} V_z) \cos \psi \]  \hspace{1cm} (2.82)
\[ V_{w_z} = -\beta_p V_x \cos \psi - \beta_p V_y \sin \psi + (V_z - \alpha_x V_x + \phi_{\text{ship}} V_y) \]

The total ship airwake components \( V_x, V_y \) and \( V_z \) vary with the direction and speed \((V_{\text{WOD}}, \psi_{\text{WOD}})\) of the ambient winds. Due to the nature of the flow around the ship, they are also dependent upon the location of interest on the ship deck. Furthermore, even at a given location on the ship deck they will vary spatially across the rotor disk. In general, the ship airwake components are also time varying. Using the Reynolds Decomposition principle [80], the total ship airwake components are split into mean (steady) and fluctuating (unsteady) parts

\[ V_x = \bar{V}_x + V_x' \]
\[ V_y = \bar{V}_y + V_y' \]  \hspace{1cm} (2.83)
\[ V_z = \bar{V}_z + V_z' \]

The airwake components can be modeled in two different manners - with simple approximations called deterministic airwake distributions; or with results from detailed
numerical simulations or test data called *general airwake distributions*. The specification of the airwake components for each method is discussed in the following sections.

**Deterministic Airwake Distributions**

In the initial blade sailing research conducted at the University of Southampton, simple approximations of the ship airwake components were made. The approximations were derived from model-scale measurements correlated with limited full-scale measurements [31-32]. The distribution of the mean airwake components across the rotor disk was originally specified with several simple analytic formulas, leading them to be named deterministic airwake distributions. In this analysis, the fluctuating flow components are assumed zero for the deterministic airwake distributions.

Four approximate distributions of the mean airwake components evolved - the horizontal, constant, step, and linear airwake distributions. For each of the airwake distributions, the mean flow velocities in the plane of the rotor can be expressed as

\[
\begin{align*}
\bar{V}_x &= V_{WOD} \cos(\psi_{WOD}) \\
\bar{V}_y &= -V_{WOD} \sin(\psi_{WOD}) 
\end{align*}
\]  

(2.84)

The approximations are differentiated by the airflow through the plane of the rotor. The horizontal distribution specifies no flow through the rotor disk

\[
\bar{V}_z = 0
\]

(2.85)

The constant distribution specifies a constant vertical airflow through the plane of the rotor disk
The parameter $\kappa$ has been called the “gust” factor [1, 33, 41-42]. Normally, the term gust is associated with a time varying velocity; but in this analysis it is meant to describe a spatial distribution of the velocity. The last two airwake distributions are intended to simulate the abeam wind condition, where the air is forced up over the windward side of the ship deck and then down off the leeward side. The step distribution specifies a constant upflow on the windward side of the rotor disk and a constant downflow on the leeward side

$$\bar{V}_z = \kappa V_{WOD}$$  \hspace{1cm} (2.86)

The linear airwake distribution specifies an upflow on the windward side of the rotor transitioning linearly to a downflow on the leeward side

$$\bar{V}_z = \kappa V_{WOD} \times \text{sgn} \left[ \sin \left( \left( \psi + \zeta_{hinge} \right) - \left( \frac{x}{2} - \psi_{WOD} \right) \right) \right]$$  \hspace{1cm} (2.87)

The linear airwake distribution specifies an upflow on the windward side of the rotor transitioning linearly to a downflow on the leeward side

$$\bar{V}_z = \kappa V_{WOD} \times \sin \left( \left( \psi + \zeta_{hinge} \right) - \left( \frac{x}{2} - \psi_{WOD} \right) \right)$$  \hspace{1cm} (2.88)

*Figure 2.2* shows a schematic of each deterministic airwake distribution.
Figure 2.2: Deterministic Airwake Distributions
Measurements of the ship airwake are often made in model-scale or full-scale tests. Typically, these measurements are taken at a discrete number of points in the plane of the rotor disk at the spot of interest on the ship deck. In addition, predictions of the ship airwake environment have become more available using modern numerical methods. Here too the airwake components are typically known at a number of discrete locations or grid points. In either case, it is desirable to utilize these more realistic ship airwake components and examine their effect on the blade behavior during startup and shutdown operations.

For the general airwake distributions, each component is specified at discrete grid points in the shaft plane as shown in Figure 2.3. The airwake components can be results from experimental testing or numerical simulations. In this method, the mean flow component and “turbulence intensity” in the x, y and z directions are specified at each of the grid points. Ideally, the number of grid points in the x and y directions is about one per foot. Linear interpolation is used to find the value of the airwake components when the blade is physically in between the discrete grid points. Currently, the variation of the ship airwake components in planes parallel to the shaft plane is assumed small. That is, when the blade is physically flapped out of the shaft plane, the airwake components are assumed the same as when the blade is physically in the shaft plane.
Given a time history of the total velocity, $V_i$, at a given location over a suitable time period, the mean flow component is simply the time average of the total flow velocity

$$
\bar{V}_i = \lim_{T \to \infty} \frac{1}{T} \int_{t_0}^{t_0+T} V_i \, dt 
$$

$i = x, y, z$ (2.89)

Where $T$ is the period of the measurement. At any given instant in time, the fluctuating flow component is then

$$
V_i' = V_i - \bar{V}_i 
$$

$i = x, y, z$ (2.90)

By definition, the time average of the fluctuating flow component is identically zero

$$
\lim_{T \to \infty} \frac{1}{T} \int_{t_0}^{t_0+T} (V_i - \bar{V}_i) \, dt = 0 
$$

$i = x, y, z$ (2.91)

For simulation purposes it is desirable to have a simpler method of specifying the fluctuating flow component than through Eqn. 2.90. The following method of specifying the fluctuating flow components originated at the University of Southampton [38]. The fluctuating flow components are synthesized from two quantities, the turbulence intensity
and a Gaussian distributed random number. The turbulence intensity, $\sigma_i$, is a time-independent metric of the flow unsteadiness. It is the root-mean-square (RMS) value of the fluctuating flow component

$$\sigma_i = \sqrt{\lim_{T \to \infty} \frac{1}{T} \int_{T_0}^{T_0+T} (V_i - \overline{V_i})^2 \, dt} \quad i = x, y, z \quad (2.92)$$

The Gaussianly distributed random number is derived from the spectrum function of the unsteady flow components. The spectrum function is a measure of how much energy is present in the frequencies of the unsteady flow. Typically, the spectrum function is calculated at several locations. Once the spectrum functions have been calculated, a single "characteristic frequency" of the flow is chosen from them. In this method, it is assumed that all the unsteady flow components have the same characteristic frequency. This characteristic frequency is the frequency at which the spectrum function has its peak. Physically it represents the frequency at which the flow has the most energy. An example of a measured spectrum function is shown in Figure 2.4 from Ref. 32. In this case, the characteristic frequency is about 20 Hz.

![Figure 2.4: Example Measured Spectrum Function](from Ref. 32)
Once the characteristic frequency is chosen, a Gaussian distributed random number, $R(t)$, is calculated [81]. The time-varying nature of the fluctuating flow components is simulated by continually updating $R(t)$ at time intervals specified by the characteristic frequency. The fluctuating flow component is then calculated from

\begin{align*}
V_x' &= \sigma_x R(t) \\
V_y' &= \sigma_y R(t) \\
V_z' &= \sigma_z R(t)
\end{align*}

(2.93)

The information on the spatial variation of the fluctuating flow components is contained within the turbulence intensity, $\sigma$, and the information on the time variation of the fluctuating flow components is contained within $R(t)$. In this method, it is assumed that the fluctuating flow components over the entire rotor disk all have the same characteristic frequency. It is also assumed that the fluctuating flow components all have a Gaussian distribution.

**Resultant Velocities**

Once the resultant velocities are known in the undeformed coordinate system, they are transformed into the deformed coordinate system for use in the aerodynamic model. The total velocity then becomes

\[ \tilde{V} = U_\xi \hat{i}_\xi + U_\eta \hat{j}_\eta + U_\zeta \hat{k}_\zeta \]

(2.94)

where
\[ U_x = -\int_0^x (v' w' + w' w') \, dx - \Omega v + v \left[ \Omega x + \left( V_x + \alpha V_x \right) \sin \psi - \left( V_x - \phi_{\text{ship}} V_x \right) \cos \psi \right] \\
+ w \left[ \alpha V_x - \phi_{\text{ship}} V_x \right] - \left( V_x \cos \psi + V_x \sin \psi \right) (1 - \beta_p w') - \left( \alpha \cos \psi + \phi_{\text{ship}} \sin \psi \right) V_z \\
+ \left( \lambda - V_x \right) (\beta_p + w') - \xi \cos \theta_0 (\Omega + v') - \eta \sin \theta_0 \Omega \hat{\phi} - \hat{w}' \right) \\
+ v' v + w' w' + \frac{1}{2} \left( V_x \cos \psi + V_x \sin \psi \right) (v'^2 + w'^2) \\
+ \phi_{\text{ship}} \left[ \left( z_{nw} + z_{rh} + w \right) \sin \psi + v' (z_{nw} + z_{rh}) \cos \psi - w' (y_{nw} + y_{rh} + x \sin \psi) \right] \] 

\[ U_y = \cos \theta_0 \left\{ v - \frac{1}{2} \Omega \int_0^x (v'^2 + w'^2) \, dx - \Omega w \beta_p + \hat{\phi} \left( \lambda + \hat{w} + \alpha V_x - \phi_{\text{ship}} V_x - V_x \right) \\
+ \Omega v' + \left( \Omega x + V_x \sin \psi - V_x \cos \psi \right) (1 - \frac{1}{2} v'^2) \\
+ \left( V_x \cos \psi + V_x \sin \psi \right) \left[ v' + \hat{\phi} \left( \beta_p + w' \right) \right] + v' \left( \alpha \cos \psi - \phi_{\text{ship}} \sin \psi + \beta_p \right) V_z \\
+ \left( \alpha \sin \psi + \phi_{\text{ship}} \cos \psi \right) V_z + v' v' + w' w' \right\} \] 

\[ \sin \theta_0 \left\{ w + \lambda + \alpha V_x - \phi_{\text{ship}} V_x - V_x + \Omega v (\beta_p + w') + \left( V_x \cos \psi + V_x \sin \psi \right) (w' + \beta_p - \hat{\phi} v') \right\} \] 

\[ - \left( \Omega x + V_x \sin \psi - V_x \cos \psi \right) \left( v' \hat{\phi} + w' \hat{\phi} \right) + w' \left( \alpha \cos \psi - \phi_{\text{ship}} \sin \psi + \beta_p \right) V_z \\
- \hat{\phi} \left( \alpha \sin \psi + \phi_{\text{ship}} \cos \psi \right) V_z + \frac{1}{2} w'^2 V_x - \hat{\phi} v + \hat{w}' v' + w' w' \left\} \right. \\
\] 

\[ \cos \theta_0 \left\{ \phi_{\text{ship}} \left[ \left( z_{nw} + z_{rh} + w \right) \cos \psi - v' (z_{nw} + z_{rh}) \sin \psi - \hat{\phi} \left( y_{nw} + y_{rh} + x \sin \psi \right) \right] \right. \\
\sin \theta_0 \left\{ \phi_{\text{ship}} \left[ -v' (z_{nw} + z_{rh}) \sin \psi - w' (z_{nw} + z_{rh}) \cos \psi \right] \right. \]
The double underlined terms are unique to this analysis because they represent the airwake components in the lateral ($y$) direction and through the plane of the rotor disk ($z$).

Normally the velocity due to the rotor speed, $\Omega x$, dominates the tangential flow.

However, in this analysis $\Omega$ can be small so the airwake components are vital.

At very low rotor speeds large regions of the rotor disk can be in reverse flow.

Because of the unusual ship airwake environment and the low rotor speeds, the angles of attack can vary from $0^\circ$ to $360^\circ$. Once the blade section velocities are known, the local blade section angle of attack is calculated from

$$\alpha = \tan^{-1} \left( \frac{-U_p}{U_T} \right) \quad (2.96)$$

Likewise, the local blade section skew angle is calculated from
\[ \Lambda = \tan^{-1}\left( \frac{U_R}{\text{sgn}(U_T) \sqrt{U_T^2 + U_P^2}} \right) \] (2.97)

Note that the arctangent function is used to calculate the angle of attack and the skew angle. These angles are shown schematically in Figure 2.5.

*Figure 2.5: Definitions of Angle of Attack and Skew Angle*

### 2.2.3.4.2 Aerodynamic Forces

The virtual work done by the aerodynamic forces \((L_{AF}, L_{AF_\xi})\) and pitching moment \((M_{AF_\xi})\) in the undeformed blade coordinate system can be expressed as

\[ \delta W_{AF} = \int_0^1 \left[ L_{AF} \delta v + L_{AF_\xi} \delta w + M_{AF_\xi} \delta \phi \right] dx \] (2.98)

The aerodynamic forces and pitching moment in the undeformed blade coordinate system are composed of contributions from the aerodynamic forces \((L_{AF_\xi}, L_{AF_\eta})\) and pitching moment \((M_{AF_\eta})\) in the deformed blade coordinate system.
The aerodynamic forces and pitching moment in the deformed blade coordinate system, in turn, are composed of contributions from the aerodynamic lift \( L \), drag \( D \) and pitching moment about the quarter-chord \( M_{c/4} \)

\[
\begin{align*}
L_{AF_z} &= -D \sin \Lambda \\
L_{AF_q} &= L \sin \alpha - D \cos \Lambda \cos \alpha \\
L_{AF_c} &= L \cos \alpha + D \cos \Lambda \sin \alpha \\
M_{AF_z} &= M_{c/4} - e_c L_{AF_z}
\end{align*}
\] (2.100)

Where

\[
\begin{align*}
L &= \frac{\gamma V^2}{6a} C_f \\
D &= \frac{\gamma V^2}{6a} C_d \\
M_{c/4} &= \frac{\gamma V^2}{6a} c C_m
\end{align*}
\] (2.101)

A schematic of the airfoil and corresponding forces and moments is shown in Figure 2.6.
Substituting Eqn. 2.101 into 2.100 yields

\[ L_{Af} = \frac{\gamma V^2}{6a} (-C_d \sin \Lambda) \]

\[ L_{Af_q} = \frac{\gamma V^2}{6a} (C_i \sin \alpha - C_d \cos \Lambda \cos \alpha) \]

\[ L_{Af_c} = \frac{\gamma V^2}{6a} (C_i \cos \alpha + C_d \cos \Lambda \sin \alpha) \]

\[ M_{Af} = \frac{\gamma V^2}{6a} \left( c C_m - e_c \left( C_i \cos \alpha + C_d \cos \Lambda \sin \alpha \right) \right) \]

The aerodynamic lift, drag and pitching moment are always referenced to the quarter-chord instead of the aerodynamic center because the angle of attack may range from 0° to 360°.

The aerodynamic lift, drag and pitching moment can be modeled using either quasi-steady or unsteady aerodynamic models. Both models will be discussed in the following sections. In either case, special provisions must be made to accommodate situations in which the local angle of attack or skew angle becomes large. In addition, the aerodynamic lift, drag, and pitching moment coefficients for a blade section with leading edge spoilers will also be discussed.

**Nonlinear Quasi-steady Aerodynamic Modeling**

In the nonlinear quasi-steady aerodynamic model, the aerodynamic forces are assumed functions of the *instantaneous* values of the angle of attack and its time derivatives. As the name implies, all the effects of unsteady forces are neglected.
Furthermore, the undeformed aerodynamic forces and pitching moment are split into circulatory (C) and noncirculatory (NC) parts

\[
L_{AF_x} = \left( L_{AF_x} \right)_C
\]

\[
L_{AF_y} = \left( L_{AF_y} \right)_C
\]

\[
L_{AF_w} = \left( L_{AF_w} \right)_C + \left( L_{AF_w} \right)_{NC}
\]

\[
M_{AF_{\phi}} = \left( M_{AF_{\phi}} \right)_C + \left( M_{AF_{\phi}} \right)_{NC}
\]  

(2.103)

The circulatory aerodynamic coefficients are modeled using curve fits to experimental data from Prouty [82] and Critzos et al [83] while the noncirculatory aerodynamic forces are modeled using an analysis similar to Johnson [44]. The circulatory aerodynamic model will be discussed first, followed by the noncirculatory aerodynamic model. As stated earlier, special provisions must be made for situations in which the angle of attack or skew angle becomes large.

**Circulatory Aerodynamic Forces**

Sectional lift, drag, and moment coefficient data can easily be found from airfoil data derived from wind tunnel measurements. The empirical relations for the lift and drag coefficients are curve fits to experimentally measured data for angles of attack ranging from 0° to 360° for a NACA 0012 airfoil [82]. Mach number effects on the stall angle are modeled. It is important to note that the following relations are only strictly applicable to a NACA 0012 airfoil section; other airfoils would require modification of the relations. For angles of attack ranging from 0° to 360°, the lift coefficient is given by
The parameters $a$, $\alpha_1$, $K_1$ and $K_2$ are Mach number dependent and are given by

\[
a = \frac{0.1}{\sqrt{1-M^2}} - 0.01M
\]

\[
\alpha_1 = 15 - 16M
\]

\[
K_1 = 0.0233 + 0.342M^{7.15}
\]

\[
K_2 = 2.05 - 0.95M
\]

In a similar manner, the drag coefficient is given by

\[
C_d = C_{d_0}
\]

\[
C_d = C_{d_0} + K_1 (\alpha - \alpha_d)^{K_2}
\]

\[
C_d = 1.03 - 1.02 \cos(2\alpha)
\]

\[
C_d = C_{d_0} + K_3 (|\alpha - 360^\circ| - \alpha_d)^{K_4}
\]

\[
C_d = C_{d_0}
\]

The parameters $C_d$, $\alpha$, $K_3$ and $K_4$ are given by

\[
C_{d_0} = C_{d_0} + \left(65.8\alpha^2 - 0.226\alpha^4 + 0.0046\alpha^6\right) \times 10^{-6}
\]

\[
\alpha = 17 - 23.4M
\]

\[
K_3 = 0.00066
\]

\[
K_4 = 2.54
\]
Finally, the pitching moment coefficient is also calculated through a curve-fit to experimental data [83]. Mach number effects are not modeled.

\[
\begin{align*}
C_m &= 0.0 & 0' < \alpha < 12' \\
C_m &= 4.69 \times 10^{-5} \alpha^2 - 1.17 \times 10^{-2} \alpha + 0.133 & 12' < \alpha < 120' \\
C_m &= -4.31 \times 10^{-7} \alpha^4 + 2.41 \times 10^{-4} \alpha^3 - 5.01 \times 10^{-2} \alpha^2 + 4.60 \times 10 \alpha - 158.24 & 120' < \alpha < 172' \\
C_m &= 0.05(\alpha - 180') & 172' < \alpha < 188' \\
C_m &= +4.31 \times 10^{-7} \alpha^4 - 3.80 \times 10^{-4} \alpha^3 + 1.25 \times 10^{-1} \alpha^2 - 18.28 \alpha + 997.56 & 188' < \alpha < 240' \\
C_m &= -4.69 \times 10^{-5} \alpha^2 + 2.20 \times 10^{-2} \alpha - 1.98 & 240' < \alpha < 348' \\
C_m &= 0.0 & 348' < \alpha < 360'
\end{align*}
\]

Values of \(C_l\), \(C_d\) and \(C_m\) at a Mach number of 0.15 are shown in Figure 2.7.

![Figure 2.7: Nonlinear Quasi-Steady Lift, Drag and Moment Coefficients](image)

An enlarged view of the small angle of attack region, where \(0' < \alpha < 25'\), is shown in Figure 2.8.
Once the lift, drag and moment coefficients are calculated, the aerodynamic forces in the deformed blade coordinate system can be calculated through Eqn. 2.102. The aerodynamic forces in the deformed blade coordinate system are then transformed into the undeformed frame via Eqn. 2.99.

**Noncirculatory Aerodynamic Forces**

As stated earlier, noncirculatory forces are modeled using an analysis similar to the one from Johnson [44]. It is important to note that these terms are written directly in the undeformed blade coordinate system. For an airfoil undergoing plunging and pitching motions, the noncirculatory lift and moment are given by

\[
\begin{align*}
(L_{AF})_{NC} &= L_2 + L_3 \\
(M_{AF})_{NC} &= a_h b L_2 - (1 - a_h) b L_3 - \frac{1}{8} \rho \pi b^4 \dot{\theta}_1
\end{align*}
\]  

(2.109)

Where the impulsive forces \( L_2 \) and \( L_3 \) are
\[ L_2 = \rho \pi b^2 \left( -\dot{\omega} - a_n b \dot{\Theta}_1 \right) \]
\[ L_3 = \rho \pi b^3 V \dot{\Theta}_1 \]  
(2.110)

And where
\[ V = \Omega x + V_x \sin \psi - V_y \cos \psi \]
\[ a_n b = -(e_c + \psi c) \]
\[ b = \psi c \]  
(2.111)

Substitution of Eqns. 2.110 and 2.111 into Eqn. 2.109 yields
\[
\left( L_{sc} \right) = \frac{\gamma c}{12a} \left[ \ddot{\omega} + (e_c + \psi c) \left( \dot{\Theta}_1 + \dot{\phi} \right) + \left( \Omega x + V_x \sin \psi - V_y \cos \psi \right) \left( \dot{\Theta}_1 + \dot{\phi} \right) \right] 
\]
\[
\left( M_{sc} \right) = \frac{\gamma c}{12a} \left[ \left( e_c + \psi c \right)^2 \ddot{\omega} + \left( e_c + \psi c \right) \left( \dot{\Theta}_1 + \dot{\phi} \right) - \left( e_c + \psi c \right) \left( \Omega x + V_x \sin \psi - V_y \cos \psi \right) \left( \dot{\Theta}_1 + \dot{\phi} \right) \right] 
\]  
(2.112)

The double underlined terms are unique to this analysis and are not included in Ref. 44.

**Special Provisions for the Nonlinear Quasi-steady Aerodynamic Model**

The noncirculatory aerodynamic forces derived in the previous section are only valid for small angles of attack and skew angles. In this analysis, it has been assumed that the delineation between large and small angles occurs at 25º. Above this region, where \(|\alpha| < 25^\circ\) and \(|\Lambda| < 25^\circ\), the noncirculatory forces are assumed zero.

**Unsteady Aerodynamic Modeling**

Even at full rotor speed, a helicopter blade encounters time-varying changes in the angle of attack due to the control inputs, the motion of the blade and the complex three-dimensional flowfield. This is especially true during the engagement and disengagement
of the rotor where the rate of change of the angle of attack can become very large. Unsteady aerodynamic effects can result in increased lift and moment coefficients at angles of attack above the static stall angle.

The unsteady aerodynamic model used in this analysis is taken from the model originally developed by Leishman and Beddoes [45, 84]. It is a semi-empirical nonlinear time-domain method suited for use in comprehensive rotorcraft analyses. Both circulatory and noncirculatory effects are included. The effects of arbitrary time-varying forces are derived using the superposition principle. For reference, the unsteady aerodynamic model used in the analysis is fully detailed in Appendix A.3.

Special Provisions for the Unsteady Aerodynamic Model

Because of the unusual ship airwake environment, unsteady aerodynamic effects are as likely to occur at negative angles of attack as at positive angles of attack. Modifications to the unsteady aerodynamic model have been made that ensure unsteady effects can be modeled for both situations. The unsteady aerodynamic model described in the Appendix has only been validated for small to moderate angles of attack and skew angles (|\(\alpha| < 25^\circ\) and |\(\Lambda| < 25^\circ\)). If the angle of attack or the skew angle is large, then the aerodynamic forces are calculated through the nonlinear quasi-steady aerodynamic model. Lastly, the unsteady aerodynamic model is not valid for very low Mach numbers. In this situation, the aerodynamic forces are also calculated with the nonlinear quasi-steady aerodynamic model.
2.2.3.4.3 Spoiler Modeling

In this analysis, the effect of spoilers on the rotor blade behavior will be investigated. Spoilers are aerodynamic devices commonly used to reduce lift and increase drag on a body. Spoilers placed near the trailing edge of an airfoil section simply increase the aerodynamic drag, while spoilers placed closer to the leading edge or near the quarter-chord of an airfoil section will drastically reduce lift and increase drag. Spoilers that extend continuously down the span of a wing, called full-span spoilers, are also known to produce large hinge moments and vibration. Spoilers that are arranged in a rake configuration, called gated spoilers, have significantly lower hinge moments and buffeting problems than full-span spoilers while still having the same aerodynamic effectiveness.

Gated, leading-edge spoilers were previously tested on the upper surface of a model NACA 4415 wing section for application to tilt-wing V/STOL aircraft in the transition phase [85]. The spoilers were mounted perpendicular to the wing surface in several configurations. Two different chord locations of 5%c and 15%c; four different spoiler heights of 2.5%c, 5%c, 7.5%c, and 10%c; and two different spoiler gate spacings of 3%c and 6%c were tested. Schematics of the spoiler configurations are shown in Figure 2.9. Sectional lift, drag, and moment coefficients were measured versus angle of attack, ranging from 0° to 90°, with and without spoilers. The results for the lift and drag coefficients for one configuration are shown in Figure 2.10. Note that essentially all the lift in the attached flow region is eliminated. The lift curve slope was decreased 60% to 70% in the attached flow region and was essentially independent of gate spacing or for
spoilers with heights above 5%c. The lift generated by the airfoil was due entirely to bluff body effects. Drag was significantly increased for angles of attack less than 25°; but above 25°, the effect on drag was negligible. The moment coefficient data was essentially coincident with and without spoilers.

![Figure 2.9: Spoiler Configurations](image)

![Figure 2.10: Measured NACA 4415 Lift and Drag Coefficients for Spoilers Located at 15%c and with a Gate Spacing of 3%c](image)
Using the data from Ref. 85 as a guide, curve-fits were estimated for a NACA 0012 airfoil with and without spoilers. The spoilers were modeled on both the upper and lower surfaces to reduce both positive and negative lift generated during engagement and disengagement operations. The lift and drag coefficients for a blade section with upper and lower surface spoilers are shown in Figure 2.11 and are estimated by

\[
C_l = 1.15 \sin(2\alpha) \\
C_d = 1.03 - 1.02 \cos(2\alpha) + 0.16 |\cos(\alpha)|
\]

Much like the experimental data in Ref. 85, all the lift generated by the airfoil is due entirely to bluff body effects. At very low angles of attack, a sizeable drag coefficient increment is incurred. Since the spoilers were modeled on both surfaces, it was estimated that twice as much drag penalty as in the experimental data was incurred. Since the spoilers were modeled on both surfaces, the moment coefficient was assumed unaffected.

![Figure 2.11: Estimated NACA 0012 Lift and Drag Coefficients](image)
Special Provisions for Small Angles of Attack and Skew Angles

If the angles of attack and the skew angles are both small (|\(\alpha| < 25^\circ\) and |\(\Lambda| < 25^\circ\)), the aerodynamic forces and pitching moment in the deformed blade coordinate system can be simplified with small angle assumptions

\[
\begin{align*}
\cos \alpha &= 1 \\
\cos \Lambda &= 1 \\
\sin \alpha &= \frac{-U_p}{U_T} \\
\sin \Lambda &= \frac{U_R}{U_T} \\
V^2 &= U_T^2
\end{align*}
\]

Substituting Eqn. 2.114 into 2.102 yields the simplified form of the aerodynamic forces and pitching moment in the deformed blade coordinate system

\[
L_{AF_i} = \frac{\gamma}{6a} \left( -c_d U_R U_T \right) \\
L_{AF_{ii}} = \frac{\gamma}{6a} \left( -c_d U_R U_T - c_d U_T^2 \right) \\
L_{AF_{i}} = \frac{\gamma}{6a} \left( c_d U_T^2 - c_d U_R U_T \right) \\
M_{AF_i} = \frac{\gamma}{6a} \left( U_T^2 m - e_c c_i U_T^2 + e_c c_d U_R U_T \right)
\]

The aerodynamic forces and pitching moment in the deformed blade coordinate system are then transferred to the undeformed blade coordinate system with Eqn. 2.99. The motion independent, dependent and nonlinear terms can then be identified.
It is of interest to determine the loads that the blade experiences during startup and shutdown operations. Originally, a curvature method was used to determine the blade loads [48-51]. For increased accuracy, a force summation method is used in this analysis. Force summation techniques for calculating the blade bending moments are generally more accurate than curvature techniques, but they are also more computationally intensive. The derivation of the blade loads is similar to the procedure detailed in Ref. 43.

At any location, $x_0$, along the blade the forces and moments per unit span in the undeformed reference frame are

$$\begin{bmatrix}
F_u \\
F_v \\
F_w \\
F_{\theta} \\
F_{\phi}
\end{bmatrix} = \int_{x_0}^{x} \begin{bmatrix}
L_u \\
L_v \\
L_w \\
M_u \\
M_v \\
M_w
\end{bmatrix} \, dx \quad (2.116)$$

The blade section forces and moments are composed of contributions from aerodynamic, inertial, and gravitational components

$$L_u = L_{AFu} + L_{Iu} \quad M_u = M_{AFu} + M_{Iu} + M_{gu}$$
$$L_v = L_{AFv} + L_{Iv} \quad M_v = v'M_{AFv} + M_{Iv}$$
$$L_w = L_{AFw} + L_{Iw} + L_{gw} \quad M_w = w'M_{AFw} + M_{Iw} \quad (2.117)$$

The aerodynamic forces and moments were derived in the previous section. The gravitational forces and moments are
\[ L_{s_v} = -mg \quad M_{s_v} = -mge_s \cos(\theta_v + \phi) \quad (2.118) \]

The inertial forces and moments are calculated from

\[
\begin{align*}
\begin{bmatrix} L_{s_v} \\ M_{s_v} \\ L_{t_v} \end{bmatrix} &= \iint_A \rho_j \hat{a}_b \, d\eta d\zeta \\
\begin{bmatrix} M_{t_v} \end{bmatrix} &= \iint_A \rho_j \hat{a}_b \times \hat{s} \, d\eta d\zeta \\
\end{align*}
\quad (2.119)
\]

Where the distance from the elastic axis to any point on the blade cross section is

\[ \hat{s} = -[v'(\eta \cos \theta_i - \zeta \sin \theta_i) + w'(\eta \sin \theta_i + \zeta \cos \theta_i)]\hat{i} + (\eta \cos \theta_i - \zeta \sin \theta_i) \hat{j} + (\eta \sin \theta_i + \zeta \cos \theta_i) \hat{k} \quad (2.120) \]

The blade acceleration \( \hat{a}_b \) is defined as

\[ \hat{a}_b = (\hat{r}_b)_{rel} + \hat{\omega}_b \times \hat{\omega}_b \times \hat{r}_b + 2 \hat{\omega}_b \times (\hat{r}_b)_{rel} + (\hat{\omega}_b)_{rel} \times \hat{r}_b \quad (2.121) \]

Where \( \hat{r}_b \) and \( \hat{\omega}_b \) were defined in Eqns. 2.46 and 2.47. Substituting yields

\[ \hat{a}_b = a_{b_v} \hat{i} + a_{b_v} \hat{j} + a_{b_v} \hat{k} \quad (2.122) \]

Where

\[
\begin{align*}
a_{b_v} &= \ddot{x}_v + \Omega^2 \beta_p z_i - \Omega^2 x_i - 2\Omega \dot{y}_i - \dot{\Omega} y_i \\
a_{b_v} &= \ddot{y}_i - \Omega^2 y_i - 2\Omega \beta_p \dot{z}_i - \dot{\Omega} \beta_p z_i + 2\Omega x_i + \dot{\Omega} x_i \\
a_{b_v} &= \ddot{z}_i - \Omega^2 \beta_p z_i + \Omega^2 \beta_p x_i + 2\Omega \beta_p \dot{y}_i + \dot{\Omega} \beta_p y_i \\
\end{align*}
\quad (2.123) \]

Next, Eqn. 2.54 is substituted into 2.123. Retaining terms to second order yields

\[
L_{t_v} = -m\left\{ -\Omega^2 x - 2\Omega \dot{v} - \dot{\Omega} v + \Omega^2 \beta_p w + e_s \left[ \Omega^2 v' - v'' + \dot{\Omega} \right] \cos \theta_o \\
+ e_s \left[ \Omega^2 \left( w' + \beta_p \right) - \dot{w}' + 2\Omega \left( \dot{\theta}_o + \dot{\phi} \right) + \ddot{\Omega} \right] \sin \theta_o \right\} \quad (2.124a) \]
\[
L_v = -m \left\{ \ddot{v} - \Omega^2 v - 2\Omega \int_0^r (v' \dot{v}' + w' \dot{w}') \, dx - 2\Omega \beta_p w - \ddot{\Omega} \beta_p w + \dot{\Omega} x \right. \\
\left. - e_g \left[ \Omega^2 + 2\Omega \dot{v}' + \ddot{\Omega} v' \right] \cos \theta_0 \right. \\
\left. - e_g \left[ (\ddot{\theta}_0 + \dot{\phi}) - \Omega^2 \dot{\phi} + 2\Omega \dot{w}' + \ddot{\Omega} w' \right] \sin \theta_0 \right\} 
\]

(2.124b)

\[
L_{\iota} = -m \left\{ \ddot{\omega} + \Omega^2 \beta_p x + 2\Omega \beta_p \dot{v} + \ddot{\Omega} \beta_p \dot{v} + \Omega^2 \beta_p w + e_g \left( \ddot{\theta}_0 + \dot{\phi} \right) \cos \theta_0 \right\} 
\]

(2.124c)

\[
M_L = -m \left\{ k_m^2 \left( \ddot{\theta}_0 + \dot{\phi} + \ddot{\Omega} \beta_p \right) + (\Omega^2 + 2\Omega \dot{v}' + \ddot{\Omega} v') (k_m^2 - k_m^2) \cos \theta_0 \sin \theta_0 \right. \\
\left. + \frac{2\Omega \dot{w}'}{\ddot{\Omega} w'} (k_m^2 \sin^2 \theta_0 + k_m^2 \cos^2 \theta_0) \right\} 
\]

(2.124d)

\[
M_{L_v} = -m \left\{ \left( \Omega^2 - \ddot{\Omega} \right) (k_m^2 - k_m^2) \cos \theta_0 \sin \theta_0 \right. \\
\left. + \left[ \Omega^2 (\beta_p + \dot{w}') - \ddot{w}' + 2\Omega \left( \ddot{\theta}_0 + \dot{\phi} + \ddot{\Omega} \beta_p \right) \right] (k_m^2 \sin^2 \theta_0 + k_m^2 \cos^2 \theta_0) \right. \\
\left. - \Omega^2 \dot{x} e_g \cos \theta_0 - \left( \Omega^2 x + 2\Omega \dot{v} + \ddot{\Omega} v' \right) e_g \sin \theta_0 \right\} 
\]

(2.124e)

\[
M_{L_v} = -m \left\{ \left[ \ddot{\omega} - \Omega^2 \dot{w} - 2\Omega \left( \ddot{\theta}_0 + \dot{\phi} \right) - \Omega^2 \beta_p \right] (k_m^2 - k_m^2) \cos \theta_0 \sin \theta_0 \right. \\
\left. + \left( \ddot{v}' - \Omega^2 v - \ddot{\Omega} \right) (k_m^2 \cos^2 \theta_0 + k_m^2 \sin^2 \theta_0) \right. \\
\left. + \left( \Omega^2 x + 2\Omega \dot{v} + \ddot{\Omega} \dot{v}' + \ddot{\Omega} w' \right) e_g \cos \theta_0 - \left( \Omega^2 \dot{x} \dot{\phi} + \ddot{\Omega} x \dot{w}' \right) e_g \sin \theta_0 \right\} 
\]

(2.124f)

The double underlined terms are unique to this analysis and are not included in Ref. 43.

The aerodynamic, gravitational, and inertial moments are then summed and integrated along the blade length via Eqn. 2.116. They are then transformed into the deformed blade coordinate system.
2.4 Finite Element Discretization

Once the variations of kinetic and strain energy and virtual work are known, the generalized Hamilton’s principle is used to formulate the blade equations of motion. The energy expressions are written for $N_{el}$ number of finite elements in each rotor blade

$$\delta \Pi = \int_{t_i}^{t_f} \left( \sum_{i=1}^{N_{el}} \sum_{j=1}^{N_{el}} \left( \delta U_{i,j} - \delta T_{i,j} - \delta W_{i,j} \right) \right) dt = 0 \quad (2.125)$$

The deflections within each of the finite elements are then spatially discretized by specifying the deflections as shape functions multiplied by nodal displacements

$$v = H_v \hat{v} \quad w = H_w \hat{w} \quad \hat{\phi} = H_{\phi} \hat{\phi} \quad (2.126)$$

Cubic Hermitian shape functions $H_w$ and $H_v$ are used to discretize the flap and lag degrees of freedom respectively, while quadratic Lagrangian shape functions $H_{\phi}$ are used to discretize the torsion degrees of freedom. Each element has three nodes – 1 node at each end and 1 node in the middle of the element. The external nodes each have flap and flap slope, lag and lag slope and torsion degrees of freedom. The internal node has a torsion degree of freedom only. The shape functions are given by

$$H_v = \begin{bmatrix} H_{v_a} \\ H_{v_b} \\ H_{v_c} \end{bmatrix}^T = \begin{bmatrix} 2s_j^3 - 3s_j^2 + 1 \\ l_j \left(s_j^3 - 2s_j^2 + s_j \right) \\ -2s_j^3 + 3s_j^2 \end{bmatrix}^T$$

$$= \begin{bmatrix} H_{w_a} \\ H_{w_b} \\ H_{w_c} \end{bmatrix}^T = H_w \quad (2.127)$$

$$H_{\phi} = \begin{bmatrix} H_{\phi_a} \\ H_{\phi_b} \\ H_{\phi_c} \end{bmatrix}^T = \begin{bmatrix} 2s_j^3 - 3s_j + 1 \\ -4s_j^2 + 4s_j \\ 2s_j^2 - s_j \end{bmatrix}^T$$
Where \( s_j \) is the nondimensional distance within the element and \( l_j \) is the length of the \( j^{th} \) element. The nodal displacements are

\[
\mathbf{v} = \begin{bmatrix} v_a \\ v'_a \\ v_b \\ v'_b \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} w_a \\ w'_a \\ w_b \\ w'_b \end{bmatrix}, \quad \hat{\mathbf{\phi}} = \begin{bmatrix} \hat{\phi}_a \\ \hat{\phi}_m \\ \hat{\phi}_b \end{bmatrix}
\]

The nodal displacements for an example finite element are shown in Figure 2.12.

Figure 2.12: Beam Finite Element

The shape functions and nodal displacements for the flap, lag and torsion degrees of freedom can be combined as

\[
\begin{bmatrix} v_a \\ v'_a \\ w_a \\ w'_a \\ \hat{\phi}_m \\ \hat{\phi}_b \end{bmatrix} = \mathbf{N} \begin{bmatrix} \mathbf{q}_{el} \end{bmatrix} = \begin{bmatrix} H_{va} & H_{va} & 0 & 0 & 0 & 0 & H_{va} & H_{va} & 0 & 0 & 0 \\ 0 & 0 & H_{va} & H_{va} & 0 & 0 & 0 & 0 & H_{va} & H_{va} & 0 \\ 0 & 0 & 0 & 0 & H_{va} & H_{va} & 0 & 0 & 0 & 0 & H_{va} \end{bmatrix} \begin{bmatrix} v_a \\ v'_a \\ w_a \\ w'_a \\ \hat{\phi}_m \\ \hat{\phi}_b \end{bmatrix}
\]

Where \( \mathbf{q}_{el} \) is the vector of elemental nodal displacements. Taking the variation yields
Each flexible finite element then has eleven degrees of freedom - 4 in flap, 4 in lag, and 3 in torsion.

2.4.1 Elemental Mass, Damping and Stiffness Matrices and Force Vector

The shape functions and nodal displacements, Eqns. 2.129 and 2.130, are then substituted into the discretized Hamilton’s principle, Eqn. 2.125. The shape functions are then integrated within each element using a six-point Gaussian quadrature method. The dependence of the variation in strain energy, variation in kinetic energy and virtual work on the blade deflections is given by

$$
\delta U_{i,j} = \delta U_{i,j} \left( \delta q_{i,j}, q_{i,j}, q_{i,j}^2 \right)
$$

$$
\delta T_{i,j} = \delta T_{i,j} \left( \delta q_{i,j}, \dot{q}_{i,j}, \ddot{q}_{i,j} \right)
$$

$$
\delta W_{i,j} = \delta W_{i,j} \left( \delta q_{i,j}, \ddot{q}_{i,j}, \dot{q}_{i,j}^2 \right)
$$

The variation in strain energy and virtual work contain nonlinear terms while the variation in kinetic energy contains linear terms only. The energy expressions can then be organized into motion dependent terms ($q_{i,j}, \dot{q}_{i,j}, \ddot{q}_{i,j}$) and motion independent and nonlinear ($q_{i,j}^2$) terms. The discretized Hamilton’s principle then becomes

$$
\delta \Pi = \int_{t_i}^{t_f} \left\{ N_k \sum_{j=1}^{N_{el}} \delta \mathbf{q}_{ij}^T \left( \mathbf{m}_{el} \ddot{\mathbf{q}}_{el} + \mathbf{c}_{el} \dot{\mathbf{q}}_{el} + \mathbf{k}_{el} \mathbf{q}_{el} - \mathbf{f}_{el} \right) \right\} \, dt = 0
$$
Where \( \mathbf{m}_{\text{el}}, \mathbf{c}_{\text{el}} \) and \( \mathbf{k}_{\text{el}} \) are the elemental mass, damping and stiffness matrices and \( \mathbf{q}_{\text{el}} \) is the elemental displacement vector. The elemental force vector, \( \mathbf{f}_{\text{el}} \), contains the motion independent and nonlinear terms.

### 2.4.2 Blade Mass, Damping and Stiffness Matrices and Force Vector

The elemental mass, damping, and stiffness matrices and force vector are then assembled for each blade

\[
\delta \Pi = \int_{t_i}^{t_f} \left[ \sum_{j=1}^{N_d} \delta \mathbf{q}_u^T \left( \mathbf{M}_u \ddot{\mathbf{q}}_u + \mathbf{C}_u \dot{\mathbf{q}}_u + \mathbf{K}_u \mathbf{q}_u - \mathbf{F}_u \right) \right] dt = 0 \tag{2.133}
\]

Where \( \mathbf{M}_u, \mathbf{C}_u \) and \( \mathbf{K}_u \) are the unconstrained blade mass, damping and stiffness matrices; \( \mathbf{F}_u \) is the unconstrained blade force vector; and \( \mathbf{q}_u \) is the vector of unconstrained blade displacements

\[
\mathbf{M}_u = \sum_{j=1}^{N_d} \mathbf{m}_{\text{el}} \quad \mathbf{C}_u = \sum_{j=1}^{N_d} \mathbf{c}_{\text{el}} \quad \mathbf{K}_u = \sum_{j=1}^{N_d} \mathbf{k}_{\text{el}} \quad \mathbf{F}_u = \sum_{j=1}^{N_d} \mathbf{f}_{\text{el}} \quad \mathbf{q}_u = \sum_{j=1}^{N_d} \mathbf{q}_{\text{el}} \tag{2.134}
\]

The kinematic constraints must be applied to the unconstrained blade equation. For an articulated or hingeless rotor, the boundary conditions are that the blade is purely cantilevered at the root. In this case, the lag deflection, lag slope, flap deflection, flap slope and twist at the root of the blade are set to zero

\[
(v_a)_{i,j=1} = (v_a')_{i,j=1} = (w_a)_{i,j=1} = (w_a')_{i,j=1} = (\phi_a)_{i,j=1} = 0 \tag{2.135}
\]

It is assumed that the hinge offsets are not zero for an articulated rotor. For a teetering rotor, the boundary conditions are that the blade is pinned in the flap degree of freedom.
and cantilevered in the lag and twist degrees of freedom. The lag deflection, lag slope, flap deflection and twist at the root are set to zero

\[
(v_i')_{i,j=1} = (v_i')_{i,j=1} = (w_i')_{i,j=1} = \left( \hat{\phi}_a \right)_{i,j=1} = 0
\]  

(2.136)

For a gimballed rotor, the boundary conditions are that the blade is pinned in the flap and twist degrees of freedom and cantilevered in the lag degree of freedom. The lag deflection, lag slope and flap deflection at the root are set to zero

\[
(v_i')_{i,j=1} = (v_i')_{i,j=1} = (w_i')_{i,j=1} = 0
\]  

(2.137)

After application of the kinematic constraints, Eqn. 2.133 becomes

\[
\delta \Pi = \int_{t_i}^{t_f} \left[ \sum_{j=1}^{N_b} \delta q^T_C (M_c \ddot{q}_c + C_c \dot{q}_c + K_c q_c - F_c) \right] dt = 0
\]  

(2.138)

Where \( M_c, C_c \) and \( K_c \) are the constrained blade mass, damping and stiffness matrices; \( F_c \) is the constrained blade force vector; and \( q_c \) is the vector of constrained blade displacements.

2.4.3 Rotor Mass, Damping and Stiffness Matrices and Force Vector

The constrained blade mass, damping, and stiffness matrices and force vector must be assembled for the entire rotor. In order to do this, the kinematic constraints must be applied to the constrained blade matrices and force vector. For an articulated or hingeless rotor, the forces and moments acting on one blade do not affect the other blades. The blade motions are not coupled, so no modifications are necessary. For teetering or gimballed rotors, the forces and moments acting on one blade do affect the
other blades. The blade motions are coupled, so the constrained blade matrices must be modified. In the case of a teetering rotor, the modification is trivial. The flap slope at the root of the second blade is simply the opposite of the flap slope at the root of the first blade

\[
(w_a')_{i=2,j=1} = -(w_a')_{i=1,j=1}
\]  

(2.139)

Taking the variation yields

\[
(\delta w_a')_{i=2,j=1} = -(\delta w_a')_{i=1,j=1}
\]  

(2.140)

For a gimballed rotor, the modifications are more complex. Both the flap slope and twist degrees of freedom at the root of each blade are coupled through the following relation

\[
(\delta w_a')_{i=1,j=1} = -(\delta w_a')_{i=1,j=1}
\]  

(2.141)

Taking the variation yields

\[
(\delta w_a')_{i=1,j=1} = -(\delta w_a')_{i=1,j=1}
\]  

(2.142)

The azimuth angle between each blade, \(\Delta \psi_i\), is given by

\[
\Delta \psi_i = \frac{2\pi}{N_b} (i - 1)
\]  

(2.143)

After application of the kinematic constraints, Eqn. 2.138 becomes

\[
\delta \Pi = \int_{t_i}^{t_f} \left[ \sum_{r=1}^{N_b} \delta q_r^T (M_r \dot{q}_r + C_r q_r + K_r \eta_r - F_r) \right] dt = 0
\]  

(2.144)
Where $M_r$, $C_r$ and $K_r$ are the transformed blade mass, damping and stiffness matrices; $F_r$ is the transformed blade force vector; and $q_r$ is the vector of transformed blade displacements.

The transformed blade mass, damping, and stiffness matrices and force vector are then assembled

$$\delta \Pi = \int_0^t \delta q^T (\tilde{M} \ddot{q} + \tilde{C} \dot{q} + \tilde{K} q - \tilde{F}) \, dt = 0 \quad (2.145)$$

Where $M$, $C$ and $K$ are the rotor mass, damping and stiffness matrices; $F$ is the rotor force vector; and $q$ is the vector of rotor displacements

$$M = \sum_{i=1}^{N_b} M_i, \quad C = \sum_{i=1}^{N_b} C_i, \quad K = \sum_{i=1}^{N_b} K_i, \quad F = \sum_{i=1}^{N_b} F_i, \quad q = \sum_{i=1}^{N_b} q_i \quad (2.146)$$

The rotor virtual displacements are then arbitrary and the discretized equation of motion becomes

$$M \ddot{q} + C \dot{q} + K q = F \quad (2.147)$$

### 2.5 Analysis Types

Three different types of analyses - an eigenanalysis, an engagement or disengagement operation, or an engagement or disengagement wind envelope - can be performed on the discretized rotor equations of motion. In the first section, the reduction of the full finite element equations of motion into modal space coordinates is presented. In the second section, the methods used to time integrate the resulting modal equations of motion are discussed.
motion to determine the rotor response is discussed. In the third section, the engagement and disengagement envelope analysis used to determine the SHOLs is explained.

2.5.1 Modal Analysis

An eigenanalysis of the rotor equations of motion determines the natural frequencies and mode shapes of the rotor. It is essential that the natural frequencies and mode shape of the analytical model correlate well with experimental data to ensure that the dynamic characteristics of the math model are representative of the actual rotor system. Furthermore, a modal analysis permits the equations of motion to be transformed into a generalized, or modal, coordinate system. The number of modes required to accurately model the blade response are much less than the number of degrees of freedom in the full finite element space model. Therefore, the modal transformation increases the computational efficiency of the analysis.

To determine the natural frequencies and mode shapes of the rotor, the undamped, free vibration response of the rotor is examined. It is important to note that for the modal analysis, the rotor is assumed to be rotating at a constant speed. To examine the effects of centrifugal force on the natural frequencies and mode shapes, the rotor speed can be anywhere from 0%NR to 100%NR. Furthermore, all aerodynamic effects are ignored. The discretized rotor equation of motion, Eqn. 2.9, then reduces to

\[ {\ddot{\mathbf{q}}} + \mathbf{K}_s \mathbf{q} = 0 \]  

(2.148)

Where the matrices \( \mathbf{M}_s \) and \( \mathbf{K}_s \) refer to the mass and stiffness matrices composed from structural quantities only. In general, they are different from the mass and stiffness
matrices calculated in the transient integration, $M$ and $K$, which may include aerodynamic and rotor acceleration terms. Equation 2.148 is a homogenous, constant coefficient, ordinary differential equation. A general solution is

$$ q = \alpha e^{i\omega t} \quad (2.149) $$

Substituting Eqn. 2.149 into Eqn. 2.148 yields

$$ \left( K - \omega^2 M \right) \alpha e^{i\omega t} = 0 \quad (2.150) $$

In this equation, $\omega$ is a constant scalar and $\alpha$ is an $N_{dof}$ length vector, where $N_{dof}$ represents the number of constrained degrees of freedom for the entire rotor. This is a classical eigenvalue problem. Values of $\omega$, called eigenvalues, must exist that produce a nontrivial solution for the vector $\alpha$. This nontrivial solution is given by

$$ \det \left( K - \omega^2 M \right) = 0 \quad (2.151) $$

The $N_{dof}$ values of $\omega$ that satisfy Eqn. 2.151 are called the natural frequencies of the system. Substitution of Eqn. 2.149 into Eqn. 2.148 for each individual natural frequency, $\omega_r$, yields

$$ M_r \omega_r^2 \alpha_r = K_r \alpha_r, \quad r = 1, 2, \ldots, N_{dof} \quad (2.152) $$

The vectors $\alpha_r$ that satisfy Eqn. 2.152 are called the eigenvectors, or mode shapes, of the system.

### 2.5.2 Transient Time Integration

The blade transient response is calculated by time integrating the discretized rotor equations of motion. Recall that the rotor acceleration, speed and azimuth angle
variations are prescribed non-periodic functions of time, so the equations of motion must be recalculated at every single time step.

In this analysis, the transient time integration is permitted in the full finite element space or modal space. A time integration in the full finite element space simply refers to the time integration of Eqn. 2.9. A time integration in modal space refers to the time integration of Eqn. 2.9 when transferred into the generalized coordinate system. In this method, a modal matrix, \( \Phi \), is assembled from the individual mode shapes calculated in an eigenanalysis

\[
\Phi = \begin{bmatrix} a_1 & a_2 & \ldots & a_{N_m} \end{bmatrix} \quad N_m \leq N_{\text{def}}
\]

(2.153)

Where \( N_m \) is the total number of modes used in the analysis. The modal matrix can be assembled using several methods. In earlier research presented in Refs. 49-50, a single blade was articulated in flap only and utilized a nonzero flap stop angle. In this case, the "modal swapping" technique was used with success. In the modal swapping technique, two separate modal matrices were used. The first modal matrix was comprised of purely articulated modes, representing the blade when not in contact with the flap stop. The second modal matrix was comprised of purely cantilevered modes, representing the blade when in contact with the flap stop. Depending upon blade/stop contact during the transient time integration, the proper modal matrix was then used or "swapped". A special procedure was required to adjust the modal amplitudes to maintain the physical shape of the blade when swapping the modal matrices. In the current analysis, multiple rotor blades can be articulated in both flap and lag directions. Furthermore for a real articulated rotor blade, the flap and lag stop angles are usually nonzero. In this case, the
simplest method is to construct a single modal matrix comprised of a combination of modes representing the blades when in contact with the flap/lag or gimbal stops, and when not in contact with the stops. This method ensures that the physical shape of the blades can be properly simulated when striking a nonzero flap/lag or gimbal stop angle. Once the modal matrix is chosen, the response of each mode combines linearly to approximate the rotor deflection according to the equation

\[ \mathbf{q} = \Phi \mathbf{e} \]  

(2.154)

Where \( \mathbf{e} \) is an \( N_m \) length vector representing the amplitudes of the mode shapes. Substitution of Eqn. 2.154 into the discretized rotor equation of motion, Eqn. 2.9, and premultiplying by \( \Phi^T \) yields the equation of motion in modal space

\[ \mathbf{M} \ddot{\mathbf{e}} + \mathbf{C} \dot{\mathbf{e}} + \mathbf{K} \mathbf{e} = \mathbf{F} \]  

(2.155)

The modal mass, damping and stiffness matrices \( \mathbf{M}, \mathbf{C}, \mathbf{K} \) and force vector \( \mathbf{F} \) are defined as

\[
\begin{align*}
\mathbf{M} &= \Phi^T \mathbf{M} \Phi \\
\mathbf{C} &= \Phi^T \mathbf{C} \Phi \\
\mathbf{K} &= \Phi^T \mathbf{K} \Phi \\
\mathbf{F} &= \Phi^T \mathbf{F}
\end{align*}
\]  

(2.156)

In the current analysis, the modal matrix is calculated at a predetermined rotor speed before the transient time integration begins. Then during a transient time integration, the modal matrix and the mode shapes are not recalculated as the rotor speed and centrifugal force change at every time step. However, the mass, damping and stiffness matrices and force vector are recalculated at every time step. This procedure is used because the recalculation of the modal matrix at every time step throughout the time integration would be computationally expensive.
Once the full finite element or modal spaces are chosen, the equations of motion can be time integrated. The fourth order Runge-Kutta, Newmark, or Bossak-Newmark integration schemes have been used in previous research. The fourth order Runge-Kutta method was the time integration method used in the engagement and disengagement analyses in Refs. 1, 32-42, and 46-47. Because it is widely used in structural dynamics its specifics will not be discussed here, but a full description can be found in Ref. 86. The Newmark integration scheme \[87\] is also widely used in structural dynamics to integrate second order linear equations of motion. It is based on the assumption that the acceleration of the system varies in a linear fashion between any two instants in time. It is defined by the following equations

\[
\begin{align*}
q_{n+1} &= q_n + \Delta t \tilde{q}_n + \Delta t^2 \left( \frac{1}{2} - \beta_N \right) \tilde{q}_n + \Delta t^2 \beta_N \tilde{q}_{n+1} \\
\dot{q}_{n+1} &= \dot{q}_n + \Delta t (1 - \gamma_N) \tilde{q}_n + \Delta t \gamma_N \tilde{q}_{n+1}
\end{align*}
\]

The parameters \(\beta_N\) and \(\gamma_N\) determine how much the acceleration at the end of the time interval enters into the velocity and displacement calculations. The parameters can be chosen to obtain desired stability characteristics, but in this analysis \(\beta_N = \frac{1}{4}\) and \(\gamma_N = \frac{1}{2}\). With those values, it is equivalent to assuming that the acceleration is a constant within the time interval. Unless \(\gamma_N = \frac{1}{2}\), spurious numerical damping is introduced, proportional to \((\gamma_N - \frac{1}{2})\). If \((\gamma_N - \frac{1}{2}) < 0\) negative damping is introduced and a self-exciting oscillation results; and if \((\gamma_N - \frac{1}{2}) > 0\) positive damping is introduced and reduces the response even without real damping in the system. The method is unconditionally stable for linear systems for \(\gamma_N \geq \frac{1}{4}\) and \(\beta_N \geq \frac{1}{4} \left( \gamma_N + \frac{1}{2} \right)^2\) \[88\].
Very high frequencies are present in large finite element models. During a time integration of such a system, artificial damping of the highest frequency modes can be very beneficial. However, artificial damping of the high frequency modes in the traditional Newmark method cannot be accomplished without reduced accuracy [89]. The Bossak-Newmark integration scheme [90], a modification of the original Newmark method, remedies this situation. In this method, an additional parameter, $\alpha_B$, is used in the third equation

$$ q_{n+1} = q_n + \Delta t \dot{q}_n + \Delta t^2 \left( \frac{1}{2} - \beta_B \right) \ddot{q}_n + \Delta t^2 \beta_B \dddot{q}_{n+1} $$
$$ \dot{q}_{n+1} = \dot{q}_n + \Delta t \left( 1 - \gamma_B \right) \ddot{q}_n + \Delta t \gamma_B \dddot{q}_{n+1} $$
$$ (1 - \alpha_B) M \dddot{q}_{n+1} + \alpha_B M \dddot{q}_n + C \ddot{q}_{n+1} + K q_{n+1} = F_{n+1} $$

(2.158)

The parameter $\alpha_B$ permits positive artificial damping of only the higher modes. The parameters can be chosen to obtain desired stability characteristics, but values of $\alpha_B = 0.1$, $\beta_B = 0.3025$ and $\gamma_B = 0.6$ were shown to yield excellent results [90] and are the values used in the current analysis.

### 2.5.2.1 Initial Conditions

To time integrate any differential equation, it is necessary to know the initial conditions of the system. In this analysis, the discretized equations of motion are second order so both the displacement and the velocity of the system are required. The initial conditions are also different for engagement and disengagement operations.

For an engagement, the rotor initial displacement, $q_0$, is found from the static form of the discretized equation of motion
\[ \mathbf{K}_0 \mathbf{q}_0 = \mathbf{F}_0 \] (2.159)

This equation is nonlinear because the aerodynamic forces acting on the rotor are functions of the rotor initial displacement and initial azimuth \( (\mathbf{K}_0 = \mathbf{K}_0(\mathbf{q}_0, \psi_0), \mathbf{F}_0 = \mathbf{F}_0(\mathbf{q}_0, \psi_0)) \). A Newton-Raphson iteration technique [86] is used to solve for the initial displacement \( \mathbf{q}_0 \). A vector-valued function \( \mathbf{G}(\mathbf{q}_0) \) is defined as

\[ \mathbf{G}(\mathbf{q}_0) = \mathbf{K}_0 \mathbf{q}_0 - \mathbf{F}_0 = 0 \] (2.160)

The function \( \mathbf{G}(\mathbf{q}_0) \) is then expressed as a first order Taylor series

\[ \mathbf{G}(\mathbf{q}_0) = \mathbf{G}(\mathbf{q}_i - \Delta \mathbf{q}) = \mathbf{G}(\mathbf{q}_i) - \frac{\partial \mathbf{G}(\mathbf{q}_i)}{\partial \mathbf{q}_i} \bigg|_{\mathbf{q}_i = \mathbf{q}_{i-1}} \Delta \mathbf{q} + O(\Delta \mathbf{q}^2) \] (2.161)

Where the vector \( \mathbf{q}_i \) is the approximation to the true initial condition \( \mathbf{q}_0 \). Neglecting the second order and higher terms yields an equation that can be solved for the increment in the displacement vector \( \Delta \mathbf{q} \)

\[ \Delta \mathbf{q} = \left[ \frac{\partial \mathbf{G}(\mathbf{q}_i)}{\partial \mathbf{q}_i} \right]^{-1} \mathbf{G}(\mathbf{q}_i) \] (2.162)

The next approximation in the total displacement vector is then

\[ \mathbf{q}_i = \mathbf{q}_{i-1} - \Delta \mathbf{q} \] (2.163)

This process is repeated until a sufficiently converged solution for \( \mathbf{q}_0 \) is obtained. The initial velocity for an engagement is simply assumed zero \( (\dot{\mathbf{q}}_0 = 0) \). For integrations in modal space, the initial modal amplitudes are given in terms of the physical displacements by

\[ \mathbf{e}_0 = (\mathbf{\Phi}' \mathbf{M}_r \mathbf{\Phi})^{-1} \mathbf{\Phi}' \mathbf{M}_r \mathbf{q}_0 \] (2.164)
For a disengagement operation, the initial displacement and velocity of the rotor are not nearly as significant as for an engagement operation. As long as the rotor speed at the beginning of a disengagement is above 50%NR, the rotor will reach a steady state response long before the critical rotor speed region is reached. For this reason, the initial conditions during a disengagement operation are simply assumed zero.

2.5.3 Engagement and Disengagement SHOL Analysis

The engagement and disengagement wind envelope analysis is a graphical tool used to determine the atmospheric wind conditions conducive to safe engagement and disengagement operations. An example of a “generic” SHOL is shown in Figure 2.13. As stated in the introduction, increasing relative WOD speeds are measured outward from the center of the circle and relative WOD directions are measured clockwise in the azimuthal direction. The combinations of WOD speeds and directions inside the grid are the safe conditions, while those outside are the unsafe. Usually, the maximum negative tip deflection is compared to some metric to determine if the conditions have exceeded the safe limits. For the H-46, the metric is the deflection required to strike the tunnel, or 55 inches. However, for other helicopters it could be the deflection determined to cause excessively high bending loads in the blades.
In the current analysis, the blade transient response is used to calculate the maximum tip deflection during an engagement or disengagement. The maximum tip deflection for each WOD speed, from 0 to 60 knots in 10-knot increments, and each WOD direction, from 0° to 360° in 30°-increments, is then plotted. Since the safe metric may vary from helicopter to helicopter, the deflections are not compared to any metric to determine if they are safe. An example of a calculated wind envelope from the current analysis is also shown in Figure 2.13. In general, the lowest blade deflections occur near the center of the circle and increase outward to the rim of the circle.

Figure 2.13: Comparison of Generic and Calculated SHOLs
Chapter 3

BASELINE RESULTS

The purpose of this chapter is to determine the level of detail required in the modeling of engagement and disengagement operations. This is accomplished by examining the transient response of a generic, or “baseline”, rotor system during rotor engagements and disengagements. There are four sections in this chapter. The level of fidelity necessary in the structural model and the aerodynamic model are examined in the first and second sections. The effect of ship airwakes and the blade initial conditions are presented in the third and fourth sections.

3.1 Baseline Rotor System Properties

The structural properties of the baseline rotor system used in the simulations are listed in Appendix B.1. The baseline rotor system is representative of an articulated rotor system for a medium-sized naval helicopter. The rotor speed profile during both rotor engagements and disengagements is assumed a simple offset sine wave

\[
\begin{align*}
\Omega(t) &= \frac{1}{4} \left( \cos \left( \frac{\pi}{20} t + \pi \right) + 1 \right) \quad \text{engagement} \\
\Omega(t) &= \frac{1}{4} \left( \cos \left( \frac{\pi}{20} t \right) + 1 \right) \quad \text{disengagement}
\end{align*}
\]

(3.1)

Where time is measured in seconds and the rotor speed is nondimensional. The rotor accelerates to or from full rotor speed in 20 seconds as shown in Figure 3.1. This rotor speed profile is very similar to the H-46 rotor speed profile.
3.2 Structural Modeling

In this section, the level of detail required in the finite element model in the full finite element space and in modal space is assessed. Computational efficiency is also examined. The addition of the lag degree of freedom to the structural model and its effects on the transient blade response are also discussed.

3.2.1 Convergence Studies

The number of finite elements and number of modes necessary to obtain a converged transient blade response is determined in this section. These quantities are important to establish because of their impact on the computational efficiency of the simulation. The proper level of detail is determined by examining the transient blade
response during a simulated rotor engagement of the baseline rotor system in a given ship airwake environment. The control system settings and parameters defining the aerodynamic environment are listed in Table 3.1. Note that the wind speed is 60 knots, which represents an extreme WOD condition.

| Table 3.1: Aerodynamic Environment and Control System Settings for Convergence Study |
|-----------------------------------------------|-----------------|
| Aerodynamic Model                            | Nonlinear Quasi-steady |
| Airwake Distribution                         | Linear |
| Gust Fraction                                | $\kappa$        | 25% |
| WOD Speed                                    | $V_{WOD}$       | 60 knots |
| WOD Direction                                | $\psi_{WOD}$    | 30° |
| Collective Pitch                             | $\theta_0$      | 4° |
| Lateral Cyclic Pitch                         | $\theta_{1c}$   | 0° |
| Longitudinal Cyclic Pitch                    | $\theta_{1s}$   | 4° |
| Blade #1 Start Azimuth                       | $\psi_0$        | 0° |

3.2.1.1 Full Finite Element Space Convergence Study

It is first necessary to determine the number of finite elements required to produce a converged blade response. The number of elements is important for two reasons. In the event of blade sailing, the rotor blades may experience large flap and lag deflections. These deflections can become as large as $30\% R$ in the worst cases. It is important to determine the number of finite elements required to accurately model blade deflections.
this large. In addition, the aerodynamic environment encountered in engagement and disengagement operations is unique. Because of the low rotor speed and unusual airflow around the ship superstructure, the rotor blades may encounter large spanwise variations in the aerodynamic forces. Increased numbers of finite elements may be required to accurately capture this unusual spanwise variation. Since flap deflections are the primary interest in the simulation of engagement and disengagement operations, the maximum (upward) or minimum (downward) flap tip deflection during the engagement was chosen as the convergence criteria.

Engagements were simulated with varying numbers of finite elements ranging from 8 to 20 and with varying time steps ranging from 1 ms to 0.1 ms. The prediction for the maximum flap tip deflection and associated error in the prediction for each case is shown in Table 3.2. The error in the flap tip deflection prediction was calculated against

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the 20-element model with a time step of 0.1 ms. A fully converged solution with less than 1% error in the flap tip deflection prediction was achieved with a 12-element model and a time step of 1.0 ms.

The transient time histories of the blade motion and loads for the converged case are shown in Figure 3.2. The blade flap and lag motions are shown in the first figure; and the blade twist, angle of attack distribution and bending moments are shown in the second figure. In each case, the values are compared to those in a pure hover condition with $C_{T}/\sigma = 0.09$, $\theta_{75} = 10^\circ$, $\theta_{180} = 0^\circ$, $\lambda = 0.0587$, and $V_{WOD} = 0$. These hover values are simply used purely for qualitative comparison. Because the control system settings are different during the engagement than for the pure hover condition, the values at the end of the engagement will not match those in the pure hover condition.

During the first 3 seconds of the engagement as the rotor is accelerating, inertia causes the blade to lag back and eventually strike the lag stop, set at $-10^\circ$. At $t = 3$ seconds, the blade begins to repeatedly flap between the droop and flap stops, set at $\pm 1^\circ$. The largest elastic deflections then occur between $t = 3$ and 8 seconds. During this period, the rotor speed is between 5%NR and 35%NR. The blade bends upwards as much as $+27.78\%R$ and downwards as much as $-22.70\%R$. The Coriolis forces generated by the excessive flapping cause the blade to repeatedly strike the lag stop. The blade is very stiff in the chordwise direction, so the majority of the lag tip deflection is due to rigid body motion of the blade about the lag hinge. The blade also experiences its largest twist angle due to inertial effects during the periods of violent flapping motion.
The blade section angles of attack between \( t = 3 \) and 8 seconds are very large, ranging from -180° to +180°. Even at \( t = 8 \) seconds after the most extreme blade flap deflections have already occurred, the angles of attack still range from 25° to 50°. Compared to the variation in the angle of attack, the influence of the elastic twist angle on the aerodynamic loads is probably small. The largest blade bending moments, both in
the normal and chordwise directions, also occur in this region. Flap bending moments reach peak values of -21610 ft-lb at the blade root and -8196 ft-lb at the blade attachment point. These transient values are 5.9 and 4.6 times higher than their values in hover of -3678 ft-lb and -1787 ft-lb. Lag bending moments reach -16430 ft-lb at the blade root and -11170 ft-lb at the blade attachment point. These transient values are 2.2 and 40 times higher than their values in hover of -7269 ft-lb and -280 ft-lb. As the blade

\[ \begin{align*} 
\text{e) Tip Twist Angle} \\
\text{f) Angle of Attack} \\
\text{g) Normal Bending Moment} \\
\text{h) Chordwise Bending Moment} 
\end{align*} \]

Figure 3.2: Converged Time Histories in the Full Finite Element Space
continues to accelerate and centrifugal forces increase, the blade deflections decrease. At \( t = 10 \) seconds, the rotor reaches 50\%NR. At this point the flap stops retract, so the blade is allowed to flap freely. However, enough centrifugal force has built up to prevent the blade deflections from becoming larger than in the low rotor speed region. Centrifugal force has also become large enough to rotate the blade forward off the lag stop. Blade bending moments decrease substantially after impacts between the blade and the flap and lag stops cease.

### 3.2.1.2 Modal Space Convergence Study

It is not always convenient to simulate engagement and disengagement operations in the full finite element space. The transient integration process involves mathematical manipulations of large matrices that are computationally expensive. A modal space transformation is commonly used to reduce the size of the matrices and improve computational efficiency. When using a modal transformation, it is necessary to determine the number of modes that are necessary to approximate the blade transient response. In this section, studies are conducted to determine the number of modes required to obtain a converged solution in modal space. Furthermore, the efficiency of the different time integration techniques discussed in Chapter 2 is also examined. In the second section, the effect of the rotor speed at which the modal matrix and mode shapes are calculated is discussed.
3.2.1.2.1 Comparison of Time Integration Algorithms

With a 12-element model chosen, engagements were simulated in modal space with varying numbers of modes. In each simulation, the modal matrix was composed of a combination of both articulated (without blade-stop contact) and elastic (with blade-stop contact) rotor modes. This is because most articulated rotor blades have nonzero flap and lag stop angles. A simple way to capture the effect of a nonzero stop angle and simulate the physical shape of the blade is with both free and elastic modes as shown in Figure 3.3. In the current analysis the first two articulated blade modes, rigid flap and lag, are used to model the response of the beam without stop contact. The rest of the modal matrix was composed of 1 to 5 elastic flap modes.

![Blade Deflection Schematic](image)

**Figure 3.3**: Schematic of Blade Deflections during Blade-Stop Contact

Since the largest blade deflections occur at approximately 25%NR, the blade modes used in the modal matrix were calculated at this rotor speed. As before, for each integrator the time steps ranged from 1.0 to 0.1 ms. In each case, the error in the tip deflection was calculated against the converged 12-element full finite element space simulation. The results for each time integration algorithm are discussed in the following sections.
Fourth Order Runge-Kutta Time Integration Algorithm

The original flap-torsion analysis presented in Refs. 41-42 used a standard fourth order Runge-Kutta time integration technique. The results for the fourth order Runge-Kutta time integration technique for the current analysis are shown in Table 3.3. The solutions marked with an asterisk denote simulations in which the transient integration became unstable. A converged solution with less than 1% error in the tip deflection was achieved with nine modes (a rigid body flap and a rigid body lag mode plus four elastic flap, two elastic lag and one elastic torsion modes) and a time step of 0.25 ms. Simulation of a 12-second rotor engagement required 2 minutes, 50 seconds of wall clock time. This equates to a ratio of 1:14.2 of simulation time to wall clock time.

<table>
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Figure 3.4 shows the time histories of the flap tip deflection and flap hinge angle for the converged case using the fourth order Runge-Kutta technique. The results for the flap hinge angle have been enlarged for closer comparison to the full finite element space solution. As expected, there is no discernable difference in the flapping response between the full finite element space solution and the modal space solution. However, there is a substantial difference in the flap hinge angle response in the modal space solution when compared to the full finite element space solution. High frequency noise, which does not exist in the full finite element space solution, is apparent when the blade contacts the droop or flap stops. The noise affects the flap hinge angle so much that the blade occasionally even flaps beyond the flap and droop stops, which is not physically possible. Although this noise does not adversely affect the flap tip deflection solution, it is undesirable.

Figure 3.4: Converged Time Histories in Modal Space Using the Fourth Order Runge-Kutta Time Integration Algorithm
Standard Newmark Time Integration Algorithm

The more recent simulations presented in Refs. 49-50 have utilized the standard Newmark time integration technique with success. The results for the standard Newmark time integration technique are shown in Table 3.4. Once again, the solutions marked with an asterisk denote simulations in which the transient integration became unstable. Compared to the Runge-Kutta method, fewer solutions became unstable. A converged solution with less than 1% error in the tip deflection was achieved with ten modes (a rigid body flap and a rigid body lag mode plus five elastic flap, two elastic lag and one elastic torsion modes) and a time step of 0.25 ms. Simulation of a 12-second rotor engagement required 2 minutes, 53 seconds of wall clock time. This equates to a ratio of 1:14.4 of

<table>
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simulation time to wall clock time, nearly identical to the Runge-Kutta method.

Figure 3.5 shows the time histories of the flap tip deflection and flap hinge angle for the converged case using the standard Newmark time integration technique. Again, the results for the flap hinge angle have been enlarged for closer comparison to the full finite element space solution. Like the Runge-Kutta results, there is no discernable difference in the flapping response between the full finite element space solution and the modal space solution. However, there is still a noticeable difference in the flap hinge angle response when compared to the full finite element space solution. The high frequency noise apparent when the blade contacts the droop or flap stops has been reduced using the Newmark method, but not eliminated. The computational efficiency of the Runge-Kutta and standard Newmark methods is nearly identical, but the standard Newmark method solution contains less high frequency noise. Therefore, the standard Newmark method was judged slightly superior to the Runge-Kutta method.

![Figure 3.5: Converged Time Histories in Modal Space Using the Standard Newmark Time Integration Algorithm](image_url)

a) Flap Tip Deflection  
b) Flap Hinge Angle
Bossak-Newmark Time Integration Algorithm

A Bossak-Newmark time integration technique has most recently been used to simulate engagements and disengagements. The results for the Bossak-Newmark technique are shown in Table 3.5. Unlike the fourth order Runge-Kutta or Newmark schemes, no simulations became unstable. A converged solution with less than 1% error in the tip deflection was achieved with nine modes (a rigid body flap and a rigid body lag mode plus 4 elastic flap, 2 elastic lag and 1 elastic torsion modes) and a time step of 0.50 ms. The converged solution was achieved with a time step twice the size of the other integrators, dramatically improving run time. Simulation of a 12-second rotor engagement required only 1 minute, 27 seconds of wall clock time. This equates to a

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ratio of 1:7.3 of simulation time to wall clock time, twice as fast as the other integrators.

Figure 3.6 shows the time histories of the flap tip deflection and flap hinge angle for the converged case using the Bossak-Newmark technique. Again, the results for the flap hinge angle have been enlarged for closer comparison to the full finite element space solution. Like the other two time integration algorithms, there is no discernable difference in the flapping response between the full finite element space solution and the modal space solution. However, the high frequency noise apparent when the blade contacts the droop or flap stops has now been almost entirely eliminated. Because the Bossak-Newmark method yields a converged solution with little high-frequency noise twice as fast as the other two methods, it was judged the superior solution and is the integration technique used to generate all the results in the next several chapters.

![Graphs showing flap tip deflection and flap hinge angle](image)

*Figure 3.6: Converged Time Histories in Modal Space Using the Bossak-Newmark Time Integration Algorithm*
3.2.1.2.2 Comparison of Mode Shapes

As stated earlier, all the modes used in the modal convergence study were calculated at a rotor speed of 25%NR where the largest blade deflections typically occur. However, in the previous research at Penn State University [41-42] the modes were calculated at 100%NR. It is of interest to compare the results of the modal convergence study using modes calculated at 100%NR. The results of the convergence study for modes calculated at 100%NR are listed in Table 3.6. The Bossak-Newmark integration scheme was used to make the comparison. Comparison to the results in Table 3.5, in which the modes are calculated at 25%NR, shows that the performance is not as good. Even the solution with five flap modes has more than 3% error when compared to the full

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<td>23.40</td>
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<tr>
<td>0.10</td>
<td>21.14</td>
<td>-23.9</td>
<td>24.76</td>
<td>-10.9</td>
<td>23.41</td>
</tr>
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</table>
finite element space solution. Clearly, it is advantageous to use modes calculated at 25%NR.

The mode shapes calculated at 25%NR and 100%NR are shown in Figure 3.7. The only appreciable difference in the shape of the modes is in the first elastic blade mode. The first elastic mode calculated at 25%NR has more curvature spread over a wider section of the blade, while the first elastic mode calculated at 100%NR has curvature localized to the root area, $x = 0.10$. There is an appreciable difference in the frequencies of each mode. Naturally, the blade natural frequencies calculated at 25%NR are much lower than when calculated at 100%NR. For example, the rigid body mode at 25%NR is $0.26\Omega_0$, while at 100%NR it is $1.03\Omega_0$.

![Graphs of mode shapes and frequencies](image)

Figure 3.7: Comparison of Mode Shapes and Natural Frequencies
3.2.2 Effect of Lag Motion

As stated in the Introduction, the previous aeroelastic analyses developed at the University of Southampton and at Penn State University were completed with flap-torsion degrees of freedom only. Recently, the lag degree of freedom has been added to the current aeroelastic analysis and it is desirable to assess its effect. The flap-lag coupling inherent in rotor blades with large amounts of pre-twist motivated the addition of the lag degree of freedom. However, flap-lag coupling may also be important to the simulation of articulated rotors, even with their typically small amounts of blade twist. Articulated rotors commonly have limited amounts of rigid body motion between the flap stops, which are generally set at approximately ±1°. However, articulated blades have a much wider range of motion in between the lag stops, which are generally set at ±10°. Coriolis force coupling was previously ignored and in this case may result in large lag deflections. Additionally, the increased tangential velocity may affect the aerodynamic forces on the blade when allowed to lag freely between the lag stops. Finally, articulated rotors utilize strong lag dampers that were also not modeled.

To assess the effect of lag motion, a flap-torsion only (FT) engagement and a flap-lag-torsion (FLT) engagement without a lag damper were simulated in the same ship airwake environment as in the previous section. A comparison of the time histories with the FLT engagement with a lag damper is presented in Figure 3.8. The largest changes in the flap tip deflection occur before $t = 6$ seconds. The peaks in the flap tip deflection at $t = 3.2$ and $t = 5.3$ seconds are under predicted by 31% and 12% with the FT model. In addition, the FT response leads the FLT response by 0.2 seconds in the low rotor speed
region. The flap tip deflection for the FLT engagement without a lag damper is much larger than with the damper. The minimum and maximum deflections increase to $+28.2\%R$ and $-27.9\%R$ from $+27.6\%R$ and $-22.3\%R$. The effect of the lag damper is easily seen in the time history of the lag hinge angle for each model. The lag damper reduces the blade lag hinge angle, especially during the periods that the largest flap deflections occur.

Due to the appreciable Coriolis forces the blade lags back and forth between the stops very quickly. This causes an additional tangential velocity, proportional to $x_{\text{hinge}}\dot\zeta$, over the blade section. This additional tangential velocity affects the aerodynamic lift generated by the blades and in turn, the blade motion. The time histories of the tangential velocity and aerodynamic lift near the blade tip (95%$R$) are also shown in Figure 3.8. Near $t = 3$ seconds, the peak lift values are 38.4 lb/ft for the FT case, 41.5 lb/ft for the FLT case, and 44.8 lb/ft for the FLT case with no flap damper. This represents increases of 8% and 17% from the original FT analysis. Near $t = 5.2$ seconds, the peak lift values are 63.6 lb/ft for the FT case, 66.7 lb/ft for the FLT case, and 75.3 lb/ft for the FLT with no flap damper. This represents increases of 5% and 18% from the original FT analysis. This example suggests that modeling the lag degree of freedom and the lag damper is important for articulated rotors as well as gimballed rotors.
3.3 Aerodynamic Modeling

To date, the majority of analyses of engagement and disengagement operations have been performed with a quasi-steady aerodynamic model. Only a small amount of research has investigated the effects of unsteady aerodynamics. This section will discuss
the effects of quasi-steady versus unsteady aerodynamic modeling in-depth by examining the transient response and engagement SHOLs of the baseline rotor system.

The original flap-torsion analysis in Refs. 41-42 briefly discussed the effects of the unsteady aerodynamic model. Engagements and disengagements were simulated in the constant and linear airwake distributions in a single WOD direction. Results of the comparison study were inconclusive. In the constant airwake distribution, tip deflections were 13% larger with the quasi-steady aerodynamic model. In the linear airwake distribution, tip deflections were 15% smaller with the quasi-steady aerodynamic model. It should be noted that the quasi-steady aerodynamic model in Refs. 41-42 was a linear model with special provisions made for the reverse flow region. In the present analysis, the quasi-steady model is nonlinear. Furthermore, the unsteady aerodynamic model in Refs. 41-42 was assumed valid for all angles of attack, skew angles, and Mach numbers. In reality, the unsteady model has only been validated for angles of attack below 22°, skew angles below 30°, and Mach numbers above 0.3. In the present analysis, the unsteady aerodynamic model is only assumed active for angles of attack and skew angles lower than 25° and Mach numbers above 0.10. It is difficult to make definite conclusions on the importance of each aerodynamic model as presented in Refs. 41-42 due to these limitations. Consequently, it is desirable to reassess the importance of nonlinear quasi-steady and unsteady aerodynamic effects with the more refined techniques used in the current analysis.

A single engagement of the baseline rotor system was simulated using the same aerodynamic conditions used in Table 3.1. The resulting time history of the flap tip
deflection during the engagement is shown in Figure 3.9. The largest peaks in the tip flap deflection occur at 3.2, 4.0, 5.2 and 5.6 seconds. The resulting flap deflections with the unsteady model are slightly lower than the quasi-steady model. The quasi-steady and unsteady models predict maximum deflections of +27.6% and +25.5% and minimum deflections of –22.3% and –19.7%, respectively. The unsteady aerodynamic model predicts peak deflections approximately 10% lower than the quasi-steady model. The response of the blade using the unsteady aerodynamic model also lags behind the response using the quasi-steady model in the low rotor speed region.

Figure 3.9: Comparison of the Time History of the Flap Tip Deflection using Quasi-steady and Unsteady Aerodynamic Models

The peaks in the flap deflection occur during the period in which the rotor is spinning at less than 20%NR; therefore, they will also tend to coincide with the periods in which the angle of attack and the skew angle become large. A comparison of the time histories of the angle of attack, skew angle, Mach number and normal force coefficient at the 75%R location for each aerodynamic model is shown in Figure 3.10. In this analysis,
the unsteady aerodynamic model is deactivated when the blade section angle of attack or skew angle become larger than 25° or when the blade section Mach number falls below 0.10. The unsteady aerodynamic model, as presented in Ref. 84, is not well suited to model the flow physics in situations where the airfoil is deeply stalled, in reverse flow or at very low speed. When the unsteady aerodynamic model is deactivated, the

**Figure 3.10**: Comparison of the Time Histories using Quasi-steady and Unsteady Aerodynamic Models
aerodynamic coefficients are calculated with the nonlinear quasi-steady model. With all the restrictions placed on its usage, the unsteady aerodynamic model is only actually active from $t = 2.4$ to $3.1$ seconds and again from $t = 4.8$ to $5.2$ seconds.

The effect of the aerodynamic models is more completely assessed by examining the engagement SHOLs for the baseline rotor system in all four deterministic airwake types. By examining the SHOLs, as opposed to a single engagement, a more comprehensive understanding of the effect of the aerodynamic model can be established.

The control system settings for the baseline rotor system are the same as in Table 3.1. The horizontal, constant, step, and linear airwake distributions were all simulated with a gust factor of 25%. The minimum and maximum tip deflections for each case (at a wind speed of 50 knots) are shown in Table 3.7. The tip deflections for the unsteady aerodynamic model are anywhere from 7% to 26% lower than the quasi-steady aerodynamic model. Note that in the case of the step airwake distribution, the minimum tip deflection is the same for each model. Because the extreme velocity field in the step

<table>
<thead>
<tr>
<th>Table 3.7: Aerodynamic Model Study</th>
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<tbody>
<tr>
<td>Minimum and Maximum Tip Deflections for $V_{WOD} = 50$ knots</td>
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<table>
<thead>
<tr>
<th></th>
<th>Quasi-steady</th>
<th>Unsteady</th>
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<tr>
<td></td>
<td>Min $w_{tip}$</td>
<td>Max $w_{tip}$</td>
</tr>
<tr>
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<td>-16.43 %R</td>
<td>+18.67 %R</td>
</tr>
<tr>
<td>Constant</td>
<td>-19.87 %R</td>
<td>+29.95 %R</td>
</tr>
<tr>
<td>Step</td>
<td>-25.24 %R</td>
<td>+29.99 %R</td>
</tr>
<tr>
<td>Linear</td>
<td>-19.40 %R</td>
<td>+23.86 %R</td>
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</table>
airwake distribution causes the angles of attack to remain large, the unsteady aerodynamic model was not even activated before the minimum tip deflection occurred.

Several trends can also be seen by examining the results in Table 3.7. The importance of modeling the vertical flow component is shown. The horizontal airwake distribution has the lowest deflections, while the constant, step, and linear airwake distributions have much larger deflections.

The actual engagement SHOLs for each of the deterministic airwake distributions using the nonlinear quasi-steady and unsteady aerodynamic models are shown in Figure 3.11 through Figure 3.14. The engagement SHOLs for the horizontal airwake distribution are shown Figure 3.11. As indicated in Table 3.7, the blade deflections for the unsteady model are slightly lower than for the quasi-steady model. For each model, the largest blade deflections occur in winds near 30°.

Figure 3.11: Effect of Aerodynamic Model on Engagement SHOLs for the Horizontal Airwake Distribution
The engagement SHOLs for the constant airwake distribution using the nonlinear quasi-steady and unsteady aerodynamic models are shown Figure 3.12. The importance of the vertical flow component is demonstrated. Blade deflections increase dramatically compared to the horizontal airwake distribution. The largest difference between the two airwake models is that lower deflections are predicted by the unsteady aerodynamic model at wind speeds of 30 to 40 knots in 30° winds.

![Figure 3.12: Effect of Aerodynamic Model on Engagement SHOLs for the Constant Airwake Distribution](image)

The engagement SHOLs for the step airwake distribution using the nonlinear quasi-steady and unsteady aerodynamic models are shown Figure 3.13. In the step airwake case, the vertical flow component changes from upward on the windward side to downward on the leeward side. Because of the variation in the vertical flow component across the rotor disk, the blade deflections increase in comparison to the constant airwake distribution. The blade deflections also show much less dependence on the wind direction for the step airwake distribution. Again, the unsteady aerodynamic model predicts lower blade deflections at higher wind speeds.
The engagement SHOLs for the linear airwake distribution using the nonlinear quasi-steady and unsteady aerodynamic models are shown in Figure 3.14. The vertical flow component varies linearly from upward on the windward side to downward on the leeward side. Because the variation in the vertical flow component across the rotor disk is less dramatic than in the step airwake distribution, the blade deflections are slightly smaller.

Figure 3.14: Effect of Aerodynamic Model on Engagement SHOLs for the Linear Airwake Distribution
lower. As with the other airwake distributions, the unsteady aerodynamic model results in slightly lower blade deflections at the higher wind speeds.

These results suggest that a nonlinear quasi-steady aerodynamic model should be used in engagement and disengagement modeling for two reasons. First, although unsteady effects certainly exist, the current nonlinear unsteady aerodynamic model has not been validated at the very high angles of attack and skew angles, or very low Mach numbers encountered in these simulations. Second, since the prediction of large blade deflections is of interest it is also more desirable to over predict blade deflections than under predict them. In the remaining engagement and disengagement simulations, the nonlinear quasi-steady aerodynamic model is utilized.

### 3.4 Ship Airwake Modeling

The majority of engagement and disengagement research completed with the aeroelastic code described in this analysis has utilized the very simple deterministic airwake types described in the previous sections. This section will explore the effects of more realistic ship airwakes on the rotor behavior. An examination of the results for the baseline rotor system in numerically computed full-scale ship airwakes is examined first. In addition, a comparison of the results of the current analysis with the wind tunnel tests completed at the University of Southampton is made. In each case, the role of the spatial variation of the mean velocity components and the effect of the fluctuating velocity components will be discussed.
3.4.1 Effect of Numerically Computed Ship Airwakes

The effects of “realistic” full-scale airwake distributions on the rotor response are determined in this section by examining the engagement SHOLs for the baseline rotor system. The chosen ship geometry is the “frigate-like” Simple Frigate Shape (SFS) configuration discussed in Chapter 1 and in previous research [71]. A US Navy FFG class frigate and the Simple Frigate Shape (SFS) are shown in Figure 3.15. Each ship has a flight deck located aft of a hangar structure. The flight deck of the SFS is 45 feet wide by 90 feet long. Clearly, the airflow over the flight deck of the both the actual frigate and the frigate-like SFS will be nonuniform because of the boxy shape of the ship superstructure.

Long et al constructed a computational model of the SFS. The unstructured computational grid of the SFS is shown in Figure 3.16. Note the clustering of grid points over the flight deck. Simulated rotor engagements of the baseline rotor at three different locations, or “spots”, on the SFS flight deck will be examined in this section. The rotor disk is located on the centerline of the ship for each spot. Spot #1 is nearest the hangar
face (30 feet behind the hangar), Spot #2 is near the middle of the flight deck (60 feet behind the hangar) and Spot #3 is at the rear of the flight deck (90 feet behind the hangar).

Using the modified PUMA code [76]-[77], Long et al calculated the mean velocity components around the chosen ship geometry. Because of restrictions on computational time, several limitations were made. The fluctuating flow components were not calculated. Neither the atmospheric boundary layer nor viscous effects were modeled. The airwake cases were also limited to wind over deck conditions of 40 and 50 knots and wind over deck directions of $0^\circ$, $330^\circ$, $300^\circ$, $270^\circ$, $240^\circ$, $210^\circ$ and $180^\circ$. The results for the mean velocity components for all the cases are shown in Appendix C. For illustrative purposes, the results for the mean airwake components for 50-knot bow and port winds are shown in Figure 3.17. Only the portion of the airwake in the plane of the rotor, assumed 16 feet above the flight deck, is shown. Values on the graphs indicate the airwake components in knots at a particular location in space. The largest gradients in

Figure 3.16: SFS Unstructured Grid and Landing Spots

a) SFS Unstructured Grid (from Ref. 76)

b) SFS Landing Spots
the flow structures, in both spatial size and magnitude, consistently occur just behind the hangar.

Figure 3.17: SFS Ship Airwake Mean Velocities
For bow winds, a recirculation zone directly behind the hangar can be identified. The gradients in the flow structures are very large within a relatively small area. For port winds, the lateral flow component accelerates substantially at the windward deck edge. The values increase from the freestream value of 50 knots to 77 knots, a 54\% increase. In addition, a large region of 15 to 20 knot upward flow components exists at the windward deck edge.

The calculated ship airwakes using the modified PUMA analysis were used as input to the rotor engagement simulation using the method described in Chapter 2. The effects of the airwake model are assessed by examining the engagement SHOLs. Since only a few airwake cases were calculated for the SFS, only a partial engagement SHOL was constructed. Control system settings for the baseline rotor system are the same as in Table 3.1. The maximum tip deflections in a 50-knot wind for the horizontal and linear deterministic airwake distributions and the SFS airwake distributions are shown in Table 3.8.

<table>
<thead>
<tr>
<th>Table 3.8: Ship Airwake Study</th>
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<tr>
<td>Minimum and Maximum Tip Deflections for $V_{WOD} = 50$ knots</td>
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<tr>
<td></td>
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<tr>
<td></td>
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<tr>
<td>Horizontal</td>
</tr>
<tr>
<td>Linear</td>
</tr>
<tr>
<td>SFS Spot #1</td>
</tr>
<tr>
<td>SFS Spot #2</td>
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<tr>
<td>SFS Spot #3</td>
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</tbody>
</table>
Figure 3.18 shows the SHOLs for the SFS landing spots. Tip deflections decrease dramatically the farther the helicopter is from the flight hangar. In addition, the tip deflections are strongly dependent on the wind direction. The size and location of the

Figure 3.18: Engagement SHOLs for the SFS Airwake Distributions
flow structures in the ship airwake change with wind direction. WOD flows from 300º to 330º and near stern flows from 180º to 210º result in much less tip deflections than for bow or near-abeam flows from 240º to 300º. At Spot #1 for a wind speed of 50 knots, the maximum deflection is -13.9%R for 330º winds and -13.3%R for stern winds; however, the maximum deflection is -21.1%R for the bow winds and -24.5%R for abeam winds. This is an increase by almost a factor of two. The same trend is evident for Spots #2 and #3. The blade deflections are also dependent on wind speed, but to a lesser extent than wind direction. For abeam winds at Spot #1, the minimum blade deflections are -15.7%R and -24.5%R at wind speeds of 40 and 50 knots. A 10-knot wind speed increase resulted in 50% more blade deflection. It is quite intuitive that blade deflections increase with wind speed; however, this is not always the case. Blade deflections can occasionally decrease with increasing wind speed. This is because the locations of the flow structures change with wind speed, not just with direction. For Spot #1 in 240º winds, the minimum deflections are -23%R and -21.3%R at wind speeds of 40 and 50 knots. Likewise, at Spot #2 in 330º winds, the maximum deflections are -12.9%R and -8.6%R. To better visualize the variation of the blade deflections with wind speed and direction, they are plotted versus wind direction in Figure 3.19.
3.4.1.1 Effect of Fluctuating Flow Components

In the previous sections, all engagement simulations were performed with mean flow components only. It was assumed that there was zero turbulence intensity over the flight deck. Obviously, this is not the case in reality. This section examines the response of the baseline rotor during an engagement to varying levels of fluctuating flow. The effect of fluctuating flow components on the response of an elastic blade has not been explored in previous research at Penn State or at the University of Southampton.

The WOD conditions for the engagement are for SFS Spot #2 with a 50-knot bow wind. Measurements in previous research showed turbulence intensities of up to 20% of the freestream over the flight deck [8, 62, 69]. In this simulation, the turbulence intensities were assumed constant across the rotor disk for all velocity components. Two
representative levels of 10% and 20% were chosen. Other measurements of the velocity spectra in previous research showed peak frequencies were from 0.1 to 5 Hz [8, 69]. In this simulation, characteristic frequencies from 0.1 to 10 Hz were used. Four rotor engagements were simulated for each combination of turbulence intensity and characteristic frequency.

The results of the study are presented in Figure 3.20. The average maximum downward flap tip deflection for the four engagements is plotted as a solid line. The vertical error bars indicate the largest and smallest tip deflections. The dashed line indicates the tip deflection predicted with zero turbulence intensity of -20.2%R. The variation in the tip deflections is largest for characteristic frequencies between 0.2 and 2 Hz. This range of frequencies is lies exactly within the range of frequencies measured in previous ship airwake tests. In this range, a turbulence intensity of 10% resulted in as much as a ±5%R, or ±25%, change in the maximum tip deflection. A turbulence intensity

![Graphs showing variation of maximum downward flap tip deflection with turbulence intensity](image)

a) 10% Turbulence Intensity  

b) 20% Turbulence Intensity

*Figure 3.20: Variation of Maximum Downward Flap Tip Deflection With Turbulence Intensity for the SFS Spot #1 Airwake Distribution*
Intensity of 20% resulted in as much as a ±9% \( R \), or ±45%, change in the maximum tip deflection. Also indicated on the plots are the natural frequencies of the blade at a rotor speed of 25%NR, where the maximum tip deflections are most likely to occur. The rigid body flap frequency at 1.23 Hz and the first elastic flap frequency at 1.67 Hz are within the range of characteristic frequencies of interest. Above 2 Hz, the fluctuating flow components have a much smaller effect on the blade response.

Example time histories of the flap tip deflection for characteristic frequencies of 0.1, 1.0 and 10 Hz are shown in Figure 3.21 through Figure 3.23. The response of the baseline blade is influenced much more by characteristic frequencies of 0.1 and 1.0 Hz than by the 10 Hz frequency. The fluctuating flow model used in this analysis, although rudimentary, indicates that the fluctuating flow components can significantly affect the blade response.

\[ \text{Figure 3.21: Time Histories of Flap Tip Deflection at a Characteristic Frequency of 0.1 Hz for the SFS Spot #1 Airwake Distribution} \]
Figure 3.22: Time Histories of Flap Tip Deflection at a Characteristic Frequency of 1.0 Hz for the SFS Spot #1 Airwake Distribution

Figure 3.23: Time Histories of Flap Tip Deflection at a Characteristic Frequency of 10 Hz for the SFS Spot #1 Airwake Distribution
3.4.2 Validation with Wind Tunnel Measurements

The deterministic ship airwake distributions described in Chapter 2 were originally synthesized with limited model-scale wind tunnel measurements correlated with a few full-scale measurements completed at the University of Southampton [31-32]. Later research at the University of Southampton focused on additional model-scale wind tunnel tests [1, 38]. In these tests, engagements and disengagements of a radio-controlled helicopter on board a model-scale ship were performed. The blade flapping response was measured and predicted. This section compares the predictions of the current analysis with these previous tests and analyses.

The wind tunnel tests at the University of Southampton were broken into two parts. In the first part, a model-scale ship was inserted in a wind tunnel to simulate abeam wind conditions. To obtain a general understanding of the flow characteristics, the airwake across the width of the ship deck was examined with a wand-tuft study. Flow measurements were concentrated at five locations spaced across the width of the ship deck, called deck positions A through E. A schematic of the flow directions and a characterization of the ship airwake are shown in Figure 3.24. Large amounts of upflow exist at the windward deck positions A and B. The leeward deck positions C, D and E lie within a recirculation zone. The tuft study showed that deck position B lied directly on the boundary of the relatively undisturbed air and the recirculation zone.
The mean flow velocities and directions were then measured at a typical rotor height with a traversing LDA. The measured mean flow velocities and flow inclination angle, relative to the horizontal, of the along-freestream (lateral) wind component are shown Figure 3.25. The measured flow velocities of the perpendicular-to-freestream (longitudinal) component were shown to be very near zero. The flow velocities over deck positions A and B are much higher and also vary much more across the rotor disk than at deck positions D or E where the mean flow velocity is much lower and more uniform. These results again indicate the division between the relatively undisturbed and unsteady, separated flow regions. The only information made available on the fluctuating flow components was that the flow characteristic frequency was 20 Hz. No information on the magnitude of the fluctuating flow components was made available.

Figure 3.24: Results of Wand-Tuft Study for Model-scale Ship Airwake Tests
In the second part of the wind tunnel tests, the modified radio-controlled model helicopter was placed on the ship deck. The model helicopter utilized a teetering rotor system with very rigid blades. Engagements and disengagements of the model rotor system were performed at the five deck positions. The time histories of the rotor speed and teeter angle were measured during the engagement and disengagement. Mechanical limiters set at angles of −11º and +23º restrained the rotor.

The characteristics of the model rotor system are given in Appendix B.2. The blades were considered completely rigid for the predictions made by the University of Southampton and the current analysis. The aerodynamic conditions and control system settings used by the current analysis are shown in Table 3.9. The actual model rotor system rotated in the clockwise direction, while in the current analysis the rotor is assumed to rotate in the counterclockwise direction. Therefore, the blade start angle and wind direction were both shifted by 180º from the values in Ref. 1.

*Figure 3.25: Measured Mean Flow in Along-Freestream Direction for Model-scale Ship Airwake Tests*
The results from wind tunnel tests and the current analysis for each of the deck positions are shown in Figure 3.26 through Figure 3.30. In each of the figures, the measured and predicted rotor response from the University of Southampton are shown on top. The predicted rotor response using the current analysis is shown on the bottom of each figure. Since no information on the magnitude of the fluctuating flow components was made available, the rotor response assuming turbulence intensities of 0% and 20% are shown.

The rotor response at deck position A, the most windward deck position, is shown in Figure 3.26. The rotor strikes the stops many times at the beginning and end of the test. Furthermore, the rotor response is influenced very little by the fluctuating flow components.

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<tr>
<td>Longitudinal Cyclic Pitch $\theta_{ls}$</td>
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</tr>
<tr>
<td>Blade #1 Start Azimuth $\psi_0$</td>
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</tr>
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</table>

Table 3.9: Aerodynamic Environment and Control System Settings for Validation Study
The rotor response at deck position B, the deck position where the center of the rotor lies directly on the boundary of the relatively undisturbed air and recirculation zone, is shown in Figure 3.27. Here the rotor response is larger than at deck position A. The rotor strikes the stops more often in the low rotor speed region and has a higher mean flapping amplitude in the high rotor speed region. Again, the fluctuating flow components have very little influence on the gross rotor behavior.

Figure 3.26: Measured and Predicted Teeter Angle at Deck Position A

The rotor response at deck position B, the deck position where the center of the rotor lies directly on the boundary of the relatively undisturbed air and recirculation zone, is shown in Figure 3.27. Here the rotor response is larger than at deck position A. The rotor strikes the stops more often in the low rotor speed region and has a higher mean flapping amplitude in the high rotor speed region. Again, the fluctuating flow components have very little influence on the gross rotor behavior.
The rotor response at deck position C is shown in Figure 3.28. Here the experimental results show more influence of the fluctuating flow components. This was to be expected, since more of the rotor disk is located within the recirculation zone. Overall, the blade flapping behavior is slightly milder than at deck position A or B.

Figure 3.27: Measured and Predicted Teeter Angle at Deck Position B
The rotor response at deck positions D and E is shown in Figure 3.29 and Figure 3.30. In general, both of the leeward deck positions show much less flapping than the other three windward deck positions. However, the influence of the fluctuating flow component is much larger. For deck position D, the rotor flapping behavior is not modeled well at all in the very low rotor speed region from $0 < t < 10$ seconds with no turbulence intensity. However, the correlation becomes much improved with the addition of 20% turbulence intensity.

![Figure 3.28: Measured and Predicted Teeter Angle at Deck Position C](image)

- a) Measured (from Ref. 38)
- b) Predicted (from Ref. 38)
- c) Predicted with 0% Turbulence Intensity
- d) Predicted with 20% Turbulence Intensity
The position of the rotor on the ship deck was found to have a substantial impact upon the rotor behavior. Deck positions nearest the windward deck edge were characterized by large amounts of blade flapping, while deck positions nearest the leeward edge in the recirculation zone were characterized by very mild amounts of blade flapping. In this region, fluctuating flow components were shown to substantially influence the rotor behavior.

Figure 3.29: Measured and Predicted Teeter Angle at Deck Position D
In this section, the effects of the initial conditions of the blade during an engagement and a disengagement operation are investigated. The initial conditions of the blade are composed of two things - the blade initial deflection $q_0$ and azimuth angle $\psi_0$. Previous modeling efforts have not addressed the effects of either variable.

**Figure 3.30**: Measured and Predicted Teeter Angle at Deck Position E

3.5 Initial Condition Modeling

In this section, the effects of the initial conditions of the blade during an engagement and a disengagement operation are investigated. The initial conditions of the blade are composed of two things - the blade initial deflection $q_0$ and azimuth angle $\psi_0$. Previous modeling efforts have not addressed the effects of either variable.
3.5.1 Effect of Blade Initial Deflection

In the previous research completed at Penn State University and the University of Southampton, the initial deflection of the blade during a rotor engagement was assumed the static deflection of the blade under gravity. The influence of aerodynamic forces on the initial deflection of the blade was completely ignored. Under this assumption when a transient integration began, the blade would “experience” a large impulsive aerodynamic force. Large amounts of blade motion resulted, even when the rotor blade was at very low rotor speeds. This effect is shown for a simulated H-46 rotor engagement in Figure 3.31 from Ref. 47. The minimum tip deflection for the entire engagement occurs at \( t = 0.5 \) seconds. At this point, blade has only rotated through an azimuth angle of 0.25º and reached a speed of 0.6%NR. It is highly unlikely the predicted deflections in this region are physically realistic.

![Figure 3.31: Time Histories of Flap Tip Deflection and Flap Hinge Angle with Assumed Initial Conditions](from Ref. 47)
In the current analysis, the effect of the aerodynamic forces has been included in the calculation of the initial position of the blade. An engagement of the baseline rotor system was simulated for cases where the initial conditions have been modeled properly and have not been modeled properly. The aerodynamic conditions during the engagement are the same as Ref. 47. A constant airwake distribution is simulated in 40-knot starboard winds. The resulting time histories of the flap tip deflection for each case are shown in Figure 3.32.

![Figure 3.32](image)

*Figure 3.32: Effect of Initial Conditions on Time History of Flap Tip Deflection*

For the case in which the initial conditions are not modeled properly, the blade is initially deflected -7.8%\(R\) by gravity only. As soon as the engagement begins, the blade is forced downward by an impulsive aerodynamic force reaching a minimum deflection of -24.4%\(R\) at \(t = 0.75\) seconds. In this amount of time, the blade has only rotated through an azimuth angle of 1.3° and reached a speed of 0.3%NR. Clearly, this situation is not physically realistic. The blade then rebounds upwards and downwards for several cycles until the rotor speed and aerodynamic forces begin to build up at \(t = 4\) seconds.
For the case where the initial conditions are properly modeled, the blade is initially deflected to -13.8%\(R\). After the engagement begins, the blade does not flap appreciably in the very low rotor speed region.

### 3.5.2 Effect of Blade Initial Azimuth Angle

Because the current aeroelastic analysis is a transient analysis, some initial conditions must be assumed at the start of the time integration. As stated earlier, the initial conditions are composed of the blade initial deflection, \(q_0\), and the blade initial azimuth angle, \(\psi_0\). The previous section demonstrated that the proper modeling of the blade initial deflection had a substantial effect on the blade response in the very low rotor speed region during an engagement operation. This section will examine the effect of the blade initial azimuth angle on the blade response.

Throughout the previous simulations, only a single blade of the 4-bladed baseline rotor was considered. In each case, the blade initial azimuth angle was assumed 0°. The responses of blades 2, 3 and 4, which start at initial azimuth angles of 90°, 180° and 270° were not considered. Both engagement and disengagement SHOLs for the other three blades are presented and compared to the engagement SHOL for blade 1 to determine the sensitivity of the rotor response to changes in the initial blade azimuth angle. If the blade response is sensitive to changes in the initial azimuth angle, then each SHOL would only be valid for each initial azimuth angle. In other words, an entire family of engagement SHOLs with blade initial azimuth angles ranging from 0° to 360° would have to be generated for each wind over deck condition to determine the worst case scenario. In this
situation, the number wind over deck/initial azimuth angle combinations would become enormous. Clearly, this would be an undesirable situation.

The engagement SHOLs for each blade in the SFS Spot #1 airwake distribution are shown in Figure 3.33. In general, the SHOLs are remarkably similar. The largest deflections consistently occur in bow, port and 240° winds while the smallest deflections consistently occur in 330° and stern winds. The minimum tip deflections for each blade are -24.5%R, -25.1%R, -25.4%R and -23.2%R, a difference of only 9% between blades.

Figure 3.33: Effect of Initial Azimuth Angle on Engagement SHOLs for the SFS Spot #1 Airwake Distribution
The minimum tip deflection occurs in port winds for blades 1 and 2 and in 240° winds for blades 3 and 4. For blade 3, the deflections are slightly increased in 300° winds and decreased in bow winds.

A time history of the rotor speed, the number of revolutions the rotor has completed and the flap tip deflections for 40-knot 300° winds are shown in Figure 3.34. Blades 2 and 3 reach their minimum deflections first at $t = 3.2$ seconds, while blades 1

![Figure 3.34: Effect of Blade Initial Azimuth Angle on Rotor Engagement for 40-knot 300° Winds](image-url)
and 4 reach theirs at $t = 4.0$ and 4.5 seconds, respectively. The flap tip deflections are also plotted during the first two rotor revolutions. Blades 2 and 3 reach their minimum deflection when the rotor has already rotated 0.33 revolutions or 120°. Blades 1 and 4 reach their minimum deflections at 0.60 and 0.86 revolutions, respectively.

The disengagement SHOLs for each blade in the SFS Spot #1 airwake distribution are shown in Figure 3.35. The SHOLs are not nearly as similar for a rotor disengagement as for a rotor engagement. For example, much smaller blade deflections

![Figure 3.35: Effect of Initial Azimuth Angle on Disengagement SHOLs for the SFS Spot #1 Airwake Distribution](image-url)
are predicted for blade 1 in 210° winds than for the other blades. Much smaller blade deflections are also predicted for blade 3 in bow winds than for the other blades.

A time history of the rotor speed, the number of revolutions the rotor has completed and the flap tip deflections for 40-knot 300° winds are shown in Figure 3.36. Blade 1 reaches its minimum deflection first at $t = 16$ seconds. Blade 4 reaches its

![Diagram](image)

a) Rotor Starting Position  

b) Rotor Speed Profile

c) Rotor Flap Tip Deflection versus Time  

d) Rotor Flap Tip Deflection versus Number of Revolutions

*Figure 3.36: Effect of Blade Initial Azimuth Angle on Rotor Disengagement for 40-knot 300° Winds*
minimum deflection at $t = 18.4$ seconds, where the rotor speed is just under $2\%NR$. At this point, the rotor is almost stopped. The flap tip deflections are also plotted during the last two rotor revolutions. Blade 4 reaches its minimum deflection when the rotor is within 0.04 revolutions, or $14^\circ$, of stopping.

In the case examined, the largest blade deflections occurred after the first one-third rotor revolution for a rotor engagement. For a rotor disengagement, the largest blade deflections occurred after the last one-half revolution. Consequently, the initial blade azimuth angle has a much smaller effect on the engagement SHOLs than on the disengagement SHOLs.

### 3.6 Summary

This chapter detailed the level of detail necessary in the structural and aerodynamic models required to accurately simulate a transient engagement or disengagement operation. Convergence studies were performed in the full finite element space and modal space. A 12-element, 9-mode blade model was shown to yield converged results. The computational efficiency of different time integration techniques was also examined. The Bossak-Newmark integration algorithm, using blade modes calculated at $25\%NR$, was shown to yield the best results. Furthermore, the effect of lag motion and the inclusion of aerodynamic forces in the initial conditions was shown to more realistically predict the rotor response.

The quasi-steady nonlinear aerodynamic model was shown to predict larger blade deflections than the unsteady aerodynamic model. Realistic ship airwake distributions
for a full-scale frigate-like shape resulted in much larger blade deflections compared to the deterministic airwake distributions. The blade response was much more sensitive to changes in wind speed and direction. The location of the rotor on the ship deck was also shown to influence the blade response. Deck locations directly behind the hangar resulted in much higher blade deflections. Fluctuating flow components were shown to influence the blade response. Analysis of the response of all four blades of the rotor system showed that the flapping response was relatively insensitive to the blade initial azimuth angle for a rotor engagement, but highly sensitive for a rotor disengagement. The current analysis was also correlated with previous model-scale wind tunnel tests completed at the University of Southampton. Good correlation existed between the measured and predicted rotor response. Once again, the position of the rotor on the ship deck was shown to significantly affect the blade response.
Chapter 4
PASSIVE CONTROL METHODS FOR ARTICULATED ROTORS

The focus of this chapter is not on the prediction of the transient rotor response, but on methods by which the rotor flapping response can be controlled or reduced. If excessive flapping could be systematically reduced, the operational restrictions placed on engagement and disengagement operations could be relaxed. Thus, helicopter operations could be conducted in more adverse weather conditions increasing the helicopter's value. Several methods to reduce the transient blade deflections are examined. Each method is a passive control technique. The first control technique focuses on altering the procedures the pilot follows during engagement and disengagement operations, while the other two techniques focus on the addition of passive control devices to the rotor blades. Instead of analyzing a generic articulated rotor system, the analysis will examine the blade response for an analytic model of an H-46 Sea Knight rotor system.

4.1 H-46 Sea Knight Rotor System Properties

The relevant structural properties of the H-46 rotor system, derived from Ref. 91, are given in Appendix B.3. The experimentally measured rotor speed profiles for an H-46 Sea Knight engagement and disengagement are shown in Figure 4.1 from Ref. 41. The rotor speed during the disengagement is shown versus the time before the rotor stops.
During an engagement, the rotor is rotating at less than 25%NR for 8 seconds, while during a disengagement the rotor is rotating at less than 25%NR for 6 seconds. In each of these regions, the centrifugal stiffening is less than 5% of its value at nominal rotor speed. Both rotor speed profiles are valid for 15 seconds and are given by

\[
\Omega(t) = \begin{cases} 
1.099 \times 10^{-6} t^6 - 4.098 \times 10^{-5} t^5 + 4.561 \times 10^{-4} t^4 \\ -1.053 \times 10^{-3} t^3 - 1.712 \times 10^{-3} t^2 + 1.331 \times 10^{-2} t 
\end{cases} \quad \text{engagement}
\]

\[
\Omega(t) = -2.459 \times 10^{-4} (t-15)^3 + 4.702 \times 10^{-3} (t-15)^2 + 2.712 \times 10^{-2} (t-15) \quad \text{disengagement}
\]

Where time is measured in seconds and the rotor speed is nondimensional.

In actual H-46 fleet operations, the same control system settings are used for every engagement and disengagement operation [41]. Pilots are instructed to use 3° of collective pitch. They are also instructed to use the “AUTO” control system setting, which tilts the rotor disks 2.5° aft at full rotor speed in zero winds. It is assumed that there is 0° of lateral cyclic pitch. These control inputs are termed the “standard” control system settings.
4.2 Rotor Response with the Standard Configuration

A series of engagement SHOLs were simulated for the H-46 with the standard control system settings. The engagement SHOLs for the horizontal airwake distribution are shown in Figure 4.2. For comparison to later results, both the minimum (downward) and maximum (upward) tip deflections are shown. As expected, the tip deflections are small for the horizontal airwake distribution. The minimum and maximum tip deflections are $-14.8\%R$ and $+11.1\%R$. The minimum tip deflection is relatively insensitive to the wind direction, but the maximum tip deflections are much larger for starboard winds than for port winds.

The engagement SHOLs for the linear airwake distribution are shown in Figure 4.3. With the vertical wind component present in the linear airwake distribution, tip deflections increase. The minimum and maximum deflections are $-18.6\%R$ and $+20.7\%R$. The tip deflection required to strike the H-46 tunnel is $-18\%R$. In this case,
the deflections were large enough to potentially result in a tunnel strike. Both the minimum and maximum tip deflections occurred in starboard winds.

![Figure 4.3: Engagement SHOLs for the Linear Airwake Distribution with the Standard Configuration](image)

The engagement SHOLs for the Simple Frigate Shape (SFS) airwake distributions are shown in Figure 4.4 through Figure 4.6. Like the baseline blade examined in Chapter 3, the largest blade flapping deflections occur at Spot #1 and reduce dramatically for the other two spots.

The engagement SHOLs for the SFS Spot #1 airwake distribution are shown in Figure 4.4. The tip deflections are largest for bow and 240° winds. The minimum tip deflection is −25.2%R for 50-knot bow winds. The maximum tip deflection is +30%R for 40-knot 240° winds. It is interesting that the tip deflections are larger for 40-knot winds than for 50-knot winds at a wind direction of 240°. In this case, the largest tip deflections in 40-knot winds are −19%R and +30%R, while in 50-knot winds they are −17.9%R and +25.3%R.
The time histories of the tip deflection and flap hinge angle for 40-knot 240º winds and 50-knot bow winds are shown in Figure 4.5. In 40-knot 240º winds, the blade experiences both large upward and downward deflections and impacts the droop and flap stops several times. The minimum deflection occurs at $t = 7.6$ seconds, when the rotor speed is just under 25%NR. This suggests that any control technique used must be active until the rotor reaches this speed. The blade behavior is quite different in 50-knot bow winds. It experiences only large downward deflections; the upward deflections are minimal. The minimum tip deflection occurs at $t = 4.5$ seconds. At this time, the rotor speed is just under 5%NR and the blade is located at an azimuth angle of 150º, or just entering the hangar wake. This suggests that the blade is forced downward by the large vertical flow components present there. In addition, note that the blade remains in contact with the droop stop during the entire period before the minimum deflection occurs.

**Figure 4.4**: Engagement SHOLs for the SFS Spot #1 Airwake Distribution with the Standard Configuration

<table>
<thead>
<tr>
<th>a) Minimum SHOL with the Standard Configuration</th>
<th>b) Maximum SHOL with the Standard Configuration</th>
</tr>
</thead>
</table>

The time histories of the tip deflection and flap hinge angle for 40-knot 240º winds and 50-knot bow winds are shown in Figure 4.5. In 40-knot 240º winds, the blade experiences both large upward and downward deflections and impacts the droop and flap stops several times. The minimum deflection occurs at $t = 7.6$ seconds, when the rotor speed is just under 25%NR. This suggests that any control technique used must be active until the rotor reaches this speed. The blade behavior is quite different in 50-knot bow winds. It experiences only large downward deflections; the upward deflections are minimal. The minimum tip deflection occurs at $t = 4.5$ seconds. At this time, the rotor speed is just under 5%NR and the blade is located at an azimuth angle of 150º, or just entering the hangar wake. This suggests that the blade is forced downward by the large vertical flow components present there. In addition, note that the blade remains in contact with the droop stop during the entire period before the minimum deflection occurs.
The engagement SHOLs for the SFS Spot #2 airwake distribution are shown in Figure 4.6. Like Spot #1, blade deflections are the largest from bow and 240° winds. The minimum and maximum tip deflections for Spot #2 are -16.2%R and +15.4%R. Note the dramatic decrease in the tip deflections a relatively short distance behind Spot #1. The maximum tip deflection is cut in half compared to Spot #1.

Figure 4.5: Time Histories of the Flap Tip Deflection and Flap Hinge Angle for the SFS Spot #1 Airwake Distribution

The engagement SHOLs for the SFS Spot #2 airwake distribution are shown in Figure 4.6. Like Spot #1, blade deflections are the largest from bow and 240° winds. The minimum and maximum tip deflections for Spot #2 are -16.2%R and +15.4%R. Note the dramatic decrease in the tip deflections a relatively short distance behind Spot #1. The maximum tip deflection is cut in half compared to Spot #1.

Figure 4.6: Engagement SHOLs for the SFS Spot #2 Airwake Distribution with the Standard Configuration
The engagement SHOLs for the SFS Spot #3 airwake distribution are shown in Figure 4.7. The minimum and maximum tip deflections are $-15.4\% R$ and $+13.8\% R$. Compared to Spot #2, there are slightly larger deflections for astern winds. In 50-knot astern winds, the maximum flap deflection is $+11.3\% R$ for Spot #2 and $+13.8\% R$ for Spot #3. Since Spot #3 is at the extreme aft end of the ship, this is likely due to the upflow component as the wind is forced upward over the ship stern.

![Figure 4.7: Engagement SHOLs for the SFS Spot #3 Airwake Distribution with the Standard Configuration](image)

**Figure 4.7:** Engagement SHOLs for the SFS Spot #3 Airwake Distribution with the Standard Configuration

4.3 Control of the Rotor Response with Collective Pitch Scheduling

In the following section, a means of reducing tip deflections requiring no physical change to the helicopter will be explored. All rotor blades airfoils have a relatively flat cross-sectional shape. Thus, they are much stiffer in the chordwise direction than in the normal direction. A simple way to increase the overall flapwise bending stiffness and
therefore to reduce flap-bending deflections is to increase the geometric pitch angle, $\theta_0$, of the blade section. In doing so, some of the larger chordwise stiffness is added to the smaller flapwise stiffness. As per the coordinate systems defined Ref. 43 and detailed in Appendix A, the effective structural stiffness of the blade in the flapwise direction is

$$E_{I_{\text{flap}}} = E_{I_{yy}} \cos^2 \theta_0 + E_{I_{zz}} \sin^2 \theta_0$$  \hspace{1cm} (4.2)

Where $E_{I_{yy}}$ is the normal stiffness and $E_{I_{zz}}$ is the chordwise stiffness.

The radial variations of the normal and chordwise stiffnesses for the H-46 are shown in Figure 4.8. The chordwise stiffness is on average 20 times larger than the normal stiffness in the blade airfoil section which begins at 25% $R$. The variation of the effective flapwise stiffness with the geometric pitch angle is also shown in Figure 4.8. The effective flapwise stiffness is shown at radial locations of 25% $R$, 50% $R$ and 75% $R$. By rotating the blade section 10º, the effective stiffness is increased by 10%, 58% and 84% over the value at zero pitch. By rotating the blade section 20º, the effective flapwise stiffness is increased by 40%, 221% and 327% over the value at zero pitch.

Figure 4.8: Structural Stiffness Distributions
A simple way to increase the geometric pitch angle, $\theta_b$, at any azimuth location is to increase the collective pitch setting, $\theta_{75}$, of the blade. Naturally, large amounts of collective pitch would be undesirable as the rotor nears high normal speed. In this method, a high collective pitch setting would only be used during the low-speed region of the engagement or disengagement. At a specified speed, the pilot would simply lower the collective pitch to its standard setting. This process is called collective pitch scheduling.

### 4.3.1 Feasibility Study

Using this method, the effective stiffness of the blade is increased appreciably. However, the aerodynamic forces will also be increased, tending to increase flap deflections. Initial studies were completed with a generic blade using a collective pitch setting of 10°. Results showed that the minimum blade deflections could be reduced, but that blade maximum deflections were often increased [92]. A new feasibility study is conducted with the H-46 blade to determine the amount of collective pitch required to stiffen the blade without overly increasing the aerodynamic forces.

A series of H-46 engagements were simulated. The two worst-case airwake distributions were investigated, the SFS Spot #1 airwakes for 40-knot 240° winds and 50-knot bow winds. Collective pitch settings from 4° to 20° were investigated. Because the minimum tip deflection can occur at rotor speeds up to 25%NR, the high collective pitch setting is kept until this speed is reached. The collective pitch is then lowered at 5° per second until the standard setting is reached. The results for the minimum and maximum tip deflections are shown in *Figure 4.9*. For the standard configuration, the minimum and
maximum tip deflections were -19%R and +30%R for 40-knot 240° winds and -25.2%R and +11.3%R for 50-knot bow winds. For a collective pitch setting of 10°, the minimum and maximum tip deflections were -17.9%R and +33.8%R for 40-knot 240° winds and -16.6%R and +15.2%R for 50-knot bow winds. For a collective pitch setting of 20°, the minimum and maximum tip deflections were –14.2%R and +33.8%R for 40-knot 240° winds and -11.9%R and +31.4%R for 50-knot bow winds. Based on this feasibility study, the 10° collective pitch case was used for all further studies. The 20° collective pitch case was more successful at reducing the minimum deflection for 50-knot bow winds, but increased the maximum deflection by too large an amount.

![Figure 4.9: Collective Pitch Scheduling Feasibility](image)

The time histories of collective pitch setting and the flap tip deflection using a collective pitch setting of 10° are shown in Figure 4.10. For 40-knot 240° winds, the first downward tip deflection is reduced by 67%. However, the second downward peak was not affected. The aerodynamic forces cause additional blade deflections in the $t = 7$ to 8 second range of the engagement. For 50-knot bow winds, the blade has been stiffened enough to reduce the minimum tip deflection by 66%.
4.3.2 Engagement SHOLs with Collective Pitch Scheduling

With the feasibility study completed, a more in-depth investigation into collective pitch scheduling is completed in multiple airwake types. Because a high collective pitch setting is being used and the aerodynamic forces are increased, both the minimum (downward) and maximum (upward) tip deflections are plotted.

*Figure 4.10: Time Histories of Collective Pitch Setting and Flap Tip Deflection*
The engagement SHOLs for the horizontal airwake distribution using collective pitch scheduling are shown in Figure 4.11. The tip deflections for the standard configuration reach only \(-14.8\%R\) and \(+11.1\%R\). The first requirement of any control technique is that the blade response not be excited for an originally benign situation. Unfortunately, this is not the case. Both the minimum and maximum tip deflections have been increased to \(-16.6\%R\) and \(+28.1\%R\). Clearly, this is an undesirable situation.

*Figure 4.11: Engagement SHOLs for the Horizontal Airwake Distribution with Collective Pitch Scheduling*
The engagement SHOLs for the linear airwake distribution using collective pitch scheduling are shown in *Figure 4.12*. The tip deflections for the standard H-46 configuration for the linear airwake distribution are $-18.6\% R$ and $+20.7\% R$. Like the horizontal airwake distribution, the blade deflections have been increased in the linear airwake distribution, to $-19.8\% R$ and $+32.5\% R$.

*Figure 4.12: Engagement SHOLs for the Linear Airwake Distribution with Collective Pitch Scheduling*
The engagement SHOLs for the SFS Spot #1 airwake distribution using collective pitch scheduling are shown in Figure 4.13. The tip deflections reach \(-25.2\%R\) and \(+30\%R\) for the standard H-46 configuration. For 50-knot bow winds, the minimum blade deflection was reduced from \(-25.2\%R\) to \(-16.6\%R\). Unfortunately, the minimum blade deflection was increased for 40-knot bow winds from \(-17.2\%R\) to \(-19.7\%R\). Furthermore, the minimum tip deflection was increased for 50-knot port winds from \(-22.4\%R\) to

![Figure 4.13: Engagement SHOLs for the SFS Spot #1 Airwake Distribution with Collective Pitch Scheduling](image-url)
-27%R. The maximum tip deflections were also increased using collective pitch scheduling, from +30%R to +36.9%R.

The engagement SHOLs for the SFS Spot #2 airwake distribution using collective pitch scheduling are shown in Figure 4.14. The tip deflections for the standard H-46 configuration reach -16.2%R and +15.4%R in 50-knot 240° winds. With the high collective pitch, the minimum tip deflection was increased from -16.2%R to -17.6%R and the maximum tip deflection was increased from +15.4%R to +29%R.
The engagement SHOLs for the SFS Spot #3 airwake distribution using collective pitch scheduling are shown in Figure 4.15. The tip deflections reach $-15.4\%R$ and $+13.8\%R$ for the standard H-46 configuration. With collective pitch scheduling, the minimum tip deflection was increased from $-15.4\%R$ to $-16.1\%R$ and the maximum tip deflection was increased from $+13.8\%R$ to $+26.6\%R$.

Figure 4.15: Engagement SHOLs for the SFS Spot #3 Airwake Distribution with Collective Pitch Scheduling
The results of this study indicate that the use of a high collective pitch setting is not a feasible way to reduce blade flap deflections. Although the blade is stiffened appreciably, the aerodynamic forces are also increased. Only a few cases were found which resulted in decreased blade deflections. The blade deflections were more often increased, especially in airwake distributions that exhibited mild flapping behavior with the standard configuration.

4.4 Control of the Rotor Response with a Flap Damper

The second passive control technique examines the effect of a discrete flap damper on the rotor response during engagements and disengagements. Such a flap damper is intended to dissipate excessive flap motions much like a lag damper dissipates excessive lag motions. This is not an entirely new idea. A special flap damper device was used on the HUP-1 to HUP-4 series tandem naval helicopters as early as the 1950’s to reduce excessive blade flapping during low rotor speed operations [93]. A picture of the HUP-2 and a schematic of the flap damper are shown in Figure 4.16. Each rotor blade had its own flap damper. Each flap damper was attached to a point just outboard of the flap hinge at one end and to a mast above the rotor hub at the other end. The flap damper was designed to become inactive above a specified rotor speed, by centrifugal force pulling on a spring/counterweight combination and rotating a series of mechanical linkages. The rotation of the mechanical linkages effectively made the flap damper inactive.
4.4.1 Feasibility Study

A feasibility study using this technique was briefly investigated in earlier flap-torsion simulations for the H-46 Sea Knight [46-47]. Unfortunately, only one WOD condition and one airwake distribution was investigated. Results showed that a flap damper 4 times the strength of the lag damper could reduce blade deflections by as much as 40%. However, it was deemed necessary to reevaluate the results of earlier flap damping simulations due to the recent improvements in the rotor analysis.

A series of rotor engagements were simulated for the H-46 using this control technique. Again, the two worst-case airwake distributions were investigated, the SFS Spot #1 airwakes for 40-knot 240° winds and 50-knot bow winds. The strength of the flap damper, $C_\beta$, was scaled in multiples of the approximate H-46 lag damper strength, $C_\zeta$, of 3500 ft-lb/(rad/s). Flap damper strengths ranging from 0 to 5 times the lag damper
strength were investigated. The flap damper was assumed inactive above 50%NR. The minimum and maximum tip deflections during each rotor engagement are shown in Figure 4.17. The tip deflections were barely reduced regardless of the strength of the flap damper. This is because the current flap stop, $\beta_{FS}$, and droop stop, $\beta_{DS}$, settings for the H-46 rotor system are $+1^\circ$ and $-1^\circ$, respectively. In this configuration, the H-46 blade can only rotate a total of $2^\circ$ between the flap and droop stops. With such a small stroke, the flap damper simply cannot dissipate much energy no matter how strong it is.

In light of the previous result, another series of rotor engagements were simulated with varying flap stop angles ranging from $+1^\circ$ to $+10^\circ$. Raising the flap stop allows the damper to have a larger stroke and dissipate more energy. The droop stop was kept at the current setting of $-1^\circ$ because lowering the droop stop would tend to result in additional downward rotation of the blade and increase the chances of a tunnel strike. The minimum and maximum flap tip deflections versus the flap stop setting during each rotor engagement are shown in Figure 4.18. Flap damper strengths of 3, 4 and 5 times the lag
damper strength are shown. For the 40-knot 240° wind case, in the current configuration the minimum and maximum flap tip deflections are $-19\%R$ and $+30\%R$. For a flap damper 3 times the strength of the lag damper, the deflections are increased as the flap stop setting was increased. In this situation, the flap damper was not able to dissipate the additional potential energy developed as the flap stop setting was raised. For flap dampers 4 and 5 times the strength of the lag damper, the minimum deflections are consistently reduced. For a flap damper 4 times the strength of the lag damper and a flap stop setting of 6°, the minimum and maximum deflections are $-15.3\%R$ and $+34.8\%R$ for 40-knot 240° winds. For the 50-knot bow wind case, in the current configuration the minimum and maximum flap tip deflections are $-25.2\%R$ and $+11.3\%R$. However, neither the minimum nor maximum tip deflection was changed by the addition of flap damping or by the raising of the flap stop angle.

![Figure 4.18](image-url)

*Figure 4.18: Minimum and Maximum Flap Tip Deflections for Varying Flap Stop Angles*
In choosing the correct flap damper/flap stop combination, it is desirable to use the minimum flap damper strength and the smallest flap stop setting that have a substantial effect of the tip deflections. Obviously, the flap damper 3 times the strength of the lag damper was not strong enough. With minimal differences in the blade response between flap damper strengths of 4 and 5 times the lag damper strength, use of the lower strength flap damper is preferable. Flap stop settings higher than 6º resulted in increased maximum tip deflections, while settings lower than 6º had only a minor effect on the minimum tip deflection. Therefore, the combination of the flap damper 4 times the strength of the lag damper and the 6º flap stop setting was chosen for the rest of the simulations. It agrees well with the results from the original flap damping feasibility analysis completed in Refs. 46-47.

The time histories of the flap tip deflection and flap hinge angle for this flap damper/flap stop combination are shown in Figure 4.19. For 40-knot bow winds, both the first and second minimum flap deflections are reduced. Note that for 50-knot bow winds, the minimum tip deflection occurs before the blade ever lifts off the droop stop. Therefore, the flap damping technique will be completely ineffective in reducing the minimum tip deflection in this case.
4.4.2 Engagement SHOLs with a Flap Damper

A more in-depth investigation into the performance of the flap damper in multiple airwake types is completed in this section. Engagement SHOLs were generated with the
flap damper and compared to the standard configuration. Because the flap stop setting has been raised and higher upward flap deflections are a possibility, both the minimum and maximum flap tip deflections are plotted.

The engagement SHOLs for the horizontal airwake distribution are shown in Figure 4.20. Since the tip deflections for the horizontal airwake distribution for the current H-46 configuration were already small, the flap damper had very little effect.

Figure 4.20: Engagement SHOLs for the Horizontal Airwake Distribution with a Flap Damper
The engagement SHOLs for the linear airwake distribution are shown in Figure 4.21. The flap damper substantially reduced the minimum tip deflections. Without the flap damper, the largest downward deflection was $-18.6\%R$. With the flap damper, it has been reduced to $-12.6\%R$. Because the flap stop setting was raised to give the flap damper a higher stroke, the maximum deflections are increased. Without the flap damper, the maximum deflection was $+20.7\%R$. With the flap damper, it has been

\[\begin{array}{c}
\text{a) Minimum SHOL with the Standard Configuration} \\
\text{b) Maximum SHOL with the Standard Configuration} \\
\text{c) Minimum SHOL with a Flap Damper} \\
\text{d) Maximum SHOL with a Flap Damper}
\end{array}\]

*Figure 4.21: Engagement SHOLs for the Linear Airwake Distribution with a Flap Damper*
increased to +29.3%R. A flap deflection of this magnitude, whether upward or downward, is probably undesirable in realistic naval operations because of the possibility of excessive bending loads in the blades.

The engagement SHOLs for the SFS Spot #1 airwake distribution are shown in Figure 4.22. The minimum tip deflections were successfully reduced for near abeam wind conditions. For 50-knot port winds, they were reduced by 33% from -22.4%R to
-14.8% \( R \). The maximum tip deflections were increased with the flap damper. For 40-knot 240° winds, the maximum flap deflection increased 16% from +30% \( R \) to +34.8% \( R \). As explained in the feasibility study, the minimum tip deflections were not reduced for bow winds because the minimum tip deflection occurs before the blade lifts off the droop stop. In this situation, the flap damper can have no effect. Unfortunately, the maximum tip deflections are also increased in each of these situations. To better visualize the results, line plots of the variation of the tip deflection with azimuth angle are shown in Figure 4.23. Tunnel strikes are successfully avoided for 40-knot 240° winds and for 50-knot 240° and port winds, although tunnel strikes are still predicted for 50-knot 300° and bow winds.

\[
\begin{align*}
\text{Max } w_{\text{tip}} \text{ (%R)} & \quad \text{Min } w_{\text{tip}} \text{ (%R)} \\
\text{H-46 Tunnel} & \quad \text{H-46 Tunnel}
\end{align*}
\]

\[
\begin{align*}
\text{Stern Winds} & \quad \text{Port Winds} \quad \text{Bow Winds} \\
\text{Stern Winds} & \quad \text{Port Winds} \quad \text{Bow Winds}
\end{align*}
\]

a) 40-knot Winds

b) 50-knot Winds

Figure 4.23: Minimum and Maximum Tip Deflections for the SFS Spot #1 Airwake Distribution with a Flap Damper
The engagement SHOLs for the SFS Spot #2 airwake distribution are shown in Figure 4.24. In general, flap damping was more successful at Spot #2 than at Spot #1. The minimum tip deflection was decreased for the 240° wind case. At a wind speed of 50 knots, the flap deflection was $-16.2\% R$ in the standard configuration and $-10.7\% R$ with the flap damper. In addition, the maximum flap deflections were not increased for any
case. Line plots of the variation of the tip deflection with azimuth angle are shown in Figure 4.25. The damper has the largest effect on the minimum tip deflection for 40 and 50-knot 240° winds.

![Diagram showing minimum and maximum tip deflections for 40-knot and 50-knot winds with and without the flap damper.]

Figure 4.25: Minimum and Maximum Tip Deflections for the SFS Spot #2 Airwake Distribution with a Flap Damper

The engagement SHOLs for the SFS Spot #3 airwake distribution are shown in Figure 4.26. The minimum tip deflections were not reduced for 40 or 50-knot winds from port to bow using the flap damper. The minimum tip deflection remained more than -15%R for the 330° wind case. However, the minimum tip deflections were successfully reduced for 240° to astern winds. In 50-knot astern winds, the minimum flap deflection was reduced from -12.1%R in the standard configuration to -6.5%R with the flap damper.
Line plots of the variation of the tip deflection with azimuth angle are shown in Figure 4.27. The damper has the largest effect on the minimum tip deflection for stern to 240° winds and has no effect on the minimum tip deflection for port to bow winds.
The use of a flap damper to reduce blade flapping during engagement operations was only partially successful. In the best cases, the flap damper reduced the minimum flap deflections by as much as 50%. However, the minimum flap deflections were not reduced in situations where the minimum tip deflection occurred before the blade ever lifted off the droop stop. In addition, the flap stop setting had to be increased to increase the stroke of the flap damper. Therefore, the maximum flap deflections were often increased as much as the minimum flap deflections were reduced. Although the chances of a tunnel strike were reduced overall, large upward blade flapping deflections are probably as undesirable as large downward flap deflections in actual naval operations.
4.5 Control of the Rotor Response with Extendable/Retractable Spoilers

In the previous sections, the reduction of excessive flapping by collective pitch scheduling or by the addition of flap damping was investigated. However, neither solution addressed the true cause of the blade sailing phenomenon. The ideal method to reduce excessive aeroelastic flapping would be to simply to reduce the excessive lift generated during low speed operations. Spoilers are well-known devices used to reduce lift. In this section, the use of extendable/retractable spoilers to reduce the blade flap deflections will be examined. Like the other passive control methods, the spoilers would only be used during the rotor speed region less than 25%NR. They are assumed to retract into the blade section under the action of a passive, centrifugal force mechanism above the specified rotor speed and in normal operating conditions. Flap stops are already designed to retract with a similar mechanism. Leading edge spoilers are examined because they are most effective at reducing the lift generated by the blade section. The spoilers are also assumed to be in a gated, or rake, configuration. In such a configuration, the hinge moments and buffeting are reduced compared to full-span spoilers without much loss in performance. The aerodynamic properties of this type of spoiler were discussed in detail in Chapter 2. A schematic of the spoilers in the extended and retracted configurations is shown in Figure 4.28. The spoilers are contained within the blade section only near the tip of the blade. Because the blade D-spar near the tip of the blade does not carry large centrifugal loads, it could be modified to contain the spoilers.
4.5.1 Feasibility Study

A major concern of the current investigation is the percentage of the blade radius that the spoilers must cover to substantially reduce blade tip deflections. A series of H-46 rotor engagements were simulated with spoilers covering the outer 10%\(R\), 15%\(R\), 25%\(R\), 50%\(R\), and 75%\(R\). The spoilers were assumed to retract into the blade section at 25%NR. Again, the chosen airwake distribution was the Spot #1 SFS airwake distribution with 40-knot 240º and 50-knot bow winds. The minimum and maximum tip deflections for each case are shown in Figure 4.29. For 40-knot 240º winds, the standard configuration had tip deflections of -19%\(R\) and +30%\(R\) while for 50-knot bow winds the standard configuration had tip deflections of -25.2%\(R\) and +11.3%\(R\). For each airwake distribution, spoilers covering the outer 10%\(R\) were just sufficient to avoid a tunnel strike. Spoilers covering the outer 15%\(R\) reduced the tip deflections to -15%\(R\) and +23%\(R\) for 40-knot 240º winds and +5.3%\(R\) and -16.2%\(R\) for 50-knot bow winds. Spoilers covering more than 25%\(R\) had a minimal impact on the tip deflections. Therefore, all further studies utilize spoilers covering the outer 15%\(R\). These results agree well with earlier feasibility studies completed in Ref. 94.
The lift at the 95%R station and tip deflection time histories are shown in Figure 4.30. The spoilers are assumed to retract into the blade section at 25%NR, which occurs at \( t = 7.8 \) seconds. Note that during the period of the engagement that the spoilers are deployed, the peak lift forces are reduced. The largest minimum tip deflections were reduced by as much as 62% so that tunnel strikes were avoided for both airwake distributions. Unlike the high collective pitch and flap damping control techniques, both the maximum and minimum tip deflections were successfully reduced. For 40-knot 240° winds, the minimum tip deflection at \( t = 5.9 \) seconds has been reduced from -17.7%\( R \) to -10.9%\( R \), or by 38%. The minimum tip deflection at \( t = 7.6 \) seconds has been reduced from -19%\( R \) to -15%\( R \), or by 21%. For 50-knot bow winds, the minimum tip deflection at \( t = 5.9 \) seconds has been reduced from -25.2%\( R \) to -16.2%\( R \), or by 36%. The maximum negative tip deflection at \( t = 7.6 \) seconds has been reduced from -18.9%\( R \) to -7.1%\( R \), or by 62%. In each case, the spoilers reduced the flapping deflections enough to prevent a tunnel strike from occurring.

*Figure 4.29: Extendable/Retractable Spoiler Feasibility*
The lag hinge angle and required torque time histories for the standard H-46 configuration and for the H-46 blade with spoilers extending over the outer 15% $R$ are shown in Figure 4.31. For 40-knot 240° winds, note the effect of including the aerodynamic forces in the blade initial conditions. In this case, the engagement starts with the blade in contact with the lag stop, set at $-12^\circ$. In each case, Coriolis forces cause the blade to repeatedly contact the lag stop resulting in the sharp peaks in the torque. For

Figure 4.30: Lift at 95%$R$ and Flap Tip Deflection Time Histories

The lag hinge angle and required torque time histories for the standard H-46 configuration and for the H-46 blade with spoilers extending over the outer 15%$R$ are shown in Figure 4.31. For 40-knot 240° winds, note the effect of including the aerodynamic forces in the blade initial conditions. In this case, the engagement starts with the blade in contact with the lag stop, set at $-12^\circ$. In each case, Coriolis forces cause the blade to repeatedly contact the lag stop resulting in the sharp peaks in the torque. For
50-knot bow winds, the blade even strikes the lead stop, set at +12°. The required torque then becomes negative, meaning the blade is accelerating the rotor. In each airwake distribution, the peak torque during the engagement is due to the lag stop impacts, not the additional aerodynamic drag of the spoilers. The smooth line in each required torque time history labeled “inertial torque” represents the torque required to simply accelerate the blade inertia on the specified rotor speed profile without any aerodynamics.

Figure 4.31: Lag Hinge Angle and Required Torque Time Histories

a) Lag Hinge Angle in 40-knot 240° Winds
b) Lag Hinge Angle in 50-knot Bow Winds
c) Required Torque in 40-knot 240° Winds
  d) Required Torque in 50-knot Bow Winds
4.5.2 Engagement SHOLs with Extendable/Retractable Spoilers

The minimum and maximum engagement SHOLs with extendable/retractable spoilers for the horizontal airwake distribution are shown in Figure 4.32. For the standard configuration, the minimum and maximum tip deflections are $-14.8\% R$ and $+11.1\% R$. For the configuration with spoilers, the tip deflections are $-12.8\% R$ and $+10.9\% R$. Without the presence of vertical winds, the effect of the spoilers is minimal.

*Figure 4.32: Engagement SHOLs for the Horizontal Airwake Distribution with Extendable/Retractable Spoilers*
The minimum and maximum engagement SHOLs using spoilers for the linear airwake distribution are shown in Figure 4.33. For the standard configuration, the minimum and maximum tip deflections are reduced to -18.6%R and +20.7%R. For the configuration with spoilers, the tip deflections are -15.6%R and +17.4%R, which prevents a tunnel strike in all conditions.

Figure 4.33: Engagement SHOLs for the Linear Airwake Distribution with Extendable/Retractable Spoilers
The minimum and maximum engagement SHOLs using spoilers for the SFS Spot #1 airwake distribution are shown in Figure 4.34. For the standard configuration, the minimum and maximum tip deflections are $-25.2\%R$ and $+30\%R$. For the configuration with spoilers, the tip deflections are reduced to $-18.5\%R$ and $+23\%R$. Line plots of the variation of the tip deflection with azimuth angle are shown in Figure 4.35. Tunnel
strikes are successfully avoided for 40-knot 240° winds and for 50-knot bow and 240° winds, while tunnel strikes are nearly avoided 50-knot port and 300° winds. Also note the maximum upward deflections are reduced for the worst cases.

![Graphical representation of tip deflections for different wind conditions](image)

**Figure 4.35**: Minimum and Maximum Tip Deflections for the SFS Spot #1 Airwake Distribution with Extendable/Retractable Spoilers

The minimum and maximum engagement SHOLs using spoilers for the SFS Spot #2 airwake distribution are shown in **Figure 4.36**. For the standard configuration, the maximum tip deflections are -16.2%R and +15.4%R. For the configuration with spoilers, the tip deflections are -14%R and +10.4%R.
Line plots of the variation of the tip deflection with azimuth angle for Spot #2 are shown in Figure 4.37.
The minimum and maximum engagement SHOLs with spoilers for the SFS Spot #3 airwake distribution are shown in Figure 4.38. For the standard configuration, the minimum and maximum tip deflections are \(-15.4\%R\) and \(+13.8\%R\). For the configuration with spoilers, the tip deflections have been reduced to \(-12.2\%R\) and \(+10.5\%R\).
Line plots of the variation of the tip deflection with azimuth angle are shown in Figure 4.39. Both the minimum and maximum tip deflections have been reduced for each of the cases.
Leading edge spoilers covering the outer 15% $R$ and retracting into the blade section at 25%NR were successful in reducing both the minimum and maximum tip deflections during engagement operations. The minimum tip deflections were reduced by as much as 62%, avoiding tunnel strikes in all but the worst airwake distributions. Because impacts between the blade and the lead and lag stops cause large spikes in the rotor torque, the increase in rotor torque from the aerodynamic drag of the spoilers was small.

4.6 Summary

Three different passive control schemes intended to reduce the blade aeroelastic flapping deflections were investigated in this chapter. The first was the use of collective
pitch scheduling during the period of the engagement in which the rotor speed was less than 25% NR. Collective pitch scheduling increases the effective flapwise stiffness of the blade, which would tend to reduce blade deflections. However, it was found that in most situations the aerodynamic forces were increased more than the blade was stiffened and larger blade deflections resulted. The second method was the use of a discrete flap damper that would also only be active during the low rotor speed region of the engagement. Such a flap damper would dissipate excessive motion in the flapping direction as a lag damper dissipates excessive motion in the lag direction. It was determined that because the standard H-46 configuration only allows 2° of motion between the flap and droop stops the flap stop setting had to be raised for the flap damper to have an appreciable effect. Minimum blade deflections were successfully reduced in this manner; however, because the flap stop setting was raised the maximum blade deflections were often increased. The third method explored the effect of extendable/retractable leading edge spoilers on the blade. It was found that spoilers covering only the outer 15% R substantially reduced both the blade minimum and maximum flapping deflections. It was also found that the torque required did not increase appreciably because the maximum rotor torque was dominated by loads incurred when the blade struck the lead or lag stops.

The numerical results of all three control techniques are summarized in Table 4.1. The collective pitch scheduling method only reduced tip deflections for a few isolated cases; in general, the blade deflections were increased. The flap damper control method was more successful at reducing the minimum tip deflections for all airwake distributions.
except at SFS Spot #1. The control method using spoilers achieved the best results; both minimum and maximum blade deflections were reduced for all airwake distributions.

### Table 4.1: Passive Control Methods Summary
Minimum and Maximum Tip Deflections for $V_{WOD} = 50$ knots

<table>
<thead>
<tr>
<th></th>
<th>Uncontrolled</th>
<th>High Collective</th>
<th>Flap Damper</th>
<th>Spoilers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$w_{tip}$ (%R)</td>
<td>$w_{tip}$ (%R)</td>
<td>$w_{tip}$ (%R)</td>
<td>$w_{tip}$ (%R)</td>
</tr>
<tr>
<td>Horizontal</td>
<td>-14.8</td>
<td>+11.1</td>
<td>-16.6</td>
<td>+28.2</td>
</tr>
<tr>
<td>Linear</td>
<td>-18.6</td>
<td>+20.7</td>
<td>-19.8</td>
<td>+32.5</td>
</tr>
<tr>
<td>SFS Spot #1</td>
<td>-25.2</td>
<td>+30.0</td>
<td>-27.0</td>
<td>+36.6</td>
</tr>
<tr>
<td>SFS Spot #2</td>
<td>-16.2</td>
<td>+15.4</td>
<td>-17.6</td>
<td>+29.0</td>
</tr>
<tr>
<td>SFS Spot #3</td>
<td>-15.4</td>
<td>+13.8</td>
<td>-16.1</td>
<td>+25.6</td>
</tr>
</tbody>
</table>
Chapter 5

FEEDBACK CONTROL OF GIMBALED ROTORS USING SWASHPLATE ACTUATION

The previous chapter examined the control of the aeroelastic flapping response of the rotor system during engagement and disengagement operations through passive methods. The two more successful methods required that physical changes be made to the rotor hub or the rotor blades. Although each method was successful at reducing the rotor response, it is undesirable to make physical changes to the rotor system because they could be expensive or maintenance intensive. Instead of controlling the rotor flapping response using passive methods, this chapter will examine controlling the rotor response using an active control technique.

Most rotorcraft utilize mechanical actuators to control the inputs to the rotor system. Future rotorcraft may also have on-board devices that can measure the motion of the rotor blades. Such systems could be combined to reduce the flapping response of the rotor. The motion of the blades could be controlled through feedback from the measured blade motions to the control system inputs. The V-22 Osprey already utilizes such a control system to limit the flapping motion of the blades during forward flight conditions.

In this chapter, a feasibility study is conducted to determine the effectiveness of using an active control system to reduce blade flapping during rotor engagements of a gimballed rotor system. Naturally, this control system would only be used during the critical rotor speed region and be shut off during normal operating conditions. Because
this is a feasibility study, several simplifying assumptions are made. Unlike the research in the earlier chapters, a rigid blade is assumed and a very simple linear quasi-steady aerodynamic model is utilized. The research presented in this chapter is an extended version of the research presented in Ref. 95.

5.1 Derivation of the Equations of Motion

In this section, the equations of motion for a rigid, gimballed rotor are derived. Unlike most articulated rotor blades, gimballed rotor blades are relatively short and stiff. In addition, the blades are rigidly attached to each other through the rotor hub and pivot as a rigid body about a universal joint. The majority of the rotor flapping motion is due to the rigid body motion of the entire rotor system about the hub, not the elastic motion of the blades. Therefore, a rigid blade assumption is much more justified for a gimballed rotor than for an articulated rotor.

The first step in deriving the equations of motion for the rotor is to consider the motion of a single blade. A free body diagram of a single blade is shown in Figure 5.1. In this figure, AF is the aerodynamic force, CF is the centrifugal force, IF is the inertial force, and M_i is the mass of the blade.
force, \( GF \) is the gravitational force and \( M_i \) is the moment about the rotor hub. If the individual blade flap angle \( \beta_i \) is small, the moment about the hub is

\[
M_i = -I_b \dot{\beta}_i - I_b \Omega^2 \beta_i + \int_0^R F_i r dr - S_b g \quad (\text{5.1})
\]

Where \( S_b \) and \( I_b \) are the first and second mass moments of inertia of the blade. The individual blade flap angle can be written in terms of the fixed-frame flap angles as

\[
\beta_i = \beta_{i_c} + \beta_{i_c} \cos \psi_i + \beta_{i_s} \sin \psi_i \quad (\text{5.2})
\]

The fixed-frame degrees of freedom, \( \beta_{i_c} \) and \( \beta_{i_s} \), are the tilt of the gimbaled rotor in the longitudinal and lateral directions and are shown in Figure 5.2.

Substituting Equation 5.2 into 5.1 yields

\[
M_i = -I_b \left[ \Omega^2 \beta_i + \left( \dot{\beta}_{i_c} + 2\Omega \beta_{i_c} + \dot{\Omega} \beta_{i_c} \right) \cos \psi_i + \left( \dot{\beta}_{i_s} - 2\Omega \beta_{i_s} + \dot{\Omega} \beta_{i_s} \right) \sin \psi_i \right] + \int_0^R F_i r dr - S_b g \quad (\text{5.3})
\]

The equation of motion for the entire system can be written by considering a top view of the rotor as shown in Figure 5.3.
Summing the individual blade moments about the rotor hub in the \( x \) and \( y \) axes yields

\[
\sum_{i=1}^{N_b} M_i \cos \psi_i + K_\beta \dot{\beta}_i = 0 \\
\sum_{i=1}^{N_b} M_i \sin \psi_i - K_\beta \dot{\beta}_i = 0
\]  \hspace{1cm} (5.4)

Where \( K_\beta \) is the stiffness of the hub springs. By substituting Eqn. 5.3 into Eqn. 5.4, the \( N_b \) individual blade equations of motion are transferred into 2 equations of motion for the fixed frame degrees of freedom \( \beta_{ic} \) and \( \beta_{is} \). The following relations are used to simplify the result

\[
\frac{2}{N_b} \sum_{i=1}^{N_b} \cos \psi_i = 0 \quad \frac{2}{N_b} \sum_{i=1}^{N_b} \sin \psi_i = 0 \\
\frac{2}{N_b} \sum_{i=1}^{N_b} \cos^2 \psi_i = 1 \quad \frac{2}{N_b} \sum_{i=1}^{N_b} \sin^2 \psi_i = 1 \\
\frac{2}{N_b} \sum_{i=1}^{N_b} \cos \psi_i \sin \psi_i = 0
\]  \hspace{1cm} (5.5)

Premultiplication of all remaining terms by \( 2/N_b \) and nondimensionalizing by \( I_b \Omega_0^2 \) yields

\[
M_s \left\{ \ddot{\beta}_{ic} \right\} + C_s \left\{ \dot{\beta}_{ic} \right\} + K_s \left\{ \beta_{ic} \right\} = \left\{ M_{ic} \right\} \\
M_s \left\{ \ddot{\beta}_{is} \right\} + C_s \left\{ \dot{\beta}_{is} \right\} + K_s \left\{ \beta_{is} \right\} = \left\{ M_{is} \right\}
\]  \hspace{1cm} (5.6)

The structural mass, damping and stiffness matrices \( M_s, C_s \) and \( K_s \) are given by

Figure 5.3: Top View of Gimballed Rotor
Where the nondimensional flap frequency of the rotor is defined as
\[
\nu^2 = \frac{K_{\beta}}{N_b I_b \Omega_0^2}
\]  

Note in this expression that the moment of inertia \(I_b\) is for a single blade while the gimbal stiffness \(K_{\beta}\) is for the entire rotor. The nondimensional moments about the rotor hub due to aerodynamic forces, \(M_{A_i}\) and \(M_{A_s}\), are derived using a simple attached flow aerodynamic model detailed in Appendix D. Using this approach, the nondimensional moments can be written as
\[
\begin{bmatrix}
M_{A_i} \\
M_{A_s}
\end{bmatrix} = -C_A \begin{bmatrix}
\beta_{i_s} \\
\beta_{s_s}
\end{bmatrix} - K_A \begin{bmatrix}
\beta_{i_s} \\
\beta_{s_s}
\end{bmatrix} + F_A
\]  

The final equations of motion are then
\[
M \begin{bmatrix}
\dot{\beta}_{i_s} \\
\dot{\beta}_{s_s}
\end{bmatrix} + C \begin{bmatrix}
\dot{\beta}_{i_s} \\
\dot{\beta}_{s_s}
\end{bmatrix} + K \begin{bmatrix}
\beta_{i_s} \\
\beta_{s_s}
\end{bmatrix} = F
\]  

Where the global mass, damping and stiffness matrices \(M\), \(C\) and \(K\) and force vector \(F\) are given by
\[
M = M_s, \quad C = C_s + C_A, \quad K = K_s + K_A, \quad F = F_A
\]  

Gimballed rotors typically have a mechanical restraint that prevents excessive gimbal tilt angles. Because in this analysis the rotor is assumed rigid, this mechanical restraint is not enforced.
5.1.1 Optimal Control Theory

Once the equations of motion have been written, they are transferred into state-space form. Because the rotor acceleration, speed, and azimuth angles are non-periodic functions of time, the resulting state-space equations are linear time-variant (LTV)

\[
\dot{x} = A(t)x + B(t)u + d(t)
\]

\[
y = Cx
\]

(5.12)

The state vector, \(x\), input vector, \(u\), and output vector, \(y\), are defined as

\[
x = \begin{bmatrix}
\beta_{lc} \\
\beta_{ls} \\
\beta_{lc} \\
\beta_{ls}
\end{bmatrix}, \quad u = \begin{bmatrix}
\theta_{75} \\
\theta_{lc} \\
\theta_{ls}
\end{bmatrix}, \quad y = \begin{bmatrix}
\beta_{lc} \\
\beta_{ls}
\end{bmatrix}
\]

(5.13)

The matrices \(A\), \(B\), \(C\) and the vector \(d\) are derived through the appropriate manipulations of Eqn. 5.10. Note that there is an additional term \(d\) in the state equation. Typically, this term is called the “disturbance” and is often used to represent external noise. That is not the case in the current analysis. Here the disturbance term is a known quantity and arises from aerodynamic forces generated by anything other than the pilot control inputs. It is composed of contributions from the blade twist, blade precone and the vertical ship airwake component \(\vec{V}_{z}\).

Because the equations of motion are time-variant, traditional frequency domain control approaches such as pole placement are ineffective. Instead, a time-domain Linear Quadratic Regulator (LQR) control method is used [96]. Using this method, a quadratic performance index, \(J\), is defined as
\[
J = \frac{1}{2} x^T(t_f) S(t_f) x(t_f) + \frac{1}{2} \int_0^{t_f} (x^T Q x + u^T R u) dt
\]  \hspace{1cm} (5.14)

Where \( S(t_f), Q \) and \( R \) are weighting matrices. It is assumed the matrices \( S \) and \( Q \) are symmetric and positive semi-definite and \( R \) is symmetric and positive definite.

Minimization of the final state, \( x(t_f) \), is achieved by setting \( S(t_f) \) large. Minimization of the time history of the rotor response is achieved by setting \( Q \) larger than \( R \), while minimization of the time history of the control inputs is achieved by setting \( R \) larger than \( Q \). The matrices \( S(t_f), Q \) and \( R \) are user-specified.

Assuming the final state \( x(t_f) \) is free, the Matrix Ricatti Equations (MRE) can then be used to solve for the time variation of \( S \)

\[
-\dot{S} = A^T S + S A - S B R^{-1} B^T S + Q
\]  \hspace{1cm} (5.15)

Where \( S(t_f) \) is given. The MRE are then solved backwards in time for \( S(t) \). The Kalman gain matrix \( K \) is then given by

\[
K = R^{-1} B^T S
\]  \hspace{1cm} (5.16)

Since the weighting matrix \( S \) varies in time, so does the Kalman gain matrix. Due to the disturbance term \( d \) in the state equation, an additional time-varying bias term exists in the MRE. The time variation of the bias term, \( v \), is given by

\[
-\dot{v} = (A - BK)^T v + S d
\]  \hspace{1cm} (5.17)

Where \( v(t_f) \) is zero. This equation is also solved backwards in time for \( v(t) \). The closed-loop optimal control vector is then given by

\[
u = -K x - R^{-1} B^T v
\]  \hspace{1cm} (5.18)
The second term in the equation is due to the disturbance term. The response of the closed-loop system is then given by

\[ \dot{x} = (A - BK)x - BR^{-1}B^T v + d \]

(5.19)

With a given initial condition \( x(t_0) \), this equation can be integrated forward in time for the optimal state trajectory.

### 5.1.2 Control System Limits

Mechanical devices called swashplate actuators are often used to control the movements of the swashplate, and hence the control system inputs. For many rotor systems, several linear hydraulic actuators are used to move the swashplate. The relation between the linear movement of the actuators and the control system inputs is given by

\[ u = T_{ac}x_{ac} \]

\[ \dot{u} = T_{ac}\dot{x}_{ac} \]

(5.20)

Where \( x_{ac} \) is a \( N_{ac} \) length vector of the displacement from a given reference of each swashplate actuator, with \( N_{ac} \) being the total number of actuators. The matrix \( T_{ac} \) is a \( 3\times N_{ac} \)-sized matrix that determines the control system inputs from the actuator displacements. A schematic of the swashplate and actuators is shown in Figure 5.4. The swashplate actuators used in rotorcraft control system typically have limits in magnitude and rate. In the time integration of the closed-loop response, if any of the calculated optimal control inputs are greater than the maximum allowable control inputs the maximum allowable control inputs are simply used. Therefore, the feasibility study was not conducted as a constrained optimization problem.
5.2 Results

Using the simple, two degree of freedom analysis outlined in the previous section, a study is conducted to determine the effectiveness of using feedback from the gimbal tilt angles to the control system inputs to minimize the flapping response of a three-bladed gimballed rotor. The time history of the rotor response and required control system effort will be examined in this section. A useful metric of the rotor response is the maximum tilt angle of the rotor. It is expressed as

$$\beta_{\text{max}} = \sqrt{\beta_{\text{tic}}^2 + \beta_{\text{is}}^2} \quad (5.21)$$

The angle at which the hub would contact the gimbal restraint is called $\beta_r$. That is, if $\beta_{\text{max}}$ is less than $\beta_r$, the hub is not in contact with the gimbal restraint. As stated earlier, contact between the gimbal and the restraint is not enforced in this model. However, the maximum gimbal tilt angle provides a useful metric as to how large the gimbal tilt must be to be considered excessive.
5.2.1 Rigid Gimbaled Rotor System Properties

The properties of the rigid blade model of the gimbaled rotor are given in Appendix C.4. The gimbaled rotor system used in the analysis is loosely based on the V-22 Osprey rotor system. The engagement rotor speed profile for the gimbaled rotor is assumed identical to the baseline rotor system and is shown in Figure 5.5. The rotor speed profile is an offset sine wave, reaching 50% NR in 10 seconds and full rotor speed in 20 seconds.

![Figure 5.5: Assumed Gimbaled Rotor Speed Variation](image)

The restraint angle for the gimbaled rotor system is 11°. It is important to note that the gimbaled rotor used in the analysis also has several limits on the allowable control system inputs. It is assumed the collective pitch has a lower limit of -7.5° and an upper limit of +54°. The lateral cyclic pitch has assumed limits of ±10°. Furthermore, it is assumed that each of the linear hydraulic actuators has maximum extension/retraction rate limits of ±7 in/s. The actuator transformation matrix for the assumed gimbaled rotor is given by
5.2.2 Optimal Control Results

Rotor engagements were simulated in each deterministic airwake distribution with a gust factor of 25\% in 30-knot bow winds. The response for the uncontrolled rotor system was generated assuming zero control inputs. For the initial analysis, the control system weights were set at values of $S(t_f) = 0$, $Q = 4I_4$, and $R = I_3$. No weighting is needed on the final state of the system because as the rotor nears full speed and full centrifugal force the gimbal tilt automatically decreases. The matrices $I_3$ and $I_4$ denote $3 \times 3$ and $4 \times 4$ identity matrices. More weight was placed on the time history of the gimbal motion than the time history of the control effort. In each figure, the time histories of the maximum and fixed frame gimbal tilt angle are shown first, and the control inputs and actuator extension rates are shown second.

The results of the simulation in the horizontal airwake distribution are shown in Figure 5.6. As expected, without a vertical airwake component the rotor flapping response is very mild. The maximum gimbal tilt angle is less than 3.5\textdegree for the uncontrolled case and less than 1.5\textdegree for the optimally controlled case. Because the rotor response is very mild, the control inputs and actuator extension rates are also well within their limits.
The results of the simulation in the constant airwake distribution are shown in Figure 5.7. With a vertical airwake component the rotor tilts much farther than in the horizontal airwake distribution. For the uncontrolled case, the rotor would initially be tilted far enough to contact the gimbal restraint. On two other occasions, at \( t = 3.2 \) and 4 seconds, the rotor contacts the gimbal restraint. For the optimally controlled rotor, the maximum gimbal tilt angle has been reduced by 50% to only 6.4º. The required control
inputs are much larger for the constant airwake distribution than the horizontal airwake distribution. The collective pitch setting just reaches its lower limit of -7.5° at $t = 3$ seconds. The actuator extension rates are also very close to reaching their limits between $t = 2$ and 4 seconds.

Figure 5.7: Time Histories for the Constant Airwake Distribution
The results of the simulation in the linear airwake distribution are shown in Figure 5.8. The rotor response is lower than for the constant airwake distribution. The uncontrolled rotor reaches a maximum gimbal tilt angle of 8.6°. For the optimally controlled rotor, the maximum gimbal tilt angle has been reduced by 56% to 3.8°. Although the rotor flapping response is lower than in the constant airwake distribution, the actuator extension rates are larger. All three actuators repeatedly reach their
maximum extension rates of ±7 in/s from $t = 3$ to 6 seconds. This is due to the nature of the airwake distribution itself. Because the vertical flow component changes from upward to downward in the linear airwake distribution, the actuators are forced to respond with a higher amplitude and speed than in the constant airwake distribution where the vertical velocity does not change across the rotor disk.

5.2.3 Relaxation of Control System Limits

In the previous simulations, the response of the gimbal was predicted while enforcing the current control system limits. In the constant airwake distribution, the minimum collective pitch setting was reached. In the linear airwake distribution, the actuator extension retraction limits were reached. If the control system limits could be relaxed, the response of the rotor system could be further minimized. This is also allows the relative weight of the matrix $Q$ to be increased. In this section, the response of the rotor with relaxed control system limits and an increased weight of $Q$ from $4I_4$ to $10I_4$ will be examined.

The rotor response in the constant airwake distribution is shown in Figure 5.9. The maximum gimbal tilt angle has been reduced from $6.4^\circ$ for the original optimal control settings to $3.9^\circ$ with the relaxed control limits and increased weighting. The lower limit on the original collective pitch setting is just briefly exceeded. The largest change is in the actuator extension rates. The rates for actuators #1 and #3 reach nearly ±10 in/s.
The rotor response in the linear airwake distribution is shown in Figure 5.10. The maximum gimbal tilt angle has been reduced from 3.8° for the original optimal control settings to 2.6° with the relaxed control limits. This represents a reduction of 70% compared to the uncontrolled rotor. The actuator extension rates increase to nearly ±10
in/s from $t = 3$ to 5 seconds. This coincides with the period in which the largest gimbal tilt angles occur for the uncontrolled rotor.

Figure 5.10: Time Histories for the Linear Airwake Distribution with Relaxed Control System Limits
5.2.4 Sub-optimal Control Results

The simulations completed in the previous sections assumed that the ship airwake in the plane of the rotor disk was completely known. Aerodynamic force terms, which are largely dependent upon the ship airwake velocities, appear in the $A$, $B$ and $d$ state matrices. These state matrices are then used to calculate the Kalman gain matrix $K$ and bias term $v$. Realistically, complete knowledge of the ship airwake environment is highly unlikely. However, most ships have anemometers that measure the relative wind speed and direction of the oncoming air. As shown schematically in Figure 5.11, these anemometers are often located above the flight deck on a mast. These anemometers could be used to obtain an estimate of the wind speed on the flight deck in the plane of the rotor disk, $V_x$ and $V_y$. It is unlikely that an estimate of the vertical velocity component $V_z$ could be made with the ship anemometer.

Figure 5.11: Schematic of Ship Anemometer
In this section, the sub-optimal response of the rotor system is investigated. In this context, the term sub-optimal is used to denote the response of the rotor in which the Kalman gain matrix $K$ and bias term $v$ are calculated assuming the in plane components are known and the vertical velocity component is zero. The original control system weights and limits are used in the investigation.

The response of the rotor system using sub-optimal control in the constant airwake distribution is shown in Figure 5.12. The maximum gimbal tilt angle using optimal control is $6.4^\circ$, while the maximum gimbal tilt angle using sub-optimal control is $8.4^\circ$. This still represents a significant reduction from the uncontrolled case.

![Figure 5.12: Time Histories for the Constant Airwake Distribution with Sub-optimal Control](image)

a) Maximum Gimbal Tilt Angle  

b) Fixed Frame Gimbal Tilt Angles

The response of the rotor system using sub-optimal control in the linear airwake distribution is shown in Figure 5.13. The uncontrolled rotor reaches a maximum gimbal tilt angle of $8.6^\circ$. The maximum gimbal tilt angle using optimal control is $3.8^\circ$, while the
maximum gimbal tilt angle using sub-optimal control is 5°, or a reduction of 42% from uncontrolled case.

![Graph](image)

**Figure 5.13**: Time Histories for the Linear Airwake Distribution with Sub-optimal Control

### 5.2.4.1 Robustness to Anemometer Error

It is well known that the ship anemometer may not necessarily give an accurate measure of the ship airwake velocities on the flight deck. The anemometer is often not physically located near the flight deck; usually it is placed on a mast above the conning towers. Furthermore, depending on the ship type the ship airwake over the flight deck may be different than at the anemometer location.
In this section, the sensitivity of the rotor response to an inaccurate anemometer reading is investigated. It is assumed that the anemometer measurement has some error relative to the actual wind speed and direction on the flight deck

\[
V_{\text{meas}} = V_{\text{WOD}} + \Delta V_{\text{WOD}} \\
\psi_{\text{meas}} = \psi_{\text{WOD}} + \Delta \psi_{\text{WOD}}
\]  

(5.23)

The incorrect information from the anemometer is then used to form the incorrect state matrices \( A_E, B_E \) and \( d_E \). These incorrect state matrices are used to calculated the incorrect Kalman gain matrix, \( K_E \), and bias vector, \( v_E \)

\[
\begin{align*}
-S_E &= A_E^T S_E + S_E A_E - S_E B_E R^{-1} B_E^T S_E + Q \\
K_E &= R^{-1} B_E^T S_E \\
-\hat{v}_E &= (A_E - B_E K_E)^T v_E + S_E d_E
\end{align*}
\]  

(5.24)

The incorrect optimal control vector, \( u_E \), is then calculated from

\[
\begin{align*}
u_E = -K_E x_E - R^{-1} B_E^T v_E
\end{align*}
\]  

(5.25)

The rotor response is then calculated using the incorrect optimal control vector \( u_E \), but the true state matrices \( A, B \) and \( d \). In other words, the rotor response \( x_E \) resulting from the inaccurate anemometer reading is calculated from the time integration of the state equation

\[
\dot{x}_E = Ax_E + B u_E + d
\]  

(5.26)

The response of the rotor system with an error in the measurement of the wind speed only is shown in Figure 5.14. The figure on the left examines the response of the rotor system when the wind speed is underestimated and the figure on the right examines the response of the rotor system when the wind speed is overestimated. In general, the maximum gimbal tilt angle is increased when the wind speed is underestimated and is
decreased when the wind speed is overestimated. For a measurement underestimation of 20 knots, the maximum gimbal tilt angle increases from 5° to 6.2°. However, this still represents a reduction of 29% from the uncontrolled case. For a measurement overestimation of 20 knots, the maximum gimbal tilt angle decreases from 5° to 4.5°.

The response of the rotor system with an error in the measurement of the wind direction only is shown in Figure 5.15. The figure on the left examines the response of the rotor system when the wind direction is measured more in the counterclockwise direction and the figure on the right examines the response of the rotor system when the wind direction is measured more in the clockwise direction. In general, the maximum gimbal tilt angle is decreased when the measured wind direction is more in the counterclockwise direction and increased when the measured wind speed is more in the clockwise direction. For a measurement underestimation of 30°, the maximum gimbal tilt angle decreases from 5° to 4.8°. For a measurement overestimation of 30° knots, the
maximum gimbal tilt angle increases from 5° to 6°. However, this still represents a reduction of 30% from the uncontrolled case.

![Graph showing time histories for linear airwake distribution with errors in the wind direction measurement]

a) Underestimation (Counterclockwise)  
b) Overestimation (Clockwise)

*Figure 5.15: Time Histories for the Linear Airwake Distribution with Errors in the Wind Direction Measurement*

5.3 Summary

This chapter examined the control of the flapping response of a rigid, gimballed rotor system during an engagement operation using feedback from the motion of the rotor to the swashplate actuators. The ship airwake model was assumed to consist of several different types of deterministic airwakes and aerodynamic forces were calculated using a simple, attached flow model. A Linear Quadratic Regulator optimal control method was used to reduce the flapping of the rotor system. With full knowledge of the ship airwake velocities, the rigid flapping response of the rotor system was reduced by as much as 56% within the current physical limitations on the control system. If the control limits were
relaxed, the flapping response of the rotor was reduced by as much as 70%. With only partial knowledge of the ship airwake velocities, the flapping response of the rotor system was reduced by as much as 42%. The sensitivity of the rotor response with errors in the ship airwake measurements was also examined. The rotor response with the errors in the ship airwake measurements of 20 knots and 30° was increased; however, this response was still at least 30% lower than without any control.
Chapter 6

CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

Aeroelastic Flap-Lag-Torsion Analysis

An analysis has been developed to predict the transient aeroelastic response of a helicopter rotor system during shipboard engagement and disengagement operations. The blade coupled flap-lag-torsion equations of motion were developed using Hamilton’s Principle and then discretized spatially using the finite element method. The analysis is capable of simulating articulated, hingeless, teetering, or gimbaled rotor systems. Aerodynamics were simulated using both nonlinear quasi-steady and time domain nonlinear unsteady models. The ship airwake environment was simulated with simple deterministic airwake distributions, with results from experimental measurements or with numerical predictions. The transient aeroelastic response of the rotor blades was then time integrated along a specified rotor speed profile.

With the aeroelastic flap-lag-torsion analysis, the response of a generic articulated rotor system was examined first. Using this model, it was shown that:

1) The Bossak-Newmark time integration method was shown to yield superior results compared to Runge-Kutta or standard Newmark time integration techniques. A
converged transient response with less high frequency noise was achieved with a time step twice as fast in terms of wall clock time as the other two integrators.

2) The addition of the lag degree of freedom was shown to significantly influence the blade response. Tip deflections were under predicted by as much as 31% without lag motion.

3) Predictions were made using both the nonlinear quasi-steady steady and unsteady aerodynamic models. The nonlinear quasi-steady model was shown to consistently yield a blade response approximately 10% larger than the unsteady model. Use of the unsteady aerodynamic model at the high angles of attack, skew angles, and low Mach numbers often encountered in engagement and disengagement operations is difficult.

4) The transient blade response was compared using the simple deterministic and numerically predicted ship airwakes for a frigate-like ship shape. The blade response was often much larger for the numerically predicted ship airwakes and was much more dependent on deck location and wind speed and direction. Fluctuating flow components in the range of frequencies measured in previous ship airwake tests were shown to influence the blade tip deflections by as much as 45%.

5) Correlation with the wind tunnel tests performed at the University of Southampton showed good correlation between the measured and the predicted blade response. The position of the rotor on the deck was again shown to significantly affect the blade response. Deck positions nearest the windward deck edge showed a larger blade response than deck positions nearer the leeward deck edge. Fluctuating flow
components were shown to improve the correlation with the results for the blade response at the leeward deck positions.

6) The proper modeling of the initial conditions was examined. The inclusion of aerodynamic forces in the solution for the initial deflection was shown to significantly affect the blade response in the low rotor speed region. Tip deflections were reduced by as much as 43% in the very low rotor speed region with the inclusion of aerodynamic forces. The effect of the initial azimuth angle was also examined. Engagement wind envelopes were not sensitive to the initial azimuth angle because the minimum tip deflections typically occurred during the first one-half to one rotor revolution. However, disengagement wind envelopes were sensitive to the initial azimuth angle because the minimum tip deflections typically occurred in the last half rotor revolution.

The control of the rotor response for an analytic model of the H-46 Sea Knight rotor system was investigated with three different passive control techniques. It was shown that:

1) The use of a high collective pitch setting in the rotor speed region below 25%NR was not successful in reducing the blade flapping response. A collective pitch setting of 10° increased the effective flapwise stiffness of the blade by as much as 84%; however, the increase in blade stiffness only reduced the flapping response of the rotor in a few isolated cases. In the majority of the cases, the aerodynamic forces were often increased enough to offset the increased stiffness and double the deflections.

2) The use of a discrete flap damper to damp excessive flap motions was investigated. At the current H-46 flap stop setting of +1°, the flap damper was ineffective
in reducing the blade response regardless of its strength because the stroke of the damper was too small. If the flap stop was raised to 6° and a flap damper 4 times the strength of the lag damper was used, the downward blade deflections were typically reduced by 10%. However, because the blade flap stop setting was raised the blade upward deflections were increased by as much as 40%.

3) The use of extendable/retractable, gated leading-edge spoilers was investigated. The spoilers were assumed to be extended only in the region of the engagement or disengagement where the rotor speed was less than 25%NR. Spoilers covering the outer 15%R of the rotor blade were shown to reduce both the downward and upward blade flapping response by as much as 30% without an increase in the rotor torque.

Rigid Gimballed Rotor Analysis

Another separate analysis was developed to investigate the active control of gimballed rotor systems. The equations of motion for a rigid, three-bladed gimballed rotor system were derived using a Newtonian force balance approach. Aerodynamic forces were simulated with a simple, linear attached flow model. The ship airwake was assumed to consist of the deterministic airwake distributions. The resulting equations of motion for the rigid roll and pitch of the rotor system were cast into state space form and time integrated along an assumed rotor speed profile. A Linear Quadratic Regulator optimal control technique was applied to the equations of motion to minimize the transient rotor response. With this analysis, it was shown that:
1) With full knowledge of the ship airwake environment, the maximum transient gimbal tilt angle was reduced by as much as 56% within the current physical limits of the control system.

2) If the physical limits of the control system were relaxed, the maximum transient gimbal tilt angle was reduced by as much as 70%.

3) With only partial knowledge of the ship airwake from a ship anemometer measurement, the maximum transient gimbal tilt angle was still reduced by as much as 42%.

4) The control technique was even successful considering errors in the ship anemometer measurements. Overestimating the wind speed on the flight deck minimized the rotor response. The gimbal tilt angle was still reduced by 30% with a wind speed measurement error of 20 knots and a wind direction measurement error of 30º.

6.2 Recommendations

1) Several interesting trends in the blade transient response were discovered when using the numerically predicted airwakes for the Simple Frigate Shape. Additional predictions of the SFS airwake, especially at the 30-knot wind speed, would be useful. Ideally, predictions of the airwake around more realistic ship shapes, especially the LHA, LHD, CVN, CV, AE or AOR, could be made and investigated. Prediction of the ship airwake around more realistic ship types would allow correlation between the aeroelastic code and actual historical tunnel strike events.
2) Currently, it is assumed that the rotor rotates in the counterclockwise direction only. The analysis should be modified to simulate either clockwise or counterclockwise rotation. This would improve the modeling of the rear rotor of tandem rotorcraft, the second rotor of tiltrotors, or most European rotorcraft.

3) For gimbaled rotors, such as the V-22 Osprey, the rotor pylon is not rigidly attached to the wing structure. In the event of a gimbal restraint impact, the entire pylon-wing structure bends as an elastic structure. Adding the pylon motion degrees of freedom to the analysis would improve the modeling of gimbaled tiltrotors.

4) Initial research has shown that the fluctuating flow components can significantly impact the rotor response. This is because the range of frequencies present in actual ship airwake measurements overlaps with the fundamental rotor frequencies when rotating at low speed. However, the current fluctuating flow model is somewhat rudimentary and should be improved.

5) In the current aeroelastic analysis, nonlinear structural terms are treated as an additional linear force terms. As the blade response becomes large, the influence of these nonlinear terms increases. Currently, the time integration techniques used in the analysis are only unconditionally stable for linear systems. It is suspected that this is the reason that the time integration can occasionally become unstable when the blade response becomes large. The use of time integrator methods for nonlinear systems should be investigated.
6) Blade and hub loads are currently predicted using a force summation technique. This method can be computationally expensive. In the future, it is suggested that a reaction force method be used to calculate blade loads.

7) Active control techniques with a rigid gimbaled rotor system were shown to have promise in reducing the rotor response. However, the aerodynamic model used in the study was very simplified. The next step in the investigation of active control techniques would be to include nonlinear aerodynamic effects. If the results are still promising, it would be advantageous to incorporate the active control techniques into the aeroelastic rotor analysis. In addition, a more rigorous examination of the stability of the system should be undertaken.
Appendix A

ADDITIONAL MODELING

A.1 Coordinate Systems

The following coordinate systems are used to define the rotor blades and hub. They are identical to those in Ref. 43 and are included here simply for reference. Rotated through the shaft angle is the hub-fixed nonrotating coordinate system, \((X_H, Y_H, Z_H)\), with corresponding unit vectors \(\hat{I}_H, \hat{J}_H, \hat{K}_H\). The \(X_H\) axis points to the rear of the rotor, the \(Y_H\) axis points to the right side of the rotor and the \(Z_H\) axis points upwards parallel to the rotor shaft. Note that only longitudinal shaft tilt, \(\alpha_s\), is considered; lateral shaft tilt is assumed zero. Assuming that the longitudinal shaft tilt angle is small, the transformation matrix is given by

\[
\begin{pmatrix}
\hat{I}_H \\
\hat{J}_H \\
\hat{K}_H
\end{pmatrix} =
\begin{bmatrix}
1 & 0 & \alpha_s \\
0 & 1 & 0 \\
\alpha_s & 0 & 1
\end{bmatrix}
\begin{pmatrix}
\hat{I}_F \\
\hat{J}_F \\
\hat{K}_F
\end{pmatrix}
\tag{A.1}
\]

Where the shaft tilt angle is positive nose down.

Also centered at the rotor hub is the hub-fixed rotating coordinate system, \((X, Y, Z)\), with corresponding unit vectors \(\hat{i}, \hat{j}, \hat{k}\). The hub-fixed rotating coordinate system rotates with the blades in the counterclockwise direction at an angular velocity of
relative to the hub-fixed nonrotating coordinate system. The transformation matrix
between the hub-fixed nonrotating and the hub-fixed rotating coordinate systems is given
by

\[
\begin{pmatrix}
\hat{i} \\
\hat{j} \\
\hat{k}
\end{pmatrix} =
\begin{bmatrix}
\cos \psi & -\sin \psi & 0 \\
\sin \psi & \cos \psi & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{pmatrix}
\hat{i}_H \\
\hat{j}_H \\
\hat{k}_H
\end{pmatrix}
\]

where \( \psi \) is the blade azimuth angle and is positive measured counterclockwise. In this
analysis the rotor speed is not a constant; therefore, the blade azimuth angle is equal to
the time integral of the rotor speed

\[
\psi = \int_0^t \Omega(t) dt + \psi_0
\]

Where \( \psi_0 \) is the azimuth angle of the master blade at the start of the engagement or
disengagement.

The undeformed blade coordinate system, \((x, y, z)\), with corresponding unit
vectors \( \hat{i}, \hat{j}, \hat{k} \) is inclined through the blade precone angle. It is attached to the
undeformed blade and is shown in Figure A.1. The x axis is coincident with the blade,
the y axis is in the plane of rotation pointed towards the leading edge of the blade and the
z axis is pointed upwards. Assuming the precone angle is small, the transformation
between these two coordinate systems is given by

\[
\begin{pmatrix}
\hat{i} \\
\hat{j} \\
\hat{k}
\end{pmatrix} =
\begin{bmatrix}
1 & 0 & \beta_p \\
0 & 1 & 0 \\
-\beta_p & 0 & 1
\end{bmatrix}
\begin{pmatrix}
\hat{i} \\
\hat{j} \\
\hat{k}
\end{pmatrix}
\]

Where \( \beta_p \) is the precone angle and is positive blade flapped up.
Figure A.1: Blade Coordinate Systems
The final coordinate system used in this analysis defines the deformed blade and is appropriately named the *deformed blade coordinate system*, \((x_\xi, y_\eta, z_\zeta)\), with corresponding unit vectors \((\hat{i}_\xi, \hat{j}_\eta, \hat{k}_\zeta)\). The \(x_\xi\) axis points out along the deformed blade axis, the \(y_\eta\) axis points towards the leading edge, and the \(z_\zeta\) axis points up through the blade cross section. The deformed blade coordinate system accounts for the displacements and rotations of the blade cross section and is also shown in *Figure A.1*. For clarity, the deformed blade cross section is shown in *Figure A.2*.

*Figure A.2: Deformed Blade Cross Section*
A point $P$ on the undeformed elastic axis undergoes deflections of $u, v, w$ in the undeformed blade coordinate system and moves to a point $P'$. It also undergoes a rotation by the total blade pitch angle $\theta_1$ (positive pitch up) about the deformed elastic axis. The transformation between the undeformed and deformed blade coordinate systems is

$$
\begin{bmatrix}
\hat{i}_\zeta \\
\hat{j}_\eta \\
\hat{k}_\zeta
\end{bmatrix} = T_{DU}
\begin{bmatrix}
\hat{i} \\
\hat{j} \\
\hat{k}
\end{bmatrix}
\tag{A.5}
$$

The transformation matrix $T_{DU}$ is derived by rotations through the Euler angles $\zeta, \beta, \theta$ shown in Figure A.3. In terms of these Euler angles, the transformation is

$$
T_{DU} = \begin{bmatrix}
\cos \beta \cos \zeta & \cos \beta \sin \zeta & \sin \beta \\
-\sin \theta \sin \beta \cos \zeta - \cos \theta \sin \zeta & \cos \theta \cos \zeta - \sin \theta \sin \beta \sin \zeta & \cos \theta \sin \beta \\
-\cos \theta \sin \beta \cos \zeta + \sin \theta \sin \zeta & -\sin \theta \cos \zeta - \sin \zeta \sin \beta \cos \zeta & \cos \theta \cos \beta
\end{bmatrix}
\tag{A.6}
$$

*Figure A.3: Blade Deformation in Terms of Euler Angles*
Using the small angle approximation, the Euler angles can be written in terms of the blade deformations with the following relations

\[
\cos \zeta = \frac{\sqrt{1-\nu'^2} - \frac{1}{2} w'^2}{\sqrt{1-w'^2}} \\
\sin \zeta = \frac{\nu'}{\sqrt{1-w'^2}} \\
\cos \bar{\beta} = \sqrt{1-w'^2} \\
\sin \bar{\beta} = w' \\
\bar{\theta} = \theta_i 
\]  
(A.7)

If the magnitudes of \( \nu' \) and \( w' \) are assumed small (\( \ll 1 \)), then the Euler angles in Eqn. A.7 can be simplified to

\[
\cos \zeta = \frac{1-\frac{1}{2} \nu'^2 - \frac{1}{2} w'^2}{1-\frac{1}{2} w'^2} \\
\sin \zeta = \frac{\nu'}{1-\frac{1}{2} w'^2} \\
\cos \bar{\beta} = 1-\frac{1}{2} w'^2 \\
\sin \bar{\beta} = w' \\
\bar{\theta} = \theta_i 
\]  
(A.8)

Substituting the above relations into Eqn. A.6 yields the final form of \( T_{DU} \)

\[
T_{DU} = \begin{bmatrix}
1-\frac{1}{2} \nu'^2 - \frac{1}{2} w'^2 & \nu' & w' \\
-v' \cos \theta_i - w' \sin \theta_i & (1-\frac{1}{2} \nu'^2) \cos \theta_i - v' \sin \theta_i & (1-\frac{1}{2} \nu'^2) \sin \theta_i \\
v' \sin \theta_i - w' \cos \theta_i & -(1-\frac{1}{2} \nu'^2) \sin \theta_i - v' \cos \theta_i & (1-\frac{1}{2} \nu'^2) \cos \theta_i
\end{bmatrix} 
\]  
(A.9)
A.2 Strain Energy

This analysis assumes that the blade acts as a Bernoulli-Euler beam, in which \( \sigma_{\eta\eta} = \sigma_{\eta\zeta} = \sigma_{\zeta\zeta} = 0 \). The strain energy of each rotor blade then is given by

\[
U_b = \frac{1}{2} \int_0^1 \oint_A \left( \sigma_{xx} \varepsilon_{xx} + \sigma_{x\eta} \varepsilon_{x\eta} + \sigma_{x\zeta} \varepsilon_{x\zeta} \right) d\eta d\zeta \, dx \tag{A.10}
\]

Substituting the linear stress-strain relations \( \sigma_{xx} = E\varepsilon_{xx}, \sigma_{x\eta} = G\varepsilon_{x\eta}, \sigma_{x\zeta} = G\varepsilon_{x\zeta} \) yields

\[
U_b = \frac{1}{2} \int_0^1 \oint_A \left( E\varepsilon_{xx}^2 + G\varepsilon_{x\eta}^2 + G\varepsilon_{x\zeta}^2 \right) d\eta d\zeta \, dx \tag{A.11}
\]

Taking the variation yields

\[
\delta U_b = \int_0^1 \oint_A \left( E\varepsilon_{xx} \delta \varepsilon_{xx} + G\varepsilon_{x\eta} \delta \varepsilon_{x\eta} + G\varepsilon_{x\zeta} \delta \varepsilon_{x\zeta} \right) d\eta d\zeta \, dx \tag{A.12}
\]

The strains are expressed as

\[
\varepsilon_{xx} = u' + \frac{1}{2} v'^2 + \frac{1}{2} w'^2 - \lambda_r \phi' + \left( \eta^2 + \zeta^2 \right) \left( \theta_0 \phi' + \frac{1}{2} \phi'^2 \right)
- \nu' \left( \eta \cos \theta_1 - \zeta \sin \theta_1 \right) - \nu'' \left( \eta \sin \theta_1 + \zeta \cos \theta_1 \right)
\]

\[
\varepsilon_{x\eta} = -\zeta \phi'
\]

\[
\varepsilon_{x\zeta} = \hat{\eta} \phi'
\]

Taking the variation of each yields

\[
\delta \varepsilon_{xx} = \delta u' + v' \delta v' + w' \delta w' - \lambda_r \delta \phi' + \left( \eta^2 + \zeta^2 \right) \left( \theta_0 + \phi' \right) \delta \phi'
- \left( \eta \cos \theta_1 - \zeta \sin \theta_1 \right) \left( \delta v' + w' \delta \theta_1 \right) - \left( \eta \sin \theta_1 + \zeta \cos \theta_1 \right) \left( \delta w'' - \nu'' \delta \theta_1 \right)
\]

\[
\delta \varepsilon_{x\eta} = -\zeta \delta \phi'
\]

\[
\delta \varepsilon_{x\zeta} = \hat{\eta} \delta \phi'
\]

In the preceding equations there are two different twist angles, where \( \phi \) can be viewed as the elastic twist about the undeformed elastic axis and \( \hat{\phi} \) can be viewed as the elastic twist about the deformed elastic axis. The relationship between the twist angles is...
\[ \phi = \hat{\phi} + \int_{0}^{x} w' v^* \, dx \quad (A.15) \]

In the following derivations, all the elastic twist angles are written in terms of the elastic twist about the deformed elastic axis. Furthermore, it is assumed that \( \hat{\phi} \) is a small angle so the following trigonometric expansion can be employed

\[
\sin \theta_1 = \sin \left( \theta_0 + \hat{\phi} \right) \equiv \sin \theta_0 + \hat{\phi} \cos \theta_0
\]

\[
\cos \theta_1 = \cos \left( \theta_0 + \hat{\phi} \right) \equiv \cos \theta_0 - \hat{\phi} \sin \theta_0
\quad (A.16)

Furthermore, the total axial deflection is composed of the elastic deflection, \( u_e \), and deflection due to radial foreshortening. In this analysis, the elastic axial deflection is neglected (\( u_e = 0 \)) so

\[
u = -\frac{1}{2} \int_{0}^{x} \left( v'^2 + w'^2 \right) \, dx
\quad (A.17)

Substituting the above expressions and retaining terms up to second order yields the expression for the blade strain energy

\[
\delta U_v = \int_{0}^{x} \left( U_v \delta v'' + U_w \delta w' + U_w \delta v'' + U_{\phi} \delta \phi' + U_{\phi} \delta \phi'' + U_{\phi} \delta \phi''' \right) \, dx
\quad (A.18)

Where

\[
U_v = v^* \left( EI_{zz} \cos^2 \theta_0 + EI_{yy} \sin^2 \theta_0 \right) + w^* \left( EI_{zz} - EI_{yy} \right) \cos \theta_0 \sin \theta_0
\]

\[
-\hat{\phi}' \hat{E} B \hat{\theta}_0 \cos \theta_0 + w'' \hat{\phi} \left( EI_{zz} - EI_{yy} \right) \cos 2\theta_0 + v'' \hat{\phi} \left( EI_{zz} - EI_{yy} \right) \sin 2\theta_0
\]

\[
\left( GJ + E B \hat{\theta}_0 \right) \hat{\phi} w'
\quad (A.19a)

\[
U_w = \left( GJ + E B \hat{\theta}_0 \right) \hat{\phi}' v^*
\quad (A.19b)
The cross-sectional properties are defined as

\[
EI_{yy} = \iint_A E\xi^2 \, d\eta \, d\zeta \\
EI_{zz} = \iint_A E\eta^2 \, d\eta \, d\zeta \\
EA = \iint_A E \, d\eta \, d\zeta \\
GJ = \iint_A G \left( \eta^2 + \xi^2 \right) \, d\eta \, d\zeta \\
EB_1 = \iint_A E \left( \eta^2 + \xi^2 \right)^2 \, d\eta \, d\zeta \\
EB_2 = \iint_A E \eta \left( \eta^2 + \xi^2 \right) \, d\eta \, d\zeta \\
EC_1 = \iint_A E\lambda_1^2 \, d\eta \, d\zeta \\
EC_2 = \iint_A E\lambda_1 \xi \, d\eta \, d\zeta
\]

(A.20)

Note that in the derivation of the strain energy, the blade cross-section has been assumed symmetric about the \( \xi-\eta \) plane. Therefore, all integrals of the form \( \iint_A \zeta^2 \eta \, d\eta \, d\zeta \) are equal to zero.

### A.3 Unsteady Aerodynamic Modeling

The unsteady aerodynamic model used in this analysis was originally developed by Leishman and Beddoes [45, 84]. The model is split into contributions from attached flow unsteady aerodynamics, nonlinear separation effects, and dynamic stall effects.
Each will be discussed in the following subsections. Provisions have been made to ensure that unsteady effects can be modeled for both positive and negative angles of attack.

A.3.1 Unsteady Attached Flow Modeling

Naturally, the first step in any aerodynamic model is the correct simulation of attached flow conditions. This section outlines the methods used to calculate the normal force, chord force and pitching moment coefficients under unsteady attached flow conditions. It also is used as input for the nonlinear separation and dynamic stall sections.

The circulatory normal force due to a series of step inputs in the angle of attack is given by

\[ C_{n}^{c} = C_{n_{0}} + C_{n_{u}} \alpha_{E_{u}} \]  \hfill (A.21)

The effective angle of attack at the \( \frac{3}{4} \) chord is defined as

\[ \alpha_{E_{u}} = \alpha_{n} - X_{n}^{(1)} - X_{n}^{(2)} \]  \hfill (A.22)

Where \( \alpha_{n} \) is the geometric angle of attack at the \( \frac{3}{4} \) chord. The deficiency functions \( X_{n}^{(1)} \) and \( X_{n}^{(2)} \), which account for the effects of the shed wake on the angle of attack, are

\[ X_{n}^{(1)} = X_{n-1}^{(1)} e^{(-h_{n-1}^{2} \Delta S)} + A_{2} \Delta \alpha_{n} e^{\left(\frac{-h_{n-1}^{2} \Delta S}{2}\right)} \]  \hfill (A.23)

\[ X_{n}^{(2)} = X_{n-1}^{(2)} e^{(-h_{n-1}^{2} \Delta S)} + A_{2} \Delta \alpha_{n} e^{\left(\frac{-h_{n-1}^{2} \Delta S}{2}\right)} \]  \hfill (A.23)
Where $\Delta S$ is the distance traveled by the airfoil in semi-chords over the time interval $n-1$ to $n$

$$\Delta S = \frac{(V_n + V_{n-1}) \Delta t}{c} \quad (A.24)$$

The change in angle of attack over the time interval is simply $\Delta \alpha_n = \alpha_n - \alpha_{n-1}$. The noncirculatory normal force due to a step input in the angle of attack is given by

$$C_{\alpha}^{i} = \frac{4k_T T_i}{M} (K_{\alpha_n} - K'_{\alpha_n})$$

$$K_{\alpha} = \frac{\alpha_n - \alpha_{n-1}}{\Delta t} \quad (A.25)$$

Where the deficiency function $K'_{\alpha_n}$ is given by

$$K'_{\alpha_n} = K'_{\alpha_{n-1}} e^{\frac{-\Delta \alpha_n}{k_T T_i}} + (K_{\alpha_n} - K_{\alpha_{n-1}}) e^{\frac{-\Delta \alpha_n}{k_T T_i}} \quad (A.26)$$

The normal force coefficient due to a step change in pitch rate is given by

$$C_{q}^{i} = \frac{k_T T_i}{M} (K_{q_n} - K'_{q_n})$$

$$K_{q} = \left( \frac{\dot{\alpha}_n - \dot{\alpha}_{n-1}}{\Delta t} \right) \frac{c}{V_n} \quad (A.27)$$

Where the deficiency function $K'_{q_n}$, which accounts for the effect of wave-like pressure disturbances, is given by

$$K'_{q_n} = K'_{q_{n-1}} e^{\frac{-\Delta q_n}{k_T T_i}} + (K_{q_n} - K_{q_{n-1}}) e^{\frac{-\Delta q_n}{k_T T_i}} \quad (A.28)$$

The time constants governing the decay of the normal force coefficient are Mach number dependent and are given by
The time decay constant $T_i$ is defined as

$$T_i = \frac{c}{a}$$  \hspace{1cm} (A.30)

The constants $A_i$ and $b_i$ are

$$A_i = 0.30 \quad A_2 = 0.70 \quad A_3 = 1.50 \quad A_4 = -0.50 \quad A_5 = 1.0$$

$$b_1 = 0.14 \quad b_2 = 0.53 \quad b_3 = 0.25 \quad b_4 = 0.10 \quad b_5 = 0.5$$  \hspace{1cm} (A.31)

The potential normal force coefficient under attached flow conditions is then

$$C_{n_p}^p = C_{n_a}^c + C_{n_p}^i + C_{n_k}^i$$  \hspace{1cm} (A.32)

The potential unsteady chord force coefficient is given by

$$C_p^p = \eta_c C_{a_e}^2$$  \hspace{1cm} (A.33)

Where $\eta_c$ is the recovery factor and is used to model the fact that a real airfoil is unable to attain 100% leading edge suction. The circulatory moment coefficient about the quarter chord due to a series of step inputs in the angle of attack is given by

$$C_{m_c} = C_{m_0} + k_0 C_{v_c} - \frac{\pi}{8 \beta} \left( \alpha_n - X_n^{(5)} \right) \frac{c}{V_n}$$  \hspace{1cm} (A.34)

The deficiency function $X_n^{(5)}$, which accounts for the induced camber due to unsteady effects, is given by

$$X_n^{(5)} = X_{n-1}^{(5)} e^{\left(-b_5 \beta \Delta \delta\right)} + A_5 \Delta \alpha_n e^{\left(-b_5 \beta \Delta \delta\right)}$$  \hspace{1cm} (A.35)
The change in the angle of attack rate is \( \Delta \dot{\alpha}_n = \dot{\alpha}_n - \dot{\alpha}_{n-1} \). The noncirculatory pitching moment coefficient due to a step change in angle of attack is

\[
C_{m_n} = \frac{-k_{\alpha_n} T_f}{M} \left[ A_3 b_3 \left( K_{\alpha_n} - K_{\alpha_{n-1}} \right) + A_4 b_4 \left( K_{\alpha_n} - K_{\alpha_{n-1}} \right) \right] \tag{A.36}
\]

Where the deficiency functions \( K_{\alpha_n} \) and \( K_{\alpha_{n-1}} \) are given by

\[
K_{\alpha_n} = K_{\alpha_{n-1}} e^{\left( -\frac{\Delta \alpha}{b k_{\alpha_n} T_f} \right)} + \left( K_{\alpha_n} - K_{\alpha_{n-1}} \right) e^{\left( -\frac{\Delta \alpha}{2 b k_{\alpha_n} T_f} \right)} \tag{A.37}
\]

The pitching moment coefficient due to a step change in pitch rate is

\[
C_{m_q} = \frac{-7 k_{\alpha_n} T_f}{12 M} \left( K_{\alpha_n} - K_{\alpha_{n-1}} \right) \tag{A.38}
\]

Where the deficiency function \( K_{\alpha_n} \) is

\[
K_{\alpha_n} = K_{\alpha_{n-1}} e^{\left( -\frac{\Delta \alpha}{k_{\alpha_n} T_f} \right)} + \left( K_{\alpha_n} - K_{\alpha_{n-1}} \right) e^{\left( -\frac{\Delta \alpha}{2 k_{\alpha_n} T_f} \right)} \tag{A.39}
\]

The time constants governing the decay of the moment coefficient are also Mach number dependent and are given by

\[
k_{\alpha_n} = \frac{0.8 (A_3 b_4 + A_4 b_3)}{b_3 b_4 (1 - M)} \tag{A.40}
\]

\[
k_{q_n} = \frac{5.6}{15 (1 - M) + 3 \pi \beta^2 A_3 b_3}
\]

The potential (total) pitching moment coefficient under attached flow conditions is then
In this derivation the impulsive forces and moments are inversely proportional to the Mach number. Therefore, at very low Mach number their contributions become large.

A.3.2 Nonlinear Separation Effects

One of the most important aspects of modeling dynamic stall is the delineation between attached and separated flow conditions. This subsection modifies the previously determined total normal force, chord force and pitching moment coefficients for leading and trailing edge separation effects. Typically, both leading and trailing edge separation effects are delayed to higher angles of attack due to the favorable effects of the pitch rate on the airfoil pressure distribution.

The first source of nonlinear behavior is leading edge separation. Since flow separation is delayed in unsteady conditions, higher suction pressures are attained near the airfoil leading edge. Thus, the airfoil is more susceptible to separation from the leading edge in unsteady conditions. However, it is desirable to define a condition in which leading edge separation will occur, independent of unsteady effects. This is accomplished by adjusting the potential normal force coefficient to remove unsteady effects. This adjusted normal force coefficient is

\[ C'_{n_u} = (C^c_{n_u} - D_{p_n}) + C^i_{n_u} + C^{ii}_{n_u} \]  

Where the deficiency function \( D_{p_n} \) is
Leading edge separation can be monitored by comparing the unsteady, adjusted normal force coefficient to the static, critical normal force coefficient.

Another source of nonlinear behavior is trailing edge separation. The trailing edge separation model is based on Kirchoff theory of flow separation over a flat plate. A flow separation point $f_n$, measured from the leading edge, is defined. A flow separation value of one means the flow is fully attached, while a value of zero means the flow is fully separated. In the steady case, the flow is considered separated when $f_n = 0.7$. The angle of attack at this point is denoted by $\alpha_1$. It has been noted that a static hysteresis effect in the separation condition occurs – the airloads are different when angle of attack is increased slowly rather than decreased slowly. The static hysteresis effect is accounted for by defining an adjusted separation angle of attack

$$\alpha_n' = \alpha_1 - \Delta \alpha_n$$ (A.44)

The parameter is defined for both positive and negative angles of attack

$$\begin{align*}
\text{if } \alpha_n \geq 0 & \quad \Delta \alpha_n' = \begin{cases} 
\Delta \alpha_1 (1 - f_{n-1})^{\frac{1}{2}} & \text{if } K_{\alpha_n} < 0 \\
0 & \text{if } K_{\alpha_n} \geq 0
\end{cases} \\
\text{if } \alpha_n < 0 & \quad \Delta \alpha_n' = \begin{cases} 
\Delta \alpha_1 (1 - f_{n-1})^{\frac{1}{2}} & \text{if } K_{\alpha_n} \geq 0 \\
0 & \text{if } K_{\alpha_n} > 0
\end{cases}
\end{align*}$$ (A.45)

Once the adjusted separation angle of attack is known, a quasi-static separation point accounting for the static hysteresis effect is defined as
Naturally, unsteady effects play a role in determining the trailing edge separation point. The adjusted normal force coefficient is used to define an effective angle of attack \( \alpha_{nf} \), which gives the same unsteady leading edge separation condition as the quasi-steady case

\[
\alpha_{nf} = \frac{C'_{n_{nf}} - C_{n_{nf}}}{C_{n_{nf}}}
\]  

(A.47)

An effective separation point, \( f'_{nf} \), accounting for the unsteady leading edge pressure response is defined as

\[
f'_{nf} = \begin{cases} 
1 - 0.3e^{-\frac{[\alpha_{nf} - \alpha_{n}]}{s_{1}}} & \text{if } |\alpha_{nf}| \leq \alpha_{1} \\
0.04 + 0.66e^{-\frac{[\alpha_{nf} - \alpha_{n}]}{s_{2}}} & \text{if } |\alpha_{nf}| > \alpha_{1}
\end{cases}
\]  

(A.48)

An additional separation point, \( f''_{nf} \), accounting for the lag in the boundary layer response due to unsteady effects is given by

\[f''_{nf} = f'_{nf} - D_{nf}
\]  

(A.49)

Where the deficiency function \( D_{nf} \) is given by

\[
D_{nf} = D_{nf-1} e^{\left(-\frac{-\Delta S}{T_f}\right)} + \left(f'_{nf} - f'_{nf-1}ight) e^{\left(-\frac{-\Delta S}{2T_f}\right)}
\]  

(A.50)

The nonlinear normal force coefficient accounting for separation effects is given by
The nonlinear pitching moment coefficient accounting for separation effects is given by

\[
C'_{m_n} = C_{m_n} + \left( \frac{1 + \sqrt{f_n^*}}{2} \right)^2 \alpha_{m_n} + C_{m_n} + C_{m_n}'
\]  
(A.51)

Finally, the nonlinear chord force coefficient accounting for separation effects is given by

\[
C'_{c_n} = C_{c_n} + \left\{ k_0 + k_1 (1 - f_n^*) + k_2 \sin \left( \pi \left( f_n^* \right)^{1/2} \right) \right\} C_{c_n}' - \frac{\pi}{8 \beta} \left( \alpha_n - \mathcal{X}_n^{(1)} \right) \frac{c}{u_n} + C_{c_n} + C_{c_n}'
\]  
(A.52)

Where the factor \( KD \) removes discontinuities in the chord force at separation.

\[ KD = D_{FD} \left( C'_{c_n} - C_{c_n} \right) \]  
(A.53)

A.3.3 Dynamic Stall

As previously stated, the phenomenon of dynamic stall describes the formation and detachment of a vortex from the leading edge of the airfoil, which significantly alters the normal force and pitching moment coefficients. Under unsteady conditions, the vortex is assumed to form at the leading edge of the airfoil but have no effect on the forces and moments. Once a critical leading edge pressure has been attained, the vortex detaches from the leading edge and is swept downstream. While the vortex is traversing the chord, it induces large changes in the normal force and pitching moment.

The increment in the vortex lift, \( C_{v_n} \), due to the existence of the vortex at the leading edge is given by
The total accumulated vortex lift, \( C^v_{n} \), is allowed to decay exponentially with time, which represents the effects of viscous diffusion and the decreasing influence of the vortex as it is shed downstream. In addition, the total accumulated vortex lift is updated by the new increment in the vortex lift and is given by

\[
C^v_{n} = C^v_{n-1} e^{-\frac{\Delta \tau}{T}} + \left( C^v_{n} - C^v_{n-1} \right) e^{-\frac{\Delta \tau}{2T}},
\]  

(A.55)

Once the critical leading edge pressure has been attained, the vortex is assumed to begin traversing the chord. While the vortex is traversing the chord, the vortex lift continues via Eqns. A.54 and A.55; however, when the vortex reaches the trailing edge the accumulation is ceased. To calculate the effect of the vortex on the pitching moment it is necessary to know its position along the chord and subsequently, its effect on the airfoil center of pressure. The position of the vortex along the airfoil chord is tracked with the parameter \( \tau_v \), measured in semi-chords. When \( \tau_v = 0 \) the vortex is at the leading edge and when \( \tau_v = T_{vl} \) the vortex has reached the trailing edge. The vortex travels significantly slower than the freestream, at approximately one-third to one-half its value. Knowing the position of the vortex, an empirical relation for the center of pressure is

\[
CP_v = 0.20 \left[ 1 - \cos \left( \frac{\pi \tau_v}{T_{vl}} \right) \right],
\]  

(A.56)

The additional pitching moment coefficient due to the aft-moving center of pressure is

\[
C^v_{m_n} = -CP_v C^v_n
\]  

(A.57)
The phenomenon of secondary vortex shedding described in Ref. 84 has not been accounted for in this analysis. At this time, it is not clear whether this phenomenon actually occurs on helicopter rotor blades.

A.3.4 Flow Reattachment after Dynamic Stall

After the vortex traverses the airfoil, the flow eventually reattaches. Reattachment has been shown to significantly affect the pitching moment but not the lift. To reattach, the normal force must be less than the critical leading-edge pressure and the pitch rate must be negative for upper surface reattachment and positive for lower surface reattachment. A quasi-steady trailing edge separation point, \( f_{n_{qs}} \), is defined as

\[
\alpha_{n} - k_{s} = \frac{1 - 0.3e^{-\frac{|\alpha_{n} - k_{s}|}{s_{i}}}}{1 - 0.3e^{-\frac{|\alpha_{n} - k_{s}|}{s_{i}}}} \quad \text{if } |\alpha_{n}| < \alpha_{1}
\]

\[
0.04 - 0.66e^{-\frac{|\alpha_{n} - k_{s}|}{s_{i}}} \quad \text{if } |\alpha_{n}| > \alpha_{1}
\]

A reattachment point, \( f_{n}' \), is defined as

\[
\text{if } \alpha_{f} \geq 0 \quad f_{n}' = \begin{cases} 
\frac{f_{n}'}{s_{i}} & \text{if } K_{\alpha} \geq 0 \\
\frac{f_{n_{qs}}}{s_{i}} & \text{if } K_{\alpha} < 0
\end{cases}
\]

\[
\text{if } \alpha_{f} < 0 \quad f_{n}' = \begin{cases} 
\frac{f_{n}'}{s_{i}} & \text{if } K_{\alpha} \leq 0 \\
\frac{f_{n_{qs}}}{s_{i}} & \text{if } K_{\alpha} > 0
\end{cases}
\]

A third separation point, \( f_{n_{'}} \), is defined as

\[
f_{n_{'}} = f_{n} - D_{f_{n_{'}}}
\]

Where the deficiency function, \( D_{f_{n_{'}}} \), is given by
\[ D_{f_n} = D_{f_n} e^{\left(\frac{-\Delta \delta}{T_f}\right)} + (f_{n+1}^{r} - f_{n+1}^{r}) e^{\left(\frac{-\Delta \delta}{2T_f}\right)} \] (A.61)

The pitching moment during reattachment is

\[ C_{m}^{f} = C_{m} + \left\{ k_{n} + k_{1} \left(1 - f_{n}^{r}\right) + k_{2} \sin \left(\pi \left(f_{n}^{r}\right)^{2}\right) \right\} C_{m}^{f} - \frac{\pi}{8\beta} \left(\alpha_{n} - X_{n}^{(s)}\right) C_{n}^{s} + C_{m}^{s} + C_{m}^{s} \] (A.62)

### A.3.5 Total Unsteady Airloads

The final normal force coefficient is simply the sum of the nonlinear and vortex induced normal force coefficients

\[ C_{n} = C_{n}^{f} + C_{n}^{v} \] (A.63)

The total chord force coefficient is

\[ C_{c} = C_{c}^{f} = \eta_{c} C_{c_{n}} \alpha_{n}^{2} \sqrt{\int f_{n}^{s} f_{n}^{s}KD} \] (A.64)

Translating them into lift and drag force coefficients yields

\[ C_{l} = C_{c_{n}} \cos \alpha + C_{l} \sin \alpha \]
\[ C_{d} = C_{n} \sin \alpha - C_{c} \cos \alpha + C_{d} \] (A.65)

The total pitching moment coefficient is simply the sum of the nonlinear and vortex induced pitching moment coefficients

\[ C_{m} = C_{m}^{f} + C_{m}^{v} \] (A.66)

It should be noted that the time modifications to the model described in Ref. 84 have not been included in this analysis.
Appendix B

ROTOR SYSTEM MODELS

B.1 Baseline Rotor System Model

The properties of the baseline articulated rotor system, representative of a medium-sized naval helicopter, are listed in Table B.1. The spanwise variations of the structural properties are shown in Figure B.1.

<table>
<thead>
<tr>
<th>Table B.1: Baseline Rotor System Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotor Type</td>
</tr>
<tr>
<td>Number of Blades</td>
</tr>
<tr>
<td>Uniform Mass Distribution</td>
</tr>
<tr>
<td>Full Rotor Speed</td>
</tr>
<tr>
<td>Radius</td>
</tr>
<tr>
<td>Chord</td>
</tr>
<tr>
<td>Flap Hinge Location</td>
</tr>
<tr>
<td>Lag Hinge Location</td>
</tr>
<tr>
<td>Pitch Bearing Location</td>
</tr>
<tr>
<td>First Flap Frequency</td>
</tr>
<tr>
<td>First Lag Frequency</td>
</tr>
<tr>
<td>First Torsion Frequency</td>
</tr>
<tr>
<td>Flap Stop Settings</td>
</tr>
<tr>
<td>Lag Stop Settings</td>
</tr>
<tr>
<td>Effective Blade Twist</td>
</tr>
<tr>
<td>Lock Number</td>
</tr>
<tr>
<td>Airfoil Type</td>
</tr>
<tr>
<td>Root Cutout</td>
</tr>
</tbody>
</table>
Figure B.1: Spanwise Variations of the Baseline Blade Properties

a) Axial Stiffness

b) Normal Stiffness

c) Chordwise Stiffness

d) Torsional Stiffness

e) Weight per Unit Length

f) Pitch Inertia

g) Shear Center to Quarter-chord Offset

h) Shear Center to Mass Center Offset
B.2 Rigid Teetering Model Rotor System

The properties of the rigid teetering model rotor system used in the University of Southampton tests are listed in Table B.2.

<table>
<thead>
<tr>
<th>Table B.2: University of Southampton Teetering Model Rotor System Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Blades</td>
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<tr>
<td>Uniform Mass Distribution</td>
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<td>Full Rotor Speed</td>
</tr>
<tr>
<td>Radius</td>
</tr>
<tr>
<td>Chord</td>
</tr>
<tr>
<td>Pitch Bearing Location</td>
</tr>
<tr>
<td>Teeter Restraint Angles</td>
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<tr>
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<td>Lock Number</td>
</tr>
<tr>
<td>Airfoil Type</td>
</tr>
<tr>
<td>Root Cutout</td>
</tr>
</tbody>
</table>

B.3 H-46 Sea Knight Rotor System Model

The structural properties of the H-46 rotor system model used in this analysis are listed in Table B.3. The spanwise variations of the structural properties derived from Ref. 91 are shown in Figure B.2. The production H-46 employs a Boeing Vertol designed V23010-1.58 airfoil section. The nonlinear quasi-steady and unsteady aerodynamic properties of the Boeing V23010-1.58 were not available for this analysis so a NACA 0012 was simulated instead.
### Table B.3: H-46 Rotor System Properties

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotor Type</td>
<td></td>
<td>Articulated</td>
</tr>
<tr>
<td>Number of Blades</td>
<td>$N_b$</td>
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</tr>
<tr>
<td>Uniform Mass Distribution</td>
<td>$m_0$</td>
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<tr>
<td>Full Rotor Speed</td>
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<td>27.65 rad/s (264 RPM)</td>
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<td>Radius</td>
<td>$R$</td>
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<td>Chord</td>
<td>$c$</td>
<td>18.75 in</td>
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<td>1.7$%R$</td>
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<tr>
<td>Lag Hinge Location</td>
<td>$e_\zeta$</td>
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<tr>
<td>Pitch Bearing Location</td>
<td>$e_\theta$</td>
<td>6.5$%R$</td>
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<tr>
<td>First Flap Frequency</td>
<td>$\nu_\beta$</td>
<td>1.02 /rev</td>
</tr>
<tr>
<td>First Lag Frequency</td>
<td>$\nu_\zeta$</td>
<td>3.89 /rev</td>
</tr>
<tr>
<td>First Torsion Frequency</td>
<td>$\nu_\theta$</td>
<td>6.05 /rev</td>
</tr>
<tr>
<td>Flap Stop Settings</td>
<td>$\beta_{FS}$</td>
<td>$\pm1^\circ$</td>
</tr>
<tr>
<td>Lag Stop Settings</td>
<td>$\zeta_{LS}$</td>
<td>$\pm12^\circ$</td>
</tr>
<tr>
<td>Effective Blade Twist</td>
<td>$\theta_{tw}$</td>
<td>-8.50$^\circ$</td>
</tr>
<tr>
<td>Lock Number</td>
<td>$\gamma$</td>
<td>7.96</td>
</tr>
<tr>
<td>Airfoil Type</td>
<td></td>
<td>Assumed NACA 0012</td>
</tr>
<tr>
<td>(Actual V23010-1.58)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Root Cutout</td>
<td></td>
<td>25%</td>
</tr>
</tbody>
</table>
Figure B.2: Spanwise Variations of H-46 Blade Properties
B.4 Rigid Gimballed Model Rotor System Properties

The properties of the rigid gimballed rotor system are listed in *Table B.4*.

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Blades</td>
<td>$N_b$</td>
<td>3</td>
</tr>
<tr>
<td>Mass Moment of Inertia</td>
<td>$I_b$</td>
<td>742.6 slug-ft$^2$</td>
</tr>
<tr>
<td>Full Rotor Speed</td>
<td>$\Omega_0$</td>
<td>41.57 rad/s (397 RPM)</td>
</tr>
<tr>
<td>Radius</td>
<td>$R$</td>
<td>19 ft</td>
</tr>
<tr>
<td>Gimbal Restraint Angle</td>
<td>$\beta_r$</td>
<td>11º</td>
</tr>
<tr>
<td>Gimbal Spring Stiffness</td>
<td>$K_\beta$</td>
<td>250 ft-lb/deg</td>
</tr>
<tr>
<td>Pitch-Flap Coupling</td>
<td>$k_p$</td>
<td>0.274</td>
</tr>
<tr>
<td>Precone</td>
<td>$\beta_p$</td>
<td>2.5º</td>
</tr>
<tr>
<td>Effective Blade Twist</td>
<td>$\theta_{tw}$</td>
<td>-47.5º</td>
</tr>
<tr>
<td>Lock Number</td>
<td>$\gamma$</td>
<td>5.48</td>
</tr>
</tbody>
</table>
Appendix C

SFS SHIP AIRWAKE PREDICTIONS

*Figure C.1* shows a top view of the SFS ship. Wind-over-deck direction is measured clockwise from starting at the bow. *Figure C.2* shows the airwake velocities for WOD directions from 180° to 0° in 30° increments in the counterclockwise direction. The velocities shown are in the plane of the rotor disk, assumed 16 ft above the flight deck. Three different landing spots are shown, each has a radius of 25 ft. Spot #1 is centered 30 ft behind the hangar wall, Spot #2 is centered 60 ft behind the hangar wall, and Spot #3 is centered 90 ft behind the hangar wall at the extreme rear of the flight deck.

*Figure C.1*: Simple Frigate Shape (SFS) Top View
\[ \vec{V}_x \]
\[ \vec{V}_y \]
\[ \vec{V}_z \]

a) \( V_{WOD} = 40 \) knots

b) \( V_{WOD} = 50 \) knots

*Figure C.2: SFS Ship Airwake Mean Velocities for \( \psi_{WOD} = 0^\circ \)
Figure C.3: SFS Ship Airwake Mean Velocities for $\psi_{WOD} = 330^\circ$

a) $V_{WOD} = 40$ knots 

b) $V_{WOD} = 50$ knots
\[ \vec{V}_x \]
\[ \vec{V}_y \]
\[ \vec{V}_z \]

a) \( V_{WOD} = 40 \) knots

b) \( V_{WOD} = 50 \) knots

Figure C.4: SFS Ship Airwake Mean Velocities for \( \psi_{WOD} = 300^\circ \)
a) $V_{WOD} = 40$ knots

b) $V_{WOD} = 50$ knots

Figure C.5: SFS Ship Airwake Mean Velocities for $\psi_{WOD} = 270^\circ$
Figure C.6: SFS Ship Airwake Mean Velocities for $\psi_{WOD} = 240^\circ$
Figure C.7: SFS Ship Airwake Mean Velocities for $\psi_{WOD} = 210^\circ$
Figure C.8: SFS Ship Airwake Mean Velocities for $\psi_{WOD} = 180^\circ$

a) $V_{WOD} = 40$ knots

b) $V_{WOD} = 50$ knots
Appendix D

AERODYNAMIC MODELING FOR FEEDBACK CONTROL

The equations of motion for the rotor in the fixed frame are given by

\[
\mathbf{M}_s \begin{bmatrix} \ddot{\beta}_t \\ \dot{\beta}_t \\ \beta_t \end{bmatrix} + \mathbf{C}_s \begin{bmatrix} \dot{\beta}_t \\ \dot{\beta}_t \end{bmatrix} + \mathbf{K}_s \begin{bmatrix} \beta_t \end{bmatrix} = \begin{bmatrix} M_{A_{\psi}} \\ M_{A_{\theta}} \end{bmatrix} \tag{D.1}
\]

Where \( M_{A_{\theta}} \) and \( M_{A_{\psi}} \) are the nondimensional moments about the rotor hub due to aerodynamic force \( F_z \) and are given by

\[
\begin{bmatrix} M_{A_{\theta}} \\ M_{A_{\psi}} \end{bmatrix} = \frac{2}{N_k I_x \Omega_0} \begin{bmatrix} \sum_{i=1}^{N_k} \int_0^R F_i r dr \cos \psi_i \\ \sum_{i=1}^{N_k} \int_0^R F_i r dr \sin \psi_i \end{bmatrix} \tag{D.2}
\]

Both the rotor motion and position contribute to the moment of the aerodynamic forces about the rotor hub. A very simple, quasi-steady, fully attached flow model for determining the aerodynamic forces is utilized. This allows the motion dependent aerodynamic contributions to be moved to the left-hand side of Eqn. D.1. A schematic of the rotor is shown in *Figure D.1.*

![Figure D.1: Gimballed Rotor Schematic](image)
A side view of a typical rotor blade section is shown in Figure D.2.

Assuming the angle of attack is small, the aerodynamic force is approximated by

\[ F_z = L \cos \alpha - D \sin \alpha = L \]

\[ L = \frac{1}{2} \rho V^2 C_l \]  

(D.3)

Furthermore, the following assumptions are made

\[ V^2 = U_T^2 + U_P^2 = U_T^2 \]

\[ C_l = C_{l_i} \alpha = C_{l_i} (\theta_0 - \phi) \]

\[ \phi = \tan^{-1} \left( \frac{U_P}{U_T} \right) = \frac{U_P}{U_T} \]  

(D.4)

Where \( \phi \) is the inflow angle. Substitution of Eqn. D.4 into D.3 yields

\[ F_z = \frac{1}{2} \rho c C_{l_i} \left( \theta_0 U_T^2 - U_P U_T \right) \]  

(D.5)

The blade pitch angle \( \theta_0 \) is a function of the control system inputs, built-in twist, and the pitch-flap coupling

\[ \theta_0 = \theta_{\gamma s} + \theta_{\psi} \cos \psi + \theta_{\psi} \sin \psi + \theta_{\alpha} \left( x - \frac{1}{2} \right) - k_p \left( \beta_i - \beta_p \right) \]  

(D.6)

The blade section velocities in the shaft plane, \( U_T \) and \( U_P \), are given by
\[ U_T = \Omega r + \nabla_i \sin \psi_i + \nabla_i \cos \psi_i \]
\[ U_P = r \beta_i + (\nabla_i \cos \psi_i - \nabla_i \sin \psi_i) \beta_i - \nabla_z \]  
(D.7)

The mean ship airwake velocities \( \nabla_x, \nabla_y \) and \( \nabla_z \) are defined using the deterministic airwake distributions described in Chapter 2. For convenience, they are restated here.

The in plane mean ship airwake velocities for each of the distributions are defined as

\[ \nabla_x = V_{\text{wod}} \cos \psi_{\text{wod}} \]
\[ \nabla_y = -V_{\text{wod}} \sin \psi_{\text{wod}} \]  
(D.8)

The out-of plane mean ship airwake velocity for each airwake distribution is given by

\[
\nabla_z = \begin{cases} 
0 & \text{horizontal} \\
V_{\text{wod}} \sin \left( \psi_i - \left( \frac{\pi}{2} - \psi_{\text{wod}} \right) \right) & \text{constant} \\
V_{\text{wod}} x \sin \left( \psi_i - \left( \frac{\pi}{2} - \psi_{\text{wod}} \right) \right) & \text{linear} 
\end{cases}
\]  
(D.9)

Where the maximum vertical velocity is given by

\[ V_{\text{wod}} = \kappa V_{\text{wod}} \]  
(D.10)

Substitution of Eqns. D.5 through D.9 into Eqn. D.2 yields

\[
\begin{bmatrix}
M_A \\
M_A^b
\end{bmatrix} = \chi \begin{bmatrix}
\sum_{i=0}^{\infty} \int_0^1 (\theta_i U_i^2 - U_i U_i) x dx \cos \psi_i \\
\sum_{i=0}^{\infty} \int_0^1 (\theta_i U_i^2 - U_i U_i) x dx \sin \psi_i
\end{bmatrix}
\]  
(D.11)

Where the blade section velocities \( U_T \) and \( U_P \) have been nondimensionalized by the full rotor speed. The integration over the blade length is carried out first, followed by the summation over the number of blades. The following relations are useful in simplifying the result
Only constant terms and integer multiples of the number of blades are transferred to the fixed frame. Using these identities, Eqn. D.11 can be written as

\[
\begin{aligned}
&\begin{bmatrix}
M_{A_i} \\
M_{A_i}
\end{bmatrix} = -C_A \begin{bmatrix}
\beta_{ic} \\
\beta_{ic}
\end{bmatrix} - K_A \begin{bmatrix}
\beta_{ic} \\
\beta_{ic}
\end{bmatrix} + F_A \\
\end{aligned}
\] (D.13)

The aerodynamic damping and stiffness matrices \(C_A\) and \(K_A\) are independent of the airwake distribution. For \(N_b = 3\), they are given by

\[
C_A = \frac{\gamma}{8} \left[ \begin{array}{c}
\Omega + \frac{2}{3} \left( \vec{r}_{ij} \sin 3\psi - \vec{r}_{ij} \cos 3\psi \right)
\end{array} \right] - \frac{4}{3} \left[ \begin{array}{c}
\vec{r}_{ij} \cos 3\psi - \vec{r}_{ij} \sin 3\psi
\end{array} \right]
\]

\[
K_A = \frac{\gamma}{8} \left[ \begin{array}{c}
\Omega + \frac{2}{3} \left( \vec{r}_{ij} \sin 3\psi - \vec{r}_{ij} \cos 3\psi \right)
\end{array} \right] - \frac{4}{3} \left[ \begin{array}{c}
\vec{r}_{ij} \cos 3\psi - \vec{r}_{ij} \sin 3\psi
\end{array} \right]
\] (D.14)

Because the vertical velocity \(\vec{V}_z\) can vary across the rotor disk, the aerodynamic force vector \(F_A\) is dependent on the airwake distribution. For \(N_b = 3\), the aerodynamic force vector for the horizontal airwake distribution is given by

\[
F_A = \frac{\gamma}{8} \left[ \begin{array}{c}
\left( \vec{r}_{ij} \sin 3\psi - \vec{r}_{ij} \cos 3\psi \right) \cos 3\psi + 2\vec{r}_{ij} \sin 3\psi + 4\Omega \vec{r}_{ij} \theta_i \\
\left( \vec{r}_{ij} \sin 3\psi - \vec{r}_{ij} \cos 3\psi \right) \sin 3\psi + 2\vec{r}_{ij} \cos 3\psi + 4\Omega \vec{r}_{ij} \theta_i \\
\left( \vec{r}_{ij} \sin 3\psi - \vec{r}_{ij} \cos 3\psi \right) \cos 3\psi - 2\vec{r}_{ij} \sin 3\psi + 4\Omega \vec{r}_{ij} \theta_i \\
\left( \vec{r}_{ij} \sin 3\psi - \vec{r}_{ij} \cos 3\psi \right) \sin 3\psi - 2\vec{r}_{ij} \cos 3\psi + 4\Omega \vec{r}_{ij} \theta_i \\
\left( \vec{r}_{ij} \sin 3\psi - \vec{r}_{ij} \cos 3\psi \right) \cos 3\psi + 2\vec{r}_{ij} \sin 3\psi + 4\Omega \vec{r}_{ij} \theta_i \\
\left( \vec{r}_{ij} \sin 3\psi - \vec{r}_{ij} \cos 3\psi \right) \sin 3\psi + 2\vec{r}_{ij} \cos 3\psi + 4\Omega \vec{r}_{ij} \theta_i \\
\left( \vec{r}_{ij} \sin 3\psi - \vec{r}_{ij} \cos 3\psi \right) \cos 3\psi - 2\vec{r}_{ij} \sin 3\psi + 4\Omega \vec{r}_{ij} \theta_i \\
\left( \vec{r}_{ij} \sin 3\psi - \vec{r}_{ij} \cos 3\psi \right) \sin 3\psi - 2\vec{r}_{ij} \cos 3\psi + 4\Omega \vec{r}_{ij} \theta_i \\
\end{array} \right]
\] (D.15)

For \(N_b = 3\), the aerodynamic force vector for the constant airwake distribution is given by
For $N_b = 3$, the aerodynamic force vector for the linear airwake distribution is given by

\[
F_x = \frac{V}{8} \left\{ \begin{array}{l}
\left[ -\left( \nabla_c - \nabla_c^* \right) \cos 3\psi + 2\nabla_c \nabla_i \sin 3\psi + \frac{1}{2} \Omega \nabla_i \right] \theta_x \\
+ \left[ \Omega^2 + \frac{1}{2} (\nabla_i + 3\nabla_i^*) + \frac{1}{2} \Omega (\nabla_i \sin 3\psi + \nabla_i \cos 3\psi) \right] \theta_x + \left[ \nabla_c \nabla_i - \frac{1}{2} \Omega (\nabla_i \cos 3\psi - \nabla_c \sin 3\psi) \right] \beta_x + 2\nabla_c \nabla_i \end{array} \right. \\
\left[ \frac{1}{2} (\nabla_c - \nabla_c^*) \sin 3\psi - 2\nabla_c \nabla_i \cos 3\psi + \frac{1}{2} \Omega \nabla_i \right] \theta_x \\
+ \left[ \nabla_c \nabla_i - \frac{1}{2} \Omega (\nabla_i \cos 3\psi - \nabla_c \sin 3\psi) \right] \theta_x + \left[ \Omega^2 + \frac{1}{2} (3\nabla_i^* + \nabla_i^* - \frac{1}{2} \Omega (\nabla_i \sin 3\psi + \nabla_i \cos 3\psi) \right] \beta_x \\
\left[ \frac{1}{2} (\nabla_c - \nabla_c^*) \sin 3\psi + \frac{1}{2} \nabla_c \nabla_i \cos 3\psi \right] \theta_x + \left[ \frac{1}{2} (\nabla_c - \nabla_c^*) \cos 3\psi + \frac{1}{2} \Omega \nabla_i \right] \beta_x + 2\nabla_c \nabla_i \end{array} \right.
\]  

\[
(D.16)
\]

\[
F_y = \frac{V}{8} \left\{ \begin{array}{l}
\left[ -\left( \nabla_c - \nabla_c^* \right) \cos 3\psi + 2\nabla_c \nabla_i \sin 3\psi + \frac{1}{2} \Omega \nabla_i \right] \theta_y \\
+ \left[ \Omega^2 + \frac{1}{2} (\nabla_i + 3\nabla_i^*) + \frac{1}{2} \Omega (\nabla_i \sin 3\psi + \nabla_i \cos 3\psi) \right] \theta_y + \left[ \nabla_c \nabla_i - \frac{1}{2} \Omega (\nabla_i \cos 3\psi - \nabla_c \sin 3\psi) \right] \beta_y + 2\nabla_c \nabla_i \end{array} \right. \\
\left[ \frac{1}{2} (\nabla_c - \nabla_c^*) \sin 3\psi - 2\nabla_c \nabla_i \cos 3\psi + \frac{1}{2} \Omega \nabla_i \right] \theta_y \\
+ \left[ \nabla_c \nabla_i - \frac{1}{2} \Omega (\nabla_i \cos 3\psi - \nabla_c \sin 3\psi) \right] \theta_y + \left[ \Omega^2 + \frac{1}{2} (3\nabla_i^* + \nabla_i^* - \frac{1}{2} \Omega (\nabla_i \sin 3\psi + \nabla_i \cos 3\psi) \right] \beta_y \\
\left[ \frac{1}{2} (\nabla_c - \nabla_c^*) \sin 3\psi + \frac{1}{2} \nabla_c \nabla_i \cos 3\psi \right] \theta_y + \left[ \frac{1}{2} (\nabla_c - \nabla_c^*) \cos 3\psi + \frac{1}{2} \Omega \nabla_i \right] \beta_y + 2\nabla_c \nabla_i \end{array} \right.
\]  

\[
(D.17)
\]
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