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NURSING STUDENTS' CONCEPTIONS OF DIMENSIONAL ANALYSIS FOR CALCULATING MEDICATION DOSAGE

A Dissertation in

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by

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ABSTRACT

This study examines how nursing students think about and make sense of dimensional analysis, a common mathematical procedure for calculating medication dosage (Curren, 2010; Greenfield et al., 2006). Participants in this study include ten pre-licensure nursing students from a small, private health sciences college located in the northeastern United States. Data were collected in two phases: (1) asynchronously through e-mail, and (2) 60-minute semi-structured, task-based interviews held through Zoom. In each of these phases, the participating students used dimensional analysis to complete intentionally designed dosage calculation tasks. Students' submitted work, and their actions and statements during the task-based interview, were analyzed using a hybrid coding scheme (Miles et al., 2020). Analytic memos were created to capture researcher reflections and facilitate the synthesis of overarching themes in the data (Maxwell, 2013). Drawing upon a recent specification of *mathematical conception* as a researcher-constructed model (Simon, 2017), nine distinct conceptions of dimensional analysis emerged from the data, including those relating to *how* the nursing students completed dosage calculations with dimensional analysis, *why* they chose to use dimensional analysis to calculate dosage, and the proportional reasoning strategies they used to support their completed dimensional analysis work. The results indicate that nursing students may utilize different approaches of dimensional analysis to complete dosage tasks, with some illustrating a more-flexible perspective on dimensional analysis. The students in this study also employed a variety of proportional reasoning strategies to make sense of their completed dimensional analysis work. These results contribute to the literature by offering novel insights into *how* and *why* nursing students utilize dimensional analysis. The nine conceptions developed in this study offer an empirically-grounded starting point for conceptualizing and articulating the ways in which nursing students

make sense of and understand dimensional analysis as a method for calculating medication dosage.

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Chapter 1

Introduction

In 1999, the Institute of Medicine released the report, *To Err is Human: Building a Safer Health System*, which brought international attention to the impact of preventable errors in medical settings. Extrapolating on data from two United States hospitals, the report claimed that more Americans die each year from medical errors (44,000) than they do from motor vehicle accidents (43,458), breast cancer (42,297), or AIDS (16,516) (Donaldson et al., 2000). In addition to providing sobering statistics and estimates for the costs associated with these errors, the report recommended the creation of a National Center for Patient Safety and encouraged members of the health care community to improve patient safety by studying how and why medical errors occur (Donaldson, 2008). Not surprisingly, in the years following the report's publication, research awards and publications on patient safety increased, including those that explored factors contributing to medical errors (Stelfox et al., 2006). In one such publication, Armitage et al. (2003) identified institutional policies and procedures, distractions in the workplace, workload and staffing issues, nurses' knowledge of medications, and – most pertinent to this study – nurses' mathematical skills, as contributing to the frequency of medication administration errors in practice. The identification of nurses' mathematical skills as a contributing factor is consistent with additional findings that weak mathematical skills and struggles to conceptualize quantitative information in clinical settings contribute to the likelihood of making a medication error in practice (Brady et al., 2009; Preston, 2004).

This raises a number of important questions about how mathematics is used in nursing practice. What mathematical competencies and skills are necessary for safe practice? In what specific situations do nurses apply these competencies and skills? What can be done to prevent these errors in practice? Studies have consistently identified a number of mathematical competencies and skills that are foundational for safe nursing practice, including: operating with whole numbers, rational numbers, percentages, and ratios; converting between systems of measurement; solving missing-value proportions; and applying basic algebraic principles to solve equations and simplify formulas (Pirie, 1987; Roberts, 1990; Young, Weeks, & Hutton, 2013). According to O'Shea (1999), nurses' proficiency in these areas of mathematics is necessary to monitor patients' intake and output, regulate intravenous fluids, and calculate the appropriate amount of medication (i.e., medication dosage) to administer to patients.

Given the importance of accurately applying mathematical skills for patient safety, much of the research literature focuses on medication dosage calculations and understanding the calculation errors in the medication administration process. This study aims to contribute to the research literature by examining how students of nursing practice, who are learning medication dosage calculations, conceptualize a common computational method used in practice and throughout the nursing education community. The sections below offer further context about medication dosage calculations and how educators have sought to support nurses' learning of the mathematics that is critical for patient safety.

Medication Dosage Calculations

A medication dosage calculation (herein referred to as *dosage calculation*) is essentially a conversion between measurements. When nurses complete dosage calculations, they are given information about how much medication a patient needs (often as a one-time amount or as a rate of administration), but it is not in the desired units that must be accurately measured and administered. The given information, often referred to as the *desired dose*, is usually presented within the framework of a medication order, which can also include instructions for how, when, and how often to administer the medication (Booth et al., 2012). Prior to performing any calculations, a nurse must interpret the medication order, consider the dosage strength or concentration of the available medication, and recall known conversion factors between the quantities that are needed for the calculation (Lesmeister, 2017).

An example of a medication order as commonly seen in educational settings is found in Figure 1-1. The order states that the patient should receive 0.32 grams ("g") of acetaminophen as an oral suspension ("oral sus"), every six hours ("q6h"), as needed for pain ("prn"). Given the desired dose of 0.32 grams, a nurse must locate the strength of the available medication (given as "160 milligrams per 5 milliliters" on the provided label) and confirm that it is consistent with how the medication is ordered to be given (a liquid for an oral suspension). Since the desired dose is a measurement of grams, and the strength of the medication incorporates a unit of milligrams, the nurse must to perform a metric conversion using the relationship between these units (i.e. 1 gram is equivalent to 1000 milligrams), before converting that result into a number of milliliters (mL) to administer to the patient. In a clinical situation, the nurse would then confirm this calculated value with a peer before preparing the medication with the appropriate tool (e.g. oral medicine cup, oral syringe). Preparing the medication with the appropriate tool is not included in Figure 1-1 and it is a step that is often omitted in dosage calculation problems in educational settings (Young, Weeks, & Hutton, 2013).

Figure 1-1

An Example of a Medication Dosage Order with the Available Medication

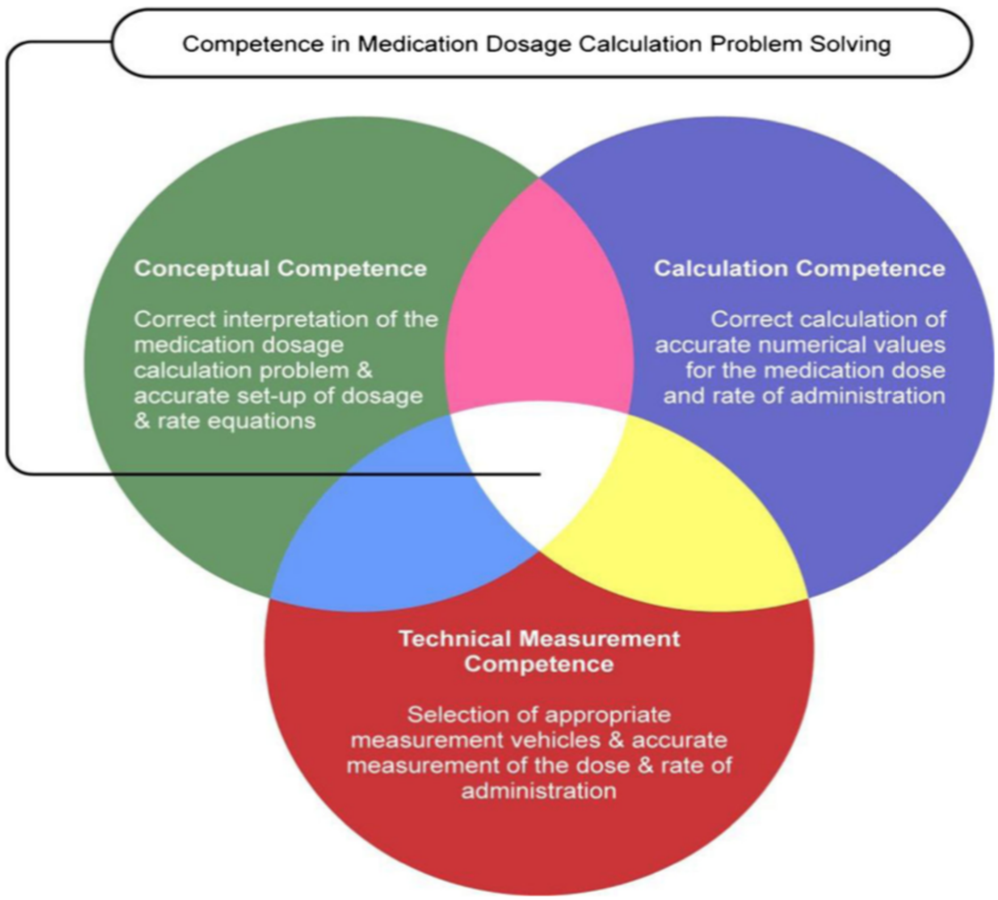
Order: acetaminophen 0.32 g oral sus q6h prn

Available Medication:

A summative view of the dosage calculation process is conceptualized in the Competence in Medication Dosage Calculation Problem Solving Model in Figure 1-2 (Coben & Weeks, 2014). This model posits that competency in dosage calculations consists of conceptualizing the clinical situation and preparing a mathematical representation describing that situation (Conceptual Competence), performing accurate arithmetic operations to calculate the desired quantity (Calculation Competence), and accurately measuring the identified dose in the appropriate administration tool (Technical Measurement Competence). As noted, the dosage example in Figure 1-1 would expect a learner to correctly interpret the nursing-specific elements so that they can be manipulated into an accurate mathematical representation.

Figure 1-2

A Model for Dosage Calculation Competence (Coben & Weeks, 2014, p. 262)



There are a number of ways that a nurse might conceptualize, and then operate with, a mathematical representation of the dosage calculation situation, including utilizing proportional reasoning strategies, a formula (referred to as "the nursing formula"), and dimensional analysis (Gilies, 2004; Wright, 2013). These three methods are often those presented in dosage calculation curriculum materials designed for students in nursing and other healthcare-focused programs (Booth et al., 2012; Lesmeister, 2017). A brief illustration of how these methods might be applied to complete the previous dosage order (Figure 1-1) is provided in Figure 1-3. These

Figure 1-3

An Oral Dosage Calculation Completed Using Three Common Methods

Proportion Method

$$\frac{0.32\ g}{X\ mg} = \frac{1\ g}{1000\ mg} \rightarrow \begin{matrix} X=320 \\ 320\ mg \end{matrix}$$

$$\frac{320\ mg}{X\ mL} = \frac{160\ mg}{5\ mL} \rightarrow \begin{matrix} X=10 \\ 10mL \end{matrix}$$

Nursing Formula

$$\frac{\text{"Desired"}}{\text{"Have"}} \times \text{"Quantity"}$$

$$\frac{320\ mg}{160\ mg} \times 5\ mL$$

$$10mL$$

Dimensional Analysis

$$\frac{0.32\ g}{1} \times \frac{1000\ mg}{1\ g} \times \frac{5\ mL}{160\ mg}$$

$$\rightarrow 10\ mL$$

methods will be discussed in more detail in Chapter 2, but it is important to note that although they are often presented as different methods, they each utilize the same underlying multiplicative relationships and result in the same amount of medication to administer: 10 mL.

Dimensional Analysis and Dosage Calculations

As will be detailed in the next chapter, dimensional analysis is a popular method for calculating medication dosage, and some pre-licensure nursing mathematics textbooks focus exclusively on its use (Craig, 2011; Curren, 2010). It has been argued that dimensional analysis is more “conceptual” in nature and incorporates more problem-solving and reasoning than other calculation methods that incorporate abstract formulas and lead to rote memorization over understanding (Arnold, 1998; Johnson & Johnson, 2002). Since dimensional analysis involves applying consistent steps and reasoning regardless of the situation or clinical context, it eliminates the need for memorizing unique formulas (Greenfield, Whelan, & Cohn, 2006). While there are often multiple ways for setting up a dosage calculation for dimensional analysis (e.g. starting with the units that need to be changed, or starting with the units that are desired at the end of the calculation), the overarching process is the same: one must multiply by conversion factors so that the resulting "unit path" results in eliminating unwanted units (Arnold, 1998). For example, as seen in Figure 1-4, multiplying "0.32 g" by the first conversion factor ("1000 mg / 1 g") results in the units of grams ("g") being eliminated; Then, multiplying by the next factor ("5 mL / 160 mg") results in the elimination of milligrams ("mg"), leaving the final, desired unit of milliliters ("mL"). Finally, all that is left is to perform the appropriate arithmetic operations to arrive at 10 mL. Alternatively, as seen in Figure 1-4, one could begin the calculation by considering the units that are desired at the end (mL). These units are found in the medication strength ("5 mL / 160 mg"), so this ratio would begin the calculation, with the proceeding conversion factors aligning so that the unit path results in canceling all other units (i.e. all units by the desired "mL").

Figure 1-4

Two Approaches to Employing Dimensional Analysis to Calculate Dosage

$$\frac{0.32\,\cancel{g}}{1} \times \frac{1000\,\cancel{mg}}{1\,\cancel{g}} \times \frac{5\,mL}{160\,\cancel{mg}} = 10\,mL$$

Starting with the units that need to be changed

$$mL = \frac{5\,mL}{160\,\cancel{mg}} \times \frac{1000\,\cancel{mg}}{1\,\cancel{g}} \times \frac{0.32\,\cancel{g}}{1}$$
$$= 10\,mL$$

Starting with the units that are desired at the end of the calculation

Results from empirical studies suggest that focusing dosage calculation instruction on dimensional analysis is associated with fewer calculation errors on dosage proficiency tests (Craig, 1993; Greenfield et al., 2006; Rice & Bell, 2005). Given these results, and considering abundant data illustrating that pre-licensure and practicing nurses often struggle with mathematics (Blas & Bath, 1992; McMullen, Jones, & Lea, 2010), it is not surprising that dimensional analysis has become a popular method for instruction on basic conversions and dosage calculations (DeMeo, 2016; Wright, 2013). Additionally, if focusing instruction on dimensional analysis is associated with fewer calculation errors, and if its use could reduce unnecessary dosage calculation errors in practice, then it could be argued that the teaching and learning of dosage calculations should focus on dimensional analysis methods.

However, it is important to note that much of the empirical literature exploring computational methods for calculating dosage relies on the analysis of quantitative data from dosage calculation tests (Wright, 2009). While studies suggest that consistent and organized problem-solving approaches like dimensional analysis can be effective for pre-licensure nursing students seeking numerical solutions to dosage calculation tasks (Blas & Bath, 1992; Craig, 1993; Greenfield et al., 2006; Rice & Bell, 2005), there are unanswered questions about *how* nursing students use these methods to obtain a solution. For example, how do students make sense of dimensional analysis as a method for calculating dosage? In what ways do they employ dimensional analysis as a rote algorithm without a connection to mathematical concepts and reasoning? The answers to these questions are especially important from a mathematics education perspective given the plentiful evidence illustrating that one's ability to obtain a numerical solution to a mathematical task does not mean that the individual applied sound mathematical reasoning, nor that they have a conceptually-supported understanding of the underlying mathematics (Erlwanger, 1973; Hiebert, 1986; National Research Council, 2001). Addressing these questions, and gathering a better understanding of how students think about and make sense of mathematical ideas, likely demands the use of qualitative research methods, including analysis of data from in-depth interviews that provide an opportunity for participants to explain their reasoning as they complete mathematics tasks (Labinowicz, 1985)

This Study

This study addresses a gap and a need in the literature by employing qualitative research methods to examine *how* nursing students think about dimensional analysis as a method for calculating medication dosage. That is, rather than comparing quantitative scores on a dosage proficiency assessment and determining whether individuals can obtain a correct numerical solution to a dosage calculation task, this study explores students' reasoning with dimensional analysis through a qualitative analysis of their mathematical work on particular dosage calculation tasks. Drawing upon a recent specification of *mathematical conception* (Simon, 2017), I aim to make sense of participating students' verbal explanations and mathematical justifications of dimensional analysis as a dosage calculation method. More specifically, this study addresses the following research question: *What are nursing students' conceptions of dimensional analysis as a method for calculating medication dosage?*

The results of this study have the potential to inform educational practices in the mathematics education and nursing education communities by providing novel insights on how students make sense of a crucial tool for performing conversions and calculating medication dosage. A better understanding of how nursing students think about dimensional analysis for calculating dosage will provide an empirically grounded foundation for improving instructional practices and curriculum materials focused on calculating medication dosage for nursing education. The focus of this study is consistent with the goals of a recently launched national initiative seeking to build connections between the mathematics education and nursing education communities to improve quantitative education practices in nursing (Hughes & Zoellner, 2019).

By integrating perspectives from mathematics education and nursing education research, this study also provides a model for future research and interdisciplinary collaboration between these seemingly distinct communities.

Conceptual Frameworks and Theoretical Perspectives

The aim of this study is to understand students' conceptions of dimensional analysis as a method for calculating medication dosage. In the following sections, I provide an overview of several theoretical perspectives on *conception* and describe why I chose a recent conceptualization of *conception* (Simon, 2017) for framing the design of this study.

Perspectives on Mathematical Conception

The term *conception* is often used as a broad construct to denote the ways in which individuals "think about" or perceive a mathematical concept (Roth & Thom, 2009). For example, Sfard (1991) uses conception to denote the "whole cluster of internal representations and associations" (p. 2) that are evoked by a concept and argues that many mathematical notions can be perceived from both operational and structural perspectives. Considering the topic of functions, an individual might think of a function as a computational process in which operations are performed on inputs. This perspective is associated with an operational conception of function. However, Sfard also posits a structural conception of function, in which an individual might think of a function as a "static relation between two magnitudes" (e.g., a set of ordered pairs) (p. 6). Others have taken a similar approach and have used conception to differentiate between ways of thinking about mathematical ideas. In APOS theory, the words *action*, *process*, *object*, and *schema* (APOS) are used to denote the mental stages or structures that an individual might possess around a mathematical idea (Arnon et al., 2014). Once again considering function as an example, an individual with an action conception of function might be able to plug numbers into an algebraic expression and perform simple calculations, but they would be limited to thinking about this as a "one-step-at-a-time" procedure (Breidenbach et al., 1992; Dubinsky & Harel, 1992). Once the individual is able to break away from the step-by-step mentality and can "think about the transformation as a complete activity" (Dubinsky & Harel, 1992; p. 85), then they might possess a process conception of function. Finally, when the individual is able to think about a function as its own object that can be manipulated, such as with function transformations and compositions, then they would exhibit an object conception of function.

Another use of conception relates more closely to an individual's subjective meanings, beliefs, and preferences around a mathematical idea (Furinghetti & Pehkonen, 2002; Lloyd & Wilson, 1998; Philipp, 2007; Thompson, 1992). Take, for example, the seminal case of Benny, a twelve-year-old boy in an Individually Prescribed Instruction (IPI) program designed for students needing remedial work in mathematics (Erlwanger, 1973). Interviews with Benny revealed that the behaviorist approaches to mathematics in the IPI program unintentionally impacted his conceptions of fractions and decimals. That is, through his repetitive interactions with math problems and an answer key, Benny could add and multiply decimals with some success. However, he also developed unique perspectives about how one should operate with decimals and fractions. In one exchange with the interviewer on converting between fractions and decimals, Benny argued that 429/100 = 5.29, 3/1000 = 1.003, and that 0.5 can be written as either 3/2 or 2/3. These are not mathematically accurate statements, however, Benny's experiences with the materials lead him to develop unique ways of thinking about mathematical relationships. It is these individual meanings, views, and beliefs about mathematical ideas that can be referred to as Benny's conceptions.

A similar construct, *concept image*, has been used to refer to all the mental pictures, properties, and characterizations in an individual's mind related to some concept (Tall & Vinner, 1981; Vinner, 1983; Vinner & Dreyfus, 1989). These researchers argue that one's concept image develops over time, and as was the case with Benny, one's experiences do not necessarily lead to consistent or mathematically accurate perspectives (Tall & Vinner, 1981).

A more detailed specification of conception extends the individualized nature to include situational factors that might impact an individual and their thinking related to a mathematical idea. Grounding their work in Brousseau's (1997) Theory of Didactical Situations and Vergnaud's (2009) Theory of Conceptual Field, Balacheff and Gaudin (2013) use conception to describe a complex system between the individual and their learning environment (referred to as a "learner/milieu" system). More specifically, they define conception as "the state of dynamical equilibrium of an action/feedback loop between a learner and a milieu under prescriptive constraints of viability" (p. 5). When learners are "disturbed by the influence of the milieu" (e.g., completing a challenging mathematical task), they will seek to return to equilibrium by "modifying the milieu and/or by engaging in learning by which [they themselves are] changed" (Brousseau & Balacheff, 2002, p. 55). Borrowing language and notation from Vergnaud's Theory of Conceptual Fields, Balacheff and Gaudin suggest that for a given problem or set of problems, the learner will employ a set of actions (operators), semiotic tools from a representation system, and a control structure to arrive at point of equilibrium. According to Balacheff and Gaudin, these four components (set of problems, set of operators, representation system, control structure) constitute a conception, and when taken together, they provide a framework for diagnosing an individual's behaviors around a mathematical idea.

Figure 1-5

A Conception of Addition: "Counting on 16 and 4" as a Prototypical Problem (Balacheff & Gaudin, 2013, p. 7)

P -- *prototype*: "You have 16 pebbles, I give you 4 more, how many do you have now?" (The numbers are given, but the collections are not present, one of the numbers must be small enough)

R -- choose the greater number, count on to determine the result.

L -- body language (finger counting), number naming, verbal counting.

Σ -- order of the number names, match of fingers to number names

As an example, consider the conception of addition suggested by Balacheff and Gaudin (Figure 1-5). For a prototypical problem (P) of "adding 16 pebbles with 4 pebbles," there are a number of actions (R) the individual might employ, such as "adding on from the greater number." There are also a number of semiotic tools (L) to help the individual represent the ideas and relationships in the problem (finger counting, number naming, verbal counting), as well as potential metacognitive behaviors (Σ) to support the individual through the problem-solving process. When these items are taken as a whole, they constitute one conception of addition that characterizes the learner/milieu system.

Another specification of conception comes from the work of Simon (2017). According to Simon, a mathematical conception is "an explanatory model used to explain observed abilities and limitations of mathematical learnings in terms of their (inferred) ways of knowing" (p. 120). This perspective differs from those previously discussed in that a mathematical conception is attributed to the researcher and it is "not a claim about what is true for the learner" (p. 120).

Through careful analysis of an individual's actions and verbalizations, a researcher can generate a model that describes what the individual appears to think, know, and understand about some mathematical idea from the researcher's perspective. This articulated model is an invention of the researcher, and its purpose is to make sense of the data generated by the learner and explain the learner's observed abilities while completing a task. Once articulated, these models of inferred understanding allow researchers and educators to "make distinctions among students… that go beyond how they perform on a particular task" (p. 131). Moreover, they have the potential to "provide a basis for claims of and specification of learning" (p. 131).

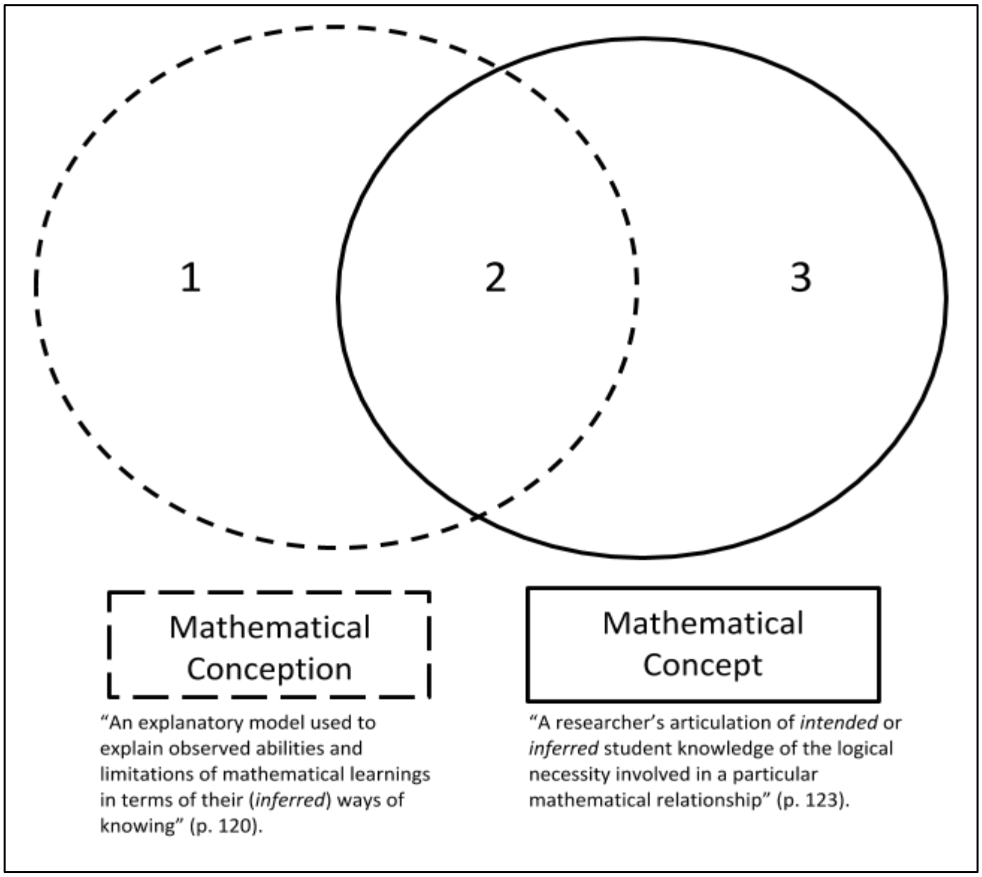
Connections Between Mathematical Conception and Mathematical Concept

Simon's (2017) specification of mathematical conception was chosen for this study because it explicitly connects with the notion of mathematical concept, and it provides researchers with rich language for conducting exploratory qualitative research. In concurrence with his specification of mathematical conception, Simon describes a mathematical concept as an invention of the researcher, stating that it is "a researcher's articulation of intended or inferred student knowledge of the logical necessity involved in a particular mathematical relationship" (p. 123).

As illustrated in the Venn diagram in Figure 1-6, it is possible for an articulated mathematical concept to be a mathematical conception if it is consistent with the model constructed by the researcher to explain the inferred knowing of the learner based on their actions and behaviors ("2" on the diagram). Given the similarities between these constructs, a researcher could articulate an intended mathematical concept, gather data from task-based activities, and examine the extent to which the inferred understandings of the participating individuals align with the articulated concept.

Figure 1-6

Relating Mathematical Conception and Mathematical Concept (Simon, 2017)



Simon (2017) suggests that a mathematical concept is more than a definition or a result of "knowing that" something is true based on inductive processes (e.g., recognizing that multiplying an integer by 6 always yields an even result). Instead, a mathematical concept is the result of a reflective abstraction, which according to the work of Piaget and colleagues, "[consists of] deriving from a system of actions or operations at a lower level" to construct a new, higher-level action (Beth & Piaget, 1966, p. 189). This type of abstraction can be contrasted with empirical abstraction, which "[consists of] deriving the common characteristics from a class of objects" (p. 189). Whereas both types of abstraction lead to the creation of knowledge by performing actions on objects, a reflective abstraction "interiorizes and coordinates these actions to form new actions and, ultimately new objects" (Dubinsky, 2002, p. 98). From this perspective, the construction of logico-mathematical knowledge -- and thus the nature of mathematical concepts -- is not the result of abstractions from one's observations, but rather "an abstraction from one's own activity" (Simon, 2017, p. 122).

As previously described, an articulated mathematical conception is an explanatory model of *inferred* knowing that emerges from closely analyzing an individual's behaviors and actions. This is different from an articulated mathematical concept, which is the intended or inferred knowing of the logical necessity of some mathematical relationship(s). To differentiate these constructs, Simon (2017) provides an example of an articulated mathematical concept describing the relationship between the size of a denominator and the size of a unit fraction: "The denominator of a unit fraction gives the number of parts of that size that make up the related whole. Equal partitioning of a whole into a greater number of parts (sharing it more ways) results in each part being smaller. Therefore, the larger the denominator is, the smaller the unit fraction must be" (p. 123). This articulated mathematical concept describes what a researcher or educator would expect a student to understand, and it captures the "logical necessity that the student would come to know" (p. 133) about the mathematical idea. Once articulated, a mathematical concept could be used as a lesson goal, assessment target, or a component in a learning trajectory (Kara, Simon & Placa, 2018; Norton, 2018; Simon, 2018).

For the purposes of this study, mathematical concepts around dimensional analysis will be articulated to serve as a conceptual foundation for designing some interview tasks and synthesizing inferences from the interview data (to be described in Chapter 3). In Chapter 2, I provide a synthesis of relevant theoretical and empirical literature informing the design of this study.

Chapter 2

Relevant Theoretical and Empirical Literature

In this chapter, I synthesize important theoretical and empirical literature pertinent to the focus of this study. I begin by describing dimensional analysis, its application for converting units in a variety of applied sciences, and research findings related to students' use of dimensional analysis. I then synthesize theoretical perspectives and empirical findings related to ratios, proportions, and students' use of proportional reasoning strategies that are pertinent to this study. Finally, I detail literature on medication dosage calculations, including theoretical perspectives and research findings on nurses' dosage calculation competency, as well as their use of dimensional analysis as a method for computing dosage.

Mathematical and Pedagogical Perspectives of Dimensional Analysis

Dimensional analysis – as its name suggests – refers to the analysis of quantities, their dimensions (e.g., length, mass, time, temperature), and their units of measure (e.g., meter, gram, seconds, kelvin) (Bridgman, 1931; Gibbings, 2011). According to Bridgman (1931), the purpose of dimensional analysis is "to give certain information about the relations which hold between the measurable quantities associated with various phenomena" (p. 17). For example, using the dimensional symbols L and T to represent theoretical quantities of length and time, respectively, one could represent the mathematical nature of other physical phenomena, such as area (L^2), velocity (LT^{-1}), and acceleration (LT^{-2}) (Gibbings, 2011; Pankhurst, 1964). Algebraic manipulations of these representations provide a means to "check the dimensional correctness" of some mathematical solution and examine the "functional dependence" of an unknown quantity on a set of physical parameters (Pankhurst, 1964, p. 16). In these instances, dimensional analysis is often applied to study the mathematical nature of physical quantities and not necessarily their units of measure (p. 13).

Dimensional analysis can also refer to a problem-solving method that specifically focuses on the units of a measured quantity. When used for performing calculations with units, including converting between different systems of measurement, dimensional analysis is referred to as unit analysis, the unit-factor method, or the factor-label method. This application of dimensional analysis is especially useful for engineers, scientists, and other individuals interested in applying model-scale results to corresponding full-scale conditions (Gibbings, 2011; Pankhurst, 1964; Sonin, 2001; Sterret, 2009). Additionally, given its widespread use in many applied sciences, dimensional analysis is frequently taught to students in secondary and post-secondary science courses, including physics, chemistry, and biology (DeMeo, 2008; Fink, 2009). It is this application of dimensional analysis that is the focus of this study.

Underlying dimensional analysis is the idea that "physical laws do not depend on arbitrarily chosen units of measurement" (Barenblatt, 1996, p. 1). For example, consider measuring the length of a desk using one foot as the basic unit of measure. Iterating this basic unit, the length of the desk in Figure 2-1 is found to be 8 feet. While re-measuring the length of the desk with a different basic unit (e.g., inch, meter, mile, etc.) would result in a different numerical value, this does not change the physical nature of the desk; it is not shorter or longer even though measuring with a new unit would result in a different final measurement. Instead, we find there is a unique relationship between the multiplicative factor between the two units of measure and the final measurements in those units. This relationship is explored further in Figure 2-2, where the desk is re-measured in inches; a measure one-twelfth (1/12) the size of a foot. By changing the unit of measure by a factor of one-twelfth (1/12), the final measurement increased by a factor of twelve (12), the reciprocal of one-twelfth (i.e $(1/12)^{-1} = 12$).

Figure 2-1

Iterating "1 ft" to Measure the Length of a Desk

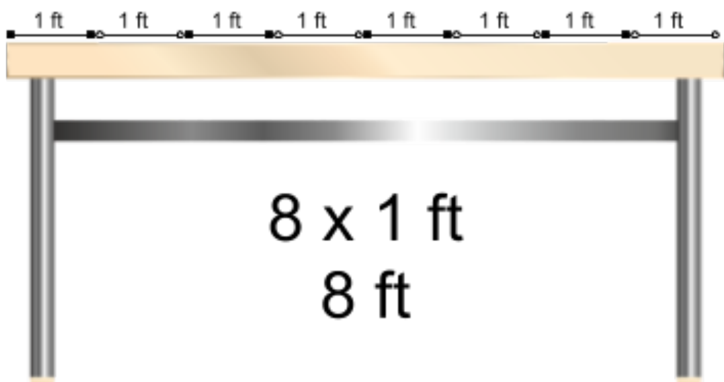
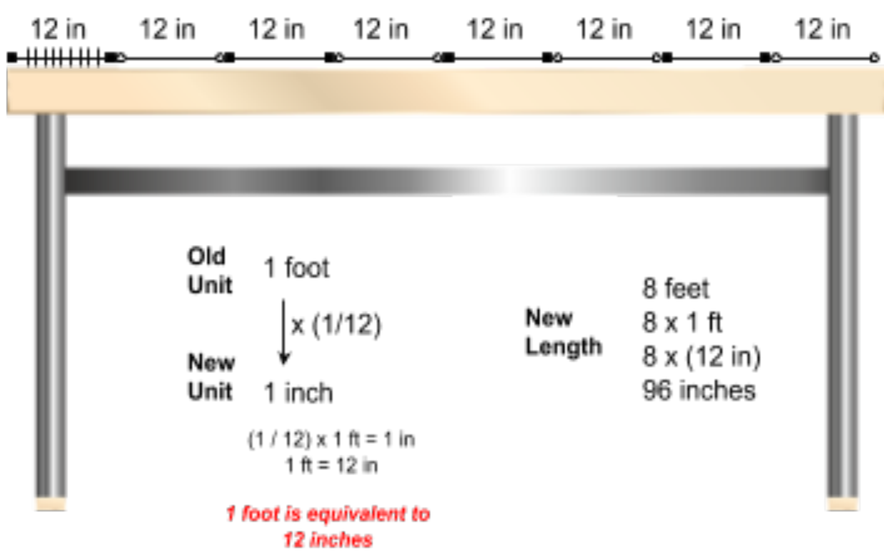


Figure 2-2

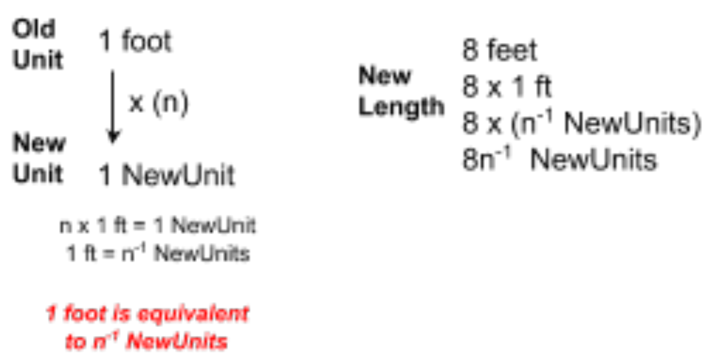
Changing the Unit of Measure by a Factor of 1/12



This relationship can be generalized by considering a change of the unit of measure from one foot to any new unit that is n-times the size of one foot (Figure 2-3). If the new unit is n-times the size of one foot (i.e., n x 1 ft = 1 NewUnit), then each foot is equivalent to n^{-1} NewUnits (1 ft = n^{-1} NewUnits). The length of the desk expressed in the new unit can be calculated by finding the length in feet and then substituting each foot with n^{-1} NewUnits. Put another way, if the desk is 8 feet long, the product "8 x 1 ft" can be rewritten as $8n^{-1}$ NewUnits (Figure 2-3). In the previous example (Figure 2-2), one inch is 1/12 the size of a foot, so the length expressed in inches is $8(1/12)^{-1} = 8(12)$, or 96 inches. This factor, n, often referred to as a *conversion factor* or *units-conversion factor*, plays an essential role in converting between measured quantities (Bridgman, 1933; Gibbings, 2011; Pankhurst, 1964; Sonin, 2001).

Figure 2-3

Generalizing the Relationship with a Unit Changed by a Factor of "n"

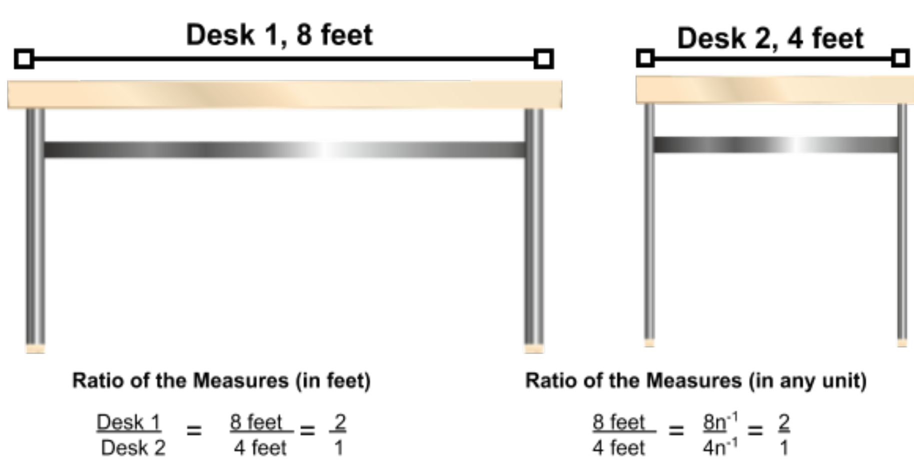


To summarize, changing a unit of measure by a multiplicative factor, n, changes the measurements expressed in that unit by the reciprocal of that factor, n^{-1}, or by some power of n^{-1}, depending on what is being measured (e.g., changing the unit of time in a measurement of acceleration would impact the new measurement by the square of n^{-1}). A direct consequence of this relationship is Bridgman's principle of absolute significance of relative magnitude, which states the ratio of the measures of any two particular quantities has an absolute significance, independent of the size of the units (Bridgman, 1933, p. 19). Using the example of two desks with lengths of 8 feet and 4 feet, respectively, the principle of absolute significance of relative magnitude tells us that the ratio of these lengths will remain constant (2:1), regardless of the chosen unit of measure (e.g., feet, inches, meters). That is, when they are measured with the same unit, the larger desk will always be two times longer than the smaller desk.

This relationship is explored further in Figure 2-4. Changing the unit of measure from feet to a unit n-times larger, results in the 8-foot desk being converted to a new length, $8n^{-1}$. Similarly, a desk that is four feet in length would be $4n^{-1}$ units when measured with a unit n-times larger than one foot. Thus, constructing a ratio with these new measures, $8n^{-1}$ and $4n^{-1}$, simplifies to 2:1. This proportional relationship provides a powerful and efficient framework for converting physical measurements into new units.

Figure 2-4

The Ratio of the Lengths of Two Desks Remaining Constant



Dimensional Analysis as a Method for Converting Units

The principles above form the foundation of dimensional analysis as a method for converting a measurement into a new unit. This conversion process requires a few important elements, including a quantity measured in some initial unit and a desired unit to convert to. Additionally, it is essential that one knows the appropriate conversion factor, or the invariant multiplicative factor connecting the two units of measure (DeMeo, 2008; Pankhurst, 1964). For example, in the previous example, it was necessary to know that one inch was equivalent to one-twelfth (1/12) of a foot prior to converting measurements from feet to inches. It would not have been possible to convert the length into an equivalent number of inches without this information (unless the desk was physically re-measured with the new unit). Knowing the multiplicative factor linking the initial and new unit of measure (i.e., units-conversion factor) means one can convert the measurement using the reciprocal of the multiplicative factor.

This method of converting units can be extended to a variety of physical quantities, including those represented as rates. Figure 2-5 illustrates how one could convert 88 feet per second (ft/sec) into an equivalent number of miles per hour (Bridgman, 1933). First, it is essential to consider the conversion factors between the initial units and desired units: a mile is 5280-times larger than the unit of a foot, and an hour is 3600-times larger when compared with the unit of a second. This means that 1 foot is equivalent to (1/5280) of a mile, and 1 second is equivalent to (1/3600) of an hour. These reciprocal values can be substituted into the units of "feet" and "sec," just as was done when converting the length of the desks. After a few algebraic manipulations, the new velocity, 60 miles per hour, is obtained by multiplying the initial velocity by (3600/5280).

Figure 2-5

Converting Between Measures of Velocity (Bridgman, 1933, p. 28)

$$88\frac{ft}{sec} = 88 \times \frac{1\ ft}{1\ \text{sec}}$$

$$= 88 \times \frac{\frac{1}{5280}\ mile}{\frac{1}{3600}\ hour}$$

$$= 88 \times \frac{3600}{5280}\frac{mile}{hour}$$

$$= 60\frac{mile}{hour}$$

The structure in this example closely resembles the ways that dimensional analysis is often presented in secondary and post-secondary textbooks (i.e., as the *unit-factor method* or *factor-label method*). That is, the specific placement and orientation of conversion factors resembles a pattern that allows one to focus on “canceling” one unit and replace it with another (DeMeo, 2008; Ellis, 2013; McClure, 1995). This domino-like pattern is even more apparent when one applies the principle of absolute significance of relative magnitude (Bridgman, 1933) to perform a multi-step conversion.

Consider the example of converting 750 grams to an equivalent number of pounds. We will assume we do not know the direct multiplicative factor linking grams and pounds, but we are certain that 1000 grams (g) is equivalent to 1 kilogram (kg), and that 1 kilogram approximately 2.2 pounds (lb). These conversion factors can be used to first convert 750 grams into an equivalent number of kilograms, and then into an equivalent number of pounds (Figure 2-

Figure 2-6

A Domino Pattern of Units Developing in a Multi-step Conversion

(A.) $$\frac{1000\ g}{750\ g} = \frac{1\ kg}{?\ kg} \qquad \frac{?\ kg}{750\ g} = \frac{1\ kg}{1000\ g}$$

(B.) $$\frac{x\ kg}{750\ g} = \frac{1\ kg}{1000\ g} \longrightarrow x\ kg = \boxed{750\ g \times \frac{1\ kg}{1000\ g}}$$

(C.) $$\frac{y\ lb}{x\ kg} = \frac{2.2\ lb}{1\ kg} \longrightarrow y\ lb = \boxed{x\ kg} \times \frac{2.2\ lb}{1\ kg}$$

(D.) $$y\ lb = x\ kg \times \frac{2.2\ lb}{1\ kg} \longrightarrow y\ lb = \boxed{750\ g \times \frac{1\ kg}{1000\ g}} \times \frac{2.2\ lb}{1\ kg}$$

(E.) $$y\ lb = 750\ \cancel{g} \times \frac{1\ \cancel{kg}}{1000\ \cancel{g}} \times \frac{2.2\ lb}{1\ \cancel{kg}}$$

6). Focusing first on converting 750 grams into kilograms, the *principle of absolute significance of relative magnitude* asserts there is a directly proportional relationship between the measurement of some object in grams and its measurement in kilograms. This means 750 grams, the conversion factor between grams and kilograms, and some unknown measurement in kilograms can be represented in multiple ways using equivalent ratios (Row A.).

Manipulating either of these proportions to isolate the unknown quantity leads to a result where the equivalent number of kilograms can be found by multiplying 750 by (1/1000) (Row B.). However, rather than performing the multiplication, "x" can be used to represent the equivalent value of kilograms, and another missing-value proportion between pounds and kilograms (Row C.) can be set up. Just as before, this proportion can be rearranged to isolate the unknown quantity of pounds ("y"). Substituting in the expression for the quantity of kilograms ("x") results in a final expression representing the number of pounds. Row E. of Figure 2-6 clearly illustrates the domino pattern of units that develops as one proceeds through the conversion process.

While being able to convert between measured units has been identified as an essential skill in a variety of applied sciences (Cohen et al., 2000; DeMeo, 2008; Pankhurst, 1964), the mathematical principles underlying dimensional analysis are often abandoned in secondary and post-secondary science curricula in favor of an algorithmic approach that focuses on positioning conversion factors to "cancel out" units (Canagaratna, 1993; Ellis, 2013). This focus on units, or labels, is often referred to as the unit-factor or factor-label method.

Correctly applying this method requires the correct placement of conversion factors from one unit to another. One way this process is presented is to take the given measurement and multiply by the appropriate conversion factor so that the given unit is in the denominator and the desired unit is in the numerator (Figure 2-7) (Brown et al., 2017; Cadle & Cadle, 1986). This results in the cancelation of the given units in favor of the new desired unit. A similar process is followed when converting one rate into another rate; the unit in the denominator is "canceled out" and converted into a desired unit when it is multiplied by the conversion factor with the old unit in the numerator and the desired unit is in the denominator (Mortimer, 2013). This systematic approach of focusing on the placement of units is viewed as an efficient and time- saving method that helps to minimize careless errors in the calculation process (Brown et al., 2017; Curran, 2011; DeMeo, 2008).

Figure 2-7

Canceling the Given Unit into a Desired Unit

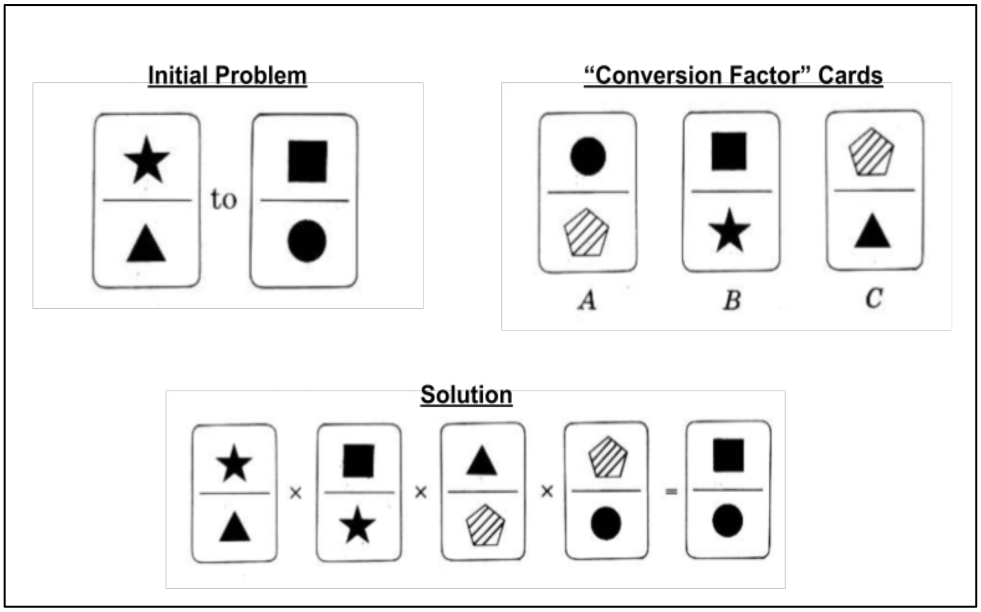
$$\boxed{\cancel{\textit{Given unit}} \times \frac{\textit{Desired unit}}{\cancel{\textit{Given unit}}} = \textit{Desired unit}}$$

Pedagogical Perspectives of Dimensional Analysis

Educators have recommended a number of instructional strategies to help students correctly apply dimensional analysis to convert units. One such strategy uses non-numerical symbols on cards to help students develop confidence in manipulating terms to cancel units (Garrett, 1980). For example, students might be asked to convert a “star / triangle” card into a “square / circle” card using three additional cards that act as conversion factors (Figure 2-8). Students work to arrange the cards so that once properly aligned, the resulting placement would lead to the initial symbols being canceled out to become the desired symbols on the solution card. Others have taken a similar approach, but with pictures of animals instead of shapes (Saitta et al., 2011).

Figure 2-8

Using Symbols to Practice Canceling Units (modified from Garrett, 1980)



The systematic nature of the factor label method has led some to create step-by-step instructions for its use (Figure 2-9). In their respective guidelines for employing dimensional analysis, both DeLorenzo (1976) and Graham (1986) provide learners with an approach that can be completed by following a simple "recipe" or flow chart. By following a short set of rules, these researchers argue that students become more proficient in solving problems and avoid using the same conversion factor twice.

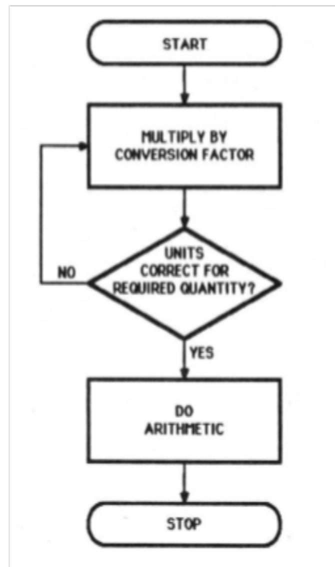
Figure 2-9

Rote Approaches to Converting Units with Dimensional Analysis

Cookbook Dimensional Analysis
DeLorenzo (1976)

1. Write down the given number with its units.
2. Write a ratio with the given unit in the denominator and the unit sought in the numerator.
3. Insert numbers into the ratio so that the numerator and denominator become equivalent
4. Multiply 1. and 3. together.

A Flowchart for Dimensional Analysis
Graham (1986)



The rote nature of these methods has led others to argue for use of other methods of calculation (Canagaratna, 1993; Cohen et al., 2000) and to caution against applying dimensional analysis until certain criteria are met (Navidi & Baker, 1984). More specifically, these individuals argue that dimensional analysis is often approached from a units-perspective (i.e., the process is guided by the units of the measurements) rather than a relations-based approach (Canagaratna, 1993) and that it would be better for students to employ dimensional analysis only when they understand the relationship among the quantities involved. From this perspective, dimensional analysis should be used as “a sophisticated way to condense the familiar reasoning process and to double check by verifying that the unwanted units cancel” (Navidi & Baker, 1984,

p. 522). Cohen et al. (2000) share this view and argue that students should be encouraged to use dimensional analysis as a way to verify that their calculations from formulas and proportional reasoning strategies make sense, rather than “slipping into meaningless symbol manipulation” (p. 1171).

A focus on canceling units is also evident in backwards, or reverse, applications of dimensional analysis. Rather than starting with a “given unit” and canceling labels to produce some “desired unit” (or converting “information given” into “information sought”) (Brown et al., 2017; McClure, 1995), individuals begin with the “units desired” and systematically work to cancel out all units (Drake, 1985; Pursell et al., 2016). For example, if we apply this reverse approach to the conversion completed in Figure 2-6 (i.e., converting 750 grams to an equivalent number of pounds), an individual would first begin by determining the “units of the answer” (pounds) and then place these units “in their correct numerator or denominator positions” (Drake, 1985, p. 414). Once the unit of pounds is set up in the numerator, the individual would use all conversion factors (1000 g = 1 kg, 2.2 lb = 1 kg) and the initial data (750 g) to create a unit path that results in the cancelation of the unwanted units (Figure 2-10).

Figure 2-10

A Conversion Problem Set Up Using a Reverse Dimensional Analysis Approach

$$? \ lb = \frac{2.2 \ lb}{1 \ \cancel{kg}} \times \frac{1 \ \cancel{kg}}{1000 \ \cancel{g}} \times \frac{750 \ \cancel{g}}{1}$$

The sequential placement of units results in the units sought (i.e., those placed in the initial position) as being the “only units remaining in the dimensional analysis set up” (Pursell et al., 2016, p. 23). Although the mathematical expressions produced in Figure 2-6 and Figure 2-10 are equivalent, the thinking involved in constructing each expression is different. With a reverse approach to dimensional analysis, individuals begin with the unit (or units) that are desired and work to cancel the sequential units that follow. It is argued that this approach provides individuals with an easy way to identify conceptual errors since all the labels (or units) -- other than the initial starting unit (or units) -- should be canceled out by the end of the “unit path” (Arnold, 1988, p. 24).

Dimensional analysis can be approached as a rote, algorithmic process (DeLorenzo, 1976; Graham, 1986), or it can be applied in ways that promote a deeper understanding of the underlying mathematical and scientific principles involved in a variety of contexts. Herron (1975) makes the case for the latter in the context of a chemistry course:

> [The factor-label method] provides an almost foolproof procedure for solving stoichiometric problems correctly without the necessity for formal thought. Furthermore -- and I consider this to be important -- the procedure organizes the chemical facts in the problem in such a way that it may lead the student to see the reasoning that characterizes the solution. At the very least, it does not interfere with the perception of the logical relationships implied in the equation and assumed in the solution of the problem. (p. 150)

Goodstein (1983) extends this idea and suggests that dimensional analysis becomes more “meaningful and satisfying” for individuals when the focus of the process is on the underlying relationships among quantities rather than the “mechanical plugging in of numbers or on rote memorization of problems" (p. 667). This can be done by reinforcing the nature of the ratios and conversion factors involved in the dimensional analysis.

DeLorenzo (1994) argues that encouraging students to use verbs, nouns, and adjectives to interpret ratios will help them to "understand what they are doing and to think more deeply during the problem-solving process" (p. 791). In a response to Navidi and Baker (1984), Maloy (1986) provides a similar argument and implores educators to focus their instruction on the mathematical relationships involved in dimensional analysis to help students see the process as logical. He states:

> We should teach our students to examine each conversion factor to see: 1) if its numerator is logically equivalent to its denominator and 2) if the labels in the numerator and denominator are aligned so as to replace old units with new ones. We should tell them that this process may be repeated as many times as necessary (using valid conversion factors) to achieve the desired result. Because of its mathematical rigor, this procedure is always sensible and never mysterious, regardless of whether the conversion is chemical in nature … or not. (p. 186)

These approaches are consistent with educators who encourage students to cautiously manipulate and contrast the various interpretations of a ratio or rate involved in each problem (Arons, 1990; Cohen et al., 2000).

Findings on the Use of Dimensional Analysis

I now briefly synthesize empirical findings on the use of dimensional analysis and highlight unanswered questions in the literature. Findings related to the use of dimensional analysis for calculating dosage are discussed later in this chapter.

Dimensional analysis has become a preferred tool for converting between units in physics, chemistry, biology, and nursing courses. A survey of over 500 secondary and post-secondary teachers found that a significant majority of respondents preferred dimensional analysis over other traditional proportional reasoning methods for solving problems in their respective science courses (DeMeo, 2008). Many of the respondents indicated a preference for dimensional analysis because it was “compact” and “concise” (p. 47), and easier for students to use when competing conversion problems. However, as DeMeo notes, a small group of educators preferred more traditional methods of calculation and viewed dimensional analysis as a rote algorithm, divorced from the underlying concepts involved in a given problem (p. 65). The results from this survey speak to the varying perspectives of dimensional analysis found in the literature: (1) dimensional analysis as a rote algorithm; (2) dimensional analysis as a method to supplement students’ understanding of underlying mathematical and scientific principles.

A few studies have explored the impact of instructional strategies on students’ use of dimensional analysis and the extent to which a particular intervention had positive effects on students’ dimensional analysis performance. In their study of 309 students enrolled in a chemistry course, Saitta et al. (2011) lead students through activities in two discussion-focused sessions throughout the semester. During these sessions, students used cards with pictures of animals to become familiar with the systematic nature of canceling units, before reflecting on how the activity translates to the more formal mathematics of dimensional analysis. The researchers argue that by practicing the process of canceling units with animal cards, students “think through and set up the problems” (p. 915) prior to completing a similar conversion on a practice worksheet. Moreover, pre- and post-test data suggest that students who completed the activity were more likely to attempt and successfully use dimensional analysis to convert units when compared to those in a control group. These results are consistent with McClure (1995). who argues that approaching dimensional analysis using domino-like analogies leads to "more rapid student mastery of the problem-solving technique and a smaller frequency of inversion of conversion factors in student calculations" (p. 1093).

Ellis (2013) implemented a similar domino-like strategy to support students' conceptual understanding of dimensional analysis in a high school chemistry course. Using an online program, *Conversionoes*, students completed conversion problems by first addressing a few questions, such as "What are you asked to do?" and "Will the final answer be a larger or smaller number?" (p. 557). After addressing these questions, the students proceeded to map out conversion factors and perform the arithmetic. Although some students struggled to perform the arithmetic on their calculators, both pre- and post-test scores and qualitative data suggested the supplemental computer program had a positive impact on students' conceptual and visual understanding of dimensional analysis. That is, students in the treatment group scored significantly higher on a dimensional analysis assessment, and interview data revealed the students valued their experiences with *Conversionoes* and the problem-solving approach to dimensional analysis.

These studies suggest that linking domino imagery with the placement of conversion factors is an effective approach for students to accurately apply dimensional analysis. However, other than Ellis (2013) who gathered some data through task-based interviews, these studies do not employ qualitative methods to explore how students think about and make sense of dimensional analysis as a process for converting units. Moreover, these studies do not address the extent to which students understand the mathematical foundations of dimensional analysis.

Perspectives on Ratio and Proportion

The goal of this study is to understand students' conceptions of dimensional analysis as a method for calculating medication dosage. The proceeding sections described dimensional analysis and included a number of phrases (e.g., ratio, invariant multiplicative factor, proportion) that were not explained in detail. The purpose of this section is to (1) provide more context on the concepts of ratio and proportion, including important theoretical and conceptual frameworks; and (2) synthesize empirical findings on students' understanding of ratio and proportion concepts.

Ratios

Ratios can represent a comparison between quantities and be expressed in a number of different ways, including with a colon (2:5), in words (two to five; 2 to 5), or as a fraction (2/5) (Lamon, 2006). With the latter representation (i.e., ratios expressed in fraction notation) comes questions about the relationship between ratios and fractions (rational numbers) and how — if at all — they differ. In their synthesis of how textbooks and teachers posit the relationship between ratios and fractions, Clark, Berenson, and Cavey (2003) describe five distinct models: (1) ratios as a subset of fractions; (2) fractions as a subset of ratios; (3) ratios and fractions as distinct sets; (4) ratios and fractions as overlapping sets; (5) ratios and fractions as identical sets. The authors provide examples to contrast each of these models, but they ultimately argue it is the context and mathematical meaning in a given problem or situation that determine the extent of the relationship between ratios and fractions. They use the term *conceptual convergence* to describe how the various meanings of ratios and fractions intersect, and they argue that students should develop an understanding of how a particular representation could be both interpreted as a ratio and as a fraction. Certain situations, such as the non-integer average of a discrete variable, would require individuals to "pull from both concepts and make connections at the intersection" of these two ideas in order to meaningfully solve problems (p. 308). For example, in order to understand and reason with the ratio "1/3 of a tablet per patient," an individual would need to see the average as both a multiplicative comparison of two specific quantities (number of tablets and number of patients) and as a number that can be operated on, just like any other rational number (Smith III, 2002). More generally, understanding and reasoning with ratios involves representing, "attending to," and "coordinating" the multiplicative relationship between the quantities in a given ratio (Lobato et al., 2010, p. 13)

The nature of the quantities in a ratio is also an important consideration in interpreting its meaning. For example, consider ratios that could be formed between various magnitudes of mass and volume, such as $\frac{10\ grams}{4\ grams}$, $\frac{2\ grams}{8\ milliliters}$, and $\frac{6\ milliliters}{3\ milliliters}$. Freudenthal (1983) uses the phrase *internal ratio* to describe a comparison of two magnitudes within the same system, such as $\frac{10\ grams}{4\ grams}$ and $\frac{6\ milliliters}{3\ milliliters}$. In this case, when *internal ratios* are interpreted as quotients, the result is a number describing the multiplicative relationship between the magnitudes (p. 183). When ratios are constructed with two differing magnitudes like $\frac{2\ grams}{8\ milliliters}$, Freudenthal describes this as "between systems" or as an *external* ratio. Additionally, when interpreted as a quotient, this type of ratio is seen as its own magnitude, such as concentration or velocity.

Ratios as Intensive Quantities

The phrases *within system* and *between system (*or *internal* and *external* ratios) are similar to the notions of *intensive* and *extensive* quantities (Howe et al., 2010; Kaput & West, 1994; Nunes et al., 2003). Citing the work of Piaget, Nunes et al. (2003) describe a quantity as being *intensive* or *extensive* depending on whether it is “susceptible” to addition. For example, if one were to consider the volume of two different saline solutions, say 5 liters and 8 liters, respectively, it would be meaningful to describe the sum of their volumes as 13 liters. This makes volume an *extensive* quantity since it is “susceptible” to addition. However, if the concentrations of the solutions were 9 milligrams of salt per milliliter and 4.5 milligrams of salt per milliliter, respectively, mixing the solutions together would not produce a saline solution with a concentration of 13.5 milligrams of salt per milliliter. In this case, concentration is not “susceptible” to addition, which means it is not an extensive quantity. Instead, this quantity is *intensive*, meaning it is the “product of two quantities” and it is measured by “a relation between two variables” (Nunes et al., 2003, p. 653).

Others have described intensive quantities as those “constituted from proportional relations” (Howe et al., 2010, p. 309). As an example, speed – constructed from the extensive quantities distance and time – is directly proportional to distance and inversely proportional to time. Kaput and West (1994) broaden this idea to describe intensive quantities as “all types of quantities typically described in our culture as rates (speed, density, price),” or more generally, any ratio constructed with two extensive quantities in the form “X per Y” (p. 239). However, as Thompson (1994) notes, constructing a ratio with two extensive quantities does not automatically mean one would conceptualize that ratio as a rate. He writes:

> When one conceives of two quantities in multiplicative comparison and conceives of the compared quantities as being compared in their independent, static states, one has made a ratio. As soon as one recognizes the situation as being that the ratio applies generally outside of the phenomenal bounds in which it was originally conceived, then one has generalized that ratio to a rate. (p. 193)

In other words, Thompson argues that a rate is a ratio that has been reflectively abstracted, meaning the relationship between the two quantities is not just describing one particular instance, but rather an abstract relationship that "gives prominence to the constancy…of the multiplicative comparison" (p. 192). Take for example an individual who is told that 2 pounds of cheese cost 7 dollars. Iterating this relationship, one reason that 4 pounds of cheese cost 14 dollars, 6 pounds of cheese cost 21 dollars, and so forth. Thompson (1994) might argue that this individual has an *internalized* ratio concept since they have shown the values of both variables vary in constant ratio of each other. However, this individual would only possess an *interiorized* ratio concept, or illustrate understanding of a rate, if they could conceptualize the relationship as 2/7 of a pound per one dollar. This view of ratio has also been referred to as a ratio as per-one conception (Johnson, 2015). In general, these perspectives suggest that how one conceptualizes a ratio is not just dependent on the types of quantities involved in the construction of the ratio (i.e., *within system* and *between system*), but it is also how the individual is thinking about the multiplicative relationship and the mental operations the individual is able to perform (Harel et al., 1994).

Ratio as Measure

Ratios can also be conceptualized as a measure of some attribute. In their study of prospective elementary teachers' conceptions of ratio, Simon and Blume (1994) asked participants to explain which land dimensions would appear most square: 75 feet by 114 feet, 455 feet by 508 feet, or 185 feet by 245 feet. In order to address such a question, the prospective teachers had to recognize that the "squareness" of each plot of land could be measured as a ratio of length to width. In this problem, the ratio is not just an invariant, multiplicative relationship between the length and width, but is itself a measure of a particular attribute, "squareness" (Simon & Placa, 2012, p. 40). According to Lobato and Thanheiser (2002), the slope of a line can also be thought of as a ratio-as-measure given that it is not just the ratio of two covarying quantities, but it is also a measure of some contextual attribute, such as velocity, density, or gas efficiency (p. 163). They suggest four components of understanding a ratio as a measure: (1) isolating the attribute that is being measured; (2) determining which quantities affect the attribute and how; (3) understanding the characteristics of a measure; and (4) constructing a ratio. This framework suggests that understanding a ratio as a measure involves more than simply constructing a ratio with two quantities; it requires an understanding of each of the covarying quantities, the specific attribute constituted by the ratio of those quantities, and how these items are inextricably linked.

Simon and Placa (2012) argue that a ratio-as-measure conception is "closely tied to the development of a functional concept of ratio" (p. 40) since one's focus is on the multiplicative relationship between the quantities and not just as an association between amounts of quantities. Johnson (2015) provides an example using the value 1.4 to describe the strength of "chocolate flavor" of a batch of hot chocolate. A batch that has this strength would have 1.4 times as many chocolate packets as cups of water (i.e., 1.4 x cups of water = packets of chocolate). An individual with a *ratio as per-one* conception -- similar to Thompson's (1994) notion of *interiorized* ratio -- would not necessarily view this relationship as the "strength," but as the association of the sets of chocolate packets and cups water (i.e., there are 1.4 packets of chocolate for every 1 cup of water) (pp. 67-68). While this is seen as a more advanced conception compared to Thompson's (1994) notion of *internalized* ratio (there are 7 packets of chocolate for every 5 cups of water), only when individuals acknowledge and operate with the fixed, multiplicative factor linking the quantities (i.e., the *constant of proportionality*) do they go beyond an association between sets (Johnson, 2015; Lamon, 2006).

Proportions

Lobato et al. (2010) state that a proportion is a "relationship of equality between two ratios" where "the ratio of two quantities remains constant as the corresponding values of the quantities change" (p. 33). This description suggests an individual's ability to reason with (and about) proportions involves supporting "claims about the structural relationship among four quantities, (say a,b,c,d) in a context simultaneously involving covariance of quantities and invariance of ratios or products" (Lamon, 2007, p. 637). Others describe proportional reasoning as pertaining to the covarying and invariant relationships among the quantities, requiring "the ability to mentally store and process several pieces of information" in order to make inferences and predictions involving "qualitative and quantitative methods of thought" (Post et al., 1988, p. 79). Proportional reasoning also includes the ability to discern a multiplicative relationship between quantities and to extend a multiplicative relationship to other pairs of quantities, including through the actions of iterating, partitioning, scaling up, and scaling down ratios (Lamon, 2006, 2007; Lobato et al., 2010).

Multiplication and multiplicative relationships play a significant role in one's understanding of and reasoning with ratios and proportional relationships. One framework for illustrating how multiplication, division, and contextual factors interact as one reasons with proportions is Vergnaud's (1983, 1988, 1994) conceptual field of multiplicative structures. According to Vergnaud (1988), the conceptual field of multiplicative structures "consists of all situations that can be analyzed as simple and multiple proportion problems and for which one needs to multiply or divide" (p. 85). Mathematically, the conceptual field of multiplicative structures incorporates notions of operation (multiplication, division), function (linear, bilinear), and other important concepts (ratio, rate, fraction, rational number, dimensional analysis). It also includes the set of schemes that an individual might employ to solve proportion problems, as well as the set of situations that evoke these concepts and schemes. It is argued that utilizing these elements to analyze student behavior using proper mathematical notation (described as studying theorems-in-action), places researchers in a better position to learn how students complete proportion problems, and understand how students' thinking develops in solving increasingly more complex problems (p. 58). Through this lens, researchers have analyzed students' strategies for solving various proportion problems, such as rate comparison and missing-value problems. In the next section, I describe this analysis further and incorporate components of Vergnaud's conceptual field of multiplicative structures (measure space notation, proportion tables) to illustrate underlying mathematical ideas in completing proportion tasks.

Strategies for Completing Missing-Value Proportion Problems

Missing-value proportion problems are a type of task requiring students to reason with proportional relationships to solve for unknown values. These problems can be represented using measure space notation to illustrate the underlying multiplicative relationships between extensive quantities (Cramer et al., 1993; Vergnaud, 1983, 1988). Consider the following example: Sam was told that 3 milliliters of a particular solution contain 5 grams of a drug; how much of the drug is contained in 18 milliliters of the same solution? As illustrated in Figure 2-11, the quantities 3 mL and 18 mL fall within the same measure space, while the quantity of grams are found in their own measure space.

Figure 2-11

A Diagram Illustrating the Relationships Between and Within Measure Spaces

Measure Space 1 M1 Milliliters	Measure Space 2 M2 Grams
3	5
18	?

Implementing a *norming* strategy to complete this task would involve thinking about this system "in relation to some fixed unit or standard"; that is, reinterpret one measure in terms of another (Lamon, 1994, p. 94). Choosing "3 mL" as the normed quantity and staying within the measure space, the student would see that a scalar multiplier of 6 is needed to produce a product of 18 mL. This same scalar multiplier would then be used within the second measure space to produce 30 grams (Figure 2-12). This particular strategy has also been referred to as the *factor-of-change* method (Cramer et al., 1993; Post et al,1988). However, if the student chose "3 mL" as the normed quantity but instead decided to relate quantities *between* measure spaces, they would be creating a relationship between the number of mL and the number of grams (Figure 2-13). This is often referred to as the *functional method* (Cramer et al., 1993; Lamon, 1994). Regardless of whether students consider the multiplicative relationship within measure spaces or between measure spaces, they would still be re-conceptualizing the situation using the same normed quantity of 3 mL (Freudenthal, 1983).

Figure 2-12

Using 3 mL as the Normed Quantity to Find the Within Measure Space Scalar Multiplier

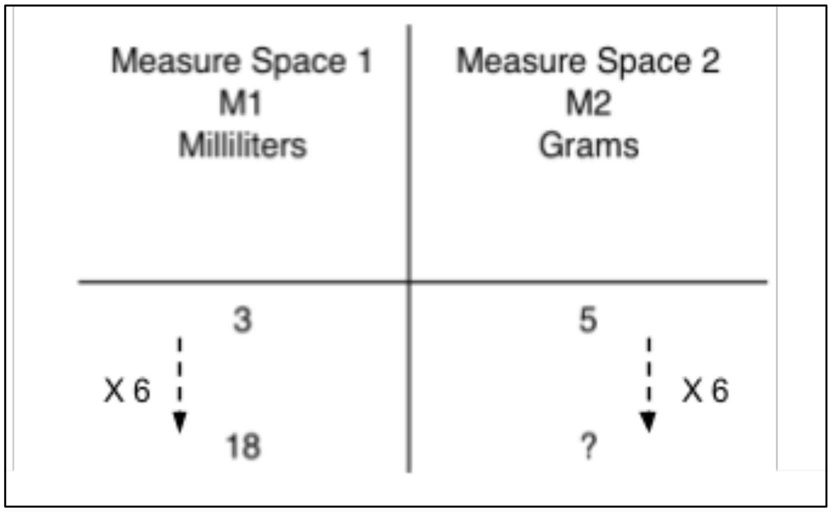
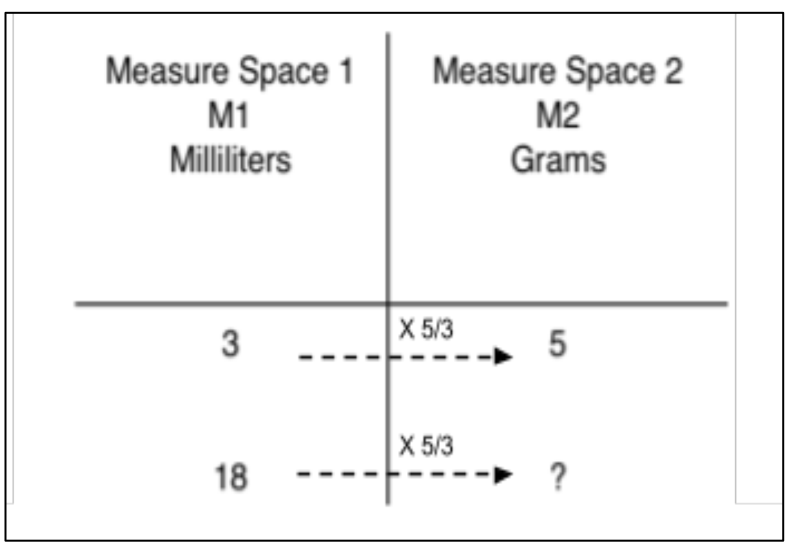


Figure 2-13

The Functional Scalar Multiplier Between Measure Spaces with 3 mL as the Normed Quantity



The process of *unitizing* could be described as a specific type of *norming*, where the student re-conceptualizes the situation not with some composite quantity (e.g., “3 mL”) but with a single unit (e.g., 1 mL) (Lamon, 1994; Steffe, 1994; 2001). When unitizing or using a unit-rate strategy, students first ask themselves, “how much (many) for one?” (Post et al.,1988, p. 81). Using the example above, a student using this approach would first consider the number of grams in each mL of solution by comparing the quantities between measure spaces (i.e., dividing 5 grams by 3 mL). This unit-rate would then be used as a functional operator to find the number of grams in 18 mL (Figure 2-13), or its reciprocal (the number of mL per gram) could be found if seeking an unknown quantity in the first measure space. This strategy is consistent with Thompson’s (1994) interiorized conception of ratio and Johnson’s (2015) ratio-as-per-one conception given that students would be constructing -- and performing mental operations with -- a ratio representing the number of grams per one. It has also been suggested that this approach to solving proportion problems can be more meaningful for students as they are often familiar with the concept of a unit rate from other contexts (e.g., shopping at the grocery store) (Cramer et al., 1989; Miller & Fey, 2000).

The use of measure space notation is helpful in that it affords educators and researchers a clear way to illustrate the multiplicative relationships among the quantities and the ways students might complete a proportion task involving those quantities. However, it is more likely that students would utilize different representations and notations to illustrate the mathematical relationships in a proportion task -- especially those representations that are common in textbooks and curricular materials (e.g., a/b = c/d) (Lobato et al., 2010). Examples of these representations are presented in Figure 2-14.

Figure 2-14

Possible Ways to Represent a Proportion with Fraction Notation

$$\frac{3\text{ mL}}{18\text{ mL}} = \frac{5\text{ g}}{30\text{ g}} \qquad \frac{18\text{ mL}}{3\text{ mL}} = \frac{30\text{ g}}{5\text{ g}} \qquad \frac{3\text{ mL}}{5\text{ g}} = \frac{18\text{ mL}}{30\text{ g}} \qquad \frac{5\text{ g}}{3\text{ mL}} = \frac{30\text{ g}}{18\text{ mL}}$$

While the first two representations compare ratios where the quantities are within the same measure space, Noetling (1980b) would refer to these ratios as *between-state* ratios. Additionally, while the last two representations are equating ratios constructed with quantities between measure spaces, Noelting (1980b) would refer to these as *within-state* ratios. In this case, Noelting appears to use the word *state* to refer to a single instance of time relating the two measure spaces. Similarly, Noetling uses *within* and *between* to describe the strategies that one might utilize to complete a proportion task. If an individual begins the problem by comparing quantities *between* states (considering the multiplicative factor between 3 mL and 18 mL), this would be characterized as a *between strategy,* and it would appear the individual is seeking information about the covariation between quantities. This is similar to the factor-of-change method and the within measure space approach described by Vergnaud. However, if the individual first operates on the quantities *within* each state, this would be considered a *within strategy* (p. 334). This is similar to the between measure space approaches described above (i.e., functional approach, unitizing, and norming).

Researchers have used the phrases *within* and *between* in different ways to describe students' approaches to thinking about proportional relationships. Whether it is reasoning within and between *measure spaces* or within and between *states*, the types of thinking that one employs to solve a proportion tasks are similar. More specifically, when considering the relationship between two quantities with a linear function, the chosen strategy and the reasoning one applies will depend on whether the individual compares quantities related to one instance of the function (i.e., the relationship or correspondence between the two variables) or whether the same variable is compared over two instances of that function (Kaput & West, 1994; Karplus et al., 1983;). As I describe below, there are many factors that impact how individuals conceptualize these relationships and how their reasoning and problem-solving strategies might differ between stages of understanding.

Findings Related to Students' Understanding Ratio and Proportion

In the next sections of this chapter, I synthesize findings on students' understanding of ratio and proportion concepts. I use Noelting's (1980a, 1980b) seminal work on students' proportional reasoning and the stages of proportional reasoning depicted in his work, to organize the literature and describe how students' make sense of and solve ratio and proportion tasks at various stages of understanding.

In his study on the development of proportional reasoning, Noetling (1980a) asked 321 subjects aged 6 to 16 years about the "orange taste" of various mixtures of orange juice and water. During one part of the experiment, subjects were presented with two trays: one containing three glasses of orange juice and 1 glass of water (3,1), and the other containing 1 glass of orange juice and 3 glasses of water (1,3). The subjects were asked which tray of glasses — once combined — would produce the stronger orange taste and why. A total of 23 different items were presented to the subjects ranging from less complex comparisons (e.g., (3,1) vs. (1,3) described above) to more complicated ones (e.g., 5 orange and 2 water vs. 7 orange and 3 water). After analyzing the subjects' responses, Noetling categorized the explanations into stages of thinking that mirrored Piaget's pre-operational (or intuitive), concrete operational, and formal operational stages of development. Noelting noted that the stages, "correspond to structured behavior and strategies capable of solving certain types of problems" and that the differences between the stages "correspond to a change in behavior with the introduction of new ways in solving problems" (p. 247). Put another way, in order to progress to a more advanced stage, individuals would need to modify old strategies or create new strategies to solve more complicated tasks. Noetling refers to these changes as qualitative changes and differentiates them from quantitative changes, which occur within a given stage as students consolidate a particular strategy to overcome changes from varying numerical quantities. When individuals are presented with more-complex tasks outside the application of their current schema, they are required to restructure their thinking and problem-solving strategies. This restructuring constitutes a qualitative change, and it differentiates the various stages of proportional reasoning.

Pre-operational or Intuition Stage of Proportional Reasoning

At the *pre-operational* or *intuition* stage, individuals are only able to make qualitative comparisons between the number of glasses of orange juice and the number of glasses of water. The children in Noetling's experiment initially had a strategy of comparing just the number of orange juice glasses, but as they progressed to higher levels in this stage, their reasoning evolved to compare both types of drink and were able to determine correct answers by relying on more sophisticated qualitative arguments. For example, when asked to compare the "orange flavor" of 2 orange juice glasses and 3 water glasses with 1 orange juice glass and 1 water glass, children at an advanced level within the intuitive stage could provide an explanation such as, "At the right there is the same amount of water and juice while at the left there is more water than juice” (Noelting, 1980a, p. 240). This comparison, although qualitative in nature, shows a reliance on additive thinking.

In their own study on children’s conceptions of ratio and proportion through “taste-related” experiments, Harel et al. (1994) found that students suggested orange juice poured in two different-sized glasses (a 4 ounce glass and a 7 ounce glass) resulted in a different orange taste. They state,

> The child’s additive world is in conflict with her or his experience with taste because when the size of a mixture’s sample is varied (made greater or smaller — definitely an additive operation) the taste of the samples (i.e., the measure of the quality of the operand) stays the same, which is against the child’s expectation. (p. 326)

In other words, in their experiences and limited view of the “quantitative world,” children associate an increase in one quantity as directly altering the “muchness” of that quantity. This is supported by other studies suggesting children struggle with conceptualizing intensive quantities, even when they are constructed using extensive quantities that they are typically familiar with (e.g., sugar and water) (Nunes et al., 2003). Their difficulty in comparing and conceptualizing a relationship between two quantities limits their ability to successfully complete tasks containing intensive quantities (Lamon, 1993; Howe et al., 2010; Nunes et al., 2003).

Concrete Operational Stage of Proportional Reasoning

At the *concrete operational* stage of proportional reasoning, Noetling’s (1980a; 1980b) subjects were able to move past purely qualitative arguments and provide explanations that showed they could operate on a given relationship. Their strategies included both *within* (comparing the glasses on one tray) and *between* (comparing similar drink type across trays) strategies, but their reliance on additive thinking still led to incorrect conclusions. For example, Noelting (1980a) remarks about Christiane, who when asked to compare 2 orange juice glasses and 3 water glasses to 1 orange juice glasses and 2 water glasses stated they were equal, because "in both there is 1 glass of water more than of juice" (p. 233). This type of additive thinking has been found in other studies of students' understanding of proportional relationships as well (Cramer et al., 1993; Hart, 1978). To suggest that these students might be at the same stage of proportional reasoning, it is essential to consider how the students are arriving at their solution. Christiane's explanation was in response to a comparison task rather than a missing-value task, and as Noetling (1980a) notes, her strategy of "compensation with estimate of remainder" (p. 233) can be quite sophisticated depending on *how* the student arrived at her or his solution.

Although Christiane's additive reasoning strategy resulted in an incorrect solution, this doesn't mean that additive strategies cannot lead to correct solutions or more advanced thinking. In fact, studies suggest that students at the *concrete operational* stage begin to understand ratio as a unit that can be operated on (Kaput & West, 1994; Lamon, 1993; Nabors, 2003; Noetling, 1980a, 1980b). Kaput and West (1994) used the notion of *coordinated build-up/build-down* (the first level in their framework of students' informal proportional reasoning) to describe the process of operating on a ratio to solve missing-value proportion problems. For example, using the example previously described in this chapter, a student could take the ratio of 3 mL to 5 mg and continuously add it to itself until they arrived at the ratio associated with 18 mL (6 mL to 10 mg, 9 mL to 15 mg, 12 mL to 20 mg, 15 mL to 25 mg, 18 mL to 30 mg). This iteration of a composite unit rather than a single unit (e.g., 3 mL compared to 1 mL) has be shown to be a significant cognitive leap for learners (Lamon, 1996; Noelting, 1980b).

At an advanced level within this stage, Noelting (1980a, 1980b) describes students who are able to move past their additive schemes and begin to conceptualize the multiplicative relationship between the orange juice and water. However, data suggest this understanding was limited as students were unable to apply their multiplicative strategies to every task. This often resulted in students relying on their additive schemes and incorporating incorrect reasoning into their problem-solving strategies. For example, some students could reason that 1 glass of orange juice and 2 glasses of water would have the same "orange taste" as 2 glasses of orange juice and 4 glasses of water by seeing that the number of glasses on the second tray was double the number of glasses on the first try. Others used a within strategy to see that the number of water glasses was double the number of orange juice glasses. However, students were not always able to implement these strategies for tasks involving more complex combinations (5 orange and 2 water vs. 7 orange and 3 water). This is consistent with Hart (1978), who found that children relied on "doubling" and "halving" ratios to complete various tasks, but that this didn't necessarily imply the children could complete other tasks requiring a different integral factor.

Cramer et al. (1993) found a similar result among 7th grade students completing missing-value problems. While many students were capable of successfully employing a within strategy when the problem involved an integral *factor-of-change*, the researchers suggest the "presence of a noninteger [sic] relationship does two things: first, it significantly decreases the level of student achievement and second, it actually changes the way in which students think about a problem" (p. 12). These researchers cite Karplus's notion of "fraction avoidance syndrome" to suggest that students will purposefully fall back on additive strategies instead of dealing with non-integer relationships.

In summary, individuals at the concrete operational stage of proportional reasoning build on their additive strategies, operate on ratios, and exhibit some multiplicative understanding of ratio and proportion. However, empirical evidence suggests that individuals' strategies for completing comparison and missing-value tasks are often not robust enough to complete tasks with non-integer relationships. Thus, when students arrive at tasks that can no longer be addressed by making quantitative adjustments to their strategies, then they must make qualitative changes to demonstrate more-advanced levels of proportional reasoning.

Formal Operational Stage of Proportional Reasoning

At the formal operational stage of proportional reasoning, individuals have an understanding of the multiplicative relationship between quantities. In comparison tasks, students are able to arrive at correct solutions by using both between and within strategies (Noelting, 1980a, 1980b). For example, when presented with 2 glasses of orange juice and 3 glasses of water versus 1 glass of orange juice and 2 glasses of water, students using a between strategy might see that the first tray has double the amount of orange juice, but less than two times the amount of water. This approach suggests that students possess an understanding of the multiplicative covariation between equivalent ratios and that they can also use additive comparisons to arrive at the “stronger orange” taste. Students using a within strategy would divide both quantities to determine a unit-ratio before making a comparison. For example, when given the task above, Diane explained her within-strategy thinking: “Because in A, theres one and a half glass of water for one glass of juice, while in B, there are two glasses of water for one glass of juice” (Noetling, 1980b, p. 340).

These strategies within the formal operational stage are consistent with Kaput and West’s (1994) second and third level of proportional reasoning for missing-value problems: the abbreviated build-up/build-down approach and the unit factor approach. The abbreviated build-up/build-down is closely related to the between strategy that the student used above. That is, the student would have an understanding of the multiplicative covariation between quantities, including an understanding that non-integer factors can be used. Similar to Diane's explanation of her within strategy, students using a unit factor approach exhibit understanding of the invariant nature between equivalent ratios.

A unit factor approach can be used to solve missing-value problems. As Nabors (2003) puts it, students "divide the unit size of the unknown quantity by the unit size of the known quantity to determine the unit factor" and then "multiply the unit factor and given total quantity to determine the total amount of the unknown quantity" (p. 140). However, Langral and Swafford (2000) note the importance of students understanding the relationships involved in the situation as well. That is, students at this level should not only able to solve tasks using appropriate notation, strategies, and procedures, but they should also have full understanding of the multiplicative relationships between the quantities in which they are working with.

Summary

In the literature, researchers differentiate between how students complete ratio and proportion tasks by describing qualitative and quantitative changes in their thinking and implementation of additive and multiplicative schemes. Karplus et al. (1983) differentiate between students' explanations during tasks as either (1) incomplete or illogical, (2) qualitative, (3) additive, or (4) proportional. Although not identical, this sequence is comparable to Noetling's (1980a, 1980b) description of the stages of proportional reasoning, and also to how Kaput and West (1994) categorized students' reasoning strategies (coordinated build up/build down, abbreviated build up/build down, unit factor). Empirical findings suggest that students' proportional reasoning strategies initially include qualitative comparisons of familiar, extensive quantities, before eventually giving way to more sophisticated additive strategies. As students begin to encounter more complex mathematical situations, additive strategies are no longer productive, thus requiring a restructuring of their schema to incorporate the multiplicative relationships between quantities. Although this might seem like a straightforward, linear trajectory, research suggests that students often struggle to adapt and restructure their thinking to tackle new problems. Educators support students' development of proportional reasoning by incorporating problems with more familiar numerical values (e.g., 2, 5, 10), supporting students' functional thinking with multiple representations, and explicitly building connections between the notion of ratios and fractions (Confrey & Scarano, 1995; Lamon, 2006; Post et al., 1988).

Medication Dosage Calculations

In this study, the participating students' actions and behaviors will be analyzed in the act of calculating medication dosage, which has been an area of study by researchers in both the mathematics education and nursing education communities. The purpose of this section is to synthesize perspectives and research on medication dosage calculations. It will be organized as such: (1) I begin by reviewing the dosage calculation process and the connections between the underlying mathematical and nursing concepts; (2) I then synthesize empirical findings related to pre-licensure and practicing nurses' numeracy and dosage calculation skills; (3) Finally, I describe conceptual perspectives and empirical findings related to the use of dimensional analysis for calculating dosage.

Mathematics and Dosage Calculations

A dosage calculation is a conversion problem requiring nurses to change the units of an ordered dose into another unit or set of units. Figure 2-15 illustrates two examples of dosage

Figure 2-15

Examples of Traditional Dosage Calculations

Patient Name: Tina Dianna		
Date	**Time**	**Physician Order**
9/20/19	1340	Amoxicillin 500 mg oral sus q6h

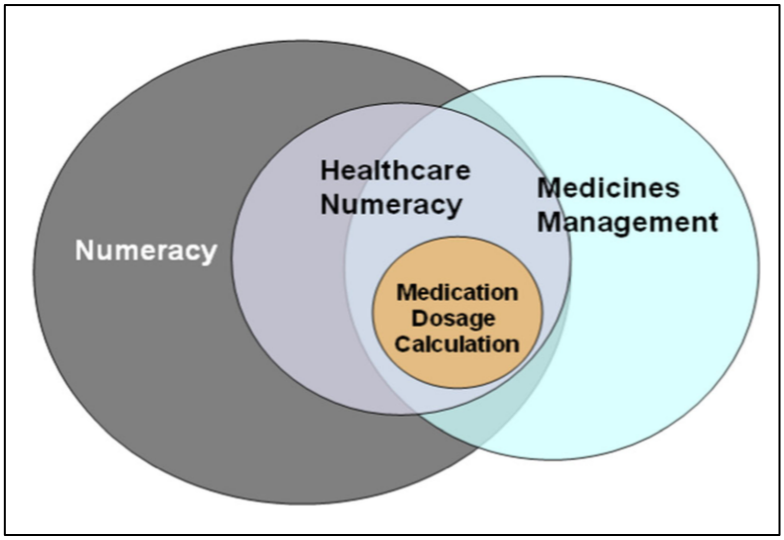
Patient Name: Jordan Kling **Weight:** 144.9 lb		
Date	**Time**	**Physician Order**
9/20/19	1550	Infuse 500 mL NS over 4 hours

calculations, one requiring a conversion between milligrams of acetaminophen and milliliters to administer and the other requiring a conversion between milliliters of normal saline per four hours into an equivalent rate in drops per minute (gtt/min).

In both of these situations, a nurse must employ a unique set of knowledge and skills to arrive at a correct numerical answer. For example, prior to performing any computations and determining the amount of medication to administer to Tina Dianna (Figure 2-15), a nurse must interpret the medication order and medication label to correctly identify the essential information for the calculation and the dosage strength (250 mg per 5 mL). For Jordan Kling's medication, the nurse must also interpret the label of the available IV tubing to recognize the equipment has a "drop factor" or calibration of 15 drops per mL. Coben and Weeks (2014) and Young et al. (2013) argue that successfully completing problems like these involves knowledge at the intersection of numeracy, health care numeracy, and medicines management (Figure 2-16). That is, calculating medication dosage involves the correct application of numeracy skills and it incorporates the application of unique mathematical relationships found in healthcare settings, including those related to the interpretation, management, and administration of various medications and equipment.

Figure 2-16

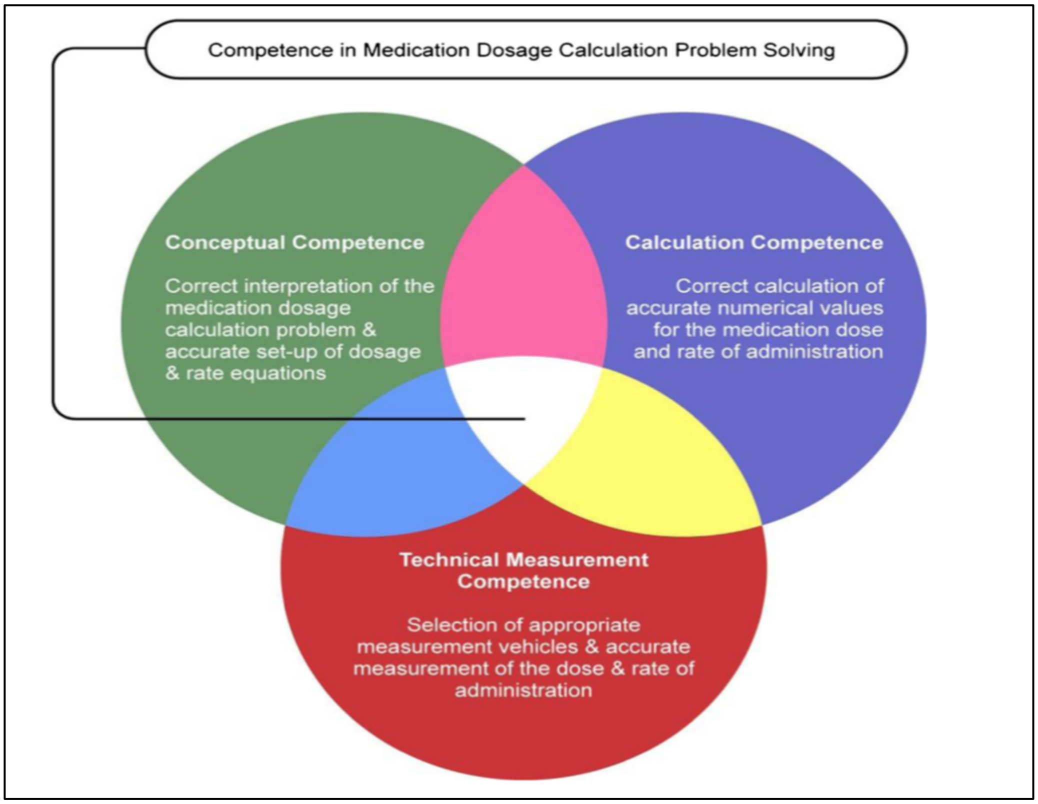
Situating Medication Dosage Calculations Within Broader Domains



The unique set of knowledge required for completing a dosage calculation is articulated further in the Medication Dosage Calculation Problem Solving Model (Figure 2-17), which frames dosage calculation competency as the intersection of conceptual competence, calculation competence, technical measurement competence (Coben & Weeks, 2014; Weeks et al., 2013). This model suggests calculation competency involves: extracting numerical information from nursing-specific artifacts and positioning the information into an appropriate mathematical expression or equation (conceptual competence); applying arithmetical operations to calculate the appropriate amount of medication to administer (calculation competence); and selecting an appropriate "measurement vehicle" (e.g. capsule, syringe, infusion pump) and accurately measuring the amount to administer (technical measurement competence) (Weeks et al., 2013; e.25).

Figure 2-17

Medication Dosage Calculation Problem Solving Model (Coben & Weeks, 2014)



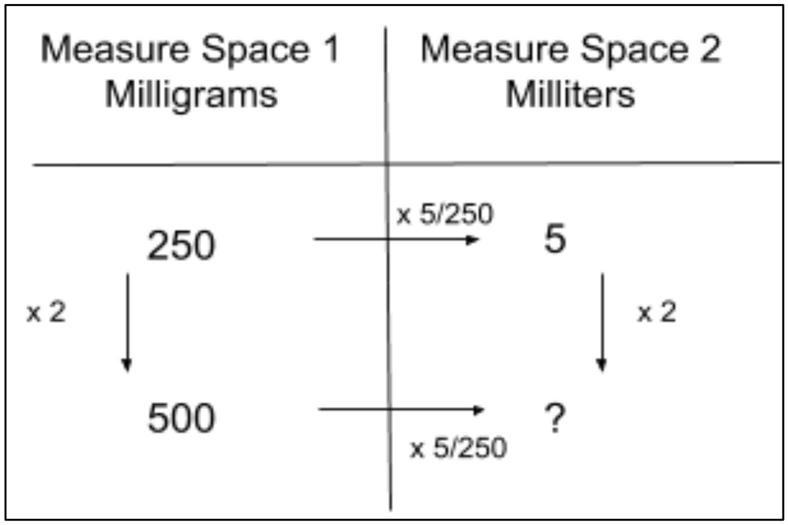
Similarly, Johnson and Johnson (2002) use “4 Cs” to describe the essential components of completing the dosage calculation process: conceptualize, convert, compute, and critically evaluate. These components suggest that mathematical skills play an important role in converting between systems of measurement and computing drug dosages, but nurses must also be able to “set up the problem correctly” (p. 82) and critically evaluate whether the process they follow and the final value they calculate make sense. Johnson and Johnson argue it is the final “C” (critically evaluate) that is often omitted in the teaching of dosage calculations, which can lead students to blindly calculating values without thinking whether their result makes sense in a clinical context.

Strategies for Completing Dosage Calculations

Calculating medication dosage requires reasoning with ratios and proportions. In the previous oral dosage calculation example (Tina Dianna, Figure 2-15), one must calculate the number of milliliters to administer based on the available strength of the medication that is available (250 milligrams per 5 milliliters). A common approach for completing this calculation is the nursing formula, which involves interpreting the problem to identify the "desired dose" (D), the "supply on hand" (H), and the "quantity of unit" (Q) (Lesmeister, 2017). Once identified, the numerical values are placed in the formula: $Amount\ to\ Administer = \frac{D}{H} \times Q$. For example, the desired dose for Tina Dianna (Figure 2-15) is 500 milligrams, the supply on hand is (250 milligrams), and the quantity that the supply is contained in (5 milliliters). With the nursing formula, the amount to administer can be found by simplifying $\frac{500}{250} \times 5$ into 10 milliliters. Others have described the components of the nursing formula with the phrases "what you want," "what you need," and "what you've got" (Coben & Atere-Roberts, 2005, p. 46), as well as "number of measures to be given," "dose prescribed," and "dose per measure" (Pirie, 1987, p. 94). Regardless of the terminology used, the nursing formula is the result of algebraically manipulating the proportion between the concentration of the available medication, the ordered dose, and the unknown amount to administer. Although this method provides nurses with a tool for calculating dosage quickly, it has been argued that this method leads nurses away from the underlying meaning of the clinical contexts in the problem (Dyjur et al., 2011; Marks et al., 2015; Wright, 2013). That is, once a nurse extracts the numerical information from the problem context, the values are "stripped from their meaning" (Wright, 2009, p. 546), and placed into the formula for context-free arithmetic calculation.

Figure 2-18

Representing an Oral Dosage Calculation with Measure Space Notation.



As missing-value proportion problems, dosage calculations can be represented with measure space notation (Cramer et al., 1993; Vergnaud, 1983; 1988). Figure 2-18 illustrates the two distinct measure spaces (milligrams and milliliters) and the multiplicative relationships within and between the given quantities in the previous oral calculation example.

As discussed earlier in this chapter, it is important to consider how individuals operate with the given quantities in these situations to analyze their understanding of proportional relationships (Kaput & West, 1994; Karplus et al., 1989; Nabors 2003). Hoyles, Noss, and Pozzi (2001) draw upon the work of Vergnaud to describe five potential strategies for completing a dosage calculation depending on how an individual operates with the given quantities (Figure 2-19). With the Scalar Operator and Functional Operator strategies, an individual is operating directly with multiplicative factors within or between measure spaces, respectively, whereas with the Unitary Method, they are unitizing or using a unit-rate strategy (Lamon, 1996; Post et al., 1988). All three strategies (Scalar Operator, Functional Operator, Unitary Method) are consistent with those Noelting (1980a; 1980b) described at the formal operational stage of proportional reasoning, or that Karplus, Pulos, and Stage (1983) described as proportional. On the other hand, the Scalar Decomposition Method more closely aligns with additive strategies, such as Kaput and West's (1994) notion of *coordinated build-up/build-down*. Rather than incorporating the multiplicative factors connecting the quantities, individuals operate with the given ratio in an additive process to arrive at the solution.

Figure 2-19

Proportional Reasoning Strategies for Calculating Dosage (Hoyles, Noss, & Pozzi, 2001)

Strategy	Measure Space Reasoning	Calculation
Scalar Operator	Within	$\dfrac{250\text{ mg}}{5\text{ mL}}$ $\begin{matrix}\xrightarrow{\times 2}\\ \xrightarrow{\times 2}\end{matrix}$ $\dfrac{500\text{ mg}}{10\text{ mL}}$
Functional Operator	Between	$\dfrac{250\text{ mg}}{5\text{ mL}}\div 50$ $\quad\dfrac{500\text{ mg}}{10\text{ mL}}\div 50$
Unitary Method	Within	$\dfrac{250\text{ mg}}{5\text{ mL}}$ $\begin{matrix}\xrightarrow{\div 250}\\ \xrightarrow{\div 250}\end{matrix}$ $\dfrac{1\text{ mg}}{0.02\text{ mL}}$ $\begin{matrix}\xrightarrow{\times 500}\\ \xrightarrow{\times 500}\end{matrix}$ $\dfrac{500\text{ mg}}{10\text{ mL}}$
Scalar Decomposition	Within	$250\text{ mg} + 250\text{ mg} = 500\text{ mg}$ $5\text{ mL} + 5\text{ mL} = 10\text{ mL}$
Rule-of-three "Cross Multiplication"	Both	$\dfrac{250\text{ mg}}{5\text{ mL}} = \dfrac{500\text{ mg}}{X}$ $\quad 250X = 500 \times 5$ $X = 10$ 10 mL

In the study of Hoyles, Noss, and Pozzi (2001), nurses were more likely to utilize scalar (within-measure space) and functional (between-measure space) strategies to calculate the appropriate dosage to administer. This result contrasts with Wright (2013), who found that a small group of senior nurses tended to prefer the nursing formula approach over the proportional reasoning strategies in Figure 2-19. Additionally, Wright notes that nurses who either employed a "single units" strategy (i.e., unitary method) or a scalar approach over the nursing formula tended to use fewer steps in their calculation and "kept the calculation within the context" of the dosage situation (p. 456).

Dosage calculation textbooks and curricular resources often instruct individuals to use other strategies to complete dosage calculations, such as cross multiplication (Booth et al., 2012; Gray-Morris, 2014). Students are often encouraged to begin with the strength of the available medication ("dosage on hand" over "dose unit"), set this equal to another ratio with the "desired dose" over the "amount to administer," and then cross multiply to solve for the unknown "amount to administer" (Booth et al., 2012, p. 273). In an oral or parenteral dosage situation, this method is only applicable if the "desired dose" and the "dosage on hand" are the same unit (e.g., a patient needs to receive 500 milligrams of a drug and the available medication is labeled with a strength incorporating milligrams). If these units do not match, then a metric conversion between grams and milligrams is required using the appropriate conversion factor (1 gram is 1000 milligrams). Employing this algorithmic process for more-advanced clinical calculations often requires multiple proportions.

Related Empirical Findings

Nurses' Mathematical Skills

A number of mathematical skills have been identified as being crucial for practicing nurses, including estimating and measuring units, interpreting equations, and operating with percentages, decimals, and ratios (Cartwright, 1996; Coben et al., 2008; Young et al., 2013). However, numerous studies over the last thirty years have raised concerns that both student nurses and registered nurses are not proficient in employing many of these basic skills (Wright, 2006). In one such study of 119 first-year student nurses, participants scored an average of 51% on a 50-item mathematics assessment covering computations with whole numbers, rational numbers, and ratios and percentages (Hutton, 1998). This included a mean score of 37.5% on ratio and percent problems and a mean score of 37.5% on problems requiring participants to multiply and divide fractions (p. 27). Similar results on a basic mathematics assessment were found in studies of second-year nursing students (Eastwood et al., 2011; Jukes & Gilchrist, 2006; McMullan et al., 2010), third-year nursing students in a baccalaureate program (Bindler & Bayne, 1984), and students enrolled throughout the first three years of a baccalaureate program (Bagnasco et al., 2016). Additional studies have found that, in general, students in nursing programs tend to be less proficient in completing basic mathematical tasks (e.g., operating with fractions, decimals, and ratios) when compared to their counterparts in non-nursing programs (Arkell & Rutter, 2012; Pozehl, 1996).

Similar findings are not exclusive to student nurses in undergraduate programs. McMullan, Jones, and Lea (2010) found 45% of a sample of registered nurses in the United Kingdom were unable to pass a numeracy test that included basic unit conversions and operations with fractions, decimals, and percentages. A study of Finnish nurses' mathematical skills found many performed poorly on tasks involving percentages and estimation, but a majority performed well on basic arithmetic tasks involving multiplication and division (Grandell et al., 2006). Additionally, in both of these studies, it was found that registered nurses received higher scores on a mathematical skills assessment compared to student nurses.

Dosage Calculation Competency

Beyond the evidence of student nurses and practicing nurses performing poorly on basic mathematical skills assessments, studies have also found these groups do not always perform well on assessments designed to evaluate dosage calculation competency. In their study of 66 junior-level baccalaureate nursing students, Blais and Bath (1992) found that 59 (89%) were unable to achieve a minimum passing score of 90% on a dosage calculation exam consisting of oral, parenteral, and IV dosage questions. McMullan (2010) found similar results with both second year nursing students and practicing nurses. On a 20-item dosage calculation assessment covering oral dosage, injections, and intravenous calculations, 92% of students (n=229) and 89% of practicing nurses (n=44) were unable to obtain a passing score of 60%. Additionally, McMullan notes that all participants in the study earned less than an 80% on the dosage calculation assessment. Another study of practicing nurses found that 23 out of 51 (45%) were unable to obtain a passing score of 85% on a dosage calculation examination without the use of a calculator. When these nurses were given a calculator, 14 (27%) were still unable to achieve a passing score (Blitz-Holtz, 1994).

Wright (2007) found that even after an instructional intervention designed to support nursing students' drug calculation skills, only 32% (n=14) were able to achieve a score of 83% or higher, and 32% (n=14) of the students had at least one-third of the problems marked incorrect. This is similar to McMullan, Jones, and Lea (2011) who found that while an e-learning tool was helpful in improving scores of second-year diploma nursing students, the students in two cohorts only earned an average score of 48.4% and 47.6% on a post-intervention dosage calculation assessment (compared to pre-intervention average scores of 41.2% and 36%, respectively).

It has been argued that dosage calculation errors and low scores on a medication dosage assessment cannot be explained solely by poor mathematical and computational skills. In Blais and Bath's (1992) analysis of student errors, 68% of all errors were *conceptual* in nature, meaning the error was either the result of the student setting up the problem incorrectly (e.g., constructed an incorrect proportion to describe the situation), or stating an improper form of administration (e.g., stated the amount to administer was 2 mg instead of 2 mL). Only 19% of all errors were labeled as *mathematical*, which included errors related to multiplication, division, and conversions between decimals and fractions. The third category of errors -- *measurement errors* -- accounted for 13% of the total errors and they occurred when students incorrectly used a conversion factor between measurement systems (metric and apothecary systems).

In their evaluation of nurses' dosage calculation competency, Fleming et al. (2014) found conceptual errors were most common. The researchers describe the participants' poor scores on IV drip rate calculations as "a conceptual issue as opposed to a mathematical issue, with participants not extracting the information exactly" (p. 58). Lesar's (1998) study at a teaching hospital provides insight into similar issues. In an evaluation of 200 errors associated with "dosage equations" (a phrase used by the researchers to describe the process of calculating dosage), it was discovered that 24% of errors were due to the individual incorrectly using the daily dose, divided dose, and/or dose frequency in the calculation process. For example, the nurse might calculate the dosage as 5 mL of medication to administer per dose, when instead it should be 5 mL per day divided into two doses, or 2.5 mL per dose. While these types of errors might be labeled as miscalculations, they cannot be solely attributed to one's mathematical or computational skills. Instead, it is the misreading and misrepresentation of essential clinical information that leads to an incorrect dosage calculation. As Wright (2012) argues, evidence from these studies, "[point] towards calculations being solved in different ways and involving different numeracy skills, which are grounded in the context of the drug administration rather than in formal arithmetic operations" (p. 343). The results suggest that research exploring individuals' dosage calculation abilities should consider more than just their mathematical skills and the numerical result that they obtain on a dosage calculation task.

Additional Findings

The dosage calculation process has been described as a social practice in which "the skills of drug calculations are embedded within the clinical context and are made sense of and solved within this practice" (Wright, 2012; p. 342). Given these unique connections, it is not surprising to find researchers who have taken different methodological approaches to studying how nurses calculate medication dosage.

In their ethnographic study of pediatric nurses in practice, Hoyles, Noss, and Pozzi (2001) found that the nurses dosage calculations on the ward were "routine and error free" (p. 22). The researchers argue this is due to the fact that the nurses' knowledge of calculating dosage is inextricably connected to the contexts and resources they experience. For example, during some episodes on the ward, the nurses attributed their calculation strategy to the specific drug they were administering, instead of relying on general formulas or other methods they were taught at their university. In a follow-up study, the researchers found nurses' mathematical and professional knowledge were interwoven in such a way that while they could coordinate their mathematical and professional knowledge to complete dosage-related tasks, they could not complete similar tasks outside of a nursing context (Noss et al., 2002).

Using a grounded theory approach, Marks et al. (2015) explored the conflicts that nursing students' face in learning the mathematics necessary for nursing practice. Rather than relying on a mathematics or dosage calculation assessment measure, the researchers gathered data through semi-structured interviews using four numeracy questions as the basis for the discussion. Although the purpose of their study was not to assess the students' dosage calculation abilities, the interviews provided rich data on the students' experiences with dosage calculations in the classroom and in clinical contexts.

There are a few important ideas that can be taken from the results of these studies. First, the findings of these studies are consistent with those who argue that educators must "dispense with reductionist approaches that focus on calculation skill development in isolation" (Weeks et al., 2013, p. 30) and instead focus on the strategies that support students' conceptual understanding of dosage calculations (Marks et al., 2016; Ramjan, 2011; Ramjan et al., 2014; Shanks & Enlow, 2011; Wright, 2012). These studies also illustrate that exploring dosage calculations with qualitative research methods (e.g., observations, interviews) can be insightful for understanding *how* individuals complete dosage tasks. That is, qualitative methods have been used to understand the unique reasoning strategies individuals employ for calculating dosage and the connections they make between underlying mathematical and nursing concepts.

Dimensional Analysis for Calculating Dosage

Dimensional analysis has become a popular method for calculating dosage as it is presented as an "easy to learn" computation method that saves time and does not require "a knowledge of simple algebra" (Carr et al., 1976, p. 1937). As described earlier in this chapter, there are multiple ways for employing dimensional analysis, including for calculating medication dosage. Craig (2011) describes two such methods: a sequential method and a random method (Figure 2-21). With both methods, the given quantity and appropriate conversion factors needed produce the wanted quantity must be identified. Once identified, the sequential approach is completed by establishing the unit path from the given quantity to the wanted quantity, and systematically placing the conversion factors so unwanted units are canceled. With the random method, the individual begins with the given quantity and focuses on canceling units to produce the wanted "without regard to a logical, sequential placement of the conversion factors" (Craig, 2011, p. 72). That is, one does not have to sequentially place the conversion factors, but instead is free to ignore the proceeding units in the unit path. This is evidence in infusion rate calculations in Figure 2-21. These calculations illustrate that a correct numerical value to a dosage problem can be obtained by following either a sequential or random placement of units, as long as the unwanted units cancel to produce the desired units.

Figure 2-20

Calculating infusion rates with the sequential and random methods (Craig, 2011, p. 177)

Sequential Method

$$\frac{0.5\ \cancel{\text{mg}}}{\text{kg}/\text{hr}} \cdot \frac{\cancel{250}\ \text{mL}}{\cancel{250}\ \cancel{\text{mg}}} \cdot \frac{1\ \cancel{\text{kg}}}{2.2\ \cancel{\text{lb}}} \cdot 132\ \cancel{\text{lb}} = \frac{0.5 \times 1 \times 132}{2.2} = \frac{66}{2.2} = \frac{30\ \text{mL}}{\text{hr}}$$

Random Method

$$\frac{7\ \cancel{\text{mcg}}}{\cancel{\text{kg}/\text{min}}} \cdot \frac{25\cancel{0}\ \text{mL}}{50\cancel{0}\ \cancel{\text{mg}}} \cdot \frac{1\ \cancel{\text{mg}}}{100\cancel{0}\ \cancel{\text{mcg}}} \cdot 8\cancel{0}\ \cancel{\text{kg}} \cdot \frac{6\cancel{0}\ \cancel{\text{min}}}{1\ \text{hr}} = \frac{7 \times 25 \times 1 \times 8 \times 6}{50 \times 10 \times 1} = \frac{8400}{500} = 16.8 \text{ or } 17\ \frac{\text{mL}}{\text{hr}}$$

Applying dimensional analysis to complete a dosage calculation can also be done with a backwards or reverse approach. Arnold (1998) describes this approach in four steps (Figure 2-22). Following these steps, an individual would not begin with the given quantity and units (as was seen with the sequential and random methods above), but instead the calculation process begins with the desired quantity and units. With the desired units as the starting factor, individuals then focus on canceling unwanted units until they arrive at the desired units in the denominator.

Figure 2-21

An Approach to Using Dimensional Analysis for Calculating Dosage (Arnold, 1998).

1. Establish the desired form of the medication (both quantity and label), always as a fraction, on the left side of the equation (e.g., ? ml/dose).
2. Begin the right (computational) side of the equation with a "starting factor" containing the desired numerator label in the numerator position (e.g., the concentration of the medicine supplied).
3. Continue the right side of the equation by arranging all other known quantities in a sequence in which all undesired denominator labels are consecutively cancelled by a succeeding undesired numerator label.
4. Stop entering data only when the desired denominator label appears and remains as the only uncancelled denominator label (the only uncancelled numerator label is entered in the starting factor).

Figure 2-22

Calculating an Infusion Rate with a Backwards Approach (Cookson, 2013, p. 60)

Prescribed: *nitrogylcerin 5 mcg/minute I.V.*
Supplied: *nitrogylcerin 50 mg in 500 mL 0.9% sodium chloride solution.*

Step 1: Label needed?	$\frac{?\ \text{mL}}{\text{hour}}$
Step 2: Same numerator.	$\frac{?\ \text{mL}}{\text{hour}} = \frac{500\ \text{mL}}{50\ \text{mg}}$
Step 3: Place to cancel.	$\frac{?\ \text{mL}}{\text{hour}} = \frac{500\ \text{mL}}{50\ \cancel{\text{mg}}} \times \frac{1\ \cancel{\text{mg}}}{1{,}000\ \cancel{\text{mcg}}} \times \frac{5\ \cancel{\text{mcg}}}{\cancel{\text{minute}}} \times \frac{60\ \cancel{\text{minutes}}}{1\ \text{hour}}$
Step 4: Do the math.	$\frac{?\ \text{mL}}{\text{hour}} = \frac{500\ \text{mL}}{50\ \cancel{\text{mg}}} \times \frac{1\ \cancel{\text{mg}}}{1{,}000\ \cancel{\text{mcg}}} \times \frac{5\ \cancel{\text{mcg}}}{\cancel{\text{minute}}} \times \frac{60\ \cancel{\text{minutes}}}{1\ \text{hour}}$ $= \frac{150{,}000}{50{,}000} = \frac{3\ \text{mL}}{\text{hour}}$

Cookson (2013) recommends a similar process for calculating dosage (seen in Figure 2-23). In this dosage example, the nurse must calculate the number of milliliters per hour (mL/hr) to administer to the patient so that they are receiving nitroglycerin at a rate of 5 micrograms per minute (mcg/min) with an available concentration of 50 milligrams in 500 milliliters. Since the desired units at the end of the calculation are milliliters per hour, the individual would begin with the given factor that incorporates the unit of milliliters (the concentration of the available medication). From there, conversion factors are sequentially placed in a unit path until the desired denominator is reached (hour). Just like with the random approach described above, the placement of factors in the unit path does not have to be sequential to arrive at a correct numerical value, however, confirming a solution and identifying mistakes can often be easily identified when the canceled units alternate between successive numerators and denominators (Curren, 2010).

Findings of Nurses' Use of Dimensional Analysis for Calculating Dosage

There is empirical evidence suggesting dimensional analysis is an effective and error-reducing method for calculating medication dosage, especially when dosage calculation instruction focuses specifically on its use. In one quasi-experimental study of 59 nursing students, Craig (1993) found that nursing students who were taught to use dimensional analysis for calculating dosage over traditional formula and ratio approaches saw significantly greater gains on a dosage calculation post-test. This is consistent with studies suggesting students' use of dimensional analysis leads to significant differences on dosage calculation assessments. Greenfield, Whelan, and Cohn's (2006) exploration of the impact of teaching a standardized dimensional analysis approach for dosage calculations found that students utilizing dimensional analysis committed less errors on a dosage calculation examination compared to those who used traditional formulas. More specifically, the researchers found that 33 of the 39 students (84.6%) in the dimensional analysis group passed the examination compared to 16 out of the 26 students (61.5%) in the formula group.

Another study of 107 nursing students in a baccalaureate nursing program found promising results from those using dimensional analysis to calculate dosage. Following a series of instructional treatments focused on using dimensional analysis, Rice and Bell (2006) report that students who used dimensional analysis on a dosage assessment demonstrated "improved ability to calculate dosages correctly" (p. 316) by committing fewer conceptual errors. Additionally, students reported higher scores on a self-perceived confidence assessment with all students in the dimensional analysis group rating themselves as "always" confident or confident "most of the time" when completing dosage calculations with dimensional analysis. The researchers argue that using dimensional analysis for calculating dosage "[empowers] students to conceptualize [the] dosage calculation" (p. 317), thus leading to fewer conceptual errors.

Turner's (2018) doctoral dissertation on the impact of a schema-based dimensional analysis workshop found that those who attended the workshop performed better on a dosage calculation exam than those who did not attend the workshop. In one cohort, 85.7% of the students who completed the dimensional analysis workshop passed the dosage exam with the mandatory score of 100%, compared to the pass rate of 66.7% for those who did not attend the workshop. Turner argues that the results suggest dimensional analysis should be taught as the sole problem-solving strategy for calculating dosage. This suggestion is consistent with Koharchik et al. (2014) and their study of junior-level nursing students' use of dimensional analysis. Their results suggest that focusing dosage calculation instruction on dimensional analysis can help students to avoid common errors, and thus improve competency rates. The researchers also found that students in two separate cohorts expressed positive comments about dimensional analysis, with a total of 151 out of 164 (92.1%) stating it was a useful tool, and 139 out of 164 (84.8%) stating that they planned to continue using dimensional analysis for dosage calculations.

Koohestani and Baghcheghi (2010) found somewhat different results in their study of 42 third semester nursing students use of dimensional analysis for IV rate calculations. In their study, half of the participants completed a workshop where they learned to calculate IV dosage using traditional methods (i.e., formulas and proportional reasoning strategies), and the other half learned to complete the calculations with dimensional analysis. An analysis of the pre- and post-test scores revealed no statistically significant differences between the two groups following the educational treatments. However, 3-months later, when participants completed another dosage calculation assessment, those in the dimensional analysis group scored significantly higher than those utilizing traditional methods. Koohestani and Baghcheghi argue that while both methods led to increased scores on students' post-test scores, seeing significantly different scores 3-months later suggests a level of "sustained learning" when using dimensional analysis for calculating dosage. Put another way, the researchers suggest that dimensional analysis provides a systematic and stable approach that remains with students following dosage calculation instruction (p. 236).

Not all studies on nursing students' use of dimensional analysis have found significant evidence to support its use. Veldman (2016) explored the relationship between instruction focused on dimensional analysis for calculating dosage and nursing students' scores on a self-efficacy assessment and found that a dimensional analysis instructional program was not any more effective in increasing students' self-efficacy levels related to their dosage calculation abilities. Kohtz and Gowda (2018) also found non-statistically significant results in their evaluation of a dimensional analysis teaching program. When compared to students in the control group, students taught to use dimensional analysis had a slightly lower passing rate on a dosage calculation assessment (61.11% to 65.12%). These students also committed errors at a higher rate with 53.49% of the dimensional analysis group committing between one to four errors, compared to 36% of those students using traditional methods.

Summary

Dimensional analysis is generally viewed as an easy-to-implement method for calculating dosage that involves a focus on canceling units. Given that it can be applied consistently in a variety of dosage contexts without having to rely on multiple formulas, it has gained popularity in the nursing community as a method for reducing unnecessary dosage administration errors. There is empirical evidence suggesting dimensional analysis can be used to improve students' dosage calculation abilities and confidence. With the exception of the study of Rice and Bell (2006), who incorporated students' comments in their analysis, studies investigating dimensional analysis as a tool for calculating dosage rely solely on quantitative measures. The literature base lacks qualitative data describing how students use dimensional analysis for calculating dosage and the extent to which they connect dimensional analysis with foundational notions of ratio and proportion.

Chapter 3

Methods and Procedures

Research Design

The design of this qualitative study aligns with the constructivist research paradigm, which posits that (1) an individual's understanding of the world is socially and experientially constructed, and (2) one's mental constructions are not more or less "true" than another's, but "less informed and/or sophisticated" (Guba & Lincoln, 1994, p. 111). Qualitative research studies designed from this epistemology often seek to understand the meanings and beliefs of the participants as they relate to a topic, setting, or some other context (Creswell & Miller, 2000; Maxwell, 2013). In such studies, it is often the goal of the researcher to elicit one's mental constructions through activities and interactions (Guba & Lincoln, 1994; Sharma, 2013). One way of eliciting such constructions is through interviews, which provide participants the opportunity to explain their thinking while engaging with and responding to carefully designed tasks (Hatch, 2010; Labinowicz, 1985). In an interview setting, researchers are able to follow participants' explanations and actions, pose questions, and seek clarification to better understand the participants' realities (Hunting, 1997; Maxwell, 2013). This allows the researcher to collect rich data that other methods (e.g., paper and pencil surveys, multiple choice examinations) are less likely to generate (Forsey, 2012).

The goal of this study is to better understand nursing students' conceptions of dimensional analysis as a method for calculating medication dosage. I use Simon's (2017) specification of conception as "an explanatory model used to explain observed abilities and limitations of mathematics learners in terms of their (inferred) ways of knowing" (p. 120). This specification suggests that in order to construct a conception from *observed* abilities, the researcher must provide participants the opportunity to describe their individual thinking and perspectives related to the mathematical idea or context of interest. In this study, the participants completed dosage calculation tasks in two different situations: (1) asynchronously completing three dosage problems sent through e-mail; (2) participating in a 60-minute semi-structured interview that afforded each participant the opportunity to describe their thinking and illustrate their individual perspectives on dimensional analysis as a method for calculating medication dosage.

Participants

An invitation to participate in a qualitative research study was sent to all active students enrolled in a nursing program at a small, private health sciences college in the northeastern United States. Out of approximately 1765 students who received the invitation, 43 completed the volunteer survey, and 15 of these individuals received and returned completed work for three dosage calculation tasks (described later in this chapter). Ten final participants agreed to continue with the second phase of the study, which consisted of a 60-minute task-based interview through Zoom.

Nine of the ten completed their mathematics requirement for their program at this institution. These individuals completed the course, *Clinical Mathematics for the Health Sciences*, which is taught by mathematics faculty[1] and is specifically designed for future nurses, surgical technicians, and other workers who might be tasked with calculating and administering medication in a clinical setting. Students taking this course explore topics in algebra and statistics, but a significant portion of the course content is focused on calculating medication

[1] At the time of this study, I was one of the mathematics faculty members at this institution.

dosage in a variety of contexts (e.g., oral medications, insulin, IV infusions, critical care, titration). Although mathematics faculty at the institution tend to portray dimensional analysis as the preferred method for calculating dosage, students are encouraged to utilize any strategy they wish, including the nursing formula and other strategies incorporating ratios and proportions.

One individual in this study, Heather, did not complete the *Clinical Mathematics for the Health Science* course as part of her program requirements. Instead, she completed her mathematics requirement at another institution that integrated dosage calculation instruction throughout the curriculum. That is, there was no specifically designed course that focused on the mathematics for medication dosage calculations, and instruction of this content was primarily delivered by nursing faculty and not mathematics faculty.

The purpose of this study is to understand nursing students' conceptions of dimensional analysis as a method for calculating medication dosage. Although there is variation in how the students learned dimensional analysis for completing dosage calculations, I did not seek to make generalizations about the potential impact different curricular paths might have on students' dosage calculation abilities or their use of dimensional analysis. The methodology employed in this study is insufficient for determining a causal relationship between curriculum and pedagogy on students' conceptions of dimensional analysis. Instead, by interviewing individuals who may have different experiences with calculating dosage with dimensional analysis, I aimed to develop a richer understanding of students' conceptions of dimensional analysis as a method for calculating medication dosage.

A summary of the participating students' pseudonyms, whether they took the *Clinical Mathematics for the Health Sciences* course, and how they received instruction related to medication dosage calculations is provided in Table 3-1 below.

Table 3-1

Summary of Participating Students in this Study

Participant (Pseudonym)	**Did the individual take *Clinical Mathematics for the Health Sciences*?**	**Instructors of Dosage Calculation Content**
Allie Maya Zoey Violet Susan Laura Betty Jade	Yes	Mathematics faculty
Heather	No	Nursing faculty

Data Collection

Data were collected in two phases: asynchronously through e-mail, and through a 60-minute semi-structured, task-based interview. Prior to their interview, each participant was e-mailed three dosage calculation tasks to complete. They were instructed to show all of their calculations, scan their completed work, and e-mail the document(s) back. Participating students were then interviewed through Zoom for approximately 60 minutes. The aim of these interviews was (1) to afford participants an opportunity to discuss the work they completed asynchronously, and (2) to gather data on their explanations and reflections as they completed tasks follow-up dosage tasks informed by researcher-articulated concepts of dimensional analysis.

Asynchronous Dosage Tasks

The purpose of sending three asynchronous tasks to participants was to gather initial data on how students employ dimensional analysis for calculating dosage. These tasks (Problems 1-3 in Appendix A) could be described as standard dosage calculation tasks, including an oral dosage calculation, a rate conversion, and a weight-based dosage calculation. Participating students were instructed to show all of their calculations on paper, scan their work, and e-mail the document(s) back. Their completed work was then used to construct an interview transcript.

Task-Based Interview

After completing the asynchronous tasks, the participating students sat down for a 60-minute task-based interview. Due to the COVID-19 pandemic, these semi-structured interviews were completed online through Zoom. Pictures of the participants' completed work from the asynchronous tasks and follow-up questions were placed on a shared whiteboard through Google Jamboard. Throughout the interview, students were instructed to scroll to a particular page of the whiteboard and react to the work or follow-up prompt provided. The tasks constructed for these interviews (Tasks 1B, 2B, 3B, 3C) are presented in Appendix B.

The purpose of the task-based interviews was to gather data on how students employ dimensional analysis for calculating dosage and to understand their perspectives on how and why dimensional analysis is a valid tool for calculating dosage. Students were asked questions about their completed work, and they were asked to complete follow-up tasks related to the three patients they saw in the asynchronous tasks.

Interview questions were influenced by the student-centered questions in Hasenbank's (2006) *Framework for Procedural Understanding*, which is based on Burke's (2002) six elements of procedural literacy (Table 3-2). According to Hasenbank and Hodgson (2007), the Framework for Procedural Understanding is "a device that teachers can use to develop lessons, examples, problems, and assessments with principles of learning mathematics procedures with understanding" (p. 5). For the purpose of this study, the student-centered questions composing the Framework for Procedural Understanding influenced the development of questions in the

Table 3-2

Interview Questions were Inspired by the work of Burke (2002) and Hasenbank (2006)

Burke's Procedural Literacies (Burke, 2002)	**Framework for Procedural Understanding (Hasenbank, 2006)**
1. The student understands the overall goal of the procedure and knows how to predict or estimate the outcome.	**a.** What is the goal of the procedure? **b.** What sort of outcome should I expect?
2. The student understands how to carry out the procedure and knows alternative methods and representations of the procedure.	**a.** How do I execute the procedure? **b.** What are some other procedures I could use instead?
3. The student understands and can communicate to others why the procedure is effective and leads to valid results.	Why is the procedure effective and valid?
4. The student understands how to evaluate the results of the procedure by invoking connections with a context, alternative procedures, or other mathematical ideas.	What connections or contextual features could I use to verify my results?
5. The student understands and uses mathematical reasoning to assess the relative efficiency and accuracy of the procedure compared with alternative methods that might have been used.	When is this the "best" procedure to use?
6. The student understands why the procedure empowers her or him as a mathematical problem solver.	What can I use this procedure to do?

interview. That is, interview questions were designed to prompt the participating students to explain their reasoning and justifications related to dimensional analysis for calculating dosage. A sample interview script is provided in Appendix C.

Additional questions in the interview were specifically designed to elicit explanations from students around two, dimensional analysis concepts articulated by the researcher (Table 3-3). Consistent with Simon (2017), each of the mathematical concepts in Table 3-3 represents my attempt to articulate the students' expected understandings related to dimensional analysis

Table 3-3

Dimensional Analysis Concepts Articulated by the Researcher with Associated Tasks

Articulated Concept	Associated Task
Dimensional Analysis Concept 1: Dimensional analysis is a method for calculating medication dosage that involves multiplying some ordered quantity, scalar values, and conversion factors in order to arrive at a new, desired quantity. That is, $(ordered \times scalar\ values \times conversion\ factors) = desired$ In dosage situations requiring multiple calculations, when the scalar values and conversion factors remain constant, the product of all scalar values and conversion factors represents an invariant multiplicative relationship between the ordered quantity and desired quantity. In these situations, when either the ordered or desired quantity increases by some multiplicative factor, the other quantity must increase by the same multiplicative factor.	Oral Dosage Calculation Adjustment (Task 1B) IV Rate Calculation Adjustment (Task 2B)
Dimensional Analysis Concept 2: Dimensional analysis is a method for calculating medication dosage that involves multiplying some ordered quantity, scalar values, and conversion factors in order to arrive at a new, desired quantity. That is, $(ordered \times scalar\ values \times conversion\ factors) = desired$ In dosage situations involving multiple calculations, when the ordered quantity and conversion factors remain constant, and one (or the product) of the scalar factors changes by some multiplicative factor, the desired quantity will change by the same multiplicative factor.	Weight-based Dosage Calculation with Weight Adjustments (Tasks 3B, 3C)

and the underlying multiplicative relationships involved in a medication dosage situation. These mathematical concepts represent *envisioned mathematical conceptions*, and once data are collected and analyzed from the task-based interviews, these envisioned conceptions can be contrasted with those constructed from observed individual behaviors and actions.

To be clear, the explicated concepts do not represent the exact language an individual is expected to use during the interview, but instead, the concepts capture the logical necessity the students could potentially know regarding dimensional analysis and the underlying multiplicative relationships involved in calculating medication dosage. With two dimensional analysis concepts articulated, specific dosage tasks and prompts were created so that data generated from the interview could be used to infer the extent to which students' conceptions of dimensional analysis are aligned with each of the *envisioned* mathematical conceptions (i.e., the explicated concepts).

In these concept-informed tasks, adjustments are made to some component of the initial dosage situation that the students saw in the asynchronous tasks. These tasks include adjustments that impact the numerical solution to the initial dosage task by some multiplicative factor. Informed by the work of Cramer, Post, and Currier (1992), these tasks incorporate both whole number and non-integer multiplicative adjustments to explore the extent to which the presence of non-integer factors might impact students' thinking and strategies.

Data Analysis

All interviews were recorded, transcribed, and imported into the software, NVivo-10 (QSR International, 2012). Transcript data were analyzed using a hybrid coding method and analytic memos (Miles et al., 2020; Saldaña, 2016).

The students' completed work for the asynchronous and interview tasks were scanned by each student and submitted through e-mail. The scanned documents informed the reflections presented in the analytic memos, and they provided a reference point during the coding process.

Analytic Memos

Analytic memos were created to capture researcher reflections and to facilitate the synthesis of overarching themes in the data (Maxwell, 2013). More specifically, an analytic memo was created for each participant and reflections were updated at three different times: (1) after the synchronous work was sent through e-mail but before the interview, (2) immediately after the task-based interview on Zoom, (3) during the analysis of the interview transcript with NVivo-10.

Throughout the entire analysis process, a separate "overarching themes" analytic memo was used to (1) document notes, observed actions, and work across all of the participants, and (2) organize emergent patterns and themes. For example, after reviewing the participants' completed work on the asynchronous tasks sent through e-mail, it was clear that some individuals employed dimensional analysis with a sequential approach, while others utilized a backwards approach. Even though the task-based interviews had not yet taken place, headers for "Sequential" and "Backwards" were added to the overarching themes memo so anticipated notes, participant explanations, and submitted work could be added. Informed by other findings in the literature, similar headers were added to account for the anticipated proportional reasoning strategies that students might employ (i.e., qualitative, additive, multiplicative). As the analysis advanced, and related explanations and work across multiple participants became more visible, additional headers were added to the overarching themes memo. For example, although "algorithmic approaches" had not been initially identified in the memo, actions and statements across multiple individual memos prompted that this be added. A similar, inductive process was used to update the list of descriptive codes during the analysis of the interview transcripts.

Hybrid Coding

Transcript data were analyzed with a provisional list of descriptive codes informed by the literature. While *a priori* descriptive codes can help a researcher to ground the data in the literature, relying solely on these items might ignore more complex themes that can emerge as the analysis progresses (Miles et al., 2020). For this reason, I used an inductive coding process and added new codes to account for emerging ideas and concepts. In particular, in-vivo codes, or participant-generated words and phrases, were labeled to capture the students' voices as well as provide insights into individual perspectives and actions (Saldaña, 2016).

As an example, at the beginning of the transcript analysis, the code "Why use" was used to categorize statements related to why the student uses dimensional analysis over other dosage calculation methods. Throughout the first few interviews, multiple students spoke of being able to "see" the placement of units in the dimensional analysis work and how this instilled confidence that their work was accurate. At this time, sub-codes were added under "Why use" to account for these, and other similar, explanations by the participating students. The final set of codes and subcodes is found in Table 3-4.

Table 3-4

Final Set of Codes with Description

Code and Subcodes	**Description**

Connections
- Non-nursing contexts
- Nursing contexts
- Other dosage calculation methods
- Underlying math
    o Proportion (other)
    o Proportion-Additive
    o Proportion-Multiplicative
        ▪ Integer
        ▪ Non-integer
    o Proportion-Qualitative
	Actions and statements related to mathematical ideas (e.g., ratios and proportional reasoning), nursing concepts and ideas (e.g., medication label, nursing protocols), or other non-nursing concepts and ideas.

Goal-Purpose	Actions and statements related to the goal or purpose of dimensional analysis.
How - Backwards - Inflexible - Knows multiple ways - Other - Random - Sequential	Actions and statements related to “HOW TO” employ dimensional analysis.
Why Use - Checks work - Confidence - Ease of use - Feelings/Emotion - “See it”	Actions and statements related to why the individual (or anyone else) might use dimensional analysis.

Capturing the students’ own words is especially important for this study as one of the goals is to specify individual conceptions of dimensional analysis. Recall that Simon (2017) posits that a conception is not a statement about some individual nor an attribute of that individual, but rather it is a researcher-generated model for describing how one might think about a particular mathematical idea. In order to construct these models, researchers must immerse themselves in the data and make small local inferences to make sense of the individual's thinking, even when an individual "[acts] in ways that are inconsistent with how the researchers would act in the situation and different from expected behaviors" (p. 132). Utilizing both a priori descriptive coding and inductive in-vivo coding provided a connection to the existing literature, and it permitted conceptions of dimensional analysis to be constructed from the students' explanations and actions.

Chapter 4

How and Why Dimensional Analysis is Used to Calculate Medication Dosage

Chapters 4 and 5 present the findings of this study. In each chapter, I articulate specific conceptions that emerged from analysis of the participating students' submitted work, and their statements and actions during the task-based interview. These conceptions represent my effort to describe what the students appear to think, know, and understand about dimensional analysis for calculating medication dosage. That is, the conceptions are researcher-constructed models that aim to make sense of the data and explain the students' observed abilities (Simon, 2017). Supporting evidence, including excerpts from interview transcripts and pictures of student work, is included to provide additional context for each conception. I conclude each chapter with a discussion of how the findings connect with and add to the literature base. I also provide suggestions for future exploration in this area.

In this chapter, I present and illustrate on five conceptions that relate to *how* and *why* students chose to use dimensional analysis to complete medication dosage tasks.

Articulated Conception 1 – Sequential Application of Dimensional Analysis

Dimensional analysis is a process for calculating dosage that begins with the units of a prescriber's order. The units of the available medication strength and conversion factors are then strategically placed in a "domino-like" pattern in the unit path to cancel out the unwanted units. The dimensional analysis calculation is complete when the remaining un-cancelled units match the desired units, which are determined by given dosage situation (i.e., the nature of the medication and how it is being administered).

Supporting Evidence

Numerous students who utilized a sequential approach to dimensional analysis stated that they “always start with” the prescriber’s order, including Allie, Betty, Heather, Violet, and Zoey. Others did not use the phrase, “always start with,” but their explanations and actions supported a sequential approach to dimensional analysis, meaning that they begin the calculation with the prescriber’s order.

After an analysis of the interview data, Laura’s transcripts revealed the strongest association with the code “Sequential.” Figure 4-1 illustrates her approach for completing the follow-up to the first dosage task for Harold Smith. In the problem, Harold was ordered to receive 0.4 milligrams of Neupogen (filgrastim) as a subcutaneous injection, and the available medication had a concentration of 300 mcg per 1 mL. As evidenced in her explanation in Figure 4-1, Laura started with the prescriber’s order (0.4 mg), considered the available medication (300

Figure 4-1

Laura’s Work and Explanations for Dosage Task 1B

Harold #1

$$\frac{0.4\,\cancel{mg}}{1} \times \frac{1000\,\cancel{mcg}}{1\,\cancel{mg}} \times \frac{1\,mL}{300\,\cancel{mcg}} = 1.3\ mL$$

1.3mL

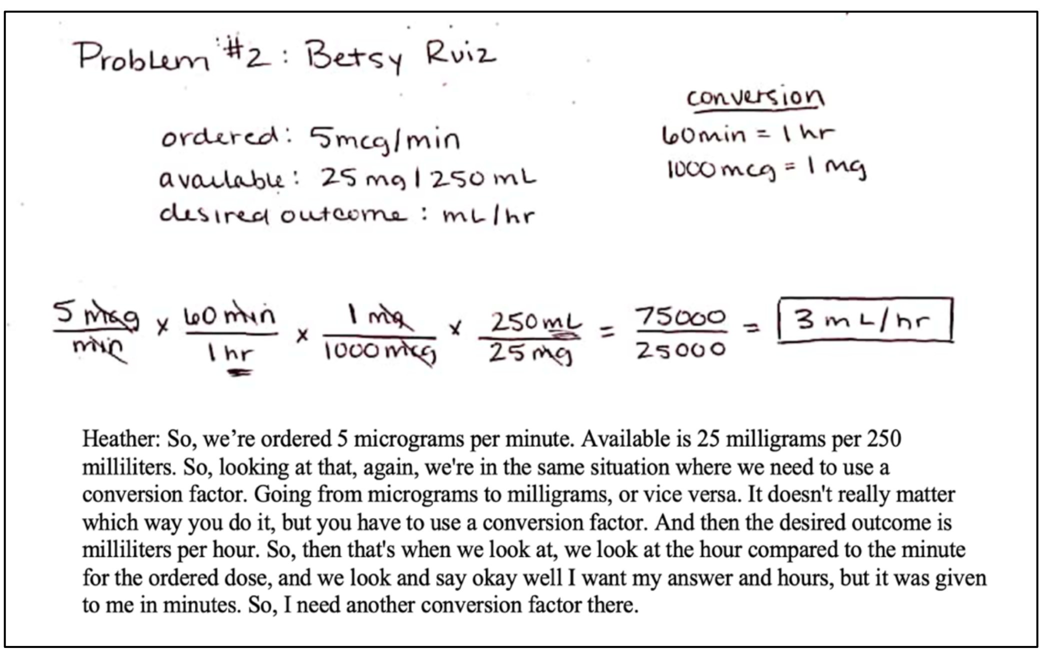
Laura: Okay, alright. So, um, the order is for that 0.4 milligrams, so I'm writing that over 1. And then my next conversion, just looking at this box here, it's saying 300 micrograms per milliliter like the other box. So, I'm writing micrograms on the top. And milligrams on the bottom, just so those milligrams are going to cancel out. And we know that there's 1000 micrograms per milligram. And then the next thing over is I want to get it into milliliters. So, I want to have milliliters on the top and micrograms on the bottom, so those will cancel out, and on the box that says 300 micrograms per 1 milliliter. And I actually go through and cross out, so I know that I'm ending up with what I want. So I crossed out my milligrams I crossed out my micrograms, I circled milliliters, because I know that's what I want and I’ll only have one of those left, and then I'm going to go through and multiply.

mcg/mL), recognized a conversion between milligrams and micrograms was needed, and placed the units in the unit path in order to arrive at the desired unit (milliliters). More specifically, Laura detailed *how* the units of sequential conversion factors must be placed to obtain the final, desired unit (mL).

Others also utilized a sequential approach and provided similar explanations and justifications of their work. In the first task requiring a conversion between rates, Heather described her process for employing dimensional analysis, stating she considered the “ordered, available, and desired outcome.” Her work and explanations for this calculation is presented in Figure 4-2. Similar to Laura, Heather explained that she started the calculation with the order, considered the available medication, and determined whether a metric conversion was required. From there, Heather considered the desired units for the given situation, mL/hr, and whether additional conversions were required. Her explanations are provided in Figure 4-2.

Figure 4-2

Heather’s Work and Explanations for Dosage Task 2A



Articulated Conception 2 — Backwards Application of Dimensional Analysis

Dimensional analysis is a process for calculating dosage that begins with the units/ratio of the available medication strength. The proper orientation of the ratio is determined by considering the desired units at the end of the calculation, which depend on the given dosage situation (i.e., the nature of the medication and how it is being administered). The units in the prescriber's order and conversion factors are then strategically placed in a "domino-like" pattern in the dimensional analysis unit path to cancel out the unwanted units. The dimensional analysis calculation is complete when all unwanted units have canceled out, or for rate calculations, when the unit in the denominator of the last factor matches the one desired for the given dosage situation.

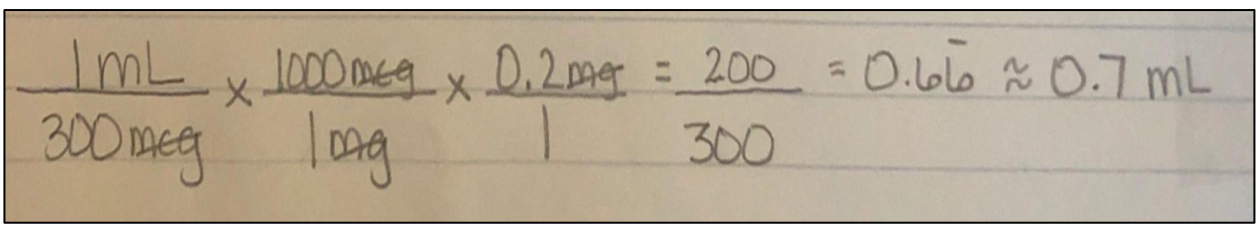
Supporting Evidence

Two students, Maya and Clair, preferred a backwards dimensional analysis approach for calculating dosage. This is different from a sequential approach in that an individual begins the procedure with the units that are desired for the given dosage situation, which are uniquely provided on the available medication. For example, in the first dosage task, Harold Smith was ordered to receive 0.2 mg of Neupogen (filgrastim) as a subcutaneous injection, and the available medication had a concentration of 300 mcg per 1 mL. Utilizing a backwards approach to dimensional analysis, both Maya and Clair started their calculation with $\frac{1\,mL}{300\,mcg}$. As Maya explained, she began the calculation with mL in the numerator because of the given dosage situation. She stated, "I know because it's an injection, it's going to be in milliliters. So, I'm trying to determine how many mLs I'm going to need for the shot."

The ways in which Maya and Clair carried out their dimensional analysis with a backwards approach depended on the dosage situation and the contextual information provided in the problem. During her interview, Clair repeatedly referred to the prescriber's order as the factor she needed to "get to" or end up with at the end of the calculation. After completing the first dosage task (work illustrated in Figure 4-3), Clair explained that since the units in the denominator of the starting factor (mcg) did not match those that she ultimately needed to get to (i.e., the prescriber's order in milligrams), then she needed to "get rid of the micrograms" with the next conversion factor. After "getting rid" of micrograms, she then multiplied by the physician's order (0.2 mg) in order to "cross out milligrams."

Figure 4-3

Clair's Work for Dosage Task 1A



In a later task, a patient was ordered to receive nitroglycerin at a rate of 5 mcg/min and the available IV bag had a strength of 25 mg per 250 mL (or equivalently, 100 mcg per 1 mL). Consistent with a backwards approach to dimensional analysis, both Maya and Clair began their dimensional analysis with the available medication and the unit of milliliters in the numerator. Additionally, they both provided similar explanations for how they carried out the procedure, including recognizing that they needed to incorporate the prescriber's order (5 mcg/min), thus bringing a unit of time into the calculation.

After recognizing that the given task required units of mL/hr at the end of the calculation, Maya began her work with the available medication (1 mL / 100 mcg) (Figure 4-4). She then explained, “I want something to cancel out the micrograms and 100, so I knew that the next the next problem had to include micrograms, which would be the five micrograms per minute.” Finally, Maya stated, “then I know that there's 60 minutes in one hour and I needed to cancel that one minute and I knew that we want it to be milliliters over hour.”

Figure 4-4

Maya’s Work for Dosage Task 2A

On hand 100mcg/1mL

$$\frac{1\,\text{mL}}{100\,\cancel{\text{mcg}}} \times \frac{5\,\cancel{\text{mcg}}}{1\,\cancel{\text{min}}} \times \frac{60\,\text{min}}{1\,\text{hr}} =$$

$$\frac{1 \cdot 5 \cdot 60}{100 \cdot 1 \cdot 1} = \frac{300}{100} = 3\,\text{mL/hr}$$

Maya’s explanations were similar to those provided by Clair and they reflect a more general approach for how they calculated rates with a backwards dimensional analysis approach. That is, after beginning the calculation with the concentration of the available medication, Maya and Clair both described focusing on the denominator of the given factor. If the unit in the denominator wasn’t the one desired, then it needed to be “canceled,” and thus would need to be in the numerator of the next conversion factor. During her interview, Clair described this process (i.e., identifying the next conversion factor in the unit path) as having a nice “flow,” and she stated, “having [the units] right next to each other, it’s an easy reminder that that's what you're trying to get rid of.”

Articulated Conception 3 — Dimensional Analysis as a Flexible Procedure for Calculating Dosage

There are multiple ways for an individual to use dimensional analysis to calculate medication dosage and arrive at an accurate value. One could begin with a different starting factor (i.e., utilize a sequential or backwards approach), order the ratios and conversion factors differently within the unit path, periodically stopping after a conversion, and/or choose to complete some calculations outside the dimensional analysis unit path. These actions do not change the fact that all necessary units will cancel, resulting in the desired units necessary for the given dosage situation.

Supporting Evidence

During the interviews, students were asked to explain their submitted work and how they complete dosage tasks with dimensional analysis. They were also asked follow-up questions prompting them to respond to whether a friend’s dimensional analysis might look different from their work. An analysis of the explanations and actions of the students revealed that a significant majority (nine out of the ten) were aware of alternate approaches of employing dimensional analysis to complete a dosage task. Put another way, it can be inferred that these individuals view dimensional analysis as flexible, meaning there is some level of flexibility in how it can be employed to calculate dosage.

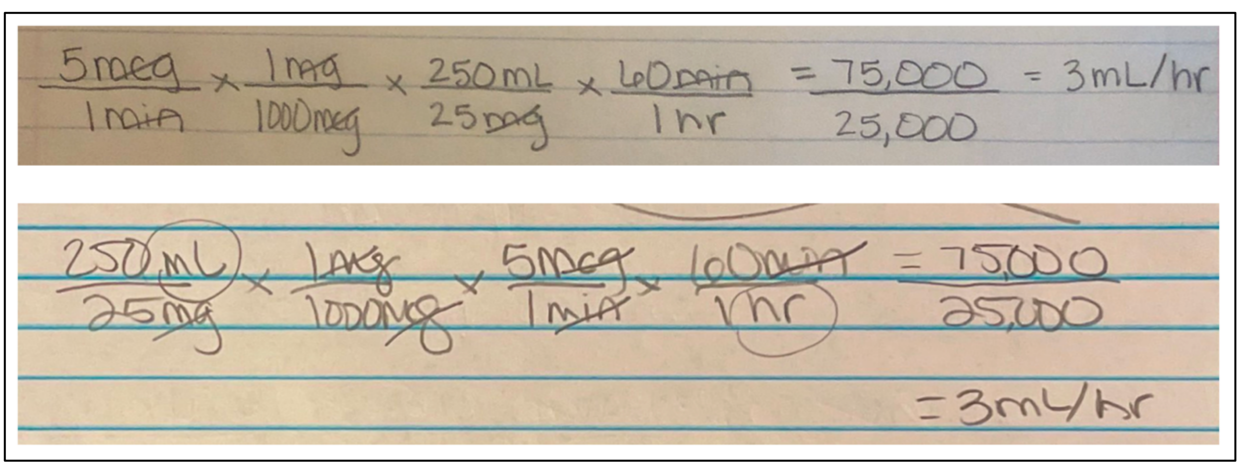
During her interview, Heather stated a preference for completing dosage calculations with a sequential approach (i.e., starting with the prescriber’s order). However, when asked whether there were alternate ways to employ dimensional analysis, Heather specifically mentioned that "you could work it backwards if you really wanted to." She further explained that although you could begin the dimensional analysis calculation with different factors, it would be essential to consider the placement of the units to ensure the calculation is accurate:

> You could have started it with what was available, but it would just be important to put your desired outcome on top. So, if I would have started with the available, medication dosage, which was 300 micrograms per milliliter, I would just put milliliter on top and then cancel everything out like I did before with using dimensional analysis. But personally, for me, I always start with was order, because it's easier for me to visualize

As discussed previously, Clair communicated her preference for a backwards approach to dimensional analysis (i.e., starting the calculation with the available medication). However, for some of the dosage calculations, Clair also chose to utilize a sequential approach. When asked to explain why she completed the second dosage task with a sequential approach, and whether a friend could have completed the calculation in a different way, Clair stated, "they could have done it backwards" and started the calculation "with what was on hand." She then proceeded to re-do the calculation illustrating this backwards approach (Figure 4-5).

Figure 4-5

Clair's Work Illustrating a Sequential and Backwards for Dosage Task 2A



Others illustrated a view of dimensional analysis as a flexible procedure by detailing the different ways that the conversion factors might be placed in the unit path. For example, when explaining her work for the second dosage task, Zoey stated that the order of the conversion factors would not matter. Zoey's work and explanation are provided in Figure 4-6.

Figure 4-6

Zoey's Work and Explanation for Dosage Task 2A

$$\frac{5\ \text{mcg}}{\text{min}} \cdot \frac{1\ \text{mg}}{1000\ \text{mcg}} \cdot \frac{250\ \text{mL}}{25\ \text{mg}} \cdot \frac{60\ \text{min}}{1\ \text{hr}} \qquad \text{mL/hr}$$

$$3\ \text{mL/hr} \qquad \frac{75000}{25000} = 3\ \text{mL/hr}$$

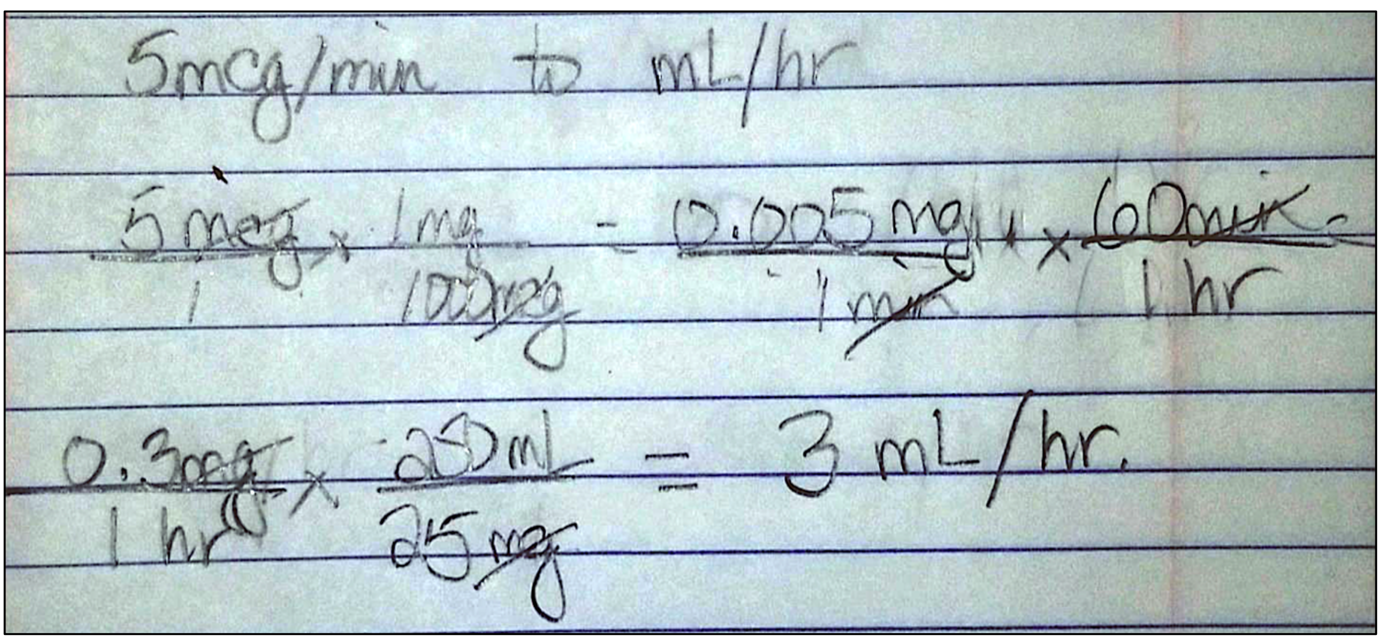
Zoey: … So, I always start with the order. And if I have to switch it to get to what's the available, then my second move is usually trying to switch it. You know, get it in the correct measurement and then I use with the available. And then I always end up with flipping the time, but you can arrange it however you'd like, as long as you end up in milliliters an hour. Like if you want to do your minutes first into hour, it doesn't matter…. Like if you had five micrograms a minute and, you know, in the end, because you have a circle down there that it is milliliters an hour, if your second move wanted to be to take the minutes, to switch minutes to hours, you could. So, you could do five micrograms over a minute times 60 minutes over one hour, you could do that second, instead of the way I did it, which always, I always try, you always kind of get your own pattern. Um, but you could do that differently. You know, you could do your, your moves can change…

When pressed about this further and asked why the order of conversion factors didn't matter, Zoey explained that "it's all about the end," meaning the desired units for the given dosage situation. When explaining why she chose 1 mg per 1000 mcg as her second factor, Zoey stated, "we're starting with micrograms over a minute, so you're going to have to do that somewhere in this problem. So, you can do it second, you can do it third, you can do it wherever you want it, but as long as you end up in the end, milliliters an hour."

Jade's explanations and actions also suggested a view of dimensional analysis as a flexible procedure. During her interview, Jade made it clear that she prefers to complete dosage calculations by "chunking" the dimensional analysis work into individual conversions – rather than performing multiple calculations, resulting in a one long unit path. An example of how Jade utilized a "chunking" approach is provided in Figure 4-7. In this example, Jade first converted 5 micrograms (from the prescriber's order, 5 mcg/min) into 0.005 milligrams. From there, she multiplied by 60 minutes over 1 hour to calculate the rate in milligrams per hour. Finally, by multiplying by the available medication strength, Jade converted the mg/hr rate into mL/hr.

Figure 4-7

Jade's "Chunking" Approach to Dimensional for Dosage Task 2A



When asked to explain her unique approach, and whether a friend might complete the calculation differently, Jade explained, “I, uh, my brain, I guess likes to take it in smaller chunks, so I could have done that in its entirety and just cross cancelled, but I like to look at things in equation form, in small format. So, I did the first step and then moved on to the next step, just all in smaller portions.” Later in the interview, Jade shared that it would not make a difference whether an individual “chunked” their work into individual conversions or completed the calculation with a longer unit path. She explained, “as long as we're setting it up the same where everything is correctly canceling each other out, as far as like milligrams, you know, hours, or minutes… everything is just multiplied across and then divided by the bottom factor. So, I guess we can, anyone could set it up in a different format.”

Articulated Conception 4 — Dimensional Analysis as a Rigid Procedure for Calculating Dosage

There is little flexibility in how one might employ dimensional analysis to calculation medication dosage. If a friend’s work doesn’t match my accurate application of dimensional analysis, it is likely that their work is not accurate.

Supporting Evidence

Although the explanations and actions of most students suggested a view of dimensional analysis as a flexible procedure (i.e., there are multiple ways to employ dimensional analysis to calculate medication dosage), there were some instances during certain interviews that would not align with this perspective. As an example, many of Maya’s explanations and actions would suggest that she views dimensional analysis as a rigid procedure, meaning that any deviations from her dimensional analysis approach would likely lead to inaccurate solutions. This particular excerpt supports this inference:

> **Daniel Ozimek:** So, let me ask it a different way, if, if your friend did this problem with dimensional analysis could their work look different?
>
> **Maya:** It could, but they might have the wrong answer.
>
> **Daniel Ozimek:** Okay, but… if all their work was correct, it would look like yours, is that correct?
>
> **Maya:** Yes, yes. It would look like mine.

Later in the interview, when promoted with an additional question about whether a friend's work could look different from her work, Maya provided a similar explanation, stating, "I don't think it would be, because dimensional analysis, I mean, at least as far as I've done, it's always looked the same." After completing the weight-based calculations, Maya emphasized her perspective on dimensional analysis:

> **Daniel Ozimek:** And so, I know I've asked this for other problems as well, but if a friend also completed this with dimensional analysis, is it possible that their work for this problem might look different?
>
> **Maya:** Not if they use dimensional analysis.
>
> **Daniel Ozimek:** So, if they use dimensional analysis, their work would look just like yours?
>
> **Maya:** Yeah.

Finally, when asked whether there were other ways to complete the given dosage task, with or without dimensional analysis, Maya explained, "Me? Personally no. I'm sure there's other ways to complete it. Um, but this is how I would have done it." This suggests that Maya is aware of additional approaches to completing dosage calculations, but when it comes to utilizing dimensional analysis, there is one, appropriate way that she knows of to complete the calculation.

Articulated Conception 5 – Why Use Dimensional Analysis to Calculate Medication Dosage.

Dimensional analysis is a logical, organized process for calculating medication dosage. When completing a dosage task with dimensional analysis, an individual places their attention on creating a “domino-like” pattern of units in the unit path to cancel unwanted units and obtain the desired units for the given dosage situation. The “domino-like” pattern of units in the unit path provides an in-the-moment check, as well as a visual artifact at the conclusion of the calculation to confirm that one’s work is accurate. As a result of these opportunities to check one’s work, utilizing dimensional analysis to calculate medication dosage can produce feelings of confidence and security.

Supporting Evidence

Multiple questions during the task-based interviews were designed to elicit actions and statements related to *why* an individual might choose to use dimensional analysis to calculate medication dosage. A consistent finding across all interviews was that the students preferred dimensional analysis in part because it is a “step-by-step” and “visual” process. As Laura explained, dimensional analysis is a “very logical sequence of taking what you and taking conversion factors to convert into what the ordered medication is that it uses.” She added, “you don't leave [anything] out when you're doing this very methodical logical step-by-step process” and it ensures you don’t “end up with the wrong information.”

Jade provided a similar explanation, stating, “I would say it's a visually appealing math process, where you can see specifically what you're canceling out, so that, you know, you're on the right track of getting to the estimates that you need, such as like milliliters per milligrams, you know, what you have to cancel out to get to that an answer. So, it visually shows you.”

Another individual, Zoey, took this idea further and suggested that the step-by-step and visual nature of dimensional analysis takes a lot of the "thinking" out of the calculation. She explained,

> I love dimensional analysis, because it, I know this is going to sound ridiculous, but it almost like takes -- I don't want to say it takes the thinking out of it -- but it keeps you organized. If you know how to use it, you know that you're going to start with your order and you're going to put what you want to cross out and get out of up top, and it kind of keeps you in the correct flow to get you down to where you want to be. So, to me, it works very well for keeping me organized in the thought process for switching units of measurement.

Allie shared a similar insight, explaining that the process of canceling units in a domino-like pattern "teaches you where to place your numbers," which means that "it's very hard to come up with the wrong answer."

Multiple students spoke about how they often reflect on their final dimensional analysis unit path to confirm their work. Betty explained, upon completion of the calculation, "if you recognize that some of the units don't match or the number is kind of odd, you can retrace your steps and see where you make a mistake and change it." Clair provided similar remarks, but also mentioned the importance of reflecting upon the information provided in the dosage task, including the order and available medication, and making sure these items are in the dimensional analysis unit path. According to Clair, if you don't "see those things" in the unit path, "then you know you've gone wrong somewhere."

During Susan's interview, the "domino-like" pattern in the dimensional analysis unit path, and the final placement of the desired unit (mL), helped her to catch a small calculation error. Figure 4-8 illustrates this error, her corrected work, and her explanation for how she knew her calculation was incorrect. Because Susan recognized the placement of milligrams in the second factor, she was unable to cancel the unit of milligrams in her first factor. Additionally, she noticed that at the conclusion of the calculation, the desired units (mL) were in the denominator, which suggested that she had made an error somewhere in her calculation.

Figure 4-8

Susan's Work for Dosage Task 1B with a Caught Mistake

$$\frac{0.4\,mg}{1} \times \frac{1\,mg}{1000\,mcg} \times \frac{300\,mcg}{}$$

$$\frac{0.4\,\cancel{mg}}{1} \times \frac{1000\,\cancel{mcg}}{1\,\cancel{mg}} \times \frac{1\,mL}{300\,\cancel{mcg}} = \frac{400}{300}\,mL = 1.3\,mL$$

Susan: So I did the .4 milligram over one, and then I want to cancel out the milligram and make it microgram, and I accidentally listed microgram on the bottom and milligram on the top, so I couldn't cancel them out and I just realized that, because when I went to list the available, the 300 microgram over one milliliter, I realized that I reversed it, because I want the milliliters on top. Does that make sense?

The analysis of the students' explanations also suggests the step-by-step and visual nature of dimensional analysis can lead to feeling a sense of confidence and security in their work. For example, during her interview, Maya shared that she prefers dimensional analysis when calculating dosage, because when applied correctly, she knows it will give her "a good answer" and that she’s “not going to make a mistake.” Violet added similar thoughts, stating that once you learn dimensional analysis and understand how to use it, that “it’s in your brain,” and “you're not mixing up stuff and trying to figure out different calculations that are most likely incorrect.”

Betty also shared insights into how dimensional analysis makes her feel when calculating medication dosage. She explained, “for me, dimensional analysis has helped me because I am able to see it. The whole thing. And I'm able to secure myself in some, somehow to see the end result, and be sure that that's what I have to give or that's what I have to set up.” In another episode during her interview, Betty shared her perspective on why it is so important for her, an aspiring nurse, to have confidence in her work. She stated, “well, I mean, because when you're working with medication, you need to make sure that you're giving the right dose. You don't want to give less or more, and when it comes to that, I always had it on the back, well not on the back, on the front of my brain that I need to make sure that that calculation is right, so I don’t kill my patients.”

Discussion

I conclude this chapter with a discussion of how the articulated conceptions and findings described above relate to and extend the existing literature about dimensional analysis.

Students’ Approaches to Dimensional Analysis

As noted in Chapter 2, multiple studies in the literature provide evidence to suggest dimensional analysis is effective for reducing errors on dosage calculation examinations (Craig, 1993; Greenfield et al., 2006; Turner, 2018). Many of these studies utilized quantitative measures, as well as treatment and control groups, to explore a variety of research questions related to dimensional analysis for calculating dosage. This is significantly different from the methods and aim of this study; however, regardless of these differences in methodology and purpose, it is important to note the similar finding of accurate, dimensional analysis work for competing dosage tasks. The nursing students in this study were extremely accurate in their use of dimensional analysis to complete medication dosage tasks. Throughout all ten interviews, there were only two instances in which an individual made a small calculation error. In both of these situations, the individual recognized and fixed the error with little to no support from the researcher.

A majority of students in this study (i.e., eight out of ten) preferred a sequential approach for completing dosage tasks with dimensional analysis, whereas two of the ten preferred a backwards approach. Both of these methods are commonly found in dosage-related curriculum materials (Arnold, 1998; Cookson, 2013; Craig, 2011). When asked to explain how they completed a dosage task with dimensional analysis, the students described a process that was thoughtful, reasoned, and connected to the contextual factors in the dosage situation. This contrasts with perspectives of dimensional analysis as a rote procedure that can be completed by following a simple “recipe” or “flowchart” (DeLorenzo, 1976; Graham, 1986). The students’ approach is markedly different from that of the Random Method (an example provided in Figure 4-9), which Craig (2011) suggests can be applied “without regard to a logical, sequential placement of the conversion factors” (p. 72).

Figure 4-9

Completing a Dosage Calculation with the Random Method (Craig, 2011, p. 177)

Random Method

$$\begin{array}{c|c|c|c|c|c|c}
7\ \cancel{mcg} & 25\cancel{0}\ mL & 1\ \cancel{mg} & 8\cancel{0}\ \cancel{kg} & 6\cancel{0}\ \cancel{min} & 7 \times 25 \times 1 \times 8 \times 6 & 8400 \\
\hline
\cancel{kg/min} & 50\cancel{0}\ \cancel{mg} & 100\cancel{0}\ \cancel{mcg} & & 1\ hr & 50 \times 10 \times 1 & 500
\end{array} = 16.8 \text{ or } 17\ \frac{mL}{hr}$$

One individual, Susan, made it clear that her approach to dimensional analysis is not one of randomness, but instead, is informed by first making sense of the information provided in the task. In this excerpt, Susan describes approaching dosage tasks like a puzzle, and she explains her thinking prior to and during the dimensional analysis process:

> Um, it's kind of like a puzzle, like you want to look at all the pieces you have. You want to figure out what is important and what's not. Like some problems will give you a weight, but you don't need a weight. So realistically, *the first thing you could focus on is what they're asking you, like what they're expecting*. So, if they're asking you, say milliliters over hours, or if they're just asking you, like an X amount of milliliters, you want to see what that end goal is, and then also look at what you have in front of you. So, *let's say you're given a bunch of just different information. You always want to look at what's ordered and what's available.* So, from there, alright, so if this is ordered and this is what I have for available, what do I have to do to both of them to get my end value?

Susan clearly articulates the importance of identifying pertinent information and thinking-through the calculations necessary to arrive at the desired units for the given dosage situation.

Betty also explained how she reasons with the information provided in a dosage task, especially when it comes to identifying sequential conversion factors for the dimensional analysis unit path. Betty shared that she often considers the phrase, "first one up and second one down" in order to reassure herself that she is completing the calculation correctly. As she put it, "it's easy to make a mistake in dimensional analysis when you don't understand where the labels should be." Both of these examples speak to how the students employed dimensional analysis with purpose and meaning, rather than as a procedure consisting of a random placement of units.

The Flexibility of Dimensional Analysis for Calculating Medication Dosage

The terms *flexible* and *rigid* (or *inflexible*) have been used in the mathematics education literature to describe the nature of a construct or mathematical idea, including mathematical conceptions and the application of procedures. As such, it is important to clarify the extent to which the findings of this study connect with these ideas.

A *flexible* conception has been characterized in different ways in the literature. In one study, Bannister (2014) characterized teachers' conceptions of function as either *flexible*, *disconnected*, or *constrained*. Individuals were said to exhibit a *flexible* conception of function if they could "[move] flexibly between constructs of process and object perspectives" (p. 229). Alternatively, if the teachers did not make connections between the process and object perspectives, or if they tended to operate with only one perspective, then these conceptions were categorized as either *disconnected* or *constrained*, respectively.

Following on the work of Lloyd and Wilson (1998), Jansen and Hohensee (2016) posit that productive conceptions are those that are both *connected* and *flexible* (p. 506). In their study, Jansen and Hohensee explored elementary pre-service teachers' conceptions of partitive division and the extent to which the conceptions were *connected* and *flexible*. Similar to Bannister (2014), Jansen and Hohensee clearly articulated a characterization of a *flexible* conception, but in the context of partitive division. More specifically, the pre-service teachers were said to hold a *flexible* conception if they were "aware it is appropriate to *partition* the dividend for whole number divisors, *iterate* the dividend for unit fraction divisors, and both *partition* and *iterate* the dividend for non-unit proper divisors" (emphasis in original) (p. 515). If the pre-service teachers were unaware of any of these notions, and thus did not illustrate a *flexible* conception, this was characterized as a *rigid* conception.

These examples are similar in that they posit a characterization of one's conception using terms such as *flexible*, *constrained*, *connected*, and *rigid*. That is, these terms are used to describe the nature or quality of an individual's mathematical conception. This is different from how "flexible" and "rigid" are being used in the articulated conceptions presented in this chapter. Rather than describing the nature or quality of the conception, these terms are used to describe one's use of dimensional analysis as a mathematical procedure. Put another way, the use of these terms in the articulated conceptions is more closely aligned with discussions in the literature around *procedural fluency*, which refers to "knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently" (National Research Council, 2001; p. 121).

Although there are differing perspectives in the literature around the nature of understanding mathematical procedures, and how they relate to conceptual understanding, there has been increased consensus that *flexibility* in applying procedures to solve problems requires building connections between conceptual and procedural knowledge (Baroody, 2003; Baroody et al., 2007; Hasenbank & Hodgson, 2007; National Research Council, 2001; Star & Rittle-Johnson, 2008). To illustrate how such connections permit flexibility, Baroody (2003) shares an analogy of a newcomer to a town:

> Initially, a newcomer's knowledge of her new hometown is rather incomplete and unconnected. She may know how to get from her house to her workplace and from her house to the grocery store. Unfortunately, if she is at work and needs to go to the grocery store, her only option is to return home… As the person explores her new hometown, she

> discovers landmarks and streets and can better see how they all fit together. As her knowledge of the town becomes more complete and interconnected, she can find her way around the town more easily. It allows her, for example, to determine the most efficient route from her workplace to the grocery store… Moreover, if this customary path is blocked, the resident's well-connected knowledge gives her the flexibility to determine the next-best route. (pp. 15-16)

In this study, the use of “rigid” to describe how one might view dimensional analysis is comparable to that of the newcomer who needs to return home before going to the grocery store. That is, the individual’s actions and explanations suggest that there is one way of employing dimensional analysis to calculate medication dosage. On the other hand, one who holds a view of dimensional analysis as flexible is aware of alternative approaches for using the procedure to complete dosage tasks.

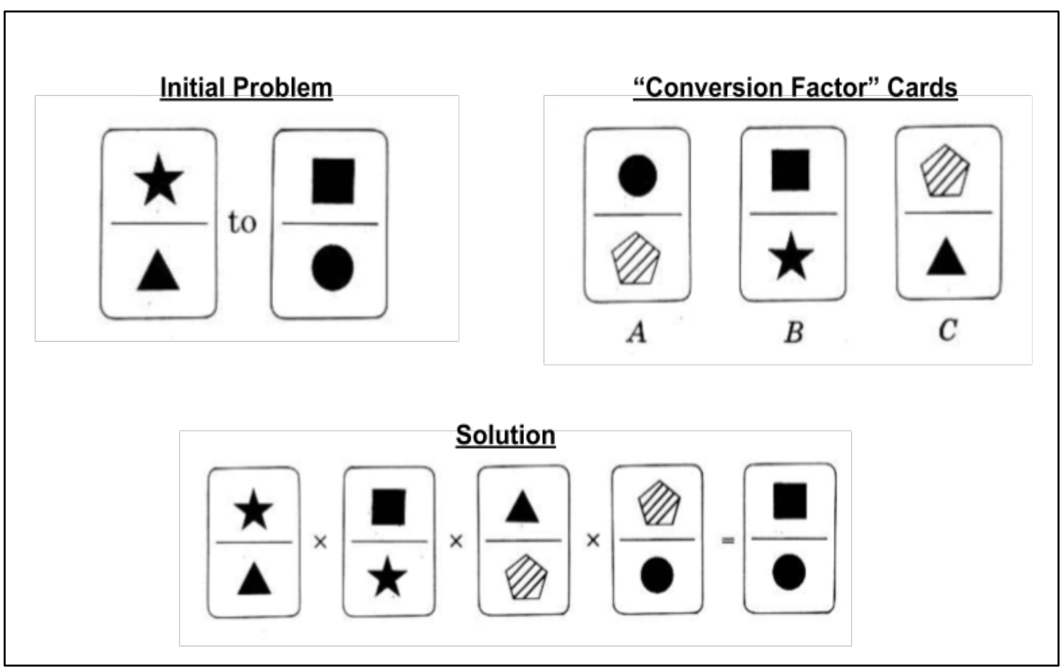
It is important to note that my purpose is not to make claims about the level or quality of understanding that an individual student might hold regarding dimensional analysis for calculating medication dosage. Rather, I posit conceptions that characterize what a *flexible* and *rigid* view of dimensional analysis might entail. Based on an analysis of student actions and explanations, one might exhibit a *flexible* view of dimensional analysis for calculating medication dosage if they recognize the procedure can be applied in different ways, such as beginning the calculation with either the order or available mediation (i.e., a sequential or backwards approach), ordering the ratios and conversion factors differently within the unit path, periodically stopping after a conversion, and/or choosing to complete some calculations outside the dimensional analysis unit path. Additionally, one might exhibit a *rigid* view of dimensional analysis for calculating dosage if they do not recognize that the procedure can be applied in these different ways.

The Visual Nature of Dimensional Analysis

Every student in this study referenced the visual alignment of units in the dimensional analysis unit path. This is not a surprising finding as the literature is rich with examples of how educators have leveraged the domino-like pattern to support students' use of dimensional analysis (Ellis, 2013; Garnett, 1980; Saitta et al., 2011). This includes the strategy from Garnett (1980), who used non-numerical symbols on cards to help students develop confidence in manipulating terms and canceling units. Figure 4-10 illustrates an example of a task where students are asked to convert a "star / triangle" card into a "square / circle" card using three

Figure 4-10

Using Symbols to Practice Canceling Units (modified from Garrett, 1980)



additional cards that act as conversion factors. Others have taken a similar approach, but with pictures of animals instead of shapes (Saitta et al., 2011). Ellis (2013) also leveraged the domino-like pattern in the unit path to support students' conceptual understanding of dimensional analysis in a high school chemistry course. Pre- and post-test scores suggested that students who engaged in an online program (*Conversionoes*), and responded to reflection questions, exhibited a deeper conceptual and visual understanding of dimensional analysis. Although the research questions posed in this study did not examine the impact of a teaching strategy on students' use of dimensional analysis, a common underlying theme is that both educators and students are aware of and tend to leverage the visual alignment of units in the dimensional analysis unit path.

Students' references to feeling secure and confident with dimensional analysis is consistent with Rice and Bell's finding (2006) that students who utilized dimensional analysis reported higher scores on a self-perceived confidence assessment. These researchers also reported that students in the dimensional analysis treatment group rated themselves as "always" confident or confident "most of the time" when completing dosage calculations with dimensional analysis. They argue that calculating dosage with dimensional analysis "[empowers] students to conceptualize [the] dosage calculation" (p. 317), thus leading to fewer conceptual errors. Although they didn't explore students' confidence with using dimensional analysis for completing dosage calculations, Koharchik et al. (2014) found students in their study viewed the method as positive. More specifically, the researchers reported that 151 out of 164 (92.1%) viewed dimensional analysis as a useful tool, and 139 out of 164 (84.8%) planned to continue using dimensional analysis for dosage calculations.

Another way the results of this study connect with the literature relates to students' descriptions of dimensional analysis as a tool that is easy to recall. As Clair put it, "when I was in math and when we learned dimensional analysis, it's just the one that stuck with me and that's the one I use." Violet shared similar remarks, stating "…if you correctly learn how to use it, and you understand which one you're starting with… it's in your brain. Then you're not mixing up stuff and trying to figure out different calculations that are most likely incorrect." These responses are consistent with Koohestani and Baghcheghi (2010), who found that after 3 months, students in their treatment group (i.e., those who used dimensional analysis) scored significantly higher on a dosage exam than students who didn't use dimensional analysis. Given these results, the researchers argue that dimensional analysis leads to a level of "sustained learning" when compared with other methods for calculating medication dosage.

Safety-Critical Contexts

When prompted to reflect on why someone would want to use dimensional analysis to complete dosage calculations, Betty shared, "when you're working with medication, you need to make sure that you're giving the right dose. You don't want to give less or more, and when it comes to that, I always had it on the back, well not on the back, on the front of my brain that I need to make sure that that calculation is right, so I don't kill my patients." The importance of these remarks cannot be understated, especially when it comes to the nursing profession, where a miscalculation could result in harming a patient or even death. Given such serious implications, Coben and Weeks (2014) argue that nursing is a *safety-critical* professional practice, and they offer several recommendations for educators who prepare students for working these contexts. For one, Coben and Weeks discuss how curriculum and instruction should be meaningful and authentic, suggesting "any disjuncture between theory and practice and between knowledge and performance may have serious consequences" (p. 260). They also recommend that educators "[make] students aware that there are many and varied ways to solve any problem." (p. 266).

When taken together, this would suggest students should be given the opportunity to engage in meaningful activities that (1) reflect authentic uses of dimensional analysis in clinical practice, and (2) posit dimensional analysis as a flexible procedure for calculating medication dosage.

Creating meaningful opportunities for students to engage in authentic contexts demands collaboration of educators from multiple disciplines (Coben & Weeks, 2014; Ozimek et al., 2021). O'Shea (1999) and Brady et al. (2009) make a similar call for collaboration across disciplines to mitigate medication administration errors in practice. Literature reviews conducted by these researchers suggest that mathematical skills are not the only contributing factor to medication administration errors; additional factors include personnel system and managerial problems, workload and staffing levels, deviations from procedures, distractions, and nurses' knowledge of medications.

Chapter 5

Dimensional Analysis, Dosage Calculations, and Connections to Proportional Reasoning Strategies

In addition to prompting students to reflect on their use of dimensional analysis for completing dosage calculations, questions during the task-based interviews were specifically designed to elicit explanations from students around two, dimensional analysis concepts articulated by the researcher (Table 5-1). Recall, these articulated concepts represent *envisioned mathematical conceptions*, and they represent an attempt to articulate the students' *expected* understandings related to dimensional analysis and the underlying multiplicative relationships involved in dosage situations (Simon, 2018). The associated tasks are presented in Appendix B., and they feature both integer and non-integer adjustments to the prescribed dosage and patient's weight in the given dosage situation. In addition to explaining their work as they completed these tasks, students were prompted to respond to whether their resulting calculation "made sense," and whether there were ways to confirm the accuracy of their calculation.

Table 5-1

Dimensional Analysis Concepts Articulated by the Researcher with Associated Tasks

Articulated Concept	Associated Task
Dimensional Analysis Concept 1: Dimensional analysis is a method for calculating medication dosage that involves multiplying some ordered quantity, scalar values, and conversion factors in order to arrive at a new, desired quantity. That is, $(ordered \times scalar\ values \times conversion\ factors) = desired$ In dosage situations requiring multiple calculations, when the scalar values and conversion factors remain constant, the product of all scalar values and conversion factors represents an invariant multiplicative relationship between the ordered quantity and desired quantity. In these situations, when either the ordered or desired quantity increases by some multiplicative factor, the other quantity must increase by the same multiplicative factor.	Parenteral Dosage Calculation Adjustment (Task 1B) IV Rate Calculation Adjustment (Task 2B)
Dimensional Analysis Concept 2: Dimensional analysis is a method for calculating medication dosage that involves multiplying some ordered quantity, scalar values, and conversion factors in order to arrive at a new, desired quantity. That is, $(ordered \times scalar\ values \times conversion\ factors) = desired$ In dosage situations involving multiple calculations, when the ordered quantity and conversion factors remain constant, and one (or the product) of the scalar factors changes by some multiplicative factor, the desired quantity will change by the same multiplicative factor.	Weight-based Dosage Calculation with Weight Adjustments (Tasks 3B, 3C)

An analysis of the students' work, actions, and explanations associated with these tasks resulted in four articulated conceptions. Recall these articulated conceptions represent my effort to describe what the students appear to think, know, and understand about dimensional analysis, particularly as they relate to using dimensional analysis to complete tasks involving an adjustment to one of the contextual factors in the dosage situation. The four articulated conceptions, as well supporting evidence, such as examples of submitted work and explanations, are presented in this chapter. I conclude the chapter with a discussion of how these results connect with the existing literature, and I also offer implications of these results and potential direction for future exploration.

Articulated Conception 6 — Qualitative Strategies for Supporting One's Completed Dimensional Analysis Work

There are often situations in which one must complete similar dosage calculations that vary by *one* contextual factor. This includes situations where the prescribed dosage or the patient's weight has changed, but all other contextual factors remain constant. One can support their dimensional analysis work in these situations by making comparisons between the adjusted quantities (i.e., identifying if the value increased or decreased). Other contextual factors may also provide insight as to whether an adjusted value is appropriate.

Supporting Evidence

In all four tasks that included an adjustment, either the prescribed dosage or the patient's weight increased. In these situations, students were accurate in calculating the new dosage, but they offered a variety of explanations when prompted to respond to whether their new value "made sense." One way that students responded to this question was to make qualitative comparisons between the adjusted values.

For example, when Maya was asked why her new adjusted dose for "Betsy" (Task 2B) made sense, she stated, "for me, it's because the physician was increasing the dose. So, it makes sense that the milliliters per hour would increase." Violet provided a similar explanation when she was asked whether it made sense that the patient weighing 32.4 kg should receive a faster infusion (97.2 mL/hr) than the patient weighing 24 kg. She stated, "Yes, because it is more than the 24 that we had last time, which was at 72, um, milliliters per hour." Betty shared similar comments, explaining, "So for this kid, I'm going to set up this, the rate of the pump at 97.2 mLs per hour. And that makes sense, because it gets a little bit bigger, so we're going to be giving a little bit more of that medication."

Betty also made sense of her calculations by considering other contextual factors in the dosage situation. In addition to making qualitative comparisons between the new and adjusted values, Betty also considered the original dose, the new dose, and the strength of the available medication to determine whether her new calculation made sense. That is, for the follow-up task where the patient (Harold) saw their dose increase from 0.2 mg to 0.4 mg (Task 1B), Betty considered the strength of the available medication (300 mcg/mL or 0.3 mg/mL) and recognized that Harold needed to receive more than the one milliliter printed on the label. She explained that her new calculation of 1.3 mL made sense, “because now it would be a little bit more than what the label is, so it makes sense that we’re giving a hundred mcg more, which makes sense.” When pressed further on this, Betty explained, “The label is 300 mcg [per] mL. Now they are asking me to give to 0.4 mg and before there were asking me to give 0.2. 0.2 is less than 300 [mcg]. In this case, they're asking me to give a little bit more than the label. The label is 300 [mcg] once you convert the 0.4, it becomes 400 [mcg].”

Articulated Conception 7 – Algorithmic Strategies for Supporting One’s Completed Dimensional Analysis Work

There are often situations in which one must complete similar dosage calculations that vary by *one* contextual factor. This includes situations where the prescribed dosage or the patient’s weight has changed, but all other contextual factors remain constant. One can support their dimensional analysis work in these situations by setting up and solving a missing-value proportion.

Supporting Evidence

Another common way that students made sense of their calculations involving an adjustment to the dosage or patient’s weight was to create and solve a missing-value proportion.

In many of these situations, students didn't necessarily discuss the multiplicative relationships within and between the various quantities, but instead, they recognized that a missing-value proportion could be used to confirm their calculation.

For example, when Susan was asked about the adjusted dose for Betsy (i.e., dosage increased from 5 mcg/min to 22 mcg/min), she knew she could create and solve a missing-value proportion. Her work and explanation are provided in Figure 5-1. Susan's explanation reflects that the initial dose and rate can be paired together in a ratio, and set equal to the new dose (22 mcg/min) and the unknown quantity (the new mL/hr rate). When asked why the proportion she created was a valid way to address the task, Susan explained, "Um, because it's like a ratio. It's

Figure 5-1

Susan's Work and Explanation for Task 2B

$$\frac{5\,\text{mcg/min}}{3\,\text{mL/hr}} = \frac{22\,\text{mcg/min}}{x\,\text{mL/hr}}$$

$$66\,\frac{\text{mcg/min}}{\text{mL/hr}} = 5\,\text{mcg/min} \times x\,\text{mL/hr}$$

$$13.2\,\text{mL/hr} = x\,\text{mL/hr}$$

$$13\,\text{mL/hr}$$

Susan: Um, so I knew the ordered, and then the given for the original problem. And since the available was the same, I did the 5, the original ordered, the 5 micrograms over minute, and the given, which is the 3 milliliters per hour. So, I did like the 5 over 3 and I did equals the new available. Or no, sorry. The new order, which is 22 micrograms per minute. And then I want to find the new rate, which is x milliliters per hour. So, then I cross multiplied. So, I ended up multiplying 3 by 22, and then it was 5x, and um, so I got 66 equals 5x, and then I divided both sides by five. So, I got 13.2 equals x milliliters per hour, but it was over 10 milliliters, so it's 13 milliliters per hour was when I got there.

like saying the same thing is like one is two as two is to four. So, like if this is the order and it's the same, or if this is the order and that's the given, it should be the same as a new order should be the same ratio as the new given." Later in the interview, Susan made it clear that she prefers using a missing-value proportion over dimensional analysis to complete dosage calculations when one of the contextual factors (i.e., the prescribed dosage or patient's weight) has changed. She explained,

> I mean, I could rewrite my entire dimensional analysis again and the only thing I would have to change is just the kilograms. So, I could write it out again and instead of 12 kilograms, I could just write 24 kilograms and I would get the same answer. Um, for me I think it's like after I know like you're original versus like the given rate. And then I get a second problem with like the equivalent, it's easy for me to look at it in ratio form.

During her interview, Zoey also illustrated an understanding of setting up and solving a missing-value proportion to calculate a new adjusted dose. In the follow-up task for Harold Smith (Task 1B), Zoey used the phrase "neighbors" to describe how the components of the missing-value proportion should be organized to calculate the new dose. She explained, "So if it was like… it's 0.2 and then you have 0.2 and that was equal to 0.67, so then this one would be underneath 0.4, you have to have the neighbors, have to be in the same, where you put things are important. 0.2, would both have to be in the numerator as 0.4, and then this one would be 0.67, and this would be an x." Zoey shared a similar explanation when calculating the new dosage for Betsy Ruiz, whose dosage increased from 5 mcg/min to 22 mcg/min. Zoey confirmed that her calculation of 13.2 mL/hr was accurate by creating and solving a missing-value proportion:

> So, I had 5 micrograms and that was 3 milliliters or, and then you have to do the neighbors that sit across from one another. And this one was 22 micrograms, and my

answer was 13.2 milliliters an hour, but I just want an “x” to see. And then I would cross multiply, 3 times 22 equals, divided by 5, equals 13.2. Mm hmm. So, it kind of works out, it checks out.

Articulated Conception 8 — Additive Strategies for Supporting One’s Completed Dimensional Analysis Work

There are often situations in which one must complete similar dosage calculations that vary by *one* contextual factor. This includes situations where the prescribed dosage or the patient’s weight has changed, but all other contextual factors remain constant. One can support their dimensional analysis work in these situations by considering *how* the adjusted factor has increased or decreased. If the prescriber’s order or patient’s weight changes, and all other contextual factors remain constant, one can consider the difference between the initial and new order/weight. This difference should coordinate with the difference between the initial problem’s desired units and the new problem’s desired units.

Supporting Evidence

In addition to utilizing qualitative comparisons and missing-value proportions to confirm and make sense of their calculations, students also considered the differences between the amounts to administer or prepare for the given dosage situation. In Task 2B, the patient’s dose increased from 5 mcg/min to 22 mcg/min, and using the available medication, Maya calculated that the infusion rate needed to increase from 3 mL/hr to 13.2 mL/hr. When asked whether the new infusion rate seemed appropriate, and if she could confirm her work in any way, Maya started to investigate the change in the ordered dose. She explained, “so essentially, the doctors increasing the order by 17 micrograms. Yeah. So, I mean, I guess we could do a problem. If we did, if we replace the 22 micrograms over 1 minute with 17 micrograms over 1 minute, maybe we would come out to a difference of what the milliliters are." Maya proceeded to use dimensional analysis to calculate the mL/hr rate associated with a dose of 17 mcg/min and found a value of 10.2 mL/hr (Figure 5-2). She confirmed that this value made sense, explaining, "if we would add the 17 micrograms, and the 5 micrograms and add what we, the solution to both of them 3 milliliters per hour plus 10.2 milliliters per hour, we would get 13.2 milliliters per hour, which is what 22 micrograms is." In a later task (Task 3C), Maya utilized a similar approach to confirm that the new infusion rate made sense for the heavier patient. More specifically, she found the difference in the patients' weights was 8.4 kg, used dimensional analysis to find this coordinated with a rate change of 25.2 mL/hr, and concluded this was accurate since it was the equivalent to the difference in the mL/hr rates for the two patients.

Figure 5-2

Maya's Dimensional Analysis Work for the Change in Dosage (Task 2B)

$$\frac{1\text{ mL}}{100\cancel{\text{mcg}}} \times \frac{17\cancel{\text{mcg}}}{1\cancel{\text{min}}} \times \frac{60\cancel{\text{mins}}}{1\text{ hr}} = \frac{1\cdot 17\cdot 60}{100\cdot 1\cdot 1} = \frac{1020}{100}$$

$$10.2\text{ mL}|\text{hr}$$

Allie also considered the differences in the patients' weights to make sense of her calculation, but she didn't utilize dimensional analysis to convert these differences into new units like Maya did. Instead, Allie recalled the first child in the weight-based calculations who weighed 12 kg and needed an infusion rate of 36 mL/hr. Allie recognized that the difference in weights for the second and third child (8.4 kg) was a bit less than the weight of the first child (12 kg), and thus the change in mL/hr rates should be a bit less than 36 mL/hr. She explained, “Well, if the first problem was 12 if my weight was 12 kilograms and I ended up with 36 milliliters per hour, and this problem, it's 32.4 kilograms… Basically, I'm adding 8.4 kilograms on to my second problem, which was 24 kilograms. So, the milliliters, it comes out in that range. It's not exactly another 36 that I'm adding milliliters, but within that range.” Although she didn’t find the exact value associated with 8.4 kg (as Maya did), Allie did incorporate an additive strategy to make sense of the work she completed with dimensional analysis.

Jade too considered the difference in weights (8.4 kg), but she struggled to make sense of this value and how it coordinated with the change in infusion rates. At first, Jade shared, “I don't know how I would test that because I'm tripped up on the 8.4,” but she continued working with the values to make sense of the calculation. She ultimately calculated the difference in mL/hr rates (25.2 mL/hr), but she didn’t make the connection of how 8.4 kg and 25.2 mL/hr related to one another. She explained,

> So, I took the, hold on, I have three different calculations here now. So, I took the 97.2 milliliters per hour, and I subtracted that from the 72 milliliters per hour. So, and that gave me 25.2 milliliters per hour. So that's a difference. I'm just trying to compare the differences, but I don't see like a connection to, to give you like a solid answer of, like, if I see a connection between them, between the answers.

Articulated Conception 9 — Multiplicative Strategies for Supporting One’s Completed Dimensional Analysis Work

There are often situations in which one must complete similar dosage calculations that vary by *one* contextual factor. This includes situations where the prescribed dosage or the patient’s weight has changed, but all other contextual factors remain constant. One can support their dimensional analysis work in these situations by considering *how* the adjusted factor has increased or decreased. If the prescriber's order or patient's weight changes, and all other factors remain constant, one can consider the multiplicative factor linking quantities within measure spaces (e.g., the initial and new order or weight), or the between measure spaces (i.e., the dosage or weight and the amount/rate to administer).

Supporting Evidence

In multiple interviews, students went beyond utilizing qualitative comparisons, missing-value proportions, and additive strategies, and spoke directly to the multiplicative relationships among the quantities in the dosage situations. In fact, for either Task 1B or Task 3B (i.e., those that saw a contextual factor increase by a multiplicative factor of 2), every individual noticed the changed quantity "doubled" when prompted whether their dimensional analysis calculation "made sense." However, when the contextual factor increased by a non-integer value (e.g., the dose in Task 2B and weight in Task 3C), many of the students did not consider the multiplicative factor of change, and those that did provided a variety of explanations when supporting their dimensional analysis calculation.

For some students, it was apparent the dose changed by some multiplicative factor, and so they estimated this value to support their calculation. For example, for Task 2B, Clair saw the dose increased from 5 mcg/min to 22 mcg/min, which she described as "a little over 4-fold." When asked to explain what she meant by "4 fold," Clair explained, "Well, 5 times 4 is 20, and you're increasing it to 22. 3 times four is 12, so… trying to explain this. It just seems like the ratio is appropriate. Looking at the difference in dosing. If that makes sense? Looks like the ratio would be appropriate. As far as a dose increase." Allie shared similar remarks, stating:

> The initial of infusion rate was 5 micrograms, and this is 22 micrograms, so I mean it's over four times that amount. So just in my head, I would figure 4, if I was just randomly guessing in my head, the original problem, I ended up with 3 milliliters. So, if I was anywhere near the ballpark, 4 times that amount would be 12, so I mean this is 22, so think I'm in the right ballpark of that, you know it's not... I think it's the right answer.

For the weight-based calculations, Jade specifically referenced that the non-integer factor presented a challenge. She explained,

> Well, the first child to the second child was doubled weight. From the second child to the third child the weight only increased 8.4 kilograms. So, I can't, I mean, is that like a third maybe? I don't know. I know that I could just double, and it was super easy the first time for the, between the second child and the first child, but… I can't, I'm stuck on the 8.4 that it's not doubled…

Laura shared a similar perspective explaining that when the weight doubled, she could "do that in my head." However, when the weight increased by a non-integer factor, she wasn't able to identify the exact, and thus couldn't precisely check her calculation. She shared, "you know, 12 verses 32.4, like it's almost 3 times as much, but not exactly… I don't know that I would utilize like a complex formula to double check my work."

Three individuals calculated the exact multiplicative factor of change, although they did not all take the same approach. For Task 2B, Heather first estimated that the dose increases by "about 4 times," sharing, "22 is about 4 times 5. So, 5 times 4 is 20, so it's about 4 times more medication ordered than previous. So that means the rate, theoretically, should be about 4 times the rate of the previous answer." When pressed whether it is possible to identify the exact value, Heather divided 22 by 5 and found the dose increased by a factor of 4.4. Later in the interview, Heather calculated the exact factor of change in the patients' weights. She explained,

> It's a ratio. Um, you can… so you can divide 32.4, which is the new child's weight, divided by 12, which was the first patient's weight, you can see that it's, the weight has increased by 2.7 times. So, we want to then take our rate from the first problem, 36, and multiply it by 2.7 times, and we should get, to make sure we have the correct answer. So, we should theoretically get our new rate, which is correct, 97.2 milliliters per hour.

When asked to explain how she might connect this relationship with the dimensional analysis work she completed, Heather shared an explanation that is very similar to the researcher-articulated concept that was used to construct the tasks and interview script. She explained,

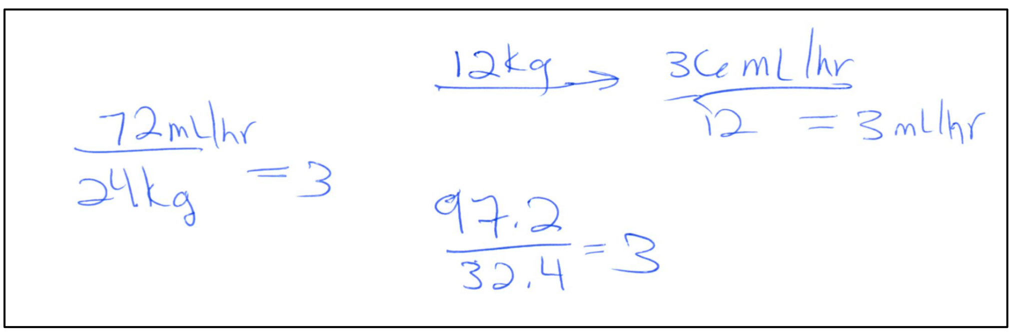
> …Everything that you're using to get your answer in the dimensional analysis is exactly the same for each patient. So, the order dose is the exact same for the patient, and the available doses is the same thing, our outcome is exactly the same thing. The only thing that changes is the patient's weight, and because only one factors changing, we can use like a ratio calculation to get our answer. So, if this patient, I wouldn't do it, but if this patient walked into the ED, and I knew that my bag was the same, so like my available dose is the same, my order dose is the same, I could just figure out like that it's, what did I say? 2.4 times greater. I could just multiply the rate of the first child by 2.4, or whatever it was, to get my answer without having to do the dimensional analysis. Just because all of the other factors are exactly the same.

Jade provided a similar explanation, first reasoning that the increase from 5 mcg/min to 22 mcg/min was between 4 and 5, stating, "roughly 5 times 4, or it's between 5 times, 5 or 4. 5 times 4 is 20, and 5 times 5 is 25 so it's kind of in between that range." When asked whether it was possible to find the exact value, Jade explained, "I could divide them, I guess. 22 divided by the initial is 4.4 and 13.2 divided by 3 is 4.4. So, they're exactly the same."

In these situations, Heather and Jade considered the covarying factor linking the increased dosage and weight. For the weight-based dosage calculations, Violet also calculated a multiplicative factor, but instead made comparisons between the patients' weights and infusion rates. That is, Violet divided each patient's weight by their respective infusion rate to find that they all resulted in a value of "3" (Figure 5-3). She explained, "so with the first question, the child's weight was 12 kilograms and then comparing that number to my answer of 36 milliliters per hour. 36 divided by 12 is 3, so I came up with 3 milliliters per hour per kilogram. And then if you multiply 3 times that 32.4 kilograms, you get 97.2."

Figure 5-3

Violet's Work Identifying a Common Multiplicative Factor in Tasks 3A, 3B, and 3C.



Discussion

Qualitative and Additive Proportional Reasoning Strategies

A significant finding from this study is that students utilize a variety of proportional reasoning strategies to support and make sense of their completed dimensional analysis work. This includes qualitative, algorithmic, additive, and multiplicative strategies, which is consistent with a number of studies that explored individuals' use of proportional reasoning strategies to complete comparison and missing-value proportion tasks.

Just as the individuals in Noelting's (1980a, 1980b) study utilized qualitative strategies to determine which set of glasses of water and orange drink would produce the strongest orange taste, individuals in this study supported their work with qualitative strategies by recognizing that the prescriber's order or patient's weight had increased. As multiple individuals explained, if these factors increase, then it makes sense that the infusion rate that the patient requires would also increase.

Other students like Maya, Allie, and Jade, utilized additive strategies to confirm their dimensional analysis work; however, their approaches were a bit different from those described in the literature. For example, Kaput and West (1994) describe the additive strategy of *coordinated build-up/build-down*, where an individual might operate with a ratio to identify like quantities (e.g., 1 mg to 10 mL is equivalent to 2 mg to 20 mL, 3 mg to 30 mL, etc.). As it relates to dosage calculations, Hoyles, Noss, and Pozzi (2001) found nurses employed a similar additive strategy, which they referred to as "scalar decomposition" (Figure 5-4). Completing dosage calculations with this approach involves iterating with the quantities of a given ratio to arrive at the desired quantity (e.g., iterating the available medication, 250 mg per 5 mL to arrive at 500 mg per 10 mL).

Figure 5-4

Scalar Decomposition Strategy for Calculating Dosage (Hoyles, Noss, & Pozzi, 2001)

$$250 \text{ mg} + 250 \text{ mg} = 500 \text{ mg}$$
$$5 \text{ mL} + 5 \text{ mL} = 10 \text{ mL}$$

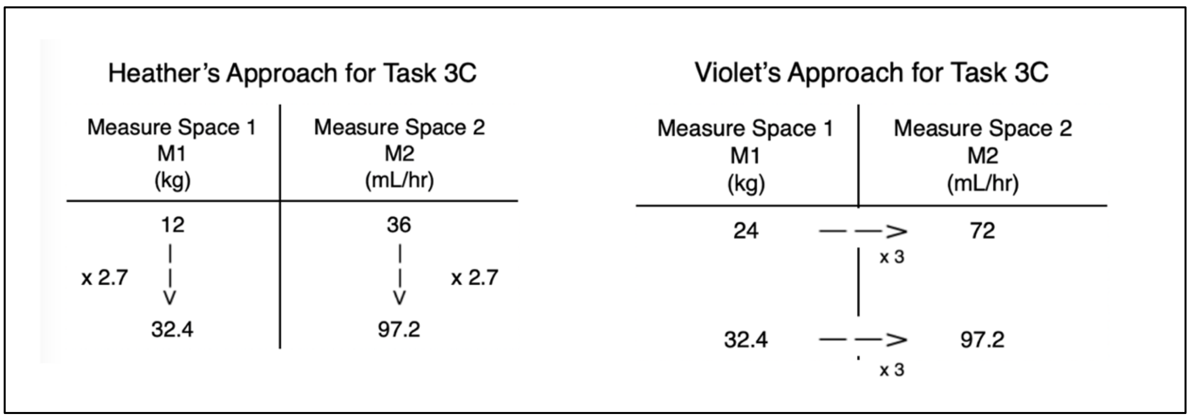
Rather than utilize this type of an approach (i.e., iterating a ratio), students instead considered the differences in changed values and confirmed that they coordinated with one another. For example, Maya recognized that from Task 2A and Task 2B the dose increased from 5 mcg/min to 22 mcg/min, which was a difference of 17 mcg/min. She then used dimensional analysis to convert 17 mcg/min to 10.2 mL/hr using the available IV bag. Maya recognized that this value (10.2 mL/hr) coordinated with the difference between infusion rates for Tasks 2A and 2B. This approach is clearly additive in nature as Maya considered the *difference* between the values rather than the multiplicative factor, but it is also different from the identified approaches for completing dosage calculations in the literature. Although their actions and explanations provide insight into their understanding of the underlying mathematical relationships involved in the situation, it is important to note that the nature of the tasks and follow-up questions in this study were different from completing traditional dosage calculation tasks. That is, students in this study had already completed their calculations with dimensional analysis, and the questions in the task-based interviews prompted them to reflect on whether their calculations were appropriate. As such, their unique approaches for checking their work are not necessarily unexpected.

Multiplicative Proportional Reasoning Strategies

Another finding that connects with the literature is the use of multiplicative proportional reasoning strategies. Two students, Heather and Violet, took different approaches to identify one of the multiplicative factors linking the quantities in the given dosage situation. More specifically, Heather's explanations closely aligned with a scalar operator strategy, and Violet's explanations aligned with the functional operator strategy (Hoyles et al., 2001; Vergnaud, 1983; Wright, 2013). These strategies differ in *how* the individual makes sense of the multiplicative relationships in the given situation. That is, an individual utilizing a scalar operator strategy considers the multiplicative factor *within* measure spaces, where as an individual utilizing a functional operator strategy considers the multiplicative factor *between* measure spaces. Figure 5-5 provides an overview of Heather and Violet's approaches using measure space notation (Cramer et al., 1993; Vergnaud, 1983, 1988).

Figure 5-5

Heather and Violet's Approaches for Confirming Their Task 3C



Another notable finding relates to the impact of non-integer multiplicative factors presented in Tasks 2B and 3C. Every individual in this study recognized that the changed quantities in Tasks 1B and 3B "doubled," or increased by a multiplicative factor of two. However, when the quantity changed by a non-integer, multiplicative value, like the dosage in Task 2B and the weight in Task 3C, only two individuals were able to find this value (Heather, Jade), and only one spoke confidently in the meaning of the value (Heather). Others relied on cross-multiplication or additive strategies to confirm their dimensional analysis work. This is consistent with Cramer et al. (1993) who found the presence of a non-integer relationship (1) significantly decreased the level of student achievement on the task, and (2) lead to students using additive strategies rather than dealing with the non-integer relationship (p. 12). Other studies have reported similar findings that suggest individuals who use “doubling” and “halving” to complete proportional reasoning tasks do not incorporate similar multiplicative approaches when a non-integer relationship is introduced (Hart, 1978; Noelting, 1980b). As Laura explained in her interview, the non-integer relationship between the adjusted quantities was “kind of an odd number, [and] it will be a little harder to compare.”

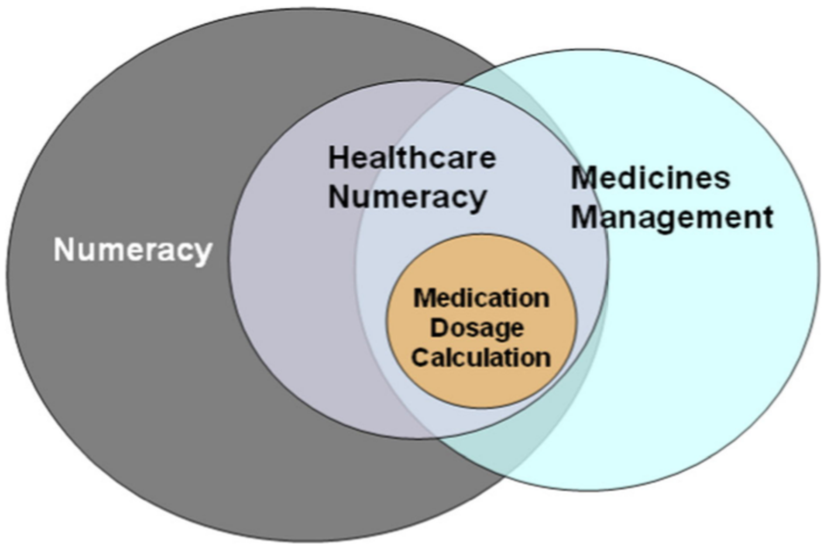
Inconsistency in Applying Strategies

Another notable finding is that some individuals did not consistency utilize proportional reasoning strategies across the tasks. For example, although she calculated the multiplicative factor linking the patient’s weight and infusion rate, Violet did not take this approach in other tasks. Additionally, Jade recognized the multiplicative factor linking the dose and rate in Task 2B, but for Task 3C, she utilized an additive strategy and considered the differences in weights and rates. A question remains as to whether Violet and Jade’s use of a particular strategy is dependent on the given dosage situation. That is, although the mathematical relationships are similar across these tasks (i.e., one contextual factor increased by a non-integer multiplicative value), to what extent does the changed contextual factor (e.g., dose versus weight) impact *how* an individual (1) makes sense of their calculation, and (2) chooses an appropriate strategy to confirm their work?

Suggesting that context might impact how one makes sense of and supports their dosage calculation would be consistent with a few perspectives in the literature. For example, as discussed in Chapter 2, Coben and Weeks (2014) and Young et al. (2013) situate medication dosage calculations at the intersection of numeracy, healthcare numeracy, and medicines management (Figure 5-6), and they argue that dosage calculation competency requires

Figure 5-6

Situating Medication Dosage Calculations Within Broader Domains



unique knowledge informed by each of these domains. This perspective is similar to Wright (2012), who posits that calculating dosage is a social practice in which "the skills of drug calculations are embedded within the clinical context and are made sense of and solved within this practice" (p. 342). Holyes, Noss, and Pozzi's (2001) study of nurses on a pediatric ward also offers evidence that nurses' dosage calculation knowledge is uniquely connected to the contexts and resources they experience.

These studies and perspectives provide insight on the knowledge that nurses' use to complete calculations *in practice*. The results of this study add to these ideas and provide evidence that the underlying factors involved in a dosage task (i.e., whether completed in practice or not) can impact how one makes sense of and calculates medication dosage.

Chapter 6

Summary and Conclusion

This final chapter provides a short summary of the findings and points of discussion from the previous two chapters. I conclude with final remarks on the aim and contribution of this study and suggested directions for future work related to ongoing mathematics and nursing education initiatives.

Conceptions of Dimensional Analysis

Preventable medication administration errors continue to persist in healthcare settings. According to the U.S. Food and Drug Administration, an estimated 100,000 reports associated with a suspected medication error are submitted to the agency each year (U.S., 2019). Given that nurses' mathematical skills have been identified as contributing factor to mediation errors (O'Shea, 1999; Brady et al., 2009), and empirical data suggest that mathematics and dosage calculation proficiency have historically been an issue in nursing education (Bindler & Bayne, 1984; Blas & Bath, 1992; Hutton, 1998; McMullan et al., 2010; Wright, 2007), it is not surprising that educators and researchers have explored ways to support nurses' mathematical understanding and dosage calculation proficiency. One such approach has been to focus dosage calculation instruction on using dimensional analysis, which is often regarded as an easy-to-implement process that can be applied consistently in a variety of dosage contexts (i.e., without having to rely on multiple formulas).

Through an analysis of ten nursing students' submitted work, actions, and statements, nine researcher-articulated conceptions of dimensional analysis were developed (Table 6-1). These conceptions provide a model how participating students made sense of dimensional analysis when calculating medication dosage. They include conceptions reflecting how the students complete dosage calculations with dimensional analysis, why they prefer it as a method for calculating dosage, and the variety of proportional reasoning strategies the students used to confirm their completed dimensional analysis work.

Table 6-1

Articulated Conceptions of Dimensional Analysis as a Method for Calculating Medication Dosage

1 – Sequential Application of Dimensional Analysis Dimensional analysis is a process for calculating dosage that begins with the unit(s) of a prescriber's order. The units of the available medication strength and conversion factors are then strategically placed in a "domino-like" pattern in the unit path to cancel out the unwanted units. The dimensional analysis calculation is complete when the remaining un-cancelled units match the desired units, which are determined by given dosage situation (i.e., the nature of the medication and how it is being administered).
2 — Backwards Application of Dimensional Analysis Dimensional analysis is a process for calculating dosage that begins with the units/ratio of the available medication strength. The proper orientation of the ratio is determined by considering the desired units at the end of the calculation, which depend on the given dosage situation (i.e., the nature of the medication and how it is being administered). The units in the prescriber's order and conversion factors are then strategically placed in a "domino-like" pattern in the dimensional analysis unit path to cancel out the unwanted units. The dimensional analysis calculation is complete when all unwanted units have canceled out, or for rate calculations, when the unit in the denominator of the last factor matches the one desired for the given dosage situation.
3 — Dimensional Analysis as a Flexible Procedure for Calculating Dosage There are multiple ways for an individual to use dimensional analysis to calculate medication dosage and arrive at an accurate value. One could begin with a different starting factor (i.e., utilize a sequential or backwards approach), order the ratios and conversion factors differently within the unit path, periodically stopping after a conversion, and/or choose to complete some calculations outside the dimensional analysis unit path. These actions do not change the fact that all necessary units will cancel, resulting in the desired units necessary for the given dosage situation.
4 — Dimensional Analysis as a Rigid Procedure for Calculating Dosage There is little flexibility in how one might employ dimensional analysis to calculation medication dosage. If a friend's work doesn't match my accurate application of dimensional analysis, it is likely that their work is not accurate.
5 – Why Use Dimensional Analysis to Calculate Medication Dosage Dimensional analysis is a logical, organized process for calculating medication dosage. When completing a dosage task with dimensional analysis, an individual places their attention on creating a "domino-like" pattern of units in the unit path to cancel unwanted units and obtain the desired units for the given dosage situation. The "domino-like" pattern of units in the unit path

provides an in-the-moment check, as well as a visual artifact at the conclusion of the calculation to confirm that one's work is accurate. As a result of these opportunities to check one's work, utilizing dimensional analysis to calculate medication dosage can produce feelings of confidence and security.
6 — Qualitative Strategies for Supporting One's Completed Dimensional Analysis Work There are often situations in which one must complete similar dosage calculations that vary by *one* contextual factor. This includes situations where the prescribed dosage or the patient's weight has changed, but all other contextual factors remain constant. One can support their dimensional analysis work in these situations by making comparisons between the adjusted quantities (i.e., identifying if the value increased or decreased). Other contextual factors may also provide insight as to whether an adjusted value is appropriate.
7 – Algorithmic Strategies for Supporting One's Completed Dimensional Analysis Work There are often situations in which one must complete similar dosage calculations that vary by *one* contextual factor. This includes situations where the prescribed dosage or the patient's weight has changed, but all other contextual factors remain constant. One can support their dimensional analysis work in these situations by setting up and solving a missing-value proportion.
8 — Additive Strategies for Supporting One's Completed Dimensional Analysis Work There are often situations in which one must complete similar dosage calculations that vary by *one* contextual factor. This includes situations where the prescribed dosage or the patient's weight has changed, but all other contextual factors remain constant. One can support their dimensional analysis work in these situations by considering *how* the adjusted factor has increased or decreased. If the prescriber's order or patient's weight changes, and all other contextual factors remain constant, one can consider the difference between the initial and new order/weight. This difference should coordinate with the difference between the initial problem's desired units and the new problem's desired units.
9 — Multiplicative Strategies for Supporting One's Completed Dimensional Analysis Work There are often situations in which one must complete similar dosage calculations that vary by *one* contextual factor. This includes situations where the prescribed dosage or the patient's weight has changed, but all other contextual factors remain constant. One can support their dimensional analysis work in these situations by considering *how* the adjusted factor has increased or decreased. If the prescriber's order or patient's weight changes, and all other factors remain constant, one can consider the multiplicative factor linking quantities within measure spaces (e.g., the initial and new order or weight), or the between measure spaces (i.e., the dosage or weight and the amount/rate to administer).

Articulated conceptions 1 and 2 reflect how students utilized sequential and backwards approaches of dimensional analysis to complete dosage tasks. Of note is that students' explanations of how they utilized dimensional analysis were reasoned and connected with the contextual factors of the dosage situation. This suggests the students' perspectives of dimensional analysis align with approaches in the literature by Goodstein (1983) and Maloy (1986), who suggest that dimensional analysis be applied with a deeper understanding of the factors and mathematical relationships in the problem. This contrasts with the perspective of dimensional as a rote procedure that is completed by following a “recipe” or “flowchart” (DeLorenzo, 1976; Graham, 1986), as well as the approach suggesting dimensional analysis can be completed “without regard to a logical, sequential, placement of conversion factors” (Craig, 2011, p. 72).

Building on how the students employed dimensional analysis to complete dosage calculations, articulated conceptions 3 and 4 posit what it means for dimensional analysis to perceived as either a *flexible* or *rigid* procedure. That is, one might exhibit a *flexible* view of dimensional analysis for calculating medication dosage if they recognize the procedure can be applied in different ways, whereas one might exhibit a *rigid* view of dimensional analysis for calculating dosage if they do not recognize that the procedure can be applied in different ways. Multiple students illustrated a view of dimensional analysis as flexible, while only one student provided explanations that aligned with a rigid view.

Articulated conception 5 speaks to *why* students in this study chose to use dimensional analysis to calculate medication dosage. Consistent with the dimensional analysis literature, students in this study emphasized the visual nature of dimensional analysis (i.e., the domino-like pattern of the units in the unit path), and how this helped them to complete the task and/or check their work at the end of the calculation. Additionally, the results of this study support other findings suggesting that dimensional analysis is associated with increased confidence in one’s calculations (Koharchik et al., 2014) and that dimensional analysis is a method that “sticks with you” for completing dosage calculations (Koohestani & Baghcheghi, 2010). This is especially important for nursing practice as it has been identified as a safety-critical practice (Coben & Weeks, 2014).

Articulated conceptions 6 through 9 speak to the proportional reasoning strategies that students employed to make sense of and confirm their dimensional analysis work. Consistent with other studies on students' use of proportional reasoning strategies (Cramer et al, 1993; Hart, 1973; Noelting, 1980b), students employed a variety of strategies, including those described as qualitative, algorithmic, additive, and multiplicative. Additionally, all of the students in this study identified when a contextual factor in the dosage situation "doubled," and they connected this with their calculated result. However, when a non-integer multiplicative factor was introduced in two of the tasks (Tasks 2B and 3C), only one student was able to identify and make sense of this value; the other students fell back to using additive, algorithmic, and qualitative strategies to confirm their calculation. This result supports the findings of other studies illustrating the impact of non-integer factors on students' proportional reasoning strategies (Cramer et al., 1993; Hart, 1973; Noelting, 1980b).

One final finding was that students' use of proportional reasoning strategies were inconsistent across the different tasks. For example, in one task involving a non-integer factor of change, Violet and Jade calculated the multiplicative factor linking the quantities; however, they did not use this approach for the other task involving a non-integer factor. Although the mathematical relationships were similar across these tasks (i.e., one contextual factor increased by a non-integer multiplicative value), their differing strategies suggest that the nature of the task (i.e., the dosage situation and the specific factor that has changed) might impact *how* an individual (1) makes sense of their calculation, and (2) chooses an appropriate strategy to confirm their work.

Implications and Future Research

There is abundant data suggesting that dimensional analysis is an effective and error-reducing method for calculating medication dosage (Craig, 1992; Greenfield et al., 2006; Rice & Bell, 2006; Turner, 2018). Studies also suggest that using dimensional analysis to calculate medication dosage is associated with greater confidence (Koharchik et al., 2014; Rice & Bell, 2006). However, a significant majority of these studies rely on quantitative measures and provide limited insight into *how* or *why* individuals choose to complete dosage calculations with dimensional analysis.

The present study, which employed qualitative methods to investigate nursing students' use of dimensional analysis as a method for calculating medication dosage, is the first to offer a deeper and more nuanced perspective of dimensional analysis for calculating medication dosage. As such, the results of this study have the potential to impact how dimensional analysis is perceived, taught, and assessed in undergraduate nursing programs. The nine distinct conceptions of dimensional analysis put forth in this study (Table 6-1) provide researchers and educators with a framework to "[examine] student understanding and [notice] key aspects of student behavior" (Simon, 2017, p. 113) related to dimensional analysis in nursing practice. Additionally, these conceptions can be used to construct "a basis for claims and specifications of learning" (Simon, 2017, p. 114), which should inform the development of dimensional analysis learning goals, instructional strategies, activities, and dosage calculation assessments.

Although these nine conceptions represent the inferred understanding of the specific participants in this study, they illustrate that nursing students can develop perspectives of dimensional analysis that might be more limiting than others (e.g., dimensional analysis as a rigid procedure versus dimensional analysis as a flexible procedure). Additionally, the results show that students utilize a variety of strategies to confirm their dimensional analysis work (e.g., considering the placement of units in the unit path and using proportional reasoning strategies in similar dosage situations), with some of the strategies being more connected to the multiplicative relationships involved in the dosage situation. It is important for mathematics and nurse educators to consider these conceptions as they develop conceptually grounded dosage calculation lessons, instructional activities, tasks, and assessments that involve students' use of dimensional analysis.

As the nursing, mathematics, and education communities continue to collaborate to improve mathematics education practices in nursing (Hughes & Zoellner, 2019; Ozimek et al., 2021), it will be important to continue investigating *how* nursing students' complete dosage calculations. The nine conceptions put forth in this study represent an initial framework to articulate how students make sense of and understand dimensional analysis as a method for calculating medication dosage; however, there are a number of areas in need of further exploration. For example, the conceptions in this study emerged from the actions and explanations of the ten participating nursing students. It will be important for researchers to explore whether similar dimensional analysis conceptions emerge from the work of other nursing students, including (1) those engaged in calculating dosage for different clinical situations (e.g., more advanced IV calculations, safe dosage calculations, total intake), and (2) individuals who are less accurate in their calculations or tend to struggle with employing dimensional analysis to calculate dosage. Additionally, given findings of other studies that illustrate the various ways that students employ calculation methods in practice (Hoyles et al., 2001; Noss et al., 2000; Noss et al., 2002; Wright, 2012; Wright, 2013), it would be important to consider whether similar conceptions emerge from an analysis of students' use of dimensional analysis in more realistic settings (e.g., in the skills lab, simulation, or clinical practice). Conducting this research would help to develop a more precise picture of how nursing students make sense of this important mathematical procedure for calculating dosage.

Finally, further research is needed to explore the impact that various perspectives of dimensional analysis might have on mitigating calculation errors on dosage examinations and medication administration errors in practice. Just as Baroody's (2003) example of the newcomer to a town who used her well-connected knowledge to find a new route to the store, one might posit that nursing students who view dimensional analysis as a *flexible* procedure might be better equipped to identify and mitigate calculation errors. Similarly, one might also hypothesize that nursing students who recognize and can operate with the multiplicative relationships in a dosage situation are also well positioned to catch errors. The results of this study are not able to support either of these claims, especially given the participating nursing students were almost entirely error free with their calculations. However, the conceptions articulated in this study offer an initial framework to ground future exploration into how nursing students employ and make sense of dimensional analysis when calculating medication dosage.

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Appendix A

Materials and Tasks Sent to Participants Prior to their Interview

Thank you for being willing to contribute to this research project.
Please read the instructions provided below. At any time, please e-mail the primary investigator, Daniel Ozimek dozimek2@pacollege.edu, with questions or concerns.

Instructions:

In this document you will find three dosage calculations.

1. Complete the three dosage calculation problems
    a. Please complete each dosage calculation on a blank piece of paper; One problem per piece of paper.
    b. **You are asked to use dimensional analysis for each of the calculations.**
    c. It is important that you organize your work and write as legibly as possible.
    d. You may use a calculator
    e. Please do not use other resources. If you need to look up a conversion that you forgot – that is fine, but please do not use online websites or your notes from math class to complete the problems.
    f. **Please complete the items by yourself and without the aid of other individuals.**
    g. **This is not a test and your work will not be graded.**

2. Take a picture of (or scan) your work
    a. When you have completed all three problems, please scan your work and/or take a picture of your work (e.g. with a smart phone, tablet, or other device).
    b. Please be sure that the scan and/or pictures are clear.
    c. It is recommended that you take a picture of each problem that you complete (so three total pictures and/or scans).

3. E-mail your completed work
    a. Please e-mail the pictures and/or scans to dozimek2@pacollege.edu.
    b. If possible, please complete the materials and return them within 3 days (72 hours)
    c. **Please save the papers containing your work. I will be asking you questions about your work during the Zoom interview that will follow.**

After submitting your completed work through e-mail, I will contact you to inquire about holding a follow-up interview through Zoom. Additional information about this interview will be provided in follow-up communication, but in the meantime, please contact me if you have questions or concerns.

Problem 1:

Patient Name: Harold Smith		
Date	Time	Physician Order
5/1/20	900	Filgrastim 0.2 mg subcut daily

We are given an order and available medication for Harold Smith.

<u>Use dimensional analysis</u> to calculate the number of milliliters (mL) to prepare for this order.

Please write legibly and show all of your work.

Problem 2

Patient Name: Betsy Ruiz	Weight: 80 kg	
Date	Time	Physician Order
5/2/20	0630	**Order:** begin nitroglycerin infusion at 5 mcg/min.

We are given an order and available IV bag for Betsy Ruiz.

<u>Use dimensional analysis</u> to calculate the appropriate infusion rate (in mL/hr) to set an IV pump.

Please write legibly and show all of your work.

Problem 3

Patient Name: Kennan Sanders Weight: 12 kg		
Date	Time	Physician Order
5/3/20	700	ampicillin 15 mg/kg/hr b.i.d.

We are given an order and available medication for Kennan Sanders.

<u>Use dimensional analysis</u> to calculate the rate the set the IV pump in mL/hr.

Please write legibly and show all of your work.

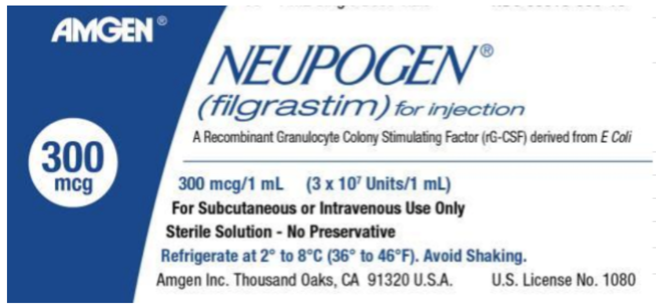
Appendix B

Tasks Completed During the Interviews

Follow Up Tasks for Harold Smith (Parenteral Dosage Adjustment)

Task 1B
On the next day, Harold's dose has changed. Determine the amount to prepare for Harold.
Describe what you are thinking prior to and during the calculations you complete.

Physician Order
Filgrastim 0.4 mg subcut daily



Follow Up Tasks for Betsy Ruiz (IV Rate Calculation with Dose Adjustments)

Task 2B

Betsy's lab values are not where the prescriber would like them, so the order is increased to 22 mcg/min. What rate should be set on the IV pump?

Describe what you are thinking prior to and during the calculations you complete.

Follow Up Tasks for Kennan Sanders (Weight-based IV Rate Calculation with Weight Adjustments)

Task 3B

Another child has arrived at the ED and was given the same order as above. The child weighs 24 kg.
Determine the rate (mL/hr) to set the IV pump.
Describe what you are thinking prior to and during the calculations you complete.

Task 3C

Yet another child has arrived at the ED and was given the same order. The child weighs 32.4 kg.
Determine the rate (mL/hr) to set the IV pump.
Describe what you are thinking prior to and during the calculations you complete.

Appendix C

Sample Interview Script

Interviewer: Thank you for your participation in this research study. I appreciate the work you have already completed, and I want to thank you for being willing to meet with me through Zoom.

The purpose of this interview is to learn a little bit more about the work you completed, and to ask a few follow up dosage calculation questions that relate to the three patients from the first three tasks.

I have prepared a whiteboard through Google that has pictures of some of your completed work, as well as some additional questions. Do you have the whiteboard open on your device?

…

Throughout the interview I will refer to the numbers attached to the slides to make sure we are referencing the same material.

Before we get started, it is important that you have scratch paper available to complete additional problems. You should also have the work you completed during the first part of this study, as well as a calculator.

Throughout the interview – and as you complete additional dosage calculations -- I will ask you to describe what you are thinking. I will also ask that you hold up the work that you completed to your webcam.

Do you have any questions before we get started?

…

TASK 1

Interviewer: Please scroll to Slide 2 on the shared whiteboard where you will see the work you completed for Harold Smith.

Can you please walk me through the work you completed? Explain how you completed this problem using dimensional analysis.

I see you started with 1 mL. over 300 mcg. Can you tell me why you started with ratio?

Is the work you completed for this problem the only way that you could use dimensional analysis to complete this problem?

Interviewer: Okay, please click over to Slide 3 on the shared whiteboard. Could you please read the problem aloud and then complete the problem on a blank piece of paper? Please explain your thinking as you complete the problem.

Looking at the value that you calculated, do you think your answer makes sense? Why? Explain.

Looking at the value that you calculated and the work you completed, is there any way to confirm that your calculation is accurate? How so?

Looking back at how you completed this problem, was there any other way you could have addressed the question?

OR Is there any other way that you could have completed this problem without dimensional analysis? Explain.

OR Suppose a friend completed this calculation using dimensional analysis, is it possible that their work might look different from yours? How so?

TASK 2

Interviewer: Please scroll to Slide 4 on the shared whiteboard where you will see the work you completed for Betsy Ruiz.

Can you please walk me through the work you completed? Explain how you completed this problem using dimensional analysis.

When you think about dimensional analysis, can you tell me how your approach to this dosage calculation is different (or similar) to our approach in the first

So in the last problem you started with the unit you wanted at the end "mL", but in this problem you started with "mcg/min" – the information given -- Can you tell me why you started with ratio?

Interviewer: Okay, please click over to Slide 5 on the shared whiteboard. Could you please read the problem aloud and then complete the problem on a blank piece of paper? Please explain your thinking as you complete the problem.

Looking at the value that you calculated, do you think your answer makes sense? Why? Explain.

Looking at the value that you calculated and the work you completed, is there any way to confirm that your calculation is accurate? How so?

Looking back at how you completed this problem, was there any other way you could have addressed the question?

OR Is there any other way that you could have completed this problem without dimensional analysis? Explain.

OR Suppose a friend completed this calculation using dimensional analysis, is it possible that their work might look different from yours? How so?

TASK 3

Interviewer: Please scroll to Page 6 on the shared whiteboard where you will see the work you completed for Kennan Sanders.

Can you please walk me through the work you completed? Explain how you completed this problem using dimensional analysis.

When you think about dimensional analysis, can you tell me how your approach to this dosage calculation is different (or similar) to your approach in the first two problems?

Is the work you completed the only way that you could use dimensional analysis to complete this problem?

Interviewer: Okay, please click over to Slide 7 on the shared whiteboard. Please read the problem aloud and then complete the problem on a blank piece of paper. Please explain your thinking as you address the problem.

Looking at the value that you calculated, do you think your answer makes sense? Why? Explain.

Looking at the value that you calculated and the work you completed, is there any way to confirm that your calculation is accurate? How so?

Looking back at how you completed this problem, was there any other way you could have addressed the question?

OR Is there any other way that you could have completed this problem without dimensional analysis? Explain.

OR Suppose a friend completed this calculation using dimensional analysis, is it possible that their work might look different from yours? How so?

Interviewer: Okay, please click over to Slide 8 on the shared whiteboard. Please read the problem aloud and then complete the problem on a blank piece of paper. Please explain your thinking as you address the problem.

Is your approach to this dosage calculation the same as the last problem? Why or why not?

Could you think of another way to complete this calculation or confirm it is accurate>

OR Is there any other way that you could have completed this problem without dimensional analysis? Explain.

OR Suppose a friend completed this calculation using dimensional analysis, is it possible that their work might look different from yours? How so?

FINAL QUESTIONS

Interviewer: Thank you very much for your responses thus far. For the final few minutes, I'd like to ask a few general questions about all of these tasks and what you think about dimensional analysis.

1. *Suppose a friend asks you, "What is dimensional analysis?" -- How would you respond to this? If you'd like, feel free to refer to the work that you completed.*

2. *Suppose a friend asks you, "How exactly do you carry out the process of dimensional analysis?" -- How would you respond to this? If you'd like, feel free to refer to the work that you completed.*

3. *Suppose a friend asks you, "Why does dimensional analysis work?" -- How would you respond to this? If you'd like, feel free to refer to the work that you completed.*

Interviewer: Do you have any questions for me?

Interviewer: Okay, thank you again for your willingness to participate in this study. I really appreciate you taking the time to complete all of these dosage calculations and answering my questions.

VITA

Daniel L. Ozimek

Education

PhD	2021 (expected)	Pennsylvania State University	Curriculum & Instruction
M.Ed.	2011	Millersville University	Mathematics
B.S.	2008	Pennsylvania State University	Mathematics
B.A.	2008	Pennsylvania State University	Economics

Professional Experience

2012 — Present	Pennsylvania College of Health Sciences, Assistant Professor, Mathematics
2012 — 2013	Kutztown University, Mathematics Instructor
2010 — 2012	Harrisburg Area Community College – Lancaster Campus, Adjunct Mathematics Instructor

Publications

Ozimek, D., Kelch, B. & Zoellner, J. (2021). *Math and statistics education for nurses* (Convening Recommendations, No. 1). Austin, TX: The Charles A. Dana Center at The University of Texas at Austin.

Ozimek, D., Ellis, M., Levy, R., Piercey, V., & Zoellner, J. (2020). Seeking community input on "Math for Nurses". *MAA Focus, 40*(1), 5. http://digitaleditions.walsworthprintgroup.com/publication/?m=7656&i=648880

Hopkins, M., **Ozimek, D.**, & Sweet, T. M. (2016). Mathematics coaching and instructional reform: Individual- and collective change. *Journal of Mathematical Behavior.* http://doi.org/10.1016/j.jmathb.2016.11.003

Select Presentations

Zoellner, J., **Ozimek, D.**, & Wendel, A. (2021, February). *Essential quantitative skills for quality and safe nursing practice*. Presented at the National Numeracy Network (NNN) 2020-2021 Annual Meeting, Austin, TX.

Ozimek, D., & Good, L. (2020, November). *Open online resources for teaching quantitative concepts in nursing*. Presented at the American Mathematical Association of Two-Year Colleges (AMATYC) 2020 Conference, Spokane, WA.

Ozimek, D., Good, L., Watson, G., & Wendel, A. (2019, August). *Mathematics and nursing: Narrowing the classroom-practice gap with authentic dosage activities*. 2019 Mathematical Association of America MathFest, Cincinnati, OH.

Honors & Awards

2020	President's Award for Innovative Teaching, Pennsylvania College of Health Sciences
2018	Mathematics Pathways Leadership Fellow, Charles A. Dana Center, University of Texas at Austin
2015	Kenneth G. Stoudt Endowed Faculty Award for Excellence in Teaching, Pennsylvania College of Health Sciences
2013	Ambassador of Student Learning Award, Kutztown University